The Interaction between a Comet and the Solar Wind

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A thesis submitted to the University of London
for the degree of Doctor of Philosophy

August 1990
Abstract

Ion ‘pickup’ is an interesting physical process which occurs when new ions are introduced into a magnetized plasma flow with a motion relative to the bulk plasma. The cometary environment provides an excellent ‘laboratory’ in which to study such processes. Ions of cometary origin are gradually accommodated into the solar wind flow. The rate of isotropization of the implanted ion velocity distribution in the solar wind frame depends on the level of ambient and self-generated turbulent waves with which the ions interact.

The work presented here studies the parallel pickup process to ascertain whether or not pitch-angle diffusion driven by the observed turbulence is fast enough to explain the development of the ion distributions. The theoretical description used is based on a quasilinear approach, and considers the implantation of cometary ions along solar wind flowlines. To make such a study requires some way of extrapolating our measurements on the Giotto trajectory into the upstream region. Models for mass-loading and turbulence are used.

A simplified mass-loading model provides the plasma flow parameters which depend upon position relative to the comet, and the cometary gas emission characteristics. The turbulent power spectrum is required for the diffusion coefficient of the wave-particle interaction. Theoretical calculations of free energy made available to upstream and downstream propagating Alfvén waves during the pickup process are used to estimate the wave intensity in the region upstream from the spacecraft track. The spectral shape is found to be reasonably constant throughout the region of interest.

A kinetic equation describing the source, convection and quasilinear velocity diffusion of the implanted cometary ions is then developed and may be solved numerically using the information outlined above. The present study finds that quasilinear theory gives a level of velocity diffusion which is indeed of the right order.
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Chapter 1

The Solar Wind

1.1 Introduction: Solar System Plasmas

On Earth, plasmas occur very rarely. In the environment of space, however, the plasma state accounts for over 90% of the volume of the universe. The term ‘plasma’ is normally used to describe a dissociated gas of electrons and positive ions, and encompasses a wide range of possible properties. The gas in the interstellar medium is excited by stellar radiation and remains in the plasma state by virtue of its low density, hence low recombination rate. However, the atmospheres of stars are almost completely ionized because the very high temperatures of the particles ensure collisions of sufficient energy to knock out electrons from the atoms. The emissions from stellar photospheres and magnetized plasmas may be detected by modern astronomical telescopes in the radio, optical, X-ray and γ-ray bands [Alfvén, 1987].

A good working definition of a plasma is [Chen, 1984]:

“A plasma is a quasi-neutral gas of charged and neutral particles which exhibits collective behaviour.”

If a gas has a sufficient level of ionization, the long-range Coulomb force dominates the physics and produces a ‘collective behaviour’ [Chen, 1984], whereby the movement of an element of plasma generates fields which affect other regions. ‘Quasi-neutrality’ results from the plasma’s ability to shield out an applied electric potential - a phenomenon known as Debye shielding. An isolated charge surface in a plasma will attract around it particles of the opposite charge such that outside these clouds there is, ideally (in the zero
1. THE SOLAR WIND

temperature case), no electric field. The Debye length, $\lambda_D$, is a measure of the shielding distance. A plasma is quasi-neutral since the electron and ion number densities are approximately equal over a distance greater than $\lambda_D$, however $\nabla \cdot E \neq 0$; electromagnetic fields exist. This is the 'plasma approximation'. The 'plasma parameter', $N_D$, is defined as the number of particles in the Debye sphere (i.e. a sphere of radius $\lambda_D$), and for statistical significance, collective behaviour requires $N_D >> 1$.

This thesis is concerned with the interaction of solar wind and cometary plasmas. A complex interaction region develops around a comet when it makes its closest approach to the Sun. Within this region there are three principal domains; a cometary ion dominated coma, a solar wind dominated upstream region, and between these a ‘mixing’ region. The behaviour of the particles is strongly species-dependent (not all species behave as fluids) and the interaction is influenced by the varying interplanetary magnetic field direction.

Historically, it was observations of comet tails that led Biermann [1951] to suggest the continuous flow of particles from the Sun (now known as the solar wind). The plasma tail of a comet does not lie along the orbit, but rather is aligned along the relative velocity vector between the comet and an outwardly flowing solar wind stream (the velocity of which can be estimated from such observations). Biermann postulated that the force involved in producing the “tail aberration” was due to the collisions of solar particles with cometary particles in the tail. He suggested that the streams from the Sun must flow continuously, rather than intermittently as had been previously believed based on the occurrence of geomagnetic storms at the Earth. The work of Biermann in turn prompted Parker [1958] to develop a theoretical model of solar coronal expansion. Parker’s theory provided estimates of solar wind velocity and temperature which were confirmed following spacecraft observations a few years later. However, the measured density was very much smaller than previously supposed by Biermann, so demonstrating that solar wind particle collisions alone are insufficient to produce the observed tail deflections. New ideas were needed.

Eddington [1910] had made a study of the changing features of comet
Morehouse. He noticed paraboloidal envelopes that formed first on the sun-comet line, and spread outwards in both directions to eventually form tail rays. But it was Alfvén [1957] who realized the important role of the magnetic field, which is strong enough to enforce fluid-like behaviour of the solar wind plasma around the comet since the ion gyroradius in the field ($< 100 \text{ km}$) is small compared with the size of the object. (For comparison, the collision mechanism of Biermann [1951] has a mean free path of $\sim 1 \text{ AU}$ and is therefore considerably less effective.) The magnetic field is ‘frozen-in’ to the solar plasma (see Section 1.3) and is dragged downstream with it. On encountering the comet the field lines become draped around the head to form the tail which acts like a windsock.

The work of this thesis is arranged as follows. Three introductory chapters present an overview of the subjects of the solar wind, cometary plasmas and the instrumentation and results of the recent spacecraft missions to comets (particularly Giotto, from which the data used in this work was obtained). In Chapter 4, a simple model of solar wind “mass-loading” with heavy cometary ions is fitted to the solar wind slow-down observed during Giotto’s approach to the comet. This analysis assumes the simplest case of cometary gas outflow and ion implantation. The process of cometary ion pick-up is considered in greater detail in Chapter 5, in particular the wave-particle interactions involved and the energy density of the Alfvén waves that this generates. The ultimate aim is to develop a numerical solution to a kinetic ion transport equation (in Chapter 7) that describes the evolution of the velocity-space distribution of the cometary ions as they are implanted into the solar wind. The equation involves a source term and a velocity diffusion term in partial differentials. The diffusion term requires as input the turbulent power spectrum in the interaction region, that is, the wave energy (Chapter 5) and the spectral shape, which is investigated by Fourier analysis in Chapter 6. The mass-loading model approach is taken as an adequate ‘first approximation’ to the ion transport, so that the bulk flow parameters obtained may be used as inputs to the partial differential equation. This is preferable to tackling its simultaneous solution with additional moment equations for the ion densities and velocities.
1.2 Expansion of the Solar Corona

The corona of the sun is so hot that the pressure of a static atmosphere cannot be contained, and the plasma extends into space. The 'solar wind' is the name given to this expanding plasma. It forms an interplanetary medium which interacts with the fields and ionized particles in the atmospheres of the planets and other bodies in the solar system. A variety of complex structures observed in the solar wind originate from solar surface features (such as loops and coronal holes) and variations in solar activity (such as flares).

The solar wind has several constituent ion species, each with different velocity-space distributions. Nevertheless it is useful to employ a simple single-fluid theory to examine the bulk outflow. A steady state, radial, spherically-symmetric expansion may be described by the following conservation equations for mass, momentum and energy [eg. Schwartz, 1985, Hundhausen, 1972]:

\[
\frac{d}{dr} (\rho u r^2) = 0
\]  

(1.1)

\[
u \frac{du}{dr} + \frac{1}{\rho} \frac{dp}{dr} = \frac{G M_\odot}{r^2}
\]

(1.2)

\[
\frac{d}{dr} (p \rho^{-\gamma}) = 0
\]

(1.3)

Where \( u \) and \( \rho \) are the solar wind flow speed and mass density (\( m_n \)) respectively, \( p \) is the total pressure of the flow, \( M_\odot \) the mass of the sun, \( G \) the gravitational constant, and \( r \) the radial distance from the centre of the sun. This analysis ignores the magnetic field (which is justified for coronal holes, but not in the region of sunspots). The mass equation simply gives:

\[
\rho u = \text{const.} \frac{1}{r^2}
\]

(1.4)

and states that the mass-flux must fall off according to \( 1/r^2 \) as it expands into an increasing spherical area. The momentum equation comes from:

\[
\rho \left( \frac{\partial}{\partial t} + u \cdot \nabla \right) u + \nabla p = \text{force}
\]

(1.5)

(see Chapter 4) for a steady-state case in 1D, assuming the gravitational field of the sun provides the only force on the particles. The energy equation (1.3) is a polytropic equation of state, where \( \gamma \) is the ratio of specific heats,
and relates to the number of degrees of freedom, \( N \), in a system by \( \gamma = (2 + N)/N \).

The momentum flux equation may be integrated to give an energy flux equation as follows:

\[
\int \left[ u \frac{du}{dr} + \frac{1}{\rho} \frac{dp}{dr} + \frac{G M_\odot}{r^2} \right] dr = \text{const.} \tag{1.6}
\]

Integrating the first term by parts gives:

\[
\int u \frac{du}{dr} dr = u^2 - \int u \, du = \frac{1}{2} u^2 \tag{1.7}
\]

To integrate the second term, consider the total derivative of \( p/\rho \):

\[
\frac{d}{dr} \left( \frac{p}{\rho} \right) = \frac{1}{\rho} \frac{dp}{dr} - \frac{p}{\rho^2} \frac{dp}{dr} \tag{1.8}
\]

Also, differentiating equation (1.3) and setting equal to zero leads to a relationship between \( dp \) and \( d\rho \),

\[
\frac{d\rho}{dr} = \frac{\rho}{\gamma p} \frac{dp}{dr} \tag{1.9}
\]

which is substituted into (1.8) to give:

\[
\frac{d}{dr} \left( \frac{p}{\rho} \right) = \frac{\gamma - 1}{\gamma \rho} \frac{dp}{dr} \tag{1.10}
\]

From this, \( 1/\rho (dp/dr) \) may be replaced in the integral:

\[
\int \frac{1}{\rho} \frac{dp}{dr} dr = \frac{\gamma}{(\gamma - 1)} \int \frac{d}{dr} \left( \frac{p}{\rho} \right) dr = \frac{\gamma}{(\gamma - 1)} \frac{p}{\rho} \tag{1.11}
\]

The third term in equation (1.6) simply gives \(-GM_\odot/r\). Finally, combining terms:

\[
\frac{1}{2} u^2 + \frac{\gamma}{(\gamma - 1)} \frac{p}{\rho} - \frac{G M_\odot}{r} = \text{constant} = E \tag{1.12}
\]

as given in [Schwartz, 1985].

From the first term in equation (1.12) it is clear that this result represents the energy per unit mass associated with the solar wind. If there is to be a flow as \( r \to \infty \) then \( E \) must be positive; the first two terms on the LHS must exceed the gravitational potential. The flow can then accelerate as it expands, while the pressure and density decrease.
In order to examine the form of the flow, the momentum equation (1.2) can be rearranged and expressed in terms of the Mach number

\[ \mathcal{M} = \left( \frac{\rho u^2}{\gamma p} \right)^{\frac{1}{2}} \]  

(1.13)

which is the ratio of the flow velocity, \( u \), to the local speed of sound,

\[ c_s = \left( \frac{\gamma p}{\rho} \right)^{\frac{1}{2}} \]  

(1.14)

The algebra is tedious and may be approached in the following way. In order to replace the \( \frac{du}{dr} \) and \( \frac{dp}{dr} \) in (1.2), consider

\[ \frac{d(M^2)}{dr} = \frac{d}{dr} \left( \frac{\rho u^2}{\gamma p} \right) = \frac{(1 - \gamma) \rho u^2 \frac{dp}{dr}}{\gamma^2 \rho^2 \frac{dr}{r}} + \frac{2 \rho u \frac{du}{dr}}{\gamma p} \]  

(1.15)

(in which (1.9) has been substituted for \( \frac{dp}{dr} \)), and from equation (1.1);

\[ \frac{u}{\gamma p} \frac{dp}{dr} \frac{du}{dr} + \frac{2 u}{r} = 0 \]  

(1.16)

(using (1.9) again). These two equations may now be used with the momentum equation to eliminate \( \frac{du}{dr} \) and \( \frac{dp}{dr} \). From equation (1.16), \( \frac{du}{dr} \) can be replaced in both (1.2) and (1.15), giving

\[ \frac{\gamma p - \rho u^2 \frac{dp}{dr}}{\gamma p \rho} = \frac{2 u^2}{r} - \frac{GM_{\odot}}{r^2} \]  

(1.17)

and

\[ - \left( \frac{(\gamma + 1) \rho u^2}{\gamma^2 p^2} \right) \frac{dp}{dr} = \frac{d(M^2)}{dr} + \frac{4 \rho u^2}{\gamma p r} \]  

(1.18)

respectively. These are then combined to eliminate \( \frac{dp}{dr} \):

\[ \frac{\gamma p}{(\gamma + 1) \rho} \left( \frac{\rho u^2 - \gamma p}{\rho u^2} \right) \left( \frac{d(M^2)}{dr} + \frac{4 \rho u^2}{\gamma p r} \right) = \frac{2 u^2}{r} - \frac{G M_{\odot}}{r^2} \]  

(1.19)

It now remains to re-express the \( \rho, u \) and \( p \) dependences. From the definition of \( \mathcal{M} \);

\[ \frac{\rho u^2 - \gamma p}{\rho u^2} = 1 - \frac{1}{\mathcal{M}^2} = \frac{\mathcal{M}^2 - 1}{\mathcal{M}^2} \]  

(1.20)

and hence

\[ \left( \frac{\mathcal{M}^2 - 1}{\mathcal{M}^2} \right) \frac{d(M^2)}{dr} = \frac{2}{r} \left[ (\gamma + 1) \left( \frac{\rho u^2}{\gamma p} - \frac{\rho G M_{\odot}}{2 \gamma p r} \right) - \frac{2 \rho u^2}{\gamma p} \left( \frac{\mathcal{M}^2 - 1}{\mathcal{M}^2} \right) \right] \]  

(1.21)
Identifying $\mathcal{M}^2$ terms:

$$\left(\frac{\mathcal{M}^2 - 1}{\mathcal{M}^2}\right) \frac{d(\mathcal{M}^2)}{dr} = \frac{2}{r} \left[ (\gamma + 1) \left( \mathcal{M}^2 - \frac{\rho}{2\gamma p} \frac{GM_\odot}{r} \right) - 2\mathcal{M}^2 + 2 \right] \quad (1.22)$$

The $\mathcal{M}^2$ components may be collected together and a factor of $[(\gamma - 1)\mathcal{M}^2 + 2]$ taken out from both the resulting terms on the RHS:

$$\left(\frac{\mathcal{M}^2 - 1}{\mathcal{M}^2}\right) \frac{d(\mathcal{M}^2)}{dr} = \frac{2}{r} \left[ (\gamma - 1)\mathcal{M}^2 + 2 \right] \left\{ 1 - \frac{(\gamma + 1)\rho}{2\gamma p} \frac{GM_\odot}{r} \right\} \quad (1.23)$$

The denominator on the right can be rearranged in terms of

$$E + \frac{GM_\odot}{r} = \frac{u^2}{2} + \frac{\gamma p}{(\gamma - 1)\rho} = \frac{(\gamma - 1)\rho u^2 + 2\gamma p}{2(\gamma - 1)\rho} \quad (1.24)$$

to give finally [Schwartz, 1985]:

$$\left(\frac{\mathcal{M}^2 - 1}{\mathcal{M}^2}\right) \frac{d(\mathcal{M}^2)}{dr} = \frac{2}{r} \left[ (\gamma - 1)\mathcal{M}^2 + 2 \right] \left\{ 1 - \frac{(\gamma + 1)}{4(\gamma - 1)} \frac{(GM_\odot/r)}{E + (GM_\odot/r)} \right\} \quad (1.25)$$

On the RHS of this equation, the term in square brackets will always be positive (for $\gamma > 1$). However, the term in curly brackets will pass through zero at a “critical radius”, $r_c$, where

$$\frac{(\gamma + 1)}{4(\gamma - 1)} \frac{(GM_\odot/r_c)}{E + (GM_\odot/r_c)} = 1 \quad (1.26)$$

from which $r_c$ is obtained [Schwartz, 1985]:

$$r_c = \frac{3}{4} \frac{5/3 - \gamma}{(\gamma - 1)} \frac{GM_\odot}{E} \quad (1.27)$$

for $(1 < \gamma < 5/3)$. At this critical distance from the sun, the LHS = 0 in equation (1.25) implies that either $d(\mathcal{M}^2)/dr = 0$ (when $\mathcal{M}^2$ passes through a max, min, or inflection) or $\mathcal{M}^2 - 1 = 0$. Where $\mathcal{M}^2 = 1$, the flow velocity equals the sound speed, so this corresponds to a transition point between sub- and super-sonic flows. The solution of $\mathcal{M}^2$ as a function of $r$ from equation (1.25) for the solar wind case is obtained from the boundary conditions of $\mathcal{M}^2 < 1$ at $R_\odot$ (observed), and $p \to 0$ as $r \to \infty$ to match the interstellar medium there. $\mathcal{M}^2 = \rho u^2/\gamma p$ increases from the value at $R_\odot$, with a smooth transition from sub- to super-sonic at $r = r_c$ [Schwartz, 1985]. The physical situation suggests an outer boundary (and transition back to sub-sonic flow) where the solar wind meets the interstellar medium at $\sim 100$ AU (where $1\text{AU}$ is the Sun-to-Earth distance).
1.3 “Frozen-in” Magnetic Field

The magnetic field of the sun is basically a dipole. The heliospheric current sheet is the boundary between outward and inward directed magnetic field lines, and is attached to the corona in the equatorial region. It has a warped shape which depends on the structure of the corona (e.g., positioning and size of sunspots, coronal holes). In what is known as the “ballerina” model, the current sheet is described as having the appearance of a spinning ballerina’s skirt, as illustrated in Figure 1.1. The solar plasma is extremely conductive and the field lines are firmly rooted to the plasma at the solar surface. Induced currents will act efficiently to prevent any relative motion between the plasma and field, which therefore convect together, and the field is said to be “frozen-in” to the flow. This can be demonstrated mathematically, to first order, by considering the following basic equations in a frame of reference that accelerates with the plasma.

Maxwell’s 2nd equation is:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

(1.28)
Ohm's law for a moving conductor is (eg. Boyd and Sanderson, 1969)

\[ j = \sigma(E + u \times B) \]  

(1.29)

where \( j \) is current density and \( \sigma \) the conductivity. Assuming an infinite conductivity plasma, \( j/\sigma = 0 \), leads to the approximate relationship:

\[ E = -u \times B \]  

(1.30)

The combination of equations (1.28) and (1.30) yields:

\[ \nabla \times (B \times u) = -\frac{\partial B}{\partial t} \]  

(1.31)

A full expansion of the cross-products gives:

\[ \nabla \times (B \times u) = B(\nabla \cdot u) - u(\nabla \cdot B) + (u \cdot \nabla)B - (B \cdot \nabla)u \]  

(1.32)

However Maxwell’s 3rd relation states that \( \nabla \cdot B = 0 \). Also, in a reference frame accelerating with the flow, \( u \) is a fixed vector and \( \nabla \cdot u = 0 \). Therefore

\[ \nabla \times (B \times u) = (u \cdot \nabla)B \]  

(1.33)

Now consider the total, or 'convective' derivative of \( B \). It is apparent from equations (1.31) and (1.33) that

\[ \frac{dB}{dt} = \frac{\partial B}{\partial t} + (u \cdot \nabla)B = \frac{\partial B}{\partial t} + \nabla \times (B \times u) = 0 \]  

(1.34)

ie the total change of \( B \) is zero in the frame moving with \( u \); the field convects with the plasma.

As the solar wind flows out from the sun, the magnetic field ‘lines’ are drawn out with it. The sun rotates with angular frequency \( \Omega_\odot \), so that the field-lines (anchored at the sun) are wrapped around and the plasma flows out radially, at an angle to the direction of the \( B \)-vector, in what has been termed a “garden-hose” fashion. Because the field and plasma convect together their motion is parallel. Thus, in the frame of reference rotating with the sun, the plasma flow velocity is tangential to the lines of \( B \) as represented in Figure 1.2, and is given by [Schwartz, 1985]:

\[ \mathbf{V} = u \hat{r} - \Omega_\odot r \hat{\phi} \]  

(1.35)
Figure 1.2: Diagram of the “Parker spiral” interplanetary magnetic field configuration in the ecliptic plane, showing the velocity components (in the co-rotating frame) of a fluid element as it leaves the Sun. (Adapted from Schwartz [1985], Bittencourt [1986].)
where \( \mathbf{f} \) and \( \phi \) are unit vectors in the direction of the polar-coordinate axes \((r, \phi)\). Since \( \mathbf{B} \) is parallel to \( \mathbf{V} \) in this frame, \( B_\phi \) and \( B_r \) components must be in the same ratio as those of \( V \), \( \text{i.e.} \):

\[
\frac{B_\phi}{B_r} = -\frac{\Omega_\odot r}{u}
\]

Thus the ratio of the components of \( \mathbf{B} \) (hence the direction of \( \mathbf{B} \)) changes with \( r \). The equation represents the "Parker spiral" (so called after Parker, 1963) sketched in Figure 1.2. Note that this is an approximation to the real situation because the plasma and field are assumed not to affect each other. An assumption of radial outflow (taken also in Section 1.2) is only suitable where the field does not significantly restrain the plasma, requiring the kinetic pressure to exceed that of the field:

\[
\frac{\rho u^2}{2} > \frac{B^2}{2\mu_0}
\]

This may be rearranged into a condition on the flow velocity, \( u > V_A \), where \( V_A = B/(\mu_0 \rho)^{1/2} \) is the local Alfvén wave speed (see Chapter 5). Inside \( 1/4 \) AU, the magnetic pressure dominates and the plasma is forced into approximate co-rotation with the sun.

### 1.4 Solar Wind Properties

There are two fluid components to the solar wind. The low velocity, high density plasma is highly variable and spatially structured, and it is interlaced with the high-velocity component in thin streams. The fast solar wind is correlated with coronal holes, magnetically open regions at the solar surface, so it has a smooth, homogeneous nature. Expansion of high-speed features and sporadic solar flare ejecta into slower moving flows has been suggested to produce some of the observed interplanetary shocks.

The solar wind is an effective plasma, with \( N_D \sim 5 \times 10^9 \), and the Debye length, \( \lambda_D \), is longer than the average spacecraft. Therefore a spacecraft should not alter the plasma properties it is hoping to measure to the extent a probe in a laboratory contained plasma does, although it may still charge up to the order of a few eV so measurements are good for ion energies greater
1. THE SOLAR WIND

Figure 1.3: Identification of ion species in the solar wind from E/q spectra at two different pulse-counting thresholds, measured by Vela 3 [Bame et al., 1968].

than this. The interplanetary medium thus provides a good 'laboratory' in which to study plasma ion populations. Some properties of the solar wind are listed in Table 1.1.

Relative abundances of ions in the solar wind can be determined from the energy-per-charge spectra that have been observed by spacecraft such as Mariner 2, Vela 3, Explorer 34 and HEOS-1 [Hundhausen, 1972]. The ions are predominantly protons, with a $\sim 4 - 5\%$ (by number) alpha particle content on average. Various heavier ion peaks are also identified, (eg. $^{16}$O, $^{12}$C, $^{14}$N), not all of which are fully ionized. Unfortunately, E/q is somewhat ambiguous (eg. a mass-4, charge-1 ion and a mass-12, charge-3 ion both have $m/q = 4$) and identification is often based on the assumption of a common expansion speed of all ions from the sun. Full consideration of ionization and recombination processes in the expanding solar corona is ideally required.

Figure 1.3 shows example spectra from Vela 3 [Bame et al., 1968]. Varying the detector pulse-counting threshold can provide crude supplementary information to aid classification of the peaks since the higher the particle energy, the larger the pulse it produces. This helps to decouple E and q. In
1. *THE SOLAR WIND*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bulk Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>velocity (u)</td>
<td>300-1000 km s(^{-1})</td>
</tr>
<tr>
<td>density (n)</td>
<td>4-10 /cc</td>
</tr>
<tr>
<td>temperature (T)</td>
<td>10(^5) K</td>
</tr>
<tr>
<td>magnetic field (</td>
<td>B</td>
</tr>
<tr>
<td><strong>Velocities (km s(^{-1}))</strong></td>
<td></td>
</tr>
<tr>
<td>( v_{\text{thermal}} ) proton</td>
<td>30</td>
</tr>
<tr>
<td>( v_{\text{thermal}} ) electron</td>
<td>1000</td>
</tr>
<tr>
<td>( V_A )</td>
<td>50</td>
</tr>
<tr>
<td><strong>Frequencies (rad s(^{-1}))</strong></td>
<td></td>
</tr>
<tr>
<td>ion cyclotron (( \Omega_i ))</td>
<td>0.5</td>
</tr>
<tr>
<td>electron ( \Omega_e )</td>
<td>1000 ( \Omega_e )</td>
</tr>
<tr>
<td>ion plasma (( \omega_p ))</td>
<td>3000</td>
</tr>
<tr>
<td>electron ( \omega_{pe} )</td>
<td>( 10^{5} )</td>
</tr>
<tr>
<td>ion collision (( 2\pi \nu_{cp} ))</td>
<td>( 2\pi \times 3 \times 10^{-7} )</td>
</tr>
<tr>
<td><strong>Timescales (s)</strong></td>
<td></td>
</tr>
<tr>
<td>ion gyroperiod</td>
<td>12</td>
</tr>
<tr>
<td>electron gyroperiod</td>
<td>( 6 \times 10^{-3} )</td>
</tr>
<tr>
<td>collision time</td>
<td>( 3 \times 10^{8} )</td>
</tr>
<tr>
<td>expansion (( \tau/u ))</td>
<td>( 3 \times 10^{5} )</td>
</tr>
<tr>
<td><strong>Lengthscales</strong></td>
<td></td>
</tr>
<tr>
<td>ion Larmor (( r_{Li} ))</td>
<td>60 km</td>
</tr>
<tr>
<td>electron ( r_{Le} )</td>
<td>1 km</td>
</tr>
<tr>
<td>mean free path, ( \lambda_{mfp} )</td>
<td>0.6 AU</td>
</tr>
<tr>
<td>Debye length, ( \lambda_D )</td>
<td>10 m</td>
</tr>
</tbody>
</table>

Table 1.1: Average Solar Wind Parameters at 1AU (from Schwartz [1985])
Figure 1.3, nearly all counts appear in trace A, but only very large pulses are recorded in C.

The solar wind electrons accompany the ions at the average bulk speed (over all species) since the plasma must remain neutral. (Positive and negative charge must leave the sun at an equal rate and thus any difference in electron expansion speed relative to the ions would lead to a difference in densities in the ion frame, hence a non-neutral situation.) The flow speed of the alpha particles is typically observed to be greater than that of the protons, by an amount of the order of the Alfvén speed, $V_A$. There have been attempts to explain this velocity difference in terms of flow and heating mechanisms involving wave processes in the sun. Such a difference would be an expected consequence of wave-particle interactions between the alpha particles and Alfvén waves in the solar wind (see Chapter 5), and it is conceivable that these alpha particles are being "picked-up" with the protons as they leave the sun.

A gas in thermal equilibrium has particles of different velocities distributed about a central, most likely velocity. The most probable distribution is described by a 'Maxwellian' function. The average energy is related to the width, or temperature of the distribution, and is $\frac{1}{2}k_BT$ per degree of freedom (obtained from the 2nd moment of the Maxwellian distribution, see eg. Chen, 1984), where $k_B$ is the Boltzmann constant and $T$ the temperature. To avoid any confusion, $E_{th} = k_BT$ is normally used. A plasma may have several temperatures simultaneously, for each species of particle present, and according to forces and constraints. In a magnetized plasma, temperatures are often unequal in directions perpendicular and parallel to the B-field because of the Lorentz force. The ion species in the solar wind have temperatures approximately proportional to mass, ie. all ions have nearly the same thermal velocities [Schwartz, 1985], where $v_{th}$ is obtained from the thermal energy:

$$E_{th} = k_BT = \frac{1}{2}m v_{th}^2; \quad v_{th} = \sqrt{\frac{2k_BT}{m}} \quad (1.38)$$

The 'fluid' approximation, much-used in describing plasma flow, relies on a high collision rate in the plasma to maintain equilibrium. It is relevant to compare the collisional mean-free-path with the scalelength over which particle distributions and macroscopic parameters vary. Spacecraft observa-
Figure 1.4: An anisotropic solar wind proton distribution (measured by Helios 2 at 0.3 AU) which is typical of high-speed solar wind streams [Marsch and Goldstein, 1983].

Cross-sections
perpendicular
parallel to B

The contours indicate that frequent collisions in the slow, dense solar wind ensure isotropic, near-Maxwellian ion distributions [Marsch and Goldstein, 1983]. However, the high speed streams are less dense and virtually collision free. In this case the temperatures (defined from the average width of the velocity distribution) differ perpendicular and parallel to the interplanetary magnetic field, as the contours of the proton distribution in Figure 1.4 (bottom panel) indicate. In the top panel, 1D cuts measured along (open circles) and across (dots) the field are shown, and the solid line is the best-fit Maxwellian to the perpendicular profile. The ions are considered to be “collision-dominated”
where they have suffered at least one impact since leaving the sun. Within the vertical lines in Figure 1.4 is the collisional regime outside of which severe deviations from a Maxwellian are to be expected. This region is smaller for higher temperature, lower density distributions.

Double-peaked ion distributions are frequently observed in the interplanetary medium [Marsch and Goldstein, 1983]. The higher speed beam often travels at around the local Alfvén speed relative to the bulk flow, in the direction parallel to the magnetic field. It has been suggested that wave-particle interactions between the ions and Alfvén waves in the solar wind may be responsible for the acceleration of these ions. In the ‘collisionless’ domain, any process that changes the distribution from Maxwellian cannot be statistically counteracted by Coulomb collisions and anisotropies can therefore develop.

1.5 Waves in the Solar wind

Most of the wave modes that are possible in the solar wind (from low-frequency MHD waves up to high-frequency plasma oscillations) are reported to have been observed. Unfortunately, the observations of a single, moving spacecraft do not allow separation of spatial and temporal features, but in many cases the mode can be inferred from polarization considerations and the plasma parameters [Schwartz, 1985].

Analysis of Helios-1 and Helios-2 observations [Denskat and Neubauer, 1983] reveals an Alfvénic turbulence of which the purest examples are essentially restricted to high speed plasma streams. (Low speed streams generally display lower amplitude Alfvén waves intermixed with non-Alfvénic structures [Belcher and Davis, 1971].) Along with these waves there is always a compressive component. Enhanced compressional fluctuations are seen at the leading edges of the plasma streams, suggesting local generation where faster streams collide with more slowly moving plasma. Denskat and Neubauer computed the magnetic field spectral power density and found that the Alfvén waves are present in the frequency range $2.4 \times 10^{-8}$ to $1.2 \times 10^{-2}$ Hz and the spectrum is best described by a power law, where the peak amplitude falls off with distance, $R$, from the Sun. The shape and steepness of these
spectra observed out to $\sim$1AU also evolve with $R$. In particular, the slope of the spectrum (see Chapter 6) is flatter closer to the Sun; the spectral index was observed between values of 0.87 and 1.15 at 0.29 AU, reaching between 1.59 and 1.69 at 0.97 AU.

Belcher and Davis [1971] presented vector correlations of $B$ and $u$ from the Mariner 5 solar wind data. Periods such as that in Figure 1.5 are commonly seen in high-velocity streams. There is a high degree of correlation between the vector components of $B$ and $u$, while there is comparatively little variation in the proton number density and total field strength (bottom two profiles). The fluctuations in $B$ are of magnitude comparable with the mean field. These features are characteristic of Alfvén waves. Though they are far from simple sinusoids, statistically there is evidence for a predominance of transverse magnetic fluctuations. The phase relationships between the $B$ and $u$ variations indicate that the Alfvén waves propagate predomi-
nantly outwards from the Sun and may therefore have been generated within the ‘Alfvénic radius’ of the Sun where $u < V_A$ and inward-propagating waves would not be convected outward. Belcher and Davis suggest that these waves are remnants of a broader spectrum, including magnetosonic modes, believed responsible for solar coronal heating. This is reasonable because Alfvén waves are weakly damped and would not have suffered serious attenuation by the time they reached 1AU [Hundhausen, 1972].
Chapter 2

Spacecraft Cometary Missions

2.1 Spacecraft Missions to Comet Halley

The return of Halley’s Comet in 1986 provided the opportunity for five spacecraft missions to study the cometary environment. In Table 2.1, some details of the spacecraft trajectories are given. The phase angle and closest approach distance, CA, are shown schematically in Figure 2.1.

During the Comet’s active period, its asymmetric outgassing into the sunward hemisphere causes an acceleration of the nucleus away from the Sun, extending the ‘Keplerian’ orbit by four days [Reinhard, 1986c]. These non-gravitational forces cannot be modelled precisely, which makes spacecraft targeting a difficult task. Giotto was the last spacecraft to encounter the comet and its aiming on final approach benefitted from the position information passed on from Vega 1 and 2. The encounters of the five spacecraft between the 6th and 14th of March 1986 fell around 4 weeks after the Halley perihelion passage (with heliocentric distance between 0.79 and 0.89 AU) when the comet was at its most active [Reinhard, 1986a]. Halley’s retrograde orbit imposes a very high flyby speed, so that close targeted spacecraft required extensive protection against dust impact damage to ensure a reasonable chance of survival.

The spacecraft carried experiments designed to study all aspects of gas and dust emission, the interaction of cometary ions with the solar wind, and the comet nucleus. The full complement of instrument types aboard the five
Table 2.1: Missions to Comet Halley

<table>
<thead>
<tr>
<th>Spacecraft</th>
<th>Date</th>
<th>Time</th>
<th>Distance (km)</th>
<th>Flyby Speed (km s(^{-1}))</th>
<th>Phase angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vega 1</td>
<td>6 Mar 86</td>
<td>07:27</td>
<td>8,500</td>
<td>79.2</td>
<td>111.2</td>
</tr>
<tr>
<td>Vega 2</td>
<td>9 Mar 86</td>
<td>07:20</td>
<td>8,500</td>
<td>76.8</td>
<td>113.4</td>
</tr>
<tr>
<td>Suisei</td>
<td>8 Mar 86</td>
<td>13:01</td>
<td>150,000</td>
<td>73.0</td>
<td>104.2</td>
</tr>
<tr>
<td>Sakigake</td>
<td>11 Mar 86</td>
<td>04:37</td>
<td>7,000,000</td>
<td>75.3</td>
<td>109.4</td>
</tr>
<tr>
<td>Giotto</td>
<td>14 Mar 86</td>
<td>00:02</td>
<td>500</td>
<td>68.4</td>
<td>107.2</td>
</tr>
</tbody>
</table>

Figure 2.1: Flyby parameters for the encounters with Comet Halley. The distance of closest approach (CA) and the phase angle are shown schematically and vary between the five spacecraft (as given in Table 2.1).
spacecraft are indicated in Table 2.2 [Reinhard, 1986b].

2.2 The Vega 1 and 2 instruments

The instruments aboard the two Vega spacecraft [Grard et al., 1986] are summarized as follows.

- The television system (TVS) is designed to observe the dimensions and albedo of the cometary nucleus and study the central coma. The system incorporates a narrow-angle camera (TVY) providing high-resolution imaging and a wide angle camera (TDN) to track the comet. Each has a set of filters giving spectral analysis capability.

- The infrared spectrometer (IKS) analyses 2.5 - 12 μm radiation from the inner coma to study chemical composition, nucleus size and thermal emission.

- The three-channel spectrometer (TKS) measures the intensity of radiation in the UV, visible, and IR ranges of the spectrum. The visible range records light diffused by the nucleus and dust particles, the UV channel measures emissions from atoms and ions in the coma and tail for composition studies, and the IR region covers radiation from molecular vibrations.

- A shield penetration detector (PHOTON) records the flux density of cometary dust particles and tests the performance of the shield.

- SP-1 and SP-2 dust particle impact detectors measure the flux and the mass distribution of dust particles. An acoustical detector records the count rate. The ion and electron clouds produced by each particle impact are collected in the plasma detectors, where the pulse magnitude is proportional to particle mass.

- The dust-particle counter and mass analyser (DUCMA) consists of a polarized polymer coated with a metallic conductor. A particle impact destroys a volume of polarized material, producing an electrical pulse
Table 2.2: Experiments on the Halley Spacecraft [Reinhard, 1986b]
(acronyms are given in the text)

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Vega 1 &amp; 2</th>
<th>Suisei</th>
<th>Sakigake</th>
<th>Giotto</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plasma in-situ measurements</td>
<td>Solar wind ions</td>
<td>PLAS MAG-1</td>
<td>ESP</td>
<td>SOW</td>
</tr>
<tr>
<td></td>
<td>Solar wind electrons</td>
<td>PLAS MAG-1</td>
<td>ESP</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Energetic particles</td>
<td>Tünde-M</td>
<td></td>
<td>EPA</td>
</tr>
<tr>
<td></td>
<td>Magnetometer</td>
<td>MISCHA</td>
<td>IMF</td>
<td>MAG</td>
</tr>
<tr>
<td></td>
<td>Plasma waves</td>
<td>APV-N, APV-V</td>
<td>PWP</td>
<td></td>
</tr>
<tr>
<td>Gas and dust in-situ measurements</td>
<td>Ion mass spectrometer</td>
<td>PLAS MAG-1</td>
<td>IMS, JPA, PICCA</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Neutral mass spectrometer</td>
<td>ING</td>
<td>NMS</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dust mass spectrometer</td>
<td>PUMA, DUCMA</td>
<td>PIA</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dust impact detectors</td>
<td>PHOTON, SP-1, SP-2</td>
<td>DID</td>
<td></td>
</tr>
<tr>
<td>Remote sensing</td>
<td>Camera</td>
<td>TVS</td>
<td>HMC</td>
<td></td>
</tr>
<tr>
<td></td>
<td>UV Camera</td>
<td>UVI</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>IR Sounder</td>
<td>IKS</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Photopolarimeter</td>
<td></td>
<td>OPE</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Three-channel spectrometer</td>
<td>TKS</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Radio science</td>
<td></td>
<td>GRS</td>
<td></td>
</tr>
</tbody>
</table>
with amplitude proportional to impact velocity (known ram velocity) and mass [Simpson et al., 1986].

- The dust mass spectrometer (PUMA) employs a time-of-flight technique to measure the chemical composition, size, and the spatial density of dust particles.

- The neutral-gas mass spectrometer (ING) measures the chemical composition of the cometary atmosphere. It consists of a field ionization detector and an electron ionization detector which use, respectively, an electron-stripping and time-of-flight mass analysis, and electron impact ionization followed by electrostatic mass analysis. The advantage of the first method is that the molecules are not dissociated, but in the second case an energy range can be chosen to discount the gas evaporated from the spacecraft.

- The plasma energy analyser (PLASMAG-1) package includes two electrostatic ion energy distribution analysers (the cometary ram analyser, CRA, and solar direction analyser, SDA [Gringauz et al., 1986a]), an electrostatic electron energy analyser, a solar direction Faraday cup (integral detector), a ram Faraday cup (RFC) for cometary ions, and a photon and particle bombardment monitor.

- The energetic particle instrument (Tunde-M) measures energy and flux of energetic cometary ions in two telescopes set at different angles to provide some angular distribution information.

- The magnetometer (MISCHA) includes a single axis sensor and a tri-axial sensor which are mounted on a boom 1 m and 2 m, respectively, from the solar panel, enabling also the spacecraft effects on the magnetic field to be examined.

- The two sensors of the plasma-wave and ion-trap experiment (APV-N) are an electric dipole consisting of two mesh spheres on a Y-shaped boom, and a Faraday cup.

- The electric-field and Langmuir-probes experiment (APV-V) consists of two spheres mounted on the two outer solar panels (11 m apart) to
form a dipole, and two cylindrical Langmuir probes, similarly mounted, to measure plasma density.

2.3 Suisei and Sakigake

The instruments carried by Suisei and Sakigake [Hirao, 1986] are listed below.

Suisei

- The ultra-violet imager (UVT) is designed to obtain a photo-mosaic of the comet’s hydrogen coma, operating in the wavelength range of the hydrogen-alpha line. Inside the coma, the camera is operated in photometer mode which converts the image data into a light-level histogram distribution [Kaneda et al., 1986].

- The solar wind experiment (ESP; energy spectrum of particles) measures the 3-dimensional velocity distributions of solar wind ions and electrons using two 270° spherical electrostatic analysers.

Sakigake

- The Plasma-wave probe (PWP) incorporates a dipole antenna for the electric field signals and a search coil to measure the magnetic field components of waves.

- The solar wind instrument (SOW) measures density, bulk velocity and temperature of solar wind ions using a Faraday cup. A modulator grid just inside the aperture grid alternates between 0 and a voltage $V$, where $V$ can be selected from ground, to transmit pulses of incoming ions with energy/$q > V$ to the ion collector.

- The interplanetary magnetic-field (IMF) experiment consists of a tri-axial, ring-core fluxgate magnetometer mounted on a 2 m boom.
Figure 2.2 - Diagram to show the comet Halley orbit (retrograde, inclined 18° with respect to the ecliptic plane) and the Giotto trajectory [Renard, 1986a].
Figure 2.3: The structure of the Giotto spacecraft [Reinhard, 1986a].

2.4 The Giotto spacecraft

Giotto was launched on the 2nd July 1985 and the cruise phase was 8 months. The orbit is illustrated in Figure 2.2 in the sun-centred system. The spacecraft is spin-stabilized at 15 rpm (4 seconds per spin), with the spin axis aligned along the relative-velocity vector. The structure of Giotto is illustrated in Figure 2.3.

A dual sheet bumper is used to protect Giotto from dust impact damage, because with impacts occurring at $\sim 68 \text{ km s}^{-1}$ (flyby velocity) a single absorbing shield would need to be thick and extremely heavy [Reinhard, 1986c]. Dust particles are completely vaporised on impact with the thin front sheet and the vapour cloud expands in the space between the shields to distribute
the impact over a large area of the rear sheet, so dissipating the energy.

A high-gain parabolic dish antenna transmits data back to the Earth. It is despun to retain a constant point direction (at 44.3° to the Giotto rearward direction) as the spacecraft spins. A deviation of more than 1° from the normal attitude would result in the loss of the downlink [Reinhard, 1986c]. Such an event could be caused by an impact with a $\geq 0.1$ g dust particle (the probability of which was estimated before launch to be a few percent [Reinhard, 1986c]) and was observed soon after closest approach when contact was lost for $\sim 20$ seconds.

A brief description of experiments on board Giotto follows.

- The Halley multicolour camera (HMC) is a narrow angle camera designed to image the nucleus. It has three detectors each with a fixed colour filter and one with an interchangeable-filter wheel for coma and dust observation [Schmidt et al., 1986].

- The dust impact detector system (DID) monitors dust impacts on the Giotto bumper shield using three piezo-electric elements (microphones) to record the shock wave, and particle mass is derived from the size of the plasma cloud produced on impact [McDonnell et al., 1986].

- The dust mass spectrometer (PIA; particulate impact analyser) accelerates to a time-of-flight mass analyser the ions formed when a dust particle disintegrates on impact at the detector's target [Kissel, 1986]. Thus the composition of individual dust particles is measured.

- The neutral mass spectrometer (NMS) consists of two sensors. In both of these the cometary neutrals are first ionized by electron bombardment. The M-analyser is a double focusing system involving an electrostatic analyser followed by a magnetic sector field and ions are focused at positions on the detector micro-channel plate according to mass/$q$ only. The electrostatic parallel-plate E-analyser focuses ions of a given $E/q$ independent of their angle of incidence to the sensor [Krankowsky et al., 1986a].

- The ion mass spectrometer (IMS) incorporates two instruments [Balsiger et al., 1986a]. The high-energy range spectrometer (HERS) mea-
sures the ion abundances and 3-dimensional velocity distributions in the outer coma. It uses a sector magnet with entrance and exit slits which fixes a particular ion normal momentum and an electrostatic deflector then acts as a mass/q analyser. The high intensity spectrometer (HIS) is optimised to operate in the inner coma where densities are high. It has a mass analyser combining electrostatic and magnetic deflection systems, and an electrostatic quadrupole spherical angle analyser.

- In brief, the Johnstone plasma analyser (JPA) [Johnstone et al., 1986a] includes a fast ion sensor (FIS), which is a 270° electrostatic analyser measuring the 3-dimensional energy distributions of the ions, and the implanted ion sensor (IIS), combining electrostatic and time-of-flight analysis to give mass discrimination but with lower time resolution. A more extensive description follows in Section 2.5.

- Two instruments form the RPA-Copernic plasma experiment [Rème et al., 1986a]. The electron electrostatic analyser (EESA) has an entrance aperture in the centre of a hemispherical electrostatic energy-selecting section and the electrons are deflected through 90° to be detected at positions on a microchannel plate ring dependent upon their polar angle of incidence. The positive ion cluster composition analyser (PICCA) measures the mass of cometary ions of assumed charge and velocity in the inner coma using also an electrostatic E/q analyser.

- The energetic particle experiment (EPONA/EPA) is designed to detect electrons and ions with energies \( \geq 20 \text{ keV} \). It consists of 3 identical small telescopes, two at 45° to the ram direction (one of which admits protons and electrons, the other admits electrons only) and one at 135°. Each telescope has two solid-state (depleted silicon) detectors [McKenna-Lawlor et al., 1986a].

- The magnetometer (MAG) has two sensors [Neubauer et al., 1986a] mounted ‘inboard’, 0.5 m, and ‘outboard’, 1.1 m away from the spacecraft on the antenna tripod. The outboard instrument is a triaxial ring-core fluxgate magnetometer with three orthogonal sensors, measuring each magnetic field component. The inboard sensor is biaxial,
Figure 2.4: Instrument positioning on the Giotto spacecraft [Reinhard, 1986a]. (Acronyms are given in the text.)
with one core and two pick-up coils. A detailed description is presented in Section 2.6.

- The optical probe experiment (OPE) is a photo-polarimeter [Levasseur-Regourd et al., 1986], aimed rearwards along the spacecraft velocity vector so that the integral column brightness can be transformed into values for a small volume of space traversed by the instrument field of view between two successive measurements. Observations of light diffused by dust grains are made in three wavelength ranges, and four specific spectral bands are used to measure line emissions from OH, CN, C$_2$ and CO$^+$. 

- The radio-science experiment (GRE) is designed to measure the columnar electron content of the comet's ionosphere by observing the phase shifts developed in two initially coherent, monochromatic radio signals of different frequencies transmitted at the spacecraft and received at the Earth [Edenhofer et al., 1986].

The location of the instruments on the Giotto spacecraft is shown in Figure 2.4.

### 2.5 JPA Experiment on Giotto

The work of this thesis primarily uses data acquired by the Johnstone plasma analyser (JPA) on board the Giotto spacecraft. The JPA instrument is therefore considered in detail in this section.

The Johnstone plasma analyser (JPA) includes two instruments (described in detail by Johnstone et al. [1987a], Johnstone et al. [1986a], Wilken et al. [1987]), together designed to cover both solar wind and cometary ions. The fast ion sensor (FIS) measures the three-dimensional energy distribution of positive ions with speed of response gained at the expense of mass discrimination. The implanted ion sensor (IIS) is a high sensitivity instrument, to enable measurement of low density distributions of heavy cometary ions in five mass groups, achieved at the expense of speed of response.
2.5.1 Fast Ion Sensor

The fast ion sensor is illustrated in Figure 2.5. Ions are admitted through an entrance aperture into a hemispherical electrostatic energy analyser consisting of two concentric deflection plates. Variable plate voltages are applied in a fixed ratio of $V_{inner}/V_{outer} = -1.18$ which gives a zero-potential ‘surface’ midway between them. The radius of curvature of the analyser path and the applied voltages selects the ion $E/q$ band, where $\delta E/E = 4.7\%$ according to analyser characteristics such as the narrow plate spacing. After 180° all hemispherical paths (for the given $E/q$) reconverge regardless of ion incidence angle to the detector, and the ions pass through an intermediate aperture to enter the 80° (not quite quadrispherical) angular dispersion sector. They then disperse according to angular incidence (see Figure 2.5) to arrive around a 160° sector microchannel plate (MCP) detector, which generates an electron cloud for each ion impact. Behind the MCP the electrons are collected on one of eight discrete metal anodes of defined angular range. Each anode is connected to a charge-sensitive pulse amplifier. The MCP can handle high count-rates of up to $\sim 2 \times 10^6$ pulses per second, per anode sector.

The (polar) acceptance angle of the detector is 160° in the plane containing the spacecraft spin axis, as shown in Figure 2.6. The 3-dimensional coverage is swept out by the detectors as the spacecraft spins. For any ion detected by the instrument, the polar angle is known from the anode registering the count and the azimuthal angle is computed from the Sun reference pulse (SRP) timing. 96% of $4\pi$ solid angle is covered, missing only a 20° cone about the velocity vector (in the ram direction).

The FIS energy range is 10 eV to 20 keV. For operation in the full ‘wide energy’ mode, the energy passband is swept over the range continuously in an exponential decay curve from the maximum to minimum in 1/16 of a spin. Successive sweeps through the same energy therefore occur at azimuthal intervals 22.5° apart. Since the acceptance of the detector is a narrow 5° in azimuth, then obviously the 22.5° are not covered each time and the energy distributions have gaps in azimuthal coverage.

The undisturbed solar wind may be confined to near parallel incidence and in such a case could be missed between wide energy mode sweeps. Thus
Figure 2.5: Schematic diagram of the Fast Ion Sensor in operation, for two example ion incident angles [Johnstone et al., 1986a].
Figure 2.6: The 3-D angular coverage of the JPA is determined by the polar acceptance angle of the instruments, which sweeps around in azimuth as the spacecraft spins.

A solar wind (SW) mode is used on alternate spins (with 8 second resolution), covering one quarter of the energy range, four times as often, in the $45^\circ \times 45^\circ$ angular sector centred on the solar direction. The FIS operation sequence has a duration of two spins where each spin divided into eight $45^\circ$ sectors, and is synchronised to the rotation according to the Sun reference pulse. During the first sector the instrument operates in the solar wind mode, making eight short energy sweeps during each of which the spacecraft and detector rotates through $5^\circ$. The field-of-view fan crosses the solar direction after $22.5^\circ$ (i.e. half way through this first spin sector). Counts are accumulated in 8 azimuthal and 4 polar angle bins, and in 30 logarithmic energy steps. For the next 15 sectors to complete the two spins, operation is in the full wide energy mode with two full energy sweeps per sector.

During the encounter, the section of the full energy range chosen in the solar wind mode was selected by an autoranging procedure that follows the peak of the counts distribution, which is normally that of solar wind protons. It places the peak in the lowest quarter of the range so that the sweep also covers the alpha particle spectrum (at twice the energy). If the peak is lost a cycling procedure takes over, which searches through the entire energy range.
Autoranging proved invaluable for following the large changes in the solar wind speed at the bow shock which occurred over a very short time period of tens of seconds. Then as the solar wind distribution broadened and the count-rate dropped, the autoranging was switched off and the energy sweep set to the lowest level by ground control at 20:20 hrs ground received time.

The fast ion sensor operated during the inbound leg and until shortly after closest approach when it ceased to return data as a result of damage sustained during the dust bombardment.

2.5.2 Implanted Ion Sensor

The implanted ion sensor (IIS) is an ion spectrometer primarily designed to measure the distributions of implanted cometary ions. It combines electrostatic energy analysis with a time-of-flight technique. The layout of the sensor is illustrated in Figure 2.7.

The IIS has an array of five sensors, each made up of a spherical-section electrostatic analyser segment (ESA) and a time-of-flight (TOF) analyser, as shown in Figure 2.7. These sensors cover an angular range of 15° to 165° in five equally spaced 10° sectors in the plane that includes the spacecraft spin axis, such that as the spacecraft rotates (see Figure 2.6), three-dimensional coverage is provided in the same way as for the FIS.

The ESA sectors have a mean radius of 50 mm and a plate spacing of 3 mm, with an energy bandwidth $\delta E/E = 10\%$. The outer plate is held at 0 V and up to -11 kV may be applied to the inner plate. Positive ions are selected in the ESA according to their $E/q$. They are then accelerated by 10 kV to pass through a thin carbon foil at the entrance of the TOF analyser, in the process of which a small fraction of their energy is transferred to secondary electrons that are deflected from the foil onto an MCP to record the 'start' signal. The 'stop' detector is an aluminium absorber and the secondary electrons in this case are released from the surface layer. The spherical concave shape of the 'stop' detector limits to ±5% the error in flight-path length caused by coulomb angular scattering of the ions with atoms in the 'start' foil.

Response of the incident ions to a given accelerating potential depends
Figure 2.7: The Implanted Ion Sensor layout, (a) in plan, showing the array of five analysers covering different ion angles of incidence at the aperture [Johnstone et al., 1986a], and (b) a perspective view [Johnstone et al., 1987a].
on \( q/\text{mass} \) and thus the selected \( E/q \) and the measured time of flight (after acceleration) over a known path length are combined to give \( m/q \). Counts are sorted into 5 mass bins using an on-board look-up table. A period of 25\( \mu \text{s} \) is required to process the signals for each event, during which time further pulses cannot be processed. A maximum time interval of 80\( \text{ns} \) is allowed between the start and stop signals before the event is discounted. Such a coincidence technique gives a high rejection of background signals, allowing low densities to be reliably measured.

The IIS operation sequence has a duration of 32 spins (128 seconds); the energy range of 86\( \text{eV}/q \) to 86\( \text{keV}/q \) is covered in 32 logarithmically spaced steps, where an entire angular distribution is acquired at one energy level every spin.

### 2.5.3 Bulk Parameters from Moments

The detector response function \( G(v) \) relates the response of the detector (counts recorded on an anode in a fixed period of time, \( dt \)) to the phase space density, \( f(v) \), of the plasma it is measuring. \( f \) is a function representing the density of ions that have velocity \( v \). The detector response is a narrow function of \( v \) which peaks at the central velocity, \( v_0 \), in the energy passband. In other words, the sensor operating at a selected energy level should respond to ions whose velocities lie in a close spread around \( v_0 \) (where the spread should be wide enough in order to record statistically significant counts at all ion densities to be measured). The ‘geometric factor’ of an instrument, given in terms of \( G(v) \) and the aperture area of the sensor, is a property independent of the energy level (\( v_0 \)) and is obtained during calibration testing (with known ion beams) prior to launch. Thus the counts recorded during operation of the instrument at encounter may be converted into \( f(v) \) using the known geometric factor.

The FIS bulk parameters are computed on the ground from moments of the energy distribution and by fitting Gaussian functions to the azimuthal and polar angle distributions separately. Accuracy is limited by the size of the velocity bins in which the data is accumulated. The data is binned according to the polar acceptance angle covered by an individual anode,
the fractional angle in spacecraft spin covered during a measurement and between measurements, and the energy width in an E/q-level bin (where it is necessary to identify the ion species and charge state). In some parts of three-dimensional velocity space the velocity bins overlap and other regions may have gaps in coverage. The results are most accurate if only one species is present in a distribution since with multiple peaks, part-merged, the counts may become confused. As the solar wind proton temperature becomes higher the distribution widens to the point where the alpha particle peak is swamped and alpha parameters cannot be derived.

Bulk parameter calculation involves integration (or summation) over the whole measured velocity distribution. Such moments are defined as follows (see also Chapter 4, Section 4.1.3 for further explanation). The ion density is simply the total number of ions in the velocity distribution:

\[ n = \int f(v) \, dv \]  

(2.1)

The fluid velocity is the average obtained by ‘adding’ all the velocities of the \( n \) ions and then dividing by \( n \):

\[ u = \langle v \rangle = \frac{1}{n} \int v \, f(v) \, dv \]  

(2.2)

The temperature is defined

\[ T = \frac{m < v_{th}^2 >}{3k_B} = \frac{m}{3k_B n} \int (v - u)^2 f(v) \, dv \]  

(2.3)

for three degrees of freedom, where the average ion thermal energy is \( \frac{1}{2}m < v_{th}^2 > = \frac{1}{2}k_B T \) per degree of freedom; \( v_{th} \) is the thermal (or random) velocity of the ion, \( v_{th} = v - u \), and \( k_B \) is the Boltzmann constant. The solar wind proton distribution has two distinct temperatures, parallel and perpendicular to the magnetic field lines. Three values are obtained for the thermal velocity aligned with the principal axes of the instrument. These axes are arbitrarily related to the magnetic field direction so that it is not possible to derive values of the parallel and perpendicular temperature. The value used is the mean of the three values. The pressure tensor is given by:

\[ p_{ij} = m \int (v_i - u_i)(v_j - u_j) f(v) \, dv \]  

(2.4)
where the subscripts $i$ and $j$ denote the components of the vectors.

The accuracy of the moment calculations in an FIS type of plasma analyser has been examined in detail by Kessel et al. [1989]. They estimate an error in the density calculation of $\pm 10\%$. The analysis provides the velocity components to a relative accuracy of $\pm 2 \text{ km s}^{-1}$ for individual measurements and to an absolute accuracy of $\pm 20 \text{ km s}^{-1}$. The accuracy of the thermal velocity is comparable to that of the velocity components, i.e., $\pm 2 \text{ km s}^{-1}$.

It is possible to derive the solar wind velocity from the IIS, but with less energy resolution and less time resolution than that from the FIS. However, the IIS operated throughout the encounter, even though suffering some damage at closest approach, and thus provides useful data for the outbound leg.

### 2.6 The Giotto Magnetometer

The magnetometer is discussed in detail here, since MAG data is used in Chapters 5 and 6 of this work.

The Giotto MAG is a fluxgate magnetometer which was designed to record fields in the range $\sim 5 \text{ nT}$ for the interplanetary magnetic field (IMF) up to several $100 \text{ nT}$ for conditions estimated near the comet. The maximum field actually observed during the mission was $65 \text{ nT}$ at 00.05 hrs on 14th March 1986 [Neubauer et al., 1987]. Higher field ranges (up to $65,536 \text{ nT}$) were included in the instrument capability for test purposes since a $\sim 50,000 \text{ nT}$ geomagnetic field exists at the Earth. A maximum sampling rate of 28.24 vectors per second was used during the encounter. This was expected to cover electromagnetic plasma waves with frequencies up to the lower hybrid resonance and, at a flyby speed of $\sim 68 \text{ km/s}$, spatial features with scalelengths as small as several ion gyroradii.

The operating principles of the fluxgate magnetometer are described as follows [Neubauer et al., 1986a; Neubauer et al., 1987]. A ferromagnetic core is periodically driven into saturation by the magnetic field generated using a periodic current in a drive coil wound around the core. The periodic signal form is of frequency $f_0$ and of suitable shape (the shape defines high-frequency components present with $f_0$). A sense coil wound around the
system is sensitive to changes in the magnetic flux (ie the differential) produced by the varying current in the drive windings (as in a transformer). It registers a voltage at the drive frequency $f_0$ with its odd harmonics only if there is no ambient magnetic field. In the presence of a field with component $H_a$, as shown in Figure 2.8, aligned along the sensitive axis of the sense coil, then even harmonics will emerge in the output. The second harmonic at $2f_0$ has an amplitude proportional to $H_a$.

In the case of the Giotto magnetometer, a ring-core of molybdenum permalloy is used. The sense coil is wound in such a way (see Figure 2.8) that the odd harmonic signals cancel out leaving only the even harmonics due to the ambient field (and a small amplitude zero offset effect). A feedback coil, controlled by the sense coil, produces a magnetic field to compensate the ambient field so the sense coil output is effectively used to detect a zero resultant field.
2. SPACECRAFT COMETARY MISSIONS

The MAG instrument consists of two magnetometer systems. The main system, MAG-1, is a triaxial fluxgate magnetometer (with three orthogonal sensors). MAG-4 is biaxial, with one axis parallel to the spacecraft spin axis and the other in the orthogonal plane. The sensors are housed in glass fibre boxes which are metallised on the inside to reduce interference. Instrument sensitivity is determined by the resistance in the feedback circuit \( R_f \) in Figure 2.8, which gives the proportionality between \( H_a \) and the output voltage \( U_a \) in the figure. The resistance may be changed by electronic switching to select one of 7 measuring ranges (5 in MAG-4). This can be done either 'manually' by command from ground control, or in an automatic ranging mode by a digital processor subprogram.

The operational modes are:

- Real time mode. Measurements are timed synchronously with the spacecraft telemetry system.
- Sensitivity calibration. Calibration fields may be applied to provide a limited in-flight check on instrument sensitivity.
- Memory modes, incorporating vector averaging (over a chosen number of spins) in order to store data over long time periods. A gap in telemetry coverage of up to 10 days could be bridged if necessary. Memory dump occurs automatically when real-time telemetry becomes available.

Both the magnetometer instruments are mounted on the Giotto antenna tripod, MAG-1 outboard and MAG-4 inbound, as seen in Figure 2.4. A protruding boom could not be used because of the risk of damage at a high flyby velocity and very close approach. Thus the additional MAG-4 sensor was included in the project to aid with handling of the spacecraft magnetic contamination problem. A maximum permissible field at the position of MAG-1 was specified for all spacecraft and experiment systems, but nevertheless, priority was given to field stability rather than a low absolute magnitude. The two despin motors of the Giotto antenna dish presented the greatest problem. These were mounted with their magnetic moments antiparallel, and after the installation of two precision compensator magnets, left a residual field of 7 nT at MAG-1.
Since the ambient field is the same at MAG-1 and MAG-4 (ignoring any features on a smaller-scale than their separation), then the difference in their readings should be purely due to the spacecraft field [Neubauer et al., 1987]. This varies with distance from the spacecraft and thus can be calculated and the measurements adjusted accordingly. The spin variation (of 4 s period) of the measurements can also be used to determine the spacecraft field in the spin plane. Any error in correction for this field will leave residual variations at the spin period in the vectors when transformed to a non-rotating frame (such as Halley-centred solar ecliptic coordinates). A further check on the spacecraft field may be made around closest approach with Giotto in the "magnetic cavity" of zero ambient field (see Chapter 3).

2.7 ICE Mission to Giacobini-Zinner

On 11th September 1985, the International Cometary Explorer (ICE) was the first ever spacecraft to encounter a comet. It passed through the tail of comet Giacobini-Zinner, 7800 km behind the nucleus, on a trajectory inclined at ~ 93° with respect to the plasma tail axis and at a flyby velocity of ~ 21 km/s [Von Rosenvinge et al., 1986; Brandt et al., 1985].

ICE was originally known as the third International Sun-Earth-Explorer (ISEE-3) and was launched in August 1978 to study the region of interaction between the solar wind and the Earth's magnetosphere. An extended mission was suggested in summer 1982, and the spacecraft left the Earth-moon system on 22nd December 1983. At the encounter with Giacobini-Zinner, ICE was ~ 50 times farther from Earth than designed to go [Von Rosenvinge et al., 1986]. In view of this, the data was relayed to Earth over two transmitters to improve power received, the bit-rate was reduced, and backup ground receiving stations were employed.

Of the 13 instruments on board the ICE spacecraft, 7 were expected to provide useful results at the comet. These are as follows.

- The plasma electron instrument [Bame et al., 1986] is a 135° spherical section electrostatic analyser with a system of secondary electron emitters and an electron multiplier. The energy range is 10 to 1000 eV.
Only 2-dimensional measurements were made during the encounter to maximize the sampling rate.

- The energetic particle anisotropy spectrometer (EPAS) measures 3-dimensional ion distributions in 8 logarithmically spaced energy channels with an average 32 second temporal resolution during encounter [Hynds et al., 1986]. Measurements are obtained using 3 identical semiconductor particle telescopes each with a 32° field of view cone, mounted at angles of 30°, 60° and 135° to the spacecraft spin axis. The instrument is unable to resolve ion species, however the observed ions are believed to be predominantly of the water group.

- The ultra-low-energy charge analyser (ULECA) uses an electrostatic deflection system with an array of solid-state detectors and has a ±30° acceptance about the ecliptic plane [Ipavich et al., 1986]. It was originally designed for the measurement of H⁺, He²⁺ and highly ionized heavy ions. Charge state composition is determined from the simultaneous measurement of $E/q$ and total $E$.

- The ion composition instrument (ICI) [Ogilvie et al., 1986] was designed for solar wind observations. It makes mass spectroscopic measurements of $m/q$ in the ranges 1.4 to 3 amu e⁻¹ and 14 to 33 amu e⁻¹ (covering the water group). The range is selected by varying the potentials applied to the plates of the energy analyser.

- The vector helium magnetometer [Brandt et al., 1985; Smith et al., 1986] obtains three triaxial measurements of the magnetic field per second.

- The radio experiment [Meyer-Vernet et al., 1985] measures the electric field power spectrum at 30 kHz to 2 MHz frequencies using two antenna systems. The 'S antenna' is a 90 m (tip to tip) dipole consisting of two thin, long wires, which detected the thermal noise due to plasma motions. The 'Z antenna' is a monopole, consisting of a single thick, short boom aligned parallel to the spacecraft spin axis, and mounted so the receiver measures the voltage difference between the boom and the
spacecraft. This recorded charged particles impacting on (or emitted from) the spacecraft skin.

- The plasma wave instrument uses a 16-channel spectrum analyser (covering frequencies 18 Hz to 100 kHz) connected to the 90 m electric antenna and an 8-channel analyser (18 Hz to 1 kHz) connected to the magnetic search coil [Scarf et al., 1986]. During encounter, one E-field spectrum was obtained per second, and one B-field spectrum every 32 seconds.
Chapter 3

Comets: Mission Results

3.1 Origin of Comets

It is generally assumed that comets were formed at the same time as the Solar System, about 4.6 billion years ago, and now occupy the 'Oort cloud' at 10,000 to 100,000 AU from the sun. This distance is sufficiently far from the sun for minimal external heating to occur. Also, self-gravitational compacting and heating (such as might occur in the early stages of planet formation) is insignificant in the case of comets, which have small masses. Therefore chemical changes are slow and the original composition of comet nuclei may be reasonably well preserved.

The comets remain in the Oort cloud until deflected into a closer orbit by gravitational perturbations perhaps caused by the movement of distant bodies in the Galaxy. Once within the Jovian capture region, if a periodic orbit around the sun is formed then mass loss occurs on every perihelion pass, so these comets have a finite active lifetime. Comets that have returned many times are generally less bright than new comets (with less dust emission), possibly as a result of the depletion of ice from the outer layers of the nucleus. Halley's comet has a period of 76.09 years and has been documented since 240 BC (the first Chinese records) but has an activity comparable to that of many newer comets. Giacobini-Zinner is a short period comet (6.5 years), and infra-red observations indicate that it is not particularly dusty.
3. COMETS: MISSION RESULTS

Figure 3.1: Sketch of the principle features of the comet Halley nucleus as seen by the Halley multicolour camera (HMC) on board the Giotto spacecraft [Keller et al., 1987].

3.2 Structure of Nucleus, Coma and Tails

At large distances from the sun, a comet is an inactive body that can generally only be resolved by Earth-based telescopes as a single point of light, if at all. When closer to the sun, the heated ice sublimes from the nucleus to form a coma which makes it impossible to observe the nucleus from the Earth. So, prior to the 1986 encounters, the size of the Halley nucleus could only be estimated from the observed brightness, and the mass on further assumption of the composition. Only line-of-sight observations of some strong molecular emission lines (at suitable wavelengths) are possible from Earth.

Images of the nucleus obtained by the Halley Multicolour Camera aboard the Giotto spacecraft revealed a single, elongated ‘peanut’-shaped nucleus, approximately $15 \times 8 \times 8$ km, with many irregular surface features [Keller
et al., 1987] as illustrated in Figure 3.1. The irregularities are possibly connected with the inhomogeneous activity. Nearly all the gas and dust emanates from a few isolated areas which become active only when illuminated by the Sun. The nucleus material is solid and appears dark. Its mean density has been estimated from the upper mass limit of $\sim 10^{17}$ g inferred from its acceleration by non-gravitational forces (such as the asymmetric outgassing). The density obtained is $\sim 0.2 \, \text{g/cm}^3$ [Rickman, 1986]. A similar value is estimated for the nucleus of comet Giacobini-Zinner, for which calculations indicate a mass of $< 2.8 \times 10^{15}$ g [Rickman et al., 1987]. The density of such comets is the lowest of any known object in the solar system.

Spacecraft observations confirmed the predicted 'Dirty Snowball' icy conglomerate model of Whipple [1950] which postulates a nucleus composed of ices of compounds such as $\text{H}_2\text{O}$, $\text{NH}_3$ and $\text{CO}_2$ with dust grains frozen into the ice. When the comet approaches the sun in its orbit, the ice is heated and sublimes from the surface, releasing dust with it. The plasma in the visible coma of the comet (the head section of up to $10^5 \, \text{km}$ in diameter) is a mixture of neutral molecules and ions, which are excited by solar radiation and emit light at characteristic wavelengths. For any particle species, the coma size is determined by the gas outflow velocity from the nucleus (which is species dependent), multiplied by the molecule ionization lifetime. This lifetime defines how far out the neutrals can reach simply because once ionized the particles are picked up into the solar wind flow back towards the comet, as will be described in the next section. The hydrogen coma is many times the size of the Sun, typically $\sim 10^7 \, \text{km}$ across, which is considerably larger than the visible coma [Keller, 1976] but because emission occurs in the UV range, it can only be observed from above the Earth's atmosphere.

A comet has two tails which can have a visible length of up to $10^8 \, \text{km}$. The large, curved dust tail appears white because visible sunlight is scattered off the grains. The plasma tail consists mainly of $\text{CO}^+$, $\text{H}_2\text{O}^+$ and $\text{OH}^+$ ions and appears blue [Johnstone, 1985] owing to the excitation wavelength of $\text{CO}^+$ which dominates in the visible range. It displays spectacular features such as filaments, rays, kinks and helices which are evidence of complex hydromagnetic phenomena. The electromagnetic forces control its behaviour and determine its streaming almost radially away from the Sun (see also
3. COMETS: MISSION RESULTS

Chapter 1). In contrast, gravity holds the dust tail in a heliocentric orbit, while the solar radiation pressure pushes it outwards to some degree. Thus the two tails are seen as separate entities.

3.3 Production and Ionization of Gas

The volatile cometary material continually sublimes from the surface of the nucleus and drifts away with its thermal velocity (heavy ions reaching $\sim 1$ km s$^{-1}$). Once released, in the simplest approximation the particles expand into increasing spherical areas ($4\pi r^2$) so that their density falls off as $1/r^2$ with distance, $r$, from the nucleus. Ionization of these neutrals will deplete this density. In the real case particles are not emitted evenly over the surface of the nucleus so the initial distribution varies accordingly, and the particles follow Keplerian trajectories about the Sun so that densities are greatest in the cometary orbit plane [Daly and Jockers, 1989]. Note that gravitational effects of the comet may be ignored since the escape velocity for a comet of mass $10^{17}$ g and approximate radius 5 km is only of the order of 1 m/s.

Close to the nucleus, in the coma, the cometary gas is dense and collision-dominated. Particle composition observations [Balsiger et al., 1986b] during the encounters with comet Halley reveal perhaps the most primitive solar system material ever analysed [Reinhard, 1988]. The material is a mixture of neutral molecules including H$_2$O, CO, CO$_2$, NH$_3$, CH$_4$ and ions of the same. 80% is water ice [Krankowsky et al., 1986b]. Spectroscopic observations of Giacobini-Zinner from the Earth (and Earth orbit) enable the identification of H, OH, NH, CN, C$_3$, C$_2$, CH, NH$_2$, OI and H$_2$O particle species [Brandt et al., 1985].

As the molecules leave the surface they participate in chemical and photochemical reactions in the collisional region within the first 1000 km or so. The parent molecules originally released from the surface of the nucleus are broken into lighter fragments by dissociation processes. The major processes involved are photodissociation and fast ion-molecular reactions [Johnstone, 1985]. Molecular composition therefore varies with distance from the source. The flow of neutral atoms and radicals eventually attains a steady outward velocity of the order of 1 km s$^{-1}$ for heavy ions, and up to 20 km s$^{-1}$ for
hydrogen [Keller, 1976], with few collisions or reactions taking place. The most important subsequent event is the ionization of the neutral particles by charge exchange or photoionization, leading to a multitude of species (singly or multiply charged) originating from the parent molecules and their dissociation products. Ionization occurs on a time scale of the order of $10^6$ s, which allows some particles time to reach distances of more than $10^6$ km from the nucleus.

The ionization processes are described as follows:

**Photoionization.** Rates are proportional to the solar wind UV flux, hence vary according to $1/R^2$ with heliocentric distance $R$. The effectiveness also depends on the opacity at the local position in the coma.

**Charge exchange** (of an electron) between cometary neutrals and solar wind ions. The rate depends on the solar wind flux and the species of neutral involved (i.e. its 'cross-section' for the reaction).

**Electron collisional ionization.** Energetic electrons may have sufficient energy to 'knock out' a bound electron from an atom or molecule.

**Critical velocity ionization** has been postulated by some authors. It is suggested to take place when a neutral's kinetic energy relative to a magnetized plasma exceeds its ionization potential. This is a collisionless interaction.

Immediately a neutral is ionized, it is subjected to the solar wind electric and magnetic fields. The magnetic field, moving out from the Sun with the solar wind plasma, produces a motional electric field in the reference frame of the newly-ionized cometary particles. From Ohm's law for a moving conductor (see Chapter 1, Section 1.3), the electric field for an infinitely conductive plasma with "frozen-in" magnetic field is

$$E = -u \times B$$

(3.1)

The force, $qE$, accelerates the new ions initially out of the ecliptic plane as a result of their velocity component $v_{L,inj}$ perpendicular to $B$. The Lorentz force, $\mathcal{F}_L = q v \times B$, then causes them to orbit about the field lines so that they are initially distributed about a ring, or torus in velocity-space.
Figure 3.2: Geometry of cometary ion pick-up in real space, showing the $E = -u \times B$ direction out of the plane, and the pick-up ion cycloidal motion caused by gyration about $B$ and $E \times B$-drift.
with radius $v_{\perp,\text{inj}}$. At this same speed, the ions $E \times B$-drift towards the comet with the magnetic field, following a cycloidal trajectory. Figure 3.2 gives a schematic representation of this in real-space. The pick-up process is considered in more detail in later chapters.

### 3.4 Plasma Regions and Boundaries

The solar wind interacts differently with an unmagnetized object such as a comet than it does with those planets in the solar system that have their own intrinsic magnetic field. In the case of a planet, a "magnetopause" forms, which is a current sheet boundary at the balance of pressure between the planetary magnetic field and the solar wind (field plus particle pressure). A planetary bow shock will form due to the object in the flow. At a comet, the "contact surface" forms where the pressure of the cometary particles balances the total pressure of the solar wind, and a bow shock forms at a relatively early stage primarily as a result of mass-loading of the solar wind with heavy cometary ions. The difference this causes is apparent on comparing the distances at which these transitions occur. The ratio of the subsolar standoff distance of the shock to that of the magnetopause for the Earth is much smaller than the ratio of the shock to contact surface distances observed at comet Halley, for example.

#### 3.4.1 Regimes at Comet Halley

At the time of the Giotto encounter, the comet was close to the heliospheric current sheet (see Chapter 1) which gave much variability. Giotto crossed it several times and hence observed magnetic field direction changes of as much as 180°.

It is possible that cometary ions were detected as far out as 28 million km upstream from Halley, according to the International Cometary Explorer (ICE) measurements [Sanderson et al., 1986], on 25th March 1986. Energetic (accelerated) cometary ions were observed out to $\sim 7.5 \times 10^6$ km at Giotto [McKenna-Lawlor et al., 1986b]. Upstream waves of cometary origin were observed in the solar wind at distances $> 2 \times 10^6$ km along the Giotto trajec-
Figure 3.3: Basic regimes of the comet-solar wind interaction region [Schmidt and Wegmann, 1982].

tory, both inbound and outbound [Neubauer et al., 1986]. The turbulence is basically Alfvénic in nature and at frequencies of the order of cometary ion gyrofrequencies shifted into the spacecraft frame [Johnstone et al., 1987]. Turbulence is generated as a result of solar wind mass-loading by cometary ions, and the power level increases as the comet is approached.

The comet-solar wind interaction region can be considered in three sections, illustrated in Figure 3.3. In the inner coma the dense cometary plasma dominates and there is little or no magnetic field. The upstream region is that which extends beyond the bow shock, where the solar wind dominates and becomes increasingly mass-loaded in its flow towards the comet. Between these two regions lies the 'mixing zone', where there is much turbulence in the sub-sonic solar wind plasma and cometary ions. Within these three regimes other structures were identified at Halley following the encounter missions (see Chapter 2). Some of the most important of these are described in Table 3.1 (where the distances from Halley given here are along the Giotto
<table>
<thead>
<tr>
<th>Inbound distance (km)</th>
<th>Region/boundary</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>~ 4.6 - 1.4 x 10^6</td>
<td>upstream region</td>
<td>backstreaming electrons, sporadic connections to comet, some cometary pickup ions</td>
</tr>
<tr>
<td>1.4 - 1.15 x 10^6</td>
<td>foreshock region</td>
<td>T_e increase, more ram ions</td>
</tr>
<tr>
<td>~ 1.16 - 1.12 x 10^6</td>
<td>BOW SHOCK</td>
<td>relatively abrupt flow speed decrease, density increase</td>
</tr>
<tr>
<td>11.5 - 8.5 x 10^5</td>
<td>outer region 1</td>
<td>large n_e increase, fluctuations, anticorrelations v_e and n_i</td>
</tr>
<tr>
<td>8.5 - 5.5 x 10^5</td>
<td>outer region 2</td>
<td>high alpha particle densities, energetic electrons, n_e,T_e anticorrelate with n_i</td>
</tr>
<tr>
<td>5.5 - 1.35 x 10^5</td>
<td>outer region 3</td>
<td>n_e,T_e,v_e decrease, more ram ions, sharp decrease in density of alphas</td>
</tr>
<tr>
<td>1.35 x 10^5</td>
<td>COMETOPAUSE</td>
<td>n_e,n_p decrease, n_i increase, v_e → 0, magnetic “pile-up” begins</td>
</tr>
<tr>
<td>~ 4.7 x 10^3</td>
<td>IONOPAUSE or contact surface</td>
<td>boundary of magnetic CAVITY (≈zero B-field)</td>
</tr>
</tbody>
</table>
Figure 3.4 - Plasma regions identified in the electron data at Giotto [Rème et al., 1986b]. The panels are, from the bottom, electron density in the energy ranges 10 eV - 30 keV and 0.8 - 3.6 keV, electron temperature and bulk velocity, and the ion counts in the ram direction.

Upstream and foreshock regions

Figure 3.4 shows the plasma regions that have been identified in electron plasma data from the RPA-Copernic electron analyser on Giotto by Rème et al. [1986b]. A large, turbulent upstream region extends from the bow shock out to around $4.6 \times 10^6$ km. Analysis of the data indicates possible sporadic connections to the comet identified by equal forward and reverse electron heat fluxes [Rème et al., 1986b]. In other words, electrons may be streaming back along magnetic field lines from the comet (in the direction opposite to the heat flux from the solar corona).

The foreshock region is characterized by steadily decreasing average ve-
locities, increasing densities of solar wind and cometary pick-up ions, an increasing level of fluctuations, and a $\geq 50\%$ increase in electron temperature at the outer boundary (1.4 x 10$^6$ km from the nucleus along the Giotto path.)

**Bow Shock**

In the case of a comet, the shock does not form as a result of deflection around an obstacle (such as the magnetic field of the Earth), but occurs prior to this stage, to accommodate rapid solar wind deceleration due to mass-loading. The 1D magnetohydrodynamic (MHD) conservation equations including a cometary mass source term imply a limiting mass-flux ratio above which they are insoluble. The mass-flux ratio, $(\rho u)/(\rho_\infty u_\infty)$, is the ratio of the local mass-flux (including source ions) to the original solar wind value, and the critical point occurs at [Wallis, 1973]:

$$\frac{\rho u}{\rho_\infty u_\infty} = \frac{\gamma^2}{(\gamma^2 - 1)}$$

from the simplest set of equations. For $\gamma = 2$ this ratio is 4/3 [Biermann et al., 1967]. This requires the addition of only a small percentage (by number density) of cometary ions into the flow because of their much greater mass compared to the solar wind mean molecular weight. Equation (3.2) is derived for the head-on approach, but since the shock is not primarily produced by deflection and the situation can be reasonably analysed in terms of a 1D flow, the equation may be applied to any streamline.

A shock is characterized by a relatively abrupt jump or discontinuity in the flow parameters (eg densities and velocities) as seen in Figure 3.5, for example, which is data from the Suisei encounter [Mukai et al., 1986]. The Halley bow shock stands between 1.12 - 1.16 x 10$^6$ km along the Giotto inbound pass on 13th March 1986 [Coates et al., 1987a; Johnstone et al., 1986b; Neubauer et al., 1986b]. The shock structure observed inbound by the Giotto JPA Fast Ion Sensor [Coates et al., 1990b; Coates et al., 1987b] and shown in Figure 3.6 is approximately 4 x 10$^4$ km thick along the spacecraft trajectory (inclined at an angle of 107.2° to the sun-comet line), corresponding to ~ 5 water group ion gyroradii. This of course assumes that the features
are permanent and observations are not merely a result of the shock moving across the spacecraft. The shock identified in the Giotto Magnetometer data [Neubauer et al., 1986b] on the outbound pass was not so clearly defined.

Figure 3.5: Plasma flow parameters observed at Halley by the Suisei plasma instrument (ESP) [Mukai et al., 1986].
Figure 3.6 - The shock structure (between S1 and S2) observed in the Giotto JPA plasma data [Coates et al., 1990b]. F denotes the foreshock and M1 and M2 are the 'mystery region' boundaries of Rème et al. [1986b].
Outer Regions

Three regions have been identified in the electron data between the bow shock and the "cometopause" [Rème et al., 1986b] as seen in Figure 3.4. In the first of these, fluctuations of the order of 1-minute periods are seen in the electron density, \( n_e \), temperature, \( T_e \), and velocity, \( v_e \). The electron density is rising, and low energy protons are detected in the RAM direction (the spacecraft/comet relative velocity direction). In the second outer region, larger variations in \( n_e \) and \( T_e \) are observed, on a longer timescale. Unexpectedly, there are significant fluxes of highly energetic electrons, field-aligned in the range 0.3-3.6 keV, so that the zone has been named the "mystery region" [Rème et al., 1986b]. Also the Giotto IMS High Energy Range Spectrometer recorded a high density of increased velocity alpha-particles [Goldstein et al., 1986]. At the boundary of Outer Region 3 there is a sharp decrease in alpha-particle density. There begins a "quiet region", characterized by more isotropic, stable, cool electron distributions, of lower and relatively constant density. More low energy RAM-direction ions are observed.

At \( \sim 5 \times 10^5 \) km from the nucleus, the main (\( \text{H}_2\text{O} \) group) heavy ion peak (observed by the Giotto JPA Implanted Ion Sensor) shifted to lower energies and a second low energy peak emerged [Thomsen et al., 1987]. The downward shift is probably produced by an overall deceleration of the bulk solar wind flow at this point. It is suggested by Thomsen et al that the higher energy peak corresponds to cometary ions implanted upstream of the bow shock, and the lower peak results from particles ionized in the slower, post-shock flow. A gap between the two occurs because the shock transition is fast such that few ions are implanted with initial velocities in the range of the jump.

Inner Regions

The existence of the "cometopause" was not anticipated and is still surrounded by much controversy. It occurs at about \( 1.35 \times 10^5 \) km on the Giotto inbound trajectory [Rème et al., 1986b] and is around \( 10^4 \) km thick according to PLASMAG-1 on Vega 2 [Gringauz et al., 1986b]. Proton and electron densities \( (n_e \text{ and } n_p) \) decrease suddenly and minima in \( T_e \) and \( v_e \) are
reached. Magnetic “pile-up” begins just inside the cometopause \[Neubauer \textit{et al}., 1986b\] along with rapidly increasing densities of RAM-direction cold cometary ion species.

At around $4.7 \times 10^3$ km from the nucleus inbound, the magnetic field drops to zero (within instrumental accuracy) at the ionopause, or contact surface, as Giotto entered the magnetic “cavity”. The cavity has a width of $\sim 8,500$ km \[Neubauer \textit{et al}., 1986b\] along the spacecraft path around closest approach.

### 3.4.2 Plasma Regimes at Giacobini-Zinner

The interaction region at comet Giacobini-Zinner is considerably smaller than that observed at Halley. The comet’s activity in 1985 reached a peak just before the ICE encounter when the gas production rate from the nucleus was $\sim 10^{29}$ molecules per second \[von Rosenvinge \textit{et al}., 1986\], in contrast with a value of $\sim 10^{30}$ s$^{-1}$ for Halley (see Chapter 4). At the time of the ICE encounter, the production rate had reduced to $\sim 2$ to $5 \times 10^{28}$ s$^{-1}$ according to observations by the International Ultraviolet Explorer (IUE) and the Pioneer Venus Orbiter. Energetic ions were first detected by the ICE Energetic Particle Anisotropy Spectrometer at distance of $\sim 1.8 \times 10^6$ km inbound and observed until $\sim 4 \times 10^6$ km outbound \[Hynds \textit{et al}., 1985\]. Large amplitude turbulence was present in a region extending to $\sim 10^5$ km either side of closest approach.

Three regimes were identified in the ion data \[Hynds \textit{et al}., 1986\], schematically illustrated in Figure 3.7. The outermost of these is an upstream “pickup region”, on a scale out to the order of $10^6$ km from the nucleus. In this region, antisolar streaming pick-up ions and backstreaming electrons are observed. A weak “shock” boundary or sharp transition of scalelength the order of a heavy ion gyroradius \[Hynds \textit{et al}., 1986; Bame \textit{et al}., 1986\] occurs at a distance of around $10^5$ km, within which lies a “massloading” interaction region (or “sheath” region) of much turbulence. Here, broad ion angular energy distributions are measured. An “inner region” inside $\sim 10^4$ km contains cold plasma, with highly draped magnetic field lines, and is characterized by reduced fluxes and complex ion distributions \[Hynds et
Figure 3.7: Schematic summary of the plasma regions observed at comet Giacobini-Zinner [Breadt et al., 1985].
al., 1986]. The magnetometer record shows evidence of a bipolar tail form with a central plasma sheet [Ogilvie, 1985]. The ICE spacecraft did not pass within the “ionopause”.

3.5 Turbulence Observed in the Vicinity of Comets

In the solar wind - comet interaction region, turbulent waves are generated in the plasma at levels well above the background in the ambient solar wind. Fluctuations observed in the magnetometer data at comet Giacobini-Zinner are of greatest amplitude at around the bow wave and decrease in intensity with increasing distance from the comet [Tsurutani and Smith, 1986]. The turbulence at comet Halley upstream of the bow shock (\(\sim 2.2 \text{ to } 1.5 \times 10^6\) km from the comet) has been analysed by Glassmeier et al. [1989] in the frequency range 1 to 100 mHz and is described as Alfvénic in character. The power in transverse fluctuations is greater than that in the compressional component. Waves propagate nearly parallel (or antiparallel) to the magnetic field lines (with elliptical or almost linear polarizations) and predominantly sunward in the solar wind frame. There is possible evidence for a downstream propagating wave in regions where the angle, \(\alpha\), between the directions of the magnetic field and the solar wind flow reaches around 90°. Spectral analysis of the turbulence shows a power-law dependence \(\sim f^{-2.0}\) of the power spectral density on frequency, \(f\) [Glassmeier et al., 1989] (see Chapter 6). Wave power spectra at Giacobini-Zinner may be fitted by \(\sim f^{-\frac{3}{2}}\) at frequencies above a main peak at \(\sim 10^{-2}\) Hz [Tsurutani and Smith, 1986]. Although the spectral shape and intensity becomes increasingly variable at large distances from the comet, no convincing gradual frequency shifts of the peaks are observed.

A peak in the wave spectra near the water group ion gyrofrequency is observed at both Halley and Giacobini-Zinner, which is suggested to be a result of water group pick-up ions driving resonant ion cyclotron waves [Glassmeier et al., 1989]. The turbulence is observed out to at least \(10^6\) km which is comparable to the lengthscale for cometary ion formation [Tsurutani and Smith,
Figure 3.8: A sample of data from Giacobini-Zinner showing waves at the water group ion cyclotron period in a region where $\alpha$ is low. High frequency fluctuations are present in the region of high $\alpha$ [Tsurutani et al., 1989].

1986] (see Chapter 4). At Halley the peak is at $\sim 7$ mHz in regions where $\alpha \sim 0$, ie when $B$ is approximately parallel to $u$, while no clear peak was identified in a spectrum of data where $\alpha \sim 90^\circ$ [Glassmeier et al., 1989]. At Giacobini-Zinner the peak occurs at $\sim 10$ mHz. An example of a period of data during which such waves occur is given in Figure 3.8. The amplitude of these approximately 100 s period electromagnetic waves reaches up to the maximum limit $\Delta B/B \sim 1$ near the bow wave. For “strong” hydromagnetic turbulence $\Delta B/B = 1$ when the particle perturbation velocity due to the wave reaches the phase velocity of the wave [Tsurutani and Smith, 1986] and beyond this point the waves are strongly damped.

At Giacobini-Zinner the plasma wave instrument [Scarf et al., 1986] registered waves at frequencies of a few hertz and at tens to 100 Hz. Oscillations of several hundred Hz to several kilohertz were present in the electric field data together with bursts near the electron plasma frequency at a few tens of kilohertz [Scarf et al., 1986]. The kilohertz electrostatic emissions appear to be modulated by the angle, $\alpha$, between $B$ and $u$ directions, and are most
intense when $\alpha \sim 0$ when they may be observed out to $\sim 1.5 \times 10^6$ km from the comet [Richardson et al., 1989]. Close to the bow wave the magnetic field direction is dramatically perturbed by the $\sim 100$ second period electromagnetic waves previously discussed above and the kilohertz electrostatic emissions are modulated by $\alpha$ accordingly. The $\alpha$-dependence of these waves suggests that they are excited by an instability associated with the pick-up distribution beam-type anisotropy. Their amplitudes are large in the inner pickup region (just upstream of the bow wave) where the heavy ion population has become reasonably isotropic (see Chapters 5 and 7) which indicates that a different pick-up population may be responsible [Richardson et al., 1989]. In particular, Brinca et al. (1989) interpret the electrostatic bursts in terms of a mode driven by pick-up photoelectrons.

Electromagnetic waves with frequencies of the order of tens of hertz are observed in the pickup region at Giacobini-Zinner out to $\sim 4.5 \times 10^6$ km [Richardson et al., 1989]. Amplitudes of these waves are also modulated by $\alpha$ and in this case strong emission occurs when $\alpha > 60^\circ$ along with high fluxes of energetic heavy cometary ions. Observations therefore suggest that the growth of the waves results from an instability associated with the anisotropic ring distribution of pick-up ions (see Chapter 5), since heavy ion distributions are anisotropic in the regions where these waves are observed [Richardson et al., 1989]. Amplitudes are lower and more variable in the inner pickup region between $\sim 0.5 \times 10^6$ km and the shock, where the $\sim 100$ s period electromagnetic waves should effectively isotropize the implanted ion distributions.

Electrostatic fluctuations at $\sim 20$ Hz are detected in the inner pickup region in addition to the electromagnetic waves of similar frequency described above [Richardson et al., 1989]. Their amplitude is modulated by the magnitude of the background magnetic field and thus varies with the compressional component of the low frequency waves, at $\sim 100$ s period.
Chapter 4

Solar Wind Mass-Loading at Comet Halley

The basic nature of the interaction between a comet and the solar wind is well known (see Chapter 3). Once the cometary neutrals are ionized they are picked up into the solar wind flow and accelerated to velocities comparable with the solar wind speed. The cometary particles gain energy and momentum at the expense of the solar wind. The solar wind is thereby decelerated and deflected, gradually at first, on encountering the farthest-reaching cometary particles a long distance upstream from the nucleus, but more dramatically as the ion density increases. This is the phenomenon of mass-loading.

4.1 Introduction: Ion Motion

4.1.1 Gyration and Drift

Equations describing the cycloidal trajectory of an ion in a magnetic field may be obtained from the equation of motion. For zero electric field, this involves the Lorentz force only [eg Chen, 1984]:

\[ m \frac{dv}{dt} = qv \times B \]

(4.1)
where $q$, $m$ are the ion charge and mass, respectively. Taking $\mathbf{B}$ along the $z$-axis, $\mathbf{B} = B\hat{z}$, the components of the acceleration are

$$\dot{v}_x = \frac{qB}{m} v_y$$

$$\dot{v}_y = -\frac{qB}{m} v_x$$

(4.2)

From these, equations for a simple harmonic motion are obtained. Taking the time derivative of the first equation and substituting for $\dot{v}_y$ from the second gives a relation in terms of $v_x$ only, and vice versa for $v_y$:

$$\ddot{v}_x = -\left(\frac{qB}{m}\right)^2 v_x$$

(4.3)

$$\ddot{v}_y = -\left(\frac{qB}{m}\right)^2 v_y$$

(4.4)

The motion is in the $x, y$ plane perpendicular to $\mathbf{B}$ and the solution may be written [Chen, 1984]

$$v_x = v_{\perp} e^{i\Omega t}$$

(4.5)

$$v_y = \pm i v_{\perp} e^{i\Omega t}$$

(4.6)

in terms of the magnitude of the orbital velocity, $v_{\perp}$, and the cyclotron frequency

$$\Omega = \frac{|q|B}{m}$$

(4.7)

in radians per second. Positively charged ions gyrate in the opposite sense to negatively charged ions and the $\pm$ denotes the sign of $q$ in the above equations. The orbital velocity is the rate at which the ion describes the 'circumference' of orbit of radius $r_L$, which is $v_{\perp} = 2\pi r_L f_{\text{orbit}} = r_L \Omega$ where $f_{\text{orbit}}$ is the orbital frequency in s$^{-1}$. The Larmor radius is thus defined:

$$r_L = \frac{v_{\perp}}{\Omega} = \frac{mv_{\perp}}{|q|B}$$

(4.8)

Note that for a given ion species the gyroperiod is independent of ion velocity, but the greater the ion momentum the larger the radius of orbit about a given $\mathbf{B}$.

The magnetic moment of a gyrating particle is defined by [eg Chen, 1984]

$$\mu_m = \frac{mv_{\perp}^2}{2B}$$

(4.9)

which is an adiabatic invariant, a constant of the periodic motion. If the magnetic field strength changes, $v_{\perp}^2$ must alter accordingly. Conservation of
energy, \( \frac{1}{2} m(v_\perp^2 + v_\parallel^2) = \text{constant} \) then implies that if an ion’s perpendicular velocity increases on encountering a stronger field, then its velocity \( v_\parallel \) along \( \mathbf{B} \) must reduce. (The principle of plasma confinement within ‘magnetic mirrors’ is based on this). Thus for \( B \)-field changes that occur smoothly, \( \mu_m \) is constant. However, if there is a jump in \( B \) that occurs on a timescale more rapid than the orbital period of the ion, then the magnetic moment of the ion may not be conserved. Rapid random fluctuations in \( B \), where the time-average \( B_0 \) is uniform (or varies only slowly), cannot change \( \mu_m \) unless there is some non-linear interaction or resonance between the ions and such fluctuations.

If there is an electric field present in the plasma medium, as in the case of the motional electric field experienced by newly ionized cometary particles in the solar wind (see Chapter 3), the equation of motion becomes [Chen, 1984]:

\[
m \frac{dv}{dt} = q (\mathbf{E} + \mathbf{v} \times \mathbf{B})
\]

(4.10)

For \( \mathbf{E} = E\mathbf{\hat{x}} \), separating \( x \) and \( y \) components (as done previously) leads to:

\[
\ddot{v}_x = -\Omega^2 v_x
\]

(4.11)

\[
\ddot{v}_y = -\Omega^2 \left( v_y + \frac{E}{B} \right)
\]

(4.12)

where \( E \) is assumed constant in time. This assumption also allows the inclusion of a further term in the time derivative on the LHS of (4.12) so that the equation becomes [Chen, 1984]

\[
\frac{d^2}{dt^2} \left( v_y + \frac{E}{B} \right) = -\Omega^2 \left( v_y + \frac{E}{B} \right)
\]

(4.13)

The \( x \) motion is once again described by equation (4.5). On comparing (4.13) with (4.4) it is apparent that \( v_y \) may be replaced by \( (v_y + E/B) \) in the solution (4.6) to give

\[
v_y = \pm i v_\perp e^{i\omega t} - \frac{E}{B}
\]

(4.14)

Thus, superimposed upon the gyroratory motion is a drift in the \(-y\) direction. The drift may be obtained from \( \mathbf{E} + \mathbf{v} \times \mathbf{B} = 0 \):

\[
\mathbf{E} \times \mathbf{B} = -(\mathbf{v} \times \mathbf{B}) \times \mathbf{B} = vB^2 - B(v \cdot \mathbf{B})
\]

(4.15)
The required drift in the $-y$ direction is perpendicular to both $E$ and $B$, so that $v \cdot B = 0$ and the drift velocity is given by [Chen, 1984]

$$v_d = \frac{E \times B}{B^2}$$

(4.16)

Note that the above may be generalized to examine the effect of any applied force, $F$, by replacing $qE$ in the above analysis by $F$.

### 4.1.2 Fluid Theory

When interactions between particles in a plasma impose collective behaviour, the bulk parameters (averaged over the individual particles) are related by the equations of fluid theory.

The change in time of any fluid property, $G$, in a frame moving with the fluid is given by the “convective derivative” [eg. Chen, 1984]:

$$\frac{dG}{dt} = \frac{\partial G}{\partial t} + \frac{\partial G}{\partial x} \frac{dx}{dt} + \frac{\partial G}{\partial y} \frac{dy}{dt} + \frac{\partial G}{\partial z} \frac{dz}{dt} = \frac{\partial G}{\partial t} + (u \cdot \nabla)G$$

(4.17)

where $\partial G/\partial t$ is the change in $G$ seen at a fixed position in space, and the second term on the RHS gives the change in $G$ as a result of the parcel of plasma moving into a new region. If $G = u$, the plasma fluid velocity, then $du/dt$ in the equation of motion may be replaced according to (4.17) to give the fluid equation:

$$\rho \left[ \frac{\partial u}{\partial t} + (u \cdot \nabla)u \right] = nq(E + u \times B)$$

(4.18)

If the random, thermal motion of the particles is to be taken into account, a pressure-gradient force, $-\nabla p$, must be added to the RHS of equation (4.18). This term may be derived by considering the net momentum carried into a plasma element by particles moving in and out of the element. The particle velocity splits into $v = u + v_{th}$, the average fluid velocity, $u$, plus a thermal velocity where $\frac{1}{2}m < v_{th}^2 = \frac{1}{2}k_BT$ and the pressure is defined $p = nk_BT = mn < v_{th}^2 >$. In the more general case the pressure tensor, $P$, is required, with components such as $P_{x,y} = mn < v_{th,x} v_{th,y} >$, allowing the transfer of $y$ momentum by a movement of particles in the $x$ direction, and similarly for all other combinations. Thus $-\nabla p$ is replaced by $-\nabla . P$. 
Another useful fluid equation is the equation of continuity, which arises from the conservation of matter. The number of particles in a plasma parcel is constant in time unless there is a net flux, \( n_u \), of particles in or out of the region (by the divergence theorem):

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n_u) = 0 \tag{4.19}
\]

If the flux is the same throughout the region, then \( \partial n/\partial t \) must equal zero. Any sources (or sinks) may be added to the right hand side.

### 4.1.3 Kinetic Theory

An ion distribution function, \( f(r,v,t) \), representing the number of particles per m\(^3\) with velocity \( v \), at position \( r \) and time \( t \), must satisfy the Boltzmann kinetic equation [eg. Chen, 1984; Boyd and Sanderson, 1969]:

\[
\frac{\partial f}{\partial t} + v \cdot \nabla f + \frac{F}{m} \frac{\partial f}{\partial v} = \left( \frac{\partial f}{\partial t} \right)_c \tag{4.20}
\]

The LHS is the convective derivative of \( f \) with respect to all of its independent variables:

\[
\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial r} \frac{dr}{dt} + \frac{\partial f}{\partial v} \frac{dv}{dt} = \frac{\partial f}{\partial t} + v \cdot \nabla f + a \frac{\partial f}{\partial v} \tag{4.21}
\]

The acceleration is \( a = F/m \) where \( F \) is the force on the particles. The RHS gives the rate of change of \( f \) due to particle collisions. When collisions may be neglected and the force \( F \) is electromagnetic, equation (4.20) becomes the Vlasov equation:

\[
\frac{\partial f}{\partial t} + v \cdot \nabla f + \frac{q}{m} (E + v \times B) \cdot \frac{\partial f}{\partial v} = 0 \tag{4.22}
\]

Fluid equations for the conservation of mass, momentum and energy may be obtained from the Boltzmann kinetic equation on multiplying through with functions \( \psi(v) \) of velocity and integrating over all velocity space [Boyd and Sanderson, 1969]:

\[
\frac{\partial}{\partial t} (n \psi) + \frac{\partial}{\partial r} \left( n \psi v \right) - \frac{n}{m} \left( F \cdot \frac{\partial \psi}{\partial v} \right) = \left( \frac{\partial}{\partial t} (n \psi) \right)_c \tag{4.23}
\]
This is the transfer equation. The force, \( \mathbf{F} \), may be velocity-dependent (as in the case of the Lorentz force, \( q\mathbf{v} \times \mathbf{B} \)). The average of any function \( \psi(\mathbf{v}) \) over velocity space is written:

\[
< \psi > = \frac{1}{n(r,t)} \int \psi(r,v,t) f(r,v,t) \, dv
\]  

(4.24)

where

\[
n(r,t) = \int f(r,v,t) \, dv
\]  

(4.25)

For the mass conservation equation, the zeroth-order moment equation with \( \psi = m \) is required, for momentum \( \psi = m\mathbf{v} \) (first order) and for energy \( \psi = \frac{1}{2}mv^2 \) (second order). Quantities such as the bulk fluid velocity

\[
u = < \mathbf{v} > = \frac{1}{n} \int \mathbf{v} f \, dv
\]  

(4.26)

and pressure

\[
P_{i,j} = mn < v_{th,i} v_{th,j}> = m \int v_{th,i} v_{th,j} f \, dv
\]  

(4.27)

can then be identified in the terms resulting from equation (4.23) for a chosen \( \psi(\mathbf{v}) \).

4.2 A Mass-Loading Model

The magnitude of the mass-loading effect on the solar wind depends on the number of cometary ions created in the flow. The density of the ions in turn depends on the characteristics of the outward flow of the neutral particles; the production rate from the nucleus, the outward flow velocity, and the ionization rate. The aim here is to fit the simplest possible model of mass-loading to the solar wind slow-down observed during Giotto's approach to comet Halley and hence obtain values of these cometary emission quantities.

4.2.1 Neutral Particle Distribution

The model assumes a spherically-symmetric distribution of a single species of particle moving radially outward from the nucleus at a constant velocity, subjected to a constant photoionization rate and a charge exchange rate
which depends on the instantaneous value of the solar wind flux. Such a model assumes an absence of gravitational effects and radiation pressure. Then the density of cometary neutrals, with radial expansion velocity $V_e$, will follow a $1/r^2$ dependence on distance, modified by an exponential to account for the loss of neutrals due to ionization:

$$N_c = \frac{Q}{4\pi V_e r^2} \times \exp \left( -\frac{\nu r}{V_e} \right)$$  \hspace{1cm} (4.28)

Here $Q$ is the neutral production rate (s$^{-1}$) at the nucleus, and the ionization rate $\nu$ is given by

$$\nu = \nu_{ph} + \sigma n_{sw} u_{sw}$$  \hspace{1cm} (4.29)

where $\nu_{ph}$ is the photoionization rate and $\sigma n_{sw} u_{sw}$ is the rate of charge exchange between the neutrals and the solar wind ion flux $n_{sw} u_{sw}$, with cross-section $\sigma$. An $N_c(r)$ curve of the above form is a good fit to the measured H$_2$O abundance from the Giotto Neutral Mass Spectrometer between $\sim$1,600 and 40,000 km from the comet [Krankowsky et al., 1986].

The data analysis of this chapter uses the H$_2$O photoionization rate $\nu_{ph} = 4.2 \times 10^{-7}$ s$^{-1}$ at a heliocentric distance of $R = 0.89$ AU, converted (according to a $1/R^2$ dependence) from the value of $3.34 \times 10^{-7}$ s$^{-1}$ at 1 AU given by Huebner and Giguere [1980]. (Note that the oxygen photoionization rate is of the same order.) The charge exchange cross-section is taken to be $\sigma = 2.1 \times 10^{-15}$ cm$^2$, which has also been used by authors such as Mukai et al. [1986], Ipatich et al. [1986]. Note that $\nu$ only enters the analysis as part of the ratio $L = V_e/\nu$ (see below, equation (4.31)). Here $\nu$ is assumed and $V_e$ is to be derived. Any error in the values chosen will be directly reflected in the value obtained for $V_e$.

At distances of more than $10^6$ km from the comet, the neutral particles will have left the nucleus more than $10^6$ s (11 $\frac{1}{2}$ days) before the observations were made. The trajectories of these particles should ideally be treated as Kepler orbits about the Sun [Daly and Jockers, 1989] since the orbital structure can create significant anisotropies in the distribution about the comet (see Figure 4.1). Calculations by Daly and Jockers show that the ion fluxes inferred from a $1/r^2$ model differ significantly from those based on the Kepler orbit model at distances beyond $3 \times 10^6$ km along the Giotto trajectory (before 12:00 spacecraft event time (SCET) on March 13th, 1986).
4. SOLAR WIND MASS-LOADING AT COMET HALLEY

Figure 4.1: Cometary neutral density and cometary ion flux for both the Kepler orbit and spherically symmetric gas outflow models, plotted against distance from Halley [Daly and Jockers, 1989].

The present analysis considers data collected between $4.7 \times 10^6$ km and $10^6$ km but the results are mainly dependent on the data collected inside $3 \times 10^6$ km. Therefore there is no attempt to take these effects into account.

4.2.2 Perpendicular Pickup

The simplest case of pickup is to be considered here, where the magnetic field, $\mathbf{B}$, is perpendicular to the solar wind flow vector, $\mathbf{u}$. Then the initial velocity of the newly-born cometary ions is entirely perpendicular to $\mathbf{B}$ and will become a purely orbital motion about the field lines. Thus the ions are picked up with complete efficiency. They are accommodated into the flow with a bulk velocity $u_i = u_{sw}$ along the negative $z$-direction as defined in Figure 4.2.

If there is a component of the solar wind velocity parallel to the magnetic field then the ions cannot acquire the parallel component immediately in the region upstream from the bow shock [Coates et al., 1989]. Their bulk flow speed in this case is less than the solar wind speed and their direction is
4. SOLAR WIND MASS-LOADING AT COMET HALLEY

Figure 4.2: Schematic representation of the Giotto fly-by geometry in Halley-centred Solar Ecliptic (HSE) coordinates, showing the integration path, \(dS\), for estimation of the implanted cometary ion flux at a spacecraft position \((x_0, y_0)\).

deflected from that of the solar wind flow vector toward the direction perpendicular to the magnetic field. These effects are neglected in this analysis and an ideal case of instant, perfect pickup is considered. The assumption of \(u_i = u_{sw}\) is a good approximation where the cometary ion velocity-space distributions have become shell-like (centred roughly on the solar wind speed), which they are downstream of \(\sim 2.5 \times 10^6\) km from the nucleus [Coates et al., 1989] as a result of pitch-angle scattering. (The parallel pickup mechanism will be studied in chapters 5 and 7).

4.2.3 Ion Flux

The flux of ions past an observation point per unit time is equal to the total ionization production per unit time upstream from the point. The production rate of cometary ions at a distance \(r\) from the nucleus is given by \(N_c \nu\). Taking an observation point \((x_0, y_0)\) on the Giotto path, the flux due to the comet is derived by integrating the ion production rate back along the trajectory of the implanted ions (a streamline in this case) [Schmidt and
where \( dS \) is the integration path as shown in Figure 4.2. This is an equation of continuity for the cometary ions. Defining a length scale
\[
L = \frac{V_e}{\nu}
\] (4.31)
the distance from the comet and the integration path can be scaled according to
\[
\begin{align*}
  r &= (x^2 + y^2)^{\frac{1}{2}} L \\
  S &= Lx ; \\
  dS &= Ldx
\end{align*}
\] (4.32)
(4.33)
Substituting these in equation (4.30), the integral becomes
\[
n_i u_i = \frac{Q}{4\pi L^2} \int_{x_0}^{\infty} \frac{1}{z^2 + y_0^2} \exp \left[ -\left( x^2 + y_0^2 \right)^{\frac{1}{2}} \right] dx
\] (4.34)
for the cometary ion flux expected at any given spacecraft position, \((x_0, y_0)\). Measurements of the solar wind flow are required in order to calculate the charge exchange rate in \(L\).

The solar wind flow is assumed to be one dimensional. This is equivalent to assuming that the flow is not deflected from its original direction, and that the speed is determined by the ions picked up along its streamline and not by the flow on neighbouring streamlines. The mass and momentum conservation equations describing the unshocked, mass-loaded solar wind flow in one dimension may be written \([Galeev et al., 1985]\):
\[
- \frac{d}{dx} \left[ \rho u f(u, \mu_m) \right] = \frac{m_i Q \nu}{4\pi V_e r^2} \exp \left( -\frac{\nu r}{V_e} \right) \delta \left( \mu_m - \frac{m_i u^2}{2B} \right) 
\] (4.35)
\[
- \frac{d(\rho u)}{dx} = \frac{m_i Q \nu}{4\pi V_e r^2} \exp \left( -\frac{\nu r}{V_e} \right) 
\] (4.36)
\[
\frac{d}{dx} \left( \rho u^2 + p_\perp + \frac{B^2}{2\mu_0} \right) = 0
\] (4.37)
assuming the new ions are instantaneously and completely picked up into the flow. \(f(u, \mu_m)\) is the one-particle velocity distribution of the cometary ions, which all have magnetic moment \(\mu_m = m_i u_\perp^2/2B\) determined by the solar
wind $u_\perp (= u)$ and $B$ values at the point of injection. $p_\perp$ is the plasma kinetic pressure, $m_i$ the mass of the cometary ions, $\rho_i = m_i n_i$ their mass density and $\rho$ is the mass density of the contaminated solar wind. (Note the sign $-d/dx$ in (4.35) and (4.36), since $u$ is a magnitude and the $x$-axis is directed away from the comet source). The $\mu_m$ dependence of $f$ in equation (4.35) is approximate, for the following reason. Although the magnetic moment may be assumed constant for a particular ion, at any given locality the cometary ions present have been picked up at different positions along the flowline. However, the integrated form of the equation gives, at any $x$, the $\rho_i u f$ representing cometary ions with only the local injection value of $\mu_m$ according to the $\delta$-function.

Combining equations (4.35) and (4.36) gives

$$\frac{d}{dx} [\rho_i u f(u, \mu_m)] = \frac{d(\rho u)}{dx} \delta\left(\mu_m - \frac{m_i u^2}{2B}\right)$$

The differentials can be equivalently parameterized in terms of $u$ along $x$ (where $d/dx \equiv du/dx \times d/du$, so a factor $du/dx$ on both sides of the equation cancels). This leads to the integral form given by Galeev et al., 1985:

$$\rho_i u f(u, \mu_m) = \int_{\infty}^{u} \frac{d(\rho u)}{du} \delta\left(\mu_m - \frac{m_i u^2}{2B}\right) du$$

(4.39)

The constant of integration in this equation is zero since there are no implanted ions at infinity ($\rho_i, \infty = 0$). Alternatively, the differentials can be written in terms of $\mu_m$, which allows easy integration using the delta-function and provides an expression for the distribution function of the new ions which is of the form:

$$f(u, \mu_m) = \frac{1}{\rho_i u} \frac{d}{d\mu_m} \left[ \rho_i \left( \frac{2B \mu_m}{m_i} \right)^{3/2} \right]$$

(4.40)

The solar wind proton contribution to the plasma kinetic pressure is negligible compared to the pressure of even a few heavy implanted ions. Thus the pressure may be derived from the cometary ion distribution function [Galeev et al., 1985]:

$$p_\perp \approx \rho_\perp i = \int_{\mu_m B_{\infty}}^{\mu_m B_{\infty}} \rho_i f(u, \mu_m) \left( \frac{\mu_m B}{m_i} \right) d\mu_m$$

(4.41)
The variation in the magnetic pressure $B^2/2\mu_0$ may be neglected. Replacing (4.40) in (4.41),

$$p_\perp = \int_{m_i u^2/2B}^{m_i u^2_{\infty}/2B_{\infty}} \left( \frac{\mu_m B}{2m_i} \right)^{\frac{1}{2}} \frac{d}{d\mu_m} \left[ \rho \left( \frac{2B\mu_m}{m_i} \right)^{\frac{1}{2}} \right] \, d\mu_m$$

(4.42)

and integrating by parts

$$p_\perp = \left[ \left( \frac{\mu_m B}{2m_i} \right)^{\frac{1}{2}} \rho \left( \frac{2B\mu_m}{m_i} \right)^{\frac{1}{2}} \right]_{m_i u^2/2B}^{m_i u^2_{\infty}/2B_{\infty}} - \int_{m_i u^2/2B}^{m_i u^2_{\infty}/2B_{\infty}} \rho \left( \frac{2B\mu_m}{m_i} \right)^{\frac{1}{2}} \frac{1}{2} \left( \frac{B}{2m_i \mu_m} \right)^{\frac{1}{2}} \, d\mu_m$$

(4.43)

leads to the following expression for $p_\perp$ (assuming $\rho u B = \text{const}$ in any flux-tube):

$$p_\perp = \frac{1}{4} \frac{\rho_{\infty} u^2_{\infty}}{u^2} - \frac{\rho u^2}{4}$$

(4.44)

Finally, substituting this into $\rho u^2 + p_\perp = \text{const}$ from equation (4.37) yields [Galeev et al., 1985]:

$$\frac{\rho u}{\rho_{\infty} u_{\infty}} = \left( \frac{4u_{\infty}}{3u} - \frac{u^2_{\infty}}{3u^2} \right)$$

(4.45)

describing the ratio of the contaminated mass-flux $\rho u$ in the vicinity of the comet to the mass-flux $\rho_{\infty} u_{\infty}$ of the undisturbed solar wind.

This equation may alternatively be derived from momentum and energy conservation equations [Biermann et al., 1967, Wallis and Ong, 1975]

$$\frac{d}{dx} \left( \rho u^2 + p \right) = 0$$

(4.46)

$$\frac{d}{dx} \left[ u \left( \frac{\rho u^2}{2} + \frac{\gamma}{\gamma - 1} p \right) \right] = 0$$

(4.47)

with the assumption of $\gamma = 2$, where $\gamma$ is the ratio of specific heats, or $\gamma = (\mathcal{N} + 2)/\mathcal{N}$ where $\mathcal{N}$ is the number of degrees of freedom. For a system in which the magnetic field confines ion velocity components orthogonal to it into a gyration, it seems reasonable to accept $\mathcal{N} = 2; \gamma = 2$. However, close to the comet, where more isotropic pickup ion distributions are developed after pitch-angle scattering, then $\mathcal{N}$ should be 3 (giving $\gamma = \frac{5}{3}$). The 'transition' from $\mathcal{N} = 2$ to $\mathcal{N} = 3$ is not clear-cut.
Equations (4.46) and (4.47) imply that

\[ \rho u^2 + p = \rho_\infty u_\infty^2 \]  

and

\[ \rho u^3 + \frac{2\gamma}{\gamma - 1} up = \rho_\infty u_\infty^3 \]  

assuming \( \rho_\infty = 0 \) since the implanted cometary ion pressure dominates. Eliminating \( p \) between equations (4.48) and (4.49) and setting \( \gamma = 2 \), the resulting expression rearranges into equation (4.45). This derivation is simple, but the \textit{Galeev et al.} equations enable the same result to be reached without explicitly assuming a value for \( \gamma \). The assumption of conservation of \( \mu_m \) made by \textit{Galeev et al.} means that the ion \( v_\perp (= u_\perp) \) must remain unchanged, in which case there can be no pitch-angle diffusion, which implies \( \gamma = 2 \).

The total ion flux at the comet is given by the solar wind component plus the cometary pick-up ion source:

\[ \rho u = \rho_{sw} u_{sw} + \rho_i u_i \]  

where we must have \( \rho_{sw} u_{sw} = \rho_\infty u_\infty \). Thus a further expression for the mass-flux ratio can be obtained dividing through by \( \rho_\infty u_\infty \):

\[ \frac{\rho u}{\rho_\infty u_\infty} = 1 + \frac{n_i u_i}{n_{sw} u_{sw}} \frac{m_i}{m_{sw}} \]  

Equation (4.45) can be rearranged to give

\[ \frac{u_\infty}{u} = 2 - \left[ 4 - 3 \times \left( \frac{\rho u}{\rho_\infty u_\infty} \right) \right]^{\frac{1}{2}} \]  

Substituting (4.51) into (4.52) and remembering \( u_i = u_{sw} = u \) is assumed at any particular observation point, an equation is obtained for the undisturbed solar wind speed far upstream from the comet,

\[ u_\infty = \left( 2 - \left[ 4 - 3 \times \left( 1 + \frac{n_i u_i}{n_{sw} u_{sw}} \frac{m_i}{m_{sw}} \right) \right]^{\frac{1}{2}} \right) \times u_{sw} \]  

in terms of the locally measured values of the solar wind speed, \( u_{sw} \), and density, \( n_{sw} \), and the calculated value of the cometary ion mass flux, \( n_i u_i \). The masses are assumed to be \( m_{sw} \sim 1.15 \text{ amu} \) for solar wind protons plus a nominal alpha particle content, and since the implanted ions are mostly of the water group, \( m_i \sim 20 \text{ amu} \). These values are not critically significant to the results which scale according to the value of the ratio \( m_i/m_{sw} \).
4.3 Data Analysis

The analysis considers Giotto JPA FIS (Fast Ion Sensor) solar wind proton data from the inbound leg upstream from the bow shock, between 05:00 and 19:00 hours spacecraft event time (SCET) on 13th March 1986. This corresponds to spacecraft distances of \( \sim 4.75 \) to \( 1.25 \times 10^6 \) km, respectively, from the nucleus. The data is displayed in Figure 4.3; \( u_x, u_y, u_z \) are the velocity components in Halley-centred Solar Ecliptic (HSE) coordinates (see Figure 4.2) and it can be seen that the flow is directed largely along the negative \( x \)-direction, from the Sun. \( u_{sw} \) is calculated as the magnitude of the combined components, and ignoring any deflection of the flow around the comet the direction is assumed to be constantly antisunward. It can be seen from the data that the solar wind speed decreases on approach to the comet as a result of the mass-loading process. This is accompanied by a density increase after a minimum at around 15:00 hrs. The decreasing trend in the density up to this point may reflect a variation in the solar wind flux itself, which will add a degree of uncertainty to the results.

The aim is to study the gross behaviour of the solar wind plasma flow during the mass-loading process. Therefore, after removing bad data spikes and interpolating gaps, the data is smoothed using a 50 point sequential averaging algorithm. The resulting speed and density profiles are displayed in Figure 4.4. These, along with the spacecraft positional information, can then be used to obtain \( u_\infty \) from equations (4.53) and (4.34). A solution is found for each data point, adding greater proportions of expected cometary ions to the increasingly mass-loaded solar wind measured along the Giotto trajectory. Thus a profile of \( u_\infty \) versus time is obtained (see Figure 4.5). The choices of \( Q \) and \( V_e \) used in the model calculation of the cometary ion flux are as yet arbitrary. In order to obtain an acceptable solution an appropriate constraint is required.

This is done by assuming that in the absence of any cometary influence, the average upstream solar wind flow velocity is constant. In order to see if this assumption is reasonable, the solar wind velocity measured by the IIS (Implanted Ion Sensor) on the inbound leg can be compared with that measured on the outbound leg, as shown in figure 4.6. (The flow velocity
Figure 4.3 - Solar wind proton bulk parameters for the period 05:00 to 19:00 SCET on 13th March 1986. The velocity components are in the HSE frame, with the incoming solar wind flow largely along negative x.
Figure 4.4: Smoothed solar wind proton speed and density profiles for the same period as in Figure 4.3. An upstream speed of $u_\infty \sim 366$ km s$^{-1}$ is overlayed.

derived from the IIS is of lower resolution than that from the FIS, however the IIS remained in operation throughout the encounter and so provides outbound data.) The two sets of measurements are very close, although there are clearly changes in the solar wind speed, both increases and decreases, occurring over periods of a few hours. The condition imposed upon $u_\infty$ is that a straight-line least-squares fit to the profile should have a zero gradient. There is no evidence of a long-term, systematic change in the velocity that would suggest a different gradient should be taken. However, this assumption is a source of uncertainty in the analysis.

Different gradients can be obtained by adjusting the model parameters, $Q$ and $V_e$, as these control the amount of cometary ion flux implanted into the flow. The effect of these variations can be seen from examples in Figure 4.7. Essentially, $V_e$ changes the scale length, while $Q$ changes the magnitude of the comet's effect. The assumed ionization rates are held fixed because they only enter the calculation in combination with the expansion velocity so there is no way of determining the two parameters separately; $V_e$ is allowed to vary.

Taking initially a fixed value of the expansion velocity, $V_e$, a range of
Figure 4.5: Profiles of parameters modelled along the Giotto trajectory using the data in Figure 4.4.
Figure 4.6 - Comparison of the inbound and outbound profiles of the solar wind speed measured by the IIS.
Figure 4.7: Effects of differing gas emission values on the $u_\infty$ profiles inferred.
values of the neutral production rate, \( Q \), is input to the model. The gradient and mean value of the resulting \( u_\infty \) profile obtained in each case are calculated and plotted against \( Q \). An example of this for \( V_e = 1 \text{ km s}^{-1} \) is given in Figure 4.8. This method reveals the value of \( Q \) required to give a zero gradient with the particular value chosen for \( V_e \), and then the mean value of \( u_\infty \) corresponding to this solution may be found. Repeating the process for other fixed \( V_e \) values provides a set of possible fits to the model. Figures 4.9a and 4.9b represent analyses on data periods 07:00 to 19:00 SCET and 12:00 to 19:00 SCET, respectively, with the difference between them being most apparent at the bottom of the curve. Two sections of data were considered in order to determine the extent to which the solution was dependent on the length of the data set. Moreover, the shorter period lies within the region for which a spherically symmetric model is well justified according to the work of Daly and Jockers [1989] discussed in Section 2.

If \( V_e \) between 1.0 and 1.5 \text{ km s}^{-1} \) is selected, i.e. between the dashed lines in Figures 4.9a and 4.9b, this also places reasonable limits on \( Q \). In fact, \( Q \) varies little with \( V_e \) in this range. Possible solutions to the model are listed in Tables 4.1a and 4.1b for the two data sets. An additional requirement is that the value of \( u_\infty \) corresponds to the measured value of the solar wind velocity at the beginning of the interval when it should be unaffected by mass-loading. A solution of \( u_\infty \sim 366 \text{ km s}^{-1} \) is overlayed on Figure 4.4 and appears to be a reasonable match to the measured speed at earlier times.
Figure 4.8 - Illustration of an example fit to the model: a combination of $Q$ and $V_e$ values to give a 'zero slope' $u_\infty$ profile in (a), and a unique average in (b).
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Figure 4.9 - Set of possible solutions resulting from analysis on the data periods (a) 07:00-19:00 SCET, and (b) 12:00-19:00 SCET.
4. SOLAR WIND MASS-LOADING AT COMET HALLEY

Table 4.1: Halley and Solar wind Properties from Mass-Loading Model Fits

<table>
<thead>
<tr>
<th>$V_e$ (km s$^{-1}$)</th>
<th>$Q$ ($\times 10^{30}$ s$^{-1}$)</th>
<th>$u_\infty$ (km s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.03</td>
<td>365.7</td>
</tr>
<tr>
<td>1.10</td>
<td>1.00</td>
<td>366.6</td>
</tr>
<tr>
<td>1.20</td>
<td>0.98</td>
<td>367.4</td>
</tr>
<tr>
<td>1.35</td>
<td>0.97</td>
<td>368.4</td>
</tr>
<tr>
<td>1.50</td>
<td>0.97</td>
<td>369.5</td>
</tr>
</tbody>
</table>

b. Possible Solutions for 072/12:00-19:00 Data

<table>
<thead>
<tr>
<th>$V_e$ (km s$^{-1}$)</th>
<th>$Q$ ($\times 10^{30}$ s$^{-1}$)</th>
<th>$u_\infty$ (km s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.82</td>
<td>355.1</td>
</tr>
<tr>
<td>1.10</td>
<td>0.81</td>
<td>356.5</td>
</tr>
<tr>
<td>1.20</td>
<td>0.81</td>
<td>357.8</td>
</tr>
<tr>
<td>1.35</td>
<td>0.81</td>
<td>360.0</td>
</tr>
<tr>
<td>1.50</td>
<td>0.83</td>
<td>361.6</td>
</tr>
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</table>

4.4 Gas Emission Characteristics

This section considers values of $Q$ and $V_e$ derived by more direct methods. The information that follows is summarised in Table 4.2.

The Vega 1 Neutral Gas Experiment (NGE) measured the neutral density as a function of distance from the nucleus [Curtis et al., 1986]. From these profiles a value of $Q = 1 \times 10^{30}$ s$^{-1}$ was deduced, assuming $V_e = 1$ km s$^{-1}$. (It is worth noting that if $V_e$ is some factor greater than this, $Q$ is increased by the same factor.) Similarly, studies of Ram Faraday Cup (RFC) neutral gas measurements from both Vega 1 and 2 [Remizov et al., 1986, Gringauz et al., 1986a] give $Q = 1.3 \times 10^{30}$ s$^{-1}$ for $V_e = 1$ km s$^{-1}$. Results from the Plasma
### TABLE 4.2: Comet Halley Gas Emission Properties

<table>
<thead>
<tr>
<th>Source</th>
<th>Date</th>
<th>km from Comet</th>
<th>$V_e$ (km s$^{-1}$)</th>
<th>$Q \times 10^{30}$ s$^{-1}$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Giotto NMS</td>
<td>13-14.3.86</td>
<td>$\lesssim 3 \times 10^4$</td>
<td>0.9 / 1.1</td>
<td>0.69</td>
<td><em>Krankowsky et al., Lämmertzahl et al.</em></td>
</tr>
<tr>
<td>Vega 1 IRS</td>
<td>6.3.86</td>
<td>$4 \times 10^4, 2 \times 10^5$</td>
<td>1.1</td>
<td></td>
<td><em>Krasnopolsky and Tkachuk</em></td>
</tr>
<tr>
<td>Vega 1 NGE</td>
<td>6.3.86</td>
<td>$10^5$</td>
<td></td>
<td>1</td>
<td><em>Curtis et al.</em></td>
</tr>
<tr>
<td>Vega 1 PID</td>
<td>6.3.86</td>
<td>$\leq 1.5 \times 10^5$</td>
<td>~1 to 4</td>
<td></td>
<td><em>Remizov et al.</em></td>
</tr>
<tr>
<td>Vega 1, 2 RFC</td>
<td>6.9.3.86</td>
<td>$10^4 - 10^6$</td>
<td></td>
<td>1.3</td>
<td><em>Remizov et al., Gringauz et al.</em></td>
</tr>
<tr>
<td>Vega 2 TKS</td>
<td>9.3.86</td>
<td>~3,000</td>
<td></td>
<td>1.4</td>
<td><em>Krasnopolsky and Tkachuk</em></td>
</tr>
<tr>
<td>IUE</td>
<td>9.3.86</td>
<td>~3,000</td>
<td></td>
<td>1.1</td>
<td><em>Krasnopolsky and Tkachuk</em></td>
</tr>
<tr>
<td>PVOUVS</td>
<td>7.3.86</td>
<td>$&lt; 10^5$</td>
<td></td>
<td>1.4</td>
<td><em>Stewart</em></td>
</tr>
<tr>
<td>KAO Pre-Perihelion</td>
<td>24.12.85</td>
<td>$&lt; 10^5$</td>
<td>0.9</td>
<td>0.1</td>
<td><em>Larson et al., Weaver et al.</em></td>
</tr>
<tr>
<td>KAO Post-Perihelion</td>
<td>22.3.86</td>
<td>$&lt; 10^5$</td>
<td>1.4</td>
<td>1</td>
<td><em>Larson et al., Weaver et al.</em></td>
</tr>
</tbody>
</table>

### TABLE 4.3: A Comparison of Models

<table>
<thead>
<tr>
<th>IMS Model (Neugebauer et al., 1989)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neugebauer et al. values $Q_H \times 10^{30}$ s$^{-1}$</td>
<td>0.97</td>
<td>1.10</td>
<td>1.39</td>
<td>1.56</td>
</tr>
<tr>
<td>JPA equivalent values $Q_H \times 10^{30}$ s$^{-1}$</td>
<td>1.06</td>
<td>1.20</td>
<td>1.51</td>
<td>1.70</td>
</tr>
<tr>
<td>Ratio $Q_H/Q$, $V_e = 1.0$ km s$^{-1}$</td>
<td>1.02</td>
<td>1.15</td>
<td>1.45</td>
<td>1.63</td>
</tr>
<tr>
<td>$V_e = 1.5$ km s$^{-1}$</td>
<td>1.09</td>
<td>1.24</td>
<td>1.55</td>
<td>1.75</td>
</tr>
</tbody>
</table>
Figure 4.10: The variability of the comet Halley OH production rate recorded by ground-based observatories [Schleicher et al., 1986].

Impact Detector (PID), which is part of the same instrument package, lead to an estimate of \( Q \approx 1 \) to \( 4 \times 10^{30} \text{ s}^{-1} \).

Krasnopolsky and Tkachuk [1986] calculate theoretical emission line growth curves (relating density of a species to intensity of emission) for optically thick conditions. They apply the work to spacecraft observations of \( \text{H}_2\text{O} \) and OH band intensities to obtain estimates of \( Q \) for the water group. Vega 1 infrared spectrometer results give \( Q_{\text{H}_2\text{O}} = 1.1 \times 10^{30} \text{ s}^{-1} \), the Vega 2 three-channel spectrometer (TKS) OH band gives \( Q_{\text{OH}} = 1.4 \times 10^{30} \text{ s}^{-1} \) and from International Ultraviolet Explorer (IUE) observations, \( Q_{\text{OH}} = 1.1 \times 10^{30} \text{ s}^{-1} \). Results from the Pioneer Venus Orbiter Ultraviolet Spectrometer (PVOUVS) between December 1985 and early March 1986 have been analysed by Stewart [1987]. The inferred water production rate on 7th March 1986 is \( Q_{\text{H}_2\text{O}} = 1.4 \times 10^{30} \text{ s}^{-1} \).

There are also ground-based observations recording the activity of comet Halley. For example, Schleicher et al. [1986] consider data from Cerro Tololo Inter-American Observatory (CTIO), Mauna Kea Observatory (MKO) and
Perth observatory, revealing daily variations in the emission of gas from the nucleus (Figure 4.10). Larson et al. [1986] and Weaver et al. [1986] study infrared spectra obtained by NASA’s Kuiper Airborne Observatory (KAO). From line widths and intensities, values for \( V_e \) and \( Q \) respectively are derived. A distinct pre- to post-perihelion asymmetry in the activity of the comet is observed, as seen in Figure 4.11. Pre-perihelion spectra yield \( V_e = 0.9 \) km s\(^{-1}\) and \( Q_{H_2O} = 0.1 \times 10^{30} \) s\(^{-1}\) for a comet heliocentric distance of \( \approx 1 \) AU, while for post-perihelion, \( V_e = 1.4 \) km s\(^{-1}\) and \( Q_{H_2O} = 1 \times 10^{30} \) s\(^{-1}\) are found for the same comet-Sun distance. (At the Giotto encounter the comet is post-perihelion, at \( \approx 0.9 \) AU.)

In situ gas composition measurements at encounter are provided by the Neutral Mass Spectrometer (NMS) on board Giotto. Krankowsky et al. [1986b] derive values of \( V_e = 0.9 \) km s\(^{-1}\) from the position of peaks in the
energy spectra and $Q_{\text{OH}} = 0.69 \times 10^{30} \text{s}^{-1}$ from density profiles and $V_e$. Also from NMS energy spectra, plots of $V_e$ versus distance from the nucleus [Lämmerzahl et al., 1986] clearly show that the neutral expansion velocity is not constant. $V_e$ increases with $r$, reaching $\sim 1.1 \text{ km s}^{-1}$ at 30,000 km from the comet. It should be remembered that because of variations in the comet’s activity, the $Q$ and $V_e$ values derived from Giotto NMS measurements are not necessarily directly applicable to the work presented in this chapter. They apply to the gas observed in the coma at the time of encounter. The implanted ions which load the solar wind outside the bow shock during the Giotto encounter will have been produced from neutrals created at the comet up to 10 days before.

The range of values in Table 4.2 are consistent with the model results in Table 4.1.

An interesting result to compare with the findings of the present analysis is the estimate of the production rate of cometary hydrogen made by Neugebauer et al. [1989] using data from the Implanted Mass Spectrometer (IMS) instrument on Giotto. They measured the profile of the cometary hydrogen density directly and fitted it with a series of models of the outflow. The value of $Q_H$ they obtain varies from $1.56 \times 10^{30} \text{s}^{-1}$ for the most complex model which assumes hydrogen atoms with three different outflow velocities, down to $0.97 \times 10^{30} \text{s}^{-1}$ for the simplest model based on a single velocity. If the source of the hydrogen is the $\text{H}_2\text{O}$, which forms 80% of the composition of the volatiles in the nucleus, then $Q_H$ should be 1.6 times the total gas production rate. Measurements of the solar wind proton density made by FIS were compared with measurements made at the same time by IMS (M. Neugebauer, private communication, 1988) in order to intercalibrate the two instruments. The comparison could be most conveniently made on data between 18:30 and 19:00 SCET on March 13th and the following ratio was obtained:

$$\text{Mean solar wind density} \frac{\text{JPA}}{\text{IMS}} = 1.09$$

Converting the IMS values of $Q_H$ to JPA equivalent values and dividing by the $Q$ values obtained in this work gives the ratios in Table 4.3. This comparison seems to favour the models 3 and 4 of Neugebauer et al. [1989]
based on multiple hydrogen outflow velocities.

A direct comparison can be made between the oxygen ion density measured by the IIS [Coates et al., 1990a] on the Giotto inbound leg, and the density inferred from the model (from equation (4.34)):

\[ n_i = \frac{Q}{u_i 4\pi L^2} \int_{x_0}^{\infty} \frac{1}{(x^2 + y_0^2)} \exp \left[-(x^2 + y_0^2)^{1/2}\right] dx \]  

(4.54)

This was evaluated with \( Q = 1 \times 10^{30} \text{s}^{-1}, V_e = 1 \text{ km s}^{-1}, \nu_{ph} = 4.2 \times 10^{-7} \text{s}^{-1}, \) a charge exchange rate of \( 4.76 \times 10^{-7} \text{s}^{-1} \) for an average solar wind flux of \( n_{sw} u_{sw} = 2.266 \times 10^{18} \text{ km}^{-2} \text{s}^{-1}, \) and local values of \( u_i = u_{sw}. \) These results are plotted against distance from the nucleus in Figure 4.12 and compare well with the direct measurements. The best-fit slope to the log-log plot of the IIS values gives an implanted ion density fall-off according to \( r^{-3.7} \) with distance, \( r, \) from the nucleus along the Giotto path [Coates et al., 1990a].

4.5 Application to Shock Profile

The bow shock at a comet occurs in order to accommodate the mass-loading of the ion flow rather than as a result of deflection of the flow around an object (as is the case at a planet with its own intrinsic magnetic field). As increasing numbers of cometary ions are added into the flow, equation (4.52) can be solved up to the point where the mass-flux ratio equals 4/3 [Biermann et al., 1967], after which a different solution must be used [Galeev et al., 1985]. This reversal point may be taken as an innermost limit to the position of the cometary bow shock. On combining equations (4.34) and (4.51) for a mass-flux ratio of 4/3, the solution is obtained from

\[ \int_{x_0}^{\infty} \frac{1}{(x^2 + y_0^2)} \exp \left[-(x^2 + y_0^2)^{1/2}\right] dx = \frac{4\pi L^2}{3Q} \frac{\rho_{sw} u_{sw}}{m_i} \]  

(4.55)

Using model-fit values of \( Q = 1 \times 10^{30} \text{s}^{-1}, L = 1.12 \times 10^6 \text{ km}, \) and a time-averaged solar wind flux of \( n_{sw} u_{sw} = 2.266 \times 10^{18} \text{ km}^{-2} \text{s}^{-1}, \) the integral is evaluated numerically at positions on a series of solar wind flow lines approaching the comet to search for solutions. The set of points obtained in this way is plotted in Figure 4.13, producing an approximate shock profile. More specifically, bow shock inner limits on the Spacecraft paths may be
Figure 4.12: Comparison of the modelled implanted heavy ion density with the measured values obtained from the IIS [Coates et al., 1990a].
Figure 4.13: Shock profile inferred from the model, drawn to scale in the HSE frame. The trajectory of Giotto and the observed shock crossings are also included.

estimated in a similar way, and compared with crossings recorded by Giotto [Coates et al., 1987a; Johnstone et al., 1986b; Neubauer et al., 1986b], Vega 1 and 2 [Klimov et al., 1986; Galeev et al., 1986a; Galeev et al., 1986b] and Suisei [Mukai et al., 1986]. The variation in the cometary gas emission and solar wind conditions between the fly-by dates will be a source of error in estimates of Vega 1, 2, and Suisei events. The results are displayed in Table 4.4. (Note that some observed values referenced here are approximates taken from plots.) The Giotto trajectory is included to scale in Figure 4.13.

In the 1D modelling presented here, the estimated shock is a result of the mass-loading only. The shape of the modelled bow shock is unlikely to be as flared as the actual profile, since the deflection of the flow around the comet has been ignored (which implies an under-estimation of the proportion of ions on the flank to those arriving head-on). However, the mass-loading process itself will in reality deflect the solar wind flow to some degree. The discrepancy between the assumed and actual flow direction is probably small enough to be neglected in the preshock region and indeed the data in Figure 4.3 confirms that the flow remains predominantly anti-sunward, at least
### TABLE 4.4: Observed and Modelled Bow Shock Crossings

<table>
<thead>
<tr>
<th>Mission</th>
<th>OBSERVED Time &amp; Distance ($\times 10^6$ km)</th>
<th>MODELLED Time &amp; Distance ($\times 10^6$ km)</th>
<th>OBSERVED Time &amp; Distance ($\times 10^6$ km)</th>
<th>MODELLED Time &amp; Distance ($\times 10^6$ km)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>INBOUND SHOCKS</td>
<td>OUTBOUND SHOCKS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GIOTTO</td>
<td>13.3.86/19:20-19:31</td>
<td>20:10</td>
<td>14.3.86/03:16</td>
<td>02:50</td>
</tr>
<tr>
<td></td>
<td>1.16 - 1.12</td>
<td>0.95</td>
<td>0.76</td>
<td>0.70</td>
</tr>
<tr>
<td>VEGA 1</td>
<td>6.3.86/03:46</td>
<td>03:57</td>
<td>6.3.86/09:30</td>
<td>09:48</td>
</tr>
<tr>
<td></td>
<td>1.01</td>
<td>1.00</td>
<td>0.6</td>
<td>0.67</td>
</tr>
<tr>
<td>VEGA 2</td>
<td>9.3.86/02:40-03:20</td>
<td>03:38</td>
<td>No Data</td>
<td>09:43</td>
</tr>
<tr>
<td></td>
<td>1.3-1.1</td>
<td>1.02</td>
<td></td>
<td>0.67</td>
</tr>
<tr>
<td>SUISEI</td>
<td>No Data</td>
<td>09:57</td>
<td>8.3.86/14:43-14:49</td>
<td>15:25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.82</td>
<td>~0.45</td>
<td>0.65</td>
</tr>
</tbody>
</table>
4. SOLAR WIND MASS-LOADING AT COMET HALLEY

in to \( \sim 1.25 \times 10^6 \) km from the nucleus on the flank.

For the sub-solar standoff (ie the shock position on the comet-sun line) the model innermost estimate is \( 5.5 \times 10^5 \) km from the nucleus. The value obtained by Rème et al. [1987] is \( 4 \times 10^5 \) km calculated from the Giotto inbound crossing assuming a paraboloidal shock surface with flaring factor, \( \alpha = 2 \). More recently, Coates et al. [1990b] fitted a paraboloid to both Giotto shock crossings to obtain \( \alpha = 1.44 \) and a sub-solar standoff of \( 5.98 \times 10^5 \) km (so that this fit lies outside the singularity estimate from the mass-loading model, as it should). An equation for the paraboloid in HSE coordinates is [Fuselier et al., 1987]:

\[
x = X_{\text{SUB}} + K(y^2 + z^2)
\]

(4.56)

where \( X_{\text{SUB}} \) is the sub-solar standoff distance and \( 1/(\alpha X_{\text{SUB}})^{1/2} = \alpha \). The flaring factor is the ratio of the shock standoff at \( 90^\circ \) (\( x = 0 \)) to that at \( 0^\circ \), ie \( X_{\text{SUB}} \) [Wallis, 1986]. A ratio of \( \alpha \sim 1.5 \) is given by the modelled profile in Figure 4.13, although it is not a paraboloid. Nevertheless, since the bow shock shape has never been measured directly, the 'standard' paraboloid form cannot be verified.

4.6 Summary

- The solar wind velocity profile on the approach to comet Halley has been fitted to a model of the mass-loading, giving a range of values for the pair of parameters \( Q, V_e \) which are consistent with values derived in independent ways. The model values of \( Q, V_e \) range from \( 1.03 \times 10^{30} \) s\(^{-1}, 1.0 \) km s\(^{-1}\) to \( 0.97 \times 10^{30} \) s\(^{-1}, 1.5 \) km s\(^{-1}\) respectively, for an assumed photoionization rate of \( 4.2 \times 10^{-7} \) s\(^{-1}\) and a cross-section of \( 2.1 \times 10^{-15} \) cm\(^2\) for charge exchange with the solar wind. Note that since \( V_e \) is only obtained in the ratio \( V_e/\nu \), any error in the value assumed for the ionization rate will give the same proportional error in \( V_e \). However, the combination of values \( Q, V_e, \) and \( \nu \) is entirely consistent with those obtained by other means.

- From the model, the calculation of an innermost bow shock profile is based on the self reversal point in the flow solution. This places the
shock on the comet-sun line at a distance of more than \(5.5 \times 10^5\) km from the nucleus.

- The model describes the 3-dimensional mass-loading near the comet, but outside the bow shock. The cometary parameters derived are useful in the analysis of processes in this region, such as the generation of turbulence by the implantation of ions (which is the subject of Chapters 5 and 6).
Chapter 5

Parallel Pick-up and Wave Energy

5.1 Parallel Pick-up Mechanism

Immediately after ionization, the velocity of the newly born cometary ions is equal to the expansion velocity they had as neutrals, \( V_e \sim 1 \text{ km s}^{-1} \) for most species. (Note that if the outflow is spherically-symmetric with no thermal spread, then in velocity-space the neutrals are evenly distributed on a shell, radius \( V_e \), centred on the comet’s velocity.) In the solar-wind frame the expansion velocity is relatively small (c.f. \( u_\infty \sim 366 \text{ km s}^{-1} \) for the Giotto encounter) and may be neglected. Therefore the ions are assumed to be injected with \(|v_{inj}| = u\), the solar wind speed at the local point of pick-up, as illustrated in Figure 5.1. Their direction is antiparallel to \( u \), so that initially \( \cos \theta = -\cos \alpha \) where \( \alpha \) is the angle between \( u \) and the interplanetary magnetic field, \( B \), and \( \theta \) is the ‘pitch-angle’ of the cometary ions.

The component of the newly born cometary ion velocity perpendicular to \( B \) is very rapidly picked up into a gyration about the B-field lines (see Chapter 4). However, there is no simple mechanism by which the ions can be accelerated parallel to the field and hence they are not completely accommodated into the solar wind flow immediately. Their \( v_\parallel \) component remains relative to \( u \), so the pick-up ion population streams against the bulk flow. This ring beam is unstable to the generation of turbulent waves [eg. Wu and Davidson, 1972; Sagdeev et al., 1986; Galeev, 1986b], and the implanted ions
Figure 5.1: Real-space and velocity-space representations of the cometary ion pick-up geometry in (a) the comet reference frame and (b) the solar wind frame neglecting neutral expansion velocity, $V_e$. 
are scattered fairly rapidly in pitch angle into an isotropic "shell" distribution in velocity-space (see Figure 5.1).

Note that the ring is initially broad to some degree due to variations in $u_{sw}$ and $B$ during the pick-up process [Lee, 1987]. The initial $|v_{inj}|$ of the cometary ion is equal to the local $u_{sw}$, i.e., the value at the point of injection. Continual pick-up in a decelerating plasma flow means that the ions are introduced at different speeds, which fills in and adds a broadening to the shell. Also during solar wind slow-down, adiabatic compression results in acceleration of the ions. These effects occur in addition to any energy diffusion due resonant wave-particle interactions (as discussed below, section 5.3).

5.2 Alfvén Waves

For electromagnetic waves, each fluctuating component of $E$ and $B$ must satisfy the wave equation [e.g. Boyd and Sanderson, 1969; Coulson and Boyd, 1979]:

$$\nabla \psi = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2}$$
(5.1)

The solution is a waveform such as

$$E = E_0 e^{i(k \cdot r - \omega t)} = E_0 \cos(k \cdot r - \omega t) + E_0 i\sin(k \cdot r - \omega t)$$
(5.2)

where $\omega$ is the angular frequency of the oscillation in radians per second and $k$ is the propagation vector, or wavenumber. $E_0$ is a constant amplitude.

The phase, $\delta$, (determined by the choice of the origin of $x$ and $t$) is sometimes incorporated into $E_0$ by allowing a complex amplitude [Chen, 1984]:

$$E = E_0 e^{i(k \cdot r - \omega t + \delta)} = E_0 e^{i\delta} e^{i(k \cdot r - \omega t)} = E_{c} e^{i(k \cdot r - \omega t)}$$
(5.3)

5.2.1 Phase and Group Velocity

The phase velocity (velocity at which a point of constant phase on a wave moves) is:

$$\frac{dx}{dt} = \frac{\omega}{k} = v_{ph}$$
(5.4)
Sometimes this may exceed the speed of light in a medium, but no information is carried unless there is some form of modulation, which travels at the group velocity, given by:
\[ v_g = \frac{d\omega}{dk} \] (5.5)

Consider a wave packet from a non-monochromatic source with central frequency, \( \omega_0 \), and central wave number, \( k_0 \). In a medium with magnetic permeability, \( \mu \), and dielectric function, \( \epsilon \), the velocity is given by \( v(k) = 1/(\mu \epsilon)^{1/2} \) for each wave-number component. If \( (\mu \epsilon)^{1/2} \) in the medium is constant, all modes propagate with the same phase velocity \( \omega_0/k_0 \). However, in a dispersive medium, \( \epsilon = \epsilon(\omega) \) and the shape of the wave packet changes as it travels. The dispersion relation, \( \omega(k) \), describes relationship between propagation vector and frequency. For the non-dispersive case, the phase velocity, \( v_{ph} \), does not vary with \( k \), and hence the group velocity \( v_g \equiv v_{ph} \).

The wave frequency \( \omega = \omega(k) \) may be split into real and imaginary parts, \( \omega = \omega_r + i\omega_i \). The wave representation is
\[ X_0 e^{i(kx - \omega_r t - i\omega_i t)} = X_0 e^{i\omega t} e^{i(kx - \omega t)} \] (5.6)

It is clear from the \( e^{i\omega t} \) term that if \( \omega_i > 0 \) the oscillation is unstable and wave growth occurs with increasing \( t \). Thus a growth rate \( = \omega_i \) can be obtained from the imaginary part of the dispersion relation.

### 5.2.2 Alfvén and Magnetosonic Modes

The propagation of hydromagnetic waves in a compressible, perfectly conducting plasma in the presence of a magnetic field may be studied by considering a suitable set of magneto-hydrodynamic (MHD) fluid equations. These will be solved by the method of linearization, by separating oscillating parameters into “equilibrium” and “perturbation” parts to first order, eg:
\[ B = B_0 + B_1 \] (5.7)
\[ u = u_0 + u_1 \quad \rho = \rho_0 + \rho_1 \] (5.8)

for the magnetic field, solar wind flow velocity and mass density, respectively.

Maxwell’s equations in free space are [eg. Chen, 1984]:
\[ \epsilon_0 \nabla \cdot \mathbf{E} = \sigma \] (5.9)
\[ \nabla \times \mathbf{E} = -\dot{\mathbf{B}} \quad (5.10) \]
\[ \nabla \cdot \mathbf{B} = 0 \quad (5.11) \]
\[ \nabla \times \mathbf{B} = \mu_0 (\mathbf{j} + \varepsilon_0 \dot{\mathbf{E}}) \quad (5.12) \]

where \( \sigma \) here is the charge density, \( q_n \), and \( \mathbf{j} \) is the current density, \( q_n \mathbf{u} \).

The momentum equation
\[ \rho \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + q_n \mathbf{u} \times \mathbf{B} \quad (5.13) \]

may be combined with Maxwell's 4th equation, replacing \( \mathbf{B} \), \( \mathbf{u} \) and \( \rho \) according to (5.7) and (5.8) above and neglecting second order terms to give [Bittencourt, 1986]
\[ \rho_0 \frac{\partial \mathbf{u}_1}{\partial t} + \nabla p + \mathbf{B}_0 \times \frac{\nabla \times \mathbf{B}_1}{\mu_0} = 0 \quad (5.14) \]

since \( u_0 \) and \( B_0 \) are constant. Similarly, the equation of continuity (mass conservation) becomes:
\[ \frac{\partial \rho_1}{\partial t} + \rho_0 (\nabla \cdot \mathbf{u}) = 0 \quad (5.15) \]

Maxwell's second equation, with \( \mathbf{E}_1 = -\mathbf{u}_1 \times \mathbf{B}_0 \) (for an infinitely conductive plasma) gives:
\[ \frac{\partial \mathbf{B}_1}{\partial t} = \nabla \times (\mathbf{u}_1 \times \mathbf{B}_0) \quad (5.16) \]

The polytropic equation of state is \( p \rho^{-\gamma} = \text{constant} \) (where \( \gamma \) is the ratio of specific heats). This may be differentiated to give \( \nabla p = \text{const.} \gamma \rho^{-1} \nabla \rho = (\gamma p/\rho) \nabla \rho = c_s^2 \nabla \rho \) where the sound speed \( c_s = (\gamma p/\rho)^{\frac{1}{2}} \) has been identified. Thus \( \nabla p \) may be replaced in terms of \( \rho \) and \( c_s \) in equation (5.14).

Since the perturbation quantities vary as \( e^{i(k \cdot r - \omega t)} \) then
\[ \frac{\partial}{\partial t} \equiv -i\omega ; \quad \frac{\partial^2}{\partial t^2} \equiv -\omega^2 \quad (5.17) \]
\[ \nabla \equiv ik ; \quad \nabla^2 \equiv -k^2 \quad (5.18) \]

The set of linearized fluid equations then becomes:
\[ -\omega \rho_0 \mathbf{u}_1 + c_s^2 k \rho_1 + \frac{k(\mathbf{B}_0 \cdot \mathbf{B}_1)}{\mu_0} - \frac{\mathbf{B}_1(\mathbf{B}_0 \cdot \mathbf{k})}{\mu_0} = 0 \quad (5.19) \]
\[ -\omega \rho_1 + \rho_0 k \mathbf{u}_1 = 0 \quad (5.20) \]
\[-\omega \mathbf{B}_1 = \mathbf{k} \times (\mathbf{u}_1 \times \mathbf{B}_0) \]  

(5.21)

From equations (5.20) and (5.21):

\[\rho_1 = \frac{\mathbf{k} \cdot \mathbf{u}_1}{\omega} \]  

(5.22)

\[\mathbf{B}_1 = \frac{\mathbf{B}_0 (\mathbf{k} \cdot \mathbf{u}_1)}{\omega} - \frac{\mathbf{u}_1 (\mathbf{k} \cdot \mathbf{B}_0)}{\omega} \]  

(5.23)

which may be substituted into (5.19) to give:

\[-\omega \rho_0 \mathbf{u}_1 + c_s^2 \mathbf{k} \frac{(\mathbf{k} \cdot \mathbf{u}_1)}{\omega} - \frac{\rho_0}{\omega \mu_0} + \frac{\mathbf{B}_0^2 (\mathbf{k} \cdot \mathbf{u}_1)}{\omega \mu_0} - \frac{(\mathbf{B}_0 \cdot \mathbf{u}_1)(\mathbf{k} \cdot \mathbf{B}_0)}{\omega \mu_0} \]

\[-\frac{(\mathbf{k} \cdot \mathbf{u}_1)(\mathbf{B}_0 \cdot \mathbf{k})}{\omega \mu_0} \mathbf{B}_0 + \frac{(\mathbf{k} \cdot \mathbf{B}_0)^2}{\omega \mu_0} \mathbf{u}_1 = 0 \]  

(5.24)

Following Bittencourt [1986], setting

\[\frac{\mathbf{B}_0}{(\mu_0 \rho_0)^{1/2}} = \mathbf{V}_A \]  

(5.25)

and rearranging, the equation becomes

\[-\omega^2 \mathbf{u}_1 + (c_s^2 + \mathbf{V}_A^2)(\mathbf{k} \cdot \mathbf{u}_1) \mathbf{k} + (\mathbf{k} \cdot \mathbf{V}_A) [(\mathbf{k} \cdot \mathbf{V}_A) \mathbf{u}_1 - (\mathbf{V}_A \cdot \mathbf{u}_1) \mathbf{k} - (\mathbf{k} \cdot \mathbf{u}_1) \mathbf{V}_A] = 0 \]  

(5.26)

Taking the dot-product of (5.26) with the cross-product \( \mathbf{k} \times \mathbf{V}_A \) [Cairns, 1985] picks out the 1st and 3rd terms, perpendicular to both \( \mathbf{k} \) and \( \mathbf{V}_A \),

\[-\omega^2 + (\mathbf{k} \cdot \mathbf{V}_A)^2 = 0 \]  

(5.27)

from which the Alfvén phase velocity is:

\[\frac{\omega}{k} = v_{ph} = \mathbf{V}_A = \frac{B_0}{(\mu_0 \rho_0)^{1/2}} \]  

(5.28)

Note that from equation (5.28), the group velocity is \( v_g = d\omega/dk = \omega/k = v_{ph} \), the phase velocity. In other words, since the phase velocity is independent of frequency, Alfvén waves are dispersionless. It is also worth remarking here that Alfvén waves are ion electromagnetic waves, and have frequencies below the ion gyrofrequency; \( \omega^2 \ll \Omega_i^2 \) [Chen, 1984]. (Light is an electron electromagnetic wave).
The dispersion relation for magnetosonic waves may be found as follows. The dot-product of (5.26) with \( k \) is

\[
(u_1.k) \left[(c_s^2 + V_A^2) k^2 - \omega^2\right] - (k.V_A)(V_A.u_1) k^2 = 0
\]

(5.29)

and the dot-product with \( V_A \):

\[-\omega^2 (u_1.V_A) + c_s^2 (k.u_1)(k.V_A) = 0\]

(5.30)

Combining these two equations by substituting for \((u_1.V_A)\) from (5.30) into (5.29) gives:

\[
(c_s^2 + V_A^2) k^2 - \omega^2 - (k.V_A)^2 \frac{c_s^2 k^2}{\omega^2} = 0
\]

(5.31)

Setting \( k.V_A = kV_A \cos \theta_k \) where \( \theta_k \) is the angle between the propagation vector and \( V_A \) (or \( B_0 \) according to (5.25)) then provides the dispersion relation [eg. Bittencourt, 1986; Cairns, 1985]:

\[
\frac{\omega^4}{k^4} - \frac{\omega^2}{k^2} (c_s^2 + V_A^2) + V_A^2 c_s^2 \cos^2 \theta_k = 0
\]

(5.32)

This is a quadratic in \((\omega^2/k^2)\), for which the solutions are

\[
\frac{\omega^2}{k^2} = \frac{(c_s^2 + V_A^2)}{2} \pm \left[(c_s^2 + V_A^2)^2 - 4 V_A^2 c_s^2 \cos^2 \theta_k\right]^{\frac{1}{2}}
\]

(5.33)

The +, - signs give the fast and slow MHD wave modes, respectively.

Consider the two extreme cases of \( \theta_k \). For propagation perpendicular to the magnetic field, \( \theta_k = 90^\circ \), \( \cos \theta_k = 0 \) and the fast and slow mode solutions are

\[
\left(\frac{\omega}{k}\right)^2 = 0
\]

(5.34)

\[
\left(\frac{\omega}{k}\right)^2 = (c_s^2 + V_A^2)
\]

(5.35)

Equation (5.35) is the magnetosonic wave (or compressional Alfvén wave). For parallel propagation, \( \theta_k = 0 \), \( \cos \theta_k = 1 \), the term within the square root in equation (5.33) becomes \( c_s^4 + V_A^4 - 2 c_s^2 V_A^2 \) which is \((c_s^2 - V_A^2)^2\) or \((V_A^2 - c_s^2)^2\) and the solution depends on whether \( V_A = B_0/(\mu_0 \rho)^{\frac{1}{2}} \) is greater than \( c_s = (\gamma p/\rho)^{\frac{1}{2}} \), or vice versa. In a particular plasma medium, this will depend on the strength of \( B_0 \) (in \( V_A \)) and the pressure (ie the temperature and ion density, where \( p = mn < v_{th}^2 >= n k_B T \)). The solutions are

\[
\left(\frac{\omega}{k}\right)^2 = c_s^2
\]

(5.36)
Figure 5.2 - Phase velocity versus propagation angle for the fast and slow MHD waves and pure Alfvén waves in cases (a) $V_A > c_s$ and (b) $c_s > V_A$. (From Bittencourt [1986].)
the longitudinal (compressional) sound wave, and

\[
\left( \frac{\omega}{k} \right)^2 = V_A^2
\]  

(5.37)

the Alfvén wave.

The complete set of solutions for phase velocity as a function of \( \theta_k \) are sketched in Figure 5.2 for the two cases; \( V_A > c_s \) and \( c_s > V_A \).

### 5.3 Velocity Diffusion and Wave Growth

The solar wind itself carries a spectrum of waves. Then, when the solar wind encounters the pick-up ions, the anisotropy of the ring-beam distribution in the solar wind frame is a source of Alfvén wave excitation [eg. Wu and Davidson, 1972; Galeev, 1986b; Sagdeev et al., 1986]. The cyclotron resonance condition for the strong wave-particle interaction is [Lyons and Williams, 1984]:

\[
\omega - k_v v_\| + n\Omega_i = 0
\]  

(5.38)

where the ion gyrofrequency is

\[
\Omega_i = \frac{qB_0}{m_i}
\]  

(5.39)

independent of ion velocity, and \( k_v \) is the component of the wave vector \( k \) along the magnetic field direction. For parallel-propagating waves then \( k = k\hat{B} \). The principle cyclotron harmonic resonance \( (n = \pm 1) \) occurs where the wave frequency in the ion reference frame is equal to the ion gyrofrequency:

\[
\omega - k v_\| = \pm \Omega_i
\]  

(5.40)

where an ion travelling at velocity \( v_\| \) in the same direction as the wave 'sees' the wave at a reduced frequency of \( \omega - k v_\| \).

For Alfvén waves propagating in any one direction only, the energy of the particles in the frame moving with the waves is conserved since the wave electric field vanishes in this frame [Terasawa, 1989]. The particles are constrained to move on a circle centred at \( \omega/k = V_A \) where the energy conservation equation for the system in the solar wind frame is [Lyons and
5. PARALLEL PICK-UP AND WAVE ENERGY

Figure 5.3: Single-wave characteristics (centred on upstream and downstream Alfvén phase velocity in the solar wind frame) along which an ion injected at $v_{inj}$ may initially diffuse in velocity-space in order to release energy to the waves. (Adapted from Terasawa [1989].)

Williams, 1984; Terasawa, 1989]:

$$\left( v_\parallel - \frac{\omega}{k} \right)^2 + v_\perp^2 = \text{const.} \quad (5.41)$$

(Surfaces in velocity-space such as these are referred to as 'single-wave characteristics'.) Thus for a unidirectional wave field, only pitch-angle scattering of the particles is possible; the overall wave-frame energy of the ions in the distribution remains unchanged. For energy diffusion to occur, it is essential to have waves propagating both parallel and anti-parallel to B. Then ions moving successively on circles centred at $+V_A$ and $-V_A$ may be accelerated or decelerated as illustrated in Figure 5.3. The rate of pitch-angle scattering depends on the intensity of the waves, but the energy diffusion will also depend on the proportions of waves travelling in each direction and occurs in conjunction with the pitch-angle diffusion.

The anisotropy of the implanted ion distribution function $F$ in the solar wind frame means that it is unstable; there is 'free energy' available, equal to the difference in the total energy density associated with the ring and shell configurations. In any diffusion process (spatial or velocity diffusion) particles diffuse from places of high density to where the density is low,
flattening the gradients in the distribution of particles. In velocity-space the ions diffuse along the single-wave characteristics towards lower values of \( F \). If the net movement of ions at a particular point in \( v \)-space is towards a higher energy position, this occurs at the expense of the waves which will be damped in the corresponding resonant region of the spectrum [Lyons and Williams, 1984]. Where the ions on average lose energy, that region of their distribution causes wave growth. As the initial pick-up ion ring-beam distribution evolves towards isotropy in the solar wind frame, the diffusion caused by velocity-space gradients reduces and the associated wave growth rate subsides [Lee, 1982; Cairns, 1985]. The resulting isotropic shell is in equilibrium under the approximation of \( V_A << u_{sw} \) [Lee, 1987], since the shells will in fact be centred on the Alfvén speeds, not on \( u_{sw} \).

Waves generated during the pick-up process propagate predominantly anti-parallel to \( B \), in the direction away from the comet, because if ‘isotropy’ is to be attained, the majority of the new ions at their injection point need to move on the \(-V_A\) single-wave characteristic in order to lose energy to the waves, as is clear from Figure 5.3. The total power level in the vicinity of a comet is the sum of the ambient solar wind level and that of turbulence generated during the ion pick-up process, \( P = P_\infty + P_{\text{implant}} \) [Gombosi, 1988]. The ions may interact with both the ambient solar wind spectrum and the self-generated waves.

### 5.4 Theoretical Calculation of Free Energy

As discussed in the previous section, the cometary ion implantation process may generate Alfvénic turbulence in the solar wind, since the initial distribution of these pick-up ions is unstable to wave growth. The greater proportion of the waves produced will propagate upstream along magnetic field lines. The aim of this section is to calculate theoretically the free energy available for upstream and downstream-propagating waves separately. In order to do this, a pick-up ring distribution is assumed to evolve towards a bispherical shell distribution, as described below.

Newly ionized cometary particles are picked up at a velocity \( v_{\text{inj}} \) in the solar wind frame initially into a ring distribution about the magnetic field
5. PARALLEL PICK-UP AND WAVE ENERGY

lines, as described in Section 5.1. The ions may diffuse along 'single-wave characteristics' in velocity space as a result of wave-particle interactions. An ion releases energy to the waves by pitch-angle scattering to a lower-energy position on such velocity-space 'shells' (which are centred on the upstream and downstream wave velocities, ±\(V_A\), in the solar wind frame, see Section 5.3). Those ions that spread around the upstream-centred shell from their point of pick-up interact with upstream-propagating waves, and similarly for the downstream case. Although some ions may be accelerated or decelerated considerably, the low energy shell portions shown in Figure 5.4 are the most likely to become populated \([\text{Galeev and Sagdeev, 1988}]\). Then it is assumed that the combined lower-energy shell-portions eventually achieve an evenly spread density of particles. The geometry of this final configuration is determined by the position of the initial ring. The difference in energy density of the ions in their initial and final distributions is the total free energy available to the waves. This is split between upstream and downstream waves depending on the proportion of ions that evolve onto each shell portion.

Two velocity-space distribution functions are required to describe the uniform density shell-portions of the bispherical configuration. From these, the particle energy densities in each partial shell are found. Hence the free energy available after complete pitch-angle scattering from the initial ring distribution can be derived.

In order to make the upstream and downstream derivations symmetric, the upstream average velocity and energy density will be obtained in the 'upstream-wave' frame where the upstream direction along \(B\) is positive, and the downstream equivalents are then similarly obtained relative to the 'downstream-wave' frame (downstream positive) simply by replacing the appropriate parameters in the equations. Thus, as illustrated in Figure 5.4, initial pick-up ion injection angles \(\theta_u\) and \(\theta_d\) are measured with respect to the field-aligned upstream and downstream directions, respectively, and the parallel-propagating Alfvén velocities are \(V_{Au} = V_{Ad} = |V_A|\). Alpha is the angle between the \(B\)-field lines and the solar wind flow vector such that

\[
0 < \alpha < 90^\circ
\] (5.42)
Figure 5.4: Sketch of velocity-space shell portion geometry in the solar wind frame. $v_{inj}$ is the initial cometary ion pick-up velocity.

(see Figure 5.4). The injection angles and shell radii are, by geometry:

\[ V_u^2 = u^2 \sin^2 \alpha + (u \cos \alpha - V_A)^2 \]  \hspace{1cm} (5.43)

\[ V_d^2 = u^2 \sin^2 \alpha + (u \cos \alpha + V_A)^2 \]  \hspace{1cm} (5.44)

\[ \sin \theta_u = \frac{u \sin \alpha}{V_u} \quad \sin \theta_d = \frac{u \sin \alpha}{V_d} \]  \hspace{1cm} (5.45)

\[ \cos \theta_u = \frac{u \cos \alpha - V_A}{V_u} \quad \cos \theta_d = \frac{-u \cos \alpha + V_A}{V_d} \]  \hspace{1cm} (5.46)

where $u$ is the solar wind speed, and is also the magnitude of the velocity vector of the initial pick-up ring distribution. Note that the downstream $V_d$ and $\theta_d$ are always greater than, or equal to (at $\alpha = 90^\circ$), the upstream $V_u$ and $\theta_u$ (as is clear from Figure 5.4), because the cometary ion injection velocity, $v_{inj}$, is upstream relative to the solar wind flow.

### 5.4.1 One-Particle Shell-Portion Distributions

For an evenly-spread pitch-angle scattered distribution, the number of particles per cubic metre (at a given position in $(x,y,z)$-space) in each shell
Figure 5.5: A 2-dimensional representation of the spherical shell portion, or 'cap', in the upstream case. $h$ is the height of the cap.

section will be a proportion of the total density depending on the relative area of these sections, with

$$n_u + n_d = n_i \quad (5.47)$$

where $n_i$ (m$^{-3}$) is the total implanted ion density and $n_u, n_d$ are, respectively, the density in the upstream, downstream shell portion, or 'cap'. The area of such a cap is $2\pi rh$, where $h$ is the height of the cap, as sketched in Figure 5.5. Consider the height of the upstream cap. From the Figure;

$$h = V_u + v_{\parallel,\text{inj}} - V_A = V_u + V_u \cos \theta_u \quad (5.48)$$

The radius, $r$, of this cap is simply $V_u$ and thus its area, $A_u$, is

$$A_u = 2\pi V_u^2(1 + \cos \theta_u) \quad (5.49)$$

and by symmetry, for the downstream cap:

$$A_d = 2\pi V_d^2(1 + \cos \theta_d) \quad (5.50)$$
The pick-up ion density on the upstream shell is given by

\[ n_u = n_i \frac{A_u}{A_u + A_d} \]  

(5.51)

which on substitution of equation (5.49) becomes

\[ n_u = n_i \frac{V_u^2(1 + \cos \theta_u)}{V_u^2(1 + \cos \theta_u) + V_d^2(1 + \cos \theta_d)} \]  

(5.52)

Similarly for \( n_d \):

\[ n_d = n_i \frac{V_d^2(1 + \cos \theta_d)}{V_u^2(1 + \cos \theta_u) + V_d^2(1 + \cos \theta_d)} \]  

(5.53)

Normalized one-particle distribution functions are required for use with \( n_u \) and \( n_d \). The integral over a spherical surface (where there is no \( \phi \) variation) in velocity space is performed by the integral operator

\[ \int d\phi \ 2\pi v^2 \sin \phi \]  

(5.54)

where \( 2\pi v^2 \sin \phi \ d\phi \) is simply the elemental area of a torus of radius \( v \sin \phi \) (see chapter 7 for a more detailed explanation). For the upstream shell portion, this is applied in the following way. Normalization requires:

\[ 1 = \int_0^\infty \int_{\theta_u}^{\pi} 2\pi v^2 \sin \phi \ C_N \delta(v - V_u) \ d\phi \ dv \]  

(5.55)

where \( C_N \) is the normalization constant and the \( \delta \)-function simply implies that all the particles must lie on the shell surface of radius \( V_u \). The \( \phi \) integral is performed from the injection angle, \( \theta_u \), to \( \phi = \pi \) (ie. over the shell portion).

This gives

\[ 1 = 2\pi C_N V_u^2 [-\cos \pi + \cos \theta_u] \]  

(5.56)

so that

\[ C_N = \frac{1}{2\pi V_u^2(1 + \cos \theta_u)} \]  

(5.57)

(Note that this is simply 1 over the area of the cap. In other words, the density of the one particle is divided evenly over this area.) Thus, the one-particle upstream and downstream shell-portion distribution functions may be written:

\[ f_u = \frac{\delta(v - V_u)}{2\pi V_u^2(1 + \cos \theta_u)} \]  

(5.58)

\[ f_d = \frac{\delta(v - V_d)}{2\pi V_d^2(1 + \cos \theta_d)} \]  

(5.59)
5.4.2 Upstream and Downstream Energies

The average velocities for the upstream and downstream shell distributions will be calculated in the frame of the respective upstream and downstream Alfvén velocity 'centres'. (The average velocity of ions in a uniform-density partial sphere in velocity-space differs from the spherical 'centre'.) The total energy density of an ion distribution depends on the frame of reference, however the derived free energy is independent of frame.

The average field-aligned cometary ion velocity for the upstream-centred partial-shell population in the \( V_A \) wave frame (with upstream positive) is obtained from the 1st moment of the normalized distribution \( f_u \):

\[
< v >_u = \int f_u \, v \, dv
\]

\[
< v >_u = \int_0^\infty \int_0^\pi \frac{\delta(v - V_u)}{2\pi V_u^2(1 + \cos \theta_u)} \, v \, 2\pi v^2 \sin \theta \, d\theta \, dv
\]

Evaluating the \( v \)-integral according to the \( \delta \)-function by replacing \( v^2 \rightarrow V_u^2 \) and \( v = V_u \cos \theta \) on the shell (for the parallel component, since the average of the random-phase perpendicular gyratory component is zero) leaves the \( \theta \)-integration:

\[
< v >_u = \frac{V_u}{(1 + \cos \theta_u)} \int_0^\pi \sin \theta \cos \theta \, d\theta
\]

Hence the average velocity of \( n_u \) pick-up ions on the upstream-wave shell portion, if scattered fully isotropically, is:

\[
< v >_u = \frac{-V_u \sin^2 \theta_u}{2(1 + \cos \theta_u)}
\]

in the wave frame centred on \( V_A \).

The kinetic energy density of the ions in this partial shell can be found in the solar wind frame from the average velocity \( < v >_u \) above added to the offset velocity of the wave frame relative to the solar wind (ie. \( V_A + < V >_u \) for upstream positive), which gives

\[
KE_{u,sw} = \frac{1}{2} m_i n_u \left( V_A - \frac{V_u \sin^2 \theta_u}{2(1 + \cos \theta_u)} \right)^2
\]

Where \( m_i \) is the mass of the cometary ions.
The thermal energy density is frame independent, since it is effectively the average energy of the ion distribution with respect to its kinetic ‘drift’:

\[ TE_u = \frac{1}{2} m_i n_u \int f_u (v - <v>_u)^2 \, dv \]  \hspace{1cm} (5.65)

where, by vector subtraction

\[ (v - <v>_u)^2 = [V_u \cos \theta - <v>_u]^2 + [V_u \sin \theta]^2 \]  \hspace{1cm} (5.66)

\[ = V_u^2 + <v>_u^2 - 2V_u <v>_u \cos \theta \]  \hspace{1cm} (5.67)

for \( v \) anywhere on the surface of the shell portion with radius \( V_u \). Replacing this in equation (5.65) and performing the \( v \)-integral using the \( \delta \)-function gives:

\[ TE_u = \frac{m_i n_u}{2(1 + \cos \theta_u)} \int_0^\pi \left\{ (V_u^2 + <v>_u^2) \sin \theta - 2V_u <v>_u \cos \theta \sin \theta \right\} \, d\theta \]  \hspace{1cm} (5.68)

The integral of \( \sin \theta \to -\cos \theta \) which, between the limits gives \( (1 + \cos \theta_u) \), and the integral of \( \cos \theta \sin \theta \to 1/2 \sin^2 \theta \) gives \( -1/2 \sin^2 \theta_u \), so that, on substituting for \( <v>_u \) from equation(5.63):

\[ TE_u = \frac{m_i n_u}{2} \left[ V_u^2 - \frac{V_u^2 \sin^4 \theta_u}{4(1 + \cos \theta_u)^2} \right] \]  \hspace{1cm} (5.69)

Note that, in the frame of the waves, the total energy density of the partial shell is, from equations (5.63) and (5.69):

\[ (KE + TE)_{\text{waveframe}} = \frac{1}{2} m_i n_u \left[ <v>_u^2 + V_u^2 - \frac{V_u^2 \sin^2 \theta_u}{4(1 + \cos \theta_u)^2} \right] \]  \hspace{1cm} (5.70)

\[ = \frac{1}{2} m_i n_u V_u^2 \]  \hspace{1cm} (5.71)

as expected, since all \( n_u \) particles have a \( V_u \) velocity “radius” from the centre of the frame (\( V_{Au} \)).

The total energy density of the \( n_u \) ions in the solar wind frame (upstream positive) is obtained by adding equations (5.64) and (5.69):

\[ U_{u \text{ shell}} = \frac{1}{2} m_i n_u \left[ V_A^2 + V_u^2 - \frac{V_A V_u \sin^2 \theta_u}{(1 + \cos \theta_u)} \right] \]  \hspace{1cm} (5.72)

The equation for the total energy density of the downstream-centred shell portion is identical in form in a downstream-positive solar wind frame:

\[ U_{d \text{ shell}} = \frac{1}{2} m_i n_d \left[ V_A^2 + V_d^2 - \frac{V_A V_d \sin^2 \theta_d}{(1 + \cos \theta_d)} \right] \]  \hspace{1cm} (5.73)
(Note that $U$ here is an energy density since $n_i$, $n_u$, $n_d$ are per cubic metre. For a closed system, e.g., a "box" of a fixed number of particles, the total energy of these could be specified in Joules. However, for a potentially infinite system, the useful units are Joules per cubic metre.)

The available free energy is simply the total energy released by the cometary ions on complete pitch-angle scattering from the initial ring distribution to the assumed bispherical shell configuration, that is in other words, the energy difference between the ring and shell. This can be evaluated for the $n_u$ and $n_d$ ions separately, as they scatter onto their respective partial shells, to give the free energy available to upstream and downstream waves. The total energy density of the ring-beam in the solar wind frame is:

$$U_{RING} = KE_R + TE_R = \frac{1}{2} m_i n_i v_{||,inj}^2 + \frac{1}{2} m_i n_i v_{\perp,inj}^2 = \frac{1}{2} m_i n_i u^2$$ (5.74)

Of these $n_i$ ions, $n_u$ scatter on upstream waves, and $n_d$ on downstream waves. (See equations (5.47), (5.52) and (5.53).) The free energies $U_{RING} - U_{SHELL}$ are:

$$E_{F,u} = \frac{1}{2} m_i n_u \left[ u^2 - V_A^2 - V_u^2 + \frac{V_A V_u \sin^2 \theta_u}{(1 + \cos \theta_u)} \right]$$ (5.75)

$$E_{F,d} = \frac{1}{2} m_i n_d \left[ u^2 - V_A^2 - V_d^2 + \frac{V_A V_d \sin^2 \theta_d}{(1 + \cos \theta_d)} \right]$$ (5.76)

Replacing the $\theta_u$, $\theta_d$ according to equations (5.43) to (5.46) gives, in terms of solar wind parameters $\alpha$ and $u$:

$$E_{F,u} = \frac{1}{2} m_i n_u \left[ \frac{V_A u^2 \sin^2 \alpha}{V_u + u \cos \alpha - V_A} + 2u V_A \cos \alpha - 2V_A^2 \right]$$ (5.77)

$$E_{F,d} = \frac{1}{2} m_i n_d \left[ \frac{V_A u^2 \sin^2 \alpha}{V_d - u \cos \alpha - V_A} - 2u V_A \cos \alpha - 2V_A^2 \right]$$ (5.78)

It is worth noting that if these equations are evaluated at any particular position in space (for example at a point on the Giotto path) using the local value of the solar wind speed, $u$, then all $n_i$ ions in the flow at this position will be taken to have been injected at the local $u$. An equation for free energy should ideally be integrated down flowlines along which the ions are implanted. The waves are assumed to convect downstream with a parcel of plasma so that the energy released does not propagate away (which is reasonable under the approximation of $u >> V_A$). In reality, waves will
propagate along the magnetic field lines (i.e., generally at an angle to \( u \)) so that there will be a 'mixing' of waves (and indeed ions, with \( v_\parallel \)) across the flowlines.

**Dependence on Solar Wind Alpha**

The angle between the solar wind flow and the magnetic field was observed to fluctuate dramatically during the close approach to comet Halley. The free energy varies with alpha. Because of the ion pick-up geometry (see previous sections) there will always be more free energy available to upstream waves, except in the special case of perpendicular pick-up when the proportions are equal. Plots of the shell portion populations, \( E_{F,u} \), \( E_{F,d} \) and the total \( E_F \) as functions of alpha are displayed in Figure 5.6. The total free energy available is greatest for \( \alpha = 0 \), the purely parallel pick-up case, and reduces to a minimum at \( \alpha = 90^\circ \) (perpendicular pick-up). A schematic diagram of the partial shell configurations for these two cases is shown in Figure 5.7.

For a fixed ratio of \( V_A \) and \( u \), the *shapes* of the curves against alpha in Figure 5.6 are fixed, since all the radii in Figure 5.4 scale with these velocities. The *magnitude* of the curve values will change with the scale.

**5.4.3 Bispherical Bulk Velocity**

The field-aligned average velocity, \( v_{sh} \), of the completely pitch-angle scattered bispherical shell distribution is given by

\[
v_{sh} = \frac{n_u}{n_i} < v >_{u,sw} + \frac{n_d}{n_i} < v >_{d,sw}
\]

(5.79)

where \( n_u/n_i, n_d/n_i \) are the respective density weightings from equations (5.52) and (5.53), and \( < v >_{u,sw}, < v >_{d,sw} \) are the field-aligned average velocities of the \( n_u, n_d \) ion populations in the solar wind frame where the downstream direction (along the solar wind flow vector) is positive, that is:

\[
< v >_{u,sw} = -(V_A + < v >_u)
\]

(5.80)

\[
< v >_{d,sw} = V_A + < v >_d
\]

(5.81)

Thus, from equations (5.63) and (5.43) to (5.46):

\[
< v >_{u,sw} = -V_A + \frac{u^2 \sin^2 \alpha}{2(V_u + u \cos \alpha - V_A)}
\]

(5.82)
Figure 5.6 - The $\alpha$-dependences of the cometary ion pick-up shell portion populations, upstream and downstream free energies and total free energy, for a fixed ratio of $V_A$ to $u$. 
5. PARALLEL PICK-UP AND WAVE ENERGY

Figure 5.7: Partial shell configurations for the cases (a) $\alpha = 90^\circ$, and (b) $\alpha = 0$ (upstream wave generation only). The dashed-line circles indicate the $|v|$-radius at injection and it is clear that more energy is released, in theory, in case (b).

\[
<v>_{d,\text{sw}} = V_A - \frac{u^2 \sin^2 \alpha}{2(V_d - u \cos \alpha - V_A)}
\]  

(5.83)

5.5 Wave Energy at Giotto

As hydromagnetic waves travel in a plasma medium, the particles and fields are disturbed into oscillation by the passing of the wave. The wave energy density is the sum of the magnetic energy in the B-field oscillation and the kinetic energy of the particles (which gives the energy density of the E-component), and may be calculated from solar wind data according to:

\[
U = U_E + U_B = \frac{1}{2} \rho <u_1^2> + \frac{1}{2} \frac{<B_1^2>}{\mu_0}
\]

(5.84)

where $\rho$ is the solar wind proton mass density, and $<u_1^2>$ and $<B_1^2>$ are the mean square amplitudes of the ion velocity and field fluctuations, where

\[
u = u_0 + u_1
\]

(5.85)

\[
B = B_0 + B_1
\]

(5.86)
The RMS (root mean square) variations of these parameters are, for example:

\[
\left( \langle u_x - u_{0,x} \rangle^2 \right)^{\frac{1}{2}} = \left( \langle u_{1,x}^2 \rangle \right)^{\frac{1}{2}}
\]

(5.87)

and similarly for the \(y\) and \(z\) components.

The Giotto FIS and magnetometer (MAG) data is split into two files containing average and 'difference' data by passing it through a 45-point Gaussian-weighted running mean filter. The Gaussian width is such that 3 mHz is the approximate frequency 'boundary' between the two files; the < 3 mHz fluctuations remain represented in the smoothed file, and the fluctuations file is obtained by subtracting the mean values from the raw data.

The \(B_1\) and \(u_1\) components are plotted in Figure 5.8 for the inbound period 12:00 to 19:00 hours spacecraft event time (SCET) on 13\(^{th}\) March, 1986. The data has been transformed into the field-aligned (FLD) coordinate system using values of the mean \(B_0\) direction and \(u\) in the smoothed file. In this system, \(y\) is downstream along the B-field lines (can be parallel or antiparallel to the B vector), \(z\) is in the \(-u \times B\) direction, and \(x\) is upstream completing a right-handed set. It is clear from the Figure that the level of turbulence increases dramatically as the comet is approached. The fluctuations are substantially smaller along the field, in the \(y\) components, as is expected for parallel-propagating transverse waves. The mean square amplitudes are calculated from this data and hence the wave energy profile along the Giotto trajectory is computed according to equation (5.84). The \(U_E\) and \(U_B\) plots in Figure 5.9 indicate that 'equipartition' of energy between the B-field and ions is a reasonable assumption.

In Figure 5.10, the parameters \(u\), \(B_0\), \(\alpha\), \(n_{sw}\) and \(V_A\) are plotted. \(\alpha\) is calculated from the dot product

\[
\frac{u \cdot B_0}{u B_0} = \cos \alpha
\]

(5.88)

and taken in the range \(0 < \alpha < 90\) degrees for use with FLD coordinates. The Alfvén velocity (given in Section 5.2.2) in terms of the magnitude of the solar wind magnetic field, \(B_0\), and ion mass density, \(\rho\), is:

\[
V_A = \frac{B_0}{(\mu_0 \rho)^{\frac{1}{2}}}
\]

(5.89)
Figure 5.8: 'Wave' components $B_1$ and $u_1$ in FLD coordinates for the period 12:00 to 19:00 SCET on March 13, 1986.
Figure 5.9 - Profiles along the Giotto path of the measured wave energy density divided between the magnetic field fluctuations ($U_B$) and the ion motion ($U_E$). The wave energy increases on approach to the comet.
Figure 5.10 - Variation of the parameters $n_{sw}$, $u_{sw}$, $B_0$, $\alpha$ and $V_A$ along the Giotto spacecraft trajectory.
As expected from this equation, $V_A$ in Figure 5.10 varies with $B_0$. $\alpha$ also follows a similar pattern from 05:00 to 11:00 hrs ($B_0$ lagging behind $\alpha$), which suggests that perhaps in this region the magnitude of $B$ fluctuates with its direction relative to $u$. The solar wind speed $u$ shows a possible correlation with $B_0$ between $\approx 07:30$ and 11:00 hrs.

The theoretical free energy available to upstream and downstream waves, $E_{F,u}$ and $E_{F,d}$ respectively, are calculated from the $u$, $\alpha$ and $V_A$ profiles using equations (5.77) and (5.78), for each data point measured along the Giotto trajectory. The implanted oxygen ion density, $n_i$, is calculated according to the mass-loading model results of Chapter 4 (from equation (4.34)). The $E_{F,u}$ and $E_{F,d}$ profiles are displayed in Figure 5.11. $E_{F,u}$ clearly accounts for the majority of the available energy, and increases rapidly in the latter part of the plot in a similar way to the $U_e$ and $U_g$ curves.

In Figure 5.12 profiles of $E_F = E_{F,u} + E_{F,d}$ and $U = U_u + U_d$ are presented (top and bottom traces, respectively). Figure 5.13 is a scatter plot of $U$ versus $E_F$. It is evident that not all the free energy available is immediately released as wave energy, particularly at early times when there are few cometary ions and the free energy is low. This supports the idea that pick-up and isotropization is not an instantaneous process and depends on the level of turbulent waves. The more turbulence there is, the more rapidly the ions are scattered in pitch angle, in the process of which further waves are generated. A polynomial fit to the data in Figure 5.13 may be computed. The simplest, adequate fit is a quadratic of the form $U = a + b(E_F) + c(E_F)^2$ where $a$, $b$ and $c$ are given in the figure. This enables an estimate of wave energy density to be modelled from the theoretical free energy at any position in space (where it may not have been measured), assuming the comet-solar wind interaction region is a steady state environment. The alpha value must be assumed constant on a particular flowline, and $n_i$ may be obtained from the results of Chapter 4.

By fitting to polynomials of order 4 between 12:00 and 19:00 hrs, $U_*$ and $E_F$ smooth profiles are generated. These are overlayed in Figure 5.12. $U_*$ extends out to a value of $\approx 0.2 \, \text{pJ m}^{-3}$ at around $10^7$ km from the comet, at about 06:00 hrs SCET on 12th March 1986 (based on the solar wind JPA plasma data assuming equipartition of wave energy between $E$ and $B$). The
Figure 5.11 - The theoretical upstream and downstream free energy profiles, $E_{Fa}$ and $E_{Fa}$, from the bispherical shell model.
Figure 5.12 - Free, released and measured wave energy density profiles with smooth fits overlayed.
Figure 5.13: Scatter plot of $U$ against $E_F$ with the fitted quadratic overlayed.

$\begin{align*}
  a &= 0.6717 \\
  b &= -0.0345 \\
  c &= 0.0232
\end{align*}$
solar wind "background" wave energy thus estimated is variable in time (as indeed are \( u_\infty \) and \( n_\infty \) over long timescales). For example, on 6th March 1986 the average turbulence level is \( \sim 0.5 \text{ pJ m}^{-3} \), and the solar wind speed is nearer \( 400 \text{ km s}^{-1} \) rather than \( u_\infty \sim 366 \text{ km s}^{-1} \) on the 12th - 13th March.

**Poynting Vector**

The Poynting vector of the waves is defined by

\[
S = E_1 \times H_1
\]  

(5.90)

where \( H_1 = B_1/\mu_0 \) in free space. The electric vector is \( E_1 = -u_1 \times B_0 \). Thus the Poynting vector may be calculated from the measured solar wind parameters:

\[
S = (-u_1 \times B_0) \times \frac{B_1}{\mu_0}
\]  

(5.91)

In the field-aligned coordinate system, \( B_0 = B_{0,y} \hat{y} \) only so that \((-u_1 \times B_0)\) cannot have a \( y \) component, \( E_{1,y} = 0 \). With \( B_{1,y} \) generally small compared to the transverse components (for parallel-propagating transverse waves) then the major component of \( S \) is \( S_y \), which is in the assumed direction of wave propagation. This assumption will later be shown to be reasonable.

The Poynting vector calculated from the measured data (equation (5.91)) may be used to obtain an estimate of wave energy, as the following derivation will show (see for example Coulson and Boyd, 1979; Bittencourt, 1986). Linearizing the 2nd Maxwell equation (see Section 5.2.2) for fluctuations \( E_1, B_1 \) of the form \( e^{i(k \cdot r - \omega t)} \) gives

\[
 i k \times E_1 = i \omega B_1
\]  

(5.92)

or

\[
 k \times E_1 = \omega \mu_0 H_1
\]  

(5.93)

This equation shows that the propagation direction is perpendicular to both \( E_1 \) and \( B_1 \), for transverse waves.

Returning to the Poynting vector, \( S = E_1 \times H_1 \), equation (5.93) may be used to substitute for \( H_1 \):

\[
S = E_1 \times \frac{k \times E_1}{\omega \mu_0}
\]  

(5.94)
The cross-product of $E_i$ with $k \times E_1$ is simply $E_i^2$ since $E_1.k = 0$. Replacing also $\omega = v_{ph}k$ where $v_{ph} = 1/(\mu_0\epsilon)^{\frac{1}{2}}$, gives:

$$S = \left(\frac{\epsilon}{\mu_0}\right)^{\frac{1}{2}} E_i^2 \hat{k} \quad (5.95)$$

The electromagnetic energy density of the wave field is [eg. Coulson and Boyd, 1979]:

$$U = \frac{1}{2} [E_1.D_1 + B_1.H_1] \quad (5.96)$$

where $D_1 = \epsilon E_1$. If equipartition of wave energy between $E$ and $B$ fields is assumed, this may be written as

$$U \equiv [\epsilon E_1^2] \quad (5.97)$$

Then equation (5.95) written in terms of $U$ from (5.97) becomes

$$S = U V_A \hat{k} \quad (5.98)$$

where the wave velocity $v_{ph} = 1/(\mu_0\epsilon)^{\frac{1}{2}}$ is simply the Alfvén speed, $V_A$, in the solar wind frame.

The magnitude of the Poynting vector is given by the wave velocity multiplied by wave energy density, and it is in the direction of wave propagation. Figure 5.14 shows a comparison of the total poynting vector magnitude and its component along the $y$-axis (in the FLD coordinate system) obtained from the measured data using equation (5.91). It can be seen that $S_y$ is a large proportion of $|S|$, hence $\hat{k}$ is primarily directed along $B$ (in the negative $y$ direction, upstream) and the assumption of parallel-propagation is justified. According to equation (5.98), the measured magnitude of the Poynting vector, (5.91), divided by the Alfvén velocity, (5.89), at points along the Giotto trajectory gives an estimate of the wave energy density profile, which is included in Figure 5.14. A scatter plot of $S/V_A$ against the $U$ values obtained from equation (5.84) is displayed in Figure 5.15. The linear correlation coefficient of 0.98 indicates that the Poynting vector estimate is reasonable. However, the $S$ vector represents the wave energy flux with a direction, which means that in terms of upstream and downstream wave energy densities, $U_u$ and $U_d$ respectively, $S$ gives:

$$\frac{|S|}{V_A} \equiv (U_u - U_d) \quad (5.99)$$
Figure 5.14 - Profiles along the Giotto trajectory of the Poynting vector magnitude, its major component, $S_y$, and the wave energy estimated from $S$. 
Figure 5.15: Linear correlation of the wave energy density estimated from the Poynting vector against the values of $U$ obtained more directly from measurements.
An estimate of the proportion of $U_u$ to $U_d$ (dependent upon $\alpha$) may be derived using this and the total measured wave energy density from equation (5.84) which is:

$$U = (U_u + U_d) \quad (5.100)$$

Solving (5.99) and (5.100) simultaneously for $U_u$ and $U_d$:

$$U_u = \frac{1}{2} \left( U + \frac{|S|}{V_A} \right) \quad (5.101)$$

$$U_d = \frac{1}{2} \left( U - \frac{|S|}{V_A} \right) \quad (5.102)$$

for upstream and downstream propagating waves in the solar wind frame. The plot in Figure 5.16 shows that $U_d \ll U_u$, which is why $S$ gives a respectably close estimate of $U$.

5.6 Energy Released and Damping

The proportion of the available free energy that has been released from the evolving ion distribution at any position along the Giotto trajectory can be estimated by comparing the initial, measured, and isotropized ion distribution bulk velocities. A schematic view of the comparison is given in Figure 5.17a. In the solar wind frame, FLD coordinates, these velocities are negative. $V_{sh}$ (equation (5.79)) is calculated from the final ‘isotropic’ v-space configuration and lies between 0 and $-V_A$ according to the value of $\alpha$ (where $0 < \alpha < 90$ degrees). The initial velocity, $v_{\|,inj}$, is simply $-u \cos \alpha$. The measured implanted oxygen ion bulk velocity, $v_{ox}$, should lie between $v_{\|,inj}$ and $v_{sh}$ for an intermediate stage in the parallel pick-up process. The profiles of $v_{\|,inj}$, $v_{ox}$, $v_{sh}$ obtained along the Giotto trajectory between 05:00 and 19:00 hrs SCET are displayed in Figure 5.17b. It appears that these velocities are all well correlated and follow the variations in $\alpha$.

The estimate of the ratio of released energy to free energy can be based on the momentum changes for the following reason. A wave-particle interaction in which the ion loses energy will release an electromagnetic Alfvén wave photon with a quantum $h \omega$ of energy [Lyons and Williams, 1984], and similarly the ion momentum change is represented in terms of the photon
Figure 5.16 - The estimated energy density proportions in upstream and downstream propagating waves. The plots are on the same scale to allow easy comparison.
Figure 5.17: (a) Schematic diagram of the comparison between $v_{\parallel,\text{inj}}$, $v_{ox}$ and $v_{sh}$ cometary ion field-aligned velocities in the solar wind frame. (b) Values of the same obtained on the Giotto path, with $\alpha$ (0 to 90°) and $f_R$ plotted in the upper two panels.
momentum $h\kappa_||$, or simply $h\kappa$ for parallel-propagating waves. Thus the ratio of energy change to momentum change is

$$\frac{\Delta E}{\Delta p} = \frac{\omega}{k} = V_A$$

(5.103)

so that the energy released as the cometary ions are pitch-angle scattered may be obtained from the momentum change using $\Delta E = m_i \Delta(n_i\nu_{||}) V_A$.

The proportion of released energy to free energy available is then:

$$f_R = \frac{m_i n_i (v_{||,inj} - v_{ox}) V_A}{m_i n_i (v_{||,inj} - v_{sh}) V_A} = \frac{(v_{||,inj} - v_{ox})}{(v_{||,inj} - v_{sh})}$$

(5.104)

for a given position in space. This is the ratio of the velocity gap between $v_{||,inj}$ and $v_{ox}$ as a proportion of the total anticipated change from $v_{||,inj}$ to the final theoretical $v_{sh}$ for the $n_i$ ions. The profile of $f_R$ is plotted in Figure 5.17, having been set to 0 or 1 where it exceeds this range.

The energy released is given by $E_R = f_R \times E_F$, and is included in Figure 5.12 (middle trace). It appears that the released energy correlates better with the measured wave energy than the free energy does, as might be expected if the energy released has indeed gone directly into the waves. Figure 5.18a displays the $E_F$, $E_R$ and $U$ profiles together in the same frame, showing the energy gaps between these parameters.

The difference between the energy released and the measured wave energy can be equated to an energy lost through 'damping' of the turbulence. Consider the small changes of the parameters in a distance $\Delta s$ on a flowline approaching the comet:

$$\Delta U \equiv \Delta (f_R E_F) - \frac{U}{\tau_D} \times \frac{\Delta s}{\text{wave velocity}}$$

(5.105)

where $\tau_D$ is a 'damping lifetime', in seconds. This equation states that the energy $\Delta U$ added to the wave spectrum in a distance $\Delta s$ equals the increase in energy released less that which is lost in $\Delta s$ due to damping. The energy lost in $\Delta s$ is represented by the energy loss per second multiplied by the number of seconds the waves take to travel this distance towards the comet.

It may be assumed that $V_A < u_{sw}$; the waves are essentially moving with the solar wind. Dividing equation (5.105) through by $\Delta s$ and taking the infinitessimal limit $\Delta s \to ds$ gives a differential equation:

$$\frac{dU}{ds} = \frac{dE_R}{ds} - \frac{U}{u \tau_D}$$

(5.106)
Figure 5.18: Comparison of $E_F$, $E_R$ and $U$ profiles in (a), and (b) the damping lifetime, $\tau_D$, obtained from them.
Smoothed (fitted) profiles of $U_a$ and $E_R$ are used in the numerical implementation of this equation. These overlay the data in Figure 5.18a. An assumption of constant $\alpha$ is implicit in the use of the smoothed files, which is desirable for the consideration of a single flowline.

The Giotto path crosses the solar wind flow, but $ds$ is required for $s$ on a single streamline. For this purpose, 'mass-loaded equivalent' positions on the Sun-comet line (as an example flowline) are obtained by comparing $n_i$. In other words, $n_i$ at Giotto is estimated from the model of Chapter 4, and then an 's' is picked out on the Sun-comet line at which $n_i$ is the same. (This assumes a 1-dimensional flow around the comet where all the flowlines are equivalent.) This method then enables the damping timescale to be investigated as the waves convect downstream with the solar wind along a flowline (in the approximation $V_A << u_{sw}$). For each data point, the difference between the new $s$ value and that of the previous point is computed to give $ds$. Similarly $dU$ and $dE_R$ are obtained, and hence $\tau_D$ is calculated. $\tau_D$ is plotted in Figure 5.18b. It can be seen that the damping is an extremely small effect at early times before the turbulent energy has started to increase much above the solar wind background level. The timescale reduces to an approximate level of $\tau_D \sim 1,000$ seconds at 17:00 to 19:00 hours on the Giotto inbound path.

The 'damped' energy is likely to be absorbed by the solar wind during its deceleration and thermalization. A detailed analysis of the possible damping mechanism and coupling to solar wind ions is beyond the scope of this thesis.

5.7 Summary

- Equipartition of wave energy density between the B-field wave fluctuations and the ion oscillations is reasonably apparent from the data considered.

- The Poynting vector is found to lie almost completely along the $-y$ axis (upstream along B) in the field-aligned coordinate system, and the estimate of the wave energy from the $S$ vector and $V_A$ is in good agreement with that measured. Thus the assumption that the waves
are primarily parallel-propagating is consolidated.

- The free energy available to upstream and downstream propagating waves during the pickup process has been calculated theoretically from the pickup geometry in velocity space and the implanted ion density given by the mass-loading model. The waves propagate predominantly upstream in the solar wind frame, according to both theoretical free energy calculations and estimations using the measured $U$ and $S$ obtained from the data. The free energy is released according to the velocity diffusion rate, dependent upon the level of turbulence already present, and hence the increase of wave energy with available free energy is non-linear. The proportion of $E_F$ that has been released at any position on the Giotto path is estimated in terms of the ratio of momentum changes of the ions; from the parallel momentum at injection to the momentum of the observed heavy ion distribution, compared to the total change expected during evolution to the final 'shell' distribution.

- Polynomial fits to the energy density profiles provide a means of estimating $U$ from $E_F$ anywhere in the vicinity of the comet.

- A damping timescale is obtained from differential calculations of the released and measured wave energy changes. The damping rate increases on approach to the comet.
Chapter 6

Wave Spectral Analysis

The turbulent power spectrum carried by the solar wind in the cometary interaction region may be approximated by a power-law with spectral index $\gamma$ [eg. Isenberg, 1987a]:

$$P_j(k) = A_j|k|^{-\gamma}$$ (6.1)

For simplicity it is assumed that the waves are parallel-propagating, Alfvénic, and are not polarized ($P_j(-k) = P_j(k)$). The intensities propagating parallel and antiparallel to $B$ are $P_+$ and $P_-$ respectively (denoted by the index, $j$).

In this chapter a study is made of the spectral shape and power amplitude from measurements at Giotto, with a view to computing the pick-up ion velocity diffusion (in chapter 7) due to particle interaction with both ambient solar wind and self-generated waves (as described in chapter 5).

6.1 The Fourier Transform

The Fourier transform (FT) of a time function, $h(t)$, is defined as

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{2\pi if t} dt$$ (6.2)

or

$$H(\omega) = \int_{-\infty}^{\infty} h(t)e^{i\omega t} dt$$ (6.3)

where the result is a function of frequency, $f$ (in Hz), or angular frequency, $\omega = 2\pi f$. Fourier analysis may be envisaged as follows. The signal $h(t)$ is 'correlated' by multiplication with sinusoids of different frequencies $e^{i\omega t} =$
cos(\omega t) + i \sin(\omega t)\) to look for ‘resonances’. The Fourier integral in equation (6.3) is taken over all time. For a signal \(h(t)\) that is a perfect sinusoid, multiplying with \(e^{i\omega t}\) of identical frequency (and phase) gives positive values for all time, and the integral is then finite and positive. However, the product of \(h(t)\) with any different Fourier frequency will be another oscillating signal (since the waves ‘beat’), and when integrated over enough wavelengths the result is zero. The frequency spectrum will consist of a single spike at the frequency of the signal. In reality the sample length is finite and the smaller the sample the less the information contained, and the lower the accuracy of the result, particularly for the longer wavelength components which will be measured over fewer cycles. The minimum frequency that can be analysed with reasonable confidence is limited to that with a period equal to the time sampled. (The zero-frequency D.C. offset component (average signal level) can always be obtained).

Any function, \(h(t)\), can be represented over a finite period by a “Fourier series” of sines and cosines such that on the addition of each successive term in the series the original function is better approximated. A complete summation of all the terms in the series tends to the original function. Because of the symmetric/antisymmetric nature (about \(t = 0\)) of the cosine/sine functions respectively, symmetric \(h(t)\) will be described by cosine series only, and antisymmetric \(h(t)\) entirely by a sine series. This of course will depend on the choice of origin of \(t = 0\). The phase of each frequency component relative to the same origin must be known if it is desired to recover the signal from its Fourier series, as Figure 6.1 illustrates. For each frequency the Fourier method multiplies the signal function by two sinusoids 90° out of phase (both contained within \(e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)\)). The relative proportion of the resulting sine and cosine ‘correlation’ at that frequency thus define the phase of that component with respect to \(t = 0\). As an example, consider a particular frequency component of phase \(\phi\), weighting \(W\):

\[
W e^{i(\omega t + \phi)} = W e^{i\omega t} e^{i\phi} = e^{i\omega t} W(\cos \phi + i \sin \phi)
\]  

(6.4)
The phase at \(t = 0\) appears in the constant multiplier \(W \cos \phi + iW \sin \phi\), which is complex. Thus the properties of any Fourier component can be represented by a complex number, \(z = a + ib\) as shown in Figure 6.2, with
Figure 6.1: Signals $a$ and $b$ in this diagram are composed of identical frequency components, $f_1$ and $f_2$. It is the different phase relationships between the components in the two cases that produce the different signals.

Figure 6.2: An illustration of the Fourier coefficients in the complex plane.
"Fourier coefficients" $a$, $b$, where the phase is given by

$$\phi = \tan^{-1}\left(\frac{b}{a}\right)$$ \hspace{1cm} (6.5)

and the amplitude of the component is:

$$W = (a^2 + b^2)^{\frac{1}{2}}$$ \hspace{1cm} (6.6)

### 6.1.1 Discrete Fourier Series

A continuous function or signal is normally measured by sampling at discrete points. For $N$ time-domain sample points, $h_k$, $k = 0, \ldots, N - 1$, the discrete Fourier transform gives the $H_n$'s ($N$ of them) where $H_n = a_n + ib_n$ in terms of the Fourier coefficients, for $n = 0, \ldots, N - 1$. The transform and inverse transform are defined by [Press et al., 1986]:

$$H_n = \sum_{k=0}^{N-1} h_k e^{2\pi ink/N}$$ \hspace{1cm} (6.7)

$$h_k = \frac{1}{N} \sum_{n=0}^{N-1} H_n e^{-2\pi ink/N}$$ \hspace{1cm} (6.8)

where the $H_n$'s need to be divided by $1/N$ before the inverse transform can be made to recover the original signal. (Note that the placing of the $1/N$ factor in the definition is a matter of choice, and sometimes appears symmetrically as $1/\sqrt{N}$ in front of both summations.) The samples must be taken at evenly spaced time intervals.

The sample spacing, $\Delta$, (in seconds) of the $N$ measured values dictates the 'Nyquist' frequency, $f_c$, that is, the highest frequency that may be analysed. To recognise a waveform, at least two measurements per wavelength are required, as sketched in Figure 6.3. Thus the Nyquist frequency is $f_c = \frac{1}{2\Delta}$. The minimum frequency such that one wavelength is measured within the time-span $N\Delta$ of the $N$ samples is given by $\frac{1}{N\Delta}$. A discrete Fourier transform $H_n$ may therefore be found for the frequencies

$$f_n = \frac{n}{N\Delta} \quad n = 0, 1, 2, \ldots, \frac{N}{2}$$ \hspace{1cm} (6.9)

(and for the negative frequencies, $n = -1, -2, \ldots, -N/2$.) If there were frequency components above $f_c$ in the original signal and these were not low-pass filtered out before sampling, then the power at these frequencies will
6. WAVE SPECTRAL ANALYSIS

Figure 6.3: Illustration of minimum and maximum frequencies that may be analysed for N discrete data points of spacing $\Delta$ (seconds) in the time domain.

be folded, or "aliased" into the $< f_c$ frequency range power obtained by the transform. An example of an apparent lower frequency as sampled in this way is shown in Figure 6.4a, and the effect of aliasing on the power spectrum is illustrated in Figure 6.4b.

For entirely real $h_k$ data points, $h_k = x_k + iy_k$ becomes $h_k = x_k$ ($y_k$ all zero), then the Fourier transform has the following property. Consider (for simplicity of expression) the continuous form:

$$H(\omega) = \int_{-\infty}^{\infty} h(t) e^{i\omega t} dt \tag{6.10}$$

The transform $H(\omega)$ can be considered split into two halves, for positive $\omega$ as above, and for negative frequencies, $H(-\omega)$. The complex conjugate of the negative frequency half is

$$H^*(-\omega) = \int_{-\infty}^{\infty} (h(t)e^{-i\omega t})^* dt \tag{6.11}$$

Since $h(t)$ is real for all $t$, then $h^* = h$. The complex conjugate of $e^{-i\omega t}$ is simply $e^{i\omega t}$, so that

$$H^*(-\omega) = \int_{-\infty}^{\infty} h(t) e^{i\omega t} dt \tag{6.12}$$

Thus a simple relationship has been derived for the transform of a real signal:

$$H(\omega) = H^*(-\omega) \tag{6.13}$$
Figure 6.4: In (a) the undersampling of a signal (top trace) produces data with an apparently lower frequency (lower trace), and (b) shows how a Fourier transform power spectrum is affected (taken from Press et al., 1986).
(where the complex conjugate of $a + ib$ is $a - ib$). This proves useful in the calculation of the transform of real data because only the positive frequency half of the coefficient pairs $a, b$ need be computed and stored.

### 6.1.2 Power Spectrum Estimation by FFT

Parseval’s theorem states that the total power in a signal is found to be the same whether computed in the frequency or time domain [Press et al., 1986]. In terms of the continuous signal and transform, this is:

$$\int_{-\infty}^{\infty} |h(t)|^2 dt = \int_{-\infty}^{\infty} |H(f)|^2 df$$

(6.14)

The discrete form is [Press et al., 1986]:

$$\sum_{k=0}^{N-1} |h_k|^2 = \frac{1}{N} \sum_{n=0}^{N-1} |H_n|^2$$

(6.15)

The “power spectral density” (PSD) is the power per unit frequency, a function of frequency. There are several different descriptions of the PSD, (see Press et al., 1986, pgs 384-385, 420-421) depending on whether the Fourier transform is defined for negative frequencies as well as positive, and according to the required normalization of the total power. For total power equal to the mean squared amplitude of the wave, then for the $N$ samples of the time signal $h(t)$:

$$\frac{1}{N} \sum_{k=0}^{N-1} |h_k|^2 = \text{"mean squared amplitude"}$$

(6.16)

The same power may be computed from the transform, $H_n$, using Parseval’s theorem to give

$$\text{Total Power} = \frac{1}{N^2} \sum_n |H_n|^2$$

(6.17)

The discrete Fourier transform puts the wave power into ‘bins’ according to equation (6.9), where $\frac{1}{N^2} |H_n|^2$ is the power per frequency bin.

The power spectral density is required such that

$$\text{Total Power} = \sum_n PSD_n \times \delta f$$

(6.18)

as shown in Figure 6.5, where $\delta f$ is the frequency width of the bin given by
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Figure 6.5: Schematic representation of a spectrum in terms of the $PSD_n$ values and the binned power where the area of the blocks of width $\delta f$ is equal to the power in that bin.

$$\delta f = \frac{1}{N\Delta}$$  \hspace{1cm} (6.19)

Thus the PSD is in units of amplitude squared per Hz, and the summation (or integration) over the whole spectrum gives the total power. Equating (6.17) and (6.18) using (6.19) gives a formula for obtaining the $PSD_n$ from the $H_n$'s:

$$PSD_n = \frac{\Delta}{N} |H_n|^2 \quad n = \frac{-N}{2}, \ldots, \frac{N}{2}$$  \hspace{1cm} (6.20)

Note that this relationship varies according to whether the $H_n$ are defined for all discrete frequencies including the negative, or for the positive side only. We have, for real $h(t)$ [Press et al., 1986]:

$$P(f) \equiv |H(f)|^2 + |H(-f)|^2 = 2|H(f)|^2 \quad 0 \leq f < \infty$$  \hspace{1cm} (6.21)

(in the continuous form). This defines the "one-sided" power spectral density. Some transform algorithms for use with real input data return only the positive frequency components in the interests of saving computing effort and space. In this case equation (6.21) should be used, and a corresponding factor of 2 will appear in equation (6.20) giving:

$$PSD_n = \frac{2\Delta}{N} |H_n|^2 \quad n = 0, 1, 2, \ldots, \frac{N}{2}$$  \hspace{1cm} (6.22)
6. WAVE SPECTRAL ANALYSIS

6.2 Wave Spectra

6.2.1 Spectral Index

Fourier transform spectral analysis is performed on the file (Figure 5.8) containing the > 3 mHz frequencies data used in the wave energy analysis of chapter 5. This data is in the field-aligned (FLD) coordinate system. Gaps are first filled using a linear interpolation between points on either side of the gap, in order to obtain an even sample spacing. The power spectral density is calculated using equation (6.22) and may be plotted against the frequency in the spacecraft frame obtained from equation (6.9). Figure 6.6 compares examples of 512-sample-point and 128-sample-point transforms of the magnetic field x-component. Both these spectra are for time periods centered on around 12:34 hrs spacecraft event time (SCET), but the 512-sample spectrum naturally covers a longer time period (namely 512 × 8 seconds, where there is an 8 second interval between the Giotto JPA data points). It can be seen that the smaller sample spectrum has a less accurate, more erratic profile, and does not extend down to as low a frequency (according to \( f_c \), see Section 6.1.1). The 3 mHz frequency is marked off on the plots.

The spectra here (in Figure 6.6) are plotted on a log-log scale. The assumed spectral shape of equation (6.1) may be examined by a linear regression analysis on the log-log plot as follows:

\[
\log P(f) \equiv \log PSD_n = \log(A_f f^{-\gamma}) = \log A_f - \gamma \log f \quad (6.23)
\]

On plotting \( \log P(f) \) against \( \log f \), the form \( y = mx + c \) may be fitted to give a slope of \( -\gamma \) and the \( y \)-intercept, \( \log A_f \). Blocks of 512 data points are taken sequentially along the Giotto trajectory and a spectrum obtained for each. The spectral index measured at that position (time) of the spacecraft is then computed. The \( \gamma \) versus time profile from the 512-sample spectra is just a smoothed version of the \( \gamma \) variations that may be obtained with the 128-sample spectra, as the example in Figure 6.7 shows. Profiles of \( \gamma \) for all the fluctuating components of the magnetic field, \( B_1 \), are given in Figure 6.8.

Power spectra for the B-field may be obtained on summing the compo-
Figure 6.6: A comparison of the $B_x$ power spectra at $\sim 12:34$ hours SCET obtained using (a) 128-sample, and (b) 512-sample Fourier transforms.
Figure 6.4 - An overlay of $\gamma$ versus time profiles for 128-sample and 512-sample $B_z$ spectra.
Figure 6.8: Spectral index profiles for the fluctuating components of B.
Table 6.1: Characteristics of B spectra measured in spacecraft frame frequencies.

There seems no evidence to suggest any general trend in $\gamma$ on approach to the comet on the basis of the results in Figures 6.8 and 6.9. A constant value of $\gamma = 2$ is reasonable and convenient for use in the following sections (and as a simplifying assumption in Chapter 7). These results are consistent with the work of Glassmeier et al. [1989] who observed power spectra at Halley which could in general be described by a power-law with $\gamma \approx 2$. Spectra at comet Giacobini-Zinner were interpreted by Tsurutani and Smith [1986] with a lower index of $5/3 (\approx 1.67)$. The quasilinear theory of the generation of Alfvénic turbulence by implantation of cometary ions (see Chapter 7) yields expressions for the PSD which also have power-law dependence on $k$, with spectral index of $\gamma = 2$ [eg. Galeev and Sagdeev, 1988; Galeev et al., 1987; Johnstone et al., 1990]. The spectrum in this case may be obtained from the wave growth rate [Galeev and Sagdeev, 1988], or by considering
Figure 6.9 - Spectral index versus time profile for B.
the wavenumber, \( k \), corresponding to particular positions on the appropriate single-wave characteristics (from the resonance condition), where the ions are assumed to diffuse from the initial ring to the final bispherical distribution and the rate of change of particle energy along the characteristic may be examined [Johnstone et al., 1990]. The result provides justification for the use of \( \gamma = 2 \) in the analysis of the quasilinear velocity diffusion in Chapter 7.

### 6.2.2 Doppler Effect

The resonance condition (see Chapter 5) for the wave-particle interaction between a cometary pick-up ion and Alfvénic turbulence in the solar wind determines that the wave frequency as seen by the ion must equal the ion’s gyrofrequency (for the principal resonance). The shifted wave frequency, \( (\pm V_A - v_{\parallel})k \), is obtained from the wave and ion velocities in the solar wind frame. Calculation of the implanted ion velocity diffusion (to be studied in Chapter 7) requires the power, \( P(k) \), of the waves with which the ions interact. In order to obtain the spectral density of the wave field as a function of \( k \) in the solar wind frame, the Doppler shift due to the Giotto spacecraft velocity must be removed from the measured frequencies.

The spacecraft may be considered to be at rest in the comet frame, in the approximation \( V_{SC} \ll u_{sw} \). Thus in the solar wind frame (downstream positive) the spacecraft velocity \( V_{SC} \) is \(-u\), with a velocity component normal to the ‘wavefronts’ of \(-u \cos \alpha\) where \( \alpha \) is the angle between \( u_{sw} \) and the magnetic field (see Figure 6.10). Then at the spacecraft, the apparent wave velocity (at which the wavefronts pass) is

\[
(\pm V_A - V_{SC}) = (\pm V_A + u \cos \alpha) = \frac{\omega_{sc}}{k},
\]

where \( \omega_{sc} \) is the frequency of the Alfvén waves as seen by the spacecraft. The required wavenumber, \( k \), in terms of measured frequency is then

\[
k = \frac{\omega_{sc}}{(\pm V_A + u \cos \alpha)} \tag{6.26}
\]

Since the spectra all span a finite period of time along the Giotto trajectory, an average value of \( \alpha \) is assumed, and \( \pm V_A \) may be reasonably neglected.
under the assumption of $V_A < u \cos \alpha$. Discrete $k_n$ are calculated from $\omega_{sc,n} = 2\pi f_n$ using equations (6.26) and (6.9) which gives

$$k_n = \frac{2\pi n}{N\Delta} \frac{1}{(u \cos \alpha)}$$  \hspace{1cm} (6.27)

so that spectra may be obtained in $k$. The $PSD_n$ are obtained in units of amplitude per wavenumber on replacing $\delta f$ by $\delta k$ in the derivation of equation (6.22):

$$PSD_{n(k)} = \frac{\Delta}{\pi N} (u \cos \alpha) |H_n|^2 \hspace{1cm} n = 0,1,2,\ldots,\frac{N}{2}$$  \hspace{1cm} (6.28)

For an assumed $\gamma = 2$, the spectra in $k$ may be fitted to $A_k k^{-2}$ giving values of $A_k$, listed in Table 6.2. Note that $A_k$ corresponding to the slightly differing measured values of $\gamma$ cannot be used with a $\gamma = 2$ slope; such a combination does not give a reasonable fit to the spectrum. This is because $A_k$ obtained from the $y$-intercept of the log-log plot may be significantly altered by a small change in the slope. Two examples of the $A_k k^{-2}$ fit for the $B$ spectra are plotted in Figure 6.11 on a linear scale.
Figure 6.11: Examples of measured B spectra at (a) 12:34 SCET and (b) 14:51 SCET, plotted on a linear scale, with the fit $A_k k^{-2}$ overlayed in each case.
### TABLE 6.2: Solar Wind Wave Power at Comet Halley

<table>
<thead>
<tr>
<th>SCET (hrs)</th>
<th>$\sum_n PSD_n \times \delta f_n$ (nT)</th>
<th>$\mu U_n$ (nT)</th>
<th>$A_k/k_L$ (nT)$^2$ $\times 10^{-8}$ m$^{-1}$</th>
<th>$k_L$, km$^{-1}$</th>
<th>$A_k/k_L$, (nT)$^2$ $\times 10^{-8}$ m$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12:34</td>
<td>0.59</td>
<td>0.80</td>
<td>4.26</td>
<td>0.78</td>
<td>0.38</td>
</tr>
<tr>
<td>13:42</td>
<td>1.00</td>
<td>1.00</td>
<td>4.78</td>
<td>8.78</td>
<td>0.78</td>
</tr>
<tr>
<td>14:51</td>
<td>1.89</td>
<td>0.80</td>
<td>12.73</td>
<td>9.3</td>
<td>1.29</td>
</tr>
<tr>
<td>15:59</td>
<td>1.91</td>
<td>1.76</td>
<td>7.17</td>
<td>7.17</td>
<td>1.40</td>
</tr>
<tr>
<td>17:07</td>
<td>4.84</td>
<td>3.55</td>
<td>11.36</td>
<td>11.36</td>
<td>4.49</td>
</tr>
</tbody>
</table>
6.2.3 Normalization

The power spectrum \( P(k) \) is normalized according to \([\text{Lee, 1982; Terasawa, 1989}]\):

\[
\langle \delta B \cdot \delta B \rangle = \int P(k) \, dk \tag{6.29}
\]

where \( \delta B (\equiv B_1 \) in Chapter 5) is the amplitude of the magnetic field fluctuations. \( P(k) \) is the sum of the power in the upstream and downstream directions. It is clear that the power-law may not be integrated over all \( k \) since as \( k \to 0 \) then \( k^{-\gamma} \to \infty \). A low frequency cut-off must be assumed. Then, (for \( \gamma > 1 \)),

\[
\int_{k_L}^{\infty} A_k k^{-\gamma} \, dk = A_k \left[ \frac{-1}{(\gamma - 1)k^{\gamma-1}} \right]_{k_L}^{\infty} = \frac{A_k}{(\gamma - 1)k_L^{\gamma-1}} \tag{6.30}
\]

where \( k_L \) is the wavenumber at low frequency cut-off. For \( \gamma = 2 \) this is

\[
\langle \delta B^2 \rangle = \frac{A_k}{k_L} \tag{6.31}
\]

Assuming an equipartition of wave energy between the \( E \) and \( B \) fields gives approximately (see Chapter 5)

\[
U_s = 2U_B = \frac{\langle \delta B^2 \rangle}{\mu_0} \tag{6.32}
\]

where \( U_s \) here is the fitted wave energy calculated in Chapter 5, which may be estimated from the theoretical free energy, \( E_F \), at any position in space. From this:

\[
\langle \delta B^2 \rangle = \mu_0 U_s \tag{6.33}
\]

Equating (6.31) and (6.33) gives:

\[
A_k = \mu_0 U_s k_L \tag{6.34}
\]

This enables \( A_k \) to be obtained via \( U_s \) from theoretical \( E_F(\alpha, u_{sw}, V_A) \) to give an estimate of \( P(k) = A_k k^{-2} \) anywhere in the vicinity of the comet (ie not necessarily along spacecraft trajectories), where there may not be any direct measurements. The \( P(k) \) can then be used in Chapter 7 to calculate the velocity diffusion of cometary ions during their transport along solar wind flowlines. (In theory, an \( A_k \) profile could itself be fitted to \( E_F \) directly,
rather than utilizing the $U_s$ fit to $E_F$ in Chapter 5. However, for accuracy the Fourier analysis requires large sections of data and so a time resolution for $A_k$ similar to that of $U_s$ may not be achievable. The spectral analysis and power-law fitting is more involved than the calculations of $U_s$, and the results for $A_k$ are arguably less reliable.)

First a value for $k_L$ must be obtained by considering the measured data. The values of $U_s$ on the Giotto trajectory may be used to calculate $k_L$ from the values of $A_k$ in Table 6.2 using equation (6.34). Assuming the spectral shape is unchanging on the approach to the comet, a constant $k_L$ may be taken. The average of the values listed in Table 6.2 is $k_L = 9.0 \times 10^{-8}$ m$^{-1}$.

It is important that $A_k$ gives the correct $P(k)$ for the range of $k$ that interact with the cometary ions (as selected by the resonance condition). The minimum wavenumber that may be generated in the diffusion process is obtained from $\omega - kv_\parallel = \pm \Omega_i$. For frequencies $\omega^2 \ll \Omega_i^2$ (as for Alfvén waves, see also Chapters 5 and 7) then $|k| = \Omega_i/v_\parallel$. The minimum $k$ is given by a maximum $v_\parallel$, which will be $u_\infty$ for the earliest implanted ions on a streamline with $\alpha = 0$. For typical values of $B_0 = 5$ to 7 nT, $m_i = 16$ to 20 amu for water group ions, and $u_\infty = 366$ km s$^{-1}$, then $k_{\text{min}} \approx 7$ to $11 \times 10^{-8}$ m$^{-1}$. (Note that the 3 mHz lower limit to the measured frequencies in the spacecraft frame is equivalent to $k \sim 6 \times 10^{-8}$ m$^{-1}$ and so the range of interest has been covered by the measurements.) It is to be expected that a spectrum of mostly pickup-ion-generated waves will extend down to the lowest generated frequency so that an integral over $k$ from infinity down to this point gives a reasonable estimate of the total power. Therefore $k_L = k_{\text{min}}$ seems a good assumption and the value of $k_L = 9 \times 10^{-8}$ m$^{-1}$ obtained from the normalization is reasonable.

Values of $A_k/k_L (= \langle \delta B^2 \rangle )$ where $k_L = 9 \times 10^{-8}$ m$^{-1}$ are listed in Table 6.2 for the spectra along the Giotto path. A comparison plot in Figure 6.12 shows the wave power as computed in the time domain, $\mu_0 U_s$ (equation (6.33)), and in the frequency domain, both for the original binned power summed using equation (6.18), and $A_k/k_L$ from the fitted spectral shape. The results are in good agreement.
Figure 6.12: Comparison of the total power calculated in the time and frequency domains.
6.3 Summary

- A power-law, $P(k) = A_k k^{-\gamma}$, with a spectral index of $\gamma = 2$ is found to be a reasonable fit to the spectrum of turbulent waves observed at comet Halley during the Giotto encounter. The time-series of spectra along the Giotto path (obtained by computing the FT on sequential blocks of data) shows no clear evidence of any general trend in $\gamma$ as the comet is approached.

- When Doppler effects are removed from the measurements, a low frequency cut-off value corresponding to wavenumber $k_L = 9 \times 10^{-8}$ m$^{-1}$ is obtained for the fit to the spectra. The spectra are normalized so that the total power equals the mean squared amplitude of the waves.

- Estimates of the total power calculated in the time domain, and in the frequency domain from both the discrete transform and the power-law fit are all found to agree well.
Chapter 7

Implanted Cometary Ion Distributions

This chapter concerns a preliminary analysis of the evolution of the velocity-space distribution of cometary ions as they are picked up into the solar wind flow. The velocity diffusion coefficient for the wave-particle interaction is computed from the wave energy and spectral analysis of Chapters 5 and 6. The cometary ion production rate and implanted number density are obtained from the results of Chapter 4.

7.1 Ion Kinetic Equation

The distribution of implanted cometary ions in the solar wind can be represented by a function $F = F(t, x, v)$, of time $t$, position vector $x$, and velocity vector $v$ relative to the mean solar wind flow, $u$. $F$ is the phase-space density distribution, with the velocities of the ions represented in the frame of the solar wind. The velocity distribution is in general anisotropic in this frame (see Chapter 5), so that $v$ is not random and provides a mean offset ion flow relative to that of the bulk solar wind ions. $F$ at any position will include all the cometary ions present in their various ‘stages’ of pick-up. In order to study the evolution of the ion distribution along solar wind flow lines in the vicinity of the comet, an ion kinetic equation of the following form must be
considered [Gombosi, 1988]:

\[
\frac{\partial F}{\partial t} + (u_i + v_i) \frac{\partial F}{\partial x_i} - \left( \frac{\partial u_i}{\partial t} + (u_j + v_j) \frac{\partial u_i}{\partial x_j} \right) \frac{\partial F}{\partial v_i} = \frac{\delta F}{\delta t} + \frac{\partial}{\partial v_i} \left( D_{ij} \frac{\partial F}{\partial v_j} \right) \tag{7.1}
\]

which is a modified Boltzmann, or Fokker-Planck equation. \((u_i + v_i) \equiv (u + v)\) is the total ion velocity along \(x\), \(ie\) in the stationary frame of the comet. (Here the vectors are expressed in component notation where \(i, j\) are coordinate subscripts, and the summation convention applies.) The terms on the R.H.S. represent the rate of change of \(F\) due to the source, \(\delta F/\delta t\), of new ions being added into the flow along \(x\), and velocity diffusion of ions due to wave-particle interactions. (Spatial diffusion is ignored). These will be discussed in following sections. The three terms on the L.H.S. comprise the convective derivative, which is obtained in the following way.

The total rate of change of \(F\) with respect to the seven independent variables \(x_i, v_i\) and \(t\) is:

\[
\frac{dF}{dt} \equiv \frac{\partial F}{\partial t} + \frac{\partial F}{\partial x_i} \frac{\partial x_i}{\partial t} - \frac{\partial F}{\partial (u_i + v_i)} \left( \frac{\partial (u_i + v_i)}{\partial t} + \frac{\partial (u_i + v_i)}{\partial x_j} \frac{\partial x_j}{\partial t} \right) \tag{7.2}
\]

\(F\) is not explicitly a function of \(u\), hence \(\partial F/\partial (u_i + v_i)\) becomes \(\partial F/\partial v_i\) which is an approximation, because \(u\) is not constant and there will be an implicit change of the velocity distribution with \(u\). (The solar wind frame is a decelerating frame of reference.) Equation 7.2 is equivalent to

\[
\frac{dF}{dt} = \frac{\partial F}{\partial t} + (u_i + v_i) \frac{\partial F}{\partial x_i} + a_i \frac{\partial F}{\partial v_i} \tag{7.3}
\]

where the time derivative of \(x_i\) is identified as the total velocity, \((u_i + v_i)\), and the rate of change of \((u_i + v_i)\) is the acceleration, \(a_i\). These apply for individual particles. Within \(F\), the \(x, v, t\) are independent orthogonal variables, since at any time, \(t\), and position, \(x\), there are ions with different velocities represented by the distribution. Equation (7.3) is part of a Taylor expansion that expresses the possible change a parcel of plasma may undergo along its trajectory in a time \(dt\), obtained from [Bittencourt, 1986]:

\[
F(t + dt, x_i + v_i dt, v_i + a_i dt, a_i + \dot{a}_i dt, ...) = F(t, x_i, v_i, a_i, ...) + \left[ \frac{\partial F}{\partial t} + v_i \frac{\partial F}{\partial x_i} + a_i \frac{\partial F}{\partial v_i} + \dot{a}_i \frac{\partial F}{\partial a_i} + ... \right] dt \tag{7.4}
\]
7. IMPLANTED COMETARY ION DISTRIBUTIONS

where terms of higher order than $\partial t^2$ have been ignored in equations (7.2) and (7.3). For the purposes of equation (7.1), where $a_i$ are not desired variables in $F$, there is no way of calculating a $du_i/dt$ term where $v_i$ and $t$ are independent variables. Thus $a_i$ in terms of $u_i$ only is used, where $u_i$, the bulk flow of the solar wind ions depends on both $x_i$ and $t$ so that the rate of change with respect to both must be considered. The convective derivative of $F$ can therefore be expressed as in equation (7.1). Note that if we identify force, $\mathcal{F}/m = a$, the total derivative of $F$ may be written:

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + (\mathbf{u} + \mathbf{v}).\nabla F + \frac{\mathcal{F}}{m} \cdot \frac{\partial F}{\partial \mathbf{v}}$$

which is recognized as a Boltzmann equation (see Section 4.1).

The cometary environment is assumed to be in a steady state. In other words the parameters governing the solar wind - comet interaction (such as $Q, V, \nu, u_\infty, n_\infty$) are assumed to be constant in time. Thus the $t$-dependence of $F$ (and hence partials with respect to $t$) in equation (7.1) are ignored. To a good approximation, it is only necessary to consider changes occurring along the direction of solar wind plasma flow ie along $s$-lines (downstream positive) parallel to $x$ in H.S.E. coordinates as illustrated in Figure 7.1. Thus changes in $u$ and $F$ between neighbouring flow lines (terms such as $\partial F/\partial y, \partial u/\partial y$) are neglected in favour of the large-scale change along $s$. Hence $u = u(s)$; any deflection of the solar wind flow is neglected, as are changes in the flux-tube area with $s$ (ie spreading or compression of the flow lines). Equation (7.1) then simplifies:

$$\left( u + v_s \right) \frac{\partial F}{\partial s} - \left( u + v_s \right) \frac{\partial u}{\partial s} \frac{\partial F}{\partial v_s} = \frac{\delta F}{\delta t} + \frac{\partial F}{\partial t} \bigg|_{w-p}$$

where $v_s$ is the component along $s$ of the ion velocity relative to $u$.

The nature of the pick-up process is such that it is convenient to express the ion distribution in the solar wind frame in terms of the ion velocity magnitude,

$$v = |\mathbf{v}| = (v_\perp^2 + v_\parallel^2)^{1/2}$$

where $v_\perp$ and $v_\parallel$ are the components perpendicular and parallel to the magnetic field, $B_0$, and

$$\mu = \cos \theta,$$
where $\theta$ is the pitch angle of the ion velocity relative to the field line direction. The $v_{\perp}$ velocity component is a gyration about the field lines (see Chapters 4, 5) which has a time-average of zero in the solar wind frame, and at any stage in the pickup process the implanted ion moves along $B$ relative to $u$ at its $v_{\parallel}$ velocity. The $s$-component (projection onto the $s$-axis) of $v_{\parallel}$ is

$$v_s = v_{\parallel} \cos \alpha = v \mu \cos \alpha$$  \hspace{1cm} (7.9)

(as used by Galeev, 1986b and Sagdeev et al., 1986, for example) where $0 < \alpha < 90$ degrees is the angle between the solar wind flow and the $B$-field (see Figure 7.1). The ion injection geometry is such that $\mu = \cos \theta = -\cos \alpha$ is initially negative for a newly-born ion in the solar wind frame. The derivative of $F$ with respect to $v_s$ in equation (7.6) is obtained from $\partial F / \partial v_{\parallel}$, which needs to be transformed from the cartesian coordinates, $(v_{\parallel}, v_{\perp})$, into the new system, $(v, \mu)$. This is done using:

$$\frac{\partial F}{\partial v_{\parallel}} = \frac{\partial F}{\partial \nu} \frac{\partial \nu}{\partial v_{\parallel}} + \frac{\partial F}{\partial \mu} \frac{\partial \mu}{\partial v_{\parallel}}$$  \hspace{1cm} (7.10)
The new variables may be expressed in terms of the old,
\[ v = \left( v_\parallel^2 + v_\perp^2 \right)^{1/2} \]  
(7.11)
\[ \mu = \cos \theta = \frac{v_\parallel}{v} = \frac{v_\parallel}{\left( v_\parallel^2 + v_\perp^2 \right)^{1/2}} \]  
(7.12)
for which the partials with respect to \( v_\parallel \) may be obtained as follows:
\[ \frac{\partial v}{\partial v_\parallel} = \frac{1}{2} \left( v_\parallel^2 + v_\perp^2 \right)^{-1/2} \cdot 2v_\parallel = \frac{v_\parallel}{v} = \mu \]  
(7.13)
\[ \frac{\partial \mu}{\partial v_\parallel} = \frac{1}{\left( v_\parallel^2 + v_\perp^2 \right)^{1/2}} - \frac{1}{2} \frac{v_\parallel}{\left( v_\parallel^2 + v_\perp^2 \right)^{3/2}} \cdot 2v_\parallel = \frac{1 - \mu^2}{v} \]  
(7.14)
These are substituted into equation (7.10) to give [Lyons and Williams, 1984]:
\[ \frac{\partial F}{\partial v_\parallel} = \frac{\partial F}{\mu \partial v} + \frac{(1 - \mu^2)}{v} \frac{\partial F}{\partial \mu} \]  
(7.15)
Now from equation (7.9), \( \partial v_\parallel = \partial v_*/\cos \alpha \), and hence
\[ \frac{\partial F}{\partial v_*} = \frac{\mu}{\cos \alpha} \frac{\partial F}{\partial v} + \frac{(1 - \mu^2)}{v \cos \alpha} \frac{\partial F}{\partial \mu} \]  
(7.16)
which is the required component of change along the \( s \)-axis relative to the solar wind flow.

Finally, replacing the \( v_* \) terms in (7.6), the steady state equation for \( F(s, v, \mu) \) becomes:
\[ \left( u + v \mu \cos \alpha \right) \frac{\partial F}{\partial s} - \left( \frac{u}{\cos \alpha} + v \mu \right) \frac{\partial u}{\partial s} \left( \frac{\partial F}{\partial v} + \frac{(1 - \mu^2)}{v} \frac{\partial F}{\partial \mu} \right) = \frac{\delta F}{\delta t} + \left. \frac{\partial F}{\partial t} \right|_{w-p} \]  
(7.17)
Expressions for the source and velocity diffusion terms now need to be formulated.

### 7.2 Source Distribution

The 'source' term in equation (7.1) is taken to include both the rate of change of \( F \) due to the addition of new cometary ions, and a loss to \( F \) as a result of charge exchange with cometary neutrals [Gombosi, 1988]:
\[ \frac{\delta F}{\delta t} = S_{inj} N_c \nu - |u + v| \frac{F}{\lambda} \]  
(7.18)
Figure 7.2: Simulation results of Gombosi [1988] for the ionization lifetime, \(1/\nu\), of cometary oxygen and hydrogen, as a function of distance from the nucleus along the subsolar flowline (for which the bow shock occurs at around \(5 \times 10^5\) km).

The first term on the R.H.S. is the source term, which is the production rate of cometary ions by ionization of the neutrals, \(N_c \nu\), (as used in the mass-loading model of Chapter 4) distributed here according to \(S_{inj}\), the initial one-particle velocity-space distribution of the new ions. The term is in units of (phase space) density per second. The cometary neutral density is a function of distance, \(r\), to the nucleus given by

\[
N_c = \frac{Q}{4\pi V_c r^2} \times \exp \left( -\frac{\nu r}{V_c} \right)
\]  

(7.19)

(as in Chapter 4.) The ionization rate is:

\[
\nu = \nu_{ph} + \sigma \left( n_{sw} u_{sw} + n_i u_i \right)
\]  

(7.20)

which includes photoionization, \(\nu_{ph}\), and ionization due to charge exchange with both the solar wind and the implanted heavy ions (with average flux \(n_i u_i\)). Plots of simulation results [Gombosi, 1988] show the ionization rate remaining constant upstream of the bow shock, Figure 7.2, and so for the purposes of this analysis it seems reasonable to take a fixed value of \(\nu\).

The second term in equation (7.18) is the loss term, in which \(\lambda\) is the charge transfer mean free path and may be written

\[
\frac{1}{\lambda} = \sigma N_c
\]  

(7.21)
in terms of the charge transfer cross-section, $\sigma$, and the neutral density, $N_c$. The magnitude of the relative velocity vector between the ion and neutral (assuming the neutrals are at rest in the frame of the comet) is $|u + v|$. This vector sum in terms of the components of $u$ and $v_\parallel$ along and perpendicular to $s$ is:

$$|u + v| = \left[ (u + \mu v \cos \alpha)^2 + (\mu v \sin \alpha)^2 \right]^\frac{1}{2}$$  \hfill (7.22)

Note that the implanted ions lost through charge exchange are in various stages of pick-up in $F$, whereas the newly created cometary ions are described by $S_{inj}$, and therefore the source and loss terms for charge exchange between cometary ions and neutrals do not cancel.

The source distribution has the form of a ring, or torus in velocity space (see Chapter 5) which is represented by [Wu and Davidson, 1972]:

$$S_{inj} = \left( \frac{1}{2\pi u^2} \right) \delta(v - u) \delta(\mu + \cos \alpha)$$  \hfill (7.23)

in which the $\delta$-functions set the initial implanted-ion velocity magnitude in the solar wind frame equal to $u$, and the initial direction antiparallel to the solar wind, $\mu = \cos \theta = -\cos \alpha$. $S_{inj}$ is a normalized distribution for one pick-up ion, since it multiplies the density in equation (7.18). Thus the integral of $S_{inj}$ over all velocity space must be equal to unity and the $(1/2\pi u^2)$ term is required in order to give this. The integration is performed in the following way.

A function $F(s, v, \mu)$ represents the distribution in velocity space of the ion density at position $s$. Since the component of the ion velocity vector perpendicular to the field, $v_\perp$, becomes a gyration about the field lines with random phase, $F$ does not vary around the axis of $B$. Thus the system in velocity space is equivalent to a simplification of spherical coordinates (see Figure 7.3) with no $\phi$ dependence, such that the $\phi$-integral reduces:

$$\int_0^{2\pi} F(s, v, \mu = \cos \theta) v \sin \theta \, d\phi = F(s, v, \mu) 2\pi v \sin \theta$$  \hfill (7.24)

where $2\pi v \sin \theta$ is simply the circumference of the torus of radius $v \sin \theta$ illustrated in Figure 7.3. The area of the elemental torus is approximated by length (circumference) times breadth $= 2\pi v \sin \theta \times v d\theta$, where $v d\theta$ is the elemental arclength subtended by angle $d\theta$. Hence the area integral over the
Figure 7.3: Schematic diagram showing infinitesimal elements of surface and volume integration for a gyrotropic distribution in velocity space.

The entire spherical surface of radius \( v \) is:

\[
2\pi \int_0^\pi F(s,v,\mu) v^2 \sin \theta \, d\theta
\]  

(7.25)

But \( \mu = \cos \theta \); \( d\mu = -\sin \theta \, d\theta \). The limits become \( \cos(0) = 1 \), \( \cos(\pi) = -1 \), and may be swapped over to remove the minus in \( d\mu \). The integration becomes

\[
F(s,v) = 2\pi \int_{-1}^1 F(s,v,\mu) v^2 \, d\mu
\]  

(7.26)

(Note that if this integral is divided by \( 4\pi \) (solid angle), the result is an average, isotropic density distribution \( F(s,v) \) of ions per unit solid angle of velocity direction, with speed \( v \), at position \( s \).) So far, integration over the velocity-space angular distribution has been performed. To integrate \( F(s,v,\mu) \) over all velocity space requires the volume integral:

\[
F(s) = 2\pi \int_0^\infty \int_{-1}^1 F(s,v,\mu) v^2 \, d\mu \, dv
\]  

(7.27)

which includes a further integral over the velocity-space “shells” of width \( dv \), radius \( v \) from 0 to \( \infty \).
The integral operator may now be applied to $S_{inj}$:

$$2\pi \int_0^\infty dv \int_{-1}^1 d\mu v^2 S_{inj} = \int_0^\infty dv \int_{-1}^1 d\mu \frac{v^2}{u^2} \delta(v - u) \delta(\mu + \cos \alpha) = 1 \quad (7.28)$$

(Note that through the $\delta$-function, $1/(2\pi v^2)$ is an equivalent normalization factor.)

The complete source term required is:

$$\frac{\delta F}{\delta t} = \frac{N_c \nu}{2\pi u^2} \delta(v - u) \delta(\mu + \cos \alpha) - \left[ (u + \mu v \cos \alpha)^2 + (\mu v \sin \alpha)^2 \right]^{\frac{1}{2}} \sigma N_c F \quad (7.29)$$

### 7.3 Quasilinear Velocity Diffusion

An equation for the rate of change of $F$ due to wave-particle interactions may be obtained from quasi-linear theory. The derivation briefly outlined below follows the analysis of a series of papers by Lee [1971; 1982], Lee and Ip [1987].

The Vlasov and Maxwell equations for a system of particles with phase space density $N(x, p, t)$ are [Lee, 1971] (in S.I. units):

$$\frac{\partial N}{\partial t} + \mathbf{v} \cdot \frac{\partial N}{\partial x} + \frac{q}{mc} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \frac{\partial N}{\partial p} = 0 \quad (7.30)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (7.31)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (7.32)$$

$$\varepsilon_0 \nabla \cdot \mathbf{E} = q n \int N \, d^3 p \quad (7.33)$$

$$\frac{\nabla \times \mathbf{B}}{\mu_0} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + q n \int N \, \mathbf{v} \, d^3 p \quad (7.34)$$

Here $p$ is a dimensionless momentum, given by $p = \bar{p}/(mc)$, where $\bar{p}$ is the real relativistic momentum of the particle of rest mass $m$.

In the quasi-linear theory the $N$, $\mathbf{B}$ and $\mathbf{E}$ are separated into uniform parts and small fluctuating wave components [Kennel and Engelmann, 1966]. $\mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B}$ (see Chapter 5) where $\mathbf{B}_0 = B_0 \hat{z}$ is the ambient magnetic field, and $N = F + \delta N$ where $F$ is the ensemble average phase space density $< N >$. There is no static $\mathbf{E}$-field ($< \mathbf{E} > = 0$) and the average charge and
current densities are zero. The ensemble average of equation (7.30) may be subtracted from the original to obtain the Vlasov equation in fluctuation components, which is then linearized, neglecting terms of second order and above in the fluctuation amplitudes. The resulting equation is transformed into \((k, \omega)\)-space using a Fourier-Laplace transform to give [Lee, 1971]:

\[
\Omega_i \frac{\partial}{\partial \phi} \delta N(k, p, \omega) + i(\omega - k \cdot v) \delta N(k, p, \omega) =
\]

\[- \delta N(k, p, 0) + \frac{q}{mc} [\delta E(k, \omega) + v \times \delta B(k, \omega)] \cdot \frac{\partial F(p, t)}{\partial p} (7.35)\]

where \(\delta/\delta x \equiv ik\) and \(\delta/\delta t \equiv -i\omega\) have been identified (see Chapter 5) and \(\phi\) is the angle between \(p_\perp\) and the \(z\)-axis in the perpendicular plane of gyration about \(B_0\). Similarly, the Maxwell equations in \((k, \omega)\)-space are:

\[
\begin{align*}
\textbf{i}k \cdot \delta \textbf{B}(k, \omega) &= 0 \\
\textbf{i}k \times \delta \textbf{E}(k, \omega) &= i\omega \delta \text{B}(k, \omega) + \delta \text{B}(k, 0) \\
\varepsilon_0 \textbf{i}k \cdot \delta \text{E}(k, \omega) &= q n \int \delta N(k, p, \omega) \, d^3p \\
\frac{\textbf{i}k \times \delta \text{B}(k, \omega)}{\mu_0} &= -i\omega \varepsilon_0 \delta \text{E}(k, \omega) - \varepsilon_0 \delta \text{E}(k, 0) + q n \int \delta N(k, p, \omega) \, v \, d^3p
\end{align*}
\]

\[(7.36 - 7.39)\]

The ensemble-average Vlasov equation for a spatially homogeneous plasma may be written in terms of an integral over \(k\) using the equivalence of \(V \delta_{k+k'} \rightarrow (2\pi)^3 \delta(k+k')\) for wavelengths in a box of volume \(V\) (3 degrees of freedom) and further integrated over \(\phi\) to give [Lee, 1971]:

\[
\frac{\partial F(p||, p_\perp, t)}{\partial t} = -\frac{q}{mc} \frac{1}{(2\pi)^3 V} \int_0^{2\pi} \frac{d\phi}{2\pi} \int d^3k
\]

\[
	imes \left\langle (\delta \text{E}(-k, t) + v \times \delta \text{B}(-k, t)) \cdot \frac{\partial \delta N(k, p, t)}{\partial p} \right\rangle (7.40)
\]

Implicit in the \(\phi\)-integration is the assumption of a very rapid particle gyroperiod (compared to the timescale on which \(F\) will change) so that a gyrotropic distribution of particles may be assumed on a macroscopic scale. In other words, a particle completes many orbits before diffusing significantly. Thus an important assumption in quasi-linear theory is that the growth-rate of the waves (which cause the diffusion) is much less than the gyrofrequency [Kennel and Engelmann, 1966]. In the 'limit of resonant diffusion', \(\gamma_k \rightarrow 0\) where
the growth rate, $\gamma_k$, is the imaginary component of complex wave frequency ($\omega = \omega_R + i\gamma_k$).

Solutions of equations (7.35) to (7.39) are extremely complex. The least stable roots are required as these waves dominate the long time evolution of the plasma. In this case the most important parallel-propagating, transverse waves are Alfvén and magnetosonic waves [Lee, 1971]. Assuming the electromagnetic and electrostatic fluctuations decouple, they may be obtained independently for all $k$. On combining equations (7.35), (7.37) and (7.39) and solving for $\delta E_\pm(k||,\omega)$ (where $\pm$ denotes the polarization) for electromagnetic waves, the E fluctuations $\delta E_\pm(k||,t)$ may then be found in terms of $k$ and $t$ by taking the inverse Laplace transform. For each frequency, both the parallel and antiparallel propagating waves are included in the Laplace $\omega$-integration. The magnetic fluctuations $\delta B_\pm(k||,t)$ are then obtained from equation (7.37) using $\delta E_\pm(k||,\omega)$ and $\delta E_\pm(k||,t)$.

An equation for $\delta N(k||,p,t)$ produced by the electromagnetic fluctuations may be computed in terms of $\delta E_\pm$ and $\delta B_\pm$ from equation (7.35), on taking the inverse Laplace transform. Substituting for $\delta N$, $\delta E_\pm$ and $\delta B_\pm$ in equation (7.40) gives, after much algebra, an equation for the evolution of the particle distribution function due to interactions with electromagnetic waves [Lee, 1971 (his equation 49, term 5); Lee, 1982]:

$$\frac{\partial F}{\partial t} = \frac{q^2}{2p_\perp} Re i \int_\Omega \sum_{j=\pm} \left( \frac{\omega_j}{k} \right)^2 \hat{G}_{R,j} \left[ \frac{p_\perp P_j(k)}{\omega_j - kv_\parallel + \Omega_i} \right] dk$$ (7.41)

where the operator is:

$$\hat{G}_{R,j} = \frac{\partial}{\partial p_\perp} + \frac{k}{\omega_j} \left( v_\perp \frac{\partial}{\partial p_\parallel} - v_\parallel \frac{\partial}{\partial p_\perp} \right)$$ (7.42)

It is clear that the magnitude of the diffusion given by equation (7.41) for a particle of velocity $v_\parallel$ will be dominated by the resonance with the wave $k$ for which $\omega_j - kv_\parallel + \Omega_i = 0$ for the principle cyclotron harmonic resonance (see Chapter 5). Thus the interaction can be limited to the resonant case for small wave growth rate conditions, when the effect of non-resonant waves on the particles becomes insignificant. In the 'limit of resonant diffusion' the following relation may be used [Cairns, 1985]:

$$\text{Im} \left( \frac{1}{\omega_j - kv_\parallel + \Omega_i} \right) \rightarrow \pi \delta(\omega_j - kv_\parallel + \Omega_i) \quad \text{as} \quad \gamma_k \rightarrow 0$$ (7.43)
Taking also the non-relativistic limit, then \( p_\perp = m v_\perp, p_\parallel = m v_\parallel \) and the ion gyrofrequency \( \Omega_i = q B_0 / \Gamma m \) becomes \( q B_0 / m \) (in S.I. units). Equation (7.41) then becomes [Isenberg, 1987a]:

\[
\frac{\partial F}{\partial t} \bigg|_{w-p} = \frac{\pi \Omega_i^2}{2 B_0^2} \sum_{j=\pm} \int_{-\infty}^{\infty} \frac{1}{v} \hat{G}_j \left[ \frac{v_j^2}{v} \left( \frac{\omega_j}{k} \right)^2 P_j(k) \delta(\omega_j - k v_\parallel - \Omega_i) \hat{G}_j F \right] dk
\]  

(7.44)

(Note the subscript \( w-p \), as used in the early part of this chapter, indicates that the \( \partial F / \partial t \) is due to wave-particle interactions.) The operator \( \hat{G}_{R,j} = v_\perp/(mv) \hat{G}_j \) has been replaced, where

\[
\hat{G}_j = \left[ \frac{\partial}{\partial v} + \left( \frac{k}{\omega_j} - \frac{\mu}{v} \right) \frac{\partial}{\partial \mu} \right]
\]  

in \( v, \mu \) coordinates. \( P_j(k) \) is the Alfvénic wave power (examined in Chapter 6) at wavenumber \( k \), frequency \( \omega_j \) where \( j (= \pm) \) denotes the direction of propagation and the sign of \( k \) gives the helicity of the wave. \( P_j(k) \) is normalized such that [Lee, 1982]

\[
< \delta B \cdot \delta B > = \int P(k) dk
\]  

(7.46)

where \( P(k) = \sum_{\pm} P_{\pm}(k) \). The waves are assumed to be unpolarized and parallel-propagating, with velocity, \( V_A \), determined by the local values of the plasma mass density, \( \rho \), and magnetic field, \( B_0 \) :

\[
\pm V_A = \frac{\omega_\pm}{k} = \pm \frac{B_0}{(\mu_0 \rho)^{\frac{1}{2}}}
\]  

(7.47)

\( V_A^2 \) may be substituted for \( (\omega_j/k)^2 \) in equation (7.44).

For the low frequencies \( \omega_j \ll \Omega_i \) considered (hydromagnetic waves), the \( \delta \)-function selects the waves for resonant interaction with an ion of field-aligned velocity \( v_\parallel \) by setting

\[
|k| = \frac{\Omega_i}{v_\parallel} = \frac{q B_0}{m v_\parallel}
\]  

(7.48)

in \( P_j(k) = A_j |k|^{-\gamma} \). It can be seen from the resonance condition that taking \( \omega_j \ll \Omega_i \) is justified for \( v_\parallel \gg \omega/k \), i.e. for \( v_\parallel \gg V_A \) [Isenberg, 1987a]. Since the solar wind velocity is superalfvénic (in the preshock region), the assumption is good for pick-up ions with small \( \theta \), large \( \mu \), but not good as the pitch-angle approaches 90°.
Integrating equation (7.44) according to the $\delta$-function gives [Isenberg, 1987a]:

$$\frac{\partial F}{\partial t}_{w-p} = \frac{\pi \Omega_i^2}{2B_0^2} V_A^2 \sum_{j=\pm} \frac{A_j}{v} \hat{G}_j \left[ \frac{v_i^2}{v_{||}} \left( \frac{\Omega_i}{v_{||}} \right)^{-1} \hat{G}_j F \right]$$ (7.49)

Assuming $\gamma = 2$ from the results of Chapter 6, and putting in $V_A^2 = B_0^2/(\mu_0 \rho)$, the equation for the rate of change of $F$ under resonant wave-particle interactions becomes:

$$\frac{\partial F}{\partial t}_{w-p} = \frac{\pi}{2\mu_0 \rho} \frac{1}{v} \sum_{j=\pm} A_j \hat{G}_j \left[ v^2(1 - \mu^2)|\mu| \hat{G}_j F \right]$$ (7.50)

Where $v_i^2 = v^2(1 - \mu^2)$ and $v_{||} = \mu v$ have been replaced in terms of $\mu$ and $v$, and the operator $\hat{G}_j$ is as given above in equation (7.45).

### 7.4 Numerical Analysis

#### 7.4.1 Method of Solution

A numerical solution is sought for the simplified cometary ion transport equation:

$$(u + \mu v \cos \alpha) \frac{\partial F}{\partial s} = \frac{\delta(v - u) \delta(\mu + \cos \alpha)}{2\pi u^2} N_c v - \left[ (u + \mu v \cos \alpha)^2 + (\mu v \sin \alpha)^2 \right]^{\frac{1}{2}} \sigma N_c F$$

$$+ \frac{\pi}{2\mu_0 \rho} \frac{1}{v} \sum_{j=\pm} A_j \hat{G}_j \left[ v^2(1 - \mu^2)|\mu| \hat{G}_j F \right]$$

where the wave-particle interaction term written in full is:

$$\frac{\partial F}{\partial t}_{w-p} = \frac{\pi}{2\mu_0 \rho v} \left\{ A_+ |\mu|(1 - \mu^2) \frac{\partial}{\partial v} \left[ v^2 \frac{\partial F}{\partial v} + v^2 \left( \frac{1}{V_A} - \frac{\mu}{v} \right) \frac{\partial F}{\partial \mu} \right] \right.$$

$$+ A_- |\mu|(1 - \mu^2) \frac{\partial}{\partial \mu} \left[ v^2 \frac{\partial F}{\partial v} + v^2 \left( \frac{1}{V_A} - \frac{\mu}{v} \right) \frac{\partial F}{\partial \mu} \right]$$

$$+ A_+ \left( \frac{1}{V_A} - \frac{\mu}{v} \right) v^2 \frac{\partial}{\partial v} \left[ |\mu|(1 - \mu^2) \frac{\partial F}{\partial v} + |\mu|(1 - \mu^2) \left( \frac{-1}{V_A} - \frac{\mu}{v} \right) \frac{\partial F}{\partial \mu} \right]$$

$$+ A_- \left( -\frac{1}{V_A} - \frac{\mu}{v} \right) v^2 \frac{\partial}{\partial \mu} \left[ |\mu|(1 - \mu^2) \frac{\partial F}{\partial v} + |\mu|(1 - \mu^2) \left( -\frac{1}{V_A} - \frac{\mu}{v} \right) \frac{\partial F}{\partial \mu} \right]$$

The basic method of solution is outlined as follows.
• Assume constant values of the cometary gas emission and ionization parameters. A constant value of $\alpha$ is taken on a given flowline.

• Obtain profiles along a flowline for $N_c$, $n_i$, $u_{sw}$, $n_{sw}$ according to the mass-loading model.

• Start with $F$ for a ring distribution of the initial $n_i(s)$ ions at $s = 0$, a long distance from the comet.

• Calculate $\partial F/\partial s$ on the LHS of equation (7.51) from the source and gradients of $F$ at each $\mu, v$ grid point (in the 2-dimensional $(\mu, v)$ array of $F(s, \mu, v)$ values at this $s$) and ‘integrate’ to obtain $F$ at the next $s$. The distribution thus evolves along the flowline and the process may be stopped at a chosen position (after enough $s$-steps) to examine the theoretical distribution there.

• The process may be repeated on a set of flowlines terminating on the Giotto path, in order to build up a picture of the evolution of $F$ as seen by the spacecraft, which can be compared with measurements.

The constant values assumed in the analysis of this chapter are the cometary gas production rate, $Q = 10^{30}$ s$^{-1}$, expansion velocity, $V_e = 1$ km s$^{-1}$, charge exchange cross-section, $\sigma = 2 \times 10^{-19}$ m$^2$, ionization rate, $\nu = 10^{-6}$ s$^{-1}$ and $u_{\infty} = 366$ km s$^{-1}$ from the mass-loading model results of Chapter 4, with $n_{sw} u_{sw} = 2.27 \times 10^{12}$ m$^{-2}$ s$^{-1}$ and $m_i = 16$ amu for implanted oxygen ions. The Alfvén velocity is taken to be $V_A = 40$ km s$^{-1}$ (based on the values plotted in Chapter 5).

For completeness, the equations from Chapter 4 for calculation of $N_c$, $n_i$, $u_{sw}$ and $n_{sw}$ profiles from the mass-loading model results are reproduced here:

\[ N_c = \frac{Q}{4\pi V_e r^2} \times \exp \left( -\frac{\nu r}{V_e} \right) \]  

\[ n_i = \frac{1}{u_{sw}} \frac{Q}{4\pi L^2} \int_{x_0}^{\infty} \frac{1}{(x^2 + y_0^2)} \exp \left[ -(x^2 + y_0^2)^{1/2} \right] \, dx \]  

\[ L = \frac{V_e}{\nu} \]  

\[ u_{sw} = u_{\infty} \times \left( 2 - \left[ 4 - 3 \times \left( 1 + \frac{n_i u_i}{n_{sw} u_{sw}} \frac{m_i}{m_{sw}} \right) \right]^{1/2} \right)^{-1} \]
\[ n_{sw} = \frac{(n_{sw} u_{sw})}{u_{sw}} \]  

(7.57)

where \( u_i = u_{sw} \) is assumed.

The power amplitudes, \( A_+ \) and \( A_- \) are estimated in the following way. The free energy, \( E_F = E_{F,u} + E_{F,d} \), available to the turbulent wave spectrum may be calculated at any point on any flowline from \( V_A, u_{sw} \) and \( \alpha \) values, using the equations derived in Chapter 5. From \( E_F \), the turbulent wave energy density, \( U_s \), is then estimated using the quadratic for \( U_s \) as a function of \( E_F \) fitted in Section 5.5. The solar wind ‘background’ waves, with energy density \( U_{bg} \sim 0.2 \text{ pJ m}^{-3} \) (see Chapter 5), are assumed to be split into equal intensity populations propagating in the directions parallel and antiparallel to the magnetic field. For the cometary-ion pick-up generated waves, the ratio of the upstream to downstream free energies may be used to obtain upstream and downstream proportions of \( U_s \) (assumed to be in the same ratio). The power spectrum is fitted in Chapter 6 such that \( A_k = A_- + A_+ = \mu_0 U_s k_L \) and hence the upstream and downstream power amplitudes, \( A_- \) and \( A_+ \) respectively, are then obtained from:

\[
A_- = \left[ \frac{U_{bg}}{2} + \left( \frac{E_{F,u}}{E_{F,u} + E_{F,d}} \right) \times (U_s - U_{bg}) \right] \mu_0 k_L 
\]  

(7.58)

\[
A_+ = \left[ \frac{U_{bg}}{2} + \left( \frac{E_{F,d}}{E_{F,u} + E_{F,d}} \right) \times (U_s - U_{bg}) \right] \mu_0 k_L 
\]  

(7.59)

where \( k_L = 9 \times 10^{-8} \text{ m}^{-1} \).

For the numerical analysis, the phase space of interest is divided into ‘bins’, all with finite widths \( \Delta \mu \), and \( \Delta v \). (Note that the volume of velocity space covered by such bins of equal \( \mu \) and \( v \) spacing is larger at higher values of \( v \)). Phase space density of cometary ions at each \( s \)-position on the flowline is put into these bins and specified in a 2-dimensional array, \( F(\mu, v) \), of values giving the number density of ions per bin, where each bin is centred on a particular \( \mu \) and \( v \) in the solar wind frame.

The lowest \( v \)-bin may be centred at a value of \( v = 0 + (\Delta v/2) \) if coverage is required from \( v = 0 \) (representing ions with velocity identical to that of the solar wind bulk velocity). For each value of \( v \) there is a one-dimensional array of bins for different values of pitch-angle. The width \( \Delta \mu \) is equal to
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2/(total number of \( \mu \) bins) since \(-1 < \mu < 1 \) where \( \mu = \cos \theta \), and the first bin centred on \(-1 + (\Delta \mu/2) \) includes ions with pitch-angles between \(-1 \) and \(-1 + \Delta \mu \).

Partial differentials may be computed using the numerical method of ‘finite differencing’ (see for example Mitchell [1969], Conte [1965], Press et al. [1986]). The gradients obtained are approximations to the ‘analytical’ case (which would be achieved in the infinitesimal limit of \( \Delta \mu, \Delta v \to 0 \)) because the accuracy is limited by the size of the bins. As an example of finite differencing, the slope of \( F(\mu, v) \) along an increment in \( v \) (at a given \( \mu \) and \( v \)) is the partial differential of \( F \) with respect to \( v \):

\[
\left( \frac{\partial F}{\partial v} \right)_{\mu,v} \equiv \frac{F_{\mu,v} - F_{\mu,v-1}}{\Delta v}
\]  

(7.60)

where \( \Delta v \) is the bin width, \( F_{\mu,v} \) denotes the value in the bin centred on \( \mu \) and \( v \), and \( F_{\mu,v-1} \) the value in the previous \( v \) bin centred at \( v - \Delta v \) and the same \( \mu \). Second derivatives (curvature) are given by calculations such as

\[
\left( \frac{\partial^2 F}{\partial \mu^2} \right)_{\mu,v} \equiv \frac{1}{\Delta \mu} \left( \frac{F_{\mu+1,v} - F_{\mu,v}}{\Delta \mu} - \frac{F_{\mu,v} - F_{\mu-1,v}}{\Delta \mu} \right) = \frac{(F_{\mu+1,v} - 2F_{\mu,v} + F_{\mu-1,v})}{(\Delta \mu)^2}
\]

(7.61)

or, for example

\[
\left( \frac{\partial}{\partial v} \left[ v \frac{\partial F}{\partial \mu} \right] \right)_{\mu,v} \equiv \frac{1}{\Delta v} \left( v^2 \frac{F_{\mu,v} - F_{\mu-1,v}}{\Delta \mu} - (v - \Delta v)^2 \frac{F_{\mu,v-1} - F_{\mu-1,v-1}}{\Delta \mu} \right)
\]

(7.62)

Since gradient calculations will use values in the bins one either side of the bin to which the gradient applies, boundary conditions are necessary to enable these calculations to be performed at the bins on the edge of the grid. The boundary conditions are specified as an extra bin beyond each edge of the range so that the required 2-dimensional velocity space grid (or array within which the solution for \( F(\mu, v) \) is sought) is ‘surrounded’ by boundary-condition bins which are reset on each \( s \)-step to correspond to the new distribution. Note that for the \( \mu \) range, where all possible pitch-angles are completely covered, these extra bins outside the range have no physical significance. Boundary conditions on \( \mu \) are required such that when complete isotropy around the shell is achieved, the gradients in \( \mu \) at the edge of the \( \mu \)-range are zero, the same as everywhere else in the range. Thus the relevant
boundary bins are each assigned a value equal to that of the adjacent bin just inside the \( \mu \)-range. Similar conditions are required for \( v \), unless enough \( v \)-space is to be covered so that \( F \) may be set to zero at the upper limit.

At the position in space chosen for \( s = 0 \), a long distance from the comet, an initial ring distribution is the starting point for \( F(s = 0, \mu, v) \), where the \( n_i(s = 0) \) ions are all assigned to the bin centred nearest to \( \mu = -\cos \alpha \), \( v = u_{sw} \). For each \( s \)-step in the numerical run along a flowline, the partial differentials of the velocity diffusion term on the RHS of equation (7.51) are calculated from \( F \) for every grid-point or bin in the 2-dimensional \((\mu, v)\)-array. To this RHS is added the 'source and loss' term. Note that in the numerical implementation of the source term, \( N_e \nu \) source ions all go into one bin and the \( 1/(2\pi u^2) \) multiplier (which would give a phase space density, rather than a density of ions per bin) is not employed. The RHS at each \( \mu, v \) is then multiplied by \( (u + \mu v \cos \alpha)^{-1} \) to give \( (\partial F/\partial s)_{\mu, v} \), which is integrated:

\[
F_{s+1, \mu, v} = F_{s, \mu, v} + \Delta s \times \left( \frac{\partial F}{\partial s} \right)_{\mu, v}
\]

(7.63)

to give a new array of \( F \) values corresponding to the next \( s \)-position.

### 7.4.2 Numerical Limitations

The velocity bin size needs to be small enough in order to maintain accuracy, and the \( s \)-steps (or distance, \( \Delta s \), between points on the flowline at which the evolution of \( F \) is evaluated) must also be small to avoid an 'overcorrection' of the distribution between steps. As an illustration of this problem, consider Figure 7.4. This shows a hypothetical example of the \( \mu \)-dependence of \( F \) in a 1-dimensional 'slice' through the velocity-space array at a fixed \( v \). At the particular \( \mu \)-bin represented by point number 3 in this figure, the second derivative of \( F \) with respect to \( \mu \) is negative, since the curvature is concave downwards. This should act to reduce the value of \( F \) in this bin for the next \( s \). At point number 2 preceding it, the curvature is positive which leads to an increase in \( F \) at this point. In this fashion, the peak should be smoothed and spread outwards as the distribution evolves. However, if the values of the double-differentials are large, then, for example at point 3, the negative adjustment to \( F \) may be larger than the value of \( F \) itself at this point such
Figure 7.4: Diagram to illustrate how 'overcorrection' may result in an oscillation of the density values in the bins.
FORWARD CENTERING

BACKWARD CENTERING

Figure 7.5: Forward and backward centering, respectively, are used in partial differential calculations of the present analysis at values of \( \mu \) less than, and greater than that of ion injection.

that on integration (see equation (7.63)) \( F \) becomes negative here at the next \( s \). This overcorrection could ultimately lead to an oscillating distribution as shown in the figure. In this case the numerical method does not achieve convergence to a solution. On the other hand, if the \( s \)-steps are kept small, then \( \Delta s \times \text{RHS} \) leads to small adjustments to \( F \) on each \( s \)-step.

Another problem may arise due to inaccuracy in assigning a gradient to a particular bin which inevitably occurs in calculation of some derivatives because bin sizes are finite. In Figure 7.5, four adjacent binned values of \( F \) are schematically represented as 'density' pixels where the darker the shading, the larger the value of \( F \) for the bin. In both of the examples shown, the double differential \( \partial^2 F/(\partial v \partial \mu) \) is obtained on subtracting the \( \mu \)-gradient of the lower pair in the figure from the gradient of the upper pair. The 'centre' of such a gradient calculation lies between the four bins, but in practice must be applied to one of them. If the resulting differential is negative, for example, then the error in relating it to the bin at which \( F \) is zero (unshaded in the figure) could cause problems such as were described above. The two
cases in the figure represent, firstly, a 'forward centering' approach, for which double differentials are (in this example)

$$\left( \frac{\partial^2 F}{\partial v \partial \mu} \right)_{\mu,v} = \frac{1}{\Delta v \Delta \mu} \left( [F_{\mu,v} - F_{\mu-1,v}] - [F_{\mu,v-1} - F_{\mu-1,v-1}] \right)$$  \hspace{1cm} (7.64)

and 'backward centering':

$$\left( \frac{\partial^2 F}{\partial v \partial \mu} \right)_{\mu,v} = \frac{1}{\Delta v \Delta \mu} \left( [F_{\mu+1,v+1} - F_{\mu,v+1}] - [F_{\mu+1,v} - F_{\mu,v}] \right)$$  \hspace{1cm} (7.65)

### 7.5 Implanted Ion Distributions

This section presents the preliminary results of the numerical evaluation of the quasilinear diffusion described by equations (7.51) and (7.52). As previously discussed, an initial ring distribution is taken at $s = 0$ and the numerical code is run along $s$ towards the comet until an end-point on the Giotto trajectory is reached. To obtain the distributions which are examined below, the starting distance is $5 \times 10^6$ km upstream from the end point. This is set by the use of 50,000 $s$-steps at $\Delta s = 10^5$ m (= 100 km) in the numerical computations. (It is worth noting that increasing the number of steps to e.g. 80,000 at the same $\Delta s$ has a virtually imperceptible effect on the final distribution. The smaller number of $s$-steps is favoured, considering the vast computing time required to calculate, at every $s$-step, the partial gradients for every bin in a two-dimensional array.)

The phase-space density is required from the binned $F$ values (which represent the sum of ion phase-space density over the bin-volume of $v$-space). Consider the volume of velocity space covered by a bin centred on $\mu, v$. An annulus subtended by an angle $\Delta \theta$ on a shell of radius $v$ (where $|\Delta \mu| = |\sin \theta \Delta \theta|$) has a surface area $2\pi v^2 \Delta \mu$ (see Section 7.2 and Figure 7.3). Hence the volume of $v$-space in the bin is $2\pi v^2 \Delta \mu \Delta v$. From this, phase space density values may be derived from the binned values

$$\text{Phase space density, } F(\mu, v) = \frac{F_{\mu,v} \text{bin}}{2\pi v^2 \Delta \mu \Delta v}$$  \hspace{1cm} (7.66)

the units of which are $m^{-6} s^3$.

Water-group ion phase-space density distributions thus obtained from the numerical analysis are plotted on ($v_\|, v_\bot$) coordinate axes as grey-scale
Figure 7.6: Development of the numerical ion distribution along a solar wind plasma flowline. $\Delta s = 10^5$ km, 29 $\mu$-bins and 19 $v$-bins were used in the numerical computations, where $\Delta v = 30$ km s$^{-1}$. (Figure continues overleaf).
Figure 7.6 (continued)
Figure 7.6 (continued)
density pixels. Figure 7.6 presents the numerical distributions obtained (in
the process of a single run) at six equally-spaced distances along the flowline
that ends at \( D = 2.0 \times 10^6 \) km from the comet on the Giotto trajectory. This
shows a combination of both the evolution of the implanted ion distribution
along a flowline according to quasilinear theory with assumptions previously
discussed, and the operation of the numerical code used in the analysis.
Examples for different flowlines are given in Figure 7.7 for the distances 2.0,
2.5, 3.0, and 4.5 \( \times 10^6 \) km from Halley on the Giotto path. An average value
of the measured \( \alpha \) over the distance between these points (from the Giotto
data) is taken for the flowline \( \alpha \) in each case. The distributions all appear
highly evolved in pitch-angle.

In order to investigate pitch-angle evolution, the 10%-width of a distribu­
tion may be defined as the total angular spread either side of the peak
out to where the phase space density has reduced to \( \frac{1}{10} \) th that of the peak.
\( F_{\mu,v} \) bins are first collapsed into \( \mu \)-bins only, by summation over all \( v \)-bins
at each \( \mu \), and the ion density in these bins is then examined. All the dis­
tributions in Figure 7.7 have the maximum 10%-width of 180° (note that
this does not mean that they are isotropic). Distributions measured by the
Giotto IIS at encounter have varying 10%-widths (obtained by averaging the
\( v \)-dependence into \( \mu \)-bins where the instrument provided incomplete coverage
of the 3-dimensional velocity-space) which are plotted in Figure 7.8 [Coates
et al., 1989].

An assumption of immediate isotropization as used in simulations by
some authors does appear sensible as far as the numerical quasilinear diffusion
(from equation (7.51)) is concerned. Results of the analysis suggest that
the pitch-angle width of the present numerical solution reaches an approx­
imately stable state at an early stage, where the new ions from the comet
(source term) are being injected into the ring as fast as ions are diffusing
round the shell from this peak. This is somewhat contrary to observations.
However, in obtaining a solution to quasilinear theory, the distribution func­
tion can be split into an isotropic part, \( F_0(x,v) \), and a small anisotropic
component, \( F_1(x,v,\theta) \), where \( F_1 \ll F_0 \) (eg. Galeev et al. [1987]), and
\( F_0 + F_1 \) is substituted into the kinetic equation. The continual creation of
new ions maintains the weak anisotropy, which in turn feeds the instabil-
Figure 7.7 - Phase space density plots of the numerically obtained theoretical distributions at distances of 2.0, 2.5, 3.0, and $4.5 \times 10^6$ km from the comet Halley nucleus along the Giotto inbound trajectory. (Continues overleaf). In all cases, the binning is as described in Fig. 7.6.
Figure 7.7 (continued)
Figure 7.7 (continued)
Figure 7.7 (continued)
Figure 7.8: A plot of the 10%-widths obtained by Coates et al. [1989] for the measured water-group ion distributions.

ity and the generation of Alfvén waves, thus maintaining the high degree of isotropy in the distribution [Galeev et al., 1987]. The theoretical energy diffusion, however, is such that the shell broadens continually as the plasma convects downstream.

Following Neugebauer et al. [1990] and Coates et al. [1990a], the 'mean width' of a distribution may be defined as follows. The average pitch-angle of the ions in the distribution (which should be close to the pick-up ion injection angle at early times) is [Press et al., 1986]

$$\bar{\theta} = \frac{\sum (\theta \cdot F_{\mu,v})}{\sum F_{\mu,v}}$$  (7.67)

where $0 < \theta < 180^\circ$ and $\sum F_{\mu,v} = n_i$ where $F_{\mu,v}$ is number density per bin rather than phase space density. Then the mean width in degrees is given by [Press et al., 1986]

$$\text{pitch angle width} = \frac{\sum (|\theta - \bar{\theta}| \cdot F_{\mu,v})}{\sum F_{\mu,v}}$$  (7.68)

This is the mean absolute deviation of the ions from the average pitch-angle and will vary up to a maximum possible value of $45^\circ$ for a completely isotropic
case (where $\bar{\theta}$ will be $90^\circ$) since the ions will be spread evenly over a range 0 to $90^\circ$ either side of $\bar{\theta}$. It is a statistically robust estimator and more useful than the 10%-widths when distributions become highly evolved.

In Figure 7.9, the crosses represent mean-width values for the measured distributions [Coates et al, 1990a] and the filled dots the results of the present analysis for the distributions in Figure 7.7. The results in this case agree reasonably well. For low values of $\alpha$, the convection term in equation (7.51) is such that some ions in the distribution (in the bins near $v_\parallel \equiv u$) have velocities, $u + v_\parallel \cos \alpha$, near to zero along the flowline. This implies that they take a very long time to travel the spatial distance $\Delta s$, in the process of which they may potentially undergo much diffusion (particularly when at close proximity to the comet). Then the adjustment to the distribution over $\Delta s$ is large, and the numerical method used may sometimes fail to converge on a reasonable solution (see also Section 7.4.2). This problem was encountered for the flowlines ending at $3.5$ and $4.0 \times 10^6$ km from the comet on the Giotto path, with $\alpha = 47^\circ$ and $34^\circ$, respectively. (Reducing $\Delta s$ even by $\sim 2$ orders of magnitude does not alleviate the problem.) If a value of $60^\circ$ is used instead, then $\theta$-widths represented by the two hollow dots in Figure 7.9 are obtained. These width values are likely to be higher than they should be, since, for a given position in space, the larger the value of $\alpha$ then the greater the pitch-angle width of the numerical solution, as the examples in Figure 7.10 show.

The ratio of $n_i \equiv \sum F_{n,v}$ to that $n_i(s)$ which should be implied on integration of the source term back along the flowline (as in the mass-loading model) given by equation (7.54) is found to be a very good measure of the reliability of the numerically obtained distributions. When the code fails, this ratio may rise dramatically to the order of a few magnitudes. For the results presented in Figures 7.6, 7.7 and 7.10, it is close to unity.

To consider the shell broadening, velocity widths of the distributions may be obtained on replacing $\theta$ by $v$ in equations (7.67) and (7.68). Results for the numerical distributions on the Giotto path are compared with the measurements [Coates et al, 1990a] in Figure 7.11. It is clear from the figure that the last two theoretical points do not coincide with the $v$-widths of the measured ion population. It is possible that this is due to the boundary-
Distance to Nucleus ($10^6$ km)

Spacecraft Event Time  (13 March 1986)

Figure 7.9 - Comparison of mean pitch-angle widths of the measured and numerical distributions on the Giotto path.
Figure 7.10 - Illustration of the effect of $\alpha$ on the numerical distributions. These solutions were obtained for the same spatial position (at $D = 2.0 \times 10^6$ km on the Giotto path) using $\alpha$ values of (a) $60^\circ$, (b) $70^\circ$, (c) $80^\circ$, and (d) $90^\circ$. (Continued overleaf).
IMPLANTED COMETARY ION DISTRIBUTIONS

Figure 7.10 (continued)
Figure 7.11 - Comparison of mean $v$-widths of the measured and numerical distributions on the Giotto path.
condition effect of an inadequate coverage of velocity space. This would only begin to take effect when the distribution has spread to the edge of the grid. Ideally, the whole of velocity-space over which the ions diffuse should be covered, which is impractical because of the processing time involved in computing partial differentials for every bin in a very large 2-D $(\mu, v)$ array at each spatial step.

Analytical solutions have been obtained for the energy-diffusion (shell broadening) by authors such as Isenberg, [1987b], whose solution for $F(x, v)$ (under the assumption of immediate isotropization) is in terms of confluent hypergeometric functions. Numerical solutions obtained in the present analysis may be qualitatively compared with those of Isenberg on summing the $F_{\mu, v}$ into $v$-bins. Figure 7.12 shows a comparison of analytical and numerical $F(v)$ curves (on a logarithmic scale) for different (arbitrary) values of the diffusion coefficient. Figures 7.12a, c and e are the results of Isenberg [1987b] and b, d and f the results of the numerical analysis of equation (7.51). The distances from the comet of the analytical solutions are given only in the ratio $x/x_\infty$ where $x_\infty$ implies a long way upstream. The values of this ratio for the plots of Isenberg are 0.9, 0.6, 0.3, 0.1, 0.07, and 0.05, respectively, for curves in order of increasing peak height in each Figure. In Figures b, d and f, the results are plotted for distances of 9.0, 6.0, 3.0, 1.0, 0.7 and 0.5 $\times$ $10^6$ km along the Sun-comet flowline (which correspond to an $x_\infty = 10^7$ km in terms of the Isenberg ratios). Note that at the first three distances, to the accuracy of the bin-size used, ion injection is at $u_{sw} \equiv u_\infty$. The position of the peaks is determined by the injection at $u_{sw}(x)$, the profile of which is given by the mass-loading model. On the other hand, Isenberg [1987b] assumes a power-law form for the solar wind speed, $u = u_\infty(x/x_\infty)\eta$ where $\eta = 0.1$, and similarly, for the wave power amplitude, $A = A_\infty(x/x_\infty)^{-\beta}$ where $\beta = 0.8$ to 0.9. After considering these points, the qualitative agreement of the analytical and numerical results is good.
7. IMPLANTED COMETARY ION DISTRIBUTIONS

Figure 7.12 - Qualitative comparisons of \( F(v) \) ion distribution curves from the numerical analysis with an analytical solution [Isenberg, 1987b]. Each plot shows results at a series of distances from the comet (see text). The diffusion coefficient increases from (a), (b) to (c), (d) to (e), (f). (Figure continues overleaf).
Figure 7.12 (continued)
Figure 7.12 (continued)
7.6 Summary

- A kinetic transport equation for the implanted cometary ions, involving a quasilinear velocity diffusion term including both pitch-angle and energy diffusion, has been solved numerically along flowlines parallel to the sun-comet line. A basic form of finite differencing is used, which has some limitations, particularly for lower values of $\alpha$ and at close proximity to the comet.

- The distributions obtained at positions along the Giotto path are fairly well developed in terms of pitch-angle spread, and their mean pitch-angle widths fit in reasonably well with those of the measured distributions.

- The mean velocity widths of the numerically obtained distributions agree well with measurements at larger distances from the comet but where the distributions become more highly evolved then boundary conditions may present a problem because of the limited $v$ coverage of the grid.

- Numerical $F(v)$ distributions at a series of distances along a flowline are found to compare well with a similar series of curves produced from an analytical solution by Isenberg [1987b]. The reasonable agreement promotes confidence in the validity of the numerical approach used.
Conclusions

The ion pickup process has been studied in detail in this thesis. A theoretical description based on a quasilinear approach is found to give the right order of pitch-angle diffusion for the observed level of turbulence. The following points arise from the analysis.

- The observed solar wind slow-down on approach to comet Halley has been fitted with a simple model of mass-loading of the flow with heavy cometary ions. The model provides a means of calculating 'first approximations' to plasma flow parameters ($u_{sw}$, $n_{sw}$ and $n_i$) as functions of position in the vicinity of the comet. Values are obtained for the upstream solar wind speed, $u_\infty$, and cometary gas emission parameters, $Q$ and $V_c/\nu$, that are found to be consistent with information from other studies.

- An innermost profile of the cometary bow shock is inferred from the self-reversal point of the MHD equations used in the model and is found to lie entirely within a recent paraboloidal fitting that used both the crossing points of the Giotto spacecraft. Inside the bow shock a different set of equations is needed and there is scope for further analysis in this area.

- The level of turbulence in the solar wind plasma increases on approach to the comet. A reasonable equipartition of wave energy density between $E$ and $B$ fields is observed. For the cometary ion pickup process, the free energy available to upstream and downstream propagating Alfvén waves in the solar wind frame may be obtained theoretically from the pickup geometry in velocity-space. A polynomial fit to the curve of measured $U$ versus $E_F$ provides a means of estimating the
turbulent energy density from the theoretically calculable free energy for any position in the comet - solar wind interaction region. Studies of the wave energy (measured, and estimated from the Poynting vector $S$) indicate that the waves are predominantly parallel-propagating, primarily in the sunward field-aligned direction in the solar wind frame. There are possibilities for future study involving a more detailed consideration of wave modes, their polarization and propagation.

- The wave spectra observed along the Giotto path may be described by a power-law form with a spectral index of $\gamma = 2$. There appears to be no clear evidence of any gradual trend in the spectral shape on approach to the comet. The estimates of the total wave power along the spacecraft trajectory calculated in the time domain and in the frequency domain (from both the discrete FT spectra and the power-law fits) are in good agreement.

- The turbulence is generated as the cometary pick-up ions lose energy in the solar wind frame as they are picked up into the flow, and the difference between the energy released (a proportion of the available free energy) and that observed may be analysed in terms of a damping timescale. As the level of turbulence increases, so does the rate of damping. The mechanism involved in the damping of the waves provides a suitable subject for future work. Energy and momentum are transferred between the cometary ions and the solar wind flow via the turbulence generated.

- A simplified ion transport equation describing the source, convection and quasilinear velocity diffusion of the heavy cometary pickup ions has been solved numerically along flowlines parallel to the sun-comet line. Inputs to the equation are the turbulent power spectrum in the interaction region and ‘first approximations’ to the bulk flow and cometary activity parameters from the mass-loading model. Results are well corroborated by the measured ion distributions (for flowline characteristics within the range of successful operating parameters of the present code). The observed turbulence in the interaction region produces ap-
approximately the correct level of velocity diffusion in the theoretical description. One very interesting point is that the anisotropy of the evolving numerical distribution function remains relatively constant, which is not in total agreement with observations, but is nevertheless consistent with a quasilinear approach. The mean pitch-angle widths of the numerically obtained distributions show reasonable agreement with those of the measured distributions, and the mean velocity widths are sensible at large distances from the comet. Numerical $F(v)$ distributions are found to compare well qualitatively with analytical results of another study.

- The most basic form of the finite differencing method has been used in the numerical computation of the ion transport described above, and there are limitations to its usefulness, particularly for lower values of $\alpha$ and at close proximity to the comet. Improving upon this (in terms of a more robust method with better properties of convergence) remains an outstanding aim for the future. Further refinements to the kinetic equation may also be considered. The inclusion of an adiabatic term would allow for the reaction of the non-resonant particles as their fluctuating kinetic energy alters with changes in wave amplitude. Also, perhaps a set of transport equations, one for each particle species present (related through their moment equations for bulk flow parameters) could be solved simultaneously.
Acknowledgements

Firstly I would like to thank my supervisor, Dr Alan Johnstone for much useful help and advice throughout the period of my study. I also extend thanks to my friends and colleagues in the Space Plasma Group, in particular Dr Andrew Coates (partly for the use of his scissors), Dr David Rodgers and Dr Ramona Kessel. Thanks also to Dr Paul Lamb for serving on my PhD panel. I am grateful to Professor Len Culhane, director of the lab, for allowing me the opportunity to work at MSSL.

It gives me great pleasure to thank my parents and family for their endless support and encouragement, and Jon Lapington for his invaluable help (not least for mending my car).

This study would not have been possible without the hard work which lead to the success of the Giotto mission, and in particular I am grateful to the institutes involved in obtaining the JPA and MAG data. I acknowledge the receipt of an SERC grant in support of my studentship.
Glossary of Publications


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Addendum

Derivation of the "Frozen-in" magnetic field condition to replace that given on page 19

Maxwell’s 2nd equation is:

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]  

(1)

Ohm’s law for a moving conductor is (eg. Boyd and Sanderson, 1969)

\[ \mathbf{j} = \sigma (\mathbf{E} + \mathbf{u} \times \mathbf{B}) \]  

(2)

where \( \mathbf{j} \) is the current density and \( \sigma \) the conductivity. Assuming an infinite conductivity plasma, \( \mathbf{j}/\sigma = 0 \), leads to the approximate relationship:

\[ \mathbf{E} = -\mathbf{u} \times \mathbf{B} \]  

(3)

Combining equations (1) and (3) gives:

\[ \nabla \times (\mathbf{u} \times \mathbf{B}) = \frac{\partial \mathbf{B}}{\partial t} \]  

(4)

Consider magnetic tubes of force [Bittencourt, 1986] enclosing magnetic flux \( \Delta \Phi_B \). For local cross-sectional area, \( S \), the magnitude of \( \mathbf{B} \) at a local point, \( P \), in the tube is \( \Delta \Phi_B/S \); the magnitude of \( \mathbf{B} \) is inversely proportional to the cross-sectional area of the tube. Now imagine a closed line \( C_1 \) at time \( t \), bounding an open surface \( S(t) = S_1 \), where the points on \( C_1 \) move at velocity \( \mathbf{u} \), and for the present, \( \mathbf{u} \) may be an arbitrary function of position and time. After a time \( \Delta t \), the closed curve and open surface become \( C_2 \) and \( S(t + \Delta t) = S_2 \). The flux of the magnetic field through surface \( S \) at time \( t \) is

\[ \Phi_B(t) = \int_S \mathbf{B}(\mathbf{r}, t) \cdot dS \]  

(5)

and the rate of change of this flux may be written [Bittencourt, 1986]:

\[ \frac{d}{dt} \left[ \int_S \mathbf{B}(\mathbf{r}, t) \cdot dS \right] = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \left[ \int_{S_2} \mathbf{B}(\mathbf{r}, t + \Delta t) \cdot dS - \int_{S_1} \mathbf{B}(\mathbf{r}, t) \cdot dS \right] \]  

(6)

Using a Taylor series expansion for \( \mathbf{B}(\mathbf{r}, t + \Delta t) \),

\[ \mathbf{B}(\mathbf{r}, t + \Delta t) = \mathbf{B}(\mathbf{r}, t) + \frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t} \Delta t + \ldots \]  

(7)
Figure A1 - The magnetic flux through an open surface which is bounded by a closed line moving with velocity \( u(r,t) \). The line is denoted by \( C_1 \) and \( C_2 \) at times \( t \) and \( t + \Delta t \), respectively. [Bittencourt, 1986].

Neglecting higher order terms in the limit \( \Delta t \to 0 \) then equation (6) becomes:

\[
\frac{d}{dt} \left( \int S B(r,t) \cdot dS \right) = \lim_{\Delta t \to 0} \left\{ \int_{S_2} \frac{\partial B(r,t)}{\partial t} \cdot dS + \frac{1}{\Delta t} \left[ \int_{S_2} B(r,t) \cdot dS - \int_{S_1} B(r,t) \cdot dS \right] \right\} \tag{8}
\]

To evaluate the RHS, consider Gauss' divergence theorem for any closed surface which gives at time \( t \),

\[
\oint B \cdot dS = \int_V \nabla \cdot B \ d^3r = 0 \tag{9}
\]

where \( \nabla \cdot B = 0 \) (Maxwell's 3rd relation). Applying this to the closed surface shown in Figure A1, defined by \( S_1, S_2 \), and the sides of the cylindrical surface between \( C_1 \) and \( C_2 \) of length \( u \Delta t \):

\[
- \int_{S_1} B(r,t) \cdot dS + \int_{S_2} B(r,t) \cdot dS - \oint_{C_1} B(r,t) \cdot [(u \Delta t) \times dl] = 0 \tag{10}
\]

Here, the minus sign in the 1st term occurs because the outwardly drawn unit normal to \( S_1 \) is in the opposite direction to that of \( S_2 \). The \(-(u \Delta t) \times dl\) is an area element (directed outwards) over which a vector element \( dl \) on the closed curve moves in time \( \Delta t \) (see Figure A1). Substituting (10) into (8) and taking the limit \( \Delta t \to 0 \)
(when \( S_2 = S_1 = S(t) \)) gives:

\[
\frac{d}{dt} \left[ \int_S \mathbf{B}(r,t) \cdot dS \right] = \int_S \frac{\partial \mathbf{B}(r,t)}{\partial t} \cdot dS + \oint_C \mathbf{B}(r,t) \cdot (\mathbf{u} \times dl) \tag{11}
\]

For the last term on the RHS, consider the vector identity \( \mathbf{B}(r,t) \cdot (\mathbf{u} \times dl) = -[\mathbf{u} \times \mathbf{B}(r,t)].dl \); then from Stokes' theorem,

\[
\oint_C [\mathbf{u} \times \mathbf{B}(r,t)].dl = \int_S \nabla \times [\mathbf{u} \times \mathbf{B}(r,t)].dS \tag{12}
\]

which may be used in (11) to give [Bittencourt, 1986]:

\[
\frac{d}{dt} \left[ \int_S \mathbf{B}(r,t) \cdot dS \right] = \int_S \left\{ \frac{\partial \mathbf{B}(r,t)}{\partial t} - \nabla \times [\mathbf{u} \times \mathbf{B}(r,t)] \right\} \cdot dS \tag{13}
\]

Now if \( \mathbf{u} \) is taken to be the fluid velocity, then from (4) and (13) the final result is [Bittencourt, 1986]:

\[
\frac{d}{dt} \left[ \int_S \mathbf{B}(r,t) \cdot dS \right] = 0 \tag{14}
\]

This says that the magnetic flux linked by the closed line (bounding \( S \)) moving with the fluid at velocity \( \mathbf{u} \) is constant. Thus the lines of \( \mathbf{B} \) are "frozen" into the plasma and are carried along with any fluid motion perpendicular to \( \mathbf{B} \) (since the component of \( \mathbf{u} \) parallel to \( \mathbf{B} \) does not contribute to \( \mathbf{u} \times \mathbf{B} \)).