Chapter 7

The Spatial Prepositions in English, Vector Grammar, and the Cognitive Map Theory

John O'Keefe

7.1 Introduction

In this chapter I wish to return to a subject that Lynn Nadel and I first addressed in our book *The Hippocampus as a Cognitive Map* (1978) nearly two decades ago. The gist of the argument presented there was as follows. Evidence from animal experiments proves strong evidence that the hippocampus, a cortical area in the mammalian forebrain, is involved in the construction of an allocentric spatial representation of the environment, what Tolman (1948) called "a cognitive map." Constructed and modified during exploration (a cognitive behavior), this map provides the animal with a representation centered on the environment and locates it within that environment. During the initial exploration of an environment and subsequently, places of interest are labeled in the map and their label and locations stored for future use; these locations can subsequently be retrieved into the map and used as goals to direct behavior. For example, if a satiated animal notices food in a location during its initial exploration of an environment, it can on a subsequent occasion use that information to satisfy a hunger need. Upon finding itself in the same environment it can retrieve the location of the food and use it to direct its behavior toward that location.

This theory can account for a substantial part of the experimental literature on the infrahuman hippocampus. In order to extend the theory to account for the human data, however, we needed to extend it in two ways. First, we had to incorporate a temporal sense into the basic map to account for the ability of humans to process and store spatiotemporal, or episodic, information. Second, we had to allow for the impressive lateralization of function that has been repeatedly demonstrated in the human brain. Neuropsychological studies had suggested that while much of the right cerebral hemisphere is specialized for "visuospatial" processing, the left side has been given over to language function. Following her dramatic demonstration with Scoville of a memory function for structures in the mesial temporal lobe (Scoville and Milner 1957), Milner showed that this memory function respected the general lateralization
of function: patients with damage restricted to the right mesial temporal lobe were amnesic for visuospatial material, whereas those with left-sided damage were amnesic for linguistic material. Evidence gathered since has strengthened this conclusion (Smith and Milner 1981, 1989; Frisk and Milner 1990).

Nadel and I suggested that this lateralization of function might be due primarily to differences between the inputs to the hippocampal map on the two sides of the human brain and not necessarily to any fundamental differences in principles of operation. The right human hippocampus would receive inputs about objects and stimuli derived from the sensory analyzers of the neocortex and attributable to inputs from the external physical world. It would operate in the same way as both right and left infrahuman hippocampuses. In contrast, the left human hippocampus would receive a new set of inputs, which would come primarily from the language centers of the neocortex and would consist of the names of objects and features and not of their sensory attributes. In addition, this “semantic map” would incorporate linear temporal information and in consequence would serve as the deepest level of the linguistic system, providing the basis for narrative comprehension and narrative memory.

However, language is clearly not reducible to the set of spatial sentences; therefore we sought to create a more general framework by following the work of Gruber (1965, 1976) and Jackendoff (1976), who noted the similarity in structure between sentences such as “The message went from New York to Los Angeles,” “The book went from Mary to the library,” “The rock went from smooth to pitted,” “The librarian went from laughing to crying.” They proposed that the parallels in surface structure reflected parallels in underlying meaning, in this case the substitution of possessional sense, identificational sense, and a circumstantial sense for the positional sense of the prototype. Nadel and I interpreted this to mean it might be possible to envisage nonphysical spaces that located items, not according to their physical location, but according to their location in these other dimensions. We suggested one such dimension might be that of influence but did not develop this notion any further.

In this chapter I would like to develop further this idea of the semantic map. In the years that have intervened since the first publication of the idea, we have learned a great deal about the working of the infrahuman cognitive map at the physiological level, and there are now several computational models available. I intend to explore the adequacy of one of these in particular (O'Keefe 1990) as the basis for a semantic map.

Before returning to the semantic map idea, it will be helpful if I elaborate some of the details of the basic theory as developed for physical space. In the cognitive map theory, entities are located by their spatial relationships to each other. Spatial relationships are specified in terms of three variables: places, directions, and distances (figure 7.1). Places are patches of an environment that can vary in size and shape
ELEMENTS FOR A MAP

\[ \begin{align*}
\text{MAP} &= \\
\text{PLACES} &\quad A & B & C \\
\text{DIRECTIONS} &\quad \angle AB & \angle AC & \angle CB \\
\text{DISTANCES} &\quad |AB| & |AC| & |CB|
\end{align*} \]

\textbf{Figure 7.1}

Cognitive maps consist of a set of place representations and the distances and directions between them. Distances and directions can be represented by vectors drawn from one place to another. In animals such as the rat, they are computed in real time on the basis of actual movements, whereas in higher mammals they may become autonomous from actual movements.

depending on the size of the environment and the distribution of features in that environment. They are located in terms of the spatial relations among the invariant features of the environment; they can also be located by their direction and distance from other places. The \textit{place code} is carried by the pattern of firing of the place cells in the cortical region called the "hippocampus." \textit{Directions} are specified as a set of parallel vectors. As with places, these can be identified in one of several ways: either as the local gradient of a universal signal such as gravity, geomagnetism, or olfactory currents, as the vector originating at a place or object and passing through another place or object (or passing through two places), or as having a specified angle to a
previously identified direction (e.g., through updating the current direction on the basis of angular head movements). For every direction there is an opposite direction, which can be marked by the negative of that vector. The direction code is carried by the pattern of firing of the head direction cells in the postsubiculum (see, for example, Tauke, Muller, and Ranck 1990), another cortical region that neighbors on the hippocampal region and is anatomically connected to it. Distances between objects or places are given by a metric. The basic unit of this metric might be derived from one of two sources: either there is a reaference signal from the motor system which estimates the distance that a given behavior should translate the animal or use is made of environmental or interoceptive inputs which result from such movements. An example of an environmental input would be a change in retinal location of visual stimuli, and an example of an interoceptive input would be a vestibular signal. In either case, the geodesic distance between two objects or places needs to be computed by, for example, gating the metric signals arising from such sources by the head-direction signals so that only movements when the animal is heading in the same direction are integrated.

A path is an ordered sequence of places and the translation vectors between them. Paths can be identified by their end places or by a distinct name. Conversely, places along the path can be identified and associated with the path. A path may be marked by a continuous feature such as an odor trail or a road but need not be.

Within this spatial framework, translations of position in an environment are specified as translation vectors whose tail begins at the origin of movement and whose head ends at the destination. Vector addition and subtraction allow journeys with one or more subgoals to be represented and integrated. Furthermore, on a journey with more than one destination the optimal or minimal path can be calculated. It is still not clear whether the spatial coordinate framework is a rectilinear or a polar one and whether the metric is Euclidean or otherwise. In recent papers, I have explored Euclidean polar models (O’Keefe 1988, 1990, 1991).

If the cognitive map theory is on the right track in its contention that the left human hippocampus is basically a spatial mapping system that has been modified to store linguistic as opposed to physical information, then it might be possible to learn something about the structure of the system by analyzing the way it represents space, linguistically. A long tradition in linguistics, recently revived within case grammar theory, postulates that the deep semantic structure of language is intrinsically spatial and that other, nonspatial, propositions are in some way parasitical on these prototypical formulas, perhaps by means of metaphorical extension of their core spatial meanings. This is the contention of a group of linguists called "locational" or "localists" (Anderson 1971; Bennett 1975). These localist theories (see Cook 1989 for a recent review) suggest that the basis for spatial sentences consists in a verb and its
associated cases. Typical cases might be agent, object, and locative, identifying the initiator of the action, the thing acted on, and the place or places of the action, respectively. In an uninflected language such as English, many of the spatial relations described in spatial sentences are conveyed by the prepositions. As Landau and Jackendoff (1993) have pointed out in their recent article, there are only a limited number of these. If this be the case then it is possible that a description of the representations set up by the spatial prepositions might provide the basis for a more general linguistics. Nadel and I speculated that the origin of language might have been the need to transmit information about the spatial layout of an area from one person to another (O'Keefe and Nadel 1978, 401n). This view suggests that at some point in their evolution hominids began to elaborate the basic cognitive map by substituting sounds for the elements in the map or for some of the sensory aspects of these elements. These maps were probably primarily transmitted as drawings in the sand or dirt with different icons standing for different environmental objects. In this way one group of a family could forage a patchy environment and report back the locations of foods to the rest of the family. Different grunts would enrich the detailed information in the map and might serve the additional purpose of acting as an encrypting device. Over time, an increase in vocabulary would eventually obviate the need for the externalized map entirely, but the neural substrate would retain the structure of the original mapping function.

In this chapter I will set out the basic framework of vector grammar and show how it accounts for many of the spatial meanings of the spatial prepositions. My thesis is that the primary role of the prepositions is to provide the spatial relationships among a set of places and objects and to specify movements and transformations in these relationships over time; these spatial relationships and their modifications can be represented by vectors.

The location of an entity within this notation is given by a vector that consists of a direction and a distance from a known location. Much of the work of the locative prepositions involves the identification of these two variables. In some cases (for example, with vertical prepositions; see below), the direction is given by an environmental signal such as the force of gravity. In most cases, however, it needs to be calculated from the spatial relationships between two or more objects or places, which specify the origin and termination (or the tail and the head) of the vector or a point along the vector. By contrast, distances are less well specified; in most cases, the metric is an interval one. One of the roles of the preposition for is to supply the necessary metric information. The space coded by the locative prepositions is a mixed polar-rectilinear one.

In this chapter I will assume (following the locationists; see above) that the prepositions in English have a spatial (or in one or two instances, temporal) sense as
their basic meaning and that the other meanings are derived by metaphor. I will concentrate on the locative prepositions and in particular those dealing with the vertical dimension, although others will also be covered. I will then extend the analysis to show how the temporal prepositions code for a fourth dimension, which differs only slightly from the three spatial ones, and how changes in state or location can be coded by the translational meanings of the same prepositions. If time can be coded by a fourth dimension, is it possible to incorporate other nonspatial relationships by higher dimensional axes as well? As a preliminary exploration of this question, I will conclude with a discussion of the metaphorical uses of the vertical stative prepositions to represent the nonphysical relations of status and control.

My primary concern in this chapter is to set out the premise that a vector notation can capture many of the basic meanings of the spatial prepositions in English. Consequently, I will not address in any detail the role of syntax in this kind of grammar. In general, the syntax of such a system will consist of a set of rules for relating the spatial structure of the deep semantic narrative to the temporal structure of the surface information transmission system. Thus, just as there is an associated motor programmer that translates information from the spatial map into instructions to the motor planning systems so that the animal can approach places containing desirable objects or avoid ones with undesirable objects, so also there is a production system for generating sentences from the map narrative. The syntactic rules specify, among other things, the order in which the elements of the narrative are to be read and how the different parts of the vector system are to be translated into surface elements as a function of the way that they are read. For example, the difference between the active and the passive voice in the surface sentence depends on the direction of travel along the underlying vector (head to tail or vice versa) relating an agent and its actions.

### 7.2 Physical Spatial Meanings of the Vertical Prepositions

In this section I shall analyze the spatial meanings of four related prepositions: below, down, under, and beneath (or underneath). Although these have antonyms (above, up, over, and on top of), I shall refer to these latter only when they contribute something extra to the discussion. The four prepositions have in common that they denote spatial relationships between entities in one linear dimension, which I shall call the "Z-dimension." They differ from each other in interesting ways that will allow us to explore the properties of the space they depict.

#### 7.2.1 Below

Let us begin with what I believe to be the most basic of the four prepositions, below. On my reading, below relates two entities \(A\) and \(B\) in terms of their relative location along the \(Z\)-direction. Consider the simple deictic sentence
(1) John is below.

Because *below* is a bipolar preposition, there must be a second suppressed term, which I shall argue is the place occupied by the speaker or the listener. John or his place is $A$, the speaker's (or listener's) place is $B$, and the relationship between them is as follows: the magnitude of the component of $A$'s place in the $-Z$-direction is greater than the magnitude of $B$'s place. In order to make the assertion in (1), or to assess its validity, we need a notation for specifying the $Z$-direction, a way of locating $A$ and $B$ along that direction, and means for assessing whether $A$ or $B$ has a larger component along that direction. The most convenient notation for accomplishing these is vector algebra.

In this notation a direction is designated by a set of parallel vectors of unlimited magnitude and unspecified metric. The location of each entity is specified by a vector drawn from an observer to the entity. This vector can be specified by a magnitude $R$ and an angle $\phi$ with the direction vector through the point observer (figure 7.2). The component of the vector $A$ along the $Z$-axis can be computed by calculating the inner product of $A$ and $Z$ given by the formula:

$$A_z = A \cos \phi,$$

where $A$ is the magnitude of $A$ and $\phi$ is the angle that $A$ makes with the $Z$-direction vector at observer (obs).

In the deictic example of sentence 1, $A$ is *below* the observer if $A_z < \text{obs}$, and *above* the observer if $A_z > \text{obs}$. The same considerations allow the observer to decide whether $A$ is below $B$ when neither is located at the observer (figure 7.3 shows this situation). Again, the question of whether $A$ is below or above $B$ can be assessed by comparing their relative magnitudes along the $Z$-axis.

If $A_z - B_z > 0$, $A$ above $B$;

If $A_z - B_z < 0$, $A$ below $B$.

Note that the relationship between $A$ and $B$ is perfectly symmetrical and that neither $A$ nor $B$ can be considered a reference entity in the deep structural representation of the relationship. Choice of one or other as the referent in the surface sentence may depend more on the topic of the discussion, the previous sentences, which of the two entities has already been located, which is easier to locate perceptually, and other considerations. The *below* relationship is a transitive one. By simple transitivity of arithmetical relations on the $Z$-dimension,

if $A_Z > B_Z$ and $B_Z > D_Z$,

$$\therefore A_Z > D_Z.$$
Figure 7.2
Vector location and the below relation. The location of an entity \( A \) can be represented by a vector drawn from the observer to that entity. The vector is characterized by a distance \( R \) and an angle \( \phi \) measured with respect to a direction \( Z \). The projection of the vector onto the \( Z \) direction is shown as \( A_Z \).

Figure 7.3
Each item \( A, B, C, \) and \( D \) has a projection onto the \( Z \)-axis. The relative lengths of the projection onto this axis determine which items are below which. In the example, \( B \) and \( C \) have identical projections and are therefore both equally below \( A \).
In figures 7.2 and 7.3, I chose to represent entities $A-D$ in an allocentric framework; that is, I assumed that they existed in an environmental framework independent of the location of the observers and that their relationship within the framework could be assessed independently of the locations of the observers. Further, I assumed that the distances from each observer to the entities was known or could be computed, for example, by movement parallax. Does this imply that the spatial relationship denoted by *below* can be computed only within an allocentric framework? Can we say anything about the constraints on frameworks that can be used?

In general, the use of *below* relies on the availability of a direction vector shared between the speaker and listener; in the case of the allocentric framework, this is provided by the universal gravity signal. There are, however, other, more limited uses of *below* that employ egocentric and object-centered directional vectors. Egocentric vectors are fixed to the body surface of the observer, and object-centered vectors are fixed to the entity or entities related. Sentences 2–5 are examples.

(2) The new planet appeared below the moon.

(3) Below this line on the page.

(4) Hitting below the belt.

(5) The label below the neck of the bottle.

The egocentric use occurs under circumstances (a) where the entities are very far away from the observer and therefore do not change relative locations with observer location; or (b) where the entities are constrained to lie on the $XZ$-plane, as on a page or a video display unit. In the former case, the conversants must ensure that they are similarly aligned to each other relative to the entities or that there is a conventional orientation relative to the gravity signal that enables the $Z$-direction to be labeled conventionally. This is most obvious with the specialized case of the parts of the human body, which are probably labeled by reference to their canonical orientation relative to gravity (see Levelt, chapter 3, this volume). The case of the bottle and similar manufactured objects that refer to body parts (back of a chair, leg of a stool) would seem to follow the same rule. In general, then, normal conversation would seem to require the use of an allocentric framework for most purposes, for the reasons pointed out by other contributors to the present volume (Levelt, chapter 3; Levinson, chapter 4). Even the ability to see things from another's point of view would appear to involve computations based on an underlying knowledge of the two observers' locations in allocentric space.

A second conclusion can be drawn about the underlying framework on the basis of our discussion of *below*. Where it is used to describe an allocentric relationship, the framework cannot be a simple polar coordinate system, but must have at least one
rectilinear axis. This follows from the simple observation that in a polar coordinate system the below relation cannot be specified by one variable alone, but requires two variables: a distance and an angle (see figure 7.2). It follows therefore that the most parsimonious theory would specify the Z-direction by a single dimension in all usages. As we shall see, this does not necessarily imply that the other two (non-Z) dimensions are also rectilinear.

We have, then, evidence for a single dimension along which entities can be located. Can we say anything more about the metric at this stage, and if so, how are distances specified along this dimension? Scales differ in the type of metric employed. Roughly, this describes the relationship of the observations or measurements to the system of real numbers. The usual categories of scales are the nominal, ordinal, interval, ratio, and absolute; they differ in the number of properties of the real number system they respect. This is most easily characterized by the types of transformations that can be applied to the assigned values without transforming the relationship of the scale to the thing measured. Nominal scales are simple classification scales in which the labels stand for the names of classes. For the purposes of the scaling, the elements within each class are considered equivalent and different from all the elements in the other classes. No other relationship among the elements is implied, and only transforms equivalent to the relabeling of the classes are allowed. Clearly, the below relationship satisfies a nominal scale. Ordinal scales consist of a series of numbers such that observations equal to each other are assigned the same number and an observation larger than another is assigned a larger number, but no significance is attached to the interval between the numbers. The relationship between numbers is transitive because \( m > n \) and \( n > p \) implies \( m > p \), and all mathematical transformations that maintain the monotonic ordering of the numerical assignments are permissible. Because it is possible to say that \( B \) below \( A \) and \( C \) below \( B \) implies \( C \) below \( A \), we are dealing with at least an ordinal scale. Interval scales are ordinal scales that, in addition, provide information about the differences between the scale values. In particular, they assert that some differences are equal to each other. For example, \( m - n = p - q \). Transformations that preserve the differences between values as well as their ordering are permissible. Specifically, the values of one scale can be multiplied by a positive constant and added to another constant without consequence to relationships.

\[
Z_2 = aZ_1 + b, \ a > 0
\]

In this linear transform, \( a \) changes the gain of the metric, and \( b \) the origin. It would appear that the below directional scale comes close to fulfilling the requirements for an interval scale. One way of testing this is to ask whether it is possible to apply the comparative operator more to the preposition and thus to derive equivalent intervals of belowness. The question is whether the comparative notion is an intrinsic part of
the meaning of below or merely an extension of it. I would argue that because it is always legitimate to ask for the relationships set out in (8), the scale is an interval one. Indeed, it may not be possible to compute the vector calculations suggested in this chapter on material ordered on less than an interval scale.

(6) A and B are below C. (nominal)

(7) A is more below C than B. (ordinal)

(8) A is as far below B as C is below D. (interval)

Compare these to

(6a) A and B are brighter than C.

(7a) *A is more brighter compared to B than to C.

(8a) *A is more brighter than B by the same amount as C is more brighter than D.

Ratio scales are interval scales that do not have an arbitrary origin. Here the only permissible transform is the gain of the scale

\[ Z_2 = aZ_1, \ a > 0. \]

In absolute scales, the final category we shall consider, no transfers are allowed and the underlying assumption is that the real number system uniquely maps onto the observations

\[ Z_2 = Z_1. \]

As we have seen already, the metric of the below relationship is at least ordinal, and probably interval. But is it ratio? Here the fact that the below relationship can be assessed from any arbitrary observation point and can use any origin suggests that it does not rely on a fixed origin but is invariant under arbitrary translations. Furthermore, it is intuitively obvious that changes in scale do not affect the relationship either. These suggest that it falls short of a ratio scale. It can, however, be elevated into a ratio or even an absolute scale by the provision of explicit metric information.

(9) a. A is twice as far below B as C is.

b. A is three feet below the surface.

7.2.2 Down (and Up)

The locative meaning of down is related to that of below in that it specifies the direction of the entity as lying in the \(-Z\)-direction. In addition, however, it requires a line or surface that is not orthogonal to the \(Z\)-direction and on which the entity is located. This line or surface is the object of the preposition down. As with below, the directional component of down is relative to another entity, which in this case is
governed by the preposition *from*. In general the preposition *from* identifies the source or tail of a direction vector. If this information is not supplied explicitly, it is assumed that the referent is the deictic location *here*.

(10) The house is down the hill (from here).

(11) Just down the tree from Sam was a large tiger.

(12) *The boat was down the ocean.*

Thus there are two reference entities: a plane or line that I shall call the "reference plane" and a place or object that I shall call the "reference entity." As long as the extended reference entity is not horizontal (perpendicular to the Z-axis) as in (12), it can be a one-dimensional line or a two-dimensional surface. Intuitively, this reference entity should be a linear or at least monotonically decreasing function of Z over the relevant range. Someone on the other side of the hill, regardless of the person’s relative — Z-coordinate, is not *down* the hill from you. Similarly, a local minimum on the slope of the hill between the entity located and the reference entity disrupts the use of *down*. To put it another way, the preposition *down* can only take as direct objects entities that have or can be treated as having monotonic slopes in the nonhorizontal plane. Applying our comparative *more* to the preposition *down*, we find, as we did with *below*, that its primitive sense is to operate on the Z-component of the relationship.

(13) John is more (farther) down the hill than Jill.

John and Jill are both located on the hill, the hill has a projection onto the Z-dimension, and John has a larger — Z than Jill. There is no interaction between the steepness of the reference plane and the sense of the preposition. This can be tested by asking the question of the three people in figure 7.4 Who is more (farther) down the hill from Jill? John or Jim?

My sense of the meaning of *down* is that neither John nor Jim is more down from Jill than the other, indicating that the non-Z-dimensions are irrelevant. However, the ability to extract the Z-component from a sloping line or surface suggests either that these can be decomposed into two orthogonal components (Z and non-Z) or that their projections onto the Z-axis can be computed. It seems, then, on the basis of our analysis of *down*, that we are dealing with at least a two-dimensional coordinate system in which one dimension is vertical and the other one or more dimensions, orthogonal to this. As with the *below/above* direction, the difference between *down* and its antonym *up* is merely a change of sign and there are no obvious asymmetries. If A is *down* from B, then B is *up* from A. The measurement scale of the Z-axis would appear to be an interval one and there is clear evidence of the absence of a true 0 or
Figure 7.4
Down measures the relationship in the Z-direction. John and Jim are equally far down the hill from Jill, despite different lateral displacements.

origin (this is relative to the reference point identified by from), and therefore the scale is not a ratio one. The scale of the other two dimensions is not clear from the two prepositions below and down because the use of the comparative operator more in conjunction with these only operates on the Z-component of the meaning. Evidence about these other dimensions can, however, be garnered from an analysis of the third of our prepositions, under.

7.2.3 Under
Under is similar to down and below in that it also codes for the spatial relationship between two entities in the Z-direction. In addition, however, it places restrictions on the location of these entities in one or two directions orthogonal to the Z-direction. If B is under A, then it must have a more negative value in the Z-dimension. In addition, however, it must have one or more locations in common in at least one orthogonal dimension (let us call them X and Y for the moment without prejudice to the question of the best representation of relationships in this plane). The projection of the entity onto the X-direction is determined in the same way as that onto the Z-direction by calculating the inner product of the vector drawn to the entity from an observer. Figure 7.5 shows this relationship for three pointlike objects. The relation depicted is conveyed by the sentences

(14) C is under A but not under B; B is not under A.

When one or more of the entities is extended in one or more of the non-Z-directions, the under relationship can be assessed by the same algorithm. For example, if the entities are extended in the XY-plane, then an overlap in any location in the
**Figure 7.5**

*Under* represents a spatial relationship in the *XY*-plane as well as the *Z*-direction. *C* is *under* *A* because it has the same *X*-length and a greater — *Z*-length. *C* is not *under* *B* because the *B*\(_X\) and *C*\(_X\) lengths differ.

*XY*-plane suffices. Note that unlike *below*, *under* is not transitive when applied to entities that are extended in the *XY*-plane. *B* *under* *A* and *C* *under* *B* does not mean that *C* is *under* *A*. Another interesting difference between *under*, on the one hand, and *down* and *below*, on the other, arises when we examine the locus of operation of the comparator *more*. Recall that when applied to *below* and *down*, *more* acted to increase the length of the *Z*-component of the vector to the entity. When applied to *under*, the effect of the comparator is not fixed but depends on the relative dimensions of the two entities. Let us leave aside for the moment the small number of usages that seem to mean that there is no intervening entity between the two relata:

(15) Under the canopy of the heavens.

(16) Under the widening sky.

The comparator cannot be applied to these usages, which I shall designate *under*\(^1\). In the more frequent usage of *under*, the comparator is more often found to operate on the orthogonal *X*-dimension than on the primary *Z*-dimension. Compare the following two sentences:
The Spatial Prepositions

Figure 7.6
Stick \( B \) is farther (more) under the table than stick \( A \) because there is a greater length of overlap with the projection onto the \( XY \)-plane.

(17) The wreck was farther under the water than expected.

(18) The box was farther under the table than expected.

Ignoring the metonymic uses of table and water, it is clear that the first usage, (17), implies a greater depth or \( Z \)-dimension, while the second, (18), implies a greater length in the \( X \)-dimension. In the first usage, which I shall designate \( \text{under}^2 \), \( \text{under} \) acts as a synonym for below, and the substitution can usually be made transparently. These usages may be confined to situations in which the upper entity is very long relative to the lower one and completely overlaps with it. It follows that any change in the lateral location of the lower one will not affect the amount of overlap, and there is no information contained in the preposition about the lateral variable. In contrast, where both relata have a limited extension in the \( XY \)-plane, \( \text{under}^2 \) is responsive to these dimensions. We can use this fact to explore the properties of the second and third dimensions of spatial language and the relations between these and the \( Z \)-dimension. Consider sentence (19) and related figure 7.6:

(19) Stick \( A \) was under the table, but stick \( B \) was even farther under it.

I read sentence (19) to mean that both sticks \( A \) and \( B \) and the table (top) have projections onto the \( XY \)-plane and these projections overlap, that is, have locations in common. Further, the magnitude of some aspect of the projection of \( B \) onto the table is greater than that of \( A \). In general, this magnitude will be a length along some vector (e.g., \( Y \) in figure 7.6) measured from the edge of the table to thefarthest edge
of the object projection. Furthermore, any differences in the projections of the objects in the Z-direction are irrelevant. Thus

(20) Box A was farther below the shelf than box B and farther under it.

Applying the comparative test to the preposition under\(^2\) reveals that the metric is the same as that for the \(-Z\)-direction, that is, an interval scale.

(21) Chair A was as far under the table as chair B.

Note that this sentence can be used even when the chairs are at right angles to each other, in which case each distance is measured from the edge of the table intersected by the chair. The sentence also confirms that both measurements are on an interval scale and that the same metric applies to each. This conclusion is strengthened by the fact that it makes sense to say

(22) Chair A was as far under the table as it was below it.

This last sentence also suggests that the meaning of under\(^2\) in the XY-plane is a distance and not an area. Evidence for this can be gained by imagining the same or different objects of different projection sizes and exploring the meaning of

(23) A farther under than B,

as these objects are positioned in different ways under a constant-size table (see figure 7.7). Figure 7.7 shows that the judgment of which objects are more under (or more under\(^2\)) does not depend on the relative proportion of the length that intersects with the reference object (B more under than A); the orientation of the objects need not necessarily be the same because the relevant length is taken from the intersection of the object with the edge of the table or from the nearest edge (C is as far under as B).

My claim that A more under\(^2\) refers to the absolute length of A might appear to be contradicted by sentences such as

(24) Mary got more under the umbrella than Jane and thus got less wet.

This clearly implies that Mary got more of herself (i.e., a greater proportion) under the umbrella. In this usage, however, it is clear that “more” modifies “Mary” rather than “under,” and does not constitute a refutation of the present proposal.

Finally, D more under\(^2\) than C in figure 7.7 suggests that when an object has two dimensions either of which could be taken into consideration, the distance under\(^2\) is taken from the longer length. It is interesting to note that, unlike the antonyms up (for down) and above (for below), over does not show complete symmetry with under\(^2\). In some subtle sense, the table is less over the chair than the chair is under\(^2\) the table. This slight asymmetry appears not to relate so much to size as to relative mobility. Consider (25) and (26):
Figure 7.7
The relationship *more under* is determined by the total length of the overlap between the two objects in the *XY*-plane and not by the proportion of the total object which is *under* (*B > A*), or the orientation of the object (*C > A*). When two objects differ in more than one dimension, *farther under* is determined by the largest dimension of each and not by the total area (*D > C*).

(25) The red car was under the street lamp.

(26) The street lamp was over the red car.

Sentence (26) is not incorrect, but less likely in most contexts. The reason for this, at least in part, may be that the places in the cognitive map are specified primarily by the invariant features of an environment and only secondarily and transiently by objects which occupy them.

7.2.4 *Beneath* (or *Underneath*)

*Beneath* (or *underneath*) has a meaning that is close to that of *under* but differs in two ways. First, it has a more restricted sense in the *XY*-plane. Whereas *under* means an overlap between the projections of the reference entity and the target entity, *beneath* means that the target entity is wholly contained within the limits of the reference entity projection. It follows that the projection of the lower entity in the *XY*-plane must be smaller than the upper. Furthermore, and in part as a consequence of this restriction, the application of the comparator *more* (or *farther*) to *beneath* operates on the *Z*-direction and not on the *XY*-plane.
(27) The red tray was farther beneath the top of the stack than the blue one.  

_Beneath_ then means that the target element is contained within the volume of space defined by its _XY_-projection through a large (or infinite) distance in the _−Z_-direction. _Underneath_ seems to have a slightly more restricted meaning in the sense of limiting the projection in the _−Z_-direction. _More underneath_ sounds less acceptable than _more beneath_ and might indicate that _underneath_ is a three-dimensional volume of space restricted to the immediate proximity of the _−Z_ or _under_ surface of the reference element.

### 7.3 Distance Prepositions

Distances are given by the preposition _for_ and the adverbials _near_ (to) and its antonym _far_ (from) as in (28) and (29).

(28) This road goes on for three miles.

(29) The house was near (far from) the lake.

_For_ gives the length of a path; _near_ and _far from_ give relative distances that are contextually dependent. In some cases, one or more of the contextual referents have been omitted. Let us begin by examining the meaning of _near_ when points are being related. O'Keefe and Nadel (1978, 8) observed that the meaning of _near_ was context-dependent, and I will pursue that line here. It follows that, with only two points, neither is _near_ (or _far from_) the other. Three points, _A, B, and C, provide the necessary and sufficient condition for use of the comparatives _nearer_ and _farther_. Note that the directions of the points from each other are not confined to the same dimension but are free to vary across all three dimensions, and that the distance is measured along the geodesic line determined by the Euclidean metric. _Near_ is not simply derived from _nearer_ but contains in addition a sense of the proportional distances among the items in question.

(30) _A_ is not near _B_ but it is nearer to _B_ than _C_ is.

The distance measure incorporated in _near_ seems to be calibrated relative to distances between the items with the smallest and largest Euclidean distance separation in the set. These items act as anchor points that control the meaning of the terms for all the others. Changing the relations of other items in the set can alter whether two items are near to or far from each other. Thus, in figure 7.8a, _B_ and _E_ are near each other, but in figure 7.8b, they are not.

Consideration of the _near/far_ relationship of two- or three-dimensional entities shows it is the surface points that are important and not any other aspect of their
Nearness is context-dependent. In (a) $A$ is not near $B$ but nearer than $C$. $E$ is near $B$ in (a) but not in (b). In (c), $B$ is nearer $A$ than $C$ is by virtue of point $x$.

shape (e.g., centroid) or mass (center of gravity). If we inspect figure 7.8c and ask which is nearer to $A$, shape $B$ or shape $C$, we will see that $B$ is, by virtue of point $x$. Finally, the presence of barriers seems not to influence our judgment of near or far, because (31) is permissible.

(31) The house is nearby, but it will take a long time to get there since we have to go the long way around.

7.4 Vertical Prepositions: Reprise

These considerations of the meanings of the vertical prepositions suggest the following conclusions:
1. Prepositions identify relationships between places, directions, and distances, or combinations of these. Static locative prepositions relate two entities; static directional prepositions relate three entities because there is always an (often implied) origin of the directional vector; and static distance prepositions also relate three entities because this is the minimum required to give substance to the comparative judgment that they imply.

2. The space mapped by the prepositions is at least two-dimensional and rectilinear in the vertical direction. The nonvertical dimension (if present) may be rectilinear, but there are also circumstances in which the two nonvertical dimensions may be expressed in polar (or other) coordinates.

3. The metric of vertical and nonvertical axes is identical because it is possible to compare distances along orthogonal axes. Interestingly, the distance between objects is calculated from the nearest surface of each entity and not from some alternative derived location such as the geometric centroid or center of mass.

4. The scale is an interval scale with a relative origin determined by one of the reference entities of the directional prepositions (usually the vector source or tail).

5. In the vertical dimension, direction can be given by the universal gravity signal, which is constant regardless of location. In the horizontal plane, nothing comparable to this signal is available and the direction vectors must be computed from the relative positions of environmental cues.

7.5 Horizontal Prepositions

The original cognitive map theory suggested that, in the horizontal plane, places could be located in several ways. Foremost among these was their relation to other places as determined by vectors that coded for distance and direction (figure 7.1). In a recent paper (O'Keefe 1990) I have suggested that the direction component of this vector is carried by the head direction cells of the postsubiculum. These cells are selective for facing in specific directions relative to the environmental frame, irrespective of the animal's location in that environment. The direction vector originating in one place or entity and running through a second can be computed by vector subtraction (see figure 7.9) of the two vectors from the observer to each of the entities, and this computation is independent of the observer's location. The resultant direction vector functions in the same way in the horizontal plane as the gravitational signal in the vertical direction. The primary difference is that, whereas the latter is a universal signal, the horizontal direction vectors are local and need to be coordinated relative to each other. This is achieved by mapping them onto the global directional system. Locative horizontal prepositions, in common with their vertical cousins, specify places in terms of directions and distances. The directions are given relative to the
The direction vector through two objects $A$ and $B$ can be computed by taking the difference between the vectors $A$ and $B$.

direction vector, and distances are given relative to the length of a standard vector drawn between the two reference entities along the reference direction.

7.5.1 Beyond
Let us begin with an analysis of the spatial meaning of the preposition beyond. As shown on the left side of figure 7.10, this specifies a three-dimensional region located by the set of vectors with a specific relationship to the reference direction and a pair of reference vectors ($\overrightarrow{AB}, \overrightarrow{AC}$) terminating on different parts of the reference object or place. The region beyond the mound is specified by the set of vectors originating at $A$ whose projection onto the direction vector (inner product) has a greater length than the larger of the two reference vectors coincident with the direction vector ($\overrightarrow{AC}$). According to this definition, it acts in a manner analogous to below in the vertical dimension. No restriction is placed on the location of the entity in the vertical direction, as can be seen from sentence (32):

(32) Jane camped beyond and above the woods.

Furthermore, the effect of the comparator more is to act on the length of the vector in the horizontal plane:

(33) The tower was farther beyond the mound than the castle.
Figure 7.10

Beyond, behind, and beside can be represented as places determined by their relation to the direction vector drawn through two reference entities and a set of reference vectors (\(\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}\)). Beyond is the set of all places with a length greater than \(AC\). Behind is a restricted subset of beyond and includes only the places with location vectors greater than \(AC\) and angle with the direction vector smaller than \(AD\). Beside represents those places having a projection onto the reference direction of magnitude greater than \(AB\) and less than \(AC\). In addition the angle with the direction vector must exceed that of \(AD\).

The opposite of beyond is the seldom-used behither, and this simply means that the location vector has a length less than the reference vector \(\overrightarrow{AB}\).

7.5.2 Behind

Behind functions in a manner analogous to under in that it places greater restrictions on location than does beyond. An object behind a reference entity is located by the set of vectors with a larger magnitude than the reference vector (\(\overrightarrow{AC}\)) but with an angle less than vector \(\overrightarrow{AD}\) (figure 7.10, center). As with under, an entity can be partially behind the reference entity, and the test for this is an overlap in the projections of the two in the \(XZ\)-plane. This need for overlap accounts for the awkwardness in using behind with referents that are not extended in the vertical dimension.

(34) ?The tree was behind the trench.

(35) ?The cottage was behind the lake.

The application of the comparator test shows further similarities. In the same way that farther under can refer to the amount of overlap in the \(XY\)-plane between two
entities separated in the vertical dimension, so farther behind can refer to greater overlap in the XZ-plane of entities separated along a horizontal reference direction.

(36) The red toy was pushed farther behind the box than the blue ball.

The source of the direction vector can be specified explicitly as the object of the preposition from.

(37) From where Jane stood, James was hidden behind the boulder.

More usually, the source is implicit, being inferable from the previous context. In sentence (37), for example, it would be legitimate to omit the first clause if the previous narrative had established that Jane had been looking for James. More often, the source of the direction vector is the implicit deictic here. In a pool game it might be the cue ball:

(38) The last red was behind the eight ball.

Familiar objects have “natural” behinds established by a vector drawn from one differential part to another, as, for example, the front to the back of a car. However, this is easily overridden by the motion of the vehicle:

(39) The car careered backward down the hill, scattering pedestrians in front of it and leaving a trail of destruction behind it.

The opposite of behind is before, or more usually in front of.

7.5.3 Beside

Beside identifies a region at the end of the set of vectors whose projections onto the reference direction fall between the reference vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ but whose angle with the reference direction is greater than that of reference vector $\overrightarrow{AD}$ (figure 7.10, right).

7.5.4 By

By is the generalized horizontal preposition and includes the meanings of before, behind, beyond, and beside with a slight preference for the latter.

7.6 Omnidirectional Prepositions

At, about, around, between, among (amid), along, across, opposite, against, from, to, via, and through locate entities in terms of their relationships to other entities irrespective of their direction in a coordinate reference framework and therefore can be used in any of the three directions. At is the general one-to-one substitution operator that locates the entity in the same place as the reference entity. About relaxes the precision
of the localization and introduces a small uncertainty into the substitution. *About* is equivalent to *at* plus contiguous places. In the cognitive map theory the size of the place fields is a function of the overall environment, and this would appear to apply to *about* as well. Therefore the area covered by *about* is relative to the distribution of the other distances in the set under consideration in the same way that the meaning of *near* depends on the distribution of the entities within the set. *Around* has at least two distinct meanings, both related to the underlying figure of a circle (i.e., the set of vectors of a constant *R* originating at an entity) with the reference entity at its center. The first meaning is that the located entity is somewhere on that circle. If it is extended, it lies on several contiguous places along the circle; if more compact, it lies at one place on the circle perhaps at the end of an arc of the circle.

(40) The shop was around the corner.

Because in almost all instances the radius of the circle is left undefined, except that it be small relative to the average interentity distances of the other members of the set, there is little to choose between the use of *about* and *around* when single entities are located. When multiple entities are located, there is the weak presumption that they all lie on the same circle when *around* is used, but not when *about* is used.

(41) Those who could not fit around the table sat scattered about the room.

*Between* locates the entity on the geodesic connecting the two reference entities. The computation is the same as that for deriving a direction vector from the subtraction of two entity vectors (see above discussion in section 7.5), except that the order in which these are taken is ignored. An equivalent definition of *between* is that the sum of the distances from each of the reference entities to the target entity is not greater than the distance between the two reference entities. Alternatively, the angle made by the vectors joining the target to each of the references should be 180°. *Among* increases the number of reference entities to greater than the two of *between*. The interesting issue here, as with many of these prepositions that use multiple reference entities, is how the reference set is defined. *Among* roughly means that the target entity is within some imaginary boundary formed by the lines connecting the outermost items of the set. But clearly the membership of the reference set itself is not immediately obvious. Consider a cluster of trees with an individual outlier pine tree some distance from the main group.

(42) He was not among the trees, but stood between the thicket and the lone pine.

This suggests that the application of the preposition *among* depends on a prior clustering operation that is necessary to determine the numbers of the reference set. *Amid* is a stronger version of *among* that conveys the sense of a location near to the center
of the reference entities. One possibility is that the centroid or geometrical center of the cluster is computed, and amid denotes a location not too far from this. The centroid is a central concept in one computational version of the cognitive map theory (O'Keefe 1990).

Across, along, and opposite are like down in that they situate an entity in terms of its relationship to a reference entity and a one- or two-dimensional feature. Two-dimensional features are usually more extended in one direction than the other. Across specifies that the vector from the reference entity to the target intersects the reference line or plane an odd number of times. Along specifies an even number (including 0) of intersections. In addition, there is the weak presumption that the distance from the target entity to the last intersection is roughly the same as from the reference entity to the first intersection; that is, both are roughly the same distance from the reference line or plane. Opposite restricts the number of intersections to one and the intersection angle to 90°.

Against specifies that the entity is in contact with the surface of the reference entity at at least one point. It is, however, not attached to it but is supported independently in the vertical dimension. In the present scheme, from and to mark places at the beginning and end of a path that consists of a set of connected places, and via and through specify some of the places along the way.

(43) Oxford Street goes from Tottenham Court Road to Marble Arch via Bond Street but doesn't pass through Hyde Park.

7.7 Temporal Prepositions and the Fourth Dimension

The incorporation of time into the mapping system is accomplished through various grammatical and lexical features. The primary grammatical features are tense, aspect, and the temporal prepositions. Because my emphasis in this chapter is on the prepositional system, I will mention tense and aspect only in passing (see Comrie 1975, 1976/1985 for detailed discussions).

In the present system, time is represented as a set of vectors along a fourth dimension at right angles to the three spatial ones. Each event is represented as a vector that is oriented with its tail to the left and its head to the right, this constraint being due to the fact that changes in time can take place in only one direction (from past to future). The location of these time events is also based on vectors and these can be oriented in either direction from a reference point, which can be the present moment of the utterance or some other time. Times future to the reference point have vectors of positive length, times past have vectors of negative length, and the present, a vector of 0 length. These different times are represented by the tenses of the verb.
The choice of the present time as a 0 reference point is traditionally called "absolute tense" while that of a nonpresent reference point, "relative tense" (see Comrie 1985 for further discussion). Because the vectors representing time are all unidimensional, lying parallel to the fourth axis, we will expect that the senses of the temporal prepositions are also unidirectional. For example, most of the temporal prepositions are similar to (diachronically borrowed from?) their homophonic spatial counterparts, but not all spatial prepositions can be so employed. The general rule seems to be that only spatial prepositions that can operate in the single, nonvertical dimension of the line can be borrowed in this way (but see the special cases around and about). As we shall see, this leaves the nonphysical vertical prepositions free to represent specialized relationships between entities.

The temporal prepositions, then, specify the location, order, and direction within the fourth dimension of the entities and events of the other three dimensions. In my brief summary I will classify them according to whether they use one or more reference points. Because the temporal dimension appears to be confined to a single axis orthogonal to the spatial axes, in the latter cases the two references are confined to that axis and are therefore collinear. My discussion of the meanings of the temporal prepositions will be based on the abstract events portrayed in figure 7.11. The upper event shows a state of affairs in which an entity occupies a vertical location before time $A$, then jumps to a new location and remains there for a short period $AB$, after which it returns to the previous location. The lower event shows a process of movement over a period of time. Let us use the sentences 44 and 45 as examples of the process $CD$ and the state $AB$, respectively.

(44) Mary moved from an apartment on the top floor to one on the floor beneath.

(45) Sarah, Mary's roommate, dropped down to tidy up the new apartment for an hour during the move.

The projection of these sequences of events onto the time axis is shown at the bottom of the figure. The punctate events $A$ and $B$, the beginning and end of the dropping down, are marked as points on the time axis. These points can be located in three ways. First, they can be placed in isolation independently of any other representation, as might occur at the beginning of a story. Second, they can be related to the present time of the speaker/listener or, third, to some other previously identified time. In these latter instances, the location vector is drawn with the tail at the reference point and the head at the located time, that is, from right to left (with a negative magnitude) if the event occurred prior to the reference point, and from left to right (with a positive magnitude) if it occurred later than the reference point.

The events themselves are states (dropping down) or processes (Mary's move) and are represented as vectors that must move from left to right (no time reversal). The
The Spatial Prepositions

![Diagram showing spatial prepositions and time.]

**Figure 7.11**
Temporal prepositions as relationships in a fourth dimension. An event such as "Sarah dropped down" is represented by a physical movement on the $Z$-axis that begins at time $A$, ends at time $B$, and is represented by vector $AB$ on the time axis. A process such as "Mary moved" has a similar representation on the time axis. The representation assumes that the events occurred in the past, but other 0 reference points could have been adopted.

three events of the top sequence (the dropping down and the presuppositions of being in and returning to the upstairs apartment, are represented on the $T$-axis by vectors $AB$, $TA$, and $BT$, respectively. The tail of the second and head of the third are left indeterminate. Here I am assuming that all events have some projection in the time domain, but that this can be ignored, for example, when the length of the event vector is short in comparison to the length of the location vectors.

The process of moving represented by vector $CD$ has a similar representation on the time line, the difference between a state and a process residing in changes in the nontime dimensions.

Referring to figure 7.11, I suggest that the meaning of the temporal prepositions is as follows. The usual representation of a process such as $CD$ is

(46) The move took place from noon to 2 p.m.

The event $CD$ has a time vector which begins at $T_C$ (noon) and ends at $T_D$ (2 p.m.). $T(CD) = T_D - T_C$, where $D$ and $C$ are the respective location vectors.
(47) The move lasted for two hours
sets the length of vector $\vec{CD}$.

(48) Sarah dropped down after Mary began moving, before Mary finished moving,
by the end of the move
sets $T_A > T_C$, $T_A < T_D$, $T_A \leq T_D$.

(49) Sarah visited the new apartment during the move
sets $T_C < T_A \leq T_B < T_D$.

Since and until are two temporal prepositions that do not have spatial homologues. Until specifies the time at which a state or process ended, whereas since specifies the time at which it began. Since has the additional restriction that the temporal reference point acting as the source of the location vectors for the event in question must be later than the event, that is, the location vectors must have negative magnitudes. This is to account for the acceptability of (50) but not (51).

(50) Mary has (had) been moving since noon.

(51) By 2 p.m. tomorrow Mary will have been moving since noon.

The simple temporals at, by, in locate an entity by reference to a single place on the fourth axis. At operates in the same way as it does in the spatial domain by substituting the place of the referent for the entity. By fixes the location of the reference point as the maximum of a set of possible places. In suggests that there is an extent of time that is considered as the referent and that contains the entity. On is somewhat more difficult; it would seem to introduce the notion of a second temporal dimension, a vertical dimension that would place the entity at a location above or alongside of the time point. About and around also suggest a second dimension. In general, however, the temporal use of on seems to be restricted to the days of the week (on Friday) and to dates (on the first of April) and is not used in any general sense. It may therefore be an idiosyncratic use to distinguish these from the pointlike hours of the day (at 5 o'clock) on the one hand and the extended months of the year (in May).

Other simple temporal prepositions give the location of the event or duration of the condition by reference to a time marker that fixes the beginning or end of the time vector. Whereas by and to set the head of the temporal vector at the reference place, before sets it to the first place to the left of that place. In neither case is the origin or tail of the vector specified. This is given as the object of from. During specifies both the head and tail of the temporal vector. An event that occurs after one time and before another occurs during the interval. The length of the vector is given by the preposition for.
The Spatial Prepositions

As with the spatial prepositions, some of the temporal prepositions require two reference points for their meaning. These include *between*, *beyond*, *past*, *since*, and *until*. *Between* two times locates the start of the event later than the first time and the end of the event before the second. The referent in *beyond* denotes the value that the head of an event vector exceeds. Because the time axis is basically a unidimensional one, the important distinction between *past* and *beyond* in the location of the entity in the orthogonal axis of the spatial domain does not apply, and the two prepositions appear to be interchangeable in most expressions.

7.8 Translation and Transformation Vectors

Once one has a temporal framework, it is possible to incorporate the notion of changes into the semantic map. These take two forms: changes in location and changes in state. The second of these relates to the circumstantial mode of Gruber (1976) and Jackendoff (1976). Both changes are represented by vectors. Changes in location of an object are represented by a vector whose tail originates at the object in a place at a particular time and ends at the same object in a different place at a subsequent time. Changes in state are represented by a vector drawn from an object at time $t$ to itself in the same location at time $t + 1$. The change is encoded in the attributes of the object. In both types of change, the origin or tail of the vector is the object of the locative preposition *from*, and the head or terminus of the vector is the location identified by the locative preposition *to*.

(52) The icicle fell from the roof to the garden.

The representation of this is shown in figure 7.12. It consists of a four-dimensional structure with time as the fourth dimension. In the figure, I have shown two spatial dimensions and one temporal dimension. The left side of the representation shows the unstated presupposition that the icicle was on the roof for some unstated time prior to the event of the sentence. As Nadel and I noted (O'Keefe and Nadel 1978), the relationship between an object and its location is read as

(53) a. The icicle was on the roof (before time $t$).
   b. The roof had an icicle on it.

The middle of the figure shows the translation vector that represents the event of the sentence, and the right hand the postsupposition that the icicle continues in the garden for some duration after the event.

(53) c. The icicle was in the garden (after time $t$).

The representation of the second type of change, the circumstantial change, also involves a vector, this time a transformation vector, where there is no change in the
Figure 7.12
Change in location of an object in the semantic map at a particular time $t$ is represented by a translation vector. In addition to the time axis, one spatial axis ($Z$) is shown. The four-dimensional object, labeled "icicle," is shown on the place labeled "roof" at all times prior to $t$ ($t^-$) and in the place labeled "garden" at all times after $t$ ($t^+$). The vertical movement between the two places at $t$ is represented by a translation vector drawn between the two places.

location of the object, but a change in one of the attributes assigned to the object. Objects are formed from the collection of inputs that occupy the same location in the map and that translocate as a bundle (see O'Keefe 1994 for a discussion of this Kantian notion of the relationship between objects and spatial frameworks). Thus each object has associated with it a list of attributes. In a circumstantial change, a vector represents the change in one of these attributes at a time $t$. Figure 7.13 shows the map representation of sentence 54.

(54) The icicle melted (= changed from hard to soft at time $t$, or changed from solid to liquid).

7.9 Metaphorical Uses of Vertical Prepositions

In the following sections, I shall explore the metaphorical uses of the vertical stative prepositions. I hope to show that they apply to two restricted domains: influence (including social influence) and social status. In the course of this discussion I shall ask some of the same questions about these metaphorical uses as I did for their physical uses: what are the properties of the spaces represented, what type of scale is used, and so on?

Section 7.9.1 will explore the metaphorical meanings of below and beneath as used within the restricted domain of social status. Section 7.9.2 will deal with under, whose
The Spatial Prepositions

TRANSFORMATION VECTOR

\[
\begin{array}{c}
\text{ICICLE} \\
\text{long} \\
\text{cold} \\
\text{solid} \\
\end{array} \rightarrow
\begin{array}{c}
\text{ICICLE} \\
\text{long} \\
\text{cold} \\
\text{liquid} \\
\end{array}
\]

\[
\text{ROOF}
\]

\[ t^- \quad t \quad t^+ \]

Figure 7.13
Changes in state of an object in the semantic map are represented by a transformation vector whose tail originates in the old property before \( t \) and whose head ends in the new property after \( t \).

semantics is more complex, but appears to be restricted to the domain of influence or control. In general, the representation of ideas such as causation, force and influence in the semantic map presents a problem. The basic mapping system appears to be a kinematic one which does not represent force relations. The closest one comes in the physical domain is the implicit notions that an entity which is vertical to another and in contact with it might exert a gravitational force on it or that an entity inside another might be confined by it. This might explain why the prepositions that convey these relationships, such as \textit{under} and \textit{in}, are used to represent influence in the metaphorical domain.

7.9.1 \textbf{Below, Beneath, and Down}
Contrast the following legitimate and illegitimate metaphorical uses of \textit{below} and \textit{under}:

(55) She was acting below (beneath) her station.
(56) She was acting under his orders.
(57) *She was acting under her station.
(58) *She was acting below his orders.

When looking at \textit{below} and \textit{beneath} within the domain of social status, the first thing to notice is that people are ranked or ordered in terms of their social status on a vertical scale. One person has a higher or lower status than another, and that status would appear to be transitive: if \( A \) has a higher status than \( B \) and \( B \) than \( C \), it follows
that $A$ has a higher status than $C$. I am ignoring here the possibility that status might be context-specific because I do not think this is reflected in the semantics of the prepositions. Now within the vertical scale of status, one can have a disparity between the value assigned to an individual act and the longer-term status. This gives rise to sentences such as

(59) John acted in a manner beneath him.

(60) That remark was below you.

A sequence of such actions, however, will result in a status change, so that

(61) Until recently that remark would have been beneath you, but now it is quite
in character.

The antonym of below/beneath in this context is above, although it is not much used.

(62) Sally was getting above her station,
but not

(63) *That remark was above you.

The use of below and beneath in this sense is restricted to reflexive status, and thus
one could not say

(64) John acted in a way beneath Sally (Sally’s station).

Thus the best model (see figure 7.14) seems to be one in which each status token is
confined to a vertical line in the status dimension, but these are free to vary in the
other dimensions such that John can move so as to be beneath himself but not
beneath Sally, but at the same time can be compared in the vertical dimension with
Sally, “His status is below hers.” Finally, note that there is no vantage point (egoce-
tric point) from which these judgments are made or which would change them (i.e.,
the speaker’s status is not relevant).

The stative preposition down seems to have almost no use in the nonphysical sense.
The closest one comes are colloquial forms of verbal ranking such as

(65) Put him down.

7.9.2 Under

Under has perhaps the most interesting use of the vertical prepositions in the meta-
phorical domain. It seems to be confined to the domain of influence or control. In The
Hippocampus as a Cognitive Map (1978), Nadel and I suggested that one of the
metaphorical domains would be that of influence. Here I will pursue the idea that this
relationship is represented by an additional “vertical” dimension (figure 7.15).
Figure 7.14
Social status is conveyed by the metaphorical use of *below* and is represented as a location on the metaphorical vertical axis.

Figure 7.15
Influence of one entity, usually an agent, over another entity or an event is represented by a superior location of the first on the vertical influence axis.
There are two homophones \((\text{under}^1 \text{ and } \text{under}^2)\), which follow different rules and which are derived from the two meanings in the physical domain:

(66) under a widening sky

(67) under the table

Compare

(68) Under the aegis of

with (66), and

(69) a. under John’s influence

b. under Sally’s control

with (67).

The first meaning of \textit{under} cannot take a comparative form.

(70) *More under the aegis of the King

is not transitive, and has no antonym.

(71) *He was above, outside of, free from the aegis of the King.

In contrast, the second meaning follows all the rules for the second physical \textit{under}^2.

(72) More under her influence every day.

But surprisingly the antonym of this \textit{under}^2 is not \textit{over} in many examples, but varies with the direct object.

(73) She was free from stress.

(74) The car was out of control.

(75) He was out from under the control of his boss.

As the last examples suggest, the referent in this meaning of \textit{under} has an extent in the vertical dimension, and to be more \textit{under} a cloud than \(X\) has the same sense of a greater overlap in the projection onto (one or more) horizontal dimension as in the physical meaning. To increase or decrease this influence requires a movement or expansion of one or the other entity in the horizontal plane, and this may require force in that direction.

(76) John was more under control than Sam.

(77) John was more under the influence of Mary than Sam.

(78) She slowly extricated Sam from Harry’s influence.
There are two types of relationships that conform to this pattern, control and influence, and these vary in the amount of freedom left to the referent object.

(79) Jane increased her influence over Harry until she had complete control.

The antonym of under\(^2\) is over.

(80) Jane's influence over John

(81) Jane lords it over John.

(82) Jane holds sway over John.

(83) a. *The King's aegis was over John.
     b. *The King held his aegis over John.

Notice that the under relationship is not transitive. John can be under Jane's influence and Jane can be under Joe's, but John is not necessarily under Joe's.

Finally, I wish to remark briefly on the fact that there appear to be two nonphysical vertical dimensions that are orthogonal to each other and to the physical vertical one. On the face of it, it does not seem obvious how they could be reduced to a single dimension because one wishes to preserve the possibility of the following types of relationship.

(84) Jack felt it necessary to act below his station in order to maintain control over Jane.

Perhaps here one should consider the possibility that overlapping representations symbolize a control or influence relationship while nonoverlapping ones stand for a status one in the same 2-D space. If this were the case, what would the Z-axis be? Perhaps the higher the status, the more possibility for control?

Finally, in terms of the scaling of the metaphorical vertical prepositions, they appear to have the same interval scale as their physical counterparts. Thus one can say:

(85) Jane is as far below Mary in status as John is above

(86) John is less under Sam's control than Jim is

and it will be easier to extricate John.

Note that, unlike the three dimensions of physical space, we cannot compare the Z-axis and the non-Z-axis directly.

(87) *John is more under Sam's control than Sam acted below himself.

Now we come to the most difficult part of the theory: the relationship between control and causation. Causation, on this reading, would be the occurrence of an event underneath the control of an agent's influence.
(88) The book went to the library.

(89) John caused the book to go to the library.

7.10 Causal Relations in the Semantic Map

Our analysis of the metaphorical use of below and under has led to the suggestion that the causal influence of one item in the map over another might be represented by relationships in the fifth dimension. If the influence of an agent over another agent or object can be represented by the location of the first above the second, then it might be possible to represent the influence of an agent over an event such as that portrayed in (90) an (91) by an action or movement along the influence dimension. Consider the closely related sentences:

(90) Mary made (caused) the icicle fall from the roof to the garden.

(91) Mary let (did not prevent) the icicle fall from the roof to the garden.

According to the present analysis, these are five-dimensional sentences, which differ in the control exerted by the agent over the event. As we saw in the previous section, influence is represented by an under\(^3\) relationship between the influencer and the influenced. The lateral overhang between the two represents the amount of control exerted, and the distance between them on the vertical dimension, the amount of influence exerted. On the simplest reading, causation is represented as a pulsatile increase in influence coincident with the physical spatial event. Figure 7.16 shows this as a momentary increase in Mary's influence to symbolize an active role in the event, while figure 7.17 shows a continuing influence but no change to symbolize a passive role in the event. The sentence

(92) Mary did not cause \(X\)

is ambiguous, with two possible underlying structures: one in which Mary has influence but the event did not happen; and the other in which the event did happen but the causal influence was not exerted by Mary. This type of representation can also capture some of the more subtle features of causal influence, because it can show how influence can selectively act on parts of the event as well as on the whole. For example, the sentence

(93) Mary made John throw down the icicle

means that both Mary and John had agentive roles in the event, but that Mary's was the superior one. This can be represented by placing Mary at a higher level than John in influence space and showing momentary synchronous changes in their locations at the time of the event. The complex influence relationship also allows for the following sentences:
Figure 7.16
Causal influence is represented by a pulsatile change in the vertical influence dimension at the same time \( t \) as the physical event.

(94) Mary allowed John to throw down the icicle.
(95) Mary allowed John to drop the icicle.
(96) Mary made John drop the icicle.

It also permits one to represent relative degrees of influence over an event in a manner analogous to that over agents or objects, as in

(97) Mary had more influence over the course of events than John,

or the idea that an event of continuing duration can have variable amounts of control at different times,

(98) Mary took over control of the event from John on Monday.

7.11 Syntactic Structures in Vector Grammar

Thus far, I have said very little about the way that surface sentences and paragraphs could be generated from the static semantic map. Nadel and I (O'Keefe and Nadel
Figure 7.17
Permissive influence is represented by the absence of change in the vertical influence dimension of the influencer during the event.

1978) likened this operation to the way in which an infinite number of routes between two places could be read off a map. Recall that the cognitive map system in animals includes a mechanism for reading information from the map as well as for writing information into the map. In particular, we postulated a system that extracts the distance and direction from the current location to the desired destination. This information can be sent to the motor programming circuits of the brain to generate spatial behaviors. The corresponding system in the semantic map would comprise the syntactic rules of the grammar. The syntactic rules operate on both the categories of the deep structures and the direction and order in which they are read. For example, reading the relationship between an influencer and the object or event influenced determines whether the active or passive voice will be used. In an important sense there are no transformation rules for reordering the elements of sentences because these are read directly from the deep structure. Given a particular semantic map, a large number of narrative strings can be generated depending on the point of entry and the subsequent route through the map. Economy of expression is analogous to the optimal solution to the traveling salesman problem.
The Spatial Prepositions

Acknowledgments

I would like to thank Miss Maureen Cartwright for her extensive help and substantive contributions to this chapter. Neil Burgess made comments on an earlier version. The experimental research that forms the basis for the cognitive map model was supported by the Medical Research Council of Britain.

Notes

1. I have deliberately chosen the term *entities* to refer to the relationships because I do not wish to limit my discussion to objects, but wish to include places, features, and so on.

2. In what follows, I have relied heavily on the classic discussion by Torgerson (1958).

3. I am assuming the geomagnetic sense is absent or so weak in humans that it is not available for spatial coding. As far as I am aware, there is no evidence for it in the prepositional system of any language.

References


