Routledge
Taylor & Francis Group

Education 3-13

International Journal of Primary, Elementary and Early Years Education

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/rett20

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To cite this article: Sarah Seleznyov, M. Adhami, A. Black, J. Hodgen & S. Twiss (2022) Cognitive acceleration in mathematics education: further evidence of impact, Education 3-13, 50:5, 564-576, DOI: 10.1080/03004279.2021.1872678

To link to this article: https://doi.org/10.1080/03004279.2021.1872678

Published online: 20 Jan 2021.

EDUCATION 3-13
2022, VOL. 50, NO. 5, 564–576
https://doi.org/10.1080/03004279.2021.1872678

Cognitive acceleration in mathematics education: further evidence of impact

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ABSTRACT

The Cognitive Acceleration (or Let's Think) approach to mathematics teaching is a Piagetian programme drawing on Vygotsky's research, developed at King's College London over 30 years ago, along with its associated professional development (PD) programme. This project sought to replicate the original studies conducted 10–15 years earlier and before much national curriculum change, through a professional development project with 41 teachers of children aged 6–12 from London in 2014. Results of pre- and post-test of mathematics attainment are reported for 232 students. Despite a shorter duration, data shows increased teacher efficacy, improved teaching and a mean gain equivalent to 2.6 months learning for benefitting students, which broadly mirrors cognitive effects of original trials with twice the duration. This evidence corroborates the impact demonstrated in several original CA research papers, and additionally details specific impact on teacher confidence and classroom practice.

ARTICLE HISTORY
Received 9 December 2020
Accepted 23 December 2020

KEYWORDS
Cognitive Acceleration; professional development; mathematics teaching; problem solving

Introduction

The Cognitive Acceleration (or Let's Think) approach is a programme that draws on the research of Piaget and Vygotsky and focuses on questioning, collaborative work, problem solving, independent learning, metacognition and challenge. This article discusses the impact evaluation of a one-year funded Cognitive Acceleration (CA) professional development programme that took place between January and December 2014 for teachers in London and presents an analysis of the evidence of its impact.

There is an increased interest in the CA approach to mathematics teaching and learning in the light of the UK government's support for a 'mastery' approach to mathematics in which all students experience the same mathematics curriculum at the same pace, instead of differentiated pathways for students of different achievement levels (Boyd 2020). In addition, recent research evidence has highlighted the crucial role played by the teacher in explicit teaching in mathematics (Hodgen et al. 2020). CA offers teachers a set of teaching material which have demonstrated their effectiveness for whole class teaching, making the publication of these 2014 findings timely and of relevance to current mathematics teaching.

The article begins with a literature review, which gives an overview of the CA programme itself and the theory supporting the cognitive processes involved in its classroom practice, going on to explore historic evidence of impact for the CA approach in mathematics. The article then goes on

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to describe the methodology for the research, providing details about the professional development project, the data collection methods, then reviewing the findings of the impact evaluation. Finally, the article compares these findings to those of previous CA studies, suggesting further avenues for research.

The Cognitive Acceleration approach

The CA approach was developed at King's College, London over 30 years ago and addressed the problem identified in the late 1970s that over half the students in secondary schools demonstrated little progress in their science learning, and could not cope with the demands of the maths and science examinations. CA offered an alternative to simply accommodating the students by offering a less demanding curriculum in the late secondary years so as to avoid students experiencing a sense of failure. That alternative was to offer cognitively challenging, flexible, but structured activities at age 11–13, to improve students' level processing and thinking skills such that they might benefit more from instruction in later years. Lessons are designed to support cognitive development by moving students progressively through Piaget's stages of development (Shayer 2003), in line with more recent evidence of the strong potential for Piagetian programmes to impact on learning (Hattie 2012).

According to Shayer and Adhami (2010), each CA lesson has three stages. Typically, these three stages are then repeated in a second episode of learning, with a more challenging agenda.

The first stage is called Concrete Preparation, and it introduces the lesson context at a level where all students in a mixed achieving class can participate. In this way, the CA approach offers a 'low threshold, high ceiling' (McClure, Woodham, and Borthwick 2011) approach in which all students can access the lesson since its initial level of conceptual difficulty sits within their own Zone of Proximal Development (Vygotsky 1980). In this Concrete Preparation stage, students are presented with a Cognitive Conflict, a problem or puzzle to be solved, asked to describe what they believe the task ahead will require, and to suggest possible strategies. In some activities, students attempt some of the simpler parts of the task in pairs, and then discuss and compare their approaches.

In the second stage, called Social Construction, students in pairs or small groups are asked to work collaboratively at their tables on the first worksheet. After a few minutes, they may present their ideas to the rest of the class, making them available for all to work with. The focus here is not on working 'neatly' but on vigorous discussion so that they have something worthwhile to share with the class. These carefully constructed sequences of cognitively challenging problems or difficulties, followed by episodes of collaborative struggle and sharing of strategies align with evidence on the value of 'desirable difficulties' (Kapur and Rummel 2012) or 'productive failure' (Bjork and Bjork 2011) learning approaches. The value of CA's collaborative groupwork for mathematical discussion is also corroborated by a number of meta-analyses of evidence of collaborative learning on attainment and attitudes (Hodgen et al. 2020) and by recent evidence on the impact of dialogic teaching in mathematics (Jay et al. 2017).

In the third Metacognition stage, students share and explore solutions as a whole class. Recent evidence of effective mathematics teaching and learning strategies highlights metacognition as one of the most effective strategies (Education Endowment Foundation 2018). In this final stage, the carefully managed discussion gives each student the opportunity to complete their own Zone of Proximal Development (Vygotsky 1980) as they explore ideas generated by other students.

For example in one lesson for Y2, students are faced with the problems of how to share out a number of bars of chocolate between a number of children. The problem is familiar and grounded in the concrete aspects of the problem, however, the cognitive challenge lies in trying to achieve an equal share when, on first sight, this does not seem possible. Solutions are found and shared. Later the students look at other opportunities for sharing and decide whether they are similar. They also decide which is better because there would be more per child or there would be an equal share. By exploring many:many relationship between two quantities, the students through this lesson move from additive to multiplicative understanding.

In line with recent research on high impact professional development for teachers, the CA professional development programme engages teachers in structured collaboration (Timperley, Kaser, and Halbert 2014) as they explore and reconstruct mathematical concepts with students through repeated cycles of live teaching experiences, and collaborative reflective discussions. It mobilises knowledge across schools (Stoll 2009) by enabling discussion amongst peers and with mathematics experts. Programmes operate over a sustained timeframe (Hallgarten, Bamfield, and McCarthy 2014) of several days across a minimum of one year.

Evidence of impact

Analysis of the long-term effects of Cognitive Acceleration in Maths Education (CAME) showed a gain in the 11 experimental schools of 0.8 O' Level grades, taken at age 16, three years after the intervention had finished (Shayer 2003). The highest gain was 1.35 grades, and the lowest 0.23 grades, from an average of 18 CAME lessons taught in Years 7 and 8 (11–13-year-olds). Intervention schools also showed a 0.51 grades gain in Science and a 0.52 gain in English.

The PCAME project (Adhami 2002) involved 11 experimental and 6 control Year 5 and 6 classes (ages 9–11). Year 5 teacher participants did not continue to teach the students in Year 6, meaning most students were taught by teachers new to CA in both years. Measured using the Piagetian Spatial Relations test, gains for study schools were 0.26 SD over 20 months.

Adey, Robertson, and Venville (2002) measured the impact of 29 CA lessons on 300 experimental and 170 control Year 1 (ages 5–6) children in experimental classes. The experimental group made significantly greater gains in cognitive development than controls, in both direct (effect size 0.47) and transfer (effect size 0.43) tests. Robertson's (2014) study of the same project showed that CA made a substantial difference to students' awareness of themselves as learners and ability to articulate ideas. Teachers made substantial changes to pedagogy, including increasing the level of challenge and providing more opportunities for metacognition.

Shayer and Adhami (2010) researched the effect of CA on 275 Year 1 and 2 students (aged 4–7) in 18 classes in two Local Authority areas. At post-test the mean effect sizes assessed by the Piagetian test Spatial Relations were 0.71 SD in one Local Authority and 0.60 SD in the other. The five participating classes achieved a median increase of 1.3 SD. The mean gains over pre-test in 2002 for all children in Key Stage 1 (age 7) English in 2004 were 0.51 SD, and at Key Stage 2 (age 11) English in 2008, the long-term effect, were 0.36 SD, an improvement of 14 percentile points, showing that the impact did not dissipate over time.

Methodology

This article discusses the impact evaluation of a one-year funded CA professional development programme that took place between January and December 2014. The Greater London Authority (GLA) aimed to improve attainment in maths and literacy by using interventions that had an impact on teacher efficacy and children's attainment. The GLA offered a grant to help primary school teachers, who were unfamiliar with the CA approach, to use this in their lessons. Teachers were helped with the CA materials and the CA approach through a professional development programme led by CA tutors. Unlike historic CA research, these tutors were not members of the CA research team but experienced CA practitioners trained by the original researchers. The efficacy of the teachers was sampled before and after the programme as was the attainment of the children.

Below we give an overview of the professional development project and report the methods, tools and results for our evaluation of the impact of CA on teacher participants and their students in this project. We then go on to present the results, and finally to discuss the implications of these findings for future research.

The professional development programme

On this professional development programme, teachers new to CA worked alongside expert CA practitioners to develop an understanding of CA pedagogies and exemplar lessons, and a deeper understanding of mathematics teaching more generally. The programme involved cross-school groups of teachers planning collaboratively, teaching and subsequently reflecting on this shared practice across 10 full days. The programme initially provided specimen lessons for specific age groups which could be adapted to the needs of classes in response to student engagement and understanding. Participating teachers also benefitted from two coaching visits by CA experts at the beginning and end of the programme to provide formative feedback and to track their progress with the approach. Teachers were recruited to these programmes through existing CA networks, for example using the CA newsletter email list and through Local Authority contacts. There was also cross-subject recruitment, whereby schools with teachers already trained in CA in another subject area were invited to participate in these mathematics programmes.

A project timeline is shown in Figure 1.

Teachers were able to access the professional development programme free of charge and received 25% of supply cover costs, subject to strong attendance. The very small number of attendance issues was due to teachers withdrawing from the programme with reasons cited as changing schools or roles, maternity leave, stress or workload. Teachers missing single sessions were offered an opportunity to catch up on what they had missed, for example by visiting another school to observe lessons or by having a catch-up session with the tutor. There were 41 participating teachers in total, 37 from primary schools, Years 2–6 (ages 6–11), and 4 from secondary schools (ages 11 upwards).

We anticipated that impact on teacher practice and student learning would initially be low as teachers struggled to adopt the new pedagogical practices (Hobbiss, Sims, and Allen 2020), but that towards the end of the professional development programme and after the programme ends, there would be an impact. We would also expect this impact to deepen as teachers reteach the CA lessons they know for a second time, in the years after the professional development programme ends: research into CA has historically shown that if the intervention is carried out with the same cohort of teachers and students for two years, there is a significant and lasting impact on student learning both within and beyond the focus subject case (e.g. Shayer 2003; Shayer and Adhami 2010). Whilst the intention was to replicate the original CA research in this project, it was not possible to replicate the longitudinal tracking of impact; therefore, we expected the impact data to be less than that found in the original research.

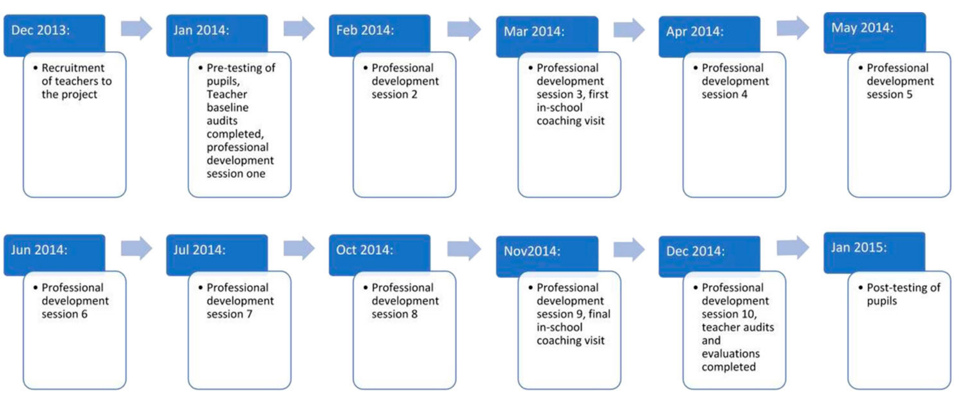

Figure 1. Project timeline.

The impact evaluation addressed the following questions:

(1) What are teacher perceptions of the impact of the different aspects of the professional development programme on their practice and is this perception reflected in changes to practice?
(2) What impact does the teaching of CA activities over one year have on mathematical outcomes for students?

Methods

Teacher perceptions: At the beginning and the end of the taught programme, teachers completed an online audit. We used Tschannen-Moran and Woolfolk Hoy's (2001) 'teacher efficacy' audit, which was shown to have a

> unified and stable factor structure and assesses a broad range of capabilities that teachers consider important to good teaching, without being so specific as to render it useless for comparisons of teachers across contexts, levels, and subjects. (2001, 801–802)

The audit was administered at programme start and end during face-to-face sessions. Additional fields for the post-survey included teachers being asked to identify one item where significant impact had occurred.

The survey asked teachers to grade themselves in terms of confidence on a nine point Lickert scale (1 = least confident, 9 = most confident) against 16 aspects of teacher self-efficacy.[a] No additional guidance was provided:

1. How much can you do to get through to the most difficult students?
2. How much can you do to help your students think critically?
3. How much can you do to motivate students who show low interest in school work?
4. How much can you do to get students to believe they can do well in school work?
5. How well can you respond to difficult questions from your students?
6. How much can you do to help your students value learning?
7. How much can you gauge student comprehension of what you have taught?
8. To what extent can you craft good questions for your students?
9. How much can you do to foster student creativity?
10. How much can you do to improve the understanding of a student who is failing?
11. How much can you do to adjust your lessons to the proper level for individual students?
12. How much can you use a variety of assessment strategies?
13. To what extent can you provide an alternative explanation or example when students are confused?
14. How much can you assist families in helping their children do well in school?
15. How well can you implement alternative strategies in your classroom?
16. How well can you provide appropriate challenges for very capable students?

[a]Efficacy in student engagement: items 1, 2, 3, 4, 6, 9, 10, and 14. Efficacy in instructional strategies: items 5, 7, 8, 11, 12, 13, 15, and 16.

We anticipated that the greatest impact on teachers would be in relation to aspects 2, 3, 4, 5, 7, 8, 10, 12, 16, as these are aspects that are directly addressed through the CA activities and the professional development programme. The design of the lesson activities intends to provide engaging real-life contexts for mathematical learning, and to enable students with diverse Zones of Proximal Development to experience mathematical success, and lesson activities are structured to enable considerable time to be devoted to students thinking critically about each other's strategies. The professional development programme involves live teaching and reflections on practice. Preparing for live teaching includes time spent carefully crafting questions and considering how best to gauge student understanding, and reflections on practice include time critically evaluating teachers' responses to student questions. We anticipated that there would be very little impact in relation to aspect 14, which was not addressed as part of the programme. We were conscious of the fact that teacher ratings of self-confidence can be unreliable as pre- and post-impact measures, since teachers may only become conscious of a shift in their practice during a professional development experience. We, therefore, anticipated a possible negative skew in results.

In addition to completing the audit at programme end, teachers were asked to write a qualitative open ended response describing their views of the programme in the form of a written text. A thematic analysis (Braun and Clarke 2006) was then carried out, hand coding to identify common themes and differences, counting instances of recurring themes to identify and understand patterns in the data.

Quality of teaching: All 32 project participants were observed teaching a CA lesson at the beginning and the end of the professional development programme. These lesson observations were carried out by experienced CA experts, initially in pairs, to moderate their judgements, then carrying out further observations alone, using the Let's Think Progression Framework (see Figure 2) which was developed specifically for this project by a group of expert CA Tutors, in order to support teacher progression as part of professional development.

We were not anticipating significant shifts in practice as our experience of running professional development would suggest that teachers only truly gain confidence in the approach in the year after the professional development. In fact, we expected some teachers to remain as Starters in the approach due to the short duration of the pre- and post-test gap.

Student outcomes: To measure the impact of the project on student learning in mathematics, the Kings College Numeracy Assessment Tests were used (Rhodes et al. 1998). Tested students were in classes taught by teachers participating in the programme.

The CA intervention involved classes from Y3 to Y7. Hence, the evaluation used age appropriate tests that have been equated across year groups and were originally developed by Askew et al. (1997) for the Effective Teachers of Numeracy study. These tests give an estimated maths age based on a form of Rasch modelling (Brown et al. 2008). Slight modifications were made to the model to account for differences in the administration of the tests in this evaluation.

Unfortunately, some questions in the tests required poster size displays that were not possible to provide electronically. Teachers were therefore instructed to miss these questions out. Two questions on tests for different years had also to be ignored due to large numbers of students misunderstanding the question. The omission of questions reduced the total number of maximum marks achievable by the test, complicating analysis of effects.

The analysis of the Kings College Numeracy Assessment Test uses age norms that have been established about the year 2000 using two nationally representative samples of around 2000 students (Brown et al. 2008). These norms act as the controls against which to measure progress, but may not be the same as current. It may be argued, based on current estimates of students' cognitive levels (Shayer, Coe, and Ginsburg 2007), that current age norms are, if anything, more likely to be lower than higher. In which case, the value added in this study is more likely to be an underestimate, rather than an overestimate, of progress.

It is a limitation of the evaluation that it was not possible to use a control group and that no account has been taken of the clustering of children in schools and classes. However, in the absence of a control group, the age norms provide a reasonably robust yardstick by which to measure whether progress was higher, or lower than would be normally expected.

The marking was carried out by teachers not involved in the project. As a reliability check, pairs of teachers checked each other's marking and aggregation of marks for a sub-sample of 15% of the students.

Data

Teacher perceptions of impact on practice

The data represents 41 matched names out of a possible 41 (Table 1). The teachers involved in the CA programme perceived a growth in efficacy of on average 0.82 on the 1–9 scale, an

	Starter	Improver	Competent	Expert
Lesson Flow • Planning • Cognitive match Responsiveness	• Closely follows outline instructions	• Some tweaking of the plan • A relevant and engaging hook • Short launch that is effective	• Adjusting the plan to the given class • Responsiveness to class at key points in the lesson	• Adapting and responsiveness throughout • Manage the flow of the key teaching features to meet the developing needs of the learners
Cognitive Content • Concepts • Reasoning • Cognitive range	• Launch and group work construction takes up most of the lesson time • Group work repeats launch • Limited range of ideas	• Launch provides language and skills to engage construction with minimal teacher support • Time for a generalisation and review includes similarities and differences between group outcomes	• The generalisation and review produce further cognitive conflict around the range of current group outcomes	• Information collected while scanning group work construction to plan the generalisation and review (orchestrated feedback)
Interactions • Teacher-pupils • Pupils-Pupil • Materials • Language	• Cognitive conflict is posed by providing verbal or non-verbal cues • Teacher-pupil interactions dominant	• Teacher mediation in response to requests for support is reflected back to the individual or the group • Some fruitful pupil-pupil interactions	• Teacher mediation that is balanced against the possible disruption to peer mediation as the groups engage to reach their agreed solution to the activity	• Curtailment of construction to ensure time for generalisation and review • Teacher mediation with struggling / fast track groups to produce further engagement in learning
Culture of Learning Attitudes to: • Exploring topics • Venturing ideas • Cooperation	• Limited attention to pupils' attitudes to learning and ways of working • General praise	• Highlighting positive pupils attitudes, with specific examples	• Attention to attitudes and values changes across the whole ability range	• Attention to attitudes and values integrated and naturally flows with other learning features

Figure 2. Let's Think Progression Framework.

Table 1. Changes in teachers' views of efficacy comparing efficacy of instruction and student engagement.

Average teacher confidence in their overall effectiveness	N	Mean	Confidence interval 95%	Standard deviation	Min	Max
Pre-test	41	6.565	5.035–8.098	0.766	5.688	7.625
Post-test	41	7.381	6.131–8.631	0.625	6.000	8.250

average increase in effect size of Cohen's $d = 0.78$. This was a statistically significant gain, $t(39) = -3.87$, $p < .01$.

The five most marked increases in individual aspects of efficacy were in teachers' ability:

- to craft good questions;
- to adjust lessons to the proper level for individual students;
- to assist families in helping their children do well in school;
- to implement alternative strategies;
- to provide appropriate challenges for very capable students.

These positive increases ranged between 0.35 and 1.74 ES (Standard deviations) perceived increase in their capabilities.

All but three of the teachers for whom there was matched data showed an overall increase in efficacy. However, the efficacy in dealing with difficult questions from students was seen as a problem by the majority of the teachers and only a few believed they had improved their approach to this problem. The CA activities differ from most mathematics teaching approaches in that they offer open ended problems, and the questions students may ask are therefore more difficult to predict, relying on teaching contingently, which has been shown to be challenging for many teachers (Evans and Ayalon 2017). We hypothesise that teachers on the project struggled with this aspect of the CA approach, and therefore saw their confidence drop as the project continued. We imagine that this confidence might have increased if we had been able to track teachers in their second year of using the materials, by which point the activities would have become more familiar to them, and predicting and preparing for students' questions easier.

All comments from teachers attending professional development programmes were positive about the professional development programme and the impact of the approach on their own practice and students' learning. Of the 41 comments in an optional open-ended text field, 22 teachers identified that the CA programme had improved classroom talk in terms of their own ability to ask the right questions and 17 felt it had improved students' ability to think critically, reason, explain and justify their answers, for example:

> Let's Think has enabled me to confidently ask a wide range of questions to deepen the children's thinking and to ask wider questions that require more than a yes or no answer. Children are now able to talk to each other in a more 'basketball' fashion rather than 'pingpong' with the teacher. … . Children are required to explain their answers and what they think/ why they think that which deepens their own understanding and allows others to work out how they got to that answer if they are unsure themselves. Let's Think questioning is applicable to lessons that aren't Lets Think or maths related and children often now start explaining their answers much deeper and 'proving' how they know across the curriculum.

> The course has helped me to pre-plan for any possible misconceptions and to change my questioning style to give those most able students moments of doubt where they have to really explain their reasoning more deeply.

> This course has helped me with my questioning because you're not always looking for the obvious answer. You need to deepen children's responses so their explanations are clear.

Eighteen out of 41 teachers noted that the CA lessons had improved motivation, engagement and confidence, especially for more challenged learners, for example:

> As the activities in Let´s Think lessons are engaging, exciting and different to more conventional lessons, I do think that more difficult students are engaged, especially as in these lessons they can have more of a voice and more discussion.

> We have a few students who have behaviour issues and who show little interest in their work. This lessons and the training have helped them become more involved in their work and the lower ability children access the work and make good progress.

Ten teachers noted that the impact on learning benefitted both low and high achieving students, for example:

> These sessions have helped the less able children to develop their confidence as well as providing opportunities for the capable students to be appropriately challenged.

> Children with SEN, or those who are switched off to maths, enjoy the CAME approach to learning and find that maths can actually be fun! Middle and higher ability children found the lessons enjoyable; the challenge for them was explaining their reasoning.

Eight teachers stated that they had moved away from providing solutions and quickly correcting misconceptions, towards valuing mistake making and leaving students to find their own solutions, for example:

> The training has helped me to think more creatively about how I teach maths, allow children to take risks and see that there is not necessarily a right answer.

> The emphasis on metacognition has encouraged me to ask questions that help children to reflect on their own learning and realise the importance of enquiry based learning, making mistakes and learning form their peers. I believe that Let´s Think is challenging for the teacher as when the children are not quite understanding or have misconceptions, the teacher needs to take care over their questions and answers so that they do not tell the answer to the child but that the child works through getting to the answer themselves.

Six teachers felt the programme had had a cross-curricular impact beyond mathematics lessons, for example:

> Taught me to employ a different style of teaching and I feel this has had an effect on my teaching in other lessons and subjects.

> I feel much more confident in using a different style of teaching, adapting my questioning and responses to children accordingly. This approach has filtered through to my delivery of other lessons in other subjects.

Impact on quality of teaching

There was a shift in practice for most teachers between the beginning and the end of the intervention. As we predicted, most teachers were Starters in the approach when they began benefitting from professional development. As we had anticipated shifts in practice were not large, the biggest shifts involved teachers moving from Starter to Improver (Figure 3).

Only one teacher in the entire cohort was identified as Competent at the start of the project, 21% as Improvers and 76% as Starters. By the end of the intervention, the majority of teachers' practice had improved, with only 20% remaining as Starters. 61% were Improvers by the end of the intervention and 17% Competent. The one Competent Maths teacher had now been identified as an Expert. This means 26% of teachers did not move at all, 67% moved up one level and 5% moved up two levels.

Impact on student outcomes

In the pre-test in the spring of 2014, we secured scripts for 938 students from 38 classes. In the post-test in the spring term of 2015 we secured scripts from 15 classes, with one class managing only half

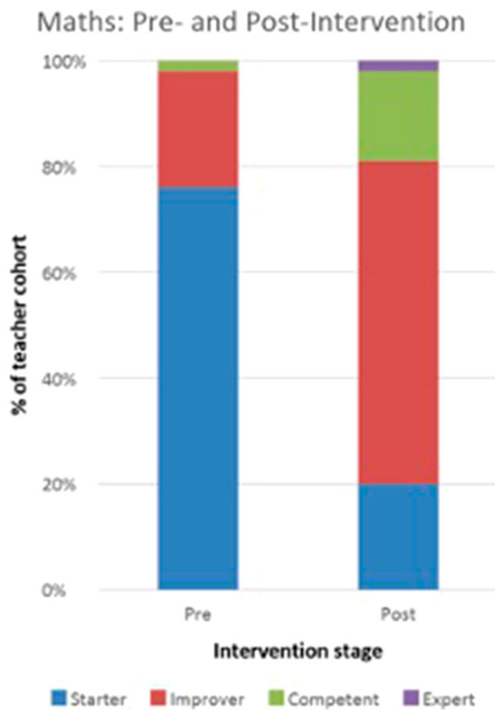

Figure 3. Teacher observation judgements pre and post-intervention.

the test items, and therefore this class was ignored in analysis. A lower number of tests were returned for impact analysis than had been hoped for, due to

- teachers changing roles and/or leaving their schools mid-project;
- difficulties maintaining contact with all involved schools in several Local Authorities when programmes ended in December 2014;
- Year 6 student cohorts transferring to multiple secondary schools, making post-tests for these students impossible to obtain;
- some sets of tests being lost in transit with the postal system.

In total, 14 complete sets of pre- and post-tests have been analysed, which represents 25% of the total possible sets. This analysis of individual students' gains was applicable to 232 students in 14 classes, across Years 2–6.

The maths age model allows an estimation of additional months learning over and above the expected progress. Comparing the maths ages in pre- and post-tests enables us to calculate the gains over expected progress, or value added, above the expected.

Based on pre-tests (administered in the Spring 2014) and post-tests (administered in the Spring term 2015), the group of children ($N = 232$) made greater progess than would normally be expected. The mean gain was equivalent to 2.6 months learning over and above their expected progress ($d =$ 0.22 95% CI: 0.11, 0.33). This is a relatively large effect for education interventions (Cheung and Slavin 2016).

The results are indicative but nevertheless reasonably robust since the age norms are well-established. The classes were taught for at least two school terms by teachers participating in the project. It is not possible to distinguish the effects of the lessons themselves from the gains in proficiency by the teachers in the professional development sessions.

The overall effect is in line or more positive than previous CA studies, all of which were based on two years' intervention. The theoretical assumption, confirmed in earlier phases of previous research, is that the effect is cumulative over the years, and is greater in the second and subsequent years. That is because higher achievement in a given topic or a school subject, like the numeracy assessed in this test, requires as a prerequisite a higher level of general reasoning developed in the thinking lessons.

This moderate-to-high result ties in with the project's theory and published research data for overall cognitive effects (e.g. Adey, Robertson, and Venville 2002; Oliver, Venville, and Adey 2012), confirming that classroom collaborative work on reasoning has an impact on conceptual understanding and numerical proficiency.

Discussion and conclusion

This was a project that achieved significant measurable impact on student learning and teacher confidence, corroborating the impact evidence produced in several of the original CA research papers, despite having much shorter duration in PD and in classroom intervention, and not being led directly by members of the CA research team. Whilst it is a limitation of the study that a control group was not used, evidence from this project also supplements the original CA findings by detailing impact on teacher confidence and practice at classroom level.

Participant teacher self-confidence grew on average by 0.82 on a 1–9 scale, an average increase of 12.4%, a statistically significant gain. The teachers believed they had most improvements in their ability to improve the level of their instructional influence. Whilst Robertson's (2014) study of CA measured changes to teacher pedagogy, this is the first CA study to systematically measure impact on teacher confidence in teaching and learning. Like Robertson's (2014) study, this new study's evidence of improved teacher confidence was corroborated by the evidence of a shift in practice for most teachers between the beginning and the end of the intervention as measured through lesson observation. Furthermore, teacher qualitative feedback highlighted in particular an impact on classroom talk; motivation, engagement and confidence, approaches towards misconceptions, and cross-curricular changes to teaching.

Pre- and post-tests in functional numeracy for 232 students in 14 Y3 to Y6 classes taught for at least two school terms by teachers participating in the project, showed a mean gain equivalent to 2.6 months learning over and above their expected progress in mathematics. Although unlike several previous CA studies (e.g. Shayer 2003; Adhami 2002; Adey, Robertson, and Venville 2002), this new study did not measure impact on achievement in subjects beyond mathematics nor measure impact longitudinally, the moderate-to-high result in mathematics over the relatively short time frame of one year ties in with the project's theory and published research data for overall cognitive effects.

Significant measurable shifts in practice or gains in teacher knowledge and student learning were not expected for this project, as we know the impact of professional development is not always visible until the year after the intervention has taken place (Kennedy 2016). It would be interesting to track these teachers as they develop their expertise in the years following the intervention.

Perhaps the greater than expected effects can be attributed to development over time of the CA professional development approach and the materials, compared to the first cohort of teachers who were using draft materials and experiencing a new and experimental professional development model. Recent improvements to the professional development programmes significantly include greater attention to the mathematics curriculum of the day, and to common errors and misconceptions amongst current cohorts of students.

It is not possible to distinguish the effects of the lessons themselves from the gains in proficiency by the teachers in the professional development sessions. Either the improved student test outcomes result from experiencing the lessons being taught by teachers who are confident in using the approach as a result of the professional development experience. Alternatively, teachers' confidence in teaching mathematics more generally has improved as a result of learning how to teach, and teaching these lessons, resulting in improved test outcomes for students.

Despite the lack of a control group, this impact evaluation does suggest that the impact seen in the original CA studies is replicable and that further replication studies might seek to explore this approach to teaching and learning, taking on board evidence of impact on teacher practice and confidence in classroom practice.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This project was funded by the Greater London Authority , through the London Schools Excellence Fund.

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