Morphology and Dynamics of Saturn’s Magnetopause

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Pour Maman et Papa;
Pour Clélia et Michael;
Et pour Noa, sa joie et ses rires déconfinés.
I, Flavien Hardy, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the work.
Abstract

This thesis describes how the structure and dynamics of Saturn’s magnetopause can be explored by modelling the interactions between the planetary environment and the solar wind.

Saturn’s magnetosphere has significant internal sources of plasma: a cold, dense equatorial population originating from the moon Enceladus, and hotter, more tenuous population in the outer magnetosphere. As solar wind particles approach this system, they experience the influence of different internal drivers, including Saturn’s magnetodisk magnetic field and the internal hot plasma. The magnetopause is, to first order, the region where the overall pressure from these internal drivers balances the solar wind dynamic pressure. We model the boundary in three dimensions by describing these interactions from considerations of pressure balance. We find that Saturn’s magnetopause becomes increasingly flattened at the poles as the system size increases, with sharp indents, or ‘cusps’, at high latitudes.

The boundary responds to sudden changes in solar and/or internal conditions by moving closer to or further from the planet. By using magnetopause crossings from the Cassini spacecraft, we derive a novel method which unites external and internal drivers in our study of magnetopause compressibility. We find that Saturn’s magnetopause responds to changes in the pressure budget in ways that depend on its size: it behaves similarly to that of Earth when it is compressed, and closer to that of Jupiter when it is expanded. More precisely, we generalise the concept of magnetopause compressibility to describe it as a function of system size at Saturn. In doing so, we also explain previous observational estimates of compressibility as distinct ‘sub-regions’ within this compressibility-size relation.
Finally, the interactions between Saturn’s magnetosphere and the solar wind flow are modulated by planetary seasons. We introduce the planet’s obliquity in our model and build a framework which describes the magnetopause under any seasonal configuration. We find that, throughout a Kronian year, the nose of the boundary traces a ‘figure eight’-shaped locus centred on the Sun-planet line. The amplitude of the locus is such that the nose is periodically displaced from the rotational equator, thus creating a clear North-South asymmetry. In particular, the polar cusps are found to move complementarily closer to the nose or terminator. We compute the magnetic field generated by the currents flowing along the magnetopause and find that its inclusion in the internal field causes the current sheet to hinge seasonally, as was evidenced by previous observational studies.
Impact Statement

This thesis presents results that contribute to advancing our knowledge in the area of space plasma physics, and in particular concerning Saturn’s magnetopause.

In chapter 4, we describe an entirely new numerical procedure to obtain a robust 3D equilibrium model of Saturn’s magnetopause. In particular, we accurately resolve the complex high-latitude cusp structure, which is particularly important for auroral studies but absent in previous empirical models. The results also confirm the observed polar flattening of the boundary and additionally show that this characteristic is modulated by system size. Our physics-based approach, published in Hardy et al. (2019), thus confirms results from previous empirical studies while shedding light on new properties regarding magnetopause structure at Saturn.

Chapter 5 describes a novel method to study magnetopause dynamics at Saturn. Our approach confirms that the boundary behaves similarly to that of Earth when compressed, and closer to that of Jupiter when expanded. Additionally, our generalised definition of magnetopause compressibility allows, for the first time, to estimate the compressibility index for a given system size. This work was published in Hardy et al. (2020) and contributes to advancing our knowledge of magnetospheric dynamics at the Gas Giants.

In chapter 6, we describe a novel framework which models the seasonal variations of Saturn’s magnetopause. Our model predicts the seasonal displacements of the nose, cusps and overall restructuring of the boundary. Our first-principle modelling approach confirms the periodic hinging of the current sheet at Saturn, which was previously evidenced by Cassini observations. The results are soon to be submitted for publication. They are particularly relevant to the studies of seasonal
effects at Saturn and their impact on the way Titan interacts with its magnetosphere — both main objectives of the Cassini mission.

The results from this thesis were presented to the magnetospheric community on numerous occasions. They also bear the fruits of many discussions with teams from other institutions, including Imperial College London and Michigan University. These collaborations contribute not only to the quality of the research itself, but also to the inter-disciplinary and multi-cultural aspect of the field.

This work contributed to the exploitation of the Cassini space mission dataset. It therefore illustrates the scientific outcome that can be achieved by such large-scale space missions. It contributes to the broad scientific objectives that may motivate future campaigns for exploring other planets and their moons.

This research also offered the wonderful opportunity to take part in pedagogical communication and outreach programs. In particular, it allowed middle school students to conduct inspiring research projects in space physics. These students being from all-girl schools, this project contributed to inspire bright and potential up-and-coming female scientists, thus combatting existing cultural bias in the field.
Acknowledgements

First and foremost, I must thank my supervisor Nicholas Achilleos for being the best supervisor one could wish for. His signature blend of patience, optimism and scientific rigour is one that I will never forget, and without which none of this work would have been possible.

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Contents

1 Introduction 19

1.1 Physics of Space Plasma 19

1.1.1 Single Particle Motion in a Magnetic Field 20

1.1.2 Collective Description 24

1.1.3 Ideal MHD and Frozen-In Field Theorem 27

1.2 The Sun and The Solar Wind 29

1.2.1 Extension of the Solar Corona 29

1.2.2 Flow Structure and Variation 31

1.3 Planetary Magnetospheres 33

1.3.1 Formation of Planetary Magnetospheres 34

1.3.2 Magnetospheres of Our Solar System 36

1.3.3 Magnetospheric Dynamics 39

1.4 Modelling the Kronian Magnetosphere 42

1.4.1 Key Drivers and Open Questions 42

1.4.2 Modelling Saturn’s Magnetopause 44

2 The Cassini-Huygens Mission 49

2.1 Mission Overview and Objectives 49

2.2 Key Instruments 51

2.2.1 Cassini Magnetometer (MAG) 51

2.2.2 Cassini Plasma Spectrometer (CAPS) 54

2.2.3 Magnetospheric Imaging Instrument (MIMI) 56

2.3 Observations Pertinent to Magnetopause Modelling 61
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.3.2</td>
<td>Dimensionality Reduction and Plasma $\beta$ - Scaling</td>
<td>112</td>
</tr>
<tr>
<td>5.3.3</td>
<td>Revisiting the Impact of Internal Plasma Pressure on System Size</td>
<td>113</td>
</tr>
<tr>
<td>5.4</td>
<td>System Size and Magnetopause Compressibility</td>
<td>114</td>
</tr>
<tr>
<td>5.4.1</td>
<td>Filtering Crossings Far From Pressure Balance</td>
<td>114</td>
</tr>
<tr>
<td>5.4.2</td>
<td>System Size and Magnetopause Compressibility</td>
<td>115</td>
</tr>
<tr>
<td>5.4.3</td>
<td>Generalising Magnetopause Compressibility to Account for the Impact of System Size</td>
<td>116</td>
</tr>
<tr>
<td>5.5</td>
<td>Summary and Conclusion of this Study</td>
<td>119</td>
</tr>
<tr>
<td>6</td>
<td>Modelling Seasonal Variability of Saturn’s Magnetopause</td>
<td>121</td>
</tr>
<tr>
<td>6.1</td>
<td>Introduction to this Study</td>
<td>121</td>
</tr>
<tr>
<td>6.2</td>
<td>Modelling Seasonal Effects</td>
<td>124</td>
</tr>
<tr>
<td>6.3</td>
<td>Position of the Sub-Solar Point</td>
<td>127</td>
</tr>
<tr>
<td>6.3.1</td>
<td>Sub-Solar Nose of the Magnetopause</td>
<td>127</td>
</tr>
<tr>
<td>6.3.2</td>
<td>Position of the Sub-Solar Nose</td>
<td>128</td>
</tr>
<tr>
<td>6.3.3</td>
<td>Seasonal Displacement of the Nose</td>
<td>129</td>
</tr>
<tr>
<td>6.4</td>
<td>Seasonal Variations of the Magnetopause</td>
<td>130</td>
</tr>
<tr>
<td>6.4.1</td>
<td>Noon-Midnight Meridional Profiles</td>
<td>130</td>
</tr>
<tr>
<td>6.4.2</td>
<td>Modelling Continuous Seasonal Variations</td>
<td>135</td>
</tr>
<tr>
<td>6.5</td>
<td>Contribution of Magnetopause Currents</td>
<td>135</td>
</tr>
<tr>
<td>6.5.1</td>
<td>Magnetopause Currents and Shielding Field</td>
<td>135</td>
</tr>
<tr>
<td>6.5.2</td>
<td>Internal Magnetic Field Structure</td>
<td>136</td>
</tr>
<tr>
<td>6.5.3</td>
<td>Shielding Field and Current Sheet Distortion</td>
<td>138</td>
</tr>
<tr>
<td>6.6</td>
<td>Conclusion to this Study</td>
<td>143</td>
</tr>
<tr>
<td>7</td>
<td>Conclusions and Perspectives</td>
<td>145</td>
</tr>
<tr>
<td>7.1</td>
<td>General Summary of Conclusions</td>
<td>145</td>
</tr>
<tr>
<td>7.1.1</td>
<td>A Physical Model of Saturn’s Magnetopause</td>
<td>145</td>
</tr>
<tr>
<td>7.1.2</td>
<td>Generalising Magnetopause Compressibility</td>
<td>146</td>
</tr>
<tr>
<td>7.1.3</td>
<td>Modelling Seasonal Variations</td>
<td>147</td>
</tr>
</tbody>
</table>
### Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.2 Future Perspectives</td>
<td>149</td>
</tr>
<tr>
<td>7.2.1 Open Questions</td>
<td>149</td>
</tr>
<tr>
<td>7.2.2 Possible Directions for Future Work</td>
<td>150</td>
</tr>
<tr>
<td>7.2.3 Context and Future Space Missions</td>
<td>153</td>
</tr>
<tr>
<td><strong>Appendices</strong></td>
<td></td>
</tr>
<tr>
<td>A Reorganisation of Magnetopause Crossings</td>
<td>155</td>
</tr>
<tr>
<td>B Considerations of Pressure Balance</td>
<td>158</td>
</tr>
<tr>
<td>C Comparison of Magnetopause Models</td>
<td>160</td>
</tr>
<tr>
<td>Bibliography</td>
<td>162</td>
</tr>
</tbody>
</table>
List of Figures

1.1 Space plasmas parameters ........................................... 25
1.2 The Parker Spiral ......................................................... 32
1.3 Formation of CIRs ....................................................... 34
1.4 Diagram of Saturn’s Magnetosphere .................................. 36
1.5 The Dungey Cycle ......................................................... 40
1.6 The Vasyliunas cycle ...................................................... 42

2.1 Cassini Mission Timeline .................................................. 50
2.2 Cassini Instruments Overview .......................................... 52
2.3 Schematic of Flux Gate Magnetometers ................................ 53
2.4 Diagram of the CAPS Instrument ....................................... 55
2.5 Diagram of the MIMI-LEMMS sensor ................................ 56
2.6 Diagram of the MIMI-CHEMS sensor ................................ 58
2.7 Diagram of the MIMI-INCA sensor ................................... 59
2.8 ENA Observations by MIMI-INCA .................................... 61
2.9 Observed magnetopause crossings by CAPS-ELS ................. 62

3.1 Coordinate System and Discretisation ................................. 67
3.2 Numerical Approximation of Partial Derivatives ..................... 68

4.1 Schematic of the $\phi \times \theta$ grid ..................................... 86
4.2 Equatorial profile of the magnetopause .............................. 89
4.3 Piece-wise approach to the noon-midnight meridian plane ........ 90
4.4 Magnetopause Profile in the Noon-Midnight meridian plane ....... 92
4.5 Construction of the initial guess surface ............................. 93
<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.6</td>
<td>First guess and final boundary</td>
<td>95</td>
</tr>
<tr>
<td>4.7</td>
<td>Iso-theta contours of equilibrium boundary</td>
<td>96</td>
</tr>
<tr>
<td>4.8</td>
<td>Modelling the effect of hot plasma pressure</td>
<td>97</td>
</tr>
<tr>
<td>5.1</td>
<td>Determination of System Size</td>
<td>107</td>
</tr>
<tr>
<td>5.2</td>
<td>Phase space visualisation of magnetopause crossings</td>
<td>109</td>
</tr>
<tr>
<td>5.3</td>
<td>Accounting for Internal Drivers</td>
<td>112</td>
</tr>
<tr>
<td>5.4</td>
<td>Crossing Distribution and Compressibility</td>
<td>115</td>
</tr>
<tr>
<td>5.5</td>
<td>Compressibility and System Size</td>
<td>118</td>
</tr>
<tr>
<td>6.1</td>
<td>Parametrisation of Seasonal Effects</td>
<td>125</td>
</tr>
<tr>
<td>6.2</td>
<td>Cartoon of a Tilted Magnetopause</td>
<td>128</td>
</tr>
<tr>
<td>6.3</td>
<td>Method to Determine the Position of the Nose</td>
<td>129</td>
</tr>
<tr>
<td>6.4</td>
<td>Seasonal Displacement of the Nose</td>
<td>131</td>
</tr>
<tr>
<td>6.5</td>
<td>North-South Seasonal Asymmetry</td>
<td>132</td>
</tr>
<tr>
<td>6.6</td>
<td>Diagram of Magnetopause Interpolation Method</td>
<td>134</td>
</tr>
<tr>
<td>6.7</td>
<td>Schematics of Magnetopause Current Contribution</td>
<td>137</td>
</tr>
<tr>
<td>6.8</td>
<td>Internal Magnetic Field during Vernal Equinox</td>
<td>140</td>
</tr>
<tr>
<td>6.9</td>
<td>Internal Magnetic Field at Northern Summer Solstice, $30 R_S$</td>
<td>141</td>
</tr>
<tr>
<td>6.10</td>
<td>Internal Magnetic Field at Northern Summer Solstice, $25 R_S$</td>
<td>142</td>
</tr>
<tr>
<td>A.1</td>
<td>Clustering of Magnetopause Crossings</td>
<td>156</td>
</tr>
<tr>
<td>A.2</td>
<td>Scaling of Magnetopause Crossings</td>
<td>157</td>
</tr>
<tr>
<td>a</td>
<td>Unscaled Crossings: Equatorial Plane</td>
<td>157</td>
</tr>
<tr>
<td>b</td>
<td>Scaled Crossings: Equatorial Plane</td>
<td>157</td>
</tr>
<tr>
<td>c</td>
<td>Unscaled Crossings: Noon-Midnight meridional Plane</td>
<td>157</td>
</tr>
<tr>
<td>d</td>
<td>Scaled Crossings: Noon-Midnight meridional Plane</td>
<td>157</td>
</tr>
<tr>
<td>C.1</td>
<td>Comparison of Magnetopause Models</td>
<td>161</td>
</tr>
</tbody>
</table>
List of Tables

1.1 Comparison of Planetary Magnetospheres . . . . . . . . . . . . . . 37
6.1 Comparison of Planetary Configurations . . . . . . . . . . . . . . . 122
Chapter 1

Introduction

The European Space Agency’s Cosmic Vision 2015-2025 decadal programme addresses four questions that are considered to be among the highest on the European and international research agenda. One of them focuses on understanding ‘how the Solar System works’, and thereby formulates our need to understand how our Sun creates its heliosphere, and how planetary environments interact with the star through the solar wind and its magnetic field. These interrogations have a wide range of applications that go beyond pure scientific curiosity. The performance and reliability of advanced technological systems is now more than ever reliant on the fluctuations of space weather, or how the impact of solar conditions cascade its way through the Earth’s magnetosphere, ionosphere and thermosphere. At other planets, the solar wind and the magnetosphere interact in ways that are specific to the planetary environment. This thesis fits in this broader context of understanding how planets interact with the Sun. In particular, it investigates the structure and dynamics of Saturn’s magnetosphere, using both in-situ measurements and physics-based computer models. In this chapter, we introduce the underlying physics behind the interactions of Saturn’s magnetosphere with the solar wind.

Planetary magnetospheres and the surrounding interplanetary medium are made up of plasma, which is the name given to a gas that has been ionised, such that atoms have been split up into negatively charged electrons and positively charged ions. These charged particles are strongly influenced by electromagnetic fields, making the roles of electrical and magnetic forces fundamental in understanding
how space plasma behaves. We will start by considering the effects of these forces on the motion of a single charged particle, before transitioning to the description of plasma as a bulk fluid. We will then follow the energy flow from the Sun – the energy source for most of the space plasma we encounter in our solar system –, its extension into the interplanetary medium through the solar wind, all the way to the magnetospheres of the Earth, Jupiter and Saturn.

1.1 Physics of Space Plasma

1.1.1 Single Particle Motion in a Magnetic Field

Let us start by considering a single particle of charge $q$ and velocity $v$ within an electrical field $E$ and a magnetic field $B$. This particle will experience a force described by the Lorentz force law

$$ F_L = q (E + v \times B), $$

expressed here in SI units; the variations of the fields $E$ and $B$ in space and time are governed by Maxwell’s equations of electromagnetism

$$ \nabla \cdot E = \frac{\rho}{\varepsilon_0} $$

(1.2)

$$ \nabla \cdot B = 0 $$

(1.3)

$$ \nabla \times E = -\frac{\partial B}{\partial t} $$

(1.4)

$$ \nabla \times B = \mu_0 J + \frac{1}{c^2} \frac{\partial E}{\partial t}, $$

(1.5)

where $J$ is the current density, $\rho = \Sigma q_i n_i$ is the charge density, $c = 1/\sqrt{\mu_0 \varepsilon_0}$ is the speed of light, and $\varepsilon_0$ and $\mu_0$ are the permittivity and permeability of free space, respectively.

If $m$ denotes the mass of the particle, the rate of change of its momentum $mv$ is given by

$$ m \frac{dv}{dt} = q (E + v \times B) + F_g, $$

(1.6)

with $F_g$ denoting the non electromagnetic forces, such as gravitational forces. Their contributions will be assumed negligible compared to the ones of the Lorentz force.
In the specific case of a uniform magnetic field \( \mathbf{B} = B \mathbf{e}_z \) and \( \mathbf{E} = 0 \), Eq. (1.6) leads to

\[
\begin{pmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{pmatrix} = \begin{pmatrix}
-\Omega \mathcal{C}_0 & 0 & 0 \\
0 & -\Omega \mathcal{C}_0 & 0 \\
0 & 0 & 0
\end{pmatrix} \begin{pmatrix}
x \\
y \\
z
\end{pmatrix},
\]

with \((x, y, z)\) denoting the cartesian coordinates of the charged particle and the dot notation being used to refer to time derivatives, and where

\[
\Omega \mathcal{C} = \frac{|q|B m}{|q|B m}.
\]

This implies that in this case the magnetic field \( \mathbf{B} \) does not affect the motion of the particle along its direction. In the direction perpendicular to the magnetic field, the charged particle gyrates around the magnetic field direction with an angular frequency \( \Omega \mathcal{C} \), called the cyclotron frequency or gyrofrequency. More specifically, positively charged particles will rotate in the left-hand sense around the magnetic field direction, and negatively charged particles will rotate in the right-hand sense. The radius \( \rho_L \) of this circular motion, called the cyclotron radius or Larmor radius, is linked to the component \( v_\perp \) of the velocity perpendicular to the field and the gyrofrequency \( \Omega \mathcal{C} \) according to

\[
\rho_L = \frac{v_\perp}{\Omega \mathcal{C}} = \frac{mv_\perp}{|q|B}.
\]

Positive ions will then generally gyrate around the magnetic field direction with a larger Larmor radius than electrons if their velocities are the same (since they are more massive) and in opposite directions. Since the Lorentz force acts perpendicular to the motion in the absence of an electric field, no work is done and the particle’s kinetic energy does not change.

If a non-vanishing electrical field \( \mathbf{E} \) is now introduced perpendicular to the uniform magnetic field \( \mathbf{B} \), the electrical force will accelerate the particle during half a gyration and decelerate it during the other half, causing the centre of its gyratory motion, called the guiding centre, to drift in a direction perpendicular to \( \mathbf{E} \). This
effect is called the ‘E-cross-B drift’ and the resulting drift velocity $v_E$ satisfies
\[ v_E = \frac{E \times B}{B^2}. \tag{1.10} \]

This velocity is independent of both the particle charge $q$ and mass $m$, meaning that it does not generate currents into the plasma.

Qualitatively, a similar drift will occur when the particle ‘sees’ significant changes in force during each gyration period. In a spatially non-uniform magnetic field $B$ for example, its variation will cause the guiding centre to drift according to the gradient drift velocity
\[ v_g = \frac{mv^2 B \times \nabla B}{2qB^3}, \tag{1.11} \]
in a direction perpendicular to both the magnetic field $B$ and the gradient of the field magnitude $\nabla B$. As opposed to the E-cross-B drift velocity from Eq. (1.10), $v_g$ now depends on the particle charge $q$, thus causing charge separation in the plasma. In the case of Saturn, the gradient $\nabla B$ is oriented towards the planet and the equatorial field $B$ points southwards: the electrons will drift westwards whereas the positively-charged ions will drift in the opposite direction, thus generating a current flowing eastwards (Guio et al., 2020).

A charged particle moving freely along a curved magnetic field line will also feel a centrifugal force causing significant changes in force during a single gyration. This force introduces a new drift for the gyratory motion characterised by the curvature drift velocity
\[ v_c = \frac{mv^2_B \times \hat{n}}{R_c qB^2}, \tag{1.12} \]
with $v^B_\parallel$ being the component of the velocity vector parallel to the magnetic field direction, $\hat{n}$ a unit vector perpendicular to $B$ pointing away from the centre of curvature, and $R_c$ the radius of curvature of the field line. Just like the gradient velocity from Eq. (1.11), the curvature drift velocity depends on the particle charge $q$ and thus also generates an azimuthal current.

Another aspect of particle motion that is common in the approximately dipolar field of magnetised planets is that of magnetic mirroring. Let us consider a charged
particle travelling along a magnetic field line from the equatorial regions to higher latitudes within Saturn’s magnetosphere. Each particle gyration forms a current loop with a magnetic moment \( \mu = IA \), where \( A = \pi \rho_L^2 \) is the area of the circle enclosed by the particle motion during one gyration, and \( I \) is the current generated by the flow of charge \( q \) during one gyration period. This current can be expressed as

\[
I = \frac{|q|}{\left(\frac{2\pi \rho_L}{v_\perp}\right)},
\]

leading to

\[
\mu = \frac{mv_\perp^2}{2B},
\]

using the definition of the Larmor radius given by Eq. (1.9). If the particle does not experience a ‘significant’ change in magnetic field during one gyration, or if the field varies under a characteristic timescale much larger than one gyration period, this magnetic moment is considered conserved and is known as the first adiabatic invariant. As the particle travels in regions of increasing magnetic field, the conservation of this magnetic moment forces the velocity component \( v_\perp \) to increase. Since a magnetic field alone cannot change the kinetic energy of the particle, this causes \( v_\parallel \) to correspondingly decrease until eventually \( v_\parallel = 0 \): at this point, known as the mirror point, the particle is reflected back towards the opposite pole. This causes charged particles to bounce up and down the magnetic field lines, with their motion reversing when the magnetic field is strong enough to reflect the motion of the guiding centre near the poles.

The reflections occur when the angle \( \alpha \) between the velocity vector and the magnetic field direction, or pitch angle, reaches 90°. Substituting \( v_\perp = v \sin \alpha \) into Eq. (1.14), one finds that the quantity \( \frac{\sin^2 \alpha}{B} \) is conserved. Given a reference magnetic field strength \( B_0 \) and a corresponding pitch angle \( \alpha_0 \), the magnetic field strength \( B_m \) at the mirror point is then determined by

\[
B_m = \frac{B_0}{\sin^2 \alpha_0},
\]
If the initial pitch angle $\alpha_0$ is small enough, the value $B_m$ of the magnetic field strength might not be achievable before the particles reach the atmosphere of the planet: they are likely to be scattered and lost via precipitation into the atmosphere. Such particles with a pitch angle $\alpha < \alpha_0$ are said to be within the loss cone.

Charged particles making up the plasma within a planet’s magnetosphere will thus interact with the magnetospheric field by gyrating around the magnetic field lines, bouncing up and down along them while also undergoing azimuthal drifts in directions that depend on their charge.

### 1.1.2 Collective Description

The previous description of single charged particle motion in a magnetic field is fundamental, but not sufficient, to describe the behaviour of space plasmas at the scale of a planetary magnetosphere. Previously, we referred to plasmas as ionised gases that are considered electrically neutral. Let us consider the spatial scales over which this assumption is satisfied.

A positively charged ion of charge $q$ embedded in a plasma will attract surrounding electrons, which will shield and reduce its electrostatic potential to give it the form

$$\Phi = \frac{q}{4\pi\varepsilon_0 r} e^{-\frac{r}{\lambda_D}}, \quad (1.16)$$

where $\lambda_D$ is the Debye length, corresponding to the characteristic scale over which a plasma influences its surrounding. For an electron-proton plasma assumed quasi-neutral, the mobile electrons form a neutralising sheath of charge characterised by

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T}{n_e e^2}}, \quad (1.17)$$

with $T$ being the plasma temperature, $k_B$ the Boltzmann constant, $e$ the elementary charge, $n_e$ the density of electrons. A collective description of a quasi-neutral ionised gas is possible if a sphere of radius $\lambda_D$ centred on an ion, known as the Debye sphere, contains a large number of shielding particles, meaning that the plasma
1.1. Physics of Space Plasma

Figure 1.1: Debye length and plasma lambda parameter plotted versus electron number density and electron temperature, for various plasmas. Original figure from Kivelson et al. [1996]; colours are added for parameters relative to the solar wind and magnetospheric plasmas.

The lambda parameter $\Lambda$ should satisfy

$$\Lambda = n_e \lambda_D^3 \propto \sqrt{\frac{T^3}{n_e}} \gg 1.$$  \hspace{1cm} (1.18)

Fig. 1.1 shows that the required combination of low density and high temperature is mostly satisfied for plasmas making up the solar wind and magnetospheres. A collective description of such plasmas is commonly adopted by considering them as a conductive fluid with a certain bulk flow velocity $\mathbf{v}$. This approach is the foundation of the theory of single fluid Magnetohydrodynamics, or MHD, which combines a fluid description of the bulk plasma with Maxwell’s equations of electromagnetism to account for its conducting properties. The relation between momentum and force for a plasma parcel is then governed by the Navier-Stokes momentum equation

$$\rho \frac{D\mathbf{v}}{Dt} = \rho_q \mathbf{E} + \mathbf{J} \times \mathbf{B} - \nabla P + \rho \mathbf{g},$$  \hspace{1cm} (1.19)
where \( \rho \) is the mass density of the fluid, \( \frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \) the convective derivative, \( \rho q \mathbf{E} + \mathbf{J} \times \mathbf{B} \) the Lorentz Force from Eq. (1.1) exerted on the plasma parcel, \( P \) is the plasma pressure and \( \mathbf{g} \) the acceleration due to gravity.

Assuming the criterion from Eq. (1.18) holds, as is mostly the case for the plasmas we are considering, \( \rho \) is small enough for the contribution of the electric field to be negligible. Similarly, the effect of the gravitational acceleration is commonly considered small compared to the other terms. The momentum equation is now reduced to

\[
\rho \frac{D\mathbf{v}}{Dt} = \mathbf{J} \times \mathbf{B} - \nabla P. \tag{1.20}
\]

\( \nabla P \) describes a pressure gradient force; the effect of the \( \mathbf{J} \times \mathbf{B} \) force can be studied by considering the Maxwell equations. From the Maxwell-Ampère law described in Eq. (1.5), the electric and magnetic field strengths are linked to the characteristic space and time scales \( L \) and \( T \) of the problem by

\[
E \sim \frac{BL}{T}. \tag{1.21}
\]

These orders of magnitude can be introduced in the Maxwell-Faraday law from Eq. (1.4) to assess the contributions of the current-displacement term

\[
\left| \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \right| \sim \frac{1}{c^2} \frac{E}{T} B/L \sim \left( \frac{L}{T} \right)^2 \sim \left( \frac{\nu}{c} \right)^2 \sim \left( \frac{v}{c} \frac{L}{T} \right)^2 = \left( \frac{v}{c} \right)^2, \tag{1.22}
\]

with \( \nu \) denoting the bulk velocity of the MHD fluid. The space plasmas we consider can be considered non-relativistic, and thus \( \frac{v}{c} \ll 1 \): the Maxwell-Ampère law reduces to Ampère’s law

\[
\nabla \times \mathbf{B} = \mu_0 \mathbf{J}. \tag{1.23}
\]

The \( \mathbf{J} \times \mathbf{B} \) force can now be written as

\[
\mathbf{J} \times \mathbf{B} = \frac{1}{\mu_0} \left( \nabla \times \mathbf{B} \right) \times \mathbf{B} = \frac{1}{\mu_0} \left( \mathbf{B} \cdot \nabla \right) \mathbf{B} - \nabla \left( \frac{B^2}{2\mu_0} \right). \tag{1.24}
\]

This expression of the volume force illustrates how a magnetic field can be consid-
ered to exert both a magnetic tension \(-\frac{1}{\mu_0}(\mathbf{B} \cdot \nabla)\mathbf{B}\) term – and a magnetic pressure gradient force \(-\nabla \left( \frac{\mu_0 B^2}{2} \right)\) term – on the plasma parcel. Qualitatively, the magnetic tension contribution acts to straighten curvatures in the magnetic field, and the magnetic pressure term acts towards reducing inhomogeneities in the field strength. The field-perpendicular component of the tension force is related to the field line curvature by
\[
\mathbf{n} \cdot \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} = -\frac{B^2}{\mu_0 R_c},
\]
with \(\mathbf{n}\) being a unit vector locally normal to the field line pointing outwards from the centre of curvature, and \(R_c\) its radius of curvature. In the rapidly rotating, disc-like outer magnetospheres of Saturn and Jupiter, this curvature force balances the combination of the centrifugal force and plasma pressure gradient force. The components of the tension and magnetic pressure gradient forces parallel to the magnetic field cancel each other out, since the overall \(\mathbf{J} \times \mathbf{B}\) force is oriented perpendicular to the field by definition.

This collective consideration of space plasmas thus allows us to describe their bulk interactions with magnetic fields, which is key in understanding the dynamics and behaviour of plasmas from the solar wind and planetary magnetospheres.

### 1.1.3 Ideal MHD and Frozen-In Field Theorem

The previous description of a fluid parcel occurred at a mesoscopic scale: large enough to contain a significant number of particles (local properties can be described by integrating over a phase-space density), but small compared to the size of the entire system. The dynamics were also assumed to be slow enough to allow local measurements of fluid properties, and the gas was considered tenuous enough to neglect the effects of collisions.

The description of a plasma as a bulk fluid thus relies on two underlying assumptions: the system is studied on a time scale much larger than an ion gyroperiod and mean free path time, and on a space scale much larger than an ion gyroradius and mean free path length. This description of large scale, slow dynamics plasmas sets the conditions for ideal MHD.
The set of MHD equations requires a relation between the current density $j$ and
the fields, which is given by Ohm’s law

$$ j = \sigma E', $$  

(1.26)

where $\sigma$ is the electrical conductivity and $E'$ the electric field experienced by the
plasma parcel in its rest frame. For the tenous space plasmas we study, the conduc-
tivity can be assumed high enough for Eq. (1.26) to reduce to

$$ E' = 0. $$  

(1.27)

If the plasma is at rest, this is consistent with the particles being free to respond
to a perturbing electric field by inducing a field which cancels the source of the
perturbation (assuming they are given enough time to respond fully). If the plasma
moves at a velocity $v$, applying the Lorentz transformation leads to

$$ E' = E + v \times B = 0. $$  

(1.28)

The combination of Eq. (1.28) with Faraday’s law from Eq. (1.4) forms the induc-
tion equation for the magnetic field $B$ in the case of collision-less plasmas

$$ \frac{\partial B}{\partial t} = \nabla \times (v \times B). $$  

(1.29)

Under the influence of the convective term on the right hand side, Alfvén showed
that the magnetic field is \textit{frozen in} to the fluid (Alfvén, [1942]): even if the size or
shape of a moving plasma fluid element changes in time, the magnetic flux threading
the parcel remains constant. This result – known as the \textit{frozen-in condition}, or
\textit{frozen-in field theorem} – is very helpful in studying the evolution of a given plasma
population, and in understanding the relationship between the solar wind and the
Interplanetary Magnetic Field discussed in section [1.2.1] However, it only holds
as long as the conditions for ideal MHD are valid, which is no longer the case
when variations of the magnetic field occur over space scales comparable to ion
gyroradii. For example, the expression for the gyroradius in Eq. (1.9) shows that energetic particles will gyrate around magnetic field lines with larger gyroradii, allowing them to be subjected to significant changes in force during each gyration and drift accordingly. The account of such energetic populations thus breaks the conditions for ideal MHD and the frozen-in theorem. This is particularly important in our modelling of Saturn’s magnetopause boundary: the Kronian magnetosphere contains significant populations of energetic plasma, which can have a very large impact on the shape and size of the magnetopause. Chapters 4 and 5 will discuss how we account for their contributions in our study of magnetosphere structure and dynamics at Saturn.

1.2 The Sun and The Solar Wind

1.2.1 Extension of the Solar Corona

In our modelling approach to how planetary magnetospheres interact with the Sun, we need to discuss how the star extends its influence in interplanetary space through a flow of ionised solar plasma called the **solar wind**.

The Sun is a main sequence star which can simply be described as being made of a dense plasma in hydrostatic equilibrium: it is fuelled by the nuclear fusion of hydrogen to form helium at its core, with the outward force related to gas and radiation pressure providing the energy necessary to keep the star from collapsing under its own weight. This thermonuclear process provides energy to the surrounding planetary systems. The outer layer of the star, or **solar corona**, is heated to temperatures of the order of $10^6$ K [Warren and Brooks, 2009] by mechanisms that are still not fully understood to this day. The coronal plasma is composed mainly of ionised hydrogen with a 4% abundance of ionised helium [Robbins et al., 1970], and being surrounded by interstellar space, it is driven outwards by significant pressure differences. The particles that are energetic enough to escape the influence of solar gravity are free to propagate into interplanetary space, making up the solar wind plasma. The solar wind expands through our solar system until it becomes so tenuous that interstellar winds cause it to slow down, its velocity becoming sub-
sonic and forming the termination shock. This boundary was first encountered by the Voyager 1 spacecraft at 94.0 AU (one Astronomical Unit corresponding to the distance from Earth to the Sun, or \(1.496 \times 10^8\) km) from the Sun in December 2004 (Decker et al., 2005); it marks the transition with the heliosheath, in which the solar wind becomes denser and hotter. The dynamical pressure eventually becomes balanced with the competing pressure from the interstellar medium at a boundary called the heliopause. The overall ‘bubble’ enclosed by the heliopause is known as the heliosphere. The Voyager 1 spacecraft crossed the heliopause to venture out of the heliosphere into interstellar space in August 2012 (Gurnett et al., 2013; Burlaga and Ness, 2014).

The properties of the solar wind plasma vary over various time scales, which will be discussed below. It is however important to first describe the nominal conditions under which the planets interact with it, especially in the context of steady-state models for these interactions. Beyond 1 AU, the solar wind flow is supersonic, streaming away from the Sun with typical speeds of \(\approx 400\) km\(s^{-1}\). The consideration of the flux of particles across concentric spheres shows that the particle number density falls as the inverse square of the radial distance from the Sun. In particular, the particle density drops from \(\approx 7\) cm\(^{-3}\) at Earth (1 AU) to \(\approx 0.07\) cm\(^{-3}\) at Saturn (9.5 AU) (Bagenal et al., 2017). This flow of solar wind particles exerts a dynamic pressure

\[ P_{SW} = \rho v^2, \quad (1.30) \]

where \(v\) is the flow velocity and \(\rho\) the plasma mass density, which can be approximated as \(\rho = 1.16m_p n\) with \(m_p\) denoting the mass of a proton, \(n\) the proton number density, and the factor 1.16 accounting for the 4% abundance of ionised helium (Robbins et al., 1970; Beard, 1967). Using the aforementioned typical values for the flow density and velocity, Eq. (1.30) leads to the solar wind exerting a nominal pressure of \(\approx 2\) nPa at Earth, and \(\approx 0.02\) nPa at Saturn.

As discussed in section 1.1.3 under the conditions for ideal MHD, the magnetic field is frozen-in to the plasma. The flow could then be controlled by the magnetic field if the field is strong enough, or inversely, the field could be con-
1.2. The Sun and The Solar Wind

vectored along the flow if it is weak enough. Let us consider the plasma $\beta$ parameter defined as

$$\beta = \frac{P}{B^2/(2\mu_0)}$$  \hspace{1cm} (1.31)

where $P$ is the plasma particle pressure and $B^2/(2\mu_0)$ is the magnetic pressure introduced in Eq. (1.24). In a low-plasma regime, i.e. $\beta \ll 1$, the magnetic field is strong enough to govern the flow; this is the case of the coronal plasma where the strong magnetic field of the Sun dominates, giving rise to complex structures such as coronal loops. As the solar wind propagates away from the Sun, the solar magnetic field strength decreases and the solar wind plasma transitions into higher beta regimes. In particular, if $\beta \gg 1$, the magnetic field becomes locked into and carried outwards by the moving solar wind plasma, where it is known as the Interplanetary Magnetic Field (or IMF).

The radial outwards flow of the solar wind is combined to the $\approx 24.5$ day rotation of the Sun, producing a plasma distribution in the shape of a spiral, known as the Parker Spiral (Parker, 1958), along the solar equatorial plane. The orientation of the frozen-in IMF thus changes with distance from the Sun: it is expected to evolve from being quasi-parallel to the flow close to the Sun – spiral angle (angle between the flow and the field) of about 20° at the orbit of Mercury (James et al., 2017) – to perpendicular to the flow very far from it – approximately 87° at Saturn (Jackman and Arridge, 2011) –, with intermediate orientations in between – close to 45° at Earth (Wilcox, 1968). This effect of the Parker Spiral on the orientation of the IMF throughout the solar system is shown in Fig. 1.2. It will affect the way planetary environments interact with the solar wind, as will be discussed in section 1.3.

1.2.2 Flow Structure and Variation

The properties of the solar wind and IMF also undergo structural variations over various timescales. Massive transient ejections of coronal plasma – known as Coronal Mass Ejections, or CMEs – can occur during periods of significant restructuring of the coronal magnetic field. They can lead to sudden, nonrecurring outbursts
1.2. The Sun and The Solar Wind

Figure 1.2: Illustration of how the Parker Spiral affects the orientation of the IMF (in orange) in the solar equatorial plane, with a solar wind velocity fixed to 400 km.s\(^{-1}\). The green dotted lines represent the orbits of Mercury, the Earth, Jupiter and Saturn.

Many of these aspects of the flow also change over the eleven year long solar cycle. The transition from the period of least solar activity – or solar minimum – to the period of maximum activity – solar maximum – describes a restructuring of the entire heliosphere. This periodical behaviour of the Sun, which can be tracked by the number of sunspots observed on its surface, is coupled to regular variations in solar wind plasma properties (Odstrcil and Pizzo, 1999; Hathaway, 2015). At solar minimum, the solar surface magnetic field can be considered nearly dipolar with a small tilt relative to the rotational axis; the average solar wind magnetic field strength is low, and coronal regions of open field lines and low plasma density – known as coronal holes – extend from polar regions to low latitudes, leading to associated high speed solar wind streams being observed near the ecliptic plane. Near solar maximum, however, the coronal holes retreat to being constrained to polar regions while the coronal field structure becomes more disorganised. The interplanetary field strength increases and fast solar wind streams are restricted to
higher solar latitudes. This recurring variation in solar wind velocity was observed by the *Ulysses* spacecraft at heliocentric distances ranging from 1.4 AU to 5.4 AU (Gosling, 2014). When coupled to the radial flow of solar wind plasma and its large-scale structure related to solar rotation, the alternating fast-slow conditions contribute to regions where faster streams ‘catch-up’ with slower streams of plasma.

The leading edge of the fast streams compresses the slower plasma ahead in order to prevent any overlap between fast and slow regions, in line with the frozen-in conditions discussed at the end of section 1.1.3. On the opposite end of the faster solar wind stream, a rarefaction wave propagates away from the compressed zone and decelerates the trailing plasma, resulting in a net transfer of momentum from fast to slow solar wind streams. Since the solar wind cools down as it flows away from the Sun, the flow is increasingly supersonic; this leads to the leading and trailing edges of the faster solar wind flow becoming increasingly steep, eventually resulting in a pair of forward and reverse shocks between the orbits of Earth and Jupiter (Gosling, 2014). This interaction is illustrated in Fig. 1.3 from Pizzo (1978); the resulting solar wind structure is usually long-lived and is thus referred to as a *Corotating Interaction Region*, or CIR. The periodic formation of CIR has been observed up until the vicinity of Saturn’s orbit by the Cassini spacecraft during the declining phase of the solar cycle (e.g. Jackman et al. (2004).

The solar wind is thus a propagation of the coronal plasma and magnetic field into the heliosphere, with properties – e.g. velocity, density, field strength – that undergo both periodic and nonrecurring variations over a broad range of time and spatial scales. Its resulting structure modulates the way it interacts with magnetic planets through the formation of their magnetospheres.

### 1.3 Planetary Magnetospheres

When the solar wind plasma encounters the environment of a magnetic planet, it forms a region of space within which the local internal magnetic field controls the motion of charged particles. This volume surrounding the planet is called a *magnetosphere* (Gold, 1959), and acts qualitatively as a ‘protective bubble’ from which
Figure 1.3: Description of the interactions between fast and slow solar wind streams leading to the formation of Corotating Interaction Regions (CIRs). Viewed from above the solar North pole; the faster stream catching up with the slower stream leads to regions of compressed and rarefied plasma. From [Pizzo (1978)].

the solar wind is mostly excluded by being diverted around its outer boundary, or magnetopause.

1.3.1 Formation of Planetary Magnetospheres

The magnetospheres of magnetic planets form due to interactions between the upstream solar wind plasma and the magnetospheric magnetic field. In our solar system, six out of eight planets are known to generate magnetic fields in their interiors due to magnetic dynamos supported by the convective motion of conductive fluids in their deep interiors. Mercury and Earth have cores of liquid iron alloys [Glatzmaier and Roberts, 1995; Christensen, 2006], while the high pressures in the interiors of Jupiter and Saturn make hydrogen behave like a liquid metal [Parker, 1979; Stevenson, 2003]; at the Ice Giants Uranus and Neptune, a mixture of water, ammonia and methane forms an internal conducting ocean [Hubbard et al., 1991].

The supersonic flow of solar wind plasma encountering such magnetic obsta-
cles forms a bow shock. The plasma is suddenly decelerated into a dynamic region of turbulent flow called the magnetosheath, in between the bow shock and the magnetosphere cavity. The magnetopause is the boundary which encloses the magnetosphere and generally separates the shocked solar wind plasma of the magnetosheath from the internal magnetospheric population.

The shape and position of the magnetosphere is governed by the interactions between competitive processes of both solar and magnetospheric origin. In a steady state description of the system, the structure of the magnetopause boundary follows, to the first order, the pressure balance between the dynamic pressure from the incident solar wind defined in Eq. (1.30), and pressure sources due to internal magnetic fields and plasma populations. In particular, Saturn’s magnetosphere is fed by different internal sources of plasma which include plumes from the satellite Enceladus, as well as Titan, the rings and the ionosphere (Blanc et al., 2015). Given the broad range of solar conditions and internal properties between the different magnetic planets of our solar system, the size and structure of the magnetopause boundary can be expected to vary from one system to the next. Common features can however be identified and are illustrated in Fig. 1.4.

The solar wind dynamic pressure compresses the magnetosphere on the day side and an extended tail is formed on the night side, thereby breaking the symmetry of a vacuum dipole magnetic field. The magnetopause boundary separates the shocked magnetised plasma of the solar wind from the magnetospheric plasma: the magnetic field strength gradient across it gives rise to a current flowing along the surface, as described by Ampère’s law in Eq. (1.5). The corresponding current system, known as magnetopause currents or Chapman Ferraro currents (Chapman and Ferraro, 1930), acts to confine the magnetic field lines within the magnetosphere and shields it from the solar wind; its contribution to magnetospheric field structure will be discussed in chapter 6. An azimuthal ring current system also surrounds the planet, due to the differential motion of charged particles subjected to the curvature and gradient drifts described by the velocities from Eq. 1.12 and 1.11; this current system extends into and merges with a magnetotail current sheet on the night side.
These current systems at Jupiter and Saturn are oriented in opposite directions to those of the Earth, since the internal magnetic field orientation is also reversed.

The polar cusps are conical regions of the magnetosphere that separate the compressed magnetic field lines closing on the day side from those swept into the tail. In these regions, the magnetospheric field changes in magnitude and direction over small spatial scales, allowing solar wind particles to be funnelled into the magnetosphere. Along the magnetospheric boundary, the polar cusps are identified by sharp, singular ‘dents’. The determination of their positions from considerations of pressure balance will be discussed in section 4.2.3.

1.3.2 Magnetospheres of Our Solar System

The characteristics of planetary magnetospheres will vary with both the properties of the solar wind and those of the local planetary environments.

For example, qualitatively, large magnetospheres can be associated to planets with strong magnetic fields (e.g. Jupiter) or systems far enough from the Sun for the tenuous solar wind to exert a weak pressure (e.g. Uranus and Neptune).
Table 1.1: Comparisons of solar wind and internal magnetic fields at Earth, Jupiter and Saturn. $Q_{\text{dip}}$ is a metric defined in Eq. (1.33) to compare the internal field to a pure dipole field. 1 AU = $1.496 \times 10^8$ km and 1 $M_{\text{Earth}} = 7.9 \times 10^{15}$ T m$^3$.

Adapted from Kivelson and Bagenal (2014).

scale of a magnetosphere is often described by the stand-off distance $R_{\text{MP}}$, which is defined as the distance separating the planet from the point of the magnetopause closest to the Sun – known as the nose, or sub-solar point. Assuming that the magnetospheric magnetic field can be approximated by a dipolar planetary magnetic field, the pressure balance at the nose can be written as

$$\rho v^2 = \frac{1}{2\mu_0} \left( B_0 \left( \frac{R_p}{R_{\text{MP, dip}}} \right)^3 \right)^2 \iff R_{\text{MP, dip}} = R_p \left( \frac{B_0^2}{2\mu_0\rho v^2} \right)^{\frac{1}{5}}, \quad (1.32)$$

where $R_{\text{MP, dip}}$ is the dipole magnetopause stand-off distance, $B_0$ the surface equatorial field of the planet, $R_p$ the planetary radius and $\rho v^2$ the solar wind dynamic pressure defined in Eq. (1.30). The resulting estimates for the size of the magnetospheres, along with the properties of Earth, Jupiter and Saturn, are given in Table 1.1. The internal fields of the planets can be compared to the simple dipole description by considering the ratio Max/Min of maximum to minimum surface field (Kivelson and Bagenal 2014): this ratio takes a value of 2 for a pure dipole field, and larger values indicate significant non dipolar contributions to the internal field. We define a simple metric

$$Q_{\text{dip}} := \frac{2}{\text{Max/Min}} \times 100 \quad (1.33)$$

so that for a pure dipolar field $Q_{\text{dip}} = 100\%$ and $Q_{\text{dip}}$ decreases as the internal field deviates from the dipole model.

Table 1.1 illustrates significant differences in the scales of the magnetospheres.
between Earth, Jupiter and Saturn. In particular, Jupiter’s magnetic moment is about 20000 times greater than that of the Earth, with typical solar wind pressure values around 0.06 nPa: this results in a much larger magnetosphere with a dipole stand-off distance of $46R_J$ ($R_J = 71492$ km denotes the radius of Jupiter). Similarly, even though the solar wind pressure is close to 100 times weaker at Saturn compared to that of Earth, the Kronian internal field is strong enough to expand the magnetosphere to $20R_S$ ($R_S = 60268$ km is the radius of Saturn) if assumed purely dipolar.

At both Jupiter and Saturn, however, the observed values for the stand-off distance are significantly larger than the scales determined assuming a dipole internal field using Eq. (1.32). This is due to their magnetospheres having important internal sources of plasma which act to inflate the magnetopause further. Enceladus, for example, orbits within Saturn’s magnetosphere at $3.95R_S$ from the planet. The moon ejects plumes of water-group molecules with mass loading rate estimates ranging from $\approx 100 \text{ kg.s}^{-1}$ (Tokar et al., 2006a) to $\approx 250 \text{ kg.s}^{-1}$ (Spencer, 2011; Bagenal and Delamere, 2011a). At Jupiter, Io orbits the planet at $5.9R_S$ and ejects sulphur dioxide into the magnetosphere at a rate reaching the order of $\approx 1000 \text{ kg.s}^{-1}$ (Bagenal and Delamere, 2011a). For both systems, the values of the mass loading rates, and hence their contributions to the structure and dynamics of the magnetospheres, were shown to vary greatly in time (Bagenal et al., 1997). The ejected material is partially ionised to form a plasma torus around each planet; it contributes to an additional plasma pressure which inflates the magnetosphere to stand-off distance values greater than the dipole estimates, as shown in table [1.1]

Moreover, these magnetospheric plasma populations also contribute to the overall structure of the magnetospheric field. As newly created ions are accelerated from Keplerian velocities to corotation with the fast-rotating magnetospheres of the Gas Giants – with a spin period of order $\approx 10$ h –, they experience a centrifugal force in the corotating frame of reference which confines them close to the rotational equatorial plane (Gledhill, 1967; Smith et al., 1974), creating a plasma sheet. This distorts the magnetospheric field into an extended structure with field lines near to the equatorial plane stretched outwards in the outer magnetosphere,
known as a magnetodisc, supported by the strong azimuthal ring current \cite{Kivelson1997, Achilleos2010}. Additionally, hot plasma populations originating from the outer magnetosphere enhance the intensity of the ring current and further contribute to the magnetodisc structures. At Saturn, the plasma sheet was found to extend up until the dayside magnetopause boundary \cite{Sergis2007} and the local energetic populations (ions with energies \( \gtrsim 3 \) keV) were characterised by plasma \( \beta \) values with orders ranging from \( \approx 0.01 \) to \( \approx 10 \) \cite{Sergis2010a, Pilkington2015}. These magnetospheric drivers greatly influence the pressure balance at the magnetopause boundary, and the modelling of their contributions is described in section 4.3.3.

1.3.3 Magnetospheric Dynamics

As discussed in section 1.3.2, the size of a steady-state magnetosphere is determined by the pressure balance between the solar wind dynamic pressure and pressure sources from magnetospheric origin. As a result, the magnetopause responds to changes in the ‘pressure budget’ by being compressed or expanded: if the solar wind pressure suddenly increases due to events – CMEs, CIRs – mentioned in section 1.2 for example, the magnetopause boundary will be displaced closer to the planet until the magnetospheric field is strong enough once again to balance the upstream ram pressure. Conversely, the magnetosphere will inflate should the solar wind pressure decrease. This simple description, albeit instructive, only considers the response of the magnetosphere to external drivers. As such, it can be considered valid under the assumption of fixed or slowly-varying internal contributors. We will further discuss adaptations for studying the dynamics of Saturn’s magnetopause in response to varying solar wind conditions in chapter 5.

Due to the dynamic nature of both solar and magnetospheric pressure sources, the boundaries of planetary magnetospheres are thus in continuous motion \cite{Kaufmann1969, Escoubet2013, Escoubet2015}; Saturn’s magnetopause, for example, has been estimated to move at a velocity of the order of 100 km.s\(^{-1}\) \cite{Masters2011}. This behaviour is dictated by the pressure balance at the magnetopause, with the boundary being displaced to compensate for any increase
or decrease in upstream dynamic pressure. This seems to be in contradiction with the steady-state nature of the magnetopause models we will develop throughout this thesis. However, one can consider that a magnetosphere disturbed from equilibrium will eventually evolve dynamically towards a new pressure balance dictated by updated pressure conditions. Steady-state models thus offer previous insights on magnetosphere evolution and the corresponding dominant drivers. Moreover, equilibrium models could also be understood as tangential or ‘osculating’ states, in describing the system accurately should any time variation be fictionally removed instantaneously. When we will use our steady-state models alongside observed magnetopause crossings from Cassini, scaling procedures will be necessary to account for the dynamic behaviour of the boundary; this will be further discussed in chapter 5.

Planetary magnetospheres are also the sites of large-scale dynamic processes. For magnetospheres which derive most of their plasma and energy from the solar wind – as is the case for Mercury and Earth – the dominant process is known as the Dungey Cycle (Dungey, 1961). At Jupiter and Saturn, the energy is mainly derived from the rapid rotation of the planet, and the plasma originates from internal sources; the main dynamical process is then known as the Vasyliunas cycle (Vasyliunas, 1983).
A diagram of the Dungey cycle is shown in Fig. 1.5. It begins when the IMF encounters the magnetospheric field at the sub-solar magnetopause and causes the magnetic field to vary on spatial scales comparable to the gyroradii of plasma particles (see step 1 in Fig. 1.5). The conditions for ideal MHD break down and reconnection between the magnetospheric field and the IMF can take place by interconnecting previously separated field lines from solar wind and planetary origins. The newly-opened field lines are swept to the night side by the solar wind flow to form an extended magnetotail (steps 2 to 5). Reconnection eventually occurs in the central plane of the tail to close open flux (step 6) and the closed field lines proceed sunwards to complete the cycle (steps 7 to 9). At Saturn, the Northwards orientation of the planetary dipole makes sub-solar reconnection related to the Dungey cycle more frequent when the IMF is also oriented Northwards (Jia et al., 2012a). However, modelling approaches suggest that the solar wind (Jia et al., 2012a) and internal drivers (Masters, 2015; Pilkington et al., 2015) are the predominant sources of large-scale restructuring for the Kronian magnetosphere. Day side reconnection was also shown to be limited to regions where the IMF and magnetospheric field are almost perfectly anti-parallel, due to the expected values for plasma $\beta$ (Swisdak et al., 2003; Masters et al., 2012).

The Vasyliunas cycle, predominant in the centrifugally driven magnetospheres of Saturn and Jupiter, is illustrated in Fig. 1.6. It takes place on closed field lines and describes the transport of plasma from the inner magnetosphere to regions further down the tail where it is eventually lost. The description of the cycle can start deep within the magnetosphere, where the planetary field is dominant: the plasma population originating from internal sources is accelerated towards co-rotation with the planet (see circular dashed line in Fig. 1.6). Strong centrifugal forces cause interchange instabilities where hot, tenuous flux tubes move inwards and inner cold, dense flux tubes move out (Southwood and Kivelson, 1989). When the magnetic pressure and plasma pressure become comparable, the flux tubes stretch radially (Kivelson and Southwood, 2005), as shown in step 1 in Fig. 1.6. These flux tubes are then swept around to the night side, where the flaring of the magnetopause allow
Figure 1.6: Schematics of the Vasyliunas cycle for the transport of plasma at rotationally driven magnetospheres. The white arrows orient the flow of plasma, the dotted line marks the region where the plasma corotates with the magnetic field. From Vasyliunas (1983).

them to be stretched further (step 2) until reconnection occurs in the median plane of the magnetotail (step 3). A transient magnetic loop structure called a plasmoid is formed (step 4) which removes plasma down the tail while conserving magnetic flux (Hill et al., 2008); the empty flux tubes are then swept back sunwards through the dawn side.

At the outer planets, the fast rotation rates make the centrifugally driven Vasyliunas cycle dominant over the solar wind driven Dungey cycle. The latter is also limited by the scales of the magnetospheres, since a longer period of time is necessary for open magnetic field lines to be convected antisunwards across the polar cap, as shown on the right of Fig. 1.5. The exact contribution of the Dungey cycle to overall magnetospheric dynamics at Saturn remains, however, relatively uncertain (Cowley et al., 2004; Southwood and Chané, 2016).

1.4 Modelling the Kronian Magnetosphere

1.4.1 Key Drivers and Open Questions

The previous sections have introduced several drivers that are key in determining magnetopause structure at Saturn through their contributions in boundary interac-
tions:

- **External drivers**: The solar wind dynamic pressure compresses the planetary field on the day side and can be linked to a large-scale oscillatory response of the magnetopause under varying upstream conditions. Comparatively, the influence of the IMF strength and orientation has been found to be negligible on the location of the boundary.

- **Internal drivers**: Magnetospheric plasma populations from internal sources lend a magnetodisc structure to the internal field, with field lines being stretched outwards along the equator. Energetic plasma populations originating from the outer magnetosphere contribute to an additional particle pressure which inflates the magnetosphere further. Under steady state considerations, the combination of this particle pressure and the magnetic pressure due to the overall magnetospheric field balances the upstream solar wind pressure at the magnetopause.

Magnetospheric observations from Cassini put forward structural and dynamical properties which opened outstanding questions related to these interactions. For example, in contrast to the Earth’s magnetopause, the Kronian magnetosphere was shown to have an elongated and flattened paraboloid shape [Pilkington et al., 2014]. This structure can be expected to vary in response to changes in magnetospheric plasma content and field structure. We have yet, however, to quantify the extent to which this flattening changes in time, nor the magnetospheric conditions which cause it, or the high-latitude structure close to the cusp. Moreover, among the main physical drivers summarised above, we do not know which ones are the most influential in reshaping a quiescent magnetopause. Their relative importance may very well vary, and studies of magnetospheric dynamics have shed light on a bimodal and transitionary behaviour of Saturn’s magnetosphere [Pilkington et al., 2015; Sorba et al., 2017; Achilleos et al., 2008]; we have yet to understand why the dominant drivers change, and how it relates to planetary seasons, variations in solar cycle phase or plasma heating due to magnetic reconnection in the tail, for example.
These fundamental questions have motivated numerous modelling approaches for the Kronian magnetopause which vary in nature, flexibility and limitations.

1.4.2 Modelling Saturn’s Magnetopause

Early studies of Saturn’s magnetopause were inspired by previous empirical work at Earth orbit \([\text{Fairfield, 1971}]\). In particular, \text{Slavin et al., (1983)} used particle and magnetic field observations from Pioneer 11, Voyager 1 and 2 to identify magnetopause crossings during their flybys of the system. The upstream solar wind pressure was estimated using a Newtonian pressure balance relation at the magnetopause. The local angular incidence of the flow was determined empirically by sampling the magnetic data to search for the direction in which it varies the least – this direction can be expected to coincide with the magnetopause normal, assuming a infinitesimally thin boundary – using Minimum Variance Analysis \((\text{Sonnerup and Cahill, 1967})\). The response of the magnetopause to varying solar wind conditions was accounted for by scaling the observed crossings to a common solar wind pressure, assuming a dipolar solar wind dependence on the boundary stand-off distance (i.e. a compressibility index of \(\alpha = 6\), as will be discussed further in chapter \([5]\)). The position of the magnetopause was then determined by fitting parametrised conical sections to the normalised crossings.

A more flexible empirical representation of the boundary was proposed by \text{Shue et al. (1997)}, with the terrestrial magnetopause being described by the functional form

\[
\begin{align*}
r &= r_0 \left( \frac{2}{1 + \cos \theta} \right)^\kappa, \\
r_0 &= a_1 (P_{SW})^{-a_2}, \\
\kappa &= a_3 + a_4 P_{SW},
\end{align*}
\]

where \((r, \theta)\) denote the polar coordinates of a point along the magnetopause from the planet, with a reference axis along the planet-Sun line; \(P_{SW}\) is the upstream solar wind dynamic pressure, \(r_0\) the boundary stand-off distance, and \(\kappa\) describes the flaring of the boundary. The coefficients \((a_i)_{1 \leq i \leq 4}\) are introduced to model the
scale and flaring of the magnetopause, as well as how its geometry responds to varying solar wind conditions. The flexibility of this model makes it still widely-used to this day; it remains, however, limited by its pure empirical nature, and by relying on smooth, axi-symmetric descriptions of magnetopause profiles.

This model was applied by Arridge et al. (2006) using flybys of Voyager 1 and 2, and the first six orbits of the Saturn-orbiter Cassini. A refined pressure balance equation was considered to confine the magnetosphere in the tail section (Petrinec and Russell, 1997), and the incidence of the solar wind flow along the magnetopause was determined analytically by calculating the local normal to the surface model using Eq. (1.34). We will explain in section 3.1.2 how we will define the surface normal in a similar way, but using numerical approximations of derivative operators to free ourselves from assuming an initial analytical model of the boundary. Applications of this model showed that Saturn’s magnetopause was more ‘elastic’ than that of the Earth when subjected to varying solar wind pressure – this property corresponds to a compressibility index inferior to the dipole value of 6; we discuss this point in more details in section 5.4.3 and Fig. 5.5. The model was also applied to studying the long-term behaviour of Saturn’s magnetopause, using 430 additional days of Cassini observations (Achilleos et al., 2008); it was found that the distribution of magnetopause stand-off distances followed a bimodal distribution with peaks at $22R_S$ and $27R_S$. A comparison between the probability density function of this distribution and the one of the solar wind pressure – observed by the Cassini Plasma Spectrometer (CAPS) ahead of orbital insertion – suggested that the internal Vasyliunas cycle described in section 1.3.3 plays a significant role in the behavioural bimodality of the Kronian magnetopause. Further refinements of the pressure balance equation followed to model the contribution of magnetospheric hot plasma population (Kanani et al., 2010).

High-latitude data observed by Cassini during high-inclination orbits between 2007 and 2009 were then used to empirically illustrate the polar flattening of Saturn’s magnetopause (Pilkington et al., 2014). The boundary was found to have the shape of flattened paraboloid, with a divergence from previous axi-symmetric mod-
els described by a 81% compression of the North-South dimensions compared to the equatorial profile. This result will be discussed further in chapter 4.4.

The aforementioned empirical models are very useful in their extensive flexibility. Their applications are, however, limited by the quantity of observed data. They also fundamentally rely on assumptions inherent to the initial functional forms they use: none of these models, for example, account for the high-latitude cusp mentioned at the end of section 1.3.1. We explain in detail how we solve for this singular structure in section 4.2.3. A completely different approach is to infer magnetopause structure from solving the full set of magnetohydrodynamic equations – e.g. Jia et al. (2012a). This method requires careful considerations of the validity of MHD conditions – especially when energetic populations play such a considerable role in magnetospheric structure –, is much more computationally intensive and relies on non physical numerical dissipation.

The model detailed in this thesis builds on a physics-based approach developed by Mead and Beard (1964) for the Earth. It consists in solving the Newtonian pressure balance equation numerically using robust boundary conditions corresponding to regions where the magnetopause position can be physically determined a priori (e.g. the sub-solar nose). We will explain in chapter 4 how the model is refined to describe the properties of the Kronian system. This model will be used in chapter 5 to study how Saturn’s magnetopause behaves in response to varying solar wind conditions, and accounting for the significant contributions of magnetospheric drivers. Seasonal effects will be introduced in chapter 6 to the study of seasonal variations of magnetopause morphology and their impact on magnetospheric field structure. This work is prefaced by two preliminary chapters: chapter 2 will briefly present the Cassini mission at Saturn, and chapter 3 will reference the main numerical methods used hereafter.

It is also important to note the abundance of observational Cassini studies which have discovered a general, quasi-periodic modulation in location of Saturn’s magnetopause, with variable period close to the rotation period of Saturn and amplitude typically of a few $R_S$ (Clarke et al., 2006; Carbary and Mitchell, 2013 and
These oscillations are thought to arise from a system of two rotating current systems, which generate perturbations in the magnetic field – and hence the plasma pressure. The physical origin of the energy required to drive these currents remains to be definitely explained, but atmospheric flow perturbations at Saturn of a ‘vortical nature’ are promising, according to theoretical studies which involve the magnetosphere-ionosphere coupling (Smith and Achilleos 2012; Jia et al. 2012b). In this work, we do not attempt to incorporate such oscillations into our magnetopause model. We note, however, that chapter 5 introduces a method for determining the influence of changes in magnetospheric internal pressure upon magnetopause morphology. A further development and refinement of this work could form the basis for future methods of modelling magnetopause oscillations in finer detail.
Chapter 2

The Cassini-Huygens Mission

2.1 Mission Overview and Objectives

Three spacecraft – Pioneer 11 in 1979, Voyager 1 in 1980 and Voyager 2 in 1981 – flew past Saturn and provided glimpses of the Kronian System and its giant moon Titan. The Voyager instruments were however unable to penetrate the moon’s thick photochemical haze, and the flybys raised numerous questions which led to the prospect of a return mission to Saturn with a dedicated probe into Titan’s atmosphere. After years of discussions between American and European partners, the Announcements of Opportunity for such a mission were released in 1989. The Saturn orbiter was named after the Italian/French astronomer Giovanni Domenico Cassini who discovered several Kronian satellites and ring features between 1671 and 1685. The Titan probe was named after the Dutch astronomer Christiaan Huygens who discovered Titan in 1655. Cassini-Huygens was launched as an international flagship mission on 15\textsuperscript{th} October 1997, with NASA’s Cassini orbiter entering orbit around Saturn in July 2004, and ESA’s Huygens probe landing on Titan on 14\textsuperscript{th} January 2005.

Cassini completed its \textit{Nominal (Prime) Mission} at Saturn in July 2008 – shown in red in Fig. 2.1 – and was extended into an \textit{Equinox Mission} until July 2010 – see blue section. During this extension, The spacecraft flew an additional 65 orbits around the planet and performed 27 flybys of Titan and 7 of the icy moon Ence- ladus. At the end of the Equinox Mission, a second extension was granted: the \textit{Solstice Mission} continued past Saturn’s Northern Summer solstice in May 2017 –
2.1. Mission Overview and Objectives

Figure 2.1: Timeline of the Cassini-Huygens mission at Saturn. Orbital Insertion occurred on July 1st 2004; the Equinox Mission started in June 2008, followed by the Solstice Mission in July 2010. The mission ended with its Grand Finale phase on September 2017. Credit: Ralph Lorenz, JHU-APL.

coloured in orange in Fig. 2.1 – and completed an additional 155 orbits at Saturn, 54 flybys of Titan and 11 flybys of Enceladus. September 15th 2017 marked the mission’s Grand Finale, as the spacecraft dived into the planet’s atmosphere, gathering data as long as the thrusters could keep its antenna pointed at Earth, before plunging and disintegrating in Saturn’s atmosphere. The extraordinary longevity of the Cassini mission – \( \approx 13 \) years exploring Saturn, with one Kronian year corresponding to \( \approx 29.5 \) years – allowed studies of seasonal variations of the system and investigations of discoveries made during earlier mission phases under previously unobserved seasonal configurations. This will, in particular, motivate our modelling of seasonal effects at Saturn described in chapter 6.

The scientific objectives of the Cassini-Huygens Prime, Equinox and Solstice missions cover a thorough study of the Kronian system and can be grouped in five categories: the planet itself, its ring system, its magnetosphere, the icy moons and Titan. In particular, among the key objectives related to Saturn’s magnetosphere and the modelling work described in this thesis, Cassini was aiming at
• **Studying magnetospheric plasma populations**, by determining the dynamics and global configuration of hot plasma populations (see sections 1.4.1 and 4.3.3) in Saturn’s magnetosphere; in parallel, *in situ* measurements would be used to study the sources of plasmas and energetic ions. Modelling these key internal drivers and their effects on magnetopause dynamics is a key objective raised in this thesis.

• **Analysing interactions between the magnetosphere and the moons** by studying how their surfaces and atmospheres are modified due to plasma and radiation bombardment. In particular, a key objective was to investigate Titan’s interactions with the magnetosphere and solar wind, as it orbits within or beyond the magnetopause. It is worth noting that this last objective requires a thorough study of the structure and dynamics of Saturn’s magnetopause, to which chapters 5 and 6 have made contributions.

• **Investigating solar and seasonal variations of the magnetosphere** by observing it over a full solar cycle (which is ≈ 40% of Saturn’s orbital period), and studying how magnetospheric structure relates to transitions from one solar minimum (beginning of Nominal Mission) to the next (end of Solstice Mission). This relates directly to the modelling of seasonal effects on magnetopause structure, magnetospheric field and current system detailed in chapter 6.

In order to reach these objectives, specific scientific instruments were selected to be part of the Cassini payload. We describe below three of these instruments that are related to the objectives mentioned above and the work presented by the following chapters.

### 2.2 Key Instruments

#### 2.2.1 Cassini Magnetometer (MAG)

The Cassini Magnetometer (MAG), highlighted in orange in Fig. 2.2, was a direct sensing instrument that measured the three vector components and magnitude of
the magnetic field outside and within Saturn’s magnetosphere. It consisted of two separate sensors mounted on a common 11-meter boom: a Fluxgate Magnetometer (FGM) placed halfway along it and a Vector/Scalar Helium magnetometer (V/SHM) at the end of it. This configuration was chosen to limit magnetic interference from other instruments aboard the spacecraft. The V/SHM sensor stopped performing as planned a year after arrival at Saturn, so we will focus on summarising the characteristics of the FGM sensor; a thorough description of the MAG instrument can, for example, be found in Dougherty et al. (2004).

Since a single flux gate sensor is only capable of measuring the magnetic field strength in the direction of its sensor coil, the FGM instrument was composed of three single-axis core flux gate sensors mounted orthogonally. They were supported by a block of glass ceramic whose low thermal expansion coefficient minimised the risk of misalignments due to changes in temperature. Each sensor making up the 0.44 kg instrument consisted of a wire coil (or drive winding in Fig. 2.3) wound around a high permeability ring core. An alternating 15.625 Hz square wave current

Figure 2.2: Schematic of the Cassini spacecraft highlighting its science instruments. The Magnetometer (MAG) is shown in orange, the Cassini Plasma Spectrometer (CAPS) in green and the Magnetospheric Imaging Instrument (MIMI) in pink. Adapted from an original figure by ESA.
2.2. Key Instruments

Figure 2.3: Schematic of a Flux Gate Magnetometer, from Miles et al. (2017). The drive coil is wound around the ring shaped core. The black and white arrows shown within the core describe the magnetic field induced by currents in the drive winding.

was flowed through the solenoid in order to induce a field shown by circular black and white arrows within the core in Fig. 2.3. In the absence of an external field, the black and white halves of the core would transition in and out of saturation simultaneously, and there would be no change in flux through the surrounding coil of wire, or sense winding. In the presence of an external field component oriented as shown in Fig. 2.3, one half of the core would become saturated faster than the other, causing a change in flux through the sense winding. A voltage would be induced in accordance with Faraday’s law, and the corresponding external magnetic field strength could be measured after applying calibration procedures to the output of the sense winding.

Cassini’s FGM sensor could be used to measure the background magnetic field with an approximate resolution of one part in 10000, and could automatically switch between four ranges depending on the expected ambient magnetic field


2.2. Key Instruments

strength: ±40 nT, ±400 nT, ±10000 nT and ±44000 nT. In situ measurements of Saturn’s magnetic field shed light on many aspects of the Kronian magnetosphere, including: atmospheric plume activity from the moon Enceladus (Dougherty et al., 2006) emanating from a liquid water ocean beneath its icy crust; and the structure and composition of the magnetosphere—in particular, the ‘bowl-shaped’ structure of Saturn’s equatorial current sheet (Arridge et al., 2008a) will be discussed in chapter 6.

discoveries about the interior of Saturn, its rings and tiny ringmoons, and the gap between the rings and the planet

2.2.2 Cassini Plasma Spectrometer (CAPS)

The Cassini Plasma Spectrometer (CAPS), shown in green in Fig. 2.2, was an instrument that investigated the plasma environment around Saturn with measurements of plasma composition, flow velocity, density and temperature. It comprised three sensors, an Electron Spectrometer (ELS), a time-of-flight Ion Mass Spectrometer (IMS) and an Ion Beam Spectrometer (IBS) mounted on a platform capable of rotating the field-of-view to compensate for the spacecraft’s three-axis stabilisation. The CAPS instrument operated as planned from the start of the mission well into the Solstice Mission, during which it stopped functioning from June 2012 due to a power anomaly. Identifications of magnetopause crossings relevant to this thesis (see chapter 5) relied on data processed by Pilkington et al. (2014) using the CAPS-ELS sensor, which will described below; a detailed description of the instrument can be found in Young et al. (2004).

CAPS-ELS was an electrostatic analyser capable of measuring electrons with energies ranging from 0.6 eV to 29 keV. Incident electrons entered a five degree aperture and would be detected only after travelling between a pair of hemispherical charged plates and reaching a Micro Channel Plate (labelled MCP in Fig. 2.4). The electrons were deflected towards the MCP by an electric field formed between the two charged plates after a given voltage was applied. This allowed the selection of electrons within a controlled energy range, and the instrument would sweep through sets of voltages in order to construct a full electron spectogram.
The angular distribution of electrons within each energy range could also be determined based on where they had finally struck the MCP. More precisely, the detector consisted of eight microchannel plate ‘pixels’ with fields of view of $5^\circ \times 20^\circ$, providing the instrument with a total field of view of $5^\circ \times 160^\circ$. The ELS sensor was mounted on top of the Ion Mass Spectrometer (labelled IMS in Fig. 2.4) in order to ensure alignment between its field of view and the IMS without obscuration from other parts of the instrument.

Observations from CAPS-ELS were used by Pilkington et al. (2014) to identify positions of the spacecraft as it transitioned in or out the magnetosphere, through the magnetopause. Section 2.3 will summarise how these magnetopause crossings were identified using in situ measurements. The corresponding database was used in chapter 5 to assess the validity of our magnetopause model (especially its high latitude structure) and study the compressibility of the boundary under varying solar wind and magnetospheric conditions.
2.2.3 Magnetospheric Imaging Instrument (MIMI)

The Magnetospheric Imaging Instrument (MIMI) is a neutral and charged particle detection system – coloured in pink in Fig. 2.2 – comprised of three sensors designed to measure energetic electrons, ions and neutrals in Saturn’s magnetosphere: the Low Energy Magnetospheric Measurement System (LEMMS), the Charge-Energy-Mass Spectrometer (CHEMS) and the Ion and Neutral Camera (INCA).

It is particularly pertinent for the study of Saturn’s magnetosphere, where internal hot plasma populations were shown to be important sources of pressure (Sergis et al., 2007; Thomsen et al., 2010). At the magnetopause for example, energetic particle populations have been found to be a significant – even sometimes dominant – driver (Kanani et al., 2010; Pilkington et al., 2015), hence the modelling approach described in section 4.3.3. Moreover, in the vicinity of the ring current in the inner magnetosphere, oxygen ion populations with energies above 10 keV were shown to contribute to most of the total particle pressure (Sergis et al., 2007). Such significant energetic particle populations can be detected and analysed by the three sensors of the MIMI instrument, which are described in details in Krimigis et al. (2004) and summarised below.
2.2. Key Instruments

2.2.3.1 Low Energy Magnetospheric Measurement System (LEMMS)

The MIMI Low Energy Magnetospheric Measurements System (LEMMS) was a sensor that consisted of a double-ended telescope with oppositely directed fields of view for separate low (left side of Fig. 2.5) and high (right side of Fig. 2.5) energy ends. The identification and measurements were based on how much energy the incoming particles lost in 11 semiconductor detectors.

The low energy end had an aperture of 15° and was designed to measure low energy ions (with energies $0.03 \text{ MeV} \leq E \leq 18 \text{ MeV}$) and electrons ($0.015 \text{ MeV} \leq E \leq 0.884 \text{ MeV}$). An internal permanent magnet produced an inhomogeneous magnetic field which made electrons strike detectors E and F depending on their incident energy. Low energy ions were less affected by the magnetic field and were measured by detectors A and B facing the aperture. A gold absorber separated each end of the instrument to stop low energy ions from penetrating detectors A and B.

The high energy end measured high energy ions ($1.6 \text{ MeV} \leq E \leq 160 \text{ MeV}$) and electrons ($0.1 \text{ MeV} \leq E \leq 5 \text{ MeV}$). Its opening angle was 30° and consisted of five stacked detectors labelled $D_1$, $D_2$, $D_{3a}$, $D_{3b}$ and $D_4$ on the right side of Fig. 2.5.

The entire instrument was shielded by a platinum cover to prevent particles with energies below 30 MeV from penetrating its outer shell. It was originally mounted on a rotating platform so that it could determine the incident directions of measured particles, but it ended up being fixed in a non-obscured direction in 2005 after a power anomaly.

Combined observations from the LEMMS and CHEMS sensors were used to determine the distributions of hot plasma populations over a wide range of latitudes within Saturn’s magnetosphere (Sergis et al., 2017). We model the contributions of these populations at the magnetopause to assess how its compressibility varies with system size in chapter 5.

2.2.3.2 Charge-Energy-Mass Spectrometer (CHEMS)

The CHEMS sensor, shown in Fig. 2.6, was designed to measure the three-dimensional distribution functions of ions to characterise the suprathermal ion populations ($E \geq 27 \text{ MeV}$) upstream of and within Saturn’s magnetosphere. The par-
particles were separated using electrostatic deflection before the mass, charge and incident energy were determined by *Time Of Flight* (TOF) and energy measurements (*Gloeckler and Hsieh, 1979*).

Ions would enter the electrostatic deflection analyser through an aperture shown on the bottom right in Fig. 2.6. The system is similar to the one of the CAPS-ELS sensor described in section 2.2.2, with the addition of ions being deflected differently for a given voltage depending on their energy-to-charge E/Q ratio. The system thus acts as an effective E/Q filter, only allowing ions with E/Q ratios within a set interval into the upcoming TOF system.

The time of flight system operates by measuring the time taken for the ion to travel between two start and stop detectors separated by 10.0 cm. The filtered ions would pass through a thin carbon foil at the entrance of the telescope, where secondary electrons would be emitted and deflected by electric fields towards one of three MCPs to generate the start signal. At the other end of the telescope, the ions would strike a silicon Solid-State Detectors (SSD), where secondary electrons were similarly emitted and guided towards another MCP to provide the stop signal. The detectors also measured the residual energy of the particle, allowing its full identification when combined with the measured time of flight and E/Q ratio. The instrument included three independent telescopes with different viewing angles to
benefit from a nearly $4\pi$ viewing geometry when the spacecraft was rolling.

Pressure moments derived from the data were used to improve empirical models of Saturn’s magnetopause with the inclusion of measurements of suprathermal particle pressure within the boundary (Kanani et al., 2010). We compare our physics-based approach to these results in our study of magnetopause compressibility at Saturn (see chapter 5 and Fig. 5.5).

2.2.3.3 Ion and Neutral Camera (INCA)

MIMI’s Ion and Neutral Camera (INCA), a diagram of which is shown in Fig. 2.7, is a time of flight detector designed to measure Energetic Neutral Atoms (ENAs) and ion populations with energies ranging from 0.007 to 3 MeV/nucleon. The energy and incident direction of the particles are determined similarly to the previously-discussed CHEMS’s TOF system.

Particles would enter the sensor from the top of Fig. 2.7 through a fan of collimator plates. When used in ‘neutral mode’, the sweeping plates would be charged with alternate potentials of up to $\pm6$ V in order to divert and exclude energetic charged particles (with energies $\leq 500$ keV). ENAs would be free to pass through and penetrate the foil layer, producing secondary electrons which are accelerated.
and steered electrostatically onto a microchannel plate. This event generated a start signal and the particle continued through the instrument – albeit scattered slightly – until it struck a second foil layer in front of a stop MCP. This allowed both the time of flight and the 2D exit position to be recorded. In parallel, some secondary electrons produced at the exit foil were guided towards a coincidence MCP where the signal produced could be used to minimise background noise. These measurements could be combined to determine the mass, energy and incident direction of the incoming particles. Their natures (e.g. Oxygen or Hydrogen, the most common species within Saturn’s magnetosphere) could also be deduced since the number of electrons generated in the foil was dependent on the mass of the particles.

When used in ‘ion mode’, the charge of the sweeping plates would be turned off and ionic species would be observed and characterised in a similar way.

At Saturn, the detection of ENAs can be used to indirectly trace the energetic ions making up the equatorial ring current. This is because Saturn’s neutral gas distribution – originating from planetary rings and moons – can interact with ion populations via collisional charge exchange. The resulting neutral particles would be free of any magnetic confinement and continue their course unperturbed. Fig. 2.8 shows ENA images in neutral hydrogen taken by MIMI-INCA in the 20-40 keV energy range at Saturn (Krimigis et al., 2007); it traces the dynamic, rotating ring current with evident longitudinal asymmetries in quasi corotation with the planet.

These in situ measurements illustrate the actual dynamic nature and structural complexity of Saturn’s ring current. In particular, the hot plasma sheet has been observed to be inflated on the day side with a latitudinal extent of up to 45°. During Southern Summer at Saturn, the ring current plasma sheet has also been observed to be tilted above the equatorial plane on the night side, with a Northwards deflection characterised by a ‘hinge’ point around 20 $R_S$. This is consistent with further empirical modelling work (Arridge et al., 2008a) and with our study of seasonal effects described in chapter 6.
2.3 Observations Pertinent to Magnetopause Modelling

To a large extent, the magnetopause separates the shocked solar wind plasma in the magnetosheath from plasma populations within the magnetosphere. This can be understood by both plasmas having high conductivities and the conditions for ideal MHD being mostly satisfied: in line with the frozen-in theorem discussed in section [1.1.3] a frozen in magnetic field excludes other fields from the plasma region, preventing solar wind and magnetospheric plasma populations from mixing. A boundary is thus required to shield the magnetosphere, as long as the conditions for ideal MHD hold.

**Figure 2.8:** Series of ENA images in neutral hydrogen taken by MIMI-INCA in the 20-40 keV energy range. The observations were made on 24th February 2007 and covered one full Saturn rotation. Saturn is at the origin, the X axis pointing towards the Sun and the Y axis pointing towards dusk. Dotted lines show the orbits of Dione (6.26 $R_S$), Rhea (8.74 $R_S$) and Titan (20.2 $R_S$). From [Krimigis et al. (2007)]
2.3. Observations Pertinent to Magnetopause Modelling

Figure 2.9: *In situ* MAG (top) and CAPS-ELS (bottom) observations over 48h starting at midnight on 3rd May 2007. Magnetopause crossings are identified by vertical magenta dotted lines and correspond to sudden changes in electron energies and Differential Energy Flux (proportional to density). The magnetic field is expressed in the Kronocentric Solar Magnetospheric coordinate system. From Pilkington et al. (2014).

As mentioned in section 1.3.3, this separation at the magnetopause is not absolute: magnetic reconnection between solar wind and magnetospheric field lines can act locally as a bridge between both plasma populations, and energetic particles may have large enough gyroradii to traverse the boundary.

Thermal plasma populations are, however, generally well separated. For a nominal field strength of about 1-10 nT at Saturn’s magnetopause for example, the gyroradius of a 10 eV electron ranges from around 11 km to 110 km. This is small compared to the typical thickness of a magnetopause current layer (Kaufmann and Konradi [1973], Berchem and Russell [1982]). Electron observations from the CAPS-ELS instrument (see section 2.2.2) are thus useful to identify positions of the spacecraft as it travelled from one region to the other through the boundary, or magnetopause crossings. Pilkington et al. (2014) discussed in detail how CAPS-ELS observations were used to identify a database of such crossings at Saturn. Since we use this crossing database to assess our magnetopause model and to apply it to the study of magnetopause dynamics at Saturn in chapter 5, we briefly summarise the method below.
Compared to the electron population in the magnetospheric plasma, the one in the magnetosheath typically tends to have densities an order of magnitude higher, and energies an order of magnitude lower. Therefore, a CAPS-ELS signature characterised by a jump in energy from \( \lesssim 100 \text{ eV} \) to a few keV can be associated to the spacecraft crossing the magnetopause to leave the magnetosheath and enter the ambient magnetospheric plasma. Such configurations can be observed in Fig. 2.9 around 02:00 and 17:00 during the first 24 hour period displayed. These crossings from magnetosheath to magnetospheric plasma are accompanied with a distinct decrease in Differential Energy Flux, which is a calibrated count rate proportional to particle density.

The radial position of that crossing from the planet, along with its corresponding Local Time and latitude, fully describe a point belonging to the magnetopause surface. We will describe in chapter 4 how we are able to infer the structure of Saturn’s magnetopause from two of the following three parameters:

- **The stand-off distance** \( R_{\text{MP}} \), measuring the distance between the nose of the magnetopause and the planet. It is a useful proxy for system size and fixes the parameters of the equatorial ring current model within the magnetosphere (Bunce et al. [2007]).

- **The solar wind pressure** \( P_{\text{SW}} \), which describes the dynamic pressure from the incident solar wind plasma.

- **The plasma beta** \( \beta \), defined in Eq. (1.31), which models the ratio of particle pressure from hot plasma populations in the outer magnetosphere to the magnetic pressure.

Section 5.2.2 will detail how we can use the observed position of a crossing to deduce \( R_{\text{MP}} \). Assuming the pressure balance is approximately satisfied at the magnetopause as the spacecraft flew by, we can then determine \( P_{\text{SW}} \) from MAG measurements of the magnetic field strength and values for \( \beta \) from MIMI suprathermal plasma pressure measurements – see section 5.2.3 for more details. A full magnetopause surface corresponding to the observed crossing is now fully defined; the
2.3. Observations Pertinent to Magnetopause Modelling

Theoretical, equilibrium value of the solar wind pressure can be compared to the empirically-determined $P_{SW}$ to assess how close the boundary actually was to pressure equilibrium, as detailed in section 5.4.1.

A similar treatment can be applied to the rest of the database in order to estimate, for each crossing, the upstream solar wind pressure and the corresponding scale of the magnetopause. As described in section 1.3.3 however, the boundary is highly dynamic: one crossing could be observed as the magnetopause is highly compressed (e.g. due to high solar wind pressure), another could correspond to a highly expanded boundary (e.g. because of high contributions from energetic magnetospheric plasma populations), and others could be observed while the magnetopause was suddenly moving in response to changes in the pressure budget. We will describe in chapter 5 how the crossing database can be ‘scaled’ to comparable configurations in both solar wind and magnetospheric conditions. Another apparent hurdle lies in the treatment of a relatively large database with magnetopause models that need to be computed individually for each crossing; we suggest an interpolation solution in chapter 6 which may be used to estimate magnetopause equilibrium model predictions by interpolating between models in a subset whose size is comparatively small, for reasons of computational expedience and practicality.
Chapter 3

Numerical Approach to Magnetopause Modelling

This chapter serves as a foundation to the magnetopause modelling work described hereafter: it presents the key numerical methods that are used and places them in the particular context of this study.

The magnetopause boundary responds to changes in both external and internal conditions. It is simultaneously compressed towards the planet by solar pressure sources (e.g. incoming solar wind dynamic pressure), and inflated outwards by the planetary magnetic field and magnetospheric hot plasma population. As a result, its structure and position can be considered – to a first order – governed by a balance between these various drivers of antagonistic effects.

Previous empirical studies assume initial analytical relationships involving these pressure sources, before using in-situ measurements and data to optimise the empirical description. For example, many studies assume a power law relation between the upstream solar wind dynamic pressure and the magnetopause stand off distance (Slavin and Holzer 1981; Arridge et al. 2006; Pilkington et al. 2014).

The approach taken in this study is fundamentally different, as it is physics-based: we use the aforementioned pressure balance equation to infer the morphology of an equilibrium magnetopause surface. This requires a numerical approach to the problem, namely discretising and formulating it into an optimisation problem. The groundwork of such an approach was laid out by Mead and Beard (1964) for the
3.1 Numerical Optimisation Towards Pressure Balance

3.1.1 Discretisation of Magnetopause Morphology

Let us adopt the planet-centred description illustrated in Fig. 3.1, where the Z-axis points towards the Sun, the Y-axis is such that the magnetic dipole of the planet $\mathbf{M}$ is contained in the plane $(OZY)$, and the X-axis completes the right-handed coordinate system. The Z, X and Y axes thus correspond to the KSM (Kronocentric Solar Magnetospheric) $X_{KSM}$, $Y_{KSM}$, $Z_{KSM}$ axes respectively.

A point $A$ on the magnetopause boundary can be positioned by its spherical coordinates $(r, \theta, \phi)$ shown in green. In the case of an aligned dipole (i.e. $\mathbf{M}$ being orthogonal to the planet-Sun direction, as shown in Fig. 3.1), some regions of space will be of particular interest:

- The nose of the magnetopause, positioned along the Sun-Planet line (OZ), is the point on the boundary for which $\theta = 0$.
- $\phi \in \{0; \pi\}$ corresponds to the equatorial plane, shown in orange.
- $\phi = \pi \in \{\pi, \frac{3\pi}{2}\}$ corresponds to the noon-midnight meridional plane, shown in green.

By considering a two-dimensional grid of discrete values for the angles $\phi$ and $\theta$

$$\text{Grid: } \{\phi_1, ..., \phi_{N_{\phi}}\} \times \{\theta_1, ..., \theta_{N_{\theta}}\},$$

(a discretised magnetopause boundary can be described by a set of radial distances $r$ at each vertex of the grid

$$\text{Magnetopause: } \{r_1, ..., r_{N_{\phi}N_{\theta}}\}.$$
At each vertex $k$ of this grid, the point of the boundary should satisfy the local pressure balance

$$\mathcal{F}_k = \Delta P_k = P_{\text{ext},k} - P_{\text{int},k} = 0$$

in the frame of reference of the magnetopause, where $P_{\text{ext},k}$ and $P_{\text{int},k}$ are the external and internal pressure sources to the magnetopause surface respectively, at vertex $k \in [1, N_\phi N_\theta]$.

The magnetopause boundary can now be defined as the solution of the vectorised equation

$$\vec{\mathcal{F}}(\vec{r}) = \mathbf{0},$$

with $\vec{\mathcal{F}} = (\mathcal{F}_k)_{1 \leq k \leq N_\phi N_\theta}$ and $\vec{r} = (r_k)_{1 \leq k \leq N_\phi N_\theta}$. In this form, the magnetopause surface corresponds to a zero of the implicit function $\vec{\mathcal{F}}$. Iterative numerical techniques derived from Newton’s method can be used to optimise an initial guess surface towards the final equilibrium boundary.

The functions $\mathcal{F}_k$ mentioned in Eqs. (3.3–3.4) describe the balance between internal and external drivers at the boundary. As such, they rely on the way these influences are modelled (e.g. planetary field, particle pressure, contributions of ring current, etc.); the final equilibrium solution is consequently intrinsically linked to these initial hypotheses. The analytical details of these functions will be given in
the following chapters, but the general approach used to solve them numerically is detailed below.

### 3.1.2 Discrete Formulation of Pressure Balance

The balance of pressure at a point along the magnetopause boundary depends on its local orientation, both in the $\phi$ and $\theta$ directions, and hence on the local normal to the boundary. This initially makes the local pressure difference mentioned in Eq. (3.3) a non-linear partial differential equation of the form

\[
\mathcal{F}\left(r_k, \left(\frac{\partial r}{\partial \theta}\right)_k, \left(\frac{\partial r}{\partial \phi}\right)_k\right) = 0.
\]  
(3.5)
At each vertex \( k = (j - 1)N_\theta + i \) of the grid, the partial derivatives can be approximated using centred finite differences

\[
\left( \frac{\partial r}{\partial \theta} \right)_k = \frac{r(k + N_\phi) - r(k - N_\phi)}{2\Delta \theta} + \mathcal{O}(\Delta \theta^2),
\]

\( 3.6 \)

\[
\left( \frac{\partial r}{\partial \phi} \right)_k = \frac{r(k + 1) - r(k - 1)}{2\Delta \phi} + \mathcal{O}(\Delta \phi^2),
\]

\( 3.7 \)

with \( \Delta \phi \) and \( \Delta \theta \) being the angular increments of the \( \phi \times \theta \) grid.

Doing so converts the differential equation Eq. (3.5) into a set of algebraic equations

\[
\mathcal{F}_k \left( r_k, r_{k+N_\phi}, r_{k-N_\phi}, r_{k+1}, r_{k-1} \right) = 0,
\]

(3.8)

with \( k \in [1, N_\phi N_\theta] \).

In practise, the finite difference elements are calculated by ‘sliding’ a mask over the initial grid, as shown in Fig. 3.2:

- The original grid is coloured in orange, at the centre of the figure.
- Sliding a mask one step to the right defines a grid – coloured in blue – of the same size, which contains values for \( r_{k+N_\phi} \) at each vertex \( k \).
- Sliding a mask one step to the left defines another grid – coloured in green – containing values for \( r_{k-N_\phi} \) at each vertex \( k \).
- Doing the same thing one step up, or down, defines two grids containing values for \( r_{k+1} \), and \( r_{k-1} \) respectively; these are coloured in purple and red.

This allows the finite elements to be calculated efficiently; for example, using Eq. (3.6), a grid containing values for \( \left( \frac{\partial r}{\partial \theta} \right)_k \) can be determined using

\[
\left( \frac{\partial r}{\partial \theta} \right)_{i,j} = \frac{\text{Blue Grid - Green Grid}}{2\Delta \theta} + \mathcal{O}(\Delta \theta^2),
\]

(3.9)

with \( 1 \leq i \leq N_\phi \) and \( 1 \leq j \leq N_\theta \).

Eq. (3.4) is now fully converted into a set of algebraic equations, which can be solved using diverse iterative numerical methods. The technique that has proved the
most efficient and that is used in the following studies is the Levenberg-Marquardt method; it is presented in the subsection below.

### 3.1.3 Optimisation Procedure and Levenberg-Marquardt Algorithm

As discussed previously, the magnetopause boundary can be described as a one-dimensional vector \( \mathbf{r} \) whose components correspond to the radial position of points of the discretised surface at each vertex of the \( \phi \times \theta \) grid. These points satisfy the local pressure balance described in Eq. (3.3), making the equilibrium boundary a solution of the overall vectorised equation (3.4).

At any vertex \( k \) of the grid, we want the pressure balance equation to be satisfied, i.e. the pressure difference \( y_k = \Delta P_k = 0 \). Given a discretised magnetopause vector \( \mathbf{r} \), the pressure difference at vertex \( k \) is estimated by \( \mathcal{F}_k \left( r_k, r_{k+1}, r_{k-1}, r_{k+N_\phi}, r_{k-N_\phi} \right) \).

At each iteration, this magnetopause vector \( \mathbf{r} \) is replaced by a corrected estimate \( \mathbf{r} + \delta \mathbf{r} \), where the corrective displacement \( \delta \mathbf{r} \) is determined through the linearisation

\[
y = 0 = \mathcal{F} (\mathbf{r} + \delta \mathbf{r}) = \mathcal{F} (\mathbf{r}) + \mathbf{J} (\mathbf{r}) \cdot \delta \mathbf{r} + O \left( \| \delta \mathbf{r} \|^2 \right),
\]

where \( \mathbf{J} \) is the Jacobian matrix

\[
\mathbf{J} = \nabla_r \mathcal{F} = \left( \frac{\partial \mathcal{F}_i}{\partial r_j} \right)_{i,j}.
\]  

(3.11)

If the partial derivatives are approximated using centred finite differences, as men-
3.1. Numerical Optimisation Towards Pressure Balance

mentioned in Eq. (3.7), the Jacobian matrix is a sparse matrix of the form

\[
\mathbf{J} = \begin{bmatrix}
\frac{\partial \mathcal{F}_1}{\partial r_1} & \frac{\partial \mathcal{F}_1}{\partial r_2} & \cdots & \frac{\partial \mathcal{F}_1}{\partial r_{N+1}} \\
\frac{\partial \mathcal{F}_2}{\partial r_1} & \frac{\partial \mathcal{F}_2}{\partial r_2} & \cdots & \frac{\partial \mathcal{F}_2}{\partial r_{N+1}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial \mathcal{F}_{N\theta}}{\partial r_1} & \frac{\partial \mathcal{F}_{N\theta}}{\partial r_2} & \cdots & \frac{\partial \mathcal{F}_{N\theta}}{\partial r_{N+1}} \\
\frac{\partial \mathcal{F}_{N\theta-1}}{\partial r_1} & \frac{\partial \mathcal{F}_{N\theta-1}}{\partial r_2} & \cdots & \frac{\partial \mathcal{F}_{N\theta-1}}{\partial r_{N+1}} \\
\frac{\partial \mathcal{F}_{N\theta-N\phi}}{\partial r_1} & \frac{\partial \mathcal{F}_{N\theta-N\phi}}{\partial r_2} & \cdots & \frac{\partial \mathcal{F}_{N\theta-N\phi}}{\partial r_{N+1}} \\
\frac{\partial \mathcal{F}_{N\theta-N\phi-N\theta}}{\partial r_1} & \frac{\partial \mathcal{F}_{N\theta-N\phi-N\theta}}{\partial r_2} & \cdots & \frac{\partial \mathcal{F}_{N\theta-N\phi-N\theta}}{\partial r_{N+1}} \\
\end{bmatrix}
\]

Let \( S \) denote the sum of square deviations

\[
S(r + \delta r) = \left\| y - \mathcal{F}(r) - \mathbf{J}(r) \cdot \delta r \right\|^2
\]

(3.13)

\[
= \left( y - \mathcal{F}(r) - \mathbf{J}(r) \cdot \delta r \right)^T \left( y - \mathcal{F}(r) - \mathbf{J}(r) \cdot \delta r \right)
\]

\[
= [y - \mathcal{F}(r)]^T [y - \mathcal{F}(r)] - 2 [y - \mathcal{F}(r)]^T J(r) \cdot \delta r + \left( \delta r \right)^T J(r)^T J(r) \cdot \delta r .
\]

We are looking for the correction \( \delta r \) that minimises the sum of square deviations \( S(r + \delta r) \), leading to

\[
\mathbf{J}^T \mathbf{J} \cdot \delta r = \mathbf{J}^T (y - \mathcal{F}(r)) = -\mathbf{J}^T (\mathcal{F}(r)),
\]

(3.14)

since \( y = 0 \), and with \( \mathbf{J}^T \) denoting the transposed Jacobian matrix. This results in a set of \((N_\theta N_\phi)\) linear equations that can be solved for \( \delta r \). If \(-90^\circ \leq \phi \leq 90^\circ\) and \(0 \leq \theta \leq 90^\circ\) in \(5^\circ\) increments for example, this corresponds to \((N_\theta N_\phi) = 703\) linear equations.

The Levenberg-Marquardt algorithm introduces an additional damping factor \( \lambda \geq 0 \) into the previous equation to consider

\[
\left( \mathbf{J}^T \mathbf{J} + \lambda I \right) \cdot \delta r = -\mathbf{J}^T \mathcal{F}(r),
\]

(3.15)
where $I$ is the identity matrix. The damping parameter $\lambda$ controls both the magnitude and direction of the correction $\delta r$, and is adjusted at each step of the procedure to ensure that each iteration is effective, i.e. $\|\mathbf{f}(\mathbf{r} + \delta \mathbf{r})\| < \|\mathbf{f}(\mathbf{r})\|$. 

In particular, if $\lambda \to +\infty$ the direction taken with $\delta r$ tends towards the steepest descent direction; if $\lambda \to 0$, it tends to the Gauss-Newton direction. At each iteration, the search direction taken by the Levenberg-Marquardt algorithm is thus an optimal combination of both methods.

In the context of the current study, it is also worth mentioning that the efficiency of the optimisation procedure is highly dependent on

- The ‘quality’ of the initial guess for $\mathbf{r}$, namely how ‘physically accurate’ the starting surface is. This means that some particular attention will have to be given to finding the ‘best’ initial guess possible to ensure convergence, as will be shown in the following chapters.

- The consistency of the boundary conditions at the periphery of the grid.

If these conditions are satisfied, the chosen initial guess-surface can be iteratively corrected until the algorithm converges to a final equilibrium magnetopause boundary.

### 3.2 Explicit Numerical Methods to Solve ODEs

#### 3.2.1 Pressure Balance Equation in Particular Planes

The equilibrium magnetopause boundary is a surface that is expected to satisfy, at every point, a balance between

- The dynamic pressure due to the normal component of the solar wind $P_{SW} \propto (v \cdot n)^2$,

- The magnetic pressure due to the tangential component of the magnetospheric field $P_{mag} \propto \|B \times n\|^2$, where $v$ and $B$ denote the solar wind velocity vector and the modelled magneto-spheric field respectively, and $n$ is the vector locally normal to the surface. The magnetopause is *implicitly* defined by the balance between these two pressure sources.
According to the implicit function theorem (Krantz and Parks, 2013), for any value of \((\theta, \phi)\) at which the Jacobian from Eq. (3.12) is invertible, there exists a neighbourhood of \((\theta, \phi)\) such that \(r = g(\theta, \phi)\) where \(g\) is a unique continuously differentiable function. This makes the pressure balance relation a Partial Differential Equation, with the components of \(n\) leading to dependencies on both \(\frac{\partial r}{\partial \theta}\) and \(\frac{\partial r}{\partial \phi}\), described by Eq. (3.5). However, the problem presents several symmetries that simplify the equation in certain planes.

Such is the case of the equatorial plane (i.e. (OXZ) in Fig. 3.1) in the configuration of an aligned dipole: it is a plane of symmetry for both the solar wind flow and the magnetic field, and the latter is also entirely normal it. This results in the magnetopause boundary itself being normal to this plane, i.e. \(\frac{\partial r}{\partial \phi}(\theta, \phi \equiv 0 \text{ mod } \pi) = 0\). In this configuration, the final surface will present a North-South symmetry with respect to this equatorial plane.

Moreover, in the general case of a tilted dipole, the magnetic noon-midnight meridional plane contains the planetary magnetic moment at all times. The field is also contained in this plane, and in particular \(B_\phi = 0\). This also causes the boundary to be normal to the plane, and thus \(\frac{\partial r}{\partial \phi}(\theta, \phi = \pm 90^\circ) = 0\). The final surface will then present a Dawn-Dusk symmetry; in the general case, the North-South symmetry only appears during the Autumnal and Vernal Equinox, as discussed in chapter 6.

In these specific planes, the pressure balance equation is thus simplified into an Ordinary Differential Equation (ODE), with dependencies on only \(\frac{\partial r}{\partial \theta}\). We will discuss below numerical methods that were used to solve these ODEs. They are of particular importance, since they will allow us to find profiles of the magnetopause in specific symmetry planes, before using these solutions as robust boundary conditions to anchor the edges of the grid in the optimisation procedure described in section 3.1.3.
3.2. Explicit Numerical Methods to Solve ODEs

3.2.2 Runge-Kutta Methods for Solving ODEs Numerically

In the specific planes where the pressure balance equation is reduced to an Ordinary Differential Equation, it can be written as

$$\frac{dr}{d\theta} = f(\theta, r), \quad r(\theta_0) = r_0,$$

where $r$ is the radial position of a point of the magnetopause boundary, $\theta$ its angular elevation relative to the nose-planet axis, $\theta_0 = 0$ and $r_0$ is the radial position of the nose from the planet; $f$ is a function that comes from solving Eq. (3.3) for $dr/d\theta$. This is an initial value problem of the form

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0,$$

with $y = r, t = \theta, t_0 = \theta_0$ and $y_0 = r_0$. The family of explicit Runge-Kutta methods are iterative numerical methods commonly used to find approximative solutions to such ODEs.

In order to solve Eq. (3.17) on a discrete time interval $t_0 < \ldots < t_n < \ldots < t_f$, a general $q^{th}$-order Runge-Kutta method consists in, for $1 \leq i \leq q$,

1. Considering a set of intermediate time values on which the solution will be propagated from the left

$$t_{n,i} = t_n + c_i h_n,$$

where $h_n = t_{n+1} - t_n$ is the time interval and $0 \leq c_i < c_{i+1} \leq 1$.

2. Use the values of the slope $p_{n,i} = f(t_{n,i}, y_{n,i})$ at previous steps to estimate the function at the next intermediate time value

$$y_{n,i} = y_n + h_n \sum_{k=1}^{i-1} a_{ik} p_{n,k},$$

with $\sum_{k=1}^{i-1} a_{ik} p_{n,k}$ describing a linear combination of the slopes at previous time steps.

3. Use this estimated value of the function to approximate the slope at the next
3.2. Explicit Numerical Methods to Solve ODEs

Intermediate time value

\[ p_{n,i} = f(t_{n,i}, y_{n,i}) \]  \hspace{1cm} (3.20)

and finally use a combination of estimates \( p_{n,i} \) to propagate the solution at the next point

\[ y_{n+1} = y_n + h_n \sum_{k=1}^{q} b_k p_{n,k} \]  \hspace{1cm} (3.21)

The method is parametrised by the nodes \( c = (c_i)_{1 \leq i \leq q} \) from Eq. (3.18), the Runge-Kutta matrix \( A = (a_{i,j})_{(i,j)} \) from Eq. (3.19) and the weights \( b = (b_i)_{1 \leq i \leq q} \) from Eq. (3.21).

In the simple case where \( q = 1 \) (i.e. no intermediary point is added within each time interval), \( c = 0 \), \( A = 0 \) and \( b = 1 \), putting Eq. (3.21) in the form

\[ y_{n+1} = y_n + h f(t_n, y_n) \]  \hspace{1cm} (3.22)

which reduces to a forward Euler Method. \( q^{th} \) order Runge-Kutta methods thus estimate the next value \( y_{n+1} \) as the present one \( y_n \) plus a weighted average of \( q \) increments, each of them being the product of the interval size \( h_n \) and an estimate of the slope. They are methods of order \( q \), meaning the local truncation error is an \( O(h^{q+1}) \) and the total accumulated error is an \( O(h^q) \). The method used in the following chapters to solve Eq. (3.16) is the Dormand-Prince method, an adaptive version of Runge-Kutta techniques.

3.2.3 Adaptive Dormand-Prince Method

Adaptive Runge-Kutta methods are designed to adapt the step size \( h_n \) so that the estimated local truncation error stays below a chosen threshold. The error is estimated by interweaving two methods with common intermediate steps, one of order \( q \), the other \( q - 1 \).

Let us consider the Dormand-Prince method which uses methods of orders 4 and 5 calculated with the same Runge-Kutta matrix and node vector. The low-order method is used to propagate the solution at each step, and the high-order method is used to estimate the error of the former.
3.2. Explicit Numerical Methods to Solve ODEs

The lower-order step is given by

\[ y_{n+1}^{(4)} = y_n + h \sum_{k=1}^{5} b_k^{(4)} p_{n,k}^{(4)}, \]  
\[ (3.23) \]

with the (4) exponent indicating the order of the method, and \( b_5 = 0 \). The real value of the local truncated error of this method would be

\[ \left| y^*_n - y_{n+1}^{(4)} \right| = \mathcal{O}(h^5), \]  
\[ (3.24) \]

where \( y^*_n \) is the real value of the function.

The order-5 method is given by

\[ y_{n+1}^{(5)} = y_n + h \sum_{k=1}^{5} b_k^{(5)} p_{n,k}^{(5)}, \]  
\[ (3.25) \]

where the (5) relates to the order of the method, and \( p_{n,k}^{(5)} = p_{n,k}^{(4)} \) since the Runge-Kutta matrix and node vectors are the same. The node coefficients from Eq. (3.23) and Eq. (3.25) are calculated to minimise the error of the fifth-order method.

The error in the low-order method mentioned in Eq. (3.24) is estimated by

\[ \left| y^*_n - y_{n+1}^{(4)} \right| \approx \left| y_{n+1}^{(5)} - y_{n+1}^{(4)} \right| = \mathcal{O}(h^5). \]  
\[ (3.26) \]

The value of this error can be controlled by adjusting and rescaling the step-size \( h \). Most adaptive time stepping routines define error thresholds with both relative and absolute tolerances. We used the Matlab suite and its ODE45 solver (Shampine and Reichelt, 1997), which estimates the local error from Eq. (3.26) at each step and adjusts the step-size so that it satisfies

\[ \left| y_{n+1}^{(5)} - y_{n+1}^{(4)} \right| \leq r \left| y_{n+1}^{(4)} \right| + a_{n+1}, \]  
\[ (3.27) \]

where \( r \) is the relative error chosen fixed to \( 10^{-4} \), and \( a_{n+1} \) is the \( n+1 \) component of
the absolute error tolerance vector, fixed to $10^{-6}$; these values led to a satisfactory trade-off between speed and accuracy.

This implementation of the adaptive Dormand-Prince method allows us to solve the pressure balance ODEs mentioned in Eq. (3.16); the solutions can then be used for high-accuracy boundary conditions in the Levenberg-Marquardt optimisation of a magnetopause guess-surface, when appropriate.
Chapter 4

Modelling Boundary Interactions and Magnetopause Structure

In this chapter, we present how the structure and position of the magnetopause can be inferred from modelling the solar wind - magnetic field interactions at Saturn. To a first approximation, these interactions can be understood as a balance between the dynamic pressure from the solar wind flow, and interior pressure sources from within the magnetosphere. We explain how we build onto a framework previously developed for boundary studies at Earth (Mead and Beard, 1964) to obtain a robust and flexible model for Saturn’s magnetopause.

As discussed in section 1.3.2, the Kronian magnetosphere is fed by significant internal plasma sources which influence both the magnetic field structure, as well as the shape and size of the magnetosphere. We explain how we model the specificities of the interior field at Saturn to extract a physics-based description of an equilibrium magnetopause boundary. We derive a novel method to solve the high-latitude indentation of the surface, or ‘cusp’, particularly important in auroral studies.

Special attention is given to the foundations of the resulting Kronian magnetopause model: when applicable, we describe the analytical methodologies for simplified configurations (e.g. for a dipolar field) before expanding them to more realistic descriptions with robust numerical methods. When appropriate, we will refer to sections of chapter 3 to provide details for these techniques.

The content of this chapter is based on the following study:
4.1 Introduction to this Study

We discussed in section 1.3.1 how the structure of a planetary magnetopause is, in a steady-state system, governed by the local balance between exterior and interior pressure sources. At Saturn, the surface enclosing the magnetosphere is in effect compressed closer to the planet by the dynamic and thermal pressure of the shocked solar wind plasma in the magnetosheath, while it is pushed outwards by the internal magnetic field and particle pressure from magnetospheric plasma populations.

This competitive interaction makes it very difficult to predict the shape of the boundary: not only is it continuously accelerated due to the highly dynamic nature of these local pressure sources (Kaufmann and Konradi, 1969; Escoubet et al., 2013, 2015), it is also the result of a complex balance between drivers of different nature and variability. Saturn’s magnetopause is thus expected to differ from that of Earth or Jupiter in its morphology and dynamics due to the specificities of the Kronian magnetic field, magnetospheric plasma populations, etc. A physics-based modelling approach to magnetopause studies offers both helpful flexibility in parameter space and potential insight in the underlying physics, within the range of the assumptions made. The dynamical behaviour of the boundary is, for example, largely governed by the magnetosphere trying to establish an equilibrium, or balance, between the internal and external sources of pressure. A steady-state magnetopause model could be considered to be a good approximation of the average location of the boundary during ‘quiet-times’ during more active periods. Predicting its shape would help in understanding its general behaviour, as well as its local structure and dynamics under different regimes of exterior/interior pressure. It would also be a powerful tool for auroral studies, or solar wind pressure estimation (as discussed further in Chapter 5).

A self-consistent approach for modelling such an equilibrium boundary was
first developed by Mead and Beard (1964) for studying the Earth’s magnetopause: the planetary field was described by a magnetic dipole normal to the solar wind flow in order to find the locus of points which satisfy pressure balance. A mix of discretisation and interpolation methods was used to extrapolate the structure radially from the position of the sub-solar nose. The surface was then corrected by iterative computations of the magnetopause surface currents and the shielding field they generate. Since then, several studies have aimed at determining the equilibrium surface under the introduction of diverse current systems (Maurice and Engle, 1995; Sotirelis and Meng, 1999; Zaharia et al., 2004).

Observations from the Cassini-Huygens mission at Saturn have motivated more empirical studies and modelling approaches of a planetary magnetopause. In the absence of an upstream solar-wind monitor, magnetopause crossings of the spacecraft were identified and used to model the shape of the boundary including variable solar wind pressure (Arridge et al., 2006; Kanani et al., 2010), as well as its behaviour in response to variations in the internal plasma pressure distributions (Achilleos et al., 2008; Pilkington et al., 2014; Sorba et al., 2017). This latter effect was shown to have significant large-scale effects on the size and shape of Saturn’s magnetopause (Pilkington et al., 2015).

Saturn’s magnetopause has also been observed to oscillate in location with an amplitude of a few planetary radii. It is believed that this is a response to a rotating system of internal currents, probably linked to magnetosphere-ionosphere coupling (Clarke et al., 2010; Hunt et al., 2018). The models we describe here do not include this periodic perturbation, but do provide an accurate reference surface which acts as an indicator of average magnetopause location, and also organises Cassini magnetopause crossings successfully (see appendix A).

We describe in this chapter the first steps of a new numerical framework aiming at delivering a comprehensive theoretical model of Saturn’s magnetopause. It is based on an updated take on self-consistent methods, while also making use of the recent work and models developed using the data from the Cassini-Huygens mission. The outline method introduces new ways to compute, visualise and as-
Section 4.2 will introduce the criterion of local pressure balance which implicitly defines the equilibrium magnetopause boundary. Section 4.3 will then describe how a steady-state, field-dominated solution can be obtained using an initial guess-surface which is constrained by specific profiles in the equatorial and noon-midnight meridional planes. In particular, we present a novel method to solve the high-latitude ‘cusp’ structure which dents the boundary along the noon-midnight cross-section of the magnetopause boundary. We then expand the model with contributions from hot plasma particle pressure and equatorial ring currents. We conclude with a summary of our results and discuss the specific applications to the Kronian system which will be addressed in more details in the following chapters.

### 4.2 Magnetopause Definition from Local Pressure Balance

#### 4.2.1 Local Solar Wind - Planetary Magnetic Field Interactions

At Saturn, the additional plasma pressure from internal plasma populations has been shown to displace the magnetopause by up to $10^{-15}$ planetary radii at constant solar wind dynamic pressure for the case of Saturn \cite{Pilkington2015}. In a plasma-depleted state in which the magnetosphere is compressed towards the planet, however, the planetary contribution to the interior field may dominate \cite{Arridge2008b,Sorba2017}. The magnetopause boundary would then be described by interactions similar to that of the Earth.

In this section, we study magnetopause equilibrium by focusing on the interior contribution of the planetary magnetic field. Further account for internal plasma pressure distribution and more complete interactions will be considered in section 4.3 and studied in chapters 5 and 6.

As discussed in section 1.4.1, the magnetopause boundary is intrinsically gov-
4.2. Magnetopause Definition from Local Pressure Balance

*Magnetopause Definition from Local Pressure Balance*

\[ P_{\text{ext}} = P_{\text{int}}, \quad (4.1) \]

with \( P_{\text{ext}} \) and \( P_{\text{int}} \) being the overall pressure just outside and inside the boundary, respectively. If we choose to first focus on the interactions between the solar wind and the magnetospheric field, Eq. (4.1) can be written as

\[ 2\mu_0 \rho (\mathbf{v} \cdot \hat{n})^2 = ||\hat{n} \times \mathbf{B}_{\text{tot}}||^2, \quad (4.2) \]

where \( \mu_0 \) is the vacuum permeability, \( \rho \) denotes the solar wind mass density, \( \mathbf{v} \) the solar wind velocity vector and \( \hat{n} \) the unit vector locally normal to the surface pointing outwards. On the right hand side, \( \mathbf{B}_{\text{tot}} \) is the total interior magnetic field at the boundary.

A comparison of the validity of this pressure balance equation with the one from Petrinec and Russell (1997) can be found in appendix B. Following the framework developed at Earth by Mead and Beard (1964), the total internal magnetic field \( \mathbf{B}_{\text{tot}} \) may be expressed as

\[ \mathbf{B}_{\text{tot}} = \mathbf{B}_{\text{planet}} + \mathbf{B}_{\text{shield}}, \quad (4.3) \]

where

- \( \mathbf{B}_{\text{planet}} \) is the intrinsic magnetic field of the planet

- \( \mathbf{B}_{\text{shield}} \) denotes the shielding field produced by the magnetopause currents flowing on the surface consistent with the discontinuity of the parallel component of the field at the boundary. It can be expressed as the sum of a surface current planar field \( \mathbf{B}_p \) due to an infinite tangential current sheet, and a curvature field \( \mathbf{B}_c \) due to the curvature of the surface; this leads to

\[ \mathbf{B}_{\text{tot}} = \mathbf{B}_{\text{planet}} + \mathbf{B}_p + \mathbf{B}_c, \quad (4.4) \]

Assuming an infinitesimally thin surface boundary, the field just outside of the mag-
4.2. Magnetopause Definition from Local Pressure Balance

The curvature field $B_c$ adds a significant contribution to the magnetospheric field structure by confining the field lines within the magnetosphere. Its introduction is necessary to obtain a magnetospheric field model consistent with the boundary structure: it will be addressed in Chapter 5. However, its influence on the position of the magnetopause surface has been shown to not be particularly significant (Mead and Beard 1964). For now, we thus consider $B_c = 0$; this remains a reasonable approach since we consider the tangential component of the internal field in the initial pressure balance described by Eq. (4.2). These considerations lead to

$$2\mu_0 \rho (v \cdot \hat{n})^2 = 4 \left\| \hat{n} \times B_{\text{planet}} \right\|^2 ; \quad (4.7)$$

The curvature field can then be considered as

$$B_{\text{out}} = B_{\text{planet}} + B_c - B_p = B_{\text{IMF}}, \quad (4.5)$$

with $B_{\text{IMF}}$ being the interplanetary field. In order for Ampère’s law to be satisfied, the surface current planar field $B_p$ must be uniform on either side of the current sheet layer with opposite directions, hence the ‘minus’ sign in Eq. (4.5). Using Eqs. (4.4–4.5), the total interior field can be expressed as

$$B_{\text{tot}} = 2(B_{\text{planet}} + B_c) - B_{\text{IMF}}. \quad (4.6)$$

This equation differs from Mead and Beard (1964)’s in its inclusion of the interplanetary magnetic field $B_{\text{IMF}}$, as there was a paucity of IMF data in the 1960’s, in the early days of the Mariner program. Assuming a typical value of $B_{\text{IMF}} \approx 1$ nT at Saturn, the contribution of the interplanetary field might initially be considered negligible when compared to the incident solar wind pressure. At points where the boundary is practically tangential to the solar wind flow, however, it may lead to a measurable effect on the shape of the surface. We choose to focus here on the day-side structure of the magnetopause boundary, where the surface intercepts sufficiently large solar wind pressure for us to neglect the IMF contribution to Eq. (4.2).
4.2. Magnetopause Definition from Local Pressure Balance

let us introduce the following scaling factors:

- a magnetic scaling factor used as a unit of magnetic field

\[
b_0 = \sqrt{2\mu_0\rho v^2}; \quad (4.8)
\]

- a unit of distance

\[
r_0 = R_p \left( \frac{2B_{\text{equ}}^2}{\mu_0\rho v^2} \right)^{\frac{1}{6}} = R_p \left( \frac{2B_{\text{equ}}}{b_0} \right)^{\frac{1}{3}}, \quad (4.9)
\]

corresponding to the location of the sub-solar point; this relationship will be further refined in Chapter 5. \(B_{\text{equ}}\) and \(R_p\) denote the planet’s equatorial surface field and radius respectively.

By dividing both sides of Eq. (4.7) by \(b_0\), the balance between the solar wind dynamic pressure and the magnetic pressure at the magnetopause boundary can be described by the dimensionless equation

\[
\left\| \hat{n} \times B_{\text{planet}}^* \right\| - \left( -\frac{1}{2} \hat{n} \cdot \hat{v} \right) = 0, \quad (4.10)
\]

where the hat symbol indicates unit vectors and the asterisk normalised variables. We adopt the coordinate system illustrated in Fig. 3.1, the solar wind velocity vector \(\mathbf{v}\) is chosen to be normal to the planetary magnetic moment, in the opposite direction of the positive Z-axis. The dipole tilt at Saturn and the corresponding seasonal effects on the magnetosphere will be addressed in chapter 6.

From this point on, unless stated otherwise, we will work with the dimensionless pressure balance equation Eq. (4.10) and the hat and asterisk symbols will be omitted. The term on the left of the minus sign corresponds to a scaled magnetic pressure, and the term in brackets is related to a scaled solar wind dynamic pressure. The following subsection describes how this pressure balance equation can be solved numerically using the method of finite differences.
4.2. Magnetopause Definition from Local Pressure Balance

Using the spherical coordinates \((r, \theta, \phi)\) illustrated in Fig. 3.1, the pressure balance described in Eq. (4.10) is discretised on the two dimensional \(\phi \times \theta\) grid shown in Fig. 4.1.

On every point of this grid, we now wish to find the radial distance \(r\) at which the solar wind dynamic pressure balances the planetary magnetic pressure. In particular, if the equilibrium boundary is implicitly defined by a relationship of the form \(r = f(\phi, \theta)\), we are looking for, at every vertex \((\phi_k, \theta_k)\), the radial distance \(r_k = f(\phi_k, \theta_k)\).

The vectors in Eq. (4.10) can be expressed in the same coordinate system:

1. The unit outward vector \(\hat{n}\) locally normal to the surface is a scaled gradient

\[
\hat{n} = \left( \hat{e}_r - \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{e}_\theta - \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \hat{e}_\phi \right) \frac{1}{\|n\|} = \left( \hat{e}_r - \frac{1}{r^2} \frac{\partial f}{\partial \theta} \hat{e}_\theta - \frac{1}{r^2 \sin \theta} \frac{\partial f}{\partial \phi} \hat{e}_\phi \right) \frac{1}{\|n\|},
\]

(4.11)
4.2. Magnetopause Definition from Local Pressure Balance

with \( r^* = f^*(\theta, \phi) = \frac{f(\theta, \phi)}{r_0} = \frac{r}{r_0} \), \( r_0 \) corresponding to the location of the subsolar point defined in Eq. (4.9).

2. \( B^*_{\text{planet}} \) is the scaled magnetic field of the planet; it is modeled by a dipolar field with its magnetic moment aligned with the Y-axis (see Fig. 3.1)

\[
B^*_{\text{planet}} = \frac{B_{\text{planet}}}{B_0} = \frac{M^*}{r^*^3} \left( -2 \sin \theta \sin \phi \, \hat{e}_r + \cos \theta \sin \phi \, \hat{e}_\theta + \cos \phi \, \hat{e}_\phi \right),
\]  

where \( M^* = \frac{B_{\text{equ}}}{B_0} \left( \frac{R_p}{r_0} \right)^3 = \frac{1}{2} \) playing the role of a normalized magnetic moment, \( B_{\text{equ}} \) and \( R_p \) denoting the planet’s equatorial surface field and radius respectively.

3. \( \hat{v} \) is the unit velocity vector of the incoming solar wind plasma stream

\[
\hat{v} = -\hat{e}_z = -\cos \theta \, \hat{e}_r + \sin \theta \, \hat{e}_\theta.
\]

The partial derivatives in Eq. (4.11) can be approximated using finite differences and the method described in section 3.1.2. The evaluation of Eq. (4.10) at each vertex \( k \) of the \( \phi \times \theta \) grid then reformulates the equation into a set of algebraic pressure balance relations (\( \mathcal{P}_k \)), and the magnetopause boundary can be defined as the solution of the vector equation described in Eq. (3.4).

This equation can be solved numerically using various iterative methods; we chose to do so using an implementation of the Levenberg-Marquardt algorithm within the MATLAB package, the details of which are given in section 3.1.3. It requires an initial guess-surface from which to start the optimisation procedure, as well as consistent boundary conditions on the edges of the grid. We explain further how these boundary conditions are determined from considering the pressure balance equation in the equatorial and noon-midnight meridional planes.

4.2.3 Equatorial and Noon-Midnight Magnetopause Profiles

As discussed in section 3.2.1, the equatorial and noon-midnight meridian planes – coloured in orange and green respectively, in Fig. 3.1 – are planes of symmetry for
the magnetopause boundary. This simplifies the expression of the normal vector
given in Eq. (4.11) and turns the Partial Differential Equation of Eq. (4.10) into an
Ordinary Differential Equation of the form
\[ h(\phi, \theta, r, \frac{\partial r}{\partial \theta}) = 0. \tag{4.14} \]

In the equatorial plane defined by \( \phi = 0 \), Eq. (4.14) can be written explicitly as
\[ \frac{\partial r}{\partial \theta} = \frac{r \left( r^6 \cos \theta \sin \theta + \varepsilon \sqrt{r^6 - 1} \right)}{1 - r^6 \sin^2 \theta}. \tag{4.15} \]

where \( \varepsilon \in \{-1; 1\} \). The physically-admissible expression of the derivative is the
one that leads to \( |\hat{n} \cdot \hat{v}| \leq 1 \) close to the sub-solar nose. Making use of Eq. (4.17) and
the flaring expression \( r = r_0 \left( \frac{2}{1 + \cos \theta} \right)^k \) from Shue et al. (1997), a Taylor expansion
of the scaled solar wind pressure close to the Sun-planet direction reads
\[ |\hat{n} \cdot \hat{v}| = \cos \theta + \frac{\sin \theta \frac{\partial r}{\partial \theta}}{r} \bigg|_{\theta = 0} = 1 + \varepsilon \sqrt{\frac{3k}{2}} \theta^2 + O(\theta^2). \tag{4.16} \]

The condition \( |\hat{n} \cdot \hat{v}| \leq 1 \) is satisfied for \( \varepsilon = -1 \), meaning that the pressure balance
relation can be expressed as
\[ \frac{\partial r}{\partial \theta} = \frac{r \left( r^6 \cos \theta \sin \theta - \sqrt{r^6 - 1} \right)}{1 - r^6 \sin^2 \theta}. \tag{4.17} \]

This explicit formulation of the equatorial pressure balance equation can be solved
numerically using the adaptive Dormand-Prince Runge-Kutta method detailed in
sections 3.2.2 and 3.2.3. This integration of Eq. (4.17) from the sub-solar nose
leads to the magnetopause profile shown in Fig. 4.2.

As also mentioned in section 3.2.1, the noon-midnight meridional plane (cor-
responding to \( \phi = 90^\circ \)) is such that the pressure balance relation similarly reduces
to a first-order ordinary equation. Contrary to the equatorial plane however, where
the local magnetic field and the normal vector \( \hat{n} \) were normal at every point, the
angle between the two vectors now varies with $\theta$. This makes it harder to solve for the position of the boundary, compared to the previous equatorial case.

Let us note $\alpha$ the angle between the solar wind flow direction and the vector $\hat{t}$ locally tangential to the surface, such that $\hat{n} \cdot \hat{v} = -\sin \alpha$. Along the Y-axis, for $\theta = 90^\circ$, the pressure balance Eq. (4.2) can be expressed as

$$\rho v^2 \sin^2 \alpha = \frac{1}{2\mu_0} \left(\frac{4M}{r^3} \sin \alpha b_0\right)^2,$$

(4.18)

with $M = \frac{1}{2}$ being the scaled magnetic moment defined in Eq. (4.12), and $b_0$ is the magnetic scale introduced in Eq. (4.8). This leads to, for $0 < \alpha < 90^\circ$,

$$\sin \alpha \left(\frac{4M}{r^3} - 1\right) = 0.$$

(4.19)

This tells us that there are two ways for the magnetopause surface to satisfy the pressure balance along the Y-axis:

- either it crosses the Y-axis with $\alpha = 0$, so that both the external and internal pressure are locally zero. Let $A_1$ be the point where the surface intersects the axis
4.2. Magnetopause Definition from Local Pressure Balance

Figure 4.3: Piece-wise integration of the pressure balance in the noon-midnight meridian plane. The dark-blue curve is the result of integrating $\frac{\partial P}{\partial \theta} = f(\theta, r)$ from the sub-solar nose, with function $f(\theta, r)$ being defined in equation 4.20. The teal curve results from integration $\frac{\partial P}{\partial \theta} = g(\theta, r)$ from point $A_2$ downstream and upstream. $A_1$ and $A_2$ are the intersections of both curves with the Y-axis introduced by Eq. (4.19). $\mathbf{M}$ and $\mathbf{v}$ denote the planetary magnetic moment and solar wind flow respectively; The distances are scaled to the stand-off distance.

in that case;

- or it crosses the Y-axis with a strictly positive inclination $\alpha > 0$, so that the intercepted external dynamic pressure balances the internal magnetic pressure. Eq. (4.19) shows that in that case, the surface must intersect the axis at point $A_2 \left(0, (4M)^{1/3}, 0 \right)$. The position of $A_2$ thus only depends on the magnetic moment $M$, and there is only one orientation of the normal vector $\hat{n}$ at this point which satisfies pressure balance.

This consideration of the pressure balance relation along the Y-axis illustrates the fact that the definition of the magnetopause boundary through Eq. (4.10) is ill-posed. Our equilibrium boundary is in fact defined by satisfying Eq. (4.10) while still intercepting the solar wind flow at every point. Both points $A_1$ and $A_2$ will prove useful to find the unique, physically admissible solution of the pressure balance relation.

Similarly to the equatorial plane, the differential equation describing the posi-
tion of the magnetopause boundary can be rewritten explicitly as

\[
\frac{\partial r}{\partial \theta} = \frac{r \cot \theta \left(2\varepsilon M - r^3\right)}{r^3 + 4\varepsilon M},
\]

(4.20)

with \(\varepsilon \in \{-1;1\}\); let \(f(r, \theta)\) and \(g(r, \theta)\) denote the expressions of the derivative for \(\varepsilon = 1\) and \(\varepsilon = -1\) respectively. The behaviour of these solutions can be studied in the neighbourhood of \(\theta = 0\) and \(\theta = 90^\circ\):

- \(f(r, \theta)\) is such that \(f(r, \theta) \to 0\) as \(\theta \to 90^\circ\), thus corresponding to a surface crossing the Y-axis at \(A_1\).

- Making use of Shue et al. (1997)'s flaring expression \(r = r_0 \left(\frac{2 \cos \theta}{1 + \cos \theta}\right)^\kappa\), a series expansion close to the sub-solar nose reads

\[
f(r, \theta) \to -\kappa \frac{\theta}{4} + \mathcal{O}(\theta^2) \to 0. \quad (4.21)
\]

This is consistent with the conditions at the sub-solar nose.

- Proceeding with another series expansion close to \(\theta = 90^\circ\),

\[
g(r, \theta) = 18 \left(\frac{2}{\kappa}\right)^2 + \mathcal{O}\left(\left(\theta - \frac{\pi}{2}\right)^2\right) \to 0 \quad \text{as} \quad \theta \to 90^\circ \quad C \in \mathbb{R}^+, \quad (4.22)
\]

thus corresponding to a magnetopause crossing the Y-axis at point \(A_2\).

The pressure balance equation can now be solved in the noon-midnight meridian plane using the following method:

- Starting from the sub-solar nose, the derivative \(\frac{\partial r}{\partial \theta} = f(\theta, r)\) is numerically integrated to an arbitrary position on the night side using the methods described in sections 3.2.2 and 3.2.3; this results in the dark-blue curve illustrated in Fig. 4.3. The solution crosses the Y-axis at point \(A_1\) where it is locally tangential to the solar wind flow, as expected from Eq. (4.21).

- From point \(A_2\), the derivative \(\frac{\partial r}{\partial \theta} = g(\theta, r)\) is numerically integrated in two directions – upstream and downstream –, resulting in the teal curve in Fig. 4.3.
4.2. Magnetopause Definition from Local Pressure Balance

Figure 4.4: Cross-section of the magnetopause boundary with the noon-midnight meridian plane. C denotes the polar cusp found at the intersection of both curves illustrated in Fig. 4.3; its position is noted $\theta_C$. $\mathcal{M}$ and $\hat{v}$ denote the planetary magnetic moment and solar wind flow respectively. The distances are scaled to the stand-off distance. In section 6.5.3, the tracing of field lines show that $C$ corresponds to the separatrix between closed and open (or at least directed anti-sunward) field lines.

This solution crosses the Y-axis at $A_2$ with a strictly positive inclination, as expected from relation 4.22.

- The two curves form a structure that includes the physically-admissible magnetopause profile we are looking for. The only way to find a solution that intersects the Z-axis at the stand-off distance while ‘facing’ the incident solar wind flow at every point is to keep the boolean subtraction illustrated in Fig. 4.4. This represents the union of two solutions, subject to the constraint that $\hat{n}$ tilts towards the Sun. A cusp structure is formed at the intersection, denoted $C$; its position is found to be $\theta_C \approx 70.1^\circ$. The nature of this polar cusp can now be understood as a requirement for satisfying a well-posed version of the pressure balance relation in Eq. (4.10).

This preliminary analysis allows us to determine the final cross-sections of the magnetopause with the equatorial and noon-midnight meridional planes – plotted in Fig. 4.2 and Fig. 4.4. These exact solutions are used as Dirichlet boundary
4.3. Dipole-Dominated to Magnetodisk Magnetopause

4.3.1 Initial Guess Surface and Final Solution

We construct the initial guess-surface to optimise by

- Fixing the left boundary $\theta = 0$ of the grid to the position of the nose of the boundary;
- Anchoring the top ($\phi = 90^\circ$) and bottom ($\phi = 0$) edges to the equatorial and noon-midnight profiles determined in section 4.2.3 and shown in Fig. 4.2-4.4;
- Generating elliptical cross-sections in between for fixed values of $\theta$.

This method is illustrated in Fig. 4.5 and results in a quarter of the entire magnetopause boundary. The final surface is obtained making use of the North-South and...
Dawn-Dusk symmetries of the aligned configuration. When the model is refined by introducing a dipole tilt in chapter 6, the North-South symmetry is broken and the optimisation is run on half of the entire surface instead.

In order to assess the efficiency of the procedure, we need to be able to judge how ‘close’ a given surface is from the equilibrium state. We do this by considering the metric defined by the log of the pressure difference $\Delta P$ between the interior and exterior, scaled to the local mean pressure $\langle P \rangle$: at a vertex $k$ of the grid, this relative error from pressure balance then corresponds to

$$\text{Error} (k) := \frac{\Delta P}{\langle P \rangle} (k),$$

(4.23)

where $\Delta P = \frac{1}{2} \left( \mathbf{n} \times \mathbf{B}_{\text{planet}}^* \right)$ denotes the local discretised pressure balance residual of Eq. (4.10); $\langle P \rangle = \frac{1}{2} \left( \mathbf{n} \times \mathbf{B}_{\text{planet}}^* \right) + (\mathbf{n} \cdot \mathbf{v}) \mathbf{P}_{\text{mean}}$ is the local average value of the solar wind and magnetic pressure on each side of the boundary.

The choice of using this estimate to assess the accuracy of the surface differs from previous studies: Mead and Beard (1964), for example, considered how small the exterior field was compared to the dipole field, or how tangential to the surface the interior field became after introduction of the shielding field. The method we have described is extremely useful to visualise how well each section of the surface quantitatively satisfies the pressure balance.

The relative error $\log_{10} |\Delta P|/|\langle P \rangle|$ on the initial guess surface is illustrated in the left panel of Fig. 4.6. By construction, the surface already satisfies the pressure balance to a satisfactory precision around the sub-solar nose area, the equatorial and noon-midnight meridian planes, thanks to the preliminary procedures detailed in section 4.2.3. There are four regions on the day-side in which the initial-guess boundary does not describe the pressure balance accurately, with a relative error close to $\approx 50\%$. This is partly explained by the fact that these regions intercept a comparatively small dynamic pressure from the solar wind flow due to the local curvature of the surface, leading to small values of the mean pressure $\langle P \rangle$. They extend to a value of $\theta$ which corresponds to the position of the cusp in the noon-
Figure 4.6: Relative error from exact pressure balance on the initial guess surface (left) and final boundary (right): the colour-bar indicates the percentage error using the indicator $|\Delta P|/|\langle P \rangle|$ defined in Eq. (4.23). The axes are scaled to the standoff distance $r_0$ defined in Eq. (4.9). The planet is at the origin of the plot, the arrow indicating its magnetic moment. The bottom and top boundaries – equatorial and noon-midnight meridian solutions, respectively – are illustrated by the orange and green dotted curves projected onto the bottom and left planes. On the initial guess-surface, the cusp in the noon-midnight meridian plane leads to a ‘join’ where the initial error peaks at around $\approx 50\%$.

midnight meridian plane.

The final equilibrium boundary, obtained by using this surface as a starting point in the optimisation procedure described in section 3.1.3, is shown in the right panel of Fig. 4.6. We can see that the adjustment of the initial surface was very efficient: at almost every point of the magnetopause boundary, the difference between the solar wind dynamic pressure and the magnetic pressure does not exceed 1% of the local mean pressure. In particular, the method was particularly effective at correcting the region surrounding the cusp, where the relative error was initially quite high. As seen by the contours shown in Fig. 4.7, this region is characterised by a progressive ‘flattening’ of the boundary close to the noon-midnight meridian, culminating at the two cusps in the Northern and Southern hemispheres. The positions of these singular structures are at this point, by construction, symmetrical with respect to the equatorial plane. It will be shown in chapter 6 that significant seasonal North-South asymmetry arises at Saturn, once the dipole tilt is introduced in the model. The amplitude of this asymmetric property peaks at the Summer and
Winter Solstices, and disappears when the dipole becomes normal to the solar wind flow at the Vernal and Autumnal Equinoxes.

The obtained equilibrium boundary can be said to be ‘dipole-dominated’, since Eq. (4.10) only accounts for the dipolar contribution to the magnetospheric field. As discussed previously however, Saturn’s magnetosphere is fed by internal sources of plasma which can significantly impact its shape and position. It is then important to model the contributions of the magnetospheric plasma populations in the pressure balance equation in order to more fully describe the Kronian system. Sub-sections 4.3.2 and 4.3.3 will address the corresponding refinements of the model. Nevertheless, it has been shown that the structure and behaviour of Saturn’s magnetosphere is similar to that of the Earth in a plasma-depleted regime (Sorba et al., 2017, Hardy et al., 2020): the dipole-dominated magnetopause boundary shown in Fig. 4.6 can still be considered to model the Kronian system properly when its magnetosphere is depleted of plasma, making it compressed closer to the planet under nominal solar wind conditions.
4.3. Dipole-Dominated to Magnetodisk Magnetopause

4.3.2 Modelling Plasma Particle Pressure

In order to model in a simple way the effect of hot plasma pressure on the position of the magnetopause surface, Eq. (4.2) can be refined into the following pressure balance relation

$$2\mu_0 \rho (\mathbf{v} \cdot \hat{n})^2 = \|\hat{n} \times \mathbf{B}_{\text{tot}}\|^2 (1 + \beta),$$

where $\beta$ is the local ratio of the hot plasma pressure to magnetic pressure.

This new equation is, in effect, equivalent to a problem governed by the initial pressure balance equation described in Eq. (4.2) where the initial stand-off distance $r_0$ of Eq. (4.9) is replaced by an effective value $(1 + \beta)^{\frac{1}{6}} r_0$. Because the work in the previous section was done with distances being scaled to the position of the sub-solar nose, the final equilibrium magnetopause boundary illustrated in Fig. 4.6 is still applicable to Eq. (4.24).

Under the assumption of a constant value for $\beta$ along the boundary, the hot...
plasma pressure thus contributes to \textit{inflating} the ‘zero-\(\beta\)’ magnetopause surface by a factor of \((1 + \beta)^{\frac{1}{6}}\). Using the initial expression for the stand-off distance \(r_0\) given in Eq. (4.9), this corresponds to a radial displacement \(\Delta r\) of the magnetopause boundary of

\[
\Delta r = r_0 \left( (1 + \beta)^{\frac{1}{6}} - 1 \right) \approx 17.8 R_S \left( (1 + \beta)^{\frac{1}{6}} - 1 \right),
\]

for typical example values of the solar wind dynamic pressure \(P_{SW} = 0.02\) nPa and equatorial field \(B_0 = 20000\) nT for Saturn, \(R_S\) denoting the planetary radius. This qualitative ‘constant-\(\beta\)’ inflation of the boundary \(\Delta r\) is plotted as a function of plasma \(\beta\) in Fig. 4.8. For \(\beta = 3\) for example, the magnetopause boundary would be pushed outwards by around 4.6 planetary radii. These simple considerations produce similar \(\Delta r\) values to the analogous observational and theoretical results of Pilkington et al. (2015) and Sorba et al. (2017) for the Kronian magnetopause, respectively.

In chapter 5, the magnetopause model will be used alongside measurements from the Cassini spacecraft to assess how the size of Saturn’s magnetosphere responds to changes in external and internal drivers. The contribution of the magnetospheric plasma population is then taken into account differently: rather than considering it as a dimensional rescaling factor, we explain how it can be accounted for using scaled, effective values for the solar wind dynamic pressure.

### 4.3.3 Modelling the Magnetodisk Structure

At Saturn and Jupiter, the dynamics of the rapidly rotating magnetospheres are driven by internal sources of plasma from the moons Enceladus and Io. The material ejected by these satellites is partially ionised, accelerated towards co-rotation with the ambient disk plasma and confined towards the rotational equator by the centrifugal force. The resulting plasma sheet and currents distort the field into a ‘disk-like’ structure known as a magnetodisk; it is characterised by field lines being stretched outwards close to the equatorial plane and is supported by an azimuthal ring current. The activity of this ring current is enhanced by a population of hotter plasma originating from the outer magnetosphere (Sergis et al., 2007).
4.3. Dipole-Dominated to Magnetodisk Magnetopause

In order to model the position of an equilibrium magnetopause at Saturn, the local pressure balance thus needs to account for the contribution of the magnetic field produced by this azimuthal ring current. For present purposes, this is done by superposing the dipolar (internal) planetary field and the field produced by a CAN-disk – in reference to Connerney, Acuña, Ness (Connerney et al., 1981, 1983) – to consider the following pressure balance relation

\[
2\mu_0 \rho (\mathbf{v} \cdot \hat{n})^2 = \| \hat{n} \times (B_{\text{planet}} + B_{\text{disk}}) \|^2 (1 + \beta) ,
\]

where \(B_{\text{planet}}\) is the dipolar planetary field and \(B_{\text{disk}}\) describes the contribution of the equatorial ring current, according to the formalism of the CAN model. The disk contribution \(B_{\text{disk}}\) is generated by a current-carrying axi-symmetric torus with a rectangular cross section; its four parameters — the inner and outer radii, the disk half-thickness and the scale for the current density it carries – are chosen to be consistent with the system’s size at Saturn (Bunce et al., 2007).

We can assess the influence of the magnetodisk structure on the shape of the magnetopause boundary by comparing the equilibrium surface obtained using Eq. (4.26), with the dipole-dominated one corresponding to Eq. (4.10).

We can, for example, estimate the ‘polar flattening’ \(F\) of the boundary by comparing the position of the terminator in the noon-midnight meridional plane \(r_{\text{NMM}}\) relative to the one in the equatorial plane \(r_{\text{Equ}}\)

\[
F = \frac{r_{\text{Equ}} - r_{\text{NMM}}}{r_{\text{Equ}}} ,
\]

before and after introducing the ring current model. For a dipole-dominated boundary, the flattening parameter is found to be \(F \approx 6.8\%\). With the introduction of the magnetodisk model, this parameter will now change with system size: for standoff distances of \(20R_S\), \(25R_S\) and \(35R_S\), the flattening parameter increases to \(8.9\%\), \(9.6\%\) and \(12.8\%\) respectively. This shows that the magnetodisk structure at Saturn causes the magnetopause to be more flattened along the North-South direction, compared to a dipole-dominated system. This is consistent with Pilkington et al.
4.4 Summary and Conclusion of this Chapter

We have described a physics-based approach to constructing an equilibrium magnetopause boundary on which the external and internal pressure sources are balanced. We explained how the local pressure balance can be used to define the steady-state magnetopause as the solution of an optimisation problem. We detailed the construction of an initial non-axisymmetric boundary, which is made possible thanks to novel studies of the pressure balance along the equatorial and noon-midnight meridian planes.

A numerical scheme was implemented to correct the initial surface and obtain a final optimised magnetopause surface that satisfies pressure equilibrium with a relative error inferior to 1%. In particular, the final boundary allowed a particularly accurate description of the high-latitude structure close to the polar cusp.

We explained how the method can be refined with the introduction of hot plasma pressure and modelled contributions of azimuthal ring currents. When applied...
plied to Saturn, the model was then shown to be in concordance with results derived from Cassini magnetospheric data. The method opens doors to numerous refinements of the model, such as: the inclusion of an ‘efficiency’ fraction for the IMF contribution to the interior magnetospheric field model (Alexeev and Belenkaya, 2005), the modelling of the magnetopause oscillating effects due to co-rotating partial ring currents evidenced by many Cassini-ENA measurements (Krimigis et al., 2007), and the modification of the plasma β distribution in the model to reflect an additional pressure from the relatively cold, centrifugally confined disk plasma (Sergis et al., 2007; Thomsen et al., 2010). It lends itself particularly well to 3D visualisations of the optimised surface morphology and its accuracy regarding exact pressure balance.

In our underlying considerations of pressure balance at Saturn’s magnetopause, we did not include the effect of magnetic reconnection. Following the Dungey cycle described in section 1.3.3, if the dayside reconnection rate exceeds the one on the nightside, the magnetopause is expected to be ‘eroded’ by the solar wind and compressed towards the planet. However, magnetic reconnection is not very efficient under the typical ~ 90° angle between the IMF and Saturn’s magnetic field (see end of section 1.2.1). Moreover, values of plasma β at the Kronian magnetopause were shown to be such that dayside reconnection only occurred when magnetic fields on either side of the boundary are almost entirely anti-parallel (Masters et al., 2012).

Preliminary applications to estimating magnetopause compressibility are discussed in appendix A with results which seem to illustrate the validity of the magnetopause model. Further applications to computing solar wind pressure estimates – when used alongside observed magnetopause crossings by the Cassini spacecraft – are detailed in chapter 5. It will be shown to be particularly useful in studying how the compressibility of the magnetopause boundary at Saturn evolves with system size.
Chapter 5

Magnetopause Dynamics and Compressibility at Saturn

We explain in this chapter how we use the magnetopause model developed in chapter 4 to study the response of the boundary to changes in solar and magnetospheric conditions at Saturn.

As discussed previously, a steady-state magnetopause corresponds to the region in space where external (i.e. of solar origin) and internal (i.e. of magnetospheric origin) pressure sources are balanced. Under that definition, relative to an arbitrary equilibrium state, the boundary can be expected to be compressed closer to the planet if the upstream solar wind pressure suddenly increases; similarly, the magnetosphere would expand should the solar wind pressure decrease. The amplitude of that response, or the displacement of the magnetopause resulting from a given change in upstream pressure, is often described by the compressibility parameter \( \alpha \). This parameter – defined and discussed in sections 5.3 and 5.4 – is qualitatively larger for ‘rigid’ magnetospheres, and smaller for more ‘elastic’ boundaries; in particular, the compressibility of Earth’s magnetopause is \( \approx 6 \), and that of Jupiter’s is closer to \( \approx 4 \).

Several limitations need to be addressed when studying magnetopause compressibility at Saturn. Firstly, the parameter has been historically introduced to study the magnetopause dynamics at Earth; the Gas Giants differ in that their magnetospheres are continually fed by significant internal sources of plasma. This changes
the structure of the magnetospheric field and impacts the dynamics of the magnetopause. Secondly, the definition of the compressibility parameter stems from a specific empirical, parametrised description of the boundary. There is a thus a need for a physics-based approach to magnetopause compressibility studies, which would be general enough to include the key magnetospheric drivers that govern magnetopause dynamics at the Gas Giants – discussed in sections 1.3.2 and 1.4.1.

This chapter addresses these issues and explain how our magnetopause model can be used alongside observed magnetopause crossings by Cassini to assess the dynamics of Saturn’s magnetopause. Its content is based on the following study:


## 5.1 Introduction to this Study

The boundary separating the internal magnetospheric plasma around a magnetized planet from the external solar wind plasma within the magnetosheath, known as the magnetopause, has been shown to be a highly dynamic system (Kaufmann and Kondrati, 1969; Masters et al., 2011; Escoubet et al., 2013). Its shape and position are the results of complex interactions between external influences (e.g. incident solar wind, IMF) and internal drivers leading to an outward pressure (e.g. magnetospheric magnetic field and plasma population). At the gas giants, the total magnetic field has a ‘disk-like’ structure (Connerney et al., 1981; Arridge et al., 2008b; Achilleos et al., 2010) due to the magnetic contribution of an extensive equatorial ring current fed internally by moon ejecta (Dougherty et al., 2006; Tokar et al., 2006b; Khurana et al., 2007; Jia et al., 2010; Kellett et al., 2010; Bagenal and Delamere, 2011b).

Recent empirical models of the magnetopause at Saturn have shown the dynamical behaviour of its magnetosphere to stand in between the relatively rigid, dipolar case at the Earth and the more elastic, compressible case at Jupiter (Arridge et al., 2006; Kanani et al., 2010; Pilkington et al., 2015; Sorba et al., 2017). Pilkington et al. (2015) have notably illustrated how the internal plasma activity can have a large-scale impact on the position and size of the boundary, and Sorba
et al. (2017) used a 2-D force balance magnetodisk model of the field (Achilleos et al., 2010) to show how the behaviour of Saturn’s magnetosphere seems to tend toward a more rigid configuration in a plasma-depleted regime, and towards a more compressible, Jupiter-like case in a plasma-loaded state. The influence of the hot plasma population on magnetospheric compressibility is still, however, not fully understood. Most studies are either purely empirical or model based, and provide an ‘average’ description of magnetospheric behaviour over very diverse internal and external conditions.

This study provides a physics-based method to determine the compressibility of the magnetopause that accounts for the variability in magnetospheric plasma activity. Section 5.2 will explain how values for the stand-off distance and upstream solar wind pressure are estimated from magnetopause crossing data. In section 5.3, we will explain how the impact of internal drivers can be taken into account in the study of magnetopause compressibility. The method is then applied to estimating the boundary compressibility at Saturn and how it varies with system size in section 5.4.

5.2 Estimating System Size and Solar Wind Pressure

5.2.1 Magnetopause Position and Pressure Balance Equation

Building on the discussions from sections 4.3.2 and 4.3.3, the size and shape of the magnetopause at the gas giants can be estimated, to first order, by solving the pressure balance between external and internal contributions

\[ P_{SW} \cos^2 \psi + P_0 \sin^2 \psi = \frac{B^2}{2\mu_0} (1 + \beta), \]  

(5.1)

where \( P_{SW} \) is the solar wind dynamic pressure, \( \psi \) denotes the angle between the local normal to the magnetopause and the solar wind flow direction; \( \beta \) is the plasma beta corresponding to the ratio of hot plasma pressure to magnetic pressure, and \( B \) is the total magnetic field strength with a magnetodisk structure. In this study, \( B \) is modelled using a magnetic dipole – aligned with the planetary rotation axis – and
an equatorial CAN-disk \cite{Connerney1981, Connerney1983}. $P_0$ denotes the isotropic thermal pressure in the solar wind – assigned a constant value of $10^{-4}$ nPa \cite{Slavin1985} – and the coefficient $\sin^2 \psi$ is introduced to avoid a complex flow velocity in the sub-solar region \cite{Petrinec1997}. It is worth noting that the CAN-disk used here to model the magnetodisk structure of the field was primarily chosen for its simplicity. It assumes a $1/r$ radial profile for the ring current density, which has been proven inaccurate by Cassini plasma and field measurements \cite{Sergeis2010b}. The current field model was still shown to organise the observed crossings fairly well \cite{Hardy2019}, but a more realistic ring current model may be considered in future work.

The numerical solution of Eq. (5.1) can be considered as representing an equilibrium magnetopause with shape and dimensions fixed by two of the three following parameters: the solar wind pressure $P_{SW}$, the plasma $\beta$ accounting for internal plasma activity, and the magnetopause stand-off distance $R_{MP}$; the parameters of the modeled equatorial ring currents – the inner and outer radii, the disk half-thickness and the current parameter $\mu_0 I_0$ depend directly on the system’s size \cite{Bunce2007}.

The dataset used to study the behaviour of the magnetosphere at Saturn consists of 1514 magnetopause crossings of the Cassini spacecraft identified using the on-board MAG (magnetometer) and CAPS-ELS (Electron Spectrometer sensor of the Cassini Plasma Spectrometer) instrument, from October 2004 to February 2013 \cite{Pilkington2015}. The trajectory of the spacecraft during this period was shown to adequately sample the mean position of the boundary, with no bias for extreme magnetospheric configurations \cite{Pilkington2014}. Seasonal distortions of the magnetopause are taken into account using the ‘general deformation method’ \cite{Tsyganenko1998}: the crossing positions are corrected appropriately to model the response of the boundary and current sheet to a dipole tilt with regards to the solar wind flow, observed at Saturn by \cite{Arridge2008b}.

Local values for the magnetic field strength $B$ and plasma $\beta$ were acquired by the spacecraft at each crossing position. In order to determine the corresponding
5.2. Estimating System Size and Solar Wind Pressure

![Illustration of the stand-off distance estimation](image)

**Figure 5.1:** Illustration of the stand-off distance estimation, given a magnetopause crossing $M$. In purple are the intersections of the crossing direction $OM$ with the equilibrium reference surfaces shown in blue; in green are the corresponding positions of the sub-solar nose. The red dashed line illustrates the mapping used to determine the system size corresponding to a crossing $M$. The axes are the ones of the orthogonal, Saturn-centered coordinate system (Kronocentric Solar Magnetospheric frame): $X_{KSM}$ points towards the Sun, and $Z_{KSM}$ is such that the $X_{KSM}−Z_{KSM}$ plane contains Saturn’s magnetic dipole. This figure corresponds to the specific case of a crossing observed in the noon-midnight meridional plane ($Y_{KSM}=0$), but the method is applicable to any crossing position.

Equilibrium solar wind pressure $P_{SW}$, it is necessary to have access to the local geometry of the boundary, as it fixes the angle $\psi$ in Eq. (5.1). The morphology of the magnetopause is itself dependent on the system size, since the equatorial ring current – and consequently the magnetodisk structure of the field – responds to how close the surface is to the planet. It is thus necessary to estimate the stand-off distance corresponding to each observed crossing, before trying to determine values of the solar wind pressure.

### 5.2.2 Magnetopause Crossings and Magnetospheric Scales

We start by solving the pressure balance equation at Saturn in order to determine a set of equilibrium magnetopause models (Hardy et al., 2019) with integer stand-off distances ranging from 15 to 40 Saturn radii ($R_S \approx 60268$ km), each with consistent
plasma disk parameters according to the results of [Bunce et al.] (2007).

The method used to determine the system size is illustrated in Fig. 5.1 in the special case of a crossing M observed in the noon-midnight meridional plane. In the general case, we determine the intersections – shown in purple – between the crossing direction $\vec{OM}$ and the reference surfaces, shown in blue. A spline function is defined to map these intersections with the matching values for the stand-off distance, shown in green along the Sun-planet line. This function is then used to estimate the system size corresponding to the observed crossing position, and the procedure is repeated throughout the entire dataset.

### 5.2.3 From System Size to Solar Wind Pressure

Now that the equilibrium system size has been determined for each crossing, local values of the solar wind pressure can be estimated. The main difficulty of this step lies in the magnetopause geometry depending on the system’s size, and us having access to a finite number – rather than a continuum – of equilibrium surfaces. This was addressed through the following procedure: at each crossing M,

- consider the two equilibrium boundaries whose scales are the closest to the stand-off distance estimate;

- these surfaces are used alongside the spacecraft measurements to solve Eq. (5.1) at M, resulting in two values for $P_{SW}$. The use of the [Hardy et al.] (2019) magnetopause model for this specific purpose is discussed in appendix B;

- the relative position of the sub-solar nose with respect to the scales of each reference surface is used to estimate the solar wind pressure at M as a weighted average.

For example, if the stand-off distance corresponding to a crossing M was found to be $22.3 R_S$, the reference surfaces of scale $22 R_S$ and $23 R_S$ would be used to determine two values for the solar wind pressure, noted $P_{SW, 22}^*$ and $P_{SW, 23}^*$ respectively. The solar wind pressure at M would then be estimated as the weighted sum $P_{SW} = 0.7 P_{SW, 22}^* + 0.3 P_{SW, 23}^*$. 
5.2. Estimating System Size and Solar Wind Pressure

Figure 5.2: Magnetopause crossings of the Saturn-orbiter Cassini spacecraft from October 2004 to February 2013, in red. The solar wind pressure estimates $P_{sw}$ were found using measurements from the spacecraft, the pressure balance described in Eq. (5.1) and a three-dimensional magnetopause model for Saturn (Hardy et al., 2019). The dark contours indicate the values of plasma $\beta$ and $P_{sw}$ required to fix the stand-off distance at specific values, assuming pressure balance. These characteristics are used to cluster the crossings in separate groups, depending on the ones they are the closest to.

Following this procedure, each magnetopause crossing can now be associated to

- An estimate for the solar wind pressure, determined using the method discussed previously; deriving these estimates is necessary in the absence of any nearby upstream solar wind monitor.

- The stand-off distance describing the size of the boundary sampled by the spacecraft (see section 5.2.2 and Fig. 5.1).

- The observed value for plasma $\beta$ – describing the relative importance of energetic particle pressure relative to the magnetic pressure.

The distribution of crossings is visualised in phase space in Fig. A.2: each red dot corresponds to one magnetopause crossing, and the stand-off distance is shown by
the black contours and colour scale. It illustrates the importance of both the solar wind pressure and the plasma $\beta$ parameter in determining the size of the system. In particular, under certain regimes, we see that internal drivers almost fully control the system size. For $P_{SW} \approx 0.4$ nPa for example, we see that the stand-off distance can vary from $\approx 17R_S$ to almost $\approx 26R_S$ under very high variations of plasma $\beta$. This is consistent with observations from Pilkington et al. (2015), where Saturn’s magnetopause has been shown to be displaced by up to $10 – 15$ planetary radii under variations of magnetospheric plasma regimes.

In order to assess the magnetopause compressibility and study the response of the magnetosphere to changes in upstream solar conditions, it thus becomes necessary to account for the variability in local plasma $\beta$. Pilkington et al. (2015) used a K-clustering algorithm to group the crossings into three clusters depending on the values of $\beta$, with a surface model that includes a 19% polar flattening (Pilkington et al., 2014) – the flattening parameter being defined as in Eq. (4.27). Though this method was able to quantify the impact of internal plasma activity on the stand-off distance, it reduced the number of crossings available to study the boundary compressibility $\alpha$ within each cluster. In particular, the uncertainty in $\alpha$ for the high-$\beta$ cluster – i.e. describing a plasma-loaded magnetosphere – was too high to illustrate any definite impact of plasma activity on magnetopause compressibility. We describe in the following section a method that reduces the number of parameters impacting system size, while accounting for the variability in internal plasma activity over the entire dataset.

### 5.3 Compressibility Estimates and Effects of Internal Drivers

#### 5.3.1 Magnetopause Compressibility and Impact of Internal Particle Pressure

Let $R_{MP}$ and $P_{SW}$ denote the stand-off distance and effective solar wind pressure of a magnetospheric state perturbed by a small change in pressure $dP_{SW}$. Assuming a
regime devoid of magnetospheric plasma, the consequent displacement of the subsolar nose \(dR_{MP}\) is assumed to satisfy, to first order,

\[
\frac{dR_{MP}}{R_{MP}} \approx -\frac{1}{\alpha} \frac{dP_{SW}}{P_{SW}},
\]

where \(\alpha\) is the compressibility parameter of the boundary; the larger the value of \(\alpha\), the smaller the impact of a change in pressure on system size, the more ‘rigid’ the magnetopause – and vice versa.

Considering infinitesimal changes in pressure, integrating Eq. (5.2) leads to the linear relationship

\[
\log R_{MP} = -\frac{1}{\alpha} \log P_{SW} + \text{cst},
\]

or the power-law

\[
R_{MP} \propto P_{SW}^{-1/\alpha}.
\]

This expression has been shown to be valid over a wide range of stand-off distance (Bunce et al., 2007; Achilleos et al., 2008), though it is affected by the magnetospheric plasma content. Given a list of crossings with consistent values for the stand-off distance \(R_{MP}\) and solar wind pressure \(P_{SW}\), the compressibility parameter \(\alpha\) could then be inferred semi-empirically from a linear fit of Eq. (5.3). The relationship found between the magnetospheric scales \(R_{MP}\) and the solar wind pressure estimates \(P_{SW}\) is shown in Fig. 5.3a.

The long ‘trailing-off’ of the crossings towards the top-right of the plane illustrates the broad range in both solar wind pressure and plasma \(\beta\), which prevents a direct determination of magnetopause compressibility over the entire dataset. This trend shows again the large impact of internal plasma activity over magnetospheric scales, consistent with previous observations showing that hot plasma dynamics are competitive with solar wind conditions in determining the system’s size (see Fig. A.2 and Pilkington et al. (2015)). This factor needs to be addressed before performing any fit to the data for determining the value of \(\alpha\).
5.3. Compressibility Estimates and Effects of Internal Drivers

![Figure 5.3: Values for the stand-off distance $R_{MP}$ plotted as a function of a) the solar wind pressure estimates $P_{SW}$, and b) the effective solar wind pressure estimates $P_{SW, eff}$ introduced in Eq. (5.6). The color bar indicates the local values of plasma $\beta$. The scaling procedure of section 5.3.2 eliminates the trend evident in figure a), making the values of $\beta$ much more evenly distributed within the cluster in figure b).](image)

### 5.3.2 Dimensionality Reduction and Plasma $\beta$ - Scaling

Let us start by noticing that the term accounting for the static thermal pressure $P_0 \sin^2 \psi$ in Eq. (5.1) only plays an important role at high-latitude positions, close to the cusp on the day-side at a latitude of $\approx 71^\circ$ (Hardy et al., 2019). Since most of our observed crossings of the Cassini spacecraft occurred at low latitudes around Saturn (with a maximum observed latitude of around $62^\circ$ and a median latitude of $\approx 6^\circ$), it is relevant to consider the approximate pressure balance equation

$$P_{SW} \cos^2 \psi = \frac{B^2}{2\mu_0} (1 + \beta).$$

(5.5)

This is equivalent to

$$P_{SW, eff} \cos^2 \psi = \frac{B^2}{2\mu_0} (1 + \beta_{ref}),$$

(5.6)

with $\beta_{ref}$ denoting a prescribed reference value of plasma beta and $P_{SW, eff} = \frac{P_{SW}}{(1+\beta)} (1 + \beta_{ref})$ an effective, scaled solar wind pressure. Thus, considering the effective pressure $P_{SW, eff}$ in place of the pressure estimates $P_{SW}$ allows us to artificially scale all the crossings to a common reference value of plasma beta $\beta_{ref}$. In
other words, under the assumption of pressure balance, \( P_{SW, eff} \) are the values of the external solar wind pressure that we would expect had all crossings been acquired with the same plasma \( \beta \). This consideration can be visualised geometrically in Fig. A.2: replacing \( P_{SW} \) with \( P_{SW, eff} \), corresponds to sliding each red crossing along the \( R_{MP} \) characteristic it lies on until we reach the horizontal line \( \beta = \beta_{ref} \).

Doing so ‘flattens’ the distribution in the vertical direction and eliminates one degree of freedom without losing any information.

For \( \beta_{ref} = 3.58 \) for example – the mean value of plasma \( \beta \) over the dataset – the relationship between the stand-off distance and the effective solar wind pressure is shown in Fig. 5.3b. The colour bar seems to indicate that the trend shown in Fig. 5.3a vanishes, and the crossings appear to cluster much closer to each other, as expected. Choosing any other value for \( \beta_{ref} \) would only displace the cluster horizontally, without disrupting the distribution shown in Fig. 5.3b.

### 5.3.3 Revisiting the Impact of Internal Plasma Pressure on System Size

Another consequence of scaling the solar wind pressure by considering \( P_{SW, eff} \) can be seen in Eq. (5.2): it may be expanded as

\[
\frac{dR_{MP}}{R_{MP}} \approx -\frac{1}{\alpha} \left( \frac{dP_{SW, eff}}{P_{SW, eff}} \right) = -\frac{1}{\alpha} \left( \frac{dP_{SW}}{P_{SW}} \right) + \frac{1}{\alpha} \left( \frac{d\beta}{1+\beta} \right) .
\]

(5.7)

In the context of the Earth’s magnetosphere – which is relatively devoid of plasma at the magnetopause (Shue et al., 1997) –, only the first term of the right hand side of Eq. (5.7) contributes to a displacement of the sub-solar nose. In this case, the magnetic field can be well approximated by a vacuum dipole and the compressibility index is found to be \( \alpha = 6 \).

The additional \( \beta \) term necessary for Saturn and Jupiter shows that an enhancement in internal plasma activity acts towards inflating the magnetosphere (note the plus sign in front) in such a way that a relative change in \( \beta \) has the same impact as a relative change in \( P_{SW} \) if \( \beta \gg 1 \). This is consistent with the large impact of plasma \( \beta \) on system size illustrated in Fig. 5.3 and A.2. Additionally, the compressibility pa-
rameter is expected to be smaller at the gas giants due to the ‘disk-like’ structure of their magnetic fields: ionised moon ejecta are accelerated towards corotation with the rapidly rotating magnetospheres, harbouring an azimuthal ring current that acts towards stretching the field lines radially outwards along the equatorial plane. This would lead to the magnetopause being more compressible when it is expanded (i.e. in a plasma-loaded state), and more similar to the dipole case as it is compressed (i.e. in a plasma-depleted state). This variability of the compressibility with regards to the system size is studied in the following section.

5.4 System Size and Magnetopause Compressibility

5.4.1 Filtering Crossings Far From Pressure Balance

In order to estimate how the magnetopause compressibility at Saturn varies depending on the system size, it is necessary to filter out the crossings that were observed while the magnetopause was not at rest in the rest frame of the planet.

To do so, at each crossing, the solar wind pressure estimates – derived from the data and the reference surfaces – can be compared to the weighted average of the values corresponding to the equilibrium surfaces of similar scales. Fig. 5.4 shows the crossings that remained after eliminating those for which the aforementioned difference in pressure was larger than 40% of the corresponding averaged equilibrium values.

Two observations can be made from Fig. 5.4: the crossings appear not to be distributed along a line, but rather along a slightly convex curve instead; this illustrates the impact of system size on magnetopause compressibility. This feature was previously hidden by the variability in plasma \( \beta \) in Fig. 5.3a, and drowned by the scatter in Fig. 5.3b; it is studied further in the next sub-section. Secondly, there is an apparent ‘flaring’ in the crossing distribution when moving towards the top-left. This could be due to the magnetosphere being less rigid when subjected to changes in solar wind pressure, as the system is expanded: the boundary is then more easily pushed away from pressure balance, and a larger number of observed crossings is thus likely to correspond to an accelerating magnetopause.
5.4. System Size and Magnetopause Compressibility

The impact of system size on magnetopause compressibility can be illustrated by performing two separate linear fits of Eq. (5.3) to the crossing distribution shown in Fig. 5.4.

The magnetopause crossings are chosen to be separated into two subsets: one corresponding to an expanded state ($R_{MP} > 24R_S$) and one corresponding to a compressed state of the magnetosphere ($R_{MP} \leq 24R_S$). In the first case, in a plasma loaded regime (and/or low dynamic pressure regime), the compressibility is found to be $\alpha = 4.51$, with a 95% confidence interval CI$_{95} = [4.31, 4.72]$; in a plasma depleted regime (and/or high dynamic pressure regime), as the boundary is pushed closer the planet, $\alpha = 5.71$, with a 95% confidence interval CI$_{95} = [5.25, 6.25]$ (the

Figure 5.4: Relationship between the stand-off distance $R_{MP}$ and the effective solar wind pressure estimates $P_{SW, eff}$ introduced in Eq. (5.6). The color bar indicates the difference in solar wind pressure $\Delta P_{SW}$ between the estimated value $P_{SW, eff}$ and the reference value from pressure equilibrium $P_{SW, ref}$; crossings with a difference smaller than 40% were kept. The dashed horizontal line indicates $R_{MP} = 24R_S$, the green and orange lines are linear fits of Eq. (5.3) to the remaining crossings, for $R_{MP} \geq 24R_S$ and $R_{MP} \leq 24R_S$ respectively. Considering the entire set of crossings leads to a compressibility $\alpha = 4.17$ and CI$_{95} = [4.08, 4.27]$. However, for $R_{MP} \geq 24R_S$, we find $\alpha = 4.51$ with a 95% confidence interval CI$_{95} = [4.31, 4.72]$; for $R_{MP} \leq 24R_S$, we find $\alpha = 5.71$ and CI$_{95} = [5.25, 6.25]$. 

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statistical interval is given as is, though the compressibility index has a physical upper-bound of 6, corresponding to a vacuum dipole case). Such a ‘bi-modal’ behavior of the magnetopause is consistent with placing Saturn’s magnetosphere in between the Earth’s, where $\alpha \approx 6$, and Jupiter’s, where $\alpha \approx 4$ (Bagenal and Delamere, 2011b). The cut-off value of $24 R_S$ also echoes with previous observation and modeling studies (Arridge et al., 2011; Sorba et al., 2017) in which a shift in behavior related to magnetic field structure was found around $\approx 22 - 25 R_S$.

It is worth noting at this stage that the precise position of this ‘bend’ in the crossing distribution – arbitrarily identified here at $R_{MP} = 24 R_S$ – is of little significance. However, it does qualitatively illustrate how the magnetopause compressibility varies with system size, and thus motivates further study in its response to changes in the position of the magnetopause.

5.4.3 Generalising Magnetopause Compressibility to Account for the Impact of System Size

In the most general case, the response of the system’s size $R_{MP}$ to changes in effective solar wind pressure can be described by an equation of the form

$$\log R_{MP} = -\phi (\log P_{SW, eff}) ,$$

(5.8)

where $R_{MP}$ denotes the magnetopause stand-off distance, $P_{SW, eff}$ the effective solar wind pressure introduced in Eq. (5.6) and $\phi$ a real function monotonically increasing on the domain considered.

The differentiation of Eq. (5.8) leads to

$$\frac{dR_{MP}}{R_{MP}} = -\phi' (\log P_{SW, eff}) \frac{dP_{SW, eff}}{P_{SW, eff}} .$$

(5.9)

Physically, the coefficient $-\phi' (\log P_{SW, eff})$ modulates a relative change in pressure $\frac{dP_{SW, eff}}{P_{SW, eff}}$ that leads to a relative displacement $\frac{dR_{MP}}{R_{MP}}$ of the magnetopause. The com-
pressibility index $\alpha$ can then be defined as

$$\alpha = \frac{1}{\varphi' \left( \log P_{SW, \text{eff}} \right)}.$$  \hfill (5.10)

If the function $\varphi$ is chosen to be a first-degree polynomial, the relationship in $\log P_{SW, \text{eff}}$ described by Eq. (5.8) is equivalent to the case of Eq. (5.3) with a constant compressibility $\alpha$. Because we expect $\alpha$ to vary with system size (as shown in Fig. 5.4), it seems necessary to introduce non-linear terms in the expression of $\varphi$.

In the case where $\varphi$ is defined as a second-degree polynomial

$$\varphi := x \rightarrow a_0 + a_1 x + a_2 x^2,$$  \hfill (5.11)

Eq. (5.10) yields a compressibility of the form

$$\alpha = \frac{1}{a_1 + 2a_2 \log P_{SW, \text{eff}}},$$  \hfill (5.12)

which hints at a hyperbolic expression for $\alpha$.

Let us then generalize this idea one step further by considering the parametric expression

$$\alpha = c_0 + \frac{1}{c_1 + c_2 \log P_{SW, \text{eff}}},$$  \hfill (5.13)

where $c_0$, $c_1$ and $c_2$ are real numbers. The additional parameter $c_0$ introduces a new degree of freedom to allow a vertical translation of the hyperbola.

The following procedure can now be performed:

- Using the expression of the compressibility $\alpha$ from Eq. (5.13), the integration of Eq. (5.10) leads to a functional form for $\varphi$.

- A fit of Eq. (5.8) to the crossing distribution shown in Fig. 5.4 provides the coefficients in the expression for $\varphi$, and thus $\alpha$.

- Using the relationship between the system size $R_{MP}$ and $P_{SW, \text{eff}}$ shown in Fig. 5.4, $\alpha$ can be plotted with respect to $R_{MP}$; this is shown in Fig. 5.5.
5.4. System Size and Magnetopause Compressibility

Figure 5.5: Estimates for the magnetopause compressibility $\alpha$ as a function of system size $R_{MP}$, for each of the magnetopause crossings shown in Fig. 5.4. The green line represents a hyperbolic fit for $\alpha$ as a function of system size (see section 3.1.3 and Eq. (5.14) for more details). The areas shaded in dark and light orange correspond to the variation in $\alpha$ associated with the 1$\sigma$ and 2$\sigma$ confidence bands, respectively. In the background are shown the values of $\alpha$ determined by Pilkington et al. (2015) (in blue, $\alpha = 5.5 \pm 0.2$), Kanani et al. (2010) (in purple, $\alpha = 5.0 \pm 0.8$), Sorba et al. (2017) (in green, $\alpha(R_{MP} < 25R_S) = 4.80 \pm 0.09$, $\alpha(R_{MP} > 25R_S) = 3.53 \pm 0.06$), and Arridge et al. (2006) (in red, $\alpha = 4.3 \pm 0.3$).

This final relationship between $\alpha$ and $R_{MP}$ is further described by fitting a hyperbolic expression of the form

$$\alpha(R_{MP}) = a + \frac{1}{bR_{MP} - d}$$

(5.14)

to the crossing distribution in Fig. 5.5. The coefficients are found to be $a = 3.83$, CI$_{95} = [3.81, 3.86]$, $b = 3.56$, CI$_{95} = [3.06, 4.06]$, $d = 13.34$, CI$_{95} = [12.74, 13.93]$; this fitted curve is plotted in green in Fig. 5.5. In particular, it is found that $\alpha(R_{MP} = 15R_S) = 5.97$ and $\alpha(R_{MP} = 35R_S) = 4.00$, which is consistent with the discussion concluding section 3.1.2. It is worth noting, however, that the uncertainties become relatively large as the system approaches either very compressed or very expanded states; this is mainly due to these extreme states being represented by a relatively
small number of observed crossings.

Fig. 5.5 also shows the values of $\alpha$ previously determined by Arridge et al. (2006), Kanani et al. (2010), Pilkington et al. (2015) and Sorba et al. (2017): within the range of stand-off distances observed at Saturn and the uncertainties cited, each of these values intersect the $2\sigma$ confidence bands shaded in light orange. In the case of Kanani et al. (2010) and Pilkington et al. (2015), we find that previous considerations of the classic linear relationship of Eq. (5.3) may have led to a slight overestimation of the mean magnetopause compressibility. The value and uncertainty for $\alpha$ determined by Arridge et al. (2006) seems to be, on average, in good agreement with our findings. In particular, Sorba et al. (2017) identified a shift in behavior around $25 R_S$, with two distinct values for the compressibility depending on whether the system is more compressed or expanded: interestingly, the value for $\alpha$ that we find at $25 R_S$ is very close to the average of the two values determined by the authors. This seems to show that their bi-modal modeling approach was able to capture the mean response of the system, though the finer behavioral structure evidenced in Fig. 5.5 was lost. For reference, a comparison of the magnetopause profiles – both in the equatorial and noon-midnight meridional planes – from the aforementioned models is shown and discussed in appendix C.

5.5 Summary and Conclusion of this Study

An extensive set of observed magnetopause crossings at Saturn was used to study the response of the planetary magnetosphere to changes in solar wind pressure. Our physics-based three-dimensional magnetopause model that includes an equatorial ring current (dependent on system size) and internal hot plasma particle pressure (with constant plasma $\beta$) was used to estimate magnetosphere scales and local values of solar wind pressure, incorporating magnetic and plasma observations from the Cassini spacecraft.

We described how the observed crossings can be scaled to a common value of plasma $\beta$ in order to account for variable local particle pressure. The compressibility of the magnetopause was studied on two sub-sets of crossings corresponding
to a compressed and expanded system, in order to qualitatively illustrate how the magnetosphere becomes more easily compressible as it expands.

The concept of magnetopause compressibility was further generalised to quantitatively account for its variation with the position of the boundary, and an analytical fit is provided to define it as a function of the stand-off distance. The resulting behaviour predicted for compressibility of the system seems to be consistent with the observed variability in magnetic field structure within Saturn’s inner and outer magnetosphere, and with recent magnetopause modelling studies – both observational and theoretical.
Chapter 6

Modelling Seasonal Variability of Saturn’s Magnetopause

6.1 Introduction to this Study

From its orbital insertion in July 2004 to its September 2017 Grand Finale, the Cassini spacecraft orbited Saturn for a significant portion of a ∼ 29.5 Earth year long Kronian year. As discussed in section 2.1 and illustrated in Fig. 2.1, the consecutive extensions of the mission allowed the exploration of Saturn’s magnetosphere under several seasonal configurations: Northern Winter, Spring and Summer. This is particularly relevant for studies of the Kronian magnetosphere, since the system is expected to undergo significant seasonal variations due to the properties of Saturn’s magnetic field.

In fact, Pioneer 11 measurements identified the planet’s internal magnetic field as an extreme configuration in the obliquity - dipole tilt parameter space (Smith et al., 1980). As described in table 6.1, the planet’s rotational equator is inclined by 26.7° relative to the ecliptic plane: this causes the angular incidence of the solar wind with respect to the rotational equator to vary seasonally over the timescale of a planetary orbital period, with maximum tilts at solstices and aligned configurations at equinoxes. Conversely, the Voyager and Cassini spacecraft confirmed that Saturn’s spin and magnetic axes are almost fully aligned (Smith et al., 1980; Ness et al., 1981, 1982; Davis and Smith, 1990; Cao et al., 2011; Dougherty et al., 2018), which would lead one to expect no diurnal periodicity related to the rotation of the
6.1. Introduction to this Study

<table>
<thead>
<tr>
<th></th>
<th>Earth</th>
<th>Jupiter</th>
<th>Saturn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbital Period (Earth Years)</td>
<td>1.0</td>
<td>11.9</td>
<td>29.5</td>
</tr>
<tr>
<td>Sidereal Day (Hours)</td>
<td>23.9</td>
<td>9.9</td>
<td>10.5</td>
</tr>
<tr>
<td>Obliquity $\Theta$</td>
<td>23.5°</td>
<td>3.1°</td>
<td>26.7°</td>
</tr>
<tr>
<td>Dipole Tilt and Sense</td>
<td>+9.9°</td>
<td>−9.4°</td>
<td>−0.0°</td>
</tr>
</tbody>
</table>

Table 6.1: Configurations of orbits and magnetic fields related to seasonal effects at Earth, Jupiter and Saturn. A sidereal day corresponds to a planet’s rotation period relative to the precessing mean vernal equinox. The obliquity describes the inclination of a planet’s rotational equator relative to the ecliptic plane. The dipole tilt is the angle between the magnetic and rotational equatorial planes; a positive sign is attributed to magnetic fields directed North at the equator. Adapted from Kivelson and Bagenal (2014).

However, additional near-planetary-period oscillations have been observed in Saturn’s magnetic field and plasma parameters (Clarke et al., 2006; Carbary and Mitchell, 2013, and references therein).

Jupiter presents the opposite case with a relatively large dipole tilt (about $-9.9^\circ$) producing diurnal variations larger than the seasonal effects caused by the comparatively small obliquity — about $3.1^\circ$ (Acuna and Ness, 1976). Other magnetic planets describe a broad range in obliquities and dipole tilts, with correspondingly varied seasonal and diurnal periodicities.

Cassini measurements provided evidence of seasonal variations of Saturn’s magnetosphere. In particular, magnetometer data corresponding to Cassini trajectories inside the magnetosphere between July 2004 and April 2006 — corresponding to a Northern Winter — identified reversals of the radial magnetic field consistent with a displacement of the current sheet from the dipole magnetic equator (Arridge et al., 2008a). This current sheet distortion was interpreted as the solar wind forcing a seasonal ‘hinging’ of the magnetic equator through its interaction with the tilted magnetosphere. In the extreme Solstice configuration, the compression of the magnetosphere leads to a normal stress acting perpendicular to the current sheet, thus displacing it Northwards of the dipole magnetic equator on the dayside. Similarly, a Northern Summer was expected to correspond to a Southwards displacement, with this distortion of the current sheet disappearing at the Vernal and Autumnal Equinoxes. However, the limited MAG data restrained the observations and
modelling work to a Northern Winter configuration. Passes of Cassini through the equatorial plane in the post- and pre-midnight sectors in 2007 and 2009 were used to show that the current sheet also exhibits a highly dynamical behaviour (see Fig. 2.8) and that there is a clear change in the tilt and latitudinal offset of the plasma sheet on the nightside, with a progressive alignment with the dipole equator towards Vernal Equinox (Sergis et al., 2011). Composite images of hydrogen and oxygen ENA obtained by the MIMI-INCA instrument (see section 2.2.3.3) between 2005 and 2015 later confirmed the seasonal ‘bowl-shape’ structure of the plasma sheet: concave Northwards near Saturn’s Northern Winter Solstice, flat near Equinox, and concave Southwards near Northern Summer (Carbary and Mitchell, 2016). The presence of a distorted current sheet at Saturn has significant implications on interactions between the planetary magnetosphere and its moons. Titan’s interactions, for example, cannot be fully described under the assumption that the moon always lies in the magnetic equator, but needs to account for the seasonal effects of a lobe-type magnetospheric magnetic field with low plasma $\beta$ instead (Arridge et al., 2008a). Additionally, the warped structure of the current sheet has a significant impact on reconnection events at the tail and associated plasmoid structure (Jia et al., 2012a).

However, a robust physical modelling of seasonal effects on planetary magnetospheres is notoriously difficult. Key aspects of the Kronian system, such as observed periodic variations of magnetospheric field and particle properties (Carbary and Mitchell, 2013), have been purposely modelled using data acquired close to Saturn Equinox to minimise the effects of planetary seasons (Sorba et al., 2018). This difficulty can be similarly sidestepped by focusing on observations where the seasonal effects are expected to be small, as was done in Arridge et al. (2006) to study the response of Saturn’s magnetopause to variable solar wind conditions. The dynamics and configuration of Saturn’s magnetosphere have also been described by global MHD simulations relying on a solar wind flow orthogonal to the planet’s rotation axis (Jia et al., 2012a). The dipole obliquity was introduced in Hansen et al. (2005), where MHD simulations have detected bow shock and magnetopause asymmetries. In particular, the magnetotail was found to be hinged near Titan’s
orbit. The previously-discussed characteristic dayside hinging of Saturn’s magnetosphere — also observed at Jupiter where it is understood to be a centrifugal effect \cite{Bridge1979, Khurana1989} — evidenced by Cassini MAG measurements \cite{Arridge2008} were used to develop an empirical ‘hinged’ magnetospheric model. This study aims at illustrating this effect at Saturn through a physics-based modelling approach.

The magnetopause model we developed in chapter 4 at Saturn, based on Mead and Beard’s \cite{Mead1964} modelling of Earth’s magnetosphere, assumes an alignment of the solar wind flow with both the magnetic and rotational equators (which for Saturn are practically coincident). In chapter 5 the positions of the observed magnetopause crossings were adjusted to account for seasonal distortions of the boundary using the ‘general deformation method’ \cite{Tsyganenko1998}. In order to study the seasonal variations of magnetopause and internal magnetic field structure, the 26.7° obliquity of Saturn needs to be taken into account. This chapter describes how such refinements of the Hardy et al. \cite{Hardy2019} model can be developed. In section 6.2 we discuss how the dipole obliquity is parametrised and included in the magnetopause model. Section 6.3 studies the resulting seasonal displacement of the nose of the boundary, and the corresponding overall structural variation of the magnetopause is determined in section 6.4. Section 6.5 finally discusses the corresponding adjustments of the interior magnetic field by considering the effects of magnetopause currents.

\section{Modelling Seasonal Effects}

Let us consider a planet at the centre of a coordinate system where the X-axis points towards the Sun, the Z-axis points northward in such a way that the planet’s magnetic axis lies in the XZ plane, and the Y-axis completes the description.

In the specific case where the magnetic dipole is aligned with the OZ axis, the system is symmetric with respect to the XZ (noon-midnight meridional plane) and XY (magnetic equator) planes. Assuming the shape of the magnetosphere is entirely described by the interactions between the incoming solar wind and the mag-
netospheric field, this would result in a magnetopause presenting both North-South and Dawn-Dusk symmetries, as is the case for the model developed in chapter 4.

Let $X_0$, $Y_0$ and $Z_0$ designate the axes in this aligned configuration.

In order to describe the seasonal variations of magnetic configurations, we need to introduce two independent angles, shown in Fig. 6.1:

- The dipole obliquity $\Theta$: the angle between the $Z_0$ axis and the magnetic dipole of the planet. If $\Theta = 0$, the system is aligned as described above; at Saturn, $\Theta \sim 26.7^\circ$ due to the planet’s obliquity and the near-perfect alignment of its magnetic and rotation axes (see table 6.1).

- The azimuthal angle $\Phi$ between the $X_0Z_0$ plane and the projection of the magnetic moment. At Saturn, $\Phi = 0^\circ, 90^\circ, 180^\circ, 270^\circ$ correspond respectively to the Northern Summer Solstice, Autumnal Equinox, Northern Winter Solstice and the Vernal Equinox.

The size and shape of the magnetosphere can be considered to be governed, to first approximation, by the interactions between the solar wind, with normalised velocity

$$v_{sw} = -e_x, 0,$$  \hspace{1cm} (6.1)

and a planetary magnetic field with a modelled magnetodisk structure, as detailed
in section 4.3.3

\[ B = B_{\text{Dip}} + B_{\text{Disk}}, \]

(6.2)

where \( B_{\text{Dip}} \) is the field of a tilted dipole

\[ B_{\text{Dip}} = \frac{3 (M \cdot e_r) e_r - M}{r^3}, \]

(6.3)

where \( e_r \) denotes the unit radial vector pointing towards the point considered, \( r \) the distance from the planet to that point, \( e_{r,0} \) the unit vector pointing towards the Sun, and \( M \) the rotated magnetic dipole, given by

\[
M = \begin{pmatrix}
\cos \Phi & -\sin \Phi & 0 \\
\sin \Phi & \cos \Phi & 0 \\
0 & 0 & 1
\end{pmatrix} \cdot \begin{pmatrix}
\cos \Theta & 0 & \sin \Theta \\
0 & 1 & 0 \\
-\sin \Theta & 0 & \cos \Theta
\end{pmatrix} (+M e_{r,0}) = +M \begin{pmatrix}
\cos \Phi \sin \Theta \\
\sin \Phi \sin \Theta \\
\cos \Theta
\end{pmatrix};
\]

(6.4)

Note that if \( \Theta = 0 \), we recover the aligned configuration from section 4.3.3. The \( B_{\text{Disk}} \) component of the magnetodisk field in Eq. (6.2) is initially modelled by an axi-symmetric ring current oriented about the rotational equator. In chapter 4, we chose to adopt the convention of orienting the magnetic moment such that it has a negative component along \( e_{r,0} \) (e.g. in Fig. 4.6), as was done originally by Mead and Beard (1964) at Earth. This had no consequence on the pressure balance equation, since the magnetic moment was aligned with the normal to the ecliptic. As we are now modelling the seasonal variations of the magnetopause at Saturn — a planet where the magnetic North pole lies in the Northern hemisphere — we adopt hereafter the Kronian configuration of a magnetic moment pointing Northwards (illustrated in Fig. 6.1).

The balance between the incident solar wind dynamic pressure and the magnetic pressure due to the field around the planet reads

\[ \frac{1}{2} (v_{\text{sw}} \cdot n) + \sqrt{1 + \beta} |B \times n| = 0, \]

(6.5)

with \( n \) denoting the vector locally normal to the magnetopause surface, and \( \beta \) the
plasma beta corresponding to the ratio of hot plasma pressure to magnetic pressure just inside the magnetopause. The cross-product $|\mathbf{B} \times \mathbf{n}|$ is used to evaluate the contribution of the tangential component of the field to the pressure balance equation, since the equilibrium boundary is expected to describe a tangential discontinuity in the magnetic field.

6.3 Position of the Sub-Solar Point

6.3.1 Sub-Solar Nose of the Magnetopause

Before inferring the structure of the magnetopause from Eq. (6.5), we make the distinction between two points:

- The point on the magnetopause boundary which lies in the direction of the Sun, i.e. on the $X_0$-axis. This point is labelled $P$ in Fig. 6.2.

- The nose of the magnetopause surface: the point of the boundary which is the closest to the Sun. If we consider the flow of solar wind particles towards the planet, it corresponds to a stagnation point at which the flow velocity is zero. This is the point labelled $N$ in Fig. 6.2.

In the assumption of an aligned dipole (i.e. $\Theta = 0$), these two points are the same; in a tilted case, a North-South asymmetry arises and the position of the nose follows the geometry of the field. In this general case, the nose of the magnetopause then lies in the direction where the orientation of the magnetic field allows the full normal flow to be balanced by the local value of the field, and for which the point is the closest to the Sun.

In other words, the nose $N$ is the point at which

1. $\mathbf{n}(N) = \mathbf{e}_{r,0}$: the solar wind incidence is normal to the surface,

2. The projection on the Sun-planet line $X_0 = r \mathbf{e}_r \cdot \mathbf{e}_{r,0}$ is maximised,

3. Eq. (6.5) is satisfied.
6.3. Position of the Sub-Solar Point

Figure 6.2: Cartoon illustrating the difference between the point P and the nose N of the magnetopause shown in green, in a Northern Summer configuration ($\Phi = 0$). The nose N is found by spanning a range of values for $\alpha$, solving for the radial distance $r$ assuming pressure balance and a normal solar wind incidence, and choosing the point for which the projection $X$ on the Sun-planet axis is the largest.

6.3.2 Position of the Sub-Solar Nose

The location of the nose $N$ of the magnetopause can then be determined through the following procedure:

- We define a list of angles $\alpha$ relative to the axis $X_0$, e.g. $[-30^\circ; 30^\circ]$ in one-degree increments. The amplitude of this range was chosen as such, because the angular offset of the nose was a priori expected to be inferior to the planet’s $26.7^\circ$ obliquity.

- For each angle $\alpha$ of the list, we solve Eq. (6.5) in the plane containing the Sun-planet axis and the magnetic moment to determine the radial distance $r$, assuming $n = e_{x,0}$.

- We search for the value of $\alpha$ at which the projection on the Sun-planet line is maximised, i.e. $\text{argmax}_\alpha \{ X_0(\alpha) = r(\alpha) \cos \alpha \}$; this is shown in Fig. 6.3. This angle describes the point on the magnetopause that is the closest to the Sun, which corresponds to the nose $N$ of the boundary.
6.3. Position of the Sub-Solar Point

Figure 6.3: Determination of the angular offset $\alpha$ of the nose $N$ relative to the Sun-planet line. Grey curves correspond to an aligned dipole (i.e. $\Theta = 0$), purple curves correspond to a tilted dipole ($\Theta = 27^\circ$, $\Phi = 0$). The pressure balance is solved for different values of $\alpha$, assuming normal incidence at the nose; the corresponding projections of the nose location on the Sun-planet line $X_0$ are shown in full-lines. The value of $\alpha$ corresponding to a maximum for $X_0$ (shown by a dot and vertical line) is the point of the magnetopause closest to the Sun, i.e. the nose $N$ shown in Fig. 6.2. The dashed curves show the results obtained by maximising the radial distance $r$ instead of the projection $X_0$. A negative value for $\alpha$ describes an offset towards the Southern hemisphere, and vice-versa.

The position of the nose $N$ of the magnetopause is thus fully determined:

- By symmetry, it lies along the plane spanned by $e_{x,0}$ — the direction of the Sun — and $M$ — the planetary magnetic moment.

- It is offset from the $X_0$ axis by $\alpha_{\text{max}} = \arg\max_{\alpha} \{X_0(\alpha) = r(\alpha) \cos \alpha\}$. This angle corresponds to the vertical dashed line marking the maximum of the solid curve, in Fig. 6.3.

- Its radial distance from the planet is $r = X_0 / \cos \alpha_{\text{max}}$.

6.3.3 Seasonal Displacement of the Nose

As shown by the grey curves in Fig. 6.3, the nose $N$ and sub-solar point $P$ collapse onto the same position along the Sun-planet axis in the case of an aligned dipole. When considering the $26.7^\circ$ tilt at Saturn, one can now expect seasonal displacements of the nose. During the Northern Summer and Winter, the absolute value of the projected magnetic moment along the solar wind flow direction is maximised; one can then predict the nose to be periodically displaced vertically in between its
6.4 Seasonal Variations of the Magnetopause

Following the procedure described in section 6.3.1, the position of the nose is determined for different values of $\Phi$ at Saturn and shown in Fig. 6.4, with the effective solar wind pressure fixed at $P_{\text{SW, eff}} = 0.01 \text{ nPa}$: from the Sun, throughout a Kronian year, the nose is found to trace a ‘figure-eight’ locus spanning $\sim 14R_S$ vertically and $\sim 4R_S$ horizontally. At the Vernal and Autumnal Equinox — $\Phi = 270^\circ$ and $\Phi = 90^\circ$ respectively — the nose lies on the Sun-Planet axis; this is consistent with the magnetic dipole lying in a plane normal to the solar wind flow direction in these two configurations. The ‘figure-eight’ locus also describes a slight radial displacement of the nose of the order of $\sim 1 - 2R_S$, as shown in the right panels of Fig. 6.4. The determination of this figure-eight will allow us to study the seasonal variations of the entire magnetopause, by ‘anchoring’ a ‘first-guess’ surface at the nose consistently with chosen values of $\Phi$. This is of particular importance in the context of the Cassini mission, since the Saturn Orbit Insertion manoeuvre occurred in July 2004 and the mission ended in September 2017, spanning part of Northern Winter (Solstice on October 2002), Spring (Equinox on August 2009) and Northern Summer (Solstice on May 2017) at Saturn.

6.4 Seasonal Variations of the Magnetopause

6.4.1 Noon-Midnight Meridional Profiles

Given a value for the seasonal azimuthal angle $\Phi$, the position of the nose is determined along the ‘figure-eight’ locus (see Fig. 6.4), and a guess-surface can be built by anchoring the equilibrium boundary obtained in the case of an aligned dipole in section 4.3.3 at this point. This starting surface is then optimised towards pressure balance by the iterative numerical method detailed in section 3.1.3.

Because the nose is now potentially offset from the Sun-planet axis, there is a need for careful transformations between three coordinate systems:

- **The Aligned System**: where the X-axis points towards the Sun, the Z-axis is normal to ecliptic pointing North, and the Y-axis completes the frame. It corresponds to the frame $(X_0, Y_0, Z_0)$ shown in Fig. 6.1.
Figure 6.4: Seasonal displacement of the nose of the magnetopause at Saturn. The colour-bar indicates values of $\Phi$ in degrees, as shown in Fig. 6.1, in particular $\Phi = 0$ corresponds to a Northern Summer (red circle, e.g. on May 2017), $\Phi = 180^\circ$ to a Northern Winter (grey circle, e.g. on October 2002), $\Phi = 90^\circ$ to an Autumnal Equinox (orange circle, e.g. May 2025) and $\Phi = 270^\circ$ to a Vernal Equinox (green circle, e.g. August 2009). Black arrows trace the seasonal displacement of the nose along the locus in the ZY plane (left panel), XZ plane (top right panel) and XY plane (bottom right). The effective solar wind pressure was fixed at 0.01 nPa.

- **The KSM (Kronocentric Solar Magnetospheric) system**: where the X-axis points towards the Sun, the Z-axis is now such that the planetary magnetic moment is contained in the XZ plane, and the Y-axis completes the frame.

- **The Nose-Dependent Frame**: in which the X-axis points towards the nose of the boundary, the magnetic moment lies in the XZ plane, and the Y-axis completes the frame. In this system, if $(r, \theta, \phi)$ denote the spherical coordinates corresponding to $(Y,Z,X)$, the nose is positioned at $\theta = 0$, and the noon-midnight meridional plane is defined by $\phi = \pm \frac{\pi}{2}$, analogous to the case of section 3.1.1.

From the determination of the position of the nose $N$, it is possible to express
6.4. Seasonal Variations of the Magnetopause

**Figure 6.5:** The left panel shows profiles of the magnetopause boundary in the noon-midnight meridional plane during a Northern Summer at Saturn ($\Phi = 0$), coloured in green. Equatorial cuts are overlaid in orange for the sake of comparison. The nose $N$ is shown in red, the black dotted line indicates the direction of the Sun, the red arrow represents the planetary magnetic moment and the model for equatorial ring currents is shown in blue. The asymmetrical positions of the cusps in the Northern and Southern hemispheres are indicated by green dashed lines. The right panel traces the seasonal positions of the nose $N$ (inner locus shown in Fig. 6.4) and its projection $P_N$ on the dipole magnetic equatorial plane, viewed from the Sun. Note that the ‘nose-dependent coordinates’ are used in the left panel, whereas the ‘aligned coordinates’ are adopted in the right panel (see section 6.4.1).

A basis of the Nose-Dependent frame $(e_{x,\text{nose}}, e_{y,\text{nose}}, e_{z,\text{nose}})$ into the KSM system

\[
e_{x,\text{nose}} = \frac{\mathbf{O}N}{ON},
\]

\[
e_{z,\text{nose}} \propto (e_{x,\text{nose}} \cdot \mathbf{M}) e_{x,\text{nose}} - \mathbf{M},
\]

\[
e_{y,\text{nose}} = e_{z,\text{nose}} \times e_{x,\text{nose}},
\]

and the transformation matrix is then

\[
P = [e_{x,\text{nose}}, e_{y,\text{nose}}, e_{z,\text{nose}}].
\]

Fig. 6.5 shows cuts of the final equilibrium boundary in the noon-midnight
meridional plane, for a Northern Summer at Saturn ($\Theta = 27^\circ$, $\Phi = 0$). The figure is set in the Nose-Dependent Frame, hence the nose in red lying on the X-axis; the direction of the Sun is indicated by a black dotted line. In green full lines are shown the profiles of the magnetopause in the Northern and Southern hemispheres; the equatorial profile ($\phi = 0$ or $\pi$) is shown in orange. The $\sim 6 R_S$ vertical displacement of the nose is consistent with the 'figure-eight' locus shown in Fig. 6.4. The inclination of the green profiles at the nose are also consistent with a normal to the surface pointing towards the Sun. The configuration of the magnetodisk model leads to a clear North-South asymmetry — both in the flattening of the magnetosphere compared to the equatorial profiles, and in the angular positions of the cusps, as shown by the green dashed lines. The cusp in the Northern hemisphere is displaced closer to the nose, and the Southern cusp trails closer to the terminator. Similarly to the nose, these two points will follow a periodic movement throughout a Kronian year, in between their Summer and Winter positions.

These seasonal displacements of the cusps are quantitatively consistent with preliminary studies made by [Maurice et al. (1996)] to model anticipated conditions relevant to the Cassini mission: it was modelled that in the hemisphere where the solar wind flow is more normal to the boundary, the cusp would be shifted closer to the nose as the angle between the solar wind incidence and the sun-planet axis increases. This can be seen in the Northern Hemisphere in Fig. 6.5, with the position of the cusp indicated by a green dashed line. In the [Maurice et al. (1996)] model however, the cusp in the second hemisphere rapidly fades and disappears; we find that it is displaced closer to the terminator instead, as shown in the Southern hemisphere in Fig. 6.5.

This movement of the cusps can be qualitatively visualised considering the incoming flow of solar wind plasma. In the context of this model, the nose of the boundary can be understood as a stationary point: the particles reach the magnetopause at a normal incidence and are stopped at this point. The particles flowing in the Northern and Southern hemispheres both travel in adverse pressure gradients, since the magnetic pressure increases in the direction of the flow. As they travel
closer to the terminator, they eventually reach a point where the magnetic pressure causes the flow to separate. In the case of a Northern Summer, the flow separation occurs closer to the nose in the Northern hemisphere, since the configuration of the field is such that the adverse magnetic pressure gradient is larger than in the Southern hemisphere.

The seasonal displacements of the nose, cusps, current sheet and overall magnetopause can at this point be visualised for any value of $\Phi$. This opens doors to a potentially powerful approach for monitoring the boundary throughout the Cassini mission: magnetopause crossings can now be assessed alongside the magnetopause surface corresponding to the configuration under which they were observed. The method could be particularly helpful in estimating values for solar wind pressure in the absence of solar wind monitors, since estimates are sensitive to the local inclination of the boundary.

However, only a finite number of equilibrium boundaries can be inferred from a discrete set of $\Phi$ values. Ideally, we would like to be able to determine the magnetopause surface corresponding to any value of $\Phi$ within a continuous range (e.g. $0 \leq \Phi \leq 180^\circ$). To this end, we turn to three-dimensional interpolation techniques.
6.4.2 Modelling Continuous Seasonal Variations

As described in section 3.1.1 and Fig. 4.1, at any point of the numerical optimisation of the guess-surface, the magnetopause is stored as a $\phi \times \theta$ grid containing values of radial distance $r$ at each vertex. Let $MP(\Phi_k)$ denote the magnetopause grid corresponding to the azimuthal seasonal angle $\Phi_k$. Fig. 6.6 illustrates the implementation of a 3D interpolation method: a finite number of magnetopause grids are computed, e.g. for $\Phi$ spanning $[0; 180^\circ]$ in $10^\circ$ increments. These grids are arranged in ascending order of $\Phi$ and are stacked on top of each other. If we are interested in the magnetopause structure corresponding to an intermediary value of $\Phi$ (as shown by the green plane in Fig. 6.6), a two-dimensional cubic-spline interpolation method is used to interpolate any vertex of the green plane using the ones from the neighbouring reference grids vertically aligned.

This way, a finite number of reference boundaries can be computed beforehand, and there is only need for the interpolated function to be evaluated for any value of $\Phi$. For each interpolated seasonal configuration of the magnetopause, the corresponding internal magnetic field can be further investigated by modelling the effects of magnetopause currents.

6.5 Contribution of Magnetopause Currents

6.5.1 Magnetopause Currents and Shielding Field

As discussed in section 1.1.1, the charged particles of the incoming solar wind plasma reaching the magnetopause interact with the magnetospheric field: they flow into regions of increasing magnetic field and are forced to return to the magnetosheath after half a gyration (Kivelson et al., 1996). As protons and electrons gyrate in opposite directions around the field, a charge separation occurs and generates a current system flowing along the boundary, known as ‘magnetopause currents’, or ‘Chapman-Ferraro’ currents (Chapman and Ferraro, 1930).

Considering a perfect shielding of the field at the magnetopause, the tangential
discontinuity at a point P of the boundary can be expressed as

\[ \mu_0 j_s(P) = n(P) \times (B_{\text{ext}}(P) - B_{\text{int}}(P)) , \]

(6.10)

with \( \mu_0 \) denoting the permeability of free space, \( j_s(P) \) the magnetopause current surface density at P, \( n(P) \) the unit vector locally normal to the boundary pointing outwards; \( B_{\text{ext}}(P) = 0 \) is the magnetic field just outside of the magnetosphere and \( B_{\text{int}}(P) \) twice the magnetospheric field at point P.

These magnetopause currents flow along the boundary in the dusk to dawn direction, forming closed-loops on the dayside and flowing around the Northern and Southern cusps identified in Fig. 6.5. Their contributions \( B_{\text{shield}} \) to the overall interior field — known as the ‘shielding field’ — can be found by integrating over the entire magnetopause surface \( MP \)

\[ B_{\text{shield}}(M) = \frac{\mu_0}{4\pi} \int \int_{P \in MP} j_s(P) dS(P) \times \frac{\vec{PM}}{PM^3} , \]

(6.11)

where \( j_s(P) \) is the current density mentioned in Eq. (6.10) and \( dS(P) \) an infinitesimal surface element around point P.

To a first approximation, this shielding field confines the magnetospheric field lines within the magnetosphere by acting towards eliminating the normal component of the interior field on the magnetopause boundary. As such, its inclusion in our modelling of the magnetospheric field is valuable: though its contribution can be expected to result in a relatively small displacement of the boundary itself (Mead and Beard, 1964), it is necessary for modelling the overall field within it. Before discussing its technical implementation in our model, it is useful to first qualitatively appreciate its effect on the field structure for a tilted magnetosphere.

### 6.5.2 Internal Magnetic Field Structure

Let us first focus on the simplified case of an aligned system with \( \Theta = 0 \); the magnetic equator then lies in the \( X_0Y_0 \) plane (see Fig. 6.1). The effect of the shielding field on the magnetospheric field structure can be qualitatively visualised by consid-
Figure 6.7: Illustrations of the effect of the magnetopause currents on the magnetodisk field structure. The left panel illustrates an aligned configuration ($\Theta = 0$) and the right panel describes a Northern Summer at Saturn. The magnetopause boundary is sketched in green, field lines of the magnetodisk field are drawn in grey, and the shielding field contributions from orthogonal projections onto the boundary are shown by green arrows. $M_0$ lies along the rotational equator, $M_1$ and $M_2$ are placed symmetrically below and above; orange arrows represent vectors locally normal to the magnetopause surface.

Figure 6.7: Illustrations of the effect of the magnetopause currents on the magnetodisk field structure. The left panel illustrates an aligned configuration ($\Theta = 0$) and the right panel describes a Northern Summer at Saturn. The magnetopause boundary is sketched in green, field lines of the magnetodisk field are drawn in grey, and the shielding field contributions from orthogonal projections onto the boundary are shown by green arrows. $M_0$ lies along the rotational equator, $M_1$ and $M_2$ are placed symmetrically below and above; orange arrows represent vectors locally normal to the magnetopause surface.

The surface elements of the boundary which contribute the most to the shielding field at each of these points are their orthogonal projections onto the magnetopause surface. At $M_0$, this contribution is oriented normal to the rotational equator, in the same direction as the local magnetodisk contribution, as shown by the grey field lines sketched in Fig. 6.7. This means that the total field is strengthened along the equatorial plane, but remains locally normal to it. The field lines are thus compressed closer to the planet, while the position of the magnetic equator does not change. At points $M_1$ and $M_2$, the added shielding field increases the vertical component of the local field by the same amount: this compresses the field lines in both hemispheres symmetrically.

This symmetry of the contribution of the magnetopause currents to the magnetospheric field is broken when the magnetodisk model is tilted. The right panel of Fig. 6.7 illustrates a Northern Summer configuration at Saturn. At a point $M_0$ along the tilted rotational equator, the contribution of the shielding field introduces
a tangential component to the local field. The magnetic equator is thus displaced Northwards, away from \( M_0 \). Due to the North-South asymmetry of the boundary shown in Fig. 6.4.1 the contributions of the shielding field at \( M_1 \) and \( M_2 \) are now also asymmetric with respect to the rotational equator: the distortion of the illustrated field lines is expected to be greater in the Northern hemisphere than in the Southern hemisphere. Moreover, the relative effect of the magnetopause currents on the interior field is larger in the outer magnetosphere, further from the planet. The displacement of the magnetic equator thus becomes correspondingly more important as the distance to the planet increases: one would expect the magnetic equator — and hence the equatorial current sheet — to curve Southwards (or Northwards for a Northern Winter configuration, similarly). This curvature of the current sheet has indeed been observed at Saturn by Arridge et al. (2008b): as a result, the azimuthally averaged current sheet geometry has been described as ‘bowl-shaped’, and an empirical hinged model of the current sheet has been shown to be in good agreement with Cassini MAG measurements. The introduction of the shielding field in our boundary calculations would thus offer a physics-based model and interpretation of dayside internal magnetic field structure.

### 6.5.3 Shielding Field and Current Sheet Distortion

Since their introduction by Chapman and Ferraro (1930), various methods have been presented to determine magnetopause current distributions and their effect on the magnetospheric magnetic field, particularly for the case of the terrestrial magnetosphere. In particular, analytical solutions were presented for particular magnetopause models and magnetic field sources (Mead [1964] Tsyganenko [1989]) and techniques were derived to minimise the mean value of the normal field component along the boundary for arbitrary magnetopause structures (Schatten et al. [1969]). These methods were applied to several empirical models at Earth (Tsyganenko 2002, 2013).

In order to model the magnetospheric field at Saturn, we start with our tilted equilibrium magnetopause obtained as described in section 6.4. This surface is the solution of Eq. (6.5), with the interior field initially described by a size-dependent
magnetodisk structure (Bunce et al., 2007) mentioned in Eq. (6.2).

The next step is to derive the magnetopause current system consistent with the tangential discontinuity of the magnetic field at the boundary using Eq. (6.10). For an interior field mostly oriented along the $Z_0$ axis (see Fig. 6.7), the current surface density is expected to flow into the page, in the dusk to dawn direction. The Biot-Savart law is then used to infer the contribution of each grid element $P$ to the overall shielding field at a point $M$ within the magnetosphere

$$\mathbf{d}B_P(M) = \frac{\mu_0}{4\pi} \mathbf{j}_s(P) \, dS(P) \times \frac{\mathbf{PM}}{PM^3},$$

(6.12)

and the total field generated at $M$ by the magnetopause currents is given by the surface integral defined in Eq. (6.11). The integral is approximated numerically from the discrete set of $\mathbf{d}B_P(M)$ using two consecutive trapezoidal integration techniques.

The contribution of the magnetopause currents to the internal magnetic field is shown in Fig. 6.8 for an Autumnal Equinox configuration at Saturn: the left panel shows field lines of the dipolar field (in green) and of the magnetodisk field (in purple). The total magnetic field, including the shielding field from Eq. (6.11), is shown in the right panel. The magnetopause resulting from the method detailed in section 6.4.1 is traced in green dotted lines, with a stand-off distance fixed at $30R_S$. Both panels illustrate the same cross-sections of the magnetosphere in the $X_{KSM}Z_{KSM}$ plane; a miniature representation of the magnetic moment orientation viewed from the Sun is shown in the top left corner. The introduction of the shielding field clearly compresses the magnetic field lines on the dayside in order to confine them within the magnetosphere. In this Equinox configuration, the magnetic equator — coloured in blue in Fig. 6.8 — remains aligned with the dipole and rotational equators. This is due to the nose $N$ coinciding with the projection $p_N$ from Fig. 6.5: the solar wind flow compresses the magnetopause in a direction tangential to the rotational equator, and no stress is applied perpendicular to the current sheet.

As shown in Fig. 6.5, the nose $N$ and its projection $p_N$ onto the rotational equatorial plane move apart away from each other from Equinox. In particular, they are the furthest from each other at a Summer or Winter Solstice. Fig. 6.9 illustrates the
Figure 6.8: Noon-midnight meridional cuts of Saturn’s magnetosphere for an Autumnal Equinox. On the left panel, field lines of the dipole field are shown in green, the ones of the magnetodisc field are coloured in purple. On the right, field lines of the total magnetospheric field are traced in red, after inclusion of the shielding field from Eq. (6.11). The magnetic equator is shown in blue, and the position of the nose $N$ of the magnetopause, shown in red, is determined as discussed in section 6.3. The boundary traced with green dotted lines is calculated following the methods detailed in sections 6.4 and 6.5.3 for a stand-off distance of 30 $R_S$. The field lines are traced by numerically integrating from points evenly-spaced along the $Z_{KSM}$ axis, at which the magnetic field is evaluated. A view of the dipole orientation from the Sun is shown in the top left corner.

field structures in the same noon-midnight meridional planes, in a Northern Summer configuration. The solar wind flow being normal to the magnetopause at the nose $N$ now results in a stress perpendicular to the rotational equator which displaces the current sheet out of the dipole equatorial plane. This effect, shown in blue in both panels, is compared to the ‘bowl-shape’ geometry from Arridge et al. (2008b), coloured in orange. The inclusion of the shielding field similarly warps the current sheet Southwards and reduces the offset seen in the left panel. The curvature of the empirical model is found to be more gradual, with our approach leading to a displacement effectively starting $\sim 4 - 5$ $R_S$ away from the magnetopause.

The parametrisation of the current sheet geometry from Arridge et al. (2008b) relies on a ‘hinging distance’ $R_H$ and a differentiable approximation to the following piece-wise description: $z(r < R_H) = 0$ and $z(r \geq R_H) = a + br$, where $z$ is the local displacement of the current sheet normal to the rotational equator, $r$ is the radial
Figure 6.9: Noon-midnight meridional cuts of Saturn’s magnetosphere for a Northern Summer. The elements of the figures are the same as the ones in Fig. 6.8. The curvature of the blue magnetic equator resulting from the inclusion of the shielding field from Eq. (6.11) (in the right panel) is described by a piece-wise linear geometry, with a hinge point $H$ positioned $\sim 27 R_S$ from the planet. It is compared to the warped current sheet geometry from Arridge et al. (2008a), shown in orange. The introduction of the shielding field reduces the offset seen in the left panel, though the distortion is less gradual than the empirical model. As discussed in the text, an iterative update of the boundary and magnetospheric field may lead to better agreement. The planet-to-nose distance was fixed at 30 $R_S$.

distance from the planet and $a, b$ are real constants. The authors found the model to be most consistent with Cassini magnetometer measurements made from February 17$^{th}$ to 21$^{st}$ 2005 with $R_H = 29 R_S$. A similar piece-wise parametrisation of the current sheet geometry is drawn in black dotted lines in the right panel of Fig. 6.9. The corresponding hinge point, labelled $H$, is found at $\sim 27 R_S$ from the planet. This value is close to the optimal hinging distance $R_H$ from the ‘bowl-shape’ model, and is consistent with the $25 − 30 R_S$ range indicated by ENA composite images (Carbary and Mitchell, 2013).

The difference between both geometries could be due to several reasons. Firstly, the current sheet distortion in our model is a direct consequence of modelling the magnetic field generated by the magnetopause currents flowing onto our equilibrium boundary. A new iterative treatment of the magnetopause which includes the updated field shown in the right panel of Fig. 6.9 might lead to slight
corrections of the magnetopause structure, and hence of the current sheet geometry. In other words, one could iteratively modify the presently used Connerney ring current model, in order to self-consistently include a curved current sheet in the final magnetospheric configuration. Another reason could be that the current sheet geometry is not only season dependent, but can also vary with system size. If the magnetopause is compressed — due to variations in solar wind or internal conditions — the relative contribution of the shielding field to the total internal magnetic field, and the corresponding current sheet distortion, will be different.

Fig. 6.10 shows the magnetospheric field structure for a magnetopause standoff distance of 25 $R_S$, in a similar Northern Summer configuration. The hinge point $H$ is now moved at $\sim 23 R_S$ from Saturn, but the distortions from both models are qualitatively similar, both describing a displacement of the current sheet of $\sim 1 R_S$ from its nominal position at $X_{KSM} = 20 R_S$. Finally, a variation is to be expected since both models differ in their underlying approach: the empirical model describes the magnetosphere by sampling it under specific conditions, whereas our physical method aims at describing the system in its nominal, equilibrium state from modelling the key magnetospheric drivers at Saturn. The comparison paints the consistent picture of a seasonal warping of the current sheet rising from a displacement of the nose of the magnetopause from the rotational equator; it also hints
6.6 Conclusion to this Study

We have extended the Hardy et al. (2019) magnetopause model to study the seasonal variations of the boundary due to Saturn’s obliquity. This is done by introducing two additional angular parameters to the model: the fixed $26.7^\circ$ dipole tilt relative to the normal to the ecliptic, and a continuously-varying azimuthal angle $\Phi$ parametrising the season (see Fig. 6.1). This approach allows us to fix the solar wind flow direction along the Sun-planet axis, as was done in the previous aligned version of the model from chapter 4.

A method was derived to determine the seasonal position of the nose of the magnetopause from considerations of pressure balance. It was found that the nose traces a ‘figure-eight’ locus with a $\sim 7 R_S$ amplitude in the $Y_{KSM}Z_{KSM}$ plane (see Fig. 6.4). At a given seasonal configuration, this point is used to anchor a realistic ‘guess-surface’ — determined in chapter 4 — which is numerically optimised with the method detailed in chapter 3 to obtain a final equilibrium boundary. The varying solar wind incidence leads to a North-South asymmetry which is maximum at a Summer or Winter Solstice, and vanishes at a Vernal or Autumnal Equinox. In particular, the positions of the polar cusps vary from their aligned positions — determined precisely in section 4.2.3 — by moving complementarily closer to the nose or terminator. An interpolation method is proposed to model magnetopause structure continuously from a discrete set of reference seasonal configurations.

As the nose travels along the seasonal ‘figure-eight’ locus, it periodically moves apart from the rotational equatorial plane. This asymmetry leads to the solar wind flow exerting a stress perpendicular to this plane, which displaces the current sheet from its nominal planar geometry. This effect is apparent when computing the magnetopause currents flowing along the surface and the shielding magnetic field they generate. The magnetic field lines are confined to within the magneto-
sphere and the current sheet is found to curve Southwards for a Northern Summer. The distortion resulting from our physical approach is qualitatively consistent with previous empirical modelling studies (Arridge et al., 2008a) and ENA observations (Sergis et al., 2011; Carbary and Mitchell, 2013). Our current sheet geometry is, however, found to be more abrupt and inherently dependent on the magnetopause stand-off distance. At Vernal or Autumnal Equinox, the nose was found to return to the rotational equatorial plane and the current sheet distortion disappears, as was expected by a hinging mechanism driven by the asymmetric action of the solar wind on the magnetopause. The model described in this study thus allows us to estimate the structure of the magnetopause and infer a more consistent magnetospheric field model, under any seasonal configuration. The interpolation technique described in section 6.4.2 is also helpful in modelling magnetospheric field structure continuously and efficiently, thus potentially paving the way towards robust constructions of magnetopause and magnetosphere ‘timelines’ over the duration of space missions.

This study of the equatorial current sheet curvature at Saturn might also offer qualitative insights as to why Jupiter’s current sheet becomes parallel to the solar wind flow on the day-side (Arridge et al., 2008b). Indeed, Jupiter’s relatively small obliquity (see table 6.1) produces much smaller seasonal effects — but larger diurnal variations. The ‘figure-eight’ locus describing the displacement of the nose away from the rotational equator (see Fig. 6.5) would have a much smaller amplitude, thus effectively leading to a quasi-symmetric contribution of magnetopause currents to magnetospheric field structure. The Jovian system differs from Saturn’s, however, in the dipole tilt relative to the rotational axis causing relatively large diurnal periodicities. Modelling these effects in a similar fashion would require the introduction of two analogous angular parameters — one fixed, and the other varying under timescales comparable to the planetary rotation period.
Chapter 7

Conclusions and Perspectives

7.1 General Summary of Conclusions

7.1.1 A Physical Model of Saturn’s Magnetopause

In this thesis, we describe the development of a physics-based numerical model for a steady-state magnetopause at Saturn, based on considerations of pressure balance. Chapter 4 explains how we build on the framework developed by Mead and Beard (1964) at Earth to infer the shape, position and structure of the boundary by modelling the contributions of key magnetospheric drivers at Saturn. Section 4.3.3 in particular details the introduction of a magnetodisk field structure and additional pressure due to hot plasma populations originating from the outer magnetosphere in the pressure balance equation, which are both significant internal drivers of the Kronian magnetosphere.

We set out a numerical procedure which allows the determination of magnetopause structure from a particular description of local pressure balance at the boundary. It relies on first resolving sections of the magnetopause where arguments of symmetry simplify the analytical problem – namely the nose of the boundary, the equatorial plane and the noon-midnight meridional plane, in the case of a non-tilted planetary dipole. Particular attention was given to resolving the complex high-latitude structure characterised by ‘indents’, or cusps, along the boundary. Section 4.2.3 describes an entirely new, semi-analytical procedure which results in solving the position of these singular points accurately.
A ‘guess-surface’ is then constructed with fixed boundary conditions corresponding to the aforementioned regions, and an iterative Levenberg-Marquardt algorithm is implemented to converge towards the equilibrium magnetopause. The framework would also be extremely useful if a refined version of the initial pressure balance equation is to be considered, since the optimised surface could be used as a new starting point for the updated procedure; this has proven to be very efficient in chapter 6 where a dipole tilt is introduced.

Given the physics-based nature of the model, the resulting magnetopause is free from any assumption inherent to initial parametric descriptions of the surface geometry. It also does not depend on the quantity of in-situ observations at hand, nor on the specific system state they may describe (e.g. a particular seasonal configuration, a transient plasma-loaded state, etc.); it remains, however, a steady-state description of a highly-dynamic system. We have used a database of observed magnetopause crossings by Cassini [Pilkington et al., 2014] to illustrate the following points:

- The model describes a non-axisymmetric structure consistent with the polar flattening evidenced by [Pilkington et al. 2014]. The amplitude of the flattening was, however, found to be smaller (we assume due to most crossings having been observed at low latitudes) and was shown to increase with system size (see section 4.3.3).

- An average magnetopause compressibility was determined: it was found to be consistent with values determined by empirical models, and allows the scaled observed crossings to cluster around a reference surface (see appendix A). A more detailed study of magnetopause compressibility at Saturn is conducted in chapter 5.

### 7.1.2 Generalising Magnetopause Compressibility

The numerical framework developed in chapter 4 was used to estimate the compressibility of Saturn’s magnetopause in chapter 5. We described a procedure to consider observed magnetopause crossings and determine the corresponding list of
7.1. General Summary of Conclusions

system size and solar wind pressure. A useful visualisation of the results in phase space (see Fig. A.2) clearly shows that the size of Saturn’s magnetosphere can be entirely governed by variations in internal drivers.

We have thus developed a way to unite internal and external drivers at Saturn. This was done by scaling the determined solar wind pressure estimates using the corresponding measured values of interior plasma beta. By considering these scaled, effective values of solar wind pressure, we indirectly account for the additional pressure due to magnetospheric hot plasma populations. The scatter initially present in the crossing distribution – due to variations in internal conditions – was found to be significantly reduced when this correction was applied (see Fig. 5.3).

A method was then constructed to assess how close the magnetopause was to pressure equilibrium when each crossing was observed. This allowed us to filter the magnetopause crossings corresponding to the spacecraft encountering a rapidly moving boundary. After eliminating these transient states, the compressibility of Saturn’s magnetopause appeared to be a function of system size: larger (i.e. more rigid) for a compressed system and smaller (i.e. more elastic) for larger values of the stand-off distance (see Fig. 5.4). In order to further characterise this size-dependent property, we established an entirely new generalisation of magnetopause compressibility which resulted in it being defined as a function of system size. In particular, Saturn’s magnetopause was shown to behave like that of Earth when compressed, and closer to that of Jupiter when expanded, with the intermediary values being consistent with previous empirical studies (see Fig. 5.5).

7.1.3 Modelling Seasonal Variations

Given the extraordinary longevity of the Cassini mission at Saturn, modelling the seasonal dynamical and structural variation of the magnetosphere is particularly pertinent to the mission’s prime objectives. In chapter 6, we refine our magnetopause model to account for the magnetospheric tilt changing due to the planet’s obliquity. In order to analyse the impact of planetary seasons on the structure of the internal magnetic field, we developed a specific modelling framework: determine the position of the nose of the magnetopause for a given seasonal configuration, infer the
structure of the magnetopause by using the previous surfaces as the new starting point in the numerical procedure, and calculate the magnetopause currents flowing along the newly-optimised boundary.

The position of the nose of the magnetopause was determined through a novel method consisting in scanning for the point on the boundary which satisfies all its expected characteristics: (i) satisfying the pressure balance equation, receiving the solar wind flow at a normal incidence, (ii) lying in the plane defined by the Sun-planet axis and the magnetic moment, (iii) all the while being the closest point to the Sun. Following this procedure, the nose was found to trace a seasonal ‘figure-eight’ locus (see Fig. 6.4) by moving away from the Sun-planet line by up to $7 \, R_S$ at a Summer or Winter Solstice. This displacement forces the nose to seasonally leave the rotational equator, thus forcing a North-South asymmetry on the entire magnetopause at Saturn.

From the position of the nose, the equilibrium boundary is determined by further optimising the solutions from chapter 4. A clear North-South asymmetry appears with the cusps in both hemispheres moving in tandem closer to the nose or terminator through the seasons (see Fig. 6.5). A numerical interpolation technique is proposed to compute a magnetopause surface at any given seasonal configuration from a discrete set of previously-calculated surfaces. Using these optimised surfaces, we calculated the contribution of the magnetopause currents to the internal magnetic field. By confining the magnetic field lines to within the magnetosphere, the resulting shielding field was shown to periodically displace the equatorial current sheet away from the rotational equator: Southwards for a Northern Summer, Northwards for a Norther Winter, with no distortion at the Vernal and Autumnal Equinox. This distorted geometry of the current sheet was shown to be quantitatively consistent with previous observations [Sergis et al. (2007); Arridge et al. (2008a); Carbary and Mitchell (2016)]. The curvature was found to be more abrupt than described in previous empirical studies (Arridge et al., 2008a) and inherently dependent on system size. The flexibility of our magnetopause model, coupled with the efficiency of the previously-mentioned interpolation methods, paves the way to-
7.2 Future Perspectives

7.2.1 Open Questions

Despite our studies at Saturn of the dynamics of its magnetosphere, and its responds to the changes in the upstream solar wind, there are still some fundamental outstanding questions about space weather at Saturn. These include:

- **How and why does the morphology of Saturn’s magnetopause change?**
  Our modelling work has confirmed that Saturn’s magnetopause has a ‘flattened paraboloid’ shape. However, we still have not quantified how the extent of this flattening changes with time, nor the corresponding physical changes in magnetospheric plasma and field which cause it.

- **What are the dominant physical drivers which determine magnetopause size, and how and why do they change?** We have established three competing factors which locate the magnetopause: the internal pressure of ‘hot’ and ‘cold’ plasma populations, the internal magnetic pressure, and the external dynamic pressure of the solar wind. We still do not know which of these are most influential in changing the magnetopause size relative to a ‘quiescent magnetopause’ state. Nor do we know how and why the dominant driver changes according to variations in solar cycle phase, planetary season and plasma heating due to internal magnetic reconnection.

- **How does the statistical distribution of observed magnetopause size change, and can it be used as a proxy for the dominant driver at any given time?** The subsolar distance $R_{MP}$ from the planet to the magnetopause indicates the boundary’s global size; the distribution of $R_{MP}$ values seen for a particular time interval can be an indicator of internal modulation of the magnetopause in addition to modulation by solar wind pressure changes (Joy et al., 2002; Achilleos et al., 2008). But we have yet to fully exploit this observable distribution as a diagnostic of the internal state of the magnetosphere itself.
The answers to these questions will not only advance our knowledge in poorly understood space weather physics at Saturn; they will also require the advent of new tools and analysis methods which would then be potentially applicable to other planets. Our current magnetopause models could be used and refined accordingly to improve our understanding of how planets interact with our Sun; we discuss below possible directions that could be taken to implement such projects.

### 7.2.2 Possible Directions for Future Work

In order to address the aforementioned science questions, the current version of my model would need to be refined in several ways. One key point is to improve the completeness of the description for the planet’s environment. At the moment, the plasma beta parameter (ratio of plasma to magnetic pressure) is assumed constant along the magnetosphere boundary: a more detailed description based on observed particle densities would vastly improve our understanding of how the Gas Giants interact with our Sun. Furthermore, connecting the upstream solar wind conditions to recent three-dimensional solar wind propagation models would constitute our missing link in propagating the observed influence of our Sun into our magnetosphere structure determination. Doing this would allow us to monitor the way planets interact with the solar wind throughout the timelines of planetary space missions. These improvements could be implemented by following the steps detailed below:

1. **Propagate observed solar wind properties into novel 3D magnetopause models**: One could expand on the magnetopause modelling work from [Hardy et al. (2019, 2020)](#) by using recently developed solar wind propagation tools [Tao et al. (2005); Shiota and Kataoka (2016)](#) in order to construct a history of upstream solar wind parameters – dynamic pressure ($P_{SW}$) and IMF – at Saturn during the Cassini mission timeline. For the sub-intervals corresponding to each individual pass of the spacecraft, a corresponding equilibrium magnetopause model could be calculated using the propagated $P_{SW}$ as the external pressure, and using the hot plasma index to quantify the internal driver (pressure carried by the energetic particle population). This index can be calculated from available plasma data as described in [Achilleos et al. (2010)](#) and [Sergis et al. (2007)](#). One constructs
an equatorial profile of hot plasma pressure in the magnetosphere and situates it within the observed range, as a function of distance, over the whole mission. This allows identification of a ‘quiet’, ‘median’ or ‘active’ plasma pressure / ring current state.

The magnetopause crossing positions observed by Cassini could be compared with those predicted by the magnetopause models, in order to quantify how close the boundary is to pressure equilibrium, within each sub-interval. Further model validation could be performed through comparison with previous Cassini/Saturn observational studies of the plasma pressure and magnetic field (Bunce et al., 2007; Achilleos et al., 2008; Arridge et al., 2008b; Pilkington et al., 2015). Additional modulation of the plasma pressure, due to centrifugal confinement of the cold particle population, could also be modelled (Achilleos et al., 2010).

2. **Refine descriptions of key magnetospheric drivers:** Once the time sequence or mission ‘history’ of magnetopause models is established and validated, one could use it to advance and improve the descriptions of the important internal drivers which determine magnetopause morphology. This could be done by determining, for each sub-interval, as many as possible of the following contextual parameters:

- **The phase of the Solar Cycle** (available for all sub-intervals)
- **The seasonal phase of Saturn’s orbit**, related to solar wind flow orientation with respect to the planet’s magnetic dipole - a factor which modulates magnetopause structure (available for all sub-intervals)
- **The presence of significant episodes of magnetic reconnection** during the sub-interval, as revealed, for example, by characteristic brightness increases in the planet’s ultraviolet auroral emissions which ‘fill in’ the regions poleward of the quiescent aurorae — i.e. the rings of emission surrounding the magnetic poles (Kimura et al., 2015; Bader et al., 2020). The available auroral data for Saturn from the Hubble Space Telescope and Cassini archives combined with in situ signatures of reconnection, should provide this context
for the majority of the sub-intervals.

By then comparing the quantified internal and external driver ‘states’ – plasma index and solar wind dynamic pressure – at each sub-interval with the corresponding contextual parameters, one would be able to fulfil the following goals:

- Identify which is the dominant driver in each sub-interval: For both equilibrium and non-equilibrium magnetopause states (as determined through model comparison with observations), one could identify whether it is the internal plasma or the solar wind which is exerting a pressure unusually large compared to its median value over the mission timeline. One would also be able to determine whether the two drivers both fall within a statistically pertinent interval (e.g. one quartile of the median) to distinguish periods of comparable influence.

- Examine the relationship between the dominant driver and contextual parameters: one could look at the periods of dominance for the different drivers, and how they relate to solar cycle, planetary season, and magnetic reconnection. In so doing, the results will help identify the physical origin of each driver, and further quantify its influence on magnetospheric dynamics at Saturn. The relation between plasma state and reconnection is particularly important, as the latter can be an important source of plasma heating. It would also be possible to use the ‘magnetopause history’ model sequence in order to quantify the statistical distribution of magnetopause size (as represented by magnetopause subsolar location – ‘standoff distance’ $R_{MP}$ – relative to Saturn). This could be done for sub-intervals with similar values of plasma index and $P_{SW}$.

The realisation of these research goals would thus allow us to answer the open questions mentioned above, which relate to the dynamical behaviour of magnetopause morphology; the physical mechanisms – ‘drivers’ – which control magnetopause morphology and dynamics; and the potential use of $R_{MP}$ distributions as a diagnostic of solar wind and/or magnetospheric physical states.
This would ensure a more complete and advanced understanding of Gas Giant-
solar wind interactions, particularly timely in light of the current Juno and future
JUICE (JUpiter ICy moon Explorer) missions.

3. **Develop open-access magnetopause monitoring tools**: An accurate descrip-
tion of solar and magnetospheric drivers, along with the account of planetary
seasonal variations [Hardy et al., 2020], could be the base of flexible, open-
access and user-friendly tools that would monitor the interactions (and conse-
quently magnetopause structure, magnetospheric field models, current sheet po-
sitions, etc.) throughout the timeline of space missions of interest. The flexible,
physics-based nature of the model would allow preliminary theoretical appli-
cations to very particular and lesser-explored worlds, such as Mercury, the Ice
Giants, Mars, and exoplanetary magnetospheres.

### 7.2.3 Context and Future Space Missions

A robust 3D description of magnetosphere boundary and interactions would have
strong and relevant links to future major mission objectives and activities.

In particular, a consistent monitoring of the magnetosphere boundary and equa-
torial current sheet at Saturn would continue to benefit the Cassini community in
assessing the impact of local energetic particle populations on the global configura-
tion of the Kronian system. The resulting magnetospheric model would additionally
contribute to the study of Titan’s cometary interactions with Saturn’s magnetosphere
and the solar wind, which is a prime objective of the Cassini mission – Titan orbits
close to Saturn’s magnetopause, and is occasionally located in the solar wind.

An application to Jupiter’s magnetosphere would also be particularly relevant
and timely in light of the JUNO and JUICE mission. It would contribute to char-
acterising the environment of Ganymede, which is known to have an intrinsic mag-
netic field, and would shed light on the moon’s unique interactions with the Jovian
magnetosphere; this is again one of the mission’s main science objectives. The
model could also potentially be applied to the moon itself: determining the struc-
ture of the Jovian magnetosphere, and considering the output as new ‘upstream’
conditions to study Ganymede’s own magnetosphere. This would pave the way towards a unique method to constrain magnetospheric conditions at Ganymede, which is though to be a laboratory for analysing the potential habitability of icy worlds.

Our study of how magnetised planets interact with the Sun is also closely related to the origin of the solar wind, solar eruptions and how they fill the heliosphere before encountering planetary environments. These major scientific questions are to be addressed by the Solar Orbiter mission; a continuation of this thesis would also contribute to advancing our broader understanding of how magnetic planets and moons interact with their host star.
Appendix A

Reorganisation of Magnetopause Crossings

In order to assess the validity of the magnetopause model developed in chapter 4, a computed surface was compared to the list of observed magnetopause crossing from Pilkington et al. (2014), mentioned in sections 1.4.2 and 2.3. As discussed in 1.3.3 however, the magnetopause is a highly dynamic system constantly responding to changes in external and internal drivers. It is then necessary to ‘scale’ the observed crossings to common solar wind and magnetospheric conditions. In other words, we need to find where the magnetopause crossings would have been, if they had been observed under the same solar wind pressure and plasma beta.

The large-scale contribution of internal drivers on system size – evidenced in (Pilkington et al., 2015) – was first accounted for by clustering the magnetopause crossings in distinct groups of similar plasma beta values. This was done using a k-clustering algorithm (Pilkington et al., 2015), the results of which are shown in Fig. A.1. The magnetopause compressibility index was estimated for each cluster using its average value of plasma beta, and by performing a linear fit of the power law related to Eq. (5.7). The average magnetopause compressibility parameter over the three groups was found to be $\alpha \approx 5.2$. Given that this estimate is independent of $\beta$ at this stage, it can be expected to be an over-estimate for regimes of high $\beta$, and conversely an under-estimate for regimes of low $\beta$.

A magnetopause crossing $P$, observed at the position $\overrightarrow{OP}_{\text{obs}}$ under the solar...
Figure A.1: Determination of the compressibility parameter $\alpha$ using observed magnetopause crossings of the Cassini spacecraft and the relationship $R_{\text{MP}} \propto P_{\text{sw}}^{-1/\alpha}$. The crossings have been clustered in three groups using a k-clustering algorithm, depending on their local values of plasma-$\beta$: $0 \leq \beta \leq 2.41$ in the dark-blue cluster, $2.43 \leq \beta \leq 6.67$ in the green-blue cluster and $6.69 \leq \beta \leq 14.45$ in the yellow cluster. The average compressibility parameter is $\alpha \approx 5.2$.

Wind pressure $P_{\text{sw}}$, would then respond to the solar wind pressure changing to $P_{\text{sw,ref}}$ by being displaced to the new position

$$\overrightarrow{OP}_{\text{scaled}} = \overrightarrow{OP}_{\text{obs}} \left( \frac{P_{\text{sw}}}{P_{\text{sw,ref}}} \right)^{\frac{1}{\alpha}}.$$  \hspace{1cm} (A.1)

The left panel of Fig. A.2 shows projections of observed magnetopause crossings in the equatorial and noon-midnight meridional planes, with a reference magnetopause surface computed with average conditions within the crossing list. The right panel shows the scaled positions of the magnetopause crossings after the procedure described by Eq. (A.1). The crossings are found to cluster much closer to the reference surface. This result, along with the estimated magnetopause compressibility being consistent with previous studies (see Fig. 5.5), illustrates the validity and efficiency of the method detailed in chapter 4. It motivated a further in-depth analysis of magnetopause compressibility under variable internal drivers at Saturn, which led to the work described in chapter 5.
Figure A.2: Distances of non-scaled (a and c) and scaled (b and d) crossings from the modelled surface. The degree of scatter is similar to the residuals found when fitting purely empirical models \cite{Pilkington2015, Arridge2006}. All the crossings with local plasma $\beta$ inferior to 15 were kept.
Appendix B

Considerations of Pressure Balance

Section 5.2.3 details how the magnetopause model developed in chapter 4 at Saturn is used to estimate the solar wind pressure corresponding to observed boundary crossings. The calculation makes use of the formalism of Petrinec and Russell (1997) to estimate the exterior solar wind pressure in the presence of a diverging magnetosheath flow.

The Petrinec and Russell (1997) condition reads:

\[ P_{\text{ext}} = K P_{\text{sw}} \cos^2 \psi + P_0 \sin^2 \psi = P_{\text{int}}, \]  

(B.1)

where \( \psi \) is the angle between the solar wind flow direction and the normal of the surface, \( P_0 \) is the static thermal pressure and \( K \) the coefficient accounting for the divergence of the incident magnetosheath flow – usually taken to be \( K = 0.8 \) (Spreiter et al., 1966). The additional \( P_0 \) term would only become important for values of \( \psi \) satisfying, for example,

\[ P_0 \sin^2 \psi > 0.2 \left( K P_{\text{sw}} \cos^2 \psi \right), \]  

(B.2)

leading to

\[ \tan^2 \psi > 0.2 K \frac{P_{\text{sw}}}{P_0} \approx 0.2 \frac{P_{\text{sw}}}{P_0}, \]  

(B.3)

\( K \) being of the order of unity.

The pressure ratio \( \frac{P_{\text{sw}}}{P_0} \) is approximately equal to the square of the upstream sonic Mach number at Saturn \( M_S \approx 14 \) (Achilleos et al., 2006; Masters et al., 2008).
leading to a limiting value $\psi_c$

$$\psi_c \approx \tan^{-1} \left( 14 \sqrt{0.2} \right) \approx 81^\circ. \quad (B.4)$$

This corresponds to a position quite far downstream. The magnetopause model developed in section 4.3.3 which relies on solving Eq. (4.26), could thus be considered valid for a large portion of the dayside magnetopause. A refined version of the model could always be implemented with considerations of a more complete pressure balance equation. This approach would be particularly warranted if we were interested in studying the structure of the magnetopause close to the terminator, or beyond on the nightside. In this case, the numerical approach detailed in this thesis would still be applicable, and the current optimised boundaries could be used as new ‘guess-surfaces’ to be optimised further.
Appendix C

Comparison of Magnetopause Models

Numerous modelling approaches have been considered to model Saturn’s magnetopause, as discussed in section 1.4.2. In chapter 5, the structure of our model was compared to the empirical work of Pilkington et al. (2014) to discuss the polar flattening of the boundary at Saturn, as well as to other empirical models from Arridge et al. (2006) and Kanani et al. (2010) to study how the magnetopause compressibility varies with system size (see Fig. 5.5). A comparison of magnetopause structure from these different models is shown in Fig. C.1.

The Hardy et al. (2019) and Pilkington et al. (2014) models are the only models which describe a non-axisymmetric surface. The results from Pilkington et al. (2014), Arridge et al. (2006) and Kanani et al. (2010) stem from an empirical approach by fitting a parametrised description to observed magnetopause crossings. The Hardy et al. (2019) model is physics-based, based on Newtonian considerations of pressure balance. The equatorial current sheet is modelled by a current-carrying axi-symmetric disk – see section 4.3.3 – defined by parameters which depend on system size (Bunce et al., 2007). This allows the model to adopt different structures for different values of the magnetopause stand-off distance. Moreover, the physics-based approach allows a more accurate description of high-latitude magnetopause structure by resolving the complex cusp indentations – the method being detailed in section 4.2.3. The model is also flexible enough to model the seasonal varia-
Figure C.1: Comparison of magnetopause positions in the equatorial (bottom half) and noon-midnight meridional planes (top half), with models from Arridge06 (green), Kanani10 (blue), Pilkington14 (red), Hardy19 (orange). The axes are the ones of the orthogonal, Saturn-centered coordinate system (Kronocentric Solar Magnetospheric frame): $X_{KSM}$ points towards the Sun, and $Z_{KSM}$ is such that the $X_{KSM} - Z_{KSM}$ plane contains Saturn’s magnetic dipole. The regions shaded in orange correspond to the current-carrying torus used to model the equatorial ring current at Saturn in Hardy19.

...tions of magnetopause structure and internal magnetic field at Saturn, as described in chapter [6]...
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