Performance of dual frequency comb channelizers for RF signal processing

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Abstract: We analyse the performance limits of dual frequency comb based photonic signal processors. We show that the relative phase noise between the two combs is critical to defining the signal-to-noise-ratio of such systems. © 2021 The Author(s)

1. Introduction

Dual optical frequency combs have been demonstrated in many contexts as effective channelizers for RF signal processing, following their original development for rapid spectroscopy [1]. The dual comb technique exploits the Vernier effect between two frequency combs of different spacing, f_{sig} and f_{LO}, to channelize a high bandwidth signal into many narrowband signals of bandwidth $\Delta f = f_{LO} - f_{sig}$. Compressive sampling for sub-noise signal detection [2], photonic analog-to-digital conversion [3, 4], and detection of sparse wideband RF signals [5] are among the demonstrated utilizations of this technique for signal processing. For these purposes, high repetition rate frequency combs are typically required, such as electro-optic combs, micro resonators, or integrated mode locked lasers. Electro-optic combs are attractive in particular since they offer easy frequency tuning, high power per comb line and can operate stably over a wide temperature range.

Despite the numerous demonstrations of the dual comb technique in signal processing, there has to our knowledge been limited discussion of the performance limits of these systems. Here, we assess the additive noise and distortion of these dual comb systems, including phase noise, modulator nonlinearity, ADC noise and photodiode noise. In particular, we highlight that the dual comb system is especially sensitive to the relative phase noise between the two frequency combs and measure the relative phase noise of a electro-optic dual comb system to verify our theoretical analysis. We also show that photodiode noise and modulator nonlinearity also become limiting factors in low phase noise scenarios.

2. Noise and distortion in a dual comb system

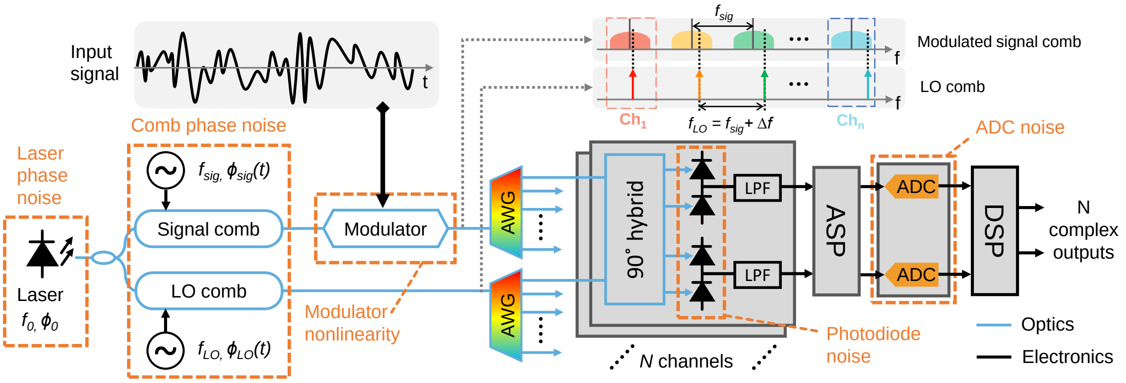

Fig. 1. Generic dual comb channelizer model with noise sources highlighted in orange. AWG, arrayed waveguide grating; DSP, digital signal processing; ASP, analog signal processing; ADC, analog to digital converter; Ch., channel.

A system diagram of a generic dual frequency comb channelizer is shown in Fig 1 with noise and distortion sources highlighted. Two frequency combs, of frequency f_{sig} and f_{LO} with corresponding phase noise ϕ_{sig} and ϕ_{LO} are generated from a seed laser of frequency f_0 and phase noise ϕ_0. The signal comb is modulated with the input signal of interest, followed by demultiplexing and mixing with the LO comb such that the LO comb forms a bank of LO oscillators that shift the input signal by $n \times -\Delta f$ in the frequency domain for the n-th channel. Detection by a bank of N low speed coherent receivers and low pass filtering of bandwidth $\Delta f/2$ allows for parallel detection of spectral slices of the original signal of bandwidth Δf.

The total achievable signal-to-noise-and-distortion-ratio (SINAD) is given by considering the contributions from stochastic noise sources 1) phase noise, 2) photodiode shot noise, 3) photodiode thermal noise, 4) ADC noise, and 5) deterministic distortion introduced by the nonlinearity of the input modulator. Limits imposed by photodiode and ADC noise are well established in the literature [6, 7], while modulator nonlinearity can be calculated for a specific input signal by applying the nonlinear transfer function of the input modulator. Assuming

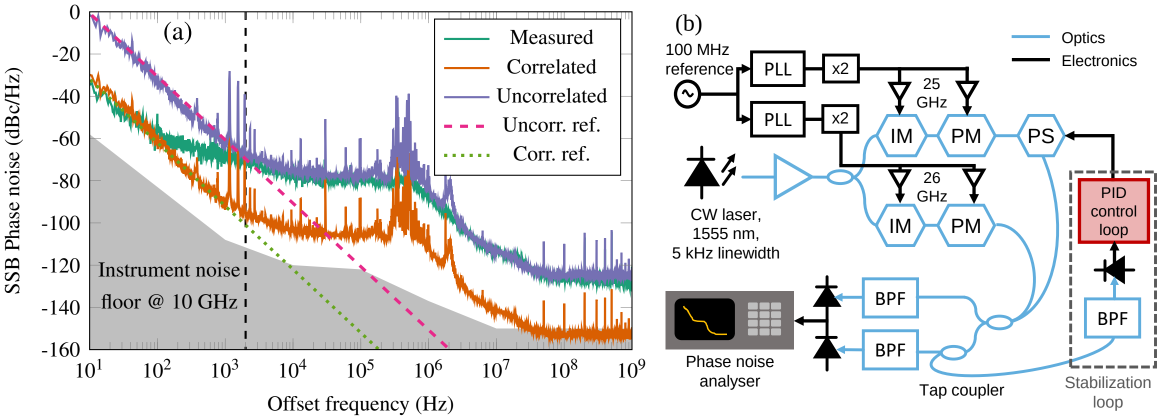

Fig. 2. (a) Phase noise measurement of an electro-optic dual frequency comb system using the setup in (b). PLL, phase locked loop; IM, intensity modulator; PM, phase modualtor; BPF bandpass filter; CW, continuous wave; SSB, single sideband; PS, phase shifter; PID, proportional integral derivative.

that the narrow linewidth laser phase noise, ϕ_0, can be cancelled at the coherent receivers via sufficient length matching, the phase noise limited SNR in the n-th channel, SNR_{pn}, is given by

$$\frac{1}{\mathrm{SNR}_{pn}} = \overline{\Delta\phi_n^2} = 2\int_0^{\Delta f/2} S_{\Delta\phi_n}(f)\mathrm{d}f \tag{1}$$

where $S_{\Delta\phi_n}(f)$ is the phase noise power spectral density in the n-th channel, given by the beating between the n-th comb lines of signal and LO comb

$$S_{\Delta\phi_n}(f) = |n(\phi_{LO}(f) - \phi_{sig}(f))|^2 = n^2\Big[\underbrace{|k_{LO}(f) - k_{sig}(f)|^2|\phi_c(f)|^2}_{\text{correlated sum}} + \underbrace{|\phi_{u,LO}(f)|^2 + |\phi_{u,sig}(f)|^2}_{\text{uncorrelated sum}}\Big]. \tag{2}$$

We presented a detailed derivation of this description in [8]. This formulation of $S_{\Delta\phi_n}(f)$ accounts for the fact that the phase noise of the combs is likely to be correlated to some degree, rather than completely independent as in previous studies. The resulting phase noise for each channel is therefore comprised of both a correlated part $\phi_c(f)$, transformed by functions $k_{sig}(f)$ and $k_{LO}(f)$ describing the mismatch between the two combs, and the uncorrelated parts $\phi_{u,LO}(f)$ and $\phi_{u,sig}(f)$ describing phase noise unique to each comb generating process [9]. This description could apply to any dual- or multi- comb generating technique providing there is some correlation between the additive phase noise of the comb generation processes.

3. Characterization of relative phase noise

To verify this model, we measured the relative phase noise of the electro-optic dual comb system shown in Fig. 2(b) which is synthesized from a common 100 MHz reference. The frequency combs were mixed in a polarization-maintaining 50/50 coupler before being optically filtered such that the n-th tone was incident on a 42 GHz balanced detector. The path lengths between the two frequency combs were matched to < 1 cm, and a feedback loop operating at ≈ 2 kHz was implemented to compensate any time varying optical path length mismatches between the two combs. A tap coupler was used to extract 1% of the light after mixing and filtered such that the central comb lines only were incident on a photodiode. The beat signal therefore between the two central combs lines was used as the input to PID loop which stabilized the optical path difference by driving a phase shifter.

The phase noise measurement for the 10th channel, i.e. the 10 GHz beat note, is shown in Fig. 2(a). Also plotted for illustration are the correlated (orange solid line) and uncorrelated (purple solid line) summations of the phase noise measured directly at the 25/26 GHz synthesizer outputs. Above 2 kHz (indicated by dashed black line), the phase noise essentially follows that predicted by the uncorrelated summation. This is because the phase noise at higher frequencies mainly results from the independent components in each PLL loop and so is uncorrrelated between the combs, i.e. $\phi_{u,LO}(f), \phi_{u,sig}(f) \gg \phi_c(f)$. Below 2 kHz however, the phase noise drops below that predicted by the uncorrelated summation and begins to follow that predicted by the correlated summation of the common reference. This verifies the prediction of (2) in that the phase noise of the dual comb system will be comprised of both correlated and uncorrelated components.

4. Discussion and performance limits

Having established the phase noise relationship for a dual comb system, we plot the worst case SINAD (i.e. for the N-th, or highest, channel) for a variety of channelizer scenarios in Fig. 3, along with specific examples that show how the various noise sources contribute. In Fig. 3(a) the SINAD is plotted vs the total optical power of the system for $N = 10, 30, 100$ channels and for relative jitter (i.e. integrated phase noise) values of 1 fs, 10 fs, 50 fs, for a

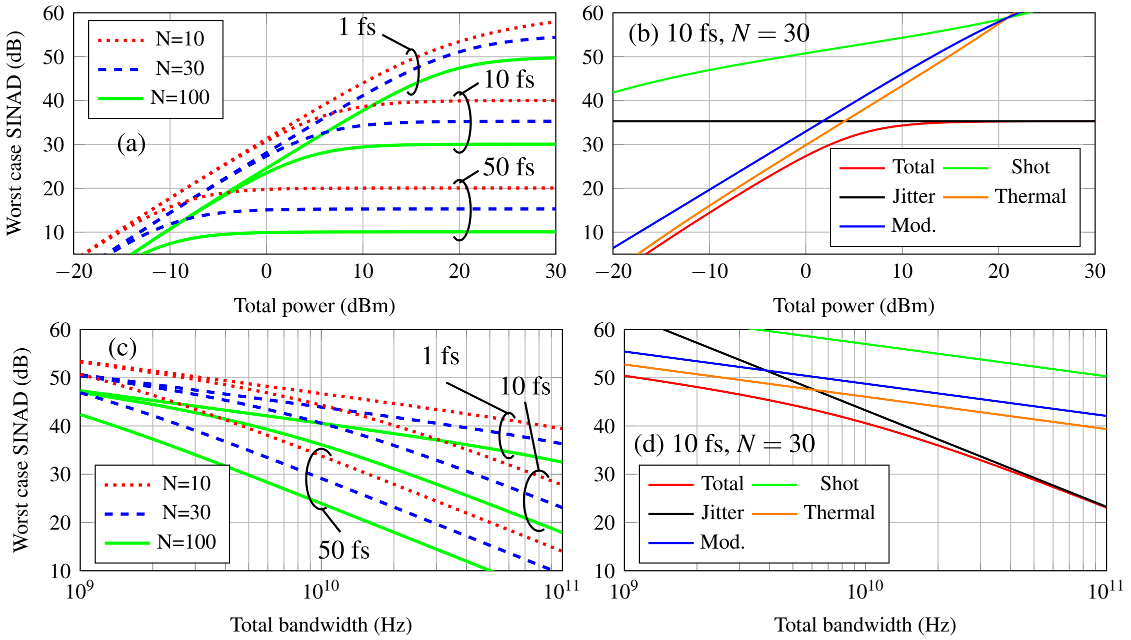

Fig. 3. (a) Total optical power vs worst case SINAD for a 25 GHz bandwidth channelizer. (b) Noise contributions of the $N = 30$, 10 fs case in (a). (c) Total bandwidth vs worst case SINAD for 10 dBm total optical power. (d) Noise contributions of the $N = 30$, 10 fs case in (c). The loss in each system (e.g. from demulitplexer and modulator insertion loss) is 6.4 dB. The photodiode load is 50 Ω in all cases. Mod., modulator nonlinearity.

25 GHz channelizer. These jitter values are full bandwidth jitter values (i.e. integrated over the total bandwidth) that assume a white phase noise and so is scaled by $1/\sqrt{N}$. The dual comb system is typically limited by the thermal noise and modulator nonlinearity tradeoff until some value (e.g. ≈ 0 dBm for the 10 fs, N=30 example shown in Fig.3(b)), above which SINAD is limited by the phase noise which is independent of the total optical power. The phase noise limit for high powers is set by not only by the total integrated jitter, but also by the total number of channels N, since it results in the relative phase noise between the combs being scaled to much higher values as it is detected further from the comb center. Increasing the number of channels also lowers the thermal noise limited SINAD, since it results in lower comb power per line.

These trends are further seen in Fig. 3(c) which plots the SINAD vs bandwidth for a fixed total optical power of 10 dBm. Moreover, it is seen that the worst case SINAD reduces as bandwidth increases: this reduction is quadratic (20 dB per decade) when phase noise limited, but reduces significantly less quickly for thermal noise limited cases. A breakdown of the noise contributions for the $N = 30$, 10 fs case shown in Fig. 3(d) illustrates these two regimes. The results in Fig. 3 highlight that minimizing the relative phase noise between the two combs is critical to achieving high bandwidth channelization using dual combs. As seen in Fig. 2 this can be accomplished by maximizing the degree of correlation between the two combs, as well as minimizing their absolute phase noise.

5. Conclusion

We investigated the performance limits of a dual frequency comb based signal channalizer and show that its SINAD is primarily limited by the relative phase noise of the dual comb. We verified our analysis by experimentally measuring the relative phase noise of an electro-optic dual comb system, and showed that minimizing relative phase noise between the two frequency combs is critical for achieving high bandwidth channelization using a dual frequency comb system.

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