Preview

Summary
Simultaneous Noise Suppression and Edge Preservation in MRI Conductivity Mapping

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Introduction

Conductivity Mapping (CM) is a new quantitative, non-invasive technique that could provide new information on tissue ion content, specifically on sodium levels\(^1\), and distinguish between brain tumour types\(^2\).

The electrical conductivity ($\sigma$) is a complicated function of the MRI phase at TE = 0 ms ($\varphi_0$) but the following differential equation is a widely-used approximation, valid in regions with slowly varying $\sigma$:

$$\sigma = (\mu_0 \omega)^{-1} \cdot \nabla^2 \varphi_0 \ [1]$$

$\mu_0$ = vacuum permeability, $\omega$ = Larmor angular frequency, and $\nabla^2$ = Laplacian operator. Calculating $\nabla^2$ using a small kernel (Fig. 2a) tends to severely amplify the noise (Fig. 3a), while larger kernels\(^3\) (Fig. 2b) induce inaccuracies and blurring near the tissue boundaries with $\sigma$ jumps (Fig. 3b).

It has been suggested that the integral form\(^4\) of Eq. 1 is more robust to noise\(^5\):

$$\sigma = (\mu_0 \omega V)^{-1} \int_S \nabla \varphi_0 \ ds \ [2]$$

$S$ is the closed surface of some kernel with volume $V$. Again, larger $V$ are expected to suppress noise but cause blurring.

Previous studies have used i) Eq. 1 with large kernels combined with either magnitude-based weighting\(^6\) (Mag) or image segmentation\(^3\) (Seg) to provide better edge preservation, or ii) Eq. 2 only with small kernels to avoid errors at the tissue boundaries\(^4,5\).

Here we implemented Eq. 2 using large kernels restricted by the magnitude (Mag) and/or the segmentation (Seg). We compared several differential- and integral-based approaches in a numerical phantom and an in-vivo brain image.

Subjects/Methods

A numerical brain phantom (Fig. 1a) with realistic conductivity and magnitude values was created from the Zubal phantom\(^7\). $\varphi_0$ was simulated using $\nabla^{-2}$ (the inverse of Eq. 1) and Gaussian noise (standard deviation = 0.05) was added to the real and imaginary parts.

Multi-echo brain images were acquired in a healthy volunteer at 3T (Fig. 1b). ASPIRE\(^8\) was used on the first
two echoes to estimate $\varphi_0$ for each channel which were then combined using scalar phase matching. The 
gray and white matter, and cerebrospinal fluid were segmented using SPM12 on the combined magnitude 
image.

![Numerical brain phantom and In-vivo brain image](image)

**Fig. 1:** The numerical brain phantom (a) and the in-vivo brain image (b) including the acquisition 
parameters.

$\sigma$ maps were calculated from both the in-vivo and simulated $\varphi_0$ using both Eq. 1 and 2 with (Fig 2.): a) small 
kernels, b) large kernels, c) Mag, d) Seg, and e) Mag+Seg.
<table>
<thead>
<tr>
<th>a) Small kernel</th>
<th>b) Larger kernel</th>
<th>c) Mag</th>
<th>d) Seg</th>
<th>e) Mag+Seg</th>
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</thead>
<tbody>
<tr>
<td>( \nabla^2 ) is calculated using the finite difference approximation. Kernel: ![3D kernel diagram]</td>
<td>( \nabla^2 ) is calculated by fitting a 3D quadratic function to all voxels within a 10 mm radius.</td>
<td>The quadratic fit from b) is weighted by the magnitude(^6) (i.e. voxels with similar magnitudes to the middle voxel get higher weights).</td>
<td>The quadratic fit from b) is performed within each segment separately.</td>
<td>c) and d) combined. The quadratic fit from b) is performed within each segment separately with magnitude weighting(^6).</td>
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<tr>
<td>The kernel radius is 12 and 15 mm for the phantom and in-vivo image respectively.</td>
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<tr>
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<th>e) Mag+Seg</th>
</tr>
</thead>
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<tr>
<td>1. ( V ) is calculated using the finite difference approximation. Kernel: ![Gradient vector]</td>
<td>1. ( V ) is calculated by fitting a 3D quadratic function to all voxels within a 6 mm radius.</td>
<td>1. The quadratic fit from b) is weighted by the magnitude(^6)</td>
<td>1. The quadratic fit from b) is performed within each segment separately.</td>
<td>1. c) and d) combined. The quadratic fit from b) is performed within each segment separately with magnitude weighting(^6).</td>
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<td>The kernel radius is 8 and 10.5 mm for the phantom and in-vivo image respectively.</td>
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</thead>
<tbody>
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<td>2. The surface integral is performed for kernel ( V ): ![Sphere integral]</td>
<td>2. The surface integral is performed for a sphere of radius = 6 mm.</td>
<td>2. Voxels with substantially different magnitudes (more than 3(^3)std(Magnitude) away) from the middle voxel are excluded from the kernel ( V ) before the surface integral is performed.</td>
<td>2. Voxels of different segments from the middle voxel are excluded from the kernel ( V ) before the surface integral is performed.</td>
<td>2. c) and d) combined. Both voxels with substantially different magnitudes and different segments are excluded from the kernel ( V ) before the surface integral is performed.</td>
</tr>
<tr>
<td>The kernel radius is 12 and 15 mm for the phantom and in-vivo image respectively.</td>
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Fig. 2: A detailed description of all conductivity mapping techniques implemented and their parameters optimised for the numerical phantom and in-vivo images used in this study.

**Discussion**

Fig. 3 shows that compared to b), all \( \sigma \) maps were substantially improved by using either Mag (white ellipses) or Seg (black ellipses) especially near the edges. Mag+Seg yielded further improvements (white arrows). The integral-based approaches always yielded much better edge preservation than the differential-based methods (white rectangles). **To sum up**, integral-based CM with Mag+Seg (Fig 2.) provided the best \( \sigma \) maps from noisy phase.
Fig. 3: All $\sigma$ maps calculated using the methods from Fig. 2. White and black ellipses, and white arrows show regions where Mag, Seg, or Mag+Seg improved the images. The white rectangles highlight the superior performance of the integral-based methods.

References


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