# A TEACHER'S USE OF DYNAMIC DIGITAL TECHNOLOGY TO ADDRESS STUDENTS' MISCONCEPTION ABOUT ADDITIVE STRATEGIES FOR GEOMETRIC SIMILARITY

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*Research has well documented that students develop a significant misconception associated with the incorrect use of additive strategies when engaging with geometric similarity (GS) tasks. Since dynamic digital technology (DDT) has the potential to support students in addressing this misconception, teachers can exploit the affordances of DDT in the classroom to accomplish it. The aim of this paper is to explore how and why a secondary mathematics teacher uses DDT in the classroom to promote students' understanding of why additive strategies are inappropriate to use for GS tasks. Drawing on the data collected, through classroom observations and post-lesson teacher interviews, the research findings indicate that the dynamic and visual nature of DDT can be used to help students realise the inappropriateness of the use of additive strategies for GS tasks.*

## INTRODUCTION

Geometric Similarity (GS) is considered a key topic in school mathematics, especially in secondary mathematics as it forms a fundamental link to numeric, geometric, and spatial forms of reasoning. However, there is consistent research evidence to suggest that GS is a notoriously hard mathematical concept that many students (even teachers) have difficulties to understand (Son, 2013). A foremost reason for this difficulty is their common misconception about the incorrect use of additive strategies for GS tasks where the use of multiplicative strategies is appropriate. Although GS involves multiplicative relationships, when engaging with GS tasks students tend to too much rely on the use of additive strategies rather than multiplicative strategies. When applying the additive strategies, they pay attention to the difference between the measurements of side lengths of mathematically similar figures thereby failing to identify a ratio relationship between them. They presume that adding the same amount to the sides of a geometric figure always yields a mathematically similar figure, whereas doing so results in eliminating the multiplicative relationship between the side lengths of similar figures and creating visual distortions.

Researchers have emphasised that students' interaction with dynamic digital technology (DDT) (e.g., dynamic geometry software (DGS)) can promote their understanding of GS (Noss and Hoyles, 1996). The mathematical environments of DDT offer important affordances including dynamically linked visual, numerical and symbolic representations of geometric figures along with measurement and dragging facilities, through which students can explore, explain and communicate the variant and invariant properties of mathematically similar figures (e.g., all pairs of corresponding sides of mathematically similar figures are related by a common multiplier or a scale factor). It is therefore important for mathematics teachers to integrate DDT into their classroom practice to support the development of students' understanding of GS. For example, by exploiting the dynamic and visual nature of DDT, teachers can aid students in addressing their misconception about the additive strategies by enabling them to recognise and understand why the use of these strategies are inappropriate for GS tasks. However, to do so efficiently, teachers need to have a range of knowledge and skills. This includes a wide and deep understanding of the underlying reasons of students' over-reliance on the use of additive strategies for GS tasks and of how to use DDT appropriately and productively to address them in the classroom. Therefore, central to understanding how DDT can be used effectively in the classroom to address the misconception is to identify and describe the associated knowledge. Nevertheless, in the literature, there appears to be no research focusing on this. To fill this gap, this paper aims at examining a secondary mathematics teacher's classroom use of a DDT tool to address students' misconceptions about the use of additive strategies for GS tasks.

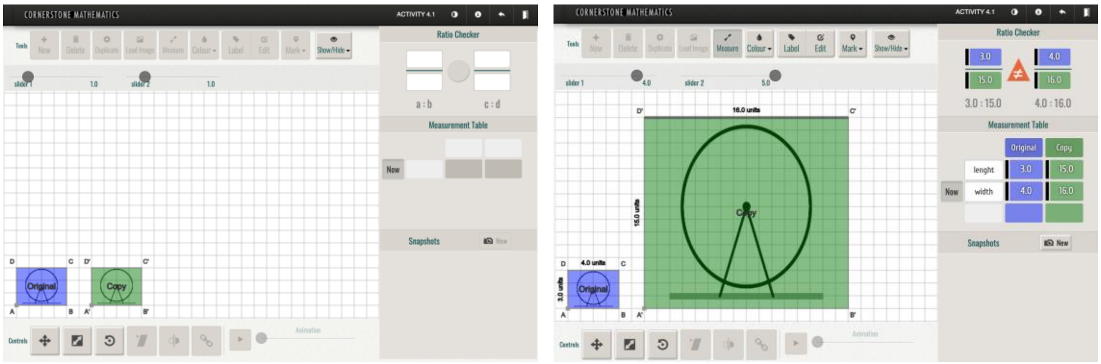
## METHODOLOGY

This paper is based on the first author's doctoral study that investigates secondary mathematics teachers' integration of DDT into classroom practices with a particular focus on GS. Three participant teachers were selected from the community of the Cornerstone Maths (CM) project, who work in secondary schools in London in the UK and have varying levels of experience teaching mathematics with technology. These teachers were deemed appropriate as case study teachers because after their involvement in the project's professional development they began to integrate a particular DDT tool, CM software, into their classroom practices for the teaching of GS in the lower secondary school mathematics (students aged 11-14 years in England**).** This paper discusses one of the three participant teachers, Jack (pseudonym), the most confident and experienced teacher of the three, both in teaching mathematics and using technology in his classroom.

Video-recorded lesson observation and audio-recorded post-lesson teacher interview were the main methods for data collection. Jack's case, conducted in November 2018, involved observing eight lessons and six interviews. In the observed lessons, Jack's 'resource system' (Ruthven, 2009) included the CM curriculum unit on GS consisting of the following materials: CM software, student workbook and teacher guide. The CM software is designed by exploiting the dynamic and multi-representational potential of digital technology with the aim of promoting students' engagement with and understanding of mathematical ideas. More specifically, it provides a set of controls and tools (e.g., scale factor and angle sliders, ratio checker, measure side lengths and angles) and various dynamically linked mathematical representations (e.g., geometric figures, tables). Following a sequence of guided activities embedded in the student workbook, students can interact with the CM software to explore the underlying mathematical concepts and relationships related to GS. One of the activities in the CM unit on GS particularly aims to address students' possible misconceptions about the incorrect use of additive strategies.

## FINDINGS

Jack adopted a carefully designed CM activity that includes two rectangles named 'original' and 'copy' (see Figure 1). In this context, 'copy' refers to a figure obtained as a result of increasing or decreasing the side lengths of an original using the length sliders, namely *slider 1* (changing the width) and *slider 2* (changing the height). With this activity, he provided students with opportunities to see what happens when scaling the length and width of a rectangle independently of each other and thereby to explore if adding the same amount to the sides of a geometric figure leads to the creation of a similar copy.




(a) (b)

Figure 1: (a) The initial version of the CM activity used by Jack; (b) The version of the activity after adding 12 units to both sides of the original rectangle as well as the use of ratio checker and measurement table

Jack began by asking students to conjecture if the following statement is true or false: I can make a mathematically similar rectangle by adding the same number to both sides. According to him, beginning the lesson with this statement is important as it may enable students to "think that this [the statement] is true and they go and check it and find it's not true and then they learn something". He then introduced a counterexample with the aim of enabling students to test their conjectures and explore the truth of them in the dynamic environment. The counterexample required students to increase the height and width of the original rectangle (the height:3, the width:4) by 12 units using the length sliders. Before students began working independently in pairs at laptops to engage with the activity involving the use of CM software, he encouraged them to produce conjectures about whether the copy rectangle would be mathematically similar after the counterexample.

Most students conjectured that the copy rectangle would be mathematically similar to the original one. However, having worked on the activity using the CM dynamic software, they changed their minds as they explored the fact that after adding the same amount to the height and width of the original, visual distortions to the London Eye appeared and the ratio of the corresponding sides between the original and copy rectangles was no longer the same. The dynamic mathematical environment provided by the CM software contributed to such exploration as students can compare the copy and the original both visually and numerically.

During the subsequent whole-class discussion, Jack underscored the key concepts and extended students' understandings of the idea that the heights and widths of the original and copy must be related multiplicatively. He first identified the numerical relationships between the side lengths of the original rectangle and copy rectangle through the measurement table. Using his desktop computer to operate the CM software 'live', he increased the copy rectangle's height and width by 12 units using the length sliders, then measured the sides of the copy rectangle and original rectangle using the measurement facilities and finally dragged the length measurements into the cells of the measurement table. The table enabled Jack to identify the multipliers for both the width and height in two different ways in the dynamic environment. First, he divided 15 by 3 and 16 by 4, which produces inequivalent numbers of 5 and 4, respectively. Second, he drew students' attention to the values on the slider 1 and slider 2, which represent the multipliers for the width and height and so help identify the multipliers. Students recognised that when adding 12 units to the width and height of the original rectangle, the values on the Slider 1 and Slider 2 become 4 and 5, respectively. In this way students explored that multipliers for the heights and widths are different numbers and therefore, the copy rectangle is not mathematically similar to the original rectangle, as the corresponding sides of mathematically similar shapes should be related by the common multiplier. Jack finished his lesson by drawing students' attention to the misconception about the incorrect use of additive reasoning in the context of GS, outlining that:

> When it comes to looking at the proportion, this [incorrect use of additive reasoning] is the biggest mistake that students make, they think they just go and add the same thing [number] to both amounts [of a rectangle] to make mathematically similar [rectangle].

## CONCLUSION

This paper has discussed the use of DDT to address students' incorrect use of the additive strategies for GS tasks.

The teacher exploited the dynamic and visual nature of the CM software in his classroom practice to help students connect height and width, scale factor and the common multiplier. For example, he manipulated geometrical figures dynamically by dragging the length sliders representing variable values to investigate the side properties of the geometric figures. Students developed an understanding that the side lengths of a geometric figure have to be multiplied by the common multiplier, in other words, by the same scale factor, in order to acquire a mathematically similar shape. They were made aware how a multiplicative or proportional strategy is appropriate for scaling a geometric figure.

Although Jack engaged with some of the dynamic and multi- representational features of the CM software such as the length sliders and measurement table and made the links between them, he did not exploit the facility of ratio checker to compare the length properties of two shapes dynamically when investigating the multiplicative relationships between the side lengths of the rectangles. This could provide students with a means to compare either the within ratios of the rectangles or the between ratios of corresponding sides, which would lead them to appreciate the use of multiplicative strategies in verifying the similarity of geometric figures. However, using the ratio checker requires complex technical and content knowledge; this may be why it was not used.

To conclude, teachers' use of DDT in the classroom could have an important role to play in enabling students to understand the necessity of the use of the multiplicative strategies for GS tasks and provide a foundation for proportional reasoning.

### References

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