A SCATTERING METHOD FOR BONE DENSITY MEASUREMENTS
WITH POLYCHROMATIC SOURCES

Thanos Koligliatis

UNIVERSITY COLLEGE LONDON

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Dedicated to my parents
Vangelis and Matina
1 page 4, line 18: for '26' read '25'
2 page 6, line 21: for '95' read '97'
3 page 6, line 22: for '96' read '98'
4 page 6, line 23: for '98' read '100'
5 page 6, line 24: for '98' read '100'
6 page 6, line 25: for '100' read '102'
7 page 6, line 26: for '101' read '104'
8 page 12, line 22: for '149-151' read '150'
9 page 12, line 24: add '152-153'
10 page 12, line 28: for '150' read '158'
11 page 15, line 4: for '181-183' read '183-184'
12 page 19, line 7: for 'examimed' read 'examined'
13 page 24, line 1: for 'semm' read 'seem'
14 page 25, line 15: for 'precludes' read 'preclude'
15 page 30, line 12: for 'Shih' read 'Shin'
16 page 35, line 3: for 'measurements' read 'measurements'
17 page 40, line 24: for 'Rosset al 1986' read 'Ross et al 1988'
18 page 41, line 4: for 'Le Blancet al' read 'Le Blanc et al'
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20 page 42, table 1.3: for 'Tradecular' read 'Trabecular'
21 page 44, line 17: for 'goverened' read 'governed'
22 page 45, figure 1.6: for 'thugh' read 'through'
23 page 47, line 13: for 'Huddleson' read 'Huddleston'
24 page 48, figure 1.8: for 'C1,C2' read 'D1,D2'
25 page 59, line 8: for 'weiht' read 'weight'
26 page 64, line 13: for 'of photon' read 'of photon'
27 page 72, line 6: for 'comptuter' read 'computer'
28 page 72, line 10: for 'smaller' read 'larger'
29 page 81, line 11: for 'Clearly mcfs' read 'Clearly mcfs'
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31 page 118: $\mu(E_s,x)$: This quantity is not defined
32 page 120, line 13: for 'thebcfs' read 'the bcfs'
33 page 144, line 21: for 'fit applied' read 'fit was applied'
34 page 155, line 2: for 'fac' read 'face'
35 page 159, line 4: for 'collimarors' read 'collimators'
36 page 171, line 1: for '100 kV_p' read '102 kV_p'
37 page 187, line 3: for 'with with' read 'with'
38 page 195, last line: for 'examixation' read 'examination'
39 page 197, line 9: for 'colimators' read 'collimators'

Compston J E, Garrahan N J, Mellish R G and Croucher P 1988 Bone structure in osteoporosis *Proc. of the Conference on osteoporosis and bone mineral measurements, Bath, UK*


Horsman A and Simpson M 1975 The measurement of sequential changes in cortical bone geometry *British Journal of Radiology* 48 471

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Mooney M J, Speller R D and Koligiatis T 1989 The influence of scatter on the accuracy of absorptiometric bone density measurements *V Mediterranean Conference on Medical and Biological Engineering, Greece*

Roos P D, Wasnich R D, Heilbrun L K and Vogel J M 1987 Definition of a spine fracture threshold based upon prospective fracture risk *Bone* 8 271

The aim of this thesis has been to investigate the use of x-ray tubes as irradiating sources in Compton scatter bone densitometry in order to diagnose early osteoporosis. The use of polychromatic sources requires that effects such as multiple scatter and beam polychromaticity are taken into account. The effect of multiple scatter was evaluated for a series of different size anthropomorphic phantoms which represent sites of interest in osteoporosis studies, namely the lower forearm, the femoral neck and the lumbar spine. It was found that multiple scatter is a highly geometrical problem and the effects of multiple scatter are least important at scattering angles between 30° (for a large examination site) and 60° (for a small examination site). For a scattering angle of 50°, typical multiple scatter correction factors (mcfs) are 0.46, 0.45 and 0.35 respectively, for the lower forearm, femoral neck and lumbar spine phantoms studied. The effect of beam polychromaticity on density measurements is much less important than that of multiple scatter. Information from the computer model was used to design and build a densitometer. Densities were measured in-vitro using anthropomorphic phantoms. Different collimator systems were studied and higher precision was achieved with multi-hole focussing detector collimators. Pre-patient photon beam filtration had only a small effect on the precision of the measurements whereas the dose was decreased by a factor of 5 under these conditions. The performance and the stability of the x-ray generator were found to be a limiting factor in the precision of the density measurements.
ACKNOWLEDGEMENTS

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CHAPTER 1

1.1 General introduction

Fractures occur in bones which are "insufficiently strong" to withstand everyday activities. They are commonly seen in the elderly and often occur due to osteoporosis (see section 1.3). Fractures of the proximal femur and lumbar vertebrae are an important cause of illness and death in the elderly.

Both density and structure contribute to the strength of the bone. Attempts have been made to assess the risk of a fracture and stochastic models relating bone loss to fracture risk assume that in adults the risk of fracture following a fall is determined by the amount of bone tissue at the fracture site. Hence, the lower the amount of bone, the higher the risk of non-traumatic fracture. Methods, therefore, have been developed in order to evaluate bone density changes in individuals and to indicate people with high fracture risk. The structural changes, on the other hand, which accompany the loss of bone due to aging and disease, are likely to be important determinants of bone strength and fracture risk.

Recent studies have shown that hormone replacement therapy (HRT) may offer protection against hip fractures. A decrease in the number of fractures will lead to less suffering and financial savings on patient treatment.

1.2 Bone

Information about bone physiology is important in understanding the changes in bone due to aging and the complications which may appear. Bone is a living tissue with a collagenous protein matrix that has been filled with mineral salts, in particular phosphates of calcium which appear mostly in the form of hydroxyapatites,
Ca\textsuperscript{2+}x\textsubscript{10-x}(H\textsubscript{3}O\textsuperscript{+})\textsubscript{2x}(PO\textsubscript{4}\textsuperscript{3-})\textsubscript{6}(OH)\textsubscript{2}. The maintenance of bone structure is based on the amount of both protein and minerals present. Bones (i.e. the skeleton) support the body, provide a store of Ca\textsuperscript{2+} and other minerals that aid in maintaining mineral homeostasis and aid the lungs and kidneys in the maintenance of acid-base balance.

### 1.2.1 Types of bone

Histologically, bone can be divided into: (i) Compact bone (cortical bone) found in the shafts of long bones and the outer surfaces of flat bones and (ii) cancellous bone (inner or trabecular bone) that makes up the trabeculae lining the marrow cavities. Mineral turnover in the skeleton is active with annual calcium turnover varying between 100% (infants) and 18% (adults). Three types of cells in bone are primarily concerned with bone formation and resorption. Osteoblasts are responsible for bone formation due to collagen secretion, osteocytes are surrounded by a calcified matrix whereas osteoclasts erode and resorb previously formed bone.

### 1.2.2 Bone diseases

There are a variety of diseases in which bone changes are affected by disorders in metabolism leading either to increased amounts of calcified bone (osteosclerotic) or more frequently to a reduction in the amount of mineral content per unit of bone matrix (osteomalacia) or to a decrease in bone mass where the ratio of mineral content to bone matrix remains the same (osteoporosis). The last disease is the most widespread disease in the western world and, thus, has attracted great attention over a long period of time.

### 1.3 Osteoporosis

Although research on osteoporosis has been going on for as long as three decades,
the disease is difficult to be defined uniquely (Nordin, 1987). Osteoporosis is described as a metabolic bone disorder where bone resorption exceeds its synthesis resulting in a thinning and weakening of the bone. It is considered as part of the aging process especially in women, characterized by spontaneous (i.e. atraumatic) or low trauma fractures of the lumbar spine, femoral neck or lower forearm. The main cause is generally accepted to be oestrogen deficiency leading to a loss of protective effect against bone resorption (Lindsay, 1976, Heaney, 1977).

1.3.1 Kinds of osteoporosis

It is necessary to differentiate between the two kinds of osteoporosis. Senile osteoporosis (Riggs et al, 1982) or simple osteoporosis (Nordin, 1987) refers to normal age related decrease in bone density whereas accelerated osteoporosis refers to a higher rate of bone density loss than that expected when age related density changes are considered. Riggs et al, 1982 called it postmenopausal osteoporosis since bone density loss accelerates in women just after the menopause.

1.3.2 Complications of osteoporosis

It has been found that bone density may contribute as much as 65% to the strength of the bone; the rest is attributed to bone structure (Compston et al, 1988). The presence of additional factors related to bone strength is responsible for the lack of a straightforward relationship between the density of the bone and its strength.

Effects associated with loss of bone density or bone mass cause much concern. Fractures give discomfort and pain and bone deformities may also develop. Severe osteoporosis may lead to loss of height due to a number of vertebral fractures whereas significant mortality rates are associated with hip fracture making it the most important accidental death among the elderly. The rate of bone density loss in an individual may be used to decide if the person examined carries a high risk of osteoporosis. If so, suitable
1.3.3 Bone density age related loss patterns

A number of methods have been used or are being developed in order to measure in-vivo bone density or bone mass loss. Measurements have been performed on a number of normal subjects in order to establish a pattern of bone density age related loss. Bone density age related loss patterns depend: (i) On the method used, reflecting mainly the quantity measured and (ii) on the site examined, reflecting bone changes at the site considered. Since cortical and trabecular bone have quite different metabolic activities with the latter being much more metabolically active, bone density loss patterns must be considered separately.

(i) Cortical bone density loss: Mazess, 1982, found that cortical bone peak density occurs between 35 and 40 years of age with an average loss of ~0.3% per year between 40 and 50 years of age. The bone loss rate increases to reach an average value of ~0.9% per year up to 75 years of age and then it decreases to ~0.3%-0.4% per year.

(ii) Trabecular bone density loss: It has been more difficult to agree on a pattern of trabecular bone density loss. This may be due to greater variation in trabecular bone changes with age between individuals. Trabecular bone density loss patterns also seem to be very dependent on the method used. Cann et al, 1985, reported an average loss of ~1.2% per year which appeared to start as early as 20-30 years of age. Pacifici et al, 1987, found a value of 1.14% per year. All these measurements were made with a quantitative computerized tomography technique (see section 1.5.1). Dual photon absorptiometry (see section 1.5.2) gave lower trabecular bone loss perhaps due to a significant contribution in the measured signal from cortical bone which is much less metabolically active. Table 1.1 gives age changes of trabecular bone measured with quantitative computerized tomography techniques.

Information based on cross-sectional studies indicates that women have lower bone mass than men of similar age and osteoporotics show an average bone density lower than that of healthy matched control groups. Thin women with poor diet carry a
Table 1.1 Rate of loss of trabecular bone in normal subjects and osteoporotic patients.

<table>
<thead>
<tr>
<th>Method</th>
<th>Site of measurement</th>
<th>Number of subjects</th>
<th>Diagnosis</th>
<th>Age range</th>
<th>Rate (% per year)</th>
<th>Workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isotope QCT</td>
<td>Distal end of tibia</td>
<td>44</td>
<td>Healthy</td>
<td>50-70</td>
<td>0.95</td>
<td>Ruegsegger et al, 1984</td>
</tr>
<tr>
<td>SEQC T</td>
<td>T₁₂-L₃</td>
<td>133</td>
<td>Healthy</td>
<td>20-80</td>
<td>1.14</td>
<td>Pacifici et al, 1986</td>
</tr>
<tr>
<td>SEQC T</td>
<td>T₁₂-L₃</td>
<td>55</td>
<td>Osteoporotics</td>
<td>42-86</td>
<td>1.62</td>
<td>Cann et al, 1985</td>
</tr>
<tr>
<td>DEQC T</td>
<td>T₁₂-L₃</td>
<td>133</td>
<td>Healthy</td>
<td>20-80</td>
<td>1.03</td>
<td>Pacifici et al, 1986</td>
</tr>
<tr>
<td></td>
<td></td>
<td>55</td>
<td>Osteoporotics</td>
<td>42-86</td>
<td>1.17</td>
<td></td>
</tr>
</tbody>
</table>

Isotope QCT, SEQC T and DEQC T techniques are described in section 1.5.1

†: T₁₂-L₃: 12th thoracic vertebra to 3rd lumbar vertebra.

††: 39% mean decrement of osteoporotic women compared to healthy.

1.3.4 The need for serial bone density measurements

Most *in-vivo* information found in the literature is from cross-sectional studies since the overall performance of most of the methods used precluded taking meaningful serial measurements on the same individual. Serial measurements, however, are very useful in assessing bone density loss due to significant overlap between normal subjects and osteoporotic patients seen in all population studies.

Bone density measurements can contribute to knowledge of bone changes with age and can indicate people at high risk from osteoporosis and possibly evaluate the course.
of treatment. In the remainder of this chapter a critical review of the methods used in bone measurement is given with special emphasis being paid to Compton scatter bone densitometry using polychromatic sources since this is the technique evaluated in this thesis.

1.4 Less popular methods of bone measurements

1.4.1 Photodensitometry

Cortical bone mineral is assessed by comparing the local optical density of an x-ray film produced by the examined bone with that obtained by a simultaneously exposed suitably designed step wedge standard (Kaene et al, 1959; Meema et al, 1964). Figure 1.1 shows the experimental arrangement used. The design of the step wedge standard is crucial. Various materials have been used but Meema et al, 1964, emphasized the advantages of using calibration materials of similar effective atomic numbers, Z_{eff}, to that of hydroxyapatite (Z_{eff}=15.9). A comparison of K_{2}HPO_{4} solutions (Z_{eff}=15.6) of various densities and an aluminium wedge (Meema et al, 1964) demonstrated the superior performance of K_{2}HPO_{4} as a bone substitute. Films must be processed automatically to allow absolute comparisons between individuals.

Sites of measurement - The need for a high contrast image, constant object thickness and accurate measurement of cortical bone thickness restrict the site of measurement to the appendicular skeleton. Meema et al, 1964, studied the proximal end of the radius where an almost parallel cortical bone zone exists. The phalanges have received attention as well (Colbert and Bachtell, 1981) due to their simple geometry and the very small amount of soft tissue present. Metacarpals and the ulna have also been examined.

Accuracy and precision - Immersion of the body part in water will provide a constant thickness object. The position of the step wedge, however, introduces a
significant error if positioned within the waterbath but away from the object examined. Inci
dent photon beam

water bath

The radius; cortical bone

Cross-section of the proximal end of the arm

Stepwedge standard

X-ray film

Figure 1.1 The experimental arrangement used in photodensitometry

Meema et al, 1964, suggested that it should be placed under the part of the body examined and close to the bone exposed but without overlapping with it. This minimises the attenuation differences between soft tissue and water. The effect of bone marrow fat introduces an error between 2\% and 7\%. Colbert and Bachtell, 1981, used more sophisticated equipment to form a 2-D density matrix from measurements on fingers. Phalanx measurements with the same equipment gave 2\% precision whereas an accuracy of ~6\% was found in in-vitro studies of the upper extremities. Dose values of ~0.4 mGy are expected.

Variations on the theme - Anderson et al, 1966, used line scans instead of selecting the mid-bone density and concluded that this improved reproducibility. A carefully taken lateral radiograph would allow the measurement of the width of the examined bone. The volume occupied by the cortical bone could then be determined and if the amount of bone mineral present is measured, cortical bone mineral density (i.e. g/cm³) can be determined.

A suitable method for the detection of osteoporosis should be able to evaluate
trabecular bone density changes in the axial skeleton with precision and accuracy close to 1%. The rather large error in accuracy and precision and the evaluation of cortical bone changes of the appendicular skeleton precludes the use of this method for the detection of osteoporosis.

1.4.2 Radiogrammetry

This was one the first attempts of “quantitative” radiology. It utilises the changes in cortical bone thickness in order to determine the change in the amount of cortical bone. It was first introduced by Virtama and Mahonen, 1960, and Barnett and Nordin, 1960.

Two quantities are measured at a fixed point along the examined bone: (i) Total bone width, D (i.e. trabecular plus cortical bone), and (ii) medullary width, d (i.e. trabecular bone only), figure 1.2. Hence, cortical bone width is given by (D-d) and if circular geometry is assumed, the mass of cortical bone can be determined. Absolute comparison of radiographs requires the same experimental conditions such as geometry and exposure factors. Fixed focus-to-film and focus-to-bone distances as well as fine grain x-ray films are used (Genant et al, 1976).

Sites of measurement - The second metacarpal at midshaft is the most common site of measurement since it is approximately circular in cross-section with the trabecular bone almost central. The hand is easy to reposition and the use of immobilizers helps repositioning and prevents bone rotation. The thin layer of adipose tissue reduces the uncertainty in the exact focus-to-bone distance and low energy spectra (i.e. 50 kVp) give clearer separation between trabecular and cortical bones. Other sites examined are the radius and the femur.

Population studies - Much of the data reported are from studies over a large number of healthy subjects and osteoporotics of both sexes. The wide bone size variability (sex, age and body size dependent), however, prevents absolute comparisons among individuals. Hence, several indices have been proposed leading to relative
cortical bone measurements which seem to be less dependent on the size of the indivi-

dual (i.e. metacarpal index, (D-d)/D%, Barnett and Nordin, 1960, cortical area to total
area, (D^2-d^2)/D^2, cortical area to surface area, (D^2 - d^2)/DL, where L is the length of the
bone, Exton-Smith et al, 1969). All indices, however, are incomplete in accounting for
body size and vary among individuals (Dequeker, 1976).

**Accuracy and precision** - The overall precision of the method largely depends
upon the accuracy in the measurement of the medullary width, d, and, to a smaller
extent, on the quality of the radiograph. Carefully followed protocols and accurate
callipers (i.e. 0.1mm) seem to reduce both intraobserver and interobserver error.
Andersen and Nielsen, 1986, reported best intraobserver error of ~2.5% whereas that of
the interobserver was ~5.8%. Changes in cortical width of ~1.5% can be detected with
95% confidence and thus a significant reduction in cortical width will be detected in
postmenopausal women in less than two years (Horsman and Simpson, 1975).

**Variations on the theme** - Meema and Meema, 1987, showed that measurements at
more than one site (i.e. second metacarpal and proximal radius) help in discriminating against postmenopausal osteoporotic women. In order to improve the precision of the method, Colbert et al, 1978, used a microdensitometer to define with greater reproducibility the boundaries of trabecular bone. Kalla et al, 1988, used an automated technique employing a digitizer. Hand radiographs were scanned and the optical density was used to define trabecular-cortical boundaries. Dose values are expected to be ~0.4 mGy.

The basic idea of radiogrammetry has been applied to CT images. Rutherford et al, 1988, used CT images of the mid-femur and studied the changes in cortical thickness. Osteoperotics were clearly separated from healthy women based on the relationship between cortical and trabecular bone fraction.

A major limitation of this technique and that of photodensitometry is that all measurements are made on cortical bone whereas trabecular bone is regarded as a much better indicator of osteoporosis (Hazan et al, 1977, Ruegsegger et al, 1984, Leichter et al, 1987). The rather large error in accuracy and precision and the evaluation of cortical bone changes of the appendicular skeleton precludes the use of this method for the detection of osteoporosis.

1.4.3 Photon absorptiometry methods

In contrast to the methods described in the previous sections where only the cortical bone was examined, photon absorptiometry techniques determine the amount of bone mineral in the beam path by taking transmission measurements of the bone at a site of interest (Cameron and Sorenson, 1963). The model used to derive the amount of bone mineral from these measurements assumes a two component system consisting of soft tissue and bone (i.e. trabecular plus cortical).

1.4.3.1 Single Photon Absorptiometry (SPA)

A well collimated gamma-ray beam aligned with a collimated NaI(Tl) scintillation
detector moves in a rectilinear mode scanning the bone at the site of examination, figure 1.3. Transmitted photons are detected and the photon intensity profile is used to derive the amount of bone mineral per unit length, $q_m$, from equation 1.1 (Cameron and Sorenson, 1963).

$$q_m = \left( \frac{g_m}{(\mu_m - \mu_s)} \right) \int_0^\infty \ln \left( \frac{I_o}{I_m} \right) \, dl$$  

1.1

g_m is the physical density of mineral, $\mu_m$ is the mass attenuation coefficient of mineral and $g_s$ and $\mu_s$ are the physical density and mass attenuation coefficient of soft tissue, respectively. Integration takes place over the entire intensity profile and $I_o$ and $I_m$ are photon intensities in water alone and in water, soft tissue and bone, respectively. Since monoenergetic sources are employed, an overall constant patient thickness must be assumed. This requires the part of the body at the site of measurement to be immersed in a water tank.

Radiation sources and sites of measurement - The most commonly used source is $^{125}\text{I}$ (27.5 keV) (Cameron and Sorenson, 1963, Christiansen et al, 1975, Christiansen and Rodbro, 1977, Wilson, 1977, Kraner et al, 1978, Richardson et al, 1986, Scheckel and Root, 1988) although $^{241}\text{Am}$ (59.5 keV) has been tried (Cameron and Sorenson, 1963). The need for an overall constant patient thickness and the use of radioisotope sources restricts this method to appendicular skeleton studies. The most commonly measured sites are the distal and proximal ends of the radius and ulna.

Accuracy and precision - The amount of adipose tissue introduces a positive bias to bone mineral measurements due to the difference in photon attenuation properties between soft tissue and adipose. Determination of the amount of bone mineral does not take into account the amount of adipose tissue present. Best accuracy is $\sim 2\%$ at $\sim 0.03$ - 0.04 mGy on measurements at the radius (Richardson et al, 1986). Corrections due to scatter contribution to the signal detected have not been considered. The precision of the method is determined by: (i) The number of transmitted photons detected, (ii) the
of trabecular and cortical bone varies along the radius and ulna (Schlenker and VonSeggen, 1976), careful repositioning is required for absolute comparison between successive examinations on the same individual. Regions of interest (ROI) are positioned and scanned in relation to some selected bone spacing values. Precisions of ~1.4% and 1.7% have been estimated in normal subjects and osteoporotic patients, respectively, on measurements at the lower forearm at a skin dose of ~30-40 µGy (Christiansen and Rodbro, 1977). Improved systems (Nuclear Data ND 1100B) claim precision of ~1% on measurements at the proximal end of the radius (Sheckel and Root, 1988).

SPA is a quick, easy to apply method with precision between 1 and 2% and
1.4.4 Scattering techniques

In contrast to the methods described in sections 1.4.1 to 1.4.3 (i.e. transmission methods), scattering techniques have the potential to separately evaluate the response of trabecular bone from that of cortical bone. Essentially this is achieved by recording information from a "critical volume" of interest located inside the scattering medium considered, figure 1.4.

1.4.4.1 Coherent to Compton scattering technique

This method is used to evaluate changes in bone composition. Although osteoporosis does not alter the bone composition, the coherent to Compton scattering technique can possibly be used to detect bone diseases such as osteomalacia. The method makes use of the strong dependence of coherent scatter on the effective atomic number of the scatterer, $Z_{\text{eff}}$, (i.e. coherent scatter $\propto (Z_{\text{eff}})^n$ with values of $n$ varying from $\sim 3$ to $\sim 7$ in biologically important materials). The use of coherent scatter alone would introduce systematic errors due to attenuation by the scatterer and any surrounding materials. The problem of attenuation is very much reduced when the ratio of coherent to Compton scatter is considered. Due to similar attenuation properties of elastically and inelastically scattered photons this ratio reduces the attenuation accuracy close to 2%. However, the evaluation of bone status at the lower forearm (i.e. poor correlation between peripheral and axial bone status, Mazess et al, 1984, Richardson et al, 1986) and the inability to separate trabecular from cortical bone limit the use of this method in clinical applications. Dual photon absorptiometry, DPA, is an alternative absorptiometry method. Since dual photon absorptiometry is the most widely used $in$-$vivo$ method, it is described in the section which deals with the currently popular methods of bone density measurement (see section 1.5.2).
When atomic form factors for coherent and Compton scatter are considered, the ratio of the differential atomic cross-sections, $R$, is proportional to $(Z_{\text{eff}})^{n-1}$

$$R = C (Z_{\text{eff}})^{n-1}$$

where $C$ depends upon the scattering angle $\theta$ and the incident photon energy, $E$. Theoretical calculations (Bradley and Ghose, 1984) and experimental measurements (Karellas et al, 1983, Gegante and Sciuti, 1985) have shown that the exponent in equation 1.2 varies with the scattering angle and the incident photon energy. Hence, it cannot be regarded as a uniquely defined quantity unless both scattering angle and incident photon energy are kept the same throughout all measurements. If so, $Z_{\text{eff}}$ may be taken as a reliable indicator of the scatterer's composition.

**Figure 1.4** The basic principle of scattering techniques. Information is recorded from the "critical volume".

*Radiation sources and optimum geometry* - Most systems used employ either an Am$^{241}$ (59.5 keV) or a Gd$^{153}$ (44, 100 keV) source with a collimated high purity Ge
detector connected to a multi channel analyser (MCA). A common problem with all scattering techniques is the determination of an optimum scattering angle based on certain criteria where practicalities must be taken into account. A compromise between counting statistics, separation of coherent and Compton peaks and sensitivity (i.e. ability to detect small changes) must be reached (Karellas et al, 1983). Scattering angles of 90° (Shih-Shen Ling et al, 1982) and ~135° (Gegante and Sciuti, 1985) have been tried in-vitro whereas a scattering angle of ~70° has been used in-vivo with an Am\textsuperscript{241} source (Greenfield et al, 1988). The low probability of coherent scatter at scattering angles greater than 20° and the use of radioisotopes limit the clinical applications of this method to small cross-sections of the patient and, thus, the appendicular skeleton only (i.e. radius, os-calcis).

**Accuracy and precision** - Effects such as changes in the surrounding tissue and the presence of fatty marrow in trabecular bone were studied by Shih-Shen Ling et al, 1982. The former introduces a positive bias whereas the latter shows a ~2.5% decrease in R for a 10% increase in fatty marrow. Studies considering the effect of multiple scatter have not been reported. Proper calibration may totally eliminate attenuation effects. In-vivo accuracy was found to be ~5% on measurements on the os-calcis. In-vitro precision was found to be ~2% but in-vivo studies on a small group of individuals showed precision of ~3.4% (Greenfield et al, 1988). The examination time for measurements on the os-calcis was ~ 10 minutes and the dose ~1.6 mGy.

Since this technique evaluates changes in the composition of the medium considered it can possibly be used in the detection of osteomalacia. Restriction of this method to the appendicular skeleton, relatively poor accuracy and precision and high dose means that the method has limited use in clinical applications.

1.4.5 Neutron activation analysis

This method is used to determine the absolute quantity of a particular element in the body (elemental analysis). Neutron activation analysis, NAA, can be used for bone
Mineralisation studies since 99% of body calcium is found in the skeleton. Hence, significant changes either in total body calcium (i.e. total body neutron activation analysis, TBNAA) or in calcium at a particular site examined (i.e. part body neutron activation analysis, PBNAA) will possibly reveal skeletal physiological changes.

The method is based on the fact that when the body is irradiated with neutrons, some body elements become radioactive. The quantity of these elements can then be determined by measuring the emission of their characteristic gamma radiation. The element of interest in bone mineral is calcium, \( ^{48}\text{Ca}(n,\gamma)^{49}\text{Ca} \). Chamberlain et al., 1968, first suggested the possible measurement of body calcium using NAA.

**Radiation sources** - Fast neutron beams have to be used with a moderator (usually a sufficiently thick perspex or polyethylene sheet) while attention must be paid to beam uniformity. Neutron beams can be provided either by radionuclide sources such as \( ^{252}\text{Cf} \) (mean neutron energy \( \sim 2 \text{ MeV} \)), \( ^{239}\text{Pu-Be}, ^{241}\text{Am-Be} \) (mean neutron energy \( \sim 4.5\text{MeV} \)), or by neutron generators or cyclotrons (higher neutron energies; 4-8 MeV (cyclotrons), 14MeV (neutron generators)). Neutron generators are more preferable than cyclotrons due to lower cost (Boddy et al., 1973). More than one neutron source has also been used in TBNAA to give a sufficiently intense neutron beam at optimum energies (Cohn et al., 1972, McNeill et al., 1973).

**The apparatus; quantitative measurements** - A whole body counter is used to detect photons emitted from \( ^{49}\text{Ca} \). It consists of a number of sensitive NaI(Tl) detectors placed both above and below the patient. The detectors can either be stationary or move at a constant speed while the subject is being scanned (i.e. scanning mode system). The spectrum taken from an unirradiated torso for the same period of time to give a measurement of background is subtracted from that obtained by the subject examined and the relative abundance of body calcium is determined by comparing the obtained spectrum with that from a suitable size phantom filled with a known concentration calcium solution.

TBNAA is used to reflect the status of the entire skeleton. Only 20% of the
skeleton, however, is trabecular bone whose changes are of great importance in the
detection of osteoporosis. Small changes in calcium at a particular site cannot be detected
with TBNAA (Cohn et al, 1972). The hand is one site of measurement that has been
chosen for PBNAA since it is less radiosensitive and measurements on the trunk have
been made as well to improve the sensitivity of the method due to the larger volume

**Accuracy and precision** - Body size must be considered (Nelp et al, 1972) as well
as sex, weight and age to allow absolute comparisons between groups of individuals.
Uniform body irradiation and corrections for body size are the major factors affecting the
accuracy of the method. When these parameters are carefully considered, phantom
measurements show an accuracy of ~5%. Precision measurements based on *in-vitro*
studies are ~2% for most of the systems whereas *in-vivo* precision determined with one
of the TBNAA systems available was ~2.9% (Nicoll et al, 1987). Cross-sectional
studies give a 1 to 1.5% bone loss rate per year in postmenopausal women.

**Dosimetry** - The radiation dose given to the patient is a major concern with this
method. Doses largely depend on the source used (i.e. incident neutron beam mean
energy) and, to a lesser extent, on the geometry and part of the body examined. In
TBNAA, radionuclide sources give dose equivalent values of ~2.8 mSv compared to
values of ~6.4 mSv given when neutron generators are employed and to those of ~13-20
mSv from cyclotrons when a quality factor of 10 is assumed. Hence, dose to the patient
is higher than that given by any other technique making this a major disadvantage of the
method.

**1.4.6 Broad band Ultrasonic Attenuation**

As ultrasonic waves propagate through a medium, absorption, scatter, reflection
and refraction at the medium boundaries take place. These effects, which in general are
different between different media, can be used to give information relevant to the status
of the bone examined (i.e. either trabecular or cortical) and possibly differentiate between healthy subjects and early osteoporotic patients.

The technique used is that of broad band ultrasonic attenuation, BUA, in which the frequency attenuation pattern of a broad frequency ultrasonic spectrum (i.e. 200kHz to 1MHz) is obtained from trabecular bone (Langton et al, 1984). Measurements depend upon the amount of bone mineral in the path of the ultrasonic beam and upon the structure of the bone.

The apparatus; sites of measurement - The os-calcis is the most common examination site because it is easily accessible and has a significant amount of trabecular bone with almost parallel lateral and medial sides. Two transducers (i.e. transmitting and receiving) were employed held at a fixed distance inside a constant temperature water bath to provide a tissue-like, air free medium between the transducers, figure 1.5. A frequency generator and analyser are connected to transmitting and receiving transducers, respectively, and an on-line computer is used for frequency analysis. Broad ultrasonic spectra are taken before and after the foot is immersed into the water bath to correct for the frequency dependence of the transducer efficiency and beam profile. Spectra subtraction will show the difference due to the presence of the examined heel. The examination time is very short in comparison to that of other methods (i.e. 10 seconds compared to 7-15 minutes) since a complete measurement takes ~1 second and,
Population studies - A decision about the bone status is based on the ultrasonic attenuation versus frequency. Langton et al, 1984, and Miller and Porter, 1988, found significant differences between measurements on young women, healthy older women and osteoporotics with femoral neck fractures. Cross-sectional studies show great dispersion (i.e. coefficient of variation, (C.V.) between 34% and 43%) but no overlap was noticed among the three groups of people studied by Langton et al, 1984, possibly due to the high attenuation frequency dependence on age and fracture related changes. An error for five successive measurements of the subject of ~1.35% was found and that for five daily measurements was ~1.40% (Langton et al, 1984). Poll et al, 1986, reported a precision of ~3.9%.

Variations on the theme - Diagnostic ultrasound has been used for in-vivo evaluation of the strength of the cortical bone in the human femur described by the elasticity modulus, E, equation 1.3

$$E = \rho v^2$$  \hspace{1cm} \text{1.3}

where \(\rho\) is the physical density of the cortex at the site of measurement and \(v\) is the ultrasound velocity in the same medium (Andre et al, 1980).

The quantity measured is the velocity of longitudinal ultrasonic waves, \(v\). A pulse-echo technique is employed and the time of flight, \(T\), that corresponds to the distance between the outer and inner surfaces of the lateral femoral cortex is measured. The cortex thickness, \(d\), is measured radiographically to be used for the calculation of the ultrasound velocity \(v\), equation 1.4

$$v = \frac{2d}{T}$$  \hspace{1cm} \text{1.4}

Patient repositioning and radiographic measurements bring the overall precision of this technique to ~4%. Significantly less femoral ultrasound velocity was found for individuals with bone disorders than for healthy subjects and this may be successfully used to assess the femoral cortical bone status.

Broad ultrasonic attenuation can be used to assess the status of the bone examined.
The great advantage of this technique compared to all other methods used in bone measurement is that it does not involve ionising radiation.

1.4.7 Summary of less popular bone measurement methods

Less popular methods of bone measurements apart from the coherent to Compton scatter technique deal with cortical bone changes (i.e. either thickness or amount of mineral) or look at effects on whole bone (i.e. trabecular plus cortical). The examination sites in all methods are restricted to the appendicular skeleton. The accuracy of these techniques varies between 2-6% and the precision between 1 and 3.4%, table 1.2. None of the methods measure trabecular bone density in the axial skeleton which is ideal for the detection of osteoporosis.

1.5 Currently popular methods of bone density measurements

With the current interest in bone density measurements, most of the techniques described in section 1.4 have either fallen into disuse or proved to be of little clinical use. Currently two methods have been shown to provide the most accurate diagnosis of osteoporosis.

1.5.1 Quantitative Computerized Tomography

In CT an image is reconstructed by taking x-ray profiles of the cross-section to be imaged at various orientations around the patient. The data displayed as the CT image are a representation of the x-ray attenuation coefficients of a series of voxels within the
Table 1.2 Summary of less popular bone measurement methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Quantity measured</th>
<th>Site of measurement</th>
<th>Precision</th>
<th>Accuracy</th>
<th>Dose</th>
<th>Workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photo-densitometry</td>
<td>Amount of cortical bone mineral</td>
<td>Proximal radius, phalanges</td>
<td>&gt;2 %</td>
<td>&gt;6 %</td>
<td>0.4 mGy</td>
<td>Meema et al, 1964; Colbert &amp; Bachtel, 1981</td>
</tr>
<tr>
<td>Radio-grammetry</td>
<td>Thickness of cortical bone</td>
<td>Second metacarpal, radius</td>
<td>&gt;2.5 %</td>
<td></td>
<td>0.4 mGy</td>
<td>Virtama, 1960; Colbert 1978; Kalla et al, 1988</td>
</tr>
<tr>
<td>SPA</td>
<td>Bone mineral per unit length</td>
<td>Radius, ulna</td>
<td>1-1.7 %</td>
<td>&gt;2 %</td>
<td>0.04 mGy</td>
<td>Cameron &amp; Sorenson, 1963; Sheckel &amp; Root, 1988</td>
</tr>
<tr>
<td>Coherent to Compton</td>
<td>Bone composition</td>
<td>Radius, os-calcis</td>
<td>~3.4 %</td>
<td>&lt;5 %</td>
<td>1.6 mGy</td>
<td>Karellas et al, 1982; Greenfield et al, 1988</td>
</tr>
<tr>
<td>NAA</td>
<td>Amount of Ca in total body or examination site</td>
<td>Whole body, Trunk, hand</td>
<td>&gt;2.9 %</td>
<td>&gt;5 %</td>
<td>15 μSv-20 mSv</td>
<td>Chamberlain et al, 1968; Nicoll et al, 1987</td>
</tr>
<tr>
<td>BUA</td>
<td>Status of bone*</td>
<td>os-calcis, femur</td>
<td>1.4-3.9 %</td>
<td></td>
<td>----</td>
<td>Langton et al, 1984</td>
</tr>
</tbody>
</table>

* The method of BUA depends upon density and structure of bone.
reconstructed image. CT has been applied to measurement of bone densities because in the energy range used almost all photon interactions that take place are due to Compton effect and Compton scatter depends upon the physical density. Thus, a direct relationship between CT values and the physical density per voxel can be established (Speller et al, 1981).

Quantitative computerised tomography (QCT) is the only transmission technique which allows separate evaluation of trabecular and cortical bone. This makes the method very attractive since the status of the trabecular bone at the site of measurement is of primary interest in bone densitometry. Since CT values can be related directly to the physical density of the scanned tissues, K$_2$HPO$_4$ solutions (i.e. trabecular bone equivalent) of variable densities (i.e. 50-200 mg/ml) and an alcohol solution (i.e. adipose tissue equivalent) are inserted into a soft tissue equivalent crescent shaped phantom (Cann and Genant phantom). If the phantom is scanned simultaneously with the patient and CT values of the K$_2$HPO$_4$ solutions are compared with those of the patient, the amount of bone mineral can be estimated (Cann and Genant, 1980).

QCT can be divided into three categories based on the type of source employed: (i) Single energy QCT (SEQCT), (ii) dual energy QCT (DEQCT) and (iii) isotope QCT. SEQCT employs a conventional x-ray tube whereas DEQCT requires an x-ray generator able to supply two kV$_p$s (i.e. 80 and 140 kV$_p$ or 75 and 125 kV$_p$). I$^{125}$ (27.5 keV) or Gd$^{153}$ (44 and 100 keV) have been used in isotope QCT.

**Sites of measurement** - Both appendicular and axial bones have been studied. Isotope QCT has been successfully used in radii and tibiae (Ruegsegger et al, 1981,1984) whereas SEQCT and DEQCT have been applied almost entirely to the axial skeleton; normally the proximal femur (Revak, 1980) and especially the lumbar spine, L$_1$-L$_4$ (Cann and Genant, 1980, Adams et al, 1982, Kalender et al, 1987).

**Accuracy** - The accuracy with which CT values have been determined defines the accuracy of this technique. CT values are affected by: (i) Beam hardening, (i.e. when polychromatic sources are employed) (ii) scatter due to overlying tissues present in the
(i) Volume averaging between adjacent objects (i.e. more important when large physical inhomogeneities exist) and (iv) assumptions related to trabecular bone composition (i.e. amount of fatty marrow in trabecular bone). Accurate corrections due to beam hardening effects can be done if the precise size and composition of the object scanned is known. Since this information is never available in vivo, assumptions about body size and composition must be made. Isotope QCT does not suffer from beam hardening effects, and also, the scatter contribution to the image is less due to smaller regions being examined. Robertson and Huang, 1986, used a second order correction algorithm (i.e. data postprocessing) to correct for beam hardening effects on CT values and found that density errors were reduced by 50-95%. Software is used to correct for the scatter contribution to the image although assumptions about body composition have been made.

An increase in the amount of fatty marrow in trabecular bone will show a decreased apparent BMC. Since SEQCT cannot determine the amount of fatty marrow, some a priori knowledge is needed. Dunhill et al, 1967, showed that a small variation exists in fatty marrow changes per year between age 40-60 and therefore a small error in bone density measurement is expected due to the presence of fatty marrow. This, however, is in contrast to large accuracy errors observed in all studies of trabecular bone with SEQCT by Mazess, 1983. DEQCT offers an alternative to this problem since it has the potential to consider the amount of fatty marrow in trabecular bone. SEQCT shows an accuracy between 6 and 9% and DEQCT from 3 to 6%.

**Precision** - The precision of QCT is defined by the reproducibility of CT values inside the volume (or area) examined. This is determined by: (i) Changes in body composition, (ii) patient repositioning and (iii) relocation of ROI.

CT number reference phantoms and careful calibration procedures can reduce errors due to changes in body composition and accurate table movement would reduce the error due to patient repositioning. Attention, however, must be paid to relocation of the ROI. The shape of the ROI and its position are important due to trabecular bone inhomogeneities. The importance of accurate relocation largely depends on the site of measurement and is related to the volume examined. Irregular-reproducible ROIs are
preferred (Kalender et al, 1987). The thickness of CT slices vary; slices as thin as 1 mm have been used in radii and tibiae whereas measurements in vertebrae seem to favour thick slices (i.e. 10 mm). Radiographic localisers help the exact repositioning of the volume scanned (Ruegsegger, 1981, 1984). Precision close to 1% can be obtained with SEQCT whereas DEQCT shows best precision of ~3%.

**Variations on the theme** - 3-D histograms of all pixels included within the vertebral cancellous bone obtained by multi CT sections give precision better than 1%. Due to a larger volume being examined, the problem of trabecular bone inhomogeneities can be overcome (Lambiase, 1987). Adams et al, 1982, suggested that the determination of an effective atomic number of the trabecular bone in the region of interest (ROI) by DEQCT gives a more accurate value than CT values in SEQCT. Rao et al, 1987, showed that trabecular bone density can be more accurately determined by DEQCT than bone mineral content alone.

SEQCT gives skin doses of at least 1.4-3.4 mGy when applied to spine and DEQCT could double these figures (Cann, 1988). The high doses given by QCT is a major problem which may prevent QCT serial studies. DEQCT has restricted use clinically due to poor precision and high dose. SEQCT with precision close to 1% allows precise in-vivo trabecular bone mineral density measurements.

\[ \sum_T b \epsilon \]

### 1.5.2 Dual Photon Absorptiometry (DPA)

Since it is only recently that x-ray sources replaced the radioisotopes used in dual photon absorptiometry, the concepts of DPA will be discussed here. The amount of bone mineral in the beam path is determined by taking transmission measurements. Since a dual energy radioisotope source is employed, the need for an overall constant patient thickness no longer exists. The amount of both soft tissue and bone mineral in the beam path can now be determined in contrast to the amount of bone mineral only, as determined with SPA.

Similar considerations to those in SPA (see section 1.4.3.1) lead to equations 1.5
and 1.6 for the determination of soft tissue and bone mineral mass along the beam path (Roos and Skoldborn, 1974). Measurements obtained from the entire examination site are used to build up an image. An ROI is defined and bone mineral measurements are evaluated in this region.

\[
m_a = \frac{(\mu_b \ln \left(\frac{I}{I_0}\right) + \mu_b \ln \left(\frac{I'}{I_{0'}}\right))}{(\mu_a \mu_b' - \mu_a' \mu_b)}
\]

\[
m_b = \frac{(-\mu_a \ln \left(\frac{I'}{I_{0'}}\right) + \mu_a' \ln \left(\frac{I}{I_0}\right))}{(\mu_a \mu_b' - \mu_a' \mu_b)}
\]

\(I\) and \(I_0\) are the attenuated and unattenuated photon intensities at energy \(E_1\) and \(I', I_{0}'\) the same quantities at energy \(E_2\). Mass attenuation coefficients at \(E_1\) and \(E_2\) are denoted by \(\mu_a\) and \(\mu_a'\) (soft tissue) and \(\mu_b\) and \(\mu_b'\) (bone mineral).

**Radiation sources and sites of measurement** - The first measurements were performed with a combination of an Am\(^{241}\) (59.5 keV) - Cs\(^{137}\) (662 keV) source by Roos and Skoldborn, 1974, whereas the most commonly used source is Gd\(^{153}\) (44 and 100 keV) (Peppler and Mazess, 1981, Le Blanc et al, 1986, Ross et al, 1988 etc.). Tothill et al, 1983, used a low activity (i.e. 0.7Ci) Gd\(^{153}\) source with a coarser beam collimation. Since there is no need for a water bath for maintaining a constant measurement thickness, more clinically relevant areas such as the femur and the lumbar spine can be studied.

**Accuracy and precision** - The cross-over effect (i.e. scattered photons which appear in a lower energy window) both in the patient and inside the detector is a factor which affects the accuracy of the method (Peppler and Mazess, 1981), in addition to the factors reported in section 1.4.3.1. A study by Mooney et al, 1989, showed that scatter may significantly affect the accuracy. Source aging seems to give an elevated bone mineral per unit path length (i.e. BMC) of an average of ~5% per year and corrections suggested by Hanson et al, 1986, reduce but do not eliminate this effect (Ross et al, 1986). The presence of adipose tissue introduces a positive bias and different software used in the analysis may give different results (Gluer et al, 1988). Krolner, 1985,
reported a T1M accuracy in lumbar spine of less than 6%.

Similar factors to those described in SPA determine the precision of this method. Identical ROIs are required between successive examinations on the same individual to allow absolute comparisons. Le Blancet al, 1986, found that bone mineral measurements expressed in g/cm² (i.e. BMD) give better precision in longitudinal studies. Precision between 1.3% and 1.5% has been reported on normal subjects (i.e. lumbar spine) at dose values of ~0.15 mGy (Dunn et al, 1980, Kronler and Pros Nielsen, 1980, Tothill et al, 1983) but it is as low as 2.6% on low BMC osteoporotics.

DPA is the most widely used in-vivo method showing good precision and accuracy at sites of primary interest in bone densitometry (i.e. femoral neck and lumbar spine). A disadvantage of this method is that measurements refer to both trabecular and cortical bone and, thus, separate evaluation of the trabecular bone is not possible.

1.5.3 Dual photon absorptiometry using x-ray sources

A different approach in bone density measurements using the principles of dual photon absorptiometry involves the use of a dual energy x-ray source, thus dual energy x-ray absorptiometry, DEXA. The great advantage of using an x-ray tube is the much higher photon flux which leads to higher transmitted count rate, finer spatial resolution, elimination of source aging effects and possibly reduced dose to the patient.

Although the use of an x-ray tube in DEXA has certain advantages, the stability of the x-ray unit may be a limiting factor and this has to be taken into account. For example, the Hologic QDR 1000 system (Vertec scientific Ltd.) uses a continuous on-line calibration of the x-ray unit by a spinning filter wheel. The filter wheel is spinning synchronously with the mains frequency and hence the tube's accelerating voltage (Cullum et al, 1989). Furthermore, since during each cycle the x-ray tube is supplied alternately with different voltages, only one energy source is present at a time and therefore cross-over effects no longer exist.

The high incident photon beam intensity allows the use of DEXA in axial skeleton density measurements (i.e. femoral neck and lumbar spine) with good precision. The
examination time is ~10 minutes which is almost half the time required with DPA techniques.

**Accuracy and precision** - Images obtained with DEXA are superior compared to those taken with DPA as a result of fine beam collimation (i.e. 1 mm collimator aperture). The improved spatial resolution allows more accurate repositioning of ROIs due to better edge detection and the scatter contribution to the image is reduced due to fine beam collimation. Typically BMD (i.e. g/cm²) is measured on the lumbar spine of normal subjects with precision better than 1% at a dose of ~ 0.12 mGy.

The effect of changes in adipose tissue has been studied by Cullum et al, 1989. Changes in the amount of adipose tissue that surrounds the soft tissue do not significantly affect the accuracy of the method but there is a negative bias in BMD measurement as the amount of fatty marrow increases.

**Table 1.3** Summary of currently popular bone density methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Quantity measured</th>
<th>Site of measurement</th>
<th>Precision</th>
<th>Accuracy</th>
<th>Dose</th>
<th>Workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isotope QCT</td>
<td>Tradecular bone mineral density</td>
<td>Radius, tibia</td>
<td>~0.5%</td>
<td>~1%</td>
<td>10-50</td>
<td>Riegsegger et al, 1981, 1984</td>
</tr>
<tr>
<td>SEQCT</td>
<td>Trabecular bone mineral density</td>
<td>Lumbar spine</td>
<td>&gt;1.5%</td>
<td>~6%</td>
<td>1.3-3.4 mGy</td>
<td>Cann and Genant, 1980, Kalender et al, 1987</td>
</tr>
<tr>
<td>DEQCT</td>
<td>Trabecular bone mineral density</td>
<td>Lumbar spine</td>
<td>&gt;3%</td>
<td>&gt;3%</td>
<td>2.5-6 mGy</td>
<td>Adams et al, 1982, Rao et al, 1987</td>
</tr>
<tr>
<td>DPA</td>
<td>Bone mineral per unit length</td>
<td>Femoral neck, lumbar spine</td>
<td>1.3-2.6%</td>
<td>&lt;6%</td>
<td>0.15 mGy</td>
<td>Roos &amp; Skoldborn, 1974, Roos et al, 1988</td>
</tr>
<tr>
<td>DEXA</td>
<td>BMC or BMD</td>
<td>Lumbar spine</td>
<td>&lt;1%</td>
<td>&lt;1%</td>
<td>~0.12 mGy</td>
<td>Cullum et al, 1989</td>
</tr>
</tbody>
</table>
Jonson et al, 1986, reported transmission measurements on the lumbar spine at more than two energies using an ultrastable x-ray unit and a solid state Ge detector enabling the amount of adipose tissue that surrounds soft tissue to be considered. The stability of the x-ray tube was a limiting factor giving an accuracy of ~5% and precision of ~2.6%.

In general, DEXA measures BMD on lumbar spine with precision better than 1% and this makes the method superior over the conventional DPA techniques. In contrast to QCT, DEXA is unable to separate trabecular bone from cortical bone.

1.6 Compton scatter bone densitometry; an alternative method

One method that has the advantages of using an x-ray tube source, can isolate trabecular bone response and also operate at low radiation dose levels is that of x-ray Compton scatter densitometry, XCSD. Before considering this technique in detail, the concepts of isotope Compton scatter densitometry will be discussed.

1.6.1 Compton scattering technique using radioisotopes

The probability of a Compton interaction depends on the number of scattering centres available in the scattering medium, and at a fixed scattering angle $\theta$ it shows only a small dependence upon the incident photon energy up to 150 keV. The electron density, $\rho_e$, is directly related to the physical density, $\rho$, by

$$\rho = N_A (Z/A)_{\text{eff}} \rho_e$$

where $N_A$ is the Avogadro number and $(Z/A)_{\text{eff}}$ is the ratio of the atomic number to mass.
The number calculated from the elemental composition of the scatterer. The dependence of \( \rho \) on \((Z/A)_{\text{eff}}\) is not very significant since for all elements except hydrogen, \((Z/A)\) is equal to 0.5 (± 0.05). Hence, Compton scattered photons \( N_s \), can directly reflect the physical density of the scatterer, equation 1.8.

\[
N_s \propto \left( \frac{1}{N_A (Z/A)_{\text{eff}}} \right) \rho
\]

Equation 1.8 is true if the effects of the attenuation of incident and scattered photons are not considered. Compton scattered photons, however, would not reflect the physical density of the medium considered unless the problem of photon attenuation is taken into account.

The problem of photon attenuation - This problem has been studied by Kennett et al., 1972, and Clarke and Van Dyk, 1973, both at high (i.e. hundreds of keV) and low (i.e. \( E \ll 511 \text{ keV} \)) incident beam photon energies.

(i) High energy incident photon beam: Instead of a single scatter measurement, an additional transmission measurement is taken at the site of examination. If a suitable energy source (i.e. secondary source) is used and a transmission measurement is made with this source, the problem of photon attenuation can be eliminated, figure 1.6. The choice of the secondary source is mainly governed by the primary source photon energy and the scattering angle. If the energy of the secondary source is equal to that carried by the scattered photons it can be shown that the physical density of the scatterer, \( \rho \), is given by equation 1.9 and is independent of the amount of tissue which surrounds the scatterer.

\[
\rho = k \left( \frac{N_{S1} N_{S2}}{N_{T1} N_{T2}} \right)^{1/2}
\]

\( N_{S1}, N_{S2} \) are scatter measurements and \( N_{T1}, N_{T2} \) are transmission measurements, shown in figure 1.6. \( k \) is a calibration constant determined with a known density scatterer.

(ii) Low energy incident photon beam: If the energy carried by the incident
When the photon beam is low, the Compton energy shift will be small and, thus, the same source can be used as a secondary one. The source will move from position 1 to position 2 and scatter and transmission measurements will be taken simultaneously at both source positions, figure 1.7.

**Radiation sources** - Clarke and Van Dyk, 1973, used a combination of Co$^{60}$ (i.e. mean energy of 1250 keV) and Cs$^{137}$ (662 keV) sources with beams at 50° and Kennett et al., 1972 used Au$^{198}$ (412 keV) and Hg$^{203}$ (279 keV) with beams at 67°. The use of these energies resulted in high doses (Garnett et al., 1973). Piper et al., 1973, carried out an evaluation of several radionuclides in the energy range between 80 keV and 200 keV. They concluded that when dose, Compton interaction probabilities and specific activity are taken into account the combination of Sm$^{153}$ (103 keV) with Tm$^{170}$ (84 keV) had the highest potential. A similar result was stated by Garnett et al., 1973.

**Sites of examination** - Due to the count rate statistics resulting from the use of radio

---

**Figure 1.6** Compton scatter densitometry. Arrangement of sources and detectors for bone density measurement with a high energy incident photon beam. S1: Primary source, S2: Secondary source. (a) Transmission measurements from sources 1 and 2 and scatter measurement from source 1. (b) Sample rotated through 180°. Scatter measurement from source 1.

- nuclides, studies are restricted to the appendicular skeleton only (distal radius, Hazan et al., 1977, os-calcis, Webber and Kennett, 1976). Leichter et al., 1978, showed that the
Trabecular bone density of the distal radius correlates well with spinal osteoporotic changes and the same conclusion was drawn by Hazan et al, 1977, with a low energy incident photon beam system.

Accuracy and precision - The accuracy of the method is limited by inherent errors related to: (i) Finite geometry, (ii) difference between transmission and scatter volumes and (iii) effects due to photon multiple scatter. The presence of an unknown amount of fatty marrow is also expected to affect accuracy.

(i) The effect of finite geometry: When the scatter volume is not infinitesimal, the attenuation factors of scatter and transmission measurements do not cancel completely which leads to a false apparent density increase (Kennett and Webber, 1976).

(ii) The effect of different transmission and scatter volumes: Both transmission and scatter volumes are shown in figure 1.8. The scatter volume, also referred to as "critical volume", is larger than the transmission volume and this gives an apparent bone density increase. The magnitude of this error depends upon the geometrical arrangement used.

(iii) The effect of photon multiple scatter: The accuracy of the method is seriously

Figure 1.7 Compton scatter densitometry. Density measurement with a low energy incident photon beam. (a) Source position 1, (b) source position 2. Transmission and scatter measurements are taken at the same time.
affected by multiple scattering effects. Different methods have been tried in order to reduce this problem. Kennett and Webber, 1976, took an analytical approach to the problem of photon multiple scatter studying a number of cylindrical samples of different size and density. An increase in either size or density of the sample gives an elevated apparent density. However, the effect of the surrounding tissues on density measurements was not studied although according to Leichter et al, 1980, the surrounding tissues contribute most to multiple scattering events.

With a monoenergetic source, multiple scatter can be significantly reduced by using a narrow energy window around the scattered photons energy, (Leichter et al, 1978, Huddleston and Sackler, 1985). Calibration curves taking account of size and density changes have been suggested to be applied to in-vivo bone density measurements. With a dual energy Compton scatter method an in-vitro accuracy of ~2% was found (Huddleston and Sackler, 1985).

The precision of this technique is limited by patient repositioning and the number of scattered counts recorded. Measurements on the distal radius may use both horizontal and vertical scatter scans to identify the site of examination and exclude any cortex (Huddleston and Bhaduri, 1979). Best long term in-vivo precision of ~1.8% has been reported by Leichter et al, 1987, on distal radius measurements.

Dosimetry - The dose given to the patient is determined by the precision required. Dose is localised by the size of the incident photon beam and is restricted to the appendicular skeleton. Kennett and Webber, 1976, estimated skin doses to be between 1.5 and 2.0 mGy on os-calcis measurements for a 1.5% precision.

In summary, Compton scatter bone densitometry with radioisotopes can measure appendicular trabecular bone density with good precision and accuracy and this makes the method attractive in clinical applications. Use of the same method in axial skeleton studies could be ideal but using radioisotopes is unlikely to allow sufficient statistical quantities in the reduced scattered count rate.
1.6.2 Compton scattering technique using x-ray sources

The possibility of using x-ray tubes as irradiating sources has been explored by Hanson and Duke, 1984. It is the much higher incident photon beam intensity compared to that of any radioisotopes available that makes the use of x-ray units attractive in clinical applications because it would lead to higher precision, reduced radiation dose and shorter examination times. The spatial resolution will also improve since x-ray tubes have focal spots between 1 and 2 mm which are smaller than the size of the radioisotopes employed in bone densitometry.

The replacement of the radioisotope source employed in the Compton scattering technique by a diagnostic x-ray tube leads to a low photon energy bone densitometry system (i.e. maximum photon energy provided by diagnostic x-ray tubes ~120 keV). Instead of using only one x-ray tube which rotates around the patient, two x-ray tubes
are employed at fixed positions, figure 1.9. Each incident photon beam is finely collimated and aligned with a collimated detector.

With reference to figure 1.9, when x-ray tube 1 irradiates, a transmission measurement is taken by detector 1 and a scatter measurement by detector 2. When x-ray tube 2 irradiates, detector 1 records scattered radiation and detector 2 transmitted photons. Thus,

\[(N_{S1} / N_{T1} \cdot N_{T2})^{1/2} = k \rho \] 

1.10

where \(N_{S1}, N_{S2}\) are scatter measurements and \(N_{T1}, N_{T2}\) are transmission measurements when tube 1 and tube 2 are on, respectively. The quantity shown in equation 1.10 can be used to determine the physical density of the scatterer, \(\rho\), inside the "critical volume" and is independent of the amount of tissue which surrounds the scatterer. \(k\) is a calibration constant determined experimentally.

### 1.7 Problems of x-ray Compton scatter bone densitometry

The accuracy of the method is affected by the same factors as those described in Compton scattering technique with radioisotopes. Equation 1.10 can be rewritten as:

\[\rho = k' (N_{S1} / N_{T1} \cdot N_{T2})^{1/2} \cdot f(a,b,c,d,\theta) \] 

1.11

where \(k'=(1/k)\) and \(f(a,b,c,d,\theta)\) is a correction factor taking into account: (i) The difference in photon path lengths and energy shift between scattered and transmitted beams and (ii) the effect of multiple scatter. Distances \(a,b,c\) and \(d\) are the transmitted and scattered path lengths and \(\theta\) is the scattering angle defined by the incident photon beams, figure 1.9. Correction factors due to differences in photon path lengths and energy shift between scattered and transmitted beams were obtained by Duke and Hanson, 1984, for a number of different geometry phantoms with filtered incident x-ray spectra.
1.7.1 Early work on the effect of photon multiple scatter

The problem of photon multiple scatter is a serious limitation since errors due to multiple scatter are one order of magnitude greater than other inherent errors such as finite geometry and the difference in transmission and scatter volumes. Since a spectrum of photon energies are employed, the choice of a narrow energy window will not eliminate the effect of multiple scattering events on density measurements. The use of an additional detector that detects photons which have undergone more than one interaction seems to correct for the effect of multiple scatter but its use is restricted to cylindrical samples, (Hanson and Duke, 1984).

Figure 1.9 X-ray Compton scatter densitometry. The experimental arrangement. T1, T2: Transmitted beams. S1, S2: Scattered beams.
An alternative approach was taken by Speller and Horrocks, 1988, where a photon transport computer model was used to study the effect of multiple scatter. Three sites of the human body were considered (i.e. lower forearm, femoral neck and lumbar spine) and correction factors were obtained at different scattering angles.

1.8 The scope of the thesis

The aim of this thesis is to investigate x-ray Compton scatter bone densitometry and to consider in particular the problems of photon multiple scatter and system design. A method of high accuracy and precision would allow small bone density changes to be detected by taking serial measurements on the same individual over a period of time.

1.8.1 Layout of the thesis

Chapter 2 deals with improvements and testing of the photon transport computer model which is used to study the effect of multiple scatter in x-ray Compton scatter densitometry.

Chapter 3 deals with factors which affect the accuracy of the method. Correction factors due to (i) difference in photon path lengths and energy shift between scattered and transmitted beams and (ii) due to the effect of multiple scatter (i.e. using a photon transport computer model) are obtained for a series of different size phantoms which represent cross-sections of the lower forearm, the femoral neck and the lumbar spine. The importance of the correction factors in bone density measurements is evaluated.

Chapter 4 deals with the design and construction of the x-ray Compton scatter bone densitometer. A number of tests were carried out in order to evaluate either the performance of individual components (i.e. x-ray units, electronics) or that of the whole x-ray Compton scatter bone densitometer.

Chapter 5 contains the experimental results. Three phantoms were used of cross-sections which represent sites of interest in bone densitometry (i.e. lower forearm,
remoral neck and lumbar spine). Different experimental conditions (i.e. incident photon beam intensity, spectrum filtration, energy windows etc.) and detector collimators were tried. Results are presented and discussed that indicate the precision and accuracy of the system. Skin dose was measured and dose distribution inside the phantoms studied was estimated.

Chapter 6 gives conclusions considering the performance of the x-ray Compton scatter bone densitometer built. Suggestions for further improvement are given and future work considering the use of the method in clinical applications is described.
2.1 General introduction

In x-ray Compton scatter densitometry the determination of bone density is based on the detection of scattered photons which reach the detector after a single scatter in bone. It is probable, however, that a photon has reached the detector after being scattered more than once, either in the bone itself or in the surrounding tissue. This comprises a multiple scattering event giving rise to the problem that will be referred to in this thesis as multiple scatter. Since information carried by a multiply scattered photon does not relate to scattering in the region of interest, the effect of multiple scatter on density measurements must be evaluated.

Multiple scatter is a highly geometrical problem. As the size of the patient increases, the probability for the same photon to undergo another scatter before it leaves the patient is higher. An additional factor to consider is that the larger the size of the patient, the greater the probability of the photon being absorbed.

2.2 Evaluating the problem of multiple scatter

In order to evaluate this problem both experimental and theoretical approaches have been tried by earlier workers. For example, improvement of the energy resolution of the detector can reduce, but not eliminate, the effect of multiple scatter on density measurements (Battista and Bronskill, 1978).

An experimental way to correct for the effect of multiple scatter has been described by Hanson and Duke, 1984, where a second detector of similar response to those employed in the bone densitometer is positioned in such a way that only photons scattered at least twice are detected. Geometrical assumptions involved in the corrections and the detector response to low count rates make this idea of limited use in practice.
A semi-experimental way to account for the effect of multiple scatter has been proposed by Kennett and Webber, 1976, where cylindrical samples which show either high or low probability for a photoelectric interaction (i.e. absorbing and non-absorbing materials, respectively) were used. Measurements were performed on different size samples of the same density and the effect of multiple scatter was evaluated. This effect was found to vary both with size and density of the sample examined. None of the samples were surrounded by any other material and, thus, the arrangement cannot be considered as a simulation of the clinical situation.

A purely theoretical way to correct for the effect of multiple scatter on density measurements is to use a computer model which samples the photon histories through the phantom studied (i.e. Monte Carlo photon transport model). Details of the spectrum of the incident photon beam, the phantom dimensions and geometry, the materials involved and the photon interactions which may occur in the energy range of interest, enable the history of a photon to be followed as it passes through a phantom. The sites and number of interactions of each photon can be found and this information can be used to study the effect of multiple scatter. Such studies must be performed by computer because of the huge number of calculations. However, they are time consuming and generally demand large amounts of computer memory and this limits their wide applicability in routine work.

In this thesis computers have been used for the evaluation of the effect of multiple scatter on x-ray Compton scatter bone densitometry, XCSD. The current chapter describes the improvements involved in the new version of the photon transport computer model and the tests carried out for the evaluation of the behaviour of the Monte Carlo program.

### 2.3 The Monte Carlo photon transport model

The photon transport computer model used phantoms that considered three sites of the human body which are of interest in bone densitometry. By changing the input data,
different experimental conditions could be simulated and parameters which affect the problem of multiple scatter evaluated.

![Diagram](image.png)

**Figure 2.1** The geometry used in the photon transport computer model.

### 2.3.1 The input data

Information about the incident photon spectrum, attenuation properties of all of the phantom materials and the geometrical arrangement must be given.

*(i) Spectral data* - The incident photon spectrum (i.e relative photon output per keV) is given and this determines the photon energy range of interest.

*(ii) Phantom materials attenuation properties* - This involves the mass attenuation coefficients (μ/ρ) at certain energies within the photon energy range of interest and the densities of the materials comprising each phantom.

*(iii) Types of interactions* - Rayleigh, Compton and photoelectric interactions
have been taken into account and the relative interaction probabilities are given at certain energies.

(iv) Geometrical considerations - (a) The dimensions of each phantom material and its position relative to the incident photon beam, (b) the collimation of the incident photon beam and (c) the geometry of the detector collimator are taken into account. Figure 2.1 shows the geometry used in the computer model. The detector rotates at a fixed distance around the phantom between 0° and 360° and information is collected in one degree intervals.

2.3.2 The operation of the Monte Carlo photon transport model

The model operates in the following manner, figure 2.2. A photon is chosen from the incident photon beam and its energy is determined from the energy spectrum using a random number. The site of interaction is determined from the linear attenuation coefficients of the materials comprising the phantom, the photon’s direction and the most probable interaction at this photon energy (i.e. relative interaction probabilities). Once the type and site of interaction have been determined, the scattering angle (if there is one) is calculated. A photon with energy less than 10 keV is no longer followed and is assumed to undergo a photoelectric interaction and be locally absorbed. Photons that are outside the phantom are considered to continue in a straight line in the same direction as that determined by their last interaction. Photons that hit the collimator of the detector are absorbed and no information carried is conveyed.

This photon transport study allows the number of interactions which take place within each phantom material to be determined and further study is then based on this information. Due to the nature of the problem, photons were grouped into two major categories: (i) Photons which have reached the detector after having been scattered once and only inside the bone, and (ii) photons that have undergone more than one interaction anywhere inside the phantom. The former comprises the signal and the latter the noise. The variation of both signal and noise with the scattering angle is studied for several different size phantoms in which a large number of photon histories are followed.
Figure 2.2 Flow chart of the photon transport computer model. N: Current photon history followed. E: Energy carried by the photon.
2.4 Improvements to the Monte Carlo photon transport model

The original versions of the computer code model were called photon scattering 1 (PS1) (Gowland, 1987) and photon scattering 2 (PS2) (Speller and Horrocks, 1988). A number of points were improved to produce the new version (PS3) of the photon transport computer model. These improvements involved: (i) Phantom materials, (ii) sampling of elastic scatter, and (iii) information collected.

2.4.1 Inclusion of adipose tissue

In the original versions of the photon transport computer model (Gowland, 1987, and Speller and Horrocks, 1988) all phantoms studied were assumed to consist of three materials; trabecular bone, cortical bone and soft tissue. Adipose tissue, however, is present at all sites of the body and attenuates a photon beam considerably less than soft tissue in the diagnostic energy range. In the current version of the model a layer of adipose tissue surrounding the soft tissue of each phantom was considered. The elemental composition of the adipose tissue was taken from Woodard and White, 1986. Its mass attenuation coefficients and relative interaction probabilities were obtained from Hubbel, 1969, and Storm and Israel, 1970. The density of the adipose tissue was assumed to be 950 kg/m$^3$ (Woodard and White, 1986). Figure 2.3 shows a cross-sectional view of one of the phantoms considered showing the additional layer of tissue that has been included in the simulation.

2.4.2 Elastic scatter

Elastic (or Rayleigh) scatter had been sampled in the original versions PS1, PS2 of the Monte Carlo photon transport model using a Thomson scattering simulation. For the
work discussed in this thesis, Rayleigh scatter replaced that of Thomson. This was done by considering the form factors for each phantom material. The form factors, $F_m$, of each of the phantom materials were calculated from the atomic form factors of all of the constituent elements weighted according to their elemental composition (Woodard and White, 1986). Equation 2.1 shows the formula used for the calculation of the form factor of a material when a "free atom" model is assumed.

$$F_m^2 = a_1 F_1^2 + a_2 F_2^2 + \ldots + a_n F_n^2$$

$F_i^2$ represents the form factor square of the element $i$ and $a_i$ is the percentage by weight of that element present in the material considered.

![Diagram of phantom materials](image)

**Figure 2.3** The phantom materials considered in PS3. The amount of adipose tissue present varies between different size phantoms.

Data were taken from Hubbel, 1969, for all of the elements of interest. Form factors of the phantom materials were calculated for momentum transfer values between 0 and 10 (T. Baba-Hamed, 1990). Two methods of elastic scatter sampling were evaluated: (i) A tabulation method (Neizel *et al*, 1985) and (ii) an inversion-rejection...
method (Williamson and Norin, 1983). The latter method was used in the Monte Carlo photon transport model discussed here.

The inversion-rejection method can be described as follows. The momentum transfer, $x$, is defined as:

$$x = \alpha k \sqrt{1 - \cos \theta}$$  \hspace{1cm} (2.2)

where $\alpha$ is the energy of the incident photon expressed in rest mass units, $k$ has a value of 29.1445 when energy is expressed in keV and $\cos \theta$ is the direction cosine relative to the incident photon direction. A value of the momentum transfer, $x$, can be taken by defining a cumulative probability as shown in equation 2.3.

$$r_n = \frac{\int_0^{x_{\text{max}}} F_m^2 \, dx}{\int_0^{x_{\text{max}}} F_m^2 \, dx}$$  \hspace{1cm} (2.3)

$F_m^2$ is the form factor square of the material considered and $x_{\text{max}} = \alpha k \sqrt{2}$. From equation 2.2 a value of $\cos \theta$ can be defined; $\cos \theta = w = 1 - (x/\alpha k)^2$. There is, therefore, a correlation between the momentum transfer defined by the cumulative probability and the scattering angle, $\theta$. If a random number, $r_{n+1}$, is generated and is less than or equal to $((1+w)^2/2)$, the value of $w$ is accepted and a scattering angle $\theta$ is then determined. Otherwise, the value of $w$ is rejected and the whole procedure is repeated. In the computer model used, the maximum value of the momentum transfer was taken as 10 and the integration defined in equation 2.3 was performed numerically using the trapezoidal rule. This was done for form factors of all of the phantom materials. If the value of the momentum transfer is greater than 10, the result is rejected and the entire procedure is repeated.

Using this method, a scattering angle $\theta$ can be determined from a random number. By generating many random numbers, different scattering angles which correspond to a Rayleigh scatter angular distribution can be obtained. Rayleigh scatter angular distributions obtained by the inversion-rejection method are shown in figures 2.4 and 2.5 for soft tissue equivalent material at fixed incident beam photon energies of 30 keV.
and 50 keV, respectively. Elastic scatter angular distributions calculated analytically when the form factors were taken into account are shown in the same figures. Figures 2.4 and 2.5 show that a photon that undergoes an elastic scatter has a high probability of being scattered by only a few degrees and this probability decreases rapidly with the scattering angle.

The angular distribution is more forwardly peaked for a 50 keV incident photon.
beam than it is for a 30 keV photon beam. Figure 2.6 compares the Thomson angular distribution (PS1, PS2) with that of Rayleigh (PS3).

![Thomson and Rayleigh scatter angular distributions at 50 keV](image)

Figure 2.6 Comparison between Thomson and Rayleigh scatter angular distributions at 50 keV. Data are normalized to the total number of scattered photons.

### 2.4.3 The collimation of the detector

In the original versions of the Monte-Carlo program PS1, PS2 two detector collimators were studied which differed considerably in size and geometry. The smaller collimator was identical to that used to collimate the incident photon beam and the larger one consisted of 50 such collimators superimposed on each other in such a way that they focussed on the irradiated bone (i.e. focussing collimator). Figure 2.7 shows the geometry of the large detector collimator.

The effect of multiple scatter when the large detector collimator was considered showed all the main features as seen with the small detector collimator. The only difference being the number of scattered photons detected in the former case was significantly increased (Speller and Horrocks, 1988). The focussing detector collimator seems more likely to be used in practice and, therefore, only this collimator is considered further in the photon transport computer model.
2.4.4 Photon interactions inside the phantom materials

In order to evaluate the importance of each of the phantom materials in terms of their contribution to the effect of multiple scatter, the number of interactions that took place inside the phantom materials was recorded as the photon beam passed through the phantom. The angle of detection was that at which the photon left the phantom (i.e. defined by the last photon interaction).

2.4.5 Dosimetric Computations

The study of a large number of photon histories can be used for evaluating the distribution of the energy deposited inside the phantom as the photon beam passes through it. Photons of energy less than 10 keV were assumed to undergo a photoelectric interaction and then be locally absorbed, assuming that all secondary electrons deposited...
their energies at the site of production. Fine spatial resolution of the energy distribution inside the phantom was obtained when 2 mm pixels were used and the energy distribution was converted into dose distribution when the mass defined by each pixel was taken into account. Results of this work are considered in section 5.10.

### 2.5 Testing the photon transport computer model

When the new version of the Monte Carlo photon transport model was completed, the following tests were carried out:

(i) *Generation of the x-ray spectrum* - selecting the initial energy of the photon in the x-ray beam. Results from this test are shown in figure 2.8 for a 100 kVp incident photon spectrum.

(ii) *Interaction types* - the part of the computer model that deals with the types of photon interactions was used to check: (a) The frequency of each type of photon interaction at certain energies. The results from soft tissue scattering medium are shown in figure 2.9; (b) Compton scatter angular distributions for monoenergetic incident photon beams.

![Figure 2.8](image.png)

*Figure 2.8* •:100 kVp spectrum (Birch et al,1979), + : 100 kVp spectrum obtained from the photon transport model.
The results from test (b) are shown in figures 2.10 and 2.11 for 50 keV and 80 keV incident photon beams, respectively. In all of these tests the results generated by the model compare well with the theoretical distributions. Since the photon transport computer model is lengthy and complicated, tests dealing with intermediate computations are not easy to compare with expected distributions. However, final scatter contributions should confirm the expected results. Thus, a symmetrical phantom was considered and
scatter information was examined when over $3.4 \times 10^7$ photon histories were followed.
The phantom consisted of four concentric materials of circular cross-section. The
dimensions and geometry of this phantom are given in table 2.1.

<table>
<thead>
<tr>
<th>Dimensions and geometry of the symmetrical phantom used to test PS3.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A and B are the lengths of the semi-major and semi-minor axes of the ellipses. All dimensions are expressed in centimeters. All tissues are concentric.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Adipose tissue</th>
<th>Soft tissue</th>
<th>Cortical bone</th>
<th>Inner bone</th>
<th>Collimator front-to-origin</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 3.0 B 3.0</td>
<td>A 2.5 B 2.5</td>
<td>A 1.0 B 1.0</td>
<td>A 0.9 B 0.9</td>
<td>CTO 8.0</td>
</tr>
</tbody>
</table>

Due to the geometry of the phantom, scatter information is expected to be symmetrical on either side of the incident photon beam. To clarify the details, figure 2.12 shows the variation with the scattering angle of the ratio of single scatter in bone to multiple scatter (i.e. signal to noise ratio, section 3.3.1). Signal to noise variation is statistically identical for negative and positive scattering angles of the same magnitude.

![Figure 2.11 Compton scatter angular distribution](image)
2.6 Experimental validation of the photon transport computer model

Multiple scatter correction factors cannot be determined experimentally. However, the variation in the number of scattered counts with the scattering angle can be measured and compared with that obtained from the photon transport computer model. For an absolute comparison, a phantom of the same dimensions and geometry as that described in the computer model has to be used. Due to practicalities related to the position of both x-ray tube and detector, the geometry described in the computer model could only be retained if a lower forearm phantom was employed.

2.6.1 The phantom used in the experimental validation of the computer model

In order to produce the correct shape and dimensions, an aluminium mould was constructed. Five aluminium elliptical cross-sections (3.0 x 2.5) cm each 12 mm thick
were made and placed one on top of the other to define an elliptical cylinder of ~ 60 mm in height. A PVC cylinder 1 cm in diameter was positioned at (1, -0.5) cm relative to the centre of the elliptical cross-sections (0, 0).

The tissue substitute material was epoxy resin based with appropriate particulate fillers (White et al, 1977). The composition of the substitute material was adjusted to fit the attenuation values and density (1.00 g/cm³) of soft tissue. The material was poured into the mould, left for ~24 hours and then put into the oven at 80°C for ~3 hours. When the mould was removed, an elliptical cross-section phantom (2.9 x 2.4) cm with a 1.0 cm cylindrical hole at (1.0, -0.5) cm remained. A thin aluminium ring (i.e. ~0.4 of a mm) was inserted in the cylindrical hole to simulate the cortical bone with medullary cavity, figure 2.13.

**Figure 2.13** The lower forearm phantom used in the experimental validation of the Monte Carlo photon transport model. Phantom size: (3.0 x 2.5) cm.

Compared to the smallest size of the lower forearm phantoms considered in the Monte Carlo photon transport model, the above phantom lacks a thin layer of adipose
tissue (i.e. soft tissue has been used instead) and the bone has a circular cross-section. The phantom dimensions and the site of the bone are the same as those used in the photon transport computer model. A 1.12 g/cm³ K₂HPO₄ solution in a plastic syringe was inserted into the aluminium lined hole to simulate trabecular bone of the required density.

2.6.2 The experimental arrangement

The experimental arrangement is shown in figure 2.14. The phantom was positioned so that the centre of the K₂HPO₄ solution was at the origin and 8 cm away from the face of the x-ray tube collimator, with the incident photon beam parallel to the ellipse minor semiaxis. A single hole detector collimator was positioned such that the x-ray beam passed through the centre of the "bone". A lead shield (3 mm thick) covered the entire active area of the detector except for a small hole aligned with the collimator. The detector was a scintillation counter, DM 1-2 (Nuclear Enterprises Ltd.), held in place behind the collimator's base so that the detector's "face-to-bone" distance remained the same throughout all measurements with the same area of its crystal irradiated at each scattering angle. The signal was fed directly to an ST7 scaler-timer, (Nuclear Enterprises Ltd.). Scattering angles up to 125° were used since the size of both detector and x-ray tube prevented taking measurements beyond that angle.

2.6.3 Experimental measurements

The measurements are discussed in two parts. First the variation in the scattered photons with scattering angle and secondly the effect of scattered path length changes on the scattered counts at different scattering angles.

(i) The number of scattered photons which reach the detector was measured at scattering angles between 10° and 125° in 5° intervals. A 100 kVₚ incident photon beam
Figure 2.14 The experimental arrangement used in the validation of the computer model, PS3.

Figure 2.14a Variation of scattered photons with the scattering angle.

Figure 2.14b The effect of scattered path length changes on the scattered counts.
was used, the same as the spectrum considered in the photon transport computer model. The intensity was 0.6 mA and the duration 10 seconds. The energy window was set at (25 - 100) keV which compares favourably with that considered in the photon transport computer model (i.e. 10 - 100 keV) when the energy resolution of the detector (~ 15%) and the multiple scatter which takes place inside the crystal are both taken into account. An overall error of at least 3% is present in all measurements, this is attributable to statistical fluctuations and errors in detector positioning. The latter is greater at small scattering angles where the scattering profile changes rapidly. At each scattering angle a background measurement was made and subtracted from the signal recorded.

Figure 2.15 is a plot of the number of scattered counts against the scattering angle compared with that obtained by the photon transport computer model for the same geometry and phantom (except for the adipose tissue layer).

![Figure 2.15 Scattered counts variation for a lower forearm phantom, (3.0x2.5) cm. +: Measured experimentally with a 100kVp spectrum, o: Obtained from the computer model, PS3.](image)

Values have been normalised at -10° and 10°. A reasonable agreement is seen at small scattering angles and a good agreement for ±50° < θ < ±120°. However, a discrepancy exists between ±30° and ±40°. This is seen only in the case where a 100 kVp spectrum has been used. It is probably due to the creation of characteristic x-rays in
the lead collimator. The use of a different incident photon spectrum is not expected to alter the scattering profiles significantly since most photons carry energies between 35 keV and 70 keV. Results from measurements taken with an 80 kVp spectrum are shown in figure 2.16.

The reasonable agreement between experimental measurements and theoretical calculations shows that the photon transport computer model behaves well. The rapid change in the scattering profile at small scattering angles, which is predicted theoretically, is attributed to the consideration of Rayleigh scatter implemented in the new version of the Monte Carlo photon transport model (PS3). However, experimental measurements at $\theta < 25^\circ$ show smaller values than those predicted by the computer model and this is probably due to the form factor values used in the simulation of Rayleigh scatter (see section 2.4.2). The behaviour of the computer model at $\sim 50^\circ$ is of primary interest since a scattering angle of $\sim 50^\circ$ is likely to be used in practice.

![Figure 2.16 Scattered counts variation for the lower forearm phantom, (3.0x2.5) cm](image)

(ii) Experimental measurements were taken in order to determine the variation in scattered counts against the path travelled by the scattered photons at different scattering
angles. Using the experimental arrangement described in figure 2.14.b scatter measurements were taken at different scattering angles with various thicknesses of soft tissue equivalent materials placed between the lower forearm phantom and the detector. Soft tissue equivalent materials placed between the phantom and the detector are expected to have a significant effect on the scattered counts. Since no significant difference was found in the variation of scattered counts against the scattering angle for different spectra, a low intensity 80 kVp incident photon spectrum was used. Exposures of thirty second duration were taken with an energy window fixed at (25 - 80) keV.

Figure 2.17 is a plot of the scattered counts measured experimentally against the scattered path length at scattering angles of 15°, 30°, 45° and 60°. In the same figure, a plot of scattered counts obtained by the computer model is shown for the lower forearm phantoms studied. Although a direct comparison cannot be made, the experimental measurements show that for the lower forearm phantom, for the angular range considered, the larger the scattering angle the smaller the variation in scattered counts with scattered path length change. This agrees with that described by the computer model.

![Figure 2.17](image_url)
2.7 Summary

In this chapter the improvements involved in the new version of the Monte Carlo program and the tests evaluating its performance were described. It can be concluded that the photon computer transport behaves well and, thus, can be used to study the effect of multiple scatter in XCSD. Results of this study are shown in chapter 3.
3.1 Evaluation of scatter signals using the Monte Carlo program (PS3)

With *in-vivo* XCSD, changes in bone density at the examination site have to be determined by successive measurements on the same individual over a period of time. Absolute comparison of bone densities is not possible unless the size of the patient at the examination site has not changed significantly, or the effect of patient size change on the density measured can be evaluated. The new version of the Monte Carlo program (PS3) was used to study photon scattering in order to evaluate the effect of multiple scatter in XCSD. All Monte Carlo programs were run on the supercomputers CRAY1S and X-MP/28 installed at University of London.

3.1.1 The phantoms studied

Phantoms which represent cross-sections of the lower forearm, the femoral neck and the lumbar spine were studied since these are sites of interest in bone densitometry. The dimensions and geometries of all phantoms considered are given in table 3.1. Only the amount of adipose tissue was varied between different size phantoms which represent the same examination site, because it is the amount of adipose tissue which is most likely to change between successive examinations on the same individual. Bone tissue and muscle are unlikely to change at these sites.
TABLE 3.1 Dimensions and geometries of phantoms studied for the evaluation of the effect of multiple scatter on bone density measurements. A and B are the lengths of the semi-major and semi-minor axes of the ellipses. All dimensions are expressed in centimeters.

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<td>14.5</td>
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<td>10.5</td>
</tr>
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</table>

3.1.2 The incident photon spectrum

Previous work by Speller and Horrocks, 1988 showed that there was no
significant difference in the effect of multiple scatter when different spectra were used within the range of 90 kV\textsubscript{p} to 140 kV\textsubscript{p} so this does not affect the choice of the spectrum. Spectra between 90 kV\textsubscript{p} and 100 kV\textsubscript{p} are likely to be used in clinical applications so that an increased scattered count rate can be achieved. Signal to noise ratios from the symmetrical phantom (see table 2.1) for a 90 kV\textsubscript{p} incident photon spectrum are statistically identical to those obtained with an 100 kV\textsubscript{p} spectrum, figure 3.1. Thus, throughout this study a 100 kV\textsubscript{p} incident photon spectrum was considered.

![Graph](image)

Figure 3.1 Signal/noise for the symmetrical phantom using PS3. +: 90 kV\textsubscript{p}, -: 100 kV\textsubscript{p} incident photon spectra.

### 3.1.3 Multiple scatter correction factors, mcfs

The results from these computations have been used to consider the corrections for the effect of multiple scatter. The basic principle can be understood if we consider the case of one x-ray tube used in scatter densitometry. The density, \( \rho \), can then be described by equation 3.1

\[
\rho = k \left( \frac{N_s}{N_t} \right)
\]

3.1

where \( k \) is a proportionality constant determined experimentally, \( N_s \) denotes the number
of scattered photons detected at the chosen scattering angle $\theta$, and $N_t$ the number of transmitted photons. Equation 3.1 describes an ideal situation where multiple scattering events are not considered. The detector, however, records all photons that hit its crystal, (intrinsic efficiency is assumed to be 100%) and they can be photons that have undergone either a single scatter in the "critical volume" in the bone, $N_s$, or more than one scatter inside the phantom, $N_m$. It can, therefore, be written that $N_s' = N_s + N_m$, where $N_s'$ denotes the total number of scattered photons recorded. Thus, equation 3.1 can be rewritten as:

$$\rho = k \left( \frac{N_s + N_m}{N_t} \right) f$$  \hspace{1cm} 3.2$$

where $f$ is the factor involved to correct for the effect of multiple scatter (i.e. multiple scatter correction factor, $mcf$). From equations 3.1 and 3.2 it can be concluded that:

$$f = mcf = \frac{N_s}{N_s + N_m}$$  \hspace{1cm} 3.3$$

Multiple scatter correction factors depend upon the experimental arrangement used, the phantom studied and the scattering angle. Information obtained from the photon transport computer model allows the determination of multiple scatter correction factors for any phantom considered.

3.2 Results from the computer model

The results are arranged in the following order: (i) General scattering profiles, (ii) results from the lower forearm phantoms, (iii) results from the femoral neck phantoms and (iv) results from the lumbar spine phantoms.

3.2.1 General scattering profiles

Figure 3.2 shows the profile of scatter information obtained when single scatter in
bone and multiple scatter interactions are plotted against the scattering angle for a lower forearm phantom of dimensions (3.0 x 2.5) cm. The scattered photons plot shows an absolute maximum at 1° decreasing rapidly with the scattering angle to reach a minimum value at ~ 90°. Single scatter is higher than multiple scatter between scattering angles ±60° and ±120°, whereas at small and large scattering angles multiple scatter predominates. Scattering profiles depend upon: (i) The angular distribution of scattered photons, (ii) the volume irradiated and (iii) the geometry of the phantom considered. This is the general form of the scatter profiles and it is these basic datasets that allow other effects to be evaluated such as the mcfs. In the following sections the individual phantoms will be considered.

![Figure 3.2 Scattering profiles from a lower forearm phantom (3.0 x 2.5) cm obtained from the photon transport model, PS3.](image)

### 3.3 Results from the lower forearm phantoms

Figure 3.3 represents a cross-section of the lower forearm phantom showing the dimensions and the sites of the materials which comprise one of the lower forearm phantoms studied. The densities of the phantom materials were: Adipose tissue, 950 kg/m³, soft tissue, 1000 kg/m³, cortical bone, 1850 kg/m³ and trabecular bone, 1120 kg/m³.
kg/m³. Over $3.5 \times 10^7$ photon histories were followed for each lower forearm phantom studied.

![Diagram of cross-section of the lower forearm phantom simulated in the photon transport computer model.](image)

**Figure 3.3** Cross-section of the lower forearm phantom simulated in the photon transport computer model.

### 3.3.1 The signal to noise ratio

Photons scattered once in the "critical volume" in the bone comprise the signal whereas those scattered more than once anywhere inside the phantom comprise the noise. Figure 3.4 shows the signal to noise ratio for a lower forearm phantom of dimensions (3.0 x 2.5) cm. In order to optimise the scattering geometry, scattering angles should be chosen where the signal to noise ratio reaches maximum values. It can be seen from figure 3.4 that maximum values appear at ± 60° and ± 120°.
3.3.2 Multiple scatter correction factors for the lower forearm phantoms

Figure 3.5 shows the variation in $m_{cfs}$ (as defined in section 2.6.3) with the scattering angle for the smallest of the lower forearm phantoms studied. Figure 3.5 shows similar features on either side of the incident photon beam but some degree of asymmetry exists due to the geometry of the phantom. The smaller the effect of multiple scatter, the closer the $m_{cfs}$ values will be to unity. It can be seen that $m_{cfs}$ have low values at small and large scattering angles, and reach maximum values at $\pm 60^\circ$. This would imply that multiple scatter is least important at these angles.

Figures 3.6 to 3.8 show how $m_{cfs}$ vary as the lower forearm phantom increases in size. The position of both x-ray tube and detector and the shape of all phantoms studied were unchanged. The densities of all phantom materials remained the same but the amount of adipose tissue in each phantom increased gradually. Clearly $m_{cfs}$ are dependent upon changes in patient size although the magnitude of these changes is different at different scattering angles.

Figure 3.4 Signal/noise from a lower forearm phantom (3.0 x 2.5) cm using PS3.
3.3.3 Selection of the optimum scattering angle

The choice of the optimum scattering angle is based on two criteria: (i) Obtaining the maximum signal to noise ratio and (ii) having the minimum variation in $mcfs$ due to
change in the patient size. The range of the mcfs encountered for phantom size changes

Figure 3.7 Comparison between mcfs of lower forearm phantoms. Upper curve: (3.0 x 2.5) cm; lower curve: (4.0 x 3.5) cm.

Figure 3.8 Comparison between mcfs of the lower forearm phantoms. Upper curve: (3.0 x 2.5) cm; lower curve: (4.5 x 4.0) cm.

between 23.6 cm² (smallest size lower forearm phantom) and 56.2 cm² (largest size lower forearm phantom) were evaluated and plotted against the scattering angle in figure 3.9. According to the first criterion, a scattering angle of either ± 60° or ± 120° must be chosen (see figure 3.4). The choice of the larger scattering angle, however, would lead
to a large variation in the mcfs with patient size changes. If the second criterion only were considered, a scattering angle of ± 20° should be chosen. In this case, however, the signal to noise ratio will be considerably reduced. For the geometry described in the Monte Carlo program, a scattering angle between ± 40° and ± 60° must be chosen so that both criteria are partially fulfilled.

![Scattering angle vs Range of mcfs](image)

**Figure 3.9** The lower forearm phantoms. Range of the mcfs encountered for phantom size changes between 23.6 and 56.2 sq. cm. Data generated from PS3.

### 3.3.4 The scatter contribution from individual tissues

The number of interactions of the detected photons which took place within each phantom material are plotted against the angle of detection for a lower forearm phantom of dimensions (3.0 x 2.5) cm, figure 3.10. The angle of detection is that at which the photon left the phantom (i.e. determined by its last interaction). The degree of scatter that each material is responsible for largely depends on its site in the phantom. Between detection angles of ± 40° and ± 140° bone is the most predominant scattering material whereas at small and large detection angles soft and adipose tissues predominate. The choice of an optimum scattering angle larger than ± 40° is expected to be affected less by changes in either soft or adipose tissues, due to their reduced effectiveness in producing
a scatter signal in the scattering angular range of ± 40° to ± 140°.

![Graph of photon interactions inside the phantom materials.](image)

Figure 3.10  Photon interactions inside the phantom materials. The lower forearm phantom ; (3.0 x 2.5) cm. Data generated from code PS3.

3.3.5 Mcfs variation with the lower forearm phantom size changes

Table 3.2 gives the mcfs for the lower forearm phantoms studied at scattering angles between ± 40° and ± 60° in two degree intervals. The error associated with each multiple scatter correction factor has been determined by the number of scattered counts recorded, which depends upon the number of photon histories followed. Figures 3.11 and 3.12 show these data plotted against size changes of the lower forearm phantoms at scattering angles of ± 40°, ± 46°, ± 50°, and ± 60°. Although the error bars are large, there is an underlying trend in the results that would allow the effects of multiple scatter in an examination of the lower forearm to be taken into account. There is no significant difference in the choice of a scattering angle between ± 40° and ± 50° when the variation of the mcfs with phantom size change is considered.

3.4 Results from the femoral neck phantoms

The femoral neck is a site of greater interest in bone densitometry than that of the
It is, therefore, important that the effect of multiple scatter is studied for a series of phantoms which represent the dimensions and geometry of the femoral neck.

The dimensions and geometries of the four femoral neck phantoms considered in PS3 are given in Table 3.1.

### TABLE 3.2 Multiple scatter correction factors at different scattering angles for the lower forearm phantoms studied. The sizes of the phantoms studied: †: (3.0 x 2.5) cm, ††: (3.5 x 3.0) cm, †††: (4.0 x 3.5) cm, ††††: (4.5 x 4.0) cm.

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<td>0.551</td>
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</tr>
<tr>
<td>±</td>
<td>±0.016</td>
<td>±0.016</td>
<td>±0.017</td>
<td>±0.018</td>
<td>±0.019</td>
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</tr>
<tr>
<td>0.498</td>
<td>0.490</td>
<td>0.503</td>
<td>0.510</td>
<td>0.517</td>
<td>0.527</td>
<td>0.533</td>
</tr>
<tr>
<td>±</td>
<td>±0.017</td>
<td>±0.017</td>
<td>±0.019</td>
<td>±0.021</td>
<td>±0.021</td>
<td>±0.021</td>
</tr>
<tr>
<td>0.463</td>
<td>0.485</td>
<td>0.478</td>
<td>0.496</td>
<td>0.511</td>
<td>0.507</td>
<td>0.506</td>
</tr>
<tr>
<td>±</td>
<td>±0.017</td>
<td>±0.018</td>
<td>±0.019</td>
<td>±0.020</td>
<td>±0.020</td>
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</tr>
<tr>
<td>0.450</td>
<td>0.446</td>
<td>0.450</td>
<td>0.476</td>
<td>0.490</td>
<td>0.486</td>
<td>0.493</td>
</tr>
<tr>
<td>±</td>
<td>±0.017</td>
<td>±0.018</td>
<td>±0.019</td>
<td>±0.020</td>
<td>±0.020</td>
<td>±0.021</td>
</tr>
</tbody>
</table>
Figure 3.11 Variation in mcf with the lower forearm phantom size change at scattering angles of -40°, -46°, -50° and -60°.

Figure 3.12 Variation in mcf with the lower forearm phantom size change at scattering angles of 40°, 46°, 50° and 60°.

Figure 3.13 represents a cross-section of the femoral neck phantom showing the dimensions and the sites of the materials which comprise the smallest of the femoral neck phantoms studied. The densities of all phantom materials were the same as the lower forearm phantom materials, and the amount of adipose tissue changed gradually between the different size phantoms. A 100 kVp incident photon spectrum was assumed to irradiate...
the phantom and over $5.5 \times 10^7$ photon histories were followed for each femoral neck phantom studied.

3.4.1 The signal to noise ratio

Figure 3.14 shows the signal to noise ratio for a femoral neck phantom of dimensions (8.0 x 7.5) cm. For the geometry described in the Monte Carlo program, signal to noise ratio reaches maximum values at $\pm 25^\circ$ and $\pm 130^\circ$ indicating that these
are the angles of optimum scattering geometry.

![Signal/noise from a femoral neck phantom, (8.0 x 7.5) cm using PS3.](image)

Figure 3.14 Signal/noise from a femoral neck phantom, (8.0 x 7.5) cm using PS3.

3.4.2 Multiple scatter correction factors for the femoral neck phantoms

Figure 3.15 is a plot of the multiple scatter correction factors against the scattering angle for the smallest of the femoral neck phantoms studied. As noted for the lower forearm phantoms, (see section 3.3.5), two maxima appear on either side of the incident photon beam which are asymmetrically positioned due to the asymmetry of the phantom. Large deviation of the \( mcfs \) from unity exists at small and large scattering angles whereas \( mcfs \) are closest to unity at \( \pm 25^\circ \).

Figures 3.16 to 3.18 show how \( mcfs \) vary with the change in the size of the femoral neck phantoms studied. As the phantom size increases, the contribution of multiple scatter to the recorded signal becomes more significant at any scattering angle. This is due to a higher probability for a photon which is scattered once in bone to undergo another scatter before it leaves the phantom. The variation in \( mcfs \) with the phantom size change is larger between scattering angles of \( \pm 160^\circ \) and \( \pm 180^\circ \) than it is between \( \pm 20^\circ \).
and ± 140°.

Figure 3.15 Variation in mfcs with the scattering angle using PS3. The femoral neck phantom, (8.0 x 7.5) cm

Figure 3.16 Variation in mfcs with phantom size change. The femoral neck phantom. Upper curve : (8.0 x 7.5) cm, lower curve : (8.5 x 8.0) cm.

3.4.3 Selection of the optimum scattering angle

As stated in section 3.4.1 the choice of the optimum scattering angle depends upon
the signal to noise variation (plotted in figure 3.14) and the variation in the mcfs with the phantom size changes.

![Graph showing variation in mcfs with phantom size change.](image)

**Figure 3.17** Variation in mcfs with phantom size change. The femoral neck phantom. Upper curve : (8.0 x 7.5) cm, lower curve : (9.0 x 8.5) cm.

The range of mcfs encountered for phantom size changes between 188.5 cm² and 330 cm² (i.e. smallest and largest femoral neck phantoms, respectively) is plotted against the scattering angle in figure 3.19. Figure 3.14 shows that scattering angles of either ± 25° or ± 130° should be chosen to optimise the scattering geometry, but the choice of the
smaller scattering angle will give a smaller variation in the mcfs with patient size change, (figure 3.19). For the femoral neck phantoms studied, a scattering angle of ±30° is the optimum.

![Graph showing the range of mcfs encountered for phantom size changes between 188 and 330 sq.cm. Data were generated from PS3.]

**Figure 3.19** The femoral neck phantoms. Range of mcfs encountered for phantom size changes between 188 and 330 sq.cm. Data were generated from PS3.

### 3.4.4 The scatter contribution from individual tissues

The number of interactions of the detected photons that took place inside each phantom material, for a femoral neck phantom of dimensions (8.5 x 8.0) cm, were plotted against the angle of detection and are presented in figure 3.20. The profile of photon interactions in bone is similar to that in the lower forearm phantom but there are differences in the profiles of the interactions in both soft and adipose tissues. This is due to the much greater volumes of these tissues present in the femoral neck investigations. Bone predominates as the scattering material between detection angles of ±20° and ±140° whereas at smaller and larger detection angles soft and adipose tissues are more predominant scattering media. Thus, a scattering angle larger than ±20° is expected to be less affected by changes in either soft or adipose tissues thicknesses.
3.4.5 $M_{cfs}$ variation with the femoral neck phantom size changes

Tables 3.3.a and 3.3.b give the multiple scatter correction factors for the femoral neck phantoms studied at scattering angles between ±26° and ±50° in two degree intervals. Figures 3.21 and 3.22 show the variation in the $m_{cfs}$ with phantom size change of the femoral neck phantoms studied at scattering angles ±26°, ±30°, ±40° and ±50°. The error associated with each $mcf$ is determined by the number of scattered counts recorded. The error bars are larger in the femoral neck study compared to those in the lower forearm due to less scattered counts detected. There is, however, an underlying trend in these results showing how $m_{cfs}$ vary with phantom size change.

3.5 Results from the lumbar spine phantoms

Physical density changes in the trabecular bone of the lumbar spine are of greatest interest in bone densitometry since the status of the lumbar spine is regarded as the most significant indicator for the early detection of osteoporosis. Phantoms which represent
TABLE 3.3.a  Multiple scatter correction factors for the femoral neck phantoms studied.

<table>
<thead>
<tr>
<th>Angle (°)</th>
<th>-26</th>
<th>-28</th>
<th>-30</th>
<th>-32</th>
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<td>Phantom size</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(8.0x7.5) cm</td>
<td>0.582</td>
<td>0.569</td>
<td>0.570</td>
<td>0.559</td>
<td>0.566</td>
<td>0.556</td>
<td>0.545</td>
<td>0.548</td>
<td>0.534</td>
</tr>
<tr>
<td>±0.021</td>
<td>±0.021</td>
<td>±0.022</td>
<td>±0.022</td>
<td>±0.022</td>
<td>±0.023</td>
<td>±0.023</td>
<td>±0.024</td>
<td>±0.023</td>
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<tr>
<td>(8.5x8.0) cm</td>
<td>0.569</td>
<td>0.559</td>
<td>0.558</td>
<td>0.551</td>
<td>0.548</td>
<td>0.543</td>
<td>0.545</td>
<td>0.544</td>
<td>0.544</td>
</tr>
<tr>
<td>±0.023</td>
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<td>±0.023</td>
<td>±0.023</td>
<td>±0.023</td>
<td>±0.024</td>
<td>±0.024</td>
<td>±0.025</td>
<td>±0.023</td>
<td></td>
</tr>
<tr>
<td>(9.0x8.5) cm</td>
<td>0.546</td>
<td>0.549</td>
<td>0.543</td>
<td>0.536</td>
<td>0.531</td>
<td>0.546</td>
<td>0.520</td>
<td>0.501</td>
<td>0.507</td>
</tr>
<tr>
<td>±0.022</td>
<td>±0.023</td>
<td>±0.023</td>
<td>±0.023</td>
<td>±0.023</td>
<td>±0.024</td>
<td>±0.025</td>
<td>±0.024</td>
<td>±0.023</td>
<td></td>
</tr>
<tr>
<td>(10.5x10.0) cm</td>
<td>0.530</td>
<td>0.530</td>
<td>0.535</td>
<td>0.504</td>
<td>0.492</td>
<td>0.506</td>
<td>0.513</td>
<td>0.483</td>
<td>0.470</td>
</tr>
<tr>
<td>±0.030</td>
<td>±0.031</td>
<td>±0.031</td>
<td>±0.030</td>
<td>±0.032</td>
<td>±0.033</td>
<td>±0.032</td>
<td>±0.032</td>
<td>±0.032</td>
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<table>
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<td>Phantom size</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(8.0x7.5) cm</td>
<td>0.545</td>
<td>0.526</td>
<td>0.494</td>
<td>0.503</td>
</tr>
<tr>
<td>±0.024</td>
<td>±0.024</td>
<td>±0.023</td>
<td>±0.023</td>
<td></td>
</tr>
<tr>
<td>(8.5x8.0) cm</td>
<td>0.503</td>
<td>0.496</td>
<td>0.492</td>
<td>0.475</td>
</tr>
<tr>
<td>±0.024</td>
<td>±0.024</td>
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<td>±0.024</td>
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</tr>
<tr>
<td>(9.0x8.5) cm</td>
<td>0.507</td>
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<td>0.491</td>
<td>0.474</td>
</tr>
<tr>
<td>±0.024</td>
<td>±0.024</td>
<td>±0.025</td>
<td>±0.024</td>
<td></td>
</tr>
<tr>
<td>(10.5x10.0) cm</td>
<td>0.465</td>
<td>0.458</td>
<td>0.424</td>
<td>0.450</td>
</tr>
<tr>
<td>±0.032</td>
<td>±0.032</td>
<td>±0.031</td>
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</table>
### TABLE 3.3.b Multiple scatter correction factors for the femoral neck phantoms studied

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<th>32</th>
<th>34</th>
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<td>Phantom size</td>
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<tr>
<td>(8.0x7.5 cm)</td>
<td>0.530 ± 0.023</td>
<td>0.533 ± 0.024</td>
<td>0.532 ± 0.024</td>
<td>0.528 ± 0.025</td>
<td>0.542 ± 0.027</td>
<td>0.524 ± 0.027</td>
<td>0.533 ± 0.028</td>
<td>0.500 ± 0.028</td>
<td>0.516 ± 0.029</td>
</tr>
<tr>
<td>(8.5x8.0 cm)</td>
<td>0.516 ± 0.025</td>
<td>0.520 ± 0.025</td>
<td>0.526 ± 0.027</td>
<td>0.531 ± 0.028</td>
<td>0.534 ± 0.029</td>
<td>0.521 ± 0.029</td>
<td>0.510 ± 0.030</td>
<td>0.501 ± 0.030</td>
<td>0.480 ± 0.029</td>
</tr>
<tr>
<td>(9.0x9.5 cm)</td>
<td>0.512 ± 0.024</td>
<td>0.513 ± 0.025</td>
<td>0.516 ± 0.026</td>
<td>0.511 ± 0.027</td>
<td>0.511 ± 0.027</td>
<td>0.501 ± 0.027</td>
<td>0.493 ± 0.029</td>
<td>0.469 ± 0.029</td>
<td></td>
</tr>
<tr>
<td>(10.5x10.0 cm)</td>
<td>0.491 ± 0.034</td>
<td>0.473 ± 0.033</td>
<td>0.503 ± 0.035</td>
<td>0.486 ± 0.036</td>
<td>0.486 ± 0.038</td>
<td>0.471 ± 0.038</td>
<td>0.432 ± 0.038</td>
<td>0.459 ± 0.040</td>
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</table>

<table>
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<tr>
<th>Angle (°)</th>
<th>44</th>
<th>46</th>
<th>48</th>
<th>50</th>
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</thead>
<tbody>
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<td>Phantom size</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(8.0x7.5 cm)</td>
<td>0.493 ± 0.029</td>
<td>0.494 ± 0.030</td>
<td>0.481 ± 0.030</td>
<td>0.439 ± 0.028</td>
</tr>
<tr>
<td>(8.5x8.0 cm)</td>
<td>0.489 ± 0.030</td>
<td>0.467 ± 0.031</td>
<td>0.458 ± 0.031</td>
<td>0.450 ± 0.031</td>
</tr>
<tr>
<td>(9.0x8.5 cm)</td>
<td>0.448 ± 0.029</td>
<td>0.471 ± 0.030</td>
<td>0.439 ± 0.030</td>
<td>0.431 ± 0.030</td>
</tr>
<tr>
<td>(10.5x10.0 cm)</td>
<td>0.453 ± 0.040</td>
<td>0.400 ± 0.038</td>
<td>0.425 ± 0.039</td>
<td>0.413 ± 0.040</td>
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</table>
cross-sections of the lumbar spine were considered and the variation in mcfs with phantom size change was evaluated. The geometries and dimensions of the four lumbar spine phantoms studied are shown in table 3.1 and the densities of all of the phantom materials were the same as in the previous phantoms. Figure 3.23 represents a
cross-section of a lumbar spine phantom of dimensions (14.25 x 11.5) cm showing the sizes and sites of the materials composing this phantom. A 100 kV_p incident photon spectrum was used and over 5.5x10^7 photon histories were followed for each lumbar spine phantom studied.

Figure 3.23 Cross-section of the lumbar spine phantom simulated in the photon transport model.

3.5.1 The signal to noise ratio

Figure 3.24 shows the signal to noise ratio against the scattering angle of a lumbar spine phantom of dimensions (13.25 x 10.5) cm. Maximum values appear at scattering angles of ± 20° and ± 160° indicating the angles of optimum scattering geometry.
3.5.2 Multiple scatter correction factors for the lumbar spine phantoms

Figure 3.25 is a plot of \( mcfs \) against the scattering angle of a lumbar spine phantom of dimensions \((13.25 \times 10.5)\) cm. Two maxima appear on either side of the incident photon beam at scattering angles of \( \pm 20^\circ \) and \( \pm 160^\circ \) with \( mcfs \) decreasing rapidly on either side. These maxima are now well separated in scattering angles compared to the smaller phantoms studied in sections 3.3 and 3.4.
Figures 3.26 to 3.28 show the variation of \( mcfs \) with the scattering angle as the lumbar spine phantom size changes. The amount of adipose tissue changed gradually between the different size phantoms studied. As the phantom size increases, the contribution of multiple scattering events to the signal detected increases at any scattering angle. The degree of variation in \( mcfs \) with phantom size change is smaller at scattering angles between \( \pm 20^\circ \) and \( \pm 60^\circ \) than it is between \( \pm 60^\circ \) and \( \pm 160^\circ \).

**Figure 3.26** Variation in \( mcfs \) with phantom size change. The lumbar spine phantom. Upper curve: (13.25 x 10.5) cm, lower curve: (14.25 x 11.5) cm.

**Figure 3.27** Variation in \( mcfs \) with phantom size change. The lumbar spine phantom. Upper curve: (13.25 x 10.5) cm, lower curve: (16.25 x 13.5) cm.
3.5.3 Selection of the optimum scattering angle

The range of \( mcfs \) encountered for phantom size changes between 437 cm\(^2\) and 786 cm\(^2\) were plotted against the scattering angle in figure 3.29. The signal to noise variation, figure 3.24, indicates a scattering angle of either ±20° or ±160° for optimum scattering geometry. The choice of the large scattering angle will lead to a large variation in \( mcfs \) with patient size change.

Since the variation in \( mcfs \) with the change of phantom size at scattering angles between ±30° and ±60° is small, any angle within this range could be chosen. Thus, a small scattering angle within this range is preferred in order to optimise the scattering geometry.

3.5.4 The scatter contribution from the various tissues

The number of interactions of the detected photons that took place within each material of a lumbar spine phantom of dimensions (14.25x11.5) cm were plotted against the angle of detection in figure 3.30. Bone is the most predominant scattering material at detection angles as large as ±50°, whereas at detection angles between ±50° and ±160°
bone is only as effective a scattering medium as soft tissue. Photon interactions in adipose tissue increase rapidly at angles larger than ±80° and adipose tissue becomes the most predominant scattering medium at scattering angles larger than ±170°. Thus, a scattering angle between ±20° and ±50° will be affected less by changes in either soft or adipose tissues thicknesses.

Figure 3.30 Photon interactions inside the phantom materials. The lumbar spine phantom (14.25 x 11.5) cm. Data were generated from PS3.
3.5.5 $M_{cfs}$ variation with the phantom size changes in the lumbar spine

Table 3.4 gives the $m_{cfs}$ for the lumbar spine phantoms studied at scattering angles between $\pm 20^\circ$ and $\pm 52^\circ$ in four degree intervals. $M_{cfs}$ for the phantoms studied are plotted in figures 3.31 and 3.32 at scattering angles of $\pm 28^\circ$, $\pm 40^\circ$, $\pm 48^\circ$ and $\pm 52^\circ$.

![Figure 3.31 Variation in $m_{cfs}$ with the lumbar spine phantom size change at scattering angles of $\pm 28^\circ$, $\pm 40^\circ$, $\pm 48^\circ$ and $\pm 52^\circ$.]

![Figure 3.32 Variation in $m_{cfs}$ with the lumbar spine phantom size change at scattering angles of $28^\circ$, $40^\circ$, $48^\circ$ and $52^\circ$.]
TABLE 3.4 Multiple scatter correction factors for the lumbar spine phantoms studied.

<table>
<thead>
<tr>
<th>Angle (°)</th>
<th>-20</th>
<th>-24</th>
<th>-28</th>
<th>-32</th>
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<th>-52</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(13.25x10.5) cm</td>
<td>0.616</td>
<td>0.590</td>
<td>0.542</td>
<td>0.490</td>
<td>0.481</td>
<td>0.463</td>
<td>0.406</td>
<td>0.451</td>
<td>0.370</td>
</tr>
<tr>
<td>±0.038</td>
<td>±0.041</td>
<td>±0.041</td>
<td>±0.042</td>
<td>±0.043</td>
<td>±0.042</td>
<td>±0.047</td>
<td>±0.037</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(14.25x11.5) cm</td>
<td>0.591</td>
<td>0.585</td>
<td>0.532</td>
<td>0.473</td>
<td>0.489</td>
<td>0.457</td>
<td>0.385</td>
<td>0.403</td>
<td>0.369</td>
</tr>
<tr>
<td>±0.047</td>
<td>±0.051</td>
<td>±0.051</td>
<td>±0.049</td>
<td>±0.052</td>
<td>±0.055</td>
<td>±0.052</td>
<td>±0.051</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(16.25x13.5) cm</td>
<td>0.536</td>
<td>0.512</td>
<td>0.500</td>
<td>0.458</td>
<td>0.394</td>
<td>0.398</td>
<td>0.434</td>
<td>0.344</td>
<td>0.333</td>
</tr>
<tr>
<td>±0.058</td>
<td>±0.060</td>
<td>±0.064</td>
<td>±0.063</td>
<td>±0.060</td>
<td>±0.067</td>
<td>±0.071</td>
<td>±0.062</td>
<td>±0.063</td>
<td></td>
</tr>
<tr>
<td>(17.25x14.5) cm</td>
<td>0.564</td>
<td>0.490</td>
<td>0.462</td>
<td>0.416</td>
<td>0.376</td>
<td>0.388</td>
<td>0.312</td>
<td>0.348</td>
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<tr>
<td>±0.073</td>
<td>±0.069</td>
<td>±0.068</td>
<td>±0.072</td>
<td>±0.072</td>
<td>±0.072</td>
<td>±0.082</td>
<td>±0.074</td>
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</tr>
</tbody>
</table>
Although trends in the mean values might be observed, the errors associated with each point indicate that there is no significant difference between scattering angles of ±30° and ±50° when considering the effect of multiple scatter.

3.5.6 The effect of the geometry of the densitometer

Since the effect of multiple scatter is highly geometrical, small differences either in the size of the phantom studied or in the geometry defined in the Compton scatter densitometer used may have a significant effect on the accuracy of the densities measured. This section demonstrates the effect a different geometry Compton scatter densitometer has on multiple scattering events. The effect of multiple scatter was evaluated for the smallest (i.e. (3.0x2.5) cm) and the largest size (i.e. (4.5x4.0) cm) lower forearm phantoms defined in table 3.1. The geometry was that defined in the Compton scatter densitometer built (see section 4.4) but a finer multi-hole focussing detector collimator than that shown in figure 5.7 was considered.

Figure 3.33 shows the signal to noise ratio for the smaller size lower forearm phantom studied. The signal to noise ratio has been considerably reduced compared to that shown in figure 3.4 because of a significant increase in the "critical volume". However, there is no significant difference in the mcfs at scattering angles smaller than 30°. The scattering geometry is optimum at ~ 60°.

![Figure 3.33 Signal to noise variation for a lower forearm phantom (3.0x2.5) cm. The geometry is that defined in the Compton scatter densitometer built.](image-url)
Figure 3.34 shows the variation in $mcfs$ with the scattering angle for the smaller lower forearm phantom studied. Similar features are seen on either side of the incident photon beam but some degree of asymmetry exists due to the geometry of the phantom. Multiple scatter is least important at $\sim 60^\circ$.

![Figure 3.34](image1.png)

**Figure 3.34** Multiple scatter correction factors for the lower forearm phantom, (3.0x2.5) cm. The geometry is that defined in the Compton scatter densitometer built.

Figure 3.35 shows the variation in $mcfs$ with the lower forearm phantom size change. Large variations with phantom size changes exist at small (less than $\pm 30^\circ$) and large scattering angles (greater than $\pm 150^\circ$) whereas there is a considerably smaller variation at $\pm 50^\circ$. Hence, for the geometry defined in this Compton scatter densitometer and for the lower forearm phantoms studied the optimum scattering angle is $\sim \pm 50^\circ$.

![Figure 3.35](image2.png)

**Figure 3.35** Comparison between $mcfs$ for the lower forearm phantoms. Upper curve: (3.0x2.5) cm, lower curve: (4.5x4.0) cm. The geometry is that defined in the Compton scatter densitometer built.
Although the multiple scatter to the signal recorded is now increased, similar features to those shown in figure 3.8 are seen in the variation of the mcfs with the scattering angle between 30° and 130°.

### 3.6 Discussion of mcfs calculation

The study of this series of phantoms, which represent different sites of the human body, revealed that there is a large variation in multiple scatter contribution to the signal recorded with the scattering angle. There is also a significant variation with phantom size showing that multiple scatter is a highly geometrical problem. The multiple scatter contribution to the signal recorded reaches minimum values at two scattering angles (one forward and one backward) for all phantoms studied. Moving from a small size phantom (i.e. lower forearm), towards a large size phantom (i.e. lumbar spine), the scattering angles at which minimum multiple scatter contribution occurs are more separated. For the lower forearm phantoms studied an additional feature is observed. This is that multiple scatter is reduced at scattering angles less than ~5°; this is attributed to the sampling of Rayleigh scatter in the Monte Carlo program.

Considering the choice of the optimum scattering angle, a forward scattering angle no larger than ± 60° must be used. A scattering angle close to ± 40° would be preferred so as to make the best compromise between optimum scattering geometry and variation in multiple scatter contribution to the signal with patient size changes.

At forward scattering angles, the variation in the multiple scatter correction factors with the phantom size change decreases while moving from a small size phantom (i.e. lower forearm), towards a large size one (i.e. lumbar spine). There is no significant variation in the multiple scatter contribution to the signal recorded with the size change of the lumbar spine phantoms at scattering angles between ± 30° and ± 50°. A change in the geometry of the Compton scatter densitometer described in the computer model will affect both the absolute values and the behaviour of the mcfs.
3.7 Explanation of the behaviour of the multiple scatter correction factors

Multiple scatter correction factors show certain features that depend upon scattering angle and phantom size. Since multiple scatter is a highly geometrical problem, the variation in mcfs can be explained in terms of the geometrical arrangement described in the computer model. The explanation of the behaviour requires a knowledge of the geometrical considerations and the angular distribution of the scattered photons.

3.7.1 Geometrical considerations

The collimation of the x-ray beam and the position of the phantom relative to the incident photon beam define the volume irradiated. The volume defined by the incident photon beam and the view of the detector collimator is the "critical volume", figure 1.8. The "critical volume" should be wholly located inside the trabecular bone examined. Since bones at different sites of the body will be examined and the extent of the trabecular bone varies, the "critical volume" must be known for each different collimation used.

The "critical volume" was calculated numerically for the experimental arrangement described in the Monte Carlo photon transport model. The detector collimator was rotated on a circle at a given distance from the origin (i.e. 8 or 20 cm) defining scattering angles between 10° and 170°. Figure 3.36 shows the variation in the "critical volume" with the scattering angle for the geometry described in table 3.1. Figure 3.37 is a plot of the variation in the amount of trabecular bone located inside the "critical volume" with the scattering angle in the lower forearm, femoral neck and lumbar spine phantoms studied. At scattering angles between 30° and 150° the "critical volume" in the femoral neck and the lumbar spine phantoms was at least 75% occupied by trabecular bone. In the lower forearm phantom, however, within the same angular range there is a significant contribution from soft tissue (i.e. as high as 75%) due to the amount of trabecular bone present and the collimation used.
Figure 3.36 The "critical volume" calculated numerically for the geometrical arrangement used in the photon transport computer model.

Interactions can take place either inside the trabecular bone or both in trabecular bone and the surrounding tissues. If a thin layer of soft tissue is assumed to surround the bone, then bone itself will be the main source of multiple scatter. If, on the other hand, the amount of tissue that surrounds the bone is considerably increased, the surrounding tissue will contribute most to multiple scatter. In this analysis, multiple
scatter is assumed to be only due to the amount of tissues which surround the bone since their contribution to the phantom is considerably greater than that of bone in any phantom studied (i.e. bone cross-sectional area is ~4.8%, ~4.4% and ~6.4% of the total area in the lower forearm, femoral neck and lumbar spine phantoms, respectively). The amount of trabecular bone irradiated is assumed to be responsible for the single scattering events. Hence, mcsfs are expected to be proportional to the quantity defined in equation 3.4.

$$\phi (\theta) = \frac{V_b}{V_b + V_{ph}} \quad 3.4$$

$V_b$ and $V_{ph}$ denote volumes shown in figure 3.38. $V_b$ represents the amount of "critical volume" inside the trabecular bone and $V_{ph}$ the volume inside the phantom defined by
the detector collimator and limited by the volume irradiated, excluding volume \( V_b \). The quantity defined in equation 3.4 varies with the scattering angle, the phantom size, geometry and the collimation used. The quantity \( \phi(\theta) \) was calculated numerically for all phantoms used and the geometrical conditions described in the computer model at scattering angles between 10° and 170° in a 5° interval. Results are plotted against the scattering angle in figures 3.39.a to 3.39.c. Figures 3.39.a to 3.39.c show different features due to the different geometry of the phantoms considered.

![Graph](image)

**Figure 3.39a** Variation in \((V_b/(V_b+V_{ph}))\) of the lower forearm phantom, (3.0x2.5) cm, for the geometrical arrangement defined in the photon transport computer model.

### 3.7.2 The angular distribution of the scattered photons

So far, only the geometrical considerations involved in multiple scatter have been discussed. However, the angular distribution of the scattered photons must be taken into account as well. The scatter angular distribution was calculated numerically for a 100 kVp incident photon spectrum when relative interaction probabilities and angular distributions of both Rayleigh and Compton scatter were considered, equation 3.5.
\[ f(\theta) = \sum_{E} \left( P_R(E) \left( \frac{d\sigma(E)}{d\theta} \right)_R + P_C(E) \left( \frac{d\sigma(E)}{d\theta} \right)_C \right) N(E) \]  

\( P_R(E) \) and \( P_C(E) \) represent probabilities of Rayleigh and Compton interactions, respectively which take place at an energy \( E \). The expressions \( \left( \frac{d\sigma(E)}{d\theta} \right)_R \) and \( \left( \frac{d\sigma(E)}{d\theta} \right)_C \) denote the angular distributions of Rayleigh and Compton interactions, respectively at an energy \( E \) and \( N(E) \) is the intensity of the incident photon spectrum.

Compton scatter angular distributions were calculated from the Klein-Nishina formula and those of Rayleigh scatter were computed using the inversion rejection method, (see section 2.4.2). Interaction data were taken from Storm and Israel, 1970 and Hubbel et al, 1969, and spectral information from Birch et al, 1979, at energy values between 10 keV and 100 keV in a 5 keV interval. The angular distribution is shown in figure 3.40.

![Figure 3.39.b Variation in (Vb/(Vb+Vph)) of the femoral neck phantom, (8.0x7.5) cm, for the geometrical arrangement defined in the photon transport computer model.](image)

### 3.8 Results from the analytical calculations

The function used to explain the variation in the multiple scatter correction factors
with the scattering angle must be described by both angular distribution of scattered photons and phantom geometry. If the effect of multiple scatter is ignored, the number of scattered photons detected will be governed by the scattered photons angular distribut-

![Graph](image-url)

**Figure 3.39.c** Variation in $\frac{V_b}{V_b + V_{ph}}$ of the lumbar spine phantom, (14.25x11.5) cm, for the geometrical arrangement defined in the photon transport computer model.

![Graph](image-url)

**Figure 3.40** Scattered photons angular distribution calculated for a 100 kVp incident photon spectrum.
the analysis described above only a fraction equal to \( \frac{V_b}{(V_b + V_{ph})} \) of the scattered counts will have undergone a single scatter in the "critical volume" in the bone and thus, equation 3.6 can explain the variation in mcfs.

\[
F(\theta) = f(\theta) \left( \frac{V_b}{(V_b + V_{ph})} \right)
\]  

3.6

The quantity \( F(\theta) \) is plotted against the scattering angle for the three phantoms studied, (lower forearm \( (3.0 \times 2.5) \) cm, femoral neck \( (8.0 \times 7.5) \) cm and lumbar spine \( (14.25 \times 11.5) \) cm), in figures 3.41.a to 3.41.c.

![Graph](image)

Figure 3.41.a "Multiple scatter correction" factors calculated analytically for a lower forearm phantom, \( (3.0 \times 2.5) \) cm.

Similar features to multiple scatter correction factors variation obtained from the photon transport model (see figures 3.5, 3.15, and 3.25) are seen. Maximum values of \( F(\theta) \) appear at scattering angles similar to those predicted by the computer model. The larger the phantom, the more obvious the appearance of the two peaks and the variation in \( F(\theta) \) on either side of scattering angles of maxima \( F(\theta) \). Of interest is the variation in \( F(\theta) \) with the phantom size change shown in figures 3.42.a to 3.42.c for the three sites.
studied. The larger the phantom, the smaller the variation in $F(\theta)$ with the phantom size change, and, at larger scattering angles, the variation in $F(\theta)$ is greater with the phantom size change than that at small scattering angles. The features shown in figures 3.42.a to 3.42.c agree with those of the multiple scatter correction factors obtained by the computer model.

Figure 3.41.b "Multiple scatter correction" factors calculated analytically for a femoral neck phantom, (8.0x7.5) cm.

Figure 3.41.c "Multiple scatter correction" factors calculated analytically for a lumbar spine phantom, (14.25x11.5) cm.
Figure 3.42.a Mcfs calculated analytically for the lower forearm phantoms. Upper curve: (3.0x2.5) cm, lower curve: (4.5x4.0) cm.

Figure 3.42.b Mcfs calculated analytically for the femoral neck phantoms. Upper curve: (8.0x7.5) cm, lower curve: (10.5x10.0) cm.

Table 3.5 compares the variation in mcfs with phantom size change computed from the Monte Carlo program with the variation predicted by F(θ). The smaller the phantom considered, the better the agreement.
Figure 3.42.c Mcfs calculated analytically for the lumbar spine phantoms. Upper curve: (14.25x11.5) cm, lower curve: (17.25x14.5) cm.

TABLE 3.5 Comparison between multiple scatter correction factors: (1) Computed from the photon transport computer model\(^1\), (2) calculated analytically considering the geometrical arrangement described in the Monte Carlo photon transport model\(^2\).

<table>
<thead>
<tr>
<th>Phantom studied</th>
<th>Scattering angle</th>
<th>Phantom size (cm(^2))</th>
<th>% difference in mcfs(^1)</th>
<th>% difference in mcfs(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower forearm</td>
<td>60°</td>
<td>23.6</td>
<td>11.5</td>
<td>13.0</td>
</tr>
<tr>
<td></td>
<td>120°</td>
<td>56.5</td>
<td>16.0</td>
<td>18.0</td>
</tr>
<tr>
<td>Femoral neck</td>
<td>40°</td>
<td>188.5</td>
<td>11.0</td>
<td>11.0</td>
</tr>
<tr>
<td></td>
<td>140°</td>
<td>330.0</td>
<td>32.0</td>
<td>23.0</td>
</tr>
<tr>
<td>Lumbar spine</td>
<td>40°</td>
<td>437.0</td>
<td>6.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>140°</td>
<td>786.0</td>
<td>33.0</td>
<td>7.0</td>
</tr>
</tbody>
</table>
3.9 Multiple scatter in bone

An inherent assumption of the model used in the explanation of the behaviour of \( mcfs \) was that the "critical volume" in bone was responsible for single scattering events whereas tissues surrounding this volume were solely responsible for multiple scatter. Multiple scatter, however, can take place in the "critical volume" located inside the bone examined and its contribution to the total amount of multiple scatter has to be evaluated.

Figure 3.43 shows the contribution of multiple scatter in bone to the total amount of multiple scatter for the extreme sizes of the lower forearm phantoms studied with the photon transport model. Bone contributes little, (i.e. less than 5%), to multiple scatter at scattering angles smaller than \( \pm 25^\circ \), increasing gradually with the scattering angle to reach a plateau between \( \pm 60^\circ \) and \( \pm 110^\circ \). The larger the phantom size, the less the multiple scatter contribution due to bone. Over 65% of multiple scatter is due to tissues which surround the "critical volume" showing that the surrounding tissues are the dominant sources of multiple scatter.

![Figure 3.43](image)

Figure 3.43 Contribution of multiple scatter in bone to the total amount of multiple scatter. Upper curve: (3.0x2.5) cm, lower curve: (4.5x4.0) cm.

In the remainder of the chapter correction factors for beam polychromaticity and difference between scattered and transmitted path lengths are calculated for the same phantoms as those simulated in the computer model.
3.10 The bias correction factors (bcf)

Duke and Hanson (Medical Physics, 1984) evaluated the effect of (i) beam polychromaticity and (ii) difference between scattered and transmitted path lengths on bone density measurements. The formula used is given by equation 3.7

\[
\text{bcf} = \left( \frac{A_1 A_2}{B_1 B_2} \right)^{1/2} \tag{3.7}
\]

where

\[
A_1 = \int_{E_{\text{max}}}^{E_{\text{max}}} (I_1(E) \exp(-\int_a^b \mu(E,x) \, dx) \exp(-\int_c^d \mu(E,x) \, dx)) \, dE \tag{3.8}
\]

\[
A_2 = \int_{E_{\text{max}}}^{E_{\text{max}}} (I_2(E) \exp(-\int_a^b \mu(E,x) \, dx) \exp(-\int_c^d \mu(E,x) \, dx)) \, dE \tag{3.9}
\]

\[
B_2 = \int_{E_{\text{max}}}^{E_{\text{max}}} (I_1(E) \exp(-\int_a^b \mu(E,x) \, dx) g(E) \exp(-\int_a^b \mu(E_s,x) \, dx)) \, dE \tag{3.10}
\]

\[
B_1 = \int_{E_{\text{max}}}^{E_{\text{max}}} (I_2(E) \exp(-\int_a^b \mu(E,x) \, dx) g(E) \exp(-\int_a^b \mu(E_s,x) \, dx)) \, dE \tag{3.11}
\]

$I_1(E)$ and $I_2(E)$ are the intensities of the x-ray spectra from x-ray tube 1 and x-ray tube 2, respectively and $\mu(E,x)$ represents the linear attenuation coefficient of the phantom at the incident photon energy, $E$. $E_{\text{max}}$ denotes the maximum energy of the spectra used and integration limits $a$, $b$, $c$ and $d$ are the distances shown in figure 3.44.

A proportionality constant determined by irradiating a well known density material will involve the probability of a Compton interaction inside the material and, thus, the contribution of factor $g(E)$ to the equations 3.10 and 3.11 will be determined by the calibration procedure. Equations 3.10 and 3.11 can then be rewritten as:

\[
B_1 = \int_{E_{\text{max}}}^{E_{\text{max}}} (I_1(E) \exp(-\int_a^b \mu(E,x) \, dx) \exp(-\int_c^d \mu(E_s,x) \, dx)) \, dE \tag{3.12}
\]

and

\[
B_2 = \int_{E_{\text{max}}}^{E_{\text{max}}} (I_2(E) \exp(-\int_a^b \mu(E,x) \, dx) \exp(-\int_c^d \mu(E_s,x) \, dx)) \, dE \tag{3.13}
\]

Assuming that both x-ray tubes give the same x-ray spectra, $I_1(E) = I_2(E) = I(E)$ and equations 3.8, 3.9, 3.12 and 3.13 can be rewritten as:
\begin{align*}
A_1' &= \int_{E_{\text{max}}}^{\infty} (I(E) \exp(-\int_{a}^{d} \mu(E,x) \, dx) \exp(-\int_{b}^{h} \mu(E,x) \, dx)) \, dE \\
A_2' &= \int_{E_{\text{max}}}^{\infty} (I(E) \exp(-\int_{a}^{d} \mu(E,x) \, dx) \exp(-\int_{c}^{f} \mu(E,x) \, dx)) \, dE \\
B_1' &= \int_{E_{\text{max}}}^{\infty} (I(E) \exp(-\int_{a}^{d} \mu(E,x) \, dx) \exp(-\int_{b}^{h} \mu(E_s,x) \, dx)) \, dE \\
B_2' &= \int_{E_{\text{max}}}^{\infty} (I(E) \exp(-\int_{a}^{d} \mu(E,x) \, dx) \exp(-\int_{c}^{f} \mu(E_s,x) \, dx)) \, dE \\
\end{align*}

The quantity defined in equation 3.18 is used to correct for photon attenuation due to the difference in the photon path length and the energy shift between transmitted and scattered beams.

\[ bcf' = \frac{(A_1' A_2')}{(B_1' B_2')}^{1/2} \]

\textbf{Figure 3.44} Path lengths of scattered and transmitted photon beams
3.11 Variation in $bcf$ with the phantom size change

Equation 3.18 was used to evaluate the bias correction factors for the same phantoms considered in the computer model for the same incident photon spectrum of 100 kV$_p$. Attenuation data of all elements of interest were taken from Hubbel, 1969, and Storm and Israel, 1970, and the mass attenuation coefficients of all of the phantom materials were calculated in a 1 keV interval from their elemental compositions based on the mixture rule.

(i) The lower forearm phantoms - Figure 3.45 shows the variation in the $bcfs$ with the scattering angle for the lower forearm phantoms of dimensions and geometries described in table 3.1. The larger the scattering angle, the greater the deviation of $bcfs$ from unity for the same phantom, and the larger the phantom the greater the value of the $bcf$ at the same scattering angle. $Bcfs$ vary from 1.021 to 1.034 between the smallest and the largest of the lower forearm phantoms studied at scattering angles of 40° and 50°.

![Figure 3.45 Variation in bcf for the lower forearm phantoms with the scattering angle. (a): (3.0x2.5) cm, (b): (3.5x3.0) cm, (c): (3.5x4.0) cm, (d): (4.0x4.5) cm.](image)

(ii) The femoral neck phantoms - Figure 3.46 shows the variation in the $bcfs$ with the scattering angle for the femoral neck phantoms studied of dimensions and geometries described in table 3.1. The behaviour of the $bcfs$ with the scattering angle is similar to
that in the lower forearm phantoms and \(bcfs\) vary from 1.017 to 1.057 between the smallest and the largest of the femoral neck phantoms studied at scattering angles of 30° and 50°.

**Figure 3.46** Variation in bcf for the femoral neck phantoms with the scattering angle, (a): (8.0x7.5) cm, (b): (8.5x8.0) cm, (c): (9.0x8.5) cm, (d): (10.5x10.0) cm.

(iii) The lumbar spine phantoms - Figure 3.47 is a plot of the \(bcfs\) against the scattering angle for the lumbar spine phantoms studied of dimensions and geometries.

**Figure 3.47** Variation in bcf for the lumbar spine phantoms with the scattering angle. (a): (13.25x10.5) cm, (b): (14.25x11.5) cm, (c): (16.25x13.5) cm, (d): (17.25x14.5) cm.
described in table 3.1. Values of \(bcfs\) vary from 1.022 to 1.075 between the smallest and the largest of the lumbar spine phantoms studied at scattering angles of 30° and 50°.

### 3.11.1 Variation in \(bcf\) with the density of the trabecular bone

In the evaluation of the \(bcfs\), the densities of all of the phantom materials remained the same. However, over successive examinations of the same individual the density of the trabecular bone is likely to decrease with a rate that varies from as low as 0.9% per year in a healthy individual to as high as 3% per year in an osteoporotic patient. For a density change in the trabecular bone of 4%, the maximum variation in the \(bcfs\) of the lower forearm phantoms studied was less than 0.10%, and that of the femoral neck and lumbar spine were 0.45% and 0.16% respectively. Since these variations are quite small compared to the required precision of the method (i.e.~1%) the exact value of the density of the trabecular bone is not significant in the evaluation of the \(bcfs\).

### 3.11.2 The effect of adipose tissue

Bias correction factors were calculated when soft tissue replaced that of adipose in all phantoms studied. This led to increased \(bcfs\) and a slightly greater variation in the \(bcfs\) with the phantom size change. Figures 3.48 to 3.50 show this variation. \(Bcfs\) have values between ~1.041 and 1.052 for the lumbar spine phantoms studied at a scattering angle of 40° showing a variation of ~1%.

### 3.11.3 The importance of \(bcfs\)

The importance of the bias correction factors in XCSD can be assessed when \(bcfs\) are compared with the correction factors due to the effect of multiple scatter, \(mcfs\). Table 3.6 gives the variation in both \(bcfs\) and \(mcfs\) with the size change of the lower
Figure 3.48 Bcf for the lower forearm phantoms at 40°. (a): Adipose tissue, (b) Soft tissue has replaced the adipose tissue.

Figure 3.49 Bcf for the femoral neck phantoms at 40°. (a): Adipose tissue, (b) Soft tissue has replaced the adipose tissue.

forearm, femoral neck and lumbar spine phantoms studied at scattering angles of 40° and 50°. The variation in the $bcfs$ with phantom size change is at least one order of magnitude less than that of the $mcfs$ at scattering angles between 20° and 60°. Moving from lower forearm to lumbar spine phantoms, at the same scattering angle the variation in $bcfs$ with phantom size change increases, whereas that of the $mcfs$ decreases. If the
Figure 3.50 Bcf for the lumbar spine phantoms at 30°. (a) Adipose tissue, (b) Soft tissue has replaced the adipose tissue

Lumbar phantom size increases from 437 cm² to 786 cm² and correction due to the effect of multiple scatter is not applied, an error of ~18% is expected in the determination of bone density at scattering angles between 40° and 50°, whereas the error implied by the bcf would be as low as 1%.

**TABLE 3.6** Variation in bcf and mcf with the size change of the phantoms studied at scattering angles of 40° and 50°.

<table>
<thead>
<tr>
<th>Phantom studied</th>
<th>Δ (bcf) 40°</th>
<th>Δ (bcf) 50°</th>
<th>Δ (Area) (cm²)</th>
<th>Δ (mcf) 40°</th>
<th>Δ (mcf) 50°</th>
<th>Δ (bcf)/cm² 40°</th>
<th>Δ (bcf)/cm² 50°</th>
<th>Δ (mcf)/cm² 40°</th>
<th>Δ (mcf)/cm² 50°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower forearm</td>
<td>0.0035</td>
<td>0.0048</td>
<td>32.9</td>
<td>0.061</td>
<td>0.066</td>
<td>1.06x10⁻⁴</td>
<td>1.46x10⁻⁴</td>
<td>1.9x10⁻³</td>
<td>2.0x10⁻³</td>
</tr>
<tr>
<td>Femoral neck</td>
<td>0.0060</td>
<td>0.0081</td>
<td>141.5</td>
<td>0.065</td>
<td>0.053</td>
<td>4.2x10⁻⁵</td>
<td>5.7x10⁻⁵</td>
<td>4.6x10⁻⁴</td>
<td>3.7x10⁻⁴</td>
</tr>
<tr>
<td>Lumbar spine</td>
<td>0.0087</td>
<td>0.0120</td>
<td>349</td>
<td>0.075</td>
<td>0.078</td>
<td>2.5x10⁻⁵</td>
<td>3.4x10⁻⁵</td>
<td>2.1x10⁻⁴</td>
<td>2.2x10⁻⁴</td>
</tr>
</tbody>
</table>
The variation in the bias correction factors with the phantom size change was studied at three sites of interest. It was found that $b_{cfs}$ vary by $\sim 1.0 \times 10^{-4}/\text{cm}^2$ of the area of the phantom cross-section, (i.e. lower forearm phantoms), reaching values between $2.5 \times 10^{-5}/\text{cm}^2$ and $3.4 \times 10^{-5}/\text{cm}^2$ at scattering angles between 40° and 50° (i.e. lumbar spine phantoms). A 1% variation in the $b_{cfs}$ would approximately correspond to an 80% change in the size of the lumbar spine phantom which is rather unlikely to happen between successive examinations of the same individual over a period of a year. Hence, the variation in the $b_{cfs}$ is expected to be much less than 1% which is within the required precision of the method and, thus, $b_{cfs}$ can be disregarded in in-vivo applications.
This chapter deals with the design and construction of the x-ray Compton scatter bone densitometer. This involves: (i) Design and construction of collimators, (ii) support system for the incident photon beam and detector alignment and (iii) tests carried out in order to evaluate the correct operation of the Compton scatter densitometer.

4.1 The design and construction of the collimators

With XCSD, the tissue examined is trabecular bone and this is performed at three sites of the body; the lower forearm, the femoral neck and the lumbar spine. The “critical volume” must be within the trabecular bone examined. Since the extent of the trabecular bone varies with the examination site, the collimation of either the x-ray beam or that of the detector must vary so that the “critical volume” is still within the trabecular bone examined. Since the collimator of either the x-ray tube or the detector cannot be replaced by a different size collimator every time the examination site changes, collimators of variable aperture were designed.

Figure 4.1 shows one of the collimators that were made. Each collimator consists of an 8 cm square base made of 5 mm of lead placed in between two plates of 6.5 mm of aluminium. In the middle of the base, the lead thickness has been increased to 20 mm and a 2 mm hole has been drilled. A brass tube 8.3 cm in length has been attached to the base of the collimator with 5 mm of lead placed inside its front part and a 2 mm hole aligned with the hole in the base. On the front part of collimator tube there is a revolving disc with three holes of different diameters (i.e. 1.0, 1.5 and 2.0 mm) drilled in 5 mm of lead and 16 mm of copper. A simple rotation could align one of these holes with the hole in the base of the collimator, resulting in a different aperture collimator.
Figure 4.1 The single hole detector collimator used in the x-ray Compton scatter densitometer. Identical collimators were used to collimate the x-ray beams.
4.2 The mounting of the collimator on the x-ray tube

The x-ray tube collimator had to be mounted on the central axis of the x-ray beam so that maximum output can be obtained. The procedure used to determine the central axis of the x-ray beam is described below, figure 4.2. The x-ray tube was supported on an optical bench and a piece of lead ~4 mm thick was fixed onto the window of the x-ray tube defining a hole of ~2 cm in diameter. A pinhole (100 μm aperture) was positioned ~2 cm away from the x-ray tube on a mechanical support with both horizontal and vertical movement. An x-ray film was placed behind the pinhole ~15 cm away and exposures were taken on the same x-ray film at different pinhole positions, as it was moved both vertically and horizontally in half millimetre intervals.

![Diagram](image_url)

**Figure 4.2** The experimental arrangement used to determine the central axis of the x-ray beam

The film was developed and all images were scanned with an automatic recording microdensitometer (i.e. Joyce Loebl, MK III CS) and the images which corresponded to the highest intensity were found. Results from both x-ray tubes are shown in figures 4.3.a to 4.4.b. It can be seen that scans marked as X3 and Y5 (x-ray tube 1) and X8
Figure 4.3.a Scans of the exposures taken from x-ray tube 1 at different positions of the pinhole-horizontal movement.
Figure 4.3.b Scans of the exposures taken from x-ray tube 1 at different positions of the pinhole-vertical movement
Figure 4.4.a Scans of the exposures taken from x-ray tube 2 at different positions of the pinhole-horizontal movement
Figure 4.4.b Scans of the exposures taken from x-ray tube 2 at different positions of the pinhole-vertical movement
and Y8 (x-ray tube 2) define the points in space where the intensities of the x-ray beams have maximum values and, therefore, these are the points where the central axes of the x-ray beams pass. Thus, these points and the position of the pinhole define the central axis of the x-ray beam.

Having determined two points in space through which the central axis of the x-ray beam passes, a laser beam was adjusted to be colinear with the axis. The collimator was then mounted on the x-ray tube so that the laser beam passed through its hole. Two images were taken, one with the entire collimator in place and a second image with only the base of the collimator mounted on the x-ray tube. Further adjustments of the collimator base relative to the x-ray tube were necessary because of the x-ray tube housing. An image taken with the entire collimator should be at the centre of an image taken with only the base of the collimator, figures 4.5 and 4.6. Although the spots shown in figures 4.5 and 4.6 were not exactly concentric, they were considered as satisfactory since any further adjustment would have a significant effect on the images taken at a distance of ~15 cm away from the collimator.

4.3 The radiation beams

As soon as the collimators were mounted on the x-ray tubes, the shape and size of both collimated x-ray beams were evaluated from images of the x-ray beams taken at certain distances from the front of each x-ray tube collimator. Figures 4.7 and 4.8 show the images of the x-ray beams from both x-ray tubes, at 2 cm intervals, for the three apertures of the x-ray tube collimator at distances varying between 16 and 46 cm from the face of each x-ray tube collimator. All images were scanned using the same microdensitometer as before and the diameters of the cross-sections of the x-ray beams were determined from the full width half maxima (FWHM).

Table 4.1 gives the variation in the size of the x-ray beams with the distance from the x-ray tube collimator. Considering the shape of the x-ray beams, images from x-ray tube 2 are more symmetric than those from x-ray tube 1 and this maybe due to a less well defined focal spot in x-ray tube 1. The sizes of the x-ray beams from both tubes are
Figure 4.5 Images taken after the mounting of the collimator on x-ray tube 1. The collimator has been adjusted relative to the x-ray tube housing. (a) 1.0 mm aperture, (b) 1.5 mm aperture, (c) 2.0 mm aperture.
Figure 4.6 Images taken after the mounting of the collimator on x-ray tube 2. The collimator has been adjusted relative to the x-ray tube housing. (a) 1.0 mm aperture, (b) 1.5 mm aperture, (c) 2.0 mm aperture.
Figure 4.7 Variation in the size of the radiation beam with the distance from x-ray tube1 collimator for the different collimation used. (a) 1.0 mm aperture, (b) 1.5 mm aperture, (c) 2.0 mm aperture.
Figure 4.8 Variation in the size of the radiation beam with the distance from x-ray tube2 collimator for the different collimation used. (a) 1.0 mm aperture, (b) 1.5 mm aperture, (c) 2.0 mm aperture.
virtually the same and thus, the same volume is irradiated by either x-ray tube.

**TABLE 4.1** Variation in the size of x-ray beams with the distance from each x-ray tube collimator. a: Results from x-ray beam 1, b: results from x-ray beam 2. The error is ~ 0.1 mm.

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<th>1.0 mm (FWHM) (mm) b</th>
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**4.4 The gantry-support system for x-ray tubes and detectors**

The x-ray Compton scatter bone densitometer consists of two well collimated x-ray tubes and two NaI(Tl) scintillation detectors facing the x-ray tubes with the patient to be...
Figure 4.9 The x-ray Compton scatter bone densitometer. The positions of both x-ray tubes and detectors are shown. The symmetrical phantom shows the position of the patient during an examination.
examined laying between the x-ray tubes and the detectors, figure 4.9. Both detectors must be aligned with the transmitted beams and minor adjustments of the scattering angle must be possible. Once the scattering angle at which the Compton scatter densitometer operates is chosen and the alignment of the detectors with the transmitted beams is completed, neither the x-ray tubes nor the detectors must be disturbed.

In order to be able to combine the x-ray tubes and the collimated detectors in an appropriate way, a suitable "gantry" was designed to accommodate the two x-ray tubes and the detectors. The design of the gantry took into consideration the examination sites of interest (i.e. primarily the femoral neck and the lumbar spine) and the convenience of the patient.

The gantry, figure 4.9, consists of two parallel plates of dural (6 mm thick), 12.5 cm apart with an allowance of 6 mm for sideways movement of both x-ray tubes. Three holes of slightly larger diameter than the x-ray tube housings were cut in both plates so that the x-ray tubes could gently rotate. The three holes allowed alternative positions for the x-ray tubes. Once the exact position of the x-ray tubes was decided, both x-ray tubes were clamped in position. The entire gantry was mounted on a steel bar and a counterbalance of ~90 kilograms was used in a conventional x-ray tube stand. The gantry could move vertically (i.e. stepper motors were employed) and the table on which the patient lays is intended to move horizontally.

4.4.1 The intersection of the x-ray beams

According to the basic principles of XCSD, the response from the same volume of trabecular bone must be taken when either x-ray tube is used and thus, the same volume must be irradiated by either x-ray beam. The size and location of the volume irradiated depend on both collimation and orientation of the x-ray beams. The former is defined by the collimators attached to the x-ray tubes and the latter by the position of the x-ray tubes on the gantry. As seen in table 4.1, both collimators define very similar sizes of transmitted beams (difference of only 0.1 mm at a distance of 27 cm away from the collimator face) and, therefore, only the position of the x-ray tubes has to be considered.

As the x-ray tubes have been positioned on the gantry, two movements can take
place; a rotational movement which enables the scattering angle to vary, and a sideways movement to allow the two x-ray beams to be in the same plane. In the geometrical arrangement used a scattering angle of 50° was defined. This was chosen from the results of chapter 3 (see section 3.3.3). A smaller scattering angle is difficult to obtain due to size and position of the x-ray tubes and the height of the intersection point of the x-ray beams. Fine adjustment of the x-ray tubes to study the intersection of the x-ray beams was based on images taken on dental x-ray films placed at a height close to that of the expected intersection point. Figure 4.10 shows the image of both x-ray beams taken at the intersection point. The distance between the front of the x-ray tube collimator and the centre of the "bone" was 23 cm and that between the the centre of the "bone" and the base of the detector collimator 27 cm.

![Image taken at the intersection point of x-ray beams.](image)

**Figure 4.10** Image taken at the intersection point of x-ray beams.

### 4.4.2 Detector collimator alignment with the transmitted beam

The alignment of each detector collimator with the transmitted beam is important for the performance of the Compton scatter densitometer. It is, therefore, necessary to be able to adjust the position of each detector collimator while the x-ray tube collimator facing it is fixed. For this reason, each detector collimator was positioned on an aluminium base designed to have two degrees of freedom when attached to the gantry.

In order to align the detector collimator with the transmitted beam, the position of
the x-ray beam had to be accurately known and, therefore, the position of the x-ray beam in space had to be determined. A laser beam, placed behind the detector collimator, was used to simulate the transmitted beam, and was adjusted to pass through the centre of the spot of the x-ray beam where it meets the aperture of the x-ray tube collimator. The detector collimator was placed on an aluminium base and adjusted so that the laser beam also passed through it. The position of each detector collimator relative to the transmitted beam was tested and fine adjustments carried out. Figure 4.11 shows the dental x-ray films taken when the alignment of each detector collimator with the transmitted beam was completed. The images were expected to be in the middle of the films.

![Figure 4.11 Images taken when the alignment of each detector collimator with the transmitted beam was completed. (a) From detector collimator 1, (b) from detector collimator 2.](image)

4.5 The shielding of the detectors

The presence of a significant amount of background radiation may degrade the performance of the Compton scatter densitometer. To ensure that background radiation is as low as possible, detector shielding is necessary.

Both detectors were placed behind their collimator base. Between the collimator
base and the face of the detector there was a lead plate (i.e. 3.2 mm thick) covering the entire detector face apart from a gap aligned with the collimator hole. A cylindrical lead ring (i.e. 3.2 mm thick) of slightly larger diameter than that of the detector crystal case surrounded the front part of the detector. In addition to that, the entire detector crystal and its collimator were surrounded by a square lead housing (i.e. 16 mm thick) leaving only the collimator hole unblocked. Figure 4.12 gives details about the shielding of each detector. Scatter measurements made with the lower forearm phantom (see section 5.2) showed a background count rate of ~ 50 cps (less than 10 % of the scattered count rate).

![Diagram of detector shielding](image)

**Figure 4.12** Cross-sectional view of the detector shielding

Attention must be paid to patient repositioning if density measurements from the same volume of the same bone are to be compared between successive examinations. The position of the patient examined is related to the intersection point of the x-ray beams which is identical relative to the position of the gantry. The entire gantry, however, can move vertically and the table on which the patient lays, horizontally. To be able to adjust either the position of the gantry or that of the table so that the same volume of the patient is irradiated, the intersection point of the x-ray beams must be accurately known. For this reason, two laser devices (i.e. Uniphase 1508) were fixed onto the
gantry with their beams one pointing vertically and the other horizontally and adjusted so that the beams meet at the intersection point of the x-ray beams.

4.6 Evaluation of the performance of the x-ray Compton scatter densitometer

A number of tests were carried out in order to evaluate the performance of the Compton scatter densitometer. Both the behaviour of individual components (i.e. x-ray units, detectors and electronics) and the performance of the Compton scatter densitometer as a whole were evaluated.

4.6.1 Measurement of x-ray spectra

Both scattered and transmitted count rates depend upon the intensity of the incident photon spectrum and the performance of the detectors and electronics used. Different kV_p spectra were measured from both x-ray units using a Ge solid state detector.

Resolution of the Ge solid state detector - The Ge solid state detector used (ORTEC) had a 16 mm diameter crystal of 8.8 mm thickness. It was operated at 1000 Volts (negative polarity) according to the manufacturer’s instructions. A Co^{57} standard source was placed on the top of the detector and data collected until the channel number corresponding to the 122 keV peak registered ~3000 counts. In order to measure the resolution of the solid state detector, a spectroscopy amplifier (CANBERRA model 2010), a high voltage supply (CANBERRA model 3100-01) and a multichannel analyser, MCA (CANBERRA 35 plus) were used. Data were transferred to an Apple microcomputer and a "least squares" fit applied on either side of the central axis of both peaks. The 122 keV peak was used for the determination of the energy resolution, since the count rate was higher, by determining the channels that corresponded to the full
width half maximum (FWHM) of this peak. The energy resolution was found to be 0.493 keV which was well within the required precision of 1 keV.

**Experimental arrangement** - The experimental arrangement used for the measurement of the spectra from both x-ray tubes is shown in figure 4.13. In order to prevent the saturation of the solid state detector, a gold pinhole (100 μm aperture) was placed in front of the detector. The intensity of both x-ray tubes was kept as low as possible and pile up rejection circuits were used in the spectroscopy amplifier and MCA. Several exposures of 5 seconds duration each were made in order to improve the precision of the measurements and spectra of 60, 70, 80, 90 and 100 kVp (i.e. according to the kVp indicator) were taken from both x-ray units. The data were transferred to an Apple microcomputer and stored on a floppy disk for further analysis.

**Figure 4.13** The experimental arrangement used for the measurement of the x-ray spectra

Data analysis consisted of three parts: (i) Conversion of counts per channel into
counts per 1 keV interval, (ii) correction of the spectrum due to the efficiency of the detector and (iii) correction of the spectrum due to the effect of the gold pinhole.

**Correction due to the efficiency of the detector** - A method to correct for the efficiency of the Ge detector to x-ray beams of incident energies up to 150 keV has been described by Fewell and Shuping, 1977. The loss of efficiency between 11.1 keV and 60 keV is due to the escape of the Ge k-x-rays from the detector, and efficiency loss for energies between 50 keV and 150 keV is due to the thickness of the detector.

Equation 4.1 was used to correct for the detector efficiency in an energy range between 11.1 keV and 60 keV

\[
I_{0,E} = \left( I_E - f_{(E + Ek)} I_{(E + Ek)} \right) / (1 - f_E) \tag{4.1}
\]

where \( I_{0,E} \) is the incident intensity at energy \( E \) and \( I_E \) is the intensity recorded at the same energy. \( f_E \) is the fraction of k-x-rays escape coefficient at energy \( E \) and \( f_{(E + Ek)} \) is this coefficient at energy \( (E + Ek) \) where \( Ek \) represents the energies of the Ge k-x-rays. \( I_{(E + Ek)} \) is the intensity recorded at \( (E + Ek) \). The values of \( Ek \) were approximated with a monoenergetic x-ray beam of 10 keV and equation 4.1 was rewritten as:

\[
I_{0,E} = \left( I_E - f_{(E + 10)} I_{(E + 10)} \right) / (1 - f_E) \tag{4.2}
\]

Equation 4.3 was used to correct for the Ge detector efficiency at energies greater than 50 keV

\[
I_{0,E} = I_E / f_p(E) \tag{4.3}
\]

where \( I_E \) represents the intensity recorded when corrections for Ge k-x-rays escape between 50 and 60 keV have been considered and \( I_{0,E} \) represents the incident intensity at \( E \). Values of \( f_p(E) \) are defined in equation 4.4

\[
f_p(E) = 1 - \exp(-\mu_en \rho x) \tag{4.4}
\]

where \( \mu_en \) is the mass energy absorption coefficient of germanium, \( \rho \) is its density and \( x \)
is the thickness of the Ge detector.

_Correction due to the effect of the gold pinhole_ - The beam reducing aperture used was made of a circular piece of lead 7.5 cm in diameter and 5 mm thick with a gold pinhole of 100 μm aperture in the middle. Due to the k-absorption edge of gold, spectra above 80 kV<sub>p</sub> showed a discontinuity over a few keV between 70 keV and 80 keV.

If there was no effect due to gold k-absorption edge, the photon output at energies greater than 70 keV would show a gradient similar to that shown in spectra of 60 kV<sub>p</sub> and 70 kV<sub>p</sub>. The gradient in a spectrum above 80 kV<sub>p</sub> can be determined from the photon output of the same spectrum at energies between 50 keV and 70 keV by using a polynomial fitting and excluding the values that correspond to gold characteristic x-rays. This principle was applied to spectra of 90 kV<sub>p</sub> and 100 kV<sub>p</sub> and the gradient determined was used to give the photon output in the energy range between 70 keV and 80 keV.

### 4.6.2 Results of x-ray spectra measurements

Spectra of 60, 70, 80, 90 and 100 kV<sub>p</sub> were taken from both x-ray units (i.e. GEC D-35 MK.2) and photon outputs against energy are plotted in figures 4.14 to 4.18 after corrections for the efficiency of the Ge detector and the gold-k-absorption edge of the pinhole were applied. Spectra of greater interest are those of 90 kV<sub>p</sub> and 100 kV<sub>p</sub>, since they are more likely to be used in XCSD. The 100 kV<sub>p</sub> spectra were compared with that given by Birch _et al_, 1979, figures 4.19 and 4.20. A shift of the measured photon output towards smaller energies appears in both 100 kV<sub>p</sub> spectra and this may be due to Compton scatter that occurs inside the detector (i.e. this effect has not been considered in the above analysis). Spectra from x-ray unit 2 at the same kV<sub>p</sub> setting showed an increased output above ~35 keV compared to those from x-ray unit 1 which might be due to slightly different kV<sub>p</sub> setting and inherent tube filtration. The mean energies of 90 kV<sub>p</sub> spectra from unit 1 and unit 2 are 41.7 keV and 43.7 keV, respectively and those of 100 kV<sub>p</sub>, 44 keV and 46 keV.
Figure 4.14 Spectra taken at 60 kVp from both x-ray units. 
+ : x-ray unit 1, — : x-ray unit 2.

Figure 4.15 Spectra taken at 70 kVp from both x-ray units. 
+ : x-ray unit 1, — : x-ray unit 2.

Figure 4.16 Spectra taken at 80 kVp from both x-ray units. 
+ : x-ray unit 1, — : x-ray unit 2.
Figure 4.17 Spectra taken at 90 kVp from both x-ray units.  
+ : x-ray unit 1, — : x-ray unit 2.

Figure 4.18 Spectra taken at 100 kVp from both x-ray units.  
+ : x-ray unit 1, — : x-ray unit 2.
Figure 4.19 Spectra at 100 kVp. +: Measured from x-ray tube 1,
- - - given by Birch et al., 1979.

Figure 4.20 Spectra at 100 kVp. +: Measured from x-ray unit 2,
- - - given by Birch et al., 1979.
Spectra from both x-ray units are quite similar. The differences mentioned above are not expected to have a significant effect on the evaluation of the effect of multiple scatter but the effect of different exposure factors on the determination of bone density must be studied.

4.6.3 The x-ray tubes output

Any displacement of the x-ray tube collimator would result in a significant reduction of the x-ray tube output and this would lead to poor scattered count rate. The outputs of both x-ray tubes were measured at the "critical volume" with different exposure factors. Figures 4.21 to 4.25 are plots of exposure values against intensity at 80, 85, 90, 95 and 100 kVp. X-ray tube 1 shows an increased output compared to that of x-ray tube 2 (as high as 14%). Since both radiation beams have very similar sizes, the difference in the output of the x-ray tubes may be either due to a slight displacement of the x-ray tube collimator relative to the central axis of the photon beam or due to fluctuating x-ray units. Further adjustment of either of the x-ray tube collimators would be too difficult since their positions are so crucial.

4.6.4 The performance of the detectors

The detectors response to count rate - The signal detected depends on the performance of the detectors with their associated electronics. Ideally, at the same exposure factors and for the same geometrical arrangement the same signal should be recorded from both combinations of detector-electronics. Since both detectors are used to measure transmitted as well as scattered radiation (i.e. low and high count rate, respectively), their performance had to be determined at both extremes.

Two Am$^{241}$ sources (i.e. 10 µCi and 200 mCi) were used to determine the response of both detectors at low and high count rates. The lower energies were set at 20 keV (or 30 keV) and the energy windows were 40 keV (or 30 keV) on both detectors.
Figure 4.21 X-ray tubes output at 80 kVp. Open circles: x-ray tube 1, closed circles: x-ray tube 2.

Figure 4.22 X-ray tubes output at 85 kVp. Open circles: x-ray tube 1, closed circles: x-ray tube 2.

Figure 4.23 X-ray tubes output at 90 kVp. Open circles: x-ray tube 1, closed circles: x-ray tube 2.
Figure 4.24 X-ray tubes output at 95 kVp. Open circles: x-ray tube 1, closed circles: x-ray tube 2.

Figure 4.25 X-ray tubes output at 100 kVp. Open circles: x-ray tube 1, closed circles: x-ray tube 2.

Figure 4.26 Detectors response to low count rate. +: Detector 1, □: detector 2. Both energy windows were set at (20-60) keV.
Figure 4.27 Detectors response to high count rate. +: Detector 1, ◇: detector 2. Both energy windows were set at (20-60) keV.

Figure 4.28 Detectors response to low count rate. +: Detector 1, ◇: detector 2. Both energy windows were set at (30-60) keV.

Figure 4.29 Detectors response to high count rate. +: Detector 1, ◇: detector 2. Both energy windows were set at (30-60) keV.
Multiple measurements of 10 seconds duration each were taken with source-to-detector fac distances varying from 18.5 to 56.5 cm. Figures 4.26 to 4.29 show the response of both detectors at low and high count rate with different energy windows.

The energy resolution of the detectors - Significant difference in the energy resolution of the detectors used in the Compton scatter densitometer would result in count rate differences under the same experimental conditions. The energy resolution of both detectors was measured using an Am$^{241}$(59.5 keV) standard source. The energy resolutions were 14.7% and 17.8% for detector 1 and detector 2, respectively.

4.6.5 Scatter and transmission measurements on a symmetrical phantom

Scatter as well as transmission measurements were compared from both combinations of detector-electronics used in the Compton scatter densitometer. If the experimental geometry and the performance of all components were the same, very similar count rates (i.e. scattered or transmitted) should be recorded by both detectors with the same exposure factors.

To ensure that the experimental geometry was the same when either x-ray tube was used, a circular phantom made of soft tissue equivalent material was used, figure 5.2. Double distilled water (1.00 g/cm$^3$) surrounded by ~2 mm cylinder of aluminium was inserted in the middle of the phantom and the phantom was positioned in the Compton scatter densitometer with its centre at the intersection point of the x-ray beams. The focussing detector collimators were used (see section 5.6) and thus, an increased scattered count rate could be achieved giving improved precision. Four mm of Cu was placed on the front of each transmission detector to prevent saturation. Both detectors were calibrated with an Am$^{241}$(59.5 keV) standard source. A ratemeter (Canberra 2081) was employed, a narrow "energy" window was selected (i.e. 5.9 to 6.0 Volts) and the fine gain adjusted so that the count rate reached maximum.

Measurements taken at different kVp settings, low intensity (i.e. 2.7 mA) and for 8
seconds duration are given in table 4.2 for the energy windows studied. For the same energy window on both detectors, an increase in the kVp setting emphasizes the difference in the ratio of scattered to transmitted counts between the two combinations of detector-electronics whereas selection of a narrow energy window towards the higher part of the spectra significantly reduces this difference.

Three factors can contribute to the difference in the scattered to transmitted counts ratio: (i) The incident photon spectra used, (ii) the difference in the energy resolution of the detectors and (iii) the difference in the response of the detectors to count rates. The different behaviour in the scattered to transmitted counts ratio with the energy windows reflects the difference in the energy resolution of the detectors. Detector 2 records more scattered photons than detector 1 for the "same" energy window and this effect is more evident for energy windows towards the lower part of the spectra. The increased number of transmitted photons recorded by detector 1 cannot compensate for the increased number of scattered photons recorded by detector 2. For energy windows above ~30keV and at kVp settings as high as 100, the relative difference in the scattered to transmitted counts ratio between the two combinations of detector-electronics is less than 25%.

4.6.6 The performance of the focussing detector collimators

According to the basic principle of Compton scatter densitometry, the use of a second source eliminates the problem of photon beam attenuation. The quantity \((S_1S_2/T_1T_2)^{1/2}\) where \(S_1, S_2\) are scatter and \(T_1, T_2\) are transmission measurements from source 1 and source 2, respectively should be independent of the amount of material that surrounds the volume whose density is measured if the effect of multiple scatter is disregarded. This principle was tested experimentally and the performance of both focussing detector collimators was evaluated.

The femoral neck phantom was positioned with its centre at the x-ray beams intersection point (i.e. phantom position, 0 mm). A \(\text{K}_2\text{HPO}_4\) solution (1.18g/cm\(^3\)) was inserted into the phantom and scatter and transmission measurements were performed
TABLE 4.2 Scatter and transmission measurements from both combinations of detector-electronics using a symmetrical phantom. 4.0 mm of Cu was placed in front of each transmission detector. S1,S2: Scatter measurements; T1,T2: Transmission measurements. DE: Energy windows. kVp1, kVp2: kVp settings of tube 1 and tube 2, respectively.

<table>
<thead>
<tr>
<th>kVp1</th>
<th>kVp2</th>
<th>T1</th>
<th>S1</th>
<th>T2</th>
<th>S2</th>
<th>ΔE</th>
<th>S1/T1</th>
<th>S2/T2</th>
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<td>84</td>
<td>17650</td>
<td>3633</td>
<td>18132</td>
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<td>87</td>
<td>87</td>
<td>26416</td>
<td>4691</td>
<td>29163</td>
<td>4062</td>
<td>(30-75) keV</td>
<td>0.182</td>
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<td>90</td>
<td>89</td>
<td>33450</td>
<td>5697</td>
<td>36878</td>
<td>4654</td>
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<td>31706</td>
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<td>90</td>
<td>89</td>
<td>44624</td>
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<td>93</td>
<td>93</td>
<td>57663</td>
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<td>108545</td>
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<td>86</td>
<td>31440</td>
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<td>30309</td>
<td>3350</td>
<td>0.110</td>
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<td>90</td>
<td>89</td>
<td>45257</td>
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<td>(40-E_max) keV</td>
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<td>5640</td>
<td>0.073</td>
<td>0.065</td>
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</tr>
</tbody>
</table>

157
from both x-ray tubes while the phantom was being shifted horizontally in a 4 mm interval. As the position of the phantom changes, transmitted and scattered path lengths vary and the effect is shown on scatter to transmission measurements taken from each x-ray tube, figure 4.30. Since for the first three positions of the phantom the x-ray beams intersection point is still inside the solution, density measurements should be constant. The change, however, in scattered to transmitted counts against the position of the phantom shows that with only one source, density measurements depend on photon beam attenuation.

![Figure 4.30 Scatter/transmission measurements taken from each x-ray tube at different positions of the phantom. A: Inside the solution, B: outside the solution.](image)

The situation is different when scatter and transmission measurements from both x-ray tubes are combined. This is shown in figure 4.31 where \( (S_1S_2/T_1T_2)^{1/2} \) is plotted against the position of the phantom. Density values are statistically the same at phantom positions between 0 and 8 mm (i.e. inside the solution) and between 16 and 24 mm (i.e. outside the solution). The average value that appears at ~12 mm coincides with the end of the volume occupied by the inserted solution (i.e. 25 mm diameter) and is due to partial volume effect.

Unless both focussing detector collimators are properly positioned and of right focal lengths, scattered counts will be collected from a different volume than that of interest and thus, density measurements will not correspond to the volume at the x-ray
beams intersection point. The higher values taken from a volume inside the solution compared to these from the rest of the femoral neck phantom (1.00 g/cm³) and the reduced value that coincides with the end of the volume occupied by the insert solution show that both detector collimators focus at the right volume as a result of proper design and positioning in the Compton scatter densitometer.

![Graph](image)

**Figure 4.31** Scatter/transmission measurements taken from both x-ray tubes at different positions of the phantom. A: Inside the solution, B: Outside the solution.

### 4.6.7 The effect of the response of the detector on the accuracy of the method

The response of each detector to different count rates may affect the accuracy of the method. Figure 4.32 shows the response of detector 1 used in the Compton scatter densitometer to low and high count rates. As the source-to-detector distance decreases, the count rate increases but the gradient becomes smaller at higher count rate. Transmission measurements lead to count rates varying between 5000 cps and 14000 cps whereas scatter measurements have much lower count rates (i.e. ~1000 cps). If in the analysis the same gradient in the detector response is assumed for both scattered and
transmitted count rates, the measured density, $\rho_{\text{meas}}$, will be greater than that expected, $\rho_{\text{exp}}$, since the transmitted counts recorded are significantly reduced.

![Figure 4.32](image)

Figure 4.32 The response of detector 1 to low and high count rates. The gradient has decreased at higher count rate. A: Scattered count rate region, B: Transmitted count rate region.

The greater the gradient difference in the detector response between the regions of scattered and transmitted count rates, the worse the accuracy. This systematic error, however, can be significantly reduced if density measurements performed at a particular site of examination are compared with those taken from a suitable size calibration phantom. At the same exposure factors, a transmission measurement taken from the calibration phantom will carry the same error because of the detector response to high count rate as that performed on the examination site.

### 4.7 Summary

The source and detector collimators were aligned and adjusted for optimum performance. The use of dual x-ray tube and detector combinations was shown to
correct for photon beam attenuation. Quality control involved output and spectral measurements of the x-ray tubes and each x-ray tube and detector pair were producing similar scattered and transmitted count rates. The next stage was the use of the x-ray Compton scatter densitometer for bone density measurements under different experimental conditions. This is shown in chapter 5.
This chapter contains the experimental results. Density measurements were performed on phantoms used in the Compton scatter densitometer under different experimental conditions. Factors which affect the precision of the results are discussed and the effect of multiple scatter is shown experimentally.

5.1 The density solutions

Potassium orthophosphate ($K_2HPO_4$) has very similar attenuation properties to hydroxyapatite (HA), Meema et al., 1964, and since bone mineral appears mostly in the form of HA, $K_2HPO_4$ can be used to simulate bone mineral. In order to cover the trabecular bone density range, $K_2HPO_4$ solutions of densities varying between 1.02 g/cm$^3$ and 1.21 g/cm$^3$ were used in experimental density measurements. A high density solution (i.e. 1.28 g/cm$^3$) was prepared first and the lower density solutions were then made by dilution. Each solution was tested by measuring the weight of an accurate volume (i.e. 0.8 cm$^3$). Solution densities were measured with 0.7% precision.

The solutions had to be stored in containers of suitable size and shape which fit in the phantoms at the location of the inner bone. In this way the same geometry is maintained throughout all measurements. Also, since air inside the solutions will lead to erroneous results, all solutions should be bubble free. Syringes (i.e. 20 ml volume) were used as containers and this allows to remove the air bubbles.

5.2 The phantoms used

Bone density measurements are based on both scattered and transmitted counts
from the site of examination, which in turn depend on the scattering properties and the size of the site examined. The lumbar spine, the femoral neck and the lower forearm are sites of interest in bone densitometry. Although they do not show significantly different attenuation properties, an obvious difference is their size. Figures 5.1 to 5.3 show the phantoms employed in the bone density measurements.

The lower forearm phantom is represented by a circular cross-section of soft tissue equivalent material 7 cm in diameter, with a circular off-centre space 2.1 cm in diameter which is surrounded by 2 mm of cortical bone equivalent material. Although its geometry is quite simplified, its size is similar to that expected. The physical density of the soft tissue of the phantom is 1.00 g/cm³.

The femoral neck phantom is represented by a circular cross-section symmetrical phantom, 21 cm in diameter composed of soft tissue equivalent material. Both the size and shape of this phantom are similar to those expected but the amount of trabecular bone is significantly less. The physical density of the soft tissue of the phantom is 1.00 g/cm³.

The lumbar spine phantom is composed of soft tissue equivalent material of size and shape similar to a lumbar spine cross-section without any anatomical details of the internal organs. These are assumed to be equivalent to soft tissue. Both of the transmitted beam path lengths at the scattering angle used in the Compton scatter densitometer are 23.5 cm. The physical density of the soft tissue of the phantom is 1.00 g/cm³.

The same density solutions were used in all phantoms. In the femoral neck and the lumbar spine phantoms a 2 mm thick aluminium ring surrounded the solution container to simulate the cortex.

5.3 Factors affecting the precision of the method

The overall precision of the method is determined by: (i) Patient repositioning, (ii) stability of the x-ray units outputs, (iii) performance of the electronics and (iv) scattered counts recorded. Density measurements performed over long time intervals will prevent
Figure 5.1  The lower forearm phantom used for density measurements in the Compton scatter densitometer. The syringe shown was used to store the different density solutions.
Figure 5.2 The femoral neck phantom used for density measurements in the Compton scatter densitometer. The syringe shown was used to store the different density solutions.
Figure 5.3  The lumbar spine phantom used for density measurements in the Compton scatter densitometer. The syringe shown was used to store the different density solutions.
overloading of the x-ray tubes and, thus, providing a more stable performance of the x-ray units.

The precision of the method, however, is limited by the scattered counts recorded (i.e. statistical precision). Each measurement is associated with a statistical error which is at least equal to ±√N, where N is the number of counts recorded. Since the percentage statistical fluctuation, (i.e. ±1/√N), decreases as the counts recorded increase, and the low count rate determines the precision of a measurement, experimental conditions which lead to increased scattered counts are desirable. Hence, high kV_p incident x-ray spectra and longer exposures are expected to improve the statistical precision.

5.4 Density measurements - Single hole detector collimators

Density measurements taken with the single hole detector collimators for the three phantoms studied are presented here. Different kV_p incident x-ray spectra were used in order to improve the statistical precision of the measurements.

5.4.1 The volume examined

The amount of scattered radiation detected depends on the electron density within the "critical volume", the tissues which surround the "critical volume", the incident photon beam intensity and the detector efficiency. For the same phantom size and with the same exposure factors, the "critical volume" will govern the scattered radiation detected. The "critical volume" was calculated numerically for the geometry defined by the single hole detector collimators used in the Compton scatter densitometer, (i.e. detector collimator length = 95 mm and detector collimator aperture = 2 mm), and found to be ~0.23 cm³. The "critical volume" is inside the volume occupied by the solution.
5.4.2 Incident photon beam filtration

Low energy photons will be preferentially absorbed inside the patient, giving an increased dose, whereas the statistical precision will remain the same. In all density measurements taken with the single hole detector collimators, both incident photon beams were filtered with 0.33 mm Cu before they entered the phantom (i.e. pre-patient beam filtration). The lower energy windows were set at ~20 to 30 keV on both detectors in order to maximize the scattered counts recorded.

5.4.3 The lower forearm phantom

Figure 5.4 is a plot of the quantity measured, \((S_1 S_2/T_1 T_2)^{1/2}\), against the density of the solutions used in the lower forearm phantom. The incident x-ray spectra were 95 kV\(_p\), tube currents of 4 mA and 6 mA for x-ray tube 1 and x-ray tube 2 respectively, and the counting duration 8 seconds. The energy windows were set at (20-95) keV on both detectors and 5.34 mm Cu filtration was used on each transmission detector (i.e. post-patient transmission beam filtration).

![Figure 5.4: Density measurements made on the lower forearm phantom with 97 kVp incident photon spectra. Correlation coefficient, \(R=0.967\)](image)
Table 5.1 shows the effect of the different incident photon spectra on density measurements. The scattered counts recorded were ~3400 and 2700 when x-ray tube 1 and x-ray tube 2 were irradiated respectively, defining a statistical precision of ~2%.

<table>
<thead>
<tr>
<th>Incident beam filtration</th>
<th>Transmission detector filtration</th>
<th>Incident x-ray spectra (kVp)</th>
<th>Energy windows (keV)</th>
<th>Linear regression analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.33 mm Cu</td>
<td>4.67 mm Cu</td>
<td>92</td>
<td>(20-92)</td>
<td>a=0.022±0.003, b=0.007±0.003</td>
</tr>
<tr>
<td>0.33 mm Cu</td>
<td>4.67 mm Cu</td>
<td>97</td>
<td>(20-97)</td>
<td>a=0.017±0.001, b=0.007±0.002</td>
</tr>
</tbody>
</table>

The coefficients a and b which appear in tables 5.1 to 5.6 are the gradient and the intercept, respectively for the linear regression analysis of \(\sqrt{S_1S_2/T_1T_2}\) against density.

### 5.4.4 The femoral neck phantom

The femoral neck phantom is considerably larger than that of the lower forearm and thus, photon attenuation will be higher leading to poorer statistical precision. Since the intensity of the incident photon beam strongly depends on the kVp setting, higher kVp incident photon spectra had to be used. Density measurements were taken with incident photon spectra of 95 kVp and 100 kVp at low tube currents and a counting duration of 8 seconds. The energy windows were set at (20-\(E_{\text{max}}\)) keV on both detectors, where \(E_{\text{max}}\) denotes the maximum photon energy, and each transmission detector was filtered with 3.33 mm of Cu.

Figure 5.5 is a plot of the quantity measured, \((S_1S_2/T_1T_2)^{1/2}\), against the density.
of the solutions used in the femoral neck phantom. The statistical precision defined by the scattered counts recorded is ~ 2.2%. Table 5.2 gives the response of the system at different incident photon spectra.

![Graph showing density measurements](image)

**Figure 5.5** Density measurements made on the femoral neck phantom with 100 kVp incident photon spectra. Correlation coefficient, $R=0.946$

**TABLE 5.2** The effect of different kVp incident x-ray spectra on density measurements made on the femoral neck phantom with the single hole detector collimators.

<table>
<thead>
<tr>
<th>Incident beam filtration</th>
<th>Transmission detector filtration</th>
<th>Incident x-ray spectra (kVp)</th>
<th>Energy windows (keV)</th>
<th>Linear regression analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.33 mm Cu</td>
<td>3.33 mm Cu</td>
<td>95</td>
<td>(20-95)</td>
<td>$a=0.022\pm0.003$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>$b=0.014\pm0.003$</td>
</tr>
<tr>
<td>0.33 mm Cu</td>
<td>3.33 mm Cu</td>
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<td>(20-100)</td>
<td>$a=0.018\pm0.002$</td>
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<td></td>
<td></td>
<td></td>
<td>$b=0.015\pm0.002$</td>
</tr>
</tbody>
</table>

**5.4.5 The lumbar spine phantom**

Using the phantom shown in figure 5.3 density measurements were made with a
100 kVp incident photon spectrum at low tube currents (i.e. 4.6 mA and 6.0 mA). The results are shown in figure 5.6. The counting duration was 8 seconds and the energy windows were set at (20-100) keV on both detectors. A 2.67 mm Cu filter was used on each transmission detector. The scattered counts recorded defined a statistical precision of \( \sim 2.5\% \).

![Graph showing density measurements](image)

**Figure 5.6** Density measurements made on the lumbar spine phantom with \( \sim 102 \) kVp incident photon spectra. Correlation coefficient, \( R=0.949 \)

### 5.5 Summary

As seen in figures 5.4 to 5.6, a linear relationship exists between the quantity measured and the density of the solutions in all phantoms used. A higher correlation exists in the smallest size phantom due to improved statistical precision and possibly more stable performance of the x-ray units. Even with high kVp incident x-ray spectra, the statistical precision for the smallest size phantom was only slightly better than 2%. Use of even higher incident photon beam intensities may improve the statistical precision determined by the small size phantom, but it is expected to have little effect on the precision defined by the larger size phantoms, although the dose given to the patient will increase. An alternative was to design detector collimators which would considerably increase the amount of scattered radiation detected at relatively low intensity spectra.
5.6 The focusing detector collimators

Evidence for improved statistical precision with suitably designed detector collimators was provided by the photon transport computer model where two types of detector collimators were simultaneously considered, (see section 2.4.3); a single hole collimator and a multi-hole focusing collimator. Since the single hole detector collimators used in the Compton scatter densitometer, (see section 4.1), were finer than those considered in the photon transport computer model, even poorer statistical precision is expected than that shown in the computer model. In order to improve the statistical precision of the method while the dose given to the patient is kept low, two focusing detector collimators were made and used in the Compton scatter densitometer.

5.6.1 Design and construction

Attention had to be paid to the focal lengths of the focusing detector collimators so that they focused on the x-ray beams' intersection point in the "critical volume". Both collimators were designed to have focal lengths of 25 cm so that they could be easily adjusted once mounted on the gantry. Collimators with too long focal lengths could not be accommodated in the densitometer due to the fixed positions of both detectors.

Both focusing detector collimators had five holes, 4 mm in diameter and 20 mm long with septa varying between 2 and 3 mm. A lead block was used and the holes were carefully drilled at the appropriate angles. Figure 5.7 shows a cross-section of one of the focusing detector collimators.

The focal length of each detector collimator was determined as follows: Two laser beams were adjusted to pass through the centres of the outer holes of the collimator. A screen, placed in front of the collimator, moved away from the collimator face until the laser beams' intersection point was met. The distance of this point from the back of the collimator was found to be close to 25 cm in both collimators.

The collimators were fixed on bases made of an aluminium-lead-aluminium plate 1.0 cm thick. The position of each base was adjusted so that the distance between the
x-ray beams intersection point and the plate on which the entire collimator was mounted was 27 cm. The same procedure as that with the single hole detector collimators was followed for the alignment of the focussing detector collimators with the incident photon beam when they were mounted on the gantry.

![Diagram](image)

**Figure 5.7** Cross-section of the focussing detector collimator used in the Compton scatter densitometer.

### 5.7 Density measurements - The focussing detector collimators

Density measurements performed with the focussing detector collimators for the three phantoms studied are presented in this section. Different kV\(_p\) incident x-ray spectra
and energy windows were used and their effect on density measurements was studied.

Preliminary density measurements taken from the lower forearm phantom with the focussing detector collimators showed statistical precision better than 1.6%. Solutions, therefore, which differ in density by at least 3% should be distinguishable. For this reason, density solutions of 1.00 g/cm$^3$, (i.e. double distilled water), 1.05, 1.095, 1.15 and 1.18 g/cm$^3$ were used in all phantoms studied. The exposure factors largely depend on the size of the phantom examined.

5.7.1 The volume examined

The "critical volume" calculated numerically for the geometry defined by the incident photon beam and the detector focussing collimator is $\sim 4.7$ cm$^3$. Over 50% of this volume is occupied by the density solution. Due to the enlarged acceptance angle and the use of a multi-hole detector collimator, the scattered count rate will be increased resulting in improved statistical precision. However, multiple scatter contribution to the signal recorded will be higher.

5.7.2 The lower forearm phantom

Due to increased scattered count rate, x-ray spectra up to 90 kV$_p$ were used at low tube currents (i.e. 4 mA and 6 mA for x-ray tube 1 and x-ray tube 2, respectively). Pre-patient beam filtration made no significant difference on the multiple scatter contribution to the signal recorded.

(i) Unfiltered incident photon beams

Figure 5.8 is a plot of the quantity measured $(S_1 S_2 / T_1 T_2)^{1/2}$, against the different density solutions used in the lower forearm phantom. The incident x-ray spectra were 85 kV$_p$, the current 4 mA and 6 mA for x-ray tube 1 and x-ray tube 2 respectively, and the
counting duration 8 seconds. Both energy windows were set at (40-85) keV and 4.67 mm Cu filtration was used on each transmission detector.

\[0.116 \quad 0.114 \quad 0.112 \quad 0.110 \quad 0.108 \quad 0.106\]

\[0.96 \quad 1.00 \quad 1.04 \quad 1.08 \quad 1.12 \quad 1.16 \quad 1.20\]

\[\text{Density (g/cm}^3\text{)}\]

Figure 5.8 Density measurements made on the lower forearm phantom with 85 kVp incident photon spectra. Correlation coefficient, \(R=0.998\)

The scattered count rates were \(\sim1400\) cps defining a statistical precision of \(\sim0.95\%\). Skin dose measured with a large volume probe dosemeter (i.e. mdh 2025 radiation monitor, \textit{Radcal Corporation}) is less than 55 µGy.

\(\text{(ii) Pre-patient filtered incident photon beams}\)

Both incident photon beams were filtered with 0.33 mm of Cu. Incident x-ray spectra between 85 kVp and 90 kVp were used with energy windows set at \((40-E_{\max})\) keV on both detectors. The counting duration was 8 seconds and 4.67 mm Cu filtration was used on each transmission detector.

Figure 5.9 is a plot of the quantity measured \((S_1 S_2/T_1 T_2)^{1/2}\), against the different density solutions used in the lower forearm phantom. Table 5.3 shows the response of the system to different incident photon spectra. The statistical precision is \(\sim1.1\%\). Skin dose has now dropped considerably due to incident beam filtration (i.e. \(\sim8\) µGy).
Figure 5.9 Density measurements made on the lower forearm phantom with 85 kVp incident photon spectra. Correlation coefficient, $R=0.995$

**TABLE 5.3** The effect of different kVp incident x-ray spectra on density measurements made on the lower forearm phantom with the focussing detector collimators.

<table>
<thead>
<tr>
<th>Incident beam filtration</th>
<th>Transmission detector filtration</th>
<th>Incident x-ray spectra (kVp)</th>
<th>Energy windows (keV)</th>
<th>Linear regression analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.33 mm Cu</td>
<td>4.67 mm Cu</td>
<td>85</td>
<td>(40-85)</td>
<td>$a=0.033\pm0.001$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$b=0.054\pm0.002$</td>
</tr>
<tr>
<td>0.33 mm Cu</td>
<td>4.67 mm Cu</td>
<td>87</td>
<td>(40-100)</td>
<td>$a=0.028\pm0.002$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$b=0.054\pm0.002$</td>
</tr>
<tr>
<td>0.33 mm Cu</td>
<td>4.67 mm Cu</td>
<td>90</td>
<td>(40-90)</td>
<td>$a=0.023\pm0.003$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$b=0.045\pm0.003$</td>
</tr>
</tbody>
</table>

5.7.3 The femoral neck phantom

Higher energy x-ray spectra than those used in the lower forearm phantom were used in order to compensate for the increased photon attenuation.
(i) Unfiltered incident photon beams

Figure 5.10 shows density measurements made on the femoral neck phantom with ~98 kVp incident x-ray spectra, at low tube currents (~6 mA) and for 8 seconds counting duration. Both energy windows were set at (40-98) keV and 4.33 mm Cu filtration was used on each transmission detector.

![Graph showing density measurements vs. density.](image)

Table 5.4 shows how the use of different incident x-ray spectra affects the density measurements. The statistical precision was 0.95% at skin dose of less than 0.2 mGy.

(ii) Pre-patient filtered incident photon beams

Both incident photon beams were filtered with 0.33 mm of Cu. Incident x-ray spectra of ~95 kVp and ~102 kVp were used and the counting duration was 8 seconds. The lower energy windows were set at 35 keV on both detectors and 4.33 mm Cu filtration was used on each transmission detector.
**TABLE 5.4** The effect of different $kV_p$ incident x-ray spectra on density measurements made on the femoral neck phantom with the focusing detector collimators.

<table>
<thead>
<tr>
<th>Incident beam filtration</th>
<th>Transmission detector filtration</th>
<th>Incident x-ray spectra (kVp)</th>
<th>Energy windows (keV)</th>
<th>Linear regression analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>---</td>
<td>4.33 mm Cu</td>
<td>92</td>
<td>(40-85)</td>
<td>$a=0.072\pm0.005$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$b=0.030\pm0.006$</td>
</tr>
<tr>
<td>---</td>
<td>4.33 mm Cu</td>
<td>98</td>
<td>(40-98)</td>
<td>$a=0.022\pm0.002$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$b=0.056\pm0.002$</td>
</tr>
</tbody>
</table>

Figure 5.11 is a plot of the quantity measured $(S_1S_2/T_1T_2)^{1/2}$, against the different density solutions used in the femoral neck phantom. The change in the incident x-ray spectrum has only a small effect on density measurements, table 5.5. Skin dose was less than 30 $\mu$Gy for statistical precision of $\sim1.1\%$.

**TABLE 5.5** The effect of different $kV_p$ incident x-ray spectra on density measurements made on the lumbar spine phantom with the focusing detector collimators.

<table>
<thead>
<tr>
<th>Incident beam filtration</th>
<th>Transmission detector filtration</th>
<th>Incident x-ray spectra (kVp)</th>
<th>Energy windows (keV)</th>
<th>Linear regression analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.33 mm Cu</td>
<td>4.33 mm Cu</td>
<td>96</td>
<td>(35-96)</td>
<td>$a=0.036\pm0.003$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$b=0.047\pm0.004$</td>
</tr>
<tr>
<td>0.33 mm Cu</td>
<td>4.33 mm Cu</td>
<td>102</td>
<td>(35-102)</td>
<td>$a=0.029\pm0.002$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$b=0.055\pm0.002$</td>
</tr>
</tbody>
</table>
5.7.4 The lumbar spine phantom

Similar incident x-ray spectra to those used in the femoral neck phantom were used in density measurements performed with the lumbar spine phantom. In this case 4 mm.

![Graph showing density measurements](image)

Figure 5.11 Density measurements made on the femoral neck phantom with 96 kVp spectra. Correlation coefficient, $R=0.983$

Cu filtration was used on each transmission detector. The lower energy windows were set at 30 to 40 keV and the counting duration was 8 seconds.

All density measurements were taken without pre-patient beam filtration in order to increase the scattered count rate and, thus, the precision of the measurements. Figure 5.12 shows density measurements made on the lumbar spine phantom with ~100 kVp incident x-ray spectra, at tube currents of ~6 mA and for 8 seconds counting duration. The energy windows were set at (30-100) keV on both detectors. The statistical precision was ~1.2%.

Table 5.6 shows the effect of the selection of different energy windows on density measurements. An increase in the lower energy window will lead to a decrease in the absolute values of the quantity measured. Skin dose was less than 0.25 mGy.
Figure 5.12 Density measurements made on the lumbar spine phantom with 100 kVp incident photon spectra. Correlation coefficient, R=0.987

**TABLE 5.6** The effect of different energy windows on density measurements made on the lumbar spine phantom with the focussing detector collimators.

<table>
<thead>
<tr>
<th>Incident beam filtration</th>
<th>Transmission detector filtration</th>
<th>Incident x-ray spectra (kVp)</th>
<th>Energy windows (keV)</th>
<th>Linear regression analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>---</td>
<td>4.0 mm Cu</td>
<td>100</td>
<td>(30-100)</td>
<td>a=0.034±0.002, b=0.047±0.003</td>
</tr>
<tr>
<td>---</td>
<td>4.0 mm Cu</td>
<td>100</td>
<td>(35-100)</td>
<td>a=0.035±0.004, b=0.060±0.004</td>
</tr>
<tr>
<td>---</td>
<td>4.0 mm Cu</td>
<td>100</td>
<td>(40-100)</td>
<td>a=0.021±0.002, b=0.049±0.002</td>
</tr>
</tbody>
</table>

Scatter measurements made with the phantoms used both with and without the density solutions in place showed that as much as 85% of the scattered counts recorded are due to scatter that takes place inside the tissues that surround the "critical volume". This is due to the detector collimation and the size of the phantoms examined and is expected to reduce the sensitivity of the system. Although the sensitivity of the system
might be reduced, densities were measured with ~2% precision. Finer detector
collimation is expected to reduce the scatter contribution from the surrounding tissues to
the signal recorded and, thus, improve the sensitivity of the system.

5.8 Density measurement dependence on incident x-ray
spectra

Scatter over transmission ratio measurements taken from the same phantom, with
the same transmission detector filtration and with similar exposure factors, show a
statistically significant variation in measured density values, see tables 5.1 to 5.5. The
effect of different kVp spectra on bone density measurements was fully evaluated and is
described here.

Scatter and transmission measurements were performed on the femoral neck
phantom with a 1.00 g/cm³ density solution, using different kVp incident x-ray spectra
and energy windows. A 4 mm Cu filter was used on each transmission detector. The
variation in $\sqrt{\frac{S_1 S_2}{T_1 T_2}}$ with the different kVp spectra and energy windows used is
shown in figures 5.13 to 5.15.

![Graph showing variation in $\sqrt{\frac{S_1 S_2}{T_1 T_2}}$ with incident photon spectrum. Energy windows: (30-Emax)]

Figure 5.13 Variation in $\sqrt{\frac{S_1 S_2}{T_1 T_2}}$ with the incident photon spectrum change. Energy windows: (30-Emax)
As seen in figures 5.13 to 5.15, a small change in the incident x-ray spectra has a significant effect on both scattered and transmitted counts recorded, which leads to different bone density values. The selection of different energy windows would not change this behaviour. It is, therefore, important that bone density measurements are either to be taken at carefully controlled exposure factors (i.e. the incident x-ray spectra) or the density dependence upon the incident photon spectra should be taken into account.

Figure 5.14 Variation in √(S1S2/T1T2) with the incident photon spectrum change. Energy windows: (35-Emax)

Figure 5.15 Variation in √(S1S2/T1T2) with the incident photon spectrum change. Energy windows: (40-Emax)
In order to explain the variation in the scattered to transmitted counts ratio, R, with the incident x-ray spectra, correction factors (fs) were calculated. fs are calculated using equation 3.18 where transmission beam filtration has also been included. These factors are similar to bias correction factors (see section 3.10) but included the transmission detector filtration effect.

Mass attenuation data for Cu were taken from Storm and Israel, 1970, and Cu density was taken as 8.96 g/cm³. Table 5.7 gives the correction factors for the femoral neck phantom for different x-ray spectra and detector filtration. The variation in these factors with the incident photon spectra change was found to be less than 1% per 10 keV when the transmission detector filtration was not taken into account, but the variation increased dramatically with detector filtration (i.e. over 5% per keV) and became more significant with low energy spectra.

<table>
<thead>
<tr>
<th>Detector filtration Cu (mm)</th>
<th>f_s</th>
<th>f_s</th>
<th>f_s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>80 kV_p</td>
<td>90 kV_p</td>
<td>100 kV_p</td>
</tr>
<tr>
<td>0.00</td>
<td>1.0790</td>
<td>1.0704</td>
<td>1.0647</td>
</tr>
<tr>
<td>2.00</td>
<td>0.0629</td>
<td>0.0956</td>
<td>0.1277</td>
</tr>
<tr>
<td>2.67</td>
<td>0.0296</td>
<td>0.0511</td>
<td>0.0748</td>
</tr>
<tr>
<td>3.33</td>
<td>0.0147</td>
<td>0.0287</td>
<td>0.0458</td>
</tr>
<tr>
<td>4.00</td>
<td>0.0075</td>
<td>0.0165</td>
<td>0.0287</td>
</tr>
</tbody>
</table>

Tables 5.8 and 5.9 give the correction factors for the lower forearm and lumbar spine phantoms used in the Compton scatter densitometer for different spectra and transmission detector filtration. At 90 kV_p and for an energy window of (30-90) keV, the scattered to transmitted counts ratio for the femoral neck phantom was 0.124 (±1.3x10⁻³). At 96 kV_p and for an energy window of (30-96) keV, R had a value of
0.095 (±7.2x10^-4) showing a 24% decrease. However, when corrections were applied, the scattered to transmitted counts ratios had values of 2.04x10^-3 and 2.14x10^-3, respectively bringing the difference down to ~4.5%.

The same considerations as those described above reduced the difference in R from 33% to 17% when measurements at 90 kV_p and 100 kV_p were compared. The difference in R that still exists may be due to the response of the detectors and the spectra considered in the calculations.

TABLE 5.8 Variation in correction factors with transmission detector filtration at different kV_p incident spectra. The lower forearm phantom was considered in the calculations.

<table>
<thead>
<tr>
<th>Detector filtration Cu (mm)</th>
<th>f_s 80 kV_p</th>
<th>f_s 90 kV_p</th>
<th>f_s 100 kV_p</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.0434</td>
<td>1.0385</td>
<td>1.0352</td>
</tr>
<tr>
<td>2.00</td>
<td>0.0445</td>
<td>0.0701</td>
<td>0.0955</td>
</tr>
<tr>
<td>2.67</td>
<td>0.0205</td>
<td>0.0366</td>
<td>0.0545</td>
</tr>
<tr>
<td>3.33</td>
<td>0.0100</td>
<td>0.0202</td>
<td>0.0329</td>
</tr>
<tr>
<td>4.00</td>
<td>0.0050</td>
<td>0.0114</td>
<td>0.0203</td>
</tr>
</tbody>
</table>

TABLE 5.9 Variation in correction factors with transmission detector filtration at different kV_p incident spectra. The lumbar spine phantom was considered in the calculations.

<table>
<thead>
<tr>
<th>Detector filtration Cu (mm)</th>
<th>f_s 80 kV_p</th>
<th>f_s 90 kV_p</th>
<th>f_s 100 kV_p</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.0850</td>
<td>1.0755</td>
<td>1.0694</td>
</tr>
<tr>
<td>2.00</td>
<td>0.0656</td>
<td>0.0993</td>
<td>0.1325</td>
</tr>
<tr>
<td>2.67</td>
<td>0.0310</td>
<td>0.0533</td>
<td>0.0778</td>
</tr>
<tr>
<td>3.33</td>
<td>0.0155</td>
<td>0.0300</td>
<td>0.0479</td>
</tr>
<tr>
<td>4.00</td>
<td>0.0079</td>
<td>0.0173</td>
<td>0.0301</td>
</tr>
</tbody>
</table>
The variation in the scattered to transmitted counts ratio with different $kV_p$ incident x-ray spectra can be explained by using scatter information obtained from the photon transport computer model. A 90 $kV_p$ incident x-ray spectrum was simulated in the computer model and ~$3.4 \times 10^7$ photon histories were followed through the symmetrical phantom defined in table 2.1, (see section 2.5). Scattered to transmitted counts ratios at $50^\circ$ had values of $2.28 \times 10^{-4}$ ($\pm 6.0 \times 10^{-6}$) and $2.39 \times 10^{-4}$ ($\pm 6.0 \times 10^{-6}$) at 100 $kV_p$ and 90 $kV_p$, respectively when the effect of transmission detector filtration was not taken into consideration. However, when transmission detector filtration, (i.e. 4 mm Cu), was taken into account, R changed dramatically due to a significant change in the transmitted counts recorded, table 5.10.

**TABLE 5.10** Variation in scattered to transmitted counts ratio, R, at 90 $kV_p$ and 100 $kV_p$ incident photon spectra with different transmission detector filtration. The symmetrical phantom, (see table 2.1), was considered in the calculations.

<table>
<thead>
<tr>
<th>Detector filtration Cu (mm)</th>
<th>$R_{100}$</th>
<th>$R_{90}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>$2.28 \times 10^{-4}$ $\pm 6.0 \times 10^{-6}$</td>
<td>$2.39 \times 10^{-4}$ $\pm 6.0 \times 10^{-6}$</td>
</tr>
<tr>
<td>2.33</td>
<td>$0.00372 \pm 1.00 \times 10^{-4}$</td>
<td>$0.0057 \pm 1.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>2.67</td>
<td>$0.00493 \pm 1.33 \times 10^{-4}$</td>
<td>$0.0078 \pm 2.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>3.00</td>
<td>$0.00640 \pm 1.72 \times 10^{-4}$</td>
<td>$0.0106 \pm 2.9 \times 10^{-4}$</td>
</tr>
<tr>
<td>3.33</td>
<td>$0.00820 \pm 2.21 \times 10^{-4}$</td>
<td>$0.0143 \pm 3.9 \times 10^{-4}$</td>
</tr>
<tr>
<td>3.67</td>
<td>$0.01060 \pm 2.84 \times 10^{-4}$</td>
<td>$0.0192 \pm 5.2 \times 10^{-4}$</td>
</tr>
<tr>
<td>4.00</td>
<td>$0.01340 \pm 3.60 \times 10^{-4}$</td>
<td>$0.0253 \pm 6.9 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

It can be concluded that the transmission detector filtration is responsible for the strong dependence of density measurements upon the incident x-ray spectra. Density measurements, however, cannot be taken without transmission detector filtration due to the very high transmitted count rate. This effect is expected to be reduced on density measurements made at the lumbar spine because of the large degree of inherent filtration.
5.9 Experimental evaluation of the effect of multiple scatter

The effect of multiple scatter on density measurements was shown experimentally for the geometry defined in the Compton scatter densitometer by studying the variation in performance when the phantom size was changed. Each phantom was positioned with the "critical volume" at the x-ray beams' intersection point. Photon beam path lengths were measured by considering the transmitted beams' entrance points and their intersection point. A number of soft tissue equivalent materials were carefully positioned on each phantom at the entrance and exit points of both incident photon beams so that the size of the phantom was effectively increased and the effect of phantom size change on density measurements was studied.

Figure 5.16 shows the position of the tissue equivalent materials relative to the incident photon beams. Double distilled water (density 1.00 g/cm³) was inserted in each phantom at the critical volume. According to information obtained from the Monte Carlo program, an increase in phantom size leads to a higher apparent density (see sections 3.3.5, 3.4.5 and 3.5.5).

![Figure 5.16](image-url)  

*Figure 5.16* The experimental arrangement used to evaluate the effect of multiple scatter in bone densitometry. The average beam path length is:

\[
\frac{((OA+OD+OC)+(OB+OC+OD))}{2}
\]
An $80 \text{kV}_p$ incident photon spectrum filtered with 0.33 mm of Cu was used and 4 mm Cu filtration for each transmission detector. The counting duration was 8 seconds with energy windows set either at (30-80) keV or at (35-80) keV on both detectors.

Results are shown in figure 5.17 where $(S_1S_2/T_1T_2)^{1/2}$ is plotted against the "average" photon path beam length shown in figure 5.16.

The larger the average photon beam path length, the greater the apparent density. The horizontal line, figure 5.17, shows the normalized response from the phantom used for a given energy window. An average variation in $(S_1S_2/T_1T_2)^{1/2}$ or density of $\sim 8\%$ per cm of the average photon beam path length is seen which is much higher than a $0.15\%$ per cm due to beam polychromaticity and difference between scattered and transmitted beams path lengths (see section 3.10). The use of a narrower energy window (i.e. (35-80) keV) led to reduced density variation with phantom size change (i.e. $\sim 5.5\%$ per cm of the average photon beam path length).

(ii) The femoral neck phantom - A $90 \text{kV}_p$ incident photon spectrum was used
with 2.7 mm Cu filtration on each transmission detector. Eight seconds duration measurements were taken with energy windows set at (25-90) keV, (30-90) keV and (35-90) keV. A variation between 1.4% and 3% per cm of the average photon beam path length is seen with the energy windows used, figure 5.18.

![Graph showing variation in \(\sqrt{S_1S_2/T_1T_2}\) with femoral neck phantom size change.](image)

**Figure 5.18** Variation in \(\sqrt{S_1S_2/T_1T_2}\) with the femoral neck phantom size change. (a) (25-90) keV, (b) (30-90) keV, (c) (35-90) keV.

**iii) The lumbar spine phantom** - In order to improve the statistical precision of the measurements, a 100 kV\(_p\) incident photon spectrum was used, whereas counting duration and filtration on each transmission detector was the same as in the femoral neck phantom. Due to practicalities related to size and position of the lumbar spine phantom, soft tissue equivalent materials were put only at the entrance point of each incident photon beam. Results are shown in figure 5.19 for energy windows set at (25-100) keV, (30-100) keV and (35-100) keV. A variation of \(~6.5\%\) per cm of the average photon beam path length is seen for photon beam path lengths up to 385 mm, with the energy windows used. Selection of different energy windows does not seem to have a significant effect on the degree of variation in the density measured with phantom size change.

It has been shown in chapter 3 that multiple scatter has an effect on serial bone density measurements if patient dimensions should change. Ideally the computed mcfs
should be used to correct the raw data to take account of these changes. An approach towards this aim has been to study the variation in $m_{efs}$ with the average photon beam path. Equation 3.3 was used to correct the raw data and figures 5.20 to 5.22 show these results using the data from sections 3.3.5, 3.4.5 and 3.5.5. The phantoms studied were not the same as those used in the densitometer but the behaviour of the $m_{efs}$ with phantom size change is sufficient to demonstrate their use.

![Graph](image)

**Figure 5.19** Variation in $\sqrt{S_{1S2/T1T2}}$ with the lumbar spine phantom size change. (a) (25-100) keV, (b) (30-100) keV, (c) (35-100) keV.

![Graph](image)

**Figure 5.20** Variation in $m_{efs}$ with the "average beam path" at $50^\circ$ for the lower forearm phantoms studied.
The change in the mcfs with the average beam path was found to be -2.6%, -1.2% and -1.4% per cm respectively, for the lower forearm, femoral neck and lumbar spine phantoms studied.

Figure 5.21 Variation in mcfs with the "average beam path" at 50° for the femoral neck phantoms studied.

Figure 5.22 Variation in mcfs with the "average beam path" at 50° for the lumbar spine phantoms studied.

5.10 Dosimetric computations

The skin dose can be measured experimentally unlike the dose deposited inside the
patient. Dose delivered to bone marrow, however, is of greater importance than skin
dose since bone marrow tissue is the most radiosensitive tissue of all.

The dose distributed inside the patient was evaluated at three examination sites
using the photon transport computer model. Energy deposition was considered to occur
at the site of secondary electrons production. All events producing secondary electrons
were assumed to contribute to dose. Either photoelectric (i.e. total energy deposited) or
Compton effects thus produced dose. Finally, photons carrying energies less than 10
keV were assumed to undergo a photoelectric interaction. Each phantom was divided
into a (150x150) matrix of 2 mm pixel size and the energy deposited inside each pixel
was evaluated for a number of photon histories followed. To be able to convert the
amount of energy deposited per pixel into dose inside the same pixel, the mass which
corresponded to each pixel had to be found.

(i) Unfiltered incident photon beams

Energy distributions inside the phantoms studied were plotted in three dimensions
using the CRAY 1S and XMP-28 facilities. The incident photon beam was a 100 kV$_p$
source and results shown in figures 5.23 to 5.25 correspond to $\sim 3.5\times10^7$ photon
histories, (skin dose $\sim 4.3$ $\mu$Gy), $\sim 5.0\times10^7$ photon histories, (skin dose $\sim 3.7$ $\mu$Gy) and
$\sim 5.5\times10^7$ photon histories, (skin dose $\sim 3.7$ $\mu$Gy), for the lower forearm, femoral neck
and lumbar spine phantoms respectively.

As seen in figures 5.23 to 5.25 the position and direction of the incident photon
beam are clearly shown by the energy distributed inside each phantom. High energy
values and, thus, doses are extended to a few pixels showing that dose is localised due
to x-ray beam collimation. The outline of the cortical bone is visible and considerably
higher doses exist inside the cortical bone due to its composition and higher density.
Deposited dose decreases proceeding through the same phantom material because of
beam attenuation.

The maximum dose deposited inside the trabecular bone, calculated from the
computer model, is increased by $\sim 40\%$ compared to the dose deposited within the soft
tissue. For the number of photon histories followed, at a scattering angle of ~50° the number of scattered counts recorded vary between ~1700 and ~250 for the lower forearm and the lumbar spine phantoms respectively, and thus, for 1% precision, maximum doses inside the trabecular bone will vary between ~25 μGy and ~120 μGy for the lower forearm and lumbar spine phantoms respectively.

(ii) Pre-patient incident photon beam filtration

In order to evaluate the effect of incident photon beam filtration on the dose distributed inside the patient, a 100 kV<sub>p</sub> incident x-ray spectrum filtered with 0.33 mm of Cu was simulated in the computer model and 1.7x10<sup>7</sup> photon histories were followed through one of the lower forearm phantoms of dimensions (3.5x3.0) cm. Results are shown in figure 5.26. The ratio of the maximum dose deposited inside the trabecular bone to the dose delivered to the skin along the central axis of the incident photon beam, calculated from the computer model, was ~1.32.

Experimental measurements showed that for the same precision the skin dose was reduced by a factor of 5 due to pre-patient photon beam filtration. The dose, therefore, delivered to the trabecular bone for 1% precision will be ~5 μGy. If pre-patient photon beam filtration has a similar effect on the dose distributed inside the lumbar spine phantom, the dose delivered to the trabecular bone for 1% precision is expected to be ~30μGy.
Figure 5.23 Energy (keV) per pixel deposited inside the lower forearm phantom. Z axis x 10
X axis x 10, Y axis x 10

Figure 5.24 Energy (keV) per pixel deposited inside the femoral neck phantom. Z axis x 10
X axis x 10, Y axis x 10
Figure 5.25 Energy (keV) per pixel deposited inside the lumbar spine phantom. Z axis \( \times 10^7 \) X axis \( \times 10 \), Y axis \( \times 10 \)

Figure 5.26 Energy (keV) per pixel deposited inside the lower forearm phantom. The incident photon beam is filtered with 0.33 mm Cu. Z axis \( \times 10^7 \) X axis \( \times 10 \), Y axis \( \times 10 \)
6.1 Conclusions and discussion

A Compton scattering technique can isolate the response of the trabecular bone and measure bone density directly. A method of high accuracy and precision will allow the determination of bone density loss on the same individual over a period of time. The use of radioisotopes, however, limits the clinical use of this technique for detection of osteoporosis because of poor precision. The much higher incident photon beam intensity of x-ray units compared to that of any radioisotope available makes the use of these units attractive in the clinical application of Compton scatter densitometry. The aim of this thesis has been to investigate the possibility of using x-ray tubes as irradiation sources in Compton scatter bone densitometry and the ability of the method to diagnose early osteoporosis.

The use of polychromatic sources requires that effects such as multiple scatter and beam polychromaticity are taken into account. In chapters 2 and 3 the effect of multiple scatter on density measurements was studied using a photon transport computer model. Once the computer model had been validated experimentally, a series of different size phantoms, which represent sites of interest in bone densitometry, were studied.

The effect of multiple scatter is highly geometrical. The size of the phantom and the geometry defined in the experimental arrangement (i.e. x-ray beam collimation, detector collimation and phantom position relative to the incident photon beam) determine the multiple scatter contribution to the signal recorded. In all phantoms studied, scattering angles were determined where the effect of multiple scatter is least important and, thus, at these angles density measurements are affected less by changes in the size or cross-sectional area of the examination site. Scattering angles between 30° (for large cross-sectional area examination sites) and 60° (for small cross-sectional area
examination sites) show a small variation with phantom size change and the signal to noise ratio is high. Hence, it is concluded that scattering angles between 30° and 60° will lead to optimum scattering geometry.

At a scattering angle of 50° the factors that correct for the contribution made by multiple scatter, the $mcfs$, vary between (0.46-0.53), (0.45-0.50) and (0.35-0.45) respectively, for size changes in the lower forearm, femoral neck and lumbar spine phantoms studied. Hence, unless the effect of multiple scatter is taken into account significant errors in experimentally measured bone densities will occur. The variation of multiple scatter with phantom size change is smaller at large examination sites and, therefore, density measurements on the femoral neck and the lumbar spine will be affected less by patient size changes. It has been shown that there is only a small dependence of the $mcfs$ on the energy spectra used.

Correction factors due to beam polychromaticity and the difference between transmitted and scattered photon path lengths ($bcfs$) were calculated for the same phantoms as described in the computer model. At 50° $bcfs$ vary between (1.031-1.034), (1.044-1.057) and (1.065-1.075) for size changes in the lower forearm, femoral neck and lumbar spine phantoms, respectively. The variation in $bcfs$ with phantom size change is much smaller compared to the variation of the $mcfs$ and, thus, it is concluded that, $bcfs$ are not important in clinical applications.

In the current work it has been shown that Monte Carlo studies enable a more realistic approach to the problem of multiple scatter to be taken. Assumptions related to the geometry of the phantoms studied and the importance of photon interactions can be considerably reduced. Since multiple scatter is highly geometrical, differences in the irradiation geometry will have an effect on the accuracy of the densities determined.

In future work, the range of the sizes of the phantoms studied should be extended to enable the evaluation of the effect of multiple scatter on density measurements even when large differences in patient sizes exist.

Information obtained from the photon transport computer model was used for the construction of the x-ray Compton scatter bone densitometer and density measurements were made on three anthropomorphic phantoms with precision of ~2%. The performance of the Compton scatter densitometer and the factors which affect the
precision of the density measurements were evaluated. X-ray spectra between 85 kV\textsubscript{p} and ~100 kV\textsubscript{p} were used in all density measurements. Examination sites with large cross-sectional areas required higher energy x-ray spectra to compensate for the increased photon attenuation.

The statistical precision of the measurements is determined by the number of scattered counts recorded which depends upon the examination site, the intensity of the incident photon beam and the detector collimation. Better precision was achieved with smaller size phantoms due to reduced statistical fluctuations.

The use of different detector colimators had a significant effect on the precision of the measurements. The use of multihole focussing detector collimators gave better precision than single, fine hole collimators because of a significantly increased scattered count rate and, therefore, these collimators are more likely to be used in clinical applications.

It was shown that an increase in the energies of the incident photon spectra reduced the sensitivity of the measurements although the statistical precision improved. Restricting measurements to narrow energy windows reduced both the sensitivity and the precision of the method because of significantly lowering the scattered count rate.

Density measurements taken with pre-patient photon beam filtration showed that this had only a small effect on the precision of the measurements whereas the dose given to the patient was considerably decreased. It is, therefore, likely that pre-patient beam filtration will be used in clinical applications.

In the current work all density measurements were made with 8 seconds exposure duration. For future work longer exposures at low tube currents would improve the precision of the measurements. This, however, will require some modification of the x-ray units since maximum exposure duration is limited to 10 seconds. Also, the use of finer multihole focussing detector collimators will reduce the scatter contribution from the surrounding tissues and, thus, improve the sensitivity of the system.

It was shown that the stability of the x-ray tube output (i.e. energy spectra) is a limiting factor to overall precision. A small change in the kV\textsubscript{p} settings has a significant effect on density measurements when transmission detector filtration is used. This filtration is necessary because of the very high transmitted count rate. The implication of
this is that large examination sites are expected to be affected less by the stability of the x-ray tube output due to the need for reduced transmission detector filtration. In future work, monitoring the output of each x-ray tube during the density measurements may be necessary to correct for energy spectra variations. Failure to correct for this effect will require the use of very stable performance x-ray units.

Short duration exposures and the variation of the x-ray tubes output precluded taking trabecular bone density measurements with precision better than 1% which is required for the diagnosis of early osteoporosis. However, the superiority of using x-ray tubes as irradiation sources is seen since bone densities were measured at low dose with 2% precision.
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