

A Review of Optimization Approaches for Controlling Water-cooled Central Cooling Systems

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Abstract: Buildings consume a large amount of energy across all sectors of society, and a large proportion of building energy is used by HVAC systems to provide a comfortable and healthy indoor environment. In medium and large-size buildings, the central cooling system accounts for a major share of the energy consumption of the HVAC system. Improving the cooling system efficiency has gained much attention as the reduction of cooling system energy use can effectively contribute to environmental sustainability. The control and operation play an important role in central cooling system energy efficiency under dynamic working conditions. It has been proven that optimization of the control of the central cooling system can notably reduce the energy consumption of the system and mitigate greenhouse gas emissions. In recent years, numerous studies focus on this topic to improve the performance of

optimal control in different aspects (e.g., energy efficiency, stability, robustness, and computation efficiency). This paper provides an up-to-date overview of the research and development of optimization approaches for controlling water-cooled central cooling systems, helping readers to understand the new significant trends and achievements in this area. The optimization approaches have been classified as model-based and data-based. In this paper, the optimization methodology is introduced first by summarizing the key decision variables, objective function, constraints, and optimization algorithms. The principle and performance of various optimization approaches are then summarized and compared according to their classification. Finally, the challenges and development trends for optimal control of water-cooled central cooling systems are discussed.

Keywords: Building energy, Energy efficiency, Central cooling system, Optimal control, System-model-based optimization, Data-based optimization.

Nomenclature		<i>Subscripts</i>	
<i>CL</i>	Cooling load (kW)	<i>a</i>	Air
<i>CP</i>	Critical point	<i>ahu</i>	Air handle unit
<i>D</i>	Vector of decision variables	<i>app</i>	Approach
<i>dc</i>	Continuous decision variables	<i>chw</i>	Chilled water
<i>ds</i>	Binary decision variables	<i>ct</i>	Cooling tower

<i>DP</i>	Differential pressure (kPa)	<i>cw</i>	Cooling water
<i>f</i>	Frequency (Hz)	<i>ex</i>	Exchanger
<i>J</i>	Objective function	<i>fan</i>	Cooling tower fan
<i>H</i>	Head (kPa)	<i>i</i>	Number of equipment
<i>N</i>	Number of equipment	<i>j</i>	Number of equipment
<i>M</i>	Mass flow rate (m ³ /h)	<i>k</i>	Number of equipment
<i>OM</i>	Operation mode	<i>pum</i>	Pump
<i>P</i>	Power consumption (kW)	<i>r</i>	Water return
<i>PLR</i>	Partial load ratio (%)	<i>rated</i>	Rated
<i>Q</i>	Cooling supply (kW)	<i>s</i>	Supply
<i>S</i>	Operation state	<i>set</i>	Setpoint
<i>T</i>	Temperature (°C)	<i>ss</i>	Substations
<i>t</i>	Time	<i>ter</i>	Terminal unit
<i>U</i>	Vector of uncontrollable variables	<i>total</i>	Sum of the variables
ΔT	Temperature difference (°C)	<i>wet</i>	Wet-bulb temperature

Abbreviation

AFSA	Artificial Fish Swarm Algorithm
AGSO	Augmented Group Search Optimization
ANN	Artificial Neural Network
B&B	Branch & Bound

BGA	Binary Genetic Algorithm
CGA	Continuous Genetic Algorithm
COC	Conventional Optimal Control
COP	Coefficient of Performance
CSA	Cuckoo Search Approach
CTBA	Camel Traveling Behavior Algorithm
DCSA	Differential Cuckoo Search Approach
DE	Differential Evolution
DMM	Data Mining Method
DOF	Degree of Freedom
DS	Differential Search
EDOC	Event-Driven Optimal Control
EIWO	Enhanced Invasive Weed Optimization
EMA	Exchange Market Algorithm
EP	Evolutionary Programming
EPSO	Elitism-based Particle Swarm Optimization
ES	Evolution Strategy
ESM	Extremum-Seeking Method
ExS	Exhaustive Search
FA	Firefly Algorithm
GA	Genetic Algorithm

GD	Gradient Descent
GRG	Generalized Reduced Gradient
HAVC	Heating Ventilation and Air Conditioning
HJ	Hooke Jeeves
HOC	Hierarchical Optimal Control
IAFSA	Improved Artificial Fish Swarm Algorithm
IAFSA	Improved Artificial Fish Swarm Algorithm
IFA	Improved Firefly Algorithm
IFFA	Improved Fruit Fly Algorithm
IGA	Improved Grasshopper Algorithm
IPSO	Improved Particle Swarm Optimization
IRBSO	Improved Ripple Bee Swarm Optimization
IWO	Invasive Weed Optimization
LM	Lagrangian Method
LMA	Levenberg-Marquardt Algorithm
MAOC	Multi-Agent Optimal Control
MAPSO	Multi-Agent Particle Swarm Optimization
MGA	Modified Genetic Algorithm
MILP	Mixed Integer non-Linear Program
MPGA	Multi-Phase Genetic Algorithm
NMS	Nelder-Mead Simplex

PSO	Particle Swarm Optimization
QN	Quasi-Newton
RLM	Reinforcement Learning Method
ROC	Robust Optimal Control
SA	Simulated Annealing
SAM	Systematic Analysis Method
SEOC	Stability-Enhanced Optimal Control
SOC	Stochastic Optimal Control
TDE	Two-stage Differential Evolution
TDOC	Time-Driven Optimal Control
TLBO	Teaching-Learning Based Optimization

1. Introduction

Buildings account for almost 40.0% of the global energy consumption and CO₂ emissions [1-3], so reducing the energy demand of buildings has become an essential component of global sustainability [4, 5]. In buildings, a large proportion of energy is consumed by the central cooling system to provide a comfortable and healthy indoor environment [6-8]. The water-cooled central cooling system is commonly used due to its high cooling capacity and energy efficiency [9, 10]. Figure 1 shows a typical water-cooled central cooling system consists of three main sub-systems, namely, the chilled water loop, the chiller plant, and the cooling water loop [11].

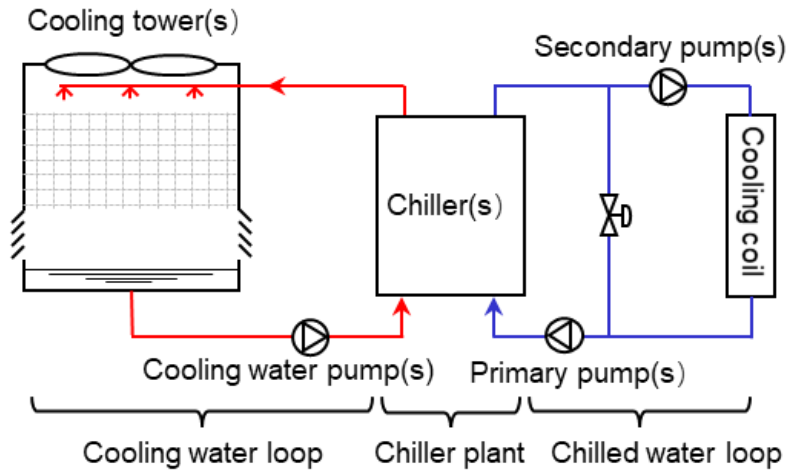


Figure 1: Basic structure of a typical water-cooled central cooling system

Due to the common use of water-cooled central cooling systems in energy-intensive buildings, improving the energy efficiency of the central cooling system is crucial for building energy conservation. Using energy-efficient equipment is an essential measure for reducing the energy consumption of the central cooling system. However, it is insufficient to minimize overall system energy consumption due to the interaction of different subsystems or components. Therefore, the control and operation become extremely important for system energy-saving [12]. In real applications, this is mainly achieved at two levels, namely, supervisory control and local control [11, 13], as shown in Figure 2. The supervisory control determines the operation mode and specifies setpoints for the local control loops. The local control adjusts the sequences and processes of relevant equipment to maintain the operation mode and setpoints determined at the supervisory control level.

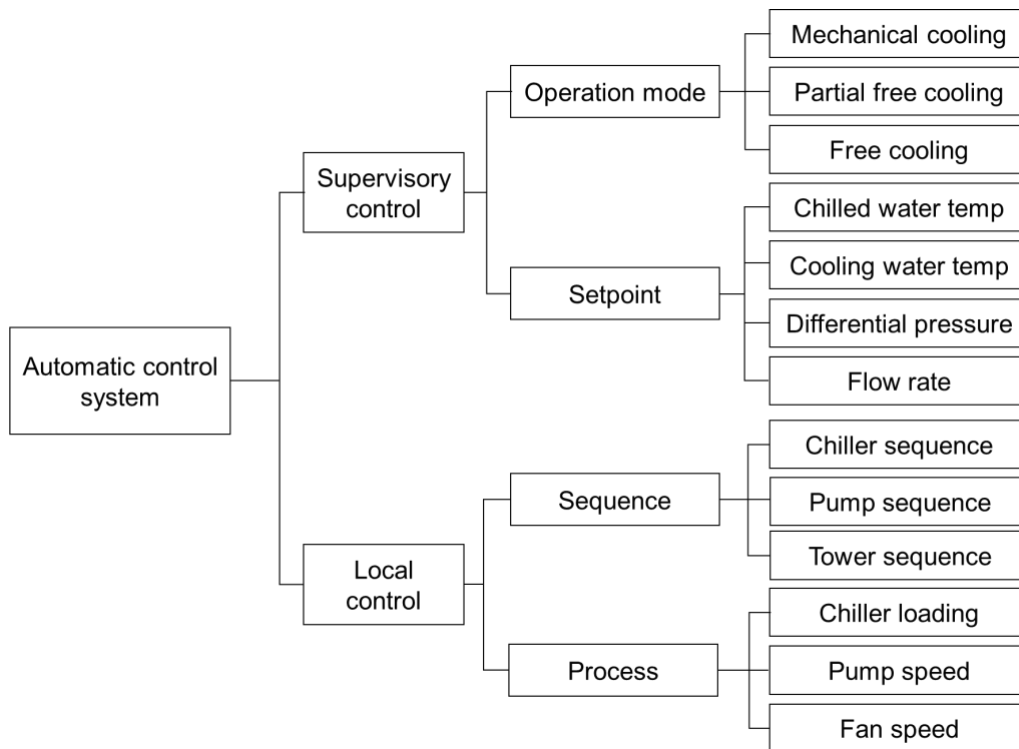


Figure 2: The control structure of water-cooled central cooling systems

In recent decades, numerous studies have focused on optimizing the control of water-cooled central cooling systems to improve system performance [14-16]. Firstly, optimizing the control of central cooling systems could significantly reduce energy consumption while maintaining a comfortable indoor environment. For example, by optimizing cooling water temperature, Huang et al. [14] reduced the total system energy demand by 9.7%. Secondly, the optimization can help to enhance the control performance (e.g., stability and robustness) [17-23]. For example, using a multiplexed optimization method, Sun et al. [18] improved the system control stability with a 50.0% reduction in tracking error of setpoints. Kumar et al. [21] employed a stochastic optimization method to mitigate constraint violations and enhance the robustness of control

by explicitly incorporating uncertainties in the optimization. The existing optimization methods can be classified into system-model-based and data-based approaches [13]. Whether a method is specified as a system-model-based or a data-based method is dependent on whether the numerical models are used to describe system dynamic behavior. This system model was used to evaluate the energy consumption or operation cost in each iteration of the optimization process [13]. The optimization process is to identify control variables that minimize the system energy use or operation costs [24, 25]. The methods that do not involve system models during the optimization process are classified as data-based optimization. The data-based optimization methods are generally based on the experience of engineers [26, 27], learned knowledge from historical data [28-31], or regulation on real systems directly [32-34].

In the literature, numerous studies talk about optimal control of the central cooling system, while only two reviews focused on this specific topic [11, 13]. Chapter 43 of ASHRAE Handbook-HVAC Applications [11] has summarized supervisory control strategies and optimization methods applied to HVAC systems. In this chapter, the general optimization approach is introduced, and the simplified near-optimal control strategies are then described in detail to guide the real application. Wang and Ma [13] have reviewed the supervisory optimal control in building HVAC systems and proposed a classification framework for optimal control methods. However, these two reviews do not

summarize the decision variables, objective functions, and constraints of the optimal control used in this specific field. Moreover, the literature cited by the two reviews is mainly before 2005. With the rapid development of technology and computer science, many advanced optimization approaches have been developed for various purposes, as well as algorithms for finding the optimum solution. Therefore, this paper aims to provide a comprehensive overview of the research and recent development of optimization approaches for water-cooled central cooling systems.

In the rest of this paper, Section 2 described the review material and methods. Section 3 introduced the general optimization methodology by summarizing the general formulation, decision variables, objectives, constraints, and optimization algorithms. Then, Section 4 reviewed the recent development of optimization approaches and compared their performance according to their classification. The challenges and potential future research directions in the optimization of water-cooled central cooling systems are discussed in Section 5. A summary of this review article is then provided in Section 6.

2. Material and Methods

2.1 Search keywords

This research aims to provide an overview of the literature on optimization approaches for improving the energy efficiency of the water-cooled central cooling system. This study used the “Web of Science” and “Scopus” scientific

databases to search relevant publications. The main search keywords included: “optimization”, “optimal control”, “central cooling system”, “chiller plant”, “chilling system”, “chilled water system”. To focus on the most recent technologies and approaches, only studies published in or after 2005 have been included.

2.2 Inclusion criteria

In the initial search, around 1897 publications were found. They were screened by reading through their titles and abstracts and restricted by following inclusion criteria: 1) only English-written peer-reviewed articles published in journals, chapters of books, and proceedings of conferences were included; 2) studies that mainly focused on optimization of system design, siting and components sizing were excluded; and 3) articles focused on air-cooled or other types central cooling system was not included, as the water-cooled central cooling system is the main object of this review. Applying these criteria, 98 papers were obtained for this review.

2.3 Data collection

The following information was extracted from the selected publications: 1) the decision variables, the objective function, and constraints used in the optimization, 2) optimization technologies and algorithms used for solving optimization problems, 3) the potential energy and cost savings, and other aspects of performance for different optimal control methods.

3. Optimization Methodology

3.1 General formulation

An optimization problem is typically defined in three parts: a set of decision variables, an objective function, and a series of constraints. The goal of optimization is to minimize or maximize the objective by varying the decision variables while satisfying a set of constraints. Equation (1) is a general objective function, that minimizes the total energy or operation cost of the target system. Equations (2) and (3) are equality and inequality constraints to which all potential solutions are subject.

$$J(f, D, U) = \min\left(\sum_{i=1}^N J_i(x_i, y_i, f_i, D_i, U_i)\right) \quad (1)$$

$$g_i(x_i, y_i, f_i, D_i, U_i) = 0 \quad (2)$$

$$h_i(x_i, y_i, f_i, D_i, U_i) \geq 0 \quad (3)$$

where J_i represents the operational cost or energy consumption of each component; f_i represents non-control variables, such as cooling demand and outdoor wet-bulb temperature; D_i and U_i are vectors of discrete and continuous control variables, which could be adjusted to minimize the objective function; x_i and y_i are input and output stream variables, respectively; g_i are equality constraints; and h_i represent inequality constraints.

3.2 Decision variables

Decision variables are input parameters that can be adjusted to minimize the objective function. The decision variables used in the optimization of the central cooling systems are summarized in Table 1, as well as their types, ranges, and

corresponding energy performance of the optimization. The decision variable could be classified into continue (e.g., $T_{chw,i}$) and discrete variables (e.g., S_i and N). In the reviewed literature, the partial load ratio (PLR_i) is the most commonly decision variable used to reduce chillers' energy consumption since chillers consume a large proportion of the system's energy. Although significant energy savings could be obtained by optimizing the PLR_i , it has not been widely applied because the PLR_i cannot be directly adjusted. Several studies [35-37] optimized the chilled water supply temperature ($T_{chw,s,i}$) of each chiller, a directly controllable parameter, to improve the energy efficiency. Cooling water supply temperature ($T_{cw,s}$) is another important decision variable as it impacts the energy consumption of both chillers and cooling towers. Due to the coupling feature of subsystems, holistic optimization, which explores the synergistic effects of multiple variables to increase whole system energy efficiency, has become a trend with an increasing number of studies on holistic optimization. In holistic optimization, $T_{chw,s}$ and $T_{cw,s}$ are the most significant decision variables and they are typically optimized simultaneously. The energy-saving potential of different decision variables has also been summarized in Table 1. It shows that the energy savings varying with system structures and weather conditions, and thus the variables should be selected carefully for various applications. Compared with subsystem optimization, holistic optimization generally exhibits greater energy-saving potential, detailed

comparative data could be found in [15, 38, 39].

Table 1. Decision variables used in the optimization of water-cooled central cooling systems (DES: daily energy savings, WES: weekly energy savings, AES: annual energy savings. Others are energy savings under specified conditions)

Subsystem	Decision Variables	Variable Type	Range of Variables	Energy Savings	Ref.
Chillers	$PLR_i, (S_i)$	Continue (Mixed)	$PLR_i \in [0.3, 1.0], S_i \in \{0, 1\}$	Case 1: 3.6-7.2% Case 2: 0.4-29.3% Case 3: 0.6-18.5%	[40-60]
	PLR_i, S_i	Mixed	$PLR_i \in [0.0, 1.0], S_i \in \{0, 1\}$	DES: 1.0-21.2%	[61, 62]
	PLR_i	Continue	$PLR_i \in [0.5, 1.0]$	0.4-25.7%	[63-68]
	Q_i	Continue	$Q_i \in [0, Q_{rated,i}]$	DES: 0.4-9.4%	[69, 70]
	CP_i	Continue	$Q_{cp,i} \in [CP_{i-1}, CP_i]$	DES: 0.5-7.8%	[15, 71]
	$T_{chw,i}, S_i$	Mixed	$T_{chw,i} \in [7.0, 13.0 \text{ }^\circ\text{C}], S_i \in \{0, 1\}$	0.7-13.8%	[22, 35-37, 72, 73]
Cooling water system	$T_{cw,s}$	Continue	$T_{cw,s} \in [15.4, 29.4 \text{ }^\circ\text{C}]$	AES: 9.4-9.7%	[14, 74]
	ΔT_{app}	Continue		AES: 4.1-15.6%	[75-77]
	$T_{cw,s}, Q_{cp,i}$	Continue	$T_{cw,s} \in [13.9, 23.9 \text{ }^\circ\text{C}]$ $Q_{cp,i} \in [Q_{rated,i}, 1.1Q_{rated,i}]$	AES: 5.6%	[71]
	$T_{cw,s}, (f_{fan}, N_{ct})$	Mixed	$T_{cw,s} \in [\max(18.0, T_{op} - 2.0), T_{op} + 2.0 \text{ }^\circ\text{C}]$, $f_{fan} \in [20, 50 \text{ Hz}], N_{ct} \in \{1, 2, 3, \dots, 11\}$	DES: 14.8-22.6%	[78, 79]
	$T_{cw,s}, f_{fan}, PLR_i, N_{ch}$	Mixed	$T_{cw,s} \in [15.5, 29.4 \text{ }^\circ\text{C}], f_{fan} \in [0.3, 1.0]$ $PLR_i \in [0.4, 1.0], N_{ch} \in \{1, 2, 3, 4\}$	16.7%	[80]
	M_{cw}, M_{ct}	Continue		2.5-8.6%	[81, 82]
	T_{cw}, Q_i, S_i	Mixed	$Q \in [0.5Q_{max}, Q_{max}]$,	DES: 14.0%	[83]

			$T_{cw,s} \in [23.9, 29.4 \text{ °C}], S \in \{0, 1\}$		
Chilled water system	$T_{chw,s}, T_{ter,s}$	Continue	$T_{chw,s} \in [5.0, 8.0 \text{ °C}], T_{a,su} \in [13.0, 19.0 \text{ °C}]$	AES: 6.7%	[84]
	$T_{chw,s}, DP_{set}, N_{chw,pum}, N_{ahu}$	Continue	$T_{chw,s} \in [5.0, 10.0 \text{ °C}]$	DES: 1.3-2.6%	[85]
	$M_{chw,j}, PLR_i$	Continue		9.9%	[86]
Whole system	$T_{chw,s}, T_{cw,s}$	Continue	$T_{chw,s} \in [5.0, 13.0 \text{ °C}], T_{cw,s} \in [18.0, 32.0 \text{ °C}]$	DES: 9.4-11.1%	[87]
	$T_{chw,s}, T_{cw,s} (T_{a,su})$	Continue	$T_{chw,s} \in [5.0, 8.0 \text{ °C}], T_{cw,s} \in [20.0, 35.0 \text{ °C}],$ $T_{a,su} \in [12.0, 18.0 \text{ °C}]$	DES: 3.5-11.8%	[18, 39, 88-91]
	$T_{chw,s}, T_{cw,s}, Q_i$	Continue		DES: 10.5-13.6%	[15]
	$T_{chw,s}, T_{cw,s}, M_{cw}$	Continue		WES: 22.0%	[17, 92]
	$T_{chw,s}, T_{chw,r}, M_{chw},$ $T_{cw,s}, T_{cw,r}, M_{cw}$	Continue	$T_{chw,s} \in [7.0, 13.0 \text{ °C}], T_{cw,s} \in [30.0, 35.0 \text{ °C}]$ $T_{chw,r} - T_{chw,s} \in [5.0, 7.0 \text{ °C}],$ $T_{cw,r} - T_{cw,s} \in [0.0, 7.0 \text{ °C}]$ $M_{chw} = \frac{Q}{4.2 \times 10^6 (T_{chw,r} - T_{chw,s})}$ $M_{cw} \in [0, 0.2 \text{ m}^3/\text{s}]$	DES: 9.1-23.3%	[93]
	$T_{chw,s}, T_{cw,s}$	Mixed		2.0-25.0%	[94, 95]
	$T_{chw,s}, T_{cw,s}, M_{cw}, S_i, PLR_i,$	Mixed	$S \in \{0, 1\}, PLR_i \in [0.3, 1.0]$	Daily cost saving: 0.2-0.5%	[96]
	$T_{chw,s}, T_{cw,s}, M_{chw}, M_{cw}, S_{ch,i}$	Mixed	$T_{chw,s} \in [6.7, 9.0 \text{ °C}], T_{cw,s} \in [28.0, 34.0 \text{ °C}]$ $M_{cw} \in [0.6M_{max}, M_{max}], S \in \{0, 1\}$	DES: 20.0-42.5%	[97]
$T_{chw,s}, T_{cw,s}, T_{a,s}, \Delta T_{cw}, OM$	Mixed	$T_{chw,s} \in [11.0, 22.0 \text{ °C}], T_{cw,s} \in [28.0, 32.0 \text{ °C}]$ $T_{a,s} \in [18.0, 27.0 \text{ °C}], \Delta T_{cw} \in [4.0, 5.0 \text{ °C}]$ $OM \in \{0, 1\}$	AES: 6.6%	[98]	
$T_{chw,r}, \Delta T_{app}, M_{chw}, M_{cw}, \Delta T_{ss}, P_s$	Continue		DES: 3.0-31.9%	[38]	

3.3 Objective functions

An objective function describes the relationship between input and output variables of the target system. Most studies addressed single-objective optimization problems, except for a few studies that considered thermal comfort together with system energy consumption [22, 72]. Various formulations of the objective function used in the optimal control of water-cooled central cooling systems are summarized in Table 2. The general objective is to minimize the total power consumption of the target system at a given instant in time, which is generally used for static optimization. The accumulated energy consumption in an optimization period is another way to define the objective function and is normally used for dynamic optimization. In several studies, maximizing the target system's energy efficiency was set as an alternate objective function to obtain a convex objective function [65, 99, 100]. Similarly, minimizing the ratio of power consumption to the cooling load (kW/ton) over the optimization period was considered as the objective function in [15, 97].

Table 2. Various formulations of the objective function

Objective	Objective function
Minimum power consumption at the current time	$J = \min\left(\sum_{i=1}^n P_{chw,i} + \sum_{j=1}^m P_{ct,j} + \sum_{k=1}^l P_{pump,k}\right)$
Minimum energy consumption in an optimization period	$J = \min\left(\int_{t_0}^{t_0+\Delta t} P_{total}(t) dt\right) \text{ or}$ $J = \min\left(\sum_{t_0}^{t_0+\Delta t} P_{total}(t) \Delta t\right)$
Maximum energy efficiency	$J = \max\left(\sum_{i=1}^n CoP_i\right)$
Minimum system kW/ton	$J = \min\left(\frac{\int_{t_1}^{t_2} P_{total} dt}{\int_{t_1}^{t_2} Q_{total} dt}\right)$

As mentioned above, an objective function is the mathematical expression of the system's dynamic behavior, and it is used to evaluate the energy consumption and the system response to variations in the control parameters.

The performance of the optimal control relies heavily on the accuracy of the system model. According to the type of knowledge used to establish the system model, they can be divided into physics-based, data-driven, and hybrid models.

3.3.1 Physics-based models

Physics-based models are based on mathematical descriptions of physical processes and include both detailed and simplified models. They have the potential to capture the system dynamics with high accuracy and reliability

within their allowed operating ranges. Validation of these models only requires a small amount of data, as they are based on the fundamental laws of heat and mass transfer. However, multiple iterations are generally required to solve the differential equations, leading to high computational costs, especially for detailed physics-based models. If the iteration process does not converge in a short control interval, the output may be unstable or undesired. These features prevent the online application of detailed physics-based models, and simplified physics-based models are commonly used in optimal control of central cooling systems.

In water-cooled central cooling systems, chillers and cooling towers are two major components with complicated heat and mass transfer processes, and their physics-based models are summarized as follows. For chillers, the simplified theoretical chiller model, Gordon-Ng model, and Carnot model are commonly used. The simplified theoretical chiller model assumes a virtual refrigeration cycle to simplify the complicated thermodynamic processes. A polynomial was used to yield the actual power consumption of the chiller based on the virtual power consumption [78, 85, 101]. The Gordon-Ng model integrates the first and second thermodynamics laws, and it is simplified to an equation that relates the CoP of chillers to water inlet temperatures of condenser and evaporator and the cooling load. This model is known as the universal model in which the parameters are inherent to each type of

compressor [102]. This model has been utilized to predict both the power consumption [40, 79] and the chillers' maximum cooling capacity [61]. In the Carnot chiller model, the COP of a chiller changes with the condensation and evaporation temperatures in the same way that the Carnot efficiency changes [14]. For cooling towers, the Braun effectiveness model and Lebrun model are commonly used. The Braun model partially uses the effectiveness-NTU model with the assumptions of linearized saturated air enthalpy and the Lewis number of unities [61, 76, 77]. The Braun model is the most popular in cooling tower modeling, but iterations are required indicating a considerable computational cost. The Lebrun model (also named the Toolkit model) takes a cooling tower as an equivalent heat exchanger. The heat transfer coefficient varies with the water and air mass flow rates, and the power consumption of a cooling tower regressed as a third-order function of the air mass flow rate [101, 103]. The Lebrun model is a simple model with a lower computation cost since no iteration is needed.

3.3.2 Data-driven models

Data-driven models statistically describe the relationship between the input and output variables without explicit knowledge of the physical processes. They cannot guarantee accurate and stable output, since their performance is dependent on the variable range covered by the training set. When the input data are outside the range of the training set, the output of a data-driven model

may exhibit large errors, potentially reducing system efficiency or disrupting the stable operation of the system. Therefore, large amounts of data are required for model training to cover various operating conditions and to achieve acceptable performance.

The data-driven models used in water-cooled central cooling systems vary from simple regression models to complex deep neural networks. A simple quadratic or cubic function of PLR_i is generally used to estimate the power consumption or COP of chillers. In this simple regression model, the impact of operating parameters, such as $T_{chw,s}$ and $T_{cw,s}$ are not considered. Some studies have added $T_{chw,s}$ and $T_{cw,s}$ to the regression models to reflect their impact on chillers' energy performance [22, 68, 73]. The ElectricEIR model consists of three regression equations that describe the capacity and efficiency under varying operating conditions and the power consumption under partial load conditions. Chillers' power consumption can be calculated from the reference conditions using these equations [71, 94, 104]. The calibration of this model requires both full-load and partial load operating data. However, chillers rarely operate at full load, which makes the model calibration a difficult task. An adaptive piecewise approximation regression model trained online with recent operation data has also been used to describe the local dynamic behavior of the target system [39]. With a simple structure, this adaptive model can be calculated quickly, which is suitable for online applications. For cooling towers,

the YorkCalc model is a simple empirical model based on varying approach temperatures, and the cooling tower energy consumption is calculated according to the affinity law [14, 71, 104]. The use of YorkCalc model is limited to the valid operating range and exceeding the limits may be problematic. In recent years, various neural networks, such as artificial neural networks (ANN) [66, 67], multi-layer perceptron networks [83, 105], and deep neural networks [106], have been employed to describe the dynamic behavior of the target system as they have good ability in processing nonlinear problems.

3.3.3 Hybrid models

A Hybrid model combines the advantages of physics-based and data-driven models. In a hybrid model, prior knowledge is used to restrict the parameters of a data-driven model or to define the structure of the model. With the restriction of prior knowledge, the variables in a hybrid model have certain physical meanings. A hybrid model built from simplified mathematical equations for the physical process would show better reliability with limited extrapolation, and the model training requires less data than that of data-driven models. Moreover, the integration of physics-based models with data-driven models in a single system model constitutes another hybrid model. In a system, components involved in complex physical processes use data-driven models, while the remaining components and the interactions between them use physics-based models. Thus, the computational time can be reduced

significantly, and the system model still has significant physical meaning. For instance, some studies have used data-driven models (such as ANNs) for chillers and cooling towers to reduce the whole-system model's complexity [17, 92, 97]. A hybrid model that combined the output error predictor with simplified physics-based chiller and cooling tower models predicts the system energy consumption with better accuracy than a single simplified system model [107]. The features mentioned above help to improve the practicality of hybrid models. However, the models' accuracy still relies strongly on the size and quality of the training data.

3.4 Constraints

The optimal control of a water-cooled central cooling system is a typical constrained optimization problem because the system or components confront various limitations. The commonly used constraints are summarized below.

3.4.1 Satisfying the cooling demand

The cooling supply of the system is required to satisfy the demand of end-users, which is generally defined by Equation (4):

$$\sum_{i=1}^N CT_{i,rated} PLR_i = CL \quad (4)$$

Satisfying the cooling demand is an equality constraint. In practice, the Lagrange multiplier method [62, 63, 94] or the penalty function method [15, 69, 71] deals with the equality constraints. The equality constraint can also be

expressed as inequality constraints with small deviation since it is difficult to obtain solutions that satisfy equality exactly. For example, some studies [15, 71] have taken the variation of the chilled water supply temperature from its setpoint as an alternative expression of the constraint to ensure that enough cooling is provided.

3.4.2 Heat and mass balances

All possible solutions are subject to heat and mass balances, which are usually expressed by equality constraints. For example, the heat exhaust from chillers is equal to the sum of the cooling supply and the chillers' power input [75, 80]. Similarly, the mass flow rate of cooling water pumps and cooling towers is equal to the chillers' cooling water mass flow rate [94].

3.4.3 Bounds of decision variables

The bounds of the decision variables define the allowable ranges of values, which are typically formulated as inequalities. These constraints are also considered as physical limitations of the equipment. For example, to avoid water freezing in the evaporator, the chilled water supply temperature cannot be too low. The upper bound of the chilled water supply temperature should maintain the indoor temperature and humidity setpoints. Some studies have defined varying bounds for the decision variables to narrow the search range and accelerate the optimization [14, 78, 108].

3.4.4 Others

In addition to the constraints mentioned above, many other constraints are used to improve the target system's performance. Limitations on the change rates of decision variables [73, 88] and the minimum online/offline time of critical equipment [62, 65, 99] have been applied in practice to enhance the operation stability of target systems.

3.5 Optimization Algorithms

An optimization algorithm is a procedure for finding an optimal or satisfactory solution that minimizes/maximizes the objective function. In this section, optimization algorithms are summarized according to their classification, as shown in Figure 3. They are divided into deterministic algorithms, meta-heuristic algorithms, and hybrid algorithms. The frequency of different algorithms used in the reviewed literature is shown in Figure 4.

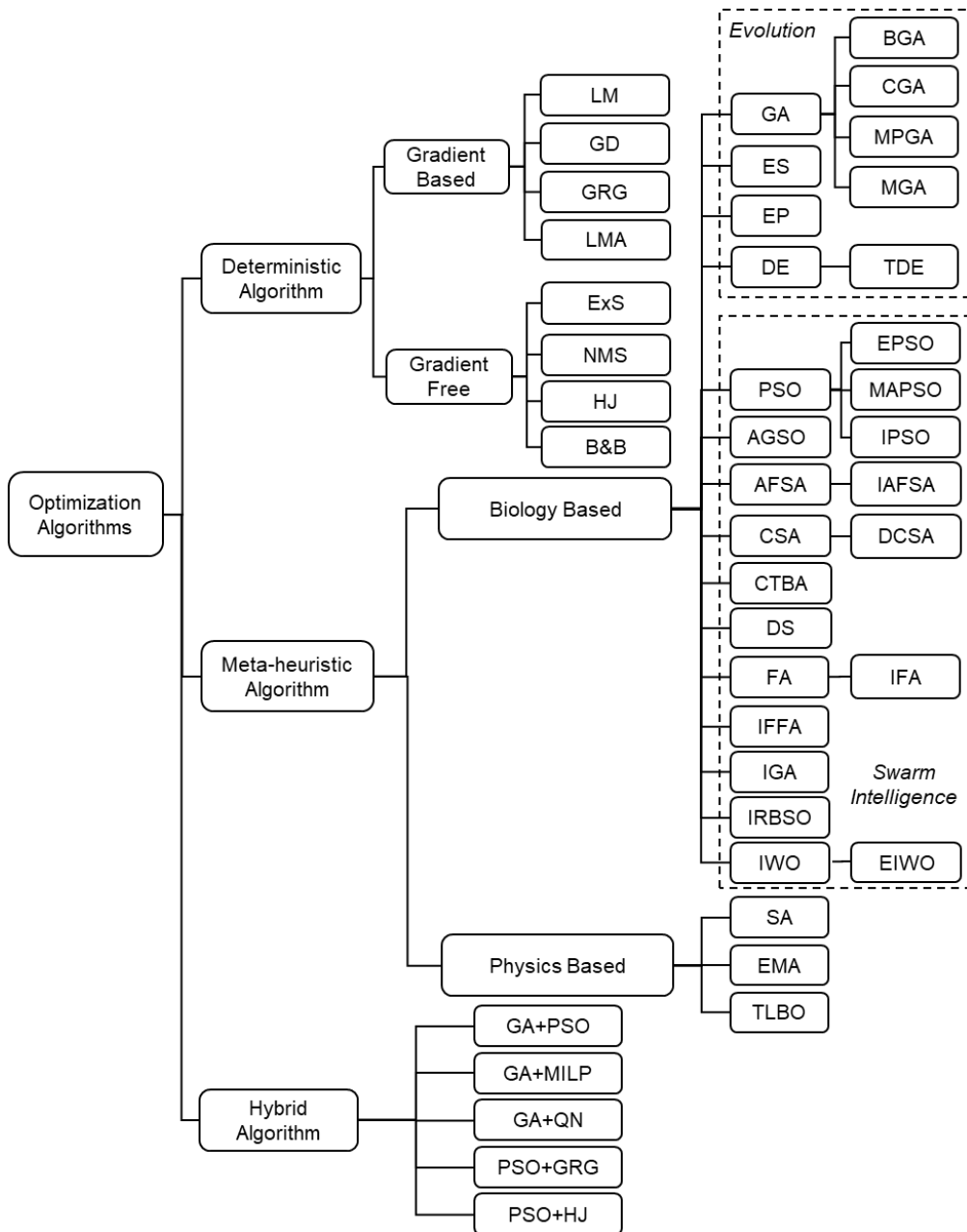


Figure 3: Classification of optimization algorithms for optimal control

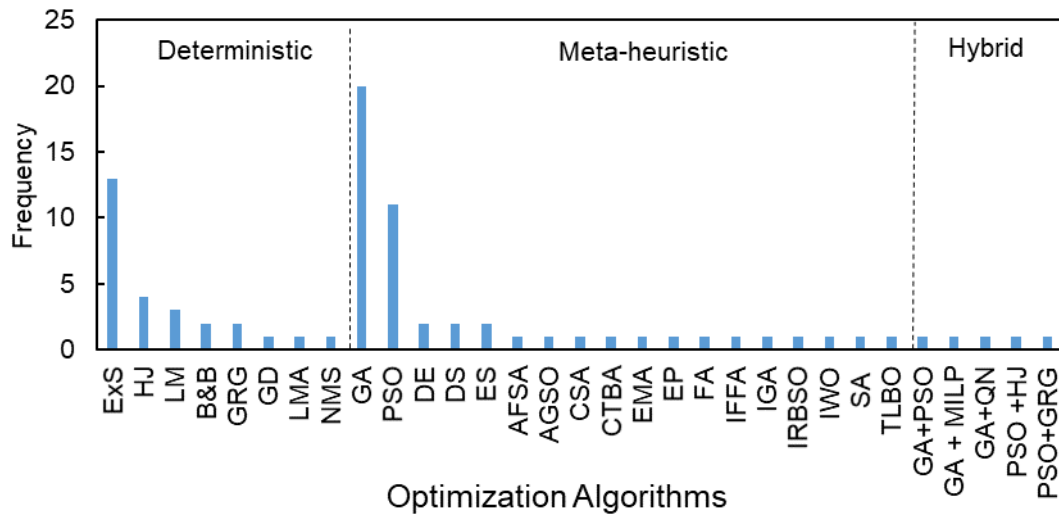


Figure 4: Frequency of different algorithms used in optimization

3.5.1 Deterministic algorithms

Deterministic algorithms use specific rules defined by rigorous mathematical formulations to find the optimum solution. These rules help the algorithm converge to a stationary point quickly, and the result of a deterministic algorithm is definite and replicable. However, the solution found may be a local optimum, because deterministic optimization algorithms look for stationary points in the response variable. For deterministic algorithms, there are two subclasses: gradient-based algorithms and gradient-free algorithms.

Gradient-based algorithms use function values and the associated gradient information to find the optimal solution. In Figure 3, the gradient-based algorithms employed in this field include the Lagrange multiplier (LM) method [49, 100], gradient descent (GD) [70], generalized reduced gradient (GRG) [77], and Levenberg-Marquardt algorithm (LMA) [109]. The LM algorithm utilizes the derivative test to seek the optimal solution. For a convex differentiable objective

function, the LM method can find the global optimum quickly. However, this algorithm is not suitable for non-convex functions, and it may not converge under certain conditions, such as low cooling demand [47, 48]. The GD is a first-order algorithm for seeking the optimal decision variables by iteratively moving in the direction of the steepest descent. In the GRG method, the decision variables are separated into independent and dependent variables, and the original optimization problem can be reduced and solved by the reduced gradient method. The LMA combines the advantages of the GD and Gauss-Newton methods by adaptively selecting the parameter updating methods from the GD and Gauss-Newton. For a convex objective function, gradient-based algorithms can find the global optimum quickly. However, a convex objective function is not guaranteed, due to the nonlinearity and highly constrained nature of central cooling systems, making it difficult for gradient-based algorithms to find the global optimum.

Gradient-free algorithms do not use any derivatives, and only the output of the objective function is iteratively compared in the optimization process. The gradient-free optimization algorithms used in this field include the exhaustive search (ExS) [18, 85], Nelder-Mead simplex (NMS) [110], Hooke-Jeeves (HJ) [14, 104], and branch-and-bound (B&B) algorithms [61, 65]. The ExS algorithm tests possible solutions sequentially to find the optimum point, and it is normally used for discrete or discretized variables. As it is simple and easy to implement,

the ExS algorithm is popularly used, as shown in Figure 4. However, its computational cost increases quickly with the number of decision variables. Some studies have introduced near-optimal points determined from prior knowledge to narrow the search range of the ExS algorithm [78, 79, 101]. The NMS is a direct search algorithm that constructs an n-dimensional simplex in the space of decision variables. The function values of vertices are evaluated, and the vertex with the highest function value is replaced with a new vertex through reflecting, contracting, or expanding the simplex. However, the NMS algorithm may fail to converge to a stationary point on nonsmoothed functions [111]. The HJ algorithm searches along each coordinate and determines the search direction through exploratory moves. This algorithm can quickly achieve a good reduction of the objective function when the discontinuities of the objective function are small [111]. The B&B algorithm performs a top-down recursive search through the instance tree formed by the branch operation, which can solve various optimization problems, including discrete and combinatorial optimization.

3.5.2 Meta-heuristic algorithms

Meta-heuristic algorithms are non-deterministic or stochastic, and the updating rules for the decision variables are typically inspired by nature [112]. In recent years, numerous meta-heuristic algorithms have been proposed and popularly used to optimize the control of the central cooling system, as 61.0% of the

review studies use this kind of algorithm. They could be classified into biology-based and physics-based algorithms according to the source of inspiration. Evolutionary and swarm intelligence algorithms are two families of biology-based optimization algorithms. Evolutionary algorithms are inspired by the evolution of species, and swarm intelligence algorithms are inspired by the collective behavior of natural systems, such as the social behavior of birds flocking. Physics-based optimization algorithms are inspired by physical processes, such as simulated annealing (SA) [46]. Among these algorithms, the genetic algorithm (GA) is the most popular, followed by the particle swarm optimization (PSO) algorithm, as shown in Figure 4. These meta-heuristic algorithms normally shown better global search ability as certain tradeoffs between randomization and local search are made to move toward searching on a global scale [112]. Thus, they are suitable for global exploration in the space of the decision variables of complex optimization problems. Compared with deterministic algorithms, meta-heuristic algorithms are less mathematically complicated, as no derivative information is required. However, their convergence speed is slower, and the accuracy and reproducibility of their solutions may be somewhat low due to random processes. Thus, improving the accuracy, convergence speed, and robustness of the algorithm has been a trend, in recent years [40, 55, 82].

3.5.3 Hybrid algorithms

Hybrid algorithms use two or more algorithms in a single optimization problem to improve search efficiency, as shown in Figure 3. In hybrid algorithms, meta-heuristic algorithms are typically employed first for global exploration, and deterministic algorithms are then used to refine the search locally to improve the accuracy of the solution and accelerate convergence. The search efficiency of hybrid optimization algorithms has been verified in various studies [38, 39, 42, 87]. For instance, the PSO+GRG algorithm achieved slightly higher energy savings than that obtained by the PSO or GRG algorithm alone. Moreover, a hybrid algorithm that integrated the GA with PSO was employed in optimal control of a typical chiller plant [83]. The GA and PSO were used to optimize binary variables and continuous variables, respectively.

4. Optimization Approaches

Optimization the control of central cooling systems determines the control variables that minimize the operational cost of the target system while providing a comfortable indoor thermal environment. According to whether a system model is used to evaluate the system energy performance in the optimization process, existing approaches are classified as system-model-based and data-based, as shown in Figure 5. In the reviewed studies, the system-model-based method is still the major solution for optimizing the control of the water-cooled central cooling system, as 83.7% of reviewed papers used this kind of approach.

In recent years, data mining and reinforcement learning methods have been gradually used in this field.

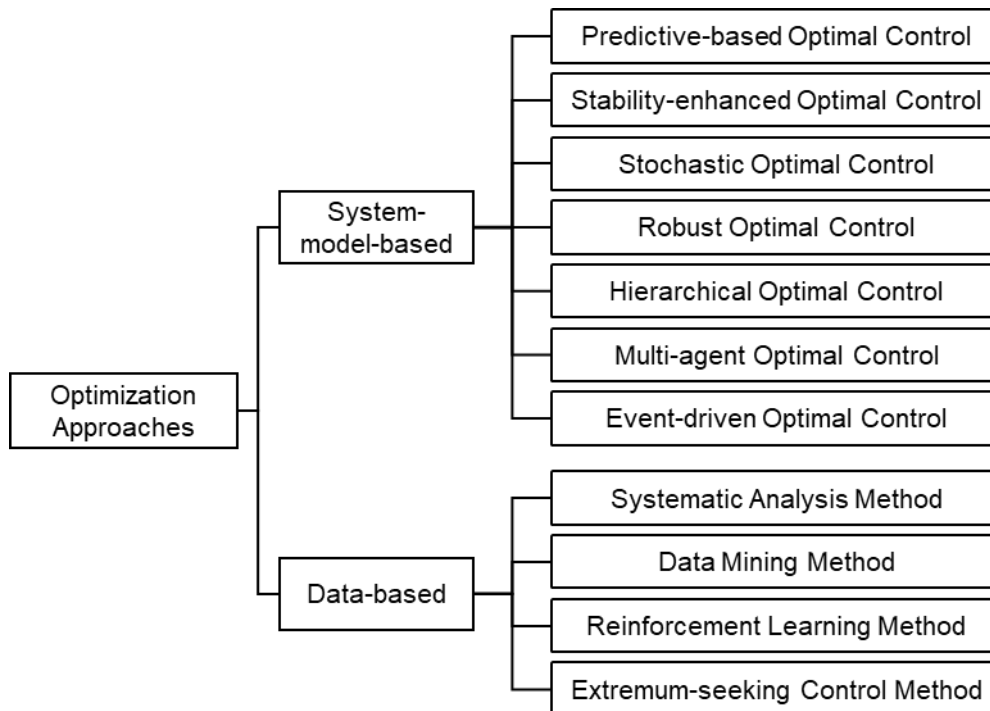


Figure 5: Classification of optimization approaches

4.1 System-model-based optimization

The system-model-based optimization is concerned with seeking the most cost-effective control variables while satisfying operation constraints, which requires a mathematical model that describes the system's dynamic behavior. In the optimization process, the system model is used to estimate the system energy consumption and the system responses to the variation of control variables. The conventional system-model-based optimization for central cooling systems aims to minimize the system energy or operation costs. The optimization is conducted periodically with the assumption that the operating conditions remain

constant in the upcoming control interval. In this process, uncertainties associated with input parameters and the system model are not considered, although they widely exist in engineering processes. Thus, conventional optimal control (COC) may not fully reveal the energy-saving potential of the target system. In the past decade, numerous advanced optimal control methods have been proposed to improve the performance of central cooling systems from different perspectives. In this section, the recent development of optimal control methods was summarized below.

4.1.1 Predictive-based optimal control (PBOC)

To further improve the energy efficiency, predictive-based optimal control determines the optimal solution from the predicted cooling demand and weather conditions over a finite prediction horizon instead of at an instant in time. Many studies have evaluated the ability of BPOC in improving the energy efficiency of central cooling systems [71, 92, 106]. For instance, Wang et al. [92] using a one-timestep-ahead optimal control method reduced the redundant energy consumption by 86.1%. Sala-Cardoso et al. [106] utilized a data-driven model for cooling demand forecasting and a deep neural network for system performance modeling to optimize the energy performance of a chiller plant, and the energy efficiency of the target system was improved by 19.5% compared with the standard real-time controller.

4.1.2 Stability-enhanced optimal control (SEOC)

In general, the static optimization is conducted independently in each control interval, which may introduce significant disturbances and thus damage the stability of the operation. This weakness has become the main challenge for applying optimal control. To avoid unstable operations, researchers have proposed several measures. Ma and Wang [108] employed a rule-based supervisor to determine the actual control settings by compromising the control stability and energy savings. To maintain the control stability, the control settings keep unchanged, if there is no significant energy-saving. Sun et al. [18] proposed a multiplexed optimization method to update the decision variables sequentially instead of simultaneously. This method reduced the energy consumption by 6.8%, which is comparable to the conventional optimal control method, and improved the control stability significantly with a 50.0% reduction in tracking error of setpoints. Based on the multiplexed optimization method, Asad et al. [19] proposed a DOF-based setpoint-resetting scheme to further improve the stability of control. Compared with the conventional scheme, the DOF-based method reduced the tracking error by 14.7-63.4% for different setpoints.

4.1.3 Stochastic optimal control (SOC)

Uncertainties are widely existent in engineering processes such as control and monitoring. In central cooling systems, uncertainties may cause inappropriate

control actions and degrade the energy performance of the target system. To reduce the impact of uncertainties on system performance, the data fusion [107, 113-115] and the multi-indexes control methods [116, 117] have been proposed to reduce uncertainty in cooling load measurement. The study [98] has shown that the impact of uncertainties on optimal control methods was more significant than on conventional control methods. To enhance control robustness, the SOC method has been proposed to manage uncertainties by directly capturing uncertainty disturbances in the optimization formulation. Li et al. [118] used a SOC method to optimize the sequencing control of chillers with measurement and modeling uncertainties. This method introduced flexibility in decision-making and improved the robustness of different control aspects by setting different threshold values. Qiu et al. [22] used the SOC method for multi-objective optimization with measurement uncertainty. The results have shown that the SOC showed slightly higher energy savings (0.7%) than deterministic optimal control and enhanced the robustness of control by canceling unnecessary starts-up. Kumar et al. [21] investigated stochastic model-predictive control in a central cooling system to mitigate uncertainties and constraint violations. This approach ultimately achieved 7.5% cost savings of the central plant and mitigated constraint violations by explicitly incorporating uncertainty in modeling.

4.1.4 Robust optimal control (ROC)

The ROC is another method that deals with uncertainties in the optimization process. This method assumes that uncertain variables belong to a limited range specified by lower and upper bound (uncertainty set). The ROC aims to make a decision that is feasible for each value in the uncertainty set, which is typically achieved by optimizing the worst-case objective function. The robustness of this method increases with the conservatism level of control. However, the system energy performance showed the opposite trend. Saeedi et al. [20] and Tian et al. [23] used this method to optimize chiller loading with uncertain cooling demand. According to their results, the energy consumption of chillers with the most conservative solution was 8.5% higher than that without uncertainty. This difference occurred because the optimum solution was obtained from the worst-case, in which the cooling demand was 10.0% higher than the deterministic case. This method has also been successfully used for optimal chiller loading with multiple uncertainties, including measurement, control, and threshold uncertainties [63]. However, the ROC problem is difficult to solve directly, and the results are very conservative, indicating low energy efficiency.

4.1.5 Hierarchical optimal control (HOC)

For large central cooling systems, a formulation of the optimization that adequately captures the thermal dynamic and physical interactions, as well as

the tight coupling between subsystems, is a complex mixed-integer nonlinear constrained problem. This complicated problem is generally decomposed into multiple subproblems using different approaches. Zhang et al. [94, 95] decomposed the complicated single optimization using a Lagrangian relaxation approach. The optimal solutions for subproblems are determined first and then coordinated through iterative updating of the Lagrangian multipliers to minimize the high-level objective function. Using this method, a near-optimal solution can be obtained at a lower computational cost. Chiam et al. [38] proposed a hierarchical framework for holistic optimization, which decomposed the problem into a master level and a slave level. The nonlinear variables are optimized at the master level, and they are treated as known parameters to linearize the objective function at the slave level. With a linearized objective function at the slave level and fewer decision variables at the master level, the computation time can be reduced significantly. This HOC approach reduced the energy consumption of a district cooling system by 3.0-31.9% with a reasonable resolution time. Similarly, Rawlings et al. [119] employed a hierarchical decomposition for model-predictive control of a large-scale commercial HVAC system. In high-level optimization, the total energy consumption of the system is approximated to reduce the complexity. In low-level optimization, the energy consumption of each subsystem is considered in detail, and they are solved in parallel to reduce overall computational time. This two-level model-predictive

control achieved cost savings of 10.0-15.0% compared with control by professional operators.

4.1.6 Multi-agent optimal control (MAOC)

The MAOC is another solution for energy system management in large-size buildings. This approach divides the optimization problem into smaller and more manageable pieces that can be solved in parallel by individual agents. Individual solutions are then handled by a coordination agent to achieve the overall objective. Cai et al. [81] proposed a MAOC framework for building energy system management. The results of a simulation case study show that the MAOC method obtained a near-optimal solution along with significant energy savings (2.5-12.2%). Jaramillo et al. [120, 121] compared the performance of different optimization algorithms under the MAOC framework in a central cooling system. According to their results, the MAOC with algorithms capable of handling mixed-integer, non-convex objective functions was able to find a near-optimal solution. With good scalability, the MAOC is more economical and easier to configure for central cooling systems [120]. However, there are some drawbacks including the requirement of equipment for additional data transfer and the tradeoff of optimality for reduced computation. Moreover, the control topology and inter-controller of multi-agent control are different from the conventional ones, and hence further research is required.

4.1.7 Event-driven optimal control (EDOC)

The conventional optimization of control is normally conducted periodically (e.g., every 15 min), referred to as time-driven optimal control (TDOC). The conventional optimal control suffers from high computational cost since the nonlinear objective function is evaluated in each iteration. The EDOC triggers optimization by predefined “events” rather than “time”. Since optimization was only conducted when one of the events occurred, the computational cost of optimal control reduced significantly [89-91]. The energy performance of event-driven optimal control is equal to or even better than that of TDOC, and it deals more successfully with aperiodic behaviors of the target system because optimizations are conducted at the right times. For example, the computation time of an EDOC was found to be 59.6-83.5% shorter than that of TDOC with a 15 min interval, and the energy-saving rate was slightly higher than that of TDOC [88]. Hou et al. [122] further improved the EDOC method by establishing an event map between event-actions and influential decision variables. When an event occurs, only the corresponding decision variable is optimized, rather than all the decision variables, to further reduce the computational cost. The computational load of the EDOC was reduced by 75.9-85.2% in comparison with TDOC with a 30 min interval in a typical air conditioning system. Meanwhile, this event-driven optimal control reduced energy consumption by 4.2-7.4% in different seasons, which is comparable to the TDOC.

4.1.8 Comparison of different optimal control methods

Table 3 briefly summarized the performance and the features of recently developed optimal control methods to help readers understand and choose the correct method. Their performance on various aspects was compared with the COC method. Among them, the PBOC method shows better energy efficiency by employing future information in the optimization process. Other methods except ROC showed comparable energy savings with the COC. The ROC exhibited strong robustness by describing uncertainty in optimization formulation and making decisions to satisfy the worst-case. Thus, the results are very conservative, indicating low energy efficiency. The SOC also showed strong robustness of control by directly capturing uncertainty disturbances in the optimization formulation. Dealing with uncertainties increases the computational complexity and computation cost of the ROC and SOC, while it is still acceptable for engineering applications. Moreover, quantification of uncertainties, such as uncertainty distribution or uncertainty set of different variables, are normally difficult to obtain in practice. The SEOC method improves the system operation stability and robustness from the time domain perspective by reducing the number of update variables and decision variables' changing rate in each control interval. The HOC and MAOC exhibit high computational efficiency and good scalability by decoupling the complicated optimization problem into manageable subproblems and solving them in

parallel. Thus, they are suitable for large-size central cooling systems. The EDOC provided excellent computational efficiency by conducting optimization only when necessary, instead of periodically. The definition of “events”, which is used to triggering optimization procedures, is crucial to the success of using EDOC, and it requires abundant prior knowledge and sophisticated techniques.

Table 3. The performance of various system-model-based optimal control methods (ISSOC: instant system state and operation condition; TER: tracking error reduction)

Methods	Input information	Driven Method	Complexity	Energy savings	Stability and Robustness	Computation time	Strength and weaknesses	Ref.
PBOC	<ul style="list-style-type: none"> • ISSOC • Forecast data 	Time	High	AES: 0.5-5.6%			•Exhibit high energy efficiency.	[71]
				Reduce 80% redundant energy of COC			•Can handle a wide variety of constraints.	[92]
				DES: 19.5%			•Require a suitable cooling demand. forecasting model and the system model.	[106]
SEOC	<ul style="list-style-type: none"> • ISSOC •Previous control setting or set-point reset scheme 	Time	Medium	DES: 6.8% (6.7%) ¹	TER: 50%	3.6s (98.3%) ²	•Enhance the stability of control.	[18]
					TER: 14.7–63.4%		•Comparable energy efficiency to COC.	[19]
				DES: 0.7-2.6%			•The control variable updating sequence and resetting scheme should be determined previously.	[108]
SOC	<ul style="list-style-type: none"> •ISSOC •Uncertainty distribution 	Time	High		Enhanced		•Show strong robustness.	[118]
				AES: 7.5% (9.7%)	Avoid constraint violation		•Near-optimal energy efficiency.	[21]
				AES: 0.7% higher	Cancel unnecessary start-up	5 min for whole year operation	•High computational complexity.	[22]
ROC	<ul style="list-style-type: none"> •ISSOC •Uncertainty 	Time	High	DES: -8.5% (worst-case)*	Risk-averse strategy		•Show strong robustness.	[20,
							•Energy efficiency decreases with the	23,

	set					conservatism level.	[63]	
						•High computational complexity.		
						•Require quantification of uncertainties.		
HOC	•ISSOC	Time	Medium	2.0-25.0%	Reasonable	3.3-11.7s	•Good scalability.	[95]
				6.0-18.0%		•High computation efficiency.	[95]	
				DES: 3.0-31.9%		•Suitable for large-scale applications.	[38]	
				AES: 10.0-15.0%		170.0s	•Only achieve near-optimal energy efficiency.	[119]
MAOC	•ISSOC	Time	High	12.2-42.7%		41.0-114.0s	•Good scalability.	[81]
				(13.3-45.1%)		•Achieve near-optimal energy efficiency.		
				3.4-11.3%		6.4-86.3s	•Require additional data transfer among agents.	[120]
				4.2-8.5%		12.5-55s	•The control topology and inter-controller are different from the conventional one.	[121]
EDOC	•ISSOC •Event map	Events	Medium	DES: 6.3-11.8%	Fewer control actions	10.5-76.5s		[88]
						(59.6-83.5%)		
				DES: 6.9-11.5 %		14.6-26.3	•Excellent computation efficiency.	[89]
						(79.1-81.0%)	•Near-optimal energy efficiency.	
				DES: 6.6-10.4%		(62.2-84.8%)	•Require abundant prior knowledge and sophisticated techniques for event-map definition.	[90]
				DES: 5.8-12.1% (5.3-9.2%)				[91]
				DES: 4.2-7.4%		1.7-2.9s		[122]
				(4.4-7.5%)		(75.9-85.2%)		

* Negative value indicates more energy consumption than the conventional optimal control method.

¹ The energy savings of the conventional optimal control method.

² The percentage of computation time reduction compared with the conventional optimal control method.

4.2 Data-based Optimization

The data-based optimization approach determines the “best” control variables without using a mathematical system model. The data-based optimization methods used to improve the system energy efficiency include 1) systematic analysis, 2) data mining, 3) reinforcement learning, and 4) extremum-seeking methods.

4.2.1 Systematic analysis method (SAM)

In the systematic analysis method, the energy performance of the whole system is first analyzed in detail, and then energy-saving strategies are proposed to increase system energy efficiency. Deng et al. [26] analyzed influential factors systematically and proposed optimal control strategies, considering internal factors, external factors, and their synergetic effect, to improve the energy performance of a high-rise office building. These strategies increased the energy efficiency of the chilled water system by 29.2%. Yu and Chan [27] examined the correlation between control variables and system COP under different operating conditions to determine which control variable should be adjusted to improve the system energy efficiency. This method reduced the energy consumption of a water-cooled chiller plant by 5.3%. Wang et al. [123] developed a near-optimal performance map from historical operation data, and the cooling load of chillers was allocated according to this periodically updated map to achieve the near-highest COP. The performance of this method was

evaluated in a simulation case, and energy reductions of 3.4-9.0% were achieved in comparison with original sequence strategies.

4.2.2 Data mining method (DMM)

Data mining refers to extracting useful information from a raw data set. In recent years, data mining methods, including cluster analysis [124, 125], decision tree [126], and association rule mining methods [16, 28, 29], have been employed to extract useful information from the historical operation data of the target system. This method could help to identify the operational problems, energy-saving potential, and near-optimal control strategies under different operating conditions. During operation, control parameters are regulated according to the extracted near-optimal control strategies. Li et al. [124] used the hierarchical cluster method to determine energy-saving potential from an analysis of the COPs of individual chillers in different clusters. In [125], a two-step clustering and odds ratio analysis method was used to detect deficient controls. Several energy-saving measures were then proposed to improve the efficiency of a multi-chiller system, including switching off unnecessary chillers, setpoint resetting, and improving the control accuracy of chilled water and cooling water temperature. Fan and Xiao [126] utilized the association rule mining method to detect operational problems and energy-saving opportunities. Zhang et al. [28, 29] used the FP-growth algorithm, an association rule mining method, to discover the operational problems hidden behind them. Zhou et al. [16] utilized

the Apriori algorithm to extract control parameters associated with high energy efficiency, and these parameters were then used as the setpoints in practical operation. Using this method reduced the energy consumption of a chiller plant by 11.6% in summer and by 13.3% in winter [16].

4.2.3 Reinforcement learning method (RLM)

RLM is a branch of machine learning techniques. The RLM not only extracts useful information from historical data, as do the data mining methods; it also learns to make decisions to maximize the reward feedback from the target system. Data mining methods are normally trained offline, whereas the RLM method typically learns online by directly interacting with the target system. In the RLM method, one or more agents are typically used to take a sequence of actions to maximize the energy efficiency of the target system. The action taken by agents is to change the setpoints of control variables such as chilled water supply temperature. The state of the target system varies with the agents' action, and a reward or penalty is fed back to the agent to reveal the quality of action. The reward or penalty is the objective of optimization, such as the COP of the target system. Using the RLM could significantly reduce the energy consumption of the target system. For example, the Q-learning algorithm, a classical reinforcement learning method, was employed to optimize the chilled water supply temperature of chillers, and the system energy efficiency of the chillers increased by 4.4% in the first cooling season [30]. This method was also

used to optimize the frequencies of both cooling tower fans and cooling water pumps. Energy reductions of 11.0% were achieved in the first cooling season after implementation, which is close to 14.0% obtained by the model-based optimization method [31]. Because of the self-learning ability, the energy-saving rate of the RLM increased to 12.0% in the second cooling season [31]. This method does not require a mathematical model for the target system, as the agent takes actions online directly using feedbacks from the real system.

4.2.4 Extremum-seeking method (ESM)

The extremum-seeking method is another model-free optimization that interacts with the target system directly. It roughly estimates the gradient of the system output by perturbing the system with a slow periodic signal, and the control inputs are adjusted along the gradient descent direction to seek the extremum point. This process is conducted online through direct interaction with the target system, and no mathematical model of the target system is required. A single-variable extremum-seeking method [32, 33] was used to optimize cooling tower fan speed and minimize the total energy consumption of chillers and cooling towers. The results showed that the total power consumption of the chillers and towers was very close to the estimated optimum solution. Meanwhile, Mu et al. [34] put forward a multivariable extremum-seeking method to simultaneously optimize the cooling tower airflow rate, cooling water flow rate, and chilled water supply temperature setpoint. This method was evaluated by

simulation and converged to a near-optimal solution. Moreover, this multivariable extremum-seeking method with a penalty function can prevent integral windup due to actuator saturation. In this method, design parameters such as the amplitude and frequency of the perturbation signal affect the convergence speed and system operation stability. Therefore, these design parameters need to be selected carefully. It should also be noted that this method normally converges to a local minimum.

4.2.5 Comparison of different data-based optimization methods

Table 4 Table 3 summarized the characteristics of different data-based optimization approaches to help readers understand and choose the correct method. All these methods could help to achieve significant energy savings. The SAM and DMM are conducted or trained offline to extract the energy-efficient rules and parameters. The SAM do not require complex system model and data analysis skills, but it relies heavily on the prior knowledge and experience of engineers. Compared with the SAM, the DMM relies less on prior knowledge, while the data size for training and computational complexity increased significantly. The RLM and ESM directly interact with the target system for determining the optimal control parameters. The energy savings by using these two methods are close to that of the system-model-based optimization approaches. However, the application of the RLM is still challenging due to its complexity and a long training period to achieve near-optimal energy efficiency.

Moreover, the decisions making by RLM may be unstable due to the inherent uncertainty and randomness, especially in the starting period of training. These unstable actions may result in deterioration of the control robustness, which prevents the application of the method in practice. Similarly, the ESM optimizes the control parameters by perturbing the system, thus the perturbation signal should be designed carefully to avoid unstable operation.

Table 4. Comparison of various data-based optimization methods

Methods	Algorithms	Data size	Energy savings	Complexity	Time use	Strengths and weaknesses	Ref.
SAM	Systematic analysis	One-day filed test (15-min) ¹	DES: 29.2%	Low		• Do not require complex system models and data analysis skills.	[26]
	data envelopment analyses	6-month (0.5-hour)	DES: 5.3%	Low		• Rely on the prior knowledge and experience of engineers	[27]
	Performance map	1-2 months	AES: 3.4-9.0%	Medium			[123]
DMM	hierarchal cluster	1-year (1-hour)		High			[124]
	2-step cluster	1-year		High		• Rely less on prior knowledge.	[125]
	Decision tree and QuantMiner	1-year		High		• Require skillful data mining technologies and a large historical operation data set.	[126]
	Apriori algorithm	4-year	DES: 11.6-13.3%	High			[16]
	FP-growth algorithm	1-year (5-min)		High			[28, 29]
RLM	Q-learning (table-based)	4-year (Online training)	AES: 4.4% (4.95%) ²	High	<1.0s (10.0s) ³	• Requires less prior knowledge of the system. • Long training time to achieve near-optimal energy efficiency.	[30]
	Q-learning (table-based)	two cooling seasons (Online training)	AES: 11.0-12.0% (14%)	High		• Actions may be unstable due to inherent uncertainty and randomness.	[31]
ESM	Standard/ Anti-windup ESM	-	0.3-5.7% (0.3-5.7%)	Medium	Settling time: 11720-13735s	• Do not require mathematical system model. • Seek the minimum point online.	[32, 33]
	Newton-Based ESM	-	14.2-26.6% (15.2-27.7%)	Medium	Settling time: 1382-1536s	• Normally converges to a local minimum.	[34]

¹ Time interval of the data collected.

² The maximum energy savings estimated by a system-model-based optimization approach.

³ The computational time of the system-model-based optimization approach.

5. Discussion and Future Work

5.1 Discussion

5.1.1 System-model-based optimization

Recently, considerable efforts (83.7% of the reviewed papers) have been devoted to system-model-based optimization, and they can improve the system energy performance from different aspects. To improve the performance of a target system, users could select optimal control methods according to their characteristics, as summarized in Section 4.1.8. For systems with significant dynamic features, such as thermal storage and dynamic energy price, a predictive-based optimal control method might be a suitable solution, as the future control input and future system response are predicted for decision making. For systems with strict control requirements, the existence of uncertainties may fail in meeting the control requirements. In this scenario, SOC and ROC are two potential solutions that take uncertainties into account in the optimization formulation. ROC provides a more conservative solution than SOC as it finds the solution to counteract the worst-case scenario. For large, complicated central cooling systems, the computation time is a crucial factor that impacts the selection of optimal control methods. The HOC and MAOC improve the computational efficiency by dividing the complex optimization into small manageable subproblems. The EDOC could significantly reduce the computation time by triggering online optimization only when necessary.

Moreover, two or more optimal control methods might be employed together to improve the overall performance of the target system. For example, the predictive-based optimal control combined with stochastic or robust optimal control might be used for systems with significant dynamic features and non-ignorable uncertainties.

5.1.2 Data-based optimization

In recent years, DMM and RLM have increasingly been used as they rely less on the expert knowledge of the system than that of the SAM. DMM normally requires a large data size for training offline (as shown in Table 4), which is suitable for existing systems with rich historical operating data. RLM interacts with the target system for online training, thus this method could be used in a newly built system or system with less historical data, while it may suffer a long training period.

5.2 Future work

By summarizing the previous literature, the following aspects were found to be the trends of the recent studies and future work directions. First, many studies on optimization of the control of central cooling systems were found to emphasize improving the control stability and robustness in addition to the system energy efficiency. Although uncertainties have a significant effect on the system's overall performance including energy efficiency, stability, and robustness of control, they are ignored in conventional optimal control. Ignoring

uncertainties in optimal control might lead to energy efficiency degradation and failure to satisfy constraints in practice. Several studies have tried to deal with uncertainties in optimizing the control of the central cooling system. In these studies, they mainly focus on dealing with the measurement and forecasting uncertainty of the cooling demand in the local control process, such as chiller sequencing control. As uncertainties widely exist in the control process on both the supervisory and the local control levels, more efforts are required to dealing with uncertainties both in local and supervisory control levels. Moreover, the uncertainties are assumed to be known in the previous studies, which are difficult to be quantified in practice due to the lack of data and the variations of system structure. Therefore, more efforts on quantification of uncertainties and dealing with uncertainties in optimization are needed in the future.

Second, another frequently discussed problem on optimal control methods is the computational efficiency, as the long computation time prevents the application of online optimal control in complex central cooling systems. In the literature, there are two directions to improve the computational efficiency of the optimization, dividing the complex optimization problems into more manageable pieces (HOC and MAOC) and conducting the optimization only when necessary (EDOC). However, they are still in the early stages of development. Studies on multi-agent optimal control have focused only on the design of the algorithm. Issues related to hardware implementation, such as

control topology design and inter-controller communication, must be considered. For event-driven optimal control, the definition of event space is the key factor for the success of using EDOC. The events and their corresponding thresholds are impacted by multiple factors, such as the system types and climate conditions. The methods of event space definition should be further investigated considering both system energy efficiency and stability of system operation.

Third, in past few years, machine learning methods have increasingly been used for optimizing the control of central cooling systems as they rely less on the expert knowledge of the system. Currently, the machine learning algorithms employed in optimal control of central cooling systems are rule-based or table-based with limited and predefined state space and action space, which sacrifices the flexibility and precision of control. Therefore, value-based or network-based machine learning methods is required to further improve the energy performance and control precision. With the development of technology, more advanced systems (e.g., renewable energy and thermal storage) are integrated with central cooling systems, which exhibit more complex dynamic properties. Therefore, machine learning methods that explore the synergistic effects of different subsystems and components, such as multi-agent RL, are worth further investigation to improve both the energy and control performance of the target system.

6. Conclusions

This study reviews the state-of-the-art optimization approaches for controlling the water-cooled central cooling systems. Two kinds of optimization approaches, including system-model-based and data-based, have been successfully applied in optimizing the control of water-cooled central cooling systems. The optimization formulation, including decision variables, objective functions, constraints, and optimization algorithms, and various optimization approaches in previous studies were summarized, which will be helpful in subsequent studies to conduct similar optimization.

For optimization formulation, frequently used decision variables are partial load ratio, cooling water temperature, and chilled water temperature. However, it is not recommended to use the partial load ratio of each chiller as the decision variable, as it is an undirect controllable variable. Due to the tight coupling feature among subsystems, holistic optimization is recommended to explore the synergistic effects of multiple variables and to achieve the global optimum. The objective function impacts the accuracy and computation time of the optimization. Hybrid models with acceptable accuracy and shorter computation time are recommended for real-time optimal control. For solving optimization problems, meta-heuristic optimization algorithms have increasingly applied in optimizing the control of the water-cooled central cooling system, since they normally exhibit good global search capability.

Based on the conducted review, the system-model-based optimization is still the major optimization approach for improving the operation energy efficiency of the central cooling system. In addition to improving system energy efficiency, enhancing the stability, robustness, and computation efficiency of optimal control methods have been the trends of recent development. The SOC and ROC exhibit strong robustness by describing uncertainty in optimization formulation. However, the computational complexity and computation cost increased. Moreover, the ROC is very conservative indicating low energy efficiency, as the ROC makes decisions that satisfy the worst-case. The HOC, MAOC, and EDOC improve the computational efficiency from different aspects, and they are suitable for large-size complex central cooling systems. However, in these methods, the control stability and robustness were not considered. Therefore, it is recommended to analyze the main control demand and compare the performance of different optimal control methods before selecting the most suitable one or combination of them.

The data-based optimization approach has been increasingly studied in recent years, especially data mining and reinforcement learning methods. Although using the machine learning methods could achieve near-optimal energy savings, the application of the machine method is still challenging due to its complexity and limited control flexibility and precision. Therefore, more efforts are required to further improve energy efficiency and control performance of

machine learning methods in optimizing the control of the water-cooled central cooling system.

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