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DEVELOPING QUESTIONS AND PROMPTS: ENGLISH PRIMARY TEACHERS' LEARNING ABOUT VARIATION THROUGH LESSON STUDY

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This paper reports doctoral work that seeks to reveal the professional learning of 12 primary teachers in England working collaboratively when developing their practices within a school-based professional development initiative. The common goal was to develop questions and prompts associated with promoting learning from variation during three iterative lesson studies. These cycles were observed through the lens of quarternary analysis to capture individuals' accounts of practice. Thematic analysis of the accounts of teachers' practices revealed four patterns of questions and prompts for promoting learning from variation: describing relationships, comparing, explaining and noticing. The findings reveal how the community's awareness of questions and prompts for pupils to generalize increased while awareness of questions and prompts that promoted 'explanation' reduced. The community's awareness of describing relationships and comparing, on aggregate, remained similar across all three lesson studies but in different proportions.

Introduction

This paper responds to the invitation to contribute to the ICMI Study 25 theme B. It provides a context for teachers working in collaboration and a methodological approach to understand teacher learning in a community of inquiry (Jaworski, 2006). The data resulted from the first author's doctoral work (who is also a primary teacher and primary teacher educator), that seeks to reveal primary teachers' professional learning in England as they develop a pedagogical approach for teaching mathematics in England that is associated with centrally-driven pedagogical reform (Department for Education, 2016). Using a modified form of Chinese lesson study (Yang & Ricks, 2012a), the common goal was to develop teachers' use of questions and prompts with their pupils to promote learning from variation (F. Gu et al., 2017). This paper reports the outcome of the first application of this methodology.

We offer a theoretical overview for how professional learning is conceptualised in the context of this study as well as an observational framework to collect teachers' accounts of other teachers' practices (Simon & Tzur, 1999). It then presents findings on how the community's awareness of questions and prompts that promote pupils to learn from variation changed over the period of three lesson studies. Finally, other possibilities to learn from the outcomes of this trial are discussed, alongside conclusions on how the study might contribute to knowledge on capturing outcomes of teachers' collaborative work.

Theoretical Overview

This section offers a theoretical perspective for teacher professional learning with respect to a particular pedagogical approach for teaching mathematics and how a collaborative community might serve as a catalyst for this professional learning to take place.

Professional learning of mathematics teachers

The notion of teacher professional learning remains a nebulous concept where little agreement within the field resides (Lerman, 2013). Some studies have situated professional learning in terms of identity (Hodgen & Askew, 2011; Potari, 2013) and others in terms of participation in a social setting (Lave & Wenger, 1991); (Skott, 2013). Using Clarke and Hollingsworth's (2002) interconnected model for professional growth as their conceptual framework, Goldsmith, Doerr, and Lewis (2014) conducted a review of literature of 106 studies over the period 1985-2008 that related to mathematics teachers' learning. They identified 10 characteristics of professional learning, one of which was changes in teachers' instructional practice. Simon and Tzur (1999) define teachers' practice as,

> "not only everything teachers do that contributes to their teaching (planning, assessing, interacting with students) but also everything teachers think about, know, and believe about what they do. In addition, teachers' intuitions, skills, values, and feelings about what they do are part of their practice. Thus, we see a teacher's practice as a conglomerate that cannot be understood by looking at parts split off from the whole" (pp. 253-254)

This definition is used in Simon & Tzur's (1999) *accounts of teachers' practice* ethnographic methodology to illuminate professional learning where teachers' practices are analysed from the perspective of the researcher. Simon and Tzur (1999) argue that such an approach offers an alternative lens for research on teacher learning in contrast to evidence that reports teachers' beliefs about their own practices. These accounts can shed light on the nature of pedagogical problems or critical incidents (Lerman, 2013) encountered by teachers. However, Simon & Tzur (1999) only report on individual teachers. They do not comment on whether their sample worked collaboratively with other teachers during their period of the study. However, the *accounts of teachers' practice* methodology (Simon & Tzur, 1999) may also be useful to consider in the context of communities of inquiry. In which case, teachers' accounts of other teachers' practices could be interpreted by the researcher to determine professional learning for that community of teachers. The next section sets out how this might be achieved using Chinese lesson study as a context for professional learning.

Teachers working as a community of inquiry in adapted Chinese lesson study

Chinese lesson study (CLS) is an iterative form of lesson study that is widely used in the collaborative work of teacher research groups (TRGs) in China to design exemplary lessons. Teachers in the group identify a common goal, usually a mathematical topic for a particular year group of pupils and collectively design a lesson which is then taught by one of the teachers in the group and watched by the others. Inter-cycle discussions between the teacher and the observers, which take place immediately after the lesson, contribute to a second iteration of the lesson plan. This lesson is then taught by another member of the group, usually a couple of weeks after the first iteration. In CLS, planning and observation of the lessons are informed by *quarternary analysis* (Yang & Ricks, 2012b). There are four features of quarternary analysis: the *key point*: identifying what is to be learned; the *difficult point*: potential misconceptions; the *critical point:* the means by which the learning of the key point is accomplished; and *learning effect*: the observer's interpretation of the pupils' understanding as a result of experiencing the lesson. Hence, this lens provides a fine-grained framework with which to capture the teachers' and researcher's *accounts of teachers' practice*.

Variation Pedagogy

Variation pedagogy is concerned with how teachers bring into focus for their learners, what is to be learned, by helping them to notice what is varied against a background of invariance (Marton & Booth, 2013) or by experiencing invariance against a background of variation (L. Gu, Huang, & Marton, 2004). Such a focus on variation has been observed as an indigenous practice (Sun, 2011) in mathematics teaching in Shanghai, known as *Bianshi* (F. Gu et al., 2017) or 'teaching with variation'. One form of Bianshi is *procedural* variation (L. Gu et al., 2004; Sun, 2013) which seeks to promote pupils' deductions of generalisations from sequences of calculations where one feature is varied at a time. E.g., by working on the calculations in the sequence: 16 + 9 = ( ); 17 + 9 = ( ); 18 + 9 = ( ) where 9 is kept invariant, the students should experience that the value of the digit in the 'ones' position of the sum of the two numbers is always 1 less than the value of the digit in the ones position of the first addend. It can be deduced that the invariant feature, 9, can be the cause of this phenomena. This type of carefully considered sequence for the purpose of deductive reasoning and generalization is not commonly used by primary teachers in England but features as part of a wider pedagogical reform happening there at the time of writing (Department for Education, 2016). The relevance of its use in England is discussed in an earlier paper (Jacques, 2018). Little, however, is reported in the literature about the explicit role the teacher plays in supporting pupils to learn from such sequences, particularly in classrooms where variation pedagogy is not an indigenous practice.

Methodology

A group of 12 primary (Year 1-6) teachers (11 teachers and 1 researcher/teacher - first author) chose to participate in a professional development (PD) initiative (workgroup) that was advertised by a regional network for mathematics PD. The advert made explicit the common goal of the workgroup: developing classroom discussion to promote learning from variation. The intention was to develop teachers' skills to orchestrate pupil/teacher discussion about the variation sequences. These skills include using questions and prompts for pupils when pupils are working on variation tasks. The teachers did not know one another prior to involvement in the programme and none had previously used procedural variation in their teaching.

A modified CLS was used by designing a lesson 'extract' that would take approximately 20 mins to teach, this was called a *learning episode*. Three iterations of the learning episode took place in one gathering. A different teacher hosted each lesson study at his/her school. The host teacher identified an area of arithmetic that his/her pupils were working on and prior to the first iterative cycle, presented some examples of pupils' work to help the group to decide on the key point, difficult and critical point (Yang & Ricks, 2012b) that would then influence the decision about the design of sequence of examples (the task). Each iterative cycle took approximately 50 minutes (planning, teaching, reflecting, redesigning) and each involved three different groups of six to eight children from the host teacher's class.

The researcher taught cycle 1, another teacher from the group taught cycle 2 and the host teacher taught cycle 3. The observing group of teachers captured accounts of practice (Simon & Tzur, 1999) using a common proforma based on the quaternary analysis framework (Yang & Ricks, 2012) referred to as a 'learning episode notes sheet' or LENS (Table 1). After each learning episode the teachers shared and compared their accounts of practice with one another and used this to redesign

the sequence and teaching. The researcher acted as facilitator and contributor to the discussion each time and also completed a LENS.

Table 1: Transcript from one teacher's LENS

Task	Examples from accounts of practice	
12 x 9 □12 x 8 13 x 8 □12 x 8 96 ÷ 6 □ 96 ÷ 8 96 ÷ 12 □96 ÷ 26	What pupils say/ do	What teacher says/ does
Key Point *Compare by thinking about the number of groups.*		What do you notice? Discuss what you can see
Difficult Point *Pupils wanting to perform the calculation*	Working out each calculation.	What do you notice? What's different? Any patterns? Arrays.
Critical Point *Representing relationship between two multiplication or two division calculations.*	Puts them into a word problem. I know that 12 x8 is smaller than 12 x 9.	Does is matter what the answer was? What is the same? It's one extra lot of 8.

Data Analysis & Results

An overview of the available data for analysis is described in Table 2. Not all teachers were present for all lesson studies.

Table 2: Attendance and data sources for each lesson study.

Lesson Study	Lesson Study 1 (LS1)				Lesson Study 2 (LS2)			Lesson Study 3 (LS3)		
Cycle Number	Cycle 1 LS1C 1	Cycle 2 LS1C 2	Cycle 3 LS1C 3	Cycle 4 LS1C 4	Cycle 1 LS2C 1	Cycle 2 LS2C 2	Cycle 3 LS2C 3	Cycle 1 LS2C 1	Cycle 2 LS2C 2	Cycle 3 LS2C 3
Participants Present	9	10	10	10	8	8	8	7	7	7
No. of LENSs	39				24			21		

These data was interrogated using a mix of thematic analysis (Braun & Clarke, 2006) and frequency analysis. The objective was to identify patterns of teachers' awareness of the questions and prompts used when promoting learning from variation during the lesson studies.

Thematic Analysis

The set of 39 LENSs (see table 1) were interrogated by hand. Notes were made of any similarities between what had been recorded by the teachers. After all of the LENSs had been read through once, this process was repeated to check the emerging patterns in teachers' accounts. Repeating this process meant that similarities and differences that were noticed later on could become the focus of attention in the LENSs that had been looked at prior to when those patterns emerged. After two rounds of the familiarisation exercise, possible codes for what the teachers had noticed were defined.

Six codes categorised the phrases the observing teachers had noticed being used in the lesson studies. The term 'phrases' includes comments, prompts or questions. This process of codifying was based on the first author's interpretation and classified according to the *purpose* of the phrases (i.e. what the teacher might have been drawing from the children. Each documented phrase was assigned uniquely to one code for the purpose of this analysis. Table 3 provides some examples of the kinds of recorded phrases documented by the observing teachers and their classification. The first four categories, describing relationship, comparing, explaining and generalising were grouped into a single theme: *navigational questions and prompts for promoting learning from variation*.

Table 3: Sample of phrases illustrating the codes

Code	Example
Describing Relationship	Are there any other links? How did this answer help you to find this answer? What happens to the answer when…?
Comparing	What's the same about…? What's different…?
Explaining	What did you notice? Can you explain…?
Generalising	Can you see a pattern? Is there another similar one you could create?
Meta-cognition	Can anyone help him? How did that help you?
Other	What does this symbol "<" mean? Can you speak a little louder?

Frequency Analysis

Lerman (2013) claims that teacher learning may occur when they experience a disequilibrium and is most likely to be evident when teachers interact or reflect on critical incidents leading to a new experience which must then connect with the the teachers existing network of knowledge, a theoretical construct Simon (2013) defines as "major assimilatory structures". He claims that these structures are difficult to change because the network is complex. Yang and Ricks' (2012) quaternary analysis illuminates disequilibrium by assuming that what the observing teachers choose to record in their accounts of practice during the learning episode, are moments of disequilibrium experienced by the observers themselves or by the teachers and pupils they are observing. However, a single experienced moment of disequilibrium may not in itself be an indicator that learning has happened. A chronological sequence of the disequilibriums experienced over the course of the PD programme is assumed to constitute a learning journey for the community of teachers.

The thematic analysis exercise revealed that some codes were accounted for by teachers more frequently than others. Therefore, the next step was to interrogate the data for the frequencies of each of the types of navigational questions and prompts associated with promoting learning from variation for each lesson study (Table 4). These aggregated data provide an account of the community of teachers' patterns of awareness of question types unique to navigating variation problems as well as other questions and prompts (e.g. meta-cognition and other).

Table 4: Frequency Distribution of question/prompt type in each lesson study (LS)

Theme	Navigational questions and prompts for PLV						
Code	Describing Relationship	Comparing	Explaining	Generalising	Meta-cognition	Other	Total phrases noticed
LS1	53	38	95	9	3	66	264
LS2	35	16	50	5	7	30	143
LS3	12	28	17	10	7	31	105

Discussion

The aggregated data for each lesson study revealed the proportion of different questions and prompts that the teachers were aware of over the course of the three lesson studies each with a different mathematical focus (Figure 1).

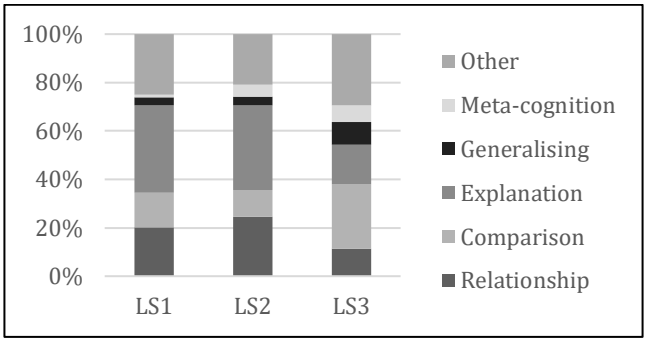

Figure 1: Proportion of question/ prompt type per lesson study

The accounts of practices generated similar profiles in the first two lesson studies. In contrast, in the third lesson study some examples of potential changes in professional learning within the community could be interpreted. The goal of promoting learning from variation is to recognise generalised features of an object of learning. The data shows that in LS1 teachers were most aware of questions and prompts to encourage the pupils to *explain* whereas they were least aware of questions and prompts associated with encouraging pupils to *generalise*. Although generalising was the category with the smallest proportion in each lesson study, by the third, the teachers were recording more instances of questions and prompts to promote generalisations and noting fewer instances of questions and prompts to promote pupils' explanations. One possibility is that the community's inter-cycle evaluation and redesign discussions drew out the need for the teachers to use more questions and prompts to promote pupils to generalise, therefore the community became sensitised to its prominence. An alternative suggestion is that the teachers used a greater number of phrases associated

with promoting generalisation over the course of the lesson studies and so increased the occurrence of these phrases. Another explanation might be a combination of the two, where the discussions not only inspired the teachers to increase the use of these types of questions and prompts but also increased awareness of them at the same time.

Similarly, the decrease in the use of questions and prompts associated with promoting pupils to use explanations might be explained in multiple ways. For instance, the high frequency in LS1 may have suggested that teachers were most sensitised to this category when observing teaching (from their existing construct of teaching). Over time the inter-cycle discussions may have shifted the community's attention to other purposes of questions and prompts. Alternatively, it may have been that the teachers used fewer questions and prompts to promote explanation in favour of other types associated with promoting learning from variation.

Before it is possible to generalize, relationships between features of a mathematical object must first be noticed. When using a task designed with procedural variation, this can be accomplished by prompting pupils to make comparisons between deliberately varied examples. Together, the two categories of describing relationships and comparing were noticed in similar proportion across all three lesson studies. Awareness of phrases associated with promoting pupils to describe relationships was highest in LS2 and lowest in the LS3 whereas the phrases associated with comparing were lowest in LS2 but highest in LS3. A transcript of the lesson might reveal possible explanations such as the content of what was being taught required the teachers to spend more time prompting pupils to compare examples in LS3 than in LS2.

Conclusion

This analysis has revealed changes in teachers' awareness of questions and prompts associated with promoting learning from variation when participating in three iterative lesson studies where the focus was on developing the use of procedural variation tasks. The teachers' accounts of teachers' practices recorded an increase in the use of questions and prompts that require pupils to make generalisations. This innovative methodology has also revealed additional data that would enhance the reliability and validity of the study which may help to understand how the collaborative work of the teachers contributed to these changes. In the next stage of the research methodology, video recordings of the lesson studies and inter-cycle discussions will be used to enable further triangulation between accounts of practice, revealing more of the actual work of the community, and supporting the researcher's interpretations and analysis.

There are also further grain-sizes of data to interrogate. For example, by using quaternary analysis, it may be possible to notice patterns in the use of the questions and prompts for the key point, difficult point and critical point. Similarly any changes noted within the community of inquiry as a whole do not take account of the contributions that individual teachers make and how that may skew the outcomes of the work of the community of inquiry. Therefore, the data should also provide a window on individual teachers' awareness of questions and prompts associated with promoting learning from variation.

The analysis was guided by accounts of practices, but not only from the researcher's perspective as suggested by Simon and Tzur (1999) but also from the collective perspectives of the teachers in a community of inquiry (Jaworski, 2006). The researcher's LENS was treated equally alongside the

teachers. By collating and interpreting these accounts, it is possible to interpret outcomes of the work of the community. It may also be useful to include pupils' accounts of practices as an additional lens, whereby the community of teachers are then able to take into account their interpretations of the pupils' experiences in the lesson study cycle.

This analysis has responded to the invitation to contribute to the ICMI Study 25 theme B. It has illuminated outcomes from a context of mathematics teacher collaboration which in turn has revealed further lines of enquiry for this study and other studies wishing to reveal professional learning of mathematics teachers working collaboratively.

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