

1                   **Periodical oscillation of particle-laden laminar flow within a tubular**  
2                   **photocatalytic hydrogen production reactor predicted by DEM simulation**

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15                   **Abstract**

16 Besides their wide existence in various industrial processes, nanoscale particle suspensions are  
17 also the important media for some emerging technologies such as photocatalytic hydrogen  
18 production. The circulating flow properties of the nanoparticles in the fluid are of great concern  
19 for their practical use. In our study, a modified experimental system was set up based on Malvern  
20 laser particle analyzer that can estimate the nanoparticle concentration and size distribution in a  
21 laminar nanoparticle circulating flow. We found that the particle concentration and size  
22 distribution were periodical oscillation with time in such flow. Understanding the oscillation  
23 mechanism is capable of promote the energy efficiency of photocatalytic hydrogen production. A  
24 simulation based on Discrete Element Method (DEM) was conducted to understand this particular  
25 oscillation mechanism by counting the single particle movement and trajectory properties in the  
26 solid-liquid suspension. The simulation results agree well with the tendency obtained by the  
27 experimental results and capable of better understanding the oscillation characteristics. The  
28 simulation results reveal that the nanoparticles tend to gather in the middle region (the higher  
29 velocity region) of the tube after several cycles. Moreover the gravity is of great significance in  
30 the circulating flow of solid-liquid suspension because the particle swarms tend to distribute a  
31 little below the axial center line of the straight tube. These obtained results are credible for  
32 understanding the nanoscale particle transport phenomenon in many natural or industrial processes.  
33 In particular, our results are helpful for the understanding and effective control of the movement

34 and distribution of photocatalyst particles in the tubular photocatalytic reactor, which is believed  
35 to significantly affect the incident radiation distribution and finally the energy conversion  
36 efficiency of the photocatalytic process.

37 **Key Words:** particle-laden flow, DEM simulation, periodical oscillation, particles trajectory

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### 39 **1. Introduction**

40 The suspensions of micro- or nanoscale particles in liquid exist widely in various fields, e.g.  
41 advanced material processing, electronic technology, chemical engineering, petroleum, food  
42 processing, waste treatment etc. It is also the important media for some emerging technologies  
43 such as the enhancement of heat transfer, photocatalytic energy conversion and environmental  
44 remediation [1-4]. In all of these processes, the particle transportation and deposition have  
45 significant impact on the properties application promotion of such kind of suspension. For  
46 example, in photocatalytic process, sedimentation of the photocatalyst which leading to ineffective  
47 light absorption must be avoided [5]. For another example, in nanofluids heat transfer  
48 enhancement, the deposition of nanoparticles will change the wettability of the heater surface, thus  
49 affecting the overall heat transfer performance [6].

50 Photocatalytic hydrogen production from water splitting using solar energy is one of the  
51 ultimate reactions to solve energy and environmental issues [7]. In order to realize the industrial  
52 application of photocatalytic hydrogen production technology, a large number of studies have been  
53 carried out in the past twenty years. And there are two main research fields, one is the exploration  
54 for efficient visible light driven photocatalyst [5,8-12], the other one is the development of high  
55 efficiency photocatalytic reactors [13-17]. As the photocatalyst has been studied for many years,  
56 and the quantum efficiency of the photocatalyst has reached 93% with noble metal loading [18]  
57 and reached 62% without noble metal loading[19], whereas the energy conversion efficiency is  
58 still not able to exceed 5% in present [20,21]. It is believed that the great difference between the  
59 above two efficiencies is due to the inefficient utilization of the light in the photocatalytic reactor.  
60 And the suspension state and transport phenomenon of photocatalytic particles in the reactor has  
61 significant impact on the absorption of light; as a result, determine the overall energy conversion  
62 efficiency of the photocatalytic reaction.

63 For the clearer understanding of the transportation phenomenon in nanoparticle flow, many

64 studies have been conducted by means of experimental and/or numerical method. In experimental  
65 study, one mainly concerns about the concentration distribution, velocity distribution and particle  
66 size distribution, and the experimental methods mainly include intrusive and non-intrusive method.  
67 Sampling method[22] and probe method[23-25] are two most frequently-used methods in the  
68 intrusive method. Although this kind of method is easy to use and intuitive, it inevitably disturbs  
69 the flow field due to the existence of sampling equipment and probe. To overcome this defect,  
70 some non-intrusive methods have been developed, such as the light technique [26], radiation  
71 technique[27], ultrasonic attenuation techniques [28], tomographic technique[29] etc. Among  
72 these techniques, light technique is the most widely used technique, for it is convenient, safe, and  
73 responsive and has simple structure. Laser diffraction (LD) technique is a kind of light technique  
74 based on the light diffraction by particles, which has become a popular method to measure the  
75 particle size and concentration distribution [30,31], due to its high speed, good reliability and high  
76 reproducibility [32].

77 Although many experimental methods have been developed, they still have some limitations,  
78 such as low accuracy, high cost, disturbance to the flow field etc. Also, Numerical methods have  
79 been employed as a very important supplementary method to give insight into the particle  
80 distribution and flow characteristics in the particle suspensions. In general, simulation of the  
81 solid-liquid suspension can be conducted by either continuum or discrete methods, depending on  
82 the time and length scales employed for solid phase. The former is often based on the two-fluid  
83 model (TFM), whereas the latter is usually based on the discrete element method (DEM). Very  
84 often, the discrete phase method (DPM) coupled with computational fluid dynamics (CFD) have  
85 been used, which gives the DPM-CFD or combined continuum and discrete model (CCDM). TFM  
86 is available to study the fluid mixture movement as a whole and the phase distribution. Hu et al.  
87 [33] investigated the effects of the slurry flow and catalyst distributions in the reactor on  
88 photocatalysis for hydrogen production with an algebraic slip mixture model (ASM). Hatami et al.  
89 [34] studied steady and unsteady magneto-hydrodynamic (MHD) Couette flows between two  
90 parallel infinite plates with a two-fluid model through numerical Differential Quadrature Method  
91 (DQM) and analytical Differential Transformation Method (DTM), respectively. The forced  
92 convection of laminar  $\text{TiO}_2$ -water nanofluid flow in a parallel plate microchannel has been studied  
93 by a modified homogeneous flow model and the dispersion model [35]. Although the TFM model

94 is used widely and suitable to most two-phase flow problems, some detailed particle-scale  
95 information might be missed, such as the trajectories and forces acting on individual particles,  
96 which are very significant to explain the mechanisms governing the complicated flow behavior.  
97 CCDM has thus been increasingly used for studying various solid-liquid flow phenomena [36].  
98 Morris and Brady [37] studied the pressure-driven flow of a non-neutrally buoyant suspension by  
99 the suspension-balance model of NB considering the particles migration which is a DEM model.  
100 Peng et al. [38] investigated the influence of primary particle size distribution (PPSD) on  
101 aggregation behavior and the resulting effect on yield stress of a concentrated colloidal suspension  
102 with DEM model. Chaumeil and Crapper [39] investigated the agglomeration and deposition on a  
103 constricted tube collector of colloidal size particles immersed in a liquid.

104 The literature review above indicate that most of the studies by DEM modeling are conducted  
105 in a one-way channel, which means the particle suspension will pass the channel for only one time  
106 in simulation. Whereas in many photocatalytic reactors, nanoparticle suspensions are in fact  
107 circulated in a loop, considering that such flowing reactor is cost-effective, easy for scale-up and  
108 for nanoparticles recycling [40-44].

109 In particular, sequencing batch reactor (SBR) is commonly used in photocatalytic technology  
110 to decrease the consumption of pump power [16,45]. And particles need to be resuspension after a  
111 period of time when the SBR photocatalytic reactor is in operation, so it is very important to know  
112 the real process of particles spreading with the suspension flow. Thus, a initial condition that  
113 particles added at the beginning was select to simulate the process. And in the circulating flow, the  
114 gravity cannot be neglected considering that the particles in solid-liquid suspension will keep  
115 moving in the reactor under the action of gravity [46]. And the equilibrium position of the  
116 particles after enough times of circulating flow should be of great interest in photocatalytic  
117 application, for it is extremely important to know the location and of particles in the photocatalytic  
118 reactor which has great impact on the light absorption. Therefore, the equilibrium position of these  
119 particles is also discussed in this study. In our previous study, an experimental research is  
120 conducted to investigate the changes of particles concentration and size distribution with time  
121 during the circulating flow. In this study, a DEM two-way coupling model was employed to obtain  
122 the details of particles migration and also the key macroscopic parameters for comparison with  
123 experimental results. This model focused on understanding the periodical oscillation of particle

124 concentration and size distribution with time in such circulating flow. The particle volume fraction,  
125 the flow field of the particle-laden flow in the circulating flow system, the particle trajectory and  
126 the particle size spatial distribution were discussed by this model. This model provides an intuitive  
127 and credible insight on the periodical oscillation mechanism of photocatalyst particles tubular  
128 photocatalytic reactor, and eventually explains incident radiation distribution and the energy  
129 conversion efficiency of the photocatalytic process.

## 130 **2. Experimental set-up and materials**

131 In our previous work, we had established an experimental set-up as has been shown in [Fig 1](#)  
132 [47]. It has six parts: main flow channel, centrifugal pump, wet sample injector, laser particle  
133 analyzer and connecting tube. The main flow channel working as test section in this study is made  
134 from acrylic glass and has an inner diameter of 30 mm and a total length of 0.5 m which is  
135 transparent. Connecting tube is the silicone hose which the inner diameter is 10mm, and the total  
136 length of it is 1.7 m. The centrifugal pump is used to provide driving power of the circulating flow.  
137 A Malvern Spraytec laser particles analyzer was used to monitor particle concentration and  
138 particles size distribution simultaneously. Degussa P25 titanium dioxide was employed as model  
139 particles and main parameters for this widely used photocatalyst, and the specifications of P25  
140 TiO<sub>2</sub> powder can be found in Ref. [48]. Again, it is worth mentioning that photocatalytic energy  
141 conversion is a very attractive technology for which one of the key issues is the well-suspension of  
142 the particles in the reactor [3,49,50]. A very low flow velocity may result in the sedimentation of  
143 particles while a too high flow velocity could lead to a significantly reduced exposure time of the  
144 photocatalyst to solar light. One must guarantee that all the useful incoming photons are used and  
145 do not escape without having intercepted a particle in the reactor [3,51]. In our design, the average  
146 volume fraction of nanoparticles is  $\varphi=1\times 10^{-4}$  in the tube. It can thus be considered as a dilute flow  
147 and the possible interactions between particles can be safely ignored during theoretical analysis.

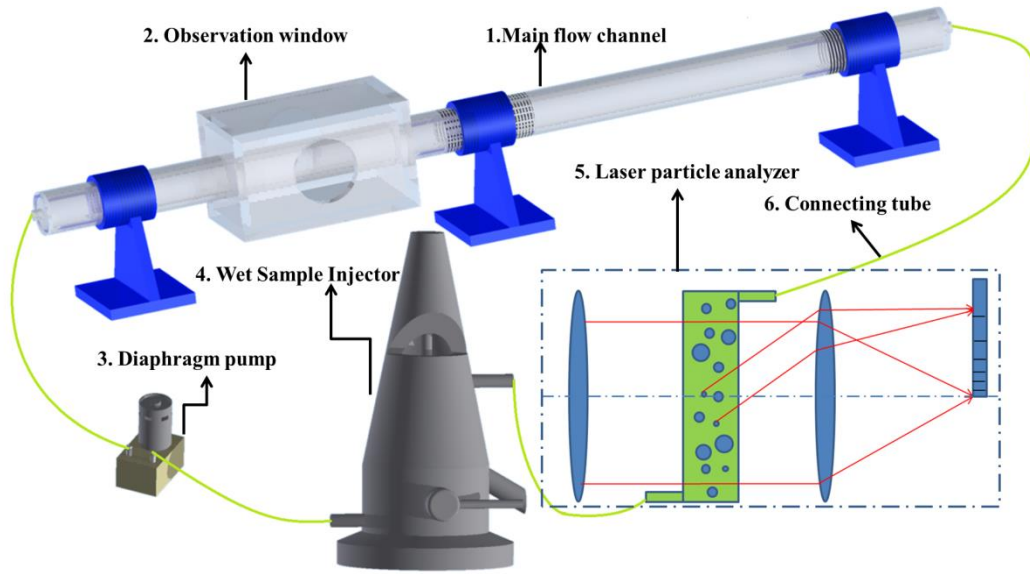


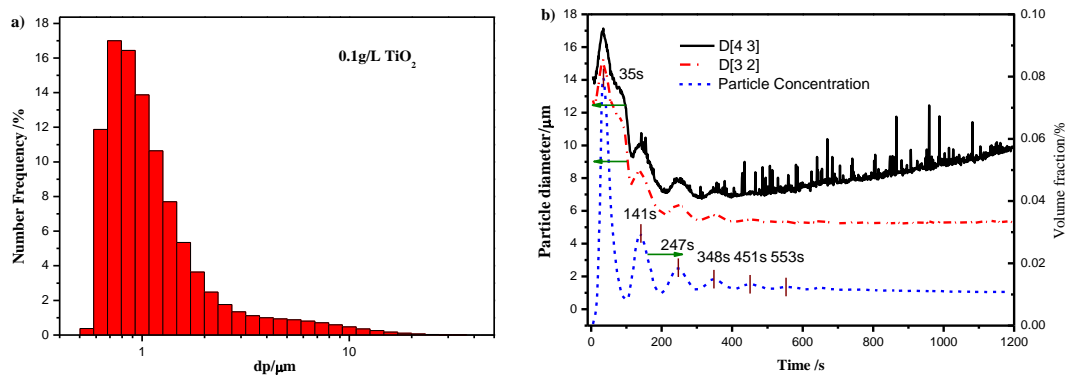
Fig.1. Schematic of experiment set-up[47]

148  
 149  
 150  
 151 Firstly, the system was filled with deionized water and circulated by pump with the flowrates  
 152 adjusted to 15 L/h. And the P25 particles are dispersed in deionized water in previous, which the  
 153 particle size distribution is showed in Fig. 2a. And it can be seen that the range of the particle  
 154 diameter is 0.50-36.87 $\mu\text{m}$ . Then the prepared P25-TiO<sub>2</sub> suspension was introduced into the wet  
 155 sample injector and the laser diffraction equipment starts to record the particles volume fraction  
 156 and size distribution simultaneously. Fig.2b shows the mean particle size and concentration versus  
 157 time in the repeatedly circulating flow. Here, the upper two curves corresponds to the variations of  
 158 two kinds of mean particle sizes, D<sub>32</sub> and D<sub>43</sub>, respectively and the bottom curve corresponds to  
 159 the variation of particles volume fraction against time. Here, D<sub>32</sub> is the mean particle size taking  
 160 into account both volume and surface area of the particles [52], while the D<sub>43</sub> is the average  
 161 particle size based on the volume moment [53]. The corresponding expression for the two average  
 162 particle sizes can be derived respectively by:

$$D_{32} = \frac{\sum V_i D_i^3}{\sum V_i D_i^2} \quad (1)$$

$$D_{43} = \frac{\sum V_i D_i^4}{\sum V_i D_i^3} \quad (2)$$

165 Obvious periodic oscillation of both mean particle sizes and particles volume fraction with time  
 166 can be noted in Fig.2b. And we can define the distance between two adjacent peaks as the period  
 167 of the oscillation curve. It can be found that the period of the three curves in the Fig. 2b is nearly  
 168 the same. The oscillation of the particle volume fraction curve with time is particularly significant  
 169 in the initial 600 seconds, and after that the particle volume fraction of the particles in suspension  
 170 tends to be a constant value, which we define this state as quasi-steady state. According to the  
 171 definition of mean particle size and  $D_{43}$ ,  $D_{43}$  is more sensitive to the large particles, while smaller  
 172 particles contribute more to the  $D_{32}$ . It is also noted in the Fig. 2b that the  $D_{32}$  and  $D_{43}$  curves are  
 173 also oscillation curves, but the tendency of them are some different comparing to the particle  
 174 volume fraction curve. It can be observed that the particle volume fraction curve fluctuates around  
 175 an average value, while the  $D_{43}$  and  $D_{32}$  curves fluctuate with an obviously downward trend.



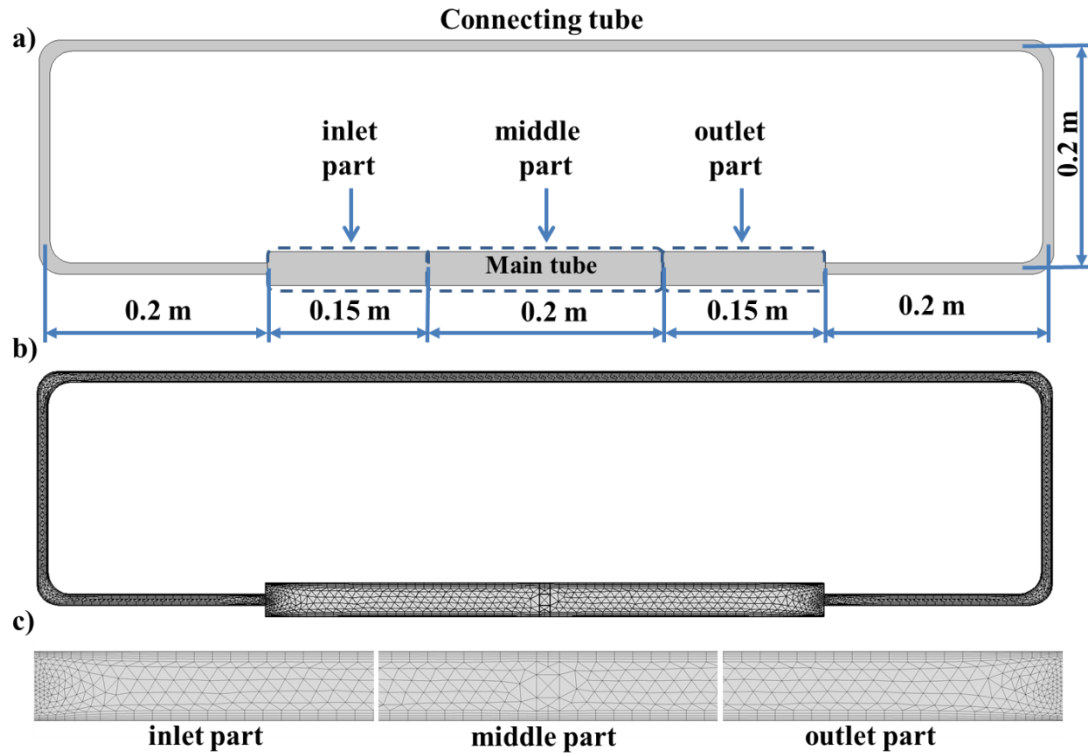
176  
 177 **Fig 2** a) Particles size distribution of the P25-TiO<sub>2</sub> powder dispersed in water, b) Variation of  
 178 various mean particle sizes and particle volume fraction with time.[47]

179  
 180 **3. Numerical simulation of the nanoparticle-laden flow**

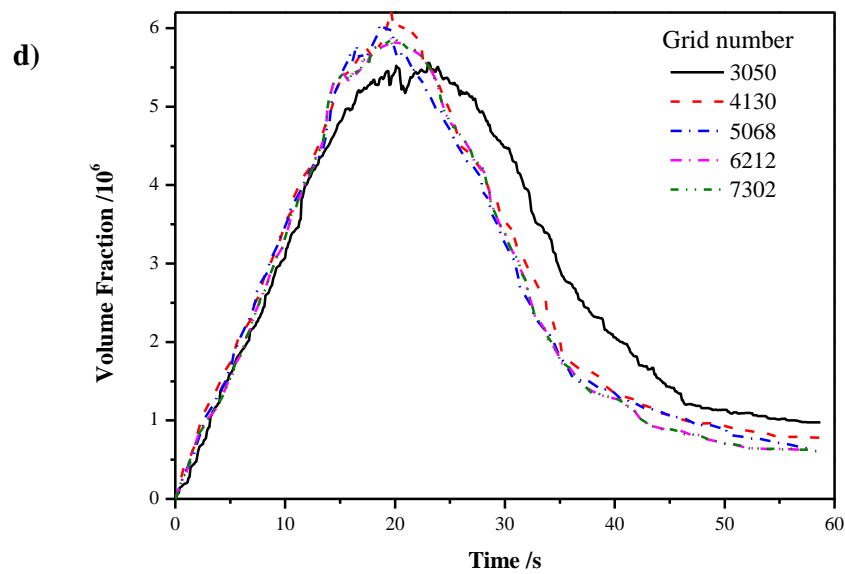
181 **3.1 Physical model for the tubular reactor**

182 As has been mentioned above, the particle volume fraction has been measured by LD method,  
 183 but only volume fraction cannot help us to know the mechanism of the particle transport  
 184 phenomenon in such a circulating flow system. To explain the transport phenomenon of the  
 185 particles in laminar circular flow, especially the details of the movement and variation of the  
 186 particles with different size during the transport process, a simulation based on the Euler-Lagrange  
 187 model was conducted. The geometry model for the simulation is showed in Fig. 3a, which is  
 188 established according to the experimental set-up, but the particle size analyze and pump are  
 189 neglected for the sake of simplicity. Here the main tube has a diameter of 30 mm with length of

190 0.5 m, and the connecting pipe has a diameter of 10 mm with length of 1.7m, which is the same  
 191 with experimental set-up. The mesh used in this simulation is showed in Fig 3b, which is obtained  
 192 from unstructured triangular grid. For the boundary layer, two layers of quadrilateral grids are  
 193 adopted near the boundary. The mesh dependency analysis based on particle volume fraction was  
 194 then conducted and the results are showed in Fig.3d. It turned out that the mesh with a grid  
 195 number of 6212 is accurate enough for this simulation and also time-efficient.



196



197

198 Fig.3 a) The Geometry model, b) computing mesh of the simulation, c) Local enlarged figure of



199 the mesh, **d)** mesh dependency analysis based on particle volume fraction

200 On the other hand, as the particles movement will be described under the Lagrange viewpoint,

201 the particles need to be traced one by one. However, it will be an extremely large amount of

202 particles in the tube in practice, for example, the total volume of the channel is about 0.423 L,

203 supposing the particles size is 3  $\mu\text{m}$  and volume fraction is 0.01%, the amount of the particles will

204 be  $2.994 \times 10^9$ , which is a too huge to cost an impossible computational resource. To solve this

205 problem, only 10,000 particles are taken into account in the simulation, and these particles can

206 present the migration principle of the particles with different size. For the convenience of

207 analyzing, we also divided the main tube into three sections, i.e. inlet, middle and outlet sections,

208 respectively. Furthermore, the properties of particles and the base fluid have significant influence

209 on the interactions between particles and liquid and/or between particles and particles. The

210 properties of particles mainly include density, shape, size, specific surface area and surface charge

211 et al. The density of the P25  $\text{TiO}_2$  particles was defined as  $4000 \text{ kg/m}^3$  according to Ref.[48], and

212 the size of particle was defined as the particle size distribution as Fig. 2a shows which was

213 obtained from the experimental data. The shape of the particles also has a significant impact on the

214 drag force between liquid and solid phase, whereas the shape is often defined as spherical in

215 numerical study to simply the computational model [54]. To solve this problem, some drag

216 correlations have been developed to modify the influence of particle shape on drag force between

217 particles and fluid [55,56]. In this study, as the particle was set as a common photocatalyst P25

218  $\text{TiO}_2$ , and the SEM of this kind of particles shows that the morphology generally exhibits spherical

219 [57], the particles in this study are all considered as spherical particles. And it needs to be further

220 pointed out that if the current computational model is needed to simulate the other non-spherical

221 particles, one merely need to add some corresponding drag correlations mentioned above. On the

222 other hand, particle aggregation is a common phenomenon in suspension especially for tiny

223 particles such as nanoparticles, and there are many influence factors of the aggregation

224 phenomenon, such as the temperature of suspension, the pH of the suspension, ions in the base

225 solution, particle size, shape, method of preparation and solid concentration [1,58]. Since the

226 particle aggregation phenomenon has noticeable impact on many applications, there have been

227 many theoretical [59-61] and experimental [62-64] studies focus on the problem. In this study,

228 aggregation was also taken into account by introducing a real particle size distribution (PSD)

229 which had been obtained from experimental data [47]. In other words, the particles aggregation  
 230 was assumed to be equilibrium state and the particles in this study are aggregated secondary  
 231 particles, thus the aggregation effect is taken into consideration in reality. Table 1 lists the  
 232 parameters of particles and base liquid used in the computational model of this study.

233 **Table 1** Some parameters of particles and base liquid used in the computational model

Parameters	Description
Particles	P25 TiO <sub>2</sub>
Base fluid	Deionized water
Density of particles	4000 kg/m <sup>3</sup>
Particle size	Using the PSD shown in <a href="#">Fig. 2a</a>
Particle quantity in the model	10000
Density of the base fluid	1000 kg/m <sup>3</sup>
Viscosity of the base fluid	1 mPa · s

234

### 235 3.2 Mathematical model

236 Considering that the particles concentration is very low in our study, to simplify the  
 237 mathematic model, we assume that the particles hardly influence the fluid field. Therefore, the  
 238 simulation can be done in two steps. Firstly, a single phase flow in the geometry was solved and  
 239 then the trajectory of particles was computed. In particular, in our case it is worth noting that the  
 240 flow region is a closed circular channel, which only has wall boundary but not has inlet or outlet  
 241 boundary. To solve this problem, the whole circular channel will be divided into two parts, the  
 242 main tube and the connecting tube. The flow direction of the circular is set to be anticlockwise, so  
 243 the right head of the connecting tube is set as the inlet boundary of the connecting tube which is  
 244 also set as the outlet boundary of the main flow tube and the left end of the connecting tube is set  
 245 as the outlet boundary which is also set as the inlet boundary of the main flow tube. The circular  
 246 flowrate is set as 15 L/h which is the same as the experimental condition.

247

248 As has been noted, in our study the trajectory of particles will be computed by solving  
 249 ordinary differential equations using Newton's law of motion. In this method, the particles are  
 250 treated as point masses. The specification of particle diameter is mostly used for size-dependent  
 251 forces, such as the drag and Brownian motion forces. As the particles concentration is very low in  
 252 the liquid phase, it can be supposed that the liquid phase affects the motion of the particles but not  
 253 vice-versa and one particle will not have obvious interactions with other particles. Thus, the

254 particle tracing equation according to Newton's second law can be expressed as [Eq. \(3\)](#):

$$255 \quad \frac{d}{dt} (m_p \mathbf{v}) = \mathbf{F}_D + \mathbf{F}_g + \mathbf{F}_B \quad (3)$$

256 Where the  $m_p$  is the mass of the particle, the  $\mathbf{v}$  is the velocity vector of particle;  $\mathbf{F}_D$ ,  $\mathbf{F}_g$  and  $\mathbf{F}_B$  are  
 257 three kinds of forces that can affect particles motion, and their calculation methods will be  
 258 introduced in detail.  $\mathbf{F}_D$  is the drag force of the fluid which is defined as:

$$259 \quad \mathbf{F}_D = \left( \frac{1}{\tau_p} \right) m_p (\mathbf{u} - \mathbf{v}) \quad (4)$$

260 where the  $\tau_p$  is the particle velocity response time,  $\mathbf{u}$  is the velocity of the fluid,  $\mathbf{v}$  is the velocity of  
 261 the particle. There are a large number of expressions for the particle response time. It depends on  
 262 the drag law, and selecting the appropriate drag law needs a consideration of the relative Reynolds  
 263 number  $Re_r$  of particles in the flow, which is given by:

$$264 \quad Re_r = \frac{\rho \|\mathbf{u} - \mathbf{v}\| d_p}{\mu} \quad (5)$$

265 where the  $\rho$  and  $\mu$  are the density and viscosity of the fluid, respectively, and  $d_p$  is the diameter of  
 266 the particles. With the dimensionless number  $Re_r$ ,  $\tau_p$  can be given by:

$$267 \quad \tau_p = \frac{4\rho_p d_p^2}{3\mu C_D Re_r} \quad (6)$$

268 where the  $C_D$  is the drag coefficient. Here, the parameter  $d_p$  is given by the particles size  
 269 distribution shown in [Fig. 2a](#). According to this particles size distribution, the range of the  $Re_r$  can  
 270 be evaluated. As the drag force drives the particles to follow the fluids movement, the term

271  $\|\mathbf{u} - \mathbf{v}\|$  that describe the difference between particle velocity and fluid velocity will be closed to  
 272 0 during the transport process, therefore the minimum value of the  $Re_r$  will be very clear to zero.

273 On the other hand, if both the term  $\|\mathbf{u} - \mathbf{v}\|$  and  $d_p$  reach the maximum value, the maximum of

274  $Re_r$  can be obtained. In this simulation, the maximum value of the  $\|\mathbf{u} - \mathbf{v}\|$  most possibly

275 happens at the initial state which the particles are static. And the maximum velocity of the fluid in

276 the main tube can be calculated according to the velocity distribution of the laminar flow, and the

277 value is 0.0118 m/s, so the maximum value of  $\|\mathbf{u} - \mathbf{v}\|$  will be 0.0118 m/s. For the maximum

278 particle size is 36.87  $\mu\text{m}$ , the maximum value of the  $Re_r$  can be evaluated according to the [Eq. 3](#), is

279 0.4203, that is, the range of the  $Re_r$  is 0-0.4203. According to the previous researches, the most  
 280 common used drag law is the Stokes drag law, but it only can be used in the case that the  $Re_r \ll 1$ .  
 281 Thus, another famous drag law called Oseen correction is used [65], which the drag coefficient is  
 282 given by:

$$283 \quad C_D = \frac{24}{Re_r} \left( 1 + \frac{3}{16} Re_r \right) \quad (7)$$

284

285  $\mathbf{F}_g$  is the gravity force which is given by:

$$286 \quad \mathbf{F}_g = m_p \mathbf{g} \frac{(\rho_p - \rho)}{\rho_p} \quad (8)$$

287  $\mathbf{F}_B$  is the Brownian Force which can describe the Brownian motion of the particles. Actually,  
 288 the Brownian motion of the particles leads to spreading of particles from regions of high particle  
 289 concentration to low concentration. The expression of the Brownian Force is:

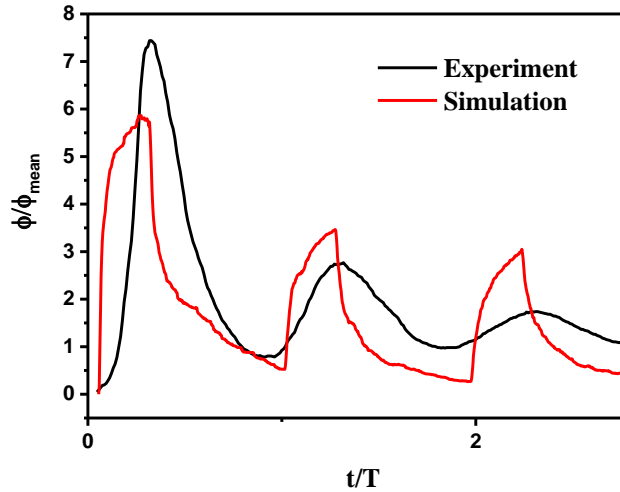
$$290 \quad \mathbf{F}_B = \zeta \sqrt{\frac{6\pi k_B \mu T d_p}{\Delta t}} \quad (9)$$

291 where the  $\Delta t$  is the time step taken by the solver,  $T$  is the absolute fluid temperature,  $k_B$  is the  
 292 Boltzmann constant and the value is  $1.3806 \times 10^{-23}$  J/K, and  $\zeta$  is a normally distributed random  
 293 number with a mean of zero and unit standard deviation which is created for each particle, at each  
 294 time step.

### 295 3.3 Model validation

296 To validate the model, the result of the variation of the particle volume with time by this  
 297 simulation is compared with the experimental result, which is showed in **Fig 4**. As has been  
 298 mentioned above, the particle volume fraction in the simulation is much smaller than the  
 299 experimental condition for the huge computational source consumption, the particle volume  
 300 fraction in **Fig 4** is normalized by dividing the mean particle volume fraction in the quasi-steady  
 301 state. Also, for it has some simplified assumptions in the physical model, so the times of the two  
 302 results are not exactly equivalent, therefore a normalized time is used by dividing the period of the  
 303 oscillation curve,  $T$ . From the **Fig 4**, one can find that the tendencies of two curves are very similar  
 304 in the first 3 periods, but it can be also seen that the continuity of the simulation curve is poor  
 305 compared with the experimental results, this may due to the particle amount is much smaller than

306 the experimental condition. However, in this study, we only want to reveal the particle movement  
 307 mechanism and how this movement results in an oscillation of particle volume fraction and mean  
 308 particle size. From this viewpoint, the result of this simulation result also exhibit a very obvious  
 309 oscillation tendency, so it can help us to understand the mechanism.



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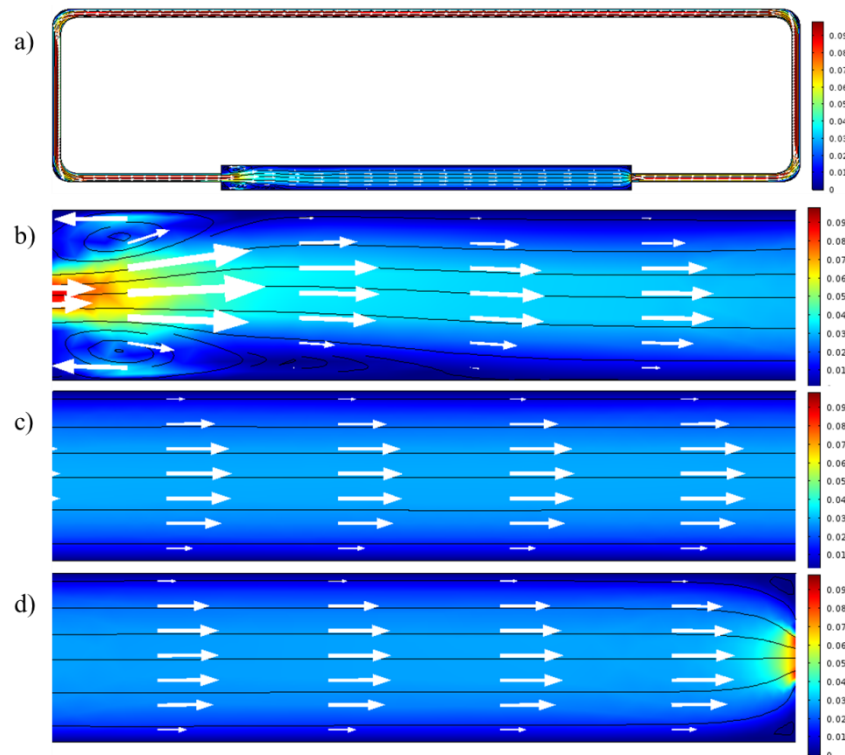
311 **Fig. 4.** Variation of particle volume fraction with time in experiment and simulation

## 312 4. Results and discussion

### 313 4.1 The flow field of the particle-laden flow in the circulating flow system

314 As the particles moving in the fluids are significantly affected by the drag force of fluid, the  
 315 flow field of the flow is believed to be extremely important for the latter analysis of particles  
 316 movement. The velocity distribution obtained by the simulation is showed in [Fig. 5](#), which is a  
 317 typical laminar tube flow. [Fig. 5b](#) shows the velocity distribution in the inlet part, and it can be  
 318 seen that the fluid flow from the thinner connecting tube into the thicker main tube. As the fluid  
 319 velocity is larger in the thinner connecting tube, there is a high velocity region in the front-end of  
 320 the inlet part, and there also appears inverse flow with two symmetrical vortexes in the top and the  
 321 bottom of this region. Then the fluid flow in the main tube gradually passes into the fully  
 322 developed flow. [Fig. 5c](#) shows the velocity distribution in the middle part, which shows the  
 323 character of fully developed flow. And the velocity distribution exhibits a parabolic distribution,  
 324 which is a typical laminar tube flow. This kind of flow has a character that the velocity in the  
 325 center of tube reaches the maximum value, while the velocity near the wall is low and the velocity  
 326 at wall is zero. That is to say, the velocity distribution shows an obvious difference in radial  
 327 direction. And [Fig. 5d](#) shows the velocity distribution in the outlet part, and it can be seen that the  
 328 velocity distribution in the front of this part is very similar to the middle part. However, it appears

329 a high velocity region in the last end of this part, for the fluid flow from the thicker main tube to  
330 the thinner connecting tube.



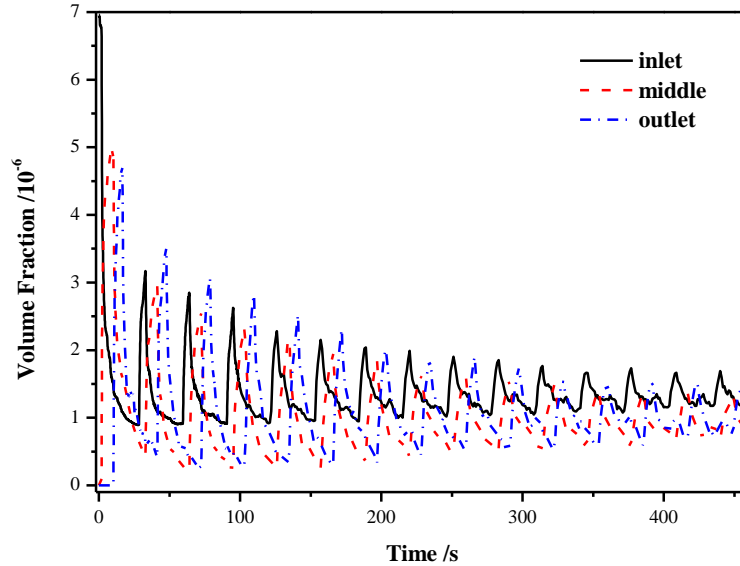
331  
332 **Fig.5** Contour of the velocity magnitude of the circular flow with streamlines **a)**the whole  
333 channel, **b)** the inlet part **c)**the middle part **d)**the outlet part, of the main tube

334

#### 335 4.2 Particle volume fraction

336 **Fig. 6** shows the variation of the average particle concentration with time during the transport  
337 process in three parts of the tube. As has been mentioned in section 3.3, the simulation result is  
338 very similar to the experimental results. The most obvious difference between the two results is  
339 that the curve of the experiment reaches a constant value in the end, whereas the curve of the  
340 simulation just holds a value in a small range. This deviation may be due to that too fewer  
341 particles are taken into account in the simulation and the disturbance from pump or other  
342 equipment used in experiment has also not been taken into account. However, as the tendency  
343 obtained from the simulation is similar to the experiment results, it is therefore reasonable to use  
344 the results of simulation to find the mechanism of the observed experimental phenomenon. From  
345 Fig 6, it can be found that the highest peak appears at the first peak of the oscillation curve in all  
346 the three part of main tube, while the height of the three peaks are different and the peak in the  
347 inlet part is the highest. It can also be seen that the peaks of the oscillation curves in different

348 parts also don't coincide in time. These phenomena may mean that the oscillation of particle  
349 volume fraction is caused by the particle movement in the circulating flow. And to verify this  
350 assumption, we will discuss the particle distribution and trajectory in the next part.



351

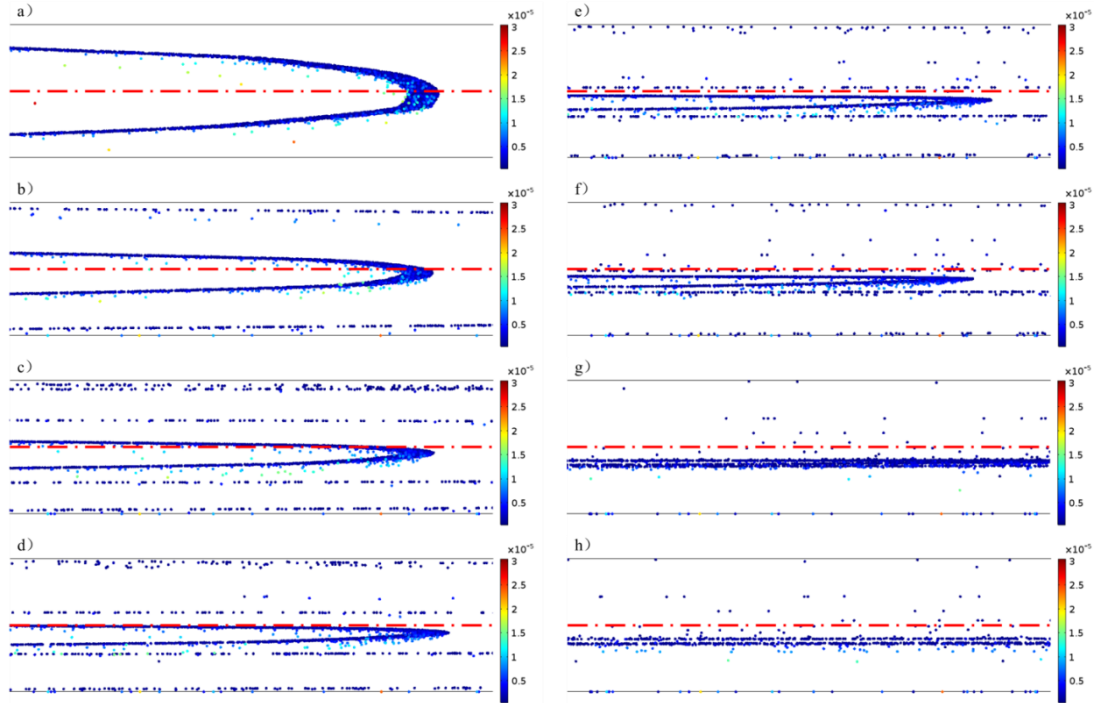
352 **Fig. 6** Variation of volume fraction of particles with time derived from the simulation

353

### 354 4.3 The particle trajectory

355 Firstly, the trajectories of particles in the times which correspond to the position of the peaks  
356 of the particle volume fraction curve are showed in [Fig 7](#). It can be found that the particles  
357 distribution exhibits a parabolic like state in the initial time which is very similar to the velocity  
358 distribution shown in the [Fig 5](#). And it can be easily found from [Fig 7a-f](#) that the particles in these  
359 peak times tend to arrange in a parabolic curve. Combined with the velocity distribution  
360 mentioned above, it can be inferred that this phenomenon is caused by the parabolic distribution of  
361 radial velocity, that is, the particles near the center will have higher velocity, while the particles  
362 near the wall will move slowly. Thus, the particles with higher velocity will pass through the  
363 whole circulating system much faster than other particles, and when they reach the counting  
364 region, there will be a relative higher particle volume fraction. This process is believed to be a  
365 good explanation for the oscillation phenomenon. Furthermore, it can be found that the parabolic  
366 shape of particles gets narrower gradually until it almost turns into a line at the quasi-steady state,  
367 and this is due to the distance between the particles with different velocity will be longer as time  
368 goes on. The narrower parabolic particles arrangement means fewer particles, which can explain  
369 the lower and lower peaks high in the oscillation curve. Also, it can be found that the position of

370 the overall parabolic shape in the tube becomes lower and lower during the transport process,  
 371 which may due to the gravity force of the particles. After several circles, particles moving in the  
 372 main tube will gather in the positions that a little lower than the axis of the main tube.



373  
 374 **Fig. 7** schematic of the particles position in the peak times and the quasi-steady state **a)**  $t=14.6s$  **b)**  
 375  $t=45.6s$  **c)**  $t=76.7s$  **d)**  $t=108s$  **e)**  $t=139s$  **f)**  $t=170s$  **g)**  $t=360s$  **h)**  $t=400s$   
 376

377 **Fig 8** shows the trajectory and evolution process of a single particle counted from the  
 378 simulation result. For there are so many particles in the tube, we just choose four particles which  
 379 we think are very representative, and the particles size are  $0.496\ \mu m$ ,  $1.117\ \mu m$ ,  $10.842\ \mu m$  and  
 380  $15.043\ \mu m$ , respectively. We compute the locations of particles in the main tube at any time. As  
 381 the particles moves in the circulating system, we also distinguished the different number of cycles  
 382 of the particle trajectory. The particle with size of  $0.496\ \mu m$  is selected to represent the little  
 383 particle and it can be found that its trajectory in the middle part is almost a straight line. At the  
 384 same time, we also found that in the first 10 cycles, the particle trajectory showed a gradual  
 385 downward trend, but after the 10<sup>th</sup> cycle, the particle trajectory remained almost unchanged. In  
 386 addition, we also selected the particles with a diameter of  $1.117\ \mu m$  and this part of particles is  
 387 treated as median-sized particles. From the **Fig 8 b)**, we can see that the trajectory of this kind of  
 388 particles is very similar to that of  $0.496\ \mu m$  particle, which looks straight in the middle part, and  
 389 will basically stabilize in one trajectory after the 10<sup>th</sup> cycle. Furthermore, we can conclude that the

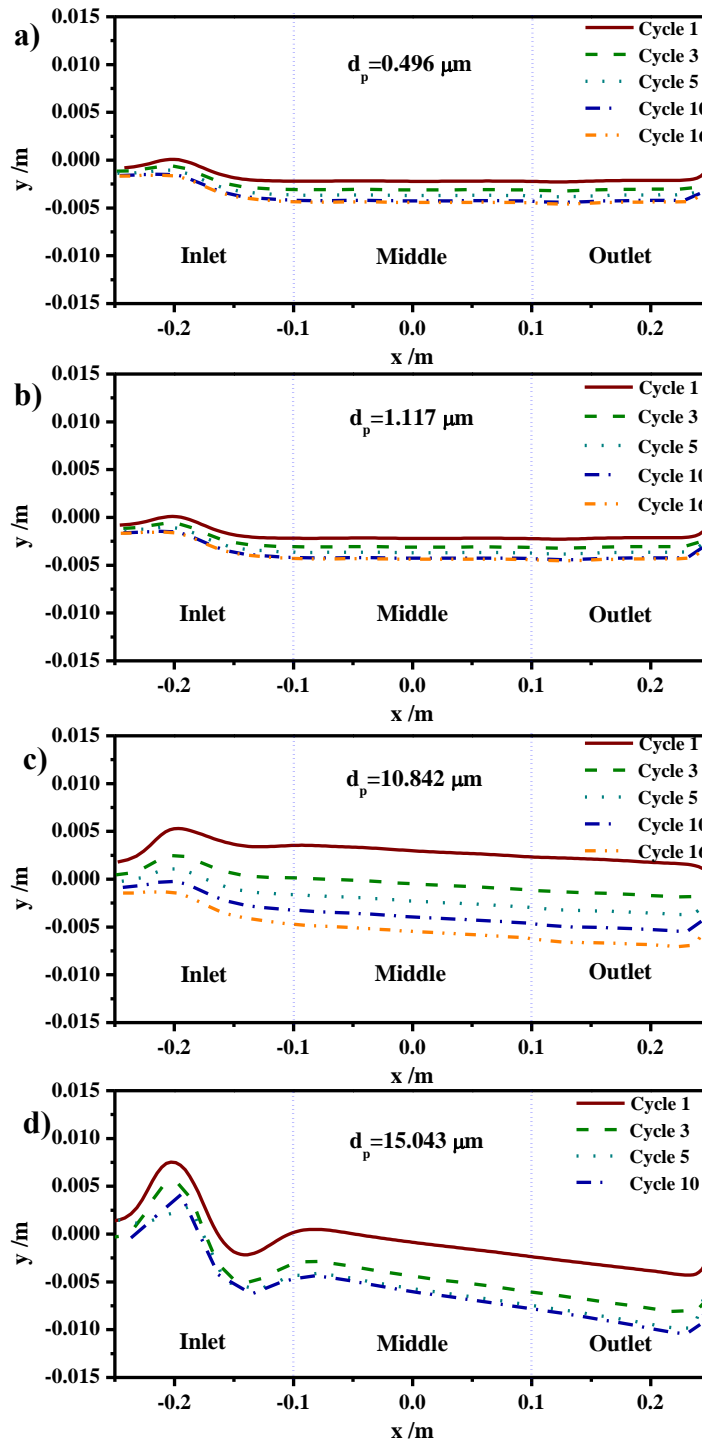


390 motion character of the last mentioned two kinds of particles is that they follow the fluid and keep  
391 moving on a straight line in the main tube, that is to say, the settling velocity generated by gravity  
392 is very small. After the discussion of small particles, it is necessary to discuss the large particle.  
393 Therefore, particle with size of 10.842  $\mu\text{m}$  is also selected. It can be found that this kind of particle  
394 is seriously affected by gravity, or the trajectory of this particle in the middle part of the main tube  
395 is in the downward inclination. And with the increase of the number of cycles, its trajectory in the  
396 main tube shows a very obvious downward trend even after 10<sup>th</sup> cycles. Finally, we chose the  
397 particle size of 15.043  $\mu\text{m}$ , because this particle is the largest particle which is still in motion at  
398 400 s in this simulation and can represent the coarse particle. It can be seen that the effect of  
399 gravity on the particle is more obvious, and the downward inclination of its trajectory in the main  
400 tube is more obvious. But for this particle, we can only take 10 cycles, because the settlement  
401 occurred before the 16<sup>th</sup> cycle, which shows that such particles in the pipeline cannot maintain a  
402 continuous circulating movement, and eventually will settle down.

#### 403 **4.4 Particle size distribution**

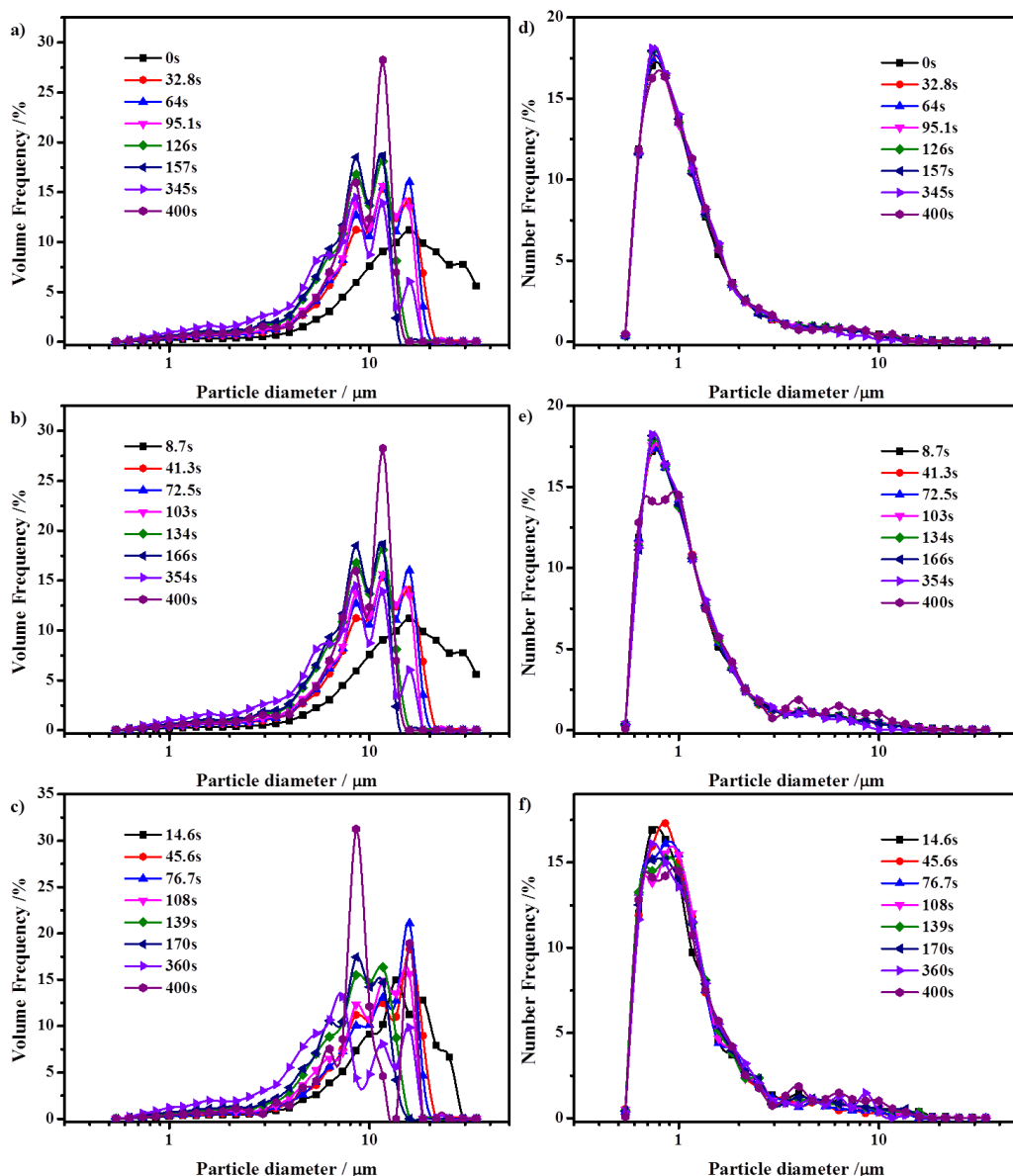
404 The particle size distribution in the different parts of the main tube counted from the simulation  
405 results is showed in [Fig. 9](#). Both of the number and volume particle size distributions are taken  
406 into account, also the three parts of the tube are computed respectively. For the convenience of  
407 description, according to the results in the section 4.3, we can call particles of 1-10  $\mu\text{m}$  as small  
408 particles, and particles of more than 10  $\mu\text{m}$  as large particles. It can be found that the volume  
409 particle size distribution is a unimodal distribution at the first peak. As the time goes on, the  
410 particle size distribution curve gradually becomes an obvious bimodal distribution. If one look at  
411 the position and corresponding size of the two peaks separately, one can see that the peak of large  
412 particles moves to the left with time, and the height of it increases obviously with time. This trend  
413 also proves that the particles with a diameter more than 15  $\mu\text{m}$  are greatly affected by gravity and  
414 easy to settle down, which results in some loss of the particles. Therefore, the particles with a  
415 diameter slightly smaller than 15  $\mu\text{m}$  occupy a higher volume fraction in the large particles, and it  
416 form the peak of large particles in the particle size distribution curve. In addition, it can be seen  
417 that the position of the peak of small particles gradually move to left with time. This phenomenon  
418 shows that the small particles almost have no loss in the transport process; as a result, volume  
419 frequency of this part of particles has obvious advantages especially when the large particles lost.

420 And it can be also seen that the particles size distribution based on number seems not have much  
 421 change on the different times of the peaks as the experiment shows, associated with Fig 7, we  
 422 further confirm that the group of particles which have the higher velocity will repeat to appear on  
 423 the region for counting, so the number particles size distribution is almost no change.



424  
 425 **Fig 8** particles motion trace in the main tube a)  $d_p = 0.496 \mu\text{m}$ , b)  $d_p = 1.117 \mu\text{m}$  c)  $d_p = 10.842 \mu\text{m}$ ,  
 426 d)  $d_p = 15.043 \mu\text{m}$

427 From the prospective of photocatalytic reaction, amount of the particles especially the smaller  
 428 particles are more benefit to the reaction efficiency. The result derived from the simulation  
 429 inspires that in the circulating system, effects of gravity is only remarkable for the large particles  
 430 which the size are more than 10  $\mu\text{m}$ , and a very low circulating flowrate such as 15 L/h is enough  
 431 to make sure the small particles keep suspending. And for the amount of the small particles are  
 432 quite larger than that of large particles, adopting a lower flowrate in the photocatalytic application  
 433 is reasonable and also energy efficient.



434  
 435 **Fig.9** Particles size distribution based on volume obtained from a) inlet part, b) middle part, c)  
 436 outlet part, and Particles size distribution based on number obtained from d) inlet part, e) middle  
 437 part, f) outlet part

438

## 439 **5 Conclusions**

440 The suspension of micro- or nanoscale particles in liquid is used widely and the circulating flow  
441 is also regularly chosen as the flow form in the application of this kind of two-phase fluid. In this  
442 study, a DEM method is used in a simulation of the circulating laminar flow system to explain the  
443 transport phenomenon of particles. It can be found the particle concentration and size distribution  
444 varies with time and appears periodical oscillation, which is related to the non-uniform velocity  
445 distribution of particles in the radial direction of tube. In this process, the particle size distribution  
446 changes from unimodal distribution to bimodal distribution, for the particles with a size less than  
447 10  $\mu\text{m}$  almost have no loss in the transport process. Furthermore, particles tend to gather in the  
448 middle region of the tube after several cycles, which is also the region with the higher velocity in  
449 the shear flow. Also, with the effects of gravity force, the integral particles group tends to  
450 distribute in the region a little lower than the center.

451 As a case study, this study mainly focus on the particle transport phenomenon in photocatalytic  
452 reactor, so the kind of particles and base liquid, operating mode, initial condition and other  
453 parameters were specified to photocatalytic reactor. However, this simulation can also be extended  
454 to other applications, such as heat transfer enhancement by nanofluid, general solid-liquid reaction  
455 et al. And it can be implemented by simply replacing some parameters or introducing some  
456 correlations in the existing model, which is believed to be useful for understanding the particle  
457 transport phenomenon in different applications.

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## 464 **References:**

465 [1] Jing DW, Song DX. Optical properties of nanofluids considering particle size distribution:  
466 Experimental and theoretical investigations. RENEW SUST ENERG REV. 2017; 78:452-65.

467 [2] Kudo A, Miseki Y. Heterogeneous photocatalyst materials for water splitting. CHEM SOC REV.  
468 2009; 38:253-78.

469 [3] Jing DW, Guo LJ, Zhao L, Zhang XM, Liu H, Li MT, et al. Efficient solar hydrogen production  
470 by photocatalytic water splitting: From fundamental study to pilot demonstration. INT J HYDROGEN

471 ENERG. 2010; 35:7087-97.

472 [4] Vajjha RS, Das DK. A review and analysis on influence of temperature and concentration of  
473 nanofluids on thermophysical properties, heat transfer and pumping power. INT J HEAT MASS TRAN.  
474 2012; 55:4063-78.

475 [5] Fajrina N, Tahir M. A critical review in strategies to improve photocatalytic water splitting  
476 towards hydrogen production. INT J HYDROGEN ENERG. 2019; 44:540-77.

477 [6] Kim SJ, Bang IC, Buongiorno J, Hu LW. Effects of nanoparticle deposition on surface wettability  
478 influencing boiling heat transfer in nanofluids. APPL PHYS LETT. 2006; 89:153107.

479 [7] KUDO A. Development of photocatalyst materials for water splitting. INT J HYDROGEN  
480 ENERG. 2006; 31:197-202.

481 [8] Luo B, Song R, Jing D. ZnCr LDH nanosheets modified graphitic carbon nitride for enhanced  
482 photocatalytic hydrogen production. INT J HYDROGEN ENERG. 2017; 42:23427-36.

483 [9] Pulido Melián E, González Díaz O, Ortega Méndez A, López CR, Nereida Suárez M, Doña  
484 Rodríguez JM, et al. Efficient and affordable hydrogen production by water photo-splitting using  
485 TiO<sub>2</sub>-based photocatalysts. INT J HYDROGEN ENERG. 2013; 38:2144-55.

486 [10] Ma Z, Cui Z, Lv Y, Sa R, Wu K, Li Q. Three-in-One: Opened Charge-transfer channel, positively  
487 shifted oxidation potential, and enhanced visible light response of g-C<sub>3</sub>N<sub>4</sub> photocatalyst through K and  
488 S Co-doping. INT J HYDROGEN ENERG. 2020; 45:4534-44.

489 [11] Fang W, Shangguan W. A review on bismuth-based composite oxides for photocatalytic  
490 hydrogen generation. INT J HYDROGEN ENERG. 2019; 44:895-912.

491 [12] Li X, Yu J, Low J, Fang Y, Xiao J, Chen X. Engineering heterogeneous semiconductors for solar  
492 water splitting. J MATER CHEM A. 2015; 3:2485-534.

493 [13] Ren YX, Zhao L, Jing DW, Guo LJ. Investigation and modeling of CPC based tubular  
494 photocatalytic reactor for scaled-up hydrogen production. INT J HYDROGEN ENERG. 2016;  
495 41:16019-31.

496 [14] Cao F, Liu H, Wei Q, Zhao L, Guo L. Experimental study of direct solar photocatalytic water  
497 splitting for hydrogen production under natural circulation conditions. INT J HYDROGEN ENERG.  
498 2018; 43:13727-37.

499 [15] Yang Y, Wei Q, Hou J, Liu H, Zhao L. Solar concentrator with uniform irradiance for particulate  
500 photocatalytic hydrogen production system. INT J HYDROGEN ENERG. 2016; 41:16040-7.

501 [16] Wei Q, Yang Y, Liu H, Hou J, Liu M, Cao F, et al. Experimental study on direct solar  
502 photocatalytic water splitting for hydrogen production using surface uniform concentrators. INT J  
503 HYDROGEN ENERG. 2018; 43:13745-53.

504 [17] Li L, Chen R, Liao Q, Zhu X, Wang G, Wang D. High surface area optofluidic microreactor for  
505 redox mediated photocatalytic water splitting. INT J HYDROGEN ENERG. 2014; 39:19270-6.

506 [18] Yan H, Yang J, Ma G, Wu G, Zong X, Lei Z, et al. Visible-light-driven hydrogen production with  
507 extremely high quantum efficiency on Pt - PdS/CdS photocatalyst. J CATAL. 2009; 266:165-8.

508 [19] Liu M, Wang L, Max Lu G, Yao X, Guo L. Twins in Cd<sub>1-x</sub>Zn<sub>x</sub>S solid solution: Highly efficient  
509 photocatalyst for hydrogen generation from water. ENERGI ENVIRON SCI. 2011; 4:1372.

510 [20] Dong Q, Fang Y, Shao Y, Mulligan P, Qiu J, Cao L, et al. Metal-free efficient photocatalyst for  
511 stable visible water splitting via a two-electron pathway. . SCIENCE. 2015; 347:967-70.

512 [21] Wang Q, Hisatomi T, Jia Q, Tokudome H, Zhong M, Wang C, et al. Scalable water splitting on  
513 particulate photocatalyst sheets with a solar-to-hydrogen energy conversion efficiency exceeding 1%.  
514 NAT MATER. 2016; 15:611-5.

515 [22] Barresi A, Baldi G. Solid dispersion in an agitated vessel effect of particle shape and density.  
516 CHEM ENG SCI. 1987; 12:2969-72.

517 [23] Huang JK, Lu YJ, Wang H. A new quantitative measurement method for mixing and segregation  
518 of binary-mixture fluidized bed by capacitance probe. CHEM ENG J. 2017; 326:99-108.

519 [24] Wang K, Liu G, Liu ZG, Wu J, Yi LT, Zhang JL, et al. Acoustic sensor approaches for sand  
520 detection in sand – water two-phase flows. POWDER TECHNOL. 2017; 320:739-47.

521 [25] Felder S, Chanson H. Phase-detection probe measurements in high-velocity free-surface flows  
522 including a discussion of key sampling parameters. EXP THERM FLUID SCI. 2015; 61:66-78.

523 [26] Nocentini M, Pinelli D, Magelli F. Dispersion coefficient and settling velocity of the solids in  
524 agitated slurry reactors stirred with multiple rushton turbines. CHEM ENG SCI. 2002; 57:1877-84.

525 [27] Roshani GH, Nazemi E, Feghhi SAH, Setayeshi S. Flow regime identification and void fraction  
526 prediction in two-phase flows based on gamma ray attenuation. MEASUREMENT. 2015; 62:25-32.

527 [28] Xu YQ, Xu CB, Guan ZC, Liu YW, Tian Y, Sheng YN, et al. Numerical simulation method of  
528 ultrasonic wave propagation in gas-liquid two-phase flow of deepwater riser. MECH SYST SIGNAL  
529 PR. 2019; 118:78-92.

530 [29] Liu L, Fang ZY, Wu YP, Lai XP, Wang P, Song K. Experimental investigation of solid-liquid  
531 two-phase flow in cemented rock-tailings backfill using Electrical Resistance Tomography. CONSTR  
532 BUILD MATER. 2018; 175:267-76.

533 [30] Black DL, McQuay MQ, Bonin MP. Laser-based techniques for particle-size measurement: A  
534 review of sizing methods and their industrial applications. PROG ENERG COMBUST. 1996;  
535 22:267-306.

536 [31] Levoguer C. Using laser diffraction to measure particle size and distribution. MET POWDER  
537 REP. 2013; 68:15-8.

538 [32] Ma ZH, Merkus HG, de Smet J, Heffels C, Scarlett B. New developments in particle  
539 characterization by laser diffraction: size and shape. POWDER TECHNOL. 2000; 111:66-78.

540 [33] Hu XW, Guo LJ. Numerical investigations of catalyst – liquid slurry flow in the photocatalytic  
541 reactor for hydrogen production based on algebraic slip model. INT J HYDROGEN ENERG. 2010;  
542 35:7065-72.

543 [34] Hatami M, Hosseinzadeh K, Domairry G, Behnamfar MT. Numerical study of MHD two-phase  
544 Couette flow analysis for fluid-particle suspension between moving parallel plates. J TAIWAN INST  
545 CHEM E. 2014; 45:2238-45.

546 [35] Hedayati F, Domairry G. Nanoparticle migration effects on fully developed forced convection of  
547 TiO<sub>2</sub> – water nanofluid in a parallel plate microchannel. PARTICUOLOGY. 2016; 24:96-107.

548 [36] Feng YQ, Yu AB. Assessment of Model Formulations in the Discrete Particle Simulation of Gas–  
549 Solid Flow. IND ENG CHEM RES. 2004; 43:8378-90.

550 [37] Morris JF, Brady JF. Pressure-driven flow of a suspension: Buoyancy effects. INT J  
551 MULTIPHAS FLOW. 1998; 24:105-30.

552 [38] Peng ZB, Doroodchi E, Evans G. DEM simulation of aggregation of suspended nanoparticles.  
553 POWDER TECHNOL. 2010; 204:91-102.

554 [39] Chaumeil F, Crapper M. Using the DEM-CFD method to predict Brownian particle deposition in  
555 a constricted tube. PARTICUOLOGY. 2014; 15:94-106.

556 [40] Jing DW, Liu H, Zhang XH, Zhao L, Guo LJ. Photocatalytic hydrogen production under direct  
557 solar light in a CPC based solar reactor: Reactor design and preliminary results. ENERG CONVERS  
558 MANAGE. 2009; 50:2919-26.

559 [41] Li D, Xiong K, Li W, Yang ZH, Liu C, Feng X, et al. Comparative Study in Liquid-Phase  
560 Heterogeneous Photocatalysis: Model for Photoreactor Scale-Up. *IND ENG CHEM RES.* 2010;  
561 49:8397-405.

562 [42] Malato S, Blanco J, Vidal A, Richter C. Photocatalysis with solar energy at a pilot-plant scale: an  
563 overview. *APPL CATAL B-ENVIRON.* 2002; 37:1-15.

564 [43] Ochoa-Gutiérrez KS, Tabares-Aguilar E, Mueses MÁ, Machuca-Martínez F, Li Puma G. A Novel  
565 Prototype Offset Multi Tubular Photoreactor (OMTP) for solar photocatalytic degradation of water  
566 contaminants. *CHEM ENG J.* 2018; 341:628-38.

567 [44] Xing Z, Zong X, Pan J, Wang LZ. On the engineering part of solar hydrogen production from  
568 water splitting: Photoreactor design. *CHEM ENG SCI.* 2013; 104:125-46.

569 [45] Cao F, Liu H, Wei Q, Zhao L, Guo L. Experimental study of direct solar photocatalytic water  
570 splitting for hydrogen production under natural circulation conditions. *INT J HYDROGEN ENERG.*  
571 2018; 43:13727-37.

572 [46] Segré G, Silberberg A. Behaviour of macroscopic rigid spheres in Poiseuille flow Part 2.  
573 Experimental results and interpretation. *J FLUID MECH.* 1962; 14:136.

574 [47] Geng J, Tang J, Wang Y, Huang Z, Jing D, Guo L. Attenuated Periodical Oscillation  
575 Characteristics in a Nanoscale Particle-Laden Laminar Flow. *IND ENG CHEM RES.* 2020;  
576 59:8018-27.

577 [48] Kalantary RR, Shahamat YD, Farzadkia M, Esrafil A, Asgharnia H. Photocatalytic degradation  
578 and mineralization of diazinon in aqueous solution using nano-TiO<sub>2</sub>(Degussa, P25): kinetic and  
579 statistical analysis. *DESALIN WATER TREAT.* 2015; 55:555-63.

580 [49] Cao F, Wei QY, Liu H, Lu N, Zhao L, Guo LJ. Development of the direct solar photocatalytic  
581 water splitting system for hydrogen production in Northwest China: Design and evaluation of  
582 photoreactor. *RENEW ENERG.* 2018; 121:153-63.

583 [50] Wei QY, Yang Y, Hou JY, Liu H, Cao F, Zhao L. Direct solar photocatalytic hydrogen generation  
584 with CPC photoreactors: System development. *SOL ENERGY.* 2017; 153:215-23.

585 [51] Jing DW, Liu H, Zhang XH, Zhao L, Guo LJ. Photocatalytic hydrogen production under direct  
586 solar light in a CPC based solar reactor: Reactor design and preliminary results. *ENERG CONVERS  
587 MANAGE.* 2009; 50:2919-26.

588 [52] Kowalczyk PB, Drzymala J. Physical meaning of the Sauter mean diameter of spherical  
589 particulate matter. *PARTICUL SCI TECHNOL.* 2016; 34:645-7.

590 [53] Wang L, Fang NF, Yue ZJ, Shi ZH, Hua L. Raindrop Size and Flow Depth Control Sediment  
591 Sorting in Shallow Flows on Steep Slopes. *WATER RESOUR RES.* 2018; 54:9978-95.

592 [54] Song D, Yang Y, Jing D. Insight into the contribution of rotating Brownian motion of  
593 nonspherical particle to the thermal conductivity enhancement of nanofluid. *INT J HEAT MASS  
594 TRAN.* 2017; 112:61-71.

595 [55] Cao Z, Tafti DK, Shahnam M. Development of drag correlation for suspensions of ellipsoidal  
596 particles. *POWDER TECHNOL.* 2020; 369:298-310.

597 [56] Liu X, Gan J, Zhong W, Yu A. Particle shape effects on dynamic behaviors in a spouted bed:  
598 CFD-DEM study. *POWDER TECHNOL.* 2020; 361:349-62.

599 [57] Jin J, Li X, Geng J, Jing D. Insights into the complex interaction between hydrophilic  
600 nanoparticles and ionic surfactants at the liquid/air interface. *PHYS CHEM CHEM PHYS.* 2018;  
601 20:15223-35.

602 [58] French RA, Jacobson AR, Kim B, Isley SL, Penn RL, Baveye PC. Influence of Ionic Strength, pH,

603 and Cation Valence on Aggregation Kinetics of Titanium Dioxide Nanoparticles. ENVIRON SCI  
604 TECHNOL. 2009; 43:1354-9.

605 [59] Song D, Hatami M, Wang Y, Jing D, Yang Y. Prediction of hydrodynamic and optical properties  
606 of TiO<sub>2</sub>/water suspension considering particle size distribution. INT J HEAT MASS TRAN. 2016;  
607 92:864-76.

608 [60] Jing D, Hu S, Zhang Y, Luo J. A modified diffusion-limited cluster aggregation model for  
609 accurate prediction of the coagulation and fragmentation process in nanoparticle suspension. J PHYS D  
610 APPL PHYS. 2019; 52:455305.

611 [61] Kumar S, Ramkrishna D. On the solution of population balance equations by discretization—I. A  
612 fixed pivot technique. CHEM ENG SCI. 1996; 51:1311-32.

613 [62] Chowdhury I, Duch MC, Mansukhani ND, Hersam MC, Bouchard D. Colloidal Properties and  
614 Stability of Graphene Oxide Nanomaterials in the Aquatic Environment. ENVIRON SCI TECHNOL.  
615 2013; 47:6288-96.

616 [63] Luo B, Song R, Jing D. Particle aggregation behavior during photocatalytic ethanol reforming  
617 reaction and its correlation with the activity of H<sub>2</sub> production. COLLOID SURFACE A. 2017;  
618 535:114-20.

619 [64] Ebini RH, Sorensen CM. Light scattering studies of the sol-to-gel transition in particulate systems.  
620 J COLLOID INTERF SCI. 2019; 556:577-83.

621 [65] Yang M, Li S, Marshall JS. Effects of long-range particle - particle hydrodynamic interaction on  
622 the settling of aerosol particle clouds. J AEROSOL SCI. 2015; 90:154-60.

623

624