THREE ESSAYS IN MACROECONOMICS

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A DISSERTATION SUBMITTED IN FULFILLMENT OF THE REQUIREMENT
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY OF
UNIVERSITY COLLEGE LONDON

JULY 2020
DECLARATION

I, Chanwoo Kim, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.
Abstract

This thesis consists of three independent papers on macroeconomics. Chapter 1 provides the introduction of the papers.

Chapter two investigates the firm-level markup to aggregate shocks and the role of intangible capital in markup determination. A markup is a key object in understanding a firm’s pricing behavior. Using a panel version of local projection, I document noble evidence that firm-level markup is countercyclical to aggregate productivity and monetary policy shock. To explain the empirical evidence, I combine Hopenhayn (1992) firm dynamics model with habit accumulation at a good level (Ravn et al., 2006). After calibrating the model to US data, I find that the model can quantitatively match the empirical evidence. Furthermore, the model endogenously matches the age-dependent growth rate and the exit rate, which the profession had difficulty with.

Chapter three asks, “how a firm responds to tax shock in the short run?”. Differently from the existing literature, I exploit narratively identified shock and study the firm-level response over the business cycle using local projection instrumental variable approach. I find that intangible and tangible investment and labor use increase, while leverage goes down. Firm revenue productivity and markup increase, but firm churn is stable in the short run. Extrapolating the estimates, I project that the effect of the 2017 tax cut is significant.

Chapter four studies the effect of forward guidance under an incomplete market. In standard New Keynesian models, there is a peculiar property called “forward guidance puzzle”: if a central bank promises to cut its policy rate from a farther future, the effect of promise strengthens. There exists debate that the introduction of an incomplete market can solve the puzzle, or it additionally requires procyclical income risk. Building on Ravn and Sterk (2020), my paper analytically proves that income risk cyclicality matters.
Impact Statement

This thesis addresses some key open questions in the academic and policy circles. Chapter two studies the effect of intangible capital accumulation in individual firms’ behavior. Using a new empirical approach, this chapter documents two new empirical evidence. First, individual firm’s markup is countercyclical to productivity and monetary policy shocks. Second, small firms have more countercyclical markups. This evidence is important since the markup is a key object in understanding the amplification of shocks by shifting the labor demand curve. Existing theories cannot explain the two pieces of evidence above. Therefore, I claim that the customer base accumulation of firms can explain the evidence. In the model, firms face a tradeoff between invest and harvest of the customer base. To expansionary shocks, firms’ invest incentive dominates harvest motive, hence, firms decrease markup to accumulate the customer base. For size-dependent response, the opportunity cost of lowering markup and the exit risk related to the demand base plays an important role. The tradeoff between invest and harvest furthermore allows the model to endogenously match the age-dependent growth rate and exit rate which the literature has difficulty matching. Hence, this chapter emphasizes the importance of intangible capital in analyzing firm-level behavior, particularly on pricing, growth, and exit decisions.

Chapter three investigates the firm-level response to corporate tax cut shocks. Understanding the effect of tax reform carries first-order importance in macro and public economics. I focus on short term firm-level responses to corporate tax shock over the business cycle. To measure the effect of tax shock precisely, I implement a panel version of the local projection instrumental variable approach. I use a firm-level tax rate as an instrument to narratively identified shock to tackle measurement errors. To tax cut shocks, firm-level intangible and tangible capital investment, and total labor costs, while leverage decrease. Measured productivity and markup increase, however, firm entry and exit rates are stable to the tax cut in the short run. As an application, I study the effect of the 2017 Tax Cuts and Jobs Act. Using the estimates from this chapter and previous research, I conclude that
the effect of the 2017 tax cuts can be more significant than literature.

Chapter four analytically shows the concrete conditions to solve forward guidance puzzle. After officially adopted as a policy tool of central banks, forward guidance has been of great interest to researchers and professionals. However, standard New Keynesian models fail to explain the quantitative effect of forward guidance (forward guidance puzzle). The puzzle states that the quantitative effect of the guidance is stronger if the authority promises the policy rate cut in a farther future. There is an open debate among researchers whether the introduction of an incomplete market per se can solve the puzzle or not. I provide analytic equations that the income risk should be procyclical to solve the forward guidance puzzle under an incomplete market. Therefore, chapter four answers the open and policy-relevant question articulately.
Acknowledements

I am grateful to my supervisor, Morten O. Ravn, for his continuous guidance and support during my studies. I admire his depth and width of knowledge as well as personality.

I am indebted to my second supervisor Vincent Sterk who provided sharp advice and useful comments.

Furthermore, I appreciate to UCL faculty members. They generously shared their time to listen to my research as well as personal difficulties, and provided brilliant ideas and emotional support. I have especially benefitted from the discussion with Wei Cui, Aureo De Paula, Ralph Lueticke, Albert Marcet, and Franck Portier.

I thank my fellow Ph.D. students. From UCL macro reading group, I learned from their presentation as well as received great feedback from my colleagues. Their presence made my studies more enjoyable and memorable. I am also thankful to the administrative staff at the Department of Economics for their support and assistance.

Lastly, and most importantly, I would like to thank my family. Without gentle encouragement and unconditional support from my wife, Kyunghwa, I would not be able to complete this thesis. Equal gratitude should be given to my daughter and parents.

I dedicate this thesis to all of them.
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Chapter 1

Introduction

This thesis consists of three independent pieces of research in macroeconomics, particularly the behavior of micro-level agents. Traditionally, macro literature focused more on aggregate level or representative agent level behavior due to various difficulties. However, with the development of new technical tools and databases, more researches focus on the behavior of micro levels such as individual households and firms. By doing so, I can answer many new questions such as the role of heterogeneity in macro-dynamics and the effect of idiosyncratic characteristics in regression.

Chapter two, Markup, Customer Base, and Firm Dynamics, studies the effect of market power from demand accumulation on markup cyclicality and lifecycle behavior of firms. Recent literature emphasizes the role of intangible capital in understanding the aggregate economy and individual agent behavior. Moreover, countercyclical markup is important in the amplification of shocks by shifting the labor demand curve. Using a panel version of local projection, I document two pieces of new empirical evidence: (i) individual firm markup is countercyclical to productivity and monetary policy shocks, (ii) smaller firms have more countercyclical markups to the shocks. Since I am not aware of any theory that explains my new facts, I propose a firm dynamics model with the customer base and endogenous entry and exit. On top of standard firm dynamics models such as Hopenhayn and Rogerson (1993), I assume that the households in my model form habits at the individual goods level (Ravn et al., 2006). The entry and exit are
endogenous similar to Clementi et al. (2014). In my model, the customer base makes the firm pricing problem dynamic, so firms compare current profit and the value of the customer base when they set markup. The incentive to invest in the customer base, or more generally intangible capital generates the results consistent with the data. Furthermore, I show that the model can endogenously match age-dependent growth and selection of firms.

The next chapter, Short-Run Firm Responses to Corporate Tax Shocks, empirically studies the firm-level response to corporate tax shocks. The effect of tax shock is one of the key questions in the profession. Relative to the existing literature, I overcome the identification problem using narratively identified shocks and investigate the firm-level response to corporate tax shock over the business cycles. To study the firm-level responses, I use a panel version of local projection with an instrument variable. For the corporate shock series, I exploit the corporate tax shock series identified by Mertens and Ravn (2013) and I use a firm-level tax rate as an instrument. I find that firms increase intangible and tangible investment, and total labor costs, whereas decrease leverage. Moreover, measured firm productivity and measured markup increase, but firm churn is stable to the tax cut. Since tax reform tends to persist, I further compare the cumulative response of tax shock in the spirit of Ramey and Zubairy (2018) to the non-cumulative response of tax shock. And I find that cumulative response shows more persistence in impulse response functions. Using the estimates from my results and existing research such as Mertens (2018), I predict that the effect of the 2017 Tax Cuts and Jobs Act is significant.

The last chapter, titled as Forward Guidance Puzzle under HANK & SAM, studies the effect of forward guidance under frictions in financial, labor, and goods market. In response to the COVID-19, the monetary policy rates of many central banks hit the lower bound and I observe some form of forward guidance. However, a standard New Keynesian model has a peculiar property called "forward

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1 For example, "The Committee expects to maintain this target range until it is confident that the economy has weathered recent events and is on track to achieve its maximum employment and price stability goals." (FOMC statement, 15 Mar 2020)
guidance puzzle"; if a central bank promises to cut its rate from a farther future, the quantitative effect of the promise is stronger. In the academic circle, there is a debate between researchers that the introduction of an incomplete market per se can solve the puzzle or the introduction of an incomplete market and procyclical income risk are necessary. By building on Ravn and Sterk (2020), I analytically show that the cyclicality of income risk is critical. The model has an incomplete financial market, search and match labor friction, and monopolistic competition in the goods market. Relative to Ravn and Sterk (2020), I use the continuous-time model so that the two contrasting income cyclicality forces, i.e. procyclical wage risk and countercyclical job loss risk, changes at the same time when a parameter of the model varies. In this setting, I derive the analytic expression for aggregate Euler equation and show that the income cyclicality critically changes the effect of forward guidance relative to representative agent models. I also find that income risk is weakly countercyclical under standard calibration of the model.
Chapter 2

Markup, Customer Base, and Firm Dynamics

2.1 Introduction

The market power of a handful of firms is increasing in the US (Autor et al., 2017; Kehrig and Vincent, 2018). The increased concentration of the market has important implications; for example, it is related to the declining labor share and the rise of markup (De Loecker et al., 2019; Eggertson et al. 2018). Different from these studies which focus on trends, I study the effect of market power on the business cycle frequency. Specifically, I attempt to relate the firm-level market power by demand accumulation to markup cyclicality.

Countercyclical markup plays a key role in the amplification of shocks in macro-models by shifting the labor demand curve\(^1\) in the direction of the shocks. Examples are firm entry and exit models (Jaimovich and Floetotte, 2008) of productivity shocks and New Keynesian models (Christiano, Eichenbaum, and Evans, 2005, Smets and Wouter, 2007) of demand shocks.

Given the importance of the markup cyclicality in macro models, researchers try to measure markup cyclicality using different approaches. However, studies

\(^1\) “Countercyclical markup is like salt in cooking” (Basu, 2016) summarizes the importance of markup in macromodels.
tend to find little agreement. Apart from the existing literature, I propose a new approach to measure the markup cyclicality: the impulse response of firm-level markup to aggregate shocks. This approach is more granular and model consistent than the existing studies. Furthermore, I study how markup cyclicality varies with the size of a firm. Then, I propose a model that is consistent with empirical evidence.

I measure the markup cyclicality to aggregate productivity and monetary policy shocks using a unique combination of existing literature. I first identify the firm-level markups from COMPUSTAT data using the production approach in the line of Hall (1986), De Loecker and Warzynski (2012), and De Loecker and Eeckhout (2017). Then, I take the identified aggregate shocks from the literature, i.e., Fernald (2014) and Gertler and Karadi (2014). Last, I employ a panel version of local projection (Jorda, 2005) using the firm-level markup and aggregate shocks to recover nonlinear impulse response functions to aggregate shocks. To explore size-dependent markup cyclicality, I use the mean group estimator.

I find that the impulse responses of a firm-level markup to aggregate productivity and monetary policy shocks are countercyclical. Furthermore, small firms have more countercyclical markup. I note that countercyclical markup in response to a productivity shock is at odds with the celebrated New Keynesian models, which predict procyclical markup. Since I am not aware of any model that can explain my empirical evidence, I propose a theory to investigate my empirical results.

My model is a firm dynamics model with customer markets and endogenous entry and exit. The key difference between my model and a standard firm dynamics model (Jovanovic, 1982; Hopenhayn, 1992) is that a firm is concerned with both its productivity and its customer base. In a standard firm dynamics model, a firm only concerns with a productivity. A customer base is a group of loyal customers that buy the product of a firm repeatedly. In other words, I model that consumers buy Nike because everyone else bought Nike. This deep

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habit preference assumption (Ravn, Schmitt-Grohe, and Uribe., 2006) provides the foundation of a demand curve that shifts outward as the customer base accumulates. Furthermore, I model that the entry and exit of a firm are subject to idiosyncratic and aggregate economic conditions. Since exit is endogenous to the amount of the customer stock a firm has, the customer base plays two roles, i.e., the demand base and insurance (Gilchrist, Schoenle, Sim, and Zakrajsek, 2017).

Firms face a dynamic tradeoff between the current profit and the future value of the customer base. The customer base is a fraction of past sales; therefore, a firm’s pricing problem becomes dynamic. Firms can invest in the customer base by charging low markup today to harvest from the customer stock by charging high markup in the future. This invest and harvest incentive is at the heart of markup determination. I note that this demand accumulation mechanism is well established in previous data (Foster, Haltiwanger, and Syversion, 2008, 2016; Hottman, Redding, and Weinstein, 2016) and models (Phelps and Winters, 1970; Arkolakis, 2010; Gourio and Rudanko, 2014).

My model can endogenously match the age-dependent growth and the selection of firms. Small firms choose to grow fast since their incentive to invest is higher than that of big firms because (a) exit risk decreases faster for a unit increase of the size for small firms and (b) “the price” of lowering markup is lower for small firms given their small customer base. This condition implies that small firms, on average, charge lower markup to grow fast. Since the entrants start small, the lifecycle markup, in general, increases with age. Although existing studies show that the demand accumulation slows down the growth of a firm (Foster et al., 2008, 2016), this paper applies the model to the data and shows that the demand mechanism can actually match the data closely without targeting any moment related to the firm growth rate. The importance of understanding lifecycle behavior of firms is documented in Haltiwanger, Jarmin, and Miranda (2013) and Fort, Haltiwanger, Jarmin, and Miranda (2013).
The model shows countercyclical markup in response to productivity shock and monetary policy shock due to the dynamic tradeoff. When there is a positive supply shock, a firm’s marginal cost is low; therefore, firms want to invest further in the customer base by lowering markup. When there is an expansionary monetary policy shock, firms decrease markup to attract more customers since the current demand is larger than the future demand. This insight is consistent with a search theoretic customer base model with an endogenous opportunity cost of search (Paciello, Pozzi, and Trachter, 2017).

I find that the markups of small revenue firms are more countercyclical to productivity and monetary policy shocks. For positive productivity shock, big firms have less incentive to decrease markup since lowering markup is more costly given a large amount of customer stock. For expansionary monetary policy shock, the exit risk of big firms decreases less than that of small firms; therefore, big firms decrease markup less than small firms. This finding implies that the aggregate response to shock is affected by a firm distribution. Hong (2019) also finds that small firms have more countercyclical markup in response to the change in GDP in the data.

(Literature Review) Given the importance of markup cyclicality in macromodels, researchers try to measure markup cyclicality using different approaches. Depending on the aggregation level of markup and the measure of the business cycle, the existing research can be summarized into three categories. The first line of research investigates the correlation between a certain measure of aggregate markup and a measure of business cycles (Domowitz, Hubbard, and Petersen, 1986; Bils, 1987; Rotemberg and Woodford, 1991). The second strand of the literature considers the correlation between firm-level markup and aggregate output (Hong, 2019). The third line of research studies the impulse response of aggregate markup to aggregate shocks (Nekarda and Ramey, 2019).

I further study size-dependent markup cyclicality in response to the two shocks. Existing studies examined the heterogeneous response of sales (Gertler and
Gilchrist, 1994; Crouzet and Mehrotra, 2018) or employment (Moscarini and Postel-Vinay, 2012). Hong (2019) is the only paper to study the size-dependent markup; however, he studied the correlation of markup to GDP.

Therefore, I propose a model that combines the firm dynamics literature with the customer base studies. The pioneering work in firm dynamics is Hopenhayn (1992). The present paper is related to the firm dynamics models with emphasis on the demand side, such as the models of Arkolakis (2010), Dinlersoz and Yorukoglu (2012), and Sedlacek and Sterk (2017). Customer base models start from Phelps and Winter (1970) and the models used to explain macro- and international economics such as Rotemberg and Woodford (1991, 1995), Drozd and Nosal (2012), Gourio and Rudanko (2014), Fitzgerald et al. (2017), and Piveteau (2019). My model contributes to this line of literature by studying the aggregate response of firm-level markup and by showing that the model can match the selection and the growth of a firm in the data.

Two independent studies explored similar environments. Hong (2019) combined deep habits and firm dynamics. My work shares the results that markup is countercyclical to aggregate productivity shock. However, I isolate the demand accumulation mechanism\(^3\) and show that the mechanism can match the lifecycle characteristics of firms. Hong, in contrast, studied the cyclical dispersion of firm-level measured productivity. Furthermore, I study the response to demand shock and financial shock on top of aggregate supply shock. Gilbukh and Roldan (2019) also studied the firm-dynamics model with demand accumulation under a product search and match environment. They found that markup is procyclical to aggregate supply and demand shocks. However, in their directed search model, the customer only considers the present value of the utility from the match while ignoring the current price of the product. This property is due to the linear utility function of the buyers and the sellers limiting the role of price as simply allocative. Moreover, due to the block recursive property, the agents’ payoff is independent of the firm distribution.

\(^3\) Hong assumes decreasing return to scale, which affects firms’ customer base acquisition.
This paper is also related to the studies on the firm growth mechanism. The pioneering study by Gibrat (1931) claimed that the proportionate speed of firm growth is independent of firm size. However, more recent studies have shown that Gibrat’s Law does not hold since small firms grow much faster than big firms (Dunn, Roberts, and Samuelson, 1988; Luttmer, 2007, 2011; Pugsley, Sedlacek, and Sterk, 2019).

(Roadmap) In the next section, I establish two pieces of new empirical evidence. To do so, I first explain the production approach to identify an individual firm’s markup. Then, I show how to measure the impulse response of markup using an individual firm’s markup decisions. After documenting the empirical evidence, I present a demand-driven firm dynamics model with endogenous entry and exit. Using the model, I show the analysis at a steady state and with aggregate shocks. Finally, I conclude.

2.2 Impulse Response of Markup to Aggregate Shocks

I use the production approach to obtain an individual firm’s markup in the line of Hall (1986) and De Loecker and Warzynski (2012). Using the identified markup from the production approach, I execute local projection (Jorda, 2005) to find the impulse response function upon aggregate shocks.

2.2.1 Production Approach

The production approach builds on the insight that, in a perfectly competitive market, the output elasticity of a variable input is equal to its expenditure share of total revenue. Therefore, the gap between the two is viewed as a markup that comes from imperfect competition. For the main result, I closely follow De Loecker and Eeckhout (2017) to estimate an individual firm’s markup.

Since the method is widely used in the recent literature, I attempt to be concise in explaining the framework. For details, please see De Loecker and Eeckhout (2017) or De Loecker and Warzynski (2012).
The advantage of the production approach is that it can be applied to the general environment since it does not require any assumption of a market structure or demand system. In addition, the methods used in this paper do not need to impose constant returns to scale. Furthermore, the method does not require observing or measuring the user cost of capital.

Consider an economy that consists of a continuum of firms that want to minimize their cost. Firm i's cost minimization problem is the following.

$$L_{it} = \min_{L_{it}, K_{it}} W_{it} L_{it} + r_{it} K_{it} - \Lambda_{it}(Q_{it}(L_{it}, K_{it}) - \bar{Q}_{it})$$

I assume that labor is a variable input. I note that I can easily extend the assumptions to include many variable inputs and many fixed or dynamic inputs. By differentiating the labor input, I obtain the optimal labor input demand condition.

$$\frac{\partial L_{it}}{\partial L_{it}} = W_{it} - \Lambda_{it} \frac{\partial Q_{it}}{\partial L_{it}} = 0$$

I note that $\Lambda_{it}$ is a measure of marginal cost. Intuitively, the above equation shows that marginal cost equals the cost for hiring one unit of labor over the marginal labor productivity ($MC_{it} = W_{it} MPL_{it}$). The definition of output elasticity to variable cost is

$$\theta_{it}^l = \frac{\partial Q_{it}}{\partial L_{it}} \frac{L_{it}}{Q_{it}} = \frac{1}{\Lambda_{it}} \frac{W_{it} L_{it}}{Q_{it}}$$

By rearranging the definition of output elasticity, I obtain an equation for markup.

$$\mu_{it} \equiv \frac{P_{it}}{MC_{it}} = MPL_{it} \frac{P_{it} Q_{it} L_{it}}{W_{it} L_{it} Q_{it}} = \theta_{it}^l \frac{P_{it} Q_{it}}{W_{it} L_{it}}$$

I need to find output elasticity ($\theta_{it}^l$) and the share of labor cost to total sales. The share of labor cost to total sales is easily found in firms' financial statements; therefore, I focus on how to recover the output elasticity of variable input from the data.

I estimate the production function to find the output elasticity of variable in-
put. Although a long line of literature on this topic exists, the main problem of estimating the production function is endogeneity between the input choice and unobserved productivity\(^5\). I use the control function approach by following Olley and Pakes (1996), Levinsohn and Petrin (2003), and Ackerberg, Caves, and Frazer (2015). The key idea of the control function approach is that I can use the economic structure to write unobserved productivity as a nonparametric function of inputs. The advantages of the control function approach are that (i) it does not require an instrument that is very difficult to find, (ii) it does not need strong assumptions such as fixed productivity or perfectly competitive input and an output market, and (iii) it allows a subset of inputs to be dynamic. For possible concerns for identification (Gandhi, Navarro, and Rivers, Forthcoming), I estimate production function with a dynamic panel approach (Blundell and Bond, 1998) for a robustness test.

For the main result of this paper, I assume an industry-specific Cobb-Douglas production function. The function may seem to be a value-added production function. However, I can interpret the function as the Leontief gross output production function, in which intermediate input is proportional to the output (Ackerberg et al., 2015)\(^6\). I also show that the result is robust to a general production function, e.g., the translog.

\[
Q_{it} = L_{it}^{\theta_l} K_{it}^{\theta_k} \exp(\omega_{it})
\]

\[
\tilde{q}_{it} = \theta_j L_{it} + \theta_k K_{it} + \omega_{it} + \epsilon_{it}
\]

where \( i \) denotes an individual firm, \( j \) denotes the industry, and \( \omega_{it} = h(l_{it}, k_{it}) \) is idiosyncratic productivity that follows an AR(1) process. The second equation is obtained by taking the log of the first equation and adding measurement error \((\epsilon_{it})^7\).

---

\(^5\) De Loecker and Eeckhout (2017) argue that the selection problem may affect capital elasticity more than labor elasticity.

\(^6\) Using this specification, the model does not suffer from the functional dependence problem. See Appendix A for details.

\(^7\) One can think of it as an independent and identically distributed (IID) productivity shock that is unknown at the point of production decision.
I estimate the production function using a two-step GMM. The first step involves purging measurement error and productivity. Specifically, I regress sales on labor, capital, time dummies, and a constant. Then, I set \( \hat{q}_{it} \) as the true output and obtain productivity \( \omega_{it} \) by calculating
\[
\omega_{it} = \hat{q}_{it} - \theta_j^l l_{it} - \theta_k^k k_{it} - \text{constant}
\]
The second step is GMM. I regress the obtained productivity on its lag: the residual \( \xi_{it} \) is the shock to productivity. Then, I use two-moment conditions\(^8\) to find two parameters.

\[
E[\xi_{it}(\hat{\theta}_j^l, \hat{\theta}_j^k)l_{i,t-1}] = 0
\]
\[
E[\xi_{it}(\hat{\theta}_j^l, \hat{\theta}_j^k)k_{i,t}] = 0
\]
I am interested in \( \theta_j^j \). The key assumptions are that the past variable input use is (i) independent of the current period productivity shock and (ii) related to the current period variable input use. The timing guarantees the first assumption, and the AR(1) process of productivity supports the second assumption.

The last step is to adjust for measurement error.
\[
\mu_{it} = \frac{P_{it}}{A_{it}} = \theta_j^l \frac{P_{it}Q_{it}}{W_{it}L_{it} \exp(\epsilon_{it})}
\]
In this approach, the Solow residual is the sum of idiosyncratic productivity and measurement error; therefore, to find the “true” quantity, I need to eliminate the measurement error component using the residual from the first stage.

### 2.2.2 Data

I choose COMPUSTAT data from Wharton Research Data Services (WRDS). The choice of COMPUSTAT is based on three characteristics. First, the database is the only publicly available source that covers firms in all industries. Second, COMPUSTAT provides detailed financial statement variables for the use of an

\(^8\) De Loecker and Eeckhout (2017) report only one moment condition related to labor. Since there are two parameters to estimate, I believe they use two moment conditions. However, the results are robust even if I use one moment condition.
empirical strategy. Third, the database covers a significant fraction of the economy. Relative to other datasets that cover manufacturing, which accounts for less than 10% of GDP, COMPUSTAT covers approximately 30% of employment (Davis, Haltiwanger, Jarmin, and Miranda, 2007). I present the data-cleaning procedure in Appendix A.

For total factor productivity (TFP) shock, I use Fernald’s (2014) utilization-adjusted productivity for the US business sector. The utilization-adjusted productivity is developed to consider the fact that standard TFP includes the change in factor use, such as labor effort and the workweek of capital. The approach finds data on inputs using careful growth accounting as in the Bureau of Labor Statistics (BLS). Unobserved utilization, i.e., labor effort and capital utilization, is estimated using hours per worker as a proxy under the assumption that firms optimize their input choice.

I use a high-frequency identification approach to identify the monetary policy (MP) shock. This approach is useful in addressing a possible forecast problem in identifying the monetary policy shock. Specifically, I use high-frequency identification shock (Ramey, 2018). Ramey claims that Gertler and Karadi’s (2015) high-frequency identification shock has serial correlation, which comes from the method that Gertler and Karadi used to convert the announcement day shocks to a monthly series. Therefore, I use Romer and Romer’s (2004) method to generate annual shocks following the suggestion of Ramey (2018). The high-frequency identification approach was pioneered by Cook and Hahn (1989) and is widely used in the literature. Under the assumption that most of the information related to monetary policy is revealed around the FOMC meeting, the approach uses the change in the bond price within a small time window. I normalize the shock so that the increase in the shock reflects the expansionary monetary policy.

---

9 I choose Eurodollar six-month future data since it has the longest sample period.

10 The procedure is the following: First, create a cumulative daily monetary policy shock series. Second, take the difference between the end-of-the-year level and the beginning-of-the-year level of the cumulative shock series.

11 For example, see Kuttner (2001), Cochrane and Piazzesi (2002), Gurkaynak, Sack, and Swanson (2005), Campbell, Evans, Fisher, and Justiniano (2012), and Nakamura and Steinsson (2018).
2.2.3 Markup Distribution

In this section, I describe how markup distributions have evolved over the last couple of decades (figure 2.1). Over, the mean and the variance of the markup distributions increase, whereas the skewness of the distributions decreases. I note that markup can even be lower than one.

Figure 2.1: Change in Markup Distribution

![Markup distribution graphs](image)

Note: Markup distributions are truncated at 1\textsuperscript{st} and 99\textsuperscript{th} percentile

2.2.4 Markup Response to Aggregate Shocks

In this section, I show the markup response to aggregate shocks. To estimate the impulse response, I first take the log of all variables except shocks, age, and market share. I then use an industry-specific\textsuperscript{12} quadratic time series to eliminate the trend for relevant firm-level variables and use a quadratic time series to remove the macrotrend for GDP. Lastly, I use a panel version of local projection (Jorda, 2005) to estimate dynamic responses of markup to aggregate shocks. All robustness exercises are summarized in the last part of this section.

\textsuperscript{12} I choose two-digit industry due to sample numbers. For robustness, I use the first difference for detrending and obtain a similar result.
Impulse Response Analysis

Using the identified shocks from the data and the literature, I find the response of markup to aggregate productivity, monetary policy, and financial shocks. Specifically, I regress

\[
\text{Markup}_{i,t+h} = \gamma_1^h + \gamma_2^h \text{Shock}_t + \gamma_3^h \text{Markup}_{i,t-1} + \gamma_4^h \text{Control}_{i,t-1} + \gamma_i^h + e_{i,t+h}
\]

\(\gamma_2\) captures the average cross-sectional percent change of markup due to an aggregate shock. The control variables are GDP, firm size, market share, age, productivity, and sales effort. Since my data are annual, I set \(h = \{0, 1, 2, 3\}\).

I find that the individual unweighted average markup is countercyclical to productivity, monetary policy, and financial shocks (figure 2.2). The dots in the figure represent the level of coefficients \((\gamma_2)\), and the lines show the 95\% confidence interval\(^{13}\). I provide the regression tables in Appendix A.

Figure 2.2: Impulse Response of Markup

![Graph showing impulse response of markup to TFP and MP shocks.](image1)

Note: Solid lines are firm-level responses, and dotted lines are 95\% percentile confidence intervals.

The left panel of figure 2.2 shows that a 1\% increase in TFP shock may cause a 0.15\% decrease in individual markup on average on impact and disappears in the next year. The right panel of figure 2.2 illustrates the response of the firm-level markup to one unit change in the six-month future Eurodollar price due to

\(^{13}\) I use heteroskedasticity and autocorrelation robust standard errors to calculate the confidence interval.
expansionary monetary policy shock. It shows that the average firm-level markup decreases by 0.5%, and the effect goes away the next year. Relative to procyclical or acyclical markup, countercyclical markup amplifies the response of output and prices since it shifts a labor demand curve to the same direction of the shock.

Size-Dependent Markup Response

In this section, I study how the markup response differs from the revenue of a firm. To proceed, I first pool the data and divide it into three bins according to the revenue of each data point. Under the assumption that revenue and customer base are positively correlated, I use revenue as a proxy for the customer base. Then, I use the mean group estimator to find the size-dependent markup responses.

I find that the markup of a smaller firm is more countercyclical to productivity and monetary policy shocks (figure 2.3). The solid lines are the average response of firms in the group, and the dotted lines are the 68th percentile confidence interval. For the TFP shocks and the monetary policy shock (the left and right panels of figure 2.3, respectively), the response of smaller firms is different from that of medium and large firms. Furthermore, the difference tends to persist for some years after. In the model section, I explain the mechanism behind the data.

Figure 2.3: Size-Dependent Response of Markup

Note: Solid lines are firm-level responses, and shaded areas are 68th percentile confidence intervals.

---

14 Here, I use current revenue. The result is robust to the use of lagged revenue.
2.2.5 Robustness Tests

This subsection provides robustness of my results. The robustness tests are executed in four aspects illustrated below. The results are in general robust.

Detrending

To test the effect of detrending method, I use the first difference. By using the first difference, I can take out firm specific trend; hence I can test the potential bias both from a quadratic trend and from an industry-specific time trend. To test, I take the first difference of the data and use ordinary least squares (OLS) to estimate the following model.

$$
\Delta \text{Markup}_{i,t+h} = \gamma_1 \text{Shock}_t + \gamma_2 \Delta \text{GDP}_{i,t-1} + \gamma_3 \text{Control}_{i,t-1} + \gamma_i + \epsilon_{it}
$$

Figure 2.4: Markup Response to Aggregate Shocks (First Difference)

Average Response

Size Dependent Response

Note: 1) Solid lines are firm-level responses and dotted lines are 95th percentile confidence band.
2) Solid lines are firm-level responses and shaded areas are 68th percentile confidence band.
Controls are sales, market share, productivity, sales effort, and age as in the main regression. Figure 2.4 shows that the results are robust although the size dependent response to aggregate TFP shock is similar across all sizes.

To test the detrending method further, I include the industry-specific time trend rather than detrending each variable directly. Specifically, I include a quadratic industry specific trend term instead of detrending each variables and the control variables are the same as above. In this approach, variables share the common trend whereas each variable has its own industry trend in the main results.

\[
\text{Markup}_{i,t+h} = \gamma_1^h \text{Shock}_t + \gamma_2^h \text{Markup}_{i,t-1} + \gamma_3^h \text{Control}_{i,t-1} + \gamma_i^h F(\text{trend}) + \epsilon_{i,t+h}
\]

Figure 2.5 shows that the results are generally robust for both average and size dependent responses. I note that the results are robust to cubic time trend.

Figure 2.5: Markup Response to Aggregate Shocks (Industry Trend)

Note: 1) Solid lines are firm-level responses and dotted lines are 95th percentile confidence band.
2) Solid lines are firm-level responses and shaded areas are 68th percentile confidence band.
Production Function

I test the robustness to specification and estimation of the production function. To test the specification of the production function, I set a flexible production function, i.e., the translog production function. I approximate the function with second-order, and I do not include the interaction term of labor and capital due to the possible measurement error of the capital. A detailed discussion can be found in Collard-Wexler and De Loecker (2016). In sum, I regress the following equation using local projection.

\[ Q_{it} = F(L_{it}, K_{it}) \exp(\omega_{it}) \]

\[ \tilde{q}_{it} = \theta_{jt}^{v1} l_{it} + \theta_{jt}^{v2} l_{it}^2 + \theta_{jt}^{k1} k_{it} + \theta_{jt}^{k2} k_{it}^2 + \omega_{it} + \epsilon_{it} \]

Figure 2.6 illustrates that the impulse responses are very similar to the main results.

Figure 2.6: Markup Response to Aggregate Shocks (Translog)

Note: 1) Solid lines are firm-level responses and dotted lines are 95th percentile confidence band.
2) Solid lines are firm-level responses and shaded areas are 68th percentile confidence band.
To deal with the identification issue raised in Gandhi, Navarro, and Rivers (Forthcoming), I estimate the production function with a dynamic panel data method (Blundell and Bond, 1998) that allows autocorrelation in the error term. Figure 2.7 shows that most impulse responses are similar to the main responses. However, there is no significant difference among different size groups in the response to aggregate monetary policy shocks.

![Figure 2.7: Markup Response to Aggregate Shocks (Blundell-Bond)](chart)

Note: 1) Solid lines are firm-level responses and dotted lines are 95th percentile confidence band.
2) Solid lines are firm-level responses and shaded areas are 68th percentile confidence band.

**Production Approach**

I further execute two tests related to the production approach. First regression is related to the concern in Karabarbounis and Neyman (2018): not adjusting the measurement error can generate a significant difference in studying the markup
Since we detrend variables, the difference in the trend may not be an issue. In line with the prior, figure 2.8 illustrates that the results are robust to this margin. However, there is no significant difference among size groups in response to the aggregate monetary policy shocks.

Figure 2.8: Markup Response to Aggregate Shocks (No Measurement Error)

Another concern for the production approach is the data choice of the variable cost. Traina (2018) claim that it is important to include "Selling, General and Administrative (SG&A)" cost in a variable cost for the markup trend. If we set variable cost as the sum of SG&A and cost of goods sold, there is no significant change in the markup trend. As I show above, the change in the trend does not

\[15\]

I further suspect that any estimation method would give robust results as long as the coefficients are fixed over time since the output elasticity is simply a scaling of the labor cost expenditure ratio.
affect the response at the business cycle frequency.

Figure 2.9: Markup Responses to Different Variable Cost

Average Response

Size Dependent Response

Local Projection

Lastly, I check whether the shock exogenous from other shocks. Specifically, I include the two shocks in one regression and check the coefficients. Figure 2.10 shows that the results are almost identical to the main impulse responses. Therefore, the shocks I use are independent each other.
2.3 Model

In this section, I propose a model that captures the salient features of the data. The model has firm dynamics with endogenous entry and exit (Hopenhayn, 1992; Clementi et al., 2014) and households’ external habit formation on individual goods (Ravn et al., 2006). I consider the external habit formation as a customer base. The customer base means a group of loyal customers who buy the good repeatedly, and, in the model, the customer base is the sum of the past sales quantity after depreciation.

The model is different from a standard firm dynamics model in demand specification (figure 2.11). In a standard firm dynamics model, a firm is a productivity and factor input such as labor and capital. In this paper, a firm is a productivity
and a customer base. Demand is derived from the households’ optimization and positively correlated with the customer base. Therefore, firms want to accumulate the customer base to grow further. To grow further, firms invest in a demand base using single prices.

Figure 2.11: Model Comparison

<table>
<thead>
<tr>
<th>Standard Model</th>
<th>Born</th>
<th>Hire more input</th>
<th>(...)</th>
<th>Maximum Size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Prod, Input&lt;sub&gt;0&lt;/sub&gt;)</td>
<td>(Prod, Input&lt;sub&gt;1&lt;/sub&gt;)</td>
<td>(...)</td>
<td>(Prod, Input&lt;sub&gt;max&lt;/sub&gt;)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>My Model</th>
<th>Born</th>
<th>Sell more goods</th>
<th>(...)</th>
<th>Maximum Size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Prod, C-base&lt;sub&gt;0&lt;/sub&gt;)</td>
<td>(Prod, C-base&lt;sub&gt;1&lt;/sub&gt;)</td>
<td>(...)</td>
<td>(Prod, C-base&lt;sub&gt;max&lt;/sub&gt;)</td>
</tr>
</tbody>
</table>

Firms’ pricing problem is dynamic since firms have to compare the value of the current profit to that of the customer base due to deep habits. In addition to deep habits, there is an endogenous exit. The endogenous exit implies that firms’ survival probability changes depending on the amount of the customer base. A firm with a greater customer base can survive longer upon a series of adverse shocks since it still has loyal customers. Therefore, the customer base serves as a demand base as well as insurance (Gilchrist et al., 2017). In the model, the insurance role of the customer base changes the planning horizon of a firm. Therefore, the value of the customer base changes, which affects markup decisions.

2.3.1 Setup

(Environment) The following three types of agents exist in the economy: a continuum of identical households, a continuum of heterogeneous incumbent firms, and a continuum of ex post heterogeneous potential entrants. Households consume a product, supply labor, and trade bonds. Firms produce goods, hire labor, set prices, and build customer capital. Incumbent firms are heterogeneous to
productivity and the customer base. Entrants are ex ante homogeneous but differ after they draw random idiosyncratic productivity.

Agents interact in the three markets: a monopolistically competitive goods market, a perfectly competitive labor market, and a complete financial market. Three idiosyncratic shocks and two aggregate shocks are considered. The idiosyncratic shocks are exogenous exit shock ($\vartheta_{it}$), operating cost shock, and productivity shock. The two aggregate shocks are TFP shock and monetary policy shock.

**Preference** Preference depends on the brand equity of each product (Ravn et al., 2006). Specifically, households form a past external habit regarding an individual product, which is often called “Catching up with the Joneses.” Households are denoted by $j \in [0, 1]$ and consume a variety of consumption goods indexed by $i \in [0, M_t], M_t < 1$.

$$U_{jt}^t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{1}{1-\sigma} \tilde{c}_{jt}^{s-\sigma} - \omega n_{jt} \right]$$

where

$$\tilde{c}_{jt} = f(\{c_{ijt}, h_{it-1}\}_i)$$

$\tilde{c}_{jt}$ is a habit-adjusted consumption bundle.

The fact that preference depends on external habit implies that there is no time consistency concern since atomistic households cannot affect the aggregate habit for each good\(^\text{16}\). I view external habit as a brand equity. Given that my dataset is firm level, external habit is more consistent with the dataset. The habit-adjusted consumption basket depends on a predetermined level of habit, which implies that a good’s market demand depends on sales history. Thus, firms compare the future benefit of the current profit to the benefit of the customer base.

**Technology** Firms produce goods using a constant return to scale technol-

\(^{16}\) See Nakamura and Steinsson (2011), Rudanko (2017) for a detailed discussion.
ogy\textsuperscript{17}, and labor is the only input.

\[ y_i = e^{A_i}e^{z_i}n_i \] (2.1)

where aggregate productivity \( A \) is an AR(1) process, idiosyncratic productivity \( z_i \) is an AR(1) process, and \( n_i \) denotes labor input. I discretize idiosyncratic productivity with seven grids.

\[ A_{i,t} = \gamma A_{i,t-1} + \epsilon_A, \epsilon_A \sim N(0,\sigma_A) \] (2.2)
\[ z_{i,t} = \gamma z_{i,t-1} + \epsilon_z, \epsilon_z \sim N(0,\sigma_z) \] (2.3)

Firms face independent and identically distributed (IID) random operating costs \( (\zeta_i) \).

\[ \zeta_i \sim N(\mu_\zeta,\sigma_\zeta) \] (2.4)

The operating cost captures any shock on a firm’s cash flow. I note that the operating cost is not a state variable since it is IID.

(Firm Dynamics) A fixed number\( (= \Psi) \) of potential goods exists (Clementi et al., 2014). Potential entrants are determined as the number of potential goods in the economy after the incumbent’s exit decision \( (M_a = \Psi - \int_0^{M_a} d\tilde{i}) \textsuperscript{18} \). The timing of events is as follows (figure 2.12). First, incumbent firms produce. Second, incumbent firms draw operating costs. Third, incumbent firms exit if they are hit by an exogenous exit shock or the value of the firm is lower than the operating costs. Fourth, entrants draw their productivity. Fifth, entrants decide to enter or not.

(Households’ Problem) Given a homothetic and weakly separable preference, one can consider households’ problems as a two-stage budget problem. In

\textsuperscript{17} This assumption guarantees that size-dependent pricing is due to the demand factor, in contrast to Hong (2019) and Gilbukh and Roldan (2017).

\textsuperscript{18} One can simplify the model slightly and assume a fixed mass of potential entrants.
the first-stage problem, households choose the amounts of habit-adjusted consumption basket ($\tilde{c}$), labor ($n^j_t$), and risk-free bonds ($b^j_t$) to maximize their discounted expected utility given the prices and aggregate state variables, $F = \{A, Q, M\}$:

$$V(F_{-1}) = \max_{\tilde{c}^j, n^j, b^j_t} \left[ \frac{1}{1-\sigma} \tilde{c}^j_{1-\sigma} - \omega n^j + \beta EV(F) \right]$$

subject to the budget constraint (Equation 2.5), and the laws of motion for other aggregate state variables (Equations 2.2, 2.6, and 2.11-13). Under the assumption that monetary authority targets real interest rates directly, I can consider $Q$ as a monetary policy shock.

$$\tilde{p}_j \tilde{c}_j + E_t\left[ \frac{b^j_t}{(1+r)e^{Q_t}} \right] = b^j + W n^j + d^j \quad (2.5)$$

$$Q_{t+1} = \gamma_Q Q_{t-1} + \epsilon_Q, \epsilon_Q \sim N(0, \sigma_Q) \quad (2.6)$$

where $\tilde{p}_j = \left[ \int_0^t (\frac{p^j_t}{b^j_{t-1}})^{1-\rho} dt \right]^{-\frac{1}{1-\rho}}$ is the habit-adjusted price and $d^j$ is the dividend.

The equilibrium conditions are

$$[\tilde{c}_j] : \quad \lambda_j = (\tilde{c}_j)^{-\sigma} \frac{1}{\tilde{p}_j}$$

$$[n^j] : \quad \lambda_j = \omega \frac{1}{W}$$

$$[b^j_t] : \quad \frac{1}{1+r} = \beta e^Q E \frac{\lambda_j}{\lambda^j}$$
where $\lambda^j$ is the Lagrange multiplier related to the budget constraint. In the second stage, households solve the following cost minimization problem given $\tilde{c}^j$ and $\{\tilde{p}_i, h_{i-1}\}_i$.

$$\min_{\{c_{ij}\}} \int_0^1 \tilde{p}_j c_{ij} \, di$$

subject to the habit-adjusted consumption bundle.

$$\tilde{c}_j = f(\{c_{ij}, h_{i-1}\})$$

(Demand Function) Using the symmetry of the households, I integrate over the individual demand function from the cost minimization problem to obtain a demand function for each good.

$$c_i = c(\frac{p_i}{\tilde{P}}, \tilde{C}, h_{i-1}) \quad (2.7)$$

where $\tilde{C} = \int \tilde{c}^j \, dj$ is the aggregate of habit-adjusted consumption and $\tilde{P} = \int \tilde{p}_j \, dj$ is the aggregate habit-adjusted price.

(Incumbent Firm’s Problem) Incumbent firms have two idiosyncratic state variables (S) and four aggregate state variables (F). The idiosyncratic state variables are its productivity and customer capital, and the aggregate state variables are two aggregate shocks (supply and demand) and the distribution of firms.

The current customer base (h) is the sum of the customer base depreciated from the last period and the fraction of the current period quantity sales.

$$h_i = (1 - \delta) h_{i-1} + \delta c_i \quad (2.8)$$

$\delta$ is a measure of how fast the customer base adjusts. Since the acquisition and the depreciation of customer capital are at the same speed, the maximum amount of the customer base is equal to the output ($h_i = y_i$).

A collection of households owns firms, and the demand function is derived
from the households’ problem. Now, I am ready to define the incumbent firm’s problem. Firms maximize the discounted stream of habit-adjusted real profit.

\[
V(z_{-1}, h_{-1}; F_{-1}) = \max_{p_i, h_i, n_i, y_i} \left\{ \frac{p_i}{P} y_i - \frac{W}{P} n_i + \max_{\text{exit, stay}} \left[ 0, -\frac{\epsilon}{P} + \frac{1}{1 + r} EV(z, h; F) \right] \right\}
\]

subject to the production function (Equation 2.1), operating cost distribution (Equation 2.4), demand function (Equation 2.7) and the laws of motion for the state variables (Equations 2.2, 2.3, 2.6, 2.8, and 2.11-13). I note that \( F = \{ M, A, Q \} \) represents aggregate state variables and \( \vartheta \) is exogenous exit shock. I also note that labor is static input given output choice.

A cut-off level of operating cost is the level that equates the habit-adjusted real operating costs and the discounted value of the next period.

\[
\frac{\zeta^*}{P} = \frac{1}{1 + r} EV(S'; F')
\]

Survival probability \( (G(\zeta^*)) \) is obtained using the property of log-normal distribution.

\[
G(\zeta^*) \equiv \Pr(\zeta \leq \zeta^*) = \Phi(\zeta \leq \frac{\log(\zeta^*) - \mu_{\zeta}}{\sigma_{\zeta}})
\]

where \( \Phi \) is a standard normal distribution.

(Entrants’ Problem) After the production and exit decision of incumbents, the potential entrants make their entry decision. Entrants draw their productivity from the long-run distribution of the idiosyncratic productivity process.

For entry, potential entrants have to choose their initial advertising to set the initial customer base. I assume that each advertisement is a posting that contains information about the presence of a good in a market. All consumers are aware of the product once the advertisement is out, but only a fraction of consumers are attracted to the good by the advertisement. Therefore, the amount of advertising labor input determines the quality of the advertisement, which determines
the initial customer base. Alternatively, one can assume that only a fraction of customers can see the advertisement, and the advertising labor input determines the amount of advertisement.

\[ h_{k,0} = \alpha_1 y_{k,a} \]  

(2.9)

The advertisement production function is in a generic form.

\[ y_{k,a} = a(z_k, n_{k,a}) \]  

(2.10)

If the expected value from entry exceeds the sum of the advertising cost, aspiring entrants enter.

\[ \hat{V}(z_k; F_{-1}) = \max_{\text{enter, not}} \{ \max_{h_{k,a},0} \{ -\frac{W}{P} n_{k,a} + \frac{1}{1+r}EV(z_i, h_{k,0}; F) \}, 0 \} \]

subject to the initial customer base condition (Equation 2.9), advertising production (Equation 2.10), and the constraints that incumbents face (Equations 2.1-8 and 2.11-13). The optimal level of the initial habit, \( h_0^*(z; F) \), is implicitly defined by equating the value of entering and not entering.

\[ \frac{W}{P} n^*(z_i, h^*_0) = \frac{1}{1+r}E_{F'}V_{t+1}(z'_i, h^*_0; F') \]

(Distribution Updating) The firm distribution \( M \) is updated by exogenous productivity shock, endogenous habit choice, and firm entry and exit. The current mass of a firm is the sum of survived incumbents and new entrants.

\[ M(S; F) = M_i(S; F) + M_e(S; F) \]

(2.11)

\[ M_i(S; F) = (1 - \theta)G(\zeta^*) \int \int 1(z = z)1(h = h^*)dM(z_{-1}, h_{-1}; F_{-1}) \]

(2.12)

\[ M_e(S; F) = M_a \int 1(z = z)1(h = h^*_0)1(\frac{W}{P} n_i \leq -\frac{\kappa}{P} + \Lambda E_{F'}V_{t+1}(z_i, h^*_0; F))dG(z) \]

(2.13)

where \( M_e \) denotes the actual entrants’ distribution, \( M_a = \Psi - M_i \) is the aspiring entrants, and \( h^*_0(z_{-1}; F) \) and \( G(\zeta^*) \) are implicitly defined by the exit and entry
2.3.2 Equilibrium

Recursive monopolistic competition equilibrium with entry and exit of firms consists of \( \{p_{it}\}_i, W_t, r_t, \{c_{ijt}\}_i, n_{jt}, b_{j,t+1}\}_j, \{V_t, p_{it}, h_{it}, y_{it}, n_{it}\}_i, \{\tilde{V}_t, y_{a,k,t}, n_{kt}, h_{0,k,t}\}_k, \)
and \( \{M_t, M_{i,t}, M_{e,t}\} \) such that

1. Households maximize their utility and observe their budget constraint.

2. Policy functions \( \{p_{it}, h_{it}, y_{it}, n_{it}\}_i \) and the exit condition solve incumbents’ problem.

3. Policy functions \( \{y_{akt}, n_{kt}, h_{0kt}\}_k \) and the entry condition solve entrants’ problem.

4. Incumbents exit if operating cost is higher than the expected next period value, i.e., exit if \( \zeta < \zeta^* \), where \( \frac{\zeta}{P_l} = \frac{1}{1+r} EV(S'; F') \)

5. Entrants enter if the value of entry is higher than the entry cost. i.e., \( \frac{W}{P} n^*(z_i, h_0^*) > + \frac{1}{1+r} EV(V_{t+1}(z_i', h_0^*; F')) \)

6. Distributions \( (M_t, M_{i,t}, M_{e,t}) \) satisfy the law of motion.

7. All markets clear.

2.3.3 Functional Forms

In this section, I specify functional forms for quantitative analysis. First, habit-adjusted consumption basket is

\[
\tilde{c}_j = \left[ \int_1^j (c_{ij} h_{i,-1}^{\theta_1}) \frac{\theta_1}{\theta_1 - 1} \frac{\rho}{\rho - 1} \frac{P}{P_l} \right]^{\frac{\rho}{\rho - 1}}
\]

where \( \theta_1 \) represents the degree of habit that is price elastic and \( \rho \) indicates the elasticity of substitution. It gives the following demand function:

\[
c_i = \left( \frac{P_i}{P} \right)^{-\rho} \tilde{c}_i h_{i,-1}^{\theta_1 (\rho - 1)}
\]
Two remarks are relevant here. First, I note that the model is equivalent to a standard real business cycle model if I turn off the habit by setting $\theta_1 = 0$. Second, the price elasticity of demand is fixed to $\rho$, unlike many models, in which the price elasticity of demand is a function of market share.

To produce advertising, firms use labor and productivity $^{19}$.

$$y_{i,a} = e^{z_i} n_{i,a}^{\alpha_2}$$

2.3.4 Computation Approach

To solve the model, I first find the steady state of the model and use the first-order perturbation to analyze the aggregate dynamics (Reiter, 2009).

To solve the model at the steady state, I discretize the state space. I choose seven grids for productivity and 100 grids for habit. The productivity grids are chosen to be equally distanced within the range of three standard deviations to both sides. For habit, I ensure that firms’ choice is within the bound, and the grid is chosen to be exponentially distanced.

I use the following a procedure to solve the model$^{20}$. First, I guess the aggregate habit stock ($\tilde{C}$). Second, I solve the incumbent firm’s problem. To solve the problem, I first approximate the value function by using the Chebyshev polynomial for computational efficiency.

$$V(z_i, h_{i,-1}; F) = \sum_{a=1}^{n_a} \sum_{b=1}^{n_h} \theta_{a,b}^v T_a(z) T_b(h_{-1})$$

With the approximated value function, I find the habit choice.

$$h^* = h \left\{ \mu \left( \frac{p_i y_i - W_n_i}{P} \right) + \max_{\text{exit, stay}} \left[ 0, -\frac{\zeta}{P} + \beta e^Q EV'(z_i', h_i; F') \right] \right\}$$

$^{19}$ I find advertising production function has a decreasing return to scale technology (Sutton, 1991, Arkolakis, 2010). This response can be due to media saturation or different tendencies to view ads among households (Grossman and Shapiro, 1984).

$^{20}$ I leverage on some of the routine from Winberry (2016) for computing the steady state.
Then, I iterate the policy function many times to find the value function. I iterate the obtained value function until it converges. Then, I approximate the value function and the habit choice function by using the Chebyshev polynomials. With the approximated value function, I solve the entrant’s problem by using the approximated value function. I update the distribution and iterate until the aggregate habit adjusted consumption ($\tilde{C}$) converges. I use collocation to approximate the Bellman equation and Gauss-Hermite quadrature to evaluate the expectation concerning idiosyncratic shocks.

To find the dynamics to aggregate shocks, I use a projection and perturbation approach (Reiter, 2009). Let

$$V(z_i, h_{i-1}; F) = \sum_{a=1}^{n_z} \sum_{b=1}^{n_h} \theta_{a,b}^u T_a(z) T_b(h_{i-1})$$

$$V(z'_i, h_i; F') = \sum_{a=1}^{n_z} \sum_{b=1}^{n_h} \theta_{a,b}^{u'} T_a(z') T_b(h)$$

I can then write the system of equations in a Schmitt-Grohe and Uribe (2004) form:

$$\text{E}_{\tilde{F}_t}[f(X_t, X_{t-1}, Y_t, Y_{t-1})] = 0$$

where $X = \{V, \tilde{C}\}$, $Y = \{M, A, Q, \varphi\}$ and $f()$ are the equations that subtract the left-hand side from the right-hand side at the system of equations in the next section. I numerically differentiate the system around the steady state to study the impulse response with respect to the aggregate shocks.

### 2.3.5 Calibration

I set the parameters in the model in three steps. I first calibrate certain parameters based on external information. I then calibrate the other parameters, except for the standard deviations of the aggregate shocks, to match the moments at the steady state. Lastly, I simulate the model to calibrate the standard deviations of aggregate shocks to match the moments.

---

21. I modify some of the routine from Bayer and Luetticke (2018) to compute aggregate dynamics.
I provide a detailed explanation for choosing specific parameter values (table 2.1). The first set of parameters is related to preference. Time is quarterly; therefore, I fix the discount factor ($\beta$) at 0.99 to set an annual interest rate of 4%. I then fix the consumption smoothing parameter at two, which is in the mid-range of Attanasio and Weber (1993). I set the habit depreciation parameter ($\delta$) to 0.04, which implies an approximately 15% depreciation in the customer base at the annual level. This level is used in Gourio and Rudanko (2014) based on the literature, in which the turnover rate for the cell phone industry is approximately 11% to 26% and the turnover rate for the banking industry is approximately 10% to 20%. The estimates for the elasticity of the substitution parameter ($\rho$) vary significantly by the type of products. I use 3.3, which is in the mid-range of median elasticity of finely separated products from Broda and Weinstein (2006). They estimate the cross-elasticity of goods using the Feenstra (1994) decomposition and data for newly introduced goods from highly disaggregated US import data.

Then, I calibrate the persistency of shocks. For idiosyncratic productivity, little agreement on the estimates exists. I regress physical productivity, which I identify from the COMPUSTAT data, on its lag. Then, I convert the coefficient to fit the quarterly frequency, which is 0.84. This value is in the range of Cooper, Haltiwanger, and Willis's (2015) estimates. For the aggregate shock persistence, I take the estimates from Smets and Wouter (2003).

I then calibrate the following eight parameters to match the moments (table 2.2). I note that all the parameters are calibrated simultaneously since all parameters affect all the moments.

The model can match the data fairly well\(^{22}\) (table 2.3). For the labor disutility parameter, I target the value of habit-adjusted real wage to be normalized to one. For the habit parameter, I aim for the markup level from the COMPUSTAT data

\(^{22}\) The estimates are from the seven- to ten-digit code level of goods.

\(^{23}\) Given the nonlinearity of the model, it is difficult to match the moments exactly.
Table 2.1: Fixed Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Explanation</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta = 0.99$</td>
<td>discount factor</td>
<td>Interest rate $\approx 4%$</td>
</tr>
<tr>
<td>$\sigma = 2$</td>
<td>intertemporal subs.</td>
<td>Attanasio and Weber (1995)</td>
</tr>
<tr>
<td>$\delta = 0.04$</td>
<td>habit depreciation</td>
<td>Gourio and Rudanko (2014)</td>
</tr>
<tr>
<td>$\rho = 3.3$</td>
<td>elasticity of subs.</td>
<td>Broda and Weinstein (2006)</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_z = 0.84$</td>
<td>idio productivity persistence</td>
<td>COMPUSTAT</td>
</tr>
<tr>
<td>$\gamma_A = 0.823$</td>
<td>AGG TFP persistence</td>
<td>Smets and Wouter (2003)</td>
</tr>
<tr>
<td>$\gamma_Q = 0.855$</td>
<td>Bond shock persistence</td>
<td>Smets and Wouter (2003)</td>
</tr>
</tbody>
</table>

Table 2.2: Matched Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega$</td>
<td>labor disutility</td>
<td>0.068</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>degree of habit</td>
<td>0.310</td>
</tr>
<tr>
<td><strong>Firms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>idio productivity std</td>
<td>0.022</td>
</tr>
<tr>
<td>$\mu_\zeta$</td>
<td>operating cost (log mean)</td>
<td>-6.195</td>
</tr>
<tr>
<td>$\sigma_\zeta$</td>
<td>operating cost (std)</td>
<td>4.546</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>advertising efficiency</td>
<td>0.143</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>advertising return to scale</td>
<td>0.153</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>potential blueprints</td>
<td>0.760</td>
</tr>
</tbody>
</table>

and match it to the COMPUSTAT equivalent firms in the model. The COMPUSTAT equivalent firms in the model are the top 30% firms in terms of labor, in which the 30% estimate comes from Davis et al. (2007). For the operating cost parameters, I target the moments related to the exit rate. Two related moments are the exit rate and the 0 to 3 year survival rate. The exit rate is obtained from business dynamics statistics (BDS). The 0 to 3 year survival rate is the average of firm birth cohort data from business employment dynamics (BED). For the advertising-related parameters, I target the ratio of entrant’s TFPQ estimated in Foster et al. (2016) and the 0 to 2 year average employment share from BDS. For the standard deviation of idiosyncratic productivity and the amount of potential goods, I target 0 to 3 year firm number share and employment share. The firm number share is from BED.

---

24 One can target other moments, for example, a 0-year employment share. The result is robust.
Table 2.3: Moments Used to Match Parameters

<table>
<thead>
<tr>
<th>Target Data Model</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>real wage</td>
<td>1.00</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td>markup level</td>
<td>1.16</td>
<td>1.14</td>
<td></td>
</tr>
<tr>
<td>entrance TFPQ</td>
<td>1.02</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>exit rate</td>
<td>10.73%</td>
<td>10.22%</td>
<td></td>
</tr>
<tr>
<td>0-3yr survival rate</td>
<td>53.85%</td>
<td>52.80%</td>
<td></td>
</tr>
<tr>
<td>0-3yr firm number share</td>
<td>31.90%</td>
<td>30.37%</td>
<td></td>
</tr>
<tr>
<td>0-3yr employment share</td>
<td>11.24%</td>
<td>12.80%</td>
<td></td>
</tr>
<tr>
<td>0-2yr employment share</td>
<td>8.63%</td>
<td>9.34%</td>
<td></td>
</tr>
</tbody>
</table>

Note: 1) Normalization
2) COMPUSTAT data to COMPUSTAT equiv. firms
3) BDS exit rate for all firms, 4) LBD average
5) BED firm numbers share, 6) BDS Employment share for firm age

(Standard Deviations of the Aggregate Shock Process) To calibrate the standard deviation of the aggregate shock parameter, I simulate the model to match two moments. I simulate the economy for 200 quarters including 30 quarters of burn-in periods. The targets are the standard deviation of GDP and hours worked. GDP is used to capture the productivity shock process, and hours worked considers the demand shock.

Table 2.4: Moments to Match Aggregate Shock Volatility

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\text{TFP}}$</td>
<td>0.028</td>
<td>std(GDP$^1$)</td>
<td>0.033</td>
<td>0.022</td>
</tr>
<tr>
<td>$\sigma_{Q}$</td>
<td>0.021</td>
<td>std(N$^1$)</td>
<td>0.013</td>
<td>0.023</td>
</tr>
</tbody>
</table>

Note: 1) Detrended using quadratic time trend after log.

2.3.6 Age Dependent Growth Rate

In this subsection, I show that the model can match untargeted moments well, especially the moments related to age. The model can endogenously match the labor growth rate of firms conditional on age (figure 2.13). This is a success of the model since literature tends to have difficulty in matching these moments.

I calculate the growth rate of each age by taking an average of each cohort at
the given age using all available cohorts (solid green line). The empirical literature such as Foster et al. (2016) finds that the demand constraint is a crucial factor in slowing firm growth. However, models with capital accumulation, for example, the New Keynesian models with capital, such as that of Ottonello and Winberry (2019), cannot match this margin. My model matches the growth rate of firms without assuming any arbitrary adjustment cost. Although I do not target any moments, the blue dashed line (the model result) closely tracks data.

2.4 Results

2.4.1 Incumbent firms’ Markup Determination

In this subsection, I present an analytic equation that describes incumbent firm’s markup decision. For simplicity, I impose full depreciation of the customer base each period ($\delta = 1$)\(^{25}\). The incumbent firm’s markup determination equation is as follows:

$$\mu_{it} = \mu^* - \theta_1 E_t \frac{1}{1 + r_{t,t+2}} \left( G(c_{st}) \frac{m_{c_{it+1}}}{m_{c_{it}}} c_{it+1} c_{it} ^{\mu_{it+1}} \right)$$

\(^{25}\) In the Appendix A, I provide the equation without the full depreciation of the customer base.
where $\mu_{it}$ is markup, $\mu^* = \frac{G}{p-1}$ is the markup in a standard model and $G(\zeta_{it+j})$ is survival probability. The equation indicates that incumbent firms charge markups by comparing the current profit and the future value of the customer base. I explore each of these factors in detail.

I first document that the markup in this model is lower than in a standard model ($\mu^*$). This is precisely due to the forward-looking term coming from introducing habit. I note that the second term is zero if I shut down the habit ($\theta_1 = 0$). Therefore, firms decrease their markup to attract more customer base relative to the standard monopolistic competitive setting. This result is slightly different from the common view that large firms charge high markup to exploit the market power. My theory shows instead that firms charge lower markup to attract more customers (Ravn et al., 2007). Since the price elasticity of demand is constant, firms never charge higher markup than the standard markup level.

The intertemporal channel (1) is akin to the one in the representative agent deep habit model (Ravn et al., 2006). Firms consider the future benefit of investing in a customer base when they set prices. Therefore, the change in the value of the future customer base affects the current markup decision. Relative to the standard deep habit model, this paper describes an additional effect that comes from the change in firm distribution.

The survival probability channel (2) exists since firms’ planning horizon changes as the exit probability varies. In contrast to Gilchrist et al. (2017), this paper describes a distribution effect, which plays an important role when there is productivity shock. To illustrate the quantitative importance of this channel, I show the steady-state level of the exit risk, i.e., $1 - G(\zeta_{it}^*)$, in figure 2.14. I first notice that the model closely matches the data even if I only target the aggregate exit rate only. Therefore, the model can match the selection of a firm. The figure furthermore shows that the exit risk quickly drops as a firm grows given the fact that young firms start small. This finding implies that the incentive for growth is much higher than in the standard model.

If I add an additive habit term to the demand function, which changes the price elasticity of demand, big firms can charge higher markups than the standard markup level.
stronger for small firms than for big firms. The yellow dash-dot line shows that the customer stock is essential in matching the data.

The productivity channel (3) shows that the high-productivity firms charge low markup. Since the productivity processes revert to the mean, firms want to accumulate the customer base when their marginal cost is lower than the long-run level. This condition implies that markup goes down when there is positive idiosyncratic productivity shock\(^{27}\) (figure 2.15). This finding is in contrast to the search theoretic customer base models such as Gourio and Rudanko (2014). In Gourio and Rudanko, the positive productivity shock increases firms’ capacity to produce, but the customer base constrains the sales due to the convex adjustment cost. Thus, firms increase both prices and increase sales efforts. This congestion effect comes from the adjustment cost from which my model is free.

The output growth channel (4) captures the change in markup as a firm approaches its optimal size. Upon favorable aggregate shocks, the gap between the customer base and the current sales decreases since firms grow faster. Therefore, this channel contributes to the procyclical markup counterbalancing the survival probability channel.

\(^{27}\) For this experiment, I assume all firms in the 2\(^{nd}\) lowest idiosyncratic productivity group to have 2\(^{nd}\) highest productivity group. Then, I calculate the percent deviation from the initial average markup level.
Figure 2.15: Markup Response to 1% Idiosyncratic Productivity Shock

![Graph showing the markup response to a 1% Idiosyncratic Productivity Shock.]

Figure 2.16 illustrates the steady-state markup level by productivity and customer stock. I first notice that the customer base effect pushes down the markup, as I see from the analytic equation. Apart from other customer base models, firms in my model charge markups even lower than one that I find from the data. I document that markup increases as the customer base increases, and more productive firms charge lower markup. Since survival probability changes drastically for small firms, markup changes sharply for small firms.

Figure 2.16: Markup Determination by Individual Firms

![Graph showing markup determination by individual firms.]

My results strengthen the empirical evidence in the existing literature. Foster et al. (2008) find that smaller businesses have higher productivity and lower
prices than bigger firms in manufacturing industries that produce highly homogenous goods. Dinlersoz and Yorukoglu (2012) also document, using a subset of manufacturing industries, for which physical quantity data are available, that price and firm size are positively related, while price and TFP are negatively associated. I note that the difference in price among different productivity levels is greater than the difference in markups.

2.4.2 Aggregate Dynamics

In this section, I analyze the impulse response function of the aggregate markup and GDP to one standard deviation shocks. Before I present the results, I note that there is an additional distribution effect that comes from aggregation.

\[ \mu_t = \sum_i \mu_{it} M_{it} \frac{p_{it} y_{it}}{\sum_i p_{it} y_{it}} \]

When there is an aggregate shock, distribution \((M_{it})\) changes and the weight \((\frac{p_{it} y_{it}}{\sum_i p_{it} y_{it}})\) changes. Due to the change in distribution, the aggregate markup varies when there is an aggregate shock even if there is no habit.

To investigate the role of each channel, I present the results after shutting down habit and endogenous exit with the benchmark case. I note that there is only a distribution effect if I shut down the habit. The survival probability channel shuts down when I set the operating cost to be zero.

(Aggregate Productivity Shock) I find countercyclical markup to aggregate productivity shock (the right panel of figure 2.17) since firms invest in the customer base using prices. When the current marginal cost is lower than the future marginal cost, firms want to expand their customer base. Therefore, firms charge lower markup to increase sales quantity. This mechanism is different from Jaimovich and Floetto (2008), in which more entry of firms causes greater competition. In my model, the exit rate is stable or slightly increases upon positive
supply shock since big firms charge lower markup, pushing out small firms. Another interesting contrast is to switching cost-type models that generate procyclical markup to productivity shocks (Gilbukh and Roldan, 2017). In Gilbukh and Roldan, given state-contingent contract and risk-neutral preference, markup plays only an allocative role; price adjusts to transmit the effect of a shock to the customers.

Figure 2.17: Aggregate Response to TFP Shock

(Monetary Policy Shock) Under the assumption that a central bank targets the real interest rate directly, I interpret bond price shock as monetary policy shock. I find that the markup is countercyclical when there is a monetary policy shock (the right panel of figure 2.18). I further observe that the output response is stronger if the markup is more countercyclical to the monetary policy shock (the left panel of figure 2.18). When the current demand is higher than the future demand, it is a good time for firms to decrease markup since firms invest in the customer base using prices. Markup is countercyclical to a demand shock in a search theoretic customer base model such as Paciello et al. (2017). The key mechanism that generates countercyclical markup is the incentive to increase the customer base in their work as well.

28 exit risk dynamics are shown in Appendix A.
29 In their model, the promised utility of the match determines everything instead of markup.
30 I note that, in the empirical section, my measure for monetary policy shock is essentially bond price shock.
(Size-Dependent Response) I analyze the response of markup dependent on the size of the customer base by comparing COMPUSTAT equivalent firms to all firms. COMPUSTAT equivalent firms are large firms in the model in terms of labor following Davis et al.’s (2007) estimates. I also note that I provide unweighted markup within the group to be consistent with the data.

I find that firm-level markups for COMPUSTAT firms (dotted red line) are countercyclical to productivity and monetary policy shocks, consistent with the data (figure 2.19). For positive productivity shock, big firms have less incentive to grow further by lowering markup since it is more costly for them given the large demand
base. Therefore, big firms decrease markup less than small firms. For positive monetary policy shocks, the survival probability channel is the underlying mechanism. Since the exit risk of small firms increases more than that of big firms, small firms charge even lower markup than big firms\textsuperscript{31}.

2.5 Extension: Financial Shock

I can include other aggregate shocks to the data analysis and the model. As an example, I include financial shock. In the empirical framework, I take the mortgage market intervention shock as a financial shock (Fieldhouse, Mertens, and Ravn, 2018). The shock is the noncyclically motivated projected portfolio change of government agencies. The development of the narrative instrumental variable follows five steps. First, Fieldhouse et al. (2018) narratively identify policy changes that significantly affect future agency portfolios. Second, they quantify agencies’ ex ante projected impact on agency mortgage holdings. Third, they pinpoint the timing of when the policies became publicly known. Fourth, they classify each policy change as either cyclically or noncyclically motivated. Fifth, they exclude the announcement with very long delay in implementations. After identifying the extent of a noncyclical portfolio change, I first sum up the amount of portfolio change in the year to create an annual series. I then scale them by dividing the shock by the amount of mortgage origination in the year.

I find that firm-level markups are countercyclical to the financial shock (figure 2.20). Furthermore, small firms have more countercyclical markup. An interesting feature is that markup goes down by a small amount for a short time.

In the model, the financial shock ($\varphi$) is a shock on a firm’s cash flow. The shock shifts the mean of the operating cost distribution.

$$\zeta_i \sim N(\varphi \mu_\zeta, \sigma_\zeta)$$

\textsuperscript{31} If I include exogenous exit and shut down the endogenous exit, the size-dependent markup response to monetary policy shock closes down. The result can be provided upon request.
Figure 2.20: Firm-level Markup Cyclicality to the Financial Shock

Note: Solid lines are firm-level responses; dotted lines represent 95th percentile confidence intervals, and shaded areas are 68th percentile confidence intervals.

\[ \phi' = \rho \phi + \epsilon \phi \]

The impulse response analysis demonstrates the similar response to the data (figure 2.21). On impact, the markup goes down and GDP goes up. However, the change is small and quickly changes in sign. This result is due to the laggard firms. Given the expansionary financial shock, inefficient firms can accumulate more customer base, which allows them to survive extended periods of time. Therefore, despite the initial boom, the GDP goes down and markup goes up since low-productivity firms charge higher markup and remain small.

Figure 2.21: Aggregate Responses to the Financial Shock

Note: The shock is normalized to one percent.
2.6 Conclusion

How does firm-level markup change with the business cycle? To answer this question, I study the aggregate dynamics upon productivity and monetary policy. I further investigate how the markup response differs according to the size of a firm.

I document that firm-level markup is countercyclical to productivity and monetary shocks. The model shows that the customer base factor plays a crucial role in generating this pattern both quantitatively and qualitatively. Furthermore, the demand accumulation mechanism amplify monetary policy shock more significantly. I then proceed one step further and show that the markup of a small firm responds more strongly to aggregate shocks. Using the model, I show that customer stock plays a key role in generating countercyclical markup in response to aggregate shocks. The model also illustrates that the size-dependent markup cyclicality comes from the size-dependent sensitivity in the value of the customer base. Moreover, I show that the proposed model can match the individual firm growth speed and turn over without any adjustment cost.

More broadly, this paper emphasizes the role of intangible capital in understanding the behavior of firms. Despite progress in this paper and other literature, identifying intangible capital empirically and understanding what it does theoretically remain important future research topics.
Chapter 3

Short-run Firm Responses to Corporate Tax Shocks

3.1 Introduction

At least from Harberger (1962), the effect of tax shocks is one of the key subjects in the profession. Thanks to the progress, we can analyze the effect of tax reform in a much-sophisticated manner. However, due to the complexity of the tax policy, there is still much to be done. For example, empirical literature tends to focus on one specific tax reform event to obtain clean identification. The cost of this approach is the applicability: it is hard to extrapolate the one event to the present reform. To fill this gap, I exploit an exogenous shock series to overcome endogeneity and study the effect of tax shocks at the firm-level over the business cycle.

I document how the cumulative change in firm-level tax due to exogenous tax shocks affects the cumulative change of the variables of interest. The empirical framework is a panel version of local projection with instrument variables (LP-IV), similar to Ramey and Zubairy (2018), and Fieldhouse, Mertens and Ravn (2018). Given the persistent effect of tax shocks, I want to capture the cumulative effect of shocks on the variables of interest in a more sophisticated manner. Furthermore, by studying firm-level data, I can test the results of literature that uses aggregate data at the micro level. As tax shocks, I use narratively identified corporate tax shocks from Mertens and Ravn (2013).
I first find the response of tangible and intangible capital investment to tax cut shocks. Thick amount of literature study the change of tangible capital investment; however, I am not aware of any literature that documents the response of intangible capital. If intangible capital changes significantly to tax shocks, I can argue that the effect of tax shock is under-estimated in the literature since it omits the important margin. For intangible capital, I use Selling, General, and Administrative (SG&A) costs following the literature, for example, Gilchrist, Schoenle, Sim, and Zakrajsek (2017). I empirically find that tax cut shocks boost intangible and tangible capital investment significantly. The response of tangible capital is quantitatively similar to literature that uses aggregate data and a VAR model such as Mertens and Ravn (2013).

I then document the response of labor and financial decision of individual firms. I find that firm’s labor decision to tax shock is relatively less studied. Hence, I show that a firm’s total labor costs increase to tax cut shocks, but the response is more delayed than investments. Hiring also increases with some delay. As firms increase labor and capital input, firms output, measured as real sales, increases accordingly.

The firm’s financial decision such as leverage and dividend payout is also of important interest. In line with the theoretical claim that firms decrease leverage to a tax cut for tax saving motive, I find that firms decrease leverage. Public economics literature finds a negative relationship between dividend payout and tax shock. However, I cannot find evidence that firms significantly increase dividends to tax cuts, possibly due to large standard errors.

I further study the response of measured revenue productivity, markup, and firm churn. To identify productivity and markup, I use the production approach (Hall 1986; De Loecker and Warzynski 2012; De Loecker and Eeckhout 2017). In the short run, I find that measured revenue productivity and markup increase, but firm churn does not change much to tax cut shock. Disproportionate literature
focuses on the change in labor productivity or firm churn in the long run. While the long-run effect is interesting, the short-run response also is of independent interest. I find that revenue productivity and markup increase one percent and 0.75 percent each in the first year. From the next year, productivity increases in a humped shape, whereas markup goes down slowly. Further investigation shows some evidence that procyclical factor use is the main reason for the initial increase in productivity and markup. I moreover document that industry-level firm entry and exit are relatively stable in the short run to tax shocks.

Using the estimates, I predict the effect of the 2017 Tax Cuts and Jobs Act (TCJA). TCJA is one of the largest tax cuts in US history. However, to my knowledge, no paper shows the firm-level effect of the tax cut using a reduced-form model. I first argue that I can implement out of sample inference using my estimates since the 2017 tax cut was unanticipated, exogenous, and similar to the past tax reform. Then, I find that my disaggregated estimate results are quantitatively similar to the Mertens and Ravn (2013, hereafter MR) estimate that uses a structural VAR model with aggregate data. Since my estimates are in line with the MR estimate, I argue that the estimate of Mertens (2018) is supported at the micro-level. Mertens (2018) finds that the 2017 tax cut can increase GDP approximately two percent increase of GDP in the first year and disappears.

This paper is related to the study of firm-level responses to corporate tax shocks. The investment response to tax shocks is a key subject of interest in the literature. Many theoretical studies focus on Tobin’s Q theory or the cost of capital, such as Hall and Jorgenson (1967), Tobin (1969), and Hayashi (1982). Caballero and Engel (1999) further introduce an adjustment cost to model the lumpiness of investment. Related empirical studies include Summers (1981), Cummins, Hassett, and Hubbard (1994), Goolsbee (1998), and Desai and Goolsbee (2004). Summers (1981) empirically studies the effect of tax policy on tangible investment using Tobin’s Q theory. Cummins et al. (1994) argue that the effect of tax policy on investment is more significant if the endogeneity of Q is treated properly. Goolsbee (1998) claims that, in the short run, higher capital demand from
the investment tax policy increases the capital price, limiting the effect of the tax cut. This paper adds to the literature by providing average firm-level responses over the business cycle to corporate income tax in a reduced form model. For intangible investment response to tax shocks, Bhandari and McGrattan (2020) argue that the inclusion of intangible capital is quantitatively crucial in analyzing fiscal policy shocks using a structural model. This paper complements Bhandari and McGrattan by adding reduced-form estimates.

The effect of tax policy on employment is relatively less studied. Monacelli, Perotti, and Trigari (2010) find a significant effect of tax shocks on unemployment using various empirical approaches. Using a VAR model with aggregate data, Colciago, Lewis, and Matyska (2017) study the effect on employment due to firm churn by tax shocks and find that the effect is delayed but significant. I also find that the response of labor use is slow but significant.

I study the effect of corporate income tax shock on a firm’s financial decisions. Heider and Ljungqvist (2015) find that leverage increases the tax hike in the US state. Ohrn (2018) finds that corporate income tax cut increases dividend payout and decreases debt. I provide additional evidence that leverage and tax shocks are positively related using new approach. However, this paper finds that the effect of corporate income tax cut to dividend payout is unclear.

I further study the short-run effect of tax shocks on productivity and firm churn. Mertens and Ravn (2011) find that tax shocks significantly affect labor productivity in the long run. Hussain (2015) documents that corporate tax shocks affect utilization unadjusted productivity in the short run with aggregate data. My results complement the literature by providing empirical estimates at the firm level. For firm entry and exit, Colciago et al. (2017) show that firm entry and exit are relatively stable using aggregate data. I study the response of firm entry and exit to tax shocks at the industry level differently from the literature.

This paper studies the effect of the 2017 tax cut. Barro and Furman (2018)
use a plain-vanilla neoclassical model to study the long-run effect of the 2017 TCJA. On the other hand, Mertens (2018) studied the short-run effect of TCJA using a reduced-form model, found a significant effect of the tax reform, and argued that the response is front-loaded. Sedlacek and Sterk (2019) and Erosa and Gonzalez (2019) show the lifecycle effect of tax shocks and emphasize the importance of firm dynamics. This paper adds to this line of literature by providing short-run firm-level responses using a reduced-form model.

The paper is related to the studies that use a narratively identified shock to investigate the effect of a tax change. Mertens and Ravn (2012) distinguish the anticipated and unanticipated tax shocks from Romer and Romer (2010). MR (2013) further refine the Romer and Romer’s unanticipated shock with additional information sources. I study the response of micro-level data such as firms or industries relative to MR and subsequent studies using the MR tax shocks. By exploiting cross-sectional data, I can control for idiosyncratic factors. Moreover, my approach is free from possible aggregation concerns.

This paper begins by explaining the data and the empirical framework. Under the framework, I demonstrate the firm’s policy change regarding investment, labor, production, and financial decisions. I then illustrate the responses of productivity, markup, and firm dynamics. Using these estimates, I provide the predicted effect of the 2017 tax cut. Finally, I conclude.

3.2 Empirical Framework

I use a carefully set reduced-form model to study the effect of tax shocks. The advantage of reduced-form models is that they rely on fewer assumptions than structural models. Given my focus on the short-run effect of the tax cut, some assumptions of the structural models, such as the expectations of future tax rates, can be hard to verify. The dynamics of the expected future tax rates are instead part of the estimation in reduced-form models.
However, the reduced form approach requires tax shocks to be unanticipated and exogenous in order to overcome endogeneity concerns. I use the narratively identified exogenous corporate shock series from Mertens and Ravn (2013). Romer and Romer (2010) investigate all major tax policy changes in the US after 1945 and set a time series of tax shocks, including personal and corporate. Since tax changes can be endogenous to the states of the economy, Romer and Romer also distinguish endogenous and exogenous changes. MR (2013) chooses exogenous tax changes from Romer and Romer and filter out anticipated tax shocks using the implementation period\(^1\) (Mertens and Ravn, 2012).

I use Jorda (2005) style local projection (LP) with panel data to study the firm-level response to the corporate tax shocks. Apart from the conventional impulse responses function (IRF) of a VAR model that gives global IRF, local projection directly estimates the IRF locally. To be specific, a VAR model provides global IRF once the model is estimated, whereas local projection estimates each point of IRF separately. Since it is free from a VAR structure in obtaining IRFs, local projection provides nonlinear impulse responses, and it is orthogonal to specification bias.

I further implement the local projection instrumental variable (LP-IV) approach to tackle the measurement error. Narratively identified shocks should be considered as the best effort to identify the shocks since it sometimes requires a difficult judgment call, as well as it only considers major events. Therefore, the use of instrumental variable helps to deal with the measurement error problem in a narratively identified shock\(^2\).

I study the cumulative responses to the tax shocks. Tax reform is often persistent, and it takes time to change firm-level decisions as well as the economic environment, such as prices or firm distribution. Therefore, studying the cumulative response of interested firm-level variables to the cumulative change in the firm-level tax rate might be a good idea. To study the cumulative effect of tax

---

1. MR further make a minor adjustment to the Romer and Romer series with more data sources.
2. MR develops proxy VAR to tackle the measurement error.
shocks, I implement Ramey and Zubairy (2018) style regression with panel data. To test the robustness, I provide the results from standard local projection with instrumental variables. The results are robust, and the cumulative responses are more persistent than non-cumulative responses as I expect.

A relatively minor but complicated issue in the approach is the confidence interval. There are three concerns in the framework. Since I use the panel data, I need to consider heteroscedasticity. Furthermore, I should use autocorrelation robust standard error due to the use of local projections. Lastly, I need to test identifying power coming from the use of an instrument. In the case of the weak instrument, I need to use the identification robust confidence interval. For the heteroscedasticity and autocorrelation robust standard error, I use Arellano (1987) clustered errors at the firm level. To test the validity of the instrument, I use the Kleibergen-Paap (2007) rk\(^3\) Wald F statistic\(^4\). As a decision rule, I follow Staiger and Stock’s (1997) rule of thumb: if the F statistics are higher than ten, I consider the instrument to be strong. For most regressions that I execute, F statistics are higher than fifty. For a strong instrument case, I provide a confidence interval using Arellano. For a few regressions that suffer from weak instrument variables, I use Anderson-Rubin (1949) identification robust confidence interval.

3.2.1 Data

I use Compustat since it is open to public use. In the US, there are pass-through entities (sole proprietorships, S corporations, and partnerships) and C-type corporations. Since C-type corporations make up a large tax revenue fraction of US firms\(^5\), the use of Compustat can be a good proxy for the aggregate results\(^6\). I further note that Compustat fits my use of corporate tax shocks. The sample consists of 26,914 firms with 344,302 data points ranging from 1950 to 2006.

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\(^3\) rk stands for rank. Literature tends to use rk statistics following the Kleibergen-Paap (2007).

\(^4\) Andrew et al. (forthcoming) recommend to use the Kleibergen-Paap F statistic for weak IV test for a single endogenous instrument. Kleibergen-Paap F statistic is equivalent to Montiel Olea and Pflueger (2013) F statistic.

\(^5\) Research such as Dyrda and Pugsley (2018) shows that C-type corporations make up approximately 60 to 90 percent of total tax receipt.

\(^6\) MR compares the aggregate response between corporate tax shocks and individual tax shocks. Hence, one can extend my results to aggregate results by combining their estimates to mine.
Summary statistics for firm-level variables are in table 3.1, and detailed explanation for the variables are in the Appendix B.

<table>
<thead>
<tr>
<th>Data</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intangible Investment¹ (XSGA)</td>
<td>166.0</td>
<td>979.5</td>
</tr>
<tr>
<td>Tangible Investment¹ (CAPX)</td>
<td>81.8</td>
<td>581.4</td>
</tr>
<tr>
<td>R&amp;D¹ (XRD)</td>
<td>39.4</td>
<td>281.8</td>
</tr>
<tr>
<td>Labor Cost¹ (COGS)</td>
<td>683.8</td>
<td>4239.6</td>
</tr>
<tr>
<td>Sales¹ (SALE)</td>
<td>980.8</td>
<td>5,668.6</td>
</tr>
<tr>
<td>Number of Employees² (EMP)</td>
<td>6.9</td>
<td>28.7</td>
</tr>
</tbody>
</table>

Note: 1) Unit: million dollar  2) Unit: Thousands

I use Selling, General, and Administrative (SG&A) as an intangible capital investment (Gilchrist et al., 2017). SG&A is the cost that is not related to the actual production, which includes R&D, human capital, brand equity, and customer relationships. As a robustness test, I measure an intangible capital investment as the sum of spending on knowledge capital and organizational capital. Specifically, I use R&D + 0.3 × (SG&A - R&D) as in Peters and Taylor (2017), and Eisfeldt and Papanikoloau (2014). To identify the stock of intangible capital, I used the following law of motion:

\[ K_{\text{intang},it} = \delta I_{\text{intang},it} + K_{\text{intang},it} \]

I set \( \delta = 0.2 \), which is in common in the literature, for example, Peters and Taylor (2017). The result is robust to the use of other depreciation parameters⁷.

My instrumental variable for the firm-level tax shocks is the firm-level tax rate. I define the firm-level tax rate as the paid tax divided by the pre-tax income, which I obtain from Compustat. The left-hand panel of figure 3.1 illustrates the time-varying mean and standard deviation of the firm-level tax rate, which I calculate using the sales weight. In line with the aggregate tax rate, the firm-level tax rate shows a downward trend. However, the standard deviation is relatively stable during the sample period. The right-hand panel in figure 1 shows a histogram of

⁷ The results can be provided upon request.
the tax rate. I set the minimum tax rate is zero, and the maximum tax rate is 100.
Hence, the masses at both ends refer to firms that are with lower or higher than
the threshold firms\(^8\). I note that attenuation bias exists if I do not trim down the
outliers.

![Figure 3.1: Firm-level Tax Rate](image)

Note: Mean tax rate and standard deviations (SD) are sales weighted.

### 3.2.2 Model

I study the cumulative effect of firm-level tax changes due to tax shocks on the
variables of interest. To do so, I build a panel data model in the spirit of Ramey
and Zubairy (2018) and Fieldhouse et al. (2018). The first step regression uses
firm-level tax rate on tax shocks and control variables. For \( h = \{0, 1, 2, 3\} \), at the
first step, I regress,

\[
\sum_{h=0}^{j} \Delta \tau_{i,t+h} = \gamma_1^h + \gamma_2^h \Delta X_{i,t-1} + \gamma_3^h \text{Shock}_t + \gamma_4^h \Delta C_{i,t-1} + \gamma_5^h + \nu_{i,t+h}
\]

where \( i \) denotes an individual firm or an industry, \( \tau \) is the tax rate, \( X_{it} \) is the
variables of interest, \( C_{it} \) is the set of control variables\(^9\). This step identifies the
change in the tax level that is due to the exogenous tax shocks.

---

\(^8\) Even if I drop these outliers, the results are robust.

\(^9\) Adding more control variables does not change the results much.
The firm-level tax rate is an ideal candidate as an instrument. First, the firm-level tax rate is closely related to tax shocks that I identify (relevance condition). The corporate tax shocks also affect firm-level decisions only through the tax rate (exclusion restriction). The instrument is strong for the most of regressions except R&D and industry level of firm entry and exit.

Control variables consist of the firm-level tax rate, dividend payout, and real GDP. The lag of the firm-level tax rate captures the firm characteristics related to taxes. Dividend payout is known to play a key role in firm behavior (Chetty and Saez, 2005). Since I use the growth rate of dividend payouts, it picks up stable firms that pay out a positive dividend. Theses firms make up about 90% of the sales in the sample. GDP is included to control for possible aggregate economic conditions such as persistent aggregate shocks or business cycles. I note that the results are robust to include more control variables.

The second step regresses the cumulative growth rate of the variables of interest on the estimated cumulative difference in the tax rate and control variables. Specifically, I regress

\[
\sum_{h=0}^{j} \Delta X_{i,t+h} = \beta_1^h + \beta_2^h \Delta X_{i,t-1} + \beta_3^h \sum_{h=0}^{j} \Delta \hat{\tau}_{t+h} + \beta_4^h \Delta C_{i,t-1} + \beta_5^h + e_{i,t+h}
\]

where \(X\) is the variables of interest, \(C_{it}\) is the set of control variables from the first step, and \(\sum_{h=0}^{j} \Delta \hat{\tau}_{t+h}\) is obtained from the first step. \(\beta_3^h\) is the multiplier of the cumulative change in firm-level variables of interest due to the cumulative change in the firm-level tax rate, that is generated by the exogenous aggregate tax reform. Cumulative impulse response functions (IRFs) represent the point estimates of \(\beta_3^h\).
3.3 Results

In this section, I present the results of the regression. In the first part, I show how individual firms react to the tax shock in terms of investment, labor use, output, and finance decisions. Then, I document the response of productivity, markup, and firm turnover. Lastly, I project the quantitative micro effect of the 2017 tax cut using the estimated results and the effective tax receipt change.

3.3.1 Firm-Level Response

(Ivestment) I first study the effect of intangible and tangible capital investment. While the response of tangible capital is well documented, I am not aware of any paper that empirically shows the response of intangible capital at the individual firm level.

Recent literature emphasizes the importance of intangible capital in measuring productivity (McGrattan, 2020), investment decision (Gourio and Rudanko, 2014), firm’s lifecycle growth (Kim, 2020), and lifecycle markup (Kim, 2020; Gilbukh and Roldan, forthcoming). Bhandari and McGrattan (2020)\(^\text{10}\) demonstrate the importance of intangible capital in studying the effect of a tax change. I document the noble empirical estimates of firm-level short-run change in intangible capital investment to the tax shock. It is important to know the response of the intangible capital to tax shocks since we can omit the important margin of the effect of the tax shocks.

I find that intangible capital investment increases with the tax cut. The dotted line shows the response of intangible capital investment, and the shaded area denotes a 95\% confidence set. The left-hand panel in figure 3.2 documents that the cumulative change in intangible capital investment to the cumulative change in the tax rate shows a humped shape. For an one-percent firm-level tax rate increase due to an aggregate tax shock, intangible capital investment increases by one percent on impact. It then increases to 1.5 percent in the next year and

\(^{10}\) Their work emphasizes a pass-through entity for intangible capital investment.
gradually decreases. Firms want to increase their intangible investment since the net cost of capital decreases to a tax cut. As demand for investment increases, the price of investment goes up; hence, the investment decreases slightly. The right panel in figure 3.2 illustrates the response of intangible capital with a more stringent definition (See section 3.2.1). The response is slightly more amplified, but the two results are similar to both qualitatively and quantitatively. Significant change of intangible capital investment implies that the effect of tax shock from existing literature may be underestimated.

I find that tangible capital investment increases more than intangible capital in the left panel of figure 3.3. On impact, tangible capital increases by two percent, and the effect is more pronounced as time passes. The plot shows that the cumulative response increases to eight percent two years after the shock, and gradually disappears. Similarly, McGrattan (2012) finds that the intangible investment is more stable than tangible investment in the Great Depression although she did not isolate the response to tax shock. I further note that the result is well within the confidence interval of Mertens and Ravn (2013) with similar humped shapes. MR uses an aggregate VAR model to see the aggregate response of key variables, including physical capital investment.
I also study the component of intangible capital. SG&A consists of Research and Development (R&D), organizational capital investment (e.g., brand equity, supply chain), and some other costs unrelated to production activity. The right panel of figure 3.3 illustrates that the response of R&D is similar to the response of intangible and tangible capital investment but with a wider confidence interval. Since the regression did not go through the weak instrument test, I use the identification robust confidence interval\(^\text{11}\) (Anderson and Rubin, 1949). On impact, R&D increases approximately 1.8 percent, and the effect peaks at the two years after the shock. Based on this result, the effect of tax shock on the component of intangible capital is not concentrated on R&D, but other components as well.

To investigate why investment increases with the tax cut, I study the relation to Tobin’s Q theory (1969). The idea of Tobin’s Q is that firms invest until each dollar spent for capital purchase raises a firm’s value one dollar if there is no tax. The advantage of using Q theory is that it requires only a firm’s financial statements to test the theory. I note that I assume the average Q, the market value of the capital stock to its replacement cost, is a good proxy for marginal Q.

\(^{11}\) Andrews, Stock, and Sun (forthcoming) recommend using the AR confidence interval in the just identified models.
My measure of adjusted average Tobin’s Q includes intangible capital similar to Peters and Taylor (2017), although the result is robust to a standard definition of Tobin’s Q\textsuperscript{12}.

\[ Q_{it}^* \equiv \frac{V_{it}}{K_{\text{tan},it} + K_{\text{intan},it}} \]

where I set \( K_{\text{tan},it} \) is the stock of tangible capital and \( K_{\text{intan},it} \) is the stock of intangible capital. The mean of \( Q_{it}^* \) is approximately 1.8 and the median of \( Q_{it}^* \) is approximately 0.8\textsuperscript{13}.

I find, in figure 3.4, that intangible adjusted \( Q_{it}^* \) increases with the tax cut shock. It documents that the cumulative change in adjusted Q increases by 0.13 and persists for a while.

Figure 3.4: Intangible Adjusted Tobin Q Response to Tax Cut

Note: Shade denotes 95% Confidence Band. Standard errors are clustered by a firm.

To claim that the tax cut increases adjusted Tobin’ Q, and in turn, that a higher Q drives more investment, I have to show the latter part. I regress the following to test the Tobin’s Q equation.

\[ \frac{X_{it}}{K_{\text{intan},it-1} + K_{\text{tan},it-1}} = a_1 + a_2Q_{it}^* + a_3, it + \omega_{it} \]

\textsuperscript{12} Interested readers can see the Appendix B for the relevant results.

\textsuperscript{13} I winsorize 5 and 95 percentile firms by \( Q^* \).
where $X_{it}$ is (in)tangible investment. Since the Q variable summarizes everything, I do not include any control variables. As table 3.2 shows, I find a positive and significant coefficient for the Q variable.

<table>
<thead>
<tr>
<th>Table 3.2: Estimated Tobin Q∗ Equation</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intangible</td>
<td>Tangible</td>
</tr>
<tr>
<td>Coefficient ($a_2$)</td>
<td>0.102**</td>
<td>0.020**</td>
</tr>
<tr>
<td>S.E</td>
<td>0.051</td>
<td>0.009</td>
</tr>
</tbody>
</table>

Note: ** denotes the coefficient is significant at 5%.

In line with the literature such as Summers (1981), and Peters and Taylor (2017), the coefficient for Q is very low, probably due to the measurement error in Tobin’s Q.

(Labor Use) Labor market response might be one of the central concerns for the tax policy analysis. However, I am not aware of any literature that studies the change of firm-level labor decisions to tax shocks. The left panel of figure 3.5 shows that the response of total labor costs, or labor use, is more delayed than that of investment in line with the finding in the literature with aggregate data (Monacelli et al., 2010; Mertens and Ravn, 2013). The shock increases labor use, up to three percent for two years, and gradually disappears. Quantitatively, the response is similar to Colciago et al. (2017), who use a structural VAR model with aggregate data, including firm entry and exit.

I further study the extent to which labor cost increase is due to the extensive margin. The left panel of figure 3.5 shows that approximately half of the labor cost change is due to the change in the number of employees. However, the response of employment is somewhat faster than the change in labor cost. In sum, the tax cut increases the firm’s labor use with some delay. Also, firms increase labor both intensive and extensive margin14.

---

14 I, however, can think of some peculiar case that wage increases so much that the change of intensive margin decreases. I do not think of this case, though.
(Output) The response of firm output is of interest. For output, I deflate firm sales with GDP deflator.

I find that real sales increases to tax cuts in figure 3.6 in line with intuition. In the first year, real sales increases by 1.5 percent and the change persists for a while. Cumulative response increases approximately up to three percent after two years and decreases. The sales changes faster than the that of labor cost signals that measured markup increases to tax cuts.
(Financial Policy) Firms’ financial response to tax cuts is an important topic, but the literature is far from a consensus on the topic. For example, Fama (2011) writes that “the big open challenge in corporate finance is to produce evidence on how taxes affect market values and thus optimal financing decisions.” My primary interest is the response of leverage to tax shocks. In the US, while interest payments are tax-deductible, returns to equity investors are not. Furthermore, dividends are taxed at the firm level and the individual level. Therefore, firms may want to use debt as their funding source to save tax. The left panel of figure 3.7 demonstrates that the story is supported in the data. Quantitatively, it demonstrates that firms decrease the market leverage by approximately one percent on impact, with one percent tax decrease, and the effect gradually disappears. Heidier and Ljungqvist (2015) find that book leverage decreases approximately 0.4 percent on impact. This level may seem smaller than my result; however, the response is close to their level if I use book leverage.

Figure 3.7: Financial Policy Change to Tax Cut

![Figure 3.7: Financial Policy Change to Tax Cut](image)

Note: Shade denotes 95% Confidence Band. Standard errors are clustered by a firm.

Differently from the existing literature that focus on the effect of dividend tax on the dividend payout policy (Auerbach, 2002; Chetty and Saez, 2005), I focus on the effect of corporate income tax shock on dividend policy. Corporate tax cuts decrease the cost of equity, or return from equity; therefore, firms may want to pay out more dividends to stabilize the return. However, the right panel of figure
3.7 illustrates that the effect is unclear; the standard error is too large although the response is quantitatively big.

### 3.3.2 Productivity and Firm Churn

In this section, I investigate the response of productivity and firm dynamics. In the long run, the literature demonstrates that labor productivity (Mertens and Ravn, 2012) and firm dynamics (Sedlacek and Sterk, 2019) increase to an unanticipated tax shock. I focus instead on the short-run effect of tax shocks.

To study the response of productivity, I need to start from identifying productivity. I use the production approach to identify idiosyncratic revenue productivity and markup at the same time (Hall, 1986; De Loecker and Warzynski, 2012; De Loecker et al., 2019). Advantages of production approach are two folds: first, it only requires firm’s financial statement. Second, the production approach is free from specifying a demand system because it exploits firms’ cost-minimization.

I find, in the left panel of figure 3.8, that measured productivity increases with the tax cut. The left-hand panel illustrates that productivity increases approximately one percent on impact. The cumulative response shows hump shape and peaks at the next year. Since it takes some time to increase productivity through investment such as R&D, the jump of productivity on impact results can be somewhat puzzling at glance. However, I believe that procyclical factor utilization can explain the increase.

I provide some evidence for current response of input utilization increases to tax cuts. My Regression equation is similar to the main regression.

\[ \Delta X_t = \kappa_1 + \kappa_2 \Delta X_{t-1} + \kappa_3 \Delta \tau_t + \kappa_4 \Delta C_{i,t-1} + \kappa_{i5} + e_{i,t} \]

where X denotes input utilization, and C is control variables (input utilization, tax level, and GDP growth).
For input utilization, I use two different measure. First, I use aggregate capacity utilization for total industry which obtained from FRED database. Second, I use aggregate electricity consumption except residential use from US Energy Information Administration. Table 3.3 documents that input utilization is procyclical to tax cuts. The first column demonstrates that capacity use increases by 2.6 percent point for one percent point decrease of the tax level due to tax cuts. The second column shows that the electricity use increases by 0.3 percent to the unit change of tax level.

<table>
<thead>
<tr>
<th>Coefficient (κ₃)</th>
<th>Capacity Use</th>
<th>Electricity Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.E</td>
<td>0.889</td>
<td>0.041</td>
</tr>
</tbody>
</table>

Note: 1) *** denotes the coefficient is significant at 1%.
2) The sign is adjusted to tax cuts.

I furthermore note that my result is similar to the literature with aggregate data and revenue productivity; for example, Hussain (2015). Hussain finds that the utilization unadjusted TFP moves by approximately 1.3 percent in the first year.

It consists of industrial, commercial, transportation, and direct use.
The right panel of figure 3.8 demonstrates that the measured markup increases with the tax cut, but less than that of the revenue productivity. After the initial jump, markup goes down gradually. Since the user cost of capital goes down to tax cuts, marginal cost decreases, in turn, markup goes up. Pro-cyclical factor use also can contribute to the increase in markup. I note that Mertens and Ravn (2013) also predict the fall in marginal cost to tax cuts. Another force comes from measured productivity. Since productivity increases, the marginal cost goes down, and markup increases.

I now study the response of entry and exit using industry-level data. Tax shock can affect firm churn significantly in the long run (Sedlacek and Sterk, 2019). I instead empirically study the response of industry level entry and exit in the short run.

To establish industry-level panel data, I distinguish firms into nine industries using SIC two-digit codes and aggregate firm-level data using sales weight. For firm entry and exit, I use Business Dynamics Survey (BDS) data. BDS is a collection of the snapshot of the March every year. To match the data to a calendar year, I use a weighted average of the current year and the previous year data. Furthermore, my sample is limited to 1977 to 2006 since firm entry and exit data exist from 1977. Then, I use my empirical model to estimate the effect of tax cuts on industry-level entry and exit.

I find, in the left panel of figure 3.9, that entry increases approximately 0.05 percent to the tax cut with a 68% confidence level. Since the regression did not pass the weak instrument test, I use identification robust confidence interval (Anderson and Rubin, 1949). The increase disappears quickly after the shock, regardless of cumulation. Based on this evidence, I conclude that the tax cut has insignificant role in firm entry in the short run.

Exit increases at a 68% confidence level to the tax cut, however not significant

I note that Monacelli et al. (2011) find stable markup to a tax shock; however, they do not document how they obtain the markup.
at a 95% level. The right panel of figure 3.9 shows that the exit increases on impact by approximately 0.15 percent before the effect disappears quickly. The result is similar to Colciago et al. (2017) with the state-level tax data. They find that the exit rate is insignificant to tax shocks when they use cross-sectional variation to identify tax shocks. Relying on these evidence, I claim that the firm exit is not significantly affected by tax shocks in the short run.

### 3.3.3 The Effect of the 2017 Tax Cuts and Jobs Act

In December 2017, President Trump signed into law a tax reform package called “Tax Cuts and Jobs Act (TCJA).” The key aspect of the reform is the massive tax cut for pass-through entities and corporations. The law also introduces full immediate expensing of equipment investment replacing depreciation allowances.

In this chapter, I focus on the effect of the tax cut on corporations. The TCJA slashed the statutory tax rate for C-type businesses from 35 percent to 21 percent. To fit into my framework, the cut should be unanticipated, exogenous, and reasonably similar to past events that model estimates. To test anticipation, I use Mertens and Ravn (2012) criterion: they classify a shock as unanticipated if a law is implemented within 90 days of passing. Through the lens of these criteria, the
2017 tax cut was unanticipated. For exogeneity, I check the state of the economy, which I believe to be good. I do not find any specific reason for the 2017 tax cut to be different from other tax reforms from post-war history. Therefore, the prediction using my estimates is suitable.

Although the tax decreased by 14 percent, the effective tax rate change differs due to progressivity and various deductions and exemptions (GAO, 2016). According to the Joint Committee on Taxation, the new law is expected to decrease the tax receipt by seven percent of corporate income, and the effect disappears slowly. In the framework of my reduced-form model, it implies that the firm-level tax rate change would be seven percent due to the aggregate tax shocks. Therefore, the predicted effect is approximately seven times my impulse responses.

The firm-level results in this section are well within the range of the results of Mertens and Ravn (2013) who use a VAR model with aggregate data. Thus, I predict that the aggregate results will be quantitatively in line with the Mertens (2018) estimates. They predict that the 2017 TCJA will increase the GDP about two percent on impact, and this effect will then disappear. This level is higher than Barro and Furman (2018) estimates that uses a standard neoclassical model which predict 0.4 percent increase of GDP after ten years. However, it would be much smaller relative to Sedlacek and Sterk (2019) who predict that the 2017 tax cut can increase GDP by forty percent in the long-run that rely on a neoclassical model with an emphasis on firm dynamics channel. However, I note that this chapter focus on the short-run estimates whereas the Barro and Furman, and Sedlacek and Sterk emphasize the long-run effect.

One caveat for the response of the tangible investment is that TCJA is implemented with the full expense of the equipment investment. Although theoretical literature shows that there is no effect of tax change under full expensing, I argue that the effective expensing rate would be different. For example, investment except equipment such as structure is not under full expensing.
3.4 Discussion and Robustness Tests

This section discusses the results of this chapter. First, I discuss the results from a standard local projection. Second, I discuss the regression results at the first stage. In other words, I show how the aggregate tax shock is related to the individual firm’s tax level. Third, I provide the robustness test results with adding other shocks. Fourth, I briefly mention the consequences of the use of Compustat.

To compare the difference between cumulative response and without cumulation response, I find the impulse responses from a standard local projection. Concrete regression equation is following.

\[
\Delta X_{i,t+h} = \alpha_1 h + \alpha_2 h \Delta X_{i,t-1} + \alpha_3 h \hat{\Delta} \tau_t + \alpha_4 h \Delta C_{i,t-1} + \alpha_5 h + e_{i,t+h}
\]

where \(X\) is the variables of interest, \(\hat{\Delta} \tau_t\) is obtained from the first step without cumulation, and \(C_{it}\) is the control variables. \(\alpha_3 h\) is the change of the variable of interest due to the current change in the tax level due to current tax shocks.

Figure 3.10-12 illustrates that the impulse responses are less persistent than cumulative results. Further, for all impulse responses, the initial response is identical which show the robustness of the results.

Figure 3.10: Response to Tax Cut without Cumulation I

SG&A Alternative Intangible
SG&A Response Intangible Investment Response

Note: Shade denotes 95% Confidence Band. Standard errors are clustered by a firm.
Figure 3.11: Response to Tax Cut without Cumulation II

Tangible R&D

Labor Cost Employment

Leverage Dividend Payout

Entry Exit

Note: Shade denotes 95% Confidence Band except R&D, entry, and exit.
For R&D, entry, and exit, it shows 68% band. Standard errors are clustered by a firm.
Then, I inspect the first stage regression. The first stage regression shows that aggregate tax shock increases individual tax levels approximately the same amount on impact. I find that the coefficient for the aggregate tax shock is around one percent, which is reasonable. It increases close to two percent in the next year, and increases gradually; however, it does not increase up to three percent. It can be since the tax shock I use captures the short-term changes in the after-tax cost of new projects (Mertens, 2018). Within firm reallocation for tax purposes can be another reason, although this may apply to multinational firms mostly.

To address a possible concern that the narrative shock that I use can be endogenous to other shocks, I add more control variables such as technology shock and government spending shock. Figure 3.13 and 3.14 show that the responses
are robust to the inclusion of other shocks. I cannot find a proper monetary policy shock series that cover my sample period; therefore, I abstract from monetary policy shock. For technology shock, I use the utilization adjusted TFP shock from Fernald (2015). For government spending shock, I use defense news spending shock from Ben Zeev and Pappa (2017) since it includes more small shocks than Ramey (2011). However, the result is similar to the use of Ramey.

Figure 3.13: Responses with More Shocks I

Note: Shade denotes 95% Confidence Band except R&D.

For R&D, it shows 68% band. Standard errors are clustered by a firm.
Figure 3.14: Responses with More Shocks II

Leverage

Dividend Payout

Entry Exit

Productivity Markup

Sale Tobin Q

Note: Shade denotes 95% Confidence Band except entry and exit.
For entry and exit, it shows 68% band. Standard errors are clustered by a firm.
The last issue comes from the use of Compustat. Compustat only covers a subset of the firms in the US economy. There exists small public firms and pass-through entities. Therefore, an additional assumption is required to extend my results to the universe of the firms\textsuperscript{17}. However, my estimates are quantitatively similar to those of Mertens and Ravn (2013), who use the same tax shock with an aggregate VAR model. Therefore, I claim that the results in this paper can be generalized to the firms with some caution.

3.5 Conclusion

This paper studies micro-level responses to aggregate tax shocks in the short run. Using narrative tax shocks and firm-level tax rate as an instrument, I document that firm-level investment, total labor costs, and output increase whereas leverage decreases. Moreover, I show that a tax cut increases measured revenue productivity and measured markup, but firm entry and exit are stable to tax shocks. Lastly, with the estimates of this paper and the previous research, I claim that the effect of the 2017 tax reform can be more significant than literature such as ten years estimates of Barro and Furman (2018).

The significant change of intangible capital investment to tax cut implies that the effect of tax shock in the existing literature can be underestimated. This is because existing literature does not take the effect of intangible capital change into account. Even if the firm-level result is similar to the aggregate results such as GDP, the claim is still valid since GDP does not take intangible capital into account properly\textsuperscript{18}.

I study the firm-level responses to corporate tax shocks and find that the result is quantitatively close to aggregate data. Given the possible bias coming from the use of aggregate data such as aggregation bias, the results in the paper confirms

\textsuperscript{17} For example, Bhandari and McGrattan (2020) show a different response between pass-through entities and public firms.

\textsuperscript{18} It is even after the recent effort to consider intangible capital such as intellectual property. See Bhandari and McGrattan (2020) for more discussion.
the findings of existing literature with aggregate data such as Mertens and Ravn (2013). Conversely, the results with aggregate data lends robustness to my results.

The paper can be extended in two aspects. Although I try to relate the evidence to structural models, the results are empirical in nature. Hence, I can build a model to explain the evidence in this paper. I can also extend the narrative shock series to the recent period to study the effect of tax shocks, including the recent tax cuts. I plan to push this paper to both margins.
Chapter 4

Forward Guidance Puzzle under HANK & SAM

4.1 Introduction

More than ten years after the great recession, the Federal Funds Rate and the policy rates in advanced countries continue to remain at a low level. This implies that I am highly likely to see another forward guidance in the next recession. Although many empirical papers show that forward guidance has been effective (Campbell, Evans, Fisher, and Justiniano, 2012, Del Negro, Giannoni, and Pater-son, 2015), theoretical work has been slow to obtain the quantitatively reasonable effect of forward guidance.

In a standard Representative Agent New Keynesian (RANK) model, Del Ne-gro et al. (2015) show that there is a “forward guidance puzzle”: if a central bank commits to decreasing its rate in the farther future, the effect of a promise becomes stronger. These scholars furthermore find that the key reason behind the puzzle is the lack of discounting for the future economic outcome.

In this paper, by building on Ravn and Sterk’s (2020) framework, I show that introducing an incomplete market with a procyclical income risk can solve the forward guidance puzzle. There have been many attempts to solve the forward guidance puzzle, such as sticky information (Carlstrom, Fuerst, and Paustian,
2012; Kiley, 2016), incomplete market models (McKay, Nakamura, and Steinson, 2017) and heterogeneous belief (Andrade, Gaballo, Mengus, and Mojon, 2019). Among these attempts, I focus on incomplete market models such as McKay et al. (2016) and Werning (2015). McKay et al. (2016) claim that introducing an incomplete market can solve the forward guidance puzzle since agents discount the future more due to precautionary saving. However, Werning (2015) argues that if an incomplete market is combined with countercyclical income risk, the forward guidance puzzle can be aggravated due to general equilibrium forces.

The model endogenizes the income risk by introducing the frictions in the goods market, labor market, and the financial market. This framework is initially developed by Ravn and Sterk (2020), who coin the term HANK&SAM\(^1\). In the model, aggregate shock changes an agent’s job-finding rate, which affects the worker’s income risk. Relative to Ravn and Sterk, this paper expresses income cyclicity as a function of deep parameters\(^2\) by using a continuous-time approach. As a result, different aspects of earning risks, such as the job-finding rate and wage flexibility, change simultaneously as a deep parameter varies.

This paper is related to many strands of literature. First, I study how idiosyncratic income risk changes the response of the aggregate variables to monetary policy shocks as in Kaplan, Moll, and Violante (2018) and Luetticke (2019). Since this line of literature inherently relies on a complicated computational method, it is relatively more difficult to see the deep mechanism of the model. This paper attempts to provide analytic expression to alleviate the stated concern.

I also explore the effect of forward guidance shock in incomplete markets as in Werning (2015) and McKay et al. (2016). This paper extends Werning in two aspects. First, I show the effect of procyclical and countercyclical income risks in a more concrete setting. Second, I provide a quantitative exercise to show how much the change in income risk affects the effect of forward guidance.

---

1 In addition to a Heterogeneous Agent New Keynesian (HANK) setting, the model adds search and match (SAM) friction

2 Ravn and Sterk posit that there is an exogenous wage flexibility parameter.
Archarya and Dogra (2020) provide analytic expressions to study the effect of income risk cyclicality in a HANK setting. They also show that countercyclical income risk aggravates the forward guidance puzzle. However, by using a Constant Absolute Risk Aversion (CARA) utility, they abstract from the marginal propensity of consumption (MPC) difference among agents.

The next section provides the model. After presenting the model, I derive the linearized aggregate Euler equation and execute comparative analysis. Then, I conclude.

4.2 Model

The model combines nominal rigidity, an incomplete financial market, and search and match labor friction as in Ravn and Sterk (2020). By adding labor market frictions in a HANK model, one can study the effect of the endogenous earning risk that comes from countercyclical job loss risk and procyclical wage change risk. In this paper, I briefly present the model.\(^3\)

4.2.1 Environment

Preference

There is a continuum of households of measure one indexed by \(i \in (0, 1)\). They consume goods, supply labor, and trade bonds. Households maximize the expected discounted sum of periodic utility.

\[
U_i = \max E_0 \int_0^\infty e^{-\rho t} \left( \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \zeta n_{i,t} \right) dt
\]

where \(c_{i,t}\) is a basket of consumption goods with constant elasticity of substitution.

\(^3\) Interested readers can refer to Ravn and Sterk (2020).
and \( \rho \) is the discount rate, \( \sigma \) is a risk aversion parameter, \( \zeta \) controls the disutility from labor supply, and \( j \) refers to each consumption product.

Households can work full time \( (n_{i,t} = 1) \) or not work at all \( (n_{i,t} = 0) \). If they are unemployed, households receive unemployment benefits \( (\vartheta) \).

Entrepreneurs

There is a continuum of entrepreneurs of measure one that owns firms. They are risk-neutral and do not trade the ownership of the businesses. I assume that entrepreneurs are out of the labor market\(^4\). Entrepreneurs pay lump-sum taxes to the government.

Production

There is a continuum of firms of measure \( M (M < 1) \) indexed by \( j \) that produce differentiated goods with labor. The production function is

\[
y_{j,t} = n_{j,t}
\]

Monetary Policy and Government

A central bank uses a Taylor rule for its normal operation. Under a liquidity trap, the monetary authority commits to the forward guidance\(^5\). After the forward guidance, the bank follows a Taylor rule. Therefore, I posit the following monetary policy rule:

\(^4\) This is innocuous because I can instead assume that there is a small number of entrepreneurs who consume profits. Even if the profits are tiny, a sufficiently low ratio of entrepreneurs will guarantee that they are out of the labor market.

\(^5\) Agents in the economy fully trust the central bank. That is, there is no credibility concern.
\[
R_t = \begin{cases} 
R^*, & \text{for } t < \tau \\
R + \delta_x \pi_t + \delta_y \theta_t + \epsilon_{R,t}, & \text{for } t \geq \tau 
\end{cases}
\]

\[
\dot{e}_{R,t} = -\lambda e_{R,t}
\]

where \( \dot{x} = \frac{dx}{dt} \) for a generic variable \( x \), \( R_t \) represents the nominal interest rate, \( \tau \) is the moment at which forward guidance is realized, \( \pi_t \) denotes the net inflation rate, and \( \lambda \) is the persistence of forward guidance.

The government collects a lump-sum tax from entrepreneurs to fund unemployment benefits.

**Labor Market**

There exists search and match friction in the labor market. Firms hire by posting a vacancy after paying some fixed cost, \( \kappa > 0 \), per unit. I assume that each firm is sufficiently large that the job-filling probability \( (q_{j,t}) \) is the fraction of vacancies that are filled. Existing matches are dismissed randomly, and new matches are produced by a matching function.

\[
m_{j,t} = \psi s_t^\alpha v_t^{(1-\alpha)} \]

(4.1)

where \( \psi \) is the matching efficiency, \( \alpha \) is the matching elasticity of the unemployed, \( s_t \) denotes job searchers, and \( v_t = \int v_{j,t} dj \) is the aggregate measure of vacancy.

The job-filling probability, \( q_t \), and job-finding rate, \( \eta_t \), are functions of market tightness, \( \theta_t \equiv \frac{\omega}{\epsilon_t} \).

\[
q_t = \frac{m_{j,t}}{v_t} = \psi \theta_t^{-\alpha} \]

(4.2)

\[
\eta_t = \frac{m_{j,t}}{e_t} = \psi \theta_t^{1-\alpha} \]

(4.3)
I note the relationship between the job-filling rate and the job-finding rate as

\[ q_t = \psi^{\frac{1}{1-\alpha}} \eta_t^{\frac{\alpha}{1-\alpha}}. \]

The employment dynamics are the following.

\[ \dot{n}_t = m_t - \delta n_t \quad (4.4) \]

**Nash Bargaining**

Wages are determined by Nash Bargaining. Households that match surplus functions differ across their idiosyncratic states; therefore, the functions are labeled with i. Firms are symmetric, and I thus do not differentiate among individual firms. The following problem determines the wage:

\[
\max \left( \nu \left( \frac{V^i_{e,t} - V^i_{u,t}}{J_t} \right)^{1-\nu} \right)
\]

where \( V^i_{e,t} \) is the employed worker’s value, \( V^i_{u,t} \) is the unemployed worker’s value, \( J_t \) is the operating firm’s surplus, and \( \nu \) is the workers’ bargaining power.

I assume that workers can meet at most one firm and that firms can meet only one worker. The surplus of the match for the firm is equal to the expected cost of hiring:

\[ J_t = \frac{\kappa}{q_t} \]

The first-order condition for Nash bargaining is as follows.

\[ (1 - \nu)(V^i_{e,t} - V^i_{u,t}) = \nu J_t \]

Then, I find the following wage equation\(^6\).

\[ w_t(\eta_t) = \left\{ \theta^{1-\sigma} + (1 - \sigma) \left[ (\rho + 1) \frac{\nu \kappa}{1 - \nu q_t(\eta_t)} + \zeta \right] \right\}^{\frac{1}{1-\sigma}} \quad (4.5) \]

\(^6\) A detailed derivation is in Appendix C
Financial Market

Households consist of a single member and cannot be perfectly insured against job uncertainty. They self-insure by using savings in a zero-dividend nominal bond, and they cannot borrow:

\[ b_{i,t} \geq 0, \]

A zero borrowing constraint assumption combined with a no government bond assumption implies that the wealth distribution degenerates to zero bond holding for all agents. The assumption can be relaxed slightly\(^7\).

4.2.2 Households’ Problem

Households maximize the sum of expected discounted periodic utility:

\[
V_i(b_{i,t}, n_{i,t}; F_t) = \max_{c_{i,t}, b_{i,t}, n_{i,t}} \mathbb{E}\left[ 0 \int_0^\infty e^{-\rho t} \left( \frac{c_{1,i,t}^{1-\sigma}}{1-\sigma} - \zeta n_{i,t} \right) dt \right]
\]

subject to a budget constraint,

\[
c_{i,t} + \dot{b}_{i,t} = w_t n_{i,t} + (1 - n_{i,t}) \theta + (R_t - \pi_t) b_{i,t}
\]

where \( F = \{ B_t, \epsilon_{R,t} \} \) is a set of aggregate state variables, and \( B_t \) is the households’ distribution over the bond and employment state. Now, I set up a Hamilton-Jacobi-Bellman (HJB) equation for the employed households.

\[
\rho V^e(b) = \frac{\epsilon e^{1-\sigma}}{1-\sigma} - \zeta + \delta (1 - \eta)(V^u(b) - V^c(b)) + \dot{b} V^e_0 (b)
\]

where \( \dot{b} = (R - \pi) b + w - c \). Then, I write an HJB Equation for the unemployed households:

\[
\rho V^u(b) = \frac{c^{1-\sigma}}{1-\sigma} + [1 - \delta (1 - \eta)](V^c(b) - V^u(b))
\]

\(^7\) One can refer to Ravn and Sterk (2020) for details.
4.2.3 Firms’ Problem

Firms operate in a monopolistically competitive goods market. Firms set the prices, $p_{j,t}$, of their products given a quadratic menu cost, $\phi$, following Rotemberg (1982). Firms maximize a discounted stream of real profits given the production function, employment process, and demand function:

$$ W_{j,t} = \max E_0 \int_0^{\infty} e^{-\rho t} \left[ \frac{p_{j,t}}{P_t} y_{j,t} - w_t n_{j,t} - \kappa v_{j,t} - \frac{\phi}{2} \left( \frac{\dot{p}_{j,t}}{p_{j,t}} \right)^2 y_t \right] dt $$

subject to

$$ y_{j,t} = n_{j,t} $$
$$ \dot{n}_{j,t} = m_{j,t} - \delta n_{j,t} $$
$$ y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{1-\gamma} y_t $$

where $P_t$ is the aggregate price and $w_t$ is the real wage. Then, I establish an HJB equation for a firm.

$$ H(p_{j,-1}, \lambda) = p_j y_j - p_j w n_j - p_j \kappa \left( \dot{n} + \delta n \right) - \frac{\phi}{2} \left( \frac{\dot{p}_j}{p_j} \right)^2 p_j y_j + \lambda \dot{p}_j $$

4.2.4 Equilibrium

I study the symmetric equilibrium where all firms choose the same price.

**Definition 1.** A recursive symmetric equilibrium is a set of policy functions $\{c, b, n, v\}$, prices $\{P, R, w, \Pi\}_t$, labor market variables $\{\eta, q, s, n\}_t$, value functions $\{V(n = 1), V(n = 0)\}$, and a distribution of agents $B_t$ such that

1. The policy functions $\{c_i, b_i, n_i\}_i$ solve the households’ problems.
2. $(v, P)$ solve the firms’ problems.
3. The goods and asset markets are clear.
4. The aggregate labor market variables evolve according to (2.1)-(2.4).
5. The wage solves the Nash bargaining problem.

6. The central bank commits to the policy rule.

7. The actual and perceived laws of motion for the state variables coincide.

4.2.5 Aggregate Consumption

Before I linearize the model, I find aggregate consumption by using the market clearing condition: 'aggregate consumption = aggregate output - hiring cost'\(^8\) (Werning, 2015). The advantage of this approach is that I abstract by directly aggregating each agent's consumption.

\[ C_t = N_t - \frac{\kappa}{q_t}[1 - (1 - \delta)N_t] \]

4.2.6 Local Dynamics

Before I linearize the model, I define the incomplete markets' wedge: \( \Theta(\eta) \equiv \delta(1 - \eta_t)[(\frac{\vartheta}{w_t})^{-\sigma} - 1] \). Notice that \( \Theta(\eta) = 0 \), if \( \delta = 0, \sigma = 1, \) or \( \vartheta = w \).

Now, I linearize the nonlinear equations\(^9\) with a Taylor expansion around the intended steady-state while assuming that the central bank targets price stability\(^10\) and vanishing liquidity \( \bar{R} = \bar{\pi} + \rho - \Theta \)\(^11\). I define \( \hat{x}_t = x_t - x \), where an \( x \) without the subscript \( t \) is a steady-state value.

---

\(^8\) A detailed derivation is in Appendix C.

\(^9\) The nonlinear equations can be found in Appendix C.

\(^10\) This model has multiple equilibria: two interior solutions (high and low inflation) and a liquidity trap. For further details, see Ravn and Sterk (2020).

\(^11\) This is the threshold interest rate level beyond which employed agents do not have any incentive to hold bonds. For an interest rate level above or below this level, agents have an incentive but are not allowed to hold bonds.
\[ \dot{\eta}_t \approx F_\eta \dot{\eta}_t + F_\sigma \dot{\pi}_t + F_R \dot{R}_t \]  
(Euler equation of the employed)

\[ \dot{\pi}_t \approx G_\eta \dot{\eta}_t + G_\sigma \dot{\pi}_t \]  
(Phillips curve)

\[ \dot{N}_t \approx H_\eta \dot{\eta}_t + H_N \dot{N}_t \]  
(Law of motion for aggregate labor)

\[ C_t \approx C_N N_t + C_\eta \eta_t \]  
(Aggregate consumption)

The detailed derivation of and expressions for the coefficients are in Appendix C. I emphasize that the first two equations form a subsystem, which allows computing the impulse response function efficiently.

The Euler equation of the employed agents, which is presented below, is the fundamental equation that I want to investigate. Since all other agents except the employed are not subject to the Euler equation, the Euler equation of the employed determines the price of bonds.

\[ \hat{c}_e t \approx \frac{w_\eta}{\sigma} (\hat{R}_t - \hat{\pi}_t) + \tilde{\Theta} \hat{\eta}_t \]
\[ \hat{\Theta} \equiv w_\eta F_\eta = w_\eta \delta (1 - \eta) \left( \frac{\delta}{w} \right)^{-\sigma} - \frac{w \delta}{\sigma} \left( \frac{\delta}{w} \right)^{-\sigma} - 1 \]

By comparing the above equation to the Euler equation under complete markets \( \hat{C}_t = \frac{C}{\sigma} [\hat{R}_t - \hat{\pi}_t] \), one can immediately see that there is an additional term, \( \tilde{\Theta} \hat{\eta}_t \). This term works as an endogenous wedge that is not present in acyclical income risk models such as RANK models or many HANK models. I name the term the income risk of the employed and \( \tilde{\Theta} \)\(^{12} \) the cyclicality of income risk, which I elaborate on in the next paragraph. Two straightforward remarks are in order: (1) if there is no job loss risk \( (\delta = 0) \), income risk is zero since \( \tilde{\Theta} = 0 \); and (2) if \( \tilde{\Theta} < 0 \), the income risk of the employed increases during a recession (i.e., countercyclical income risk).

The cyclicality of income risk, \( \tilde{\Theta} \), captures both procyclical and countercyclical

\(^{12}\) Ravn and Sterk (2020) define it an incomplete market wedge.
income risk (Ravn and Sterk, 2020). The first term represents procyclical income risk\(^{13}\). This term describes how much current employees’ earnings will change if they are employed in the next period. This term is also discounted by the probability that an employee will be employed in the future. The second term represents countercyclical job loss income risk. Therefore, the term denotes the change in the demand for precautionary saving. Notice that this term goes to zero if there is no job loss risk \( \delta = 0 \) or \( \vartheta = w \). If the countercyclical risk is stronger than the procyclical risk, there is additional amplification in consumption relative to complete markets.

4.2.7 Benchmark Calibration

To quantitatively evaluate the model, I calibrate the parameters, as summarized in table 4.1. Time is monthly, and I set \( \rho = 0.007 \), which implies an annual discount factor of 0.92\(^{14}\). The consumption loss upon job loss ratio (\( g \)) is 0.82, which is in the middle of the standard range. The labor disutility parameter (\( \zeta \)) is 0.06 and yields a steady-state wage level of approximately 0.83. The target job-filling rate (0.69) is calculated by Davis et al. (2013). I set the job-separation rate to 0.04, which is close to the historical average and yields a steady-state job-finding rate of 0.43 given an unemployment rate of 5\%. Because of the steady-state job-finding rate, match efficiency, \( \psi \), is calculated to be 0.546 to match the steady-state job-filling rate. Workers’ bargaining power and the matching elasticity for the unemployed is calibrated at 0.5. I set \( \phi \) to 96, which implies that the average firm changes its price every five months. The central bank promises to decrease the policy rate by 25 basis points in 6 quarters. The monetary policy rule parameters are set to be stronger than standard, as emphasized by Ravn and Sterk (2020). The persistence of the forward guidance shock parameter is set to 0.5, which corresponds to a quarterly autoregressive coefficient of 0.61.

\(^{13}\) I implicitly assume that the real wage is procyclical.

\(^{14}\) This value will give steady-state real interest rates close to zero due to the precautionary saving motive.
Table 4.1: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Explanation</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>Household risk aversion</td>
<td>Standard</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.007</td>
<td>Household patience</td>
<td>Annual discount factor $\approx 0.92$</td>
</tr>
<tr>
<td>$g$</td>
<td>0.82</td>
<td>Replacement ratio</td>
<td>Standard</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.06</td>
<td>Labor disutility</td>
<td>Steady-state wage $\approx 0.83$</td>
</tr>
<tr>
<td>Labor Market</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.04</td>
<td>Job separation rate</td>
<td>Data</td>
</tr>
<tr>
<td>$u$</td>
<td>0.05</td>
<td>Steady-State unemployment rate</td>
<td>Data</td>
</tr>
<tr>
<td>$q$</td>
<td>0.69</td>
<td>Steady-State job filling rate</td>
<td>Davis et al. (2013)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.546</td>
<td>Steady-State match efficiency</td>
<td>Job-filling rate $= 0.69$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.50</td>
<td>Worker’s Bargaining power</td>
<td>Standard</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.50</td>
<td>Elasticity w.r.t unemployed</td>
<td>Standard</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.14</td>
<td>Hiring cost</td>
<td>4% of quarterly wage bill</td>
</tr>
<tr>
<td>Firm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>96</td>
<td>Price adjustment cost</td>
<td>Bils and Klenow (2004)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>6</td>
<td>Market power</td>
<td>Markup $= 20%$</td>
</tr>
<tr>
<td>Monetary policy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_\tau$</td>
<td>3</td>
<td>Taylor rule coefficient for inflation</td>
<td>-</td>
</tr>
<tr>
<td>$\delta_\phi$</td>
<td>0.3</td>
<td>Taylor rule coefficient for tightness</td>
<td>-</td>
</tr>
<tr>
<td>$\tau$</td>
<td>36</td>
<td>Forward guidance horizon</td>
<td>3 years</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.5</td>
<td>Persistence of forward guidance</td>
<td>AR(1) coef $\approx 0.61$</td>
</tr>
</tbody>
</table>

4.3 Results

4.3.1 Aggregate Euler Equation

To compare the effect of monetary policy shocks to the effect in RANK models, I need to obtain the aggregate Euler equation. The necessary assumption to have a closed-form aggregate Euler equation is that all jobs are resolved at every moment ($\delta = 1$). To match the labor market turnover rate, I further assume that a fixed fraction ($p$) of unemployed agents are hired without any cost every second. In the sense that an existing match is similar to capital, this assumption is not uncommon in the literature.

$$N_t = p + (1 - p)\eta_t$$ (4.6)

where I set $p \approx 0.83$ to match $\eta \approx 0.43$ under $N = 0.95$. After some algebra, I obtain the aggregate Euler equation as below. See Appendix C for the derivation.
\[ \hat{C}_t = \Lambda [\hat{R}_t - \hat{\pi}_t] + F_{\eta} \hat{C}_t \] (4.7)

where

\[
\Lambda \equiv \frac{1}{\sigma} \frac{C_{\eta} w}{w_{\eta}}
\]

\[
F_{\eta} \equiv \delta (1 - \eta) \left( \frac{\vartheta}{w} \right)^{-\sigma} - \frac{1}{\sigma} \frac{w \delta}{w_{\eta} \eta} \left( \frac{\vartheta}{w} \right)^{-\sigma - 1}
\]

The aggregate Euler equation resembles the Euler equation of the employed agents. Therefore, all the intuition from the Euler equation of the employed workers transfers to the aggregate Euler equation.

It is clear from the expression that there is an additional term compared to Representative Agent New Keynesian (RANK). I study how \( \Lambda \) and \( F_{\eta} \) are different from RANK models and how they change as the deep parameters of the model vary.

### 4.3.2 What Drives Income Risk Cyclicality?

To the best of my knowledge, there is no consensus on the cyclicality of income risk. Therefore, I show how each parameter affects the cyclicality of income risk \( (F_{\eta}) \) by using partial derivatives. To economize the notation, I define \( g \equiv \left( \frac{\vartheta}{w} \right) \), \( A \equiv (g^{-\sigma} - 1) \), \( B \equiv -\delta \sigma (1 - \eta) g^{-\sigma} + \frac{w}{w_{\eta}} g^{-\sigma - 1} \), and \( C \equiv g^{-\sigma} \ln(g) \).

**Proposition 1.** Higher bargaining power, lower unemployment benefits, a higher risk aversion, and a higher hiring cost make income risk more procyclical.

\[
\left( \frac{\partial F_{\eta}}{\partial \nu}, \frac{\partial F_{\eta}}{\partial \vartheta}, \frac{\partial F_{\eta}}{\partial \sigma}, \frac{\partial F_{\eta}}{\partial \kappa} > 0 \right)
\]

1. **Bargaining power** \( (\nu) \):
   \[
   \frac{\partial F_{\eta}}{\partial \nu} = -\frac{1}{\sigma} A \left( \frac{\partial (w/w_{\eta})}{\partial \nu} \right) > 0
   \]

2. **Unemployment benefits** \( (\vartheta) \):
   \[
   \frac{\partial F_{\eta}}{\partial \vartheta} = \frac{\partial F_{\eta}}{\partial g} \frac{\partial g}{\partial \vartheta} = [B - \frac{\delta}{\sigma} A \left( \frac{\partial (w/w_{\eta})}{\partial g} \right)] \frac{1}{w} > 0
   \]

---

15 \[ \frac{\partial (w/w_{\eta})}{\partial \nu} = \frac{\vartheta \kappa}{\nu} \frac{1}{1 + \rho \frac{1 - \alpha}{\alpha} [(1 - \sigma) \frac{\partial w}{\partial \nu} \frac{1 - \nu}{\nu} - w^{1 - \sigma} \frac{1}{\nu^2}] < 0 \]
16 \[ \frac{\partial w}{\partial \nu} = -w^{(1 - \sigma) \frac{\vartheta}{\eta}} (1 + \rho) \frac{1}{\nu^2} < 0 \]
17 In this exercise, I fix the steady-state wage.
18 \[ \frac{\partial w/w_{\eta}}{\partial g} = \frac{1}{(1 - \sigma) \frac{\vartheta}{\eta} \frac{1}{w_{\eta} \eta} (1 + \rho) \frac{1}{\nu^2} < 0 \]
3. Risk aversion ($\sigma$): \[
\frac{\partial F_\eta}{\partial \sigma} = \frac{w}{\sigma w_\eta} \left[ \frac{1}{\sigma} A + C \right] + \frac{1}{\sigma} A \frac{\partial (w/w_\eta)}{\partial \sigma} > 0.
\]

4. Hiring cost ($\kappa$): \[
\frac{\partial F_\eta}{\partial \kappa} = \delta (1 - \eta) g^{\sigma-1} \frac{\partial w}{\partial \kappa} - \frac{w_\kappa}{w_\eta} g^{\sigma-1} \frac{\partial A}{\partial \kappa} > 0.
\]

Figure 4.1 illustrates how much income cyclicality varies as the deep parameters change. I recall that there exists a procyclical wage change risk and countercyclical job loss risk. Changes in each parameter vary the effects of the two-income risks. Higher bargaining power places greater weight on the procyclical wage change risk than the countercyclical job loss risk. Higher unemployment benefit decreases the job loss risk since the consumption loss upon job loss decreases. Higher risk aversion weakens countercyclical income risk since households put more emphasis on procyclical wage change risk\(^{22}\). A higher hiring cost implies greater wage flexibility since firms are more reluctant to hire workers, which contributes to procyclical income risk\(^{23}\).

Figure 4.1: Income Risk Cyclicality by Different Parameterization

4.3.3 Monetary Policy Shock under Exogenous Income Risk

Under acyclical income risk ($F_\eta = 0$), the only difference from the complete markets’ Euler equation is the sensitivity of aggregate consumption to the real interest rate change, which I define as \[\Lambda \equiv \frac{C_n}{C} \frac{w}{w_\eta}.\]

\[\begin{align*}
\frac{\partial (w/w_\eta)}{\partial \sigma} &= \frac{\partial (w/w_\eta)}{\partial \sigma} = \frac{w}{\sigma w_\eta} \left[ \frac{1}{\sigma} A + C \right] + \frac{1}{\sigma} A \frac{\partial (w/w_\eta)}{\partial \sigma} > 0 \\
\frac{\partial w}{\partial \kappa} &= \frac{w_\kappa}{w_\eta} \frac{\partial w}{\partial \kappa} - \frac{w_\kappa}{w_\eta} \frac{\partial w}{\partial \kappa} = \left[ \frac{1}{\alpha \kappa} - \frac{1}{\alpha \kappa} \right] \\
\text{It can be subject to calibration though.} \\
\text{It also can be subject to calibration though.}
\end{align*}\]
Proposition 2. Higher bargaining power, lower unemployment benefits, a higher risk aversion, and a higher hiring cost decrease the sensitivity of aggregate consumption to interest rate changes ($\frac{\partial \Lambda}{\partial \nu}, \frac{\partial \Lambda}{\partial u}, \frac{\partial \Lambda}{\partial \sigma}, \frac{\partial \Lambda}{\partial \kappa} < 0$).

1. Bargaining power ($\nu$): $\frac{\partial \Lambda}{\partial \nu} = \frac{C_{\eta}}{C} \left( \frac{\partial (w/w_{\eta})}{\partial \nu} \right) < 0$.

2. Unemployment rate ($u$): $\frac{\partial \Lambda}{\partial u} = \frac{\partial (C_{\eta}/C)}{\partial u} \frac{24 w}{w_{\eta}} + C_{\eta} \frac{\partial (w/w_{\eta})}{\partial u} < 0$.

3. Risk aversion ($\sigma$): $\frac{\partial \Lambda}{\partial \sigma} = \frac{C_{\eta}}{C} \frac{\partial (w/w_{\eta})}{\partial \nu} < 0$.

4. Hiring cost ($\kappa$): $\frac{\partial \Lambda}{\partial \kappa} = \frac{\partial (C_{\eta}/C)}{\partial \kappa} \frac{27 w}{w_{\eta}} + \frac{C_{\eta}}{C} \frac{\partial (w/w_{\eta})}{\partial \kappa} < 0$.

Figure 4.2 summarizes the results graphically. The sensitivity of aggregate consumption to real interest rates differs due to the wage channel ($\frac{w}{w_{\eta}}$) and the aggregate consumption channel ($\frac{C_{\eta}}{C}$). Higher bargaining power weakens the wage channel since it only increases wage flexibility. A higher unemployment rate diminishes the wage channel and amplifies the aggregate consumption channel. However, the change in the wage channel is stronger than the change in the aggregate consumption channel; thus, the overall effect of monetary policy decreases. A higher hiring cost mitigates both the wage channel and the aggregate consumption channel.

Higher wage elasticity dampens the effect of monetary policy since the employed households are more willing to smooth their consumption. Given the zero-borrowing constraint, the difference in the precautionary saving motive changes how much real interest should move to clear the market. In this sense, marginal propensity to consume is not one for all households even if households consume all of their income\(^{29}\).

\[ \frac{\partial (C_{\eta}/C)}{\partial \kappa} = \frac{1}{w_{\eta}} \left( \frac{\partial C_{\eta}}{\partial \kappa} - \frac{1}{C_{\eta}} \frac{\partial C_{\eta}}{\partial \kappa} \right) = \frac{1 - \alpha}{\alpha} \frac{q}{\kappa} \left( \frac{C_{\eta}}{C} \right) \left( 1 - \delta \right) \]

\[ \frac{\partial (w/w_{\eta})}{\partial \kappa} = \frac{1}{w_{\eta}} \left( \frac{\partial w}{\partial \kappa} - \frac{1}{w_{\eta}} \frac{\partial w}{\partial \kappa} \right) = \frac{1 - \alpha}{\alpha} \frac{q}{\kappa} \left( 1 - \delta \right) \]

\(24\) \(\frac{\partial (C_{\eta}/C)}{\partial u} = \frac{1}{(1-p)} \left( 1 - \sigma - \frac{w}{w_{\eta}} \frac{2a - 1}{1 - a} \right) \]

\(25\) \(\frac{\partial (w/w_{\eta})}{\partial u} = \frac{1}{(1-p)} \left( 1 - \sigma - \frac{w}{w_{\eta}} \frac{2a - 1}{1 - a} \right) \]

\(26\) \(\frac{\partial (w/w_{\eta})}{\partial \sigma} = \left( \rho^{\sigma} - \sigma \ln \rho + (\rho + 1) \frac{w}{w_{\eta}} (1 - \frac{q}{\kappa}) \right) > 0 \]

\(27\) \(\frac{\partial (C_{\eta}/C)}{\partial \kappa} = \frac{1}{1 - \alpha} \left( \frac{\partial C_{\eta}}{\partial \kappa} - \frac{1}{C_{\eta}} \frac{\partial C_{\eta}}{\partial \kappa} \right) \]

\(28\) \(\frac{\partial (w/w_{\eta})}{\partial \kappa} = \frac{1}{w_{\eta}} \left( \frac{\partial w}{\partial \kappa} - \frac{1}{w_{\eta}} \frac{\partial w}{\partial \kappa} \right) = \left( \frac{1 - \alpha}{\alpha} \frac{q}{\kappa} \right) \left( 1 - \delta \right) \]

\(29\) That is, consumption equals income in equilibrium.
4.3.4 Forward Guidance

Income Risk Channel and Forward Guidance

The income risk channel operates keenly on forward guidance. By assuming a liquidity trap $\hat{R}_t = \hat{\pi}_t = 0$, I shut down the sensitivity to the real interest rate channel. Then, the aggregate Euler equation becomes $\hat{C}_t = F_\eta \hat{C}_t$. This equation immediately demonstrates propositions 2 and 4 of Werning (2015).

**Proposition 3.** According to Werning (2015), under countercyclical income risk and a liquidity trap, households compound future news. If income risk is acyclical, the effect of forward guidance is equivalent to this effect under complete markets.

Impulse Response Function

I show how the key variables respond to a forward guidance shock. The shock is a committed promise to lower the policy rate by 25 basis points after 36 months. I remind the reader that income risk is countercyclical ($F_\eta \approx -0.05$) under the benchmark calibration. To have the impulse responses, I first calculate the response of the key variables to a contemporaneous monetary policy shock. Since I only consider the subsystem of two jump variables at this stage, I regard it as the responses of two jump variables when forward guidance shock is realized. Then, I back out the initial response of the job-finding rate and inflation by using the response to ordinary monetary policy shock as a terminal point of the system.
of differential equations. Finally, I calculate the time paths of consumption and labor supply by using the time paths of the job finding rate and inflation rate.

**Proposition 4.** Countercyclical income risk amplifies the effect of forward guidance, whereas procyclical income risk reduces the effect of forward guidance relative to the RANK models.

As the left panel of figure 4.3 shows, the job-finding rate and inflation increase following a forward guidance shock. As the two jump variables increase, the two diffusion processes, i.e., labor supply and consumption, also increase with a lag. The right panel of figure 4.3 illustrates that the effect of forward guidance is stronger relative to the complete markets under countercyclical income risk. The responses from acyclical income risk are equivalent to RANK models since I shut down the sensitivity to the real interest rate channel.

**Figure 4.3: Impulse Response Functions to Forward Guidance Shock**

4.4 Conclusion

This paper shows that the effect of forward guidance in an incomplete market model with countercyclical income risk can be stronger than Representative Agent New Keynesian models. I further show how the income risk and, therefore, the effect of monetary policy can be changed depending on the deep parameters of the model. This feature is helpful in comparing the effect of earning risk cyclicality.

---

30 I adjust for the wage change risk to compare with the RANK models.
on different countries.

To obtain an analytic solution, I impose a tight borrowing constraint assumption. One can study the model further by relaxing the assumption and be more quantitative as in Ravn and Sterk (2017). Another interesting topic is to study the prediction of the model empirically. There have been many attempts to study the cyclicality of income risk such as Guvenen et al. (2014). However, not much investigation is conducted to relate income cyclicality to the effect of monetary policy, probably due to difficulty in obtaining the data. I will work on this topic in future research.
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Appendix A

Appendix for Chapter 2

A.1 Data

I follow De Loecker, Eeckout, and Unger (2018) to clean the data. Specifically, I use the firm-level financial variables of all US-listed public firms from Wharton Research Data Services (WRDS). The sample period is from 1950 to 2017, and I allow entry and exit within the period. I use the industry format and eliminate the firms that do not report the NAICS industry code. Firms without key variables to estimate the production function (sales, cost of goods, and capital) are excluded from the sample. Additionally, I eliminate the firms with higher than a 99th percentile and first percentile of labor cost share, where the percentiles are calculated for each year. I deflate all variables with a GDP deflator.

A.2 Production Function Assumptions

In this section, I show how Leontief gross output production function translates to Cobb-Douglas function. I assume industry-specific Leontief gross production function that the output is proportional to the intermediate input use.

\[ Q_{it} = \min \{ (L_{it}^{\theta_L} K_{it}^{\theta_K}) \exp(\omega_{it}), \alpha_M M_{it} \} \]
where $i$ denotes individual firms, $j$ denotes industry, $\omega_{it}$ is idiosyncratic productivity. At the optimum, I have

$$ Q_{it} = (L_{it}^{\theta_L} K_{it}^{\theta_K}) \exp(\omega_{it}) = \alpha_M M_{it} $$

Since it is hard to find appropriate intermediate input in COMPUSTAT, I use

$$ Q_{it} = (L_{it}^{\theta_L} K_{it}^{\theta_K}) \exp(\omega_{it}) $$

A.3 Markup Trend

In this section, I investigate the discussion related to the measurement error raised by Karabarbounis and Neiman (2018). The left panel of figure A.1 shows that I have difficulty in replicating their results leveraging on the code of De Loecker and Warzynski (2012). Further to the result, I show that I can generate an almost identical result (the right panel of figure A.1) if I use only one moment condition as it is written on DLE. The small difference may come from the update of COMPUSTAT data or the difference in the data cleaning procedure. However, I do not take stance in the markup trend since I find the rise of markup with dynamic panel approach (Blundell and Bond, 1998)\(^1\).

![Figure A.1: Markup Trend](image)

Note: The left panel uses two moment conditions whereas the right panel uses the condition related to labor only.

The measurement error affects aggregate markup in two aspects. First, the

\(^1\) This result is available upon request.
measurement error distorts the inverse of the cost-share of productive firms. Given the industry-specific production function, the inverse of the cost share of a high-measurement-error firm is larger in the first stage. Therefore, the markup of high-measurement-error firms is inflated \( \mu_{it} \equiv \frac{P_{it}}{K_{it}} = \beta_{it} \frac{P_{it}Q_{it}}{P_{it}V_{it} \exp(\epsilon_{it})} \). However, the green dotted line in figure A.2 shows that the first channel itself is not important since low-measurement-error firms cancel each other. Second, the weights on high-measurement-error firms are different. Since I use the sales-weighted average to aggregate markup, the sales of high-measurement-error firms become larger if I do not adjust for the measurement error.

\[
\mu_t = \sum_i^N \mu_{it} \frac{P_{it}Q_{it}}{\exp(\epsilon_{it})} / \left( \sum_i^N \frac{P_{it}Q_{it}}{\exp(\epsilon_{it})} \right)
\]

The purple dashed line in figure A.2 illustrates that this weight channel accounts for half of the increase. The other half of the increase falls on the multiplicative effect of two channels.

Figure A.2: Markup Trend Decomposition

A.4 Regression Tables

I provide some regression tables for the main results. I can provide the regression tables for all other results upon request.
### Table A.1: Regression Table of TFP Shock

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LightCyan TFP</td>
<td>-0.158***</td>
<td>0.014</td>
<td>0.062***</td>
<td>-0.033***</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.019)</td>
<td>(0.019)</td>
<td>(0.020)</td>
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<tr>
<td>L.Markup</td>
<td>0.601***</td>
<td>0.403***</td>
<td>0.320***</td>
<td>0.235***</td>
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<td></td>
<td>(0.010)</td>
<td>(0.013)</td>
<td>(0.015)</td>
<td>(0.017)</td>
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<tr>
<td>L.PROD</td>
<td>-0.138***</td>
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<td>-0.136***</td>
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<tr>
<td></td>
<td>(0.013)</td>
<td>(0.020)</td>
<td>(0.024)</td>
<td>(0.028)</td>
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<tr>
<td>L.SALE</td>
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<td>-0.015***</td>
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<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
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<td>(0.003)</td>
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<td>L.MS</td>
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<td>L.SG&amp;A</td>
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<td>(0.002)</td>
<td>(0.002)</td>
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<td>0.144***</td>
<td>0.163***</td>
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<td></td>
<td>(0.008)</td>
<td>(0.012)</td>
<td>(0.014)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>N</td>
<td>192,218</td>
<td>174,881</td>
<td>159,462</td>
<td>146,337</td>
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</table>

Note: * p < 0.1, ** p < 0.05, *** p < 0.01, standard errors in parentheses

### Table A.2: Regression Table of MP Shock

<table>
<thead>
<tr>
<th></th>
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<th>(2)</th>
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<tbody>
<tr>
<td>LightCyan MP</td>
<td>-0.536***</td>
<td>0.198</td>
<td>-0.028</td>
<td>-0.171*</td>
</tr>
<tr>
<td></td>
<td>(0.087)</td>
<td>(0.103)</td>
<td>(0.105)</td>
<td>(0.102)</td>
</tr>
<tr>
<td>L.Markup</td>
<td>0.541***</td>
<td>0.342***</td>
<td>0.205***</td>
<td>0.127***</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.017)</td>
<td>(0.021)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>L.PROD</td>
<td>-0.123***</td>
<td>-0.331***</td>
<td>-0.238***</td>
<td>-0.028</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.032)</td>
<td>(0.032)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>L.SALE</td>
<td>-0.027***</td>
<td>-0.023***</td>
<td>-0.019***</td>
<td>-0.017***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>L.MS</td>
<td>-0.117***</td>
<td>-0.169**</td>
<td>-0.206**</td>
<td>-0.278***</td>
</tr>
<tr>
<td></td>
<td>(0.043)</td>
<td>(0.070)</td>
<td>(0.088)</td>
<td>(0.097)</td>
</tr>
<tr>
<td>L.AGE</td>
<td>0.001***</td>
<td>0.000**</td>
<td>-0.000**</td>
<td>-0.000*</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>L.SG&amp;A</td>
<td>0.012***</td>
<td>0.010***</td>
<td>0.008***</td>
<td>0.010***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>L.GDP</td>
<td>0.096***</td>
<td>0.284***</td>
<td>0.415***</td>
<td>0.317***</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.025)</td>
<td>(0.029)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>N</td>
<td>110,226</td>
<td>99,783</td>
<td>90,571</td>
<td>82,973</td>
</tr>
</tbody>
</table>

Note: * p < 0.1, ** p < 0.05, *** p < 0.01, standard errors in parentheses
A.5 Incumbent Firm’s Markup Determination

I can derive the full analytic equation for incumbents’ markup determination. I first set Lagrangian function for incumbents’ problem.

\[ L = p_i y_i - w_n + \max_{\text{exit, stay}} \left[ 0, -e^\xi + \frac{1}{1 + r} EV(z, h; \Gamma) \right] + \lambda_n (e^x e^A n - y) + \lambda_h [(1 - \delta) h_{-1} + \delta y - h] + \lambda_c \left[ p^{-p} h_{-1}^{(1-p)} \bar{c} - y \right] \]

FOCs are following:

[B] : \[ -w + \lambda_n e^x e^A = 0 \] (A.1)

[y] : \[ p + \lambda_n + \lambda_h \delta - \lambda_c = 0 \] (A.2)

[h] : \[ \beta EV_h - \lambda_h = 0 \] (A.3)

[p] : \[ y - \lambda_c p_{it}^{-1} p h_{-1}^{(1-p)} \bar{c} = 0 \] (A.4)

I notice that \( \lambda_n \) is marginal cost and \( \lambda_c = \frac{1}{p} p_{it} \) from equation (16). Plug those into equation (14) and rearrange to get the below.

\[ \delta \lambda_h = \left( \frac{1 - \rho}{\rho} \right) p_{it} + mc_{it} \] (A.5)

Using envelope condition, I obtain

\[ \lambda_h = \frac{1}{1 + r} G(\zeta_{it}) [\lambda_h' (1 - \delta) + \lambda_h' \theta_1 (\rho - 1) \frac{y'}{h}] \] (A.6)

Forward iterate equation (18) and use equation (17) to obtain,

\[ \left( \frac{1 - \rho}{\rho} \right) p_{it} + mc_{it} = \left( \frac{\rho - 1}{\rho} \right) \delta \theta_1 \frac{1}{1 + r} G(\zeta_{it}) \sum_{j=1}^{\infty} (1 - \delta)^{j-1} \Pi_{j=1}^{\infty} \frac{1}{1 + \eta_{it+j}} G(\zeta_{it+j}) p_{it+j} \frac{y_{it+j}}{h_{it+j-1}} \] (A.7)

Divide equation (19) by \( mc_{it} \) and multiply \( \frac{\rho}{1 - \rho} \) on both sides to obtain
\[ \mu_{it} = \mu^* - E_t A_{it} \]

\[ A_{it} = \delta \theta_1 \sum_{j=1}^{\infty} (1 - \delta)^{j-1} \Pi_j \left[ \frac{1}{1 + r_{t,t+j}} G(\zeta_{it+j}^*) \right] m_{c_{it+j}} \frac{y_{it+j}}{m_{c_{it}}} \frac{\mu_{it+j}}{\pi_{it+j-1}} > 0 \]

A.6 Exit Risk Dynamics

This section shows how exit risk changes when there is aggregate shocks. I find that exit risk is stable with respect to positive technology shock. It implies that the value of a firm is stable when there is positive aggregate productivity shock since markup goes down. By contrast, the exit rate decreases for expansionary monetary policy shock since demand increases.

Figure A.3: Change in Exit Risk upon Aggregate Shocks

A.7 Additive Habit

In this section, I depart from the constant elasticity of substitution (CES) demand assumption by adding additive habit. I generalize the Ravn et al. (2006) preference by including additive habit (maniacs) and multiplicative habit (loyal customers) at the same time. Functional form for habit-adjusted consumption basket is now

\[ \tilde{c}_j = \left[ \int_i^t \left( c_{ij} \eta_i - \theta_2 h_i \right) e^{\frac{-\lambda}{\sigma}} di \right] \eta^{\frac{\sigma}{\sigma+1}} \]
which gives following demand function:

\[ c_i = \left( \frac{P_i}{P} \right)^{-\rho} \tilde{C} h_i^{\theta_1 (\rho - 1)} + \theta_2 h_i^{1 - \theta_1} \]

where \( \theta_2 \) represents the degree of habit that is price inelastic. Then the incumbents problem is the following.

\[
V(S_{-1}; F_{-1}) = \max_{p_i, h_i, n_i, y_i} \left\{ \frac{p_i}{P} y_i - \frac{W}{P} n_i + \max_{\text{exit, stay}} \left[ 0, -\frac{\zeta}{P} + \Lambda EV(S; F) \right] \right\}
\]

subject to

\[ y_i = \left( \frac{P_i}{P} \right)^{-\rho} \tilde{C} h_{i-1}^{\theta_1 (\rho - 1)} + \theta_2 h_{i-1}^{1 - \theta_1}, \quad \frac{p_i}{P} \leq \bar{p} \]

and production function (equation 2.1), operating cost distribution (equation 2.4), and the law of motions for the state variables (equations 2.2, 2.3, 2.6, 2.8, and 2.11-13). I need to assume that a maximum price exits\(^2\) since there is completely price inelastic demand. The maximum level of price is set to be high\(^3\). I calibrate \( \theta_2 \) using an additional moment\(^4\).

In this specification, firms price elasticity is a weighted sum of the price elastic habit part and price inelastic habit part. Therefore, firms price elasticity changes as a firm grows. This channel adds additional effect of channel. However, the result is similar\(^5\).

**A.8 Value Function and Policy Function**

Figure A.4 shows that there exists a value function that is a fixed point of the incumbent firms’ problem.

\(^2\) One may relax this assumption slightly using a Logit function that the probability of dropping habit increases as relative price increases. In this case, I need more parameters to match.

\(^3\) I set the relative price cannot exceed 2.5.

\(^4\) For the results in this section, I use 0-2 year firm number share. I repeat the entire calibration process, and the model can match data fairly well.

\(^5\) I provide the result upon request.
Figure A.4: Value Function and Policy Function

Value function

- Low productivity
- High productivity

Habit Choice Policy

- Low productivity
- High productivity
Appendix B

Appendix for Chapter 3

B.1 Data

This section provides details of my data. I deflate all nominal variables using GDP deflator except capital. For capital, I use the relative price of capital to deflate the variable. For firm-level data, I use the following items from Compustat.

Table B.1: Details of Data

<table>
<thead>
<tr>
<th>Data</th>
<th>Compustat item</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital investment</td>
<td>Capital Expenditure (CAPX)</td>
</tr>
<tr>
<td>Research &amp; Development</td>
<td>R&amp;D (XRD)</td>
</tr>
<tr>
<td>Value of a Firm</td>
<td>Market Value of Outstanding Equity + The Book Value of Debt - the firm’s current asset.</td>
</tr>
<tr>
<td>Replacement Cost of Capital</td>
<td>Book Value of Gross Capital (PPEGT)</td>
</tr>
<tr>
<td>Labor Cost</td>
<td>Cost Of Goods Sols (COGS)</td>
</tr>
<tr>
<td>Employment</td>
<td>Number of Employees (EMP)</td>
</tr>
<tr>
<td>Output</td>
<td>Sale (SALE)</td>
</tr>
<tr>
<td>(Market) Leverage</td>
<td>(Long term and Current Debt) / (Total Asset - Book Equity + Market Value of Common Equity)</td>
</tr>
<tr>
<td>Dividend</td>
<td>Total Dividend Payout (DVT)</td>
</tr>
<tr>
<td>Payout Ratio</td>
<td>(Dividend + Stocks Purchase) / Lag of Total Asset</td>
</tr>
</tbody>
</table>

B.2 More Robustness Tests

In this section, I demonstrate my results are robust to a different definition of variables or individuals. First, I use the conventional definition of Tobin Q using
tangible capital only.

\[ Q_{it} = \frac{V_{it}}{K_{\text{tan},it}} \]

Figure B.1: Tobin Q Response

Note: Shade denotes 95% Confidence Band. Standard errors are clustered by a firm.

I further provide the selected SIC 3 digit sectoral regression results for robustness checks. Other results are available upon request. First, figure B.2 shows the cumulative response of investment.

Figure B.2: Sectoral Response to Tax Cut

Note: Shade denotes 95% Confidence Band. Standard errors are clustered by a firm.

Figure B.3 demonstrates the cumulative response of total labor costs.
B.3 Regression Tables

This section provides selected the regression tables for the cumulative response regressions. All regression tables can be provided upon request. I note that DVT means total dividend, and $\Delta_{t+0}^h$ is a time difference between $X_{t+h} - X_{t+0}$.

Table B.2: Cumulative Response of Intangible Capital Investment

<table>
<thead>
<tr>
<th>$\Delta_{t+0}^h TaxRate_i$</th>
<th>$\Delta_{t+0}^h SG&amp;\text{A}$</th>
<th>$\Delta_{t+0}^h SG&amp;\text{A}$</th>
<th>$\Delta_{t+0}^h SG&amp;\text{A}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_{t+0}^h TaxRate_i$</td>
<td>-1.028**</td>
<td>-1.583***</td>
<td>-1.575***</td>
</tr>
<tr>
<td></td>
<td>(0.287)</td>
<td>(0.289)</td>
<td>(0.283)</td>
</tr>
<tr>
<td>$L.\Delta_{t+0}^h TaxRate$</td>
<td>-0.445***</td>
<td>-0.747***</td>
<td>-0.761***</td>
</tr>
<tr>
<td></td>
<td>(0.133)</td>
<td>(0.146)</td>
<td>(0.145)</td>
</tr>
<tr>
<td>$L.\Delta_{t+0}^h DVT$</td>
<td>2.483***</td>
<td>3.394***</td>
<td>4.463***</td>
</tr>
<tr>
<td></td>
<td>(0.267)</td>
<td>(0.384)</td>
<td>(0.452)</td>
</tr>
<tr>
<td>$L.\Delta_{t+0}^h SG&amp;\text{A}$</td>
<td>-5.448***</td>
<td>-10.534***</td>
<td>-14.241***</td>
</tr>
<tr>
<td></td>
<td>(1.694)</td>
<td>(1.598)</td>
<td>(1.866)</td>
</tr>
<tr>
<td>$L.\Delta_{t+0}^h GDP$</td>
<td>0.800***</td>
<td>1.066***</td>
<td>0.980***</td>
</tr>
<tr>
<td></td>
<td>(0.050)</td>
<td>(0.078)</td>
<td>(0.091)</td>
</tr>
<tr>
<td>$N$</td>
<td>92,897</td>
<td>86,088</td>
<td>79,910</td>
</tr>
<tr>
<td>Fixed Effect</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>F stat$^1$</td>
<td>56.421</td>
<td>96.111</td>
<td>122.228</td>
</tr>
</tbody>
</table>

Note: * p<0.1, ** p<0.05, *** p<0.01, standard errors in parentheses

( ) are standard errors. Standard errors are clustered at the firm level.

1) Kleibergen-Paap Wald F stat. If F is higher than 10, instrument is strong.
Table B.3: Cumulative Response of Tangible Capital Investment

<table>
<thead>
<tr>
<th></th>
<th>$\Delta^0_0$ CAPX</th>
<th>$\Delta^1_0$ CAPX</th>
<th>$\Delta^2_0$ CAPX</th>
<th>$\Delta^3_0$ CAPX</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta^0_0$ TaxRate</td>
<td>-2.106***</td>
<td>-5.100***</td>
<td>-7.355***</td>
<td>-4.357***</td>
</tr>
<tr>
<td></td>
<td>(0.656)</td>
<td>(0.639)</td>
<td>(0.840)</td>
<td>(0.733)</td>
</tr>
<tr>
<td>L. $\Delta^0_0$ TaxRate</td>
<td>-0.846***</td>
<td>-2.435***</td>
<td>-3.608***</td>
<td>-2.093***</td>
</tr>
<tr>
<td></td>
<td>(0.306)</td>
<td>(0.324)</td>
<td>(0.428)</td>
<td>(0.375)</td>
</tr>
<tr>
<td>L. $\Delta^0_0$ DVT</td>
<td>6.848***</td>
<td>5.832***</td>
<td>6.526***</td>
<td>4.311***</td>
</tr>
<tr>
<td></td>
<td>(0.616)</td>
<td>(0.924)</td>
<td>(1.228)</td>
<td>(0.994)</td>
</tr>
<tr>
<td>L. $\Delta^0_0$ CAPX</td>
<td>-28.638***</td>
<td>-42.120***</td>
<td>-45.694***</td>
<td>-45.577***</td>
</tr>
<tr>
<td></td>
<td>(0.667)</td>
<td>(0.805)</td>
<td>(0.930)</td>
<td>(0.745)</td>
</tr>
<tr>
<td>L. $\Delta^0_0$ GDP</td>
<td>3.348***</td>
<td>2.385***</td>
<td>0.406</td>
<td>-0.097</td>
</tr>
<tr>
<td></td>
<td>(0.124)</td>
<td>(0.204)</td>
<td>(0.282)</td>
<td>(0.247)</td>
</tr>
</tbody>
</table>

| N            | 99,821             | 92,974             | 86,753             | 81,053             |
| Fixed Effect | Yes                | Yes                | Yes                | Yes                |
| F stat$^1$   | 59.317             | 132.421            | 122.453            | 95.143             |

Note: * p<0.1, ** p<0.05, *** p<0.01, standard errors in parentheses
( ) are standard errors. Standard errors are clustered at the firm level.
1) Kleibergen-Paap Wald F stat. If F is higher than 10, instrument is strong.
2) CAPX is physical capital investment.

Table B.4: Cumulative Response of R&D

<table>
<thead>
<tr>
<th></th>
<th>$\Delta^0_0$ R&amp;D</th>
<th>$\Delta^1_0$ R&amp;D</th>
<th>$\Delta^2_0$ R&amp;D</th>
<th>$\Delta^3_0$ R&amp;D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta^0_0$ TaxRate</td>
<td>-1.700</td>
<td>-2.263***</td>
<td>-3.036***</td>
<td>-1.654**</td>
</tr>
<tr>
<td></td>
<td>(1.337)</td>
<td>(0.809)</td>
<td>(1.109)</td>
<td>(0.683)</td>
</tr>
<tr>
<td>L. $\Delta^0_0$ TaxRate</td>
<td>-0.809</td>
<td>-1.150***</td>
<td>-1.553***</td>
<td>-0.784**</td>
</tr>
<tr>
<td></td>
<td>(0.660)</td>
<td>(0.426)</td>
<td>(0.587)</td>
<td>(0.349)</td>
</tr>
<tr>
<td>L. $\Delta^0_0$ DVT</td>
<td>2.513***</td>
<td>3.953***</td>
<td>3.277***</td>
<td>4.305***</td>
</tr>
<tr>
<td></td>
<td>(0.714)</td>
<td>(0.945)</td>
<td>(1.192)</td>
<td>(1.050)</td>
</tr>
<tr>
<td>L. $\Delta^0_0$ R&amp;D</td>
<td>-18.898***</td>
<td>-25.033***</td>
<td>-28.160***</td>
<td>-29.889***</td>
</tr>
<tr>
<td></td>
<td>(1.990)</td>
<td>(1.760)</td>
<td>(2.238)</td>
<td>(1.978)</td>
</tr>
<tr>
<td>L. $\Delta^0_0$ GDP</td>
<td>0.693***</td>
<td>0.660***</td>
<td>0.225</td>
<td>0.110</td>
</tr>
<tr>
<td></td>
<td>(0.136)</td>
<td>(0.199)</td>
<td>(0.252)</td>
<td>(0.207)</td>
</tr>
</tbody>
</table>

| N            | 29,292             | 26,728             | 24,440             | 22,371             |
| Fixed Effect | Yes                | Yes                | Yes                | Yes                |
| F stat$^1$   | 4.800              | 20.577             | 16.261             | 26.592             |

Note: * p<0.1, ** p<0.05, *** p<0.01, standard errors in parentheses
( ) are standard errors. Standard errors are clustered at the firm level.
1) Kleibergen-Paap Wald F stat. If F is higher than 10, instrument is strong.
Table B.5: Cumulative Response of Tobin Q

<table>
<thead>
<tr>
<th></th>
<th>$\Delta_0^o\text{TobinQ}$</th>
<th>$\Delta_1^o\text{TobinQ}$</th>
<th>$\Delta_2^o\text{TobinQ}$</th>
<th>$\Delta_3^o\text{TobinQ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_0^o\text{TaxRate}_i$</td>
<td>-0.132***</td>
<td>-0.155***</td>
<td>-0.027***</td>
<td>-0.122***</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td>(0.022)</td>
<td>(0.010)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>$L.\Delta_0^o\text{TaxRate}$</td>
<td>-0.062***</td>
<td>-0.078***</td>
<td>-0.014***</td>
<td>-0.063***</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.011)</td>
<td>(0.005)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>$L.\Delta_0^o\text{DVT}$</td>
<td>-0.000</td>
<td>-0.059*</td>
<td>-0.030*</td>
<td>-0.114***</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.035)</td>
<td>(0.017)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>$L.\Delta_0^o\text{TobinQ}$</td>
<td>-0.250***</td>
<td>-0.325***</td>
<td>-0.412***</td>
<td>-0.348***</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.024)</td>
<td>(0.022)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>$L.\Delta_0^o\text{GDP}$</td>
<td>-0.026***</td>
<td>-0.026***</td>
<td>-0.023***</td>
<td>-0.024***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.006)</td>
<td>(0.003)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>$N$</td>
<td>62,787</td>
<td>57,686</td>
<td>53,266</td>
<td>49,272</td>
</tr>
<tr>
<td>Fixed Effect</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$F\text{ stat}^1$</td>
<td>49.552</td>
<td>68.427</td>
<td>74.156</td>
<td>69.181</td>
</tr>
</tbody>
</table>

Note: * p<0.1, ** p<0.05, *** p<0.01, standard errors in parentheses

Table B.6: Cumulative Response of Total Labor Costs

<table>
<thead>
<tr>
<th></th>
<th>$\Delta_0^o\text{COGS}$</th>
<th>$\Delta_1^o\text{COGS}$</th>
<th>$\Delta_2^o\text{COGS}$</th>
<th>$\Delta_3^o\text{COGS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_0^o\text{TaxRate}_i$</td>
<td>0.284</td>
<td>-1.154***</td>
<td>-2.936***</td>
<td>-2.294***</td>
</tr>
<tr>
<td></td>
<td>(0.275)</td>
<td>(0.244)</td>
<td>(0.337)</td>
<td>(0.346)</td>
</tr>
<tr>
<td>$L.\Delta_0^o\text{TaxRate}$</td>
<td>0.157</td>
<td>-0.537***</td>
<td>-1.439***</td>
<td>-1.114***</td>
</tr>
<tr>
<td></td>
<td>(0.127)</td>
<td>(0.121)</td>
<td>(0.169)</td>
<td>(0.175)</td>
</tr>
<tr>
<td>$L.\Delta_0^o\text{DVT}$</td>
<td>2.553***</td>
<td>4.527***</td>
<td>5.412***</td>
<td>5.795***</td>
</tr>
<tr>
<td></td>
<td>(0.278)</td>
<td>(0.387)</td>
<td>(0.526)</td>
<td>(0.549)</td>
</tr>
<tr>
<td>$L.\Delta_0^o\text{COGS}$</td>
<td>-8.208***</td>
<td>-14.998***</td>
<td>-20.567***</td>
<td>-22.061***</td>
</tr>
<tr>
<td></td>
<td>(1.276)</td>
<td>(1.400)</td>
<td>(1.561)</td>
<td>(1.674)</td>
</tr>
<tr>
<td>$L.\Delta_0^o\text{GDP}$</td>
<td>1.359***</td>
<td>1.550***</td>
<td>1.019***</td>
<td>0.559***</td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
<td>(0.076)</td>
<td>(0.121)</td>
<td>(0.127)</td>
</tr>
<tr>
<td>$N$</td>
<td>124,518</td>
<td>116,182</td>
<td>108,428</td>
<td>101,303</td>
</tr>
<tr>
<td>Fixed Effect</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$F\text{ stat}^1$</td>
<td>69.304</td>
<td>168.963</td>
<td>175.474</td>
<td>149.028</td>
</tr>
</tbody>
</table>

Note: * p<0.1, ** p<0.05, *** p<0.01, standard errors in parentheses

( ) are standard errors. Standard errors are clustered at the firm level.

1) Kleibergen-Paap Wald F stat. If F is higher than 10, instrument is strong.
2) COGS denotes Cost of Goods Sold.
Table B.7: Cumulative Response of Employment

<table>
<thead>
<tr>
<th></th>
<th>$\Delta_0^{\text{EMP}}$</th>
<th>$\Delta_1^{\text{EMP}}$</th>
<th>$\Delta_2^{\text{EMP}}$</th>
<th>$\Delta_3^{\text{EMP}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_0^{\text{TaxRate}}$</td>
<td>-0.739***</td>
<td>-1.155***</td>
<td>-1.478***</td>
<td>-0.700**</td>
</tr>
<tr>
<td></td>
<td>(0.269)</td>
<td>(0.218)</td>
<td>(0.266)</td>
<td>(0.275)</td>
</tr>
<tr>
<td>$L.\Delta_0^{\text{TaxRate}}$</td>
<td>-0.321***</td>
<td>-0.553***</td>
<td>-0.726***</td>
<td>-0.333**</td>
</tr>
<tr>
<td></td>
<td>(0.124)</td>
<td>(0.109)</td>
<td>(0.134)</td>
<td>(0.139)</td>
</tr>
<tr>
<td>$L.\Delta_0^{\text{DVT}}$</td>
<td>1.648***</td>
<td>2.509***</td>
<td>3.404***</td>
<td>3.927***</td>
</tr>
<tr>
<td></td>
<td>(0.253)</td>
<td>(0.369)</td>
<td>(0.447)</td>
<td>(0.490)</td>
</tr>
<tr>
<td>$L.\Delta_0^{\text{EMP}}$</td>
<td>-5.845***</td>
<td>-12.208***</td>
<td>-16.618***</td>
<td>-17.989***</td>
</tr>
<tr>
<td></td>
<td>(1.011)</td>
<td>(1.410)</td>
<td>(1.738)</td>
<td>(1.999)</td>
</tr>
<tr>
<td>$L.\Delta_0^{\text{GDP}}$</td>
<td>0.517***</td>
<td>0.639***</td>
<td>0.303***</td>
<td>0.290***</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td>(0.067)</td>
<td>(0.097)</td>
<td>(0.106)</td>
</tr>
</tbody>
</table>

$N$ | 108,218 | 100,182 | 92,844 | 86,182 |

Fixed Effect | Yes | Yes | Yes | Yes |

F stat$^1$ | 52.789 | 153.630 | 145.750 | 122.849 |

Note: * p<0.1, ** p<0.05, *** p<0.01, standard errors in parentheses

( ) are standard errors. Standard errors are clustered at the firm level.

1) Kleibergen-Paap Wald F stat. If F is higher than 10, instrument is strong.

2) EMP denotes employment.

Table B.8: Cumulative Response of Sale

<table>
<thead>
<tr>
<th></th>
<th>$\Delta_0^{\text{Sale}}$</th>
<th>$\Delta_1^{\text{Sale}}$</th>
<th>$\Delta_2^{\text{Sale}}$</th>
<th>$\Delta_3^{\text{Sales}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_0^{\text{TaxRate}}$</td>
<td>-1.423***</td>
<td>-2.147***</td>
<td>-2.901***</td>
<td>-1.520***</td>
</tr>
<tr>
<td></td>
<td>(0.292)</td>
<td>(0.265)</td>
<td>(0.317)</td>
<td>(0.281)</td>
</tr>
<tr>
<td>$L.\Delta_0^{\text{TaxRate}}$</td>
<td>-0.640***</td>
<td>-1.040***</td>
<td>-1.444***</td>
<td>-0.734***</td>
</tr>
<tr>
<td></td>
<td>(0.135)</td>
<td>(0.132)</td>
<td>(0.160)</td>
<td>(0.142)</td>
</tr>
<tr>
<td>$L.\Delta_0^{\text{DVT}}$</td>
<td>2.740***</td>
<td>3.230***</td>
<td>3.924***</td>
<td>4.317***</td>
</tr>
<tr>
<td></td>
<td>(0.332)</td>
<td>(0.429)</td>
<td>(0.523)</td>
<td>(0.464)</td>
</tr>
<tr>
<td>$L.\Delta_0^{\text{Sales}}$</td>
<td>-4.019***</td>
<td>-12.568***</td>
<td>-15.878***</td>
<td>-18.232***</td>
</tr>
<tr>
<td></td>
<td>(1.568)</td>
<td>(1.958)</td>
<td>(2.056)</td>
<td>(2.067)</td>
</tr>
<tr>
<td>$L.\Delta_0^{\text{GDP}}$</td>
<td>0.951***</td>
<td>1.076***</td>
<td>0.654***</td>
<td>0.514***</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.085)</td>
<td>(0.111)</td>
<td>(0.097)</td>
</tr>
</tbody>
</table>

$N$ | 129,070 | 120,714 | 112,916 | 105,697 |

Fixed Effect | Yes | Yes | Yes | Yes |

F stat$^1$ | 71.844 | 174.280 | 185.299 | 158.077 |

Note: * p<0.1, ** p<0.05, *** p<0.01, standard errors in parentheses

( ) are standard errors. Standard errors are clustered at the firm level.

1) Kleibergen-Paap Wald F stat. If F is higher than 10, instrument is strong.
Table B.9: Cumulative Response of Leverage

<table>
<thead>
<tr>
<th>$\Delta_0^i TaxRate$</th>
<th>$\Delta_1^i Lev$</th>
<th>$\Delta_2^i Lev$</th>
<th>$\Delta_3^i Lev$</th>
<th>$\Delta_4^i Lev$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_0^i TaxRate$</td>
<td>1.086***</td>
<td>0.712***</td>
<td>0.056</td>
<td>0.602***</td>
</tr>
<tr>
<td></td>
<td>(0.223)</td>
<td>(0.093)</td>
<td>(0.064)</td>
<td>(0.096)</td>
</tr>
<tr>
<td>$L.\Delta_0^i TaxRate$</td>
<td>0.511***</td>
<td>0.357***</td>
<td>0.030</td>
<td>0.308***</td>
</tr>
<tr>
<td></td>
<td>(0.104)</td>
<td>(0.046)</td>
<td>(0.032)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>$L.\Delta_0^i DVT$</td>
<td>0.637***</td>
<td>1.192***</td>
<td>1.472***</td>
<td>1.928***</td>
</tr>
<tr>
<td></td>
<td>(0.147)</td>
<td>(0.138)</td>
<td>(0.118)</td>
<td>(0.161)</td>
</tr>
<tr>
<td>$L.\Delta_0^i Lev$</td>
<td>-0.047***</td>
<td>-0.253***</td>
<td>-0.362***</td>
<td>-0.366***</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.010)</td>
<td>(0.008)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>$L.\Delta_0^i GDP$</td>
<td>0.622***</td>
<td>0.839***</td>
<td>0.721***</td>
<td>0.695***</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.031)</td>
<td>(0.024)</td>
<td>(0.039)</td>
</tr>
</tbody>
</table>

$N$ 97,451 90,167 83,536 77,349
Fixed Effect Yes Yes Yes Yes
F stat $^1$ 30.504 109.819 127.986 99.402

Note: * p<0.1, ** p<0.05, *** p<0.01, standard errors in parentheses
( ) are standard errors. Standard errors are clustered at the firm level.
1) Kleibergen-Paap Wald F stat. If F is higher than 10, instrument is strong.
2) Lev implies leverage.

Table B.10: Cumulative Response of Dividend Payout

<table>
<thead>
<tr>
<th>$\Delta_0^i PAYOUT$</th>
<th>$\Delta_1^i PAYOUT$</th>
<th>$\Delta_2^i PAYOUT$</th>
<th>$\Delta_3^i PAYOUT$</th>
<th>$\Delta_4^i PAYOUT$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_0^i TaxRate$</td>
<td>-6.941</td>
<td>-1.640*</td>
<td>-1.734*</td>
<td>-3.144**</td>
</tr>
<tr>
<td></td>
<td>(4.776)</td>
<td>(0.849)</td>
<td>(0.973)</td>
<td>(1.411)</td>
</tr>
<tr>
<td>$L.\Delta_0^i TaxRate$</td>
<td>-3.038</td>
<td>-0.775**</td>
<td>-0.829*</td>
<td>-1.527**</td>
</tr>
<tr>
<td></td>
<td>(2.082)</td>
<td>(0.389)</td>
<td>(0.466)</td>
<td>(0.684)</td>
</tr>
<tr>
<td>$L.\Delta_0^i Payout$</td>
<td>-0.477***</td>
<td>-0.489***</td>
<td>-0.492***</td>
<td>-0.499***</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.013)</td>
<td>(0.008)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>$L.\Delta_0^i GDP$</td>
<td>-0.108</td>
<td>-0.405</td>
<td>-0.273</td>
<td>-0.634</td>
</tr>
<tr>
<td></td>
<td>(0.180)</td>
<td>(0.269)</td>
<td>(0.242)</td>
<td>(0.397)</td>
</tr>
</tbody>
</table>

$N$ 230,172 208,862 189,732 172,524
Fixed Effect Yes Yes Yes Yes
F stat $^1$ 27.673 109.819 127.986 99.402

Note: * p<0.1, ** p<0.05, *** p<0.01, standard errors in parentheses
( ) are standard errors. Standard errors are clustered at the firm level.
1) Kleibergen-Paap Wald F stat. If F is higher than 10, instrument is strong.
2) Payout means dividend payout ratio.
### Table B.11: Cumulative Response of Productivity

<table>
<thead>
<tr>
<th></th>
<th>$\Delta_0^t \text{Prod}$</th>
<th>$\Delta_1^t \text{Prod}$</th>
<th>$\Delta_2^t \text{Prod}$</th>
<th>$\Delta_3^t \text{Prod}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_0^t \text{TaxRate}_i$</td>
<td>-1.065***</td>
<td>-1.374***</td>
<td>-1.230***</td>
<td>-0.713***</td>
</tr>
<tr>
<td></td>
<td>(0.166)</td>
<td>(0.163)</td>
<td>(0.134)</td>
<td>(0.091)</td>
</tr>
<tr>
<td>$L.\Delta_0^t \text{TaxRate}$</td>
<td>-0.496***</td>
<td>-0.698***</td>
<td>-0.629***</td>
<td>-0.364***</td>
</tr>
<tr>
<td></td>
<td>(0.077)</td>
<td>(0.083)</td>
<td>(0.068)</td>
<td>(0.047)</td>
</tr>
<tr>
<td>$L.\Delta_0^t DVT$</td>
<td>0.043</td>
<td>-0.377*</td>
<td>-0.422**</td>
<td>-0.708***</td>
</tr>
<tr>
<td></td>
<td>(0.147)</td>
<td>(0.224)</td>
<td>(0.212)</td>
<td>(0.143)</td>
</tr>
<tr>
<td>$L.\Delta_0^t \text{Prod}$</td>
<td>-2.040</td>
<td>-14.577***</td>
<td>-24.566***</td>
<td>-17.062***</td>
</tr>
<tr>
<td></td>
<td>(3.181)</td>
<td>(4.429)</td>
<td>(4.203)</td>
<td>(3.066)</td>
</tr>
<tr>
<td>$L.\Delta_0^t GDP$</td>
<td>0.060**</td>
<td>-0.006</td>
<td>-0.013</td>
<td>-0.095***</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.048)</td>
<td>(0.046)</td>
<td>(0.028)</td>
</tr>
<tr>
<td>$N$</td>
<td>75,447</td>
<td>70,279</td>
<td>65,613</td>
<td>61,278</td>
</tr>
<tr>
<td>Fixed Effect</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$F$ stat$^1$</td>
<td>42.629</td>
<td>74.591</td>
<td>91.477</td>
<td>71.685</td>
</tr>
</tbody>
</table>

Note: * p<0.1, ** p<0.05, *** p<0.01, standard errors in parentheses.  
( ) are standard errors. Standard errors are clustered at the firm level.  
1) Kleibergen-Paap Wald $F$ stat. If $F$ is higher than 10, instrument is strong.  
2) Prod denotes productivity.

### Table B.12: Current Response of Utilization

<table>
<thead>
<tr>
<th></th>
<th>(1) $\Delta_0^t TCU$</th>
<th>(2) $\Delta_0^t ELEC$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_0^t \text{TaxRate}_i$</td>
<td>-2.628***</td>
<td>-0.314***</td>
</tr>
<tr>
<td></td>
<td>(0.889)</td>
<td>(0.041)</td>
</tr>
<tr>
<td>$L.\Delta_0^t \text{TaxRate}$</td>
<td>-1.149***</td>
<td>-0.151***</td>
</tr>
<tr>
<td></td>
<td>(0.389)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>$L.\Delta_0^t TCU$</td>
<td>-0.066</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.085)</td>
<td></td>
</tr>
<tr>
<td>$L.\Delta_0^t ELEC$</td>
<td></td>
<td>0.325***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.009)</td>
</tr>
<tr>
<td>$L.\Delta_0^t GDP$</td>
<td>0.415***</td>
<td>-0.236***</td>
</tr>
<tr>
<td></td>
<td>(0.160)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>$N$</td>
<td>172,975</td>
<td>172,578</td>
</tr>
<tr>
<td>Fixed Effect</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$F$ stat$^1$</td>
<td>8.737</td>
<td>59.074</td>
</tr>
</tbody>
</table>

Note: * p<0.1, ** p<0.05, *** p<0.01, standard errors in parentheses.
<table>
<thead>
<tr>
<th></th>
<th>$\Delta_0^0$Markup</th>
<th>$\Delta_1^1$Markup</th>
<th>$\Delta_2^2$Markup</th>
<th>$\Delta_3^3$Markup</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_0^0$TaxRate$_i$</td>
<td>-0.762***</td>
<td>-0.800***</td>
<td>-0.505***</td>
<td>-0.440***</td>
</tr>
<tr>
<td></td>
<td>(0.136)</td>
<td>(0.114)</td>
<td>(0.086)</td>
<td>(0.091)</td>
</tr>
<tr>
<td>L.$\Delta_0^0$TaxRate</td>
<td>-0.359***</td>
<td>-0.411***</td>
<td>-0.261***</td>
<td>-0.230***</td>
</tr>
<tr>
<td></td>
<td>(0.063)</td>
<td>(0.058)</td>
<td>(0.044)</td>
<td>(0.046)</td>
</tr>
<tr>
<td>L.$\Delta_0^0$DVT</td>
<td>-0.102</td>
<td>-0.330**</td>
<td>-0.243*</td>
<td>-0.405***</td>
</tr>
<tr>
<td></td>
<td>(0.132)</td>
<td>(0.168)</td>
<td>(0.138)</td>
<td>(0.150)</td>
</tr>
<tr>
<td>L.$\Delta_0^0$Markup</td>
<td>-19.971***</td>
<td>-28.032***</td>
<td>-34.569***</td>
<td>-32.686***</td>
</tr>
<tr>
<td></td>
<td>(1.486)</td>
<td>(1.769)</td>
<td>(1.635)</td>
<td>(1.647)</td>
</tr>
<tr>
<td>L.$\Delta_0^0$GDP</td>
<td>-0.013</td>
<td>-0.062**</td>
<td>-0.050**</td>
<td>-0.047*</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.030)</td>
<td>(0.025)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>$N$</td>
<td>75,447</td>
<td>70,279</td>
<td>65,613</td>
<td>61,278</td>
</tr>
<tr>
<td>Fixed Effect</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>F stat$^1$</td>
<td>43.003</td>
<td>75.271</td>
<td>91.995</td>
<td>71.244</td>
</tr>
</tbody>
</table>

Note: * $p<0.1$, ** $p<0.05$, *** $p<0.01$, standard errors in parentheses
( ) are standard errors. Standard errors are clustered at the firm level.
1) Kleibergen-Paap Wald F stat. If F is higher than 10, instrument is strong.
Appendix C

Appendix for Chapter 4

C.1 Global Dynamics

I derive the Euler equation by solving a Hamilton-Jacobi-Bellman (HJB) equation. For Phillips curve, I solve a current value Hamiltonian equation.

C.1.1 Deriving Euler Equation

Set up the HJB Equation:

\[
\rho V^e(b) = \frac{e_1}{1 - \sigma} - \zeta + \delta(1 - \eta)[V^u(b) - V^e(b)] + \dot{b}V^e_b(b)
\]

where \( \dot{b} = (R - \pi)b + w - c. \)

Find the optimality conditions:

1. \( \frac{\partial \rho V^e(b)}{\partial c_e} = c^{-\sigma} - V^e_b(b) = 0 \)

2. \( dV^e_b(b) = V^e_{bb}(b)\dot{b} \)

Differentiate the HJB equation with respect to the bond and substitute out using the first order condition:
\[
\rho V_v^e(b) = \delta (1 - \eta) [V_v^u(b) - V_v^e(b)] + (R - \pi)V_v^e(b) + \hat{b} V_{v_{0b}}^e(b)
\]

\[
(\rho - R + \pi) V_v^e(b) = \delta (1 - \eta) [V_v^u(b) - V_v^e(b)] + \hat{b} V_{v_{0b}}^e(b)
\]

\[
(\rho - R + \pi) c_e^{-\sigma} = \delta (1 - \eta) [c_u^{-\sigma} - c_e^{-\sigma}] + d(c_e^{-\sigma})
\]

Use \(d(c_e^{-\sigma}) = -\sigma c_e^{-\sigma - 1} \dot{c}_e\), divide by \(c_e^{-\sigma}\), and rearrange:

\[
\frac{\dot{c}_e}{c_e} = \frac{1}{\sigma} \left( (R_t - \pi_t - \rho) + \delta (1 - \eta) \left( \frac{\partial}{\partial c_e} \right)^{-\sigma} - 1 \right)
\]

C.1.2 Deriving the Phillips Curve

Firms maximize nominal profit:

\[
E_0 \int_0^\infty e^{-\rho t} \left[ p_{j,t} y_{j,t} - P_t w_{j,t} - P_t \kappa v_{j,t} - \phi^2 (\dot{p}_{j,t})^2 y_t \right]
\]

Set up the current value Hamiltonian and use \(y_{j,t} = \left( \frac{p_{j,t}}{P_t} \right)^{-\gamma} y_t\) to substitute out \(y_{j,t}\):

\[
H(p_{j,-1}, \lambda) = P \left( \frac{p_{j}}{P} \right)^{1-\gamma} - P w \left( \frac{p_{j}}{P} \right)^{-\gamma} Y - P \kappa \left( \dot{n} + \delta n \right) - \frac{\phi}{2} \left( \frac{\dot{p}_{j,t}}{p_{j,t}} \right)^2 P Y + \lambda \dot{p}_{j}
\]

Find the optimality conditions:

1. \(\frac{\partial H}{\partial p_{j}} = -\phi \pi Y + \lambda = 0\)

2. \(\dot{\lambda} = \rho \lambda - H_{p_{j}} = \rho \lambda - [(1 - \gamma) \left( \frac{p_{j}}{P} \right)^{-\gamma} Y + \gamma w \left( \frac{p_{j}}{P} \right)^{-\gamma} Y + \gamma \delta \left( \frac{p_{j}}{P} \right)^{-\gamma} Y + \phi (\dot{p}_{j})^2 Y]\)

Use symmetry \(p_{j} = P\) and differentiate the first condition with respect to time.

1. \(\dot{\lambda} = \phi \pi Y + \phi \pi \dot{Y}\)

2. \(\dot{\lambda} = \rho \lambda - [(1 - \gamma) Y + \gamma w Y + \gamma \delta \frac{\kappa}{q} Y + \phi \pi^2 Y]\)

Substitute out \(\lambda\) and \(\dot{\lambda}\):

\[
\phi \pi Y + \phi \pi \dot{Y} = \rho (\phi \pi Y) - [(1 - \gamma) Y + \gamma w Y + \gamma \delta \frac{\kappa}{q} Y + \phi \pi^2 Y]
\]
Divide by $\phi Y$ and rearrange:

$$\dot{\pi} = \rho_\pi - \pi \frac{\dot{Y}}{Y} - \pi^2 - \frac{\gamma}{\phi} \left[ \frac{1 - \gamma}{\gamma} + w + \frac{\kappa}{q} \right]$$

### C.1.3 Finding the Wage Equation

Set up the HJB equations for the employed and unemployed:

$$\rho V^c(b) = \frac{c^1_{e}}{1 - \sigma} - \zeta + \delta (1 - \eta)(V^u(b) - V^c(b)) + \dot{b}V^c_e(b)$$

$$\rho V^u(b) = \frac{c^1_{u}}{1 - \sigma} + [1 - \delta (1 - \eta)](V^c(b) - V^u(b))$$

Impose the constraints and subtract two equations:

$$\rho (V^e - V^u) = \frac{1}{1 - \sigma} (w^{1 - \sigma} - \vartheta^{1 - \sigma}) - \zeta - (V^e - V^u)$$

$$(\rho + 1)(V^e - V^u) = \frac{1}{1 - \sigma} (w^{1 - \sigma} - \vartheta^{1 - \sigma}) - \zeta$$

$$w^{1 - \sigma} = (1 - \sigma)[(\rho + 1)(V^e - V^u) + \zeta] + \vartheta^{1 - \sigma}$$

Use the first order condition from Nash Bargaining \((V^e - V^u = \frac{\nu}{1 - \nu q})\),

$$w_t(\eta_t) = \{\vartheta^{1 - \sigma} + (1 - \sigma)[(\rho + 1)\frac{\nu}{1 - \nu q}(\eta_t) + \zeta]\}^{\frac{1}{1 - \sigma}}$$

### C.1.4 Nonlinear System of Equations

Now I can write down the system of non-linear equations.

1. Euler Equation for the employed:
$$\dot{\pi} = \frac{\pi}{\phi} \left[ (R_t - \pi_t - \rho) + \delta (1 - \eta_t) \left( \frac{\sigma}{\vartheta_t} \right)^{-\sigma} - 1 \right]$$

2. Phillips Curve:
$$\dot{\pi} = \pi_t(\rho - \pi_t - \frac{\dot{Y}}{Y}) - \frac{\gamma}{\phi} \left[ \frac{1 - \gamma}{\gamma} + w_t + \frac{\kappa}{q} \right]$$

3. Employment:
$$n_t = m_t - \delta n_t$$

4. Wage determination:
$$w_t(\eta_t) = \{\vartheta^{1 - \sigma} + (1 - \sigma)[(\rho + 1)\frac{\nu}{1 - \nu q}(\eta_t) + \zeta]\}^{\frac{1}{1 - \sigma}}$$

5. Aggregate consumption:
$$C_t = N_t - \frac{\kappa}{q} \eta_t [1 - (1 - \delta)N_t]$$
I define the incomplete markets wedge: \( \Theta(\eta) \equiv \delta(1 - \eta) \left[ \left( \frac{\vartheta}{w_t} \right)^{-\sigma} - 1 \right] \). Notice that \( \Theta(\eta) = 0 \), if \( \delta = 0 \), \( \sigma = 1 \), or \( \vartheta = w \).

C.2 Local Dynamics

I linearize the nonlinear equations using Taylor expansion around \( \pi = 0 \)\(^1\) and \( R = \pi + \rho + \Theta \)\(^2\). I define \( \hat{x}_t = x_t - x \), where an \( x \) without the subscript \( t \) is a steady-state value.

\[
\begin{align*}
\hat{\eta}_t & \approx F_\eta \hat{\eta}_t + F_\pi \hat{\pi}_t + F_R \hat{R}_t \\
\hat{\pi}_t & \approx G_\eta \hat{\eta}_t + G_\pi \hat{\pi}_t \\
\hat{N}_t & \approx H_\eta \hat{\eta}_t + H_N \hat{N}_t \\
C_t & \approx C_N N_t + C_\eta \hat{\eta}_t
\end{align*}
\]

where

\(^1\) This model has multiple equilibria: at a low job-finding rate, a zero interest rate, and the intended steady state. In the intended state, there can be two equilibria, depending on the inflation level. For details, see Ravn and Sterk (2020).

\(^2\) There are many equilibria that degenerate the asset distribution. I choose a borderline case.
\[ F_\eta = \delta (1 - \eta) \left( \frac{\vartheta}{w} \right)^{-\sigma} - \frac{1}{\sigma} \frac{w \delta}{w_\eta} \left( \frac{\vartheta}{w} \right)^{-\sigma} - 1 \]

\[ F_\pi = -\frac{w}{\sigma w_\eta} < 0 \]

\[ F_R = \begin{cases} 0, & t < \tau \\ \frac{w}{w_\eta^\sigma}, & t \geq \tau \end{cases} \]

\[ \tilde{R}_t = \begin{cases} \phi_\pi \hat{n}_t + \frac{\phi_\pi}{1 - \alpha} \hat{\eta}_t, & \text{if } t > \tau \\ \epsilon_R, & \text{if } t = \tau \\ -\gamma^{t - \tau} \epsilon_R, & \text{if } t < \tau \end{cases} \]

\[ G_\eta = -\frac{\gamma}{\phi} \left[ w_\eta + \frac{\kappa}{\alpha q} \frac{\alpha}{1 - \alpha} \right] \]

\[ G_\pi = \rho > 0 \]

\[ H_\eta = 1 - (1 - \delta)N > 0 \]

\[ H_N = -\eta - \delta + \eta \delta < 0 \]

\[ C_N = 1 + \frac{\kappa}{q} \eta (1 - \delta) > 1 \]

\[ C_\eta = -\frac{\kappa}{q} \frac{1}{1 - \alpha} \left[ 1 - (1 - \delta)N \right] < 0 \]

\[ w_\eta = w^\sigma (1 + \rho) \frac{\nu}{1 - \nu} \frac{\alpha}{1 - \alpha} \frac{\kappa}{q} \]

\[ \text{and } \tau \text{ is the realization timing of forward guidance.} \]

### C.3 Derivation of the Aggregate Euler Equation

From the definition of aggregate consumption,

\[ C_t^A \equiv (\beta + (1 - \beta) \eta_t) c_t + \Omega_t, \]

where \( \Omega_t = y_t - w_t n_t - \kappa v_t - \frac{\phi}{2} (\hat{\pi}^2) \) is profit.
Substitute out profit and use $y_t = N_t$ and $c_t^e = w_t$:

$$C_t^A = N_tw_t + y_t - w_tN_t - \kappa v_t - \frac{\phi}{2} \left( \frac{P_t}{p} \right)^2$$

using $v_t = \frac{m_t}{q_t}(1 - \beta)$ and $q_t = \psi^{\frac{1}{1-\alpha}} \eta_t^{\frac{\kappa}{\alpha}}$,

$$C_t^A = \beta + (1 - \beta) \eta_t - (1 - \beta) \kappa \psi^{\frac{1}{1-\alpha}} \eta_t^{\frac{1}{\alpha}}$$

I use Taylor expansion around steady state with $\pi = 0$:

$$\hat{C}_t^A = (1 - \beta)[1 - \frac{1}{1 - \alpha} \frac{\kappa}{q} \hat{\eta}_t], \quad \text{(C.1)}$$

From the linearized Euler Equation for the employed:

$$\hat{\eta}_t = \frac{1}{\sigma w_t} [\hat{R}_t - \hat{\pi}_t] + F_{\eta} \hat{\eta}_t$$

Use equation (3) to obtain the aggregate Euler equation.

$$\hat{C}_t^A = \frac{1}{\sigma w_t} [\hat{R}_t - \hat{\pi}_t] + F_{\eta} \hat{C}_t^A$$

### C.4 Finding Eigenvalues of the Subsystem

The algebra for calculating eigenvalues is as follows:

$$A - \lambda I = \begin{vmatrix} F_\eta + \frac{\phi_\eta}{1-\alpha} F_R - \lambda & F_\pi + \phi_\pi F_R \\ G_\eta & G_\pi - \lambda \end{vmatrix}$$

$$= \lambda^2 - (F_\eta + \frac{\phi_\eta}{1-\alpha} F_R + G_\pi)\lambda + G_\pi(F_\eta + \frac{\phi_\eta}{1-\alpha} F_R) - (F_\pi + \phi_\pi F_R)G_\eta = 0$$

$$\Leftrightarrow \lambda = \frac{F_\eta + \frac{\phi_\eta}{1-\alpha} F_R + G_\pi \pm \sqrt{(F_\eta + \frac{\phi_\eta}{1-\alpha} F_R + G_\pi)^2 - 4\{G_\pi(F_\eta + \frac{\phi_\eta}{1-\alpha} F_R) - (F_\pi + \phi_\pi F_R)G_\eta\}}}{2}$$

$$= \frac{F_\eta + \frac{\phi_\eta}{1-\alpha} F_R + G_\pi \pm \sqrt{(F_\eta + \frac{\phi_\eta}{1-\alpha} F_R - G_\pi)^2 + 4G_\eta(F_\pi + \phi_\pi F_R)}}{2}$$
C.5 Monetary Policy Shock under Endogenous Income Risk

The precautionary motive of households varies as income risk changes along the business cycle. This endogenous change in the precautionary saving motive generates income risk transmission channel for monetary policy. If income risk is countercyclical, agents save less following monetary policy cut due to the low precautionary saving motive, thereby further boosting consumption. To formally show the income risk channel, I set $\tau = t_0$ to model a current-period monetary policy shock. To maintain tractability, I focus on the subsystem of job finding rate and inflation. Then, I show how to aggregate consumption changes numerically with impulse response functions.

### Income Risk Cyclicality and the Effect of a Monetary Policy Shock

From the linearized subsystem, I can write everything as a function of a monetary policy shock. Let $\hat{\eta}_t = \Gamma_\eta \epsilon_{R,t}$ and $\hat{\pi}_t = \Gamma_\pi \epsilon_{R,t}$, and assume $\lambda > 0$.

\[
\begin{align*}
\hat{\eta}_t &= \Gamma_\eta \epsilon_{R,t} = -\Gamma_\eta \lambda \epsilon_{R,t} = (F_\eta + F_R \frac{\phi_\eta}{1-\alpha}) \Gamma_\eta \epsilon_{R,t} + (F_\pi + \phi_\pi F_R) \Gamma_\pi \epsilon_{R,t} + \epsilon_{R,t} \\
\hat{\pi}_t &= \Gamma_\pi \epsilon_{R,t} = -\Gamma_\pi \lambda \epsilon_{R,t} = G_\eta \Gamma_\eta \epsilon_{R,t} + G_\pi \Gamma_\pi \epsilon_{R,t}
\end{align*}
\]

After rearrangement, I obtain

\[
\begin{align*}
\Gamma_\eta &= \frac{-\lambda - G_\pi}{(\lambda + F_\eta + F_R \frac{\phi_\eta}{1-\alpha})(\lambda + G_\pi) - G_\eta(F_\pi + F_R \phi_\pi)} \\
\Gamma_\pi &= \frac{G_\eta}{(\lambda + F_\eta + F_R \frac{\phi_\pi}{1-\alpha})(\lambda + G_\pi) - G_\eta(F_\pi + F_R \phi_\pi)}
\end{align*}
\]

Given the analytic solution, I show how the effect of monetary policy changes as income risk cyclicality differs.

**Proposition 5.** Countercyclical income risk amplifies, whereas procyclical income risk reduces the effect of the monetary policy shock. Acyclical income risk shows that...
The effect of the monetary policy shock is equal to that in RANK models.

\[
\frac{\partial \Gamma_\eta}{\partial F_\eta} = \frac{-\lambda + G_\pi}{\left[\left(\lambda + F_\eta + F_{R1 + \phi}\right) - \left(F_\pi + F_{R\phi}\right)G_\eta\right]^2} < 0
\]

\[
\frac{\partial \Gamma_\pi}{\partial F_\eta} = \frac{\lambda + G_\pi}{\left[\left(\lambda + F_\eta + F_{R1 + \phi}\right) - \left(F_\pi + F_{R\phi}\right)G_\eta\right]^2} < 0
\]

The above two equations clearly show that more countercyclical income risk (lower \(F_\eta\)) implies a higher \(\Gamma_\eta\) and \(\Gamma_\pi\). Therefore, the effect of the monetary policy shock is amplified when income risk is countercyclical.

**Impulse Response Functions**

I confirm the analytic results for aggregate consumption and inflation from figure C.1. It shows that the effect of the monetary policy shock is 30\% stronger when income risk is countercyclical (\(F_\eta = -0.5\)) than when it is procyclical (\(F_\eta = 0.5\)) for both aggregate consumption and inflation.

**Figure C.1: Impulse Response Functions to Monetary Policy Shock**

**C.6 Determinacy**

Determinacy in a New Keynesian model concerns how the interest rate should move to rule out self-fulfilling equilibria. In RANK models, the nominal rate should

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3 For this experiment, I directly set \(F_\eta\), while the other parameters remain unchanged.
move more than one for one to output gap or/and the inflation gap. The existence of endogenous income risk can relax or strengthen this condition since there is additional precautionary saving channel.

In the model I presented above, the dynamics are determined by the first two equations. Once the paths of the job finding rate and inflation rate are determined by the Euler equation and Phillips curve, aggregate labor supply and consumption will be determined trivially as long as aggregate labor supply reverts to the mean.\(^4\). Hence, I only consider the two main equations, which I rewrite as below:

\[
\begin{bmatrix}
\hat{\eta}_t \\
\hat{\pi}_t
\end{bmatrix} = A \begin{bmatrix}
\hat{\eta}_t \\
\hat{\pi}_t
\end{bmatrix} + \begin{bmatrix}
\epsilon_{R,t} \\
0
\end{bmatrix}
\]

where

\[
A = \begin{bmatrix}
F_\eta + \frac{\phi_\theta}{1-\alpha} F_R, & F_\pi + \phi_\pi F_R \\
G_\eta, & G_\pi
\end{bmatrix}
\]

Since both variables are jump variables, I obtain determinacy if and only if both eigenvalues of A are positive. After some algebra, I find following equations for the two eigenvalues:

\[
2\lambda_1 = F_\eta + \frac{\phi_\theta}{1-\alpha} F_R + G_\pi + \sqrt{(F_\eta + \frac{\phi_\theta}{1-\alpha} F_R - G_\pi)^2 + 4G_\eta(F_\pi + \phi_\pi F_R)}
\]

\[
2\lambda_2 = F_\eta + \frac{\phi_\theta}{1-\alpha} F_R + G_\pi - \sqrt{(F_\eta + \frac{\phi_\theta}{1-\alpha} F_R - G_\pi)^2 + 4G_\eta(F_\pi + \phi_\pi F_R)}
\]

For convenience, I assume \(\phi_\theta = 0\) and \(G_\pi = \rho\).

**Proposition 6.** If income risk is countercyclical enough \((-F_\eta > G_\pi\)), it is not possible to obtain determinacy under \(\phi_\theta = 0\).

The proposition above casts doubt on inflation targeting, which is widely held in many central banks. It is possible for the economy to go wildly wrong if a central

\(^4\) \(H_N\) is negative, so it indeed reverts to the mean.
bank responds only to inflation. In RANK, first diagonal of A is zero, so it does not matter a central bank responds to either inflation or labor market condition. In HANK with endogenous income risk, a central bank must target labor market condition to ensure the stability of the economy.

**Proposition 7.** Ravn and Sterk (2020): Depending on the cyclicality of income risk, the determinacy conditions differ as below.

1. For procyclical income risk \((F_\eta > 0)\), the real part of \(\lambda_1\) would be positive. Moreover, the real parts of \(\lambda_2\) would be positive, unless \(F_\pi + \phi_\pi F_R = \frac{\nu}{\sigma_\eta}(\phi_\pi - 1) \ll 0\). I therefore can have a unique equilibrium even if \(\phi_\pi < 1\) in contrast to RANK models.

2. For acyclical income risk \((F_\eta = 0)\), the determinacy condition is identical to that of RANK models since it requires \(\phi_\pi > 1\).

3. For countercyclical income risk \((F_\eta < 0)\), the determinacy conditions are more stringent. When it systemically responds to income risk \((\phi_\theta > 0)\), a central bank should respond strong enough \((F_\eta + \frac{\phi_\eta}{1-\alpha}F_R + G_\pi > 0\), and \(F_\eta + \phi_\pi F_R\) should not be too negative).

The two propositions can be depicted graphically as in figure C.2. The first graph shows that I can have determinacy even with \(\phi_\pi < 1\) under procyclical income risk \((F_\eta > 0)\). The second graph illustrates that determinacy can be obtained only if a monetary authority responds directly to income risk under the model setting. The third graph shows that there is indeterminacy when a central bank does not systemically respond to income risk even if it has a very strong stance on inflation.
Figure C.2: Income Risk Cyclicality and Determinacy