THE INVESTIGATION OF OPACITY IN THE
JET TOKAMAK DIVERTOR REGION

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Abstract

The purpose of this thesis is to investigate Lyman line absorption by deuterium atoms in the divertor region of the JET tokamak for four high density, low temperature, detached plasma pulses. A collisional radiative model of deuterium level populations has been used to estimate the extent of Lyβ radiative absorption in the divertor along the same line of sight as a VUV spectrometer. This uses a first order escape probability method to evaluate the line escape probabilities and gives a self consistent model of the level populations and radiation field. These results are compared with experimental measurements of the branching ratio of Lyβ to Dα from the VUV spectrometer and various visible diagnostics. Both the theoretical and experimental results agree that opacity reduces the level of Lyβ emission from the divertor plasma.

The effects of opacity on the ionisation and power balance of the plasma are examined for various conditions. The results of this investigation are compared with other theoretical work in the field. It is shown that the levels of opacity are not great enough to significantly alter the ionisation and power balance of the plasma for the conditions presently being created within the JET tokamak.

The population code requires information about the background plasma. This can be provided by either a fluid code or an ‘onion-skin’ plasma simulation. Both models are used in this investigation and their levels of accuracy are compared. Finally, a brief investigation into the level of opacity in a future tokamak, ITER, is carried out using predicted plasma profiles. It is shown that opacity levels in the divertor region of the ITER tokamak could match those of JET and by creating highly detached plasmas could easily exceed these levels.
Acknowledgements

Throughout the course of this study I have been privileged in having two exceptional supervisors, Dr Lorne Horton at the JET project and Dr Peter Storey at University College London. Their advice, guidance and insight have been invaluable during this work. I must also thank all my friends and colleagues at JET and UCL, especially Awi Gondhalekar, Kerry Lawson, Hans Lingertat, Alberto Loarte, Akko Maas, Costanza Maggi, Ray Monk, Mike Stamp, Peter Stangeby and Paul Thomas.

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<td>AU</td>
<td>Arbitrary Units</td>
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<tr>
<td>ADAS</td>
<td>Atomic Data Analysis Structure</td>
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<tr>
<td>ASDEX</td>
<td>Axial Symmetric Divertor Experiment</td>
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<tr>
<td>CCD</td>
<td>Charge Coupled Device</td>
</tr>
<tr>
<td>D-D</td>
<td>Deuterium-Deuterium</td>
</tr>
<tr>
<td>D-T</td>
<td>Deuterium-Tritium</td>
</tr>
<tr>
<td>DIII-D</td>
<td>Doublet III-Divertor Tokamak</td>
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<td>D$\alpha$</td>
<td>Balmer-Alpha</td>
</tr>
<tr>
<td>D$\gamma$</td>
<td>Balmer-Gamma</td>
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<tr>
<td>ECE</td>
<td>Electron Cyclotron Emission</td>
</tr>
<tr>
<td>ECRH</td>
<td>Electron Cyclotron Resonance Heating</td>
</tr>
<tr>
<td>ELM</td>
<td>Edge Localised Mode</td>
</tr>
<tr>
<td>FWHM</td>
<td>Full Width Half Max</td>
</tr>
<tr>
<td>ICRH</td>
<td>Ion Cyclotron Resonance Heating</td>
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<tr>
<td>IERM</td>
<td>Intermediate Energy R-Matrix</td>
</tr>
<tr>
<td>ITER</td>
<td>International Thermonuclear Experimental Reactor</td>
</tr>
<tr>
<td>JET</td>
<td>Joint European Torus</td>
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<tr>
<td>LCFS</td>
<td>Last Closed Flux Surface</td>
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<tr>
<td>LIDAR</td>
<td>Light Detection and Ranging</td>
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<tr>
<td>LTE</td>
<td>Local Thermodynamic Equilibrium</td>
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<tr>
<td>Ly$\alpha$</td>
<td>Lyman-Alpha</td>
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<tr>
<td>Ly$\beta$</td>
<td>Lyman-Beta</td>
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<tr>
<td>MCP</td>
<td>Micro-Channel Plate</td>
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<td>NLTE</td>
<td>Non-Local Thermodynamic Equilibrium</td>
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<tr>
<td>O-S</td>
<td>Onion-Skin</td>
</tr>
<tr>
<td>PDA</td>
<td>Photo Diode Array</td>
</tr>
<tr>
<td>Paschen-Alpha</td>
<td>Paschen-Alpha</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>SOL</td>
<td>Scrape-Off Layer</td>
</tr>
<tr>
<td>SPRED</td>
<td>Survey, Poor Resonance, Extended Domain</td>
</tr>
<tr>
<td>VUV</td>
<td>Vacuum Ultra-Violet</td>
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<tr>
<td>1-D</td>
<td>One-Dimension</td>
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Chapter 1

Introduction

1.1 Thermonuclear Fusion and the JET Project

The majority of material in the universe is in the form of ionised gas or plasma, ranging from the low density low temperature plasma of the interstellar medium to the high temperature dense plasmas found in the interiors of stars. These latter conditions produce the ideal environment for thermonuclear fusion and a source of power for the Sun and stars.

As the Earth’s fuel reserves are decreasing, this method of producing energy is becoming highly attractive and many projects have been set up in an attempt to recreate these fusion reactions in the laboratory.

The process of nuclear fusion releases energy by the fusion of light nuclides such as deuterium, tritium, helium-3 and lithium. Examples are

\[
\begin{align*}
D + T & \rightarrow ^4\text{He} \ (3.52 \text{ MeV}) + n \ (14.06 \text{ MeV}) \quad (1.1) \\
D + D & \rightarrow ^3\text{He} \ (0.82 \text{ MeV}) + n \ (2.45 \text{ MeV}) \quad (1.2) \\
D + ^3\text{He} & \rightarrow ^4\text{He} \ (3.67 \text{ MeV}) + n \ (14.06 \text{ MeV}) \quad (1.3)
\end{align*}
\]
Deuterium comprises 0.015% of the hydrogen in water and is therefore easily obtainable. Tritium, on the other hand, has to be manufactured. As lithium is available in large quantities the most convenient way of manufacturing tritium is via the processes given in Equations 1.4 and 1.5. In these processes a lithium blanket thermalises the neutrons for heat production and breeds tritium.

\[ ^6\text{Li} + n \rightarrow ^7\text{T} + ^4\text{He} + 4.8 \text{ MeV} \quad (1.4) \]

\[ ^7\text{Li} + n \rightarrow ^7\text{T} + ^4\text{He} + n - 2.5 \text{ MeV} \quad (1.5) \]

Although deuterium is more widely available the D-D reaction requires higher temperatures and the D-T reaction has the largest cross-section and energy release. Consequently, the costs of a higher temperature reactor must be compared to those of breeding and reproducing tritium when deciding whether a D-D or a D-T reactor should ultimately be aimed for.

For a D-T reactor (Figure 1.1) a lithium blanket surrounding the plasma slows down the neutrons, which carry away the majority of the energy from the reaction, converting its kinetic energy into heat energy. The electrically charged helium becomes trapped in the plasma where it deposits its energy, thus making the fusion reactor igniting. Furthermore, the lithium blanket creates the tritium via the reactions in Equations 1.4 and 1.5 and the high temperature lithium gives up its heat to a heat exchanger which generates electric power.

To provide sufficient energy to overcome the Coulomb repulsion of positively charged nuclei, thus enabling them to fuse together, high temperatures are necessary. Consequently, for the D-T reaction the gaseous fuels need to be heated to temperatures of at least $10^6$K. The fusion reactions must therefore produce more energy than it takes to produce and maintain the hot plasma.

The power created by fusion ($P_f$) is distributed between the $\alpha$-particles ($P_\alpha$) and the neutrons ($P_n$) according to the relative masses of the particles. Hence neutrons, having the least mass, carry most of the power (four-fifths to be precise) away from the plasma, whilst the $\alpha$-particles, which remain inside the plasma, retain a smaller fraction of the power (the remaining fifth). In a reactor the neutron power can be converted from heat into electrical power to refuel further fusion. The efficiency of the conversion from thermal to electric power is denoted by the coefficient $\eta$. 
In addition to the power lost via the neutrons there is further power loss from the plasma from other processes such as convection, conduction \((P_{ce})\) and radiation \((P_r)\), as described by Miyamoto (1989).

Convective and conductive power, or energy loss of the plasma per unit time, goes mainly to the target plates and partially to the walls of the machine. It is given by

\[
P_{ce} = \frac{3nkTV}{\tau}
\]  

(1.6)

where \(n\) is the plasma density, \(k\) is the Boltzmann constant, \(T\) is the plasma temperature, \(V\) is the plasma volume and \(\tau\) is the energy confinement time.
There are two forms of radiative power loss; these being bremsstrahlung and line radiation. The former is produced by the collision of an electron with an ion in the plasma and can be calculated from

$$P_b = 1.5 \times 10^{-38} Z^2 n^2 V (T/e)^{1/2} \text{ W}$$  \hspace{0.5cm} (1.7)

where \( Z \) is the ratio of the ion charge to electron charge.

Line radiation is produced mainly by impurities and to a lesser extent by deuterium causing a power loss \( P_l \). The general radiative power loss term incorporates both bremsstrahlung and impurity radiation, given as

$$P_r = P_l + P_b$$  \hspace{0.5cm} (1.8)

Lawson's condition (Equation 1.9) describes the energy balance in a fusion reactor taking into consideration the efficiency \( \eta \) of thermal to electric energy conversion.

$$P_r + P_{cc} = \eta (P_n + P_r + P_{cc})$$  \hspace{0.5cm} (1.9)

To be able to use thermonuclear fusion as a source of energy it is necessary that the energy needed to maintain the reaction is less than \( \eta \) times the energy released from the plasma, including the thermonuclear fusion energy.

An igniting reactor has not yet been produced and consequently additional heating is needed to maintain fusion. The Joint European Torus (JET) tokamak, shown in Figure 1.2, does not utilise the energy lost to refuel fusion, as a reactor does, but relies entirely on various heating mechanisms such as radio-frequency (RF), ohmic and neutral beam heating, which will be described in Chapter 5.
The heating power \( P_h \) necessary to sustain fusion can be expressed in the power balance equation (Wesson, 1997).

\[
P_h + P_a = P_L \tag{1.10}
\]

where \( P_L \) is the total power lost from the plasma and can be expressed as

\[
P_L = P_n + P_t + P_{ic} \tag{1.11}
\]

In a reactor a fraction of this energy, depending on the thermal to electric conversion efficiency, would contribute to \( P_h \).

For a fusion reaction to occur, according to Equation 1.10, the following triple product must be satisfied

\[
n_i \tau T_i > 5 \times 10^{21} \text{m}^3\text{sKeV} \tag{1.12}
\]
To obtain this for a magnetically confined plasma, such as that created in a tokamak, the individual parameters should take the values

\[
\begin{align*}
\text{Central ion temperature} & \quad T_i = 10-20\text{KeV} \\
\text{Central ion density} & \quad n_i = 2.5 \times 10^{20} \text{m}^{-3} \\
\text{Energy confinement time} & \quad \tau = 1-2\text{s}
\end{align*}
\]

Due to the high temperatures needed for fusion it is essential that the plasma is confined from the walls of the vacuum chamber. This is done to reduce melting of the wall tiles and impurity contamination from sputtering. Sputtering occurs when high energy particles strike the wall of the tokamak causing erosion and hence impurity contamination.

In magnetic confinement fusion, magnetic fields are used to isolate the plasma from the walls of the chamber. In a tokamak this is achieved by three magnetic field components, as shown in Figure 1.3.

![Figure 1.3. Schematic diagram of the magnetic field configuration in the JET tokamak.](image)
The toroidal magnetic field, around the major axis of the tokamak, is produced by a set of coils around the minor circumference (Figure 1.4). This is the principal magnetic field, however a poloidal magnetic field is also necessary to balance the plasma pressure. In a torus the toroidal magnetic field gets weaker with increasing major radius, due to the increasing radius of curvature. The gradient in the magnetic field causes transverse electron and ion movement. However, the electrons and ions move in opposite directions. In order to oppose this charge separation an electric field forms. The combination of the magnetic and electric fields leads to a force perpendicular to these fields, which causes a particle drift. Hence, a poloidal field is necessary to compensate for this force. Transformer action produces a large current flow through the plasma in the toroidal direction which creates the poloidal field.

![Figure 1.4. Schematic of the coils used to produce the magnetic fields in the JET tokamak.](image)

The combination of the toroidal and poloidal magnetic fields produce a helical magnetic field which prevents the plasma from touching the walls of the vacuum vessel. A set of hoopcoils produce a third toroidal magnetic field component which shapes and stabilises the position of the plasma.

Ohmic heating provides enough energy to produce and initially heat the plasma. The large electric current, flowing through the plasma, creates the poloidal field. However, as the
temperature of the plasma increases the electrical resistance of the plasma decreases, and hence ohmic heating cannot attain the temperatures necessary for fusion.

Consequently, an additional heating method is necessary to overcome this problem. There are two such heating systems in JET. The first is neutral beam heating, where a beam of charged hydrogen or deuterium, or possibly even helium (\(^3\)He and \(^4\)He), ions are accelerated to high energies towards the plasma. These charged particles are then neutralised by being passed through a gas. The resulting neutral atoms are then able to cross the magnetic field and, via collisions, transfer their energy to thermal energy in the plasma. The second additional heating system is RF heating. In this process, high power RF waves transfer energy to the plasma by oscillating with the same frequency at which the ions or electrons gyrate in the magnetic field. There are two forms of RF heating, these being ion cyclotron resonance heating (ICRH) and electron cyclotron resonance heating (ECRH). The former process causes ion oscillation and is the method used in JET, the latter process causes electron oscillation.

The project has achieved a core plasma triple product value of greater than \(1 \times 10^{21}\) m\(^3\)sKeV. A central ion temperature of 18KeV has been achieved and an energy confinement time of 1s. It should be noted, however, that each term in the triple product does not peak simultaneously, as an increase in one typically results in a drop in the others. This is due to pressure limitations brought about by instabilities such as Edge Localised Modes (ELMs), which are described in Chapter 5. As a result of these instabilities, increases in the temperature tend to cause a decrease in the density and vice-versa.

There is a large contrast between the core plasma and the divertor plasma. In the divertor region (shown in Figure 1.5) the densities are higher and electron temperatures lower than that of the core.

The divertor, a system of coils and targets, is necessary to reduce target erosion and sputtering effects. Erosion causes low Z impurities, such as carbon and beryllium, which form the tiles of the tokamak, to be released into the plasma which dilutes the number of effective ions available in the plasma for productive fusion reactions.
Two methods of reducing the heat load on the targets are tilting the target plate to increase the effective area and flux expansion, which is the spreading of magnetic field lines near the target. However, despite the benefits of flux expansion the power density on the tiles is still likely to be too high unless there is a significant loss of energy across the field lines (a volume sink).

The divertor is a closed system with large interactive cooled areas which provides a solution to this problem. This is possible because in this cooled region a high neutral deuterium density builds up due to neutral recycling (described in Chapter 2). Interaction of the plasma flux with the neutral deuterium particles reduces the power flux reaching the target by two processes. The first is charge exchange which reduces the momentum of the ion particles. The second process can be explained by considering an electron, which flows towards the
target, colliding with a neutral atom. The collision reduces the momentum of the colliding electron, transferring this energy into exciting the neutral atom’s trapped electron. This trapped electron then decays by radiation, releasing a photon in any direction, over $4\pi$. Hence the energy reaching the target is reduced considerably, thereby causing a volume sink. The majority of the power that does reach the target has been radiated and scattered, by the interaction, which enables the heat load of the plasma to be absorbed over a much larger area of the target plates.

Typical divertor parameter values produced are ion and electron densities of approximately $10^{20}\text{m}^{-3}$, neutral deuterium densities of approximately $10^{18}\text{m}^{-3}$ and temperatures of $5\text{eV}$, although larger density, lower temperature plasmas are also produced. These larger densities and lower temperatures are favoured in the divertor region to minimise sputtering effects and impurity contamination and also favour opaque plasmas.

Opaque plasmas in the divertor region of a tokamak could be useful to reduce the light energy and the speed of particles hitting the target plates of the tokamak, therefore reducing sputtering target erosion. However, lack of knowledge on the effects of opacity can be harmful since it can possibly lead to inaccurate plasma modelling and misleading results when measuring power balance and ionisation and recombination levels.

1.2 Previous Studies of Opacity Effects within Magnetically Confined Fusion Plasmas

The majority of research involving opacity has been applied to stellar atmospheres and space physics. Investigations into the effect of opacity in fusion have been limited mainly to the field of laser produced plasmas, with very little research being carried out for magnetically confined plasmas. However, with increasing tokamak sizes the likelihood of opacity effects occurring becomes higher.

Opacity becomes an issue when electron and ion densities are of order $10^{20}\text{m}^{-3}$, the neutral deuterium density is of order $10^{18}\text{m}^{-3}$ and the temperature is below $5\text{eV}$. These conditions are being satisfied individually in small areas of the JET divertor region for short periods. The question is, are these conditions being satisfied simultaneously over a large enough area to
cause significant levels of absorption? More research into the extent of opacity and its effects on the surrounding plasma environment is necessary to answer this question.

A rare example of a study into opacity in a tokamak has been undertaken by Wan et al. (1995), where the importance of line and continuum radiation on the power balance of a high-recycling radiative divertor is assessed.

This task is performed using a non-local thermodynamics equilibrium (NLTE) code, CRETIN, which employs a one-dimensional configuration using parameters outside of the magnetic separatrix. Further details of the code and the methods Wan employs to perform this complex task are given in Wan et al. (1995).

In summary, Wan's investigation shows that when introducing density and temperature profiles that are consistent with those in a high-density plasma near the divertor plate into CRETIN, the plasma is optically thick to Lyα line radiation. This reduces the power flux to the divertor plate. It also shows that ionisation and recombination rates depend strongly on electron temperature, electron density, neutral density, emissivity and opacity of the radiation field and that an optically thick Lyα line can disturb the ionisation balance of the cold divertor plasma.

Another more recent example of research into the effect of opacity on a tokamak plasma was undertaken by Behringer (1997). In this, a collisional-radiative model (the ADAS208 atomic physics code) is employed to calculate excitation, ionisation and recombination coefficients for neutral hydrogen, for optically thin and thick plasma parameters. Using this information, a comparison is then made between the ionisation balance and hydrogen line emission of an optically thin and optically thick plasma.

Behringer shows that the opacity of Lyα leads to an increase in the level of ionisation by almost an order of magnitude and to a similar increase in the degree of hydrogen line radiation.
1.3 Thesis Outline

The primary aim of the research in this thesis is to determine whether plasmas with a significant level of opacity are created in the divertor region of the JET tokamak.

The first part of this thesis (Chapters 2 to 4) deals with the theoretical aspect of this study. In this, real plasma pulse parameters, in addition to those of simulated plasma pulses from the JET tokamak, are modelled using two different methods; these being, a fluid code (EDGE2D) and an ‘onion-skin’ (O-S) model. Basic descriptions of these models and the codes which accompany them in the plasma modelling process are given in Chapter 2.

Chapter 3 and 4 describe the population code, a collisional-radiative model, which calculates line emissivities and determines, according to the input parameters produced by the plasma modelling process, whether a significant amount of line absorption occurs. The former chapter describes the collisional aspect of the code, whilst Chapter 4 describes the basics of opacity theory and how this theory is incorporated into the population code.

In order to validate the theoretical investigation, an experimental study is undertaken into the level of absorption that is measured by diagnostics for the same plasma pulses as those being modelled in the theoretical investigation. The description of this experimental examination and the analysis of the results produced from it are given in Chapter 5.

Chapter 6 provides the results of the theoretical investigation and a comparison is made with those determined from the experimental study. The secondary aim of this thesis is to examine the effects of opacity on a plasma. Although more basic than the aforementioned investigation, an attempt is made to determine whether opacity can alter the ionisation and power balance of a plasma. This covers similar ground to that of Wan and Behringer and is also described in Chapter 6. This chapter also includes a general study into the effects of changing various plasma parameters on its opacity.

Finally, Chapter 7 includes a brief investigation into the possible levels of opacity that we might expect to find in the larger tokamaks of the future. This is achieved by examining predicted physical conditions in the International Thermonuclear Experimental Reactor (ITER) divertor and predicting the levels of Lyβ absorption as a result of these conditions.
Chapter 2

Modelling the Background Plasma and Impurity Transport in the JET Tokamak

2.1 Introduction

In this chapter we are considering how to model the divertor plasma by utilising information from specific JET plasma pulses. These pulses are chosen at particular time slices to represent low temperature divertor plasmas with extreme changes in density, which may cause opacity effects. The modelling procedure, described shortly, is performed until it provides a plasma background that is consistent with diagnostic data and can be used as input to the population code. The population code, described in Chapters 3 and 4, provides information on spectral line absorption and can be used, in conjunction with experimental data, to determine the extent of opacity in the divertor region.
2.2 The Plasma Modelling Procedure

The aim of modelling the plasma is to produce a complete two-dimensional (2-D) description of the edge and divertor plasma which is consistent with available diagnostic measurements. Plasma conditions vary dramatically over short distances and so a method of storing plasma information in small partitions is necessary. The plasma models calculate and produce sets of parameters each representing an area of the plasma called a cell, which is not a real entity within the tokamak but a convenient method of storing data.

The cell parameters of this plasma can then be easily averaged in a selected region vertically above the target plates and between the separatrix and the wall. This averaging process is necessary before data is entered into the population code since, in our model, it considers uniform slabs of plasma. The parameters that undergo the averaging process are the electron temperature and density, the neutral density, the effective neutral temperature and the neutral velocity shifts in three orthogonal directions.

To describe the plasma modelling procedure in its simplest terms three basic sources of information are required; these being a plasma solver, an impurity solver and a neutral model. The impurity solver consists of a program which calculates the impurity properties given the properties of the background plasma. Similarly, the neutral model is a program which determines the neutral properties of a plasma in accordance with the background plasma. These models interact until they are consistent with each other and a plasma model can be provided. Figure 2.1 shows a flow chart which outlines the main stages in the plasma modelling procedure.
There are two methods of modelling the background plasma. The first uses EDGE2D, a 2-D fluid code, the second method uses an O-S model which maps probe data in one-dimension (1-D) along magnetic field lines. Both methods provide plasma parameters in a form which can be used in DIVIMP, a divertor impurity code. However, EDGE2D has a more detailed approach to modelling the plasma, as will be described later in this chapter, and provides a wider range of parameters.

EDGE2D consists of an impurity ion solver, a plasma solver and a neutral model and solves the fluid equations for the background plasma and the impurities simultaneously. It accesses information from the impurity model and a neutral model (NIMBUS) and is iterated many times until the background plasma is consistent with the impurity and neutral information that it has been provided with. It then sends the background plasma information such as plasma density, drift velocity, background ion and electron temperatures, and the electric field at each grid point, to be put in a standardised format to an external file. From here this information can be accessed by DIVIMP, which consists of a plasma solver and the neutral model NIMBUS. In this case, however, DIVIMP is used simply to provide formatting.
The neutral information can be read from EDGE2D but it is actually easier to recalculate it by one call to NIMBUS. Although NIMBUS has already been used in the formation of the EDGE2D background plasma, the raw neutral information is not passed onto DIVIMP at this stage. Instead DIVIMP accesses the neutral information directly from NIMBUS, when it is needed, for the recalculation of the background plasma.

EDGE2D has a variety of applications in addition to being able to model a pure deuterium plasma. A radiated power prescription, average impurity ion model or a full impurity model can also be accessed. This full impurity model, although using a detailed level of physics and therefore being among the most accurate available, uses a considerable amount of computer time and is consequently not always the most practical model. Similarly, a Monte Carlo impurity solver is available in DIVIMP which is capable of treating all ionisation stages of an impurity separately or an average charge impurity species can be considered. However, this impurity solver is not used for the plasmas being modelled in this thesis, since a pure deuterium plasma is being assumed.

The second method of modelling the plasma background is via the O-S model. This consists solely of a plasma solver which uses probe data that has been incorporated into the input file. It also requires access to neutral information from NIMBUS which is then iterated within DIVIMP until a realistic background plasma has been obtained.

Both EDGE2D and the O-S model require magnetic geometry. This is provided in the form of a grid, shown in Figure 2.2.

The unusual grid shape is due to the fact that it follows magnetic field lines. It is necessary to model transport according to these lines as parallel and perpendicular transport are completely different. The plasma flows freely along the magnetic field lines and 1-D classical transport is usually assumed. By considering collisional, or neo-classical, effects it is possible for a particle, via a collision, to jump from one orbit to another causing cross-field transport. Although neo-classical transport is considered in the calculation of perpendicular transport, it does not reproduce the level of perpendicular transport observed experimentally. Hence, an anomalous factor is added to the neo-classical term and adjusted until it produces a level of perpendicular transport which forces the scrape-off layer (SOL) to be of the same thickness as observed experimentally. This additional term is considered to represent the perpendicular transport caused by turbulence induced transport.
Figure 2.2. Computational grid used in EDGE2D and DIVIMP with (a) showing the entire tokamak cross-section and (b) a close up of the divertor region.
The grids are calculated using the results of a 2-D magnetic equilibrium code (EFIT). A grid divides the plasma into cells defined by their centre, the vertices of their boundaries and by elemental volumes. The information in the grid file includes the time slice, the coordinates of the magnetic axis, co-ordinates of the X-point, toroidal field and a table of values at each of the grid points which define the plasma's structure.

It is not possible to use the O-S model to solve for the core plasma. This is because the O-S model uses target probe data at each end of the open flux surfaces in the SOL and follows that field line, mapping the changes due to conservation equations along the way. It is therefore obvious that this method of modelling cannot apply to the closed flux surfaces of the core where probe data is not available. The core plasma, therefore, has to be defined within DIVIMP by specifying the rate of change of densities and temperatures inwards from the last closed flux surface (LCFS) using data from core diagnostics such as Thomson scattering, interferometry and electron cyclotron emission (ECE).

The EDGE2D solutions, however, do apply within the separatrix as well as in the SOL since the fluid model maps perpendicular as well as parallel transport and can therefore follow conditions across the flux surfaces and into the core. In doing this it assumes perpendicular transport to be constant throughout the plasma.

2.3 Descriptions of the Modelling Codes

2.3.1 NIMBUS

NIMBUS is a neutral code which employs the Monte Carlo method to follow the series of processes that a hydrogenic and impurity neutral undergoes during transport in a plasma. A background plasma must be specified in order to trace neutral profiles. After a sufficient number of neutrals have been followed, profiles of the rate densities for various processes are generated using weighted sums. These profiles include hydrogenic ionisation, carbon sputtering and charge exchange as well as the density and temperature of neutrals for both the hydrogenic and impurity species.

Neutral recycling is a process where incident plasma ions, mainly protons, are neutralised at the target plates due to ion-surface interactions. Depending on the ion energy and the
material of which the target is constructed, a substantial amount of the ion flux is scattered back as atoms. In steady state conditions the target material saturates and so the protons that are not backscattered are re-emitted as low energy neutrals, in this case deuterium molecules, with their kinetic energy depending on the target surface temperature. These deuterium neutrals pass through the collisionless plasma sheath into the dense, hot boundary plasma, where they are ionised. The recycling process can then repeat itself.

2.3.2 EDGE2D

The development in this section follows the work of Taroni et al. (1992) and Keilhacker et al. (1991). EDGE2D is a 2-D fluid code that treats transport along and perpendicular to field lines in the SOL, including that of impurities. It is assumed that the deuterium and impurity ions can be described entirely by a set of fluid equations of conservation of particles, momentum and energy. Hydrogenic ions and the ionisation stages of an impurity are also incorporated in the analysis and the assumption is made that all of these have the same temperature, $T_i = T_e$, where $T_i$ and $T_e$ are the impurity and ion temperatures, respectively. The electron temperature $T_e$, however, is taken to be different to that of the ions. Toroidal symmetry is assumed with $\theta$ being a co-ordinate in the poloidal direction, $\phi$ being a co-ordinate in the toroidal direction and $\rho$ being a radial co-ordinate orthogonal to $\theta$. This geometry system for the torus is shown in Figure 2.3.

![Figure 2.3. Schematic diagram of the geometry of the torus showing the co-ordinate system used.](image-url)
The fluid equations are written in the poloidal plane with the vector and tensor quantity components running parallel to the magnetic field lines and perpendicular to the magnetic surfaces. The metrics in the co-ordinates are given in Equation 2.1.

$$ds^2 = H^2_\rho d\rho^2 + H^2_\theta d\theta^2 + R^2 d\phi^2$$  \hspace{1cm} (2.1)

where $R$ is the distance from the axis of symmetry, known as the major radius, and $\phi$ is the toroidal angle. Equation 2.1 expresses an orthogonal system of curvilinear co-ordinates with $H_\rho$, $H_\theta$ and $R$ being the metric projections, or scale factors. These factors are what the differentials of the co-ordinates need to be multiplied by to obtain values for distances (Boas, 1973). The angles $h_\rho = \frac{B_\rho}{B}$ and $h_\theta = \frac{B_\theta}{B}$, where $B$ is the magnetic field vector, and the volume element is given as $Hd\phi d\rho d\theta = RH_\rho H_\theta d\rho d\theta d\phi$.

The fluid equations (Taroni et al., 1992) are given below in the form of the conservation equations of parallel ion momentum, ion density, electron energy and ion energy, respectively, where subscript $e$ refers to electrons, $i$ to hydrogenic ions, $z$ to impurity ions with a charge number $Z$ and $\alpha$ to both hydrogenic and impurity ions.

$$\frac{\partial}{\partial t} (m_\alpha n_\alpha v_\alpha) + \frac{1}{H} \frac{\partial}{\partial \rho} \left( \frac{H}{H_\rho} m_\alpha n_\alpha v_{\alpha,\rho} v_\alpha \right) + \frac{1}{H} \frac{\partial}{\partial \theta} \left( \frac{H h_\theta}{H_\theta} m_\alpha n_\alpha v_{\alpha,\theta} \right) =$$

$$- \frac{h_\theta}{H} \frac{\partial}{\partial \theta} \sum_{\alpha,1} + \sum_{\alpha,\rho} + Z_e n_e E + R_{ae} + \sum_{\alpha,z} R_{ae} + F_\alpha$$ \hspace{1cm} (2.2)

$$\frac{\partial n_\alpha}{\partial t} + \frac{1}{H} \frac{\partial}{\partial \rho} \left( \frac{H}{H_\rho} n_\alpha v_{\alpha,\rho} \right) + \frac{1}{H} \frac{\partial}{\partial \theta} \left( \frac{H h_\theta}{H_\theta} n_\alpha v_{\alpha,\theta} \right) = S_\alpha \hspace{1cm} (2.3)$$

38
\[
\frac{3}{2} \frac{\partial \rho_e}{\partial t} + \frac{1}{H} \frac{\partial}{\partial \rho} \left[ \frac{H}{H_\rho} \left( \frac{5}{2} \sum p_e v_{\epsilon,\rho} - \kappa_{\epsilon,\rho} \frac{1}{H_\rho} \frac{\partial T_e}{\partial \rho} \right) \right] + \frac{1}{H} \frac{\partial}{\partial \theta} \left[ \frac{H h_\theta}{H_\theta} \left( \frac{5}{2} p_e v_e - \kappa_e \frac{h_\theta}{H_\theta} \frac{\partial T_e}{\partial \theta} \right) \right] =
\]

\[
v_{\epsilon,\rho} \frac{1}{H_\rho} \frac{\partial \rho_e}{\partial \rho} + v_e \frac{h_\theta}{H_\theta} \frac{\partial \rho_e}{\partial \theta} + \eta^2 - Q_{ei} - \sum \alpha Q_{ea} + Q_e
\]  
(2.4)

\[
\frac{3}{2} \frac{\partial \sum p_a}{\partial t} + \frac{1}{H} \frac{\partial}{\partial \rho} \left[ \frac{H}{H_\rho} \left( \frac{5}{2} \sum p_a v_{a,\rho} - \kappa_{a,\rho} \frac{1}{H_\rho} \frac{\partial T_a}{\partial \rho} \right) \right] +
\]

\[
\frac{1}{H} \frac{\partial}{\partial \theta} \left[ \frac{H h_\theta}{H_\theta} \left( \frac{5}{2} \sum p_a v_a - \kappa_{\epsilon,\rho} \frac{h_\theta}{H_\theta} \frac{\partial T_a}{\partial \theta} \right) \right] =
\]

\[
\sum \alpha v_{a,\rho} \frac{1}{H_\rho} \frac{\partial \rho_a}{\partial \rho} + \sum \alpha v_{a,\rho} \frac{h_\theta}{H_\theta} \frac{\partial \rho_a}{\partial \theta} + Q_{ai} + \sum \alpha Q_{ea} + Q_i
\]  
(2.5)

Here, \(v_{\epsilon,\rho}\) and \(v_{a,\rho}\) are the components of the electron and ion velocities in the \(\rho\) direction and are given by diffusive models with particle diffusion coefficients proportional to the irregular electron heat diffusion coefficient \(\chi_{\epsilon,\rho} = \chi_{\epsilon,\rho} n_e\). A Bohm-like model of the thermal conductivity coefficients \(\kappa_{\epsilon,\rho}\) and \(\kappa_{a,\rho}\) is used and it is assumed that these values are approximately equal. The components of the electron and ion velocities in the radial direction, \(v_{\epsilon,\rho}\) and \(v_{a,\rho}\), include the classical terms \(\frac{\partial T_e}{\partial \rho}\) and \(\frac{1}{H} \frac{\partial}{\partial \theta} \left( \frac{H R h_\theta h_\theta}{H_\theta} m_n v_i^2 \right)\), and classical electron and ion heat conductivities across magnetic surfaces are also included. Other classical terms to be included are \(\Pi_{\alpha,\beta}\) and \(\Pi_{\alpha,\rho}\), the interspecies friction and thermoelectric forces in \(R_{\alpha}\) and \(R_{\alpha,\beta}\), and the thermal exchange terms \(Q_{ea}\) and \(Q_{ei}\). These terms are discussed in more detail in Keilhacker et al. (1991). The third term on the right hand side of Equation 2.4 defines the electric heating.

The source term \(S_{\alpha}\) \(F_{\alpha}\) (the friction with hydrogenic neutrals), \(Q_e\) (the heat source or sink due to neutrals, impurity radiation and transfer from the main plasma) and \(Q_i\) (the ion heat source due to neutrals and transfer from the main plasma) are found from the temperature and density profiles of the neutrals. These neutral profiles are provided by NIMBUS, which also supplies the deuterium radiation. It is EDGE2D, however, that adds the impurity radiation.
The parallel component of electron velocity $v_e$ can be defined by

$$v_e = \frac{1}{n_e} \left( n_i v_i + \sum_z n_z Z v_z - \frac{J}{e} \right)$$

(2.6)

where $J$ is the parallel component of the current density.

It is assumed that ions reach the velocity of sound, or Mach 1, at the target plates

$$v_\alpha = c_{sat} = \left( \frac{Z_a T_e + T_i}{m_\alpha} \right)^{1/2}$$

(2.7)

This assumption arises from a combination of the Bohm Criterion (Bohm, 1949) and research carried out into the plasma sheath conditions by Stangeby (1986). The former of these uses sheath equations to demonstrate that the ion velocity at the plasma/sheath interface must be greater than or equal to the acoustic sound speed for $T_i = 0$. The latter uses plasma equations to show that, whilst adhering to a steady-state solution, the ion drift velocity cannot exceed the sound speed $c_s$. It should be mentioned that, although EDGE2D adheres to this theory during the course of this investigation, it has since been discovered that it is possible to achieve a supersonic parallel velocity at the magnetic pre-sheath entrance (Stangeby, 1995b).

Additional target plate conditions are defined in the electron and ion energy flux equations given as

$$\frac{5}{2} n_e v_e T_e - \chi_e \frac{1}{H_\theta} \frac{\partial T_e}{\partial \theta} = \beta_e n_e v_e T_e$$

(2.8)

$$\frac{5}{2} n_i v_i T_i - \chi_i \frac{1}{H_\theta} \frac{\partial T_i}{\partial \theta} = \beta_i n_i v_i T_i$$

(2.9)

Typical values for ion and electron energy transmission are $\beta_i \approx 2.5$ and $\beta_e \approx 3.5$, respectively.
Conditions are also given at the flux surface separating the boundary region from the internal plasma region and on the outermost flux surface. These surface conditions can be given by

\[
\frac{\partial u}{\partial \rho} + bu = c
\]  

(2.10)

where \( u \) describes any of the dependent variables. The values of \( a, b \) and \( c \) are well determined according to the temperatures and hydrogenic density at the inner surface.

In steady state conditions and without the presence of impurities in the inner plasma region, a zero flux condition can be expected or a combination of free streaming conditions (where \( a = 1, b \neq 0, c = 0 \)) for different ion charges to hold at the inner surface. Despite having little effect on the results it is also assumed that there is negligible parallel velocity shear at both surfaces, all quantities take a small value at the outermost surface, or alternatively negligible fluxes can be assumed.

2.3.3 The ‘Onion-Skin’ Plasma Model

The following description of the O-S model follows the work of Monk et al. (1995). This model uses experimental Langmuir probe data (including single and triple probes) to obtain the boundary conditions at the divertor target plates, these being electron temperature \( T_{e0} \), ion temperature \( T_{i0} \) and density \( n_{e0} \). The electron and ion temperatures are assumed to be equal, which is highly probable in the case of high density plasmas.

In general, it is not necessarily true that \( T_{e0} \) and \( T_{i0} \) are the same. Electrons, due to their greater conductivity, tend to lose less heat when travelling to the target and can therefore have a greater target temperature than ions. Conversely, electrons lose energy by exciting a trapped electron which then decays by radiation or by ionising. Interactions such as these can culminate in the target ions having the greater temperature. However, it is extremely difficult to maintain this difference in high density plasmas because the increased collision rate tends to equilibrate the particle temperatures (the \( Q_e \) and \( Q_i \) terms in Equation 2.4 and 2.5).
Parallel plasma transport along the magnetic field lines is considered dominant and so the fluid equations are solved in this one-dimension along each of the separate flux tubes. Neutral sources, however, must be treated in two dimensions, so the plasma model is iterated with NIMBUS (the 2-D neutral transport code) for each SOL flux tube to incorporate the interaction of the plasma with recycled neutrals and to solve the continuity equation for their corresponding particle source terms. Due to the particle flux to the divertor target plates being fixed, this being the main source of recycled neutrals, fast convergence is obtained and therefore very few iterations are necessary. It is due to each flux tube being treated as a separate layer that provides the model with its name the ‘onion-skin’ or O-S model.

Plasma flow in the SOL is due to electrostatic and pressure-gradient forces that arise along the magnetic field in this region. Almost immediately after ionisation, the electrons, due to their lighter mass, rush towards the target and charge it up negatively. Plasmas dislike charge differences and so, as in Debye shielding, the negatively charged plate repels electrons and attracts ions, which creates a positively charged sheath, illustrated in Figure 2.4 from Marchand and Lauzon (1992). This sheath has a width of approximately a few Debye lengths which shields the charged plate.

![Figure 2.4. Profiles of the electron density (n_e), the Mach number (M) and the electrostatic potential (e\phi) in the divertor region where the abscissa represents the target plate. Figure from Marchand and Lauzon (1992).](image-url)
An ambipolar flow of ions and electrons occurs, where the ion flux leaving the plasma equals that of the electron flux. This requires a potential of $V_{se} = \frac{3kT_e}{e}$ at the sheath edge, due to the difference in ion to electron mass ratio.

Shielding, however, is not perfect and an electric field does extend from the sheath into the pre-sheath region (Figure 2.4) attracting the pre-sheath ions towards the target. This sink of ions in the pre-sheath creates a pressure-gradient which also encourages the plasma flow towards the target. Therefore, the forces carrying the ions are the electrostatic force and the pressure-gradient. However, the electric field in the pre-sheath repels the electrons away from the target and consequently the pressure-gradient is cancelled by the electric force leaving the electrons, with their small masses, to obey the Boltzmann distribution.

From this information, the momentum conservation for ions can be given as

$$nm_i \frac{dv}{dx} = \frac{-dp_i}{dx} + enE - m_i \nu S_p - S_{mom}$$  \hspace{1cm} (2.11)

where $S_p$ is the ion particle source. The first term on the right hand side of Equation 2.11 is the pressure-gradient and the second term is the electrostatic force. The third term denotes the drag exerted by newly ionised ions, since the ions are approximately stationary at the time of being ionised and therefore need to be pulled up to speed at the expense of the plasma flux velocity. The final term is the momentum loss due to ion-neutral collisions, as calculated by the Monte Carlo neutral code, NIMBUS. It is assumed in the conservation equations for the O-S model that ion viscosity and momentum sources can be neglected.

The momentum conservation equation for electrons (Equation 2.12) can be written in the same way but all the terms apart from the pressure-gradient and electrostatic force can be neglected since they are relatively small due to the electron to ion mass ratio.

$$\frac{dp_e}{dx} = enE$$  \hspace{1cm} (2.12)

Combining these two equations with the mass conservation equation
and the definition of the sound speed, which is the particle flow speed at the target, in terms of the electron and ion temperatures

\[ c_s^2 = \frac{k(T_e + T_i)}{m_i} \]  

(2.14)

and integrating with respect to the variables of distance, temperature, density and velocity from the target conditions to those at a distance \( s \) from the target gives the following equation for the momentum conservation

\[ n(s)kT_e(s) + kT_i(s) + m_v(s)^2 = 2n,k(T_{\alpha} + T_{\nu}) + \int_0^s S_{\text{mom}}(s')ds' \]  

(2.15)

where the subscript \( t \) denotes the target, \( n \) is the ion density (also equal to electron density), \( k \) is the Boltzmann's constant, \( v \) is the ion parallel velocity and \( s \) is the distance along a ring, or flux tube. It can be shown, from Equation 2.15, that for the particular case where the momentum loss due to ion-neutral collisions is small that the target pressure is half the pressure as the plasma enters the SOL from the core.

Another equation necessary in the description of the plasma movement and conditions in the SOL is the particle conservation equation. This can be given in terms of the rate of change of flux or mass conservation defined in Equation 2.13. Again integrating from the target conditions to those at a distance \( s \) away from the target gives the mass conservation equation in the form it is used in the O-S model.

\[ n(s)v(s) = n_t c_s + \int_0^s S_p(s')ds' \]  

(2.16)

The final conservation equation necessary to describe the plasma is the energy balance equation for electrons and ions. It is assumed that heat transport is dominated by electron conduction and so convection is not considered. Consequently, parallel heat conduction is balanced with radiated power to give the equation

\[ S_p = \frac{d(nv)}{dx} \]  

(2.13)
\[ d(q_{\text{cond}}A) = -\frac{P}{V} A\, ds \]  \hspace{1cm} (2.17)

where \( q_{\text{cond}} \) is the conducted power per unit area, \( A \), \( V \) and \( ds \) are the cross-sectional area, volume and length of the flux tube element, respectively, and \( P \) is the radiated power per unit area. It is assumed that \( A \) is constant and the conducted power can be defined as

\[ q_{\text{cond}} = -K_0 T^{\frac{1}{2}} \frac{dT}{ds} \]  \hspace{1cm} (2.18)

where \( K_0 \) is the Spitzer conductivity.

Using these assumptions, Equation 2.17 now becomes

\[ d\left(-K_0 T^{\frac{1}{2}} \frac{dT}{ds}\right) = -\frac{P}{V} ds \]  \hspace{1cm} (2.19)

This can now be integrated with the limits as shown

\[ \int_{\frac{P}{A}}^{-K_0 T^{\frac{1}{2}} \frac{dT}{ds}} d\left(-K_0 T^{\frac{1}{2}} \frac{dT}{ds}\right) = \int_{0}^{s} \frac{P}{V} ds \]  \hspace{1cm} (2.20)

where \((P/A)\) is the conducted power per unit area at the target and \( s \) is the distance along the flux tube. After integration this becomes

\[ K_0 T^{\frac{1}{2}} \frac{dT}{ds} = \frac{P}{A} + \int_{0}^{s} \frac{P}{V} ds \]  \hspace{1cm} (2.21)

Integrating again over the same limits gives

\[ K_0 \int_{\tau_{n,0}}^{\tau_{n,i}} T^{\frac{1}{2}} dT = \int_{0}^{s} \frac{P}{A} ds + \int_{0}^{s} ds \left[ \int_{0}^{s} \frac{P}{V} ds \right] \]  \hspace{1cm} (2.22)
which, after integrating and reformatting, leads to the energy conservation equation for electrons and ions

$$T_{e,i} = \left[ T_{e0,i} \frac{\gamma^2}{2} + \frac{7}{2K_0} \left( \frac{P}{A} s + \int_0^s \frac{P(s''') ds'''}{V} \right) \right]^{\gamma^2} \tag{2.23}$$

The boundary conditions for the energy equation are

$$\left( \frac{P}{A} \right)_{et} = (5kT_{et}) n_{et} c_s \tag{2.24}$$
$$\left( \frac{P}{A} \right)_{it} = (2kT_{it}) n_{it} c_i \tag{2.25}$$

It is assumed that the ion and electron target densities and temperatures are equal and so the parallel heat flux to the target plates is obtained using these sheath conditions combined to give

$$q_p = 7n_i c_s kT_i \tag{2.26}$$

where \(n_i\) and \(T_i\) denote the target density and temperature, respectively, for ions and electrons.

The O-S model is based on these steady state conservation equations for classical transport along the magnetic field lines which incorporate these effects and balance these forces.

The similarities between the conservation equations in the O-S model and those of EDGE2D become apparent when certain terms are eliminated from Equations 2.2 to 2.5. Since the O-S model describes a steady state 1-D plasma with transport only parallel to the magnetic field, the time derivative and radial terms should be ignored in the fluid model. Furthermore, the O-S model does not treat impurities, or ion species other than deuterium, and so these terms can also be neglected. The particle conservation equations for the two models, Equations 2.3 and 2.16, already have identical terms. It can be seen that in the case of momentum and energy conservation a further elimination of the
interspecies friction, thermoelectric forces, thermal exchange, electric heating and the convection terms. Equation 2.2 resembles Equation 2.11 and Equations 2.4 and 2.5 resemble Equation 2.19.

An advantage of the O-S model over the fluid code to model the background plasma is the minimisation of uncertainty at the targets, the most physically important and sensitive regions. In addition to this, cross-field transport is included implicitly in the experimental Langmuir probe data defining the boundary conditions and, as previously mentioned, convergence to the solutions is rapid needing very few iterations.

The O-S model does, however, have its disadvantages in comparison with the fluid code EDGE2D; these being that the latter incorporates far more detailed physics using 2-D hydrodynamic equations (accounting for both parallel and perpendicular transport). For its input, it uses power going into the SOL and density at the edge of the SOL which are less uncertain than the target parameters, measured by Langmuir probes, required as input to the O-S model. Target predictions of the electron temperature are often too high and the ion temperature is not measured at all.

EDGE2D also has a self consistent impurity model and background and although it takes longer to converge and needs many iterations, it provides a radiation profile which is consistent with the background plasma solution, as the radiated power is consistently included in the electron energy conservation Equation 2.4.

### 2.4 Modelling Impurity Transport

Impurity contamination is a major obstacle in the path to generate nuclear fusion. The extent of contamination and its effects in the varying tokamak conditions is one of the most time consuming and vital studies being carried out at JET. Impurity contamination leads to an increase in power and radiation losses via bremsstrahlung and line radiation and dilutes the number of ions available for the fusion process.

The preventative measure which has been applied to combat this problem is the installation of a divertor. This is a system of coils and targets which is used to isolate the plasma-solid interactions from the hot central plasma. JET divertor target plates are composed of the low Z impurities beryllium and carbon. These are eroded by the impact
of the particle flux which travels parallel to the magnetic field, directly towards the targets, culminating in sputtering of beryllium and carbon back into the plasma.

A divertor is a preferable approach to shape and stabilise the plasma rather than a limiter because the solid surface of the limiter would be in contact with the main core plasma which would promote impurity leakage into the core, whereas the divertor reduces this contact and therefore minimises impurity contamination.

Neutral recycling also aids the reduction of impurity leakage into the main plasma as the impurities from the target get trapped in the cyclic motion of the deuterium neutrals being released from the plate only to be ionised and dragged back down to the target.

In the previous section the various codes used in the plasma modelling procedure were discussed and it was mentioned that impurities are modelled either using EDGE2D or via DIVIMP. DIVIMP is essentially a simulation code used to model and follow the transport of an impurity ion in the edge plasma region of the tokamak. It does this using the Monte Carlo method and assumes classical transport parallel to the magnetic field lines and anomalous diffusion perpendicular to the field lines.

It is highly important, when modelling impurities, to get the balance correct of the force on the impurity ion, as slight changes in the balance can have catastrophic effects on impurity leakage into the main plasma. The definition for the force on an impurity ion, in DIVIMP, is given by

\[
F_{\text{tot}} = -\frac{1}{\nu} \frac{dP}{ds} + m \frac{(v_D - v)}{\tau_e} + Z e E + \alpha_e \frac{dT_e}{ds} + \beta_i \frac{dT_i}{ds}
\]

where the lack of subscript denotes an impurity value. The five terms, in order, are the impurity pressure gradient force per particle, the friction with the background plasma (flowing at a velocity \(v_D(s)\) and with a Spitzer stopping time \(\tau_e\)) the electric force and the electron and ion temperature gradient forces, respectively, with \(T_e(s)\) and \(T_i(s)\) given and \(s\) measured along \(B\), where \(s = 0\) at the target and \(s = L\) halfway along the ring.

The most damaging effects of impurity contamination arise when leakage into the main plasma occurs. Studies performed by Stangeby and Elder (1995a) have shown that if the frictional force (FF) and the ion temperature gradient force (FiG) are zero there will be a
plateau or constant value of impurity density $n_p$ for $s > s_{inj}$, where $s_{inj}$ is the ion injection point. From this ion leakage to the core could occur. Taking the opposite approach, if $FF$ and $FiG$ were not zero, the plateau would not exist. In this instance if $|FF| < |FiG|$ then $n(s)$ will rise to disastrous levels where leakage to the core will be enormous. Consequently, a necessary, but not solely adequate, condition to prevent leakage is the situation where $|FF| > |FiG|$. This can be seen in Figure 2.5 where region $A$ is the prompt-loss region, extending from the target to the source injection point, region $B$ displays a density profile for $|FF| > |FiG|$ and region $C$ shows the opposite case where $|FiG|$ is dominant.

![Figure 2.5](image)

**Figure 2.5.** Impurity density profiles along the magnetic field line (a) for parallel diffusion only and (b) allowing for friction, temperature-gradient forces and parallel-diffusion. Figure from Stangeby and Elder (1995a).
Fortunately, the typical situation that arises is one where $|\mathbf{F}_I| > |\mathbf{F}_G|$. It can be seen that low temperatures are required to prevent leakage. In fact a prescription can be given for zero leakage, assuming $Z = 4$ ions, as

$$T_0 \leq 2.4 \times 10^{8} (n_0 S_0)^{1/2}$$  \hspace{1cm} (2.28)

Figure 2.6 shows $n(s)$ profiles as a function of ring length for various temperatures and shows that, although low temperatures cause high impurity densities in the target region, they lower the impurity content further along the ring where leakage to the core could occur.

![Figure 2.6. C$^+$ profile versus the distance along a ring, or poloidal field line, for varying ion temperatures at the target where the electron density is $10^{20}$ m$^{-3}$, the perpendicular confinement time is 1s. Here 0.00m and 72.29m represent the positions of the inner and outer target plates. Figure from Stangeby and Elder (1995a).](image)

Furthermore, even a small amount of cross-field transport can prevent impurity build up and therefore leakage occurring as can be seen in Figure 2.7.
Figure 2.7. Impurity profile versus the distance along a ring, or poloidal field line, with and without cross-field transport calculated by DIVIMP. Here 0.00m and 72.29m represent the positions of the inner and outer target plates. Figure from Stangeby and Elder (1995a).

It is also usually considered that if \( n(s)/n_i \) (Figure 2.5) is small enough, for example less than 0.01, then leakage is not a practical problem.

2.5 Typical Characteristics of the Plasmas to be Modelled

Since this thesis is based on the investigation of opacity in the divertor region of the tokamak the type of plasma that is of interest is one where a large amount of absorption is possible. Obviously this would increase with increasing absorber density and since the neutrals are responsible for the absorption of radiation, this study is focused on high neutral density plasmas. These high divertor density plasmas are associated with low temperatures often culminating in a detached plasma, which will be described later in this chapter.

Density ramp experiments, or experiments with large density variations are also necessary, to enable a comparison to be made of the absorption in relation to changing density, thereby confirming that the absorption is due to opacity effects.
The values of target parameters that would justify an investigation into the opacity of a plasma pulse are electron densities of the order of $10^{20} \text{m}^{-3}$, neutral densities of approximately $10^{19} \text{m}^{-3}$ and electron temperatures of less than 5eV. The pulse should also display a reduction in the branching ratio of Lyman-beta ($\text{Ly}\beta$) to Balmer-alpha ($\text{D}\alpha$) radiation of 20% or more with increasing density. Therefore, the types of plasma under investigation are high density, low temperature plasmas.

2.5.1 A Description of Detached Plasmas and Momentum Loss

As mentioned in Section 2.3, neutral recycling occurs predominantly in the divertor region. The number of elastic and charge exchange collisions that a recycling neutral experiences before ionisation ($N$), for constant $T_i$ and $T_e$, is defined in Equation 2.29 (Stangeby, 1993).

\[
N = \left( \frac{\sigma V_{iz} + \sigma V_{div}}{2} \right) \left[ \frac{1}{2} \frac{\sigma V_{i-n}}{\sigma V_{iz}} + \sigma V_{i-n} \left( \frac{\sigma V_{div}}{\sigma V_{iz}} \right) \right]
\]  

(2.29)

where $\sigma V_{iz}$ is the ionisation rate (assumed to be equal for atoms and molecules), $\sigma V_{div}$ is the dissociation rate, $\sigma V_{i-n}$ is the elastic collision rate for $D_2$ on $D^+$ and $\sigma V_{i-n}$ is the charge exchange rate.

Figure 2.8 depicts graphs of $N$ for various electron temperatures using Equation 2.29 for various values of ion to electron temperature ratio.

This figure shows that in a uniform plasma at low $T_e$ such as 3eV, similar to that in the target region, a recycling neutral undergoes a large number of collisions, greater than 100, before being ionised.
At the low electron temperatures created in the divertor plasma, the recycling neutrals could, via elastic and charge exchange collisions with the downward flow of ions, cause momentum loss as considered in Equation 2.15. This results in an increase in the ion particle confinement time and in order to balance the conservation laws the plasma pressure above the target increases. If a significant momentum loss and peak pressure drop, by a factor of approximately 5-10, occurs and the target electron temperature is less than 5eV then a peak density occurs away from the target and the plasma actually detaches from the solid surface, hence the name 'detached' plasma. This results in a low saturation current and decreased power to the divertor target.

Certain queries arise under these conditions where the plasma loses contact with the target, hence posing problems when trying to model this plasma state. The first is a question of power balance and determining exactly what has happened to the power if such diminished amounts are reaching the target. This question is answered by assuming that the power is radiated, especially since impurity radiation is usually required to lower the divertor temperature to the point where detachment can occur. Surprisingly, it seems that neutrals probably do not contribute to removing power from the SOL, since most of
this power would have to be dissipated upstream of the collisional zone by radiation for a collisional or momentum effect to occur.

The second problem is similar to the first but is a matter of pressure balance since the target pressure is much smaller than that of the mid-plane value. An explanation that has been provided for this is that elastic ion-neutral collisions remove the plasma momentum and energy and therefore account for this pressure drop.

The final enigma is, ‘where are the electric charges going, if not directly to the target?’ The ion saturation current at the target is extremely low and so flow must be greatly reduced. However, one is led to believe that plasma, composed of charged pairs, would flow freely along the open field lines in the SOL and towards the target. The explanation for this is that the charged particle flow to the planes is greatly reduced by the aforementioned ion-neutral momentum/energy loss collisions. This increases the plasma particle confinement time and, although volume recombination is typically not fast enough in this environment to explain the current drop, a prolonged journey to the target provides a better chance of volume recombination occurring.

This statement is supported by an investigation carried out by Loarte (1996) to model divertor detachment using 2-D plasma edge codes. Here it is shown that a decrease in the integrated ion flux to the targets is only reproduced by the codes if recombination is incorporated within them. However, it should be stressed that, in some cases, the divertor electron temperature was predicted at 1eV, which is much lower than the measured value of 3-5eV which implies that these measurements may be in error. This issue is discussed later in this section.

Loarte’s investigation, uncovered further evidence to support the occurrence of recombination during detachment when looking at the Dα emission, using deuterium visible spectroscopy. This showed extremely low values of the ratio of the divertor total ion flux to Dα, in some cases as low as 0.2, compared with more typical values of 25. Yet again, this low ratio can only be reproduced by the codes if recombination is included. One final piece of evidence to support recombination is that an increase in the ratio of Balmer-gamma (Dγ) to Dα is observed during detachment. This implies recombination is taking place to populate the excited states of deuterium, when electron collisions are least likely. However, when recombination is incorporated within the codes, although it shows...
an increasing trend to match that of the experimental data, it also dramatically overestimates the D\(\gamma\)/D\(\alpha\) ratio (Figure 2.9).

![Graph showing Normalised D\(\gamma\)/D\(\alpha\) ratio for a density scan to detachment in a JET Ohmic discharge. Figure shows comparison between computed values from EDGE2D, both including and excluding recombination, with experimental data. Figure from Loarte (1996).](image)

**Figure 2.9.** Normalised D\(\gamma\)/D\(\alpha\) ratio for a density scan to detachment in a JET Ohmic discharge. Figure shows comparison between computed values from EDGE2D, both including and excluding recombination, with experimental data. Figure from Loarte (1996).

It appears volume recombination is still not an adequate answer to explain the reduction of ion flux reaching the targets. This is because, even in very low temperature environments with electron temperatures of 1eV and ion densities of 10\(^{19}\) m\(^{-3}\), the volume recombination time is approximately 0.1s, which is long compared with the time it takes for ions to pass through the region. It seems that volume recombination could only work at an adequate rate, to explain the loss of electric charge phenomena, if the electron temperatures were to drop to values of less than 1eV.

As previously mentioned, detachment is associated with an increased particle confinement time. Although it may seem that this would reduce the rate at which the divertor can eliminate the impurities, in fact the rate at which the target impurities are created could possibly be lessened due to reduced sputtering which may counteract the lower impurity flushing efficiency (Stangeby, 1993). However this is a delicate balance and not yet completely resolved.
To better model a detached plasma, the O-S model has been modified to incorporate the aforementioned momentum loss due to the ion-neutral collisions as calculated by the Monte Carlo neutral code NIMBUS (Cupini et al., 1983), and can be seen in the momentum conservation Equation 2.15. The final term in this equation denotes the decrease in pressure due to momentum loss.

For detached plasma simulations, this modified model does show an increased density just away from the target as predicted (Figure 2.10). However the density increase does not reproduce the measured level of bremsstrahlung in the divertor. More detailed calculations, using the fluid code, predict a larger momentum loss and better reproduce experimental measurements (Figure 2.11).

The discrepancy in the two simulations is thought to be related to errors in the interpretation of electron temperature measurements with the target Langmuir probes (Gunther, 1995). This is very worrying when using this information to model the plasma background since opacity increases linearly with absorber density (neutral deuterium density), so locations with the greatest densities suffer the most radiative absorption. Typically high neutral density plasmas are associated with low temperatures which are extremely difficult to measure accurately. Consequently, the types of plasmas under investigation in this thesis are amongst the most complicated to model.

In a dense, cold divertor plasma close to or in a state of detachment, the electron temperatures (derived from the slopes of current-voltage-characteristics at the floating point using the formula $T_e = eI_{n}^+ (dV/dI_{n})$) appear to be overestimated considerably when comparing them to predictions from SOL code calculations for this type of discharge. An example of the magnitude of this discrepancy is 12eV in contrast to 2eV and can be seen in Figure 2.12.
Figure 2.10. O-S model simulations of a detached plasma showing the effect of ion-neutral collisions in the SOL. Figure from Lovegrove et al. (1995).

Figure 2.11. Fluid model simulations of a detached plasma showing the effect of ion-neutral collisions in the SOL. Figure from Lovegrove et al. (1995).
This occurs because a Langmuir probe operates by applying a voltage to a network of different types of resistors such as probe-tip ($R_{ps}$) and return sheaths ($R_{rs}$), cross-field ($R_{\perp}$) and longitudinal ($R_{||}$) resistances of the plasma, and measuring the current through it. In classical single probe theory, only $R_{ps}$ is considered, whilst $R_{\perp}$ and $R_{||}$ are considered to be zero. However, a voltage drop does occur at $R_{\perp}$, $R_{||}$ and $R_{rs}$, especially at low temperatures where the Spitzer resistivity is higher, and so by ignoring these voltage drops, the electron temperature $T_e$ is always overestimated.

This problem is being investigated by Gunther (1995) and a step towards solving this is to apply non-ambipolar fluid theory to the plasma between the probe-tip Debye sheath and the adjacent and opposite return sheaths. The main weapon being used in this investigation to combat inaccuracies in $T_e$ is the way it deals with plasma resistivity (finite Spitzer conductivity) which is strongly associated with a highly collisional, cold divertor plasma.

The model considers a flush mounted probe in a 100% recycling scrape-off plasma, which is therefore sustained entirely by the ionisation of neutrals. Other assumptions made are constant flux, constant electron and ion temperatures and zero viscosity.
In summary the electron temperature measurements taken with the target Langmuir probes, in this thesis, could be six times larger than reality. An error of this magnitude can cause large distortions in opacity predictions.

It should be mentioned that the probe data, used to model the plasma in the research of this thesis, has not had the version of the non-ambipolar fluid theory that assumes a resistive plasma applied to it.

For the purpose of this thesis, it is sufficient to conclude that there are still uncertainties, so the strategy is to reduce the uncorrected temperatures by a constant factor until other diagnostic measurements such as bremsstrahlung and Dα are reproduced by the model.

This chapter has dealt with the methods used in experimental modelling of the divertor plasma. This process is necessary to produce realistic information on the background plasma to be used as input parameters to a population code, as shown in Figure 2.1. This code uses the plasma information provided to produce level populations and absorption spectra to determine the extent of opacity in the divertor plasma. The following two chapters describe how the population code performs this complicated task and also goes on to explain how the plasma model is linked to the population code in a way that complies with the geometry of the divertor region.
Chapter 3

The Population Code

3.1 Introduction

The population code is an extremely versatile program that was initially written with the intention of being applied to stellar atmospheres. It calculates transition rates and cross-sections for a wide range of atomic processes and uses this information to evaluate the population of various energy states.

A considerable amount of modification has been necessary to produce a code that can be applied to the dynamic divertor plasma conditions under consideration in this thesis. The majority of these changes are related to geometry and plasma motion and will therefore be dealt with in Chapter 4. However, one difference that is discussed in this chapter is the atomic processes that need to be considered.

In this chapter we show how the equations of statistical equilibrium are derived and solved, following the approach of Burgess and Summers (1969), Storey (1972) and Summers (1977). The number of equations that need to be solved are infinite and so a brief explanation is given of the reduced matrix method used in the code.
Values for transition rates need to be substituted into the statistical equilibrium equations to obtain values of the populations of various energy states. The methods employed to calculate the transition rates and cross-sections for the atomic processes are discussed. No interaction between matter and the radiation field is considered here but is discussed in the following chapter.

### 3.2 Atomic Processes Incorporated in the Population Code

As previously mentioned the population code was initially written to deal with the diverse conditions of stellar atmospheres which require a large number of atomic processes to be considered. The environment of the plasma in the divertor of the JET tokamak does not demand the consideration of such a wide range of atomic processes. Those that are necessary are

(i) Collisional excitation and de-excitation by protons and electrons between bound states

\[
X^+(a) + e \rightarrow X^+(a') + e
\]

\[
X^+(a) + H^+ \rightarrow X^+(a') + H^+
\]  

(3.1)

with the corresponding net collisional rate coefficient between \(a\) and \(a'\) being \(C_{aa'}\) and transition rate of \(N(a)C_{aa'}\) where \(N(a)\) is the number density of ions \(X^+(a)\).

(ii) Spontaneous emission of radiation

\[
X^+(a) \rightarrow X^+(a') + h\nu
\]  

(3.2)

with a radiative transition probability of \(A_{aa'}\) and where \(h\nu = \Delta E_{aa'}\) is the energy difference between \(X^+(a)\) and \(X^+(a')\).

The following bound-free and free-bound processes are also considered

(iii) Radiative recombination

\[
X^{+Z+1}(b) + e(E) \rightarrow X^+(a) + h\nu
\]  

(3.3)
with a radiative recombination rate of \( N_e N(b) \alpha_r(b,a) / \text{cm}^3/\text{s} \) where \( h\nu = E + \epsilon(a) \) with \( E \) being the free electron energy and \( \epsilon(a) \) the ionisation energy of \( X^{+Z}(a) \).

(iv) Three-body recombination

\[
X^{+Z}(b) + e + e \rightarrow X^{+Z}(a) + e + h\nu
\]

with a transition rate of \( N_e N(b) \gamma(b,a) / \text{cm}^3/\text{s} \). It is clear that only one electron density is incorporated in this expression for the transition rate whereas the transition depends on the presence of two electrons. This is because, in this particular case and the reverse process of collisional ionisation, \( \gamma(b,a) \) does not represent the rate coefficient but the product of the electron density and the rate coefficient.

(v) Collisional ionisation

\[
X^{+Z}(a) + e \rightarrow X^{+Z+1}(b) + e + e
\]

with the corresponding collisional ionisation rate of \( N(a) \gamma(a,b) / \text{cm}^3/\text{s} \).

It should be mentioned that the ionisation balance is not solved by the population code. To do this it would be necessary to incorporate the effect of plasma transport, neutral and ion flows, and the effects of charge exchange with deuterium ions, in addition to the above atomic processes. Instead the ionisation balance is solved in the transport code, EDGE2D, and then used in the population code.

3.3 The Statistical Equilibrium Equation

For the purpose of this thesis, a pure deuterium plasma is being modelled. This simplifies the atomic processes discussed in the previous section since there is only one recombining ion state and conversely, only one state into which ionisation can occur and so consequently, the \( b \), which is the energy state of a second atom, can be neglected.
In this section any process to or from the continuum will be denoted by \( c \) whilst \( a \) and \( a' \) represent the quantum numbers \( nl \) and \( n'l' \) respectively.

The atomic processes can be combined to form a set of time dependent equations for the populations \( N(nl) \), where the population of the state \( X^*(nl) \) is \( N(nl) \) /\( \text{cm}^3 \) and the number of ions \( X^+Z^+ \) is \( N_+ /\text{cm}^3 \).

\[
\dot{N}(nl) = N_+N_e[\alpha_e(c,nl) + \gamma(c,nl)] + \sum_{n'l'(n'l'(\neq nl))} N(n'l')C_{n'l',nl} + \sum_{n'l'(n'l'(> nl))} N(n'l')A_{n'l',nl} - \\
N(nl)[\gamma(nl,c) + \sum_{n'l'(n'l'(< nl))} C_{nl,n'l'} + \sum_{n'l'(n'l'(> nl))} A_{nl,n'l'}]
\]  

(3.6)

The total transition rate coefficient between the two states \( nl \) and \( n'l' \) can be defined as \( D_{nl,n'l'} \) such that the transition rate is \( N(nl)D_{nl,n'l'} \). Similarly, the transition from bound state \( nl \) to the continuum, or the process of ionisation, has a total rate given by \( N(nl)D_{nl,c} \) and inversely for the opposite process of recombination the total rate is given by \( D_{c,nl} \). Therefore, we can define the total depopulation rate coefficient for a particular state as \( D_{nl,nl} \). This incorporates collisional excitation and de-excitation, spontaneous emission of radiation and collisional ionisation. For each ingoing transition to the state \( nl \), the transition rate can be defined as \( D_{n'l',nl} \) including collisional excitation and de-excitation and spontaneous emission of radiation.

These assumptions can be incorporated into Equation 3.6 to give

\[
\dot{N}(nl) = D_{c,nl} + \sum_{n'l'(n'l'(\neq nl))} N(n'l')D_{n'l',nl} - N(nl)D_{nl,nl}
\]  

(3.7)

where

\[
D_{nl,nl} = \sum_{n'l'(n'l'(\neq nl))} D_{nl,n'l'} + D_{nl,c}
\]  

(3.8)

\[
D_{c,nl} = N_+N_e[\alpha_e(c,nl) + \gamma(c,nl)]
\]  

(3.9)
It is also assumed that the states \( nl \) are degenerate with respect to \( l \) and that redistribution of the angular momentum occurs. If this redistribution is completely effective, the populations \( N(nl) \) can be given by the Boltzmann distribution

\[
N(nl) = N(n)\frac{(2l+1)}{n^2}
\]  

(3.10)

where

\[
N(n) = \sum_l N(nl)
\]  

(3.11)

Summing Equation 3.7 over all \( l \) gives

\[
\dot{N}(n) = \sum_l D_{c, nl} + \sum_{l,l' \neq n} \frac{(2l'+1)}{n'^2} N(n')D_{n', nl} - \sum_l \frac{(2l+1)}{n^2} N(n)D_{nl, nl}
\]  

(3.12)

We can now define the in going transition rate of a transition from state \( n' \) to state \( n \) as

\[
D_{n', n} = \sum_{l,l'} \frac{(2l'+1)}{n'^2} D_{n', nl'}
\]  

(3.13)

and the outgoing transition rate as

\[
D_{n, n'} = -\sum_l \frac{(2l+1)}{n^2} D_{nl, nl}
\]  

(3.14)

and the total recombination rate as

\[
D_{c, n} = \sum_l D_{c, nl}
\]  

(3.15)

These give

\[
\dot{N}(n) = D_{c, n} + \sum_{n'} N(n')D_{n', n}
\]  

(3.16)
or in matrix form

\[
\dot{\mathbf{N}} = \mathbf{D}_c + \mathbf{D}\mathbf{N} \tag{3.17}
\]

In steady state \( \dot{\mathbf{N}} = 0 \) for all \( n \) and so

\[
\mathbf{N}^{eq} = -\mathbf{D}^{-1} \mathbf{D}_c \tag{3.18}
\]

where \( \mathbf{N}^{eq} \) are the steady state populations. Therefore Equation 3.17 becomes

\[
\mathbf{D}^{-1}\dot{\mathbf{N}} = \mathbf{N} - \mathbf{N}^{eq} \tag{3.19}
\]

Generally, the rate of depopulation of state \( n \), or \( D_{n,n} \), is far greater than that of the ground state, \( D_{1,1} \), causing the excited state to have a smaller population and the time scale for variation of the \( n \) state population to be shorter than that of the ground state.

As a whole, the populations of the accumulative \( n \) states are in quasi-equilibrium such that \( \dot{\mathbf{N}}(n) = 0 \). Consequently, any change leading to a variation in \( N(1) \) causes the populations of the other energy levels to compensate for this within a time \( \tau \approx \tau_{n} \).

Since the time scale for variations of \( N(1) \) (defined as \( 1/D_{1,1} \)) is much greater than that of the excited levels (\( 1/D_{n,n} \)), we can assume that \( \dot{\mathbf{N}}(n) = 0 \) for times greater than \( 1/D_{n,n} \) and so the excited levels are considered to be in equilibrium with respect to the ground state. Therefore the equation for the ground state can be given by

\[
(D^{-1})_{11} \dot{N}(1) = N(1) - N^{eq}(1) \tag{3.20}
\]

This assumption is necessary to derive expressions for the recombination and ionisation coefficients. Since the excited states are less highly populated than the ground state, the ionisation and recombination rates for the ion \( X^{+2} \) can be given as

\[
\dot{N}(1) = -\dot{N}(X^{+2}) = N_e N(X^{+2}) \alpha_{cr} - N_e N(1) S_{cr} \tag{3.21}
\]
where $\alpha_{cr}$ and $S_{cr}$ are the collisional radiative recombination and ionisation coefficients, respectively. These can be described by comparing Equations (3.20) and (3.21) as

$$S_{cr} = \frac{-1}{N_e (D^{-1})_{11}}$$  \hspace{1cm} (3.22)

$$\alpha_{cr} = \frac{-N^{eq}(1)}{N_e N(X^{+2+})(D^{-1})_{11}}$$  \hspace{1cm} (3.23)

The solution to the equilibrium Equation 3.18 is necessary to evaluate these. This solution is simplified by the factor $b(n)$ which relates the solutions $N(n)$ to those of the Saha equation for a particular series $N_s(n)$

$$\frac{N_s(n)}{N_e N_s} = \frac{\omega(n)}{2 \omega_s} \left( \frac{h^2}{2 \pi m k T} \right)^{1/2} e^{-\nu(n)/kT}$$  \hspace{1cm} (3.24)

where $\omega_s$ is unity in the case of deuterium.

In the version of the population code that is being used for this thesis, the section of the code which iterates to produce a value for the ratio of neutral to ionised deuterium, depending on the level populations, is suppressed in order to force a value for this ratio. If we wish the ratio of neutral to ionised deuterium to be equal to $f$ then we replace the equation for the population of the ground state by

$$\sum_{n=1}^{\infty} \frac{N_n}{N_s} = f$$  \hspace{1cm} (3.25)

which in terms of the departure coefficient $b(n)$, discussed shortly, is

$$\sum_{n=1}^{\infty} \frac{N_s(n)}{N_s(1)} b(n) = \frac{N_s f}{N_s(1)}$$  \hspace{1cm} (3.26)

It is more convenient to work with $b(n)$, the departure of the population from its thermodynamic equilibrium value, defined by
\[ N(n) = b(n)N_s(n) \]  
\[ \text{(3.27)} \]

Substituting these into Equation 3.16 we get

\[ \dot{b}(n)N_s(n) = D_{v,n} + \sum_{n'} b(n')N_s(n')D_{n,n} \]
\[ \text{(3.28)} \]

Dividing through by \( N_s(n) \) gives

\[ \dot{b}(n) = \frac{D_{v,n}}{N_s(n)} + \sum_{n'} b(n') \frac{N_s(n')}{N_s(n)} D_{n,n} \]
\[ \text{(3.29)} \]

The new rate terms can be defined as

\[ D^* = \frac{D_{v,n}}{N_s(n)} \]
\[ \text{(3.30)} \]

and

\[ D^* = \frac{D N_s(n')}{N_s(n)} \]
\[ \text{(3.31)} \]

Equation 3.29 can now be expressed in terms of a matrix equation using the rate expressions given in Equations 3.30 and 3.31.

\[ \dot{b} = D^*_v + D^* b \]
\[ \text{(3.32)} \]

In steady state, when \( \dot{b} = 0 \)

\[ b^{eq} = -D_v^{-1} D^* \]
\[ \text{(3.33)} \]

Substituting this into Equation 3.32 to eliminate \( D^*_v \) gives

\[ D_v^{-1} \dot{b} = b - b^{eq} \]
\[ \text{(3.34)} \]
Using the same assumption that the excited levels are all in equilibrium with respect to the ground state, the equation for the ground state is given as

\[(D^{-1})_{11} \dot{b}(1) = b(1) - \dot{b}^q(1)\]  \hspace{1cm} (3.35)

and Equation 3.21 still applies and can be rewritten in the form

\[\dot{b}(1)N_s(1) = N_e N(X^{*z+1}) \alpha_{cr} - N_e b(1)N_s(1)S_{cr}\]  \hspace{1cm} (3.36)

By eliminating \(\dot{b}(1)\) between Equations 3.35 and 3.36 and comparing similar terms gives

\[S_{cr} = \frac{-1}{N_e(D^{-1})_{11}}\]  \hspace{1cm} (3.37)

\[\alpha_{cr} = \frac{-b^q(1)N_s(1)}{N_e N(X^{*z+1})(D^{-1})_{11}}\]  \hspace{1cm} (3.38)

The departure coefficient \(\dot{b}^q\) is a far easier vector to work with than the population \(N^q\) and so the program solves the equilibrium matrix equations in terms of \(\dot{b}^q\) and converts these values back into population values when this information is needed via Equations 3.27. The ionisation and recombination coefficients are found in a similar way but using Equations 3.37 and 3.38 for final conversion.

In the case of \(n\) being larger than some critical value, \(n_{cr}\), the populations of the states will tend to those expected in thermodynamic equilibrium from the Saha equation, as collisional effects become dominant, and so

\[b^q(n) \xrightarrow{n \to \infty} 1\]  \hspace{1cm} (3.39)

The critical quantum number, \(n_{cr}\), is reached when collisional processes are dominant over radiative processes. This varies depending on the plasma parameters, such as density and temperature. As the plasma density and temperature increases, so does the collisional rate. The cross-section for an \(n\) to \(n+1\) transition is approximately equal to \(\pi r_n^2\), where \(r_n\) is the radius of the \(n\)-th orbit, and \(r_n \propto n^2\). It follows that the rate coefficient is directly proportional
to \( n^4 \). However, for radiative decay from a state \( n \), the rate coefficient is inversely proportional to \( n^5 \) (Equation 3.58). Therefore, although the radiative term dominates at low \( n \)-states due to its extra \( n \)-factor, as the density and temperature of the plasma increase, collisional processes become more common and at a particular value, \( n_a \), the \( n^4 \) term dominates.

The matrix is equivalent to a set of simultaneous equations extending to infinity and so has to be reduced to a more manageable size. There are many techniques available to perform this task but the one selected in this case is a splining procedure. This process of interpolation is done by assuming that the solutions \( b(n) \) can be interpolated from a subset of the infinite numbers describing the entire population structure. We refer to this subset as the nodal values and the departure coefficients for all levels can be expressed in terms of the nodal values using the following equation

\[
b(z) = \sum_{i=1}^{N} a_i b(n_i)
\]

(3.40)

where \( b(z) \) is the departure coefficient of energy level \( z \) in the original matrix, \( i \) is the term in the new compressed matrix and \( N \) is the size of the new compressed matrix. The coefficients, \( a_i \), are determined using the splining procedure which entails expressing the change between the nodal points of the new compressed matrix in terms of a cubic function of the terms of the original matrix which fall between these points. One of these functions is formed between each pair of nodal points. It is then assumed that the first derivative of the two functions on either side of a nodal point are equal. Using this assumption the coefficients of each function can be found. After performing this process for every nodal point a function is formed where every point in the original matrix can be expressed in terms of every nodal point, as shown in Equation 3.40. The end points are dealt with by assuming that as \( n_i \) tends to infinity, \( b(n_i) \) tends to unity and when \( n=1 \), the second derivative of the function at that point is zero.

This method therefore allows non-nodal equations to be eliminated. After the reduction has been carried out, the finite matrix equation can be solved to find \( b^{0q}(n) \) of each energy level. However to do this the transition rates must be known.
3.4 Cross-sections and Rate Coefficients for Atomic Processes

(i) Collisional Excitation and De-Excitation by Protons and Electrons

The transition rate for these atomic processes can be defined by

\[
C_{n,n'}^{e,p} = \int Q_{n \rightarrow n'}^{e,p}(v)vf(v)dv
\]  \(\text{(3.41)}\)

where the superscripts \(e\) and \(p\) represent incident electrons and protons, respectively, and \(Q_{n \rightarrow n'}^{e,p}\) denotes the process cross-section. The integration is taken over all incident particle velocities \(v\), such that \(1/2(m_0v^2) \geq \Delta E_{n,n'}\), where \(m_0\) is the reduced mass of the colliding system and the incident particles are assumed to have a Maxwellian distribution of velocities, given as

\[
f(v)dv = 4\pi \left(\frac{m_0}{2\pi kT_{e,p}}\right)^{\frac{3}{2}} \frac{1}{v^2} e^{-\frac{m_0v^2}{2kT_{e,p}}} dv
\]  \(\text{(3.42)}\)

The most prominent and important collisions are those between electrons and neutrals. Because the neutrals are so much heavier than the electrons they can be considered stationary and so only the electron velocity is taken into account. This enables Equation 3.41 to be alternatively defined in terms of, and integrated over, energy by defining \(E = 1/2(m_0v^2)\) to give

\[
C_{n,n'}^{e,p} = \left(\frac{kT_{e,p}}{2\pi m_e}\right)^{\frac{3}{2}} \left(\frac{m_e}{m_0}\right)^{\frac{3}{2}} \int \frac{Q_{n \rightarrow n'}^{e,p}}{\Delta E_{n,n'}} (E)e^{-\frac{E}{kT_{e,p}}} \frac{E}{kT_{e,p}} d\left(\frac{E}{kT_{e,p}}\right)
\]  \(\text{(3.43)}\)

where it is assumed that \(T_e = T_p\) and \(m_e/m_0\) is considered to be unity for the case of electrons colliding with neutrals.

In calculating the rates between \(n\) and \(n'\) only electron collisions are considered except for the case when \(n' = n \pm 1\), where both electron collisional rates \((N_e C_{n,n'=1}^{e,p})\) and proton collisional
rates \(N_pC_{n,n,z_i}^n\) are considered, as proton collisions are less effective for the case of large energy changes. Here the relationship between the electron and proton densities in the tokamak divertor plasma are taken to be \(N_p = 0.90N_e\), although this is variable depending on the level of impurities within the plasma. This is, however, a typical value given the content of carbon and beryllium present in the divertor region of the tokamak.

To form the matrix element \(D_{n,n'}\) needed for the solution of the equilibrium equations, \(\sum_{n'\neq n} C_{n,n'}\) must be evaluated. Although nodal \(n\) values are selected, as discussed earlier, to compress the matrix, the possible transitions to \(n'\) from each \(n\), are infinite. This infinite sum is evaluated directly for transitions between the \(n\) and the \(n'\) states, where \(n' \leq p = n+200\). The remainder is approximated by an integral, as can be seen in the following equation:

\[
\sum_{n'\neq n} C_{n,n'} = \sum_{n=n_0}^{n-1} C_{n,n'} + \sum_{n'=n+1}^{p} C_{n,n'} + \frac{1}{2} C_{n,p+1} + \int_{n=p+1}^{\infty} C_{n,n'} \, dm \tag{3.44}
\]

where \(n_0\) is the ground state.

From Equations 3.41 and 3.43, it can be seen that to calculate the transition rates for this process, it is necessary to know the cross-section for the transition.

In a dense laboratory plasma, such as that under investigation, collisional and radiative processes determine recombination rates and the spectra observed. For low \(n\) levels radiative transitions are more rapid whereas for high levels, collisions are more rapid. Therefore the extent of recombination depends on the intermediate levels where these rates are comparable.

Two methods are used to evaluate cross-sections and transition rates, each covering a different range of \(n\). The first approach adopts the method described by Percival and Richards (1978). This applies to the collisional region of \(n, n' \geq 5\) for the hydrogenic atom or ion. However, in this thesis it is also used for the initial and final quantum number of 4, even at the risk of larger errors, since there was no other method that could be applied to this level.
Bohr's correspondence principle states that quantum mechanics is in agreement with classical physics when studying transitions between highly excited states because the energy difference between such adjacent levels is so small that they are effectively continuous.

Semi-empirical cross-sections and rates are obtained using classical and semi-classical methods for transitions between highly excited levels of positively charged hydrogenic ions and hydrogen via electron impact.

The cross-section (Gee et al., 1976) is derived from calculations based upon the strong coupling correspondence principle (Percival and Richards, 1970) and is given by

\[
\frac{Z^2 \bar{E} \sigma(n \rightarrow n')}{n^4 \pi a_0^2} = ADL + FGH
\]  

(3.45)

where \( \bar{E} = E/R \), \( E \) being the incident electron energy and \( R \) the Rydberg unit of energy. \( Z \) is the nuclear charge in units of the electron charge and \( n^4 \pi a_0^2 \) is the geometric cross-section where \( a_0 \) is the Bohr radius and where

\[
A = \frac{8}{3s} \left( \frac{n'}{sn} \right) (0.184 - 0.04/s^{2/3}) \left( 1 - \frac{0.2s}{nn'} \right)^{2s}
\]  

(3.46)

\[
s = n' - n > 0
\]  

(3.47)

\[
D = \exp \left[ -Z^2 \left/ \left( nn' \bar{E}^2 \right) \right. \right]
\]  

(3.48)

In general

\[
L(\bar{E}) = \ln \left[ \frac{1 + (\bar{E}/E_1)^2}{1 + (\bar{E}/E_2)^2} \right]
\]  

(3.49)

However, specific values of \( E_1 \) and \( E_2 \) are chosen to ensure the cross-section is of the correct form in the high energy region where \( \bar{E} > 1 \) and in the intermediate energy region where \( 2Z^2/n < \bar{E} < 1 \). Modifications are then made to reflect the \( Z = 1 \) situation, for the case of electrons on hydrogen. Therefore, the \( L \) function becomes
\[ L(E) = \ln \left[ \frac{1 + 0.53E^2nm/Z_i^2}{1 + 0.4E} \right] \] (3.50)

Also,

\[ F = \left[ 1 - 0.3sD/(nn') \right]^{1+2s} \] (3.51)

\[ G = \frac{1}{2} \left( \frac{E_{n^2}}{Z_n n'} \right)^3 \] (3.52)

\[ H = C_1(x_-, y) - C_1(x_+, y) \] (3.53)

\[ C_1(x, y) = \frac{x^2 \ln(1 + 2x/3)}{2y + 3x/2} \] (3.54)

\[ x_+ = 2Z_i \left( \frac{n^2}{E} \left( \sqrt{2 - n^2/n'^2} \pm 1 \right) \right) \] (3.55)

\[ y = \left[ 1 - \frac{D \ln 18s}{4s} \right]^{-1} \] (3.56)

The first term on the right hand side of Equation 3.45, \( ADL \), is valid for high energies in the range \( E > 2Z_i/n \) and contains the quantum effects such as the cross-section depending logarithmically on energy. Alternatively, the second term \( FGH \) is the classical contribution to the equation and has no logarithmic dependence on the energy.

Cross-sections are valid for incident electron energies, \( E \), in the range

\[ \frac{Z_i^2}{n^2} \frac{E}{R} \ll 137^2 \] (3.57)

Typically the error in this cross-section is 6% but as \( E/R \) approaches the lower end of the range this error can increase up to an error of 20%, at worst, for incident energies greater than the ionisation energy.
Similarly the rate coefficient is valid in the temperature range

\[ 1.6 \times 10^5 \frac{Z_2^3}{n^2} \frac{T}{k} \ll 3 \times 10^9 \] (3.58)

with its greatest error of 25% occurring at the lower temperature boundary but with a more moderate and typical error of 6% as the temperature increases.

The second method of calculating the transition cross-sections is a quantum approach using matrix mechanics and covers the quantum states \(1 \leq n \leq 3\). The matrix method employed is that of the R-matrix theory, introduced in nuclear physics by Wigner (1946a,b) to describe resonance reactions. Since then its use has been extended to describe atomic and molecular processes such as the electron collisions with atoms, ions and molecules, as under investigation in this section. The R-matrix theory, applied to these particular processes, is described by Burke et al. (1987), and will be summarised here briefly.

The R-matrix considers separately two atomic regions, one falling within the radius of the target particle (which incorporates \(N+1\) electrons), the other falling outside (comprising the single free electron). Due to the complexity of electron exchange and correlation effects between the scattered electrons in the internal region, a detailed quantum mechanical solution is required, using a configuration interaction expansion of the total wave function. In the external region, however, close proximity reactions such as electron exchange with the target can be ignored. Consequently, the electron is considered to scatter away to the long-range multi-pole potential of the target. In this situation it is appropriate to use an asymptotic expansion or perturbation approach to find a solution. The R-matrix theory then links the solutions in the two regions to give values of cross-sections or collision strengths.

In a recent development of Burke et al. (1987), the R-matrix theory has been updated to incorporate a third region, which falls inside the radius of the target particle so that the inner region is now divided in two sub-regions. This is called the intermediate energy R-matrix (IERM) theory. Here, the radius of the inner sub-region just envelopes the charge distribution of the target states of the \(N\)-electron core, this is treated similarly to the previous approach used for the inner region. The radius of the outer sub-region envelopes the charge distribution of the states of all \(N+1\) electrons. Here only the outer valence electron, the scattered electron and their interaction are considered and can be represented by the less complicated single-centre wave functions. In the external region either one or two electrons
can be present depending on whether excitation or ionisation is being considered to take place. As before the scattered electron moves into the long-range multi-pole of the target, but this time it could be in a bound state as well as a continuum state. This enables the R-matrix basis states to cover the infinite number of channels available at intermediate energies, which would not be incorporated in a close coupling expansion.

Diagonalisation of the Hamiltonian produces an R-matrix. This can then be combined with the T-matrix energy averaging technique introduced by Burke et al. (1981) to obtain values for the cross-sections or, in this case, collision strengths.

The values for the cross-sections are obtained from collision strengths for the transitions. However, to enable the collision strengths to be calculated easily without re-using the complicated R-matrix method, a least squares fit is applied to the data. In this case, the least-squares fit procedure of Scholz (1990) is used to calculate the collision strength \( \gamma \). This is based upon Scott's (1989) calculations of \( 1s \rightarrow 2s \) and \( 1s \rightarrow 2p \) scattering processes, which are believed to be the most accurate at present.

For the \( 1s \rightarrow 2p \) transition this effective collision strength is given by

\[
\gamma_{2p} = \sum_{i=0}^{M} b_i T^i
\]  

(3.59)

where \( M = 5 \) and \( T \) is in Rydbergs.

The \( 1s \rightarrow 2s \) transition collision strength has a logarithmic exponential term added to this which takes effect at low temperatures and can be seen below

\[
\gamma_{2s} = \sum_{i=0}^{N} b_i T^i + b_{N+1} \ln(b_{N+2} T) e^{-b_{N+3} T}
\]  

(3.60)

where \( N = 4 \).

The parameters \( b_6 \) and \( b_7 \) are taken from the values given by Scholz (1990) and are given along with the other parameters in Table 3.1 (Callaway, 1994).
### Table 3.1

Table giving the $b_i$ coefficients in Equations 3.54 and 3.55 for various transitions. Table from Callaway (1994).

#### (A)

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<td>0.25501041</td>
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<td>-4.25196238</td>
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<td>1.26871040</td>
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<td>0.00724136</td>
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<tr>
<td>$b_9$</td>
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#### (C)

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<thead>
<tr>
<th></th>
<th>$2s-3s$</th>
<th>$2s-3p$</th>
<th>$2s-3d$</th>
<th>$2p-3s$</th>
<th>$2p-3p$</th>
<th>$2p-3d$</th>
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The $n=1\rightarrow2$ transitions have been researched the most and are therefore more accurately known than the other hydrogen transitions. For $1s\rightarrow n=3$ transitions Equations 3.59 and 3.60 are still used to evaluate the transitions collision strength but for the case of $1s\rightarrow3p$, $M=6$ and for the $3s$ and $3d$ excitations, $N=6$. Similarly for $n=2\rightarrow3$ transitions, Equation 3.59 is used with $M=6$.

(ii) The transition probability for spontaneous emission between states $n$ and $n'$ is given (Baker and Menzel, 1938) as

$$A_{n,n'} = \frac{A_0 g_{n,n'} Z^4}{n'n^3(n^2-n'^2)}$$  \hspace{1cm} (3.61)

where

$$A_0 = \frac{16\alpha^4 c}{3\pi\sqrt{3}a_0} = 1.574 \times 10^{10} \text{ s}^{-1}$$ \hspace{1cm} (3.62)

where $\alpha$ is a spline structure constant and $g_{n,n'}$ is the Gaunt factor, which is typically of order unity, but is more precisely given by the expression

$$g_{n,n'} = n\sqrt{\frac{(n'-n)^2+2n'}{(n'+n)^2+2n'}} \frac{nn'}{n'-n} \Delta(n',n)$$  \hspace{1cm} (3.63)

where

$$\Delta(n',n) = \left[ F(-n',-n'+1,1,x) \right]^2 - \left[ F(-n',-n+1,1,x) \right]^2$$ \hspace{1cm} (3.64)

where $F$ is a confluent hypergeometric function.
In the case where \( n \gg n' \), Menzel and Pekeris (1935) derived an asymptotic expansion for \( g_{n,n'} \), which was corrected by Burgess in 1959 to give

\[
g_{n,n'} = 1 - \frac{0.178 \left( 1 + \frac{n'^2}{n^2} \right)}{\left( 1 - \frac{n^2}{n'^2} \right)^{2/3} (n')^{2/3}} + 0.0496 \frac{1 - \frac{4n'^2}{3n^2} + \frac{n'^4}{n^4}}{\left( 1 - \frac{n^2}{n'^2} \right)^{2/3} (n')^{2/3}}
\]  

(3.65)

For 21 selected values of \( n-n' \leq 40 \), \( g_{n,n'} \) has been evaluated to 0.01% using Equations 3.63 and 3.64 and has been calculated to an accuracy of 0.05% for intermediate values of \( n' \) by the method of linear interpolation, described earlier. Equation 3.65 has been used to calculate \( g_{n,n'} \) for \( n-n' > 40 \).

(iii) Radiative Recombination

In the conditions of thermodynamic equilibrium, the number of photo-ionisations from a particular state can be equated to the total number of radiative and induced radiative recombinations to that state. This equation then gives the transition coefficients in terms of the photo-ionisation cross-section, which can be used generally even if conditions do not support thermodynamic equilibrium, as in this thesis. From this the radiative recombination coefficient can be defined as

\[
\alpha_r(n) = \frac{8\pi N_s(n)}{c^2} \int_{\nu_b}^{\nu_f} \nu^2 a_n(\nu) e^{-h\nu/kT} d\nu
\]

(3.66)

where \( a_n(\nu) \) is the photo-ionisation cross-section for a state \( n \) and, as before, can be defined in terms of the cross-section of an \( nl \) state by the following

\[
a_n(\nu) = \sum_{l=0}^{2l} \frac{(2l+1)}{n^2} a_{nl}(\nu)
\]

(3.67)
where the velocities of the free electrons have a Maxwell distribution which corresponds to the temperature $T$. The threshold frequency for these processes can be given by $h v_0 = I(n)$ and the photo-ionisation cross-section for a hydrogenic ion of nuclear charge $Z$ is defined as

$$a_n(v) = \frac{2^6 a_m a_e^2}{3\sqrt{3}} \frac{n}{Z^2} \left(1 + n^2 \varepsilon\right)^{-3} g_n(n, \varepsilon)$$  \hspace{1cm} (3.68)$$

where

$$h v = Z^2 I_{\mu} \left(\frac{1}{n^2 + \varepsilon}\right)$$  \hspace{1cm} (3.69)$$

$\varepsilon$ is the free electron energy in Rydbergs divided by $Z^2$ and $g_n(n, \varepsilon)$ is a Gaunt factor of order unity. However, this factor has been replaced with an asymptotic expansion (Seaton, 1959) before substitution and integration of Equation 3.63 to find the recombination coefficients. Flower and Seaton (1969) used the above method to calculate recombination coefficients in a computer programme and obtained an accuracy better than 1%.

(iv) and (v) Three-body Recombination and Collisional Ionisation

The relationship between the three-body recombination coefficient and the collisional ionisation coefficient for a state $n$ is given as

$$\gamma(c, n) = \frac{\omega(n)}{2} \left(\frac{h^2}{2\pi m k T}\right)^{3/2} e^{I(n)/kT} \gamma(n, c)$$  \hspace{1cm} (3.70)$$

The ionisation rate is found using the binary encounter method for all states except the ground state. The binary encounter method is a classical approach in which the collision between incident and bound electrons is equated to a collision between two free electrons and is a good method to use for high energies. It is valid for situations where the impact parameter is comparable to that of the electron’s orbit radius and the velocity of the incident particle is much greater than that of the bound electron. These conditions ensure that neither of the particles are affected by the remainder of the ion during the collision. Burgess and Percival (1968) consider direct and exchange scattering and the effect of their interference on each other in arriving at a definition of the ionisation cross-section.
\[ Q(E) = \frac{\pi e^4}{E} \left[ \left( \frac{1}{U} - \frac{1}{E} \right) + \frac{2U}{3} \left( \frac{1}{U^2} - \frac{1}{E^2} \right) + \log_e \frac{E}{E+U} \right] \]  

(3.71)

where \( U \) is the binding energy of the target electron. The ionisation coefficient is then defined as

\[ \gamma(n,c) = \frac{2}{3} \left( \frac{2U}{3kT} - 1 \right) - \frac{1}{2} \left( \frac{E_1(U/kT)}{kT} \right)^2 e^{U/kT} \]  

(3.72)

This equation can now be substituted into Equation 3.70 and the three-body recombination rate can be obtained from this. This method of calculating the ionisation cross-sections is used for all but the ground state.

The values for the ionisation cross-sections from the ground state, presently being used in the population code, are those of Rudge and Schwarz (1966a) which are calculated using the Born exchange approximations. Further study into the ionisation cross-sections from the ground state means that more recent recommended values are available for various energies (Bell et al., 1982). The ionisation cross-sections already being used in the population code are compared with these recommended values. Figure 3.1 shows a graphical comparison.
Figure 3.1. Comparison between (a) recommended values of the ionisation cross-sections from the ground state (Bell et al., 1982) and (b) those being used in the population code.

Figure 3.1 demonstrates that the ionisation cross-sections, already being used in the population code, match the recommended values for the low energies that are being considered in the divertor region. Even at energies of an order 100eV, which are much greater than the temperatures in the JET divertor region, the recommended values of ionisation cross-sections and those used in the population code differ by less than 10%.
Chapter 4

The Opacity Investigation

4.1 Introduction

In the previous chapter, the methods adopted by the population code to obtain values for transition rates and cross-sections for various line absorption and emission processes, were discussed. This enabled the populations of the various states to be calculated. However, despite its name the population code is used not only to evaluate line populations but also to apply the radiative transfer equation to obtain values for line emissivities and therefore to measure levels of opacity.

It does this by extending the collisional radiative model of deuterium level populations to incorporate radiative absorption effects and estimate the extent of radiative absorption in the divertor plasma. This new approach utilises a first order escape probability method (Hummer and Rybicki, 1982a) to evaluate the line escape probabilities and gives a self consistent model of the level populations and radiation field in a plane parallel slab of uniform temperature and density.

This chapter introduces the basic theories of opacity and goes on to describe how the population code combines the collisional radiative model with that of radiative absorption. It
then goes on to describe the geometric modelling of the plasma and how this information is assembled to determine the extent of opacity in a plasma.

The majority of the opacity theory outlined in the following section is obtained from Hummer and Storey (1992), Bohm-Vitense (1993) and Mihalas (1978).

4.2 The Theory of Opacity

When radiation interacts with matter, two distinct processes may occur. The first of these is scattering, causing a redistribution of energy, and incorporates Thompson, Compton and Rayleigh scattering.

The second is absorption, both continuous absorption including free-free and bound-free transitions (which are relatively rare in JET), and line absorption or bound-bound transitions.

The main difference between these processes is that, by definition, a scattering process is one in which a photon interacts with a scattering centre and emerges in a new direction. Hence, energy is not converted into kinetic energy of the gas particles, as in the case of absorption, instead the energy of a scattering process goes into altering the direction of the photon.

In the absorption process, the photon is destroyed and its energy is either partially or entirely converted directly into the thermal kinetic energy of the gas and hence has a greater connection to its local thermodynamic properties. Another process that occurs, and hasn’t yet been mentioned, is that of emission. This is the opposite of absorption and involves the transfer of energy from kinetic energy of the gas into the radiation field.

Some examples of absorption processes are photo-ionisation, photo-excitation, free-free absorption. Some examples of the opposite process of emission are radiative recombination and bremsstrahlung.

Some of the emission processes have been discussed in the previous chapter and although all the above processes are incorporated within the population code, they do not all apply to the conditions in the JET divertor plasma.
The removal of energy from the radiation field by matter can be described by the extinction coefficient, or opacity. This is a combination of both absorption and scattering but can be loosely described as the total absorption coefficient.

To create a model that analyses the extent of opacity in a plasma, a radiative transfer equation must be applied where the thermodynamic state of the gas and the distribution function of the radiation field must be specified simultaneously by solving the coupled equations of transfer and statistical equilibrium. In the following section the transfer equation is derived.

### 4.2.1 The Transfer Equation

When light passes through a gas or source of continuous opacity, its intensity, or number of photons per unit area, is diminished. The change in intensity along a path length $ds$ is given by

$$dI_\lambda = -k_\lambda I_\lambda ds$$

(4.1)

where $I_\lambda$ is the intensity of the beam (or energy per unit area per unit time per unit wavelength interval per unit solid angle) at wavelength $\lambda$ and $k_\lambda$ is the extinction or absorption coefficient, which, due to the variety of possible transitions, can become a complicated function of wavelength. $dI_\lambda$ is negative since the intensity is decreasing. Equation 4.1 leads to

$$d(ln I_\lambda) = -k_\lambda ds$$

(4.2)

Integrating both sides gives

$$ln[I_\lambda(s)] - ln[I_\lambda(0)] = - \int_0^s k_\lambda ds = - \int_0^s d\tau_\lambda = - \tau_\lambda(s)$$

(4.3)

where $\tau_\lambda$ is the optical depth at wavelength $\lambda$, defined by
\[ d\tau = k_\lambda ds \quad \Rightarrow \quad \tau = \int_0^s k_\lambda ds \quad (4.4) \]

If the value of the optical depth is much less than unity, the source of opacity is said to be optically thin at wavelength \( \lambda \) and if the optical depth is much greater than unity the source of opacity is optically thick. The geometrical depth through which the light can travel before being absorbed is inversely proportional to the absorption coefficient. Thus, if absorption is sufficiently high, it is not possible to detect light emitted from the deeper layers of the material.

When rearranged, Equation 4.3 becomes

\[ I_\lambda (s) = I_\lambda (0) e^{-\tau(s)} \quad (4.5) \]

Before continuing further, a proper definition of intensity \( I_\lambda \) should be given. Consider a beam of light of energy \( E_\lambda \) in a wavelength band \( d\lambda \) passing through an area \( d\sigma \) into a cone of solid angle \( d\omega \) as seen in Figure 4.1.

\[ \text{Figure 4.1. The amount of energy } E_\lambda \text{ in a wavelength band } \Delta \lambda \text{ passes through an area } d\sigma \text{ into a cone with opening } d\omega. \text{ Figure from Böhm-Vitense (1993).} \]
The intensity is then given as

\[ I_\lambda = \frac{E_\lambda}{d\omega d\delta d\lambda} \]  

(4.6)

Now, by bringing angle dependence into this discussion it can be shown that if the energy were to pass through the same surface at an angle \( \theta \) with respect to the normal of this surface area, as seen in Figure 4.2, the effective beam width is reduced by a factor of \( \cos \theta \) and the intensity defined as

\[ I_\lambda = \frac{E_\lambda}{\delta\omega\delta\phi \cos \theta \delta\lambda} \]  

(4.7)

\[ \begin{figure}[h] 
\centering 
\includegraphics[width=0.5\textwidth]{figure4_2.png} 
\caption{The amount of energy passing through an area \( d\sigma \), at an angle \( \theta \) to the normal, is reduced by a factor of \( \cos \theta \). The area \( d\sigma' \) is perpendicular to the inclined beam. When the light beam passes through \( d\sigma \) at an angle \( \theta \) its effective beam width is reduced since the area of the plane that it hits is reduced according to \( d\sigma' = d\sigma \cos \theta \). Figure from Böhm-Vitense (1993).} 
\end{figure} \]

When a beam of radiation passes through a volume of gas, its energy is reduced by absorption of an amount

\[ dE_\lambda = -k_\lambda E_\lambda ds = -k_\lambda ds I_\lambda d\omega d\lambda d\sigma \]  

(4.8)

and increased by emission from the volume \( dV = d\sigma ds \) by an amount
where \( j_\lambda \) is the emission coefficient, the amount of energy emitted per unit time per unit volume into unit solid angle \( \Delta \omega = 1 \) per unit wavelength band \( \Delta \lambda = 1 \). Therefore the total change in energy is

\[
dE_\lambda = j_\lambda ds \, d\omega \, d\lambda \, d\sigma
\]

By cancelling \( d\omega \, d\lambda \, d\sigma \) and rearranging we get a general expression for the radiative transfer equation

\[
\frac{dl_\lambda}{k_\lambda ds} = \frac{dl_\lambda}{d\tau_\lambda} = -I_\lambda + \frac{j_\lambda}{k_\lambda} = -I_\lambda + S_\lambda
\]

where \( S_\lambda \) is the source function which is the ratio of the emission to absorption coefficients.

In the plane parallel case, it is now assumed that the length of path \( s \), and therefore the optical depth, is dependent on the angle of emission due to the geometry, or boundaries of the plane being considered. When referring to the optical depth of a plane, one is generally referring the optical depth along the path which is normal to the plane. The optical depth along path \( s \), at an angle \( \theta \) to the normal, can be related to the optical depth along the normal, \( \tau_d(t) \), as shown in Figure 4.3.

![Figure 4.3](image-url)  
**Figure 4.3.** The ray of light, with initial intensity \( I_{\lambda_0} \), passes along path \( s \) which is inclined to the plane’s perpendicular by an angle \( \theta \). Figure from Böhm-Vitense (1993).
From this it can be seen that

\[ ds = \sec \theta \, dt \] (4.12)

where \( \theta \) is the angle between the light beam and the path of shortest length from the source to the edge of the atmosphere or slab under observation (the zenith distance).

Substituting this into Equation 4.3 gives

\[ \tau_\lambda(s) = \sec \theta \int_0^t k_\lambda \, dt = \sec \theta \tau_\lambda(t) \] (4.13)

Therefore Equation 4.5 now becomes

\[ I_\lambda(s, \theta) = I_\lambda(0) e^{-\sec \theta \tau_\lambda(t)} \] (4.14)

The transfer equation for the plane parallel case, Equation 4.11, can now be expressed in its simplest form as

\[ \mu \left( \frac{\partial I_\lambda}{\partial \tau_\lambda} \right) = S_\lambda - I_\lambda \] (4.15)

where \( \mu = \cos \theta \), \( \theta \) is the angle between the direction of the photon beam and the normal to the planar surface and \( \tau_\lambda \) represents the optical depth at wavelength \( \lambda \) along the normal to the plane and can be defined as

\[ d\tau_\lambda = -d\tau_\lambda \sec \theta \] (4.16)
4.2.2 The Mean Intensity

The specific intensity \( I_\lambda \), used extensively in the previous section, is angle and wavelength (or frequency) dependent. The specific intensity averaged over all angles is denoted as \( J_\lambda \). Averaging this again over the wavelength range provides the mean intensity (\( \bar{J} \)), which can be defined as the straight average of the specific intensity over all angles and wavelengths.

This section obtains a solution for the mean intensity of the transfer equation in the escape probability approximation, closely following the development in Hummer and Storey (1992). For a more comprehensive look at the derivation in this section, further references to study are Hummer and Kunaz (1980), Hummer and Rybicki (1982b), Hummer and Rybicki (1985) and Mihalas (1978).

Complete redistribution is assumed, this is discussed later in Section 4.2.5, and under these circumstances the following expression for the source function applies

\[
S_L = \frac{N_n A_{n' n}}{N_n B_{n'} - N_n' B_{n' n}}
\]

where \( A_{n' n}, B_{n' n} \) and \( B_{n'} \) are the Einstein coefficients for spontaneous emission, induced emission and induced absorption, respectively and the subscript \( L \) denotes the spectral line.

To obtain an expression for the source function in terms of the mean intensity (Hummer, 1969), requires a more detailed look at the absorption and emission coefficients. In doing this it is assumed that radiative and collisional transitions occur in both directions, atoms that are excited in a non-resonant manner decay with a frequency distribution identical to that of the absorption profile \( \phi(\nu) \), polarisation is neglected and stimulated emission is treated as negative absorption.

With these assumptions the absorption coefficient can be expressed, in terms of frequency rather than wavelength, as

\[
k_\nu = \frac{\hbar \nu \phi}{4\pi} \left( N_n B_{n'} - N_n' B_{n' n} \right) \phi(\nu)
\]
where $\phi(v)dv$ is the probability that a photon with frequency in the interval $v, v + dv$ will be absorbed and $v_0$ is the line centre frequency.

Similarly, the energy emitted per unit volume, per unit time in the frequency interval $dv$ into the element of solid angle $d\Omega$ about direction $s$, can be expressed as

$$
\epsilon_{\nu}(s)dv d\Omega = \left\{ N_n B_{n\nu} \frac{A_{n\nu}}{A_{n\nu} + B_{n\nu} J + C_{n\nu}} \left( \oint I(v') R(v',s) dv' d\Omega' \right) \right\} \\
\quad \quad \quad \quad \quad \quad \quad \quad + \left\{ N_n C_{n\nu} \frac{A_{n\nu}}{A_{n\nu} + B_{n\nu} J + C_{n\nu}} \phi(v) \right\} \frac{h \nu_0}{4\pi} dv d\Omega
$$

(4.19)

where

$$
J = \oint I(v') \phi(v') dv' d\Omega' = \frac{1}{4\pi}
$$

(4.20)

and $C_{n\nu}$ and $C_{n\nu}$ are the collisional rate constants, including the electron density factor. Collisional excitation and de-excitation are the only non-radiative atomic processes incorporated in Equation 4.19 because a two-level atom is being considered. It should be mentioned, however, that any other form of non-resonant excitation, such as radiative recombination, three-body recombination, spontaneous emission and collisional excitation and de-excitation could be included in the first factor of the second term of the equation, Hummer (1969). Also in Equation 4.19 $R(v',s')$ is the redistribution function. When a photon with frequency and direction within the elements $dv'$ and $d\Omega'$ about $v'$ and $s'$, is absorbed and followed by the emission of a photon in the same spectral line but with a frequency and direction within the elements $dv$ and $d\Omega$ about $v$ and $s$, then the probability of this event occurring can be represented by $R(v',s') dv' d\Omega' dv d\Omega$. The redistribution function is normalised such that

$$
\oint \oint R(v',s') dv' d\Omega' dv d\Omega = 1
$$

(4.21)

and because each absorbed photon must be emitted, the absorption profile can be given as
The absorption profile is therefore normalised also such that

\[ \int \phi(\nu) d\nu = 1 \]  \hspace{1cm} (4.23)

Returning to Equation 4.19, the first term defines radiative excitation and comprises three factors which can be described, respectively, as the rate at which radiative excitation raises the atom's energy state from level \( n \) to level \( n' \), the fraction of atoms leaving level \( n' \) via spontaneous radiative transitions and the third factor is a normalised function of frequency and direction depending functionally on the radiation field. The second term of Equation 4.19 corresponds to non-resonant excitation and is also comprised of three factors. The second factor is identical to that of the first term, whilst the first and third factor define the rate at which atoms are collisionally excited from the lower to the upper state and a normalised frequency function characteristic of non-resonant excitation, respectively.

As mentioned earlier in this chapter, the source function is the ratio of the emission to absorption coefficients or

\[ S(\nu) = \frac{\varepsilon_{\nu}(s)}{k_{\nu}} \]  \hspace{1cm} (4.24)

Therefore, substituting Equations 4.18 and 4.19 into Equation 4.24 and eliminating \( N_{n} \) and \( N_{n'} \) using the equation of statistical equilibrium for a two-level atom

\[ N_{n} \left( B_{nn'} \bar{J} + C_{nn'} \right) = N_{n'} \left( A_{n'n} + B_{nn'} \bar{J} + C_{n'n} \right) \]  \hspace{1cm} (4.25)

gives an expression for the source function in terms of the mean intensity

\[ S(\nu s) = (1 - \epsilon) \frac{4\pi}{\phi(\nu)} \int_{0}^{\infty} R(\nu s', vs) (\nu s') d\nu d\Omega' + \epsilon B(\nu_{o}, T_{e}) \]  \hspace{1cm} (4.26)

where \( B(\nu_{o}, T_{e}) \) is the Planck function at the line-centre frequency for the local electron temperature and
\[ \varepsilon = \frac{C_{n,n}^n}{\left[ C_{n,n}^n + A_{n,n} \left[ 1 - \exp\left( -\frac{h v_0}{k T} \right) \right]^{-1} \right]} \] (4.27)

where \( \varepsilon \) is the probability per scattering of a photon being removed from the line. In our model, in which only opacity effects in the Lyman transitions are considered, it is the probability that a photon is emitted in any other transition than a Lyman transition. For Lyman transitions, with the exception of Ly\( \alpha \), it has a value of 0.1 - 0.2.

Now

\[ J(\nu') = \frac{1}{4\pi} \int I(\nu')d\Omega' \] (4.28)

By assuming an isotropic radiation field, Equation 4.28 becomes \( J(\nu') = I(\nu') \) and therefore Equation 4.26 becomes

\[ S(\nu) = (1 - \varepsilon) \frac{1}{\phi(\nu)} \int R(\nu', \nu) J(\nu')d\nu' + \varepsilon B \] (4.29)

where

\[ R(\nu', \nu) = 4\pi \int R(\nu', \nu') d\Omega' \] (4.30)

From Equation 4.22 it can be seen that

\[ \int_0^\nu R(\nu, \nu')d\nu' = \phi(\nu) \] (4.31)

With the assumption of complete redistribution, there is no relation between the absorption or emission frequency so the probability of absorption at frequency \( \nu' \) followed by emission at frequency \( \nu \) is the product.

\[ R(\nu', \nu) = \phi(\nu')\phi(\nu) \] (4.32)
A more comprehensive outline of the assumptions made in this integration is given in Hummer (1969). Substituting Equation 4.32 into Equation 4.29 gives

\[ S(v) = (1 - \epsilon) \int_0^\infty \phi(v) J(v) dv + \epsilon B \]  

(4.33)

From Equations 4.20, 4.28 and 4.33 it can be seen that

\[ S(v) = (1 - \epsilon) \bar{I} + \epsilon B \]  

(4.34)

From this point, the line source function will be used instead of the frequency source function. Consequently, it will depend on the optical depth rather than frequency. The last term of Equation 4.34 can be re-expressed as \( G(\tau) \) which is the photon creation rate by line processes (regarded as known). The line source function can therefore be expressed as

\[ S_L(\tau) = (1 - \epsilon) \bar{J}(\tau) + G(\tau) \]  

(4.35)

where

\[ \bar{J}(\tau) = \int dx J_x(\tau) \]  

(4.36)

and \( x \) is the frequency displacement from the line centre in thermal Doppler units (which measure the ratio of the frequency from the line centre to the Doppler width). The general result for the solution of the transfer equation for \( \bar{J} \) is (Hummer and Storey, 1992)

\[ \bar{J}(\tau) = \int_0^\tau d\tau' \bar{K}_1(\tau, \tau') S_L(\tau') \]  

(4.37)

where

\[ \bar{K}_1(\tau, \tau') = \frac{1}{2} \int dx \phi^2(x) \mathcal{E}_1 \left\{ \phi(x) \Delta(\tau - \tau') \right\} \]  

(4.38)
where $\tau'$ is a dummy variable optical depth. Although it measures the optical depth to a point on the ray, it is not a measurement of the optical depth, $\tau$, to the point being investigated for opacity. $E_1$ is a standard exponential integral.

In the first order approximation, which is used in the population code, it is assumed that the absorption source is the same as the emission source and so the variation of $S_i$ is slow compared to that of $\tilde{K}_1$ for $\tau \approx \tau'$. Hence, $S_i(\tau)$ can be replaced with $S_i(\tau)$ which can then be taken out of the integral in Equation 4.37, enabling it to be solved easily to give

$$J(\tau) = S_L(\tau)[1 - P_e(\tau, T)]$$

(4.39)

where $P_e$ is the escape probability function and $T$ is the optical thickness. The escape probability function, derived in Appendix A, can be expressed as

$$P_e(\tau, T) = \frac{1}{2} \left[ \tilde{K}_2(\tau, a) + \tilde{K}_2(T - \tau, a) \right]$$

(4.40)

where

$$\tilde{K}_2(\tau, a) = \int_{-\infty}^{\infty} dx \phi(a, x) E_2 \left\{ \tau (\phi(a, x)) \right\}$$

(4.41)

and

$$\phi(a, x) = \frac{a}{\pi^{\frac{3}{2}}} \int_{-\infty}^{\infty} \frac{dt e^{-t^2}}{(x - t)^2 + a^2}$$

(4.42)

where $E_2$ is a standard exponential integral and $\phi(a, x)$ is the general expression for the Voigt function which is a convolution of the Lorentzian and Doppler profile expressions. Here, $a$ is the damping constant, which specifies the relative importance of the Lorentzian and Gaussian components and $t = 2x\sqrt{(\ln 2)}$ and $x$ is the frequency parameter, defined in Equation 4.53. Any profile function could used in place of Equation 4.42, depending on which absorption or emission profile is best suited to the spectral lines under investigation. This will be discussed in Section 4.2.5.
The expression for the escape probability, in Equation 4.40, applies to a plasma model with a uniform slab of infinite extent in two dimensions. This is not the same as the model being used in this thesis and hence a modified expression for the escape probability, Equations 4.60 and 4.61 which are given later in this chapter, are used in the population code. However, the escape probability expressed in Equation 4.40 is used in a testing capacity to validate of the results obtained via the population code. It can be shown (Appendix A) that Equation 4.40 is derived from Equation 4.46, assuming simple geometric factors.

In the equations of statistical equilibrium, the bound-bound radiative rates can be grouped into the form

\[ R_{n'\rightarrow n} = N_{n'}A_{n'\rightarrow n} + \bar{J}(N_{n}B_{n'\rightarrow n} - N_{n}B_{n'\rightarrow n'}) \]  

(4.43)

By substituting Equation 4.17 into 4.39, \( \bar{J} \) can be expressed in terms of populations, Einstein coefficients and the escape probability as shown

\[ \bar{J} = \left(1 - P_e\right) \frac{N_{n'}A_{n'\rightarrow n}}{N_{n}B_{n'\rightarrow n} - N_{n}B_{n'\rightarrow n'}} \] 

(4.44)

Substituting Equation 4.44 into 4.43 gives the bound-bound radiative rates in the form of

\[ R_{n'\rightarrow n} = N_{n'}A_{n'\rightarrow n}P_e(\tau_{n'\rightarrow n}) \]  

(4.45)

It can be seen from this, that the original values of the bound-bound radiative rates in the statistical equilibrium equations should be multiplied by the escape probability to incorporate the first order escape probability treatment of radiative transfer into the collisional radiative model.

It is assumed that the bulk of the population is in the ground state, hence it is only necessary to treat the effect of the optical depth in the Lyman-lines, this assumption will be explained in detail in the following sections of this chapter. Using Equation 4.45 for the Lyman-lines, the statistical equilibrium equations can be solved to calculate the populations of the various energy states. From this, the code can calculate the emissivities of the various lines.
4.2.3 Escape Probability and the ‘Slab-Stack’ Model

Before describing the method of evaluating the escape probability within the population code, it is essential to give a brief description of the geometric model, shown in Figure 4.4.

This consists of a stack of uniform rectangular slabs each with different physical conditions to represent a section of the divertor plasma under investigation. A more detailed description is given in section 4.2.6 of this chapter. However, it is sufficient for now to mention that emission and absorption profiles are assumed to be Doppler with the width depending on the neutral temperature of the slab, and Doppler shifts between slabs, caused by neutral velocity gradients, are considered. The assumption of Doppler profiles will be discussed in Section 4.2.5.

The single-flight escape probability at a frequency $\nu$ and from a point in one of the slabs, along a ray, can now be defined as

$$P_e(\nu, \alpha, \beta) = e^{-\tau}$$

(4.46)

where $\alpha$ and $\beta$ are the angles shown in Figure 4.4.

The optical depth at a particular frequency is the sum of the individual slabs optical thickness, as shown
\[ \tau_v = \sum_i \tau_v(i) \]  \hspace{1cm} (4.47)

where \( i \) denotes the slab number.

The optical depth can be defined as a product of the absorption coefficient, ray path length and the profile function, which we assume to be Doppler (as discussed in section 4.2.5).

\[ \tau_v(i) = \frac{k_i l_i}{\sqrt{\pi}} e^{-\left(\frac{v - v_0}{\Delta v_{\text{Diss}}}\right)^2} \]  \hspace{1cm} (4.48)

where \( k_i \) and \( l_i \) are the mean absorption coefficient (averaged over the line width) and ray length through slab \( i \), respectively. \( v_0 \) is the line centre frequency in the absorbing slab \( i \) and \( \Delta v_{\text{Diss}} \) is the Doppler width in the absorbing slab, in frequency units.

Now

\[ \Delta v_i = v_0 - v_0^i \]  \hspace{1cm} (4.49)

where \( v_0 \) is the line centre frequency in the emitting slab and \( \Delta v_i \) is the frequency shift between the absorbing and emitting slab, which can be expressed in terms of the velocity shift between the slabs by the expression

\[ \Delta v_i = \frac{v_0 d v_i}{c} \]  \hspace{1cm} (4.50)

where \( dv_i \) is the velocity shift between the emitting and absorbing slabs which is calculated using the equation

\[ dv_i = (v_{x0} - v_{y0}) \sin \alpha \sin \beta + (v_{y0} - v_{x0}) \cos \alpha + (v_{y0} - v_{y0}) \sin \alpha \cos \beta \]  \hspace{1cm} (4.51)

Here, the subscripts \( i \) and 0 denote the absorbing and the emitting slab, respectively and the angles \( \alpha \) and \( \beta \) are as shown in Figure 4.4. The velocities of the neutral plasma flux in the \( x \),
y and z direction of the tokamak, as seen in Figure 4.4, are provided by the plasma modelling procedure described in Chapter 2.

Rearranging Equation 4.49 gives

\[ \nu - \nu_{0i} = \nu - \nu_0 + \Delta \nu_i \]  (4.52)

where \( \nu \) is some frequency on the profile under investigation.

Re-expressing the exponential term of Equation 4.48 gives

\[ \frac{v - v_{0i}}{\Delta \nu_{Di}} = \frac{v - v_{0i}}{\Delta \nu_D} \frac{\Delta \nu_D}{\Delta \nu_{Di}} \]  (4.53)

where \( \Delta \nu_D \) and \( \Delta \nu_{Di} \) are the Doppler widths of the emitting and absorbing slabs, respectively.

The Doppler width \( \Delta \nu_D \) can be expressed as

\[ \Delta \nu_D = \frac{v_0}{c} \sqrt{\frac{2kT}{m_D}} \]  (4.54)

where \( m_D = 2 \) (the mass of deuterium in a.m.u.).

Substituting Equation 4.52 into Equation 4.53 gives

\[ \frac{v - v_{0i}}{\Delta \nu_{Di}} = \left( \frac{v - v_0 + \Delta \nu_i}{\Delta \nu_D} \right) \frac{\Delta \nu_D}{\Delta \nu_{Di}} \]  (4.55)

The frequency parameter, \( x \), can be defined as

\[ x = \frac{v - v_0}{\Delta \nu_D} \]  (4.56)
Using this expression and Equation 4.55, equation 4.48 becomes

\[ \tau_s(i) = \frac{-k_i}{\sqrt{\pi}} l_i e^{-\left(\frac{\Delta V_D}{\Delta V_{Di}}\right)^2} \left(\frac{\Delta V_i}{\Delta V_{Di}}\right)^2 \]  

(4.57)

Assuming Doppler profiles implies

\[ \left(\frac{\Delta V_D}{\Delta V_{Di}}\right)^2 = \frac{T}{T_i} \]  

(4.58)

where \( T \) is the temperature of the emitting slab and \( T_i \) is the temperature of the absorbing slab. Substituting this into Equation 4.57 gives the expression for the optical depth which is used in the population code

\[ \tau_s(i) = \frac{-k_i}{\sqrt{\pi}} l_i e^{-\left(\frac{\Delta V_i}{\Delta V_{Di}}\right)^2} \]  

(4.59)

Equation 4.59 is substituted into Equation 4.47 to sum the optical depths over the slabs. This frequency dependent optical depth is then substituted into Equation 4.46 to give the escape probability of a photon at a particular frequency in the line and in one direction out of the slab, \( P_e(\nu, \alpha, \beta) \). The population code calculates the average escape probability over all angles and frequencies. An expression for the average escape probability is

\[ \overline{P_e} = \int\int \int \phi(\nu) P_e(\nu, \alpha, \beta) \frac{\sin \alpha}{4\pi} \, d\beta \, d\alpha \, d\nu \]  

(4.60)

This integration is evaluated in the population code using the expression

\[ \overline{P_e} = \frac{1}{4\pi} \left( \frac{3}{8} \right)^2 i_\alpha i_\beta \sum_\nu \sum_\alpha w(\alpha) \sum_\beta w(\beta) e^{-\tau_e} \]  

(4.61)

where \( i_\alpha \) and \( i_\beta \) are increments of the angles \( \alpha \) and \( \beta \) through 180° and 360° respectively. The sum over each of the angles is done using Simpson’s 3/8 – rule which incorporates the weights \( w(\alpha) \) and \( w(\beta) \). The sum over frequency utilises the Gaussian Laguerre quadrature.
method where \( w(\nu) \) is the weighting factor for each term, which incorporates the line profile and frequency increment \( \phi(\nu)d\nu \) at each frequency.

Here the photon escape probability is calculated to the boundaries of the ‘slab-stack’ model. This is achieved by calculating the path length \( (l) \) of the ray through each slab of the model for a particular angle of emission. This value is incorporated into Equation 4.59, which gives an expression for the optical depth of a slab for a particular frequency and a specific angle of emission. The optical depth is summed over all slabs according to Equation 4.47 and then substituted into Equation 4.61 in order to calculate the average escape probability over all frequencies and angles of emission, to the boundaries of the ‘slab-stack’ model.

![Spectrometer Line of Sight](image)

Figure 4.5. Several equidistant points of emission, along the same line of sight as the VUV spectrometer, are selected for use in the population code. This method enables a direct comparison to be made between the theoretical model and the diagnostic data.

The value for the average escape probability is calculated, via Equation 4.61, for several equidistant emission points along a particular ray path within the ‘slab-stack’ model, as shown in Figure 4.5. This is done in order to simulate the line of sight of the VUV spectrometer, as will be described shortly. The value of the average escape probability, for each of these points, is incorporated into the statistical equilibrium equations, as described in Section 4.2.2, to obtain values for the level populations. Any photons other than Lyman photons are assumed to have an escape probability of unity in the population code. This,
along with the assumption that the ground state population is constant for the conditions under analysis, removes the need for iteration between the values of the level populations and the radiation field, or escape probability, within the code. An investigation into the effects of opacity on the ionisation balance of the plasma is undertaken and described in Chapter 6. The results to this study indicate that it is a good approximation to assume that the ground state population is constant for the conditions in the JET divertor.

The average escape probability is used to determine values for the level populations as described. However, to ensure consistency between the theoretical and experimental investigations, a second additional escape probability is calculated. This is the photon escape probability along the same line of sight as that under observation by the VUV spectrometer. The angles \((\alpha \text{ and } \beta)\) at which the spectrometer line of sight passes through the 'slab-stack' model, are determined so as to align the theoretical model with that of the experimental procedure. A number of emission points are selected along this line of sight, as shown in Figure 4.5. The optical depth is calculated from one emission point to the next and summed to give the total optical depth, \(\tau_o\), between two extreme boundaries of the 'slab-stack' model in the direction of the ray. This value can then be incorporated into Equation 4.62, which gives an expression for the photon escape probability averaged over frequency for a specific angle of emission.

\[
P_e(\alpha, \beta) = \sum \nu w(\nu) e^{-\tau_o} \quad (4.62)
\]

Here \(w(\nu)\) is a frequency weighting factor. This photon escape probability can be used to calculate various spectral line emissions along the same line of sight as the VUV spectrometer and is used to obtain the branching ratio values given in Section 6.4.1. This is done by calculating the intensity of light at frequency \(\nu\) emitted in a specific direction and can be defined as

\[
I_\nu = \int_{s_i}^{s_f} j_\nu P_e(\nu, \alpha, \beta) ds \quad (4.63)
\]
where $s_1$ and $s_2$ are the boundaries of the plasma along the ray path. The emission coefficient can be defined as

$$j_v = N_a \cdot A_{a,n} \cdot h \cdot v_{n,n} \cdot \phi(v)$$ (4.64)

Hence, Equation 4.63 can be expressed as

$$I_v = \int_{s_1}^{s_2} N_a \cdot A_{a,n} \cdot h \cdot v_{n,n} \cdot \phi(v) \cdot P_e(v, \alpha, \beta) \, ds$$ (4.65)

The specific intensity of a particular spectral line can be expressed as

$$I_L = \int I_v \, dv = \int_{s_1}^{s_2} N_a \cdot A_{a,n} \cdot h \cdot v_{n,n} \cdot \int_{\text{line}} \phi(v) \cdot P_e(v, \alpha, \beta) \, ds$$ (4.66)

Now, substituting Equation 4.62 into Equation 4.66, where the profile function $\phi(v)$ is incorporated within the weighting factor $w(v)$, gives

$$I_L = \int_{s_1}^{s_2} N_a \cdot A_{a,n} \cdot h \cdot v_{n,n} \cdot P_e(v, \alpha, \beta) \, ds$$ (4.67)

This integration is actually calculated by summing values of intensity at various equidistant points in the slab along the same direction as a light ray, as shown in Figure 4.5. Therefore

$$I_L = \sum_{k=1}^{M} N_n^k \cdot A_{a,n} \cdot h \cdot v_{n,n} \cdot P_e^k(v, \alpha, \beta) \Delta s^k$$ (4.68)

where $k$ represents an emission point along the ray and $M$ is the maximum number of emission points along the ray. The distance between two emission points is $\Delta s^k$. The average escape probability over all angles $\bar{P}_e$ is incorporated implicitly in Equation 4.68 since it is used in the calculation of the level populations $N_n^k$.

In the divertor region of the JET tokamak electron densities are of the order of $10^{20} \, m^{-3}$, with temperatures in the region of 1eV to 3eV and neutral deuterium to ion density ratio of
approximately 0.1. The typical values of Lyα optical depth that are produced by these conditions are approximately unity.

The code assumes the majority of the population to be in the ground state because decay to \( n = 1 \) is almost instantaneous. Typical values of the population distribution for levels \( n = 2 \) and \( n = 3 \) relative to level \( n = 1 \), under these conditions, are \( 4.7 \times 10^4 \) and \( 1.8 \times 10^6 \), respectively.

Figure 4.6 shows escape probabilities for the Lyman transitions as a function of \( n \). Each curve is labelled with the Lyα optical thickness of the gas, \( \tau_{\text{Ly} \alpha} \) as a power of ten.

![Figure 4.6. Loss probability versus principal quantum number \( n \), for Lyα optical thickness, \( \tau_{\text{Ly} \alpha} = 10^2, 10^3, 10^4, 10^5, 10^6 \) for \( \beta \) (the ratio of continuum opacity per unit Doppler width to the line opacity) = 0. Figure from Hummer and Storey (1992).](image)

Figure 4.6 shows a situation without continuous opacity, which is the case under investigation in JET. The population code does have a method of treating spectral line loss due to continuous absorption, in addition to the loss due to single flight escape. However, it will be seen in Equation 4.79 that as \( n \) increases the Lyman cross-section drops dramatically and it would require optical depths in the region of \( 10^3 \) for continuous absorption to occur in the case of a hydrogen, or in this case deuterium, atom. Continuous absorption is possible at lower optical depths in case of other atoms, but since impurity levels are low in the JET
The tokamak, there are negligible sources of continuous absorption and so its effects are not considered within the code.

The level of continuum emission is proportional to the effective charge which can be expressed as

\[ Z_{\text{eff}} = \frac{n_D + \sum q_z^2 n_z}{n_e} \quad (4.69) \]

where \( D \) and \( Z \) represent deuterium and impurity ions, respectively, \( q_z \) is the impurity ion charge, \( n_z \) is the impurity ion density, \( n_D \) is the deuterium ion density and \( n_e \) is the electron density. According to the charge neutrality of a plasma, the overall electron charge must equal that of the ions which implies

\[ n_e = n_D + \sum q_z n_z \quad (4.70) \]

Rearranging this equation gives

\[ n_D = n_e - \sum q_z n_z \quad (4.71) \]

Substituting Equation 4.71 in Equation 4.69 gives the expression

\[ Z_{\text{eff}} = 1 + \frac{\sum q_z (q_z - 1) n_z}{n_e} \quad (4.72) \]

In the core of the JET tokamak, \( C^{6+} \) is typically the dominant impurity and in this situation it is reasonable to assume that it is the only impurity, hence \( q_z = 6 \). The value of \( Z_{\text{eff}} \) for the JET core plasma is typically 2. Substituting these values into Equation 4.72 gives the ratio of the electron density to the \( C^{6+} \) density \( n_C \) as

\[ \frac{n_e}{n_C} = 30 \quad (4.73) \]
Hence the impurity content in the core is negligible with respect to the electron density. Although the electron to carbon ratio in the divertor is not identical to that in the core, it can be considered to be similar. This low impurity level renders the level of continuous absorption insignificant. In addition to this, photo-ionisation has a small cross-section compared with line absorption. The CII ion is the most probable impurity in the divertor. If we assume the levels of the CII ion are populated according to a Boltzmann distribution, the optical depth of a continuous absorption process is 0.002. This value is an overestimation as the population of the relevant excited levels is overestimated by a factor of, at least, 10^5.

It can be assumed, therefore, that the ratio of continuous to line absorption (β) is zero in the JET divertor region. Figure 4.6 shows that for β = 0, the escape probabilities increase with n so the lines either remain optically thick or become optically thin with increasing n. The following section derives this dependency of n on the Lyman-line optical depth.

### 4.2.4 Optical Depth

It can be seen from Equation 4.4 that optical depth is directly proportional to the absorption coefficient. Now, the absorption coefficient is also proportional to the cross-section of a particular transition

\[
k_v = \sigma_v \sum n_u
\]  

(4.74)

where \( \sigma_v \) is the cross-section at a particular frequency \( v \) in the line and \( n_u \) is the absorber density. Normally there are many different types of absorber which need to be summed up, however the only absorber being considered in this case is neutral deuterium. The sum over all absorber atoms \( N \) can be defined as

\[
N = \sum n_u
\]  

(4.75)

A similar expression to that of Equation 4.74 defines the mean absorption coefficient \( \bar{k} \) in terms of the mean cross-section \( \bar{\sigma} \) and \( N \).
\[ \bar{k} = \bar{\sigma}N \]  
(4.76)

The mean cross-section can be defined in terms of the cross-section for a specific frequency in the expression

\[ \bar{\sigma} = \frac{\int \sigma_d d\nu}{\Delta V_D} \]  
(4.77)

where the Doppler width can be defined as

\[ \Delta V_D = \frac{v_0}{c} \left[ \frac{2kT}{M} \right]^{1/2} \]  
(4.78)

and where \( M \) is the mass of the absorber atom.

Now,

\[ \int \sigma_d d\nu = \frac{\pi e^2 f_{ij}}{mc} \]  
(4.79)

where \( f_{ij} \) is the oscillator strength of a transition between energy levels \( i \) and \( j \) which can be given in terms of the Einstein coefficient \( B_j \) by the expression

\[ f_{ij} = \frac{h \nu_{ij} mc}{4\pi^2 e^2} B_j \]  
(4.80)

Substituting Equation 4.79 into Equation 4.77 gives an expression for the mean cross-section in terms of the transition oscillator strength and the Doppler width

\[ \bar{\sigma} = \frac{\pi e^2 f_{ij}}{mc\Delta V_D} \]  
(4.81)

Substituting this into to Equation 4.76 gives an expression for the mean absorption coefficient

\[ \bar{k} = \frac{N\pi e^2 f_{ij}}{mc\Delta V_D} \]  
(4.82)
A similar expression to that of Equation 4.4 defines the mean optical depth $\bar{\tau}$ in terms of the mean absorption coefficient and the path length $l$ that the photon travels through the plasma.

$$\bar{\tau} = \bar{k}l$$

(4.83)

It is this mean optical depth which is calculated in the population code. Combining Equations 4.83 and 4.76 shows that the optical depth is proportional to the cross-section

$$\tau_v \propto \sigma_v$$

(4.84)

In Lyman transitions the following relation also applies approximately

$$\frac{d\sigma}{dn} \propto \frac{1}{n^3}$$

(4.85)

where $\sigma$ is the cross-section of the entire Lyman line including all the frequencies in its range. This is because the energy of state $n$ is given by

$$E_n = -\frac{R}{n^2}$$

(4.86)

where $R$ is the Rydberg constant. The derivative of this energy gives the density of states

$$dE_n = \frac{2Rdn}{n^3} \Rightarrow \frac{dn}{dE_n} \propto n^3$$

(4.87)

Now, the cross-section per unit energy $\sigma_E$ is approximately independent of energy (especially for high $n$) and the cross-section per quantum number can be defined as

$$\frac{d\sigma}{dn} = \frac{d\sigma}{dE} \frac{dE}{dn}$$

(4.88)

Therefore, from Equations 4.87 and 4.88 we get Equation 4.85. Substitution of Equation 4.84 into Equation 4.85 gives
Consequently, in a plasma, as $n$ increases, the optical depth in the Lyman transitions decreases.

There are two extreme cases of optical thickness for Lyman line photons that can be defined in terms of the single flight escape probability $P_e$. Case A describes a photon single flight escape probability of unity, where all the photons escape. Here, there is zero absorption of Lyman line photons and this situation is described as being optically thin (Hummer and Storey, 1992).

Case B describes the opposite situation of complete optical thickness, where $P_e$ is zero, again, this only applies to Lyman lines. In this case, it is assumed that the rate of depopulation of excited states, by emission of Lyman photons, is exactly balanced by the rate of absorption of Lyman photons. This situation occurs when a Lyman photon scatters enough times to create a high probability of being destroyed by collisional processes such as collisional excitation, de-excitation or ionisation with electrons or being converted into two or more other photons by radiative processes such as spontaneous emission of radiation or stimulated absorption of a photon, whilst remaining close to the optical depth at which it was formed by electron cascading. So, a photon absorbed sufficiently close to its point of emission, or in a region of the same optical depth can be regarded as never having been emitted since it remains within the blackbody field that it was formed.

### 4.2.5 Line Profiles

Opacity depends greatly on the shape, width and area of the line profile in question. The shape of line profiles and the extent of their broadening are partially determined by the frequency, or wavelength, dependence of the emission and absorption coefficients. In this section, some well known line profiles, such as Lorentzian and Doppler will be discussed along with less familiar profiles such as Voigt profiles and those subjected to the effects of Zeeman splitting and collisional broadening.

The absorption profiles that will be described firstly are the Lorentzian, Doppler and Voigt profiles. The latter is formed by both natural and Doppler broadening and is therefore a
convolution of the Lorentzian and Doppler profile. The Voigt profile is defined in Equation 4.42. The Lorentzian profile, shown in Figure 4.7, has a natural broadening of the line profile, created by the uncertainty principle, which means that energy states are not infinitely sharp and therefore lines are not infinitely narrow. Also, for a transition there is not only an uncertainty in the frequency of one state but two, for each energy state in the transition.

The Doppler profile is created by Doppler broadening, where the Doppler width is defined in Equation 4.54. Here the wavelength or frequency of the light emitted by a moving atom is shifted. Convolving this shift with a Maxwellian distribution of particle velocities gives the complete Doppler broadened profile (Figure 4.7).

The combination of the Lorentzian and Doppler profile to form the Voigt profile can be seen in Figure 4.7. This has a core with a high absorption coefficient, implying a large optical depth and low photon escape. However, the Lorentzian wings of the profile have a low absorption coefficient and consequently the probability of photon escape is much greater here.

A photon is most likely to be emitted in the Doppler core of a line, from where it can enter the wing by a random walk process of absorption and re-emission as long as the photon
remains within the profile long enough for this migration to the wings to occur. In most cases this is not possible since radiative branching and $n$-changing collisions occur and the photon is removed from the profile rapidly before it has time to reach the wings. This is referred to as photon destruction and consequently the Voigt profile is an overestimation of the photon population in the Lorentzian wings. To solve this, it has been found that a Doppler rather than a Voigt profile is more accurate for cases where the probability of photon destruction is relatively high, typically 0.1–0.2.

The exception to this rule is the case of Ly$\alpha$ in a medium with a high optical depth. Ly$\alpha$ has the lowest probability of $n$-changing collisions occurring and branching cannot occur. Consequently, the photon remains in the core long enough to have sufficient scattering collisions for the photon to enter the wings. For Ly$\alpha$ a Voigt profile is a good estimation of the absorption profile.

In this situation, where the first resonance line has a large optical depth, it is essential to use the correct redistribution function in the transfer equation for this line (Hummer, 1969). This is due to the large number of scatterings that the photon undergoes, causing the difference between partial and complete redistribution to be significant. When the number of scatterings is small and the photon is unlikely to migrate from the core, it is sufficient to assume a complete redistribution with Doppler broadening, such that the absorbed photon can be emitted with any frequency within the profile enabling the absorption and emission profile to be Doppler. This assumption holds for all but the first resonance line. In the case of Ly$\alpha$, a partial redistribution with the Voigt profile would be preferable since the frequency change that occurs within the wings of the profile tends to be less. However, since the Lyman lines are treated together in the code and it is assumed that the number of scatterings in Ly$\alpha$ is never large in the JET tokamak, it is easier to assume a Doppler profile with complete redistribution for all the Lyman lines, including Ly$\alpha$.

Another characteristic of the Lyman line profile is overlapping (Van Blerkom and Hummer, 1968). Above a critical principal quantum number ($n_c$), as the series reaches it limit and the cross-section per unit energy becomes dominant, the line width exceeds the line separation, leading to overlap of the Lyman series which results in a continuum optical depth.

It was shown in the previous section that in the Lyman series, for example, Ly$\alpha$ is the most susceptible to absorption and goes optically thick first. Then each Lyman line, with increasing $n$ goes optically thick, in turn, as the surrounding conditions increase the
probability of absorption, i.e. increasing densities and decreasing temperatures. It is therefore true that, with increasing \( n \), the Lyman series can go from being optically thick to being optically thin. However it is theoretically possible that as \( n \) increases further the series can return to being optically thick, due to merging of the spectral lines. It must be stressed, however, that this situation is not of any relevance for the conditions inside a tokamak as it would require optical depths of approximately \( 10^6 \) in Ly\( \alpha \), as shown in Figure 4.6.

It has been established that the preferred profile to assume for use in the population code is the Doppler profile. However, the effects of magnetic fields and collisions on the profile have not yet been taken into account. The issue of the effects of a magnetic field on the line profile will be addressed first.

Every electron has an intrinsic magnetic moment, which when subjected to an external magnetic field can cause a slight shift in the energy of the atomic states. This is called the Zeeman effect which according to the quantum values \( m_l \) and \( m_s \) causes the line to have multiple components shifted in wavelength which can broaden the profile (Bransden and Joachain, 1991). When a transition takes place between two quantum levels, under the influence of the Zeeman effect, the resulting profile is said to suffer from Zeeman splitting. Figure 4.8 shows a D\( \alpha \) line profile produced in the JET tokamak which displays strong-field Zeeman splitting.

![Figure 4.8](image)

**Figure 4.8.** A schematic plot of a D\( \alpha \) line profile displaying Zeeman splitting obtained from a JET plasma pulse.
Not all cases of Zeeman splitting are so visibly obvious and the only way of detecting the extent of Zeeman splitting is to calculate its magnitude. To do this it is necessary to know whether to apply the theory of the weak-field Zeeman effect or the strong-field Zeeman (or Paschen-Back) effect. This question is resolved by evaluating the ratio of the Zeeman energy shift to the fine-structure energy shift. If this ratio is less than unity then a the weak-field theory can be applied. Conversely, if the ratio is much greater than unity, then the strong-field theory must be applied. This procedure is carried out in order to assess the relative strength of an external magnetic field and the atom’s internal magnetic field.

In order to carry out this assessment, the Zeeman energy shift can be defined as the product of the Bohr magneton ($\mu_B=9.274\times10^{-24}\text{JT}^{-1}$) and the magnetic field $B$. The states with the largest fine-structure splitting are $p_{1/2}$ and $p_{3/2}$. For these states, the fine-structure splitting for $n=2$ and $n=3$ has a magnitude of 0.365cm$^{-1}$ and 0.108cm$^{-1}$, respectively (Bransden and Joachain, 1991). These values can be converted into Joules by multiplication with the ratio of the non-relativistic ionisation potential of atomic hydrogen for an infinite mass ($2.17991\times10^{18}\text{J}$) to Rydberg’s constant for infinite nuclear mass ($1.09737\times10^{5}\text{cm}^{-1}$). This calculation yields values for the fine-structure energy splitting for $\text{Ly}\alpha$ and $\text{Ly}\beta$ of $7.25\times10^{-24}\text{J}$ and $2.15\times10^{-24}\text{J}$, respectively.

For a magnetic field of 2.7T, which is the relevant value for the region of interest in the JET tokamak, the Zeeman energy shift is approximately $2.504\times10^{-23}\text{J}$. The ratios of the Zeeman energy to that of the fine-structure energy for $\text{Ly}\alpha$ and $\text{Ly}\beta$ are approximately 3.45 and 11.65, respectively. Since this ratio is low for the $\text{Ly}\alpha$ line, it would be appropriate to treat both weak-field and strong-field Zeeman effects. However, for the $\text{Ly}\beta$ line, it is clear that only the strong-field Zeeman effects need to be considered.

The strong field energy shift can be defined as

$$\Delta E_z = \mu_B B (m_z + 2m_s)$$  \hspace{1cm} (4.90)

where $m_z$ and $m_s$ are quantum numbers associated with the $z$-components of the orbital angular momentum and the spin angular momentum, respectively. In order to evaluate Equation 4.90 it is necessary to examine the possible transitions. The selection rules for a transition are $\Delta m_z=0$ and $\Delta m_s=0,\pm 1$. Table 4.1 gives the values of the orbital angular
momentum $l$, $m_l$, $m_s$ and $m_l+2m_s$ for the allowed transitions between quantum levels $1s$ and $3p$. Figure 4.9 shows the possible Ly$\beta$ transitions under the strong-field Zeeman effect.

<table>
<thead>
<tr>
<th>State</th>
<th>$n$</th>
<th>$l$</th>
<th>$m_l$</th>
<th>$m_s$</th>
<th>$m_l+2m_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1s$</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
<td>-1/2</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1/2</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>$3p$</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>1/2</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-1/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>-1</td>
<td>1/2</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>-1</td>
<td>-1/2</td>
<td>0</td>
<td>-2</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.1. Table giving $n$, $l$, $m_l$, $m_s$, and $m_l+2m_s$, for a particular quantum state.

![Figure 4.9](image.png)

Figure 4.9. Possible Ly$\beta$ transitions of hydrogen in a strong magnetic field.
The accumulative value of \( m_i + 2m_s \) for a particular transition is the difference between its value for the upper and lower state. These values are given, for the allowed Ly\( \beta \) transitions, in Table 4.2.

<table>
<thead>
<tr>
<th>( m_i )</th>
<th>( m_s )</th>
<th>( m_i )</th>
<th>( m_s )</th>
<th>( m_i + 2m_s )</th>
<th>( m_i + 2m_s )</th>
<th>( \text{Resultant} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/2</td>
<td>0</td>
<td>1/2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1/2</td>
<td>0</td>
<td>1/2</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-1/2</td>
<td>0</td>
<td>-1/2</td>
<td>0</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>1/2</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>-1/2</td>
<td>0</td>
<td>-1/2</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>-1/2</td>
<td>0</td>
<td>-1/2</td>
<td>-2</td>
<td>-1</td>
<td>-1</td>
</tr>
</tbody>
</table>

**Table 4.2.** Table giving the resultant value of \( m_i + 2m_s \) for the allowed Ly\( \beta \) transitions.

It can be seen from Table 4.2 that there are three Ly\( \beta \) line components at 0,±1. This is the famous Lorentzian triplet. It should be mentioned that if fine-structure splitting were being considered, then each of the pairs of lines would also be split. However, for the Ly\( \beta \) line in a field of 2.7T this effect would be negligible (approximately a twelfth of the triplet separation).

It can be deduced from these calculations that the value of the total separation of the two extreme line components is 2. Hence substituting this value into Equation 4.90 gives \( \Delta E_z = 5.0 \times 10^{-23} \) J for a Ly\( \beta \) line at 2.7T.

In Chapter 5, the experimental analysis of JET plasma pulses will be undertaken. The emphasis will be on two plasma pulses in particular these being pulses 34355 and 35405. For these pulses the magnetic field, in the region of interest, is approximately 2.70±0.05T. Hence, a Ly\( \beta \) transition occurring in the presence of a magnetic field of this magnitude displays a Zeeman energy splitting of approximately \( 5.0 \times 10^{-23} \) J.

Variations in the magnitude and direction of the magnetic field relative to the atoms alters the extent of Zeeman splitting. Incorporating a full treatment of the Zeeman effect into the population code is an extremely complex procedure. A simple way of accounting for the
Zeeman effect is to broaden the energy width of the Doppler profile by $\Delta E_Z$. The FWHM Doppler energy width can be expressed as

$$
\Delta E_D = \frac{2E_0}{c} \sqrt{\frac{2kT_0 \ln 2}{M}}
$$

(4.91)

where $E_0$ is the line centre energy, $T_0$ is the neutral temperature and $M$ is the mass of the absorber atom, which is neutral deuterium in the JET divertor. The mass of deuterium in cgs units is $3.3406 \times 10^{-21}$ grams and the line centre energy for Ly$\beta$ is $1.94 \times 10^{-18}$ J. Therefore the Doppler width of a Ly$\beta$ line profile can be expressed in terms of the neutral temperature by the relation

$$
\Delta E_D = 9.88 \times 10^{-25} T_0^{1/2}
$$

(4.92)

By substituting neutral temperature of 1eV (11600K) and 5eV (58000K) into this equation gives FWHM Doppler widths of $1.06 \times 10^{-22}$ J and $2.38 \times 10^{-22}$ J, respectively. Hence the relative values of the Zeeman width to the Doppler width at neutral temperatures of 1eV and 5eV are 0.47 and 0.21, respectively. Unfortunately, because the peak values of the Doppler profile do not correspond to those of the Zeeman components, the Doppler and Zeeman widths can't just be added to produce a new profile width. Instead, we construct a numerical profile by convolving the three components of the Lorentz triplet with a Doppler profile of a given temperature. The FWHM of the resultant profile ($\Delta E_m$) is then obtained. For neutral atom temperatures of 1eV and 5eV the ratio of $\Delta E_m/\Delta E_D$ is 1.11 and 1.02, respectively. Figure 4.10b shows the modified Ly$\beta$ line profile for a magnetic field of 2.7T and a neutral atom temperature of 1eV, which are typical parameters for a high density plasma in the JET divertor region. Figure 4.10a shows the Zeeman components contributing to the modified profile.
In the population code, the Doppler width is controlled by the input value of the neutral temperature. Hence the modified Doppler width can be created by modifying the neutral temperature according to Equation 4.92. Hence by replacing $\Delta E_D$ and $T_0$ with $\Delta E_M$ and $T_M$, (the new modified neutral temperature) respectively in Equation 4.88 a new expression for the modified neutral temperature is given

$$T_M = \left( \frac{\Delta E_M}{9.88 \times 10^{-25}} \right)^2 \tag{4.93}$$

This method of treating Zeeman broadening is utilised to modify the neutral temperatures, obtained from the plasma modelling procedure, for plasma pulses 34355 and 35405. The original and modified temperatures are represented in bar charts in Chapter 5. The population code is run for both sets of neutral temperature to investigate the effects of Zeeman broadening on the opacity of the JET divertor plasma. The results of this investigation are given in Chapter 6.

In high density plasmas, such as those being investigated in this thesis, particle collisions are frequently occurring. According to Heisenberg’s uncertainty principle, the degree of
uncertainty in the energy or frequency of a collision increases with increasing collision rate. This uncertainty in the energy and frequency broadens the line profile and it is necessary to know whether this broadening is significant with respect to the width of the line profiles created for the plasma conditions being considered in this thesis.

The collisional line profile is Lorentzian in shape and can be expressed as (Seaton, 1987)

\[
\phi_c = \frac{\left( \frac{\gamma}{2\pi} \right)}{\left( 2\pi v - 2\pi v_0 \right)^2 + \left( \frac{\gamma}{2} \right)^2}
\]  

(4.94)

where \( \gamma \) is the width of the profile. It should be mentioned that in this equation an expression for the shift in the line centre frequency of the profile is omitted, since only the magnitude of line broadening is of interest here.

The peak of the profile occurs when \( v = v_0 \), therefore by substituting this into Equation 4.94 gives the profile peak in terms of the profile width as shown in Equation 4.95.

\[
\phi_{\text{peak}} = \frac{2}{\pi \gamma}
\]  

(4.95)

Hence the value of the profile at the FWHM is

\[
\phi_{\text{FWHM}} = \frac{1}{\pi \gamma}
\]  

(4.96)

Substituting Equation 4.96 into Equation 4.94 gives a value for the frequency shift from the line centre as

\[
(\nu - \nu_0) = \frac{\gamma}{4\pi}
\]  

(4.97)
The FWHM of the profile is double the energy shift given in Equation 4.97. Hence the FWHM can be expressed as

\[ \text{FWHM} = \frac{1}{2\pi} \]  

(4.98)

The width of the collisional line profile \( \gamma \) can be expressed as

\[ \gamma = n_e \langle \nu Q_o \rangle \]  

(4.99)

where \( n_e \) is the electron density and \( \langle \nu Q_o \rangle \) is a rate coefficient for collisions. For an electron density of \( 5 \times 10^{14} \text{cm}^{-3} \), which is greater than any electron density achieved in the JET divertor plasma, the population code gives the rate coefficients for collisions out of energy states \( n=1 \) and \( n=3 \) as \( 1.87 \times 10^{-10} \text{cm}^3 \text{s}^{-1} \) and \( 3.42 \times 10^{-6} \text{cm}^3 \text{s}^{-1} \). Since the collision rate out of energy state \( n=1 \) is negligible with respect to that of energy state \( n=3 \), the accumulative rate coefficient for the production or absorption of Ly\( \beta \) lines is approximately \( 3.42 \times 10^{-6} \text{cm}^3 \text{s}^{-1} \). Substituting this rate coefficient and the electron density into Equation 4.99 yields \( \gamma = 1.71 \times 10^6 \text{Hz} \). Substituting this value into Equation 4.98 gives a value for the FWHM of the collisional profile of \( 2.72 \times 10^6 \text{Hz} \), which converts to a wavelength FWHM of \( 1.2 \times 10^4 \text{Angstrom} \). A typical Doppler energy FWHM value for the temperatures of neutral deuterium in the JET divertor is \( 1.5 \times 10^{22} \text{J} \). Dividing this by Planck's constant yields a frequency FWHM of \( 2.26 \times 10^{11} \text{Hz} \). This is equivalent to a wavelength FWHM of \( 0.101 \text{Angstrom} \). Hence the ratio of the wavelength FWHM values of the collision profile to the Doppler profile is approximately \( 1.2 \times 10^{-3} \). It can therefore be concluded that collisional broadening has an insignificant effect on the line profile near the line centre and consequently is not treated in the population code.

### 4.2.6 Geometrical Modifications

One of the main modifications to the population code has been in the way that it models the tokamak geometry and changing conditions within the divertor region. The original model comprised a single plane parallel slab of uniform temperature and density. This was not ideal
since the parameters in the divertor vary considerably over short distances. To solve this problem a new model, shown earlier in Figure 4.4, has been employed.

This consists of a finite stack of uniform rectangular slabs, each one having a different set of plasma parameters. Hence, it imitates the changing environment in the dynamic divertor plasma far more accurately. Using this model also allows more versatility to consider effects, previously ignored in the code such as velocity gradients. Doppler shifts between slabs, caused by neutral velocity gradients, are now accounted for in all three dimensions, as described in Section 4.2.3. Emission and absorption profiles are assumed to be Doppler broadened and the code now allows for a variation in the profile width from slab to slab.

The initial model incorporated a slab of infinite extent, although this was a reasonable treatment for stellar atmospheres, it did not accurately represent the confines of the tokamak. Modifications were made to use slabs of a finite length, which are selected for each case. For the plasma pulses investigated in this thesis slabs of height 2cm, length 15cm and infinite breadth are used. Figure 4.11 shows a simulated electron density profile, it displays the typical variation trend of parameters in the divertor region.

![Figure 4.11. Electron density profiles for a simulated JET plasma pulse depicting that the density contours run mostly parallel to the separatrix. The units of the electron densities in this figure are m⁻³.](image)
It can be seen that the contours of constant density form slab like layers of uniform density. Hence, the code has been altered to let the ‘slab-stack’ base run parallel to the separatrix so that it follows the contours. The differences between the two models can be seen in Figure 4.12.

![Figure 4.12](image)

**Figure 4.12.** Cross-section of the JET divertor region, depicting data storage cells and comparisons between the original ‘single-slab’ model, which is used in the ionisation and power balance investigation (in Chapter 6), and the improved ‘slab-stack’ model used to obtain the branching ratios (given in Chapter 6).

Before describing the final modification, a description is needed of how the code uses this model. The single flight escape probability from the whole stack, for a photon emitted at an arbitrary point in the ‘slab-stack’, is calculated by numerically averaging over all frequencies and angles of emission.

The final modification extended the use of several emission points along the same line of sight as the spectrometer being used to observe Lyβ emissivities in the slabs. The alteration was made for the purpose of comparison of the codes results with that of the experimental data, as described earlier.
The parameters for each slab are from one of two transport codes, described earlier. Both codes work on the same grid structure, dividing the plasma edge and divertor region into cells as shown in Figure 4.12.

The plasma parameters from the plasma models, up to a selected height vertically above the target and between the separatrix and the wall, are averaged and fed into the population code to form the uniform slabs of plasma. The height and width of the slabs can be varied as a parameter, trading off extra plasma depth with lower average densities in the slab.
Chapter 5

Experimental Analysis

5.1 Introduction

The first step in determining whether a plasma may display significant signs of opacity is provided by experimental measurements. In the preliminary stage of the experimental investigation, spectrometer data providing Lyman and Balmer line emissivities are used in order to see whether a significant amount of absorption may have occurred in a particular shot. If so, a further study is made to see whether density, temperature and power changes occurring within the tokamak, or geometrical effects could explain the observed line emissions. Only when all these paths have been explored, and the criterion for high opacity plasmas is fulfilled, and causes other than opacity have been eliminated, does further investigation proceed.

Diagnostic information is necessary to define boundary conditions, which enables the background plasma and neutral impurity distributions to be calculated using the transport and neutral models described in Chapter 2. In addition, cross checks of predictions of the simulations with diagnostic measurements are essential in order to validate the model.
In this chapter the main diagnostics used in this investigation, along with diagnostic measurements and their accuracy are described. A basic comparison is made with the results obtained via the two plasma models, the O-S model and EDGE2D, for one of the pulses being used in this investigation. From this, the decision as to which model is to be used as input to the population code and the plasma information that it provides for this input will be given and discussed.

5.2 Diagnostics Used to Measure Spectral Lines

The only reliable test for the opacity of a particular spectral line involves looking at the ratio of its emissivity to that of an optically thin line with the same upper level, called a branching ratio. Using a branching ratio to test for opacity eliminates the need for accurately calibrated data as absolute values of the ratio are not important, only the variation of the ratio over time is necessary. This is fortunate since the calibration of Lyman line data has error bars of approximately 50% and the plasma modelling procedures have errors of the same magnitude.

The line most susceptible to absorption is Lyα, however a branching ratio of Lyα cannot be taken. Hence, the spectral line under investigation in this thesis is Lyβ, the next most likely candidate for absorption. The variation over time of the ratio of Lyβ to the optically thin Dα allows a measure of opacity without the need for a plasma model. Comparing a plasma at a time when it is expected to display low levels of opacity with a time when it has a higher density and is expected to suffer from a greater degree of opacity shows the full extent of Lyβ line absorption.

The emissivity of these spectral lines can be measured by the various diagnostics described in the following section.

5.2.1 The Combined Visible/VUV Spectrometer

The emissivities of Lyα, Lyβ and Dα can be measured using the combined visible and vacuum ultra-violet (VUV) spectrometer (Wolf et al., 1995). This enables the observation of time-resolved impurity spectra with a moderate spectral resolution.
This diagnostic actually utilises three spectrometers, a double SPRED (survey, poor resolution, extended domain) spectrometer (Figure 5.1) which comprises two VUV spectrometers, covering the wavelengths from 100 to 1700 Angstrom, and a spectrometer which employs a fibre and lens system and observes the visible wavelength range along the same line of sight. Unfortunately, data is only available for a limited number of pulses which reduces the choice of plasma pulses available for analysis using this diagnostic.

Figure 5.1. The double SPRED spectrometer, showing how the visible spectrometer is incorporated to achieve the same line of sight. Figure adapted from Wolf (1995).
The position of the double SPRED spectrometer and its approximate line of sight can be seen in Figure 5.2. While in principle the line of sight can be varied either from shot to shot or even during a pulse, the line of sight is fixed for the data used in this thesis, as shown in Figures 5.10 and 5.12.

Figure 5.2. The position of the double SPRED spectrometer within the tokamak and its spatial coverage of the divertor (indicated by the dashed line). Figure from Wolf (1995).
A brief description of the double SPRED spectrometer follows, a detailed description is given in Wolf (1995).

The double SPRED spectrometer combines two fixed toroidal gratings mounted back to back in the same vacuum chamber. These gratings have groove densities of 450 g/mm and 2105 g/mm with corresponding wavelength ranges of 180-1500 Ångstrom and 140-440 Ångstrom, respectively. The latter operates at an angle of incidence which is 1.3° larger than that of the 70.6° used for the 450 g/mm grating. Detector positions can be moved in the focal plane so as to change the spectral coverage. This adjustment is approximately 50 Ångstrom in either direction for the 2105-SPRED and 250 Ångstrom in the upward direction for the 450-SPRED, which ensures reasonable signal strengths down to approximately 100 Ångstrom.

The spectrometers, although inclined to each other by an angle of 0.6°, are designed so that their lines of sight meet on the divertor target plates. This can be seen, along with the position of the gratings, in Figure 5.1, which shows the structure and components of the spectrometer. The spectrometers have a combination of a micro-channel plate (MCP) image intensifier and a phosphor screen coupled to a 2048-pixel linear photo diode array (PDA) by a fibre-optic image conduit.

The double SPRED (which has a spot size of approximately 10 cm) and visible spectrometer (which has a spot size of approximately 20 cm) are shielded from neutrons and γ-rays by a minimum of 15 cm of stainless steel between the plasma and the detectors and 5 cm elsewhere. The entrance slits, seen in Figure 5.1, are 25 µm wide and 2.5 mm high. However, a baffle between the entrance slits and the grating reduces the ruled area of the grating used to 3 mm x 18 mm. These relative dimensions provide a spatial resolution in the poloidal direction of approximately 110 mm.

The spectral resolution depends on the illumination of the grating. To obtain values of spectral resolution, a line fit (Equation 5.1) is performed, which uses a formula with the versatility to represent either a Gaussian or Lorentzian function (Fraser, 1970) depending on a profile parameter, \( \alpha \).
\[ F(p) = \frac{1}{\left(1 + \left(2^{2\alpha^2} - 1\right)\left(2^{\frac{p - p_0}{\Delta p_{1/2}}}\right)\right)^{1/\alpha^2}} \]  

(5.1)

where \( \Delta p_{1/2} \) is the FWHM in pixels. \( F(p) \) changes continuously with \( \alpha \) from a Gaussian profile at \( \alpha=0 \) to a Lorentzian profile at \( \alpha=1 \). The measured relation between \( \alpha \) and the line width is shown in Figure 5.3 for the two instruments.

![Figure 5.3. Line profile parameter \( \alpha \) versus line width. Figure from Wolf (1995).](image)

The fitted line widths are converted into a spectral resolution using a factor of approximately 1.13, which depends weakly on \( \alpha \). This factor is obtained by applying the Rayleigh criterion to Equation 5.1. Note that because line shape changes, the line width cannot be solely used as an accurate measure of the spectral resolution. According to the criterion, this is defined as the separation between two neighbouring lines of equal intensity sufficient for their summed intensities between the lines to fall to 80% of the peak values.

The spectral resolution is shown in Figure 5.4, as a function of wavelength.
The shortest achievable exposure time that will allow the full spectral range to be observed is 11ms. This time resolution is adequate enough to highlight separate low frequency ELMs, which only last for approximately 100μs but can be separated by a time of greater than 11ms. However it is not possible to follow the evolution of the emission during the ELM.

ELMs are edge instabilities and are common in the high confinement (H-mode) plasmas. They cause a redistribution of particles and a loss of energy on a short time scale of less than 1ms. Strong peaks of deuterium radiation characterise ELMs. Most of this radiation, approximately 95%, is due to Lyα emission but a significant amount of this energy loss is in the form of Lyβ and Dα emission. Time traces that display ELMs can be extremely awkward to analyse due to the large peaks and troughs in radiation. Problems with geometry effects can also cause large fluctuations in a time trace, as will be discussed later in this chapter. The problem is dealt with by taking a hand estimated mean through the oscillations and considering this to be the average emissivity.

The position of the visible spectrometer system can be seen in Figure 5.1, which shows a generalised indication of the line of sight of the instrument.
Light is transmitted, via quartz fibres, from the torus to be analysed by a Czerny-Turner type spectrometer, using a beam-splitter. The layout of a Czerny-Turner spectrometer can be seen in Figure 5.5.

![Figure 5.5. Layout of a Czerny-Turner spectrometer. Figure from Maas (1995).](image)

Light from the fibres is focussed onto the entrance slits of the spectrometer using lenses. Once inside the spectrometer, this beam is reflected by a spherical mirror onto a grating. Here, light is diffracted by a wavelength dependent angle where the position of light at a particular wavelength depends on the groove density of the grating (600g/mm) and the angle at which the light falls onto it. The wavelength range covered can be altered by turning the grating. The total wavelength range covered is from 3800 Angstrom to 8000 Angstrom, however, only a range of approximately 100 Angstrom can be covered at any particular time.

After diffraction, another spherical mirror focuses the light onto the exit slit of the spectrometer where a field lens produces a parallel beam which is then focussed, by another lens, onto a two dimensional frame transfer CCD (Charged Coupled Device) detector which records the information (Ravich, 1987).

This visible spectrometry system is characterised by dispersion, instrument functions and calibration. The wavelength dispersion is obtained by knowing the wavelength separation of the two neighbouring lines produced by a neon or mercury lamp. Several line pairs are needed to encompass the wavelength range of the visible spectrum. Calibration can be carried out when the extent of dispersion is obtained. This can be achieved using Equation 5.2 (Jenkins and White, 1981), which gives an expression for the spectral dispersion in terms of the wavelength, grating parameters, instrumental factors and dispersion.
\[
\frac{d\lambda}{dp} = \frac{dx}{dp mfg} \left( \sqrt{\cos^2 \phi - \left( \frac{mg\lambda}{2} \right)^2} - \frac{mg\lambda}{2} \tan \phi \right)
\] (5.2)

where \( g \) is the grating's groove density, \( \phi \) is the angle shown in Figure 5.5, \( f \) is the focal length of the spectrometer, \( m \) is the order in which the grating is used, \( dx \) is the spatial interval (or spatial resolution) given by the size of the pixel (or detector channel), \( dp \). For the CCD camera used in this diagnostic instrument, \( dv/dp=22.5\mu m/pixel \). Figure 5.6 shows the dispersion of the visible spectrometer as a function of wavelength.

Performing a line fit for Figure 5.6 gives a value for the focal length \( f \) of the spectrometer to be 0.747633 m and a value of angle \( \phi \) of 6.693467°.

A tungsten calibration lamp is utilised to perform an intensity calibration of the spectrometer. A complete calibration is performed, with an accuracy of between 5-10%, in
wavelength increments of 100 Angstrom in the visible range. Figure 5.7 shows the
calibration factor as a function of wavelength.

![Graph showing calibration factor versus wavelength for the visible spectrometer.](image)

Figure 5.7. Calibration factor versus wavelength for the visible spectrometer. Figure from Maas (1995).

### 5.2.2 The CCD Cameras

CCD cameras are another available method to determine $D_\alpha$ emission in the divertor region. These comprise of three spectroscopic thermoelectrically cooled cameras. They provide direct quantitative measurements of the radiation intensity in the divertor region with a high spatial resolution of 3mm and a reasonable time resolution of 5ms and cover the radial profile of the divertor. The accuracy of the CCD cameras is approximately 20%. Data taken by the cameras is compressed to one-dimension by averaging the pixels toroidally. The lines of sight are illustrated in Figure 5.8.
The CCD cameras are able to measure many line intensities but not simultaneously. The cameras differentiate between the different spectral lines with the use of interference filters. In the case of Dα a filter with an approximate width of 1nm, centred on the spectral line, is placed in front of the camera.

The main advantage of using the CCD cameras is that they have excellent spatial resolution and so can be used to average the line integrated Dα emission over the same line of sight and time span as the VUV spectrometer, hence providing an accurate branching ratio.

5.2.3 The Dα and Visible Light Monitors

This diagnostic observes light over a wide range of lines of sight. Twelve lines are used to scan the major radius of the divertor target in sections. These lines of sight and their spot size, of approximately 3.3cm, can be seen in Figure 5.9.
Light is collected along these lines of sight through the plasma and fed into quartz optic fibres. Light from some fibres is then split and passed through narrow band interference filters, being detected by photo-multiplier tubes, or passed into a visible survey spectrometer similar to the one described above. For the purpose of this investigation the Dα filters, similar to those on the CCD cameras, are used to obtain measurements of the Dα emission and the photo-multiplier tubes are used as detectors.

5.3 Analysis of Plasma Pulses

Two different density ramp pulses have been selected for the initial examination of opacity due to the high densities obtained and the availability of visible/VUV spectrometer measurements. Pulses 34857 and 34859 are both heated ohmically by applying a constant current to the plasma. Deuterium gas puffing steadily increases the density of the plasma. As a result of this, the energy has to be split among more particles and so the temperature of the plasma drops. The resistivity of the plasma to the imposed current increases with the decreasing temperature and causes heating. This heating is very strong at low temperatures but is less effective at higher temperatures because the resistance of the plasma depends on
The steady increase in the input power as the plasma cools can be seen in Figures 5.11 and 5.13.

The increase in the density, due to deuterium gas puffing, causes an increase in the radiative power. The radiative power is proportional to density squared and has a stronger dependence on the density than the input power. Hence, at a certain density, the difference between the input power and the radiative power becomes so small that a thermal instability occurs. This leads to an abrupt termination of the discharge, called a disruption, as shown in Figures 5.11 and 5.13. It is clear from this discussion that there is an upper restriction on the density achievable called the density limit.

The plasma current and toroidal magnetic field applied to both pulses 34857 and 34859 are approximately 2MA and 2T, respectively. Figures 5.10 and 5.11 illustrate the magnetic geometry and the time traces (showing the variation of the input and radiative powers of the plasma in addition to the peak electron density and temperature), respectively, for pulse 34857. In Figure 5.10, it can be seen that the separatrix strike points hits the vertical plate of the divertor.

The second pulse (34859) is similar, but with the strike points on the horizontal plate of the divertor. Its magnetic geometry and time traces can be seen in Figures 5.12 and 5.13, respectively.
Figure 5.10. The magnetic geometry for vertical target pulse 34857. The estimated visible and VUV spectrometer viewing lines are also shown.

Figure 5.11. The time traces of an ohmic detached plasma on the vertical target plates.
Figure 5.12. The magnetic geometry for the horizontal target pulse 34859. Also shown are the estimated visible and VUV spectrometer viewing lines.

Figure 5.13. The time traces for an ohmic detached plasma on the horizontal target plates.
The measurements of the total radiative power and the radiative power at the X-point, in the time traces, are made with bolometers with an accuracy of 10%. There are two horizontal bolometers positioned on the side of the tokamak and one vertical bolometer positioned at the top of the tokamak. Of the horizontal bolometers, one is angled so that it looks diagonally across the plasma in the upper half of the tokamak, the other is angled so that it looks diagonally across the plasma in the lower half of the tokamak. The bolometer that looks across the top half obtains a profile of the radiation which is then integrated and doubled to give the bulk radiative power. The radiative power at the X-point and from the divertor is defined as the difference between the total radiative power, measured using the vertical bolometer, and the bulk radiation.

The vertical bolometer is positioned at the top of the tokamak and looks down through the core plasma to the target with 14 channels, or viewing chords, that fan out, as shown in Figure 5.14. This bolometer is also used to measure the radiated power along the lines of sight depicted in this figure. Measurements from this instrument are used for comparisons with theoretical model prediction of the radiated power later in this chapter.

![Figure 5.14](image_url)
A LIDAR (light detection and ranging) Thomson Scattering system has been used to provide the electron temperature and density time traces, of the core plasma, with accuracies of 10% and 5%, respectively. The LIDAR system looks horizontally through the core plasma. This process involves transmitting a short laser pulse through the plasma and recording the backscattered light as a function of time using a fast detection and recording system. Analysis of the scattered spectrum at each time point gives values for the electron temperature and density (Wesson, 1997).

For the two pulses under investigation, Lyβ and Dα line profiles from the visible/VUV spectrometer, looking down the lines of sight as shown in Figures 5.10 and 5.12, have been fitted, at all times, to give total intensities as shown in Figure 5.15. The Lyβ has an estimated fitting error of 10% and the Dα has an even smaller error, given in Figure 5.15. The ratio of these lines has been taken and can also be seen in Figure 5.15. It should be mentioned that arbitrary units are expressed as \( a.u \) in this figure and throughout the thesis.

![Figure 5.15](image)

**Figure 5.15.** Line integrated signals for pulses with the strike point on (a) the vertical target (pulse 34857) and (b) the horizontal target (pulse 34859). (i) Dα emission, (ii) uncertainty in the fit to the Dα emission, (iii) Lyβ emission, and (iv) the ratio of the two signals as a measure of the divertor’s opacity.
It can be seen that for the first shot there is a notable drop in the ratio of approximately 50%, which is well outside the fitting errors described above. In the second shot, both the D\(\alpha\) and the Ly\(\beta\) show oscillations as the strike points are swept across the horizontal target.

The plasma strike points are swept over the target plates at 4Hz and with an amplitude of up to 10cm. This increases the effective wetted area of the target, therefore reducing the heat load and target erosion, which causes impurity contamination. Figure 5.16 shows the position of the strike points for the two extremes of the plasma sweep for pulse 34355.

![Figure 5.16](image)

**Figure 5.16.** The magnetic geometry showing the two extreme positions of the strike point during the process of plasma sweeping for a horizontal target pulse 34355 at times (a) 49.5s and (b) 49.6s, respectively.

Returning to Figure 5.15, it can be seen that the oscillations in the VUV signal are larger than those in the visible signal, due to the fact that the two systems have different acceptance cones and the resulting ratio then also oscillates making interpretation difficult. Existing Ly\(\beta\) and D\(\alpha\) line profiles from Figure 5.15a are used to estimate the importance of the different viewing geometries, as will now be described.

Changes in the divertor plasma's opacity can also be assessed by comparing the Ly\(\beta\) signal to an average of the D\(\alpha\) spatial profile, measured by the CCD camera, over the VUV acceptance cone. The resulting ratio for the first plasma pulse is illustrated in Figure 5.17. It can be seen that the ratio is quite similar to that in Figure 5.15a, but showing a reduced drop of 30%.
The fact that the Dα emission is seen to oscillate less than the Lyβ emission as the plasma strike point is swept suggests that the acceptance cone for the visible channel of the visible/VUV spectrometer is greater than that for the VUV channel. Using the radial profile of the Dα emission from the CCD camera, it is possible to assess the effect of geometry on these signals. For plasma pulses 34357 and 34359, this Dα radial profile is averaged over a 20cm spot size, the acceptance cone size of the spectrometer’s visible channel, and a 10cm spot size, the acceptance cone size of the spectrometer’s VUV channel. The ratio of the two averages is shown in Figure 5.18 for the two shots considered.
Figure 5.18. Ratios of CCD camera profiles averaged over the acceptance cone of the visible channel of the VUV spectrometer to the visible channel measurement for pulses with the strike point on (a) the vertical target and (b) the horizontal target.

It can be seen from Figure 5.18b that changes in the emission profile as the plasma is swept do explain the variations seen in the horizontal target shot (c.f. Figure 5.15b) as the numerator and the denominator of the ratio in Figure 5.18b are identical in every respect, apart from the acceptance cone that they are averaged over.

Nevertheless, using the same averaging process, the ratio of averages for the vertical case (Figure 5.18a) does not reproduce the drop seen in the experimental ratio of Lyβ/Dα (Figure 5.15a). Hence this process of eliminating the cause of oscillations in the branching ratio has not explained the decrease in levels of Lyβ emissivity with time, which lessens the likelihood of geometrical effects being the cause of the drop in the branching ratio of Figure 5.15a.

It must be said that the characterisation of the VUV spectrometer is not complete. In particular, shifts in the wavelength position of spectral lines have been observed during a shot. This may be due to under-filling of the grating, which would be most apparent in the horizontal plate discharges where the spectrometer views large spatial emission gradients. It is not clear exactly how this would affect the effective calibration of the VUV system but it
typically alters the line intensity by less than 5%. Although this problem can cause an increase in the magnitude of the oscillations for the time history of the Lyβ emission, the inaccuracy occurs in short cycles, as the plasma sweeps across the target. It does not affect the overall trend of the line emission over the entire period of the plasma pulse and can therefore not explain the drop in branching ratios observed in Figures 5.15 and 5.17. To ensure that this is true for the plasma pulses in this investigation the Lyβ profile is examined for a time at the beginning and the end of each plasma pulse, when the plasma sweep is at the same position in its cycle. It is found that there is a negligible difference in the wavelength of the profile. An example of this can be seen in Figure 6.3 for plasma pulse 35405. It seems likely from these studies that the changes in the ratios of Lyβ/Dα are mainly due to transfer of photons from Lyβ to Dα and therefore to absorption in the Lyβ line.

For further analysis and more evidence of opacity a second pair of plasma pulses have been selected for analysis. As will be described in the following section, the first pair of plasma pulses could only be modelled using the O-S model. This is because the time taken to obtain a reasonable plasma model using EDGE2D is considerably greater than using the O-S model. Since time is such an issue in an investigation of this magnitude, a choice has to be made as to which plasma pulses should be prioritised for a detailed analysis. The second pair of plasma pulses, as will be described in detail shortly, have not only ohmic heating but additional heating also. The advantage of this is that, after the additional heating is switched on, the input power remains roughly constant, which is not true for plasmas being heating by ohmic heating alone. Since one of the parameters used as input to EDGE2D is the input power, it is much easier to model plasmas with additional heating. Hence the second pair of plasma pulses are selected to be modelled using EDGE2D.

These plasma pulses are similar to the first in that they are also density ramp pulses, the first with the strike point on the vertical target (pulse 35405), the second with the strike point on the horizontal target (pulse 34355). However, these plasma pulses both have neutral beam heating in addition to ohmic heating. In this heating process, a neutral beam is transmitted into the plasma and its neutral particles become ionised. This produces fast ions that get slowed down by Coulomb collisions, during which time energy is passed to the plasma particles, heating them in the process. At a high neutral beam injection velocity the electron heating dominates initially. However, as the beam ions slow down, the heating is transferred predominantly to plasma ions (Wesson, 1997).
When applying a heating method in addition to ohmic heating, it is often possible to achieve a high confinement plasma (H-mode). This describes a plasma where the confinement time is greatly increased, typically by a factor of two. H-mode plasmas are characterised by ELMs which are short lived instabilities causing bursts of radiation. Plasma pulse 35405 has a toroidal current of approximately 2MA and a toroidal magnetic field of approximately 2T. Despite the neutral beam heating, it is a low confinement mode (L-mode) plasma. In L-mode plasmas the confinement of the plasma degrades with increased heating, hence the plasma cannot be confined for too long a period. Plasma pulse 34355 fluctuates from being an L-mode to an H-mode plasma and it also has a plasma current of 2MA and a toroidal magnetic field of approximately 2T.

The magnetic geometry and time traces for the vertical target shot 35405 can be seen in Figures 5.19 and 5.20, respectively.

Figure 5.19. The magnetic geometry for the vertical target pulse 35405. Also shown is the estimated VUV spectrometer viewing line.
Figure 5.20. The time traces of the vertical target pulse 35405.

The magnetic geometry and time traces of the horizontal target shot 34355 can be seen in Figures 5.21 and 5.22, respectively.

Figure 5.21. The magnetic geometry for the horizontal target pulse 34355. Also shown is the estimated spectrometer viewing line.
It can be seen from Figure 5.20 that at 51.5s the input power to pulse 35405 is approximately 1.2MW, about the same as that at the earlier times in the previous plasma pulses, 34857 and 34859. However, unlike the previous pulses, additional neutral beam heating is switched on at 52.0s, causing the input power to shoot up to 4.8MW. Since neutral beam heating is not temperature dependent the input power remains constant thereafter. Figure 5.22 shows the time traces of the horizontal target shot and it can be seen from the rise in the input power that additional neutral beam heating is switched on at 50.0s for this plasma pulse.

Apart from a slight increase at the point where the input power shoots up, both shots display a reasonably steady increase in the total radiative power and the radiative power dissipated at the X-point. These help to determine how much power is going to the divertor and thus the divertor temperature. For the vertical target shot these parameters start at approximately 0.5MW at 51.5s and increase to about 2.2MW and 1.4MW, respectively. This closely follows the trend observed in the plasma pulses 34857 and 34859. Figure 5.22 shows that the plasma pulse 34355, although having similar values of the total radiative power and radiative power at the X-point at 49.5s, has a greater increase in these parameters to the values of approximately 4MW and 2.6MW, respectively, at 54.5s.
All the plasma pulses demonstrate a steady increase in the peak electron density. However, Figures 5.11 and 5.13 show that for the first pair of shots this density increases from approximately $2 \times 10^{10} \text{m}^{-3}$ to $5 \times 10^{10} \text{m}^{-3}$. This behaviour is very similar to that of pulse 34355 but pulse 35405 has a slightly higher peak electron density throughout, starting at approximately $3 \times 10^{10} \text{m}^{-3}$ and rising to approximately $6 \times 10^{10} \text{m}^{-3}$ at 56.0s.

It should be pointed out that these density and temperature readings are taken at the plasma core, not in the divertor region where the temperatures are considerably lower than the core values. However, the time traces of conditions in the core often reflect similar trends to those in the divertor region. For example, if the core density increases throughout the duration of a pulse the target density also tends to increase.

Before selecting an appropriate time slice, in a particular pulse, to represent an optically thin or potentially optically thick plasma for modelling, the target Langmuir probe data were studied. Unfortunately, in the case of pulse 34355, the quality and quantity of Langmuir probe data available is inadequate for this investigation. Consequently, it is necessary to rely on the plasma core parameters and the branching ratio alone for this pulse.

Fortunately, for the case of pulse 35405, reasonable Langmuir probe data are available. These data show that the lowest electron density, when averaged over all probes and the major radius, occurs at the time 51.5s and has a value of $4.1 \times 10^{18} \text{m}^{-3}$. The average electron temperature reading given by the probes at this time is 5.8eV. The maximum temperature reading in the divertor is 6.5eV, at 52.5s, at which time the electron density reading is $1 \times 10^{19} \text{m}^{-3}$. When selecting a time slice likely to correspond to a low optical thickness, the highest temperatures and lowest densities are sought after. In this case the lowest electron density and highest electron temperature occur at different times. It is assumed that density is a greater representative of the plasma's opacity. Therefore, the time slice which has the lowest reading of electron density, 51.5s, is selected to represent an optically thin plasma for pulse 35405.

The time with the highest electron density and lowest electron temperature in pulse 35405 is selected as the time slice to represent a potentially optically thick plasma. The target Langmuir probes readings of temperature and density agree on 56.0s where the electron temperature is 5eV and the electron density is $2.6 \times 10^{19} \text{m}^{-3}$. 

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Returning to plasma pulse 34355, with only the plasma core parameters to aid the process of selecting a time slice, it is worthwhile considering the similarities between the parameter changes in this pulse and that of 35405. Consequently, although the plasma core temperature reading at 51.5s is the maximum and would therefore be an ideal time selection to represent an optically thin plasma, the lowest electron density occurs earlier at 49.5s.

The relation between core and divertor densities is stronger than that of temperature between the two regions. Hence the time which displays the lowest electron density in the core, 49.5s, is selected for the time slice likely to correspond to a plasma with a low optical thickness in the horizontal target pulse. Just as for pulse 35405, the process of selecting a time, in pulse 34355, to represent a potentially optically thick plasma is much simpler since the largest electron density and lowest electron temperature correspond at time 54.5s.

Neither of these pulses have visible spectrometer data from the visible/VUV spectrometer available. However, as discussed previously, the advantages of having Lyβ and Dα emissivities along the same line of sight are negated by the problems of different acceptance cones and the area of the grating reached by the light. Hence, the remaining visible diagnostics offer data which are just as promising as that from the visible/VUV spectrometer for branching ratios.

The visible light monitors provide an initial Dα line profile which, along with the line profile of Lyβ from the VUV spectrometer, have been fitted to give total intensities. Branching ratios of these are taken for pulse 35405 and 34355, as before. Unfortunately, as for the case of the horizontal target plasma pulse 34859, large oscillations occur in the branching ratio of horizontal target pulse 34355 to such an extent that the analysis of the ratio is virtually impossible. Again this oscillatory characteristic could be due to sweeping of the plasma over two instruments with different lines of sight or acceptance cones, as before, or it could be due to ELMs.

The line intensities and branching ratio of pulse 35405 and 34355 are shown in Figure 5.23. Here, the Lyβ has an estimated fitting error of 5% and the Dα profile has an accuracy of 10%.
Figure 5.23. Line integrated signals for pulses with the strike point on (a) the vertical target (pulse 35405) and (b) the horizontal target (pulse 34355). (i) Dα emission, (ii) Lyβ emission, and (iii) the ratio of the two signals as a measure of the divertor’s opacity.

It can be seen from Figure 5.23a that there is a drop in the ratio of approximately 55% for plasma pulse 35405, which is far greater than the fitting error for the lines and accuracy level of the Dα profile. By taking a central line through the large oscillations in the branching ratio of pulse 34355, Figure 5.23b, it is possible to see a drop of approximately 60% between times 49.5s and 54.5s.

The same technique of comparing the Lyβ signal to the average of the Dα spatial profile over the VUV acceptance cone can be used to assess the importance of the various viewing geometries. These branching ratios for each pulse can be seen in Figure 5.24.
Figure 5.24. Ratio of Ly$\beta$ from the VUV divertor spectrometer, to D$\alpha$ from the CCD camera, averaged onto the same line of sight for (a) the vertical target pulse 35405 and (b) the horizontal target pulse 34355.

It can be seen from this that the drop in the branching ratio, of approximately 50% for the first pulse 35405, Figure 5.24a, is in good agreement with the initial spectrometer measurement of the branching ratio which shows a 55% drop over the same time period, Figure 5.23a.

The branching ratio for the horizontal target pulse 34355, however, is again suffering from large oscillations. This makes interpretation of the data less reliable. However, a drop in the branching ratio is clearly visible in Figure 5.24b and so by using data along the centre of the oscillations, it can be seen that a drop in the branching ratio of approximately 50% occurs. Again this is in good agreement with the drop in branching ratio of 60% observed using the visible light monitors, Figure 5.23b.
5.4 The Plasma Models

In this final section comparisons are made between the two plasma models discussed in Chapter 2 and the improvements made to the O-S model are put to the test.

It was mentioned in Chapter 2 that at high densities, such as those in the divertor region, studies show that a momentum loss is caused by collisions of ions with recycling neutrals (Stangeby, 1993). This results in higher plasma pressures above the target plate, which increases the density in this region. The culmination of these effects is a peak density away from the target, in preference to the usual peak target density which is typical in an attached plasma. Chapter 2 also explains how the O-S model has been modified to incorporate the momentum loss due to ion-neutral collisions as calculated by the Monte Carlo neutral code NIMBUS. For detached plasmas, this modified model does show an increased density just away from the target as predicted. This can be seen in Figure 2.10 for the plasma pulse 34857. However, despite this higher density, the model does not reproduce the measured level of bremsstrahlung in the divertor.

The fluid code, using more detailed calculations, does reproduce the experimental data, such as bremsstrahlung, more accurately and predicts a larger momentum loss, as shown in Figure 2.11. The discrepancy in the two simulations is thought to be related to errors in interpretation of the electron temperature measurements with the target Langmuir probes (Gunther 1995), as described in Chapter 2.

To give an idea of the differences between the two methods of predicting the target temperature, a comparison is made for the plasma pulse 35405 at time 56.0s. Here, the average temperature reading using the target Langmuir probes is approximately 5eV whereas the EDGE2D prediction of the average target temperature is approximately 1eV. As will be seen, in Figures 6.1 and 6.2 in Chapter 6, this temperature difference can alter the outcome of an opacity prediction quite dramatically at higher densities.

In an attempt to compare the two plasma models, further analysis has been carried out on one of the plasma pulses (35405). The modelling procedure for pulse 35405 is carried out for two time slices, as described earlier. The first time slice (51.5s) has a relatively low divertor density and the second time slice (56.0s) has a relatively high divertor density. The success of the plasma modelling procedure is tested by comparing specific parameters, resulting from the modelling process, with those obtained via direct diagnostic measurements. This
involves comparing Dα emission, bremsstrahlung (including both free-free and free-bound emission) and radiated power measurements. The results which demonstrate the closest modelling attempt using the O-S model are given in Figure 5.25.

It can be seen from Figure 5.25 that there is a large degree of disagreement between the diagnostic measurements and the O-S model predictions. This is thought to be due to inaccuracies with Langmuir probe measurements of low electron temperatures, which are used to provide input to the O-S model. This issue is described in Section 2.5.1. Unfortunately, this has a considerable impact on the quality of results, which renders this method of plasma modelling unsuitable for low temperature plasmas. Hence, although the O-S model is reliable when used for attached plasmas, EDGE2D is better for modelling the plasma pulses being studied in this thesis.
Figure 5.25a. Comparison between diagnostic data (solid lines) and theoretical predictions of $D\alpha$ emission using the O-S model (broken lines) for a vertical target pulse (35405) at (i) time 51.5s when the plasma is optically thin and (ii) time 56.0s when the plasma is potentially optically thick.
Figure 5.25b. Comparison between diagnostic data (solid lines) and theoretical predictions of radiated power using the O-S model (broken lines) for a vertical target pulse (35405) at (i) time 51.5s when the plasma is optically thin and (ii) time 56.0s when the plasma is potentially optically thick.
Figure 5.25c. Comparison between diagnostic data (solid lines) and theoretical predictions of bremsstrahlung using the O-S model (broken lines) for a vertical target pulse (35405) at (i) time 51.5s when the plasma is optically thin and (ii) time 56.0s when the plasma is potentially optically thick.
Figure 5.26a. Comparison between diagnostic data (solid lines) and theoretical predictions of Dα emission using EDGE2D (broken lines) for a vertical target pulse (35405) at (i) time 51.5s when the plasma is optically thin and (ii) time 56.0s when the plasma is potentially optically thick.
Figure 5.26b. Comparison between diagnostic data (solid lines) and theoretical predictions of radiated power using EDGE2D (broken lines) for a vertical target pulse (35405) at (i) time 51.5s when the plasma is optically thin and (ii) time 56.0s when the plasma is potentially optically thick.
Figure 5.26c. Comparison between diagnostic data (solid lines) and theoretical predictions of bremsstrahlung using EDGE2D (broken lines) for a vertical target pulse (35405) at (i) time 51.5s when the plasma is optically thin and (ii) time 56.0s when the plasma is potentially optically thick.
The results displaying the closest modelling attempt using EDGE2D are shown in Figure 5.26. Here a pure deuterium plasma is assumed because there is not much sputtering for this pulse and using this assumption produces a better match for the Dα emission when compared with diagnostic data.

It should be mentioned that the inner target region is the area being observed in this investigation, since it tends to have plasmas with larger densities and lower temperatures. This is for two reasons, both caused by the geometry of the torus. The first reason being that the outer edge of the torus has approximately twice the surface area of the inner side, therefore allowing more power transport out from the core plasma and down to the outer target plates. The second reason is due to an effect called the ballooning type mode, which is caused by outwardly directed pressure gradients. The pressure pushes against a convex wall of plasma on the inner edge of the torus which tends to nullify the effect of the pressure. However, on the outer edge the pressure meets a concave wall of plasma, which is not able to resist the force exerted upon it. With this, the likelihood of instabilities and turbulence increases thereby creating greater temperatures in the outer SOL. It is often difficult to produce a good plasma model of both the inner and outer target areas simultaneously, consequently, priority will be given to the inner region.

It can be seen from Figures 5.25 and 5.26 that EDGE2D best models the Dα emissivity both in form and magnitude for the higher density plasma, at the later time in the pulse. The power is also modelled more effectively by EDGE2D for this time, as is the bremsstrahlung but the benefits of using EDGE2D rather than the O-S model are not as prominent as in the case of other parameters.

Comparing the two models for the first time slice of the shot does not enable such a clear decision to be made. However, since both sets of results are similar in their qualitative and quantitative predictions of the diagnostic data, it is decided that the EDGE2D model will be used since it is consistent with that used for the latter time slice.

Although EDGE2D provides a reasonable plasma model, it can be seen in Figure 5.26 that for the first time slice, it overpredicts the Dα emission by a factor of approximately 4 and for the second time slice it underpredicts the Dα emission by a factor of approximately 2. Unable to resolve these discrepancies easily using EDGE2D, another method is employed which involves modifying the neutral and ion densities. The neutral deuterium density and
ion density can be crudely related to the emission of Dα, depending on which processes are producing the emission.

The earlier times of the plasma pulses have higher temperatures whereas the later time slices of the plasma pulses have low temperatures. At low temperatures the energy level n=3 is predominantly populated by recombination. However, as the temperature increases collisional excitation from the ground state becomes more prominent.

If excitation is producing the Dα emission then the neutral deuterium density can be considered to be directly proportional to the emission. Hence by multiplying the deuterium densities, predicted by EDGE2D, by a factor of 0.25 and 2 for the first and second times, respectively, it would be possible to produce a more accurate plasma model. However, if the Dα is produced almost entirely by recombination then the level of Dα emission is proportional to the product of the ion and electron densities, which are considered equal in this model. Since the fraction of ions, or electrons, to neutrals is constant in the population code, it is still necessary to modify the neutral density by the same correction factor as that of the ions, in order to keep the fraction consistent. Consequently both the ion and neutral density need to be multiplied by the square root of the correction factor when Dα emission is produced predominantly by recombination.

After further analysis, it is found that for pulse 35405, the contribution to Dα emission is predominantly from excitation for the first time slice (51.5s). At the second time slice (56.0s) the contribution to Dα emission is almost entirely from recombination.

Hence at time 51.5s the neutral deuterium density, and consequently the fraction of neutral to ion density, predicted by EDGE2D are multiplied by the correction factor of 0.25. However, at time 56.0s the neutral deuterium density and ion density, as predicted by EDGE2D, are multiplied by the correction factor of √2, whilst the fraction of neutral to ion density remains unchanged. The ultimate parameters to be used as input to the population code for pulse 35405 can be seen in Figure 5.27. In these figures, each bar represents one slab of the ‘slab-stack’ model, described in Chapter 4. The level of the individual bar describes the average value of a particular parameter within the area of the slab.

As mentioned previously, EDGE2D is also used to model the plasma pulse 34355 at times 49.5s, when the electron density is low, and 54.5s when the electron density is high and the temperature is low. The comparisons with diagnostic data can be seen in Figure 5.28.
Figure 5.27a. Parameters used in the 'slab-stack' model of the population code for plasma pulse 35405 at time 51.5s. Each vertical bar represents a slab. The parameters are (i) neutral deuterium temperature and (ii) electron and ion temperature.
Figure 5.27a. Parameters used in the 'slab-stack' model of the population code for plasma pulse 35405 at time 51.5s. Each vertical bar represents a slab. The parameters are (iii) neutral deuterium density and (iv) electron and ion density.
Figure 5.27a. Parameters used in the ‘slab-stack’ model of the population code for plasma pulse 35405 at time 51.5s. Each vertical bar represents a slab. The parameters are (v) neutral deuterium to deuterium ion ratio and (vi) neutral velocity in the X direction (parallel to the separatrix).
Figure 5.27a. Parameters used in the 'slab-stack' model of the population code for plasma pulse 35405 at time 51.5s. Each vertical bar represents a slab. The parameters are (vii) neutral velocity in the Z direction (perpendicular to the plane of the separatrix) and (viii) neutral velocity in the Y direction (parallel to the toroidal field).
Figure 5.27b. Parameters used in the 'slab-stack' model of the population code for plasma pulse 35405 at time 56.0s. Each vertical bar represents a slab. The parameters are (i) neutral deuterium temperature and (ii) electron and ion temperature.
Figure 5.27b. Parameters used in the ‘slab-stack’ model of the population code for plasma pulse 35405 at time 56.0s. Each vertical bar represents a slab. The parameters are (iii) neutral deuterium density and (iv) electron and ion density.
Figure 5.27b. Parameters used in the ‘slab-stack’ model of the population code for plasma pulse 35405 at time 56.0s. Each vertical bar represents a slab. The parameters are (v) neutral deuterium to deuterium ion ratio and (vi) neutral velocity in the X direction (parallel to the separatrix).
Figure 5.27b. Parameters used in the 'slab-stack' model of the population code for plasma pulse 35405 at time 56.0s. Each vertical bar represents a slab. The parameters are (vii) neutral velocity in the Z direction (perpendicular to the plane of the separatrix) and (viii) neutral velocity in the Y direction (parallel to the toroidal field).
Figure 5.28a. Comparison between diagnostic data (solid lines) and theoretical predictions of Dα emission using EDGE2D (broken lines) for a vertical target pulse (34355) at (i) time 49.5s when the plasma is optically thin and (ii) time 54.5s when the plasma is potentially optically thick.
Figure 5.28b. Comparison between diagnostic data (solid lines) and theoretical predictions of radiated power using EDGE2D (broken lines) for a vertical target pulse (34355) at (i) time 49.5s when the plasma is optically thin and (ii) time 54.5s when the plasma is potentially optically thick.
The absence of a bremsstrahlung measurement for the first time slice is due to the lack of a signal being detected. However, the plasma modelling of the first time slice is not as important as that of the later time. This is due to the fact that, as the plasma tends towards becoming optically thick, it is more sensitive to changing parameters. In the optically thin conditions of the first time slice, the levels of Lyman absorption are hardly altered by parameter changes, within reason, and always give a similar value for the branching ratio. Hence the modelling of the second time slice takes priority in this thesis.

Here, it can be seen that, as before, EDGE2D overpredicts the D\(\alpha\) emission for the first time slice by a factor of 4. However, for the second time slice EDGE2D goes from underpredicting to overpredicting the D\(\alpha\) emission with the increasing values of the major radius. Since the spectrometer line of sight for this shot centres on R=2.55m, it seems that, in this region, EDGE2D gives similar D\(\alpha\) values to those observed by the spectrometer. Therefore, the deuterium density for this particular time slice does not need to be altered.

As for plasma pulse 35405, an investigation into the nature of the D\(\alpha\) production is undertaken. It is found that, as for the vertical target pulse, the main contribution to D\(\alpha\)
emission comes from excitation at time 49.5s and recombination at 54.5s. Consequently, for the first time slice it is necessary to multiply the neutral deuterium density and fraction of neutral to ion density, as predicted by EDGE2D, by the correction factor of 0.25. The ultimate parameters, for plasma pulse 34355, used as input to the population code can be seen in Figure 5.29.

In Chapter 4, we describe how the Zeeman effect could be incorporated in to the population code by modifying the input neutral deuterium temperatures using Equation 4.90. This procedure is carried out for the neutral deuterium temperatures, obtained from EDGE2D, of plasma pulse 35405 at times 51.5s and 56.0s and plasma pulse 34355 at times 49.5s and 54.5s. The modified temperatures are given in Figures 5.30 and 5.31.

In the following chapter the results from the use of this data in the population code will be given and comparisons made between the changes in the branching ratios as predicted by this theoretical model and the changes observed by the diagnostics.
Figure 5.29a. Parameters used in the ‘slab-stack’ model of the population code for plasma pulse 34355 at time 49.5s. Each vertical bar represents a slab. The parameters are (i) neutral deuterium temperature and (ii) electron and ion temperature.
Figure 5.29a. Parameters used in the ‘slab-stack’ model of the population code for plasma pulse 34355 at time 49.5s. Each vertical bar represents a slab. The parameters are (iii) neutral deuterium density and (iv) electron and ion density.
Figure 5.29a. Parameters used in the ‘slab-stack’ model of the population code for plasma pulse 34355 at time 49.5s. Each vertical bar represents a slab. The parameters are (v) neutral deuterium to deuterium ion ratio and (vi) neutral velocity in the X direction (parallel to the separatrix).
Figure 5.29a. Parameters used in the 'slab-stack' model of the population code for plasma pulse 34355 at time 49.5s. Each vertical bar represents a slab. The parameters are (vii) neutral velocity in the Z direction (perpendicular to the plane of the separatrix) and (viii) neutral velocity in the Y direction (parallel to the toroidal field).
Figure 5.29b. Parameters used in the ‘slab-stack’ model of the population code for plasma pulse 34355 at time 54.5s. Each vertical bar represents a slab. The parameters are (i) neutral deuterium temperature and (ii) electron and ion temperature.
Figure 5.29b. Parameters used in the ‘slab-stack’ model of the population code for plasma pulse 34355 at time 54.5s. Each vertical bar represents a slab. The parameters are (iii) neutral deuterium density and (iv) electron and ion density.
Figure 5.29b. Parameters used in the ‘slab-stack’ model of the population code for plasma pulse 34355 at time 54.5s. Each vertical bar represents a slab. The parameters are (v) neutral deuterium to deuterium ion ratio and (vi) neutral velocity in the X direction (parallel to the separatrix).
Figure 5.29b. Parameters used in the 'slab-stack' model of the population code for plasma pulse 34355 at time 54.5s. Each vertical bar represents a slab. The parameters are (vii) neutral velocity in the Z direction (perpendicular to the plane of the separatrix) and (viii) neutral velocity in the Y direction (parallel to the toroidal field).
Figure 5.30. Modified neutral deuterium temperatures to account for Zeeman splitting effects for plasma pulse 35405 at times (a) 51.5s and (b) 56.0s.
Figure 5.31. Modified neutral deuterium temperatures to account for Zeeman splitting effects for plasma pulse 34355 at times (a) 49.5s and (b) 54.5s.
Chapter 6

Theoretical Analysis and Interpretation of Experimental Results

6.1 Introduction

In this chapter, the effects of varying plasma parameters on a plasma's opacity are investigated using the population code. The branching ratio of Ly$\beta$ to D$\alpha$ is examined as a function of varying plasma temperature, neutral deuterium density and electron density. The results of this study provide a quick and simple indication of the approximate level of opacity expected for a particular set of conditions, which is also useful for tokamak plasmas other than those of JET.

The parameters representing the plasma pulses 35405 and 34355, and used as input to the population code, are used to test the validity of the investigation into the effect of varying parameters on the plasmas opacity, as outlined in the previous paragraph. This comparison is also done to provide a preliminary idea of the expected level of opacity occurring for these particular pulses.
The branching ratios of Lyβ to Dα, as predicted by the population code for the plasma pulses under investigation, are given. A comparison is then made between these branching ratios and those measured in the experimental analysis of the previous chapter.

Finally in this chapter, a brief study to examine the effects of opacity and varying plasma parameters on the ionisation and power balance of the plasma is undertaken.

6.2 Effects of Varying Parameters on a Plasma’s Opacity

The effects of increasing individual plasma parameters, such as the neutral density, on the levels of line absorption are reasonably simple to determine. However, electron and ion densities and temperatures as well as neutral temperatures contribute to the overall levels of line absorption occurring in a plasma.

The effects of temperature on opacity are due to line profile broadening. As the temperature increases, so do the widths of the line profiles but the area under a particular line profile remains the same. Hence, although the line profile is spread out over a wider range of frequencies, the absorption coefficient is less and consequently this decreases the opacity.

Accumulatively, all these contributing factors somewhat complicates an investigation in the effects of varying a particular parameter. Hence, separate investigations are undertaken, each one examining the effects on opacity of varying a single plasma parameter.

The ‘slab-stack’ model, described in Chapter 4, is used in the production of these results, with parameters varying from slab to slab. The parameters used and the way in which they vary from one slab to the next is determined by studying the parameters of JET plasmas.

In an attempt to form a more realistic plasma model, various typical density and temperature profiles, in the divertor region of the JET tokamak, are examined to find general trends. This study shows that, although temperatures remain reasonably constant over the divertor region, electron and neutral deuterium densities vary greatly. In order to conform to the density trends, the plasma is modelled on the ‘slab-stack’ concept. Here ten slabs, each of 2cm thickness, are used. Five of these assume a neutral deuterium density of $1 \times 10^{19}$ m$^{-3}$, four slabs have a density of half this value at $5 \times 10^{18}$ m$^{-3}$ and the final slab has a density which is a
quarter that of the peak density, with a value of $2.5 \times 10^{18} \text{m}^{-3}$. The average density is then taken over all slabs to provide a neutral deuterium density of $7.25 \times 10^{18} \text{m}^{-3}$. The electron density is also modelled using this method.

An assumption made in this model is that the neutral deuterium temperature is equal to the electron and ion temperature. Typically, the temperature of neutral deuterium is slightly greater than that of the electrons and ions, especially in plasmas with a high opacity. This is mainly due to neutral recycling, described in Chapter 2, which increases the temperature of the neutrals. The deuterium molecules dissociate into deuterium atoms and, due to the Frank-Condon principle, the energy used to bond the deuterium molecule is transformed into kinetic energy of the deuterium atoms, increasing their temperature. The difference in temperature between the electrons, ions and the neutrals is usually less than 1eV and so in a simple model such as the one being used for this part of the investigation, it is reasonable to assume that they have the same temperature.

The range of parameters selected stretch beyond those obtainable in the JET divertor region. This is intentional so that the results can be used for other tokamak plasmas as an initial indicator to the potential levels of opacity.

The population code is used to find the effect on Ly$\beta$ line absorption of increasing the neutral deuterium density of the plasma for fixed electron temperatures and electron densities. In order to simultaneously examine the effect of varying the electron temperature and density, this procedure is carried out several times, with electron temperatures ranging from 0.5eV to 40eV and electron densities ranging from $7.25 \times 10^{15} \text{m}^{-3}$ to $7.25 \times 10^{20} \text{m}^{-3}$. The results to this investigation are represented twice, each with a different graphical approach. The first (Figure 6.1) shows the effect of a varying electron density on the opacity of a plasma for a fixed temperature. This procedure is carried out for a range of fixed electron temperatures, each represented in an individual graph. The second graphical representation (Figure 6.2) shows the effect of varying the electron density on the opacity of a plasma for a fixed electron density. Again this procedure is carried out for a range of fixed electron densities, each on a separate graph.
Figure 6.1. Branching ratio of Lyβ/Dα versus average neutral density for various average electron densities (depicted in the key at the top of the page) and an electron temperature of (a) 0.5 eV and (b) 1 eV.
Figure 6.1. Branching ratio of Lyβ/Dα versus average neutral density for various average electron densities (depicted in the key at the top of the page) and an electron temperature of (c) 2eV and (d) 5eV.
Figure 6.1. Branching ratio of Lyβ/DA versus average neutral density for various average electron densities (depicted in the key at the top of the page) and an electron temperature of (e) 20eV and (f) 40eV.
Figure 6.2. Branching ratio of Lyβ/DA versus average neutral deuterium density for various temperatures (depicted in the key at the top of the page) and an average electron density of (a) $7.25 \times 10^{17} \text{m}^{-3}$ and (b) $7.25 \times 10^{18} \text{m}^{-3}$. 
Figure 6.2. Branching ratio of Lyβ/De versus average neutral deuterium density for various temperatures (depicted in the key at the top of the page) and an average electron density of (c) $3.625 \times 10^{17}$ m$^{-3}$ and (d) $7.25 \times 10^{17}$ m$^{-3}$. 
Figure 6.2. Branching ratio of Lyβ/Δv versus average neutral deuterium density for various temperatures (depicted in the key at the top of the page) and an average electron density of (e) $3.625 \times 10^{20} \text{m}^{-3}$ and (f) $7.25 \times 10^{20} \text{m}^{-3}$.
It can be seen from Figures 6.1 and 6.2 that as the neutral density increases, the branching ratio of Ly\(\beta\) to D\(\alpha\) drops from a value of 8.1 until it becomes an almost completely optically thick plasma and tends to a branching ratio value of zero.

In Figure 6.1, it can be seen that a particular electron density is not represented across the full range of neutral densities in the graphs. The decision to limit this range has been made in order to only consider realistic values for the ratio of neutrals to electrons. However, to enable an examination into the effects of varying the electron density on a plasma’s opacity, ‘cross-over’ points (when crossing over from one electron density value to another) are plotted. At these points the branching ratios for two different electron densities, but the same neutral density, are given. It can be seen from the smooth nature of the graphs in Figure 6.1 that the branching ratios at the ‘cross-over’ points are approximately the same for both electron densities. Hence, it can be concluded that the electron density has a negligible effect on the opacity of a plasma.

Figure 6.2 shows the effects of electron temperature on the opacity of a plasma. It can be seen that the lower temperature plasmas do not require such large neutral densities as the higher temperature plasmas before becoming optically thick. This is because at lower temperatures the absorption coefficient for a spectral line is greater, as described earlier in this section.

Figures 6.1 and 6.2 also demonstrate that line absorption is most sensitive to the neutral deuterium density. This is simply because the neutral deuterium is the absorber atom.

### 6.3 Preliminary Analysis of the Opacity of the Plasma Pulses Using a Simple ‘Single-Slab’ Model

In the previous chapter, the parameters for two time slices of two plasma pulses are given in the form of bar charts, Figures 5.26 and 5.28. The parameters are averaged over all slabs and the temperature units are converted from degrees Kelvin to eV in order to be consistent with the graphs in Figures 6.1 and 6.2. The results can be seen in Table 6.1. It should be mentioned that the velocity shifts from Figures 5.26 and 5.28 are not included into this part of the investigation. The reason for this is because it can be assumed that these velocity shifts play an insignificant role in effecting the opacity of a plasma. This assumption is tested by
running the population code with the input parameters for plasma pulse 35405 at time 56.0s, with velocity gradients of zero in all directions. The Ly/βDα branching ratio value obtained from this procedure is 5.04, which is identical the branching ratio value obtained when the velocity gradients are included.

<table>
<thead>
<tr>
<th></th>
<th>Plasma pulse 35405</th>
<th>Plasma pulse 34355</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>51.5s</td>
<td>56.0s</td>
</tr>
<tr>
<td>Neutral deuterium density (m⁻³)</td>
<td>3.385 × 10¹⁵</td>
<td>5.195 × 10¹⁵</td>
</tr>
<tr>
<td>Electron (and ion) temperature (eV)</td>
<td>8.504</td>
<td>1.667</td>
</tr>
<tr>
<td>Unmodified neutral deuterium temperature (eV)</td>
<td>9.708</td>
<td>2.379</td>
</tr>
<tr>
<td>Modified neutral deuterium temperature to incorporate the Zeeman effect (eV)</td>
<td>9.897</td>
<td>2.604</td>
</tr>
<tr>
<td>Electron (and ion) density (m⁻³)</td>
<td>1.501 × 10¹⁹</td>
<td>1.494 × 10²⁰</td>
</tr>
<tr>
<td>Neutral deuterium to deuterium ion ratio</td>
<td>1.907 × 10⁻⁵</td>
<td>1.336 × 10⁻¹</td>
</tr>
</tbody>
</table>

Table 6.1. Table giving the average parameter values for pulses 35405, at times 51.5s and 56.0s, and 34355, at times 49.5s and 54.5s.

It can be seen from Table 6.1 that the average parameters obtained at the earlier times, when the conditions favour an optically thin plasma, are not fully represented in Figures 6.1 and 6.2. This is because the emphasis of investigation is towards the potentially optically thick plasmas which are produced at the later times in these plasma pulses. However, it is obvious by observing the general trends of the graphs in Figures 6.1 and 6.2 that the parameters at the earlier times of the plasma pulses would not reduce the branching ratio of Ly/βDα to below a value of 8.0.

By first examining the parameters of the vertical target plasma pulse 35405 at a time of 56.0s, it can be seen that the best graphs for comparison in Figures 6.1 and 6.2 are Figure 6.1c and Figure 6.2e. It should be mentioned that these graphs do not provide
branching ratios for the exact values of neutral density and electron temperature given in Table 6.1. For a neutral density of $5.195 \times 10^{18} \text{ m}^{-3}$, Figures 6.1c and 6.2e give branching ratio values of approximately 5.0 and 5.2, respectively. Considering that these graphs do not represent exactly the same parameters as those given in Table 6.1, these branching ratios are in excellent agreement.

A second inaccuracy with this method is that, in producing the graphs in Figures 6.1 and 6.2, the electron, ion and neutral deuterium temperatures are taken to be equal. As can be seen in the bar charts of Figures 5.26 and 5.28, the plasma models, although assuming the electron and ion temperatures to be equal, have different values for the neutral deuterium temperatures.

The largest problem with this method of determining opacity effects is in the way the averaging is done. It can be seen in Table 6.1 that for plasma pulse 35405 at 56.0s, the average value for the neutral deuterium density is $5.195 \times 10^{18} \text{ m}^{-3}$ and the average electron, or ion, density is $1.494 \times 10^{20} \text{ m}^{-3}$. Hence the ratio of the average deuterium density to the average electron density is $3.477 \times 10^{2}$. This is very different from the average value of the neutral deuterium to deuterium ion ratio of $1.336 \times 10^{-1}$, given in Table 6.1. Through the process of averaging the parameters of a stack of slabs across one big slab of uniform plasma, a distorted result has been created. However, in this case the temperatures are the same throughout the slabs for each case and only one parameter is varied at a time, which almost eliminates the problem.

The same examination is now undertaken for pulse 34355 at time 54.5s. It can be seen that the best graphs for comparison in Figure 6.1 and 6.2 are Figure 6.1c and Figure 6.2e. As before, these graphs do not provide branching ratios for the exact values of neutral density and electron temperature given in Table 6.1. For a neutral density of $3.617 \times 10^{18} \text{ m}^{-3}$, Figures 6.1c and 6.2e give branching ratio values of approximately 6.0 and 5.8, respectively. Again these branching ratios are in excellent agreement.

The best test for the reliability of Figures 6.1 and 6.2 is by comparing the branching ratios, obtained from them, to those obtained directly from the population code in conjunction with the 'slab-stack' model containing the bar chart parameters of Figures 5.26 and 5.28. The results of this process will be given in Section 6.4 enabling this comparison to be made in Section 6.4.3.
6.4 Comparison between Theoretical and Experimental Results

In this section the validity of the population code is put to the test by using the parameters of the plasma models for the pulses under investigation as input to the code. This produces branching ratios of Ly$\beta$/D$\alpha$ for the two time slices of each pulse, one at a time when the plasma is expected to be optically thin and another at a time where the plasma has a greater optical thickness.

The difference in ratio between the two times indicates the amount of Ly$\beta$ radiation that has been absorbed, which assists in determining whether a significant amount of line radiation absorption is occurring for the plasma to be defined as being optically thick.

In addition to making comparisons with the branching ratios obtained in the previous section, they will also be compared to those obtained by diagnostic measurements, as given in Chapter 5.

6.4.1 Theoretical Population Code Results

The plasma pulses being investigated in the most detail are 35405 and 34355 in preference to pulses 34857 and 34859. This is firstly because the latter are modelled using the O-S model rather than the more accurate EDGE2D transport code that is used to model pulses 35405 and 34355. The reason behind this is that pulses 35405 and 34355 have additional heating which provides a constant input power, making them easier to model using EDGE2D. This is described in more detail in Chapter 5. Secondly the 'slab-stack' model did not have the facility of computing the emission along a specific line of sight, as described in Section 4.2.3 and shown in Figure 4.5, when pulses 34857 and 34859 were under analysis. Finally, the plasma modelling process is only performed on the vertical target plasma pulse, 34857, because the diagnostic information for the horizontal target pulse 34859 demonstrates large oscillations, as shown in Figure 5.13b, which makes comparison with this data difficult. Hence, a more detailed description and analysis is reserved for plasma pulses 35405 and 34355 whilst only a brief analysis of pulse 34857 is given.
Table 6.2. Table giving the drop in the Lyβ/Dα branching ratio obtained by various experimental and theoretical methods for pulses 34857, 35405 and 34355.

<table>
<thead>
<tr>
<th>Method used to obtain branching ratio</th>
<th>Pulse 34857</th>
<th>Pulse 35405</th>
<th>Pulse 34355</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measure using the VUV/Visible spectrometer</td>
<td>50</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Measured using the VUV spectrometer and visible light monitors</td>
<td>-</td>
<td>55</td>
<td>60</td>
</tr>
<tr>
<td>Measured using VUV spectrometer and CCD cameras</td>
<td>30</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Theoretical prediction from the population code without the Zeeman effect</td>
<td>14</td>
<td>38</td>
<td>23</td>
</tr>
<tr>
<td>Theoretical prediction from the population code with the Zeeman effect</td>
<td>-</td>
<td>36</td>
<td>22</td>
</tr>
</tbody>
</table>

Pulse 34857

After using the plasma model parameters of pulse 34857 as input to the population code, the values for the branching ratio of Lyβ/Dα at times 12.0s and 19.0s are 8.09 and 6.96, respectively. This gives a 14% drop in the ratio over this time.

In Chapter 5 the branching ratio of Lyβ/Dα, produced using the visible/VUV spectrometer, Figure 5.13a, displayed a drop of approximately 50%. Similarly, the drop in this branching ratio, using a Dα profile from the CCD camera, Figure 5.14, is approximately 30%.
Clearly, the population code produces a branching ratio drop which is significantly smaller than those observed experimentally. However, it should emphasised that for this pulse the result produced by the population code does not represent the level of Lyβ absorption along the same line of sight as the VUV spectrometer.

**Pulses 35405 and 34355**

The drop in the LyβDα ratio over time, as predicted by the population code, for the plasma pulses 35405 and 34355 are now presented. Both of these pulses benefit from a more detailed and accurate investigation. This includes testing the plasma modelling, by EDGE2D, with diagnostic measurements, in Figures 5.25 and 5.27, for its validity and making compensatory corrections wherever possible. As described in Chapter 4, the branching ratio values are calculated along the same line of sight as the VUV spectrometer. This increases the reliability of comparisons between the experimental and theoretical results.

The values for the branching ratio, as calculated by the population code, without considering the Zeeman effect, for plasma pulse 35405 at times 51.5s and 56.0s are 8.08 and 5.04, respectively. This gives a 38% drop in the branching ratio.

When incorporating the effects of Zeeman splitting, as described in Chapter 4, by modifying the neutral deuterium temperatures that are fed into the population code, the branching ratio is increased. This increase is not noticeable for the optically thin plasma at time 51.5s, which still has a branching ratio of 8.08. However, the plasma with a greater optical thickness, at the later time of 56.0s, the branching ratio is increased to a value of 5.14. Hence the drop in the branching ratio throughout between the two times for plasma pulse 35405 is 36%. It can be deduced from this that the Zeeman effect acts to reduce the opacity of a plasma by a small amount, by broadening the line profile and reducing the value of the absorption coefficient.

The branching ratio of LyβDα, produced using the VUV spectrometer and visible light monitors, as shown in Figure 5.20a, displays a drop of 55% over the same time. Similarly, the branching ratio observed using the Dα profile from the CCD camera, shown in Figure 5.21a, gives a drop of 50% between the times of 51.5s and 56.0s. The latter of the two ratios is considered more reliable, since the CCD camera looks over the same acceptance cone as the VUV spectrometer, as described in Chapter 5. Hence the experimental measurement of the drop in the branching ratio between 51.5s and 56.0s for plasma pulse 35405 is
approximately 50%. Since the data from the VUV spectrometer and CCD cameras have an accuracy of approximately 10% and 20%, respectively, the overall accuracy of the branching ratio is approximately 22%. However, since the change in the branching ratio over a period of time is being examined, and not the absolute value, it is highly likely that the errors occurring in the results would be approximately consistent throughout the plasma pulse. Hence the change in the branching ratio with time should not have such a larger error bar.

The population code provides branching ratio values of 8.09 and 6.22 for plasma pulse 34355 at times 49.5s and 54.5s, respectively. These give a drop of 23% in the branching ratio.

The incorporation of modified neutral deuterium temperatures to simulate the effect of Zeeman splitting, increases the branching ratio for the plasma with the greater optical thickness, at time 54.5s, to a value of 6.30. As for plasma pulse 35405, the branching ratio remains unchanged under the influence of the Zeeman effect, for the optically thin plasma at the earlier time of 54.5s in the plasma pulse 34355. Hence the drop in the branching ratio from 54.5s to 56.0s in plasma pulse 34355, when accounting for the Zeeman splitting, is 22%. This, once again, clearly indicates that the Zeeman effect is only mildly significant in reducing the opacity of a plasma.

The VUV spectrometer and visible light monitors display a branching ratio drop of approximately 60%, shown in Figure 5.20b, over the same time period. When using the CCD camera to provide the $\text{D} \alpha$ emission profile, the drop in branching ratio between 49.5s and 54.5s is approximately 50%. As described earlier, the latter of the two ratios is considered more reliable. Hence the experimental measurement of the drop in the branching ratio between 49.5s and 54.5s for plasma pulse 34355 is approximately 50%.

For both plasma pulses 35405 and 34355 the population code underestimates the drop in the branching ratio with respect to the experimental data. The discrepancies between the results are most probably due to inaccuracies with the complex plasma modelling procedure. In Chapter 5 we discussed how the neutral and ion densities, predicted by EDGE2D, needed to be multiplied by a correction factor for certain time slices of these plasma pulses. This correction factor carries a substantial error which is extremely hard to determine, as can be seen by from Figure 5.26a(ii) and Figure 5.28a(ii). An estimate of the error to the densities used in the population code for the later time slices of plasma pulse 35405 and 34355 is 50%. Since the optical depth is directly proportional to the neutral density, it would also be
altered by 50%. For plasma pulses 35405 and 34355 this would give the drop in the branching ratios an approximate range of (36±15)% and (22±9)%. These error bars, combined with the errors due to averaging techniques and the diagnostic errors, mean that the theoretical and experimental results are in reasonable agreement. Further discussion of the inaccuracies in this investigation and ways of reducing them are discussed in Section 6.4.2 and Section 6.4.4. Both the theoretical and experimental investigations strongly indicate that a significant level of absorption is occurring in the divertor region of the JET tokamak.

6.4.2 Future Improvements

Although the plasma modelling, by EDGE2D, is reasonable for these plasma pulses, there is room for improvement, which will reduce the error bars of the investigation quite considerably. Geometrical effects can play a role in distorting experimental results, an example of this is seen in the large oscillations observed in the line integrated signals, in Figure 5.20b, of the horizontal target pulse of 34355.

The method of line fitting for these pulses could also be improved. The method employed to fit the emission line profiles at present works on the basis of identifying the line centre and integrating over a range of ±5 pixels from the line centre. Line merging of Lyβ with OVI occurs at certain times in the pulses which poses a problem. Fortunately, because this line merging does not occur throughout the entire pulse but only in a few random places, it does not affect the overall change in the branching ratio. However, it does increase the magnitude of oscillations at the times when merging occurs, which makes it difficult to determine the exact value of the branching ratio at this point.

Figure 6.3 shows the Lyβ line profile for a time towards the beginning and the end of plasma pulse 35405. Both times are selected so that the position of the plasma, as it swept across the target, is the same in both cases. This figure demonstrates that, not only is the line centre of the profile the same at both times but also that neither of the line profiles suffer from merging with the adjacent line. Hence, although the method employed to fit the line profiles could be improved so that it eliminates the effects of line merging and shifting of the line centre throughout the duration of the pulse, it is still a reasonable method to employ. The flaws that occur during the process of line fitting do not provide an explanation or contribute to the drop in branching ratio that is observed for the plasma pulses under investigation.
Figure 6.3. Ly$\beta$ line profiles for plasma pulse 35405 at times (a) 51.5s and (b) 55.5s, both taken at times where the plasma is in the same central position of a plasma sweep.
6.4.3 Validity of the ‘Single-Slab’ Model

In Section 6.2, it was described how a very basic and simplistic ‘single-slab’ plasma model has been developed to imitate a divertor plasma. This model is used because the plasmas produced in tokamaks vary widely and so a generalised guide to how changing parameters alter the opacity of a plasma is an excellent utility to have. The validity of this model will now be tested against the more accurate model developed using plasma parameter information from EDGE2D for pulse 35405 and 34355.

The Lyβ/Dα branching ratio values provided, in Section 6.2, for average parameters similar to those of plasma pulse 35405 at time 56.0s are 5.0 and 5.2, depending on the parameter being altered. In the previous section, the branching ratio for this pulse at the same time, given by the population code using parameters derived from the EDGE2D modelling method, is 5.04. It is clear that these results are in good agreement.

When comparing average parameter data for pulse 34355 at time 54.5s, with that of the ‘single-slab’ model, the two branching ratio values obtained are 6.0 and 5.8. The branching ratio value, for this pulse at time 54.5s, given by the population code, using the EDGE2D parameters, is 6.22. These results are in reasonable agreement.

Hence, it can be concluded that this simple ‘single-slab’ method is reasonably reliable when used to make a quick indication of the approximate level of opacity. However, when making a more detailed analysis, the more accurate plasma modelling method should be used. Although the ‘single-slab’ model and the detailed ‘slab-stack’ model produce similar results for the parameters being studied in this investigation, other parameter profiles may vary dramatically in the JET tokamak. Therefore it may not be possible to represent these parameters correctly using the ‘single-slab’ model.

6.4.4 Summary of Results

The main pursuit of this thesis is to determine whether a significant amount of line absorption is occurring in the JET divertor plasma. Typically, if approximately 20% of a particular line is being absorbed, this is considered as a significant level of opacity, as it is something which could be measured using a branching ratio technique.
The minimum drop in the level of line absorption predicted is 22%, which is the theoretical result produced for pulse 34355, when accounting for Zeeman splitting. The experimental branching ratio detected using the VUV spectrometer and the CCD camera for pulse 34355 and pulse 35405 shows a drop in the branching ratio of 50%. This amount of line absorption would be classed, not only as significant in terms of providing a useful diagnostic, but also as optically thick.

The average values of Lyβ optical thickness for plasma pulses 35405 at time 56.0s and 34355 at time 54.5s, according to the population without the Zeeman effect modifications, are approximately 0.31 and 0.25, respectively. It was described in Section 5.2 that although it is easier and more accurate to test for opacity using a branching ratio of Lyβ/\(\text{D}\alpha\), the line most susceptible to absorption is in fact Lyα. The actual value for the average optical thickness of Lyα for plasma pulses 35405 at time 56.0s and 34355 at time 54.5s are approximately 1.3 and 1.0, again without the Zeeman effect modifications. When the effects of Zeeman splitting are incorporated into the population code, the average Lyα optical thickness for plasma pulses 35405 at 56.0s and 34355 at time 54.5s are 1.26 and 0.93, respectively. Hence both plasma pulses 34355 can be defined as balancing on the border of being significantly optically thick to Lyα. It should be mentioned, however, that the modifications to account for Zeeman splitting were performed on the Lyβ line. If these modifications accounted for the Lyα line we would expect the values of Lyα optical thickness to be slightly smaller.

The values of optical thickness are obtained from the same theoretical process as the branching ratio values. Although in reasonable agreement, the drop in branching ratio values, for the pulses under investigation, are underestimated with respect to the diagnostic measurements. It follows from this that the values of optical thickness given for these pulses are also possibly underestimated. The possible causes of these inaccuracies could be from either the experimental or theoretical investigation, or more likely an accumulative effect from both.

The main area of concern in the theoretical investigation is the accuracy of the method used to model the plasmas. The problems that arise when using a transport code to model such a complicated environment, as that of the detached divertor plasma, are highlighted in Chapter 5. Here, it is mentioned that the levels of Dα emission are often underestimated,
which implies inaccuracies in the predicted values of the neutral deuterium and ion densities. The techniques used to compensate for these discrepancies in the modelling process are also outlined in Chapter 5.

The process of averaging the plasma parameters in the JET divertor region to form a 'slab-stack' model also creates the possibility of inaccuracies. The branching ratio values obtained are calculated along a particular line of sight in the 'slab-stack' model. This line of sight is meant to represent the path, along which, the spectrometers look. It is highly likely that at many points along this line the spectrometers look through regions where the independent plasma densities differ from that of the average slab densities of the 'slab-slack' model, which depend on the surrounding density values. This discrepancy can distort the results. The only way to reduce this problem is to create a model with shorter slabs which are not only stacked up into a column but are also side by side in rows. This however, did not seem a necessary step to take, considering that the plasma density and temperature profiles seem reasonably constant along the magnetic field lines, shown in Figure 4.11, therefore displaying slab-like qualities, as described in Chapter 4.

The experimental investigation suffers from relatively small inaccuracies that could only cause a heavy data distortion accumulatively. The main problems that arise are mostly due to geometry effects, such as comparing data from diagnostics with different viewing cones. However, it is shown in Chapter 5 that these effects do not explain the drop in the observed branching ratio that is displayed in the results. The method employed to fit the spectral line also allows for inaccuracies, due to line merging and shifts in the spectral line centre as the plasma is swept. However, the line merging problem is infrequent and the plasma sweeping effect occurs many times throughout the duration of a plasma pulse and hence does not explain the observed steady drop in the branching ratio. Another source of error, in the experimental investigation, is the accuracy of the instruments used to measure data. These values are given, along with a description of the diagnostics in Chapter 5.

Despite inaccuracies in some of the results produced, one thing is clear. That is, for both plasma pulses 35405 and 34355 the experimental and theoretical investigation support the conclusion that the levels of Lyβ absorption occurring are certainly significant. Also, the Lyα line has values of optical thickness which indicate that the divertor plasmas of pulse 35405 at time 56.0s and pulse 34355 at 54.5s can be considered to be optically thick.
6.5 The Effects of Opacity on the Ionisation and Power Balance of a Plasma

In this thesis so far, the emphasis of the investigation has been in determining whether significant levels of line absorption are occurring in the JET divertor. The results, given in the previous section, suggest that the level of opacity for some of the plasma pulses is significant. The next stage of the investigation deals with the effects of opacity on both the ionisation and the power balance of the plasma.

6.5.1 The Ionisation Balance Investigation

In order to examine the effect of opacity on the ionisation of the plasma, it is obvious that the population code needs to generate its own values of the ratio of neutral deuterium to ions. It was described in Section 3.3 how this value has been set in order for the population code to be consistent with the plasma models provided by EDGE2D. In order to do this the section of the code, which iterates to produce the ratio of neutrals to ions, was suppressed. This suppression is now reversed to allow the code to iterate onto a value for the neutral to ion ratio that is consistent with the input parameters.

A problem encountered when using the population code for the ionisation balance investigation is that the code does not account for neutral and ion flows in the plasma or charge exchange. After studying raw data of the excitation and recombination rates for plasma pulses 35405 and 34355, it has been found that for the early high temperature, low density time slices of the pulse the plasma is less ionised than predicted by the population code, presumably due to neutral flows into the divertor region. Conversely, for the later, high density, low temperature time slices the plasma is more ionised than predicted by the population code, presumably due to ion flows. To obtain a correct ionisation balance calculation, it is necessary that the population code incorporates terms which allow for ion and neutral flows.

This problem has been dealt with by calculating correction factors to the ionisation and recombination rates to compensate for the neutral and ion flows, respectively. The value of the correction factor is calculated in order to provide a selected value for the ratio of neutrals...
to ions, for a completely optically thin plasma, which is consistent with the values found in the divertor.

The way in which the correction factors are incorporated into the ionisation and recombination rates in Equation 3.21 is now described. Assuming that there is an ionisation balance, then $\dot{N}(1) = 0$ and Equation 3.21 becomes

$$\frac{N(1)}{N(X^{\#\#})} = \frac{\alpha_{cr}}{S_{cr}} = f_i$$

(6.1)

where $f_i$ is the initial fraction of neutrals to ions for the provided parameters, before compensation for ion and neutral flows. The selected value for the fraction of neutrals to ions, for an optically thin plasma, is represented by $f$. To obtain this value for the ratio, correction factors to the ionisation and recombination rates are calculated. For a low temperature the population code neglects ion flows, which can be incorporated by enhancing the ionisation rate. Hence, the ionisation rate needs altering by an amount $\Delta S_{cr}$, to produce the required fraction of neutrals to ions

$$f = \frac{\alpha_{cr}}{S_{cr} + \Delta S_{cr}} = \frac{\alpha_{cr}/S_{cr}}{1 + \Delta S_{cr}/S_{cr}} = \frac{f_i}{1 + \Delta S_{cr}/S_{cr}}$$

(6.2)

For high temperatures the population code neglects neutral flows, which can be incorporated by increasing the recombination rate. The recombination rate needs to be altered by an amount $\Delta \alpha_{cr}$, to produce $f$, where

$$f = \frac{\alpha_{cr} + \Delta \alpha_{cr}}{S_{cr}} = f_i + \frac{\Delta \alpha_{cr}}{S_{cr}}$$

(6.3)

The necessary correction factors, $\Delta S_{cr}$ or $\Delta \alpha_{cr}$, are then fed into the code along with the rest of the input data, and the code iterates normally to provide values of the neutral to ion ratio.

To examine the effects of increasing opacity, a simple single slab model is used with a finite height but an infinite width and breadth. The initial slab height is set to an extremely low value of $2 \times 10^3 \text{cm}$ in order to represent a plasma of very low optical thickness. A value for the ratio of neutrals to ions is fed into the code and the code calculates the correction factor
to either the ionisation or recombination rate so that after iteration, the ratio of neutrals to ions is the same as that chosen. In basic terms the ratio of neutrals to ions is set for the optically thin plasma. Then the code is run for increasing values of slab thickness up to \(2 \times 10^6\) cm, to represent a range of optical thicknesses, whilst keeping all other input parameters the same, including the correction factor for the ionisation and recombination rates. For each value of the slab thickness the ratio of neutral to ion populations is generated and this can be seen plotted as a function of Ly\(\alpha\) optical thickness in Figure 6.3 for the first three energy states of neutral deuterium. Here the populations of the first three energy levels are represented as \(n_1\), \(n_2\) and \(n_3\), respectively and the parameters are selected to roughly represent the parameters of a high density, low temperature JET divertor plasma. Hence all temperatures are taken to be 1eV, the electron density is set to \(1 \times 10^{20}\) m\(^{-3}\) and the initial value of the fraction of neutrals to ions for a completely optically thin plasma is set to \(1 \times 10^{-1}\).

\[
T = 1\text{ eV}, n_e = 1 \times 10^{20}\text{ m}^{-3}, f = 1 \times 10^{-1}
\]

![Graph showing ratios of energy level populations](image)

**Figure 6.4.** Ratios of the first three energy level populations to the ion population versus log(Ly\(\alpha\) optical thickness) for a plasma with a temperature of 1eV, an electron density of \(1 \times 10^{20}\) m\(^{-3}\) and a neutral deuterium to deuterium ion ratio of \(1 \times 10^{-1}\).

Figure 6.4 shows that, although most of the neutral population resides in the ground state, the population in the third energy state is greater than that of the second for an optically thin plasma. This is surprising, since the populations usually decrease with increasing energy.
levels. However, at low temperatures populations are governed by radiative decays and population inversions can occur below the collision limit. At higher temperatures, collisional excitation from \( n=1 \) dominates which causes the population to fall as \( n \) increases.

This population inversion is also enhanced because, at low temperatures, the collisions that do occur do not produce enough energy to collisionally excite ground state electrons to level \( n=2 \), but do produce enough energy to collisionally excite electrons from level \( n=2 \) to level \( n=3 \). Hence it is possible that the population of energy level \( n=3 \) can be greater than that of level \( n=2 \) in these circumstances.

At a Ly\( \alpha \) optical thickness of unity, when the plasma begins to go optically thick, there is a drop in \( n=1 \). This drop in the ground state population is due to the fact that Lyman photons are becoming more and more difficult to produce, effectively, with increasing optical thickness. Consequently, the population of the \( n=2 \) state rises, from where excitation and ionisation can occur more readily.

In terms of an escape probability model, the \( n=2 \) to \( n=1 \) escape probability decreases and therefore so does the radiative decay rate from level \( n=2 \), causing an increase in the population of level \( n=2 \). This increase enhances the overall rate of collisional ionisation since the ionisation rate from \( n=2 \) is much greater than that from \( n=1 \). The enhanced ionisation from \( n=2 \) causes the total neutral fraction to fall as shown in the behaviour of \( n/n^+ \) in Figure 6.4.

It can be seen from Figure 6.4 that although there is initially a steady increase in the second level population, this eventually flattens off with increasing optical thickness. As the Ly\( \alpha \) optical thickness rises, the escape probability falls and consequently the rate of radiative transitions from level \( n=2 \) to the ground state also falls until collisional processes from level \( n=2 \) become dominant. From this point, the rise in the second level population gets less until it completely stops.

The increase in the optical thickness has the same effect on the population of level \( n=3 \), as it has on level \( n=2 \), but to a lesser extent. Consequently a slight increase occurs in the population of level \( n=3 \) as the plasma starts to go optically thick.

Figure 6.4 shows that, despite a slight compensatory increase in the populations of energy levels \( n=2 \) and \( n=3 \), the large drop in the population of level \( n=1 \) with respect to the ion
population means that the ratio of neutrals to ions drops by a factor of 10, approximately, when going from an optically thin to an optically thick plasma. Hence increasing opacity can effect the ionisation balance of the plasma.

This implies that, not considering for the moment the radiative effects in the plasma modelling process of EDGE2D, the predicted values of neutral density could quite possibly be incorrect for opaque plasmas. It should be mentioned that this is only a tentative conclusion because the plasma transport will change as it is coupled to the neutral sources. A proper treatment requires the fluid code to be coupled with the radiation transport in the plasma.

The high density, low temperature plasmas in the JET tokamak, similar to those examined in this thesis, typically have a Lyα optical thickness of approximately 1.0. It can be seen in Figure 6.4 that for this optical thickness there should be very little effect on the ionisation balance of the plasma. However, this optical thickness is at the maximum value it can reach before the ionisation balance of the plasma is altered. Hence, any plasma produced with higher densities and lower temperatures than the ones examined in this thesis will display a change in the ionisation balance.

The effect of the increase in opacity on the populations of the neutral atom is also investigated for three varying input parameters. The first test examines the effect of changing the neutral deuterium density on the ionisation balance. The values for the neutral deuterium density, selected for investigation, are $1 \times 10^{18} \text{m}^{-3}$ (Figure 6.5a) and $5 \times 10^{19} \text{m}^{-3}$ (Figure 6.5b). In both cases the temperature and electron density are at 1eV and $1 \times 10^{20} \text{m}^{-3}$, the same as in Figure 6.4.

Figure 6.5 shows that increasing the neutral density affects the magnitude of $n_i/n_+$, which is a direct result of increasing the neutral density in the ground state. However the change in this value with increasing optical thickness remains a factor 10, approximately. The ratio of neutrals in levels $n=2$ and $n=3$ with the ion density remains the same because, at low temperatures, there is not enough energy available to couple ground state electrons to higher energy levels by collisional excitation. Hence, the energy levels are populated by recombination into higher levels. However, the recombination rate is fixed throughout. A change in the population of the higher energy states would be observed, therefore, only for higher temperatures enabling collisional excitation from the ground state to occur.
A similar investigation is performed to look at the effect varying electron and ion densities on the ionisation balance of the plasma (Figure 6.6). This is done for electron and ion densities of $5 \times 10^{19} \text{ m}^{-3}$ (Figure 6.6a) and $5 \times 10^{20} \text{ m}^{-3}$ (Figure 6.6b). The neutral deuterium density is selected to maintain a fraction of the neutral to ion ratio of $1 \times 10^4$, for a completely optically thin plasma, and the temperature is 1eV.

Figure 6.6 shows that increasing the electron density doesn’t affect the ratio $n_i/n_e$. However, the overall magnitude of $n_i/n_e$ and $n_j/n_e$ are greater for higher electron and ion densities, especially at lower values of optical thickness. This is because, at higher electron and ion densities the amount of collisional excitation and recombination increases, therefore even if the energy available in a low temperature plasma is not great enough to excite ground state electrons to higher energy levels, then at least recombination into higher levels is possible. As the optical thickness increases, it becomes possible to achieve ionisation from, not only the higher energy levels, but also from the second and third. This causes the flattening out of the rise in these level populations, observed in Figure 6.6, and is the reason why the change in the populations of levels $n=2$ and $n=3$, with increasing electron and ion density, is greater for the optically thin plasmas.

The final parameter being tested for its effect on the ionisation balance is temperature (Figure 6.7). For the same electron density and neutral deuterium density used in Figure 6.4, the effect of varying the temperature to 3eV (Figure 6.7a) and 5eV (Figure 6.7b) is examined. The first and most noticeable change to be observed with increasing temperature is that the population of level $n=2$ exceeds that of level $n=3$. This is because at higher temperatures there is enough energy to couple the ground state to the second energy state, which increases its population. Since the majority of electrons reside in the ground state, it means that it is possible to increase the population of the second energy level above that of the third. However, there is an increase in the third energy level, albeit smaller than that of the second, with increasing temperature.
Figure 6.5. Ratios of the first three energy level populations to the ion population versus log(Lyα optical thickness) for a plasma with a temperature of 1 eV, an electron density of $1 \times 10^{20} \text{m}^{-3}$, and a neutral deuterium to deuterium ion ratio of (a) $1 \times 10^{-2}$ and (b) $5 \times 10^{-1}$. 
Figure 6.6. Ratios of the first three energy level populations to the ion population versus log(Lyα optical thickness) for a plasma with a temperature of 1 eV, a neutral deuterium to deuterium ion ratio of $1 \times 10^4$ and an electron density of (a) $5 \times 10^{19} \text{m}^{-3}$ and (b) $5 \times 10^{20} \text{m}^{-3}$. 
Figure 6.7. Ratios of the first three energy level populations to the ion population versus log(Lyα optical thickness) for a plasma with an electron density of $1\times10^{20}$ m$^{-3}$ and a neutral deuterium to deuterium ion ratio of $1\times10^{11}$ and a temperature of (a) 3eV and (b) 5eV.
6.5.2 The Power Balance Investigation

The power balance investigation uses the same procedure as that used in the ionisation balance investigation, except the Dα emission and total emission are investigated as a function of the Lyα optical thickness. The emission occurs at the mid-plane of the slab and at right angles to the surface. The total emission includes all possible line transitions up to $n=10$. A test run using levels up to $n=30$ was undertaken to assess the error introduced by only accounting for a discrete number of levels. The difference in the two cases was less than 1%.

As before, all Balmer emissions are assumed to have an escape probability of unity, or in other words zero optical thickness. In a real plasma the Balmer series would eventually become optically thick also, causing photons to emerge primarily as Paα.

As in Figure 6.4, the power balance is investigated for an electron density of $1 \times 10^{20} \text{m}^{-3}$, a neutral deuterium density of $1 \times 10^{19} \text{m}^{-3}$ and a temperature of 1eV. The results can be seen in Figure 6.8.

![Figure 6.8](image)

**Figure 6.8.** The total emissivity and Dα emissivity, up to and including state $n=10$, versus log(Lyα optical thickness) for a plasma with a temperature of 1eV, an electron density of $1 \times 10^{20} \text{m}^{-3}$ and a neutral deuterium to deuterium ion ratio of $1 \times 10^{19}$. 

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Figure 6.8 shows that as the plasma becomes optically thick the total emissivity, and therefore power escaping from the plasma decreases but the amount of Dα contributing this, increases.

This increase in Dα emissivity is to be expected because, as the plasma becomes optically thick to the Lyman lines, photons can, effectively, no longer be produced by a transition to the ground state. Hence the population of the energy levels above the ground state increases, especially \( n=2 \) and \( n=3 \), and photons are produced by a transition to energy state \( n=2 \), or by a Balmer transition. Since the higher energy states are not as highly populated as the lower states, the dominant transition becomes that of Dα.

Transitions to \( n=2 \) do not produce as much energy as those to the ground state. Consequently, although there is no great difference in the number of photons being produced to that of an optically thin plasma, the energy of each photon is not as great. Hence the total emissivity of an optically thick plasma is less than that of an optically thin plasma. For the parameters examined in Figure 6.8, this drop in power is of an order of a factor of 10, approximately.

This additional power heats the plasma, or in other words as the plasma goes optically thick, a certain amount of control in cooling the plasma is lost. This is a great concern, especially in the divertor region because higher temperatures can cause greater impurity contamination from high velocity impacts with the target plates. It also means that, by not incorporating a radiative model in the EDGE2D code, the predicted temperatures and impurity levels are inaccurate for extreme cases of optical thickness.

The high density, low temperature plasmas in the JET tokamak have values of Lyα optical thickness of approximately 1. Hence, as can be seen from Figure 6.8 the power balance of the plasma remains unaltered. However, if the values of Lyα optical thickness achieved were any greater, the power balance would be affected by opacity.

The same test for the effect of varying parameters on the power balance is performed as in the ionisation balance investigation. For each corresponding case the same parameters are used. Figure 6.9 shows the effect of a changing neutral density on the power balance. It can be seen from this that the magnitude of both the total emissivity and the Dα emissivity remain unchanged as the neutral density is altered. This is because at low temperatures, such as 1eV, the \( n=2 \) population is determined by recombination, not excitation. Hence there is no
dependence on neutral density, in this situation, only on electron and ion density. However, for higher temperatures, where collisional excitation from \( n=1 \) determines the \( n=2 \) population, the reverse is true and the D\( \alpha \) and total emission should depend on the neutral density.

Similarly Figure 6.10 shows the effect of increasing electron density on the power balance of a plasma. This shows that the magnitude of both the total and D\( \alpha \) emissivity increases with the increasing electron density. This is because emissivity is proportional to the product of the electron and ion density, or in this case the electron density squared. It can be seen from Figure 6.10, that as the electron density is increased by a factor of 10, the total emissivity is increased by an approximate factor of 100, which complies with this rule.
Figure 6.9. The total emissivity and Dα emissivity, up to and including state $n=10$, versus log(Lyα optical thickness) for a plasma with a temperature of 1eV, an electron density of $1 \times 10^{20} \text{ m}^{-3}$ and a neutral deuterium to deuterium ion ratio of (a) $1 \times 10^{-2}$ and (b) $5 \times 10^{-1}$. 
Figure 6.10. The total emissivity, up to and including state $n=10$, and $\Delta \alpha$ emissivity versus log($\text{Ly} \alpha$ optical thickness) for a plasma with a temperature of 1 eV, a neutral deuterium to deuterium ion ratio of $1 \times 10^{-4}$ and an electron density of (a) $5 \times 10^{17}$ m$^{-3}$ and (b) $5 \times 10^{18}$ m$^{-3}$.
Figure 6.11. The total emissivity, up to and including state $n=10$, and $\Delta \alpha$ emissivity versus log(Ly\(\alpha\) optical thickness) for a plasma with an electron density of $1\times 10^{20}$ m\(^{-3}\), a neutral deuterium to deuterium ion ratio of $1\times 10^{1}$ and a temperature of (a) 3eV and (b) 5eV.
Figure 6.11 illustrates the change in emissivities with increasing temperature. Again, as the temperature increases the magnitude of both emissivities increase. This is because collisional processes (collisional excitation) increase and therefore more atomic processes are occurring at higher temperatures than at lower temperatures which leads to an overall increase in the level of emission.

The most interesting change that occurs with increasing temperature is the drop in the total emissivity that occurs when going from an optically thin to an optically thick plasma. Until now, the power balance investigation has shown that this drop in emissivity, with increasing optical thickness, is approximately a factor of 10. However, at 3eV and 5eV the drop in power with increasing optical thickness is a factor of 100, approximately. This is because, at higher temperatures collisional excitation, in addition to radiative recombination, populates $n=2$ which leads to an increase in Ly$\alpha$ emission. Since the drop in the total emission is due almost entirely to the absorption of Ly$\alpha$, it follows that a decrease in the production of Ly$\alpha$ leads to an increase in the drop of the total emissivity when going from an optically thin to an optically thick plasma.

This drop is basically the difference between the emissivity of Ly$\alpha$ for an optically thin plasma and the emissivity of D$\alpha$ for an optically thick plasma. This can be expressed as

$$n_2 \left( \tau_{L_{\alpha}} \rightarrow 0 \right) A_{21} h \nu_{21} - n_1 \left( \tau_{L_{\alpha}} \rightarrow \infty \right) A_{12} h \nu_{32}$$

(6.4)

where $A_{21} (4.699\times10^8 \text{s}^{-1})$ and $A_{12} (4.410\times10^7 \text{s}^{-1})$ are the Einstein coefficients for Ly$\alpha$ and D$\alpha$ transitions. It can be seen, when comparing Figure 6.4 and Figure 6.7b, that for plasmas which are almost completely optically thin, the population of $n=2$ is greater for higher temperatures than it is for low temperatures. Hence, the expression in Equation 6.4 becomes much larger with increasing temperature.
6.5.3 Summary of the Results to the Ionisation and Power Balance Investigations

The investigation into the effects of opacity on the ionisation and power balance of the plasma show that opacity does play a significant role in altering certain plasma parameters.

Plasmas which are completely optically thick can show a drop in the ratio of neutrals to ions of approximately an order of magnitude. This has the effect of decreasing the level of neutrals in the plasma which tends to neutralise the opacity effects. Hence a negative feedback mechanism occurs. The values of $n_i/n_e$ given in the figures incorporate this feedback effect, since an iterative procedure is used to determine the ionisation balance in the presence of absorption. At each iteration of the ionisation balance, the neutral fraction and hence opacity from the previous iteration are used. This procedure converges after a few iterations. By not including radiative processes when modelling the plasma, and therefore ignoring opacity effects, there is a danger of inaccurately predicting, not only the neutral particle source terms but also the energy loss terms which are derived from these sources.

In addition to this, the opacity also affects the power balance of the plasma. It has been described that, for plasma with large values of optical thickness, the drop in power emitted from the plasma can be as much as an order of magnitude, when compared with that of an optically thin plasma. This power stays in the plasma and reduces the cooling rate of the plasma leading to higher divertor temperatures which can also affect the levels of target erosion and impurity contamination. Hence, not including a radiative transport model in EDGE2D, means that inaccurate plasma parameters would be obtained for an optically thick plasma.

It has been shown that the plasmas displaying the greatest levels of opacity in JET, generally have a Lyα optical thickness of approximately unity. For this level of opacity the effects of opacity on the ionisation and power balance are small. However, any further increase in the Lyα optical thickness would produce a change to the ionisation and power balance.
6.6 Comparisons with Previous Research

This is a good point to reflect and compare the results to the investigation undertaken in this thesis with those of the previous research projects into this field, introduced in Chapter 1. In order to perform this comparison, a little more detail is needed of the codes used and the assumption made in these studies.

It was mentioned in Chapter 1 that Wan (Wan et al., 1995) uses a non-local thermodynamic equilibrium code, CRETIN, which uses a one-dimensional configuration, incorporating parameters outside the magnetic separatrix from a deuterium gas-puff shot in DIII-D. These parameters are scaled up for the ITER tokamak, which is approximately twice the size of the DIII-D tokamak.

CRETIN is a multi-dimensional code which accounts for the atomic processes of electron-ion and ion-ion collisions, photo-ionisation, photo-excitation and Auger processes. The rates for the inverse processes are calculated using balance equations. Radiative processes treated in the code are bound-bound, bound-free and free-free processes. CRETIN uses a diffusion model to follow the transport of neutral particles, which assumes the flux of neutrals re-entering the plasma from the boundary, via the process of recycling (described in Chapter 2), is proportional to the ion flux at the boundary. The neutral diffusion is determined by charge exchange and ionisation processes.

In Wan's study, a pure hydrogen plasma is assumed, its density and temperature profiles are fed into the code and used in the calculation of ion, atom and radiation distribution. These input profiles provide total hydrogen ion and neutral densities ranging along the SOL from $4 \times 10^{14}$ cm$^{-3}$ at the target plate to $8 \times 10^{13}$ cm$^{-3}$ near the core. The electron temperature at the target is taken as 2 eV with an unsteady increase to 40 eV next to the core.

Three approaches are used to study the effects of opacity on the ionisation and power balance of the divertor plasma. These utilise three different transport models which treat the situations where (i) the plasma is optically thin to Ly$\alpha$, (ii) a full treatment of the Ly$\alpha$ line by detailed line transfer is used, where the optical depth of the Ly$\alpha$ line is approximately 30 and (iii) a detailed line transfer treatment with many hydrogen lines up to $n=6$ level is assumed.
Figure 6.12 shows the spatial profiles of neutral and ion densities for the first two cases, the dashed and solid lines being used for cases (i) and (ii), respectively. In this figure x=0cm and x=40cm denote the positions of the target and the core, respectively. It should also be mentioned that the ionisation mean free path for neutrals is approximately 50cm. The shaded area depicts the ionisation front, where the electron temperature increases from 2eV to 20eV.

This figure shows that treating the Lyα line as being optically thick, causes an increase in the ion population. This is due to excitation of the ground state electrons, by the trapped Lyα in the cold region, and subsequent collisional ionisation, as described in Section 6.5.1. Thus the ionisation balance can be altered in this way.

The spatial distribution of the directional line fluxes for the three cases under investigation is shown in Figure 6.13, where the dashed and solid lines represent fluxes radiating towards the target and back to the core, respectively. The line flux can be defined as the power per unit area from a particular spectral line, in this case Lyα.
By comparing Figure 6.13b to 6.13a, it can be seen that by treating Lyα as optically thick, the line fluxes going towards the target are not only of lesser magnitude but also drop off dramatically in the cold region due to absorption of the Lyα line. When making a similar comparison for the line fluxes travelling towards the core, it can be seen that the overall magnitude of the flux is less for case (ii) than for case (i). It should be mentioned that the line fluxes are integrated from the target to the point of emission. Therefore, as length increases, the probability of absorption of the Lyα line also increases. This factor, along with the absorption due to high neutral densities, outweighs the emission from the high neutral densities. Hence, from x=4cm there is decrease in the line flux. By comparing case (ii) with case (iii) it can be seen that there are not a great deal of change and Wan therefore concludes that Lyα is the dominant line in a pure hydrogen plasma.
Wan demonstrates that high density, low temperature plasma parameters from a deuterium gas-puff shot in DIII-D, when scaled up for the ITER tokamak, produce plasmas which are optically thick to Ly$\alpha$ radiation. It is also shown that this has an effect on the ionisation and power balance of the plasma.

The discoveries made in this thesis agree with Wan’s conclusions. However, there are differences in the methods of research which will now be described. The conditions which are being modelled for this thesis are slightly different for those used in Wan’s study. The ratio of neutral hydrogen density to electron, or ion, density is approximately 0.1 in the JET divertor, in comparison to the ratio of order 2 at the target, considered in the above study. Electron temperatures and densities are of approximately the same order of magnitude.

Continuum absorption is not considered to play a large role in opacity effects in the JET divertor, due to low impurity levels, and is therefore neglected. Both studies, however, base their models on a pure hydrogen or deuterium plasma.

Another similarity between the two investigations is that they both model neutral transport, albeit by different methods. In Wan’s study, neutral transport is modelled within the code, CRETIN, whereas in the investigation in this thesis, profiles of neutral densities from a plasma model are fed directly into the population code. The codes used to model the plasma are described in Chapter 2. CRETIN uses a full radiative transfer approach to finding the populations and emissivities of the various atomic energy states of hydrogen in contrast to the simpler first order escape probability approach applied in the population code in this thesis.

There is a difference between the way that the line profiles are treated. Wan assumes a Voigt profile and in doing so allows for photon escape from the profile’s wings. We, on the other hand, assume the Doppler profile with escape only occurring from the core of the profile. In addition, we allow for Doppler shifting due to the bulk motion of the plasma and include velocity gradients throughout the plasma, therefore providing an accurate model of the dynamic plasma.

Wan’s study uses predicted profiles and simulated data similar to a deuterium gas-puffing shot in the DIII-D tokamak and scales the parameters to those of ITER, in order to predict the effects of opacity. In contrast, we model two real experimental JET plasma pulses to determine whether opacity effects are a real issue in JET and can thus make direct
comparisons with diagnostic data so as to validate the results. Although these detailed 'slab-stack' models of the plasma pulses are not used directly in the ionisation and power balance investigation, a better idea of the average parameters to use in the simpler 'single-slab' model is obtained via the detailed plasma modelling procedure of the JET plasma pulses.

A comparison will now be made with Behringer's (1997) research, introduced in Chapter 1. Behringer utilises the ADAS208 collisional-radiative model to calculate excitation, ionisation and recombination coefficients for hydrogen atoms under optically thin and optically thick plasma conditions. The Atomic Data Analysis Structure (ADAS) is a combination of atomic physics codes and data which are used to provide appropriate derived atomic data for diagnosis and modelling (Summers, 1994).

ADAS208 is a code which treats the optical thickness of a line by introducing an escape factor. This factor is calculated for the Axial Symmetric Divertor Experiment (ASDEX) Upgrade density limit plasma conditions, where ASDEX Upgrade is a tokamak experiment based in Germany. The escape factor is multiplied by the transition probabilities of an optically thin line to reduce this probability. This is the same approach used in the population code, described in Chapter 4. Similar assumptions apply when calculating this escape factor. These are that the calculated ionisation and recombination rate coefficients demonstrate the process of stepwise ionisation and recombination into excited levels, which are a function of Lyman line optical thickness.

The escape factor ($\Theta$) is determined by the relative difference of the emitted and the absorbed line radiation powers $E$ and $G$, respectively, such that

$$\Theta = \frac{E - G}{E} = 1 - \frac{G}{E}$$  \hspace{1cm} (6.5)

The absorbed power $G$ is proportional the spectral radiance $I_d(\lambda)$ and the local effective absorption coefficient $k(\lambda)$. Here, $k'$ represents the true absorption coefficient $k$ minus the coefficient for stimulated emission. The total emission and absorption power within a spectral line is obtained by integrating the spectral emission and absorption over the wavelength $\lambda$ and solid angle $\Omega$. In order to do this the spectral line profiles $P_d(\lambda)$ are required. The spectral absorption coefficient can be expressed in terms of the spectral line profile as given
\[ k(\lambda) = n_j \frac{g_i \lambda^4}{g_j} \frac{A_{ij}}{c} \frac{1}{8\pi} P_{\lambda}(\lambda) \]  

(6.6)

where \( g_i \) and \( g_j \) are the statistical weights and \( A_{ij} \) is the Einstein coefficient.

The line profiles are calculated according to the relevant line broadening mechanism in the plasma. Doppler broadening, according to the neutral temperature \( T_0 \) and the relative ion mass \( \mu \) of the emitters, is adopted in this case. Hence the absorption coefficient in the line centre can be written as

\[ k(0) = 1.08 \times 10^{-15} n_j \lambda f_{ji} \sqrt{\mu/T_0} \]  

(6.7)

where \( f_{ji} \) represents the absorption oscillator strength, which can be defined as

\[ f_{ji} = 1.5 \times 10^{-14} \lambda^2 \frac{g_i}{g_j} A_{ij} \]  

(6.8)

The emitted power \( E \), after the double integration over wavelength and solid angle, is simply the product of \( 4\pi \) and the line emission coefficient \( \varepsilon_\lambda \). Hence, by incorporating the newly calculated expressions for the absorbed and emitted power into Equation 6.5, the following expression for the escape factor is obtained

\[ \Theta = 1 - \frac{\int \int k'(\lambda) I_\lambda(\lambda, \Omega) d\lambda d\Omega}{4\pi \varepsilon_\lambda} \]  

(6.9)

The spectral radiance is calculated by the equation of radiative transfer, Equation 4.11, using the spectral emission coefficient \( \varepsilon_\lambda \) as a function of the plasma geometry. Assuming that the plasma extends from a position \( s=0 \) to \( s=a \), where \( I_\lambda=0 \), than the radiance can be expressed as

\[ \frac{dl_\lambda}{ds} = \varepsilon_\lambda - k' I_\lambda \to I_\lambda(0) = \int_0^a \varepsilon_\lambda(s) \exp \left[ -\int_0^s k'(s') ds' \right] ds \]  

(6.10)
The escape factor is ultimately obtained by integrating over the spectral line profile and over all directions. Hence a complete radiative transfer approach is utilised to obtain the escape factor. An assumption made in this calculation, however, is that stimulated emission is neglected, therefore enabling $k' = k$, which is a fair assumption for the Lyman lines in the VUV spectrum. Behringer also assumes the ground state density to be constant, rendering the absorption coefficient $k$ to be constant.

The geometric model Behringer uses is a cylinder of infinite length and of radius $R = 5\text{cm}$. In this, a parabolic spatial profile is used for the radial dependence of the line emission coefficients and the spectral radiance is then calculated by integration by parts.

The first task performed by the code is the calculation of escape factors for hydrogen and for conditions relevant to ASDEX Upgrade divertor plasmas. The parameters used for this are a neutral temperature $T_0$ of 1eV and a neutral ground state density $n_0$ of $5\times10^{13}\text{cm}^{-3}$. The transition probabilities and wavelengths of the hydrogen lines are obtained from files in the ADAS database. The resulting escape factors are presented as a function of the optical depth of the line centre. This is a product of the line centre absorption coefficient $k(0)$ and the cylinder radius $R$. The results are presented in Figure 6.14, where the vertical dashed lines show the resulting line centre optical depths of the hydrogen Lyman lines and their corresponding escape factors, in these conditions.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure6.14}
\caption{The escape factor $\xi$ on the axis of a plasma cylinder of radius $R$ versus the optical depth in the line centre $k(0)R$. The vertical dashed lines represent the optical depths of the first four Lyman lines. Figure from Behringer (1997).}
\end{figure}
Ionisation and recombination rate coefficients, $S$ and $\alpha$ respectively, are used to investigate the effects of opacity on the ionisation and power balance of the plasma. The hydrogen ionisation and recombination rate coefficients are calculated by the collisional radiative code ADAS208.

Figure 6.15 shows the hydrogen ionisation rate coefficients $S$, calculated by ADAS208, as a function of electron density. This is done for electron temperatures of 1eV and 2eV and for an optically thin plasma and two varying degrees of optically thick plasma. The ground state neutral hydrogen density of the two optically thick plasmas are $1\times10^{13}\text{cm}^{-3}$ and $5\times10^{13}\text{cm}^{-3}$. The processes of radiative and three-body recombination are incorporated in the calculation of these results.

Figure 6.16 shows the recombination rate coefficients $\alpha$, as a function of electron density. The same conditions are used as those in Figure 6.15.

![Figure 6.15. Ionisation rate coefficients $S$ for hydrogen versus electron density, as calculated by ADAS208. An optically thin case and two increasingly optically thick cases with $n_0=1\times10^{13}\text{cm}^{-3}$ and $n_0=5\times10^{13}\text{cm}^{-3}$ are represented. Figure from Behringer (1997).](image-url)
ADAS208 calculates the collisional-radiative population coefficients for excited states of a specific atom or ion. There are four contributions to the population, these being electron impact excitation from the ground state and metastable levels, recombination of free electrons and ions in their ground or metastable states, charge exchange of the ions in their ground or metastable state with neutral hydrogen (or helium, if required) and ionisation of ions in the ground or metastable states. These processes are then incorporated to form an equilibrium equation, which is a similar approach to that used in this thesis. However, charge exchange is not accounted for in the atomic processes incorporated in the population code because it is a resonant process which only occurs, in a significant capacity, between identical levels of deuterium atoms. Therefore the overall ion and neutral density remains unchanged.

The solutions to the equilibrium equations provide values for the populations of the excited levels, which can then be used to calculate line emissivities.

Figure 6.17 shows the photon emissivity coefficient for Dα versus the electron temperature from excitation and recombination, respectively. This is done for two cases, one where radiative recombination is considered in addition to three-body recombination and one where only three-body recombination is accounted for. The electron density is $10^{14}\text{cm}^{-3}$ and the neutral density of the optically thick plasma is $5\times10^{13}\text{cm}^{-3}$.
Figure 6.17. Photon emissivity coefficient for $D\alpha$ as a function of $T_e$ and for optically thin and optically thick ($n_0=5\times10^{13}\text{cm}^{-3}$) cases with $n_e=10^{14}\text{cm}^{-3}$. Contributions from (a) excitation and (b) recombination. Figure from Behringer (1997).
Although the way in which Behringer’s results are produced and presented are different to the approach used in this thesis, as will be described shortly, he concludes from these results that opacity does effect the ionisation balance and the power balance of the plasma. This is in agreement with the findings made in the ionisation and power balance investigations performed in this thesis.

Although these different investigative methods agree that opacity does effect the ionisation and power balance of the plasma, it is not immediately clear that they agree on the magnitude of this effect for certain plasma conditions. Making a direct comparison between the two methods is complicated task, since there are many differences between the two approaches, which will be described shortly.

In order to attempt a simple comparison between the two sets of results, a comparison of the optical depths is made using Figure 6.14. From this it can be seen that for a plasma with a temperature of 1eV, a neutral density of $5 \times 10^{13}$ cm$^{-3}$, the optical depth in the line centre of the Ly$\beta$ line for a cylinder of radius 5cm is approximately 4. The number of neutrals that a photon could encounter on its way out of the cylinder is just the product of the neutral density and the radius, which is $2.5 \times 10^{14}$ cm$^{-2}$. In the ‘slab-stack’ model of height 20cm, used in this thesis, this value would correspond to a neutral density of $2.5 \times 10^{14}$ cm$^{-2}$/20cm, or $1.25 \times 10^{13}$ cm$^{-3}$. Assuming that the electron density is $1.25 \times 10^{14}$ cm$^{-3}$, it can be seen from Figure 6.1e, that in these conditions the branching ratio of Ly$\beta$/D$\alpha$ is approximately 2. It would take 3 to 4 scattering events to produce such a branching ratio, since the probability of a D$\alpha$ photon being emitted is 1/9 compared to a Ly$\beta$ photon which has a 8/9 probability of being emitted for a particular scattering event. The optical depth is directly proportional to the number of scatterings and it can actually be assumed that the optical depth of the Ly$\beta$ line has a value of approximately 3 to 4, in the given conditions. This agrees with the value obtained from Figure 6.14. Hence the optical depths that the population code produces crudely agree with those of Behringer.

It is slightly more complicated task to make comparisons between the two ionisation balance investigations. In this thesis, the ionisation balance is performed by holding all parameters constant, except the neutral deuterium density and the plasma dimensions. This provides results which show the change in the neutral to ion ratio as a function of optical depth. In Behringer’s work, the neutral hydrogen and plasma dimensions are held constant whilst the
electron density is varied and the code, ADAS208, produces ionisation and recombination rates.

The difference in graphical representation of results also poses a problem when trying to make a comparison between the two ionisation balance investigations. Neutral populations to ion ratios are presented in this thesis, rather than ionisation and recombination rates, as an indication to the effects of opacity on the ionisation balance of the plasma.

In order to make the closest comparison possible between the two investigative methods, the differences between the geometric models used to define the boundaries of the plasma, must be examined. As mentioned, Behringer uses a cylindrical model, as opposed to the slab model used in the population code. It is, however, possible to simulate the results of the cylinder model within the slab model. This can be done by calculating a value for the height of the slab so that the two geometric models produce the same escape probability for a photon emitted at the centre of each model. A detailed discussion of this calculation is given in Appendix B.

The Lyα mean optical depth for a photon emitted on the axis of a cylinder of plasma with a radius of 5cm, a temperature of 1eV and a neutral hydrogen density of $5 \times 10^{13} \text{cm}^{-3}$ can be calculated from an expression which combines Equations 4.75, 4.80 and 4.82 to give

$$\tau = \frac{\pi e^2}{mc} \frac{N l f_{ij}}{\Delta V_D}$$

(6.11)

In cgs units $(\pi e^2/mc)=0.02654$. The Doppler width for a Lyα profile, with a central line frequency of $2.467 \times 10^{15} \text{Hz}$, a temperature of 1eV, or 11600K can be calculated for a deuterium atom from Equation 4.54. This gives a Doppler width of $8.052 \times 10^{10} \text{Hz}$. The oscillator strength for this transition is 0.416. Substituting these values, in addition to the neutral hydrogen density and cylinder radius, into Equation 6.11 gives a Lyα mean optical depth of 34.3. It can be seen from Table B.1 that for this mean optical depth the ratio of $R$ to $z$ is 1.06. Hence, under the conditions given, a slab with a half height of 4.71cm is required to produce the same mean escape probability of a cylinder with a radius of 5cm.

In an attempt to simulate the conditions which produced the ionisation and recombination rate coefficients in Figures 6.15 and 6.16 and for an electron density of $5 \times 10^{14} \text{cm}^{-3}$, the
population code is run with a total slab height of 9.42 cm, a temperature of 1 eV, an electron density of $5 \times 10^{14} \text{cm}^{-3}$ and an initial neutral density of $5 \times 10^{13} \text{cm}^{-3}$, before iterating. The correction terms which allow for ion and neutral flows in the population code are neglected.

For these conditions, the population code gives values for the ionisation and recombination rate coefficients of $9.83 \times 10^{-13} \text{cm}^3 \text{s}^{-1}$ and $7.45 \times 10^{-13} \text{cm}^3 \text{s}^{-1}$, respectively. These give a ratio for the recombination to ionisation rate coefficient of approximately 0.76.

For an optically thin plasma the population code gives values for the ionisation and recombination rate coefficients of $1.26 \times 10^{-13} \text{cm}^3 \text{s}^{-1}$ and $2.95 \times 10^{-12} \text{cm}^3 \text{s}^{-1}$, respectively. These give a ratio for the recombination to ionisation rate coefficient of approximately 23.4.

Figures 6.15 and 6.16 show that for electron densities of approximately $5 \times 10^{14} \text{cm}^{-3}$ and for a temperature of 1 eV, and neutral hydrogen density of $5 \times 10^{13} \text{cm}^{-3}$, the ionisation and recombination rate coefficients are approximately $4.5 \times 10^{-13} \text{cm}^3 \text{s}^{-1}$ and $1.3 \times 10^{-12} \text{cm}^3 \text{s}^{-1}$, respectively. These produce a ratio of the recombination rate coefficient to the ionisation rate coefficient of approximately 2.89.

For the optically thin plasma, it can be seen, from Figures 6.15 and 6.16, that for an optically thin plasma the ionisation and recombination rate coefficients are approximately $1 \times 10^{-13} \text{cm}^3 \text{s}^{-1}$ and $2.7 \times 10^{-12} \text{cm}^3 \text{s}^{-1}$, respectively. These produce a ratio of the recombination rate coefficient to the ionisation rate coefficient of approximately 27.

It is clear from these results that the two models are in reasonable agreement on the ratio of the recombination to ionisation rate coefficients for the optically thin plasma. This is not true, however, for the optically thick case. Here Behringer shows that the opacity of Ly$\alpha$ leads to an increase in the degree of ionisation by approximately an order of magnitude. The population code, however, shows that for identical conditions this opacity effect leads to an increase in the ionisation by approximately a factor of 30. Hence, the population code predicts greater opacity effects than the method employed by Behringer. To get a better idea of the difference in the predicted opacity levels, the slab, used in the population code, would have to be reduced to a height of approximately 2 cm to achieve the same ratio of the recombination to ionisation rate coefficients, that is obtained by Behringer.
Many factors could influence the different results produced by the two investigations. The first is obviously the geometry. Although, an attempt has been made to reduce the differences between the two models, discrepancies still exist.

Another possible difference is that Behringer’s radiative transfer model could be producing larger escape factors, for a particular optical depth, than the population code. To investigate this possibility another discrepancy between Behringer’s method and the population code needs to be highlighted. This is that different types of optical thickness are used, which doesn’t affect the ionisation balance comparisons but is relevant when comparing values of the optical depth. In the population code, the optical depth calculated is the mean value, which can be expressed in terms of the mean cross-section such that

\[ \bar{T} = lN\bar{\sigma} \]  \hspace{1cm} (6.12)

where

\[ \bar{\sigma} = \frac{\sigma_{tot}}{\Delta \nu_D} \]  \hspace{1cm} (6.13)

and where \( \sigma_{tot} \) is the total cross-section which can be defined as

\[ \sigma_{tot} = \frac{\pi e^2}{mc} f_{ij} \]  \hspace{1cm} (6.14)

When Behringer refers to the optical depth, it is the line centre optical depth which has a cross-section \( \sigma_0 \). This can be expressed as

\[ \sigma_0 = \frac{\sigma_{tot}}{\Delta \nu_D \sqrt{\pi}} \]  \hspace{1cm} (6.15)

Hence, it can be seen from Equations 6.13 and 6.15 that

\[ \sigma_0 = \frac{\bar{\sigma}}{\sqrt{\pi}} \]  \hspace{1cm} (6.16)
Hence the optical depths, which are directly proportional to cross-section, that Behringer uses should differ from those calculated in the population code by a factor of 1.77. It should be mentioned, however, that the values for the optical depth of the cylinder given in Table B.1 are mean optical depths. These are not provided by ADAS208, they are obtained by hand calculations using the expression in Equations 4.54, 4.81 and 4.82.

In Figure 6.14, it can be seen that for a plasma with a neutral density of \(5 \times 10^{13} \text{cm}^{-3}\) and a temperature of 1eV, the Ly\(\alpha\) line has a line centre optical thickness of approximately 20. Hence, according to the above argument, the equivalent mean optical thickness should have a value of \(1.77 \times 20\). Hence when the population code, using the same input parameters, produces a Ly\(\alpha\) optical thickness of 35.4, the escape probability should be the same as that given in Figure 6.14. In Figure 6.14, the escape factors for the Ly\(\alpha\) and Ly\(\beta\) lines are approximately 0.04 and 0.40, respectively. The population code produces values for the Ly\(\alpha\) and Ly\(\beta\) escape factor of approximately 0.008 and 0.08, respectively. These values are exactly five times less than Behringer’s escape factor. This difference could explain the fact that the population code predicts larger opacity induced changes to the ionisation balance than Behringer’s approach.

The final comparison that needs to be made between the two investigations is the power balance. It can be seen from Figure 6.17 that the D\(\alpha\) radiation is enhanced by almost an order of magnitude by the opacity of Ly\(\alpha\).

Figures 6.9, 6.10 and 6.11 show the power balance results produced using the population code. These figures show an increase in the D\(\alpha\) radiation with increasing Ly\(\alpha\) optical depth. Hence, there is an agreement with Behringer’s work that the opacity of Ly\(\alpha\) effects causes an increase in D\(\alpha\) emission. However, the magnitude of this increase appears to be less than that calculated by Behringer, according the results of this thesis.

It is difficult to compare the extent of the increase in D\(\alpha\) emission with optical depth because Behringer plots the D\(\alpha\) photon emissivity, the number of photons coming out of a particular volume per second, as a function of temperature in Figure 6.17. In contrast, Figures 6.9, 6.10 and 6.11 plot the D\(\alpha\) energy coming out of a particular volume per second as a function of the Ly\(\alpha\) optical thickness. In an attempt to simulate the conditions in Figure 6.17 the population code is run with a slab of height 9.42cm, an electron density of \(1 \times 10^{14} \text{cm}^{-3}\), a temperature of 1eV for a neutral density of \(5 \times 10^{13} \text{cm}^{-3}\), to represent the optically thick case,
and a neutral density of $5 \times 10^4 \text{ cm}^{-3}$ to represent an optically thin plasma. The increase in the emissivity of $D\alpha$ when going from the optically thin to the optically thick case is of the order $10^3$.

The increase in the level of $D\alpha$ emission when going from a plasma with a low optical thickness to a one with a high optical thickness is much greater according to the population code, than it is from Behringer’s study. Again, the population code predicts greater opacities than that of Behringer’s. Behringer states that ‘a parabolic spatial profile is used for the radial dependence of the line emission coefficient’. This could imply that the ionisation and recombination coefficients quoted by Behringer are averaged in some way over the cross-section of the cylindrical plasma. If so, points nearer the surface of the cylinder would have a greater escape probability than those on the axis. Behringer (1997) does not give enough information to finally resolve this question.

It has been high-lighted in this Chapter that geometry plays a major role in the production of opacity. It is interesting therefore to consider, the levels of opacity that will be achieved in the large tokamaks planned for the future. In Chapter 7, this issue is addressed, albeit briefly, giving a very short investigation into the levels of opacity that we might expect in ITER.
Chapter 7

Conclusions and The Future

7.1 Introduction

This final chapter serves more as a beginning than an end, for there is much work still to be done in discovering the influence of opacity in the world of tokamak fusion. This thesis acts merely as another step on this journey of discovery. Hence this chapter is devoted to not only summarising the most prominent results produced in this thesis, but also to exploring what can be done to improve this work in the future.

The first section of this chapter deals directly with the issue of future tokamaks by making an estimated calculation of the level of opacity that we may see in the next planned major tokamak, ITER.

7.2 Opacity in the ITER Tokamak

With the growing demand for larger tokamaks, the likelihood of achieving higher opacity plasmas is increasing. The next major tokamak project ITER (shown in Figure 7.1) is twice the size of the JET tokamak and it is planned to be built and fully functioning in about 25 years.
In order to determine the levels of opacity that we might expect in the divertor region of the ITER tokamak, knowledge of its geometry and the parameters of the plasma that will be produced are needed. Predictions of these parameters are provided for defined areas called cells which accumulatively form a grid for the cross-section of the tokamak. This is similar to the cell system to that used to form the grid in the JET tokamak, described in Chapter 2.

For each cell the neutral and electron densities and the electron, ion and neutral temperatures are available. This information comes from numerical studies of the expected performance of the ITER divertor (Kukushkin, 1987) and simulates a high density low temperature partially-detached plasma.

The data are averaged, according to the method used for the JET plasmas, and compiled to form the ‘slab-stack’ model. In order to perform the averaging process, knowledge of the area

Figure 7.1. Schematic diagram of the ITER tokamak cross-section. Figure from Poucet and Saji (1996).
of each cell is required. This information is not provided but is calculated using the co-
ordinates of the cell centres, which are supplied. These co-ordinates are given in terms of the
major radius and the height of the tokamak. An assumption is made to simplify the
calculation of the cell areas. This is to consider that the edges of a cell fall exactly half way
between the cell centre and that of the adjacent cells.

The base of the stack of slabs is taken to lie along the separatrix on the inner leg of the
divertor, as it does when modelling plasmas in the JET divertor. The length of the slabs is
60cm, which is exactly four times the length used for the JET plasmas, and runs from the
target up, parallel to the separatrix. This region is selected for analysis as it has the highest
values of neutral density, the peak occurs along the separatrix at approximately 30cm from
the target. The height of the stack of slabs is only 4cm, consisting of four 1cm slabs. This
height is a lot less than that used to represent JET plasmas because the inner leg of the ITER
tokamak is a more tapered with respect to the separatrix than that of JET. Hence, although the
ITER tokamak is generally bigger than JET, the width of the SOL in the target region is much
smaller. Another reason why the height of the JET plasmas, modelled in this thesis, is larger
is because the area of plasma being studied did not extend all the way down to the target.

The JET plasmas examined throughout this thesis can be described as being fully detached.
When this situation occurs, a region of high neutral density exists near the X-point a
significant distance above the target plates. At the X-point the SOL is naturally broader,
resulting in a wider region of high density.

The population code calculates the ratio of emissivities of Ly\(\beta\) to D\(\alpha\) along a particular line of
sight, described in Chapter 4. The angle of the line is inclined at 45° to the separatrix, so that
it represents an instrument looking almost vertically down, from the top of the tokamak,
through the slabs.

The calculated value of drop in the branching ratio, from that of an optically thin plasma, is
approximately 22%. This demonstrates that there is a significant level of absorption occurring
in the ITER plasma, similar to the amount of absorption as calculated for the JET plasma
pulse 34355. This is because, although the neutral density predicted for parts of the divertor
region are larger than those modelled in JET, the volume containing a high neutral density in
the ITER tokamak is smaller than that for JET. Although the ITER tokamak is twice the size
of the JET tokamak, the SOL in the divertor region, where the high neutral density occurs,
tapers inwards towards the target plates, therefore reducing the volume.
It should be mentioned here that, since the ITER tokamak is still in the planning stages, the predicted data, provided for the investigation described, can only be considered as an estimate of what the plasma parameters might be in one mode of operation. This simulation probably doesn’t represent the highest divertor density plasma that can be achieved in the ITER tokamak. A fully detached plasma is still an option for ITER, which will force the high neutral density region further up the divertor leg, where the SOL width is greater. This might lead to significantly higher absorption.

Wan’s investigation (Wan et al., 1995) scaled parameters from a high density, low temperature DIII-D pulse up for the ITER tokamak. A brief comparison between the parameters used by Wan and those predicted for the ITER tokamak is now possible. Wan considered a plasma with the peak electron density of $4 \times 10^{14}$ cm$^{-3}$ and a peak neutral density of $3 \times 10^{14}$ cm$^{-3}$ occurring at the target. The simulated ITER pulse has a peak electron density of approximately $2.3 \times 10^{15}$ cm$^{-3}$ and a peak neutral density of $8.0 \times 10^{13}$ cm$^{-3}$ occurring at 30 cm from the target. Wan uses a plasma with a lower electron temperature which is 2 eV between the target and the peak neutral density. Conversely, the simulated ITER plasma has an average electron temperature of approximately 4 eV over the same region. Since opacity is more sensitive to neutral densities and plasma temperatures than it is on electron densities, it is clear that Wan investigates plasmas with higher opacities than those predicted for ITER. However, as already mentioned, the simulated ITER parameters represent just one mode of operation. Therefore it is quite possible that the conditions investigated by Wan, may be produced in the ITER tokamak.

7.3 Conclusions

The main aim of this thesis has been to determine whether plasmas in the divertor region of the JET tokamak suffer from a significant level of opacity. This issue has been tackled by carrying out a theoretical and an experimental investigation. Both investigations examine the changes that occur in the branching ratio of Lyβ to Dα with time for four selected plasma pulses as their density increases and temperature decreases. The theoretical results showed a drop in this branching ratio of up to 36% when going from an optically thin plasma to a high density, low temperature detached plasma. This is less than the typical drop in the branching ratio of 50% provided via experimental measurements. However both investigations agree that opacity is reducing the level of Lyβ line emissivity from the divertor region of the JET tokamak by a significant amount. This implies that even larger reductions in the emissivity of
the Lyα line must also be occurring in this region due to opacity effects. The results presented in this thesis therefore indicate that it is no longer reasonable to assume that all divertor plasmas are optically thin.

The possible effects that opacity may have on the ionisation and power balance of plasmas, covering a wide range of parameters, have been examined and compared with other theoretical work carried out in this field. The results show that the ionisation rate of the plasma tends to increase with opacity and the amount of total power escaping from the plasma is reduced, with most of the Lyman photons being converted to Dα photons. The plasma pulses modelled in this thesis have a typical Lyα optical thickness of unity and for this optical thickness it is shown that the effects of opacity on the ionisation and power balance are negligible. In fact a Lyα optical thickness of approximately 6 is required to reduce the ratio of neutrals to ions by approximately 20% and the total emissivity by approximately 30%. However, since the theoretical investigation underpredicts the level of opacity with respect to the experimental investigation, it is possible that slight changes in the ionisation and power balance of the divertor plasma could be occurring and therefore this should be taken into account when modelling the plasma pulses, as a precautionary measure.

This thesis also incorporates a brief study into the levels of opacity that we may expect in the larger tokamaks of the future. Predicted parameter profiles of a high density, low temperature, partially detached plasma from the ITER tokamak have been used to perform this task. The plasma parameters are incorporated into the ‘slab-stack’ model of the population code, in the same way as those of the JET plasma pulses, to determine the drop in the branching ratio of Lyβ to Dα with respect to an optically thin plasma. The results showed a 22% drop in this branching ratio, which is of a similar magnitude to those produced by the JET plasmas. By increasing the detachment to a similar level as that occurring in the JET plasmas the region of high neutral density would be forced further up the leg of the divertor, which is much wider than that of the JET divertor, thereby causing a larger increase to the drop in the branching ratio.

Finally, during the process of modelling the JET plasma pulses, a comparison has been made between two plasma simulation codes, these being a 2-D fluid code, EDGE2D, and the ‘onion-skin’ model. Both codes provide excellent models for the plasma throughout the majority of the tokamak, however the divertor region is an extremely complex and difficult region to model due to the close proximity to the target. In this region the plasma becomes a gas, impurities are produced by sputtering and various atomic and molecular processes such
as neutral recycling, dissociation, charge exchange, ionisation and recombination are all occurring within a small volume of the machine. It has been found that in these conditions EDGE2D provides the better plasma models as it incorporates more physics within its conservation equations. Significant uncertainties still remain, particularly with respect to the plasma transport in the divertor, and it is still not possible to obtain a completely satisfactory match between model and experiment.

7.4 Future Work and Improvements

The work in this thesis is not just a self contained study but a basis on which to develop new, improved techniques to further the knowledge of opacity effects in tokamak plasmas. In order to ease this task, a brief summary of the main weaknesses of this investigation will now be presented along with some suggestions for their improvement.

The main weakness in this investigation is the plasma modelling procedure. Although the code used to perform this task, EDGE2D, is quite sophisticated, it is stretched to its limits when trying to model the high density, low temperature detached divertor plasmas being studied. This is due to the importance, in the divertor plasma, of a larger number of atomic and molecular processes and to the uncertainties in the plasma transport in the SOL and divertor of tokamak plasmas. The modelling process, which is time consuming in itself, needed to be repeated several times in order to obtain the closest plasma models shown in this thesis. Further time spent on this process could be beneficial in producing a more accurate model of the plasma pulses.

The EDGE2D code does not include a radiative model to account for opacity effects. It has been shown that the levels of opacity are significant, although we have also shown that this shouldn’t affect the ionisation and power balance of the plasma for the conditions in the JET tokamak. However, the investigation performed to examine the effects of opacity on the ionisation and power balance was generalised and quite basic. It would be interesting to see whether this was true for a real JET plasma by comparing the parameters produced from an EDGE2D run, which incorporated a radiative model, and the present parameters produced by EDGE2D.

The need for plasma models could, in principle, be eliminated altogether if a large number of VUV and visible spectrometers could be positioned so as to view the divertor plasma from
different angles and different positions. Unfortunately, this is not a realistic proposition for tokamaks given the small size scales in the divertor and the restricted access. Nevertheless, more spectrometer data would facilitate a more detailed method of testing plasma models.

The branching ratios obtained by direct measurements from the spectrometers, visible light monitors and CCD cameras suffered the least inaccuracies. However, these would have been minimised further if a VUV/visible spectrometer with identical viewing cones were available. Errors arising from line merging and line fitting have been found to be insignificant.

Only a few plasma pulses had the necessary detailed diagnostic information required to perform a complete investigation into the opacity of the JET divertor plasmas. In addition to this, only high density, low temperature, detached plasma pulses with a large density and temperature variation can be used. In accordance with these criteria, many plasma pulses were deemed as not suitable for further investigation. It would be interesting to carry out a further opacity study on any more suitable plasma pulses produced since those investigated in this thesis.

A study of the opacity effects over a wider spectral range such as the entire Lyman series, rather than just Lyβ, would also be beneficial. However, accurately calibrated spectral measurements would be necessary for such an investigation.

The process of averaging the JET plasma parameters in the divertor region to form a 'slab-stack' model can distort data. This is of particular concern when calculating the amount of absorption occurring along a specific line of sight because the parameters along this line of sight are weighted according to the surrounding parameters. A suggestion for improving this situation would be to create a 3-D 'grid-model' rather than a 2-D 'slab-model'. The 'grid-model' would divide the slab up into a selected number of cells, each one representing a small volume of the divertor region, which in principle could be the same as the grid system shown in Figure 2.2.

The population code incorporates a first order escape probability method to simplify the calculation of photon escape probabilities out of the 'slab-stack' model. In this it is assumed that the source function at the point of absorption is the same as that at the point of photon emission. This approach could be compared with other more sophisticated methods such as a second order escape probability method, which assumes a gradient in the source function between the point of emission and absorption, or a complete radiative transfer treatment, in
which the atomic level populations and radiation field are treated self consistently at all points
in the plasma.

The method employed to account for Zeeman effects, in this thesis, has been to broaden the
Doppler profile to the same width as a profile which is suffering from Zeeman splitting. This
is a reasonable approach, considering the complexity of the Zeeman splitting process and the
minimal effect that it seems to have on opacity in the conditions being considered. However,
as a future project it would be beneficial to incorporate Zeeman effects accurately in the
population code.

The best case scenario for opacity investigations of the future would be to incorporate the
aforementioned improvements and perform the examination on a large tokamak, such as the
proposed ITER tokamak, for a large number of high density, low temperature, highly
detached plasma pulses.
Appendix A

The single flight escape probability of a photon along a ray, out of a slab of infinite extent (Figure A.1) can be expressed as

\[ P_e(\nu, \theta) = e^{-\tau_\nu(\theta)} \]  

(A.1)

where \( \tau_\nu(\theta) \) is the optical thickness at frequency \( \nu \) and angle \( \theta \).

Figure A.1. A schematic showing the photon escape paths out of a slab of infinite extent.
The mean escape probability over all possible directions from the slab can be given as

\[
P_e(T) = \frac{1}{4\pi} \left[ \int_0^{2\pi} \sin\theta d\theta \int_0^\infty \phi(v) e^{-\tau(v)} dv + \int_0^{2\pi} \sin\theta d\theta' \int_0^\infty \phi(v) e^{-\tau'(v)} dv \right]
\]

(A.2)

where \( T \) is the optical thickness of the slab and \( \phi(v) \) is the frequency dependent line profile. In the upper emission cone, shown in Figure A.1, the optical thickness can be expressed as

\[
\tau_v(\theta) = \tau \phi(v) \sec \theta
\]

(A.3)

and in the lower emission cone the optical thickness can be expressed as

\[
\tau_v(\theta') = (T - \tau) \phi(v) \sec \theta'
\]

(A.4)

where angle \( \theta' \) is shown in Figure A.1. Using these expression for optical thickness, the average escape probability can now be expressed as

\[
P_e(T) = \frac{1}{2} \left[ \int_0^{2\pi} \sin\theta d\theta \int_0^\infty \phi(v) e^{-\tau(v) \sec \theta} \frac{du}{u^2} + \int_0^{2\pi} \sin\theta d\theta' \int_0^\infty \phi(v) e^{-\tau'(v) \sec \theta'} \frac{du}{u^2} \right]
\]

(A.5)

Now, by letting \( u = \sec \theta \) it is possible to derive the following expression

\[
\sin \theta d\theta = \frac{du}{u^2}
\]

(A.6)

Substituting this expression into Equation A.5 gives

\[
P_e(T) = \frac{1}{2} \left[ \int_0^\infty \phi(v) dv \int_0^\infty \frac{du}{u^2} e^{-\tau(v) \sec \theta} + \int_0^\infty \phi(v) dv \int_0^\infty \frac{du}{u^2} e^{-\tau'(v) \sec \theta'} \right]
\]

(A.7)

By using the expression

\[
E_u(z) = \int_1^\infty \frac{e^{-ut}}{t^z} dt
\]

(A.8)
Equation A.7 can be put in the same format as Equation 4.40, as shown below

\[
P_s(\tau, T) = \frac{1}{2} \left[ \int_0^\infty \phi(\nu) E_2 (\tau \phi(\nu)) d\nu + \int_0^\infty \phi(\nu) E_2 ((T - \tau) \phi(\nu)) d\nu \right] \tag{A.9}
\]
Appendix B

The mean escape probability $\bar{P}_e$ can be defined in terms of the escape probability in a particular direction by the expression

$$\bar{P}_e = \int \frac{P_e(\gamma) d\Omega}{4\pi} = 2 \int_0^{\pi/2} e^{-k\ell} \frac{2\pi \sin \gamma}{4\pi} d\gamma. \tag{B.1}$$

Therefore

$$\bar{P}_e = \int_0^{\pi/2} e^{-k\ell} \sin \gamma d\gamma. \tag{B.2}$$

It can be seen from Figure B.1a that for the slab model $l=z/cos\gamma$. Allowing $u=cos\gamma$ and substituting the expression for $l$ into Equation B.2 gives an expression for the mean escape probability from the slab is

$$\bar{P}_{eslab} = \int_0^1 e^{-kz/u} du. \tag{B.3}$$

Similarly, it can be seen from Figure B.1b that for the cylindrical model $l=R/sin\gamma$. Hence the mean escape probability from the cylinder can be expressed as

$$\bar{P}_{ecyl} = \int_0^1 e^{-kR/\sqrt{1-u^2}} du. \tag{B.4}$$
Figure B.1. Schematic showing the geometry of (a) the 'single-slab' model and (b) the cylindrical model.

Allowing Equation B.3 to equal Equation B.4 gives an expression for the half of the slab height \( z \) in terms of the radius of the cylinder, that allows both models to have the same escape probabilities and optical depths. This expression is

\[
\int_{0}^{l} e^{-z/\mu} du = \int_{0}^{R} e^{-kr/\sqrt{1-u^2}} du
\]

\[_{S.5}^{248}\]
Table B.1 gives values for the ratio of $R$ to $z$ for various values of the cylinder's optical depth.

<table>
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<th>Mean optical depth of the cylinder</th>
<th>$R/z$</th>
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<td>40</td>
<td>1.05</td>
</tr>
</tbody>
</table>

**Table B.1.** Table giving ratio of $R$ to $z$ for the corresponding mean Ly$\alpha$ optical depth of the cylinder.
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