Chapter 16: A MOOC for adult learners of mathematics and statistics: Tensions and compromises in design

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Abstract

There are many adults with low mathematical/statistical knowledge who would like to enhance that understanding. There are insufficient teachers to respond to the level of need and so innovative solutions must be found. In the UK, the Ufi Charitable Trust has funded a project to develop a free open online course to offer motivated adults access to powerful ideas. We reflect on the tensions and compromises that emerged during its design. More specifically, referring to data collected from users, we consider the challenge of developing resources that will support heterogeneous students from unknown backgrounds, who may have already been failed by the conventional educational system and who will have no interactive tutor support within this course.

A MOOC FOR ADULT LEARNERS OF MATHEMATICS AND STATISTICS TENSIONS AND COMPROMISES IN DESIGN

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There are many adults with low mathematical/statistical knowledge who would like to enhance that understanding. There are insufficient teachers to respond to the level of need and so innovative solutions must be found. In the UK, the Ufi Charitable Trust has funded a project to develop a free open online course to offer motivated adults access to powerful ideas. We reflect on the tensions and compromises that emerged during its design. More specifically, referring to data collected from users, we consider the challenge of developing resources that will support heterogeneous students from unknown backgrounds, who may have already been failed by the conventional educational system and who will have no interactive tutor support within this course.

1. ADULT LEARNERS OF MATHEMATICS AND STATISTICS

Open online courses (MOOCs) are sometimes posed as offering an educationally and cost-effective way of enabling adults new opportunities to improve their grasp of a particular subject, without needing to enrol on a face-to-face course, and at a much lower cost per learning outcome than for an equivalent taught course. MOOCs, for example those provided under the banners of FutureLearn, Coursera, EdX or Udacity, are typically at or approaching undergraduate degree level. But within the general adult population there are major gaps in knowledge at much lower levels. Mathematical and statistical knowledge is a case in point. Khan Academy (www.khanacademy.org) provides open online resources for learning mathematics, though these are closely tied to the traditional 'schools' curriculum.

A substantial proportion of the adult population (reported as 29% in BIS 2013 p. 38) have basic numeracy, but are not confident users of key mathematical ideas in life and work, at, in UK terms, Level 2[1]. Statistically literate behaviour is, according to Gal (2002), predicated on the activation of five knowledge bases, including mathematics, and a cluster of supporting dispositions. Statistical literacy, defined as "the ability to interpret, critically evaluate, and communicate about statistical information and messages" (Gal 2002, p. 1), is one critical but neglected skill area that needs to be addressed if adults are to become more informed citizens and recognise the value of statistics in making decisions (Wallman 1993).

In 2013, a consortium comprising Calderdale College, UCL's Institute of Education, and the awarding organization OCR secured funding from the Ufi Charitable Trust to develop a free open

[1] In the UK system, Level 2 is the standard expected of school students by age 16. BIS (2013 p. xxviii) notes that individuals "with skills below Level 2 may not be able to compare products and services for the best buy, or work out a household budget." This describes a floor for level 2 but at the higher end, it is expected that individuals will be able to solve multi-stage problems. For data handling, this would mean dealing with descriptive rather than inferential statistics and calculating probabilities for combined events but not formalising algebraically as distributions.

online course for self-motivated adults who want to improve their appreciation of mathematics and statistics at Level 2. The course, being free to the user, needed to be sustainable at very low cost per learner. The course "Citizen Maths: Powerful Ideas in Action" (www.citizenmaths.com) has been developed over the last 30 months, through four main iterations, during which period several thousand people have signed up for it. Citizen Maths covers five powerful ideas in mathematics: proportion, measurement, pattern, representation, and uncertainty. The focus of this paper is on the tensions and compromises in the design of the sections of the course on representation (powerful because it underpins data, graphs, distributions, sampling and bias), and, to a lesser extent, on uncertainty (powerful because it underpins probability, risk, odds, large and small scale effects). We illustrate the design of the course through example activities drawn from representation and uncertainty. In fact, these activities speak to several of the five suggestions to work with learners in ways that are different from the typical instructional methods as proposed by Gal (2002). We explain those connections when reporting the example activities below.

As the course has developed, we have collected information to evaluate and inform each iteration of the resource. To review the challenges, the following data has been collected and will be used:

- Information about participants at sign up including their purposes in taking part
- Data about participant completion of each element of Citizen Maths
- Participant views of the various activities via rating scales and additional qualitative comments

In this paper, we discuss four design challenges often arising when the constraints of the course conditions competed with our pedagogic ideals. The challenges addressed are: how to offer resources to heterogeneous students; how to engage students without tutor support; how to promote meaningful mathematics; and how to assess. For each of the four challenges, we discuss the issues involved and evaluate our response in light of the data collected.

2. CHALLENGE 1: HOW TO OFFER RESOURCES TO HETEROGENEOUS STUDENTS

2.1. Discussion of Challenge 1

In a conventional teaching situation, the classroom teacher or tutor will often know the students in a quite personal way but, even if this is not the case, there will be a number of known characteristics amongst the students. In the case of our MOOC, little is known about the students. We anticipated that our students, as adults, will have been through elementary and high school mathematics, taught with a focus on the skills, methods and concepts set out in typical school curricula. Although research gives some general pointers to what typical learners know and understand at Level 2, further complexity was added to the design challenge because we could expect our learners to be heterogeneous in what, as experienced adults, they might bring to the course.

We therefore took an early view that there was little point in offering a course that focused on those same methods, skills and concepts in the system that had already failed our students. Instead of focusing hard on techniques when we knew little about what techniques the students might already have grasped, we aimed to focus on a few powerful ideas that might give our students some insight into how the discipline can 'get stuff done' for them (Pratt 2012). In this respect, we were influenced by Papert's ideas on how students should learn to mathematize before following a formal course in mathematics (Papert 1972).

Each powerful idea in Citizen Maths is structured according to different ways in which it might be experienced, called 'Powerful-Ideas-In-Action' (PIAs). Each PIA consists of three or four activities, in which the student learns how the mathematical idea might be useful for them in their personal, social, occupational or scientific lives – the same contexts used by PISA (www.oecd.org/pisa/). Here are two examples of activities from each of the PIAs in 'Representation', which broadly speaking focuses on statistical literacy. Students meet the PIA, 'interpreting data', for example, in the contexts of: (i) how voting translates into seats in the Houses of Parliament and (ii) using statistics on violence and alcohol; both contexts can be considered societal. Students meet 'interpreting charts', for example, in the context of: (i) trends in media communications and (ii) how your household income compares to the rest of the country; both contexts could both be regarded as personal. Students use statistics when 'comparing groups' in the context, for example, of: (i) how many people live in the same house in different regions of the country (societal) and (ii) of battery lifetimes for different usage of mobile phones (personal). Similarly, students work on the Uncertainty powerful idea through the PIAs, 'making decisions', 'playing games' and 'creating or using simulations' in personal, societal and occupational contexts.

It is worth noting that various powerful ideas present different challenges when attempting to create opportunities for the idea to be seen as useful in the students' everyday or working life. How one interprets statistical data is highly dependent on the context whereas other areas of mathematics are often presented formally as if they were context-free. Cobb and Moore (1997) argued that, although mathematicians draw on context for motivation, their ultimate concern is with abstract patterns. In contrast, patterns in data analysis only have value according to how those patterns interweave with a contextual story line. Indeed, Wild and Pfannkuch (1999) have noted the fundamental importance of context in statistical thinking, when they depicted such thinking as emanating from the raw materials of statistical knowledge, information in data and context knowledge.

Our approach in Citizen Maths has been somewhat radical insofar as we have attempted to design activities that present proportion, measurement and pattern as contextually meaningful as those in representation and uncertainty; in so doing, we recognize that this aspect of the project has been especially challenging.

2.2. Evaluating our response to Challenge 1

Prior to registering to use Citizen Maths, participants are encouraged to undertake a pre-course self-assessment which is intended to make sure that the individuals who do take part have some understanding of the level of mathematics involved and the commitment needed. By the end of February 2017, there were just under 19,000 individuals who had completed a pre-course self-assessment of whom just under 10,000 had gone on to register for the course.

What can we say about these individuals?

The age profile of participants (see Fig. 16.1) shows a good spread across age groups with modal group being in their 30s. There was a smaller percentage of the participants under 20 although it should be noted that the sign-up process required individuals to be older than 16 and thus the available cohort is smaller than other age groups.

The gender split is weighted towards women (see Fig. 16.2). This is consistent with broader educational participation, for example Bosworth and Kersely (2015) noted that there were 53.9% females to 40% males involved in apprenticeship programmes in the academic year 2012/13.

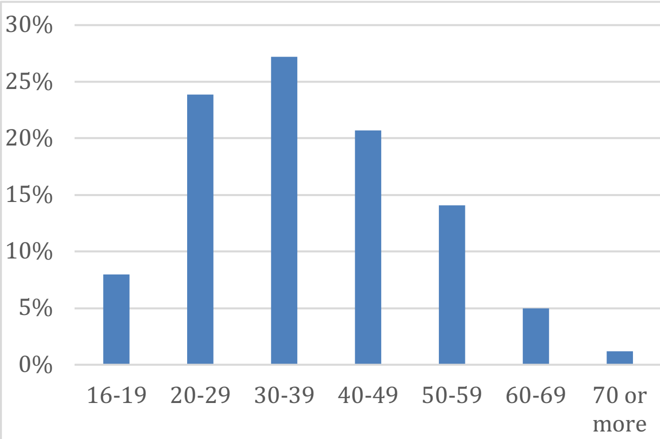


Figure 16.1: Age profile of participants in Citizen Maths

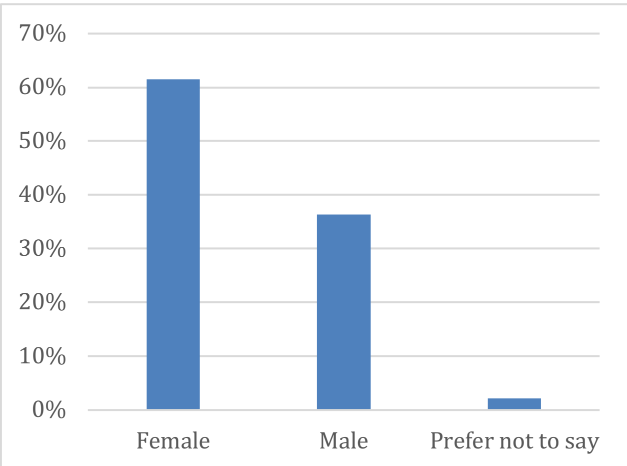


Figure 16.2: Gender profile of participants in Citizen Maths

We asked the participants to identify the level of their highest qualification in any subject and for mathematics. The largest group did identify qualifications that were at NQF level 2 and below with significant numbers at Advanced level (level 3) and Higher education levels (level 4 and above). This includes those who have successfully progressed in non-mathematical subjects but have yet to achieve in mathematics.

We asked individuals to identify their reasons for taking part in Citizen Maths (see Fig. 16.3). It is worth noting that a good proportion of those signing up to Citizen Maths could be described as 'interested professionals' who have already succeeded at mathematics and have taken part in order to see how useful the resource is for others. Nevertheless, these are not a majority and as time moves on, and participant numbers increase, they are becoming an ever-smaller proportion of the total.

Overall, Citizen Maths has attracted males and females across ages who feel that they need to improve their mathematics and these individuals do have a range of other qualifications (although most have not achieved NVQ level 3). Whilst we do have data about some other characteristics, it might be also be useful to know how participants are spread in relation to income or by profession. This might help us to understand for whom the course is most and least effective and to make changes. The problem is that the more questions that are asked, the larger is the barrier to participation.

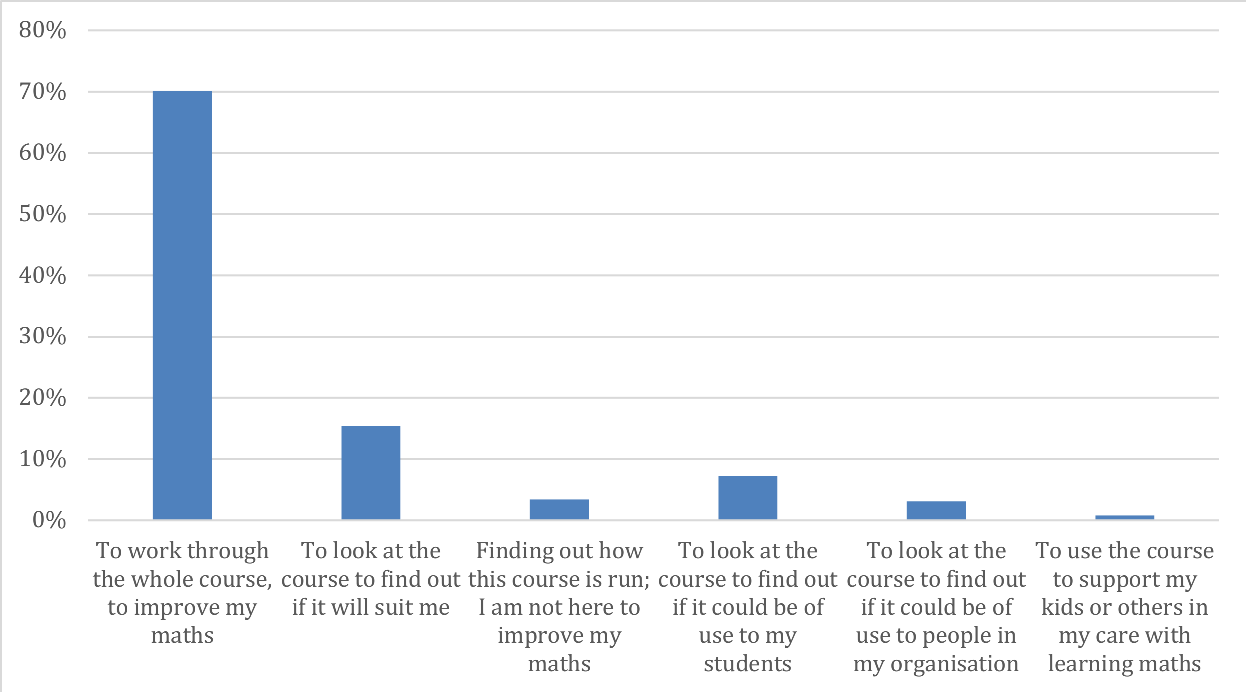


Figure 16.3: The goals of the starters of Citizen Maths as selected when registering

We have noted some of the characteristics of those who have signed up to Citizen Maths and noted the mix. What is missing here is the extent to which the resource may or may not meet the needs of this group. In the evaluation of our response to Challenge 2 we summarise the feedback about the effectiveness of the course from users.

3. CHALLENGE 2: HOW TO ENGAGE STUDENTS WITHOUT TUTOR SUPPORT

3.1. Discussion of challenge 2

Teachers in conventional classrooms are able to offer a personal level of interest and empathy with their students. In Citizen Maths there is no tutor present in real time. Moreover, the course aims to be sustainable in the future without the presence of a tutor. We decided to make the course feel personal by adopting techniques used by Peter Norvig and Sebastian Thrun in their very successful 2011 open online course "An introduction to artificial intelligence". We used two 'to-camera' tutors, each of whom would introduce and develop particular PIAs. We make regular use of short videos, sometimes involving the talking head of one of the tutors but often showing their hands or computer screen as they develop the mathematical idea in real time. Although no real-time interaction with students was possible, we hoped this would help to develop some intimacy, almost as if the tutor were talking directly with the student in their own home (Guo et al. 2014).

A limitation of this approach is that it is not possible to produce video that is equally engaging and correctly paced for a diverse set of students. The 'feature', however, that people can, if they wish, skip through videos or indeed repeat them may be an important benefit but with the associated risk that a student who skips through videos might miss a key learning point.

We recognized that these students already would have some initial motivation for joining the course and it was important to maintain engagement or eagerness that might be available at the outset.

Face-to-face teachers might use their own personality to push through times when their class is less engaged. The best we could offer was to design purposeful tasks and this became very important in our approach in the light of other limitations.

Designing tasks that are seen as purposeful by the learners such that the learner comes to appreciate the power of the mathematics in that context is far from trivial, even in conventional classrooms (Ainley et al. 2006). Noss and Hoyles (1996) discuss what they call the ‘play paradox’; when a designer builds an environment that supports playful activity, the designer loses some control over what the learner might in fact learn. The teacher in a conventional classroom is unlikely to escape this same tension. With a clear obligation to a curriculum, teachers have to manage a corresponding ‘planning paradox’ (Ainley et al. 2006) as they attempt to inspire engagement without losing focus on that curriculum. This tension was alleviated to some extent in designing Citizen Maths by avoiding a highly prescriptive curriculum. We aligned ourselves to a philosophy in which the aim was to introduce students to the power of mathematics and statistics within their personal and social contexts. This felt less constraining than a commitment to a curriculum, especially one that might emphasize skills and techniques.

In discussing how to enhance statistical literacy, Gal (2002) proposed that a novel way to work with learners might be to focus on understanding results from polls, samples, and experiments as reported in the media. In fact, in one Citizen Maths activity, the student is required to predict the number of seats that a political party will gain, given the results of a prior opinion poll. The activity is first introduced by Noel-Ann, one of the two to-camera tutors, who describes the results of an opinion poll conducted on the day of the general election, and Noel-Ann then introduces an app, specially designed for the course. The app allocates seats randomly to the main political parties, according to probabilities set by the opinion poll results (see Fig. 16.4). The app introduces an element of playfulness as the student can run the app several times and perhaps note that the number of seats allocated varies though there is a limit to the extent of the variation. Utts (2003) has emphasised the importance of helping students to understand that variability is natural.

Of course, there is a danger that the student may miss this key learning point. In a conventional classroom, the teacher is able to assess the extent to which the student needs support in recognizing the nature of the random fluctuation. The best we can do in the MOOC context is to include a self-assessment quiz and offer a review of the lesson. In this example, Noel-Ann shows how she used the app and places some emphasis on the key learning points. Even so, the activities often felt more prescriptive than might have been the case with a teacher working with a class face-to-face, who could respond on the fly to what the students were doing.

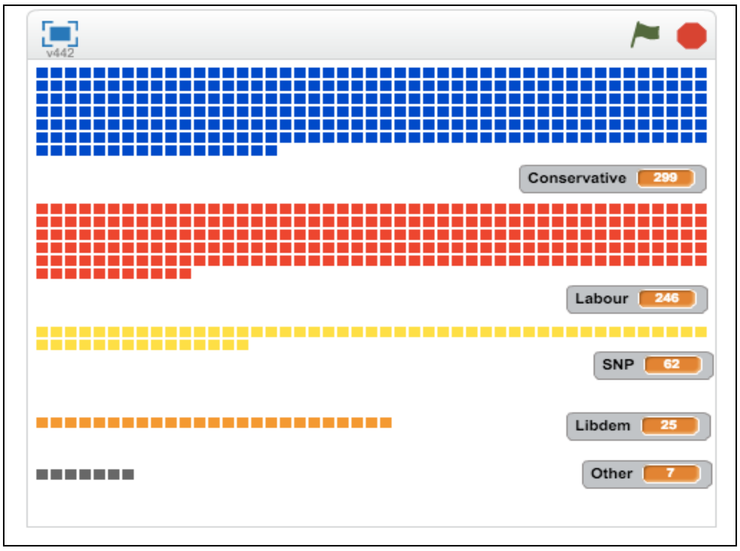


Figure 16.4: The app allocates randomly seats to each of the main political parties.

The above example is one of many where we adopted the use of technology to instil a sense of playfulness, which we hoped would encourage sustained engagement. Whereas the above activity involves using a specially designed app, we exploited technology in a variety of other ways. For example, an activity on interpreting crime figures offers the data in the form of a spreadsheet. The data shows the number of violent incidents each year and how many of those were carried out by offenders under the influence of alcohol or drugs. The students are invited to explore this data to investigate possible relationships between crime and the use of alcohol or drugs. We see this example of analysing crime figures as responding to another of the five suggestions by Gal (2002) to work with learners in new ways. The crime figures activity offers an opportunity to develop a critical stance by supporting beliefs with statistical information. In fact, according to the analysis by Watson and Callingham (2003), the use of critical thinking signals the two highest of six levels of statistical literacy (the highest level also requires the use of mathematical expertise such as proportional thinking). A second example is in the playing games PIA of the 'Uncertainty' powerful idea. The students are given a simulation of a slot machine (Fig. 16.5). They are told that slot machines are illegal if they do not pay out in the long term more than a stated proportion of its income, in this case 85%. The activity might support understanding probabilistic aspects of statements about risk and side effects (another of the five new areas in which work might be done with learners, as proposed by Gal 2002). In addition to playing the slot machine game, the student can open up the app and look inside at the coding, written in Scratch (https://scratch.mit.edu/).

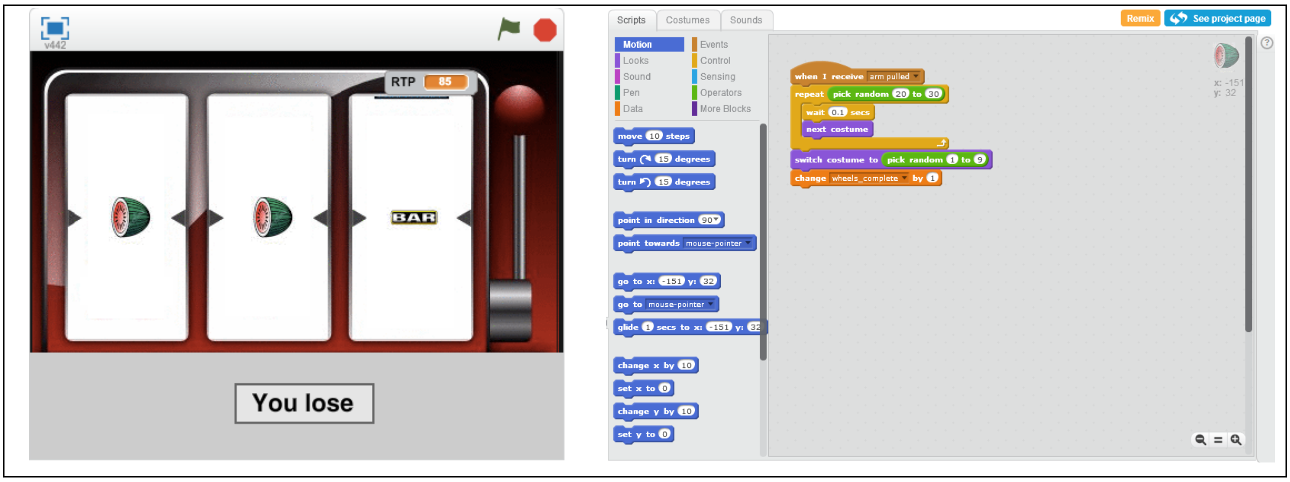


Figure 16.5: Screenshots from slot machine activity. Students are asked the following question: does the slot machine payback at least 85% of its income?

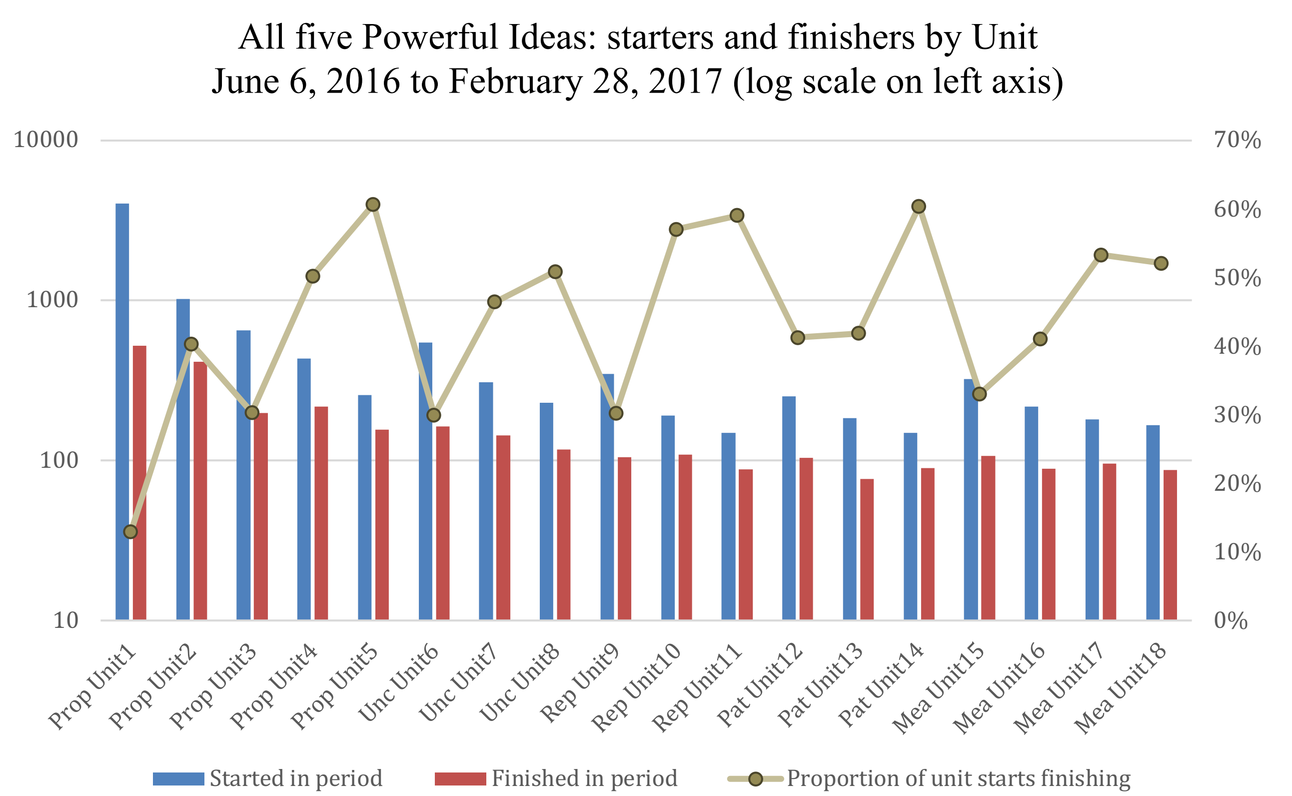
Being able to open up the app creates new dimensions for an inquisitive student, and builds on earlier stages of Citizen Maths which introduce Scratch programming as a way of revealing hidden mathematics (Noss and Hoyles 1996). Of course, a student who gets out of their depth in this course will have less support to recover than one in a conventional situation. For example, we have noted students becoming frustrated when programming in Scratch because of not knowing simple remedies to problems such as how to clear the screen. Such problems would be resolved trivially by a live tutor. On the other hand, the student can always return to the original app, and try again, although this is not necessarily an ideal solution. In later iterations of the course, we have developed on-screen help and refined its design to improve usability.

There are a number of learning points in this activity. The students may appreciate the power of knowing the underlying probabilities, as can be found by examining code, rather than just running the simulation. They can gain some understanding of how they may win in the short term but they will inevitably lose in the long term if they keep playing. This latter point, as is often the case in Citizen Maths, has social as well as statistical importance.

3.2. Evaluating our response to challenge 2

We have collected a range of information that we can use to assess engagement. These include the extent to which various part of the course are completed, the views of participants on whether they have met their goals, and feedback from the course 'widgets' on various components of the course.

Figure 16.6 shows a snapshot of course completion during the stage at which all elements of the Citizen Maths course were available. We see that there is drop off on unit completion as the course progresses with relatively small numbers completing all 18 units (less than 100). It is important to note that these data were collected in a 17-week period whereas we expect a typical participant to take as much as 50 weeks to complete the course. As such we expect the unit completion rates to be a little higher as time progresses.



Prop = proportion, Unc = uncertainty, Rep = representation, Pat = pattern, Mea = measuring (the five Powerful Ideas)

Figure 16.6: Graph showing a snapshot of the numbers of participants starting and finishing the different elements of the course between June 6, 2016 and February 28, 2017 (Note the left hand scale is logarithmic and is used for the starter / finisher bars, the right hand scale is linear and is used to display the proportion of finishers to starters for each unit.)

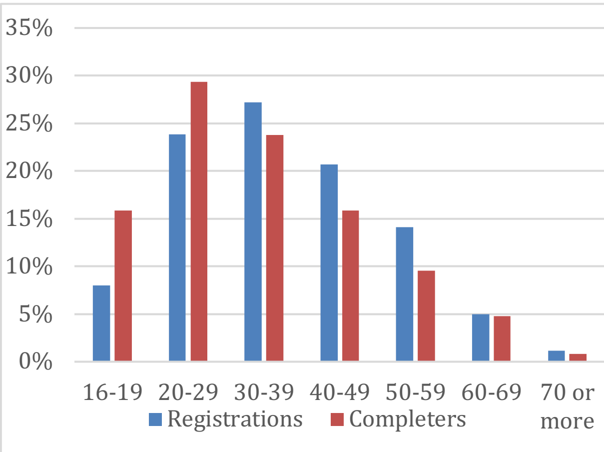


Figure 16.7 Proportion of registrations and completers by age

Figure 16.8 Proportion of registrations and completers by gender

The breakdown of completers by age profile (Fig. 16.7) and gender (Fig. 16.8) compared to registrations is evidence that the course does engage our different groups. The age profile of completers is a little younger than the registrations although all groups have completers. The gender breakdown of completers is similar to that of registrations with a slightly higher proportion of female completers. It is also worth noting that the proportion of completers who identified that they wished to 'work through the whole course, to improve my maths' was higher than registrations (85.6% completion compared to 70.1% registrations). This is important as it shows that the course has engaged its target audience more than 'interested professionals'.

Unit 1 Mixing	2.50
Unit 2 Comparing	2.73
Unit 3 Scaling	2.46
Unit 4 Sharing	2.71
Unit 5 Trading off	2.73
Unit 6 Making decisions	2.71
Unit 7 Playing	2.83
Unit 8 Simulating	2.67
Unit 9 Interpreting data	2.70
Unit 10 Interpreting charts	2.80
Unit 11 Comparing groups	2.84
Unit 12 Appreciating	2.80
Unit 13 Tiling	2.76
Unit 14 Constructing	3.06
Unit 15 Reading scales	2.86
Unit 16 Converting	2.73
Unit 17 Estimating	2.83
Unit 18 Quantifying	2.97

Figure 16.9 Aggregate feedback from the "Rate this lesson" widget

How does the snapshot data (Fig. 16.6) compare to other MOOCs? Doug Clow (2013) notes that progression through MOOCs tends to display a decaying feature, the funnel, far more dramatic than in traditional, face-to-face courses. The author draws together data from a range of sources to illustrate the effect. For example, Clow notes that the "first MIT course, Circuits and Electronics, attracted over 150,000 participants, but "fewer than half look at the first problem set", and only 7,157 passed, or about 5%" (Daniel 2012 cited in Clow 2013: 187).

These results suggest that the course is broadly in line with other MOOCs in terms of engagement and completion. Indeed, those who completed each powerful idea were asked to rate the extent to which their goals were met on a scale of 1 to 5 (1 being "not at all" to 5 "completely") and were very positive about the course. For the Uncertainty section of the course, the overall figure was just short of 4 and for representation the views averaged 3.5.

At the bottom of each lesson in Citizen Maths is an optional "Rate this lesson" widget, allowing learners rapidly to provide high level feedback as to the utility of the lesson on a five point (0 to 4) scale (0 being "not at all useful" to 4 "extremely useful").

Figure 16.9 shows the aggregate feedback for the 18 course units, with the three units within, respectively Uncertainty (6-8) and Representation (9-11) highlighted.

This data shows that learners using the feedback widget are on average reasonably satisfied with the usefulness of Citizen Maths, with some small variations between successive units. Of course, it must be remembered that widget scores are being provided by a "survivor" population; the further into the course a learner is, the more they might be expected to find course units useful because dissatisfied learners would tend to cease to engage with the course.

At the powerful idea level the data is broken down by age, gender, and goal in Figure 16.10. Again, we have evidence that across the different groups that we have identified there are participants that have engaged positively with the course.

		Age						
	All	16-19	20-29	30-39	40-49	50-59	60-69	70+
Uncertainty (Units 6 to 8)	2.73	2.92	2.71	2.62	2.74	2.68	2.83	2.56
Representation (Units 9-11)	2.77	2.99	2.84	3.04	2.57	2.70	2.66	2.28

		Gender		Goal*	
	All	Female	Male	Goal 1	Goals 2-6
Uncertainty (Units 6 to 8)	2.73	2.75	2.71	2.77	2.61
Representation (Units 9-11)	2.77	2.82	2.64	2.96	2.32

*Goals as in Figure 16.3. Goal 1 is "to work through the whole course, to improve my maths"

Figure 16.10 Aggregate feedback from the "Rate this lesson" widget broken down by age, gender, and goal.

The completers of each powerful idea were also asked to judge the extent to which they engaged with the course elements. The summary (see Fig. 16.11) shows a broadly positive picture.

Which of the following options best sums up your engagement with …?					
I carefully watched every video and did every activity, sometimes more than once	I worked through all the content, but sometimes my attention wandered	I skimmed through, mainly to see what was there rather than to engage closely with the course	I sampled what was there, focusing on the things that interested me	None of these options apply in my case	I would prefer not to say
Uncertainty					
36%	32%	27%	5%	0%	0%
Representation					
32%	32%	27%	5%	5%	0%

Figure 16.11 Summary of feedback from users about how engaged they felt by parts of Citizen Maths

There was some differentiation in how each activity type was viewed by participants.

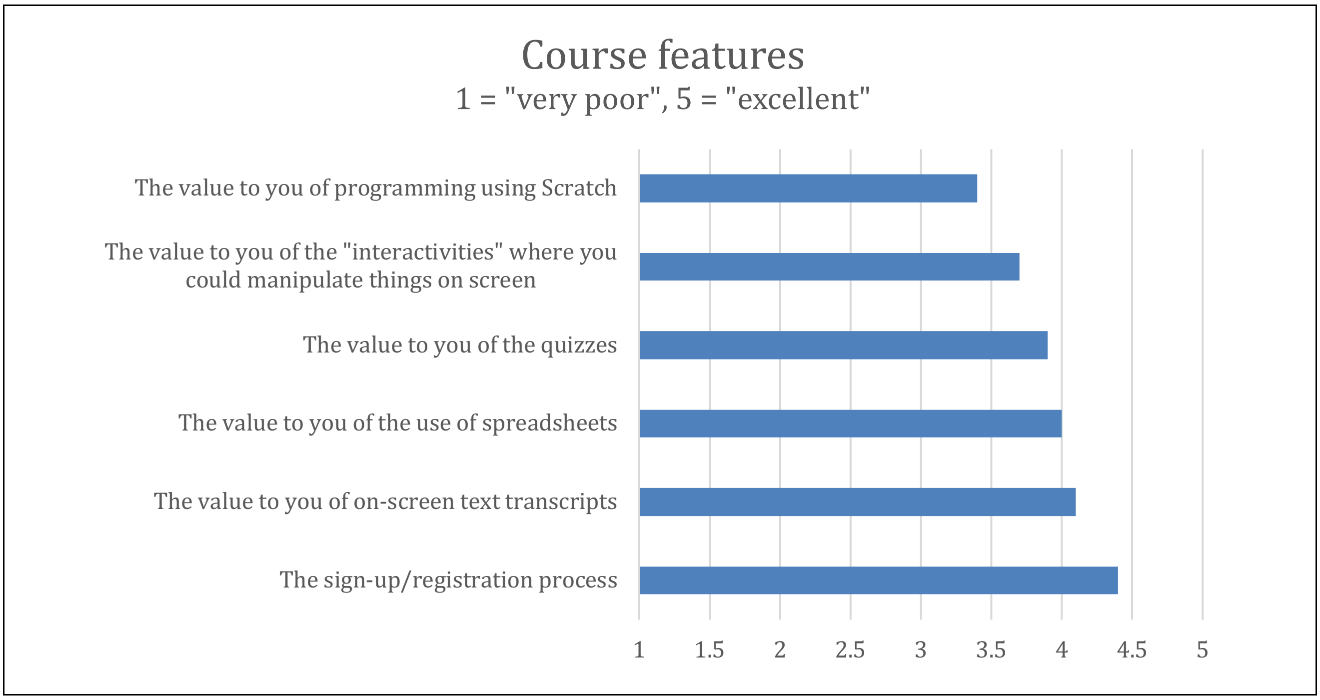


Figure 16.12 Feedback on Citizen Maths course features by users

Overall, we see that as the course progresses there is an attrition rate of some concern. Nevertheless, we are optimistic that once the full course has been running for long enough to make a considered judgement, the attrition rate will not be out of line with many other open online courses (see Clow 2013). This is most likely to be true for learners who have made a proper start rather than for those who have merely "dipped in" to take a look. Of those who complete the relevant sections of the course, there is evidence that a good proportion engage well with the various aspects of the course. Of course, we should not be complacent; these positive views are from those who have completed various sections of the course. Those that drop out have not had the chance to contribute to these views. Also, the participants are not positive across the board and there are some potential concerns that could be addressed. For example, there are some participants who are unconvinced of the value of programming with Scratch.

4. CHALLENGE 3: HOW TO PROMOTE MEANINGFUL MATHEMATICS

4.1. Discussion of challenge 3

Many online mathematics learning resources, such as those in the Khan Academy, are closely tied to traditional topic areas – for example, arithmetic, algebra, geometry – and either assume that students will find these meaningful or that other human learning support will provide context and purpose. Citizen Maths aims to communicate the purpose of mathematics in a way that will make it intrinsically meaningful. The challenge to design purposeful activities in a tutor-free environment is further complicated by the need to help the student engage and make sense of the mathematics. In a conventional classroom, effective teachers continuously monitor the student's actions and step in, as necessary, to clarify or offer alternative ways of thinking about the mathematics. Although we have no such opportunity in Citizen Maths, we are dealing with students who, as adult learners, are experienced members of society and will have a range of prior experiences. There is of course some difficulty in exploiting these experiences when the backgrounds of the students will inevitably be so variable. We explored in Citizen Maths solutions in which the technology was adaptive to how the students responded to the challenges. Although such technology is improving rapidly, we were unconvinced that the technical adaptive systems were sufficiently advanced as yet to make effective recommendations to the student in the holistic (as opposed to skill-based) approach we were adopting.

We were aware of the literature on the authenticity trap. Lave's (1988) early work on situated cognition had led to a discussion of the need to create authentic learning experiences; but, it is not possible to take an authentic experience into the classroom, or indeed into a MOOC, because the act of doing so transforms the task, which is no longer authentic. However, research (Nunes et al. 1993) has shown that knowledge is not so much trapped in the situation where that knowledge first became available but is rendered meaningful by that situation. We set out to find situations that were likely to be familiar, even if not directly experienced, and use them to introduce the student to the power of mathematics and statistics in those situations.

The 'interpreting charts' PIA of Citizen Maths has, as one focus, learning about styles and biases in reporting and advertisements, one of the five suggestions for new areas of work with learners proposed by Gal (2002). The following example from this PIA illustrates how it is at the same time possible to engage with charts in personally meaningful ways. This example draws on an applet created by the Institute of Fiscal Studies (www.ifs.org.uk/wheredoyoufitin/). The applet asks the user to enter a few basic facts about themselves, such as their household income, and generates a histogram that shows in which percentile out of the population as a whole the user's net income lies (see the shaded bar in Fig. 16.13 to the left of £300 per week).

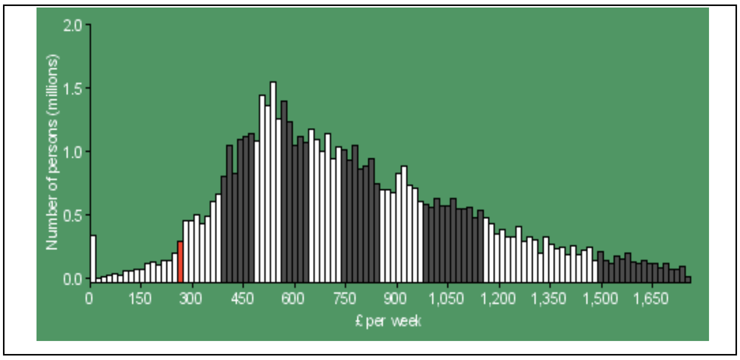


Figure 16.13: This user's income is well below that of most households in the population

In this example, we clearly positioned the learner as the active person at the centre of the task as we imagined they would enter data about their own household. We intended that the familiarity of the context would help the student to interpret the histogram. In other situations the learner might feel more like an onlooker. Consider the position of a male student in the following example taken from the 'making decisions' PIA of 'Uncertainty'. The student is given data about the number of women out of 1000 who receive a positive result from a mammogram, used to screen for breast cancer. They are also given data about the proportion of times that the screening machine gives a positive result when the woman does not have breast cancer (a false positive). The result is that, of the women who get a positive mammogram result, more than ten times as many do not have breast cancer as the number that do. Although the calculations are not difficult, many people find this result surprising. This situation will be familiar to many students, though may be felt more personally by female than by male students.

This is an example where in a sense we piqued curiosity by courting controversy, a well-known trick that face-to-face teachers use but which we are also able to exploit. The problem is that there is no teacher to support students when the controversy is too upsetting. Might this be problematic for students whose near relative is suffering from breast cancer? Or is there advantage insofar as the student will be less exposed if working with the materials from home on their own rather than in front of peers? We asked such questions in various focus groups. Views were divided but on balance it was felt that the opportunity to raise social awareness should not be missed. After all, it is reasonable to suppose that the use of statistics to support or deny claims during the study of sensitive and controversial topics might promote critical thinking and a broad appreciation of the value of statistics. In the light of these views, we have included such situations in the course, partly because the controversy can be engaging but also because they demonstrate the power of mathematics and statistics to inform such debate. We acknowledge though that this decision might be limiting were the course to be used as part of a blended course or within a learning centre, since controversy raises potentially embarrassing scenarios in collective learning situations that such courses might seek to avoid.

4.2. Evaluating our response to Challenge 3

One of the key elements of Citizen Maths was the selection of activities that draw upon a range of scenarios in order to engage learners in meaningful mathematics. We noted the example of income data drawn from the Institute of Fiscal studies in 'interpreting' and the decision-making aspect of uncertainty using data around mammograms. What evidence do we have to evaluate the extent to which these activities might be perceived as meaningful?

One thing to note is the completion data and positive views of the completers noted above displaying a good engagement by a number of participants. Nevertheless, this does not necessarily mean that the participants feel the mathematics has been meaningful. What we do have are some of the comments made by those using the feedback 'widgets'. A small number added qualitative comments, many of which were fairly short 'I did / didn't find this useful'. Others made some more interesting, and mostly positive, comments.

> I really enjoyed this section, I was able to do the calculations quickly & easily. I have even been able to grasp rounding and will be using this method to help me in future tasks.
>
> I feel I have learnt how to read and interpret data on both on a spreadsheet and even without the aid of a spreadsheet. This lesson has been the most enjoyable, clear and easiest to understand.
>
> This is a good app to use to interpret household data. I enjoyed working through the lessons.

And even when the comments suggest some less positive outcomes, there is also evidence of meaningful engagement.

> I found this part very difficult to follow but have taken notes so I will be practising this session again.

Of course, there are some more negative comments around some activities. In commenting on a dice simulation activity.

> I did not like this activity as it is something I won't need.

Aside from the comment suggesting that the activity is not meaningful to the learner, going back to challenge 2 and the lack of tutor support, there is no opportunity for us to intervene to discuss this further. A discussion space would offer an opportunity for other users to be involved in debates around such utility issues. Interestingly a more positive comment on the ‘pass the pigs’ game suggests that meaningful activity can be a very personal issue

> Definitely got me thinking as I have played the Pass the Pigs so this helped me to construct mental images to accompany the statements about the probability.

5. CHALLENGE 4: HOW TO ASSESS

5.1. Discussion of Challenge 4

In designing the structure of the course, we have followed the PISA methodology (www.oecd.org/pisa/pisaproducts/Draft%20PISA%202015%20Mathematics%20Framework%20.pdf) when categorizing activities in terms of content (PISA uses four categories), content topics (15 topics), mathematical processes (3 categories), mathematical capabilities (7 categories), and context (see the four categories above). By using these categories to profile our activities, we have been able to monitor coverage of both mathematical content and the processes without closely prescribing a curriculum, which might have hindered our large grain-size aims.

Citizen Maths does not lead directly to a qualification, although we have collaborated with the awarding organization, OCR, a partner in the project, to track compatibility (or otherwise) between their Cambridge Progression Level 2 units (www.ocr.org.uk/qualifications/by-type/cambridge-progression/) and Citizen Maths. A lack of availability of a direct qualification presumably does not meet some students’ desires but our focus has been on students who were motivated to gain a sense of mathematics as a discipline, perhaps as a precursor to working towards a qualification.

Teachers of conventional courses are able to adopt methods of formative assessment to respond to their students within lessons and between lessons in ways not open to us in Citizen Maths. For example, although the course offers a suggested sequence of activities, this preordained sequence might not be suited to some learners because of their prior knowledge or interests. Because there is no tutor to assess whether the proposed sequence should be broken, the course is left open for students to make that decision themselves in the light of progress. Our approach instead was to offer several types of short assessment tasks, aimed at helping the student to make up their own mind whether they have properly understood the content.

These assessment tasks frequently offer multiple-choice questions. When the student gives the wrong answer, responses attempt to give some hint about where they may have gone wrong. When the student gives the correct answer, the response gives a full explanation of the answer in case the student obtained the correct answer through incorrect reasoning. Where possible the wrong options in the multiple choice questions are chosen to reflect common errors that students make though, in Citizen Maths, the student’s wrong responses can only be used for formative assessment by the student and not by a teacher, as recommended by Wiliam (2014).

For example, the activity on interpreting data about incidents of crime, described above, is followed by the question “What is the percentage of alcohol related incidents in 2006/2007?” The correct answer is 54%. It is anticipated that some students will mistakenly refer to the data where the use of drugs (rather than alcohol) is apparent. Such a mistake would lead to an answer of 21%. If a student gives that answer, the following response is given by way of a hint, “This is the percentage of drug related incidents. Use the data for alcohol related”.

Multiple-choice questions are not always appropriate to assess learning points. For example, a number of activities in the course have a range of solutions that are not easily captured by a system of offering multiple choices. In other cases, the essence of the task lies in the process of doing it, rather than in the outcome. It is not perhaps surprising that this type of activity is quite prevalent when the aim of the course is to engage students in work that reveals the power of mathematics. In such cases, the onus is on the student to watch the to-camera tutor review the task and decide for themselves whether they have understood it sufficiently. One of the powers that Citizen Maths and other MOOCs have that is not available in the conventional classroom is that students can easily return as many times as they want to a video of the tutor's explanation or to a task they have already done. So, we hope that a student who is not satisfied that they have properly understood the activity after watching the tutor is able to return to the introductory video and the task itself.

5.2. Evaluating our response to Challenge 4

We have no direct evidence for the extent to which these elements of the course have been effective. The multiple-choice questions are typical activities within lessons, not a separate element. During the development of the course, we chose to collect data on the lesson level rather than on individual components. As such we cannot claim any particular success for the responses to participant choices except to note that the sessions overall have achieved good ratings, the quizzes are noted as a good feature (see Figure 16.12) and there are positive comments from users about the value of the quizzes. One learner describing the positive features of Citizen Maths noted that with the quizzes there is a "summary of the question you've just answered and whether you got it correct, and if you haven't, an explanation of why it's not correct".

6. FINAL COMMENTS

We accept that some of our solutions have been compromises in the sense that they often do not match our pedagogic ideals, which we have learned through research and teaching in conventional classrooms. Some colleges of further education are using our materials as additional resources and in those cases a tutor will typically be able to support the learning process. On the other hand, it is not realistic to expect the shortfall in Level 2 mathematical understanding across the population to be remedied in conventional ways. The 2011 Skills for Life survey (BIS 2012) estimated that some 72% of the population of England were below Level 2 and the 2012 international survey of adult skills (PIACC) (BIS 2013), while using different level descriptors, was consistent with this level of need. The existing educational structure of further, adult and vocational education could not cater for anything close to numbers on this scale. This problem may be even more acute in the case of statistical literacy, where the shortage of appropriately knowledgeable teachers is widely reported (see, for example in Batanero et al. 2011) and where a better-informed populace would advantage individuals and society (see, for example, Gal 2002).

Designing the structure and the content of Citizen Maths has been an interesting challenge and we have described some of those challenges in this paper. By focussing on powerful ideas rather than specific small grain-size techniques, we have placed emphasis on a broad interpretation of numeracy and statistical literacy. While it could be argued that such a change in emphasis would benefit teaching and learning in conventional classrooms, it seems critically important in a MOOC-based approach where the students are heterogeneous and sufficiently mature to bring rich experience to the learning environment.

We believe that the use of contextualised activities actualises those powerful ideas in familiar and meaningful ways whether those tasks are fundamentally mathematical or statistical in nature. Of course, in the representation and uncertainty PIAs, the role of context potentially extends beyond motivation and meaningfulness to opportunities for developing understanding of the contextually-dependent transitions from design to data capture, from exploring data to identifying patterns and towards the end of the statistical problem-solving activity from results to their interpretation (Cobb and Moore, 1997). We note though that the transition from design to data would be challenging to implement in a MOOC and has not as yet been the focus for an activity in Citizen Maths.

We have experienced affordances in the use of a MOOC that are common with the use of technology in teaching and learning statistics and probability in classroom contexts: (i) the facility to handle complex computations can help the student to focus on the underlying conceptual challenge and not be distracted by the difficulties of carrying out calculations; (ii) the repetition of trials digitally enables data to be collected quickly so that variation in data can be explored; (iii) the use of computer-based simulations allows the learner to explore situations which might not otherwise be accessible to the student; (iv) programming the computer allows the learner to try out models of solutions and then modify those proposed solutions according to feedback; (v) dynamic graphical displays tends to support visualisation of mathematical and statistical concepts.

Of course, we do not argue that MOOCs such as Citizen Maths offer as good a learning experience as can be provided by teachers, but we have noticed some advantages: (i) students can work intensively over a short spaces of time or spread their work over longer periods to fit learning around other commitments; (ii) they can learn at their own pace by repeating lessons or specific videos as often as they like; (iii) students can make choices about what content to study and in which order, which might suit those students who are aware of particular strengths and weaknesses in mathematics or statistics; (iv) it is possible to access the resources remotely rather than needing to travel to a particular location to engage in learning; (v) some students might appreciate being able to learn on their own in order to avoid embarrassment should they feel inadequate in some way or if the topic were in some way sensitive or controversial (though of course there are also distinct advantages to collaborative learning, which is more natural in a conventional classroom).

As social platforms continue to improve it is possible that MOOCs will in the future be able to support collaborative learning in ways that have not been built into Citizen Maths. Some might argue that future research might provide a better understanding of conceptual development in mathematics and statistics with opportunities for artificially intelligent support structures. Our experiences suggest however that when the focus is on broad interpretations of numeracy and statistical literacy, the grain size of what needs to be learned is large enough for us to believe that society is still some way from being able to develop such systems.

The data collected so far on users offers some broadly positive feedback about the extent to which we met these challenges. Sustaining Citizen Maths in the future will be another challenge. Some additional finance will be needed as shortcomings in the current course will inevitably be identified and the use of data files will quickly need to be updated. Fundamental changes in technology will offer new solutions, including better ways to personalize the experience, making old technologies look obsolete. Citizen Maths was never going to be an ideal solution but the process has enhanced our knowledge about what seems to work and what does not, knowledge which should help us be ready for that time.

Endnote

This paper is narrowly focused on the interests of Topic Study Group 15. But a wider range of issues appears when creating (maths) MOOCs. Several of these are described on the Citizen Maths web site, at https://citizenmaths.com/, and, in particular, on the Citizen Maths blog, at https://citizenmaths.com/blog.

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