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INDONESIAN MATHEMATICS TEACHERS' KNOWLEDGE OF CONTENT AND STUDENTS OF AREA AND PERIMETER OF RECTANGLE

Wahid Yunianto[1], Rully Charitas Indra Prahmana[2], Cosette Crisan[3]

[1]SEAMEO QITEP in Mathematics, Jl. Kaliurang, Condongcatur, Depok, Sleman, Yogyakarta 55281, Indonesia
[2]Universitas Ahmad Dahlan, Jl. Pramuka 42, Pandeyan, Umbulharjo, Yogyakarta, Indonesia
[3]University College London, Gower St, Bloomsbury, London WC1E 6BT, United Kingdom
Email: rully.indra@mpmat.uad.ac.id

Abstract

Measuring teachers' skills and competencies is necessary to ensure teacher quality and contribute to education quality. Research has shown teachers competencies and skills influence students' performances. Previous studies explored teachers' knowledge through testing. Teachers' knowledge of the topic of area-perimeter and teaching strategies has been assessed through testing. In general, items or tasks to assess mathematics teacher knowledge in the previous studies were dominated by subject matter knowledge problems. Thus, it seems that the assessment has not fully covered the full range of teacher knowledge and competencies. In this study, the researchers investigated mathematics teachers' Knowledge of Content and Students (KCS) through lesson plans developed by the teachers. To accommodate the gap in the previous studies, this study focuses on KCS on the topic of area-perimeter through their designed lesson plans. Twenty-nine mathematics teachers attended a professional development activity voluntarily participated in this study. Two teachers were selected to be the focus of this case study. Content analysis of the lesson plan and semi-structured interviews were conducted, and then data were analyzed. It revealed that the participating teachers were challenged when making predictions of students' possible responses. They seemed unaware of the ordinary students' strategies used to solve maximizing area from a given perimeter. With limited knowledge of students' possible strategies and mistakes, these teachers were poorly prepared to support student learning.

Keywords: Knowledge of Content and Students, Mathematics Teacher, Area and Perimeter, Teachers' Skills and Competencies

Abstrak

Mengukur keterampilan dan kompetensi guru diperlukan untuk memastikan kualitas guru dan berkontribusi pada kualitas pendidikan. Penelitian ini menunjukkan bahwa komptensi dan keterampilan guru mempengaruhi performa siswa. Penelitian sebelumnya telah mengkaji pengetahuan gru melalui tes. Pengetahuan guru pada topik keliling-luas dan strategi pembelajaran juga telah dikaji melalui tes. Pada umumnya, banyaknya soal pada tes didominasi oleh soal-soal tentang pengetahuan subjek yang diajarkan. Oleh karena itu, asesmen seperti ini belum mencakup kesuluruhan pengetahuan dan kompetensi guru. Pada studi ini, peneliti menginvestigasi pengetahuan guru matematika tentang KCS pada rencana pelaksanaan pembelajaran yang mereka kembangkan. Untuk mengakomodasi kesenjangan pada penelitian sebelumnya, penelitian kali ini berfokus pada pengetahuan tentang konten dan siswa (KCS) pada topik keliling-luas pada rencana pelaksanaan pembelajaran. Dua puluh Sembilan guru matematika yang sedang mengikuti pelatihan peningkatan kompetensi secara suka rela mengikuti penelitian ini. Dua guru matematika menjadi fokus penelitian studi kasus ini. Konten analisis dan interview semi terstruktur dilakukan dan datanya dianalisis. Terungkap bahwa peserta ini mengalami tantangan dalam memprediksi kemungkinan respon yang diberikan siswa. Mereka belum menyadari strategi siswa yang biasanya digunakan untuk menyelesaikan persoalan memaksimalkan luas dari keliling yang ditentukan. Dengan pengetahuan yang terbatas pada kemungkinan strategi siswa dan kesalahan siswa, guru ini kurang siap dalam mendukung siswanya

Kata kunci: Pengetahuan tentang Materi dan Siswa, Guru Matematika, Luas dan Keliling, Keterampilan dan Kompetensi Guru

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Shulman (1986) refers to Pedagogical Content Knowledge (PCK) as the ways of representing and formulating the subject that is understandable to others. Research have shown that student achievements are more affected by PCK than Subject Matter Knowledge (SMK) as the quality of instruction is related to PCK (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill, Ball, & Schilling, 2008). As the use of SMK terminology varies, SMK in this paper refers to common content knowledge (CCK) which is part of SMK (see Figure 1).

> *Hill, Ball and Shilling (2008), in seeking to conceptualize the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking, proposed the following domain map for mathematical knowledge for teaching (see Figure 1) (White, et al., 2012, p.394).*

One specific aspect of PCK is the Knowledge of Content and Students (KCS). KCS is 'knowledge that combines knowing about students and knowing about mathematics (Ball, Thames, & Phelps, 2008, p. 401). It consists of anticipating what students are likely to think about, what they could find confusing or complicated, and what students are expected to do mathematically to complete the chosen task.

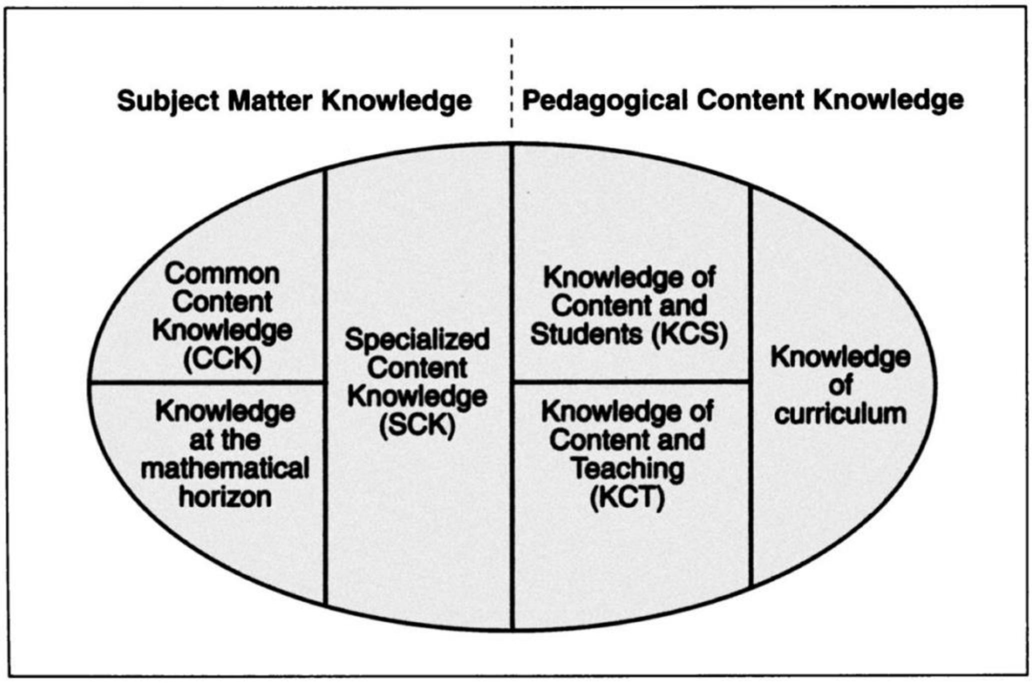


Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

There are some teacher assessment models which measure knowledge for teaching. The Teacher Education and Development Study in Mathematics (TEDS-M) is one of the international assessments intended for pre-service mathematics teachers (Tatto et al., 2012). Some researchers assert that the Assessment of Teachers' PCK could be done through micro-teaching (Setyaningrum, Mahmudi, & Murdanu, 2018; Ünver, Özgür, & Güzel, 2020). In the case of pre-service teachers, they have challenges with student's thinking, mistakes and responding (Korkmaz & Şahin, 2019; Setyaningrum et al., 2018; Ünver et al., 2020). It makes sense as they have limited teaching experiences or even have not taught yet. For in-service teachers, Baumert and Kunter (2013) developed instruments to measure teacher's professional competence (COACTIV). The COACTIV adopted the three main core knowledge CK, PCK and PK from Shulman's work and extended it.

As one of the ways, testing is used to assess teachers. The Ministry of Education and Culture (MoEC) of the Republic of Indonesia has also implemented Teacher Competency Tests (TCT) to evaluate teachers' knowledge. The result of this assessment is both to evaluate teachers and to provide support for them (Widodo & Tamimudin, 2014). However, the content of this assessment is commonly dominated by SMK, in this case within the mathematical problems. It seems that the PCK has not been measured fully through this wide assessment. Another study using testing faced challenges in measuring teachers' knowledge (Fauskanger, 2015). An interesting finding of a study of pre-service teachers is that they possessed higher PCK scores than SMK from limited test items (Kristanto, Panuluh, & Atmajati, 2020). A case study in South Korea revealed that teachers with sufficient SMK of a certain competence/ topic faced challenges in incorporating KCS and KCT of that topic (Lee, Capraro, & Capraro, 2018). Therefore, testing to measure teachers' knowledge still face challenges.

Lesson plans are considered to play an important role in teaching and learning. Having a good lesson plan is important in ensuring that learning would take place during the lesson (Jones & Edwards, 2010). Academics argue that the key determinant of success in teaching is the effectiveness of planning and how well a plan is carried out in the classroom. Effective lesson planning considers possible classroom problems and how to tackle them adequately (Jones & Edwards, 2010). In the common Japanese lesson plan, it contains detailed instruction so that teachers can easily understand it when reading it (Nakahara & Koyama, 2000). Japanese lesson plans also include possible student solutions and errors. The blackboard is also carefully planned. Called, 'Bansho', which anticipates and tries to elicit student mathematical thinking and student thinking schema for solving the given problems.

In developing lesson plans, teachers integrate their knowledge, such as subject matter knowledge and pedagogical content knowledge (An, Kulm, & Wu, 2004; Burns & Lash, 1988; Simon, 1995). A study in Australia revealed that the teacher, in planning a lesson, gave attention to students' engagement (Clarke, Clarke, Roche, & Chan, 2015). The students' engagement involves a choice from many pedagogical strategies, all designed to motivate the students to engage with the topic. It has been shown by several studies that novice teachers improved their PCK by teaching and preparing to teach (Turnuklu & Yesildere, 2007). There is a reciprocal relationship between teacher thought process (including planning) and teachers actions, the latter much influenced by the former (Clark & Peterson, 1986; Superfine, 2008). In other words, teacher classroom practices are influenced by a complex mix of teacher beliefs, attitudes knowledge and intentions Therefore, arguably it is possible to look at teacher lesson plans to investigate their knowledge. The illustration of a model of teacher knowledge and planning can be seen in Figure 2.

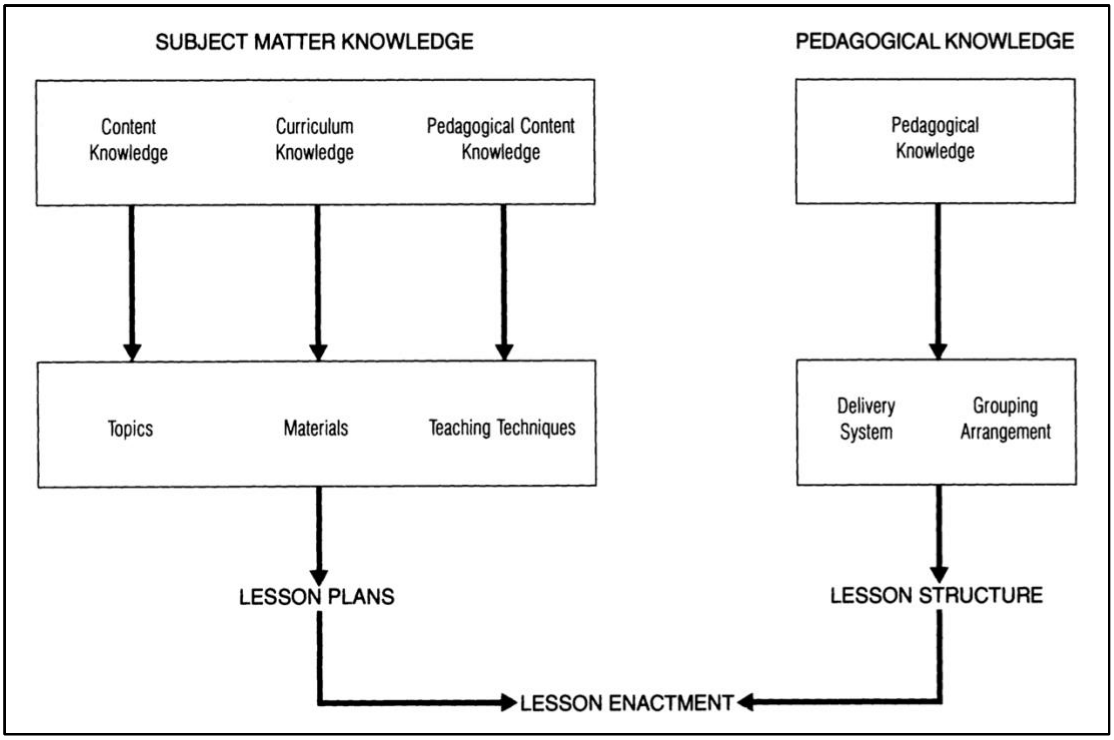


Figure 2. Model of teacher knowledge and planning (Burns & Lash, 1988, p. 382)

Carle (1993) has investigated several student misconceptions related to the area-perimeter topic. A meta-analysis of research has shown some student misconceptions on area measurement was due to area being taught together with perimeter causing many students to confuse area and perimeter (Watson, Jones, & Pratt, 2013; Cavanagh, 2007). Cavanagh (2007) studied Australian Year 7 secondary students and reported students experienced difficulties dealing with area concepts because of the above confusion with perimeter. As a consequence, students used slant and perpendicular height interchangeably. Zacahros & Chassapis, (2012) reported Greek Year 6 elementary students added the base plus the height instead of multiplying base with height to find the area of a rectangle. Özerem (2012) reported that seventh year secondary school students in Cyprus had a number of misconceptions due to a lack of knowledge related to geometry, resulting in them using the wrong formula. This lack of understanding of the concept of area resulted in students memorizing the formulas. Students who learn through manipulating area seem likely to avoid misconceptions on area measurement (Watson et al., 2013). It seems to make sense as they could manipulate and observe what changes happen by reshaping a figure (Yunianto, 2015).

It has been shown that SMK and PCK of mathematics teachers influenced student performance (Baumert et al., 2010). Thus, we should not expect teachers to deliver mathematics well if they do not have mastered it and do not understand how to teach it. Kow and Yeo (2008) explored the importance of SMK and PCK in the topic of area-perimeter from the planning of the lesson to its delivery. It was found that teachers with strong SMK and PCK provided more freedom to students to approach the task. Baturo and Nason (1996) evaluated first-year teacher education student understanding of subject matter knowledge in the domain of area measurement and uncovered many misconceptions. Success was related to their experience of learning the topic. John (2006) argued that novice teachers have difficulty making predictions about student responses and how to respond to unpredicted situations they encountered. In line with this, lack of mathematics pedagogical content knowledge of the teacher potentially lead to students having misconceptions (Kow & Yeo, 2008).

This study intends to focus on a part of PCK, the KCS within lesson plans on the topic of area-perimeter of a rectangle. It is necessary to obtain a fuller insight into teacher knowledge as it influence students' performance. Beside testing, there might be alternative way such as lesson plans to investigate teachers' knowledge. How are mathematics teachers prepare their lesson plans and how is PCK integrated in their lesson plans? How are the KCS integrated in the lesson plans? In the next section, the ways of gaining this insight will be discussed and the strategies used in collecting and analyzing the data. Furthermore, the results and discussion sections will describe the KCS evident in the lesson plans and the interviews with the respondents.

METHOD

This research involved humans and has been approved by IOE research ethics of University College London (IOE.researchethics@ucl.ac.uk) as this is a part of completion of the first author's dissertation. This study administrated a case study approach. This approach suits this study as it does not seek to generalize the findings but to gain deeper insight into the issue (Denscombe, 2010; Yin, 2014). The research subjects were the mathematics teachers in Yogyakarta and its surrounding registered themselves to participate on PD organized by SEAMEO QITEP in Mathematics. Some teachers teach across multi-grades. The first researcher who was facilitating one of the sessions asked the participants to develop a lesson plan as part of the whole PD. It was done somewhere in the middle of all complete sessions. As it is a case study, the researchers examined two selected lesson plans of two mathematics teachers. The remaining lesson plans have not been analyzed due to time limitation. The sample was chosen from twenty-nine teachers who attended a professional development (PD) session, and two teachers were selected for the lesson plan analysis and interview. Additionally. these teachers were selected based on their teaching experience; at least five years. The interview scenario was a semi-structured interview, and the two teachers were interviewed together. The two teachers who had been interviewed were a female teacher and a male teacher. They have different years of teaching experience. The female teacher teaches in a city while the male teacher teachers in a rural area. Participation in this study was voluntarily. The Indonesian mathematics teachers attending this PD were teaching grade 7 to grade 9. The topic that would be taught was area and perimeter for grade 7. The "Gold Rush/Mining" task was selected. This task was chosen because it is a problem-solving task and has several ways to be solved on area-perimeter of a rectangle (see Figure 3). Additionally, the complete Gold Rush activity showed the mistakes that students might do. Thus, it is considered as a good activity to be explored to understand how teachers prepare this activity.

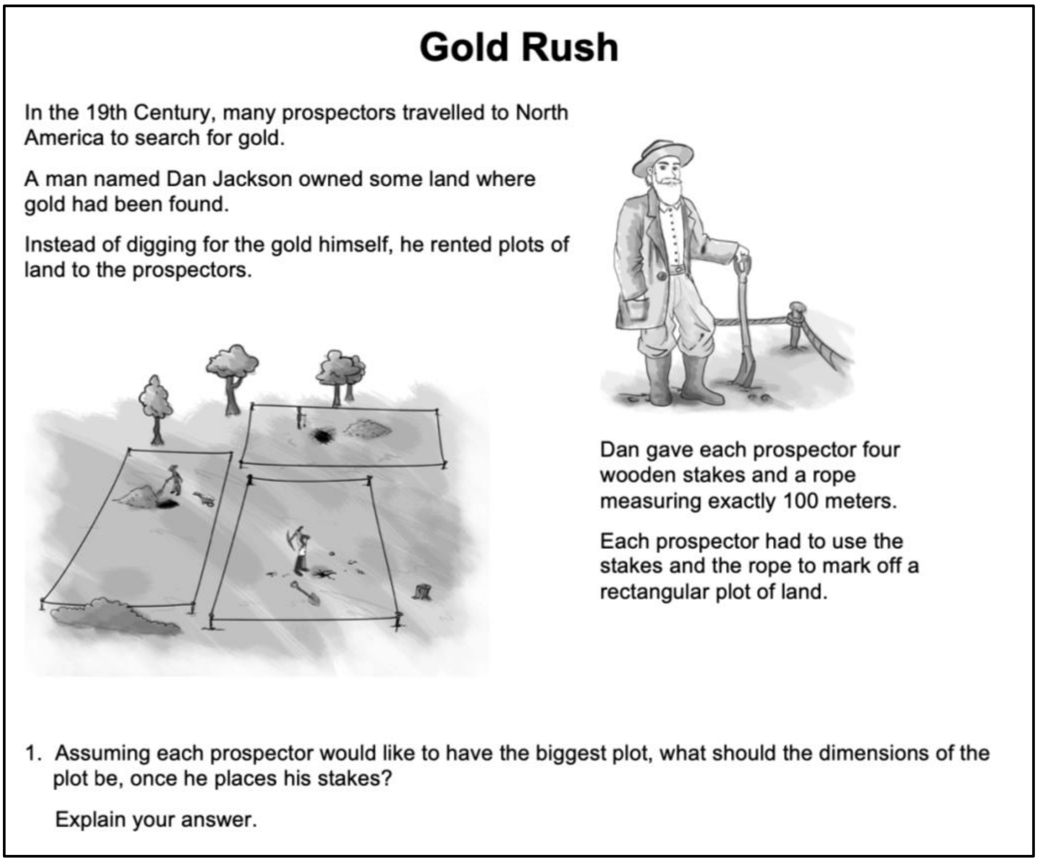


Figure 3. The Gold Rush problem (https://www.map.mathshell.org/download.php?fileid=1637)

To analyze the lesson plans, the researchers used content analysis. This method has the 'potential to disclose many hidden aspects of what is being communicated through the written text' (Denscombe, 2010, p. 282). From the lesson plan, the researcher would investigate to what extent the teachers' knowledge of students' conceptions and misconceptions is reflected in their written lesson plans (Table 1). The two lesson plans were coded to find themes by classifying instructions and KCS integrated in the lesson plans.

Table 1. Knowledge of Content and Student (KCS) (Ball et al., 2008, p. 401)

No.	Knowledge of Content and Student
1.	The ability to anticipate what students are likely to think and what they will find confusing
2.	The ability to predict what students will find interesting and motivating when choosing a task
3.	The ability to anticipate how students are likely to solve a given task and whether they will find it easy or difficult
4.	The ability to hear and interpret students' emerging and incomplete thinking

By using Table 2, it is easy to differentiate instructions' categories. These themes were useful in providing information on what the lesson plans contained. It focused on whether or not, the teachers included information about what students would do to the task (KCS). The data were presented descriptively.

The two lesson plans were coded and analyzed. There were three types of instructions to refer to with the codes. First, general instruction (GI) is where the teacher gives students instructions in a general way. This type of instruction is relatively simple, short and contains the doer(s) and their actions (verb) but leads to some mysteriousness (unclear). The second type of instruction is specific instruction with no detail (SIND). This refers to specific action, which has more information than GI but lacks detail in necessary aspects. The last type of instruction is specific instruction with detail information (SID). This instruction provides more detail and clearer information. Some forms of SID are short and require no detail, as it can be found easily or understood easily in other parts of the text. Looking through the instruction types, the researcher seeks evidence of KCS on the lesson plans (Table 2).

Table 2. Coding for instructions

Code	Example 1	Example 2
GI	Teacher asks a question to students	Teacher asks students to present their work
SIND	Teacher asks a question to students about their strategy.	Teacher asks two groups to present their work
SID	Teacher asks a question to students about their strategy. "what did you do and How did you do it? How are you convinced with your strategies?	Teacher asks two groups with different strategies to present their work starting with the group with less sophisticated strategy.

The two teachers were also interviewed to gain more insight. They were interviewed together (focus-group interview). The researcher wanted to clarify what was written on the lesson plans and why. Through a semi-formal interview style, data were collected through voice recording as well as video recording. From the records, data were transcribed and analyzed.

RESULTS AND DISCUSSION

Using the codes, the lesson plans revealed some interesting findings. Teachers 1 (T1) and Teachers (T2) have different proportions of the use of the instructions (Table 3). The percentage is from type of instruction per total instructions written on the lesson plans.

Indonesian teachers follow the prescribed template of a lesson plan by MoEC. The template consists of three main parts namely; introduction, main and closure. It also consists learning goals and how teachers and students would do in the classroom.

Table 3. Proportions of the instructions

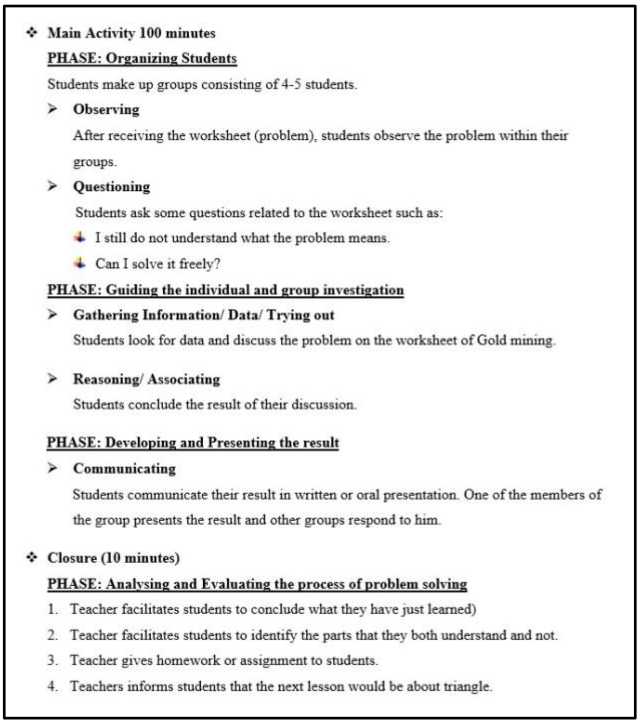
Instruction	**T1**	**T2**
GI	8 (35%)	6 (31.6%)
SIND	6 (26%)	7 (36.8%)
SID	9 (39%)	6 (31.6%)
Total	**23 (100%)**	**19 (100%)**

Based on the partition T1 used more instruction in the introduction and has less instruction in the main body. Interestingly, T2 has more instructions in the Main body with detailed information. Compared to T1, T2 had fewer total instructions, and detailed instructions (SID). From T2's SID, there were several instructions that provided information relating to PCK (Table 4).

Table 4. Comparison of Instructions

Code	**Introduction**		**Main**		**Closure**	
	T1	T2	T1	T2	T1	T2
GI	2	0	3	4	3	2
SIND	3	1	3	3	0	3
SID	7	2	1	4	1	0
Total	**12**	**3**	**7**	**11**	**4**	**5**

T1 put more details of what students would ask to her on her lesson plan. For instance: 'Can I solve it freely?' has been put on her lesson plan. This is a proof of PCK in the lesson plan, but not specific to KCS.



- **Main Activity 100 minutes**

PHASE: Organizing Students

Students make up groups consisting of 4-5 students.

- **Observing**

  After receiving the worksheet (problem), students observe the problem within their groups.

- **Questioning**

  Students ask some questions related to the worksheet such as:
  - I still do not understand what the problem means.
  - Can I solve it freely?

PHASE: Guiding the individual and group investigation

- **Gathering Information/ Data/ Trying out**

  Students look for data and discuss the problem on the worksheet of Gold mining.

- **Reasoning/ Associating**

  Students conclude the result of their discussion.

PHASE: Developing and Presenting the result

- **Communicating**

  Students communicate their result in written or oral presentation. One of the members of the group presents the result and other groups respond to him.

- **Closure (10 minutes)**

PHASE: Analysing and Evaluating the process of problem solving

1. Teacher facilitates students to conclude what they have just learned)
2. Teacher facilitates students to identify the parts that they both understand and not.
3. Teacher gives homework or assignment to students.
4. Teachers informs students that the next lesson would be about triangle.

Figure 4. Teacher 1 Lesson Plan

In addition, the way she would organize the discussion are provided in detail. This would provide information to other readers/ teachers how the classroom discourse was managed (Figure 4). On the phase of guiding the individual and group investigation which be rich of KCS. In this lesson plan, detail ways of students might solve it or make mistakes and how to facilitate it have not been depicted.

The T2 lesson plan of rectangle using Gold Rush task depicted detailed information about a possible student strategy (KCS). Figure 5 shows that T2 considered one strategy that students would utilize by asking students to make a table. T2 prompted students to make a table and gave an example to start with simple numbers. Within that table students would investigate the largest area by filling the lengths and widths that added to 100. More interestingly, two examples with easy numbers were provided to support students. Therefore, T2's instruction can be understood as providing a method to solve the task, with much support given to students.

Main Activity

- ☑ Teacher divide students into groups
- ☑ Teacher delivers the worksheet to be discussed
- ☑ Teacher facilitates the learning processes

- For the first question, students are asked to make a table by filling up the length column and determine the width to make 100 m.
  for instance, p=10, l=… m
  p = 15 m, l = ...m, then the area = …. p=15 m, l= … m etc
  Students determine the largest area by themselves
- For the second question, after students have solved the largest area for one miner, then how if it is for 2 miners?
  Next, if the ropes of the 2 miners are joined, and continue like the first question, what will be the largest area?
  How if you continue doing this for 3 miners and 4 miners until n miners?

Figure 5. Teacher 2 Lesson Plan of Gold Rush

After finding the largest area of the rectangle, students had to find the largest area by joining two miners' ropes and how would they join it. T2 also offered questions for students, revealing the organization on their lesson plan. T2 has also provided students actions in Figure 6.

- Students evaluate and make generalisation into questioning.
  - ☑ Teacher asks students to present in front of the class
  - ☑ Other students respond the presenter

Figure 6. T2's lesson plan on organizing the classroom discussion

Students were expected to evaluate and generalize during discussion. Although it was unclear what kind of evaluations and generalizations would be made. It would be clear if he put, for instance, that the generalization would be that 'the largest area would always be a square'. This generalization might come out from students. In addition, it was not clear how T2 would organize the presentation, or which group would present first. If there were two groups with different strategies or different conclusions, it is not clear how it would be organized.

Teachers T1 and T2 have more than five-years teaching experience each. Based on the questionnaire and interview, their schools are different in terms of location and students' background. These teachers themselves employed different abilities in solving the Gold Mining problem (Figure 3). From the conversation below, it seems that they have three correct strategies or less to solve it: T1-Ms. Excel integration and T2 -table, quadratic function and graph. However, there is a significant difference between the two teachers. T1 allowed the students to solve the task freely (students' own ways).

The interview with Teacher 1 showed that she has the ability to solve the problem.

R : Are there other ways T1?
T1 : Yesterday, I just did that one.
T1 : …just let students find the ways to solve it …. Then, I will let them know that there are some ways to solve it. I give that opportunity to students

This teacher (T1) would allow her students to approach the task in their own ways. However, T2 had a different way of letting students approach the task, providing only one strategy.

*T2 : To me, I could do it directly because **I already knew it** but to students if I want to students to learn it, **I make a table for them**. If the table is not made, students will find it difficult to solve it for students in my school.*
R : So, you (T2), induce them by using the table?
T2 : Yes, by the table.
R : What do you think, how many ways to solve it?
*T2 : To me, I did one way I know it directly it would be a square**. I knew it already**. But for students, **with table**, students will measure the perimeter, area, so if the length is 5, how long is the width, if the length is 10, how long is the width, and.., they will list it, this is how I let them learn. If I do not do it they will have no clue to solve it.*

From the transcript of T2, he seemed to only allow his students to use one strategy. He believed that his students would not be able to approach the task without inducing the table. He has had previous experiences where students were unable to complete a similar task.

*T2 : I have tried several times an easier task, for instance, given the perimeter of a rectangle and how big is the area, changing from the perimeter to area, I let them do it and facilitated them, but students were not able. For the story problem, the reading comprehension, the task asks to go to the East, most of my students go to the West (**metaphor**).*
T2 : However, I have thought only one strategy, which is global to solve a task. ... I, I... know at least I understand my students' characteristic so that it will be difficult for my students. ... It is not possible to come up if I let them to do it freely. ... I am so careful to give it the various strategies because students would get confuse

To know how to solve the mathematical task, these teachers tried the problem themselves. During the interview, T2 seemed to be familiar with the task and had three ways of finding the answer. Meanwhile, T1 only thought of one strategy.

T2 : *By using the strategy of making rectangles with certain sizes and order them and estimate the biggest area.*
T2 : *To me, I did one way I know it directly it would be a square. I knew it already*
T2 : *…instead of table, we can make the variable x, then I will be a quadratic function,*
R : *Are there other ways to solve it?*
T2 : *For the time being, not yet, making rectangles and to the square*
R : *Do you think there are still other ways to solve that problem?*
T2 : *I could use the graph …*

To some extent, from the lesson plan, T2 gave students a global strategy (table) to solve the task based on his previous experiences, although there is no guarantee that students would continue to have the same issues with the task (Figure 5). However, by giving the students the strategy, he inadvertently is making the students dependent on him. Whereas, from the lesson plan, T1 is helping the students to make decisions themselves (Figure 4). From the interview evidence, the two teachers have different abilities in solving the task and differ on the approaches they offer to their students.

In relation to students' possible mistakes and misconceptions, it seems that these teachers had some ideas as to what their students would find difficult.

T1 : *The task has missing information, it should be more, and some students would think that. So that they* ***have not thought*** *yet the possible ways to solve it. In average, students can directly solve it with possible ways to do. They can find it directly.*
T1 : *100. Maybe* ***they thought that*** *that's the only think they know.*
R : *… So, they would answer it 100, possibly*
T1 : *Yeah, possibly*
T2 : *… for those who did not understand,* ***they would not know what 100*** *m rope is to with the perimeter. So that the concept of perimeter, for those who understood, they already make it but later* ***they would not think*** *the rectangles can be varied.*
T2 : *Students* ***would confuse*** *the meaning of maximum, which is the largest,* ***they have not thought about it.*** *So that students' thinking is not yet there. Their thinking is still circulated on the perimeter not yet the perimeter to area and from area to find maximum area.*

Teachers also have ways of responding to students' mistakes, prompted by the researcher (Figure 7). The researcher proposed a possible mistake by a student of which the shape looks like a rectangle 25 x 26,5.

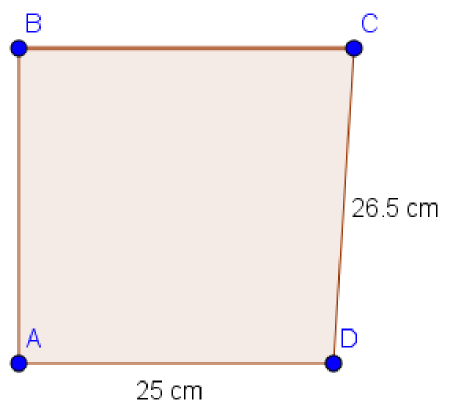


Figure 7. A student's possible mistake proposed by the researcher

If faced with a student mistake that they have not thought of before, both teachers seemed to engage thoughtfully with the scenario presented and sought ways of supporting students in addressing the mistake. Rather than telling a student their answer was incorrect, they asked what the task wants, and told them to check whether the shape is a rectangle or not.

R	*:*	*If it happens if you see this (showing)*
T1	*:*	*I would ask students back to try it then you calculate it as what being asked to you*
R	*:*	*They have not yet known the result!*
T1		*Try, try it, by trialing they would know that it is different, this one is more, and that one is like that, ...*
R	*:*	*T2, what if your students did this? what would you do?*
T2	*:*	*I would check it first, is it correct or not, the shape is a rectangle or not, they said that it is not, so I asked whether the perimeter is 100 cm or not. So, by knowing that it is a rectangle, the length would be equal, and the width would be equal (opposite sides), so that the perimeter would be 100 cm...*

In this study, the lesson plans facilitated an insight into teachers' knowledge. In this case, it showed teacher's pedagogical knowledge as well as PCK. Lesson plans can contain rich information on how the lesson is expected to be carried out. This is potential data to be used for assessing teachers' knowledge. How the teachers organize and manages the classroom, task, and the discussion would be depicted in the lesson plans. This resonates with Burns and Lash (1988) and Simon (1995) who argue that in developing lesson plans, teachers integrate their knowledge, such as SMK and PCK. On the other hand, experienced teachers may not use paper planning (written lesson plan) or just outlines as they have knowledge of what will work best (Butt, 2008; Jones & Edwards, 2010). In addition teachers also do mental planning for the lesson plans and the lesson plans are not written (Borko, Livingston, & Shavelson, 1990). The dynamics of a classroom are very fluid, and a teacher must adjust to that fluidity while following the plan. It is rare for a lesson to go exactly to plan. Yet, the execution of the lesson plan determines the effectiveness of the lesson (Kow & Yeo, 2008). In Japanese lesson plans, they contain more detailed instructions (Nakahara & Koyama, 2000) which shows more information about teachers knowledge. In contrast, the two case of teachers in this study, have not yet shown detailed instructions but more in general instruction.

Teachers have different ways of supporting students to solve tasks (Yeo, 2008). Students' performance is more affected from teachers' PCK (Baumert et al., 2010). However, SMK is basis knowledge for teachers (Shulman, 1986; Turnuklu & Yesildere, 2007). It is not usual that teachers teach 'something' before mastering the subject matter thus reducing the possibility of teaching effectively (Turnuklu & Yesildere, 2007). The teachers in this study were able to solve the task and had some ways to respond to students when they made mistakes in solving the given task (possessing SMK and PCK). However, these results are not generalizable. The limited sample was not chosen randomly and as these teachers came from relatively developed areas in Java and have at least five years teaching experiences they are not representatives of the wider Indonesian teaching population. Mathematics teachers in this study might not show detail information on their lesson plans and have not fully been aware of integrating PCK on developing their lesson plans. This study might not cover all mathematics teachers' PCK profile in Yogyakarta or broadly in Indonesia. However, this study has provided an interesting glimpse into one part of the very complex decision and knowledge processes that are involved in teacher pedagogical knowledge.

CONCLUSION

This study indicates that it is possible to assess teachers' KCS of a specific topic through analysis of the lesson plans when supported by interviews. There is evidence that these teachers had some knowledge about student strategies and misconceptions about the area-perimeter of rectangle topic, and that this knowledge was not necessarily fully integrated into their lesson plans. When prompted to think about possible misconception, the teachers found that it was challenging. Understanding possible misconceptions, making predictions and the anticipation of student responses would help teachers to be better prepared in facing the situations during teaching. Developing problem solving skills and autonomy among students requires teachers to stop providing a particular way (limiting students' strategies) but rather provide an environment where students are able to choose strategies, to make mistakes and to explore. Training for teachers could be more supportive in providing pedagogy that promotes such an environment. Additionally, this study explored a rectangle topic, the result might vary in different topics. Therefore, further investigation on different topic could be conducted. This study is not generalizable as it used limited research subjects.

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