INELASTIC RESPONSE AND EFFECTIVE DESIGN
OF ASYMMETRIC BUILDINGS UNDER STRONG
EARTHQUAKE LOADING

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A THESIS SUBMITTED TO THE UNIVERSITY OF LONDON FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY

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OCTOBER 1991
ABSTRACT

Asymmetric building structures arise frequently in civil engineering practice as a result of architectural or other requirements. Collapse and widespread severe damage of buildings associated with the torsional response created by structural asymmetry or other forms of irregularity have been observed repeatedly in past strong earthquakes. This highlights the inadequacies in current earthquake resistant design and construction of asymmetric buildings in seismic prone regions. This thesis investigates the inelastic behaviour of asymmetric buildings under strong earthquake motions and provides guidelines and recommendations for their effective design. These guidelines are widely applicable, offer conservative estimates of design loadings for individual resisting elements, and retain simplicity for ease of code implementation.

A comprehensive review and critical assessment of previous studies are presented in which the different approaches and contradictory conclusions reached are identified. Prior to inelastic dynamic studies, an inelastic failure mode analysis of single-storey asymmetric buildings under static monotonic loading is carried out. The latter analysis helps to explain the model dependency of the inelastic torsional response and clarifies the influence of key system parameters.

Based on single-storey building models, the model dependency of the inelastic torsional response is studied further by carrying out inelastic earthquake response analysis. A suitable analytical model is developed, which provides conservative estimates of the inelastic torsional effects, represents a wide range of actual buildings, is simply defined and facilitates the straightforward interpretation of results. A thorough inelastic dynamic parametric study is carried out employing this model, leading to
improved understanding of the inelastic behaviour of asymmetric buildings to strong earthquake motions and the influence of key system parameters on the inelastic seismic response.

The torsional provisions of current earthquake resistant design building codes from Europe, New Zealand, Canada, Mexico and the United States are rigorously evaluated based on the analysis of the single-storey building model. In particular, an assessment is made of the adequacy of the linear elastic modal analysis procedure, which is specified in most codes as an alternative approach to the equivalent static force procedure for the design of asymmetric buildings when certain regularity conditions are not satisfied. Inadequacies in the code torsional provisions and the linear elastic modal analysis procedure are identified. Improvements to these code provisions are sought and a new unified approach for torsional design is proposed.

A multistorey regularly asymmetric frame building model is also developed, and the code torsional provisions and the linear elastic modal analysis method are re-evaluated based on this model. The results of this study reinforce the conclusions obtained from analysis of the single-storey model and provide insight into the effect of inelastic torsional coupling in the various storeys of the building. A new equivalent static force procedure is proposed for the design of multistorey regularly asymmetric frame buildings. As a result of its inadequacies, it is suggested that linear elastic modal analysis be deleted as an alternative method for design of these buildings.
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Notation

\( a \) = building dimension parallel to the direction of the ground motion
\( a/v \) = peak ground acceleration to velocity ratio of the ground motion
\( b \) = building dimension perpendicular to the direction of the ground motion
\( d \) = distance between adjacent elements
\( e_a \) = accidental eccentricity
\( e_{D1} \) = primary design eccentricity
\( e_{D2} \) = secondary design eccentricity
\( e_p \) = strength eccentricity
\( e_p^* \) = strength eccentricity ratio \( e_p/r \)
\( e_s \) = stiffness eccentricity
\( e_s^* \) = stiffness eccentricity ratio \( e_s/r \)
\( F_{max} \) = maximum static load the structure can withstand
\( F_t \) = concentrated lateral load applied at the top of the building
\( F_y \) = total yielding strength
\( f_{yi} \) = yielding strength of element \( i \)
\( h_i \) = height of floor \( i \) above the foundation
\( K_y \) = total lateral stiffness
\( K_{0r} \) = total torsional stiffness about the centre of rigidity
\( k_i \) = lateral stiffness of element \( i \)
\( m \) = total mass of a floor deck
\( R \) = force reduction factor
\( r \) = radius of gyration of a floor deck about the centre of mass
\( T_y \) = uncoupled lateral period
\( V_0 \) = design base shear
\( v \) = lateral movement of a floor deck
\( \dot{v}_g(t) \) = ground acceleration input
\( W \) = total weight of the building
\( W_i \) = weight of floor \( i \)
\( x_i \) = \( x \)-ordinate of element \( i \)
\( \theta \) = torsional displacement of a floor deck about the centre of rigidity
\( \mu \) = displacement ductility demand
\( \xi \) = damping ratio
\( \Omega \) = uncoupled torsional to translational frequency ratio \( \omega_\theta/\omega_y \)
\( \omega_y \) = uncoupled translational frequency
\( \omega_\theta \) = uncoupled torsional frequency
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ACKNOWLEDGEMENTS

My sincere and profound thanks go first to Dr A. M. Chandler, my supervisor, for his supervision of the research work presented in this thesis, his encouragement, his contributions in our joint publications and in numerous hours of stimulating and constructive discussion between us, and for his valuable comments and suggestions on the manuscript.

I also wish to acknowledge Dr R. H. Bassett of the Department of Civil and Environmental Engineering of University College London for his excellent comments and suggestions on the work presented in Chapter 6 of this thesis.

The joint financial support from the British Council and the State Education Commission of the People’s Republic of China under the Technical Co-operation Training Award scheme, which has made my study in the United Kingdom possible, is greatly appreciated.

Thanks are also due to my fellow research students of the structures group for their contributions in maintaining a friendly and academic atmosphere during the past few years.

My gratitude also goes to my wife, Liping, for her encouragement, understanding and patience. Without her support this undertaking could not have been achieved.
CHAPTER 1

Torsional Coupling in Seismic Response of Building Structures:
Introduction

1.1 Background to earthquake engineering research

Earthquakes have been one of nature's greatest hazards to life throughout the recorded history of human beings. They are unique in many aspects. Firstly, unlike floods, hurricanes and other natural catastrophes, the mechanisms that invoke tectonic earthquakes are the least understood of the natural hazards and hence earthquake prediction is extremely difficult with present knowledge. Long-term earthquake predictions, which are commonly based on earthquake-recurrence intervals determined from historical records of past earthquake events in a seismic-prone region, usually have significant uncertainties. For instance, the Parkfield region, which is situated on the San Andreas fault midway between San Francisco and Los Angeles, has experienced six moderate earthquakes (magnitudes 5.5-6) since 1850, occurring in 1966, 1934, 1922, 1901, 1881 and 1857, with an average recurrence time of about 22 years (Fifield 1990, Stuart 1990 and Wyss et al. 1990). Based on these dates alone, a long-term prediction has been made by engineering seismologists that the next moderate quake was expected to occur in 1988 with an uncertainty of ± 5 years. Such an earthquake has not yet happened. Recent developments in earthquake prediction have indicated that, by seeking precursory anomalies, intermediate-term earthquake predictions are possible. Wyss et al. (1987) claimed that they had successfully predicted the May 1986 Stone Canyon earthquake on the San Andreas fault by applying the seismic quiescence hypothesis, which presumes that quiescence precedes large mainshocks. On this basis,
they predicted in May 1985 that such an earthquake would occur within a year. A moderate quake of magnitude 4.6 affected the Stone Canyon region on 31 May 1986, occurring exactly on the specified segment of the fault. Having observed the quiescence during the last 4 years and by comparing the similarities of quiescence patterns at Stone Canyon and at Parkfield, Wyss et al. (1987) concluded that the seismic quiescence hypothesis is also applicable to the Parkfield area and thus claimed that the next moderate earthquake would occur in March 1991 with an uncertainty of ±1 year (Wyss et al. 1990, Stuart 1990). This prediction remains to be proved.

Successful short-term earthquake predictions with an uncertainty of a few weeks or even days are still extremely difficult to achieve because of the lack of physical understanding of the earthquake rupture process and an accurate and detailed mathematical model for predicting earthquakes. In comparison, complex atmospheric models are employed to forecast the weather and coming typhoons, hurricanes and other natural hazards, and the use of these models gives much greater confidence of exactly when and where such events will occur. The first (and probably the only) successful short-term prediction of a major earthquake was the forecast of the Haicheng earthquake of 1975, which was of magnitude 7.3 on the Richter scale and affected a highly populated industrial area in the northeastern People's Republic of China. Chen et al. (1988) reported that in January of 1975, having observed precursory anomalies, seismologists in China predicted that an earthquake of magnitude 6 would occur in this region as soon as late January or February 1975. At 10:30 a.m. on 4 February 1975 a warning was issued by the local government to the public, and people and domestic animals were immediately evacuated. Rescue teams and vehicles were organised and emergency supplies were prepared. At 7:36 p.m. of the same day, an earthquake of magnitude 7.3 struck the Haicheng region. As a result of the warning, thousands of lives were saved. This has been regarded throughout the world as the first successful attempt to forecast a major
earthquake (Chen et al. 1988, Clough 1977). As a result of such difficulties in short-term predictions, earthquakes also pose a great psychological threat to people living in seismic prone areas.

Earthquake-triggered hazards are almost entirely associated with man-made structures, i.e. the collapse and failure of buildings, dams, bridges and other works, which result in great loss of life and damage to the regional economy. For instance, on 28 July 1976, one of the most devastating earthquakes in world history (which had a magnitude of 7.8 on the Richter scale) occurred in Tangshan in the northeastern part of China. In a matter of seconds an industrial city of a million people was reduced to rubble. About 240,000 people perished, including 7,000 families which were completely obliterated (Chen et al. 1988). Recently on 17 October 1989, the latest strong earthquake on the San Andreas fault in North America, which had a magnitude of 7.1, hit the San Francisco Bay area in the United States. It caused considerable widespread damage to buildings and bridges in this region. A total of 64 deaths and an economic loss of about $10 billion were attributed to this earthquake (Mitchell et al. 1990). About 105,000 houses and 320 apartment buildings suffered some form of damage (Bruneau 1990).

Earthquakes therefore present a most challenging and fascinating design problem to the civil engineering discipline. With the present state of knowledge of earthquake prediction, the only method to ensure safety of life is the design and construction of earthquake resistant structures in seismically vulnerable regions. Structural engineers must ensure that, firstly, structures in seismic regions be designed to resist without collapse the most severe earthquake they may experience in their life and therefore guarantee the safety of life and the evacuation of the occupants. Secondly, in order to minimize the loss of property, these structures must be designed to resist moderate earthquakes without significant structural damage so that they are repairable and can be
strengthened for future use after sustaining a moderate earthquake. However, the probability that a given structure will be shaken by such strong earthquakes is extremely low. Moreover, unlike loading due to gravity and wind, the loading on a structure due to an earthquake is dependent on the dynamic characteristics of the structure itself. Thus, structural engineers should pursue an optimum design approach to provide the required seismic resistance at the lowest possible additional costs.

Understanding the behaviour of structures during earthquakes is the key to providing guidelines for optimum earthquake resistant design and is also the basis for the formulation of code design regulations. With these objectives in mind, this Ph.D. thesis is aimed primarily at increasing the understanding of the inelastic behaviour of torsionally asymmetric building structures under severe earthquake ground excitations, in which buildings are excited well into the inelastic range and are in some cases near the threshold of failure and collapse. Secondly it is aimed to provide guidelines and recommendations for an optimum, effective earthquake resistant design approach for such buildings. The criteria adopted for the development of such an approach are that it should be widely applicable, retain simplicity and provide reasonably conservative estimates of the design earthquake loading for the individual elements of such buildings.

1.2 Lessons from recent strong earthquakes

The collapse and severe damage of structures during past earthquakes have constantly demonstrated the mistakes made in structural design and construction, and the inadequacies of some regulations of building codes for earthquake resistant design. A great many lessons have been taught to the earthquake engineering community. The Mexico City earthquake of 19 September 1985 is considered to be a typical example of recent earthquakes having great engineering significance. Earthquake resistant design provisions were first introduced in Mexico as early as 1942 in the Federal District
Building Code, and up-dated building codes for earthquake resistant design have been introduced subsequently, including the Emergency Regulations issued after the 28 July 1957 Mexico City earthquake, and the 1966 edition and 1976 edition of the Mexico Federal District Building Code (National University of Mexico 1977, henceforth referred to as the Mexico 76 code). Despite this, the 1985 earthquake still caused collapse and severe damage to many buildings constructed after 1957, according to the reports of several field investigation teams (Rosenblueth and Meli 1986, Mitchell et al. 1986, Popov 1987, Meli 1988 and Navarrete et al. 1988). About 10,000 people died and 200,000 people lost their homes as a result of this earthquake (Mitchell et al. 1986). 210 buildings collapsed and thousands were damaged (Rosenblueth and Meli 1986). Of these collapsed or severely damaged buildings, about 70% were built after 1957, when the Emergency Regulations were introduced. This high percentage of collapsed or severely damaged buildings designed in accordance with earthquake resistant design codes suggests that the code provisions employed in Mexico City were inadequate, even when they were strictly adhered to.

The 1985 Mexico City earthquake has taught the earthquake engineering profession many lessons. Among them, the extent to which torsional effects have contributed to building collapse and severe structural damage is of particular importance. All post-earthquake field investigation reports have identified torsional effects to be one of the major causes of failure. A statistical survey conducted by Rosenblueth and Meli (1986) revealed that 15% of the cases of failure in Mexico City were caused by pronounced asymmetry in stiffness, and 42% of the buildings that collapsed or suffered severe damage were corner buildings which had solid masonry walls on two perpendicular sides and masonry walls with open facades on the other two sides facing the streets. There were other buildings not at corners which also failed in torsion due to asymmetric layout of masonry in-fill walls.
Navarrette (1988) reported following the 1985 Mexico City earthquake that "a large number of corner buildings suffered heavy damage due to torsional oscillations produced by the contribution of asymmetrical non-structural walls, ... ". Meli (1988) also stated that "a remarkable number of corner buildings failed, generally having brick walls on two sides and open facades on the others. This is an extreme case of torsion induced by non-symmetric distribution of walls that greatly increased the forces in some column lines and contributed to their failure."

In his report on the same earthquake, Popov (1987) indicated that "a large number of buildings experienced either severe damage or total collapse due to torsional effects caused by the fact that the centres of mass and resistance did not coincide. In this category, examples among corner buildings with two rigid walls and two open sides were found. Likewise, even buildings having a rectangular plan can have lateral-torsional problems. For example, in some major steel buildings having moment resisting frames, unsymmetrically located braced elevator shafts may have contributed to the ultimate collapse".

In addition to field investigations of the Mexico City earthquake, theoretical analysis has been carried out by Chandler (1986) based on an elastic single storey building model. He concluded "one lesson to be learnt from the recent Mexico City earthquake is that the Mexico earthquake code torsional design recommendations for asymmetric buildings do not include a sufficient safety margin."

Field investigations after the 17 October 1989 San Francisco earthquake also concluded that torsional coupling played an important part in causing structural damage. Mitchell et al. (1990) reported "torsion played a significant role in a number of buildings of different types of construction in which damage was observed" and "there were
many examples of damage resulting from vertical irregularity in stiffness and strength. These included soft stories, discontinued shear walls. "Such vertical irregularities often result in irregularities in plan, giving rise to torsional eccentricities."

Apart from the observations of post-earthquake field investigations and theoretical analyses, instrumentally recorded building earthquake response has also been studied and quantified. Hart et al. (1975) carried out quantitative studies on the responses of 13 buildings in the city of Los Angeles to the 9 February 1971 San Fernando earthquake. The earthquake responses of these buildings were recorded on strong motion accelerometers which the city of Los Angeles began to install in approximately 100 buildings in 1966. Hart et al. (1975) quantified the contribution of the torsional response mode to the displacement and acceleration responses of these buildings. They found that at the edge of the buildings the ratio of the contribution from the torsional response mode to that of the translational response mode ranges from 0.21 to 1.16 for displacement response and from 0.41 to 1.91 for acceleration response. Therefore, they concluded that building torsional response to this earthquake was significant and strongly influenced by building asymmetry and the torsional motion of the ground.

1.3 Causes of torsional building response

Buildings exhibit coupled torsional and translational response to lateral ground motion input if their centres of floor mass do not coincide with their centres of rigidity at floor levels. In this thesis, the building's centres of rigidity at floor levels are defined as the set of points at the floor levels through which the set of applied lateral forces will not produce rotation of the floor slabs. This phenomenon can be clearly demonstrated by observing the response of a single-storey building model shown in Fig. 1.1. When this building model is shaken by the lateral earthquake ground motion \( \ddot{v}_g(t) \), the resultant inertial force, which acts through the centre of mass, CM, of the floor deck and
has a value of \( m\ddot{y}_x(t) \) (where \( m \) is the total mass of the floor), results in a torque about the centre of rigidity, \( \text{CR} \). This torque, which equals the total inertial force multiplied by the distance between \( \text{CM} \) and \( \text{CR} \) called the stiffness or static eccentricity, \( e_s \), in turn invokes the torsional movement \( \theta \) of the floor deck about \( \text{CR} \). Therefore, the building displays coupled lateral and torsional response to the earthquake input. Because of the contribution of the torsional response mode, the total deformation (and hence the total element internal force) of the two edge elements may be increased considerably at some points in time or decreased at others, depending on whether the torsional response is in-phase or out-of-phase with the translational response at the instant under consideration.

Furthermore, torsional motions may occur even in nominally symmetric buildings, in which \( \text{CM} \) and \( \text{CR} \) coincide with each other. This is due to two main reasons, namely accidental eccentricity and torsional ground input, as discussed below.

The sources giving rise to accidental eccentricity include the difference between the assumed and actual distributions of mass and stiffness, asymmetric yielding strength and patterns of non-linear force-deformation relationships, and differences in coupling of the structural foundation with the supporting soil or rock. Pekau and Guimond (1990) have evaluated the seismic torsionally coupled response of yielding single-storey initially symmetric structures (Fig. 1.2), which respond initially only in translation. The torsional response mode is induced after yielding by a shifting of the centre of rigidity caused by changing element stiffness. The latter effect originates from the variation in the yielding strength of lateral resisting elements (Fig. 1.3(a)) or by dissimilar post-yielding element force-deformation relationships (Fig. 1.3(b)).

Torsional input may also arise in some cases from the rotational component of the ground motion about a vertical axis and differences in the coupling of the structural foundation with the supporting soil or rock.
Traditionally, structural engineers employ the separate planar model approach (Chopra and Newmark 1980) for the analysis of building response (such as the internal forces and deformations) to various forms of loading. In the planar model approach, the planar resisting structures (shear walls and/or frames) in the two orthogonal horizontal directions are analysed separately for the effects of the in-plane horizontal component of ground motion. Computer programs such as SAKE (Otani 1974), DRAIN-2D (Kanaan and Powell 1973) and IDARC (Kunnath, and Reinhorn et al. 1990) employ this model and have been developed for both elastic and inelastic earthquake response analysis of buildings. However, in view of the above discussions, the planar model is very limited in application and may result in significant errors in some cases even when the eccentricities of the centres of floor mass with respect to centres of rigidity are small. Because the torsional motion creates additional internal forces and deformations in certain earthquake load-resisting elements of the structure, this effect must be taken into account. In order to account for torsional coupling effects, a three dimensional model in which a building is modelled as an assemblage of plane frames and shear walls connected by floor slabs has been employed in computer programs such as SUPER-ETABS (Maison and Neuss 1983), DRAIN-TABS (Guendelman-Israel and Powell 1977) and IDARC-3D (Kunnath and Reinhorn et al. 1989).

1.4 The necessity for this Ph.D. research programme

As discussed in Section 1.3, torsional response coupled with the translational response of buildings has the effect of increasing the deformation and strength demand in certain earthquake load-resisting structural elements. This may in turn result in a serious pounding problem between buildings and excessive structural and non-structural damage or even collapse. Many factors contribute to the torsional response including the structural layout, the vertical and horizontal distribution of mass, stiffness and
strength, the dynamic properties of the structures and the characteristics of earthquake ground excitations. In recent years, the study of torsional effects in asymmetric building structures has been a challenging and attractive subject to many engineering researchers (see Chandler 1988 for a comprehensive bibliography). However, most of the existing studies are based on elastic structural models. A comprehensive review and summary of existing elastic studies has been carried out by Chandler (1988), in which the need for research on inelastic response of asymmetric building structures to strong earthquake motions has been highlighted.

As stated earlier, the presently accepted philosophy of earthquake resistant design is to ensure that structures resist with slight or no damage moderate intensity earthquakes, and to provide a large measure of resistance to prevent collapse or failure when a severe earthquake occurs. Therefore, all seismic building codes allow buildings to exhibit inelastic response during a strong earthquake by utilising the structural ductility and energy dissipation capacities to resist the earthquake loading. Thus, structural elements are expected to be excited well into the inelastic range under strong earthquake shaking.

However, although current code provisions allow inelastic deformations to develop in earthquake resisting elements, they do not require a non-linear inelastic analysis to be carried out for the design of buildings due to the complexity and expense of such an analysis. Instead, code provisions only require the designers to conduct a linear elastic analysis based on the elastic earthquake response spectrum of a single-degree-of-freedom system. This analysis results in an elastic strength demand for the individual elements of a structure. This elastic strength demand is then reduced by dividing it by a force reduction factor leading to the actual strength capacity, which is usually a fraction of the elastic strength demand. This design strategy presumes that by selecting suitable constant force reduction factors according to the ductility and energy
dissipation capacities of different structural materials (steel, reinforced concrete or masonry, for instance), and different structural forms (ductile space frame, shear wall, etc.), the structural damage caused by the expected major earthquake will be within acceptable limits. Results from a theoretical study by Zhu et al. (1989) have shown that this approach is applicable to single-degree-of-freedom systems. Furthermore, observations in past earthquakes suggest that this design strategy usually leads to satisfactory performance of buildings during major earthquakes if the distribution of mass and stiffness of the building is symmetric in plan and more or less uniform vertically. Nevertheless, lessons from past strong earthquakes (Section 1.2) have indicated that those buildings having either asymmetry in plan or set-backs, and those with non-uniformly distributed stiffness along the height such as soft storeys and discontinued shear walls, when designed by this strategy have often suffered severe structural damage or even collapse. This implies that such a design strategy is inadequate for the design of these buildings.

In asymmetric buildings, the nonlinear hysteretic behaviour due to yielding, unloading and reloading of structural elements, as well as the stiffness and strength deterioration phenomenon exhibited to a significant extent in reinforced concrete structures under cyclic loading, shifts the vibration periods of the building and leads to changes in the relative stiffness of the resisting elements, which in turn shifts the centres of rigidity of the building. This phenomenon will affect the torsionally coupled response of inelastic building structures and trigger different behaviour from that exhibited by linear elastic systems. Popov (1987) has concluded that "the engineering profession must rapidly move in the direction of capacity design with realistic force levels. The use of the linear elastic approach for solution of critical seismic problems is outdated".

In a similar context, Rosenblueth and Meli (1986) have concluded in their report on the 1985 Mexico City earthquake that "both stiffness and strength deterioration
accentuated the importance of torsion due to nonlinear behaviour, uneven distribution of effective load factors in storey shear, P-Δ effects, and foundation performance". It has also been speculated that very high torsional vibrations, significantly higher than those predicted by the linear theory, might have taken place in buildings responding to the earthquake (Esteva 1987).

Although it is well known that the response of symmetric and regular buildings to earthquake loading is easier to predict and that the ability of these buildings to survive a strong earthquake is much better than asymmetric and irregular buildings, the designers are often still faced with decisions which compromise structural symmetry and regularity, in order to accommodate functional and aesthetic needs. As a result, serious and widespread damage associated with torsional response created by structural asymmetry and vertical irregularity, which invokes horizontal torsional eccentricities, has been observed repeatedly in past major earthquakes. Lessons from these earthquakes suggest that the individual resisting elements of such buildings must be designed with realistic load levels, which are specified taking into account the different behaviour caused by yielding from that predicted by linear elastic analysis, and that the load-resisting elements be carefully detailed to ensure ductile response. Therefore, research on torsional effects in asymmetric building structures in the inelastic range near the threshold of failure is urgently needed by the earthquake engineering profession. A new codified design approach, which takes account of both elastic and inelastic torsional coupling effects, should be sought and adopted for the practical design of asymmetric buildings, to reduce the likelihood of failure or collapse due to structural asymmetry in future earthquakes.
1.5 Objectives of research programme

1.5.1 Parametric investigation of the inelastic torsional behaviour of asymmetric buildings under strong earthquakes

Many factors affect the inelastic earthquake response of asymmetric buildings. These include the distribution of mass, stiffness and strength; the number, location and orientation of the lateral load resisting elements; the lateral period or frequency; the uncoupled torsional to lateral frequency ratio, and the overall strength with respect to the elastic strength demand. Understanding the inelastic behaviour of asymmetric buildings is the basis for providing effective earthquake resistant design of such structures. Aimed towards such an objective, this research programme therefore includes an investigation of the influence of the above key system parameters on the inelastic response of asymmetric buildings, by carrying out a comprehensive inelastic dynamic parametric study (see Chapter 4).

1.5.2 Evaluation of the torsional provisions in current aseismic building codes

The torsional provisions in aseismic building codes determine the horizontal distribution of earthquake loading among the resisting elements. In order to achieve the objectives of earthquake resistant design stated earlier and reduce the likelihood of severe structural damage or collapse due to structural asymmetry in plan, the adequacy of the torsional provisions in current major aseismic building codes needs to be examined in the context of satisfactory control over additional displacement ductility demand caused by structural asymmetry, and consistent protection for both symmetric and asymmetric structures against structural damage (see Chapter 5). The high
percentage of collapsed or severely damaged buildings arising due to structural asymmetry in the 1985 Mexico City earthquake suggests that inadequacies exist in the torsional provisions of the Mexico 76 code and these need to be identified. In view of the similarities between the torsional provisions of this code and those specified in codes elsewhere, an assessment of the torsional provisions in current aseismic building codes is necessary. The aims are, firstly, to identify in what way and to what extent these code torsional provisions are inadequate, and secondly to form a basis for effective improvements of current code torsional provisions.

### 1.5.3 Recommendation of a new unified procedure for effective design of multistorey asymmetric buildings

Based on the parametric study and the evaluation of code torsional provisions as indicated in Sections 1.5.1 and 1.5.2 above, further studies have been carried out aimed at formulating a new equivalent static force procedure for the design of multistorey regularly asymmetric frame buildings (see Chapters 5 and 6). This new design procedure is intended to provide satisfactory control over additional structural damage due to structural asymmetry, and to offer consistent protection for both symmetric and asymmetric buildings against structural damage when buildings are excited well into the inelastic range. At the same time, it should ensure that asymmetric buildings will suffer only minor or even no structural damage when responding to moderate earthquakes. Thus, this research programme seeks a new design procedure which satisfies the present earthquake resistant design philosophy, is widely applicable, retains simplicity for ease of code implementation, provides conservative estimates of the design loading for individual lateral load resisting elements, and finally leads to relatively small increases in costs compared with conventional design of symmetric buildings, that is, represents an optimum and effective design procedure.
1.6 Organization of thesis

Chapter 2 presents a comprehensive historical review and assessment of previous studies on the inelastic seismic response of asymmetric buildings. It summarises the approaches employed and results achieved by different researchers, including the publications from this Ph.D. research programme. It then gives a critical assessment of the differences in research approaches employed in the various studies and the resulting variations in the conclusions reached.

Chapter 3 gives the results of the inelastic static parametric study of single-storey asymmetric buildings loaded by a static monotonic force until collapse. This failure mode analysis helps to explain the model dependency of the inelastic torsional response and the influence of system parameters, prior to carrying out a full inelastic dynamic analysis.

Chapter 4 deals with the inelastic dynamic parametric study of asymmetric buildings to strong earthquake motions based on a single-storey building model. The influence of key system parameters on the inelastic response of individual resisting elements is examined.

The evaluation of torsional provisions of current earthquake resistant design codes from Mexico, New Zealand, Europe, Canada and the United States is presented in Chapter 5. The evaluation is carried out employing the single-storey building model. Particular attention is paid to the adequacy of the linear elastic modal analysis, which is employed in most codes as an alternative to the torsional provisions of the equivalent static force procedure. Also presented in Chapter 5 is the study aimed at the improvement of code torsional provisions, which leads to a new unified approach for torsional design.
Chapter 6 presents studies on the inelastic seismic response and effective design of multistorey regularly asymmetric frame buildings. Code torsional provisions and the linear elastic modal analysis procedure are re-evaluated based on a multistorey frame building model. Special attention is focussed on the inelastic seismic response of columns of various storeys along the height of the building. A new equivalent static force procedure is developed and proposed for the design of such buildings.

Chapter 7 summarises the contributions, design guidelines and recommendations for code provisions made in this thesis. It also describes how the state-of-the-art of research in this subject has been influenced by this Ph.D. research programme and gives proposals for future research.
Figure 1.1 Torsional coupling in asymmetric buildings

Figure 1.2 Structural model with accidental strength eccentricity

(after Pekau and Guimond 1990)
Figure 1.3 Hysteretic behaviour of elements for (a) strength variation; (b) unbalanced stiffness degradation (after Pekau and Guimond 1990)
CHAPTER 2

Review of Studies on Inelastic Response of Asymmetric Buildings to Strong Earthquakes

2.1 Historical review

The study of torsionally coupled seismic response and associated earthquake resistant design of asymmetric building structures has continuously been an interesting and attractive research area since the mid-1970’s. At the early stage of research, studies were concentrated on the elastic earthquake response analysis of asymmetric buildings. Later, and particularly since the mid-1980’s, research on the inelastic torsionally coupled seismic response of asymmetric buildings has been carried out concurrently with elastic studies. In recent years, research work in this area has generally been focussed on inelastic studies.

Systematic and thorough parametric studies on elastic torsional coupling effects in asymmetric buildings subjected to seismic motions have been carried out based on theoretical as well as some limited experimental approaches. Theoretical research methods include the linear elastic modal response spectrum analysis approach using idealised smooth response spectra, as employed by Kan and Chopra (1977a, 1977b), Tso and Dempsey (1980) and Tsicnias and Hutchinson (1981), and the time history analysis approach using actual earthquake records, as employed by Chandler and Hutchinson (1986, 1987, 1988a and 1988b) and Rutenberg and Pekau (1987). Experimental building model tests on an earthquake simulator have been carried out by Maheri, Chandler and Bassett (1991). The achievements of many of these studies have
been summarised and reviewed by Chandler (1988).

Irvine and Kountouris (1980) pioneered the research on inelastic torsional coupling. They investigated the inelastic seismic response of a simple single-storey monosymmetric building model having two identical resisting elements oriented parallel to the direction of earthquake input. The eccentricity was caused by the offset of the centre of mass from the centre of rigidity, the latter located midway between the two resisting elements as shown in Fig. 2.1. They found that the peak ductility demand of the worse affected element, which is the one closer to the centre of mass, is insensitive to either the eccentricity or the uncoupled torsional to translational (lateral) frequency ratio. It was also concluded that the difference in ductility demand between symmetric and eccentric structures remains small.

Kan and Chopra (1981a, 1981b) carried out parametric studies on the inelastic response of a single storey one-way eccentric structural model subjected to the 1940 El Centro earthquake record. The force-deformation relationship of the resisting elements was assumed to be elastic-perfectly plastic, and the yielding displacement of each individual resisting element was taken to be the same. The multi-element model was simplified to a single element model (Fig. 2.2) by defining a circular yield surface in terms of the shear and torque acting on the system at and about the centre of rigidity, respectively. The effects of torsional coupling were characterised by the lateral translation of the centre of mass, the rotation of the floor about the vertical axis through it, and the ratio of the total vector displacement of the corner columns to the displacement of the centre of mass (Fig. 2.3). The system parameters of this simple single-storey, one-element model were (i) the stiffness eccentricity ratio \( e_s/r \), where \( e_s \) is the stiffness or static eccentricity, namely the distance between the centres of mass and rigidity, and \( r \) is the mass radius of gyration of the floor about the centre of mass; (ii) the uncoupled torsional to translational frequency ratio \( \Omega = \omega_t/\omega_s \); (iii) the uncoupled lateral
period $T_x$; and (iv) the damping ratio. Since all elements are assumed to have the same yielding displacement, an element's yielding strength is proportional to its elastic stiffness. Therefore, the model's centre of strength PC (plastic centroid), defined as the point at the floor level through which the resultant of the total element resisting forces act when all elements are loaded to yielding, coincides with the centre of rigidity CR. Kan and Chopra concluded that the effects of torsional coupling in the inelastic range depend significantly on the uncoupled torsional to translational frequency ratio, being most pronounced for systems with this ratio close to unity (Fig. 2.4). They also found that for systems with the uncoupled frequency ratio larger than 2, the effect of torsional coupling on system and column deformations increases with increasing eccentricity ratio, but for systems with uncoupled frequency ratios smaller than 2, the effects of torsional coupling are complicated, giving no apparent systematic trends (Fig. 2.5). Finally, Kan and Chopra (1981a) concluded that after the initial yielding, the system has a tendency to yield further primarily in translation and behave more and more like an inelastic single-degree-of-freedom system, responding primarily in translation.

Tso and Sadek (1985) and Bozorgnia and Tso (1986) carried out parametric studies to investigate the inelastic behaviour and the sensitivity of response parameters to system parameters of a simple single-storey one-way eccentric structural model. This model was subjected to earthquake ground excitations. The step-by-step integration approach was employed to carry out numerical analysis of the inelastic earthquake response of the model. The model used in their studies consists of a rigid rectangular floor deck of mass $m$ supported by three resisting elements in the direction of the ground motion (Fig. 2.6). This model is statically indeterminate and the changes of the stiffness eccentricity and the uncoupled torsional to translational frequency ratio were obtained by adjusting the relative elastic stiffness of the resisting elements and the distance $h$ of the two edge elements from the centre of mass. The force-displacement relationship of the elements was assumed to be either bi-linear or bi-linear degrading.
All elements were assumed to have the same yielding displacement (Fig. 2.7). The peak element displacement ductility demand and the displacement at the flexible edge were chosen as the response parameters. The system parameters were defined to be (i) the stiffness eccentricity ratio, (ii) the uncoupled torsional to translational frequency ratio, (iii) the uncoupled lateral period and (iv) the excitation level parameter, which was the ratio of the elastic strength demand to the strength capacity of the system assuming the system was symmetric.

The results of extensive studies show that, firstly, unlike the results of elastic studies, the coincidence of the uncoupled torsional and translational (lateral) frequencies does not lead to abnormally high peak inelastic responses and that the element ductility demand is not sensitive to the uncoupled frequency ratio, with no systematic trends identified from the results (Fig. 2.8). Secondly, significant rotational motion is involved at the instant when the peak ductility demand is reached (Fig. 2.9), which implies that the eccentric system does not respond primarily in translation when it is excited well into the inelastic range as concluded by Kan and Chopra (1981a). Thirdly, these studies (Tso and Sadek 1985, Bozorgnia and Tso 1986) showed that eccentricity has a large effect on both element ductility demand (Fig. 2.10) and the flexible edge displacement. Finally, it was concluded that the effect of asymmetry on the element ductility demand and on the flexible edge displacement is most pronounced for stiff systems with low yield strength relative to the elastic strength demand (Figs. 2.8, 2.10).

Syamal and Pekau (1985) studied the inelastic response of a single-storey monosymmetric building model with four resisting elements, subjected to sinusoidal ground acceleration, employing the Kryloff-Bogoliuboff method. Their single storey model consisted of two elements parallel to the direction of the ground acceleration input and another two elements perpendicular to it (Fig. 2.11). The resisting elements were assumed to be bi-linear hysteretic. The eccentricity was caused by the unbalanced
stiffness of the elements oriented parallel to the direction of the ground acceleration. The yield displacements of these two lateral elements were taken to be equal and thus the yield strengths were proportional to their elastic stiffnesses. The system parameters characterising the properties of the model were (i) the bi-linear coefficient of the resisting elements, (ii) the eccentricity ratio, (iii) the torsional to translational frequency ratio of the corresponding torsionally uncoupled system, (iv) the amplitude of ground acceleration and (v) the damping ratio. The response parameters were the peak displacement ductility demand of the resisting elements and the response amplitudes of the lateral and torsional displacements of the system.

A parametric study was carried out by Syamal and Pekau (1985) to investigate the influence of system parameters on response parameters. They found that in contrast to the results of elastic parametric studies, the structure does not exhibit pronounced inelastic torsional coupling when the uncoupled torsional and lateral frequencies are close and the eccentricity is small (Fig. 2.12). They also found that the element peak ductility demand appears to be most pronounced for torsionally flexible structures, and that the peak ductility demand of the element at the flexible edge of the structure increases rapidly with increase in eccentricity (Fig. 2.13). For elements at the stiff edge, the ductility demand decreases only slowly with increase in eccentricity ratio (Fig. 2.13). Therefore, seismic building codes which reduce force requirements for these elements with increasing eccentricity ratio appear to underestimate substantially the actual behaviour. Finally, Syamal and Pekau (1985) found that although the uncoupled frequency ratio does not affect significantly the ductility demand of the element at the stiff edge, the ductility demand of the element at the flexible edge is critically affected by this parameter (Fig. 2.12).

In a later study, Tso and Bozorgnia (1986) studied the maximum dynamic flexible edge displacement and the inelastic deformation of the resisting element at the flexible
edge of a single-storey monosymmetric structural model subjected to uni-directional
ground excitations. This model was the same as that employed in earlier studies by Tso
and Sadek (1985) and Bozorgnia and Tso (1986). The concept of effective eccentricity,
which was introduced for elastic one-way eccentric structures to evaluate the effect of
asymmetry on the lateral displacement at the flexible edge of asymmetric buildings
(Dempsey and Tso 1982), was generalised for inelastic systems. The aim was to match
the maximum dynamic displacement at the flexible edge of the asymmetric building to
the edge displacement of the same building subjected to an equivalent static lateral load
applied at a distance from the centre of rigidity equal to the effective eccentricity. In
order to minimize the dependence of results on any individual record, six earthquake
records were considered and the inelastic effective eccentricity was calculated by
averaging the effective eccentricities obtained for these records. It was concluded that
except for short period structures having low yield strength relative to the elastic
strength demand, the concept of effective eccentricity can be extended to the response
of inelastic systems and the elastic effective eccentricity curves can provide a
reasonable or conservative estimate of inelastic effective eccentricity. Hence these
curves can be used to estimate the edge displacement and element deformation of
inelastic systems.

Bruneau and Mahin (1987) (see also Bruneau and Mahin 1990) carried out studies
on the torsionally coupled inelastic seismic response of single-storey structures having
only two resisting elements oriented in the direction of the ground excitation. They
considered two types of eccentric systems: an initially symmetric system in which the
two resisting elements have identical elastic stiffness but different yielding strength, and
an initially eccentric system in which the two resisting elements have different elastic
stiffness giving rise to an initial eccentricity. It was further assumed that the two
resisting elements in the initially eccentric system have equal yielding displacement.
This assumption leads to the element yielding strength being directly proportional to its elastic stiffness and thus the coincidence of the centre of rigidity CR with the centre of strength PC.

For initially symmetric systems, Bruneau and Mahin (1987) considered four simple cases of unequal element strength distribution, the strength of the two elements being 0.8Fy and Fy, Fy and 1.2Fy, Fy and 1.5Fy, and Fy and 2Fy. Ten values of the uncoupled lateral period, ranging from 0.1 to 2 seconds, and six uncoupled torsional to translational frequency ratios were considered in their parametric study. They compared the inelastic response of such systems with that of the corresponding symmetric system having the same lateral period, and equal element stiffness and strength (Fy and Fy). It was found that the weak element ductility demand is higher and the strong element ductility demand is lower than that of the corresponding symmetric system. They also found that the weak element ductility demand increases with increasing differences in yield strength, and that higher values of the uncoupled torsional to translational frequency ratio always lead to higher ductility demand of the weak element.

For initially eccentric systems, Bruneau and Mahin (1987) developed a procedure by which the weak element’s ductility demand can be predicted by employing an equivalent single-degree-of-freedom (SDOF) system which has a period equal to that of the translation-dominated mode of the initially eccentric system and the same yield displacement as that of the resisting elements of the initially eccentric system. The earthquake input applied to the equivalent SDOF system should be scaled so that the peak elastic displacement of the SDOF system equals the maximum elastic weak element displacement of the initially eccentric system, subject to the same earthquake motion.
In 1987, the Mexico Federal District Building Code made significant changes to the torsional design provisions. As a result, the centre of strength is required to be close to the centre of rigidity, and linear elastic modal analysis is not allowed to be used to determine the horizontal distribution of storey shears. These changes were based on the work conducted by Gomez et al. (1987), which is written in Spanish and not generally available. It was later briefly referred to by Esteva (1987). Gomez et al. (1987) investigated the inelastic seismic response of single-storey mass-eccentric systems, which have three resisting elements with equal stiffness in the direction of earthquake motion but an asymmetric distribution of mass (Fig. 2.14). The element strength was specified according to the provisions of the Mexico 76 code and the east-west component of the earthquake motion recorded at the SCT station during the 1985 Mexico City earthquake was employed as the earthquake input. They found that the provisions in this code were inadequate, resulting in large ductility demand for the element located at the opposite side to the centre of mass, measured from the centre of rigidity (element 1). As a result, the findings of Gomez et al. (1987) were adopted in the 1987 edition of the Mexico Federal District Building Code (Gomez and Garcia-Ranz 1988), henceforth referred to as the Mexico 87 code, requiring a minimum strength eccentricity leading to a centre of strength positioned close to the centre of rigidity. These changes were made primarily in order to reduce the ductility demand of the element at the stiff side of asymmetric structures.

Chandler and Duan (1990) gave a comprehensive review of previous studies in this research area and summarised the different approaches employed and contradictory results obtained by the different researchers. A static inelastic parametric analysis based on a single-storey monosymmetric building model under monotonically increasing loading was also carried out, in order to identify the failure mode and to clarify the influence and relative importance of the key system parameters on the displacement ductility demand of the resisting elements. This analysis also helped to explain the
model dependency of the inelastic torsional response of asymmetric buildings. This study forms a part of the Ph.D. research programme and is presented in Sections 2.1 to 2.3 and in Chapter 3.

Tso and Ying (1990) studied the additional seismic inelastic deformation of lateral load resisting elements caused by structural asymmetry in eccentric buildings, based on the same single-storey monosymmetric model as used in previous studies (Tso and Sadek 1985, Bozorgnia and Tso 1986, Tso and Bozorgnia 1986). Unlike previous studies in which the element’s strength was assumed to be directly proportional to its elastic stiffness, leading to coincident centres of rigidity and strength, Tso and Ying (1990) specified the strength distribution among resisting elements according to the code torsional provisions from Canada, Mexico (1976 edition and 1987 edition), New Zealand and the United States. Their results indicated that in general buildings designed in accordance with code provisions have the centre of strength close to the centre of mass (that is, small strength eccentricity), with the exception of those designed according to the Mexico 87 code.

Normalising the ductility demand of resisting elements of asymmetric buildings with respect to that of a reference symmetric SDOF system having a lateral period equal to the uncoupled lateral period of the asymmetric buildings, Tso and Ying (1990) concluded that if element strength is specified based on these codes, again with the exception of the Mexico 87 code, structural asymmetry caused by uneven distribution of element stiffness does not lead to significant differences in ductility demand for any resisting element. It was also concluded that the torsional provisions of the Mexico 76 code already lead to satisfactory control of additional ductility demand, which is contradictory to the observations in post-earthquake field investigations after the 1985 Mexico City earthquake.
Tso and Ying (1990) also emphasised that the resisting element located at the flexible edge, that is the element at the opposite side to the centre of rigidity with respect to the centre of mass, is the critical element since the additional displacement of this element is of the order of 2 to 3 times that obtained based on the reference symmetric structure. This finding is particularly valuable for urban planning and structural design to avoid pounding between buildings during earthquakes. They also concluded that the Mexico 87 code leads to substantial increases in the overall strength of asymmetric buildings. The displacement ductility demand of the element at the stiff edge is greatly reduced because of the requirement that the centre of strength be close to the centre of rigidity. However, the improvement in decreasing the displacement ductility demand and the maximum displacement of the element at the flexible edge is only marginal.

Concurrent to the study of Tso and Ying (1990), Duan and Chandler (1990) and Chandler and Duan (1991a) (see Chapter 5) have reached fundamentally contradictory conclusions to those of Tso and Ying regarding the identity of the critical element and the adequacy of some code torsional provisions. Duan and Chandler (1990) emphasised that it is the element at the stiff edge (that is the element located at the same side as the centre of rigidity with respect to the centre of mass), which is traditionally considered as favourably affected by torsion, that suffers substantially more severe structural damage (based on peak displacement ductility demand) than corresponding symmetric structures. Also, the maximum displacement ductility demand of the element at the flexible edge, traditionally considered to be unfavourably affected by torsion, is generally lower than that of corresponding symmetric structures. Furthermore, Duan and Chandler (1990) concluded that when calculating the strength demand of the element at the stiff edge, the code provision specifying the (secondary) design eccentricity equal to the static eccentricity \(e_s\), if the accidental eccentricity is not included, is inadequate. As a result, they have recommended that this design
eccentricity be changed as $0.5e$, if excluding accidental eccentricity, which is of the same form as that of the 1990 edition of the National Building Code of Canada (Associate Committee on the National Building Code 1990), henceforth referred to as NBCC 90.

In a later study, Chandler and Duan (1991a) discussed three fundamentally important issues regarding the adequacy of approaches employed in research. They include, firstly, the validity of including the additional torque given by an accidental eccentricity of 0.1 or 0.05 times the building plan dimension $b$ parallel to the direction of eccentricity in the specification of element strength in the analytical model, as is the case in Tso and Ying (1990). Secondly, the need to employ localised earthquake records or records with dissimilar frequency contents was discussed. Finally, the necessity to carry out inelastic analysis and to present the results over a wide period range was emphasised. Chandler and Duan (1991a) have concluded that in this context Tso and Ying (1990) have, firstly, misleadingly included the accidental eccentricity in the specification of element strength since they have carried out inelastic dynamic analysis ignoring any such uncertainties in the distribution of mass, element stiffness and strength as well as the rotational component of the ground motion. Secondly, they did not employ localised earthquake records or records having varying frequency contents. And finally, they did not carry out analysis and present results in the full period range.

Taking the Mexico 76 and the Mexico 87 codes as examples, Chandler and Duan (1991a) reinforced their conclusions (Duan and Chandler 1990) and further concluded that the torsional provisions of the Mexico 76 code are inadequate, since they substantially underestimate the strength demand of the element at the stiff edge, and that the torsional provisions of the Mexico 87 code are overly conservative. Studies have
also been carried out by Chandler and Duan (1991a) to seek improvements to the form of the Mexico 87 code and to provide a unified, widely applicable approach for torsional design (see Chapter 5).

Goel and Chopra (1990, 1991a and 1991b) carried out a comprehensive parametric study on the inelastic response of single-storey asymmetric plan systems. They investigated the effect of different system parameters on the inelastic response parameters, which include the maximum normalised lateral and torsional displacements at and about the centre of rigidity, the normalised maximum element deformation, and the peak displacement ductility demand among the resisting elements. The system parameters were taken as (i) the torsional stiffness distribution (with or without transverse resisting elements oriented perpendicular to the direction of earthquake excitation), (ii) the number of resisting elements in the direction of the earthquake motion, (iii) asymmetric distribution of mass or stiffness, (iv) over-strength factor, (v) the strength eccentricity, (vi) the stiffness eccentricity, (vii) the uncoupled torsional to translational frequency ratio, and (viii) the strength reduction factor. The major findings and conclusions from these three studies are summarised as follows:

1. The inelastic response of asymmetric systems is influenced significantly by the contribution to the torsional stiffness arising from the resisting elements oriented perpendicular to the direction of earthquake motion. These elements affect substantially the response of short-period, acceleration sensitive systems but their influence is small for medium-period, velocity sensitive and long-period, displacement sensitive systems. As a result, a single storey monosymmetrical building model with two elements parallel and two perpendicular to the direction of earthquake motion was adopted as the most realistic and appropriate for studies of inelastic torsional coupling effects.
2. The number of resisting elements oriented parallel to the direction of earthquake motion has little influence on the response of systems with similar stiffness and strength eccentricities. It also has an insignificant effect on the lateral and torsional displacements at and around the centre of rigidity, for systems which have much smaller strength eccentricity than stiffness eccentricity. However, the maximum displacement ductility demand can be significantly affected in the latter systems.

3. Mass-eccentric and stiffness-eccentric systems may have different inelastic response when measured in terms of element displacement ductility demand. Therefore, Goel and Chopra (1990) concluded that since plan asymmetry in most buildings arises from asymmetric distribution of stiffness and not of mass, the mass-eccentric system should not be employed. Furthermore, they also concluded that the requirement of a minimum strength eccentricity in the Mexico 87 code (in order to reduce the ductility demand) may be unnecessary for most buildings because this provision is based on studies employing mass-eccentric systems.

4. Over-strength, signifying that the total strength of the asymmetric plan system is higher than that of the corresponding symmetric system, reduces significantly the lateral response, element deformation and element displacement ductility demand in short-period, acceleration sensitive asymmetric buildings. Goel and Chopra (1990) further found that conclusions made in earlier studies which assumed equal total strength between asymmetric buildings and corresponding symmetric buildings are not directly applicable to code designed buildings. This is because code designed asymmetric buildings are generally stronger than corresponding symmetric buildings (Tso and Ying 1990). Furthermore, results obtained in earlier studies which commonly assumed equal strength and stiffness eccentricities are
also not directly applicable to code designed asymmetric buildings, because these buildings usually have strength eccentricity much smaller than stiffness eccentricity (Tso and Ying 1990).

5. Lateral stiffness becomes zero because of yielding of the lateral resisting elements parallel to the direction of earthquake input whilst the transverse elements still remain elastic and hence contribute to the torsional stiffness. As a result, the system effectively becomes torsionally rigid. Therefore, when the system is excited more and more into the inelastic range, it behaves more and more like the corresponding symmetric system.

6. The rotational response about the centre of mass increases with increasing stiffness eccentricity and decreasing uncoupled torsional to translational frequency ratio.

2.2 Summary of different approaches employed and contradictory conclusions reached

It is clear that in this research area investigators have employed different models and approaches to carry out their studies and that as a result contradictory conclusions have been obtained. The five main features of these variations are summarised below.

2.2.1 Structural models

Most researchers have used single storey, monosymmetric structural models to carry out their studies. The floor deck is considered rigid in its own plane with its mass supported by massless planar resisting elements. The structural elements are assumed to be inextensible and their torsional stiffness about the vertical axes passing through them is neglected.
However, differences exist in the number of resisting elements parallel to the direction of earthquake motion and whether or not transverse elements are included in the models. Kan and Chopra (1981a, 1981b) conducted their parametric studies based on a simplified model having only one resisting element with a circular yielding surface which specifies the interaction between base shear and torque. Irvine and Kountouris (1980), and Bruneau and Mahin (1987) used a two-element model; Tso and Sadek (1985), Tso and Bozorgnia (1986), Gomez et al. (1987), Tso and Ying (1990), Duan and Chandler (1990), and Chandler and Duan (1991a) used a three-element model; whereas Syamal and Pekau (1985), and Goel and Chopra (1990, 1991a and 1991b) have employed a four element model, which has two elements parallel and two perpendicular to the direction of earthquake excitation.

Differences also exist in the definition and calculation of the values of some of the parameters. Irvine and Kountouris (1980), and Gomez et al. (1987) considered eccentricity to be the result of eccentric mass, whilst the remaining researchers regarded eccentricity to be the result of uneven distribution of stiffness among of the structural elements. In the definition of the uncoupled torsional frequency, Kan and Chopra (1981a, 1981b), Bruneau and Mahin (1987), Syamal and Pekau (1985), and Tso and Sadek (1985) defined it about the vertical axis through the centre of mass, Irvine and Kountouris (1980), Tso and Bozorgnia (1986), Bozorgnia and Tso (1986), and Tso and Ying (1990) specified it about the vertical axis through the centre of rigidity, whereas Duan and Chandler (1990), Chandler and Duan (1991a), and Goel and Chopra (1990, 1991a and 1991b) considered the torsional stiffness about the centre of rigidity but took the mass moment of inertia about the centre of mass. In order to obtain different values for the uncoupled torsional to translational frequency ratio, Bruneau and Mahin (1987) adjusted the value of the mass radius of gyration of the floor slab, however all other researchers achieved this by changing the distances of the resisting elements from the geometric centre of the floor slab.
Different response parameters were also used to characterise the torsional effects. Kan and Chopra (1981a, 1981b) employed the ratio of the total vector deformation of the corner columns to the lateral displacement of the centre of mass (Fig. 2.3). Irvine and Kountouris (1980), Syamal and Pekau (1985), Bruneau and Mahin (1987), Duan and Chandler (1990), and Chandler and Duan (1991a) employed the peak displacement ductility demand of the resisting elements. Tso and Sadek (1985), Bozorgnia and Tso (1986), Tso and Bozorgnia (1986), and Tso and Ying (1990) employed both the peak displacement ductility demand and the flexible edge displacement. Goel and Chopra (1990, 1991a and 1991b) adopted the peak lateral and torsional displacements at and about the centre of rigidity, and the peak element displacement ductility demand.

### 2.2.2 Uncoupled torsional to translational frequency ratio

In elastic studies of torsional coupling in earthquake-excited asymmetric buildings, all results show that structures with a small stiffness eccentricity experience pronounced torsional coupling when the uncoupled torsional to translational frequency ratio ($\Omega$) is close to unity. Nevertheless in inelastic studies, different researchers have reached contradictory conclusions about the effect of this parameter. Kan and Chopra (1981a, 1981b) found that inelastic torsional coupling is significantly sensitive to $\Omega$, the effect being most pronounced for systems with $\Omega$ close to unity. However, all other researchers found that unlike elastic studies, the coincidence of the uncoupled torsional and translational frequencies does not lead to significantly high inelastic responses of torsionally coupled systems with a low eccentricity compared with other systems.

Contradictory conclusions have also been reached concerning the sensitivity and overall trends of the peak element displacement ductility demand to the value of $\Omega$. Bruneau and Mahin (1987) found that the peak element displacement ductility demand is not very sensitive to $\Omega$, but increases systematically with the increase of $\Omega$. Results
from the studies of Tso and Sadek (1985) and Bozorgnia and Tso (1986) show however that the element displacement ductility demand is not sensitive to the frequency ratio and no systematic trend of changes of this response parameter with changes in $\Omega$ can be identified from the results (Fig. 2.8). Syamal and Pekau (1985) found that, for sinusoidal ground excitation, although the ductility demand of the strong element is not very sensitive to $\Omega$, the ductility demand of the weak element is critically affected by $\Omega$, particularly for torsionally flexible systems having a large eccentricity. Their results show that in all cases the ductility demand of the strong element falls slightly with the rise of $\Omega$. For small eccentricities the ductility demand of the weak element remains constant with changes of $\Omega$. However, for moderate to large eccentricities it drops rapidly with the increase of $\Omega$ (Fig. 2.12). A recent study carried out by Duan and Chandler (1990) concluded that $\Omega$ is not a critical system parameter affecting the element displacement ductility demand, which decreases slightly with increasing value of $\Omega$. Goel and Chopra (1990, 1991a and 1991b) found that in a yielding system, the instantaneous value of the uncoupled torsional to translational frequency ratio $\Omega$ varies with time and that the centre of rigidity may move further away from the centre of mass or abruptly shift to the opposite side. Therefore, the pronounced effect of torsional coupling in elastic systems with $\Omega = 1$ is reduced by inelastic behaviour. They also found that the system torsional displacement decreases with increasing $\Omega$.

### 2.2.3 Effect of eccentricity on inelastic response

In elastic studies, all results show that as the stiffness eccentricity ratio ($e/r$) increases, the effect of torsional coupling on the earthquake forces increases; namely the base shear and the lateral deformation decrease, whilst the torque and the torsional deformation increase. The conclusions from inelastic studies are more uncertain. Kan and Chopra (1981a, 1981b) concluded that the effects of torsional coupling on system
and column deformations depend on the eccentricity in a complicated manner, with no apparent systematic trends except for systems with $\Omega \geq 2$. In this case, the effects of torsional coupling increase with increasing eccentricity ratio (Fig. 2.5). Irvine and Kountouris (1980) found that the peak element ductility demand is independent of the eccentricity ratio. They showed that differences in element ductility demand between asymmetric and symmetric structures and between structures with different eccentricities remain small.

On the other hand, Tso and Sadek (1985), Tso and Bozorgnia (1986), and Syamal and Pekau (1985) found that the element ductility demand is very sensitive to eccentricity. An increase of over 100 per cent in the element displacement ductility demand was found to be not uncommon for systems with a large eccentricity when compared with the response of systems with a small eccentricity. Furthermore, Tso and Sadek (1985) showed that eccentricity has the effect of increasing the flexible edge displacement of the structure by a factor of up to three, demonstrating that eccentricity is a critical parameter controlling the inelastic response of asymmetric structures to earthquake excitations. Duan and Chandler (1990) found that the effect of eccentricity on element displacement ductility demand is significant, being pronounced in the very short-period region for ground motions having a high peak ground acceleration to velocity ratio ($a/v$ ratio), in the very short-period and short-period regions for ground motions having an intermediate $a/v$ ratio and in the medium-period and long-period regions for ground motions having a low $a/v$ ratio.

2.2.4 Methods for specifying the strength of resisting elements

Early studies assumed that all resisting elements have the same yielding displacement, leading to coincident centres of strength and rigidity. Moreover, the combined strength of resisting elements of asymmetric buildings was taken as equal to
that of the corresponding symmetric buildings. In more recent studies carried out by Gomez et al. (1987), Tso and Ying (1990), Duan and Chandler (1990) and Chandler and Duan (1991a), element strength has been specified in accordance with code provisions. The effect of over-strength as discussed earlier, which means that code designed asymmetric buildings are stronger than corresponding symmetric buildings, has therefore been taken into account. As indicated by Goel and Chopra (1990), results from earlier studies are not directly applicable to code designed asymmetric buildings because such structures have their strength eccentricity much smaller than their stiffness eccentricity and have a higher total strength than the corresponding symmetric buildings, the latter causing beneficial effects in asymmetric buildings.

When specifying element strength based on codes, different approaches again arise in dealing with the accidental eccentricity specified in code provisions to allow for the uncertainties and the torsional component of ground motion not explicitly accounted for in design. Gomez et al. (1987) included the accidental eccentricity in calculating element strength and considered an uncertainty of 10 per cent of the building dimension perpendicular to the direction of earthquake motion about the location of the centre of mass. Tso and Ying (1990) also included the accidental eccentricity in specifying element strength but at the same time ignored any uncertainties and the influence of rotational ground motion in their analysis. Duan and Chandler (1990) and Chandler and Duan (1991a) excluded the accidental eccentricity when determining element strength and also excluded uncertainties and the rotational ground motion when carrying out the inelastic dynamic analyses, leaving the uncertainties and the rotational ground motion to be accounted for by the subsequent application of the accidental eccentricity.
2.2.5 Critical resisting element and code torsional provisions

Tso and Ying (1990) reached the conclusion that as in the case of elastic torsional coupling, the inelastic response of the element at the flexible edge is again the most critical and that all codes considered are adequate in this respect. However, Gomez et al. (1987), Duan and Chandler (1990), and Chandler and Duan (1991a) have revealed that yielding in asymmetric buildings leads to significantly different behaviour in inelastic systems than that in elastic systems, and that the element at the stiff edge is the more critical in inelastic asymmetric structures. The latter studies further concluded that the torsional provisions in some codes are inadequate and proposed new code torsional provisions. The findings of Gomez et al. (1987), Duan and Chandler (1990) and Chandler and Duan (1991a) are supported by the study carried out by Goel and Chopra (1990), in which they identified that in systems with $\epsilon_r = 0$, representative of code designed buildings, the largest element displacement ductility demand occurs in the element at the stiff edge.

2.3 Discussion and critical assessment of previous studies

2.3.1 Model dependency of inelastic earthquake response of asymmetric structures

The review of previous studies given in Sections 2.1 and 2.2 shows that the interpretation of results from inelastic analysis of torsional coupling effects in asymmetric structures is a complicated issue. In the case of elastic studies using various analytical models and employing either the time history analysis or the smoothed response spectrum modal analysis, all studies reached similar conclusions about the effects of the various system parameters on the elastic response of torsionally coupled structures. The situation is different in the case of inelastic studies, because unlike
elastic studies, the inelastic response of asymmetric structures is model dependent. This phenomenon can be clearly demonstrated by considering the single-storey, monosymmetric structural model shown in Fig. 2.15. It is assumed that the earthquake acceleration input is uni-directional and parallel to the y-axis; hence only two degrees of freedom are concerned, namely the lateral displacement of the centre of rigidity (CR), \(v\), and the rotational movement of the floor slab about the vertical axis through CR, \(\theta\). Thus, in the elastic range, if damping is neglected, the equations of motion can be written as:

\[
\begin{bmatrix}
    m & me_s \\
    me_s & mr^2(1+e^*_{s2})
\end{bmatrix}
\begin{bmatrix}
    \ddot{v} \\
    \theta
\end{bmatrix}
+ \begin{bmatrix}
    \Sigma k_i & 0 \\
    0 & \Sigma k_i x_i^2
\end{bmatrix}
\begin{bmatrix}
    \dot{v} \\
    \theta
\end{bmatrix}
= - \begin{bmatrix}
    m\ddot{v}_s \\
    me_s \ddot{\theta}
\end{bmatrix}
\]  

(2.1)

in which \(m\) is the mass of the floor deck, \(r\) is the radius of gyration of the deck about the vertical axis through the centre of mass (CM), \(e^*_s\) is the stiffness eccentricity ratio \(e/s\), and \(k_i\) is the lateral stiffness of element \(i\) parallel to the y-axis, \(\Sigma k_i\) is the structure’s lateral or translational stiffness, \(K_v\), and \(\Sigma k_i x_i^2\) is the structure’s torsional stiffness about the centre of rigidity, \(K_{\theta}\).

Eqn. (2.1) can be re-written as follows:

\[
\begin{bmatrix}
    1 & e^*_s \\
    e^*_s & 1+e^*_s e^{*2}_s
\end{bmatrix}
\begin{bmatrix}
    \ddot{v} \\
    r \ddot{\theta}
\end{bmatrix}
+ \omega^2 \begin{bmatrix}
    1 & 0 \\
    0 & \Omega^2
\end{bmatrix}
\begin{bmatrix}
    \dot{v} \\
    r \dot{\theta}
\end{bmatrix}
= - \begin{bmatrix}
    \ddot{v}_s \\
    e^*_s \ddot{\theta}
\end{bmatrix}
\]  

(2.2)

in which \(\Omega = \omega_\theta/\omega_v\) is the torsional to translational frequency ratio of the corresponding torsionally uncoupled system (Fig. 2.16), in which \(e_s=0\) but \(m, r, K_v,\) and \(K_{\theta}\) remain the same as for the torsionally coupled asymmetric system (Fig. 2.15). This torsionally uncoupled system can be viewed physically by reinstalling the floor slab of the structure so that the centre of mass of the floor coincides with the centre of rigidity. It is apparent from eqn. (2.2) that the elastic system response parameters, \(v\) and \(r\theta\), are uniquely...
determined by the system parameters, $e^*, \Omega, \omega$, the damping ratio and the earthquake input. Systems having the same values of $e^*, \Omega, \omega$, and damping ratio, and subjected to the same earthquake excitation, will have the same elastic responses $v$ and $r\theta$.

For analysis of inelastic response, the equations of motion can be written in incremental form:

$$[m] \{ \Delta \dot{v} \} + [K(t)] \{ \Delta v \} = - \left[ \begin{array}{c} \Delta \dot{v} x \\ e, \Delta \dot{v} y \end{array} \right]$$

(2.3)

in which

$$[m] = \begin{bmatrix} 1 & e^* \\ e^* & 1 + e^{*2} \end{bmatrix}$$

(2.4)

$$\{ \Delta \dot{v} \} = \left[ \begin{array}{c} \Delta \dot{v} x \\ r \Delta \dot{\theta} \end{array} \right]$$

(2.5)

$$[K(t)] = \begin{bmatrix} \sum k_i(t) & \sum k_i(t)x_i \\ \sum k_i(t)x_i & \sum k_i(t)x_i^2 \\ mr & mr^2 \end{bmatrix}$$

$$= \omega^2(t) \begin{bmatrix} 1 & s^*(t) \\ s^*(t) & \Omega^2(t) \end{bmatrix}$$

(2.6)

In eqn. (2.6), $\omega(t)$ and $\Omega(t)$ may be considered as the instantaneous translational circular frequency and torsional to translational frequency ratio at time $t$; $s(t)$ is the instantaneous position of the centre of rigidity at time $t$; $s^*(t)=s(t)/r$.

From eqns. (2.3) to (2.6), it is clear that the inelastic response not only depends on the system parameters, $e^*, \Omega$ and $\omega$, but also on the number, location, force-deformation
relationship and the yield strength of the individual resisting elements. Therefore, the inelastic response is highly model dependent. This model dependency explains why contradictory conclusions have been drawn by various researchers employing different analytical models, as summarised in Section 2.2.

Since the inelastic response of asymmetric structures is model dependent, an assessment of the analytical models employed by various researchers and the results obtained has the highest priority for further research work. The validity of an analytical model should be decided after consideration of the following factors:

1. Reliability for ensuring conservative estimates of the effect of inelastic torsional coupling on structural response parameters

2. Realistic representation of a range of actual building structures, and ease of interpretation of the results, and

3. Simplicity of model definition and subsequent analysis.

In view of the above criteria, although the single-element model employed by Kan and Chopra (1981a, 1981b) is simple and enables straightforward parametric studies on the global lateral and torsional system responses, it is however too simple to provide a complete insight into the inelastic response of the individual resisting elements. This makes the application of the single element model very limited and unsuitable for the analysis of inelastic torsional effects.

The two-element model employed by Irvine and Kountouris (1980) and Bruneau and Mahin (1987) can predict both the global response of the system and the displacement ductility demand of the structural elements, whilst at the same time it retains simplicity. However, results given by Tso and Sadek (1985) and Goel and Chopra (1990) have indicated that the two-element model does not provide conservative
estimates for the element displacement ductility demand compared with those obtained from multi-element models. Even in Chapter 6 of Bruneau and Mahin (1987), in which the displacement ductility demand of resisting elements in an initially symmetric two-element model was compared with that of initially symmetric multi-element models, the displacement ductility demand obtained from the two-element model is in many cases significantly lower than that obtained from multi-element models. Moreover, the two-element model is statically determinate having no redundant elements and is therefore not encouraged in building design. Therefore, the two-element model is also somewhat unrealistic and thus is not the most suitable for inelastic analysis of torsional effects.

The three-element model employed by Tso and Sadek (1985), Tso and Bozorgnia (1986), Bozorgnia and Tso (1986), Gomez et al. (1987), Duan and Chandler (1990), and Chandler and Duan (1991a) seems overall to be the most suitable. This model is simple and statically indeterminate, the structural form encouraged in earthquake resistant design (Dowrick 1987), and therefore represents a more realistic model. In Chapter 4, it will be shown that the three-element model can provide satisfactory estimates of the peak element displacement ductility and hysteretic energy ductility demands compared with multi-element models.

Including transverse resisting elements oriented perpendicular to the direction of earthquake input as in Goel and Chopra (1990, 1991a and 1991b) is considered inappropriate. Goel and Chopra (1990, 1991a and 1991b) included transverse resisting elements in their analytical model and claimed that this model is a more realistic one. However, they still assumed the earthquake input to be uni-directional, ignoring the transverse component of the ground motion. Therefore, the transverse elements remain elastic most of the time during the response and contribute considerably to the structure's torsional stiffness. The building therefore tends to behave as though it were
torsionally rigid. Since in reality the transverse component of the earthquake motion may also excite these resisting elements into the inelastic range, the torsional stiffness contributed from these elements can be ignored. Therefore, when the ground motion is assumed to be uni-directional, the transverse resisting elements should not be taken into account.

Since plan asymmetry in most buildings arises from asymmetric distribution of stiffness and not of mass, it is concluded that the mass-eccentric system should not be employed.

2.3.2 Definition and influence of the uncoupled torsional to translational frequency ratio

There are two alternative methods for varying the uncoupled torsional to translational frequency ratio $\Omega$ whilst keeping the other system parameters constant. The first is to change the radius of gyration of the floor deck, $r$, and the second is to adjust the distances of the resisting elements from the geometric centre of the floor plan. In their study, Bruneau and Mahin (1987) adopted the first approach. Changing the value of $r$ is equivalent to changing the aspect ratio of the building. This is rarely possible in practice because of constraining architectural requirements. In order to vary $\Omega$, a practical engineering solution is to adjust the distribution of the stiffness rather than the aspect ratio of the building. Furthermore, reducing the value of $r$ whilst fixing other parameters (particularly the system’s torsional stiffness) in order to increase $\Omega$ decreases the rotational inertia, $m r^2$, of the floor slab and therefore reduces the system’s capacity of torsional resistance. This gives rise to greater rotational movement.

Because Bruneau and Mahin (1987) fixed the distances of the resisting elements to the geometric centre of the floor deck, a higher torsional response leads to a higher
deformation of the weak element in initially symmetric systems. Hence in their study the weak element displacement ductility demand always increases with increasing value of $\Omega$. This differs from the finding of Tso and Sadek (1985), Bozorgnia and Tso (1986), and Syamal and Pekau (1985) who adopted the second method, in which a higher value of $\Omega$ corresponds to a higher torsional stiffness. This is achieved by increasing the distances of the resisting elements to the geometric centre of the floor plan. In this approach, increasing the value of $\Omega$ has two contradictory effects on the element displacement ductility demand. On the one hand, a higher torsional stiffness makes the structure torsionally stiffer, therefore reducing the system's torsional response. On the other hand, the effect of the rotational floor motion on the element deformation increases with increasing distances of the element to the geometric centre of the floor plan. Consequently, the effect of $\Omega$ on the element displacement ductility demand is complicated. The studies conducted by Tso and Bozorgnia (1986), and Syamal and Pekau (1985) disagreed on the effect of $\Omega$ on the peak displacement ductility demand of the weak element. The former concluded that $\Omega$ is a less critical parameter for the displacement ductility demand of weak element, namely the element at the flexible edge, whilst the latter found that although the displacement ductility demand of the strong element, namely the element at the stiff edge, is not influenced significantly by this parameter, the weak element displacement ductility demand is affected critically by $\Omega$, particularly for torsionally flexible systems with a large eccentricity.

The main reason why the above studies reached different conclusions about the effect of $\Omega$ on the weak element displacement ductility demand appears to be that the range of the parameters, $\Omega = 0.8 - 1.2$, $e^* = 0.25$, chosen by Tso and Bozorgnia (1986) for their parametric study is not wide enough to reveal the full trends. In particular, they omitted the range of parameters representing the characteristics of torsionally flexible structures having a high eccentricity ($\Omega < 0.8$ and $e^* > 0.25$). Further research on this topic is therefore needed before final conclusions are drawn. The direct comparison of
results obtained in Bozorgnia and Tso (1986) and Syamal and Pekau (1985) is made more difficult by the different reference points chosen to define the rotational movement, hence leading to different definitions of $\Omega$. In deriving the equations of motion for single-storey, monosymmetric structural models, there are two approaches to define the rotational movement of the floor deck. The first is to define it about the centre of mass (CM); the alternative is to define it about the centre of rigidity (CR). These two approaches lead to different definitions of the uncoupled torsional to translational frequency ratio, written as $\Omega_m$ if the reference point is CM and $\Omega_r$ if the reference point is CR. In addition, Rutenberg and Pekau (1987) suggested a third definition as follows:

$$\Omega^2 = \frac{\omega_0^2}{\omega_r^2} = \frac{K_\theta m}{J_{0m} K_y} = \frac{K_{0r}}{K_y r^2}$$  \hspace{1cm} (2.7)

in which $J_{0m}$ is the mass moment of inertia of the floor slab about CM. This definition has later been adopted by Duan and Chandler (1990), Chandler and Duan (1991a), and Goel and Chopra (1990, 1991a and 1991b), and is adopted also in this thesis.

The relationships between $\Omega_m$, $\Omega_r$ and $\Omega$ can be derived using the following two equations:

$$J_{0r} = J_{0m} + me_x^2$$  \hspace{1cm} (2.8)

$$K_{0m} = K_{0r} + K_y e_z^2$$  \hspace{1cm} (2.9)

Thus
\[
\Omega_m^2 = \frac{\omega_m^2}{\omega_y^2} \\
= \frac{K_m m}{J_m K_y} \\
= \frac{(K_\theta + K_y e_s^2)}{K_y r^2} \\
= \Omega^2 + e_s^{*2} \quad (2.10)
\]

Similarly

\[
\Omega_r^2 = \frac{\Omega^2}{(1 + e_s^{*2})} \quad (2.11)
\]

Therefore, \( \Omega_r \leq \Omega \leq \Omega_m \) and the relationship between \( \Omega_m \) and \( \Omega_r \) is given by:

\[
\Omega_m = [\Omega_r^2(1 + e_s^{*2}) + e_s^{*2}]^{1/2} \quad (2.12)
\]

\[
\Omega_r = \left[\frac{\Omega_m^2 - e_s^{*2}}{1 + e_s^{*2}}\right]^{1/2} \quad (2.13)
\]

The relationship between \( \Omega_m \) and \( \Omega_r \) is plotted in Fig. 2.17 for selected values of \( e_s^{*} \).

The model employed by Bozorgnia and Tso (1986) with the system parameters \( e_s^{*} = 0.25 \) and \( \Omega_r = 0.8 - 1.2 \) has the corresponding \( \Omega_m = 0.862 - 1.262 \). Results from Syamal and Pekau (1985) show that for systems having these parametric values, the weak element displacement ductility demand is not sensitive to \( \Omega_m \), but for smaller values of \( \Omega_m \) or larger values of \( e_s^{*} \), the weak element displacement ductility demand drops rapidly with increasing value of \( \Omega_m \) (Fig. 2.12).
A detailed discussion on the advantages and disadvantages of choosing CM or CR as the reference point has been given by Bruneau and Mahin (1987). In previous studies, the choice of the reference centre to define the rotational motion seems to have been a matter of the researcher's preference. However, as indicated in Section 2.1.3 of Bruneau and Mahin (1987), if CM is chosen as the reference centre and the uncoupled frequency ratio is defined as $\Omega_m$, some combinations of the stiffness eccentricity and the uncoupled frequency ratio (relating to a low uncoupled frequency ratio $\Omega_m$ and a large eccentricity ratio $e^*$) have no physical meaning, that is no real physical systems can be represented since the system's first eigenpair does not exist. If CR is chosen as the reference centre and the uncoupled frequency ratio is defined as either $\Omega_c$ or $\Omega$, all combinations of system parameters are possible, having corresponding physical systems. Thus, for studies based on single-storey models CR may be a better choice than CM, and $\Omega$ seems to be a better definition than $\Omega_m$, bearing in mind that in the case of single-storey buildings the centre of rigidity is easily defined and the corresponding torsionally uncoupled system having a torsional to translational frequency ratio $\Omega$ is also clearly and easily defined. However, it should be noted that for multistorey buildings, especially irregular structures, the locations of the centres of rigidity are load dependent and may vary significantly from floor to floor (Stafford Smith and Vezina 1985, Cheung and Tso 1986).

2.3.3 Stiffness and strength eccentricities

The contradictory conclusions concerning the sensitivity of element displacement ductility demand to the system's eccentricity ratio ($e/r$), as summarised in Section 2.2.3, may also be considered to be the result of the different model definitions employed by the various researchers. Studies based on a single-element model (Kan and Chopra 1981a, 1981b) reached the conclusion that the effect of the eccentricity ratio on the
corner element’s deformation is complicated and that there are no apparent trends. Study based on a two-element, mass eccentric model (Irvine and Kountouris 1980) has shown that the element displacement ductility demand is independent of the eccentricity ratio, whilst studies based on three-element models (Tso and Sadek 1985, Bozorgnia and Tso 1986, Gomez et al. 1987, Tso and Ying 1990, Duan and Chandler 1990, and Chandler and Duan 1991a) show the opposite effect. Since most building structures have more than two resisting elements, the single-element and the two-element models seem to be over-simplified and underestimate the effect of eccentricity on the element displacement ductility demand, hence giving non-conservative results.

In elastic analysis only one eccentricity parameter, namely the stiffness eccentricity ratio $e^*$, needs to be considered. But in inelastic analysis, asymmetric element yielding strength and varying nonlinear force-deformation relationships also give rise to torsional coupling. Because yielding changes the stiffness of the resisting elements and moves the centre of rigidity away from the yielding element, systems with initially symmetric stiffness but asymmetric yielding strength will exhibit torsional coupling as soon as the first yielding occurs. There are three sources which lead to an asymmetric distribution of a system’s strength, namely different types of structural elements (mixed frames and shear walls, for instance) employed to resist the lateral loads, uncertainties in calculating the yielding strength of the structural elements, and statistical variations of the strength of the materials used. The strength eccentricity is expected to have a major effect on the inelastic response of asymmetric buildings to earthquake excitation. Therefore, in inelastic studies, the system’s strength eccentricity must be taken into account as a separate parameter, even for systems having an initially symmetric distribution of stiffness.

Irvine and Kountouris (1980) studied a two-element model with symmetric stiffness and strength but with an asymmetric distribution of mass. All the other earlier
studies considered the strength eccentricity to be equal to the stiffness eccentricity by assuming that all elements have the same yielding displacement, as shown in Fig. 2.7. Thus, in the models employed in these latter studies an element’s yielding strength is proportional to its stiffness. Later, Bruneau and Mahin (1987) studied some very simple cases of the inelastic response of initially symmetric two-element models having unequal yielding levels, without taking the strength eccentricity as a separate parameter. Sadek and Tso (1989) introduced the strength eccentricity concept and proposed to use it as a measure of the degree of asymmetry. Goel and Chopra (1990, 1991a and 1991b) adopted the strength eccentricity ratio \( (e_p/r, \text{ where } e_p \text{ is the strength eccentricity}) \) as an independent system parameter. They found that the relative values of the strength and stiffness eccentricities can influence significantly the response behaviour of asymmetric systems. Therefore, in investigating the influence of the various system parameters on the inelastic response of asymmetric buildings, the strength eccentricity ratio must be taken as an independent system parameter.

2.3.4 Accidental eccentricity in code torsional provisions

The accidental eccentricity \( (e_a) \) in code torsional provisions is intended to account for uncertainties in design and the influence of the rotational component of the ground motion. Therefore, if such effects are absent in analysis, the accidental eccentricity should not be included in the specification of element strength but instead be reserved to account subsequently for uncertainties and the rotational component of the earthquake motion, as is the case in Duan and Chandler (1990), Chandler and Duan (1991a) and this thesis. This issue will be further discussed in Section 5.3.2, which presents detailed quantitative results. The approach employed in Gomez et al. (1987), as summarised in Section 2.2.4, is adequate. However, since the real location of the centre of mass may
have an offset of $\pm e_a$ from its nominal position, the amount of work needed to implement such proposals is twice that needed in the procedure adopted by Duan and Chandler (1990) and Chandler and Duan (1991a).

2.4 Conclusions

Because of the various differences and inadequacies in the analytical models and approaches employed by previous researchers, the inelastic behaviour of asymmetric buildings to strong earthquake motions has not been well understood and the influence of the various system parameters on the inelastic torsionally coupled response of asymmetric buildings are still not clarified. A comprehensive study on these fundamental issues based on the use of an adequate analytical model is therefore clearly needed. Based on Section 2.3, it can be concluded that an adequate analytical model should have at least three resisting elements parallel to the direction of earthquake motion and should not include transverse resisting elements if the ground motion is assumed to be uni-directional. The stiffness and strength eccentricity ratios should be taken as independent system parameters. Such an analytical model, subjected to earthquake records with different frequency contents, has been employed in Chapter 4, which presents the results of a thorough inelastic dynamic parametric study.

Based on a better understanding of the inelastic behaviour of asymmetric buildings to strong earthquake motions and the influence of the various system parameters, code torsional provisions can then be properly evaluated and appropriate changes to code provisions can be recommended. As stated in Sections 2.1 and 2.3.4, the study carried out by Tso and Ying (1990) mistakenly included the accidental eccentricity in the specification of element strength and therefore led to misleading results. Chapter 5
demonstrates the need for code torsional provisions be re-evaluated and modified in order to lead to satisfactory inelastic seismic performance of asymmetric buildings, and makes proposals for appropriate revisions.

All existing studies are based on single-storey building models. It is well known that if the response of a multistorey building is dominated by its fundamental vibration mode, results obtained from analysis based on a SDOF system having a period equal to the fundamental period of the multistorey building are also applicable to the multistorey building. However, in medium-period and long-period multistorey buildings, higher modes also contribute considerably to the total response. When structures are excited well into the inelastic range, yielding leads to significantly increased periods. This period shifting effect increases the importance of the higher modes. Therefore, conclusions based on analysis of a single-storey building model will not necessarily be directly applicable to multistorey asymmetric buildings. As a result, studies based on a multistorey asymmetric building model should also be carried out, as presented in Chapter 6.
Figure 2.1 Two-element mass-eccentric structural model (after Irvine and Kountouris 1980)

Figure 2.2 Single-storey structural model: (a) idealised single-storey system; (b) its equivalent single-element model (after Kan and Chopra 1981a, 1981b)
Figure 2.3 Rectangular plan and its displaced configuration (after Kan and Chopra 1981a)

Figure 2.4 Ratios of corner column deformation $u_m$ to the lateral displacement $u_{xm}$ at CM for systems with $a/b=1$ and $e/r=0.4$ subjected to the El Centro S00E record (after Kan and Chopra 1981a)
Figure 2.5 Ratios of corner column deformation $u_{im}$ to the lateral displacement $u_{xm}$ at CM for systems subjected to the El Centro S00E record: (a) $\Omega = 2$ and $a/b=1$; (b) $\Omega = 1$ and $a/b=1$ (after Kan and Chopra 1981a)
Figure 2.6 Plan view of the single-storey monosymmetric three-element structural model (after Bozorgnia and Tso 1986)

Figure 2.7 Element force-deformation relationship (after Bozorgnia and Tso 1986)

(continued over leaf)
Figure 2.8 Effect of the uncoupled torsional to translational frequency ratio on the displacement ductility demand of element 3; $e/r=0.25$ (after Bozorgnia and Tso 1986)

PERIOD $T = 1$ sec

$e/D = 0.05$

$\Omega = 0.8$  $\Delta = 3.5$  $\Delta = 4.2$

$\Omega = 1.0$  $\Delta = 3.3$  $\Delta = 3.9$

$\Delta = 4.2$  $\Delta = 2.3$

$\Omega = 1.4$  $\Delta = 3.3$  $\Delta = 3.9$  $\Delta = 2.5$  $\Delta = 5.0$

$\Delta = U/U_y$

- UNLOADING
- YIELD

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Figure 2.9 Position of the structure at the instant of reaching the maximum flexible edge displacement (after Tso and Sadek 1985)
Figure 2.10 Effect of the eccentricity ratio $\bar{e}=e/r$ on the displacement ductility demand of element 3; $\Omega = 1.2$ (after Bozorgnia and Tso 1986)
Figure 2.11 Plan view of the single-storey monosymmetric structural model (after Syamal and Pekau 1985)

Figure 2.12 Effect of $\Omega$ on the displacement ductility demand $\bar{\delta}$ of elements 1 and 2 for varying eccentricity ratio $\bar{\epsilon} = e/r$ (after Syamal and Pekau 1985)

Figure 2.13 Effect of the eccentricity ratio $\bar{\epsilon} = e/r$ on the displacement ductility demand of elements 1 and 2, and the translational and torsional system displacements $\bar{\Lambda}_x, \bar{\Lambda}_\phi$: (a) $\Omega = 0.5$; (b) $\Omega = 1.0$; (c) $\Omega = 1.5$ (after Syamal and Pekau 1985)
Figure 2.14 Plan view of the single-storey, three-element, mass-eccentric structural model (after Gomez et al. 1987)

Figure 2.15 Plan view of the single-storey monosymmetric structural model

Figure 2.16 Plan view of the corresponding torsionally uncoupled system
Figure 2.17 Relationship between $\Omega_r$ and $\Omega_m$
CHAPTER 3

*Inelastic Static Behaviour of Single-storey Asymmetric Buildings under Monotonic Loading*

**3.1 Introduction**

It was demonstrated in Chapter 2 that the inelastic response of asymmetric buildings to strong earthquake excitations is model dependent and that more parameters influence the inelastic response than those influencing the elastic response. In addition to the stiffness eccentricity ratio, the uncoupled torsional to translational frequency ratio of the corresponding torsionally uncoupled system, the uncoupled lateral period, and the viscous damping, which are the system parameters determining the elastic response of asymmetric buildings, the inelastic seismic response of these buildings also depends on the number, location and post-yielding force-deformation relationship of the resisting elements, the system’s total yielding strength relative to the elastic strength demand and the distribution of this total yielding strength, as characterised by another system parameter, namely the strength eccentricity ratio.

Therefore, selecting an adequate analytical model and employing it for a comprehensive investigation of the influence of the various system parameters on the inelastic torsionally coupled response requires a considerable amount of analytical work. Prior to this computationally expensive numerical solution of the inelastic dynamic earthquake response, it is enlightening to carry out an inelastic failure mode analysis of asymmetric single-storey buildings subjected to a monotonically increasing static force acting through the centre of mass. This static failure mode analysis is a
simple and efficient technique to predict the inelastic seismic response behaviour and to identify potential failure mechanisms. Such an analysis also helps to explain the model dependency of the inelastic seismic response of asymmetric buildings and to clarify the influence of the various system parameters on the inelastic response parameters. In the case of single-storey models, closed form solutions can be obtained with relative ease. This method has been employed in the recently developed computer program IDARC-3D for inelastic three-dimensional earthquake response analysis of reinforced concrete buildings (Kunnath and Reinhorn 1989), to provide a means for assessing design requirements and consequently to change appropriate parameters in order to achieve a desired sequence of component or element yielding. Static monotonic load analysis has also been carried out by Bozorgnia and Tso (1986) and Tso and Bozorgnia (1986) to give an insight into the behaviour of the models considered.

In this chapter, single-storey building models having two or three resisting elements parallel to the direction of the applied static force are employed. The static force is monotonically increased through the centre of mass until the structure collapses. The collapse mode and the critical resisting element (which has the largest displacement ductility demand among the elements) are identified. The influence of the various system parameters on the maximum lateral load-resisting capacity and on the maximum displacement ductility demand of the critical element is investigated quantitatively.

3.2 Characteristics of the models employed

The models employed in the static parametric study are single-storey and monosymmetric, having two or three resisting elements oriented parallel to the direction of the applied load as shown in Fig. 3.1. The floor slab is assumed to be rigid in its own plane and the mass is uniformly distributed on the floor. The resisting elements are considered to be massless and inextensible. An element’s torsional stiffness about the
vertical axis through it and its translational stiffness perpendicular to its own acting plane are neglected. To simplify the analysis, the force-deformation (end shear and end lateral displacement) relationship of the elements is assumed to be elastic-perfectly plastic.

There are three reference centres associated with each model, namely the centre of mass CM, the centre of rigidity CR and the centre of strength PC (plastic centroid) characterising the distribution of mass, stiffness and strength of the model, respectively. CM coincides with the geometric centre of the floor plan in all cases. For single-storey building models, the centre of rigidity coincides with the centre of stiffness, which is defined as the point in the plan of the floor deck about which the sum of the first moment of the lateral stiffness of the resisting elements is zero. Consequently, the location of CR can be determined from:

\[
X_{cr} = \frac{\sum k_i x_i}{\sum k_i} = 0
\]  
(3.1)

\[
y_{cr} = 0
\]  
(3.2)

and the stiffness or static eccentricity \( e_s \) is defined as the offset of CR from CM. The stiffness eccentricity ratio is defined as \( e_s^* = e_s/r \), where \( r \) is the radius of gyration of the floor deck about CM, which is assumed constant.

The centre of strength, PC, is defined as the point through which the resultant of all the element forces acts when all elements are loaded to their yield strengths. The coordinates of PC can be found by taking the first moment of the yield strength of all resisting elements about the origin of coordinates as follows:

\[
x_{pc} = \frac{\sum f_{yi} x_i}{\sum f_{yi}}
\]  
(3.3)

\[
y_{pc} = 0
\]  
(3.4)
and the strength eccentricity $e_p$ is defined as the offset of PC from CM. The strength eccentricity ratio $e_p^* = e_p/r$.

The system’s total yield base shear is defined as $F_y = \sum f_{yi}$, where $f_{yi}$ is the yield strength of the $i$th element. The system’s total lateral stiffness $K_y = \sum k_i$, where $k_i$ is the lateral stiffness of the $i$th element in the $y$ direction. The system’s torsional stiffness about CR, $K_{tor} = \sum k_i x_i^2$. Eccentricities are caused by the unbalanced stiffness and strength of the two edge elements. Changes in the stiffness eccentricity ratio $e_y^*$ and the torsional stiffness $K_{tor}$ are achieved by adjusting the difference in stiffness and the distance to CM ($d$) of the two edge elements. For example, a higher value of $K_{tor}$ combined with a constant value of $e_y^*$ can be achieved by increasing the distance $d$ and reducing the difference in the lateral stiffness of the two edge elements. In this thesis, changes in $\Omega$ are obtained by adjusting the value of the torsional stiffness $K_{tor}$ as opposed to changing $r$.

3.3 Determination of element properties from system parameters

3.3.1 Two-element model

For the two-element model shown in Fig. 3.1(a), $k_i$, $f_{yi}$ and $d$ are uniquely determined by the values of the system parameters, as follows:

$$f_{y1} = \frac{1}{2} F_y \left( 1 + \frac{e_p^* r}{d} \right)$$  \hspace{1cm} (3.5)

$$f_{y2} = \frac{1}{2} F_y \left( 1 - \frac{e_p^* r}{d} \right)$$  \hspace{1cm} (3.6)

$$k_1 = \frac{1}{2} K_y \left( 1 + \frac{e_y^* r}{d} \right)$$  \hspace{1cm} (3.7)
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\[ k_2 = \frac{1}{2} K_\gamma \left( 1 - \frac{e_x^* r}{d} \right) \]  \hspace{1cm} (3.8)

\[ d = r(\Omega^2 + e_x^*)^{\frac{1}{3}} \]  \hspace{1cm} (3.9)

3.3.2 Three-element models

Unlike the two-element model, \( k_\gamma \), \( f_\gamma \), and \( d \) of three-element models cannot be determined uniquely by the system parameters. For a given set of system parameters, many combinations of \( k_\gamma \), \( f_\gamma \), and \( d \) are possible. In this thesis, both single-step and double-step variations of stiffness distribution between the three elements are considered, but only a double-step element strength variation is considered for initially symmetric systems. In the case of a double-step variation of element stiffness or yielding strength, one edge element (element 1) is set to a high stiffness or strength value, the other edge element (element 3) to a low stiffness or strength value, and the element at the centre (element 2) has the mean value of stiffness or strength. In the case of a single-step stiffness variation, the element at the centre (element 2) and the flexible edge element (element 3) are set to have the same stiffness and the stiff edge element (element 1) is set to have a higher value of stiffness. Thus, in initially symmetric systems, all elements have the same stiffness while element strength \( f_\gamma \) is determined by:

\[ f_{\gamma 1} = F_\gamma \left( \frac{1}{3} + \frac{1}{2} e_x^* r \right) \]  \hspace{1cm} (3.10)

\[ f_{\gamma 2} = \frac{1}{3} F_\gamma \]  \hspace{1cm} (3.11)

\[ f_{\gamma 3} = F_\gamma \left( \frac{1}{3} - \frac{1}{2} e_x^* r \right) \]  \hspace{1cm} (3.12)
For systems having a double-step variation of stiffness distribution, element stiffness $k_i$ and the distance between adjacent elements $d$ are specified by:

\begin{align*}
    k_1 & = K_y \left( \frac{1}{3} + \frac{1}{2} \frac{e^*_r}{d} \right) \\
    k_2 & = \frac{1}{3} K_y \\
    k_3 & = K_y \left( \frac{1}{3} - \frac{1}{2} \frac{e^*_r}{d} \right) \\
    d & = r \sqrt{\frac{3}{2} \left( \Omega^2 + e^*_{y2} \right)}
\end{align*}

For systems having a single-step variation of stiffness distribution, element stiffness $k_i$ and the distance between adjacent elements $d$ are specified by:

\begin{align*}
    k_1 & = \frac{1}{3} K_y \left( 1 + \frac{2e^*_r}{d} \right) \\
    k_2 & = \frac{1}{3} K_y \left( 1 - \frac{e^*_r}{d} \right) \\
    k_3 & = \frac{1}{3} K_y \left( 1 - \frac{e^*_r}{d} \right) \\
    d & = \frac{1}{4} r \sqrt{24\Omega^2 + 25e^*_{y2} - e^*_{y1}}
\end{align*}

3.4 Results of static parametric study

A monotonically increasing static concentrated load $F$ is applied to each model through CM and parallel to the $y$-axis. Closed form solutions for the ultimate values of the static load $F$ (denoted as $F_{\text{max}}$) which the model can withstand and the maximum displacement ductility demand of the resisting elements have been obtained as presented
3.4.1 Results from the two-element model

The two-element model is a statically determinate system. Because of this, the distribution of the load $F$ between the two elements is independent of the distribution of stiffness. Moreover, as soon as the weak element yields, the system becomes a mechanism and collapses (Fig. 3.2), assuming the post-yielding force-deformation relationship of the elements to be elastic-perfectly plastic. It is therefore not possible for plasticity to develop in the material of the elements. Hence, the maximum element displacement ductility demand, which is defined as the ratio of the element's peak displacement to the yield displacement, is 1.0 irrespective of the values of $e^*_s$ and $e^*_p$. After the first yielding, a dynamic problem exists and is therefore beyond the scope of the present static analysis.

The maximum value of the lateral load $F$ is:

$$F_{\text{max}} = F_y \left(1 - \frac{e^*_p r}{d}\right)$$

$$= F_y \left(1 - \frac{e^*_p}{\sqrt{\Omega^2 + e^*_s^2}}\right) \quad (3.21)$$

To demonstrate the effect of $e^*_p$ and $\Omega$ on $F_{\text{max}}$, eqn. (3.21) has been plotted in Figs. 3.3 and 3.4. In Fig. 3.3, it is assumed that the element yield strength is proportional to its stiffness (see Fig. 3.5), therefore $e^*_s = e^*_p$. In Fig. 3.4, $e^*_s$ is taken to be zero, representing initially symmetric systems (Fig. 3.6). From Figs. 3.3 and 3.4, it is clear that in static analysis both the strength eccentricity and the uncoupled frequency ratio significantly affect the ultimate load the structure can withstand. The effect of strength eccentricity is particularly pronounced for torsionally flexible structures ($\Omega = 0.7$); in
this case, $F_{\text{max}}$ drops rapidly with increasing $e_p^*$. In all cases, $F_{\text{max}}$ increases with increasing value of $\Omega$. The effect of $\Omega$ on $F_{\text{max}}$ is more critical for structures having moderate to large strength eccentricities ($e_p^* \geq 0.6$).

3.4.2 Results from the three-element models

The three-element models are statically indeterminate. Following first yielding, the system becomes statically determinate and can still sustain an increasing load $F$. When a second element yields, the system becomes a mechanism and the static load reaches its ultimate value $F_{\text{max}}$. At this stage, there has been some development of material plasticity in the element which yields first, and hence its ductility demand is larger than 1.0. Therefore, in inelastic static analysis of the effect of torsional coupling on the displacement ductility demand of resisting elements, the two-element model underestimates the displacement ductility demand compared with three-element models since in the former case no development of plasticity is possible.

3.4.2.1 Initially symmetric systems

Initially symmetric systems are chosen to demonstrate the effect of the strength eccentricity and remove the effect of the stiffness eccentricity on $F_{\text{max}}$ and the element displacement ductility demand. In initially symmetric systems, all three elements have the same elastic stiffness but unbalanced yielding strength, as shown in Fig. 3.7. Therefore, CR coincides with CM but PC does not, that is, $e_r^* = 0, e_p^* \neq 0$. Before yielding, the system does not exhibit torsional motion, all three elements having the same translational displacement. The weakest element (element 3) yields first, following which the resisting force in element 1 remains constant in order to maintain equilibrium ($\sum M_{cm} = 0$). Element 1 becomes stationary and the floor deck rotates
around element 1. The resisting force in element 2 increases with the increase of the load until it yields. When element 2 yields, the structure becomes a mechanism and collapses (Fig. 3.8). The ultimate value of the load is given by:

\[
F_{\text{max}} = 2f_{y3} + f_{y2}
= F_y \left(1 - \frac{e_r^*}{d}\right)
= F_y \left(1 - \frac{e_p^*}{\sqrt{\frac{3}{2}}(\Omega^2 + e_r^*)}\right)
= F_y \left(1 - \frac{e_p^*}{\Omega \sqrt{\frac{3}{2}}}\right)
\]  

Eqn. (3.22) has been plotted in Fig. 3.9. The maximum displacement ductility demand of element 3 (the weakest element) is:

\[
\mu_3 = 1.0 + \frac{\sqrt{6} e_p^*}{\Omega - e_p^* \sqrt{\frac{3}{2}}}
\]  

Eqn. (3.23) has been plotted in Fig. 3.10. Discussion of the results shown in Figs. 3.9 and 3.10 is given in Section 3.5.1.

3.4.2.2 Initially eccentric systems with a double-step variation of stiffness distribution

(a) Systems with balanced yielding strength

Systems with balanced yielding strength are studied to demonstrate the effect of the stiffness eccentricity and to remove the effect of the strength eccentricity on \(F_{\text{max}}\) and the element displacement ductility demand. In this case, the distribution of strength is
symmetric, PC coincides with CM but CR does not, that is, $e_{p}^{*} = 0, e_{r}^{*} \neq 0$. All three elements have the same yielding strength but a double-step variation of stiffness, as shown in Fig. 3.11. The ultimate load the structure can withstand is $F_{\text{max}} = F_\gamma$.

The equilibrium condition $\sum M_{cm} = 0$ requires that $F_1 = F_3$. Because the floor slab is rigid in its own plane, the displacement of element 2 is $y_2 = \frac{1}{2}(y_1 + y_3)$, and hence in the elastic range the resisting force in element 2 is:

$$F_2 = k_2 y_2$$

$$= \frac{1}{3} K \gamma \left( \frac{F_1}{k_1} + \frac{F_3}{k_3} \right)$$

$$= F_3 \frac{1}{6} K \gamma \left( \frac{1}{k_1} + \frac{1}{k_3} \right)$$

$$= F_3 \frac{1}{6} K \gamma \frac{k_1 + k_3}{k_1 k_3} \quad (3.24)$$

Let $k_1 = \alpha K_\gamma, \ 0 < \alpha < 1$. From the assumption $k_1 + k_3 = \frac{2}{3} K_\gamma$, the following relationship is obtained:

$$F_2 = F_3 \frac{1}{\frac{1}{3} \alpha (\frac{2}{3} - \alpha)} \quad (3.25)$$

Now, $9\alpha (\frac{2}{3} - \alpha) < 1.0$ if $\alpha > \frac{1}{3}$. Furthermore, if $\alpha = \frac{1}{3}$ then $9\alpha (\frac{2}{3} - \alpha) = 1.0$ and in this case $e_{r}^{*} = 0$, that is the system is symmetric. Therefore, in all cases $F_2 \geq F_3 = F_1$. As a result, element 2 yields first. After the yielding of element 2, the resisting forces in element 1 and 3 remain equal and increase with the increasing load until these two elements yield simultaneously and the structure collapses (Fig. 3.12).
The displacement ductility demand of elements 1 and 3, $\mu_1 = \mu_3 = 1.0$ when $F$ reaches $F_{\text{max}}$. The maximum displacement ductility demand occurs in element 2 and its value is:

$$\mu_2 = 1.0 + \frac{3e_1^*}{2\Omega^2 - e_2^2}$$  \hspace{1cm} (3.26)

Eqn. (3.26) has been plotted in Fig. 3.13 and the results are discussed in Section 3.5.1.

(b) Systems with equal stiffness and strength eccentricities

In this case, the yielding displacements of the resisting elements are equal and the yield strengths of the elements are proportional to their stiffnesses, as shown in Fig. 3.14.

Because the elements have the same yielding displacement, the element furthest away from CR (element 3) yields first. Then, the resisting force in element 1 remains constant because of the equilibrium requirement $\Sigma M_{\text{en}} = 0$. Therefore, element 1 becomes stationary and the floor slab rotates about element 1 until element 2 yields. The structure then collapses (Fig. 3.15). The ultimate value of the applied static load is:

$$F_{\text{max}} = 2f_{y3} + f_{y2}$$

$$= F_y \left( 1 - \frac{e_3^*}{d} \right)$$

$$= F_y \left( 1 - \frac{e_1^*}{\sqrt{\frac{3}{2}(\Omega^2 + e_2^2)}} \right)$$  \hspace{1cm} (3.27)
Eqn. (3.27) has been plotted in Fig. 3.16. The maximum displacement ductility demand occurs in element 3,

$$
\mu_3 = 1.0 + 6 \left( 1 - \frac{e_s^*}{\sqrt{2} (\Omega^2 + e_s^{*2})} - \frac{\Omega^2}{\Omega^2 + e_s^{*2} + e_s^* \sqrt{2 (\Omega^2 + e_s^{*2})}} \right) 
$$

(3.28)

Eqn. (3.28) has been plotted in Fig. 3.17 and the results are discussed in Section 3.5.1.

### 3.4.2.3 Initially eccentric systems with a single-step variation of stiffness distribution

(a) Systems with balanced yielding strength

In systems with a single-step variation of stiffness but balanced strength, the distribution of strength is uniform and symmetric, and elements 2 and 3 have equal stiffness but lower than that of element 1 as shown in Fig. 3.18.

Because of the equilibrium condition $\Sigma M_{cm} = 0$, the forces in elements 1 and 3 are always equal, $F_1 = F_3$. Since the floor deck is assumed to be rigid in its own plane, the translation of element 2 is $y_2 = \frac{1}{2} (y_1 + y_3)$, and hence in the elastic range the resisting force in element 2 is:

$$
F_2 = k_2 y_2 \\
= k_2 \frac{1}{2} (y_1 + y_3) \\
= k_2 \frac{1}{2} \left( \frac{F_1}{k_1} + \frac{F_3}{k_3} \right)
$$
\[
\frac{1}{2} F_3 \left( \frac{k_2}{k_1} + \frac{k_2}{k_3} \right) = \frac{1}{2} F_3 \left( 1 + \frac{k_2}{k_1} \right) \tag{2.29}
\]

Since \( k_2 = k_3 < k_1 \), it follows that \( 1 + \frac{k_2}{k_1} < 2 \) and hence \( \frac{1}{2} \left( 1 + \frac{k_2}{k_1} \right) < 1 \). Therefore \( F_2 < F_3 = F_1 \). Consequently, elements 1 and 3 yield simultaneously while element 2 remains elastic, and the structure can still sustain an increased lateral load. After the first yielding of elements 1 and 3, the centre of rigidity shifts to the position of the centre of mass and the floor deck translates without further rotation (Fig. 3.19) until element 2 yields. At this point a failure mechanism forms and the structure collapses.

The maximum lateral load the structure can sustain is \( F_y \). The displacement ductility demand of element 2 is 1.0. Element 1 has the highest displacement ductility demand, which is:

\[
\mu_1 = 1 + \frac{6e^*_x}{\sqrt{24\Omega^2 + 25e_x^{*2}} - 5e^*_x} \tag{3.30}
\]

The displacement ductility demand in element 3 is:

\[
\mu_3 = 1 + \frac{6e^*_x}{\sqrt{24\Omega^2 + 25e_x^{*2}} + 7e^*_x} \tag{3.31}
\]

Eqns. (3.30) and (3.31) have been plotted in Fig. 3.20 and Fig. 3.21, respectively.
(b) Systems with equal stiffness and strength eccentricities

In this case, each element’s strength is proportional to its stiffness and all three elements have the same yielding displacement. Elements 2 and 3 have the same stiffness and strength, while the stiffness and strength of element 1 are higher than those of elements 2 and 3, as illustrated in Fig. 3.22.

Because all elements have equal yielding displacements, the one most unfavourably affected by torsion, element 3, reaches its yield displacement first. Then, the resisting force in element 1 remains unchanged and equal to the yielding strength of element 3 because of the equilibrium requirement $\Sigma M_{cm} = 0$. Since the yielding strength of element 1 is higher than that of element 3, element 1 stays in the elastic range and remains stationary with further loading beyond the first yielding of element 3. Further displacement of the floor deck consists of rotation around element 1 only at this stage of loading (Fig. 3.23). The failure mechanism forms and the structure collapses when element 2 yields. The maximum lateral load is:

$$F_{max} = 2f_{y3} + f_{y2}$$

$$= 3f_{y3}$$

$$= F_y \left(1 - \frac{e_r^*}{d}\right)$$

$$= F_y \left(1 - \frac{4e_r^*}{\sqrt{24\Omega^2 + 25e_r^{*2} - e_r^*}}\right)$$

(3.32)

Element 2 just reaches yielding. The highest displacement ductility demand occurs in element 3:

$$\mu_3 = 1 + 6 \left[1 - \left(\frac{\sqrt{24\Omega^2 + 25e_r^{*2} - e_r^*}}{\sqrt{24\Omega^2 + 25e_r^{*2} - 5e_r^*}}\right)\left(\frac{4\Omega^2}{4\Omega^2 + 3e_r^{*2} + e_r^*\sqrt{24\Omega^2 + 25e_r^{*2}}}\right)\right]$$

(3.33)
Eqns. (3.32) and (3.33) are plotted in Figs. 3.24 and 3.25 respectively and the results are discussed in Section 3.5.1.

3.5 Discussion and conclusions

3.5.1 Discussion on the results obtained from three-element models

Figs. 3.9, 3.16 and 3.24 show that the system’s strength eccentricity ratio and the uncoupled frequency ratio are critical parameters affecting the ultimate value of the static load the structure can withstand. Increasing the system’s torsional stiffness, i.e., increasing the value of $\Omega$, always has the effect of increasing the system’s capacity to withstand the static load. The effect of the uncoupled frequency ratio on $F_{\text{max}}$ is more pronounced for structures with moderate to large strength eccentricity ratios ($e_p^* > 0.6$). $F_{\text{max}}$ always declines with increasing value of the strength eccentricity. For torsionally flexible systems ($\Omega = 0.7$), a moderate value of the strength eccentricity ratio ($e_p^* = 0.6$) reduces the system’s lateral load resisting capacity to about 40 to 50 per cent of its total yield base shear (Figs. 3.9, 3.16 and 3.24). Comparing these figures with Figs. 3.3 and 3.4, it is clear that results obtained from both two-element and three-element models are similar, but the results from three-element models indicate a larger $F_{\text{max}}$ compared with those from the two-element model, illustrating the effect of development of material plasticity on the structure’s capacity to resist lateral loading. Therefore, in inelastic static failure mode analysis of asymmetric buildings, the two element model underestimates both the peak element displacement ductility demand and the structure’s capacity to resist lateral load.

Fig. 3.10 demonstrates the effect of the strength eccentricity ratio and the uncoupled frequency ratio on the maximum displacement ductility demand of initially symmetric systems. It can be seen that both these system parameters significantly affect
the maximum element displacement ductility demand. The maximum displacement ductility demand increases rapidly with the increase in the strength eccentricity ratio but reduces significantly with increasing uncoupled torsional to translational frequency ratio. The effect of $\Omega$ is more pronounced for systems with moderate to large strength eccentricities than for systems with a small strength eccentricity. Increasing the value of $\Omega$ also has the effect of reducing the influence of the strength eccentricity on the maximum displacement ductility demand; the element displacement ductility demand becomes less sensitive to the strength eccentricity for torsionally stiff systems ($\Omega \geq 1.5$) compared with torsionally flexible systems ($\Omega = 0.7$) and systems with moderate values of $\Omega$ (in the range 1.0-1.3).

Figs. 3.13 and 3.20 illustrate the effect of the stiffness eccentricity ratio and the uncoupled frequency ratio on the maximum displacement ductility demand of systems with a symmetric distribution of yielding strength ($e^* = 0$). In structures with a single-step stiffness variation, the maximum displacement ductility demand occurs at the stiff edge element (element 1) whilst it occurs at the element at the centre (element 2) in structures with a double-step variation of stiffness. In both cases, the element at the flexible edge (element 3), which is considered to be the most unfavourably affected element by torsional coupling in elastic studies, is not the critical element in static inelastic failure mode analysis of structures having an asymmetric distribution of stiffness but a symmetric distribution of strength. When the structure collapses, the flexible element (element 3) has just reached the threshold of yielding in a double-step stiffness variation system or has been slightly loaded into the inelastic range as shown in Fig. 3.21 in a single-step stiffness variation system. In both cases, the stiffness eccentricity ratio significantly affects the maximum displacement ductility demand of structures with a symmetric distribution of strength, being more pronounced for structures having moderate to large stiffness eccentricity ratios ($e^* > 0.6$) and small to moderate values of the uncoupled torsional to translational frequency ratio
Again, the maximum displacement ductility demand decreases with increasing $\Omega$. The effect of $\Omega$ is particularly pronounced for structures having moderate to large stiffness eccentricity ratios ($e_* > 0.6$).

Figs. 3.17 and 3.25 illustrate the influence of the eccentricity ratio and the uncoupled frequency ratio on the maximum displacement ductility demand for systems having equal strength and stiffness eccentricity ratios. In this case, the maximum element displacement ductility demand occurs in the flexible edge element (element 3) and is sensitive to both the eccentricity and the uncoupled frequency ratios. The effect of eccentricity is more pronounced in the range of small to moderate eccentricities ($e_* = e_p < 0.6$). In the range of large eccentricities ($e_* = e_p > 0.8$), element displacement ductility demand is not as sensitive to the eccentricity ratio. Larger values of $\Omega$ always correspond to a smaller displacement ductility demand, the effect being more significant for systems with moderate to large eccentricities.

### 3.5.2 Conclusions and implications for analysis and design

Inelastic static behaviour of asymmetric buildings under monotonic loading has been studied to investigate the failure mode, the ultimate lateral load-carrying capacity and the maximum element displacement ductility demand. The analysis carried out in Section 3.4 has clearly identified the structure’s failure mode and the identity of the critical resisting element when it is loaded to the threshold of collapse. The discussion in Section 3.5.1 has clarified the influences of the key structural parameters on the ultimate load carrying capacity and the maximum element displacement ductility demand. This inelastic static monotonic study provides a basis for further research into the inelastic earthquake response analysis of asymmetric buildings when they are loaded by strong ground motions well into the inelastic range near the threshold of collapse.
The two-element model underestimates both the ultimate lateral load-carrying capacity of the structure and the maximum displacement ductility demand of the resisting elements. As a result, it is concluded that the two-element model is not an adequate model and should not be employed in further studies analysing the inelastic dynamic response of asymmetric buildings to strong earthquake motions.

There are systematic trends in the influence of the uncoupled torsional to translational frequency ratio on the structure’s ultimate lateral load-resisting capacity and the maximum element displacement ductility demand. Larger values of $\Omega$ always lead to higher values of the ultimate load the structure can withstand and a lower maximum element displacement ductility demand. The influence of $\Omega$ is particularly pronounced for structures having moderate to large stiffness or strength eccentricity ratios. It is strongly recommended that torsionally flexible structures ($\Omega < 1.0$, such as buildings having central cores) which have large strength or stiffness eccentricities are to be avoided in design practice, and that designers locate the cores for lifts to the periphery of the building and check explicitly the value of the uncoupled torsional to translational frequency ratio.

The strength eccentricity in initially symmetric structures affects strongly the ultimate lateral load-resisting capacity of these structures and the maximum element displacement ductility demand. In asymmetric hybrid buildings in which mixed lateral load-resisting elements (frames and shear walls) are used to resist the lateral load and to provide lateral stiffness, the stiffness of these elements can be adjusted by the designers to achieve symmetry satisfying the requirements in earthquake resisting design codes for structural configuration. However, because of the design and detailing provisions in the material codes, such as the minimum reinforcement ratios in reinforced concrete structures, the strength distribution among these elements may be left significantly asymmetric. Therefore, it is desirable that designers check explicitly the strength
eccentricity ratio in initially stiffness symmetric structures when buildings are designed for the ultimate limit state, which at the present is not required by earthquake resistant design building codes. If necessary, the strengths of the resisting elements should be adjusted in order to shift the centre of strength as close as possible to the centre of rigidity. Hence, a refined design can be achieved.

The stiffness eccentricity ratio influences significantly the maximum element displacement ductility demand in stiffness asymmetric buildings having a symmetric distribution of strength. This category of buildings represents buildings designed in accordance with the torsional provisions of building codes. According to Tso and Ying (1990), code torsional provisions lead to much smaller strength eccentricity than stiffness eccentricity in stiffness asymmetric buildings. In these buildings, irrespective whether the variation of element stiffness is double-step or single-step, the flexible edge element (element 3), which is considered to be the critical element in elastic dynamic analysis, is no longer the critical element. Rather, the stiff edge element (element 1) or the centre element (element 2) becomes the critical. This behaviour in inelastic analysis is fundamentally different from that indicated in elastic analysis.

Structures having equal stiffness and strength eccentricities ($e_s^*=e_p^*$) behave quite differently from structures having the strength eccentricity much smaller than the stiffness eccentricity ($e_p^*=0$). The former case has been extensively studied in the early studies of inelastic torsional coupling effects and the flexible edge element has been found to be the critical element, which is also the case in this study. Since the latter case is representative of code-designed buildings, it should receive more attention in further inelastic earthquake response studies.
Figure 3.1 Plan view of single-storey monosymmetric structural models employed in static parametric analysis: (a) 2-element model; (b) 3-element model

Figure 3.2 Behaviour of the 2-element model under static monotonic lateral load

Figure 3.3 Effect of the eccentricity ratio \( e^*_s = e^*_f \) on \( F_{max} \) for the 2-element model with equal strength and stiffness eccentricities
Figure 3.4 Effect of the strength eccentricity ratio on $F_{\text{max}}$ for the initially symmetric 2-element model

Figure 3.5 Stiffness and strength distribution of the 2-element model with equal stiffness and strength eccentricities

Figure 3.6 Stiffness and strength distribution of the initially symmetric 2-element model

Figure 3.7 Stiffness and strength distribution of the initially symmetric 3-element model
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Figure 3.8 Behaviour of the initially symmetric 3-element model under static monotonic lateral load

Figure 3.9 Effect of the strength eccentricity ratio on $F_{\text{max}}$ for the initially symmetric 3-element model

Figure 3.10 Displacement ductility demand of element 3 of the initially symmetric 3-element model under static monotonic load

Figure 3.11 Stiffness and strength distribution of the 3-element model with a double-step stiffness variation and balanced yielding strength
Figure 3.12 Behaviour of the 3-element model with a double-step stiffness variation and balanced yielding strength under static monotonic load

Figure 3.13 Displacement ductility demand of element 2 of the 3-element model with a double-step stiffness variation and balanced yielding strength under static monotonic load

Figure 3.14 Stiffness and strength distribution of the 3-element model with a double-step stiffness variation and equal stiffness and strength eccentricities

Figure 3.15 Behaviour of the 3-element model with a double-step stiffness variation and equal stiffness and strength eccentricities under static monotonic load
Figure 3.16 Effect of the eccentricity ratio \((e^*_p = e^*_s)\) on \(F_{\text{max}}\) for the 3-element model with a double-step stiffness variation and equal stiffness and strength eccentricities.

Figure 3.17 Displacement ductility demand of element 3 of the 3-element model with a double-step stiffness variation and equal stiffness and strength eccentricities.

Figure 3.18 Stiffness and strength distribution of the 3-element model with a single-step stiffness variation and balanced yielding strength.

Figure 3.19 Behaviour of the 3-element model with a single-step stiffness variation and balanced yielding strength under static monotonic load.
Figure 3.20 Displacement ductility demand of element 1 of the 3-element model with a single-step stiffness variation and balanced yielding strength.

Figure 3.21 Displacement ductility demand of element 3 of the 3-element model with a single-step stiffness variation and balanced yielding strength.

Figure 3.22 Stiffness and strength distribution of the 3-element model with a single-step stiffness variation and equal stiffness and strength eccentricities.

Figure 3.23 Behaviour of the 3-element model with a single-step stiffness variation and equal stiffness and strength eccentricities under static monotonic load.
Figure 3.24 Effect of the eccentricity ratio ($\varepsilon^* = \varepsilon^*$) on $F_{\text{max}}$ for the 3-element model with a single-step stiffness variation and equal stiffness and strength eccentricities.

Figure 3.25 Displacement ductility demand of element 3 of the 3-element model with a single-step stiffness variation and equal stiffness and strength eccentricities.
CHAPTER 4

Effect of System Parameters on The Inelastic Seismic Response of Single-storey Asymmetric Buildings

4.1 Introduction

As discussed in Chapter 2, previous studies have employed analytical models which have various important deficiencies. For example, earlier studies all assumed that a structure's centre of rigidity CR and its centre of strength PC coincide with each other. This deficiency makes the conclusions of such studies restricted to this particular category of structures and hence unable to reveal realistic trends of the influence of the strength and stiffness eccentricities, because buildings generally have separated centres of rigidity and strength. In particular, code designed buildings have their centres of strength close to their centres of mass rather than their centres of rigidity. Also, recent inelastic dynamic parametric studies carried out by Goel and Chopra (1990, 1991a) have inappropriately included the resisting elements oriented perpendicular to the direction of earthquake motion in their analytical model, as discussed below. For these and other reasons, the inelastic behaviour of asymmetric buildings to strong earthquake motions has not been well understood and the influence of the various system parameters on the inelastic torsionally coupled response of asymmetric buildings has still to be clarified.

The static failure mode analysis carried out in Chapter 3 has helped to develop understanding of the inelastic behaviour of asymmetric buildings when loaded well into the inelastic range until collapse and also helps to clarify the influence of the key system
parameters on the maximum element displacement ductility demand. The applicability of the conclusions drawn in Chapter 3 to the inelastic dynamic earthquake response of asymmetric buildings remains to be examined and tested by carrying out inelastic time history analysis of asymmetric buildings to earthquake loading.

Firstly, this chapter attempts to investigate the model dependency of the inelastic earthquake response of asymmetric buildings by employing the two-element and three-element models described in Chapter 3 as well as a four-element model to investigate the effect of the number of resisting elements on the inelastic response of asymmetric buildings. A five-element model is also employed to study the influence of the torsional stiffness contributed from the resisting elements oriented perpendicular to the direction of earthquake motion. The response of mass-eccentric systems is also studied and the results compared with those of stiffness-eccentric systems. A suitable analytical model is then selected by comparing results obtained on the basis of different models, and this is used to study the influence of the key system parameters on the inelastic torsional response.

4.2 Analytical models

4.2.1 Models considered

Five idealised building models have been employed in this chapter, as illustrated in Fig. 4.1. They are, the two-element stiffness-eccentric model (Fig. 4.1(a)), the three-element stiffness-eccentric model (Fig. 4.1(b)), the four-element stiffness-eccentric model (Fig. 4.1(c)), the stiffness-eccentric model with resisting elements oriented perpendicular to the direction of ground motion (Fig. 4.1(d)), and the three-element mass-eccentric model (Fig. 4.1(e)).
These idealised models are single-storey and monosymmetric. The distribution of mass, stiffness and strength is symmetric about the x-axis but may be asymmetric about the y-axis. The floor deck is rectangular with a typical aspect ratio a/b equal to 1/3 and is assumed to be rigid in its own plane. The resisting elements (frames and/or shear walls) are considered to be massless and inextensible. An element's torsional stiffness about the vertical axis through the element itself, and the lateral stiffness perpendicular to its acting plane are both neglected. The post-yielding force-deformation relationship of each element is taken as bi-linear hysteretic with inclusion of the Bauschinger effect and 3 per cent strain hardening, as shown in Fig. 4.2. The horizontal earthquake motion is assumed to be uni-directional and is applied parallel to the y-axis. The models are assumed to be built at stiff soil sites. The foundation is considered to be rigid, and therefore the earthquake motion is identical along the foundation without including a rotational component.

The decision to specify the buildings' aspect ratio a/b to be equal to 1/3 can be justified by examining the relationship between the building's dimension b and the radius of gyration of the floor deck r, as shown in Table 4.1.

<table>
<thead>
<tr>
<th>Aspect ratio a/b</th>
<th>Relationship between b and r</th>
</tr>
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<tbody>
<tr>
<td>1/2</td>
<td>b = 3.098r</td>
</tr>
<tr>
<td>1/3</td>
<td>b = 3.286r</td>
</tr>
<tr>
<td>1/4</td>
<td>b = 3.361r</td>
</tr>
<tr>
<td>a/b → 0</td>
<td>b → 3.464r</td>
</tr>
</tbody>
</table>

Table 4.1 Relationship between b and r for various aspect ratios
It can be seen that the coefficient relating \( b \) and \( r \) of a rectangular floor plan is nearly constant for all aspect ratios in the range typical of engineering practice. Therefore, it can be concluded that results obtained from structural models having a rectangular layout and an aspect ratio of 1/3 are generally applicable to those having a rectangular layout but different aspect ratios.

### 4.2.2 System parameters

Let \( k_{iy} \) represent the elastic lateral stiffness of the \( i \)th element oriented parallel to the \( y \)-axis and \( k_{jx} \) represent the elastic stiffness of the \( j \)th element, if any, oriented parallel to the \( x \)-axis. The total lateral stiffnesses of the structure parallel to the \( y \)-axis and the \( x \)-axis are:

\[
K_y = \sum_i k_{iy} \tag{4.1}
\]
\[
K_x = \sum_j k_{jx} \tag{4.2}
\]

The centre of rigidity \( CR \) is defined as the point at the floor level, through which a static force must be applied in order to induce pure translation of the floor deck without torsion. For single-storey systems, the centre of rigidity \( CR \) coincides with the centre of stiffness \( CS \). The centre of stiffness \( CS \) is the point in the plane of the floor deck about which the sum of the first moment of the lateral stiffness becomes zero. In this chapter, the origin of the co-ordinates is located at \( CR \). Therefore,

\[
x_{cr} = \frac{\sum k_{iy} x_i}{K_y} = 0 \tag{4.3}
\]
\[
y_{cr} = 0 \tag{4.4}
\]

The stiffness eccentricity or static eccentricity \( e_x \) is the offset between \( CR \) and \( CM \).
The centre of strength or plastic centroid PC will be recalled as the point in the plane of the floor deck through which the resultant of the yielding forces of all the resisting elements acts. Let $f_{iy}$ and $f_{jx}$ represent the yielding strength of the $i$th element parallel to the y-axis and that of the $j$th element parallel to the x-axis (if any), respectively. Then, the total yielding strengths parallel to the y-axis and the x-axis are $F_y = \sum_i f_{iy}$ and $F_x = \sum_j f_{jx}$, respectively. The location of the centre of strength is:

$$x_{pc} = \frac{\sum_i f_{iy} x_i}{F_y}$$  \hspace{1cm} (4.5)$$

$$y_{pc} = 0$$  \hspace{1cm} (4.6)$$

The strength eccentricity $e_p$ is the distance between PC and CM.

The elastic torsional stiffness of the structure about CR is:

$$K_{\theta} = \sum_i k_{iy} x_i^2 + \sum_j k_{jx} \left(\frac{1}{2} h\right)^2$$  \hspace{1cm} (4.7)$$

The corresponding torsionally uncoupled structure is defined as the one which has coincident CR and CM, i.e. $e_s=0$, but has the same values of $m$, $K_y$, $K_x$, and $K_{\theta}$ as the torsionally coupled structure under consideration. This system can be visualised by reinstalling the floor deck such that CM is located at the same point as CR.

The uncoupled translational and torsional frequencies of the torsionally coupled structure are expressed as the respective frequencies of the corresponding torsionally uncoupled structure. Thus, the translational frequencies are:

$$\omega_y = \sqrt{\frac{K_y}{m}}$$  \hspace{1cm} (4.8)$$

$$\omega_x = \sqrt{\frac{K_x}{m}}$$  \hspace{1cm} (4.9)$$
and the torsional frequency is:

\[ \omega_0 = \sqrt{\frac{K_0}{m r^2}} \]  \hspace{1cm} (4.10)

The uncoupled torsional to translational frequency ratio \( \Omega \) is defined as the appropriate ratio for the corresponding torsionally uncoupled structure. Therefore,

\[ \Omega = \frac{\omega_0}{\omega_y} = \sqrt{\frac{K_0}{K_y r^2}} \]  \hspace{1cm} (4.11)

The viscous damping \( \xi \) of the structure is considered to be 5 per cent of the critical damping for all vibration modes. The damping value of 5 per cent of critical is realistic for frame and frame-wall structures and is the assumed or reference damping value in specifying the elastic design spectra in most aseismic building codes.

### 4.2.3 Design spectrum

The total yielding strength of the structure \( F_y \) is taken to be equal to the design base shear determined in accordance with the Newmark-Hall spectrum (Newmark and Hall 1982). Hence:

\[ F_y = V_0 = \frac{m_s(\xi, T_y)}{R} \]  \hspace{1cm} (4.12)

in which \( s_s(\xi, T_y) \) is the 5 per cent damped elastic Newmark-Hall median spectrum (50% probability level), as shown in Fig. 4.3. \( R \) is the force reduction factor or the ductility factor which takes the displacement ductility and the energy dissipation capacities of the resisting elements into account and therefore reduces the strength capacity of the structure relative to the elastic strength demand specified by \( m_s(\xi, T_y) \).
The force reduction factor $R$ is specified in this chapter as either 2 or 4. The value $R=2$ results in the structure being excited slightly into the inelastic range whilst the value $R=4$ is near the upper bound of values specified in earthquake resistant design codes and represents structures shaken well into the inelastic range near the threshold of failure. The inelastic design spectrum is flat if the uncoupled lateral period $T_y$ is less than the second corner period $T_2$ and its variation is inversely proportional to the uncoupled lateral period if $T_y$ exceeds $T_2$. The effective force reduction factor used in the period range lower than the first corner period $T_1$ is smaller than that used for periods larger than $T_1$. Therefore, this approach leads to less strength reduction for buildings having fundamental lateral periods lower than $T_1$ than for those having such periods higher than $T_1$. In this thesis, $T_1$ is equal to 0.17 seconds, a value suggested by Mohraz et al. (1972). The second corner period $T_2$ can be determined by considering the peak ground acceleration ($a$) and velocity ($v$) of the earthquake motion and the spectrum amplification factors for acceleration ($2.12$ in the case of the median spectrum) and for velocity ($1.65$ in the case of the median spectrum). Therefore, at the second corner period, the following relationship exists:

$$2.12a = 1.65v \frac{2\pi}{T_2}$$

(4.13)

Hence,

$$T_2 = 4.89 \frac{v}{a}$$

(4.14)

At stiff soil sites, a typical value of the peak ground acceleration to velocity ratio ($a/v$ ratio) is about 1.0 g/(m/s). Substituting this $a/v$ ratio into eqn. (4.14) with $a$ in units of m/s$^2$, $T_2$ is approximately equal to 0.5 seconds. As a result, $T_2$ is taken herein to be
0.5 seconds. This value is appropriate for earthquake motions at stiff soil sites as considered in this thesis and is also consistent with the value specified in earthquake resistant design codes for stiff soil sites.

It should be noted that the above approach of specifying the total yielding strength as a function of the structure’s uncoupled lateral period is consistent with code provisions. The trend of the variation of the total yielding strength is similar to that commonly specified in codes. Consequently, this approach will lead to results which are more relevant to buildings designed in accordance with code provisions. In an alternative approach, instead of employing the widely used Newmark-Hall type elastic design spectrum, Goel and Chopra (1990, 1991a and 1991b) have employed the elastic response spectrum of certain specific ground motions and have used a constant force reduction factor over the entire period range. While such an approach is convenient to contrast the results obtained from elastic and inelastic response analysis of structures under the same ground motion, it is not consistent with code provisions and therefore not directly applicable to design practice. This approach may result in unrealistic inelastic structural response, particularly in the short period region, as discussed in detail in Section 4.7.

### 4.2.4 Specification of element properties by system parameters

The structure’s total lateral stiffness parallel to the direction of ground motion is determined by its uncoupled lateral period $T_y$:

$$K_y = m \omega^2$$

$$= m \left( \frac{2\pi}{T_y} \right)^2$$  \hspace{1cm} (4.15)
The total yielding strength is set to be equal to the design base shear which is calculated based on the Newmark-Hall spectrum and the structure's uncoupled lateral period $T_y$:

$$F_y = V_0 = \frac{mS_a(\xi, T_y)}{R}$$  \hspace{1cm} (4.16)

in which $\xi = 5\%$ of critical and $S_a(\xi, T_y)$ is as shown in Fig. 4.3.

4.2.4.1 Two-element model

For the two element model shown in Fig. 4.1(a), the properties $k_i$, $f_{yi}$ (i=1, 2) and $d$ are uniquely determined by the values of the system parameters, as follows:

$$f_{y1} = \frac{1}{2}F_y \left(1 + \frac{e^*r}{d}\right)$$  \hspace{1cm} (4.17)

$$f_{y2} = \frac{1}{2}F_y \left(1 - \frac{e^*r}{d}\right)$$  \hspace{1cm} (4.18)

$$k_1 = \frac{1}{2}K_y \left(1 + \frac{e^*r}{d}\right)$$  \hspace{1cm} (4.19)

$$k_2 = \frac{1}{2}K_y \left(1 - \frac{e^*r}{d}\right)$$  \hspace{1cm} (4.20)

$$d = r(\Omega^2 + e^*_r)^\frac{1}{2}$$  \hspace{1cm} (4.21)

4.2.4.2 Three-element models

As discussed in Section 3.3.2, the element properties of three-element models cannot be specified uniquely by the system parameters. One additional assumption is
needed. In this chapter, the distribution of element strength is considered to have a
double-step variation only (see eqns. 4.22 to 4.24). Both single-step and double-step
variations of the element stiffness distribution are considered. Therefore, the element
strength distribution is specified by:

\[
f_{y1} = F_y \left( \frac{1}{3} + \frac{\varepsilon_F r}{2 d} \right)
\]

(4.22)

\[
f_{y2} = \frac{1}{3} F_y
\]

(4.23)

\[
f_{y3} = F_y \left( \frac{1}{3} - \frac{\varepsilon_F r}{2 d} \right)
\]

(4.24)

The element stiffness distribution is defined by the following equations:

(a) Systems with a double-step stiffness variation

\[
k_1 = K_y \left( \frac{1}{3} + \frac{\varepsilon_F r}{2 d} \right)
\]

(4.25)

\[
k_2 = \frac{1}{3} K_y
\]

(4.26)

\[
k_3 = K_y \left( \frac{1}{3} - \frac{\varepsilon_F r}{2 d} \right)
\]

(4.27)

\[
d = r \sqrt{\frac{3}{2} (\Omega^2 + \varepsilon_F^2)}
\]

(4.28)

(b) Systems with a single-step stiffness variation

\[
k_1 = \frac{1}{3} K_y \left( 1 + \frac{2 \varepsilon_F r}{d} \right)
\]

(4.29)
4.2.4.3 Four-element model

The element properties in the four-element model cannot be determined uniquely by the system parameters. Two additional assumptions have to be made. In this chapter, only single-step stiffness and strength variation is considered for the four-element model. The two elements at the stiff side are considered to have the same stiffness and strength and the two elements at the flexible side are also considered to have the same stiffness and strength. Therefore, the stiffness and strength of the resisting elements of the four-element model are:

\[ f_i = f_2 = \frac{1}{4} F_y \left( 1 + \frac{e^*_r}{d} \right) \]  \hspace{1cm} (4.33)

\[ f_3 = f_4 = \frac{1}{4} F_y \left( 1 - \frac{e^*_r}{d} \right) \]  \hspace{1cm} (4.34)

\[ k_1 = k_2 = \frac{1}{4} K_y \left( 1 + \frac{e^*_r}{d} \right) \]  \hspace{1cm} (4.35)

\[ k_3 = k_4 = \frac{1}{4} K_y \left( 1 - \frac{e^*_r}{d} \right) \]  \hspace{1cm} (4.36)

\[ d = \frac{1}{4} r \left( \sqrt{24\Omega^2 + 25e^*_s^2 - e^*_s} \right) \]  \hspace{1cm} (4.32)
4.2.4.4 Building model with transverse resisting elements

For the building model with transverse resisting elements, two additional assumptions are needed to specify the distribution of element stiffness and strength. One assumption relates to the relative translational stiffness in the x- and the y-directions. The other is concerned with the contribution of the transverse elements to the system's total torsional stiffness, i.e., the locations of these elements. In this chapter, the uncoupled translational frequencies in the x- and y-directions, \( \omega_x \) and \( \omega_y \) respectively, are taken to be equal, as also assumed by Goel and Chopra (1991a and 1991b). As a result, the total translational stiffnesses \( K_x \) and \( K_y \) have the same value, and the total yielding strengths are also equal, \( F_x = F_y = V_0 \).

Assuming that the two transverse elements are located near the periphery of the building with a distance of \( h \) (Fig. 4.1(d)) between them, the contribution of these two elements to the structure's total torsional stiffness is \( \frac{1}{4} K_y h^2 \). The maximum value of \( h \) is \( a \) (see Fig. 4.1(d)) and the maximum contribution of the two transverse elements to the total torsional stiffness is then \( \frac{1}{4} K_y a^2 \). For a rectangular floor deck with an aspect ratio \( a/b = 1/3 \), its radius of gyration \( r \) is approximately 0.913a. Therefore, assuming that the two transverse elements are located near the periphery of the building leads to the assumption that \( h = r = 0.913a \) and that the contribution of the two transverse elements to the structure's total torsional stiffness is \( \frac{1}{4} K_y r^2 \). The stiffness and strength distributions among the three resisting elements parallel to the direction of earthquake motion are assumed to have a double-step variation. Thus,

\[
d = r \sqrt{\frac{4}{5} (\Omega^2 + e_{x}^{2})}
\]

(4.37)
\[ k_1 = K_y \left( \frac{1}{3} + \frac{1}{2} \frac{e^*_r}{d} \right) \]  (4.38)

\[ k_2 = \frac{1}{3} K_y \]  (4.39)

\[ k_3 = K_y \left( \frac{1}{3} - \frac{1}{2} \frac{e^*_r}{d} \right) \]  (4.40)

\[ k_4 = k_5 = \frac{1}{2} K_y \]  (4.41)

\[ f_1 = F_y \left( \frac{1}{3} + \frac{1}{2} \frac{e^*_r}{d} \right) \]  (4.42)

\[ f_2 = \frac{1}{3} F_y \]  (4.43)

\[ f_3 = F_y \left( \frac{1}{3} - \frac{1}{2} \frac{e^*_r}{d} \right) \]  (4.44)

\[ f_4 = f_5 = \frac{1}{2} F_y \]  (4.45)

\[ d = r \sqrt{\frac{3}{2} (\Omega^2 + e^*_r^2 - 0.25)} \]  (4.46)

\[ h = r \]  (4.47)

### 4.2.4.5 Mass-eccentric model

In mass eccentric structures, plan asymmetry is caused by an asymmetric distribution of mass as opposed to stiffness (Fig. 4.1(e)). The centre of rigidity CR is located at the geometric centre of the floor deck and the centre of mass CM has an offset \( e_s \) from CR. The centre of strength PC is situated with an offset \( e_p \) from CM. Similar to stiffness-eccentric systems, \( e_s \) and \( e_p \) are called the stiffness eccentricity and
the strength eccentricity, respectively. The strength distribution among the resisting elements is considered to be a double-step variation. Therefore, element stiffness and strength can be determined from the system parameters as:

\[
k_1 = k_2 = k_3 = \frac{1}{3} K_y
\]

\[
f_1 = F_y \left( \frac{1}{3} + \frac{1}{2} \left( \frac{e_s^* - e_p^*}{d} \right) \right)
\]

\[
f_2 = \frac{1}{3} F_2
\]

\[
f_3 = F_y \left( \frac{1}{3} - \frac{1}{2} \left( \frac{e_s^* - e_p^*}{d} \right) \right)
\]

\[
d = r \sqrt{\frac{3}{2} \Omega}
\]

4.3 Ground motion input

In order to study the inelastic response of asymmetric buildings to ground motion excitations of different characteristics and to reduce the dependence of the results on any selected earthquake record, three records recorded on stiff soil sites from western United States strong-motion earthquake events have been selected as ground motion inputs for the purposes of this chapter. Table 4.2 summarises the key characteristics of these records, and their time histories are plotted in Fig. 4.4.

These records represent three categories of earthquake ground motions:

(a) Ground accelerations exhibiting large amplitude, high frequency oscillations in the strong motion phase of the earthquake record with a high peak ground acceleration to peak ground velocity ratio \((a/v > 1.2g/(m/s))\) and very high spectral acceleration values for structures having very short fundamental periods \((T_y \leq 0.25 \text{ sec.})\) but very
Table 4.2 Key Characteristics of Selected Earthquake Records

<table>
<thead>
<tr>
<th>Site</th>
<th>Earthquake event</th>
<th>Component direction</th>
<th>Strong motion duration (sec.)</th>
<th>Max. acc. a (g)</th>
<th>Max. vel. v (m/s)</th>
<th>a/v ratio (g/m/s)</th>
<th>Site soil type</th>
</tr>
</thead>
<tbody>
<tr>
<td>3470 Wilshire Blvd.</td>
<td>San Fernando</td>
<td>N00E</td>
<td>30</td>
<td>0.136</td>
<td>0.223</td>
<td>0.61</td>
<td>Stiff soil</td>
</tr>
<tr>
<td>El Centro</td>
<td>Imperial Valley</td>
<td>S00E</td>
<td>30</td>
<td>0.312</td>
<td>0.325</td>
<td>0.96</td>
<td>Stiff soil</td>
</tr>
<tr>
<td>Cholame Shandon No. 5</td>
<td>Parkfield</td>
<td>N85E</td>
<td>44</td>
<td>0.467</td>
<td>0.26</td>
<td>1.82</td>
<td>Stiff soil</td>
</tr>
</tbody>
</table>

low spectral acceleration values for short periods (0.25 sec. \( \leq T_s \leq 0.50\) sec.) and for medium and long periods. The N85E component of Cholame Shandon No. 5, Parkfield record is a typical example of this category.

(b) Ground motions exhibiting an intermediate \( a/v \) ratio (\( 0.8<(g/(m/s)) \leq a/v \leq 1.2g/(m/s) \)) and acceleration spectra similar to the standard Newmark-Hall type design spectrum; the S00E component of the El Centro, Imperial Valley record being an example of this category.

(c) Ground motions containing a few severe, long duration acceleration pulses having a very low \( a/v \) ratio (\( a/v<0.8g/(m/s) \)) and very high spectral acceleration values
for medium (0.5 sec. ≤ T_y ≤ 1.0 sec.) and long (T_y > 1.0 sec.) periods. The N00E component of the 3470 Wilshire Boulevard, San Fernando record is a typical example of this category.

In this chapter, the above records have been scaled to the same peak ground acceleration (0.3g). The 5 per cent damped elastic acceleration response spectra of the selected records, normalised to this common peak ground acceleration, together with the 5 per cent damped elastic and inelastic design spectra suggested by Newmark and Hall (1982) and associated with the force reduction factor R=4, have been plotted in Fig. 4.5.

### 4.4 Equations of motion

Assuming the earthquake acceleration input is uni-directional and parallel to the y-axis, then only two degrees of freedom are concerned, namely the lateral displacement of CR in the y-direction, v, and the rotation of the floor slab about the vertical axis through CR, θ. For analysis of the inelastic nonlinear response, the equations of motion can be written in incremental form:

\[
[M] \begin{bmatrix} \Delta \ddot{v}(t) \\ r \Delta \ddot{\theta}(t) \end{bmatrix} + [C(t)] \begin{bmatrix} \Delta \dot{v}(t) \\ r \Delta \dot{\theta}(t) \end{bmatrix} + [K(t)] \begin{bmatrix} \Delta v(t) \\ r \Delta \theta(t) \end{bmatrix} = \begin{bmatrix} \Delta \ddot{v}_g(t) \\ r \Delta \ddot{\theta}_g(t) \end{bmatrix}
\]

in which, [M], [C(t)] and [K(t)] are the mass matrix, and the instantaneous damping and stiffness matrices, respectively.

The mass matrix is defined as:

\[
[M] = \begin{bmatrix} 1 & e_s^* \\ e_s^* & 1 + e_s^{**} \end{bmatrix}
\]

and the stiffness matrix is given by:
The Rayleigh type damping matrix, proportional to both the mass and the instantaneous stiffness matrices, is considered in this study:

\[
[C(t)] = c_1 [M] + c_2 [K(t)]
\]

where \(c_1\) and \(c_2\) are constants determined by the two coupled natural vibration periods \(T_1\) and \(T_2\) of the system and the damping ratios \(\xi_1\) and \(\xi_2\) associated with its two vibration modes:

\[
c_1 = \frac{4\pi}{T_1 + T_2} \xi_1 \quad (4.57)
\]

\[
c_2 = \frac{T_1 T_2}{\pi(T_1 + T_2)} \xi_2 \quad (4.58)
\]

In this study, the damping for each of the two modes is considered to be 5 per cent of critical damping. Thus, \(\xi_1 = \xi_2 = 0.05\).

The governing equations of motion, eqn. (4.53), are solved using a step-by-step numerical integration method. The time interval of the numerical integration is taken to be 1/25 of the shorter of the two initial natural periods, or the time increment of the digitization of an earthquake record if smaller. The numerical calculation is carried out using a computer program written for inelastic earthquake response analysis of three-dimensional buildings — DRAIN-TABS (Guendelman-Israel and Powell 1977).
The inelastic response time history of individual resisting elements is obtained from this program and subsequently used to determine the inelastic seismic response parameters, as defined below.

4.5 Inelastic seismic response parameters

As summarised in chapter 2, existing studies have employed different response parameters to characterise the inelastic response of asymmetric buildings to strong earthquake motions. The maximum global lateral and torsional displacements at and about the centre of rigidity, the maximum displacement ductility demand of resisting elements, the maximum flexible edge displacement and the maximum deformation of resisting elements have been employed by various researchers. Unlike elastic response analysis which is concerned mainly with the elastic deformation and hence the elastic strength demand of resisting elements, inelastic seismic response analysis pays more attention to the assessment of structural damage of resisting elements than to the global structural displacement or deformation of resisting elements. Therefore, response parameters related to the assessment of structural damage are employed in this chapter to characterise the inelastic response of asymmetric buildings.

The maximum displacement ductility demand of the resisting elements, defined as the ratio of the maximum displacement to the yield displacement, has been widely employed as a measure of the damage sustained by structural elements, and is a parameter traditionally used to characterise the inelastic response in earthquake resistant design. It is also both simple in concept and easy to calculate.

However, in view of the cyclic nature of dynamic response, structural damage associated with inelastic seismic response is dependent upon many factors, of which the maximum displacement ductility demand is but one index. Others include the size and
number of hysteretic loops formed during the inelastic seismic response, the number of yielding excursions, and the total hysteretic energy which has been dissipated in the resisting elements.

There are two patterns of structural damage associated with the cyclic behaviour of the inelastic seismic response. One is characterised by excessively large inelastic deformation far beyond the yielding deformation, associated with a few large cycles of load reversal. The other corresponds to a large number of cycles of load reversal having deformation magnitudes slightly larger than the yielding deformation. The former case is typical of damage caused by large inelastic deformation and is described by the maximum displacement ductility. The latter implies low-cycle fatigue type damage and is quantified by the normalised hysteretic energy ductility as proposed by Mahin and Bertero (1981).

The normalised hysteretic energy ductility is defined as the total hysteretic energy dissipated in a lateral load resisting element, normalised to twice the maximum elastic energy absorbed when the loaded element just reaches the threshold of yielding, as shown in Fig. 4.6(a). Hence:

\[ \mu_E = \frac{E_H}{R_y U_y} \]  \hspace{1cm} (4.59)

in which \( \mu_E \) is the normalised hysteretic energy ductility, \( R_y \) is the yield resistance (force or moment) and \( U_y \) is the yield deformation.

This normalised hysteretic energy ductility can also be interpreted as dividing the total hysteretic energy dissipated in a resisting element by the amount of energy in a standard hysteretic cycle to obtain the number of equivalent cycles required to dissipate the same amount of energy. The standard hysteretic cycle is shown in Fig. 4.6(b).
Both the maximum displacement ductility and the normalised hysteretic energy ductility are employed in this chapter to characterise the inelastic seismic response of resisting elements in asymmetric buildings. These two parameters complement each other in estimating the structural damage sustained by the resisting elements. A study carried out by Loh and Ho (1990) to investigate methods of assessment of seismic damage based on different hysteretic rules suggests that these two parameters represent important, virtually independent indices for predicting structural damage.

Observing the response patterns of short-period and long-period structures, it can be concluded that in both cases the maximum displacement ductility is a critical indicator of damage. The normalised hysteretic energy ductility however is less important for long-period structures which generally undergo only a small number of yield cycles, and hence the maximum displacement ductility governs the structural damage and design of these buildings. However, the normalised hysteretic energy ductility in conjunction with the maximum displacement ductility are important parameters for the prediction of damage of short-period buildings, since such buildings usually experience a large number of yield cycles.

4.6 Effect of the number of resisting elements

This section examines the effect of the number of resisting elements oriented parallel to the direction of ground motion on the inelastic response of the resisting elements. The maximum displacement ductility demand and the maximum normalised hysteretic energy ductility demand of resisting elements are compared for single-storey building models having two, three and four resisting elements in the direction of earthquake motion. These building models have identical system parameters $T_y$, $\Omega$, $e_s^*$, $e_p^*$, $R$, and $\xi$, and are subjected to the same earthquake ground motions. The three-element model is assumed to have a double-step stiffness variation.
The maximum displacement ductility demand and the maximum normalised hysteretic energy ductility demand occur in the stiff edge element, element 1, in two-element and three-element models and in element 2 of the four-element model (see Fig. 4.1). These response parameters are presented in Figs. 4.7 to 4.10 for two earthquake records, namely El Centro S00E and 3470 Wilshire Blvd. N00E. The Cholame Shandon No. 5 N85E record has a high a/v ratio, and when scaled according to its peak ground acceleration has very low spectral acceleration values in the short-period, medium-period and long-period regions as shown in Fig. 4.5. Comparing the elastic response spectrum of this record and the inelastic design spectrum corresponding to R=4 in Fig. 4.5, it is apparent that buildings designed with a force reduction factor R=4, except for those in the very short-period region, will be excited only slightly into the inelastic range by this record. As will be shown in Sections 4.8.3 and 4.8.4, except at very short periods, plan asymmetry has little effect on the inelastic response of asymmetric buildings subjected to records having a high a/v ratio. Therefore, results corresponding to the Cholame Shandon No. 5 N85E record are not presented in this and some later sections.

The results have been plotted as functions of the uncoupled lateral period $T_y$ in the form of response spectra over the period range 0.1 sec. to 2.0 sec., which covers the fundamental period of buildings having up to approximately 20 storeys. Two types of systems are considered, namely initially eccentric systems ($e^*_x \neq 0, e^*_y = 0$, representing code designed asymmetric buildings) and initially symmetric systems ($e^*_x = 0, e^*_y \neq 0$, representing buildings with symmetric distribution of stiffness but unbalanced strength). These systems are used to examine separately the influence of the stiffness and strength eccentricity ratios.

It is apparent that the inelastic response of asymmetric buildings is dependent on the number of resisting elements oriented in the direction of earthquake motion. The
two-element model underestimates both the maximum displacement ductility demand and the normalised hysteretic energy ductility demand in both initially eccentric and initially symmetric systems, compared with those of three- and four-element models. The two-element model underestimates the normalised hysteretic energy ductility demand more significantly than the maximum displacement ductility demand.

It can also be seen that there is very little difference between results obtained from three- and four-element models. Employing more than three resisting elements in the analytical model not only complicates the system definition, requiring additional assumptions regarding the distribution of stiffness and strength, but also leads to results which are dependent on the form of such assumptions. This defect makes it difficult to generalise conclusions based on models having a particular assumption regarding the distribution of stiffness and strength. Figs. 4.7 to 4.10 also indicate that very little benefit can be achieved by employing more than three elements in the analytical model. The three-element model can provide conservative and satisfactory predictions of structural response and possible damage of the resisting elements. Moreover, the three-element model is simple in definition, needing only one additional assumption to define the model uniquely from the system parameters. Therefore it is concluded that, as far as the number of resisting elements is concerned, the three-element model adequately satisfies the criteria proposed in Section 2.3.1.

4.7 Effect of resisting elements transverse to ground motion

Most buildings have load resisting elements aligned along two perpendicular directions, termed here lateral and transverse, in order to resist the two horizontal components of the ground motion. At present, in design practice, the seismic forces in the two orthogonal directions are assumed to act separately, parallel to each of these two
horizontal directions. Therefore, most existing studies have assumed that the earthquake motion is uni-directional and the transverse component of the earthquake motion has therefore not been considered.

If the earthquake input is considered to be uni-directional, the resisting elements oriented transverse to the direction of the earthquake motion do not contribute significantly to the lateral stiffness of the building, since the stiffness of these elements (frames and/or shear walls) orthogonal to their acting planes can be neglected. However, these elements contribute significantly to the building's torsional stiffness if they remain elastic during the response. Goel and Chopra (1991a) included these transverse elements in their single-storey building model and found that when the uni-directional earthquake motion excites the lateral elements into the inelastic range (which results in the structure's lateral stiffness becoming zero) the transverse elements remain elastic most of the time during the response and hence contribute significantly to the structure's torsional stiffness. Therefore, the structure becomes torsionally rigid and responds more and more in translation like a single-degree-of-freedom system with decreasing torsional response. Goel and Chopra (1990) also concluded that systems in the short-period, acceleration sensitive region are affected significantly by the contribution to the system's torsional stiffness provided by the transverse elements, but that these elements have little influence on systems in the medium period, velocity sensitive and in the long period, displacement sensitive regions. This was attributed to the fact that structures in the short-period region undergo much yielding whereas those in the medium-period and long-period regions experience less yielding.

As discussed in Section 2.3.1, including transverse elements and assuming that the earthquake input is uni-directional as in Goel and Chopra (1990, 1991a and 1991b) is inappropriate, because in reality the transverse component of the earthquake motion also excites these resisting elements into the inelastic range and hence the torsional
stiffness contributed from these elements can be ignored. Therefore, when the ground motion is assumed to be uni-directional, the transverse resisting elements should not be taken into account.

Nevertheless, it is still enlightening to study the influence of the transverse elements on the structure’s maximum displacement ductility demand and the maximum normalised hysteretic energy ductility demand of asymmetric buildings, when the earthquake motion is assumed to be uni-directional. Figures 4.11 and 4.12 illustrate the inelastic response of the three-element model (Fig. 4.1(b)) with a double-step stiffness variation and that of the model having transverse elements (Fig. 4.1(d)). All system parameters, except the existence or absence of the transverse elements, are identical. The results shown in Figs. 4.11 and 4.12 are for initially eccentric systems \( (e_r^* \neq 0, e_p^* = 0) \) which represent code-designed buildings. In these systems, the maximum inelastic response parameters both occur in the stiff edge element (element 1).

It is apparent from Figs. 4.11 and 4.12 that the transverse elements have very little influence on both the maximum displacement ductility demand and the maximum normalised hysteretic energy ductility demand of asymmetric structures. The difference between results based on the three-element model without transverse elements and those based on the model with transverse elements is very small over the full period range considered.

This finding contradicts that of Goel and Chopra (1990), summarised above, in relation to stiff, short-period systems. This difference can be attributed to the inadequate approach employed by Goel and Chopra (1990) in specifying the total base shear for structures in the short-period, acceleration sensitive region as discussed in Section 4.2.3. Rather than using a flat design spectrum, as commonly employed in design practice, Goel and Chopra (1990) specified a fraction, 0.25, of the elastic strength demand as the
base shear of asymmetric buildings. This approach leads to declining strength capacity with decreasing period in the short-period, acceleration sensitive region and therefore gives much higher displacement ductility demand in the short-period region. In their study (Goel and Chopra 1990), the displacement ductility demand of short-period, acceleration sensitive structures is in the neighbourhood of 700 which is an unrealistic value even for buildings constructed using modern materials and designed in accordance with current earthquake resistant design building codes. This very high displacement ductility means that short-period structures undergo excessive yielding, and hence the contribution from the transverse elements to the system’s torsional stiffness is highly significant. Therefore, the influence of transverse elements has been exaggerated in the short-period, acceleration sensitive region in the study carried out by Goel and Chopra (1990).

Figures 4.13 and 4.14 show the influence of the transverse elements on the inelastic response of initially symmetric structures. In these systems, the maximum inelastic response parameters occur in the flexible edge element (element 3). It can be seen that the maximum displacement ductility demand of the two models is very similar over the full period range, but the maximum normalised hysteretic energy ductility demand of the model with transverse elements is significantly higher than that of the model without transverse elements in the long-period, displacement-sensitive region. However, since the design and structural damage of long-period, displacement sensitive buildings are governed mainly by the maximum displacement ductility demand, the three-element model without transverse elements still provides satisfactory estimates for structural damage and strength demand of the lateral elements.

Based on this investigation, it can be concluded that the influence of the transverse elements is not significant in estimating structural damage of asymmetric buildings. The three-element model without transverse elements can provide satisfactory estimates of
the inelastic seismic response in terms of the maximum displacement ductility demand and the maximum normalised hysteretic energy ductility demand of resisting elements in asymmetric buildings. Including transverse elements in the analytical model gives very little advantage in estimating the inelastic seismic response of the lateral elements. Furthermore, the analytical model with transverse elements cannot be defined uniquely by the system parameters. Two more assumptions regarding their translational stiffness and their contribution to the structure's total torsional stiffness, defined by the location of these elements, are needed. Such additional assumptions hinder the drawing of generalised conclusions of relevance to design situations.

4.8 Effect of mass, stiffness, and strength distributions

4.8.1 Stiffness-eccentric and mass-eccentric systems

This section investigates the inelastic response of stiffness-eccentric and mass-eccentric systems. The objectives are firstly to obtain a better understanding of the inelastic seismic behaviour of these two asymmetric systems and then to provide insight into the reasons why previous researchers reached contradictory conclusions based on stiffness-eccentric and mass-eccentric models.

Figures 4.15 to 4.18 compare the inelastic responses of stiffness-eccentric and mass-eccentric structures having the same system parameters and subjected to the same earthquake ground motions. In Figs. 4.15 and 4.16, the centre of strength PC coincides with the centre of mass CM, but both PC and CM have an offset of $e_s$ from the centre of rigidity CR, i.e., $e_s \neq 0, e_r = 0$. In both systems, the maximum displacement ductility and the maximum normalised hysteretic energy ductility occur in the stiff edge element (element 1). However, the values of both inelastic response parameters for the stiff edge element of the mass-eccentric system are substantially higher than those of the
stiffness-eccentric system. The values of inelastic response parameters for the flexible edge element (element 3) are very similar for both systems. This observation explains why Chandler and Duan (1991a) have found that the new torsional provision in the Mexico 87 code imposing a minimum strength eccentricity is overly conservative and hence unnecessary. The torsional provisions of this code require that the centre of strength be close enough to the centre of rigidity, in order to increase the strength capacity of the elements at the stiff side and hence reduce the inelastic response of these elements. It is based on the study of Gomez et al. (1987) who have employed a single-storey mass-eccentric model with three elements parallel to the direction of the earthquake motion, as shown in Fig. 4.1(e). Therefore, the inelastic response of the stiff edge element in the study of Gomez et al. (1987) is substantially higher than that in the study of Chandler and Duan (1991a), the latter study employing a stiffness-eccentric model as shown in Fig. 4.1(b).

Figures 4.17 and 4.18 show the inelastic response of stiffness-eccentric and mass-eccentric systems having the same system parameters and equal stiffness and strength eccentricities, $e^* = e^{**} \neq 0$. The inelastic response of systems having equal stiffness and strength eccentricities has been investigated intensively in earlier studies, but contradictory conclusions have been drawn, as summarised in Section 2.2.3. It can be seen that when CR and PC coincide with each other, mass-eccentric and stiffness-eccentric systems again behave differently. The maximum displacement ductility and the maximum normalised hysteretic energy ductility occur in the flexible edge element in both systems. Although the values of the inelastic response parameters for the stiff edge element are almost identical in the two systems, the response parameters of the flexible edge element of the mass-eccentric system are significantly lower than those of the corresponding stiffness-eccentric system. This finding explains why Tso and Sadek (1985), Bozorgnia and Tso (1986), and Syamal and Pekau (1985) have concluded that eccentricity critically influences the maximum displacement
ductility of the flexible edge element based on a stiffness-eccentric model, whilst Irvine
and Kountouris (1980) have found that the maximum displacement ductility of the
flexible element is not sensitive to eccentricity based on a mass-eccentric model with
two identical resisting elements.

In most buildings, the distribution of mass on the floor slabs is more or less
uniform, leading to a symmetric distribution of mass. However, the distribution of
stiffness can be substantially unbalanced resulting in significant asymmetry because of
architectural and functional reasons. For instance, many buildings have large openings
for entrances at the sides facing the streets, and at the rear sides the walls are continuous
without large openings, and many others have asymmetric distribution of shear walls
and frames. Therefore, plan asymmetry arises in most buildings from an asymmetric
distribution of stiffness rather than mass. As a result, the stiffness-eccentric system is a
more realistic model representing the majority of actual buildings, and hence should be
employed in the study of inelastic response of asymmetric buildings.

4.8.2 Double-step and single-step variations of stiffness distribution

The three-element model with a single-step stiffness variation has been employed
extensively by Tso and Sadek (1985), Bozorgnia and Tso (1986), Tso and Bozorgnia
(1986), and Tso and Ying (1990). The three-element model with a double-step stiffness
variation was first proposed by Chandler and Duan (1990) and subsequently employed
by Duan and Chandler (1990) and Chandler and Duan (1991a). Compared with the
system with a single-step stiffness variation, the system with a double-step stiffness
variation requires elements 1 and 3 to be located further away from CM in order to
provide the same torsional stiffness, because in the latter case a larger portion of the
total lateral stiffness is located at the centre of the structure than in the former case. For
a given stiffness eccentricity ratio $\varepsilon^*$, the difference between the lateral stiffness of the
stiff edge element (element 1) and that of the flexible edge element (element 3) is larger for the single-step stiffness variation than for the double-step stiffness variation. This difference is expected to lead to variations in the inelastic response of the two types of three-element model.

This section studies this issue in order to provide the basis for the generalisation of conclusions based on one particular assumption. The two systems have the same system parameters, $T_y$, $\Omega$, $e_s^*$, $e_p^* = 0$, $R$, and $\xi$, and are subjected to the same earthquake excitations. Figures 4.19 and 4.20 show the displacement ductility demand and the normalised hysteretic energy ductility demand of the two systems in the form of response spectra plotted over the period range up to 2.0 seconds. In both cases, the maximum displacement ductility demand and the maximum normalised hysteretic energy ductility demand occur in the stiff edge element (element 1). It can be seen that the maximum displacement ductility demand of the two systems are similar. The maximum normalised hysteretic energy ductility of the single-step system is, in general, slightly higher than that of the double-step system. However, the difference is small and the double-step system can still provide satisfactory estimates of the maximum hysteretic energy dissipation demand of asymmetric buildings.

The above observations demonstrate that the assumption dependency of the inelastic response of the three-element model is not significant and that the double-step stiffness variation and the single-step stiffness variation can be used interchangeably in examining the inelastic response of resisting elements in asymmetric buildings.

4.8.3 Effect of stiffness eccentricity

In previous sections, the model dependency of the inelastic earthquake response of asymmetric buildings has been thoroughly investigated, based on various different
structural models. It is apparent from the results that the three-element, stiffness-eccentric model with a double-step stiffness variation provides satisfactory and conservative estimates of the inelastic response of resisting elements, represents a wide range of actual buildings, is simply defined and facilitates the straightforward interpretation and generalisation of results. Hence, in this and the following sections, such a model will be employed to study the influence of the stiffness eccentricity, the strength eccentricity, the uncoupled torsional to translational frequency ratio, the uncoupled lateral period, the characteristics of the earthquake motion, and the force reduction factor on the inelastic response of asymmetric buildings.

Strength-symmetric systems (e_p*=0) are employed to investigate the influence of stiffness eccentricity and remove the effect of the strength eccentricity on the inelastic torsionally coupled response of asymmetric buildings. The displacement ductility demand and the normalised hysteretic energy ductility demand of the resisting elements (elements 1 and 3) are presented in Figures 4.21 and 4.22 in the form of response spectra in the period range 0.1 to 2.0 seconds, for two values of stiffness eccentricity ratio e_t*=0.3 and 0.6, and for fixed values of e_p*=0, Ω = 1.0, R=4, and ξ = 5%. These results are presented for three earthquake records, namely 3470 Wilshire Blvd. N00E, El Centro S00E, and Cholame Shandon No. 5 N85E. The inelastic response of the corresponding symmetric system, which has the same T_r, R and ξ, is also presented for the purpose of comparison.

Figures 4.21 and 4.22 clearly indicate that the inelastic response of code designed asymmetric buildings, in which the distribution of strength is approximately symmetric (e_p*=0), is fundamentally different from the elastic response based on linear theory, when such buildings are excited well into the inelastic range. In elastic studies, the element at the flexible edge (element 3) is the critical element which is the most unfavourably affected by torsional coupling. This implies that its deformation and
elastic strength demand increase most unfavourably compared with that of the corresponding torsionally uncoupled system. On the other hand, the element at the stiff edge is the one most favourably affected. Torsional coupling causes its deformation and elastic strength demand to increase marginally or even to decrease, compared with the corresponding torsionally uncoupled system. As a result, current code torsional provisions specify an increased strength for the elements at the flexible side and allow designers to make use of the possible beneficial effect of torsional coupling on the elements at the stiff side by reducing the strength capacity of these elements.

In all early studies, as discussed in Section 2.2.4, the researchers did not employ the strength eccentricity as an independent system parameter or specify the element yielding strength in accordance with code torsional provisions. Instead, they assumed that all elements have the same yielding deformation, the yielding strength of an element being proportional to its elastic stiffness. Therefore, the flexible edge element has the lowest yielding strength and the highest displacement ductility demand because of the unfavourable effect of torsional coupling on this element. On the other hand, the stiff edge element has the highest yielding strength and the lowest displacement ductility demand, lower than that of the corresponding symmetric structure. The inelastic behaviour of this category of asymmetric structures having equal stiffness and strength eccentricities is similar to the elastic behaviour of asymmetric structures. Consequently, Tso and Bozorgnia (1986) have concluded that the effective design eccentricity based on linear elastic earthquake response analysis of asymmetric buildings is generally applicable to this category of asymmetric buildings when they are excited well into the inelastic range.

However, as stated earlier, asymmetric buildings which are designed according to code torsional provisions, as is common in current design practice, are characterised by near-zero strength eccentricity. In this case, the stiff edge element is the critical element
suffering the most severe structural damage under strong earthquake ground motions. Both the displacement ductility demand and the normalised hysteretic energy ductility demand of the stiff edge element are substantially higher, and those of the flexible edge element are significantly lower, than those of the corresponding symmetric structures.

To pursue this issue further, the amount of hysteretic energy dissipated by individual resisting elements is presented in Figure 4.23 as percentages of the total hysteretic energy dissipated in the structure. If the distribution of stiffness and strength in the structure is symmetric and uniform, each element dissipates the same amount of hysteretic energy. Figure 4.23 indicates that in code designed asymmetric buildings, hysteretic energy dissipation is not distributed evenly among the resisting elements. The element at the stiff edge dissipates a substantially larger amount of hysteretic energy than the elements at the centre and at the flexible edge. Consequently, when code designed asymmetric buildings are excited well into the inelastic range, the structural damage and the hysteretic energy dissipation tend to be concentrated in the elements at the stiff side.

The stiffness eccentricity ratio $e^* = e/r$, the uncoupled lateral period $T_y$, and the spectral characteristics of the earthquake motion (determined partly by the a/v ratio) are important factors influencing the inelastic response of asymmetric buildings. The effect of stiffness eccentricity on the inelastic response of the resisting elements increases significantly with increasing stiffness eccentricity ratio. The effect is most pronounced in a wide period range covering both the medium-period (0.5 sec. $< T_y \leq 1.0$ sec.) and the long-period ($T_y > 1.0$ sec.) regions, for earthquake records having a low a/v ratio (such as 3470 Wilshire Blvd. N00E). For records having an intermediate a/v ratio (El Centro S00E), the effect is most significant in the very short-period (0 $< T_y \leq 0.25$ sec.) and short-period (0.25 sec. $< T_y \leq 0.5$ sec.) regions, and only in the very short-period region for ground motions having a high a/v ratio (Cholame Shandon No. 5 N85E).
It can be seen from Fig. 4.5 that if earthquake records are normalised to a common peak horizontal acceleration, those having a high a/v ratio have very high spectral acceleration values in the very short-period region but relatively low spectral acceleration values in all other period regions, approaching for longer periods the inelastic design spectrum associated with a force reduction factor of 4.0. Thus, only those buildings whose fundamental lateral period is in the very short-period region will be excited well into the inelastic range. Those whose fundamental period is not in the very short-period region will be excited to around the threshold of yielding. Therefore, the influence of stiffness eccentricity on the inelastic response of asymmetric buildings in the short-, medium- and long-period regions is not significant, as shown in Figures 4.21(c) and 4.22(c) for records having a high a/v ratio. Because most buildings are multistorey and have their fundamental periods in the short-, medium-, or long-period regions, if earthquake records are normalised according to their peak ground horizontal acceleration, it is not necessary to employ records having a high a/v ratio in studying the inelastic response of asymmetric buildings excited well into the inelastic range by strong earthquake motions.

When responding elastically and inelastically to earthquake motions, code designed buildings exhibit different behaviour. This observation can be explained by considering the shifting, or the instantaneous location, of the centre of rigidity caused by yielding of the resisting elements. Under strong earthquake loading, the resisting elements are frequently loaded into the inelastic range and unloaded back to the elastic range, resulting in frequent abrupt changes of element lateral stiffness. Because of torsional coupling in asymmetric buildings, the process of yielding into the inelastic range, unloading back into the elastic range, and re-yielding into the inelastic range does not occur simultaneously for the two edge elements (elements 1 and 3). Therefore, unlike elastic structures in which the location of CR remains unchanged, the centre of rigidity of an inelastic asymmetric building shifts abruptly from one side of CM to the
other. This behaviour results in a cancellation or reduction of torsional effects. Consequently, yielding leads to reduced torsional coupling effects in inelastic structures compared with elastic structures and hence inelastic asymmetric structures behave more like a single-degree-of-freedom system than elastic asymmetric structures. That is why in elastic studies, the element at the flexible edge is the critical element, whilst in the study of the inelastic response of code designed asymmetric buildings the element at the stiff edge is the critical element for design against structural damage.

The findings that code designed asymmetric buildings experience less torsional coupling and that the element at the stiff edge is the critical element suffering the most severe structural damage provide information to develop guidelines for improving code torsional provisions. In early studies of the inelastic earthquake response of asymmetric buildings, researchers made the inadequate assumption that the centre of strength PC coincides with the centre of rigidity CR, which leads to similar behaviour between elastic and inelastic asymmetric buildings. Because of these historical reasons, research on torsional coupling in asymmetric buildings has been focussed mainly on estimating the deformation, elastic strength demand and displacement ductility demand of the element at the flexible edge. Very little attention has been paid to the element at the stiff edge. Only two studies, carried out by Rutenberg and Pekau (1987) and Chandler and Hutchinson (1988a), have addressed in detail the deformation and the elastic strength demand of the element at the stiff edge. Both studies have identified that current code torsional provisions underestimate the elastic strength demand of the stiff edge element for torsionally flexible asymmetric buildings ($\Omega \leq 1.0$), but provide satisfactory results for torsionally stiff asymmetric buildings ($\Omega > 1.0$). Each of these studies proposed simplified design formulae for estimating the design loading of the stiff edge element. Results of the inelastic study carried out in this section indicate that the strength capacity of the element at the stiff edge should be considerably increased, in order to reduce the concentration of structural damage in the stiff edge element and to provide
consistent protection to the resisting elements at the stiff side and the flexible side. Such an approach should also aim to give consistent control over structural damage in both symmetric and eccentric structures.

### 4.8.4 Effect of strength eccentricity

In the static failure mode analysis in Chapter 3, it was found that the strength eccentricity greatly affects the lateral load carrying capacity of initially symmetric structures and the displacement ductility of the weak element (element 3). This section investigates the influence of the strength eccentricity on the inelastic earthquake response of initially symmetric buildings \((e_p^*=0)\) with an asymmetric distribution of strength. Figures 4.24 and 4.25 illustrate the displacement ductility demand and the normalised hysteretic energy ductility demand of the strong element (element 1) and weak element (element 3), respectively, for two values of the strength eccentricity ratio, \(e_p^*=0.15\) and 0.3, and fixed values of \(e_s^*=0\), \(\Omega=1\), \(R=4\), and \(\xi=5\%\). The same three earthquake records as in previous results are used as the ground motion input and the inelastic response of the corresponding symmetric structures having the same values of the system parameters \(T_y\), \(\xi\) and \(R\) is also presented for comparison.

It is apparent that the strength eccentricity critically affects the inelastic response of initially symmetric buildings. The displacement ductility demand and the normalised hysteretic energy ductility demand of the weak element increase very significantly with the increase of the strength eccentricity. The effect of the strength eccentricity is most pronounced in the very short-period region for earthquake motions having a high a/v ratio (such as Cholame Shandon No. 5 N85E), and in the very short-period and short-period regions for motions having an intermediate a/v ratio (El Centro S00E). However, for motions having a low a/v ratio (3470 Wilshire Blvd. N00E), the effect is significant over the full period range. Again, earthquake motions having a high a/v ratio
do not significantly affect the torsionally coupled inelastic response of buildings in the short-, medium- and long-period regions. Because most buildings have their fundamental period in these ranges, if earthquake records are normalised in accordance with their peak ground acceleration, it is sufficient to employ only those having intermediate and low a/v ratios as the ground motion input in studying the inelastic earthquake response of asymmetric buildings.

The findings in this section reinforce the conclusions drawn in Chapter 3, based on inelastic static monotonic failure mode analysis. Because the inelastic response of the weak element in initially symmetric structures increases rapidly compared with that of symmetric structures \((e_r^* = e_p^* = 0)\) even when the strength eccentricity ratio is small, caution should be taken when designing stiffness symmetric hybrid structures. In designing hybrid structures in which mixed structural systems (frames, shear walls and cores) are used to resist the lateral load, designers can in many cases achieve a balanced distribution of stiffness to satisfy code requirements of symmetry. However, the distribution of strength may be left substantially unbalanced because of the difficulties in achieving a symmetric strength distribution when different structural systems are employed. At present, building codes do not require designers to check the strength eccentricity in stiffness symmetric buildings. However, in view of the findings of this investigation, it is strongly recommended that designers check the position of the centre of strength in stiffness symmetric buildings and locate it as close as possible to the centre of rigidity by appropriately adjusting the distribution of strength.
4.9 Effect of the uncoupled torsional to translational frequency ratio

The uncoupled torsional to translational frequency ratio characterises a structure’s torsional stiffness relative to its translational stiffness. This system parameter may therefore be considered as an index of a building’s structural configuration. Buildings having central shear cores for lifts, as shown in Fig. 4.26(a), tend to have values of $\Omega$ less than unity and therefore are classified as torsionally flexible buildings. Those having a more or less uniform translational stiffness distribution, as shown in Fig. 4.26(b), tend to have values of $\Omega$ nearly equal to unity and are classified as buildings having an intermediate torsional stiffness. Finally, buildings in which shear walls are located near the periphery, for example 'box' structures as shown in Fig. 4.26(c), have values of $\Omega$ significantly larger than unity and are therefore termed torsionally stiff buildings.

In elastic studies, $\Omega$ has a critical effect on the torsionally coupled response of asymmetric buildings. In particular, buildings having a small stiffness eccentricity and $\Omega$ close to unity experience greatly amplified torsional response. However, in inelastic studies, different researchers have reached contradictory conclusions about the effect of $\Omega$ on the inelastic response of such buildings, as discussed in Section 2.2.2. The static monotonic failure mode analysis carried out in Chapter 3 of this thesis indicated that $\Omega$ only critically influences the displacement ductility demand of resisting elements in torsionally flexible asymmetric buildings. In torsionally stiff buildings and those with an intermediate torsional stiffness, the displacement ductility demand of resisting elements is not sensitive to $\Omega$ but decreases systematically with increasing values of $\Omega$. The validity of these conclusions drawn in Chapter 3 should be examined in the light of results from inelastic dynamic earthquake response analysis of asymmetric buildings.
The maximum displacement ductility demand and the maximum normalised hysteretic energy ductility demand are presented in Figures 4.27 to 4.30 in the form of response spectra for three values of $\Omega$, namely $\Omega=0.7$, 1.0 and 1.3, representing torsionally flexible buildings, buildings with an intermediate torsional stiffness and torsionally stiff buildings respectively, and for fixed values of $R$ and $\xi$. The inelastic response of the corresponding symmetric structures having the same values of system parameters $T_y$, $R$ and $\xi$ is also presented for comparison.

Figures 4.27 and 4.28 illustrate the inelastic response of initially asymmetric buildings, $\epsilon_*=0.3$, but with a symmetric strength distribution, $\epsilon_p^*=0$, representing code designed buildings. The maximum displacement ductility demand and the maximum normalised hysteretic energy ductility demand occur in the stiff edge element (element 1) for all three values of $\Omega$. In general, both response parameters fall with increasing values of $\Omega$. The values of the inelastic response parameters for $\Omega = 0.7$ are much higher than those corresponding to $\Omega = 1.0$ and 1.3. There is very little difference between the maximum displacement ductility demand of structures with $\Omega=1.0$ and 1.3, but the difference between the maximum hysteretic energy ductility demand is more significant.

Figures 4.29 and 4.30 show the inelastic response of initially symmetric buildings, $\epsilon_*=0$, but with an asymmetric distribution of strength, $\epsilon_p^*=0.3$. In this case, the maximum displacement ductility demand and the maximum normalised hysteretic energy ductility demand occur in the weak element, element 3. It is apparent that increasing the value of $\Omega$ generally has the effect of decreasing the inelastic response of the weak element. Again, the values of the inelastic response parameters for $\Omega = 0.7$ are much higher than those corresponding to $\Omega = 1.0$ and 1.3. Both the maximum displacement ductility demand and the maximum normalised hysteretic energy ductility demand are essentially the same for systems having $\Omega = 1.0$ and those having $\Omega = 1.3$. 


The above observations indicate that unlike elastic response, asymmetric buildings do not experience greatly amplified torsionally coupled response if the uncoupled torsional to translational frequency ratio is close to unity, when they are excited well into the inelastic range. Higher values of $\Omega$ generally lead to lower inelastic responses. The effect of $\Omega$ on the inelastic response is critical if $\Omega$ is smaller than unity but is not significant if $\Omega$ exceeds unity. Therefore, the parameter $\Omega$ critically influences the inelastic response of torsionally flexible structures but does not significantly affect that of torsionally stiff structures and those with an intermediate torsional stiffness. Syamal and Pekau (1985) also reached this conclusion based on a model with proportional element stiffness and strength.

It is also clear that, similar to the effect of stiffness and strength eccentricities, the effect of $\Omega$ on the inelastic response of asymmetric buildings is most pronounced over a wide period range covering the medium and long-period regions for records having a low $a/v$ ratio (3470 Wilshire Blvd. N00E) and in the very short and short-period regions for records having an intermediate $a/v$ ratio (El Centro S00E).

Based on the study carried out in this section, it can be concluded that torsionally flexible asymmetric buildings are the most vulnerable to severe structural damage and ought to be avoided in design practice. Lateral load resisting elements should be located uniformly along two orthogonal axes or near the periphery of the building. Shear cores for lifts should be located symmetrically near the periphery rather than at the centre of the building. Since the inelastic response of torsionally stiff buildings is close to that of buildings having an intermediate torsional stiffness, a structural model associated with $\Omega = 1.0$ can provide accurate and conservative results of the inelastic response of these two categories of asymmetric buildings.
It is of significance and interest in this context to examine the range in which the values of $\Omega$ of real buildings vary. Hart et al. (1975) studied the recorded response of 6 reinforced concrete buildings and 1 steel building in the Los Angeles area in southern California to the 9 February 1971 San Fernando earthquake. These buildings range from 7 to 22 storeys and have fundamental translational periods between 1.03 and 3.43 seconds. The fundamental periods of those vibration modes dominated by translation in the two orthogonal principal directions and those dominated by torsion were found, and 14 values of the fundamental torsional to translational frequency ratios were obtained. These ratios range from 1.04 to 1.79 with an average value of 1.28 and 7 ratios (50%) below 1.2. These periods are for torsionally coupled vibration modes, or in other words actual periods of these buildings. As shown in Fig. 3 of Chandler and Hutchinson (1987), the actual periods of asymmetric buildings are more separated than the uncoupled periods. Therefore, the average uncoupled torsional to translational frequency ratio is lower than 1.28 and there are at least 7 ratios below 1.2. It is reasonable to assume that most buildings are torsionally stiff, having their uncoupled torsional to translational frequency ratios greater than but close to unity. Consequently, a building model with $\Omega = 1.0$ can represent a wide range of actual buildings and gives accurate and conservative predictions of the inelastic response of asymmetric buildings to strong earthquake motions.

The conclusions drawn in this investigation are contradictory to those reached by Bruneau and Mahin (1987) who also studied the inelastic response of initially symmetric buildings with unbalanced strength, subjected to earthquake excitations. They concluded that the weak element displacement ductility demand increases systematically with increasing values of $\Omega$. The reason for this contradiction has been pointed out by Chandler and Duan (1990 and 1991b), namely Bruneau and Mahin mistakenly reduced the value of the radius of gyration $r$ of the floor in order to increase the value of $\Omega$, whilst the locations and the relative strength (0.8$F_y$ and 1.0$F_y$, 1.0$F_y$ and
1.5Fy, etc.) of the resisting elements were kept intact. This approach not only increases the value of Ω but also increases the value of the strength eccentricity ratio $e^*_p = e_p/r$.

The results presented in this section have shown that the weak element displacement ductility demand increases rapidly with increasing strength eccentricity ratio. This explains why Bruneau and Mahin (1987) reached the above misleading conclusion, which implies that buildings should be designed to be torsionally flexible in order to avoid excessive structural damage.

### 4.10 Effect of the force reduction factor

The force reduction factor quantifies a structure's strength capacity relative to its elastic strength demand determined from the elastic design spectrum. A larger force reduction factor implies a lower strength capacity compared with the elastic strength demand and hence greater displacement ductility and energy dissipation demands. In the long-period, displacement sensitive region, the value of the force reduction factor is close to the displacement ductility demand of a single-degree-of-freedom (SDOF) system, since the lateral deformation of a long-period, displacement sensitive SDOF system is controlled by the peak ground horizontal displacement and is the same for both elastic and inelastic structures, being unaffected by the force reduction factor. The force reduction factor is widely employed in the base shear provisions of earthquake resistant design codes in Europe, Canada, Mexico and the United States and accounts for the ductile behaviour of structures. The design base shear is obtained by dividing the elastic load level by the force reduction factor R which is assigned different values according to the construction material and the structural systems employed, allowing for varied displacement ductility and energy dissipation capacities.

This section studies the influence of the force reduction factor on the inelastic response of asymmetric buildings. The displacement ductility and the normalised
hysteretic energy ductility demands are presented in Figures 4.31 to 4.34 as response spectra for two types of buildings, namely initially eccentric \((e^*_x = 0.3, e^*_y = 0)\) and initially symmetric \((e^*_x = 0, e^*_y = 0.3)\) structures. The value of \(R\) is taken to be either 2 or 4. The inelastic response increases very significantly with the increase of the force reduction factor, implying that structures are excited more and more into the inelastic range when their strength capacities are correspondingly reduced from the elastic strength demand. The effect of \(R\) on the inelastic response is dependent on the vibration period of the structure, being most pronounced in the very short-period, acceleration sensitive region.

Comparing the inelastic response of asymmetric structures with that of symmetric structures, it is apparent that the difference between the inelastic response of asymmetric and symmetric structures becomes larger when they are excited more and more into the inelastic range. The increase in the difference of displacement ductility demand is more significant than that of the normalised hysteretic energy ductility demand. As the structures are excited more and more into the inelastic range, the effect of stiffness and strength eccentricities on the inelastic response of asymmetric buildings becomes more critical. The additional inelastic response due to stiffness or strength eccentricities increases with increasing force reduction factor. Hence increasing the force reduction factor not only increases significantly the overall inelastic response but also accentuates the effect of stiffness and strength eccentricities on the inelastic response of asymmetric buildings.

4.11 Summary and conclusions

This chapter has presented the results of a detailed parametric study of the inelastic earthquake response of asymmetric buildings, based on five different structural models. The results have highlighted the model dependency of the inelastic response of
asymmetric buildings and the influence of the key system parameters. The observations in this study contribute to and improve significantly the understanding of the behaviour of asymmetric buildings to strong earthquake motions and provide guidelines for the assessment of design procedures for such buildings (see Chapter 5). The findings and conclusions made in this study are summarised as follows:

1. The inelastic torsional response of asymmetric buildings is model dependent when they are excited by strong earthquake motions well into the inelastic range. The inelastic response not only depends on the system parameters, \( T_y, \Omega, e_z^*, e_p^*, R \) and \( \xi \) and the ground motion input, but also on the number, location, orientation, and the post-yielding force-deformation relationship of the resisting elements, and the source of the plan asymmetry (stiffness-eccentric or mass-eccentric).

2. The extent of the inelastic earthquake response of asymmetric buildings generally increases with increasing number of resisting elements in the direction of earthquake motion. The two-element model significantly underestimates the inelastic earthquake response of asymmetric buildings compared with the three- and four-element models. However, the results based on the three-element model are very similar to those based on the four-element model. Therefore, the three-element model is not only simple in definition but can also provide satisfactory and conservative results of the inelastic earthquake response of asymmetric buildings. Employing more than three elements in the direction of earthquake motion gives very little benefit, but results in more complex system definition. Therefore, the three-element model is adequate for studying the inelastic earthquake response of asymmetric buildings.
3. The resisting elements oriented transverse to the direction of earthquake motion have very little influence on the inelastic response of the resisting elements parallel to the direction of earthquake motion. Including transverse elements in the analytical model not only has very little benefit compared with the model having three elements parallel to the direction of earthquake motion without any transverse elements, but also leads to more complex system definition and difficulties in drawing generalised conclusions. Moreover, in view of the fact that these transverse elements are also excited well into the inelastic range by the transverse component of the earthquake motion and can therefore contribute very little to the system's torsional stiffness, the approach of including these elements in the analytical model and at the same time assuming the ground motion to be uni-directional is inappropriate. Therefore, the three-element model without transverse elements is the most suitable for further study.

4. Stiffness-eccentric and mass-eccentric systems behave differently under earthquake excitations. If the centre of strength PC coincides with the centre of mass CM, mass-eccentric systems exhibit significantly higher inelastic response than stiffness-eccentric systems. The opposite is true if PC coincides with the centre of rigidity CR. These findings explain why existing studies have reached contradictory conclusions based on stiffness-eccentric or mass-eccentric systems. Since most asymmetric buildings are stiffness-eccentric, mass-eccentric building models should not generally be employed in studying the inelastic earthquake response of asymmetric buildings.

5. The assumption dependency of the inelastic response of the three-element model is not significant. The double-step stiffness variation model and the
single-step stiffness variation model are interchangeable for the purpose of examining the inelastic response of asymmetric buildings. As a result of this and previous conclusions, the three-element model with a double-step stiffness variation is an appropriate model and is employed throughout the inelastic dynamic parametric study.

6. The inelastic earthquake response of stiffness asymmetric but strength symmetric buildings, typical of code-designed asymmetric buildings, is fundamentally different from the elastic response of asymmetric buildings. Because of the abrupt shifting of the centre of rigidity due to yielding and unloading of resisting elements, there is some cancellation in the torsional motion. Therefore, code designed asymmetric buildings respond more in translation when excited well into the inelastic range than when responding elastically. Traditionally, the element at the stiff edge is considered to be favourably affected and the element at the flexible edge is considered to be unfavourably affected by torsional coupling in elastic studies, and also in earlier inelastic studies which have assumed proportional element stiffness and strength. However, when code designed asymmetric buildings are excited well into the inelastic range, the stiff edge element becomes the critical one, experiencing much higher inelastic response than the corresponding symmetric buildings. In contrast, the response of the flexible edge element is generally substantially lower than that of the corresponding symmetric buildings.

7. The stiffness eccentricity ratio is an important system parameter influencing the inelastic response of asymmetric buildings. The inelastic response of the stiff edge element increases substantially with the increase of the stiffness
eccentricity ratio. Therefore, code torsional provisions should be modified such that the design strength capacity of this element is increased significantly.

8. The strength eccentricity ratio is a critical system parameter affecting the inelastic response of initially symmetric buildings but with an asymmetric distribution of strength. The inelastic response of the flexible edge element increases very significantly with the increase of the strength eccentricity ratio. As a result, designers are strongly recommended to check the location of the centre of strength of initially stiffness symmetric buildings and to refine the design, if necessary, to shift the centre of strength as close as possible to the centre of rigidity.

9. The fundamental uncoupled lateral period and the frequency content of the earthquake motion also significantly affect the inelastic response of asymmetric buildings. If earthquake records are scaled according to their peak horizontal acceleration, the effect of stiffness and strength eccentricities is pronounced in the very short-period region for motions having a high a/v ratio, in the very short and short-period regions for records having an intermediate a/v ratio and in the medium and long-period regions for records having a low a/v ratio. Since most buildings are multistorey and have their fundamental lateral periods in the short, medium or long-period regions, earthquake motions having a high a/v ratio are unlikely to cause significantly more severe structural damage to asymmetric buildings than to symmetric buildings.

10. The uncoupled torsional to translational frequency ratio $\Omega$ is a critical system parameter influencing the inelastic response of a certain class of asymmetric
buildings. In general, the inelastic response of asymmetric buildings decreases with increasing value of the uncoupled torsional to translational frequency ratio. The effect of $\Omega$ is most pronounced for torsionally flexible asymmetric buildings. However, this system parameter does not significantly affect the inelastic response of asymmetric buildings with $\Omega \geq 1.0$. Therefore, a building model with $\Omega = 1.0$ can provide conservative results of the inelastic response of asymmetric buildings with $\Omega \geq 1.0$. Torsionally flexible asymmetric buildings with low values of $\Omega$ undergo much higher inelastic response than symmetric buildings. Consequently, it is strongly recommended that torsionally flexible asymmetric buildings should be avoided in design practice. Hence the load resisting elements should be located uniformly along two orthogonal axes or near the periphery of the building. Shear cores for lifts should be located symmetrically near the periphery of the building rather than at the centre.

11. The force reduction factor $R$ also has a significant effect on the inelastic response of asymmetric buildings. Increasing the force reduction factor not only increases rapidly the overall inelastic response but also amplifies the effect of stiffness and strength eccentricities. Therefore, it is strongly recommended that when calculating the design base shear based on the elastic design spectrum and the force reduction factor, symmetric and asymmetric buildings should be treated differently. Adequate allowance should be provided for asymmetric buildings by employing a smaller force reduction factor for asymmetric buildings than for symmetric buildings.

12. The conclusions reached in Chapter 3 based on static monotonic failure mode analysis agree extremely well with the above conclusions based on a detailed inelastic dynamic parametric study. Static monotonic failure mode analysis is
therefore a simple and efficient technique to identify the failure mode, the critical element, and the post-yield behaviour of buildings under strong earthquake motions.
Figure 4.1 Idealised building models
Figure 4.2 Force-deformation relationship of resisting elements

Figure 4.3 Newmark-Hall design spectrum

(a) 3470 Wilshire Blvd., San Fernando record (N00E component)

(b) El Centro, Imperial Valley record (S00E component)

(c) Cholame Shandon No. 5, Parkfield record (N85E component)

Figure 4.4 Time histories of selected earthquake records
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Inelastic design spectrum, $R=4$

Newmark-Hall elastic spectrum

Cholame Shandon No.5 N85E
3470 Wilshire Blvd. N00E
El Centro S00E

Figure 4.5 5% damped elastic acceleration spectra of selected earthquake records and the 5% damped median Newmark-Hall elastic and inelastic design spectra

- Force $R$
- Deformation $U$

(a) Definition of the normalised hysteretic energy ductility, $\mu_E = \frac{E_H}{R_yU_y}$

(b) Definition of the standard hysteretic cycle

Figure 4.6 Definition of the normalised hysteretic energy ductility and the standard hysteretic cycle
Figure 4.7 Maximum displacement ductility demand of asymmetric buildings with two, three and four resisting elements oriented in the direction of ground motion: $e^* = 0.6$, $e^* = 0$, $\Omega = 1.0$, $R=4$, $\xi = 5\%$

Figure 4.8 Maximum normalised hysteretic energy ductility demand of asymmetric buildings with two, three and four resisting elements oriented in the direction of ground motion: $e^*_n = 0.6$, $e^*_p = 0$, $\Omega = 1.0$, $R=4$, $\xi = 5\%$
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Figure 4.9 Maximum displacement ductility demand of initially symmetric buildings with two, three and four resisting elements oriented in the direction of ground motion: \( \varepsilon_s^* = 0, \varepsilon_p^* = 0.3, \Omega = 1.0, R=4, \xi = 5\% \)

Figure 4.10 Maximum normalised hysteretic energy ductility demand of initially symmetric buildings with two, three and four resisting elements oriented in the direction of ground motion: \( \varepsilon_s^* = 0, \varepsilon_p^* = 0.3, \Omega = 1.0, R=4, \xi = 5\% \)
Figure 4.11 Maximum displacement ductility demand of asymmetric buildings with and without transverse resisting elements: $e^*_r = 0.6$, $e^*_p = 0$, $\Omega = 1.0$, $R=4$, $\xi = 5\%$

Figure 4.12 Maximum normalised hysteretic energy ductility demand of asymmetric buildings with and without transverse resisting elements: $e^*_r = 0.6$, $e^*_p = 0$, $\Omega = 1.0$, $R=4$, $\xi = 5\%$
Figure 4.13 Maximum displacement ductility demand of initially symmetric buildings with and without transverse resisting elements: $e_r^* = 0$, $e_p^* = 0.3$, $\Omega = 1.0$, $R=4$, $\xi = 5\%$

Figure 4.14 Maximum normalised hysteretic energy ductility demand of initially symmetric buildings with and without transverse resisting elements: $e_r^* = 0$, $e_p^* = 0.3$, $\Omega = 1.0$, $R=4$, $\xi = 5\%$
Figure 4.15 Maximum displacement ductility demand of stiffness-eccentric and mass-eccentric buildings: $e_* = 0.3, e^* = 0, \Omega = 1.0, R=4, \xi = 5\%$

Figure 4.16 Maximum normalised hysteretic energy ductility demand of stiffness-eccentric and mass-eccentric buildings: $e_* = 0.3, e^* = 0, \Omega = 1.0, R=4, \xi = 5\%$
Figure 4.17 Maximum displacement ductility demand of stiffness-eccentric and mass-eccentric buildings: $e_s^* = 0.3$, $e_p^* = 0.3$, $\Omega = 1.0$, $R=4$, $\xi = 5\%$

Figure 4.18 Maximum normalised hysteretic energy ductility demand of stiffness-eccentric and mass-eccentric buildings: $e_s^* = 0.3$, $e_p^* = 0.3$, $\Omega = 1.0$, $R=4$, $\xi = 5\%$
Figure 4.19 Maximum displacement ductility demand of asymmetric buildings with a double-step or a single-step variation of stiffness distribution: $e_r^* = 0.6$, $e_p^* = 0$, $\Omega = 1.0$, $R=4$, $\xi = 5\%$

Figure 4.20 Maximum normalised hysteretic energy ductility demand of asymmetric buildings with a double-step or a single-step variation of stiffness distribution: $e_r^* = 0.6$, $e_p^* = 0$, $\Omega = 1.0$, $R=4$, $\xi = 5\%$
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Figure 4.21 Effect of the stiffness eccentricity ratio on the maximum displacement ductility demand of asymmetric buildings: $e^* = 0.3, 0.6$, $e_p^* = 0$, $\Omega = 1.0, R=4, \xi = 5\%$

(a) El Centro S00E

(b) 3470 Wilshire Blvd. San Fernando

N00E

(c) Cholame Shandon No. 5 Parkfield

N85E

(continued overleaf)
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Figure 4.22 Effect of the stiffness eccentricity ratio on the maximum normalised hysteretic energy ductility demand of asymmetric buildings: 

(c) Cholame Shandon No. 5 Parkfield N85E

Figure 4.23 Percentages of the total hysteretic energy dissipated by individual resisting elements in asymmetric buildings: 

(e) El Centro S00E

(b) 3470 Wilshire Blvd. San Fernando N00E

Note: The diagrams show the effect of the stiffness eccentricity ratio on the maximum normalised hysteretic energy ductility demand. The percentages of the total hysteretic energy dissipated by individual resisting elements in asymmetric buildings are also presented. The parameters used are: eccentricity ratio $\varepsilon_e = 0.3, 0.6$, coefficient $\Omega = 1.0$, $R=4$, $\xi = 5\%$. The diagrams illustrate the distribution of energy dissipation across different elements under varying translational periods.
Figure 4.24 Effect of the strength eccentricity ratio on the maximum displacement ductility demand of initially symmetric buildings: $e_r^* = 0, e_p^* = 0.15, 0.3, \Omega = 1.0, R = 4, \xi = 5\%$

(continued overleaf)
Figure 4.25 Effect of the strength eccentricity ratio on the maximum normalised hysteretic energy ductility demand of initially symmetric buildings: $e^*_c = 0, e^*_p = 0.15, 0.3, \Omega = 1.0, R=4, \xi = 5\%$

(a) Plan view of a building with a central shear core ($\Omega < 1.0$)
(b) Plan view of a building with uniformly distributed column mesh ($\Omega = 1.0$)
(c) Plan view of a 'box' building with peripheral shear walls ($\Omega > 1.0$)
Figure 4.27 Effect of the uncoupled torsional to translational frequency ratio on the maximum displacement ductility demand of asymmetric buildings: $e^*_t = 0.3$, $e^*_p = 0$, $\Omega = 0.7, 1.0, 1.3$, $R = 4$, $\xi = 5\%$

Figure 4.28 Effect of the uncoupled torsional to translational frequency ratio on the maximum normalised hysteretic energy ductility demand of asymmetric buildings: $e^*_t = 0.3$, $e^*_p = 0$, $\Omega = 0.7, 1.0, 1.3$, $R = 4$, $\xi = 5\%$
Figure 4.29 Effect of the uncoupled torsional to translational frequency ratio on the maximum displacement ductility demand of initially symmetric buildings: $e_t^* = 0$, $e_p^* = 0.3$, $\Omega = 0.7, 1.0, 1.3$, $R=4$, $\xi = 5\%$

Figure 4.30 Effect of the uncoupled torsional to translational frequency ratio on the maximum normalised hysteretic energy ductility demand of initially symmetric buildings: $e_t^* = 0$, $e_p^* = 0.3$, $\Omega = 0.7, 1.0, 1.3$, $R=4$, $\xi = 5\%$
Figure 4.31 Effect of the force reduction factor on the maximum displacement ductility demand of asymmetric buildings: $\varepsilon^* = 0.3, \varepsilon_r^* = 0, \Omega = 1.0, R=2, 4, \xi = 5\%$

Figure 4.32 Effect of the force reduction factor on the maximum normalised hysteretic energy ductility demand of asymmetric buildings: $\varepsilon^* = 0.3, \varepsilon_r^* = 0, \Omega = 1.0, R=2, 4, \xi = 5\%$
Figure 4.33 Effect of the force reduction factor on the maximum displacement ductility demand of initially symmetric buildings: $e^* = 0$, $e^* = 0.3$, $\Omega = 1.0$, $R=2, 4$, $\xi = 5\%$

Figure 4.34 Effect of the force reduction factor on the maximum normalised hysteretic energy ductility demand of initially symmetric buildings: $e^* = 0$, $e^* = 0.3$, $\Omega = 1.0$, $R=2, 4$, $\xi = 5\%$
**CHAPTER 5**

*Evaluation and Improvement of The Torsional Provisions of Current Aseismic Building Codes: A New Unified Design Approach*

**5.1 Code approach for aseismic torsional design: introduction**

Torsional coupling in asymmetric buildings results in an additional torque applied simultaneously with the seismic lateral load at each floor level. This torque leads to increased deformation and strength demand in certain resisting elements. The value of the additional torque can be roughly estimated by simply multiplying the seismic lateral load by the distance between the centre of mass CM and the centre of rigidity CR of the floor level being considered, known as the stiffness eccentricity or static eccentricity $e_s$. Comprehensive analytical studies of the torsional effects based on elastic single-storey models employing either the smoothed response spectrum linear elastic modal analysis approach (Kan and Chopra 1977a, 1977b, Tso and Dempsey 1980, Tsicnias and Hutchinson 1981) or the time history approach (Chandler and Hutchinson 1986, 1987, 1988a, 1988b, and Rutenberg and Pekau 1987) have been carried out in the last decade. These studies have concluded that the dynamic nature of the seismic torsionally coupled response of asymmetric buildings has the effect of reducing the seismic lateral loading but at the same time amplifying the torque compared with that given by the product of the lateral load and the static eccentricity. This effect is most pronounced when the static eccentricity is small and the uncoupled torsional to translational frequency ratio is close to unity.
To account for the torsional effects arising in asymmetric buildings, most of the current major aseismic building codes include torsional design provisions. Code torsional provisions usually prescribe two alternative analytical procedures for the specification of design load for the individual resisting elements. They are the equivalent static force procedure and the linear elastic modal analysis procedure.

5.1.1 The equivalent static force procedure

The equivalent static force procedure applies to regular buildings in which the centres of floor mass and the centres of rigidity at floor levels lie approximately on two vertical lines. In this procedure, the fundamental lateral period is calculated based on empirical formulae and the design base shear is obtained from the design spectral value corresponding to the estimated fundamental lateral period. The design spectrum is given by the code base shear provision. Most current major aseismic building codes specify a design lateral load combined with a design torque at each floor level. The design torque is the product of the design lateral load $V_y >$ and the design eccentricity $e_D$, see Fig. 5.1 in the case of a single-storey asymmetric building.

All the current major aseismic building codes provide conservative estimates of the earthquake lateral load by ignoring the torsional effects, and specify two design eccentricities, called the primary, $e_1D$, and the secondary, $e_2D$, design eccentricities respectively (whichever is more unfavourable for the element being designed). These account for the increased or the decreased strength demand in certain elements. The design eccentricities $e_1D$ and $e_2D$ define the locations through which the design lateral load must be applied to induce the design torque about a vertical axis through the centre of rigidity CR of the floor being considered. Most current aseismic building codes specify primary and secondary design eccentricities in the form:
\[ e_{D1} = e_{d1} + e_a \]  
\[ e_{D2} = e_{d2} - e_a \]  

(5.1)  
(5.2)

in which, \( e_{d1} \) and \( e_{d2} \) are called the dynamic eccentricities which take into account the dynamic amplification of the static eccentricity. The dynamic eccentricities \( e_{d1} \) and \( e_{d2} \) usually take the form of \( e_d = \alpha e_s \), in which \( \alpha \) is equal to or greater than unity for the primary design eccentricity and equal to or smaller than unity for the secondary design eccentricity. The second term in eqns. (5.1) and (5.2), \( e_a \), which usually takes the form of \( \beta b \), is the accidental eccentricity intended to account for all uncertainties in design and the rotational component of the ground motion which is not explicitly considered in design. The accidental eccentricity is usually a fraction of the dimension of the building, \( b \), measured perpendicular to the direction of the earthquake input, and \( \beta \) is a coefficient defining the accidental eccentricity as a proportion of \( b \).

Various codes take different forms for the specification of the values of \( e_a \) and \( \beta \). A summary of the primary and secondary design eccentricities specified in current major aseismic building codes is given in Table 5.1.

**Table 5.1 Aseismic building code regulations for design eccentricities**

<table>
<thead>
<tr>
<th>Building codes</th>
<th>Primary design eccentricity</th>
<th>Secondary design eccentricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexico 76</td>
<td>1.5e_s+0.1b</td>
<td>1.0e_s-0.1b</td>
</tr>
<tr>
<td>New Zealand 89</td>
<td>1.0e_s+0.1b</td>
<td>1.0e_s-0.1b</td>
</tr>
<tr>
<td>Eurocode 8</td>
<td>1.0e_s+e_1+0.05b</td>
<td>1.0e_s-0.05b</td>
</tr>
<tr>
<td>NBCC 90 (Canada)</td>
<td>1.5e_s+0.1b</td>
<td>0.5e_s-0.1b</td>
</tr>
<tr>
<td>UBC 88 (USA)</td>
<td>1.0e_s+0.05b</td>
<td>0</td>
</tr>
</tbody>
</table>
In Table 5.1, the term $e_t$ in the Eurocode 8 formula is the additional eccentricity taking into account the dynamic amplification of the static eccentricity $e_s$. The equations for calculating $e_t$ are given in Section 5.3.5.

### 5.1.2 The linear elastic modal analysis procedure

The linear elastic modal analysis procedure is also called the dynamic analysis procedure in some codes and it is specified to be generally applicable to all buildings, whether regular or irregular in form. The exception is the Mexico 87 code (Gomez and Garcia-Ranz 1988), which does not employ this procedure. Most codes state explicitly that if certain regularity conditions are not satisfied, modal analysis rather than the equivalent static force procedure should be used.

In modal analysis, the actual vibration periods and mode shapes of asymmetric buildings are calculated by solving an eigenvalue problem:

$$[K] \{\phi_i\} = \omega_i^2 [M] \{\phi_i\} \quad (5.3)$$

in which $[K]$ and $[M]$ are, respectively, the global stiffness and mass matrices, $\omega_i$ is the $i$th natural frequency and $\{\phi_i\}$ is the $i$th vibration mode of the building. The modal response corresponding to the $i$th mode is calculated based on the design spectrum given in various codes, together with $\omega_i$ and $\{\phi_i\}$. Then, the modal responses are combined employing either the Square Root of the Sum of Squares (SRSS) procedure or the Complete Quadratic Combination (CQC) procedure to obtain an approximate value of the total response. The deformation and therefore the design loading of the individual resisting elements can be calculated based on this approximated total response.
5.1.3 The necessity to evaluate and improve code torsional provisions

It is apparent that current code analysis procedures for earthquake resistant design are based largely on the linear elastic theory. It is expected that these analysis procedures, in particular the linear elastic modal analysis, can provide satisfactory estimates for the deformation and strength demand of individual resisting elements of asymmetric buildings if they are responding elastically to small or moderate earthquakes.

As discussed in Section 1.4, the present philosophy of earthquake resistant design allows buildings to be excited well into the inelastic range by utilising the ductility and energy dissipation capacities of structures when resisting a strong earthquake. However, as mentioned above the code analysis procedures are based on the linear elastic theory and hence generally a non-linear inelastic analysis is not required by present code regulations. As studied in Chapter 4, the non-linear hysteretic behaviour due to yielding of the resisting elements influences the torsionally coupled response of asymmetric buildings and triggers different behaviour from that predicted by the linear elastic theory. Lessons from past strong earthquakes have demonstrated that some of these code torsional provisions are inadequate. Therefore, these design procedures need to be evaluated and appropriately improved.

The results presented in Chapter 4 revealed that in code designed asymmetric buildings, the resisting elements at the stiff side are the critical elements, and hence it was recommended that their strength capacities should be increased substantially. The former conclusion was based on the assumption that the combined strength capacity of an asymmetric building is the same as that of the corresponding symmetric building. This assumption is required in order to carry out the inelastic parametric study, in which one system parameter is changed independently and individually whilst keeping all
other system parameters constant. However, it is well known that the modal analysis procedure leads to strength reductions, and the equivalent static force procedure leads to strength increases in asymmetric buildings, when compared with corresponding symmetric structures. The latter result arises because of the assignment of the more unfavourable value determined by either the primary or the secondary design eccentricities as the strength capacity of the resisting elements. In Section 4.10, it was shown that increasing the total strength capacity can significantly reduce the inelastic response, together with the influence of stiffness and/or strength eccentricities. Therefore, when evaluating code torsional provisions, the actual value of the total strength capacity should be used and the assumption referred to above should be removed.

Based on the single storey three-element asymmetric building model, this chapter firstly evaluates the code torsional provisions. The base shear and torsional provisions of various codes are rigorously adhered to, and localised earthquake records or records with different frequency contents and a/v ratios are used as the ground motion input. The questions addressed include whether or not the code torsional provisions provide consistent protection to resisting elements at the flexible side and at the stiff side, and whether or not they offer consistent control over structural damage for symmetric and eccentric buildings. Secondly, improvements to code torsional provisions are sought and a new unified approach for the design of asymmetric buildings is proposed.

5.2 Analytical model

In Chapter 4, the three-element stiffness-eccentric model with a double-step stiffness variation has been shown to be the most appropriate model, since it is simple in definition, is representative of a wide range of actual asymmetric buildings and offers
satisfactory and conservative estimates of the inelastic response of such buildings. Therefore, this model is again employed in this chapter to evaluate and improve the code torsional provisions.

In this chapter, the model’s system parameters and the specification of element properties, with the exception of the design base shear and the yielding strength of the resisting elements, are the same as those given in Section 4.2.4.2(a). The design base shear $V_{y0}$ is determined from the inelastic design spectrum specified in various codes and using the uncoupled lateral period of $T_y$ of the structure. The yielding strength of the resisting elements is specified in accordance with various code torsional provisions, that is, by applying the design base shear $V_{y0}$ at distances from the centre of rigidity $CR$ equal to the primary and the secondary design eccentricities, whichever produces the more unfavourable strength demand for the element under consideration.

In view of the reasoning given above for the introduction of the accidental eccentricity, and bearing in mind that in the present theoretical study there are neither uncertainties in the specification of structural and element properties, nor is there a rotational component of the ground motion, the accidental eccentricity has not been included in the specification of the elements’ yielding strength.

The yielding strength of elements 2 and 3, for which the primary design eccentricity applies, is

$$F_{y2} = \frac{V_{y0}}{K_y} k_2 \left( 1 + \frac{e_{d1}}{\Omega^2 r^2} e_z \right) \tag{5.4}$$

$$F_{y3} = \frac{V_{y0}}{K_y} k_3 \left( 1 + \frac{e_{d1}}{\Omega^2 r^2} (e_z + d) \right) \tag{5.5}$$

The yielding strength of element 1, for which the secondary design eccentricity usually applies, is
\[ F_{y1} = \frac{V_{yo}}{K_y} k_i \left( 1 - \frac{e_{d2}}{\Omega^2 \rho^2} (d - e_s) \right) \] (5.6)

In torsionally flexible structures with a large stiffness eccentricity, the primary design eccentricity expression gives a higher strength demand for element 1 than the secondary design eccentricity expression. In this case, the strength capacity of element 1 should be determined by the primary design eccentricity expression (see Chandler and Hutchinson 1988a for the conditions under which the secondary design eccentricity expression provides the more unfavourable design loading for elements at the stiff side, corresponding to various codes).

Because of the application of both the primary and the secondary design eccentricities, and the yielding strength of element 1 being assigned the more unfavourable value, the sum of the yielding strength of all elements \( F_y = \sum_{i=1}^{3} F_i \) is significantly larger (by up to 50%, see Fig. 5.27) than the design base shear \( V_{yo} \). The total strength variation corresponding to various codes, given as functions of the stiffness eccentricity, has been studied in detail by Tso and Ying (1990).

As concluded in Chapter 4, torsionally flexible buildings should be avoided in design practice and the inelastic response of torsionally stiff asymmetric buildings (\( \Omega > 1.0 \)) and those with an intermediate torsional stiffness (\( \Omega = 1.0 \)) is not sensitive to the uncoupled torsional to translational frequency ratio. A model with this system parameter equal to unity can provide accurate and conservative results for the inelastic response of these latter two categories of asymmetric buildings. Therefore, the uncoupled torsional to translational frequency ratio \( \Omega \) of the model is set to unity without sacrificing generality.

The equations of motion are the same as in eqn. (4.53) in Chapter 4. The viscous damping is assumed to be 5 per cent for both modes. The equations of motion are
solved using the step-by-step numerical integration method. The time interval of the numerical integration is taken to be $1/25$ of the shorter of the two initial elastic natural periods, or the time increment of the digitization of an earthquake record if smaller. Again, the numerical calculation is carried out using the afore-mentioned computer program for inelastic earthquake response analysis of three-dimensional buildings — DRAIN-TABS (Guendelman-Israel and Powell 1977).

In Chapter 4, it was shown that in most cases the two inelastic response parameters, namely the displacement ductility demand and the normalised hysteretic energy ductility demand, of the resisting elements of both symmetric and asymmetric buildings show similar trends. Furthermore, the present design philosophy is based on the displacement ductility rather than the hysteretic energy ductility. One example is the force reduction factor widely employed in aseismic building codes. The force reduction factor is an indicator of the global or system ductility (which is different from local or member ductility) of structures. For single-degree-of-freedom structures the system ductility can be directly related to the force reduction factor, but for multistorey buildings this relationship is more complex. Another example is the maximum allowable storey drifts presently specified in aseismic building codes. These provisions also represent a form of displacement ductility limit, in the sense that the maximum deformation is restricted to an acceptable value. Therefore, in this and the following chapter which evaluate and propose improvements to the code torsional provisions, the displacement ductility demand alone is employed as the inelastic earthquake response parameter of the resisting elements.
5.3 Evaluation of the torsional provisions of current major aseismic building codes

5.3.1 Assessment of the Mexico 76 code torsional provisions

In view of the high percentage of collapsed buildings or buildings severely damaged caused by plan asymmetry in the 1985 Mexico City earthquake, and the similarities between the Mexico 76 code torsional provisions and those specified in codes elsewhere (see Table 5.1), it is enlightening and of interest to evaluate first the Mexico 76 code torsional provisions. The adequacy of these provisions is examined in the light of the twin objectives of consistent protection against structural damage for both symmetric and eccentric buildings and the satisfactory control of additional displacement ductility demand arising due to plan asymmetry. This evaluation provides valuable insight into the nature and the extent of the inadequacies of these provisions and therefore forms the basis for proposed improvements to the code torsional provisions.

The design base shear $V_{y0}$ is determined according to the Mexico 76 code design spectral acceleration, $a$, as shown in Fig. 5.2, expressed as a fraction of the acceleration of gravity, for group B buildings in Zone III. The force reduction factor Q (referred to as the ductility reduction factor in the Mexico 76 code) is assigned the value 4.0. Group B buildings are those having normal importance. Zone III encompasses the lake bed region in Mexico City and has the highest design spectral ordinates. A force reduction factor of 4 is the most commonly used value for ductile moment-resisting frames and is the highest factor employed in the Emergency Regulations (Rosenblueth 1986) issued after the 1985 Mexico City earthquake. Although a Q factor of 6 is allowed in the Mexico 76 code, it was rarely used in practice (Hadjian 1989) and was deleted in the Emergency Regulations. The design base shear $V_{y0}$ is calculated as follows:
\[ V_{\varphi} = \frac{Wa}{Q} \]  

(5.7)

in which \( W \) is the weight of the floor deck, and \( a/Q \) equals 0.06 in the period range \( 0 < T_y \leq 3.3 \) seconds (see Fig. 5.2).

The element strength is specified in accordance with the Mexico 76 code design eccentricity formulae:

\[ e_{D1} = 1.5e_s + 0.1b \]  

(5.8)

\[ e_{D2} = 1.0e_s - 0.1b \]  

(5.9)

However, for the reasons outlined in Section 5.2 the accidental eccentricity 0.1b is not included herein when specifying the element strength. Therefore, the yielding strength of the resisting elements are:

\[ F_{y1} = \frac{V_{\varphi}}{K_y} k_1 \left( 1 - \frac{e_s}{\Omega^2 r^2} (d - e_s) \right) \]  

(5.10)

\[ F_{y2} = \frac{V_{\varphi}}{K_y} k_2 \left( 1 + \frac{1.5e_s}{\Omega^2 r^2} e_s \right) \]  

(5.11)

\[ F_{y3} = \frac{V_{\varphi}}{K_y} k_3 \left( 1 + \frac{1.5e_s}{\Omega^2 r^2} (e_s + d) \right) \]  

(5.12)

The uncoupled torsional to translational frequency ratio \( \Omega \) is equal to unity. The viscous damping is assumed to be 5 per cent of critical for both modes. The east-west component of the record obtained at SCT, Mexico City lake bed, during the 19 September 1985 earthquake is employed as the ground motion input. The time history of this record is shown in Fig. 5.3 and its 5% damped elastic acceleration response spectrum, together with the Mexico 76 code design spectra, are plotted in Fig. 5.4.
Fig. 5.5 presents the displacement ductility demand of elements 1 and 3 of asymmetric buildings and that of the corresponding reference symmetric buildings, over the period range 0.1-2.0 sec. The corresponding reference symmetric buildings are single-degree-of-freedom systems having the same lateral periods as the uncoupled lateral periods of the asymmetric buildings, and a total strength equal to the design base shear \( V_{yo} \). Three stiffness eccentricity values are considered, namely 0.1b, 0.2b, and 0.3b. These stiffness eccentricity values represent asymmetric buildings having small, moderate and large stiffness eccentricities, respectively.

The results clearly indicate that the element at the stiff edge (element 1) is the critical element which is likely to suffer much more severe structural damage than the element at the flexible edge (element 3) or the corresponding reference symmetric building. This observation reinforces the findings made in Chapter 4. The results also demonstrate that the Mexico 76 code torsional provisions are inadequate, even for buildings having a small stiffness eccentricity (\( e_s = 0.1b \)). These torsional provisions do not offer consistent protection to both symmetric and eccentric buildings and do not provide consistent control of structural damage to resisting elements at the flexible side and those at the stiff side. Although the total strength of asymmetric buildings is significantly higher than that of the corresponding reference symmetric buildings, for instance an increase of 15 per cent of the base shear in the case of a moderate stiffness eccentricity (\( e_s = 0.2b \), see Table II of Tso and Ying 1990), which leads to the beneficial effect of reducing the translational response of the resisting elements, the Mexico 76 code torsional provisions still substantially underestimate the strength demand of element 1.

The Mexico City SCT1 east-west record has a very low a/v ratio of 0.28 g/(m/s). It can be seen clearly that the effect of plan asymmetry on the additional displacement ductility demand of element 1 and the reduced displacement ductility demand of
element 3 is pronounced in the medium-period \((0.5 \text{sec.} < T_y \leq 1.0 \text{sec.})\) and in the long-period \((T_y > 1.0 \text{sec.})\) regions. Again, this observation agrees with that made in Chapter 4.

The above conclusions are in contrast to those of Tso and Ying (1990), who concluded that the Mexico 76 code provisions (without modification) lead to satisfactory control of the additional displacement ductility demand. Tso and Ying (1990) also concluded that stiffness-eccentric buildings with strength distributions based on the code torsional provisions of Canada, New Zealand and the United States will result in only minor additional displacement ductility demands on the structural elements, and that element 3 is the critical element whose displacement can be up to two or three times that of the corresponding reference symmetric buildings. They concluded that the latter effect could lead to a serious pounding problem between adjacent structures.

### 5.3.2 Discussion on the accidental eccentricity and other factors influencing the inelastic performance of asymmetric buildings

A number of issues have contributed to the contradictory conclusions outlined in the previous section. Firstly, the validity of including the additional torque given by an accidental eccentricity of 0.1 or 0.05 times the building plan dimension \(b\) parallel to the direction of the eccentricity is questionable, when specifying the element strengths in the analytical model. This affects the assessment of the results of inelastic dynamic analysis carried out without consideration of any uncertainties or the rotational ground motion, as in Tso and Ying (1990). It should be borne in mind that the accidental eccentricity is intended to account for such effects.
In Tso and Ying (1990), the specification of the total strength and the strength eccentricity of the analytical models is based on observations of the general trends of the variation of these quantities as functions of $e_{n}$, as determined by the different codes. Tso and Ying (1990) defined two models representing the torsional design provisions of Canada, New Zealand and the United States (generic model 1), and those of Mexico 76 (generic model 2). In deriving the total strength and strength eccentricity of these models, as well as specifying the relationship of strength between element 2, the element at the centre, and element 3 at the flexible edge, Tso and Ying (1990) included the accidental eccentricity in the design eccentricity expressions. However, in their inelastic dynamic analysis, there are neither any uncertainties regarding the real values of the stiffness and strength eccentricities, nor is consideration given to any rotational components of ground motion. Therefore, their results are inevitably non-conservative and tend to mask the effect of eccentricity on the additional element displacement ductility demand.

In a previous paper by Cheung and Tso (1986a) studying the elastic response of multistorey eccentric buildings, the accidental eccentricity of 0.1$b$ was also included in calculating the code static torque and the resulting static shear in resisting elements. Subsequently, Stafford Smith and Vezina (1986) pointed out in their discussion of the former paper that much better results showing the discrepancies between the codified static approach and dynamic analysis would have been achieved if only the actual eccentricity had been used and the accidental eccentricity not included in calculating the code static torque, in view of the omission of the accidental eccentricities from the dynamic analysis. This point has been clearly proven by the results provided in the reply of Cheung and Tso (1986b). Comparing the results given by Cheung and Tso (1986a and 1986b), it is obvious that including the accidental eccentricity in the design
eccentricity expressions is misleading when assessing the adequacy of code torsional provisions, particularly when neither uncertainties nor rotational ground motion are present in the dynamic analysis.

In view of the purpose of specifying the accidental eccentricity in the code design eccentricity expressions (as stated explicitly in the commentary of NBCC 90), and given that both uncertainties and the rotational ground motion are omitted in the dynamic analysis, a practical approach as employed in this thesis is not to include the accidental eccentricity in the code design eccentricity expressions when specifying element strengths. This approach retains the additional torque due to the accidental eccentricity for dealing appropriately with uncertainties and the possible effect of rotational ground motion. The magnitude of the accidental eccentricity needed to account for the latter effects is an issue to be studied elsewhere (see Rutenberg and Heidebrecht 1985 and Pekau and Guimond 1990). It is noted that in Tso and Ying (1990), if the accidental eccentricity had not been included, the static equilibrium model rather than the generic model 1 would have been the representative model designed in accordance with the New Zealand and the United States code torsional provisions.

The effect on element displacement ductility demand arising from the inclusion of the accidental eccentricity in the design eccentricity expressions is also studied in this section. Fig. 5.6 shows the displacement ductility demand of elements 1 and 3, for structures designed in accordance with the Mexico 76 code base shear and torsional provisions but including an accidental eccentricity $e_a$ of zero, 0.05$b$ and 0.1$b$ in the design eccentricity expressions. Three values of the stiffness eccentricity $e_s$ are considered as in previous results, that is, $e_s$ equals 0.1$b$, 0.2$b$ and 0.3$b$. Other parameters and the ground motion input are the same as stated above. It can be seen that including the accidental eccentricity in specifying element strength strongly affects the displacement ductility demand of element 1 at all stiffness eccentricities but only
slightly affects that of element 3, particularly for structures with moderate ($e_s=0.2b$) and large ($e_s=0.3b$) stiffness eccentricities. Judging from the long dashed curve ($e_s=0.1b$, element 1) in Figs. 5.6(a) and 5.6(b), one may arrive at the same conclusion as that of Tso and Ying (1990) stated above in relation to the Mexico 76 code. However, for conservative design, allowance should be given to the possible uncertainties and the rotational ground motion which are not considered in the dynamic analysis, and judgement based on the solid curve ($e_s=0$, element 1) or the short dashed curve ($e_s=0.05b$, element 1) in Figs. 5.6(b) and 5.6(c), which clearly indicate the inadequacy of the Mexico 76 code torsional provisions for this element. Consequently, the approach of including the accidental eccentricity in the design eccentricity expressions when carrying out inelastic dynamic analysis can lead to misleading results.

The second issue to be addressed is the need to use localised earthquake records or records having dissimilar frequency contents in the dynamic analysis. In order to assess the validity of code provisions applicable to a particular region, localised records should be used, whenever they are available. For instance, when evaluating the Mexico 76 code provisions, which are intended for use in the Federal District of Mexico primarily encompassing Mexico City, use should be made of records obtained from Mexico City, for example in the 1985 earthquake, as is the case herein. In order to generalise the applicability of conclusions drawn from dynamic analysis to other locations or regions, records having dissimilar frequency contents, or shapes of response spectra, and varying peak ground acceleration to velocity (a/v) ratios should be used. Tso and Ying (1990) employed eight records in their study, each having an intermediate a/v ratio and response spectrum shape similar to that of the standard Newmark-Hall type design spectrum. This approach limits the generalisation of the conclusions drawn from their inelastic dynamic analysis.
The third issue discussed here is the necessity to carry out inelastic analysis and present results over the full period range relevant to most actual buildings. It has been demonstrated in Chapter 4 and in Section 5.3.1 that when structures are designed on the basis of a Newmark-Hall type design spectrum with a total strength capacity substantially lower than the elastic strength demand, the effect of eccentricity on the increased or decreased displacement ductility demand of resisting elements is pronounced in different period regions for ground motions having low, intermediate and high a/v ratios. For instance, if records with intermediate a/v ratios are used as the ground motion input, the effect of eccentricity on the additional displacement ductility demand of element 1 is most pronounced in the very short-period (\(T_y < 0.25\) sec.) and the short-period (\(0.25\) sec. \(\leq T_y < 0.5\) sec.) regions. It is expected that Tso and Ying (1990) would have observed a more pronounced effect of stiffness eccentricity on the additional displacement ductility demand of element 1 if they had presented results for this element over the whole period range rather than for one period only, namely \(T_y=0.5\) sec.

### 5.3.3 Evaluation of the Mexico 87 code torsional provisions

The Mexico 87 code incorporates some radical changes compared with the Mexico 76 code torsional provisions, as described in detail by Esteva (1987) and Gomez and Garcia-Ranz (1988). The new regularity conditions now require that the force reduction factor \(Q\) be reduced by 20 per cent if \(e_x\) exceeds 0.1b. The new requirements added to the torsional provisions now specify that the strength eccentricity be at least \(e_x-0.2b\) if \(Q \leq 3\) and at least \(e_x-0.1b\) if \(Q > 3\), and that the centres of stiffness and strength be on the same side with respect to the centre of mass. The unique feature of the torsional provisions of the Mexico 87 code is that, unlike all other codes, it does not allow a linear elastic modal analysis to be carried out to deal with torsional coupling effects in asymmetric buildings. Instead, this code specifies explicitly that the
The effect of torsional eccentricity and accidental eccentricity shall be considered by the static analysis procedure outlined above. Modal analysis is used only to calculate the storey shears and in doing so, stiffness eccentricities at all floor levels are ignored. These new requirements are intended primarily to increase the strength capacity of elements at the stiff side and therefore to provide more protection to these elements against structural damage. This is in line with the results and recommendations given in Section 5.3.1 above.

The new Mexican code requirements are based on the research of Gomez et al. (1987) published in Spanish, which is also briefly presented by Esteva (1987), translated into English. In contrast to the present study and that of Tso and Ying (1990), Gomez et al. (1987) carried out inelastic dynamic analysis based on a single-storey three-element model having a symmetric distribution of stiffness but an asymmetric distribution of mass, and concluded that having the strength eccentricity much smaller than the stiffness eccentricity leads to excessive displacement ductility demand. Therefore, it was recommended that the centre of strength be close to the centre of rigidity, in accordance with the regulations given above. Tso and Ying (1990) stated that this is in contrast to the finding of Sadek and Tso (1989), who concluded that the maximum element displacement ductility demand decreases with a system’s decreasing strength eccentricity. This issue is clarified by the observation that Sadek and Tso (1989) were referring to the element at the flexible edge whilst Gomez et al. (1987) were apparently referring to the element at the stiff edge. Furthermore, systems based on the recommendations of Gomez et al. (1987), namely that PC should be close to CR, are different from the stiffness proportional model (PC coincides with CR) employed by Tso and Ying (1990). The approach adopted by Gomez et al. (1987) requires the element strength be specified firstly according to code design eccentricity expressions and then the strength of element 1 be increased to shift the centre of strength PC sufficiently close to the centre of rigidity CR. In contrast, the element strength in the
stiffness proportional model as studied by Tso and Ying (1990) is determined simply by applying the design base shear through the centre of rigidity, which ignores the torsional shear produced by the applied torque and therefore obviously underestimates the strength demand and leads to excessive displacement ductility demand of element 3.

In view of the significant changes introduced in the Mexico 87 code as the above discussion, a key issue arises. This concerns the effectiveness of the new provisions in providing satisfactory control over additional structural damage due to uneven distribution of stiffness. Evaluation of the Mexico 87 code torsional provisions is carried out in this section based on the same three element model as described in Section 4.2.4.2(a). The calculation of the design base shear is in accordance with the modified design spectra for group B buildings in zone III and the appropriate regularity conditions specified in the Mexico 87 code. The design spectral ordinates for group B buildings in Zone III in the Mexico 87 code, as illustrated in Fig. 5.7, have been increased by 67 per cent compared with those in the Mexico 76 code. The specification of element strength is achieved firstly by applying the design base shear at distances from the centre of rigidity CR equal to the design eccentricity expressions specified in the Mexico 87 code (which are the same as those in the Mexico 76 code, see eqns. (5.8) and to (5.9)) but excluding the accidental eccentricity 0.1b, and then increasing the strength of element 1 to shift the centre of strength towards element 1 in order to satisfy the new requirements. Here, the Q factor is again taken as 4.0, which is the highest value permitted in the Mexico 87 code, and the requirement that the strength eccentricity should be at least $e_r - 0.1b$ is interpreted as meaning equal to $e_r - 0.1b$, as in Tso and Ying (1990). Other parameters and the ground motion input are as given in Section 5.3.1 above.

Fig. 5.8 shows the displacement ductility demand of elements 1 and 3 over the period range of 0.1 to 2 seconds. The displacement ductility of element 1 decreases
rapidly with the increase of the stiffness eccentricity but the corresponding decrease of the displacement ductility of element 3 is only marginal. These findings agree with those shown in Fig. 6 of Tso and Ying (1990). Because of the additional requirements, there is a large increase in the strength of element 1 and hence a large increase in the overall strength as shown in Table II of Tso and Ying (1990). This leads to an unnecessarily conservative displacement ductility demand for element 1. As a result, unlike the Mexico 76 code, the torsional provisions in the Mexico 87 code are overly conservative. This over-conservatism is due to the use of a mass-eccentric model by Gomez et al. (1987). As studied in Section 4.8.1, if \( e_p \approx 0 \), the inelastic response of the stiff edge element in mass-eccentric systems is significantly higher than that in stiffness-eccentric systems. As a result, the Mexico 87 code torsional provisions are found to be overly conservative when assessed with reference to stiffness-eccentric systems, which are more common in practice than their mass-eccentric counterparts.

5.3.4 Evaluation of the New Zealand 89 code torsional provisions — modal analysis procedure

In contrast to the Mexico 87 code, the 1989 edition of the New Zealand Code of Practice for General Structural Design and Design Loadings for Buildings (Standards Association of New Zealand 1989), henceforth referred to as the New Zealand 89 code, sets severe restrictions for the application of the equivalent static force procedure and relies strongly on the linear elastic modal analysis procedure for the analysis of asymmetric buildings. The modal analysis procedure can be applied to all buildings. However, the static force procedure can only be used if buildings are regular both vertically and horizontally and there is no significant rotation of the floor. The latter condition is called the torsional stability condition which implies a small stiffness eccentricity or a high torsional stiffness (the latter implies a high value of \( \Omega \)). The New
Zealand 89 code quantitatively defines this condition such that under the application of the equivalent static loading at a point 0.1b from the centre of mass at each floor level, the difference in the lateral displacement between the two sides of the structure at any level in the direction of the loading does not exceed half the sum of the lateral displacement of the two sides in the same direction, as shown in Fig. 5.9. To satisfy this condition, asymmetric buildings must have either a small stiffness eccentricity or a high value of the uncoupled torsional to translational frequency ratio, as given for typical building properties in Table 5.2.

Table 5.2 Torsional stability condition of the New Zealand 89 code

<table>
<thead>
<tr>
<th>Stiffness eccentricity es</th>
<th>Required* uncoupled torsional to translational frequency ratio Ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>es = 0.05b</td>
<td>Ω ≥ 1.13</td>
</tr>
<tr>
<td>es = 0.1b</td>
<td>Ω ≥ 1.27</td>
</tr>
<tr>
<td>es = 0.2b</td>
<td>Ω ≥ 1.47</td>
</tr>
<tr>
<td>es = 0.3b</td>
<td>Ω ≥ 1.59</td>
</tr>
</tbody>
</table>

* for application of the static force procedure

The base shear formula of the New Zealand 89 code is:

\[ V_{yo} = C(T_y)W \]

\[ = C_0(T_y, \mu)RZW \]  

(5.13)

in which \( W \) is the total weight of the building; \( R \) is the risk factor taken to be unity for buildings with normal occupancy or usage, as in this study; \( Z \) is the zone factor which is assigned a value of 0.8 in this section, this being the highest value in the New Zealand
89 code implying a site with severe seismicity and a peak site ground acceleration of 0.32g, and $C_0(T_y, \mu)$ is the seismic response factor which is a function of $T_y$ and the target global displacement ductility $\mu$ as illustrated in Fig. 5.10 for stiff soil sites.

Unlike all other codes, the New Zealand 89 code does not employ the force reduction factor $R$ to obtain the inelastic design base shear from the elastic design spectrum. Instead, the inelastic design spectra for different levels of global displacement ductility are obtained by adjusting the inelastic base shear relative to the elastic response spectrum ($\mu = 1.0$), in order to achieve an average displacement ductility demand of a SDOF system (for six strong earthquake records) close to each target ductility level (Hutchison et al. 1986). The latter approach has the advantage of achieving a uniform displacement ductility demand over the entire period range, whilst the former one leads to much higher displacement ductility demand and normalised hysteretic energy ductility demand in the very short and short-period regions than in the medium and long-period regions, as shown in Chapter 4 and Section 5.3.1.

The New Zealand 89 code specifies that for vertically and horizontally regular buildings, if the above torsional stability condition is satisfied, the equivalent static force procedure can be used and the equivalent static forces are applied through points $\pm 0.1b$ from the centre of mass at each floor. Therefore, the design eccentricity expressions can be written as:

$$e_{D1} = 1.0e_x + 0.1b$$  \hspace{1cm} (5.14) \\
$$e_{D2} = 1.0e_x - 0.1b$$  \hspace{1cm} (5.15)

The secondary design eccentricity expression, eqn. (5.15), is identical to that of the Mexico 76 code. As has been evaluated in Section 5.3.1, this provision is inadequate since it substantially underestimates the strength demand of the elements at the stiff side when asymmetric buildings are excited well into the inelastic range. When asymmetric
buildings are responding elastically to a moderate earthquake, the New Zealand 89 code primary design eccentricity expression, eqn. (5.14), underestimates the elastic strength demand of the elements at the flexible side, because the adverse effect of torsion on these elements due to the dynamic amplifying effect of the static eccentricity is most pronounced for asymmetric buildings having a small stiffness eccentricity and $\Omega$ close to unity. For these buildings, the static force method is applicable according to the New Zealand 89 code.

If the above torsional stability condition is not satisfied, a modal analysis has to be carried out. The rest of this section examines the validity of the modal analysis procedure for the design of asymmetric buildings which are excited well into the inelastic range.

A target displacement ductility of 6, which is the highest allowable in the New Zealand 89 code, is chosen. The three-element stiffness-eccentric building model with a double-step stiffness variation, as described in Section 4.2.4.2(a), is analysed by the linear elastic modal analysis method employing the elastic design spectrum ($\mu = 1.0$) in the New Zealand 89 code (see Fig. 5.10). The modal analysis is carried out using the enhanced version of the computer program SUPER-ETABS (Maison and Neuss 1983), for elastic dynamic analysis of three-dimensional building systems. The elastic strength demand of the individual resisting elements is obtained directly from the program. As is well known, the combined elastic strength of the resisting elements calculated by modal analysis is lower than that given by the equivalent static force method, eqn. (5.13), using the uncoupled lateral period and the elastic design spectrum ($\mu = 1.0$). In accordance with the New Zealand 89 code specification, the strength of each of the resisting elements is scaled down by a fixed proportion, to obtain the strength for each element and to achieve a total combined strength equal to that given by eqn. (5.13) using the uncoupled lateral period and the target displacement ductility of 6.
Having determined the strength distribution of the analytical model, inelastic time history analysis has been carried out for two earthquake records, namely the El Centro S00E and the 3470 Wilshire Blvd. N00E records. These records have been scaled to a common peak ground acceleration of 0.32g. Fig. 5.11 presents the displacement ductility demand of elements 1 and 3 of asymmetric buildings for three stiffness eccentricity values, 0.1b, 0.2b and 0.3b respectively. The results for the reference symmetric SDOF systems are also included for comparison. It is apparent that modal analysis and the equivalent static force method result in the same qualitative and quantitatively similar inelastic performance of asymmetric buildings. The element at the stiff edge, element 1, is again the critical element experiencing much higher inelastic response than the reference symmetric system and the displacement ductility demand of the element at the flexible edge is correspondingly much lower than that of the reference symmetric system. Modal analysis can lead to satisfactory inelastic performance of asymmetric buildings having a small stiffness eccentricity ($e_s \leq 0.1b$), but results in very poor inelastic performance of asymmetric buildings having moderate to high stiffness eccentricities ($e_s \geq 0.2b$). It is concluded therefore that the modal analysis procedure and the New Zealand 89 code torsional provisions are inadequate for the design of highly asymmetric buildings when they are shaken well into the inelastic range.

Fig. 5.11 also indicates that the New Zealand 89 code base shear provisions do result in a more or less uniform displacement ductility distribution over the entire period range, especially for symmetric buildings and asymmetric buildings with a small stiffness eccentricity. It is therefore recommended that code-drafting authorities in other countries should adopt the New Zealand code practice for specifying the design base shear, as described in Hutchison et al. (1986).
5.3.5 Evaluation of the Eurocode 8 torsional provisions

Eurocode No 8: Design for structures in seismic regions (Commission of the European Communities, May 1988 edition), henceforth referred to as Eurocode 8, will in due course become a statutory earthquake resistant design code for structures in seismic regions in European Economic Community member countries and will ultimately replace the present national earthquake resistant design codes of the Community nations. It is the only one among current major aseismic building codes which considers the aspect ratio and the structure's torsional to translational stiffness ratio in its torsional provisions. In the case of a single-storey building, this ratio can be related directly to the uncoupled torsional to translational frequency ratio \( \Omega \). The design eccentricity expressions are as follows:

\[
e_{D1} = 1.0e_s + e_1 + 0.05b \\
e_{D2} = 1.0e_s - 0.05b
\]

in which \( e_1 \) is the additional eccentricity taking account of the dynamic torsional coupling amplification effect on the stiffness eccentricity. This additional eccentricity is assigned the lower of the following two values:

\[
e_1 = 0.1(a + b)\sqrt{10e_s/b} \leq 0.1(a + b) \\
e_1 = \frac{1}{2e_s}\left[r^2 - e_s^2 - \rho^2 + \sqrt{(r^2 + e_s^2 - \rho^2)^2 + 4e_s^2\rho^2}\right]
\]

In eqn. (5.19), \( r \) is the radius of gyration of the floor slab and \( \rho^2 \) is the structure's torsional to translational stiffness ratio, that is, \( \rho^2 = K_{\theta}/K_y \).

Eurocode 8 is also unique among current major aseismic building codes since it provides a satisfactory approximation, \( e_s + e_1 \), for the dynamic eccentricity \( e_{d1} \) obtained
by elastic time history analysis (Chandler and Hutchinson 1987, Chandler 1988). Similar to the New Zealand 89 code, Eurocode 8 also limits the application of the above static force method. The static force procedure is applicable to regularly asymmetric buildings having a small stiffness eccentricity or a large torsional stiffness. The regularity requirements of Eurocode 8 demand that the plan configuration of buildings should be compact and regular and that stiffness and mass properties of buildings are approximately uniformly distributed over the building's height. The centres of stiffness of the individual storeys should lie approximately on one vertical line and the centres of mass of the individual floor slabs should also lie on one vertical line. Furthermore, Eurocode 8 requires that at any storey the distance between the centre of stiffness and the centre of mass should not exceed 0.15\( \rho \), in which \( \rho \) is the square root of that storey's torsional to translational stiffness ratio. If these conditions are not satisfied, then the static force procedure is not allowed, and a modal analysis must be carried out.

Although the Eurocode 8 primary design eccentricity expression \( e_s+e_1 \) matches the results of elastic dynamic analysis well, the provisions do have significant shortcomings. Some confusion exists regarding their application in practice, in addition to their complexity which is obvious when comparing with the other code formulae listed in Table 5.1. Firstly, the condition \( e_s \leq 0.15\rho \) is difficult to verify or evaluate in the case of multistorey buildings, and furthermore it is over restrictive or even superfluous because the Eurocode 8 primary design eccentricity expression \( e_s+e_1 \) matches the results of dynamic analysis very well for torsionally stiff asymmetric buildings, and is conservative for torsionally flexible buildings for all realistic stiffness eccentricity values \( 0 \leq e_s \leq 0.5b \) (Chandler and Hutchinson 1987, Chandler 1988). Therefore, it is strongly recommended that this restriction be removed.

As mentioned above, the Eurocode 8 design eccentricity expressions and the restrictive conditions for their use are not directly applicable to multistorey buildings.
Confusion arises regarding the definition of the structure’s torsional to translational stiffness ratio in eqn. (5.19). For a single storey building, this ratio is clearly and easily defined. However, for a multistorey building, either an estimate of the overall stiffness of the building in torsional and in translation may be required, for which there is no well established procedure or guidance available, or in the case of regularly asymmetric buildings in which the torsional to translational stiffness ratio of individual storeys is constant throughout the height of the building, an estimate can be made on a storey-to-storey basis. Furthermore, the Eurocode 8 regularity condition requiring that the centre of stiffness of individual storeys be located approximately on one vertical line and that the distance between the centre of stiffness and the centre of mass at any level should not exceed 15 per cent of the square root of that storey’s torsional to translational stiffness ratio may be erroneous in the case of certain types of multistorey building.

Although in a single storey building, the centre of stiffness and the centre of rigidity refer to the same point, this is not the case in a multistorey building. As defined by Stafford Smith and Vezina (1985) and Cheung and Tso (1986a), given a set of lateral loads, the centres of rigidity are the set of points at floor levels such that when the given set of lateral loads passes through them, no rotation of the floors will occur. The locations of the centres of rigidity are therefore generally load dependent and are different from the centres of stiffness of individual storeys determined on a floor-to-floor basis, considering only the resisting elements directly supporting the floor slab in question. In particular, in asymmetric hybrid buildings in which wall-frame interaction is significant, the centres of stiffness of individual storeys may lie on one vertical line, but the centres of rigidity at floor levels are scattered from a vertical line. For the design of asymmetric buildings, of most importance are the locations of the centres of rigidity at floor levels and the distances between these points and the corresponding centres of mass of floor slabs, namely the values of stiffness eccentricity. Consequently, the above regularity conditions of Eurocode 8 should be changed firstly
in order to specify that the centres of rigidity at floor levels corresponding to the given set of lateral loads should be located approximately on one vertical line, and secondly to remove the restriction on the distance between the centres of stiffness and mass.

The design base shear \( V_{y0} \), according to Eurocode 8, is given by

\[
V_{y0} = \alpha s \beta(T_y)W
\]  

(5.20)

where \( \alpha \) is the ratio of the site peak ground acceleration to the acceleration of gravity, chosen herein to be 0.3; \( s \) is the soil parameter, being 1.0 for soil profiles A (rock or extended layers of very stiff soil) and B (stiff soil), or 0.8 for soil profile C (soft soil); \( W \) is the weight of the building; \( \beta(T_y) \) is the normalised design spectrum specifying the variation of the design base shear with respect to the uncoupled lateral period \( T_y \), as shown in Fig. 5.12. In Fig. 5.12, \( q \) is the force reduction factor, or the structural behaviour factor as defined in Eurocode 8, which takes into account the structure's energy dissipation capacity through inelastic ductile behaviour. Except at very short periods \( T_y \leq T_1 \), the design base shear is obtained simply by dividing the elastic strength demand by \( q \). The structural behaviour factor can also be considered as the target displacement ductility allowed by Eurocode 8 for certain types of structural systems. In this section, the structural behaviour factor is taken to be 4, a commonly employed value for ductile steel and reinforced concrete structural systems.

Three records from European strong motion earthquake events have been selected as ground motion input for the assessment of the Eurocode 8 torsional provisions. These records have different frequency contents and have low (Romania), intermediate (Thessaloniki) and high (Patras) a/v ratios respectively. Table 5.3 summarises the key characteristics of these records and their time histories are plotted in Fig. 5.13. The 5% damped elastic acceleration response spectra of these selected records, normalised to a
common peak ground acceleration of 0.3g are plotted in Fig. 5.14. Also shown are the 5% damped inelastic design spectra stipulated by Eurocode 8, taking the structural behaviour factor \( q=4 \) and the parameter \( \alpha=0.3 \).

Table 5.3 Key characteristics of selected European earthquake records

<table>
<thead>
<tr>
<th>Site</th>
<th>Date</th>
<th>Component</th>
<th>Soil Type</th>
<th>Strong motion duration (sec.)</th>
<th>Max. Acc. (g)</th>
<th>Max. Vel. (m/s)</th>
<th>Max. Disp. (m)</th>
<th>a/v ratio (g/m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patras</td>
<td>29/1/74</td>
<td>Trns.</td>
<td>A</td>
<td>4</td>
<td>0.296</td>
<td>0.062</td>
<td>0.014</td>
<td>4.74</td>
</tr>
<tr>
<td>Thessaloniki</td>
<td>20/6/78</td>
<td>Horiz.A</td>
<td>B</td>
<td>20</td>
<td>0.316</td>
<td>0.343</td>
<td>0.134</td>
<td>0.922</td>
</tr>
<tr>
<td>Romania</td>
<td>4/3/77</td>
<td>NS</td>
<td>C</td>
<td>22</td>
<td>0.203</td>
<td>0.808</td>
<td>0.291</td>
<td>0.252</td>
</tr>
</tbody>
</table>

The inelastic response of the reference symmetric systems and asymmetric buildings to these earthquake records are presented in Fig. 5.15 for three stiffness eccentricity values, 0.1b, 0.2b and 0.3b respectively. It is apparent that, similar to the Mexico 76 code, the Eurocode 8 torsional provisions substantially underestimate the strength demand of the resisting element at the stiff edge. Despite the significant increase of the total yielding strength of asymmetric buildings over the design base shear of the corresponding symmetric buildings (by approximately 20%, see Fig. 5.27), such provisions do not offer consistent protection to symmetric and asymmetric buildings against structural damage, leading to excessively high displacement ductility demand for the stiff-side elements of asymmetric buildings compared with that of symmetric buildings.
Again, Fig. 5.15 clearly illustrates the effect of the stiffness eccentricity, the uncoupled lateral period $T_y$, and the $a/v$ ratio of the ground motion on the additional displacement ductility demand of asymmetric buildings. The additional displacement ductility demand of element 1 rises rapidly with the increase in the stiffness eccentricity, being pronounced in the very short-period region for ground motions having a high $a/v$ ratio (Patras Tran.), in the very short-period and short-period regions for those having an intermediate $a/v$ ratio (Thessaloniki Horiz. A), and in the medium-period and long-period regions for those having a low $a/v$ ratio (Romania NS).

### 5.3.6 Evaluation of the Canadian code torsional provisions

NBCC 90 has some unique features compared with other building codes. First of all, it is the only code whose base shear formula is linked to the peak ground velocity, $v$, rather than the peak ground acceleration, $a$, as in all other codes. The base shear formula in NBCC 90 is:

$$v_{y,0} = \frac{V_e}{R} U$$

(5.21)

in which $V_e$ is the elastic strength demand, $R$ is the force reduction factor and $U=0.6$. The elastic strength demand $V_e$ is linked to the site peak horizontal velocity as follows:

$$V_e = vSIFW$$

(5.22)

where $v$ is the zonal velocity ratio which is the ratio of the zonal peak ground velocity to a velocity of 1m/sec, $S$ is the seismic response factor specifying $V_e$ as a function of the fundamental lateral period, $I$ is the importance factor, $F$ is the foundation factor and $W$ the weight of building.
NBCC 90 specifies three branches in the very short-period ($T < 0.25$ sec.) and short-period ($0.25$ sec. $\leq T < 0.5$ sec.) regions for the seismic response factor $S$ (see Fig. 5.16), in order to deal with ground motions having low, intermediate, and high $a/v$ ratios. This approach has the advantage of leading to consistent control over structural damage when structures are excited by ground motions with dissimilar frequency contents, whilst structural damage is highly variable if the base shear formula is simply linked to the peak ground acceleration (Zhu et al. 1988 and 1989) as prescribed by all other codes.

Secondly, NBCC 90 is the only code which does not allow dynamic analysis to be used as an alternative to the static approach given by the base shear formula eqn. (5.21) for the determination of the base shear, as discussed by Heidebrecht and Tso (1985, 1986) and Ferahian (1986). It only requires a dynamic analysis to be carried out to determine the vertical and the horizontal distribution of the static base shear in irregularly asymmetric buildings in which the centres of mass and rigidity do not lie approximately on two vertical lines. For analysis of regularly asymmetric buildings, both the equivalent static force procedure and dynamic analysis are allowed to be used to determine the vertical and the horizontal distribution of the static base shear. The reason for this restriction on the application of dynamic analysis is that dynamic analysis usually results in a lower base shear compared with that calculated using the equivalent static force procedure. Other codes specify dynamic analysis as an alternative approach to the equivalent static force formula without imposing any limits to the use of the dynamic analysis, but require that the total combined strength be scaled up to at least 90 per cent (the Uniform Building Code of the United States) or 100 per cent (New Zealand 89 code) of the base shear calculated from the static base shear formula. However, the Mexico 87 code and Eurocode 8 do not require such a modification to the results of dynamic analysis.
Thirdly, compared with Eurocode 8, or the New Zealand 89 and Mexico 76 codes, NBCC 90 allows less reduction for the design loading of the resisting elements at the stiff side as a result of the secondary design eccentricity expression. These elements are traditionally considered favourably affected by torsion and therefore their design loading is allowed to be decreased by the above mentioned codes. The design eccentricity expressions of NBCC 90 are:

\[
e_{D1} = 1.5e_s + 0.1b \\
e_{D2} = 0.5e_s - 0.1b
\]

Unlike the New Zealand 89 code and Eurocode 8, NBCC 90 does not require that asymmetric buildings have a small stiffness eccentricity or a high torsional stiffness for the application of the static force procedure. It only requires that the centres of mass and centres of rigidity at different floor levels lie approximately on two vertical lines. In other cases, a dynamic analysis should be carried out to determine the torsional effects.

The displacement ductility demand of resisting elements in asymmetric buildings designed in accordance with the NBCC 90 base shear and torsional provisions are presented in Fig. 5.17. These buildings are assumed to be built on stiff soil sites in a zone with moderately high seismicity (the foundation factor \( F=1.0 \), for a site in Zone 4 defined according to zonal velocity, with \( Z_v=4 \) and \( v=0.2 \)). The buildings analysed consist of steel or reinforced concrete ductile moment-resisting space frame systems (the force reduction factor \( R=4 \), the highest in NBCC 90), and have normal importance (the importance factor \( I=1.0 \)). Three strong motion records from earthquake events in the United States recorded on stiff soil sites are employed as the ground motion input. They are Cholame Shandon No. 5 N85E Parkfield (high a/v ratio), El Centro S00E Imperial Valley (intermediate a/v ratio), and 3470 Wilshire Blvd. N00E San Fernando (low a/v ratio). The time histories and the 5 per cent damped elastic response spectra of
these records scaled to a common peak ground acceleration of 0.3g are shown in Figs. 4.4 and 4.5 respectively. In this section, these three records have been scaled to a common peak velocity of 0.2m/sec. The 5 per cent damped elastic response spectra of these records scaled to this common peak ground velocity, and the design spectra specified in NBCC 90 corresponding to $v=0.2$, $F=1.0$, $I=1.0$ and the force reduction factor $R=4$ are shown in Fig. 5.18. In the period range $T_y \leq 0.5$ sec., the base shear $V_y$ of the buildings is determined by the branch $Z_y > Z_r$ (see Fig. 5.16) when considering the Cholame Shandon No. 5 N85E record as input motion, branch $Z_y = Z_r$ when considering the El Centro S00E record and branch $Z_y < Z_r$ when considering the 3470 Wilshire Blvd. N00E record.

Because NBCC 90 allows less reduction of the design loading for elements at the stiff side of the building, the code leads to slightly better inelastic performance of asymmetric buildings compared with the Mexico 76 code and Eurocode 8. The displacement ductility demand of element 1 in buildings having a small stiffness eccentricity ($e_z = 0.1b$) is about the same as that of the corresponding symmetric reference systems. However, the NBCC 90 code torsional provisions are not adequate for buildings having moderate and large stiffness eccentricities ($e_z = 0.2b$ and $0.3b$). They still result in excessively high additional displacement ductility demand for element 1 in these buildings.

5.3.7 Evaluation of the torsional provisions of the Uniform Building Code of the United States

Unlike other countries which impose a single building code for nationwide use, the United States has several documents specifying provisions for earthquake resistant design of buildings, both regional and national. The 1988 edition of the Recommended Lateral Force Requirements and Tentative Commentary (Structural Engineers
Association of California 1988), henceforth referred to as SEAOC 88, is intended for use in California. The 1988 edition of the Uniform Building Code (International Conference of Building Officials 1988), henceforth referred to as UBC 88, and the 1988 edition of the Provisions for Development of Seismic Regulations for New Buildings recommended by the National Earthquake Hazards Reduction Program (1988) (henceforth referred to as NEHRP 88) are for use across the United States. Among these three, UBC 88 is a building code and the other two are reference documents covering earthquake resistant design only. UBC 88 incorporates most of the provisions in SEAOC 88 with only minor modifications, changing the local character of SEAOC 88 to suit national use by adding a zone factor and the corresponding seismicity map to cover the entire United States (Luft 1989).

This section assesses the torsional provisions of UBC 88. Unlike all other codes, UBC 88 explicitly states in the general design requirements (Sentence 2 of Section 2303) that the design loading of structural elements should not be decreased due to torsional effects. The design eccentricity formulae of UBC 88 can therefore be written as:

\[ e_{D1} = 1.0e_s + 0.05b \]  
\[ e_{D2} = 0 \]  

(5.25)  
(5.26)

The design base shear is determined as follows:

\[ V_{yo} = \frac{ZIC}{R_w} W \]  

(5.27)

where

\[ 0.075 \leq C = \frac{1.25S}{T_y^2} \leq 2.75 \]  

(5.28)
The parameter $C$ is the period-dependent numerical coefficient derived from smoothed earthquake design response spectra and is shown in Fig. 5.19. $S$ is the soil factor, $Z$ the zone factor, $I$ the importance factor and $R_w$ the force reduction factor, the latter ranging from 4 to 12. There are two main reasons for this comparatively large force reduction factor specified by UBC 88, as discussed by Uang (1991). UBC 88 recognizes that structures usually have a significant reserve of strength beyond the minimum required by codes and therefore takes advantage of structural overstrength to reduce further the seismic design loading by an overstrength factor $O_s$. This is in addition to reduction due to consideration of a structure’s displacement ductility and energy dissipation capacities. Furthermore, UBC 88 is a building code based on the working stress design method in which the working stress (or allowable stress) is a fraction of the material’s yielding stress. Therefore, the seismic design loading is reduced again by a factor $Y (>1.0)$ to reach the working stress loading. As a result, the force reduction factor of UBC 88 can be expressed as:

$$R_w = R_{\mu}O_s Y$$

(5.29)

in which $R_{\mu}$ is the ductility reduction factor similar to the force reduction factor employed in other codes. UBC 88 considers the beneficial contribution of structural overstrength, it nevertheless does not require designers to check whether or not sufficient overstrength does exist. Therefore, the base shear provisions of UBC 88 may lead to insufficient strength capacity if the ultimate strength is not checked.

Fig. 5.20 shows the displacement ductility demand of resisting elements in asymmetric buildings responding inelastically to the Cholame Shandon No. 5 N85E, El Centro S00E and 3470 Wilshire Blvd. N00E records. These three records are scaled to a common peak ground acceleration of 0.4g (the value specified for the Californian coastal region). The displacement ductility demand of the corresponding reference
symmetric buildings is also shown in Fig. 5.20 for comparison. These buildings are
designed in accordance with the UBC 88 base shear (corresponding to the force
reduction factor \( R_w = 12 \)) and torsional provisions and are assumed to be located on stiff
soil sites (the soil factor \( S = 1.0 \)) in Zone 4, which has the highest seismicity (the zone
factor \( Z = 0.4 \) implying a peak ground acceleration of 0.4g) and have normal occupancy
(the importance factor \( I = 1 \)). It is clear that because UBC 88 does not allow any
reduction of design loading due to torsion, it leads to satisfactory inelastic performance
of asymmetric buildings. The displacement ductility demand of element 1 is close to
that of the corresponding reference symmetric structures for small \( (e_s = 0.1b) \), moderate
\( (e_s = 0.2b) \) and large \( (e_s = 0.3b) \) stiffness eccentricities and for earthquake records having
different a/v ratios. Consequently, the UBC 88 torsional provisions are adequate for the
design of asymmetric buildings when they are excited well into the inelastic range.

However, the UBC 88 torsional provisions are not adequate when asymmetric
buildings are responding elastically to moderate earthquakes. The primary design
eccentricity expression, eqn. (5.25), does not provide any allowances for the dynamic
amplification of the static eccentricity \( e_s \), which is pronounced due to elastic torsional
coupling when the static eccentricity is small and the uncoupled torsional to
translational frequency ratio is close to unity.

5.4 Study and recommendation of a new unified approach for
torsional design

The results presented in Section 5.3 have shown that the torsional provisions of
current major building codes have various inadequacies. Those in Mexico 76, New
Zealand 89, Eurocode 8, and NBCC 90 substantially underestimate the strength demand
of the resisting elements at the stiff side of the structure and lead to excessively high
additional displacement ductility demand of these elements over that of corresponding
reference symmetric buildings. In contrast, the torsional provisions in the Mexico 87 code have included additional requirements which correctly increase the strength capacity of the elements at the stiff side. These new requirements result in much lower displacement ductility demand of asymmetric buildings compared with that of corresponding reference symmetric systems. However, in view of the large increase in the overall strength (by a factor of up to 2.5), the Mexico 87 code provisions have been found to be overly conservative. Only the torsional provisions in UBC 88 provide consistent protection to both symmetric and asymmetric buildings against structural damage. Nevertheless, they are inadequate if asymmetric buildings are responding elastically to moderate earthquakes, significantly underestimating the elastic strength demand of the resisting elements at the flexible side.

There are two main objectives in earthquake resistant design, namely to ensure safety of life and to protect property. Recognizing these objectives, the present earthquake resistant design philosophy requires that buildings in seismic prone regions be designed to resist a major earthquake without collapse of failure and to resist moderate earthquakes without structural damage. Furthermore, these objectives should be achieved at the lowest possible additional costs. Otherwise, the objective of protecting property would be undermined. In the light of the above discussion, neither the static torsional provisions in any of the current major aseismic building codes, or the dynamic modal analysis method is adequate for the effective design of asymmetric buildings, which are common in engineering practice due to architectural and functional reasons. In order to prevent failure due to structural asymmetry and to offer effective and consistent control over structural damage to both symmetric and asymmetric buildings, there is a need to develop new recommendations for a refined design approach which is widely applicable, offers conservative estimates for the design loading of individual resisting elements, retains simplicity for ease of code implementation and results in relatively low additional cost.
Such a design approach should lead to satisfactory elastic as well as inelastic seismic performance of asymmetric buildings. It should give conservative estimates for the elastic strength demand of individual resisting elements and result in displacement ductility demand for asymmetric buildings which is lower or around that of symmetric buildings. When determining the design loading for elements at the flexible side, which are the critical elements in the case of the torsionally coupled elastic response of asymmetric buildings, adequate allowance must be given to account for the dynamic amplifying effect of the static eccentricity $e_s$ due to torsional coupling. An amplification factor of 1.5, as suggested in the Mexico 76 and 87 codes and in NBCC 90 and which is based on the early work of Rosenblueth and Elorduy (1969) (see also Newmark and Rosenblueth 1971), is overly simplistic and fails to account for the important influence of the frequency ratio $\Omega$ on elastic torsional coupling. Therefore, in many cases it underestimates the elastic strength demand of elements at the flexible side, and in particular, for asymmetric buildings having small to moderate static eccentricity and $\Omega$ close to unity. However, such a provision matches the results of elastic dynamic analysis reasonably well for buildings with moderate to large static eccentricities.

On the other hand, the Eurocode 8 primary design eccentricity expression, as evaluated by Chandler and Hutchinson (1987), matches the results of elastic dynamic analysis very well at all static eccentricity values if $\Omega \geq 1$ and provides conservative estimates for the elastic strength demand of the resisting elements at the flexible side in torsionally flexible asymmetric buildings ($\Omega < 1.0$). However, this design eccentricity expression is too complex and has difficulties in application to practical design situations, as discussed in Section 5.3.5.

Rutenberg and Pekau (1987) evaluated the mean plus one standard deviation of the dynamic peak displacement of resisting elements at the flexible edge, corresponding to five strong earthquake records, and hence determined the lateral design loading. This
was then compared with that produced by the design base shear and torque determined by the primary design eccentricity. Rutenberg and Pekau found that the required eccentricity amplification factor is in the range of 2.5 to 3 for small eccentricities and approaches unity for large eccentricities. Adopting a similar approach, Chandler and Hutchinson (1988b) proposed the following formula as the primary design eccentricity expression:

\[ e_{D1} = (2.6 - 3.6e_s/b)e_s + 0.1b \]  

(5.30)

This design eccentricity expression gives an estimate for the peak displacement of the flexible edge element which is close to the mean plus one standard deviation of the peak dynamic flexible edge displacement obtained from analysis of the response to seven strong earthquake records. It corresponds to an amplification factor of 2.6 at small eccentricities and approaches unity at large eccentricities. It is also simple in form and straightforward in application. Therefore, it is adopted in this section as the recommended primary design eccentricity expression.

In view of earlier results and conclusions, the adequate specification of the design loading for elements at the stiff side is the key issue to be addressed in this section. Section 5.3.4 demonstrated that the linear elastic modal analysis procedure is inadequate for specifying the design loading for resisting elements at the stiff side of asymmetric buildings having moderate to large stiffness eccentricities and which are excited well into the inelastic range. One possible solution to this issue is to remove the option to employ linear elastic modal analysis for this purpose and to improve the equivalent static force procedure by modifying the secondary design eccentricity expression, and the base shear provisions if necessary, to increase the design loading for resisting elements at the stiff side. The secondary design eccentricity expression usually controls the design loading of these elements. In the case of elastic response, Rutenberg and
Pekau (1987) found that except for torsionally flexible asymmetric buildings, the following secondary design eccentricity formula provides a good estimate for the dynamic peak displacement, and hence the elastic strength demand, of elements at the stiff side:

\[ e_{p2} = 0.5e_s - 0.1b \]  \hfill (5.31)

It should be noted that eqn. (5.31) is exactly the same as that suggested by NBCC 90. It is expected that the design eccentricity expressions suggested in eqns. (5.30) and (5.31) can give satisfactory elastic performance of asymmetric buildings having \( \Omega \geq 1.0 \), whilst torsionally flexible asymmetric buildings should be avoided in design, as concluded in Chapters 3 and 4.

The inelastic performance of asymmetric buildings designed in accordance with the above recommended design eccentricity expressions is an issue of considerable interest and is therefore studied in this section. The displacement ductility demands of asymmetric buildings with the design base shear \( V_{y0} \) determined in accordance with the Mexico 87 code design spectrum with the force reduction factor \( Q=4 \) (see Fig. 5.7), and the strength distribution determined employing the above recommended design eccentricity expressions but excluding the accidental eccentricity 0.1b, are shown in Fig. 5.21. The Mexico City SCT1 EW record (see Figs. 5.3 and 5.4 for its time history and elastic response spectrum) is used as the ground motion input. It has been widely recognized that this earthquake record is an abnormal one which has an extremely low a/v ratio of 0.28g/(m/s) and a strong periodical pattern with a dominant period of about 2 seconds. Employing this record as the ground motion input leads to conclusions applicable to the lake bed district of Mexico City but which may not necessarily be applicable to other seismic prone regions. Hence, other more typical records are also employed in the following part of this section in order to achieve a widely applicable
design approach.

The displacement ductility demand of the corresponding reference symmetric buildings, which have fundamental lateral periods equal to the fundamental uncoupled lateral periods of the asymmetric systems and total strengths equal to the design base shears determined by the above mentioned design spectrum, is also shown in Fig. 5.21 for comparison. It can be seen that the above recommended design eccentricity expressions (eqns. (5.30) and (5.31)) lead to satisfactory inelastic performance of asymmetric buildings having a small stiffness eccentricity ($e_s = 0.1b$). The displacement ductility demand of element 1 in asymmetric buildings having moderate ($e_s = 0.2b$) and large ($e_s = 0.3b$) stiffness eccentricities is still excessive in the long period region ($T_y > 1.0$ sec.) compared with that of the corresponding symmetric buildings.

The inelastic performance of asymmetric buildings having moderate and large stiffness eccentricities can be improved by adequately increasing the yielding strength of resisting elements at the stiff side. This could be achieved by changing the factor 0.5 in eqn. (5.31) to appropriately smaller values corresponding to moderate and large stiffness eccentricities. However, this approach would undermine the simplicity of the form of the secondary design eccentricity expression as recommended in eqn. (5.31), which already leads to adequate elastic response of resisting elements at the stiff side for any value of the stiffness eccentricity (Rutenberg and Pekau 1987) as well as producing satisfactory inelastic performance of these elements for buildings with a small stiffness eccentricity. It is therefore advantageous to preserve the form of the secondary design eccentricity as recommended in eqn. (5.31). An alternative way of increasing the design loading for elements at the stiff side is to reduce the force reduction factor in the base shear provisions for asymmetric buildings having moderate to large stiffness eccentricities. Chapter 4 has concluded that reducing the force reduction factor, hence increasing the total strength of asymmetric buildings, not only
decreases the overall inelastic response of resisting elements but also decreases the
effect of plan asymmetry on the additional displacement ductility demand of elements at
the stiff side. Therefore, such an approach is adopted in this section.

The displacement ductility demands of asymmetric buildings having moderate
\(e_s=0.2b\) and large \(e_s=0.3b\) stiffness eccentricities respectively, together with that of
the corresponding reference symmetric buildings, are presented in Fig. 5.22. The design
base shear \(V_{yo}\) of these asymmetric buildings is firstly calculated in accordance with the
Mexico 87 code design spectrum corresponding to the force reduction factor \(Q=4\) (see
Fig. 5.7). Then, the base shear values are increased by 10 per cent, 25 per cent and 50
per cent, respectively. Finally, the yielding strength of resisting elements is determined
by applying the increased base shear at distances equal to the design eccentricity
expressions recommended in eqns. (5.30) and (5.31) but excluding the accidental
eccentricity 0.1b. The Mexico City SCT1 EW record is again used as the ground motion
input.

Fig. 5.22 clearly demonstrates that whilst a 10 per cent increase in the design base
shear is adequate for asymmetric buildings with a moderate stiffness eccentricity
\(e_s=0.2b\), Fig. 5.22(a)), it nevertheless still leads to an unacceptably large increase in the
displacement ductility demand of element 1 in the long period region \(T_y>1.0 \text{ second}\)
for buildings with a large stiffness eccentricity \(e_s=0.3b\), Fig. 5.22(b)), when compared
with the response of the corresponding reference symmetric buildings. On the other
hand, although a 50 per cent increase in the design base shear results in satisfactory
inelastic performance of element 1 for buildings with a large stiffness eccentricity
\(e_s=0.3b\), Fig. 5.22(b)), it is overly conservative for buildings with a moderate stiffness
eccentricity \(e_s=0.2b\), Fig. 5.22(a)). An increase of 25 per cent in the design base shear
generally results in reasonably conservative and hence satisfactory inelastic
performance of asymmetric buildings. Only for buildings with a large stiffness
eccentricity \( (e_s = 0.3b) \) and in the period region \( T_y > 1.5 \) seconds are the results slightly unconservative. Therefore, a 25% increase in the design base shear for asymmetric buildings having moderate and large stiffness eccentricities is adequate. This recommendation coincides with the regularity requirement in the Mexico 87 code, which specifies that if the stiffness eccentricity exceeds 0.1b, then the force reduction factor \( Q \) should be multiplied by a factor of 0.8, which is equivalent to increasing the design base shear by 25 per cent.

The new equivalent static force procedure for the design of asymmetric buildings studied and recommended in this section can therefore be summarised as the specification of a pair of design eccentricity expressions as in eqns. \( (5.30) \) and \( (5.31) \) and the proposal of a regularity condition which specifies that if the stiffness eccentricity exceeds 0.1b the design base shear be increased by 25 per cent. The recommended secondary design eccentricity expression, eqn. \( (5.31) \), is the same as that in NBCC 90 and the proposed regularity condition coincides with that of the Mexico 87 code.

This new procedure has been shown in this section to be applicable for earthquake resistant design in the Federal District of Mexico. In order to generalise this procedure for application in other countries, the remainder of this section studies the applicability of this new procedure for aseismic design in Canada, the United States, New Zealand and Europe. The inelastic response of asymmetric buildings having small \( (e_s = 0.1b) \), moderate \( (e_s = 0.2b) \) and large \( (e_s = 0.3b) \) stiffness eccentricities as well as that of the corresponding reference symmetric buildings are studied and presented in Figures 5.23 to 5.26 in the form of response spectra in the period range of 0.1-2.0 seconds. These buildings are designed in accordance with the base shear provisions, as specified in NBCC 90, UBC 88, the New Zealand 89 code and Eurocode 8, and the above recommended new procedure for torsional design. Local strong motion earthquake
records (three western Californian records in the case of NBCC 90 and UBC 88, three European records in the case of Eurocode 8), or records with varying frequency contents and a/v ratios (two western Californian records in the case of the New Zealand 89 code, namely, the El Centro S00E and the 3470 Wilshire Blvd. N00E records) are used as the ground motion input. These records are the same as those employed in Sections 5.3.4 to 5.3.7 and their time histories and 5 per cent damped elastic response spectra are plotted in Figures 4.4, 4.5, 5.13 and 5.14, respectively.

Figures 5.23 to 5.26 indicate that in all cases the above recommended new design approach results in conservative and satisfactory inelastic performance of asymmetric buildings compared with that of the corresponding reference symmetric buildings. The approach provides consistent protection to both symmetric and asymmetric buildings against structural damage when applied in conjunction with the respective base shear provisions of aseismic building codes from Mexico, Canada, the United States, New Zealand and Europe. In view of its effectiveness in predicting both the elastic and inelastic performance of asymmetric buildings and its widespread applicability, this design procedure can be regarded as a unified design approach serving as a general guideline for the specification of element strength capacity in asymmetric buildings.

It is enlightening to examine the additional increase in the total strength in asymmetric buildings compared with the design base shear $V_{se}$ in symmetric buildings due to the torsional provisions in current major aseismic building codes and the above recommended new unified approach for torsional design. Fig. 5.27 illustrates the variation of the total strength as functions of the stiffness eccentricity ratio $e_s^* = e_s/b$, corresponding to different codes and the new procedure recommended in this section. The total strength in Fig. 5.27, $F_{y1} + F_{y2} + F_{y3}$, is calculated based on the three-element model employed in this chapter corresponding to a fixed value of $\Omega = 1.0$. In Fig. 5.27, the accidental eccentricity 0.1b or 0.05b has not been included. Otherwise, a further
increase of about 20 per cent (see Tso and Ying 1990) should be added to all values. In this case, even for symmetric structures, there is an increase of about 20 per cent in the total strength (see Tso and Ying 1990) over the design base shear $V_{yr}$.

It is apparent that the total strength corresponding to the new design procedure recommended in this section has been greatly reduced from that required by the Mexico 87 code at moderate and large stiffness eccentricities. The recommended new procedure leads to about the same increase in the total strength as that required by Eurocode 8, NBCC 90 and UBC 88 at small stiffness eccentricities. At moderate to large stiffness eccentricities, the new procedure requires more increase in the total strength (by about an additional 25 per cent) compared with UBC 88 and NBCC 90. However, this further increase is justified by the ability of this approach to achieve satisfactory elastic and inelastic performance of asymmetric buildings having moderate to large stiffness eccentricities. Consequently, unlike the Mexico 87 code, the new procedure recommended in this section results in a relatively low and hence more acceptable additional increase in the total strength for asymmetric buildings.

5.5 Conclusions and recommendations

This chapter has rigorously evaluated the torsional provisions of current major aseismic building codes by adhering to the base shear and torsional provisions of individual codes and employing local strong motion earthquake records or records with different frequency contents and a/v ratios in the inelastic dynamic analysis. Particular attention has been paid to the adequacy of the linear elastic modal analysis procedure for the design of asymmetric buildings when they are excited well into the inelastic range. A new unified procedure for the design of asymmetric buildings has been investigated and recommended. The following conclusions and recommendations can be made, based on the study carried out in this chapter.
1. The torsional provisions of the Mexico 76 code, Eurocode 8, NBCC 90 and the New Zealand 89 code (the latter using modal analysis) are inadequate for the design of asymmetric buildings when they are excited well into the inelastic range, since they substantially underestimate the strength demand of resisting elements at the stiff side. If the element strengths in asymmetric buildings are specified in accordance with these codes, the element at the stiff edge is the critical element which suffers significantly more severe damage (as measured by standard response parameters) than the corresponding symmetric buildings. The displacement ductility demand of the element at the flexible edge is usually lower than that of the corresponding symmetric structures.

2. The Mexico 87 code torsional provisions have included additional requirements which correctly increase the strength capacity of elements at the stiff side. However, in view of the large increase in the total strength and the very low inelastic response of asymmetric buildings compared with that of the corresponding symmetric buildings, the Mexico 87 code torsional provisions are overly conservative and result in unnecessarily large additional costs for the design of asymmetric buildings.

3. Among the current major aseismic building codes evaluated in this chapter, only UBC 88 does not allow any reduction of element design loading due to the favourable effect of torsion and therefore leads to satisfactory inelastic performance of asymmetric buildings. However, the UBC 88 torsional provisions are inadequate if asymmetric buildings respond elastically to moderate earthquakes, giving no allowance for the effect of dynamic
amplification of the static eccentricity. Therefore, these provisions underestimate the elastic strength demand of resisting elements at the flexible side of the structure.

4. The approach of including the accidental eccentricity in code design eccentricity expressions to specify element strength but ignoring uncertainties and the rotational ground motion in inelastic dynamic analysis gives misleading results. This explains why Tso and Ying (1990) have reached conclusions contradictory to those drawn by Duan and Chandler (1990), Chandler and Duan (1991a) and those given in this chapter. In theoretical studies, if neither uncertainties nor the rotational ground motion is present, the accidental eccentricity should not be included in the design eccentricity expressions to specify element strength.

5. This chapter recommends a new unified static force procedure for torsional design of regularly asymmetric buildings in which the centres of mass of the floor slabs and the centres of rigidity at floor levels lie on two vertical lines. The procedure is as follows:

Step 1: Calculate the structure’s fundamental uncoupled lateral period using the simplified, empirical methods suggested in building codes.

Step 2: Calculate the stiffness eccentricity. In the case of regularly asymmetric buildings, the stiffness eccentricity at all floor levels has the same value and is load independent.

Step 3: Calculate the design base shear $V_{yo}$ based on the design spectrum specified in building codes and the uncoupled lateral period obtained in step 1. If the stiffness eccentricity calculated in step 2 is greater
than 10 per cent of the building dimension perpendicular to the direction of earthquake motion, $V_{yo}$ should be increased by 25 per cent.

Step 4: Calculate the storey shears in accordance with the code-specified equivalent static force procedure.

Step 5: Calculate the storey torques about the centre of rigidity according to one of the following two design eccentricity expressions measured from the centre of rigidity:

$$e_{d1} = (2.6 - 3.6e_f/b)e_s + 0.1b$$ \hspace{1cm} (5.32)

$$e_{d2} = 0.5e_s - 0.1b$$ \hspace{1cm} (5.33)

The design eccentricity which induces the more severe design loading for the resisting element concerned should be employed.

6. The above recommended procedure for torsional design has been proven to be an effective approach which leads to satisfactory elastic and inelastic performance of asymmetric buildings, offers consistent protection to both symmetric and asymmetric buildings against structural damage, is widely applicable, retains simplicity for ease of code implementation and results in relatively low and acceptable increases in the total strength compared with that of the corresponding symmetric buildings. Therefore, this new procedure can be regarded as a general guideline for torsional design and is recommended to be incorporated in aseismic building codes to be applied in design practice.
7. On the basis this chapter, it is recommended that the modal analysis procedure be deleted in aseismic building codes as an alternative method to the equivalent static force procedure for the design of regularly asymmetric buildings. Aseismic building codes usually specify two alternative methods for structural analysis under earthquake loading, namely the equivalent static force procedure and the linear elastic modal analysis procedure. As a result, the limit of application of these two methods is an important issue for designers. Different codes adopt quite different approaches to address this issue. The Mexico 87 code does not allow the modal analysis procedure to be used to deal with torsional coupling. It only allows modal analysis to be used to calculate the storey shears. Torsional coupling should be dealt with solely by the equivalent static force procedure according to the Mexico 87 code. In contrast, all other codes and especially the New Zealand 89 code, impose strict conditions restricting the use of the equivalent static force procedure to deal with torsional coupling and encourage the modal analysis procedure to be used as a generally applicable method to deal with structural asymmetry and irregularity.

Unlike the equivalent static force procedure, modal analysis in general results in a lower total strength for asymmetric buildings than that of the corresponding symmetric buildings. Some codes require that the design loading obtained by modal analysis be scaled up to obtain a total strength which is at least 90 per cent of the static base shear. Because of this, NBCC 85 and subsequently NBCC 90 have deleted modal analysis as an alternative method to the static procedure to calculate the base shear. This chapter has revealed that the linear elastic modal analysis procedure is inadequate for the
design of asymmetric buildings when they are excited well into the inelastic range, even if the total strength obtained by modal analysis has been scaled up to be the same as that of the corresponding symmetric structures.

A solution for achieving satisfactory inelastic performance of asymmetric buildings without carrying out an inelastic dynamic analysis can only be reached by improving the static force procedure rather than relying on linear elastic modal analysis. In addition, the costs and complexity involved in carrying out a modal analysis are much higher than that associated with the static force procedure. As a result, it is strongly recommended that the linear elastic modal analysis procedure be deleted as an alternative method for the structural analysis of regularly asymmetric buildings under strong earthquake loading.
Figure 5.1 Idealised single-storey building model and the storey design eccentricity $e_0$

Figure 5.2 Design spectra of the Mexico 76 code for group B buildings in Zone III

Figure 5.3 Time history of the Mexico City SCT1 record (EW component)

Figure 5.4 5% damped elastic response spectrum of the Mexico City SCT1 record EW component and comparison with the Mexico 76 code design spectra
Figure 5.5 Maximum displacement ductility demand of resisting elements of buildings designed in accordance with the Mexico 76 code base shear and torsional provisions (excluding accidental eccentricity)

(a) $e_s=0.1b$

(b) $e_s=0.2b$

(continued overleaf)
Figure 5.6 Maximum displacement ductility demand of buildings designed in accordance with the Mexico 76 code base shear and torsional provisions (including in the latter case $e_a=0.05b$ and $0.1b$)

Mexico City SCT1 EW

(c) $e_a=0.3b$

Figure 5.7 Design spectra of the Mexico 87 code for group B buildings in Zone III

Figure 5.8 Maximum displacement ductility demand of buildings designed in accordance with the Mexico 87 code base shear and torsional provisions (excluding accidental eccentricity)
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Figure 5.9 Torsional stability condition of the New Zealand 89 Code

Figure 5.10 Elastic and inelastic design spectra of the New Zealand 89 Code

Figure 5.11 Maximum displacement ductility demand of resisting elements of buildings designed in accordance with the New Zealand 89 code base shear and torsional provisions (modal analysis)
Figure 5.12 Normalised elastic and inelastic design spectra of Eurocode 8

(a) Romania NS  
(b) Thessaloniki Horiz. A  
(c) Patras Tran.

Figure 5.13 Time histories of selected European strong motion earthquake records
Figure 5.14 5% damped elastic response spectra of selected European strong motion earthquake records (scaled to a common peak ground acceleration of 0.3g) and the elastic and inelastic design spectra of Eurocode 8 corresponding to $\alpha = 0.3$ and $q=4$.
Figure 5.15 Maximum displacement ductility demand of resisting elements of buildings designed in accordance with the Eurocode 8 base shear and torsional provisions (excluding accidental eccentricity)

Figure 5.16 Seismic response factor $S$ of NBCC 90

(a) 3470 Wilshire Blvd. N00E

(b) El Centro S00E

(continued overleaf)
Figure 5.17 Maximum displacement ductility demand of resisting elements of buildings designed in accordance with NBCC 90 base shear and torsional provisions (excluding accidental eccentricity)

(c) Cholame Shandon No. 5 N85E

Figure 5.18 5% damped elastic response spectra of selected western Californian strong motion earthquake records and the inelastic design spectra (force reduction factor R=4) of NBCC 90

Figure 5.19 Numerical coefficient C in the base shear provision of UBC 88
Figure 5.20 Maximum displacement ductility demand of resisting elements of buildings designed in accordance with the base shear and torsional provisions of UBC 88 (excluding accidental eccentricity)

Figure 5.21 Maximum displacement ductility demand of resisting elements of buildings designed in accordance with the Mexico 87 code inelastic design spectrum (force reduction factor $Q=4$) and the recommended design eccentricity expressions (eqns. (5.30) and (5.31) but excluding accidental eccentricity)
Figure 5.22 Maximum displacement ductility demand of resisting elements of buildings designed in accordance with the Mexico 87 code inelastic design spectrum (force reduction factor Q=4) with an increased design base shear of 10%, 25% and 50%, respectively, and the recommended design eccentricity expressions (eqns. (5.30) and (5.31) but excluding accidental eccentricity)
Figure 5.23 Maximum displacement ductility demand of resisting elements of buildings designed in accordance with the NBCC 90 base shear provisions and the recommended new static procedure (excluding accidental eccentricity)

(a) 3470 Wilshire Blvd. N00E
(b) El Centro S00E
(continued overleaf)
Figure 5.24 Maximum displacement ductility demand of resisting elements of buildings designed in accordance with the UBC 88 base shear provisions and the recommended new static procedure (excluding accidental eccentricity)

Figure 5.25 Maximum displacement ductility demand of resisting elements of buildings designed in accordance with the New Zealand 89 code base shear provisions and the recommended new static procedure (excluding accidental eccentricity)
Figure 5.26 Maximum displacement ductility demand of resisting elements of buildings designed in accordance with the Eurocode 8 base shear provisions and the recommended new static procedure (excluding accidental eccentricity)

Figure 5.27 Variation of the total strength \((F_1+F_2+F_3)\) as functions of the stiffness eccentricity according to various codes and the recommended new static procedure (excluding accidental eccentricity)
CHAPTER 6

Inelastic Seismic Response and Effective Design of Multistorey Regularly Asymmetric Frame Buildings: A New Equivalent Static Force Procedure

6.1 Code approach for analysis of multistorey asymmetric buildings: introduction

Multistorey, high-rise buildings are one of the striking features of modern civilisation. In the past century, industrialisation has led to a large concentration of population in cities and an acute shortage of land in urban areas. As a result, the number of slender and tall buildings has been greatly increased all over the world. Moreover, high-rise architecture is continually changing. Regular and prismatic shapes and configuration have given way to terraced, set-back, and splayed elevations due to architectural and functional reasons.

From the point of view of a structural engineer, the design of tall buildings is governed primarily by lateral loads due to wind or earthquakes. In this respect, the above-mentioned architectural features are sources giving rise to coupled lateral and torsional response of building structures. Unlike the case of single-storey buildings, the design and analysis of multistorey buildings involve consideration of the vertical distribution of the earthquake lateral load in addition to its horizontal distribution arising due to torsional coupling. For this purpose, building codes usually provide two alternative methods, namely the equivalent static force procedure and the linear elastic modal analysis procedure to be employed in design practice.
6.1.1 The equivalent static force procedure

The equivalent static force procedure is straightforward to apply, simple in form and requires little computational effort. It is applicable to regular buildings in which the centres of floor mass and the centres of rigidity at floor levels lie approximately on two vertical lines and the distributions of mass and stiffness are nearly uniform over the height of the building. In this method, the fundamental period of the building is estimated using empirical formulae specified in codes and the total seismic load, called the base shear, is calculated based on this period and the design spectrum given in building codes. Then simple formulae are employed to distribute this base shear both vertically and horizontally.

6.1.1.1 Vertical distribution of earthquake load

The equivalent static force procedure assumes that a building’s earthquake response is dominated by its fundamental vibration mode. In calculating the base shear, it is further assumed that the building’s total mass, instead of the effective mass corresponding to the first mode, vibrates in the first mode. In general, the effective mass corresponding to the first mode is approximately 60 per cent to 80 per cent of the total mass (Clough and Penzien 1975). Since the contributions to the total response from modes higher than the first mode are significantly lower than that from the first mode, the above assumption results in a higher base shear than that obtained based on modal analysis. Therefore, the static procedure generally leads to a conservative estimate of the base shear when compared with the modal analysis procedure.

Furthermore, most building codes assume that the fundamental mode shape is a straight line. This assumption is based on the observation of the vibrations of a large number of regular buildings which demonstrate that the fundamental mode shape is
generally close to a straight line (Clough and Penzien 1975). This assumption leads to a linear distribution of the seismic lateral load along the height of the building as shown in Fig. 6.1 and defined in eqn. (6.1):

$$F_i = V_0 \frac{W_i h_i}{\sum_{j=1}^{N} W_j h_j}$$

in which $F_i$ is the seismic lateral load at floor level $i$, $V_0$ the base shear, $W_i$ the weight of floor $i$, $h_i$ the height of floor $i$ above the base, and $N$ the number of storeys of the building. This approach has been adopted by Eurocode 8 and the Mexico 87 code.

The above approach fails to account in quantitative terms for the contribution to the building’s total response of the higher modes, which tend to influence the total response of the upper storeys more significantly than the lower storeys. Furthermore, for tall buildings, which are relatively flexible and have their fundamental periods in the long-period region, the fundamental mode significantly deviates from a straight line and lies approximately between a straight line and a parabola with a vertex at the base (Chopra and Newmark 1980, Gupta 1990). In these buildings, the influence of higher modes on the total response of the upper storeys is more significant than in stiff, short-period buildings. Whilst the linear distribution of the earthquake lateral load is adequate for short-period buildings, it nevertheless underestimates the earthquake load at a building’s upper storeys when the first mode shape deviates from a straight line, and consequently the influence of higher modes cannot generally be ignored.
To account for the effect of higher modes and a longer fundamental period, UBC 88, NBCC 90 and the New Zealand 89 code require that a concentrated force $F_t$ be applied at the top of the building and the remaining force $(V_0 - F_t)$ be distributed linearly along the height of the building as shown in Fig. 6.2. In UBC 88 and NBCC 90, the value of $F_t$ is given by:

$$F_t = 0.07T_y V_0 \leq 0.25V_0$$  \hspace{1cm} (6.2)$$

in which $T_y$ is the fundamental lateral period of the building. The value of $F_t$ may be taken as zero if $T_y \leq 0.7$ sec. In this case, the earthquake load distribution is identical to the linear distribution. In the New Zealand 89 code, the value of $F_t$ is given a constant value $0.08V_0$ in all cases, to encourage simplicity.

The Applied Technology Council document, Tentative Provisions for the Development of Seismic Regulations for Buildings (Applied Technology Council 1978), henceforth referred to as ATC3, recommended the following formula for the vertical distribution of the earthquake lateral load:

$$F_i = V_0 \frac{W_i h_i^k}{\sum_{j=1}^{N} W_j h_j^k}$$  \hspace{1cm} (6.3)$$

where $k$ is a numerical coefficient ranging from 1 to 2. If $k=1$, eqn. (6.3) is identical to eqn. (6.1), resulting in a linear distribution. If $k=2$, it gives a parabolic distribution of the earthquake lateral load as shown in Fig. 6.3, leading to higher earthquake loads at the upper storeys. ATC3 suggests that if $T_y < 0.5$ seconds, $k=1$, if $T_y > 2.5$ seconds, $k=2$, and $k$ varies linearly if $0.5$ seconds $\leq T_y \leq 2.5$ seconds, providing a simple transition between the two extreme cases.
Having determined the vertical distribution of the earthquake lateral load acting at floor levels, the storey shears can be calculated simply by summing the lateral forces acting at the floor levels above the storey under consideration. In a similar manner, the storey overturning moments can also be calculated.

### 6.1.1.2 Horizontal distribution of earthquake load

The horizontal distribution of the earthquake lateral load is determined by the code torsional provisions (see Section 5.1.1 for details). In the case of multistorey asymmetric buildings, there are two alternative approaches employed by codes in determining the torsional moments acting on the building, namely the floor eccentricity approach and the storey eccentricity approach (Tso 1990).

In the floor eccentricity approach, the torque acting at a floor level is calculated as the product of the lateral force acting at the same floor level and the design floor eccentricity at that floor level, as shown in Fig. 6.4. The design floor eccentricity is a function of the floor stiffness eccentricity $e_s$ (see Table 5.1), which is the horizontal distance between the centre of mass of that floor slab and the centre of rigidity at that floor level. The centres of rigidity are defined as the set of points at each of the floor levels through which the given set of lateral forces induce only translation of the floor diaphragms. In general, the locations of the centres of rigidity at floor levels depend on the vertical and horizontal distributions of stiffness of the structure and the vertical distribution of the applied lateral load. Having determined the lateral forces and torques acting at floor levels, the internal forces of the individual structural elements can be calculated by the theory of static structural analysis. When calculating the floor torques, the primary or the secondary design eccentricity expression should be used such that the most unfavourable internal forces are obtained for each element under consideration. This approach is adopted in the New Zealand 89 code.
In the storey eccentricity approach, a cut is made at each of the storey levels. The storey torque, rather than the floor torque, is calculated. Its value equals the product of the storey shear and the design storey eccentricity, as shown in Fig. 6.5. The design storey eccentricity is a function of the storey stiffness eccentricity $e_s$ (see Table 5.1), which is the horizontal distance between the location of the resultant of all lateral forces acting above the storey being considered and the shear centre of that storey. The storey shear centres are defined as the set of points at each of the storey levels through which the resultant of the element shears pass when the given set of lateral forces pass through the centres of rigidity, hence giving no rotation of the floor diaphragms. The storey shear centres are also dependent on the horizontal and vertical distributions of stiffness and the vertical distribution of the lateral load. When calculating the storey torques, the primary or the secondary design eccentricity expression should be used such that for a structural element in the storey being considered, the most unfavourable internal forces are obtained. The storey eccentricity approach is adopted in Eurocode 8, NBCC 90, UBC 88 and the Mexico 87 code.

Unlike the case of single-storey buildings, the centres of rigidity at floor levels and the storey shear centres are not always the same set of points. However, Tso (1990) has shown that the above two approaches are equivalent if the above floor and storey stiffness eccentricity concepts are employed. For buildings with orthogonal framing, Stafford Smith and Vezina (1985) and Tso (1990) have also demonstrated that the locations of the centres of rigidity and the shear centres can easily be determined using standard two-dimensional structural analysis computer programs such as those readily available in design offices.
6.1.2 The linear elastic modal analysis procedure

The modal analysis procedure is the same for both single-storey and multistorey buildings, as summarised in Section 5.1.2. Except for the Mexico 87 code, all other major aseismic building codes encourage designers to use modal analysis in order to determine the vertical and horizontal distributions of the earthquake lateral load. In particular, for analysis of irregularly asymmetric buildings, the use of modal analysis rather than the equivalent static force procedure is required by these codes. As discussed in Section 6.1.1.1, the total lateral force or strength at a building’s base obtained from modal analysis is usually lower than the base shear determined from the static force procedure. Hence, some codes require that the strength of all structural elements be scaled up by the same proportion such that the total strength at a building’s base is at least equal to (NBCC 90 and the New Zealand 89 code) or at least 90 per cent (UBC 88) of the base shear determined by the static force procedure. However, Eurocode 8 and the Mexico 87 code do not require such a scaling up of element strength.

6.2 The need to employ a multistorey asymmetric building model in the present study

Most previous studies on the earthquake response of asymmetric buildings, in both the elastic and inelastic ranges, as well as Chapters 3 to 5 of this thesis, are based on single-storey asymmetric building models. In studying the elastic earthquake response of regularly asymmetric buildings in which the centres of mass and the centres of rigidity lie on two vertical lines, a single-storey asymmetric building model is sufficient to investigate the effects of torsional coupling.

Kan and Chopra (1977b) and Hejal and Chopra (1989) have demonstrated that the maximum response quantities of a regularly asymmetric multistorey building can be
determined exactly by combining the maximum response quantities of the corresponding torsionally uncoupled multistorey building \((e_i = 0\), a building visualised by re-installing all floor slabs such that CM coincides with CR at all floor levels), and those of an associated torsionally coupled single-storey building. Hejal and Chopra (1989) have shown that in regularly asymmetric buildings, the stiffness eccentricity values at all floor levels are the same and also that the uncoupled torsional to translational frequency ratios \(\Omega\) associated with all modal pairs (one dominated by translation and the other by torsion) are identical.

Hejal and Chopra (1989) also showed that the maximum response quantities corresponding to the \(i\)th modal pair of the torsionally coupled multistorey building can be calculated accurately by combining the maximum response quantities corresponding to the \(i\)th mode of the corresponding torsionally uncoupled multistorey building and those of the corresponding torsionally coupled single-storey system which has the same stiffness eccentricity ratio \(e_i^* = e_i/r\). In addition, the uncoupled torsional to translational frequency ratio \(\Omega\) of the corresponding single-storey system must be identical to that of the torsionally coupled multistorey building, and the uncoupled lateral period should be equal to the \(i\)th mode period of the torsionally uncoupled multistorey building. The total maximum response quantities of the multistorey asymmetric building can be obtained by combining those corresponding to the first few modal pairs. Therefore, if an adequate estimate of the maximum elastic response quantities of a regularly asymmetric multistorey building can be obtained by considering only the first few, for instance the first 3, modal pairs of vibration, this problem is then simplified to the analysis of the corresponding torsionally uncoupled multistorey building, considering the first 3 translational modes, and the analysis of 3 single-storey asymmetric systems.
However, the above procedure, though simple and easy to implement, cannot be applied to the inelastic earthquake response of regularly asymmetric multistorey buildings. In this case, the structure’s response is nonlinear and inelastic, and hence the structure’s vibration periods and mode shapes change with time. As a result, the normal co-ordinate uncoupling of the equations of motion no longer exists (Clough and Penzien 1975). The principle of modal superposition, upon which the above procedure is based, is only valid over a very short time increment in which the structure’s dynamic properties can be assumed constant, but not valid in predicting the structure’s inelastic response in the entire response history.

A single-storey asymmetric building model is sufficient for studying the inelastic earthquake response of regularly asymmetric multistorey buildings only if the response is dominated by the first modal pair of vibration, although the periods and modal shapes associated with the first modal pair change with time. In this case, the conclusions drawn in Chapters 3 to 5 apply to regularly asymmetric multistorey buildings. However, in a multistorey building, the contribution of higher modes to the building’s total response increases due to period elongation because of yielding in the resisting elements. In explaining the numerous upper storey collapses during the 1985 Mexico City earthquake, Villaverde (1991) has found that because of the development of plastic hinges at the bottom of the columns at the first storey and at most beam ends, the natural periods of the structure elongate and thus, during some time intervals, the structure’s instantaneous third mode period is in resonance with the dominant period of the incoming ground motion and thus the structure vibrates predominantly during some time intervals in its instantaneous third mode. As a result, a failure mechanism involving only the collapse of the structure’s upper storeys is generated by the development of plastic hinges in its upper storey columns, induced by storey shears that have exceeded those for which the structure was designed. Therefore, in order to
account adequately for the contribution of higher modes, a multistorey model is needed to study the inelastic earthquake response and effective design of regularly asymmetric multistorey buildings.

This chapter employs a multistorey building model to investigate the inelastic earthquake response of regularly asymmetric multistorey frame buildings designed in accordance with the lateral and torsional provisions of several current major aseismic building codes. Thus, code provisions for the design of asymmetric buildings are re-evaluated based on such a multistorey asymmetric building model. In particular, the adequacy of the modal analysis procedure which is widely relied upon by most codes to deal with structural asymmetry and irregularity is investigated. Special attention is focused on the variation of column displacement ductility demand along the height of the building. A new recommended equivalent static force procedure is developed and studied.

6.3 Idealised multistorey regularly asymmetric frame building model

6.3.1 Model description

Many multistorey buildings can be characterised by the following features: (i) all floors have the same geometry in plan, (ii) the locations of columns in all storeys are the same, and (iii) the distribution of stiffness along the height of the building is nearly uniform. Considering these common features, the idealised multistorey regularly asymmetric frame building model, as illustrated in Fig. 6.6, is assumed to have the following properties:

(1) The model is multistorey and monosymmetric. The distribution of mass, stiffness and strength is symmetric about the x-axis but may be asymmetric about the y-axis.
(2) The floors are rectangular with a typical aspect ratio $a/b$ equal to $1/3$, rigid in their own plane, and supported on massless axially inextensible columns. The centres of mass of all floors lie on a vertical line. All floors have the same mass $m$ and radius of gyration $r$ about the vertical axis passing through their centres of mass.

(3) There are three planar frame elements oriented parallel to the $y$-axis, the direction of ground motion. Transverse frames are excluded for the reasons given in Chapter 4. The frames are assumed to have stiffness in their acting planes only.

(4) The flexural stiffness of columns and beams is uniform along the height of the building. Furthermore, the flexural stiffness of beams is considered to be very stiff relative to that of columns, so that each frame can be treated as a "shear beam" for computational purposes. Hence, the lateral stiffness matrices of all frames are proportional to each other, termed proportional framing. As a result, the centres of rigidity at all floor levels of the building model lie on a vertical line (Stafford Smith and Vezina 1985, Cheung and Tso 1986a), separated by a distance $e_s$, the stiffness eccentricity, from the vertical line passing through the centres of floor mass, as shown in Fig. 6.6. Buildings having this feature are called regularly asymmetric buildings. The uncoupled torsional to translational frequency ratios associated with each of the modal pairs are also equal to each other (Hejal and Chopra 1989).

(5) The post-yielding moment-curvature relationship of all beams and columns is assumed to be bi-linear hysteretic with inclusion of the Bauschinger effect and 3 per cent of strain hardening, as shown in Fig. 6.7. Yielding of beams
and columns is defined in terms of the yielding moments in pure bending, at the ends of beams and columns. Axial force-bending moment interaction in columns is ignored.

In order to investigate the effect of the fundamental lateral period on the inelastic earthquake response of multistorey asymmetric buildings, three models having 3, 5, and 8 storeys corresponding to typical fundamental uncoupled lateral periods of 0.3, 0.5 and 1.0 seconds, respectively, are employed in this study.

6.3.2 Column sidesway mechanism versus beam sidesway mechanism

There are two possible failure mechanisms of moment resisting frames when excited well into the inelastic range by earthquake motions, namely the column sidesway and the beam sidesway mechanisms (Park and Paulay 1975), as illustrated in Fig. 6.8. Obviously, the beam sidesway mechanism is the preferred one to utilise the structure's displacement ductility and energy dissipation capacities. The column sidesway mechanism, on the other hand, should be avoided if possible, because this mechanism is associated with large interstorey drifts and hence is more likely to result in problems of instability or total collapse of the structure. Furthermore, the compression load in columns significantly reduces their ductility capacity compared with that of beams. Therefore, the column sidesway mechanism also leads to a brittle failure mode, rather than a ductile failure mode as exhibited by the beam sidesway mechanism.

Most building codes require that a capacity design procedure (Park and Paulay 1975) be applied to the earthquake resistant design of frames. A "strong column weak beam" design philosophy, which specifies that at a beam-column joint the sum of column moment resisting strength in both directions, clockwise and anti-clockwise, be
higher than the sum of beam moment resisting strength, as shown in Figs. 6.9(a) and 6.9(b), is applied in design practice to decrease the probability of plastic hinges forming at column ends, except at the bottom ends of the first storey columns. Whilst this design philosophy can delay the formation of plastic hinges at column ends, it nevertheless is not possible to prevent plastic hinges forming at the column ends due to the influence of higher modes of vibration, as discussed by Park and Paulay (1975) and Villaverde (1991). The influence of higher modes leads to two effects. Firstly, the point of contraflexure in a column, which usually exists close to the mid-point of the column, may move close to one beam-column joint or even lie outside the storey height, causing the column to be in single curvature during some time intervals of the response. This effect results in a plastic hinge forming at the opposite end of the column (Park and Paulay 1975). Secondly, at some time intervals, the structure responds predominantly in its instantaneous second or third vibration mode because of the period elongation effect caused by yielding at beam ends, as previously discussed (Villaverde 1991). Therefore, an idealised beam sidesway mechanism as shown in Fig. 6.8(b) is very difficult to achieve in practice and the possible (although undesirable) column sidesway mechanism should therefore be considered, as in the present study.

Increasing the beam moment resisting strength at beam-column joints may have undesirable adverse effects by causing plastic hinges to form at column ends earlier than at beam ends, therefore increasing the inelastic response of columns. This phenomenon can be illustrated by the inelastic earthquake response analysis of the typical three-storey frame shown in Fig. 6.10. This frame has a beam to column flexural stiffness ratio of 3 to 1, which is appropriate to many prototype buildings, and a fundamental period of 0.38 seconds. The design base shear is determined according to the 5 per cent damped Newmark-Hall design spectrum with a force reduction factor of 4.0 (see Fig. 4.3). The base shear is then distributed linearly along the height of the frame. Fig. 6.11 shows the column yielding strengths calculated assuming that the
points of contraflexure lie at the mid-height of columns. The El Centro S00E record scaled to a peak ground acceleration of 0.3g is used as the ground motion input. The beam moment resisting strength is taken to be either 0.7, 1.0, or 1.3 times the sum of the column moment resisting strength at beam-column joints. The maximum inelastic responses, in terms of the maximum inelastic bending moments and shear forces at the ends of the columns and beams, are given in Fig. 6.12. This example clearly indicates that the inelastic response of columns increases with increasing beam to column strength ratio. In an extreme case, in which the beams are very strong relative to the columns and hence the beams do not yield, an idealised column sidesway mechanism as shown in Fig. 6.8(a) is achieved and the worst possible inelastic response of the columns is obtained. Since the analytical model should provide conservative estimates of the inelastic response of the critical resisting elements, which in the case of ductile moment-resisting frames are the columns, the analytical model employed in this chapter is assumed to develop a column sidesway mechanism. However, in design practice, beneficial effects can be obtained and the column response can be reduced compared with that determined in this chapter by applying the "strong column weak beam" philosophy in order to delay and reduce the probability of plastic hinge formation at the column ends.

6.3.3 Specification of column stiffness and strength

6.3.3.1 Column stiffness

The torsionally uncoupled multistorey system corresponding to the actual torsionally coupled multistorey building model is defined as a multistorey system with coincident centres of mass and rigidity at all floor levels but all other properties identical to the torsionally coupled multistorey building under consideration. As stated earlier, this torsionally uncoupled multistorey system can be visualised by re-installing
all floor slabs such that the centres of mass of floors coincide with the centres of
torsional rigidity, as shown in Fig. 6.13. The lateral and torsional vibration periods of the
torsionally uncoupled system are defined as the uncoupled lateral and torsional periods
of the actual torsionally coupled multistorey building.

Since the vertical distribution of stiffness is uniform, the total lateral interstorey
stiffness $K_{yi}$ ($i=1, 2, 3, ..., N$, where $N$ is the total number of storeys in the building) is
the same at all storeys. Given the value of the fundamental uncoupled lateral period $T_y$,
$K_{yi}$ can be determined by solving the eigenproblem:

$$[K_y] \{\phi_i\} = \omega_y^2[M] \{\phi_i\} \quad (6.4)$$

where $[K_y]$ is the system’s lateral stiffness matrix, $[M]$ the mass matrix, and $\{\phi_i\}$ the
modal shape corresponding to the $i$th vibration mode. In the case of the three-storey
model, for instance, $[K_y]$ and $[M]$ are expressed as:

$$[K_y] = \begin{bmatrix} K_{y1} + K_{y2} & -K_{y2} & 0 \\ -K_{y2} & K_{y2} + K_{y3} & -K_{y3} \\ 0 & -K_{y3} & K_{y3} \end{bmatrix} \quad (6.5)$$

in which $K_{y1} = K_{y2} = K_{y3}$, and

$$[M] = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} \quad (6.6)$$

where $M_1 = M_2 = M_3$. 

Having determined the total lateral interstorey stiffnesses \( K_{yi} \), \( i=1, 2, 3, \ldots, N \), the horizontal distribution of stiffness and therefore the column stiffnesses can be determined on a storey-to-storey basis. The procedure is the same as for single-storey models and hence the equations are identical to those given in eqns. (4.25) to (4.27).

### 6.3.3.2 Column strength

The 5 per cent damped Newmark-Hall median (50% probability level) elastic response spectrum, as shown in Fig. 4.3 scaled to a peak ground acceleration of 0.3g, is chosen as the elastic design spectrum representing the elastic strength demand. When following the equivalent static force procedure, the base shear is calculated in accordance with the inelastic design spectrum corresponding to a force reduction factor \( R=4.0 \), as shown in Fig. 4.3, and the structure’s fundamental uncoupled lateral period. Then, the base shear is distributed vertically in accordance with the procedure specified in Eurocode 8, NBCC 90, UBC 88 and the Mexico 87 code, respectively, as described in Section 6.1.1.1.

In the case of the 3-storey and the 5-storey models, all codes assume a linear distribution of the base shear along the height of the building. For the 8-storey model having a fundamental uncoupled lateral period of 1.0 second, NBCC 90 and UBC 88 require a concentrated force of 7 per cent of the base shear to be applied at the top of the building. However, in the Mexico 87 code, for buildings under 60 metres in height, and in Eurocode 8, for buildings with a fundamental uncoupled lateral period less than twice the second corner period (\( T_2 \) in Fig 5.12) in the design spectrum, a linear distribution of the earthquake lateral load along the height of the building is also allowed. Therefore, a linear distribution of the base shear is again employed for the 8-storey model when designed in accordance with the Mexico 87 code and Eurocode 8. Then a cut is made at each of the storey levels. Storey shears are calculated by summing the earthquake lateral
forces acting at floor levels above the storey being considered. Storey torques acting about the storey shear centres are obtained as the products of the storey shears and the design eccentricity, as described in Sections 6.1.1.2 and 5.1.1. Again, the accidental eccentricity has been excluded when calculating the storey torques. By this procedure, the most unfavourable horizontal shear force acting on each column is determined. Finally, by assuming that the points of contraflexure in the columns are located at the mid-height points, the column moment resisting strengths at the beam-column joints are obtained.

If the linear elastic modal analysis procedure is employed, the lower few vibration periods $T_i$ and modal shapes $\{\phi_i\}$ are first computed by solving the appropriate eigenproblem. The maximum response quantities corresponding to the $i$th mode are determined based on the 5 per cent damped Newmark-Hall median elastic response spectrum and the period $T_i$. The total maximum response quantities are then computed employing the Complete Quadratic Combination (CQC) procedure. The numerical computation is carried out employing the program for three dimensional elastic analysis of building systems SUPER-ETABS (Maison and Neuss 1983). Finally, the lateral force resisting strength and the moment resisting strength of all columns at the beam-column joints are scaled down by the same proportion such that the total lateral force resisting strength of the first storey equals the inelastic base shear calculated by the static procedure.

The beams are assumed to be very stiff and very strong compared with columns for the results presented in this chapter.
6.4 Equations of motion

There are two degrees of freedom associated with each of the floor diaphragms, namely the lateral displacement of CR in the y direction, $v_i$, and the rotation of the floor diaphragm about the vertical axis passing through CR, $\theta_i$. The damped inelastic nonlinear equations of motion of a regularly asymmetric multistorey building subjected to ground acceleration $\ddot{y}_g(t)$ along the y-axis can be written in incremental form as:

$$ [M] \{\ddot{u}(t)\} + [C(t)] \{\dot{u}(t)\} + [K(t)] \{u(t)\} = -[M] \{u_b\} \ddot{y}_g(t) $$

in which $[M]$, $[C(t)]$, $[K(t)]$ are the mass, instantaneous damping, and instantaneous stiffness matrices respectively, $\{u(t)\}$ is the displacement vector consisting of $v_i$ and $\theta_i$ at all floors, and $\{u_b\}$ is a vector consisting of the static displacements $v_i$ and $\theta_i$ (i=1, 2, 3, ..., N) when the base undergoes a unit displacement in the direction of the y-axis.

The mass matrix is:

$$ [M] = \begin{bmatrix} [M_1] & [0] & \ldots & [0] \\ [0] & [M_2] & \ldots & [0] \\ \vdots & \vdots & \ddots & \vdots \\ [0] & [0] & \ldots & [M_N] \end{bmatrix} $$

(6.8)

where $[M_i]$, i=1, 2, 3, ..., N, is the mass matrix of the ith floor:

$$ [M_i] = \begin{bmatrix} m_i & e_i m_i \\ e_i m_i & m_i (r^2 + e_i^2) \end{bmatrix} $$

(6.9)

The structure’s instantaneous stiffness matrix is obtained from:
where \([K_i(t)], i=1, 2, 3, ..., N,\) is the instantaneous stiffness matrix of the \(i\)th storey and is given by:

\[
[K_i(t)] = \begin{bmatrix}
\sum j k_j(t) & \sum j k_j(t)x_j \\
\sum j k_j(t)x_j & \sum j k_j(t)x_j^2
\end{bmatrix}
\]  

(6.11)

in which \(k_j(t)\) is the instantaneous lateral stiffness of the \(j\)th column at the \(i\)th storey, and \(x_j\) is the x-ordinate of this column.

The displacement vector \(\{u(t)\}\) is defined as follows:

\[
\{u(t)\} = \begin{bmatrix}
\{u_1(t)\} \\
\{u_2(t)\} \\
... \\
\{u_N(t)\}
\end{bmatrix}
\]  

(6.12)

in which \(\{u_i(t)\}, i=1, 2, 3, ..., N,\) consists of the lateral and torsional displacements of the \(i\)th floor at and about CR, respectively:

\[
\{u_i(t)\} = \begin{bmatrix}
v_i(t) \\
\theta_i(t)
\end{bmatrix}
\]  

(6.13)

Accordingly, \(\{u_b\}\) is expressed as:
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The matrix \[ C(t) \] is considered to be a Rayleigh type damping matrix, being proportional to both mass and instantaneous stiffness matrices:

\[
[C(t)] = c_1[M] + c_2[K(t)]
\]  
(6.15)

in which \( c_1 \) and \( c_2 \) are constants determined by the damping ratios \( \xi_1 \) and \( \xi_2 \), together with the actual periods \( T_1 \) and \( T_2 \) corresponding to the first modal pair of the asymmetric multistorey building:

\[
c_1 = \frac{4\pi}{T_1 + T_2} \xi_1
\]  
(6.16)

\[
c_2 = \frac{T_1 T_2}{\pi (T_1 + T_2)} \xi_2
\]  
(6.17)

In this chapter, the viscous damping for each mode of the first modal pair is taken to be 5 per cent of critical damping; hence, \( \xi_1 = \xi_2 = 0.05 \).

The nonlinear equations of motion, eqn. (6.7), are solved by the step-by-step numerical integration method. The time interval of the numerical integration is selected to be small enough to ensure stable and accurate numerical integration of response contributed from at least the first three modal pairs of the asymmetric multistorey building.
6.5 Ground motion input and the inelastic response parameter

Two strong motion earthquake records are selected as ground motion input, namely the El Centro S00E record and the 3470 Wilshire Blvd. N00E record. These two records are representative of earthquake motions having an intermediate and a low a/v ratio, respectively. Key characteristics of these two records have been shown in Table 4.2. The time histories of these two records have been plotted in Fig. 4.4. In this chapter, these two records are scaled to a common peak ground acceleration of 0.3g. Their 5 per cent damped elastic response spectra have been plotted in Fig. 4.5.

In a ductile moment resisting frame, one of the most important indicators of seismic damage to structural elements (columns and beams) is the curvature ductility demand $\mu_c$, which is defined as the ratio of the maximum curvature reached at a plastic hinge to the curvature when this hinge starts developing, as shown in Fig. 6.14. Another important inelastic seismic response parameter in a moment resisting frame is the maximum interstorey drift. Most codes require that the value of this response parameter be limited to an acceptable level, since an excessive interstorey drift may easily result in increased p-Δ effects, structural instability, and even total collapse.

If a column sidesway mechanism has been developed in such a frame, then the column curvature ductility demand and the maximum interstorey drift are directly related to the interstorey displacement ductility demand $\mu$. Let $\Delta_{max}$ and $\Delta_y$ denote the maximum and the yielding interstorey drift, and $\phi_{max}$ and $\phi_y$ denote the maximum and the yielding curvature at the plastic hinges of columns. Then, referring to Fig. 6.8(a), the maximum interstorey drift can be expressed as:

$$\Delta_{max} = \Delta_y + \theta_p h$$  \hspace{1cm} (6.18)
where $\theta_p$ is the rotation of the plastic hinges at column ends, and is in turn equal to $(\phi_{\text{max}} - \phi_y)l_p$, in which $l_p$ is the equivalent plastic hinge length. Assuming that the points of contraflexure in columns are located at the mid-height points, the yielding interstorey drift can be expressed as:

$$\Delta_y = \frac{\phi_y h^2}{6}$$

(6.19)

Hence, the interstorey displacement ductility demand can be calculated as follows:

$$\mu = \frac{\Delta_{\text{max}}}{\Delta_y} = \frac{\Delta_y + \theta_p h}{\Delta_y} = 1 + \frac{6l_p}{h} \left[ \frac{\phi_{\text{max}} - \phi_y}{\phi_y} \right] = 1 + \frac{6l_p}{h} (\mu_c - 1)$$

(6.20)

Therefore, the column curvature ductility demand:

$$\mu_c = \frac{(\mu - 1) h}{6l_p} + 1$$

(6.21)

and the maximum interstorey drift:

$$\Delta_{\text{max}} = \mu \Delta_y$$

(6.22)

For instance, assuming $h$ and $l_p$ are 10 and 0.5 times the column depth, respectively, if the interstorey displacement ductility demand is 4.0, then the column curvature ductility demand is about 11. This example indicates clearly that local
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(column curvature) ductility demand may be much higher than the global (interstorey displacement) ductility demand if a column sidesway mechanism develops in a frame. Since the column curvature ductility demand and the maximum interstorey drift can be directly related to the global (interstorey displacement) ductility demand, which is in turn directly related to code provisions specifying earthquake loading, this chapter employs the interstorey displacement ductility demand as the characteristic inelastic earthquake response parameter of asymmetric multistorey buildings.

6.6 Code provisions for design of regularly asymmetric multistorey buildings: a re-evaluation

Inelastic dynamic time history analysis has been carried out to investigate the inelastic earthquake response of multistorey regularly asymmetric frame buildings. These buildings are designed in accordance with the lateral and torsional provisions of current major building codes, as described in Section 6.3.3, or the modal analysis procedure. It is intended that the adequacy of code provisions and the modal analysis procedure should be re-evaluated based on the results of such a study. These buildings are assumed to have an uncoupled torsional to translational frequency ratio $\Omega$ equal to unity and are excited by the El Centro S00E and the 3470 Wilshire Blvd. N00E records, scaled to a common peak ground acceleration of 0.3g, as described above.

The distributions of the interstorey displacement ductility demand over the height of the frame located at the stiff edge (element 1) are presented in Figs. 6.15 to 6.23 for three fundamental uncoupled lateral periods, namely 0.3 seconds (3-storey model), 0.5 seconds (5-storey model) and 1.0 second (8-storey model) respectively, and three stiffness eccentricity values, 0.1b, 0.2b and 0.3b, representing small, intermediate and large stiffness eccentricities respectively. Similar to single-storey asymmetric buildings, element 1 in a multistorey regularly asymmetric frame building designed in accordance
with code provisions or the modal analysis procedure is generally the critical element. In most cases, the interstorey displacement ductility demand of this element is significantly higher than that of the corresponding symmetric buildings and that of the frame at the flexible edge (element 3). Only in buildings with intermediate and large stiffness eccentricities and designed according to the Mexico 87 code does the interstorey displacement ductility of element 3 exceed that of element 1 (see Fig. 5.8). In general, the interstorey displacement ductility demand of element 3 is lower than that of the corresponding symmetric buildings and is therefore not presented herein.

The interstorey displacement ductility demand of multistorey symmetric frame buildings is also presented in Figs. 6.15 to 6.23, for purposes of comparison. These buildings have the same fundamental lateral periods as those of the asymmetric buildings and identical stiffness for the three lateral load resisting frames, among which the storey shears are uniformly distributed. In determining the storey shears of multistorey symmetric frame buildings, three approaches have been employed, namely the modal analysis procedure, the linear vertical distribution of the base shear and finally a concentrated force at the top of the building plus a linear distribution of the remainder of the base shear along the height (the latter for the 8-storey model only). In order to facilitate comparison, the storey shear at the first storey has been kept identical to the base shear from the equivalent static force procedure.

It can be seen from the results of the 8-storey symmetric model (Figs. 6.21 to 6.23) that the effect of the top concentrated force in reducing the interstorey displacement ductility demand of the upper storeys is significant compared with results corresponding to modal analysis and the linear distribution of the base shear. This is because the design storey shears of the upper storeys have been increased substantially due to the application of the top force. For symmetric buildings, the modal analysis procedure is adequate, resulting in a nearly constant interstorey displacement ductility demand with
values around the target displacement ductility of 4, except in the first storey. The results corresponding to the linear distribution of the base shear lie between those corresponding to modal analysis and the static procedure with a linear distribution of the base shear plus a top force, being close to the former in the upper storeys and approaching the latter in the lower storeys (Figs. 6.21 to 6.23). The above observations suggest that the static procedure with a linear distribution of the base shear plus a top force is the preferred approach, being conservative and simple in application.

Figs. 6.15 to 6.23 indicate that the conclusions drawn in Chapter 5, in which code torsional provisions and the modal analysis procedure were evaluated based on a single-storey asymmetric building model, generally apply also to multistorey regularly asymmetric frame buildings. The Eurocode 8 provisions result in a very significant increase in the interstorey displacement ductility demand of element 1 of buildings having an intermediate ($e_s=0.2b$) or a large ($e_s=0.3b$) stiffness eccentricity. They are also inadequate even for buildings with a small stiffness eccentricity ($e_s=0.1b$). On the other hand, the UBC 88 provisions, which do not allow any reduction in the strength capacity of the resisting elements at the stiff side, are adequate in all cases. The modal analysis procedure and the NBCC 90 provisions are adequate only for asymmetric buildings having a small stiffness eccentricity, leading to large increases in the interstorey displacement ductility demand of element 1 for buildings having an intermediate or a large stiffness eccentricity. The Mexico 87 code provisions are adequate for buildings having a small stiffness eccentricity and are over conservative for columns in the lower storeys of element 1 in buildings having an intermediate or a large stiffness eccentricity.

However, as shown in Figs. 6.22 and 6.23, it should be noted that for long-period asymmetric frame buildings with an intermediate or a large stiffness eccentricity (the 8-storey model, with $T_r=1.0$ second, $e_s=0.2b$ or 0.3b), the Mexico 87 code provisions result in an increasing interstorey displacement ductility demand in element 1 with
storey level, approaching those of the corresponding symmetric multistorey buildings designed either in accordance with the static procedure with a linear distribution of the base shear or by the modal analysis procedure. Therefore, for the upper storeys of such buildings, the Mexico 87 code provisions are no longer over conservative as in the case of single-storey asymmetric buildings, although they lead to large increases in the total strength in all storeys. This can be attributed to the approach of a simple linear distribution of the base shear over height as employed in the Mexico 87 code, and the increased influence of higher modes on the inelastic response of the upper storeys in long-period multistorey buildings. In taller buildings approaching 60 metres in height, which is the limit specified in the Mexico 87 code for the application of the static procedure, the fundamental uncoupled lateral period is likely to be in the range of 1.5 to 2.0 seconds and hence the influence of the higher modes would be even more significant. Therefore, if an intermediate or a large stiffness eccentricity exists in such a building, the Mexico 87 code provisions tend to be overconservative for columns in the lower storeys of frames at the stiff side, and in contrast, tend to underestimate the strength demand of columns in the upper storeys of these frames.

Figs. 6.15 to 6.23 also show that when buildings are designed in accordance with the modal analysis procedure or the equivalent static force procedure specified in Eurocode 8, NBCC 90, and the Mexico 87 code, the effect of the higher modes on the inelastic response of the upper storeys increases rapidly with increasing stiffness eccentricity and the fundamental uncoupled lateral period of the building. Therefore, in long-period multistorey asymmetric frame buildings designed on the basis of the above mentioned code provisions or the modal analysis procedure, the columns in the upper storeys of frames at the stiff side are more vulnerable to structural damage than those in the lower storeys.
This observation provides one possible explanation for the numerous upper storey collapses during the 1985 Mexico City earthquake and supplements the results given by Villaverde (1991) as summarised in Section 6.3.2. According to a survey of the damage to buildings caused by the 1985 Mexico City earthquake carried out by Rosenblueth and Meli (1986), upper storey collapses were observed in 79 buildings, representing 38 per cent of the 209 cases of collapse. Among these 79 buildings, the majority are those with 8 storeys (15 cases), 7 (14 cases), 5 (13 cases), and 9 (11 cases) storeys, respectively, and 75 per cent were constructed after 1957 (Villaverde 1991) when earthquake resistant design and detailing provisions were first incorporated in the Mexico Federal District Building Code. The finding in this study suggests that structural asymmetry firstly results in the deformed profile of the building being more complex, secondly accentuates the significance of higher modes and finally may lead to a failure mechanism involving the collapse of the upper storeys only, as indicated in Fig. 8 of the study carried out by Villaverde (1991).

Once again, the modal analysis procedure has been shown in this chapter to be inadequate for the design of multistorey regularly asymmetric buildings having an intermediate or a large stiffness eccentricity. It is adequate only for multistorey symmetric buildings and those asymmetric buildings in which the stiffness eccentricity is small \((e < 0.1b)\). Strictly speaking, the modal analysis method is applicable to analysis of linear elastic systems only. The extension of this procedure to the analysis of inelastic, nonlinear systems, such as earthquake resistant buildings responding inelastically to strong earthquake motions, is based on the assumption that nonlinear structural response can be determined to an acceptable degree of accuracy by linear analysis of the building (Chopra and Newmark 1980). However, when an asymmetric building is excited well into the inelastic range, its dynamic properties change. Its periods elongate and the centres of rigidity shift. As a result, asymmetric buildings exhibit fundamentally different inelastic behaviour than that based on linear elastic
theory, as concluded in Chapter 4. Hence, the above assumption is no longer valid in the case of asymmetric buildings. Subsequently, studies in Chapter 5 and this chapter have indicated that the modal analysis procedure is inadequate for design of asymmetric buildings if they are expected to be excited well into the inelastic range by strong earthquake motions.

A study carried out by Shahrooz and Moehle (1990) investigating the inelastic seismic response and design of setback plane frames also found that the modal analysis procedure is inadequate. They carried out inelastic earthquake response analysis of plane frames, each having a setback dividing the frame into a tower and a base. Both the modal analysis and the UBC 88 static force procedures were employed for design of these setback plane frames. Various levels of setbacks and tower-to-base area ratios were considered. They found that modal analysis and the UBC 88 static force procedure both result in similar values and distribution of the column curvature ductility demand, and that both these methods are inadequate to prevent concentration of structural damage in columns near the setback level. They also concluded that there is no apparent advantage in the use of the modal analysis procedure to design setback frames. Similar to the study in Chapter 5 of this thesis, they proposed a modified static force procedure that amplifies design forces in the tower in order to prevent damage concentration in setback frames.

However, most current building codes, except the Mexico 87 code, still consider the modal analysis procedure to be generally applicable. In particular, these codes rely on the modal analysis procedure for analysis of buildings having a large stiffness eccentricity or having horizontal or vertical irregularities. If setbacks or soft storeys are present in buildings, or if the stiffness eccentricity exceeds certain limits (as stated in Eurocode 8 and implicitly in the New Zealand 89 code), the use of the static force procedure is not allowed by most codes. Instead, the modal analysis procedure is
required to be employed. This gives designers the impression that the modal analysis procedure is the "remedy" for design of asymmetric and irregular buildings without the need to question its limits of application. Studies in this chapter and Chapter 5 of this thesis and that by Shahrooz and Moehle (1990) have shown that in many situations this is not the case.

A more appropriate solution is to modify the static force procedure as in Chapter 5 of this thesis and also in Shahrooz and Moehle (1990). Adequate criteria should be established to classify asymmetric and irregular buildings. When such a modified static force procedure is shown to be adequate for design of a particular category of asymmetric or irregular buildings, it should be adopted in design practice and the modal analysis procedure should then be deleted as an alternative approach for analysis of this category of buildings. Similar steps have already been taken in NBCC 90, whereby the modal analysis procedure has been deleted as an alternative approach to the static force procedure for determining the base shear, and in the Mexico 87 code, which does not allow the modal analysis procedure to be used to consider torsional effects, namely the horizontal distribution of the earthquake lateral load.

6.7 Effective design of multistorey regularly asymmetric frame buildings: a new equivalent static force procedure

The analyses of inelastic earthquake response presented in Section 6.6 have indicated that with the exception of UBC 88 all the other codes considered herein are inadequate for design of multistorey regularly asymmetric frame buildings. Furthermore, the results in Section 6.6 have shown once again that the modal analysis procedure by no means leads to better inelastic performance of asymmetric buildings. As has been indicated in Section 5.3.7, the UBC 88 provisions are inadequate for serviceability limit state design of asymmetric buildings, which requires buildings to
behave elastically and without damage when responding to moderate earthquakes. An improved design procedure which ensures satisfactory elastic and inelastic performance of asymmetric buildings is evidently needed.

The unified approach for torsional design proposed in Chapter 5 and based on a single-storey asymmetric building model can be used to develop an improved static force procedure for design of multistorey regularly asymmetric frame buildings. Firstly, the applicability of this unified approach to torsional design of multistorey regularly asymmetric frame buildings should be examined. The 5-storey model is employed for this purpose. Code provisions for the vertical distribution of the earthquake lateral load, namely a linear distribution of the base shear in the case of the 5-storey model, and the proposed unified approach for torsional design are employed jointly for determining the column yielding strengths.

The interstorey displacement ductility demand over the building height, for columns of elements 1 and 3, is presented in Fig. 6.24 for three stiffness eccentricity values, 0.1b, 0.2b and 0.3b, and for two earthquake records, El Centro S00E and 3470 Wilshire Blvd. N00E. It is apparent that the proposed unified design approach leads to a lower interstorey displacement ductility demand of element 3 compared with that of the corresponding symmetric building. However, it does not ensure satisfactory inelastic performance of element 1 of asymmetric buildings having a large stiffness eccentricity ($e_s = 0.3b$). When the stiffness eccentricity is large, the interstorey displacement ductility demand of element 1 in the upper storeys (see Fig. 6.24(a), corresponding to the El Centro S00E record) and the intermediate storeys (see Fig. 6.24(c), corresponding to the 3470 Wilshire Blvd. record) is still excessive compared with that of the corresponding symmetric building. Otherwise, for buildings having a small ($e_s = 0.1b$) or an intermediate ($e_s = 0.2b$) stiffness eccentricity, the proposed approach leads to satisfactory inelastic performance for both elements 1 and 3 of asymmetric buildings.
The above observation indicates that unlike elastic studies, a single-storey model is not on its own sufficient to investigate completely the inelastic torsional effects in multistorey regularly asymmetric buildings. Period elongation due to yielding in inelastic structures amplifies the contribution of higher modes to the structure's total response. As a result, conclusions drawn on the basis of a single-storey model agree very well with the results for the lower storeys, but may be substantially different from those of the intermediate or the upper storeys of multistorey regularly asymmetric buildings. In view of the objectives of applying a concentrated force at the top of a building and its significant effect in reducing the inelastic response of the upper storeys as illustrated in Figs. 6.21 to 6.23, this top force is obviously needed for short-period and intermediate-period buildings. This top force has been considered by many codes to be unnecessary. For long-period buildings, code specified values of this top force may need to be increased.

The 5-storey model having a large stiffness eccentricity ($e_s=0.3$) has been re-designed by applying a concentrated force of either 5, 10, or 20 per cent of the base shear, at the top of the building, and distributing the remainder of the base shear linearly over the height. The proposed approach for torsional design is again employed for the horizontal distribution of storey shears. The interstorey displacement ductility demand of columns of element 1 over the height of the building is shown in Fig. 6.25. It can be seen again that the application of the top concentrated force reduces significantly the inelastic response of columns in upper storeys, slightly decreases that of columns in the intermediate storeys, and has essentially no effect on the inelastic response of columns in the lower storeys. While a value of 5 per cent of the base shear for the top concentrated force is not sufficient, a value of 20 per cent of the base shear is slightly overconservative. The results corresponding to a top concentrated force of 10 per cent
of the base shear are close to those of the corresponding symmetric building. Therefore, for the 5-storey model, specifying a value of 10 per cent of the base shear for the top concentrated force is adequate.

The 8-storey model and the 3-storey model, which have fundamental uncoupled lateral periods of 1.0 second and 0.3 seconds respectively, have been re-designed by applying a concentrated force equal to 10 per cent of the base shear at the top of the building and distributing the remainder of the base shear linearly over the height. The horizontal distribution of storey shears is again determined in accordance with the torsional design approach proposed in Chapter 5. The interstorey displacement ductility demand of columns in elements 1 and 3 over the height is illustrated in Figs. 6.26 and 6.27 for three stiffness eccentricity values, namely 0.1b, 0.2b and 0.3b, and for two earthquake records, namely El Centro S00E and 3470 Wilshire Blvd. N00E. It is apparent that the above modified static force procedure leads to satisfactory inelastic performance of elements 1 and 3 of both models.

Hence, this study recommends that for buildings having a fundamental lateral period $T_y \leq 1.0$ second, a concentrated force equal to at least 10 per cent of the base shear should be applied at the top of the building. The remainder of the base shear should be distributed linearly over the height of the building. This recommendation is in line with the provision in the New Zealand 89 code, which specifies that a top concentrated force equal to 8 per cent of the base shear should be applied at the top of any building, irrespective of its fundamental lateral period. For taller buildings with longer fundamental lateral periods, the value of the top concentrated force should be increased depending on the value of the fundamental lateral period. For simplicity, and considering the formula given by NBCC 90 and UBC 88, the value of the top
concentrated force can be assumed to be proportional to the building’s fundamental uncoupled lateral period. Therefore, this study recommends the following formula for determining the value of the top concentrated force:

\[ F_t = 0.1T_yV_{yo} \geq 0.1V_{yo} \]  \hspace{1cm} (6.23)

Compared with the value suggested in NBCC 90 and UBC 88, eqn. (6.23) results in an increase in \( F_t \) of 3 per cent of the base shear if \( T_y \geq 1.0 \) second. For buildings having \( T_y \leq 0.7 \) seconds, for which \( F_t \) should be taken as zero according to NBCC 90 and UBC 88, the increase in \( F_t \) required by eqn. (6.23) is substantial, with a total increase of 10 per cent of the base shear. The adequate upper bound of \( F_t \) is an issue for future study. At present, the value suggested by UBC 88 and NBCC 90, namely \( 0.25V_{yo} \), may be taken as the upper bound of \( F_t \).

6.8 Conclusions and recommendations

Based on a multistorey frame building model, this chapter has investigated the influence of a building’s higher vibration modes on its inelastic response and has re-evaluated the adequacy of the provisions of current aseismic building codes and the modal analysis procedure for design of multistorey regularly asymmetric frame buildings. By employing a multistorey asymmetric building model, the code lateral as well as torsional provisions, which determine both the vertical and horizontal distributions of the seismic lateral load, can be evaluated. A study has also been carried out to seek and recommend a new equivalent static force procedure which ensures satisfactory inelastic performance of multistorey regularly asymmetric frame buildings. The following conclusions and recommendations can be made based on the study of this chapter.
1. Conclusions drawn in Chapter 5, in which code torsional provisions and the modal analysis procedure were evaluated based on a single-storey asymmetric building model, generally apply also to multistorey regularly asymmetric frame buildings.

2. The influence of a multistorey regularly asymmetric building’s higher vibration modes on the inelastic response of the upper storey columns of frames at the stiff side increases rapidly with the increase in the stiffness eccentricity and the building’s fundamental uncoupled lateral period. Period elongation due to yielding results in an increasingly more complex deformed profile of such buildings and the upper storey columns of the stiff side frames become more and more vulnerable to structural damage compared with those in the lower storeys. This conclusion provides one explanation for the numerous upper storey collapses during the 1985 Mexico City earthquake.

3. The application of a concentrated force at the top of a building, coupled with a linear distribution of the remainder of the base shear over the height of the building decreases the inelastic response of the upper storey columns very significantly compared with the modal analysis procedure and a linear distribution of the total base shear. In designing short-period and medium-period asymmetric buildings, for which this top concentrated force has been considered unnecessary by many current building codes, this top concentrated force is essential in reducing the inelastic response of the upper storey columns.

4. The Mexico 87 code prescribes a linear distribution of the base shear over the height for buildings up to 60 metres high. This approach substantially underestimates the strength demand of the upper storey columns in medium-period and long-period buildings. As a result, the conclusion drawn
in Chapter 5 based on a single-storey asymmetric building model that the Mexico 87 code provisions are overly conservative applies to short-period multistorey asymmetric buildings, but only to the lower storey columns of medium-period and long-period multistorey regularly asymmetric frame buildings. The inelastic response of the upper storey columns of elements at the stiff side of such buildings increases significantly with the increase in the fundamental uncoupled lateral period and approaches that of the corresponding symmetric building, in spite of a large increase in the total storey strength in all storeys over that of the corresponding multistorey symmetric building. Hence, for long-period multistorey regularly asymmetric frame buildings, the Mexico 87 code tends to overestimate the strength demand of the lower storey columns, and in contrast, tends to underestimate the strength demand of the upper storey columns of frames located at the stiff side.

5. Unlike the case of elastic studies, a single-storey asymmetric building model is not sufficient on its own to investigate completely the inelastic torsional coupling effects in multistorey regularly asymmetric buildings. A multistorey asymmetric building model should therefore be employed.

6. Based on the results of this chapter, a new equivalent static force procedure for design of multistorey regularly asymmetric frame buildings has been recommended. It can be implemented by the following steps:

Step 1: Calculate the structure’s fundamental uncoupled lateral period using the simplified, empirical methods suggested in building codes.

Step 2: Determine the locations of storey shear centres at each of the storey levels and then calculate the stiffness eccentricity at each storey
level by following the steps described by Stafford Smith and Vezina (1985) and Tso (1990). In the case of regularly asymmetric buildings, the stiffness eccentricity at all storey levels has the same value and is load independent.

Step 3: Calculate the design base shear \( V_{yo} \) based on the design spectrum specified in building codes and the uncoupled lateral period obtained in step 1. If the stiffness eccentricity calculated in step 2 is greater than 10 per cent of the building dimension perpendicular to the direction of earthquake motion, \( V_{yo} \) should be increased by 25 per cent.

Step 4: Apply a concentrated force \( F_t \) at the top of the building. The value of \( F_t \) should be taken as:

\[
0.1V_{yo} \leq F_t = 0.1T_y V_{yo} \leq 0.25V_{yo} \tag{6.24}
\]

Then, the remainder of the base shear is distributed linearly over the height of the building:

\[
F_i = \frac{W_i h_i}{\sum_{j=1}^{N} W_j h_j} (V_{yo} - F_t), i = 1, 2, 3, ..., N \tag{6.25}
\]

where \( N \) is the total number of storeys in the building.

Step 5: Make a cut at each of the storey levels. Calculate the storey shears by summing the lateral forces acting at the floor levels above the storey under consideration.
Step 6: Calculate the storey torques about the storey shear centres at each of the storey levels according to one of the following two design eccentricity expressions measured from the storey shear centres:

\[ e_{D1} = (2.6 - 3.6e_s/b)e_s + 0.1b \]  
\[ e_{D2} = 0.5e_s - 0.1b \]

The one which induces the more severe design loading for the resisting element concerned should be employed.

7. Chapter 5 and this chapter have shown that the above recommended new equivalent static force procedure for design of multistorey regularly asymmetric buildings is an effective approach which leads to satisfactory elastic and inelastic performance of asymmetric buildings, offers consistent protection to both symmetric and eccentric buildings against structural damage, is widely applicable, retains simplicity for ease of code implementation and results in relatively small and hence acceptable increases in the total strength compared with that of the corresponding symmetric buildings. Therefore, this new equivalent static force procedure can be regarded as a general guideline for design of multistorey regularly asymmetric frame buildings and is recommended for incorporation into aseismic building codes applied in design practice.

8. The linear elastic modal analysis procedure has once again been found inadequate for design of asymmetric buildings if such buildings are expected to be excited well into the inelastic range by strong earthquake motions. A study by Shahrooz and Moehle (1990) has also found that the modal analysis procedure is inadequate and has no apparent advantage over the static force procedure in designing setback plane frame structures. Buildings having plane
asymmetry or horizontal or vertical irregularities should be classified by appropriate criteria. Improved static force procedures should be sought for effective design of each category of buildings and the modal analysis procedure should then be deleted as an alternative design method.

9. Multistorey asymmetric buildings can be classified as regular and irregular according to the locations of the centres of mass and the centres of rigidity at floor levels. For design of regularly asymmetric frame buildings, the static force procedure recommended above has been shown to be an effective procedure. Therefore, this chapter strongly recommends that the modal analysis procedure be deleted as an alternative approach for design of this category of buildings. For effective design of irregularly asymmetric buildings, further research is needed. It is suggested that until adequate criteria and improved static force procedures are established the modal analysis procedure may still be kept as an option for initial design of this category of buildings and that if possible, inelastic earthquake response analyses be carried out to identify possible damage concentration and inadequacies at the initial design stage. A refined design can then be obtained on this basis.
Chapter 6 Multistorey regularly asymmetric buildings

Figure 6.1 Linear distribution of the lateral earthquake load over the height of a building

Figure 6.2 A concentrated force applied at the top of a building plus a linear distribution of the remainder of the base shear over the height of the building

Figure 6.3 Parabolic distribution of the earthquake lateral load over the height of a building

Figure 6.4 Floor torques and lateral forces acting at floor levels
Figure 6.5 Storey shears and storey torques

Figure 6.6 Idealised multistorey regularly asymmetric frame building model

Figure 6.7 Inelastic moment-curvature relationship of beams and columns
Figure 6.8 Column sidesway and beam sidesway mechanisms

(a) Column sidesway mechanism

(b) Beam sidesway mechanism

Figure 6.9 "Strong column weak beam" design philosophy

(a) \( M_{c1} + M_{c2} > M_{b1} + M_{b2} \)

(b) \( \bar{M}_{c1} + \bar{M}_{c2} > \bar{M}_{b1} + \bar{M}_{b2} \)
Figure 6.10 A three-storey plane frame model

Figure 6.11 Lateral earthquake load and column moment resisting strength

(a) $\frac{\sum M_{ai}}{\sum M_{ci}} = 0.7$

(b) $\frac{\sum M_{ai}}{\sum M_{ci}} = 1.0$

(continued overleaf)
Figure 6.12 Maximum inelastic internal forces of columns and beams of the three-storey plane frame having various beam to column strength ratios, subjected to the El Centro S00E record.

(c) \( \frac{\sum M_{ui}}{\sum M_{di}} = 1.3 \)

Figure 6.13 Definition of the corresponding torsionally uncoupled building system.

Figure 6.14 Definition of the curvature ductility demand.
Figure 6.15 Maximum interstorey displacement ductility demand of element 1 of the 3-storey model having a small stiffness eccentricity and designed in accordance with various code provisions and the modal analysis procedure: $e_s = 0.1b$, $R = 4.0$, $\Omega = 1.0$, $\xi = 0.05$

Figure 6.16 Maximum interstorey displacement ductility demand of element 1 of the 3-storey model having an intermediate stiffness eccentricity and designed in accordance with various code provisions and the modal analysis procedure: $e_s = 0.2b$, $R = 4.0$, $\Omega = 1.0$, $\xi = 0.05$
Figure 6.17 Maximum interstorey displacement ductility demand of element 1 of the 3-storey model having a large stiffness eccentricity and designed in accordance with various code provisions and the modal analysis procedure: $e_s = 0.3b$, $R = 4.0$, $\Omega = 1.0$, $\xi = 0.05$

Figure 6.18 Maximum interstorey displacement ductility demand of element 1 of the 5-storey model having a small stiffness eccentricity and designed in accordance with various code provisions and the modal analysis procedure: $e_s = 0.1b$, $R = 4.0$, $\Omega = 1.0$, $\xi = 0.05$
Figure 6.19 Maximum interstorey displacement ductility demand of element 1 of the 5-storey model having an intermediate stiffness eccentricity and designed in accordance with various code provisions and the modal analysis procedure: $e_s=0.2b$, $R=4.0$, $\Omega = 1.0$, $\xi = 0.05$

Figure 6.20 Maximum interstorey displacement ductility demand of element 1 of the 5-storey model having a large stiffness eccentricity and designed in accordance with various code provisions and the modal analysis procedure: $e_s=0.3b$, $R=4.0$, $\Omega = 1.0$, $\xi = 0.05$
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1.3 Interstorey Displacement Ductility Demand

Symmetric, modal analysis Symmetric, static procedure, $F_t=0.07TV_o$
Symmetric, static procedure, $F_t=0$ Modal analysis NBCC 90
Mexico 87 Eurocode 8 UBC 88

(a) El Centro S00E (b) 3470 Wilshire Blvd. N00E

Figure 6.21 Maximum interstorey displacement ductility demand of element 1 of the 8-storey model having a small stiffness eccentricity and designed in accordance with various code provisions and the modal analysis procedure: $e_s=0.1b$, $R=4.0$, $\Omega = 1.0$, $\xi = 0.05$

(a) El Centro S00E (b) 3470 Wilshire Blvd. N00E

Figure 6.22 Maximum interstorey displacement ductility demand of element 1 of the 8-storey model having an intermediate stiffness eccentricity and designed in accordance with various code provisions and the modal analysis procedure: $e_s=0.2b$, $R=4.0$, $\Omega = 1.0$, $\xi = 0.05$
Figure 6.23 Maximum interstorey displacement ductility demand of element 1 of the 8-storey model having a large stiffness eccentricity and designed in accordance with various code provisions and the modal analysis procedure: $e_s=0.3b$, $R=4.0$, $\Omega=1.0$, $\xi=0.05$
Figure 6.24 Maximum interstorey displacement ductility demand of elements 1 and 3 of the 5-storey model designed in accordance with a linear distribution of the base shear over the height and the unified approach for torsional design recommended in Chapter 5: $e_s = 0.1b, 0.2b,$ and $0.3b$, respectively, $R = 4.0$, $\Omega = 1.0$, $\xi = 0.05$.

Figure 6.25 Maximum interstorey displacement ductility demand of element 1 of the 5-storey model having a large stiffness eccentricity and designed by applying a concentrated force $F_t$ equal to zero, 5 per cent, 10 per cent, and 20 per cent of the base shear, respectively, plus a linear distribution of the remainder of the base shear together with the unified approach for torsional design as recommended in Chapter 5: $e_s = 0.3b$, $R = 4.0$, $\Omega = 1.0$, $\xi = 0.05$.
Figure 6.26 Maximum interstorey displacement ductility demand of elements 1 and 3 of the 8-storey model designed in accordance with the recommended new equivalent static force procedure: $\epsilon = 0.1b, 0.2b,$ and $0.3b,$ respectively, $R = 4.0, \Omega = 1.0, \xi = 0.05$
Figure 6.27 Maximum interstorey displacement ductility demand of elements 1 and 3 of the 3-storey model designed in accordance with the recommended new equivalent static force procedure: $e_\gamma=0.1b$, $0.2b$, and $0.3b$, respectively, $R=4.0$, $\Omega = 1.0$, $\xi = 0.05$
CHAPTER 7

Concluding Remarks

7.1 Summary of conclusions and recommendations

Inelastic seismic response and earthquake resistant design of asymmetric buildings has been an attractive research subject receiving intensive studies in recent years. Previous studies have contributed to this research subject, as reviewed in Section 2.1. Being independent and concurrent with other recent studies, this thesis has firstly identified the contradictory conclusions drawn by various researchers and the inadequacies in the approaches employed in their studies, as discussed and assessed in Section 2.3. The model dependency of the inelastic torsional response of asymmetric buildings to strong earthquake loading has been emphasised.

Secondly, inelastic static monotonic failure mode analysis and inelastic earthquake response analysis have been carried out employing various single-storey monosymmetric structural models. Hence, the model dependency of the inelastic torsional response of asymmetric buildings has been clarified and a suitable analytical model developed. This model is simple in definition and facilitates straightforward interpretation of results, is representative of a wide range of actual buildings, and provides conservative estimates of the inelastic response of the critical resisting elements.

Thirdly, based on this model, a comprehensive inelastic dynamic parametric study has been carried out. The influence of the various system parameters and the characteristics of the earthquake input motions on the inelastic earthquake response of resisting elements has been determined.

Finally, provisions associated with the equivalent static force procedure of current earthquake resistant design building codes from Europe, New Zealand, Canada, the United States and Mexico, together with the dynamic modal analysis procedure, have been rigorously evaluated employing the above-mentioned single-storey asymmetric
building model and a multistorey regularly asymmetric frame building model. Inadequacies in the code provisions and the modal analysis procedure have been identified. A new and effective equivalent static force procedure has been developed and recommended for design practice.

The contributions of this thesis to the study of the inelastic seismic response and design of asymmetric buildings can be summarised by the following set of conclusions and recommendations:

1. The inelastic earthquake response of asymmetric buildings generally increases with the number of lateral load-resisting elements. The three-element model has been shown to be the most appropriate for studying inelastic torsional coupling effects, since it is both simple and reasonably conservative.

2. The inclusion of transverse resisting elements in the analytical model but assuming that the earthquake motion is uni-directional is inappropriate and can lead to erroneous results.

3. Since plan asymmetry in buildings arises primarily from unbalanced stiffness rather than mass, the stiffness-eccentric model should be employed in analytical studies. The inelastic response of mass-eccentric systems is higher than that of stiffness-eccentric systems when the centre of mass CM coincides with the centre of strength, but the reverse applies when the centre of strength coincides with the centre of rigidity.

4. The inelastic earthquake response of the 3-element model is relatively insensitive to the assumption of a double-step or a single-step stiffness variation. The former has been used in this study, including the evaluation and improvement of code torsional provisions.

5. The inelastic earthquake response of code-designed buildings is fundamentally different from their elastic response. Such buildings respond more in translation when excited well into the inelastic range than when responding elastically. As a result, the resisting elements at the stiff side
respond more critically than those at the flexible side, in comparison with corresponding symmetric buildings. This effect is accentuated in structures having large stiffness eccentricities.

6. In initially symmetric buildings with unbalanced distribution of yielding strengths, the critical elements are those at the flexible side. Their response increases significantly with the magnitude of the strength eccentricity, which also leads to important reductions in the structure’s lateral load carrying capacity. In design, whenever possible the strengths of individual resisting elements should be adjusted to give coincident or nearly coincident centres of strength and rigidity. This is particularly important when different lateral load-resisting systems are used to achieve a balanced stiffness distribution.

7. The fundamental uncoupled lateral period and the frequency content of the earthquake input motion, described by the a/v ratio, both influence significantly the inelastic response of asymmetric buildings. It is concluded that records with different frequency contents should be employed and that the structural period should be varied over a wide range when studying the inelastic response and aseismic design of asymmetric buildings.

8. The effect of the uncoupled torsional to translational frequency ratio \( \Omega \) is most pronounced for torsionally flexible asymmetric buildings (where this parameter is smaller than unity), giving much higher inelastic response than corresponding symmetric buildings. Therefore, it is strongly recommended that torsionally flexible asymmetric buildings be avoided in design practice, by locating the lateral load-resisting elements uniformly along the two principal axes or near the periphery of the building. Similarly, shear cores for lifts should be located symmetrically near the periphery rather than at the centre of the building. In general, the inelastic response of asymmetric buildings decreases with increasing value of \( \Omega \), and is relatively insensitive to this parameter when \( \Omega \) is greater than unity. For this latter case, a structural model with \( \Omega=1 \) provides sufficiently conservative results.
9. Decreasing the total strength of asymmetric buildings relative to the elastic strength demand of the corresponding symmetric buildings not only increases rapidly the overall inelastic response but also amplifies the effect of stiffness and strength eccentricities. For these reasons, a smaller force reduction factor is recommended for the design of asymmetric buildings.

10. Static monotonic failure mode analysis is a simple and efficient technique to identify the failure mode, the critical element, and the post-yielding behaviour of buildings under strong earthquake motions. It is recommended that whenever possible, designers carry out such an analysis and if necessary refine the original design.

11. The application of the equivalent static torsional provisions of major building codes, and the modal analysis procedure, is inadequate for the inelastic design of asymmetric buildings since the critical stiff-side elements suffer significantly more severe structural damage than corresponding symmetric buildings. However, the displacement ductility demand of the elements at the flexible side is usually lower than that of the corresponding symmetric structures.

12. The provisions of the Mexico 87 code give special protection to the stiff-side elements and as a result are overly conservative for these elements in short-period regularly asymmetric buildings and for the lower storey columns of these elements in medium-period buildings. For long-period buildings, the Mexico 87 provisions overestimate the strength demand of the lower storey columns, but the reverse is true for the upper storey columns of the stiff-side elements.

13. Unlike other codes, UBC 88 does not allow any reduction of element design loading due to favourable effects of torsion and therefore leads to satisfactory inelastic performance of asymmetric buildings. The flexible-side elements in such buildings may however be substantially under-designed in conditions of elastic response to moderate earthquakes.
14. If neither uncertainties nor possible rotational components of ground motion are present in the analysis, element strengths should be specified without including the accidental eccentricity in the design eccentricity provisions.

15. The influence of higher vibration modes on the inelastic response of the upper storey columns of frames at the stiff side increases significantly with the stiffness eccentricity and the building’s fundamental uncoupled lateral period. This may explain the numerous upper storey collapses resulting from the 1985 Mexico City earthquake.

16. In equivalent static design, the application of a concentrated force at the top of a building decreases the inelastic response of the upper storey columns very significantly compared with the modal analysis procedure and a simple linear distribution of the total base shear. This top concentrated force is essential to reduce the inelastic response of the upper storey columns in short-period and medium-period asymmetric buildings, but is not included in several current building codes.

17. Unlike elastic studies, a single-storey structural model is not on its own sufficient to investigate completely the inelastic torsional coupling effects in multistorey regularly asymmetric buildings. A multistorey structural model should therefore be employed for this purpose.

18. Based on the results of this thesis, a new equivalent static force procedure for design of multistorey regularly asymmetric frame buildings has been recommended. It can be implemented by the following steps:

   Step 1: Calculate the structure’s fundamental uncoupled lateral period using the simplified, empirical methods suggested in building codes.

   Step 2: Determine the locations of storey shear centres at each of the storey levels and then calculate the stiffness eccentricity at each storey level by following the steps described by Stafford Smith and Vezina.
(1985) and Tso (1990). In the case of regularly asymmetric buildings, the stiffness eccentricity at all storey levels has the same value and is load independent.

Step 3: Calculate the design base shear $V_{yo}$ based on the design spectrum specified in building codes and the uncoupled lateral period obtained in step 1. If the stiffness eccentricity calculated in step 2 is greater than 10 per cent of the building dimension perpendicular to the direction of earthquake motion, $V_{yo}$ should be increased by 25 per cent.

Step 4: Apply a concentrated force $F_t$ at the top of the building. The value of $F_t$ should be taken as:

$$0.1V_{yo} \leq F_t = 0.1Ty_{yo} \leq 0.25V_{yo} \quad (7.1)$$

Then, the remainder of the base shear is distributed linearly over the height of the building:

$$F_i = \frac{W_i h_i}{\sum_{j=1}^{N} W_j h_j} (V_{yo} - F_i), i = 1, 2, 3, \ldots, N \quad (7.2)$$

where $N$ is the total number of storeys in the building.

Step 5: Make a cut at each of the storey levels. Calculate the storey shears by summing the lateral forces acting at the floor levels above the storey under consideration.

Step 6: Calculate the storey torques about the storey shear centres at each of the storey levels according to one of the following two design eccentricity expressions measured from the storey shear centres:

$$e_{D1} = (2.6 - 3.6e_{y} / b)e_s + 0.1b \quad (7.3)$$

$$e_{D2} = 0.5e_s - 0.1b \quad (7.4)$$

The one which induces the more severe design loading for the resisting element concerned should be employed.
19. The new equivalent static force procedure outlined in para. 18 above leads to satisfactory elastic and inelastic performance of multistorey regularly asymmetric frame buildings. It also offers consistent protection against structural damage for both symmetric and eccentric buildings, is widely applicable, retains simplicity for ease of code implementation and results in relatively small and hence acceptable increases in the total strength compared with that of corresponding symmetric buildings. Therefore, this new equivalent static force procedure can be regarded as a general guideline for design of multistorey regularly asymmetric frame buildings and is recommended for incorporation into aseismic building codes applied in design practice.

20. Regularly asymmetric buildings shaken well into the inelastic range cannot be adequately designed using linear elastic modal analysis. It is strongly recommended that this approach be deleted as an alternative to equivalent static force methods for analysis of such buildings.

21. For irregularly asymmetric buildings, it is suggested that modal analysis be kept as an option at the initial design stage. Whenever possible, an inelastic earthquake response analysis should be carried out to identify possible damage concentration and inadequacies in the initial design. A refined design can then be implemented on this basis.

7.2 Proposals for further research

7.2.1 Inelastic seismic response and effective design of high-rise regularly asymmetric shear wall buildings

The inelastic seismic response and effective design of multistorey regularly asymmetric frame buildings have been studied in Chapter 6, in which the resisting elements have been modelled as ductile moment-resisting frames exhibiting a "shear beam" behaviour. Many multistorey buildings, in particular high-rise buildings, utilise shear wall elements to resist the lateral load. Shear walls display a "flexural beam" type
of behaviour which is different from the "shear beam" type of behaviour exhibited by moment-resisting frames. Therefore, the inelastic seismic response and effective design of high-rise regularly asymmetric shear wall buildings is an area worthy of further research. The applicability of the above-recommended new equivalent static force procedure to shear wall buildings needs to be evaluated. An effective equivalent static force procedure applicable to high-rise regularly asymmetric shear wall buildings may need to be developed. An analytical model similar to those in Chapter 6 but consisting of shear wall elements can be employed for this purpose. The computer program IDARC-3D (Kannath and Reinhorn 1989), a program for the inelastic three-dimensional seismic response analysis of reinforced concrete buildings, can be used for carrying out the numerical analysis.

7.2.2 **Elastic and inelastic earthquake response and effective design of irregular multistorey buildings**

Irregular structures arise frequently in engineering practice as a result of architectural requirements. Such buildings commonly consist of asymmetrically located shear walls and frames exhibiting wall-frame interaction, known as hybrid structures, which are examples of horizontally irregular multistorey buildings. Buildings having "soft storeys" resulting from large openings at the ground floor level, and those with discontinued or staggered shear walls, are examples of vertically irregular structures. Those with setbacks have both horizontal and vertical irregularities. Although it is well known that the response of symmetric and regular buildings to earthquake loading is easier to predict and that the ability of these buildings to survive a strong earthquake is much better than eccentric and irregular buildings, designers are often faced with decisions which compromise structural symmetry and regularity, in order to accommodate functional and aesthetic needs. As a result, severe and widespread damage associated with torsional response created by structural asymmetry and irregularity has been observed repeatedly in past major earthquakes. Unfortunately, very
little research has been carried out on this subject. The guidance given in current major
aseismic building codes for earthquake resistant design of such buildings is sparse, and
lacks clarity and consistency.

In general, a modal analysis is required to be carried out by codes for analysis of
irregular multistorey buildings. However, this thesis has shown that the modal analysis
procedure is inadequate for analysis of buildings having plan asymmetry. A study
carried out by Shahrooz and Moehle (1990) has also indicated that modal analysis is
inadequate and has no apparent advantage over the static force procedure for analysis of
plane frame structures with setbacks, exhibiting vertical irregularity. Therefore, modal
analysis is obviously not an adequate or even appropriate approach for analysis of
irregular buildings.

Further research should be carried out to establish adequate criteria in order to
classify irregular buildings. Improved equivalent static force procedures or design
guidelines should be sought for effective design of each category of irregular buildings.

7.2.3 Inelastic seismic response and effective design of asymmetric
and irregular buildings under bi-axial earthquake excitation

In order to simplify analysis, aseismic building codes specify that resisting
elements oriented in two orthogonal directions are allowed to be designed separately,
considering only the earthquake motion parallel to the elements. Most previous studies
and that carried out in this thesis have considered uni-directional earthquake motion
input only. The accuracy of this approach and the influence of the transverse elements
and corresponding component of the earthquake motion should be studied in future
research by employing a structural model with transverse elements and subjected to
bi-directional earthquake excitation. The results of such a study may subsequently be
used to improve the above-mentioned uni-directional design approach, by modifying the
uni-directional earthquake lateral design loads to account for the influence of the
transverse elements and ground motion.
References


Structural Engineers' Association of California (1988). "Recommended lateral force requirements and tentative commentary." Seismology Committee, Structural Engineers' Association of California, San Francisco, California, U.S.A.


