# On the performance of priority rules for the stochastic resource constrained multi-project scheduling problem


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## Abstract

The majority of research studies the resource constrained multi-project scheduling problem in a deterministic environment, regardless of the uncertainty nature of the environment. In this paper, we assume that the activity duration is a stochastic variable, and propose two new robustness measures to analyse the performance of priority rules under a stochastic environment. A full factorial experiment is designed to solve the problem and investigate the relationship between project characteristics and the performance of priority rules. Furthermore, a trade-off relationship between the quality and robustness is investigated and the best priority rules are recommended from both a project and portfolio manager's perspective.



*Keywords:* Multi-projects scheduling; Priority rules; Stochastic duration; Robustness


## 1. Introduction

The *resource-constrained multi-project scheduling problem* (RCMPSP) has been a research topic for decades, emerging in a wide variety of problem types and solution procedures. In a deterministic environment, the RCMPSP generates a baseline schedule with the aim of optimizing the performance of a set of projects under the constraints of precedences and resources. However, due to the dynamic nature of the real-world environment, project scheduling may be subject to considerable uncertainties. For example,


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activities may take longer or shorter than initially expected, resources may become temporarily unavailable, new activities may have to be included or dropped, etc. These disruptions cause the deviation of the actual execution from the original planned schedule and may lead to undesirable side-effects, such as having to change agreements with subcontractors, accumulating inventory costs, dealing with employee malcontent, etc. Furthermore, in contrast with single project scheduling, resources are shared by multiple projects in a portfolio, and a small disruption in one activity may result severe impacts on the performance of the entire portfolio.almost 95% of the time is spent on revising baseline schedules due to uncertainty changes. Therefore, effectively dealing with these uncertainties is becoming an important challenge for project and portfolio managers. Based on the survey of [17], almost 95% of the time is spent on revising baseline schedules due to uncertainty changes. According to [37], up to 90%, by value, of all projects are carried out in the multi-projects context, that even a small improvement in management would bring enormous benefits. Although there has been a lot of research on uncertainty in single project scheduling, for the multi-project case, however, it is rather scarce. Thus, in this paper, we concentrate on the stochastic version of the RCMPSP and assume activity duration as a random variable following certain probability distribution.

Generally, two basic strategies are classified to solve the RCMPSP. Firstly, all subprojects can be aggregated into a single large project with a single start and finish node, effectively reducing the problem to the traditional RCPSP [18]. This method, however, possess obvious drawbacks [10]. The major disadvantage is that a lot of detail regarding the individual projects is lost. Moreover, the method implicitly assumes that the delay penalties are identical for all projects in the portfolio which is rather unrealistic [25]. Alternatively, the sub-projects can be considered separately, each having a distinct start and finish node that we call it the *multi-project* (MP) method. It has been noted that the MP method has a lot more potential for improvement [19]. Hence, the MP approach will be used in this research.

For the MP method, exact and heuristic approaches are two general methods used to solve the problem. Because of the NP-hard property of the RCMPSP [29], exact methods are limited to solving small problem instances [9, 13, 38, 45, 19]. Meta-heuristics, such as, genetic algorithms [18, 24], simulated annealing [7], tabu search [14], particle swarm optimisation [30] as well as combinations of traditional meta-heuristics [8], improve the efficiency of the performance, but often need a huge computational efforts to solve problems. Therefore, priority rule based heuristics are thus frequently suggested, which are also the focus of this research. These techniques have a number of advantages when compared to the meta-heuristic solution techniques. Firstly, these techniques are generally computationally inexpensive [22], and can therefore be applied to large scale scheduling problems. Because of this, they are frequently used in commercial scheduling software [19]. Secondly, priority rules do not require an explicit schedule to be formulated and adhered to, and only a preference ordering of the activities is required in combination with a mechanism that iterates over the activities [4]. Thirdly, these priority rules add little to no overhead when applied in stochastic environments since they do not require explicit rescheduling and are able to cope with changes in the expected durations.

Taking advantage of the successful application of the robustness concept in resource-constrained project scheduling problem to address uncertainty [20], we utilize this robustness concept in this paper and measure the stability of the baseline when encountering uncertain events. 20 priority rules taken from [4] are adopted and testified for the stochastic version of the RCMPSP under two kinds of objectives: quality measures and robustness measures. Each measure is classified to project lateness and portfolio lateness. An extensive simulation based experiment is designed to demonstrate the performance of the 20 priority rules. Based on the results, we analyse the influence of project characteristics and the relationship between the two kinds of objectives. Recommendations are given for both project and portfolio managers based on different project characteristics and uncertainty levels.

The remainder of this paper is organized as follows. In the next section, we provide an overview of the origin and state of the solution methodologies for the RCMPSP. Section 3 presents the basic RCMPSP model and the two kinds of objective functions. In section 4, solution procedures are given. Computational experiment and results are presented in section 5 and 6. Section 7 concludes the paper and offers some suggestions for future research.

## 2. Literature review

In this section, the literature is reviewed first for a deterministic environment and then from an uncertainty perspective.

### *2.1. Priority rules for deterministic scheduling*

There is abundant of literature on priority rules for deterministic environment, however, none of a single priority rule performs the best under every instances. Therefore, researches are mainly focusing on investigating the relationship between priority rules and problem characteristics, including resource utilization, precedence constraints, due date setup and objective measures [36, 12]. [16] was the first discussing the modelling of a complete multi-project scheduling system and proposing methods for assigning due dates to incoming projects and priority rules for sequencing individual jobs. Three efficiency measurements, project slippage, resource utilization and in-process inventory are considered to minimize the total project delay. The most important conclusion of this work is that the priority rule *Minimum Slack* (MINSLK) obtains the best efficiency with the three response variables. [28] study the multi-resource problem and develop an efficient resource price based priority rule that obtain good performance with the objective of minimising weighted tardiness cost. [31] develop a multi-criteria heuristic to improve resource allocation for both time-related and time-unrelated criteria. [23] extend the classical RCMPSP to a joint resource-constraint case. A resource transfer cost is incurred when resource is removed from one project and reassigned to another, or from one job to another within the same project. The objective is minimising the multi-project duration for the single-project approach or the mean project duration for the multi-project approach. A priority rule based solution procedure is proposed and obtain good results, especially the rule based on resources, for example, the *Maximum Total Work Content* (MAXTWK).

Some authors generate new priority rules through the combination between existing ones in literature. [41] propose two new rules, the minimum *Critical Ratio first* (CR) and the minimum weighted *Latest Start time and Scheduling Activity time first* (LSSA), where LSSA is formed by combining the *Minimum Latest Start Time* (MINLST) [35] and the *Shortest Activity from Shortest Project* (SASP) [26]. Through comparison with 15 existing rules, the results show that the LSSA and CR rules rank first and second, respectively. [32] divide the MAXTWK rule into two parts according to project and activity sorting criteria, and by varying the activity part with other rules thus obtaining a new two-phase priority rule. The new proposed rules outperform the classical ones such as the MAXTWK and the SASP. [33] combine the two activity costing methods of [28] with resource pricing schemes, obtaining three Bottleneck Dynamic (BD) priority rules, *BD with Myopic activity costing* (BD-MC), *BD with Global activity costing and Uniform resource pricing* (BD-GC-U) and *BD with Global activity costing and Dynamic resource pricing* (BD-GC-D).

It should be noted that the rules used in above literature are mostly based on single projects. They are, however, somewhat unsuitable for multi-projects. Therefore, some authors propose new measures that can better reflect the nature of the multi-project characteristics. For example, [26] provide a categorisation process for existing heuristic rules using two measures including the *Average Resource Load Factor* ($ARLF$) and the *Average Utilisation Factor* ($AUF$). 6 new priority rules are proposed and the SASP and the MAXTWK rule show to be the best ones in minimizing the project delays. Furthermore, [27] and [25] extend the objective to include unequal project delay penalties and devise four new penalty related rules, namely the *Maximum Duration and the Penalty* (DURPEN), the *Maximum Penalty* (MAXPEN), the *Maximum Total Duration Penalty* (MAXTOP), and the *Slack and Penalty* (SLKPEN). They conclude that the priority rule MAXPEN performs best for minimizing the sum of the project weighted delays.

[3, 4] analyse the drawback of the parameters for a significant different case and propose two new measures named the *Normalized ARLF* ($NARLF$) and the *Modified AUF* ($MAUF$) to better reflect the resources distribution among activities. Based on the new problem characteristics, a comprehensive experiment involving 12,320 test problems is conducted to investigate the performance of 20 priority rules. Recommendations for the best priority rules are given from both project and portfolio managers' perspective. The priority rules *Minimum Worst Case Slack* (MINWCS) and the priority rule that combines the MAXTWK and the MINLST rules in a two-phased *Total Work Content & Latest Start Time* (TWK-LST) rule show their superiority to other rules in general. [5] extend the problem to highly iterative (cyclical) projects. Experimental results show that the best priority rules for iterative project portfolios differ significantly from those for acyclical ones, and that the best priority rules at the project level differ from those at

the portfolio level. The best rule depends on project and portfolio characteristics such as network density, iteration intensity, resource loading profile, and amount of resource contention. In particular, by amplifying the effects of iteration, high-density networks hold dramatically different implications for iterative projects. However, only small instances with 20 activities are reported.

[44] propose an easy and quick learning process to determine the best priority rule for each instance regardless of previous knowledge about the instance types and applied for any instance size. The analysis was carried out with 34 popular priority rules in 26 benchmarking problems. It is demonstrated that the selection of the most appropriate priority rule is extremely relevant to the instance, even when any meta-heuristic is used to solve the problem. However, it consumes a lot of computational time. [6] incorporate ten different priority rules to variable neighbourhood search algorithm. The experiment shows that the widely advocated rules such as MINSLK, SASP and LST do not perform well, while the Random Selection Rule (RSR) emerges the best.

### *2.2. Priority rules for stochastic scheduling*

In contrast with the research in deterministic environment, the literature on stochastic RCMPSP is relatively scarce. [48] examines the performance of 13 dispatching rules for executing a resource-constrained project whose estimated activity durations may differ from the actual activity durations. The dispatching rules are tested in environments characterized by three factors, namely, the order strength of the precedence relationship, the level of resource availability and the level of estimation errors in the activity durations. The results show that project environment affects only the performance differences but not the grouping of the best dispatching rules. Taking from actual firm data, [42] develop a heuristic scheduling and control model for the RCMPSP. The method in [26] is used to select the appropriate priority rules and an update routine is proposed to monitoring the activities during execution. The method can be applied to any multi-project scheduling problem.

Besides activity duration, some other types of uncertainties are also studied, for example, dynamic arrival of project. [49, 50] study the synthetic impact of resource allocation rules, resource transfer rules and activity scheduling rules on the performance of project mean flow time, tardiness, and lateness within a dual-level management structure. A central resource pool manager assigns resources to projects, whereas each project manager schedules jobs within his/her project using the allocated resources. [15] examined the performance of five resource allocation heuristics and four strategies to assign due dates to the projects with dynamic project arrival. The computational experience shows that the priority rule *First Come First Served* (FCFS) with the strategy *Scheduled Finish Time Due Date* (SFTDD) rule is the best algorithm for minimising the mean completion time, the mean lateness, the standard deviation of lateness and the total tardiness. [11] also prove its effectiveness by comparing it with a pre-emption priority rule derived based on the critical chain approach. [2] design a computational experiment based on the work of [15]. Their purpose is to find the combination of due date rule, scheduling heuristic, and pre-emption policy that performs best in minimizing mean flow time, mean absolute lateness, and weighted lateness. This work shows that the priority rules FCFS and MINSLK with the *Due Date* (DD) rule obtain the best performance. [1] extend the situation further by examining the impact of the *Learning, Forgetting, and Relearning* (LFR) on project completion time when pre-emption is allowed. Different scheduling, pre-emption and resource reassignment rules are tested. The results show that the worst performing rules are those that attempt to maintain high resource utilization and the best performing rules are based on activity criticality and resource learning. [34] develop a nonlinear mixed-integer programming model for simultaneously planning, scheduling and managing multiple projects where activity duration and resource requirements are uncertain. To solve the problem, the *Earliest Start Time* (EST) and *Most Total Successors* (MTS) rules are used to optimize the resources and schedules respectively.

By utilizing the priority rule in heuristic algorithms, [46] propose a strategy utilising existing efficient priority rules to reduce the solution space for the stochastic RCMPSP. A Markov decision processes model is constructed to represent the uncertainty of available resources and dynamic programming is used to find the suboptimal strategy to minimise the expected total tardiness penalty. The results show that the *Cost Over Time* (COVERT) and the *Apparent Tardiness Cost* (ATC) rules generate good performance in most cases, and COVERT performs better in large scale case. [51] discuss the RCMPSP with project priorities and schedule robustness under uncertain activity durations. A discrete bi-objective decision model is formulated to solve the problem. The results show that the problem parameters indeed have evident impacts on the robustness and makespan of projects. However, all project characteristics that are used are taken from single project scheduling field.

## 3. Problem definition

### *3.1. Basic RCMPSP model*

The basic RCMPSP consists of a set of projects $l$, ($l$=1,2,...,$L$), each of which has a set of activities $i$, ($i$=1,2,...,$N_l$) that is constrained by precedence relations within its own project and renewable resources within multi-projects. Pre-emption is not allowed. We assume the presence of a set of renewable resource types $K_l$ with availability $R_k$ in project $l$ ($k$=1,2,...,$K_l$), and activity $i$ in project $l$ requires a per-period amount $r_{ilk}$ of resource type $k$. The duration of activity $i$ is characterised by $d_{il}$. A feasible schedule operating within this environment has to satisfy a number of constraints:

$$s_{il} + d_{il} \leq s_{jl}, \forall j \in S_{il}, \forall i \in N_l, \forall l \in L \tag{1}$$

$$s_{0l} = 0, \forall l \in L \tag{2}$$

$$\sum_{\forall l \in L} \sum_{\forall i \in A_t} r_{ilk} \geq R_k, \forall k \in K_l, \forall t \in T \tag{3}$$

where $s_{il}$ represents the starting time of activity $i$ of project $l$, $S_{il}$ represents the set of successors for activity $i$ of project $l$, $A_t$ the set of activities in progress at time $t$ and $T$ represents the maximum time horizon. Equations 1 and 2 ensure that the precedence relationships of the projects are respected (the first activity of a project is always a dummy activity). The resource limitations in time $t$ are enforced by Equation 3, which aggregates the resource demands of the various activities and projects.

### *3.2. Stochastic activity duration*

One of the key objectives of this research is to compare the robustness of priority rules in a stochastic environment. Hence, the duration of activities is specified as a stochastic distribution rather than a deterministic estimation. As common practice in stochastic project scheduling, triangular distributions were used to represent the activity duration: $d_{il} \sim \mathit{Triangular}(a, b, c)$, where $a$ is the shortest (optimistic) duration, $b$ is the longest (pessimistic) duration and $c$ is the most likely duration [47]. The parameters of the triangular distributions used for the activity durations are summarised in Table 1 and visualised in Figure 1. Five different risk levels are used to represent different degrees of uncertainty with a maximum deviation ($DEV_{max}$) ranging from 10% to 50%. All the distributions are skewed to the right to represent the generally higher potential of delays rather than decreases in activity duration. To make sure that activity duration do not deviate too much, we have multiplied $DEV_{max}$ by 0.5 and 1.5, respectively. The optimistic and pessimistic duration ($a$ and $b$) are thus calculated based on the following two equations:

$$a = c - 0.5 \cdot c \cdot DEV_{max} \tag{4}$$

$$b = c + 1.5 \cdot c \cdot DEV_{max} \tag{5}$$

Table 1: Risk levels for the triangular distribution

| $RL$ | $DEV_{max}$ | $a$ | $c$ | $b$ |
|---|---|---|---|---|
| 1 | 10% | $0.95c$ | $c$ | $1.15c$ |
| 2 | 20% | $0.90c$ | $c$ | $1.30c$ |
| 3 | 30% | $0.85c$ | $c$ | $1.45c$ |
| 4 | 40% | $0.80c$ | $c$ | $1.60c$ |
| 5 | 50% | $0.75c$ | $c$ | $1.75c$ |

### *3.3. Optimization objectives*

Two optimisation objectives are taken into account: quality and robustness. Conceptually, quality is a measure of the time needed to complete the (sub-)project(s). Robustness on the other hand is a measure of how consistent the performance of a solution method is when faced with stochastic activity durations.

Two metrics are defined for each of the objectives in order to take into account different perspectives when optimising multi-projects. The first perspective is typically

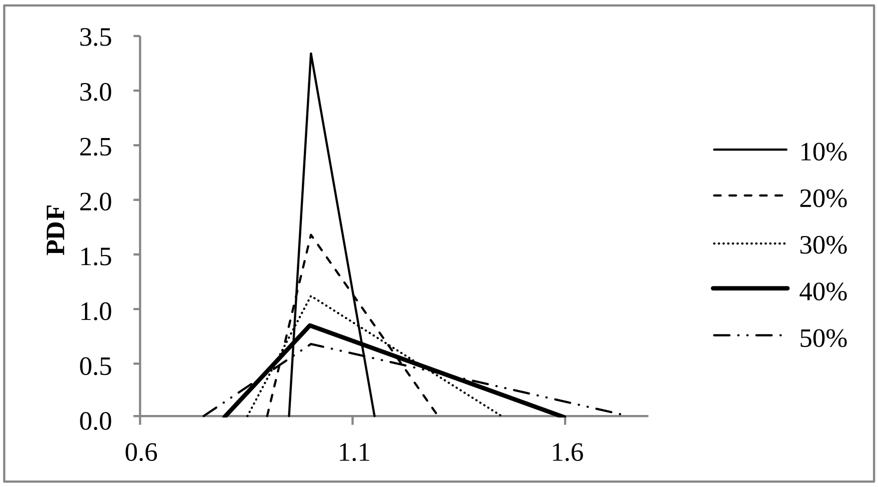


Figure 1: Probability density function for triangular distributions

that of the project manager who is responsible for a single project within this portfolio and wants to ensure that individual projects can perform adequately. The alternative perspective is that of the portfolio manager accountable for the complete portfolio who is more concerned with the performance and robustness of the complete set of projects and who is perhaps more willing to sacrifice the performance of individual projects in order to improve the performance of the portfolio.

### *3.3.1. Schedule quality*

Similar to the approach taken by [4], the critical path duration of the subprojects ($CP_l$) is used as a point of reference to calculate the quality metrics. This critical path duration is compared to the actual duration ($AD_l$) of the subprojects. Two quality measures $Q1$ and $Q2$ are defined as follows:

$$Q1 = \frac{1}{|L|} \sum_{l \in L} \frac{AD_l - CP_l}{CP_l} \tag{6}$$

$$Q2 = \frac{\max_{l \in L} AD_l - \max_{l \in L} CP_l}{\max_{l \in L} CP_l} \tag{7}$$

The first metric ($Q1$) measures the average relative delay of the individual sub-projects, weighing the importance of each project's delay equally. The second metric ($Q2$) on the other hand focusses on the performance of the aggregated multi-project by measuring only the longest critical path (i.e. the shortest possible duration of the multi-project) versus the actual duration of the execution of all projects.

### 3.3.2. Robustness

Analogously to the quality metrics, two robustness metrics are introduced. These metrics compare the deterministic performance (using the $c$ value of the triangular distribution as the activity duration) to the performance as observed when simulating stochastic activity durations. For each project, $AD_l$ represents the deterministic project duration and $SAD_l$ represents the average project duration derived from Monte Carlo simulations with random activity durations. Using these two quantities $R1$ and $R2$ are introduced as robustness metrics:

$$R1 = \frac{1}{|L|} \sum_{l \in L} \frac{SAD_l - AD_l}{AD_l} \tag{8}$$

$$R2 = \frac{\max_{l \in L} SAD_l - \max_{l \in L} AD_l}{\max_{l \in L} AD_l} \tag{9}$$

The first metric ($R1$) measures the average relative deviation of the simulated subproject duration compared to the expected project duration when using deterministic estimates for the duration of the activities. Again, each of the subprojects is weighted equally. The second robustness metric ($R2$) focusses on the robustness of the complete multi-project, rather than the robustness of the individual subprojects. This measure examines how the multi-project duration evolves with stochastic activity duration.

## 4. Solution procedure

Priority-rule-based heuristics are made up of two components, a *schedule generation scheme* (SGS) and a *priority rule*. In this section, we first describe how priority rules are used in order to obtain a scheduling solution (section 4.1) and then give an overview of all the priority rules tested in this research (section 4.2).

### 4.1. Schedule generation scheme

[21] has proposed two methods for creating schedules based on priority lists: the serial and parallel schedule generation scheme. The former calculates activity priorities and then iterates over the activities allocating the activity with the highest priority at the earliest possible time. The parallel schedule generation scheme on the other hand iterates over the time periods and if necessary recalculates the priorities of the activities as the scheduling progresses.

In line with the majority of proceeding research on the multi-project scheduling problem [4], this research uses the parallel schedule generation scheme. The reason for this is that this approach generally outperforms the serial schedule generation scheme or the combination of the serial and parallel scheduling schemes when used in multi-projects, especially when activity scales are beyond 300 on average [32]. Moreover, this technique is also better adjusted for operational use in a stochastic setting where the actual schedule is created organically as the project progresses and it may be more important to update the respective activity priorities as the project progresses.

*4.2. Priority rules*

A lot of priority rules for both single and multi-project scheduling have been proposed in literature. For this research, we use the same set of twenty priority rules as was used in the research by [4]. For the reader's convenience these priority rules are listed in Table 2. The second column takes advantage of the $\alpha/\beta/\chi/\delta$ classification of the priority rules in [22]. But differently, $\alpha$ has the value $S$ or $M$ for a single- or multi-project respectively. $\beta$ indicates whether the information used to calculate the priority is linked to the activity ($A$), project ($P$) or resources ($R$). $\chi$ represents the use of static or ($S$) or dynamic ($D$) priority rules. Finally, $\delta$ indicate if a priority rule is local ($L$) or global ($G$).

Table 2: Priority rules designed for the stochastic RCMPSP

| **Priority Rule** | **Classification** | **Description** |
|---|---|---|
| 1.MINSLK: Minimum slack | S,M/A/D/G | $\min(SLK_{il})$ with $SLK_{il} = LS_{il}$ - $\max(ES_{il},\ t)$. Where $ES_{il}$ and $LS_{il}$ represent the earliest and latest start time respectively and $t$ is the current time period |
| 2.MAXSLK: Maximum slack | S,M/A/D/G | $\max(SLK_{il})$ |
| 3.SASP: Shortest activity from shortest project | S,M/A,P/S/G | $\min(f_{il})$, with $f_{il} = CP_l + d_{il}$. |
| 4.LALP: Longest activity from longest project | S,M/A,P/S/G | $\max(f_{il})$ |
| 5.MINTWK: Minimum total work content | S,M/A,R/D/G | $\min\ (\sum_{k=1}^{K} \sum_{i \in AS_l} d_{il} r_{ilk} + d_{il} \sum_{k=1}^{K} r_{ilk})$ with $AS_l$ the set of already planned activities of project $l$ |
| 6.MAXTWK: Maximum total work content | S,M/A,R/D/G | $\max(\sum_{k=1}^{K} \sum_{i \in AS_l} d_{il} r_{ilk} + d_{il} \sum_{k=1}^{K} r_{ilk})$ |
| 7.TWK-LST: MAXTWK & earliest late start time (2 phase rule) | S,M/A,R/D/G | Use min $(LS_{il})$ as a tie breaker for the MAXTWK rule. |
| 8.TWK-EST: MAXTWK & earliest early start time (2 phase rule) | S,M/A,R/D/G | Use min $(ES_{il})$ as a tie breaker for the MAXTWK rule. |
| 9.FCFS: First come first serve | S/A/S/G | $\min(ES_{il})$ |
| 10.SOF: Shortest operation first | S/A/S/L | $\min(d_{il})$ |
| 11.MOF: Maximum (longest) operation first | S/A/S/L | $\max(d_{il})$ |
| 12.RAN: Random | | Random selection of activities |
| 13.EDDF: Earliest due date first | S/A/S/G | $\min(LS_{il})$ |

Table 2: (continued)

| Priority Rule | Classification | Description |
|---|---|---|
| 14.LCFS: Last come first serve | S/A/S/G | $\max(ES_{il})$ |
| 15.MAXSP:Maximum schedule pressure | S/A/D/G | max $(\frac{t-LF_{il}}{d_{il}W_{il}})$ with $W_{il}$ equal to the fraction of the duration of activity $i$ of project $l$ still remaining. |
| 16.MINLFT: Minimum late finish time | S/A/S/G | $\min(LF_{il})$ |
| 17.MINWCS: Minimum worst case slack | S/A,R/D/G | $\min(LS_i - \max[E_{(i,j)} \vert (i,j) \in AP_t])$ where $E_{(i,j)}$ is the earliest time to schedule activity $j$ if activity $i$ is started at time $t$, and $AP_t$ is the set of eligible activities at time $t$ [22] |
| 18.WACRU: Weighted activity criticality & resource utilization | S/A,R/S/G | max $(w\sum_{q=1}^{N_i}(1+SLK_{iq})^{-\alpha}+(1-w)\sum_{k=1}^{K}\frac{r_{ik}}{R_{Max,k}})$ where $N_i$ is the number of immediate successors of the $i$th activity, $SLK_{iq}$ is the slack of the $q$th immediate successors of the $i$th activity and both $w$ and $\alpha$ are weights set to 0.5. |
| 19.MS: Maximum total successors | S/A/S/G | max $(TS_{il})$ with $TS_{il}$ the total number of successors of the $i$th activity of project $l$. |
| 20.MCS: Maximum critical successors | S/A/S/G | Max $(CS_{il})$ with $CS_{il}$ the number of critical successors of the $i$th activity of project $l$ |

## 5. Dataset

To test the performance of the various priority rules described in section 4.2, a new dataset has been constructed. This section gives an overview of the nature of this dataset, as well as the manner in which this dataset has been created. Table 3 gives an overview of the parameters which have been used to create the dataset.

The dataset consists of 1,260 projects which are combined into 420 project portfolios (i.e. 3 projects per portfolio: $|L| = 3$). Each of these projects contains 30 activities ($|N_l| = 30$), and 4 different resource types are used in all of the portfolios ($|K| = 4$). The first step in the data generation procedure is the creation of network structures. This is done by using the RanGen2 tool created by [43], which uses the serial/parallel ($SP$) indicator [39, 40] in order to create topologically diverse network

Table 3: Parameters setup

| Variable | Meaning | Value(s) |
|---|---|---|
| $\|L\|$ | Projects in portfolio | 3 |
| $\|N_l\|$ | Activities per project | 30 |
| $\|K\|$ | Resource types | 4 |
| $SP$ | Serial-Parallel indicator | $\{S, M, P\}$ |
| $NARLF$ | Resource loading factor | -3,-2,-1,0,1,2,3 |
| $MAUF$ | Resource conflicts factor | 0.6,1.0,1.4 |
| $\delta^2_{MAUF}$ | Difference in degree of resource conflicts | 0,0.25 |
| $c$ | Most likely activity duration | $U[1, 9]$ |
| $r_{ilk}$ | Resource usage of activity $i$ in project $l$ for resource type $k$ | $U[1, 9]$ |

structures. This indicator is defined as $(m-1)/(N_l-1)$ with $m$ the maximal progressive level of the project network. With $SP = 1$ ($m = N_l$), the activities of the generated network are all in series and with $SP = 0$ ($m = 1$), all activities are in parallel. Three different ranges have been specified: $P \in [0.1, 0.3]$, $M \in [0.35, 0.65]$ and $S \in [0.7, 1.0]$. These ranges indicate projects that are relatively parallel ($P$), serial ($S$) or which have an intermediate structure ($M$). Next, the individual projects are combined into project portfolios, each consisting of three individual projects. To do this, all possible combinations of the $SP$-categories of the projects are taken into account: $\{SSS, PPP, MMM, SSP, SSM, PPS, PPM, MMS, MMP, SMP\}$, resulting in 42 portfolios in each of these categories. Once the structure of the portfolios has been determined, the most likely activity duration $c$ and resource requirement $r_{ilk}$ are iteratively generated from a uniform distribution between 1 and 9 to satisfy specified project characteristics. For generating resources, two indicators have been used. The first parameter is the $NARLF$, that measures possible peaks in resource demand, and the second parameter is the $MAUF$ (two versions are used), which compares the resource demand to resource availability [4].

The $NARLF$ gives insight in how (un)evenly resource demand is spread over the duration of the project. A value of zero indicates that the resource demand is perfectly spread across the duration of the project. A negative value of the $NARLF$ indicates a relative high resource demand at the start of the project, whereas a positive value of the $NARLF$ indicates a relatively high resource demand at the end of the project. Equation 10 shows how the value of the $NARLF$ is calculated.

$$NARLF = \frac{1}{L \cdot CP_{max}} \sum_{l=1}^{L} \sum_{t=1}^{CP_l} \sum_{k=1}^{K_l} \sum_{i=1}^{N_l} Z_{ilt} X_{ilt} \frac{r_{ilk}}{K_{il}} \tag{10}$$

with

$$CP_{max} = \max\{CP_1, CP_2, \ldots, CP_L\} \tag{11}$$

$$Z_{ilt} = \begin{cases} -1 & t \leq CP_l/2 \\ 1 & t > CP_l/2 \end{cases} \tag{12}$$

$$X_{ilt} = \begin{cases} 1 & \text{if activity } i \text{ of project } l \text{ is executed at time } t \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

Whereas the $NARLF$ only indicates how evenly the resources are spread over the duration of the project, the $MAUF$ is an indication of the scarcity of a specific resource. To obtain the $MAUF$ value for the project, the utilisation is first calculated for each individual resource (Equation 14). The resource utilisation of the complete project is then simply calculated as the maximum $MAUF_k$ for the individual resources (Equation 15).

$$MAUF_k = \frac{1}{|T|} \sum_{l=1}^{|L|} \sum_{t=1}^{|T|} \sum_{i=1}^{N_l} \frac{r_{ilk} \cdot X_{ilt}}{R_k} \tag{14}$$

$$MAUF = \text{Max}(MAUF_1, MAUF_2, \ldots, MAUF_K) \tag{15}$$

Because only the maximal utilization factor is taken into account in order to determine the $MAUF$ for the project, some information is lost. To mitigate this, [3] have proposed a new metric which measures to what degree the $MAUF_k$ of the individual resources deviates from the $MAUF$. The metric is denoted as and is calculated as $\delta^2_{MAUF}$shown in Equation 16.

$$\delta^2_{MAUF} = \frac{\sum_{k=1}^{K} (MAUF - MAUF_k)^2}{|K|} \tag{16}$$

A value of zero for the indicator signifies that all the resources are equally scarce. As the value of this metric increases, it indicates that some resources are substantially less restricted than the most restricted resource. Ceteres paribus, this scenario should result in a lower or equal problem delay.

These three parameters ($NARLF$, $MAUF$ and $\delta^2_{MAUF}$) are used to define the nature of the resources within the portfolio. As shown in table 4 these parameters can take seven, three and two different possible values respectively. This results in a total of 42 possible combinations ($7 \cdot 3 \cdot 2$) of these parameters. As mentioned above there are 42 portfolios for each possible combination of serial-parallel indicators for the sub-projects. Hence, for each of these projects one of these 42 unique combinations of the resource parameters is randomly assigned.

## 6. Computational experiments

### *6.1. General results for quality and robustness*

Considering the five uncertainty levels in Table 1, we divide them into two categories for their similar variation tendency, i.e., the lower case with $DEV_{max}$ varying from 10% to 20% and the higher case with $DEV_{max}$ varying from 30% to 50%. Based on this classification, we deliver the quality and robustness trade-off performance of the 20 priority rules from (a) the project manager perspective ($Q1\&R1$) and (b) the portfolio manager perspective ($Q2\&R2$) under the two uncertainty cases, shown in Figure 2 and Figure 3. In each Figure, the $x$-axis represents the quality measure and the $y$-axis denotes the robustness measure. The graph is divided into four quadrants by the average value depicted in dashed line. Ideally, the priority rules in the third quadrant near to the origin as close as possible are the best on both criteria. Rules in the first quadrant located far away from the origin are the worst and the remaining ones are preferable to either the quality or the robustness measure.

Comparing Figure 2 and Figure 3, the performance of the priority rules for the higher uncertainty case is more concentrated near the average value than the lower case, and this is especially true for the portfolio plot where most priority rules are located near the cross point of the quality and robustness average lines and with almost no rule in the third quadrant. This suggests that the beneficial effect of priority rules decrease for portfolio managers when the uncertainty is high. Moreover, it is much easier for project managers to make decisions without the consideration of the uncertain environment and the objective preference. In addition, with the increase of uncertainty levels, the quality improves slightly but the robustness deteriorates to a much larger extent.

For a clear representation, we depict the two Pareto frontiers in Figure 4 based on the lower case, i.e. the $DEV_{max}$ equal to 10% or 20%. Similar results can be obtained for the higher case. The figure shows that priority rules SASP, MINLFT, MINSLK, EDDF, MINWCS and FCFS constitute the Pareto set for the project managers while MS, MCS, LALP, EDDF, MINSLK, MAXSP, MINWCS, MINLFT, FCFS and SASP for the portfolio managers. For both project and portfolio managers, the quality measures are more sensitive than the robustness measures. For example, in case of $Q1$ and $R1$, from the quality best rule (SASP) to the robustness best rule (FCFS), the quality decrease of 27.1% leads to an improvement of 16.2% for the robustness. But for $Q2\&R2$, the MS decreases 62.8% on quality and delivers an improvement of only 14.6% on robustness for the SASP. Therefore, it is advantageous to choose the priority rule based on quality measures.

For the $Q1$ measure in Figure 2(a), it can be seen that the priority rule SASP performs the best among all other rules. The worst rules are MS and MCS. However, the results are the opposite for the quality measure $Q2$. In this case, the MC, MCS dominate other priority rules with obvious distinction. The performance of the best rule SASP now declines sharply and stands for the worst priority rule.

For robustness measure $R1$ in Figure 2(a), the best priority rule is FCFS, and the LCFS performs the worst. For $R2$ in Figure 2(b), the FCFS is ranked as second best and the SASP emerges to be the best priority rule. The MS delivers the worst results.

The superiority of SASP confirms the recommendations of [26], but conflict with the results in [4]. [26] strongly recommend the priority rule SASP and MINSLK, while [4] found that the TWK-LST priority rule performs significantly well. The underlying reason for the conflict may stem from two aspects. First is the uncertainty environment. Since activity durations are random variables, arranging activities as early as possible can avoid the impact of project delays to a certain extent. Therefore, those priority rules that involve early finish time are more appropriate, for example, the SASP, MINLFT and the EDDF. In the above two papers, however, no uncertainty has been incorporated. Second, different data sets are used. Different from the experimental setups in [4], we generated the multi-projects through the combination of single projects based on the problem indicator $SP$. Therefore, it is reasonable to have different results.

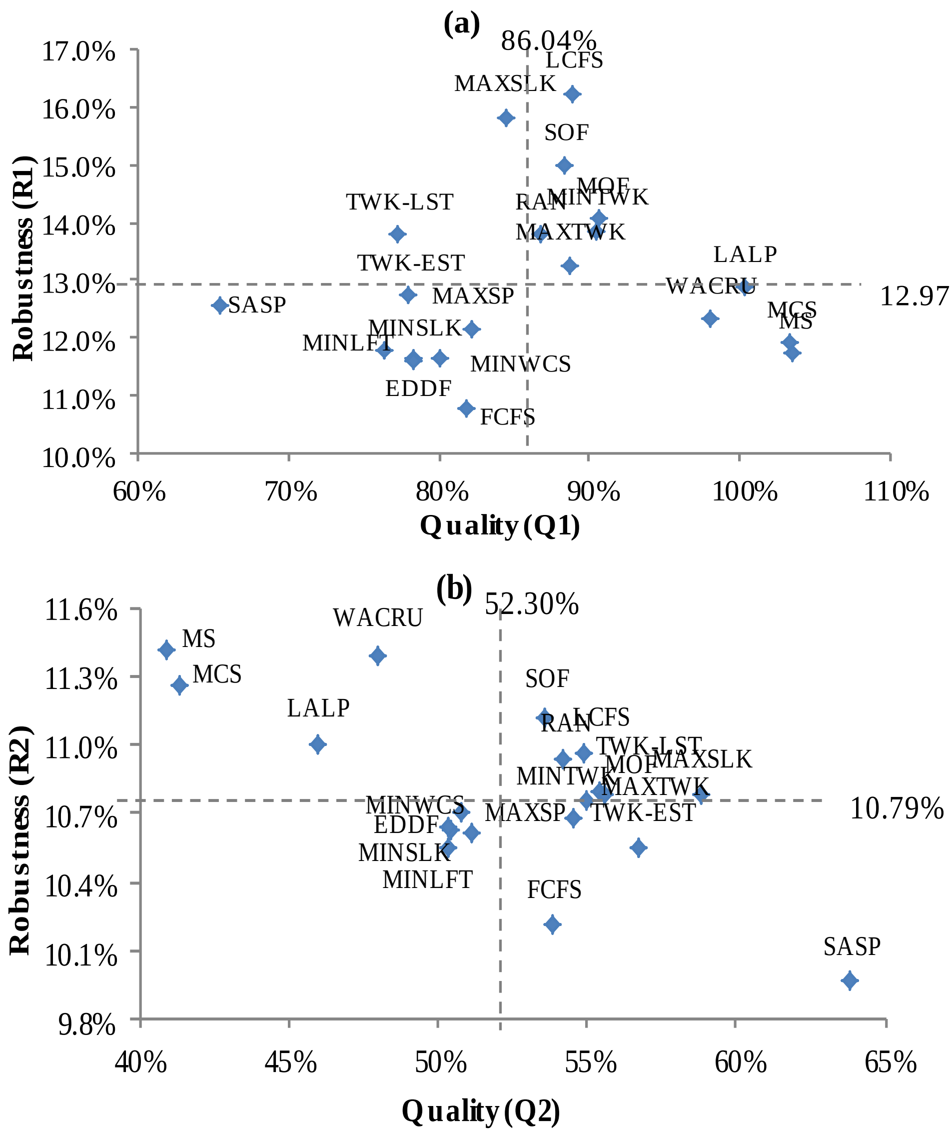


Figure 2: Trade-off relationship between quality and robustness under lower uncertainty levels: (a) $Q1\&R1$ and (b) $Q2\&R2$

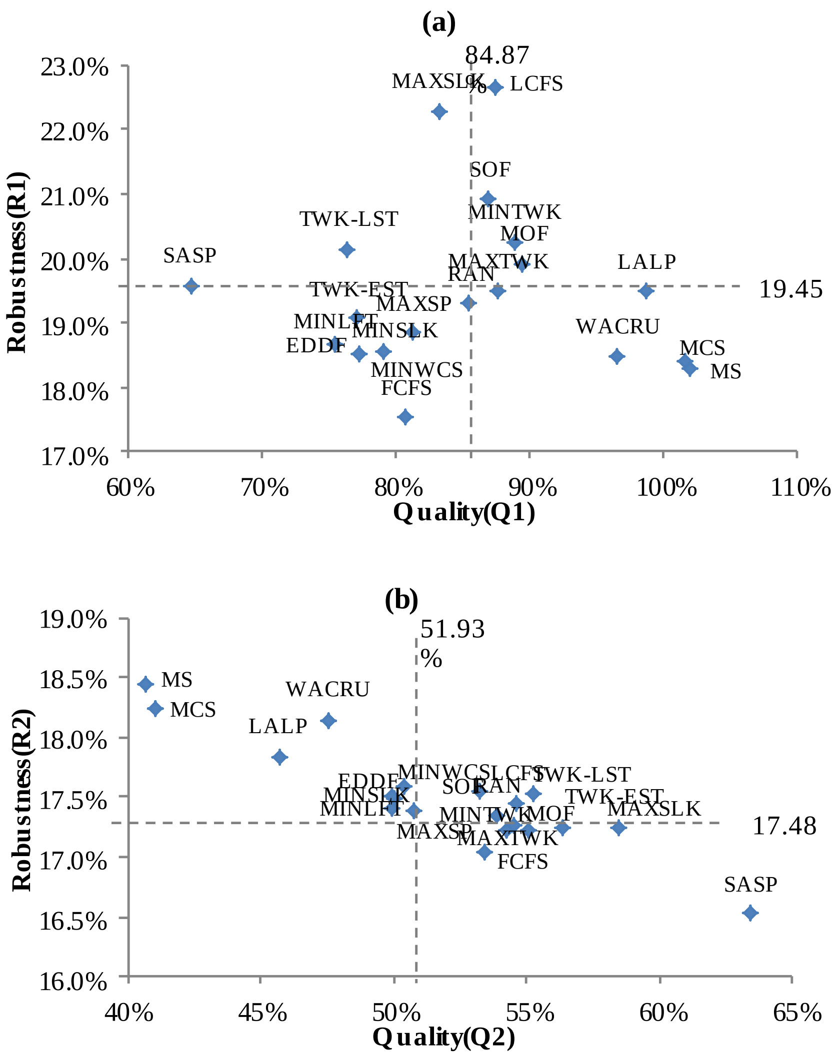


Figure 3: Trade-off relationship between quality and robustness under higher uncertainty levels: (a)$Q1$&$R1$ and (b)$Q2$&$R2$

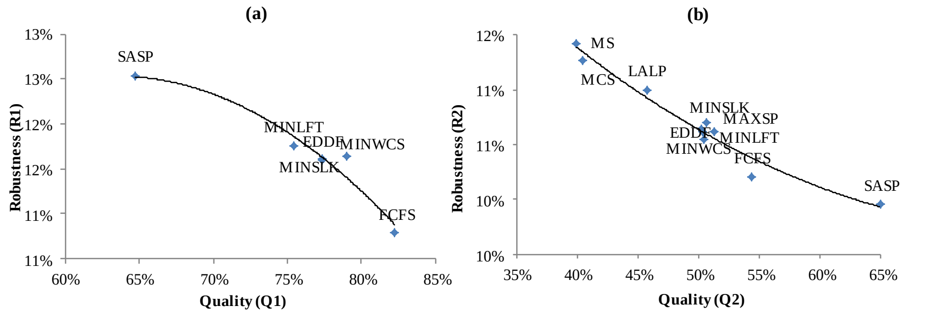


Figure 4: Pareto frontier for the quality and robustness: (a) $Q1\&R1$ and (b) $Q2\&R2$

### *6.2. Sensitivity analysis*

Figure 5 shows the main effects of the project parameters $DEV_{max}$, $SP$, $NARLF$, $MAUF$ and $\delta^2_{MAUF}$ on the performance of the priority rules for both the quality and robustness measures. The $y$-axis on the left are used to measure the quality values, and the right $y$-axis are used for the robustness values. Since the 20 priority rules show a similar variation tendency, we merely take the average as an illustration.

First, the $DEV_{max}$ plot in Figure5(a) indicates that the quality measures $(Q1, Q2)$ are not very sensitive to uncertainty levels. For increasing $DEV_{max}$, the performance improves but with only a slight difference. Contrarily, the performance of the robustness measures deteriorates almost linearly. Since the robustness measures describe the difference between the stochastic and deterministic performance, the higher the uncertainty is, the longer the activity prolongs, and thus a higher project delays.

Second, Figure 5(b) shows the impact of $SP$. It is noteworthy that the smallest $Q1$ is observed for a multi-project that consists of three projects with the same network type. The results deteriorate when two of the three projects are of the same type and the other of a different type. The peak is reached when the two different networks are the most divergent, for example, $SSP$ and $PPS$. This phenomenon can be explained by the following. For a serial network, precedence constraints play a major role, few activities are executed simultaneously and less delay is caused. However, including parallel network types in multi-projects, activities cannot be operated concurrently due to the resource constraints and thus more delayed activities will emerge. Moreover, the critical path for a parallel project is remarkably shorter than that for a serial project. Therefore, including a parallel network actually increases the chance of activity delay for the original multi-project and deteriorates the performance of the quality measure. This effect is even more obvious when two absolutely different networks exist in the multi-project.

For $Q2$, however, a completely different pattern can be observed. The best results, interestingly, are obtained with multi-project that consists of at least one parallel network and no serial network, for example, $PPP$, $PPM$ and $MMP$, and the worst all contains at least one type of serial network, such as $SSS$, $SSP$,$SSM$ and $MMS$. Since

$Q2$ is particularly focused on the longest project, once there is one type of serial network in a multi-project, the maximum actual duration $AD_l$ will be longer than that without serial network, leading to a higher project delay. However, if there are mainly parallel projects in the portfolio, the precedence constraints are relatively relaxed and delays caused by resource constraints will distribute equally over all activities and thus no extremely longer project appear. In addition, the delay of longer projects will be compensated by shorter projects, making the maximum $AD_l$ deviate not too much and deliver better results.

For the robustness measure $R1$ and $R2$, their behaviour is similar to each other, although the effect is more outspoken for $R1$ than for $R2$. However, it is interesting to see that the plots of $R1$ and $R2$ are totally contrary to that of $Q2$. For different levels of $SP$, $Q2$ needs parallel networks to absorb the project delay in longer projects and decrease the longest project duration as much as possible, while $R1$ and $R2$ need to decrease the number of parallel networks in order to avoid the impacts of delay to more succeeding activities caused by parallel networks. Therefore, the plot for $Q2$ will be opposite to the plot for $R1$ and $R2$. In addition, compared with the quality measures that are calculated based on the critical path, the elements $SAD_l$ and $AD_l$ in the robustness measures are both constrained by precedences and resources. Therefore, the influence of resources and precedence constraints is largely weakened and the difference is merely impacted by the uncertainty of activity durations. That is why the variation in robustness is lower than that in the quality measures. Similarly, the same phenomenon can also be found in parameters $NARLF$ and $MAUF$ in Figure 5(c)$\sim$(d).

Third, from the $NARLF$ interaction plot in Figure 5(c), for the quality measures in general, multi-projects with a negative $NARLF$ deliver inferior results to multi-projects with a positive $NARLF$, and $Q1$ improves significantly than $Q2$. Since a negative $NARLF$ indicates that most of the resources are consumed at the front of the project, which will induce more downstream activities to be delayed due to the snowball effect. This finding confirms the observation of [4]. The insensible variation in $Q2$ can be explained by the fact that most delays are absorbed by shorter projects, making the delay in the longest project negligible, for example, a multi-project with two shorter projects and one long project. For the robustness measures, however, there is no obvious distinction found among different levels of $NARLF$. The robustness seems to increase first with a much smaller distinction and then decrease when $NARLF$ is more than 2.

Fourth, Figure 5(d) illustrates the $MAUF$ interaction plot. For both quality measures $Q1$ and $Q2$, a higher $MAUF$ is always accompanied by a poor performance, and the increasing slope become steep when $MAUF$ grows. Comparatively for robustness measures $R1$ and $R2$, although the performance deteriorates likewise for increasing $MAUF$, the slope is decreasing gradually. It is easy to understand that the highly constrained resources lead to more activities to be delayed and therefore, the worse the performance of the priority rules.

Finally, we compare the two levels of $\delta^2_{MAUF}$ shown in Figure 5(e). It is observed that with the increase of $\delta^2_{MAUF}$ , the performance of the quality and the robustness measures improves but robustness measures change little, and the shapes of $Q1$ and $Q2$,

$R1$ and $R2$ are the same. This is logical since a higher $MAUF$ variability indicates that problems are constrained by fewer types of resources, decreasing the impact of delays caused by resource constraints. Therefore, the results will become better.

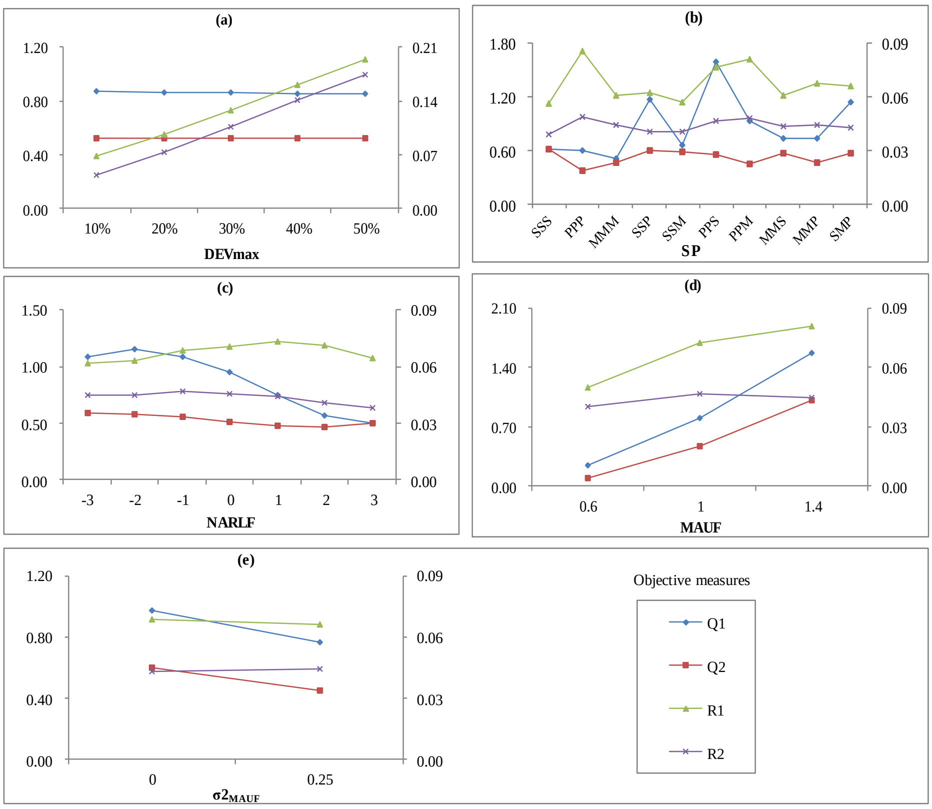


Figure 5: Analysis of parameters on quality and robustness measures

### 6.3. Best recommendation

When choosing an appropriate priority rule, project and portfolio managers should pay more attention on a priority rule that performs well on both the quality and the robustness criteria. In the first section, we have given the set of rules shown on Pareto frontier, but a decision for which one to choose is still a question that needs to be answered. For this reason, we sequence the 20 priority rules based on the average of the four objective measures, shown in Table 4. First, we standardize the original objective value of the priority rule to an integer ranged from 1 to 20 based on its position among the 20 priority rules and then calculate its mean value over all instances, and the results are listed from the second to the fifth column. The column "Avg." and "Std." are obtained by averaging the mean and variance of the four measures with equal weight. The column labelled "Global Ranking" sequences the 20 priority rules from the best to the worst according to the value in the column "Avg.". In order to compare the difference between project managers and portfolio managers, we also give the ranking separately, shown in the columns "Project" and "Portfolio".

From the values under each measure, it can be concluded that no priority rule performs always the best or the worst. This stems from the fact that different characteristic of projects will benefit from different rules. However, sometimes we cannot specifically tell the detailed characteristics of the environment. Therefore, a rule that performs well under most cases is recommended. Comparing the general results with the lower and higher cases in first section, although uncertainty influences the value of objectives, the rank of the best priority rules remain unchanged. In conclusion, the EDDF is the best to recommend on average and it is also performs the best for project managers. For portfolio managers, it is ranked as the fourth best, while the MS is here the clear winner. Since MS performs distinctly better for the $Q2$ (=2.98), it makes the $R2$ (=11.97) insensible when calculating the average of the two measures. However, the SASP rule, although lying on the Pareto frontier, it is ranked the last for the portfolio managers, even worse than the RAN rule. In order to compare the performance of the priority rules statistically, a Tukey's HSD test have been performed to compare the 20 priority rules in a pairwise way in order to find out if significant difference exists between each other (hence, all pairwise comparisons are made). The significance level is set at 5% and the 20 rules are classified into different clusters separated by horizontal lines in Table 4. The priority rules in the same cluster have a similar performance (i.e. no significant differences between them) while the priority rules between clusters have a different performance. As an example, the priority rules in the first cluster (EDDF, MINSLK and MINLFT) for the global ranking index all perform equally well, and perform better than all the other rules belonging to other clusters. Hence, priority rules in the first cluster perform better than rules in the second cluster, and the rules of the second cluster perform better than rules in the third cluster, and so on, but with each cluster, each rules performs equally well.

### *6.4. Additional experiments*

In the previous experiments, the results were obtained by experiments using triangular distributions with 5 levels of risk, as shown in Table 1. In this section, a short summary is given why these 5 risk levels and triangular distributions have been used by extending the set of experiments and conclude that the results do not vary significantly.

#### *6.4.1. More risk levels*

It should be noted that the 5 risk levels have been chosen after some initial experiments with more extensive $DEV_{max}$ values, ranging from 10% to 180% in steps of 10% (i.e. 18 levels of risk rather than 5). However, our experiments showed that the

Table 4: Average sequence of the 20 priority rules

| Rule | $Q1$ | $Q2$ | $R1$ | $R2$ | Avg. | Std. | Global Ranking | Project | Portfolio |
|---|---|---|---|---|---|---|---|---|---|
| FCFS | 11.51 | 12.55 | 6.61* | 9.18 | 9.96 | 4.57 | EDDF | EDDF | MS |
| SOF | 12.78 | 12.11 | 13.26 | 10.98 | 12.28 | 5.36 | MINSLK | MINLFT | MCS |
| MOF | 12.58 | 12.93 | 11.94 | 10.25 | 11.92 | 5.42 | MINLFT | MINSLK | LALP |
| MINSLK | 6.55 | 7.54 | 8.48 | 10.12 | 8.17 | 4.36 | MINWCS | SASP | EDDF |
| MAXSLK | 12.13 | 15.31 | 13.42 | 10.23 | 12.77 | 6.09 | MAXSP | MINWCS | MINSLK |
| SASP | 4.84* | 18.52 | 10.41 | 8.75* | 10.63 | 4.74 | MS | FCFS | MINLFT |
| LALP | 13.64 | 6.28 | 10.99 | 11.03 | 10.48 | 5.61 | MCS | TWK-LST | MINWCS |
| MINTWK | 13.37 | 13.15 | 12.21 | 10.49 | 12.31 | 5.45 | FCFS | MAXSP | MAXSP |
| MAXTWK | 11.86 | 12.18 | 11.15 | 10.14 | 11.33 | 5.31 | LALP | TWK-EST | WACRU |
| RAN | 13.07 | 12.76 | 13.78 | 11.49 | 12.77 | 3.05** | TWK-LST | MS | FCFS |
| EDDF | 6.31 | 7.34 | 8.37 | 10.13 | 8.04** | 4.34 | SASP | MAXTWK | MAXTWK |
| LCFS | 10.35 | 12.43 | 14.07 | 10.70 | 11.89 | 6.23 | WACRU | WACRU | SOF |
| MAXSP | 8.33 | 8.76 | 9.82 | 10.20 | 9.28 | 4.73 | TWK-EST | MCS | LCFS |
| MINLFT | 5.98 | 7.94 | 8.75 | 10.10 | 8.19 | 4.38 | MAXTWK | LCFS | MOF |
| WACRU | 13.68 | 7.59 | 9.60 | 11.69 | 10.64 | 5.28 | LCFS | MOF | MINTWK |
| TWK-LST | 7.30 | 13.50 | 10.84 | 10.42 | 10.51 | 5.31 | MOF | LALP | TWK-LST |
| TWK-EST | 8.89 | 14.70 | 9.68 | 9.86 | 10.78 | 5.24 | SOF | MAXSLK | RAN |
| MS | 14.32 | 2.98* | 8.67 | 11.97 | 9.49 | 4.56 | MINTWK | MINTWK | TWK-EST |
| MCS | 14.40 | 3.18 | 9.19 | 11.79 | 9.64 | 4.80 | MAXSLK | SOF | MAXSLK |
| MINWCS | 8.13 | 8.25 | 8.78 | 10.49 | 8.91 | 4.66 | RAN | RAN | SASP |

Note:$*$ means the best value under each measure; $**$ means the best value on average;

differences between the results from 10% to 50% were much more significant than for the experiments with $DEV_{max}$ above 60%. Table 5 displays the ranking comparison between the two cases for each priority rule. "Case1" is the benchmark results with the original 5 uncertainty levels, and the "Case2" is obtained with above mentioned 18 uncertainty levels. The table shows that only minor changes occur, and we therefore conclude that under increasing uncertainty levels, the best performing rules remain the same. As an example, the cluster of EDDF, MINLFT and MINSLK, or the cluster of MS and MCS is the same for both cases. For some other priority rules that perform less good, their ranking differs a little bit, but no fundamental changes could be detected (no major changes). We therefore believe that using the five uncertainty levels is sufficient to explain the relative ranking of priority rules for the problem under study.

Table 5: Comparison between the two cases for rules ranking

| Global Ranking | | Project | | Portfolio | |
|---|---|---|---|---|---|
| **Case1** | **Case2** | **Case1** | **Case2** | **Case1** | **Case2** |
| EDDF | EDDF | EDDF | MINLFT | MS | MCS |
| MINSLK | MINSLK | MINLFT | EDDF | MCS | MS |
| MINLFT | MINLFT | MINSLK | MINSLK | LALP | EDDF |
| MINWCS | MAXSP | SASP | SASP | EDDF | MINSLK |
| MAXSP | MINWCS | MINWCS | MAXSP | MINSLK | LALP |
| MS | MCS | FCFS | TWK-LST | MINLFT | MINLFT |
| MCS | MS | TWK-LST | FCFS | MINWCS | MAXSP |
| FCFS | FCFS | MAXSP | MINWCS | MAXSP | WACRU |
| LALP | WACRU | TWK-EST | TWK-EST | WACRU | MINWCS |
| TWK-LST | TWK-LST | MS | MAXTWK | FCFS | FCFS |
| SASP | LALP | MAXTWK | WACRU | MAXTWK | MAXTWK |
| WACRU | TWK-EST | WACRU | MOF | SOF | LCFS |
| TWK-EST | SASP | MCS | SOF | LCFS | SOF |
| MAXTWK | MAXTWK | LCFS | MCS | MOF | MOF |
| LCFS | MOF | MOF | MINTWK | MINTWK | RAN |
| MOF | SOF | LALP | MS | TWK-LST | MINTWK |
| SOF | RAN | MAXSLK | RAN | RAN | TWK-LST |
| MINTWK | MINTWK | MINTWK | LALP | TWK-EST | TWK-EST |
| MAXSLK | LCFS | SOF | LCFS | MAXSLK | MAXSLK |
| RAN | MAXSLK | RAN | MAXSLK | SASP | SASP |

### *6.4.2. Other distributions*

The previous experiments all made use of the triangular distributions to model uncertainty under a left-skewed, symmetrical and right-skewed mode, each with two additional parameters with values equal to 0.5 and 1.5 (cf. equations 4 and 5). These parameters have been used to guarantee that the activity durations are generated without substantially exceeding the mode $c$. Any two parameters that satisfy $b - c > c - a$ could have been used. In order to investigate the impact of the two parameters on the performance of the 20 priority rules, we have additional experiments on other values, as shown in Table 6, in which the second and third rows represent the values of the two parameters. In addition, we have also experimented with the beta and uniform distribution to model uncertainty, where the minimum and maximum durations are the same with that of the benchmark triangular distribution, shown in the last two columns, labelled by "Beta" and "Uniform". The table shows that for various set of parameter combination and even for the beta or uniform distribution, the global ranking of rules differs only a little bit, and especially for the best performing set of rules, there almost no changes. But for some worse-performing rules, for example LCFS, it deteriorates and becomes even worse than the rule RAN, but the variation range is still acceptable (i.e. they go from the sixth cluster to the seventh or at most the eighth cluster). Since we are more interested in the best and robust priority rules rather than the worse ones, we believe that the use of these parameters is satisfactory to present our results.

## 7. Conclusion

In this paper, we mainly explore the performance of priority rules for the resource constrained multi-project scheduling problem with uncertain activity durations. Besides quality measures used in existing literature, new robustness criteria are also proposed to measure the performance distinction between deterministic and stochastic environment from both project and portfolio perspectives. Five uncertainty levels are used to illustrate the impact of uncertainty on the performance of priority rules. A full factorial experiment is conducted on 1,260 projects and results are given.

First, the overall performance of the priority rules are analysed, and based on the trade-off relationship between the quality and the robustness measures, we obtain the corresponding Pareto frontiers for project and portfolio managers. Generally, the performance of the higher uncertainty levels is much concentrated to the average value than the lower case. Moreover, project managers can choose the appropriate priority rules among the SASP, MINLFT, EDDF, MINWCS, MINSLK and FCFS. While portfolio managers have a much wider set of options, including MS, MCS, LALP, EDDF, MINSLK, MINLFT, MINWCS, MAXSP, FCFS and SASP. Separately for each measure, SASP wins for $Q1$ while MS and MCS dominates for $Q2$. For robustness measures, FCFS performs well for $R1$, whereas SASP delivers the best for $R2$. These results show that the performance of priority rules differs significantly according to the objectives used. Therefore, one can hardly tell which priority rule is the best for all cases. Second, the influence of problem characteristics is examined on the average performance of the priority rules. Interestingly for $SP$, $NARLF$ and $MAUF$, the multi-projects that obtain the best results for $R1$ and $R2$ are almost the same and opposite with that for quality measure $Q2$. Finally, the best recommendation is given considering the four types of measures and the results are compared with that for separate project and portfolio managers. In conclusion, EDDF is the best rule for the average of the four objectives and also the best for project managers. MS is the best for portfolio managers.

In future research, more priority rules that perform well in uncertainty environment

Table 6: Comparison of rules ranking for different set of parameters

| **Benchmark** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 0.5 | 0.6 | 0.7 | 0.9 | 1.0 | 1.5 | 2.0 | 0.1 | Beta | Uniform |
| 1.5 | 1.6 | 1.7 | 1.9 | 2.0 | 2.0 | 3.0 | 1.1 | | |
| EDDF | EDDF | EDDF | EDDF | EDDF | EDDF | EDDF | EDDF | EDDF | EDDF |
| MINSLK | MINSLK | MINSLK | MINSLK | MINSLK | MINSLK | MINSLK | MINLFT | MINSLK | MINLFT |
| MINLFT | MINLFT | MINLFT | MINLFT | MINLFT | MINLFT | MINLFT | MINSLK | MINSLK | MINLFT |
| MINWCS | MINWCS | MINWCS | MINWCS | MINWCS | MINWCS | MINWCS | MINWCS | MINWCS | MINWCS |
| MAXSP | MAXSP | MAXSP | MAXSP | MAXSP | MS | MAXSP | MAXSP | MS | MAXSP |
| MS | MS | MS | FCFS | MS | FCFS | MS | MS | MCS | MS |
| MCS | MCS | FCFS | MS | FCFS | MCS | MCS | FCFS | MAXSP | MCS |
| FCFS | FCFS | MCS | MCS | MCS | MAXSP | FCFS | MCS | FCFS | FCFS |
| LALP | LALP | TWK-LST | TWK-LST | TWK-LST | TWK-LST | WACRU | LALP | LALP | TWK-LST |
| TWK-LST | TWK-LST | LALP | WACRU | WACRU | LALP | TWK-LST | TWK-LST | WACRU | WACRU |
| SASP | WACRU | WACRU | LALP | LALP | WACRU | LALP | SASP | TWK-LST | LALP |
| WACRU | SASP | TWK-EST | TWK-EST | TWK-EST | TWK-EST | TWK-EST | TWK-EST | SASP | SASP |
| TWK-EST | TWK-EST | SASP | SASP | SASP | SASP | SASP | WACRU | TWK-EST | TWK-EST |
| MAXTWK | MAXTWK | MAXTWK | MAXTWK | MAXTWK | MAXTWK | MAXTWK | MAXTWK | MAXTWK | MAXTWK |
| LCFS | MOF | MOF | MOF | MOF | MOF | MOF | MOF | MOF | MOF |
| MOF | SOF | SOF | RAN | SOF | SOF | RAN | MINTWK | MINTWK | SOF |
| SOF | MINTWK | MINTWK | SOF | RAN | MINTWK | SOF | SOF | SOF | MINTWK |
| MINTWK | RAN | RAN | LCFS | MINTWK | RAN | MINTWK | RAN | LCFS | RAN |
| MAXSLK | LCFS | LCFS | MINTWK | LCFS | LCFS | LCFS | LCFS | RAN | LCFS |
| RAN | MAXSLK | MAXSLK | MAXSLK | MAXSLK | MAXSLK | MAXSLK | MAXSLK | MAXSLK | MAXSLK |

should be studied and compared with current results. Moreover, other types of uncertain factors can be considered, for example, dynamic project arrival, stochastic resource availability and etc. Moreover, all the results in the research are obtained based on the simulation with self-generated multi-project instances. It is therefore significant to compare the results with real-life multi-projects to investigate the effects of the best priority rules for both quality and robustness measures.


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