Financing Entrepreneurial Production: Security Design with Flexible Information Acquisition*

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Abstract

We propose a theory of security design in financing entrepreneurial production, positing that the investor can acquire costly information on the entrepreneur's project before making the financing decision. When the entrepreneur has enough bargaining power in security design, the optimal security helps incentivize both efficient information acquisition and efficient financing. Debt is optimal when information is not very valuable for production, whereas the combination of debt and equity is optimal when information is valuable. If, instead, the investor has sufficiently strong bargaining power in security design or can acquire information only after financing, equity is optimal. (*JEL* D82, D86, G24, G32, L26)

*We thank Itay Goldstein (the Editor), two referees, John Campbell, Mark Chen, Peter DeMarzo, Darrell Duffie, Emmanuel Farhi, Diego Garcia, Mark Garmaise, Simon Gervais, Daniel Green, Barney Hartman-Glaser, Ben Hebert, Steven Kaplan, Arvind Krishnamurthy, Josh Lerner, Stephen Morris, Christian Opp, Marcus Opp, Adriano Rampini, David Robinson, Hyun Song Shin, Andrei Shleifer, Alp Simsek, Jeremy Stein, S. Viswanathan, Michael Woodford, and John Zhu. We also thank seminar and conference participants at Berkeley Haas, Duke Fuqua, Harvard, Minnesota Carlson, MIT Sloan, Northwestern Kellogg, Peking University, Stanford GSB, UNC Kenan-Flagler, University of Hawaii, University of Toronto, University of Vienna, WFA, Wharton Conference on Liquidity and Financial Crises, Finance Theory Group, Toulouse TIGER Forum, SFS Cavalcade, CICF, SAIF Summer Institute of Finance, and SAET Conference. Earlier versions of this paper have been circulated under the titles "Venture Finance under Flexible Information Acquisition" and "Security Design in a Production Economy under Flexible Information Acquisition." All errors are our own.

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What is the optimal security design when the investor can acquire costly information about the entrepreneur's project before making the financing decision? In reality, many professional investors are better able than the entrepreneur to acquire information and thus to assess a project's uncertain market prospects, drawing upon their industry experience. For instance, start-ups seek venture capital (VC), and most venture capitalists are themselves former founders of successful start-ups, so they may be better able to determine whether new technologies match the market.[1] However, it is less known how the entrepreneur should optimally design the security when the investor's (1) information acquisition and (2) subsequent financing decision are both endogenous, which circumstance is empirically relevant for the finance of small private businesses that account for the majority of corporates. Our paper offers a tractable framework to address this question. It provides a theory of the use of debt and nondebt securities. In particular, we show under what conditions debt or nondebt securities will be optimal. These results are consistent with the empirical evidence regarding the finance of different types of entrepreneurial businesses.

In our model, an entrepreneur (she) has the potential to produce a project that requires a fixed investment. She has no initial resource, but she can design and offer a security to a potential investor (he) in exchange for financing. Facing the security offer, the investor can acquire costly information about the project's uncertain cash flow before making the financing decision.

Although production, that is, the creation of social surplus, depends on potential information acquisition and the subsequent financing, two sources of friction come from the separation of security design (by the entrepreneur) and information acquisition and financing (by the investor). First, the investor may not acquire information efficiently. Second, the investor may not make the financing decision efficiently after his endogenous information acquisition. Therefore, the objective of security design is to appropriately incentivize efficient information acquisition and then an efficient financing decision by the investor.

Our model predicts standard debt and the combination of debt and equity[2] as optimal securi-

[1] More generally, research of investors' potential information advantages dates back to Knight (1921) and Schumpeter (1942). Apart from extensive anecdotal evidence, recent empirical literature (Chemmanur, Krishnan, and Nandy 2011; Kerr, Lerner, and Schoar 2014) has also identified information advantage by various types of institutional investors.

[2] The formal mathematical definitions of debt and equity and the combination of debt and equity in our framework are given in Sections 2.1 and 2.2. In defining debt and equity, we focus on the qualitative aspects of their cash flow rights but ignore the aspects of control rights. Specifically, debt means the security pays all the cash flow in low states but has a constant face value in high states, whereas equity means the security and its residual both strictly increase in the fundamental. Consistent with reality, debt is also more senior than is equity in our framework.

ties in different circumstances. When the project's ex ante market prospects are good and not very uncertain, the optimal security is debt, which does not induce information acquisition. Notably, the expected overall payment of the debt strictly exceeds the initial investment requirement. The prediction of debt is consistent with the evidence that conventional start-ups and mature private businesses heavily rely on plain vanilla" debt finance from investors, such as relatives, friends, and traditional banks (see Berger and Udell 1998; Kerr and Nanda 2009; Robb and Robinson 2014).

The intuition of the optimality of debt is reflected in the design of the debt's *shape* and *level*.[3] On the one hand, in this case, as the benefit of information does not justify its cost, the entrepreneur finds it optimal to deter costly information acquisition, and debt fulfills this role because its flat *shape* minimizes the investor's incentive to acquire information. Hence, the flat *shape* of the debt is designed to help incentivize *efficient information acquisition*, which in this case happens to be not to acquire any information. On the other hand, because the investor has the option to acquire information and thus obtain an information rent, the entrepreneur must grant a high enough overall payment so that the offer (and thus the project) will not be rejected. In other words, the face value of the debt needs to be high enough so that the offer (and thus the project) will not be rejected. Hence, the face value, which determines the *level* of the debt contract, is designed to incentivize the *efficient financing decision*.

In contrast, when the project's ex ante market prospects are obscure, the optimal security is the combination of debt and equity that induces the investor to acquire information. Regarding cash flow rights only, this is equivalent to participating convertible preferred stock. This prediction is consistent with empirical evidence that the combination of debt and equity has been frequently used in financing more innovative and less transparent projects conducted by young firms (Brewer, Genay, Jackson, and Worthington 1996; Berger and Udell 1998). Participating convertible preferred stock also accounts for half of the contracts between entrepreneurs and venture capitalists

[3]Throughout the paper, we use the concepts *shape* and *level* literally, but, to be more specific, shape means how the payment of the optimal security varies across different states when the limited liability constraint is not binding; intuitively, it reflects whether the optimal security is "flat" or "steep" (when the limited liability constraint is not binding) and in what sense it is steep (whether it is increasing or decreasing and how quickly it is increasing or decreasing). And level means at what underlying cash flow θ the optimal security deviates from the 45° line (i.e., the limited liability constraint); intuitively, it captures how generous the overall payment of the security is given any fixed shape, and it also captures the face value of the debt component of the optimal security. It is worth noting that, the 45° line part of the optimal security mechanically follows the limited liability constraint, so that it is natural not to consider that exogenous constraint as a part of the security's shape (for instance, the shape of any debt is flat despite it follows the 45° line in bad states).

(Kaplan and Stromberg 2003).

The intuition of the optimality of the combination of debt and equity is also reflected in its *shape* and *level*. On the one hand, in this case, the entrepreneur wants to induce the investor to acquire information only if the investor screens in (out) a potentially good (bad) project.[4] That is, any project with a strictly higher ex post cash flow should have a strictly better chance to be financed ex ante. Only when the investor's payment is strictly higher (lower) in good (bad) states does the investor have the right incentive to distinguish between these different states, because he is more willing to finance the project when his payment is higher. Therefore, an equity component with payments that are strictly increasing in the underlying cash flow is offered, encouraging the investor to acquire adequate information to distinguish between any different states.[5] In this sense, the steep and strictly increasing *shape* of the equity component is designed to help incentivize *efficient information acquisition*. On the other hand, because the endogenous information advantage gives the investor an information rent, the entrepreneur must also make the overall payment of the security high enough to ensure that the investor will not directly reject the offer without information acquisition. Given the optimal shape of the equity component, this high enough overall payment is guaranteed by the debt component with a high enough face value. Hence, the *level* of the debt component incentivizes the *efficient financing decision*.

The approach of flexible information acquisition, following Yang (2015),[6] helps (1) characterize the detailed properties of the optimal securities under general conditions[7] and (2) capture endogenous information acquisition and financing decision simultaneously. Flexible information acquisition means that the investor can choose any possible information structure. Intuitively, it captures not only how much but also what kind of information the investor acquires through state-contingent attention allocation.[8] Information is costly, so the investor will only acquire

[4]Our model features a continuous state, but we use the notions of good and bad projects to help develop our intuition.

[5]However, notice that it is not optimal for the entrepreneur to offer all the cash flows, that is, the entire 45° line, to the investor in our baseline model, because doing so would leave the entreprenuer with nothing. In other words, the optimal security must deviate from the 45° at some point, which is consistent with the endogenously determined level of the security. This point will be illustrated later in greater detail.

[6]It is based on the literature of rational inattention (Sims 2003) but has a different focus.

[7]Our model is built over continuous states and does not have any distributional assumptions or usual technical restrictions on the feasible security space.

[8]The traditional approach of exogenous information asymmetry does not capture these incentives. Recent models of endogenous information acquisition do not capture such flexibility of incentives adequately, because they only

payment-relevant information for guiding financing decisions, given the security offered. For instance, debt, with its flat shape, is less likely than equity to prompt information acquisition as the payments are constant over states (when the limited liability constraint is not binding) so that there is no point in differentiating them. In contrast, equity holders are willing to differentiate good states from bad ones, as they benefit from the upside payments. Overall, security design determines how the investor acquires information in equilibrium, which directly reflects how he wants to finance the project. This mechanism, in turn, helps pin down the optimal securities for the entrepreneur in different scenarios.

The friction in our model comes from the separation between security design (by the entrepreneur) and information acquisition as well as the subsequent financing (by the investor). Thus, the investor may not internalize all the benefits from information acquisition and financing. This in turn relies on two assumptions that (1) the entrepreneur has enough bargaining power in the process of designing the security and (2) the investor acquires information before making the financing decision. If any assumption is violated, using equity to sell all the cash flows of the potential project to the investor is optimal.[9] We view these two assumptions reasonable in entrepreneurial productions and will discuss the plausibility and limitation of them in detail.

This paper contributes to the security design literature by modeling both (1) endogenous information acquisition and (2) endogenous financing by investors in a production economy.[10,11] In the security design literature, the closest papers to us are those that feature investors' (buyers') information advantage in a production setting.[12] But endogenous information acquisition and financing decision are generally modeled separately so far.

On the one hand, existing models that consider investors' endogenous information acquisition

consider the amount or the precision of information (see Veldkamp 2011, for a review).

[9]Depending on which of the two assumptions is violated, the resultant equity transaction in equilibrium is subtly different in terms of which party obtains the surplus. We elaborate on the two cases in Sections 2.3 and 2.4.

[10]For other theoretical work that features the effects of investors' information advantage, but not security design, see Bond, Edmans, and Goldstein (2012).

[11]A small, but burgeoning, security design literature considers individual investors' endogenous information acquisition and financing decisions in an exchange economy (Dang, Gorton, and Holmstrom 2015; Yang 2017). These models feature a seller selling an asset in place and show that debt is the only optimal security because it deters endogenous adverse selection. They do not fit our setting of financing entrepreneurial production.

[12]More research in the security design literature features information advantage by the seller (entrepreneur), but not by the buyer (investor). Some predict debt as optimal to deter adverse selection (Myers and Majluf 1984; Gorton and Pennacchi 1990; DeMarzo and Duffie 1999). Others predict nondebt securities (including equity and convertibles) as optimal in various circumstances (see Nachman and Noe 1994; Chemmanur and Fulghieri 1997; Chakraborty and Yilmaz 2011; Chakraborty, Gervais, and Yilmaz 2011; Fulghieri, Garcia, and Hackbarth 2016).

typically feature binary states, and the financing decision is exogenous in the sense that a good (bad) project, known to the investor after information acquisition, will always be financed (rejected) for sure. Notably, Boot and Thakor (1993) and Fulghieri and Lukin (2001) consider a competitive public equity market in which investors endogenously acquire information, which can be then aggregated in a price. They show that the entrepreneur optimally designs a high payment in the good state because it encourages information aggregation, which in turn helps the entrepreneur signal its own type.[13] Our model has a different focus on an entrepreneurial private firm that does not have access to a public equity market but may still face an informationally sophisticated investor. Additionally, our model can handle continuous states of cash flows and state-contingent information acquisition, and, in doing so, helps deliver more detailed predictions of both the shape and the level of the optimal securities.

On the other hand, Inderst and Mueller (2006) consider how optimal security design promotes efficient financing decisions by an investor, but the investor is endowed with private information so that there is no endogenous information acquisition.[14] There, debt is optimal because its 45° line part mitigates the investor's underinvestment problem, whereas levered equity is optimal because its flat part mitigates the investor's overinvestment. In our model, levered equity is never optimal. Instead, debt is optimal, because its flat shape helps deter costly information acquisition (when unnecessary), whereas the combination of debt and equity is optimal, because its strictly increasing shape helps incentivize information acquisition (when valuable). Our optimal security also incentivizes information acquisition and financing simultaneously; the level of the debt component is designed to be sufficiently high to incentivize an efficient financing decision. Thus, our model can provides an explanation for the popularity of the combination of debt and equity in financing entrepreneurial production, where the investor actively acquires information about a proposed project rather than just relies on endowed information from his past experience.

Our model also contributes to the venture contract design literature by focusing on one specific role of venture capitalists, pre-investment screening, which is captured by our modeling of endogenous information acquisition followed by endogenous financing. The existing venture

[13] Hennessy (2013) considers uninformed sellers in a framework similar to ours but with additional specifications on uninformed buyers; he shows that the optimality of equity remains.

[14] A related literature considers how security design interacts with the aggregation of investors' private information endowment in an auction setting; see DeMarzo, Kremer, and Skrzypacz (2005); Axelson (2007) and Garmaise (2007).

contract design literature covers aspects such as control right allocation (Hellmann 1998; Kirilenko 2001), staging (Admati and Pfleiderer 1994; Cornelli and Yosha 2003; Repullo and Suarez 2004), and exiting (Hellmann 2006). One popular explanation for venture contracts focuses on entrepreneur moral hazard and investor monitoring (Ravid and Spiegel 1997; Bergemann and Hege 1998; Schmidt 2003; Casamatta 2003), which follows the insight of Innes (1990) that the entrepreneur should have enough skin in the game to curb its own moral hazard problem. So far, this literature has not yet focused on deal screening by the investors, which is shown by recent survey evidence as the most important factor contributing to value creation in venture financing (Gompers, Gornall, Kaplan and Strebulaev 2016).[15,16] In addition, existing models in the venture contract design literature typically focus on one class of optimal security, but not on explaining why different types of securities may be optimal in different scenarios.

A new strand of literature on the real effects of rating agencies (see Opp, Opp, and Harris 2013; Kashyap and Kovrijnykh 2016) is also relevant. On behalf of investors, the rating agency screens an uninformed firm. Information acquisition may improve social surplus through ratings and the resultant investment decisions. They do not consider security design as we do.

1 The Model

1.1 Financing entrepreneurial production

Consider a production economy with two dates, $t = 0, 1$, and a single consumption good. There are two agentsan entrepreneur lacking financial resources and a deep-pocket investorboth of whom are risk neutral. Their utility function is the sum of consumptions over the two dates: $u = c_0 + c_1$, where c_t denotes an agent's consumption at date t. The subscripts E and I indicate the entrepreneur and the investor, respectively.

The financing process of the entrepreneur's risky project is as follows. To initiate the project

[15]Gompers, Gornall, Kaplan and Strebulaev (2016) have explicitly documented in their abstract that "[W]hile deal sourcing, deal selection, and post-investment value-added all contribute to value creation, the VCs rate deal selection as the most important of the three." In their definition, "deal selection" corresponds to investor information acquisition and screening in our model, whereas "post-investment value-added" corresponds to investor monitoring, that is, solving the standard entrepreneur moral hazard problem.

[16]An reinterpretation of our model is that the entrepreneur needs to design a security to monitor the investor and solve the investor's moral hazard problem because the investor's information acquisition is his hidden effort and the sole contributor to surplus in our model. Thus, this reinterpretation provides an alternative perspective to see the difference between our channel and the standard entrepreneur moral hazard channel.

at date 0, the underlying technology requires an investment of $k > 0$. If financed, the project generates a nonnegative verifiable random cash flow θ at date 1. The project cannot be initiated partially. Hence, the entrepreneur must raise k, by selling a security to the investor at date 0. The payment of a security at date 1 is a mapping $s(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ such that $s(\theta) \in [0, \theta]$ for any θ. We focus only on the cash flow of projects and securities rather than the control rights.

Security design, information acquisition and financing happen sequentially, but both at date 0. The agents have a common prior Π on the potential project's future cash flow θ, and neither party has any private information ex ante.[17] The entrepreneur designs the security, and then proposes a take-it-or-leave-it offer to the investor, asking for a fixed investment k. Facing the offer, the investor acquires information about θ in the manner of rational inattention (Sims 2003; Woodford 2008; Yang 2015 2017), updates beliefs on θ, and then decides whether to accept the offer to finance the project. The information acquired is measured by reduction of entropy. The information cost per unit reduction of entropy is μ. We elaborate this information acquisition process in more detail in subsection 1.2.

Three implicit assumptions are important in the setting. First, the entrepreneur owns the project but cannot undertake it without external finance. This is a common assumption in the corporate finance literature,[18] and is consistent with the empirical evidence that entrepreneurs and private firms are often financially constrained (Evans and Jovanovic 1989; Holtz-Eakin, Joulfaian, and Rosen 1994). It implies that the investor's endogenous financing is crucial, and thus the entrepreneur needs to incentivize it through security design.

Second, the entrepreneur has bargaining power *in the process of designing the security*. This assumption is also common in the security design literature, including papers focusing on venture capital financing (Admati and Pfleiderer 1994; Hellmann 1998). It is consistent with the evidence in Gompers, Gornall, Kaplan and Strebulaev (2016) that even when an entrepreneur contracts with sophisticated investors such as venture capitalists, most contractual terms are subject to bargaining, and the entrepreneur has strong bargaining power over many of them. It is also

[17] We can interpret this setting as the entrepreneur may still have some private information about the future cash flow, but does not have effective ways to signal this to the investor. Signaling has been extensively discussed in the literature and is already well understood, so we leave it aside.

[18] See Tirole 2006 for an overview. Alternatively, the entrepreneur may have capital but cannot acquire information, so she may hire an information expert to improve the investment decision. This alternative situation boils down to a consulting problem. A large literature on the delegation of experimentation (e.g., Manso 2011) considers consulting problems in corporate finance, but this is beyond the scope of this paper.

consistent with more recent evidence in Evans and Farre-Mensa (2017) that documents a time-series increase in entrepreneur's bargaining power in the last two decades. We acknowledge other evidence suggesting that entrepreneurs in reality may not have strong bargaining power relative to the investors, for example, Hall and Woodward (2010) find that entrepreneurs on average obtain a very small surplus in excess of their labor market outside options, and Kaplan and Stromberg (2003) suggest that venture capitalists structure the securities. Thus, in Section 2.3, we consider a model extension with a general allocation of bargaining power between the entrepreneur and the investor, and we also clarify in what sense our assumption may be viewed as consistent with various evidence.

Third, underlying the time line is the assumption that the investor can acquire information *before* his financing decision. If the investor believes that the project is good enough, then he willing to finance the project. We view this time line plausible because nothing can prevent sophisticated investors from acquiring information or screening projects before providing finance, and they indeed do so (Chemmanur, Krishnan, and Nandy 2011; Kerr, Lerner, and Schoar 2014; Opp 2016). It implies that the investor's endogenous information acquisition before financing is also crucial, and thus the optimal security design needs to incentivize it as well. It also implies that the investor benefits from his endogenous information rent, which effectively contribute to his overall bargaining power in terms of sharing the social surplus. Also, to clarify the role of this assumption, Section 2.4 considers a reversed time line in which the investor can only acquire information after making a financing decision.

Together, the three above assumptions set forth the key friction in our production economy. They imply that information acquisition and financing are important for efficient production, and thus the entrepreneur designs the security to incentivize both, but she also wants to retain as much cash flow as possible and can indeed do so because of her bargaining power. This friction implies that although the optimal security helps promote efficient information acquisition and financing, it may not necessarily achieve the first best. As Sections 2.3 and 2.4 will show, when this friction is effectively removed, the socially efficient outcome can be achieved.

To further set the scope of this paper, it is worth noting which other aspects of finance in the production economy are abstracted away. First, to focus on pre-investment screening, we set aside moral hazard and the allocation of control rights. To set them aside is not unusual when hidden information is important in security design (see DeMarzo and Duffie 1999, for a justification). Second, consistent with the security design literature, we do not allow for partial financing or endogenous investment scale choice. Because our theory can admit any prior distribution, a fixed investment requirement in fact enables us to capture projects with differing natures in an exhaustive sense. Third, we do not model the staging of finance. We interpret the cash flow θ as already incorporating the consequences of investors' exiting, and each round of investment may be mapped to our model separately with a different prior. Last, we do not model competition or strategic interaction among multiple investors. The last two points pertain to the structure of the financial markets, which is interesting but would significantly change the focus of the current paper, so we leave it for future research.

1.2 Flexible Information Acquisition

We elaborate the approach of flexible information acquisition, following Woodford (2008) and Yang (2015), which means that the investor who acquires information can choose any information structure, and the information cost is proportional to the expected entropy reduction.

We first characterize the information structure. Consider an investor who chooses a binary action, $a \in \{0, 1\}$, where $a = 1$ denotes financing, whereas $a = 0$ no financing. The investor receives a payment $u(a, \theta)$, where $\theta \in \mathbb{R}_+$ is the fundamental, distributed according to a continuous probability measure Π over $\mathbb{R}_+$. Before making the financing decision, the investor can acquire information flexibly. In particular, the nature of the binary decision problem implies that the investor always chooses a binary-signal information structure where each signal corresponds to an action recommendation.[19] Specifically, any such information structure can be represented by a measurable function of θ, $m(\cdot) : \mathbb{R}_+ \to [0, 1]$, the probability of observing signal 1 if the true state is θ, so that the investor's decision problem amounts to choosing a function $m(\cdot)$. As elaborated later, the investor will always choose an $m(\cdot)$ such that it is optimal for him to follow the action recommendation, that is, his optimal action is 1 (or 0) when the signal is 1 (or 0). By choosing

[19] In general, the investor can choose any information structure. But he always prefers binary-signal information structures in binary decision problems; otherwise, he must incur a waste of information cost without contributing to the information content. This is standard observation in the rational inattention literature; it is formally stated in the binary action context in Woodford (2008) and Yang (2015).

different functional forms of $m(\cdot)$, the investor can make the signal correlate with the fundamental in any arbitrary way.[20] Intuitively, for instance, if his payment is sensitive to fluctuations of the state within some range $A \subset \mathbb{R}_+$, he would pay more attention to this range by making $m(\theta)$ covary more with θ in A.

The new approach of modeling information structure fits the economic forces underlying our research question and offers two unique advantages compared to traditional ways of modeling information acquisition. First, the function $m(\cdot)$ allows us to simultaneously capture both endogenous information acquisition and endogenous financing decision, which is our key departure from the existing security design literature and is hard to achieve parsimoniously by more traditional modeling approaches. On the one hand, conditional on a cash flow θ, $m(\theta)$ is the conditional probability of the project being financed, which captures endogenous financing decision. On the other hand, the absolute value of the first-order derivative $|dm(\theta)/d\theta|$ captures the investor's state-dependent intensity of information acquisition, which captures the intensity of endogenous information acquisition. Intuitively, when $|dm(\theta)/d\theta|$ is larger, the investor acquires more information around θ and thus better differentiates the nearby states. Because $m(\cdot)$ embodies a natural interpretation of screening, which accounts for both acquiring information and financing, we call $m(\cdot)$ a *screening rule* in what follows.

Second, this approach also allows us to generate more detailed predictions regarding the shape of the securities and to work with arbitrarily feasible securities over continuous states and without parametric distributional assumptions. The essence of flexible information acquisition is that it captures not only how much but also what kind of information an investor acquires. This is important because in reality the entrepreneur can design the security's payment structure arbitrarily, and thus the investor will pay different attention to different aspects of the project in screening it. This therefore calls for an equally flexible modeling account of screening to capture the interaction between the shape of the securities and the incentives to allocate attention.

We then specify the cost of information acquisition. Like in Woodford (2008) and Yang (2015), the amount of information conveyed by a screening rule $m(\cdot)$ is defined as the expected reduction of uncertainty through observation of the signal, where the uncertainty associated with

[20] Technically, this allows the investor to obtain signals drawn from any conditional distribution of the fundamental.

a distribution is measured by Shannon's entropy $H(\cdot)$. This reduction from the investor' prior entropy to expected posterior entropy can be calculated as:

$$I\left(m(\cdot)\right) = \mathbb{E}\left[g\left(m\left(\theta\right)\right)\right] - g\left(\mathbb{E}\left[m(\theta)\right]\right) \ ,$$

where $g\left(x\right) = x \cdot \ln x + (1-x) \cdot \ln\left(1-x\right)$, and the expectation operator $\mathbb{E}(\cdot)$ is θ under the probability measure Π.[21] Denote by $M = \{m(\cdot) \in L\left(\mathbb{R}_+, \Pi\right) : \theta \in \mathbb{R}_+, m\left(\theta\right) \in [0,1]\}$ the set of binary-signal information structures, and $c(\cdot) : M \to \mathbb{R}_+$ the cost of information. The cost associated with a screening rule $m(\cdot)$ is assumed to be proportional to the expected reduction in entropy:

$$c\left(m(\cdot)\right) = \mu \cdot I\left(m(\cdot)\right) \ ,$$

where $\mu > 0$ is the cost of information acquisition per unit of reduction of entropy.[22]

An implicit assumption underlying the information cost is that all expected entropy reductions of the same magnitude have the same cost. In other words, it is equally costly for the agent to acquire information around any state. This represents a theoretical benchmark that no state is more special than others. In reality, some investors may find it less costly to acquire information about some certain states perhaps due to their state-dependent expertise. In Section 2.2, we briefly discuss to what extent such state-dependent expertise may affect our results.

Built on flexible information acquisition, the investor's problem is to choose a functional form of $m(\cdot)$ to maximize the expected payment less the information cost. We characterize the optimal screening rule $m(\cdot)$ in the following proposition. We denote $\Delta u(\theta) = u(1,\theta) - u(0,\theta)$, which is the payoff gain of taking action 1 over action 0 when the state is θ. We also assume that $\Pr\left[\Delta u\left(\theta\right) \neq 0\right] > 0$ to exclude the trivial case where the investor is always indifferent between the two actions. The proof is in Yang (2017) (see also Woodford 2008, for an earlier treatment).

[21] Formally, we have

$$I(m(\cdot)) = H(\Pi) - \int_x H(\Pi(\cdot|x))\Pi_x dx \, ,$$

where Π denotes the prior, x the signal received, $\Pi(\cdot|x)$ the posterior distribution, and Π_x the marginal probability of signal x. Under a binary-signal structure, a standard calculation yields the result above.

[22] Following the literature of rational inattention, the functional form of the information cost is not a crucial driver of our qualitative results. See Yang (2015) for discussions on related properties of this cost function. In particular, although the cost $c(m(\cdot))$ is linear in the expected entropy reduction $I(m(\cdot))$, it does not mean it is linear in information acquisition. Essentially, the expected reduction in entropy $I(m(\cdot))$ is a nonlinear functional of the screening rule $m(\cdot)$ and the prior Π, microfounded by the information theory.

Proposition 1. *Given u, Π, and $\mu > 0$, let $m^*(\cdot) \in M$ be an optimal screening rule and*

$$\bar{\pi}^* = \mathbb{E}\left[m^*(\theta)\right]$$

be the corresponding unconditional probability of taking action 1. Then

a. The optimal screening rule is unique

b. There are three cases for the optimal screening rule:

a) $\bar{\pi}^* = 1$, that is, $Prob[m^*(\theta) = 1] = 1$ if and only if

$$\mathbb{E}\left[\exp\left(-\mu^{-1} \cdot \Delta u(\theta)\right)\right] \leqslant 1\,; \tag{1.1}$$

b) $\bar{\pi}^* = 0$, i.e., $Prob[m^*(\theta) = 0] = 1$ if and only if

$$\mathbb{E}\left[\exp\left(\mu^{-1} \cdot \Delta u(\theta)\right)\right] \leqslant 1\,;$$

c) $0 < \bar{\pi}^* < 1$ and $Prob[0 < m^*(\theta) < 1] = 1$ if and only if

$$\mathbb{E}\left[\exp\left(\mu^{-1} \cdot \Delta u(\theta)\right)\right] > 1 \text{ and } \mathbb{E}\left[\exp\left(-\mu^{-1} \cdot \Delta u(\theta)\right)\right] > 1\ ; \tag{1.2}$$

in this case, the optimal screening rule $m^*(\cdot)$ is determined by the equation

$$\Delta u(\theta) = \mu \cdot \left(g'\left(m^*(\theta)\right) - g'\left(\bar{\pi}^*\right)\right) \tag{1.3}$$

for all $\theta \in \mathbb{R}_+$, where

$$g'(x) = \ln\left(\frac{x}{1-x}\right)\ .$$

Proposition 1 fully characterizes the investor's possible optimal decisions of information acquisition. Cases a and b correspond to the scenarios of optimal action 1 or 0. These two cases do not involve information acquisition. They correspond to the scenarios in which the prior is extreme or the cost of information acquisition is sufficiently high. But Case c, the more interesting one, involves information acquisition. In particular, the optimal screening rule $m^*(\cdot)$ is not constant in this case, and neither action 1 nor 0 is optimal ex ante. This case corresponds to the scenario where the prior is not extreme, or the cost of information acquisition is sufficiently low. In Case c, where information acquisition is involved, the investor equates the marginal benefit of information with its marginal cost, as indicated by condition (1.3). In doing so, he chooses the shape of $m^*(\cdot)$ according to the shape of payoff gain $\Delta u(\cdot)$ and the prior Π.[23]

2 Security Design

We consider the entrepreneur's security design problem. Denote the optimal security of the entrepreneur by $s^*(\cdot)$. The entrepreneur and the investor play a sequential Bayesian game. Concretely, the entrepreneur designs the security, and then the investor screens the project given the security designed. Hence, we apply Proposition 1 to the investor's information acquisition problem, given the security, and then solve backward for the entrepreneur's optimal security. To distinguish this from the general decision problem in Section 1.2, we denote the investor's optimal screening rule as $m_s(\cdot)$, given the security $s(\cdot)$; hence the investor's optimal screening rule is now denoted by $m_s^*(\cdot)$.

We formally define the equilibrium as follows.

Definition 1. *Given u, Π, k and $\mu > 0$, the sequential equilibrium is defined as a combination of the entrepreneur's optimal security $s^*(\cdot)$ and the investor's optimal screening rule $m_s(\cdot)$ for any generic security $s(\cdot)$, such that*

a. the investor optimally acquires information given any generic security $s(\cdot)$: $m_s(\cdot)$ is prescribed by Proposition 1,[24] *and*

b. the entrepreneur designs the optimal security $s^*(\cdot)$ that maximizes the expected payment:

$$\mathbb{E}[m_s(\theta) \cdot (\theta - s(\theta))] \,.$$

According to Proposition 1, there are three possible investor behaviors, given the entrepreneur's optimal security. First, the investor may optimally choose not to acquire information and simply accept the security as proposed. This implies that the project is certainly financed.

[23]See Woodford (2008) and Yang (2015 2017) for more examples on this decision problem.

[24]The specification of belief for the investor at any generic information set after information acquisition is implied by Proposition 1, provided the definition of $m_s(\cdot)$.

Second, the investor may optimally acquire some information, induced by the proposed security, and then accept the entrepreneur's optimal security with a positive probability. In this case, the project is financed with a probability that is positive but less than one. Third, the investor may simply reject the security without acquiring information, implying that the project is certainly not financed. All of the three cases can be accommodated by the equilibrium definition. This third case, however, represents the outside option of the entrepreneur, who can always offer nothing to the investor and drop the project. Accordingly, we focus on the first two cases. The following lemma helps distinguish the first two cases of equilibrium from the third. All proofs are relegated to Appendix A.

Lemma 1. *The project can be financed with a positive probability if and only if*

$$\mathbb{E}\left[\exp(\mu^{-1}\cdot(\theta-k))\right] > 1\,. \tag{2.1}$$

Lemma 1 is an intuitive investment criterion. It implies that the project is more likely to be financed if the prior of the cash flow is better, if the initial investment k is smaller, or if the cost of screening μ is lower. When condition (2.1) is violated, the investor will reject the proposed security, whatever it is.

The following Corollary 1 implies that, in the baseline model, the entrepreneur will never propose to concede all the cash flows to the investor if the project is financed. This corollary is straightforward but worth emphasizing, in that it helps illustrate the key friction by showing that the interests of the entrepreneur and the investor are not aligned.

Corollary 1. *When the project can be financed with a positive probability, $s^*(\cdot)=\theta$ is not an optimal security.*

In what follows, we assume that condition (2.1) is satisfied, and characterize the entrepreneur's optimal security, focusing on the first two types of equilibria with a positive screening cost $\mu>0$.

2.1 Optimal security without inducing information aquisition

In this subsection, we consider the case in which the entrepreneur's optimal security is accepted by the investor without information acquisition. In other words, the entrepreneur finds that the cost of screening does not justify its benefit and thus wants to design a security to deter it. Concretely, this means $Pr\left[m_s^*(\theta)=1\right]=1$. We first consider the investor's problem of screening, given the entrepreneur's security, then characterize the optimal security.

Given a security $s(\cdot)$, the investor's payoff gain from accepting rather than rejecting it is

$$\Delta u_I(\theta) = u_I(1,\theta) - u_I(0,\theta) = s(\theta) - k\,, \text{ for any } \theta\,. \tag{2.2}$$

According to Proposition 1 and conditions (1.1) and (2.2), any security $s(\cdot)$ that is accepted by the investor without information acquisition must satisfy

$$\mathbb{E}\left[\exp\left(-\mu^{-1}\cdot\left(s(\theta)-k\right)\right)\right] \leqslant 1\,. \tag{2.3}$$

If the left-hand side of inequality (2.3) is strictly less than one, the entrepreneur could lower $s(\theta)$ for some θ to increase the expected payoff gain without affecting the investor's incentives. Hence, condition (2.3) always holds as an equality in equilibrium.

By backward induction, the entrepreneur's problem is to choose a security $s(\cdot)$ to maximize the expected payoff

$$u_E(s(\cdot)) = \mathbb{E}\left[\theta - s(\theta)\right]$$

subject to the investor's information acquisition constraint

$$\mathbb{E}\left[\exp\left(-\mu^{-1}\cdot\left(s(\theta)-k\right)\right)\right] = 1\,,$$

and the feasibility condition $0 \leqslant s(\theta) \leqslant \theta$.

In this case, the entrepreneur's optimal security is a debt. We characterize this optimal security by the following proposition, along with its graphical illustration in Figure 1.

Proposition 2. *If the entrepreneur's optimal security $s^*(\cdot)$ induces the investor to accept the security without acquiring information, then it takes the form of a debt:*

$$s^*(\cdot) = \min\left(\theta, D^*\right)$$

where the unique face value D^* is determined in equilibrium. In particular, we have

$$\mathbb{E}[s^*(\theta)] > k\,.$$

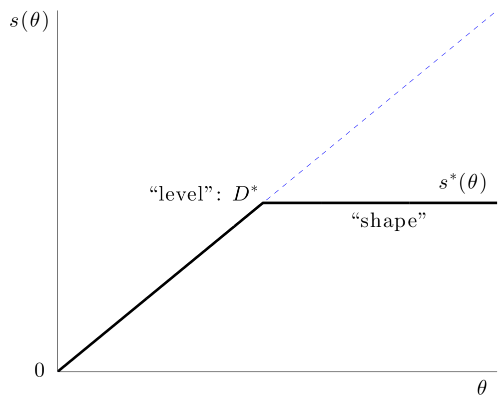

Figure 1: The unique optimal security without information acquisition

It is intuitive that debt with a high enough face value is the optimal means of finance when the entrepreneur does not want to induce information acquisition. First, it is the flat *shape* of debt that incentivizes *efficient information acquisition*, which in this case is to not acquire any information. Specifically, the optimal security must minimize the investor's incentive to acquire information to the extent at which he does not want to acquire information. This implies that the optimal security should be as flat as possible when the limited liability constraint is not binding, which leads to debt.

Second, the expected overall payment $\mathbb{E}[s^*(\theta)]$, which also captures the face value and level of the debt, must exceed the investment requirement k. This difference $\mathbb{E}[s^*(\theta)] - k$ exists because the investor has the option to acquire information and thus enjoys an information rent, which forces the entrepreneur to grant a sufficiently high overall payment and thus a sufficiently high face value. Otherwise, the investor will reject the offer and not finance the project. Therefore, the *level* of debt helps incentivize the *efficient financing decision*.

The optimality of debt here accounts for the real-world scenarios in which new projects are financed by fixed-income securities. When a project's market prospects are clear and thus extra information is less useful, it is optimal to deter or mitigate investor's costly information acquisition by resorting to debt. Empirical evidence suggests that many conventional businesses and less revolutionary start-ups relying heavily on plain vanilla debt finance from investors such as relatives, friends, and traditional banks (e.g., Berger and Udell 1998; Kerr and Nanda 2009; Robb and Robinson 2014), as opposed to more sophisticated financial contracts.

The optimality of debt described here resembles that in Yang (2017), but the underlying channel has subtle differences. Yang (2017) considers security design with flexible information acquisition in a comparable exchange economy. In that model, a seller has an asset in place and proposes a security to a more patient buyer to raise liquidity. The buyer can acquire information about the asset's cash flow before purchasing. There, information is always bad: it does not guide any production but always induces adverse selection. Thus, debt is always optimal because it offers the greatest mitigation of the buyer's adverse selection. In the present production economy, however, information is socially beneficial because it helps screen in (out) good (bad) projects and thus guides efficient investment decisions, while it is still costly to the entrepreneur because of the information rent that emerges from the investor's endogenous information advantage. When its benefit does not justify the cost, the entrepreneur optimally designs debt to deter information acquisition. Rather, when its benefit exceeds the cost, debt is no longer optimal, as we will show below. Overall, in both papers, debt is optimal when the entrepreneur wants to deter information acquisition, but the reason she wants to do so is not exactly the same.

2.2 Optimal security inducing information acquisition

Here, we characterize the entrepreneur's optimal security that induces the investor to acquire information. In this case, the entrepreneur finds screening desirable and designs a security to incentivize it. According to Proposition 1, this means $Prob\,[0 < m_s(\theta) < 1] = 1$, that is, the investor will finance the project with positive probability, but not certainty.

Again, according to Proposition 1 and conditions (1.2) and (2.2), any generic security $s(\cdot)$ that induces the investor to acquire information must satisfy

$$\mathbb{E}\left[\exp\left(\mu^{-1}\left(s\left(\theta\right)-k\right)\right)\right] > 1 \tag{2.4}$$

and

$$\mathbb{E}\left[\exp\left(-\mu^{-1}\left(s\left(\theta\right)-k\right)\right)\right] > 1\ , \tag{2.5}$$

Given such a security $s(\cdot)$, Proposition 1 and condition (1.3) also prescribe that the investor's optimal screening rule $m_s(\cdot)$ is uniquely pinned down by the functional equation:

$$s(\cdot) - k = \mu \cdot \left(g'(m_s(\cdot)) - g'(\overline{\pi}_s)\right), \tag{2.6}$$

where

$$\overline{\pi}_s = \mathbb{E}\left[m_s(\theta)\right]$$

is the investor's unconditional probability of accepting the security. In what follows, we denote by $\overline{\pi}_s^*$ the unconditional probability induced by the entrepreneur's optimal security $s^*(\cdot)$.

We derive the entrepreneur's optimal security backwards. Taking into account the investor's response $m_s(\cdot)$, the entrepreneur chooses a security $s(\cdot)$ to maximize

$$u_E(s(\cdot)) = \mathbb{E}\left[m_s(\theta) \cdot (\theta - s(\theta))\right] \tag{2.7}$$

subject to (2.4), (2.5), (2.6), and the feasibility condition $0 \leqslant s(\theta) \leqslant \theta$. [25]

We use a variational approach to solve this problem. Intuitively, we impose the condition that the entrepreneur should not benefit from any deviation from the optimal security. Let

$$s(\cdot) = s^*(\cdot) + \alpha \cdot \varepsilon(\cdot) \tag{2.8}$$

be a perturbation of the optimal security, where $\varepsilon(\cdot)$ can be any arbitrary measurable function of θ over $\mathbb{R}_+$.

Lemma 2. *The entrepreneur's marginal expected payoff from adding arbitrage cash flows $\varepsilon(\cdot)$ to the optimal security $s^*(\cdot)$ is given by*

$$\left.\frac{du_E(s(\cdot))}{d\alpha}\right|_{\alpha=0} = \mathbb{E}[r(\theta) \cdot \varepsilon(\theta)],$$

where

$$r(\cdot) = -m_s^*(\theta) + \mu^{-1} \cdot \left(g''(m_s^*(\theta))\right)^{-1} \cdot (\theta - s^*(\theta) + w^*) \tag{2.9}$$

[25] Again, the entrepreneur's individual rationality constraint $\mathbb{E}\left[m_s(\theta) \cdot (\theta - s(\theta))\right] \geqslant 0$ is automatically satisfied.

is the Frechet derivative (a function of θ) that measures the entrepreneur's marginal benefit from varying $s^*(\cdot)$,[26] *and w^* is a constant determined in equilibrium.*

The two terms in $r(\cdot)$, shown on the right-hand side of (2.9), reflect the key trade-off that the entrepreneur faces when designing the security. The first term captures the direct effect of raising $s^*(\theta)$ for any θ disregarding the induced change in $m_s^*(\theta)$. This term is always negative, because increasing $s^*(\theta)$ reduces the entrepreneur's residual claim. The second term captures the indirect effect of raising $s^*(\theta)$ for any θ through the induced change in $m_s^*(\theta)$. Intuitively, this term should be positive, because increasing $s^*(\theta)$ helps incentivize the investor's information acquisition and financing decision, the effect of which is summarized by the change in $m_s^*(\theta)$. The two effects compete with each other and help pin down the shape of the optimal security.

Based on the trade-off above, the Frechet derivative naturally leads to the entrepreneur's first-order condition. We have

$$r^*(\theta) \begin{cases} \leqslant 0 & \text{if} \quad s^*(\theta) = 0 \\ = 0 & \text{if} \quad 0 < s^*(\theta) < \theta \\ \geqslant 0 & \text{if} \quad s^*(\theta) = \theta \end{cases} .$$

By the definition of $r(\cdot)$ (2.9) and the fact that $g''(x) = x^{-1}(1-x)^{-1}$, the first-order condition is equivalent to

$$(1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \begin{cases} \leqslant \mu & \text{if} \quad s^*(\theta) = 0 \\ = \mu & \text{if} \quad 0 < s^*(\theta) < \theta \\ \geqslant \mu & \text{if} \quad s^*(\theta) = \theta \end{cases} . \tag{2.10}$$

Based on the first-order condition, we first characterize the shape of the optimal security. Notably, we argue that it helps incentivize efficient information acquisition, and we illustrate to what extent it does so. To do this, we first solve for the "unconstrained" part of the optimal security, which essentially determines the shape of the optimal security where the feasibility condition $0 \leqslant s(\theta) \leqslant \theta$ is not binding. We denote the solution by $\widehat{s}(\cdot)$. We also denote the corresponding screening rule by $\widehat{m}_s(\cdot)$. The unconstrained part will represent the equity component of the eventually optimal security.

[26] In more mathematical terms, $r(\cdot)$ is the functional derivative used in calculus of variations, which is itself a function. It is analogous to the derivative of a real-valued function of a single real variable but generalized to accommodate functions on Banach spaces.

Lemma 3. *In an equilibrium with information acquisition, the unconstrained part of the optimal security* $\widehat{s}(\cdot)$ *and its corresponding screening rule* $\widehat{m}_s(\cdot)$ *are determined by the following two functional equations*

$$\widehat{s}(\cdot) - k = \mu \cdot \left(g'\left(\widehat{m}_s(\cdot)\right) - g'\left(\overline{\pi}_s^*\right)\right), \tag{2.11}$$

and

$$(1 - \widehat{m}_s(\cdot)) \cdot (\theta - \widehat{s}(\cdot) + w^*) = \mu, \tag{2.12}$$

where $\overline{\pi}_s^*$ *and* w^* *are two constants determined in equilibrium.*

Lemma 3 exhibits the relationship between the unconstrained part $\widehat{s}(\cdot)$ and the corresponding screening rule $\widehat{m}_s(\cdot)$. Condition (2.11) directly follows condition (2.6), which specifies how the investor responses to the unconstrained part by adjusting his screening rule. On the other hand, condition (2.12) follows the entrepreneur's first-order condition (2.10) in the case of $0 < s^*(\theta) < \theta$. It indicates the entrepreneur's optimal choices of payments across states, given the investor's screening rule. In equilibrium, $\widehat{s}(\cdot)$ and $\widehat{m}_s(\cdot)$ are jointly determined. We can characterize their monotonicity in the following lemma.

Lemma 4. *In an equilibrium with information acquisition, the unconstrained part of the optimal security* $\widehat{s}(\cdot)$ *and the corresponding screening rule* $\widehat{m}_s(\cdot)$ *satisfy*

$$\frac{\partial \widehat{m}_s(\theta)}{\partial \theta} = \mu^{-1} \cdot \widehat{m}_s(\theta) \cdot (1 - \widehat{m}_s(\theta))^2 > 0 \text{ , for any } \theta \tag{2.13}$$

and

$$\frac{\partial \widehat{s}(\theta)}{\partial \theta} = 1 - \widehat{m}_s(\theta) \in (0,1) \text{ , for any } \theta\,. \tag{2.14}$$

Lemma 4 prescribes three predictions about the shape of the unconstrained part of the optimal security and its associated screening rule. First, condition (2.13) implies that the corresponding optimal screening rule $\widehat{m}_s(\cdot)$ is strictly increasing. Second, condition (2.14) implies that the unconstrained part $\widehat{s}(\cdot)$ is also strictly increasing. These are because, per Proposition 1, we have $Prob[0 < \widehat{m}_s(\theta) < 1] = 1$ in this case, and thus the right-hand sides of (2.13) and (2.14) are positive. Third, it follows immediately that the residual of the unconstrained part, $\theta - \widehat{s}(\cdot)$, as a function of θ, is also strictly increasing.

The three predictions reveal the intuition underlying the investor's information acquisition and the entrepreneur's motive in incentivizing it. First, when the entrepreneur finds it optimal to induce information acquisition, it should benefit the entrepreneur, which is true only if the investor screens in a potentially good project and screens out bad ones. In other words, a better project must have a strictly higher probability to be screened in. This implies that the screening rule $\widehat{m}_s(\cdot)$ should be more likely to generate a good signal and to result in a successful finance when the cash flow θ is higher, while more likely to generate a bad signal and a rejection when θ is lower. Therefore, $\widehat{m}_s(\cdot)$ should be strictly increasing in θ.

Second, to induce a strictly increasing screening rule $\widehat{m}_s(\cdot)$, the optimal unconstrained part $\widehat{s}(\cdot)$ must be strictly increasing in θ as well, according to condition (2.11). Intuitively, this monotonicity reflects the dependence of production on information acquisition: the entrepreneur is willing to better compensate the investor in the event of higher cash flows to encourage efficient information acquisition. This monotonicity also gives the unconstrained part of the optimal security a natural equity interpretation. In particular, this equity component is designed to help incentivize efficient information acquisition, and thus the investor acquires adequate information to distinguish between any different states. Note that this monotonicity result relies on the assumption underlying the entropy-based information cost that it is equally costly to acquire information about any state. If the investor finds it less costly to acquire information about some certain states, it is natural to expect the resultant optimal security to become flatter on those states.[27]

Third, the residual of the optimal unconstrained part, $\theta - \widehat{s}(\cdot)$, also strictly increases in θ in states with high cash flows. In other words, $\widehat{s}(\cdot)$ is dual monotone when it deviates from the 45° line in states with high cash flows. Intuitively, the dependence of production on information acquisition implies that the investor would obtain all underlying cash flows. However, the entrepreneur's bargaining power allows her to retain some surplus, and thus she will not propose all the cash flows to the investor, because doing so would leave her with nothing. This conflict is mitigated in a mutually compromised, but most efficient, way: the entrepreneur rewarding the investor more but also retaining more in better states. As a result, $\widehat{s}(\cdot)$ deviates further from the

[27]Formally showing this point requires adding more parametric and distributional assumptions, which would make the model less tractable and is beyond the scope of this present paper.

45° line in better states. This deviation literally captures the entrepreneur's retained benefit. And economically, it reflects the degree to which the allocation of cash flow is not perfectly efficient, which in turn comes from the separation of the entrepreneur's bargaining power in security design and the investor's information acquisition.

Overall, Lemma 4 suggests that the eventually optimal security consists of an unconstrained equity component, the dual monotonic *shape* of which is designed to help incentivize *efficient information acquisition*. Next, Proposition 3 fully characterizes the optimal security $s^*(\cdot)$ and suggests that it further consists of a debt component, the *level* of which is design to help incentivize the *efficient financing decision*. Figure 2 shows the payment structure.

Proposition 3. *If the entrepreneur's optimal security $s^*(\cdot)$ induces the investor to acquire information, then it takes the following form of a combination of debt and equity:*

$$s^*(\cdot) = \begin{cases} \theta & \text{if} \quad 0 \leqslant \theta \leqslant \widehat{\theta} \\ \widehat{s}(\theta) & \text{if} \quad \theta > \widehat{\theta} \end{cases},$$

where $\widehat{\theta} > k$ and the unconstrained part $\widehat{s}(\cdot)$ satisfies:

i) $\widehat{\theta} < \widehat{s}(\theta) < \theta$ for any θ;

ii) $0 < d\widehat{s}(\theta)/d\theta < 1$ for any θ.

Finally, the corresponding optimal screening rule satisfies $dm_s^*(\theta)/d\theta > 0$ for any θ.

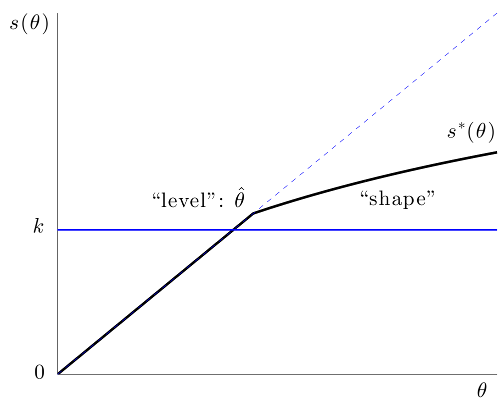

Figure 2: The unique optimal security with information acquisition

In Proposition 3, the characterization of the unconstrained equity component $\widehat{s}(\cdot)$ directly follows Lemma 4; what is new is the characterization of a constrained part when the cash flow is low. Because this constrained part follows the 45° line until a strictly positive threshold $\widehat{\theta}$, it admits a natural debt interpretation with a face value $\widehat{\theta}$. Notably, Proposition 3 further shows that the face value $\widehat{\theta}$ must be strictly greater than the investment requirement k.

The debt component, in particular, the associated level, of the optimal security is designed to help incentivize the efficient financing decision. When information is desirable, the endogenous information advantage gives the investor an information rent. Thus, the overall payment of the optimal security should be high enough (given its shape) in order to avoid the offer being rejected. To achieve this, the entrepreneur must give the investor a debt component with a sufficiently high face value. It is worth noting that the debt component here plays a subtly different role than the optimal debt in Proposition 2: there the flat shape of debt is designed to deter costly information acquisition, while here the level of the debt makes sure that the overall payment of the optimal security is high enough and thus the investor will not reject the offer.

Proposition 3 is consistent with empirical evidence regarding the popularity of the combination of debt and equity when information acquisition is likely to be desirable. Brewer, Genay, Jackson, and Worthington (1996) examine young firms in the U.S. government-sponsored Small Business Investment Companies program (commonly known as SBIC) and find that firms with less transparent projects are likely to issue the combination of debt and equity to the same investor, which case accounts for 26% of their whole sample. Looking at a more representative sample of private firms, Berger and Udell (1998) also suggest that younger and more innovative firms are more likely to be financed by both external debt and equity at the same time.

The combination of debt and equity as proscribed in Proposition 3 also resembles participating convertible preferred stock, with $d\widehat{s}(\theta)/d\theta$ defined as the conversion rate. In reality, such a security grants holders the right to receive both the face value and their equity participation as if it was converted, in the event of a public offering or sale.[28] This prediction is consistent with the empirical evidence of venture contracts documented in Kaplan and Stromberg (2003), who find that 94% of all financing contracts are convertible preferred stock,[29] among which 40% are participating. Participating preferred stocks are more popular than straight convertible preferred stocks in earlier investment rounds when the project faces more uncertainty and thus investor

[28] Compared to equity (common stock), debt and preferred stock are identical in our model, because the model only features two tranches and no dividends.

[29] If we include convertible debt and the combination of debt and equity, this number increases to 98%.

screening is more necessary, consistent with our model predictions. For brevity, in what follows we refer to the optimal security in this case as convertible preferred stock or the combination of debt and equity, and use the two terms interchangeably.

Finally, comparison of the production with an exchange economy helps show why our model can predict both debt and nondebt securities. In a production economy, costly information contributes to the output, whereas in an exchange economy it only affects the reallocation of existing resources. As discussed earlier, in an exchange economy as modeled in Dang, Gorton, and Holmstrom (2015) and Yang (2017), information is always socially wasteful, and it is always optimal to discourage information acquisition. In the present paper, however, the entrepreneur and the investor jointly tap the project's cash flow if the investor accepts the proposed security. Thus, the present model features a production economy in which the social surplus may depend positively on costly information. As a result, the entrepreneur may want to design a security that encourages the investor to acquire information favorable to the entrepreneur and then finance the project, which justifies the combination of debt and equity.

2.3 Allocation of bargaining power in designing the security

As illustrated in Proposition 3, the equity component deviates from the 45° line as the cash flow increases, which makes the resultant optimal security not fully efficient in incentivizing the investor's information acquisition when it is desirable. The feature stems from the entrepreneur's bargaining power in security design. To better understand this point, we extend the baseline model to consider more general allocation of bargaining power between the entrepreneur and the investor.

We consider a bargaining parameter $1 - \alpha$ capturing the entrepreneur's bargaining power (and α capturing that of the investor) *in the process of security design.* Suppose a third party in the economy knows α, designs the security and proposes it to the investor. Facing the offer, the investor acquires information according to the security and then decides whether or not to accept this offer. The third party's objective function is an average of the entrepreneur's and the investor's utilities, weighted by the bargaining parameter of each. When $\alpha = 0$, this extension reduces to our baseline model.

To clarify this bargaining parameter α, we highlight that it captures the allocation of bargaining power in the process of designing the security, but not necessarily the *overall* bargaining power in terms of the ultimate ability to share the social surplus. In other words, $\alpha = 0$, like in the baseline model, does not suggest that the investor obtains a zero surplus in equilibrium. Similarly, $1 - \alpha > \alpha$ does not suggest that the entrepreneur obtains a higher surplus than does the investor. This is because only the investor can acquire information and finance the project, the resultant endogenous information rent contributes to the investor's overall bargaining power in terms of sharing the total social surplus even if $\alpha = 0$. In this sense, we view both our baseline model and this extension consistent with the evidence in Opp (2016) that informationally sophisticated investors such as venture capitalists can capture a great share of surplus, and their information advantage indeed contributes to their share of surplus. Therefore, we focus on changes in α and interpret that the entrepreneur's bargaining power becomes weaker as α increases and stronger as α decreases in this extension.

We also note that this bargaining parameter α does not capture which party is more likely to literally draft the contractual terms of a security in reality. Instead, both our baseline model and the extension offer an equilibrium view of security design based on the strategic interaction between the entrepreneur and the investor.

The derivations for the results are the same as those used in the baseline model. In this setting, the third-party's objective function, that is, the payoff gain, is

$$u_T(s(\cdot)) = \alpha \cdot (\mathbb{E}[(s(\theta) - k) \cdot m(\theta)] - \mu \cdot I(m)) + (1 - \alpha) \cdot \mathbb{E}[(\theta - s(\theta)) \cdot m(\theta)] \, .$$

We can show that, with information acquisition, the equation that governs information acquisition is still the same as condition (2.6):

$$s\left(\cdot\right) - k = \mu \cdot \left(g'\left(m_s\left(\cdot\right)\right) - g'\left(\overline{\pi}_s\right)\right) \, ,$$

while the Frechet derivate that characterizes the optimality of the unconstrained part of the optimal security becomes

$$r(\cdot) = (2\alpha - 1) \cdot m(\theta) + (1 - \alpha) \cdot \mu^{-1} \cdot m(\theta) \cdot (1 - m(\theta)) \cdot (\theta - \widehat{s}(\theta) + w)\,.$$

The following two propositions characterize the optimal security in the general setting.[30]

Proposition 4. *Consider the bargaining power parameter α:*

i). when $0 \leqslant \alpha < 1/2$ and if information acquisition happens in equilibrium, the unconstrained part $\widehat{s}(\cdot)$ and the corresponding screening rule $\widehat{m}_s(\cdot)$ satisfy

$$\frac{d\widehat{s}\,(\theta)}{d\theta} = \frac{1 - \widehat{m}_s\,(\theta)}{1 - \frac{\alpha}{1-\alpha}\widehat{m}_s(\theta)} \in (0,1),\ \textit{for any}\ \theta$$

and

$$\frac{d\widehat{m}_s\,(\theta)}{d\theta} = \frac{\mu^{-1} \cdot \widehat{m}_s\,(\theta) \cdot (1 - \widehat{m}_s\,(\theta))^2}{1 - \frac{\alpha}{1-\alpha}\widehat{m}_s(\theta)} > 0,\ \textit{for any}\ \theta\,,$$

and all the results from Proposition 1 to Proposition 3 still hold;

ii). when $1/2 \leqslant \alpha \leqslant 1$, the optimal security is $s^*(\cdot) = \theta$, that is, an equity that is backed by all the cash flows of the potential project.

The first part of Proposition 4 suggests that as the entrepreneur's bargaining power becomes weaker than in the baseline model, but not sufficiently weak, the qualitative results remain. However, a comparison to Lemma 4 suggests that the slope of both the equity component and the corresponding screening rule becomes steeper when the entrepreneur's bargaining power becomes weaker. This is intuitive: the entrepreneur becomes less able to retain a benefit in designing the security, leading to a more generous payment schedule to the investor and thus more efficient information acquisition. As documented by Gompers, Gornall, Kaplan and Strebulaev (2016), entrepreneurs usually have bargaining power in the process of security design even facing sophisticated investors like venture capitalists, but the level of bargaining power may vary. Our model thus generates new testable predictions regarding how the allocation of cash flow rights changes if the allocation of bargaining power changes between the entrepreneur and the investor.

On the contrary, when the entrepreneur's bargaining power becomes sufficiently weak, the second part of Proposition 4 suggests the entrepreneur sell an equity that represents all the cash

[30] The proofs for the extended model follow those for the baseline model closely, so we omit them for brevity.

flows of the potential project. In this case, the investor as the information producer does internalize all the benefits from information acquisition. In other words, as the investor's bargaining power in security design becomes sufficiently strong, he can effectively lift the friction in our economy and make socially efficient information acquisition and production decisions. As suggested by Aghion and Tirole (1994) and Rajan (2012), when the entrepreneur's bargaining power becomes weaker in a firm's life cycle, selling a company as a whole to an outside investor becomes more common and desirable, consistent with the predictions in this model extension.

A special case is that the investor can both design the security and acquire information. In this sense, our result in this extension nests the result of equity as the optimal security in Manove, Padilla, and Pagano (2001). They consider a two-state security design model in which an entrepreneur needs to raise capital from a monopolistic bank to finance a project, and the bank can both design the security and acquire information through costly state verification. They show that a sufficiently high payment in the good state, representing an equity, can incentivize fully efficient information acquisition.

2.4 Alternative time line of information acquisition and finance

The time line that the investor can acquire information *before* his financing decision is also crucial in driving our security design results. Consider an alternative time line in which the investor can acquire information only *after* the financing decision. It can be easily shown that:

Proposition 5. *Under the alternative time line, the optimal security is $s^*(\cdot) = \theta - p^*$, in which $p^* > 0$ is set so that the investor obtains zero profit.*

Proposition 5 suggests that the optimal security is still an equity backed by all the cash flows of the potential project, but the entrepreneur should sell it at a positive lump-sum price such that the investor obtains zero surplus. By doing this, the entrepreneur ensures that the investor will choose the efficient information acquisition strategy and make an efficient investment decision, thus maximizing social surplus. Then, by setting an upfront lump-sum price, p^*, the entrepreneur retains the maximal surplus. In this case, the detailed information structure does not matter anymore, and, likewise, the security design becomes less relevant.

The key to understand Proposition 5 is the entrepreneur's overall bargaining power becomes too strong in the sense that she can prevent the investor from acquiring any information before the financing decision, essentially removing the friction in the security design process. Thus, the socially efficient outcome is achieved and entrepreneur also captures all the social surplus.

In practice, however, it is common and reasonable for investors to have the option of acquiring information about the project before the financing decision, and investors indeed do so (Chemmanur, Krishnan, and Nandy 2011; Kerr, Lerner, and Schoar 2014; Opp 2016). This option gives rise to the investor's endogenous information rent, which effectively contributes to the investor's overall bargaining power in terms of sharing the social surplus even if the entrepreneur designs the security in the first place.[31] In reality, this option also justifies the time line and the security design results in the baseline model.

3 Optimal Securities in Different Circumstances

Two natural questions emerge: given the characteristics of the production economy, when is debt optimal? And when is the combination of debt and equity optimal? We focus on the baseline model and the cases in which the project can be financed with a positive probability, that is, when condition (2.1) is satisfied.

3.1 Net present value dimension

We first investigate how the optimal security varies when the ex ante net present value (NPV) is different, which is one of the most natural dimensions to measure the market prospects of a project.

Proposition 6. *Consider the ex ante NPV (i.e., $\mathbb{E}[\theta] - k$) of the project:*

i). if $\mathbb{E}[\theta] - k \leqslant 0$, the optimal security $s^*(\cdot)$ is convertible preferred stock; and

ii). if $\mathbb{E}[\theta] - k > 0$, $s^*(\cdot)$ may be either convertible preferred stock or debt.

When the project has a zero or negative NPV, convertible preferred stock is the only type of optimal security. In this case, the investor will never finance the project without acquiring

[31] Again, this argument helps justify the fact that the entrepreneur's overall bargaining power is not too strong in the baseline model, even if the entrepreneur designs the security.

information, because doing this incurs an expected loss even if the entrepreneur promises the entire cash flow. However, when the investor acquires information, the probability of financing the project becomes positive because a potentially good project can be screened *in*. Hence, the entrepreneur is better off by proposing convertible preferred stock to encourage screening in.

On the other hand, when the project has a positive NPV, convertible preferred stock may still be optimal, but the aim is to encourage the investor to screen *out* a potentially bad project. In this case, the entrepreneur can finance the project with probability one by proposing debt with a sufficiently high face value (as information rent) to deter information acquisition. However, such certain financing may be too costly because it leaves too little for the entrepreneur. Instead, the entrepreneur may retain more by offering a less generous convertible preferred stock and inviting the investor to acquire information. Doing so results in financing with a probability less than one, but the entrepreneur's total expected profit could be higher because a potentially bad project may be screened out. This ultimately justifies convertible preferred stock as the optimal choice. If reaching a certain financing is not too costly, and the benefit from information acquisition and the resultant screening out is not high enough, the entrepreneur may find it optimal to propose debt to simply deter costly information acquisition.

Although intuitive, one limitation of this NPV dimension is that we cannot fully determine whether debt or convertible preferred stock is optimal when the ex ante NPV is positive.[32] The next subsection investigates how the optimal security changes when the severity of the friction in the economy varies, which helps reveal the model mechanism at a more fundamental level.

3.2 The friction dimension

In our baseline model, production and security design is performed by the entrepreneur while information acquisition and financing by the investor. This physical separation is always present and unchanged regardless of any exogenous characteristics of the economy. Hence, the severity of the friction is naturally reflected in the extent to which production depends on information acquisition and the subsequent financing; the friction is more (less) severe in the sense that

[32] Although we view it as a limitation of the NPV dimension, we emphasize that it does not necessarily suggest a shortcoming of our model, which is designed to be free from parametric or distributional assumptions. We can fully determine the optimal security under a given positive ex ante NPV if we impose mild parametric and distributional assumptions, as numerically shown in Section 4.

production depends more (less) on information acquisition and financing, given the separation. However, our model does not feature any parametric or distributional assumptions, so one challenge we face is to find a measure for the friction. To overcome this challenge, we rely on the standard definition of social efficiency, which is naturally linked to the notion of friction.

Definition 2. *An optimal security in the baseline economy achieves social efficiency if and only if the induced optimal screening rule $m_s^*(\cdot)$ maximizes the expected social surplus:*

$$\mathbb{E}[m(\theta) \cdot (\theta - k)] - \mu \cdot I(m(\cdot)), \tag{3.15}$$

which is the difference between the expected profit of the project and the cost of information, both of which are functions of the screening rule $m(\cdot)$.

Intuitively, if the optimal security can help the underlying economy achieve social efficiency, we view the friction to be not severe because it can be effectively eliminated by the optimal security design. In contrast, if even the optimal security design cannot achieve social efficiency, we view the friction of the underlying economy to be severe. Along this friction dimension measured by the achievability of social efficiency, we have the following result.

Proposition 7. *In the baseline production economy:*

i) the optimal security $s^*(\cdot)$ is debt if and only if friction in the economy is not severe, i.e., the optimal security achieves efficiency; and

ii) $s^*(\cdot)$ is convertible preferred stock if and only if the friction is severe, i.e., even the optimal security cannot achieve efficiency.

This result offers a clear dichotomy between the two types of optimal securities depending on how severe the friction in the economy is. If the friction is severe (i.e., the dependence of production on information acquisition and financing is strong), information acquisition is worthwhile and thus convertible preferred stock is optimal. In contrast, if the friction is not severe (i.e., the dependence of production on information acquisition and financing is weak), information acquisition does not justify its cost and thus debt is optimal.

Our predictions help unify the empirical evidence of the financing of entrepreneurial businesses. Debt financing is popular for conventional projects, when information is not very useful and thus the friction is not severe. Instead, financing with convertible preferred stock (or the combination of debt and equity) is common for innovative projects, especially in the early rounds, when information is crucial and thus the friction is severe.

4 Comparative Statics of the Optimal Security

To provide further intuition, we impose mild parametric and distributional assumptions and look at numerical comparative statics on the optimal securities for two empirical dimensions: the profitability of the project and its uncertainty.[33] When the environment varies, the role of screening changes, and the way in which the entrepreneur incentivizes information acquisition and financing changes accordingly, producing different optimal securities. We still focus on the baseline model and the cases in which the project can be financed with a positive probability, that is, when condition (2.1) is satisfied.

4.1 Project profitability

First, we consider the effects of variations in the project's profitability on the optimal security $s^*(\cdot)$, holding constant the project's market prospects (i.e., the prior distribution of the cash flow θ), and the cost of screening, μ. Thus, a decrease in the investment requirement k implies that the project is more profitable ex ante.

Figure 3 shows the results. The investment k takes three increasing values: 0.4, 0.475, and 0.525. When $k = 0.4$, the optimal security is debt; for the two other projects with larger k, one with positive and one with negative ex ante NPV, it is convertible preferred stock. Notably, the face value $\widehat{\theta}$ and the conversion ratios $d\widehat{s}(\theta)/d\theta$ of the convertible preferred stock are both increasing in k. For the prior of the cash flow θ, we take a normal distribution with mean 0.5 and standard deviation 0.125, and then truncate and normalize this distribution to the interval $[0, 1]$. The screening cost μ is fixed at 0.2.

The comparative statics for the profitability of the project serve as a detailed illustration of

[33] We are not aware of any analytical comparative statics pertaining to functionals. An analytical comparative statics requires a complete order, which is not applicable for our security space. Even for some ordered characteristics of the optimal security, for instance, the face value, analytical comparative statics are not achievable. Thus, we rely on numerical results to deliver intuitions and leave analytical work to future research. Numerical analysis in our framework is tractable but already technically intensive because we need to solve a system of functional equations. The algorithm and codes are available on request.

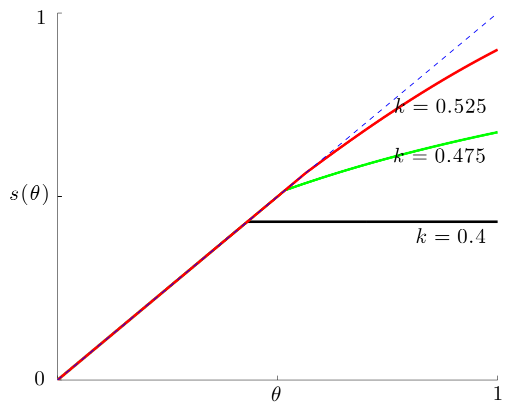

Figure 3: Change in investment with $\mathbb{E}[\theta] = 0.5, \mu = 0.2$

Propositions 6 and 7. When the project is sufficiently profitable ex ante ($k = 0.4$), the friction is not severe and the project will be financed by debt without inducing screening. When the project looks mediocre in terms of its profitability but still has a positive ex ante NPV ($k = 0.475$), the friction becomes severe, and information acquisition becomes worthwhile to screen bad projects out, so that convertible preferred stock becomes optimal. When the project is not ex ante profitable in the sense that its NPV is negative ($k = 0.525$), the friction is more severe, and the only way for the entrepreneur to obtain financing is to propose convertible preferred stock and expect a potentially good project to be screened in. For this type of project screening is more valuable, and hence the entrepreneur is willing to compensate the investor more generously to induce more effective screening, as seen in Figure 3.

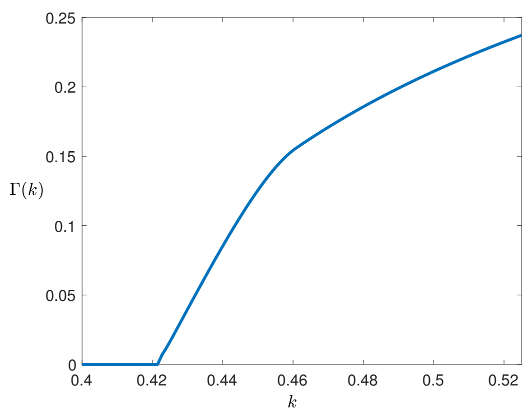

Figure 4: Efficiency loss with $\mathbb{E}[\theta] = 0.5, \mu = 0.2$

To further illustrate the friction in the economy, in Figure 4, we further plot the relative efficiency loss $\Gamma(k)$, defined as the percentage difference between the first-best social surplus in a hypothetical centralized economy and the equilibrium social surplus in the baseline model, against the investment requirement k. When k is small, the friction is accordingly not severe. Consistent with Propositions 6 and 7, in this case it is optimal not to acquire information and the optimal debt contract achieves first-best allocation, that is, there is no efficiency loss. In contrast, when k becomes larger, the friction accordingly becomes larger. As shown in Proposition 7, the optimal convertible preferred stock in this case cannot help achieve the first best, and the relative efficiency loss becomes larger as k becomes larger.

4.2 Project uncertainty

We then consider how varying the degree of the project's uncertainty affects the optimal security $s^*(\cdot)$. Concretely, we consider different prior distributions of the cash flow θ with the same mean, ranked by second-order stochastic dominance.[34] We also hold constant the investment requirement k and the cost of screening μ. Note that the effect of varying uncertainty cannot be accounted for by any argument involving risks, because both the entrepreneur and the investor are risk neutral. Instead, we still focus on friction and the role of screening to explain these effects.

Interestingly, the comparative statics for uncertainty depend on the sign of the project's ex ante NPV. As implied by Proposition 6, the role of screening differs when these signs differ. This further leads to different patterns of comparative statics when the degree of uncertainty varies.

First, we consider projects with positive ex ante NPV and increasing uncertainty. The results are shown in Figure 5, where the upper-left panel illustrates the priors of the cash flow θ, and the right panel illustrates the evolution of the optimal security. When the project is the least uncertain, the optimal security is debt. For more uncertain projects convertible preferred stock becomes optimal, while the face value $\widehat{\theta}$ and the conversion ratios $d\widehat{s}(\theta)/d\theta$ are both increasing in uncertainty. For the priors, we take normal distributions with mean 0.5 and standard deviations 0.125 and 0.25, and then truncate and normalize them to the interval $[0,1]$. We also construct a third distribution, in which the project is so uncertain that the cash flow has a greater probability of taking extreme values in $[0,1]$. The investment is $k=0.4$, and the cost of screening is $\mu=0.2$.

[34] The project's uncertainty can be measured other ways. For comparative statics, we find a partial order of uncertainty over the space of distributions, while keeping the project's ex ante NPV constant. Second-order stochastic dominance seems like a natural choice for this purpose.

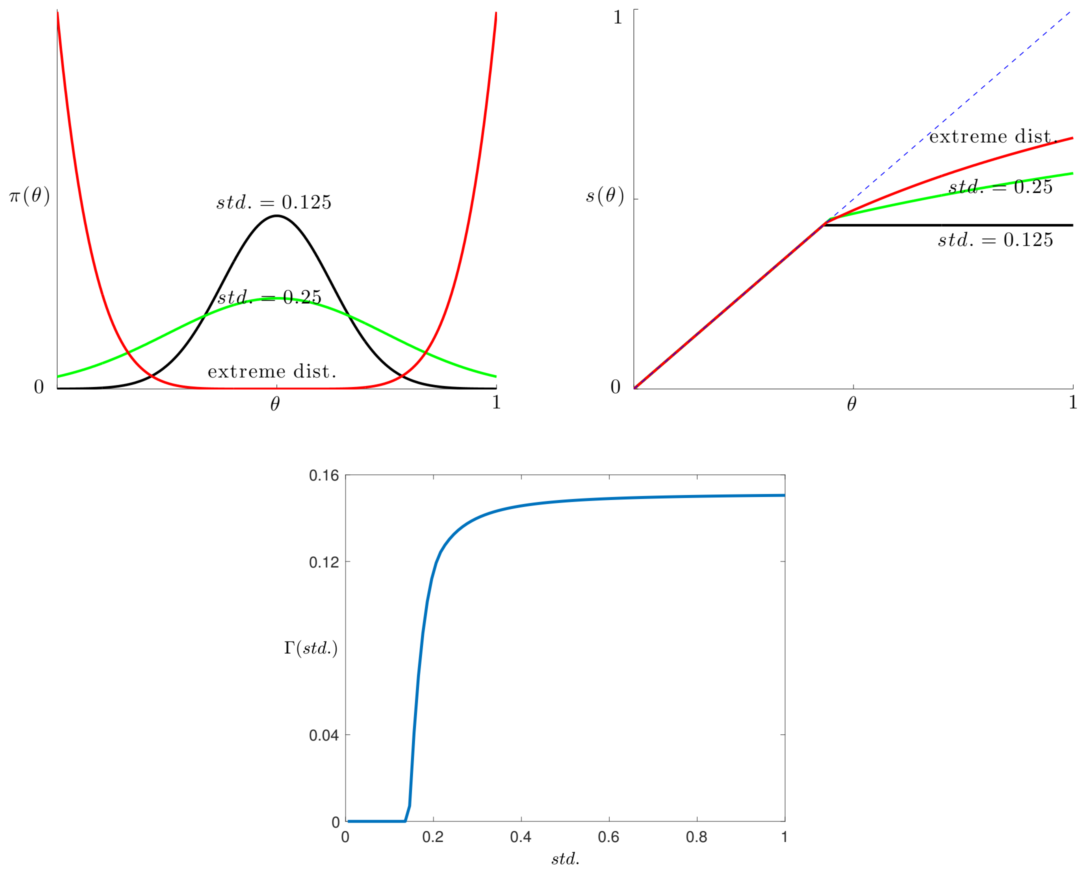

Figure 5: Change in uncertainty: $k = 0.4 < \mathbb{E}[\theta] = 0.5$, $\mu = 0.2$

The comparative statics in the upper right panel of Figure 5 demonstrate how varying uncertainty affects screening out bad projects, given positive ex ante NPV. Similarly, in the lower panel of Figure 5, we plot the relative efficiency loss against project uncertainty. When the project is least uncertain, it is least likely to be bad, which implies that screening out is least relevant and debt financing is accordingly optimal. In this case there is no efficiency loss, consistent with a friction being not severe. When uncertainty increases, the project is more likely to be bad, and screening out becomes more valuable. Hence, the entrepreneur finds it optimal to propose a more generous convertible preferred stock to induce screening out. In this case, the relative efficiency loss becomes larger as the project becomes more uncertain, reflecting a more severe friction despite the optimal convertible preferred stock.

Next, we consider projects with negative ex ante NPV, focusing on those that may be financed with a positive probability due to screening in through convertible preferred stock. Figure 6 shows the results, where both the face value $\widehat{\theta}$ and the conversion ratio $d\widehat{s}(\theta)/d\theta$ of the convertible preferred stock are decreasing in uncertainty. The priors are generated in the same way as in Figure 5. The investment is $k = 0.525$, and the cost of screening is $\mu = 0.2$.

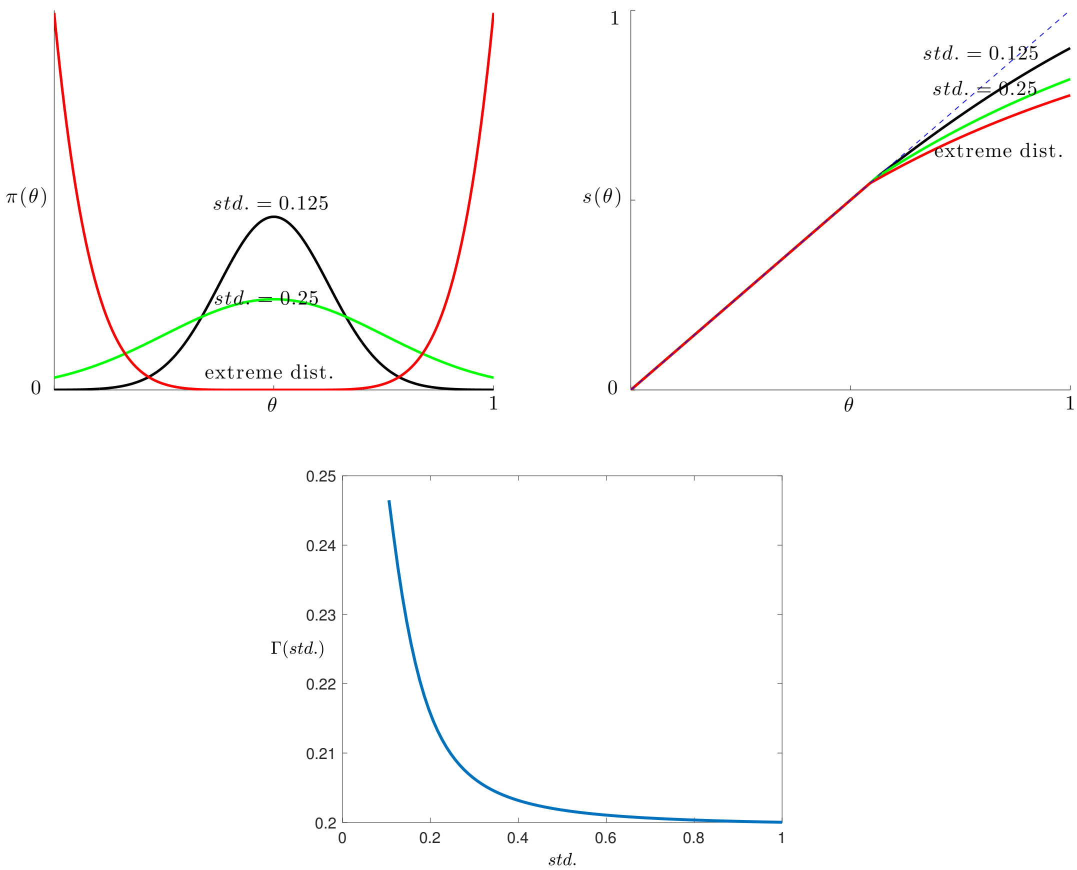

Figure 6: Change in uncertainty: $k = 0.525 > \mathbb{E}[\theta] = 0.5$, $\mu = 0.2$

The comparative statics in this case are also intuitive, according to the role of screening in. Given negative ex ante NPV, the investor screens in potentially good projects. In contrast to the positive-NPV case, here the increase in uncertainty means that the ex ante negative-NPV project is more likely to be good. Thus, acquiring costly information to screen in a potentially good project becomes less necessary, and, thus, the friction becomes less severe. Therefore, the entrepreneur wants to propose a less generous convertible preferred stock for less costly screening. Not surprisingly, the resultant convertible preferred stock moves away from the 45° line when the project is more uncertain. In this case, the relative efficiency loss becomes smaller as the project becomes more uncertain, reflecting the less severe friction.

5 Conclusion

Why are some projects financed by debt while other by nondebt securities? We propose a theory of security design in financing entrepreneurial production that posits that the investor can acquire costly information on the entrepreneur's project before making the financing decision. The key friction is that real production depends on information acquisition but the entrepreneur's bargaining power in the process of security design prevents the investor from internalizing all the benefits from information acquisition and financing. When the entrepreneur has some bargaining power, debt is optimal when information is not valuable for production, while the combination of debt and equity is optimal when information is valuable. However, when the investor has sufficiently strong bargaining power or can only acquire information after the financing decision, an equity that is backed by all the cash flows of the potential project is optimal, which effectively lifts the friction in this economy. These predictions are consistent with the empirical facts regarding the finance of entrepreneurial businesses.

This paper contributes to the security design literature in several respects, as well as to the broader corporate finance and contract design literature. By using the modeling approach of flexible information acquisition, we can (1) model endogenous information acquisition and endogenous finance decisions simultaneously and (2) work with arbitrary securities over continuous states while dispensing with usual distributional assumptions. Consequently, our results are general and can better explain which aspects of the optimal security design help encourage efficient information acquisition and which aspects help encourage efficient financing decisions.

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Appendices

Appendix A Deviation and Proofs

Proof of Lemma 1. We first prove the "only if" part. Suppose that

$$\mathbb{E}\left[\exp(\mu^{-1}(\theta - k))\right] \leqslant 1\,.$$

According to Proposition 1, even if the entrepreneur proposes all the future cash flow to the investor, the investor will still reject the offer without acquiring information. Because $s(\theta) \leqslant \theta$, the project cannot be initiated in this case.

Then we prove the "if" part. Let $t \in (0, 1)$. Because $\mathbb{E}\left[\exp(\mu^{-1}(t \cdot \theta - k))\right]$ is continuous in t, there exists $t < 1$ such that

$$\mathbb{E}\left[\exp(\mu^{-1}(t \cdot \theta - k))\right] > 1\,.$$

Hence, according to Proposition 1, the security $s_t(\cdot) = t \cdot \theta$ would be accepted by the investor with a positive probability. Moreover, let $m_t(\cdot)$ be the corresponding screening rule. Because $s_t(\cdot)$ would be accepted with a positive probability, $m_t(\cdot)$ cannot be always zero. Hence, the entrepreneur's expected payment is $\mathbb{E}[(1-t) \cdot \theta \cdot m_t(\theta)]$, which is strictly positive.

The security $s_t(\cdot)$ is a feasible security. Hence, the optimal security $s^*(\cdot)$ also will be accepted with a positive probability and deliver a positive expected payment to the entrepreneur. This concludes the proof. $\square$

Proof of Corollary 1. The proof is straightforward following the above proof of Lemma 1. Proposing $s^*(\cdot) = \theta$ gives the entrepreneur a zero payment, while proposing $s_t(\cdot) = t \cdot \theta$ constructed in the proof of Lemma 1 gives a strictly positive expected payment. This suggests that $s^*(\cdot) = \theta$ is not optimal. $\square$

Proof of Proposition 2. The Lagrangian of the entrepreneur's problem is

$$\mathscr{L} = \mathbb{E}\left[\theta - s(\theta) + \lambda \cdot \left(1 - \exp\left(\mu^{-1} \cdot (k - s(\theta))\right)\right) + \eta_1(\theta) \cdot s(\theta) + \eta_2(\theta) \cdot (\theta - s(\theta))\right] ,$$

where λ, $\eta_1(\cdot)$ and $\eta_2(\cdot)$ are multipliers.

The first-order condition is

$$\frac{d\mathscr{L}}{ds(\theta)} = -1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (q - s(\theta))\right) + \eta_1(\theta) - \eta_2(\theta) = 0 \,. \tag{A.1}$$

First, we consider a special case that allows us to solve the optimal security. If $0 < s(\theta) < \theta$, the two feasibility conditions are not binding. Thus, $\eta_1(\theta) = \eta_2(\theta) = 0$ for any θ, and the first-order condition is simplified as

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - s(\theta))\right) = 0 \text{ for any } \theta \,.$$

By rearrangement, we obtain

$$s(\theta) = k - \mu \cdot \ln(\lambda^{-1} \cdot \mu) \text{ for any } \theta \,. \tag{A.2}$$

We denote by D^* the right-hand side of (A.2), which is irrelevant for θ. By definition, we have $D^* > 0$. Also, it is straightforward to have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - D^*)\right) = 0 \,. \tag{A.3}$$

In what follows, we characterize the optimal solution $s^*(\cdot)$ for different regions of θ.

First, we consider the region of $\theta > D^*$. We show that $0 < s^*(\theta) < \theta$ in this region by contradiction.

If $s^*(\theta) = \theta > D^*$, we have $\eta_1(\theta) = 0$ and $\eta_2(\theta) \geqslant 0$. From the first-order condition (A.1), we obtain

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - \theta)\right) = \eta_2(\theta) \geqslant 0\,. \tag{A.4}$$

On the other hand, as $\theta > D^*$, we have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - D^*)\right) > -1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - \theta)\right)\,. \tag{A.5}$$

Conditions (A.3), (A.4), and (A.5) construct a contradiction. So we must have $s^*(\theta) < \theta$ if $\theta > D^*$.

Similarly, if $s^*(\theta) = 0$, we have $\eta_1(\theta) \geqslant 0$ and $\eta_2(\theta) = 0$. Again, from the first-order condition (A.1), we obtain

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot k\right) = -\eta_1(\theta) \leqslant 0\,. \tag{A.6}$$

On the other hand, as $D^* > 0$, we have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - D^*)\right) < -1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot k\right)\,. \tag{A.7}$$

Conditions (A.3), (A.6), and (A.7) construct another contradiction. So we must have $s^*(\theta) > 0$ if $\theta > D^*$.

Therefore, we have shown that $0 < s^*(\theta) < \theta$ for $\theta > D^*$. From the discussion above for this specific case, we conclude that $s^*(\theta) = D^*$ for $\theta > D^*$.

We then consider the region of $\theta < D^*$. We show that $s^*(\theta) = \theta$ in this region.

Because $s^*(\theta) \leqslant \theta < D^*$, we have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - s^*(\theta))\right) > -1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - D^*)\right)\,. \tag{A.8}$$

From condition (A.3), the right-hand side of this inequality (A.8) is zero. Together with the first-order condition (A.1), the inequality (A.8) implies that $\eta_1(\theta) = 0$ and $\eta_2(\theta) > 0$. Therefore, we have $s^*(\theta) = \theta$ in this region.

Also, from the first-order condition (A.1) and condition (A.3), it is obvious that $s^*(D^*) = D^*$.

In sum, the entrepreneur's optimal security without inducing the investor to acquire information features a debt with face value D^* determined by condition (A.2).

We need to check that there exists $D^* > 0$ and the corresponding multiplier $\lambda > 0$ such that

$$\mathbb{E}\left[\exp\left(-\mu^{-1} \cdot (\min(\theta, D^*) - k)\right)\right] = 1\,, \tag{A.9}$$

where D^* is determined by condition (A.2).

Consider the left-hand side of condition (A.9). Clearly, it is continuous and monotonically decreasing in D^*. When D^* is sufficiently large, the left-hand side of (A.9) approaches $\mathbb{E}\left[\exp\left(-\mu^{-1} \cdot (\theta - k)\right)\right]$, a number less than one, which is guaranteed by condition (2.3) and by the feasibility condition $s(\theta) \leqslant \theta$. On the other hand, when $D^* = 0$, it approaches $\exp\left(\mu^{-1} \cdot k\right)$, which is strictly greater than one. Hence, there exists $D^* > 0$ such that condition (A.9) holds.

Moreover, from condition (A.2), we also know that D^* is continuous and monotonically increasing in λ. When λ approaches zero, D^* approaches negative infinity, while when λ approaches positive infinity, D^* approaches positive infinity as well. Hence, for any $D^* > 0$ there exists a corresponding multiplier $\lambda > 0$.

Suppose $D^* \leqslant k$. It is easy to see that this debt would be rejected by the investor due to Proposition 1, a contradiction.

Finally, by condition (2.3) again, because the optimal security $s^*(\theta)$ satisfies

$$\mathbb{E}\left[\exp\left(-\mu^{-1} \cdot (s^*(\theta) - k)\right)\right] = 1\,,$$

Jensen's inequality implies that $\mathbb{E}[s^*(\theta)] > k$ given $\mu > 0$. This concludes the proof. □

Proof of Lemma 3. We derive the entrepreneur's optimal security $s^*(\cdot)$ and the corresponding unconstrained part $\widehat{s}(\cdot)$ through variational methods. Specifically, we characterize how the entrepreneur's expected payment responds to the perturbation of the optimal security.

Let $s(\cdot) = s^*(\cdot) + \alpha \cdot \varepsilon(\cdot)$ be an arbitrary perturbation of the optimal security $s^*(\cdot)$. Note that the investor's optimal screening rule $m_s(\cdot)$ appears in the entrepreneur's expected payoff

$u_E(s(\cdot))$, according to condition (2.7), and it is implicitly determined by the proposed security $s(\cdot)$ through the equation (2.6) for any θ. Hence, we need to first characterize how $m_s(\cdot)$ varies by the perturbation of $s^*(\cdot)$. Taking the derivative for α at $\alpha = 0$ for both sides of (2.6) leads to

$$\mu^{-1}\varepsilon(\cdot) = g''(m_s^*(\cdot)) \cdot \left.\frac{\partial m_s(\theta)}{\partial \alpha}\right|_{\alpha=0} - g''(\overline{\pi}_s^*) \cdot \mathbb{E}\left.\frac{\partial m_s(\theta)}{\partial \alpha}\right|_{\alpha=0} .$$

We take the expectation of both sides for θ and obtain

$$\mathbb{E}\left[\left.\frac{\partial m_s(\theta)}{\partial \alpha}\right|_{\alpha=0}\right] = \mu^{-1} \cdot \left(1 - \mathbb{E}\left[\left(g''(m_s^*(\theta))\right)^{-1}\right] \cdot g''(\overline{\pi}_s^*)\right)^{-1} \cdot \mathbb{E}\left[\left(g''(m_s^*(\theta))\right)^{-1} \varepsilon(\theta)\right] .$$

Combining the above two equations, for any perturbation $s(\cdot) = s^*(\cdot) + \alpha \cdot \varepsilon(\cdot)$, the investor's screening rule $m_s(\cdot)$ is characterized by

$$\begin{aligned} \left.\frac{\partial m_s(\theta)}{\partial \alpha}\right|_{\alpha=0} &= \mu^{-1} \cdot \left(g''(m_s^*(\theta))\right)^{-1} \varepsilon(\theta) \\ &\quad + \frac{\mu^{-1} \cdot (g''(m_s^*(\theta)))^{-1} \cdot \mathbb{E}\left[(g''(m_s^*(\theta)))^{-1} \varepsilon(\theta)\right]}{(g''(\overline{\pi}_s^*))^{-1} - \mathbb{E}\left[(g''(m_s^*(\theta)))^{-1}\right]} \text{ for any } \theta . \end{aligned} \tag{A.10}$$

We interpret condition (A.10). The first term of the right-hand side of (A.10) is the investor's local response to perturbation $\varepsilon(\cdot)$. It is of the same sign as the perturbation $\varepsilon(\cdot)$. When the payment of the security increases at state θ, the investor is more likely to accept the security at this state. The second term measures the investor's average response to perturbation $\varepsilon(\cdot)$ over all states. It is straightforward to verify that the denominator of the second term is positive due to Jensen's inequality. As a result, if the perturbation increases the investor's payment on average over all states, then the investor is more likely to accept the security.

Now, we can calculate the variation of the entrepreneur's expected payoff $u_E(s(\cdot))$, according to condition (2.7). Taking the derivative of $u_E(s(\cdot))$ for α at $\alpha = 0$ leads to

$$\left.\frac{\partial u_E(s(\cdot))}{\partial \alpha}\right|_{\alpha=0} = \mathbb{E}\left[\left.\frac{\partial m_s(\theta)}{\partial \alpha}\right|_{\alpha=0} (\theta - s(\theta))\right] - \mathbb{E}\left[m_s^*(\theta) \cdot \varepsilon(\theta)\right] . \tag{A.11}$$

Substitute (A.10) into (A.11), and we obtain

$$\left.\frac{\partial u_E(s(\cdot))}{\partial \alpha}\right|_{\alpha=0} = \mathbb{E}\left[r\left(\theta\right)\cdot\varepsilon\left(\theta\right)\right] \ , \tag{A.12}$$

where

$$r\left(\cdot\right) = -m_s^*\left(\theta\right) + \mu^{-1}\cdot\left(g''\left(m_s^*\left(\theta\right)\right)\right)^{-1}\cdot\left(\theta - s^*\left(\theta\right) + w^*\right) \tag{A.13}$$

and

$$w^* = \mathbb{E}\left[\left(\theta - s^*\left(\theta\right)\right)\frac{g''\left(\overline{\pi}_s^*\right)}{g''\left(m_s^*\left(\theta\right)\right)}\right]\left(1-\mathbb{E}\left[\frac{g''\left(\overline{\pi}_s^*\right)}{g''\left(m_s^*\left(\theta\right)\right)}\right]\right)^{-1} \ .$$

Note that w^* is a constant that does not depend on θ and will be endogenously determined in the equilibrium. As defined in the main text, $r\left(\cdot\right)$ is the Frechet derivative of the entrepreneur's expected payoff $u_E(s(\cdot))$ at $s^*(\cdot)$, which measures the marginal contribution of any perturbation to the entrepreneur's expected payoff when the security is optimal.

To further characterize the optimal security, we discuss the Frechet derivative $r(\cdot)$ in detail. Recall that the optimal security would be restricted by the feasibility condition $0 \leqslant s^*(\theta) \leqslant \theta$. Let

$$A_0 = \left\{\theta \in \Theta : \theta \neq 0, s^*\left(\theta\right) = 0\right\} \ ,$$

$$A_1 = \left\{\theta \in \Theta : \theta \neq 0, 0 < s^*\left(\theta\right) < \theta\right\} \ ,$$

$$A_2 = \left\{\theta \in \Theta : \theta \neq 0, s^*\left(\theta\right) = \theta\right\} \ .$$

Clearly, $\{A_0, A_1, A_2\}$ is a partition of $\Theta\backslash\left\{0\right\}$. Because $s^*\left(\theta\right)$ is the optimal security, we have

$$\left.\frac{\partial u_E(s(\cdot))}{\partial \alpha}\right|_{\alpha=0} \leqslant 0$$

for any feasible perturbation $\varepsilon\left(\cdot\right)$.[35] Hence, condition (A.12) implies

$$r\left(\theta\right)\begin{cases}\leqslant 0 & \text{if } \theta \in A_0\\ = 0 & \text{if } \theta \in A_1\\ \geqslant 0 & \text{if } \theta \in A_2\end{cases} \ . \tag{A.14}$$

[35] A perturbation $\varepsilon(\theta)$ is feasible for $s^*(\theta)$ if there exists $\alpha > 0$ such that for any $\theta \in \Theta$, $s^*\left(\theta\right) + \alpha\cdot\varepsilon\left(\theta\right) \in [0, \theta]$.

According to Proposition 1, when the optimal security $s^*(\cdot)$ induces the investor to acquire information, we have $0 < m_s^*(\theta) < 1$ for all $\theta \in \Theta$. Hence, condition (A.14) can be rearranged as

$$\frac{r(\theta)}{m_s^*(\theta)} = -1 + \mu^{-1} \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \begin{cases} \leqslant 0 & \text{if } \theta \in A_0 \\ = 0 & \text{if } \theta \in A_1 \\ \geqslant 0 & \text{if } \theta \in A_2 \end{cases} . \tag{A.15}$$

Recall condition (2.6), given the optimal security $s^*(\cdot)$, the investor's optimal screening rule $m_s^*(\cdot)$ is

$$s^*(\cdot) - k = \mu \cdot \left(g'(m_s^*(\cdot)) - g'(\overline{\pi}_s^*)\right) , \tag{A.16}$$

where

$$\overline{\pi}_s^* = \mathbb{E}\left[m_s^*(\theta)\right]$$

is the investor's unconditional probability of accepting the optimal security $s^*(\cdot)$. Conditions (A.15) and (A.16), as a system of functional equations, jointly determine the optimal security $s^*(\cdot)$ when it induces the investor's information acquisition.

Finally, when we focus on the unconstrained part $\widehat{s}(\cdot)$ of the optimal security, note that is would not be restricted by the feasibility conditions. Hence, the corresponding Frechet derivative $r(\cdot)$ always would be zero at the optimum. On the other hand, the investor's optimal screening rule would not be affected. As a result, conditions (A.16) and (A.15) become

$$\widehat{s}(\cdot) - k = \mu \cdot \left(g'(\widehat{m}_s(\cdot)) - g'(\overline{\pi}_s^*)\right) ,$$

where

$$\overline{p}_s^* = \mathbb{E}\left[m_s^*(\theta)\right] ,$$

and

$$(1 - \widehat{m}_s(\cdot)) \cdot (\theta - \widehat{s}(\cdot) + w^*) = \mu ,$$

where

$$w^* = \mathbb{E}\left[(\theta - s^*(\theta)) \frac{g''(\overline{\pi}_s^*)}{g''(m_s^*(\theta))}\right] \left(1 - \mathbb{E}\left[\frac{g''(\overline{\pi}_s^*)}{g''(m_s^*(\theta))}\right]\right)^{-1} ,$$

in which $\overline{p}_s^*$ and w^* are two constants that do not depend on θ. This concludes the proof. $\square$

Proof of Lemma 4. From Lemma 3, $(\widehat{s}(\cdot), \widehat{m}_s(\cdot))$ satisfies the two Equations (2.11) and (2.12). By condition (2.12), we obtains

$$\widehat{m}_s(\cdot) = 1 - \frac{\mu}{\theta - \widehat{s}\,(\cdot) + w^*}\ . \tag{A.17}$$

Substituting (A.17) into (2.11) leads to

$$\mu^{-1}\left(\widehat{s}(\cdot) - k\right) = g'\left(\frac{\mu}{\theta - \widehat{s}\,(\cdot) + w^*}\right) - g'\left(\overline{\pi}_s^*\right)\ .$$

Taking the derivatives of both sides of the above functional equation for θ leads to

$$\begin{aligned}
\mu^{-1} \cdot \frac{d\widehat{s}\,(\theta)}{d\theta} &= g''\left(\widehat{m}_s\,(\theta)\right) \cdot \frac{d\widehat{m}_s\,(\theta)}{d\theta} \\
&= g''\left(\widehat{m}_s\,(\theta)\right) \cdot \frac{\mu \cdot \left(1 - \frac{d\widehat{s}(\theta)}{d\theta}\right)}{\left(\theta - \widehat{s}\,(\theta) + w^*\right)^2} \\
&= \frac{1 - \frac{d\widehat{s}(\theta)}{d\theta}}{\theta - \widehat{s}\,(\theta) + w^* - \mu}\ , \text{ for any } \theta\,,
\end{aligned}$$

where we use

$$g''\,(x) = \frac{1}{x\,(1-x)}$$

while deriving the third equality. Rearrange the above equation, and we obtain

$$\begin{aligned}
\frac{d\widehat{s}\,(\theta)}{d\theta} &= \frac{\mu}{\theta - \widehat{s}\,(\theta) + w^*} \\
&= 1 - \widehat{m}_s\,(\theta)\,, \text{ for any } \theta\,,
\end{aligned}$$

where the last equality follows (A.17).

Again, taking the derivatives of both sides of the above functional equation for θ leads to

$$\begin{aligned}
\mu^{-1} \cdot \frac{d\widehat{s}\,(\theta)}{d\theta} &= g''\left(\widehat{m}_s\,(\theta)\right) \cdot \frac{d\widehat{m}_s\,(\theta)}{d\theta} \\
&= \frac{1}{\widehat{m}_s\,(\theta)\left(1 - \widehat{m}_s\,(\theta)\right)} \cdot \frac{d\widehat{m}_s\,(\theta)}{d\theta}, \text{ for any } \theta\,.
\end{aligned}$$

Hence

$$\begin{aligned}\frac{d\widehat{m}_s(\theta)}{d\theta} &= \mu^{-1}\cdot\widehat{m}_s(\theta)\cdot(1-\widehat{m}_s(\theta))\cdot\frac{d\widehat{s}(\theta)}{d\theta}\\ &= \mu^{-1}\cdot\widehat{m}_s(\theta)\cdot(1-\widehat{m}_s(\theta))^2\,,\ \text{for any }\theta\,.\end{aligned}$$

This completes the proof. □

In the following, we prove Proposition 3 by establishing five useful lemmas.

Proof of Proposition 3.

Lemma 5. *Three possible relative positions between the unconstrained part $\widehat{s}(\cdot)$ and the feasibility constraints $0 \leqslant s(\theta) \leqslant \theta$ may occur in equilibrium, in the $\theta \sim s(\theta)$ space:*

i) $\widehat{s}(\cdot)$ intersects with the $45°$ line $s=\theta$ at $(\widehat{\theta},\widehat{\theta})$, $\widehat{\theta}>0$, and does not intersect with the horizontal axis $s=0$;

ii) $\widehat{s}(\cdot)$ goes through the origin $(0,0)$, and does not intersect with either the $45°$ line $s=\theta$ or the horizontal axis $s=0$ for any $\theta\neq 0$;

iii) $\widehat{s}(\cdot)$ intersects with the horizontal axis $s=0$ at $(\widetilde{\theta},0)$, $\widetilde{\theta}>0$, and does not intersect with the $45°$ line $s=\theta$.

Proof of Lemma 5. From Lemma 4, it is easy to see that the slope of $\widehat{s}(\cdot)$ is always less than one. Hence, Lemma 5 is straightforward. □

In the three different cases, the actual optimal security $s^*(\cdot)$ will be constrained by the feasibility condition in different ways. For example, $s^*(\cdot)$ will be constrained by the $45°$ line $s=\theta$ in Case a, while, by the horizontal axis, $s=0$ in Case c. By imposing the feasibility conditions, we have the following characterization for $s^*(\cdot)$:

Lemma 6. *In an equilibrium with information acquisition, the corresponding optimal security $s^*(\cdot)$ satisfies*

$$s^*(\theta)=\begin{cases}\theta & \textit{if} \quad \widehat{s}(\theta)>\theta\\ \widehat{s}(\theta) & \textit{if} \quad 0\leqslant\widehat{s}(\theta)\leqslant\theta\\ 0 & \textit{if} \quad \widehat{s}(\theta)<0\end{cases},$$

where $\widehat{s}(\theta)$ is the unconstrained part of the optimal security.

Proof of Lemma 6. We proceed by discussing three cases.

Case 1: We show that $\widehat{s}(\theta) > \theta$ implies $s^*(\theta) = \theta$, for any θ.

Suppose $s^*(\theta) < \theta$. Then we have $s^*(\theta) < \widehat{s}(\theta)$. Because both $(s^*(\theta), m_s^*(\theta))$ and $(\widehat{s}(\theta), \widehat{m}_s(\theta))$ satisfy condition (2.6), we must have $m_s^*(\theta) < \widehat{m}_s(\theta)$. Therefore,

$$\begin{aligned}
\frac{r(\theta)}{m_s^*(\theta)} &= -1 + \mu^{-1} \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \\
&> -1 + \mu^{-1} \cdot (1 - \widehat{m}_s(\theta)) \cdot (\theta - \widehat{s}(\theta) + w^*) \\
&= 0\,,
\end{aligned}$$

which implies $s^*(\theta) = \theta$, a contradiction.

Note that, the logic for the inequality above is as follows. Because $(\widehat{s}(\theta), \widehat{m}_s(\theta))$ satisfies condition (2.12), we must have $\theta - \widehat{\theta} + w^* > 0$. Hence, $\widehat{s}(\theta) > s^*(\theta)$ implies that

$$\theta - s^*(\theta) + w^* > \theta - \widehat{s}(\theta) + w^* > 0\,.$$

Also, by noting that

$$1 - m_s^*(\theta) > 1 - \widehat{m}_s(\theta) > 0\,,$$

we obtain the inequality above.

Hence, we have $s^*(\theta) = \theta$ in this case.

Case 2: We show that $\widehat{s}(\theta) < 0$ implies $s^*(\theta) = 0$ for any θ.

Suppose $s^*(\theta) > 0$. Then we have $s^*(\theta) > \widehat{s}(\theta)$. By similar argument we know that $m_s^*(\theta) > \widehat{m}_s(\theta)$. Therefore,

$$\begin{aligned}
\frac{r(\theta)}{m_s^*(\theta)} &= -1 + \mu^{-1} \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \\
&< -1 + \mu^{-1} \cdot (1 - \widehat{m}_s(\theta)) \cdot (\theta - \widehat{s}(\theta) + w^*) \\
&= 0\,,
\end{aligned}$$

which implies $s^*(\theta) = 0$. This is a contradiction. Hence, we have $s^*(\theta) = 0$ in this case.

Case 3: We show that $0 \leqslant \widehat{s}(\theta) \leqslant \theta$ implies $s^*(\theta) = \widehat{s}(\theta)$ for any θ.

Suppose $\widehat{s}(\theta) < s^*(\theta)$. Then similar argument suggests $r(\theta)/m_s^*(\theta) < 0$, which implies $s^*(\theta) =$

$0 < \widehat{s}(\theta)$. This is a contradiction.

Similarly, suppose $s^*(\theta) < \widehat{s}(\theta)$. Similar argument suggests that $r(\theta)/m_s^*(\theta) > 0$, which implies $s^*(\theta) = \theta > \widehat{s}(\theta)$. This is, again, a contradiction. Hence, we have $s^*(\theta) = \widehat{s}(\theta)$ in this case.

This concludes the proof. □

Lemma 6 is helpful because it tells us how to construct an optimal security $s^*(\cdot)$ from its corresponding unconstrained part $\widehat{s}(\cdot)$. Concretely, $s^*(\cdot)$ will follow $\widehat{s}(\cdot)$ when the latter is within the feasible region $0 \leqslant s \leqslant \theta$. When $\widehat{s}(\cdot)$ goes out of the feasible region, the resultant optimal security will follow one of the feasibility constraints that is binding.

We apply Lemma 6 to the three cases described in Lemma 5. This gives the three potential cases of the optimal security $s^*(\cdot)$, respectively.

Lemma 7. *In an equilibrium with information acquisition, the optimal security $s^*(\cdot)$ may take one of the following three forms:*

i) when the corresponding unconstrained part $\widehat{s}(\cdot)$ intersects with the $45°$ line $s = \theta$ at $(\widehat{\theta}, \widehat{\theta})$, $\widehat{\theta} > 0$, we have

$$s^*(\theta) = \begin{cases} \theta & \text{if} \quad 0 \leqslant \theta < \widehat{\theta} \\ \widehat{s}(\theta) & \text{if} \quad \theta \geqslant \widehat{\theta} \end{cases} ;$$

ii) when the corresponding unconstrained part $\widehat{s}(\cdot)$ goes through the origin $(0,0)$, we have $s^*(\theta) = \widehat{s}(\theta)$ for $\theta \in \mathbb{R}_+$;

iii) when the corresponding unconstrained part $\widehat{s}(\cdot)$ intersects with the horizontal axis $s = 0$ at $(\widetilde{\theta}, 0)$, $\widetilde{\theta} > 0$, we have

$$s^*(\theta) = \begin{cases} 0 & \text{if} \quad 0 \leqslant \theta < \widetilde{\theta} \\ \widehat{s}(\theta) & \text{if} \quad \theta \geqslant \widetilde{\theta} \end{cases} .$$

Proof of Lemma 7. Apply Lemma 5 to Lemma 6, then Lemma 7 is straightforward. □

Lemma 7 shows that the optimal security $s^*(\cdot)$ takes different shapes in the three potential cases. In Case a, $s^*(\cdot)$ follows a debt in states with low cash flows but increases in states with high cash flows. In Case c, $s^*(\cdot)$ has zero payment in states with low cash flows, while is an increasing function in states with high cash flows. Case b lies in between as a knife-edge case.

We proceed by determining whether these three potential cases are valid solutions to the entrepreneur's problem in equilibrium with information acquisition. Interestingly, not all the three cases can occur in equilibrium.

Lemma 8. *If the entrepreneur's optimal security $s^*(\cdot)$ induces the investor to acquire information in equilibrium, then it must follow Case a in Lemma 7, which corresponds to a participating convertible preferred stock with a face value $\widehat{\theta} > 0$.*

Proof of Lemma 8. We prove by contradiction. Suppose that the last two cases in Lemma 7 can occur in equilibrium. Hence, there exists a $\widetilde{\theta} \geqslant 0$, such that $s^*(\theta) = 0$ when $0 \leqslant \theta \leqslant \widetilde{\theta}$ and $s^*(\theta) = \widehat{s}(\theta)$ when $\theta > \widetilde{\theta}$.

Note that, $s^*(\cdot)$ is strictly increasing when $\theta > \widetilde{\theta}$. Also, because we focus on the equilibrium with information acquisition, there must exist a θ'' such that $s^*(\theta'') > k$; otherwise, the optimal security would be rejected without information acquisition. Therefore, there exists a $\theta' > \widetilde{\theta}$ such that $s^*(\theta') = \widehat{s}(\theta') = k$. Recall condition (2.11), we have

$$m_s^*(\theta') = \bar{\pi}_s^* .$$

Moreover, because we have $s^*(\theta') \in (0, \theta')$, we have

$$\begin{aligned} 0 = r(\theta') &= -m_s^*(\theta') + \mu^{-1} \cdot m_s^*(\theta') \cdot (1 - m_s^*(\theta')) \cdot (\theta' - s^*(\theta') + w^*) \\ &= -\bar{\pi}_s^* + \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - k + w^*) \\ &= \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - k) + \mathbb{E}[r(\theta)] \ , \end{aligned}$$

where

$$\begin{aligned} \mathbb{E}[r(\theta)] &= -\bar{\pi}_s^* + \mu^{-1} \left(\mathbb{E}\left[\frac{(\theta - s(\theta)) \cdot g''(\bar{\pi}_s^*))}{g''(m(\theta))} \right] / g''(\bar{\pi}_s^*) + w^* \mathbb{E}\left[\frac{1}{g''(m(\theta))} \right] \right) \\ &= -\bar{\pi}_s^* + \mu^{-1} \left(w^* \cdot \left(1 - \mathbb{E}\left[\frac{g''(\bar{\pi}_s^*)}{g''(m(\theta))} \right] \right) / g''(\bar{\pi}_s^*) + w^* \mathbb{E}\left[\frac{1}{g''(m(\theta))} \right] \right) \\ &= -\bar{\pi}_s^* + \frac{\mu^{-1} w^*}{g''(\bar{\pi}_s^*)} \\ &= -\bar{\pi}_s^* + \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot w^* \ . \end{aligned}$$

We can express the expectation term $\mathbb{E}[r(\theta)]$ in another way. Note that, for any $\theta \in [0, \widetilde{\theta}]$, by definition we have

$$
\begin{aligned}
r(\theta) &= -m_s^*(\theta) + \mu^{-1} \cdot m_s^*(\theta) \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \\
&= -\widehat{m}_s(\widetilde{\theta}) + \mu^{-1} \cdot \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \cdot (\theta - 0 - \widetilde{\theta} + \widetilde{\theta} + w^*) \\
&= r(\widetilde{\theta}) - \mu^{-1} \cdot \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \cdot (\widetilde{\theta} - \theta) \\
&= -\mu^{-1} \cdot \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \cdot (\widetilde{\theta} - \theta) \ .
\end{aligned}
$$

Also, as $s^*(\theta) = \widehat{s}(\theta)$ for any $\theta > \widetilde{\theta}$, we have $r(\theta) = 0$ for all $\theta > \widetilde{\theta}$. Hence,

$$
\mathbb{E}[r(\theta)] = -\mu^{-1} \cdot \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (\widetilde{\theta} - \theta) d\Pi(\theta) \ .
$$

Therefore, we have

$$
\begin{aligned}
\mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - k) &= -\mathbb{E}[r(\theta)] && \text{(A.18)} \\
&= \mu^{-1} \cdot \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (\widetilde{\theta} - \theta) d\Pi(\theta) \ . && \text{(A.19)}
\end{aligned}
$$

Now we take the tangent line of $s^*(\cdot)$ at $\theta = \widetilde{\theta}$. The tangent line intersects $s = k$ at $\widetilde{\theta}'$, which is given by

$$
\frac{k}{\widetilde{\theta}' - \widetilde{\theta}} = \left. \frac{ds^*(\theta)}{d\theta} \right|_{\widetilde{\theta}} = 1 - \widehat{m}_s(\widetilde{\theta}) \, .
$$

Hence, we have

$$
\widetilde{\theta}' = \widetilde{\theta} + \frac{k}{1 - \widehat{m}_s(\widetilde{\theta})} \ . \tag{A.20}
$$

Also, note that we have shown that for any $\theta \geqslant \widetilde{\theta}$, we have

$$
\frac{ds^*(\theta)}{d\theta} = \frac{d\widehat{s}(\theta)}{d\theta} = 1 - \widehat{m}_s(\theta) = 1 - m_s^*(\theta) \, .
$$

Hence,

$$
\frac{d^2 s^*(\theta)}{d\theta^2} = -\mu^{-1} \cdot m_s^*(\theta) \cdot (1 - m_s^*(\theta))^2 < 0 \, .
$$

Therefore, $s^*(\cdot)$ is strictly concave for $\theta \geqslant \widetilde{\theta}$, and we also have $\widetilde{\theta}' < \theta'$. Consequently, by condition (A.20) and then conditions (A.18) and (A.19), we have

$$\begin{aligned}
\bar{\pi}_s^* \cdot (1-\bar{\pi}_s^*) \cdot \left(\widetilde{\theta} + \frac{\widehat{m}_s(\widetilde{\theta})}{1-\widehat{m}_s(\widetilde{\theta})} \cdot k\right) &= \bar{\pi}_s^* \cdot (1-\bar{\pi}_s^*) \cdot (\widetilde{\theta}' - k) \\
&< \bar{\pi}_s^* \cdot (1-\bar{\pi}_s^*) \cdot (\theta' - k) \\
&= \widehat{m}_s(\widetilde{\theta}) \cdot (1-\widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (\widetilde{\theta} - \theta) d\Pi(\theta) \ .
\end{aligned}$$

On the other hand, by Jensen's inequality, we know that

$$\bar{\pi}_s^* \cdot (1-\bar{\pi}_s^*) > \mathbb{E}\left[m_s^*(\theta) \cdot (1-m^*(\theta))\right] \ .$$

Therefore, we have

$$\begin{aligned}
\widehat{m}_s(\widetilde{\theta}) \cdot (1-\widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (\widetilde{\theta} - \theta) d\Pi(\theta) &> \bar{\pi}_s^* \cdot (1-\bar{\pi}_s^*) \cdot \left(\widetilde{\theta} + \frac{\widehat{m}_s(\widetilde{\theta})}{1-\widehat{m}_s(\widetilde{\theta})} \cdot k\right) \\
&> \mathbb{E}\left[m_s^*(\theta) \cdot (1-m^*(\theta))\right] \cdot \left(\widetilde{\theta} + \frac{\widehat{m}_s(\widetilde{\theta})}{1-\widehat{m}_s(\widetilde{\theta})} \cdot k\right) \ .
\end{aligned}$$

Expand the expectation term above and rearrange, we obtain

$$\begin{aligned}
& \widehat{m}_s(\widetilde{\theta})^2 \cdot k \cdot Prob[\theta \leqslant \widetilde{\theta}] + \int_{\widetilde{\theta}}^{+\infty} m_s^*(\theta) \cdot (1-m_s^*(\theta)) d\Pi(\theta) \cdot \left(\widetilde{\theta} + \frac{\widehat{m}_s(\widetilde{\theta})}{1-\widehat{m}_s(\widetilde{\theta})} \cdot k\right) \\
< \ & \widehat{m}_s(\widetilde{\theta}) \cdot (1-\widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (-\theta) d\Pi(\theta) \\
\leqslant \ & 0 \ .
\end{aligned}$$

Nevertheless, the left-hand side of the above inequality should be positive, which is a contradiction. This concludes the proof. □

Lemma 9. *In the optimal security $s^*(\cdot)$, there must be $\widehat{\theta} > \theta' = k$.*

Proof of Lemma 9. First, note that $s^*(\cdot)$ is strictly increasing and continuous. Also, note that there exists a θ'' such that $s^*(\theta'') > k$; otherwise, the offer will be rejected without information acquisition.

Therefore, there exists a unique θ' such that $s^*(\theta') = k$, which ensures that $m_s^*(\theta') = \bar{\pi}_s^*$, and

$$\begin{aligned} r(\theta') &= -\bar{\pi}_s^* + \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - s^*(\theta') + w^*) \\ &= \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - s^*(\theta')) + \mathbb{E}[r(\theta)] \ . \end{aligned}$$

Note that $\mathbb{E}[r(\theta)] > 0$ and $\theta' - s^*(\theta') \geqslant 0$, we have $\theta' < \widehat{\theta}$. As $\theta' = s^*(\theta') = k$, it follows that $\widehat{\theta} > \theta' = k$. This concludes the proof. $\square$

Together with the lemmas already established, combining Lemma 8 and Lemma 9 immediately leads to Proposition 3. $\square$

Proof of Proposition 6. When we have $\mathbb{E}[\theta] \leqslant k$ and

$$\mathbb{E}\left[\exp(\mu^{-1}(t \cdot \theta - k))\right] > 1 \, ,$$

according to Proposition 1, even if the entrepreneur proposes all the future cash flow to the investor, the security would induce the investor to acquire information and accept it with a positive (but less than one) probability. The only optimal security for this case is convertible preferred stock. This concludes the proof. $\square$

Proof of Proposition 7. To facilitate the proof, we first characterize a frictionless centralized economy to help benchmark the friction in the corresponding decentralized economy. In the centralized economy, u, Θ, Π, k and μ are given as the same. However, we assume that the entrepreneur has sufficient initial wealth and can also acquire information flexibly to screen the project. Here, production still depends on information acquisition and financing, but they are aligned. Thus, there is no friction and security design is irrelevant. The entrepreneur's problem is whether to undertake the project directly without acquiring information, to abandon it without acquiring information, or to acquire information before making a decision. The entrepreneur's payoff gain from undertaking the project rather than abandoning it is

$$\Delta u_I(\theta) = u_I(1, \theta) - u_I(0, \theta) = \theta - k \, , \text{ for any } \theta \, .$$

We denote an arbitrary screening rule in the centralized economy by $m_c(\cdot)$ and the optimal screening rule by $m_c^*(\cdot)$. Thus, the entrepreneur's problem in the centralized economy is to choose $m_c^*(\cdot)$ that maximizes

$$\mathbb{E}[m_c(\theta) \cdot (\theta - k)] - \mu \cdot I(m_c(\cdot))\,. \tag{A.21}$$

By construction, the entrepreneur's objective (A.21) in the centralized economy is exactly the expected social surplus in the decentralized economy (3.15). This immediately leads to the following result.

Lemma 10. *An optimal security in the decentralized economy achieves efficiency if and only if the associated optimal screening rule $m_s^*(\cdot)$ satisfies*

$$Prob[m_s^*(\theta) = m_c^*(\theta)] = 1\,,$$

where $m_c^*(\cdot)$ is the optimal screening rule in the corresponding centralized economy.

Proof of Lemma 10. The "if" part is straightforward, following the definition of efficiency. The "only if" part is ensured by the fact that the optimal screening rule is always unique given an arbitrary security, established in Proposition 1. □

We then state another useful lemma. It allow us to focus on the first two types of equilibria for welfare analysis.

Lemma 11. *A project is initiated with a positive probability in the decentralized economy if and only if it is initiated with a positive probability in the corresponding centralized economy.*

Proof of Lemma 11. With the objective function (A.21) in the centralized economy, the entrepreneur's optimal screening rule $m_c^*(\cdot)$ is characterized by Proposition 1. Specifically, the investor will initiate the project without information acquisition, i.e., $Prob[m_c^*(\theta) = 1] = 1$ if and only if

$$\mathbb{E}[\exp(-\mu^{-1} \cdot (\theta - k))] \leqslant 1\,,$$

will skip the project without information acquisition, i.e., $Prob[m_c^*(\theta) = 0] = 1$ if and only if

$$\mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] \leqslant 1\,,$$

and will initiate the project with probability $0 < \bar{\pi}_c^* < 1$, $\bar{\pi}_c^* = \mathbb{E}[m_c^*(\theta)]$, if and only if

$$\mathbb{E}[\exp(-\mu^{-1} \cdot (\theta - k))] > 1 \text{ and } \mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] > 1\,,$$

in which $m_c^*(\cdot)$ is determined by the functional equation:

$$\theta - k = \mu \cdot (g'(m_c^*(\cdot)) - g'(\bar{\pi}_c^*))\,.$$

It is straightforward to observe that, the project is initiated with a positive probability in the frictionless centralized economy if and only if

$$\mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] > 1\,. \tag{A.22}$$

Note that, condition (A.22) is the same as condition (2.1) in Lemma 1 that gives the investment criterion in the corresponding decentralized economy. This concludes the proof. □

We continue the proof of Proposition 7. By Lemma 11, once we prove the "only if" parts of both cases of debt and convertible preferred stock, the "if" parts will be proved simultaneously.

First, consider the case when $s^*(\cdot)$ is debt. In this case, we have $Prob[m_s^*(\theta) = 1] = 1$, and

$$\mathbb{E}[\exp\left(-\mu^{-1} \cdot (s^*(\theta) - k)\right)] \leqslant 1\,,$$

both from Proposition 1. Because $s^*(\theta) < \theta$ when $\theta > D^*$, it follows that

$$\mathbb{E}[\exp\left(-\mu^{-1} \cdot (\theta - k)\right)] \leqslant 1\,,$$

which implies that $Prob[m_c^*(\theta) = 1] = 1$, also by Proposition 1. Hence, we know that

$$Prob[m_s^*(\theta) = m_c^*(\theta)] = 1\,,$$

which suggests that $s^*(\cdot)$, as debt, achieves efficiency, according to Lemma 10.

Second, consider the case when $s^*(\cdot)$ is convertible preferred stock that induces information acquisition. In this case, we have $Prob[0 < m_s^*(\theta) < 1] = 1$, and

$$\mathbb{E}[\exp\left(-\mu^{-1} \cdot (s^*(\theta) - k)\right)] > 1\,,$$

again both from Proposition 1. Because $s^*(\theta) < \theta$ when $\theta > \widehat{\theta}$, the relationship between $\mathbb{E}[\exp\left(-\mu^{-1} \cdot (\theta - k)\right)]$ and 1 is ambiguous. If

$$\mathbb{E}[\exp\left(-\mu^{-1} \cdot (\theta - k)\right)] \leqslant 1\,,$$

we have $Prob[m_c^*(\theta) = 1] = 1$, and information acquisition is not induced in the centralized economy. It follows that

$$Prob[m_s^*(\theta) = m_c^*(\theta)] \neq 1\,.$$

Otherwise, if

$$\mathbb{E}[\exp\left(-\mu^{-1} \cdot (\theta - k)\right)] > 1\,,$$

suppose we also have $Prob[m_s^*(\theta) = m_c^*(\theta)] = 1$, then according to condition (A.16), we have

$$Prob[s^*(\theta) = \theta] = 1\,,$$

which violates Corollary 1. This is a contradiction. As a result, from Lemma 10, we know that $s^*(\cdot)$, as convertible preferred stock, cannot achieve efficiency. This concludes the proof. $\square$