Challenges in Modelling Optical Fibres for Spatial Division Multiplexing

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***Abstract*—We review recent studies of the nonlinear interference in spatial division multiplexing systems. Different solution methods of the multimode Schrödinger equation are compared, highlighting the accuracy of a stochastic solution method including distributed mode coupling.**

***Index Terms*—Spatial Division Multiplexing, fibre Nonlinearity, Linear Mode Coupling, Digital-Back Propagation.**

I. INTRODUCTION

Spatial division multiplexing (SDM) over multi-mode/-core fibres has emerged as a solution to overcome the capacity limit of single-mode fibres (SMFs) [1]. However, the multitude of spatial modes introduces new impairments, namely: group delay (GD) spread [2] given the interplay between differential mode delay (DMD) and linear mode coupling (LMC), inter-mode nonlinear effects [3], and mode dependent loss [4]. Chief among these is the LMC that plays a crucial role at controlling the GD spread, MDL accumulation and the efficiency of the overall nonlinear interactions. Here we study the most common models for nonlinear transmission in SDM fibres over a wide range of LMC and DMD scenarios.

II. METHODS

The models proposed for SDM fibres often use assumptions valid only for extreme LMC regimes. A fibre link operates in the strong coupling (SC) regime for transmission distances L much larger than the coupling length L_c, in the weak coupling (WC) regime for $L \ll L_c$, and in the intermediate coupling regime otherwise ($L \backsim L_c$). While L_c is quantified as the length for which the accumulated LMC XT reaches $XT(L_c) = [e^2 - 1]/[e^2 + 1]$. And, the XT after an arbitrary length z is quantified as $XT(z) = \sum_{v \neq m}[P_v(z)/P_m(z)]$, where $P_v(z)$ is the average power in mode v, and m is the launch mode (and the one that most likely to couple to others).

Nonlinear transmission modelling in SDM fibres involves solving the coupled nonlinear Schrödinger equation (CNLSE) in [5]. Its numerical integration can be achieved considering the three CNLSE operators, dispersion, LMC and nonlinearity, acting independently for a sufficiently short integration step. In such case, the LMC operator can be resolved in two ways: (i) numerically, having to generate random coupling matrices every step with a given coupling strength; (ii) analytically via new Manakov equations derived by averaging the nonlinear operator over all possible LMC realisations. And, in the numerical approach there are 2 main variants referred here as lumped LMC and distributed LMC.

1) Lumped LMC Modelling: in a multi-section model, LMC among non-degenerate modes is included by introducing random unitary matrices every section, with a section length just longer than L_c such that in average $XT = 0$ dB [2]. This approach is very convenient in the linear power regime allowing matching the analytical predictions for GD statistics [2] provided that L_c is $\ll$ than the dispersion length and the walk-off length. In the nonlinear power regime, there is an additional requirement to the applicability of the lumped LMC model, L_c must be $\ll$ the nonlinear effective length ($\backsim 1/\alpha$, where α is the attenuation coefficient)

2) Distributed LMC Modelling: a semi-analytical model capable of describing the LMC for fibres operating in the intermediate regime has been proposed in [6]. In this model all LMC is assumed to arise from core-cladding imperfections which are discretised by dividing the fibre in multiple sections, each with a random displacement of the core center position. The LMC strength is set using a fixed radial displacement and a random azimuth displacement given by a uniform distribution. In this way, introducing a random amount of LMC that in average approximates the desired level.

3) Manakov Equations: averaging the CNLSE over all possible occurrences of the LMC operator for the WC- and SC-regimes. In the SC-regime, the averaging considers random full coupling between all modes [3] and assumes that all modes propagate with similar GD (this is, DMD should not be higher than a few ps/km). In the WC-regime, the CNLSE is averaged considering randomly coupled modes within each mode group and neglecting inter-group coupling.

III. RESULTS AND DISCUSSION

This section reviews the comparison of the models discussed above for full system simulation [7]. A symmetric implementation of the split-step Fourier method is used to solve the CNLSE, the step size is adapted to keep the local error smaller than 10^{-5}. Transmission simulations consider: a few-mode fibre with 12 polarisation modes; spans of 35 km to minimize the total energy requirement [8]; an optical super-channel with

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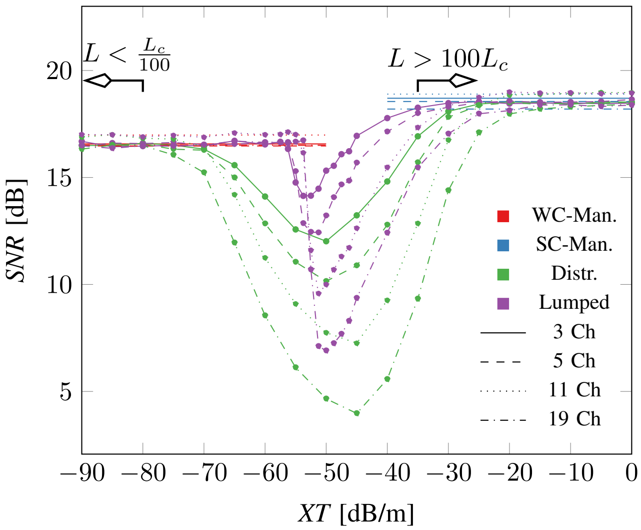
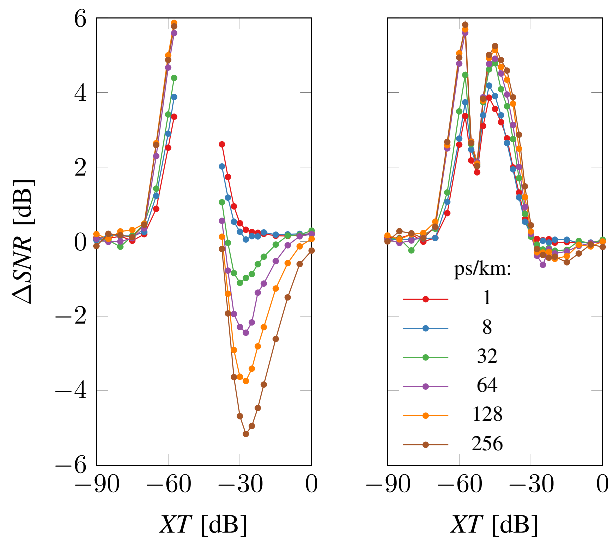


Fig. 1. SNR as a function of XT at 0 dBm/ch with DMD = 0 ps/km, for: 3 channels over 15 spans, 5 channels over 12 spans, 11 channels over 8 spans and 19 channels over 7 spans. Data points averaged over 10 repetitions.



(a) WC- and SC-Manakov. (b) Lumped LMC.

Fig. 2. SNR difference as a function of XT using the distributed LMC model as reference (0 dBm/ch, 3 channels and 15 spans) for different models.

a varying number of WDM channels (per mode) modulated with 14 Gbaud polarization-multiplexed 16QAM, 14.1 GHz spaced. Additional details on the fibre and simulation setup in [7]. The figure of merit in the following is the minimum signal-to-noise-ratio (SNR) among the 12 polarization modes in the center channel. The SNR is evaluated as in [9]: $E[|X|^2]/E[|X-Y|^2]$, X and Y represent the transmitted and received symbols, respectively.

Fig. 1 shows SNR as a function of XT, in the nonlinear regime 0 dBm/ch and absence of DMD, for: 3 channels over 15 spans, 5 channels over 12 spans, 11 channels over 8 spans and 19 channels over 7 spans. The results in Fig. 1 show an excellent agreement between the Manakov models and the lumped LMC and the distributed LMC models in the extreme LMC regimes. However, in the intermediate LMC regime (-70 dB/m to -30 dB/m), out of Manakov applicability, the lumped LMC and the distributed LMC models are found to be in qualitative agreement but not quantitative. More importantly, both models capture a performance dip with XT before the SC-regime is reached. This can be understood noting that as XT increases into the intermediate LMC regime, additional phase rotations (from LMC) allow inter-mode four-wave-mixing phase matching to be achieved for more frequency combinations than it would be possible in the absence of LMC - degrading performance without significant averaging of the nonlinear coefficients. In this case the additional nonlinear penalty grows significantly with the number of WDM channels, as Fig. 1 shows. Eventually, by increasing XT towards the SC-regime, fast random rotations of the hyper-polarization state of the field along the fibre reduce the efficiency of the overall nonlinear process, averaging the nonlinear coefficients, improving performance. In the following the distributed LMC model is taken as the reference; the lumped LMC model assumptions introduce an artificial step degradation for $XT \approx -55$ dB/m.

To further evaluate the applicability of the different methods, Fig. 2 shows the SNR difference in dB ($\Delta SNR = SNR_x - SNR_{dist.}$, x equals WC/SC-Manakov or lumped LMC) as a function of XT for a wide range of DMD and XT values. In general, it can be seen that accuracy degrades with DMD. Fig. 2(a) shows that WC-Manakov generates accurate results even for $DMD > 100$ ps/km. However, Fig. 2(a) shows that SC-Manakov accuracy quickly degrades with DMD. Finally, Fig. 2(b) shows that the lumped LMC model is able to accurately model propagation in the SC-regime even for $DMD > 100$ ps/km [10].

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