Multi-Hazard Vulnerability Assessment  
of Unreinforced Masonry Structures  

Thesis submitted to University College London  
for the Degree of Doctor of Philosophy  
in Structural and Earthquake Engineering  

by  

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Declaration

I, Valentina Putrino, confirm that the work presented in this thesis is my own.

Where information has been taken from other sources, I confirm that this has been indicated in this thesis.

Signed: Valentina Putrino

Date: 15/05/2021
Abstract

The purpose of this research study is to develop a methodological framework for the multi-hazard vulnerability assessment of unreinforced masonry structures (URMs) undergoing seismic, flood and wind loading. To date, the two main challenges related to multi-hazard vulnerability assessment are 1) the substantial discrepancy in the level of advancement of single-hazard vulnerability assessment procedures, specifically in relation to the complexity of analytical model used to correlate the level of damage caused to buildings to the hazard component and 2) the conceptual differences in the definition of single-hazard fragility curves to be used to conduct multi-hazard damage assessment in a commensurate manner.

Therefore, research effort is still required to develop a harmonized analytical model able to relate the behaviour of masonry structures subjected to earthquake, flood and wind hazard to the corresponding levels of damage to unreinforced masonry structures, to define a common structural parameter for the derivation of single-hazard fragility functions which also allow for damage comparisons between these distinct perils. The framework proposed in this work carries out the assessment at a wall level. The hazard and the exposure components of the vulnerability assessment procedure are taken as inputs for the development of a kinematic model based on revised Yield Line Theory concepts. The main elements of added novelty are the inclusion of the contribution of torsional effects generated at unit level caused by the application of horizontal loadings, and a more refined computation of the crack pattern, defined on the basis of the geometry of the wall and the geometry of the units.

Given that several configurations of admissible crack patterns can be identified for the same wall layout subjected to horizontal loading, an optimization routine is built to find, by means of Limit State Analysis, the minimum load required to produce failure corresponding to a specific crack pattern, and the maximum value of the performance variable, defined as the ratio between the demand imposed by the loading and the capacity of the system itself, for the collapse limit state.

Such parameter, representative of the strength capacity of the system is then used to derive single-hazard fragility function to conduct collapse assessment. These curves are extracted by considering the variability of the asset, and hence focus on the aleatory aspect of the exposure component, rather than considering the uncertainties associated with each of the hazard’s intensity measure. The variance considered includes geometry, materials, presence of opening and boundary conditions.

Comparisons on resulting fragility functions are drawn across seismic, flood and wind hazard, to establish relevance of the above parameters and sensitivity of the fragility functions.

The framework is applied to the case study area of the Philippines, to prove the feasibility of the approach proposed.
**Impact statement**

Multi-hazard vulnerability assessment of URM buildings is a widely studied topic within the field of earthquake and conservation engineering. This thesis provides with a simple, reliable, and straightforward analytical procedure to assess the vulnerability of unreinforced masonry structure in multi-hazard scenarios, which are becoming increasingly more common due to the well-known climate change issues we are experiencing.

Among the main advantages, this procedure allows to extract results which can be used to derive fragility function which are commensurate, therefore allowing for comparison across different perils facilitating the prioritization of resources and the decision-making process of involved stakeholders. The method can easily be extended and tailored to other perils, and be customized to the specific case study analysed, thus proving flexible and applicable in the wider research context.

Finally, this study addresses and intellectual challenge by going beyond the state-of-the-art single-hazard vulnerability assessment methods for URM and has resulted in conference papers and ongoing publications.
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I entered the door of UCL in September 2014 and since then I have met a myriad of people: some were transient, some more permanent, few turned to become very important. More importantly, I have also realised that each one of these people has shaped and enriched my life, to the extent that I would not be the same Valentina if I would had never embarked on this journey.

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**Group A:** Walls with supports on 3 sides and 1 top side free

**Group B:** Walls with supports on 3 sides and 1 side free

**Group C:** Walls with supports on 4 sides

**Group D:** Walls with openings

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**Introduction**

- Multi-hazard fragility assessment results
- Choice of regression models for fragility functions derivation
- Single-hazard fragility curves: Earthquake
- Group A: Walls with supports on 3 sides and 1 top side free
- Group B: Walls with supports on 3 sides and 1 side free
- Group C: Walls with supports on 4 sides
- Group D: Walls with openings

**Boundary conditions**

- Geometric and Material characteristics
- Wind
- Conclusions

**Comparison between Approach_S and Approach_D**

- Detailed method (Approach_D)

**Conclusions**

- Comparison of the proposed method with EC6
- Comparison of the p

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**Loading schematization**

- Principle of least work equations: simplified procedure (Approach_S)
- Detailed method (Approach_D)
- Comparison between Approach_S and Approach_D

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**Geometric and Material characteristics**

- Generation of the walls’ taxonomy
- Boundary conditions

**Comparison of proposed method and ELS model**

- Comparison of the proposed method with experimental tests on masonry brick walls
- Comparison of the proposed method with EC6
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1. Introduction

1.1. Preamble

Unreinforced Masonry (URM) structures are the most widespread building typologies around the world, housing people in regions which are very often – and increasingly more frequently – struck by natural hazards.

There are a number of reasons that explain the inherent vulnerability of URM buildings subjected to the action exerted by natural hazards, such as the quality of construction and the level of connection between the URM walls and the horizontal structures, which – if combined to the increasing frequency of the natural events considered and their associated intensity measures, result in high level of damage often leading to collapse (Sorrentino et al., 2017).

As a result of the climate change now being observed on earth, it is becoming progressively evident that natural hazards such as flood and cyclonic winds – which in the past were perceived as isolated local or regional events – have now become a problem characterized by a global scale (Van Aalst, M. K. (2006)). This new awareness is reflected in the declaration of the Sendai Framework for Disaster Risk Reduction, which clearly states that ‘[...] disaster risk reduction practices need to be multi-hazard and multi-sectorial-based, inclusive and accessible in order to be efficient and effective’ (United Nations, 2015). Figure 1-1: Global map of earthquake, flood, and cyclone winds for return periods of 475 years, 500 years and 500 years respectively (UNEP/GRID-Geneva and UNISDR) shows a map of the distribution of the three natural hazards which are known to cause greatest damage to URM structures, earthquake, flood and cyclonic wind, whose expected intensities at any location on the globe are reported with similar return periods.

Figure 1-1: Global map of earthquake, flood, and cyclone winds for return periods of 475 years, 500 years and 500 years respectively (UNEP/GRID-Geneva and UNISDR)
The extent of overlap between the three perils shown in Figure 1 proves that, if more research efforts are required to further investigate the vulnerability of URMs, these efforts should be devoted towards the development of multi-hazard assessment procedures able to analytically correlate these structures’ performance to the different hazards intensities in such a way as to ensure that the results obtained are commensurate and comparable in a single common framework. However, the joint examination and quantification of multiple hazards is a very difficult task to achieve, since natural hazards exhibit a wide range of characteristics and are currently analysed by very different models (Kappes et al., 2012).

Typically, hazard assessment methods deal with each type of hazard separately, hence the modelling of the hazard acting on the investigated structure and the resulting damage tend to be quantified differently, thus lacking in homogeneity of both the methods used and the results obtained (Schoepfer et al., 2018).

At the hazard analysis stage, the problems relate to the differences concerning the impact indicators and impact metrics used to study the effects on humans and assets (Papathoma-Köhle et al., 2011). Since neither impact indicators nor impact measures are directly comparable, additional steps are required to achieve the desired harmonization. This highlights that there is a need for a consistent approach to assess vulnerability of masonry structures to different hazards so that the calculation of the risk is commensurate across the hazards and decisions can be taken on the basis of multi-hazard risks ranked on a single common scale.

1.2. Research aims and objectives

The scope of this study is to propose a procedure for the multi-hazard vulnerability assessment of URMs structures. The three sources of hazard investigated in this study are considered independently acting on the structure, thus being neither co-occurring nor triggered by the same source.

Among the categories of URMs, the focus of this study is on structures made of brick/blocks bound in mortar, due to the relative ease of representation of regular specimens in comparison to walls made of units of uneven dimension or with irregular joints dimensions. Brick walls make for a suitable test subject to apply the multi-hazard vulnerability assessment framework presented in this work. Moreover, the availability of laboratory tests and the ease of implementation of brick-and-mortar layouts in computer software helps the validation of the procedure proposed.

The main objective of this research study is to address some of the conceptual challenges that are raised by the multi-hazard analytical vulnerability assessment of historic masonry structures. The first challenge relates to finding an analytical model able to describe in a rigorous and unified manner the behaviour of URMs undergoing horizontal loadings of different nature, which should
guarantee to be applicable to different walls’ layout and boundary conditions. In this work, the Yield Line method of analysis, a long-established and effective means of estimating the maximum load sustainable by a concrete slab or plate (Johansen, K. W. (1962)), is chosen. Several adjustments and assumptions are needed to ensure the applicability of the method to inhomogeneous systems such as URMs. The core of the thesis is devoted to the development of a methodological framework that allows the application of yield line theory to URM walls subjected to earthquake, flood and wind hazard, by simulating the different hazard loads in a homogeneous and commensurate way. Although conceptually representable as horizontal forces acting on the same wall, seismic, flood and wind loads are very different in nature, exploiting their action with a very different dynamic pattern. The intrinsic need of homogenization required to build up the analytical procedure proposed implies a certain degree of simplification to find a baseline set of required information, valid across the three perils, which represents an additional challenge to consider.

Once the method is defined and results are extracted in terms of walls’ structural performance, the second challenge of this thesis is to prove that the analytical procedure developed can be effectively used to conduct fragility assessment, thus enabling a homogenised quantification of collapse probability, which is consistent and commensurate across the three perils considered and can be employed to tailor the decision-making process in any multi-hazard environment. A major gap exists when considering the different levels of advancement in single-hazard vulnerability assessment methods, which hinders the harmonization process needed to develop a multi-hazard vulnerability assessment framework: while a plethora of approaches is available in the field of seismic vulnerability assessment, flood, and wind analytical vulnerability assessment methods applicable to masonry structures are less common and poorly detailed.

The procedure is applied at wall level, with the aim of providing solutions at building and at compound scales. To this end, sixteen different walls configurations clustered in four main groups are investigated, characterized by different restraint conditions, opening layouts, global and local geometric parameters.

1.3. Thesis structure

This thesis is composed of nine chapters and three appendices.

Chapter 2 contains a review of the relevant literature focusing on vulnerability assessment methods tailored to URMs structures subjected to seismic, flood and wind loadings. The chapter begins with a general overview of the empirical and analytical available methods and progresses onto the concept of derivation of fragility functions for vulnerability assessment. The objective of this section is to identify potential areas of research that need enhancement when the aim is to conduct multi-hazard vulnerability
assessment, to highlight the shortcomings and difficulties in finding a suitable homogenization method, as well as to identify the knowledge gaps that require further investigation.

Chapter 3 contains a review of methods for the prediction of the ultimate load capacity of URMs under horizontal loading. The main aim pursued throughout the chapter is to assess the need to establish a failure criterion to establish the interaction between the moment capacities in the two orthogonal directions, determining the mechanics of biaxial bending. The second part of the chapter is dedicated to the review of the modelling techniques and methods to assess the OOP failure of URMs, to ascertain to which extent a unified hazard-independent model can be developed to determine comparable results across different perils.

Chapter 4 presents the proposed framework to conduct multi-hazard vulnerability of URMs, the various inputs needed to define hazard and exposure and how these inputs feed into the main core of this study, which focuses entirely on the development of a kinematic model based on revised Yield Line Theory concepts, aiming at determining the analytical parameter representative of the response of the URM walls used to derive homogenised single hazard fragility curves. Such non-dimensional parameter of structural response, representative of the strength capacity of the system, and needed to link the threshold defining collapse to the relative intensity measure, is defined as performance variable $\chi$.

Chapter 5 details the steps required to build the kinematic model proposed for the determination of the performance variable $\chi$ and fundamental for the derivation of single-hazard fragility curves. Starting from the loading schematization of the three perils investigated in this study, this Chapter focuses also on detailing the crack pattern modelling alternative, highlighting how the global geometric aspect ratio of the wall affects the shape of the resulting crack pattern. The chapter also includes the full set of work equilibrium equations based on Yield Line (YL) Theory concepts (defined as Approach_S which stands for simplified approach) used to define the performance variable $\chi$ for the wall configuration used as example to describe the procedure. A further refinement of Approach_S which accounts for the torsional effects developing at the single brick/bloc unit scale when subjected to horizontal loading acting out-of-plane is also provided, and defined as Approach_D (i.e., detailed approach). The chapter includes a comparison between the two approaches to highlight the gained level of accuracy when going from a coarser and less refined approach that considered the crack pattern at wall scale (i.e., Approach_S) to a more detailed assessment conducted at brick scale (i.e., Approach_D).

Chapter 6 is dedicated to discussing the steps followed to generate the walls’ taxonomy and to present the set of validations conducted before reaching the final aim of the thesis, i.e., the fragility assessment. Since the focus of this study is to prove the efficacy of the kinematic model to describe the behaviour of URMs under different loading conditions, fragility functions (which are covered entirely in Chapter 7) are extracted by considering the variability of the model rather than focusing on the hazard intensity.
As mentioned, the crack pattern depends on the geometry of the wall, therefore two building samples are generated and used in this study, which are identical in the local geometric parameters and material properties but differ in the overall wall aspect ratio (i.e., L/H ratio). The chapter includes the details of the sixteen boundary condition configurations considered and how the geometric and material characteristics of the two walls configurations have been determined. Ranges of parameters variability are defined in accordance with existing building taxonomy repositories and, where information is lacking, Code prescriptions are used. To ascertain the validity of the results obtained with the proposed analytical procedure, three validations are conducted: 1) a comparison with Eurocode 6, to verify the extent of discrepancy between the assumptions made on the loading profile; 2) a comparison against a well-established experimental tests campaign on brick wall subjected to seismic loading to verify the level of discrepancy in terms of structural capacity of the wall and 3) a comparison with the Discrete Element software ELS to validate the layout of the crack pattern and the position of the line of maximum displacement.

Chapter 7 is dedicated to present the steps to derive single-hazard fragility curves. After having presented the regression method chosen to conduct fragility assessment, single-hazard fragility curves are derived, following a categorization made by hazard and by group. More general sets of fragility curves are also generated, to allow for comparisons across walls’ groups characterised by different aspect ratio.

Chapter 8 provides with an application of the proposed analytical method to a real casestudy located in the Philippines, to prove the feasibility of the analytical method proposed and the ease of implementation when a suitable building sample is made available.

Finally, Chapter 9 summarizes the main findings of the thesis and concludes by recommending the future directions of investigation.

Appendix 1 includes all working equations to determine the $\chi$ value for each of the sixteen wall configurations considered in this study.

Appendix 2 details the calculations for the comparison between failure load calculated following the EC6 procedure and the Approach_D procedure.

Appendix 3 details the Philippines Sample (PH_Sample).

An overview of the thesis outline is provided in Figure 1-2.
Figure 1-2: Overview of the thesis outline

2.1. Introduction

The aim of this first section of the literature review is to identify existing analytical methods to assess the vulnerability of URM structures to different sources of natural hazard, namely earthquake, flood, and wind and to evaluate the current state-of-the-art on multi-hazard vulnerability assessment procedures, to understand existing gaps which set the motivation of this research study.

To be able to successfully conduct risk assessment of buildings, a truly integrated analysis of the three ‘ingredients’, namely hazard vulnerability and exposure, becomes essential (Crichton, D. (1999)).

When dealing with more than one peril at the time, it is of fundamental importance to understand:

- The level of advancement of the single-hazard assessment procedures, including quantitative measures of intensity relevant to structural response,
- The knowledge of the response of the structure undergoing the specific action exerted by the single hazards considered and the commonalities among these responses,
- The most appropriate way to best represent the damage extent of the structure as a function of the hazard measure (IM) and the building response.

Given the discrepancy very often observed in the level of advancement of single-hazard assessment procedures, the real challenge in the development of any multi-hazard framework is to find the knowledge gaps and establish the number of assumptions needed to assure the proposed method can be univocally applicable to all perils considered.

The following section (i.e., 2.2) provides an overview of existing analytical methods to compute the response of URM structures subjected to seismic, flood and wind loading. Since the focus of this study is on the development of an analytical procedure, empirical and hybrid procedures are not included in this review.

Section 2.3 deals with the current state-of-the-art about comprehensive approaches to multi-hazard risk assessment.

Section 2.4 focuses on the review of fragility functions, statistical tools for carrying out vulnerability assessment of a given structural system, thus highlighting the main elements needed to build these curves. The following subsections provide a review of these elements for each single hazard, to frame the choices considered suitable for the purpose of the proposed overall framework.
2.2. Capacity Assessment Methods

There are several different methods reported in literature aiming at detailing the various aspects to consider when assessing the capacity of URMUs subjected to seismic, flood and wind loadings. This represents a key step to conduct fragility assessment as a fragility function (the details of which are provided in following sections) can be interpreted as the cumulative distribution function of the capacity of an asset to resist an undesirable limit state (Porter, K. (2015)).

A very important criterion that can be used to distinguish more comprehensive and exhaustive capacity assessment approaches is whether the method is purely empirical, analytical or hybrid. However, for the purpose of the present study only the analytical assessment methods are investigated. These rely on mechanical models of representative building typologies to describe what physically happens to the building when subjected to a given loading profile, and, through their algorithms, these procedures aim at analysing the building response in structural terms. Their use is preferable when construction details and information on building typologies are reliable and when both experimental works and record of damage are available to validate the results and calibrate the procedure.

The following sections are dedicated to summarizing the current key points of seismic, flood and wind analytical assessment methods to determine the capacity of URM structures, the main assumptions, the advantages and shortcomings.

2.2.1. Seismic Capacity Assessment Methods

Seismic vulnerability can be described as the susceptibility of a given structural asset to get damaged due to seismic ground shaking. In terms of assessment, each method is characterized by a specific way of correlating the hazard component to the level of damage experienced by the buildings.

The increasing development of more refined attenuation equations for the derivation of hazard maps in terms of spectral ordinates (instead of Modified Mercalli Intensity (MMI) or Peak Ground Acceleration (PGA) commonly employed in empirical vulnerability assessment frameworks), has promoted the use of analytical methods as they allow for the estimation of fragility functions with direct correlation to the seismicity of the area under investigation. Among these methods, Capacity Spectrum Methods (CSM), are nonlinear static procedures capable of providing predictions of response variables under a given hazard. In the case of seismic hazard, CSM estimate the seismic performance by comparing seismic capacity (in the form of a pushover curve) and demand intensity (in the form of a response spectra) and the graphical intersection of the two curves approximates the response of the structure. More specifically, by converting the base shear and roof displacement from a non-linear pushover curve and superimposing an earthquake demand curve, the nonlinear pushover becomes a capacity spectrum. By determining the point where the capacity spectrum “breaks through” the earthquake demand, it is
possible to estimate the spectral acceleration, displacement and damage that may occur for a specific structure responding to a given earthquake (Freeman, S. A. (2004)). When applied to large sets of buildings, this procedure allows for the derivation of fragility curves and damage scenario which are site-dependent and linked to the hazard levels. The damage levels are directly related to the Engineering Demand Parameters (EDP), expressed in terms of drift or displacement.

The HAZUS 99 framework (FEMA (1999)) proposed by Kircher et al., (1997) and the N2 method for new and existing reinforced concrete (RC) buildings, proposed by Fajfar, P., & Gašperšič, P. (1996) fall within the category of CSM.

A method specifically tailored to assess the vulnerability of historic URMs undergoing to seismic ground motion, is the FaMIVE procedure developed by D’Ayala, D., & Speranza, E. (2003). Starting from the study conducted on the case study of Lisbon (D’Ayala et al., (1997)), the approach based on limit state analysis concepts and involving the equilibrium of “macro” blocks, was further implemented including the CSM procedure within the original “mechanism approach”, as outlined in D’Ayala (2005) while also incorporating within the procedure the effects of strengthening devices and repair interventions aiming at reducing the vulnerability of the building investigated.

Within the Capacity Spectrum Method (CSM) approaches, there is the Mechanics-Based Seismic Risk Assessment method (MeBaSe) proposed by Restrepo-Velez, L. F., & Magenes, G. (2004). Starting from the formulation of the structural capacity and response in terms of mechanics concepts for both in-plane (IP) and out-of-plane (OOP) failure mechanisms (however only restricted to simple one-way bending mechanisms), considering four limit states (LS1=no damage to LS4=collapse) for the former, and the limit states based on the tri-linear model for non-linear behaviour of masonry walls developed by Doherty et al., (2002) and Griffith et al., (2003) to describe the one-directional OOP response of an URM walls based solely on the geometry, mass and boundary conditions (Ferreira et al., (2014)). More specifically, the tri-linear constitutive model shows the relationship between the force required to initiate the incipient rocking motion and the ultimate static displacement at the point of instability, requiring only three values of displacement (Δ1 controlling the initial stiffness reduction, Δ2 controlling the strength reduction and Δf which represents the maximum stable displacement) and one value of F0, which is the maximum force at the threshold of overturning (Velez, L. F. R. (2003)).

Along the same line of the N2 method, Borzi et al., (2008) proposed the Simplified Pushover-Based Assessment Method (SP-BELA), originally developed to assess the vulnerability of RC buildings then adapted to also define the structural capacity of URM buildings. From the assessment of URMs failure due to in-plane forces, the method aims at deriving vulnerability curves, with the main difference being that the pushover curves are built by using randomly generated dataset defined through Monte Carlo
simulation, thus implying that SP-BELA is a more probabilistic framework compared to others and yet confined to IP failure load only.

The seismic vulnerability of masonry buildings can also be estimated by means of more sophisticated CSMs based on Finite Elements (FE) models, with the aim of capturing the heterogeneous material properties of masonry modelled as a periodic arrangement of units and mortar joints and treated as a layered material. Several variations of the same approach are available in literature, all aiming at considering specific characteristics of the aggregate itself (i.e., different stiffness of mortar joints, different aspect ratio of units and joints). However, several assumptions are also required to carry on with the analysis such as the reduction of the joints to interfaces, the consideration of perfect continuity between units and mortar (which is never the case when dealing with existing masonry structures undergoing aging effects) or the alternative way of defining the horizontal strengths through “two-steps homogenization”, i.e. only the units with bed joints are modelled and the units with head joints are derived through second homogenization process (Giordano et al., (2002)).

The TREMURI software developed by Lagomarsino et al., (2013), seats among the methods based on an Equivalent Frame Approach for the nonlinear seismic analysis of masonry buildings, which – as the name suggests - relies on the idealization that each masonry wall is subdivided into sets of masonry panels in which deformation and nonlinear response are concentrated and a set of rigid portions which connect the deformable ones (Belmouden, Y., & Lestuzzi, P. (2009)). The method accounts for the presence of horizontal diaphragms (e.g., timber floors or roof), which are simulated via orthotropic membrane elements, and accounts for the nonlinear evolution of the lateral response of 3D masonry buildings through an algorithm that includes the deterioration of the base shear caused by increasing lateral displacements after the attainment of peak strength. However, the TREMURI procedure only assesses the in-plane wall behaviour, provided that proper connections prevent the activation of OOP mechanisms, in favour of the development of the building global response.

Other models adopting the Equivalent Frame Approach such as the one proposed by Belmouden, Y., & Lestuzzi, P. (2009) make use of the discretization of structural components, i.e., piers and spandrels, into series of ‘slices’ of material to better represent the heterogeneous bricks and mortar “one-phase material” behaviour and the non-linearity associated with it. More specifically, “one-phase materials” are characterized by the tendency to behave as a linear behaviour in compression and a brittle behaviour with modest tensile resistance at failure (Binda et al., (1988)). Although the model allows to consider some of the most relevant features of masonry structural behaviour such as the structural wall coupling, the zero-moment location shifting, the interaction between axial force and bending moment, the interaction between axial forces and shear forces and the prediction of failure modes, it is still hindered by some important limitations, such as the smeared crack assumption. According to such assumption,
the cracked section is a continuum and can be treated as such even when defining stress-strain relationship: however, the converse drawback occurs since the underlying assumption of displacement continuity conflicts with the realism of a physical discontinuity of the crack itself (Rots, J. G., & Blaauwendraad, J. (1989)).

Discrete element models (DE) derive from an inverse perspective than FE models. The masonry is modelled as an assembly of distinct entities (i.e., masonry units) interacting along their boundaries (Lemos, J. V. (2007)). In his review work Lemos, (2007) lists the distinctive features and advantages of DE models over FE models, such as:

- The representation of point contact hypothesis: this implies that the interaction between blocks is represented by a set of contact points and each contact force is a function of the relative block displacement at that point.
- The constitutive behaviour of contacts between units: there is more flexibility in defining the ‘forms’ in which the contact between units happens. Rigid and soft contact are the two main types.
- The block representation: the simplest and most common way of representing the mechanical behaviour of block is to represent it as rigid body (alternatively to FE) to allow full separation between blocks and to continue into the large displacement regime.

Applied Element Methods (AEM), an improved variation of DE methods allows to model the structure as an assembly of elements connected to each other by means of distributed deformable springs (Adhikari, R., & D’Ayala, D. (2019)). In AEM masonry is modelled through a micro-modelling technique. The connection between units is realised through two types of springs, namely unit or element, and joint: the former guarantee the connection between masonry elements, the latter simulate the equivalent properties of mortar. Although DE methods prove to solve some of the conceptual issues characterizing the FE methods, the accuracy of the stress distribution and, more in general of the model itself, highly depends on the number of points considered and how precise the discretization in units is done, proportionally affecting the number of analysis needed and the computational efforts required.

Within the available analytical methods to compute the capacity of URM structures subjected to seismic action, Non-Linear Time History Analysis (NLTHA) seats among the ones able to capture at best the full seismic loading process (from the initial state, through the non-linear behaviour under service loading, up to the strongly non-linear behaviour leading to collapse). However, they require a whole set of assumptions to model the heterogeneous combination of bricks/blocks and mortar interaction (usually modelled as a ‘simile-material’ with approximated properties), and they are also computationally very expensive, therefore not considered as an option in the framework of multi-hazard vulnerability assessment.
The review of the analytical methods to conduct seismic capacity assessment of URMs is focused on Capacity Spectrum Based Methods, which aim at linking the ground shaking to the building damage through quantitative parameters representative of the building capacity. These methods prove effective in accomplishing the purpose while keeping the computational efforts required limited and therefore allowing to be applicable at large scale. Macro-element approaches based on limit state analysis concepts are also the most suitable when aiming at developing mechanical models that have proven to represent the global masonry behaviour, while giving the chance to be further refined to simulate the ‘discrete’ nature of masonry.

2.2.2. Flood Capacity Assessment Methods

Although the increasing number of catastrophic events has risen the attention to monitor and to measure the flood damage to understand the impact to URM structures (D’Ayala et al (2006)), the majority of studies available in literature are still qualitative/empirical in nature, being more focused on assessing the wider range of general losses due to the non-structural damage caused by flood-induced damages (Pace, C. E. (1988), Custer, R., & Nishijima, K. (2015), Holicky, M., & Sykora, M. (2010), Mebarki et al., (2012)) rather than the more specific physical damage on structures, for which accurate information on water depth and velocity are required (Kelman, I., & Spence, R. (2004)).

The flooding actions causing damage to buildings due to their impact can be classified according to the relevance and predictability of their occurrence (Kelman, I., & Spence, R. (2004)), and can be categorized as follows:

- Lateral pressure imposed to walls due to water depth differential between inner and outer part of the building.
- Lateral pressure due to water velocity.
- Buoyancy.
- Water depth in slow-rise floods.

In terms of assessment procedure, the damage caused by flooding is usually estimated from available data such as surveys, insurance claims, historical catalogues or any combination of these, and computed in terms of monetary losses, (Nadal et al (2010), Scawthorn, et al (2006)), and represented through stage-damage curves, else defined as loss-functions, which correlate the impact of flood forces acting on structures to the specific flood characteristic considered, such as flood water depth (depth of inundation), flood water velocity, year and duration of flooding, sediment and contents, flooded area covered and flood warning system (Kreibich et al., (2009), Kelman, I., & Spence, R. (2004), Smith, D. I. (1994)).
The review conducted by *Pregnolato et al., (2015)* highlights that the two ways to obtain vulnerability and fragility functions for flood, are either 1) by evaluating statistical data of loss/damage values from past events (empirical methods) or expert judgment (synthetic methods (*Nadal et al., (2010)*)); 2) by combining empirical and synthetic approaches, thus using hybrid methods such as HAZUS-MH Flood Estimation Model (*FEMA, (2003)*).

Among the few mechanical analytical methods aiming to define the capacity of the walls against water depth there are *Kelman (2003)*, who modelled the damage at component level in an effort to define discrete damage states which combine damage to water depth and water velocity; *De Risi et al., (2013)*, authors of the VISK platform, a GIS-compatible software which performs micro-scale flood risk assessment for building stocks located in homogeneous urban areas which are assumed to be characterized by similar building characteristics (i.e. wall thickness, height of the building, presence of barriers in front of openings, size of openings); the vulnerability of the portfolio of buildings is evaluated in terms of fragility functions for specific limit states (i.e., serviceability (SE), life safety (LS) and structural collapse (CO), expressed in terms of critical flooding height) and the flooding risk is obtained by integrating the flooding hazard map and the fragility functions, over the range of possible flooding heights, for the entire buildings’ portfolio. Analytical work on the OOP load capacity of masonry walls subjected to non-uniform hydraulic lateral load was conducted by *Herbert, D. M. (2013)* and *Herbert et al., (2018)*, proving that Yield Line Theory concepts can be applied to determine the failure mechanisms of walls with various restraining conditions. Within the framework of limit state analysis, *Milanesi et al., (2018)* proposed a method to assess the structural vulnerability of traditional masonry buildings in alpine areas exposed to flash floods (i.e., sudden events caused by the response of the basins to intense precipitation or dam failure). The study emphasises the attention on the stability of the external walls at ground level since their failure might lead to the collapse of the supported walls and floors. The final aim of the work proposed by *Milanesi et al., (2018)* was to obtain families of dimensionless curves defined as ‘stability curves’, which are representative of the support conditions, the wall geometric and material characteristics (among which the slenderness ratio, the wall thickness, the number of floors, the wall aspect ratio and the ratio between fluid and masonry density) and the percentage of wall impacted by the flow. Stability thresholds obtained from the limit analysis approach were then compared with the results of nonlinear numerical analyses of same models developed by using ABAQUS, with implementation of various sets of material properties and geometrical layouts. The comparison between the two methods allowed for the definition of a benchmark for the proposed approach and to test the reliability of the assumptions used to ease the computations. The combined approach proposed by *Milanesi et al., (2018)* offers an economical and realistically achievable assessment based on a simplified model used to determine the physically based estimate of the vulnerability to flash flood which couples limit state analysis with flood hazard data obtained via hydraulic modelling.
To summarize, this review has shown that the majority of work available on the topic of flood vulnerability assessment is empirical. The reviewed analytical works which focus on URMs utilize limit state analysis, thus proving that – to that the limit state approach – is the most promising to conduct both seismic and the flood capacity assessment of URMs. The next section focuses on discussing the wind capacity assessment methods.

2.2.3. Wind Capacity Assessment Methods

The devastation caused by hurricanes proves them to be one of the most significant natural hazards affecting people living in shorelines, especially in high wind-prone regions (Vanek, C. M., 2010). The main culprit of such damage-rise is the increasing exposure in risk-prone regions, caused by population moving from inland to coastal areas. Given the losses affecting the insurance companies and governments and the level of disruption caused to general public, many research initiatives have risen to promote the investigation of damage mitigation measures and the development of loss prediction models of structures built in high-wind regions (Shanmugam et al., 2009). However, there is a rather sparse literature regarding available methods to assess the structural damage caused by extreme wind loading to URMs, and even lesser that encompasses the specific aspect of damage to the buildings’ walls, while great emphasis is currently devoted to the understanding and the vulnerability assessment of roof elements. This is because the damage to the vertical walls due to extreme wind speeds is usually associated to the failure of the roof, which consequently provokes the damage to the structure.

In terms of available codes, Section 6 of EC6 (CEN, 2010) is devoted to the determination of the ultimate resistance of URMs under vertical and horizontal (wind) loading, the latter simulated as an uniformly distributed load calculated per unit area. The moment of resistance is calculated in vertical and horizontal direction and then checked against the bending moment per unit length determined in the direction parallel and perpendicular to the direction of loading, including bending moment coefficient accounting for edge restraints and wall aspect ratio.

According to Pita et al., (2012), there are 5 types of methodologies available in literature to conduct wind vulnerability assessment, namely 1) methodologies based on past-loss data; 2) methodologies based on damage data; 3) heuristic approaches based on expert opinions; 4) methods based on the extent of physical damage which can be recorded on site; 5) numerical simulation of wind-building interaction.

Given the purpose of the present study, only methods 4) and 5) are included in the current review. More specifically, component-based methods 4) were developed as a more realistic alternative to enhance data models by assessing the vulnerability within an engineering framework completed with expert opinions and based upon information of structural resisting components. To further improve the component-based approach Holmes, J. (1996) and Leicester (1983) proposed an alternative cumulative probability
function able to better estimate the correlation of failure modes of individual structural elements by means of including load paths and progressive stages of damage toward final collapse.

In terms of analytical assessment methods (5), simulation vulnerability models enhanced the physical models by means of a probabilistic simulation of the wind-structure interaction and a more realistic assessment of the hazard. The three main improvements in using these methods are 1) a more precise estimation of the wind-load effects on the building; 2) a more realistic assessment of the impact of debris induced damage on structural components; 3) the use of Monte Carlo simulation to assess the variability of results obtained when different components’ properties are considered (Walker, G. R. (2011), Pita et al., (2014)).

With a sample of more than 20000 years of simulated hurricane activity, the HAZUS-MH Hurricane Model (Hazus-MH, FEMA (2009)) can be considered the most well-established simulation-based tool to carry out wind vulnerability assessment (Vickery et al., (2006)). The model contains the first database of surface roughness, which is one of the most critical aspect to consider when modelling wind effects, damage and consequent losses to building components (Vickery et al., 2006). In the HAZUS-MH, the modelling approach focuses on the damage to external structural elements and cladding including roofs, windows, joint failures and wall failures, to both wood and masonry structures. The model uses a load and resistance methodology to estimate damage to structures subjected to extreme wind loads and debris impact. Losses are computed by means of a combined empirical and cost-estimation technique which is based on building damage states governed by the performance of the building envelope, ranging from 0 (no damage) to 4 (complete destruction) (Vickery et al., 2006)).

A research programme aiming at assessing the load capacity of masonry walls subjected to uniform wind and hydraulic loading was conducted by Herbert et al., (2011), to allow the determination of the ultimate load capacity of the wall panels, the crack pattern at failure and the identification of the processes that occur during flood and wind loading. The application of the load was simulated through air bags pushing 1/6 scaled masonry wall panels made of three different types of masonry scaled units (i.e., autoclave aerated blocks, bricks and medium density blocks) cast in place with three different types of mortar (i.e. M6, M4, and M2 classes). The deflection of the wall was allowed to stabilize before the next increment was applied. It was observed that all specimens cracked horizontally, with an initial crack extending along the length of the wall. Given the symmetry in restraining conditions, it was observed that vertical cracks were also following a symmetric pattern, running from the position of the horizontal cracks towards the corners of the wall. This confirms the idealized crack pattern failure predicted by Yield Line Method approach. The main element of innovation added in the simulation was the use of digital image correlation (DIC) to record the specimen displacement both in horizontal and vertical component but also the formation of cracks leading to out-of-plane failure. In particular, the understating
of the exact value of load to be applied to generate the first crack could represent a way to understand
the behaviour of the wall and address in a more meaningful way the repair techniques proposed. The
scope of the experimental testing campaign of Herbert et al., (2011) was to compare the results with
the Yield Line theory detailed in EC6 (CEN (2010)), used to determine the theoretical failure loads.

More recently, a framework to perform the wind vulnerability assessment of priority heritage buildings
in the Philippines was proposed by D'Ayala et al., (2016), which follows very closely the procedure
outlined in the HAZUS-MH Hurricane Model. The physical damage to buildings subjected to extreme
wind is modelled using an engineering-based load and resistance approach. As the wind-induced loads
acting on the building are computed, the physical damage is estimated in terms of failure of the building
envelope components. The limit states of interest are related to the breach of the building roof, which is
closely related to the performance objective to minimize the overall building damage. The two modes
of failure identified are: 1) pull-out failure, in which the fastener is pulled out from the holding member
or material penetrated due to withdrawal load; 2) pullover failure, in which the roof panel fails due to
the shear while the fastener is still intact with the holding member. In fact, once the failure of a single
fastener occurs, the load is distributed to the surrounding fasteners causing the failure to propagate
through the panel; similarly, there is a strong correlation between panel removal and subsequent damage
to the building structure. In particular, the uplift resistance of the panel depends on the properties of the
panels and the fasteners, thus implying that pull-out and pullover functions depend on several variables,
defined as state variables (D'Ayala et al., (2016), Song, B., et al. (2019)). Given the high level of
uncertainty in defining the performance of the roof structural elements, the probability of failure of the
overall roof cannot be determined in close form. It is rather preferred to use a hybrid method that
combines the data collected on site, Monte Carlo simulations of both the resistance of the structural
elements constituting the roof and the two failure modes as defined. Fragility functions defined as the
probability of pull-out/pullover failure versus basic wind speed, where developed for each building
surveyed using the considered limit states.

From the review of the wind vulnerability assessment methods, it appears clear that there is yet no model
developed to assess the behaviour of historic URMs under such loading profile and therefore showing
the knowledge gap which requires to be filled.

The following section deals with outlining the current state-of-the-art approaches for multi-hazard
vulnerability, to the aim of highlighting where the proposed methodology stands and the added novelty
which that could introduce.
2.3. Comprehensive approaches to conduct multi-hazard capacity assessment

The concept of multi-hazard approach made its first appearance in the Agenda 21 Conference in Rio de Janeiro (UNEP, 1992) and then in the Johannesburg Plan (UN, 2002) and the Hyogo Framework for action have put forward relevant perspectives of the “multi-hazard” concept. From there, the initiatives focused on analysing the multiple risks arising from different hazards and affecting the exposed elements have been progressively increasing (Gallina, 2016, Wang et al., 2020).

Multi-hazard vulnerability falls within the remit of the broader concept of multi-risk, for which various methodologies have been developed at national and international level, especially after the sharp increase in natural and man-made disasters (Schmidt-Thomé, 2006). Of relevance to the present study are the studies focusing on determining analytical models that aim at linking the physical action exerted from different hazards to a given asset typology, while also accounting for the fact that the exposed elements could be differently vulnerable to different perils’ actions. However, multi-hazard tools such as HAZUS-MH (FEMA, Hazus Releases, 2007), provide single hazard analysis without providing any analytical measure nor an index that could serve to rank the consequences in terms of damage to the asset typology of interest in a commensurate manner. The MATRIX project (Multi-Hazard and MulTi-RiSsK Assessment Methods) sets out to tackle some of the issues associated with multi-hazard and multi-risk and focusing on earthquakes, landslides, volcanos, tsunamis, wildfires, storms, and fluvial and coastal flooding (Carpignano, A., et al., 2009).

The review papers of Kappes et al., (2012), Gallina, (2015), Tilloy et al., (2019) and Wang et al., (2020) endeavour to clarify some of the challenges related to the topic of multi-hazard risk assessment. From their reviews it appears evident that – depending on the field of application – the wide concept of multi-hazard often narrows down to either the hazard, the exposure, or the vulnerability components of the risk matrix (Tyagunov et al., (2014)) and for each of these subbranches of assessment, various methods and definition can be found (Wang et al., 2020). Moreover, additional complications may come from hazards interrelationships (Tilloy, et al., (2019)) and cascading effects (Kappes et al., (2012)), often leading to difficulties in discerning discrepancies in advancement of single-hazard vulnerability assessment.

The overall outcome of the review has shown that there is lack of homogenised analytical methodologies which aim at establishing mechanical models that correlate the capacity of the system to the intensity of the peril considered, leaving scope for further research, representing the novelty of the research proposed. This works in fact, sits within the framework of multi-hazard vulnerability assessment as it aims at keeping the perils independently acting against the asset typology of interest, whilst keeping the focus on the damage caused by the external action onto the structural typology of interest.
The following sections introduce very briefly the concept of fragility curves as the tools to quantify the probability of damage as a function of the intensity measure of the hazard of interest, a review of the most appropriate fitting techniques to generate these functions to conduct multi-hazard assessment, and a review of the main parameters needed to define fragility curves for seismic, flood and wind hazard appropriately tailored for historic URMs.

2.4. Review of fragility functions for masonry structures

To effectively conduct vulnerability assessment, fragility curves are needed, with the aim of predicting the structural performance of the buildings analyzed for a given hazard scenario.

In synthesis, fragility functions represent the probability that the specific asset class will reach or exceed predefined damage states, given a range of hazard intensities. Mathematically they can be expressed as:

\[ P(DS \geq d_{s_i}|IM) \]  

where:
- P is the probability of reaching/exceeding a given limit state.
- DS is the damage state of the asset class being assessed.
- \( d_{s_i} \) is the pre-defined damage state.

To define fragility functions based on analytical structural assessment, the three main parameters that need to be defined are:
- The intensity measure (IM) which represents the hazard loading on the structure.
- The Damage States (DSs) that can capture the potential mechanisms causing failure to the structure(s) under consideration. Each DS indicates a threshold of a given engineering demand parameter (EDP) chosen for the assessment.
- The function linking the probability of reaching/exceeding the DS at the given IM.

In a multi-hazard framework, as the IMs are strictly dependent on the type of hazard considered, an effort must be made to find representative EDPs, and therefore the DSs, that is able to describe the behavior of the structures undergoing damage under different loading profiles while allowing for comparison across different perils.

2.4.1. Intensity Measures (IMs)

With reference to the identification of the seismic hazard of a region, the two different approaches that can be used are 1) probabilistic seismic hazard approaches (PSHA) (Cornell (1968)) and 2) deterministic scenario hazard approach (DSHA) (Costa, (1993)), directly correlated to the events occurred in the region. The latter typically assigns a maximum earthquake magnitude for a particular seismic source, often referred to as the ‘maximum credible earthquake’, and, based on the minimum
distance from the site to the fault source, the level of ground shaking at the site is estimated (Woo, (2002)). DSHA approaches develop earthquake scenarios defined by location and magnitude to derive ground motions at the site, from which the controlling event is determined, by means of attenuation relationships. These approaches are deemed to be more relevant for the assessment of unreinforced masonry structures, mainly because, if the seismicity of the area is well known, a credible and reliable earthquake scenario can be developed and directly correlated to the building stock performance (D’Ayala & Ansal, (2012)). The definition of the criteria for damage estimation are mainly dependent upon the type of structural model used or the numerical approach employed for structural analysis (Jaiswal, Wald, & D’Ayala, 2011). In terms of IMs, the ones generally used to derive analytical fragility functions for masonry structures are:

- Peak ground acceleration (PGA) used in the case of acceleration-sensitive structures such as masonry structures (Rota, M., Penna, A., & Magenes, G. (2010), D’Ayala, D. F. (2005), Lagomarsino S and Cattari S (2014)).
- Peak ground velocity (PGV) preferred in cases of intermediate-period structures.
- $\text{Sa}(T) / \text{Sd}(T)$ (spectral acceleration / spectral velocity) used when assuming the structure as a single-degree-of-freedom (SDoF) system, characterized by period (T) and damping (Putrino, V., & D’Ayala, D. (2019)).

One of the major issues affecting the definition of reliable analytical fragility relationship in the case of flood is associated to the lack of completeness, misclassification or insufficient sample size that very often characterizes post-flood loss and damage databases (Pregnolato et al., (2015)). The most common types of intensity measures (IM) reported in literature and relevant to the purpose of this work are water depth (m) and velocity of flow (m/s) (Herbert et al., (2018), De Risi et al., (2013)). Among the listed, water depth is the most common variable due to the ease of measurement in the field, the reliability of the output from the numerical inundation simulation standpoint, reportedly being considered a straightforward mean of correlation with the damage to structural and non-structural components of the buildings affected.

Quantitative means of damage estimations able to relate the wind speeds to the five hurricane categories specified in the Saffir-Simpson (SS) Scale for structural engineering purposes, have been provided by the Commentary of ASCE 7 Standards (2013), in which is adopted the definition that each SS category is associated with 1-min average wind speed in the scale (Pinelli et al., (2004), Simiu et al., (2006), FEMA (2009)). Wind speed is the IM mostly adopted in literature, hence the one considered most suitable for the current study. Other estimates were provided by Simiu et al., (2006) such as the ratio between peak 3-s gust speed at 1-min wind speeds at 10 m above open-terrain exposure to 1-mins speed at 10 m above open water.
2.4.2. Engineering Demand Parameters (EDP)

Engineering Demand Parameters (EDPs) are structural response quantities that are used to estimate damage to structural and non-structural components and systems (Whittaker, A., Deierlein et al., (2004)). EDPs can also be interpreted and used as damage descriptors of the given building typology undergoing a given hazard condition. By simulating the behavior of the structure investigated under seismic ground motion, hydrostatic or wind pressure, it is possible to describe the structure response in terms of deformations, accelerations or other response quantities, which can be correlated to Damage Measures (DMs) (i.e. means to describe the condition of the structure resulting from the imposed demand) to determine Decision Variables (DVs) to ‘quantify’ the damage and to interpret it within a more general risk management decision framework.

According to FEMA P-58-6 (2012a), the most used EDPs in the case of seismic vulnerability assessment are 1) peak transient story drift; 2) residual story drift; 3) peak floor acceleration. All three EDPs are computed with respect to the two orthogonal directions.

Depending on the type of action the structure is undergoing (i.e., whether is shear or flexure), the choice of the parameter that best represents the deformation demand in structural and components terms may vary (Lotfy et al., (2019)).

In flexural-dominated walls, the wall aspect ratio can have significant influence on the fragility parameters. Lotfy et al., (2019) have proposed a slightly modified version of EDP defined as a normalized deformation ratio given as a ratio of the lateral drift to the wall aspect ratio.

In flood vulnerability assessment there is no such clear definition of EDP as in the case of seismic assessment. With the aim of assessing the structural vulnerability against debris flow, Haugen, E. D., & Kaynia, A. M. (2008) proposed a model that follows the HAZUS principles developed for seismic assessment. More specifically, the EDP adopted is the structure’s maximum lateral displacement caused by debris flow impact and that is used for the derivation of fragility curves.

In the work conducted within the European FP7 project Climate Change and Urban Vulnerability in Africa (CLUVA) Jalayer et al., (2014), De Risi et al., (2013) defined a structural performance variable which is given as the ratio between the critical demand exerted by the load to the capacity of the system, indicated as Y, a measure of strength. For incremental values of critical water height defined as $h_Y$, the resulting values $Y$ are compared thus providing $Y- h_Y$ data points that form the incremental flood height analysis curve, used to found by interpolation, the critical water height value corresponding to the onset of the limit state identified as $Y=1$.

In their experimental campaign conducted to test the effect of OOP hydrostatic loading on single skin masonry walls, Herbert et al., (2018) consider the OOP deflection at the center of the panel to evaluate the cracking and to relate the corresponding water level causing collapse.
Very limited work was found in literature on the assessment of wind vulnerability of masonry buildings. Most available studies focus on the vulnerability of the roof and its connections to the building envelope rather than on the damage caused to the main structural components. The only available study found to date Herbert et al., (2011) refers to an experimental campaign of scaled 1/6 masonry walls made of bricks and blocks tested against uniform wind loading and non-uniform hydraulic load. As per the flood loading mentioned in previous section, the authors consider the OOP deflection at the centre of the panel to evaluate both cracking and corresponding magnitude of wind loading causing collapse.

In a multi-hazard fragility assessment framework, such as the one proposed for this work, the aim is to ‘compare’ the investigated building typology’s performance undergoing different external loading conditions and to rank the results in a commensurate manner. The choice of the most appropriate EDP also depends on the type of mechanical model adopted to determine the performance of the system and consequent failure mode. The possible choices considered most suitable for the are discussed in Chapter 4. An EDP defined in terms of strength performance (i.e., force) rather than displacement, suits better the purposes of multi-hazard vulnerability assessment frameworks as it is easier to define across different perils, although limiting the understanding of the ductility of the system investigated.

2.4.3. Structural Taxonomy

In order to assess the impact of any hazard on the built environment, it is essential to know the structural systems of the typology investigated and their performance under the considered hazard, the engineering standards adopted and the location and distribution of vulnerable assets in the area of interest.

A very detailed review of available global building inventories for earthquake loss estimation and risk management was provided by Jaiswal, et al., (2010). In their work, the source data are classified according to three classes of ‘quality’ namely, high, medium and low. ‘High’ quality refers to data compiled by engineers through field visit and/or relying on local engineering experts; ‘medium’ quality data, refers to data based on general field data and not based on engineering standards, ‘low’ quality, refers to data coming from non-engineering agencies.

The Earthquake Engineering Research Institute (EERI), the World Housing Encyclopaedia (WHE), the U.S. Geological Survey (USGS) and the Prompt Assessment of Global Earthquake for Response (PAGER) have compiled ‘high’ quality databases with the aim of improving the understanding and classification of the building inventory and collapse vulnerability of non-US construction types worldwide (D’Ayala et al., (2010)). More specifically, the World Housing Encyclopaedia developed by EERI focuses on residential buildings in terms of construction type and occupancy and covers more than 37 counties and 110 residential construction types (Jaiswal, K., & Wald, D. J. (2009)).
To the ‘medium’/‘low’ category belongs the UN Database (UN, (1993)), with data for more than 44 countries and a catalogue of exterior walls materials, and the Census of Housing made of data compiled from housing census statistics.

Similar effort in compiling the HAZUS-MH building stock database was carried out by the U.S. FEMA (Barthel et al., (1998)). This taxonomy was built upon the building classes proposed within the framework of rapid visual screening with the aim of evaluating life-safety performance level and immediate occupancy.

As part of the Global Earthquake Model (GEM) Brzev et al., (2013) provide a scheme for classification of buildings worldwide, based on 13 main parameters which define the ‘genome’ of the building taxonomy. Among the listed characteristics there are: 1) orientation of the building, 2) material of the lateral load-resisting system, 3) lateral load-resisting system, 4) height, 5) date of construction or retrofit, 6) occupancy, 7) position within the block, 8) plan shape, 9) structural irregularities; 10) material of exterior walls, 11) roof (material, attributes, covering), 12) floors (material, attributes, covering), 13) foundation (material, attributes, covering).

To date, the development of building taxonomies for flood vulnerability assessment is still at its very early stages. Based on the findings coming from the earthquake engineering field (Brzev, et al., (2013)), Blanco-Vogt et al., (2015) proposed a methodology framework for the assessment of flood vulnerability and damage to buildings and critical infrastructures in Dresden, Germany, by means of remote sensing data and GIS analysis. Among the planimetric and elevation information, the building size, presence of elevation plinths, elongation, roof form, adjacency with other buildings (how many sides exposed to open space) and compactness (in case of clusters) are the main factors to include.

An important and challenging task of wind hazard assessment relates to the understanding and computation of damage caused by the wind action to the building envelope (Baheru et al., (2012)). To date, great emphasis is given to understanding the damage to roofing elements and to the connections between roof and building envelope, as it is perceived that the damage to the weakest link causes the un-usability – and hence the damage - of the whole structure (Song, et al., 2019). However, after the breach in the building envelope, it is reportedly known that wind forces can effectively continue to create damage until full collapse of the building is reached (Pita et al. (2012)). No wind-tailored taxonomy was found in literature to define the vulnerability of the building envelop to wind loading for masonry structures.

Finally, an example of building taxonomy developed with the aim of conducting multi-hazard risk assessment is the GED4ALL taxonomy by Silva et al., (2018), an exhaustive classification system considering various natural hazards, namely earthquake, volcanoes, floods, tsunamis, storms, cyclones and drought, (Dabbeek, J., & Silva, V. (2020)). According to this study the 9 building physical features
relevant for multi-hazard risk assessment are 1) the type of lateral load resisting system and material, 2) the date of construction, 3) the height of the building, 4) the roof shape and material, 5) the wall material, 6) the presence of openings and in particular number, size, and type of opening; 7) the ground floor hydrodynamics and plan orientation; 8) the foundation type and the 9) occupation type.

As highlighted in this section, the level of accuracy of the existing taxonomies developed to define the vulnerability of URMs to seismic, flood and wind hazards is both very disproportionate and non-homogenised. Inventories for earthquake loss estimation are arguably more detailed than others for flood and wind. The reasons for such a discrepancy relate to a) the fact that in the case of wind, great attention is usually paid to the roof system rather than to the damage that the wind loading can cause to the walls, and b) in the case of flood the current efforts to established simplified procedures looking at defining the vulnerability of URM walls to the impact of water is still not as much investigated as it is in the case of seismic hazard.

The ‘genome’ provided in the GEM taxonomy represents the most comprehensive effort available in literature. However, in an analytical approach to multi-hazard vulnerability, the geometric characteristics of the walls, the opening layout and a detailed understanding of the material properties of the masonry units (bricks/block) and mortar represent the minimum requirements needed to ‘define’ the system-wall. As the primary elements needed to characterize the system-wall are established, all that relates to the definition of the system-wall’s boundary conditions becomes necessary to understand the damage extent the system can sustain and how it is going to crack. To this end, the type of connection to the foundations and to the roof system, the connection with adjacent walls and the presence of restraining elements is essential data. On the basis of this review, Chapter 6, section 6.2 provides the detailed list of parameters pertaining to geometry, material and constraints, deemed necessary to develop the proposed analytical procedure.

### 2.4.4. Fragility functions derivation method

The function used to model the damage fragility to earthquake, flood and wind is the lognormal Cumulative Distribution Function (CDF), reportedly found to provide a good representation of earthquake damage fragility (Singhal, A., & Kiremidjian, A. S. (1996)) and with significant number of precedents for seismic risk modelling (Kennedy et al., (1980), Lallemand et al., (2015), Bradley, B. A., & Dhakal, R. P. (2008)). As examples of lognormal distributions working well to obtain cumulative fragility functions for large samples of buildings to determine their resistance to seismic action see Putrino, V., D’Ayala, D. (2019). The main advantages in preferring CDF functions over other types of functions are the constrain that the probability of reaching the damage state considered is bounded between 0 and 1, thus assuring that all IMs considered are greater than 0 and that no-damage corresponds to IM equal to 0. This becomes particularly relevant to the current study, in which only one damage state
is considered, namely collapse. Lognormal cumulative distribution functions are characterised by median value and standard deviation and appears in the following mathematical form:

\[ P(\text{DS} \geq d_{s_i}|\text{IM}) = \Phi \left( \frac{\ln(\text{IM}) - \theta_{\text{DS}|\text{IM}}}{\beta} \right) \]

With \( \Phi \) being the standard normal cumulative distribution function, \( \theta_{\text{DS}|\text{IM}} \) the lognormal mean of the generic structural response conditioned to the IM, and \( \beta \) is the lognormal standard deviation of \( \text{DS|IM} \). The dispersion term \( \beta \) is directly linked to the various uncertainties that are contained in the prediction of damage state DS given an IM, which can either be aleatory or epistemic (Bradley, B. A. (2010), Dolšek, M. (2012)). As discussed in D’Ayala et al., (2015), the dispersion parameter \( \beta \) can be decomposed into specific uncertainty sources, which can be categorised as follow:

- Uncertainties in the demand imposed to the system \( \beta_{\text{demand}} \).
- Uncertainties related to the definition of the capacity of the system such as modelling uncertainties or variability in the mechanical or geometrical properties \( \beta_{\text{capacity}} \).
- Uncertainties associated with the definition of the damage thresholds \( \beta_{\text{DS}} \).

The single sources of uncertainties should be combined to obtain a total dispersion \( \beta \) of the fragility functions, which according to D’Ayala et al., (2015), can take the form:

\[ \beta = \sqrt{\beta_{\text{demand}}^2 + \beta_{\text{capacity}}^2 + \beta_{\text{DS}}^2} \]

When fitting a lognormal distribution to a set of observations (such as the number of collapse cases given a value of IM), the goal is to identify the lognormal distribution parameters \((\theta, \beta)\) so that the fitted distribution predicts probabilities that are consistent with the observed fractions of IM causing collapse at each IM level. The specific choice of IM, damage scale and regression analysis provide a base for the classification of the obtained fragility curves. To obtain the parameters of the lognormal distribution, several methods have been considered.

Based on the main assumption that the IM at the damage threshold of interest (e.g., collapse), is known for all the data points, the Method of Moment (MM) can be used. According to this method, the parameters such as mean and standard deviation found are such that the resulting distribution has the same moments as the sample moments of the data points (Baker, J. W. (2015), Lallemant et al., (2015)). The MM fitting technique is preferred when testing experimental samples until failure and very often used when conducting Incremental Dynamic Analysis (IDA) (Vamvatsikos, D., & Cornell, C. A. (2002)), a procedure used to assess the structural performance under seismic loading which produces a set of IM values associated with the onset of collapse for each ground motion. The probability of
collapse at a given IM level, $im_i$, can be estimated as the fraction of records for which collapse occurs at a level lower than $im_i$. The CDF curve describing the resulting collapse fragility curve is given as follows:

$$P(C|IM = im_i) = \Phi \left( \frac{\ln (im_i/\theta)}{\beta} \right)$$

More specifically $P(C|IM = im_i)$ is the probability that a given hazard with IM = $im_i$ will cause the structure to collapse; $\Phi$ is the CDF; $\theta$ is the IM level with 50% probability of collapse and $\beta$ is the standard deviation of ln IM. Values of $\theta$ and $\beta$ are found through the following equations:

$$\ln \theta = \frac{1}{n} \sum_{i=1}^{n} \ln IM_i$$  

$$\beta = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (\ln(IM_i/\theta))^2}$$

With $n$ being the generic set of analysis considered tested to failure, and IM, being the IM associated with the onset of collapse for the $i$th value of IM considered.

Another way to fit a fragility function to bounding EDP data is to perform a Sum of Square Error (SSE) fit to the binary failure data, which allows to estimate the parameters of the fragility curve without estimating the sample moments (as required in MM method). The error $\varepsilon^2$ between the probabilities predicted by the fragility function and the fractions observed by the data is defined as in Eq. 2.7:

$$\varepsilon^2 = \frac{1}{n} \sum_{i=1}^{n} \left( f_i - \Phi \left( \frac{\ln(im_i/x_m)}{\beta_r} \right) \right)^2$$

With $n$ being the generic set of analysis considered tested to failure; $f_i$ being the failure indicator for specimen $i$; $im_i$ being EDP to which specimen $i$ was subjected; $x_m$ being the estimated median EDP and $\beta_r$ being the logarithmic standard deviation of the fragility function given a particular value of $x_m$.

According to this method, the aim is to find $x_m$ and $\beta_r$ such that the error $\varepsilon^2$ is the minimum (Porter, K., Kennedy, R., & Bachman, R. (2006)). Mean $\theta$ and standard deviation $\beta$ are defined as reported in Eq. 2.8:
\[
\{\theta, \beta\} = \arg\min_{\theta, \beta} \sum_{j=1}^{m} \left( \frac{z_j}{n_j} - \Phi \left( \ln \left( \frac{im_j}{\theta} \right) / \beta \right) \right)^2
\]

2.8

with \( \frac{z_j}{n_j} \) being the observed ratio of collapses at IM=imj. This method transforms the observed fractions of collapse so that the linear regression can be used to estimate the fragility function parameters. Among the main advantages, the SSE method is based on the actual EDP values in the data set, and it has been proven to be rather stable even with a reduced number of points (Cornell, C. A., et al (2002)). However, as noted by Lallemant et al., (2015), a model that minimizes the least square error implies that the normal distribution of the error prediction and the dispersion in the error (i.e., the expectation of the squared deviation of the random variable from its mean) are independent from the prediction itself. These two implications could further result in poor fit of the curve in both low and high portions of the resulting fragility curve.

A further comparison is made with the Maximum Likelihood Estimation method (MLE). The main concept behind the MLE approach is to estimate the parameters of the distribution that maximize the probability of occurrence of the observed data. MLE was extensively used and validated in binomially distributed cases (i.e., exceedance/non-exceedance data or data bounded between 0 and 1) (Shinozuka et al., (2000), Baker, J. W., & Cornell, C. (2005), Straub, D., & Der Kiureghian, A. (2008)), thus including the collapse/no collapse case which is of relevance for the current study. The main assumption of this technique is that at each IM level the structural analysis produces some number of collapses out of a total number of IM considered (i.e., ground motion, height of the water, wind speed), thus introducing some level of uncertainty in the determination of the IM causing the given damage state (Lallemant et al., (2015), Baker, J. W. (2015)). Moreover, the underlying assumption of the MLE is that the system investigated remains intrinsically the same (i.e., very limited modelling dispersion), and the number of collapses computed within the total number of analysis will only depend on the variation of the IM under consideration.

The review proposed in this section serves as background study for the following Chapters 4 dedicated to the methodology framework and Chapter 7 dedicated to present the rationale to derive single-hazard fragility curves, the latter of which also includes the reasons in support to the regression method considered the best fit for this study.

2.5. Conclusions

The main aim of this chapter is to identify the current state-of-the-art analytical methods to assess the vulnerability of URM to earthquake, flood, and wind, thus identifying studies which focus on
correlating the hazard component to the damage extent, intended as a loss of performance in structural terms. Particular attention has been paid to simplified methods over more complex and computationally expensive procedures as they are easier to tailor and based on a limited number of parameters needed to be defined. Given the substantial discrepancy in the level of development and the detail of analysis achieved in single-hazard vulnerability assessment procedures, the review has highlighted the common aspects needing consideration when aiming to define a homogenized analytical procedure such as the one proposed in this study. The review has shown that numerous studies are available in the field of seismic vulnerability assessment of non-engineered structures, while works in the field of flood and wind vulnerability assessment are still very modestly advanced, challenging the development of comprehensive analytical methods able to conduct multi-hazard vulnerability assessment and to carry out the analysis in a commensurate manner across these hazards.

Capacity Spectrum Methods that aim at linking the seismic IM to the building damage through quantitative parameters representative of the building capacity have proven effective in accomplishing the purpose with limited computational effort and therefore allowing to be applicable to large building stocks. Among the CSM, macro-element approaches-based limit state analysis concepts are the most suitable especially when aiming to develop mechanical models to represent the global masonry behaviour, while giving the chance to be further refined to simulate the ‘discrete’ nature of masonry. Such concepts, already thoroughly investigated in the field of earthquake engineering, can easily be extended to flood and wind vulnerability assessment, thus being a favourable option towards the homogenisation required to develop multi-hazard vulnerability frameworks.

The review of the available methods for multi-hazard vulnerability assessment specifically tailored to assess masonry structures has shown an existing gap. The general concept of defining a common analytical parameter to use for comparison as a ‘measure’ to correlate IM and level of fragility across different perils allow not only to keep the different hazards independent, but also to extend the procedure to other perils which are currently not included in this work.

The review of the main ‘ingredients’ needed to derive single-hazard fragility functions proves that homogenization is required among the three perils considered, to be able to find a common ground for comparison. As far as the intensity measure (IM) is concerned, there is the need of defining a common Engineering Demand Parameter (EDP) to link the hazard component to the level of damage caused to the structure, to then be able to conduct fragility assessment.

The three main gaps emerging from this review can be summarised as follows:

1) The lack of a ‘common’ analytical assessment method able to describe the behaviour of unreinforced masonry structures undergoing different types of horizontal loadings such as earthquake, flood and wind. This requires a review of the current state-of-the-art structural assessment methods, which
will be conducted in Chapter 3, with the aim of setting the scene for the analytical method proposed in this study.

2) The current lack in the definition of an engineering demand parameter (EDP) common to the three perils, which validly allows for the derivation of comparable single-hazard fragility curves. More specifically, such EDP must be a quantitative parameter able to relate the IM to the level of damage and – since it comes as the output of the analytical method proposed – it is thoroughly investigated in Chapter 5.

3) Finally, regarding the choice of the regression method for the derivation of single-hazard fragility curves, the review provided in Section 2.4.4 has highlighted the advantages and limitations of three of the main fitting techniques considered the most suitable for the derivation of fragility curves for seismic, flood and wind hazard, namely the Method of Moments (MM), the Sum of Square Error (SSE) and the Maximum Likelihood of Estimation (MLE), setting the premises to define the adopted method, which will be thoroughly discussed in Chapter 7.

The next chapter focuses on providing with a review of the state-of-the art methods for the prediction of the ultimate load capacity of URMs under horizontal out-of-plane loading, which represents the second part of the literature review.
3. Methods for the prediction of the ultimate load capacity of URMs under horizontal OOP loading. A review.

3.1. Introduction

When Unreinforced Masonry (URM) buildings are subjected to lateral loading due to seismic shaking, floods, or windstorms, they experience a combination of IP and OOP response as shown in Figure 3-1.

![Diagonal Bending](image)

**Figure 3-1:** Typical failure patterns in URMs subjected to lateral loading

The orientation of the internal stresses within a wall and the resulting crack pattern developed is dictated by the load position, the type of restraint conditions and the wall aspect ratio. In the case of weak connections with side edges, the walls subjected to out-of-plane loading undergo uniaxial bending resulting in cracks that run parallel to the panel’s supports and the axis of internal bending. If the connection with side edges is strong, the walls undergo biaxial bending, whereby the internal flexural stresses act in both the horizontal and vertical directions. As a result, two-way walls develop crack patterns which exhibit a combination of vertical, horizontal, and diagonal crack lines. Consequently, the internal moments along the different types of crack lines consist of a combination of flexure (normal stresses) and torsion (shear stress). For completeness, Figure 3-1 also reports the type of cracks developed by walls subjected to in-plane loading, including vertical settlements, as identified by Giardina et al., (2012), although these are not the focus of this research study.
Given the complexities in defining the composite actions that determine the formation of a diagonal crack, most of the available methods assume that diagonal moments result from a composite contribution of horizontal and vertical moments, to which the stabilizing action of the wall weight is added. As far as the method of analysis to calculate these moments and forces is concerned, the assessment of the panel strength is dependent upon the criterion adopted for the failure of the material. A more detailed overview of the different failure criteria available in literature, and especially of the ones that account for the torsional component in the determination of the moment capacity of diagonal cracks, is provided in Section 3.2.

As the side related to the mechanics of masonry material is discussed, section 3.3 provides a review of the relevant analytical assessment methods available in literature which are used to model the capacity of URM walls subjected to out-of-plane loading. Finally, Section 3.4 is dedicated to draw the conclusions of this chapter and of the literature review section of the thesis.

3.2. Masonry failure criterion and interaction between flexural and torsional failure

As far as the panel dimensions and the support conditions are concerned, URMs (brick or block) walls subjected to OOP loading experience failure modes which depend on the strengths of the units and the mortar bond (Page, A. W. (1981), Griffith, M. C., Lawrence, S. J., & Willis, C. R. (2005)).

![Figure 3-2: Mechanics of diagonal bending as a combination of vertical bending, horizontal bending and torsion along the bed joints.](image)

Figure 3-2 shows the diagonal bending as a combination of flexure (vertical and horizontal) and torsion. Whilst there is no difficulty in the determination of the distribution of moments and the failure criterion for brickwork spanning in just one direction (Duarte, R. B. (1993)), at present, no accurate mathematical solution is available that can predict cracking or failure of laterally loaded masonry panels under biaxial bending, thus hindering the development of a failure criterion able to describe the complex interaction of horizontal and vertical bending and to quantify the distribution of moments throughout these structures. The traditional failure criterion of Tresca, Von Mises and Rankine are mainly applicable to ductile or, to some extent, brittle isotropic materials (i.e., characterized by same moment capacities in
the two orthogonal directions), while proved to be not applicable to masonry in bending, due to its brittle nature characterized by strength and stiffness orthotropies (Sinha et al., 1997).

The main problem arising when investigating the bi-axial flexural behaviour of wall panels subjected to lateral loading pertains to the determination of the real distribution of moments in a highly redundant structure like an anisotropic plate, which is found to be characterized by a limited – but not negligible – tensile strength capacity but also – and more importantly – by the shear strength capacity.

The initial assumption of no interaction between principal moments at failure and orientation of the joints was first questioned in the late ’70 by Baker, L. R. (1982) who attempted the definition of a failure criterion for brickwork, by subjecting single joint to vertical and horizontal moments simultaneously. With his experiments, he proved that – after cracking – brick panels exhibit a reserve of strength, which is due to the redistribution of moments from the weaker vertical direction to the stronger horizontal direction (Duarte, R. B. (1993)). In his experiments, he demonstrated that the failure criterion for combined vertical and horizontal moments is given approximately by an elliptical expression (Eq. 3.1), clarifying that the assumption of no-interaction was both incorrect and unconservative.

\[
\left( \frac{M_v}{M_{vc}} \right)^2 + \left( \frac{M_h}{M_{hc}} \right)^2 = 1 
\]  

With \( M_h \) and \( M_v \) being the moments at failure in biaxial bending and \( M_{hc} \) and \( M_{vc} \) being the uniaxial moment capacities.

Baker’s model however did not account for the stiffness orthotropy, namely the influence of the ratio of the modulus of elasticity in the two orthogonal directions. Sinha, B. P., et al (1997) proposed an updated version of the biaxial failure envelope based on an experimental program on small specimen tests and simultaneous application of both vertical and horizontal moments and results were compared to a twin finite element method model, built considering masonry as a linear homogeneous orthotropic material. Two sets of wallets were tested under a 4-point loading system to obtain the strength in the two orthotropic directions, namely normal and parallel to bed joints. Rankine maximum stress theory was used to obtain the failure load (Fenner, R. T. (1989), Sinha, B. P., et al (1997)), according to which failure takes place in the direction that reaches its ultimate moment of resistance first. A refined failure envelope which shows the enhancement of the strength in the weaker direction (i.e., vertical) was proposed and reported in Eq. 3.2.

\[
\left( \frac{M_v}{M_{vc}} \right)^2 - 0.75 \frac{M_h}{M_{hc}} \left( \frac{M_v}{M_{vc}} \right)^2 - 0.25 \frac{M_h}{M_{hc}} \frac{M_v}{M_{vc}} + \left( \frac{M_h}{M_{hc}} \right)^2 = 1 
\]  

3.2
The shape of the envelope is highly less conservative than the one proposed by Baker and goes beyond the case of no interaction as shown in Figure 3-3, retrieved from Sinha, B. P., et al (1997), and indicating $\frac{M_c}{M_{cc}}$ as the weaker direction and $\frac{M_h}{M_{hc}}$ as the stronger direction. Sinha’s model – however – did not account for any torsional strength component in the definition of the envelope.

![Figure 3-3: Biaxial failure envelope proposed by Sinha, B. P., et al (1997)](image)

The first mention of the importance of considering torsional strength in determining the horizontal bending strength of masonry is to be attributed to Candy, C. C. (1988), whose work was later continued by Samarasinghe, W., & Lawrence, S. J. (1994). Through their tests on brick-mortar couplets set to capture the behaviour of bed joints subjected to torsion about an axis normal to the bed face with various levels of pre-compression stress, they found out that a considerable resistance is provided by the torsional shear capacity of bed joints. In their paper they did not provide with an updated equation of the failure envelop, however they found out that, after cracking, the residual torsional moment capacity is a function of the normal stress and that the torque resistance capacity of a bed joint increases linearly as the pre-compression stress increases.

More focused on determining the displacement in the diagonal failure lines as a combination of bending torsion and shear, the experimental tests conducted by Hagsten, L.G. & Nielsen, M.P. (2000) on the interface between bricks and mortar of walls subjected to out-of-plane loading, have proven that the ductility of the brick panels steams from the diagonal cracks, which are also recognized to form after horizontal cracks and usually at the same time of vertical cracks, thus fundamental to determine the overall strength capacity of the panel.

In his PhD thesis, Willis, C. R. (2004) first individuated the four contributions to the diagonal moment capacity $M_d$ in the direction parallel and perpendicular to bed joints, namely the flexural tensile strength of the bed joint and of the head joint, the torsional capacity of the bed joint and of the head joint, and
then modelled their interaction in consideration of Hagsten, L.G. & Nielsen, M.P. (2000) experiments, which highlighted that torsion and bending are influenced by the angle of orientation of the masonry section (Brincker, R. (1979)). For a single mortar join under diagonal bending, as the level of flexure increases, tensile stresses are induced on the plane normal to the torsional behaviour, decreasing as a result the potential torsional capacity of the bed joint, and vice versa. It has also been demonstrated that, for the torsional behaviour of a bed joint, the ultimate shear stress increases approximately linearly with both bond strength and compressive stress, thus making the expected linear interaction between flexure and torsion of a mortar joint, effectively difficult to predict. Willis proposed three types of interaction of the mechanisms contributing to the diagonal moment $M_{di}$, namely no interaction, linear interaction, and elliptical interaction — a representation of which is provided in Figure 3-4.

![Figure 3-4](image)

**Figure 3-4** Biaxial bending failure criterion with three envelopes: no interaction, elliptical interaction, linear interaction, retrieved from Willis, C. R. (2005)

As the name suggests — no interaction implies that the flexural behavior has no effect on the torsional behavior: in this case failure occurs once either of the two moments reaches its respective capacity; there is linear interaction if the sum of the ratios of flexural and torsional moment to respective strengths is unity at failure as reported in Eq. 3.3.

$$\frac{M_j}{M_{j(u)}} + \frac{T_j}{T_{j(u)}} = 1 \quad 3.3$$

And finally, there is elliptical interaction if the sum of the squares of the moments over strengths is unity at failure as reported in Eq. 3.4.

$$\left(\frac{M_j}{M_{j(u)}}\right)^2 + \left(\frac{T_j}{T_{j(u)}}\right)^2 = 1 \quad 3.4$$

This implies a mutually weakening influence between the orthogonal moment capacity at the point of failure.
Building on Willis’ work, Vaculik, J. (2012) proposed a generalised symmetric failure envelope described by Eq. 3.5:
\[
\left( \frac{M_j}{M_{j(u)}} \right)^{1/n} + \left( \frac{T_j}{T_{j(u)}} \right)^{1/n} = 1
\] 

3.5

Where \( n \) can assume any value greater than 0, as \( n=1 \) represents the linear envelope and \( n=0.5 \) gives an elliptical envelope. As \( n \) tends to 0, the weakening effects of the two orthogonal failure modes become less significant and \( n=0 \) represents the case of no interaction between failure modes, where failure occurs once either moment reaches its respective capacity.

Vaculik’s findings were reconfirmed by laboratory experiments of more recent studies such as Graziotti, et al. (2019), Sharma, et al., (2021), who conducted novel characterization tests to evaluate the response of masonry bed joints under combined torsion and compressions. Although the merit of the latest works mentioned is to have refined and perfectioned the techniques to better characterize the mechanics of the problem, the core assumptions related to the interaction between torsion and flexure remain substantially unvaried, proving that not much improvement has been made on the specific topic of the definition of a more refined masonry failure criterion.

This section has endeavoured to investigate the mechanics of the brick/block units of URM walls subjected to lateral loading and undergoing to biaxial bending. These walls prove to behave like plates, delimited by diagonal cracks which are characterized by a complex interaction between flexure and torsion. Through experimental tests and continuous research, these interactions were then translated into equations, which constitute the body of failure envelopes presented in this section.

In the following section, a review of the methods to assess the OOP failure of masonry walls is provided.

3.3. Modelling techniques and methods to assess the OOP failure of URM walls

It is widely known that masonry is a construction material exhibiting distinct directional properties due to its anisotropic nature and the fact that is an assemblage of units and mortar joints, which act as a plane of weakness (Lourenço, P. B. (2002), Cecchi et al., (2007)). The approach towards its numerical representation may focus on the modelling of individual components, namely units and mortar, which is indicated as micro-modelling approach, or the masonry as a composite, which is instead indicated as macro-modelling. Depending on the level of accuracy and the simplicity desired, it is possible to choose either 1) detailed micro-modelling, 2) simplified micro-modelling or 3) macro-modelling techniques. With the first approach unit and mortar in the joints are represented by a continuum element, whereas the unit-mortar interface is represented by a dis-continuum element. The material properties of each individual component are accounted for and it is assumed that the interface between unit and mortar
represents the crack/slip plane. With this approach the interaction of units, mortar and interface are studied under a magnifier lens. The second approach assumes that each joint – consisting of mortar and two unit-mortar interfaces - is lumped into an average interface, whilst the units are kept as single elements. Masonry is therefore considered as a set of elastic blocks bonded by fracture lines at the joints. The third approach does not distinguish between individual units and joints but treats masonry as an anisotropic homogeneous continuum.

Depending on the choice of the modelling approach, the level of information needed to define the model and the computational effort required, there are several methods that can be used for the prediction of either the ultimate displacement or the ultimate load bearing capacity of URM walls loaded out-of-plane, the former belonging to the displacement-based formulations, the latter to the force-based formulations (Ferreira et al., (2014), Godío, M., Beyer, K. (2018)), which are of relevance to this study.

Although computationally very intensive, methods based on finite element analysis have seen an increase in use throughout years, especially when conducting detailed micro-scale analysis. However, they rely upon knowledge of precise values of material properties, they are difficult to validate due to the lack of availability of comprehensive experimental results, but also due to the intrinsic complexity in formulating anisotropic inelastic behaviour of masonry, which often require the adoption of different inelastic criteria for tension and compression (Lourenço, P. B. (2002), Agnihotri, et al., (2013)).

As reported in Section 2.2.1, DE methods have proven reportedly more advantageous than FE methods, as they do not require the definition of the homogenized ‘simile’ material, allowing to maintain the discrete nature of the masonry assemblage of units and to control the constitutive behaviour through the springs placed within the layers of contact between units. To the family of DEM belongs the AEM, whose main characteristics have been widely discussed in Chapter 2 and also the Macro-Distinct Element Model (M-DEM), which have recently been employed for the simulation of in-plane cyclic behaviour of URMs as reported in Malomo, D., DeJong, M.J. (2021). In the M-DEM framework, FE homogenized macro-blocks are connected by discrete spring interfaces, whose layout is determined a priori as a function of the masonry texture. In-plane and diagonal shear failure mechanisms, as well as flexural damage are accounted for by the discrete spring interface.

Along the same line, Pantò, B., et al., (2017) proposed a 3D macro-model intended to simulate the combined in-plane and out-of-plane behaviour of URM walls against experimental results and FEM simulations (Lourénço, et al., (1997)). The model, which represent a nontrivial extension of a plane macro-element introduced by Calió, et al., (2012) conceived for the simulation of the in-plane linear response of masonry walls, is based on a three-dimensional macro-element whose kinematics is governed by seven degrees of freedom only. The mechanical behaviour of the element is based on a fiber discretization approach that adopts basic material parameters and the performance of the macro-
element strategy is assessment by means of nonlinear static analyses performed on masonry walls for which both numerical and experimental results are available in literature. In their model, the plane macro-element consists of a pinned quadrilateral element made with four rigid edges in which a diagonal link is connected to the corners to simulate the shear behaviour. The restraint conditions are simulated via a series of springs denoted as the ‘interface’ acting orthogonally and longitudinally to the direction of the panel. Such layout allows to simulate the in-plane failure mechanism. The spatial (3D) macro-element is characterized by longitudinal springs ruling the in-plane sliding motion and two additional shear-sliding springs to control the out-of-plane sliding mechanism and the torsion around the axis perpendicular to the plane of the interface.

With the aim of evaluating the performance of force-based and displacement-based seismic assessment methods for the life-safety limit state check of OOP loaded URM masonry walls, Godio, M., Beyer, K. (2018) proposed an application to verify the differences between these two approaches by evaluating URM wall strips of unitary width, which represent a simple yet important case study for practitioners. The URM walls were modelled by discrete elements using UDEC 6.0 (Itasca, (2014)), with masonry units modelled as rigid blocks characterized by infinite strength and effective height equivalent to the thickness of the mortar layer plus the unit height, and masonry deformation concentrated in the joints. The models were validated against results from experimental tests for both static and dynamic loading conditions. The study did also carry out a parametric analysis to investigate the effects of four selected parameters on the static and dynamics response of the wall stripes, namely the elastic modulus of masonry, the height-to-thickness ratio, the effective thickness of the walls and the level of applied axial load. The choice of these parameters was motivated by other studies which highlighted their influence in determining the OOP response of URM walls in both static and dynamic loading conditions (Griffith et al., (2004), Meisl et al., (2007), Godio, M., Beyer, K. (2018)).

On the other hand, limit state analysis approaches have been widely used for the analysis at failure of masonry structures at a larger scale of assessment/design, because they require only a limited number of material parameters to build the model and provide limit multipliers of loads, failure mechanisms and, at least on critical sections, the stress distribution at collapse (Cecchi et al., (2007)), thus making them more appropriate for assessment at building scale.

From the very first formulation of the plasticity theory proposed by Heyman, J. (1966), several adjustments have been made as the two main assumption of the theory, namely 1) the lack of tensile strength of the mortar and 2) the not allowable sliding failure occurring within the joints, have been proven wrong by several authors (Mendes, N. (2015), Sutcliffe et al., (2001), D'Ayala, D., & Speranza, E. (2003)), and, as shown in previous section, also by extensive tests campaigns. Moreover, due to the ease of implementation, most codes today specify the ultimate strength of two-way spanning
URM walls according to procedures which are based on rigid plastic analysis, including the Eurocode, former British Code, the Canadian Code, and the Australian Code (Vaculik, J. (2012)).

Any plastic analysis begins with the arbitrary selection of a collapse mechanisms which must be compatible with the boundary conditions and the wall aspect ratio. For the chosen mechanism, the lateral capacity of the wall is evaluated form the principle of energy conservation (Lovegrove, R. (1988)), by equating the internal and external virtual work. An inherent aspect of plastic analysis is that it requires an accurate assumed collapse mechanism to produce an accurate strength prediction. Since – in theory – there are an infinite number of possible collapse mechanism shapes that may form given a set of restraints and wall aspect ratios, the critical – and the correct – mechanism is the one that occurs under the smallest load, producing the minimum collapse load factor based, thus being often defined as the upper bound solution of the limit state analysis theorem, as any chosen mechanism can only provide an upper limit to the strength of the critical mechanism. The critical solution finding process requires 1) the consideration of all possible and plausible collapse mechanisms for the given set of restraints and aspect ratio, and 2) the definition of the shape finding variables (such as length, height, and crack angle) to be found through optimization, in order to minimize the failure load.

There are numerous adaptations of plastic analysis theory which have been developed for calculating the ultimate load capacity of two-way URM walls. These include the traditional yield line analysis, first introduced by Johansen, K. W. (1962), the fracture line method discussed in Sinha, B. P. (1978), and Sinha, B. P. (1980), the failure line method discussed in Drysdale, R. G., & Baker, C. (2003) and Baker et al., (2005) and the virtual work method (Lawrence, S., & Marshall, R. (2000)). Among the cited applications of the plastic theory, the yield line theory based on the simultaneous attainment of moment capacities along the various cracks can be considered the most unconservative among the methods reported and, as discussed in Section 3.2, also unconservative and incorrect Baker, L. R. (1982). Although all these methods share two main commonalities:

- The characteristic shape of the mechanism featuring a combination of vertical, horizontal, and diagonal cracks.
- The involvement of the wall as a ‘whole’ in the collapse mechanism.

There also two main distinctions between these methods:

- The method used to calculate the moment capacities along the cracks.
- The treatment of the diagonal crack slope either as an independent variable or related to the brick/block shape.

The yield line method makes the highest strength predictions of the various plastic analysis methods cited, as it assumes full moment capacities to be achieved along all cracks present in the mechanism.
The diagonal moment capacity is defined as the sum of the contributions of flexural moment capacity along the vertical and horizontal direction and the angle of diagonal crack is treated as an independent variable, so that it can be optimized to minimize the failure load.

The fracture line method proposed by Sinha, B. P. (1978) is in all aspects similar to the standard yield line method, with the only exception of assuming that the internal moments in vertical and horizontal direction are distributed according to the stiffness orthotropy ratio, namely the ratio of the two Young Modulus of elasticity in the two orthogonal directions.

The so-called failure line method, included in the Canadian Code (Standard, C. S. A. (2004)), represents an improved version of standard yield line analysis, which – by making use of an iterative of failure mechanism finding process – discards the formation of early cracks, thus resulting in a strength prediction which is lower than the standard yield line theory. Furthermore – for conservatism – it treats any supported edges as being simply supported, regardless of the actual edge fixity (Drysdale, R. G., & Baker, C. (2003)).

The formulation included in the Australian Standard for masonry structures (Standard, Australia (2018)) find their scientific background in the work of Lawrence, S., & Marshall, R. (2000) later adopted, revised, and improved through modelling and experimental campaigns by Vaculik, J. (2012). The two-way bending formulation considered accounts for free/supported/rotationally restrained edges, for the presence of opening, for diagonal bending moment capacity and for joint and unit geometry, thus making this one of the most comprehensive approaches of the group of limit state analysis approaches. Accounts are also made for the joint spacing, the tensile strength of the units, for horizontal bending and mortar failure of the head joints. Among the most important aspects considered, the torsional behaviour of the joints is individuated as one of the most relevant parameters in determining the out-of-plane load resistance and properly included in the computation of the horizontal moment capacity. In his work also, a clearer and neater distinction is made of the two limit states that any wall section undergoes when subjected to OOP load:

- The ‘ultimate’ condition when masonry is assumed to be initially uncracked. The wall moment capacity relies on material strength and the contribution from vertical compression.
- The ‘residual’ condition, representative of the case when the masonry is fully cracked and possesses zero bond strength. The one and only resistance of the wall comes from the acting vertical axial stress.

These correspond to two different computations of wall strength and two physical models adopted. Vaculik, J. (2012) focused his attention exclusively on developing a model for seismic loading, whilst conducting an extensive experimental campaign to validate the results obtained from his models.
Finally, it is worth mentioning retrospectively the work of Martini, K. (1998). Although comparatively older than other studies mentioned in this section, in his work the author proposed a comparison between a finite element model of a two-way spanning wall simply supported on the side edge and sitting on a rigid base at the bottom and unsupported (free) at the top and a twin adaptation which follows the principles of yield line theory. The former was built following the ‘block-interface’ approach, thus using elastic volume elements to model the masonry material and surface-contact elements to model the mortar joints with a constant value of friction. The latter attempted to reproduce the crack pattern observed in the finite element model, simulating a post-cracking mechanism. For horizontal cracks, the upper bound is simply the overturning moment due to vertical loads, whilst the vertical cracks are instead ruled by the friction between horizontal surfaces at the bed joints between courses. With this comparison Martini, K. (1998) highlighted that, whilst there is good agreement in terms of shape of the pattern, the yield line model results in a lower prediction of the failure load, thus highlighting the need for tuning the definition of the contributions to the mechanisms of horizontal moment transfer and, similarly, the need to find appropriate value of friction coefficient. As shown in this section, several authors conducted investigation following these gaps. However, the most important contribution provided by his research relates to having recognized the difference in computational burden required to undertake the block-interface approach when compared to more expeditious methods such as the ones based on yield line theory, thus highlighting – once again - the advantage in using the latter to assessing large portfolios of URM structures.

3.4. Conclusions

This chapter covered the review of the methods for the prediction of the ultimate load capacity of URMs under out-of-plane loading, from a twofold perspective.

The first section aimed at providing an understanding of the mechanics of bricks/blocks, the type of internal interactions developing in the case of two-way bending walls, the torsional and flexural contributions and their interrelation, and the mathematical equations which define the failure envelope, needed to describe these complex interactions.

The second section aimed at providing a review of the methods available in literature based on the concept of plate theory and limit state analysis accounting for the anisotropic nature of the material, which are deemed to be sufficiently accurate for the determination of the strength capacity of URM walls subjected to OOP loading, whilst also accounting for the gaps and limitation in the understanding of the mechanics and the material interactions mentioned in Section 3.2.

This review provides the base for the determination of the analytical assessment model proposed in this thesis, which, streaming from the well-established examples available in literature and mainly focused
on the determination of the strength capacity of URM s subjected to seismic loading, aims at extending the procedure to assess other types of horizontal loading such as flood and wind, whilst accounting for the complexities related to the bricks/blocks interactions discussed in Section 3.2.

The following chapter is in fact dedicated to summarizing the findings of the two chapters of literature review and to present the methodology of the analytical model proposed in this thesis.
4. Methodology for the Multi-Hazard Capacity Assessment of URMs

4.1. Gaps and key issues

The review of methods for the assessment of the capacity of URMs to earthquake, flood, and wind loading, conducted in Chapter 2 has identified a wide range of approaches and studies, which demonstrate the significant technical developments that contribute to the topic. However, a number of issues and research gaps have emerged:

- There is a substantial level of discrepancy in availability of analytical approaches to establish the capacity of URM walls subjected to OOP loading between earthquake, flood and wind loading. Whilst there are many studies focusing on the first peril, there is lack of detailed approaches for wind and especially for flood.
- There is a lack of a simplified and unique method that allows to simulate the three hazards with the same analytical approach, which is simple enough to be applied to large number of cases with modest data needed to derive fragility functions applicable to URM typologies.
- There are conceptual differences in the definition of fragility curves and the parameters needed to derive them, namely Intensity Measures (IMs), Engineering Demand Parameters (EDPs), damage thresholds, and fitting techniques, in relation to the three different hazards.

The literature review (Chapter 2) has shown that there are two main frameworks that can be followed, namely a multi-layer single-hazard assessment, i.e. the events are assessed individually and the results do not account for any interaction/overlap of events (Kappes & Keiler, (2012)), or multi-hazard assessment, i.e. cases where simultaneous and possibly independent events occur in the same area and time window, or cascading events, where a given hazard event may trigger a secondary hazard event. (Gill, J. C., & Malamud, B. D. (2016), Gallina et al., (2016)). Among others, the work of Choine et al., (2015) is particularly relevant. Despite focusing on the multi-hazard risk of road infrastructures, the independent treatment of the hazard assessment models linked through a 'pinch point' variable is a concept that can be applied within the broad context of multi-hazard fragility assessment of any set of structures. More specifically, as long as the chosen analytical model is able to represent the response of the structure under different environmental loading profiles, by means of one single analytical parameter representative of the building response and independent of the type of loading considered, then the three hazards can be kept separate and then compared in terms of probability of failure, i.e., by comparing fragility functions.

Regarding the definition of the parameters needed to derive fragility functions, namely the Intensity Measure, the Engineering Demand Parameter, and the limit states thresholds, the review of the literature in Chapter 2 has shown that:
As far as the IMs are concerned:

- For seismic loading: PGA is commonly used in the case of acceleration-sensitive structures such as masonry buildings, characterized by relatively low natural period (Douglas et al., (2015), Lagomarsino, S., & Cattari, S. (2015)).
- For flood loading: the review of available studies has shown that floodwater depth and floodwater velocity are the two main IMs used to correlate flood hazard to physical damage extent to masonry structures (Kelman, I., & Spence, R. (2004), Herbert, D. M. (2013), Herbert et al., (2018), De Risi et al., (2019)).

As far as the EDPs are concerned:

- For seismic loading: inter-storey drift is one of the most common EDPs used in literature according to FEMA P-58-6 (2018). However, the base shear capacity coefficient, is one of the most used indicators for the seismic performance of masonry buildings (Tomažević, M., & Klemenc, I. (1997), D’Ayala, D. F. (2005)).
- For flood loading: currently, available standards provide no guidance for evaluating the nonuniform hydraulic (water) lateral load capacity of masonry (Herbert, (2013), Herbert et al (2018)). In their work Kelman, I., & Spence, R. (2003) established that – for a slab (or wall) to rotate and therefore, to start forming a mechanism, the resisting moment based on the weight supported by the slab (or wall) above the yield line must be overcome. Similarly, Herbert et al (2018) referred to the work of Kelman, I., & Spence, R. (2003) and Martini, K. (1998), adopting the hypothesis that the lateral capacity of the wall depends purely on the vertical load in the vertical direction and the friction coefficient in the horizontal direction. De Risi, R., et al (2013) and Jalayer, F., et al (2016) have instead determined the capacity in terms of critical water height, indicating a volume of water which – if applied to the structure – causes the loss of out-of-plane bearing capacity.
- For wind loading: according to EC6 (CEN, (2010)), the load bearing capacity of laterally loaded URM walls can be determined by assuming arching effects (Martens, D. R. W., & Vermeltfoort, A. T. (2017)). Neglecting the deformation of the wall, the distributed load-bearing capacity of an unreinforced masonry wall is calculated as function of the compressive strength of masonry in the direction of the load forming the arch thrust, the wall thickness and the distance from the rigid supports (either as a length or as a height).

As far as the limit states thresholds, namely the values of the selected response parameter at which the building enters a given damage state, are concerned:
For seismic loading: three limit states are usually defined for masonry structures, namely elastic (cracking) limit, maximum resistance, and ultimate state, where the strength capacity deteriorates up to 80% as mentioned in Tomaževič, M., & Klemenc, I. (1997), Tomaževič, M. (1999), NTC (2018), Lu et al., (2016). These limit states correspond to the performance levels of damage limitation, life safety and collapse prevention in EC8 (CEN (2004)).

For flood loading: one limit state is usually defined for structures undergoing flooding, namely collapse, and they it is usually defined in terms of flooding height and corresponding to the failure of the bearing structure, collapse of the walls, loss of supports for the roof or loss of the load bearing capacity due to prolonged contact with water and deterioration. (De Risi et al., (2013), Mazzorana et al., (2014), Jalayer et al., (2016), De Risi et al., (2019)).

For wind loading: the limit states available in literature for buildings subjected to high wind speeds usually refer to the failure of roof and roof components (D'Ayala et al., (2016), Song et al., (2019)) or to the breach of openings (windows and/or doors) leading to roof and wall failure (Vickery et al., (2006)) which are however not relevant to the considered thesis. According to EC6, CEN (2010)), for masonry structures, the ultimate limit state and serviceability limit state shall be considered for all aspects of the structures, including ancillary components in the masonry. Section 6.3 of the EC6 reports that – for URM walls subjected to lateral loading, the moment of resistance per unit length or height of the wall, must be greater than the resultant of load applied to the wall.

With respect to the choice of the most appropriate analytical methods to conduct multi-hazard vulnerability assessment for masonry structures, the literature review of structural assessment methods, reported in Chapter 3, has shown that simplified macro-element models based on kinematic principles, characterised by a limited number of parameters to describe the limit states of the structures are preferable, due to the following advantages:

- Ease of implementation for different walls layouts, openings dimensions and positions and edges restraints.
- Limited computational efforts needed compared to more detailed FE models, which are computationally less affordable and very often require many material parameters and more detail geometric data.
- Generic, so that it can be used and reproduced by professional engineers with reasonable level of structural background education.

As far as modelling the masonry structural response is concerned:

- Although firstly used to calculate the plastic collapse of flat and thin plates of rigid-perfectly plastic material when transversely loaded in bending (Johansen, K. W. (1962), Bauer, D., & Redwood,
(1987)) the Yield Line method proves to be a suitable approach to determine the lateral resistance of walls subjected to uniformly distributed load as included in EC6 (CEN (2010)). However, application to flood vulnerability assessment is limited (Milanesi, et al., (2018), Herbert, (2013), Herbert et al., (2014), Herbert et al., (2018)). In support of the choice of this method, the following advantages, discussed in depth in Chapter 3, are summarized below:

- Ease of representation of the effects caused by the hazards with simple loading schemes.
- Ease of incorporation of openings and vertical loads within the calculation.
- Ease to predict the crack pattern and to implement the torsional effect developing in the bed joints, for a more accurate determination of the out-of-plane load resistance.

The limitations pertaining to such method have also been identified:

- Pre-assuming a crack pattern implies that the whole analysis in conducted in the assumption that the pattern is the correct one (beside being feasible and in line with boundary conditions and wall aspect ratio). However, there are always several other crack patterns which could comply with the same needed requirements thus posing the challenge related to the uniqueness of the solution found.
- In terms of boundary condition and associated moment capacity along the wall edges, the method assumes there is homogeneity all along. This implies that there are difficulties in modelling ‘intermediate’ cases, which might be representative of restraint conditions which are not entirely representative only of fix supports, simple supports or free edges.
- Accounting for the presence of openings when defining the crack pattern and consequent moments of resistance might result in approximations of the true load distribution.

4.2. Proposed framework

Given the shortcomings of the state-of-the-art with respect to the objectives of the thesis and the difficulties in reconciling fragility to different hazard types, a methodological framework for the multi-hazard fragility assessment of unreinforced masonry structures is proposed in this work, shown in Figure 4-1.; with the specific intent of harmonizing both the modelling of the structural behaviour of URMs under earthquake, flood and wind and consequently deriving fragility functions for each hazard to allow for comparison of expected losses given specific intensities of the hazards at a site.
Figure 4-1: Proposed methodological framework.
To this aim, the characterization of both the hazard sources and the exposure component are treated as input to feed into the analytical method developed which represents the overarching objective of this research study. To define the harmonized analytical model able to correlate the damage of URMs due to earthquake, flood and wind loading to the intensity measures characterising the three perils considered, two main input are required, namely the hazard sources, which needs to be characterized to determine the IMs representative of each hazard type (as discussed in Chapter 2), and the exposure details, to define the genome of the building typology, as discussed in Section 2.4.3. In consideration of the level of discrepancy characterizing the exposure databases of earthquake, flood and wind hazard assessment, the set of minimum required information to define the analytical model must be defined and the three main types of data required are 1) the geometric parameters and wall scale and brick scale, 2) the material properties influencing the determination of the capacity of the URM system considered and finally 3) the configurations of the system, both in terms of opening layouts and restrain conditions. Starting from the revision of the shared database of the World Housing Encyclopaedia (Brzev et al., 2004) and GEM (Brzev et al., 2013), ranges of geometric and material parameters are found also from the literature on experimental campaigns carried on brick walls (Vaculik, J. (2012), Tomaževič, M. (2009)) and in existing Building Codes. Further details of this step are provided in Section 6.2. Since the mentioned taxonomies are all related to seismic vulnerability assessment, one of the aims and the efforts of this methodology was to identify the set of geometric and material characteristics needed to define the capacity of URMs, irrespective of the peril considered.

Having defined the two inputs required, the kinematic analytical model can be developed. The assessment is conducted at the scale of the individual wall. From the 3D building scale, a 2D model is obtained by simply considering each masonry unit as an assembly of individual walls, each characterized by specific sets of boundary conditions and openings (size and position). Depending on their position within the building layout, various combination of 4-sides-restrained or 3-sided-restrained walls are defined. The degree of restrain provided by the presence of horizontal structures (i.e., floor and/or roofs) is simulated with different level of fixity of the upper and lower horizontal edges of the walls. A detailed list of all layouts considered in this study is provided in Chapter 5. The loading profiles are applied individually to the wall and ‘simplified’ as knife edge loads applied at different positions of the wall. This assumption facilitates the generalisation of the set of equations needed to determine the term related to the external work $W_e$ in the application of virtual work principles to the kinematic approach in limit analysis. In this way, irrespective of the environmental hazard considered, for any given magnitude the load application will only differ by its position with respect to the base of the wall. Depending on the number of supports and free edges of the wall layout, several crack pattern can be assumed, which must be compatible with restraints and loading profiles. To determine the internal work $W_i$, YL theory concepts are used, adopting two levels of detail. Firstly, a simplified approach is used (Approach_S),
which is based on the assumption that the two bending moment capacity in horizontal and vertical
direction can be expressed as the ratio of the two flexural strengths in the two orthogonal directions and
the pattern is solely a function of the global geometric ratio of the wall. Approach_D is then proposed
as a more detailed approach, which accounts for 1) the torsional effects developing at the scale of the
single brick/bloc unit as a consequence of the horizontal loading acting out-of-plane and 2) a more
accurate definition of the crack pattern as a function of the brick/blocks and mortar shape ratio.
Approach_S is used in the ‘screening phase’ of all hypothesized patterns to determine the feasible and
possible ones which are then analysed through Approach_D, which identifies more accurately the value
of loading that determines the collapse of the wall for a given geometry, opening layout and set of
restraints conditions. An optimization routine is created to find the minimum load to be applied to
produce failure, resulting in a specific collapse load pattern identified by a nondimensional geometric
parameter and a value of collapse load factor λ. The two approaches are detailed in Chapter 5.

In analytical terms, the λ factor is found by applying the Virtual Work Method to the Mechanism
Approach and by equating the work done by external forces (i.e., the ultimate load λq which causes
collapse) to the internal capacity of the wall, given as a sum of vertical and horizontal moment capacity.
As the λ factor is the value of load causing collapse, its inverse represents the structural performance
variable χ, defined in terms of critical demand to capacity ratio. Each χ value indicates the response of
the wall for the given IM considered, thus linking the value of the hazard measure to the capacity of the
structure, for the following derivation of fragility functions.

Given the ultimate limit state approach, and the assumption of rigid perfectly plastic constitutive model
for the material, only characterised by an ultimate strength in tension, the only damage state that can be
identified is the collapse. The structural limit state of collapse is described as the critical value of PGA,
floodwater depth and wind speed which for a given configuration and material characteristics, yields a
value of χ=1, i.e., when demand is equal to capacity.

Given the approach chosen, the fragility functions are derived on the basis of the variability of geometry,
boundary conditions and material parameters, determining a value of maximum capacity, for each
combination of this set of parameters. This will provide the range of IM for each specific hazard, that
can be attained for each wall type, at ultimate capacity, as it will be detailed in Chapter 6. For each IM,
the probability of the system (i.e., wall) of reaching collapse (Pc) is defined as the probability that the
Demand (Q) is greater than the Capacity (R), which results in having χ>1, as formalised in Eq. 4.1:

\[ P_c = P \left( \frac{Q}{R} > 1 \right) \]  \hspace{1cm} (4.1)

In this approach the variability associated with the demand (Q) is not considered, the only source of
random variability being the model (R), therefore involving the material and geometric parameters of
the wall, given a set of constraints. The following step of this methodology focuses on determining the performance levels and consequent values of structural performance variable $\chi$ for different groups of walls (i.e., each group refers to a given set of boundary conditions) by conducting a parametric analysis of the global and local geometric characteristics and material properties within the ranges identified as realistic input. Rather than using random sampling to generate the variability in input, as it can produce combinations of values for the variables, that are not realistic, the analysis is carried out using the ‘nominal range sensitivity approach’ (NRSA), thereby evaluating the effect of each parameter varying it within given ranges in combinations of parameters that reproduce real walls patterns. Further details on the ranges considered and the rationale of the simulations is provided in Chapter 6, Section 6.2.

Before proceeding towards the second main aim of this research study, namely the derivation of single-hazard fragility functions, three validations of the analytical method proposed are provided. Firstly, the model is compared against the Eurocode 6 procedure, to evaluate the level of accuracy achieved when moving from the uniformly distributed loading profile to the line loading. Secondly, the results obtained from the proposed model, in the case of seismic action, are compared to a set of available results coming from a relevant experimental campaign available in literature. Lastly, a simulation of a specific wall layout is done with the Extreme Loading for Structure (ELS) software based on the Discrete Element (DE) method, to determine the level of accuracy of the proposed methods when compared to more advanced numerical tools.

Single-hazard fragility functions, defined as the probability of reaching/exceeding collapse for a given IM, are derived as a function of PGA, floodwater depth and wind speed and correspondent performance variables, the EDP representative of the wall strength capacity, are derived for each group of walls. Since fragility derivation methods and related assumptions vary greatly between the different hazard types, Chapter 7, section 7.2 details the regression method chosen to fit the purposes of this study, which lead to the determination of all the single-hazard fragility curves sets included in Section 7.3.

Finally, an application of the proposed methodology is provided in Chapter 8, to prove the ease of implementation and of customization of the procedure developed when a real building sample is made available.

4.3. Conclusions

This chapter has detailed the proposed methodological framework to conduct multi-hazard vulnerability assessment of URMs undergoing to seismic, flood and wind loading, highlighting the input sources, the core assumptions and presenting the flowchart with the steps to follow describing both the analytical method and the results that allow the comparison across the three perils considered in this research study. Following chapters are dedicated to providing the details of the analytical method (Chapter 5), the
generation of the walls’ taxonomy and the validation of the analytical procedure proposed with existing Codes, experimental tests and available software (Chapter 6) the derivation of fragility curves (Chapter 7), and finally the application to a relevant case study (Chapter 8).
5. Derivation of proposed kinematic models

5.1. Introduction

The review conducted in Chapter 3 has highlighted the advantages of using Yield Line theory to derive work equilibrium equations to assess the out-of-plane failure of URM walls and how such concepts, initially developed for the prediction of ultimate flexural strength of reinforced concrete slabs, can be extended and adapted to evaluate the structural behaviour of masonry wall under various loading conditions, thus becoming useful in a multi-hazard vulnerability assessment framework. In particular, the kinematic or upper-bound theorem is considered, according to which the load factor $\lambda$ calculated on the basis of a kinematically compatible mechanisms is greater or equal to the collapse load factor $\lambda_c$ that corresponds to the limit state of the structure before it collapses (Heyman, J. (1966), Sinha, B. P. (1978), Lovegrove, R. (1988)).

Section 5.2 details the rationale of the loading schematization, to determine, from the 3D seismic, flood and wind loading shapes, the 2D linear knife edge load, as well as the set of assumptions used to homogenize this step across the three perils, in consideration of the representation of these actions provided in relevant Codes.

Section 5.3 shows the modelling alternatives to define the admissible crack patterns that can develop for a given set of boundary conditions of the wall and the resulting pattern-to-loading interactions for the determination of the external work component of the work equilibrium equation. An example is provided to show all the possible assumptions that can be made when ‘pre-assuming’ a pattern and how these affect the results obtained.

To determine the wall’s internal moment capacity, YL theory concepts conventionally applied to orthotropic concrete slabs can be employed, although this neglects the information of the masonry fabric. As a result, a simplified approach defined as Approach_S, is developed considering a generic linear yield line and the coordinated moment capacity in the horizontal and vertical direction are factorized by the ratio of the two flexural strengths in the two orthogonal directions. However, these concepts can also be further improved by including – within the horizontal moment capacity – the torsional effects developing at the scale of the single brick/bloc unit when subjected to horizontal loading acting out-of-plane, thus providing a more accurate estimation of the collapse capacity of the wall. This is included in the proposed detailed approach, Approach_D. Moreover, the crack pattern defined through Approach_D is based on the geometry of the brick/bloc unit and is verified across the whole height of the wall to be able to provide the crack pattern which maximise the work-equilibrium equation, therefore identifying the ‘true’ collapse load factor, in limit analysis terms (Sinha, B. P. (1978)).
Section 5.4 focuses on detailing the set of work equilibrium equations based on Approach_S, for all crack patterns, modelling alternatives and perils considered, with the aim of highlighting the advantages in adopting the line load schematization discussed in Section 5.2.

Section 5.5 presents Approach_D and the optimization routine to determine, among the possible crack patterns proposed in section 5.3, the one corresponding to the minimum collapse load factor λ hence the maximum value of performance variable χ.

Finally, in Section 5.6, the differences between the two approaches S and D, are discussed by comparing the λ factors and the crack patterns obtained with the two methods for the same loading and constraint conditions.

5.2. Loading schematization

This section details the method followed to represent the three loading actions on the wall, with reference to existing building codes. As reported in EC6 (CEN (2010)), YL theory concepts are employed for the design of masonry walls, relying on the flexural strength of masonry to provide bending moment coefficients. However, the EC6 prescriptions, presented as tables of coefficients, are valid for walls subjected to uniformly distributed loads (such as wind loading): therefore, one of the novelties introduced in this study is to extend the same approach to loading profiles with different shapes, i.e., triangular in the case of seismic and flood loading. To define an approach common to all three hazards considered, and simple enough to allow assessment of large number of cases for fragility analysis purposes, the ‘real’ distribution of seismic, flood and wind loading over a wall panel is considered as its resultant horizontally distributed knife edge load applied at different wall heights, depending on the loading shape considered. Such simplification allows to solve the equilibrium equations without incurring into computationally demanding mathematical integrations also making the procedure easily applicable to other loading profiles exerting a lateral action which could cause out-of-plane failure, such as soil pressure or ash flows. Regarding the determination of the load pattern to best simulate the seismic action through static incremental horizontal forces, several viable options are reported in literature (Lagomarsino, S., & Cattari, S. (2015)). For the purpose of the present study for which very regular wall layouts are considered, the triangular load pattern represents the preferable option, as it ensures that the calculated seismic masses (in all parts of the wall) are involved in the computation of the inertial forces applied to the wall system.

In accordance with EC8 prescriptions (CEN (2004)), the capacity of the system to resist seismic action is determined by performing an elastic analysis based on a response spectrum reduced with respect to the elastic one by means of the behaviour factor q. For a given location, the horizontal component of the seismic action is defined through the set of equations included in section 3.2.2.5 of the EC8 (CEN
Depending on the period of the structure (determined with a simplified formula, see Eq. 5.1), the soil type of the investigated location and the values of the parameters describing the recommended elastic response spectra (included in Table 3.2 and Table 3.3 of the Code (CEN (2004))), the value of spectral design acceleration $S_d(T)$ is computed. For masonry structures, assumed to belong to the low-dissipative structural behaviour class, the value of $q$ factor is equal to 1.5 (see Table 6.1 of EC8) (CEN (2004)). As reported in section 4.3.3.2.2 of EC8 (CEN (2004)), the period of the structure and the seismic base shear force $F_b$ for the horizontal direction are determined in Eq. 5.1 and Eq. 5.2

\[ T_1 = C_1 H^2 \]  

\[ F_b = S_d(T_1)ma \]

With $S_d(T_1)$ being the horizontal component of the design spectrum at period $T_1$, $T_1$ is the fundamental period of vibration of the building for lateral motion in the direction considered, $m$ is the total mass of the building (wall in this case) considered and $\alpha$ is the correction factor (which accounts for the effective modal mass of the structure), taken equal to 1.

To define the horizontal component of the seismic action $S_d$ the equations provided in the EC8 (Section 3.2.2.5) must be used. To the aim of maintaining the proposed procedure as generic as possible, the soil type has been omitted (S). Moreover, it is also assumed that the brick/block walls considered belong to stiff structures, therefore likely to be subjected to the maximum amplification factor of 2.5, typical of the plateau portion of the response spectrum. Eq. 3.14 of EC8 (CEN, (2004)) is therefore used to define $S_d(T)$, without the inclusion of the soil type as presented in Eq. 5.3.

\[ S_d(T_1) = a_g \cdot 2.5/q [g] \]  

The remaining terms in Eq. 5.3 are $a_g$ the seismic acceleration causing failure and $q$ the behaviour factor.

(a) 3D loading profile, (b) equivalent 2-D knife edge load [KEL].

Figure 5-1: Earthquake loading on the representative wall: (a) 3D loading profile, (b) equivalent 2-D knife edge load [KEL].
Section 4.3.3.2.3 of EC8 (CEN, 2004) provides guidance on how to translate the base shear into a set of lateral inertia forces and how to distribute them over the height of the structure. Within the field of application of the equivalent static lateral force method (Beer, M., et al. 2015), the predominant response of low-rise symmetric buildings subjected to earthquake loading is governed by the first mode shape, therefore the first-mode drift pattern is normally taken proportional to the building elevation, from the base or above the top of a rigid basement, resulting in the commonly termed ‘inverted triangular’ pattern of lateral forces – although in reality it is just the drifts that have an ‘inverted triangular’ distribution and the pattern of forces depends also on the distribution of masses. The resultant of that ‘inverted triangular’ profile is placed at 2/3 from the base of the building/wall considered, as also proposed in ASCE/SEI 7-10 (Charney, F. A. 2015). To obtain the resulting KEL, applied at 2/3 of the height of the wall (H), shown in Figure 5-1:

\[ q_{\text{seismic}} = \frac{F_b}{L_{\text{wall}}} \quad [kN/m] \quad 5.4 \]

\( q_{\text{seismic}} \) is considered distributed along 1 m length of wall (L).

For what concerns flooding load, the hydrostatic pressure component of the lateral force is considered, as reported in Eq. 5.5:

\[ p_{\text{flood}} = h \rho_{\text{water}} [kN/m^2] \quad 5.5 \]

with \( h \) being the depth of the water in m; \( \rho_{\text{water}} \) being density of water in kg/m\(^3\) and \( g \) being the gravity acceleration in m/s\(^2\). Note that \( h \) is not necessarily equal to \( H \) (height of the wall).

![Figure 5-2](image)

**Figure 5-2:** Flood loading on the representative wall: (a) 3D loading profile, (b) equivalent 2-D knife edge load [KEL].

For simplicity, the position of the flood KEL in Figure 5-2 indicates the specific case in which the height of the water (\( \beta H \)) is equal to the height of the wall (H). The corresponding KEL, applied at 1/3 of the height of the water (\( \beta H \)) is represented in Figure 5.2 b) and given by Eq. 5.6:

\[ q_{\text{flood}} = \frac{1}{2} (\beta H)^2 g \rho_{\text{water}} \quad [kN/m] \quad 5.6 \]
with $0 < \beta \leq 1$, being a scaling factor that allows to determine the relationship between the depth of water and the height of the wall, such that $\beta H = h_{\text{water}}$.

$q_{\text{flood}}$ is considered along 1m length of wall ($L$).

Regarding the wind load, EC1, section 5.1 (BSI, (2005a)), defines the value of wind pressure on external surfaces $w_e$ as reported in Eq.5.7:

$$w_e = q_p(z_e)c_{pe} \tag{5.7}$$

Where $q_p$ is the peak velocity pressure at a reference height $z_e$ and $c_{pe}$ is the pressure coefficient for external pressure. The reference heights $z_e$ for windward walls depends on the plan of the building considered. Recommended values of external pressure coefficients for vertical walls of rectangular plan buildings are detailed in Table 7.1 of EC1 (BSI, (2005a)), according to which a value of 1 indicates the worst possible conditions and it is the one adopted herein.

Regarding the $q_p(z_e)$, EC1 section 4.5 (BSI, (2005a)) defines the peak velocity pressure at height $z_e$, which includes mean and short-term velocity fluctuation, as reported in Eq. 5.8:

$$q_p(z) = [1 + 7I_v(z)] \frac{1}{2} \rho_{\text{air}}V^2(z) = c_e(z)q_b \tag{5.8}$$

Where $I_v(z)$ indicates the turbulence intensity at height $z$, defined as the standard deviation of the turbulence divided by the mean, which is finally incorporated in the $c_e(z)$ factor, which is the exposure factor given as reported in Eq. 5.9:

$$c_e(z) = q_p(z)/q_b \tag{5.9}$$

And $q_b$ represent the basic velocity pressure given in Eq. 5.10:

$$q_b(z) = \frac{1}{2} \rho_{\text{air}}V^2(z) \tag{5.10}$$

With $\rho_{\text{air}}$ value suggested on 1.2 kg/m$^3$

The final value of $q_{\text{wind}}$ is defined in Eq. 5.11:

$$w_e = q_{\text{wind}} = q_p(z_e)c_{pe} = c_e(z)q_b c_{pe} = c_e(z)\frac{1}{2}V^2\rho_{\text{air}}H \quad [kN/m] \tag{5.11}$$

with $H$ being the height of the wall and $q_{\text{wind}}$ being the wind loading for unit length of wall ($L$).

A representation of the wind pressure on the wall is provided in Figure 5-3.
Figure 5-3: Wind loading on the representative wall: (a) 3D loading profile, (b) equivalent 2-D knife edge load (KEL).

Table 5-1 summarizes respectively the types, the equivalent simplified, unified formula for their determination, and points of application of the three loadings considered.

<table>
<thead>
<tr>
<th>Loading Type</th>
<th>Load Value</th>
<th>Point of Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{\text{seismic}}$</td>
<td>$\left( \frac{2.5}{1.5} \right) \alpha a_g H \rho_{\text{brick}}$</td>
<td>$\frac{2}{3} H_{\text{wall}}$</td>
</tr>
<tr>
<td>$q_{\text{flood}}$</td>
<td>$\frac{1}{2} (\beta H)^2 \rho_{\text{water}}$</td>
<td>$\frac{1}{3} \beta H_{\text{water}}$</td>
</tr>
<tr>
<td>$q_{\text{wind}}$</td>
<td>$c_e(z) \frac{1}{2} g V^2 \rho_{\text{air}}$</td>
<td>$\frac{1}{2} H_{\text{wall}}$</td>
</tr>
</tbody>
</table>

Note that, $c_e(z) = 1$ i.e., turbulence effects are not considered.

Also, while for the seismic and wind load the load line remains at a fixed distance from the base of the wall, the position of the flood KEL varies with increasing depth of water up to the full height of the wall, until the failure load is found, thus affecting the height of its line of application.

The simplification adopted on the loading schematization proves useful to speed the analysis and obtain commensurable results with a limited analytical burden. By defining a consistent approach to the definition of the hazards’ action, the set of equations remain unvaried, and the results can easily be determined in terms of performance variable, a structural performance indicator representative of the loading capacity of the structure.

5.3. Definition of crack patterns: modelling alternatives

After having detailed the loading schematization, the corresponding crack patterns shall be assumed, which must also be compatible to the boundary conditions of the wall. As discussed in Chapter 3, several patterns can be identified for the same set of loading and restraints of the wall.
To exemplify the rationale to define the patterns, a wall with no openings simply supported on the four edges is used and shown in Figure 5-4

![Figure 5-4: Boundary condition of the wall](image)

To be considered ‘admissible’, a crack pattern should comply to the set of rules, which underpin yield line theory and its extension to masonry walls (Sinha et al., 1997, Burgess, I. (2017)) and can be summarized as follows:

- Axes of rotation lies along lines of support.
- Crack lines between adjacent rigid portions must pass through the point of intersection of the axes of rotation of each of those portions.
- Crack lines must end at the wall boundary.
- Continuous supports (fixed edges) repel crack lines, while simply supported edges attract crack lines.
- The total number of axes of rotation must be equal to the total number of crack lines.

Within these assumptions, and for the wall shown in Figure 5-4, two ‘admissible’ shapes of crack patterns (CP), namely crack pattern 1 (CP1) and crack pattern 2 (CP2) for each of the loading profiles discussed in previous section, as shown in Figures 5.5, 5.6, and 5.7.

![Figure 5-5: Admissible failure patterns due to seismic load: CP1 (a), CP2 (b)](image)
These differs from each other in that CP1 has a fifth vertical crack, besides the diagonals, and CP2 has a fifth horizontal crack, besides the diagonals. The presence of such cracks depends on the relative brick shape ratio and wall shape ratio, as crack develop stepwise along bed and head joints, as well as the ratio of the two flexural strengths in the two coordinate directions.

In order to conduct the analysis, for each crack pattern, some further assumptions are required to define the coordinate y, determining the position of the point/s of convergence of diagonal cracks. These are functional to establish not only the inclination followed by the diagonal cracks themselves, which affect the computation of the wall moment capacity and the internal work, but also the ‘interaction’ between the established pattern and the line of application of the KEL, which, conversely, influences the computation of the external work. As mentioned in Section 5.2, the increasing height of water considered in the case of flood loading case causes a continuous pattern re-adjustment, hence both the shape of the crack pattern and the consequent KEL-to-cracks interaction show – as reference – the height of the water (βH) instead of the height of the wall (H).

For each KEL several configurations of crack patterns CP1 and CP2 have been considered and summarized in Table 5.2.

**Figure 5-6:** Admissible failure patterns due to flood load: CP1 (a), CP2 (b)

**Figure 5-7:** Admissible failure patterns due to wind load: CP1 (a), CP2 (b)
Table 5-2: CP1 and CP2 patterns for earthquake, flood, and wind loading. The solid lines indicate the cracks which remain fixed from the upper/lower wall’s edge; the dashed line indicate the various options considered for each crack pattern configuration.

**Earthquake**

**Flood**

Note: $\beta = 1 \rightarrow \beta H = H$
Since the KELs are respectively applied at 2/3H, βH/3 and H/2, it is chosen to verify all patterns forming at these wall’s heights irrespectively of the loading type considered.

Regarding the shape of the crack patterns three main assumptions are considered:

- Assumption 1 refers to the position of points 1 and 2 for CP1 and point 3 and 4 for CP2 (i.e., points of convergence of upper and lower diagonal lines) with respect to the line of application of the KEL. More specifically, the CP1-1s configurations assume the upper point of convergence is placed at 2/3H: three possible positions of the lower point of convergence are hypothesized, namely 1/3H and H/2 and 2/3H. On the contrary, CP1-2 configurations include the cases in which the lower point of convergence of diagonal cracks is fixed at 1/3H and the upper point slides down toward the base of the wall from 2/3H to H/2 to 1/3H.

- Assumption 2 refers to the length of vertical crack line in CP1 (i.e., crack line defined between 1-2) with respect to the H of the wall and length of horizontal crack line in CP2 (i.e., crack line defined between 3-4) with respect to the L of the wall. CP2-1 depicts the case in which there is no horizontal line forming and the diagonal cracks converge in the middle of the wall length. Finally, CP2-2 includes the cases in which the points of convergence of diagonal cracks meet either at 2/3H, H/2 or 1/3H, while also forming a horizontal crack.

- Given that the KEL lines of application of all three hazards vary between 1/3H and 2/3H, the mid-vertical crack line (in the case of CP1) and the height of the horizontal crack line (in the case of
CP2) are constrained between 1/3H and 2/3H. The distance of points 3 and 4 from the vertical edges of the wall can reach a maximum of L/2.

Geometrically, the CP1 patterns are more likely to form when the walls are narrower and slender (i.e., L/H<1), while the CP2 patterns are more likely to form when the walls are predominantly longer than tall (L/H>1).

As reported in Chapter 3, Section 3.1, and given that a high ratio of brick’s to bond’s strength is assumed in order to promote stepped failure over line failure, to maintain the ability of vertical edges to provide a path for the horizontal load, the angle of crack follows the ratio defined by the height of brick to staggering length, from the closer corner of the wall to the KEL position and then it is assumed to follow the reverse path towards the top other corner of the wall, on the same side.

In the case of flood loading, given the relationship between hydrostatic pressure and depth of water, and as the analysis implemented is discretised as increasing steps corresponding to the height of each brick course, two possible scenarios are considered, namely 1) the crack pattern is dependent of the height of water, hence it develops only within the portion subjected to the load, 2) the crack pattern is independent of the height of the water, hence it develops across the full height and length of the wall. These two options are more thoroughly investigated in following chapters.

Since, conceptually, there are an infinite number of ‘graphically’ admissible crack pattern shapes that may be applied to a particular wall layout, the critical or ‘correct’ mechanism is the one that requires the least amount of energy to form and occurs under the smallest load and therefore resulting in the smallest collapse load factor. Being defined as ‘upper bound’ approach, any chosen mechanism can only provide an upper limit of collapse load factor corresponding to the critical or ‘true’ mechanism. Finding the critical solution not only requires the various types of collapse mechanisms to be considered and to be verified with the Principle of Least Work (Lovegrove, R. (1988)), but within each mechanism, the shape-defining variables such as the length of cracks, the position within the wall height and the diagonal crack angles effectively become independent variables that need to be optimized in order to minimize the failure load.

As mentioned in Section 5.1, there are two approaches that can be adopted to conduct this search, namely Approach_S and Approach_D. The following sections discuss the aspects common to both procedures and specify the assumptions of each approach in detail.

### 5.4. Principle of least work equations: simplified procedure (Approach_S)

As the loading schematization and the crack pattern are defined, the lateral pressure resistance of the wall is evaluated by applying the Principle of Least Work, therefore equating the internal and external
virtual work. The energy equation equates the work done by the external force $W_e$ to the energy required to produce the cracks $W_i$ as reported in Eq. 5.12:

$$W_e = W_i$$  \hspace{1cm} 5.12

The external work is obtained as the sum of the product of the KEL value $q$ per unit length factorized by $\lambda$, the collapse load factor, times the lengths of these load portions and times the linear virtual displacements $\delta(x,y)$ as reported in Eq. 5.13:

$$W_e = \lambda \sum q(x) \cdot l_{KEL} \cdot \delta$$  \hspace{1cm} 5.13

The arbitrary maximum value of the displacement field $\delta(x,y)$ is chosen as 1, in correspondence to the points of convergence of the wall portions. All the other intermediate points displace of a quantity linearly proportional to $\delta = 1$.

The internal work is given as the sum of the energy contributions from the various cracks present in the mechanism, therefore defined as the product of the moment capacity per unit length $m$, the length of the crack and the virtual rotation $\theta$, namely the total relative rotation between adjacent portions being separated by a crack, equal to the rotation of each portion of the wall with respect to their axis of rotation as reported in Eq. 5.14:

$$W_i = \sum m \cdot l_{crack} \cdot \theta$$  \hspace{1cm} 5.14

Approach_S follows the same procedure used to analyse orthotropic concrete slabs (i.e., slabs with different amount of reinforcement in the two orthogonal directions) to analyse brickwork panels exhibiting different strength and stiffness orthotropies (Sinha, B. P. (1978)). More specifically, orthotropic slabs can be simplified to corresponding isotropic slabs defined as ‘affine’, by modifying the geometry and assuming an equal ratio of reinforcement for both directions, thus adopting affine transformation. In his approach for the ultimate load analysis of orthotropic masonry panels subject to lateral loading, Sinha, B. P. (1978) proposed a way of linking the two wall moment capacities (similarly to the reinforcement areas of concrete slabs) through the parameter $\mu$, defined as the ratio between the two flexural strengths in the two directions perpendicular $f_{xk1}$ (a) and parallel $f_{xk2}$ (b) to bed joints as shown in Figure 5-8 and reported in Eq. 5.16.

![Figure 5-8](image)

Figure 5-8: Flexural tensile strength perpendicular (a) and parallel (b) to bed joints
\[ m_h = \frac{m_v}{\mu} \quad \text{5.16} \]

The \( f_{xk1} \) and therefore \( m_v \), increases with the depth of the course with respect to the height of the wall. To account for such variation in Approach_S, the internal moment capacity at a wall scale is defined as an average of the \( m_v \) at brick level across the wall courses and is given as a sum of the \( f_{xk1} \) and the vertical stress induced by the weight of the wall calculated at the specific brick/bloc course considered. As the \( W_i \) is defined, the ultimate scope is to determine the collapse load factor \( \lambda \), the multiplier of loads which causes collapse, as reported in Eq. 5.17:

\[ \lambda = \frac{W_i}{W_e} \quad \text{5.17} \]

In accordance with Yield Line theory’s principles, when a wall is subjected to a KEL, the point of convergence of cracks is usually placed in correspondence to the KEL itself, as the point of maximum displacement will belong to that line. Therefore, in favour of generalising the approach to all loading profiles, irrespective of the loading type considered, the position of \( y \) along the height of the wall (H) is checked for \( 1/3 \) H, \( 1/2 \) H and \( 2/3 \) H, thus representing the main assumption of the proposed simplified approach. For the CP1 cases, given the symmetry of the pattern with respect to the length of the wall, the equations show to have one unknown term only and that is the height \( y \) of the point of convergence. Minimising the virtual work equations, provide a unique feasible solution for \( y \) and the corresponding value of the collapse load factor \( \lambda \), which is a minimum. On the other hand, when assuming CP2 patterns, the work equations are characterized by two unknown parameters to determine the position of the point of convergence of diagonal crack lines, namely the coordinate \( y \) along the height of the wall and the coordinate \( x \) from the wall vertical sides, or the angle that the crack forms with the vertical edge. The equations are therefore solved in two steps, first by assigning values of \( y \) and then by calculating the corresponding value of \( x \). The process repeats until the minimum \( \lambda \) is found, providing the optimal solution. Table 5-3 includes the symbols used in the work equilibrium equations detailed in Table 5-4 to Table 5-6 for CP1, and from Table 5-7 to Table 5-9 for CP2. Crack pattern 1 (CP1) for earthquake loading (E) is taken as example. The first index of each term of the equations indicates the hazard considered, while the second index indicates the crack pattern considered. Finally, the last number refers to the modelling alternative 1 and 2.

**Table 5-3:** List of symbols used to define the Work Equilibrium Equations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_{ie,CP1-1} )</td>
<td>Internal work_LoadingProfile_CrackPattern_Modelling alternative</td>
</tr>
<tr>
<td>( W_{ie,CP1-1} )</td>
<td>External work_LoadingProfile_CrackPattern_Modelling alternative</td>
</tr>
<tr>
<td>( \lambda_{E,CP1-1} )</td>
<td>CollapseLoadFactor_LoadingProfile_CrackPattern_Modelling alternative</td>
</tr>
</tbody>
</table>
Table 5-4: Work Equations for earthquake CP1-1 and CP1-2

**Earthquake case CP1-1 (point 1 moves towards point 2)**

Conditions $y_1 = \frac{1}{3}H; y_1 = \frac{1}{2}H; y_1 = \frac{2}{3}H$

$$W_{ie.cp1-1} = \sum m \cdot \text{length}_{crack} \cdot \theta = m_v \frac{1}{y} + 2m_h \frac{2}{L} + m_v \frac{3}{H}$$

$$m_h = m_v / \mu$$

$$W_{ie.cp1-1} = m_v \frac{(L^2 \mu H + 3L^2 \mu y + 4H^2 y)}{y \mu L}$$

$$W_{ie.cp1-1} = \sum (q \cdot \text{length}_{KEL} \cdot \delta)$$

$$W_{ie.cp1-1} = \frac{\lambda q L}{y \mu L} \frac{L}{2} 2 \frac{L}{L}$$

$$\lambda_{ie.cp1-1} = \frac{2m_v (L^2 \mu H + 3L^2 \mu y + 4H^2 y)}{q \mu L (H - y - D_{1-2})}$$

**Earthquake case CP1-2 (point 2 moves towards point 1)**

Conditions $y_2 = \frac{1}{3}H$(from wall base); $y_2 = \frac{1}{2}H$(from wall base)

$$W_{ie.cp1-2} = \sum m \cdot \text{length}_{crack} \cdot \theta = m_v \frac{1}{y} + 2m_h \frac{2}{L} + m_v L \frac{1}{(H - y - D_{1-2})}$$

$$m_h = m_v / \mu$$

$$W_{ie.cp1-2} = m_v \frac{(L^2 \mu (H - y - D_{1-2}) + L^2 \mu y + 4Hy(H - y - D_{1-2}))}{y \mu L (H - y - D_{1-2})}$$

$$W_{ie.cp1-2} = \sum (q \cdot \text{length}_{KEL} \cdot \delta)$$

$$A: (H - K) = L/2; (H - y); \rightarrow A = ((H - K)L)/(2(H - y))$$

$$W_{ie.cp1-2} = 2 \lambda q \left( \frac{(H - K)L}{2(H - y)} \right) \frac{2}{2(H - y)} + \lambda q \left( \frac{L - 2(H - K)L}{2(H - y)} \right) (H - K) \frac{1}{(H - y)}$$

$$\lambda_{ie.cp1-2} = \frac{2m_v (L^2 \mu (H - y - D_{1-2}) + L^2 \mu y + 4Hy(H - y - D_{1-2})) (H - y)^2}{q \mu L (H - y - D_{1-2}) (2L(H - K)(H - y) - L(H - K)^2)}$$

Conditions $y_2 = \frac{2}{3}H$(from wall base)

$$\lambda_{ie.cp1-2} = \frac{2m_v L^2 \mu (H - y - D_{1-2}) + L^2 \mu y + 4Hy(H - y - D_{1-2})}{q \mu L^2 (H - y - D_{1-2})}$$
Table 5-5: Work Equations for flood CP1-1 and CP1-2

**Flood case CP1-1 (point 1 moves towards point 2) – (β = 1 → βH = H)**

*Conditions: 𝑦₁ = 𝐻/3*

\[ W_{IF,CP1-1} = W_{IE,CP1-1} = m_v \left( L^2\mu H + 3L^2\mu y + 4H^2y \right) / \gamma y L H \]

\[ W_{IF,CP1-1} = W_{IE,CP1-1,CONDITION}, \quad y_1 = \frac{1}{3} H; \quad y = \frac{1}{2} H \]

\[ \lambda_{FC,CP1-1} = \frac{2m_v(L^2\mu(H - y - D_{1-2}) + L^2\mu y + 4Hy(H - y - D_{1-2})}{\gamma y L^2(H - y - D_{1-2})} \]

*Conditions: \( y_1 = 1/2H; \quad y_1 = 2/3H \)

\[ W_{IF,CP1-1} = W_{IF,CP1-2} \]

\[ W_{IF,CP1-1} = \lambda_{FC,CP1-1} \frac{(KL KL) 2}{4y L} + \lambda q (L - 2 KL 2y L), \quad K \frac{1}{y} = \lambda q(2KLy - K^2L) \]

\[ W_{IF,CP1-2} = W_{IF,CP1-2} \]

\[ (L^2\mu(H - y - D_{1-2}) + L^2\mu y + 4Hy(H - y - D_{1-2}) \lambda_q(2KLy - K^2L) \]

\[ \lambda_{FC,CP1-1} = \frac{2m_v(L^2\mu y(H - y - D_{1-2}) + L^2\mu y^2 + 4Hy^2(H - y - D_{1-2})}{\mu L(H - y - D_{1-2})(2KLy - K^2L)} \]

**Flood case CP1-2 (point 2 moves towards point 1) – (β = 1 → βH = H)**

*Conditions: \( y_2 = \frac{H}{3}; \quad y_2 = \frac{H}{2}, y_2 = \frac{2H}{3} \) (all \( y_2 \) are from wall base)*

\[ W_{IF,CP1-2} = W_{IE,CP1-1} = \frac{\lambda q L}{2 L} \]

\[ \lambda_{FC,CP1-2} = \frac{2m_v(L^2\mu H + 3L^2\mu y + 4H^2y)}{q \gamma y L^2 H} \]

\[ W_{IF,CP1-2} = W_{IE,CP1-2}, \quad y_2 = \frac{1}{2} H (from \ wall \ base); \quad y_2 = \frac{2}{3} H (from \ wall \ base) \]

\[ \lambda_{FC,CP1-1-2} = \frac{2m_v(L^2\mu H - y - D_{1-2}) + L^2\mu y + 4Hy(H - y - D_{1-2})}{q \gamma y L^2 L} \]

\[ \lambda_{FC,CP1-2} = \frac{2m_v L^2\mu(H - y - D_{1-2}) + L^2\mu y + 4Hy(H - y - D_{1-2})}{\gamma y L^2(H - y - D_{1-2})} \]
Table 5-6: Work Equations for wind CP1-1 and CP1-2

**Wind case CP1-1 (point 1 moves towards point 2)**

Conditions: \( y_1 = \frac{1}{3H}; y_1 = \frac{1}{2H} \)

\[
\begin{align*}
W_{iw,CP1-1} &= W_{ie,CP1-1} \\
W_{ew,CP1-1} &= W_{ef,CP1-1} \\
W_{iw,CP1-2} &= W_{ie,CP1-1} \\
\lambda_{w,CP1-1} &= \frac{2m_v(L^2\mu(H - y - D_{1-2}) + L^2\mu y + 4Hy(H - y - D_{1-2}))}{q yL^2(H - y - D_{1-2})} \\
\text{Condition} &= y_1 = 2/3H; \quad W_{iw,CP1-1} = W_{ie,CP1-1} \\
\lambda_{w,CP1-1} &= \frac{2m_v(L^2\mu y(H - y - D_{1-2}) + L^2\mu y^2 + 4H^2 y^2(H - y - D_{1-2}))}{\mu L(H - y - D_{1-2})(2LK - K^2L)} \\
\end{align*}
\]

**Wind case CP1-2 (point 1 and 2 meet at H/2)**

Conditions: \( y_1 = \frac{1}{2H} \)

\[
\begin{align*}
\lambda_{w,CP1-1} &= \frac{2m_v(L^2\mu(H - y - D_{1-2}) + L^2\mu y + 4H^2y(H - y - D_{1-2}))}{q yL^2(H - y - D_{1-2})} \\
\end{align*}
\]
Wind case CP1-2 (point 2 moves towards point 1)

Conditions \( y_2 = \frac{1}{3}H \) (from wall base);

\[
W_{\text{W,CP1-2}} = \sum m \cdot \text{length} \cdot h_{\text{crack}} \cdot \theta = m_v \alpha \frac{1}{y} + 2m_h \frac{L}{y} + m_v \frac{L}{H - y - D_{1-2}}
\]

\( m_h = m_v / \mu \)

\[
W_{\text{W,CP1-2}} = m_v \frac{(L^2 \mu(H - y - D_{1-2}) + L^2 \mu y + 4H \mu(H - y - D_{1-2}))}{\mu L(H - y - D_{1-2})}
\]

\[
W_{\text{w,CP1-2}} = \sum (q \cdot \text{length}_{\text{w,KE}} \cdot \delta)
\]

\( A: (H - K) = L/2; (H - y); \rightarrow A = ((H - K)L)/(2(H - y)) \)

\[
W_{\text{w,CP1-2}} = 2\lambda \mu \frac{(1-H)(L(H-K)L)}{2(H-y)} 2(L - 2(H-K)L) \left( \frac{H-K}{(H-y)} \right) \frac{1}{(H-y)}
\]

\[
W_{\text{w,CP1-2}} = \frac{2m_v}{q} \frac{(L^2 \mu(H - y - D_{1-2}) + L^2 \mu y + 4H \mu(H - y - D_{1-2}))}{\mu L(H - y - D_{1-2})} L(H - y) - L(H - K)^2
\]

Condition \( y_2 = 2/3H; y_2 = 1/2H \)

\[
W_{\text{W,CP1-2}} = W_{\text{IE,CP1-2}} = W_{\text{IF,CP1-2}}
\]

\[
W_{\text{w,CP1-2}} = \frac{W_{\text{w,CP1-2}}}{W_{\text{w,CP1-2}}}
\]

\[
\lambda_{\text{W,CP1-2}} = \frac{2m_v}{q} \frac{(L^2 \mu(H - y - D_{1-2}) + L^2 \mu y + 4H \mu(H - y - D_{1-2}))}{\mu L(H - y - D_{1-2})}
\]

Table 5-7: Work Equations for earthquake CP2-1 and CP2-2

Earthquake case CP2-1

Condition = \( y = \frac{1}{3}H; y = \frac{1}{2}H \)

\[
W_{\text{IE,CP2-1}} = \frac{m_v L}{y} + 2m_h \frac{H}{x} + m_v L \frac{1}{H - y}
\]

\( m_h = m_v / \mu \)

\[
W_{\text{IE,CP2-1}} = m_v \frac{(L(H-y)\mu + 2H \mu(H-y) + L \mu y \mu)}{\mu y \mu(H-y)}
\]

\[W_{\text{IE,CP2-1}} \rightarrow A: (H - K) = x; (H - y); \ A = x \frac{(H - K)}{H - x}; B = L - 2A \]

\[
W_{\text{IE,CP2-1}} = 2\lambda \mu \left( \frac{(H-K)x}{2(H-y)} \right) 2\mu \frac{L}{(H-y)} \frac{1}{(H-y)} + \lambda \mu \left( \frac{L}{2(H-K)} \right) \left( \frac{H-K}{(H-y)} \right) \frac{1}{(H-y)}
\]

\[
W_{\text{IE,CP2-1}} = \frac{\lambda \mu}{(H-y)^2} \frac{(L(H-y)(H-K) - x(H-K)^2)}{(H-y)^2}
\]

\[
W_{\text{IE,CP2-1}} = \frac{W_{\text{IE,CP2-1}}}{W_{\text{IE,CP2-1}}}
\]

\[
\lambda_{\text{IE,CP2-1}} = \frac{m_v}{q} \frac{(L \mu(H-y)^2 + 2H \mu(H-y)^2 + L \mu y \mu(H-y))}{\mu L \mu(H-y) - \mu y^2 \mu(H-K)^2}
\]

\[
d\lambda/dx \rightarrow 0
\]
• assign values to y values comprised in a range
• solve 1st order differential equation; find value of x, determine \( \min \lambda_{E.CP2-1} \)

Condition = \( y = \frac{2}{3} H \)

\[
W_{E.E.CP2-1} \rightarrow A: (H - K) = x; (H - y) = \frac{x(H - K)}{H - y};
\]

\[
W_{E.E.CP2-1} = 2\lambda q \left( \frac{(H - K)x(H - K)x}{(H - y) \cdot 2(H - y)} \right)
\]

\[
W_{E.E.CP2-1} = \lambda q \left( \frac{(H - K)^2 x}{(H - y)^2} \right)
\]

\[
W_{E.E.CP2-1} = W_{E.E.CP2-1}
\]

\[
m_v \frac{L(H - y)x\mu + 2Hy(H - y) + Lxy\mu}{xy\mu(H - y)} = \frac{\lambda q ((H - K)^2 x)}{(H - y)^2}
\]

\[
\lambda_{E.CP2-1} = \frac{m_v (Lx\mu(H - y)^2 + 4yH(H - y)^2 + Lxy\mu(H - y)^2)}{yx^2\mu(H - K)^2}
\]

\[
d\lambda /dx \rightarrow 0
\]

• assign values to y values comprised in a range
• solve 1st order differential equation; find value of x, determine \( \min \lambda_{E.CP2-2} \)

Earthquake case CP2-2

Condition = \( y = \frac{1}{3} H; y = \frac{1}{2} H \)

\[
W_{E.E.CP2-2} = m_vL \frac{1}{y} + 2m_nH \frac{1}{x} + m_vL \frac{1}{H - y}
\]

\[
m_b = m_v/\mu
\]

\[
W_{E.E.CP2-2} = m_v \left( L(H - y)x\mu + 2Hy(H - y) + Lxy\mu \right)
\]

\[
W_{E.E.CP2-2} = \frac{L(H - y)(H - K) - x(H - K)^2}{(H - y)^2}
\]

\[
W_{E.E.CP2-2} = \frac{L(H - y)(H - K) - x(H - K)^2}{(H - y)^2}
\]

\[
m_v \frac{Lx\mu(H - y)^2 + 2Hy(H - y)^2 + Lxy\mu(H - y)}{Lxy\mu(H - K)(H - y) - yx^2\mu(H - K)^2}
\]

\[
d\lambda /dx \rightarrow 0
\]

• assign values to y values comprised in a range
• solve 1st order differential equation; find value of x, determine \( \min \lambda_{E.CP2-2} \)
Table 5-8: Work Equations for flood CP2-1 and CP2-2

Flood case CP2-1: $\beta = 1 \rightarrow \beta H = H$

Condition = $y = 2/3H$

$W_{IE,CP2-1} = W_{IE,CP2-1}$

$W_{ee,CP2-2} \rightarrow A: K = x/3$; $A = \frac{Kx}{3}$; $B = L - 2A$

$W_{ee,CP2-2} = 2\lambda q \left( \frac{Kx}{y} \frac{1}{x} + \lambda \left( L - 2 \frac{Kx}{3} \right) \frac{K}{y} \right)$

$W_{ee,CP2-2} = \frac{LKy - xK^2}{y^2}$

$W_{IE,CP2-2} = W_{ee,CP2-2}$

$m_v \left( (H - y)x\mu + 2Hy(H - y) + Lxy\mu \right) = \lambda \frac{LKy - xK^2}{y^2}$

$d\lambda / dx \rightarrow 0$

- assign values to $y$ values comprised in a range
- solve 1st order differential equation; find value of $x$, determine $\min \lambda_{CP2-2}$
Flood case CP2-2: \( \beta = 1 \rightarrow \beta H = H \)

\[\begin{align*}
\text{Condition} &= y = \frac{1}{2} H; \ y = \frac{2}{3} H \\
W_{e_{CP2-1}} &\rightarrow A: K = x; \ y = x \left( \frac{xK}{y} \right); \ B = L - 2A \\
W_{e_{CP2-1}} &= 2\lambda q \left( \frac{Kx}{y} \right) + \lambda q \left( L - 2 \frac{Kx}{y} \right) (K) \\
W_{e_{CP2-1}} &= \frac{Lxy - K^2x}{y^2} \\
W_{e_{CP2-1}} &= \frac{Lxy - K^2x}{y^2} \\
\lambda_{e_{CP2-2}} &= \frac{m_v (Lxy - K^2x)}{q Lxy - y^2x^2K^2} \\
d\lambda / \text{d}x &\rightarrow 0
\end{align*}\]

- assign values to y values comprised in a range
- solve 1st order differential equation; find value of x, determine min \( \lambda_{e_{CP2-2}} \)

\[\begin{align*}
\text{Condition} &= y = \frac{1}{3} H \\
A: K = x; \ y = x \left( \frac{xK}{y} \right); \ B = L - 2A \\
W_{e_{CP2-2}} &= 2\lambda q \left( \frac{Kx}{y} \right) + \lambda q \left( L - 2 \frac{Kx}{y} \right) (K) \\
W_{e_{CP2-2}} &= \frac{Lxy - K^2x}{y^2} \\
W_{e_{CP2-2}} &= \frac{Lxy - K^2x}{y^2} \\
\lambda_{e_{CP2-2}} &= \frac{m_v (Lxy - y^2 + 2y^2H(H - y) + Lxy^2\mu)}{q Lxy - y^2x^2K^2} \\
d\lambda / \text{d}x &\rightarrow 0
\end{align*}\]

- assign values to y values comprised in a range
- solve 1st order differential equation; find value of x, determine min \( \lambda_{e_{CP2-2}} \)

\[\begin{align*}
\text{Condition} &= y = 1/2H; \ y = 2/3H; \\
A: K = x; \ y = x \left( \frac{xK}{y} \right); \ B = L - 2A \\
W_{e_{CP2-2}} &= 2\lambda q \left( \frac{Kx}{y} \right) + \lambda q \left( L - 2 \frac{Kx}{y} \right) (K) \\
W_{e_{CP2-2}} &= \frac{Lxy - K^2x}{y^2} \\
W_{e_{CP2-2}} &= \frac{Lxy - K^2x}{y^2} \\
\lambda_{e_{CP2-2}} &= \frac{m_v (Lxy - y^2 + 2y^2H(H - y) + Lxy^2\mu)}{q Lxy - y^2x^2K^2} \\
d\lambda / \text{d}x &\rightarrow 0
\end{align*}\]

- assign values to y values comprised in a range
- solve 1st order differential equation; find value of x, determine min \( \lambda_{e_{CP2-2}} \)
### Table 5-9: Work Equations for wind CP2-1 and CP2-2

#### Wind case CP2-1

*Condition* = $y = \frac{1}{3}H$

$$W_{w, cp2-1} = m_v \frac{L}{y} + 2m_h \frac{1}{x} + m_v \frac{L}{H - y}$$

$m_h = m_v / \mu$

$$W_{w, cp2-1} = m_v \frac{(L(H - y)x\mu + 2Hy(H - y) + Lxy\mu)}{xy\mu(H - y)}$$

$W_{w, cp2-1} \rightarrow A: (H - K) = x: (H - y); \quad A = x \frac{(H - K)}{H - y}; \quad B = L - 2A$

$$W_{w, cp2-1} = 2\lambda q \left( \frac{(H - K)x(H - K)x}{(H - y)} \right) + \lambda q \left( L - 2 \frac{(H - K)x}{(H - y)} \right) \frac{(H - K)}{(H - y)^2}$$

$$W_{w, cp2-1} = \frac{(L(H - y)(H - K) - x(H - K)^2)}{(H - y)^2}$$

$m_v \frac{(L(H - y)x\mu + 2Hy(H - y) + Lxy\mu)}{xy\mu(H - y)} = \frac{\lambda q}{q} \frac{(L(H - y)(H - K) - x(H - K)^2)}{(H - y)^2}$

$$\lambda_{w, cp2-1} = m_v \frac{(Lx\mu(H - y)^2 + 2yH(H - y)^2 + Lxy\mu(H - y))}{Lx\mu(H - y)(H - y) - yx^2\mu(H - K)^2}$$

$d\lambda / dx \rightarrow 0$

- assign values to $y$ values comprised in a range
- solve 1st order differential equation; find value of $x$, determine $\min \lambda_{w, cp2-1}$

*Condition* = $y = \frac{1}{2}H$

$$W_{w, cp2-1} \rightarrow A: (H - K) = x: (H - y); \quad A = x \frac{(H - K)}{H - y};$$

$$W_{w, cp2-1} = 2\lambda q \left( \frac{(H - K)x}{(H - y)} \right)$$

$$W_{w, cp2-1} = \lambda q \frac{(H - K)^2x}{(H - y)^2}$$

$$W_{w, cp2-1} = W_{w, cp2-1}$$

$m_v \frac{(L(H - y)x\mu + 2Hy(H - y) + Lxy\mu)}{xy\mu(H - y)} = \lambda q \frac{(H - K)^2x}{(H - y)^2}$

$$\lambda_{w, cp2-1} = m_v \frac{(Lx\mu(H - y)^2 + 2yH(H - y)^2 + Lxy\mu(H - y))}{yx^2\mu(H - K)^2}$$

$d\lambda / dx \rightarrow 0$

- assign values to $y$ values comprised in a range
- solve 1st order differential equation; find value of $x$, determine $\min \lambda_{w, cp2-1}$
From the equations presented in previous tables it becomes easy to determine which crack pattern is the most likely to form, which also corresponds to be minimum collapse load factor $\lambda$. It is sufficient to have the two main geometric characteristics of the IWs, namely the wall length and wall height, the position of KEL point of application and the distance from the base (K) or top of the wall (H-K) and the ratio of the two flexural strength in the direction perpendicular and parallel to bed joint, as these are needed to determine $\mu$ and, consequently the relationship between the two moments of capacity (as reported in Eq. 5.12). To this end, two layouts of the same wall simply supported on all four edges, shown in Figure 5-4, are used.

More specifically, Index Wall 1 (IW1), for which the typical crack pattern is of CP1 type, is characterized by a H/L ratio greater than 1 (i.e., wall predominantly taller than wider) and Index Wall 2 (IW2), for which the typical crack pattern is of CP2 type, is characterized by a H/L ratio smaller than 1.
(i.e., wall predominantly wider than taller). The walls are assumed to follow the English bond arrangement presented in Figure 5-9. The geometric and material characteristics needed are defined in Table 5-10. Note that length and height of the two IWs are determined based on a finite number of bricks and mortar joints, the dimensions of which are also reported, although not required to conduct the calculations.

![Figure 5-9: English bond arrangement](image)

**Table 5-10: Geometrical Characteristics of two generic IWs to verify Approach_S**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall Length IW1</td>
<td>3.055</td>
<td>m</td>
</tr>
<tr>
<td>Wall Height IW1</td>
<td>4.5</td>
<td>m</td>
</tr>
<tr>
<td>Wall Length IW2</td>
<td>4.465</td>
<td>m</td>
</tr>
<tr>
<td>Wall Height IW2</td>
<td>3</td>
<td>m</td>
</tr>
<tr>
<td>Brick Unit Length</td>
<td>0.215</td>
<td>m</td>
</tr>
<tr>
<td>Brick Unit Height</td>
<td>0.065</td>
<td>m</td>
</tr>
<tr>
<td>Brick Unit Width</td>
<td>0.102</td>
<td>m</td>
</tr>
<tr>
<td>Flexural tensile strength perpendicular to bed joint ($f_{xk1}$)</td>
<td>0.20</td>
<td>MPa</td>
</tr>
<tr>
<td>Flexural tensile strength parallel to bed joints ($f_{xk2}$)</td>
<td>0.40</td>
<td>MPa</td>
</tr>
<tr>
<td>Orthogonal strength ratio ($\mu$)</td>
<td>0.5</td>
<td>--</td>
</tr>
</tbody>
</table>

Table 5-11 and Table 5-12 include the values of collapse load factors $\lambda$ of the two crack patterns modelling alternatives CP1-1 and CP1-2, for the two index walls IW1 and IW2 respectively. In conducting the calculations, the value of moment capacity $m_v$ and load $q$ are kept as constants and the three assumed positions of point of maximum displacement $y$ are checked. Worth reiterating that, whilst in the case of IW1 the only unknown is $y$, in the case of IW2, to find the minimum value of $\lambda$ and assumption on the position of $x$, indicating the distance of the point of convergence of cracks from the vertical sides of the wall, is required, which ranges between 0 and L/2 in the case of CP2-2.

The results shown in Table 5-11 and Table 5-12 prove that the minimum value of $\lambda$ is always found in correspondence of the $y$ which coincide with the point of application of the KEL, thus confirming the YL theory principles. Symmetric layouts show to be characterized by the same value of $\lambda$ factor. The small discrepancy which can be observed in the case of CP2-2 (only when comparing flood and
earthquake) is due to the fact that for the flood case, the internal moment capacity terms benefits from the weight of the courses above the position of the KEL, which increases the contribution to the \( M_v \), thus resulting in a \( \lambda \) factor slightly bigger.

**Table 5-11:** Approach_S: \( \lambda \) factors for IW1, CP1 configurations

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( D_{1-2} )</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CP1-1</strong></td>
<td>( \frac{1}{3}H )</td>
<td>5.19</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{6}H )</td>
<td>4.96</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>4.85</td>
</tr>
<tr>
<td><strong>CP1-2</strong></td>
<td>( \frac{1}{3}H )</td>
<td>5.19</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{6}H )</td>
<td>5.58</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>6.47</td>
</tr>
<tr>
<td><strong>CP1-1</strong></td>
<td>( \frac{1}{3}H )</td>
<td>5.19</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{6}H )</td>
<td>5.84</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>6.47</td>
</tr>
<tr>
<td><strong>CP1-2</strong></td>
<td>( \frac{1}{3}H )</td>
<td>5.19</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{6}H )</td>
<td>4.96</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>4.85</td>
</tr>
<tr>
<td><strong>CP1-1</strong></td>
<td>( \frac{1}{3}H )</td>
<td>5.19</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{6}H )</td>
<td>4.96</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>5.73</td>
</tr>
<tr>
<td><strong>CP1-2</strong></td>
<td>( \frac{1}{3}H )</td>
<td>5.19</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{6}H )</td>
<td>4.96</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>5.73</td>
</tr>
<tr>
<td><strong>CP1-1</strong></td>
<td>( \frac{1}{3}H )</td>
<td>5.19</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{6}H )</td>
<td>5.58</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>5.73</td>
</tr>
<tr>
<td><strong>CP1-2</strong></td>
<td>( \frac{1}{3}H )</td>
<td>5.19</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{6}H )</td>
<td>5.58</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>5.73</td>
</tr>
</tbody>
</table>

73
### Table 5-12: Approach_S: \( \lambda \) factors for IW2, CP2 configurations

<table>
<thead>
<tr>
<th>CP2-1</th>
<th>CP2-2</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="CP2-1 Diagram" /></td>
<td><img src="image2" alt="CP2-2 Diagram" /></td>
</tr>
<tr>
<td>[ y = \frac{1}{3} H \rightarrow \lambda = 7.21 ]</td>
<td>[ y = \frac{1}{3} H \rightarrow \lambda = 7.51 ]</td>
</tr>
<tr>
<td>[ y = \frac{1}{2} H \rightarrow \lambda = 5.70 ]</td>
<td>[ y = \frac{1}{2} H \rightarrow \lambda = 6.46 ]</td>
</tr>
<tr>
<td>[ y = \frac{2}{3} H \rightarrow \lambda = 5.40 ]</td>
<td>[ y = \frac{2}{3} H \rightarrow \lambda = 6.26 ]</td>
</tr>
</tbody>
</table>

Table 5-11 and Table 5-12 also shows that even in the case of CP2, the case where both the crack pattern and KEL position are symmetric result in the smallest \( \lambda \) factor among the whole set. Adopting Approach_S helps discarding the least probable CP, thus being a very useful tool for initial screening. However, to properly characterize the pattern and to have a more accurate estimation of the collapse load factor and hence – the load causing collapse - a more detailed analysis of factors contributing to define the wall moment capacity is needed. Streaming from the more simplified approach, a more detailed approach – Approach_D – is presented in the following section.

### 5.5. Detailed method (Approach_D)

Starting from the conventional YL theory concepts adopted to derive the work equilibrium equations presented in the previous section, a more detailed approach defined as Approach_D is introduced, which differs from the simplified approach in two main aspects, namely:

- A more refined definition of the crack pattern defined at the scale of the brick/block unit instead of for unit length of crack, thus resulting in cracks that follow the stepped failure pattern and the shape of the brick/block unit and mortar, as shown in Figure 5-10 a).
- A more refined definition of the wall moment capacity by means of the inclusion of the torsional effects to determine the horizontal moment capacity developing as a consequence of the application of horizontal loading acting out-of-plane, as shown in Figure 5-10 b).
Considering these two aspects, Approach_D is set up to define each contribution to internal and external work at the brick/block scale. Such approach is therefore more accurate in determining the angle of crack forming with the vertical support, and therefore also affects the interaction between the KEL and the crack pattern, resulting in a more accurate definition of the external virtual work.

![Figure 5-10: a) Stepped failure pattern; b) vertical and horizontal components of internal moment capacity](image)

The computational burden added with Approach_D lies on the fact that each wall course along the height of the wall is checked as potential location of maximum displacement corresponding to the minimum collapse load multiplier, therefore requiring an optimization routine (OR) which aims at finding the optimal solution in a more expeditious and efficient manner. The baseline equations embedded in the OR are the ones presented in Approach_S, modified in the following way:

- The definition of the $y$ parameter of the equation (for both CP1 and CP2), which in Approach_D corresponds to the summative height of each single masonry course; therefore, it can only assume a set of discrete values.
- The definition of the $x$ parameter of the equation (for CP2), which also can only assume discrete values to the length of brick to height of course ratio and to the staggering pattern.
- The relationship between the two bending moment capacities in the two orthogonal directions, which is now defined with reference to the geometry of the brick/block unit considered and the additional torsional component at brick/bloc unit interface level.

The architecture of the OR is detailed in Figure 5-11, which maps the logic tree approach followed to determine the parameters needed for the calculations and it is formed of four different parts, namely:

- Phase I: Identification of the wall moment capacities components.
- Phase II: Implementation of parametric equations for generic crack patterns, given geometry.
• Phase III: Computation of parametric equations of Wi and We and optimisation of the lower bound solution.
• Phase IV: Outputs ($\lambda$, $\chi$, $y$, $x$) for the optimal solution.

The cells in the flowchart are numbered progressively (e.g. [5.14]) to refer to the underlying mathematical formulations, which are also included in the present section.
Figure 5-11: Architecture of the OR
In Phase I the primary inputs are defined and divided into four categories, namely geometric data, (i.e. length, height and thickness of the wall, length and height of the brick unit used, thickness of the mortar joint), material properties to characterize the wall (i.e. density of bricks, flexural tensile strength of mortar, orthogonal strength ratio, friction coefficient); constants to determine the external horizontal loading action (i.e. gravity, density of water, density of air) and IMs characterizing the hazard considered in each case (i.e. PGA, βH and V reference or design values).

From the geometric data, the geometric secondary inputs are defined, namely:

- Staggering ratio corresponding to the net-half length of the brick unit + the mortar header joint given as:
  \[ s_u = s_{\text{unit}+\text{joint}} = (l_u + t_j)/2 \] 
  5.18

- No. units in the length of the wall:
  \[ L/2s_u \] 
  5.19

- No. units within the height of the wall:
  \[ H/(h_u + t_j) \] 
  5.20

It is assumed that the thickness of horizontal and vertical joints is the same:

- \( Z_1 \) module to determine the moment capacity along the vertical direction:
  \[ Z_1 = (t_{\text{wall}}^2 \cdot s_u)/6 \] 
  5.21

- \( Z_2 \) module to determine the flexural component of the moment capacity along the horizontal direction:
  \[ Z_2 = (t_{\text{wall}}^2 \cdot (h_u + t_j))/6 \] 
  5.22

- \( K_{\text{be}} \) – the torque coefficient to determine the torsional component of the moment capacity along the horizontal direction. For the purpose of this study, the formula of Timoshenko & Goodier, (1970) revised by Vaculik, J. (2012) is adopted:
  \[ k_{b(T\&G)} = \frac{1}{3} \left( \frac{1}{\pi^2} \sum_{n=1,3,5,...}^{\infty} \frac{1}{n^4} \tanh \left( \frac{n\pi r}{2} \right) \right) \]
  \[ - \frac{1}{8} \left( \frac{1}{\pi^2} \sum_{n=1,3,5,...}^{\infty} \frac{1}{n^2} \cosh \left( \frac{n\pi r}{2} \right) \right) \] 
  5.23

From the material characteristics the following parameters are derived:

- The characteristic initial shear strength of masonry, under zero compressive stress \( f_{v0k} \) used to derive the torsional component of the moment capacity on horizontal joints is a function of the flexural tensile strength of masonry in the direction perpendicular to bed joints \( f_{xK1} \) factorized by \( \gamma \).
\[ f_{vk0} = \gamma f_{xk1} \]  

- The flexural tensile strength of masonry in the direction parallel to bed joints \( f_{xk2} \), needed to calculate the flexural moment capacity \( m_h \) relates to the flexural tensile strength of masonry in the direction perpendicular to bed joints \( f_{xk1} \) needed to calculate the flexural moment capacity \( m_v \): Eq. 5.25:

\[ f_{xk2} = f_{xk1}/\mu \]  

While the relationship between the two strengths provided in Approach_S depends only on the \( \mu \) factor and so is the relationship between the two moment capacities, in Approach_D the relationship linking the moment capacities in the two orthogonal directions depends on the geometry of the brick/block unit and is specified as

\[ m_h = \frac{m_v}{\xi} \]  

- The \( \xi \) factor is defined in Eq. 5.27:

\[ \xi = \frac{(h_{\text{unit}} + t_{\text{joint}})}{s_{\text{unit+joint}}} \frac{1}{\mu} \]  

with \( h_{\text{unit}} \) being the height of the brick/block unit, \( t_{\text{joint}} \) being the thickness of the mortar joint, and \( s_{\text{unit+joint}} \) being the staggering ratio equal to half of the length of unit and mortar joint. The \( \xi \) factor accounts for the fact that the two surfaces over which the stresses in the two orthogonal directions develop have different surface area.

In order to determine the shear strength at each course needed to compute the torsional capacity the wall weight for unit length of wall can be computed at each wall course as reported in Eq. 5.28

\[ p_{\text{wall}}(y) = \rho_{\text{masonry}} \cdot (H - y) \cdot t \]  

with \( y \) indicating the brick course considered, starting from the base of the wall towards the top and the term \( (H-y) \) being the portion of wall above the course considered. The wall weight is then used to compute the vertical axial stress \( \sigma_v \) reported in Eq. 5.29

\[ \sigma_v = \frac{p_{\text{wall}}(y)}{t} \]  

from which the shear strength at each wall course level \( y \) can be obtained following the formulations proposed in EC6 (CEN (2010)) and Tomaževič, M. (2009), reported in Eq. 5.30:

\[ \tau_{um}(y) = \gamma f_{mt} + \phi \sigma_v \]  

As reported by several authors (Atkinson, R. H., et al (1989), Riddington, J. R., & Ghazali, M. Z. (1990), Vasconcelos, G., & Lourenço, P. B. (2009)), the shear strength of masonry under moderate
normal stress is given by the Coulomb criterion reported in Eq. 5.30. More specifically, \( y f_{int} \) represents the shear strength at zero vertical stress usually denoted as cohesion while \( \phi \) is the coefficient of friction.

The total flexural strength perpendicular to bed joints at each wall course level \((y)\) can be obtained from eq. 5.31:

\[
f_{skTOr}(y) = (f_{sk1} + \sigma_v)
\]

Therefore, the flexural and torsional moment capacities in the two orthogonal directions can be calculated as follows:

- the flexural moment capacity along the bed joints – at unit scale
  \[
mv_{unit}(y) = f_{sk1TOr} \cdot Z_1
\]

- the flexural moment capacity along the bed joint – at wall scale
  \[
  Mv_{wall}(y) = mv_{unit} \cdot (L/s_u)
\]

- the torsional moment capacity on the bed joint – at unit scale
  \[
  mh\, torsion\, \text{unit}(y) = \tau_{um} k_{be t_u^3}
\]

- the torsional moment capacity on the bed joint – at wall scale
  \[
  Mh\, torsion\, wall = \sum_{i=1}^{n} \tau_{um}(n) k_{be t_u^3};
  \]
  with \( n = \) number of courses

- the flexural moment capacity parallel to bed joints (along the head joints) – at unit scale
  \[
  mh\, flexure\, \text{unit} = f_{sk2} \cdot Z_2
  \]

- the flexural moment capacity parallel to bed joints (along the head joints) – at wall scale
  \[
  Mh\, flexure\, wall = \sum_{i=1}^{n} (f_{sk2} \cdot Z_2)
  \]
  with \( n = \) number of courses

This set of equations completes Phase I of the OR. The computation of the component moments at the scale of the single unit (brick or block), allows the procedure the flexibility to be applied to any crack pattern, geometry of walls and boundary conditions, while complying with the assumption that the cracks will run along the joints and that the strength capacity is determined by the bond at the interface between brick and mortar. This is also particularly useful for non-symmetrical conditions, either due to geometry or boundary conditions, as it allows to reduce the number of variables determining the crack pattern.

Phase II is dedicated to check the ‘feasible’ crack patterns after the screening process conducted through
the Approach_S, which – as shown in previous section – helps determining that the position of maximum displacement \( y \) is always located in correspondence to the position of the KEL. The feasible and admissible patterns are ‘re-adjusted’ to comply with the geometry of the brick/block unit considered, thus ensuring that 1) the angle of crack forming between the diagonal cracks and the vertical wall edges of the wall, 2) the consequent rotation of each wall portion bounded between the cracks and 3) the displacement of each portion of KEL crossing the pattern, all follow the geometric shape ratio of the single unit. To this aims two geometric assumptions are adopted, such as:

- The diagonal crack can only span between the vertical support and half of the wall length progressing across the mortar layer path, aiming to cover the wall portion comprised between the horizontal support of the wall and the point of application of the KEL considered. Given the nature of the loading profiles and their different points of application, the OR is built to choose in a flexible manner which edge of the wall to prioritise when considering the progression of the crack. This means that, in the case of the earthquake loading, the crack starts progressing from the top corners towards the centre. Conversely, when considering the flood loading the OR prioritize the crack formation from the bottom corner towards the centre. In the case of wind loading, given the loading symmetric position with respect to the height of the wall, top and bottom corners are treated with equal priority, as the KEL is assumed to be placed at the middle of the wall height.

- The angle forming between the ‘return’ crack, namely the one progressing backwards to the other corner on the same side, and the vertical edge of the wall can:
  - Either remain the same: for this option to take place, it is assumed that the return crack mirrors the diagonal path forming by the bottom crack, thus implying that a vertical crack forms to connect the two diagonal cracks as shown in Figure 5-12: Diagonal cracks and angle of cracks options for a) CP1-IW1 and b) CP2 a). This pattern is typical of walls predominantly taller than longer, which tend to develop a CP1.
  - Either vary, as shown in Figure 5-12 b). This pattern is typical of walls predominantly longer than taller, which tend to develop a CP2.

![Figure 5-12: Diagonal cracks and angle of cracks options for a) CP1-IW1 and b) CP2-IW2](image-url)
In phase III of the OR shown in Figure 5.11, the internal and external work contributions are determined at the brick/block unit level, starting from the general definition of $W_e$ and $W_i$ provided respectively in Eq. 5.13 and Eq. 5.14. Depending on the assumptions made on the angle of rotation with the vertical side of the wall detailed in Phase II, the terms $y$ and $x$ get consequently determined and the pattern is defined, as a result of the optimization routine.

The internal work at the level of the brick/block unit is defined as follows:

$$W_i(y) = \left[ mv_{\text{unit}}(y) \cdot \left( \frac{L}{s_u} \right) \cdot \frac{1}{y} \right] + \left[ mv_{\text{unit}}(y) \cdot \left( \frac{L}{s_u} \right) \cdot \frac{1}{(H - y)} \right] + 2 \left[ mh_{\text{torsion \ unit}}(y) + mh_{\text{flexure \ unit}}(y) \right] \cdot \left( \frac{H}{s_u} \cdot \frac{1}{x} \right)$$

5.38

Without loss of generality, Eq. 5.38 refers to the case of a wall simply supported on its four sides and with symmetric conditions. In Eq. 5.38, each term can be updated in relation of the specific boundary condition considered.

With regards to the external work contribution, for each increment in the number of courses and for each crack pattern hypothesized, the KEL portions A and B can be expressed as a function of the coordinate $x$ of the point of convergence of diagonal crack lines, measured from the vertical support of the wall, and for each discrete value of $y$, as reported in Eq. 5.39, 5.40 and 5.41 and represented in Figure 5-13:

$$A \rightarrow if \ y < H_{KEL} \rightarrow W_{te}(y) = \left[ x \cdot (H - H_{KEL})/(H - y) \right]$$

5.39

$$A \rightarrow if \ y > H_{KEL} \rightarrow W_{te}(y) = \left[ H_{KEL} \cdot \frac{x}{y} \right]$$

5.40

$$B = L - 2A$$

5.41

With $H_{KEL}$ being the location of the point of application of the KEL.

---

**Figure 5-13**: Crack pattern – KEL interaction example
As the terms of the work equilibrium equations are defined at brick/block unit scale, Phase IV is dedicated to determining the outputs, namely the collapse load factor $\lambda$, the corresponding position of maximum displacement $y$ within the height of the wall and the performance variable $\chi$.

In the optimization of parametric equations, $\lambda$ is the load multiplier for a given wall capacity and loading condition: values of $\lambda>1$ indicate that the wall has not failed, values of $\lambda<1$ indicate that the wall fails when subjected to the specific loading as it indicates that the external work is greater than the internal work; values of $\lambda=1$ determine the failure threshold for a given wall configuration, thus determining the value of the applied load and therefore of the specific hazard intensity measure (IM) that will cause collapse and the threshold of ultimate capacity. Given a geometry and strength characteristics, the optimization routine of the parametric analysis determines the values of IM for which $\lambda=1$. As the $\lambda$ is determined, the factor $\chi$ can be determined, which provides instead with an indication of the strength capacity of the wall given the flexural behaviour and is defined as reported in Eq. 5.42:

$$
\chi = \frac{1}{\lambda} = \frac{W_e}{W_i}
$$

![Figure 5-14: Coded environment for case of flood loading](image)

The OR mapped in Figure 5.11 was implemented and coded using VBA. Figure 5-14: Coded environment for case of flood loading provides with an example of the coded environment for the case
of flood loading. After inputting the primary geometrical and material characteristics of the wall investigated, the ‘single run’ button allows to find the min $\lambda = 1$, corresponding to the IM which causes collapse, the parameter $\chi$ and also the corresponding $y$ indicating the point of maximum displacement. Each button allows to run other realizations of the wall: for instance, the ‘RUN REALIZATIONS for L wall’ generates a number of walls which vary the parameter wall length whilst keeping the other parameters constant. The reasons and the rationale followed to generate different realizations starting from a single wall are detailed in Chapter 6.

5.6. Comparison between Approach_S and Approach_D

Having detailed Approach_D in the previous section, and all the parameters needed to conduct the parametric analysis to find the $\lambda$ factor, the $\chi$ factor, the $y$ position of maximum displacement and the IM causing collapse, a comparison is conducted between Approach_S and Approach_D, to be able to appreciate the increased level of accuracy achieved by adopting the latter procedure. The same two wall configurations presented in Section 5.4, used to verify the assumptions of Approach_S, are also used herein. However, to implement Approach_D several other geometric and material parameters are needed, which, for completeness, are all included in Table 5-13.

Table 5-13: Geometrical Characteristics of two generic IWs to compare Approach_S and Approach_D

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall Length IW1</td>
<td>3.055</td>
<td>m</td>
</tr>
<tr>
<td>Wall Height IW1</td>
<td>4.5</td>
<td>m</td>
</tr>
<tr>
<td>Wall Length IW2</td>
<td>4.465</td>
<td>m</td>
</tr>
<tr>
<td>Wall Height IW2</td>
<td>3</td>
<td>m</td>
</tr>
<tr>
<td>Wall thickness</td>
<td>0.215</td>
<td>m</td>
</tr>
<tr>
<td>Brick Unit Length</td>
<td>0.215</td>
<td>m</td>
</tr>
<tr>
<td>Brick Unit Height</td>
<td>0.065</td>
<td>m</td>
</tr>
<tr>
<td>Brick Unit Width</td>
<td>0.102</td>
<td>m</td>
</tr>
<tr>
<td>Mortar Thickness bed joint</td>
<td>0.01</td>
<td>m</td>
</tr>
<tr>
<td>Mortar Thickness head joint</td>
<td>0.01</td>
<td>m</td>
</tr>
<tr>
<td>Density of brick</td>
<td>17</td>
<td>kN/m$^3$</td>
</tr>
<tr>
<td>Flexural tensile strength perpendicular to bed joint ($f_{x1}$)</td>
<td>0.20</td>
<td>MPa</td>
</tr>
<tr>
<td>Flexural tensile strength parallel to bed joints ($f_{x2}$)</td>
<td>0.40</td>
<td>MPa</td>
</tr>
<tr>
<td>Orthogonal strength ratio ($\mu$)</td>
<td>0.5</td>
<td>--</td>
</tr>
<tr>
<td>Friction coefficient ($\phi$)</td>
<td>0.5</td>
<td>--</td>
</tr>
<tr>
<td>Shear strength of masonry ($f_{vk0}$)</td>
<td>0.32</td>
<td>MPa</td>
</tr>
<tr>
<td>Value of torsion coefficient ($k_{bs}$)</td>
<td>0.2</td>
<td>--</td>
</tr>
</tbody>
</table>
With regard to the material properties reported in the Table 5-13, the value $f_{xk1}=0.20$ MPa and $f_{xk2}=0.40$ MPa, are taken from EC6 Section 3.6.3 (CEN (2010)), as these are indicated as the upper bound characteristic value of clay bricks masonry; from their ratio, the value of orthogonal strength $\mu$ is obtained. The friction coefficient value used for this comparison is taken from Section 3.6.2 of EC6 (CEN, (2010)). Following the Eq. 5.24, the characteristic shear strength of masonry under zero compressive stress proposed by EC6 (CEN (2010)) ranges between 0.1 and 0.3 MPa. However, the experimental campaigns conducted by Vaculik, J. (2012) provides a mean value of shear strength under zero compressive strength equal to $1.6 f_{xk1}$. In this study, the latter estimation obtained from experimental works is adopted. Regarding the torsion coefficient, Eq. 5.19 is used.

To the aim of conducting a meaningful comparison between Approach_S and Approach_D, it is necessary to define sets of IMs such that the magnitudes of the three KEL ($q_{seismic}$, $q_{flood}$ and $q_{wind}$) acting on the walls are commensurate. Assuming a wind speed $V$ of 30 m/s, a value within the range bounding category 1 of damage in the Saffir-Simpson (SS) Hurricane Scale (Simpson, R. H., & Saffir, H. (1974)), corresponding values of PGA and $\beta H$ are adjusted to fit the purpose of the comparison. With reference to the equations included in Section 5.2, Table 5-14 includes the IMs adopted for the current comparison which result in commensurate earthquake, flood and wind KELs.

<table>
<thead>
<tr>
<th>CP &amp; IW</th>
<th>IM</th>
<th>Value</th>
<th>Unit</th>
<th>Load Value formula</th>
<th>Load value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGA</td>
<td>0.92 g</td>
<td>$q_{seismic} = \frac{2.5}{15} \alpha a_g g H t \rho_{brick}$</td>
<td>25.66 kN/m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CP1 – IW1</td>
<td>$\beta$</td>
<td>0.51</td>
<td></td>
<td>$q_{flood} = \frac{1}{2} (\beta H)^2 g \rho_{water}$</td>
<td>25.94 kN/m</td>
</tr>
<tr>
<td>V</td>
<td>30 m/s</td>
<td>$q_{wind} = c_e(z) \frac{1}{2} g V^2 H \rho_{air}$</td>
<td>25.31 kN/m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PGA</td>
<td>0.92 g</td>
<td>$q_{seismic} = \frac{2.5}{15} \alpha a_g g H t \rho_{brick}$</td>
<td>16.81 kN/m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CP2 – IW2</td>
<td>$\beta$</td>
<td>0.62</td>
<td></td>
<td>$q_{flood} = \frac{1}{2} (\beta H)^2 g \rho_{water}$</td>
<td>16.79 kN/m</td>
</tr>
<tr>
<td>V</td>
<td>30 m/s</td>
<td>$q_{wind} = c_e(z) \frac{1}{2} g V^2 H \rho_{air}$</td>
<td>16.87 kN/m</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5-15 includes the $\lambda$–$y/H$ curves (i.e. position of maximum displacement of the point of convergence of cracks normalized to the height of the wall) obtained by applying both Approach_D and Approach_S, whilst Table 5-16 includes the results of the same comparisons in terms of $\lambda$ factor and % of error calculated in the computation of wall moment capacity, to better quantify the increased level of accuracy achieved by the latter approach.

Note that the patterns reported in Table 5-15 are exclusively the ones resulting in the smallest $\lambda$ factors among all modelling alternatives presented in Table 5.2, Section 5.3, as also verified through the screening process conducted by implementing Approach_S, reported in Table 5-11 and Table 5-12.
Table 5-15: Comparison between Approach_S and Approach_D for crack patterns CP1-IW1 and CP2-IW2

<table>
<thead>
<tr>
<th>Loading/CP/IW</th>
<th>$\lambda$ Approach_S</th>
<th>$\lambda$ Approach_D</th>
<th>$W_i$ (kNm m) Approach_S</th>
<th>$W_i$ (kNm m) Approach_D</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth/CP1-IW1</td>
<td>0.527</td>
<td>0.945</td>
<td>20.324</td>
<td>36.634</td>
<td>44.85%</td>
</tr>
<tr>
<td>Flood/CP1-IW1</td>
<td>0.533</td>
<td>0.963</td>
<td>20.691</td>
<td>37.162</td>
<td>44.32%</td>
</tr>
<tr>
<td>Wind/CP1-IW1</td>
<td>0.578</td>
<td>1.028</td>
<td>20.331</td>
<td>36.638</td>
<td>44.50%</td>
</tr>
<tr>
<td>Earth/CP2-IW2</td>
<td>0.341</td>
<td>0.558</td>
<td>16.443</td>
<td>26.881</td>
<td>38.83%</td>
</tr>
<tr>
<td>Flood/CP2-IW2</td>
<td>0.557</td>
<td>0.974</td>
<td>24.630</td>
<td>43.052</td>
<td>42.79%</td>
</tr>
<tr>
<td>Wind/CP2-IW2</td>
<td>0.364</td>
<td>0.561</td>
<td>13.736</td>
<td>21.144</td>
<td>35.03%</td>
</tr>
</tbody>
</table>

Noticeably, the curves representing CP1-IW1 reported in Table 5-15: Comparison between Approach_S and Approach_D for crack patterns CP1-IW1 and CP2-IW2a) and b) follow very similar trend and similarly, the values of min $\lambda$, which as reported in Table 5-16: Error! Reference source not found., are all around the value of 1, therefore indicating that — for the specific IMs chosen, the wall has just — or are about — to fail. When looking instead at the same results obtained with Approach_S there is a very clear underestimation of wall moment capacity, resulting in smaller $\lambda$ values and therefore much lower IMs would be needed to get the wall to collapse. Moreover, the plateau region which can be observed
in Figure 5-16 a) and b), is to be referred to the presence of a vertical crack and its length in comparison to the H wall. Along these cracks, bounded within 0.23 H to 0.66 H in the case of earthquake, within 0.23 H and 0.5 H in the case of wind and 0.23 and 0.4 in the case of flood, the only contribution to the wall moment capacity derives from the bending moment generated by the f\(k_2\) and the torsional effects in the case of Approach_D, whilst it is only dependent on the bending moment generated by the f\(k_2\) in the case of Approach_S. Notably, the reason why the plateaus start from 0.23 H is to be ascribed to the dimension of the brick unit chosen: for the diagonal crack to form and to progress towards the middle of the wall, the portion of wall height covered is only 23%. In the case of CP2-IW2, reported in Table 5 15: Comparison between Approach_S and Approach_D for crack patterns CP1-IW1 and CP2-IW2 c) and d), there is a neat difference between earthquake and wind curves compared to the flood curve. Firstly, whilst the \(\lambda\) values for the former two are far smaller than 1, indicating that the wall has already failed for the defined IMs, the value of \(\lambda\) flood is just below 1, thus proving that \(\beta=0.61\) represents the threshold of water height that leads the wall to collapse. Linked to this observed discrepancy is the shape of the crack pattern, and the extent to which the cracks develop within the wall. As proven in previous sections, the point of maximum displacement is usually located in correspondence to the position of the KEL. Given the nature of the aspect ratio of IW2, the diagonal cracks have more length of wall available to form – which for this L/H ratio chosen the available length for the diagonal cracks to form is equal to 2.25 m - thus generating an angle of crack which is much wider in the case of wind and earthquake than is in the case of flood. Consequently, the rotations between adjacent portions being bounded by a crack (which directly affect the determination of the wall moment capacity as shown in Eq. 5.14) – obtained as the inverse of the distance from the vertical edge of the wall (i.e. distance x introduced in Table 5 8 and Table 5 9 are considerably smaller in the case of earthquake and wind than in the case of flood. This explains, in mathematical terms, the reason why – even though the magnitudes of the \(q_{\text{loading}}\) have been chosen commensurately – the CP2-IW2 for the flood case provides results which are noticeably different compared to the other cases. Finally, Table 5-17 provides the comparison between Approach_S and Approach_D in terms of crack pattern shape and position of point of maximum displacement \(y_\lambda\) (indicated with dots), for earthquake, flood, and wind loading. The graphs also include the distance from the lower point of convergence of diagonal cracks, indicated as \(y_{\text{low_PoC}}\), which in the case of IW1 layouts, becomes fundamental to draw the crack pattern.
Table 5-17 Comparison between Approach_S and Approach_D results in terms of min $\lambda$ and $y_{max}$

**Earthquake**

<table>
<thead>
<tr>
<th>Approach_S</th>
<th>Approach_D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_\lambda = 0.66H$</td>
<td>$y_\lambda = 0.66H$</td>
</tr>
<tr>
<td>$y_{low_{PoC}} = 0.23H$</td>
<td>$y_{low_{PoC}} = 0.23H$</td>
</tr>
</tbody>
</table>

**Flood**

<table>
<thead>
<tr>
<th>Approach_S</th>
<th>Approach_D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_\lambda = 0.416H$</td>
<td>$y_\lambda = 0.416H$</td>
</tr>
<tr>
<td>$y_{low_{PoC}} = 0.23H$</td>
<td>$y_{low_{PoC}} = 0.23H$</td>
</tr>
</tbody>
</table>

**Wind**

<table>
<thead>
<tr>
<th>Approach_S</th>
<th>Approach_D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_\lambda = 0.57H$</td>
<td>$y_\lambda = 0.57H$</td>
</tr>
<tr>
<td>$y_{low_{PoC}} = 0.23H$</td>
<td>$y_{low_{PoC}} = 0.23H$</td>
</tr>
</tbody>
</table>
The comparison between Approach_S and Approach_D conducted in this section has proven that, irrespective of the approach used, the failure pattern shape does not vary. In the case of earthquake, for both the IW1 and IW2 configurations the point of maximum displacement coincides with the KEL position. In the case of flood, the y position is shifted above the line load and so is the case for wind IW1 configuration. The shift observed increases the further the KEL moves towards the base of the wall, thus indicating that the weight of the wall itself plays a stabilizing role against the OOP load action. Moreover, as shown in Table 5-15 a) and b) the curves are characterized by a plateau region, which indicates all the points along the vertical crack line which – by the nature of the bricks’ sizes chosen – runs from 0.23H wall until 0.66H wall. All these points display the same (or very similar) quantity – therefore the position indicated in the graph represent the absolute maximum of all points displacing of the same proportion of \( \delta \), which, as mentioned, is also shifted due to weight of the bricks’ courses.

On the other hand, the accuracy in determining the \( \lambda \) factor and the computation of wall capacity are substantially affected by the choice of the approach used. It is therefore demonstrated that, a more simplified approach can be useful to conduct the initial screening of the feasible collapse mechanisms hypothesized for a given set of restraints and boundary conditions and to expeditiously discard the less likely patterns to form, however a more detailed approach is needed to precisely estimate the various contributions determining the wall moment capacity and – consequently – to be able to estimate its response when subjected to specific types of loading conditions.

5.7. Conclusions

This chapter presented the analytical model developed to simulate the behaviour of brick/block masonry structures subjected to seismic, flood and wind loading, thus detailing step 3 of the multi-hazard assessment framework proposed in this study. It has been shown that conventional Yield Line Theory concepts, typically employed to investigate the behaviour of concrete slabs against distributed loads, can be also used to evaluate the out-of-plane behaviour of masonry walls undergoing horizontal loading profiles of various nature and shape. The assumptions required to simulate the seismic, flood and wind loading applied to two approaches of different level of refinement, prove that a common and homogenised framework can be developed, ensuring that the approach adopted in EC6 (CEN (2010)) for wind loading can be extended to other loading profiles.
Equations are then presented to verify all the crack patterns modelling alternatives, which are found compatible to the loading profiles and the boundary conditions of the wall, to demonstrate the ease of implementation and the reduced computational effort required to implement these across the three perils. It is then shown how the conventional Yield Line Theory concepts, which prove effective in determining the system behaviour at wall scale, can be further improved by moving the analysis at the brick/block unit scale and by including the torsional component of the wall moment capacity developed as a result of the horizontal actions imposed by the three loadings, therefore the simplified Approach_S is compared to the more detailed Approach_D to highlight the advantages and the required computational efforts needed to reach to a more accurate estimation of the collapse load factor and consequently a more refined estimation of the wall moment capacity. Approach_D is then extended exclusively to the modelling alternatives that have been proved admissible through Approach_S. To the aim of efficiently checking all these screened options, an optimization routine (OR) which incorporates the loading schematization assumptions, the crack patterns alternatives, and the more refined computations of the capacity of the system based on Approach_D is presented, including all the equations implemented and the geometric constraints imposed to determine the patterns which should comply to the geometry of the units constituting the wall. Finally, a comparison between the two approaches is provided, to highlight the influence of the torsional component in determining the overall wall moment capacity and the specific use of each of the two approaches.

The following chapter is dedicated to the determination of the building sample, necessary to conduct fragility assessment and to the validations of the proposed procedure against the relevant Code, against relevant experimental tests available in literature and against a twin model built by using the discrete element (DE) modelling software ELS. The aim of these comparisons is twofold: to assess the extent of variation in the results obtained from the application of the proposed procedure and to enhance confidence in generating the sets of results presented in following chapters.
6. Determination of building sample and validation of proposed procedure

6.1. Introduction

One of the major gaps highlighted in the state-of-the-art is the different level of advancement of the single-hazard vulnerability assessment methods in relation to seismic, flooding and cyclonic effects, often hindering the harmonization needed to develop a well-structured multi-hazard vulnerability assessment approach.

The review of the state-of-the-art methods to conduct vulnerability assessment of historic URMs subjected to seismic, flood and wind loadings reported in Chapter 2 has shown that there is still a substantial level of discrepancy among these approaches, which reflects onto the definition of the parameters needed to derive fragility functions. A full fragility function model includes both the variability in the demand to and in the capacity of the system. In the present study the emphasis is on determining the applicability of the Yield Line theory through the application of the kinematic approach (detailed in Chapter 5) to be an effective and simple method to represent the variance in response of walls with different geometry, boundary conditions and material characteristics. Therefore, in this study, fragility functions are produced from lognormal distributions of different responses of many walls’ models with different characteristic, rather than the more conventional approach of using one model and varying the value of the intensity measure, usually pursued for seismic fragility assessment (Baker, J. W. (2015), Lallemant et al., (2015)). This is justified by the fact that, while for seismic fragility analysis there is a consolidated literature that provides robust methods for considering the variation in response due to the randomness of the hazard, similar methods are not readily available for flooding or wind analytical fragility analysis. Therefore, the fragility functions are derived here by generating variability in the exposure component (i.e., the walls) by varying the values of all the parameters identified in section 5.5 and summarised in Table 5-13.

This chapter is dedicated to discussing the steps followed to derive single-hazard fragility curves to allow for comparison across the three hazards investigated in this research study and to establish the relevance of the parameters used for their derivation.

A general flowchart is provided to help detailing the steps followed to generate single-hazard fragility curves to allow for comparison across the three hazards (Fig. 6.1). Section 6.2 is dedicated to the walls’ taxonomy generation: this provides the walls’ configurations considered in this study, the method used to generate the variance in each parameter, and the literature consulted to justify the range of values chosen for both the geometry and material properties.

The analytical method detailed in Chapter 5 proves that it is possible to overcome the gaps found in the literature which relate to modelling the behaviour of URMs subjected to earthquake, flood and wind
loading by using a single approach common to all hazards. However, before applying it to a large sample to derive fragility functions, which are the main focus of Chapter 7, a set of validations of the proposed analytical approach is provided, to verify how it compares to the current state-of-the-art procedures.

Section 6.3 includes a validation against the procedure contained in EC6 (CEN (2010)) to determine the resistance of masonry walls to lateral loading. Since the EC6 procedure focuses on walls subjected to uniformly distributed load, the comparison focuses on the case of wind loading.

Section 0 discuss a comparison with experimental studies available in literature by Vaculik, J. (2012), conducted on full scale URM walls subjected to face loading using a system of airbags, appropriately supported at the vertical and horizontal edges to ensure that under face loading the wall undergoes two-way bending. The purpose of this testing is to determine the response of the specimens to simulated seismic loading, hence the comparison is drawn with the seismic loading condition of the analytical procedure.

Finally, Section 6.5 compares the proposed analytical procedure with a detailed modelling of the wall using the Extreme Loading for Structure (ELS) software, based on the Discrete Element (DE) method, to determine the level of accuracy of the methods when compared to advanced numerical tools. Since the software can simulate different loading profiles and loading actions, results for all 3 hazards are discussed in this section.
Figure 6-1: Flowchart detailing the derivation of single-hazard fragility curves
6.2. Generation of walls’ taxonomy

As stated in the introduction, the focus of the study is to verify the analytical method proposed and to extract fragility curves by considering the variability of the building sample, hence focusing on aleatory aspect of the exposure component of the methodological framework presented in Chapter 4. The kinematic approach adopted for determination of the performance variable $\chi$ is the one discussed in Section 5.5, namely Approach_D, defined as a more refined approach compared to the simplified method based on conventional YL Theory concept which neglects the torsional component of the wall moment capacity and considered instead the geometry of the pattern at wall scale.

To this aim, the first step is to define the population of analysed buildings that a given fragility function represents ([D’Ayala, D., et al (2015)]) both in terms of 1) boundary conditions, 2) geometric characteristic and 3) material characteristics.

To generate variability in the sample of walls considered in this study, for the set of parameters in the model defining the response of the walls it is necessary to determine their range of variation, and then to adopt an appropriate sampling approach. Similar to the concept of “one index building” proposed by the GEM guidelines, ([D’Ayala, D., et al (2015)]), for each configuration it is possible to define an “index wall” for which the geometric configuration, dimension characteristics, mechanical characteristics are defined by a chosen value and a range determined by a lower and an upper bound, as to reach a realistic distribution of characteristic membership and to account for the wall-to-wall variability.

To date, the repositories of buildings and their performance against natural hazards events are still being updated and the process is still on-going. The GED4ALL taxonomy ([Silva, V., et al (2018)]) endeavours to define a uniform taxonomy for structures, by extending the GEM taxonomy v2.0 ([Brzev, S., et al (2013)]), which was created considering a wide spectrum of existing taxonomies such as the PAGER-STR, and WHE and mainly focusing on earthquake, by implementing attributes which are relevant to other perils. Although very comprehensive in its structure, the GED4ALL (and neither of the other existing taxonomy repositories) does not provide with a list of the parameters (geometric and material) needed to build an analytical model which focuses on defining the capacity of the structure against different types of natural hazards. Table 6.1 includes the building features that reflect the expected vulnerability against earthquake, flood, and wind loading.

To generate the variability in the sample, the one-at-a-time approach is chosen. The approach consists of varying each of the parameters in input across the entire range of plausible values while holding all others in their reference value, to evaluate the effects on the model output caused by each input variation ([Cullen, A. C., Frey, H. C. (1999), Frey, H., & Patil, S. R. (2002)]).
<table>
<thead>
<tr>
<th>Feature</th>
<th>Description</th>
<th>Modelling of the feature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral load resisting</td>
<td>The type of material (bricks/blocs), the mortar quality, the bricks/blocks</td>
<td>The material characteristics affect the determination of the wall moment capacity in the horizontal and vertical directions as well as the torsional strength.</td>
</tr>
<tr>
<td>system</td>
<td>density and the mortar strengths</td>
<td></td>
</tr>
<tr>
<td>Height</td>
<td>The height of the building is usually detailed in terms of presence of</td>
<td>Each floor is modelled as an independent wall element, and any type of connection with horizontal structures is translated through sets of boundary conditions.</td>
</tr>
<tr>
<td></td>
<td>basement, ground floor level and height above grade.</td>
<td></td>
</tr>
<tr>
<td>Roof</td>
<td>Type of connection between the roof and the wall and material of the</td>
<td>The model simulates the presence of any portion of roof laying above the wall as additional weight acting on the wall, while the type of roof-to-wall connection is simulated through the boundary conditions.</td>
</tr>
<tr>
<td></td>
<td>roof structure and cover.</td>
<td></td>
</tr>
<tr>
<td>Openings</td>
<td>Number, size and position of openings within the length and height of wall</td>
<td>The presence of openings is accounted for as a ‘void’, thus affecting the overall wall moment capacity. When considering the load action, it is assumed the openings are sealed.</td>
</tr>
<tr>
<td>Foundation type</td>
<td>Defines the stability of the building exposed to lateral displacement.</td>
<td>Depending on the type of connection with the super-structures, the foundations are modelled through boundary conditions, namely as a simple support if the wall-to-foundation connection is weak or as a fix support if the wall-to-foundation connection is strong. Presence of settlements are not considered</td>
</tr>
</tbody>
</table>

The difference in the model output due to the change in input variable is referred to as the sensitivity or swing weight of the model to that particular input variable (Morgan, M. G., Henrion, M., & Small, M. (1990)). The main inconvenience of using the ‘one-at-the-time’ (OAT) method is that interdependency among the whole set of parameters cannot be detected since the variables are not changed simultaneously (Critchfield, G. C., Willard, K. E., & Connelly, D. P. (1986), Saltelli, A., & Annoni, P. (2010), Morio, J. (2011)). Nevertheless, this approach allows to generate a sample of walls which covers the range of values identified as most typical, as explained in the following subsections, therefore producing the variation in response, necessary to generate the fragility functions. On the other hand, the approach is useful to determine the pattern of sensitivity of the results, within the variables’ space. In addition, a convenient advantage of the OAT method in deterministic models is that allows a clearer identification of the relevance of the parameter to the response and, as this is currently a new approach, this is a critical output that helps validate it (Saltelli, A., & Annoni, P. (2010)).
6.2.1. Boundary conditions

In Chapter 5 the kinematic approach including crack pattern, loading and corresponding structural response in terms of flexural behaviour and collapse capacity was developed with reference to a rectangular wall simply supported on its four sides, considering how the crack pattern varies when the geometric aspect ratio of the wall (H/L) changes. In this section other boundary conditions are considered, and a taxonomy of walls is generated, with the aim of including the sets which best represents wall bays within a single façade. The layouts considered in this case study have been grouped according to the number and the position of free edges within the wall (Group 1 and Group 2), according to the type of restraint characterizing the 4 edges (Group 3) and the number and position of openings (Group 4); to account for any variation within the same Group, letters and numbers were used. An example is reported in Figure 6-2: Corresp. The façade of the building considered is assessed in terms of global aspect ratio, openings layout and number of bays – which can be obtained by considering the number of walls perpendicular to the façade considered. The focus is on the ground floor of this building, which is built in masonry, while the first storey, built in timber, is only considered in terms of loading on the lower storey, but not in terms of the limit state analysis. Each bay is considered as a separate wall panel and the rationale followed to establish the type of restraint conditions for each edge of the 3 bays in Figure 6-2 is reported in detail in Table 6-2.

![Figure 6-2](image)

**Figure 6-2:** Correspondence between real façade and corresponding wall layouts realization – Casa Gorordo Museum, Cebu, Philippines
Table 6-2: Summary of the main characteristics of the wall configurations considered – reference to flowchart in Fig 6.1.

<table>
<thead>
<tr>
<th>Wall group</th>
<th>Wall Configuration Label</th>
<th>Restraints Conditions</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group A:</td>
<td>A1 3 sides simply</td>
<td>3 sides simply supported,</td>
<td>Single storey or top storey of building (SS/TS) with limited rotational restraint lateral (LRRL) connections, no connections with roof elements, lightweight roof cover, limited rotational restraint (LRR) foundation.</td>
</tr>
<tr>
<td></td>
<td>1 side (top) free</td>
<td>1 side (top) free</td>
<td></td>
</tr>
<tr>
<td>3 sides, top</td>
<td>A2 2 sides</td>
<td>2 sides simply supported,</td>
<td>SS/TS building, LRRL connections, no connections with roof elements, lightweight roof cover, rigid foundations or slabs (RF/S).</td>
</tr>
<tr>
<td>2 sides, top</td>
<td>fixed</td>
<td>1 side fixed</td>
<td></td>
</tr>
<tr>
<td>2 sides, top</td>
<td>(1 side free)</td>
<td>1 side free (lateral)</td>
<td></td>
</tr>
<tr>
<td>2 sides, top</td>
<td>A3 3 sides fixed</td>
<td>3 sides fixed supported,</td>
<td>SS/TS, full rotational restraint lateral connection (FRRL), no connection with roof elements, lightweight roof cover, RF/S.</td>
</tr>
<tr>
<td>3 sides, top</td>
<td>supported, 1 side</td>
<td>1 side fixed</td>
<td></td>
</tr>
<tr>
<td>2 sides, top</td>
<td>(top) free</td>
<td>(1 side free)</td>
<td></td>
</tr>
<tr>
<td>3 sides, top</td>
<td>B1 3 sides simply</td>
<td>3 sides simply supported,</td>
<td>Corner SS/TS building, LRRL connection to roof elements.</td>
</tr>
<tr>
<td>3 sides, top</td>
<td>supported, 1 side</td>
<td>1 side free (lateral)</td>
<td></td>
</tr>
<tr>
<td>3 sides, top</td>
<td>B2 2 sides simply</td>
<td>2 sides simply supported,</td>
<td>Corner SS/TS building, FRRL connection to roof elements.</td>
</tr>
<tr>
<td>multi</td>
<td>supported, 1 side</td>
<td>1 side fixed</td>
<td></td>
</tr>
<tr>
<td>storey, top</td>
<td>(lateral)</td>
<td>1 side free (lateral)</td>
<td></td>
</tr>
<tr>
<td>3 sides, top</td>
<td>B3 3 sides fixed</td>
<td>3 sides fixed</td>
<td>Corner lower storey of a multi-storey (LSMS) building, FRRL connection to roof elements and RF/S.</td>
</tr>
<tr>
<td>3 sides, top</td>
<td>supported, 1 side</td>
<td>1 side free (lateral)</td>
<td></td>
</tr>
<tr>
<td>3 sides, top</td>
<td>B4 2 sides simply</td>
<td>2 sides simply supported,</td>
<td>Corner SS/TS, LRR connection to roof elements, RF/S.</td>
</tr>
<tr>
<td>3 sides, top</td>
<td>supported, 1 side</td>
<td>1 side fixed</td>
<td></td>
</tr>
<tr>
<td>multi</td>
<td>(base)</td>
<td>1 side free (lateral)</td>
<td></td>
</tr>
<tr>
<td>3 sides, top</td>
<td>B5 1 side simply</td>
<td>1 side simply supported,</td>
<td>Corner SS/T, FRRL connection to roof elements RF/S.</td>
</tr>
<tr>
<td>multi</td>
<td>supported, 2 sides</td>
<td>2 sides fixed</td>
<td></td>
</tr>
<tr>
<td>multi</td>
<td>lateral (free)</td>
<td>1 side lateral (free)</td>
<td></td>
</tr>
<tr>
<td>3 sides, top</td>
<td>C1 4 sides simply</td>
<td>4 sides simply supported,</td>
<td>Intermediate storey (IT) or SS building, LRRL connections, LRR to roof element, LRR to foundations.</td>
</tr>
<tr>
<td>4 sides, top</td>
<td>supported, 1 side</td>
<td>1 side free</td>
<td></td>
</tr>
<tr>
<td>4 sides, top</td>
<td>C2 2 sides simply</td>
<td>2 sides simply supported,</td>
<td>IT/SS building, FRRL connections, FRR roof and F/S</td>
</tr>
<tr>
<td>4 sides, top</td>
<td>supported, 2 sides</td>
<td>2 sides fixed</td>
<td></td>
</tr>
<tr>
<td>4 sides, top</td>
<td>C3 4 sides fixed</td>
<td></td>
<td>IT/SS, FRR connections, FRR roof and F/S</td>
</tr>
<tr>
<td>4 sides, top</td>
<td>C4 3 sides simply</td>
<td>3 sides simply supported,</td>
<td>IT/SS, LRRL connections, LRR roof and FRR F/S</td>
</tr>
<tr>
<td>4 sides, top</td>
<td>supported, 1 side</td>
<td>1 side free</td>
<td></td>
</tr>
</tbody>
</table>
Each of the wall configurations presented in Table 6.1 represents a schematization of a possible constraint condition of an individual storey building façade. For instance, in the case of Fig. 6.2, the two external walls would be represented as an altered configurations B3, i.e., three sides fixed and 1 side free with opening, and the wall placed in the middle corresponds to a configuration D1 altered to represent the fixities of configuration C3.

### 6.2.2. Geometric and Material characteristics

To determine the geometric and material characteristics of typical masonry buildings, real data from field survey complimented with data from experimental testing campaigns would be necessary. However, since the focus of the present study is to demonstrate the applicability of the analytical method proposed over a large range of cases, the values of geometric characteristics such as length and height of the wall, length and height of the brick units and thickness of the wall, have been taken from the taxonomy repositories available in literature (i.e. GEM taxonomy first level attributes (Brzev, S., et al (2013)) validated by field work investigation databases collected by the author (Putrino, V., & D’Ayala, D. (2019), Putrino, V., & D'Ayala, D., (2019,b))). Table 6-3 summarizes the typical ranges of wall length and height, thickness of the wall, dimension of bricks of an individual storeys URM residential building as reported in Putrino, V., & D'Ayala, D., (2019, b)).

Regarding the material properties (i.e., flexural tensile strength in both directions and their ratio, friction coefficient and brick density) relevant to determine the moment capacities contributions discussed in Approach_D (Section 5.5), central values and range are obtained from existing literature. Sources are experimental, and consider a varied range of units, mortar type and mix, and testing conditions. Table 6-4 summarize the sources consulted and the ranges of values found for each parameter, and their coefficients of variation (CoV) when provided.
Table 6-3: Geometric characteristics ranges

<table>
<thead>
<tr>
<th>Geometric parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>3.00 m to 6.00 m</td>
</tr>
<tr>
<td>Wall height</td>
<td>2.00 m to 6.00 m</td>
</tr>
<tr>
<td>Wall thickness</td>
<td>0.102 m to 0.317 m</td>
</tr>
<tr>
<td>Unit length</td>
<td>0.190 m to 0.250 m</td>
</tr>
<tr>
<td>Unit height</td>
<td>0.050 m to 0.150 m</td>
</tr>
</tbody>
</table>

Table 6-4: Material characteristics ranges and related sources in literature

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Reference</th>
<th>Range</th>
<th>Mean Value</th>
<th>CoV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{xk2} ) (MPa)</td>
<td>CEN (2010), Mann, W. (1991); Sinha, B. P., et al. (1979); Hendry, A. W. (1973), Zhou, Z., et al. (2008)</td>
<td>0.2 to 1.5</td>
<td>0.80</td>
<td>0.30</td>
</tr>
<tr>
<td>Density ( \rho ) (kN/m(^3))</td>
<td>Zhou, Z., et al. (2008), Noor-E-Khuda, S., &amp; Albermani, F. (2019).</td>
<td>14.04 to 21.35</td>
<td>17</td>
<td>0.075</td>
</tr>
</tbody>
</table>

According to the studies reported herein, the flexural tensile strength in the direction perpendicular to bed joints \( f_{xk1} \) varies from 0.1 MPa to 0.85 MPa with average CoV of 0.26, while the flexural tensile strength parallel to bed joints, \( f_{xk2} \) has a range 0.2 to 1.5 MPa with average CoV of 0.30. The test campaign conducted by Zhou et al., (2008), on hydraulic lime mortared brickwork, with attention on the mortar mix, has reported that flexural strength generally increases with lime mortar strength. However, for the purpose of the present studies, the values of \( f_{xk1} \) and \( f_{xk2} \) are respectively 0.2 MPa and 0.4 MPa, opting for the ranges provided in EC6 (CEN (2010)). In addition to what is included in Table 6.2, the orthogonal strength ratio (i.e., the ratio of parallel to perpendicular plane of failure flexural strength) resulting from their test series brickwork varied between 0.21 and 0.45, proving to be comparable to the 0.33 used in the BS 5628-1:2005 and the EC6 National Annex (CEN (2010)).
friction coefficient $\phi$ ranges between 0.2 and 0.8. Zhou et al., (2008) have also focused their attention on determining ranges of density of brick and come up with ranges spanning between 14 and 21 kN/m$^3$.

As already discussed, shear strength values found in literature are the combination of cohesion (or initial shear strength under 0 normal stress) and friction coefficient as reported in Eq. 5.30, both greatly dependent on the type of unit used, its moisture content, porosity, strength and composition of the mortar and the nature of the interface (Amadio, C., & Rajgelj, S. (1991)). Bond wrench tests conducted by Zhou et al., (2008) have provided values of shear bond strength ranging between 0.11 MPa and 0.27 MPa determined in accordance with BS EN 1052-3:2002, while results from Sarangapani et al., (2005) range between 0.054 MPa and 0.265 MPa and CoV between 0.2 and 0.35. When considering instead solid bricks (Amadio, C., & Rajgelj, S. (1991)) the initial shear resistance under 0 compressive stress can reach values ranging between 0.65 N/mm$^2$ or 0.58 N/mm$^2$ (Binda et al., 1994)). From the literature it is also found that the $f_{sk1}$ can relate to the shear strength under 0 normal stress. More specifically, depending on the type of brick/block unit and mortar type chosen, such a relationship varies between 1.6 times (Vaculik, J. (2012)) up to 2 times (Tripathy, D., & Singhal, V. (2019)). The former relationship is adopted in this study and therefore not reported in Table 6-4.

As the mean values across the available sources are determined, and coefficient of variation are derived from the same sources, the mathematical relationship provided in CEN (2016) are used to determine the upper and lower bounds of the $f_{sk1}$, $f_{sk2}$, $\phi$ and density $\rho$, required to build up the sample of walls and conduct the parametric analysis, in the hypothesis that characteristic values obtained from experimental literature are determined as the 5% fractile values. Table 6-5 reports the equations used and results obtained.

**Table 6-5**: Flexural Tensile Strengths and Friction coefficient ranges upper and lower bounds

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Equation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{sk1} = f_{m}$ (MPa)</td>
<td>$f_{sk1lower} = 0.2 - 1.96 \times (0.26 \times 0.2)$</td>
<td>$0.098 , MPa \approx 0.1 , MPa$</td>
</tr>
<tr>
<td></td>
<td>$f_{sk1upper} = 0.2 + 1.96 \times (0.26 \times 0.2)$</td>
<td>$0.301 , MPa \approx 0.3 , MPa$</td>
</tr>
<tr>
<td>$f_{sk2}$ (MPa)</td>
<td>$f_{sk2lower} = 0.4 - 1.96 \times (0.30 \times 0.4)$</td>
<td>$0.165 , MPa \approx 0.16 , MPa$</td>
</tr>
<tr>
<td></td>
<td>$f_{sk2upper} = 0.4 + 1.96 \times (0.30 \times 0.4)$</td>
<td>$0.635 , MPa \approx 0.63 , MPa$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$\phi_{lower} = 0.5 - 1.96 \times (0.30 \times 0.5)$</td>
<td>$0.206 , MPa \approx 0.2 , MPa$</td>
</tr>
<tr>
<td></td>
<td>$\phi_{upper} = 0.5 + 1.96 \times (0.30 \times 0.5)$</td>
<td>$0.794 , MPa \approx 0.8 , MPa$</td>
</tr>
<tr>
<td>$\rho$ (kN/m$^3$)</td>
<td>$\rho_{lower} = 17 - 1.96 \times (0.075 \times 17)$</td>
<td>$14.50 , \frac{kN}{m^3} \approx 14 , kN/m^3$</td>
</tr>
<tr>
<td></td>
<td>$\rho_{upper} = 17 + 1.96 \times (0.075 \times 17)$</td>
<td>$19.49 , \frac{kN}{m^3} \approx 20 , kN/m^3$</td>
</tr>
</tbody>
</table>
Regarding the orthogonal strength ratio $\mu$ – representing the ratio $f_{\text{sk1}}/f_{\text{sk2}}$ – the full range recommended by EC8 (CEN (2005)) is considered, ranging between 0.1 and 1, the latter being representative of the isotropic-like behaviour. The mean value considered when varying all other parameters is 0.5, to reflect the ranges provided in EC6 (CEN, (2010)) and the relationship between the two strengths. Table 6-6 summarizes the characteristics of the ‘index wall’ and corresponding ranges adopted for all wall configurations. The lower and upper bound values of wall length and the wall height have also been established on the basis of respectively a finite number of half brick lengths plus mortar and a finite number of courses established considering the height of the brick plus one layer of mortar of the UK standard brick. With regard to the wall thickness, the brick length and brick height, these should be linked to the size of the unit selected as well as the wall arrangement selected. However, all over the world and throughout the ages, bricks have been manufactured to different dimensions, which yield different lengths, heights and wall thickness, in a modular way. To ensure that the current method is applicable to the majority of units sizes including stone blocks or concrete blocks which might have entirely different sizes leading to entirely different masonry patterns, the ranges of variation of these three geometric parameters are kept within average upper and lower bounds of common types of bricks/blocks sizes included in Zhou et al., (2008).

Finally, given that the geometric aspect ratio $H/L$ rules the determination of the crack patterns alternatives CP1 and CP2 discussed in Section 5.3 and investigated in Section 5.6, it is established to determine two index wall samples, namely Index Wall 1 (IW1) constituted by walls with $H/L$ ratio greater than 1 (i.e. predominantly taller than wider) and Index Wall 2 (IW2) made by walls with $H/L$ ratio smaller than 1 (i.e. predominantly wider than taller). Beside the global aspect ratio, IW1 and IW2 are characterised by the same local geometric characteristics and material properties, indicated in the second column of Table 6-6.

Table 6-6: Summary table with values and ranges of geometrical parameters and material properties to conduct the sensitivity analysis

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Wall layout</th>
<th>Parameter</th>
<th>Type</th>
<th>Category</th>
<th>Chosen Value</th>
<th>Lower &amp; Upper Bound</th>
<th>Size of set</th>
</tr>
</thead>
<tbody>
<tr>
<td>WL</td>
<td>IW1</td>
<td>Wall</td>
<td>Length</td>
<td>Geometric</td>
<td>3.055 m = 26 (half brick + mortar)</td>
<td>2.9375 m = 25 (half brick + mortar joint) \n 5.99 m = 51 (half brick + mortar joint)</td>
<td>93 total \n 31 for WH=WL/0.8 \n 31 for WH=WL/0.7 \n 31 for WH=WL/0.6</td>
</tr>
<tr>
<td>WH</td>
<td></td>
<td>Wall</td>
<td>Height</td>
<td>Geometric</td>
<td>4.5 m = 60 courses + mortar</td>
<td>1.95 m = 26 courses + mortar joint \n 4.5 m = 60 courses + mortar joint</td>
<td>78 total \n 26 for WL=0.8WH \n 26 for WL=0.7WH \n 26 for WL=0.6WH</td>
</tr>
<tr>
<td></td>
<td>Opening 1 length (for configuration D1)</td>
<td>Geometric</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>----------------------------------------</td>
<td>-----------</td>
<td>---</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O1L</td>
<td>Geometric</td>
<td>WL*0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O1H</td>
<td>Geometric</td>
<td>WL*0.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O2L</td>
<td>Geometric</td>
<td>WL*0.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O2H</td>
<td>Geometric</td>
<td>WL*0.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Opening 2 length (for configuration D2)</th>
<th>Geometric</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>O1L</td>
<td>Geometric</td>
<td>WL*0.5</td>
<td></td>
</tr>
<tr>
<td>O1H</td>
<td>Geometric</td>
<td>WL*0.7</td>
<td></td>
</tr>
<tr>
<td>O2L</td>
<td>Geometric</td>
<td>WL*0.2</td>
<td></td>
</tr>
<tr>
<td>O2H</td>
<td>Geometric</td>
<td>WL*0.3</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WL</th>
<th>Wall Length</th>
<th>Geometric</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.465m = 38 (half brick + mortar)</td>
<td>WL*0.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.99 m = 51 (half brick + mortar joint)</td>
<td>WL*0.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.9375 m = 25 (half brick + mortar joint)</td>
<td>WL*0.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>93 total</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>31 for WH= 0.8 WL</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>31 for WH= 0.7 WL</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>31 for WH= 0.6 WL</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WH</th>
<th>Wall Height</th>
<th>Geometric</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 m = 40 courses + mortar</td>
<td>WH*0.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.95 m = 26 courses + mortar joint</td>
<td>WH*0.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.5 m = 60 courses + mortar joint</td>
<td>WH*0.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>78 total</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>26 for WL = WH/0.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>26 for WL = WH/0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>26 for WL = WH/0.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wt</th>
<th>Wall Thickness</th>
<th>Geometric</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>IW1 &amp; IW2</td>
<td>0.215 m</td>
<td>Wt*0.8</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BL</th>
<th>Brick unit Length</th>
<th>Geometric</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>IW1 &amp; IW2</td>
<td>0.215 m</td>
<td>BL*0.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.215 m</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BH</th>
<th>Brick unit Height + joint</th>
<th>Geometric</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>IW1 &amp; IW2</td>
<td>0.065m + 0.01 m mortar joint</td>
<td>BH*0.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.065 m</td>
<td>BH*0.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.130 m</td>
<td>BH*0.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Noticeably, the dimensions of openings for both IW1 and IW2 are a function of the WL (and therefore WH) of the specific realization considered, thus ensuring they get updated every time the WL/WH ratio changes. As a default, it is decided to keep the height of the opening bigger than the length (for both IW1 and IW2) and, more importantly, the opening dimensions are designed to guarantee that enough space for piers and spandrels remain for the crack pattern to realistically form within each wall realisation considered. As the ranges and number of increments are defined, the OAT method can be applied. For each configuration of boundary conditions, two wall samples of 264 realisations are created as per Table 6-6 and the variability in ultimate capacity is then determined for each of the three hazards, using the OR defined in Chapter 5. The intensity measures chosen for each hazard are peak ground acceleration (PGA) for earthquake, floodwater depth ($\beta H$), and wind speed (V), respectively. Table 6-7 shows the ranges of existence of the IMs used to derive the fragility functions.

**Table 6-7** Summary table of IMs ranges adopted in the study

<table>
<thead>
<tr>
<th>IM</th>
<th>Range</th>
<th>Step of IM’s increments</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGA</td>
<td>0 g to 3.00 g</td>
<td>0.075 g</td>
</tr>
<tr>
<td>$\beta H$</td>
<td>0 m to H wall</td>
<td>equal to h brick</td>
</tr>
<tr>
<td>V</td>
<td>0 m/s to 150 m/s</td>
<td>5 m/s</td>
</tr>
</tbody>
</table>

As discussed in Chapter 4 and in dedicated sections of the literature, the definition of different damage states and corresponding values of EDP capacities which are commensurable across the three different hazards considered is an outstanding issue. Given the ultimate limit state approach and the assumption of rigid perfectly plastic constitutive model for the material only characterized by ultimate strength in tension, only the collapse damage state is defined, which occurs when the demand imposed to the system is greater than the capacity of the system itself, else represented by the performance variable $\chi$ attaining
a value of 1 discussed in Section 5.5. As reported in Figure 6-1, for each wall boundary condition and each realisation of set of parameters through the OAT sampling, by performing the optimisation routine of Approach_D (Section 5.5), a duplet of $\chi = 1$ value and corresponding IM causing collapse is defined, thus identifying the performance point (PP) for the specific wall configuration and crack pattern. Single-hazard fragility curves can then be generated by performing regression over the set of duplets by adopting a fitting technique which appropriately fits the dataset, as discussed in the review in section 2.4.4.

6.3. Comparison of the proposed method with EC6 – wind loading

This section is dedicated to providing a comparison between the proposed analytical approach and the method to determine the lateral resistance outlined in EC6 (CEN (2010)). Given that the EC6 procedure is developed for uniformly distributed loads, the comparison focuses on the wind loading case of the proposed procedure. Section 5.6 has already highlighted the importance of considering the torsional component of the horizontal moment capacity for a better estimation of the wall moment capacity. This section aims at 1) comparing both Approach_S, (Section 5.4), and Approach_D, (Section 5.5), against the Code (CEN (2010)) approach to verify which of the two compares better to the existing Standards and by which extent and 2) estimating the loss of accuracy in moving from the uniformly distributed load representation adopted in the Code to the KEL load distribution adopted in this study. Comparisons are made in terms of the collapse load factor $\lambda$. The configuration IW2 simply supported on four edges, (see Figure 5-4), used in Section 5.6 for the comparison between Approach_D and Approach_S, is used and the geometric and material characteristics adopted to detail the wall are summarized in Table 6-8.

Table 6-8: Geometric and material properties of wall for EC6 and Approach_S/Approach_D comparisons

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall Length</td>
<td>4.465</td>
<td>m</td>
</tr>
<tr>
<td>Wall Height</td>
<td>3</td>
<td>m</td>
</tr>
<tr>
<td>Wall thickness</td>
<td>0.215</td>
<td>m</td>
</tr>
<tr>
<td>Brick Unit Length</td>
<td>0.215</td>
<td>m</td>
</tr>
<tr>
<td>Brick Unit Height</td>
<td>0.065</td>
<td>m</td>
</tr>
<tr>
<td>Brick Unit Width</td>
<td>0.102</td>
<td>m</td>
</tr>
<tr>
<td>Mortar Thickness bed joint</td>
<td>0.01</td>
<td>m</td>
</tr>
<tr>
<td>Mortar Thickness head joint</td>
<td>0.01</td>
<td>m</td>
</tr>
<tr>
<td>Density of brick</td>
<td>17</td>
<td>kN/m$^3$</td>
</tr>
<tr>
<td>Flexural tensile strength parallel to bed joint ($f_{k1}$)</td>
<td>0.20</td>
<td>MPa</td>
</tr>
<tr>
<td>Flexural tensile strength orthogonal to bed joints ($f_{k2}$)</td>
<td>0.40</td>
<td>MPa</td>
</tr>
<tr>
<td>Orthogonal strength ratio ($\mu$)</td>
<td>0.5</td>
<td>--</td>
</tr>
<tr>
<td>Friction coefficient ($\phi$)</td>
<td>0.5</td>
<td>--</td>
</tr>
<tr>
<td>Shear strength of masonry ($f_{k0}$)</td>
<td>0.32</td>
<td>MPa</td>
</tr>
<tr>
<td>Value of torsion coefficient ($k_{be}$)</td>
<td>0.06</td>
<td>--</td>
</tr>
<tr>
<td>Wind speed</td>
<td>27.77</td>
<td>m/s</td>
</tr>
</tbody>
</table>
The crack pattern configuration considered for the proposed comparison is CP2_0.5, namely the one characterized by the point of convergence of diagonal cracks located at $\frac{1}{2}H$. (Table 5-12: Approach_S: $\lambda$ factors for IW2, CP2 configurations, Section 5.4) shown in Error! Reference source not found.. According to Approach_S, CP2_0.5 is the one corresponding to the min $\lambda$, while also being the pattern assumed by EC6 (CEN (2010)).

![Figure 6-3: Crack pattern used for comparison with EC6](image)

The magnitude of the wind loading $q_{\text{wind}}$ is determined by adopting the Eq.5.11 reported in Section 5.2. Following the Code procedure, to determine the demand on the walls, a bending moment coefficient defined as $\alpha_2$ is required, which depends on orthogonal strength ratio, wall aspect ratio and edge support conditions. The resultant applied moment per unit length with plane of failure parallel to bed joint is:

$$M_{Ed1} = \mu \alpha_2 q_{\text{wind}} H^2$$  \hspace{1cm} \text{6.1}

The applied moment per unit length with plane of failure orthogonal to bed joint is:

$$M_{Ed2} = \alpha_2 q_{\text{wind}} L^2$$  \hspace{1cm} \text{6.2}

Eq. 6.1 and 6.2 would normally include load safety factors which are not considered in this instance as the wall is checked for its ultimate capacity. The resisting moment, in the directions parallel and perpendicular to bed joints are, respectively:

$$M_{Rd1} = f_{sk1} \cdot Z_1$$  \hspace{1cm} \text{6.3}

$$M_{Rd2} = f_{sk2} \cdot Z_2$$  \hspace{1cm} \text{6.4}

With $Z_1$ and $Z_2$ being determined as shown in Eq. 6.5 and 6.6, respectively.

$$Z_1 = lt^2/6$$  \hspace{1cm} \text{6.5}

$$Z_2 = ht^2/6$$  \hspace{1cm} \text{6.6}

Table 6-9 summarises the main steps to conduct the comparison between the EC6 (CEN, (2010)) procedure, the Approach_S which neglects the torsion contribution and the Approach_D, which includes the torsional effects.
Table 6-9 Comparison between EC6, Approach_S and Approach_D

<table>
<thead>
<tr>
<th>Approach_S (no torsion)</th>
<th>EC6</th>
<th>Approach_D (with torsion)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_2 = 0.0389$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{Ed1} = 0.2742 \text{ kNm m}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{Ed2} = 1.2147 \text{ kNm m}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\min \lambda = 1.2923$</td>
<td></td>
<td>$\min \lambda = 2.4188$</td>
</tr>
<tr>
<td>$M_{Ed1} = 1.5408 \text{ kNm m}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{Ed2} = 3.0816 \text{ kNm m}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_1 = 5.6195$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_2 = 2.563$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \lambda = \min (\lambda_1, \lambda_2) \]

Find a new $\alpha_{2*}$, that corresponds to $\lambda = 2.4188$

Find $\lambda = 2.4188$

\[ \lambda = \frac{M_{Ed2}}{M_{Ed2}} = 2.4188 \rightarrow M_{Ed2} \]

Find $\alpha_{2*} \rightarrow 0.07654$

Find $\alpha_{2*} \rightarrow 0.0409$

Compare $\frac{\alpha_2}{\alpha_{2S}} \rightarrow 1.963$ (EC6 vs Approach_S comparison)

Compare $\frac{\alpha_2}{\alpha_{2D}} \rightarrow 1.05$ (EC6 vs Approach_D comparison)

As shown in Table 6-9, there is a substantial discrepancy between $\lambda_{EC6}$ and $\lambda_{A_S}$ which results in $\frac{\alpha_2}{\alpha_{2S}} \cong 2$, proves that Approach_S, considering only the contribution of the flexural strengths, is too conservative. On the contrary, when including the torsional component of strength within the overall computation of the horizontal moment capacity, the discrepancy between $\lambda_{EC6}$ and $\lambda_D$ reduces to $\frac{\alpha_2}{\alpha_{2D}} \cong 1$, proving that 1) the assumption on the loading shape (i.e. from the uniformly distributed load to the line load) has a very marginal impact in terms of the obtained results whilst 2) the inclusion of torsional effects plays a very key role in determining the wall capacity against out-of-plane loadings. Although comparatively small, the 5% difference which can be observed in the comparison of the results obtained from the EC6 and Approach_D procedures is not negligible, and it is to be attributed to the bending moment coefficient $\alpha_2$ which depends on 1) the orthogonal ratio; 2) the degree of fixity at the edge of the panel; 3) the height to length ratio of the panel.

Even though the wall panel used in the present comparison is simply supported along all four edged, a small degree of fixity along the vertical sides of the wall still develops as a result of the cohesive strength of mortar due to the arrangement of masonry units that restraints their movements and the friction between the components, which prevents relative rotation. Such contributions are not explicitly accounted for in the $\alpha_2$ parameter, while in Approach_D these are expressed through the friction coefficient and the $f_{sk1}$. Therefore, it can be assumed that, within the process of deriving a new $\alpha_{2*}$ to
match the two \( \lambda \) values of both approaches, the overall mismatch in the results obtained derives from the way in which cohesive strength and friction contributions are accounted for.

**Table 6-10: Comparison between EC6 and Approach_D**

<table>
<thead>
<tr>
<th>Wall configuration</th>
<th>Failure Load EC6 *strongest direction ** weakest direction</th>
<th>Resulting wind speed</th>
<th>Failure Load proposed procedure</th>
<th>Resulting wind speed</th>
<th>% of error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>* 5.05 kN/m(^2) ** 2.28 kN/m(^2)</td>
<td>19.12 m/s</td>
<td>4.225 kN/m(^2)</td>
<td>26 m/s</td>
<td>26.43%</td>
</tr>
<tr>
<td></td>
<td>* 10.01 kN/m(^2) ** 4.51 kN/m(^2)</td>
<td>26.87 m/s</td>
<td>8.556 kN/m(^2)</td>
<td>37 m/s</td>
<td>27.38%</td>
</tr>
<tr>
<td></td>
<td>* 5.185 kN/m(^2) ** 2.342 kN/m(^2)</td>
<td>19.35 m/s</td>
<td>3.306 kN/m(^2)</td>
<td>23 m/s</td>
<td>15.83%</td>
</tr>
<tr>
<td></td>
<td>* 6.40 kN/m(^2) ** 2.92 kN/m(^2)</td>
<td>21.60 m/s</td>
<td>5.076 kN/m(^2)</td>
<td>28.5 m/s</td>
<td>24.21%</td>
</tr>
<tr>
<td></td>
<td>* 8.26 kN/m(^2) ** 3.77 kN/m(^2)</td>
<td>24.56 m/s</td>
<td>5.329 kN/m(^2)</td>
<td>29.2 m/s</td>
<td>15.88%</td>
</tr>
<tr>
<td></td>
<td>* 8.80 kN/m(^2) ** 3.97 kN/m(^2)</td>
<td>25.21 m/s</td>
<td>5.625 kN/m(^2)</td>
<td>30 m/s</td>
<td>15.95%</td>
</tr>
<tr>
<td></td>
<td>* 17.56 kN/m(^2) ** 7.96 kN/m(^2)</td>
<td>35.70 m/s</td>
<td>11.56 kN/m(^2)</td>
<td>43 m/s</td>
<td>16.97%</td>
</tr>
</tbody>
</table>

Further comparisons are made between the proposed procedure and the EC6 procedure, with the aim of determining the level of discrepancy in the load leading the wall to collapse. All configurations included in Roberts, J. J., & Brooker, O (2007) which are also common to the configurations included Table 6-2, are checked and reported in Table 6-10. The EC6 procedure assumes a crack pattern which forms following a pre-established angle, while - as mentioned in previous sections - the Approach_D procedure
allows to find the optimized crack pattern based on the units’ dimensions, thus justifying the discrepancies observed in terms of obtained failure load. In general, the minimum value of load causing failure computed by using the EC6 procedure is between 16% and 27.5% smaller than the one computed by using Approach_D. More details on the calculations reported in Table 6-10 are included in Appendix 2.

6.4. Comparison of the proposed method with experimental tests on masonry brick walls

Beside validating the proposed analytical procedure against the prescriptions included in EC6, a further validation against experimental results is proposed in this section, with the aim of comparing the values of λ factor and hence enhance the confidence level when deriving the parameter χ used to generate the fragility functions. The experimental campaign chosen for this purpose is the one conducted by Vaculik, J. (2012). The main reasons in support of this choice are:

- The wall layout in Vaculik’s study effectively simulates a two-way spanning specimen supported at its horizontal and vertical edges and thus responding to biaxial bending when subjected to face loads, as well as the full-rotational fixity imposed along the vertical edges by means of return walls.
- The non-dimensional load multiplier λ is defined as the ratio of the wall’s lateral force resistance to the self-weight thus providing a straightforward mean of comparison with the minimum acceleration that cause collapse, obtained from the optimization routines presented as Approach_D developed in this study.

The testing campaign conducted by Vaculik, J. (2012) is twofold, with a first stage focusing on testing the ultimate strength capacity of the wall by applying monotonic loading using airbags until the wall reaches its ultimate load capacity and a cyclic testing to evaluate the strength degradation of the wall. Specifications of the wall tested (labelled as wall ‘S2’ in Vaculik, J. (2012)) in terms of geometric and material characteristics are provided in Table 6-11.

The boundary conditions of the wall used for the comparison are presented in Fig 5.4.

Table 6-11 Geometrical and Material Characteristics of the ‘S2’ wall Vaculik, J. (2012)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall Length</td>
<td>3.96</td>
<td>m</td>
</tr>
<tr>
<td>Wall Height</td>
<td>2.494</td>
<td>m</td>
</tr>
<tr>
<td>Wall thickness</td>
<td>0.110</td>
<td>m</td>
</tr>
<tr>
<td>Brick Unit Length</td>
<td>0.230</td>
<td>m</td>
</tr>
<tr>
<td>Brick Unit Height</td>
<td>0.076</td>
<td>m</td>
</tr>
<tr>
<td>Brick Unit Width</td>
<td>0.125</td>
<td>m</td>
</tr>
<tr>
<td>Mortar bed joint Thickness</td>
<td>0.01</td>
<td>m</td>
</tr>
</tbody>
</table>
Density of brick 19 kN/m³
Flexural tensile strength (fₓk1) 0.520 MPa
Orthogonal strength ratio (µ) 0.25 --
Friction coefficient (ϕ) 0.576 --
Compressive strength of masonry 13.6 MPa
Gravity loading, axial stress effect 0.10 MPa

The crack pattern obtained by Vaculik at the conclusion of the ultimate strength test, shown from the internal face of the wall is reported in Figure 6-4 a), also including the plan view of the tested wall. Such crack pattern corresponds to an altered version of CP2, as shown in Figure 6-4 b). In consideration of the laboratory setup and of the way in which the out-of-plane load is applied to the wall, it is assumed that the corresponding seismic KEL is placed at ½ H instead of 2/3H, as per the assumption of triangular distribution of acceleration, for seismic loading, the crack pattern obtained is the CP2_0.5H, whereby the points of convergence of diagonal cracks (i.e., point3 and 4) are located at ½ H.

![Figure 6-4: Crack pattern compared: a) Vaculik, J. (2012) experimental test; b) CP2_0.5 Approach_D](image)

Given the results obtained in section 6.3, only Approach_D is used for this comparison (see Section 5.5).

![Figure 6-5: a) Wall S2 from Vaculik, J. (2012) b) λ factor determined via Approach_D applied to CP2 0.5](image)

The non-dimensional load multiplier in Vaculik, J. (2012) (i.e., λVaculik which is plotted on the right side...
of the chart in Figure 6-5 a) is defined as the ratio of the wall’s lateral force capacity to self-weight, and it represent the multiplier of the acceleration of gravity which will cause collapse. The peak value of $\lambda_{\text{Vacuik}} = 2.25 \, \text{g}$ (Figure 6-5 a)), is used as input a (g) in Approach_D, while considering also a vertical stress 0.1 MPa caused by the vertically applied load. The collapse multiplier $\lambda_{A,D}$ obtained is 1.07, indicating a difference of 7% with the test, which is within the admissible threshold used for calibration of numerical model with respect to experimental or analytical solutions.

6.5. Comparison of proposed method and ELS model

Finally, a comparison is carried out with the outputs of a micro-modelling analysis using a Discrete Element (DE) Method software Extreme Loading for Structures (ELS) (ASI (2018)) for all three loading conditions considered in the Approach_D. The index wall 2 (IW2) is chosen to discuss the reliability of the underlying assumptions of the proposed method when compared to more computationally expensive means of analysis. The geometric and material properties used to characterise the wall are the and ones introduced in section 6.3, Table 6-8. To better evaluate the results presented in this section, it is important to highlight the main differences between the two modelling approaches. In ELS, the mortar layer and the two unit-mortar interfaces are lumped into a zero-thickness joint: this translates in modelling the thickness joint as a very thin layer (i.e., 0.001 m). One main difference between the two approaches is the treatment of the mortar joint connecting the masonry units. In ELS, two sets of springs (Fig. 6.7) are used, namely the “joint” springs representing the mortar plus unit-mortar interfaces and the “unit” springs, representing the continuity within the bricks simulated as half units; these springs are characterized by two different force-displacement laws able to capture the elastic and the post-elastic behaviour of the masonry bond and of the unit material, respectively (ASI (2018), Mayorca, P., & Meguro, K. (2003)). More specifically, the deformation extent of these springs is governed by a parameter defined as separation strain, which define the limit of post-elastic behaviour in tension and shear.

![Figure 6-6: Schematic of the simplified macro-modelling of masonry using AEM retrieved from Adhikari, R., & D’Ayala, D. (2019)](image-url)

Given that the proposed procedure is only able to capture the condition of the wall at failure, the separation strain in ELS is set to be small, thus indicating that, when the ratio between capacity and
demand is equal to 1, the springs stop elongating and can be considered as they have effectively failed. In terms of edge restraint, given that the direction of the loading is \( y \), the simulation of the 4-sides simply support condition implies that the translation is restricted in \( x \) and \( z \) directions and no translation is allowed in the \( y \) direction in any of the four sides. Rotation is instead always allowed, except at the base of the wall, where it is imposed that the rotation in \( z \) direction is restricted for stability purposes. Correspondently, the proposed procedure allows to adjust the moment capacities along each side of the wall. To simulate the degree of restraint at the base of the wall, the vertical moment capacity is considered twice as it would normally be for a single support, while the two contributions acting horizontally (i.e., \( M_{h\text{ torsion \_wall}} \) and \( M_{h\text{ flexure \_wall}} \)) provide one contribution each. More specifically, reference to Eq. 5.38 is made and the ad-hoc new Eq. 6.7 is also presented below to clarify this specific case.

\[
W_i(y) = 2 \left[ m_{\text{unit}}(y) \cdot \left( \frac{L}{s_u} \right) \cdot \frac{1}{y} \right] + \left[ m_{\text{unit}}(y) \cdot \left( \frac{L}{s_u} \right) \cdot \frac{1}{(H-y)} \right] + 2 \left[ (mh\text{ torsion}_{\text{unit}}(y) + mh\text{ flexure}_{\text{unit}}(y)) \cdot \left( N \cdot \frac{1}{x} \right) \right] 6.7
\]

Following subsections deal with each hazard individually; conclusive remarks are provided at the end of the three subsections to highlight similarities and quantify the differences between the ELS simulations and the proposed analytical approach.

6.5.1. Earthquake

As mentioned in Section 5.2, in the present study the seismic action is represented by the triangular load pattern with resultant KEL applied at 2/3 of the height of the wall. To best simulate the loading profile in ELS, the KEL was converted to uniform pressure and spread across a region formed of 8 courses – i.e. 4 courses above and 4 courses below KEL line of application, resulting in a loading area of 0.6 m height spread across 4.5 m length, as shown in Figure 6-7. Spreading the load across a wider area helped preventing very localized failure, in favour of allowing the wall moment capacities to engage before reaching collapse. In terms of restraints, the translation along the \( Y \) direction is impeded in all four sides with an additional impeded translation along the \( Z \) direction added at the base of the wall to simulate more realistically the presence of a foundation supporting the wall. Fig 6.8 a), b) and c) shows the crack patterns obtained from ELS and Approach_D (Fig 6.8 d)), respectively. Spreading the load across a wider area helped preventing very localized failure, in favour of allowing the wall moment capacities to engage before reaching collapse. In terms of restraints, the translation along the \( Y \) direction is impeded in all four sides with an additional impeded translation along \( Z \) direction added at the base of the wall.
to simulate more realistically the presence of a foundation supporting the wall. Fig 6.8 a), b) and c) shows the crack patterns obtained from ELS and Approach_D (Fig 6.8 d), respectively.

Figure 6-7: ELS locally distributed uniform pressure – credits to Ms A. Parammal Vatteri

Figure 6-8: Comparison of crack patterns earthquake loading a) b) c) ELS and d) proposed procedure
Table 6-12 Comparison of results obtained via ELS and the proposed procedure in terms of height (y) and lengths (x₁ and x₂) of convergence of crack patterns – earthquake loading

<table>
<thead>
<tr>
<th>Method used</th>
<th>y value</th>
<th>x₁ and x₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELS</td>
<td>0.696</td>
<td>0.334</td>
</tr>
<tr>
<td>Proposed Procedure</td>
<td>0.651</td>
<td>0.342</td>
</tr>
<tr>
<td>% error</td>
<td>3.02 %</td>
<td>2.20 %</td>
</tr>
</tbody>
</table>

Table 6-12 reports the values obtained respectively via ELS and via the proposed procedure corresponding to a value of ground acceleration causing failure of 1g. The comparison of the failure patterns shown in Figure 6.8 a) and b) and the results presented in Table 6-12 show a very close correspondence between the two models, both in terms of height of the position of maximum displacement and distance of the points x₁ and x₂ representing the converging points of the diagonal cracks. From Figure 6.8 a) appear clear that the lower portion of the wall in close proximity to the ground does not rotate nor displace, and the cracks do not spread in that region, starting instead from around the fourth course of bricks. Such a condition is also replicated in the proposed procedure, hence the small percentage of error in the patterns found.

6.5.2. Flood

To simulate as most precisely as possible the behaviour of the wall under flood loading, two comparisons have been made, which were useful to demonstrate that the shape of the crack pattern depends on two main characteristics, namely the thickness of the wall and the flexural strength fₓk₁. To this aim, two wall profiles are simulated, the Index Wall (IW) – which is of an IW2 aspect ratio – and a modified version indicated as IW*, the characteristics of which are reported in Table 6-13, including the main references where to find the specific values used to characterize the walls.

Table 6-13: Geometric and material properties of IW and IW*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>IW (same as Table 6-8)</th>
<th>IW*</th>
<th>Unit</th>
<th>References for IW*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall Length</td>
<td>4.465</td>
<td>4.465</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>Wall Height</td>
<td>3.00</td>
<td>3.00</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>Wall thickness</td>
<td>0.215</td>
<td>0.130</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>Brick Unit Length</td>
<td>0.215</td>
<td>0.230</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>Brick Unit Height</td>
<td>0.065</td>
<td>0.075</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>Brick Unit Width</td>
<td>0.102</td>
<td>0.102</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>Mortar Thickness bed joint</td>
<td>0.010</td>
<td>0.010</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>Mortar Thickness head joint</td>
<td>0.010</td>
<td>0.010</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>Density of brick</td>
<td>17.00</td>
<td>19.00</td>
<td>kN/m³</td>
<td></td>
</tr>
<tr>
<td>Description</td>
<td>Value 1</td>
<td>Value 2</td>
<td>Unit</td>
<td>Author</td>
</tr>
<tr>
<td>-----------------------------------------------------------------</td>
<td>-----------</td>
<td>-----------</td>
<td>---------</td>
<td>----------------</td>
</tr>
<tr>
<td>Flexural tensile strength parallel to bed joint ($f_{xk1}$)</td>
<td>0.200</td>
<td>0.520</td>
<td>MPa</td>
<td>Vaculik, J.</td>
</tr>
<tr>
<td>Friction coefficient ($\phi$)</td>
<td>0.500</td>
<td>0.576</td>
<td>--</td>
<td>(2012)</td>
</tr>
<tr>
<td>Shear strength of masonry ($f_{sk0}$)</td>
<td>0.320</td>
<td>0.832</td>
<td>MPa</td>
<td></td>
</tr>
<tr>
<td>Value of torsion coefficient ($k_{be}$)</td>
<td>0.06</td>
<td>0.17</td>
<td>--</td>
<td></td>
</tr>
</tbody>
</table>

The crack pattern is initially hypothesized to run across the full wall height. ELS simulates the load as uniform hydrostatic pressure along the wall surface area and the analysis are run for increasing steps of water, which ultimately get the wall to failure. The restraints characterizing the edges are the same as the earthquake case, hence the translation along the Y direction is impeded in all four sides with an additional impeded translation along the Z direction added at the base of the wall to simulate more realistically the presence of a foundation supporting the wall. The comparison between ELS and the proposed procedure is done when the deformed shape and consequent crack pattern is found. Figure 6-9 reports the deformed shapes and crack patterns of both IW and IW* found by ELS and the proposed procedure.
**Table 6-14**: Comparison of results obtained via ELS and the proposed procedure in terms of height (y) and lengths (x₁ and x₂) of convergence of crack patterns – flood loading

<table>
<thead>
<tr>
<th>Method used</th>
<th>Height of water causing collapse</th>
<th>y value</th>
<th>x₁ and x₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELS IW</td>
<td>0.576</td>
<td>0.375</td>
<td>0.286</td>
</tr>
<tr>
<td>Proposed Procedure IW</td>
<td>0.6</td>
<td>0.35</td>
<td>0.274</td>
</tr>
<tr>
<td>% error</td>
<td>4.00 %</td>
<td>6.66 %</td>
<td>4.26 %</td>
</tr>
<tr>
<td>ELS IW*</td>
<td>0.5</td>
<td>0.34</td>
<td>0.279</td>
</tr>
<tr>
<td>Proposed Procedure IW*</td>
<td>0.54</td>
<td>0.32</td>
<td>0.26</td>
</tr>
<tr>
<td>% error</td>
<td>7.4 %</td>
<td>5.88 %</td>
<td>6.81 %</td>
</tr>
</tbody>
</table>

Even though the global wall aspect ratio is the same for both IW and IW*, the behaviour under increasing steps of water is very different. IW is a much thicker wall laid following the English bond arrangement, therefore modelled having alternate courses of headers and stretchers thus providing with more resistance against out-of-plane loading, but also characterized by a modest wall moment capacity in both directions. IW* is comparatively a much slender wall, modelled as a one-layer of headers but with a much greater flexural moment capacity perpendicular to bed joints (i.e., \( fxk1_{IW*} = 2.6 \, fxk1_{IW} \)). To further remark the difference between the two walls, the orthogonal strength ratio of IW* is purposely defined to be greater than the one of IW (i.e., \( \mu_{IW*} = 2 \, \mu_{IW} \)), to ensure that the difference in flexural strength \( fxk2 \) can be properly accounted for. The water height causing the IW and IW* configurations to reach collapse is found to be within the same range of values, and equal to 0.576 H in the case of IW and 0.5H in the case of IW*. When considering the crack pattern shapes, the former wall tends to be characterized by a wider and more widespread deformation which involves the full height, extending to the upper corners of the wall: such a behaviour is to be ascribed to a smaller difference between \( fxk1 \)
and f_{xk2} resulting in a wider redistribution of forces. Due to its slenderness, the latter wall is instead more prone to deformation: nonetheless, since the IW* wall is also much stronger due to a higher value of flexural strength, it tends to deform more in the elastic range and to break in a brittle manner when reaching the critical water height, thus justifying the localized development of the failure pattern and the fact that the upper wall portion remains completely uncracked, thus resulting in a steeper slope of the return cracks which follow a wider angle. In both cases there is a substantial good agreement between the ELS model, and the walls simulated with the proposed procedure, both in terms of crack pattern shape, length of horizontal crack, height of point of maximum displacement y and distance of the diagonal cracks (x_1 and x_2) from the vertical supports, with an overall percentage of error below 7.5%.

In both cases, the critical water height producing failure is slightly higher than the ELS model, namely 0.6 H in the case of IW and 0.54 H in the case of IW*. To account for the partial rotational restraint due to the effect of the bottom edge, the wall moment capacity M_v – linked to the f_{xk1} - is factorized of an additional 100% of its original value. Such increment is exclusively applied in the lower portion (from the base to the point of application of the KEL), in consideration that the first 4 courses of wall do not mobilize (indicated with red dashed line in Figure 6-9, with no further increment when computing the moment capacity of the upper wall portion. In the case of IW*, the proposed procedure is tailored to account of a reduced wall area within which the cracks develop. Consequently, the wall portions’ rotations and the KEL-to-crack pattern interaction are computed in consideration of a reduced H of the wall, which coincide with βH. An additional vertical moment capacity contribution in considered in this latter case, to simulate the formation of the horizontal crack located at βH as indicated in Figure 6-9 by the dotted red line. Finally, it can be said that the results provided in Table 7-14 confirm that the case of local failure reproduced with the IW* shows that the length of the cracks diagonal cracks is shorter, therefore the internal moment capacity Wi is smaller and the λ is smaller; in the case of IW instead, the greater extension of the cracks means global mechanism with greater internal capacity, greater Wi hence greater λ, therefore, for the same λ, higher water height supported by the same wall.
6.5.3. Wind

The wind loading was applied in ELS as a uniformly distributed pressure over the whole wall.

![Wind loading diagram](image)

**Figure 6-10:** Comparison of crack patterns wind loading: a) b) c) ELS and d) proposed procedure

The wind speed value found to get collapse is 24.5 m/s which is within the range of wind speeds included in grade 1, deemed to cause visible damage, in the Saffir-Simpson Hurricane Wind Scale (1974).

Table 6-15 reports the values of y, x₁ and x₂ obtained respectively via ELS and via the proposed procedure that determine the pattern.

**Table 6-15** Comparison of results obtained via ELS and the proposed procedure in terms of height (y) and lengths (x₁ and x₂) of convergence of crack patterns – wind loading

<table>
<thead>
<tr>
<th>Method used</th>
<th>y value</th>
<th>x₁ and x₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELS</td>
<td>0.625</td>
<td>0.336</td>
</tr>
<tr>
<td>Proposed Procedure</td>
<td>0.625</td>
<td>0.340</td>
</tr>
<tr>
<td>% error</td>
<td>0.00 %</td>
<td>1.3 %</td>
</tr>
</tbody>
</table>

Both ELS and proposed procedure match in terms of both y value and distance of points of convergence from the sides of the wall.
6.6. Conclusions

As one of the main purposes of this research is to determine the applicability of the proposed kinematic model detailed in Chapter 5 and its efficacy in describing the behaviour of URM walls under different sources of loading in a homogenized manner, the twofold aim of this chapter is 1) to detail the rationale followed to generate the buildings samples which serve to move to the final step of this research, namely the multi-hazard fragility assessment and 2) to validate the proposed method with other well-established procedures and other methods of analysis to ascertain the reliability of the results which are included in the following chapter. To the first aim, since this work focuses on the variability of the exposure and of the model rather than concentrating on the hazard component and hence the intensity measure (IM), the set of geometric and material characteristics is derived from available pertinent sources from the literature, both from the EC6 (CEN (2010)) and from experimental tests. As discussed in previous chapters, two building samples – defined as Index Walls (IW) - are required, as the crack patterns alternatives that can generate vary in accordance with the wall aspect ratio, as reported in Chapter 5. The variability in the response needed to generate fragility functions of the two IWs is obtained by implementing the one-at-the-time OAT method. Further details of the regression method adopted for fragility assessment are given in Chapter 7.

To the second aim, three types of validations are conducted, namely 1) to investigate the percentage of error when transitioning from a procedure which models the load as a uniformly distributed action to a procedure based assuming that the same load can be simulated as an equivalent line load with specific point of application dependent on the loading profile, 2) to verify the percentage of error in the determination of the collapse load factor when comparing results obtained from experimental tests and the more theoretical results obtained via the proposed procedure, 3) to understand the precision of the proposed analytical method in determining the shape and the position of the main key elements needed to determine the crack pattern in comparison to more detailed – and more computationally demanding – Discrete Elements (DE) methods such as ELS.

The first validation has shown that the computation of failure load is generally more accurate when adopting the Approach_D proposed, proving that the inclusion of the torsional component counts for an increase that varies between 16% to 27.5%, which is to be considered substantial.

The comparison with Vaculik’s experimental tests allows to remark the confidence in results obtained in terms of $\lambda$ factor, as the percentage of error in this case is less than 1%.

Finally, the comparison with ELS allowed to fine-tune the proposed analytical procedure in terms of determination of shape and length of cracks of the obtained crack pattern. The comparison required several iterations to understand how to treat the fixities and the simple supports and where to start considering the crack pattern formation. As these aspects were clarified – the discrepancy in terms of
shape and length of cracks fell respectively within 3% and 2% in the case of earthquake, 4% and 6.7% in the case of flood and 0% and 1.3% in the case of wind.

The following chapter is dedicated to present the results of this research study. To the aim of deriving fragility curves and to conduct multi-hazard fragility assessment, the first section is dedicated to discussing the choice of fitting techniques that better suits the purposes of this research study, then, single-hazard fragility curves are derived to highlight the influence of the restraining condition in determining the samples’ fragility to the three perils investigated in this study.
7. Multi-hazard fragility assessment results

7.1. Introduction

This chapter is dedicated to the derivation of single-hazard fragility curves representative of the groups of walls with different restraints conditions detailed in Table 6.1, and the comparison of the single hazard fragility functions to determine the most damaging loading profile caused by seismic, flood and wind hazards. Section 7.2 is dedicated to discussing the choice of regression and fitting techniques that best suit the purposes of this research study, in consideration of the existing conceptual differences that characterise the three hazards considered and with reference to the review of regression methods reported in Section 2.4.4, to determine fragility functions.

On the assumption that the geometry, the material properties of the sample of walls investigated are available, and the boundary conditions are also clearly discernible, single-hazard fragility functions are derived for each of the three hazards considered. In section Sections 7.3, 7.4 and 7.5 fragility functions are presented for earthquake, flood and wind, respectively, for the 4 groups of wall constraints discussed in Table 6.2, considered separately. A premise is provided on the assumptions made on the opening’s geometry in group D. These first set of fragility functions are generated for the two index-walls IW1 and IW2, and their corresponding crack patterns, separately. To achieve this, each wall realization in the OAT simulation is defined in such a way as to guarantee that the overall aspect ratio of IW1 and IW2 is maintained. For each of the above groups a second set of functions, defined as a restraint condition-dependent (RC-D) is also included, irrespective of the geometry ratio. These sets are governed by the assumption that the geometry of the wall cannot be determined, and that the boundary conditions are the main discriminant of the response. Moreover, with reference to the flood hazard, as introduced in Section 6.5.2, both the options of crack patterns dependent (DEP) and independent (IND) of the water height increments are considered, to better highlight the difference in structural behaviour of the same walls when considering local and global failure mechanisms.

Similarly, when boundary conditions are not clearly recognisable due to data gathering limitations (i.e., when plasters or finishes hide the key elements that allow the surveyor to understand with certainty the type of connection between building units within a cluster of buildings), the geometric aspect ratio is the discriminant of the assessment and Index Wall dependent (IW-D) can be derived. Section 7.3 is entirely dedicated presents these curves initially, categorized by group and then as a cumulative, highlighting the differences in fragility of the two index walls considered in this study. The section concludes with fragility curves which are both restraint condition-independent and geometry-independent (Geometry & Restraints INDependent (G&R-IND) curves) to show the baseline of dispersion if none of the parameters’ range is predefined, and therefore simulating the higher level of
uncertainty. Finally, Section 7.7 draws the conclusive remarks which lead to the Chapter 8, dedicated to the case study.

7.2. Choice of regression models for fragility functions derivation

As anticipated in Chapter 2 Section 2.4.4, to quantify the vulnerability of the buildings sample presented in Section 6.2 to a given deterministic hazard scenario, fragility functions must be derived. Fragility curves are lognormal probability distribution functions (PDF) reportedly found to appropriately represent earthquake damage fragility (Singhal, A., & Kiremidjian, A. S. (1996)). Lognormal PDF guarantee to exclude negative values, and there is a significant number of precedents examples in seismic risk modelling (Kennedy et al., (1980), Lallemant et al., (2015), Bradley, B. A., & Dhakal, R. P. (2008)). The review of the type of functions used to derive fragility curves provided in Chapter 2 Section 2.4.4 also includes a review of fitting techniques considered suitable for the type of dataset derived in this study. Within the same section, reference is also made to the main source of uncertainties defining the overall dispersion parameter $\beta$ are reported in Eq. 2.3. As the epistemic uncertainty associated with the hazard does not represent the focus of this study, the uncertainties $\beta_{demand}$ is not taken into consideration. The uncertainty in the capacity definition of the couples of capacity-demand points that determine the damage thresholds is also not considered, as only the collapse damage state is considered, and this is defined using a strength criterion. The variability in strength is accounted for in the capacity of the walls. Therefore, the only source of dispersion in the sample, considered in this study, depends on the $\beta_{capacity}$ associated with the variability of the geometric, constraints and material characteristics of the wall.

Considering the method used to derive the variance in the sample of walls presented in Chapter 6, Section 6.2.2, namely the OAT method, the Sum of Square Error (SSE) is chosen as the most appropriate regression analysis method, for the following reasons:

- It fits binary failure data (collapse/ no collapse).
- It is not affected by the total number of IM considered (as it occurs when using the Maximum Likelihood Estimation (MLE) method).
- It is not affected by the significant geometry variation imposed by the wall aspect ratio, as is the case for the buildings sample considered in this study.

In consideration of the fact that the fragility curves are built by performing regression of IM measures which have all caused the walls considered to fail, the mean value $\theta$ and the associated standard deviation value $\beta$ are calculated as reported in Eq. 7.1 and 7.2:
\[ \theta = \exp \left( \frac{1}{n} \sum_{i=1}^{n} \ln I_M \right) \]  \hspace{1cm} 7.1

\[ \beta = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left( \ln \left( \frac{I_M}{\theta} \right) \right)^2} \]  \hspace{1cm} 7.2

Having defined the rationale to build the sample of walls in Section 6.2.2 and the fitting technique that suits the purposes of this research study, fragility curves are generated for the wall configurations categorized and detailed in Table 6.2, in consideration of the crack patterns alternatives discussed in Section 5.3, the geometric and material parameters listed in Table 6.6 and subjected to the ranges of IMs’ increasing steps detailed in Table 6.7. Mean \( \theta \) and standard deviation \( \beta \) values are also reported for each group in summary tables.

7.3. Single-hazard fragility curves: Earthquake

The present section includes the fragility curves derived for the 4 groups of walls configurations, subjected to earthquake loading. The C1 configuration, namely the 4-sides simply supported layout with no openings, is reported independently as the reference layout used throughout the thesis, used to explain the derivation of the kinematic model (reported in Chapter 5) and coincidentally used for all the validations of the method against the Code, the experimental tests and the ELS model discussed in Chapter 6. Figure 7-1 shows the IWs and CPs associated to wall layout C1. The taller and narrower aspect ratio characterizing IW1 appears to be less vulnerable to the seismic KEL located in the upper portion of the wall in comparison to a shorter and larger wall configuration such as IW2. Such result is governed by the flexural tensile strength stronger component \( f_{xk2} \). The longer the vertical sides are, the more influent the component \( f_{xk2} \) becomes, thus allowing for a greater seismic acceleration sustained.

![Figure 7-1: Earthquake Case: Wall configuration C1 – simply supported on 4 sides](image)
Table 7-1: Earthquake Case: Mean and SD of CP1- IW1, CP2- IW2 and RC-D Curve, Wall Configuration C1 reports the values of mean and standard deviation for the two geometric configurations and for the restraint condition-dependent curve. The dispersion in the case of IW1 is around 40% greater than IW2 demonstrating that there is wider variation of PGA values causing collapse across the IW1 sample (i.e., from 0.20 PGA (g) to 2.83 PGA (g), whilst the PGA (g) determining the failure of IW2 configurations ranges between 0.275 to 1.55.

Table 7-1: Earthquake Case: Mean and SD of CP1- IW1, CP2- IW2 and RC-D Curve, Wall Configuration C1

<table>
<thead>
<tr>
<th>CP-IW</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP1-IW1</td>
<td>0.773</td>
<td>0.609</td>
</tr>
<tr>
<td>CP1-IW2</td>
<td>0.556</td>
<td>0.365</td>
</tr>
<tr>
<td>RC-D</td>
<td>0.654</td>
<td>0.527</td>
</tr>
</tbody>
</table>

The following subsections present the fragility functions for each group configuration and compare these to the reference case C1.

7.3.1. Group A: Walls with supports on 3 sides and 1 top side free

Table 7-2 includes the three wall layouts in Group A and the median and standard deviation describing the fragility functions for the two index walls and for the restraint sensitive condition, for each layout. Within a real cluster of buildings, the Group A walls represent single storeys or top storeys buildings with no connection with roof elements, lightweight roof cover, and various lateral restraint conditions and foundation connections.

Table 7-2 Earthquake Case: Mean and SD of IW1, IW2 and RC-D Curves, Wall Group A

<table>
<thead>
<tr>
<th>Group A</th>
<th>Reference sketch</th>
<th>Configuration</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td><img src="image1" alt="A1 Reference sketch" /></td>
<td>IW1</td>
<td>0.518</td>
<td>0.748</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IW2</td>
<td>0.422</td>
<td>0.451</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>0.466</td>
<td>0.622</td>
</tr>
<tr>
<td>A2</td>
<td><img src="image2" alt="A2 Reference sketch" /></td>
<td>IW1</td>
<td>0.579</td>
<td>0.726</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IW2</td>
<td>0.509</td>
<td>0.428</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>0.542</td>
<td>0.594</td>
</tr>
<tr>
<td>A3</td>
<td><img src="image3" alt="A3 Reference sketch" /></td>
<td>IW1</td>
<td>0.858</td>
<td>0.647</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IW2</td>
<td>0.804</td>
<td>0.365</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>0.829</td>
<td>0.558</td>
</tr>
</tbody>
</table>
In the case of Group A, only one crack pattern type has been considered, namely the one with diagonal cracks starting from the base of the wall, reaching towards the centre and running towards the top free edge as a vertical crack.

Figure 7-2 and Figure 7-3 show the fragility curves for the three wall layouts for index walls IW1 and IW2 affected by the same crack pattern CP. It is noticeable that the IW2 shape ratio is substantially more vulnerable than the IW1 shape ratio and that the effect of side restraints (A3) is more pronounced for IW2. In both cases the effect of rotation fixity at the base (A2) is modest but more effective in IW2.

Figure 7-4 shows the comparison of the restraint sensitive fragility for each constraint layout compared with the fragility function for the reference case C1 without considering the shape ratio. The restraint conditions A1 and A2 are weaker than C1, indicating that even adding a fixity at the wall base (as in the case of A2) does not substitute the increase in terms of strength capacity achieved by the displacement restraint (DR) provided by the connection to roof element characterizing the wall configuration C1. However, a three-sides fixed supported wall shows a greater seismic capacity compared to the four-sides simply supported case, as demonstrated by Figure 7-4.
7.3.2. Group B: Walls with supports on 3 edges and 1 side free

Group B includes walls with three edges restrained and one side edge free, thus simulating the boundary conditions of single storey or top storey corner buildings, or walls with big and large intermediate openings breaking the continuity of the wall itself, with various degrees of connection with the roof structure, foundation, and adjacent buildings. Irrespective of the wall shape ratio IW1 or IW2, only one CP has been considered, namely the one with diagonal cracks starting from the supported side, meeting at the centre of the wall and running horizontally towards the free edge. For this layout five different configurations of constraints are considered. As shown in Table 7-3, the IW1 configuration has a considerably higher mean value of PGA (g) in comparison to IW2, irrespective of the supports’ constraint configuration, up to a maximum of 52.38% more in the case of the wall constraint configuration B2. Figure 7-7 compares the restraint conditions-dependent fragility curves for each configuration in this group, including the configuration C1. In general, this layout leads to higher fragility, irrespective of the constraint’s configuration, except for configuration B3, whose mean closely matches C1’s. Although B5 might seem an overall much stronger configuration due to its fixities on the base and lateral edges, the lower standard deviation, proves it to be slightly more vulnerable under higher PGA (g) values. Such a trend can be justified by the asymmetric behaviour of the B3 configuration and the positive contribution of the vertical support for C1.

Table 7-3 Earthquake Case: Mean and SD of IW1, IW2 and RC-D curves, Wall Group B

<table>
<thead>
<tr>
<th>Group B</th>
<th>Reference sketch</th>
<th>Configuration</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>![Reference sketch]</td>
<td>IW1</td>
<td>0.473</td>
<td>0.417</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IW2</td>
<td>0.273</td>
<td>0.394</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>0.360</td>
<td>0.490</td>
</tr>
<tr>
<td>B2</td>
<td>![Reference sketch]</td>
<td>IW1</td>
<td>0.819</td>
<td>0.429</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IW2</td>
<td>0.390</td>
<td>0.273</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>0.577</td>
<td>0.538</td>
</tr>
<tr>
<td>B3</td>
<td>![Reference sketch]</td>
<td>IW1</td>
<td>0.905</td>
<td>0.401</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IW2</td>
<td>0.581</td>
<td>0.336</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>0.724</td>
<td>0.431</td>
</tr>
<tr>
<td>B4</td>
<td>![Reference sketch]</td>
<td>IW1</td>
<td>0.512</td>
<td>0.409</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IW2</td>
<td>0.314</td>
<td>0.313</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>0.401</td>
<td>0.438</td>
</tr>
<tr>
<td>B5</td>
<td>![Reference sketch]</td>
<td>IW1</td>
<td>0.863</td>
<td>0.416</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IW2</td>
<td>0.445</td>
<td>0.318</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>0.602</td>
<td>0.485</td>
</tr>
</tbody>
</table>
7.3.3. Group C: Walls with supports on 4 sides

Group C includes walls supported on 4 sides, with 6 different combinations of simple and fix supports, which aim at representing intermediate storeys or single storey buildings, as shown in Table 7-4. In this case different collapse patterns are associated to the two shape ratios IW1 and IW2 as shown in table 7.3-4. Irrespective of the geometry of the wall, i.e., whether it is IW1 or IW2 type, the presence of vertical fix constraints is what makes the substantial difference in determining the mean value of the capacity of the two groups, configurations C1, C4 and C6, being substantially weaker than configurations C2, C3 and C5. Such a difference is to be attributed to the doubling of the vertical component of the moment capacity, governed by the higher flexural strength component $f_{xk2}$, combined with the increased torsional contribution. In Figure 7-8, relative to IW1, the difference between the two groups is more substantial than in Figure 7-9, relative to IW2, whereby only configuration C3 is substantially stronger than the others. The same trend visible in Figure 7-10, which includes the restraint condition dependent fragility functions irrespective of the shape ratio.
Table 7-4 Earthquake Case: Mean and SD of CP1-IW1, CP2-IW2 and RC-D curves, Wall Group C

<table>
<thead>
<tr>
<th>Group C</th>
<th>Reference sketch</th>
<th>Configuration</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>![Sketch]</td>
<td>CP1-IW1</td>
<td>0.773</td>
<td>0.609</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2-IW2</td>
<td>0.555</td>
<td>0.365</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>0.654</td>
<td>0.527</td>
</tr>
<tr>
<td>C2</td>
<td>![Sketch]</td>
<td>CP1-IW1</td>
<td>1.413</td>
<td>0.451</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2-IW2</td>
<td>1.020</td>
<td>0.365</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>1.183</td>
<td>0.438</td>
</tr>
<tr>
<td>C3</td>
<td>![Sketch]</td>
<td>CP1-IW1</td>
<td>1.450</td>
<td>0.447</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2-IW2</td>
<td>1.210</td>
<td>0.360</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>1.314</td>
<td>0.412</td>
</tr>
<tr>
<td>C4</td>
<td>![Sketch]</td>
<td>CP1-IW1</td>
<td>0.830</td>
<td>0.595</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2-IW2</td>
<td>0.633</td>
<td>0.364</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>0.722</td>
<td>0.508</td>
</tr>
<tr>
<td>C5</td>
<td>![Sketch]</td>
<td>CP1-IW1</td>
<td>1.360</td>
<td>0.463</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2-IW2</td>
<td>0.963</td>
<td>0.377</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>1.127</td>
<td>0.450</td>
</tr>
<tr>
<td>C6</td>
<td>![Sketch]</td>
<td>CP1-IW1</td>
<td>0.900</td>
<td>0.455</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2-IW2</td>
<td>0.805</td>
<td>0.372</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>0.850</td>
<td>0.463</td>
</tr>
</tbody>
</table>

Figure 7-8 Earthquake Case: Fragility curves CP1-IW1 for Group C

Figure 7-9 Earthquake Case: Fragility curves CP2-IW2 for Group C
7.3.4. Group D: Walls with openings

Group D includes the walls with openings, whose dimensions are detailed in Table 6-6. Configuration D1 and D2 are a variation of wall configuration C1 as they are characterized by the same side restraints, namely simply supports on all 4 sides. As per Group C, different collapse patterns are associated to the two shape ratios IW1 and IW2. The corresponding fragility functions are detailed in Figure 7-11, Figure 7-12 and Figure 7-13.

Table 7-5 Earthquake Case: Mean and SD of CP1-IW1, CP2-IW2 and RC-D curves, Wall Group D

<table>
<thead>
<tr>
<th>Group D</th>
<th>Reference sketch</th>
<th>Configuration</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td><img src="image1.png" alt="Reference sketch" /></td>
<td>CP1-IW1</td>
<td>0.419</td>
<td>0.693</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2-IW2</td>
<td>0.269</td>
<td>0.473</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>0.338</td>
<td>0.659</td>
</tr>
<tr>
<td>D2</td>
<td><img src="image2.png" alt="Reference sketch" /></td>
<td>CP1-IW1</td>
<td>0.345</td>
<td>0.550</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2-IW2</td>
<td>0.218</td>
<td>0.403</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>0.274</td>
<td>0.533</td>
</tr>
</tbody>
</table>

The following assumptions are made in developing the calculation of the walls’ capacity:

- In the computation of the external work done by the loading, the openings are not considered as voids, thus implying that they are sealed and do not allow water or wind to pass through, neither they cause loss of wall mass, which is of high importance in determining the seismic loading. Therefore, even if the KEL crosses the border of an opening, no reduction is applied to the load magnitude computation as a result of the line load-to-opening interaction.
- The presence of openings is accounted instead when computing the wall moment capacity, as it assumed that there is an “interruption” - and therefore a change in terms of material - between the
wall and the openings systems, thus making the two parts behaving differently under applied loadings. In the current research study, the attention is exclusively focused on the behaviour of the walls rather than on any wall-to-opening interaction. More specifically, since each moment capacity contribution in the horizontal and vertical directions is computed at brick level (following the rationale of Approach_D), the presence of an opening is considered in terms of bricks missing, therefore allowing to discount the equivalent contribution from the overall wall moment capacity computation, both in the vertical and horizontal directions. The same assumptions are adopted also for flood and wind fragility assessment.

As shown Figure 7-11 and Figure 7-12, the procedure is able to capture the presence of openings which result in a shift of the overall fragility curve towards the left, indicating that configurations with more openings are more fragile. Noticeably IW1 configuration is stronger than IW2, whilst also being characterised by a greater uncertainty in the sample considered. Finally, the comparison with wall configuration C1 reported in Figure 7-13 shows the extent of increased fragility caused by the presence of openings, as – ultimately – the wall configuration C1 and the wall configurations D1 and D2 are characterized by the same type of restraining condition – namely 4-sides simply supported.
7.4. Single-hazard fragility curves: Flood

The present subsection includes the fragility curves derived for the 4 walls’ groups subjected to flood loading. In consideration of the comparison conducted with ELS discussed in Section 6.5.2, both the crack patterns dependent (CP (DEP)) and independent (CP (IND)) of the water height level (βH) are considered. Figure 7-14 shows the fragility curves derived for the case in which the wall develops a crack pattern only within the submerged region of the wall, whilst Figure 7-15 shows the curves generated under the assumption that the whole wall engages in forming a crack pattern, independently of the water depth considered.

![Figure 7-14](image1.png)

**Figure 7-14** Flood – water height dependent case: Wall configuration C1 – simply supported on 4 sides

![Figure 7-15](image2.png)

**Figure 7-15** Flood – water height independent case: Wall configuration C1 – simply supported on 4 sides
Figure 7-16 Flood - Wall configuration C1 comparison between water height dependent and water height independent curves

Figure 7-16 presents the restraint dependent (RC-D) curves of both sets together. When considering the global mechanism there is a perceivable shift in fragility towards the right, indicating that the wall can sustain an increasing depth of water, and this is due to the involvement of the moment capacity components along the vertical sides across the whole wall height. The curves tend to overlap for values of $\beta H > 3$ m, indicating that as the threshold beyond which the difference between global and local crack pattern is no more perceivable. In accordance to Figure 7-16, the local mechanism should – in theory – be the most likely to happen in all the realizations considered in this current study. However, as reported by Bellazzi et al., (2020) and Cavaleri et al., (2020), the latest studies on the behaviour of masonry walls under tsunami loading is usually based on the assumption that the crack pattern forms within the whole specimen. Local failures must be however considered in the context of dynamic high velocity stream actions as reported in Drdácký, M. F. (2010), a work focused on proving that, irrespective of the volume of water transported, high-velocity streams can have an extremely dangerous impact on structures integrity, especially when the wall thickness is very modest compared to the other global geometric parameters such as length and height. Figure 7-17, Figure 7-18 and Figure 7-19 present the comparison between the fragility curves representative of the IW1 and IW2 configurations, when considering both the water height dependent (CP DEP) and water height independent (CP IND) cases, including a third water height dependent (CP DEP) curve representative of a sample characterized by a reduced wall thickness (i.e. = 0.130 m instead of 0.215 m as reported in Table 6-6). This latter case is linked to the comparison provided in Section 6.5.2, where the proposed methodology’s results were compared to the ELS simulations. From the comparison with the ELS model it appeared clear that walls characterized by smaller thicknesses are more likely to experience localised failure rather than global failure.
In this section the aim is to prove these results also in terms of fragility: to a reduction in thickness equal to 39.53% corresponds a sensible shift in fragility towards the left of the graphs which is visible across
all three plots, proving that thinner and more slender walls such as the IW1s are more subjected to localised failure and the lower the IM needed to get the wall to collapse. Table 7-6 reports the values of mean and SD for all cases presented in previous plots, and the reduction (in percentage) of IM at 50% probability of failure. The configuration with reduced wall thickness needs a smaller value of $\beta_H$ to get to collapse in the case of IW2, in line with the fact that IW2 is a weaker configuration compared to IW1.

7.4.1. Group A: Walls with supports on 3 sides and 1 top side free

Table 7-7 includes the values of mean and standard deviation for the three wall layouts of Group A. as mentioned in the case of earthquake, only one crack pattern here is considered. The overall trend followed by these curves within the same group the progression shown in the graphs, re-confirm what shown in Section 7.3.1 for earthquake, thus to an increasing level of connection corresponds a higher IM needed to get the wall to collapse. However, in the case of flood, the presence of a top restraint (such as in C1) marks the essential difference in comparison to A1 and A2, as the 4-sides restraints provide an overall higher resistance towards a line load which – at most – is always concentrated in the lower third of the wall height. When adding the fixities at the vertical edges (A3), there is a sensitive shift towards the right of the graph and the presence of a top restraint becomes negligible.

Figure 7-20 to Figure 7-25 show the fragility curves for the group considered and include the comparison with layout C1.

Table 7-7: Flood Case: Mean and SD of IW1, IW2 and RC-D Curves, Local and Global Mechanisms - Wall Group A

<table>
<thead>
<tr>
<th>Group A</th>
<th>Reference sketch</th>
<th>Configuration</th>
<th>$\beta_H$ DEP *</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td><img src="image1" alt="Image" /></td>
<td>IW1</td>
<td>*</td>
<td>2.157</td>
<td>0.239</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>**</td>
<td>2.220</td>
<td>0.236</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IW2</td>
<td>*</td>
<td>1.435</td>
<td>0.224</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>**</td>
<td>1.446</td>
<td>0.221</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RD-C</td>
<td>*</td>
<td>1.704</td>
<td>0.328</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>**</td>
<td>1.734</td>
<td>0.309</td>
</tr>
<tr>
<td>A2</td>
<td><img src="image2" alt="Image" /></td>
<td>IW1</td>
<td>*</td>
<td>2.223</td>
<td>0.218</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>**</td>
<td>2.230</td>
<td>0.220</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IW2</td>
<td>*</td>
<td>1.509</td>
<td>0.212</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>**</td>
<td>1.509</td>
<td>0.212</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RD-C</td>
<td>*</td>
<td>1.737</td>
<td>0.289</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>**</td>
<td>1.855</td>
<td>0.288</td>
</tr>
<tr>
<td>A3</td>
<td><img src="image3" alt="Image" /></td>
<td>IW1</td>
<td>*</td>
<td>2.879</td>
<td>0.193</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>**</td>
<td>2.880</td>
<td>0.195</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IW2</td>
<td>*</td>
<td>1.857</td>
<td>0.196</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>**</td>
<td>1.873</td>
<td>0.196</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RD-C</td>
<td>*</td>
<td>2.281</td>
<td>0.289</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>**</td>
<td>2.291</td>
<td>0.293</td>
</tr>
</tbody>
</table>
7.4.2. Group B: Walls with supports on 3 sides and 1 side free

This subsection deals with walls of Group B, including the values of mean and standard deviation of the five different boundary conditions characterizing the Group. Figure 7-26 and Figure 7-27 show the fragility curves for IW1 configurations; Figure 7-28 and Figure 7-29 present the IW2 cases whilst Figure 7-30 and Figure 7-31 include the restraint condition-dependent comparisons with C1 wall layout.
In both the IW dependent plots, the trend is always in line with the concept that – to a stronger wall characterised by an increasingly higher level of edge fixities, corresponds a decrease in fragility, and consequently, also an increase in water depth causing collapse. In the case of IW2 configurations, the fragility curves of walls B2 (i.e. the wall with 2 sides simply supported (base and top), 1 vertical side fixed and 1 lateral side free) and B5 (i.e. the wall with 2 sides fixed (foundation and vertical side) 1 side simply supported (top) and 1 lateral side free) tend to overlap towards the upper tail part, proving that – for increasing values of water depth, the contribution of the fixity at the base of the wall becomes negligible in comparison to the contribution provided by the fixity on the vertical side. Such trend is also visible in the case when the wall is prevalently taller than wider - as it is the case of IW1 - and it is assumed that the pattern develops only within the area subjected to the load, the wall configuration B5 shows to behave very similarly to the wall configuration C1, to the extent in which the resulting fragility curves, almost overlap.
This is an indicator of the fact that, when computing the depth of water causing collapse by accounting for progressive water steps in the assumption that only the wall portion subjected to the load engages in forming a mechanism, having 2 simply supported vertical sides is almost equivalent to having 1 vertical fixed edge effectively, as the two restraint configurations provide with the same contribution in the overall computation of Internal Work. Different is the case in which it is assumed that the pattern forms throughout the whole wall independently from the water depth: in that case, as shown in Figure 7-31, the influence of the 4 supports is greater than having one fixed vertical edged and one free vertical edge.
7.4.3. Group C: Walls with supports on 4 sides

This section summarises results for walls characterised by supports on all four sides. Table 7-9 includes mean and standard deviation values characterizing the fragility curves for this group. Figure 7-32 and Figure 7-33 show the sets of fragility curves for IW1. Figure 7-34 and Figure 7-35 show the sets of fragility curves for IW2 configurations. In both cases, to the stronger wall configurations correspond fragility curves shifted towards higher $\beta_H$ values. Such trend is also remarked by the RC-Dependent curves reported in Figure 7-36 and Figure 7-37 Flood Case: RC-D fragility curves for Group C - $\beta_H$ IND.

Table 7-9: Flood Case: Mean and SD of CP1-IW1, CP2-IW2 and RC-D Curves, Local and Global Mechanisms - Wall Group C

<table>
<thead>
<tr>
<th>Group C</th>
<th>Reference sketch</th>
<th>Configuration</th>
<th>$\beta_H$ DEP *</th>
<th>$\beta_H$ IND **</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td></td>
<td>CP1-IW1</td>
<td>* 2.224</td>
<td>** 2.355</td>
<td>0.207</td>
<td>0.208</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2-IW2</td>
<td>* 1.622</td>
<td>** 1.869</td>
<td>0.078</td>
<td>0.082</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RD-C</td>
<td>* 1.821</td>
<td>** 2.088</td>
<td>0.287</td>
<td>0.193</td>
</tr>
<tr>
<td>C2</td>
<td></td>
<td>CP1-IW1</td>
<td>* 2.131</td>
<td>** 2.298</td>
<td>0.089</td>
<td>0.085</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2-IW2</td>
<td>* 2.977</td>
<td>** 2.999</td>
<td>0.167</td>
<td>0.169</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RD-C</td>
<td>* 2.600</td>
<td>** 2.490</td>
<td>0.182</td>
<td>0.212</td>
</tr>
<tr>
<td>C3</td>
<td></td>
<td>CP1-IW1</td>
<td>* 2.276</td>
<td>** 2.291</td>
<td>0.173</td>
<td>0.175</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2-IW2</td>
<td>* 2.278</td>
<td>** 2.531</td>
<td>0.070</td>
<td>0.071</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RD-C</td>
<td>* 3.109</td>
<td>** 3.121</td>
<td>0.161</td>
<td>0.163</td>
</tr>
<tr>
<td>C4</td>
<td></td>
<td>CP1-IW1</td>
<td>* 1.751</td>
<td>** 1.944</td>
<td>0.080</td>
<td>0.079</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2-IW2</td>
<td>* 2.094</td>
<td>** 2.094</td>
<td>0.154</td>
<td>0.154</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RD-C</td>
<td>* 2.679</td>
<td>** 2.692</td>
<td>0.168</td>
<td>0.169</td>
</tr>
<tr>
<td>C5</td>
<td></td>
<td>CP1-IW1</td>
<td>* 2.094</td>
<td>** 2.199</td>
<td>0.103</td>
<td>0.094</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2-IW2</td>
<td>* 2.341</td>
<td>** 2.414</td>
<td>0.181</td>
<td>0.176</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RD-C</td>
<td>* 2.401</td>
<td>** 2.424</td>
<td>0.165</td>
<td>0.171</td>
</tr>
<tr>
<td>C6</td>
<td></td>
<td>CP1-IW1</td>
<td>* 1.920</td>
<td>** 2.128</td>
<td>0.804</td>
<td>0.805</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2-IW2</td>
<td>* 2.136</td>
<td>** 2.243</td>
<td>0.175</td>
<td>0.171</td>
</tr>
</tbody>
</table>
7.4.4. Group D: Walls with openings

This section is dedicated to present the set of fragility curves generated for the two wall configurations belonging to Group 4, and results in terms of Mean and SD are summarised in Table 7-10. As mentioned in Section 7.3.4, the assumptions on the openings’ geometry are kept the same also for flood assessment. The discrepancy between D1 and D2 is greater in the case of water independent case, and that can be justified by the fact that – notwithstanding the presence of openings, the engagement of the whole wall implies that the overall moment capacity is greater, therefore the $\beta H$ causing collapse is comparatively
higher across the sample than the water dependent case. Such trend is followed in both IW1 and IW2 configuration, reported respectively in Figure 7-38 Figure 7-39 and Figure 7-40 and Figure 7-41, and reiterated in Figure 7-42 Figure 7-43, which show the restraint-condition dependent fragility curves.

Table 7-10: Flood Case: Mean and SD of CP1-IW1, CP2-IW2 and RC-D Curves, Local and Global Mechanisms - Wall Group D

<table>
<thead>
<tr>
<th>Group D</th>
<th>Reference sketch</th>
<th>Configuration</th>
<th>$\beta_H$ DEP *</th>
<th>$\beta_H$ IND **</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td></td>
<td>CP1-IW1</td>
<td>1.454</td>
<td>0.305</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D1</td>
<td></td>
<td>CP2-IW2</td>
<td>0.854</td>
<td>0.319</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D1</td>
<td></td>
<td>RD-C</td>
<td>1.116</td>
<td>0.378</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D2</td>
<td></td>
<td>CP1-IW1</td>
<td>1.283</td>
<td>0.294</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D2</td>
<td></td>
<td>CP2-IW2</td>
<td>0.729</td>
<td>0.355</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D2</td>
<td></td>
<td>RD-C</td>
<td>0.966</td>
<td>0.431</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Figure 7-38 Flood Case: Fragility curves IW1-CP1 for Group D - $\beta_H$ DEP](image)

![Figure 7-39 Flood Case: Fragility curves IW1-CP1 for Group D - $\beta_H$ IND](image)

![Figure 7-40 Flood Case: Fragility curves IW2-CP2 for Group D - $\beta_H$ DEP](image)

![Figure 7-41 Flood Case: Fragility curves IW2-CP2 for Group C - $\beta_H$ IND](image)
7.5. Single-hazard fragility curves: Wind

The present section deals with the fragility curves derived for the 4 walls’ groups subjected to wind loading. Table 7-11 includes the mean and SD representative of the curves reported in Figure 7-44 Wind Case: Wall configuration C1 – simply supported on 4 sides, which presents the IWs and CPs associated to wall layout C1.

**Table 7-11 Wind Case: Mean and SD of CP1-IW1, CP2-IW2 and RC-D Curve, Wall Configuration C1**

<table>
<thead>
<tr>
<th>CP - IW</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP1-IW1</td>
<td>28.103</td>
<td>0.268</td>
</tr>
<tr>
<td>CP2-IW2</td>
<td>23.749</td>
<td>0.199</td>
</tr>
<tr>
<td>RC-D</td>
<td>25.685</td>
<td>0.248</td>
</tr>
</tbody>
</table>

Similarly, to the case of earthquake loading (i.e., Figure 7-1), the taller and narrower aspect ratio characterizing CP1 shows to be stronger than IW2 configuration. Beside all reasons listed when discussing the earthquake loading, the position of the line load within the wall height plays a substantial role in defining how the specific IW reacts to the applied out-of-plane loading action.

**Figure 7-44 Wind Case: Wall configuration C1 – simply supported on 4 sides**
The dispersion in the case of IW1 is around 25.74% greater than IW2 demonstrating that there is wider variation of wind speed (V) values causing collapse across the IW1 sample (i.e., from 15 m/s to 60 m/s), whilst the velocity causing failure in the case of IW2 configurations ranges between 13 m/s to 41 m/s. The following subsections present the fragility functions for each group of walls’ layouts, including a comparison with C1 wall layout as per previous loading cases.

7.5.1. Group A: Walls with supports on 3 sides and 1 top side free

The current subsection presents the fragility curves derived for Group A, representative of single storeys or top storeys buildings with no connection with roof elements, lightweight roof cover and various lateral restraint types and foundation connections. Table 7-12 Wind Case: Mean and SD of IW1, IW2 and RC-D Curves, Wall Group A reports the mean and standard deviation values which help understanding the change in overall fragility linked to the change in boundary conditions.

<table>
<thead>
<tr>
<th>Group A</th>
<th>Reference sketch</th>
<th>Configuration</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td><img src="image1.png" alt="Image" /></td>
<td>IW1</td>
<td>23.968</td>
<td>0.352</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IW2</td>
<td>21.539</td>
<td>0.217</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>22.666</td>
<td>0.294</td>
</tr>
<tr>
<td>A2</td>
<td><img src="image2.png" alt="Image" /></td>
<td>IW1</td>
<td>25.438</td>
<td>0.339</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IW2</td>
<td>23.113</td>
<td>0.204</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>24.190</td>
<td>0.280</td>
</tr>
<tr>
<td>A3</td>
<td><img src="image3.png" alt="Image" /></td>
<td>IW1</td>
<td>31.066</td>
<td>0.308</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IW2</td>
<td>29.858</td>
<td>0.210</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>30.393</td>
<td>0.259</td>
</tr>
</tbody>
</table>

Figure 7-45 and Figure 7-46 show the curves derived for IW1 and IW2 respectively. The IW1 configuration is noticeably stronger than IW2, for the same probability of collapse of 50%, both A1 IW1 and A2 IW1 requires an IM which is roughly 10% greater than A1 IW2 and A2 IW2.
In the case of configuration A3 instead, such increase drops to 1.85%, demonstrating that in this latter case, the wall behaviour to the out-of-plane loading is independent of the restraint condition considered. As observed in the case of earthquake and flood loading, even in the case of wind loading, the wall configuration C1 proves to be stronger than A1 and A2 configurations, but weaker than the three-sides fixed support case, as shown in Figure 7-47.

7.5.2. Group B: Walls with supports on 3 sides and 1 side free

This section covers the fragility assessment of Group B.

Table 7-13 Wind Case: Mean and SD of CP1-IW1, CP2-IW2 and Restraint-Sensitive Curve, Wall Group 2

<table>
<thead>
<tr>
<th>Group B</th>
<th>Reference sketch</th>
<th>Configuration</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td><img src="image1" alt="Reference sketch" /></td>
<td>IW1</td>
<td>21.064</td>
<td>0.292</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IW2</td>
<td>15.311</td>
<td>0.208</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>17.959</td>
<td>0.299</td>
</tr>
<tr>
<td>B2</td>
<td><img src="image2" alt="Reference sketch" /></td>
<td>IW1</td>
<td>26.899</td>
<td>0.279</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IW2</td>
<td>18.747</td>
<td>0.204</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>23.378</td>
<td>0.303</td>
</tr>
</tbody>
</table>
Figure 7-48 reports the fragility curves extracted for IW1s. From the graph it appears that there is a visible change in structural behaviour between wall configuration B1 and B4, thus highlighting that, in the context of simply supported cases, the presence of a fixed support at the base of the wall does affect the determination of the wall moment capacity and – similarly – it affects the IM that causes collapse. When dealing instead with configurations with a fixed vertical support such as B2, B3 and B5, the fixity at the base or at the top of the wall is comparatively less influential than it would be in the case of B1 and B4. That is to be attributed to the aspect ratio of the IW1 configurations, which – by nature – are more influenced by the vertical support contribution than the horizontal support contribution. A very different trend can be instead observed in the case of IW2, reported in Figure 7-49, where there is a clear distinction between additional fixities at the base and at the vertical sides, which shows to be correlated with increasing IM values causing failure.

<table>
<thead>
<tr>
<th></th>
<th>IW1</th>
<th>IW2</th>
<th>RC-D</th>
</tr>
</thead>
<tbody>
<tr>
<td>B3</td>
<td>28.377</td>
<td>25.510</td>
<td>26.858</td>
</tr>
<tr>
<td>B4</td>
<td>23.171</td>
<td>16.872</td>
<td>19.772</td>
</tr>
<tr>
<td>B5</td>
<td>27.661</td>
<td>22.139</td>
<td>24.693</td>
</tr>
</tbody>
</table>

**Figure 7-48** Wind Case: Fragility curves IW1 for Group B

**Figure 7-49** Wind Case: Fragility curves IW2 for Group B
More specially, the pattern followed by the curves is such that B1 – which is the weakest configuration of the whole group is clearly undergoing to higher percentage of damage than B4, which has a fixity at the base. More relevant than the fixity at the base is the fixity along the vertical side, as B2 is clearly stronger than B4. The two fixities characterizing B5 allow the same wall configuration to sustain and IM equal to 9.9% in correspondence of the same % of damage equal to 50%, whilst B3 is – as expected – the strongest configuration of the whole group. Figure 7-50 reports the restraint-sensitive curves for the entire Group B, with the additional C1 curve to allow for comparison between the current group and the 4-sides simply supported case. As observed already in the case of earthquake (i.e., Figure 7-7) the C1 configuration is reportedly the closest in structural behaviours terms to the configuration B3, else the strongest configuration of the whole group.

7.5.1. Group C: Walls with supports on 4 sides

This section presents the fragility assessment conducted for Group C, dedicated to wall configurations characterised by supports on all four sides. Table 7-14 includes values of mean and SD characterizing the 6 configurations included in the group.

<table>
<thead>
<tr>
<th>Group C</th>
<th>Reference sketch</th>
<th>Configuration</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td><img src="image" alt="Sketch" /></td>
<td>CP1-IW1</td>
<td>28.104</td>
<td>0.268</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2-IW2</td>
<td>23.749</td>
<td>0.199</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RD-C</td>
<td>25.685</td>
<td>0.248</td>
</tr>
<tr>
<td>C2</td>
<td><img src="image" alt="Sketch" /></td>
<td>CP1-IW1</td>
<td>35.417</td>
<td>0.221</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2-IW2</td>
<td>30.749</td>
<td>0.195</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RD-C</td>
<td>32.588</td>
<td>0.217</td>
</tr>
</tbody>
</table>
Following a very similar trend already observed in the case of earthquake loading, Figure 7-51 reports the fragility curves for IW1. The IW1 configurations prove to be stronger than IW2 ones, reported instead in Figure 7-52. The influence of the fixities along the vertical sides is evident in both IW1 and IW2 cases: whilst there is a perceivable distinction between C5 and C2 in the case of IW1, such discrepancy tends to reduce in the case of IW2, where the horizontal fixities assume a greater influence in the overall fragility of the wall configurations.

All graphs representative of Group C show that there is a clear relationship between the IM values causing collapse and the number of support fixities of the six configurations considered.

<table>
<thead>
<tr>
<th></th>
<th>CP1-IW1</th>
<th>CP2-IW2</th>
<th>RD-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>C3</td>
<td>36.628</td>
<td>35.335</td>
<td>35.870</td>
</tr>
<tr>
<td>C4</td>
<td>28.850</td>
<td>24.708</td>
<td>26.535</td>
</tr>
<tr>
<td>C5</td>
<td>33.009</td>
<td>29.447</td>
<td>30.961</td>
</tr>
<tr>
<td>C6</td>
<td>30.285</td>
<td>27.373</td>
<td>28.681</td>
</tr>
</tbody>
</table>

**Figure 7-51** Wind Case: Fragility curves CP1-IW1 for Group C

**Figure 7-52** Wind Case: Fragility curves CP2-IW2 for Group C
7.5.2. Group D: Walls with openings

This section is dedicated to present the set of fragility curves generated for the wall configurations of Group D, characterized by the presence of openings. Table 7-15 reports the values of mean and standard deviation characterizing the fragility curves presented in following Figures.

Table 7-15 Wind Case: Mean and SD of CP1-IW1, CP2- IW2 and RC-D curves, Wall Group D

<table>
<thead>
<tr>
<th>Group D</th>
<th>Reference sketch</th>
<th>Configuration</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td><img src="image1.png" alt="Reference sketch" /></td>
<td>CP1-IW1</td>
<td>17.831</td>
<td>0.263</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2- IW2</td>
<td>14.267</td>
<td>0.224</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>16.261</td>
<td>0.280</td>
</tr>
<tr>
<td>D2</td>
<td><img src="image2.png" alt="Reference sketch" /></td>
<td>CP1-IW1</td>
<td>16.647</td>
<td>0.215</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CP2- IW2</td>
<td>13.184</td>
<td>0.194</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RC-D</td>
<td>15.096</td>
<td>0.252</td>
</tr>
</tbody>
</table>

Figure 7-53 Wind Case: RC-D fragility curves for Group C

Figure 7-54 Wind Case: Fragility curves CP1- IW1 for Group D

Figure 7-55 Wind Case: Fragility curves CP2- IW2 for Group D
Similar to earthquake and flood loadings shown in previous sections, the trend followed by the fragility curves representative of Group D, both in the case of IW1 configurations – included in Figure 7-54- and in the case of IW2 configurations – reported in Figure 7-55 – indicate an overall reduced wall moment capacity and a resulting increased level of fragility also in the case of wind loading. Figure 7-56 summarizes the restraint-dependent curves D1 and D2 in comparison with configuration C1, proving – once again – that walls with a smaller number of bricks (due to the presence of openings) are weakened therefore their overall fragility shifts towards left, thus proving that smaller IM values are needed to get these walls to failure.

7.6. Comparisons of fragility curves generated for all groups and all hazards

After having defined the single-hazard fragility curve for each individual wall configuration within each of the 4 group of walls characterized by specific boundary conditions and wall aspect ratios, this section focuses on presenting three other types of fragility curves, derived from the same sample of walls but aiming to incorporate increasing levels of uncertainty in the definition of the geometric characteristics and the level of restraints. Subsection 7.6.1 focuses on presenting the restraint-condition dependent (RC-D) fragility curves, generated following the assumption that the geometric aspect ratio of the walls is an unknown. Subsection 7.6.2 reports the curves generated in the assumption that the boundary conditions are unknown, therefore defined as index wall dependent (IW-D) fragility curves and finally subsection 7.6.1 concludes the comparisons by reporting the fragility curves which are both geometry and restraints independent (G&R-IND), representing the baseline of dispersion if none of the parameters range is predefined, and therefore simulating the higher level of uncertainty.
7.6.1. Restraint condition-dependent (RC-D) fragility curves for earthquake, flood, and wind hazard

The current subsection deals with the derivation of restraint condition-dependent (RC-D) fragility curves. It must be noted that for the walls in Group A and Group B, only one crack pattern is hypothesized, whilst for the walls in Group C and D two crack patterns are assumed. For each group, the dashed line indicates the configuration showing the weakest behaviour, corresponding to the weakest level of restraint (i.e., 3 simple supports at top edge free in the case of Group A, three simple supports and side edge free in the case of Group B, 4 simple supports in the case of Group C and the wall with 2 openings in the case of Group D), whilst the solid line indicates the strongest wall configuration (i.e. 3 fixed sides in the case of Group A and B, four fixed edges in the case of Group C and the wall with one opening in the case of Group D). To facilitate the comparison between the 16 wall configurations analysed in this study, summary tables are included reporting the mean IM value and standard deviation and organized in ascending order.

Table 7-16 reports the mean values of the restraint-dependent curves in ascending order. Figure 7-57 shows a summary of all restraint-dependent fragility curves presented throughout Section 7.3 to provide with an overview of the different wall configurations’ behaviours subjected to seismic loading.

Table 7-16: Earthquake case: Mean and SD of RC-D fragility curves in ascending order

<table>
<thead>
<tr>
<th>Wall configuration</th>
<th>Reference sketch</th>
<th>Mean (PGA (g))</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>D2</td>
<td></td>
<td>0.274</td>
<td>0.533</td>
</tr>
<tr>
<td>D1</td>
<td></td>
<td>0.338</td>
<td>0.659</td>
</tr>
<tr>
<td>B1</td>
<td></td>
<td>0.360</td>
<td>0.490</td>
</tr>
<tr>
<td>B4</td>
<td></td>
<td>0.401</td>
<td>0.439</td>
</tr>
<tr>
<td>A1</td>
<td></td>
<td>0.466</td>
<td>0.622</td>
</tr>
<tr>
<td>A2</td>
<td></td>
<td>0.542</td>
<td>0.594</td>
</tr>
<tr>
<td>B2</td>
<td></td>
<td>0.577</td>
<td>0.538</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----</td>
<td>-----</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>B5</td>
<td></td>
<td>0.602</td>
<td>0.485</td>
</tr>
<tr>
<td>C1</td>
<td></td>
<td>0.654</td>
<td>0.527</td>
</tr>
<tr>
<td>C4</td>
<td></td>
<td>0.722</td>
<td>0.508</td>
</tr>
<tr>
<td>B3</td>
<td></td>
<td>0.724</td>
<td>0.431</td>
</tr>
<tr>
<td>A3</td>
<td></td>
<td>0.829</td>
<td>0.558</td>
</tr>
<tr>
<td>C6</td>
<td></td>
<td>0.850</td>
<td>0.463</td>
</tr>
<tr>
<td>C5</td>
<td></td>
<td>1.127</td>
<td>0.438</td>
</tr>
<tr>
<td>C2</td>
<td></td>
<td>1.183</td>
<td>0.438</td>
</tr>
<tr>
<td>C3</td>
<td></td>
<td>1.314</td>
<td>0.412</td>
</tr>
</tbody>
</table>

Figure 7-57: Earthquake Case: Summary of RC-D fragility curves
The configurations D1 and D2 are clearly the weakest among the 16 layouts considered, with D1 configuration able to sustain a value of PGA (g) 18.93% bigger than D2. Seismic forces are inertia forces which depend on the mass of the structure: the smaller the mass the smaller is the wall moment capacity and the smaller the IM needed to get to failure. The three-sides simply supported case with 1 side free B1 is instead the weakest configuration among the 14 remaining layouts, followed by the three-side simply supported case with top free edge A1, the latter being able to sustain a PGA (g) 22.75% bigger to get to same 50% probability of reaching collapse. There is an overlap between configuration B3 and B4 and D1 which proves the influence of the vertical support in resisting higher values of PGA, counterbalancing the presence of openings. C1 configuration is clearly stronger than both A1 and B1, but weaker than B3 (able to sustain a value of PGA (g) 9.67% bigger than C1) and A3, able to sustain a value of PGA (g) 21.10% bigger than the 4-sides simply supported case C1. Finally, the C3 configuration sets as the strongest configuration among the ones considered in this study, requiring a value of PGA (g) 36% bigger than configuration A3 to reach the same 50% probability of collapse.

Table 7-17 reports the mean values of the restraint-dependent fragility curves presented throughout Section 7.4, and, more specifically, the water height independent (RC-IND) curves, in ascending order.

<table>
<thead>
<tr>
<th>Wall configuration</th>
<th>Reference sketch</th>
<th>Mean ($\beta_H$ (m))</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>D2</td>
<td></td>
<td>0.969</td>
<td>0.432</td>
</tr>
<tr>
<td>D1</td>
<td></td>
<td>1.176</td>
<td>0.411</td>
</tr>
<tr>
<td>B1</td>
<td></td>
<td>1.373</td>
<td>0.289</td>
</tr>
<tr>
<td>B4</td>
<td></td>
<td>1.521</td>
<td>0.261</td>
</tr>
<tr>
<td>A1</td>
<td></td>
<td>1.734</td>
<td>0.309</td>
</tr>
<tr>
<td>B2</td>
<td></td>
<td>1.757</td>
<td>0.267</td>
</tr>
<tr>
<td>B5</td>
<td></td>
<td>1.831</td>
<td>0.245</td>
</tr>
<tr>
<td>A2</td>
<td></td>
<td>1.855</td>
<td>0.288</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>----</td>
<td>-------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>B3</td>
<td></td>
<td>1.996</td>
<td>0.235</td>
</tr>
<tr>
<td>C1</td>
<td></td>
<td>2.088</td>
<td>0.154</td>
</tr>
<tr>
<td>C4</td>
<td></td>
<td>2.094</td>
<td>0.154</td>
</tr>
<tr>
<td>B3</td>
<td></td>
<td>1.989</td>
<td>0.213</td>
</tr>
<tr>
<td>C6</td>
<td></td>
<td>2.243</td>
<td>0.141</td>
</tr>
<tr>
<td>C5</td>
<td></td>
<td>2.152</td>
<td>0.283</td>
</tr>
<tr>
<td>A3</td>
<td></td>
<td>2.291</td>
<td>0.289</td>
</tr>
<tr>
<td>C5</td>
<td></td>
<td>2.414</td>
<td>0.176</td>
</tr>
<tr>
<td>C2</td>
<td></td>
<td>2.490</td>
<td>0.212</td>
</tr>
<tr>
<td>C3</td>
<td></td>
<td>2.805</td>
<td>0.186</td>
</tr>
</tbody>
</table>

**Figure 7-58** Flood Case: Summary of Summary of RC-D fragility curves
Figure 7-58 Flood Case: Summary of includes the RC-D fragility curves for flood loading. Similar to the earthquake case, even in the case of flood the configurations with openings prove to be the weakest among the 4 groups, immediately followed by the three-sides simply supported case with 1 side free B1 and its slight variation B4, which is characterized by the fixed edge at the base. A1 configuration follows immediately after, proving to be only slightly stronger than the B4 configuration, as it benefits from the two vertical supports instead of just one, but lacks in top support. This comparison clearly shows that – when considering the engagement of the whole wall under flood loading and the formation of a crack pattern which extend thought the whole wall area, the vertical side fixity plays a more influent role for lower values of $\beta_H$, whilst the symmetry in restraints – such as in the A1 case – proves stronger in resisting higher $\beta_H$ values in uneven restraints configurations.

There is an overlap in configuration A1 and A2, B2 and B5 and that is due to the fact that the overall quantity of restraints of these configurations is commensurate, as two sides simply supported provide – somehow the same restraining action of one fixity and similar is the restraining action of the uneven configuration B5 in comparison to A2. It is also important to consider that these curves aim to provide with a general outlook of the fragility of two very different walls configurations, namely IW1 and IW2, and consequently, with an ‘approximated’ mid-ground solution. Finally, the C configurations are clearly shown to be stronger compared to all other groups, with the only exception of A3 cutting across the whole left side of the graphs and stretching towards C3, which is the strongest configuration of the whole set.

To conclude the RC-D single-hazard fragility curves comparison, Table 7-18 reports the summary mean and standard deviation values for the case of wind loading followed by Figure 7-59.

**Table 7-18:** Wind case, Mean and SD of RC-D fragility curves in ascending order

<table>
<thead>
<tr>
<th>Wall configuration</th>
<th>Reference sketch</th>
<th>Mean (V (m/s))</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>D2</td>
<td>![D2 Sketch]</td>
<td>15.096</td>
<td>0.252</td>
</tr>
<tr>
<td>D1</td>
<td>![D1 Sketch]</td>
<td>16.261</td>
<td>0.280</td>
</tr>
<tr>
<td>B1</td>
<td>![B1 Sketch]</td>
<td>17.959</td>
<td>0.299</td>
</tr>
<tr>
<td>B4</td>
<td>![B4 Sketch]</td>
<td>19.772</td>
<td>0.297</td>
</tr>
<tr>
<td>A1</td>
<td>![A1 Sketch]</td>
<td>22.666</td>
<td>0.294</td>
</tr>
<tr>
<td>Location</td>
<td>V (m/s)</td>
<td>Probability</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>---------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>22.377</td>
<td>0.303</td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>24.190</td>
<td>0.281</td>
<td></td>
</tr>
<tr>
<td>B5</td>
<td>24.693</td>
<td>0.271</td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>25.685</td>
<td>0.248</td>
<td></td>
</tr>
<tr>
<td>C4</td>
<td>26.535</td>
<td>0.251</td>
<td></td>
</tr>
<tr>
<td>B3</td>
<td>26.858</td>
<td>0.242</td>
<td></td>
</tr>
<tr>
<td>C6</td>
<td>28.681</td>
<td>0.224</td>
<td></td>
</tr>
<tr>
<td>C5</td>
<td>30.961</td>
<td>0.221</td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>32.588</td>
<td>0.217</td>
<td></td>
</tr>
<tr>
<td>C3</td>
<td>35.871</td>
<td>0.161</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 7-59** Wind Case: Summary of Summary of RC-D fragility curves
As shown for the earthquake and flood cases, also in the case of wind loading the weakest configurations are the ones with openings, followed by Group B, and in particular configuration B1, B4 and B2. The latter almost identically overlaps with configuration A1, showing how the two configurations can result in a very similar probability of failure when considered in relation to a load applied symmetrically to the wall height. The trend of the remaining curves follows sequentially with almost no overlap, with the only exception of configuration A3 crossing configuration C6 and C5 and proving that, for higher wind speed intensities (i.e., higher than 36 m/s) the influence of the top restraint in determining the resistance of the wall is almost negligible, thus making configuration A3 behaving almost as configuration C2. Similar to the earthquake and flood case, C3 proves to be the strongest configuration among the 16 layouts considered. The following section deals with the index wall dependent fragility curves.

### 7.6.2. Index wall – dependent (IW-D) fragility curves for earthquake, flood, and wind hazard

The current subsection deals with index wall-dependent (IW-D) fragility curves, namely the curves generated irrespectively of the boundary conditions, for which the geometric aspect ratio becomes the discriminant for conducting fragility assessment. For each wall group detailed in Table 6-2, three sets of curves are generated, one for each index-wall (i.e., IW1 and IW2), and a third curve generated by performing regression of all IMs causing failure of both IWs. Summary tables are included to facilitate comparison across the groups. Table 7-19 includes the mean values of the IW-D curves in the case of earthquake loading.

### Table 7-19: Earthquake case: Mean and SD of IW-D fragility curves

<table>
<thead>
<tr>
<th>Group</th>
<th>IW1</th>
<th>SD</th>
<th>IW2</th>
<th>SD</th>
<th>IW1-IW2 % of variation</th>
<th>IW-IND</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.631</td>
<td>0.741</td>
<td>0.557</td>
<td>0.524</td>
<td>11.72%</td>
<td>0.591</td>
<td>0.640</td>
</tr>
<tr>
<td>B</td>
<td>0.688</td>
<td>0.499</td>
<td>0.387</td>
<td>0.425</td>
<td>43.75%</td>
<td>0.516</td>
<td>0.545</td>
</tr>
<tr>
<td>C</td>
<td>1.064</td>
<td>0.591</td>
<td>0.833</td>
<td>0.457</td>
<td>21.71%</td>
<td>0.935</td>
<td>0.538</td>
</tr>
<tr>
<td>D</td>
<td>0.382</td>
<td>0.657</td>
<td>0.269</td>
<td>0.608</td>
<td>29.58%</td>
<td>0.304</td>
<td>0.608</td>
</tr>
</tbody>
</table>

Figure 7-63 shows an overlap between Group A and Group B, which demonstrate that, when considering IW1 configuration, the presence of a top support (which characterise the walls belonging to Group B) helps sustaining comparatively higher PGA (g) values compared to the configurations with top edge free such as the ones of Group A. For PGA (g) values greater than 0.88 the presence of lateral supports becomes increasingly more influential in determining the % of failure, thus allowing walls of Group A to reach higher PGA (g) values with lower % of failures than Group B. Figure 7-61 includes the curves representing IW2 configurations, which are able to sustain smaller PGA (g) compared to IW1.
More specifically, the greatest extent of variation between the mean values of the IWs is observed in the case of Group B, for which there is a difference equal to 43.75%, followed by Group D with 29.58%, Group C with a difference of 21.7% and finally Group A with a difference of 11%. What can be noticed from Table 7-19 is that – by going from a more accurate estimation of the fragility defined per restraint condition, to a more generic fragility curve which is derived irrespectively of the restraint, the mean values of the resulting curves change even more proportionally. Figure 7-65 presents the IW-IND (index wall independent) fragility curves including, the weakest and strongest configurations of each Group. Table 7-20 includes the mean values of the IW-D curves in the case of flood loading.

**Table 7-20: Flood case: Mean and SD of IW-D fragility curves**

<table>
<thead>
<tr>
<th>Group</th>
<th>IW1</th>
<th>SD</th>
<th>IW2</th>
<th>SD</th>
<th>IW1-IW2 % of variation</th>
<th>IW-IND</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.405</td>
<td>0.248</td>
<td>1.593</td>
<td>0.235</td>
<td>33.76%</td>
<td>1.943</td>
<td>0.317</td>
</tr>
<tr>
<td>B</td>
<td>1.743</td>
<td>0.299</td>
<td>1.583</td>
<td>0.289</td>
<td>9.179%</td>
<td>1.673</td>
<td>0.293</td>
</tr>
<tr>
<td>C</td>
<td>2.594</td>
<td>0.212</td>
<td>2.067</td>
<td>0.125</td>
<td>20.31%</td>
<td>2.302</td>
<td>0.199</td>
</tr>
<tr>
<td>D</td>
<td>1.366</td>
<td>0.363</td>
<td>0.831</td>
<td>0.306</td>
<td>39.16%</td>
<td>1.065</td>
<td>0.417</td>
</tr>
</tbody>
</table>
Figure 7-63: Flood Case: IW1-D fragility curves

Figure 7-64: Flood Case: IW2-D fragility curves

Figure 7-65: Flood Case: IW-IND fragility curves

Figure 7-63 includes the curves for IW1 configurations. In the case of flood, the configurations with openings are substantially less resistant than other configurations, especially the ones characterized by supports on both vertical edges (i.e., Group A and Group C). The mean values reported in previous table confirm that the difference in mean between Group D and Group A is equal to 43.20% and the difference in mean between Group D and Group C is 47.34%. Figure 7-64 reports instead the trend of curves for the IW2 configuration. The plot shows that there is a noticeable difference between the Group D layouts and the others, which prove to be all more resistant to higher depths of water. Moreover, the plot shows that when the value of βH is greater than 2/3 of the wall H of configuration IW2- Group B has a lower mean but a greater standard deviation – almost 38.75% greater than C. Figure 7-65 includes the four IW-IND curves with the weakest and the strongest configuration of the same four wall groups considered. Interestingly, whilst the presence of a top free edge (i.e., Group A) has a substantial effect in the case of seismic action, it does not seem to have the same detrimental effect in the case of the flooding action.
Finally, Table 7-21 includes the IW-D fragility curves for each group configuration and the IW-IND curve representative of the behaviour of the whole groups in the case of wind loading.

Table 7-21: Wind case: Mean and SD of IW-D fragility curves

<table>
<thead>
<tr>
<th>Group</th>
<th>IW1</th>
<th>SD</th>
<th>IW2</th>
<th>SD</th>
<th>IW1-IW2 % of variation</th>
<th>IW-IND</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>26.477</td>
<td>0.352</td>
<td>24.557</td>
<td>0.252</td>
<td>7.25%</td>
<td>25.436</td>
<td>0.305</td>
</tr>
<tr>
<td>B</td>
<td>23.783</td>
<td>0.295</td>
<td>19.382</td>
<td>0.278</td>
<td>16.61%</td>
<td>22.050</td>
<td>0.317</td>
</tr>
<tr>
<td>C</td>
<td>31.547</td>
<td>0.261</td>
<td>28.255</td>
<td>0.237</td>
<td>10.43%</td>
<td>29.672</td>
<td>0.254</td>
</tr>
<tr>
<td>D</td>
<td>17.430</td>
<td>0.270</td>
<td>13.659</td>
<td>0.213</td>
<td>21.63%</td>
<td>15.423</td>
<td>0.271</td>
</tr>
</tbody>
</table>

The trend followed by both IW1 and IW2 configuration presented in Figure 7-66 and Figure 7-67 and summarized in Figure 7-68, shows that there is no substantial overlap between curves, which prove the very dissimilar behaviour of the four groups under a KEL positioned at mid wall height. Substantially weaker are the configurations with openings, in line to what is observed in previous plots.

Figure 7-66: Wind Case: IW1-D fragility curves

Figure 7-67: Wind Case: IW2-D fragility curves

Figure 7-68: Wind Case: IW-RC-D fragility curves

To follow, Group B has the weakest behaviour among the other configurations and that is due to the lack of vertical side support characterizing the B configurations. More specifically, the IW1’s mean is 11.76% smaller than Group A and 25.94% than Group C, whilst IW2’s mean is 21.07% smaller than
the mean of Group A and 31.25% smaller than the mean of Group C. The three subsets of curves presented in this section are share one common aspect, namely the fact that IW1 is always stronger than IW2 and that the SD of the former is always greater than the SD of the latter configuration. Also, in the case of earthquake the greater IW1-IW2 percentage of variation is observed for the configurations belonging to Group B, followed by the configuration with openings D, the 4-sides supported cases of Group C and finally the Group A, thus highlighting the role played by the lack of 1 vertical support in determining the resisting capacity of the wall in the case of seismic loading, and how this aspect is exacerbated by the wall aspect ratio. Conversely, in the case of flood, the greatest level of discrepancy between IW1 and IW2 is observed in the case of Group D, clearly showing that the presence of openings highly influences the performance of the two different walls configurations, then configuration A, C and finally B, proving that independently of the wall aspect ratio, the configuration is consistent in the limited IM which is able to bear before reaching collapse. Finally, in the case of wind loading, the greatest discrepancy between IW1 and IW2 is observed in the case of walls with openings and the least in the case of wall with top free edge. The following subsection deals with the geometry and restraint independent (G&R-IND) fragility curves.

7.6.1. Geometry and Restraint – independent (G&R-IND) fragility curves for earthquake, flood and wind hazard

The current subsection includes the geometry and restraint independent (G&R-IND) fragility curves, for earthquake, flood and wind hazard. These curves are built in the assumption that neither the aspect ratio of the walls nor the boundary conditions can be known, thus representing the baseline of complete uncertainty of a sample of data. These curves are built by performing regressions of all IMs causing collapse of both IW1 and IW2 of all four groups of wall layouts included in Table 6-2.

Table 7-22: Mean and SD of G&R- IND fragility curves for earthquake, flood and wind hazard

<table>
<thead>
<tr>
<th>Hazard</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earthquake</td>
<td>0.611 (PGA(g))</td>
<td>0.671</td>
</tr>
<tr>
<td>Flood</td>
<td>1.811 (βH (m))</td>
<td>0.351</td>
</tr>
<tr>
<td>Wind</td>
<td>23.965 (V (m/s))</td>
<td>0.357</td>
</tr>
</tbody>
</table>

Table 7-22 reports the mean and SD values of the geometry and restraint independent curves included in Figure 7-69, Figure 7-70 and Figure 7-71. To facilitate the understanding of where these curves fit within the four considered wall groups, the IW-IND curves are also reported. Noticeably, in the case of earthquake, the G&R-IND curve seems to follow almost identically the trend of the IW-RC-D indicating Group A, which means that the average of IM causing failure across the whole groups is very similar to the average of IMs causing failure of Group A itself. Conversely, the Flood and Wind G&R-IND curves
cut across the four groups, allowing the curves to lay within the lower and upper bounds marked by the weakest Group D and the strongest Group C. According to the reported values of SD, seems clear that the dispersion in the case of walls subjected to earthquake loading is much greater than in the case of flood and wind, thus proving that in the former case, the influence played by restraint conditions and shape ratios is far greater than in the latter cases.

**Figure 7-69**: Earthquake Case: G&R-IND fragility curve

**Figure 7-70**: Flood Case: G&R-IND fragility curve

**Figure 7-71**: Wind Case: G&R-IND fragility curves

### 7.7. Conclusion

As one the main purposes of this research study is to be able to derive meaningful fragility curves by performing regression of the IM values obtained from the application of the proposed analytical procedure designed to deal with different types of hazard by adopting the methodology presented in Chapter 4, detailed in Chapter 5 and validated in Chapter 6, the threefold aim of this chapter is to:

- Derive single-hazard fragility functions built in full knowledge of geometry and restraints conditions, to allow for fragility assessment across groups of walls subjected to the same type of peril, hence the same type of horizontal action.
• Derive restraint-conditions dependent fragility curves, which allow for comparison in situations where the geometry of the walls investigates is not clearly discernible whilst the type of restraint is very clear.

• Derive fragility curves which are valid for walls of which the restraint conditions are not clearly visible, whilst the geometric aspect ratio represents the discriminant of the investigation.

With respect to the first aim, sections 7.3, 7.4 and 7.5 show in detail the curves for each wall group and each peril considered. The general trend reported in the section is that to an increase in number of supports and fixities corresponds a reduction in terms of fragility of the wall layouts considered. A common aspect to all perils is that the configurations which are taller and narrower (L/H <1 = IW1) are generally less vulnerable than wall configurations characterized by an aspect ratio L/H >1 (IW2) and the dispersion characterizing the IW1 sample is generally higher than the one of IW2s. This latter aspect is more predominant in the case of earthquake and flood than the case of wind, hence can be inferred that it is strictly correlated to the position of the KEL with respect to the geometry of the wall.

The proposed analytical procedure allows to determine functions whose standard deviation can be correlated to the level of knowledge that the assessor has of the building sample investigated. The restraint-condition dependent fragility curves are representative of a sample characterized by a high level of reliability, for which the assessor is in full knowledge of all boundary conditions. By looking at the weakest (D2) and the strongest (C) configurations across the sample reported in Table 7-16, 7-17, 7-18, the highest level of dispersion is observed in the case of earthquake, followed by flood and wind. Note that these curves are IW independent. In cases in which the restraints are not clearly visible, the geometry represents the discriminant of assessment. The highest level of dispersion between IW1 and IW2 in the case of earthquake loading is observed in the A group configuration, whilst the highest percentage of variation in terms of λ causing collapse is registered in the B configuration, hence proving that the absence of a top restraint affects the overall wall behaviour in conjunction with the change in aspect ratio, whilst the lack of side restraint is the most influential parameter affecting the failure capacity of the subgroup. In the case of flood, the greatest dispersion is observed in the case of C group, whilst the greater discrepancy in λ is for the D configuration, proving that the presence of openings is quite substantial in the determination of the overall vulnerability of the group. Finally in the case of wind the highest dispersion is – as for earthquake – registered in the case of configurations A, whilst the greatest λ variation is registered in the case of Group C.

Geometry and restraint conditions independent fragility curves allow a very general type of assessment which could be used as preliminary means of comparison of vulnerability of a building stock to a given hazard scenario and corresponds to a scenario where the knowledge of sample is very scarce.
In the following chapter, the analytical procedure proposed in this study is applied to a real building sample located in the Philippines. Fragility curves representative of the Philippines sample are derived and compared to the generic fragility curves reported in this current chapter, to prove the novelty and the applicability of the proposed method to conduct multi-hazard vulnerability assessment.
8. Case study

8.1. Introduction

This chapter is dedicated to the application of the proposed methodology to assess the vulnerability of masonry structures subjected to seismic, flood and wind loading, to a case study, to prove the feasibility of the method proposed and the ease of implementation when a building sample is made available. After the 2013 earthquake that struck Bohol Island in Central Visayas and the super Typhoon Yolanda, several centuries-old heritage structures in Bohol and Cebu islands in the Philippines were severely damaged, among which churches, convents, heritage houses, watchtowers, and schools.

These events raised the need to carry out a multi-hazard vulnerability assessment of heritage buildings, many of which were irretrievably lost in the disasters. The Philippines Department of Tourism (DoT) expressed the urgent need to improve the resilience of these types of structures – defined as Priority Buildings - to natural disasters to ensure that their cultural, historical, and economic value is sustained and continues to contribute to the overall development of the areas where they are located. At the request of DoT, the World Bank, with funding from the Global Facility for Disaster Reduction and Recovery (GFDRR) financed a detailed vulnerability assessment of selected cultural heritage structures to identify, prioritize and provide initial cost estimates for risk reduction investments for structural strengthening and restoration. The project was run in collaboration between University College London, ARS Progetti SPA, Italy, University of Santo Tomas Graduate School Center for Conservation of Cultural Property and Environment in the Tropics, Manila, and specialist engineers from the De La Salle University, Manila, resulting in a number of publications of which the author of the thesis is co-author (D’Ayala, D., Galasso, C., Putrino, V. et al (2016), D’Ayala, D., Galasso, C., Putrino, V., Fanciullacci, D., Barucco, P., Fanciullacci, V., ... & Yu, K. (2016)), and a manual for professional architects and engineers aiming at providing a method to assess the vulnerability of the selected heritage buildings to multiple hazards. More specifically, the author of the thesis was primarily involved in the field data gathering and in charge of the seismic vulnerability assessment of these buildings, under the supervision of Prof. D’Ayala.

Three different procedures were applied to fulfill the aim of assessing the multi-hazard vulnerability of the Philippines heritage stock investigated in the project. The seismic vulnerability assessment was conducted by means of the analytical macro-element approach defined as FaMIVE Procedure (D’Ayala, D., & Speranza, E. (2003), D’Ayala, D. F. (2005), D’Ayala, D. (2013)), specifically tailored to include the hybrid timber-masonry building typologies typical of the Philippines Heritage stock. The flood vulnerability of the Philippines’ Priority Buildings was carried out through an assessment of eight quantifiable, yet qualitative vulnerability parameters (level of protection, age, typology, condition,
number of storeys, construction, footprint, land slope), to which a range of attributes ranging from 3 to 5 was determined, to each one of whom a Vulnerability Rating (VR) was assigned, ranging from 10 to 100. The final vulnerability was obtained from summating each attribute value for the parameters contributing to the vulnerability to give a total ranging between 100 and 800, used to classify the building within a 4 classes vulnerability scale. The physical damage to the roof system due to high wind speed was modelled using an engineering-based probabilistic load and resistance approach, and the vulnerability determined by comparing the roofs’ uplift resistances (modelled probabilistically) and wind loads considered deterministically and modelled according to the National Structural Code of the Philippines. Noticeably, the three procedures were aiming at determining the multi-hazard vulnerability of the Priority Heritage buildings in a non-homogenised manner, thus leaving us facing the difficulty of making the results comparable across hazards, hence justifying the development of the proposed procedure and the effort of applying it to the same building sample, once again.

Pertinently, a selection of these heritage buildings is used in this chapter, and a building sample – defined as Philippines Sample (PH_Sample) – is determined.

Section 8.2 provides a description of the geometric and material characteristics of the PH_Sample, focusing on highlighting the differences/similarities with the taxonomy of walls proposed in Chapter 6.2.

Section 8.3 reports the fragility curves derived for the PH_Sample following the procedure proposed in this work and, with reference to the seismic, flood and wind hazard intensity at the site, discussed in D’Ayala, D., Galasso, C., Putrino, V. et al (2016), an assessment of the percentage of failure of buildings within the PH_Sample is provided and discussed. Then, a set of comparisons of these obtained curves with: 1) the very general Geometry and Restraints – IND fragility curve for earthquake, flood and wind loading, 2) the IW-D fragility curves built on the knowledge of the geometric aspect-ratio of the wall and irrespective of the type of restraint conditions, and 3) the single-hazard fragility curves reported in Section 7.3, 7.4, and 7.5.

Section 8.4 draws the conclusive remarks to lead to the final chapter of this thesis.

8.2. The buildings sample: heritage houses, convents, and schools in the Philippines

As mentioned in the introductory section, the main areas investigated were the two islands of Cebu and Bohol. To the aim of applying the proposed analytical procedure to conduct multi-hazard vulnerability assessment, only a proportion of these building sample was selected, focusing the attention on residential buildings, convents, the school, and the blockhouse, as the geometric aspect ratios of the walls constituting this latter group were deemed to be more compatible with the walls’ sample generated in
Chapter 6.2. Figure 8-1 reports the locations of the investigated buildings and – highlighted - are the ones constituting the PH_Sample.

![Map of Philippines](image1)

![Map of Philippines](image2)

1) Manila – St Augustin Church
2) Carcar – Cebu Island: Casa Gorordo, Balay Nga Tisa, Balay Nga Daku, Dispensario; St. Catherine of Alexandria Parish Church & Convent
3) Carcar – Cebu Island: Pardo Church
4) Carcar – Cebu Island: Dalaguete Church
5) Boljoon – Cebu Island: Church, Convent, School & Blockhouse
6) Baclayon – Bohol Island: Church and Tower
7) Albuquerque – Bohol Island: Church & Convent
8) Cortes – Bohol Island: Church & Convent
9) Dimiao – Bohol Island: Church & Convent
10) Punta Cruz – Bohol Island Tower
11) Panglao – Bohol Island: Church & Tower
12) Loboc – Bohol Island: Church, Tower, Mortuary Chapel

**Figure 8-1:** Locations of investigated buildings and highlighted PH_Sample

Convents and heritage houses in the Philippines are usually two-storey buildings (as shown in Figure 8-2 left), with a masonry ground floor and a lighter upper storey made of ‘tabique pampango’, namely thin walls made of wood, which are instead used as the living spaces and built as open spaces, generally constructed following the wattle and daub method, and timber vertical walls, coated with lime mixed with sand as shown in Figure 8-2 left (ARS, CCCPET, UCL EPICentre, UDLS, (2016)).

![Convent](image3)

**Figure 8-2:** Bohol – Cortes Convent: view of the façade (left); close-up of tabique pampango (right)
The horizontal structures between ground and first floor are usually made of timber elements jutting out. Depending on the horizontal structure with respect to the façade considered, the type of connection between the two structural elements may vary, i.e. with wooden beams sitting on top of the masonry walls without strong connection as shown in Figure 8-10 left or via beams breaking the continuity of the walls as shown in Figure 8-10 right.

**Figure 8-3**: Bohol – Cortes Convent: close-ups of the connection between ground floor and horizontal structures

Timber posts are usually used to support the roof; the main roofing system is usually constituted very elaborated timber trusses and purlins supporting cladding made of light metal sheets as shown in Figure 8-4 a), connected to the vertical walls through various types of wood connections (Figure 8-4 b)c)).

**Figure 8-4**: Bohol – Cortes Convent: a) timber truss system covering the kitchen area, b) c) roof system-to-wall connections
Figure 8-5 shows two examples of Heritage House (left) and convent (right), the upper floors of which are made of wood elements finished with white painting.

Figure 8-5: Carcar, Cebu: Balay Nga Tisa House (left), St. Catherine of Alexandria Convent (right)

As visible from Figure 8-5, very often these buildings are characterised by a rectangular shape, with very long front walls and much shorter orthogonal side walls. Series of internal orthogonal partitions allow to break-down the front walls in series of interconnected bays characterised by various types of restraint conditions. These restraint conditions also depend on the direction of the horizontal structural system separating the ground floor from the upper floors, the presence/absence of strong connections at the edges of the façade such as quoins, the presence of restraining elements such as ring beams and the direction of the roof elements. Moreover, depending on the position and the dimension of openings within the same bays, these can be further subdivided in more portions characterized by free side edges.

Table 8-1 reports the rationale adopted to find a correspondence between the facades surveyed on site and the taxonomy of walls simulated with the proposed procedure.

Table 8-1: Correspondence between field surveyed façades and degree of side and top restraints of the proposed analytical model

<table>
<thead>
<tr>
<th>Facade surveyed in the field</th>
<th>Corresponding model</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. orthogonal walls behind the surveyed façade</td>
<td>No. bays +1</td>
</tr>
<tr>
<td>Connection with foundations – weak</td>
<td>Horizontal base edge simply supported</td>
</tr>
<tr>
<td>Presence of strong connections (quoins) at vertical edges</td>
<td>Vertical side fix-support</td>
</tr>
<tr>
<td>Absence of strong connection at vertical edges</td>
<td>Vertical side simply supported</td>
</tr>
<tr>
<td>Horizontal structures parallel to façade</td>
<td>Horizontal top edge simply supported</td>
</tr>
<tr>
<td>Horizontal structures orthogonal to façade</td>
<td>Horizontal top edge fix-support</td>
</tr>
<tr>
<td>Roof structures parallel to façade – heavy roof/ presence of ring beam</td>
<td>Horizontal top edge simply supported</td>
</tr>
<tr>
<td>Roof structures orthogonal to façade heavy roof/ presence of ring beam</td>
<td>Horizontal top edge fix-support</td>
</tr>
<tr>
<td>Roof structures parallel or orthogonal to façade – light metal sheet</td>
<td>Top free edge</td>
</tr>
</tbody>
</table>
Following the rationale reported in Table 8-1, each façade of the PH_Sample was first subdivided in number of corresponding bays, then a preliminary sensitivity test was conducted to establish the crack pattern that determines the lowest capacity, given the openings pattern. To accommodate the cases in of bays with various levels of edge restraint and simultaneous presence of openings, both Group B and Group D configurations have been further developed from the original cases reported in Table 6-2. Table 8-2 and Table 8-3 report on the left the sketch realized from the site survey and on the right the subdivision in corresponding bays, of the east façade of the heritage house Balay Nga Tisa and the east façade of St. Catherine of Alexandria Convent respectively.

**Table 8-2: Carcar-Balay Nga Daku-1.1.E1**

<table>
<thead>
<tr>
<th>Wall surveyed in the field</th>
<th>Corresponding model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bay 1: B Group modified with opening</td>
<td></td>
</tr>
<tr>
<td>Bay 2: D Group</td>
<td></td>
</tr>
<tr>
<td>Bay 3: B Group modified with opening</td>
<td></td>
</tr>
</tbody>
</table>

**Table 8-3: Carcar-Cebu-Convent-1.2.E**

<table>
<thead>
<tr>
<th>Wall surveyed in the field</th>
<th>Corresponding model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bay 1 &amp; 3: B Group modified with opening</td>
<td></td>
</tr>
<tr>
<td>BAY 2: D Group modified with 3 openings</td>
<td></td>
</tr>
</tbody>
</table>

Following the rationale described in Table 6-2 and Table 8-3, the PH_Sample is constituted of 29 facades, 3 of which reporting an upper floor made of masonry, which have been translated to 92 bays.
The full list of bays is included in Appendix 2, resulting in the PH_Sample bays configuration breakdown presented in Figure 8-6. It is assumed that, at ground floor, mechanisms of the Group A do not form. As weak as the connection between ground floor and upper floor might be, it is never assumed that ground floor bays could behave similarly to walls with top free edge. It is instead considered likely for Group A mechanisms to form in the case of upper floor, and more specifically in the case in which the roof system is laid parallel to the bay considered and the roof cover is made of light metal sheets (as it is the case for Bohol-Cortes-Convent-1.2. detailed in Appendix 2). It is important to highlight that the crack patterns assumed for each bay represent very often one of the many other possible crack patterns configurations which can develop, therefore the results presented in the following sections are conditional to these initial choices.

![Configuration breakdown](image)

**Figure 8-6: PH_Sample bays configurations and breakdown of bays with and without openings**

With reference to the masonry fabric and construction details, the site survey has revealed that the walls of the investigated buildings were made of two independent masonry leaves bounding an internal core made of loose sandy material characterized by a very scarce level of connection across the core thickness and between the outer layers as shown in Figure 8-7.

![Site survey](image)

**Figure 8-7: Bohol – Cortes Church: shoring applied to side of the narthex showing the multi-leaves stone blocks masonry and the core material**

On this basis, it was decided to account only for the thickness of the external layer, as it is assumed that only the outer leaf provides the out-of-plane load bearing capacity against seismic, flood and wind action. The site survey highlighted that very often there is the great variation of stone blocks dimensions.
even within the same façade. It is therefore decided to report information of stone blocks units’ length, height, and staggering ratio for each façade in Appendix 3. However, a summary table with the main ranges of the assumed geometric and material characteristics of the wall bays is also reported herein, to allow for comparison with the characteristics reported in the wall taxonomy sample, discussed in Chapter 6.2 and used to derive the fragility functions presented in Chapter 7.

Table 8-4: Geometric characteristics ranges of the PH_Sample

<table>
<thead>
<tr>
<th>Geometric parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>2.75 m to 13.51 m</td>
</tr>
<tr>
<td>Wall height</td>
<td>2.05 m to 5.70 m</td>
</tr>
<tr>
<td>Wall thickness</td>
<td>0.100 m to 0.250 m</td>
</tr>
<tr>
<td>Unit length</td>
<td>0.200 m to 0.400 m</td>
</tr>
<tr>
<td>Unit height</td>
<td>0.180 m to 0.450 m</td>
</tr>
<tr>
<td>Staggering ratio</td>
<td>0.050 m to 0.250 m</td>
</tr>
<tr>
<td>Opening length</td>
<td>1.00 m to 3.20 m</td>
</tr>
<tr>
<td>Opening height</td>
<td>1.00 m to 4.55 m</td>
</tr>
</tbody>
</table>

One very important difference to highlight between the two samples (i.e., the wall’s taxonomy and the PH_Sample) is the staggering ratio, the parameter that rules the determination of the angle of crack. Whilst in the case of the generic sample presented in Chapter 6.2 the staggering ratio was treated as a resulting parameter always equal to half of the unit length, as reported in Chapter 5, Section 5.5, Eq. 5.18, in the PH_Sample the staggering ratio is an independent datum. Noticeably, for some of the walls surveyed, the staggering ratio is equal to 0.05 m, thus proving an inherent weakness of the wall due to the arrangement of the stone blocks. As per the material properties of the masonry walls of the PH_Sample, in consideration of the level of damage surveyed, lack of laboratory testing and characteristic values obtained from national building codes, it is decided to use the lower bounds of the ranges proposed in Table 6-4, as reported in Table 8-5.

Table 8-5: Material characteristics values of the PH_Sample

<table>
<thead>
<tr>
<th>Material Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{k1} = f_{mt}$ (MPa)</td>
<td>0.1</td>
</tr>
<tr>
<td>$f_{k2}$ (MPa)</td>
<td>0.2</td>
</tr>
<tr>
<td>Coefficient of friction $\phi$</td>
<td>0.3</td>
</tr>
<tr>
<td>Density $\rho$ (kN/m$^3$)</td>
<td>14.00</td>
</tr>
</tbody>
</table>

Having defined the geometric and material parameters needed to apply the proposed analytical procedure, the value of IM causing collapse for each of the 92 bays can be determined. Fragility curves for earthquake, flood and wind are then derived by performing regression of IMs leading the bays to collapse following the same method reported in Section 7.2. The following section reports the fragility
curves representative of the PH_Sample, and the comparisons with the fragility curves derived for the walls’ taxonomy and defined in previous Chapter.

8.3. PH_Sample fragility curves and comparisons with single-hazard fragility curves, IW-D and Geometry & Restraints -IND curves

The present section includes the fragility curves derived for the PH_Sample subjected to earthquake, flood and wind loading. As reported in D’Ayala, D., Galasso, C., Putrino, et al., (2016), the assessment of the different hazard levels for the heritage sites has been carried out by using several state-of-the-art references, including data and studies from the Philippines Institute of Volcanology and Seismology (PHIVOLCS) and the Philippines Atmospheric Geophysical and Astronomical Service Administration (PAGASA) and the Nationwide Operational Assessment of Hazards (NOAH) website. The summary of the hazard assessment for the heritage sites in Cebu and Bohol is reported in Table 8-6.

Table 8-6: Hazard assessment for Heritage Structures

<table>
<thead>
<tr>
<th>Location</th>
<th>PGA (g)*</th>
<th>Flood height (m)**</th>
<th>Wind Speed (knots)***</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bohol</td>
<td>0.3</td>
<td>1.5</td>
<td>45</td>
</tr>
<tr>
<td>Cebu</td>
<td>0.3</td>
<td>1.5</td>
<td>45</td>
</tr>
</tbody>
</table>

* Corresponding to a return period of 475 years.

** Corresponding to rainfall scenarios with return period of 100-150 years.

*** Value taken from the (NOAH) website, indicating the average wind speed of the tracks of tropical cyclones that made landfall in the Philippines between January 1966 and December 2016; 1 knot= 0.514 m/s → 45 knot= 23.15 m/s.

Table 8-7 reports the mean and standard deviation values of the PH_Sample fragility curves, and the probability of damage attained when considering the hazard values at the site.

Table 8-7: PH_Sample fragility curves: Mean and SD

<table>
<thead>
<tr>
<th>Hazard</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earthquake</td>
<td>0.293 PGA (g)</td>
<td>0.736</td>
</tr>
<tr>
<td>Flood</td>
<td>1.134 βH (m)</td>
<td>0.294</td>
</tr>
<tr>
<td>Wind</td>
<td>16.642 V (m/s)</td>
<td>0.360</td>
</tr>
</tbody>
</table>

Figure 8-8 Figure 8-9 and Figure 8-10, report the fragility curves for the PH_Sample, generated for earthquake, flood and wind hazard respectively, including the IM indicating the hazard at the site (in green). The PH_Sample is more vulnerable to flood loading with a probability of damage of 82% in correspondence to 1.5 m of water, followed by an 80% probability of undergoing failure due to wind
speed and a 50% probability when subjected to 0.3 PGA (g). These percentages, however, must be read in knowledge of the fact that only the external leaf of masonry walls has been accounted for the load bearing resistance of the PH_Sample.

The PH_Sample’s fragility curves for earthquake, flood and wind loading have also been compared to the sets of fragility curves generated for the walls’ taxonomy built to test the proposed multi-hazard analytical procedure. Since the global aspect ratio of the bays constituting the PH_Sample is of the type L/H>1, the comparison reported herein refer to configurations IW2 exclusively.

**Figure 8-8:** Earthquake Case: Fragility curves of PH_Sample

**Figure 8-9:** Flood Case: Fragility curves of PH_Sample

**Figure 8-10:** Wind Case: Fragility curves of PH_Sample

**Figure 8-11:** Earthquake Case: Comparison between IW2-Ds and PH_Sample curves

**Figure 8-12:** Flood Case: Comparison between IW2-Ds and PH_Sample curves
Figure 8-13 Wind Case: Comparison between IW2-Ds and PH_Sample curves

Figure 8-11, Figure 8-12 and Figure 8-13 report the comparison between the PH_Sample and the IW2-D fragility curves for seismic, flood and wind loading respectively. The fragility curves indicating the overall PH_Sample behaviour are all sensibly shifted towards the left of the graph, proving that – notwithstanding most of the PH_Sample configurations present fixities at their edges - the presence of openings and the reduced flexural tensile strength of the masonry have played a noticeable role in increasing the fragility of the sample. One aspect common to all graphs is that the PH_Sample for which all geometric and masonry material properties are known, is much weaker if compared to the range applied to the parametric study, whereby many of the considered configuration did not have openings. As simplistic as this might seem, this proves the efficacy of the proposed analytical procedure to capture sensibly the refined level of information provided with the PH_Sample, in which each bay is different than all others within the same sample. The comparison between PH_Sample to the IW2-D curves shows a very different behaviour of the same sample, depending on the hazard considered. In the case of earthquake, the PH_Sample curve cuts across the Group D and Group B curves, with 50% probability of collapse reached at 0.293 PGA(g) and a much greater dispersion compared to the two other mentioned curves, proving that – in the specific instance of considering the seismic KEL, the influence played by the vertical restraints is greater than the role played by the flexural tensile strength. In the case of flood, the PH_Sample curve is sensibly stronger than the IW2-D Group D curve, built in the assumption that the walls have openings but are also simply supported and with a mean $\beta_H$ value of 0.831 (m). The PH_Sample is however much more fragile compared to the IW2-D Group B, proving the influence played by the presence of large openings in the sample and the influence the full-to-void ratio plays in the case of flood loading when compared to walls with no openings. Similarly, in the case of wind, the PH_Sample curve is placed almost in the middle between IW2-D Group D and IW2-D group B, the two configurations which represent 83% of the whole PH_Sample as reported in Figure 8-6: PH_Sample bays configurations and breakdown of bays with and without openings. The fragility of the PH_Sample is therefore influenced almost in equal measure by the presence of openings and the reduced flexural tensile strength of masonry: the curve reports a mean value of 16.642 (m/s), which is comprised between
13.184 (m/s) of IW2-D Group D and 19.381 (m/s) of IW2-D group B and a SD value which is 41.27 %
bigger than IW2-D Group D and 23.26% bigger than IW2-D group B (which are available at Table 7-21), thus highlighting the increased level of dispersion of the PH_Sample compared to the one of two Groups.

The following comparisons are performed between the single-hazard fragility curves detailed in Chapter 7 and specific subgroups of the PH_Sample which refer to subjected to earthquake, flood and wind loading. Starting with the earthquake case, Figure 8-14, Figure 8-15 and Figure 8-16: report the comparisons between fragility curves derived for the PH_Sample subgroups B, C and D&B with openings, accounting respectively for the 8%, 14% and 75% of the whole sample, and the fragility curves for the corresponding groups B, C and D of the parametric analysis, presented in Section 7.3.2,7.3.3 and 7.3.4.

Table 8-8 includes the values of Mean and SD to allow for comparison between the three subgroups considered, and the fragility curve representative of the whole PH_Sample. The fragility curve built
for the subgroup of bays similar to the B Group shows to be placed between wall configuration B2 and B5.

**Table 8-8:** Earthquake Case: Comparison between PH_Sample subgroups

<table>
<thead>
<tr>
<th>PH_Sample subgroup</th>
<th>Mean (PGA (g))</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>B configurations without openings</td>
<td>0.399</td>
<td>0.414</td>
</tr>
<tr>
<td>C configurations</td>
<td>0.417</td>
<td>0.503</td>
</tr>
<tr>
<td>D &amp; B configurations with openings</td>
<td>0.271</td>
<td>0.785</td>
</tr>
<tr>
<td>PH_Sample</td>
<td>0.293</td>
<td>0.736</td>
</tr>
</tbody>
</table>

Even though the values of flexural tensile strength are very low, this subgroup of the PH_Sample has fix supports at the vertical and top edges, thus justifying the fact that the resulting fragility curve is comparatively stronger than the weakest configuration of the B Group, namely B1. On the contrary, the influence of flexural tensile strength as well as the wall thickness proves to be higher in ruling the overall fragility of the PH_Sample’ subgroup of C configurations. Figure 8-15 shows that when comparing the former to the generic Group C generated for walls with an average range of $f_{sk}=0.20$ MPa and a wall thickness of 0.215 m – the fragility curve shifts towards the far left of the plot, thus indicating that the subgroup is more fragile than the weakest configuration of the C Group, namely C1. Finally, the comparison between the PH_Sample subgroup with openings (i.e., D and B layouts) and the Group D shows that the former group is slightly stronger than the latter for higher IMs (i.e., PGA (g) > 0.28), and the observed decrease in fragility is to be attributed to the fixities of the bays of the PH_Sample. A trend similar to the case of earthquake loading can be observed also in the case of flood loading. Figure 8-17 reports the fragility curves derived for the PH_Sample subgroup of B configurations without openings. The values of $\beta_H$ causing the collapse of these walls is comprised within the narrow range that goes from 1.48 m to 1.67 m and given the very limited number of walls of the subgroup, the SD value, reported in Table 8-9, is very small, thus justifying the almost vertical fragility curve representing the subgroup.

![Figure 8-17: Flood Case: Comparison between PH_Sample (subgroup of B configurations without openings) and Group B](image1)

![Figure 8-18: Flood Case: Comparison between PH_Sample (subgroup of C configurations) and Group C](image2)
Figure 8-19: Flood Case: Comparison between PH_Sample (subgroup of D & B configurations without openings) and Group D.

Table 8-9: Flood Case: Comparison between PH_Sample subgroups

<table>
<thead>
<tr>
<th>PH_Sample subgroup</th>
<th>Mean (βH (m))</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>B configurations without openings</td>
<td>1.535</td>
<td>0.041</td>
</tr>
<tr>
<td>C configurations</td>
<td>1.442</td>
<td>0.230</td>
</tr>
<tr>
<td>D &amp; B configurations with openings</td>
<td>1.066</td>
<td>0.280</td>
</tr>
<tr>
<td>PH_Sample</td>
<td>1.134</td>
<td>0.249</td>
</tr>
</tbody>
</table>

Figure 8-18: Flood Case: Comparison between PH_Sample (subgroup of C configurations) and Group C shows the fragility curve representative of the portion of PH_Sample walls which would form a crack pattern of the types reported in the Group C. Similar to the case of earthquake, also in the case of flood the overall behaviour of the subgroup is weaker than the weakest Group C configuration, reiterating the importance of geometric and material properties, namely thickness and flexural tensile strength in the determination of the overall capacity of these configurations. Contrary to the earthquake loading, the comparison between the PH_Sample subgroup of D&B configurations with openings, prove a general stronger behaviour compared to the 4-sides simply supported configurations of the Group D, thus proving that the influence of the restraints makes a clearer difference in determining the fragility of the sample subjected to flood loading.

Finally, Figure 8-20 Figure 8-21 and Figure 8-22 report the same comparisons discussed for the case of wind loading and Table 8-10: Wind Case: Comparison between PH_Sample subgroups details the mean and standard deviation of the subgroups considered. The main difference in the trend reported pertains the case of subgroup B, which – contrarily to the case of seismic and flood loading - is reportedly weaker than B1 soon after the threshold of V=14 m/s is reached. Also, similarly to the flood case, the fragility curve for this subgroup is particularly steep, as the IMs causing the configurations to collapse are bounded within the very narrow range comprised between 11 and 17 m/s. Subgroup C and D&B with openings behave as already observed in previous cases.
The comparisons presented in previous Figures aim at demonstrating that to the shift observed in the geometric and material parameters of the PH_Sample corresponds a shift in fragility. Moreover, by going from the comparison with the general fragility curve, representative of a generic sample and generated irrespective of the geometric aspect ratio, to increasingly more refined fragility curves such as the single-hazard fragility curves derived by accounting for geometry and restraint conditions, the level of mismatch tends to reduce in favour of a more accurate comparison between the general sample and the case study sample.
8.4. Conclusions

In this chapter an application of the proposed methodology to assess the vulnerability of masonry structures subjected to seismic, flood and wind loading to a real case study is provided.

In consideration of the hazards registered at the site of investigation the PH_Sample is susceptible to 80% of failure in the case of flood and wind, and 50% probability of failure in the case of earthquake. When breaking down these percentages into subgroups of walls and comparing these with more general fragility curves, the previous proportions readjust – in line with the fact that a more accurate knowledge of the sample is in place. Note that these high proportions of failure are due to the assumptions made on the poor flexural strength capacity of the sample, the very small staggering ratio surveyed (which rules the angle of crack) and the effective thickness of the wall considered as participating in the determination of the wall moment capacity.

Overall, from the determination of the building sample to the derivation of single-hazard fragility curves, the sections of this chapter aimed at providing the clear rationale followed to conduct fragility assessment of any case study deemed relevant in the context of multi-hazard vulnerability of URMs structures, thus proving the feasibility and the ease of application/tailoring of the analytical procedure proposed and detailed in this thesis. However, the proposed procedure shows to be greatly affected by the input data used, hence reiterates the importance of the knowledge of the sample.

The following and final chapter of the thesis focuses on providing the general conclusions and proposed future works.
9. Conclusions and Future Works

9.1. Summary of Results and Conclusions

The concept of multi-hazard approach made its first appearance in the Agenda 21 Conference in Rio de Janeiro (UNEP, 1992) and then in the Johannesburg Plan (UN, 2002). From there, the initiatives focused on analysing the multiple risks arising from different hazards and affecting many other exposed elements have been progressively increasing (Gallina, 2016)), the most notable examples being the HAZUS 99 (FEMA, 1999) in the United States, based on a standardized methodology and software program for estimating potential losses from earthquake, flood and wind, and the MATRIX project (Multi-HAazard and MulTi-RIsK Assessment Methods) in Europe, set out to tackle some of the issues associated with multi-hazard and multi-risk and focusing on earthquakes, landslides, volcanos, tsunamis, wild fires, storms and fluvial and coastal flooding (Carpignano et al., 2009).

However, none of the methods cited and currently available endeavour to providing information on the intrinsic vulnerability of URMs to multiple sources of natural hazards in relation to their constitution and structural quality, and this is due to two main challenges:

- The substantial level of discrepancy in the advancement of single-hazard vulnerability assessment procedures, with emphasis on the analytical model used to correlate the intensity measure to the level of damage caused to building.
- The conceptual differences in the definition of single-hazard fragility curves to be used to conduct multi-hazard damage assessment in a commensurate manner.

Therefore, an analytical procedure has been proposed here for the derivation of single-hazard fragility functions, which present the twofold merit of (1) relating the behaviour of masonry structures subjected to seismic, flood and wind hazard, to the corresponding levels of damage, and (2) defining a common structural parameter for the derivation of single-hazard fragility curves which also allow for damage comparisons between these distinct perils.

The proposed framework carries out the assessment at a wall level and accounts for the hazard and the exposure components taken as inputs.

The kinematic model is based on revised Yield Line theory concepts and includes the contribution of torsional effects generated at unit (brick/block) level, due to the application of out-of-plane loadings.

The assessment is conducted at wall level and – when the wall is constituted by different bays – a preliminary assessment of the number of individual bays is required. For each bay, a plausible crack pattern is identified, which is compatible to geometry and boundary conditions.
Given that several configurations of admissible crack patterns can be identified for the same wall layout, an optimization routine is built to find, by means of limit state analysis, the minimum load required to produce failure corresponding to a specific crack pattern and to the maximum value of performance variable, defined as the ratio between the demand imposed by the loading and the capacity of the system itself.

Each performance variable, representative of the strength capacity of the system, is linked to a value of IM causing collapse.

By performing regression of all values of IMs causing collapse of all the walls constituting the sample assessed, single-hazard fragility curves are extracted, by considering the variability of the asset, and hence focusing on the aleatory aspect of the exposure component, rather than considering the uncertainties associated with each hazard intensity measure. The variance considered includes geometric, materials, presence of opening and boundary conditions.

Comparisons on resulting fragility functions are drawn across the three hazards considered to establish relevance of the above parameters and sensitivity of the fragility functions.

To prove the ease of implementation, the proposed procedure has been applied to a real case study in the Philippines, one of the few examples of multi-hazard prone country where the author had the chance to collect field data. By applying the proposed procedure, multi-hazard fragility curves were derived. Thanks to prior studies on the hazard and exposure focusing on the same study area, deterministic values of seismic, flood and wind speed IMs were available and an assessment of probabilities of collapse was conducted, thus highlighting which of the three considered perils is more impactful and more damaging.

9.2. Impact

A robust methodological framework for the multi-hazard vulnerability assessment of URM structures, with the emphasis on the derivation of single-hazard fragility curves has been developed, thus proving that homogenize results can be obtained. While the present work has focused on URMs potentially exposed to earthquake, flood and wind loading, the proposed procedure is generic enough to be applied to other perils such as mudflows or ash flows and – more in general – any type of peril whose loading profile is known enough to be simplified as a knife-edge load with specific point of application.

The steps already conducted towards fragility analysis have led to the compilation of a table of crack patterns configurations for the three perils, which may represent one of the few examples in literature in the case of wind and one of the most complete in the context of flood assessment. This however also reveals the need of further efforts to expand the proposed table, incorporating other wall layout configurations, especially in the case of presence of openings.
The main evidence that can be observed from the case study relates to the flexibility, customizability and ease of implementation of proposed analytical procedure, which requires very few – but key – parameters both in terms of geometry, material properties, and hazard parameters to be able to provide with a ranking and an understanding of the most damaging peril among the ones considered in a very timely manner, without requiring complicated or time consuming models to be built nor detailed analysis to be processed. Based on these evidences, it is believed that the results and the proposed method have potential to become of particular interest not just for research purposes, but also in the context of rapid assessment for decision makers and stakeholders.

9.3. Recommendations for future works

While the present work has been mainly devoted to detail the methodological framework to conduct quantitative damage assessment of URMs subjected to seismic, flood and wind hazard, the lack of other studies aiming towards the same direction has represented both the main incentive to conduct this study and the main difficulty in validating it. Although the three validations proposed in this study have positively confirmed that the proposed analytical procedure provides results which are in close proximity with the ones obtained by these authors/codes/tools, it is still recognized that further comparison with experimental tests, especially in the case of flood and wind hazard - the two fields for which hazard assessment methods focusing on URMs is far less mature than seismic hazard - would greatly enhance the confidence in the results obtained for these perils.

With reference to the developed procedure, since the aim was to verify that each assumption made was univocally applied across the three hazards, some simplifications where adopted.

Fundamental to the definition of the equivalent single degree of freedom (SDOF) system is the definition of the period of the structure. The current model adopts the simplified equation reported in EC8 and included in Eq. 5.1, (Eq. 4.6 in EC8) as the focus of the procedure was to determine a strength-related performance variable. In aiming at identifying also a ductility-focused performance parameter which is indicative of the capacity of the system, a more refined computation of the effective stiffness of the wall, which involves the estimated elastic modulus of the structure, the second moment of inertia and the cross-sectional area, as well as the variation in the thickness along the height of the wall would help refining the accuracy of the results obtained.

With reference to the choice of the most appropriate EDP to relate the extent of damage experienced by the structure to the corresponding IM of interest, the current model, which is based on the kinematic approach of limit analysis, has adopted an EDP in terms of strength capacity, as it is easier to define across the three perils considered. However, such choice represents a limitation in the understanding of
the ductility of the system investigated, for which there could be scope for further investigation, especially if laboratory testing becomes available.

To the aim of deriving fragility functions which would be able to compare damage across the three horizontal actions coming from the hazards considered, only one damage state was used, namely collapse, as it is a clear state to individuate across the three perils and is defined as the loss of loadbearing capacity of the URM system under the OOP loading. As per the validation discussed previously, the availability of laboratory tests could also help defining intermediate damage states forming prior to collapse, helping individuating thresholds of strength performance and also displacement which could be associated to IMs level to derive more comprehensive sets of fragility curves, whilst guarantying to be comparable across perils.

Finally, in relation to the derivation of fragility curves, a discussion on the choice of the Sum of Square Error (SSE) method was provided both within the literature review chapter (i.e., Section 2.4.4) and also reiterated in Section 7.2. Although not considered in this research study, all different sources of epistemic uncertainties related to the hazard analysis for the conditioning IM, the uncertainties in the modelling assumptions and the uncertainties around the input parameters used to build the numerical model of the system, which are due to lack of – again – experimental tests - are all significantly important elements widely recognized to affect the epistemic uncertainty in single and multi-hazard performance assessment (Bradley, B. A. (2010), Apel, H., Merz, B., & Thieken, A. H. (2008) Li, Y., & Ellingwood, B. R. (2006)). This reflects onto the results obtained in terms of fragility curves, independently from the fitting technique adopted. Since the primary scope of this thesis was to prove that the analytical method proposed provides with meaningful results which can be used to build fragility curves aiming at comparing damage for deterministic hazard scenarios (as shown with the case study), the choice of the regression technique to derive fragility curves was deemed sufficiently appropriate.

However, it is acknowledged that the natural progression of this work is to account for the many sources of epistemic uncertainties listed above, which could lead to choose a different – and more appropriate – fitting technique and hence resulting in different – and more accurate – set of resulting fragility curves.

Among the future works already planned and with reference to the methodological flowchart presented in Figure 4-1, the author is planning to investigate the case of co-occurring perils (e.g. flood and wind which co-occur in the case of a hurricane), which requires the adaptation of the developed procedure to account for the presence of multiple OOP loadings acting co-concurrently – which affect the demand on the structure, nor its capacity - and a crack pattern shape which accounts for multiple KELs positioned at different wall heights. The shape of the crack pattern requires an extra step of ‘crack pattern-finding optimization’ as a result of the KEL pair, thus resulting in a new $\lambda$ factor which – due to the increase in demand $Q$ - is smaller than the $\lambda$ factor found when each hazard is applied individually. A new
performance variable $\chi$ will be defined, representative of the ‘multi-hazard’ Q/R ratio, which effectively mimics the multi-hazard scenario and the reduced capacity of URM samples subjected to co-occurring perils.

Fragility curves will then become fragility surfaces, representative of the conjunct probability of collapse caused by co-occurring ‘pair’ of events. Examples of fragility surface have been found for masonry structures subjected to permanent ground displacements and earthquakes (Negulescu et al., 2014), whilst none is currently available for the triplet of perils investigated in this research study, thus highlighting already the gap that this future work will cover.
Appendix 1: Work Equations

Group A – Configuration A1

Earthquake case IW1-IW2

\[
\text{Condition } = y < \frac{2}{3}H; \\
W_{\text{IE, IW1-2}} = m_v L \frac{1}{y} + 2m_h H \frac{2}{L} \\
m_h = m_v/\mu \\
W_{\text{IE, IW1-2}} = m_v (L\mu + 2Hy) \\
W_{\text{EE, IW1-2}} = \frac{\lambda q L}{2} \times \frac{L}{2} \times \frac{2}{L} \\
W_{\text{EE, IW1-2}} = \frac{W_{\text{EE, IW1-2}}}{(L\mu + 2Hy)} \\
m_v \frac{LH}{\mu y} = \frac{2}{L} \\
\lambda_{\text{IE, IW1-2}} = \frac{2m_v (L^2\mu + 4yH)}{q L^2 y\mu} \\
\text{Condition } = y > \frac{2}{3}H; \\
W_{\text{IE, IW1-2}} = m_v \left(\frac{L\mu + 2Hy}{y}\right) \\
W_{\text{EE, IW1-2}} = 2m_v \left(\frac{L^2\mu + 4yH}{L^2 y\mu}\right) \\
\lambda_{\text{IE, IW1-2}} = \frac{2m_v (L^2\mu + 4yH)}{q L^2 y\mu} \\
d\lambda/dy \rightarrow 0
\]

- assign values to \(y\) comprised in a range
- determine \(\min \lambda_{\text{IE, IW1-2}}\)

Flood case IW1-IW2 (\(\beta = 1 \rightarrow \beta H = H\))

\[
\text{Condition } = y < \frac{1}{3}H; \\
W_{\text{IF, IW1-2}} = m_v L \frac{1}{y} + 2m_h H \frac{2}{L} \\
m_h = m_v/\mu \\
W_{\text{IF, IW1-2}} = m_v \left(\frac{L\mu + 2Hy}{y}\right) \\
W_{\text{EF, IW1-2}} = \frac{\lambda q L}{2} \times \frac{L}{2} \times \frac{2}{L} \\
W_{\text{EF, IW1-2}} = \frac{W_{\text{EF, IW1-2}}}{(L\mu + 2Hy)} \\
m_v \frac{LH}{\mu y} = \frac{2}{L} \\
\lambda_{\text{IF, IW1-2}} = \frac{2m_v (L^2\mu + 4yH)}{q L^2 y\mu} \\
\text{Condition } = y > \frac{1}{3}H; K = \beta H/3 \\
W_{\text{IF, IW1-2}} = m_v \left(\frac{L\mu + 2Hy}{y}\right) \\
\]
Wind case IW1-IW2

\[ W_{ew,IW1-2} \rightarrow A: K = L/2; y; \qquad A = \frac{KL}{2y}; B = L - 2A \]
\[ W_{ew,IW1-2} = 2\lambda q \left( \frac{KL}{2y} \right) \left( \frac{2}{4y} \right) \frac{K}{L} \text{I}\left( L - 2 \frac{KL}{2y} \right) K = \frac{1}{y} \]
\[ W_\text{I} \text{W1-2} = \lambda q \left( \frac{2LKy - K^2L}{2y} \right) \]
\[ W_{iw,IW1-2} = W_{ew,IW1-2} \]
\[ m_v y = \lambda q \left( \frac{L_\mu + 2Hy}{y} \right) \]
\[ \lambda W_{IW1-2} = \frac{2m_v (L_\muY + 4y^2H)}{q} \]
\[ d\lambda/dy \to 0 \]
- assign values to y values comprised in a range
- solve 1st order differential equation; find value of x, determine \( \min \lambda_{W,IW1-2} \)

Condition = \( y < \frac{1}{2}H; \)
\[ W_{iw,IW1-2} = m_v L \frac{L}{y} + 2m_b H \frac{2}{L} \]
\[ m_b = m_v / \mu \]
\[ W_{iw,IW1-2} = m_v \left( \frac{L_\mu + 2Hy}{y} \right) \]
\[ \lambda W_{IW1-2} = \frac{2m_v (L_\muY + 4y^2H)}{q} \frac{L^2 y}{L^2 y} \]
\[ \text{Condition} = y > \frac{1}{2} \]
\[ W_{iw,IW1-2} = m_v \left( \frac{L_\mu + 2Hy}{y} \right) \]
\[ W_{ew,IW1-2} \rightarrow A: K = L/2; y; \qquad A = \frac{KL}{2y}; B = L - 2A \]
Configuration A2
Earthquake case IW1-IW2

\[
\text{Condition } y < \frac{2}{3} H; \\
W_{ie,IW1-IW2} = 2 m_v L \frac{1}{y} + 2 m_h L \frac{2}{L} \\
m_h = m_v / \mu \\
W_{ie,IW1-IW2} = m_v \left( 2L \mu + 2Hy \right) y \mu \\
W_{ie,IW1-IW2} = \lambda q L \frac{L}{2} \times \frac{2}{L} \\
W_{ie,IW1-IW2} = W_{ie,IW1-IW2} - W_{ie,IW1-2} \\
m_v \left( 2L \mu + 2Hy \right) = \lambda q L \frac{L}{2} y \mu \\
\lambda_{e,IW1-IW2} = 2 m_v \left( 2L \mu + 4yH \right) \frac{y \mu}{q \ L^2 y \mu} \\
\text{Condition } y > \frac{2}{3} H; \\
W_{ie,IW1-IW2} = \frac{2}{L} \left( 2L \mu + 2Hy \right) y \mu \\
W_{ie,IW1-IW2} = A: K = L/2; y; A = \frac{KL}{2y}, B = L - 2A \\
W_{ie,IW1-IW2} = 2 \lambda q \left( \frac{KL}{2y} \right) L + \lambda q \left( L - 2 \frac{KL}{2y} \right) \frac{1}{y} \\
W_{ie,IW1-IW2} = \lambda q \left( 2LK - K^2L \right) \frac{2y}{2} \\
W_{ie,IW1-IW2} = W_{ie,IW1-IW2} - W_{ie,IW1-2} \\
m_v \left( 2L \mu + 2Hy \right) = \lambda q \frac{2y}{2} \left( 2LK - K^2L \right) \frac{2y}{2} \\
\lambda_{e,IW1-IW2} = 2 m_v \left( 2L \mu Y + 4yH^2 \right) \frac{y \mu}{q \ L^2 K \mu - K^2L^2 \mu} \\
d \lambda / dy \rightarrow 0 \\
\bullet \text{ assign values to } y \text{ values comprised in a range} \\
\bullet \text{ determine min } \lambda_{e,IW1-IW2}
\]

Flood case IW1-IW2 (\( \beta = 1 \rightarrow \beta H = H \))

\[
\text{Condition } y < \frac{1}{3} H; \\
W_{ef,IW1-IW2} = 2 m_v L \frac{1}{y} + 2 m_h L \frac{2}{L} \\
m_h = m_v / \mu \\
W_{ef,IW1-IW2} = m_v \left( 2L \mu + 2Hy \right) y \mu \\
W_{ef,IW1-IW2} = \lambda q L \frac{L}{2} \times \frac{2}{L} \\
W_{ef,IW1-IW2} = W_{ef,IW1-IW2} - W_{ef,IW1-2} \\
m_v \left( 2L \mu + 2Hy \right) = \lambda q L \frac{L}{2} y \mu \\
\lambda_{e,IW1-IW2} = 2 m_v \left( 2L \mu + 4yH \right) \frac{y \mu}{q \ L^2 y \mu} \\
\text{Condition } y > \frac{1}{3} H; K=\beta H/3 \\
W_{ef,IW1-IW2} = \frac{2}{L} \left( 2L \mu + 2Hy \right) y \mu \\
W_{ef,IW1-IW2} = A: K = L/2; y; A = \frac{KL}{2y}, B = L - 2A \\
W_{ef,IW1-IW2} = 2 \lambda q \left( \frac{KL}{2y} \right) L + \lambda q \left( L - 2 \frac{KL}{2y} \right) \frac{1}{y} 
\]
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**Configuration A3**

---

**Earthquake case IW1-IW2**

\[
W_{eIw1-2} = \frac{\lambda q (2LKy - K^2L)}{2y}
\]

\[
W_{fIw1-2} = W_{eIw1-2}
\]

\[
m_v = \frac{(2L\mu + 2Hy)}{y\mu} = \frac{\lambda q (2LKy - K^2L)}{2y}
\]

\[
\lambda_{fIw1-2} = q \frac{2m_v (2L^2\mu Y + 4y^2H)}{2L^2K\mu Y - K^2L^2\mu}
\]

\[
d\lambda/dy \to 0
\]

- assign values to y values comprised in a range
- determine min \( \lambda_{fIw1-2} \)

**Wind case IW1-IW2**

\[
W_{wIw1-2} = \frac{2m_vL}{y} + 2m_hH^2/L
\]

\[
m_h = m_v/\mu
\]

\[
W_{wIw1-2} = m_v \frac{(2L\mu + 2Hy)}{y\mu}
\]

\[
W_{ewIw1-2} = \frac{\lambda q L}{2} \times \frac{L^2}{2} \times \frac{L^2}{L}
\]

\[
W_{iwIw1-2} = W_{ewIw1-2}
\]

\[
m_v = \frac{(2L\mu + 2Hy)}{y\mu} = \frac{\lambda q L}{2}
\]

\[
\lambda_{wIw1-2} = q \frac{2m_v (2L^2\mu Y + 4yH)}{L^2y^2\mu}
\]

\[
\lambda_{wIw1-2} = \frac{2m_vL^2}{L} + 2m_hH^2/L
\]

\[
d\lambda/dy \to 0
\]

- assign values to y values comprised in a range
- determine min \( \lambda_{wIw1-2} \)
\[ m_v \frac{(2L\mu + 4Hy)}{y\mu} = \frac{\lambda qL}{2} \]

\[ \lambda_{E,W1-2} = 2m_v \frac{(2L\mu + 8yH)}{q L^2 y\mu} \]

**Condition** \( y > \frac{2}{3} H; \)

\[ W_{E,W1-2} = m_v \frac{(2L\mu + 4Hy)}{y\mu} \]

\[ W_{E,E,W1-2} \rightarrow A: K = L/2; y; \ (2L\mu + 8yH) = 2\lambda q \left( KL \frac{2}{2y} \right) + \frac{\lambda q}{L} L \left( \frac{2}{2y} \right) K \frac{1}{y} \]

\[ W_{E,W1-2} = \lambda q \frac{(2LKY - K^2L)}{2y} \]

\[ W_{E,W1-2} = W_{E,W1-2} \]

\[ m_v \frac{(2L\mu + 4Hy)}{y\mu} = \frac{\lambda qL}{2} \]

\[ \lambda_{E,W1-2} = 2m_v \frac{(2L\mu Y + 8y^2H)}{q 2L^2 K \mu - K^2L^2} \]

\[ d\lambda/dy \rightarrow 0 \]

- assign values to \( y \) comprised in a range
- determine \( \min \lambda_{E,W1-2} \)

**Flood case IW1-IW2 (\( \beta = 1 \rightarrow \beta H = H \))**

\[ Condition \ y < \frac{1}{3} H; \]

\[ W_{F,W1-2} = 2m_v \frac{L}{y} + 4m_b H \frac{2}{L} \]

\[ m_b = m_v/\mu \]

\[ W_{F,W1-2} = m_v \frac{(2L\mu + 4Hy)}{y\mu} \]

\[ W_{F,E,W1-2} = \frac{\lambda qL}{2} \times \frac{L}{2} \times 2 \frac{2}{L} \]

\[ W_{F,W1-2} = W_{F,W1-2} \]

\[ m_v \frac{(2L\mu + 4Hy)}{y\mu} = \frac{\lambda qL}{2} \]

\[ \lambda_{F,W1-2} = 2m_v \frac{(2L\mu Y + 8yH)}{q 2L^2 K \mu - K^2L^2} \]

**Condition** \( y > \frac{1}{3} H; K = \beta W3/ \)

\[ W_{F,W1-2} = m_v \frac{(2L\mu + 4Hy)}{y\mu} \]

\[ W_{F,E,W1-2} \rightarrow A: K = L/2; y; \ (2L\mu + 8yH) = 2\lambda q \left( KL \frac{2}{2y} \right) + \frac{\lambda q}{L} L \left( \frac{2}{2y} \right) K \frac{1}{y} \]

\[ W_{F,W1-2} = \lambda q \frac{(2LKY - K^2L)}{2y} \]

\[ W_{F,W1-2} = W_{F,E,W1-2} \]

\[ m_v \frac{(2L\mu + 4Hy)}{y\mu} = \frac{\lambda qL}{2} \]

\[ \lambda_{F,W1-2} = 2m_v \frac{(2L\mu Y + 8y^2H)}{q 2L^2 K \mu - K^2L^2} \]

\[ d\lambda/dy \rightarrow 0 \]

- assign values to \( y \) comprised in a range
- determine \( \min \lambda_{F,W1-2} \)
Wind case IW1-IW2

Condition = \( y < \frac{1}{2}H \);

\[
W_{WiW_1-2} = 2m_vL \frac{1}{y} + 4m_bH \frac{2}{H}
\]

\( m_b = m_v/\mu \)

\[
W_{WiW_1-2} = y \mu
\]

\[
W_{ew,W_1-2} = \frac{\lambda q L}{2} \times \frac{1}{2} \times \frac{2}{L}
\]

\[
W_{iw,W_1-2} = \frac{W_{ew,W_1-2}}{(2L\mu + 4Hy)}
\]

\[
m_v = y \mu
\]

\[
W_{iw,W_1-2} = \frac{2m_v(2L^2\mu + 8yH)}{q}
\]

\[
W_{iw,W_1-2} \rightarrow A: K = L/2; y; A = \frac{KL}{2y}; B = L - 2A
\]

\[
W_{ew,W_1-2} = 2\lambda q \frac{(KLKL)}{2y} \times 4y \frac{2}{L} + \lambda q \left( L - 2 \frac{KL}{2y} \right)^{-1}
\]

\[
W_{iw,W_1-2} = \frac{W_{ew,W_1-2}}{(2L\mu + 4Hy)}
\]

\[
m_v = \frac{\lambda q (2LKy - K^2L)}{2y}
\]

\[
\lambda_{w,W_1-2} = \frac{2m_v(2L^2\mu + 8y^2H)}{q} \times L^2Ky - K^2L^2\mu
\]

\[
d\lambda/dy \rightarrow 0
\]

- assign values to \( y \) comprised in a range
- determine \( \min \lambda_{w,W_1-2} \)

Group B – Configuration B1

Earthquake case IW1-IW2

Condition = \( y = \frac{1}{3}H; y = \frac{1}{2}H; y = \frac{2}{3}H \);

\[
W_{ie,IW_1-2} = m_vH \frac{1}{y} + m_bH \frac{1}{x} + m_vL \frac{1}{H-y}
\]

\( m_b = m_v/\mu \)

\[
W_{ie,IW_1-2} = \frac{m_v}{xyu(H-y)}
\]

\[
W_{ee,IW_1-2} \rightarrow A: (H-K) = x; (H-y); A = \frac{x}{H-y}; B = L - A
\]

\[
W_{ee,IW_1-2} = \lambda q \left( \frac{(H-K)x(H-K)x}{(H-y)} \frac{1}{x} + \lambda q \left( \frac{L - (H-K)x}{(H-y)} \right) \frac{H-K}{(H-y)} \right)
\]

\[
W_{ee,IW_1-2} = \lambda q \frac{2L(H-y)(H-K) - x(H-K)^2}{2(H-y)^2}
\]

\[
W_{ie,IW_1-2} = W_{ie,CP2-1}
\]

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Wind case IW1-IW2 ($\beta = 1 \to \beta H = H$)

Flood case IW1-IW2 ($\beta = 1 \to \beta H = H$)
Configuration B2
Earthquake case IW1-IW2

Condition = \( y = \frac{1}{3} H; y = \frac{1}{2} H; y = \frac{2}{3} H; \)

\[ W_{ie,IW1-2} = m_v L \frac{1}{y} + 2 m_b H \frac{1}{x} + m_v L \frac{1}{H - y} \]

\[ m_b = m_v / \mu \]

\[ W_{ie,IW1-2} = m_v \frac{L (H - y) x \mu + 2 H y (H - y) + L x y \mu}{x y \mu (H - y)} \]

\[ W_{ee,IW1-2} = \lambda q \left( \frac{(H - K)x (H - K)x}{(H - y)^2} \right) + \lambda q \left( \frac{L - (H - K)x}{(H - y)} \right) (H - K) \]

\[ W_{ee,IW1-2} = \lambda q \frac{2 L (H - y)(H - K) - x(H - K)^2}{2(H - y)^2} \]

\[ \lambda_{e,IW1-2} = \frac{2 m_v (L x \mu(H - y)^2 + 2 y H (H - y)^2 + L x y \mu(H - y))}{2 L x y \mu(H - K)(H - y) - y x^2 \mu(H - K)^2} \]

\[ d \lambda / dx \rightarrow 0 \]

- solve 1\textsuperscript{st} order differential equation; find value of \( x \), determine \( \min \lambda_{e,IW1-2} \)

Flood case IW1-IW2 (\( \beta = 1 \rightarrow \beta H = H \))

Condition = \( y = \frac{1}{3} H; y = \frac{1}{2} H; y = \frac{2}{3} H; \)

\[ W_{IF,IW1-2} = m_v L \frac{1}{y} + 2 m_b H \frac{1}{x} + m_v L \frac{1}{H - y} \]

\[ m_b = m_v / \mu \]

\[ W_{IF,IW1-2} = m_v \frac{L (H - y) x \mu + 2 H y (H - y) + L x y \mu}{x y \mu (H - y)} \]

\[ W_{IF,IW1-2} = W_{ee,IW1-2} \]

\[ W_{IF,IW1-2} = \lambda q \frac{2 L (H - y)(H - K) - x(H - K)^2}{2(H - y)^2} \]

\[ W_{IF,IW1-2} = W_{ef,IW1-2} \]

\[ \lambda_{f,IW1-2} = \frac{2 m_v (L x \mu(H - y)^2 + 2 y H (H - y)^2 + L x y \mu(H - y))}{2 L x y \mu(H - K)(H - y) - y x^2 \mu(H - K)^2} \]

\[ d \lambda / dx \rightarrow 0 \]

- assign values to \( y \) values comprised in a range
- solve 1\textsuperscript{st} order differential equation; find value of \( x \), determine \( \min \lambda_{f,IW1-2} \)

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Wind case IW1-IW2

\[ \text{Condition} = y = \frac{1}{3} H; y = \frac{1}{2} H; y = \frac{2}{3} H; \]

\[ W_{w, IW1-2} = m_{v} \frac{1}{y} + 2m_{b}H \frac{1}{x} + m_{h}L \frac{1}{H - y} \]

\[ m_{h} = m_{v}/\mu \]

\[ W_{w, IW1-2} = m_{v} \frac{(L(H - y)x\mu + 2Hy(H - y) + Lxy\mu)}{xy\mu(H - y)} \]

\[ W_{w, IW1-2} = W_{w, IW1-2} \]

\[ W_{w, IW1-2} = \frac{(2L(H - y)(H - K) - x(H - K)^2)}{2(H - y)^2} \]

\[ W_{w, IW1-2} \rightarrow A: (H - K) = x; (H - y); A = x \frac{(H - K)}{H - y}; B = L - A \]

\[ W_{e, IW1-2} = \frac{\lambda q}{2Lx\mu(H - K)(H - y) - xy^2\mu(H - K)^2} \]

\[ \frac{\partial \lambda}{\partial x} \rightarrow 0 \]

- assign values to y values comprised in a range
- solve 1st order differential equation; find value of x, determine \( \lambda_{W, IW1-2} \)

Configuration B3
Earthquake case IW1-IW2

\[ \text{Condition} = y = \frac{1}{3} H; y = \frac{1}{2} H; y = \frac{2}{3} H; \]

\[ W_{e, IW1-2} = 2m_{v} \frac{1}{y} + 2m_{b}H \frac{1}{x} + 2m_{v}L \frac{1}{H - y} \]

\[ m_{h} = m_{v}/\mu \]

\[ W_{e, IW1-2} = m_{v} \frac{(2L(H - y)x\mu + 2Hy(H - y) + 2Lxy\mu)}{xy\mu(H - y)} \]

\[ W_{e, IW1-2} \rightarrow A: (H - K) = x; (H - y); A = x \frac{(H - K)}{H - y}; B = L - A \]

\[ W_{e, IW1-2} = \frac{\lambda q}{2Lx\mu(H - K)(H - y) - xy^2\mu(H - K)^2} \]

\[ W_{e, IW1-2} = \lambda q \frac{(2L(H - y)(H - K) - x(H - K)^2)}{2(H - y)^2} \]

\[ W_{e, IW1-2} = W_{e, IW1-2} \]

\[ m_{v} \frac{(2L(H - y)x\mu + 2Hy(H - y) + 2Lxy\mu)}{xy\mu(H - y)} = \lambda q \frac{(2L(H - y)(H - K) - x(H - K)^2)}{2(H - y)^2} \]
Flood case IW1-IW2 ($\beta = 1 \rightarrow \beta H = H$)

Condition: $y = \frac{1}{3} H; \ y = \frac{1}{2} H; \ y = \frac{2}{3} H$

$W_{iw, IW1-2} = 2m_v L \frac{1}{y} + 2m_h H \frac{1}{x} + 2m_v L \frac{1}{H - y}$

$m_h = m_v/\mu$

$W_{iw, IW1-2} = m_v \frac{(2L(H - y) x \mu + 2Hy(H - y) + 2Lxy \mu)}{xy \mu (H - y)}$

$W_{eW, IW1-2} = W_{eE, IW1-2}$

$\lambda_{IW, IW1-2} = \frac{2m_v (2Lxy \mu(H - y)^2 + 2yH(H - y)^2 + 2Lxy \mu(H - y))}{q (2Lxy \mu(H - K)(H - y) - xy^2 \mu(H - K)^2)}$

$d\lambda/dx \to 0$

- assign values to $y$ values comprised in a range
- solve 1st order differential equation; find value of $x$, determine $\min \lambda_{IW, IW1-2}$

Wind case IW1-IW2

Condition: $y = \frac{1}{3} H; \ y = \frac{1}{2} H; \ y = \frac{2}{3} H$

$W_{iw, IW1-2} = 2m_v L \frac{1}{y} + 2m_h H \frac{1}{x} + 2m_v L \frac{1}{H - y}$

$m_h = m_v/\mu$

$W_{iw, IW1-2} = m_v \frac{(2L(H - y) x \mu + 2Hy(H - y) + 2Lxy \mu)}{xy \mu (H - y)}$

$W_{eW, IW1-2} = W_{eE, IW1-2}$

$\lambda_{IW, IW1-2} = \frac{2m_v (2Lxy \mu(H - y)^2 + 2yH(H - y)^2 + 2Lxy \mu(H - y))}{q (2Lxy \mu(H - K)(H - y) - xy^2 \mu(H - K)^2)}$

$d\lambda/dx \to 0$

- assign values to $y$ values comprised in a range
- solve 1st order differential equation; find value of $x$, determine $\min \lambda_{IW, IW1-2}$
Configuration B4
Earthquake case IW1-IW2

Condition = \( y = \frac{1}{3}H; y = \frac{1}{2}H; y = \frac{2}{3}H; \)

\[
W_{IE,IW1-2} = 2m_v L \frac{1}{y} + m_h H \frac{1}{x} + m_v L \frac{1}{H - y}
\]

\( m_h = \frac{m_v}{\mu} \)

\[
W_{IE,IW1-2} = m_v \frac{(2L(H - y)xu + Hy(H - y) + Lxy\mu)}{xy\mu(H - y)}
\]

\( W_{EE,IW1-2} \rightarrow A: (H - K) = x: (H - y); \ A = x \frac{(H - K)}{H - y}; \ B = L - A \)

\[
W_{ee,IW1-2} = \lambda q \left( \frac{(H - K)x(H - K)x}{(H - y)} \right) + \lambda q \left( \frac{L - (H - K)x}{(H - y)} \right) (H - K)
\]

\[
W_{ee,IW1-2} = \lambda q \frac{(2L(H - y)(H - K) - x(H - K)^2)}{2(H - y)^2}
\]

\[
\frac{d\lambda}{dx} \rightarrow 0
\]

- assign values to \( y \) values comprised in a range
- solve 1st order differential equation; find value of \( x \), determine \( \min \lambda_{E,IW1-2} \)

Flood case IW1-IW2 (\( \beta = 1 \rightarrow \beta H = H \))

Condition = \( y = \frac{1}{3}H; y = \frac{1}{2}H; y = \frac{2}{3}H; \)

\[
W_{IF,IW1-2} = 2m_v L \frac{1}{y} + m_h H \frac{1}{x} + m_v L \frac{1}{H - y}
\]

\( m_h = \frac{m_v}{\mu} \)

\[
W_{IF,IW1-2} = m_v \frac{(2L(H - y)xu + Hy(H - y) + Lxy\mu)}{xy\mu(H - y)}
\]

\( W_{EF,IW1-2} = W_{IE,IW1-2} \)

\[
W_{ef,IW1-2} = \lambda q \left( \frac{(2L(H - y)(H - K) - x(H - K)^2)}{2(H - y)^2} \right)
\]

\( \lambda_{IJW1-2} = \frac{(2Lxu(H - y)^2 + y(H - y)^2 + Lxy\mu(H - y))}{q \ (2Lxy\mu(H - K)(H - y) - yx^2\mu(H - K)^2)} \)

\[
\frac{d\lambda}{dx} \rightarrow 0
\]

- assign values to \( y \) values comprised in a range

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• solve 1st order differential equation; find value of x, determine 
min \( \lambda_{F, IW1-2} \)

Wind case IW1-IW2

\[
\begin{align*}
\text{Condition} & = y = \frac{1}{3}H; y = \frac{1}{2}H; y = \frac{2}{3}H; \\
W_{iw,IW1-2} & = 2m_vL \frac{1}{y} + m_hH \frac{1}{x} + m_vL \frac{1}{H - y} \\
m_h & = m_v/\mu \\
W_{iw,IW1-2} & = m_v \frac{(2L(H - y)x\mu + Hy(H - y) + Lxy\mu)}{xy\mu(H - y)} \\
W_{ew,IW1-2} & = W_{ew,IW1-2} \\
W_{ew,IW1-2} & = \frac{\lambda q}{2(H - y)^2} \left( 2L(H - y)(H - K) - x(H - K)^2 \right) \\
\lambda_{w,IW1-2} & = 2m_v \frac{(2Lx\mu(H - y)^2 + yH(H - y)^2 + Lxy\mu(H - y))}{q} \\
\frac{d\lambda}{dx} & \rightarrow 0
\end{align*}
\]

Configuration B5

Earthquake case IW1-IW2

\[
\begin{align*}
\text{Condition} & = y = \frac{1}{3}H; y = \frac{1}{2}H; y = \frac{2}{3}H; \\
W_{ie,IW1-2} & = 2m_vL \frac{1}{y} + 2m_hH \frac{1}{x} + m_vL \frac{1}{H - y} \\
m_h & = m_v/\mu \\
W_{ie,IW1-2} & = m_v \frac{(2L(H - y)x\mu + 2Hy(H - y) + Lxy\mu)}{xy\mu(H - y)} \\
W_{ee,IW1-2} & = W_{ee,IW1-2} \\
W_{ee,IW1-2} & = \frac{\lambda q}{2(H - y)^2} \left( 2L(H - y)(H - K) - x(H - K)^2 \right) \\
W_{ee,IW1-2} & = \frac{\lambda q}{2(H - y)^2} \left( 2L(H - y)(H - K) - x(H - K)^2 \right) \\
\lambda_{e,IW1-2} & = 2m_v \frac{(2Lx\mu(H - y)^2 + 2yH(H - y)^2 + Lxy\mu(H - y))}{q} \\
\frac{d\lambda}{dx} & \rightarrow 0
\end{align*}
\]
• assign values to y values comprised in a range
• solve 1st order differential equation; find value of x, determine $\min \lambda_{E,IW1-2}$

**Flood case IW1-IW2 ($\beta = 1 \rightarrow \beta H = H$)**

Condition = $y = \frac{1}{3} H; y = \frac{1}{2} H; y = \frac{2}{3} H$;

$W_{IW1-2} = 2m_v L \frac{1}{y} + 2m_h H \frac{1}{x} + m_v L \frac{1}{H - y}$

$m_h = m_v/\mu$

$W_{IW1-2} = m_v \frac{(2L(H - y)x\mu + H y(H - y) + L x y \mu)}{xy \mu(H - y)}$

$W_{eW1-2} = W_{eW,IW1-2}$

$W_{eW1-2} = \lambda q \frac{(2L(H - y)(H - K) - x(H - K)^2)}{2(H - y)^2}$

$$d\lambda/dx \to 0$$

• assign values to y values comprised in a range
• solve 1st order differential equation; find value of x, determine $\min \lambda_{F,IW1-2}$

**Wind case IW1-IW2**

Condition = $y = \frac{1}{3} H; y = \frac{1}{2} H; y = \frac{2}{3} H$;

$W_{IW1-2} = 2m_v L \frac{1}{y} + 2m_h H \frac{1}{x} + m_v L \frac{1}{H - y}$

$m_h = m_v/\mu$

$W_{IW1-2} = m_v \frac{(2L(H - y)x\mu + 2H y(H - y)^2 + L x y \mu(H - y))}{q (2L x y \mu(H - K)(H - y) - y^2 x \mu(H - K)^2)}$

$W_{eW1-2} = W_{eW,IW1-2}$

$W_{eW1-2} = \lambda q \frac{(2L(H - y)(H - K) - x(H - K)^2)}{2(H - y)^2}$

$$d\lambda/dx \to 0$$

• assign values to y values comprised in a range
• solve 1st order differential equation; find value of x, determine $\min \lambda_{W,IW1-2}$
Configuration B6
Earthquake case IW1-IW2

\[ \text{Condition} = y = \frac{1}{3} H; y = \frac{1}{2} H; y = \frac{2}{3} H; \]
\[ W_{\text{IE.IW1-2}} = 2m_v \frac{1}{y} + m_n \frac{1}{x} + 2m_v \frac{1}{H - y} \]
\[ m_n = m_v / \mu \]
\[ W_{\text{IE.IW1-2}} = m_v \frac{(2L(H - y)xu + Hy(H - y) + 2Lxyu)}{xyu(H - y)} \]
\[ W_{e_{\text{IE.IW1-2}}} = \lambda q \frac{(H - K)x(H - K)x}{(H - y)^2} + \lambda q \frac{L}{(H - y)^2} \]
\[ W_{e_{\text{IE.IW1-2}}} = \lambda q \frac{(2L(H - y)(H - K) - x(H - K)^2)}{2(H - y)^2} \]
\[ \text{d} \lambda / \text{d}x \rightarrow 0 \]

- assign values to y values comprised in a range
- solve 1st order differential equation; find value of x, determine \( \min \lambda_{\text{IE.IW1-2}} \)

Flood case IW1-IW2 (\( \beta = 1 \rightarrow \beta H = H \))

\[ \text{Condition} = y = \frac{1}{3} H; y = \frac{1}{2} H; y = \frac{2}{3} H; \]
\[ W_{f_{\text{IW1-2}}} = 2m_v \frac{1}{y} + m_n \frac{1}{x} + 2m_v \frac{1}{H - y} \]
\[ m_n = m_v / \mu \]
\[ W_{f_{\text{IW1-2}}} = m_v \frac{(2L(H - y)xu + Hy(H - y) + 2Lxyu)}{xyu(H - y)} \]
\[ W_{e_{\text{FW1-2}}} = W_{e_{\text{IE.IW1-2}}} \]
\[ W_{e_{\text{FW1-2}}} = \lambda q \frac{(2L(H - y)(H - K) - x(H - K)^2)}{2(H - y)^2} \]
\[ \text{d} \lambda / \text{d}x \rightarrow 0 \]

- assign values to y values comprised in a range
- solve 1st order differential equation; find value of x, determine \( \min \lambda_{f_{\text{FW1-2}}} \)
Wind case IW1-IW2

\[ \text{Condition} = y = \frac{1}{3} H; y = \frac{1}{2} H; y = \frac{2}{3} H; \]

\[ W_{\text{w.IW1-2}} = 2m_vL \cdot \frac{1}{y} + m_h \cdot \frac{1}{x} + 2m_vL \cdot \frac{1}{H - y} \]

\[ m_h = m_v/\mu \]

\[ W_{\text{e.IW1-2}} = \frac{m_v}{x} \cdot (2L(H - y)\mu + H \mu y + 2Lxy\mu) \]

\[ W_{\text{e.IW1-2}} = W_{\text{w.IW1-2}} \]

\[ \lambda_{\text{w.IW1-2}} = \frac{2m_v \cdot (2Lx\mu(H - y)^2 + yH(H - y)^2 + 2Lxy\mu(H - y))}{q \cdot 2Lx\mu(H - K)(H - y) - yx^2\mu(H - K)^2} \]

\[ d\lambda/dx \to 0 \]

- assign values to y values comprised in a range
- solve 1st order differential equation; find value of x, determine \( \min \lambda_{\text{w.IW1-2}} \)

Group C – Configuration C2

Earthquake case CP1-1

\[ \text{Conditions} y = \frac{1}{3} H; y = \frac{1}{2} H; y = \frac{2}{3} H \]

\[ W_{\text{e.CP1-1}} = \sum m \times \text{length}_{\text{crack}} \theta = m_vL \cdot \frac{1}{y} + 4m_h \cdot \frac{2}{L} + m_vL \cdot \frac{3}{H} \]

\[ m_h = m_v/\mu \]

\[ W_{\text{e.CP1-1}} = m_v(2L^2\mu H + 3L^2\mu y + 8H^2y) \]

\[ W_{\text{e.CP1-1}} = \frac{q \times \text{length}_{\text{KEEL}} \times \delta}{\gamma \mu LH} \]

\[ W_{\text{e.CP1-1}} = \frac{2m_v}{2} \times \frac{L}{2} \times \frac{2}{L} \]

\[ W_{\text{e.CP1-1}} = W_{\text{e.CP1-1}} \]

\[ m_v = \frac{(2L^2\mu H + 3L^2\mu y + 8H^2y)}{\gamma \mu LH} \]

\[ \lambda_{\text{e.CP1-1}} = \frac{2m_v}{q} \cdot \frac{(2L^2\mu H + 3L^2\mu y + 8H^2y)}{\gamma \mu L^2H} \]
Conditions $y = \frac{1}{3} H$; $y = \frac{1}{2} H$

$W_{IE,CP1-2} = \sum m \times \text{length}_{\text{crack}} \theta = m_s L \frac{1}{y} + 4 m_b H \frac{2}{L} + m_v L \frac{1}{(H - y - D_{1-2})}$

$m_b = m_v / \mu$

$W_{IE,CP1-2} = m_v (L^2 \mu (H - y - D_{1-2}) + L^2 \mu y + 8 H y (H - y - D_{1-2}))$

$W_{eE,CP1-2} = \sum (q \times \text{length}_{KEL} \times \delta)$

$A: (H - K) = L/2; (H - y); \rightarrow A = ((H - K) L) / (2(H - y))$

$W_{eE,CP1-2} = 2 k q \left( (H - K) L / (2(H - y)) \right)^2 L + \lambda q \left( L - 2 \right) \frac{(H - K) L}{2(H - y)} (H - K) \frac{1}{(H - y)}$

$\lambda_{CP1-2} = \frac{2 m_v (L^2 \mu (H - y - D_{1-2}) + L^2 \mu y + 8 H y (H - y - D_{1-2})(H - y)^2)}{2 y \mu L (H - y - D_{1-2})(H - K) - (H - K)^2}$

---

**Flood case CP1-1 ($\beta = 1 \rightarrow \beta H = H$)**

Conditions $y = 1/3 H$

$W_{IF,CP1-1} = W_{IE,CP1-1}$

$W_{eF,CP1-1} = W_{IE,CP1-1 \text{ CONDITION}} \ y = \frac{1}{3} H; y = \frac{1}{2} H$

$W_{IF,CP1-1} = W_{eF,CP1-1}$

$\lambda_{F,CP1-1} = \frac{2 m_v (L^2 \mu (H - y - D_{1-2}) + L^2 \mu y + 8 H y (H - y - D_{1-2}))}{2 y \mu L (H - y - D_{1-2})}$

$W_{IF,CP1-1} = W_{eF,CP1-1}$

$A: K = L/2; y; \rightarrow A = KL/2 y$

$W_{eF,CP1-1} = 2 k q \left( KL / 2 y \right)^2 \frac{L^2 \mu y + 8 H y (H - y - D_{1-2})}{2 y \mu L (H - y - D_{1-2})}$

$W_{IF,CP1-1} = W_{eF,CP1-1}$

$(L^2 \mu (H - y - D_{1-2}) + L^2 \mu y + 8 H y (H - y - D_{1-2})) = \lambda q (2 L K y - K^2 L)$

$m_v = \frac{\lambda q (2 L K y - K^2 L)}{2 y \mu L (H - y - D_{1-2})}$

$\lambda_{F,CP1-1} = \frac{2 m_v (L^2 \mu (H - y - D_{1-2}) + L^2 \mu y + 8 H y (H - y - D_{1-2}))}{2 y \mu L (H - y - D_{1-2})}$

---

**Flood case CP1-2 ($\beta = 1 \rightarrow \beta H = H$)**

Conditions $y = H / 3$

$W_{IF,CP1-2} = W_{eE,CP1-1} = \frac{\lambda q L}{2} \times \frac{L}{2} \times \frac{2}{L}$

$W_{IF,CP1-2} = W_{IE,CP1-2 \text{ CONDITION}} \ y = \frac{1}{3} H; y = \frac{1}{2} H$

$\lambda_{F,CP1-2} = \frac{2 m_v (L^2 \mu (H - y - D_{1-2}) + L^2 \mu y + 8 H y (H - y - D_{1-2}))}{2 y \mu L (H - y - D_{1-2})}$
Wind case CP1-1

Conditions = $y = \frac{1}{3}H$; $y = \frac{1}{2}H$

$$\lambda_{W,CP1-1} = \frac{2m_w(L^2\mu(H - y - D_{1,2}) + L^2\mu y + 8Hy(H - y - D_{1,2})}{q}$$

$$m_w = \frac{y\mu L^2(H - y - D_{1,2})}{2}$$

$$\lambda_{W,CP1} = \frac{2m_w(L^2\mu y(H - y - D_{1,2}) + L^2\mu y^2 + 8Hy^2(H - y - D_{1,2})}{q}$$

Wind case CP1-2

Conditions $y = \frac{1}{3}H$;

$$W_{WCP1-2} = m_w(L^2\mu(H - y - D_{1,2}) + L^2\mu y + 8Hy(H - y - D_{1,2}))$$

$$W_{w,WCP1-2} = 2g\left(\frac{H}{H - y}\right) + \lambda Q\left(L - 2\frac{(H - K)\mu}{2(H - y)}\right)^2$$

$$W_{WCP1-2} = W_{w,WCP1-2}$$

Condition = $y = 2/3H$; $y = 1/2H$

$$\lambda_{W,CP1} = \frac{2m_w(L^2\mu(H - y - D_{1,2}) + L^2\mu y + 8Hy(H - y - D_{1,2})}{q}$$

Earthquake case CP2-1

Condition = $y = \frac{1}{3}H$; $y = \frac{1}{2}H$

$$W_{E,CP2-1} = m_w(L(H - y)x + 4Hy(H - y) + Lyx)$$

$$m_w = \frac{m_w}{\mu}$$

$$W_{E,CP2-1} = m_w(L(H - y)x + 4Hy(H - y) + Lyx)$$

$$W_{E,CP2-1} = A: (H - K) = x: (H - y); A = x: \frac{(H - K)}{H - y}; B = L - 2A$$

$$W_{E,CP2-1} = \frac{2\lambda Q}{L - 2\frac{(H - K)\mu}{2(H - y)}}$$

$$W_{E,CP2-1} = \frac{L}{(H - y)}(H - K) - x(H - K)^2$$

$$W_{E,CP2-1} = \frac{1}{(H - y)^2}$$

$$W_{E,CP2-1} = W_{E,CP2-1}$$

$$m_w = \frac{m_w}{(H - y)x + 4Hy(H - y) + Lyx} = \frac{\lambda Q}{(H - y)^2}$$

$$\lambda_{E,CP2-1} = \frac{W_{E,CP2-1}}{(H - y)x + 4Hy(H - y) + Lyx}$$

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\[
\begin{align*}
\frac{d\lambda}{dx} &\to 0 \\
& \text{assign values to } y \text{ values comprised in a range} \\
& \text{solve 1\textsuperscript{st} order differential equation; find value of } x, \text{ determine } \min \lambda_{E.CP2-1} \\
\text{Condition } &= y = \frac{1}{3} H \\
W_{E.CP2-1} &\rightarrow A: (H - K) = x: (H - y); \ A = x \frac{(H - K)}{H - y}; \\
W_{E.CP2-1} &= 2\lambda y \left( \frac{(H - K)x (H - K)x}{2(H - y)} \right) \frac{1}{x} \\
W_{E.CP2-1} &= \lambda y \left( \frac{(H - K)x}{(H - y)^2} \right) \\
W_{E.CP2-1} &= \frac{m_y}{(L(H - y)H + 4H_H - H(H - y) + L_HH)} = \lambda y \left( \frac{(H - K)^2x}{(H - y)^2} \right) \\
\lambda_{E.CP2-1} &= \frac{m_y (L_HH(H - y)^2 + 4H_H - H(H - y)^2 + L_HH(H - y)^2)}{q} \\
\frac{d\lambda}{dx} &\to 0 \\
& \text{assign values to } y \text{ values comprised in a range} \\
& \text{solve 1\textsuperscript{st} order differential equation; find value of } x, \text{ determine } \min \lambda_{E.CP2-1} \\
\text{Earthquake case CP2-2} \\
\text{Condition } &= y = \frac{1}{3} H; \ y = \frac{1}{2} H \\
W_{E.CP2-2} &= \frac{m_y L_y + 4m_H H + m_H L + \frac{1}{x} W_{E.CP2-2}}{H - y} \\
&= \lambda y \left( \frac{L(H - y)(H - K) - x(H - K)^2}{(H - y)^2} \right) \\
\lambda_{E.CP2-2} &= \frac{m_y (L_HH(H - y)^2 + 4H_H - H(H - y)^2 + L_HH(H - y)^2)}{q} \\
\frac{d\lambda}{dx} &\to 0 \\
& \text{assign values to } y \text{ values comprised in a range} \\
& \text{solve 1\textsuperscript{st} order differential equation; find value of } x, \text{ determine } \min \lambda_{E.CP2-2} \\
\text{Condition } &= y = 2/3H \\
W_{E.CP2-2} &= \frac{W_{E.CP2-1}}{W_{E.CP2-1}} \\
W_{E.CP2-2} &= \frac{W_{E.CP2-2}}{W_{E.CP2-1}} \\
W_{E.CP2-2} &= \frac{W_{E.CP2-2}}{W_{E.CP2-1}} \\
m_y \left( \frac{L(H - y)H + 4H_H(H - y) + L_HH}{x} \right) = \lambda y \frac{L_Ky - xK^2}{y^2} \\
\lambda_{E.CP2-2} &= \frac{m_y (L_HH(H - y)^2 + 4H_H - H(H - y)^2 + L_HH(H - y)^2)}{q} \\
\frac{d\lambda}{dx} &\to 0 \\
& \text{assign values to } y \text{ values comprised in a range} \\
& \text{solve 1\textsuperscript{st} order differential equation; find value of } x, \text{ determine } \min \lambda_{E.CP2-2} \\
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Flood case CP2-1 $\beta = 1 \rightarrow \beta H = H$

\[
\lambda_{F,CP2-1} = \frac{m_v (Lx\mu(H-y)^2 + 4yH(H-y)^2 + Lxy\mu(H-y)^2)}{qyH^2} \frac{\mu}{(H-K)^2}
\]

\[d\lambda/dx \rightarrow 0\]

- assign values to $y$ comprised in a range
- solve 1st order differential equation; find value of $x$, determine $\min \lambda_{F,CP2-1}$

Flood case CP2-2 $\beta = 1 \rightarrow \beta H = H$

\[
\lambda_{F,CP2-2} = \frac{m_v (Lx\mu(H-y)^2 + 4xH(H-y)^2 + Lxy\mu(H-y))}{q} \frac{\mu}{Lxy(H-K)(H-y) - yx^2\mu(K^2(H-y))}
\]

\[d\lambda/dx \rightarrow 0\]

- assign to $y$ values comprised in a range
- solve 1st order differential equation; find value of $x$, determine $\min \lambda_{F,CP2-2}$

Wind case CP2-1

\[
\lambda_{W,CP2-1} = \frac{m_v m_v (Lx\mu(H-y)^2 + 4y^2H(H-y)^2 + Lxy\mu(H-y)^2)}{q} \frac{\mu}{Lx(H-K)(H-y) + y}\]

\[d\lambda/dx \rightarrow 0\]

- assign to $y$ values comprised in a range
• solve 1\textsuperscript{st} order differential equation; find value of $x$, determine min $\lambda_{W\cdot CP2-1}$

Condition = $y = \frac{1}{2}H$

$W_{eW\cdot CP2-1} \rightarrow A: (H - K) = x: (H - y); A = x \frac{(H - K)}{H - y};$

$W_{eW\cdot CP2-1} = 2\lambda q \left( \frac{(H - K)x(H - K)\lambda}{2(H - y)} \right) \frac{1}{x}$

$W_{eW\cdot CP2-1} = \lambda q \left( \frac{(H - K)^2 x}{(H - y)^2} \right)$

$W_{iW\cdot CP2-1} = W_{eW\cdot CP2-1}$

$m_v \left( L(H - y)x\mu + 4Hy(H - y) + Lxy\mu \right) = \lambda q \left( \frac{(H - K)^2 x}{(H - y)^2} \right)$

$\lambda_{W\cdot CP2-1} = \frac{m_v (Lx\mu(H - y)^2 + 4yH(H - y)^2 + Lxy\mu(H - y))}{q(x\mu(H - y) - yx^2\mu(H - K)^2)}$

$\frac{d\lambda}{dx} \rightarrow 0$

- assign values to $y$ comprised in a range
- solve 1\textsuperscript{st} order differential equation; find value of $x$, determine min $\lambda_{W\cdot CP2-1}$

Wind case CP2-2

Condition = $y = 1/3H; y = 1/2H$

$W_{iW\cdot CP2-2} = W_{iE\cdot CP2-1} = W_{iE\cdot CP2-2}$

$W_{eW\cdot CP2-2} \rightarrow A: (H - K) = X: (H - y); A = x \frac{(H - K)}{H - y}; B = L - 2A$

$W_{eW\cdot CP2-2} = W_{eE\cdot CP2-2} = W_{eE\cdot CP2-2}$

$W_{iW\cdot CP2-2} = W_{eW\cdot CP2-2}$

$\lambda_{W\cdot CP2-2} = \frac{m_v (Lx\mu(H - y)^2 + 4yH(H - y)^2 + Lxy\mu(H - y))}{Lx\mu(H - y) - yx^2\mu(H - K)^2}$

Condition = $y = 2/3H$

$W_{iW\cdot CP2-2} = W_{iE\cdot CP2-2} = W_{iE\cdot CP2-2}$

$m_v \left( L(H - y)x\mu + 4Hx(H - y) + Lxy\mu \right) = \lambda q \left( \frac{(H - y)(H - K) - x(H - K)^2}{(H - y)^2} \right)$

$W_{eW\cdot CP2-2} = W_{eE\cdot CP2-2} = \lambda q \left( \frac{(H - y)(H - K) - x(H - K)^2}{(H - y)^2} \right)$

$W_{eW\cdot CP2-2} = W_{iW\cdot CP2-2}$

$\lambda_{W\cdot CP2-2} = \frac{m_v (Lx\mu(H - y)^2 + 4yH(H - y)^2 + Lxy\mu(H - y))}{Lx\mu(H - y) - K^2x^2\mu(H - y)}$

$\frac{d\lambda}{dx} \rightarrow 0$

• assign values to $y$ comprised in a range
• solve 1\textsuperscript{st} order differential equation; find value of $x$, determine min $\lambda_{W\cdot CP2-2}$
Group C – Configuration C3
Earthquake case CP1-1

Conditions \( y = \frac{1}{3} H; \ y = \frac{1}{2} H; \ y = \frac{2}{3} H \)

\[
W_{ie,CP1-1} = \sum m \times \text{length}_{\text{crack}} \theta = 2m_vL \frac{1}{y} + 4m_hH \frac{2}{L} + 2m_vL \frac{3}{H}
\]

\( m_h = m_v/\mu \)

\[
W_{ie,CP1-1} = m_v \left( 2L^2\mu H + 6L^2\mu y + 8H^2y \right)
\]

\[
\frac{y\mu L}{W_{ie,CP1-1}}
\]

\[
\lambda_{e,CP1-1} = 4m_v \left( L^2\mu H + 3L^2\mu y + 4H^2y \right)
\]

\[
\frac{q}{y\mu L^2H}
\]

Earthquake case CP1-2

Conditions \( y = \frac{1}{3} H; \ y = \frac{1}{2} H \)

\[
W_{ie,CP1-2} = \sum m \times \text{length}_{\text{crack}} \theta = 2m_vL \frac{1}{y} + 4m_hH \frac{2}{L} + 2m_vL \frac{3}{H}
\]

\( m_h = m_v/\mu \)

\[
W_{ie,CP1-2} = m_v \left( 2L^2\mu (H - y - D_{1-2}) + 2L^2\mu y + 8Hy(H - y - D_{1-2}) \right)
\]

\[
\frac{y\mu L}{W_{ie,CP1-2}}
\]

\[
\lambda_{e,CP1-2} = 4m_v \left( L^2\mu (H - y - D_{1-2}) + 3L^2\mu y + 4H^2y(H - y - D_{1-2}) \right)
\]

\[
\frac{q}{y\mu L^2(H - y - D_{1-2})}
\]
Flood case CP1-1 ($\beta = 1 \rightarrow \beta H = H$)

Conditions $y = H/3$

$W_{IF,CP1-1} = W_{EE,CP1-1} = \frac{\lambda q L}{2} \times \frac{L}{2} \times \frac{2}{L}$

$W_{IF,CP1-1} = W_{IF,CP1-1 \text{ condition } y_1 = 1/3 H; y = 1/2 H}$

$\lambda_{F,CP1-1} = \frac{4m_q (L_1^2 \mu H + 3L_2^2 \mu y + 4H^2 y)}{q y \mu L^2 (H - y - D_{1-2})}$

Flood case CP1-2

Conditions $y = 1/3 H$

$W_{IF,CP1-2} = W_{IF,CP1-1}$

$W_{IF,CP1-2} = W_{IF,CP1-2 \text{ condition } y_1 = 1/3 H; y = 1/2 H}$

$\lambda_{F,CP1-2} = \frac{4m_q (L_1^2 \mu H + 3L_2^2 \mu y + 4H^2 y)}{q y \mu L^2 (H - y - D_{1-2})}$

Wind case CP1-1

Conditions $y = \frac{1}{3} H; \frac{1}{2} H$

$\lambda_{W,CP1} = \frac{4m_q (L_1^2 \mu (H - y - D_{1-2}) + L_2^2 \mu y + 4H^2 y (H - y - D_{1-2}) (H - y) - (H - K)^2)}{q 2 y \mu L (H - y - D_{1-2}) ((H - K) (H - y) - (H - K)^2)}$

Condition $y_1 = 2/3 H$

$\lambda_{W,CP1-1} = \frac{2m_q (L_1^2 \mu y (H - y - D_{1-2}) + L_2^2 \mu y^2 + 4H^2 y (H - y - D_{1-2}) (H - y - D_{1-2}) (H - y - D_{1-2}) (2LKY - K^2 L))}{q \mu L (H - y - D_{1-2}) (2LKY - K^2 L)}$

Conditions $y_1 = \frac{1}{2} H$

$\lambda_{W,CP1-1} = \frac{4m_q (L_1^2 \mu (H - y - D_{1-2}) + L_2^2 \mu y + 4H^2 y (H - y - D_{1-2}) (H - y - D_{1-2}) (H - y - D_{1-2}) (H - y - D_{1-2}) (y \mu L^2 (H - y - D_{1-2}) )}{q y \mu L^2 (H - y - D_{1-2})}$
Wind case CP1-2

Conditions $y_2 = \frac{1}{3} H$ (from wall base);

$$\lambda_{W,CP1-2} = \frac{4m_v (L^2 \mu (H - y - D_{1-2}) + L^2 \mu y + 4Hy(H - y - D_{1-2}) (H - y)^2)}{y \mu (H - y - D_{1-2})}$$

Condition = $y_2 = 2/3H; y_2 = 1/2H$

$W_{W,CP1-2} = W_{E,CP1-2} = W_{F,CP1-2}$

$W_{W,CP1-2} = W_{E,CP1-2}$

$W_{W,CP1-2} = W_{F,CP1-2}$

Earthquake case CP2-1

Condition = $y = \frac{1}{3} H; y = \frac{1}{2} H$

$$\lambda_{E,CP2-1} = \frac{2m_v (Lx \mu (H - y)^2 + 2yH(H - y)^2 + Lxy \mu (H - y))}{Lxy \mu (H - K)(H - y) - yx^2 \mu (H - K)^2}$$

$d\lambda/dx \to 0$

- assign values to $y$ comprised in a range
- solve 1st order differential equation; find value of $x$, determine $\min \lambda_{E,CP2-1}$

Condition = $y = \frac{2}{3} H$

$$\lambda_{E,CP2-1} = \frac{2m_v (Lx \mu (H - y)^2 + 4yH(H - y)^2 + Lxy \mu (H - y)^2)}{yx^2 \mu (H - K)^2}$$

$d\lambda/dx \to 0$

- assign values to $y$ comprised in a range
- solve 1st order differential equation; find value of $x$, determine $\min \lambda_{E,CP2-1}$

Earthquake case CP2-2

Condition = $y = \frac{1}{3} H; y = \frac{1}{2} H$

$$\lambda_{E,CP2-2} = \frac{2m_v (Lx \mu (H - y)^2 + 2yH(H - y)^2 + Lxy \mu (H - y))}{Lxy \mu (H - K)(H - y) - yx^2 \mu (H - K)^2}$$

$d\lambda/dx \to 0$

- assign values to $y$ values comprised in a range
- solve 1st order differential equation; find value of $x$, determine $\min \lambda_{E,CP2-2}$
Flood case CP2-1: $\beta = 1 \rightarrow \beta H = H$

Condition: $y = \frac{1}{3}H$

$\lambda_{F,CP2-1} = \frac{2m_v(L\mu(H - y)^2 + 2yH(H - y)^2 + Lxy\mu(H - y))}{q \frac{Lxy\mu(H - K)(H - y) - yx^2\mu(H - y)}{}}$

$d\lambda/dx \rightarrow 0$
- assign values to $y$ comprised in a range
- solve 1st order differential equation; find value of $x$, determine $\min \lambda_{F,CP2-1}$

Condition: $y = \frac{1}{2}H; y = \frac{2}{3}H$

$\lambda_{F,CP2-1} = \frac{2m_v(L\mu(H - y)^2 + 2yH(H - y)^2 + Lxy\mu(H - y))}{q \frac{Lxy\mu(H - K)(H - y) - yx^2\mu(H - K)^2}{}}$

Flood case CP2-2

Condition: $y = 1/3H$

$\lambda_{F,CP2-2} = \frac{2m_v(L\mu(H - y)^2 + 2xH(H - y)^2 + Lxy\mu(H - y))}{q \frac{Lxy\mu(H - K)(H - y) - yx^2\mu(H - K)^2}{}}$

$d\lambda/dx \rightarrow 0$
- assign values to $y$ comprised in a range
- solve 1st order differential equation; find value of $x$, determine $\min \lambda_{F,CP2-2}$

Condition: $y = 1/2H; y = 2/3H$

$\lambda_{F,CP2-2} = \frac{2m_v(L\mu(H - y)^2 + 2yH(H - y)^2 + Lxy\mu(H - y))}{q \frac{Lxy\mu(H - y) - K^2x^2\mu(H - y)}{}}$

$d\lambda/dx \rightarrow 0$
- assign values to $y$ comprised in a range
- solve 1st order differential equation; find value of $x$, determine $\min \lambda_{F,CP2-2}$
Wind case CP2-1

Condition = \( y = \frac{1}{3}H \);
\[
\lambda_{W.CP2-1} = \frac{2m_v}{q} \left( Lx\mu(H - y)^2 + 2y(H - y)^2 + Lxy\mu(H - y) \right)
\]
\[d\lambda/dx \to 0\]
- assign values to \( y \) comprised in a range
- solve 1\textsuperscript{st} order differential equation; find value of \( x \), determine \( \min \lambda_{W.CP2-1} \)

Wind case CP2-2

Condition = \( y = \frac{1}{2}H \)
\[
\lambda_{W.CP2-2} = \frac{2m_v}{q} \left( Lx\mu(H - y)^2 + 2y(H - y)^2 + Lxy\mu(H - y)^2 \right)
\]
\[d\lambda/dx \to 0\]
- assign values to \( y \) comprised in a range
- solve 1\textsuperscript{st} order differential equation; find value of \( x \), determine \( \min \lambda_{W.CP2-2} \)

Condition = \( y = \frac{2}{3}H \)
\[
\lambda_{W.CP2-2} = \frac{2m_v}{q} \left( Lx\mu(H - y)^2 + 2y(H - y)^2 + Lxy\mu(H - y) \right)
\]
\[d\lambda/dx \to 0\]
- assign values to \( y \) comprised in a range
- solve 1\textsuperscript{st} order differential equation; find value of \( x \), determine \( \min \lambda_{W.CP2-2} \)
Group C – Configuration C4
Earthquake case CP1-1

Conditions $y = \frac{1}{3}H; y = \frac{1}{2}H; y = \frac{2}{3}H$

$W_{E,CP1-1} = \sum m \times \text{length}_{\text{crack}} \theta = 2m_v L \frac{1}{y} + 2m_n H \frac{2}{L} + m_v L \frac{3}{H}$

$W_{E,CP1-1} = W_{eE,CP1-1}$

$\lambda_{E,CP1-1} = \frac{2m_v (2L^2 \mu H + 3L^2 \mu y + 4H^2 y)}{q \mu L^2 H}$

d$\lambda/dx \to 0$

- assign values to $y$ comprised in a range
- solve $1^{st}$ order differential equation; find value of $x$, determine $\min \lambda_{E,CP1-1}$

Earthquake case CP1-2

Conditions $y = \frac{1}{3}H; y = \frac{1}{2}H$

$W_{E,CP1-2} = \sum m \times \text{length}_{\text{crack}} \theta = 2m_v L \frac{1}{y} + 2m_n H \frac{2}{L} + m_v L \frac{1}{(H - y - D_{1-2})}$

$m_h = m_v/\mu$

$W_{E,CP1-2} = m_v \left( \frac{2L^2 \mu (H - y - D_{1-2}) + L^2 \mu y + 4H (H - y - D_{1-2})}{q \mu L (H - y - D_{1-2})} \right)$

$W_{E,CP1-2} = \sum (q \times \text{length}_{\text{KEL}} \times \delta)$

$A: (H-K) = L/2; (H-y); \rightarrow A = \frac{(H-K)L}{4(H-y)}$

$W_{E,CP1-2} = 2Lq \left[ \frac{(H-K)L}{2(H-y)} \right] + \lambda q \left[ \frac{(H-K)1}{2(H-y)} \right]$ (H-K) \frac{1}{(H-y)}$

$W_{E,CP1-2} = W_{eE,CP1-2}$

$\lambda_{E,CP1-2} = \frac{2m_v (2L^2 \mu (H - y - D_{1-2}) + L^2 \mu y + 4H (H - y - D_{1-2}) (H - y)^2)}{q \frac{2yL (H - y - D_{1-2}) ((H-K)(H-y) - (H-K)^2)}{}}$

d$\lambda/dx \to 0$

- assign values to $y$ comprised in a range
- solve $1^{st}$ order differential equation; find value of $x$, determine $\min \lambda_{E,CP1-2}$
Flood case CP1-1

\[
\frac{d\lambda}{dx} \to 0
\]

\[
\begin{align*}
\lambda_{CP1-1} &= \frac{2m_v(2L^2\mu(H - y - D_{1-2}) + L^2\mu y + 4Hy(H - y - D_{1-2}))}{\mu L^2(H - y - D_{1-2})} \\
\text{Conditions} &= y = H/3, y = 1/2H, y = 2/3H
\end{align*}
\]

**Assign values to y comprised in a range**

**Solve 1st order differential equation; find value of x, determine min \( \lambda_{W,CP1-1} \)**

---

Flood case CP1-2

\[
\begin{align*}
\lambda_{CP1-2} &= \frac{2m_v(2L^2\mu(H - y - D_{1-2}) + L^2\mu y + 4Hy(H - y - D_{1-2}))}{\mu L^2(H - y - D_{1-2})} \\
\text{Conditions} &= y = \frac{1}{3}H, y = \frac{1}{2}H, y = \frac{2}{3}H
\end{align*}
\]

**Assign to y values comprised in a range**

**Solve 1st order differential equation; find value of x, determine min \( \lambda_{W,CP1-2} \)**
Wind case CP1-1

\[ \text{Conditions } = y = 1/3; y = 1/2 \]
\[ \lambda_{W,CPI-1} = \frac{2m_y(2L^2\mu(y - D_{1-2}) + L^2\mu y + 4Hy(y - D_{1-2}))}{q \mu y L^2(H - y - D_{1-2})} \]

Wind case CP1-2

\[ \text{Conditions } = y_1 = \frac{1}{2H} \]
\[ \lambda_{W,CPI-1} = \frac{2m_y(2L^2\mu(y - D_{1-2}) + L^2\mu y + 4Hy(y - D_{1-2}))}{q \mu y L^2(H - y - D_{1-2})} \]

Earthquake case CP2-1

\[ \text{Conditions } = y = 1/3; y = 1/2 \]
\[ W_{IE,CP2-1} = 2m_y \frac{L}{y} + 2m_h \frac{H}{x} + m_b \frac{1}{H - y} \]
\[ m_k = m_y / \mu \]
\[ W_{IE,CP2-1} = m_y \left( \frac{2L(H - y)x\mu + 2Hy(H - y)}{y\mu y(H - y)} \right) \]
\[ W_{EE,CP2-1} = A; (H - K) = x; (H - y); A = x \frac{(H - K)}{H - y}; B = L - 2A \]
\[ W_{EE,CP2-1} = 2\lambda q \left( \frac{(H - K)x}{y(H - y)} \right) L + \lambda q \left( L - 2 \frac{(H - K)x}{y(H - y)} \right) (H - K) \]
\[ W_{EE,CP2-1} = \lambda q \left( \frac{(H - y)(H - K) - x(H - K)^2}{(H - y)^2} \right) \]
\[ W_{IE,CP2-1} = W_{IE,CP2-1} \]
Earthquake case CP2-2

Condition = $y = \frac{1}{3} H$; $y = \frac{1}{2} H$

$W_{E.E.CP2-2} = 2m_a L \frac{1}{y} + 2m_b H \frac{1}{x} + m_v L \frac{1}{H - y}$

$m_b = m_v / \mu$

$W_{E.E.CP2-2} = m_v \frac{(2L(H - y)xu + 2Hy(H - y) + Lxyu)}{xyu(H - y)}$

$W_{E.E.CP2-2} = 2\lambda q \left( \frac{(H - K)x(H - K)x}{(H - y)} \right) \frac{1}{x}$

$d\lambda / dx \to 0$

• assign values to $y$ comprised in a range
• solve 1st order differential equation; find value of $x$, determine $\min \lambda_{E.E.CP2-2}$
Condition = $y = 2/3H$

\[ W_{e, CP2-2} = W_{e, CP2-1} \]

$W_{e, CP2-2} \to A: K = x; A = Kx/y; B = L - 2A$

\[ W_{e, CP2-2} = 2\lambda q \left( \frac{Kx}{y} \right) \frac{1}{x} + \lambda q \left( L - 2 \frac{Kx}{y} \right) \frac{1}{y} \]

\[ W_{e, CP2-2} = \lambda q \frac{LKy - xK^2}{y^2} \]

\[ W_{e, CP2-2} = \frac{W_{e, CP2-2}}{2} \frac{(2L(H - y)xu + 2Hy(H - y) + Lxy\mu)}{xyu(H - y)} = \lambda q \frac{LKy - xK^2}{y^2} \]

\[ \lambda_{e, CP2-2} = \frac{m_v}{q} \frac{(2Lxu(H - y)^2 + 2yH(H - y)^2 + Lxy\mu(H - y))}{Lxyu(H - K)(H - y)} \]

\[ \frac{d\lambda}{dx} \to 0 \]

- assign to $y$ values comprised in a range
- solve 1st order differential equation; find value of $x$, determine \[ \min \lambda_{e, CP2-2} \]

**Flood case CP2-1:** $\beta = 1 \to \beta H = H$

Condition = $y = 1/3H$

\[ W_{f, CP1-2} = m_v \frac{1}{L} + 2m_v \frac{1}{H - y} + m_v \frac{1}{H - y} \]

\[ m_v = \frac{m_v}{\mu} \]

\[ W_{f, CP1-2} = m_v \frac{(2L(H - y)xu + 2Hy(H - y) + Lxy\mu)}{xyu(H - y)} \]

\[ W_{f, CP1-2} \to A: (H - K) = x; (H - y); A = x \frac{(H - K)}{y} \]

\[ W_{f, CP1-2} = 2\lambda q \left( \frac{(H - K)x(H - K)x}{(H - y)} \right) \frac{1}{x} \]

\[ W_{f, CP1-2} = \lambda q \frac{(H - K)^2 x}{y} \frac{y}{(H - y)^2} \]

\[ W_{f, CP1-2} = \frac{W_{f, CP1-2}}{2} \frac{(2L(H - y)xu + 2Hy(H - y) + Lxy\mu)}{xyu(H - y)} = \lambda q \frac{(H - K)^2 x}{y} \frac{y}{(H - y)^2} \]

\[ \lambda_{f, CP1-2} = \frac{m_v}{q} \frac{(2Lxu(H - y)^2 + 2yH(H - y)^2 + Lxy\mu(H - y)^2)}{Lxyu(H - K)(H - y)} \]

\[ \frac{d\lambda}{dx} \to 0 \]

- assign to $y$ values comprised in a range
- solve 1st order differential equation; find value of $x$, determine \[ \min \lambda_{f, CP1-2} \]
Flood case CP2-2: $\beta = 1 \rightarrow \beta H = H$

\[ W_{if,CP2-2} = W_{if,CP2-2} \]

\[ A: (H-K) = x: (H-y); \quad A = x \frac{(H-K)}{(H-y)}; \quad B = L - 2A \]

\[ W_{ef,CP2-2} = \lambda q \frac{(L(H-y)(H-K) - x(H-K)^2)}{(H-y)^2} \]

\[ W_{if,CP2-2} = W_{if,CP2-2} \]

\[ m_v \left( 2Ly(1-H-y)^2 + 2xH(H-y)^2 + LxH(H-y) \right) \]

assign to values $y$ comprised in a range

solve 1st order differential equation; find value of $x$, determine $\min \lambda_{CP2-2}$

\[ Condition = y = 1/3H; \]

\[ W_{if,CP2-1} = W_{if,CP2-2} \]

\[ W_{if,CP2-1} = W_{if,CP2-1} \]

\[ m_v \left( \frac{(2Ly(H-y)^2 + 2yH(H-y)^2 + Lx(H-y))}{q} \right) \]

assign to values $y$ comprised in a range

solve 1st order differential equation; find value of $x$, determine $\min \lambda_{CP2-2}$

\[ Condition = y = 1/3H; \]

\[ W_{if,CP2-1} = W_{if,CP2-1} \]

\[ W_{if,CP2-1} = W_{if,CP2-1} \]

\[ m_v \left( \frac{(2Ly(H-y)^2 + 2yH(H-y)^2 + Lx(H-y))}{q} \right) \]

assign values to $y$ comprised in a range

solve 1st order differential equation; find value of $x$, determine $\min \lambda_{CP2-1}$

\[ Condition = y = 1/3H; \]

\[ W_{if,CP2-1} = W_{if,CP2-1} \]

\[ W_{if,CP2-1} = W_{if,CP2-1} \]

\[ m_v \left( \frac{(2Ly(H-y)^2 + 2yH(H-y)^2 + Lx(H-y))}{q} \right) \]

assign values to $y$ comprised in a range

solve 1st order differential equation; find value of $x$, determine $\min \lambda_{CP2-1}$

\[ Condition = y = 1/3H; \]

\[ W_{if,CP2-1} = W_{if,CP2-1} \]

\[ W_{if,CP2-1} = W_{if,CP2-1} \]

\[ m_v \left( \frac{(2Ly(H-y)^2 + 2yH(H-y)^2 + Lx(H-y))}{q} \right) \]

assign values to $y$ comprised in a range

solve 1st order differential equation; find value of $x$, determine $\min \lambda_{CP2-1}$
Condition = \( y = \frac{1}{2} H \)

\[
W_{\text{ew,cp2-1}} \rightarrow A: (H - K) = x: (H - y); \quad A = x \frac{(H - K)}{H - y};
\]

\[
W_{\text{ew,cp2-1}} = 2\lambda q \frac{(H - K)x (H - K)x}{(H - y)} \frac{1}{x}
\]

\[
W_{\text{ew,cp2-1}} = \lambda q \frac{(H - K)^2 x}{(H - y)^2}
\]

\[
W_{\text{iw,cp2-1}} = W_{\text{ew,cp2-1}}
\]

\[
m_v (2L(H - y)xu + 2Hy(H - y) + Lxy\mu) = \frac{\lambda q ((H - K)^2 x)}{(H - y)^2}
\]

\[
\lambda_{\text{w,cp2-1}} = \frac{m_v (2Lx\mu(H - y)^2 + 2yH(H - y)^2 + Lxy\mu(H - y)^2)}{q yx^2 \mu(H - K)^2}
\]

\[
d\lambda/dx \rightarrow 0
\]

- assign to \( y \) values comprised in a range
- solve 1\(^{\text{st}}\) order differential equation; find value of \( x \), determine \( \min \lambda_{\text{w,cp2-1}} \)

Wind case CP2-2

Condition = \( y = 1/3H; \quad y = 1/2H \)

\[
W_{\text{iw,cp2-2}} = W_{\text{iw,cp2-1}} = W_{\text{iw,cp2-2}}
\]

\[
W_{\text{ew,cp2-2}} \rightarrow A: (H - K) = X: (H - y); \quad A = x \frac{(H - K)}{H - y}; B
\]

\[
= L - 2a
\]

\[
W_{\text{ew,cp2-2}} = W_{\text{ew,cp2-2}} = W_{\text{ew,cp2-2}} = W_{\text{ew,cp2-2}}
\]

\[
W_{\text{lw,cp2-2}} = W_{\text{lw,cp2-2}}
\]

\[
\lambda_{\text{w,cp2-2}} = \frac{m_v (2Lx\mu(H - y)^2 + 2yH(H - y)^2 + Lxy\mu(H - y))}{q Lxy\mu(H - y) - yx^2 \mu(H - K)^2}
\]

Condition = \( y = 2/3H \)

\[
W_{\text{iw,cp2-2}} = W_{\text{iw,cp2-2}} = W_{\text{lw,cp2-2}}
\]

\[
= m_v (2L(H - y)xu + 2Hx(H - y) + Lxy\mu)
\]

\[
\frac{xu\mu(H - y)}{(H - y)^2}
\]

\[
W_{\text{ew,cp2-2}} = W_{\text{ew,cp2-2}} = W_{\text{ew,cp2-2}}
\]

\[
= \lambda q \frac{(L(H - y))(H - K) - x(H - K)^2}{(H - y)^2}
\]

\[
W_{\text{ow,cp2-2}} = W_{\text{ow,cp2-2}}
\]

\[
\lambda_{\text{w,cp2-2}} = \frac{m_v (2Lx\mu(H - y)^2 + 2yH(H - y)^2 + Lxy\mu(H - y))}{q Lxy\mu(H - y) - K^2 x^2 \mu(H - y)}
\]

\[
d\lambda/dx \rightarrow 0
\]

- assign values to \( y \) comprised in a range
- solve 1\(^{\text{st}}\) order differential equation; find value of \( x \), determine \( \min \lambda_{\text{w,cp2-2}} \)
Group C – Configuration C5
Earthquake case CP1-1

Conditions: \( y = \frac{1}{3}H; \ y = \frac{1}{2}H; \ y = \frac{2}{3}H \)

\[
W_{ie,CP1-1} = \sum m \times \text{length}_{crack} \theta = 2m_vL\frac{1}{y} + 3m_hH\frac{2}{L} + m_vL\frac{1}{H}
\]

\( m_h = m_v/\mu \)

\[
W_{ie,CP1-1} = m_v\left(2L^2\mu H + 3L^2\mu y + 6H^2y\right)\frac{1}{y\mu L H}
\]

\[
W_{ee,CP1-1} = \sum (q \times \text{length}_{kel} \times \delta)
\]

\[
W_{ie,CP1-1} = W_{ee,CP1-1}
\]

\[
m_v = \frac{\lambda q L}{\nu \mu L H}
\]

\[
\lambda_{EC1-1} = \frac{2m_v(2L^2\mu H + 3L^2\mu y + 6H^2y)}{q \nu L^2 H}
\]

Earthquake case CP1-2

Conditions: \( y = \frac{1}{3}H; \ y = \frac{1}{2}H \)

\[
W_{ie,CP1-2} = \sum m \times \text{length}_{crack} \theta = 2m_vL\frac{1}{y} + 3m_hH\frac{2}{L} + m_vL\frac{1}{H} \left(\frac{H-y-D_{1-2}}{H-y}\right)
\]

\( m_h = m_v/\mu \)

\[
W_{ie,CP1-2} = m_v\left(2L^2\mu(H-y-D_{1-2}) + L^2\mu y + 6H(y-y-D_{1-2})\right)\frac{1}{\nu \mu L(H-y-D_{1-2})}
\]

\[
A: (H-K) = L/2; (H-y) \rightarrow A = ((H-K)L)/(2(H-y))
\]

\[
W_{ee,CP1-2} = 2\lambda q \left(\frac{(H-K)L}{2(H-y)}\right)\frac{1}{4(H-y)} + \lambda q \left(L - 2\frac{(H-K)L}{2(H-y)}\right)\left(\frac{1}{H} - \frac{1}{(H-y)}\right)
\]

\[
\lambda_{EC1-2} = \frac{2m_v2L^2\mu(H-y-D_{1-2}) + L^2\mu y + 6H(y-y-D_{1-2})(H-y)^2}{q\nu \mu L(H-y-D_{1-2})(H-K)(H-y)-(H-K)^2}
\]
Flood case CP1: \( \beta = 1 \rightarrow \beta H = H \)

Conditions: \( y_1 = H/3 \)

\[
\lambda_{F,CP1-1} = \frac{2m_v (2L^2 \mu (H - y - D_{1-2}) + L^2 \mu y + 6Hy(H - y - D_{1-2}))}{q \ y \mu L^2 (H - y - D_{1-2})}
\]

Conditions: \( y_1 = 1/2H; \ y_2 = 2/3H \)

\[
\lambda_{F,CP1-1} = \frac{2m_v (2L^2 \mu y(H - y - D_{1-2}) + L^2 \mu y^2 + 6Hy^2(H - y - D_{1-2}))}{q \ \mu L(H - y - D_{1-2})(2LKy - K^2L)}
\]

Flood case CP1-2

Conditions: \( y_2 = \frac{H}{3} \) (from wall base)

\[
W_{F,CP1-2} = W_{e,CP1-1} = \frac{\lambda gL}{2} \frac{L^2}{L}
\]

\[
\lambda_{F,CP1-2} = \frac{2m_v (2L^2 \mu H + 3L^2 \mu y + 6H^2 y)}{q \ y \mu L^2 H}
\]

\[
W_{F,CP1-2} = W_{e,CP1-2}; \ y_2 = \frac{1}{2}H \) (from wall base); \ y_2 = \frac{2}{3}H \) (from wall base)

\[
\lambda_{F,CP1-2} = \frac{2m_v 2L^2 \mu (H - y - D_{1-2}) + L^2 \mu y + 6Hy(H - y - D_{1-2})}{q \ y \mu L^2 (H - y - D_{1-2})}
\]
Wind case CP1-1

\[ y_1 = \frac{1}{3H}; \quad y_1 = \frac{1}{2H} \]

\[
W_{\text{IW,CPI-1}} = 2m_L \frac{1}{y} + 3m_H \frac{2}{L} + m_v L \left( \frac{1}{H - y - D_{1-2}} \right)
\]

\[
W_{\text{IW,CPI-1}} = m_v \frac{(2L^2\mu(H - y - D_{1-2}) + L^2\mu y + 6H y(H - y - D_{1-2}))}{y \mu L(H - y - D_{1-2})}
\]

\[
W_{\text{EW,CPI-1}} = 2\lambda q \left( \frac{(H - K)L}{2(H - y)} \right) \frac{2}{L} + \lambda q \left( L - \frac{2(H - K)L}{2(H - y)} \right) \frac{1}{(H - K)} \frac{1}{y \mu L^2(H - y - D_{1-2})}
\]

\[
\lambda_{\text{W,CPI-1}} = \frac{2m_v (2L^2\mu y(H - y - D_{1-2}) + L^2\mu y + 6H y^2(H - y - D_{1-2}))}{\mu L(H - y - D_{1-2}) (2LK - K^2 L)}
\]

Wind case CP1-2

\[ y_2 = \frac{1}{3H}; \quad y_2 = \frac{1}{2H} \]

\[
W_{\text{IW,CPI-2}} = W_{\text{IE.CPI-2}}
\]

\[
W_{\text{EW,CPI-2}} = W_{\text{EF.CPI-2}}
\]

\[
W_{\text{IW,CPI-2}} = W_{\text{EW.CPI-2}}
\]

\[
\lambda_{\text{W,CPI-2}} = \frac{2m_v (2L^2\mu y(H - y - D_{1-2}) + L^2\mu y + 6H y^2(H - y - D_{1-2})) (H - y)}{\mu L(H - y - D_{1-2}) (2LK - K^2 L) (H - K)^2 L}
\]

\[ Condition = y_2 = \frac{1}{2H}; \quad y_2 = \frac{1}{2H} \]

\[
W_{\text{IW,CPI-2}} = W_{\text{IE.CPI-1}}
\]

\[
W_{\text{EW,CPI-2}} = W_{\text{EF.CPI-1}}
\]

\[
W_{\text{IW,CPI-2}} = W_{\text{EW.CPI-2}}
\]

\[
\lambda_{\text{W,CPI-1}} = \frac{2m_v (2L^2\mu y(H - y - D_{1-2}) + L^2\mu y + 6H y^2(H - y - D_{1-2}))}{\mu L(H - y - D_{1-2}) (2LK - K^2 L)}
\]
Earthquake case CP2-1

Condition = \( y = \frac{1}{3}H; y = \frac{1}{2}H \)

\[
W_{IE,CP2-1} = 2m_vL \frac{1}{y} + 3m_hH \frac{1}{x} + m_vL \frac{1}{H - y}
\]

\( m_h = m_v/\mu \)

\[
W_{IE,CP2-1} = m_v \frac{(2L(H - y)x\mu + 3y(H - y) + Lxy\mu)}{xy\mu(H - y)}
\]

\( W_{ee,CP2-1} \to A: (H - K) = x: (H - y); \ A = x \frac{(H - K)}{H - y}; B = L - 2A \)

\[
W_{ee,CP2-1} = 2\lambda q \left( \frac{(H - K)x(H - K)x}{(H - y)^2} \right) + \lambda q \left( L - 2 \frac{(H - K)x}{(H - y)} \right) (H - K) \frac{(H - K)}{(H - y)}
\]

\[
W_{ee,CP2-1} = \lambda q \left( \frac{(L(H - y)(H - K) - x(H - K)^2)}{(H - y)^2} \right)
\]

\[
\lambda_{CP2-1} = m_v \frac{(2Lx\mu(H - y)^2 + 3yH(H - y)^2 + Lxy\mu(H - y))}{q x y^2 \mu(H - K)^2}
\]

\[ d\lambda/dx \to 0 \]

- assign values to y values comprised in a range
- solve 1st order differential equation; find value of x, determine \( \min \lambda_{CP2-1} \)

Condition = \( y = \frac{2}{3}H \)

\[
W_{ee,CP2-1} \to A: (H - K) = x: (H - y); \ A = x \frac{(H - K)}{H - y};
\]

\[
W_{ee,CP2-1} = 2\lambda q \left( \frac{(H - K)x(H - K)x}{(H - y)^2} \right)
\]

\[
W_{ee,CP2-1} = \lambda q \left( \frac{(H - K)^2 x}{(H - y)^2} \right)
\]

\[
W_{ee,CP2-1} = \lambda q \left( \frac{(H - K)^2 x}{(H - y)^2} \right)
\]

\[
\lambda_{CP2-1} = m_v \frac{(2Lx\mu(H - y)^2 + 3yH(H - y)^2 + Lxy\mu(H - y))}{q x y^2 \mu(H - K)^2}
\]

\[ d\lambda/dx \to 0 \]

- assign to y values comprised in a range
- solve 1st order differential equation; find value of x, determine \( \min \lambda_{CP2-1} \)
Earthquake case CP2-2

\[
W_{iE.CP2-2} = 2m_b \frac{1}{y} + 3m_aH \frac{1}{x} + m_\ell \frac{1}{H-y}
\]

\[
m_b = m_\ell/\mu
\]

\[
W_{iE.CP2-2} = m_v \frac{(2L(H-y)x + 3Hy(H-y) + Lxy\mu)}{xy\mu(H-y)}
\]

\[
W_{EE.CP2-2} \rightarrow A: (H-K) = x; (H-y); A = x \frac{(H-K)}{H-y}; B = L - 2A
\]

\[
W_{EE.CP2-2} = 2\lambda q \left( \frac{(H-K)x(H-K)x_1}{H-y} \right) + \lambda q \left( L - 2 \frac{(H-K)x}{H-y} \right) \frac{(H-K)}{(H-y)}
\]

\[
W_{EE.CP2-2} = \frac{\lambda q}{(H-y)^2} \frac{(L(H-y)(H-K) - x(H-K)^2)}{(H-y)}
\]

\[
\lambda_{E.CP2-2} = \frac{m_v(2Lx\mu(H-y)^2 + 3yH(H-y)^2 + Lxy\mu(H-y))}{q \frac{Lxy\mu(H-K)(H-y) - xy^2\mu(H-K)^2}{y^2}}
\]

\[
d\lambda/\partial x \rightarrow 0
\]

- assign to \( y \) values comprised in a range
- solve 1st order differential equation; find value of \( x \), determine \( \min \lambda_{E.CP2-2} \)

\[
W_{iE.CP2-2} = W_{iE.CP2-1}
\]

\[
\lambda_{E.CP2-2} = \frac{m_v(2Lx\mu(H-y)^2 + 3yH(H-y)^2 + Lxy\mu(H-y))}{q \frac{Lxy\mu(H-K)(H-y) - xy^2\mu(H-K)^2}{y^2}}
\]

\[
d\lambda/\partial x \rightarrow 0
\]

- assign to \( y \) values comprised in a range
- solve 1st order differential equation; find value of \( x \), determine \( \min \lambda_{E.CP2-2} \)
Flood case CP2-1: $\beta = 1 \rightarrow \beta H = H$

\[
\text{Condition } = y = \frac{1}{3} H
\]

\[
W_{e,CP2-1} = 2m_v\frac{1}{y} + 3m_yH\frac{1}{x} + m_vL\frac{1}{H-y}
\]

\[
m_h = m_v/\mu
\]

\[
W_{e,CP2-1} = m_v \frac{(2L(H-y)x\mu + 3Hy(H-y) + Lxy\mu)}{xy\mu(H-y)}
\]

\[
W_{e,CP2-1} \rightarrow A: (H-K) = x; (H-y); A = x \frac{(H-K)}{H-y};
\]

\[
W_{e,CP2-1} = 2\lambda q \frac{(H-K)x(2(H-y))}{x(H-y)} \frac{1}{x}
\]

\[
W_{e,CP2-1} = \lambda q \frac{(H-K)^2x}{(H-y)^2}
\]

\[
W_{f,CP2-1} = W_{e,CP2-1}
\]

\[
m_v \frac{(2L(H-y)x\mu + 3Hy(H-y) + Lxy\mu)}{xy\mu(H-y)} = \lambda q \frac{(H-K)^2x}{(H-y)^2}
\]

\[
\lambda_{e,CP2-1} = m_v \frac{(2Lx\mu(H-y)^2 + 3yH(H-y)^2 + Lxy(H-y)^2)}{xy^2\mu(H-K)^2}
\]

\[
d\lambda/dx \rightarrow 0
\]

- assign values to $y$ comprised in a range
- solve 1st order differential equation; find value of $x$, determine $\min \lambda_{e,CP2-1}$

\[
\text{Condition } = y = \frac{1}{2} H; y = \frac{2}{3} H
\]

\[
W_{e,CP2-1} \rightarrow A: (H-K) = x; (H-y); A = x \frac{(H-K)}{H-y}; B = L - 2A
\]

\[
W_{e,CP2-1} = 2\lambda q \frac{(H-K)x(2(H-y))}{x(H-y)} \frac{1}{x}
\]

\[
+ \lambda q \left( L - 2 \frac{(H-K)x}{(H-y)} \right) \frac{(H-K)}{(H-y)}
\]

\[
W_{e,CP2-1} = \lambda q \frac{(L(H-y)(H-K) - x(H-K)^2)}{(H-y)^2}
\]

\[
W_{f,CP2-1} = W_{e,CP2-1}
\]

\[
m_v \frac{(2L(H-y)x\mu + 3Hy(H-y) + Lxy\mu)}{xy\mu(H-y)} = \lambda q \frac{(L(H-y)(H-K) - x(H-K)^2)}{(H-y)^2}
\]

\[
\lambda_{e,CP2-1} = m_v \frac{(2Lx\mu(H-y)^2 + 3yH(H-y)^2 + Lxy(H-y)^2)}{Lxy\mu(H-K)(H-y) - xy^2\mu(H-K)^2}
\]

\[
d\lambda/dx \rightarrow 0
\]

- assign values to $y$ comprised in a range
- solve 1st order differential equation; find value of $x$, determine $\min \lambda_{e,CP2-1}$

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Flood case CP2: $\beta = 1 \rightarrow \beta H = H$

Condition = $y = 1/3H$

$W_{f,CP2-2} = W_{ie,CP2-2}$

$A: (H - K) = x: (H - y); \ A = x \frac{(H - K)}{(H - y)}; B = L - 2A$

$W_{e,CP2-2} = \frac{\lambda q \ (L(H - y)(H - K) - x(H - K)^2)}{(H - y)^2}$

$W_{f,CP2-2} = W_{e,CP2-2}$

$m_v \ (2L(H - y)x\mu + 3Hy(H - y) + Lxy\mu) = \frac{xy\mu(H - y)}{(H - y)^2} = \frac{\lambda q \ (L(H - y)(H - K) - x(H - K)^2)}{(H - y)^2}$

$\lambda_{f,CP2-2} = \frac{m_v \ (2Ly\mu(H - y)^2 + 3xH(H - y)^2 + Lxy\mu(H - y))}{q} \frac{Lxy\mu(H - y)(H - y) - xy^2\mu(H - K)^2}{(H - y)^2}$

$d\lambda/dx \rightarrow 0$

- assign values to $y$ comprised in a range
- solve 1st order differential equation; find value of $x$, determine min $\lambda_{f,CP2-2}$

Flood case CP2: $\beta = 1 \rightarrow \beta H = H$

Condition = $y = 1/2H; y = 2/3H$

$W_{f,CP2-2} = W_{ie,CP2-2}$

$W_{e,CP2-2} = W_{e,CP2-2} = \frac{\lambda q \ (L(H - y)(H - K) - x(H - K)^2)}{(H - y)^2}$

$W_{f,CP2-2} = W_{e,CP2-2}$

$\lambda_{f,CP2-2} = \frac{m_v \ (2Ly\mu(H - y)^2 + 3xH(H - y)^2 + Lxy\mu(H - y))}{q} \frac{Lxy\mu(H - y) - K^2x^2\mu(H - y)}{(H - y)^2}$

$d\lambda/dx \rightarrow 0$

- assign values to $y$ comprised in a range
- solve 1st order differential equation; find value of $x$, determine min $\lambda_{f,CP2-2}$
Wind case CP2-2

Condition = \( y = \frac{1}{3}H \);

\[ W_{w,\text{CP2-1}} = 2m_w L \frac{1}{y} + 3m_h H \frac{1}{x} + m_w L \frac{1}{H - y} \]

\[ m_h = m_w / \mu \]

\[ W_{w,\text{CP2-1}} = m_w \frac{(2L(H - y)x\mu + 3Hy(H - y) + Lxy\mu)}{xy\mu(H - y)} \]

\( W_{w,\text{CP2-1}} \rightarrow A: (H - K) = x: (H - y); \ A = x \frac{(H - K)}{H - y}; \ B = L - 2A \)

\[ W_{w,\text{CP2-1}} = 2\lambda q \frac{((H - K)x \ (H - K)x \ \frac{1}{x})}{(H - y)^2} \]

\[ + \lambda q \left( \frac{L - 2(H - K)x}{(H - y)} \right) \frac{(H - K)}{(H - y)} \]

\[ W_{w,\text{CP2-1}} = \lambda q \frac{(L(H - y)(H - K) - x(H - K)^2)}{(H - y)^2} \]

\[ \frac{d\lambda}{dx} \rightarrow 0 \]

- assign values to \( y \) values comprised in a range
- solve 1st order differential equation; find value of \( x \), determine \( \min \lambda_{w,\text{CP2-1}} \)

Condition = \( y = \frac{1}{2}H \)

\( W_{w,\text{CP2-1}} \rightarrow A: (H - K) = x: (H - y); \ A = x \frac{(H - K)}{H - y}; \)

\[ W_{w,\text{CP2-1}} = 2\lambda q \frac{((H - K)x \ (H - K)x \ \frac{1}{x})}{(H - y)^2} \]

\[ W_{w,\text{CP2-1}} = \lambda q \frac{(H - K)^2 x}{(H - y)^2} \]

\[ W_{w,\text{CP2-1}} = W_{w,\text{CP2-1}} \]

\[ m_w \frac{(2L(H - y)x\mu + 3Hy(H - y) + Lxy\mu)}{xy\mu(H - y)} = \lambda q \frac{(H - K)^2 x}{(H - y)^2} \]

\[ \lambda_{w,\text{CP2-1}} = m_w \frac{(2Lx\mu(H - y)^2 + 3y(H - y)^2 + Lxy\mu(H - y)^2)}{yx^2\mu(H - K)^2} \]

\[ \frac{d\lambda}{dx} \rightarrow 0 \]

- assign values to \( y \) comprised in a range
Wind case CP2-2

Condition = y = 1/3H;  y = 1/2H

\[ W_{\text{CP2-2}} = W_{\text{IF.CP2-1}} = W_{\text{IE.CP2-1}} \]

\[ W_{\text{CP2-2}} \rightarrow A: (H - K) = X; (H - y);  A = x \frac{(H - K)}{H - y}; B = L - 2A \]

\[ W_{\text{CP2-2}} = W_{\text{EF.CP2-2}} = W_{\text{EF.CP2-2}} = W_{\text{EE.CP2-2}} \]

\[ W_{\text{CP2-2}} = W_{\text{EF.CP2-2}} \]

\[ \lambda_{\text{CP2-2}} = \frac{m_v (2Lx \mu (H - y)^2 + 3yH(H - y)^2 + Lxy\mu (H - y))}{Lxy\mu (H - y) - yx^2 \mu (H - K)^2} \]

Condition = y = 2/3H

\[ W_{\text{CP2-2}} = W_{\text{IF.CP2-2}} = W_{\text{IE.CP2-2}} \]

\[ W_{\text{CP2-2}} = W_{\text{EF.CP2-2}} \]

\[ W_{\text{CP2-2}} = W_{\text{EF.CP2-2}} \]

\[ W_{\text{CP2-2}} = W_{\text{IF.CP2-2}} \]

\[ \lambda_{\text{CP2-2}} = \frac{m_v (2Lx \mu (H - y)^2 + 3yH(H - y)^2 + Lxy\mu (H - y))}{Lxy\mu (H - y) - K^2x^2 \mu (H - y)} \]

\[ d\lambda / dx \to 0 \]

- solve 1st order differential equation; find value of x, determine min \( \lambda_{\text{CP2-2}} \)

Group C – Configuration C6

Earthquake case CP1-1

Conditions y = 1/3H; y = 1/2H; y = 2/3H

\[ W_{\text{IE.CP1-1}} = \sum m \times \text{length}_{\text{crack}} \theta = 2m_v L \frac{1}{y} + 2m_v H \frac{2}{L} + 2m_v J \frac{3}{H} \]

\[ m_h = m_v / \mu \]

\[ W_{\text{IE.CP1-1}} = m_v \frac{2L^2 \mu H + 6L^2 \mu y + 4H^2 y}{y \mu LH} \]

\[ W_{\text{EE.CP1-1}} = \sum (q \times \text{length}_{\text{KEL}} x \delta) \]

\[ W_{\text{EE.CP1-1}} = \frac{\lambda q L}{2} \times \frac{L}{2} \times \frac{2}{L} \]

\[ W_{\text{IE.CP1-1}} = W_{\text{EE.CP1-1}} \]
Earthquake case CP1-2

\[ m_v (2L^2 \mu H + 6\mu y + 4H^2 y) = \frac{\lambda q L}{y \mu L H} \]
\[ \lambda_{E,CP1-1} = \frac{2m_v (2L^2 \mu H + 6L^2 \mu y + 4H^2 y)}{q} \]

Conditions: \( y = \frac{1}{3} H; y = \frac{1}{2} H \)

\[ W_{E,CP1-2} = \sum m \times \text{length}_{\text{crack}} \theta = 2m_v L \frac{1}{y} + 2m_h H \frac{L}{2} + 2m_e L \frac{1}{(H - y - D_{1-2})} \]
\[ m_h = m_v / \mu \]
\[ W_{E,CP1-2} = m_v (2L^2 \mu (H - y - D_{1-2}) + 2L^2 \mu y + 4H(y(H - y - D_{1-2})) / y \mu L(H - y - D_{1-2}) \]
\[ W_{E,CP1-2} = \sum (q \times \text{length}_{EL} \times \delta) \]
\[ A: (H - K) = L/2; (H - y); \rightarrow A = ((H - K)L)/(2(H - y)) \)

\[ W_{E,CP1-2} = 2\lambda q \left( \frac{(H - K) \mu (H - K) L}{2(H - y)} \right)^2 + \lambda q \left( L - 2 \frac{(H - K) L}{2(H - y)} \right) \left( H - K \right) \frac{1}{(H - y)} \]

[Diagram of earthquake case CP1-2]

Flood case CP1-1: \( \beta = 1 \rightarrow \beta H = H \)

Conditions: \( y = 1/3; y = 1/2 \)

\[ W_{F,CP1-2} = W_{E,CP1-2} \]

\[ W_{E,CP1-2} = W_{E,CP1-2} \text{ condition } y = \frac{1}{3}, y = \frac{1}{2} \]

\[ W_{F,CP1-2} = W_{F,CP1-2} \]

\[ \lambda_{F,CP1-2} = \frac{2m_v (2L^2 \mu (H - y - D_{1-2}) + 2L^2 \mu y + 4H(y(H - y - D_{1-2}))}{y \mu L(H - y - D_{1-2})} \]

Conditions: \( y = 1/3; y = 1/2 \)

\[ W_{F,CP1-2} = W_{E,CP1-2} \]

\[ W_{E,CP1-2} = W_{E,CP1-2} \]

[Diagram of flood case CP1-1]
Flood case CP1-2: $\beta = 1 \rightarrow \beta H = H$

\[
\begin{align*}
\frac{m_v \left(2L^2 \mu (H - y - D_{1-2}) + 2L^2 \mu y + 4Hy(H - y - D_{1-2})\right)}{\mu L(H - y - D_{1-2})} &= \frac{q(2Lk_y - K^2L)}{2y^2} \\
\lambda_{F,CP1-2} &= \frac{2m_v(2L^2 \mu y(H - y - D_{1-2}) + 2L^2 \mu y^2 + 4Hy^2(H - y - D_{1-2})}{q} \mu L(H - y - D_{1-2})(2Lk_y - K^2L)
\end{align*}
\]

Conditions $= y_2 = \frac{H}{3}$ (from wall base)

\[
\begin{align*}
W_{If,CP1-2} &= W_{ie,CP1-1} = \frac{\lambda q L}{2} \frac{2}{L} \\
\lambda_{F,CP1-2} &= \frac{2m_v(2L^2 \mu H + 6\mu y + 4H^2y)}{q} \frac{y\mu L^2H}{y\mu L^2H} \\
W_{If,CP1-2} &= W_{ie,CP1-2}; y_2 = \frac{1}{2}H (from wall base); y_2 = \frac{2}{3}H (from wall base) \\
\lambda_{F,CP1-2} &= \frac{2m_v(2L^2 \mu (H - y - D_{1-2}) + 2L^2 \mu y + 4Hy(H - y - D_{1-2})}{q} \frac{y\mu L^2(H - y - D_{1-2})}{y\mu L^2(H - y - D_{1-2})}
\end{align*}
\]

Wind case CP1-1

\[
\begin{align*}
\text{Conditions} &= y = 1/3H; y = 1/2H \\
W_{iw,CP1-1} &= W_{ie,CP1-1} \\
W_{ew,CP1-1} &= W_{ef,CP1-1} \\
W_{iw,CP1-1} &= W_{ew,CP1-1} \\
\lambda_{W,CP1-1} &= \frac{2m_v(2L^2 \mu (H - y - D_{1-2}) + 2L^2 \mu y + 4Hy(H - y - D_{1-2})}{q} \frac{y\mu L^2(H - y - D_{1-2})}{y\mu L^2(H - y - D_{1-2})}
\end{align*}
\]

\[
\begin{align*}
\text{Conditions} &= y_1 = \frac{1}{2H} \\
\lambda_{W,CP1-1} &= \frac{2m_v(2L^2 \mu (H - y - D_{1-2}) + 2L^2 \mu y + 4Hy(H - y - D_{1-2})}{q} \frac{y\mu L^2(H - y - D_{1-2})}{y\mu L^2(H - y - D_{1-2})}
\end{align*}
\]

Condition = $y = 2/3H$;

\[
\begin{align*}
W_{iw,CP1-1} &= W_{ie,CP1-1} \\
W_{ew,CP1-1} &= W_{ef,CP1-1}
\end{align*}
\]
Wind case CP1-2

\[
W_{\text{CP1-1}} = W_{\text{w,CP1-1}}
\]

\[
m_v = \frac{(2L^2\mu(H - y - D_{1-2}) + 2L^2\mu y + 4Hy(H - y - D_{1-2})}{\mu L(H - y - D_{1-2})} = \frac{\lambda q (2LK y - K^2 L)}{2y^2}
\]

\[
\lambda_{\text{w,CP1-1}} = \frac{2m_v (2L^2\mu y(H - y - D_{1-2}) + 2L^2\mu y^2 + 4Hy^2(H - y - D_{1-2})}{\mu L(H - y - D_{1-2}) (2LK y - K^2 L)}
\]

Conditions \( y = \frac{1}{3} H \);

\[
W_{\text{w,CP1-2}} = \sum m \times \text{length}_{\text{crack}} \theta = 2m_v \frac{1}{y} + 2m_h \frac{H^2}{L} + 2m_v \frac{1}{(H - y - D_{1-2})}
\]

\[
m_h = m_v / \mu
\]

\[
W_{\text{w,CP1-2}} = m_v (2L^2\mu(H - y - D_{1-2}) + 2L^2\mu y + 4Hy(H - y - D_{1-2})}{\mu L(H - y - D_{1-2})}
\]

\[
W_{\text{e,CP1-2}} = \sum (q \times \text{length}_{\text{EEL}} \times \delta)
\]

\[
A: (H - K) = L/2; (H - y); A = ((H - K) L) / (2(H - y))
\]

\[
W_{\text{e,CP1-2}} = 2\lambda q \left( \frac{(H - K) L (H - K) L}{2(H - y)} \right)^2 + \lambda q \left( L - 2 \frac{(H - K) L}{2(H - y)} \right) \left( H - K \right) \frac{1}{(H - y)}
\]

\[
W_{\text{w,CP1-2}} = W_{\text{w,CP1-1}}
\]

\[
\lambda_{\text{w,CP1-2}} = \frac{2m_v (2L^2\mu(H - y - D_{1-2}) + 2L^2\mu y + 4Hy(H - y - D_{1-2})(H - y)^2}{2y^2 H y^2 (H - y - D_{1-2}) (H - K)(H - y) - (H - K)^2}
\]

Condition \( y = 2/3H; y = 1/2H \);

\[
W_{\text{w,CP1-2}} = W_{\text{e,CP1-2}} = W_{\text{ef,CP1-2}}
\]

\[
W_{\text{w,CP1-2}} = W_{\text{e,CP1-2}} = W_{\text{ef,CP1-2}}
\]

Earthquake case CP2-1

Condition \( y = \frac{1}{3} H; y = \frac{1}{2} H \);

\[
W_{\text{e,CP2-1}} = 2m_v \frac{1}{y} + 2m_h \frac{1}{x} + 2m_v \frac{1}{H - y}
\]

\[
m_h = m_v / \mu
\]

\[
W_{\text{e,CP2-1}} = m_v \frac{(2L(H - y) x \mu + 2Hy(H - y) + 2Lxy) \mu}{xy \mu (H - y)}
\]

\[
W_{\text{e,CP2-1}} \rightarrow A: (H - K) = x; (H - y); A = x \frac{(H - K)}{H - y}; B = L - 2A
\]
Earthquake case CP2-2

\[
W_{e.e.cP2} = 2\lambda q \left( \frac{(H - K) x (H - K)x}{(H - y)} \right) \frac{1}{x} + \lambda q \left( L - 2 \frac{(H - K)x}{(H - y)} \right) \frac{(H - K)}{(H - y)} \\
W_{e.e.cP2} = \lambda q \left( \frac{L(H - y)(H - K) - x(H - K)^2}{(H - y)^2} \right) \\
W_{e.e.cP2} = W_{e.e.cP2-1} \\
m_v \left( 2(H - y)x\mu + 2Hy(H - y) + 2Lxy\mu \right) \frac{x\mu}{xy\mu(H - y)} = \lambda q \left( \frac{(H - K)^2x}{(H - y)^2} \right) \\
\lambda_{e.cP2-1} = m_v \left( 2Lx\mu(H - y)^2 + 2yH(H - y)^2 + 2Lxy\mu(H - y)^2 \right) \frac{y\mu x^2(H - K)^2}{q} \\
d\lambda / dx \to 0 \\
\cdot \text{ assign values to } y \text{ comprised in a range} \\
\cdot \text{ solve 1st order differential equation; find value of } x, \text{ determine } \min \lambda_{e.cP2-1}

Condition = y = \frac{2}{3}H \\
W_{e.e.cP2-1} \to A; (H - K) = x; (H - y); A = x \frac{(H - K)}{H - y} \\
W_{e.e.cP2-1} = 2\lambda q \left( \frac{(H - K)x(H - K)x}{(H - y)} \right) \frac{1}{x} \\
W_{e.e.cP2-1} = \lambda q \left( \frac{(H - K)^2x}{(H - y)^2} \right) \\
W_{e.e.cP2-1} = W_{e.e.cP2-1} \\
m_v \left( 2L(H - y)x\mu + 2Hy(H - y) + 2Lxy\mu \right) \frac{x\mu}{xy\mu(H - y)} = \lambda q \left( \frac{(H - K)^2x}{(H - y)^2} \right) \\
\lambda_{e.cP2-1} = m_v \left( 2Lx\mu(H - y)^2 + 2yH(H - y)^2 + 2Lxy\mu(H - y)^2 \right) \frac{y\mu x^2(H - K)^2}{q} \\
d\lambda / dx \to 0 \\
\cdot \text{ assign to } y \text{ values comprised in a range} \\
\cdot \text{ solve 1st order differential equation; find value of } x, \text{ determine } \min \lambda_{e.cP2-1}

Earthquake case CP2-2

\[
\text{Condition} = y = \frac{1}{3}H; y = \frac{1}{2}H \\
W_{e.e.cP2-2} = 2m_L \frac{1}{y} + 2m_H \frac{1}{x} + 2m_L \frac{1}{H - y} \\
m_v = m_v / \mu \\
W_{e.e.cP2-2} = m_v \left( 2L(H - y)x\mu + 2Hy(H - y) + 2Lxy\mu \right) \frac{x\mu}{xy\mu(H - y)}
\]
\( W_{ee,CP2-2} \rightarrow A: (H - K) = x: (H - y); \ A = x \frac{(H - K)}{H - y}; B = L - 2A \)

\[
\begin{align*}
W_{ee,CP2-2} &= 2\lambda q \left( \frac{(H - K)x(H - K)x}{(H - y)} \right) - \frac{1}{x} \\
&+ \lambda q \left( L - 2 \frac{(H - K)x}{(H - y)} \right) \\
W_{ee,CP2-2} &= \lambda q \frac{(L(H - y)(H - K) - x(H - K)^2)}{(H - y)^2}
\end{align*}
\]

\( W_{ee,CP2-2} = W_{ee,CP2-1} \)

\[
m_v \frac{xy(\mu(H - y))}{\lambda q} = \frac{L(H - y)(H - K) - x(H - K)^2}{(H - y)^2}
\]

\[
\lambda_{CP2-2} = \frac{m_v \left(2Lx\mu(H - y)^2 + 2yH(H - y)^2 + 2Lxy\mu(H - y)\right)}{q \left(Lxy\mu(H - K)(H - y) - yx^2\mu(H - K)^2\right)}
\]

\( d\lambda/dx \rightarrow 0 \)

- assign to \( y \) values comprised in a range
- solve 1st order differential equation; find value of \( x \), determine \( \min \lambda_{CP2-2} \)

**Condition:** \( y = 2/3H \)

\( W_{ee,CP2-1} = W_{ee,CP2-1} \)

\( W_{ee,CP2-2} \rightarrow A: K = x: y; \ A = \frac{Kx}{y}; B = L - 2A \)

\[
W_{ee,CP2-2} = 2\lambda q \left( \frac{Kx}{y} \right) - \frac{1}{x} + \lambda q \left( L - 2 \frac{Kx}{y} \right) \\
W_{ee,CP2-2} = \lambda q \frac{LKx - xK^2}{y^2}
\]

\( W_{ee,CP2-2} = W_{ee,CP2-2} \)

\[
m_v \frac{2(H - y)x\mu + 2Hy(H - y) + 2Lxy\mu}{xy(H - y)} = \lambda q \frac{LKx - xK^2}{y^2}
\]

\[
\lambda_{CP2-2} = \frac{m_v \left(2Lx\mu(H - y)^2 + 2yH(H - y)^2 + 2Lxy\mu(H - y)\right)}{q \left(Lxy\mu(H - K)(H - y) - K^2x^2\mu(H - y)\right)}
\]

\( d\lambda/dx \rightarrow 0 \)

- assign to \( y \) values comprised in a range

solve 1st order differential equation; find value of \( x \), determine \( \min \lambda_{CP2-2} \)

**Condition:** \( y = \frac{1}{3}H \)

\( W_{ee,CP2-1} = m_v L - \frac{1}{y} + 2m_h L - \frac{1}{x} + 2m_v L - \frac{1}{H - y} \)

\( m_h = \frac{m_v}{\mu} \)

\[
W_{ee,CP2-1} = m_v \left(2L(H - y)x\mu + 2Hy(H - y) + 2Lxy\mu\right)
\]
\[ W_{EF.CP2-1} \rightarrow A: (H - K) = x: (H - y); \quad A = x \frac{(H - K)}{H - y}; \]
\[ W_{EF.CP2-1} = 2\lambda q \left( \frac{(H - K)x}{(H - y)} \right) \frac{1}{2(H - y)} \]
\[ W_{EF.CP2-1} = \lambda q \left( \frac{(H - K)^2x}{(H - y)^2} \right) \]
\[ W_{FCP2-1} = W_{EF.CP2-1} \]
\[ m_x \frac{(2L(H - y)x\mu + 2Hy(H - y) + 2Lx\mu)}{xy\mu(H - y)} = \frac{\lambda q ((H - K)^2x)}{(H - y)^2} \]
\[ \lambda_{FCP2-1} = \frac{m_x (2Lx\mu(H - y)^2 + 2yH(H - y)^2 + 2Lx\mu(H - y)^2)}{q} \]
\[ \frac{d\lambda}{dx} \rightarrow 0 \]
- assign to y values comprised in a range
- solve 1st order differential equation; find value of x, determine \( \min \lambda_{FCP2-1} \)

\[ \text{Condition} = y = \frac{1}{2} H; \quad y = \frac{2}{3} H \]
\[ W_{EF.CP2-1} \rightarrow A: (H - K) = x: (H - y); \quad A = x \frac{(H - K)}{H - y}; B = L - 2A \]
\[ W_{EF.CP2-1} = 2\lambda q \left( \frac{(H - K)x}{(H - y)} \right) \frac{1}{2(H - y)} \]
\[ + \lambda q \left( L - 2 \frac{(H - K)x}{(H - y)} \right) \frac{(H - K)}{(H - y)} \]
\[ W_{EF.CP2-1} = \lambda q \left( \frac{(H - y)(H - K) - x(H - K)^2}{(H - y)^2} \right) \]
\[ W_{FCP2-1} = W_{EF.CP2-1} \]
\[ m_x \frac{(2(H - y)x\mu + 2Hy(H - y) + Lx\mu)}{xy\mu(H - y)} = \frac{\lambda q ((H - y)(H - K) - x(H - K)^2)}{(H - y)^2} \]
\[ \lambda_{FCP2-1} = \frac{m_x (2Lx\mu(H - y)^2 + 2yH(H - y)^2 + 2Lx\mu(H - y)^2)}{Lxy\mu(H - K)(H - y) - xy^2\mu(H - K)^2} \]
\[ \frac{d\lambda}{dx} \rightarrow 0 \]
- assign values to y comprised in a range
- solve 1st order differential equation; find value of x, determine \( \min \lambda_{FCP2-1} \)
Flood case CP2-2: $\beta = 1 \rightarrow \beta H = H$

Condition = $y = 1/3H$

$W_{f,CP2-2} = W_{e,CP2-2}$

$A: (H - K) = x: (H - y); \ A = x \frac{(H - K)}{(H - y)}; B = L - 2A$

$W_{e,CP2-2} = \lambda q \frac{(L(H - y)(H - K) - x(H - K)^2)}{(H - y)^2}$

$W_{f,CP2-2} = W_{e,CP2-2}$

$d\lambda/dx \rightarrow 0$

- assign to y values comprised in a range
- solve 1st order differential equation; find value of x, determine $\min \lambda_{F,CP2-2}$

Wind case CP2-1

Condition = $y = 1/2H; y = 2/3H$

$W_{f,CP2-2} = W_{e,CP2-2}$

$W_{e,CP2-2} = W_{e,CP2-2} = \lambda q \frac{(L(H - y)(H - K) - x(H - K)^2)}{(H - y)^2}$

$W_{f,CP2-2} = W_{e,CP2-2}$

$\lambda_{F,CP2-2} = \frac{m_v}{q} \frac{2Ly(H - y)^2 + 2yH(H - y)^2 + 2Lxy(H - y)}{Lxy(H - y) - K^2x^2H(H - y)}$

$d\lambda/dx \rightarrow 0$

- assign to y values comprised in a range
- solve 1st order differential equation; find value of x, determine $\min \lambda_{F,CP2-2}$

1.5

Wind case CP2-1

Condition = $y = \frac{1}{3}H$

$W_{v,CP2-1} = 2m_vL \frac{1}{y} + 2m_h \frac{1}{x} + m_vL \frac{1}{H - y}$

$m_h = m_v/\mu$

$W_{v,CP2-1} = \frac{m_v}{q} \frac{(2L(H - y)xy + 2Hy(H - y) + Lxy)}{xy(H - y)}$

$W_{w,CP2-1} \rightarrow A: (H - K) = x: (H - y); A = x \frac{(H - K)}{H - y}; B = L - 2A$
\[ W_{ew.cp2-1} = 2 \lambda q \frac{(H-K)x (H-K)x}{(H-y)} \frac{1}{2(H-y)} x + \lambda q \left( L - 2 \frac{(H-K)x}{(H-y)} \right) (H-K) \]

\[ W_{ew.cp2-1} = \lambda q \frac{(L(H-y)(H-K) - x(H-K)^2)}{(H-y)^2} \]

\[ W_{iw.cp2-1} = W_{ew.cp2-1} \]

\[ m_v \frac{(2(H-y)xu + 2Hy(H-y) + Lxyu)}{xy(H-y)} = \lambda q \left( \frac{(H-K)}{(H-y)} \right) (H-K) \]

\[ \lambda_{w,cp2-1} = \frac{m_v (2Lxyu(H-y)^2 + 2Hy(H-y)^2 + 2Lxyu(H-y))}{q Lxyu(H-y) - yx^2\mu(H-K)^2} \]

\[ d\lambda/dx \rightarrow 0 \]

- assign values to y comprised in a range
- solve 1\(^{st}\) order differential equation; find value of x, determine \( \min \lambda_{w,cp2-1} \)

**Condition** = \( y = \frac{1}{2} H \)

\[ W_{ew.cp2-1} \rightarrow A: (H-K) = x; (H-y); A = x \frac{(H-K)}{H-y}; \]

\[ W_{ew.cp2-1} = 2 \lambda q \frac{(H-K)x (H-K)x}{(H-y)} \frac{1}{2(H-y)} x \]

\[ W_{iw.cp2-1} = W_{ew.cp2-1} \]

\[ m_v \frac{(2L(H-y)xu + 2Hy(H-y) + 2Lxyu)}{xyu(H-y)} = \lambda q \frac{(H-K)^2x}{(H-y)^2} \]

\[ \lambda_{w,cp2-2} = \frac{m_v (2Lxyu(H-y)^2 + 2Hy(H-y)^2 + 2Lxyu(H-y)^2)}{q yx^2\mu(H-K)^2} \]

\[ d\lambda/dx \rightarrow 0 \]

- assign values to y comprised in a range
- solve 1\(^{st}\) order differential equation; find value of x, determine \( \min \lambda_{w,cp2-1} \)

**Condition** = \( y = 1/3 H; \ y = 1/2 H \)

\[ W_{iw.cp2-2} = W_{ie.cp2-1} = W_{ie.cp2-1} \]

\[ W_{ew.cp2-2} \rightarrow A: (H-K) = X; (H-y); A = x \frac{(H-K)}{H-y}; B = L - 2A \]

\[ W_{ew.cp2-2} = W_{e,e.cp2-2} = W_{e,e.cp2-2} \]

\[ W_{iw.cp2-2} = W_{e,cp2-2} \]

\[ \lambda_{w,cp2-2} = \frac{m_v (2Lxyu(H-y)^2 + 2Hy(H-y)^2 + 2Lxyu(H-y))}{q Lxyu(H-y) - yx^2\mu(H-K)^2} \]}
Condition = $y = 2/3H$

$$W_{w,CP2-2} = W_{f,CP2-2} = W_{e,CP2-2}$$
$$= m_v \frac{(2L(H - y)x\mu + 2Hx(H - y) + 2Lxy\mu)}{xy\mu(H - y)}$$

$$W_{eW,CP2-2} = W_{eF,CP2-2} = W_{eE,CP2-2}$$
$$= \lambda q \frac{(L(H - y)(H - K) - x(H - K)^2)}{(H - y)^2}$$

$$\lambda_{W,CP2-2} = \frac{m_v}{q} \frac{(2Lx\mu(H - y)^2 + 2yH(H - y)^2 + 2Lxy\mu(H - y))}{Lxy\mu(H - y) - K^2x^2\mu(H - y)}$$

$$d\lambda/dx \rightarrow 0$$

- assign values to $y$ comprised in a range
- solve 1st order differential equation; find value of $x$, determine $\min \lambda_{W,CP2-2}$
Appendix 2: Calculation of failure load and comparison between EC6 and Approach D

With reference to the geometric and material properties listed in Table 6-8, reported here are the calculations to determine the failure load of the wall configurations included in Table 6-10.

<table>
<thead>
<tr>
<th>Wall configuration</th>
<th>Corresponding bending moment coefficient from EC6 from Roberts, J. J., &amp; Brooker, O. (2007)</th>
<th>Calcs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpolation to find $\alpha_2$</td>
<td>$\alpha_2 = 0.056 + \frac{(0.6718 - 0.5)(0.073 - 0.056)}{0.75 - 0.5} = 0.0676$</td>
<td>$M_{ud1} = 1.5408$ MPa $M_{ud2} = 3.0816$ MPa $M_{ud1} = 0.5 \cdot 0.0676 \cdot W_k \cdot 3^2$ $M_{ud2} = 0.0676 \cdot W_k \cdot 4.465^2$ $W_{k1} = 1.5408$ $0.5 \cdot 0.0676 \cdot 3^2 = 5.05; \text{Wind Speed 28.44 m/s}$ $W_{k2} = 0.0676 \cdot 4.465^2 = 2.28; \text{Wind Speed 19.12 m/s}$</td>
</tr>
<tr>
<td>Interpolation to find $\alpha_2$</td>
<td>$\alpha_2 = 0.028 + \frac{(0.6718 - 0.5)(0.037 - 0.028)}{0.75 - 0.5} = 0.0342$</td>
<td>$M_{ud1} = 1.5408$ MPa $M_{ud2} = 3.0816$ MPa $M_{ud1} = 0.5 \cdot 0.0342 \cdot W_k \cdot 3^2$ $M_{ud2} = 0.0342 \cdot W_k \cdot 4.465^2$ $W_{k1} = 1.5408$ $0.5 \cdot 0.0342 \cdot 3^2 = 10.01; \text{Wind Speed 40.03 m/s}$ $W_{k2} = 0.0342 \cdot 4.465^2 = 4.519; \text{Wind Speed 26.868 m/s}$</td>
</tr>
<tr>
<td>Interpolation to find $\alpha_2$</td>
<td>$\alpha_2 = 0.042 + \frac{(0.6718 - 0.5)(0.077 - 0.042)}{0.75 - 0.5} = 0.066$</td>
<td>$M_{ud1} = 1.5408$ MPa $M_{ud2} = 3.0816$ MPa $M_{ud1} = 0.5 \cdot 0.066 \cdot W_k \cdot 3^2$ $M_{ud2} = 0.066 \cdot W_k \cdot 4.465^2$ $W_{k1} = 1.5408$ $0.5 \cdot 0.066 \cdot 3^2 = 5.185; \text{Wind Speed 28.80 m/s}$ $W_{k2} = 0.066 \cdot 4.465^2 = 2.342; \text{Wind Speed 19.35 m/s}$</td>
</tr>
</tbody>
</table>
### Wall support condition K

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( A/ )</th>
<th>( 0.50 )</th>
<th>( 0.75 )</th>
<th>( 1.00 )</th>
<th>( 1.25 )</th>
<th>( 1.50 )</th>
<th>( 1.75 )</th>
<th>( 2.00 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.001</td>
<td>0.019</td>
<td>0.021</td>
<td>0.022</td>
<td>0.023</td>
<td>0.024</td>
<td>0.025</td>
<td>0.026</td>
</tr>
<tr>
<td>0.50</td>
<td>0.001</td>
<td>0.020</td>
<td>0.021</td>
<td>0.022</td>
<td>0.023</td>
<td>0.024</td>
<td>0.025</td>
<td>0.026</td>
</tr>
<tr>
<td>0.10</td>
<td>0.001</td>
<td>0.019</td>
<td>0.020</td>
<td>0.021</td>
<td>0.022</td>
<td>0.023</td>
<td>0.024</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Interpolation to find \( \alpha_2 \)

\[
\alpha_2 = 0.035 + \frac{(0.6718 - 0.5)(0.061 - 0.035)}{0.75 - 0.5} = 0.0535
\]

\[M_{A1} = 1.5408 \text{ MPa} \]

\[M_{A22} = 3.0816 \text{ MPa} \]

\[M_{A1} = 0.5 \cdot 0.0535 \cdot W_k \cdot 3^2 \]

\[M_{A22} = 0.0535 \cdot W_k \cdot 4.465^2 \]

\[W_k = \frac{1.5408}{0.5 \cdot 0.0535 \cdot 3^2} = 6.397; \text{Wind Speed 32.15 m/s} \]

\[W_k = \frac{3.0816}{0.0535 \cdot 4.465^2} = 2.916; \text{Wind Speed 21.60 m/s} \]

### Wall support condition L

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( A/ )</th>
<th>( 0.50 )</th>
<th>( 0.75 )</th>
<th>( 1.00 )</th>
<th>( 1.25 )</th>
<th>( 1.50 )</th>
<th>( 1.75 )</th>
<th>( 2.00 )</th>
</tr>
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<tbody>
<tr>
<td>1.00</td>
<td>0.005</td>
<td>0.017</td>
<td>0.020</td>
<td>0.023</td>
<td>0.026</td>
<td>0.029</td>
<td>0.032</td>
<td>0.035</td>
</tr>
<tr>
<td>0.50</td>
<td>0.005</td>
<td>0.016</td>
<td>0.019</td>
<td>0.022</td>
<td>0.025</td>
<td>0.028</td>
<td>0.031</td>
<td>0.034</td>
</tr>
<tr>
<td>0.10</td>
<td>0.005</td>
<td>0.015</td>
<td>0.018</td>
<td>0.021</td>
<td>0.024</td>
<td>0.027</td>
<td>0.030</td>
<td>0.033</td>
</tr>
</tbody>
</table>

Interpolation to find \( \alpha_2 \)

\[
\alpha_2 = 0.027 + \frac{(0.6718 - 0.5)(0.048 - 0.027)}{0.75 - 0.5} = 0.041
\]

\[M_{A1} = 1.5408 \text{ MPa} \]

\[M_{A22} = 3.0816 \text{ MPa} \]

\[M_{A1} = 0.5 \cdot 0.041 \cdot W_k \cdot 3^2 \]

\[M_{A22} = 0.041 \cdot W_k \cdot 4.465^2 \]

\[W_k = \frac{1.5408}{0.5 \cdot 0.041 \cdot 3^2} = 8.26; \text{Wind Speed 36.36 m/s} \]

\[W_k = \frac{3.0816}{0.041 \cdot 4.465^2} = 3.77; \text{Wind Speed 24.56 m/s} \]

### Wall support condition E

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( A/ )</th>
<th>( 0.50 )</th>
<th>( 0.75 )</th>
<th>( 1.00 )</th>
<th>( 1.25 )</th>
<th>( 1.50 )</th>
<th>( 1.75 )</th>
<th>( 2.00 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.002</td>
<td>0.015</td>
<td>0.019</td>
<td>0.022</td>
<td>0.026</td>
<td>0.030</td>
<td>0.034</td>
<td>0.039</td>
</tr>
<tr>
<td>0.50</td>
<td>0.002</td>
<td>0.014</td>
<td>0.018</td>
<td>0.021</td>
<td>0.025</td>
<td>0.029</td>
<td>0.033</td>
<td>0.038</td>
</tr>
<tr>
<td>0.10</td>
<td>0.002</td>
<td>0.013</td>
<td>0.017</td>
<td>0.020</td>
<td>0.024</td>
<td>0.028</td>
<td>0.033</td>
<td>0.038</td>
</tr>
</tbody>
</table>

Interpolation to find \( \alpha_2 \)

\[
\alpha_2 = 0.028 + \frac{(0.6718 - 0.5)(0.044 - 0.028)}{0.75 - 0.5} = 0.0389
\]

\[M_{A1} = 1.5408 \text{ MPa} \]

\[M_{A22} = 3.0816 \text{ MPa} \]

\[M_{A1} = 0.5 \cdot 0.0389 \cdot W_k \cdot 3^2 \]

\[M_{A22} = 0.0389 \cdot W_k \cdot 4.465^2 \]

\[W_k = \frac{1.5408}{0.5 \cdot 0.0389 \cdot 3^2} = 8.78; \text{Wind Speed 37.48 m/s} \]

\[W_k = \frac{3.0816}{0.0389 \cdot 4.465^2} = 3.973; \text{Wind Speed 25.214 m/s} \]

### Wall support condition H

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( A/ )</th>
<th>( 0.50 )</th>
<th>( 0.75 )</th>
<th>( 1.00 )</th>
<th>( 1.25 )</th>
<th>( 1.50 )</th>
<th>( 1.75 )</th>
<th>( 2.00 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.004</td>
<td>0.019</td>
<td>0.023</td>
<td>0.027</td>
<td>0.031</td>
<td>0.035</td>
<td>0.039</td>
<td>0.043</td>
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<tr>
<td>0.50</td>
<td>0.004</td>
<td>0.018</td>
<td>0.021</td>
<td>0.024</td>
<td>0.028</td>
<td>0.031</td>
<td>0.035</td>
<td>0.039</td>
</tr>
<tr>
<td>0.10</td>
<td>0.004</td>
<td>0.017</td>
<td>0.020</td>
<td>0.023</td>
<td>0.026</td>
<td>0.029</td>
<td>0.032</td>
<td>0.035</td>
</tr>
</tbody>
</table>

Interpolation to find \( \alpha_2 \)

\[
\alpha_2 = 0.014 + \frac{(0.6718 - 0.5)(0.022 - 0.014)}{0.75 - 0.5} = 0.0194
\]

\[M_{A1} = 1.5408 \text{ MPa} \]

\[M_{A22} = 3.0816 \text{ MPa} \]

\[M_{A1} = 0.5 \cdot 0.0194 \cdot W_k \cdot 3^2 \]

\[M_{A22} = 0.0194 \cdot W_k \cdot 4.465^2 \]

\[W_k = \frac{1.5408}{0.5 \cdot 0.0194 \cdot 3^2} = 17.56; \text{Wind Speed 53.14 m/s} \]

\[W_k = \frac{3.0816}{0.0194 \cdot 4.465^2} = 7.967; \text{Wind Speed 35.70 m/s} \]
Appendix 3: PH_Sample: from the façades to the corresponding bays and the relevant sets of geometric characteristics

### Carcar-Cebu-Convent-1.2.E

<table>
<thead>
<tr>
<th>Wall surveyed in the field</th>
<th>Corresponding model</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>

#### Geometric characteristics

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>32.00</td>
<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>10.60</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>3.25</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>3.75</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.15</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.20 x 0.25 x 0.15</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.20</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on facade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

#### Opening Dimensions

<table>
<thead>
<tr>
<th>width (m) of each opening (1st floor)</th>
<th>height (m) of each opening (1st floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
</tr>
<tr>
<td>3 3 1.2 2 1.2 3 3 0</td>
<td>1.6 1.6 1.5 2.6 1.5 1.6 1.6 0</td>
</tr>
</tbody>
</table>

### Carcar -Cebu-Convent-1.2.N

<table>
<thead>
<tr>
<th>Wall surveyed in the field</th>
<th>Corresponding model</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>

235
Bay 1: B Group modified with opening

Bay 2: D Group

### Geometric characteristics

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>26.00</td>
<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>13.00</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>3.25</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>3.75</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.20</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.30 x 0.18 x 0.06</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.20</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

### Width (m) of each opening (1st floor)

<table>
<thead>
<tr>
<th>Width (m) of each opening (1st floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

### Height (m) of each opening (1st floor)

<table>
<thead>
<tr>
<th>Height (m) of each opening (1st floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
</tr>
<tr>
<td>1.45</td>
</tr>
</tbody>
</table>

### Bohol - Cortes-Convent-1.2.S

Wall surveyed in the field  
Corresponding model

Bay 1: D Group
### Geometric characteristics

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>31.60</td>
<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>7.90</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>2.70</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>3.30</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.25</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.30 x 0.25 x 0.25</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.15</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width (m) of each opening (1ˢᵗ floor)</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
<th>1.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>height (m) of each opening (1ˢᵗ floor)</td>
<td>1.1</td>
<td>1.2</td>
<td>1.3</td>
<td>1.4</td>
<td>1.5</td>
<td>1.6</td>
<td>1.7</td>
<td>1.8</td>
</tr>
</tbody>
</table>

**Bohol -Cortes-Convent-1.2.W**

- Wall surveyed in the field
- Corresponding model

**Bay 1 GF: D Group**
### Geometric characteristics

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>10.60</td>
<td>m</td>
</tr>
<tr>
<td>Bay length (2 extra internal walls)</td>
<td>3.53</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>2.70</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>3.30</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Masonry</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.25</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.30 x 0.25 x 0.25</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Overall thickness of 2nd floor</td>
<td>1.20</td>
<td>m</td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf 2nd floor</td>
<td>0.25</td>
<td>m</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width (m) of each opening (2nd floor)</th>
<th>height (m) of each opening (2nd floor)</th>
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</thead>
<tbody>
<tr>
<td>1.1</td>
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</tr>
<tr>
<td>1.20</td>
<td>1.20</td>
</tr>
<tr>
<td>2.50</td>
<td>2.50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width (m) of each opening (1st floor)</th>
<th>height (m) of each opening (1st floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>1.2</td>
</tr>
<tr>
<td>1.15</td>
<td>1.65</td>
</tr>
<tr>
<td>0.9</td>
<td>2</td>
</tr>
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</table>
Carcar-Casa Gorordo-1.1.E

Wall surveyed in the field

Corresponding model

Wall length
Bay length
Height façade first floor
Height façade second floor
Second floor material
Thickness of masonry wall external leaf
Average size of external units (l x h x s)
Density of masonry
Thickness of LBS
Density of LBS
Horizontal structure weight on façade
Weight of roof on façade

<table>
<thead>
<tr>
<th>Geometric characteristics</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>38.00</td>
<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>9.50</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>2.50</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>3.40</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.15</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.35 x 0.25 x 0.15</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width (m) of each opening (1st floor)</th>
<th>height (m) of each opening (1st floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>1.1</td>
</tr>
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<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>1.3</td>
<td>1.3</td>
</tr>
<tr>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>1.8</td>
<td>1.8</td>
</tr>
<tr>
<td>1.45</td>
<td>1.05</td>
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<tr>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td>1.05</td>
<td>1.90</td>
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<tr>
<td>1.05</td>
<td>1.90</td>
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<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Carcar-Casa Gorordo-1.1.S

Wall surveyed in the field

Corresponding model

Bay 1-2-3a: B Group modified with opening

Bay 3b-4: B Group modified with opening

Bay 1-3: B Group modified with opening
### Geometric characteristics

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>12.20</td>
<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>4.06</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>2.50</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>3.40</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.30 x 0.25 x 0.10</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width (m) of each opening (1st floor)</th>
<th>height (m) of each opening (1st floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>1.2</td>
</tr>
<tr>
<td>1.35</td>
<td>1.45</td>
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</tbody>
</table>

### Carcar-Casa Gorordo-1.1.W

- Wall surveyed in the field
- Corresponding model

Bay 1: B Group modified with opening
### Geometric characteristics

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
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<tbody>
<tr>
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</tr>
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<td>Height façade first floor</td>
<td>2.50</td>
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</tr>
<tr>
<td>Height façade second floor</td>
<td>3.40</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.30 x 0.30 x 0.10</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width (m) of each opening (1st floor)</th>
<th>height (m) of each opening (1st floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>1.2</td>
</tr>
<tr>
<td>2.80</td>
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</table>

**Carcar-Balay nga Tisa-1.1.E**

Wall surveyed in the field  
Corresponding model
### Geometric characteristics

<table>
<thead>
<tr>
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<th>Value</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>11.00</td>
<td>m</td>
</tr>
<tr>
<td>Bay length (1 extra internal walls)</td>
<td>5.50</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>2.05</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>3.35</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.30 x 0.60 x 0.15</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.20</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width (m) of each opening (1st floor)</th>
<th>height (m) of each opening (1st floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>1.1</td>
</tr>
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<td>1.35</td>
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<tr>
<td>1.45</td>
<td>1.20</td>
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### Carcar-Balay nga Tisa-1.1.S

- Wall surveyed in the field
- Corresponding model

Bay 1&2: B Group modified with opening
### Geometric characteristics

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<tr>
<td>Bay length</td>
<td>9.92</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>2.05</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>3.35</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.30 x 0.40 x 0.10</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.20</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

### Width (m) of each opening (1st floor)

<table>
<thead>
<tr>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
<th>1.8</th>
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<tr>
<td>1.90</td>
<td>1.06</td>
<td>1.06</td>
<td>1.06</td>
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<td>0.00</td>
<td>1.04</td>
<td>1.04</td>
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### Height (m) of each opening (1st floor)

<table>
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<th>1.1</th>
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<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
<th>1.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.04</td>
<td>1.04</td>
<td>1.20</td>
<td>1.04</td>
<td>1.04</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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**Carcar-Balay nga Daku-1.1.E1**

Wall surveyed in the field

Corresponding model

---

Bay 1: B Group modified with opening

Bay 2: D Group modified with 3 openings
Geometric characteristics

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
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<tbody>
<tr>
<td>Wall length</td>
<td>12.50</td>
<td>m</td>
</tr>
<tr>
<td>Bay length (2 extra internal walls)</td>
<td>4.16</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>3.20</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>2.50</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.30 x 0.20 x 0.15</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.20</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width (m) of each opening (1st floor)</th>
<th>height (m) of each opening (1st floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
</tr>
<tr>
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Carcar-Balay nga Daku-1.1.N3

Wall surveyed in the field

Corresponding model
### Geometric characteristics

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<tr>
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<th>Value</th>
<th>Unit</th>
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</thead>
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</tr>
<tr>
<td>Bay length</td>
<td>9.10</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>3.20</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>2.50</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.14</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.34 x 0.20 x 0.14</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.20</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

### Width (m) of each opening (1st floor)

1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8

### Height (m) of each opening (1st floor)

1.50 0.32 0.00 0.00 0.00 0.00 0.00 1.90 0.22 0.00 0.00 0.00 0.00 0.00

### Carcar-Balay nga Daku-Cultural-1.1.N2

- **Wall surveyed in the field**
- **Corresponding model**

Bay 1: B Group modified with opening

### Geometric characteristics

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<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>6.55</td>
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<tr>
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<td>3.20</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>2.50</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wooden load bearing structure (LBS)</td>
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### Carcar-Balay nga Daku-Cultural-1.1.N1

**Wall surveyed in the field**

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<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>6.55</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>3.20</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>2.50</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.14</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.34 x 0.20 x 0.14</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.20</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
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**Cebu-Boljoon-School-1.4.S**

**Wall surveyed in the field**

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<tbody>
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<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>6.55</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>3.20</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>2.50</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.14</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.34 x 0.20 x 0.14</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.20</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
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### Geometric Characteristics

<table>
<thead>
<tr>
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<th>Value</th>
<th>Unit</th>
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<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>5.5</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>3.00</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>2.90</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.40 x 0.50 x 0.12</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.20</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

### Width (m) of Each Opening (1st Floor) vs. Height (m) of Each Opening (1st Floor)

<table>
<thead>
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<th>height (m)</th>
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<tbody>
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<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>1.2</td>
<td>1.60</td>
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<td>1.3</td>
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<td>1.4</td>
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<tr>
<td>1.5</td>
<td>0.00</td>
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<tr>
<td>1.6</td>
<td>0.00</td>
</tr>
<tr>
<td>1.7</td>
<td>0.00</td>
</tr>
<tr>
<td>1.8</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### Cebu-Boljoon-School-1.4.E

- Wall surveyed in the field
- Corresponding model

Bay 1: B Group modified with opening
### Geometric characteristics

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>13.00</td>
<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>4.33</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>3.00</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>2.90</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.40 x 0.50 x 0.12</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.20</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Width (m) of each opening (1st floor)</th>
<th>Height (m) of each opening (1st floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
</tr>
<tr>
<td>1.60 1.30 1.60 0.00 0.00 0.00 0.00</td>
<td>1.60 1.90 1.60 0.00 0.00 0.00 0.00</td>
</tr>
</tbody>
</table>

**Cebu-Boljoon_Blockhouse-1.3.N**

Wall surveyed in the field

Corresponding model

Bay 1 & 2 GF: C Group
### Geometric characteristics

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>8.70</td>
<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>4.35</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>5.70</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>3.30</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>masonry</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.40 x 0.50 x 0.12</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width (m) of each opening (2nd floor)</th>
<th>height (m) of each opening (2nd floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
</tr>
<tr>
<td>1.95 0.00 0.00 0.00 0.00 0.00 0.00 0.00</td>
<td>2.80 0.00 0.00 0.00 0.00 0.00 0.00 0.00</td>
</tr>
</tbody>
</table>

**Cebu-Boljoon_Blockhouse-1.3.E**

Wall surveyed in the field

Corresponding model
<table>
<thead>
<tr>
<th>Geometric characteristics</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>15.00</td>
<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>7.50</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>5.70</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>3.30</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Masonry</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.40 x 0.50 x 0.12</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

<p>| | | | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>width (m) of each opening (2nd floor)</td>
<td>1.1</td>
<td>1.2</td>
<td>1.3</td>
<td>1.4</td>
<td>1.5</td>
<td>1.6</td>
<td>1.7</td>
<td>1.8</td>
<td>1.1</td>
<td>1.2</td>
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<td>0.00</td>
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<td>2.80</td>
<td>2.80</td>
<td>2.80</td>
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<tr>
<td></td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

|                           | 1.1    | 1.2    | 1.3    | 1.4    | 1.5    | 1.6    | 1.7    | 1.8    | 1.1    | 1.2    | 1.3    | 1.4    |
|                           | 1.10   | 1.10   | 0.00   | 0.00   | 0.00   | 0.00   | 0.00   | 0.00   | 1.10   | 1.10   | 0.00   | 0.00   |

---

**Cebu-Boljoon_Blockhouse-1,3,W**

Wall surveyed in the field

Corresponding model

Bay 1&3 UF: B Group

Bay 2&4 UF: B Group
Bay 1 GF: D Group

Bay 2 GF: C Group

<table>
<thead>
<tr>
<th>Geometric characteristics</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>15.00</td>
<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>7.50</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>5.70</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>3.30</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.40 x 0.50 x 0.12</td>
<td>m</td>
</tr>
<tr>
<td>Density of masonry</td>
<td>14.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on facade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width (m) of each opening (2nd floor)</th>
<th>height (m) of each opening (2nd floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
</tr>
<tr>
<td>1.90 1.90 0.00 0.00 0.00 0.00 0.00 0.00</td>
<td>1.90 3.00 0.00 0.00 0.00 0.00 0.00 0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width (m) of each opening (1st floor)</th>
<th>height (m) of each opening (1st floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
</tr>
<tr>
<td>1.85 0.00 0.00 0.00 0.00 0.00 0.00 0.00</td>
<td>3.5 0.00 0.00 0.00 0.00 0.00 0.00 0.00</td>
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</tbody>
</table>

Cebu-Boljoon-Convent-1.2.W2

Wall surveyed in the field

Corresponding model

Bay 1&2: D Group
### Geometric characteristics

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>16.50</td>
<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>8.25</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>3.77</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>2.92</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.28 x 0.26 x 0.08</td>
<td>m</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.20</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.50</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width (m) of each opening (1st floor)</th>
<th>height (m) of each opening (1st floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>1.2</td>
</tr>
<tr>
<td>1.1</td>
<td>1.2</td>
</tr>
<tr>
<td>1.50</td>
<td>0.90</td>
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</table>

#### Cebu-Boljoon-Convent-1.2.N

Wall surveyed in the field

Corresponding model

Bay 1,2,4,5: C Group

### Geometric characteristics

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>37.50</td>
<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>7.50</td>
<td>m</td>
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<tr>
<td>Height façade first floor</td>
<td>3.77</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>2.92</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.28 x 0.26 x 0.08</td>
<td>m</td>
</tr>
</tbody>
</table>
Thickness of LBS: 0.20 m
Density of LBS: 10.00 kN/m³
Horizontal structure weight on facade: 0.50 kN/m²
Weight of roof on facade: 3.00 kN/m²

<table>
<thead>
<tr>
<th>width (m) of each opening (1st floor)</th>
<th>height (m) of each opening (1st floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
</tr>
<tr>
<td>2.05 0.00 0.00 0.00 0.00 0.00 0.00 0.00</td>
<td>2.20 0.00 0.00 0.00 0.00 0.00 0.00 0.00</td>
</tr>
</tbody>
</table>

Cebu-Boljoon-Convent-1.2.W1
Wall surveyed in the field
Bay 1&2: D Group

Cebu-Boljoon-Convent-1.2.S_2
Wall surveyed in the field

<table>
<thead>
<tr>
<th>Geometric characteristics</th>
<th>Value</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>8.15</td>
<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>8.15</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>3.77</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>2.92</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.28 x 0.26 x 0.08</td>
<td>m</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.20</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on facade</td>
<td>0.50</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width (m) of each opening (1st floor)</th>
<th>height (m) of each opening (1st floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
</tr>
<tr>
<td>2.05 0.00 0.00 0.00 0.00 0.00 0.00 0.00</td>
<td>2.20 0.00 0.00 0.00 0.00 0.00 0.00 0.00</td>
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### Geometric characteristics

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<th>Value</th>
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<td>Wall length</td>
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<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>13.10</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>3.77</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>2.92</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.28 x 0.26 x 0.08</td>
<td>m</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.20</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.50</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

### Cebu-Boljoon-Convent-1.2.S_1

Wall surveyed in the field

Corresponding model
Bay 4: B Group modified with opening

<table>
<thead>
<tr>
<th>Geometric characteristics</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>33.10 m</td>
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</tr>
<tr>
<td>Bay length</td>
<td>6.62 m</td>
<td></td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>3.77 m</td>
<td></td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>2.92 m</td>
<td></td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.10 m</td>
<td></td>
</tr>
<tr>
<td>Average size of external units (1 x h x s)</td>
<td>0.28 x 0.26 x 0.08 m</td>
<td></td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.20 m</td>
<td></td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00 kN/m$^3$</td>
<td></td>
</tr>
<tr>
<td>Horizontal structure weight on facade</td>
<td>0.50 kN/m$^2$</td>
<td></td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00 kN/m$^2$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width (m) of each opening (1$^{st}$ floor)</th>
<th>height (m) of each opening (1$^{st}$ floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>1.2</td>
</tr>
<tr>
<td>1.07</td>
<td>0.9</td>
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</table>

Bohol-Alburquerque-Convent-1.2.W

<table>
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<tr>
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<th>Value</th>
<th>Unit</th>
</tr>
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<tbody>
<tr>
<td>Wall length</td>
<td>35.35 m</td>
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</tr>
<tr>
<td>Bay length</td>
<td>11.78 m</td>
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</tr>
<tr>
<td>Height façade first floor</td>
<td>5.70 m</td>
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</table>
### Bohol-Alburquerque-Convent-1.2.S

**Wall surveyed in the field**

<table>
<thead>
<tr>
<th>Corresponding model</th>
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</thead>
</table>

<table>
<thead>
<tr>
<th>Height façade second floor</th>
<th>3.30 m</th>
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</thead>
<tbody>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.15 m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.38 x 0.18 x 0.08 m</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.20 m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00 kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on facade</td>
<td>0.24 kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00 kN/m²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width (m) of each opening (1st floor)</th>
<th>height (m) of each opening (1st floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>1.2</td>
</tr>
<tr>
<td>3.20</td>
<td>3.20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Geometric characteristics</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>Bay length</td>
<td>9.88</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>5.70</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>3.30</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.15 m</td>
<td></td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.38 x 0.18 x 0.08 m</td>
<td></td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.20</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on facade</td>
<td>0.24 kN/m²</td>
<td></td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

Bay 1, 2 & 3: D Group
**Bohol-Dmiao - Convent-1.2.N**

Wall surveyed in the field

<table>
<thead>
<tr>
<th>Geometric characteristics</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall length</td>
<td>15.20</td>
<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>7.60</td>
<td>m</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>3.31</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>4.77</td>
<td>m</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
<td></td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.15</td>
<td>m</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
<td>0.25 x 0.40 x 0.05</td>
<td>m</td>
</tr>
<tr>
<td>Thickness of LBS</td>
<td>0.10</td>
<td>m</td>
</tr>
<tr>
<td>Density of LBS</td>
<td>10.00</td>
<td>kN/m³</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
<td>kN/m²</td>
</tr>
</tbody>
</table>

**Bohol-Dmiao - Convent-1.2.E**

Wall surveyed in the field

<table>
<thead>
<tr>
<th>width (m) of each opening (1st floor)</th>
<th>height (m) of each opening (1st floor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
</tr>
<tr>
<td>1.20 2.87 1.20 0.00 0.00 0.00 0.00 0.00</td>
<td>1.60 4.55 1.60 0.00 0.00 0.00 0.00 0.00</td>
</tr>
<tr>
<td>Geometric characteristics</td>
<td>Value</td>
</tr>
<tr>
<td>-----------------------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>Wall length</td>
<td>27.02</td>
</tr>
<tr>
<td>Bay length</td>
<td>13.51</td>
</tr>
<tr>
<td>Height façade first floor</td>
<td>3.31</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>4.77</td>
</tr>
<tr>
<td>Second floor material</td>
<td>Wooden load bearing structure (LBS)</td>
</tr>
<tr>
<td>Thickness of masonry wall external leaf</td>
<td>0.15</td>
</tr>
<tr>
<td>Average size of external units (l x h x s)</td>
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<tr>
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</tr>
<tr>
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<td>10.00</td>
</tr>
<tr>
<td>Horizontal structure weight on façade</td>
<td>0.24</td>
</tr>
<tr>
<td>Weight of roof on façade</td>
<td>3.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>width (m) of each opening (1st floor)</th>
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</thead>
<tbody>
<tr>
<td>1.1</td>
<td>1.2</td>
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<tr>
<td>1.50</td>
<td>2.00</td>
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</table>

**Bohol-Dmiao - Convent-1.2.S**

Wall surveyed in the field

Corresponding model
### Geometric characteristics

<table>
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<tr>
<th></th>
<th>Value</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>Wall length</td>
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<td>m</td>
</tr>
<tr>
<td>Bay length</td>
<td>6.63</td>
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<tr>
<td>Height façade first floor</td>
<td>3.31</td>
<td>m</td>
</tr>
<tr>
<td>Height façade second floor</td>
<td>4.77</td>
<td>m</td>
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<td>m</td>
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<td>m</td>
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<td>kN/m³</td>
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<td>Weight of roof on façade</td>
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<td>1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8</td>
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<td>2.10 1.50 1.50 2.10 1.50 1.50 0.00 0.00</td>
</tr>
</tbody>
</table>
Bibliography


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