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Dynamic Development Contests

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Public, private, and not-for-profit organizations find advanced technology and product development projects challenging to manage due to the time and budget pressures, and turn to their development partners and suppliers to address their development needs. We study how dynamic development contests with enriched rank-based incentives and carefully-tailored information design can help these organizations leverage their suppliers for their development projects while seeking to minimize project lead time by stimulating competition among them. We find that an organization using dynamically-adjusted flexible rewards can achieve the minimum expected project lead time at a significantly lower cost than a fixed-reward policy. Importantly, the derived flexible-reward policy pays the minimum expected reward (i.e., achieves the first best). We further examine the case where the organization may not have sufficient budget to offer a reward that attains the minimum expected lead time. In this case, the organization uses the whole reward budget and supplements it with strategic information disclosure. Specifically, we derive an optimal information disclosure policy whereby any change in the state of competition is disclosed immediately with some probability that is weakly increasing over time. Our results indicate that dynamic rewards and strategic information disclosure are powerful tools to help organizations fulfill their development needs swiftly and cost effectively.

Key words: Contest, Development, Information Design, Innovation, Technology, Tournament

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1. Introduction

Organizations worldwide face the challenges of developing advanced technologies and products under time and budget pressures. These pressures are more pronounced in time-sensitive development endeavors with concerns such as national security, critical health care, and first-mover advantage (Stalk 1988). To overcome time and budget pressures, organizations increasingly leverage their supply base to tackle demanding problems that require rigorous work with multiple major milestones at the shortest lead time. For instance, the US Department of Defense (DoD) seeks to develop hypersonic (faster than five times the speed of sound) missiles as swiftly as possible by leveraging suppliers like Lockheed Martin and Raytheon. Similarly, Tesla induces major battery suppliers such as Panasonic, LG, and Samsung to develop the next generation 4680 Battery Cell at the earliest lead time to stay ahead of the electric vehicle (EV) industry (INSIDE EVs 2021).[1] The main challenge of such development settings is to respond to competitive and market pressures with demanding milestones and compressed timelines by inducing suppliers to make significant resource investments. In this paper, we study how organizations can go beyond contractual development, and benefit from a "development contest" approach and its advanced elements such as rank-based incentives and strategic information disclosure to stimulate development effort from competing suppliers to minimize project lead time while keeping the cost of incentives in check.

To see the important needs and challenges of development contests, consider the development of weapon systems by DoD, which we studied closely. While individual DoD programs have their own unique contextual adaptations, we discern an archetypal program abstracted from the hypersonic weapons development program. Driven by the success of rival nations in demonstrating their hypersonic missiles with recent tests, DoD is racing to develop its own hypersonic weapons such as missile and interceptor systems (The Washington Post 2022).[2] The development process of such weapons, however, is quite complex and requires the successful completion of major phases such as the development of a proof of concept and system validation (CRS Report 2023). To economize on resources and to accelerate launch, DoD leverages major defense industrial base (DIB) firms such as Lockheed Martin and Raytheon, engaging them in winner-takes-all competitions and rewarding the winner of each contest with a substantial supply contract. Each contest has a clear scope: achieve hypersonic speed and sufficient height and range. DIB firms participate in such a contest and race to complete the development project at the earliest lead time with the hope of winning the DoD supply contract that can be worth billions of dollars. While managing such a contest, DoD needs to be mindful of contestants' intellectual property and cannot share any technical details of the deliverables with any third party (CRS Report 2022). However, DoD also holds the authority to fully control the information flow about whether and when certain milestones are completed in the interest of balancing confidentiality and competition (CNN 2022b). While there are minor variations in how these contests are implemented in different contexts, the overarching goal seems to be achieving the targets in the shortest time possible.

There are a number of key distinguishing aspects of development contests as exemplified above:

1. **Lead-Time Minimization as a Key Goal in a Contest Setting.** The development organization (hereafter, "principal") is responding to a market or competitive imperative and engaged in sourcing a product (e.g., a hypersonic missile or an EV battery) at a quality level acceptable

[1] According to Larson and Gray (2014), in moderate to high-technology industries, a six-month delay in a new product launch time can result in a 35% loss of market share.

[2] In the backdrop of rival nations creating a "Sputnik moment" by launching hypersonic weapons before the US, Vice Admiral Johnny Wolfe, director of the Navy's Strategic Systems program remarked: "The need was not there. The need is now there, which is why we've got a sense of urgency to get after this" (CNN 2022a).

to the use case (e.g., with hypersonic speed and suitable for the proposed combat setting) and is focused on minimizing project lead time while keeping the cost of incentives in check. Lead-time minimization may not be a major priority for exploratory innovation contests, where the focus is to leverage the contestants to maximize the quality of the best solution(s) or contestants' total efforts. In addition, the use of the contest approach in minimizing lead time goes beyond traditional operations research (scheduling) approaches focused on complex internal projects.

2. **Expert-Sourcing.** Due to complexity of problems and the enormity of investments necessary, the principal in a development contest typically works with a few strategic suppliers (hereafter, "agents") that have a proven expertise in the specific technologies and are able to make significant resource investments. While such expert sourcing is involved in its own ways, it also offers the principal some additional commitment power and flexibility to control the information flow. Specifically, the principal who adjudicates the completion of certain milestones has the ability to decide whether, when, and how to disclose this information.[3] For example, DoD strategically withheld information about a successful test by Lockheed Martin in the hypersonic weapons program (CNN 2022b). The long-term relationship between the principal and suppliers also induces the principal to share credible and truthful information, because any attempt to fool the suppliers may be detrimental to the principal's reputation, the quality of the relationship with suppliers, and long-run profits (Fudenberg and Levine 2009).
3. **Complex Multi-Phase Structure.** The development setting described above (such as hypersonic missiles or EV batteries) goes beyond simpler projects such as designing logos, so effective management entails incentivizing agents through a development process with multiple major milestones (such as proof of concept and system validation in hypersonic missiles). Importantly, each agent has to complete these milestones independently and cannot bypass a certain milestone, and the principal cannot bring any of these agents "up to speed," either. Specifically, due to complexity involved in development settings, suppliers often draw on their proprietary knowledge to achieve ambitious functional targets. As sharing of their deliverables may also spill over this proprietary knowledge, a principal such as DoD has an obligation to keep the technical details or technologies used by agents to complete a milestone confidential (CRS Report 2022).

Due to these characteristics, development contests are fundamentally different from more exploratory innovation contests. Yet, they also share the common property of all contests: the

[3] The principal can easily prevent agents from sharing information by asking them to sign confidentiality agreements. Even when the principal does not impose such restrictions, in most development settings, only the principal can credibly verify the successful completion of a milestone and any information revealed by agents will be cheap talk. In certain development settings such as vaccine development, however, the completion of certain milestones (e.g., preclinical) may be published by agents. This situation can be captured using the full information disclosure policy in our model framework. We shall show that it is in the best interest of the principal to control the flow of information by imposing appropriate restrictions in order to strategically manage agents' incentives.

tendency to engage in and benefit from competition among contestants without having to pay all of them. We extend and adapt the contest paradigm to development settings and contribute additional novel features such as enriched rank-based incentives and carefully-tailored information design to stimulate competitive effort from suppliers.

Although development contests have the potential for significant economic and societal impact, they are not without challenges. First, as agents tackle a difficult problem, there is a danger that some agents will lose interest in the absence of sufficient progress. Thus, an effective mechanism should dynamically keep agents' incentives alive to minimize the development lead time. Second, major development projects require substantial investment from agents so the cost of incentivizing effort from agents is significant. Considering that cost of incentives is usually covered by taxpayers' money or donations, an effective mechanism should avoid overpaying agents. Therefore, we aim to understand how contest organizers can design such contests for maximum efficiency – to minimize lead time of development while keeping the cost of incentives in check.

To capture the key characteristics of a dynamic development contest, we seek to build a parsimonious stylized model where two agents compete to complete a two-stage development project by exerting costly effort over a continuous time frame (we show in the Online Appendix how our results scale to more than two agents). Successful completion of a stage (success) for an agent arrives at a random point in time where the rate of arrival for each success increases with the agent's effort. An agent's success is not observable by another agent so it is up to the principal whether and when to share this information. The contest ends when one of the agents achieves two successes at which point this winning agent is given a pre-determined reward. The principal commits to a *reward schedule* about how this reward changes over time and an *information disclosure policy* which specifies how the principal will disclose information throughout the contest (e.g., no or full information sharing, probabilistic information sharing) at the beginning of the contest.

We first focus on the problem of a principal with no budget constraint. We establish that such a principal can utilize a fixed-reward dynamic contest that does not change the reward over time to solicit agents to exert their best efforts and attain the absolute minimum expected lead time. Yet, we also find that such a principal significantly overpays agents. Specifically, by utilizing a carefully designed flexible-reward schedule that increases the reward over time, the principal can still achieve the absolute minimum expected lead time with 20% less cost of incentives/reward on average, which could mean substantial savings in development settings that can cost hundreds of millions of dollars. Better yet, a principal who conditions the contest's reward on the state in which the contest ends can simplify the reward schedule tremendously while reaping all the benefits of the flexible-reward design. Specifically, under any information disclosure policy, the principal can organize an optimal flexible-reward contest consisting of just two reward levels — a smaller (resp., larger) reward if the loser of the contest has not achieved (resp., also achieved) partial progress (we use the terms "partial progress" and "first success" interchangeably) — to attain the absolute minimum expected lead time by paying the absolute minimum expected reward (i.e., the first best).[4] Importantly, we prove that the first-best outcome can be achieved if and only if the principal's budget is greater than a certain threshold.

We next analyze a budget-constrained principal that cannot set a sufficiently large reward to achieve the absolute minimum expected lead time. In this case, the principal can organize a fixed-reward contest that uses all of the reward budget and utilize information as a strategic commodity to motivate suppliers. We first show that either too much or too little information is disclosed by canonical information disclosure policies proposed in the extant literature such as no information disclosure, full information disclosure (e.g., Halac et al. 2017), cyclic information disclosure with periodic updates about the state of partial progress every fixed periods of time (e.g., Bimpikis et al. 2019, Ely et al. 2022), and deterministic delay policy that commits to share partial progress after a fixed delay (e.g., Ely 2017). By harnessing insights generated from the above analysis and a dynamic Bayesian persuasion approach, we propose a probabilistic state disclosure policy, which we call PSD, wherein the principal commits to disclose the state of partial progress (stochastically) at a constant rate after a pre-determined initial silent period where no information is disclosed. PSD improves upon the above mainstream policies as it provides the ideal amount of incentives after the initial silent period. While this policy helps us tease out the benefit of probabilistic disclosure, it still shares a common shortcoming of the mainstream policies: it wastes incentives early in the contest because it utilizes the same approach of state-based information disclosure.

By taking a fundamentally different approach, we characterize an optimal information disclosure policy, which we call $PCSD$, that minimizes the expected lead time of the contest by optimally calibrating the flow of information and smoothing agents' incentives over time. As a novel approach, $PCSD$ probabilistically discloses the change of state in the competition rather than the state itself. Specifically, the principal commits to disclose any change in the state of partial progress immediately with some weakly increasing time-dependent probability (and no other time). In addition to proposing $PCSD$, we find that by utilizing a flexible-reward schedule on top of $PCSD$, the principal can reduce the cost of incentives without hindering project lead time. Our results suggest that enriched rank-based incentives and carefully-tailored information design can be powerful tools to incentivize development efforts of suppliers without overpaying them. Accordingly, we generate useful managerial insights in terms of when information disclosure and flexible rewards are most effective and how these instruments can be utilized to motivate suppliers.

[4] We further extend these results in §EC.2 of the Online Appendix by characterizing an optimal flexible-reward contest that minimizes the project lead time at the first-best cost to a setting with more than two agents and show that its cost savings benefits over the optimal fixed-reward contest is increasing with the number of agents.

Besides these important managerial insights, our work makes significant technical contributions by bringing together the already involved frameworks of dynamic contests and dynamic information design. While dynamic information design in itself is involved, Ely (2017) designs optimal information disclosure policies in single-agent settings without competition. Yet, as emphasized in the survey by Horner and Skrzypacz (2017) and a concurrent recent study by Ely et al. (2022), characterizing fully optimal information disclosure policies in dynamic environments involving competition is not an easy endeavor as the disclosure policy is a high-dimensional object. Probably for that reason, prior dynamic contest studies (e.g., Halac et al. 2017, Bimpikis et al. 2019) focus on *well-performing* disclosure policies without claiming any optimality. We contribute to this literature by proposing an optimal information disclosure policy that harnesses a dynamic Bayesian persuasion approach with a unique feature of using probabilistic change-of-state disclosures. Our analysis uncovers several important tradeoffs and insights that may be of use to future studies in other applications of dynamic information design.

Finally, our paper makes some conceptual contributions by expanding the scope and applications of contests to go beyond exploratory innovation contests, which have largely focused on maximizing the number and quality of solutions for a broadly defined problem given a fixed time frame. Unlike the more open-ended exploration settings, development project settings have tighter focus and scope, and the goal is to complete the project in the least amount of time. Therefore, the development setting involves less exploration of uncertain domains and more exploitation of fundamental knowledge to develop cutting-edge products and services (March 1991). We demonstrate how the contest approach can be effectively utilized in development settings.

2. Related Literature

Our paper is related to four streams of literature: (i) races with no scope for information design, (ii) innovation contests with no scope for information sharing, (iii) contests with one-time or dynamic information sharing, and (iv) Bayesian persuasion.

Our work relates to the race literature in the sense that agents compete in time. Following the early work of Loury (1979), Dasgupta and Stiglitz (1980), and Lee and Wilde (1980) on static R&D races, Harris and Vickers (1987) present a model of dynamic race as a discrete time tug-of-war with uncertainty between two agents where a prize is won by the first agent to achieve a given lead over her rival. Cao (2014) considers a similar problem in continuous time with two agents and shows that effort provision increases in competition intensity. Our results echo similar effort provision behavior by agents in a dynamic contest which ends as soon as the first agent completes a multi-stage task. In particular, the supplier with no progress loses incentives to exert any effort if she realizes that her competitor has progressed to the second stage. One key distinguishing factor between our work and the above literature is that in our model the principal controls the flow of information mid-contest whereas in the above papers the agents fully observe each other's progress.

A majority of papers in the race literature apply the dynamic race framework to study patent races with a high-level social planner (e.g., government) comparing some general patent rules rather than optimizing a specific race (e.g., Perry and Vincent 2002, Fershtman and Markovich 2010, Judd et al. 2012). Thus, a major difference between a dynamic race and a dynamic development contest is the presence of a principal organizer that determines specific contest rules to optimize the latter. As discussed in §1, organizations such as DoD act as principal organizers in their dynamic development contests by determining the reward schedule and information disclosure policies. Accordingly, our setting allows the use of enriched rank-based incentives and carefully-tailored information design that cannot exist in a race environment.

Our work has some connection to the innovation contest literature in the sense that a principal organizer incentivizes a group of agents with rank-based incentives. This literature is pioneered by Taylor (1995) and Terwiesch and Xu (2008) who study how many participants to let in a contest. In a more general framework, Ales et al. (2017, 2021) derive conditions for the optimality of winner-takes-all and open-entry contests. Extensions to heterogeneous agents, internal innovation contests, and multiple attributes are considered by Körpeoğlu and Cho (2018), Nittala et al. (2022), and Hu and Wang (2021), respectively. More recent work investigates procurement decisions (Chen et al. 2022), participation (Stouras et al. 2022), duration (Korpeoglu et al. 2021), curation (Khorasani et al. 2020a), supplier collaboration (Shalpegin et al. 2020), teamwork (Candoğan et al. 2020), and the impact of running parallel contests (Körpeoğlu et al. 2022, Stouras et al. 2020). While this literature has provided valuable insights on innovation contests that aim to crowdsource the best-quality solution to a problem, we are interested in analyzing a dynamic development contest that differs from innovation contests in three key properties discussed in §1. These distinct properties allow us to consider new degrees of freedom available to development organizations including flexible rewards and information design due to their dynamic strategic interactions with a small number of development partners. Moreover, we study the design of incentives in a dynamic multi-stage framework to show how an organization can encourage its suppliers and development partners to reduce project lead time by exerting their best efforts without overpaying them.

To study the role of information sharing in contests, a relatively recent stream of research has been focusing on how to provide interim performance feedback in a two-period, two-agent framework (e.g., Aoyagi 2010, Ederer 2010, Goltsman and Mukherjee 2011). Mihm and Schlapp (2019) apply this framework to innovation contests and add to this stream by considering a private disclosure policy in addition to full and no information disclosure policies. Schlapp and Mihm (2018) build on this work and prove that, in a wide class of feedback policies, there can be no feedback policy that outperforms pre-committed truthfulness. Private feedback in the form of screening intermediate submissions of $n \geq 2$ solvers is considered by Khorasani et al. (2020b) who show that different contest environments require different ways of balancing screening specificity and sensitivity. Our continuous-time framework enables us to add to this literature by uncovering insights on how an organization should choose the rate and timing of information disclosure, rather than giving a one-time interim feedback, in order to dynamically manage incentives of suppliers.

Our model is closer to a few papers that study dynamic information disclosure in contests using a continuous-time framework. Halac et al. (2017) consider an exploratory experimentation framework similar to the one in Bonatti and Horner (2011) (who examine moral hazard in teams) to study a dynamic contest where agents compete to obtain a single-stage innovation whose feasibility is initially unknown and the principal wishes to maximize the probability of obtaining the innovation. They show that in a setting with full information disclosure, a winner-takes-all policy dominates others. Building on this framework, Bimpikis et al. (2019) study a two-stage exploratory innovation contest with a separate reward for each stage where two agents compete and the feasibility of the first stage is initially unknown. They show that allocating the reward across stages can aid the principal in disseminating positive news regarding the project's feasibility to agents, thereby encouraging increased effort in the contest.[5] They also show that full information disclosure after an initial silent period outperforms no, full, or constant probabilistic disclosure under certain conditions. Unlike the uncertain exploratory environments mentioned above, our paper concentrates on development settings that prioritize utilizing existing knowledge to swiftly and cost-effectively create products and services. We study if, when (Theorem 1), and how (Theorems 1 and 2) flexible rewards and strategic dynamic information disclosure can *optimally* manage incentives. Bimpikis et al. (2019) also examine a winner-takes-all setting with one final reward and no uncertainty about first-stage feasibility (similar to our environment) and show that a cyclic disclosure policy may dominate no or full disclosure. We expand on their work by obtaining a novel optimal information disclosure policy in this case without imposing any restriction on the disclosure policy.

Recently, Ely et al. (2022) study a fixed-reward contest where agents exert effort to obtain a single success, and derive mechanisms that maximize the expected total effort over the duration of the contest. Their mechanisms do not necessarily minimize lead time though because prolonging the contest to collect larger total effort may be in the best interest of the principal.

We make three key contributions to this scant literature. First, we show that using carefully-designed flexible rewards not only leads to substantial savings compared to fixed rewards, it also

[5] Unlike Bimpikis et al. (2019), we consider a single reward given to the first agent to complete both stages. However, we extend our results in §EC.2.1 to a case where the principal gives rewards in two stages, and show that our fully optimal information disclosure policy can be easily extended and remains optimal in this setting. We then numerically verify that allocating the entire budget to the final reward is optimal in multi-stage development settings.

helps the principal achieve the absolute minimum expected lead time by paying the absolute minimum expected reward (i.e., the first best). This result has important implications in any development setting where time and budget are of the essence. Second, we demonstrate the value of strategic probabilistic disclosure as a non-monetary incentive instrument in competitive settings by proposing a mechanism PSD that combines probabilistic disclosure with the intuition of mechanisms proposed in the literature. Third, and most importantly, we characterize an optimal information disclosure policy (i.e., $PCSD$) that minimizes project lead time when the principal is budget-constrained. Importantly, this optimal policy only discloses the change in the state of partial progress as opposed to disclosing the state of competition as in prior contest studies.

Lastly, our work is related to the growing literature of Bayesian persuasion pioneered by Rayo and Segal (2010) and Kamenica and Gentzkow (2011). The bulk of the work on Bayesian persuasion focuses on static information design where a principal shares information with each agent only once (e.g., Rayo 2013, Bergemann and Morris 2019, Kamenica 2019). Indeed, there is a growing literature that applies this static framework to operational problems (see de Véricourt et al. 2021, Drakopoulos et al. 2021, Küçükgül et al. 2022, Candogan 2020, and references therein). Recently, Ely (2017) introduces a dynamic persuasion mechanism where the principal dynamically shares information with agent(s) based on an exogenously given state of a stochastic process.[6] Our framework is much more complex because the state of the stochastic process (i.e., the belief of each agent about her opponent's partial progress) that the principal bases its information design on is not exogenous. Instead, it is endogenously determined by actions of both agents as well as any prior information shared with them. In addition, in Ely (2017), the (short-lived) agent chooses to work at any instant myopically. As a result, the deterministic delay policy is optimal since it maximizes the principal's instantaneous payoff in every period. However, in our model, agents are long-lived vying to complete a two-stage task. This implies that their incentives must be kept alive dynamically as they determine their efforts by considering available information and both instantaneous and forward-looking incentives which depend on their future efforts and their opponent's efforts.

3. Model Development

Consider a setting where an organization ("principal") aims to incentivize a small group of expert firms ("agents") to complete a difficult multi-stage task as fast as possible by rewarding the agent who completes the task (i.e., all stages) first. Here we are interested in analyzing the impact of partial progress (see §1 for examples) on agents' incentives, so as common in the related literature (e.g., Bimpikis et al. 2019, Mihm and Schlapp 2019), we take the minimal model with two stages

[6] The key idea in dynamic Bayesian persuasion is to examine how an informed principal can persuade a set of players over time to take desirable actions (in our case, exert their best efforts) by influencing their beliefs.

and two agents $\{i, -i\}$. This setting fits our dynamic development contest framework well because such contests feature a small number of expert suppliers and a few major milestones (see §1). Time (indexed by t) runs continuously, and the contest can last over a potentially infinite horizon.

As is standard in the contest literature reviewed in §2, we consider a winner-takes-all contest where the first agent to complete both stages wins the contest and is given a reward R_t. Unlike the standard contest framework where this reward is fixed (i.e., $R_t = R$), we consider a more general rank-based mechanism where the reward can potentially change over time. We refer to the "standard" contest mechanism as a *fixed-reward* contest and ours as a *flexible-reward* contest.

For an agent, successful completion of a stage (hereafter, "success") arrives with a Poisson process, and the agent can boost the arrival rate by exerting costly effort. Specifically, agent i who has achieved $k \in \{0,1\}$ successes while her opponent has achieved $l \in \{0,1\}$ successes (if known by agent i) privately chooses effort $x^i_{k,l,t} \in [0,1]$ at each instant t with an instantaneous cost $cx^i_{k,l,t}$ for a constant $c > 0$ (e.g., Bonatti and Horner 2011, Ely 2017, Halac et al. 2017, Bimpikis et al. 2019), and a success in a stage arrives with a Poisson process with instantaneous probability $\lambda x^i_{k,l,t}$, where λ is the "achievability" parameter that is inversely proportional to how difficult a stage is.[7] (In §EC.2.4 of the Online Appendix we show how our model can accommodate different Poisson arrival rates for different stages of the contest.) The contest ends upon the arrival of the second success (i.e., completion of the second stage) for an agent at any time t and the winner receives the current posted reward R_t. We denote by T the random date at which the contest ends.

Denote by $V^i_{k,l,t}$ the expected utility (hereafter "continuation payoff") of agent i who has achieved $k \in \{0,1\}$ successes while her opponent has achieved $l \in \{0,1\}$ successes (if known by agent i) at any moment t. Then at any time t, each agent i anticipates the efforts of her opponent and chooses her effort levels from time t onward to maximize her expected utility

$$V^i_{k,l,t} = \max_{x^i_{k,l,\tau}} \mathbb{E}\left[R_T.\mathbb{1}_{\{i\ wins\}} - \int_t^T cx^i_{k,l,\tau} d\tau\right]. \tag{1}$$

As is common in the contest literature, we assume that all parties are risk-neutral (e.g., Halac et al. 2017, Bimpikis et al. 2019, Ely et al. 2022). We also assume that agents do not discount time because considering discounting complicates the expressions without providing any new insights (e.g., Halac et al. 2017, Mihm and Schlapp 2019, Ely et al. 2022). However, in §EC.2.3 of the Online Appendix, we show how our results can be generalized to a setting where all parties discount future payoffs. Also, we normalize the agents' outside option to zero without loss of generality.

Consistent with the literature on dynamic contests with information disclosure (e.g., Halac et al. 2017, Bimpikis et al. 2019, Ely et al. 2022), we assume that successful completion of a stage by an

[7] Alternatively, we can assume a model where at each instant t of continuous time, each agent decides whether to work ($x_{k,l,t} = 1$) or shirk ($x_{k,l,t} = 0$). Then, any intermediate effort level between 0 and 1 in our model can be interpreted as a randomization between the two pure strategies in the new model.

Assumptions	References
Two stages, two agents, winner-takes-all	Ederer (2010), Goltsman and Mukherjee (2011), Mihm and Schlapp (2019)
Poisson arrival rate, linear cost of effort	Bonatti and Horner (2011), Ely (2017), Halac et al. (2017), Bimpikis et al. (2019), Ely et al. (2022)
Principal has commitment power and observes successes	Halac et al. (2017), Bimpikis et al. (2019), Ely et al. (2022)

Table 1 Summary of the main assumptions in our model.

agent is only observable to that agent and the principal, and that only the principal can credibly disclose information about the status of agents' progress (i.e., whether each agent is in the first or second stage). This assumption is sensible given that an agent can easily misrepresent her partial progress (i.e., success in the first stage) to other agents. As a result, the only party who can credibly confirm the completion of a stage is the principal.[8] To ensure that agents can interpret the presence (or lack) of any information, the principal specifies its information disclosure policy to agents at the outset of the contest (e.g., Halac et al. 2017, Bimpikis et al. 2019, Ely et al. 2022). We begin by analyzing several mainstream information disclosure policies including full information disclosure where the principal commits to disclose any success upon its arrival, no information disclosure where the principal does not share any information, cyclic information disclosure where the principal stays silent during fixed-length cycles and discloses full information at the end of each cycle, and more strategic disclosure policies with deterministic or stochastic delay before characterizing an optimal policy. Note that whenever an agent does not know her opponent's state under a disclosure policy, we drop the corresponding index l (e.g., $x^i_{k,l,t}$ becomes $x^i_{k,t}$). Table 1 summarizes the key assumptions in our model, which are also standard in the literature.

The principal aims to minimize the expected lead time of the contest while also minimizing the reward necessary to achieve this goal. The standard approach in the contest literature is to assume that the principal can boil down agents' performance and reward components into a single dimensional profit function. Although this assumption makes sense in settings where agents compete in solution quality, generating such a single dimensional profit function may be hard to achieve when agents compete in time. Thus, instead of assuming such a single dimensional profit function, we take a lexicographic approach that focuses on the expected lead time first and the expected reward second.[9] Minimizing the expected project lead time in the context of

[8] As we show in §4.2, an agent always prefers her partial progress (i.e., success in the first stage) to be disclosed to her opponent because her opponent is discouraged by this information. Therefore, there is no cause for an agent to conceal partial progress from the principal. For the same reason, an agent has an incentive to falsely disclose partial progress to her opponent so without the approval of the principal, such information is not credible.

[9] This approach captures the context of our development applications discussed in §1. For example, promptly sourcing hypersonic systems are among the highest priorities of DoD to ensure battlefield dominance (DOD News 2021).

managing internal R&D operations has been studied in the Operations literature by considering various operational aspects such as overlapping product development (e.g., Ha and Porteus 1995, Krishnan et al. 1997). Here, we seek to understand how the principal can achieve the same goal in development settings by incentivizing external suppliers with appropriate reward and information policies. Specifically, we check if the principal can achieve the absolute minimum expected lead time $\underline{T}$ by inducing both agents to exert full effort throughout the contest. If $\underline{T}$ is achievable, we aim to find a reward schedule that yields $\underline{T}$ at the minimum expected reward by solving

$$\min_{R_t} \ \mathbb{E}\left[R_T.\mathbb{1}_{\{i \ or \ -i \ wins \ at \ T\}}\right] \ \text{s.t.} \ \mathbb{E}\left[T.\mathbb{1}_{\{i \ or \ -i \ wins \ at \ T\}}\right] = \underline{T}. \tag{2}$$

We show in §4 that $\underline{T}$ is achievable if and only if the principal has sufficient funds. If the principle is budget-constrained (formally defined in §5) with a low reward budget $\overline{R}$, then it is not possible to achieve $\underline{T}$. We cover this case in §5, where we compare different information disclosure policies and then characterize the one that yields the minimum expected lead time $\mathbb{E}\left[T.\mathbb{1}_{\{i \ or \ -i \ wins \ at \ T\}}\right]$ by using the whole reward budget. We study a principal with sufficient funds in §4 and a budget-constrained principal in §5. All proofs are presented in the Online Appendix.

4. A Principal with Sufficient Funds

In §4.1, we present a benchmark (first best) under which the principal achieves the absolute minimum expected lead time at the lowest possible cost by assuming observable and contractible effort. In the following sections, we use this benchmark to measure the performance of our contest mechanisms with unobservable effort. In §4.2, we characterize the optimal fixed-reward contest under full information disclosure. In §4.3 and §4.4, we derive optimal (first best) flexible-reward contests under full and no information disclosure policies, respectively. In §4.5, we present a simple necessary and sufficient condition for achieving the first-best outcome using rank-based incentives.

4.1. First-Best Contract with Observable Effort

As a form of benchmark, we first identify the absolute minimum reward the principal should give to achieve the absolute minimum expected lead time. Consider a case wherein the principal can observe agents' efforts and specify their effort path as long as it is individually rational (i.e., each agent's expected utility when exerting the designated effort is weakly higher than her outside option normalized to zero). In this case, instead of organizing a contest, the principal can offer each agent an individually rational contract that pays for her cost of effort, and hence induce both agents to exert full effort until one agent completes both stages. This contract achieves the absolute minimum expected lead time $\underline{T} \equiv 5/(4\lambda)$ as derived in (EC.1) of the Online Appendix. As a result, if the principal offers each agent $5c/(4\lambda)$, then it will be individually rational for agents to accept such a contract and exert full effort until one achieves two successes. Thus, the minimum required compensation to agents to achieve $\underline{T}$ can be calculated by multiplying $5c/(4\lambda)$ with 2.

Proposition 1. *There exists an individually rational "first-best" contract that achieves the minimum expected lead time $\underline{T}$ with the minimum required compensation of $\underline{R} \equiv 5c/(2\lambda)$ to agents.*

Proposition 1 characterizes the lower bound for the budget required to induce both agents to exert full effort at all times. Yet, this lower bound is achieved under the assumption that agents' efforts are observable, which is rarely the case in practice. Thus, in the remainder of this section, we assume that agents' efforts are unobservable, and investigate how much reward is needed to complete a two-stage task in a development contest in the shortest possible time.

4.2. Full Information Disclosure with Fixed Reward

In this section, we analyze a contest where the principal gives a fixed reward R and commits to disclose any success upon its arrival. Here, each agent knows her opponent's progress at any instant. We analyze the agent's problem by moving backward on the state of the game where the states are defined by the number of successes of the agents. If both agents have already achieved one success, agent i's continuation payoff from any time t onward is:

$$V^i_{1,1,t} = \max_{x^i_{1,1,\tau}} \int_t^\infty x^i_{1,1,\tau}(\lambda R - c) e^{-\int_t^\tau \lambda(x^i_{1,1,s} + x^{-i}_{1,1,s})ds} d\tau. \tag{3}$$

The intuition of (3) is as follows. If agent i chooses effort $x^i_{1,1,\tau}$ during interval $(\tau, \tau + d\tau)$, she incurs a cost $cx^i_{1,1,\tau}d\tau$, and achieves a success with probability $\lambda x^i_{1,1,\tau}d\tau$ earning the fixed reward R. This is conditional on no agent yet achieving the second success by time τ, which happens with probability $e^{-\int_t^\tau \lambda(x^i_{1,1,s} + x^{-i}_{1,1,s})ds}$. If the principal aims to induce both agents to exert full effort at all times to attain the absolute minimum expected lead time $\underline{T}$, then we shall have $V^i_{1,1,t} = \frac{1}{2}(R - \frac{c}{\lambda})$.

Next, consider the state of the game with a leader (an agent with one success) and a laggard (an agent with no success). The laggard's continuation payoff from any time t onward is given by:

$$V^i_{0,1,t} = \max_{x^i_{0,1,\tau}} \int_t^\infty x^i_{0,1,\tau}(\lambda V_{1,1,\tau} - c) e^{-\int_t^\tau \lambda(x^i_{0,1,s} + x^{-i}_{1,0,s})ds} d\tau. \tag{4}$$

Here, the laggard anticipates to receive a continuation payoff $V_{1,1,\tau}$ if she achieves a success (hence progresses to the second stage) and zero if her opponent achieves a success (hence wins the contest). Obviously, the laggard is willing to exert any effort only if her continuation payoff upon achieving a success compensates her cost of effort. Therefore, the principal needs to specify a fixed reward weakly greater than $3c/\lambda$ (so that $V_{1,1,t} \geq c/\lambda$) to keep the laggard working. We next show that this minimum fixed reward is enough to encourage full effort by both agents at all times.

Proposition 2. *Under full information disclosure, the minimum fixed reward needed to achieve the minimum expected lead time $\underline{T}$ is $R = 3c/\lambda$.*

Without a sufficiently large reward, an agent in the first stage is at risk of being discouraged when her opponent proceeds to the second stage, because her chance of winning the reward declines.

Therefore, the principal has to offer the minimum fixed reward $3c/\lambda$ to keep the laggard's continuation payoff upon success at c/λ. Yet, with this large fixed reward, the principal overpays the leader and she receives a continuation payoff $V_{1,0,t} = 3c/(2\lambda)$ as derived in the Online Appendix. This large reward also delivers an ex-ante expected surplus $V_{0,0,0} = c/(4\lambda)$ to each agent, which shows that the principal leaves money on the table as compared to the first-best contract.

In the next section, we investigate whether the principal can do better than the fixed-reward contest by designing a flexible-reward contest instead.

4.3. Full Information Disclosure with Flexible Reward

One can deduce from the previous section that under full information disclosure, each agent's effort provision decision and continuation payoff depend solely on the state of agents' successes in the contest (we show this formally in the proof of Proposition 3). Observe that a contest may end under two states of the game: (i) a case where the leader obtains the second success before the laggard obtains any success and (ii) a case where both agents have already obtained one success and one of them achieves the second success. Let $R_{2,0}$ and $R_{2,1}$ denote the contest reward in each case. We next show how the principal can achieve the first-best outcome using a flexible-reward contest by giving sufficient incentives to the laggard without overpaying the leader.

PROPOSITION 3. *Under full information disclosure, a flexible-reward contest with $R_{2,0} = 2c/\lambda$ and $R_{2,1} = 3c/\lambda$ achieves the minimum expected lead time $\underline{T}$ by paying the first-best expected reward $\underline{R}$.*

From §4.2, we know that the principal must set $R_{2,1} = 3c/\lambda$ to encourage the laggard to exert effort. However, with flexible rewards, the leader does not need to be overpaid. Specifically, the reward the leader will receive before the laggard achieves any success (i.e., before the leader loses her lead, see case (i) above) is $R_{2,0} = 2c/\lambda$. On the other hand, the reward a winner will receive after both agents achieve the first success is $R_{2,1} = 3c/\lambda$ (i.e., after the leader loses her lead, see case (ii) above). This reward schedule achieves the absolute minimum expected lead time $\underline{T}$ by eliciting full effort at all times from both agents and gives the minimum necessary reward.[10]

To see why the reward schedule in Proposition 3 offers the first-best expected reward $5c/(2\lambda)$, note that with probability 1/2, the contest ends before the arrival of any success for the laggard and the principal pays $R_{2,0} = 2c/\lambda$; and with probability 1/2, the contest ends after the arrival of the first success for the laggard and the principal pays $R_{2,1} = 3c/\lambda$. A key feature of this design which makes it practically appealing is its simplicity. The policy can easily be implemented by offering

[10] It is worth noting that a lead-time minimizing contest induces agents to work as much as possible as early as possible (i.e., without delay). Note that this design is different from an effort-maximizing contest (e.g., Ely et al. 2022) where the principal may benefit from inducing the leader to pause effort and resume later once the laggard catches up in order to elicit larger total effort from agents.

a guaranteed reward of $2c/\lambda$ with the option to increase the reward if multiple agents progress to the second stage. The flexible-reward schedule is quite impactful as well because the fixed-reward schedule spends at least 20% more money on average than the flexible-reward schedule to achieve the same contest outcome. It is worth noting that the dominance of flexible-reward schedule is not restricted to the case with two agents. In fact, as we show in §EC.2.5 of the Online Appendix, the flexible-reward contest provides even larger cost savings when there are more than two agents.

4.4. No Information Disclosure with Flexible Reward

We next characterize the optimal flexible-reward contest when the principal does not disclose any information. Here, each agent forms a belief about her rival's progress and updates this belief over time. Let p_t^i be the probability that agent i assigns at time t to the event that her opponent has already achieved first success. As time passes, the only information agent i receives is whether the contest is still ongoing as her rival has yet to achieve the second success. Thus, by Bayes' rule, p_t^i evolves according to (for derivation, see the proof of Proposition 4 in the Online Appendix)

$$dp_t^i = \lambda(1-p_t^i)(x_{0,t}^{-i} - p_t^i x_{1,t}^{-i})dt, \tag{5}$$

with the boundary condition $p_0^i = 0$ where $x_{0,t}^{-i}$ denotes the opponent's effort at time t conditional on not having achieved a success yet, and $x_{1,t}^{-i}$ denotes her effort at time t conditional on having achieved the first success. Intuitively, and as shown in (5), the probability that each agent's opponent already advanced to the second stage, p_t^i, is increasing in the opponent's anticipated first-stage effort $x_{0,t}^{-i}$ and decreasing in the opponent's anticipated second-stage effort $x_{1,t}^{-i}$.

Recall that $V_{k,t}^i$ is the continuation payoff of each agent i who has achieved $k \in \{0,1\}$ successes by time t. Then, the maximization problem for agent i after obtaining the first success is given by:

$$V_{1,t}^i = \max_{x_{1,\tau}^i} \int_t^\infty x_{1,\tau}^i(\lambda R_\tau - c)e^{-\int_t^\tau \lambda(x_{1,s}^i + p_s^i x_{1,s}^{-i})ds} d\tau, \tag{6}$$

where R_τ is the specified reward if the contest ends at time τ. (6) can be interpreted as follows. If agent i chooses effort $x_{1,\tau}^i$ during interval $(\tau, \tau + d\tau)$, she incurs a cost $cx_{1,\tau}^i d\tau$ and achieves a success with probability $\lambda x_{1,\tau}^i d\tau$, earning R_τ. This is conditional on the probability that none of the agents yet achieved a second success by time τ; i.e., $e^{-\int_t^\tau \lambda(x_{1,s}^i + p_s^i x_{1,s}^{-i})ds}$. Anticipating a continuation payoff of $V_{1,\tau}^i$ upon achieving the first success at time τ, agent i's continuation payoff from time t onward before achieving any success can be expressed as follows:

$$V_{0,t}^i = \max_{x_{0,\tau}^i} \int_t^\infty x_{0,\tau}^i(\lambda V_{1,\tau}^i - c)e^{-\int_t^\tau \lambda(x_{0,s}^i + p_s^i x_{1,s}^{-i})ds} d\tau. \tag{7}$$

The above expression can be interpreted similar to (6). As derived in condition (EC.21) in the Online Appendix, an agent with no success finds it optimal to exert full effort if and only if her additional utility upon the arrival of her first success is weakly greater than c/λ. Proposition 4 characterizes the optimal flexible-reward schedule under no information disclosure.

Proposition 4. *Under no information disclosure, a flexible-reward contest with $R_t = (2+p_t)c/\lambda$ where $p_t = \lambda t/(1+\lambda t)$ achieves the minimum expected lead time $\underline{T}$ by paying the first-best expected reward $\underline{R}$.*

Under no information disclosure, an agent i who is failing to achieve any success will strengthen the belief that her rival has already achieved one success. This reduces the expected utility of progressing to the second stage over time, and hence reduces the incentives for this agent to spend effort in the first stage. To restore incentives, the principal offers a gradually increasing flexible-reward schedule $R_t = (2+p_t)c/\lambda$, where p_t is the equilibrium belief of each agent about the progress of her rival. This enables the principal to achieve the first-best outcome (expected lead time $\underline{T}$ by paying the expected reward $\underline{R}$) for two reasons. First, it gives the minimum necessary reward to persuade the agent with no success to work when her rival (is likely to have) progressed to the second stage. Second, it avoids overpaying agents when they have sufficient incentives to work.

4.5. A Necessary and Sufficient Condition to Achieve First Best

We now generalize our findings in the previous sections and present the main result of §4.

Theorem 1. *Under any information disclosure policy, there exists a first-best flexible-reward contest that attains the minimum expected lead time $\underline{T}$ by paying the minimum expected reward $\underline{R}$ if and only if the principal's budget $\overline{R} \geq \frac{3c}{\lambda}$.*

Theorem 1 shows that a principal with a sufficiently large budget achieves the first-best outcome using an appropriate flexible-reward scheme irrespective of the information disclosure policy. Specifically, as we show in the proof of Theorem 1, the principal can achieve the first best by making the reward contingent on the state where the contest ends: (i) with a reward $R_{2,0} = 2c/\lambda$ if the leader wins before the laggard achieves first success and (ii) $R_{2,1} = 3c/\lambda$ if an agent wins after both agents achieve first success. While proving that this policy works under any information disclosure policy is more nuanced, the intuition is similar to that of Proposition 3.

Theorem 1 also shows that when the principal's budget $\overline{R}$ is strictly less than $3c/\lambda$ (i.e., principal is budget-constrained), the first-best outcome is not attainable. This is because under any information disclosure policy, an agent with no success eventually builds a strong enough belief that her opponent is in the second stage so that exerting full effort for such an agent is no longer incentive compatible due to insufficient reward. In this case, the principal can utilize information disclosure as a non-monetary incentive mechanism to minimize the expected lead time of the contest by keeping such an agent working as long as possible. Therefore, we next focus on the role of information disclosure policy to help a budget-constrained principal.

5. Using Information Design for Development Contests

In this section, we study the use of information design, another potential lever for principals engaging in development contests. Such an approach can come in handy for a budget-constrained principal who may fall short of the optimal reward amount derived in the previous section. Observe that, with no competition, the principal needs a minimum budget of $2c/\lambda$ for the second success to be attainable (c/λ for each success), and $3c/\lambda$ is the necessary budget to achieve first best with two competing agents. For the rest of the analysis, we assume $2c/\lambda < \overline{R} < 3c/\lambda$, and we name a principal facing this limitation a *budget-constrained* principal. By holding the reward fixed, we study how the principal can use information disclosure to incentivize agents. In §5.1, we examine the role of strategic information disclosure. In §5.2, we discuss how and why a probabilistic disclosure policy improves upon mainstream disclosure policies. In §5.3, we characterize a fully optimal information disclosure policy that minimizes the project expected lead time when the principal is budget-constrained. Finally, in §5.4 we show how flexible rewards can be combined with our optimal probabilistic disclosure policy to further reduce the project cost without hindering its lead time.

5.1. Information as a Strategic Commodity

We shall start our discussion with the observation that an agent who has already achieved one success is easy to incentivize because she is already encouraged by the fact that she needs only one more success to obtain the reward. As we shall see throughout this section, such an agent always exerts full effort irrespective of the information disclosure policy. In contrast, an agent with no success is at risk of becoming discouraged over time once she realizes (or believes) that her rival has already progressed to the second stage. To understand the impact of partial progress on the incentives of agents and establish the strategic value of information disclosure, we first analyze the problem faced by a budget-constrained principal that discloses no information throughout the contest. Notice that unlike §4.4, the principal does not have sufficient funds to gradually increase the reward up to $3c/\lambda$ to mitigate an agent's reduced incentives caused by the threat that her rival is in the second stage. Therefore, in the unique symmetric equilibrium of the contest, an agent with no success only exerts full effort as long as monetary incentives are sufficient, and lowers her effort level after a while when incentives are missing. The equilibrium is characterized below.

Proposition 5. *When the principal is budget-constrained, and under no information disclosure, there exists a unique symmetric equilibrium in which an agent with no success exerts full effort until time* $t_r = \frac{p_r}{\lambda(1-p_r)}$ *where* $p_r = \frac{\lambda\overline{R}}{c} - 2$. *After* t_r, *she reduces her effort level to* $p_r (< 1)$. *An agent who has achieved one success exerts full effort until the end.*

Condition (EC.21) in the Online Appendix implies that exerting full effort is incentive compatible for an agent with no success if and only if she earns an additional utility of at least c/λ upon the arrival of her first success. While the agent is working, the expected utility of progressing to the second stage diminishes over time as she strengthens her belief that her opponent already achieved one success. It turns out that, a budget-constrained principal can only induce an agent with no success to exert full effort until time t_r where her belief reaches $p_r = \frac{\lambda\overline{R}}{c} - 2$. After that, monetary incentives are not sufficient to justify full effort, so each agent with no success reduces her effort to p_r and keeps this effort level. To see why the equilibrium effort becomes p_r, note that p_r is the belief level that keeps an agent indifferent between exerting any effort level. When the agent's belief is below p_r (which happens if the opponent exerts effort smaller than p_r after t_r), she exerts full effort but when it is above p_r (which happens if the opponent exerts effort larger than p_r after t_r), she exerts zero effort. Thus, after t_r, the unique symmetric equilibrium effort is p_r, which keeps the agent's belief at the threshold and holds her expected continuation payoff after obtaining the first success at c/λ such that the agent remains indifferent between exerting any effort at each instant.

It is worth noting that the time threshold t_r and the belief threshold and reduced effort level p_r are increasing in the size of the budget $\overline{R}$ and the achievability parameter λ, and decreasing in the marginal cost of effort c. Also, as we discuss above, each agent has sufficient incentives to exert full effort until the end after obtaining the first success (see (EC.23) in the Online Appendix).

While no information disclosure elicits full effort before time t_r, the reduced effort provision by an agent with no success after time t_r hints the benefit of using information strategically to reduce the project lead time. Let us therefore consider the full information disclosure policy. Recall from §4.3 that the principal should offer a reward of $3c/\lambda$ to the laggard once she realizes that her opponent is in the second stage. Obviously, a budget-constrained principal does not have access to such a high reward. Therefore, upon the arrival of the first success, the laggard quits immediately (see Proposition EC.1 in the Online Appendix). This is in contrast to no information disclosure policy where the principal never loses the laggard, but an agent with no success reduces her effort level to $p_r < 1$ after time t_r even if her opponent has not achieved any partial progress. Our results echo the empirical findings of Lemus and Marshall (2021) who study extreme disclosure policies (no versus full), and find that disclosing information discourages a laggard, but the lack of information creates uncertainty regarding how much effort is needed to remain competitive.

In a recent study, Bimpikis et al. (2019) build on this intuition and suggest a cyclic information disclosure policy comprising cycles of *silent periods* during which no information is disclosed with full disclosure at the end of each cycle. This design somehow blends the key benefits of the two extreme cases by: i) hiding any partial progress during each silent period where incentives are sufficient to elicit full effort, and ii) disclosing full information at the end of each cycle to replenish agents' incentives (in case both agents have zero success) and to avoid reduced effort levels. We next present the equilibrium under the cyclic information disclosure policy in our setting.

PROPOSITION 6. *When the principal is budget-constrained, and commits to cyclic information disclosure with silent periods of length $t_r = \frac{p_r}{\lambda(1-p_r)}$ where $p_r = \frac{\lambda\overline{R}}{c} - 2$, there exists a unique symmetric equilibrium in which both agents exert full effort until time t_r. At the end of the first cycle:*
(i) If no agent has made any partial progress by time t_r, the contest resets and the next silent period with length t_r begins in which both agents exert full effort.
(ii) If only one agent has made partial progress at the end of one cycle, the agent with no success quits and the agent with one success exerts full effort until the end.
(iii) If both agents have made partial progress at the end of one cycle, both agents exert full effort until the end.

While the cyclic information policy improves upon the full information policy by delaying the laggard's stopping time with the help of silent periods, it does not necessarily improve upon the no information policy (see Figure 2). Indeed, no information policy provides *too little* information after t_r, and cyclic information policy disseminates *too much* information at the end of each cycle, which may hurt the principal by inducing the laggard to quit after learning her rival's partial progress. Thus, information is a strategic commodity whose flow must be carefully managed.

5.2. Probabilistic Information Disclosure Policy

In this section, we discuss how we can improve upon the above information disclosure policies by combining the insights from the previous section with a dynamic Bayesian persuasion approach (e.g., Ely 2017, Orlov et al. 2020, Ely et al. 2022). The idea is to calibrate the flow of information to dynamically keep agents' incentives alive. In that sense and to address the issue of too little (much) information under no (full/cyclic) disclosure policy, we can consider a "*probabilistic state disclosure*" policy (hereafter ***PSD***) as follows. The principal discloses no information to agents (i.e., implements a silent period) until time $t_r = \frac{p_r}{\lambda(1-p_r)}$ where agents' beliefs hit a threshold $p_r = \frac{\lambda\overline{R}}{c} - 2$. At any instant $(t + dt)$ for time $t \geq t_r$, if the first success *has arrived by* t, the principal commits to disclose this information with probability γdt, where $\gamma \equiv \lambda(1-p_r)/p_r = 1/t_r$. Here, γ represents the constant rate at which the principal (stochastically) discloses the *state* of partial progress in the competition after the silent period. In this setting, the belief of an agent i who has not achieved any success about her rival's progress after the silent period evolves according to

$$dp_t^i = (1 - p_t^i)(x_{0,t}^{-i}\lambda - p_t^i x_{1,t}^{-i}\lambda - p_t^i\gamma)dt. \tag{8}$$

This law of motion, which is derived in the proof of Proposition 7, illustrates how the principal can hold the agent's belief constant once it reaches the threshold p_r by promising to probabilistically disclose partial progress using γ while granting a continuation payoff of c/λ upon success to evoke full effort from this agent until partial progress is disclosed. Proposition 7 describes the equilibrium.

PROPOSITION 7. *When the principal is budget-constrained and commits to PSD, an agent who has not achieved a success exerts full effort until she obtains her first success, or her opponent obtains her second success, or the principal discloses the opponent's partial progress. An agent who has achieved one success exerts full effort until the end.*

PSD enables the principal to tune the rate of information disclosure after t_r to persuade an agent with no success to spend full effort for a longer period of time, on average. The principal is indeed facing a trade-off by sharing information regarding partial progress. A higher rate of disclosure stimulates greater effort from an agent with no success which reduces the expected lead time of the contest, but it also increases the probability of losing a laggard and prolonging the contest's expected lead time if partial progress is disclosed. PSD disseminates information such that the bare minimum of incentives are provided to sustain full effort after t_r by granting the minimum continuation payoff of c/λ upon achieving a success. This way the principal does not disclose too much information as in the case of full/cyclic information disclosure and can substantially improve the reduced effort level of the agents under no information disclosure.

PROPOSITION 8. *PSD yields an expected lead time of $(5+e^{-2\lambda t_r})/(4\lambda)$, which is strictly lower than the expected lead times under no, full or cyclic information disclosure policies.*

A key benefit of PSD is that it probabilistically delays the disclosure of partial progress. Specifically, once the silent period is over, the principal discloses partial progress after a stochastic delay with rate γ to keep an agent with no success incentivized for longer. Naturally, one might wonder if deterministic delay in information sharing can also have the same effect. The short answer is no. We discuss the details in §EC.2.2 of the Online Appendix and summarize the intuition here.

To elicit full effort from agents in the deterministic delay policy, the principal must commit to disclose any progress at most t_d periods after its arrival, where t_d is characterized in (EC.50) of the Online Appendix. Thus, the principal is effectively setting an initial silent period of t_d. Yet, the duration of silent period is shorter than the one under PSD (i.e., $t_d < t_r$, see Proposition EC.3 of the Online Appendix). To understand why, consider an agent i with no success choosing her effort at the end of the silent period. This agent trades off the benefit of achieving partial progress with the cost of additional effort. The more likely it is for agent i's opponent to give up after agent i's partial progress, the more agent i benefits from partial success. Under PSD, there is a chance that the principal discloses agent i's partial progress at any instant after t_r whereas under deterministic delay, agent i knows that there is no such chance until t_d periods after its arrival. This implies PSD

enables the principal to increase the benefit of success and extend the silent period by providing larger incentives for an agent i who has not achieved any success early in the contest. At the same time, PSD grants a lower surplus to an agent who obtains partial progress early in the contest by reducing the benefit of success as the principal informs the laggard to quit (stochastically) only after t_r, whereas under deterministic delay, the laggard will quit after t_d $(< t_r)$ periods of delay.

The key insight of this section is that probabilistic state disclosure helps the principal smoothen incentives over time and extend the period over which agents are willing to work. But, how can the principal optimally manage the flow of information during the contest to provide the bare minimum of incentives at each instant? We characterize the optimal information disclosure next.

5.3. Optimal Information Disclosure Policy

So far, we have shown that PSD outperforms all other canonical information disclosure policies by probabilistically disclosing the state of partial progress (using γ). While this design provides the bare minimum of surplus (c/λ) after time t_r to persuade an agent with no success to work, it provides too much incentives for an agent who succeeds before t_r. Specifically, by committing to disclose the state of partial progress (and hence informing the laggard to quit) after time t_r using γ, the principal increases the benefit of an early success in the competition. In other words, this design wastes some incentives that may otherwise be used in other states to incentivize effort.

To address this problem caused by the state disclosure policy, we consider a "*probabilistic change-of-state disclosure*" policy (hereafter ***PCSD***) characterized by a time-dependent parameter $\phi_t \in [0,1]$ where the principal has the following commitment. If the first success arrives during interval $(t, t+dt)$, then the principal discloses this information at instant $(t+dt)$ with probability ϕ_t. Here, ϕ_t represents the probability of announcing any *change* in the state of partial progress. In this setting, the law of motion for the belief of an agent i who has not achieved any success about her rival's progress is given by (for derivation, see the proof of Theorem 2 in the Online Appendix)

$$dp_t^i = (1-p_t^i)[x_{0,t}^{-i}\lambda - p_t^i x_{1,t}^{-i}\lambda - (1-p_t^i)x_{0,t}^{-i}\lambda\phi_t]dt. \tag{9}$$

The principal aims to choose ϕ_t at each instant to provide the optimal amount of information to induce agents to make their best efforts. It turns out that the optimal information disclosure policy must grant agents the minimum continuation payoff upon achieving partial progress that respects agents' incentive compatibility constraints. This, in turn, induces agents to exert full effort for a longer period of time, on average, and minimizes the project expected lead time. The main theorem of this section shows how the principal can tune ϕ_t to optimally manage incentives over time.

Theorem 2. *The following probabilistic change-of-state disclosure policy, which we call PCSD, minimizes the expected lead-time of the contest when the principal is budget-constrained:*
(Phase 1) The principal discloses no information to the agents up to time $\underline{t} = \frac{\underline{p}}{\lambda(1-\underline{p})}$ *where*

$$\underline{p} = \begin{cases} 0 & \text{if } \frac{2c}{\lambda} < \overline{R} \le \frac{7c}{3\lambda}, \\ \frac{3\overline{R} - 7c/\lambda}{\overline{R} - c/\lambda} & \text{if } \frac{7c}{3\lambda} < \overline{R} < \frac{3c}{\lambda}. \end{cases} \tag{10}$$

(Phase 2) At each instant $(t + dt)$ *after* $\underline{t}$*, the principal discloses partial progress with probability*

$$\phi_t^* = \begin{cases} \frac{\frac{4c/\lambda}{\overline{R}-c/\lambda} - 3 + p_t}{1 - p_t} & \text{if } \underline{t} \le t < \overline{t}, \\ 1 & \text{if } t \ge \overline{t}, \end{cases} \tag{11}$$

if it arrived during interval $(t, t + dt)$ *where* p_t *is the unique solution to the ordinary differential equation (ODE)*

$$\dot{p}_t = \lambda(1 - p_t)^2(1 - \phi_t^*) = \lambda(1 - p_t)(4 - 2p_t - \frac{4c/\lambda}{\overline{R} - c/\lambda}), \tag{12}$$

with boundary conditions $p_{\underline{t}} = \underline{p}$ *and* $p_{\overline{t}} = \overline{p} \equiv \frac{2(\overline{R}-2c/\lambda)}{\overline{R}-c/\lambda}$.

(Equilibrium) Under PCSD, an agent who has not achieved a success exerts full effort until she obtains her first success, or her opponent obtains her second success, or the principal discloses the opponent's partial progress. An agent who has achieved one success exerts full effort until the end.

We now explain the mechanism of our optimal design using Figure 1. The optimal disclosure policy starts with a silent period of length $\underline{t}$ (phase 1). During this phase, if an agent i achieves the first success, her partial progress will never be disclosed ($\phi_t^* = 0$ for $t < \underline{t}$). Despite anticipating this behavior, agent i is still willing to exert full effort in phase 1 as incentives are sufficiently high early in the contest. To see this, observe in Figure 1 that the expected continuation payoff upon achieving the first success at time t can be expressed as $V_{1,t} = p_t V_{1,1,t} + (1 - p_t)V_{1,0,t}$. At time $t < \underline{t}$, it is more likely that the rival is in the first stage (p_t is low) and the expected utility of progressing to the second stage is closer to $V_{1,0,t}$. Therefore, even if the laggard keeps working until the end (which makes $V_{1,0,t} = \frac{3}{4}(\overline{R} - \frac{c}{\lambda}) > \frac{c}{\lambda}$ if and only if $\overline{R} > \frac{7c}{3\lambda}$), agent i still earns an additional utility greater than her cost of effort (i.e., $V_{1,t} - V_{0,t} > \frac{c}{\lambda}$ for $t < \underline{t}$) upon first success. Thus, agents keep working in phase 1. Note that a principal with a larger budget can keep agents incentivized for a longer period of time so the length of phase 1 increases with $\overline{R}$ ($\underline{t} = 0$ if $\overline{R} \le \frac{7c}{3\lambda}$).

In phase 2 (after $\underline{t}$), the principal commits to inform the agents of any change in the state of partial progress with probability ϕ_t^* which is increasing over time since the agents put more weight

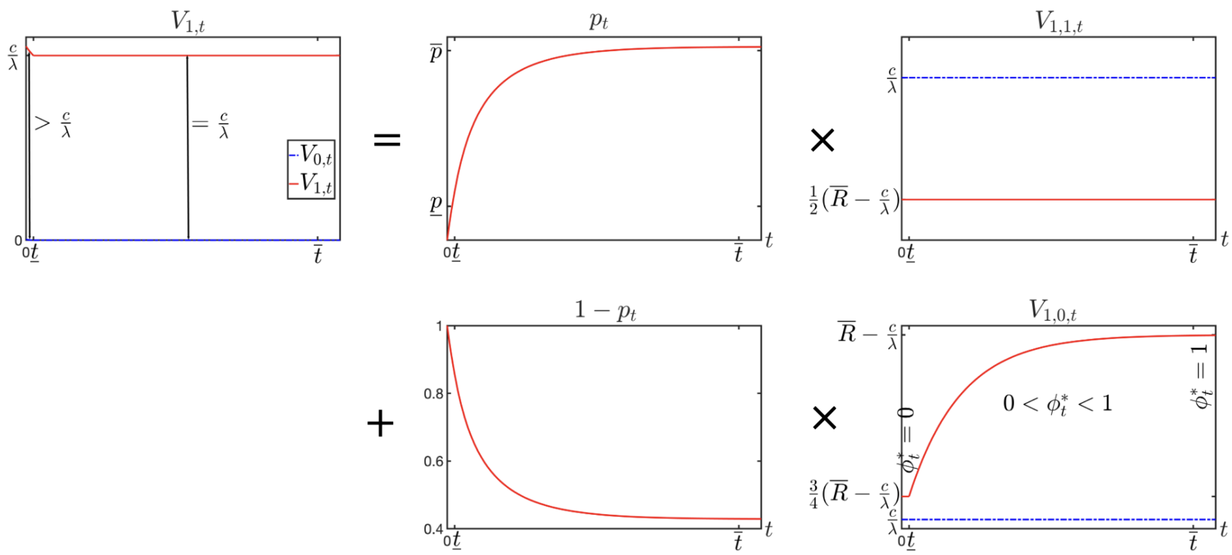

Figure 1 How optimal disclosure manages incentives. Setting: $c = \lambda = 1$, $\overline{R} = 2.4$, $\underline{t} = 0.17$ and $\overline{t} = 6.55$.

on their rival's partial progress probability (both p_t and ϕ_t^* are increasing on $[\underline{t}, \overline{t})$. After $\overline{t}$, $\phi_t^* = 1$ which holds $p_t = \overline{p}$). This way the principal gives the bare minimum of incentives to an agent i with no success to exert full effort by promising to immediately inform her rival with probability ϕ_t^* (and hence ensuring payoff $V_{1,t} = \frac{c}{\lambda}$ upon first success). If this happens and the rival has yet to achieve first success, the rival quits and agent i receives $V_{1,0,t|\text{rival quits}} = (\overline{R} - \frac{c}{\lambda})$. Otherwise, her rival keeps spending full effort until the end, leaving agent i with an expected continuation payoff of $V_{1,1,t} = \frac{1}{2}(\overline{R} - \frac{c}{\lambda})$ or $V_{1,0,t|\text{rival works}} = \frac{3}{4}(\overline{R} - \frac{c}{\lambda})$ depending on whether the rival has obtained any success by t or not, respectively. Probability ϕ_t^* is chosen such that agent i weakly prefers to exert full effort at any instant. Finally, after $\overline{t}$ and when agent i becomes very pessimistic about her chances of winning the reward, the principal commits to disclose any change of state immediately ($\phi_t^* = 1$). This holds the agent's belief about her rival's progress constant while providing sufficient incentives for this agent to keep spending full effort by anticipating that if she succeeds and her rival has yet to achieve first success, her rival will quit immediately.

The above analysis reveals how *PCSD* relinquishes the minimum expected continuation payoff to each agent at any instant during the contest — a property that any state disclosure policy fails to achieve. As illustrated in Figure 1, for all $t \geq \underline{t}$ where the first success arrived, $V_{1,t} = \frac{c}{\lambda}$ which is the minimum necessary continuation payoff to stimulate first-stage effort and for all $t < \underline{t}$ where $V_{1,t} > \frac{c}{\lambda}$, the principal induces both agents to keep spending full effort until the task is complete. Therefore, the minimum continuation payoff is granted. By optimally managing incentives at all times and extracting more surplus from the agents without discouraging them, the principal can stimulate competition and minimize the development project lead time as depicted in Figure 2.

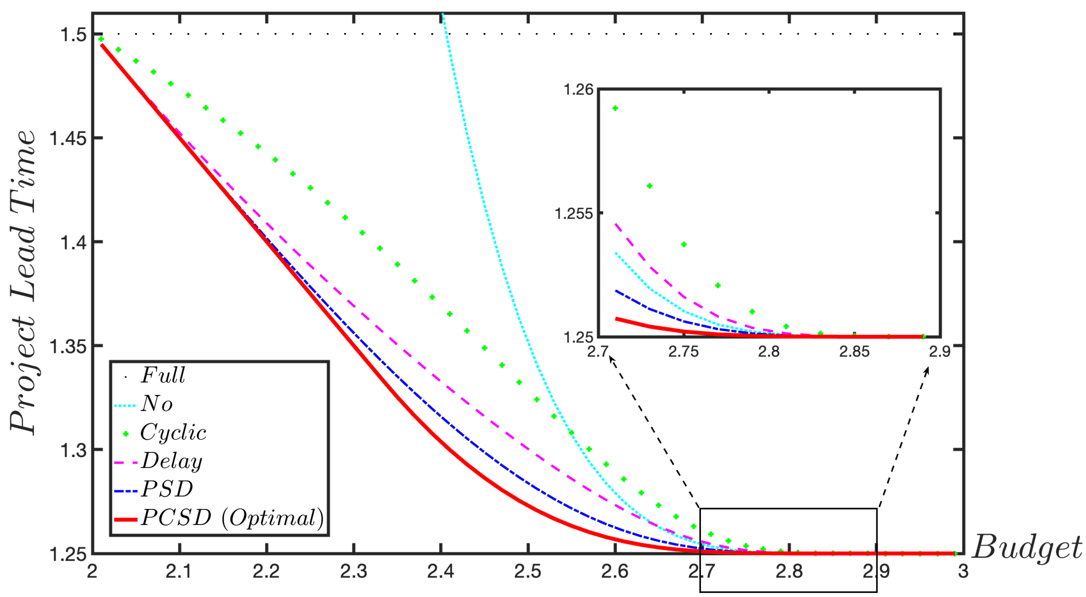

Figure 2 Optimal probabilistic change-of-state disclosure minimizes expected project lead time. Setting: $c = \lambda = 1$.

Theorem 2 provides several key technical contributions and managerial insights. First, prior literature reviewed in §2 mainly considers the disclosure of state rather than immediate change of state. For instance, the cyclic policies proposed by Bimpikis et al. (2019) or Ely et al. (2022) disclose the history (state) of the contest outside silent periods. We utilize this idea in PSD and show that adding probabilistic disclosure leads to significant improvement. However, by adopting a novel disclosure strategy in $PCSD$, we characterize a fully optimal information disclosure policy which strategically discloses change of state (captured by ϕ_t^* in our optimal design) and minimizes the expected lead time of a *multi-stage* development contest. We hope this novel approach can guide future research in dynamic Bayesian persuasion applications.

Second, Theorem 2 shows that probabilistic change-of-state disclosure can optimally smoothen incentives over time. To see this, note that by committing to disclose the change of state at time t with probability ϕ_t^*, the principal does not increase the benefit of obtaining a success before t because past partial progress will never get disclosed. PSD, deterministic delay, and other canonical information disclosure policies fail to achieve this property, hence relinquish more surplus to agents.

It is worth noting that in characterizing $PCSD$, we assume the principal can commit to the information disclosure policy (which is standard in the information design literature, see Ely et al. 2022). This commitment power results from the long-term relationship between the principal and the expert development partners as discussed in §1.[11] Building strong partnership with strategic suppliers helps the principal benefit from additional levers such as information design.

5.4. Probabilistic Change-of-State Disclosure with Flexible Reward

In §5.3, we obtain an optimal disclosure policy by holding the reward fixed to focus on the role of information disclosure. We can further improve our fixed-reward $PCSD$ design by reducing the expected reward of the contest with no impact on the agents' equilibrium behavior by incorporating flexible rewards. Recall that under $PCSD$ and when $\overline{R} > \frac{7c}{3\lambda}$, the principal discloses no information up to time $\underline{t}$ where monetary incentives are sufficient to motivate full effort. Only after phase 1, the principal starts disclosing information to satisfy each agent's incentive compatibility constraint. In other words, in phase 1, agents earn an excess of surplus which can be extracted using flexible rewards without any impact on their incentives. Proposition EC.2 in the Online Appendix states that the principal can achieve the same equilibrium outcome as in the fixed-reward $PCSD$ with a flexible-reward schedule according to $R_{2,1} = \overline{R}$ and $R_{2,0,t} = \frac{7c/\lambda - \overline{R} - p_t(\overline{R} + c/\lambda)}{2(1-p_t)}$ if $t < \underline{t}$, and $R_{2,0,t} = \overline{R}$ if $t \geq \underline{t}$, where t is the time at which the first success is obtained and $p_t = \lambda t/(1+\lambda t)$.

The goal is to not overpay agents during the early stages of the contest where incentives are sufficiently high. In our proposed design, the reward the leader will receive before the laggard achieves any success is less than $\overline{R}$ and increasing in the arrival time of the first success (t) if $t < \underline{t}$ and is equal to the budget constraint $\overline{R}$ if $t \geq \underline{t}$. On the other hand, the reward a winner will receive after both agents achieve the first success is $R_{2,1} = \overline{R}$. This reward schedule provides some cost-saving opportunities for the principal when $\overline{R} > \frac{7c}{3\lambda}$ as indicated in Figure 3. Notice that the expected lead time is strictly decreasing in $\overline{R}$ as a higher reward budget helps the principal postpone the probabilistic disclosure phase (phase 2). For the same reason, the principal benefits from allocating her entire budget after time $\underline{t}$. Combining probabilistic disclosure with flexible reward can help the principal reduce both the expected lead time and the expected cost of reward in the contest.

6. Conclusion

Organizations worldwide face the challenges of developing advanced technologies and products under time and budget pressures, and turn to their expert suppliers to tackle complex problems. In this paper, we study how such an organization can effectively organize a dynamic development contest to stimulate development effort from a small set of competing suppliers to minimize the lead time of a multi-stage project while keeping the incentive budget in check.

[11] In the absence of such commitment, a budget-constrained principal would be unable to credibly communicate with agents mid-contest due to the principal's interest in agents working until the task is complete. Then, the unique symmetric equilibrium of the resulting game would be identical to the one in Proposition 5.

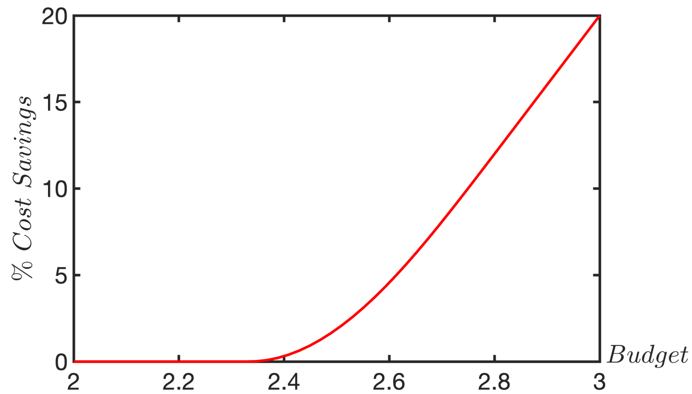

Figure 3 Percentage of cost savings under $PCSD$ with flexible reward over fixed reward. Setting: $c = \lambda = 1$.

Inspired by the current literature and practice, we sought to build a parsimonious stylized model of a dynamic development contest where two agents compete to complete a two-stage development project by exerting costly effort over a continuous time frame. Successful completion of a stage for an agent arrives at a random point in time where the rate of arrival for each success increases with the agent's effort. An agent's success is not observable by another agent so it is up to the principal whether, when, and how to share this information. The contest ends when one of the agents successfully completes both stages and hence wins a pre-determined reward. At the outset of the contest, the principal commits to a reward schedule that determines how the reward will change over time and an information disclosure policy which specifies how the principal will disclose information throughout the contest.

We establish that a principal with no budget constraint can utilize a flexible-reward schedule to achieve the absolute minimum expected lead time by giving 20% less reward on average than a fixed-reward schedule. Under any information disclosure policy, it is optimal for the principal to adopt a flexible-reward schedule by setting a guaranteed reward amount upfront with the promise of a larger reward if multiple agents achieved partial progress.[12] Importantly, this lead-time minimizing flexible-reward schedule pays the minimum expected reward (i.e., achieves the first-best outcome). We next analyze how a budget-constrained principal can utilize information as an incentive tool. By harnessing a dynamic Bayesian persuasion approach, we characterize an optimal (lead-time minimizing) information disclosure policy in which the principal does not share any information for a pre-determined initial time window (which may not exist if budget is low); and then discloses

[12] While we see examples in practice where reward varies based on time or state of a competition (e.g., Google Lunar XPrize contest featured a time-contingent prize for the first private firm to land a vehicle on the Moon, and later increased the prize purse from \$30 to \$40 million after observing some promising progress from participants), our research of the practice did not reveal any development contest where the winner reward varies based on the state in which the competition ends. Our paper offers a normative recommendation about the potential of adopting such a state-based reward policy which may not be all that difficult to implement with only two possible reward levels.

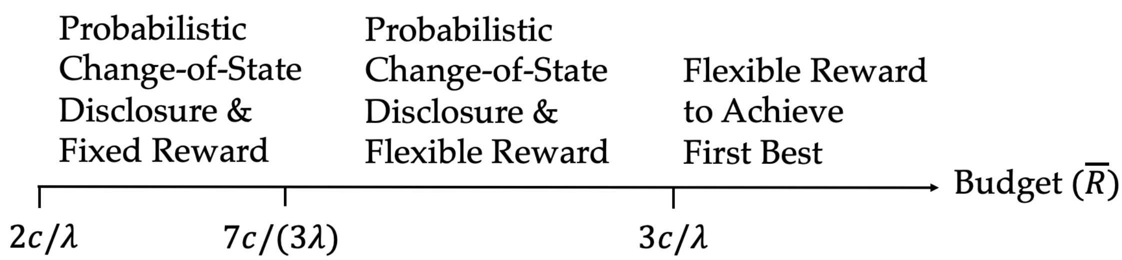

Figure 4 Managerial framework for designing dynamic development contests with two suppliers.

any change in the state of partial progress with some weakly increasing time-dependent probability. We then find that by utilizing a flexible-reward policy during the initial no-disclosure period, the principal can reduce the cost of incentives without hindering project lead time.

Our results, summarized in Figure 4, indicate that enriched rank-based incentives and carefully-tailored information design can be powerful tools to incentivize development efforts of suppliers without overpaying them. With development contests such as hypersonic missiles running into billions of dollars and racing against time, the flexible reward coupled with information disclosure can potentially achieve substantial savings in lead time as well as project budget (to the tune of hundreds of millions of dollars).

Our analysis opens up several interesting future research directions. First, as a first step to understanding dynamic development contests, we have abstracted away from features such as skills heterogeneity, learning by doing, or the uncertainty regarding the feasibility of the first or second stage, but extending our work by containing such features can be interesting research avenues. Adding these features would make the analysis more involved but can provide useful further insights on designing development contests. Second, an interesting research to pursue would be to see how our proposed methods work when success has a quality measure that can be improved over time rather than taking the form of a breakthrough. Third, while we focus on the expected lead time and reward, considering the variability in such metrics could be an interesting future research avenue. Fourth, our optimal policy characteristics also provide new opportunities for empirical and experimental research. Recently Mostagir et al. (2021) run a laboratory experiment to investigate the impact of full and no information disclosure policies on the agents' behavior and the principal's outcome. In our model, besides considering full and no information, we analyze and compare cyclic, deterministic delay and probabilistic policies to show the value of information design and highlight its impact on incentives. Studying these policies experimentally could be an interesting research direction. Finally, we take the perspective of the principal and aim to improve the principal's objective as much as possible. This requires keeping agents working as much as possible as early in the contest as possible. This also means that any effort for a non-winning solution will end up being wasted. From this perspective, our contest design may not necessarily be socially efficient.

An interesting future research avenue is to find a mechanism that considers the social welfare, including both the principal's and agents' objectives.

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Online Appendix

EC.1. Proofs

Proof of Proposition 1: We first calculate the expected duration of a contract in which both agents exert full effort at all times. We solve the problem by backward induction on the state of the game where the states are defined by the number of success for each agent. First, consider the state when both agents have already achieved one success, then the expected arrival time for the second success is given by:

$$\int_0^\infty 2\lambda t e^{-2\lambda t} dt = -te^{-2\lambda t}\Big|_0^\infty + \int_0^\infty e^{-2\lambda t} dt = -\frac{1}{2\lambda} e^{-2\lambda t}\Big|_0^\infty = \frac{1}{2\lambda}.$$

Next, consider the state of the game with a leader (an agent with one success) and a laggard (an agent with no success). Then the expected arrival time for the second success can be expressed as:

$$\int_0^\infty \left[\lambda t + \lambda\left(t + \frac{1}{2\lambda}\right)\right] e^{-2\lambda t} dt = \frac{1}{2\lambda} + \int_0^\infty \frac{1}{2} e^{-2\lambda t} dt = \frac{1}{2\lambda} + \frac{1}{4\lambda} = \frac{3}{4\lambda},$$

where with instantaneous probability λ the leader may obtain the second success at time t or the laggard may hit the first success at time t (proceeding to the above-mentioned state) in which case the expected duration of the contract is $t + 1/(2\lambda)$. Finally, considering the state when neither of the agents has one success, the expected duration of the contract is as follows:

$$\int_0^\infty 2\lambda\left(t + \frac{3}{4\lambda}\right) e^{-2\lambda t} dt = \frac{1}{2\lambda} + \int_0^\infty \frac{3}{2} e^{-2\lambda t} dt = \frac{1}{2\lambda} + \frac{3}{4\lambda} = \frac{5}{4\lambda}. \tag{EC.1}$$

Clearly, given the cost of effort at each instant, the principal has to offer each agent at least $5c/(4\lambda)$ so that each agent's ex-ante expected payoff is non-negative. ■

Proof of Proposition 2: To derive the symmetric pure-strategy Nash equilibrium with full effort, let us fix agent $-i$'s effort $x_{k,l,t}^{-i} = 1$ for all k, l, and t and find conditions under which agent i optimally chooses $x_{k,l,t}^{i} = 1$ for all k, l, and t. For notational simplicity, we drop the superscript i. Consider the state of the game where both agents have already achieved one success, using (3) we can write:

$$V_{1,1,t} = \max_{x_{1,1,\tau}} \int_t^\infty x_{1,1,\tau}(\lambda R - c) e^{-\int_t^\tau \lambda(x_{1,1,s}+1)ds} d\tau.$$

The agent's problem is an infinite horizon problem, so it is stationary. Thus, we can drop the subscript t and write the equivalent Bellman equation for the agent's problem as follows:

$$V_{1,1} = \max_{x_{1,1}} \left\{x_{1,1}(\lambda R - c)dt + (1 - \lambda x_{1,1}dt - \lambda dt)V_{1,1}\right\},$$

Note that $-cx_{1,1}dt$ denotes the agent's cost of effort within the time interval $(t, t+dt)$, while $x_{1,1}\lambda dt$ denotes the probability that a success arrives within $(t, t+dt)$, in which case the agent receives R. On the other hand, the probability that her opponent achieves the second success in that time interval is λdt, and in this case, the agent receives zero reward. With probability $(1 - \lambda x_{1,1}dt - \lambda dt)$, neither the agent nor her opponent achieves the second success, in which case the contest continues and the agent anticipates to receive a continuation payoff of $V_{1,1}$ due to stationarity. Simplifying the above expression and dividing both sides by dt, we obtain the following Hamilton-Jacobi- Bellman (hereafter HJB) equation for the agent's problem:

$$0 = \max_{x_{1,1}} \left\{ \underbrace{-cx_{1,1}}_{cost} + \underbrace{\lambda x_{1,1}(R - V_{1,1})}_{benefit} - \underbrace{\lambda V_{1,1}}_{externality} \right\}. \tag{EC.2}$$

First, second, and third terms reflect the agent's flow cost of effort, her flow benefit from effort, and the externality imposed by her opponent's effort, respectively. Since the HJB in (EC.2) is linear in $x_{1,1}$, it can be concluded that $x_{1,1} = 1$ is optimal if and only if

$$R - V_{1,1} \geq \frac{c}{\lambda}. \tag{EC.3}$$

The above condition implies that each agent finds it optimal to work if the principal rewards the agent with an additional utility of at least c/λ upon the arrival of a success.

Next, consider the state of the game where agent i is the leader with one success and agent $-i$ is the laggard with no success. Bellman and HJB equations for agent i can be expressed as follows:

$$\begin{aligned} V_{1,0} &= \max_{x_{1,0}} \{x_{1,0}(\lambda R - c)dt + \lambda V_{1,1}dt + (1 - \lambda x_{1,0}dt - \lambda dt)V_{1,0}\} \\ &\Rightarrow 0 = \max_{x_{1,0}} \{x_{1,0}(\lambda R - c - \lambda V_{1,0}) + \lambda (V_{1,1} - V_{1,0})\}, \end{aligned} \tag{EC.4}$$

The first line admits a similar interpretation as in the previous case, except that if the laggard (agent $-i$) obtains a success, the leader agent i receives a continuation payoff equal to $V_{1,1}$. From (EC.4), we can derive the Incentive Compatibility (hereafter IC) constraint for agent i which tells us that $x_{1,0} = 1$ is incentive compatible if and only if

$$R - V_{1,0} \geq \frac{c}{\lambda}. \tag{EC.5}$$

When agent i is the laggard with no success and agent $-i$ is the leader with one success, we can rewrite agent i's problem in (4) as follows:

$$\begin{aligned} V_{0,1} &= \max_{x_{0,1}} \{x_{0,1}(\lambda V_{1,1} - c)dt + (1 - \lambda x_{0,1}dt - \lambda dt)V_{0,1}\} \\ &\Rightarrow 0 = \max_{x_{0,1}} \{x_{0,1}(\lambda V_{1,1} - c - \lambda V_{0,1}) - \lambda V_{0,1}\}, \end{aligned} \tag{EC.6}$$

which implies that exerting full effort for the laggard is optimal if and only if the following IC constraint holds:

$$V_{1,1} - V_{0,1} \geq \frac{c}{\lambda}. \tag{EC.7}$$

Finally, before the arrival of any success, the continuation payoff of agent i is given by:

$$\begin{aligned} V_{0,0} &= \max_{x_{0,0}} \{x_{0,0}(\lambda V_{1,0} - c)dt + \lambda V_{0,1}dt + (1 - \lambda x_{0,0}dt - \lambda dt)V_{0,0}\} \\ &\Rightarrow 0 = \max_{x_{0,0}} \{x_{0,0}(\lambda V_{1,0} - c - \lambda V_{0,0}) + \lambda (V_{0,1} - V_{0,0})\}. \end{aligned} \tag{EC.8}$$

From (EC.8), exerting $x_{0,0}=1$ is incentive compatible for agent i if and only if

$$V_{1,0}-V_{0,0}\geq\frac{c}{\lambda}. \tag{EC.9}$$

We are now ready to show that $R=3c/\lambda$ is the minimum required fixed reward to induce agent i (and by symmetry agent $-i$ as well) to exert full effort at all times, hence achieving the minimum expected lead time $\underline{T}$. From (EC.7), $V_{1,1}\geq c/\lambda$ since $V_{0,1}$ has to be non-negative. Also, from (EC.2), under full effort, one can verify that $V_{1,1}=\frac{1}{2}\left(R-\frac{c}{\lambda}\right)$. Combining these together, we require $\frac{1}{2}\left(R-\frac{c}{\lambda}\right)\geq\frac{c}{\lambda}$, which boils down to $R\geq 3c/\lambda$. Thus, we need $R=3c/\lambda$ at the minimum to ensure that (EC.7) is satisfied, and hence it is incentive compatible for the laggard to exert full effort.

It remains to show that $R=3c/\lambda$ satisfies all IC constraints. It is straightforward to check that (EC.3) is satisfied, that is $R-V_{1,1}=\frac{3c}{\lambda}-\frac{c}{\lambda}>\frac{c}{\lambda}$. Plugging in the value of $V_{1,1}=c/\lambda$ into (EC.6), we find that $V_{0,1}=0$ and so the IC constraint in (EC.7) for the laggard is binding. Similarly, plugging in the value of $V_{1,1}=c/\lambda$ into (EC.4), it can be concluded that $V_{1,0}=3c/(2\lambda)$ and so the IC constraint in (EC.5) for the leader is satisfied since $R-V_{1,0}=\frac{3c}{\lambda}-\frac{3c}{2\lambda}=\frac{3c}{2\lambda}>\frac{c}{\lambda}$. Finally, plugging in the values of $V_{1,0}=3c/(2\lambda)$ and $V_{0,1}=0$ into (EC.8), one can verify that $V_{0,0}=c/(4\lambda)$ and so the IC constraint in (EC.9) for each agent is satisfied as $V_{1,0}-V_{0,0}=\frac{3c}{2\lambda}-\frac{c}{4\lambda}=\frac{5c}{4\lambda}>\frac{c}{\lambda}$. ■

Proof of Proposition 3: Consider a flexible-reward contest with $R_{2,0}=2c/\lambda$ and $R_{2,1}=3c/\lambda$ where the principal commits to disclose any success upon its arrival. Similar to the previous case, we analyze the problem by moving backward on the state of the game where the states are defined by the number of successes of the agents. Let us fix agent $-i$'s effort $x^{-i}_{k,l,t}=1$ for all k, l, and t and find conditions under which agent i optimally chooses $x^{i}_{k,l,t}=1$ for all k, l, and t. Consider the state of the game where both agents have already achieved one success. The Bellman equation and the corresponding HJB for agent i's problem can be expressed as follows:

$$\begin{aligned}V_{1,1}&=\max_{x_{1,1}}\{x_{1,1}(\lambda R_{2,1}-c)dt+(1-\lambda x_{1,1}dt-\lambda dt)V_{1,1}\}\\ \Rightarrow 0&=\max_{x_{1,1}}\{x_{1,1}\left(\lambda R_{2,1}-c-\lambda V_{1,1}\right)-\lambda V_{1,1}\},\end{aligned} \tag{EC.10}$$

where we use the fact that the winner receives $R_{2,1}$ in this state of the game. From (EC.10), we can derive that $x_{1,1}=1$ is optimal if and only if

$$R_{2,1}-V_{1,1}\geq\frac{c}{\lambda}. \tag{EC.11}$$

Next, consider the state of the game with a leader and a laggard. The Bellman equation and the corresponding HJB for the leader's problem (which we assume to be agent i) can be written as:

$$\begin{aligned}V_{1,0}&=\max_{x_{1,0}}\{x_{1,0}(\lambda R_{2,0}-c)dt+\lambda V_{1,1}dt+(1-\lambda x_{1,0}dt-\lambda dt)V_{1,0}\}\\ \Rightarrow 0&=\max_{x_{1,0}}\{x_{1,0}\left(\lambda R_{2,0}-c-\lambda V_{1,0}\right)+\lambda\left(V_{1,1}-V_{1,0}\right)\},\end{aligned} \tag{EC.12}$$

where we use the fact that the winner receives $R_{2,0}$ in this state of the game. From (EC.12), we can derive the IC constraint for the leader which tells us that $x_{1,0} = 1$ is incentive compatible if and only if

$$R_{2,0} - V_{1,0} \geq \frac{c}{\lambda}. \tag{EC.13}$$

Similarly, we can express the Bellman equation and the corresponding HJB for the laggard's problem (assuming to be agent i) as follows:

$$\begin{aligned} V_{0,1} &= \max_{x_{0,1}} \{x_{0,1}(\lambda V_{1,1} - c)dt + (1 - \lambda x_{0,1}dt - \lambda dt)V_{0,1}\} \\ \Rightarrow 0 &= \max_{x_{0,1}} \{x_{0,1}(\lambda V_{1,1} - c - \lambda V_{0,1}) - \lambda V_{0,1}\}, \end{aligned} \tag{EC.14}$$

which implies that exerting full effort for the laggard is optimal if and only if the following IC constraint holds

$$V_{1,1} - V_{0,1} \geq \frac{c}{\lambda}. \tag{EC.15}$$

Finally, before the arrival of any success, the continuation value of each agent is given by:

$$\begin{aligned} V_{0,0} &= \max_{x_{0,0}} \{x_{0,0}(\lambda V_{1,0} - c)dt + \lambda V_{0,1}dt + (1 - \lambda x_{0,0}dt - \lambda dt)V_{0,0}\} \\ \Rightarrow 0 &= \max_{x_{0,0}} \{x_{0,0}(\lambda V_{1,0} - c - \lambda V_{0,0}) + \lambda (V_{0,1} - V_{0,0})\}. \end{aligned} \tag{EC.16}$$

From (EC.16), exerting $x_{0,0} = 1$ is incentive compatible for each agent if and only if

$$V_{1,0} - V_{0,0} \geq \frac{c}{\lambda}. \tag{EC.17}$$

We now verify that the proposed flexible-reward schedule in Proposition 3 satisfies all of the above IC constraints and spends the minimum first-best expected reward. Given (EC.15), we can see that $V_{1,1} = c/\lambda$ is the minimum required continuation payoff to incentivize the laggard to put full effort. From (EC.10), we know that $V_{1,1} = \frac{1}{2}\left(R_{2,1} - \frac{c}{\lambda}\right)$. Thus, the principal has to specify a reward $R_{2,1} = 3c/\lambda$ in order to satisfy $V_{1,1} = c/\lambda$. Given these values, it is straightforward to check that the IC constraint in (EC.11) is satisfied, that is $R_{2,1} - V_{1,1} = \frac{3c}{\lambda} - \frac{c}{\lambda} > \frac{c}{\lambda}$. Also, plugging in the value of $V_{1,1} = c/\lambda$ into (EC.14), we obtain that $V_{0,1} = 0$ and so the IC constraint for the laggard is binding. Next, from (EC.17), we can conclude that $V_{1,0} = c/\lambda$ is the minimum required continuation payoff to motivate the agents to exert effort. Plugging in this value into (EC.12), $R_{2,0} = 2c/\lambda$ is needed to satisfy the HJB. It follows that the IC constraint in (EC.13) is indeed binding for the leader as $R_{2,0} - V_{1,0} = \frac{2c}{\lambda} - \frac{c}{\lambda} = \frac{c}{\lambda}$. Finally, given $V_{1,0} = c/\lambda$ and $V_{0,1} = 0$, we conclude by (EC.16) that $V_{0,0} = 0$ which shows that the last IC constraint in (EC.17) is also binding, that is $V_{1,0} - V_{0,0} = \frac{c}{\lambda} - 0 = \frac{c}{\lambda}$. Therefore, full effort is incentive compatible at all times and $\underline{T}$ can be achieved.

To calculate the expected reward of this flexible-reward contest, note that when both agents have already obtained one success, the expected reward of the contest is $R_{2,1} = 3c/\lambda$. When there is a leader and a laggard, the expected reward can be computed as follows:

$$\int_t^\infty \lambda \left(\frac{2c}{\lambda} + \frac{3c}{\lambda}\right) e^{-2\lambda(\tau - t)} d\tau = \frac{5c}{2\lambda}.$$

To interpret the above equation note that if the leader obtains her second success, the reward is $R_{2,0} = 2c/\lambda$ and if the laggard obtains her first success, the state of the game transitions to the case where both agents have already obtained one success and the reward is adjusted upward to $R_{2,1} = 3c/\lambda$. Finally, the ex-ante expected reward of the contest is given by:

$$\int_0^\infty 2\lambda\left(\frac{5c}{2\lambda}\right)e^{-2\lambda t}dt = \frac{5c}{2\lambda}. \blacksquare$$

Proof of Proposition 4: First, we derive (5). Note that, by Bayes' rule, the probability that agent i assigns at time $t+dt$ to the event that her opponent has succeeded once, given p_t^i, can be expressed as follows:

$$p_{t+dt}^i = \frac{p_t^i(1 - x_{1,t}^{-i}\lambda dt) + (1 - p_t^i)x_{0,t}^{-i}\lambda dt}{p_t^i(1 - x_{1,t}^{-i}\lambda dt) + 1 - p_t^i},$$

where the numerator is the probability that the game has not ended yet given that the opponent is in the second stage, and the denominator is the total probability that the contest has not finished yet. The law of motion can be obtained by subtracting p_t^i from both sides, dividing by dt, and taking the limit as $dt \to 0$.

To derive the symmetric pure-strategy Nash equilibrium with full effort, we shall fix the opponent's effort $x_{k,t}^{-i} = 1$ for all k and t and try to find conditions under which agent i best-responds by choosing $x_{k,t}^i = 1$ for all k, t.

Consider the problem faced by an agent who has not yet achieved a success. Dropping the superscript i in (7) by using the symmetry of agents, the equivalent Bellman equation for the agent's problem is as follows:

$$V_{0,t} = \max_{x_{0,t}}\left\{-cx_{0,t}dt + x_{0,t}\lambda V_{1,t}dt + (1 - x_{0,t}\lambda dt - p_t\lambda dt)V_{0,t+dt}\right\}. \tag{EC.18}$$

Note that $cx_{0,t}dt$ denotes the agent's cost of effort within the time interval $(t, t+dt)$, while $x_{0,t}\lambda dt$ denotes the probability that a success arrives within $(t, t+dt)$, in which case the agent receives a continuation payoff, $V_{1,t}$. On the other hand, the probability that her opponent is in the second stage and achieves a success in this time interval is $p_t\lambda dt$, and in that case, the agent receives a continuation value of zero. With probability $(1 - x_{0,t}\lambda dt - p_t\lambda dt)$, neither the agent achieves a success, nor does her opponent achieve the second success, in which case the contest continues and the agent anticipates to receive her continuation payoff, $V_{0,t+dt}$. Given that we have an infinite horizon dynamic model with no deadline, from (EC.18) one can verify that the continuation payoff solely depends on the probability p_t rather than time itself. Thus, we can define a stationary Bellman function $V_{k,p}$ for $k \in \{0,1\}$ that does not depend on time but depends on the current state of p_t. Let p be a state variable that corresponds to the probability that each agent assigns to the fact that her opponent is in the second stage under no information disclosure. Then, we can express each agent's continuation payoff as $V_{k,p}$. Thus, we can rewrite (EC.18) as follows:

$$V_{0,p} = \max_{x_{0,p}}\left\{-cx_{0,p}dt + x_{0,p}\lambda V_{1,p}dt + (1 - x_{0,p}\lambda dt - p\lambda dt)V_{0,p+dp}\right\}. \tag{EC.19}$$

Using a Taylor expansion (Ito's Lemma), we have

$$V_{0,p+dp} \simeq V_{0,p} + V_{0,p}^{'} dp = V_{0,p} + \lambda(1-p)^2 V_{0,p}^{'} dt,$$

where we have used that $x_{k,t}^{-i} = 1$ and $dp = \lambda(1-p)^2 dt$ according to (5). Substituting this expression into (EC.19), dropping the terms of the order dt^2 (since $dt^2 \simeq 0$), canceling terms and dividing both sides by dt, we obtain the following HJB equation for the agent's problem:

$$0 = \max_{x_{0,p}} \left\{ \underbrace{-cx_{0,p}}_{cost} + \underbrace{x_{0,p}\lambda\left(V_{1,p} - V_{0,p}\right)}_{benefit} - \underbrace{\lambda\left[pV_{0,p} - (1-p)^2 V_{0,p}'\right]}_{externality} \right\}. \tag{EC.20}$$

Note that the first term reflects the agent's flow cost of effort, the second term reflects her flow benefit from effort, and the third term captures the externality imposed by her opponent's effort. Since the HJB in (EC.20) is linear in $x_{0,p}$, we conclude that $x_{0,p} = 1$ is optimal if and only if

$$V_{1,p} - V_{0,p} \geq \frac{c}{\lambda}. \tag{EC.21}$$

The above IC constraint implies that an agent with no success finds it optimal to work if the principal rewards the agent with additional utility of at least c/λ upon the arrival of a success.

Next, consider the problem faced by an agent who has achieved one success as formulated in (6). Since the continuation payoffs of agents depend on the state variable p rather than time, the principal's problem is also stationary (i.e., independent of t) and hence it is optimal for the principal to choose a reward schedule that depends only on p. In other words, an agent who achieves two successes first is rewarded R_p, where p is her belief about her opponent's progress. As a result, after dropping the superscript i in (6) by using the symmetry of agents, the corresponding Bellman equation for the agent's problem is given by:

$$V_{1,p} = \max_{x_{1,p}} \{-cx_{1,p}dt + x_{1,p}\lambda R_p dt + (1 - x_{1,p}\lambda dt - p\lambda dt)V_{1,p+dp}\}$$

which using the previous techniques gives us the following HJB eqution

$$0 = \max_{x_{1,p}} \left\{ \underbrace{-cx_{1,p}}_{cost} + \underbrace{x_{1,p}\lambda\left(R_p - V_{1,p}\right)}_{benefit} - \underbrace{\lambda\left[pV_{1,p} - (1-p)^2 V_{1,p}'\right]}_{externality} \right\}. \tag{EC.22}$$

Since the HJB in (EC.22) is linear in $x_{1,p}$, we conclude that $x_{1,p} = 1$ is optimal if and only if

$$R_p - V_{1,p} \geq \frac{c}{\lambda}. \tag{EC.23}$$

We are now ready to prove that full effort is incentive compatible at all times given the proposed flexible-reward schedule in Proposition 4. First, notice that when we fix the opponent's effort $x_{k,t}^{-i} = 1$ for all t and solve (5) with initial condition $p_0^i = 0$, we obtain $p_t = \lambda t/(1+\lambda t)$ as stated in the proposition. Second, note that if an agent with no success receives a continuation payoff $V_{1,p} = c/\lambda$, $\forall p$, by substituting this value into the integral form of the agent's problem in (7), we obtain $V_{0,p} = 0$. Hence, (EC.21) is always binding. Moreover, if $V_{1,p} = c/\lambda$, the flexible-reward schedule $R_p = (2+p)c/\lambda$ always satisfies (EC.23). Plugging in $R_p = (2+p)c/\lambda$ into (EC.22), it can be verified that $V_{1,p} = c/\lambda$ for all p is a solution. Finally, plugging in $V_{1,p} = c/\lambda$ into (EC.20), one can verify that $V_{0,p} = 0$ for all p is a solution. Therefore, the design is incentive compatible at all times and achieves $\underline{T}$.

Finally, we can verify that this design spends the first-best expected reward. To show this, we compute the expected reward that the principal has to pay under this design. Denote by $R_{k,l}$ the principal's expected payout conditional on the first agent having achieved $k \in \{0,1\}$ successes, and the second agent having achieved $l \in \{0,1\}$ successes. Let us consider the state of the game when both agents have already achieved one success, then the expected payout is given by:

$$R_{1,1,t} = \int_t^\infty 2\lambda \left(2 + \frac{\lambda\tau}{1+\lambda\tau}\right) \frac{c}{\lambda} e^{-2\lambda(\tau-t)} d\tau = \frac{3c}{\lambda} - \frac{2c}{\lambda} e^{2(1+\lambda t)} \int_{2(1+\lambda t)}^\infty \frac{e^{-x}}{x} dx,$$

where the first equality can be interpreted as follows: if any agent obtains the second success during interval $(\tau, \tau + d\tau)$ which happens with probability $2\lambda d\tau$, the principal has to pay $(2 + p_\tau)c/\lambda$ to the winner, provided that none of the agents have already obtained the second success by time τ which is captured by the term $e^{-2\lambda(\tau-t)}$, and the second equality is obtained by change of variables.

Next, consider the state of the game with a leader and a laggard. Then the expected payout can be computed as follows:

$$\begin{aligned}
R_{1,0,t} &= \int_t^\infty \lambda \left[\left(2 + \frac{\lambda\tau}{1+\lambda\tau}\right) \frac{c}{\lambda} + R_{1,1,\tau} \right] e^{-2\lambda(\tau-t)} d\tau \\
&= \int_t^\infty \left(2 + \frac{\lambda\tau}{1+\lambda\tau}\right) c e^{-2\lambda(\tau-t)} d\tau + \int_t^\infty \lambda R_{1,1,\tau} e^{-2\lambda(\tau-t)} d\tau \\
&= \frac{1}{2} R_{1,1,t} + \int_t^\infty \lambda \left[\frac{3c}{\lambda} - \frac{2c}{\lambda} e^{2(1+\lambda\tau)} \int_{2(1+\lambda\tau)}^\infty \frac{e^{-x}}{x} dx \right] e^{-2\lambda(\tau-t)} d\tau \\
&= \frac{3c}{\lambda} - \frac{c}{\lambda} e^{2(1+\lambda t)} \int_{2(1+\lambda t)}^\infty \frac{e^{-x}}{x} dx - 2c e^{2(1+\lambda t)} \int_t^\infty \int_{2(1+\lambda\tau)}^\infty \frac{e^{-x}}{x} dx d\tau \\
&= \frac{2c}{\lambda} + \frac{c}{\lambda} e^{2(1+\lambda t)} (1 + 2\lambda t) \int_{2(1+\lambda t)}^\infty \frac{e^{-x}}{x} dx.
\end{aligned}$$

Finally, starting from time zero, the expected reward of the contest is given by:

$$\begin{aligned}
R_{0,0,0} &= \int_0^\infty 2\lambda R_{1,0}(t) e^{-2\lambda t} dt \\
&= \int_0^\infty \left[4c e^{-2\lambda t} + 2c e^2 (1 + 2\lambda t) \int_{2(1+\lambda t)}^\infty \frac{e^{-x}}{x} dx \right] dt \\
&= \frac{2c}{\lambda} + \frac{c}{2\lambda} = \frac{5c}{2\lambda}. \blacksquare
\end{aligned}$$

Proof of Theorem 1: We first prove that under any information disclosure policy, there exists a flexible-reward contest that attains the absolute minimum expected lead time at the first-best cost if $\overline{R} \geq \frac{3c}{\lambda}$. To see this, note that the principal can achieve this goal by organizing a flexible-reward contest similar to the one in Proposition 3 by committing to pay the winner $R_{2,0} = 2c/\lambda$ when one agent achieves the second success before the other agent obtaining any success and $R_{2,1} = 3c/\lambda$ if the second success is obtained when both agents have already succeeded once. Since the principal has commitment power and observes successes, conditioning the reward schedule on the state of the contest in which it ends under any information disclosure policy is feasible. To verify that this reward schedule induces both agents to spend full effort at all times under any information disclosure policy, we fix agent $-i$'s effort to 1 at all times, and prove that agent i best-responds by playing the same strategy. Notice that an agent i with one success holding a belief p about her rival's partial progress finds it optimal to spend full effort if and only if $[pR_{2,1}+(1-p)R_{2,0}]-[pV_{1,1}+(1-p)V_{1,0}]\geq c/\lambda$ (similar to condition (EC.23)), where $V_{1,1}=\frac{1}{2}(R_{2,1}-\frac{c}{\lambda})$ and $V_{1,0}=\frac{1}{2}(R_{2,0}-\frac{c}{\lambda})+\frac{1}{4}(R_{2,1}-\frac{c}{\lambda})$ if she spends full effort in equilibrium. Under this reward schedule, we obtain $V_{1,1}=V_{1,0}=\frac{c}{\lambda}$ and hence the incentive compatibility condition for this agent is indeed slack, implying that full effort is incentive compatible at all times. Next, consider an agent i with no success holding a belief p about her rival's progress. Spending full effort for agent i is optimal if and only if $V_{1,p}-V_{0,p}\geq c/\lambda$ (similar to condition (EC.21)), where $V_{1,p}=pV_{1,1}+(1-p)V_{1,0}=\frac{c}{\lambda}$ under this flexible-reward schedule. In addition, $V_{0,p}=pV_{0,1}+(1-p)V_{0,0}=0$ given that $V_{0,1}=\frac{1}{4}(R_{2,1}-\frac{c}{\lambda})-\frac{c}{2\lambda}=0$ and $V_{0,0}=\frac{1}{4}(R_{2,0}-\frac{c}{\lambda})+\frac{1}{4}(R_{2,1}-\frac{c}{\lambda})-\frac{3c}{4\lambda}=0$ under full effort provision and this reward schedule. Thus, the incentive compatibility condition for this agent is binding. Putting these together, in equilibrium, both agents exert full effort at all times which minimizes the contest's expected lead time. Moreover, this contest spends the first-best expected reward as the principal pays $2c/\lambda$ or $3c/\lambda$ each with probability $1/2$ in this design.

Next, we prove that given an information disclosure policy, if there exists a first-best (flexible-reward) contest that attains the absolute minimum expected lead time $\underline{T}$ at the minimum cost $\underline{R}$, we must have that $\overline{R}\geq\frac{3c}{\lambda}$. Suppose not, that is $\overline{R}<\frac{3c}{\lambda}$, and there exists a lead-time minimizing first-best contest which we denote by $\mathbb{C}$. Start with the observation that agents should always exert full effort in $\mathbb{C}$ because otherwise it is not possible to achieve the absolute minimum expected lead time $\underline{T}$. We first claim that if a contest achieves the absolute minimum expected lead time $\underline{T}$ by paying the first-best reward, it must be the case that, in equilibrium, each agent's ex-ante expected payoff is zero. To see this, note that the sum of agents' surplus in every first-best contest is

$$V^{i}_{0,0,0}+V^{-i}_{0,0,0}=\mathbb{E}\left[R_T.\mathbb{1}_{\{i\ or\ -i\ wins\}}\right]-2c\underline{T}$$
$$\Leftrightarrow\mathbb{E}\left[R_T.\mathbb{1}_{\{i\ or\ -i\ wins\}}\right]=V^{i}_{0,0,0}+V^{-i}_{0,0,0}+2c\underline{T}, \tag{EC.24}$$

where T is the random termination time of the contest. Notice that the left-hand side in (EC.24) admits its minimum value $\underline{R}$ (i.e., the first-best expected reward) if and only if the right-hand side admits its lower bound which implies that each agent, in contest $\mathbb{C}$, must earn zero ex-ante expected utility. Following this observation, consider an agent i with no success holding a belief p_t at time $t>0$ about her rival's partial progress. We claim that $0<p_t<1$. If not, then this means that agent i receives full information about her rival's partial progress at time t. But then if the opponent obtains a success by t, agent i quits immediately, contradicting first-best assumption, because her continuation payoff under the full effort provision in the first-best contest would be

$$V_{0,1,t} = \int_t^\infty (\lambda V_{1,1,\tau} - c)e^{-2\lambda(\tau - t)} d\tau < 0. \tag{EC.25}$$

The inequality above follows because $\overline{R} < \frac{3c}{\lambda}$ and hence $V_{1,1,\tau} = \frac{1}{2}\mathbb{E}[R_T] - \frac{c}{2\lambda} < \frac{1}{2} \times \frac{3c}{\lambda} - \frac{c}{2\lambda} = \frac{c}{\lambda}$ owing to the fact that each agent wins the expected reward with equal probability and the second success is arrived after $\frac{1}{2\lambda}$ periods of time, on average, which costs each agent $\frac{c}{2\lambda}$.

Let $V_{0,t}$ be agent i's continuation payoff under contest $\mathbb{C}$ at time t (where she holds a belief p_t about her rival's progress). We next prove that $V_{0,t} = 0$ for all $t > 0$. Recall from (EC.24) that $V_{0,0,0} = 0$. If $V_{0,t} > 0$ at some t, there is a profitable deviation for agent i where she exerts no effort until time t and then starts exerting full effort and receives a strictly positive expected utility than the equilibrium under $\mathbb{C}$, which is a contradiction. Similarly, we can argue that in contest $\mathbb{C}$ and at any t, $V_{1,t} = \frac{c}{\lambda}$. To see this note that $\frac{c}{\lambda}$ is the minimum necessary continuation payoff to induce an agent with no success to work. Now suppose there is an interval $(t', t' + dt)$ during which $\mathbb{E}[V_{1,t}] > \frac{c}{\lambda}$. Then, agent i with no success can again deviate and earn strictly positive surplus by exerting full effort only during this interval and shirking at all other times (if the agent succeeds, she earns $\lambda\mathbb{E}[V_{1,t}] - c > 0$). Hence, a contradiction. Thus, in contest $\mathbb{C}$, we have $V_{0,t} = 0$ and $V_{1,t} = \frac{c}{\lambda}$ for all t.

Finally, under full effort provision in contest $\mathbb{C}$ we can write

$$V_{0,t} = (\lambda V_{1,t} - c)dt + \lambda(p_t \times 0 + (1 - p_t) \times V_{0,1,t})dt + (1 - 2\lambda dt)V_{0,t+dt}.$$

To understand the above expected continuation payoff for an agent i with no success, note that if agent i exerts full effort during $(t, t + dt)$, she receives $(\lambda V_{1,t} - c)dt$ and if her opponent exerts full effort and obtains a success, the contest ends if this is her second success or agent i receives $V_{0,1,t}$ if this is her first success. Otherwise, the contest continues. Plugging in $V_{0,t} = V_{0,t+dt} = 0$ and $V_{1,t} = \frac{c}{\lambda}$ in the above equation, we obtain $V_{0,1,t} = 0$ which cannot be true by (EC.25). Thus, we have a contradiction, and a first-best contest $\mathbb{C}$ cannot exist if $\overline{R} < \frac{3c}{\lambda}$. ■

Proof of Proposition 5: We prove the proposition in multiple steps.

Step 1: *We first verify that the strategy of the agents in the proposition forms a symmetric equilibrium.*

To check this, we fix the strategy of agent $-i$ to the proposed one in the proposition and verify that agent i best-responds by playing the same strategy. First, using condition (EC.23), it is easy to verify that exerting full effort is incentive compatible for an agent with one success for all p since $V_{1,p}$ can not exceed $\overline{R} - c/\lambda$. Given that agent $-i$ exerts effort $x_{0,t}^{-i} = p_r$ for $t \geq t_r$, and $x_{1,t}^{-i} = 1$, by (5) we obtain $\dot{p}_t^i = 0$. As a result, $p_t^i = p_r$ for $t \geq t_r$. Following this observation, note that if agent i with no success, holding a belief p_r, receives a continuation payoff $V_{1,p_r} = c/\lambda$, by substituting this value into the integral form of the agent's problem in (7), we get $V_{0,p_r} = 0$. Hence, the incentive compatibility condition in (EC.21) is binding for all $t \geq t_r$ implying that agent i is indifferent between any level of effort and so exerting $x_{0,p_r} = p_r$ is optimal. Plugging in $\overline{R} = (2+p_r)c/\lambda$ into (EC.22), it can be verified that $V_{1,p} = c/\lambda$ is a solution for all $t \geq t_r$ where $p_t = p_r$. Finally, to prove that exerting full effort is optimal for agent i with no success for all $p < p_r$, we move backward from time t_r associated with belief p_r and prove that if the agent finds it optimal to exert strictly positive effort at any belief p' where $p \leq p' \leq p_r$ (i.e., if $V_{1,p'+dp} - V_{0,p'+dp} \geq c/\lambda$), then we have $V_{1,p} - V_{0,p} \geq c/\lambda$ implying that exerting full effort is optimal at belief $p - dp$. This can be seen by the following analysis:

$$V_{1,p} - V_{0,p} =$$
$$-cdt + \lambda\overline{R}dt + (1 - \lambda dt - p\lambda dt)V_{1,p+dp} + cdt - \lambda V_{1,p}dt - (1 - \lambda dt - p\lambda dt)V_{0,p+dp} \geq$$
$$\lambda\overline{R}dt + (1 - \lambda dt - p\lambda dt)\frac{c}{\lambda} - \lambda V_{1,p}dt \geq \frac{c}{\lambda},$$

where the last inequality results from the fact that

$$V_{1,p} = \int_t^\infty (\lambda\overline{R} - c)e^{-\int_t^\tau \lambda(1+p_s)ds}d\tau \leq \int_t^\infty (\lambda\overline{R} - c)e^{-\lambda(1+p)(\tau-t)}d\tau$$
$$= \frac{\lambda\overline{R} - c}{\lambda(1+p)} \leq \overline{R} - (1+p)\frac{c}{\lambda},$$

where the first line results from the fact that p_t is weakly increasing and the second line holds if and only if $(2+p)c/\lambda \leq \overline{R}$ which is satisfied for $p \leq p_r$. This verifies the equilibrium. Next, we prove the uniqueness of the symmetric equilibrium.

Step 2: *Let p_r solve $(2+p_r)c/\lambda = \overline{R}$. Under no information disclosure, there is no symmetric equilibrium in which an agent with no success exerts full effort at some $p > p_r$.*

First note that $x_{0,p} < 1$ for some p. This is because as p approaches 1, $V_{1,p}$ approaches $V_{1,1} = \frac{1}{2}(\overline{R} - c/\lambda) < c/\lambda$ given the budget constraint, where we use the fact that an agent with one success exerts full effort at all times. Then, suppose t is the first time at which the belief of an agent with no success reaches its maximum level (p_{max}) in a symmetric equilibrium, and $p_{max} > p_r$. Let us focus on a region where the belief is strictly increasing and reaches p_{max} for the first time. If the agent exerts full effort at p_{max}, by (5) p strictly increases which is a contradiction. Therefore, we must have $V_{1,p_{max}+dp} - V_{0,p_{max}+dp} \leq c/\lambda$. This condition implies that exerting zero effort is optimal at belief p_{max}. Then, we consider the following two cases:

- $V_{1,p_{max}+dp} - V_{0,p_{max}+dp} = c/\lambda$: To find the agent's optimal effort at belief $p_{max} - dp$, given that $x_{0,p_{max}} = 0$, we write the following:

$$V_{1,p_{max}} - V_{0,p_{max}} =$$
$$(\lambda\overline{R} - c)dt + (1 - \lambda dt - p_{max}\lambda dt)V_{1,p_{max}+dp} - (1 - p_{max}\lambda dt)V_{0,p_{max}+dp} =$$
$$V_{1,p_{max}+dp} - V_{0,p_{max}+dp} + \left[\lambda\overline{R} - c - \lambda V_{1,p_{max}+dp} - p_{max}\lambda\left(V_{1,p_{max}+dp} - V_{0,p_{max}+dp}\right)\right]dt$$
$$= \frac{c}{\lambda} + (\lambda\overline{R} - c - \lambda V_{1,p_{max}+dp} - p_{max}c)dt < \frac{c}{\lambda},$$

where the last inequality results from the fact that $V_{1,p_{max}+dp} \geq c/\lambda$ and $\overline{R} = (2+p_r)c/\lambda < (2+p_{max})c/\lambda$. Thus, an agent with no success exerts zero effort at belief $p_{max} - dp$ and by (5) p decreases which is a contradiction.

- $V_{1,p_{max}+dp} - V_{0,p_{max}+dp} < c/\lambda$: From the previous case, we know that an agent with no success must exert full effort at belief $p_{max} - dp$ which requires $V_{1,p_{max}} - V_{0,p_{max}} \geq c/\lambda$. By continuity of $V_{1,p} - V_{0,p}$, we conclude that $V_{1,p_{max}} - V_{0,p_{max}} = c/\lambda$. Doing the same analysis as before, we find that the agent finds it optimal to put zero effort at belief $p_{max} - 2dp$ which violates the assumption that the agent's belief is strictly increasing in this region.

Step 3: *Let p_r solve $(2+p_r)c/\lambda = \overline{R}$. Under no information disclosure, there is no symmetric equilibrium in which an agent with no success does not exert full effort at some $p < p_r$.*

Suppose τ is the first time that the agent with no success does not exert full effort. Let $t > \tau$ be the first time at which the belief of an agent with no success in the equilibrium reaches its minimum level (p_{min}) and $p_{min} < p_r$. Let us focus on a region where the agent's belief is strictly decreasing and reaches p_{min} for the first time. If the agent exerts zero effort at belief p_{min}, by (5) p strictly decreases which is a contradiction. Therefore, we must have $V_{1,p_{min}+dp} - V_{0,p_{min}+dp} \geq c/\lambda$. This condition implies that exerting full effort is optimal at belief p_{min}. Then we consider two cases:

- $V_{1,p_{min}+dp} - V_{0,p_{min}+dp} = c/\lambda$: To find the agent's optimal effort at belief $p_{min} - dp$, given that $x_{0,p_{min}} = 1$, we can write the following:

$$\begin{gathered} V_{1,p_{min}} - V_{0,p_{min}} = \\ (\lambda\overline{R} - c)dt + (1 - \lambda dt - p_{min}\lambda dt)V_{1,p_{min}+dp} - (\lambda V_{1,p_{min}+dp} - c)dt - (1 - \lambda dt - p_{min}\lambda dt)V_{0,p_{min}+dp} = \\ V_{1,p_{min}+dp} - V_{0,p_{min}+dp} + \left[\lambda\overline{R} - \lambda V_{1,p_{min}+dp} - (1+p_{min})\lambda\left(V_{1,p_{min}+dp} - V_{0,p_{min}+dp}\right)\right] dt = \\ \frac{c}{\lambda} + \left[\lambda\overline{R} - \lambda V_{1,p_{min}+dp} - (1+p_{min})\,c\right] dt > \frac{c}{\lambda}, \end{gathered}$$

where the last inequality results from the fact that

$$\begin{aligned} V_{1,p} = \int_t^{\infty} (\lambda\overline{R} - c)e^{-\int_t^{\tau} \lambda(1+p_s)ds} d\tau \leq \int_t^{\infty} (\lambda\overline{R} - c)e^{-\lambda(1+p_{min})(\tau - t)} d\tau \\ = \frac{\lambda\overline{R} - c}{\lambda(1+p_{min})} \leq \overline{R} - (1+p_{min})\frac{c}{\lambda}, \end{aligned}$$

where the first line results from the fact that p_{min} is the minimum belief in the equilibrium and the second line holds since $(2+p_{min})c/\lambda \leq \overline{R}$. Therefore, the agent exerts full effort at belief $p_{min} - dp$ and by (5) p strictly increases which is a contradiction.

- $V_{1,p_{min}+dp} - V_{0,p_{min}+dp} > c/\lambda$: From the previous case, we know that an agent with no success must exert zero effort at belief $p_{min} - dp$ which requires $V_{1,p_{min}} - V_{0,p_{min}} \leq c/\lambda$. By continuity of $V_{1,p} - V_{0,p}$, we conclude that $V_{1,p_{min}} - V_{0,p_{min}} = c/\lambda$. Doing the same analysis as before, we find that the agent finds it optimal to put full effort at belief $p_{min} - 2dp$ which violates the assumption that the agent's belief is strictly decreasing in this region.

From steps 2 and 3, we conclude that the symmetric equilibrium in the proposition is unique. ■

Proof of Proposition 6: We build on the proofs of Propositions 5 and EC.1. As before, let us fix the strategy of agent $-i$ to the proposed one in the proposition and verify the best response of agent i. Consider the very last instant of the first cycle at which the belief of agent i reaches p_r. The IC condition (EC.21) implies that full effort is optimal for an agent with no success if and only if $V_{1,p} - V_{0,p} \geq c/\lambda$. We can rewrite this condition at time t_r associated with belief p_r as follows:

$$p_r V_{1,1} + (1-p_r) V_{1,quit} - (1-p_r) V_{0,0} \geq c/\lambda. \tag{EC.26}$$

The above condition can be interpreted as follows: if agent i obtains her first success at t_r, her expected continuation payoff is given by $p_r V_{1,1} + (1-p_r)V_{1,quit}$ anticipating that the principal discloses full information at the end of the cycle. Therefore, with probability p_r her opponent has already made partial progress which in that case they keep working until the end and the continuation payoff is $V_{1,1}$, or her opponent quits if she has not obtained any success and the continuation payoff is $V_{1,quit}$. On the other hand, if agent i does not succeed at t_r, she quits if her opponent has progressed to the second stage. Otherwise, the contest and the beliefs reset and a new cycle begins with a continuation payoff of $V_{0,0}$.

Given that an agent with one success always puts full effort until the end, we know $V_{1,1} = \frac{1}{2}(\overline{R} - c/\lambda)$ and $V_{1,quit} = \overline{R} - c/\lambda$. Moreover, the upper bound for $V_{0,0}$ is given by $V_{0,0}^F$, where F stands for full information, which is the continuation payoff if full information is provided during each cycle. To see this, suppose that full information is provided during each cycle. We consider two cases: i) if agent i obtains the first success, her opponent immediately quits. This leads to a higher continuation payoff than the case of silent period where the opponent keeps working until the end of the cycle; ii) if agent i's opponent obtains the first success, agent i's best response is to quit. However, in a silent period, agent i earns a negative ex-post payoff. Therefore, the upper bound for $V_{0,0}$ is given by $V_{0,0}^F = \frac{1}{2}(\overline{R} - 2c/\lambda)$. Plugging in these values into (EC.26), it is easy to verify that the condition is binding implying that full effort is optimal.

Finally, to prove that exerting full effort is optimal for agent i with no success during each cycle where $p < p_r$, we can show that if the agent finds it optimal to exert strictly positive effort at any belief p' where $p \leq p' \leq p_r$ (i.e., if $V_{1,p'+dp} - V_{0,p'+dp} \geq c/\lambda$), then we have $V_{1,p} - V_{0,p} \geq c/\lambda$ implying that exerting full effort is optimal at belief $p - dp$. This can be seen by the following analysis:

$$V_{1,p} - V_{0,p} =$$
$$\int_t^{t_r} (\lambda\overline{R} - c) e^{-\int_t^\tau \lambda(1+p_s)ds} d\tau + \left[p_r V_{1,1} + (1-p_r)V_{1,quit}\right] e^{-\int_t^{t_r} \lambda(1+p_s)ds}$$
$$- \int_t^{t_r} (\lambda V_{1,\tau} - c) e^{-\int_t^\tau \lambda(1+p_s)ds} d\tau - (1-p_r)V_{0,0} e^{-\int_t^{t_r} \lambda(1+p_s)ds}$$

$$= \int_t^{t_r} \lambda\left(\overline{R} - V_{1,\tau}\right) e^{-\int_t^\tau \lambda(1+p_s)ds} d\tau + \left[p_r V_{1,1} + (1-p_r)V_{1,quit} - (1-p_r)V_{0,0}\right] e^{-\int_t^{t_r} \lambda(1+p_s)ds} \geq \frac{c}{\lambda},$$

where the last inequality can be verified after plugging in the values of $V_{1,p}$, $V_{1,1}$, $V_{1,quit}$, and $V_{0,0}$ into the above expression, computing the above integral and some tedious algebra. ■

Proof of Proposition 7: We already show that an agent with one success finds it optimal to put full effort if and only if

$$\overline{R} - V_{1,p} \geq \frac{c}{\lambda}, \tag{EC.27}$$

which always holds as $V_{1,p} \leq \overline{R} - c/\lambda$ for all p. Next, consider the continuation payoff of an agent i with no success from any time t (after the silent period) onward as follows:

$$V_{0,t}^i = \max_{x_{0,\tau}^i} \int_t^\infty x_{0,\tau}^i(\lambda V_{1,\tau}^i - c) e^{-\int_t^\tau [x_{0,s}^i \lambda + p_s^i x_{1,s}^{-i} \lambda + p_s^i \gamma]ds} d\tau, \tag{EC.28}$$

where by choosing effort $x_{0,\tau}^i$ during interval $(\tau, \tau + d\tau)$, the agent incurs a cost $cx_{0,\tau}^i d\tau$ and if a success arrives, she enters the second stage and enjoys a continuation payoff of $V_{1,\tau}^i$. If her opponent completes the task during interval $(\tau, \tau + d\tau)$, agent i receives zero reward. Moreover, if the principal discloses partial progress of agent i's opponent, agent i quits and receives zero utility because her continuation payoff upon the arrival of her first success falls below c/λ. To see the evolution of belief in (8) note that, by Bayes' rule, the probability that agent i assigns at time $t + dt$ to the event that her opponent has succeeded once, given p_t^i, can be expressed as follows:

$$p_{t+dt}^i = \frac{p_t^i(1 - x_{1,t}^{-i}\lambda dt - \gamma dt) + (1 - p_t^i)x_{0,t}^{-i}\lambda dt}{p_t^i(1 - x_{1,t}^{-i}\lambda dt - \gamma dt) + 1 - p_t^i},$$

where the numerator is the probability that the game has not ended yet and no information is received given that the opponent is in the second stage, and the denominator is the total probability that the contest has not finished yet and no information is disclosed. The law of motion can be obtained by subtracting p_t^i from both sides, dividing by dt, and taking the limit as $dt \to 0$.

Let us fix the strategy of agent $-i$ to the proposed one in the proposition and verify that agent i best-responds by playing the same strategy if $t \geq t_r$. Using p as the state variable, consider the Bellman equation for the maximization problem of agent i with no success as follows:

$$V_{0,p} = \max_{x_{0,p}} \left\{-cx_{0,p}dt + x_{0,p}\lambda V_{1,p}dt + [1 - x_{0,p}\lambda dt - p\lambda dt - p\gamma dt]V_{0,p+dp}\right\}.$$

Using the same techniques as before, we can derive the following HJB equation:

$$0 = \max_{x_{0,p}} \{-cx_{0,p} + x_{0,p}\lambda\left(V_{1,p} - V_{0,p}\right) - p\lambda V_{0,p} - p\gamma V_{0,p} + (1-p)[\lambda - p\lambda - p\gamma]V_{0,p}'\}.$$

Therefore, the IC constraint for an agent with no success implies that $x_{0,p} = 1$, if and only if

$$V_{1,p} - V_{0,p} \geq c/\lambda, \tag{EC.29}$$

which is similar to (EC.21). To derive the expected continuation payoff of agent i, holding a belief p, upon the arrival of her first success, we can write:

$$V_{1,p} = pV_{1,1} + (1-p)V_{1,0}. \tag{EC.30}$$

where $V_{1,1} = \frac{1}{2}(\overline{R} - c/\lambda)$ is the expected continuation payoff if the opponent has already progressed to the second stage, and $V_{1,0}$ is the expected continuation payoff if the opponent has not progressed to the second stage. Given the value of $\gamma = \lambda(1-p_r)/p_r$ under PSD, by (8) we obtain $p_t = p_r$ remains constant after the initial silent period. Therefore, at any threshold belief p_r, we have:

$$V_{1,0} = \int_t^{\infty} \left[(\lambda\overline{R} - c) + \lambda \frac{1}{2}\left(\overline{R} - \frac{c}{\lambda}\right) + \gamma\left(\overline{R} - \frac{c}{\lambda}\right) \right] e^{-(2\lambda+\gamma)(\tau-t)} d\tau,$$

given that during interval $(\tau, \tau + d\tau)$, the leader puts full effort and earns in expectation $(\lambda\overline{R} - c)d\tau$, or the laggard may achieve her first success (given her full effort strategy in equilibrium) in which case agent i's continuation payoff is $\frac{1}{2}(\overline{R} - c/\lambda)$, or partial progress may be disclosed, in that case the leader gets $(\overline{R} - c/\lambda)$. Taking the above integral, we obtain:

$$V_{1,0} = \frac{3\lambda + 2\gamma}{2(2\lambda+\gamma)}\left(\overline{R} - \frac{c}{\lambda}\right). \tag{EC.31}$$

Therefore, to verify that (EC.29) holds at any $t \geq t_r$, it is enough to verify this condition at the threshold belief p_r as follows:

$$V_{1,p_r} = p_r V_{1,1} + (1-p_r)V_{1,0} = p_r \frac{1}{2}\left(\overline{R} - \frac{c}{\lambda}\right) + (1-p_r)\frac{3\lambda + 2\gamma}{2(2\lambda+\gamma)}\left(\overline{R} - \frac{c}{\lambda}\right) = \frac{c}{\lambda}, \tag{EC.32}$$

where the last equality results from substituting $\gamma = \lambda(1-p_r)/p_r$ and $\overline{R} = (2+p_r)c/\lambda$. Also using (*EC*.28), we obtain $V_{0,p_r} = 0$ for $t \geq t_r$ and hence spending full effort is incentive compatible for all $t \geq t_r$. It remains to show that full effort is incentive compatible for an agent with no success for $t < t_r$. To prove this, we can move backward in time to show that if an agent with no success finds it optimal to spend full effort at any belief p' where $p \leq p' \leq p_r$, then exerting full effort is optimal at belief $p - dp$. This can be seen from the following:

$$\begin{aligned} V_{1,p} - V_{0,p} &= -cdt + \lambda\overline{R}dt + (1 - \lambda dt - p\lambda dt)V_{1,p+dp} + cdt - \lambda V_{1,p}dt - (1 - \lambda dt - p\lambda dt)V_{0,p+dp} \\ &\geq \lambda\overline{R}dt + (1 - \lambda dt - p\lambda dt)\frac{c}{\lambda} - \lambda V_{1,p}dt \geq \frac{c}{\lambda}. \end{aligned}$$

To show the last inequality, we need to show that

$$V_{1,p<p_r} \leq \overline{R} - (1+p)\frac{c}{\lambda}.$$

We prove this in two steps. First, we prove that

$$\begin{aligned} & V_{1,p<p_r} \leq \frac{1}{1+p}\left(\overline{R} - \frac{c}{\lambda}\right) \\ \Leftrightarrow\ & V_{1,p} = pV_{1,1} + (1-p)V_{1,0} = p\frac{1}{2}\left(\overline{R} - \frac{c}{\lambda}\right) + (1-p)\left(\frac{3}{4} + \frac{\gamma}{4(2\lambda+\gamma)}e^{-2\lambda(t_r-t)}\right)\left(\overline{R} - \frac{c}{\lambda}\right) \\ & \leq \frac{1}{1+p}\left(\overline{R} - \frac{c}{\lambda}\right) \Leftrightarrow p\frac{1}{2} + (1-p)\left(\frac{3}{4} + \frac{\gamma}{4(2\lambda+\gamma)}e^{-2\lambda(t_r-t)}\right) \leq \frac{1}{1+p}. \end{aligned}$$

To show the last inequality, we can drop the term $e^{-2\lambda(t_r-t)}$ and substitute for γ to see that

$$\begin{aligned} p\frac{1}{2} + (1-p)\left(\frac{3}{4} + \frac{\gamma}{4(2\lambda+\gamma)}e^{-2\lambda(t_r-t)}\right) &\leq p\frac{1}{2} + (1-p)\left(\frac{3}{4} + \frac{\lambda(1-p_r)/p_r}{4(2\lambda+\lambda(1-p_r)/p_r)}\right) \\ &\leq p\frac{1}{2} + (1-p)\left(\frac{3}{4} + \frac{\lambda(1-p)/p}{4(2\lambda+\lambda(1-p)/p)}\right) = \frac{1}{1+p}. \end{aligned}$$

In the second step, we prove that

$$\frac{1}{1+p}\left(\overline{R}-\frac{c}{\lambda}\right)\leq \overline{R}-(1+p)\frac{c}{\lambda}$$

which holds if and only if $(2+p)c/\lambda \leq \overline{R}$ which is satisfied for $p \leq p_r$. Therefore, an agent with no success puts full effort until she succeeds, or the contest ends, or partial progress is disclosed. ■

Proof of Proposition 8: We prove the theorem in multiple steps. To gain insights for why our proposed PSD improves upon other canonical disclosure policies, we prove a more general result. Suppose the principal commits to disclose information about any partial progress at constant rate $\lambda(x_0-p_r)/p_r$ after t_r so that in equilibrium an agent with no success reduces her effort to $x_0 \geq p_r$ for all $t \geq t_r$. Notice that no information disclosure is a special case with $x_0 = p_r$ and $\gamma = 0$, and PSD is a special case with $x_0 = 1$ and $\gamma \equiv \lambda(1-p_r)/p_r$ for all $t \geq t_r$.

Step 1: *We calculate the expected lead time of the contest under PSD.*

Denote by $T_{k,l,t}$ the expected lead time of the contest when one agent has obtained k successes and the other one has obtained l successes from any time t onward. Let us consider the state of the game when both agents have already obtained one success. Then the expected arrival time for the second success is given by:

$$T_{1,1,t}=\int_t^\infty 2\lambda(\tau-t)e^{-2\lambda(\tau-t)}d\tau=\frac{1}{2\lambda}.$$

Here, information disclosure does not affect the outcome since both agents exert full effort until the end. Next, consider the state of the game with a leader (an agent with one success) and a laggard (an agent with no success). Then, the expected lead time of the contest from any time $t \geq t_r$ can be expressed as follows:

$$\begin{aligned} T_{1,0,t\geq t_r} &= \int_t^\infty \left[\lambda(\tau-t)+x_0\lambda\left(\tau-t+\frac{1}{2\lambda}\right)+\frac{\lambda(x_0-p_r)}{p_r}\left(\tau-t+T_{1,quit,\tau}\right)\right]e^{-\left(\lambda+x_0\lambda+\frac{\lambda(x_0-p_r)}{p_r}\right)(\tau-t)}d\tau \\ &= \frac{2+p_r}{2\lambda\,(1+p_r)}, \end{aligned} \tag{EC.33}$$

where $T_{1,quit,\tau}$ is the expected arrival time for the second success once the principal discloses that the leader has made partial progress and the laggard quits, namely,

$$T_{1,quit,t}=\int_t^\infty \lambda(\tau-t)e^{-\lambda(\tau-t)}d\tau=\frac{1}{\lambda}.$$

(EC.33) can be interpreted as follows: conditional on reaching to any instant τ, the leader exerts full effort and if she succeeds the contest ends at $\tau-t$, or the laggard who is putting x_0 effort may achieve her first success and in that case the contest's expected lead time is $\tau-t+1/(2\lambda)$, or information may be disclosed by the principal and in that case the laggard quits and the contest ends by the leader at $\tau-t+1/\lambda$ in expectation. Interestingly, $T_{1,0,t\geq t_r}$ is independent of x_0. Next, for any $t<t_r$, the expected lead time is given by:

$$T_{1,0,t<t_r}=\int_t^{t_r}\left[\lambda(\tau-t)+\lambda\left(\tau-t+\frac{1}{2\lambda}\right)\right]e^{-2\lambda(\tau-t)}d\tau+\left(t_r-t+\frac{2+p_r}{2\lambda\,(1+p_r)}\right)e^{-2\lambda(t_r-t)}$$

$$= \frac{3}{4\lambda} + \frac{1-p_r}{4\lambda(1+p_r)} e^{-2\lambda(t_r - t)}, \tag{EC.34}$$

where we use the fact that no information is disclosed by the principal before t_r. Finally, the ex-ante expected lead time of the contest for any $t \geq t_r$ can be expressed as follows:

$$T_{0,0,t\geq t_r} = \int_t^\infty 2x_0\lambda\left(\tau - t + T_{1,0,\tau\geq t_r}\right) e^{-2x_0\lambda(\tau-t)} d\tau$$

$$= \int_t^\infty 2x_0\lambda\left[\tau - t + \frac{2+p_r}{2\lambda(1+p_r)}\right] e^{-2x_0\lambda(\tau-t)} d\tau = \frac{1+p_r+x_0(2+p_r)}{2x_0\lambda(1+p_r)}, \tag{EC.35}$$

where we use that an agent with no success exerts effort x_0 after t_r, and for any $t < t_r$ is given by:

$$T_{0,0,t<t_r} = \int_t^{t_r} 2\lambda\left(\tau - t + T_{1,0,\tau<t_r}\right) e^{-2\lambda(\tau-t)} d\tau + \left(t_r - t + T_{0,0,t_r}\right) e^{-2\lambda(t_r-t)}$$

$$= \int_t^{t_r} 2\lambda\left[\tau - t + \frac{3}{4\lambda} + \frac{1-p_r}{4\lambda(1+p_r)} e^{-2\lambda(t_r-\tau)}\right] e^{-2\lambda(\tau-t)} d\tau + \left[t_r - t + \frac{1+p_r+x_0(2+p_r)}{2x_0\lambda(1+p_r)}\right] e^{-2\lambda(t_r-t)}$$

$$= \left[\frac{2(1+p_r) - x_0(1+3p_r) + 2x_0\lambda(1-p_r)(t_r-t)}{4x_0\lambda(1+p_r)}\right] e^{-2\lambda(t_r-t)} + \frac{5}{4\lambda}, \tag{EC.36}$$

given that both agents exert full effort before t_r.

Under PSD, we have $x_0 = 1$ after t_r. Also, $p_r = \lambda t_r/(1+\lambda t_r)$. Plugging in these values into (EC.36), the expected lead time of the contest under PSD is given by:

$$T_{0,0,0} = \left[\frac{1-p_r+2\lambda(1-p_r)t_r}{4\lambda(1+p_r)}\right] e^{-2\lambda t_r} + \frac{5}{4\lambda} = \frac{1}{4\lambda} e^{-2\lambda t_r} + \frac{5}{4\lambda}. \tag{EC.37}$$

Step 2: *We prove that PSD dominates no information disclosure.*

This immediately follows from the previous step. We already show that $T_{1,0,t}$ is independent of x_0. This means the expected lead time of the contest from any time t onward once the first success is obtained is the same across any design with constant information disclosure of rate $\lambda(x_0 - p_r)/p_r$ that stimulates constant effort x_0 after t_r in the equilibrium. However, according to (EC.36), $T_{0,0,0}$ is decreasing in x_0 and probabilistic encouragement design ensures that $x_0 = 1$ as long as both agents have zero success which results in the minimum expected lead time within this class of contests. Notice that no information disclosure or any disclosure with a rate lower than γ fails to encourage full effort and hence is dominated by the probabilistic encouragement design. Finally, we can compute the expected lead time of the contest under no information disclosure by plugging in $x_0 = p_r$ into (EC.36).

Step 3: *We prove that PSD dominates full information disclosure.*

This step is easy to verify. Note that under full information, the laggard quits upon the arrival of the first success at any time t. Therefore, the expected lead time in this case is given by:

$$T^F_{0,0,0} = \int_0^\infty 2\lambda\left(t + \frac{1}{\lambda}\right) e^{-2\lambda t} dt = \frac{3}{2\lambda},$$

where F stands for full information. However, under probabilistic encouragement design, the principal delays the stopping time of the laggard by t_r periods of time on average if success arrives after time t_r (given that $\gamma = 1/t_r$) and by $2t_r - t$ periods of time on average if success arrives at any time $t < t_r$. It is easy to see that $T^F_{0,0,0} < \left(5 + e^{-2\lambda t_r}\right)/(4\lambda)$.

Step 4: *We prove that PSD dominates cyclic information disclosure.*

During the first cycle in a design with cyclic information disclosure, if the first success arrives at time $t < t_r$, both agents put full effort during the cycle and the laggard quits at time t_r at the end of the cycle. Therefore, we can write:

$$T^C_{1,0,t<t_r} = \int_t^{t_r} \left[\lambda(\tau - t) + \lambda\left(\tau - t + \frac{1}{2\lambda}\right)\right] e^{-2\lambda(\tau-t)} d\tau + \left(t_r - t + \frac{1}{\lambda}\right) e^{-2\lambda(t_r - t)} = \frac{3}{4\lambda} + \frac{1}{4\lambda} e^{-2\lambda(t_r - t)},$$

where C stands for cyclic information disclosure. Given this, the ex-ante expected lead time of the contest under cyclic information disclosure is given by:

$$T^C_{0,0,0} = \int_0^{t_r} 2\lambda\left(t + \frac{3}{4\lambda} + \frac{1}{4\lambda} e^{-2\lambda(t_r - t)}\right) e^{-2\lambda t} dt + (t_r + T_{0,0,t_r})\, e^{-2\lambda t_r}$$

$$\Rightarrow T^C_{0,0,0} = \frac{t_r\ e^{-2\lambda t_r}}{2\left(1 - e^{-2\lambda t_r}\right)} + \frac{5}{4\lambda}, \tag{EC.38}$$

where we use the fact that $T^C_{0,0,0} = T^C_{0,0,t_r}$ as the game resets at time t_r. However, under PSD, information is disclosed at least t_r periods on average after the success is arrived. It is easy to check that $T^C_{0,0,0} > \left(5 + e^{-2\lambda t_r}\right)/(4\lambda)$. Thus, PSD dominates cyclic disclosure. ■

Proof of Theorem 2: We first verify the equilibrium under $PCSD$ and then prove that this information disclosure policy minimizes the contest's expected lead time. As before, an agent with one success finds it optimal to spend full effort at all times since $V_{1,p} \le \overline{R} - c/\lambda$ for all p. Next, consider the continuation payoff of an agent i with no success from any time $t \ge \underline{t}$ (phase 2) onward:

$$V^i_{0,t} = \max_{x^i_{0,\tau}} \int_t^{\infty} x^i_{0,\tau}(\lambda V^i_{1,\tau} - c) e^{-\int_t^{\tau}[x^i_{0,s}\lambda + p^i_s x^{-i}_{1,s}\lambda + (1-p^i_s) x^{-i}_{0,s}\lambda\phi_s] ds} d\tau. \tag{EC.39}$$

To see the evolution of belief in (9) note that, by Bayes' rule, the probability that agent i assigns at time $t + dt$ to the event that her rival has succeeded once, given p^i_t, can be expressed as follows:

$$p^i_{t+dt} = \frac{p^i_t(1 - x^{-i}_{1,t}\lambda dt) + (1 - p^i_t) x^{-i}_{0,t}\lambda dt(1 - \phi_t)}{p^i_t(1 - x^{-i}_{1,t}\lambda dt) + (1 - p^i_t)[x^{-i}_{0,t}\lambda dt(1 - \phi_t) + 1 - x^{-i}_{0,t}\lambda dt]},$$

where the numerator is the probability that the contest has not ended yet and no information is disclosed given that the opponent has succeeded once, and the denominator is the total probability that the contest has not finished yet and no information is disclosed. The law of motion can be obtained by subtracting p^i_t from both sides, dividing by dt, and taking the limit as $dt \to 0$.

Let us fix the strategy of agent $-i$ to the proposed one in the equilibrium and verify that agent i best-responds by playing the same strategy. Using p as the state variable, consider the Bellman equation for the maximization problem of agent i with no success as follows:

$$V_{0,p} = \max_{x_{0,p}} \{-c x_{0,p} dt + x_{0,p}\lambda V_{1,p} dt + [1 - x_{0,p}\lambda dt - p\lambda dt - (1-p)\lambda\phi_p dt] V_{0,p+dp}\}.$$

Using the same techniques as before, we can derive the following HJB equation:

$$0 = \max_{x_{0,p}} \{-c x_{0,p} + x_{0,p}\lambda\left(V_{1,p} - V_{0,p}\right) - p\lambda V_{0,p} - (1-p)\lambda\phi_p V_{0,p} + (1-p)[\lambda - p\lambda - (1-p)\lambda\phi_p] V'_{0,p}\}.$$

Therefore, the IC constraint for an agent with no success implies that $x_{0,p} = 1$, if and only if

$$V^{\phi}_{1,p} - V^{\phi}_{0,p} \ge c/\lambda, \tag{EC.40}$$

where the superscript ϕ refers to $PCSD$. To derive the expected continuation payoff of agent i, holding a belief p, upon the arrival of her first success after $\underline{t}$ (in phase 2), we can write:

$$V_{1,p}^{\phi} = pV_{1,1}^{\phi} + (1-p)V_{1,0}^{\phi}, \tag{EC.41}$$

where $V_{1,1}^{\phi} = \frac{1}{2}(\overline{R} - c/\lambda)$ is the expected continuation payoff if the opponent has already progressed to the second stage, and

$$\begin{aligned} V_{1,0}^{\phi} &= \phi\left(\overline{R} - \frac{c}{\lambda}\right) + (1-\phi)\int_t^{\infty}\left[(\lambda\overline{R} - c) + \lambda\frac{1}{2}\left(\overline{R} - \frac{c}{\lambda}\right)\right]e^{-2\lambda(\tau - t)}d\tau \\ &= \phi\left(\overline{R} - \frac{c}{\lambda}\right) + (1-\phi)\frac{3}{4}\left(\overline{R} - \frac{c}{\lambda}\right), \end{aligned}$$

is the expected continuation payoff if the opponent has not progressed to the second stage given that if the principal immediately discloses the change of state, the rival quits and agent i receives $(\overline{R} - c/\lambda)$, otherwise, during each interval $(\tau, \tau + d\tau)$, the leader puts full effort and earns in expectation $(\lambda\overline{R} - c)d\tau$, or the laggard may achieve her first success (given her full effort strategy in the equilibrium) in which case agent i's continuation payoff is $\frac{1}{2}(\overline{R} - c/\lambda)$. Under the proposed $PCSD$, we have $\phi_p = [\frac{4c/\lambda}{\overline{R}-c/\lambda} - 3 + p]/(1-p)$ for $p \in [\underline{p}, \overline{p})$. Given the assumption that the rival spends full effort in the equilibrium and after substituting for ϕ_p in (9), we obtain (12) for the evolution of p_t. Notice that p_t and ϕ_t are strictly increasing in time for $t \in [\underline{t}, \overline{t})$. From $t \geq \overline{t}$, $\phi_t = 1$ which holds the belief constant at $\overline{p}$ from $\overline{t}$ onward. Given the value of ϕ_p and the above equations, it is straightforward to verify that $V_{1,p}^{\phi} = c/\lambda$ for any $p \geq \underline{p}$. Using (EC.39), $V_{0,p}^{\phi} = 0$, and hence spending full effort is incentive compatible for agent i for all $t \geq \underline{t}$.

Finally, the exact same argument in the proof of Proposition 5, step 1 can be provided to prove that exerting full effort is optimal for agent i with no success for all $p < \underline{p}$ where $\underline{p} < p_r$, by showing that if the agent finds it optimal to exert strictly positive effort at any belief p' where $p \leq p' \leq \underline{p}$, then exerting full effort is optimal at belief $p - dp$. Therefore, an agent with no success puts full effort until she succeeds, or the game ends, or partial progress is disclosed. This verifies the equilibrium.

Next, we prove that this design minimizes the contest's expected lead time. Note that the expected lead time is the sum of the expected time *until* the arrival of the first success and its expected time *after* the arrival of the first success until the contest ends. Observe that $PCSD$ minimizes the expected time until the arrival of the first success as both agents exert full effort until the first success arrives. Thus, we shall show that $PCSD$ also minimizes the expected time after the first success until the contest ends. Fix an arbitrary contest and observe that upon the arrival of the first success at time t associated with belief p, we can write $V_{1,p} = pV_{1,1} + (1-p)V_{1,0,p}$ where $V_{1,p}$ is the expected continuation payoff of an agent who just succeeded and $V_{1,1} = \frac{1}{2}(\overline{R} - \frac{c}{\lambda})$ (is a constant) since both agents spend full effort after achieving the first success under any design. We claim that $V_{1,0,p} = (\lambda\overline{R} - c)T_{1,0,p}$ where $V_{1,0,p}$ is the expected continuation payoff of an agent who just achieved the first success at time t conditional on her rival still being in the first stage and $T_{1,0,p}$ is the expected time between the end of the contest and the arrival of the first success at t (associated with belief p). To prove this claim, we can write

$$\begin{aligned} V_{1,0,p} &= (\overline{R}-\frac{c}{\lambda}) - \int_t^\infty x_{0,\tau}\lambda\frac{1}{2}(\overline{R}-\frac{c}{\lambda})e^{-\int_t^\tau \lambda(1+x_{0,s})ds}d\tau \\ &= (\lambda\overline{R}-c)[\frac{1}{\lambda} - \int_t^\infty x_{0,\tau}\lambda\frac{1}{2\lambda}e^{-\int_t^\tau \lambda(1+x_{0,s})ds}d\tau] = (\lambda\overline{R}-c)T_{1,0,p}. \end{aligned} \tag{EC.42}$$

To understand the first equality above, note that $(\overline{R}-\frac{c}{\lambda})$ is the continuation payoff of an agent with one success in the absence of any opponent. In the presence of an opponent and during any interval $(\tau, \tau+d\tau)$, if the leader succeeds, she loses none of this continuation payoff, but if her opponent succeeds (for any effort level of an agent with no success in the equilibrium), the leader loses $\frac{1}{2}(\overline{R}-\frac{c}{\lambda})$ as she needs to compete with her rival in the second stage (recall that $V_{1,1}=\frac{1}{2}(\overline{R}-\frac{c}{\lambda})$). The second equality follows by factoring out the term $(\lambda\overline{R}-c)$. The third equality results from the definition of $T_{1,0,p}$ where the expected duration of a contest after the arrival of the first success with only one agent is given by $\frac{1}{\lambda}$. In the presence of a laggard and during any interval $(\tau,\tau+d\tau)$, if the leader succeeds, the expected duration does not change, but if the laggard succeeds, the expected duration reduces by $\frac{1}{2\lambda}$ owing to the fact that two agents are working full time until the task is complete which takes $\frac{1}{2\lambda}$ on average (a reduction of $\frac{1}{2\lambda}$ compared to $\frac{1}{\lambda}$). Thus, we prove our claim.

Following the above arguments, if we show that $PCSD$ minimizes $V_{1,p}$ for all p, it follows that $PCSD$ also minimizes $V_{1,0,p}$ and accordingly $T_{1,0,p}$ for all p. We next show this result.

First, notice that when $\overline{R} \le \frac{7c}{3\lambda}$, phase 1 does not exist under $PCSD$ (i.e., $\underline{t}=\underline{p}=0$). The principal chooses ϕ_t such that $V_{1,p}=c/\lambda$ for all p which in turn keeps $V_{0,p}=0$ for all p. Indeed, this is the minimum necessary continuation payoff $V_{1,p}$ at each instant to incentivize an agent with no success to work; otherwise, the contest does not proceed to state $\{1,0\}$. This proves our claim in this case.

Second, suppose $\overline{R} > \frac{7c}{3\lambda}$. First, consider an agent with no success holding a belief $p > \overline{p} = \frac{2(\overline{R}-2c/\lambda)}{\overline{R}-c/\lambda}$ ($\overline{p}$ is defined in the theorem). The optimal action for this agent is to quit because even if she succeeds, her payoff is not sufficient to compensate her for her cost of effort as indicated below:

$$V_{1,p} = pV_{1,1} + (1-p)V_{1,0} \le p\frac{1}{2}(\overline{R}-\frac{c}{\lambda}) + (1-p)(\overline{R}-\frac{c}{\lambda}) < \frac{c}{\lambda},$$

where the first inequality results from the facts that an agent with one success always spends full effort (therefore, $V_{1,1}=\frac{1}{2}(\overline{R}-\frac{c}{\lambda})$) and the maximum value of $V_{1,0}$ is obtained if we assume that the rival immediately quits (therefore, $V_{1,0}=\overline{R}-\frac{c}{\lambda}$). Second, consider an agent with no success holding a belief $\underline{p} \le p \le \overline{p}$. Under $PCSD$ and for all such p, $V_{1,p}=c/\lambda$ which is the bare minimum continuation payoff to incentivize any effort in the first stage. Thus, $PCSD$ minimizes $V_{1,p}$ in this region for all p. Finally, consider an agent with no success holding a belief $p<\underline{p}$. Under $PCSD$ and for all such p, $V_{1,p}$ is strictly greater than c/λ but it is the minimum continuation payoff possible as, under $PCSD$, this agent puts full effort until the end and the principal will never disclose this partial progress to her rival, inducing the rival to keep spending full effort until the contest ends. Thus, $PCSD$ minimizes $V_{1,p}$ in this region, too. Putting these together, $PCSD$ grants the minimum surplus to an agent who obtains a success when $\overline{R} > \frac{7c}{3\lambda}$ (and when $\overline{R} \leq \frac{7c}{3\lambda}$).

In conclusion, $PCSD$ minimizes the expected lead time of the contest after the arrival of the first success. Since it achieves the same goal before the arrival of the first success, this policy minimizes the project expected lead time. ■

EC.2. Additional Results and Robustness Checks

PROPOSITION EC.1. *When the principal is budget-constrained and commits to full information disclosure, there exists a unique symmetric equilibrium in which both agents exert full effort until the first success arrives. After that, the laggard quits and the leader puts full effort until the end.*

Proof of Proposition EC.1: Incentive compatibility conditions (EC.3) and (EC.5) show that an agent with one success finds it optimal to put full effort until the end. Given this observation, we have $V_{1,1} = \frac{1}{2}(\overline{R} - c/\lambda) < c/\lambda$. Immediately, from IC condition (EC.7), it can be concluded that the laggard quits. Using this observation, we obtain $V_{1,0} = \overline{R} - c/\lambda$ and $V_{0,0} = \frac{1}{2}(\overline{R} - 2c/\lambda) \geq 0$. Therefore, IC condition (EC.9) is satisfied as $V_{1,0} - V_{0,0} = \frac{1}{2}\overline{R} > c/\lambda$. ■

PROPOSITION EC.2. *When the principal is budget-constrained, and commits to $PCSD$ with a flexible reward according to $R_{2,1} = \overline{R}$ and $R_{2,0,t} = \frac{7c/\lambda - \overline{R} - p_t(\overline{R} + c/\lambda)}{2(1-p_t)}$ if $t < \underline{t}$, otherwise $R_{2,0,t} = \overline{R}$ if $t \geq \underline{t}$, where t is the time at which the first success is obtained, $p_t = \lambda t/(1+\lambda t)$, and $\underline{t}$ is defined in Theorem 2, an agent who has not achieved a success exerts full effort until she obtains her first success, or her opponent obtains her second success, or the principal discloses the opponent's partial progress. An agent who has achieved one success exerts full effort until the end.*

The amount of cost savings (CS) relative to the optimal $PCSD$ contest with a fixed reward is

$$CS = \int_0^{\underline{t}} (2\lambda I_t) e^{-2\lambda t} dt, \tag{EC.43}$$

where

$$I_t = \int_t^{\infty} \lambda(\overline{R} - R_{2,0,t}) e^{-2\lambda(\tau - t)} d\tau = \frac{1}{2}(\overline{R} - R_{2,0,t}). \tag{EC.44}$$

Proof of Proposition EC.2: In the proof of Theorem 2, we already show that $V_{1,p}^{\phi} = c/\lambda$ for $t \geq \underline{t}$ where $R_{2,1} = R_{2,0} = \overline{R}$ which makes $V_{0,p}^{\phi} = 0$ for $t \geq \underline{t}$ and hence exerting full effort is incentive compatible. For $t < \underline{t}$ and with the proposed flexible reward in the Proposition, one can verify that

$$\begin{aligned} V_{1,p}^{\phi} &= pV_{1,1}^{\phi} + (1-p)V_{1,0}^{\phi} \\ &= p\frac{1}{2}(\overline{R} - \frac{c}{\lambda}) + (1-p)\frac{1}{2}[R_{2,0,t} - \frac{c}{\lambda} + \frac{1}{2}(\overline{R} - \frac{c}{\lambda})] = \frac{c}{\lambda}, \end{aligned}$$

where t is the time at which the first success is obtained and p is the associated belief with that time. Therefore, (EC.40) is binding implying that an agent with no success finds it optimal to exert full effort. In addition, the reward structure is such that the incentive compatibility condition is slack for an agent with one success. Indeed, this design leaves no surplus for the agents while inducing them to exert the same level of effort under $PCSD$.

To understand expressions (EC.43) and (EC.44) note that the principal can save money by paying a lower reward $R_{2,0,t}$ compared to $\overline{R}$ if the first success arrives at $t < \underline{t}$, and the leader obtains her second success before the laggard obtains any success. The expressions measure this value. ■

EC.2.1. Splitting the Reward between Stages

In this section, we consider and analyze the possibility of splitting the reward between stages for a budget-constrained principal ($2c/\lambda < \overline{R} < 3c/\lambda$) to see if any improvement in lead-time minimization can be obtained. It is worth noting that for a case with no budget restriction, an interim reward will not be useful because giving a single final reward already achieves the first best.

Because any intermediate reward given publically leads to full information disclosure, which we show to be suboptimal since the laggard quits immediately, we shall focus on the case where the interim reward is given privately. Our analysis indicates that Theorem 2 can be easily extended to settings where the rewards for the two stages are separated, like in Bimpikis et al. (2019). Specifically, we prove that our probabilistic change-of-state information disclosure policy ($PCSD$) remains optimal when the principal specifies $\alpha\overline{R}$ for the first agent who completes stage one and $(1-\alpha)\overline{R}$ for the first agent who completes the task (i.e., both stages). As we shall see in the proof of Theorem EC.1, any feasible splitting must satisfy $0 \le \alpha \le (\overline{R} - c/\lambda)/\overline{R}$, because otherwise, the project cannot be completed.

Theorem EC.1. *When the principal is budget-constrained and commits to splitting the reward between stages according to the above rule for a given α, the following probabilistic change-of-state disclosure policy ($PCSD$) minimizes the expected lead-time of the contest:*

(Phase 1) The principal discloses no information to the agents up to time $\underline{t} = \frac{\underline{p}}{\lambda(1-\underline{p})}$ where

$$\underline{p} = \begin{cases} 0 & \text{if } \dfrac{2c}{\lambda} < \overline{R} \le \dfrac{7c}{(3+\alpha)\lambda}, \\[2ex] \dfrac{(3+\alpha)\overline{R} - 7c/\lambda}{(1+3\alpha)\overline{R} - c/\lambda} & \text{if } \dfrac{7c}{(3+\alpha)\lambda} < \overline{R} < \dfrac{3c}{\lambda}. \end{cases} \tag{EC.45}$$

(Phase 2) At each instant $(t+dt)$ after $\underline{t}$, the principal discloses partial progress with probability

$$\phi_t^* = \begin{cases} \dfrac{\frac{c}{\lambda}(7-p_t) - \overline{R}[3+\alpha(1-3p_t) - p_t]}{(1-p_t)[(1-\alpha)\overline{R} - \frac{c}{\lambda}]} & \text{if } \underline{t} \le t < \overline{t}, \\[2ex] 1 & \text{if } t \ge \overline{t}, \end{cases} \tag{EC.46}$$

if it arrived during interval $(t, t+dt)$ where p_t is the unique solution to the ordinary differential equation (ODE)

$$\dot{p}_t = \lambda(1-p_t)^2(1-\phi_t^*), \tag{EC.47}$$

with boundary conditions $p_{\underline{t}} = \underline{p}$ and $p_{\bar{t}} = \overline{p} \equiv \frac{2(\overline{R}-2c/\lambda)}{(1+\alpha)\overline{R}-c/\lambda}$.

(Equilibrium) Under $PCSD$, an agent who has not achieved a success exerts full effort until she obtains her first success, or her opponent obtains her second success, or the principal discloses the opponent's partial progress. An agent who has achieved one success exerts full effort until the end.

Proof of Theorem EC.1: Fix a development contest with an intermediate reward $\alpha\overline{R}$ for the first agent to complete stage one and a final reward $(1-\alpha)\overline{R}$ for the first agent to complete both stages. Note as before that the IC condition for an agent with one success to spend strictly positive effort can be expressed as $(1-\alpha)\overline{R} - V_{1,p} \geq c/\lambda$ which always holds as long as $(1-\alpha)\overline{R} \geq c/\lambda$. If, on the other hand, $(1-\alpha)\overline{R} < c/\lambda$, an agent with one success has no incentives to work and hence the project will never be completed. Therefore, we must have $\alpha \leq (\overline{R} - c/\lambda)/\overline{R}$ which induces an agent with one success to keep exerting full effort until the project is complete. We now verify the equilibrium under $PCSD$ by fixing the strategy of agent $-i$ to the proposed one in the equilibrium and showing that agent i best-responds by playing the same strategy. Consider the continuation payoff of an agent i with no success from any time $t \geq \underline{t}$ (phase 2) onward. Using a similar approach as in the proof of Theorem 2, the IC constraint for this agent implies that $x_{0,p} = 1$, if and only if:

$$V_{1,p}^{\phi} - V_{0,p}^{\phi} = pV_{1,1}^{\phi} + (1-p)(\alpha\overline{R} + V_{1,0}^{\phi}) - V_{0,p}^{\phi} \geq c/\lambda, \tag{EC.48}$$

where the first equality is obtained by expanding, $V_{1,p}^{\phi}$, the expected continuation payoff of agent i, holding a belief p, upon the arrival of her first success after $\underline{t}$ (in phase 2). In particular, $V_{1,1}^{\phi} = \frac{1}{2}[(1-\alpha)\overline{R} - c/\lambda]$ is the expected continuation payoff if the opponent has already progressed to the second stage, and

$$\begin{aligned} V_{1,0}^{\phi} &= \phi[(1-\alpha)\overline{R} - \frac{c}{\lambda}] + (1-\phi)\int_t^{\infty}\left[\lambda(1-\alpha)\overline{R} - c + \lambda\frac{1}{2}[(1-\alpha)\overline{R} - \frac{c}{\lambda}]\right]e^{-2\lambda(\tau-t)}d\tau \\ &= \phi[(1-\alpha)\overline{R} - \frac{c}{\lambda}] + (1-\phi)\frac{3}{4}[(1-\alpha)\overline{R} - \frac{c}{\lambda}], \end{aligned}$$

is the expected continuation payoff if the opponent has not progressed to the second stage given that if the principal immediately discloses the change of state, the rival quits (since $\frac{1}{2}[(1-\alpha)\overline{R} - c/\lambda] < c/\lambda, \forall\alpha$) and agent i receives $[(1-\alpha)\overline{R} - c/\lambda]$, otherwise, during each interval $(\tau, \tau + d\tau)$, the leader puts full effort and earns in expectation $[\lambda(1-\alpha)\overline{R} - c]d\tau$, or the laggard may achieve her first success (given her full effort strategy in the equilibrium) in which case agent i's continuation payoff is $\frac{1}{2}[(1-\alpha)\overline{R} - c/\lambda]$. The evolution of p_t in (EC.47) is obtained from (9) given the assumption that the rival spends full effort in the equilibrium. Notice that p_t and ϕ_t^* are strictly increasing in time for $t \in [\underline{t}, \overline{t})$. From $t \geq \overline{t}$, $\phi_t^* = 1$ which holds the belief constant at $\overline{p}$ from $\overline{t}$ onward. Given the value of ϕ_p^* and the above equations, it is straightforward to verify that $pV_{1,1}^{\phi} + (1-p)(\alpha\overline{R} + V_{1,0}^{\phi}) = c/\lambda$ for any $p \geq \underline{p}$. This makes $V_{0,p}^{\phi} = 0$, and by (EC.48) spending full effort is incentive compatible for agent i for all $t \geq \underline{t}$. Finally, the exact same argument in the proof of Proposition 5, step 1 can be provided to prove that exerting full effort is optimal for agent i with no success for all $p < \underline{p}$. Therefore, an agent with no success puts full effort until she succeeds, or the game ends, or partial progress is disclosed. This verifies the equilibrium.

Next, we prove that this design minimizes the contest's expected lead time using a similar approach as in the proof of Theorem 2. Note that the expected lead time is the sum of the expected time *until* the arrival of the first success and its expected time *after* the arrival of the first success until the contest ends. Observe that $PCSD$ minimizes the expected time until the arrival of the first success as both agents exert full effort until the first success arrives. Thus, we shall show that $PCSD$ also minimizes the expected time after the first success until the contest ends. Observe that upon the arrival of the first success at time t associated with belief p, the expected continuation payoff of an agent who just succeeded is $V_{1,p} = pV_{1,1} + (1-p)[\alpha\overline{R} + V_{1,0,p}]$ where $V_{1,1} = \frac{1}{2}[(1-\alpha)\overline{R} - \frac{c}{\lambda}]$ (is a constant) since both agents spend full effort after achieving the first success under any design. We can also show that $V_{1,0,p} = [\lambda(1-\alpha)\overline{R} - c]T_{1,0,p}$ where $V_{1,0,p}$ is the expected continuation payoff of an agent who just achieved the first success at time t conditional on her rival still being in the first stage and $T_{1,0,p}$ is the expected time between the end of the contest and the arrival of the first success at t (associated with belief p). To see this, it is enough to note that

$$\begin{aligned} V_{1,0,p} &= [(1-\alpha)\overline{R} - \frac{c}{\lambda}] - \int_t^{\infty} x_{0,\tau}\lambda\frac{1}{2}[(1-\alpha)\overline{R} - \frac{c}{\lambda}]e^{-\int_t^{\tau}\lambda(1+x_{0,s})ds}d\tau \\ &= [\lambda(1-\alpha)\overline{R} - c][\frac{1}{\lambda} - \int_t^{\infty} x_{0,\tau}\lambda\frac{1}{2\lambda}e^{-\int_t^{\tau}\lambda(1+x_{0,s})ds}d\tau] = [\lambda(1-\alpha)\overline{R} - c]T_{1,0,p}. \end{aligned} \tag{EC.49}$$

Thus, if we show that $PCSD$ minimizes $V_{1,p}$ for all p, it follows that $PCSD$ also minimizes $V_{1,0,p}$ and accordingly $T_{1,0,p}$ for all p. The proof follows a similar argument as in the proof of Theorem 2. First, notice that when $\overline{R} \leq \frac{7c}{(3+\alpha)\lambda}$, phase 1 does not exist under $PCSD$ (i.e., $\underline{t} = \underline{p} = 0$). The principal chooses ϕ_t such that $V_{1,p} = c/\lambda$ for all p which in turn keeps $V_{0,p} = 0$ for all p, and this is the minimum necessary continuation payoff $V_{1,p}$ at each instant to incentivize an agent with no success to work; otherwise, the contest does not proceed to state $\{1,0\}$. This proves our claim in this case.

Second, suppose $\overline{R} > \frac{7c}{(3+\alpha)\lambda}$. Consider an agent with no success holding a belief $p > \overline{p} = \frac{2(\overline{R} - 2c/\lambda)}{(1+\alpha)\overline{R} - c/\lambda}$ ($\overline{p}$ is defined in the theorem). The optimal action for this agent is to quit because even if she succeeds, her payoff is not sufficient to compensate her for her cost of effort as indicated below:

$$V_{1,p} = pV_{1,1} + (1-p)[\alpha\overline{R} + V_{1,0}] \leq p\frac{1}{2}[(1-\alpha)\overline{R} - \frac{c}{\lambda}] + (1-p)[\alpha\overline{R} + (1-\alpha)\overline{R} - \frac{c}{\lambda}] < \frac{c}{\lambda},$$

where the first inequality results from the facts that an agent with one success always spends full effort (therefore, $V_{1,1} = \frac{1}{2}[(1-\alpha)\overline{R} - \frac{c}{\lambda}]$) and the maximum value of $V_{1,0}$ is obtained if we assume that the rival immediately quits (therefore, $V_{1,0} = (1-\alpha)\overline{R} - \frac{c}{\lambda}$). Second, consider an agent with no success holding a belief $\underline{p} \le p \le \overline{p}$. Under $PCSD$ and for all such p, $V_{1,p} = c/\lambda$ which is the bare minimum continuation payoff to incentivize any effort in the first stage. Thus, $PCSD$ minimizes $V_{1,p}$ in this region for all p. Finally, consider an agent with no success holding a belief $p < \underline{p}$. Under $PCSD$ and for all such p, $V_{1,p}$ is strictly greater than c/λ but it is the minimum continuation payoff possible as, under $PCSD$, this agent puts full effort until the end and the principal will never disclose this partial progress to her rival, inducing the rival to keep spending full effort until the contest ends. Thus, $PCSD$ minimizes $V_{1,p}$ in this region, too. Putting these together, $PCSD$ grants the minimum surplus to an agent who obtains a success when $\overline{R} > \frac{7c}{(3+\alpha)\lambda}$ (and when $\overline{R} \le \frac{7c}{(3+\alpha)\lambda}$).

In conclusion, $PCSD$ minimizes the expected lead time of the contest after the arrival of the first success. Since it achieves the same goal before the arrival of the first success, this policy remains optimal and minimizes the project expected lead time even when the principal splits the reward between stages. ■

Theorem EC.1 shows the robustness of our finding to the case with an interim reward. Yet, one might question whether splitting rewards is in fact desirable. We answer this question numerically. Specifically, we consider 200 instances of $\overline{R}$ in the region $(2c/\lambda, 3c/\lambda)$, and show that in *all of these instances*, the optimal lead-time minimizing contest is a $PCSD$ design that allocates the entire budget to the final stage/reward (*i.e.*, $\alpha = 0$). The results are illustrated in Figure EC.1.

The intuition for the suboptimality of interim rewards is as follows. Recall that, in our development context, the key challenge for a budget-constrained principal is to encourage an agent who is failing to obtain any success over time to keep exerting effort. To mitigate this discouragement, the principal employs monetary and non-monetary (information design) incentives to minimize the project lead time. However, assigning a portion of the reward to the intermediate stage only amplifies the early contest incentives, specifically benefiting an agent who assigns a high chance to achieving the first success. Yet, such an agent already has sufficient incentives in the absence of any interim reward. However, an agent who fails to achieve a success for a while has incentive issues because she believes her opponent is likely to have progressed to the second stage. For such an agent, an interim reward has no value because she assigns low probability to getting the interim reward. On the contrary, interim reward reduces incentives for such an agent by diverting some of the attainable final reward to an unattainable interim reward. To mitigate the loss of incentives, the principal needs to disclose more information to persuade such an agent, which is suboptimal. Therefore, giving interim rewards is not desirable in development contests.

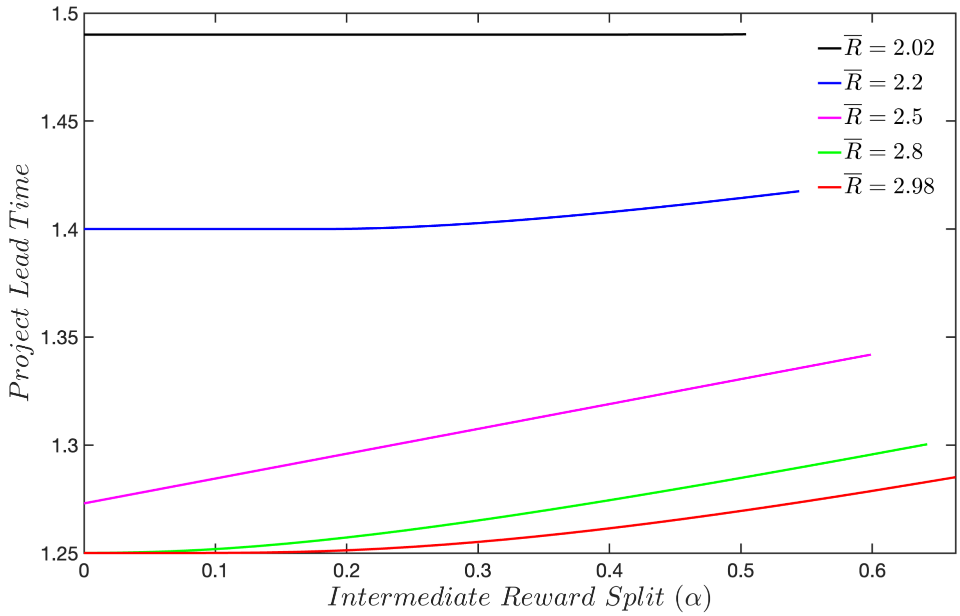

Figure EC.1 No splitting of reward ($\alpha = 0$) combined with probabilistic change-of-state disclosure ($PCSD$) minimizes lead time. Setting: $c = \lambda = 1$.

EC.2.2. Probabilistic vs Deterministic Delay

In this section, we will compare deterministic and probabilistic delay in sharing partial progress.

Consider a case where the principal commits to disclose any partial progress with t_d periods of delay. This mechanism leads to an initial silent period of length t_d during which agents' beliefs drift upward to p_d according to (5). During interval $(t_d, t_d + dt)$, if the principal announces partial progress, an agent with no success (i.e., the laggard) quits. Otherwise, each agent's belief remains constant at p_d as she realizes that no progress has been made during the initial dt period of the contest, akin to a contest that starts at dt instead of time 0. Intuitively, the principal wishes to extend this delay as long as possible to keep the laggard working. Thus, we consider the following deterministic design policy in which an agent with no success spends full effort until information is disclosed by the principal, derive the unique symmetric equilibrium under this policy, and compare the contest's expected lead time under this policy with that under PSD.

Proposition EC.3. *Suppose that the principal is budget-constrained. In the deterministic delay design, the principal commits to disclose partial progress after a delay of length t_d given by*

$$(1 + e^{-2\lambda t_d})(\overline{R} - c/\lambda) = 2(1 + \lambda t_d)(3c/\lambda - \overline{R}). \tag{EC.50}$$

Furthermore, under the deterministic delay design:

(i) An agent who has not achieved a success exerts full effort until she obtains her first success, or her opponent obtains her second success, or the principal discloses the opponent's partial progress.

An agent who has achieved one success exerts full effort until the end.

(ii) Delay length $t_d < t_r = \frac{p_r}{\lambda(1-p_r)}$ *where* $p_r = \frac{\lambda \overline{R}}{c} - 2$.

(iii) The expected lead time of the contest is given by $(5 + e^{-2\lambda t_d})/(4\lambda)$*, which is strictly larger than the expected lead time under PSD.*

Proof of Proposition EC.3: Part (i). As before, we fix the strategy of agent $-i$ to the proposed one in the theorem and verify that agent i best-responds by playing the same strategy. Condition (EC.21) implies that full effort is optimal for an agent with no success if and only if $V_{1,p} - V_{0,p} \geq c/\lambda$. We use this condition to pin down t_d. Consider the very last instant during the initial silent period of length t_d at which an agent with no success finds it optimal to work. Then, we must have:

$$V^d_{1,p_d} = p_d V^d_{1,1} + (1-p_d) V^d_{1,0} = \frac{c}{\lambda}, \tag{EC.51}$$

where $p_d = \lambda t_d/(1+\lambda t_d)$ is the belief of agent i at time t_d anticipating the equilibrium behavior of agent $-i$. We use the superscript d to refer to a design with delay. Given that the principal discloses any progress with a delay t_d, agent i's belief remains constant for $t \geq t_d$. Therefore, at any threshold belief $p = p_d$, we can write:

$$V^d_{1,0} = \int_t^{t+t_d} \left[(\lambda \overline{R} - c) + \lambda \frac{1}{2}\left(\overline{R} - \frac{c}{\lambda}\right)\right] e^{-2\lambda(\tau - t)} d\tau + e^{-2\lambda t_d}\left(\overline{R} - \frac{c}{\lambda}\right),$$

given that during interval $(\tau, \tau + d\tau)$, the leader puts full effort and earns $(\lambda\overline{R} - c)d\tau$, or the laggard may achieve her first success (given her full effort strategy in the equilibrium) and the continuation payoff is $\frac{1}{2}(\overline{R} - c/\lambda)$. If neither the leader achieves the second success, nor does the laggard achieve her first success, the principal discloses progress t_d periods of time after its arrival and in that case the laggard quits and the leader gets $(\overline{R} - c/\lambda)$. Taking the above integral, we obtain:

$$V^d_{1,0} = \left(\frac{3}{4} + \frac{1}{4} e^{-2\lambda t_d}\right)\left(\overline{R} - \frac{c}{\lambda}\right). \tag{EC.52}$$

Substituting the above value and $V^d_{1,1} = \frac{1}{2}(\overline{R} - c/\lambda)$ into (EC.51) and simplifying the equation, we find that t_d must solve (EC.50). Finally, to prove that exerting full effort is optimal for agent i with no success for all $p < p_d$, we move backward from time t_d associated with belief p_d and prove that if the agent finds it optimal to exert strictly positive effort at any belief p' where $p \leq p' \leq p_d$ (i.e., if $V_{1,p'+dp} - V_{0,p'+dp} \geq c/\lambda$), then we have $V_{1,p} - V_{0,p} \geq c/\lambda$ implying that exerting full effort is optimal at belief $p - dp$. This can be seen from the following:

$$\begin{aligned} V^d_{1,p} - V^d_{0,p} &= -cdt + \lambda \overline{R} dt + (1 - \lambda dt - p\lambda dt) V^d_{1,p+dp} + cdt - \lambda V^d_{1,p} dt - (1 - \lambda dt - p\lambda dt) V^d_{0,p+dp} \\ &\geq \lambda \overline{R} dt + (1 - \lambda dt - p\lambda dt)\frac{c}{\lambda} - \lambda V^d_{1,p} dt \geq \frac{c}{\lambda}. \end{aligned}$$

To show the last inequality, we need to show that

$$V^d_{1,p} \leq \overline{R} - (1+p)\frac{c}{\lambda}.$$

We prove this in two steps. First, we prove that

$$V^d_{1,p} \le \frac{1}{1+p}\left(\overline{R}-\frac{c}{\lambda}\right)$$
$$\Leftrightarrow V^d_{1,p} = pV^d_{1,1} + (1-p)V^d_{1,0} = p\frac{1}{2}\left(\overline{R}-\frac{c}{\lambda}\right) + (1-p)\left(\frac{3}{4}+\frac{1}{4}e^{-2\lambda t_d}\right)\left(\overline{R}-\frac{c}{\lambda}\right) \le \frac{1}{1+p}\left(\overline{R}-\frac{c}{\lambda}\right)$$
$$\Leftrightarrow p\frac{1}{2} + (1-p)\left(\frac{3}{4}+\frac{1}{4}e^{-2\lambda t_d}\right) \le \frac{1}{1+p} \Leftrightarrow 1+e^{-2\lambda t_d} \le \frac{2}{1+p}.$$

To show the last inequality, we use that $p=\lambda t/(1+\lambda t)$ and $t \le t_d$. Thus, it is enough to show that

$$1+e^{-2\lambda t_d} \le 1+e^{-2\lambda t} \le \frac{2(1+\lambda t)}{1+2\lambda t} \Leftrightarrow 1-(1+2\lambda t)e^{-2\lambda t} \ge 0,$$

and the last inequality holds given that the left-hand-side is increasing in t and at $t=0$, it is binding. In the second step, we prove that

$$\frac{1}{1+p}\left(\overline{R}-\frac{c}{\lambda}\right) \le \overline{R}-(1+p)\frac{c}{\lambda}$$

which holds if and only if $(2+p)c/\lambda \le \overline{R}$ which is satisfied for $p \le p_d$. This verifies the equilibrium.

Part (ii). To prove that $t_d < t_r$, first notice that the left-hand-side of (EC.51) is strictly decreasing in t_d. To see this, note that

$$\frac{\partial V^d_{1,p_d}}{\partial t_d} = \frac{\partial p_d}{\partial t_d}V^d_{1,1} - \frac{\partial p_d}{\partial t_d}V^d_{1,0} + (1-p_d)\frac{\partial V^d_{1,0}}{\partial t_d} < 0,$$

where the above inequality holds since $\partial p_d/\partial t_d > 0$, $V^d_{1,1} < V^d_{1,0}$ and $\partial V^d_{1,0}/\partial t_d < 0$ according to (EC.52). Following this observation, suppose that $t_d = t_r$. Then, we can show that $V^d_{1,p_r} < c/\lambda$ implying that $t_d < t_r$. To see this, recall from (EC.32) that under PSD we have:

$$V^r_{1,p_r} = p_r V^r_{1,1} + (1-p_r)V^r_{1,0} = \frac{c}{\lambda},$$

where superscript r refers to the PSD. Therefore, to prove that $V^d_{1,p_r} < c/\lambda$, it is enough to show that

$$V^d_{1,0} < V^r_{1,0} \Leftrightarrow \left(\frac{3}{4}+\frac{1}{4}e^{-2\lambda t_r}\right)\left(\overline{R}-\frac{c}{\lambda}\right) < \frac{3\lambda+2\gamma}{2(2\lambda+\gamma)}\left(\overline{R}-\frac{c}{\lambda}\right).$$

Using the fact that $\gamma = 1/t_r$ and further simplifying the above inequality, we need to show:

$$1-(1+2\lambda t_r)e^{-2\lambda t_r} > 0,$$

which always holds for $t_r > 0$. This completes the proof of part (ii).

Part (iii). We first calculate the expected lead time of the contest when the principal discloses partial progress with a delay of length t_d. Denote by $T^d_{k,l,t}$ the expected lead time of the contest when one agent has obtained k successes and the other one has obtained l successes from any time t onward under a design with delay. Consider any time t when the first success arrives. Then, the expected lead time of the contest from t can be expressed as follows:

$$T^d_{1,0,t} = \int_t^{t+t_d}\left[\lambda(\tau-t)+\lambda\left(\tau-t+\frac{1}{2\lambda}\right)\right]e^{-2\lambda(\tau-t)}d\tau + \left(t_d+\frac{1}{\lambda}\right)e^{-2\lambda t_d} = \frac{3}{4\lambda}+\frac{1}{4\lambda}e^{-2\lambda t_d}, \tag{EC.53}$$

where we use the fact that the laggard quits once the principal discloses progress t_d periods after its arrival. Using the above expression, the ex-ante expected lead time of the contest is:

$$T^d_{0,0,0} = \int_0^\infty 2\lambda \left(t + \frac{3}{4\lambda} + \frac{1}{4\lambda} e^{-2\lambda t_d}\right) e^{-2\lambda t} dt = \frac{1}{4\lambda} e^{-2\lambda t_d} + \frac{5}{4\lambda}. \quad \text{(EC.54)}$$

Recall that the expected lead time of the contest under PSD is given by $\frac{1}{4\lambda} e^{-2\lambda t_r} + \frac{5}{4\lambda}$. Part (iii) of the theorem follows from Part (ii) where we show that $t_d < t_r$. ■

EC.2.3. Discounting

In this section, we extend Proposition 1, Proposition 3 which also extends Theorem 1, and Theorem 2 to the case where the principal and the agents discount future payoffs at rate $r > 0$. Generalizations of other results follow in a similar fashion and are available upon request from the authors.

EC.2.3.1. First-Best Contract with Observable Effort and Discounting

PROPOSITION EC.4. *There exists an individually rational "first-best" contract that achieves the minimum expected lead time $\underline{T}$ with the minimum required compensation of $\frac{2c}{2\lambda + r} + \frac{4\lambda c(3\lambda + r)}{(2\lambda + r)^3}$ to agents.*

Proof of Proposition EC.4: Each agent incurs a flow cost of c while working during the contract. Given this, consider the state when both agents have already achieved one success. Then, each agent's expected cost in such a contract from any time t is given by:

$$\int_t^\infty 2\lambda \left(\int_t^\tau c e^{-r(s-t)} ds\right) e^{-2\lambda(\tau - t)} d\tau = \frac{c}{2\lambda + r}.$$

Next, consider the state of the game with a leader and a laggard. Then, each agent's expected cost from any time t can be expressed as follows:

$$\int_t^\infty \left[\lambda \int_t^\tau c e^{-r(s-t)} ds + \lambda \left(\int_t^\tau c e^{-r(s-t)} ds + \frac{c}{2\lambda + r} e^{-r(\tau - t)}\right)\right] e^{-2\lambda(\tau - t)} d\tau = \frac{c(3\lambda + r)}{(2\lambda + r)^2}.$$

Finally, each agent's ex-ante expected cost is given by:

$$\int_0^\infty 2\lambda \left(\int_0^t c e^{-rs} ds + \frac{c(3\lambda + r)}{(2\lambda + r)^2} e^{-rt}\right) e^{-2\lambda t} dt = \frac{c}{2\lambda + r} + \frac{2\lambda c(3\lambda + r)}{(2\lambda + r)^3}.$$

Multiplying the above value by 2 gives us the first-best cost of the principal. ■

EC.2.3.2. Full Information Disclosure with Flexible Reward and Discounting

PROPOSITION EC.5. *Under full information disclosure, a flexible-reward contest with $R_{2,0} = \frac{c(2\lambda + r)}{\lambda^2}$ and $R_{2,1} = \frac{c(3\lambda + r)}{\lambda^2}$ achieves the minimum expected lead time $\underline{T}$ with the first-best expected reward of $\frac{2c}{2\lambda + r} + \frac{4\lambda c(3\lambda + r)}{(2\lambda + r)^3}$.*

Proof of Proposition EC.5: Consider a flexible-reward contest with $R_{2,0} = \frac{c(2\lambda + r)}{\lambda^2}$ and $R_{2,1} = \frac{c(3\lambda + r)}{\lambda^2}$ where the principal commits to disclose any success upon its arrival. Let us fix agent $-i$'s effort $x^{-i}_{k,l,t} = 1$ for all k, l, and t and find conditions under which agent i optimally chooses $x^i_{k,l,t} = 1$ for all k, l, and t. Consider the state of the game where both agents have already achieved one success. The Bellman equation and the corresponding HJB for agent i's problem can be expressed as follows:

$$\begin{aligned} V_{1,1} &= \max_{x_{1,1}} \{x_{1,1}(\lambda R_{2,1} - c)dt + (1 - \lambda x_{1,1}dt - \lambda dt - rdt)V_{1,1}\} \\ \Rightarrow 0 &= \max_{x_{1,1}} \{x_{1,1}(\lambda R_{2,1} - c - \lambda V_{1,1}) - (\lambda + r)V_{1,1}\}, \end{aligned} \tag{EC.55}$$

where we use the fact that the winner receives $R_{2,1}$ in this state of the game. From (EC.55), we can derive that $x_{1,1} = 1$ is optimal if and only if $R_{2,1} - V_{1,1} \geq c/\lambda$. Next, consider the state of the game with a leader and a laggard. The Bellman equation and the corresponding HJB for the leader's problem (which we assume to be agent i) can be written as:

$$\begin{aligned} V_{1,0} &= \max_{x_{1,0}} \{x_{1,0}(\lambda R_{2,0} - c)dt + \lambda V_{1,1}dt + (1 - \lambda x_{1,0}dt - \lambda dt - rdt)V_{1,0}\} \\ \Rightarrow 0 &= \max_{x_{1,0}} \{x_{1,0}(\lambda R_{2,0} - c - \lambda V_{1,0}) + \lambda(V_{1,1} - V_{1,0}) - rV_{1,0}\}, \end{aligned} \tag{EC.56}$$

where we use the fact that the winner receives $R_{2,0}$ in this state of the game. From (EC.56), we can derive the IC constraint for the leader which tells us that $x_{1,0} = 1$ is incentive compatible if and only if $R_{2,0} - V_{1,0} \geq c/\lambda$. Similarly, we can express the Bellman equation and the corresponding HJB for the laggard's problem (assuming to be agent i) as follows:

$$\begin{aligned} V_{0,1} &= \max_{x_{0,1}} \{x_{0,1}(\lambda V_{1,1} - c)dt + (1 - \lambda x_{0,1}dt - \lambda dt - rdt)V_{0,1}\} \\ \Rightarrow 0 &= \max_{x_{0,1}} \{x_{0,1}(\lambda V_{1,1} - c - \lambda V_{0,1}) - (\lambda + r)V_{0,1}\}, \end{aligned} \tag{EC.57}$$

which implies that exerting full effort for the laggard is optimal if and only if $V_{1,1} - V_{0,1} \geq c/\lambda$. Finally, before the arrival of any success, the continuation value of each agent is given by:

$$\begin{aligned} V_{0,0} &= \max_{x_{0,0}} \{x_{0,0}(\lambda V_{1,0} - c)dt + \lambda V_{0,1}dt + (1 - \lambda x_{0,0}dt - \lambda dt - rdt)V_{0,0}\} \\ \Rightarrow 0 &= \max_{x_{0,0}} \{x_{0,0}(\lambda V_{1,0} - c - \lambda V_{0,0}) + \lambda(V_{0,1} - V_{0,0}) - rV_{0,0}\}. \end{aligned} \tag{EC.58}$$

From (EC.58), exerting $x_{0,0} = 1$ is incentive compatible for each agent if and only if $V_{1,0} - V_{0,0} \geq \dfrac{c}{\lambda}$.

We now verify that the proposed flexible-reward schedule in Proposition EC.5 satisfies all of the above IC constraints and spends the minimum first-best expected reward. We know that $V_{1,1} = c/\lambda$ is the minimum required continuation payoff to incentivize the laggard to put full effort. From (EC.55), we know that $V_{1,1} = \frac{\lambda R_{2,1} - c}{2\lambda + r}$. Thus, the principal has to specify a reward $R_{2,1} = \frac{c(3\lambda + r)}{\lambda^2}$ in order to satisfy $V_{1,1} = c/\lambda$. Given these values, the IC constraint $R_{2,1} - V_{1,1} \geq c/\lambda$ is satisfied. Also, plugging in the value of $V_{1,1} = c/\lambda$ into (EC.57), we obtain that $V_{0,1} = 0$ and so the IC constraint for the laggard is binding. Next, we know that $V_{1,0} = c/\lambda$ is the minimum required continuation payoff to motivate the agents to exert effort from the beginning. Plugging in this value into (EC.56), $R_{2,0} = \frac{c(2\lambda + r)}{\lambda^2}$ is needed to satisfy the HJB. It follows that the IC constraint for the leader is satisfied as $R_{2,0} - V_{1,0} \geq \frac{c}{\lambda}$. Finally, given $V_{1,0} = c/\lambda$ and $V_{0,1} = 0$, we conclude by (EC.58) that $V_{0,0} = 0$ which shows that the last IC constraint $V_{1,0} - V_{0,0} = \frac{c}{\lambda} - 0 = \frac{c}{\lambda}$ is binding. Therefore, full effort is incentive compatible at all times which means this design achieves the minimum expected lead time $\underline{T}$.

To calculate the expected reward of the contest with flexible reward, note that when both agents have already obtained one success, the expected reward of the contest with discounting is given by

$$\int_t^\infty 2\lambda\left[\frac{c(3\lambda+r)}{\lambda^2}e^{-r(\tau-t)}\right]e^{-2\lambda(\tau-t)}d\tau=\frac{2c(3\lambda+r)}{\lambda(2\lambda+r)}.$$

When there is a leader and a laggard, the expected reward can be computed as follows:

$$\int_t^\infty \lambda\left[\frac{c(2\lambda+r)}{\lambda^2}+\frac{2c(3\lambda+r)}{\lambda(2\lambda+r)}\right]e^{-(2\lambda+r)(\tau-t)}d\tau=\frac{c}{\lambda}+\frac{2c(3\lambda+r)}{(2\lambda+r)^2}.$$

Finally, the ex-ante expected reward of the contest is given by:

$$\int_0^\infty 2\lambda\left[\frac{c}{\lambda}+\frac{2c(3\lambda+r)}{(2\lambda+r)^2}\right]e^{-(2\lambda+r)t}dt=\frac{2c}{2\lambda+r}+\frac{4\lambda c(3\lambda+r)}{(2\lambda+r)^3},$$

which is the first-best expected reward. ■

EC.2.3.3. Optimal Lead-Time Minimizing Development Contests with Discounting

THEOREM EC.2. *The following probabilistic change-of-state disclosure design, which we call PCSD, minimizes the expected lead-time of the contest when the principal is budget-constrained: (Phase 1) The principal discloses no information to the agents up to time* $\underline{t}=\frac{\underline{p}}{\lambda(1-\underline{p})}$ *where*

$$\underline{p}=\begin{cases}0 & \text{if } \dfrac{c(2\lambda+r)}{\lambda^2}<\overline{R}\le\dfrac{c(2\lambda+r)}{\lambda^2}+\dfrac{c}{3\lambda+r},\\[2ex] \dfrac{3\lambda+r}{\lambda}-\dfrac{(2\lambda+r)^2c}{\lambda^2(\lambda\overline{R}-c)} & \text{if } \dfrac{c(2\lambda+r)}{\lambda^2}+\dfrac{c}{3\lambda+r}<\overline{R}<\dfrac{c(3\lambda+r)}{\lambda^2}.\end{cases}\tag{EC.59}$$

(Phase 2) At each instant $(t+dt)$ *after* $\underline{t}$, *the principal discloses partial progress with probability*

$$\phi_t^*=\begin{cases}\dfrac{\frac{c}{\lambda}-p_t\frac{\lambda\overline{R}-c}{2\lambda+r}-(1-p_t)\frac{\lambda\overline{R}-c+\lambda\frac{\lambda\overline{R}-c}{2\lambda+r}}{2\lambda+r}}{(1-p_t)[\frac{\lambda\overline{R}-c}{\lambda+r}-\frac{\lambda\overline{R}-c+\lambda\frac{\lambda\overline{R}-c}{2\lambda+r}}{2\lambda+r}]} & \text{if } \underline{t}\le t<\overline{t},\\[2ex] 1 & \text{if } t\ge\overline{t},\end{cases}\tag{EC.60}$$

if it arrived during interval $(t,t+dt)$ *where* p_t *is the unique solution to the ordinary differential equation (ODE)*

$$\dot{p}_t=\lambda(1-p_t)^2(1-\phi_t^*),\tag{EC.61}$$

with boundary conditions $p_{\underline{t}}=\underline{p}$ *and* $p_{\overline{t}}=\overline{p}\equiv\frac{(2\lambda+r)[\lambda(\lambda\overline{R}-c)-(\lambda+r)c]}{\lambda^2(\lambda\overline{R}-c)}$.

(Equilibrium) Under PCSD, an agent who has not achieved a success exerts full effort until she obtains her first success, or her opponent obtains her second success, or the principal discloses the opponent's partial progress. An agent who has achieved one success exerts full effort until the end.

Proof of Theorem EC.2: Verifying the equilibrium under *PCSD* is straightforward and follows the steps provided in the proof of Theorem 2. Hence, we omit this part. Here, we prove that this design minimizes the contest's expected lead time. Note that the expected lead time of the contest is the sum of the expected time *until* the arrival of the first success and its expected time *after* the arrival of the first success until the contest ends. Observe that $PCSD$ minimizes the expected time until the arrival of the first success as both agents exert full effort until the first success arrives. Thus, we shall show that $PCSD$ also minimizes the expected time after the first success until the contest ends. Fix an arbitrary contest and observe that upon the arrival of the first success at time t associated with belief p, we can write $V_{1,p} = pV_{1,1} + (1-p)V_{1,0,p}$ where $V_{1,p}$ is the expected (discounted) continuation payoff of an agent who just succeeded, $V_{1,1} = \frac{\lambda\overline{R}-c}{2\lambda+r}$ (is a constant) since both agents spend full effort after achieving the first success under any design, and $V_{1,0,p}$ is the expected (discounted) continuation payoff of an agent who just achieved the first success at time t conditional on her rival still being in the first stage. We claim that $V_{1,0,p} = (\lambda\overline{R}-c)T_{1,0,p}$ where $T_{1,0,p}$ is the expected (discounted) time between the end of the contest and the arrival of the first success at t (associated with belief p). To better understand the principal's objective function in the case of discounting, assume that the principal incurs a flow cost of 1 as long as the contest is running. The principal aims to minimize this cost which is equivalent to the lead-time minimization objective in the original model. The difference here is that this flow cost is discounted over time at rate r. To prove our claim, we can write

$$\begin{aligned}V_{1,0,p} &= \frac{\lambda\overline{R}-c}{\lambda+r} - \int_t^\infty x_{0,\tau}\lambda\Big(\frac{1}{\lambda+r} - \frac{1}{2\lambda+r}\Big)(\lambda\overline{R}-c)e^{-r(\tau-t)-\int_t^\tau \lambda(1+x_{0,s})ds}d\tau \\ &= (\lambda\overline{R}-c)\Big[\frac{1}{\lambda+r} - \int_t^\infty x_{0,\tau}\lambda\Big(\frac{1}{\lambda+r} - \frac{1}{2\lambda+r}\Big)e^{-r(\tau-t)-\int_t^\tau \lambda(1+x_{0,s})ds}d\tau\Big] = (\lambda\overline{R}-c)T_{1,0,p}.\end{aligned} \tag{EC.62}$$

To understand the first equality above, note that $\frac{\lambda\overline{R}-c}{\lambda+r}$ is the (discounted) continuation payoff of an agent with one success in the absence of any opponent. In the presence of an opponent and during any interval $(\tau,\tau+d\tau)$, if the leader succeeds, she loses none of this continuation payoff, but if her opponent succeeds (for any effort level of an agent with no success in the equilibrium), the leader loses $\frac{\lambda\overline{R}-c}{\lambda+r} - \frac{\lambda\overline{R}-c}{2\lambda+r}$ as she needs to compete with her rival in the second stage (recall that $V_{1,1} = \frac{\lambda\overline{R}-c}{2\lambda+r}$). The second equality follows by factoring out the term $(\lambda\overline{R}-c)$. The third equality results from the definition of $T_{1,0,p}$ where the expected discounted cost of running the contest after the arrival of the first success with only one agent is given by $\frac{1}{\lambda+r}$. In the presence of a laggard and during any interval $(\tau,\tau+d\tau)$, if the leader succeeds, the expected discounted cost does not change, but if the laggard succeeds, the expected discounted cost reduces by $\frac{1}{\lambda+r} - \frac{1}{2\lambda+r}$ owing to the fact that two agents are working full time until the task is complete. Thus, we prove our claim.

Following the above arguments, if we show that $PCSD$ minimizes $V_{1,p}$ for all p, it follows that $PCSD$ also minimizes $V_{1,0,p}$ and accordingly $T_{1,0,p}$ for all p. It can be shown that $PCSD$ indeed minimizes $V_{1,p}$ for all p by following the same steps as in the proof of Theorem 2. This completes the proof. ■

EC.2.4. Different Poisson Arrival Rates for Different Stages

In this extension, we consider a case where the hazard rate of success in stage 1 is λ_1 and in stage 2 is λ_2. Proposition 3 can be extended to accommodate different hazard rates in a straightforward manner to show that a flexible-reward contest with $R_{2,0} = c/\lambda_1 + c/\lambda_2$ and $R_{2,1} = 2c/\lambda_1 + c/\lambda_2$ induces both agents to exert full effort at all times. This contest ends with probability 1/2 before the arrival of any success for the laggard in which case the principal spends $R_{2,0} = c/\lambda_1 + c/\lambda_2$; and with probability 1/2, the contest ends after the arrival of the first success for the laggard (the state when both agents have obtained one success) in which case the principal pays $R_{2,1} = 2c/\lambda_1 + c/\lambda_2$. Thus, the expected reward of the contest equals the first-best expected reward $3c/(2\lambda_1) + c/\lambda_2$. Similarly, the result in Proposition 4 can be extended to this setting if the principal gradually increases the reward schedule over time according to $R_t = (1 + p_t)c/\lambda_1 + c/\lambda_2$ where the equilibrium belief of each agent about the partial progress of her opponent, p_t, is given by

$$\frac{\lambda_1 \left[e^{(\lambda_1-\lambda_2)t} - 1\right]}{\lambda_1 e^{(\lambda_1-\lambda_2)t} - \lambda_2} = p_t. \tag{EC.63}$$

Therefore, a principal with sufficient funds, with access to $2c/\lambda_1 + c/\lambda_2$, can find an appropriate flexible-reward schedule for any information disclosure policy that attains the absolute minimum expected lead time at the minimal cost of incentives. We next introduce the updated PSD in a case with different Poisson arrival rates (other results can be generalized similarly).

Definition EC.1. The "*probabilistic state disclosure design*" prescribes no information to the agents up to time t_r that solves

$$\frac{\lambda_1 \left[e^{(\lambda_1-\lambda_2)t_r} - 1\right]}{\lambda_1 e^{(\lambda_1-\lambda_2)t_r} - \lambda_2} = p_r, \tag{EC.64}$$

where p_r solves

$$(1 + p_r)c/\lambda_1 + c/\lambda_2 = \overline{R}. \tag{EC.65}$$

After that it discloses any partial progress with rate $\gamma_r = (\lambda_1 - p_r\lambda_2)/p_r$.

The following proposition describes the equilibrium under this design which is identical to the one in Proposition 7.

Proposition EC.6. *When the principal is budget-constrained, and commits to probabilistic state disclosure design, an agent who has not achieved a success exerts full effort until she obtains her first success, or her opponent obtains her second success, or the principal discloses the opponent's partial progress. An agent who has achieved one success exerts full effort until the end.*

Proof of Proposition EC.6: Let us fix the strategy of agent $-i$ to the proposed one in the proposition and verify that agent i best-responds by playing the same strategy. We know that an agent with one success finds it optimal to put full effort if and only if $\overline{R} - V_{1,p} \geq c/\lambda_2$ which always holds as long as $\overline{R} \geq c/\lambda_2$ since $V_{1,p} \leq \overline{R} - c/\lambda_2$. Next, notice that the belief of an agent i with no success about the partial progress of her opponent evolves according to:

$$dp_t^i = (1-p_t^i)(\lambda_1 - p_t^i\lambda_2 - p_t^i\gamma_t)dt. \tag{EC.66}$$

Using p as the state variable and applying the same techniques as before, we can derive the following HJB equation for the maximization problem of agent i:

$$0 = \max_{x_{0,p}} \left\{-cx_{0,p} + x_{0,p}\lambda_1\left(V_{1,p} - V_{0,p}\right) - p\lambda_2 V_{0,p} - p\gamma_p V_{0,p} + (1-p)(\lambda_1 - p\lambda_2 - p\gamma_p)V_{0,p}'\right\}.$$

Therefore, the IC constraint for an agent with no success implies that $x_{0,p} = 1$, if and only if $V_{1,p} - V_{0,p} \geq c/\lambda_1$. The expected continuation payoff of agent i, holding a belief p, upon the arrival of the first success is $V_{1,p} = pV_{1,1} + (1-p)V_{1,0}$, where $V_{1,1} = \frac{1}{2}(\overline{R} - c/\lambda_2)$ is the expected continuation payoff if the opponent has already progressed to the second stage, and $V_{1,0}$ is the expected continuation payoff if the opponent has not progressed to the second stage. Given the probabilistic rate of information disclosure $\gamma_r = (\lambda_1 - p_r\lambda_2)/p_r$, by (EC.66) we obtain $p_t = p_r$ remains constant for $t \geq t_r$. Therefore, at any threshold belief $p = p_r$, we have:

$$V_{1,0} = \int_t^\infty \left[\left(\lambda_2\overline{R} - c\right) + \lambda_1\frac{1}{2}\left(\overline{R} - \frac{c}{\lambda_2}\right) + \gamma_r\left(\overline{R} - \frac{c}{\lambda_2}\right)\right] e^{-(\lambda_1+\lambda_2+\gamma_r)(\tau-t)}d\tau,$$

given that during interval $(\tau, \tau + d\tau)$, the leader puts full effort and earns $(\lambda_2\overline{R} - c)d\tau$, or the laggard may achieve her first success (given her full effort strategy in the equilibrium) and the continuation payoff is $\frac{1}{2}(\overline{R} - c/\lambda_2)$, or partial progress may be disclosed and in that case the leader gets $(\overline{R} - c/\lambda_2)$. Taking the above integral, we obtain:

$$V_{1,0} = \frac{\lambda_1 + 2\lambda_2 + 2\gamma_r}{2(\lambda_1 + \lambda_2 + \gamma_r)}\left(\overline{R} - \frac{c}{\lambda_2}\right). \tag{EC.67}$$

Using this value, we can write

$$V_{1,p_r} = p_r V_{1,1} + (1-p_r)V_{1,0} = p_r\frac{1}{2}\left(\overline{R} - \frac{c}{\lambda_2}\right) + (1-p_r)\frac{\lambda_1 + 2\lambda_2 + 2\gamma_r}{2(\lambda_1 + \lambda_2 + \gamma_r)}\left(\overline{R} - \frac{c}{\lambda_2}\right) = \frac{c}{\lambda_1}, \tag{EC.68}$$

where the last equality results from substituting $\gamma_r = (\lambda_1 - p_r\lambda_2)/p_r$ and $\overline{R} = (1 + p_r)c/\lambda_1 + c/\lambda_2$. Hence, $V_{1,p_r} = c/\lambda_1$ for $t \geq t_r$. This implies $V_{0,p_r} = 0$ for $t \geq t_r$ and hence the IC constraint for an agent with no success is binding. Finally, the exact same argument in the proof of Proposition 5, step 1 can be provided to prove that exerting full effort is optimal for agent i with no success for all $p < p_r$, by showing that if the agent finds it optimal to exert strictly positive effort at any belief p' where $p \leq p' \leq p_r$, then exerting full effort is optimal at belief $p - dp$. Therefore, an agent with no success puts full effort until she succeeds, or the game ends, or partial progress is disclosed. ■

We conclude this extension by showing that our proposed probabilistic design dominates the two extremes of information disclosure for any pair of λ_1 and λ_2. Generalization of other results follow in a similar fashion and are available upon request from the authors.

Theorem EC.3. *The expected lead time under PSD is given by (EC.72) which is strictly lower than the expected lead times under no and full information disclosure policies.*

Proof of Theorem EC.3: We prove the theorem in multiple steps. As before, we prove the result for a more general class of contests in which the principal commits to disclose information about any partial progress at constant rate $(\lambda_1 x_0 - p_r\lambda_2)/p_r$ after t_r so that in equilibrium an agent with no success reduces her effort to $x_0 \geq p_r$ for all $t \geq t_r$. It is straightforward to show that no information disclosure is a special case with $x_0 = p_r\lambda_2/\lambda_1$ and $\gamma_t = 0$, and probabilistic state disclosure design is a special case with $x_0 = 1$ and $\gamma_r = (\lambda_1 - p_r\lambda_2)/p_r$ for all $t \geq t_r$.

Step 1: We first calculate the expected lead time of the contest under PSD. Denote by $T_{k,l,t}$ the expected lead time of the contest when one agent has obtained k successes and the other one has obtained l successes from any time t onward. Let us consider the state of the game when both agents have already obtained one success. Then the expected arrival time for the second success is given by:

$$T_{1,1,t} = \int_t^\infty 2\lambda_2(\tau - t)e^{-2\lambda_2(\tau-t)}d\tau = \frac{1}{2\lambda_2}.$$

Here, information disclosure does not affect the outcome since both agents exert full effort until the end. Next, consider the state of the game with a leader and a laggard. Then, the expected lead time of the contest from any time $t \geq t_r$ can be expressed as follows:

$$T_{1,0,t\geq t_r} = \int_t^\infty \left[\lambda_2(\tau-t) + x_0\lambda_1\left(\tau - t + \frac{1}{2\lambda_2}\right) + \frac{\lambda_1 x_0 - p_r\lambda_2}{p_r}\left(\tau - t + T_{1,quit,\tau}\right)\right] \times e^{-\left(\lambda_2 + x_0\lambda_1 + \frac{\lambda_1 x_0 - p_r\lambda_2}{p_r}\right)(\tau-t)}d\tau = \frac{2+p_r}{2\lambda_2\left(1+p_r\right)}, \quad \text{(EC.69)}$$

where $T_{1,quit,\tau}$ is the expected arrival time for the second success once the principal discloses that the leader has made partial progress and the laggard quits, namely,

$$T_{1,quit,t} = \int_t^\infty \lambda_2(\tau-t)e^{-\lambda_2(\tau-t)}d\tau = \frac{1}{\lambda_2}.$$

Equation (EC.69) can be interpreted as follows. Conditional on reaching to any instant τ, the leader exerts full effort and if she succeeds the contest ends at $\tau - t$, or the laggard who is putting x_0 effort may achieve her first success and in that case the contest's expected lead time is $\tau - t + 1/(2\lambda_2)$, or information may be disclosed by the principal and in that case the laggard quits and the contest ends by the leader at $\tau - t + 1/\lambda_2$ in expectation. Interestingly, $T_{1,0,t\geq t_r}$ is independent of x_0. Next, for any $t < t_r$, the expected lead time is given by:

$$\begin{aligned} T_{1,0,t<t_r} &= \int_t^{t_r} \left[\lambda_2(\tau-t) + \lambda_1\left(\tau - t + \frac{1}{2\lambda_2}\right)\right]e^{-(\lambda_1+\lambda_2)(\tau-t)}d\tau + \left(t_r - t + \frac{2+p_r}{2\lambda_2\left(1+p_r\right)}\right)e^{-(\lambda_1+\lambda_2)(t_r-t)} \\ &= \frac{\lambda_1 + 2\lambda_2}{2\lambda_2(\lambda_1+\lambda_2)} + \frac{\lambda_1 - p_r\lambda_2}{2\lambda_2(\lambda_1+\lambda_2)(1+p_r)}e^{-(\lambda_1+\lambda_2)(t_r-t)}, \end{aligned} \quad \text{(EC.70)}$$

where we use the fact that no information is disclosed by the principal before t_r. Finally, the ex-ante expected lead time of the contest for any $t \geq t_r$ can be expressed as follows:

$$\begin{aligned} T_{0,0,t\geq t_r} &= \int_t^\infty 2x_0\lambda_1\left(\tau - t + T_{1,0,\tau\geq t_r}\right)e^{-2x_0\lambda_1(\tau-t)}d\tau \\ &= \int_t^\infty 2x_0\lambda_1\left[\tau - t + \frac{2+p_r}{2\lambda_2\left(1+p_r\right)}\right]e^{-2x_0\lambda_1(\tau-t)}d\tau = \frac{\lambda_2(1+p_r) + x_0\lambda_1(2+p_r)}{2x_0\lambda_1\lambda_2(1+p_r)}, \end{aligned} \quad \text{(EC.71)}$$

where we use that an agent with no success exerts effort x_0 after t_r, and for any $t < t_r$ is given by:

$$\begin{aligned}
T_{0,0,t<t_r} &= \int_t^{t_r} 2\lambda_1 \left(\tau - t + T_{1,0,\tau<t_r}\right) e^{-2\lambda_1(\tau-t)} d\tau + \left(t_r - t + T_{0,0,t_r}\right) e^{-2\lambda_1(t_r-t)} \\
&= \int_t^{t_r} 2\lambda_1 \left[\tau - t + \frac{\lambda_1+2\lambda_2}{2\lambda_2(\lambda_1+\lambda_2)} + \frac{\lambda_1 - p_r\lambda_2}{2\lambda_2(\lambda_1+\lambda_2)(1+p_r)} e^{-(\lambda_1+\lambda_2)(t_r-\tau)}\right] e^{-2\lambda_1(\tau-t)} d\tau \\
&\quad + \left[t_r - t + \frac{\lambda_2(1+p_r)+x_0\lambda_1(2+p_r)}{2x_0\lambda_1\lambda_2(1+p_r)}\right] e^{-2\lambda_1(t_r-t)} \\
&= \left[\frac{-1}{2\lambda_1} - \frac{\lambda_1+2\lambda_2}{2\lambda_2(\lambda_1+\lambda_2)} + \frac{\lambda_1(\lambda_1-p_r\lambda_2)}{(\lambda_2-\lambda_1)\lambda_2(\lambda_1+\lambda_2)(1+p_r)} + \frac{\lambda_2(1+p_r)+x_0\lambda_1(2+p_r)}{2x_0\lambda_1\lambda_2(1+p_r)}\right] e^{-2\lambda_1(t_r-t)} \\
&\quad - \frac{\lambda_1(\lambda_1-p_r\lambda_2)}{(\lambda_2-\lambda_1)\lambda_2(\lambda_1+\lambda_2)(1+p_r)} e^{-(\lambda_1+\lambda_2)(t_r-t)} + \frac{\lambda_1^2+3\lambda_1\lambda_2+\lambda_2^2}{2\lambda_1\lambda_2(\lambda_1+\lambda_2)}, \qquad \text{(EC.72)}
\end{aligned}$$

given that both agents exert full effort before t_r.

Under PSD, we have $x_0 = 1$ after t_r. Also, p_r is given by (EC.63). Plugging in these values into (EC.72) gives us the expected lead time of the contest under PSD.

Step 2: We next prove that PSD dominates no information disclosure. This result follows from the previous step. We already show that $T_{1,0,t}$ is independent of x_0. This means the expected lead time of the contest from any time t onward once the first success is obtained is the same across any design with constant information disclosure of rate $(\lambda_1 x_0 - p_r\lambda_2)/p_r$ that stimulates constant effort x_0 after t_r in the equilibrium. However, according to (EC.72), $T_{0,0,0}$ is decreasing in x_0 and PSD ensures that $x_0 = 1$ as long as both agents have zero success, which results in the minimum expected lead time within this class of contests. Notice that no information disclosure or any disclosure with a rate lower than γ_r fails to encourage full effort and hence is dominated by PSD. Finally, we can compute the expected lead time of the contest under no information disclosure by plugging in $x_0 = p_r\lambda_2/\lambda_1$ into (EC.72).

Step 3: We finally prove that PSD dominates full information disclosure. Note that under full information, the laggard quits upon the arrival of the first success at any time t. However, under PSD, the principal delays the stopping time of the laggard by $1/\gamma_r$ periods of time on average if success arrives after time t_r and by $(t_r - t + 1/\gamma_r)$ periods of time on average if success arrives at any time $t < t_r$. The result directly follows from the fact that the laggard works for a longer duration under probabilistic design. ■

EC.2.5. Optimal Flexible-Reward Contest with n Agents

In this section, we show that a flexible-reward schedule is even more beneficial when more agents participate in the contest. We start our analysis by first characterizing the optimal flexible-reward schedule under full information disclosure that induces n agents to exert full effort to complete a two-stage task. Then, we compare our proposed optimal design with the optimal fixed-reward contest. The following remark along with Figure EC.2 formally highlight that the benefit of flexible rewards is increasing with the number of agents.

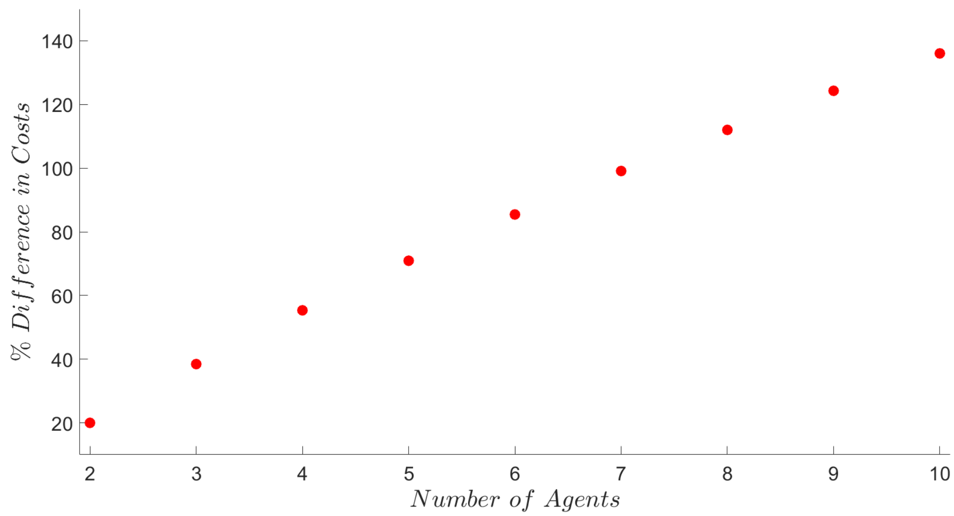

Figure EC.2 Percentage difference between the average rewards under fixed-reward and flexible-reward schedules.

Remark EC.1. In a two-stage development contest with n agents, the principal can implement an optimal flexible-reward schedule under full information disclosure that achieves the absolute minimum expected lead time by paying the absolute minimum expected reward. The cost savings relative to the optimal fixed-reward contest is increasing in the number of agents.

Proof of Remark EC.1: Consider a flexible-reward contest under full information disclosure in which the principal offers a guaranteed reward of $2c/\lambda$ with the option to increase the reward by c/λ per each additional agent progressing to the second stage. This means if exactly m out of n agents have already achieved one success, the reward for the winner will be $(m+1)c/\lambda$. The optimality of this contest can be seen from the fact that agents' and principal's combined surplus is the same across any design that achieves the absolute minimum expected lead time $\underline{T}$ by inducing full effort at all times. Therefore, the principal's surplus is maximized when the agents' surplus is minimized. The proposed flexible-reward schedule in this case minimizes the agents' surplus by keeping the continuation payoff of an agent with no success equal to zero (her outside option) and the continuation payoff of an agent with one success equal to c/λ which is the bare minimum utility to incentivize first-stage effort. Therefore, this design maximizes the principal's surplus and hence is optimal.

To see why this design keeps the continuation payoff of an agent with one success equal to c/λ, notice that if all agents have already achieved one success, the continuation payoff of each agent under full effort is equal to

$$V_{\underbrace{1,1,\dots,1}_{n}} = \int_0^\infty [\lambda(n+1)\frac{c}{\lambda} - c]e^{-n\lambda t}dt = \frac{c}{\lambda}.$$

Now, suppose this holds for the case when exactly $m+1$ agents have already achieved one success. By induction, when m agents have already achieved one success, we can show that the continuation payoff of each agent with one success is c/λ as follows

$$V_{\underbrace{1,1,...,1}_{m},\underbrace{0,0,...,0}_{n-m}} = \int_0^\infty [\lambda(m+1)\frac{c}{\lambda} - c + (n-m)\lambda\frac{c}{\lambda}]e^{-n\lambda t}dt = \frac{c}{\lambda}.$$

Given the above result, it immediately follows that this design keeps the continuation payoff of an agent with no success equal to zero.

Next, we calculate the expected first-best reward of the contest in this case when n agents exert full effort at all times to complete a two-stage task. Denote by $R_{n,s}$ the principal's expected payout when exactly s agents have not achieved any success. Let us consider the state when all agents have achieved one success (i.e., $s=0$). Then the expected reward of the contest is given by $R_{n,0} = (n+1)c/\lambda$. Now, let us guess that the expected reward is given by

$$R_{n,s} = \frac{c}{n\lambda}\sum_{j=0}^{s}\frac{(n-j)(n-j+1)(s!)}{n^{(s-j)}(j!)}.$$

Notice that the above equation holds for $s=0$. Towards proving the result by induction on s, we can express the expected reward of the contest when exactly $s+1$ agents are in the first stage as follows

$$R_{n,s+1} = \int_0^\infty \left[(n-s-1)\lambda(n-s)\frac{c}{\lambda} + (s+1)\lambda R_{n,s}\right] e^{-n\lambda t}dt = \frac{c}{n\lambda}\sum_{j=0}^{s+1}\frac{(n-j)(n-j+1)[(s+1)!]}{n^{(s+1-j)}(j!)},$$

where the first equality results from the observation that if any one of the $n-s-1$ agents in the second stage obtains another success, the contest ends and the reward is $(n-s)c/\lambda$, and if any one of the $s+1$ agents in the first stage succeeds, the expected payout is $R_{n,s}$, and the second equality follows by substituting $R_{n,s}$ and collecting terms. Thus, our guess is verified.

Finally, considering the state when neither of the agents has one success, the expected first-best reward of the contest is equal to:

$$R_{n,n} = \frac{c}{n\lambda}\sum_{j=0}^{n}\frac{(n-j)(n-j+1)(n!)}{n^{(n-j)}(j!)}. \tag{EC.73}$$

Now, consider a fixed-reward contest that induces all agents to exert full effort at all times. Such a contest requires the minimum reward of $(n+1)c/\lambda$ to give sufficient incentives for working to an agent with no success while all the other agents have progressed to the second stage. This can be seen by the fact that this reward makes $V_{\underbrace{1,1,...,1}_{n}} = c/\lambda$ which is the minimum continuation payoff to stimulate effort.

Finally, the fixed-reward contest spends $[[(n+1)c/\lambda]/R_{n,n} - 1] \times 100\%$ more than the optimal flexible-reward contest which one can (tediously) verify that this amount is increasing in n. ■