Transport and interaction effects in low-dimensional semiconductor heterostructures

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Declaration

I, Elias Peraticos confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.
Acknowledgements

Initially I would like to thank Prof. Sir Michael Pepper for being my supervisor and giving me the opportunity to join his group. Additionally, I would like to thank Dr. Sanjeev Kumar for his invaluable help with the cryostat systems and with long discussions about the data collected. Furthermore, special thanks to Prof. Afif Siddiki who was always available for discussions on theoretical concepts via Skype and for the great hospitality at his university and home-town of Istanbul resulting in scientific and cultural exchanges. I would like to thank Dr. Yilmaz Gul for the fabrication of the InGaAs/InAlAs devices. A huge thank you to Dr Kostas Rogdakis who went out of his way to read my thesis and give me constructive feedback. In addition, I would like to say a big thank you to Dr Wei Liu for always being an excellent support throughout the process of the PhD and welcoming me in the group, for the late night discussions in the lab about our work and his friendship. Thank you to the EPSRC for providing the financial support. I would also like to say a huge thank you to my closest friends, for putting up with me when I was complaining about things not working in the lab. Last but not least, a big thank you to my parents and my sisters for providing great support and who have encouraged me throughout the process every single day.
Abstract

The work in this thesis is related to the strongly correlated electron interaction effects taking place within 2D and 1D electron systems in III-V semiconductor devices. Initially the work conducted on GaAs/AlGaAs was to investigate the effects of incompressible/compressible strips on the transverse and longitudinal resistance. Effects due to these strips lead to resistance anomalies in the transverse resistance, when the incompressible strips are in the evanescent regime, as described by the screening theory. In the study described in this thesis, such anomalies are observed not just for integer states, but for fractional states as well, i.e. $\nu = 4/3, 3/2, 5/3, 8/3, 3, 10/3, 7/2$ and 5, which have been predicted theoretically but not studied experimentally. Additionally, longitudinal resistance hysteresis was noticed which increases in size with lower-valued $\nu$ and by increasing the constriction of the quasi-1D channel within the system. The relaxation times of the longitudinal resistance within these hysteretic areas were found to be linked to two mechanisms with $\tau_1$ and $\tau_2$ being in the order of $10^2$ s and $10^6$ s. These hysteretic loops were discussed in terms of dynamic nuclear polarisation and Ising ferromagnetism and the screening theory. It is discussed that the latter provides a better fit in the explanation of the hysteresis.

Further studies were conducted using the GaAs/AlGaAs device, but instead the transverse voltage was measured while setting up the system in measuring the quasi-1D conductance. By varying the Magnetic field it was found that various oscillations were measured indicating unusual features. These oscillations seem to be linked to the $\nu$'s in the 2DEG regions and the change in the peaks height and position are linked to the constriction size within the quasi-1D channel. These effects seem to be related to the crossings of Landau levels. In addition when spin-polarisation is enhanced a set of peaks increase in size compared to others, which is why it is thought that they are linked to enhanced spin polarisation within the constriction. By applying source-drain bias it was found that peaks that seem to be linked to even filling factors tend to disappear with positive voltage bias but for negative voltage bias, peaks related to both odd and even filling factors seem to persist. This is explained by scattering being induced due to quasi-elastic inter-Landau-level scattering as well as through the spin gap model. The data from these two chapters will provide a better understanding on the physical phenomena taking place and how the Landau levels and electron-electron interactions affect the behaviour of the systems. Furthermore the
oscillations observed at 3.25 T are thought that they could be linked to the Aharonov-Bohm effect.

Finally an InGaAs/InAlAs device was used to study the interaction effects due to perpendicular magnetic field and spin-orbit interactions within a quasi-1D channel. While applying lateral voltage bias within the quasi-1D channel, the asymmetric voltage on the split-gates acts as a type of electric Stern-Gerlach apparatus inducing spin-splitting as well as Rashba-spin-orbit-coupling (RSOC). As a consequence exotic phases occur which lead to fractional conductance states appearing within the system, which are enhanced by increasing the magnetic field. Such states are the 5/2 and 12/5 fractional states which if proven to be non-Abelian can help in the creation of topological fault tolerant quantum computers. These fractional conductance states are thought to be a consequence of backscattering and umklapp scattering. Also some of the fractional conductance states noticed have been observed experimentally in other 2D systems, and the fact that fractional states like 3/2, 5/2 and 7/2 weaken and strengthen with certain perpendicular magnetic fields and asymmetric voltages within the system are explained by the RSOC inducing in-plane magnetic field components in the system leading to similar behaviour observed in 2D systems in tilted B-field setups. In the InGaAs/InAlAs system though more fractional states appear in higher sub-band levels, compared to the measurements conducted in other two-dimensional systems, which is thought to be due to the intrinsically stronger spin-orbit coupling InGaAs/InAlAs systems have. These fractional states would be highly valuable for spintronic devices and the construction of quantum computers by utilising lower perpendicular magnetic fields than what is required in conventional 2D systems.
Impact Statement

Work conducted for this thesis was done by using III-V semiconductor devices. These are highly compatible to use with the silicon-based devices used in most electronic devices in our time. Therefore, understanding in depth, the behaviour of these devices and how to manipulate them could lead to exciting integrations in electronic systems and possibly lead to the creation of new quantum technologies. Initially with respect to the study discussed in Chapter 5, the importance is that resistance overshooting effect although theorised for fractional state it has not been observed experimentally in transverse resistance measurements. The importance of this is that it indicated that fractional states could also be explained by the screening theory which explains the formation of incompressible and compressible strips. Therefore, this work could possibly be an additional indicator to the highly theorised incompressibility of fractional states, resulting in an increased interest to further study and understand this phenomenon. In Chapter 6 the transverse voltage measured for the conductance set up has shown oscillations which correspond to the filling factors, but in addition to the extra peaks that seem to appear. These peaks can be manipulated by using the split-gates and source-drain bias. These extra peaks are thought to correspond to places of Landau level crossings and the source-drain seems to encourage quasi-elastic inter-Landau-level scattering. Understanding how a 1D constriction in a 2D system can interact with the Landau levels in order to manipulate them could be useful in quantum technology. Finally the fractional states observed in the 1D conductance in the InGaAs/InAlAs system, as discussed in Chapter 7 could lead to useful manipulation of these states and more interestingly the possible non-Abelian states of 5/2 and 12/5 for fault tolerant topological quantum computers. Overall quantum technologies could provide excellent methods in having high precision measurements and exponentially faster and highly secure computers. This would be of use for national security reasons. Additionally, the evolved computational power would enhance simulations and modelling. For example, some ways it could help is by manipulating larger datasets to provide accurate weather predictions faster and efficiently, extremely fast drug designs and genome sequencing.
Publications


E. Peraticos, S. Kumar, M. Pepper, A. Siddiki, I. Farrer, D. Ritchie, T. Mitchell and J. Griffiths, *Resistance hysteresis in the integer and fractional quantum Hall regime* - In progress.

E. Peraticos, S. Kumar, Y. Gul, M. Pepper, I. Farrer, D. Ritchie, T. Mitchell and J. Griffiths, *Fractional Conductance in InGaAs/InAlAs quantum wire* - In progress.

Work Presented at conferences

Condensed Matter and Quantum Materials, St Andrews, United Kingdom, Poster, E. Peraticos, S. Kumar, M. Pepper, I. Farrer, D. Ritchie, T. Mitchell and J. Griffiths, *Enhanced magnetic hysteresis in the Quantum Hall regime in the 2D-1D regime*, (July 2019).

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<tr>
<td>IS</td>
<td>incompressible strip</td>
</tr>
<tr>
<td>CS</td>
<td>compressible strip</td>
</tr>
<tr>
<td>2DE(G/S)</td>
<td>two-dimensional electron (gas/system)</td>
</tr>
<tr>
<td>B</td>
<td>magnetic field</td>
</tr>
<tr>
<td>ν</td>
<td>filling factor</td>
</tr>
<tr>
<td>LL</td>
<td>Landau level</td>
</tr>
<tr>
<td>N</td>
<td>Landau level index</td>
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<tr>
<td>DNP</td>
<td>dynamic nuclear polarisation</td>
</tr>
<tr>
<td>h</td>
<td>Planck constant</td>
</tr>
<tr>
<td>(\hbar)</td>
<td>reduced Planck constant</td>
</tr>
<tr>
<td>(k_B)</td>
<td>Boltzmann constant</td>
</tr>
<tr>
<td>(m_e)</td>
<td>mass of electron</td>
</tr>
<tr>
<td>(m_e^*)</td>
<td>effective mass of electron</td>
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<tr>
<td>e</td>
<td>elementary charge</td>
</tr>
<tr>
<td>(\lambda_F)</td>
<td>Fermi wavelength</td>
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<tr>
<td>(k_F)</td>
<td>Fermi wavevector</td>
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<tr>
<td>(l_B)</td>
<td>magnetic length</td>
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<tr>
<td>(E_F)</td>
<td>Fermi energy level</td>
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<td>(R_{xy})</td>
<td>transverse resistance</td>
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<tr>
<td>(R_{xx})</td>
<td>longitudinal resistance</td>
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<tr>
<td>(W_{IS})</td>
<td>width of incompressible strip</td>
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<tr>
<td>(I/F)QHE</td>
<td>(integer/fractional) quantum Hall effect</td>
</tr>
<tr>
<td>(V_{sg})</td>
<td>split-gate voltage</td>
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<td>(V_{tg})</td>
<td>top-gate voltage</td>
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<td>fractional quantum Hall states</td>
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<td>(bulk) electron density of 2DEG</td>
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<td>(\mu_e)</td>
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<tr>
<td>(V_{xy})</td>
<td>transverse voltage</td>
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<tr>
<td>g</td>
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<tr>
<td>g*</td>
<td>effective (enhanced) g-factor</td>
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<tr>
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<tr>
<td>(r_{cf})</td>
<td>cyclotron radius for composite fermions</td>
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<tr>
<td>BT</td>
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Chapter 1

Introduction

1.1 History of low-dimensional systems

From the early twentieth century the study of semiconductor properties has led to a fascinating growth in the understanding and manipulation of such materials. This lead to the current global multi-billion industries within the field of computers, radio-telecommunications, satellites and other fields [1].

The transistor was an important milestone to the great success of the industry. The success of producing a device with increased output power at Bell Labs in the late 1940s was the beginning of a rapid increase in a new field in physics. Transistors switched to silicon-based metal-oxide-semiconductors field-effect transistors (MOSFETs) through the 1950s proving to be highly successful even 70 or so years after its development. Furthermore there was the development of the high-electron-mobility transistor (HEMT) using heterostructures of GaAs/AlGaAs during 1979 with enhanced properties, discussed later within the thesis, at Bell Labs [2, 3].

The property which proved to be so successful in these types of field-effect transistors (FETs) was the formation of the two-dimensional electron gas (2DEG), first shown through research conducted by IBM with MOSFETs [3, 4]. In addition, the studies on 2DEGs have led to the observation of the quantum Hall effect and the study of other interesting physical phenomena [2, 3, 5]. The properties and formation of a 2DEG will be discussed further on within Chapter 2.

Lower dimensional systems have also attracted the interest of the scientific community due to their intriguing properties and possible applications. One-dimensional and zero-dimensional electron gas systems, could potentially help in realising a quantum computer. Thornton et al. were the first to
produce a 1D system by reducing the 2DEG system using split gates [6]. However, the quantization of conductance in units of $2e^2/h$ was not observed until the late 1980s, where 1D systems had dimensions less than the mean free path of the electrons [7, 8].

It is evident that the study of systems with reduced dimensions, less than 3D, have revealed a lot of fascinating properties and will probably reveal even more. As the field is further penetrated through extensive research by the scientific community one could expect a better understanding of the effects within the quantum-regime of our world.

1.2 Motivation and Thesis Outline

1.2.1 Motivation

As the number of transistors per square inch increased on integrated circuits, due to them becoming smaller and smaller in size, the scientific community had reached the breaking point of classical diffusion transport. Consequently, understanding the properties of the quantum realm became of great importance in order to keep up with Moore’s law [4, 9–11] and with the ever increasing demand of computer processing power.

Therefore, the goal of the thesis is to highlight certain effects that arise from e-e interactions in quantum systems and hopefully help in an improved understanding of this realm, in order to harness its power for the production of quantum computers.

Since the discovery of the IQHE and the FQHE the predominant theory for the edge states has been the 1D Landauer-Büttiker formalism. However, this does not explain all phenomena observed within these regimes as it fails to take into account the screening and the modified effects by strong magnetic fields. So the one of the goals of this work was to study how the screening theory and the formation of compressible and incompressible strips affect the behaviour of various 2DEG systems by taking into account the geometry of the system. Understanding how the self-consistent screening theory affects various systems by taking into account carrier density distribution as well as the geometry can help unlock more options in using various 2DEG systems in quantum metrology. Such phenomena as anomalous transverse resistance and longitudinal hysteretic resistance are better explained by using the self-consistent screening theory.

Additionally, by studying the transverse voltage by setting up the system in measuring the conductance, as is done in Chapter 6, the behaviour of the Landau levels was able to be studied as it was reflected in the transverse
voltage measurements. Such behaviour as is Landau level crossings and the effects of spin-polarisation by using a quasi-1D constriction in conjunction with the 2D regimes, will help understand how the Landau levels are affected by such conditions as well as with varying source-drain bias. Therefore, by understanding such phenomena in more detail can help in advancing quantum technologies.

Finally as fractional conductance states have been noticed in systems like GaAs/AlGaAs and strained germanium on silicon devices, the idea in Chapter 7 was to explore whether such states would appear in InGaAs/InAlAs devices. Such fractional states could help in spintronic devices and accessing multiple fractional states by tuning the split-gates only without varying the magnetic field which makes it more costly or by using relatively small magnetic fields which are easier to achieve. The asymmetry in the split-gate voltage and the high intrinsic spin-orbit coupling of InGaAs/InAlAs have been shown to reveal interesting physical phenomena which can lead to fascinating spintronic devices.

1.2.2 Thesis Outline

The thesis will start with the theory of lower dimensional systems in Chapter 2 and the fabrication process of such devices in Chapter 3. In addition, the techniques for conducting the measurements with these devices and the data received will be discussed in Chapter 4. Finally in Chapters 5 and 6 the data presented will be of studies conducted on GaAs/AlGaAs heterostructures. That is resistance overshooting in transverse resistance and hysteretic behaviour in the longitudinal resistance and then understanding the oscillatory behaviour in the transverse voltage measurements, respectively. Finally Chapter 7 will be related to measurements on InGaAs/InAlAs 1D quantum wires and the fractional conductances observed. The thesis will then conclude in Chapter 8.
Chapter 2

Theory

2.1 Mesoscopic Systems

Mesoscopic systems refer to any systems that lie between the macroscopic and microscopic regime, that is systems that can be explained using classical physics and quantum physics, respectively [4, 10–12]. In mesoscopic systems novel effects take place, changing the performance of the devices. The point at which a system is defined as entering the mesoscopic regime depends on which property of the system is studied [4, 10, 13]. The most common factors are:

1. The electronic mean free path $l_e$,

2. The de Broglie wavelength at the Fermi Energy, $E_F$, i.e. the Fermi wavelength $\lambda_F$.

When the lengths of these quantities become comparable to the size of the system, ballistic transport and quantisation are achieved, respectively. However, the length scales of these properties is heavily dependant on scattering effects [11, 14, 15]. Therefore a brief description of these effects shall be given before giving more details on the various length scales dominating the properties of mesoscopic systems.

The main scattering mechanisms are lattice scattering and impurity scattering. Lattice scattering occurs due to atoms in the crystalline lattice structure thermally vibrating [10, 14, 15]. As temperature increases so do the amplitudes of the atomic vibrations [14]. This is the reason why the measurements performed and discussed in the chapters to follow were conducted at low temperatures ($\sim 10$ mK), in order to reduce lattice scattering and increase the average electron mobility.
Impurity scattering arises due to ionised dopant impurities present within the heterostructure [10, 14, 15]. That is Coulomb interactions between the ionised impurities and the electron, cause the electron’s trajectory to be altered by deflecting it. This effect increases with dopant concentration. However, impurity scattering is more prominent at low temperatures as opposed to lattice scattering. In addition to these mechanisms, scattering is further classified as an elastic or inelastic process [14, 15].

The $l_e$ is the quantity responsible for the transition of the transport of electrons from a diffusive manner to a ballistic one. It is defined as the average distance travelled by an electron between successive scattering events [10] given by,

$$l_e = v_F \tau$$  \hspace{1cm} (2.1)

where $\tau$ is the average time between the scattering events and $v_F$ is the Fermi velocity which can be expressed as,

$$v_F = \frac{\hbar k_F}{m^*_e}$$  \hspace{1cm} (2.2)

Therefore, $l_e$ can be expressed as the Fermi wavelength of the electrons carrying current [10],

$$\lambda_F = \frac{\hbar}{\sqrt{2m^*_e E_F}}$$  \hspace{1cm} (2.3)

where $m^*_e$ is the effective electron mass and $E_F$ is the Fermi energy. If $\lambda_F$ is comparable in size with the size of the structure studied, the electrons kinetic energies quantise [4, 10]. This is the principle by which 2-dimensional, 1-dimensional and 0-dimensional electronic structures are formed.

For the case where the length, $l$ and width, $w$, of the channel are larger than the $l_e$, see Fig. 2.1(a), the transport is diffusive in its nature. That means that it acts like a semi-classical metal or semiconductor governed by the Boltzmann transport equation (BTE) [4, 10, 16]. In Fig. 2.1(b), $w$, is smaller than $l_e$ but $l$, is still relatively long. This is the quasi-ballistic regime. As a result there is quantization in one of dimensions of the carriers’ motion, but it still has a diffusive character in the other directions. The purely ballistic regime governs when, as seen in Fig. 2.1(c), both $l$ and $w$ are shorter than $l_e$ [4, 10].
Figure 2.1: Schematic diagram of the various ways an electron travels through a channel depending on its length ($l$) and width ($w$). (a) Shows the diffusive character of electrons conducting within a channel of $l > l_e$ and $w \gg l_e$, (b) shows that for dimensions similar in size to $l_e$ the system enters a quasi-ballistic regime, and (c) shows that for $l$ and $w$ smaller than $l_e$ the electron travels ballistically through the channel. The red circles are scattering centres.

2.2 Two-dimensional Electron Gas (2DEG)

2.2.1 Introduction

A 2-DEG is a gas of electrons with bi-dimensional properties (within x-y plane) while it is confined by a potential in the third dimension (z direction). As a consequence of the potential confinement, quantisation takes place along the dimension of the restriction applied to the electron gas. This forms
a 2D plane, normal to the confining potential and the electron gas’ motion is characterized by a 2D k-vector [4, 17]. This type of electron gas can be produced by using semiconductor structures. These could also be used to create confinement in more dimensions, forming one and zero-dimensional systems, like nanowires and quantum dots respectively, by using field-effect transistors (FETs), like MOSFETs and HEMTs [1, 10, 17].

2.2.2 Producing a 2DEG

A 2DEG is produced at the interfaces of heterostructures used for FETs. The operation of these types of transistors rely on interface effects, using electric fields [4, 10, 17]. Therefore, the electrical conductivity within the device can be controlled. The two main FETs, as mentioned above, are the MOSFETs and HEMTs, most commonly fabricated with Si and GaAs, respectively.

The heterostructures used to produce the 2DEG should possess specific properties in order to enhance performance of the HEMTs. The layered stacked formation of various semiconductors in order to form a heterostructure have intrigued the scientific community due to the amazing properties this kind of structure provides. Due to the different electronic band structure of the various semiconductors used their band gaps can be manipulated by forming quantum wells or barriers, through band engineering. As a result, the field-effect transistor and semiconductor laser, example of both electronic and optical applications, have been developed due to such structures [10].

The study of designing heterostructures was developed due to the acquired knowledge of band properties at or near the interface of a heterojunction. It is at the proximity of interfaces of the materials within a heterostructure that the most interesting phenomena take place, making them the active regions of scientific studies. The surface-roughness scattering, however, in addition to defect scattering can affect the electronic properties of such devices, by reducing their electron mobility and mean free path [10]. Therefore, their growth is of outmost importance as the interfaces have to be almost perfect. Methods like molecular-beam epitaxy (MBE) and metal-organic chemical vapour deposition (MOCVD), are used for performing this delicate task but at a significant increase in cost of production [10, 18, 19]. The two basic requirements for an ultimate heterostructure [10] are:

- The materials used are of similar crystal structure, and
- There is minimal or non-existent strain within the structure.
Figure 2.2: Schematic diagram of a 2DEG formed in a GaAs/AlGaAs heterostructure. The 2DEG is formed by electrons donated by the donors. AlGaAs has a larger band gap than GaAs, the band structure shifts in order to equilibrate the Fermi level, $E_F$. This as a result forms the triangular well in which the 2DEG is formed.

An example of a 2DEG being formed in a GaAs/AlGaAs structure is shown in Fig. 2.2. Initially a layer of a GaAs, is placed next to an AlGaAs layer, which is an alloy of AlAs and GaAs, with an energy band gap larger than GaAs [17]. As can be seen in Fig. 2.2 by having this composition, a triangular potential well is formed at the interfaces of the AlGaAs and GaAs, with discrete energy levels. A typical alloy composition of AlAs and GaAs is Al$_{0.3}$Ga$_{0.7}$As The relative position of each layer is adjusted by having the chemical potential being equal within the entire system. The AlGaAs layer is doped with Silicon, which acts as an electron donor, by using modulation doping in order to reduce scattering [10, 14, 17]. The electrons move into the potential well where they are trapped, thus forming the 2DEG. As the donor atoms have an unpaired electron, which is weakly bound to the Si atom, it has higher chances of moving into the conduction band. From there if it succeeds to fall within the potential well, it will remain trapped in there. As more electrons are trapped in the well, the 2DEG is formed. The first electrons most-likely travel into the well, directly through the Al-GaAs layer, but as the electron repulsion increases with increasing number
of electrons within the well, this becomes less feasible \[10, 17\]. Therefore, the electrons eventually start diffusing sideways, until they can finally enter the well through a particular contact.

As mentioned earlier in the Heterostructure section, AlGaAs and GaAs have compatible lattice structures, so the strain is negligible. In addition due to the reduced scattering by the donors, due to modulation doping, the electrons have enhanced mobility, in the ranges of \(100 – 1000 \text{ } m^2 V^{-1} s^{-1}\) and relatively long mean free paths of the order of \(10 – 10^2 \mu m\).

Using MBE the structure is grown on a semi-insulating GaAs substrate, that is a boule of GaAs is grown in excess arsenic, introducing arsenic antisite defects (arsenic atom at a gallium atom site) within the crystal lattice. As a result the defects pin the Fermi level, at approximately, the centre of the electronic bandgap, therefore reducing the carrier concentration of the GaAs crystal \[10, 20\]. This crystal is similar to the intrinsic crystal, but is easier to achieve in practice. The buffer layer is also grown, epitaxially, so defects from the substrate are isolated, in addition to acting as a suitable surface for growing the active layers on top of it, due to its smoothness. Similarly the spacer layer acts as an insulating layer between the channel and the donors.

### 2.2.3 From 2D to 1D

This can be achieved by many methods but the most common is by depositing metal gates on the GaAs/AlGaAs heterostructure \[2, 6, 10, 20\]. Then by applying a negative voltage on the gates a narrow channel is formed, by repelling the electrons underneath the gates, thus forming a wire. By changing the bias voltage on the gate the width of the wire can be altered. In section 2.3 a discussion on the current and conductance in a 1D system will take place. Finally in Fig. 2.3 an example of a split-gate/top gate system can be seen, used in forming such 1D devices in semiconductor heterostructures.

### 2.3 Electron Transport in 2D systems

#### 2.3.1 Drude Model

Drude formulated a model for which an electron gas was formed by unbound electrons. The electron’s motion was in accordance to the kinetic theory of gases \[4, 14, 16, 20\]. Although this model fails to represent the quantum mechanical properties of the electrons, it is a good starting point in understanding the basics. The current density is expressed as,
Figure 2.3: Schematic diagram of a GaAs/AlGaAs heterostructure fabricated into a HEMT. Ohmic contacts are used to have direct access with the 2DEG and the split-gates are used to deplete the electrons below them by applying a negative voltage on them. Note the 2DEG is formed at the AlGaAs and GaAs interface. n-AlGaAs is the layer within which the donors exist.

\[ j = -nev_d \]  (2.4)

If no external electric field \( E \) is present then the \( v_d \) is zero. Between each scattering events a random electron would gain a velocity of \( eE\tau/m^*_e \). Since the average is taken and electrons are scattered, the initial velocity of the electrons is not important. The average \( v_d \) is given by,

\[ v_d = -\frac{eE\tau}{m^*_e} \]  (2.5)

Therefore by combining 2.4 and 2.5 one obtains,

\[ j = \left( \frac{ne^2\tau}{m^*_e} \right) E \]  (2.6)
From equation 2.6 the Drude conductivity $\sigma$ is given by the term in the parenthesis, simplifying it to $j = \sigma E$.

The average drift velocity gained by the electrons in the presence of an external electric field is defined as, the mobility of the electrons, $\mu$. It is related to $v_d$ by the equation

$$v_d = \mu E$$

which can be re-written by using 2.5 as,

$$\mu = \frac{e\tau}{m_e^*}$$

The mobility is related to the mean free path of the electrons and therefore is an important parameter as it defines the ballistic properties of the system as described earlier in section 2.1 [4, 14]. The aforementioned model is completely derived with classical mechanics. However, classical physics breaks down at the scales associated with the devices mentioned in the project, and quantum mechanics start to dominate [4, 14, 20].

2.3.2 The Hall Effect

The Hall effect is used in order to determine the carrier density of the sample. By applying a magnetic field in the z-direction, $B_z$, and an electric field in the x-direction, $E_x$, the Hall experiment can be obtained, see Fig. 2.4, for the experimental set up [10, 14].

As the electrons flow along the x-direction a Lorentz Force is felt by the electrons along the y-direction, expressed as,

$$F_L = e(v \times B)$$

This effect forces the electrons to shift direction and therefore having an induced electric field, $E_H$, formed in the y-direction [10, 14, 17]. The electrons accumulate at the Hall bar’s edges, as seen in Fig. 2.4. When the force from the induced $E_H$, $F_H = eE_H$, is equal to the $F_L$, then the system is said to be in equilibrium.

The cyclotron angular frequency, $\omega_c$ and radius of cyclotron orbit, $r_c$ are expressed as,
Figure 2.4: Schematic diagram of a Hall experiment. An electric field, $E_x$ and magnetic field, $B_z$, is applied on the hall bar. The voltage along the longitudinal direction, $V_L$ and along the transverse direction, also known as Hall voltage, $V_H$, is indicated. The electric field, $E_H$, is the induced electric field produced causing the segregation of the positive and negative charged particles within the hall bar.

\[
\omega_c = \frac{eB}{m_e^*} \quad \text{(2.10)}
\]

\[
r_c = \frac{v}{\omega_c} \quad \text{(2.11)}
\]

The electrons motion in the system is conveyed by the equation,

\[
F = -e [E + (v \times B)] - \frac{m_e^*v}{\tau} \quad \text{(2.12)}
\]

In equilibrium the force equals to zero, therefore the x and y constituents of the force are broken down into,

\[
-e(E_x + v_yB) - \frac{m_e^*v_x}{\tau} = 0 \quad \text{(2.13)}
\]

\[
e(-E_H + v_xB) - \frac{m_e^*v_y}{\tau} = 0 \quad \text{(2.14)}
\]
As the resultant force on the electron $eE_H$ balances the $F_L$, the $v_y$ can be set to zero. Therefore, eqs. 2.13 and 2.14 can be simplified to give the longitudinal and transverse resistivities by dividing the fields, $E_x$ and $E_H$ by $j_x$ as,

$$
\rho_{xx} = \frac{1}{n_e} \frac{e \mu}{(2.15)} \\
\rho_{xy} = \frac{-B}{n_e} (2.16)
$$

The importance of expressing $\rho_{xy}$ in this format is that the equation does not depend on any material parameters and is only dependent on the carrier density [10]. In addition by conducting this experiment at low temperatures, means that the $\rho_{xy}$ is almost constant. Furthermore by using this method it can be found whether the carriers responsible for the effect measured are due to electrons or holes, since the sign of $E_H$ changes respectively [10]. By using the equations from the previous section and the fact that $\rho = 1/\sigma$, the values of $n$ and $\mu$ can be calculated.

### 2.3.3 Landau Levels

In the previous section the influence of a perpendicular magnetic field on a 2DEG was presented, with the electrons having trajectories which are affected from the Lorentz force and exhibit some form of cyclic motion. However, if the magnetic field is weak the electrons do not perform complete cyclotron orbits, as they are most likely scattered and have their trajectories altered again. As the magnetic field increases though this changes the story, as cyclotron orbits are successfully achieved, and have different energy levels, $E_n$ [4, 10, 11, 17]. The Schrödinger equation for free electrons is given by,

$$
\frac{1}{2m_e^*} \left[ (p + eA)^2 \right] \psi = E \psi 
$$

(2.17)

In order to simplify the case we choose the Landau gauge, i.e. $A = (-yB, 0, 0)$ [10]. Hence, Eq. 2.17 can be re-written as,

$$
\frac{\hbar^2}{2m_e^*} \left[ \left( \frac{1}{i} \frac{\partial}{\partial x} - \frac{eB}{\hbar} y \right)^2 - \frac{\partial^2}{\partial y^2} \right] \psi = E \psi
$$

(2.18)

with the ansatz of
\[ \psi \propto e^{ikx} \phi(y) \] (2.19)

Eq. 2.18 is modified to,

\[
\frac{\hbar \omega_c}{2} \left[ -iB \frac{\partial^2}{\partial y^2} + \left( \frac{y}{l_B} - lk_B \right)^2 \right] \phi = E\phi
\] (2.20)

which is can be illustrated as a simple harmonic oscillator equation. The magnetic length, \( l_B \) is defined as,

\[ l_B = \sqrt{\frac{\hbar}{eB}} \] (2.21)

The \( \phi \) oscillator function is established as the Landau levels, LLs, which are discrete energy levels separated by \( \hbar \omega_c \) and are spin degenerate [4, 10, 11, 17]. The energies are given by,

\[ E_n = \left( n + \frac{1}{2} \right) \hbar \omega_c \] (2.22)

where \( n = 0, 1, 2, \ldots \). In an ideal case when a perpendicular \( B \) is applied on the 2DEG and it does not have any disorder, the LLs are described by delta functions of energy as seen in Fig. 2.5(a). The degeneracy is given by,

\[ \rho(E) = \frac{1}{2\pi l_B^2} = \frac{eB}{\hbar} \] (2.23)

In a real device the impurities would cause the LLs to broaden forming the extended states, as seen in Fig. 2.5(b). These states contribute to the current flowing in the device. However, the localised states have no contribution to the total current of the device.

The filling factor \( \nu \) which indicates the total number of filled LLs, is given by,

\[ \nu = \frac{\hbar n}{eB} \] (2.24)
2.3.4 Quantum Hall Effect

The QHE was discovered in 1980 by K. von Klitzing et al. \cite{2, 5}, by observing the quantisation of the Hall resistance in the 2DEGs of Si-MOSFETs.
The quantisation was noticed as plateaus which appeared at integer filling factors, $\nu = 1, 2, 3, 4, \ldots$. That is the plateaus appeared at the values given by the resistance equation of,

$$\rho_{xy}(B) = \frac{1}{\nu} \frac{h}{e^2}$$

(2.25)

This effect was then noticed with time that it is observed in all types of materials, as long as there is a two-dimensional electron gas [10]. The fact that this is so accurate has led to the standardisation of resistance by using the $\nu = 1$ quantum Hall plateau, defined as $R_K = 25812.807 \Omega$ [23, 24]. This type of $\rho_{xy}$ quantisation is defined as the integer quantum Hall effect, seen in Fig. 2.6 [10, 17]. There is an additional phenomenon known as the fractional
quantum Hall effect, discovered by Tsui et al. [10, 17, 25], in 1982. The difference is that the $\nu$ can take rational numbers as values.

The QHE is also related to magneto-oscillation of the longitudinal resistivity $\rho_{xx}$, also seen in Fig. 2.6. These oscillations are defined as the Shubnikov-de Haas oscillations [10]. These oscillations however, shall be discussed in more detail in section 2.3.6.

The integer QHE within a classical edge state model, is comprehended as a rise in spatial separation of forward and back-going states as a perpendicular magnetic field is present. As a result edge states form semicircular orbits, as the electrons are reflected by the sample’s boundaries and therefore not
allowing a full circular orbit to be completed [10, 14]. The electrons skip into the next orbitals, see Fig. 2.7. Consequently current is carried in one direction at one of the sample’s edges whereas at the other edge current flows in the opposite direction, see Fig. 2.7. This spatial separation ensures that there is negligible backscattering between the currents carried in opposing directions [14]. In Fig. 2.8, the spatial positions of the edge states can be seen at the points at which the $E_F$ level intersects the Landau levels. The current carried can be considered as 1D quantum channels at each Landau level expressed as [10],

$$I = \frac{e}{h}(\mu_1 - \mu_2) \quad (2.26)$$

where $\mu_1$ and $\mu_2$ are the chemical potentials of the respective edges (ohmic contacts) on either side of the quantum channel. Depending on the number of Landau levels present below the $E_F$, the magnitude of the current will change.

The energy gaps between each Landau level give rise to the plateaus observed in the 4-terminal measurements, an example seen in Fig. 2.8. With increasing magnetic field the Landau levels shift towards higher energy levels. As one Landau level shifts above the $E_F$, the resistance changes and a new plateau appears due to the energy band gap between the next Landau level [10, 26, 27]. The accuracy of the plateaus is so accurate that it lead to the integral quantum Hall-effect.

### 2.3.5 Reflection of Edge States

By adding a split-gate in the middle of a Hallbar one can study the correlation of the number of edge states to the number of occupied LLs by reflecting the edge states by using the split-gate, see Fig. 2.9. At a certain B-field the occupied LLs are N within the bulk of the 2DEG. However, while voltage is applied on split-gates they act as reflectors and therefore allowing only M edge states to pass through the quasi-1D channel, that is only M LLs are occupied inside the split-gate region. As the current, $I$, passes through the system, that is between ohmic contacts 1 and 4, as seen in Fig. 2.9, one can define the following resistivities,

$$R_{14,26} = \frac{V_2 - V_6}{I} = \frac{h}{Ne^2} \quad (2.27)$$
Figure 2.9: As the split-gate is activated certain edge states within the 2D bulk regions of the Hallbar are reflected. Therefore if N edge states are present in the bulk only M edge states pass through the split-gate. (Referenced from [29]).

\[ R_{14,25} = \frac{V_2 - V_5}{I} = \frac{h}{Me^2} \]  

\[ R_{14,25} = \frac{V_2 - V_3}{I} = \left( \frac{1}{M} - \frac{1}{N} \right) \frac{h}{e^2} \]  

Due to the reflection of these edge states ‘fractional’ quantisation can be measured as shown in Eqn. 2.29, which was also observed experimentally [28]. This system can also be used to detect inter-edge-state scattering by measuring reduced Hall resistance [29, 30].

2.3.6 Shubnikov-de Haas oscillations

The Shubnikov-de Haas effect is related to the \( \rho_{xx} \), the longitudinal resistivity, measured with the 4-terminal measurements [4, 10]. The quantum Hall effect is weak along this direction, therefore \( \rho_{xx} \) oscillates, as the magnetic field modulates the density of states, weakly, but \( \rho_{xx} \) does not vanish, as seen in Fig. 2.8 [10]. As magnetic field is changed and as a consequence the
electron density, the density of states at the Fermi level and the screening properties of the electron gas oscillate.

The electron density of the 2DEG in relation to the voltage applied on a surface-gate, is conveyed by the equation [14],

$$\Delta n_{2D} = \frac{\epsilon}{e d} \Delta V_g$$  \hspace{1cm} (2.30)

where $d$ is the distance between the surface gate and the 2DEG and $\epsilon$ is the dielectric constant of the material. The Fermi level is shifted up or down with respect to the Landau levels by varying the gate voltage $V_g$.

The density $n_{2D}$ can remain constant by having the LLs shift instead of the Fermi level by varying the magnetic field perpendicular to the plane of the 2DEG. As the magnetic field is increased the $\nu$ decreases and as a consequence the number of occupied Landau levels decreases as well [10, 14]. Therefore the density of states given by, $n_L = eB/h$, increases within each Landau level occupied. However, the total density of the electrons within the 2DEG, $n_{2D}$, remains unchanged. The LLs spacing in between them also increases as the magnetic field is increased, and the electrons drop down to lower energy LLs, by depopulating the levels which rise above the Fermi level.

The minimum of the oscillations noticed in Fig. 2.8 is when the Fermi level lies in the middle of the energy gap between two succeeding Landau levels. The carrier density can be measured through these oscillations by using the equation of their period given by,

$$\Delta \left( \frac{1}{B} \right) = \frac{g_s e}{n_s h}$$  \hspace{1cm} (2.31)

where $g_s$ is the spin degeneracy.

### 2.3.7 Fractional Quantum Hall Effect

As mentioned earlier FQHE is when the filling factors are defined by fractional numbers. In certain cases when there is weak disorder present, plateaus at these fractional values in the units $e^2/h$ appear. The first example of such a fractional state was the $\nu = 1/3$ by Tsui et al. [25] at magnetic fields three times larger than those at which the $\nu = 1$ integer plateau occurred. Therefore, only $1/3$ of the lowest LL is occupied. The fact that a well defined Hall plateau was observed indicated that electrons travelled through the sample
and there is an excitation gap since $\sigma_{xx}$ tends to zero. This was highly unexpected. The physics for this phenomenon is dominated by the Coulomb repulsion between the electrons.

Laughlin provided initially a solution by defining an ansatz for $N$ quasiparticles with a $\nu = 1/m$, where $m$ is an odd integer [31]. For small $N$ values the wave-function provided a large overlap, numerically, to the true solution [32]. However, even though it is limited to particular fractional filling factors and is not an exact solution, it is thought to provide a good insight to the physical properties of these states, in a qualitative manner [32]. A year later it was suggested by Arovas et al. [33] that the Laughlin states’ fractionally charged quasiparticle excitations were anyons [33, 34].

Another explanation for these fractional states was presented by Jain [35] and further enhanced by Halperin et al. [36] with composite fermions. The theory suggests that composite fermions are quasiparticles which consist of an electron capturing more than one magnetic vortices. This lead to the idea that the fractional states of electrons are the equivalent of an IQHE of CF. Therefore, electrons of $\nu = 1/3, 2/5$ and $3/7$ for example act similarly to the $\nu = 1, 2$ and $3$ states through the CF reference frame.

However, there are still certain even denominator fractional states that are still not explained by the theories [37], so there is still an uncertainty on the true nature of the fractional states.

2.4 Quantum Conductance in 1D systems

2.4.1 Landauer Formula

In order to measure the conductance of a mesoscopic system an understanding of the transmission coefficient, $T(E)$ as a function of energy, $E$ in a 1D system is required. The $T(E)$ is defined according to the relation between the energy, $E$, of the electrons and the height of a potential barrier, $U_0$, see Fig. 2.10, which can have three possible values, depending on the energy parameters [27, 38]. These transmission coefficients are derived as:

a. 

\[
E < U_0 \rightarrow T_{1D}(E) = \left[ 1 + \frac{U_0^2}{4E(U_0 - E)} \sinh^2(\kappa_2 L) \right]^{-1} \quad (2.32)
\]

b. 

\[
E = U_0 \rightarrow T_{1D}(E) = \left[ 1 + \frac{mL^2U_0}{2\hbar^2} \right]^{-1} \quad (2.33)
\]
Figure 2.10: Schematic diagram of a square potential barrier of potential height, $U_0$. An incident plane wave is coming from the left hand-side and partially reflected while it continues to transmit on the right hand-side. The parameters $t$ and $r$ are the factors of the original amplitude of the wave for the ones being transmitted and reflected, respectively. Adapted from reference [27].

c. 

\[ E > U_0 \rightarrow T_{1D}(E) = \left[ 1 + \frac{U_0^2}{4E(U_0 - E)} \sin^2(k_2L) \right]^{-1} \]  \hspace{1cm} (2.34)

where $k_2 = \sqrt{2m(E - U_0)/\hbar^2}$ and $\kappa_2 = \sqrt{2m(U_0 - E)/\hbar^2}$. Fig. 2.11 is a plot of the 1D transmission coefficient against the $E/U_0$ ratio where the ballistic length resonances can be seen [27, 38].

The $T(E)$ has to be converted in a quantity which can be measured with ease, therefore it needs to be expressed with the current-voltage relation [4]. Imagine the barrier as seen in Fig. 2.12 surrounded by a Fermi sea of electrons. By applying a positive bias voltage, $V$ to the right hand side, the energy is lowered by $-eV$ [4]. The distribution changes and therefore each side has a different quasi-Fermi level (Fermi levels are defined when a system is at equilibrium), $\mu_L$ and $\mu_R$ on the left and right side, respectively, that is $\mu_L - \mu_R = eV$.

To measure the current the assumption that the potential is constant throughout the leads is made. Consequently plane waves are used to describe the wave functions of the leads [4]. The current in one dimensional systems can be found by calculating the current due to electrons striking the barrier from the left, $I_L$ and that due to electrons arriving from the right, $I_R$. It is
important to define the positive direction of the conventional current flow, in this case from left to right. The current $I_L$ can be defined as,

$$I_L = 2e \int_0^\infty f[E(k), \mu_L]v(k)T(k) \frac{dk}{2\pi} \tag{2.35}$$

where $E$ is the electrons energy, $k$ is the wavenumber which only takes positive values as only the electrons incident on the barrier from the left side are considered, the Fermi function $f[E(k), \mu_L]$ is the probability that each state is occupied, governed by the Fermi level, $\mu_L$, $v(k)$ is the velocity of the electrons which converts the charge into current density and finally the $T(k)$ is the transmission coefficient, which gives the probability of the incident electrons passing through the barrier. Any electrons which are reflected are not considered in this case [4]. As integration over energy is easier the following expression is used

$$dk = \frac{dk}{dE}dE = \frac{1}{\hbar v}dE \tag{2.36}$$

to convert Eq. 2.35 in terms of energy, $E$, as follows,

$$I_L = 2e \int_{U_L}^\infty f(E, \mu_L)vT(E)\frac{dE}{2\pi\hbar v} = \frac{2e}{\hbar} \int_{U_L}^\infty f(E, \mu_L)T(E)dE \tag{2.37}$$
Figure 2.12: Schematic diagram of a Fermi sea of electrons surrounding a barrier. A positive bias is exists on the right side of the barrier. Having a small bias in an ideal case as shown in (a) the bands are flat immediately outside the barrier. In case (b), which is more realistic the bands curve, due to an accumulation layer on the left and a depletion layer on the right. Finally for case (c), if a bias is large enough, the electrons on the right make no contribution to the current [4].

where $U_L$ is the bottom of the band in the left hand side lead. Notice that the velocity cancels out, which is important as it emphasises the conductance quantisation. As the higher energy states have a higher velocity, they do not carry more current because their reduction in density of states cancels this out [4]. The equation for the $I_R$ current is exactly the same only there is a change in sign, given by,

$$I_R = -\frac{2e}{\hbar} \int_{U_R}^{\infty} f(E, \mu_R) T(E) dE \quad (2.38)$$

From the theory it is known that the $T(E)$ from both left and right side are the same, which is why the same $T(E)$ is seen in Eqs. 2.37 and 2.38. However, since the electrons on the right hand side are at much lower energies than on the left, the electrons within the range $U_R$ to $U_L$ cannot propagate to the left hand side as there are no corresponding energy states on the left hand side. As a result $U_L$ is taken as the lower limit for both equations [4]. The overall current expression is found by adding 2.37 and 2.38.
Figure 2.13: Schematic diagram of a one-dimensional quantum conductor. Between the two thermal reservoirs (ohmics), an elastic scattering potential, \( U(x, V) \), exists. The electrochemical potentials of \( \mu_1 \) and \( \mu_2 \) are the left and right potentials, respectively, of the ideal conductor. By applying a voltage, \( V \), a shift between the electrochemical potentials, \( \mu_L \approx \mu + eV \) in the left reservoir and \( \mu_R = \mu \) in the right reservoir, is created. Note that \( \mu_l \neq \mu_1 \) and \( \mu_r \neq \mu_2 \) [27].

\[
I = I_L + I_R = \frac{2e}{\hbar} \int_{U_L}^{\infty} [f(E, \mu_L) - f(E, \mu_R)]T(E)dE. \tag{2.39}
\]

By assuming that the bias is very small and by using Taylor series the difference in Fermi functions can be expanded to the lowest order [4]. Also by substituting \( \mu_L = \mu + \frac{1}{2}eV \) and \( \mu_R = \mu - \frac{1}{2}eV \), the following equation is obtained,

\[
f(E, \mu_L) - f(E, \mu_R) \approx eV \frac{\partial f(E, \mu)}{\partial \mu} = -eV \frac{\partial f(E, \mu)}{\partial E} \tag{2.40}
\]

where \( \mu \) is the Fermi level at equilibrium. This equation is obtained because the \( \frac{\partial f(E, \mu)}{\partial \mu} \) is only the difference of \( E-\mu \). The current can then be expressed as,
\[ I = \frac{2e^2V}{h} \int_{U_L}^{\infty} \left( -\frac{\partial f}{\partial E} \right) T(E) dE. \] (2.41)

Since Ohm’s law applies, as the applied voltage is directly proportional to the current for a small bias, the equation above can be rewritten using \( G = I/V \), as

\[ G = \frac{2e^2}{h} \int_{U_L}^{\infty} \left( -\frac{\partial f}{\partial E} \right) T(E) dE. \] (2.42)

Furthermore at very low temperatures, the Fermi function is sharper than features in \( T(E) \) and therefore \( -\frac{\partial f}{\partial E} \) can be substituted by \( \delta(E-\mu) \) \([4]\) and as a result the equation is further simplified to,

\[ G = \frac{2e^2}{h} T(\mu). \] (2.43)

This result is somehow counter intuitive in respect to classical understanding, as it indicates that for perfect system with \( T(\mu) = 1 \), the conductance will be \( G = \frac{2e^2}{h} \) irrespective of its length. This has brought a debate within the scientific community and shall therefore be addressed in the following section.

### 2.4.2 Landauer-Büttiker Formula

The case described earlier is solely for a single-channel case. However, in reality a system has multiple-channels, with different quantum modes propagating through the quantum wire and this does not alter the quantised conductance.

Let’s define the number of propagating modes as \( M(E) \), it follows through vigorous proofs, which can be found in detail in references \([4, 11, 16, 39]\), that the quantum conductance can be expressed as,

\[ G = \frac{2e^2}{h} MT. \] (2.44)

The contacts have a multiplicity of modes due to the fact that they are larger than the channel. The one-dimensional channel constrains the number of modes to carrier degeneracy. Therefore, due to these restrictions in the
number of modes to carrier degeneracy the quantum resistance does not change whether it is measured at the contacts or in the channel as it is restricted by the contact resistance of the ohmic contacts [11]. This equation though is assuming that the transmission through each channel is the same and it is perfect, i.e. $T = 1$. However, in a real device there are different transmission probabilities for each channel therefore, the summation of the various transmission probabilities for each channel needs to be considered. Additionally, in order to take into account the scattering between different modes the current in the $i^{th}$ lead is altered to,

$$I_i = \frac{e}{h}[(N_i - R_{ii})\mu_i - \sum_{j \neq i} T_{i,j}\mu_j].$$

(2.45)

where $N_i$ is the total number of 1D-channels in the $i^{th}$ lead and $\mu_i$ and $\mu_j$ are chemical potentials in leads $i$ an $j$ respectively, $R_{ii}$ is the reflection coefficient within the same lead and $T_{i,j}$ is the transmission coefficient between different leads. This formula derived by Landauer and Büttiker only holds true if the electrons are coherent and the transmission coefficient can be calculated by Schrödinger’s equation.
Chapter 3

Device Fabrication

3.1 Introduction

The realisation of a quasi-1D quantum wire can be achieved by confining the 2-DEG formed in a GaAs/Al$_x$Ga$_{1-x}$As or a InGaAs/InAlAs heterostructure by forming split-gates on their surface [6, 10]. Modulation-doped wafers were used for the fabrication of the HEMTs. The various fabrication steps of the devices will be briefly explained in sections 3.2-3.6, and in the final section 3.7, the whole fabrication process will be put in order. For the fabrication of the devices most of the steps where carried out in a cleanroom of ISO 14644-1 class 6 (Class 1000) [40, 41], in order to reduce pollutants affecting the quality of the devices fabricated [18, 19].

3.2 Scribing and Cleaving

A Karl Suss RA120 Automatic scriber was used to scribe the pieces of wafer, according to the size needed to process. By changing the angle and the pressure of the stylus, the force by which the wafer is scribed can be adjusted. The wafers were grown along the <100> plane and the scribing is done along the major flat of the wafer which is along the [110] or [110], [18, 42]. After scribing the wafers are then cleaved.

3.3 Cleaning Wafer

The wafers are then cleaned from any debris. This is achieved by placing the wafer pieces in a beaker with acetone for 10 minutes and sonicating them in a water bath [20]. This is then repeated in methanol and isopropyl
alcohol (IPA), after which the wafer is dried with nitrogen, N\textsubscript{2}, gas [20]. After cleaning, inspection under the microscope takes place to ensure there are no contaminants left on the surface of the wafer. If so the process is repeated until no contaminants are present.

### 3.4 Lithography

In the micro- and nanofabrication process lithography has been evolved into a method of patterning shapes, as small as a few nanometres in size, on semiconductors. These patterns are used to fabricate complex structures by using resists, which are organic compounds. The pattern is transferred on the resist through photons or electrons, with photo-lithography or electron-beam lithography, respectively [18, 19]. Finally by using various chemicals, developers, the pattern is exposed and can be used in the next steps of the fabrication process. More information will be provided in the next sections on the two main lithographic methods using photons and electrons, i.e. photo-lithography and electron-beam lithography.

#### 3.4.1 Optical lithography/Photo-lithography

The outcome of the pattern transferred on the semiconductors depends on the type of resist used, which could be either positive or negative [10, 18, 19]. With positive resist the area exposed to radiation will have its chemical bonds broken down and therefore becoming soluble in a chemical developer. However, with negative resist the area exposed to radiation will harden and only the surrounding areas will be soluble, as can be seen in Figure 3.1. These patterns are then used for subtractive or additive processes [10, 18], that is to remove a certain layer from the wafer or to deposit a new material, e.g. evaporate metal, see Figure 3.2. In the fabrication of the devices discussed in this work ultraviolet (UV) light was used to expose the desired pattern on a positive photoresist. Positive resist is used, as it has more advantages. It offers higher resolution and assists in having better outcomes during lift-off processes, due to better edge profiles [20, 43]. This is a consequence of negative resist swelling when being developed [43]. The resist is spun on the wafer using a spin coater and after that a chromeplated mask, with the pattern required present on it, is used to transfer the design on the photoresist. This is done by exposing the areas of the mask not chromeplated, see Figure 3.3.

Initially the wafer is placed on a hotplate for a minute at 115°C, in order to remove any excess water present on the wafer surface due to humidity.
It is then positioned on a puck with a small aperture diameter so as to achieve better suction with the vacuum pump, attached to the spin coater. LOR 3B resist is deposited on the wafer using a pipette. It is ensured that no bubbles are present when depositing the resist on the wafer in order to achieve a coating as homogeneous as possible. In addition it is made certain that the resist is deposited in such a way so as to cover the whole surface of the wafer. The resist is spun at 2000 rpm for 60 s. After spin coating the wafer, a tissue wetted with acetone is used to clean the underside of the wafer so as to remove any excess resist that drips under it. This is to make sure the wafer is lying flat on the platform of the mask aligner, afterwards. After spin coating, the wafer is placed on a hot plate at 175 °C for 10 min to bake it [44]. This removes any excess water and solvents and

![Diagram](Image)

Figure 3.1: Flow diagram showing the effect of a positive and negative photoresist.
therefore hardens the resist. Note however that excess baking could cause cross-linking of the resist and making stripping off difficult, by either having prolonged baking or too high temperature on the hotplate. Then the wafer is spun again with S1805 resist at 5500 rpm for 60 s and then put on a hot plate for 60s at 115 °C [20, 45]. The bi-layer process is used in order to have
a good undercut, to make sure the lift-off process, described later, is easier after evaporating the metals [18, 46], see section 3.5.1.

The wafer is then placed on the platform of the mask aligner (Karl Suss MJB3 Mask Aligner). The pattern on the mask is aligned with the wafer and then pressed in contact with the mask. The exposure time is set for the duration of UV light being shone on the resist. When the exposure finishes, the substrate is placed in MF319 and left in there for 35 s. Then the wafer is washed in a deionised water bath, to stop the developer from reacting further with the resist. Finally the wafer is dried with N\textsubscript{2} gas.

The exposure and developing time of the wafer and resist is very important as it can alter the shape of the pattern transferred. Underexposure could result in the radiation not reacting sufficiently with the resist, thus leaving excess behind [18]. Overexposure can remove too much of the resist after developing [18]. In addition it should be noted that edge beading is a common problem that could affect the effectiveness of the photo-lithography procedure. To reduce this, high spin speeds and larger pieces of wafer are used [18]. Finally the patterns transferred are designed to be located at the central region of the wafer so as to avoid the edge-beaded areas, where the resist thickness can vary greatly.

3.4.2 Electron Beam Lithography (EBL)

Similarly to photo-lithography this process is used to transfer patterns onto the wafer by using a resist. However, with the EBL smaller resolutions are achieved, of the order of a few nanometres.

The resist used is sensitive to energetic electrons and similarly to photolithography can be positive or negative resist [18]. The resist used for the devices, discussed later, is positive and is made of polymethyl methacrylate, PMMA. If the resist used is PMMA 950K-A4, for example, then 950K represents the molecular weight of the resist, i.e. 950,000 g/mol and A4 means that the PMMA is dissolved in 4% anisole [47, 48]. PMMA has a resolution of \( \sim 0.01 \mu \text{m} \) and has low sensitivity to light. The bi-layer process is used to coat the wafer, that is two layers of a resist are used. This is usually done with two different resists in order to improve the undercut of the pattern and therefore have a higher success during lift-off after evaporating metals. In addition the bi-layer process is used so as to reduce proximity effects. This is achieved as the bottom layer acts as the substrate in this case and therefore is the main source of backscattering electrons. As a result only electrons with sufficiently high energies can penetrate the second layer and therefore produce finer structures [18]. The PMMA used for each feature,
i.e. split-gate, cross-linking and top-gate are shown in Table 3.1. To develop the pattern after exposure a solution of methyl isobutyl ketone (MIBK), IPA and Methyl Ethyl Ketone (MEK) with a ratio of 5:15:1, respectively, is used. The developing time is 8 seconds, and then the wafer is dipped in IPA for 15-30 s to cease further development. Finally PMMA can be used as a dielectric between multiple gate patterns as explained in a latter section of the report. This is achieved by cross-linking the PMMA by using a higher dosage [20, 49]. This is due to the fact that after a certain dosage level the PMMA rather than breaking down by irradiating it with an e-beam, the molecules start to fuse instead by forming cross-links. Resulting in a material which is very difficult to remove even by adding it in acetone. The advantages of using this method is the ease and high resolution of obtaining sub-micrometer features, as opposed to other dielectric growth methods for multilevel devices, e.g. PECVD [18, 49].

Table 3.1: Information on the parameters used to spin-coat the samples with PMMA in order to define features on them with EBL.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Layer</th>
<th>Resist</th>
<th>Spin Speed (rpm)</th>
<th>Spinning Time (s)</th>
<th>Hot Plate Temperature (°C)</th>
<th>Hot Plate Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Split- &amp; Top-gates</td>
<td>1st</td>
<td>PMMA 100K A6</td>
<td>8000</td>
<td>60</td>
<td>180</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2nd</td>
<td>PMMA 950K A1 (anisole: MIBK, 1:5)</td>
<td>8000</td>
<td>60</td>
<td>180</td>
<td>3</td>
</tr>
<tr>
<td>Crosslink</td>
<td></td>
<td>PMMA 950K A4</td>
<td>4000</td>
<td>60</td>
<td>180</td>
<td>3</td>
</tr>
</tbody>
</table>

Process

Before spin coating the wafer, it should be given a quick rinse with IPA and dried with N₂. The chip is spun with PMMA according to the parameters in Table 3.1. After coating the wafer, it is placed in the EBL to pattern the various features, that is the split-gates, cross-linked PMMA and the top-gate, for which the corresponding parameters are given in Table 3.2. Notice in Table 3.2 that for the cross-linking part, a larger aperture and lower energy beam is used. This was rendered as crucial in order to reduce the exposure time. The pattern is designed in software like AutoCAD and modified to a suitable file format for the EBL system to recognise it. After patterning the wafer, it is removed from the EBL system and developed as mentioned in the previous section for the split-gates and top-gates. However, after cross-linking, MIBK:IPA:MEK is NOT used and instead the wafer is placed in acetone in order to remove the resist that has not been cross-linked.
Table 3.2: Information on the parameters used with the EBL system in order to transfer the patterns required.

<table>
<thead>
<tr>
<th></th>
<th>Split-gate</th>
<th>Crosslinking</th>
<th>Top-gate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (keV)</td>
<td>30</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>Aperture (µm)</td>
<td>20</td>
<td>120</td>
<td>20</td>
</tr>
<tr>
<td>Dose Factor</td>
<td>1.2</td>
<td>500</td>
<td>1.4</td>
</tr>
<tr>
<td>Area Dosage</td>
<td>300</td>
<td>420</td>
<td>300</td>
</tr>
<tr>
<td>Curve Dosage</td>
<td>300</td>
<td>420</td>
<td>300</td>
</tr>
<tr>
<td>Line Dosage</td>
<td>900</td>
<td>N/A</td>
<td>900</td>
</tr>
</tbody>
</table>

Then it is dipped in IPA to remove the acetone and dried with N₂, ready to proceed with the following steps.

3.5 Wet-etching

Wet etching was used to define the mesa structures in order to isolate the devices, i.e. the 2DEG, from its surroundings [18].

The etching occurs by using chemicals which initially oxidise the surface and then dissolve it. Resulting in the removal of gallium and arsenic atoms [18]. Therefore, requires an oxidiser and dissolving agent. For fabricating these devices, a solution of sulfuric acid:hydrogen peroxide:water, $\text{H}_2\text{SO}_4$:H$_2$O$_2$:H$_2$O, with the ratio of 1:8:120 [20]. The $\text{H}_2\text{O}_2$ acts as the oxidising agent and the $\text{H}_2\text{SO}_4$ as the dissolving agent [18].

The etching depth depends on the location of the 2DEG within the wafer. If it is located at 90 nm from the surface the depth of the etching is aimed to be around 130 nm for GaAs/AlGaAs and 180 nm for InGaAs/InAlAs, in order to ensure complete isolation of the 2DEG within the mesa structure.

Additionally the temperature and the time allowed for the mixture to stand, might change the etching rate [18]. The process followed is as follows:

a. The $\text{H}_2\text{O}_2$ was mixed in with the water in the ratios described earlier. Subsequent to this the $\text{H}_2\text{SO}_4$ was added to the water mixture. The solution was thoroughly stirred and left for at least an hour so as to allow the solution to reach an equilibrium due the fact that the
reaction is exothermic.

b. Solution covered with a watch glass, so as to not have selective evaporation.

c. After leaving the solution to stand for sometime, the wafer with the photoresist was dipped for 5 s in the solution.

d. Quenching of the etching took place by adding the wafer in a DI water bath. Then dried with N₂.

e. The etch rate was checked by measuring the profile height of the mesa structures using a surface profiler, DektakXT. It should be \( \sim 10 \text{ nm/s} \).

f. The wafer with the photoresist was dipped again into the solution for the remaining time required to etch to the depth needed, dependant on the etch rate.

g. The etching was quenched once more using DI water and dried.

h. The DektakXT was used to check the profile height of the Mesa.

i. Once the required depth has been reached removal of the photoresist, capping the mesa structure, was done by immersing it in acetone for at least 60 s.

j. Finally the acetone is removed by submerging the wafer in a beaker of IPA and dried with N₂.

### 3.5.1 Ohmic Contacts

**Thermal Evaporation**

For the ohmics to be fabricated, thermal evaporation of layered nickel, Ni, gold-germanium, AuGe and gold, Au are needed. Nickel is used as a wetting agent in order to increase the adhesion of AuGe on the wafer. The gold layer on top of the AuGe is added so as to reduce the sheet resistance of the ohmic contacts [18, 50]. Furthermore it improves the bonding required for packaging the device, see section 3.6. Finally it also improves the morphology of the contacts. The process followed for depositing the metal layers on the wafer using thermal evaporation is as follows:

a. After etching the mesa structures and defining the ohmic pads with photo-lithography, as explained in sections 3.5 and 3.4.1, respectively,
the wafer was cleaned using an HCl to H$_2$O (1:2) solution. This was to remove any oxides formed on the surface and therefore reducing the adhesion of the metals evaporated. The wafer was dipped in the solution for 10 s and then quenched the reaction of the HCl with the wafer’s surface by immersing the wafer in a water. The HCl does not react strongly with the resist, leaving it stable for the evaporation.

b. The wafer pieces were then placed on the mounting plate of the evaporator as soon as possible so as to reduce any oxide being formed on the surface.

c. The system was pumped down to a pressure of the order of 10$^{-7}$ mbar.

d. Using the knurled knob, see Figure 3.4, the materials used for the evaporation were selected. The parameters for each material were set in the programme of the thermal evaporator. The metals were evaporated in the following order, Ni, AuGe, Au with thickness of 30 nm, 150 nm and 100 nm, respectively.

e. The mounting plate is rotated and the shutter was ensured to be shut, initially.

f. The power button is set to LT and then the current is slowly, slowly increased until the metal starts melting, as can be seen from the viewing window, at which point the shutter is opened and the current is adjusted until 0.1 nm/s deposition rate has been established. As the metal evaporates this might need readjusting. The reason for having slow rate of deposition is to increase the homogeneity of the layers of metal being deposited.

g. As the metal thickness goal was reached the shutter is closed, so as to stop any residual flow of metal reaching the wafer and then the current is turned off.

h. After this, the next material to be evaporated was set by using the knurled knob again. Then step 6-7 were repeated, until all the metals had been deposited in the order aforementioned.

i. Eventually when all the metal layers are deposited turn off the power by switching from LT to zero and allow the system to cool down before removing the samples.
Figure 3.4: Thermal evaporator (A306 Edwards), used for the metallisation of the ohmic contacts. The knurled knob is used to control what materials are being evaporated, when the evaporator current is turned on [46].

**Lift-off**

Lift off took place in order to remove the excess metal deposited on the areas other than the defined ohmic structures. This is done by placing the wafer coated in the Ni/AuGe/Au layers in a beaker with acetone which "eats away" the resist and with it removing the metal layers deposited on it [18]. Therefore all is left behind is the metal deposited directly on the wafer. The acetone was allowed to react for several hours and if needed a pipette was used to agitate the resist in order to aid lift-off, by spraying it with acetone. After lift-off the samples are rinsed in IPA and dried with N₂ gas.

**Annealing**

As mentioned earlier, the fabrication of ohmic contacts is of utmost importance for reduced contact and sheet resistance within the device. This is achieved with AuGe deposited on the wafer as part of the ohmic metal layers. In order to have good quality contacts the Ni/AuGe/Au layers have to diffuse within the wafer to form the contacts [18]. Such a result can be attained by rapid thermal processing. The equipment used in the cleanroom
for this was the SSI Inc Solaris 150 Rapid Thermal Processor. The annealing of the metals takes place by heating up the metals and wafer in the system, encouraging reactions to occur between the metals and the GaAs surface. Consequently surface states are formed, increasing the quality of the ohmic behaviour of the contacts. The annealing takes place in N\textsubscript{2} gas rich environment, therefore formation of oxides on the surface is reduced as the heat is increased. The temperature is increased in an interval of 30 s to 400\textdegree C and remains at that temperature for 60 s, after which it is ramp down to 200\textdegree C and then allowed to cool down slowly with time. The resistance is usually measured as 40-50 k\textohm across the device.

### 3.5.2 Insulating the Metallic Gates

For the InGaAS/InAlAs sample an extra step was added before the split-gates and top-gates were fabricated, which is the deposition of SiO\textsubscript{2} using Plasma Enhanced Chemical Vapour Deposition technique (PECVD). This is due to the fact that InGaAs has a low Schottky barrier so the SiO\textsubscript{2} is used to isolate the mesa structure of the Hall-bar from the split-gates. About 50 nm of SiO\textsubscript{2} is deposited.

In addition to the SiO\textsubscript{2} layer, about 35 nm cross-linked PMMA was added in order to prevent the development of parallel conduction. This can occur due to the SiO\textsubscript{2} layer sometimes modifying the bandgap structure [51].

As the SiO\textsubscript{2} covers the whole device photolithography is used again in the same way as described earlier in order to expose the ohmic contacts again. After this process buffered hydrofluoric acid, HF is used by dipping the sample in it for 30 s. This removes the SiO\textsubscript{2} layers on top of the ohmics. The resistance fo the sample is then tested to see if all the insulating material has been removed from the ohmic contacts.

### 3.5.3 Split-gates and Top-gates

The metallisation of the split-gates and top-gates, is done using an electron-beam evaporator. The reason for using this method of metallisation is because it has higher accuracy on the thickness of the material deposited and it has lower material wastage as only the area were the e-beam strikes the metal targets are evaporated [52]. Therefore, since the features of the split-gates and to-gates are of the order of a few 100s of nanometres, this method is preferred.

The metals deposited are in the order of titanium and then gold, of 20 and 100 nm thickness, respectively. Similarly to the thermal evaporator, the
material is selected for the evaporation and then the e-beam is turned on and the material is evaporated at a rate of 0.1 nm/s. Then the shutter is open to expose the samples and as soon as the required layers are deposited, the shutter is closed and the power turned off.

3.5.4 Optical Gates

The metallisation of the Optical Gates is also done with the same process as the split-gates and top-gates. To define optical gates optical lithography is used, the same process as described for the ohmic contacts. Once again the HCl solution is used, as for the ohmic contacts, to clean the exposed areas for the evaporation of the metals to form the optical gates. After the evaporation lift-off takes place, see section 3.5.1

3.6 Packaging

In order to load the samples fabricated, they have to be packaged in Leadless Chip Carriers (LCCs). To package the devices, they have to be scribed and cleaved to fit the LCCs. In order to secure them in the LCC, GE low temperature varnish is used to glue the backside of the sample to the LCC. The varnish is then allowed to dry and harden in order for the device to stick well to the LCC. Then the device is bonded to the LCC using a wedge bonder. The device is then ready to load into the cryostats and ready to load.

3.7 Full Fabrication Process in Order

3.7.1 GaAs/AlGaAs heterostructure

a. First the wafer is scribed and cleaved into manageable pieces for making processing easier.

b. The pieces of wafer for fabrication are cleaned.

c. Photo-lithography is used to define the mesa structure using photoresist

d. Wet Etching is used to remove the exposed wafer in order to isolate the 2-DEG within the mesa from its surrounding environment.

e. The areas of the mesa structure that are to be developed into ohmic pads are defined using photolithography.
f. Metals are thermally evaporated on the developed areas of the mesa structure to form the ohmic pads.

g. Rapid thermal processing in order to form ohmic contacts with the 2-DEG.

h. The fine structures of the split gates and top-gates are formed using electron-beam lithography, with a crosslink step in between.

i. Electron-beam evaporation is used to deposit the metals for the fine structures.

j. Optical gates are then formed, using once more photo-lithography.

k. The final e-beam evaporation takes place in order to deposit the metals for the optical gates.

l. Scribing and cleaving is used to separate all the mesa structures and device fabricated on the wafer pieces.

m. Each chip is packaged in a lead-less chip carrier (LCC).

n. Finally using a gold, Au, thread wedge bonder, bonds are formed from the LCC to the ohmic contacts and optical gates of the chip.

3.7.2 InGaAs/InAlAs heterostructure

A similar procedure is used for the InGaAs/InAlAs sample. The only difference is the deposition of SiO\textsubscript{2} layer as mentioned earlier. More details on the fabrication of these devices can be found in Ref. [53].

3.8 Devices Fabricated

3.8.1 GaAs/AlGaAS Wafer

The wafer used was obtained by our collaborators at the Semiconductor Physics group at the Cavendish Laboratory. The wafer used was T604 and the device G1 was used for the measurements obtained in Chapter 5 and 6. The structure of the heterostructure can be seen in the schematic Fig. 3.5.
Figure 3.5: GaAs/AlGaAs T604 wafer heterostructure.

Figure 3.6: InGaAs/InAlAs W436 wafer heterostructure.
3.8.2 InGaAs/InAlAs Wafer

The wafer used was obtained by our collaborators at the Semiconductor Physics group at the Cavendish Laboratory. The wafer used was W436 and the device In1 was used for the measurements obtained in Chapter 7. The structure of the heterostructure can be seen in the schematic Fig. 3.6.

![Diagram of Hallbar design](image)

Figure 3.7: Design used for defining the ohmics and the optical gates for the Hallbar. The parts with grey lines were used to define the initial mesa structure and the blue line structures were used to defined the optical gates during the photolithography.
3.8.3 Device Design

The design used for both the G1 and In1 devices is shown in Figs. 3.7 and 3.8. Figure 3.7 shows the full design of the chip, with the grey lines shapes indicating the pattern defined with the first photolithography mentioned in the previous sections used for the fabrication of the mesa and the blue lined shapes are the patterns used to fabricate the optical gates again by using photolithography. The design for the fabrication of the split-gates and top-gates using EBL are shown in Fig. 3.8. The red, green and yellow lined shapes, indicate the split-gates, the crosslinked PMMA and top-gates, respectively. The channel formed with the split-gates was fabricated with dimensions of 400 by 700 nm. Figures 3.9 and 3.10 show photos taken under the microscope showing the split-gates and top-gates as well as the finished packaged device for G1.

![Design Diagram](image)

Figure 3.8: Design used for fabricating the split- (red) and top-gates (yellow) by using the EBL as well as the crosslinked PMMA (green) between the two features.
Figure 3.9: Magnified view of split- and top-gates while coated with PMMA before defining the optical gates by using photolithography.

Figure 3.10: Final GaAs/AlGaAs (device G1 used for the measurements in Chapter 5 and 6) as seen from a microscope after fabrication and packaging it in an LCC.
Chapter 4

Experimental Techniques and Data

4.1 Two-Terminal Set-up for Conductance measurements

The quantum conductance plateaus were obtained by using a dilution refrigerator of the model Triton Oxford Instruments. The set up for the circuit used is shown in Fig. 4.1. The generator used was the a Krohn-Heit 4420b ultra-pure sine wave analogue signal generator model. It was set up to obtain a 77 Hz, a.c. voltage signal of 1 V, amplitude. The lock-in amplifier was a Stanford Reaseach System, model SR830 DSP. The signal was directed to a voltage divider, see Fig. 4.1 reducing the voltage to 10 µV, which was applied through a break out box onto the sample. The reason for reducing the voltage is to not overheat the sample, otherwise cooling down the sample for reducing thermal smearing of the Fermi function becomes pointless [10].

The input signal was placed on the left edge ohmic and the output signal was measured from the right edge ohmic. The sample’s output signal was then passed through a current to voltage preamplifier.

The current pre-amplifier’s output voltage is finally sent back to the lock-in amplifier. The lock-in amplifier then references the signal with respect to the 77 Hz frequency that was injected through the circuit and distinguishes the output voltage of the circuit. This detection from the lock-in amplifier is possible due to a small-bandwidth low pass filter. This measurement is then sent through a GPIB interface to a computer with which the data is recorded and further analysis can be done through software like Matlab or LabView. The group uses “Cryomass” which is a programme developed on
4.2 Four-Terminal Set-up for Resistance measurements

For measuring the $R_{xx}$ and $R_{xy}$ in a 2DEG system, in order to study it in the quantum-Hall regime, the 4-Terminal set up used is shown in Fig. 4.2. The same signal generator is used with the same set up as the 2-Terminal circuit. However, the signal is sent through a 100 MΩ resistor before entering the device through the left ohmic. The reason for this is that by using a large resistor the current passing through the circuit is constant but the voltage across the device is very small compared to the voltage across the resistor, so any small voltage changes in the system will be more sensitive to detect with the sensing probes. Then by using the 4 voltage probes,
the difference between the two-pairs can provide the measurements for $R_{xx}$ and $R_{xy}$ through the voltage pre-amplifiers and the lock-in amplifiers before being collected by the computer. Similarly to the 2-Terminal set-up the measurements were calibrated by using a 10 kΩ resistor so each pair of probes measured 10 kΩ as well.

4.3 Dilution Refrigerator

This type of system uses the unique properties of the $^3$He-$^4$He mixture described in Refs. [10, 54] in order to reach temperatures of a few millikelvin. The dilute phase ($^3$He poor) can be considered as the vapour phase of the concentrated phase ($^3$He rich). However, the concentrated phase is less dense than the dilute phase therefore it will float on the top of the diluted phase. Therefore, by pumping the system the temperature can be decreased [10, 54]. The pump removes the $^3$He atoms from the dilute phase (mostly $^3$He is removed as it is lighter than $^4$He). This affects the $^3$He atoms in the concentrated phase by causing them to evaporate and therefore cooling down the system. However, $^3$He atoms cannot be pumped out of the system indefinitely, therefore $^3$He is injected back into the system [54]. This process can be seen in Fig. 4.3.

The $^3$He-$^4$He mixture resides in the mixing chamber, as seen in the Fig. 4.3. The dilute phase is connected to the still through a tube.

Figure 4.2: Schematic diagram of a four-terminal set up in order to measure the $R_{xx}$ and $R_{xy}$ within a 2DEG.
still the $^3$He atoms are pumped out and are pumped back in the mixing chamber in order to maintain a constant closed loop for the cooling of the system to occur [54]. In addition the condensed $^3$He which is pumped into the mixing chamber is connected to the outgoing $^3$He from the chamber via heat exchangers, establishing a constant gradient of heat flow from the ingoing $^3$He to the outgoing $^3$He [54], as seen in Fig. 4.3. The still also has a heater connected to it in order to establish a constant vapour pressure, otherwise the circulation of the mixture will stop. The still is maintained at a temperature of $\approx 600$ mK as it is a good compromise in having mostly $^3$He atoms being pumped out of the system, but at the same time maintain a relatively high vapour pressure in order to have a fast cooldown [10, 54].

The measurements in this thesis were conducted in a dry dilution refrigerator. The difference of a dry dilution refrigerator, is that the inner vacuum chamber is not dipped directly in a $^4$He liquid bath in order to fill up the 1 K pot [54]. The 1K pot is not needed in the dilution refrigerator as a heat exchanger is used instead to further condense the ingoing $^3$He, as mentioned earlier. However the condensation by the extra heat exchanger and the restricted tubing size (impedance) in the ingoing $^3$He line means that a higher pressure exists, but this is compensated by having extra pumping

Figure 4.3: Schematic Diagram of the cooling process in a dilution refrigerator, showing the pumping of the dilute phase of $^3$He from the mixing chamber through the still. The concentrated phase is in orange and the dilute phase is blue. Image taken from Ref. [54]
power [54]. The advantage of a dry dilution fridge is that the turnaround of changing a sample is much faster and cool down takes only a few hours as opposed to a wet fridge which requires a couple of days. The sample cools down to a few milli-Kelvin by having the sample holder placed in the vacuum on the outside part of the mixing chamber and having it thermally anchored to it [54].
Chapter 5

Anomalous resistance (overshooting) and hysteresis in GaAs/AlGaAs 2DEG

5.1 Introduction

5.1.1 Motivation

The discovery of the quantum Hall effect (QHE) was of monumental importance to the scientific community. It provided the ability to study the quantisation effect within systems, through macroscopic measurements. By applying an increasing perpendicular magnetic field to a Hall bar with current passing through it, the quantisation of the Landau levels is observed on a macroscopic level, explained in more detail in Chapter 2. This phenomenon in conjunction with Zeeman splitting and many-body interactions leads to further quantised states to appear with two distinct regimes existing, that is the integer and fractional quantum Hall effect (IQHE/FQHE). Furthermore, by discovering the IQHE and FQHE the possibilities for advancement in technology had opened up by utilising the power of these phenomena for quantum technologies, as well as providing a more suited standardised unit for resistance in terms of the von Klitzing constant [55].

The predominant theory to explain the IQHE consisted of edge states described by the 1D Landauer-Büttiker formalism [56, 57], which travelled ballistically through the system. However, this does not adequately explain the quantum Hall picture observed experimentally [57]. Phenomena like the anomalous transverse resistance (resistance overshooting) in the QHE mea-
surements and the hysteretic-like behaviour in the longitudinal resistance, explained in more detail later within this chapter, are still not well understood. Theoretically, issues arise with the edge state picture, as it fails to account for screening and the modified behaviour under strong magnetic field [56]. The strong magnetic field alters the electron density of states [56, 57], resulting in the electron screening becoming highly dependent on the filling factor, $\nu$, as it decreases from its bulk value to zero at the edge of the 2DEG. In this chapter, we delve into these phenomena and attempt to explain their behaviour in a qualitative and quantitative manner.

5.1.2 Chapter Summary

The chapter is structured in the following way: initially the characteristics of the device and the measurement methods are presented in section 5.2. Afterwards in section 5.3, the theory part follows, explaining the resistance overshooting effect and the hysteretic behaviour with the various mechanisms involved for the spin relaxation of the nuclei and the electrons. Furthermore the presentation of the experimental data and a discussion on their meaning takes place in section 5.4. Finally the chapter concludes in section 5.5.

5.2 Sample and Methods

A GaAs/Al$_{0.33}$Ga$_{0.67}$As heterostructure, grown by molecular-beam epitaxy, with a high-mobility 2DEG was used to fabricate the device, see Fig. 5.1. The device used for this chapter is G1 for which more information can be found in Chapter 3. It consists of the cap layer made of GaAs, then a layer of AlGaAs doped with Si and the spacer layer also consisting of AlGaAs. At 100 nm below the surface and the interface of the GaAs and AlGaAs spacer layer the 2DEG is formed. It was estimated to have an electron (carrier) density in the dark/light of $n_0 = 1.6 \times 10^{11} \text{ cm}^{-2}$ ($4.5 \times 10^{11} \text{ cm}^{-2}$) and a mobility of $\mu_e = 0.37 \times 10^6 \text{ cm}^2/\text{Vs}$ ($1.05 \times 10^6 \text{ cm}^2/\text{Vs}$). A red LED was used to illuminate the device (for $\sim 10$ s in steps of 2 s and by applying -10 V) [10, 53]. The Hall bar was fabricated by the methods explained in Chapter 3. The geometry of the Hall bar and the size of the metallic split-gates and top-gates can be seen in Fig. 5.2, which were fabricated through lithographic methods. The magnetotransport measurements took place within a dilution refrigerator at 10 mK and by using the low-frequency lock-in amplifier technique, see Chapter 4.
The quasi-one-dimensional, quai-1D, conductance in the electron gas versus voltage applied on split-gates, $V_{sg}$, can be seen in Fig. 5.3, which was measured using the 2-Terminal method, refer to Chapter 4. This was used as the reference for the $V_{sg}$ applied in the measurements described in the following sections of this chapter. The longitudinal, $R_{xx}$ and transverse, $R_{xy}$ resistance of the sample was measured using the 4-terminal method, see Chapter 4. The ohmics and split-gates used for the set-up are shown in Fig. 5.4.

![Figure 5.1: Schematic diagram of the GaAs/AlGaAs heterostructure used for the device fabrication. The elements present in each layer are labelled, as well as the distance of the 2DEG from the surface, $d_{2degsrf}$ and the thickness of the wafer, $d_{sample}$. S.I. GaAs is a semi-insulating GaAs layer acting as the substrate. (The schematic is not to scale)
Figure 5.2: Schematic of chip used for this chapter, G1. In the top left image a top-down view is shown. The dark yellow rectangles are the ohmic contacts and the bright yellow structures are the Ti/Au optical gates used to create contact to the split-gates and top-gates seen in the zoomed in pictures in the top right and bottom pictures. The width of the Hall bar $W_H = 80\ \mu m$ (as seen in the top-right figure) and it’s length is $L_H = 1400\ \mu m$. The width and length of the quasi-1D channel formed by the split-gates are $400 \times 700\ \text{nm}$, respectively. For the rightmost split-gate pair, the top-gate was removed from the animation in order to have clarity of the split-gate design. The semi-transparent red-pink boxes represent the dielectric (PMMA) used to insulate the top-gate from the split-gates.
Figure 5.3: $G$ versus $V_{sg}$, with the signature quantised conductance plateaus for a 1D system. $V_{tg} = 0$ V, $B = 0$ T and $T \approx 10$ mK.
Figure 5.4: Schematic of the 4-Terminal setup used for the measurements, as explained in Chapter 4. $V_{sg1}$ and $V_{sg2}$ correspond to the split-gate pair used for the measurements and $V_{tg}$ is the corresponding top-gate for that pair.
5.3 Theory

5.3.1 Anomalous transverse resistance (overshooting)

Introduction

Resistance overshooting is the observation of non-monotonic increase of Hall resistance at the lower end of the magnetic field of the quantised plateaus, forming at integer filling factors, \( \nu \), which are defined by the number of occupied quantised (spin resolved) Landau levels (LL) below the Fermi energy, see Fig. 5.5.

Various explanations have been provided over the years for this phenomenon, but fail to administer strong evidence for their validity. The first suggestion was due to non-ideal probes [58–60], which result in unequal current distribution, among the edge channels. Other suggestions were the decoupling between the magnetic field of the two edge states associated with the top-most spin-split Landau level [61] and the non-equilibrium population of electrons at sample boundaries due to bulk properties, causing scatter-
Figure 5.6: Sketch of the 2DEG lying in the \((x, y)\) plane and how it changes its landscape to compressible (grey) and incompressible (white) regions. The local current densities \(j_x\) driven by local electrostatic potential gradients in the \(y\) direction are indicated by arrows, when the system is at thermal equilibrium. (Modified image from Ref. [70])

The screening theory on the other hand, which takes into account the Coulomb interactions between charged carriers, illustrates this phenomenon within a framework which matches the experimental observations and the changes that occur in the system due to the strong magnetic fields [63–67]. A qualitative explanation was initially provided by Beenakker [68] and Chang [69] which considered the electron gas being divided into strips which alternate between incompressible (ISs) and compressible states (CSs). However, Beenaker stated then, that the same principles would not apply for fractional states [68]. On the contrary, Chklovskii et al. [56] argued that these states can successfully explain the QHE. Today though it is understood that a similar model can explain these states as well, by taking into consideration the composite Fermion (CF) model. A representation of how the incompressible strips (ISs) and compressible strips (CSs) are formed within the 2DEG can be seen in Fig. 5.6.

The CSs and ISs are formed at magnetic fields for a spatially constrained 2DES, resulting in the Fermi energy alternating between a position that overlaps with the Landau level and regions where it lies between the consecutive energy levels, respectively. The latter, IS is comparable to an insulat-
ing region which is formed at the plateaus of the quantum Hall resistance, corresponding to specific filling factors and separated by the conducting, metal-like, CSs. From the screening theory the overshooting is thought to be a consequence of the decaying IS near the edges of the plateaus, i.e. they become evanescent and co-exist with ISs from adjoining filling factors which are also in the evanescent regime, leading to current leakage and causing the overshoot effect.

A more quantitative explanation on the effect of screening on the edge states in the presence of a magnetic field, was given by Chklovskii et al. [56], for both integral and fractional states. The model states that for overshooting to occur, the ISs have to enter the evanescent regime. That is the ISs widths, $W_{IS}$, must satisfy the condition $l_B < W_{IS} < \lambda_F$. The length scales defining this regime are the magnetic length, $l_B$ and the Fermi wavelength, $\lambda_F$. The former is given by $l_B = \sqrt{\hbar/eB}$, with $\hbar$ being the reduced Planck’s constant, $e$ the elementary charge and $B$ is the magnetic field and the latter is defined as $\lambda_F = \sqrt{2\pi/n_0}$, with $n_0$ being the bulk electron density of the 2DEG. If the $W_{IS}$ is less than the $l_B$ then the ISs collapse. If the $W_{IS} > \lambda_F$, then the ISs are well defined. However, within this framework the calculations are performed in a non-self consistent manner. Additionally assumptions like the 2DEG being located at the plane of $z = 0$ and the electrons being depleted by in-plane metallic gates on the same plane and generally having an oversimplified picture on the boundary conditions, lead to an unrealistic model with wide ISs [57, 65].

Subsequent work by Lier and Gerhardts [71] and Siddiki et al. [72] combat this by modifying the Chklovskii model with self-consistent calculations and taking into account Hall bar geometry and chemical etching of the mesa structures, matching to a great extent the experimental work in the regime of the IQHE. Examples of experimental work on the IQHE can be found in the works of Sailer et al. [63] and Kendirlik et al. [73], where Si/SiGe and GaAs/AlGaAs heterostructures have been used, respectively. Furthermore through the theoretical work by Salman et al. [65] the model is extended to include fractional states. Experimentally the behaviour of incompressible strips for fractional states, has been studied before, but only in the context of magneto-capacitance [74] and edge magneto-plasmon [75] measurements. Consequently one of the aims of this Chapter is to present the application of this framework in terms of ISs for fractional states, but in the context of magneto-resistance measurements and specifically the effect on Hall resistance overshooting. A comparison on the Hall resistance anomalies found not only in the IQH regime but in the FQH regime is to take place between the theory of self-consistent calculations with experimental data obtained
Figure 5.7: Density profile of the sample is shown as a plot of the density ratio $n_{el}(x)/n_0$ versus $x/W_H$. The solid lines are for the Hall bar’s width, $W_H$, and various $t$-parameters. The solid lines are for a Hallbar of $W_H = 80 \mu m$ and the dashed one for one with $W_H = 0.5 \mu m$. The inset shows a zoomed-in section of the density profile for the $W_H = 80 \mu m$ Hall bar. For each $W_H$ the effect of a couple of $t$-parameter values are also shown, i.e. for $t = 1$ and $10$.

and shown in section 5.4.

**Electron density model**

In order to calculate $W_{IS}$, first the depletion layer’s width, $l_d$, i.e. the area between the edge of the mesa and the boundary of the 2DEG which is a compressible region, has to be calculated. This is given by the empirical formula defined by Salman et al. [57] as,

$$l_d = \frac{\pi}{n_0a_B^2} \left( \frac{d_{2deg-srf}}{d_{sample}} \right) \left[ c_3 - \frac{d_{dnr-srf}}{c_3a_B^2} e \left( \frac{-d_e}{10a_B} \right) \right]$$

(5.1)
From Eq. 5.1, $a_B^*$ is the effective Bohr radius given by $a_B^* = \frac{\hbar^2}{m^*_e c^2}$, with an effective mass, $m^*_e = 0.067$, calculated using the method described in Ref. [76], and a permittivity for GaAs given by $\epsilon = \epsilon_r \epsilon_0 = 12.4 \epsilon_0$, with $\epsilon_0$ being the permittivity of free space [65]. The constant $c_3$ is referenced as $\sim 4.5$ from Salman et al. [57]. In addition $d_{2\text{deg}-\text{sr}f} = 100$ nm, is the depth of the 2DEG from the surface of the mesa, $d_{\text{sample}} = 500 \text{ nm}$, is the thickness of the wafer, $d_{\text{dnr}-\text{sr}f} = 60$ nm, is the depth at which the donors are located from the surface and $d_e = 130$ nm is the depth by which the sample was chemically etched in order to form the mesa for the Hall bar. For this sample the depletion length was estimated as $l_d \approx 20$ nm.

Then by using the self-consistent calculations which take into account the geometry of the Hall bar the electron density distribution is estimated by [65],

$$n_{el}(x) = n_0 \left(1 - e^{-(|x-W_H-l_d|)/t}\right) \tag{5.2}$$

where $x$ is the position along the width, $W_H = 80 \mu m$, of the Hall bar and $t$ is an empirical parameter which specifies the slope of the electron density profile. From Salman et al. [57] it is defined as $t \approx 10a_B^*$. An example of how the density profile varies with different Hall bar widths and $t$-parameter values is shown in Fig. 5.7. As seen for values of $t = 1a_B^*$ the slope is more steep compared to the case where $t = 10a_B^*$. Also notice how for the narrower Hall-bar of $L = 0.5 \mu m$ the density profile has a large curvature and a relatively larger depletion length. The local filling factor at both the integer and fractional ISs is specified by

$$\nu(x_{k,f}) = \pi l_B^2 n_{el}(x_{k,f}) = \{k,f\}, \tag{5.3}$$

where $k = 1, 2, 3, \ldots$, as it represents the integer values and $f$ takes fractional values corresponding to the respective fractional states [65]. Furthermore by using Eqs. 5.2 and 5.3 the expression for the central position of the incompressible strips can be established as,

$$x_{k,f} = |W_H - l_d| + t ln(1 - \{k,f\}/\nu_0) \tag{5.4}$$

if $\{k,f\} < \nu_0$ condition holds with $\nu_0 = \pi l_B^2 n_0$, being the bulk filling factor [65].
Finite wave-functions and integer states’ widths

The analytical expression for calculating the ISs widths with integer filling factors is provided by [56, 65],

\[ a_k = \sqrt{\frac{2e\Delta E}{\pi^2e^2dn_{el}(x)/dx|_{x=x_k}}} \]  \hspace{1cm} (5.5)

were \( dn_{el}(x)/dx|_{x=x_k} \) is the derivative of the density, \( \Delta E \) is the energy gap between the adjacent quantised levels. The expression can be modified by taking into consideration the Thomas Fermi approximation (TFA) and the modified density profile, as explained in Ref. [65, 77], to give the equation,

\[ a_k^{TFA} = \sqrt{\frac{4\alpha_k a_B^* l}{\pi(\nu_0 - k)}} \]  \hspace{1cm} (5.6)

where \( \alpha_k \) is a dimensionless strength parameter and gives the ratio of energy gap, \( \Delta E \) between consecutive filling factors (different for odd and even \( \nu \)), by taking into account the Zeeman energy \( g^* \mu_B B \) and the cyclotron energy, \( \hbar \omega_c \), with \( \omega_c \) being the cyclotron radius of the electrons [56, 65]. The \( \alpha_k \) parameter is given as [57],

\[ \alpha_k = \frac{\Delta E_k}{\hbar \omega_c} = \begin{cases} \frac{(g^* \mu_B B)/\hbar \omega_c}{\hbar \omega_c}, & k = \text{odd} \\ \frac{(\hbar \omega - g^* \mu_B B)/\hbar \omega_c}{\hbar \omega_c}, & k = \text{even} \end{cases} \]  \hspace{1cm} (5.7)

The effective Landé \( g^* \)-factor, \( g^* \), used in the calculations was estimated as \( \sim 11 \) using the method described in Refs. [78, 79]. Additionally \( \mu_B \) is the Bohr magneton.

It should be noted that by using the TFA to calculate the widths of the ISs, the finite widths of the wave-functions are neglected. Therefore, this method of calculating \( W_{IS} \) is feasible for only slow changing potentials on the magnetic length scale. Contrary to this for the condition \( a_k^{TFA} \lesssim l_B \) the TFA is invalid. In order to counterbalance this, Refs. [57, 77, 80] proposed the use of the quasi-Hartree approximation (QHA), as finite widths of the wave-functions can be included through substitution of the delta functions TFA with Landau wave-functions. By doing this though the energy eigenvalues are still described as in TFA. The widths of the ISs, within the QHA, can be approximated by [57, 77, 80],

\[ a_k^{QHA} = \left(1 - \frac{l_B}{a_k^{TFA}}\right) a_k^{TFA} \]  \hspace{1cm} (5.8)
By comparing the widths calculated by the two different approximations, with the $l_B$ scale, three regimes can be identified. The first regime is under the condition that $a_k^{TFA} < l_B$ is satisfied and the cyclotron motion of the electron looses its quantisation, hence the system exhibits classical Hall effect characteristics [65]. In contrast under the conditions of $a_k^{QHA} > l_B$, the IS with filling factor $\nu = k$ becomes wider than the extent of the wave, resulting in the bulk and the opposing sample edges to decouple and the IQHE is observed [65]. However, this is only as long as the $\nu_0 < k$. Last but not least, under the circumstances that $a_k^{TFA} < l_B < a_k^{QHA}$ the IS enters the evanescent phase [65]. As mentioned earlier, in this situation the electrons are able to tunnel across the strip, with the back-scattering being enhanced. Consequently the Hall resistance displays a deviation from the quantised resistance value, i.e. overshooting. Similarly this theoretical concept applies for the fractional states as well, but with slight modifications, which are to be discussed in the following subsection.

**Fractional states’ widths**

For the fractional states the CF picture by Jain [35] is used for this study. The filling factors of electrons and CF are linked by the equation [65],

$$\nu = \frac{\nu^*}{2p\nu \pm 1}$$  \hspace{1cm} (5.9)

where $\nu^*$ stands for the filling factor of the CF and $p$ is an integer determining the order of the fractional state [35, 65]. The energy gap expression for the fractional filling factors is given by [81],

$$\Delta f = c_f \frac{e^2}{d l_B}$$  \hspace{1cm} (5.10)

where $c_f$ is a coefficient determined by the corresponding filling factor. The value of $c_f$ ranges between 0.06 and 0.11 for the fractional states 1/3 and 2/3 [81]. For simplicity the value for the fractional states calculations performed within this paper is taken as $c_f = 0.11$ and set $p = 1$. Note that from the literature the difference in size of the energy gaps calculated using the various values of $c_f$ within the aforementioned range is negligible in our calculations, so it does not influence the results discussed later [65]. By substituting Eq. 5.2 and Eq. 5.10 in Eq. 5.5, one obtains
\[
T^{FA} = \sqrt{\frac{4l_B c_f t}{\pi(\nu_0 - f)}} \tag{5.11}
\]

which provides the IS width for fractional states.

In the work by Chklovskii et al. [56] it is proposed that similar regimes, to the ones mentioned earlier for the integral states, occur by comparing the \(W_{IS}\) of fractional states with the \(l_B\) scale. On the other hand, in Ref. [65], it was recommended that the comparison should occur with the cyclotron radius, \(r_{cf}\) of the CFs instead of \(l_B\). The \(r_{cf}\) for the CF can be estimated by,

\[
r_{cf} = l_B \sqrt{2(2p\nu^* + 2p + 1)} \tag{5.12}
\]

This was based on later theories by Chklovskii, Ref. [82], on the formation of fractional edge states using the CF theory. If this method of comparison is used then a normalised cyclotron radius, \(r_c = R_c / \sqrt{2} = l_B \sqrt{\frac{2n-1}{2}}\), were \(n\) is the Landau level index, for the integer states should be used as well, for a more sustainable comparison between integer and fractional states. Hence, identically to the previous subsection the evanescent IS for fractional states exist within the regime of \(a_f^{TFA} < r_{cf} < a_f^{QHA}\). This aspect on which length scale should be used for the comparison, shall be discussed in more detail, in relation to the data presented, in a later section.

**Current and ISs**

For the regime where the ISs are well established the current flows entirely within them. However, when the ISs break down the current flows increasingly in the bulk of the system. The Hall resistance is given by the equation

\[
\rho_{xy}^\nu = \frac{V_{Hall}}{I_0} = \frac{h}{e^2 \nu} \tag{5.13}
\]

which is derived in detail in Ref. [63, 65], with \(I_0\) being the current flowing along the Hall bar. From the equation it can be seen that as the IS breaks down the current density decreases and thus the Hall resistance must drop. Nonetheless, when more than one ISs are in the evanescent phase then the current density increases locally, leading to an increase in the Hall resistance,
as the local filling factor $\nu(x)$ also increases [65]. Therefore, for the case that more than one evanescent ISs co-exist, the Hall resistance is modified to,

$$\rho_{xy}^{\nu} = \frac{h}{e^2} \left( \frac{1}{\nu_1 \beta_1} + \frac{1}{\nu_2 \beta_2} \right)$$

(5.14)

where $\nu_1$ and $\nu_2$ are consecutive filling factors corresponding to an IS with $\nu_2$ having a higher $\nu$ value, i.e. it is observed at a lower magnetic field than $\nu_1$. Finally the factors $\beta_1$ and $\beta_2$ are proportionality constants that correspond to the distribution of the total current with respect to the filling factors associated with them.

### 5.3.2 Longitudinal resistance hysteresis

By studying the IQHE and the FQHE in more detail, it quickly becomes evident that the quantum Hall effect is rich in spin related phenomena. Such phenomena are the spin transitions of FQHS [83–85], skyrmionic spin excitations [86, 87] and ferromagnetic spin ordering [83, 88–90]. An indication for some of these phenomena is the hysteretic-like behaviour observed within the longitudinal resistance measurements during 4-Terminal measurements. However, understanding some of these observations and cross-referencing them with theory is a difficult task. This is due to the complex nature of electronic and nuclear spins. A possible mechanism is due to electronic spin coupling to nuclear spin via hyperfine interactions, which although weak it can alter the electronic spin properties, to a great extent [91]. Such behaviour is described as dynamic nuclear polarisation (DNP). Other reasons for DNP to occur are due to magnetic impurities and spin-orbit interactions [91]. However, for heterostructures of high quality growth, magnetic impurity effects can be neglected, as is in our case. Therefore to distinguish between hyperfine interactions and spin-orbit interactions, the relaxation times, $\tau$, needs to be considered. For hyperfine interactions the relaxation time will be approximately $\tau \geq 25$ s, so for any shorter times, spin-orbit interactions are the usual suspects [91]. Additionally explanations for the hysteretic phenomenon have been given due to Ising spin-glass [88] or Ising quantum Hall ferromagnets like behaviour [83, 90].

Finally an explanation for the hysteretic behaviour has also been given in terms of the formation of ISs and the screening theory, by Siddiki et al. [92]. It is pointed out that from their model it is observed that when at the plateau regime of the QHE, where the ISs are formed, the position of the ISs and the potential distribution change rapidly. In order for such changes
Figure 5.8: (a) Plot of transverse resistance, $R_{xy}$, versus perpendicular magnetic field (B), is shown. The dashed lines indicate the level at which the corresponding filling factors, $\nu$, labelled, are supposed to be observed. (b) The $R_{xx}$ versus magnetic field is shown. The position of the corresponding filling factors are indicated with the vertical dashed lines on the curve. $V_{sg} = 0$ V, $V_{tg} = 0$ V, $I = 10$ nA and $T \approx 10$ mK.

to occur in the system, the electrons need to be transported across the ISs (e.g. by thermal activation). In reality though these rapid changes are not easily achieved. This is due to the fact that the ISs are not connected to the ohmic contacts and the Fermi level does not exist within the IS
for the plateau regime. This could possibly result in a very slow electron density relaxation, even for the slightest of changes in the magnetic field [92]. Furthermore on this topic shall be discussed in the following section.

5.4 Experimental Data and Discussion

5.4.1 Anomalous transverse resistance

As seen in Fig. 5.8, plateaus which correspond to integer and fractional filling factors are seen in the transverse resistance data. However, overshoot anomalies are noticed, for the fractional states 4/3, 3/2, 5/3, 8/3, 10/3 and 7/2 and the integer ν = 3 and 5. As seen from Fig. 5.8(a), the plateaus for the fractional states, 4/3, 3/2, 5/3, 8/3 are well above the dashed lines which correspond to these filling factors. For filling factors 10/3 and 7/2, although no distinct plateaus are seen in the $R_{xy}$, there are striking minima which seem to occur in the $R_{xx}$ measurements that correspond to these fractional states. The reason for the plateaus, corresponding to these fractional states, not being observed is attributed to the fact that there is a large overshooting effect which seems to occur between the magnetic field range of 2.8 and 3.1, which encapsulates these fractional states as well as the integer ν = 3 state. An important observation is that the magnetic field positions of the minima observed in the longitudinal resistance measurement match the magnetic field positions of where the maximum width of the incompressible strips are located, by using the Eqs. 5.6 and 5.8 for the integer ISs and the equivalent ones for the fractional states, as seen in Figs. 5.8(b) and 5.9. Note however, that the longitudinal resistance does not reach zero value, this could indicate reflection and scattering taking place, thus increasing the resistance measured or could be due to charge build up.

In Fig. 5.9 the changes in the $W_{IS}$ for the various filling factors are illustrated through the crescents as the B increases. The $W_{IS}$ of odd integer filling factors are represented by the red coloured crescents. Similarly the $W_{IS}$ of an even and fractional ν are represented by blue and green crescents, respectively. It should be noted that as the B is decreased the crescent becomes narrower as the ISs enter the evanescent regimes. As the ν increases in value (i.e. the B decreases) the maximum $W_{IS}$ decreases, resulting in a decrease in plateau widths for $R_{xy}$. In addition an important feature is that the curvature of the crescents decreases. Consequently the overlapping of the ISs decreases by reducing the B until no overlapping takes place eventually. In this case no overshooting is noticed for ν > 5. This is due to the fact that with decreasing magnetic field the Zeemann splitting looses it’s strength and
Figure 5.9: $R$ versus $B$ at $BT$. The crescents represent the evolution of the IS widths for the integral and fractional states. Red coloured ones are for odd integer states, blue for even integers and green for fractional states. The right axis indicates the position and width variation of the ISs within the Hallbar. The dashed magenta lines correspond to the expected resistance values for each filling factor. $V_{sg} = 0$ V, $V_{tg} = 0$ V, $I = 10$ nA and $T \approx 10$ mK.

electron-electron interactions become less prominent. Also observe how the green crescents all overlap to a significant degree with IS of integer $\nu$, which is the reason why enhanced overshooting is noticed for these states.

Similarly in Fig. 5.10, the evolution of the widths of the ISs can be seen, for the odd integer filling factors in red colour, even integers in blue and green for the fractional states. The solid lines represent the evolution of the ISs’ widths as calculated using the TFA and the dashed lines the ones using QHA. The black dashed line is the $\lambda_F$, the solid black line is the $r_{cf}$ and the dashed pink line is the $l_B$. By studying the figure it can be seen that for the regime, $l_B < a_k^{TFA}$, $a_k^{QHA} < \lambda_F$, i.e. the evanescent regime of the ISs,
there are significant overshoot effects occurring. Also, notice how the more evanescent ISs overlap the higher the overshooting, which can be seen as an example for the $\nu = 3$, $10/3$ and $7/2$. The relatively large overshoot noticed for $\nu = 3$ is in agreement to measurements stated by Kendirlik et al. [73].

Furthermore note that our data seem to be in contrast to a suggestion that the $r_{cf}$ should be considered as the minimum length scale in defining the evanescent regime, as opposed to the $l_B$. This is illustrated in the example of $\nu = 5/3$ where if the $r_{cf}$ was considered it would suggest from our calculations in Fig. 5.10 that no overshooting should occur as there are no other ISs overlapping the evanescent region of the IS of $\nu = 5/3$, for example, as seen in the translucent orange shaded shaded area in Fig. 5.10. However, it can be seen from Fig. 5.8(a) that overshooting does take
Figure 5.11: $R_{xy}$ versus $B$, at varying temperatures. The upper and lower insets show zoomed in sections of the graph. The red arrows show the evolution of the $\nu = 5/3$ and $3$ plateaus, respectively. The dashed black lines are the expected values for the filling factor labelled. $V_{sg} = 0$ V, $V_{tg} = 0$ V and $I = 10$ nA.

place so defining the evanescent regime with the $r_{cf}$ does not explain the 
overshooting. Using the $l_B$ though to define the evanescent regime it can be seen in the translucent purple shaded region that there is overlapping of ISs within this region specifically for $\nu = 4/3$, $3/2$, $5/3$, $1$. Therefore the overshooting observed for $5/3$ as an example is better explained by using the $l_B$ definition rather than the $r_{cf}$ as suggested by Ref. [65].

Additionally by varying the temperature of the sample the evolution of the plateaus was examined, see Fig. 5.11. The temperature was varied from base temperature (BT) to 2 K. As can be seen from the upper inset of Fig. 5.11, by increasing the temperature the overshooting is suppressed with the weak plateaus for $\nu = 4/3$ and $3/2$ vanishing almost instantly with the increase in temperature. Temperature also leads to a steeper change of transverse resistance between $\nu = 1$ and $\nu = 2$ plateaus. However, the $5/3$
fractional state seems to persist for higher temperatures. Initially it seems that it flattens out at 500 mK and drops in resistance value, but still retains an enhanced Hall resistance compared to what is expected theoretically. By 800 mK it seems to drop down to a value that matches the dashed black line, marking the expected value and eventually smears out by 2 K.

Draw your attention to the fact that the 3/2 fractional state seemed to break down just below 300 mK. This matches experimental data from Fu et al. [93]. In addition it could be of vital importance that the 5/3 fraction survives at higher temperatures. The reason is that from Ref. [93] it is suggested that their might be a possible interplay between the bulk filling factor of 5/3 and the formation of the edge 3/2 fractional state. Although their results are not conclusive, on the origin of the 3/2, it is interesting to see that a possible interplay between the two states could be reflected in this data as well, due to the subtle similarities between the temperature dependence and due to the similarities between Ref. [93] and the split-gate data explained later on, seen in Figure 5.14. However, as stated by Fu et al. [93], explaining any edge reconstruction is a highly complicated field and at the present time their is no definitive explanation on a possible mechanism.

Additionally note how the right hand side of the $\nu = 2$ plateau also drops down to match the expected value for this $\nu$ in the upper inset of Fig. 5.11. Similarly the plateau corresponding to $\nu = 3$, seems to flatten out as the temperature is increased, but requires the temperature to reach 2 K in order for this to be achieved. Also, in contrast to the 5/3 state, the enhanced resistance seems to increase with increasing temperature with the plateau moving closer to the resistance value of the 7/3 fractional state. Once more, this temperature dependence of the $\nu = 3$ is in agreement to a previous study by Kendirlik et al. [73]. This was attributed to the fact that the IS for $\nu = 3$ is much narrower than the $\nu = 2$ IS, which does match our theoretical predictions, see Fig. 5.10 and 5.12. It is suggested that due to this at the lower magnetic field end of the $\nu = 3$ plateau the bulk strip is narrower than or of comparable size to the edge strip of $\nu = 2$ IS. This as a result leads to the overshoot being enhanced as the temperature increases and the $\nu = 3$ IS breaks down, the $\nu = 2$ IS overpowers it. Moreover, in further compliance to Ref. [73], the overshoot effect is only seen at the low field end of the odd integer filling factors. This is explained as being a consequence of the alternating gap size between the integer states being $\Delta E_{\text{even}} \gg \Delta E_{\text{odd}}$ [73].

Further studies took place by leaving the temperature constant at BT and varying the current across the Hall bar. Similarly to the temperature dependence the overshoot effect, diminishes as the plateaus flatten out by
increasing the current from 10 nA to 4550 nA. However, the overshoot effect corresponding to the fractional states $\nu = 10/3$ and $7/2$ and the integer state $\nu = 3$ does not increase in resistance value as the current is increased but rather drops down towards the expected resistance value corresponding to $\nu = 3$, i.e $R_K/\nu$, where $R_K$ is the von Klitzing constant equal to $R_K = h/e^2$.

Although this might be in contrast to the temperature measurements it is once again in agreement to the measurements from Kendirlik et al. [73]. Initially for the $\nu = 5/3$ state the plateau flattens out completely and drops to its $3R_K/5$ resistance value by the current value of 640 nA, but still persists for higher current values but its resistance value keeps dropping. In addition the $4/3$ fractional state plateau persists up to the current of 100 nA. This can be seen in Figure 5.13, where the black dashed lines indicate the evolution for some of the filling factor plateaus which exhibit overshooting behaviour. For the $\nu = 3$ state the maximum current, i.e. 4550 nA, had to be applied in order for the plateau to become completely flat. However, it should be noted that above the 640 nA current although the plateaus are flatter their corresponding resistance falls below the defined $R_K/\nu$ value, for the plateaus.
Figure 5.13: $R_{xy}$ versus $B$, for different sample currents. The dashed black lines show the evolution of the filling factors stated. The current of 10 nA corresponds to $V_{Hdc} \sim 260 \, \mu\text{V}$ and the 4550 nA corresponds to $V_{Hdc} \sim 0.1 \, \text{V}$. $V_{sg} = 0 \, \text{V}$, $V_{tg} = 0 \, \text{V}$ and $T \approx 10 \, \text{mK}$.

corresponding to $\nu < 3$. This could be due to the higher currents breaking down the IS. It should be noticed as well that the DC Hall voltage ($V_{Hdc} = R_K I$) for the $I = 4550 \, \text{nA}$ it is estimated as being $\sim 0.1 \, \text{V}$ for $\nu=1$. This is a large voltage, therefore quantum Hall breakdown takes place especially at the high B-field values, as noticed in Fig. 5.13. That is the current is a lot larger than the critical value so quantization ceases to exist [94].

One could suggest that this change in the behaviour of the plateaus by increasing the current is due to an increase in the bulk current density $j = I/W_H$ [63]. However, this is in contrast with the follow up data presented in Figure 5.14. Here the current was fixed at 10 nA and the temperature was held at BT but the size of the channel was altered by depleting the electrons under the split-gates on which the voltage, $V_{sg}$ was applied, resulting in an increase in current density. From Figure 5.14 it can be seen that
by increasing the $V_{sg}$ the overshooting is enhanced for all the integer and fractional states mentioned earlier and even seems to create an overshooting effect for the $\nu = 5$ state. Therefore thinking that the average bulk current density $j$ has an effect on the overshoot effect would be rather naive. Therefore it is concluded that the current $I$ alone, is the decisive parameter on the evolution of the overshooting. This is compatible with the work by Sailer et al. [63], with the only difference being that the constrictions were achieved lithographically rather through electrostatic methods as in our case. Rather the fact that the overshoot effect increases by decreasing the channel size, indicates that anomalous increase in the Hall resistance is not related to the bulk of the 2DEG [63].

As mentioned earlier it should be noticed how the plateau of the $3/2$ state becomes more prominent for $V_{sg} = -1.94$ V and the plateau for $\nu = 4/3$ dissapears while the plateau for the $5/3$ state persists even close to pinch-off values of $V_{sg}$.

The data fits well with the idea of the screening theory, that is as the electrons leak out of an IS within the evanescent regime, the electron gas will be heated up locally. The electrons will then scatter in the nearby compressible region at the low magnetic field end of a Hall plateau. The first IS to be affected will be the outer one as the overshoot is destroyed with increasing sample current. Nevertheless, the inner IS is less affected, leading to a preserved Hall plateau. Secondly another consequence of the increased currents through the sample is the tilting of the potential landscape [95] in the out-of-linear-response regime. Both evanescent ISs at one edge boundary become wider at the expense of both ISs at the edge boundary on the other side. For a certain current amplitude, the narrowest outer IS breaks down, resulting in a breakdown of the overshoot [63].

The channel’s size dependence on the overshoot can be explained as a consequence of the current starting to leakout from the inner IS at the lower magnetic end of the plateaus and it redistributes to the adjacent resistance minimum [63]. As the channel is further constricted, less current flows in the bulk. Therefore more current is confined within the adjacent IS which is in an evanescent state and is at a local resistance minimum. The result is that the overshoot amplitude increases due to the redistribution of the current between the evanescent IS and the bulk [63].
Figure 5.14: Upper panel shows $R_{xy}$, versus $B$, for different $V_{sg}$ and the lowest panel is $R_{xx}$, versus $B$, for different $V_{sg}$. $I = 10$ nA, $V_{tg} = 0$ V and $T \approx 10$ mK.
Also by using Eq. 5.14 and comparing the areas overlapping within the evanescent regime the amount of overshooting can be estimated to a good approximation. For example for $\nu = 5/3$ the overshoot at the B were the evanescent regime starts was measured as $17390 \text{ k}\Omega$. By calculating the areas of the overlapping currents for $\nu = 5/3$, that is taking into account the overlapped areas of $\nu = 1, \nu = 4/3, \nu = 3/2$ and $\nu = 5/3$, from Figure 5.10, and assuming the sum of the areas enclosed for each IS is proportional to the respective current for each IS, one can estimate the overshooting for $\nu = 5/3$ as $\sim 17607 \text{ k}\Omega$. The discrepancy is small enough to be considered a good match between the two values. From the table A2 in Appendix A the resistance values for the overshot quantum states, estimated from the overlapping areas, can be seen. Overall, the maximum mismatch between the measured resistance and the estimated resistance from the overlapping areas is $\sim 11 \%$, which is still relatively small.

The reason for the estimated resistance being less than the measured one could be due to the fact that more current leakage occurs, than theoretically predicted due to impurities or other factors, which could create more scattering within the system. For the cases that the estimated resistance was found in excess to the measured resistance, that could be attributed to a small extent to overestimation using the trapezoid rule for calculating the areas enclosed but still mainly due to other factors which are not considered in the theoretical model. Overall though the method of using the areas for estimating the overshoot resistance is to a great extent a precise method, further confirming that defining the evanescent regime as defined by the model is in good agreement with the experimental data. For a more detailed explanation on how the values of the overshoot resistance were calculated from the areas overlapping in Fig. 5.10, see Appendix A.

5.4.2 Longitudinal resistance hysteresis

A brief discussion on the hysteretic-like areas that seem to appear at the integer Hall plateaus of $\nu = 1, 2$ and 4 is to take place. As seen from Fig. 5.15 and it’s insets, for the longitudinal resistance measurements there are two traces the dashed line and the solid line which correspond to the rate of change of the magnetic field, $dB/dt > 0$ and $dB/dt < 0$, respectively. These loops seem to follow a clockwise direction, as opposed to hysteresis due to ferromagnets, except for $\nu = 4$ where a double loop is formed with the first one in a clockwise direction (at lower B-values) and then flips into a loop of anticlockwise direction (at higher B-values).

Interestingly they seem to be centred around the same magnetic field
values calculated for the filling factors, using the calculations for the IS widths. These loops seem to shrink in size as the temperature is increased and eventually disappear by 1 K, see Fig. 5.16. As for changing the sample current, it can be seen in Figs. 5.17 and 5.18 that the area of the loop is enhanced as the current in increased up to \( \sim 1200 \, \text{nA} \) and then diminishes and vanishes at the maximum current value, for all \( \nu = 1, 2 \) and 4. Furthermore as the \( V_{sg} \) is increased up to \(-2.81 \, \text{V}\) (near pinch-off regime), the hysteretic loop areas increase in size, in an exponential fashion, see Fig. 5.19. In addition it should be noticed that for a fixed \( V_{sg} \) and by varying the \( V_{tg} \) similar behaviour is also noticed see Fig. 5.20.
Figure 5.16: $R_{xx}$ versus perpendicular B, showing the change in $R_{xx}$ and its hysteretic behaviour, as the temperature is increased from BT to 2 K. The dashed line represents the evolution of $R_{xx}$ for increasing perpendicular B and the solid line for decreasing B. $I = 10$ nA, $V_{sg} = 0$ V and $V_{tg} = 0$ V.
Figure 5.17: $R_{xx}$ versus B, showing the change in $R_{xx}$ and its hysteretic behaviour, as the current is varied from 10 nA to 4500 nA. The solid line represents the evolution of $R_{xx}$ for increasing B and the dashed line for decreasing B. $V_{sg} = 0$ V, $V_{tg} = 0$ V and $T \approx 10$ mK.
Figure 5.18: Zoomed-in sections of $\nu = 4, 2$ and $1$ from Fig. 5.17
Figure 5.19: $R_{xx}$ versus perpendicular $B$, showing the change in $R_{xx}$ and its hysteretic behaviour, as the $V_{sg}$ is increased, consequently increasing the constriction within the quasi-1D channel. The solid line represents the evolution of $R_{xx}$ for increasing $B$ and the dashed line for decreasing $B$. $I = 10$ nA, $V_{tg} = 0$ V and $T \approx 10$ mK.
Figure 5.20: $R_{xx}$ versus perpendicular B. The system is set at fixed $V_{sg} = -2.12$ V and the $V_{tg}$ is varied. Consequently, the change in hysteretic behaviour is shown as the constriction of the quasi-1D channel increases. The solid line represents the evolution of $R_{xx}$ for increasing B and the dashed line for decreasing B. $I = 10$ nA and $T \approx 10$ mK.
Figure 5.21: $R_{xx}$ versus B. The hysteretic loop across the $\nu = 2$ is shown. The dashed line is for increasing B and the solid line for decreasing B. The outer loop in light-blue indicates the full hysteretic loop while the smaller loops in green, brown, yellow and red, correspond to loops within the larger envelope of hysteresis. The inner loops are separated to two distinct regions: (1) with large change in resistance and (2) with a slow change in resistance. The arrows indicate the overall flow of the hysteretic loops. The inner loops were formed by stopping the B sweep while it was increasing and then reversing the sweep direction within the enclosed hysteretic loop. $I = 10$ nA, $V_{sg} = -2.12$ V, $V_{tg} = -0.70$ V and $T \approx 10$ mK.

In order to further study the hysteretic behaviour, the larger loop was broken down to smaller hysteresis loops, see Figs. 5.21, 5.22, 5.23 and 5.24. For Figs. 5.21 and 5.23 magnetic field was increased across the hysteretic loops for $\nu = 2$ and 1, respectively and then was instantaneously stopped, while in the hysteresis regime. Then the direction of the magnetic field was immediately reversed. Similarly the same measurements were conducted.
Figure 5.22: $R_{xx}$ versus $B$. The hysteretic loop across the $\nu = 2$ is shown. The dashed line is for increasing $B$ and the solid line for decreasing $B$. The outer loop in light-blue indicates the full hysteretic loop while the smaller loops in red and green, correspond to loops within the larger envelope of hysteresis. The inner loops are separated to two distinct regions: 1 with large change in resistance and 2 with a slow change in resistance. The inset shows a zoomed in section. The arrows indicate the overall flow of the hysteretic loops. The inner loops were formed by stopping the $B$ sweep while it was decreasing and then reversing the sweep direction within the enveloping hysteretic loop. $I = 10$ nA, $V_{sg} = -2.12$ V, $V_{tg} = -0.70$ V and $T \approx 10$ mK.

with the opposite magnetic field directions. That is the magnetic field was decreasing and then immediately changed to a sweep with increasing magnetic field, see Figs. 5.22 and 5.24. Overall the smaller loops can be broken down to two distinct regions, 1 and 2. The 1 region has a rapid change in $dR_{xx}/dB$ and 2 is a slow change in $dR_{xx}/dB$ which follows a similar trend with the larger hysteretic loop. However, the minor loops enter the slow regime before reaching the opposite side of the larger hysteretic loop.
Figure 5.23: $R_{xx}$ versus B. The hysteretic loop across the $\nu = 1$ is shown. The dashed line is for increasing B and the solid line for decreasing B. The outer loop in light-blue indicates the full hysteretic loop while the smaller loops in red, magenta, black and green, correspond to loops within the larger envelope of hysteresis. The inner loops are separated to two distinct regions: ① with large change in resistance and ② with a slow change in resistance. The arrows indicate the overall flow of the hysteretic loops. The inner loops were formed by stopping the B sweep while it was increasing and then reversing the sweep direction within the enveloping hysteretic loop. $I = 10$ nA, $V_{sg} = -2.12$ V, $V_{tg} = 0$ V and $T \approx 10$ mK.

In addition the relaxation time of the hysteretic behaviour was studied for the $\nu = 1$ and 2 at various magnetic fields along the hysteretic loops. For $\nu = 2$ the magnetic-field was swept in an increasing manner and stopped at a specific magnetic field, e.g. B = 4.2, 4.4, 4.6, 4.8 and 5.0 T. The $R_{xx}$ was then recorded over a period of 4 hours. Similarly for $\nu = 1$ the same method was followed measuring $dR_{xx}/dt$ at fixed values of B = 8.5, 9.1, 9.4, 9.6 and 10.5 T. The measurements were also conducted by initially fixing the $V_{sg}$ and $V_{tg}$ at -2.12 V and -0.70 V, respectively. The relaxation behaviour of
Figure 5.24: $R_{xx}$ versus B. The hysteretic loop across the $\nu = 1$ is shown. The dashed line is for increasing B and the solid line for decreasing B. The outer loop in light-blue indicates the full hysteretic loop while the smaller loops in black, magenta and red, correspond to loops within the larger envelope of hysteresis. The inner loops are separated to two distinct regions: (1) with large change in resistance and (2) with a slow change in resistance. The inset shows a zoomed in section. The arrows indicate the overall flow of the hysteretic loops. The inner loops were formed by stopping the B sweep while it was decreasing and then reversing the sweep direction within the enveloping hysteretic loop. $I = 10$ nA, $V_{sg} = -2.12$ V, $V_{tg} = 0$ V and $T \approx 10$ mK.

$R_{xx}$ can be seen in Figs. 5.25 and 5.26. Similarly to the minor loops there seem to be two regimes of relaxation in $R_{xx}$, a fast one initially and then a slow one. Additionally it should be noticed that for the measurements taken at magnetic fields which are closer to the central part of the hysteresis loop, they have a larger drop in resistance as opposed to the measurements.
Table 5.1: Relaxation times $\tau_1$ and $\tau_2$ for the corresponding B-values associated with the hysteretic loops of $\nu = 1$ and $\nu = 2$ as estimated from the trend-lines fitted in Figs. 5.25 and 5.26.

<table>
<thead>
<tr>
<th>$\nu = 2$</th>
<th>$B$ (T)</th>
<th>$\tau_1$ (s)</th>
<th>$\tau_2$ ($\times 10^{-6}$ s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.2</td>
<td>1790</td>
<td>3.48</td>
</tr>
<tr>
<td></td>
<td>4.4</td>
<td>648</td>
<td>16.5</td>
</tr>
<tr>
<td></td>
<td>4.6</td>
<td>60</td>
<td>1.88</td>
</tr>
<tr>
<td></td>
<td>4.8</td>
<td>653</td>
<td>12.7</td>
</tr>
<tr>
<td></td>
<td>5.0</td>
<td>1460</td>
<td>40.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\nu = 1$</th>
<th>$B$ (T)</th>
<th>$\tau_1$ (s)</th>
<th>$\tau_2$ ($\times 10^{-6}$ s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8.5</td>
<td>6920</td>
<td>9.11</td>
</tr>
<tr>
<td></td>
<td>9.1</td>
<td>345</td>
<td>1.57</td>
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<td></td>
<td>9.4</td>
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<td>0.88</td>
</tr>
<tr>
<td></td>
<td>9.6</td>
<td>1430</td>
<td>2.47</td>
</tr>
<tr>
<td></td>
<td>10.5</td>
<td>1979</td>
<td>9.99</td>
</tr>
</tbody>
</table>

Conducted at B-values which are towards the edges of the hysteretic loop. In order to fully understand whether any spin relaxation processes take place within the system, further studies using the nuclear magnetic resonance (NMR) technique will need to be employed by future members of the group.

Furthermore, by fitting a trend-line to the measured data in Figs. 5.25 and 5.26 with an equation of the format,

$$a e^{(-t/\tau_1)} + b e^{(-t/\tau_2)} \quad (5.15)$$

where $a$ and $b$ are constants the relaxation time parameters $\tau_1$ and $\tau_2$ where estimated. These values for each B-field value and for the traces shown in Figs. 5.25 and 5.26 can be found in Table 5.1. Generally it is noticed that $\tau_1$ is smaller than $\tau_2$ with the former being of the order of a few tens of seconds to a few hundreds of seconds for both $\nu = 1$ and $\nu = 2$ and $\tau_2$ being in the order of at least a few $10^6$ s. As seen from Table 5.1 Relaxation times for both $\tau_1$ and $\tau_2$ tend to decrease for the B-fields located in the central points of the hysteretic loop, which would match the behaviour noticed in Figs. 5.21, 5.22, 5.23 and 5.24, as well as the fact that a faster and slower relaxation mechanism seem to take place.

A discussion about the various interpretations shall take place on the
mechanisms associated to the relaxation times in such phenomena. As mentioned in the theory section 5.3, one of the mechanisms associated with such behaviour is dynamic nuclear polarisation (DNP) due to hyperfine interactions [91]. The wafer used for this study is of high quality MBE-grown GaAs/AlGaAs material, therefore magnetic impurities are not considered. It should be noticed that the time-scales for the spin relaxation for our system is larger than a few 10s of seconds, which is too large to consider spin-orbit interactions and therefore hyperfine interactions are the most-likely cause in terms of DNP. That is the spin flip of a nucleus in the lattice of the GaAs (flip-flop process) is caused by an electron spin-flip, [91]. As a result part of the current flowing which is spin-resolved produces DNP at the edges of the system.

Another theory which is mentioned in the literature is due to phase transitions in the hysteretic loop, which form a spin-glass-like [88] phase. This however has no substantial experimental evidence other than the fact that the $R_{xx}$ relaxation behaviour seems to follow similar trend to magnetisation relaxation trends in spin-glasses or low-dimensional ferromagnets [88]. Therefore, there are still missing links that need to be clarified on a microscopic level about the hysteretic behaviour and should be further studied, in order to verify such a mechanism.

Such hysteretic-behaviour has also been noticed by Piazza et al. [90] for integer filling factors. They proposed that quantum Hall ferromagnetism is observed with a first-order phase transition, invoking the Ising model, where Landau level crossings create a broken-symmetry ground state with similar behaviour to isotropic, easy-axis two-dimensional ferromagnets [90]. However, they observe this behaviour just for even integer states ($\nu = 2, 4$) and their proposed theory only explains the observation of hysteretic behaviour for even states. This is in contrast with our observations of hysteretic behaviour in an odd integer state as well, i.e. $\nu = 1$. It should be noted though that in their case a 60 nm wide GaAs quantum well, sandwiched between two layers of $Al_{0.25}Ga_{0.75}As$ was used, therefore the difference in heterostructure could contribute in the discrepancies.

In addition to this study further work by Cho et al. [89], Smet et al. [83] and Eom et al. [88], also studied hysteretic behaviour but in fractional states which have a $\nu < 1$. From these studies it is also suggested that multidomain structures are formed due to time varying resistance measurements over the hysteretic regions. Ruhe et al. [96] and Budantsev et al. [97] propose that these effects are more likely associated to non-equilibrium currents (NECs). For the work in Ref. [96], a micromechanical cantilever magnetometer was used and although hysteretic loops were noticed for integer filling factors,
they were only for even numbered ones, as well.

An other important disparity with our data is that the hysteretic loops for their study flow in an anticlockwise direction [96], which is the opposite effect of what is seen in our case, that is a clockwise direction, that is in agreement with the work from Budantsev et al. [97]. For Budantsev et al. [97] there have been observations of hysteretic behaviour at not only even integer numbers but at odd ones as well. Also from their work it is shown that similar behaviour is noticed with varying temperatures and constriction sizes of the channel. The constrictions though were defined lithographically rather than with electrostatic methods using split-gates, as is our case. Budantsev et al. [97] suggested a theory using incompressible and compressible states and therefore matches the picture that is supported by the screening theory and the overshoot effect data presented in this chapter. It was suggested by them that as the magnetic field is varied an azimuthal electric field, is induced in the system causing a radial current density between the edges of the sample and the centre. This in turn creates a radial electric field, driving NECs. That is as \( \frac{dB}{dt} > 0 \), the electric field vortex creates an electron outflow from the edge to the bulk, resulting in a decrease in the area occupied by the incompressible strip and the ISs retreat from the edge of the mesa. Consequently as the constriction is increased by also increasing negatively the \( V_{sg} \), the opposite edge channels are brought closer together, leading to a higher backscattering and finally an increase in resistance which is also noticed in our results [97]. Vice versa when \( \frac{dB}{dt} < 0 \) the electric field formed pushes electrons from the bulk into the sample edges, resulting in the incompressible strips occupying more space and shifting closer to the mesa edges. Therefore, the opposite edge currents move further away from each other, compared to the equilibrium state. This reduced the backscattering taking place and the resistance is suppressed [97].

As for the enhanced hysteresis by increasing \( V_{sg} \) and \( V_{tg} \) a scattering effect is thought to be the cause for this, as suggested by Ruhe et al. [96] and Budantsev et al. [97].

However, the most simple explanation is given by Siddiki et al. [92]. As explained in the theory section it is suggested that due to the presence of ISs it makes it difficult for the system to cope with rapid changes of potential distribution, as the ISs are not connected to the ohmic contacts. Therefore, even with the slightest magnetic field change the potential distribution changes rapidly, but the electrons are slow to respond, therefore forming the hysteretic effect. This mechanism would also explain as to why we see the hysteretic loop centred on the B-values which correspond to the position at which the IS has the largest thickness. Also this could illustrate as to why at
these B-values the hysteresis is almost at its widest part. Additionally this would also explain as to why the hysteresis is larger for $\nu = 1$ than $\nu = 2$ and $\nu = 4$ in that order. As seen from Fig. 5.12 the ISs for the corresponding filling factors also follow a similar ratio difference in their widths. Finally as for the reason why the hysteresis is not observed for $\nu = 3$, could possibly be, as is the case for the large overshooting, due to the evanescent regime, making the ISs for $\nu = 3, 10/3$ and $7/2$ not well defined, and therefore altering the transition behaviour of the system for the changes in potential. Therefore no hysteresis is observed for $\nu = 3$. However, as stated earlier two loops of opposing flow directions seemed to be linked around the $\nu = 4$ regime, see Fig. 5.15. On closer inspection though it can be noticed that the leftmost loop is centred around the corresponding $B = 2.3$ T for $\nu = 4$ and the centre for the rightmost loop seems to be located at $\sim 2.6$ T which corresponds to $\nu = 7/2$. Although at the moment the flip in hysteretic loop direction, is not clear at the moment, there is the possibility that the interaction between the IS for the fractional and integer states within the region change the normal distribution of the potential. Furthermore, through this observation it raises the question whether the hysteretic effects noticed in studies mentioned previously at fractional states could also possibly be a consequence of IS rather than an exotic ferromagnetic or spin-glass phase transition.
Figure 5.25: $\frac{dR_{xx}}{dt}$ measured over a period of 14400 s (4 hours) at specific B values shown on the top-right corner of each plot. The B-values are spread across the hysteretic loop measured for $\nu = 2$. The blue points represent the data measured and the red line is the fit used of the form shown in Eq. (5.15). $I = 10$ nA, $V_{sg} = -2.12$ V, $V_{tg} = 0$ V and $T \approx 10$ mK.
Figure 5.26: \( \frac{dR_{xx}}{dt} \) measured over a period of 14400 s (4 hours) at specific B values shown on the top-right corner of each plot. The B-values are spread across the hysteretic loop measured for \( \nu = 1 \). The blue points represent the data measured and the red line is the fit used of the form shown in Eq. (5.15). \( I = 10 \, \text{nA}, \, V_{sg} = -2.12 \, \text{V}, \, V_{tg} = 0 \, \text{V} \) and \( T \approx 10 \, \text{mK} \).
As for the relaxation of $R_{xx}$ over time the behaviour could possibly be a consequence of the electron-density relaxing over time and therefore explained in the same framework as mentioned in the previous paragraph, rather than due to spin-relaxations. Therefore it is noted how this mechanism does seem to tie in a more universal manner the hysteretic and overshooting effects via the screening theory and the formation of IS and CS. Nevertheless, further studies are required to test this.

5.5 Conclusion and Future Work

To conclude, the theory on the resistance anomaly (overshooting) effect and the longitudinal resistance hysteresis has been presented followed by experimental data. The overshooting data have been explained to a good approximation by the screening theory, that is by the formation of IS/CS in the 2DEG. Finally various mechanisms have been discussed for the hysteretic behaviour. To verify though whether this hysteretic behaviour could be linked to a spin glass state or some other ferromagnetic like state, measurements comparing the effect of zero field cooling and field cooling on the hysteresis should be conducted, similarly to Ref. [98]. If there is a significant change in the structure of the hysteretic behaviour by cooling the device at zero magnetic field and after the device is warmed up and cooled back down again but in the presence of a magnetic field, this would be strong evidence that a spin glass state is present. Nonetheless, the screening theory also seems to provide an explanation, which would provide a more universal explanation for both the phenomena. However, as mentioned further studies need to be conducted, as discussed in the previous section, in order to verify experimentally if this is the case. Nonetheless the screening theory seems to be a vital tool within the quantum Hall regime.
Chapter 6

Magneto-resistance in 1D constriction in GaAs/AlGaAs

6.1 Introduction

An excellent method of studying the electric and magnetic properties of a material is by measuring its magneto-resistance, that is the likelihood of a material to change its electrical resistance while an external magnetic field is applied. Such a method is used within this chapter in order to characterise the behaviour of a GaAs/AlGaAs 2DEG sample with a 1D constriction, defined electrostatically by split-gates. Using a modified setup to the previous chapter, the interaction between the 1D constriction and the 2D regions on either side of it were studied. As discussed in later sections of this chapter, signatures of spin polarisation are reflected within the data, as well as possible LL crossing due to multiple sub-bands and an enhanced g* increasing the prominence of spin-splitting. The section that will follow is that of Methods in section 6.2. The Experimental Data will be presented in section 6.3 and followed by a Discussion in section 6.4, were various theories and comparisons with the literature will be presented. Finally the chapter is concluded in section 6.5 were future experiments are also discussed.
6.2 Methods

The device used was the same as in Chapter 5, that is G1 for which more information can be found in Chapter 3. The setup for the measurement is shown in Fig. 6.1. An a.c. voltage of $\sim 1$ V at 77 Hz was used as the input signal for the measurement. The longitudinal conductance, $\sigma_{xx}$ was then measured using the 2-Terminal set up, see Chapter 4. However, in addition to the $\sigma_{xx}$ the transverse voltage, $V_{xy}$ of the mesa was also measured, using the same set up as measuring the $R_{xy}$ in a 4-Terminal measurement. However, as the system was calibrated for the conductance measurement the $V_{xy}$ is in arbitrary units and is used more as an indicator of the system’s behaviour. The device was estimated to have an electron (carrier) density of $n_0 = 2.4 \times 10^{11}$ cm$^{-2}$ and a mobility of $\mu_e = 0.13 \times 10^6$ cm$^2$/Vs.

6.3 Experimental Data

Initially the quasi-1D conductance in the electron gas versus $V_{sg}$ can be seen in Fig. 6.2, which was measured using the 2-Terminal method, refer to Chapter 4. This was used as the reference for the $V_{sg}$ applied in the measurements described later on within this section. For the purposes of
Figure 6.2: $G$ vs $V_{sg}$, with the signature quantised conductance plateaus for a 1D system as obtained by using the G1 device explained in Chapter 3. The x’s mark the points used as $V_{sg}$ references for the magneto-resistance measurements within this chapter. $V_{tg} = 0.0$ V, $B = 0.0$ T and $T \sim 10$ mK this chapter $G_0$ will be defined with $V_{sg} \sim -1.715$ V, $0.5G_0 \sim -1.745$ V and $2G_0$ corresponds to $\sim -1.570$ V. It should be noticed that the first plateau which should correspond to $2e^2/h$ is slightly below this value which is why $G_0$ is defined at a G value lower than $2e^2/h$. This is thought to be due to scattering effect as series resistance was corrected [99].

Furthermore using the 4-Terminal method, the behaviour of the device in the quantum Hall regime was studied, as seen in Fig. 6.3, without $V_{sg}$ being applied. Using the theory from the previous chapter, the position of the ISs with respect to the B were estimated as seen in Table 6.1. The bulk g-factor for GaAs was used in these calculations, i.e. 0.44. Note Fig. 6.3 that the splitting of the Landau levels due to the Zeeman effect stops for $\nu > 8$. However, in Table 6.1 $\nu = 9$ is also recorded. This will be used as a reference point within the discussion, further ahead within the
Figure 6.3: $R_{xx}$ (blue) and $R_{xy}$ (red) versus $B$. The filling factors are indicated for each plateau in $R_{xy}$ measurement. Device used was G1 as described in Chapter 3. $V_{sg} = 0.0$ V, $V_{tg} = 0.0$ V and $T \sim 10$ mK.

chapter. Additionally note that the $\nu = 3/2$ is indicated as well. The existence of this fractional state is not well established as a plateau within the $R_{xy}$ measurement, but some indentation is present around the $B \sim 3.2$ T regime. Also, a slight dimpling within the $R_{xx}$ trace is an indication to the possible presence of fractional states. This region is of importance as it shows unusual behaviour in the data presented further on in this section. Note however, that the longitudinal resistance does not reach zero value, this could indicate reflection and scattering taking place, thus increasing the resistance measured or could be due to charge build up.

While using the setup explained in section 6.2 and the $V_{sg}$ values from Fig. 6.2, the following data were obtained. Figure 6.4 shows how $\sigma_{xx}$ varies with $B$ for various $V_{sg}$’s and Fig. 6.5 shows how $V_{xy}$ varies with $B$. From the $\sigma_{xx}$ measurement, the first and most obvious observation is the expected reduction in the conduction value as $B$ increases. This is because the re-
Table 6.1: The B-values for the corresponding $\nu$.

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>B (T)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>4.88</td>
</tr>
<tr>
<td>3/2</td>
<td>3.25</td>
</tr>
<tr>
<td>2</td>
<td>2.44</td>
</tr>
<tr>
<td>3</td>
<td>1.63</td>
</tr>
<tr>
<td>4</td>
<td>1.22</td>
</tr>
<tr>
<td>5</td>
<td>0.98</td>
</tr>
<tr>
<td>6</td>
<td>0.81</td>
</tr>
<tr>
<td>7</td>
<td>0.70</td>
</tr>
<tr>
<td>8</td>
<td>0.61</td>
</tr>
<tr>
<td>9</td>
<td>0.54</td>
</tr>
<tr>
<td>10</td>
<td>0.49</td>
</tr>
<tr>
<td>11</td>
<td>0.44</td>
</tr>
<tr>
<td>12</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Resistence increases with increasing B. Additionally as the $V_{sg}$ is increased the initial value of $\sigma_{xx}$ at $B = 0$ T is also reduced as the constriction in the quasi-1D channel, formed by the split-gates, is enhanced. On further inspection though unusual features seem to appear very faintly between the values of 3.1 T to 3.5 T within the regime of $V_{sg} = -1.72$ V to -1.75 V, which is indicated by the black arrow, see Fig. 6.4.

Similar features seem to appear around the same region under the same conditions when studying the $V_{xy}$ in terms of B, see Fig. 6.5. Furthermore two consecutive peaks that appear $\sim 1.4$ T seem to change in amplitude as the $V_{sg}$ is increased. The right peak is larger than the left one at $V_{sg} = 0.00$ V. At $V_{sg} = -1.30$ V and -1.57 V the right peak becomes less prominent and by $V_{sg} = -1.720$ V, i.e. $\sim G_0$ the two peaks seem to have same amplitude. By $V_{sg} = -1.75$ V the left peak becomes dominant with the right peak almost vanishing. In order to study these features further measurements were conducted at $V_{sg}$ values close to the pinch-off of the 1D channel and with a higher resolution of $V_{sg}$’s used, i.e -1.715 V<$V_{sg}$<-1.770 V. Finally the background of the traces was removed in order to study the oscillations in more detail. This was achieved by breaking the traces into smaller magnetic field ranges. This can be seen in Figs. 6.6 - 6.16. The data from these figures are summarised in Table 6.2, followed by a discussion and explanation on their origins.
Figure 6.4: $\sigma_{xx}$ vs B. The traces for various $V_{sg}$ values are shown from 0.000 V to -1.750 V. The black arrow is to indicate an unusual feature noticed around 3.1-3.5 T between $0.5G_0$ and $G_0$. The traces are offseted by $+25\mu S$. $V_{tg} = 0.0$ V and $T \sim 10$ mK.
Figure 6.5: $V_{xy}$ vs B. The traces for various $V_{sg}$ values are shown from 0.000 V to -1.750 V. The black dashed lines indicates regions of interest. (1) is the region where oscillations appear between $G_0$ and 0.5$G_0$. The (2) region is one of the regions where behaviour similar to spin-polarisation is noticed. The traces were offseted by $\Delta V_{xy} = +25$ (a.u.). $V_{tg} = 0.0$ V and $T \sim 10$ mK.
<table>
<thead>
<tr>
<th>B (T)</th>
<th>$\nu$</th>
<th>Observations</th>
<th>Figures</th>
<th>Notes</th>
</tr>
</thead>
</table>
| 0.0 - 0.6 | 10, 12 | - Peaks split as $V_{sg}$ increases  
- Left peak shrinks and right peak increases as $V_{sg}$ increases  
- The valley formed is centred around the B-value corresponding to the filling factor in Table 6.1  
- For $G<0.5G_0$ only the right peak remains | 6.6-6.7 | Left peak for $\nu = 12$ not well defined |
| 0.6 - 1.7 | $3<\nu<8$ | - A central peak appears between odd and even filling factors, with two valleys on either side (see $7<\nu<8$ and $5<\nu<6$)  
- The right valley is deeper initially but with increasing $V_{sg}$ the left valley becomes deeper.  
- For $0.5G_0<G<G_0$ the two valleys start to equilibrate  
- $G<0.5G_0$ the left valley becomes prominent, while the right valley diminishes in depth  
- Both valleys eventually disappear near pinch-off  
- For the case between even and odd filling factors (see $6<\nu<7$ and $4<\nu<5$), two peaks appear near the B-value of the filling factors with a central valley. The right peak corresponds to the even $\nu$ and the left peak to the odd $\nu$ | 6.8 - 6.10 | For the $4<\nu<5$ the central valley is weak and disappears by $G\sim G_0$. Instead a weak peak appears, which shifts to the left and merges with the original left peak as the $V_{sg}$ increases |
- Right peak is the prominent peak initially, but as $V_{sg}$ increases the left peak's amplitude increases and the right one decreases
- Near $G=0.5G_0$ the peaks almost equilibrate
- For $G<0.5G_0$ the left peak is the prominent peak. The right peak diminishes
- Near $G=0.5G_0$ the peaks almost equilibrate

- For the $3<\nu<4$ regime a rather different scenario appears than with the odd to even filling factors described further above. In this case there is not one central peak between the filling factor regime, but two. These peaks as well as the central valley located between them, seem to behave in a similar manner as the two peaks noticed for the cases of $6<\nu<7$. Likewise the left and right valley (on the extreme sides of the two central peaks) behave in a very similar manner to the valley discussed for the case $7<\nu<8$. 

<p>| 6.8 - 6.9 and 6.11 | 117 |</p>
<table>
<thead>
<tr>
<th>ν</th>
<th>ν&lt;3</th>
<th>Two peaks present between 2&lt;ν&lt;3, but in this case the left peak is the dominant one at $V_{sg} = 0.00$ V. The behaviour of the peaks though is similar with increasing $V_{sg}$, that is the left peak increases and shifts to the left and the right one vanishes</th>
<th>6.12 - 6.13</th>
</tr>
</thead>
<tbody>
<tr>
<td>ν&lt;2</td>
<td>Oscillations appear in the regime where $G &lt; G_0$ with one peak located where $\nu = 3/2$ is expected. For this region though there are multiple peaks appearing close by and although they seem to behave similarly to the previous sections, i.e. some peaks increase while others decrease, no obvious pattern seems to appear.</td>
<td>6.14 - 6.15</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>The peak indicated in Fig. 6.16 is very broad and shifted to the right of the expected position for $V_{sg} = 0.00$ V, but eventually shifts further to the left as the $V_{sg}$ increases aligning with the expected B-value position.</td>
<td>6.16</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.2: Summary of $V_{sg}$ data.
Figure 6.6: $V_{xy}$ vs B with the background removed. The traces for various $V_{sg}$ values are shown from 0.00 V to -1.78 V. The black dashed lines indicate regions of interest. (1) and (2) are regions where splitting of the peaks seem to occur by increasing $V_{sg}$. This is only noticed for the region where $\nu > 8$. The traces are off-setted by $V_{xy} = +5$ (a.u.). $V_{tg} = 0.0$ V and $T \sim 10$ mK.

It should be noticed that the filling factors mentioned in the previous figures and Tables. 6.1 and 6.2, as well as in the following figures mentioned in this chapter, are the filling factors that are present within the 2D regions of the system, not the constriction within the split-gate area.

Furthermore, these measurements were repeated, but the $V_{sg}$ fixed at -1.725 V and varying the $V_{tg}$ from 0.00 V to -0.45 V. The behaviour noticed for the same features discussed earlier seems to be the same as varying the $V_{sg}$, see Figs. 6.17 - 6.20. This is expected as the constriction and consequently density are varied similarly to when the $V_{sg}$ was increased. The -0.45 V was established as being the pinch-off $V_{tg}$ of the system at this configuration.
Figure 6.7: $V_{xy}$ vs B with the background removed. The traces for various $V_{sg}$ from -1.715 V to -1.770 V. The black dashed lines indicates regions of interest. ① and ② are regions where splitting of the peaks seems to occur within the regime of $0 < G < G_0$. The $V_{sg}$ is increased in increments of -0.005 V. This is only for the region where $\nu > 8$. They are off-setted by $\Delta V_{xy} + 5$ (a.u.). $V_{tg} = 0.0$ V and $T \sim 10$ mK.
Figure 6.8: $V_{xy}$ vs B with the background removed. The traces for various $V_{sg}$ values are shown from 0.00 V to -1.78 V. The black dashed lines indicates regions of interest. The black arrows show the expected values of $\nu$'s. The traces are off-setted by $V_{xy} = +5$ (a.u.). $V_{tg} = 0.0$ V and $T \sim 10$ mK.
Figure 6.9: \( V_{xy} \) vs B with the background removed. The traces for various \( V_{sg} \) values are shown from -1.715 V to -1.770 V. The \( V_{sg} \) is increased in increments of -0.005 V. The black dashed lines indicate regions of interest. The black arrows show the expected values of \( \nu \)'s. The blue dashed line indicates the 0.5\( G_0 \) point. The traces are off-setted by \( V_{xy} = +5 \) (a.u.). \( V_{tg} \) = 0.0 V and \( T \sim 10 \text{ mK} \).
Figure 6.10: $V_{xy}$ vs B with the background removed. The traces for various $V_{sg}$ values are shown from -1.715 V to -1.770 V. The $V_{sg}$ is increased in increments of -0.005 V. The black dashed lines indicates regions of interest. The black arrows show the expected values of $\nu$'s. The blue dashed line indicates the $0.5G_0$ point. The traces are off-setted by $V_{xy} = +5$ (a.u.). $V_{tg} = 0.0$ V and $T \sim 10$ mK.
Figure 6.11: $V_{xy}$ vs B with the background removed. The traces for various $V_{sg}$ values are shown from -1.715 V to -1.770 V. The $V_{sg}$ is increased in increments of -0.005 V. The black dashed lines indicates regions of interest. The black arrows show the expected values of $\nu$'s. The blue dashed line indicates the 0.5$G_0$ point. The traces are off-setted by $V_{xy} = +5$ (a.u.). $V_{tg}$ = 0.0 V and $T \sim 10$ mK.
Figure 6.12: $V_{xy}$ vs B with the background removed. The traces for various $V_{sg}$ values are shown from 0.00 V to -1.78 V. The black dashed lines indicate regions of interest. The black arrows show the expected values of $\nu$‘s. The traces are off-setted by $V_{xy} = +5$ (a.u.). $V_{tg} = 0.0$ V and $T \sim 10$ mK.
Figure 6.13: $V_{xy}$ vs $B$ with the background removed. The traces for various $V_{sg}$ values are shown from -1.715 V to -1.770 V. The $V_{sg}$ is increased in increments of -0.005 V. The black dashed lines indicates regions of interest. The black arrows show the expected values of $\nu$'s. The blue dashed line indicates the $0.5G_0$ point. The traces are off-setted by $V_{xy} = +2.5$ (a.u.). $V_{tg} = 0.0$ V and $T \sim 10$ mK.
Figure 6.14: $V_{xy}$ vs B with the background removed. The traces for various $V_{sg}$ values are shown from 0.00 V to -1.78 V. The black dashed lines indicates regions of interest. The black arrows show the expected values of $\nu$'s. The traces are off-setted by $V_{xy} = +5$ (a.u.). $V_{tg} = 0.0$ V and $T \sim 10$ mK.
Figure 6.15: $V_{xy}$ vs $B$ with the background removed. The traces for various $V_{sg}$ values are shown from -1.715 V to -1.770 V. The $V_{sg}$ is increased in increments of -0.005 V. The black dashed lines indicates regions of interest. The black arrows show the expected values of $\nu$’s. The blue dashed line indicates the $0.5G_0$ point. The traces are off-setted by $V_{xy} = +10$ (a.u.). $V_{tg} = 0.0$ V and $T \sim 10$ mK.
Figure 6.16: $V_{xy}$ vs B with the background removed. The traces for various $V_{sg}$ values are shown from 0.00 V to -1.78 V. The black dashed lines indicate regions of interest. The black arrows show the expected values of $\nu$'s. The traces are offset by $V_{xy} = +5$ (a.u.). $V_{tg} = 0.0$ V and $T \sim 10$ mK.
Figure 6.17: $V_{xy}$ vs B with the background removed. The traces for various $V_{tg}$ values are shown from -0.00 V to -0.45 V. The $V_{tg}$ is increased in increments of -0.05 V. The black dashed lines indicates regions of interest. The black arrows show the expected values of $\nu$'s. The traces are off-setted by $V_{xy} = +2.5$ (a.u.). $V_{sg} = -1.725$ V and $T \sim 10$ mK.
Figure 6.18: $V_{xy}$ vs B with the background removed. The traces for various $V_{tg}$ values are shown from -0.00 V to -0.45 V. The $V_{tg}$ is increased in increments of -0.05 V. The black dashed lines indicates regions of interest. The black arrows show the expected values of $\nu$'s. The traces are off-setted by $V_{xy} = +2.5$ (a.u.). $V_{sg} = -1.725$ V and $T \sim 10$ mK.
Figure 6.19: $V_{xy}$ vs B with the background removed. The traces for various $V_{tg}$ values are shown from -0.00 V to -0.45 V. The $V_{tg}$ is increased in increments of -0.05 V. The black dashed lines indicates regions of interest. The black arrows show the expected values of $\nu$’s. The traces are offsetted by $V_{xy} = +2.5$ (a.u.), $V_{sg} = -1.725$ V and $T \sim 10$ mK.
Figure 6.20: $V_{xy}$ vs B with the background removed. The traces for various $V_{tg}$ values are shown from -0.00 V to -0.45 V. The $V_{tg}$ is increased in increments of -0.05 V. The black dashed lines indicates regions of interest. The black arrows show the expected values of $\nu$’s. The traces are off-setted by $V_{xy} = +10$ (a.u.). $V_{sg} = -1.725$ V and $T \sim 10$ mK.
Additionally measurements were conducted by fixing the $V_{sg}$ at -1.725 V and varying B at fixed source-drain bias, $V_{sd}$, values which was varied by increments of +0.2 mV in the range of -2.0 mV < $V_{sd}$ < +2.0 mV. The data from this measurement can be seen in Fig. 6.21. The dashed blue lines represent the $V_{sd} = 0.0$ mV. Summary of the observations for Figs. 6.22 - 6.26, is shown in Table 6.3.

<table>
<thead>
<tr>
<th>B (T)</th>
<th>$\nu$</th>
<th>Observations</th>
<th>Figures</th>
</tr>
</thead>
</table>
| 0.0 - 0.6 | 10, 12 | -Double peak along black-dashed lines at $V_{sd} = 0.0$ V, which remain present in the range of -1.4 mV < $V_{sd}$ < 1.2 mV  
-At extreme values of $V_{sd}$ only right peak remains  
-The right peak shifts to higher B-values as the $V_{sd}$ increases for both polarities, but diminishes in amplitude | 6.22 |
| 0.6 - 1.7 | $3<\nu<8$ | -For odd to even filling factors ($7<\nu<8$ and $5<\nu<6$) the right valley shifts to upwards and becomes shallower as the $V_{sd}$ approaches extreme values. In contrast the left valley shifts downwards and becomes deeper.  
-The position of the central peak does not change.  
-For both case the central peak seems to merge with the peaks for the odd filling factors.  
-Furthermore a point of interest is that the even filling factor peaks of 6 and 4 seem to shift leftwards by increasing $V_{sd}$ in both the negative and positive direction and merge with the higher odd filling factor (7 and 5, respectively) and then merge with the corresponding central peaks discussed earlier or are on the verge or merging with them.  
-The merging of the peak seems to eventually shift towards the position corresponding to the odd $\nu$. | 6.23 |
<table>
<thead>
<tr>
<th>Range</th>
<th>Description</th>
<th>Page</th>
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<tr>
<td>3&lt;ν&lt;4</td>
<td>For the range between 3&lt;ν&lt;4 the two central peaks behave slightly different than the central peaks between other odd and even filling factor pairs. In this case the left peak remains present and strong through all the ( V_{sd} ), but the right peak seems to vary differently. By moving along the negative values of ( V_{sd} ), the right peak although it reduces in amplitude it does not disappear. However, when moving along the positive values the right peak diminishes relatively quickly and disappears, while forming a valley instead. On the contrary the left peak seems to be enhanced.</td>
<td>6.23</td>
</tr>
<tr>
<td>2&lt;ν&lt;3</td>
<td>Around ( \nu = 2 ) no peak seems to appear at ( V_{sd} = 0.0 \text{ mV} ) along dashed-line (1). However, as the ( V_{sd} ) is increased in both the negative and positive directions, a peak centred around it, appears rapidly. Eventually it vanishes though at extreme ( V_{sd} ) values. Another peak is present towards the left of ( \nu = 2 ). This peak appears to weaken and merge with the peak around ( \nu = 2 ) to form one broad peak instead for positive voltages, but for negative it seems to weaken but eventually splits into two peaks.</td>
<td>6.24</td>
</tr>
<tr>
<td>1&lt;ν&lt;2</td>
<td>The peaks that appear at ( V_{sd} = 0.0 \text{ mV} ) seem to flip with increasing ( V_{sd} ) for both the negative and positive direction. That is the peaks turn into valleys and the valleys into peaks, but disappear rather quickly.</td>
<td>6.25</td>
</tr>
</tbody>
</table>
The peak around 5.2 T quickly evolves into a valley and splits into two peaks at around 4.8 T and 5.7 T, with the slightest application of $V_{sd}$ especially for negative values.

<table>
<thead>
<tr>
<th>$V_{sd}$</th>
<th>$V_{xy}$ (a.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2 \text{ V}$</td>
<td>800</td>
</tr>
<tr>
<td>$+2 \text{ V}$</td>
<td>600</td>
</tr>
<tr>
<td>$0 \text{ V}$</td>
<td>400</td>
</tr>
</tbody>
</table>

Table 6.3: Summary of $V_{sd}$ data.

![Graph](image)

Figure 6.21: $V_{xy}$ vs B with the background removed. The traces for various $V_{sd}$ values are shown from -2.0 mV to +2.0 mV. The $V_{sd}$ is increased in increments of +0.2 mV. The black arrow indicates the area of interest around 3.25 T. The blue dashed line represents the trace with $V_{sd} = 0.0 \text{ mV}$. The traces are offset by $V_{xy} = +30 \text{ (a.u.)}$. $V_{sg} = -1.725 \text{ V}$ and $T \sim 10 \text{ mK}$.

Finally Temperature dependence measurements were conducted by setting up the system with $V_{sg} = -1.725 \text{ V}$ and $V_{sd} = 0.0 \text{ mV}$. An overall reduction in the oscillatory features is noticed, as seen in Figs. 6.27 - 6.32, by 1 K. Also note that the main peaks remaining are the even filling factor ones,
Figure 6.22: $V_{xy}$ vs B with the background removed. The traces for various $V_{sd}$ values are shown from -2.0 mV to +2.0 mV. The $V_{sd}$ is increased in increments of +0.2 mV. The black arrow indicates the $\nu$’s of interest. The blue dashed trace represents the trace with $V_{sd} = 0.0$ mV. The vertical blue dashed lines show how the right peak of the regions of interest shift with varying $V_{sd}$ values. The traces are off-setted by $V_{xy} = +5$ (a.u.). $V_{sg} = -1.725$ V and $T \sim 10$ mK.

except for $\nu<3$. This is expected as the Zeeman splitting breaks down at higher temperatures and spin degeneracy is restored.

As mentioned earlier for $\nu<3$ there are few additional peaks that are still visible even at $T = 1$ K. In Fig. 6.30 a peak appears very dimly around $B \sim 2.44$ T. However, additional peaks also appear to the left and to the right of this peak. The peak to the right appears to increase in amplitude and merge with the small central peak. The one the left though remains fixed at $B \sim 2$ T, but decreases in amplitude. A possible explanation for the two peaks on the right is due to scattering effects. The one on the left could be related to $\nu=3$ but is shifted to the right due to scattering effects, or inter-LL effects,
Figure 6.23: $V_{xy}$ vs B with the background removed. The traces for various $V_{sd}$ values are shown from -2.0 mV to +2.0 mV. The $V_{sd}$ is increased in increments of +0.2 mV. The black arrow indicates the $\nu$’s of interest. The blue dashed trace represents the trace with $V_{sd} = 0.0$ mV. The traces are off-setted by $V_{xy} = +5$ (a.u.). $V_{sg} = -1.725$ V and $T \sim 10$ mK.

as explained in the following section. In Fig. 6.32 there is a peak around 5.26 T at BT but shifts to $\sim 4.9$ T which is close to the B-value calculated for $\nu = 1$. Finally the amplitude of this peak reduces with temperature but does not completely disappear. Note that in this case the $\nu$ is odd but due to the stronger Zeeman effect at higher B-values it is expected that the $\nu$ will still remain relatively strong even at 1 K temperatures. This seems to apply for $\nu=3$ but is not true for higher-valued odd $\nu$’s.

6.4 Discussion

To start with the measurements with the varying $V_{sg}$ shall be discussed. For the lower B-values the peaks of the $V_{xy}$ oscillations seem to correspond
Figure 6.24: $V_{xy}$ vs B with the background removed. The traces for various $V_{sd}$ values are shown from -2.0 mV to +2.0 mV. The $V_{sd}$ is increased in increments of +0.2 mV. The black arrow indicates the $\nu$'s of interest. The blue dashed trace represents the trace with $V_{sd} = 0.0$ mV. The traces are off-setted by $V_{xy} = +5$ (a.u.). $V_{sg} = -1.725$ V and $T \sim 10$ mK.

to the position of the filling factors. The peak along (2) in Figs. 6.6 and 6.7 split into two peaks as the $V_{sg}$ in increased. This is also true for the (1) peak. The fact that for both cases by increasing $V_{sg}$ the right peak increases, seems to be an indication that spin polarisation is taking place. For (1) the splitting occurs even at $V_{sg} = 0.0$ V but with (2) it is only noticeable as the $V_{sg}$ increases. A possible reason for this is that at (1) the Zeeman effect, although not noticeable in terms of the QHE plateaus in the $R_{xy}$ measurements in Fig. 6.3, it would have a stronger effect for (1) than (2) as the B is larger. This could explain the larger shift in peaks along (1)
Figure 6.25: $V_{xy}$ vs $B$ with the background removed. The traces for various $V_{sd}$ values are shown from -2.0 mV to +2.0 mV. The $V_{sd}$ is increased in increments of +0.2 mV. The black arrow indicates the $\nu$ of interest. The blue dashed line represents the trace with $V_{sd} = 0.0$ mV. The traces are off-setted by $V_{xy} = +10$ (a.u.). $V_{sg} = -1.725$ V and $T \sim 10$ mK.

However, the lack of odd $\nu$ in the QHE measurement at $V_{sg} = 0$ V could indicate that Zeeman splitting is enhanced as the $V_{sg}$ is increased, possibly due to enhanced electron-electron interactions. Another point of interest is that the right peak shifts towards values which would correspond to $\nu = 9$ and $\nu = 11$. The reason for this possible increase in the right peak as the $V_{sg}$ increases, that is the constriction increases, could be due to spin polarisation as explained by Jaksch et al. [100]. Similar behaviour has been seen in focusing experiments in Refs. [101–103]. As for the fact that the peaks shift to B-values close to the odd filling factor $\nu = 9$ and 11 would be a consequence of the spin polarisation as each energy level formed by Zeeman splitting of a LL will have an affinity to a certain spin orientation, so if the conductance of spin-down electrons is favoured from constricting
Figure 6.26: $V_{xy}$ vs B with the background removed. The traces for various $V_{sd}$ values are shown from -2.0 mV to +2.0 mV. The $V_{sd}$ is increased in increments of +0.2 mV. The black arrow indicates the $\nu$’s of interest. The blue dashed trace represents the trace with $V_{sd} = 0.0$ mV. The traces are off-setted by $V_{xy} = +5$ (a.u.). $V_{sg} = -1.725$ V and $T \sim 10$ mK.

the channel then only one of the Zeeman energy levels associated with a specific LL will be occupied, in this case the ones with odd filling factors. Also, the fact that there is no obvious splitting at $V_{sg} = 0.0$ V for the $\nu = 10$ and 12 initially but appear afterwards, could be that electron-electron interactions are enhanced with increased spatial quantum confinement. This could possibly enhance the Zeeman effect which is why only these two peaks split whereas peaks at lower magnetic fields, other than the reduction in their amplitudes, do not seem to be perturbed position-wise by increasing $V_{sg}$.

For Figs. 6.8 and 6.9, the (4) (in Fig. 6.10 (1)) the left peak is the one that increases in amplitude at the expense of the right peak as the $V_{sg}$ is
increased. Similar behaviour is noticed for \( \tilde{2} \). However, take note of the fact that in both cases the shift in the peaks and increase of their amplitudes also corresponds to odd filling factors, \( \nu = 7 \) and \( 5 \), as mentioned earlier with the peaks close to even filing factor values diminishing as \( V_{sg} \) increases. Additionally an important feature is that for \( V_{sg} \) values close to \( 0.5 \frac{G}{0} \) the left peak becomes prominent, while the second peak is negligible in size. This also matches the theory and data from \([103]\). What is different though is that in addition to these features in the range of 0.6 to 1.2 T there are extra peaks present along\( \tilde{3} \) and \( \tilde{5} \). These peaks though seem to be linked by one common feature they are located in the order of an odd to even filling factor. That is between \( 5 < \nu < 6 \) and between \( 7 < \nu < 8 \). The exact mechanism for the development of this feature while conducting measurements in this setup is not clearly understood. One idea is that of crossing of energy levels as shown in Fig. 6.33, takes place. That is from the literature it is discussed..
that in the case where two-subbands are present in the 2DEG, that could result in spin-split LLs from the two different sub-bands to cross, see Fig. 6.33. These crossings as a result form ring-like structures within the density-B plane, as shown in Fig. 6.33. As a result if the system is set up at a specific density shown by the black dashed horizontal lines, in Fig. 6.33 by increasing B the system will pass through two separate regions corresponding to $\nu = 6$ due to the ring structure. Therefore multiple oscillations will show up in the measurement between the expected filling factor positions. In this case the $V_{xy}$ acts as a reflection probe of the conductance signal so the peaks correspond to the filling factors. However, a difference in this data and that from Ellenberger et al. [104] study is that in the $R_{xy}$ data at $V_{sg} = 0.0$ V in Fig. 6.3 there are no additional features, as in their study. Another difference between this data and that from Davies et al. [105] the system is
Figure 6.29: $V_{xy}$ vs B with the background removed. The traces are taken at various temperatures in the range of $\sim$10mK - 1000 mK. The black arrows represent the $\nu$'s of interest. The traces are off-setted by $V_{xy} = +5$ (a.u.). $V_{sg} = -1.725$ V and $V_{tg} = 0.0$ V.

setup to have two quantum wells with a thin barrier between the two but both seem to act as one system and in a similar behaviour to Ellenberger et al. [104] and Zhang et al. [106]. Therefore it could be assumed that the density of the 2DEG could be ideal for crossing over of spin-split energy levels from different LLs in this case. Therefore the central peak in the centre of the odd and even $\nu$'s could correspond to the ring-like feature that forms due to LLs crossing see (b) and (d) in Fig. 6.33. This could also explain why the peak shifts to the odd $\nu$ as the $V_{sg}$ is increased. However, as mentioned what are the exact interaction effects or enhanced exchange effects cannot be resolved from the data, so future work is required in order to further study and in order to comprehend this behaviour. Finally an interesting feature is that as the magnetic field increases the peaks corresponding to the $nu$'s seem to shift from the calculated value obtained from the method described
Figure 6.30: $V_{xy}$ vs B with the background removed. The traces are taken at various temperatures in the range of $\sim 10$ mK - 1000 mK. The black arrows represent the $\nu$'s of interest. The traces are off-setted by $V_{xy} = +5$ (a.u.).

$V_{sg} = -1.725$ V and $V_{tg} = 0.0$ V.

in the previous chapter. The precision seems to vary significantly, by $< 1$ to 8 % and does not seem to have some pattern. This though could also be explained by how the LLs evolve with B and the possibility of crossing could cause some shifting in the central position of the peaks.

By further probing Figs. 6.8 and 6.9 at higher B’s it is noticed that the arrows indicating the $\nu = 3$ and 4 do not seem to align to a close proximity with any peaks. However, what seems to occur is that they are located in the centre of two peak. That is for $\nu = 4$ is expected to be located between the right peak of $\nu = 3$ in Fig. 6.9 and for $\nu = 3$ to
Figure 6.31: $V_{xy}$ vs B with the background removed. The traces are taken at various temperatures in the range of $\sim$10mK - 1000 mK. The black arrows represent the $\nu$'s of interest. The traces are off-setted by $V_{xy} = +5$ (a.u.). $V_{sg} = -1.725$ V and $V_{tg} = 0.0$ V.

be between the right peak of (1) in Fig. 6.9 and the left peak present in Figs. 6.12 and 6.13. However, there seems to be one common pattern the odd and even filling factor mentioned earlier. The right peak of (2) in Fig. 6.9 diminishes as the $V_{sg}$ increases (similar to even $\nu$) and the left peak in Fig. 6.12 enhances as $V_{sg}$ increases (similar to odd $\nu$). So this could indicate that the two peaks along the (1) in Fig. 6.8 could be similar in nature to the peaks along (3) and (5) in Fig. 6.8 as explained earlier. However, the fact that two peaks appear instead of one does complicate things. A possible explanation for this could be provided again by Ellenberger et al. [104], Davies et al [105] and Zhang et al. [106]. The two spin-split energy levels corresponding to different LLs cross which can be noticed in their data. In an adapted schematic in Fig. 6.33 this can be seen in (c) and (e) plots of this figure. Such behaviour would explain the two peaks present as well as
the shift in their measured values compared to the expected ones, in Table 6.1. Therefore a possibility is that since the left peak of \( \nu \) in Fig. 6.9 seems to behave and evolve similarly to the left peak in Fig. 6.12, could indicate that they are both related to \( \nu = 3 \), as is the case for \( \nu = 5 \) in Fig. 6.33(e). Likewise the right peaks of \( \nu \) and of \( \nu \) in Fig. 6.9 evolve in a similar manner and could be linked to \( \nu = 4 \), similarly to the case of \( \nu = 6 \) in Fig. 6.33(e). So as the \( V_{sg} \) increases the peaks for the \( \nu = 3 \) (odd) enhance and the peaks for the \( \nu = 4 \) (even) diminish. However, it should be noticed that the studies in the aforementioned papers were conducted by measuring the \( R_{xx} \) in a 4-Terminal set up as opposed to the data here where the \( V_{xy} \) and \( \sigma_{xx} \) were measured in a 2 Terminal set up. Also they used a setup which either had a bilayer of 2DEGs in their system [105] and studying the overall interactions between the layers or in the case of Zhang
Figure 6.33: Red and blue pair of spin-split Landau levels are shown with $|i, n, s\rangle$, representing the sub-band, LL and spin quantum numbers, respectively. The quantum numbers mentioned as well as the $\nu$’s are used as an example in the schematic representations. (a) The pair of LLs cross over causing the regions of certain $\nu$ to break up in smaller regions when seen in the Energy - B plane. The yellow regions correspond to even $\nu$’s and the green ones to odd $\nu$’s. The latin numerals are the cross-section points. (b) and (c) Shematic of LLs in Density - B plane (adapted from [104]). The horizontal dashed lines indicate how the density could affect the appearance of the peaks in the $V_{xy}$ measurements in this study, seen in the schematics of (d) and (e). In the case of (b) an extra peak corresponding to $\nu = 6$ appears in (d) due to the ring structure formed from the LL crossings. However, in the case of (c) there are two extra peaks which appear corresponding to $\nu = 5$ and $\nu = 6$ in (e).

et al. [106] and Ellenberger et al. [104] the system was configured in having two sub-bands in the 2DEG. The results though are similar, as in they all observe additional maxima and minima between the filling factors. However,
the explanation as to why these energy level crossing takes place is still not clear. They all agree that they are density dependent and that there is an enhanced effect at high magnetic fields, possibly due to an enhanced g-factor [104]. In Ref. [105] they say that the effect is due to a combination of wave-function delocalisation, sub-band locking and charge transfer between the sub-bands. Whereas in [106] they state that these structures cannot be explained through a single-particle picture and therefore believe that they arise due to ferromagnetic states in the ground states. On the other hand a study by Ellenberger et al. [104] contradicts this by showing that this behaviour can be explained by a single particle model.

From our study it is difficult to compare our data with the literature as the measurements were not conducted at different densities and the sample does not possess a global gate or back-gate in order to alter the 2DEGs bulk density. It should be clarified though that these models are for 2DEGs and not quasi-1D systems, which is why it makes it difficult to compare sufficiently these models with our system. Also as mentioned they conduct their measurements using a 4 Terminal resistance set up as opposed to the study presented here. Nonetheless, an important feature in the data is how by changing the constriction in the split-gates the behaviour of the peaks is affected. This could be similarly to Davies et al. [105] in the sense that the density within the constriction is altered with varying $V_{sg}$ which then acts as a second interacting layer, though in a series configuration, as in their bilayer system. That is the interaction effects between the quasi-1D and 2D sections of the system are affected. Also by taking into account the theory from Ref. [100] and the data from Refs. [101–103] the affects in the change of the peaks can be explained by the change in density of the spin-up or spin-down electron density due to spin polarisation within the quasi-1D, which then interacts with the 2D part of the system.

Similarly the evolution of the peak corresponding to $\nu = 2$ in Figs. 6.12 and 6.13 and $\nu = 1$ in Fig. 6.16 could be explained with this theory. Another important feature to consider in Figs. 6.13 and 6.16 is the enhanced noise that seems to appear for the traces at $V_{sg}$ near $G_0$ but then disappears near pinch-off. One possible explanation for this could be due to the enhanced exchange interactions at this region as the spin-down and spin-up conduct differently from the 1D channel and therefore when injected in the 2D system, reflection and transmission might be affected causing noise to be present in the signal. As for the oscillatory behaviour in Figs. 6.14 and 6.15 and in general the whole area between $\nu = 2$ and 1 it seems to be more complicated to describe. Although a possible explanation for the extra feature is that as the $V_{sg}$ increases the ISs for the filling factors $\nu =$
2, 3/2 and 1 interact strongly causing similar energy level crossings as mentioned but further studies are required. Finally similar behaviour with the top-gate measurements as with the split-gate measurements seems to take place which as mentioned is expected due to similar effects achieved on the quasi-1D channel by applying voltage on the top-gate as to the application of voltage on the split-gates.

Another explanation could be due to the Aharonov-Bohm effect (AB) as there seems to be some periodicity. If we take the periodicity to be approximately 0.12 T from Fig. 6.25 and utilising the fact that the magnetic quantum flux \( \Phi_0 = \pi r^2 \Delta B = \hbar/2e \) then \( r \) is estimated to be \( \sim 74 \) nm. This radius size could correspond in size with the area enclosed within the split-gate at the particular voltages being applied, hence acting as a ring for an AB to manifest within the measurements [107, 108]. That is some of the edge states are reflected as they try to pass through the split-gate whereas the rest manage to pass through. The ones that enter the split-gate region form a ring-path as they follow tunnelling paths around the saddle point formed within the split-gate [108].

For the second part of our measurements with the varied \( V_{sd} \) being applied changes in the peak values and their positions are also noticed. However, the difference is that the \( V_{sg} \) is constant to allow \( G \sim G_0 \). A possible explanation for the evolution of the peaks is quasi-elastic inter-Landau-level scattering (QUILLS). That is the \( V_{sd} \) applied alters the shape of the LLs and consequently their position with respect to the Fermi energy [109], see Fig. 6.34. As the \( V_{sd} \) is increased the LLs on one side come closer to each other, see left side of Fig. 6.34, encouraging electrons to scatter between the LLs. On the opposite side though any electron transition that takes place is via phonons and/or photon emission. This induces scattering of electrons between different LLs which could explain the shifts in the positions of the peaks [109]. A detailed study on how the LLs are altered by \( V_{sd} \) was conducted by Panos et al. [109] using low-temperature scanning force microscopy. They concluded that by increasing the bias, the ISs on either side of the Hall-bar evolve in different ways. This leads to the current being re-distributed in one IS instead. As a result with the large Hall fields, there is an induced electrical breakdown in the system, changing the magneto-resistance behaviour of the system. It should be noticed though that in their work for simplification they do not consider any spin-splitting due to Zeeman effect, an important factor for the data presented in this chapter. In our case we could not verify whether similar behaviour was noticed as the group does not possess a low-temperature scanning force microscope. However, such behaviour could be verified by such a setup with future collaborations.
Figure 6.34: Schematic diagram of the samples cross section showing the Landau level schematic diagram over the sample cross section. The schemata represent the situation in figure 4(b) for (a) no bias, (b) 10 mV, and (c) 50 mV bias. Incompressible regions are located where the electrochemical potential drawn in red is positioned between the Landau levels. The empty, half filled, and fully filled circles represent the filling of the Landau levels. While (b) resembles the result of linear response theory, (c) deviates strongly. At the side where intrinsic and extrinsic current have the same direction—for our case, the left side—a dominant incompressible stripe with enhanced electric field evolves. Breakdown mechanisms like quasi-elastic inter-Landau-level scattering (QUILLS) or heating can take place here. For the right side, the Landau level bending is reduced, approaching flat level conditions. Referenced from [109]
with groups or facilities that have access to such technology. So for now this remains just a theoretical suggestion for our system with further work required to verify it.

Another important feature that should be taken into account is that the source-drain bias affects the 1D systems behaviour by changing the position of the source’s chemical potential, $\mu_s$. This is noticed in transverse focusing experiments [103]. A spin-gap forms between the energy levels of the spin-up and spin-down electrons in the quasi-1D channel. Therefore by changing the $V_{sd}$ the relative position of the $\mu_s$ can affect the type of spin-resolved electrons injected in the 2D part of the system. This would also explain the various changes in the peaks present and the observed asymmetry in the data between the negative and positive $V_{sd}$ for the peaks on either side of the \textcircled{1} in Fig. 6.23 and in Fig. 6.24 along \textcircled{2}. In these particular cases there seems to be two peaks that are persistent at the most negatively bias voltage but in the opposite spectrum only one peak is present. However, in other section there seems to by an almost symmetric behaviour. So it is suggested that at higher magnetic fields the 1D plays a more crucial role as the assymetry
is a signature of spin polarisation [103]. Nonetheless as mentioned earlier, from this data it is difficult to extract the exact behaviour. Overall though it seems that for $B < 1.2$ T there is a symmetric behaviour as the $V_{sd}$ is varied from negative to positive, with the peaks merging and shifting towards the odd $\nu$ values and the even $\nu$'s vanish. The peaks which are possibly attributed to LL crossings also seem to merge or are on the verge of merging with other peaks and shift to odd $\nu$'s. This could be explained that as $V_{sd}$ increases QUILLS takes place and the structures due to the crossings change shape and merge. Also the fact that the odd $\nu$'s are the dominant ones for higher $V_{sd}$ could be an indication that electrons scatter from spin-split LLs of even filling factor to odd ones, thus overpopulating them. As for $B > 1.2$ T it is possible that exchange interactions are more significant within the quasi-1D which could lead to the positive $V_{sd}$ shifting the source and drain chemical potentials within the quasi-1D channel in order to allow one of the spin-states of electrons to transmit through the channel (either spin-up or spin-down). While the negative $V_{sd}$ shifts the chemical potentials to a point where both spins transmit, see Fig. 6.35. This would lead to the symmetry anomaly noticed for example along $\nu$ in Fig. 6.23.

Finally for the temperature data it can be seen in Fig. 6.27 that as the temperature increases the oscillations decrease. This can be seen in more detail in Figs. 6.28 and 6.29 where the additional peaks that appear as discussed earlier an the peaks corresponding to the odd filling factors break down and the peaks shift towards the $B$ values for the even filling factors. This is expected as the spin-splitting is suppressed as the temperature increases due to thermal energy smearing. In Fig. 6.30 the peak along the $\nu = 3$ although it reduces in amplitude it still is present, which is a peak that is thought to be associated with $\nu = 3$. The other intriguing part is that the $\nu = 2$ peak which corresponds to $\nu = 2$ vanishes quite quickly with increasing temperature but a peak on its right along $\nu = 3$ increases rapidly with increasing temperature and is seen to overpower the one from $\nu = 2$. The reason for these observations in this figure are thought to be linked to the fact that the magnetic fields are larger so there is an enhanced Zeeman splitting which preserves the presence of the odd $\nu = 3$. However, as the temperature increases interaction effects smear out, breaking down any extra features which appear between the $2 < \nu < 3$. Therefore only two peaks evolve as the temperature increases for the $\nu = 2$ and 3. The shifts are most likely due scattering effects. Similarly for Fig. 6.31 the peaks rapidly reduce and disappear without any shifting. This is somewhat unusual as the peaks were expected to shift as the temperature increases due to breakdown of the
quantum states present. However, the fact that they persist in the same position, adding further complexity to the analysis and interpretation of the data. Last but not least it is noticed that by increasing the temperature to 1 K the peak corresponding to \( \nu = 1 \) shifts to the left towards the expected value from Table 6.1 but still pertains a relatively strong peak presence which is expected as the temperature is not high enough to break-down the Zeeman interaction that takes place at the relatively high magnetic fields.

6.5 Conclusion and Future Work

To conclude unusual peaks were noticed during transverse voltage, \( V_{xy} \) and longitudinal conductance \( \sigma_{xx} \) measurements. By studying their behaviour and evolution by varying parameters like \( V_{sg} \), \( V_{sd} \) and temperature, it is concluded that the evolutions of the oscillations peaks in both amplitude and position could indicate the crossing of spin-split LLs from various sub-bands leading to additional peaks other than positions expected. Finally it is suggested that complex mechanisms take place for the evolution of the peaks which possibly include QUILLS and interacting effects between the quasi-1D and the 2D electron system present, such as spin polarisation in the quasi-1D. However, as mentioned earlier the mechanisms cannot be fully explained with the current data, so further work should be conducted to test the suggested theories.

Initially experiments should be conducted with a device fabricated from the same wafer but with the addition of a back-gate or global-gate, in order to control the 2DEG’s density similarly to the literature, [104–106]. Therefore the experiments can be repeated for both the QHE measurements and the \( V_{xy} \) data for different densities in a controlled manner as opposed to using a red-LED, in order to be able to map the greyscale of the Density - B plane for the system. Therefore any formation of ring-like structures due to LL crossings would appear in the greyscale and verify such behaviour, by varying the same parameters as mentioned in this chapter. Furthermore, hydrostatic pressure measurements can be used to provide proof for the existence of more than one sub-bands present in the system. The increase of pressure alters the Shubnikov-de Haas oscillations of the system [110, 111], and with the combination of magnetic fields applied at various angles the sub-band structure and their individual density can be studied [110, 111].

Additionally a low-temperature scanning force microscopy could be used in order to scan the changes in the IS/CSs for different source-drain bias voltages and for various \( V_{sg} \), in order to compare with the literature [109].
These scans should take place in the central part of the two 2DEG sections on either side of the split-gate and scanned at the entrance and exit if the quasi-1D channel, that is at close proximity to the split-gate in order to study any interactions that could arise at the transition point between the 2D to quasi-1D electron systems.

Finally temperature measurements should be conducted with a 4-Terminal resistance measurement in order to be able to obtain the effective g-factor value of the system to see by how much it may have enhanced and therefore could provide further information on the LL crossings taking place.

To conclude similarly to the previous chapter the importance of understanding the behaviour of IS/CS in the 2DEG and consequently there interactions with quasi-1D systems are not well understood and therefore it is highlighted in this chapter the importance of further studies in understanding in depth their behaviour and providing a universal theory on the effects observed.
Chapter 7

1D-Fractional states in InGaAs/InAlAs

7.1 Introduction

In this chapter the effects of asymmetric $V_{sg}$ and consequently asymmetric electric field within the split-gate, on the quasi-1D conductance are to be discussed. Specifically the spin polarisation of the electron system, forming multiples of $0.5G_0$ and other possible fractional states within the system. Through the literature it is assumed that the possible reason for this is due to spin-orbit interactions, SOI. Understanding the effects of SOI has gained a lot of interest in the recent years, as it opens exciting possibilities in developing spintronic devices by using solely electric-fields or to reduce the strength of the B required to manipulate certain quantum spin-related states. In section 7.2 the sample’s structure, the system’s characteristics and the method used for the measurements are briefly described. This is followed by the Theory section, 7.3, were the spin-orbit interactions are discussed and the possible reasons for the appearance of fractional states in quasi-1D systems is discussed. After that in section 7.4, the experimental results are presented and accompanied by a discussion and comparison of the data with the literature. The chapter is then concluded and has some future work presented in the final section 7.5.

7.2 Sample and Methods

The device fabricated and used for the study in this chapter is In1 for which more information can be found in Chapter 3. It is a $\text{In}_{0.75}\text{GaAs/In}_{0.75}\text{AlAs/GaAs}$
quantum well heterostructure, see Fig. 7.1. The reason for this is to study the appearance of 1D fractional states in such a system and compare it with data from the literature which had been conducted in other 2D systems [112, 113].

Its electron (carrier) density was calculated in the dark as $n_0 = 2.3 \times 10^{11}$ cm$^{-2}$ and its mobility as $\mu_e = 0.30 \times 10^6$ cm$^2$/Vs using the method described in [10, 114]. Initially the device was flashed for 6 s at -10 V with a red LED and its new carrier density and mobility were calculated as $n = 4.9 \times 10^{11}$ cm$^{-2}$ and $\mu = 0.25 \times 10^6$ cm$^2$/Vs using the data from the 4-Terminal measurement, Chapter 4, in Fig. 7.2. The main measurement conducted was sweeping the $V_{sg}$ multiple times but each time setting up the split-gates in having a fixed voltage difference $\Delta V_{sg}$. This set up can be described as being analogous to an electric Stern-Gerlach [115, 116]. Therefore spin-degeneracy can be manipulated by altering the asymmetric voltage between the split-gates due to Rashba SOIs.
Figure 7.2: $R_{xx}$ and $R_{xy}$ vs B in red and blue traces, respectively. The device used is In1 as described in Chapter 3. The filling factors are indicated for each plateau. $V_{sg} = 0.0$ V, $V_{tg} = 0.0$ V and $T \sim 10$ mK

### 7.3 Theory

#### 7.3.1 Spin-Orbit Interactions

The SOI, is the interaction between a particle’s spin angular momentum $S$, in this case an electron and its orbital angular momentum $L$ while inside a potential, leading to the $L \cdot S$ [117], [118]. This is a relativistic interaction. This effect causes the electron’s energy levels within a system to change in value. In solids there are two types of SOI [117]:

a. symmetry-independent and

b. symmetry-dependent.

The symmetry-independent ones are present in all crystal types and their source is from SOI in atomic orbitals [117]. InGaAs has a higher intrinsic spin orbit than GaAs due to the presence of In [119]. However, the symmetry-dependent ones are only present in crystals that break inversion symmetry [117, 118]. The latter are the type of SOI that are to be discussed in more detail in this chapter as they are the most relevant to this study. These are:

a. Rashba is a type of SOI which is due to surface-induced-asymmetry (SIA), by asymmetric inversion of the potential creating the confinement.
b. Dresselhaus, i.e. SOI which occurs due to lack of symmetry invariance in the bulk of the crystal, otherwise known as bulk-induced-asymmetry (BIA)

For the case of the Rashba spin-orbit coupling (RSOC) in a 2D system its Hamiltonian, $H_{R2D}$ is given by,

$$H_{R2D} = \frac{\alpha}{\hbar} (\sigma_y p_x - \sigma_x p_y) \quad (7.1)$$

where $\alpha$ is the RSOC coefficient determining the strength of the SOI and $p_{i=x,y}$ is the electron momentum and $\sigma_{i=x,y}$ is the spin Pauli matrix. This can be further simplified for 1D into,

$$H_{R1D} = \frac{\alpha}{\hbar} (\sigma_y p_x) \quad (7.2)$$

Finally the Dresselhaus Hamiltonian is given by,

$$H_{D2D} = -\frac{\beta}{\hbar} (\sigma_x p_y + \sigma_y p_x) \quad (7.3)$$

where $\beta$ is the corresponding coefficient of SOI strength for the Dresselhaus interactions. The importance of these effects is that due to the SOI an extra magnetic field, $B_{SOI}$ is induced in the system, changing the effective B and therefore the energy levels in the system. For RSOC the direction of the $B_{ext}$ is important on the overall effect of the SOI on the energy levels present in a 1D system. An example of the effects of different aligned magnetic fields is shown in Fig. 7.3. In Fig. 7.4 the effect of perpendicular B is shown for an InAs nanowire and how the Zeeman effects and the RSOC affects the LLs and consequently the behaviour of the system, causing spin polarisation to be noticed, [120]. By taking into account the Dresselhaus SOC behaviour as observed in Fig. 7.5, the parabolic behaviour of the energy levels can be further modified. For the case that $\alpha = \beta$, the energy bands retain their crossing points and therefore spin injection can take place [121–123], see Fig. 7.5(a). However, for the case of $\alpha \neq \beta$ anti-crossing points evolve, therefore the channel acts as a non-ballistic channel due to the spin part of the wire eigenstates being wave vector dependent [121]. This behaviour is due to the elastic and/or inelastic scattering processes which affect the wave vector of the electrons creating a random transmission of spin states. This is in opposition to the case of $\alpha = \beta$ where the spin preservation takes place. An important aspect of using 1D systems for manipulating the RSOC is due to the fact that the $\alpha$ parameter’s strength can be manipulated by the gate voltage applied. Nitta et al. [124] observed in InGaAS/InAlAs
Figure 7.3: (a) and (d) are schematics on how the parabolic confinement and band structure behave for a III-V semiconductor, showing a single parabola (a) and the typical $2e^2/h$ conductance plateaus, when $B_{ext} = 0$ T. (b) and (e) shows the effect of the $B_{ext}$ applied in the same direction as the $B_{SOI}$ due to RSOC on the band structure and the conductance plateaus in (e). The sub-figures (c) and (f) are for the case of $B_{ext}$ applied in a perpendicular direction to the $B_{SOI}$ due to RSOC and their corresponding band structure and conductance plateaus. (Taken from Ref. [117]).

system, see Fig. 7.6, that by increasing the gate voltage in the positive direction the strength weakens but for negative voltages it seems to increase at two different rates. A slower increasing rate is observed for less negative
Figure 7.4: (a) Splitting of LLs of the 2DEG in the three regimes: (i) bare LLs with a cyclotron frequency, $\omega_c$, (ii) LL + ZM (LL splitting by Zeeman energy $\Delta_Z$) and (iii) LL + ZM + SOC (LL splitting by the Zeeman effect and the RSOC in a strong B). Electron states $(n,\sigma)$ are characterised by the LL orbital quantum number $n$ and spin quantum number $\sigma$. $\Delta E_0$ and $\Delta E_1$ are the splittings of $n = 0$ and $n = 1$ LLs, respectively, and $\Delta_{ESO}(n,\sigma)$ is the effective RSOC energy in the regime (iii). (c) Schematic DOS of the two lowest LLs $(n = 0, 1)$ of a 2DEG in the three regimes. The DOS shows the spin splitting energies of $\Delta Z$, $\Delta E_0$ and $\Delta E_1$. (Taken and modified from Ref. [120]).

voltages and a higher rate of increase for larger gate voltages. However, for GaAs/AlGaAS systems it has been noticed by Kunihashi et al. [125] that for positive gate voltages their is a linear rate of increase initially and then it saturates.

The Dresselhaus SOC parameter, $\beta$ is dependent on the a linear and cubic term which are material dependent, quantum well width dependent and strain dependent, which are explained in detail in Ref. [126]. The cubic term is of great importance for InGaAs/InAlAs devices as the strain is higher due to greater lattice mismatch as opposed for GaAS/AlGaAS devices [126]. The calculation of the $\alpha$ and $\beta$ parameters, can be done either
Figure 7.5: Quantum wire dispersions $\epsilon(k)$ in the presence of both Rashba and Dresselhaus SOI. For equal spin-orbit strengths $\alpha = \beta$ (a) the dispersions are parabolic with no anticrossings. For differing coupling strengths $\alpha \neq \beta$ (b) the bands are nonparabolic and avoided crossings occur. (Taken from Ref. [121]).

by monitoring the precession frequency of the spins as a function of an in-plane electric field [126, 127] or by observing the angular dependence of the electrons' spin precession on their direction of motion with respect to the crystal lattice via optical methods, [128]. However, both are methods which are not available within the group's lab therefore were not calculated. The importance of SOI has regained the interest of the scientific community due to the plethora of applications for which SOI can be used for spin-FET by tuning the $\alpha$ and $\beta$ parameter accordingly [115, 121] and to obtain other exotic phases [115].

7.3.2 1D Fractional States

These states were observed by using split-gates to form quasi-1D structures in germanium 2D layers [112] and in GaAs/AlGaAs heterostructures [130]. The former concludes that a possible reason for this is due to fractional quantisation with charges $e/2$ and $e/4$. That is as the confinement is relaxed the system extends in the second dimension leading to the ground state in disappearing and allowing complex states as the one described earlier to evolve. The latter had noticed fractional states without any magnetic field which was unusual as this violated the time-reversal symmetry. This fact attracted the attention of Shavit et al. [129] with a possible explanation being due to backscattering and umklapp scattering. Such features had
been predicted theoretically and the importance of SOI was emphasised as well as the need for strong e-e interactions [129, 131, 132]. The reason for the interest in such systems is that fractional states can be manipulated and produced in an easier manner by using a 1D system as opposed to a 2D system, and with fractional states being controlled either with electric fields only (as in [130]) or by using lower B-fields to achieve fractional states which would otherwise require large magnetic field values in a 2D system. In this chapter we present the effects of SOI on an InGaAs/InAlAs heterostructure as well as the fractional states noticed that do not correspond only to Laughlin states but are made of composite fermions states as well as more exotic states as is the 5/2 fractional state. These will be presented in more detail in the following section, with a discussion to follow.

Before continuing to the next section, some results from Ref. [129] will be presented as they will be used in the discussion later. First from the theory and the simulations its shown in Fig. 7.7(a) that at constant temperature but at varying interband separations there are peaks appearing on
the left hand-side of the plateau of the fractional state $\frac{2}{5} \times (e^2/h)$ modelled, which are also seen experimentally in Refs. [112, 130]. From the model it is explained as regimes which have a chemical potential value lower than the corresponding one, therefore the conductance attempts to reach conductance value which is at a non-fractional value. However, due to the finite temperature this is suppressed forming these peaks [129]. One can think of it as a 1D “conductance overshooting” effect.

Additionally it is suggested that the plateaus expected theoretically for certain fractional states may not be universal and vary due to finite temperature. Again this can be seen in Fig. 7.7(b) as the temperature is increased from left to right and the plateau corresponding to $\frac{2}{5} \times (e^2/h)$ shifts to higher conductance values [129].
Figure 7.8: $G$ vs $V_{sg}$ with varying $V_{tg}$ from 0.00 V to -3.50 V from left to right. The $V_{tg}$ was increased at increments of -0.02 V. The blue traces are for guidance in order to emphasise certain quantum states and the evolution of the plateaus. The black arrows and the corresponding values are the conductance values in units of $e^2/h$. $B = 0.0$ T.

### 7.4 Experimental Data and Discussion

Initially from Fig. 7.8 the $V_{sg}$ was swept symmetrically with varying $V_{tg}$. The $V_{tg}$ was varied from 0 V to -3.5 V and at $B = 0.00$ T. As can be seen the left-most blue trace was conducted at $V_{tg} = 0.00$ V. The conductance plateaus observed are at $4 \times (e^2/h)$ and $(9/5) \times (e^2/h)$. It should be noticed that the $(9/5) \times (e^2/h)$ was expected at $2 \times (e^2/h)$, but this is thought to be either due to impurities causing scattering or due to SOI modifying the $e$-$e$ interactions leading to a change in the energy values. An another possibility is that this feature is due to the “0.7($2e^2/h$) anomaly”, but it is manifesting at a higher value possibly due to scattering as mentioned earlier. As the $V_{tg}$ is varied to more negative values, it is noticed that by the second and
Figure 7.9: G vs \(V_{sg1}\) with varying \(\Delta V_{sg}\) from 0.00 V to -2.00 V from right to left. The \(\Delta V_{sg}\) was increased at increments of -0.02 V. The blue traces are for guidance in order to emphasise certain quantum states and the evolution of the plateaus. The black arrows and the corresponding values are the conductance values in units of \(e^2/h\). The dashed lines are traces used for further \(V_{tg}\) measurements, explained in the chapter. \(V_{tg} = 0.0\) V, \(B = 0.0\) T.

In the fourth blue trace the lowest plateau-like feature seems to increase in value again and has a peak forming on its left side, while the \(4\times(e^2/h)\) plateau has weakened significantly. The \((7/5)\times(e^2/h)\) eventually evolves to the \(2\times(e^2/h)\) plateau and the \(4\times(e^2/h)\) state evolves to the smaller value of \((10/3)\times(e^2/h)\). The peak envelope of oscillatory behaviour noticed between the third and fifth blue traces is thought to be due to the chemical potential not reaching the corresponding integer value, in this case \(2\times(e^2/h)\), as described in the previous section and in Fig. 7.7(a). By further increasing
Figure 7.10: \( G \) vs \( V_{sg1} \) with varying \( \Delta V_{sg} \) from 0 V to +1.00 V from left to right. The \( \Delta V_{sg} \) was increased at increments of +0.02 V. The blue traces are for guidance in order to emphasise certain quantum states and the evolution of the plateaus. The black arrows and the corresponding values are the conductance values in units of \( e^2/h \). \( V_{tg} = 0.0 \) V, \( B = 0.0 \) T.

The behaviour of keeping the \( V_{tg} = 0.00 \) V and varying the \( V_{sg} \) in order as to have an asymmetric behaviour and therefore varying the channel laterally, thus changing the shape of the channel and the energy bands was studied through the following measurement. By changing the asymmetry on the \( V_{sg} \) the RSOC can be changed as well. In Figs. 7.9 and 7.10 the \( \Delta V_{sg} \) was varied from 0.00 V to -2.00 V at increments of -0.02 V and from 0.00 to +1.00 V at increments of +0.02 V, respectively. An interesting observation is that by changing the \( \Delta V_{sg} \) negatively the first plateau observed at \((9/5)\times(e^2/h)\) which was observed in Fig. 7.8 increases in value gradually and settles at an
Figure 7.11: $G$ vs $V_{sg1}$ with fixed $\Delta V_{sg} = -0.38$ V and $V_{tg}$ increased from 0.00 V to -2.00 V from left to right. The $V_{tg}$ was increased at increments of -0.02 V. The blue traces are for guidance in order to emphasise certain quantum states and the evolution of the plateaus. The black arrows and the corresponding values are the conductance values in units of $e^2/h$. $B = 0.0$ T.

The integer value of $2 \times (e^2/h)$ by the second blue trace from the right. Similarly the third blue trace from the right has the $(39/10) \times (e^2/h)$ evolve to the integer value of $4 \times (e^2/h)$ though in this case the evolution happens with the peak oscillatory behaviour mentioned earlier. Another observation is that the plateaus observed at the integer values of $4 \times (e^2/h)$ and $2 \times (e^2/h)$ weaken and strengthen as the $\Delta V_{sg}$ increases negatively. By the final blue trace at $\Delta V_{sg} = -2.00$ V the $4 \times (e^2/h)$ state disappears and only the $2 \times (e^2/h)$ remains. The dashed traces represent the offset voltage values of $\Delta V_{sg} = -0.38$ V, -1.02 V, -1.26 V and -1.88 V which were used in further measurements seen in Figs. 7.11 - 7.14.

In Fig. 7.10, we observe a different behaviour, were the two plateaus
Figure 7.12: G vs $V_{sg1}$ with fixed $\Delta V_{sg} = -1.02$ V and $V_{tg}$ increased from 0.00 V to -2.00 V from left to right. The $V_{tg}$ was increased at increments of -0.02 V. The blue traces are for guidance in order to emphasise certain quantum states and the evolution of the plateaus. The black arrows and the corresponding values are the conductance values in units of $e^2/h$. $B = 0.0$ T.

observed at $\Delta V_{sg} = 0.00$ V (first blue trace), remain almost constant until they finally disappear and no plateaus appear by $\Delta V_{sg} = +1.00$ V, with the $4 \times (e^2/h)$ plateau being the first one to disappear. It is important to notice that the $(9/5) \times (e^2/h)$ plateau does not evolve to an integer value, i.e. $2 \times (e^2/h)$ as opposed to the negative asymmetry mentioned earlier. This is thought to be due to the confinement shape being altered and thus favouring the formation of this energy state.

As mentioned earlier the first dashed trace from the right was taken from Fig. 7.9 at $\Delta V_{sg} = -0.38$ V and $B = 0.00$ T with the $V_{tg}$ varied from 0.00 V to -2.00 V at increments of $\Delta V_{tg} = -0.02$ V, see Fig. 7.11. In general it is noticed that the overall plateau evolution follows a similar trend to
Figure 7.13: G vs $V_{sg1}$ with fixed $\Delta V_{sg} = -1.26$ V and $V_{tg}$ increased from 0.00 V to -2.00 V from left to right. The $V_{tg}$ was increased at increments of -0.02 V. The blue traces are for guidance in order to emphasise certain quantum states and the evolution of the plateaus. The black arrows and the corresponding values are the conductance values in units of $e^2/h$. $B = 0.0$ T.

In the case with $\Delta V_{sg} = 0.00$ V, but the states corresponding to the plateaus have shifted in value. This could indicate that RSOC may have had a small effect on the system for this asymmetry by shifting the quantum states but is not strong enough to change the overall trend of their evolution. Initially what is noticed is that the 19/11 fractional state increases as the $V_{tg}$ increases, that is the density decreases (but confinement increases), raising the value to 9/5 at the second blue trace from the left. Simultaneously the $4 \times (e^2/h)$ shoulder-like plateau weakens and almost vanishes by this point. By further increasing the $V_{tg}$ though it seems to evolve in a week 15/4 fractional state. The lower plateau also drops in value and although weak it seems to remain around 3/2 fractional state. Though from the fourth
Figure 7.14: $G$ vs $V_{sg1}$ with fixed $\Delta V_{sg} = -1.88$ V and $V_{tg}$ increased from 0.00 V to -2.00 V from left to right. The $V_{tg}$ was increased at increments of -0.02 V. The blue traces are for guidance in order to emphasise certain quantum states and the evolution of the plateaus. The black arrows and the corresponding values are the conductance values in units of $e^2/h$. $B = 0.0$ T.

blue trace and onwards this state gets stronger as it flattens out as observed by the deeper red colour but continuously evolves to $2 \times (e^2/h)$ by the fifth trace. The evolution though happens by forming small peaks on the left-hand side of the traces until the plateau flattens out eventually, indicating that there is “conductance overshooting”. However, this state then seems to weaken out and disappears by the final $V_{tg} = -2.00$ V. In contrast a higher fractional state at 7/2 appears by the 6th (right-most) blue trace but it is constantly evolving before vanishing at the highest $V_{tg}$. The fact though that the states seem to evolve with flat-like plateaus rather than having peaks formed, could indicate that this evolution happens in a more stable fashion with the chemical potential matching the fractional state observed.
For the $\Delta V_{sg} = -1.02$ V trace at $B = 0.00$ T and varying the top-gate voltage the evolution trend of the plateaus can be seen in Fig. 7.12. In this case the $\Delta V_{sg} = -1.02$ V has stabilised the lower plateau at the integer G-value of $2 \times (e^2/h)$ at $V_{tg} = 0.00$ V. The $4 \times (e^2/h)$ state is still present although weaker. By increasing $V_{tg}$ the $G = 2 \times (e^2/h)$ state becomes weaker by the second blue trace from the left, but the $G = 4 \times (e^2/h)$ becomes stronger but shifts upwards in value towards $(21/5) \times (e^2/h)$. Then both states seem to grow weak and almost vanish. However by the time the $V_{tg}$ is increased to reach the third blue trace, a new plateau starts to form at $(7/4) \times (e^2/h)$ but by further increasing $V_{tg}$ this state disappears and new states reappear at $(16/5) \times (e^2/h)$, although weak and at $4 \times (e^2/h)$ which is more well defined (i.e. more flat). By further increasing the $V_{tg}$ we can see at the fifth blue trace that the $(16/5) \times (e^2/h)$ is well defined and the $4 \times (e^2/h)$ plateau starts to weaken. An interesting feature is that the particular $(16/5) \times (e^2/h)$ seems to be a combination of the previous $(16/5) \times (e^2/h)$ state becoming stronger and an evolution taking place from the previous $4 \times (e^2/h)$ plateau with the traces in between presenting signatures of “conductance overshooting”. As the $V_{tg}$ further increases the the latter fractional state vanishes and a very weak state at $6/5$ appears at the highest $V_{tg} = -2.00$ V.

By further increasing the $\Delta V_{sg} = -1.26$ V and varying the $V_{tg}$ the same way as previously, it is noticeable in Fig. 7.13 that more fractional states appear as we reduce the density of the system by increasing the $V_{tg}$. As before the state $2 \times (e^2/h)$ is present at $V_{tg} = 0.00$ V but instead of $4 \times (e^2/h)$ the $39/10$ state is observed. By increasing the $V_{tg}$ the $5/3$ and $7/2$ states appear by the second blue trace and remain though weakly present in the third trace. The $4 \times (e^2/h)$ plateau appears as well. Similarly to the $16/5$ state in Fig. 7.12, the $7/2$ state in Fig. 7.13 becomes stronger in the fourth blue trace as the $4 \times (e^2/h)$ plateau from the third blue trace evolves into the $7/2$ with the “conductance overshooting” behaviour noticed again. By further increasing $V_{tg}$ though all fractional states associated with $\nu$'s > 2 vanish with only the integer $\nu = 3$ state appearing at $V_{tg} = -2.00$ V. In the mean time between the fifth and sixth blue traces more fractional states appear. Initially the $7/4$ and $5/4$ states appear. However, by increasing the $V_{tg}$ they evolve into the $10/7$ and $6/5$ states with the $2 \times (e^2/h)$ plateau reappearing. Important feature is that the $10/7$ state seems to form with strong “conductance overshooting” peaks forming, forming a beat-like pattern especially when the $G = 2 \times (e^2/h)$ plateau appears. The fact that these “conductance overshooting” peaks seem to be more prominent in evolutions between states where one of them is an integer state seems to further validate the theory stated in Ref. [129] and shown in Fig. 7.7.
Finally for the case of $\Delta V_{sg} = -1.88$ V it is noticed that in this case for $V_{tg} = 0.00$ V the integer states of $G = 2 \times (e^2/h)$ and $4 \times (e^2/h)$ do not appear rather a plateau-like feature appears at $9/5$ which by further increasing the $V_{tg}$ becomes weaker in strength by the second blue trace but a plateau at $G = 4 \times (e^2/h)$. As the $V_{tg}$ increases the $9/5$ state disappears by the third blue trace, but in addition to the $G = 4 \times (e^2/h)$ state the $69/20$ appears. Then by further increasing the $V_{tg}$ these two states disappear by the fourth blue trace, with the $11/3$ state appearing instead as well as the $2 \times (e^2/h)$ and the $(4/3) \times (e^2/h)$. Increasing the $V_{tg}$ results in the $4/3$ state to evolve to the $8/7$ state but disappears by the time $V_{tg} = -2.00$ V. The $11/3$ disappears. However, the integer state at $2$ evolves into the $39/25$ and eventually $7/5$ state by increasing the $V_{tg}$. Similarly this happens while observing “conductance overshooting” peaks. In addition by increasing the $V_{tg}$ the $2 \times (e^2/h)$ reappears but weakens and drops in value to $19/10$ by the $V_{tg} = -2.00$ V. Finally the integer state at $3 \times (e^2/h)$ appears at the higher $V_{tg}$ values and remains stable.

The main important feature from these experiments is that by fixing the $V_{tg} = 0.00$ V and varying $\Delta V_{sg}$ at $B = 0.00$ T the asymmetry has a very small effect on the values of plateaus observed for the negative asymmetry, but rather has an effect on the strength of these plateaus as they weaken and strengthen in definition as the asymmetry increases negatively. However, by varying the $V_{tg}$ in addition to the asymmetry seems to have a more pronounced effect. The $V_{tg}$ reduces the density while increasing the confinement which consequently seems to encourage the observation of more fractional states. However, the amount of fractional states observed is also influenced by the asymmetry as at higher negative asymmetric voltages, e.g. $\Delta V_{sg} = -1.88$ V, more fractional states are observed as opposed to the case of $\Delta V_{sg} = -0.38$ V, when the $V_{tg}$ increases. Therefore we can observe that the density of the system as well as the SOI due to $\Delta V_{sg}$ has an important role to play in the development of fractional states. An other important aspect is that the fractional states observed are in majority odd denominator states but even denominator states are also observed. More about this will be discussed at the end of this section.

In the following Figs. 7.15 - 7.20 the effect of increasing the perpendicular B from 0.00 to 2.00 T and keeping it fixed, while the asymmetry of the $V_{sg}$ was varied from 0.00 to -2.00 V, shall be studied.

Initially the B was set at 0.50 T, see Fig. 7.15. The main difference observed from the case at $B = 0.00$ T in Fig. 7.9 is that the fractional state of $(9/5) \times (e^2/h)$ increases in value to $2 \times (e^2/h)$. In addition the plateaus present do not vary in value as the asymmetry increases as at $B = 0.00$
Figure 7.15: G vs $V_{sg}$ with $\Delta V_{sg}$ varied from 0.00 V to -2.00 V, from right to left, and $V_{tg}$ being fixed at 0.00 V. The $\Delta V_{sg}$ was increased at increments of -0.02 V. The B was fixed at 0.50 T in the perpendicular direction. The blue traces are for guidance in order to emphasise certain quantum states and the evolution of the plateaus. The black arrows and the corresponding values are the conductance values in units of $e^2/h$. $V_{tg} = 0.0$ V.

That is the both the $2 \times (e^2/h)$ and $4 \times (e^2/h)$ plateaus remain constant throughout the measurement.

At B = 1.00 T more interesting quantum states seem to appear, see Fig. 7.16. To start with at $\Delta V_{sg} = 0.00$ V in addition to the integer plateaus at $G = 2$ and $4 \times (e^2/h)$, a plateau at $3 \times (e^2/h)$ is also noticed. This is thought to be due to spin polarisation. In addition though to these integer states a fractional state appears at $29/7$. By increasing the $\Delta V_{sg}$ the fractional state and the plateau at $4 \times (e^2/h)$ start to converge and by $\Delta V_{sg} = -0.76$ V, the second blue trace from the right, they both merge. By further increasing the asymmetry the only plateau that remains well defined is the $2 \times (e^2/h)$ plateau whereas the rest become weaker and disappear by the third blue
Figure 7.16: G vs $V_{sg1}$ with $\Delta V_{sg}$ varied from 0.00 V to -2.00 V, from right to left, and $V_{tg}$ being fixed at 0.00 V. The $\Delta V_{sg}$ was increased at increments of -0.02 V. The B was fixed at 1.00 T in the perpendicular direction. The blue traces are for guidance in order to emphasise certain quantum states and the evolution of the plateaus. The black arrows and the corresponding values are the conductance values in units of $e^2/h$. $V_{tg} = 0.0$ V.

trace at $\Delta V_{sg} = -1.66$ V. However, what seems to occur is that a plateau at $1 \times (e^2/h)$ starts to appear as SOI increases causing spin polarisation, with the increased asymmetry. Furthermore, the fractional state at 14/5 starts to appear although weak it remains stable as the $\Delta V_{sg}$ increases. Finally by $\Delta V_{sg} = -2.00$V further fractions appear at 7/2 and 11/3. The 7/2 state though has oscillatory behaviour. An interesting observations is that all the fractional states seem to evolve from higher conductance values before they start forming.

For Fig. 7.17 more fractional states appear at B = 1.25 T. For $\Delta V_{sg} = 0.00$ V the same integer states at $2 \times (e^2/h)$ and $4 \times (e^2/h)$ appear as with B = 1.00 T. However, the $3 \times (e^2/h)$ state shifts to a lower value of 26/9...
fractional state and in opposition the 29/7 state shifts to a higher value at 17/4. By further increasing $\Delta V_{sq}$ the states of $2 \times (e^2/h)$, $(26/9) \times (e^2/h)$ and $4 \times (e^2/h)$ remain constant by the second blue trace from the right. The 17/4 state though disappears and instead the fractional state at 40/9 appears as well as the integer state at $1 \times (e^2/h)$ starts to appear although very weak. Increasing the asymmetry through the third and blue trace indicates that the $2 \times (e^2/h)$ still remains strong and the $1 \times (e^2/h)$ state becomes stronger.

Any other fractional states that appear at higher values are not stable and keep evolving without being well defined which is why they are not labelled in the figure. The additional increase in the $\Delta V_{sq}$ though brings to our attention the formation of stable fractional states at 8/3, 3, 10/3, 7/2 and
Figure 7.18: G vs $V_{sg}$ with $\Delta V_{sg}$ varied from 0.00 V to -2.00 V, from right to left, and $V_{tg}$ being fixed at 0.00 V. The $\Delta V_{sg}$ was increased at increments of -0.02 V. The B was fixed at 1.50 T in the perpendicular direction. The blue traces are for guidance in order to emphasise certain quantum states and the evolution of the plateaus. The black arrows and the corresponding values are the conductance values in units of $e^2/h$. $V_{tg} = 0.0$ V.

18/5. The $7/2$ state for the $B = 1.25$ T case is a lot weaker than for the $B = 1.00$ T but appears at similar $\Delta V_{sg}$ values. Also, an important feature to notice is that at $B = 1.25$ T the plateau for $3 \times (e^2/h)$ appears at the higher values of $\Delta V_{sg}$ as opposed to the $B = 1.00$ T which appears from the $\Delta V_{sg} = 0.00$ V.

Further increasing the B to 1.50 T provides the measurement shown in Fig. 7.18. Once again the $2 \times (e^2/h)$ and $4 \times (e^2/h)$ plateaus are present at $\Delta V_{sg} = 0.00$ V. Although at this B field it seems that the $1 \times (e^2/h)$ state is starting to weakly appear as well. However, the $3 \times (e^2/h)$ plateau is still not visible but the fractional state of $26/9$ in B = 1.25 T measurement reduces its value to $14/5$. By increasing negatively $\Delta V_{sg}$ the $14/5$ plateau
Figure 7.19: $G$ vs $V_{sg1}$ with $\Delta V_{sg}$ varied from 0.00 V to -2.00 V, from right to left, and $V_{tg}$ being fixed at 0.00 V. The $\Delta V_{sg}$ was increased at increments of -0.02 V. The B was fixed at 1.75 T in the perpendicular direction. The blue traces are for guidance in order to emphasise certain quantum states and the evolution of the plateaus. The black arrows and the corresponding values are the conductance values in units of $e^2/h$. Green dashed lines are to indicate a crossing feature that seems to occur. $V_{tg} = 0.0$ V.

becomes stronger as does the $1 \times (e^2/h)$ plateau. However, it eventually ends up having an oscillatory behaviour around the 14/5 value before disappearing at the third blue trace, from the right. By increasing the $\Delta V_{sg}$ other fractional states that appear initially are the 37/9, as well as the 13/3. Between the third and fourth blue traces an unusual increase from the 14/5 to the value of $3 \times (e^2/h)$, by oscillating, takes place before then dropping down to the 12/5 state which remains stable for a significant amount of asymmetry. With more asymmetry the $1 \times (e^2/h)$ plateau becomes stronger and the $2 \times (e^2/h)$ vanishes, while the 4/3 state appears briefly at the fifth blue trace. Simultaneously a new oscillatory and well defined state appears.
Figure 7.20: G vs $V_{sg}$ with $\Delta V_{sg}$ varied from 0.00 V to -2.00 V, from right to left, and $V_{tg}$ being fixed at 0.00 V. The $\Delta V_{sg}$ was increased at increments of -0.02 V. The B was fixed at 2.00 T in the perpendicular direction. The blue traces are for guidance in order to emphasise certain quantum states and the evolution of the plateaus. The black arrows and the corresponding values are the conductance values in units of $e^2/h$. Green dashed lines are to indicate a crossing feature that seems to occur. $V_{tg} = 0.0$ V.

which seems to diverge into two states at 26/9 and 3 with the 26/9 showing the signs of “conductance overshooting”. Towards the higher values of $\Delta V_{sg}$ the 10/3 state also appears with “conductance overshooting”. Finally the 1, 3/2 and 19/5 states appear with the 1 being well defined, but the other fraction states being more weakly defined.

In the case of $B = 1.75$ T, Fig. 7.19 the $1 \times (e^2/h)$ appears from the $\Delta V_{sg}$ = 0.00 V trace and remains strong until the fourth blue trace, from the right. However, it then seems to evolve into a lower value state and stabilises at 10/11. Similarly to the $B = 1.50$ T measurement the $2 \times (e^2/h)$ plateau remains strong in its presence for quite a large range of $\Delta V_{sg}$ before
it vanishes and evolves into the 3/2 state. The interesting feature with the 3/2 state is that it seems to stabilise initially but then disappears at the third blue trace and reappears after the fourth blue trace and remains stable until the final trace at $\Delta V_{sg} = -2.00$ V. This last reappearance though seems to be weakly linked to the 12/5 state which forms between the second and third blue traces. In addition to the 12/5 fractional state, the 5/2 and 13/5 states also seem to appear, weakly though, in the same region as the 12/5 state. The 5/3 state appears very weakly at the highest ranges of $\Delta V_{sg}$. Now as for the 14/5 state similarly to the 3/2 states it appears, disappears and reappears again, though at different intervals and with a stronger, more defined presence. That is it appears from the beginning of the measurements, as well as the $3 \times (e^2/h)$ state, and then disappears before the second blue trace. Then a plateau-like feature reappears at 26/9 which remains stable but gradually evolves into the 14/5 state, as the $\Delta V_{sg}$ increases. What is interesting is that at the higher $V_{sg}$ asymmetry values the 26/9-14/5 plateau transition seems to be weakly linked to some oscillatory behaviour, see the third, fourth and fifth blue traces, which take place around the $G = 3 \times (e^2/h)$ value. The troughs of the oscillations seem to be tangential to the aforementioned plateaus and the crests are located close to the 31/10 state. The importance of these features is that this could be indicating resonant tunnelling between the states [132–135], which could be linked to the “conductance overshooting” mentioned earlier. Additionally, a weak fractional state at 13/4 appears at the last few traces. The 11/3 state appears with an oscillatory behaviour, but as can be seen in the figure above $G = 3 \times (e^2/h)$ there are a lot of oscillatory regions with very unstable states which is why they have not been labelled as this would be too presumptuous to define them. However, as was mentioned in Ref. [129] the reason for these large variations and evolutions could be that the particular fractional states might not have universal values in the quantum-Hall regime, but rather are dependent on the dynamics of the system, which could also lead to a non-stable transition between the various states, leading to the resonance-like oscillations. An other explanation for the oscillations could also be due to Fabry-Perot interferences [136, 137]. That is weak reflections exist in the system while the electrons are transiting between the various energy levels, hence the transmitted and reflected electrons interfere causing oscillations to appear within the conductance. The reflections could be due to impurities or due to Coulomb repulsions between the electrons. These need to be tested by coupling the system with an Aharonov-Bohm ring as Fabry-Perot interferences are Coulomb dominated, so they will obscure any coherent interference signatures due to the Aharonov-Bohm effect [138].
Finally for the B = 2.00 T, in Fig. 7.20 the plateaus at G = 1, 2 and 4 \times (e^2/h) are present from the ∆V_{sg} trace. Interestingly the 4\times(e^2/h) plateau remains present throughout the measurement, though seems to be defined by two states which are so close they almost overlap, making it difficult to distinguish the exact values or whether some crossing of energy levels seems to take place. The integer plateau at 3\times(e^2/h) briefly appears, though not well defined, with oscillatory signatures around the sixth blue trace, from the right. Now as for the states at 2 and 1 similar behaviour to the 1.75 T measurement is observed. However, this time the fractional state evolving from the 1\times(e^2/h) plateau has the value of 11/13. An important point to notice for both B = 1.75 T and 2.00 T measurements are the green dashed lines which indicate a crossing point, where the 1x(e^2/h) state crosses with weak features that seem to evolve from higher G values. At this crossing point it is noticed how the density of the traces changes. That is before the crossing point the density of traces per ∆V_{sg} is higher than after. One suggestion for this could be that the rate of change of the RSOC α-parameter, changes at this \text{V}_{sg} point. Another suggestion is that the crossing may occur possibly due to the Dresselhaus β-parameter matching the value of the α, as explained in Fig. 7.5. This however, needs to be tested by measuring the α and β values utilising the methods described in Refs. [126–128]. The 1/7 state is also important which seems to evolve from the crossing point in Fig. 7.20. This state has been discussed in the literature, Ref. [139], that it could arise due to the coexistence of a charge density wave (CDW) state and a quantum liquid state. The appearance (disappearance) of this state is discussed that it could be due to the liquid state being strong (weak) enough to get through the sample and with the CDW state being small (big) in size in order to allow the liquid state to pass through [139]. Other states that seem to appear at the higher ∆V_{sg} values are the 9/7 and 13/9 states, with the former one possibly evolving from the 24/11 state. An important feature is that the 3/2 state seems to still be quite stable and relatively strong at similar ∆V_{sg} values at B = 1.50 T, 1.75 T and 2.00 T. This is also true for the 7/2 state but in a less pronounced way for B = 1.00 T and 1.25 T. However, the position of the 5/2 fractional state seems to vary greatly with respect to the ∆V_{sg} values, as the B is increases from 1.75 T to 2.00 T. However, it should be noticed that the 5/2 state at B = 2.00 T is not exactly present as a clear, even though weak, plateau as in the B = 1.75 T measurement. Instead it is thought that it exists due to the oscillations which take part having the crests aligned along the 5/2 point and the troughs along the 26/11 state, as seen from the third blue trace onwards in Fig. 7.20, caused by resonance tunnelling. Additionally this 5/2
state seems to shift to larger $\Delta V_{sg}$ values for the higher B measurement.

This could be explained through observations from other studies in 2D systems in varying $B_{ext}$ applied at different angles, $\theta$ with respect to the normal of the 2DEG, have shown a dependence of the 3/2, 5/2 and 7/2 state on the strength of the perpendicular and/or in-plane magnetic components of the applied B [34, 140–142]. These studies have shown that the 3/2 [141] and 7/2 [140, 142] states are stronger at larger $\theta$ than the 5/2 which seems to have an optimal strength for a particular $\theta$, in the case of Ref. [140] at $\theta = 39^\circ$ before it starts diminishing. In the case of this study, although the B was kept constant in the perpendicular set up, the fact that there was the $V_{sg}$ asymmetry present within the system resulted to RSOC being present therefore adding an in-plane component to the effective B of the system. By taking into account the literature especially for the 5/2 and 7/2 states, which have been studied in more detail, there seems to be a large dependence on the asymmetry and the $B_{ext}$ and consequently the effective B. However, more experimental work as well as theoretical understanding as to the exact magnitude of the effective B and how it is broken down to its perpendicular and in-plane components for the InGaAs/InAlAs system. The effects need to be compared in both 2D and quasi-1D systems, as the literature provided for these states is for 2D systems in tilted B, especially in GaAs/AlGaAs for the 5/2 and 7/2 [34, 140, 142] and for 3/2 some studies additionally on MgZnO/ZnO [141]. Last but not least other fractional states that seem to occur at $B = 2.00$ T are the 9/4, the 13/5 and 14/5 and the 26/7. A brief observation is that once again the 14/5 state seems to appear at the initial $\Delta V_{sg}$ values before disappearing and then reappearing at the larger $\Delta V_{sg}$ values, as for $B = 1.75$ T. Also, with the exception of the 3/2 state, it seems to be the only other fractional state which seems to be consistently present, especially for the lower $\Delta V_{sg}$ values, for a relatively large B range that is between 1.50 T and 2.00 T.

Overall an important feature is that most of the oscillations mentioned earlier seem to be enhanced with asymmetric voltages across the split-gates. Then the variation of $V_{tg}$ and B seems to add an extra factor to this, as the increase in these parameters makes the presence of the oscillations more noticeable. Additionally a further observation that is of interest is that for the case of $B = 1.75$ T it is noticed that the 12/5, 5/2 and 13/5 have been noticed simultaneously and across the same asymmetry values. This is important as these features have been noticed as filling factors in a GaAs/AlGaAs 2DEG when there system was configured at a fixed $\theta = 60^\circ$ and there $B_{ext}$ was varied from 3.3 to 3.6 T. The fact that an observation of these states can be established at almost half the magnetic field being externally applied and
can be easily accessed by varying the $V_{sg}$ with small increments could lead in an easier way to access the highly discussed $\nu = 5/2$ state [34, 140–148], as well as the $12/5$ [34] state which are predicted in being non-Abelian states and could lead to the development of a fault tolerant topological quantum computer. More details on the importance of the non-Abelian states in forming such quantum computers and why the $5/2$ and $12/5$ state are so important is explained in extensive detail by Nayak et al. [34] and of course in the 1990s Moore-Read paper [149]. So although this might be presumptuous, it can be stated preliminarily that these 1D states could be relatable to the filling factors in 2D systems, but it should be expressed that more measurements and tests need to be conducted in order to verify whether they can be considered as interchangeable features, as well as whether SOI or lateral confinement are the most important parameters.

Moreover as mentioned earlier, a plethora of fractional states seem to appear in this study. Most of them have been seen in various materials and measurements, with Refs. [37, 147, 148, 150] summarising the most commonly seen. There have been many discussions on the nature of fractional states in the quantum-Hall effect, with the main ones being proposals by Laughlin on fractionally charged quasiparticles [31] and by Jain with the CF theory [35], as discussed in Chapter 2. However, for the 1D case the main suggestion that seems to emerge from various sources is the utilisation of Luttinger liquid. This idea seems to be in consistency with our data as it explains the appearance of the various fractional states by utilising theoretically the effects of backscattering and the umklapp processes. At the same time the fact that the resonances observed in these measurements as well as in other experimental works in Ref. [112, 130] seem to be well explained by these features. However, an important parameter that has to be clarified is that in this study a perpendicular $B$ was used as opposed to Gul et al. [112] and Kumar et al. [130]. This, could be why in this case higher fractional states are noticed as the perpendicular field induces the formation of the LLs. Similarly to to [120] the LLs as well as Zeeman splitting should be taken into consideration. Though in this theoretical model [120] backscattering as well as Dresselhaus SOI are not taken into consideration. Further work is required using the methods described in [126–128] to measure SOI parameters for the Rashba and Dresselhaus effects. Finally as can be notice for the smaller $\Delta V_{sg}$ values the traces seem to be closer together as opposed for the larger $\Delta V_{sg}$. This is thought to be a consequence of the RSOC parameter $\alpha$ changing its strength as $\Delta V_{sg}$ increases which would match the literature, [115]. Although it should be stated that in our case the $V_{sg}$ is asymmetric whereas in the literature [115], the $\alpha$ was measured with respect
to a global/back-gate.

Nonetheless although there are still a lot of grey areas in this field and further work experimentally and theoretically will be required to understand these features, these type of systems could prove to be of great use in constructing quantum computers in the future.

7.5 Conclusion and Future Work

Utilising asymmetric voltages, $\Delta V_{sg}$ and by applying $V_{tg}$ and a perpendicular B, fractional states in 1D conductance have been noticed. The importance of SOI in forming these states is also discussed. The formation of these states can possibly be explained by Luttinger liquids as explained in the literature, where scattering effects could lead to fractional states being formed without B. However, by utilising relatively small B values and asymmetry in the 1D channel, more exotic fractional states, like the 5/2 and 12/5 states appear. If proven to be non-Abelian, these states could be used for building a fault tolerant topological quantum computer.

Future work to be conducted is to repeat the measurements by varying the B-field at different angles to see how the plateaus react. Also, shot noise experiments should be conducted as these could reveal possible fractional quasiparticles that might play a role in certain of the fractional states [151].
Chapter 8

Conclusion

In this thesis work on studying the effects of interactions and transport in low-dimensional III-V semiconductor systems are presented. Initially some theory and fabrication processes are presented in the second and third chapter as well as experimental techniques in chapter 4.

For the work in Chapter 5 and 6 a GaAs/AlGaAs heterostructure was used. In the former chapter the effect of an electrostatically controlled constriction in the 2DEG was used to study the effect of anomalous resistance in the transverse resistance and the hysteresis effect in the longitudinal resistance measurements. A discussion on how the formation of incompressible and compressible strips was discussed and their influence on these effects. Also we present the results on how anomalous resistance in the transverse direction has been noticed not only in integer filling factors, but in fractional states as well, which seems to have been only theoretically speculated before.

In Chapter 6 the measurement of the transverse voltage while having the Hall-bar set-up in a 2-Terminal conductance measurements, has revealed peaks and troughs in the oscillations which were not expected. The evolution of these features by increasing the 1D constriction, were explained through the interactions of spin-split Landau levels from two sub-bands present within the system and the interaction of spin-polarisation in the quasi-1D channel and the 2DEG on either side. By crossing over, the LLs create these extra oscillatory features, in addition to the ones expected for specific filling factors at a particular magnetic field value. In addition the effect of source-drain on these transverse voltage oscillations were discussed, where there modification at extreme source-drain bias values thought to be a consequence of QUILLS.

Finally in Chapter 7 the effects of SOI with and without the presence of
a perpendicular B was discussed and how various fractions seem to occur in the 1D conductance study. A possible candidate for these states is the use of Luttinger liquids but there are still a lot of grey areas which need to be addressed so no final conclusion can be made until further studies can be conducted.
Appendix A

In this appendix detailed calculations on how the areas under the curve for the $W_{IS}$ versus $B$, for the overshooting study in Chapter 5, are presented. By assuming that the areas enclosed in the evanescent regime are proportional to the current flowing through each corresponding IS, a comparison can take place between the measured overshooting resistance and the approximate distribution of the current within each IS. Consequently, an estimated value of the resistance overshooting for the corresponding filling factor can occur. As an example the $\nu = 5/3$ is used to explain the detailed calculations. Finally the tables A1 and A2 show the data collected for the different filling factors.

Initially as can be seen in Fig. A1 the blue shaded areas represent the evanescent regions of the quantum states with filling factors $\nu = 1, 4/3, 3/2$ and $5/3$ which overlap within the $5/3$ fractional state, thus contributing to its resistance overshooting. Using the trapezium rule the areas of these shaded regions can be calculated. An example for the area enclosed in the evanescent regime for $\nu = 5/3$ is shown and explained in Fig. A2. Then by calculating the areas, $A_{\nu}$, for each filling factor (blue shaded regions), it is assumed that the sum of the areas is proportional to the total current flowing in the system, $I_0$. Therefore by calculating $A_{\nu}/A_{tot,\nu}$ one can estimate roughly the current distribution within each IS. By modifying Eq. 5.14 in Chapter 5, the following equation can be obtained,

$$\left(\frac{A_{\nu_1}}{A_{tot,\nu}}\right)\left(\frac{1}{\nu_1}\right) + \left(\frac{A_{\nu_2}}{A_{tot,\nu}}\right)\left(\frac{1}{\nu_2}\right) = x\left(\frac{h}{e^2}\right)$$

(A1)

where $A_{\nu_1}, A_{\nu_2}$ correspond to one of the blue shaded areas enclosed within the evanescent regime of the $\nu$, which exhibits overshooting, for the respective filling factors $\nu_1$ and $\nu_2$. $A_{tot,\nu}$ is the sum of all the areas enclosed. Then a rough estimation of the overshooting resistance, $R_{Area}$ can be calculated by,

$$R_{Area} = xR_K.$$  

(A2)

Therefore the overshooted resistance value $R_{Area}$ can be compared with the measured resistance $R_{xy}^M$ to see whether there is a good approximation between them. Also we can compare $R_{Area}$ and $R_{xy}^M$ with the expected values of resistance for the corresponding $\nu$, $R_{xy}^T$. All these values can be seen in both Tables A1 and A2.
Figure A1: The blue shaded areas in (a)-(d) are the areas enclosed for the IS of the quantum states 5/3, 1, 3/2 and 4/3, respectively, while in the evanescent regime and are overlapping. Therefore, they all contribute to the resistance overshooting in the 5/3 fractional state.
Figure A2: (a) The blue shaded region is the enclosed area calculated for the 5/3 fractional state when in the evanescent regime. (b)-(e) The grey shaded regions are the various areas calculated using the trapezium rule in MATLAB. The sum of the areas in (b) and (c) were subtracted from the sum of the areas in (d) and (e), in order to calculate shaded area in (a).
\[
\begin{array}{cccccccc}
\nu = 4/3 \\
Ar_{1/3} & Ar_{4/3} & Ar_{Total} & \text{---} & \text{---} & Ar_1/Ar_{Total} & Ar_{4/3}/Ar_{Total} & \text{---} \\
(x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\
13.640 & 18.159 & 31.799 & \text{---} & \text{---} & 0.429 & 0.571 & \text{---} \\
\nu = 3/2 \\
Ar_{1/3} & Ar_{4/3} & Ar_{3/2} & Ar_{5/3} & Ar_{Total} & \text{---} & Ar_{4/3}/Ar_{Total} & Ar_{5/3}/Ar_{Total} \\
(x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & \text{---} & \text{---} & \text{---} \\
11.870 & 6.105 & 14.871 & 32.846 & \text{---} & 0.361 & 0.186 & 0.453 \\
\nu = 5/3 \\
Ar_{1/3} & Ar_{4/3} & Ar_{3/2} & Ar_{5/3} & Ar_{Total} & \text{---} & Ar_{4/3}/Ar_{Total} & Ar_{5/3}/Ar_{Total} \\
(x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & \text{---} & \text{---} & \text{---} \\
3.043 & 2.038 & 5.111 & 12.492 & 22.684 & 0.134 & 0.090 & 0.225 & 0.551 \\
\nu = 8/3 \\
Ar_{1/3} & Ar_{4/3} & Ar_{3/2} & Ar_{5/3} & Ar_{Total} & \text{---} & Ar_{4/3}/Ar_{Total} & Ar_{5/3}/Ar_{Total} \\
(x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & \text{---} & \text{---} & \text{---} \\
0.078 & 4.657 & 5.354 & 10.089 & \text{---} & 0.008 & 0.462 & 0.531 & \text{---} \\
\nu = 3 \\
Ar_{1/3} & Ar_{4/3} & Ar_{3/2} & Ar_{5/3} & Ar_{Total} & \text{---} & Ar_{4/3}/Ar_{Total} & Ar_{5/3}/Ar_{Total} \\
(x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & \text{---} & \text{---} & \text{---} \\
5.270 & 0.031 & 6.688 & 11.989 & \text{---} & 0.440 & 0.002 & 0.558 & \text{---} \\
\nu = 10/3 \\
Ar_{1/3} & Ar_{4/3} & Ar_{3/2} & Ar_{5/3} & Ar_{Total} & \text{---} & Ar_{4/3}/Ar_{Total} & Ar_{5/3}/Ar_{Total} \\
(x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & (x \times 10^{-9} \text{ Tm}) & \text{---} & \text{---} & \text{---} \\
2.545 & 4.835 & 3.437 & 10.817 & \text{---} & 0.235 & 0.447 & 0.318 & \text{---} 
\end{array}
\]
### Table A1

Areas calculated for each IS in the evanescent regime which overlap in the same magnetic field range and therefore contribute to the overall resistance for the particular \( \nu \) mentioned. \( A_\nu \) is the area corresponding to the particular \( \nu \). \( A_{\text{Total}} \) is the sum of all \( A_\nu \) for the corresponding overshoot region.

<table>
<thead>
<tr>
<th>( \nu = \frac{7}{2} )</th>
<th>( Ar_2 ) (x 10(^{-9}) Tm)</th>
<th>( Ar_3 ) (x 10(^{-9}) Tm)</th>
<th>( Ar_{10/3} ) (x 10(^{-9}) Tm)</th>
<th>( Ar_{7/2} ) (x 10(^{-9}) Tm)</th>
<th>( Ar_{\text{Total}} ) (x 10(^{-9}) Tm)</th>
<th>( Ar_2/Ar_{\text{Total}} )</th>
<th>( Ar_3/Ar_{\text{Total}} )</th>
<th>( Ar_{10/3}/Ar_{\text{Total}} )</th>
<th>( Ar_{7/2}/Ar_{\text{Total}} )</th>
</tr>
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<tbody>
<tr>
<td>1.735</td>
<td>3.091</td>
<td>1.953</td>
<td>3.107</td>
<td>9.886</td>
<td>0.176</td>
<td>0.313</td>
<td>0.198</td>
<td>0.314</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \nu = 5 )</th>
<th>( Ar_4 ) (x 10(^{-9}) Tm)</th>
<th>( Ar_5 ) (x 10(^{-9}) Tm)</th>
<th>( Ar_{\text{Total}} ) (x 10(^{-9}) Tm)</th>
<th>( Ar_4/Ar_{\text{Total}} )</th>
<th>( Ar_5/Ar_{\text{Total}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.847</td>
<td>2.104</td>
<td>2.951</td>
<td>—</td>
<td>0.287</td>
<td>0.713</td>
</tr>
</tbody>
</table>
Table A2: Filling factors with observed resistance overshooting. \( B_T \) is the theoretical magnetic field value calculated using the screening theory. The \( B_T \) is at the value where the evanescent regime begins, i.e. where the \( \lambda_F \) and \( a_Q^{\text{QHA}} \) intersect. The \( R_{xy}^T \) is the expected value for the particular filling factor. The \( R_{xy}^M \) is the transverse resistance with overshooting measured at the \( B_T \) value. The \( R_{\text{Area}} \) is the transverse resistance with overshooting estimated through the areas under the curves. The ratio \( (R_{\text{Area}}/R_{xy}^M)-1 \) is the percentage difference between the the two resistance values.

<table>
<thead>
<tr>
<th>( \nu )</th>
<th>( B_T ) (T)</th>
<th>( R_{xy}^T = R_K/\nu ) (( \Omega ))</th>
<th>( R_{xy}^M ) (( \Omega ))</th>
<th>( R_{\text{Area}} ) (( \Omega ))</th>
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