

# PLANNING TO TEACH LOWER SECONDARY MATHEMATICS WITH DYNAMIC MATHEMATICAL TECHNOLOGY: QUALITY FEATURES OF LESSON PLANS

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*Set within the context of the longitudinal Cornerstone Maths project in England, we adapt Thomas and Hong's theoretical framework (mathematical) 'pedagogic technology knowledge' (MPTK, Thomas & Hong, 2013) to explore teachers' espoused knowledge to teach with dynamic mathematical technology in lower secondary mathematics. We conclude a set of eight 'quality features' of such plans, and highlight how each of these features can provide a dynamic insight into teachers' MPTK development over time.*

*Keywords: Dynamic mathematical technology; linear functions; algebraic variable, geometric similarity; mathematical pedagogic technology knowledge (MPTK)*

## INTRODUCTION

The Cornerstone Maths project, which began in England in 2011 has researched the design, implementation and impacts (on both students and teachers) of a set of three digitally enhanced curriculum units for lower secondary mathematics: algebraic variable; linear functions; and geometric similarity. These are three topics that are considered hard to teach and for which a body of evidence exists to suggest that dynamic mathematical technologies can enhance students' understandings. This earlier work is widely reported (Clark-Wilson & Hoyles, 2017; Clark-Wilson, Hoyles, & Noss, 2015; Clark-Wilson, Hoyles, Noss, Vahey, & Roschelle, 2015; Hoyles, Noss, Vahey, & Roschelle, 2013). In this paper, we focus on a strand of work that is motivated by the research question: What mathematical pedagogic technology knowledge is desirable for teachers to integrate dynamic visual technologies in their teaching of these concepts? This required an articulation of such knowledge and a methodological design that would have legitimacy within the context of a 15-month long professional development project.

## THEORETICAL FRAMEWORK

### Conceptualising teacher knowledge

A major shortcoming of widely adopted frameworks that conceptualise teacher knowledge such as Ball et al's 'Mathematical Knowledge for Teaching' (MKT, Ball, Hill, & Bass, 2005) and Rowland et al's 'knowledge quartet' (Rowland, Huckstep, & Thwaites, 2005) is that they have not evolved from researching teaching scenarios in which dynamic mathematical technologies were present. We define 'dynamic mathematical technologies' as those that offer different mathematical representations (geometric shapes, graphs, tables, algebraic expressions) that teachers and pupils can manipulate and by doing so, engage with the underlying mathematical concepts and relationships. Consequently, such frameworks pay no attention to the particular aspects of a teachers' knowledge for planning and teaching lessons with such technologies.

Hence, we looked to a broader framework that includes MKT, but also conceives knowledge as a dynamic construct that considers both cognitive and affective aspects of knowledge and that had emanated from research into teachers' developing use of DMT in classrooms. We adopted a

framework developed by Thomas and colleagues, ‘(Mathematical)<sup>1</sup> Pedagogical Technology Knowledge’, (Thomas & Hong, 2013; Thomas & Palmer, 2014), henceforth we call MPTK, as shown in Erreur : source de la référence non trouvée.

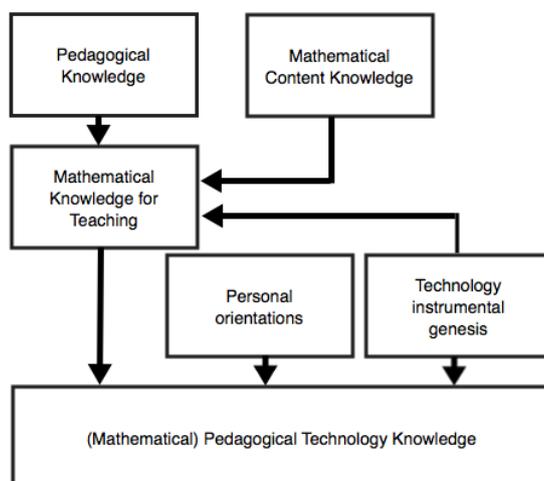


Figure 1 Components of (Mathematical) Pedagogical Technology Knowledge (Thomas & Hong, 2013)

This is a theoretical construct with the following components:

- **Pedagogical knowledge:** First suggested by Shulman (1987), this is a teacher’s knowledge of the ‘broad principles and strategies of classroom management and organization that appear to transcend subject matter’ (p. 8).
- **Mathematical content knowledge:** A teacher’s own knowledge of mathematics.
- **Mathematical Knowledge for Teaching (MKT):** This combination of a teacher’s pedagogical knowledge alongside their mathematical content knowledge was first defined as MKT by Ball, Hill and Bass (2005).
- **Personal orientations:** The teachers’ affective variables, that is their goals, attitudes, dispositions, beliefs, values, tastes and preferences, as described by Schoenfeld (2011, p.29), also incorporating their perceptions of the nature of mathematical knowledge and how it should be learned (with and without technology).
- **Technology instrumental genesis:** Rooted in activity theory, this is the process through which the teacher makes actions and decisions through which the technological tool is adapted to accomplish a particular mathematical task (Drijvers & Trouche, 2008; Guin & Trouche, 2002). Furthermore, for teachers, this genesis incorporates the development of the teachers’ understanding of pupils’ processes of instrumental genesis, whereby pupils become familiar with the affordances of the technology and can begin to use it in mathematically productive ways (Haspekian, 2005).

### Landmark activities

An important construct that underpinned both our theoretical framing and informed our research methodology is that of ‘landmark activities’. We define landmark activities as those which provoke

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<sup>1</sup> Thomas & Hong do not include Mathematical in their description of PTK, as, we conjecture their ‘overarching world’ is Mathematics. For clarity, we add Mathematical to the PTK, so henceforward call it MPTK.

a rethinking of the mathematics or an extension of previously held ideas – the ‘aha’ moments that show surprise - and provide evidence of pupils’ developing appreciation of the underlying concept. Although each curriculum unit includes several potential landmark activities, the research team selected one activity from each unit, which became the focus for teachers’ planning, teaching and subsequent reflection within our adapted ‘lesson study’ approach. In this way, the landmark activities act as boundary objects for the study (A more detailed account of landmark activities can be found in Clark-Wilson, Hoyles, & Noss, 2015).

## **METHODOLOGY**

The project involved 209 teachers from 48 London secondary schools in the period Jan 2014 - July 2016. Teachers were either self-selecting or nominated by their school and they or their schools chose for them to be involved in up to three cycles of professional development that enabled them to plan, teach and evaluate a landmark activity from each of the three Cornerstone Maths curriculum units. Some schools opted to send the same teacher(s) to two or three of these cycles. Briefly, the teachers participated in a one-day face to face session that: introduced them to the curriculum topic; supported them to become instrumented with the CM software; provided an opportunity for a lesson planning activity; and inducted them to the project’s online community. The teachers were also invited to give their ethical consent for their data to be used within the study (n=111, 53%).

During the lesson planning exercise, which the teachers carried out in pairs, a common lesson planning proforma was adopted that captured the following information:

- Contextual information about the class: (age, ability level):
- Teacher’s preparation notes:
- Pupils’ prior experience/skills with the software:
- Key intended learning outcomes for the pupils:
- Description of the planned phases of the lesson that included the teachers intended actions and the anticipated pupils’ responses to these.

Furthermore, a critical aspect of the methodology was that all teachers shared their lesson plans within the project online community, what was visible to all participants. The teachers were actively encouraged to review each other’s plans and to adopt or adapt the text, as they thought useful. The teachers were encouraged to produce the best plan possible, although it was understood that, as they taught the CM curriculum tasks that preceded the identified landmark activity, they would most probably want to review and revise their plan in the light of this classroom experience.

The lesson plans were evaluated according to the following set of ‘desirable’ features:

Feature 1: Describes teachers’ actions and questions (not involving the DMT).

Feature 2: Describes pupils’ actions on DMT.

Feature 3: Supports pupils in their instrumental genesis of the DMT, as appropriate to the activities.

Feature 4: Refers to the mathematical concept at stake (i.e. variables, functions, geometric objects).

Feature 5: Describes acting on and connecting mathematical representations.

Feature 6: Uses mathematical vocabulary.

Feature 7: Uses technological/contextual vocabulary.

Feature 8: Includes planned teacher use of the DMT.

These eight features had been developed a priori by the researchers as a means to arrive at a ‘quality score’ of between zero and eight for each plan, depending on whether the plan included particular feature. Hence it was possible to arrive at quantitative indications of quality in addition to the more obvious qualitative analysis that could be deduced from the plans.

## FINDINGS

82% of the teachers surveyed (n=111) reported that they had never or only occasionally used a mathematical technology in their key stage 3 teaching, with only 33% reporting that they felt confident or very confident to do so. From this data, we conclude that, for many of the teachers, the lesson plans were their first ever plan for this type of lesson.

The analysis of the teachers’ lesson plans provided an insight their MPKT as they prepared to teach the lessons. We begin by presenting our findings with respect to the first of the three CM curriculum topics (algebraic variable) to highlight the nature of the resulting data and then describe the cross-topic analysis that led to a more general set of outcomes.

### Algebraic variable

Twenty-eight lesson plans that had been produced in pairs and trios by 74 teachers were analysed and the frequencies of each feature is shown in Table 2.

Feature of lesson plan	Frequency	% (n=28 plans)
1. Explicit descriptions of teachers’ actions/questions	16	57%
2. Explicit descriptions of pupils' actions on DMT during the lesson	9	32%
3. Appreciation of pupils’ instrumental knowledge (i.e. prior skills with software, progression of skills in lesson)	12	43%
4. Explicit reference to variables (i.e. creating, naming, acting on)	11	39%
5. Explicit reference to acting on reps (i.e. dragging/moving sliders)	17	61%
6. Explicit use of mathematical vocabulary	18	64%
7. Explicit use of technological/contextual vocabulary	18	64%
8. Includes planned plenary phases that involved teacher use of software	5	18%

Table 2 Algebraic variable: Summary of lesson plan analysis (28 Lesson plans)

An exemplification of high quality planning for the algebraic variable research lesson in relation to each of the desirable features (taken from the complete set of lesson plans) is provided in Table 3.

Feature of lesson plan	Exemplification from teachers’ plans
1. Explicit descriptions of teachers’ actions/questions	“Encourage pupils to play the pattern again and ask does it correspond to your pattern if you change the number of blocks?”

2. Explicit descriptions of pupils' actions on DMT during the lesson	"Encourage students to use slider - ask them how you can make both sliders move at the same time. What will they need to do the variables?"
3. Appreciation of pupils' instrumental knowledge (i.e. prior skills with software, progression of skills in lesson)	"Remind students how to use the software – recap Investigation 1. i.e. Blocking and patterning. (Lock student screens)"
4. Explicit reference to variables (i.e. creating, naming, acting on)	"Ensure all pupils start to introduce a variable for their blocks ('unlock' the no of blocks column)".
5. Explicit reference to acting on reps (i.e. dragging/moving sliders)	"[Ask] What is the purpose of the slider? What impact is it having when you slide along the bar?"
6. Explicit use of mathematical vocabulary	"[Ask] How can we check if our orange and green blocks increase in the same way?"
7. Explicit use of technological/contextual vocabulary	"Ask students to create a table snapshot, starting from 1 block. What do students notice about the total number of lights?"
8. Includes planned plenary phases that involved teacher use of software	"demonstrate how the Blocks and Pattern should have been made. What does the slider do?"

Table 3 Algebraic variable: Exemplification of the features of high quality lesson plans

The plans were of a highly variable quality and it was notable that only five plans included six or more of the desirable features, which suggests that the teachers had very little prior experience of a lesson planning approach that emphasised their own actions and words, rather than solely a plan of what their pupils would be expected to do. Within the plans, approximately two thirds of the plans included references to actions on the dynamic slider and approximately one fifth of the lessons plans included a planned plenary phase that involved the teacher's use of the software.

### Development of lesson plans over time

The above process was replicated for the two subsequent curriculum topics (linear functions and geometric similarity) and distributions of the quality scores produced as shown in Figures 2, 3 and 4.

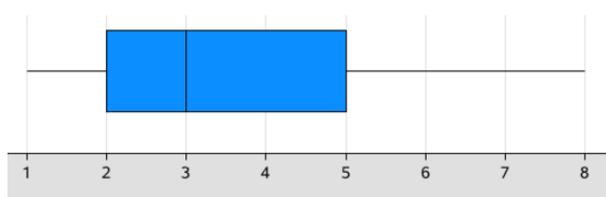


Figure 2 Algebraic variable: Distribution of quality scores for lesson plans ( $n=27$ ,  $\bar{x} = 3.9$   
SD = 1.8)

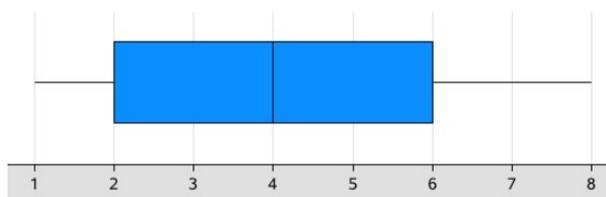


Figure 2 Linear functions: Distribution of quality scores for lesson plans ( $n=42$ ,  $\bar{x} = 4.2$ )

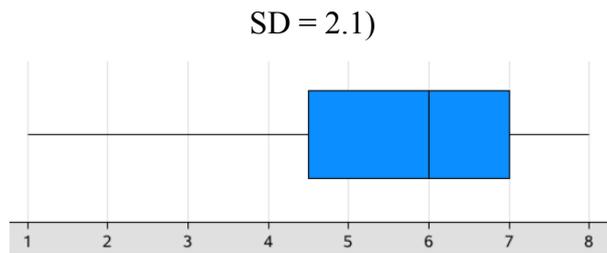


Figure 3 Geometric similarity: Distribution of quality scores for lesson plans (n=21,  $\bar{x}$  = 5.5, SD = 1.8).

These mean average and median quality scores show very clearly the development in the quality of the teachers' lesson plans over time as they participated in the repeated PD cycles as both scores increased. This is substantiated by the qualitative analysis of the lesson plan text, examples of which will be shared in the conference presentation. A summary table of the frequencies of quality features across the three topic areas is also informative (Table 4).

	Teachers' actions and	Pupils' actions on DMT	Instrumental genesis Supports for pupils'	Concept Focus on mathematical	Actions on representations	Uses maths vocabulary	Uses technical and/or	DMT
Algebraic variable (n=28)	57%	43%	32%	39%	61%	64%	64%	18%
Linear functions (n=42)	69%	38%	45%	57%	26%	62%	31%	24%
Geometric similarity (n=21)	95%	33%	86%	71%	62%	86%	71%	43%

Table 4 Comparison of lesson plan quality features by topic

The nature of the individual landmark activities did provoke a need for teachers to plan in ways that might privilege particular features, i.e. the geometric similarity landmark activity required increasingly more detailed definitions, which would by necessity privilege the use of mathematical language. However, given that the three PD cycles took place over the period of a year, the following conclusions can be made:

- Teachers became much more aware of the need to plan what they were going to do and say during the lessons and particularly during the whole class plenaries around the important mathematical ideas.
- Teachers became more mindful of the need to provide support for pupils to make sense of the DMT such that they could use it in mathematically productive ways beyond only the initial lessons (i.e. support the pupils' instrumental genesis more explicitly).
- Teachers paid increasing attention to the mathematical concept at stake.

- Teachers were more explicit in their plans to convene whole class plenaries to focus on the mathematics at stake (with more teachers considering how they would use the DMT to support this work).

## DISCUSSION

The teachers' lesson plans provide an insight into their espoused MPTK. Furthermore, the features of the lesson plans can be mapped to the components of the teachers' knowledge as shown in Figure 2.

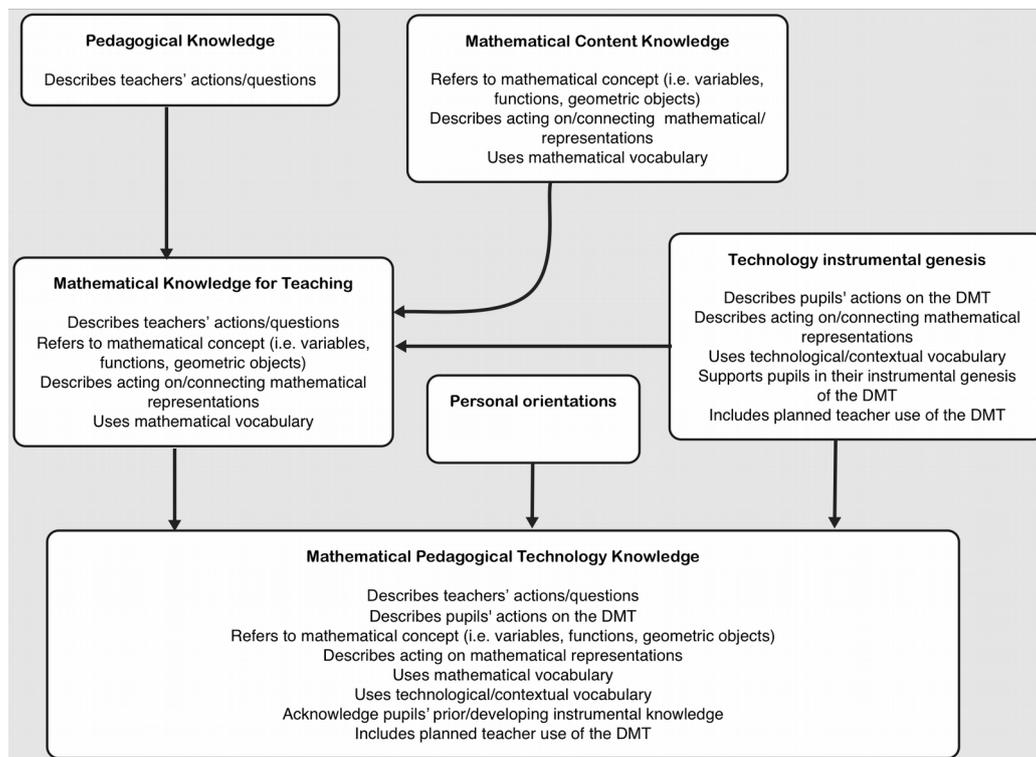


Figure 2 Features of CM landmark activity lesson plans and their relationship to a teacher's MPTK.

This provides an indication of the key elements of planning lessons with technology that concern the development of pupils' instrumental geneses – a significant element of teachers' knowledge that should be developed within teacher education and professional development programmes.

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