Liquid crystal display as a polarisation filter

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Abstract

This thesis is concerned with researching novel applications for a commercially produced twisted nematic liquid crystal television display (TNLCTVD).

TNLCTVDs work by changing the polarisation of polarised light. The polarisation change is dependent upon the voltage applied to the display, and can be converted to an intensity change by viewing the display through a polariser (analyser). The amount of polarisation change is experimentally quantified in this thesis, and is used in two main applications. Firstly the LCTVD can be used as a Stokes polarimeter. The (unknown) polarisation of light passing through the display can be determined by recording the intensity transmitted by a fixed analyser when four separate voltages are applied to the LCTVD. These intensities are used to compute the Stokes vector of the incident light. This Stokes polarimeter has no moving parts, and is easily calibrated to cope with different environmental conditions, such as temperature, as well as different wavelengths of light. The polarimeter is shown to be accurate in determining the full state of polarisation, and can distinguish between partially linearly polarised and elliptically polarised light.

The second application is to use the LCTVD or a single pixel TNLC cell to read spatially varying patterns of polarisation. Some security marks have been patented which are produced using a spatially patterned birefringent material. These emit a pattern of polarisation that can be read using the LC security mark reader. This reader is shown to be accurate in distinguishing between complex polarisation varying marks which only differ in optical path lengths by 25nm, and it can detect a rotation of the mark of only 2°. The sensitivity of the system can be customised to meet the requirements of the end user.
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Several companies were very generous with their assistance: Philips Research Laboratories in Redhill very kindly supplied the LCTVD and single pixel LC cell which were used throughout this work. Andy Pearson from Philips (always on the end of an e-mail!) was extremely patient and helpful with his technical knowledge, and Anette Hultaker of Uppsala University very kindly sent me a copy of her licentiate thesis on ITO films. The birefringent film used for the prototype security marks, and the LC polymer used to make the 'Hull' security marks was kindly supplied by Merck, and both Richard Harding and Mark Verrall from Merck deserve kind thanks for their help. Rolic Research provided the photographic quality birefringent security marks that were invaluable when testing the security mark reader, and GSI kindly provided the superbright LED. Perry Jackson at Hull University was an angel for helping me make the 'Hull' security marks.

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Finally, I must mention the tremendous support and understanding I have received from the one and only David Parkins. He has had to put up with me ignoring him for the computer on more times than I care to admit, and deserves a huge thanks. He is a very special person and every day I realise how lucky I am to have met him.
### Nomenclature and abbreviations used

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>φ</td>
<td>Phase difference ($φ = φ_x - φ_y$)</td>
</tr>
<tr>
<td>α</td>
<td>Azimuth angle of polarisation ellipse</td>
</tr>
<tr>
<td>θ</td>
<td>Angle of analyser (with respect to x axis)</td>
</tr>
<tr>
<td>λ</td>
<td>Wavelength</td>
</tr>
<tr>
<td>ω</td>
<td>Angular frequency ($= 2\pi f$, where $f$ is the frequency)</td>
</tr>
<tr>
<td>a ($=E_x$)</td>
<td>Amplitude of $x$ vibration</td>
</tr>
<tr>
<td>b ($=E_y$)</td>
<td>Amplitude of $y$ vibration</td>
</tr>
<tr>
<td>a'</td>
<td>Half length of major axis of polarisation ellipse</td>
</tr>
<tr>
<td>b'</td>
<td>Half length of minor axis of polarisation ellipse</td>
</tr>
<tr>
<td>C</td>
<td>Number of comparison points (used in security mark verification)</td>
</tr>
<tr>
<td>d</td>
<td>Thickness of optical layer</td>
</tr>
<tr>
<td>e</td>
<td>Ellipticity of polarisation ellipse ($= a'^2 / b'^2$)</td>
</tr>
<tr>
<td>E</td>
<td>Electric vector, or error score used in security mark verification, depending on context</td>
</tr>
<tr>
<td>exp</td>
<td>Exponential</td>
</tr>
<tr>
<td>GL</td>
<td>Grey level (applied to LCTVD or measured from the image of the security mark)</td>
</tr>
<tr>
<td>i</td>
<td>$\sqrt{-1}$</td>
</tr>
<tr>
<td>$I_{\text{max}}$</td>
<td>Maximum intensity measured on rotation of analyser</td>
</tr>
<tr>
<td>$I_{\text{min}}$</td>
<td>Minimum intensity measured on rotation of analyser</td>
</tr>
<tr>
<td>$I_\theta$</td>
<td>Intensity measured through analyser at angle $\theta$</td>
</tr>
<tr>
<td>k</td>
<td>Wave constant ($= 2\pi / \lambda$)</td>
</tr>
<tr>
<td>LC</td>
<td>Liquid crystal</td>
</tr>
<tr>
<td>LC(TV)D</td>
<td>Liquid crystal (television) display</td>
</tr>
<tr>
<td>n</td>
<td>Refractive index</td>
</tr>
<tr>
<td>$n_e$</td>
<td>Extraordinary refractive index</td>
</tr>
<tr>
<td>$n_o$</td>
<td>Ordinary refractive index</td>
</tr>
<tr>
<td>$n_s$</td>
<td>Refractive index as determined by index indicatrix</td>
</tr>
<tr>
<td>$\Delta n$</td>
<td>Birefringence ($= n_e - n_o$, or $n_e - n_o$)</td>
</tr>
<tr>
<td>$\Delta nd$</td>
<td>Retardance (may be multiplied by $2\pi$)</td>
</tr>
<tr>
<td>PA</td>
<td>Parallel aligned (nematic)</td>
</tr>
<tr>
<td>QWP</td>
<td>Quarter wave plate</td>
</tr>
<tr>
<td>$R^2$</td>
<td>Correlation coefficient (least squares fit of data to model)</td>
</tr>
</tbody>
</table>
Nomenclature

<table>
<thead>
<tr>
<th>SLM</th>
<th>Spatial light modulator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>TN</td>
<td>Twisted nematic (liquid crystal)</td>
</tr>
<tr>
<td>TRS</td>
<td>Transmitted radiance sinusoid</td>
</tr>
<tr>
<td>$z$</td>
<td>Distance travelled by wave (along $z$ axis)</td>
</tr>
</tbody>
</table>

$$\begin{bmatrix} J_1 + iJ_2 \\ J_3 + iJ_4 \end{bmatrix}$$  
Jones vector ($J_1$, $J_2$, $J_3$ and $J_4$ are real numbers)

$$\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}$$  
Stokes vector ($S_1$, $S_3$, $S_4$ and $S_3$ are real numbers)
'Light is characterized by its intensity, wavelength, and polarization.'

Collett, 1993, p.5
Chapter 1

Introduction

1.1 Introduction to polarisation

Despite the interference experiments carried out by Thomas Young (1773-1829), it was not until the experiments of Fresnel and Arago in 1818 that it was discovered that light had no longitudinal components, but consisted only of two transverse vibrations [Collett, 1993, p.22]. These vibrations are perpendicular to each other. For convenience, the axis system can be chosen so that these vibrations are along the $x$ and $y$ axes, and the light is propagating in the $z$ direction. The combination of these transverse components of light leads to the polarisation ellipse, but it was not until Sir George Stokes published his papers in 1852, that an adequate mathematical description could be given to describe the various states of polarisation that can occur. The mathematical description of polarised light is discussed in Chapter 2. The Stokes vectors are still used today, and have an advantage of being experimentally defined because they are constructed around the measurements of intensity, not amplitudes. They will be used in this thesis, particularly in Chapter 6.

Polarised light is ‘pure’ in the sense that it is monochromatic, and the position of the electric vector, which is formed from the combination of the two transverse components, can be predicted at any point in space or time (assuming the absolute phase of the components is also known). Despite this apparent simplicity, the human eye is insensitive to the state of light polarisation, and special equipment is needed to reveal its true nature. The interaction of polarised light with certain forms of matter, such as liquid crystals, reveals fascinating details about the structure of the matter. These hidden details include the refractive index of a
reflecting material and its orientation. Use of polarisation for image analysis will be discussed in Chapter 4. Information can also be encoded as a polarisation pattern. This pattern is invisible unless special polarisation decoding apparatus is used. This will be discussed in Chapters 7-9.

The author’s enthusiasm for the subject of this thesis was inspired by the fact that the polarisation of light is so fundamental a property, and is affected by so many things (optical activity, scattering, reflection etc), but humans are completely insensitive to it. In addition to the wavelength and intensity of light, polarisation can carry additional information [Wolff, 1997] to which humans are not privy without special equipment (at the least a polarising sheet). This thesis aims to make that information acquirable using a commercially produced liquid crystal display.

1.2 Motivation for this work

Liquid crystal displays (LCDs) are commercially available for many applications. These include simple passive matrix addressed displays, such as those used in digital watches and calculators, and the more complex actively addressed displays which are used in displays for computer and television screens. The mass market for LCDs results in them being produced at relatively low cost, and this has led to researchers looking for alternative uses of these displays. The high cost of commercial spatial light modulators (around £9000 [Hamamatsu, 2000]- see Chapter 10) has encouraged researchers to use modified liquid crystal television displays (LCTVDs) for spatial light modulation since approximately 1985 [Aiken & Bates, 2000; Wolff, 1997]. A LCD works by changing the polarisation of (polarised) light passing through it, and this thesis looks at using a LCTVD as a polarisation filter. This varies from the use of a LCTVD as a spatial light modulator because polarisation is changed by the phase difference between the orthogonal components of the light mentioned above. Spatial light modulators are concerned with the phase of one area of the incident wavefront compared with another, regardless of its polarisation state.

Two main applications are considered:

1) Using the LCTVD as a complete polarimeter. An algorithm is presented which enables unknown input polarisation to be determined by recording the intensity of light transmitted through the LCTVD and a fixed analyser. The state of polarisation can be

---

1 Some individuals are slightly sensitive to rotating linear polarisation and can see the phenomenon called Haidinger’s brushes. However, this is only in very specialised conditions, and is not universal.
calculated from intensity measurements taken at four voltages of the LCTVD\(^{ii}\). This avoids the need for any mechanically rotating parts.

2) The LCTVD can also be used as a security mark reader for authenticating security marks which emit a known pattern of polarisation. Polarisation varying security marks are difficult to counterfeit, partly because they cannot be reproduced photographically. The large number of patents that have been published detailing the production of such marks indicates the usefulness of a polarisation varying security mark. However, there have been no such publications that detail an automatic method of authenticating these marks. This thesis describes how this can be done using a LCTVD or a single LC cell. This is discussed in Chapters 7-9.

When a LCTVD is used as a display, the polarisation change is converted to an intensity change by viewing the display through a (second) linear polariser. However, this conversion of polarisation change to intensity change means that the phase information (which partly determines the state of polarisation) is lost. This can be likened to converting a three dimensional image to a two-dimensional photograph. This work looks at the use of a commercially produced liquid crystal television display (LCTVD) in the polarisation, rather than the intensity, domain. Using a commercially available device has the advantage that the device is easily reproduced, and done so at lower cost than if it were custom made.

1.3 Outline of the thesis

The thesis is arranged in three main sections:

Part I (Chapters 1-3) outlines the background behind liquid crystals and polarisation. Nematic liquid crystal displays only work with polarised light. The theory of polarisation, including the mathematical treatment of birefringence and how polarised light can be produced and manipulated is therefore discussed first in Chapter 2. What a liquid crystal is, why liquid crystals are so special, and how liquid crystal displays work is covered in Chapter 3.

Part II (Chapters 4-6) discusses the background theory, experimental work, optimisation and algorithm behind using the LCTVD as a polarisation filter to determine unknown polarisation. If light of unknown (full or partial) polarisation is passed through the LCTVD and a fixed analyser, the LCTVD can be used as a ‘Stokes-meter’. This means that by simply applying four voltages to the LCTVD and measuring the intensity of the light transmitted by the analyser, the polarisation of the incident light can be determined. This enables the full polarisation state to be measured without any rotating components.

\(^{ii}\) One intensity measurement is needed for each of the four Stokes parameters. To measure only the degree of linear polarisation 3 intensity measurements are required (see Chapter 6).
comparison of a method that uses Jones calculus with that using Mueller calculus is discussed in Chapter 6.

Part III (Chapters 7-9) discuss the use of the LCTVD or a single LC cell (which is very similar in construction to the LCTVD, but only consists of one pixel) as part of a reader which can authenticate polarisation varying security marks. As a precaution against counterfeiting, it may be desirable to attach a mark to an article that indicates its authenticity. Such a mark could be made to emit a spatially varying pattern of polarisation when it is illuminated with polarised light. This pattern would be invisible to the unaided eye and difficult to counterfeit. Part III will detail the use of the LCTVD (or single LC cell) as part of an automatic verification system for these marks, the sensitivity of which could be customised to meet the needs of the user. The production of such marks, together with the advantages and disadvantages of such a system is discussed.

The thesis ends with conclusions and suggestions for future work (Chapter 10).
Chapter 2

Polarisation

2.1 Introduction

2.1.1 Layout of this chapter

This chapter will begin by describing what polarised light is, and the different forms of polarisation that can occur. It will then (s.2.3) describe different methods (including Jones vectors and Stokes vectors) for mathematically representing these different polarisations. The methods of producing polarised light will be discussed in s.2.4, together with how these methods can be represented mathematically, to interact with the representations of polarised light discussed in s.2.3. The measurement of polarisation will be covered in s.2.5 and s.2.6 and the merits of different techniques will be discussed. Section 2.7 outlines how the intensity of the polarisation ellipse can be normalised, and s.2.8 deals with the distinction between partially linearly polarised and fully elliptically polarised light.

2.1.2 Polarised, unpolarised and partially polarised light

A light wave is described as being polarised if the position of its electric (or magnetic) field vector can be predicted in time (i.e. the variation of the end point of the field vector is regular – [Born & Wolf, 1999, p.619]). Strictly monochromatic light is always polarised [Born & Wolf, 1999, p.619]. Alternatively, polarised light can be thought of as light whose vibration pattern exhibits preference, either as to its transverse direction, or to the handedness associated with it [Shurcliff, 1962 p.2]. As is conventional in most optical texts, the vibration
Chapter 2 Polarisation

of the magnetic vector, which is always orthogonal to the electric vector, will not be explicitly mentioned in this thesis. Any references in this thesis to the field vector or its vibration should be taken as referring to the electric field vector.

If the polarisation state changes more rapidly than the speed of observation, the light is called partially polarised, or unpolarised depending on the time-averaged behaviour of the polarisation state [Yeh & Gu, 1999, p.34]. Unpolarised light occurs when the end point of the electric vector moves quite irregularly, and the light shows no preferential directional properties when resolved in different directions (at right angles to the direction of propagation). Partially polarised light is light that is neither completely polarised, nor completely unpolarised. This thesis is mainly concerned with light which is completely or almost completely polarised (i.e. the unpolarised component, if present, is very small compared to the polarised component) and this will be described in more detail. Partial polarisation will be discussed briefly in this chapter for completeness and because in some circumstances it can be confused with elliptically polarised light (see s. 2.3.8 and 2.8).

2.1.3 Types of polarisation

The state of polarisation of light depends on the relative amplitudes of the transverse components of the electric field along the $x$ and $y$ co-ordinate axes, and the phase difference between them, $\phi$. The most general description of polarisation is elliptical polarisation, because the tip of the electric vector traces out an ellipse in time (or space). This will be described in s.2.2 below. Linear and circular polarisations are special forms of elliptical polarisation. They occur when $\phi$ is an exact multiple of $\pi$, or an odd multiple of $\pi/2$ respectively. For the light to be circularly polarised, as well as the phase difference being an odd multiple of $\pi/2$, the amplitudes of the $x$ and $y$ components have to be equal.

Completely linearly polarised light can be extinguished by passing it through an ideal polariser whose transmission axis is perpendicular to the plane of vibration of the electric vector of the incident light. This principle can be extended to cover all polarised light, by defining light as being fully polarised if a quarter wave plate (QWP) (see s. 2.4.2 below) followed by an ideal polariser can be used to extinguish the beam [Mansuripur, 2000].

2.1.4 Co-ordinate axes definition

The co-ordinate axes are defined as a right handed set, with the $x$ vertical (positive direction upwards), $y$ horizontal (positive direction to the right), and with light travelling along the $z$ direction with the source behind the observer. This is illustrated in Figure 2-1. When a polariser is set at $0^\circ$, the electric vector vibrates along the $x$ axis.
Chapter 2 Polarisation

2.2 Elliptical polarisation

The two components of a light wave vibrating along the $x$ and $y$ axes can be represented as [Born & Wolf, 1999, p.25 et seq.]:

\[ E_x = a \cos(\omega t - kz + \frac{\phi}{2}) \]  
\[ E_y = b \cos(\omega t - kz - \frac{\phi}{2}) \]

Where $a$ and $b$ are the amplitudes of the $x$ and $y$ components respectively, $k$ is the wave constant ($=2\pi/\lambda$), $\omega$ is the angular frequency ($=2\pi f$, where $f$ is the frequency in hertz), and $t$ is the time in seconds. If the absolute phase of the $x$ component is defined as $\phi_x$ and that of the $y$ component is $\phi_y$, then $\phi$ is the phase difference between the orthogonal vibrations (defined so $\phi = \phi_y - \phi_x$). When $\sin \phi$ is negative, the $x$ vibration leads the $y$, so the light is defined as right-handed (see Appendix 2A). This represents the tip of the electric vector tracing a clockwise rotation when the observer looks towards the source [Azzam & Bashara, 1979, p.39]. Adding these two vibrations leads to the equation of an ellipse oriented at an angle $\alpha$ to the co-ordinate axis [Born & Wolf, 1999, p.25 et seq.] - equation 2-3. It can be seen that the sign of $\phi$ does not alter this equation.

\[ \left(\frac{E_x}{a}\right)^2 + \left(\frac{E_y}{b}\right)^2 - 2 \cos \phi \left(\frac{E_x}{a}\right)\left(\frac{E_y}{b}\right) = \sin^2 \phi \]  

This is illustrated in Figure 2-2. $\alpha$ is defined so $-180^\circ \leq \alpha \leq 180^\circ$. An azimuth angle of $+30^\circ$ is equivalent to one of $-150^\circ$.

---

1 This is valid because in the relevant equations (namely equation 2-5 and equation 2-6) $\cos \alpha$ or $\sin \alpha$ is either squared, in which case the sign doesn’t matter, or multiplied by $\sin \theta$ or $\cos \alpha$. In the latter case either both the sine and cosine have the same sign, or one is negative when the other is positive; so dividing (or multiplying) one by the other will give the correct sign. As an example, taking a value of $\alpha$ to be $+60^\circ$ is equivalent to taking it to be $-120^\circ$. At $60^\circ$, both sine and cosine are positive, and at $-120^\circ$ both are negative, so dividing one by the other will be positive.
Chapter 2 Polarisation

Figure 2-2 Ellipse oriented at an angle alpha to the coordinate axes

The azimuth angle of the ellipse, \( \alpha \), is given by equation 2-4 [Born & Wolf, 1999, p.28].

\[
\alpha = \frac{1}{2} \tan^{-1}\left(\frac{2ab \cos \phi}{a^2 - b^2}\right)
\]

equation 2-4

The magnitudes of the major and minor axes of the ellipse can be calculated from the original amplitudes of the vibrations, and the phase difference between them (equation 2-5 and equation 2-6 [Collett 1993, p.28]).

\[
\cos^2 \alpha \frac{a^2}{a^2} + \sin^2 \alpha \frac{b^2}{b^2} + \frac{2 \cos \alpha \sin \alpha \cos \phi}{ab} = \frac{\sin^2 \phi}{(a')^2}
\]

equation 2-5

\[
\sin^2 \alpha \frac{a^2}{a^2} + \cos^2 \alpha \frac{b^2}{b^2} + \frac{2 \cos \alpha \sin \alpha \cos \phi}{ab} = \frac{\sin^2 \phi}{(b')^2}
\]

equation 2-6

In this thesis, polarisation is defined using the azimuth angle of the ellipse, \( \alpha \), and its ellipticity, \( e \), defined so that \( e = \frac{a^2}{b'^2} \). It should be noted that this is the square of the reciprocal of the definition in some textbooks.

2.3 Representations of polarisation

Two mathematical representations of polarised light have been developed. The first originated in the 1850s and was developed by G.G. Stokes [Hecht, 1998, pp.366-368; Azzam & Bashara, 1979, pp.55 et seq.], and the second was developed in 1941 by R. Clark Jones [Hecht, 1998, pp.368-369]. Each method has its advantages and disadvantages: Stokes vectors
are more mathematically complex than Jones vectors because the Mueller calculus that is needed to transform one \([4 \times 1]\) Stokes vector into another involves \([4 \times 4]\) matrices, whereas the matrices needed to transform the \([2 \times 1]\) (complex) Jones vectors are only \([2 \times 2]\).

Additionally, because Stokes vectors do not include phase information, they do not describe coherent light. However, the Stokes representation can be used when the light is not completely polarised, and the intensity based definition means that it is possible to look at a Stokes vector and immediately have some knowledge about the state of polarisation (see s.2.3.7).

Jones calculus does not describe partially polarised light, or optical systems which depolarise. This disadvantage can be circumvented if partially polarised light is thought of as a combination of completely polarised light and completely unpolarised light [Clarke & Grainger, 1971, p.31; Chen & Wolff, 1998). It is also not normally easy to tell the state of polarisation simply by looking at a Jones vector - calculations have to be performed on the vector before the exact state of polarisation is known. Jones calculus is covered in [Jones, 1941] and [Jones, 1942]. A complete discussion of the difference between Stokes and Jones vectors can be found in [Azzam & Bashara, 1979, pp.13-65].

Many papers have covered the modelling of liquid crystal cells using Jones matrices (such as [Gooch & Tarry, 1975], [Raynes. & Tough, 1985], [Raynes, 1987]). A Jones matrix model of the LCTVD, which has been developed by researchers at UCL, is used in this work [Kilpatrick et al., 1998]. Consequently Jones calculus is initially used in this thesis. However, Stokes vectors will also be used in Chapter 6, so this representation is also discussed in this chapter (section 2.3.7).

2.3.1 Jones vectors

Jones calculus represents a polarised beam of light by a column vector whose elements are the two components \((E_x \text{ and } E_y)\) of the electric vector. The vectors express the amplitudes and phases of the \(x\) and \(y\) components individually. It is understood that the physical displacements of the \(E\) vector are given by the real parts of the two elements [Robson, 1974]. Using the notation in equation 2-1 and equation 2-2 the Jones vector describing the polarisation would be:

\[
E = \begin{bmatrix}
  a e^{i\phi_x} \\
  b e^{i\phi_y}
\end{bmatrix}
\]

The amplitude and phase of the waves are preserved using the complex notation. If the exact phases of the constituent waves are not needed, the phase of the \(x\) component can be removed from the vector, by dividing both components by \(\exp(i\phi_x)\), to get:
Chapter 2 Polarisation

\[ E = e^{i\phi} \begin{bmatrix} a \\ be^{-i\phi} \end{bmatrix} \]  
\text{equation 2-8}

The 2x1 complex Jones vector has four components: \[ \begin{bmatrix} J_1 + iJ_2 \\ J_3 + iJ_4 \end{bmatrix} \]. To normalise this vector, the components can all be divided by \[ \sqrt{J_1^2 + J_2^2 + J_3^2 + J_4^2} \] [Hecht, 1998, pp.366-368].

2.3.2 Use of Jones vectors

Jones vectors are a mathematical tool for analysing the effect of various (non-depolarising) optical components on the passage of polarised light. To determine the polarisation represented by each vector, the amplitude and phase information needs to be re-introduced to the complex vector, and from this ellipticity, azimuth angle and sense of rotation of the polarisation ellipse can be calculated. The methods for converting a Jones vector \((a, b, a', e')\) to \(\alpha\) and \(e\), and back again are covered below:

2.3.3 Calculation of ellipticity and azimuth angle from Jones vector

The Jones vector \[ \begin{bmatrix} \text{Re}(x) + \text{Im}(x) \\ \text{Re}(y) + \text{Im}(y) \end{bmatrix} \] allows the phase of the \(x\) and \(y\) components to be calculated directly from the Argand diagram of the individual phasors (equation 2-9).

\[ \phi_x = \tan^{-1} \left( \frac{\text{Im}(x)}{\text{Re}(x)} \right) \]  
\text{equation 2-9}

The amplitudes of the components are calculated by taking the square root of the phasor multiplied by its complex conjugate. The azimuth angle of the ellipse can now be calculated using equation 2-4. Similarly, the major and minor axes can be calculated using equation 2-5 and equation 2-6.

2.3.4 Calculation of Jones vector components from ellipticity and azimuth angle

The mathematical procedures for calculating the Jones vector components are detailed in Appendix 2B. The ratio \(a'/b'^2\) can be calculated using:

\[ \frac{a^2}{b^2} = \frac{b'^2 \sin^2 \alpha + a'^2 \cos^2 \alpha}{b'^2 \cos^2 \alpha + a'^2 \sin^2 \alpha} \]  
\text{equation 2-10}
To calculate the phase difference, $\phi$, equation 2-4 can be written:

$$\tan 2\alpha = \frac{2b \cos \phi}{a} + m\pi$$

Therefore,

$$\cos \phi = \tan 2\alpha \left( \frac{1 - \frac{b^2}{a^2}}{\frac{b}{a}} \right)$$

Equation 2-12 contains the ratio $a/b$, which can only be calculated by taking the square root of equation 2-10. This results in a sign ambiguity. This can be resolved if equation 2B-9 is divided by equation 2B-8 (equations in Appendix 2B):

$$-\frac{b \cos \phi}{a} = \cos \alpha \sin \alpha \frac{(b^2 - a^2)}{a^2 \cos^2 \alpha + b^2 \sin^2 \alpha}$$

If the sense of rotation of light is measured, the sign of $\cos \phi$ is known. Equation 2-13 therefore enables the sign of $a/b$ can be deduced. Changing the sign of both imaginary components in the Jones vector changes the sense of the light, whilst retaining the original ellipticity and azimuth angle.

It is a simple matter to calculate the Jones vector components from $a$, $b$ and $\phi$, as the complex phasors are simply $(a \exp i\phi/2)$ and $(b \exp -i\phi/2)$. The real part of the $x$ component, $A$, is simply $(a \cos \phi/2)$ and the imaginary part, $B$, is $(a \sin \phi/2)$. The same can be done for the $y$ phasor. Absolute phase does not affect polarisation, so, in most of the calculations in this thesis the Jones vectors are arranged so that the phase of the $x$ component is zero.

### 2.3.5 The complex polarisation variable

One method of representing the polarisation of light, which will be used in this thesis, is that described by Azzam & Bashara [1979, pp.39-49; and 1972]. This is to use the complex polarisation variable, $\chi$ with the basis states of polarisation as the $R$ and $L$ states. Instead of using the linear $x$ and $y$ orthogonal components, as described above, $\chi$ represents the ratio of the circular complex Jones vectors (equation 2-14).

$$\chi = \frac{|E_x|}{|E_y|} \exp i(\phi - \phi)$$

where $\phi$ and $\phi$ are the phases of the $R$ and $L$ orthogonal components. Then
Chapter 2 Polarisation

\[ |\chi| = \frac{|E_r|}{|E_i|} \quad \text{and} \quad \arg(\chi) = \phi - \phi_i \]

An advantage of this representation is that, when the Argand diagram is plotted, the state of polarisation represented by points on the diagram can be seen easily. Linear polarisation states are represented by a concentric circle about the origin, with a radius of 1. The origin and infinity represent left and right circular polarisations respectively. Polarisations which are left handed all fall within the circle of linear polarisations, and those which are right handed all fall outside it (see Figure 2-3). Concentric circles represent loci of polarisations with equal ellipticities, and radial lines represent polarisations with equal azimuth angles.

![Figure 2-3 - The circular complex polarisation variable (from Azzam & Bashara, 1979, p.42)](image)

The familiar values of azimuth angle, \( \alpha \), and ellipticity, \( e \), can be found from manipulation of the definition of the Jones vector with \( r \) and \( l \) as its basis states - i.e.

\[
\begin{bmatrix} E_r \\ E_i \end{bmatrix} = \begin{bmatrix} E_{r} \exp i\phi_r \\ E_{i} \exp i\phi_i \end{bmatrix} \quad \text{equation 2-15}
\]

To calculate \( \chi \) from the known values of \( \alpha \) and \( e \) we use the relationship:

\[
\chi = \tan(e + \frac{\pi}{4}) \exp(i(-2\alpha)) \quad \text{equation 2-16 [Azzam & Bashara, 1979, p.40]}
\]
The azimuth angle can be found directly from the plot, because $\alpha = -\frac{1}{2} \arg(\chi)$ [Azzam & Bashara, 1979, p.40], so

$$\alpha = \frac{1}{2} \tan^{-1} \left( \frac{\text{Im}(\chi)}{\text{Re}(\chi)} \right)$$  \hspace{1cm} \text{equation 2-17}

Azzam and Bashara define the ellipticity angle, $\varepsilon$, so that $\tan \varepsilon = \frac{b'}{a'} (= \frac{1}{\sqrt{\varepsilon}})$. The sign of $\varepsilon$ indicates the sense of rotation, with a positive value representing right handed and a negative value indicating left handed polarisation.

The real and imaginary parts of $\chi$ can be found directly by substituting in equation 2-16, ensuring that the correct signs for $\varepsilon$ are used to represent the sense:

$$\text{Re}(\chi) = \frac{1 + \sqrt{\varepsilon}}{\sqrt{\varepsilon} - 1} \cos(2\alpha) \quad \text{and} \quad \text{Im}(\chi) = \frac{1 + \sqrt{\varepsilon}}{\sqrt{\varepsilon} - 1} \sin(2\alpha)$$  \hspace{1cm} \text{equation 2-18}

equation 2-18 enables the polarisation to be plotted on a graph such as Figure 2-3.

2.3.6 The Poincaré sphere

Poincaré introduced a representation of states of polarisation that consists of a sphere whose points are in a one-to-one correspondence with all the different states of polarisation. It can be considered as a three dimensional version of the circular complex $\chi$ plane as represented in Figure 2-3. Use of the Poincaré sphere to represent polarisation is well covered in optical textbooks [Azzam & Bashara, 1979, p.47 et seq.] and will not be repeated here.

The relationship between the $\chi_n$ representation and the Poincaré sphere is such that the concentric circles of equal ellipticity and the radial lines of equal azimuth angle in Figure 2-3 correspond to the circles of latitude and the lines of longitude on the Poincaré sphere. The circle of linear polarisation states corresponds to the Poincaré sphere equator.

2.3.7 Stokes vectors

Unlike the phase and amplitude representation of Jones vectors, Stokes vectors are purely intensity based. We assume that $I_0$ is the total intensity of the wave, and $I_\alpha$, $I_\beta$, $I_\gamma$ and $I_\delta$ are the intensities transmitted by ideal linear polarisers placed at $0^\circ$, $90^\circ$, $45^\circ$ and $-45^\circ$ respectively and $I_r$ and $I_l$ are the intensities transmitted by polarisers which transmit $r$ and $l$.

Let $\tan(\varepsilon + \frac{\pi}{4}) = A$. Then $\chi = \cos(2\alpha) - i\sin(2\alpha) ; \tan((\varepsilon + \frac{\pi}{4}) = \frac{\tan \varepsilon + 1}{1 - \tan \varepsilon}$ so $\tan((\varepsilon + \frac{\pi}{4}) = \frac{1 + \sqrt{\varepsilon}}{\sqrt{\varepsilon} - 1}$
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states. All the filters transmit 50% of the incident intensity if the light is completely unpolarised. The 4 Stokes parameters are given by [Azzam & Bashara, 1979, p.57]:

\[
\begin{align*}
S_0 &= I_0 = I_r + I_p = I_{45} + I_{-45} = I_r + I_l \\
S_1 &= I_r - I_p \\
S_2 &= I_{45} - I_{-45} \\
S_3 &= I_r - I_l.
\end{align*}
\]

The intensity based definitions of the Stokes vector mean that it is possible to have some idea of the polarisation state from inspecting the vector. As some examples, a Stokes vector of \([1,1,0,0]^T\) represents light completely polarised along the \(x\) axis, and one of \([1,0,0,-1]^T\) represents completely polarised left circular light. A Stokes vector of \([2,1,1,0]^T\) can be envisaged as light which is linearly polarised and has equal components along the \(x\) axis and +45° directions. Further detail of the relationship between Stokes parameters, ellipticity, azimuth angle and the cartesian components is included in Appendix 2C.

As mentioned above, Stokes vectors allow the description of partially polarised light, which will be discussed next.

### 2.3.8 Partial Polarisation

If \(S_0 = S_1^2 + S_2^2 + S_3^2\) the wave is totally polarised, but if \(S_0 > S_1^2 + S_2^2 + S_3^2\) the wave is partially polarised, or unpolarised, and the degree of polarisation, \(P\), can be defined as [Collett, 1993, p.52]:

\[
P = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}.
\]

Partially polarised light can be considered as a sum of an unpolarised and a totally polarised component, so that \(S = S_u + S_p\), where \(S = [S_0, S_1, S_2, S_3]^T\), \(S_u = [S_0, \sqrt{S_1^2 + S_2^2 + S_3^2}, 0, 0, 0]^T\) as there is no preference to any particular form of polarisation, and \(S_p = [\sqrt{S_1^2 + S_2^2 + S_3^2}, S_1, S_2, S_3]^T\).

### 2.4 Producing and changing polarisation

It has already been shown that the polarisation of a light wave is dependent on the relative amplitudes and phases of its orthogonal components (equation 2-3 and equation 2-4). Therefore, if either of these components is changed, the polarisation will change. Amplitude can be changed by absorbing (or reflecting) one component preferentially over the other. A polariser which transmits \(r\) states consists of a quarter wave plate with its fast axis vertical, followed by a linear polariser with its transmission axis at 45°. The quarter wave plate transforms right handed light to linearly polarised light along 45°.

---

\(^{iii}\) A polariser which transmits \(r\) states consists of a quarter wave plate with its fast axis vertical, followed by a linear polariser with its transmission axis at 45°. The quarter wave plate transforms right handed light to linearly polarised light along 45°.

\(^{iv}\) The Stokes vectors are written as their transposes to save space.
linear polariser works by absorbing one component and transmitting the other. Phase can be changed by retarding one component relative to the other. Birefringent materials change polarisation by changing the phase difference between the orthogonal components.

To achieve the full range of polarisation change it is necessary to be able to alter both the amplitude of the orthogonal components of the light, and the phase difference between them.

2.4.1 Changing amplitude

2.4.1.8 Transmission

A linear polariser is dichroic in that it selectively absorbs one of the two orthogonal linear components of an incident beam. Polaroid is made by stretching a sheet of clear polyvinyl alcohol so that the long hydrocarbon molecules become aligned. The sheet is then dipped in iodine, which impregnates the plastic and attaches to the long molecules [Hecht, 1998, p.330; Shurcliff, 1962, p.51 et seq.; Yeh & Gu, 1999, p.71]. The conduction electrons associated with the Iodine can move along the chains so the Polaroid sheet acts like a molecular version of the wire grid polariser. Hence the component of an E wave that is parallel to the molecules drives the electrons and is strongly absorbed. The transmission axis of the Polaroid is therefore perpendicular to the direction in which the film was stretched.

Dichroic materials may also be called absorboanisotropic [Shurcliff, 1962, p.44] because they absorb polarised components to different extents. These materials can be thought of in a similar way to birefringent materials (see later) because they divide the incident ray into two polarised components. Whereas a birefringent material refracts these two rays to different extents, absorboanisotropic materials absorb them to different extents. A linear polariser can be characterised by its extinction ratio, defined as the ratio of $T_1/T_2$, where $T_1$ is the transmission with polarisation parallel to the transmission axis and $T_2$ is the transmission with polarisation perpendicular to the transmission axis.

The behaviour of polarisers can also be explained by considering their complex refractive indices ($n+ik$). The imaginary part of the refractive index represents the extinction coefficient of the material. A good polariser should have a significant difference between the extinction coefficients of the ordinary and the extraordinary refractive indices, so that the unwanted polarisation component is strongly absorbed. A discussion of this can be found in Yeh & Gu [1999, pp.70-71]. Transmission coefficients for various types of Polaroid at different wavelengths are given in Collett [1993, p.476].
2.4.1.b Polariation by reflection

When light, at non-normal incidence is reflected from a dielectric, the polarisation changes. Using Maxwell's equations, Fresnel derived the reflection coefficients for light polarised in and perpendicular to the plane of incidence, $F_p$ and $F_s$ respectively [Hecht, 1988, pp.112-113]. He showed why, at one angle of incidence, (the Brewster angle), all the reflected light is polarised perpendicular to the plane of incidence. The proportion of light reflected which is polarised parallel and perpendicular to the plane of incidence varies depending on the angle of incidence and the refractive index of the reflector. This will not be considered in detail here, but is discussed in Appendix 2D, and in many optical textbooks (such as [Shurcliff, 1962, p.79]).

2.4.1.c Summary of changing amplitude

Changing amplitude by either absorption or reflection will only cause a limited change in polarisation state, and the effect of the two is different: incident light of whatever polarisation will emerge linearly polarised from a piece of Polaroid, as only one component is transmitted. Upon reflection, elliptically polarised light can remain elliptically polarised even though the amplitude of one component is attenuated more than the other (so the azimuth angle and ellipticity will change according to equation 2-4 - equation 2-6). It will only become linearly polarised at the Brewster angle. This distinction is important when considering the experiments that were performed in double pass, although in practice when the incident angle is less than about 20° there is little effect on polarisation (see Chapter 4, figure 4.3).

It is also useful to realise that purely changing the amplitude of one (or both) of the orthogonal polarisation components may cause an apparent rotation of the plane of polarisation. This can be seen when using a piece of Polaroid: if incident light is polarised at, say 30° to the transmission axis of the material, the emergent beam will be 'rotated' by 30° (although its intensity will be significantly reduced). This should not be confused with optical activity, where although the emergent polarisation may be rotated, its amplitude will not be (significantly) reduced.

2.4.2 Changing phase - birefringence

The use of birefringent materials occurs throughout this work: twisted nematic liquid crystal displays operate because they are birefringent and therefore can change the polarisation of light passing through them. This will be covered in more detail in Chapter 3. In addition, the security marking which is described in Chapter 8 is made of a birefringent material.
2.4.2.a  Birefringent materials

Birefringent materials have different refractive indices to different polarisations, and may also be called refractoanisotropic [Shurcliff, 1962, p.65]. This causes incident light falling on the birefringent material to give rise to two refracted waves [Born and Wolf, 1998, pp.684 et seq.].

The incident ray is separated into two orthogonally polarised components, an ordinary ray, which obeys the usual law of refraction ″, and an extraordinary ray that does not ″. These components may be linearly polarised (for a linearly birefringent material) or circularly or elliptically polarised. The test security marking mentioned later in this work is made of linearly birefringent material, so this will be discussed in detail.

A birefringent material has (at least) two values of refractive index, the major and minor principal refractive indices. A material with only two principal refractive indices is called uniaxial. This is the type that will be used in this work. The optic axis of the material is defined as the direction of propagation (relative to the crystal lattice) for which light of all vibrational directions travels at the same speed [Clarke & Grainger, 1971, p.73].

The change of refractive index with direction of light propagation can be visualised by the use of a uniaxial indicatrix, where the lengths of the (major and minor) axes represent the extraordinary and ordinary refractive indices \((n_e \text{ and } n_o)\) [Collett, 1993, p.444]. A positive (i.e. \(n_e > n_o\)) uniaxial indicatrix is shown in Figure 2-4. For a particular angle of incidence, the refractive index is calculated by taking the direction of the incident light (relative to the optic axis) to the centre of the figure, and then dropping perpendiculars to the surface [Shurcliff, 1962, p.68] (shown by \(n_e\) and \(n_o\) in the figure). The length of these normals indicates the refractive indices that are encountered by the orthogonal components of the light. The largest refractive index, \(n_e\) is encountered when light is incident orthogonal to the optic axis. Light travelling along the optic axis, \(z\) has no change in its polarisation \((n_e = n_o)\).

---

"i.e. \(\sin i/\sin r = n\), and the beam lies in the plane of incidence,

"the ray is not always in the plane of incidence, and its velocity is a function of its direction of travel through the material."
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Optic axis

Figure 2-4 Index ellipsoid for a uniaxial (liquid) crystal.

The effect a birefringent material has upon light passing through it differs considerably depending on whether the principal plane (the plane containing the optic axis) is parallel to the incident face of the material. For completeness, the behaviour of a birefringent material when its optic axis is not parallel to the incident face of the material is covered in Appendix 2E. The effect when the principal section is parallel to the incident face is discussed here, as this is the arrangement used in retarders and the security marking discussed in Chapter 8. The special case of twisted nematic liquid crystals (TNLCs) will be discussed in s. 2.4.2.b.

When the birefringent material is arranged so that the principal plane is parallel to the surface of the material (i.e. in the xy plane), light travelling along the z direction (i.e. normal to the surface) will always have its E vector vibrating in the principal plane. Therefore its orthogonal components can always be considered as being parallel or perpendicular to the optic axis (Figure 2-5).

Figure 2-5 Principal plane parallel to incident face of birefringent crystal. Light incident normally on the crystal will always have its orthogonal components vibrating parallel and perpendicular to the optic axis.
Light will travel through the material undeviated, and if the incident light is unpolarised no effect will be seen as the emergent light will also be unpolarised. If the incident light is polarised, the two orthogonal components will travel through the crystal at different velocities. They will recombine on emergence from the crystal, but with a different relative phase than they had when incident on the crystal, and hence a different polarisation. The resultant polarisation depends on the incident polarisation, the retardance of the material (\(\Delta n\), where \(\Delta n = n_r - n_o\)), and the relative amplitudes of the components parallel and perpendicular to the optic axis (which can be varied by rotating the birefringent material in the \(xy\) plane).

2.4.2.b Birefringence and liquid crystals

The exact configuration of a twisted nematic liquid crystal display (TNLCD) will be discussed in Chapter 3 (s.3.4.2), but the effect of the birefringence of the TNLCD on polarisation will be discussed here. The anisotropy of nematic liquid crystals causes them to be birefringent. Nematic liquid crystals are uniaxial, but differ from crystalline waveplates in their effect upon polarised light because the configuration of the TN structure means that as voltage is applied across the LC cell, the optic axis tilts towards or away from the source, rather than merely rotating in the \(xy\) plane.

In a TNLC device, as the director (optic axis) tilts (with increasing voltage), the index indicatrix tilts about the y (or x) axis (i.e. towards or away from the source), and so the length \(n_x\) in Figure 2-4 changes. The varying refractive index for a given direction, \(\psi\), is defined by [Yeh & Gu 1999, p.63]:

\[
n_x(\theta)^2 = \left( \frac{\cos^2 \psi + \sin^2 \psi}{n_0^2 + n_r^2} \right)
\]

The change in refractive index, \(n_x\) causes a change in the effective birefringence, \(\Delta n (\Delta n = n_r - n_o)\). This causes a change in phase difference between the two rays travelling parallel and perpendicular to the optic axis. Therefore there is a change in polarisation with voltage.

2.4.2.c The different use of birefringence and liquid crystals in this thesis

Liquid crystals are used in two separate ways in this thesis: Firstly, a twisted nematic liquid crystal, as described in Chapter 3, is used either in a pixellated display or as a single pixel cell. As discussed in s.2.4.2.b, the application of a voltage to the LC cell causes a change in polarisation of light passing through it. This is caused by the re-orienting of the optic axis changing the effective birefringence. However, the application of voltage does not make the
optic axis rotate in the $xy$ plane (it twists in the $xy$ plane and tilts out of it – see s.3.4.2), the amplitudes of the components of the incident light which are parallel and perpendicular to the optic axis of the LC do not change. The change in polarisation is caused by the change in $n$, (equation 2-21).

Secondly, a LC polymer is used to make the security marking described in Chapter 8. This nematic material is also birefringent, but is arranged in such a way so that its optic axis varies spatially across the marking in the $xy$ plane. This too changes polarisation, but it does so not by changing the birefringence, $\Delta n$, but by changing the orientation of the optic axis ($\theta$ in equation 2-23).

### 2.4.2.2 Jones matrix of a birefringent layer

The effect of a birefringent layer upon polarised light can be represented by a Jones matrix, $\mathbf{M}$ [Chandrasekhar, 1992, p.215]:

$$
\mathbf{M} = \begin{bmatrix}
    \exp\left(\frac{2\pi n_x d}{\lambda}\right) & 0 \\
    0 & \exp\left(\frac{2\pi n_y d}{\lambda}\right)
\end{bmatrix}
$$

This can be simplified to:

$$
\mathbf{M} = \exp\left(\pi(n_x + n_o)d \lambda\right) \begin{bmatrix}
    \exp\left(i\frac{\delta}{2}\right) & 0 \\
    0 & \exp\left(-i\frac{\delta}{2}\right)
\end{bmatrix}
$$

where $\delta = \frac{2\pi\Delta n d}{\lambda}$

If the birefringent layer is rotated by an angle $\theta$, the Jones matrix can be transformed by the rotation transformation:

$$
\mathbf{M}_{\theta} = \begin{bmatrix}
    \cos\theta & -\sin\theta \\
    \sin\theta & \cos\theta
\end{bmatrix} \begin{bmatrix}
    \exp\left(\frac{i\delta}{2}\right) & 0 \\
    0 & \exp\left(-\frac{i\delta}{2}\right)
\end{bmatrix} \begin{bmatrix}
    \cos\theta & -\sin\theta \\
    \sin\theta & \cos\theta
\end{bmatrix}
$$

so

$$
\mathbf{M}_{\theta} = \begin{bmatrix}
    \cos^2\theta \exp\left(i\frac{\delta}{2}\right) + \sin^2\theta \exp\left(-i\frac{\delta}{2}\right) & i\sin\left(\frac{\delta}{2}\right)\sin2\theta \\
    i\sin\left(\frac{\delta}{2}\right)\sin2\theta & \cos^2\theta \exp\left(-i\frac{\delta}{2}\right) + \sin^2\theta \exp\left(i\frac{\delta}{2}\right)
\end{bmatrix}
$$

The Jones matrix of a (non-depolarising) optical system can be converted to a Mueller matrix [Lui & Chipman, 1998, following Azzam, 1979] and this is used in Chapter 6.

---

\(^{\text{vii}}\) This rotation assumes that the incident light is still perpendicular to the principal plane of the birefringent material. This is different to the tilting of the index ellipsoid which has been discussed in s.2.4.2.b
2.5 Polarisation measurement: theory

This section will describe how polarisation can be quantified and how these quantities can be derived from experimentally determined measurements of intensity through a linear polariser (analyser). How the quantification of polarisation can vary depending on the experimental method used to measure it is discussed in s.2.6.

2.5.1 Experimental quantification of polarisation

In the experimental work described in this thesis, polarisation was quantified using the analyser position at which the maximum intensity was measured, ($\alpha$), the ratio of the maximum and minimum intensities ($a'^2$ and $b'^2$) (Figure 2-2), and the sense of rotation of the electric vector. The absolute phase of the wave was not measured because this is not necessary to define the state of polarisation.

2.5.2 Intensity transmitted by a rotating analyser

As an analyser is rotated (in the $xy$ plane,) about the $z$ axis, the intensity transmitted by it varies in a sinusoidal fashion [Clarke & Grainger 1971, p.27]. The equation of this is:

$$I = I_{\text{min}} + (I_{\text{max}} - I_{\text{min}})\cos^2(\theta - \alpha)$$

Equation 2-24\textsuperscript{viii}

Where $I_{\text{min}}$ and $I_{\text{max}}$ are the minimum and maximum intensities transmitted by the analyser, $\alpha$ is the azimuth angle of the ellipse of polarisation, and $\theta$ is the angle of the analyser, as shown in Figure 2-6 below.

\textsuperscript{viii} Further manipulation and explanation of this equation will be seen in Chapter 7 (s.7.3.2).
If the light is fully polarised, equation 2-24 becomes \[ I = b^2 + (a^2 - b^2) \cos^2 \gamma \], where \( \gamma = \theta - \alpha \), and \( a^2 \) and \( b^2 \) are the intensities transmitted along the major and minor axes of the ellipse respectively. In Figure 2-6, \( \theta \) is negative and \( \alpha \) is positive (using the sign convention in Figure 2-1). A graph of the variation in intensity with analyser position is shown in Figure 2-7.

Expanding and rearranging

\[ I = b^2 + a^2 \cos^2 \gamma - b^2 (1 - \sin^2 \gamma) \]

so \( I = a^2 \cos^2 \gamma + b^2 \sin^2 \gamma \)  

equation 2-25
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It can be seen that the sum of the intensities transmitted by an analyser at an angle \( \gamma \), and \((90-\gamma)\) is equal to \(a'^2+b'^2\), and this represents the total strength of the elliptical vibration as discussed in s. 2.7.

If a graph such as Figure 2-7 is obtained by experimentally recording intensity transmitted by an analyser as it is rotated, the parameters \( \alpha \) and \( e \) can be found from analysis of the figure (as \( \theta \) is known). This is further discussed in sections 2.6.2 and 2.6.3.

2.6 Polarisation measurement: experiment

In order to determine the most effective method of determining the polarisation of light, several methods were used to measure polarisation (both with and without a liquid crystal display in place). The aim was to find a relatively quick and easy method of determining polarisation that was reproducible.

2.6.1 \( I_{\text{max}}/I_{\text{min}} \) method

The method initially used to measure polarisation in this work consisted of passing (polarised) light through an analyser to a photodetector and rotating the analyser by hand until the photodetector read a maximum reading. The reading and the angle of the analyser were noted, and this procedure was repeated to find the minimum intensity. This gave a direct value for \( I_{\text{max}} \) and \( I_{\text{min}} \) and \( \alpha \).

The sense was determined by inserting a quarter wave plate into the path of the light, so that its fast axis was coincident with the azimuth angle of the ellipse of polarisation. In this configuration, the quarter wave plate produced linearly polarised light, the orientation of which was determined by the sense of rotation of the original ellipse. This can be understood by considering the elliptical light as the combination of linearly polarised light (along the azimuth angle of the ellipse) and circularly polarised light. The linearly polarised light is unaffected by the quarter wave plate aligned with its axis of vibration, and the circularly polarised light is transformed to linearly polarised. The combination of these gives the rotation of the output (still linearly polarised). If the sense of rotation was left the azimuth angle was rotated in a positive direction

\footnote{A quarter wave retarder, oriented with its fast axis along the axis of the ellipse has a Jones matrix of
\[
\begin{bmatrix}
1 & 0 \\
0 & i
\end{bmatrix}
\] in the ellipse co-ordinate system. When this is multiplied with the Jones vector for the ellipse (with an effective azimuth angle of zero degrees) the resultant is a new polarisation ellipse with a positive azimuth angle if the original ellipse was left handed, and a negative azimuth angle if the original ellipse was right handed.}
This method, although easy to implement suffered from several limitations:

a) It was difficult to determine accurately where the maximum and minimum positions were when rotating the analyser because the detector reading was not always static. This occurred particularly when the light was more circularly than linearly polarised (i.e. the ellipticity, \( e \) was low). To counteract this, the analyser was initially rotated and the magnitude of the maximum (or minimum) was noted. Then the analyser was rotated an equal number of times in a clockwise and an anticlockwise direction until this reading was obtained, and the azimuth angle noted. The true value of the azimuth angle was taken as the mean of the readings.

b) Measuring just the maximum and minimum intensities only allowed two points on the intensity curve (Figure 2-7) to be determined. This could lead to inaccuracies, particularly when the ellipticity is low because the gradient of the intensity variation is reduced, which makes the exact positions of the maximum and minimum intensities difficult to pinpoint.

c) When the minimum intensity was very low it was difficult to get an exact minimum reading because the noise in the system was sometimes as great as the reading itself. This lead to the ellipticity of light which was nearly linearly polarised (i.e. high ellipticity) being inaccurately measured.

2.6.2 \( I_0, I_{45} \) and \( I_{90} \) method

Lawrence Wolff [Wolff et al., 1997] has developed an elegant method for measuring the first 3 Stokes vectors of polarisation using two twisted nematic liquid crystal cells in sequence. This is equivalent to rotating the analyser to measure \( I_0, I_{45} \) and \( I_{90} \) on the transmitted radiance sinusoid (TRS). The equations used are:

\[
\tan 2\alpha = \frac{I_0 + I_{90} - 2I_{45}}{I_{90} - I_0} \quad \text{(if } I_{90} < I_0 \text{)}; \quad \text{(if } I_{45} < I_0 \text{)} \quad \theta = \theta + 90^\circ \text{ else } \theta = \theta - 90^\circ
\]

and partial (linear) polarisation, \( P = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{I_{90} - I_0}{(I_{90} + I_0)\cos 2\alpha} \)

These can be derived from the coherency matrix in Born & Wolf [1998, p.545; Chen & Wolff, 1998]. If instead of the unpolarised component, \( I_{\min} \) represents the intensity along the minor axis of the (totally polarised) ellipse, manipulation of equation 2-27 enables an equation for ellipticity, \( e \) to be derived: 

\[
e = \frac{-1 - P}{P - 1}
\]

In this thesis, equations 2-26 and 2-28 were used to calculate \( \alpha \) and \( e \) from intensity measurements taken with the analyser at 0°, 45° and 90°. It was found that, for most polarisation ellipses, the results obtained were comparable to those obtained using the \( I_{\max}/I_{\min} \).
method. However, it transpired that as this method relied on taking only 3 intensity readings (even though they were repeated several times) it was not significantly better than the $I_{\text{max}}/I_{\text{min}}$ method. Although Wolff [1994] found that using more than 3 readings did not add significant extra accuracy, the author of this thesis agreed with [Wallace et al., 1999] who noted that, although only three points are necessary to reconstruct the TRS, the use of more points gives a more accurate reconstruction. The author of this thesis found that when the azimuth angle approached 45° the values for $I_0$ and $I_{\text{oo}}$ were nearly equal, and this compounded the errors in the calculation of $\epsilon$, although this may have been caused by irregularities in the polariser (see s.2.6.3.c). Although, on a cursory examination, this method appeared to be adequate, it was concluded that the most reliable and accurate way to measure the polarisation ellipse was to take intensity measurements with the analyser in several positions, and find the best fit through these points. This will be discussed next.

2.6.3 Linear regression of transmitted radiance sinusoid method

The methods described in s.2.6.1 and s.2.6.2 measure the TRS using a limited number of analyser positions (two or three respectively). Following the difficulties encountered using these methods, experiments were conducted to discern how many intensity readings (i.e. how many analyser positions) were required to accurately describe the TRS.

2.6.3.a Method used to determine accuracy of measurement

Linearly polarised light was passed through an analyser to a photodetector. The intensity recorded by the detector was noted. The analyser was then rotated and the process was repeated. This was done for each position of the analyser as it was rotated from 0-180° in 10° steps. The intensity measured was plotted against $\cos^2\gamma$ ($\gamma=\theta-\alpha$) where $\theta$ is the angle the transmission axis of the analyser makes with the x axis (Figure 2-6) x. Ideally this should be a straight line (from equation 2-24), with a gradient of $(a^2+b^2)$ and intercept of $b^2$. Linear regression was used to find the value of $\alpha$ that gave the best straight line to fit the experimental data. This was done by altering values of $\alpha$, in 0.25° steps, so that the correlation coefficient, $R^2$, was nearest to 1. $R^2$ is a measure of the fit between the experimental data and the regression line. If there is a perfect fit, $R^2=1$. For $m$ data points:

---

x This procedure was then repeated with the liquid crystal television display (described in Chapter 5), and the single pixel liquid crystal cell (see Chapter 6) to ensure that the measurements were unaffected by the liquid crystal system.
\[ S_t = \sum_{i=1}^{n \text{ points}} (y_i - \bar{y})^2 \quad \text{and} \quad S_r = \sum_{i=1}^{n \text{ points}} (y_i - f(x_i))^2 \]

then

\[ R^2 = \frac{S_t - S_r}{S_t} \quad \text{equation 2-29} \]

\( y \) is the mean of the experimental data points \((y_i)\). The variance, \( S_t \), considers the spread of the data around the mean (as opposed to the spread around the regression model). \( S_r \) quantifies the deviation of the experimental data from the regression model. The magnitude of \( S_r \) depends upon the scale of the data, so the correlation coefficient is normalised by dividing it by \( S_t \). As the fit of the regression model to the data improves, \( S_r \) comes closer to zero (as does the standard error discussed in s.9.3.1), and \( R^2 \) approaches 1.

If two values of \( \alpha \) gave the same value of \( R^2 \), the one which was an exact number, or an exact half number of degrees was chosen (i.e. if the same value of \( R^2 \) was obtained for \( \alpha=35.25^\circ \) or \( 35.5^\circ \), then the value of \( 35.5^\circ \) would be preferred). The gradient of the regression line was \( a'^2 - b'^2 \) (this was equal to \( I_{\max} - I_{\min} \) if the light was fully polarised) and the intercept was \( b'^2 \) \((I_{\min})\). The ellipticity could therefore be calculated from the equation of the regression line.

Initially, intensity measurements were taken with the analyser rotated in \( 10^\circ \) steps from \( 0^\circ \) to \( 180^\circ \). It was assumed that this gave the optimum accuracy for measuring the polarisation ellipse. Values for \( e \) and \( \alpha \) were then calculated using fewer intensity measurements, and the results compared with the optimum results.
2.6.3.b Results

Using a linearly polarised input the results are plotted in Figure 2-8 below.

**Intensity vs \( \cos^2 \gamma \)**. Input polarisation linearly polarised (0 deg). No liquid crystal

Figure 2-8 shows the results taken on two separate occasions, using an input polarisation of 0°. The value of \( \alpha \) which gave the largest value of \( R^2 \) was -1.25° for both cases (although a value of -1.5° gave an equal value of \( R^2 \) for the readings taken on 08/02/00). As the experimental data does not exactly fit the trendline the exact equation of the trendline will depend on which intensity readings are used.

The values of \( R^2 \), \( \alpha \) and \( e \) for various numbers of intensity readings are presented in Table 2-1. This is for the data taken on one day only. Additional data (for readings taken on a separate day, and for an alternative input polarisation of -30°) is presented in Appendix 2F, as are those for when the LC device was inserted into the path of the beam, so that the light was not linearly polarised.
2.6.3.c Discussion: accuracy of azimuth angle and ellipticity measurements

Table 2-1 shows that although there is very little variation in the calculated values for azimuth angle (always about -1.5°), there is a considerable variation in the values obtained for the ellipticity (which should be infinite) for linearly polarised light. This shows the difficulty involved in accurately measuring linearly polarised light. When the light is linearly polarised, the gradient of the graph in Figure 2-8 is steep and any slight error in positioning the analyser will translate to a significant error in the intensity measured through it. Table 2-1 demonstrates the range of values for ellipticity that can be obtained using different methods of measurement even if the system is very simple. The fact that some of the calculated values of ellipticity were negative (because the intercept on the graph was negative) indicates that some errors are present.

When the full range of measurements are averaged the calculated value of \( e \) is -1351. When this is recalculated using just one quadrant (0-90° or 90-180°), the values are quite different. It can also be seen that relying on just 3 intensity readings gives different values of ellipticity depending on which 3 are taken. It was noticed that the intensity measured with the analyser at 0° was not always identical to that with the analyser at 180°. This may have been caused by slight irregularities in the analyser: because the beam did not necessarily pass through the centre of the analyser, rotating the analyser caused the beam to pass through different portions of it. The effect of this should be reduced as more readings are taken.
Chapter 2 Polarisation

Examining the results obtained when the output polarisation was not linearly polarised (i.e. when the LCTVD or LC single pixel cell were inserted into the path of the beam) removed the inherent difficulties involved in measuring ellipticities near infinity. This enabled a more comprehensive estimation of the intensity readings necessary to get a reliable measure of the polarisation ellipse.

2.6.3.d Conclusions

Since polarisation parameters are calculated using the ratio of intensities, any error in measurement (which does not decrease proportionally with decreasing intensity) will have a dramatic result [Liang et al., 1996]. If the ‘correct’ parameters of the polarisation ellipse are those which are found from taking a variety of intensity readings with the analyser rotated across its 180° range, it appears sufficient to take readings every 20°. The important factor appears to be to ensure that the analyser is rotated across its whole range, rather than being confined to a quadrant (as in the \(I_0, I_{1\alpha}, I_{10}\) method discussed in section 2.6.2 above). It was concluded that the most effective method was to measure the polarisation ellipse by rotating the analyser between 0° and 180°, taking intensity readings at 20° intervals. This was taken as the ‘gold standard’ with which other measurement methods were compared (e.g. when assessing the results from the LCTVD Stokes polarimeter discussed in Chapter 6).

2.7 Intensity normalisation

The time averaged intensity (squared amplitude [Born & Wolf, 1999, p.825]) of an optical field can be defined in terms of the lengths of the major and minor axes of the ellipse, \(a'\) and \(b'\) (Azzam & Bashara 1979, p.8, Born & Wolf, 1999, p.28, Collett, 1993, p.298), so:

\[ I = a'^2 + b'^2 = a^2 + b^2 \]  

\text{equation 2-30}

The ellipticity, \(e\), is defined as the ratio of the square of the major and minor axes of the ellipse, rather than the absolute values of \(a'\) and \(b'\). This was because the absolute values for \(a'\) and \(b'\) could vary depending on factors such as: whether or not the light was exactly circularly polarised before it passed through the polariser (see figure 5-1), which part of the LCTVD the beam passed through (see s. 5.6.6), which part of the diffraction pattern from the LCTVD was viewed (s.5.6.3) and the position of the photodetector in relation to the beam. This variation, however, was not important in this thesis because the absolute values of intensity do not affect the polarisation. In order to compare experimental measurements of intensity (calculated from \(a\) and \(e\) (see equation 2-25)) with calculated values, the
experimental value for $e$ was converted to normalised values for $a'$ and $b'$ so that the total intensity was equal to 1 (i.e. $a'^2 + b'^2 = 1$).

### 2.8 Elliptically vs. partially linearly polarised light

It should be mentioned that all of the methods described above, with the exception of measurement of all four Stokes parameters, do not enable light which is elliptically polarised to be distinguished from that which is partially linearly polarised. The equations for transmission through an analyser are the same for both partially polarised and fully elliptically polarised light ($I_{\text{min}}$ may contain an unpolarised component, or may be purely the intensity along the minor axis of the ellipse). The only way these two forms of polarised light can be distinguished is to place a retarding element in the path of the beam [Clarke & Grainger, 1971, p.120] \(^\text{x1}\) (this enables the fourth Stokes parameter, $S_3$, to be determined [Chen & Wolff, 1998]). The retarding element only changes the polarisation (and therefore the intensity which is transmitted by the analyser placed after it) of any polarised component, leaving any unpolarised component unaltered. The two forms of $I_{\text{min}}$ can then be distinguished ($b'^2$ or $b'^2 + I_{\text{unpolarised}}$). This distinction is very important, as it is because the LCTVD acts as a retarding component, and does not purely rotate the plane of polarisation, that it can be used to measure the full state of polarisation. The use of the LCTVD as a complete Stokes polarimeter (rather than a device that purely measures the first three, linear, Stokes vectors) is presented in Chapter 6.

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\(^{\text{x1}}\) An exception to this may be if a polarisation grating is used, to produce a diffraction pattern. This is described by Gori [1999] and discussed in Chapter 4, s.4.4.1.a
Chapter 3  

Liquid crystals

3.1 Introduction: liquids, crystals and liquid crystals

The difference between liquids and crystals is that in a crystal the molecules are regularly spaced, with translational symmetry. Liquids however, are completely disordered and the molecules are free to move. Liquid crystals (LCs) possess physical properties that are intermediate between conventional liquids and crystals: the LC is fluid like, but there is a degree of orientational order exhibited by the molecules. It is this orientational order which makes the liquid crystal phase interesting. The amount of order in a LC is quite small relative to that of a crystal [Collings & Hird, 1998, p.1] but it is enough to lead to properties that can be harnessed in a variety of applications. The preferred direction of orientation of the molecules is called the director (see s.3.2).

Liquid crystal phases occur in many different forms: for example, lyotropic liquid crystals become liquid crystalline when they are dissolved in a solvent (usually water), and are used as detergents. In contrast, thermotropic LCs have a LC phase that occurs over a certain temperature band. Outside that band they may be solid, isotropic liquid or gaseous. Thermotropic LCs are the type of LCs which are used in LC displays (LCDs) and these are the only type of LC which will be considered in this thesis. A description of the different types of liquid crystals is found in [Collings & Hird, 1998].

As can be expected from the definition of a thermotropic LC, its behaviour depends upon its temperature. When the temperature is too low, like most liquids, the viscosity increases as a result of lower molecular kinetic energy [Yeh & Gu, 1999, pp.13-15]. This
increases the time taken for the director to reorient itself with an applied electric field. This reorientation is crucial to the operation of a LCD. When the temperature is too high the birefringence of the LC decreases, reducing the ability of the LCD to change the polarisation of incoming light. This change of polarisation is also crucial to the operation of a LCD. It is therefore vital that any LCD is operated within the correct temperature range. The anisotropies of nematic liquid crystals (NLCs), together with their variation with temperature will now be discussed.

3.2 Nematic liquid crystals

Nematic liquid crystals consist of rod-like molecules. The director is shown diagrammatically in Figure 3-1.

![Diagrammatic representation of nematic liquid crystal molecules, showing the director](image)

An order parameter is defined to quantify the amount of orientational ordering in the NLC such that [Yeh & Gu, 1999, p.10]

\[
S = \frac{\langle 3 \cos^2 \theta - 1 \rangle}{2}
\]

Where \( \theta \) is the angle the molecules make with the director, and \( \langle \rangle \) indicates a time average (as the molecules are free to move). In thermotropic LCs, the order parameter is highly temperature dependent. In the isotropic phase, the order parameter is zero.

It should be mentioned that, in a NLC sample, the director is not perfectly uniform [Collings & Hird, 1998, p.31], but varies with position. The sample consists of many domains of NLC, which causes the light to be scattered and the sample to appear milky [Yeh & Gu, 1999, p.10]. There are also many points where the director is undefined. This is indicated by the presence of defects in the sample. In order to harness the properties of NLCs in display applications it is necessary to arrange the director so it is uniform throughout the sample (i.e. it

\[\text{Nematic is Greek for thread-like}\]
consists of a single domain). This is usually done by treating the surface of the LC cell with an alignment layer, which may be created mechanically by rubbing, or by optical methods. These methods will be discussed briefly in s. 3.4.1 below.

### 3.3 Anisotropy of Liquid Crystals

The rod-like shape of the molecules, coupled with the degree of ordering, gives rise to anisotropies within the NLC. The LC has mechanical anisotropy (viscosity and elastic forces), as well as electromagnetic anisotropies (optical birefringence and dielectric anisotropy).

#### 3.3.1 Viscosity

The viscosity of LCs depends on the orientation of the director in relation to the flow direction. The three viscosities, sometimes called the Miesowicz coefficients are called $\eta_1$ (director perpendicular to the flow and parallel to the velocity gradient), $\eta_2$ (director parallel to the flow), and $\eta_3$ (director perpendicular to the flow and perpendicular to the velocity gradient). $\eta_1$ is much greater than the other two [Collings & Hird, 1998, p.18]. These will not be discussed further here.

#### 3.3.2 Birefringence (Optical Anisotropy)

NLCs are birefringent. The birefringence ($\Delta n$) is related to the order parameter, so as the thermotropic LC is heated towards its isotropic phase, the birefringence reduces. An example of this is shown in Figure 3-2, but the exact variation in refractive indices with temperature is individual to the particular LC.
Chapter 3 Liquid crystals

It can be seen from Figure 3-2 that the principal refractive indices $n_e$ and $n_o$ have different temperature dependencies. In this particular case, $n_e$ decreases strongly with increasing temperature, but there is only a slight change in $n_o$ with temperature. The birefringence disappears suddenly at the clearing temperature [Demus et al., 1998, p.132]

3.3.3 Changes in a LC with application of an electric field (dielectric anisotropy)

NLCs have a different dielectric constant perpendicular ($\varepsilon_{\perp}$) and parallel ($\varepsilon_{\parallel}$) to the director. This dielectric anisotropy ($\Delta\varepsilon=\varepsilon_{\|}-\varepsilon_{\perp}$) leads to a reorientation of the director when an electric field, $E$ is applied along the $z$ axis [de Gennes & Prost, 1993, p.134]. As for birefringence, the dielectric anisotropy is highly temperature dependent, and this is shown in (Figure 3-3).
The similarities between Figure 3-2 and Figure 3-3 are not surprising, as the two indices of refraction of a LC equal the square root of the corresponding relative permittivities.

### 3.3.4 Energetic Anisotropy

When an electric field is applied to a NLC, the director reorientates itself so it assumes the position of minimum free energy. This means that the molecules attempt to rotate to align themselves with the applied electric field. However, the viscosity and elastic forces within the LC compete with the electric field to keep the director in its 'resting' position. The director position that results depends upon the relative effects of the external electric field and these internal forces which are present in the LC. In a NLC these elastic forces are splay, twist and bend and are defined as:

\[
\text{Splay} = \frac{1}{2} K_s (\nabla \cdot \hat{n})^2 \\
\text{Twist} = \frac{1}{2} K_T (\hat{n} \cdot (\nabla \times \hat{n}))^2 \\
\text{Bend} = \frac{1}{2} K_B |\hat{n} \times (\nabla \times \hat{n})|^2
\]

where \(\nabla \cdot \hat{n} = \text{div } \hat{n}\) and \(\nabla \times \hat{n} = \text{curl } \hat{n}\) [Yeh & Gu, 1999, p.404]. In these equations \(\hat{n}\) is the vector representing the director orientation and \(\hat{n}\) is the unit vector along this direction.

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ii A similar effect occurs on the application of a magnetic field, because of the anisotropy of the magnetic susceptibilities parallel and perpendicular to the director.
K_1, K_2, and K_3 are the respective Frank elastic constants which describe the 'stiffness' of the LC, and are generally different from each other [Collings & Patel, 1997, p.9]. (\(n\) and \(-n\) are indistinguishable [deGennes & Prost 1993, p.100]). Combining these it is seen that if the director has the unit vector \(\hat{n}\) the free energy per unit volume (F_v) of a non-chiral nematic liquid crystal can be written as [deGennes & Prost 1993, p.102]:

\[
F_v = \frac{1}{2} K_1 [\nabla \cdot \hat{n}]^2 + \frac{1}{2} K_2 [\hat{n} \cdot (\nabla \times \hat{n})]^2 + \frac{1}{2} K_3 |\hat{n} \times (\nabla \times \hat{n})|^2
\]

In an unswitched twisted nematic LC (see s. 3.4.2 below), splay and bend equate to zero, so twist is the only elastic force present [Collings & Hird, 1998, p.32]. The behaviour of the director in a TNLCD is governed by the combination of the electric field applied and the elastic forces. The Freedericksz transition occurs when the force exerted by the electric field is greater than the forces that keep the twisted structure. At this point the twist begins to straighten (see below).

### 3.4 Liquid Crystal Displays

The anisotropies of the NLC, coupled with its fluidity, are used in LCDs. The dielectric anisotropy means that applying an electric field can change the orientation of the director, and the optical anisotropy means that the material is birefringent. The optic axis depends on the director orientation, and this can therefore be varied by the application of an external electric field. The polarisation of incident (polarised) light can therefore be changed (s.2.4.2). If the display is viewed through crossed (or parallel) polarisers this polarisation change is converted to an intensity change. There are several types of LC which are used in displays. The only type that will be covered in this thesis is the twisted nematic LCD. A comparison between the twisted nematic LCD and a parallel aligned nematic spatial light modulator will be made at the end of this chapter (s.3.5).

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Figure 3-4 Principle deformations in liquid crystals (after [Collings & Patel 1997, p.10])
3.4.1 Alignment layers

For the LC to be useful in a display, the molecules must be oriented in the required direction. This is usually achieved by treating the surface of the container (the LC cell), so that the molecules have a particular orientation. This ensures a single domain, and so reduces the scattering seen in an unaligned sample. In the device studied in this thesis, the inside surfaces of the glass cells are treated with polyimide, which is then mechanically rubbed to produce tiny grooves in the surface. When the LC cell is then filled with LC, the nematic molecules align along the rubbing direction. Much work has been done on explaining how a NLC aligns on a rubbed alignment layer (e.g. [Chung et al., 2000]). One theory is that it is energetically unfavourable for the long LC molecules to be distorted, so they lie along the grooves. The alignment layer also controls the pretilt: in the TNLCD, to avoid an ambiguity of twist direction, it is necessary to anchor the LC molecules to the substrate so that they are tilted slightly away from, rather than being in the plane of, the substrate. This tilt away from the substrate (the pretilt) leads to one twist sense of rotation being preferred rather than the other. The size of this pretilt angle depends on the exact monomer structure of the polyimide alignment layer [Stohr & Samant, 1999].

Alignment layers are used in two areas in this thesis: firstly in the LCD as described above, and secondly to pattern the LC polymer when making a test security mark. The making of the security marks will be described in Chapter 8.
3.4.2 The twisted nematic liquid crystal display

A TNLCD is formed by aligning the director so that it twists through 90°, like a spiral staircase, from one surface of the cell to the other. This is shown in Figure 3-5 below.

![Figure 3-5 Twisted nematic liquid crystal cell: no voltage applied. Cell appears bright through crossed polarisers.](image1)

![Figure 3-6 Twisted nematic liquid crystal cell: voltage applied. Cell appears dark through crossed polarisers.](image2)

The cell is placed between crossed polarisers so that light entering the cell is linearly polarised with its axis of vibration (of the electric vector) along the director. At the boundary of the cell, therefore, the incident light is polarised along the optic axis of the LC. However, the twist in the director causes the optic axis to rotate throughout the cell. A problem of this type can be solved by considering the TNLC cell to be a stack of birefringent layers, each with its optic axis rotated through a small angle [Jones, 1942]. If the cell is thick enough (see s.3.4.4), this has the effect of rotating the plane of polarisation so that it follows the twist in the director [Chandrasekhar, 1992, p.217]. For a 90° twisted cell, the light emerges linearly polarised with its plane of polarisation parallel to the director on the back of the cell. The light is then transmitted by the second polariser and the cell appears bright.

The TNLC cell is switched by coating the surfaces of the glass (between the alignment layer and the glass) with a transparent conductor, such as indium tin oxide (ITO). This allows an electric field to be applied across the LC. When no voltage is applied the cell appears bright (although not completely clear, as some light is lost by the polarisers as well as from reflection from the surfaces of the cell). When voltage is applied, the twist in the director straightens, as it tilts along the applied electric field (although the molecules near the surface remain in place because they are anchored by the polyimide layer). The plane of polarisation
is no longer rotated by the cell. It therefore appears dark when viewed through the crossed polarisers. Intermediate voltages produce intermediate degrees of straightening of the twist (caused by the tilting of the director), and these are used to produce grey levels. The uniformity of the tilt of the director depends upon the voltage applied: at low voltages, the twist is approximately uniform throughout the cell, but at high voltages a tilt of 90° (to the cell surface) occurs in a small region near the centre of the cell [Yeh & Gu, 1999, p. 360].

3.4.3 *Freedericksz Transition*

In most cases interactions between LC molecules and boundaries, such as that of a glass plate, have a large effect [Collings & Hird, 1998, p.32]. When no external field is applied, the influence of the boundary, coupled with the internal forces in the LC determine the position of the director. This can be seen in the “off” state of the TNLC cell shown in Figure 3-5. If an external field is applied which acts to distort this position it can be shown [Chandrasekhar 1992, p.98 et seq.] that below some threshold (magnetic or) electric field strength, no distortion takes place. The field strength at which this distortion begins to occur is called the Freedericksz transition. Above this threshold the director starts to rotate from its undistorted state toward the direction of the electric field. Figure 3-7 shows the sharp transition at the threshold voltage for a LC cell where the director is aligned parallel with the surface of the cell (i.e. homogeneous alignment). \( \theta_d \) represents the angle the director at the midpoint of the cell has rotated under the influence of the electric field, \( E \). \( E_t \) represents the threshold field at which director distortion begins.

![Figure 3-7](image.png)

Figure 3-7 To show Freedericksz transition for a parallel aligned LC cell with no pretilt. Angle of director at midpoint, \( \theta_d \) (deg), as a function of electric field (applied perpendicular to the direction of rubbing (in plane of cell)). From [Collings & Hird 1998, p.207].

The threshold field strength depends upon the boundary conditions of the LC cell, its thickness and temperature. It does not vary significantly with any pretilt of the molecules [Fraser, 1978]. A full derivation of \( E_t \) can be found in [Collings & Hird 1998, p207].
3.4.4 Thickness of TNLCDs

Except for a few specific exceptions, (e.g. the 'Gooch & Tarry' minima discussed below), the 'ideal' operation of a TNLC cell as a pure polarisation rotator only occurs if the cell is thick enough to fulfil the so-called Mauguin limit \([\text{Mauguin 1911}]\). This is where

\[ d\Delta n \gg \frac{\lambda}{2} \]

where \(d\) and \(\Delta n\) are the thickness and birefringence of the liquid crystal layer, and \(\lambda\) is the wavelength of light. The switching time of a liquid crystal cell is proportional to the square of the thickness of that cell \(^{\text{iii}}\). To achieve fast switching rates, modern liquid crystal displays are often made thinner than the Mauguin limit. The display being used in this project has a thickness of 4.3\,\mu m, with \(n_e\) of 1.5946 and \(n_o\) of 1.4876. \(\Delta n d = 460.1\,\text{nm}\). This does not fall within the Mauguin criteria. This means that linearly polarised light becomes elliptically polarised on propagating through the (unswitched) structure. A description of this, based around Jones calculus, is found in \([\text{Yeh & Gu, 1999, pp.122-123 and 163-165}]\).

Gooch & Tarry \([\text{Gooch & Tarry, 1975}]\) calculated the transmitted intensity, \(T(u)\) as a function of the Mauguin parameter \(u\), where

\[ u = \frac{\pi d \Delta n}{\theta \lambda} \]

Here, \(\theta\) is the twist angle of the twisted nematic cell in radians \((\theta \leq \pi/2)\). Calculations were based on a TNLC cell placed between parallel polarisers (i.e. both polarisers oriented parallel to the input director). The intensity of light transmitted by a polariser which is aligned perpendicular to the director at the exit surface of the cell (i.e. parallel to the input director) becomes:

\[ T = \frac{\sin^2 \left[ \theta \left( \sqrt{1 + u^2} \right) \right]}{1 + u^2} \]

It can be seen from equation 3-5 that the transmitted intensity varies with \(u\), which is a function of both wavelength and cell thickness. A plot of equation 3-5, for several twist angles (wavelength 633nm) and for a twist angle of 90° for wavelength 555nm (the peak human eye sensitivity) is shown in Figure 3-8.

\(^{\text{iii}}\) Switching on time, \(T_{\text{on}} \propto \frac{\gamma_i d^2}{\varepsilon_0 \Delta \varepsilon (U_{\text{on}}^2 - U_{\text{thresh}}^2)}\) and switching off time, \(T_{\text{off}} \propto \frac{\gamma_i d^2}{\varepsilon_0 \Delta \varepsilon U_{\text{thresh}}^2}\), where \(\gamma_i\) is the rotational viscosity, \(U_{\text{on}}\) is the operating voltage, and \(U_{\text{thresh}}\) is the Freedericksz threshold voltage. \([\text{Tarumi et al., 1992}]\).
Calculated transmission of a twisted nematic structure as a function of cell thickness, d

Figure 3-8 Calculated transmission of a twisted nematic structure as a function of cell thickness. From [Gooch & Tarry, 1975].

The minima in Figure 3-8 indicate when the transmission through an analyser placed parallel to the input director is lowest, and for these minima the polarisation state of light (at that particular wavelength) is not changed by the twisted structure [Demus et al., 1998, p.201], it is purely rotated. These values of d are the same as those for which the cell transmits maximum intensity when the analyser is placed perpendicular to the input director (normally white mode). For large values of d it can be seen that the fluctuations in intensity in Figure 3-8 reduce, and this is when the cell is operating in the Mauguin limit. In practice, modern displays operate under a compromise of conditions depending on the desired response time of the cell, the wavelength(s) being used and the contrast desired from the display (which may be different for different wavelengths, as the human eye is not equally sensitive to all wavelengths). Similar calculations for twist angles of over 90° were published in 1988 [Raynes, 1988].

The above discussion indicates how important it is that the thickness of a liquid crystal display is accurately controlled. For the HeNe wavelength of 632.8nm and the LCTV display used in this work \( u=1.4537 \). The first minimum in the Gooch & Tarry curve above occurs...
when \( u = \sqrt{3} \) (=1.732). To achieve this first minimum (or maximum when the analyser is perpendicular to the input director), \( d \) would have to be 5.123\( \mu \)m (see Figure 3-8). This is considerably thicker than the 4.3\( \mu \)m of the actual display, so for a wavelength of 632.8nm linearly polarised light will emerge elliptically polarised.

### 3.4.5 Optical modelling of TNLCDs

Even if the TNLCD is made thick enough so that, in its off state, it merely rotates the plane of linearly polarised light, this will not happen when the display is partially switched. The transmission of light by a TNLCD has been modelled by different researchers in different ways [Ong, 1988]. In this thesis the TNLCD was modelled as a Jones matrix. This was computed by previous researchers at UCL [Kilpatrick et al., 1998] by considering the display as consisting of a series of thin layers (as in [Yoshida & Kelly, 1997]). The director orientation was assumed to be constant for each layer, and it was assumed that each layer was oriented at a known angle to the adjacent layers. The director profile of each layer was specified by the manufacturer. The display was divided into 50 layers of equal thickness. By using a formula similar to equation 2.23 and summing the effect of each of the layers, a Jones matrix for the whole structure was obtained. The Jones matrix could be recalculated to vary incident wavelength, liquid crystal thickness and refractive indices as required.

### 3.4.6 Addressing of liquid crystal displays

A liquid crystal responds to the root-mean-square (r.m.s.) voltage across each pixel [Yeh & Gu ,1999, p.249]. r.m.s. behaviour is obtained providing that the response time of the LC is many times larger than the period of the driving waveform [Demus et al., 1998, p.205].

#### 3.4.6.a Direct addressing

This occurs if each pixel on the LCD is addressed by an individual connection. It is used for low information content displays. The single TNLC cell used in this thesis is directly addressed.

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\( \text{\textsuperscript{iv}} \) From equation 3-5, intensity zero and maximum values of \( T \) are given by \( \theta \sqrt{(1+u^2)} = m \frac{\pi}{2} \). The maximum transmission (through parallel polarisers) is given when \( m \) is 0 or even, and minimum transmission is when \( m \) is odd. If \( \theta \) is 90\( ^\circ \), the minima occur when \( u = \sqrt{3}, \sqrt{15}, \sqrt{35} \) etc.
3.4.6.b Multiplexing

If the pixels of a standard VGA display (480 x 640 pixels), as used in this project were each individually addressed the number of connections required would be 307201 (i.e. number of pixels + 1). To reduce this number of connections displays of this nature are generally multiplexed. A description of how multiplexing affects the performance characteristics of a LCD is given in Appendix 3A.

3.4.6.c Active matrix operation

The TNLCD is not bistable, therefore as soon as the driving electric field is switched off, or reduced, the structure will relax to its off state. The display is addressed in rows, so by the time the whole display has been addressed, the voltage required to switch the beginning of the display on will have reduced. If there are $N$ rows, and the frame time is $T$, then the voltage pulse is only applied to each row for a duration of $T/N$. By the time the display is refreshed, the LC will have begun to relax to its off state.

This difficulty is overcome by actively addressing the pixels using transistors or diodes. An active device (transistor or diode) is situated in each pixel (i.e. at the crossovers between each row and column). When the pixel is selected, the active device switches on to charge the LC capacitor. The active device then switches off, so that LC remains in its charged state until the next frame. Each row is selected only for a time $T/N$ but the diode or transistor keeps the LC pixel in its addressed state until the next pulse of voltage is delivered. Each pixel is isolated from other pixels by the active devices and so the voltages remain constant while other pixels are being electrically addressed. The display used in this project uses diodes to address the pixels. A discussion of active matrix operation can be found in LCD textbooks (eg. [Yeh & Gu, 1999, p.252 et seq.]).

3.4.6.d Field and row inversion addressing

Addressing a TNLCD with direct current induces permanent dipoles on the molecules, which degrades the performance of the LCD. Using alternating current (a.c.) to drive the display eliminates this. The simplest way to ensure that the LCD is addressed with a.c. is to invert the polarity of the LC drive signal every time a pixel is addressed. This can be achieved by inverting the drive signal polarity once every field, and this is known as field inversion. However it is very difficult to ensure complete symmetry in the drive voltages of opposite polarities. This can lead to a 25Hz flicker being seen in the image (if the display is addressed at 50Hz). The flicker can be prevented by ensuring that the drive signal has an opposite phase.
on alternating rows of the display [Powell et al., 1988]. The eye only perceives the combined effect of flicker on several rows, and because the flicker on alternate rows has opposite polarities the effect cancels out [Knapp, 1999]. The drive signal for field inversion, single and double row inversion is shown in Figure 3-9.

```

Field inversion

Pixel polarity in odd fields

Pixel polarity in even fields

+ + + + +
+ + + + +
+ + + + +

- - - - -
- - - - -
- - - - -

Single row inversion

Pixel polarity in odd fields

Pixel polarity in even fields

+ + + + +
- - - - -
+ + + + +

- - - - -
- + + + +
- + + + +

Double row inversion

Pixel polarity in odd fields

Pixel polarity in even fields

+ + + + +
- - - - -
+ + + + +

- - - - -
- + + + +
- + + + +
```

Figure 3-9 Methods of a.c. addressing of LCTVDs. Addressing rate 50Hz, 625 rows [Edwards, 1998].

The TNLCD used in this thesis has a switch which enables it to be addressed either using field inversion, or single row inversion. It was always used in single row inversion mode.

### 3.5 Comparison of TNLCD performance with that of a parallel aligned nematic device

The LC units discussed in this thesis are twisted nematic in design, because these are widely available for use in displays. However, the principles discussed in both the Stokes polarimeter (Part II), and the security mark reader (Part III) could be equally applied to a parallel aligned nematic LC device. This section will discuss the difference between the two types of LC arrangement. Both of the applications discussed in this thesis work by detecting a change of light intensity (transmitted by an analyser) produced by a change in voltage applied to the LC. It will be shown that this does not always occur when a PALC cell is used, and so the TNLC configuration is more suitable for these purposes.
3.5.1 Description and uses of parallel aligned LC spatial light modulator

The difference between a parallel aligned (PA) LC and a TNLC cell is that in a PALC cell the optic axis does not rotate throughout the cell. Therefore, the LC has homogenous alignment when the cell is unswitched [Collings & Hird, 1998, p.181], as shown in Figure 3-10.

When the cell is switched, the director re-orientates itself in response to the applied field, as does the director in the TNLC. The effect of this is that the optic axis rotates out of the plane of the cell (Figure 3-11). This means that the effective birefringence \( \Delta n = n_e - n_o \) changes with voltage, but the projection of the optic axis on the x-y plane (which is the plane of the incident E field vibrations) does not.

The fact that the director does not rotate in the x-y plane causes some fundamental differences between the optical properties of the PALC and TNLC cells. These differences will be outlined in this section. For simplicity, the following calculations assume that when voltage is applied to the PALC cell, all the molecules reorient along the electric field. However, this is not actually the case in practice, as for a PALC cell of thickness 8.5\( \mu \)m even with a peak to peak voltage of 20V some LC layers of molecules close to the electrodes remain in their initial position (as also happens with a TNLC cell) [Pain et al., 1997].
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3.5.2 When the transmitted intensity is independent of LC voltage

The fact that the optic axis does not rotate throughout the plane of a PALC cell means that the plane of polarisation of the incident light is oriented at a fixed angle to the optic axis of the LC layer. Therefore, if light is linearly polarised along the optic axis for one voltage of the PALC cell, it will also be linearly polarised along the optic axis for all other voltages of the cell. Both the Stokes polarimeter, and the security mark reader rely on detecting a change in intensity transmitted by an analyser caused by a change in LC voltage. This will only happen if the polarisation is changed by the LC cell (or LCTVD), and (from the definition of optic axis,) will not happen if the incident light is polarised parallel or perpendicular to the optic axis (s.2.4.2). The rotation of the optic axis in a TNLC cell means that phase only modulation (i.e. no intensity change with voltage) cannot be achieved using linearly polarised light [Moreno et al. 1998]. Careful selection of the polariser and analyser orientations can minimise the change in intensity transmitted with TNLC cell voltage, but cannot eliminate it. With a LCTVD similar to the one used in this thesis, a minimum contrast ratio of 1:1.32 over the voltage range was found using linearly polarised light [Gardner et al. 1999]. This can be reduced to about 15% across the voltage range if elliptically polarised light is used for input and output states [Moreno et al., 1998] \(^*\). This is because the polarisation eigenstates for a twisted nematic display are elliptically polarised \(^vi\) [Pezzaniti & Chipman, 1993]. The major axes of these eigenstates are aligned along the director axes at the incident surface [Moreno et al., 1998], and rotate through the twist angle as they propagate through the TNLCD. The ellipticity of the eigenvectors changes with the voltage of the display [Davis et al., 1998; Moreno et al., 1998]. Therefore, even if the input polarisation is an eigenpolarisation for the LCTVD at one voltage, it will not be so for another voltage. This is why the intensity variation with voltage cannot be eliminated completely for a TNLCD.

3.5.3 Orientation of PALC cell and analyser

The effect of a PALC cell upon polarised light is equivalent to that of a birefringent layer. This can be represented by a Jones matrix [Davis et al., 1999], as given in equation 2-23, which is reproduced below:

\(^*\) The elliptical input was generated using a linear polariser and QWP and the output light was passed through a second QWP and analyser before being detected.

\(^vi\) The eigenvectors become more linearly polarised as \(\Delta n d/\lambda\) increases, [Davis et al., 1998], so may be linearly polarised for older LCDs which were thicker than modern ones. If the display fulfils the Mauguin limit its eigenpolarisations are linear.
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\[ M_\theta = \begin{bmatrix} \cos^2 \theta \exp(i\frac{\delta}{2}) + \sin^2 \theta \exp(-i\frac{\delta}{2}) & i \sin(\frac{\delta}{2}) \sin 2\theta \\ i \sin(\frac{\delta}{2}) \sin 2\theta & \cos^2 \theta \exp(-i\frac{\delta}{2}) + \sin^2 \theta \exp(i\frac{\delta}{2}) \end{bmatrix} \]  

equation 2-23

If the optic axis of the PALC cell is aligned with the x-axis (i.e. \( \theta = 0^\circ \) in equation 2-23) then \( M_\theta \) becomes

\[ M_\theta = \begin{bmatrix} \exp(i\frac{\delta}{2}) & 0 \\ 0 & \exp(-i\frac{\delta}{2}) \end{bmatrix}. \]  

Multiplying this by the input Jones vector gives:

\[ M_\theta = \begin{bmatrix} \exp(i\frac{\delta}{2}) & 0 \\ 0 & \exp(-i\frac{\delta}{2}) \end{bmatrix} \begin{bmatrix} J_1 + iJ_2 \\ J_1 + iJ_4 \end{bmatrix} = \begin{bmatrix} J_1 \cos \frac{\delta}{2} - J_2 \sin \frac{\delta}{2} + i(J_1 \sin \frac{\delta}{2} + J_2 \cos \frac{\delta}{2}) \\ \text{immaterial} \end{bmatrix} \]  

equation 3-6

If the intensity, \( I_o \), transmitted by an analyser placed along the x-axis is required, the y component of the Jones vector on the right hand of equation 3-6 can be ignored. \( I_x \) is equal to the x Jones component multiplied by its complex conjugate. i.e.:

\[ I_x = J_1^2 \cos^2 \frac{\delta}{2} + J_2^2 \sin^2 \frac{\delta}{2} - 2J_1J_2 \cos \frac{\delta}{2} \sin \frac{\delta}{2} + J_1^2 \sin^2 \frac{\delta}{2} + J_2^2 \cos^2 \frac{\delta}{2} + 2J_1J_2 \sin \frac{\delta}{2} \cos \frac{\delta}{2} \]

So, \[ I_x = J_1^2 \left( \cos^2 \frac{\delta}{2} + \sin^2 \frac{\delta}{2} \right) + J_2^2 \left( \sin^2 \frac{\delta}{2} + \cos^2 \frac{\delta}{2} \right) = J_1^2 + J_2^2 \]

Therefore, if the optic axis of the PALC is aligned with the x-axis, the intensity transmitted by an analyser aligned with the x axis is only dependent upon the incident polarisation. The intensity of light transmitted does not depend upon the voltage applied to the PALC cell (which changes \( \delta \)).

Similarly, if the optic axis and analyser are rotated, but remain parallel or perpendicular to each other, the intensity transmitted by the analyser will not change with applied PALC voltage. This lack of intensity variation is why PALC SLMs are used for phase only modulation.

Therefore, if either the input polarisation is along (or perpendicular to) the optic axis of the PALC cell, or the analyser and optic axis are parallel (or perpendicular) to each other there will be no intensity variation with voltage using a PALC cell. If either of these situations occurs a PALC cell would not be suitable for the applications proposed in this thesis (although the latter situation can be avoided by careful experimental design).
3.5.4 Contrast performance of parallel aligned NLC cell compared with TNLCTVD

As can be expected from the preceding section, the change in intensity transmitted through an analyser with voltage applied to a PALC cell will depend strongly upon the polarisation of the incident light. Figure 3-12 shows how the theoretical intensity transmitted by an analyser placed along the x-axis will change with director orientation (tilt) for six different input polarisations. It assumes a PALC cell of the same birefringence and thickness as the commercially produced TNLCTVD used in this thesis, with a brushing direction of 45° (i.e. similar to the TNLCTVD used).

![Director tilt/intensity curve for PALC cell.](image)

It can be seen that the closer the input polarisation is to the brushing direction (optic axis), the less the intensity changes.

A similar set of curves to Figure 3-12 for the TNLCTVD is shown in Figure 3-13 (although the x-axis is voltage rather than director tilt).
Figure 3-13 Voltage/intensity curves for LCTVD

It can be seen that although there is less intensity variation with voltage for some polarisations than there is with the PALC, there is no incident polarisation at which the intensity transmitted does not change at all with LC voltage (as discussed in s.3.5.2).

The fact that, for a TNLCD, there is no polarisation for which there is no intensity variation with voltage is an advantage for the applications proposed in this thesis, because it is this change in intensity with LC voltage which allows the input polarisation to be determined, or the security mark to be distinguished. This is further discussed in s.10.2.
Chapter 4

**Literature survey and applications for a polarisation filter**

4.1 Introduction

Unlike some animals [Rowe et al., 1995; Shashar et al., 1996], human eyes are generally insensitive to polarisation. However knowledge of the polarisation of light can give us additional information about optical scenes. Light may be partially polarised by scattering, so that measurement of polarisation can supply information about the scattering medium. The state of polarisation can also be changed on reflection, (s.2.4.1b) so knowledge of the state of polarisation can lead to extra information being gleaned about the refractive index of a reflector, or the angle of incidence. In the inverse problem of identifying targets using radar, the use of polarisation information makes a system more reliable [Baltes, 1980].

The theory of polarisation and its measurement has been covered in Chapter 2 (s.2.5). This part of the thesis (Chapters 4-6) will describe how a commercially produced LCTVD can be used as part of a device to measure polarisation. Chapter 5 will describe how the polarisation change produced by the LCTVD was characterised, and Chapter 6 will detail how this is used to measure unknown input polarisation.

---

1 Some individuals are sensitive to the Haidinger brush phenomenon, where 'brushes' can be seen if a rotating polariser is viewed under the appropriate conditions.
This chapter will discuss the work that has been done by other researchers on measuring polarisation, and some applications of these polarimeters. It will begin by outlining the arrangement of the LCTVD polarimeter that is described in Chapter 6 so that this can be compared to the approaches used by other researchers. This device can measure the state of polarisation without any mechanical parts, which is an advantage because systems with moving optical parts are slow, and show image alignment and quality differences as they are tuned [November & Wilkins, 1995]. There may also be registration problems [Wolff, 1997].

The chapter is arranged so that the discussion starts with partial polarimeters (i.e. those which only measure the linearly polarised component of light) and follows with complete polarimeters, which enable the full polarisation state to be measured. Applications for a polarisation-measuring device will then be covered. The chapter will conclude with a discussion of the use of a single LCTVD as a polarisation filter, and its advantages over other methods.

4.2 Polarimeter proposed in this thesis using a commercially produced LCTVD, or a single pixel TNLC cell

It will be shown in Chapter 6 that the complete state of polarisation of an incident beam of light can be determined by passing the beam through the LCTVD and a fixed analyser to a detector. Four separate voltages are applied to the LCTVD and the intensity transmitted by the analyser is measured for each voltage. From these intensities, the four Stokes vectors of the incident light can be calculated. The layout of the polarimeter is as shown in Figure 4-1.
4.3 Partial polarimeters

4.3.1 Non liquid crystal polarimeters

Hauge [1980] gives a description of various types of partial and full Stokes polarimeters. Polarimeters can be classified depending on whether or not they have rotating components.

4.3.1.a With rotating components

The simplest method to measure the degree of partial polarisation is to rotate an analyser in front of a detector and measure the intensity at three or more points. This has been discussed in detail in Chapter 2 (s.2.5.2), and will not be considered further.

4.3.1.b Without rotating components

Several methods have been developed to avoid the necessity of having to rotate an analyser in the path of the beam. These include splitting the incident beam into four, using two Wollaston prisms [Geyer et al., 1996]. This splits the beam into E vector orientations of 0°, 45°, 90° and 135° from which the first three Stokes components can be found.

4.3.2 Liquid crystal polarimeters: Liquid crystal polarisation camera

Using two single liquid crystal cells in front of a CCD camera, Wolff & Mancini, [1992] have developed a camera that can measure degrees of linear polarisation without any moving parts. The LC polarisation camera consists of two LC cells and a fixed analyser in front of a conventional CCD camera [Wolff et al., 1997].
Chapter 4 Literature survey and applications for a polarisation filter.

The LCs are twisted nematic, and are made so that one of them has a twist of 45°, and the other of 90°. Both have a switching time of 16ms [Shashar et al., 1994]. When unswitched the LC cells rotate the plane of polarised light by 45° and 90° respectively, and by 0° when an a.c. voltage is applied across them. The intensity of each component of the light (at 0°, 45° and 90°) is sampled for each pixel of the CCD camera. This information is then converted into a visual display of the polarisation at each pixel. A fast switching liquid crystal cell was necessary, as the measurements are taken in sequence. The azimuth angle, intensity and partial polarisation at each pixel are computed from the equations given in s.2.6.2. To increase the frequency at which polarisation images can be acquired, an alternative design has been suggested which uses a polarising beamsplitter between a single TNLC cell (45° twist) and two CCD cameras [Wolff, 1994; Wolff & Andreou, 1995]. This has the advantage that only two, rather than three, sets of polarisation images must be taken in sequence, but has the disadvantage that the CCD cameras must be accurately aligned to ensure pixel registration. In order to maintain this registration in a mobile unit, the apparatus would have to be firmly anchored to the housing. The polarising beam splitter negates the need for a linear polariser in front of the CCD cameras.

In addition to having no moving parts, the LC method can switch far more quickly than it is possible to rotate a polariser. The technique also removes the large errors which can occur in polarisation measurement when using a mechanically rotated polariser at portions of scenes where there is a significant local variation in intensity due to shading or edges [Wolff, 1997].

The LCTVD used in this thesis does not rotate the plane of polarisation by as much as 90°. This is because the LCTVD is too thin to fulfil the Mauguin limit (s.3.4.4). Therefore, the LCTVD does not purely rotate the plane of polarisation - it also induces a phase difference.
between the orthogonal components of the electric field. Consequently a method similar to Wolff's cannot be utilised using the LCTVD. A method of calculating input polarisation from intensity measurements taken through a fixed analyser with the LCTVD at four voltages is presented in Chapter 6.

The use of the phase retarding property of the LCTVD enables the fourth Stokes vector to be measured, which cannot be done with Wolff's unmodified LC polarisation camera. A modification of the LC polarisation camera has been reported [Chen & Wolff, 1998] whereby a quarter wave plate (QWP) is inserted in front of the 2 LC cells. This enables the phase of the polarised light to be calculated (provided the incident polarisation is not parallel or perpendicular to the plane of incidence). This involves an additional step (i.e. repeating the measurements with and without the QWP in place). This QWP modification was not used for most applications because the applications for which the camera was used rarely required the fourth Stokes parameter to be measured [Wolff, 1997]. The security marks discussed in Part III of this thesis produce a pattern of polarisation which is not necessarily linearly polarised. For accurate authentication of the security marks, therefore, a reader which can distinguish partially polarised from elliptically polarised light is required.

4.4 Full polarimeters

Generally, in order for the fourth Stokes parameter to be measured, some phase retarding element must be incorporated into the polarimeter. One exception to this appears to be the method proposed by Gori [1999] which is based on diffraction and will be discussed in s.4.4.1.a and 4.4.3.

4.4.1 Non liquid crystal based polarimeters

4.4.1.a With rotating elements

Aspnes [1976] describes a polarimeter that uses a rotating analyser and compensator, with the compensator rotating at 1/3 the speed of the analyser. Alternatively, a rotating QWP, followed by a fixed polariser can be used [Berry et al., 1977]. A novel method using a single circular polariser is described by Collett [1984]: the intensity is measured through a circular polariser (placed at 0°, 45° and 90°), and then again with it flipped over and placed at any angle of rotation. This method is elegant in its simplicity, but still suffers from the problems associated with moving optical elements.

An interesting theoretical method uses a polarisation grating (i.e. a component made of strips of a linear polariser so that the polarisation orientation varies periodically) [Gori,
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1999]. Such a polarisation grating could be constructed using a LCD. When light is passed through this grating it is found that the polarisation of the diffraction pattern produced varies depending on which order is measured. One half of the impinging field emerges with its state of polarisation preserved (the central order), but the orders $1$ and $-1$ are spatially modulated to produce left and right circularly polarised beams respectively. The amplitudes of the $R$ and $L$ circularly polarised waves depend on the polarisation state of the incident field. If the light emergent from the grating is then passed through a linear polariser oriented at first $0^\circ$ and then $45^\circ$, the four Stokes parameters can be calculated from the intensities of the central order and first (positive and negative) orders. In practice, this must be done in the far field [Someda, 1999]. An advantage of this method is that because it contains no retarding elements it is wavelength independent, (but the position of the diffracted orders would vary with wavelength – this can be overcome by using suitable extended detectors [Gori, 1999]). The method assumes that the orientation of the polarising strips that constitute the grading varies linearly with direction along the $x$ axis, and in practice this may be difficult to achieve. No experimental results for this method had been published by spring 2001.

4.4.1.b Without rotating elements

The necessity for rotating parts can be avoided if electro-optic rotators are used. Azzam [1978] describes a polarimeter which uses a QWP surrounded by two Faraday cells which give equal and opposite rotation. This has the same effect as rotating the QWP in front of a fixed analyser. Alternatively two piezooptical birefringence modulators can be used as the birefringent layers in front of the fixed analyser [Boyer et al., 1979]. These are more stable to frequency variation than optical retardation plates. The analyser can be dispensed with if the beam is split into two parts by a polarising beamsplitter [Itoh et al., 1986].

Another option is to use beamsplitters and Wollaston prisms to divide the beam into four separate beams. The intensity of these beams can be used to determine the four Stokes parameters [Azzam, 1982; Nikolova et al., 1992]. This has the advantage of simultaneous, rather than sequential, measurement of the polarisation. All these methods need far more complex apparatus than the polarimeter described in Chapter 6.

4.4.2 Liquid crystal polarimeters

4.4.2.a With rotating components

The main use of LC components is to rotate the plane of polarisation, so there are few LC polarimeters that need rotating components in addition to the LC. A polarimeter that uses
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a liquid crystal variable retarder to measure two of the second three Stokes parameters simultaneously is reported by Furst et al. [1996]. This is a hybrid between a full and partial polarimeter: the state of linear polarisation can be measured without any rotating components, but if the analyser is rotated the final Stokes parameter can be found.

4.4.2.b Without rotating components

November & Wilkins [1995] used a combination of nematic LCs, as variable retarders, and ferroelectric LCs, as polarisation rotators, in their polarimeter. (An achromatic design can be obtained using three NLCs at 0°, 45° and 90°, followed by a ferroelectric LC at any orientation.) Schirmer et al. [1998] used a series of NLC cells in sequence. In a similar way to the liquid crystal polarisation camera discussed above (s.4.3.2.a), these cells are switched sequentially, and the intensity of light transmitted by a fixed analyser is recorded. In both of these designs, the NLC cells are parallel aligned, not twisted, and are therefore not commercially mass produced, as is the LCTVD used in this thesis.

It should also be pointed out that the greater the number of LC cells used in sequence, the greater the loss of light from reflections from the LC cell surface. The ITO coating of each LC cell yields a loss of about 10% (of the amplitude) [Schirmer et al., 1998]. The method presented in this thesis only has one LC element, so if the single LC cell is used (as opposed to the LCTVD – see Chapter 6), less light is lost through reflections than if several LC elements were used.

4.4.3 Wavelength dependence

All polarimeters that use a birefringent component are liable to be affected by wavelength fluctuations. This is because refractive index varies with wavelength [Born & Wolf, 1999, p.95], and the resultant polarisation depends upon the parameter $\Delta n d/\lambda$. This affects the accuracy of the results because the polarimeter is calibrated for a retardance at one wavelength, but the actual retardance at the different wavelength will be different. This will affect the measurement of the fourth Stokes parameter. One way to address this is to calibrate the polarimeter by measuring the intensity transmitted through a QWP, and then repeat this with the QWP rotated through 90° [Kihara, 1994]. The intensity transmitted through a QWP oriented at angle $\beta$ and a polariser (analyser) at angle $\theta$ depends upon the incident polarisation, the angles $\beta$ and $\theta$, and the retardance of a retarder, $\Delta$ [Clarke & Grainger, 1971, p.124]. The equation for the intensity transmitted is:
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\[ I = 0.5 \left\{ S_0 + S_1 \left[ \cos 2\beta \cos 2(\theta - \beta) + \cos \Delta \sin 2(\theta - \beta) \right] + \right. \\
\left. S_1 \left[ \sin 2\beta \cos 2(\theta - \beta) + \cos \Delta \cos 2\beta \sin 2(\theta - \beta) \right] + S_1 \sin \Delta \sin 2(\theta - \beta) \right\} \]

Equation 4-1

\( \Delta \) is defined as \((\phi_n - \phi_p)\) [Clarke & Grainger, 1971, p.105]. To analyse the effect of residual retardance on the transmitted intensity, the retarder is defined as a QWP, with a residual retardance, \( \rho \), so that \( \Delta = (90^\circ + \rho) \). The trigonometric identities enable the equation quoted by Kihara to be derived:

\[ I = 0.5 \left\{ S_0 + S_1 \left[ \cos 2\beta \cos 2(\theta - \beta) + \sin \rho \sin 2\beta \sin 2(\theta - \beta) \right] + \right. \\
\left. S_1 \left[ \sin 2\beta \cos 2(\theta - \beta) - \sin \rho \cos 2\beta \sin 2(\theta - \beta) \right] + S_1 \cos \rho \sin 2(\theta - \beta) \right\} \]

Equation 4-2 [Kihara, 1994]

When \( S_j \) is measured in the conventional manner, by positioning the QWP at 0° and taking intensity measurements with the analyser at 45° and 135°, the value obtained for \( S_j \) depends upon the value of \( S_j \) and \( \rho \). However, if the QWP is rotated to 45° and -45° and the analyser is kept at 0° this dependence upon \( S_j \) can be removed. This occurs because, (where the first orientation represents \( \beta \) and the second represents \( \theta \), i.e. \( I(45,0) = \text{QWP at 45°, analyser at 0°} \)) if \( I(45,0) \) is subtracted from \( I(-45,0) \): \( S_j = \frac{I(-45,0) - I(45,0)}{\cos \rho} \).

The value for \( \rho \) can also be found if \( S_j \) is non zero. This method is shown to be more accurate than the conventional circular polariser method, but an accurate value for \( S_j \) cannot be obtained when there is no polarised component along the x axis [Kihara, 1994].

An alternative method is to try and reduce dispersion by using more than one retarding layer: Schirmer’s achromatic polarimeter uses five LC cells. The LC materials are chosen to have differing dispersions. This gave reliable results for the wavelength region 500-800nm. The effect of a polychromatic source upon the accuracy of the LCTV polarimeter is discussed in s.6.5.2.

4.5 Benefits of the polarimeter proposed in this thesis

The polarimeter proposed in this thesis is considerably simpler than the full polarimeters described above. It only has one LC element, which reduces the reflections due to multiple LC layers, and, being based on a commercially produced device, can be produced relatively cheaply. The pixellation of the display could lead to the spatial alteration of

\[ ^{ii} \text{This is instead of measuring the intensity transmitted through the conventional circular polariser.} \]
polariation, after the input polarisation state had been determined using the LCTVD polarimeter. This could be used, for example, in highlight reduction (see s. 4.6.2.a below). The LCTVD polarimeter can be very simply calibrated before each use, to reduce the effect of ambient temperature changes as well as wavelength effects. It also has a much wider field of view (in the region of +/-30° - see s.6.5.3) than some other polarimeters, such as the Advanced Stokes Polarimeter [Elmore et al., 1992], which is limited to about 1°. The LCTVD polarimeter relies on sequential addressing of four voltages onto the LCTVD, so cannot operate in real time, unlike the polarimeters which split the beam into separate components. The switching time of the LC depends upon several factors such as temperature, start voltage and finish voltage, but the materials used in commercial displays commonly have switching times in the region of 15-30ms [Pearson, 2001]. The relaxation time of the LC is slower, in the region of 50 ms. Assuming the voltage applied to the LC is increased (so only the switching time is required), a set of four readings can be taken in around 0.1s. The LC must then have time to relax to its unswitched state before the next set of readings was made. It should be possible to take six sets of four readings per second, which is suitable for slowly changing applications. The applications of such a polarisation filter will now be discussed.

4.6 Applications:

The applications that will be covered here are:

i) Scene analysis
ii) Reducing spot reflections/false edges
iii) Enhancing underwater visibility
iv) Medical imaging

4.6.1 Scene analysis

Many methods, collectively known as shape from X, have been proposed for reconstructing a scene. These include shape from shading and texture. Detail of these techniques can be found in [Trucco & Verri, 1998]. Knowledge of the polarisation of light can also be used in scene analysis. This is because the polarisation of light reflected from a surface depends upon the refractive index, surface roughness and orientation of the surface.

4.6.1.a Introduction: specular and diffuse reflection

Specular and diffuse reflections behave differently: specular reflection partially polarises light more than diffuse reflection, and the colour behaviour of the two types of
reflection is also different. The degree of partial polarisation caused by specular reflection is dependent on the angles of incidence and reflection, and the refractive index (real and complex) of the material. If the degree of initial polarisation and angles of incidence and reflection are known, measuring the polarisation of the reflected light will enable a measure of the refractive index of the reflecting surface to be calculated. This can be useful in distinguishing military targets, and for examining printed circuit boards, where the metallic components can be distinguished [Wolff et al., 1997]. A metal coated with translucent dielectric (such as on a circuit board) gives specularly reflected light which is a mixture of strongly partially polarised light from the dielectric and less strongly partially polarised light from the metal.

4.6.1.b Methods of analysing reflected light

Several methods of analysing the reflected light have been published: The difference in colour absorption between diffuse and specular reflection has been used to distinguish between the two types of reflections [Tominaga, 1991]. This method was adequate for plastics, paints, ceramics, vinyls, tiles, fruits, leaves and woods, but had difficulty describing metals, cloths and papers. A polarisation based reflectance model using artificially produced circularly polarised light to look at glossy objects was published by Koshikawa [1979]. This method did not work well in real situations because it assumed that the reflection process is purely specular and was restricted to perfectly smooth dielectric materials.

Wolff ([Wolff, 1987; Wolff & Boult, 1991]) developed a polarisation reflectance model which he later called the Fresnel Reflectance Model. This expresses the total reflected polarisation state in terms of the sum of the polarisation states for the diffuse and specular components of reflection. The technique was developed from the Torrance and Sparrow reflection model [Torrance & Sparrow, 1967] which accurately predicted the reflectance properties of homogeneous isotropically rough surfaces for both dielectrics and conductors. The original Fresnel equations give the polarisation of the specular reflection, and Wolff's model predicts the polarisation of the diffuse component. A limitation of this method is that it uses initially unpolarised light whereas almost all naturally occurring light outdoors and underwater, scattered and reflected, as well as light in most indoor environments is partially linearly polarised ([Waterman, 1981], quoted in [Wolff, 1997]).

4.6.1.c Determining refractive index or conductivity

Using the Fresnel Reflectance model, refractive index or conductivity can be gauged, if the angle of incidence is between 30° and 80°. This is done by measuring the quantity
$I_{max}/I_{min}$ of the reflected light [Wolff, 1990]. This approximates the Fresnel ratio of $F_x/F_y$ and varies between 1 and 2 for most metals, whereas for dielectrics this ratio is above 3 [Wolff & Andreou, 1995] \(\text{iii}\). This method assumes that over each highlight, the diffuse reflected component is constant, and that the Fresnel ratio is constant. The Fresnel ratio is sensitive to angle of incidence, and therefore except for very smooth surfaces, highlights will include points with different surface normals, and therefore different incident angles [Nayar et al., 1997]. A solution is to use colour and polarisation simultaneously [Nayar et al., 1997], where the specular colour of each image point is computed independently. However, this model assumes that the specular component is polarised, whereas the diffuse component is not, which for large angles of incidence is not accurate.

If the incident light is fully or partially linearly polarised, a phase-based method can be used [Chen & Wolff, 1998]. This utilises the fact that linearly polarised light becomes elliptically polarised upon reflection from a metal, and the orientation and the ellipticity of the ellipse depends on the phase shift between the parallel and perpendicular components of the electric fields [Clark et al., 1997]. However, for a dielectric the phase change is either $\pi$ (for incident angles up to Brewster’s angle) or zero degrees, for angles over Brewster’s angle [Hecht, 1998, p.116]. Linearly polarised light therefore remains linearly polarised. This method complements the Fresnel reflectance model, which is used when the incident light is unpolarised. It avoids the necessity of having to measure the full Stokes vector by merely detecting a change in the azimuth angle, $\alpha$, of reflected light. If the reflected light is (partially or fully) linearly polarised, the azimuth angle is unchanged, but if it is elliptically polarised, $\alpha$ will change, for any specular angle of incidence. $\alpha$ is measured using the liquid crystal polarisation camera described in s.4.3.2.a. Further detail of these methods is given in Appendix 4A (s.4A.1)

Measurement of the full Stokes parameters of reflected (linearly polarised) light can be used to aid material classification. This is because reflection from a metal leads to a higher value of $S_3$ than does reflection from a dielectric. In addition, the higher amount of off-specular reflection from a dielectric leads to a lower degree of polarisation, $P$ for light reflected from a dielectric compared to light reflected from a metal [Liang et al., 1999]. The effect of reflections on polarisation is documented in [Wallace et al., 1999] where the use of polarisation information to distinguish between first and higher order reflections is demonstrated.

\(\text{iii}\) For a dielectric, in cases where the specular angle of incidence is close to the Brewster angle, $I_{max}/I_{min}$ considerably underestimates the Fresnel ratio, but is still $>3$ and so is a reliable distinguisher of conductivity.
4.6.1.d Surface quality assessment

Polarisation information has been used to aid target recognition for military use (the antiballistic glass used on military vehicles has particular polarisation properties [Wolff, 1995]), as well as to identify scrape damage on vehicles [Wolff, 1997]. Diffuse reflection from paint will significantly depolarise incident linearly polarised light, while linear polarisation reflected from exposed metal will remain almost completely linearly polarised, especially at normal incidence.

4.6.1.e Gaining surface orientation information

Figure 4-3 shows a plot of the Fresnel ratio \( F_n / F \) versus incident angle for specular reflection.

\[
\begin{align*}
\text{Specular angle of incidence, } \theta &< 60^\circ \\
\text{(from Nayar et al., 1997)}
\end{align*}
\]

It can be seen from this that if any two of the three variables: the polarisation of the reflected beam, the refractive index and the angle of incidence are known, the third unknown can be calculated. Figure 4-3 shows that using the Fresnel ratio to gauge the angle of incidence leads to an ambiguity, because on either side of the Brewster angle there are two values of the Fresnel ratio for each angle of incidence. This can be resolved by assuming that the angle of incidence is <60° [Nayar et al., 1997]. There is also a problem in that, for near normal incidence (<20°) the Fresnel components, \( F_\perp \) and \( F_\parallel \) do not differ sufficiently.
An advantage of using a polarisation based method to determine surface orientation is that the Fresnel ratio can be measured without knowledge of the angle of incidence. Another advantage is that for object points from which there is combined diffuse and specular reflection, polarisation-based methods can determine surface orientation constraints independent of the nature of the diffuse reflection [Wolff & Boult, 1991]. This is important, as not all surfaces are perfect Lambertian diffusers. A disadvantage of polarisation based methods is the requirement for a significant specular component of reflection.

4.6.2 Image enhancement

4.6.2.1 Reducing unwanted reflections and highlights

Speculally reflected light may obscure detail in an object. If this is reduced, or eliminated, the detail will be more visible. Nayar et al. [1997] have shown that using colour and polarisation enables accurate highlight removal. Limitations of this model are given in Appendix 4A (s.4A.2).

A method of highlight removal involving two TNLC cells in front of a fixed polariser and camera has been reported by Fujikake et al. [1998a and 1998b]. The TNLC cells have twist angles of 45° and 90° and are switched independently to rotate the plane of polarisation in a similar way to the liquid crystal polarisation camera in s.4.3.2.a. above. Specular highlights were reduced by rotating the plane of polarisation as near as possible to be absorbed by the analyser. More than 80% of linearly polarised incident light was removable at incident angles from 35°-70°. However, as the plane of polarisation could only be rotated by 0°, 45°, 90° or 135°, it was not possible to remove all of the highlights. To reduce the highlights further, additional LC cells with smaller twist angles could be used.

The pixellated structure of the LCTVD used in this thesis should enable reflections to be reduced selectively, across the field of view, rather than changing the plane of polarisation across the whole field as reported by Fujikake. Fujikake's method does not apply varying voltage to the LC cells - they are either fully switched, or fully unswitched. It is hoped that by applying intermediate grey levels to the LCTVD, rather than just switching the LC on and off, finer control of the state of polarisation of the scene will be achieved, which will lead to the reflections being more effectively reduced. Use of the LCTVD as such a polarisation filter would also be less bulky, having just a single LC device in front of the camera.
4.6.2.b Scene edge detection

It is important to be able to distinguish a bright highlight from a scene discontinuity. Imaging systems, especially those operating in unstructured environments, need to be able to distinguish real edges from false ones caused by reflections. Consider a curved, shiny surface such as a mug. A bright highlight may be observed down the centre of the mug, and could be interpreted by a robot as an additional edge. Measuring the polarisation characteristics of the highlight can help to resolve this ambiguity, and also enable primary reflections to be distinguished from secondary ones [Nayar et al., 1995; Clark et al., 1997].

4.6.2.c Enhancing underwater visibility

In underwater situations, most light is partially polarised. This is partly due to reflection from the surface, and partly due to scattering by the water molecules, and particles suspended in the water. The orientation of polarisation changes according to the angle of the sun, and the nature of the reflecting surface. Most marine animals appear to reflect unpolarised light and therefore it is an advantage to be able to distinguish between the polarised background and unpolarised light reflected from the animal, as this increases the contrast. (This is in contrast to the situation above where the background is unpolarised, but the specular reflection from the object is partially polarised). A technique of polarisation-difference imaging has been developed to increase the contrast of objects viewed through scattering media [Tyo et al., 1996]. This has also been used to image subsurface structures of the human body [Demos & Alfano, 1997]. It was found that reflected light which was parallel to the input polarisation imaged surface structures, and reflected light which was perpendicular to the input polarisation, contained information about subsurface structures (as it was backscattered).

4.6.2.d Medical uses

Polarisation can be used in biomedical imaging [Demos et al., 1997]: one of the major obstacles in optical tomography is the multiple scattering which takes place during light propagation through the tissue. This degrades the image information. Polarisation may be applied in the imaging of biomedical media; for example polarised light propagating in normal human breast tissue depolarises less than in malignant breast tissue. The polarisation is preserved to different degrees depending on the type of tissue and the wavelength of the propagating light. It is possible that the polarisation filter described in this thesis may be used to produce medical pictures to visualise such abnormalities.
Chapter 4 Literature survey and applications for a polarisation filter.

Polarisation-sensitive optical coherence tomography has been used to measure thermal damage to histological samples of medical tissue [Schoenenberger et al., 1998]. Tissue has inhomogeneities, which cause linear and circular anisotropy. When the temperature of the tissue is raised above a melting threshold, the collagen molecules unfold, and some of the tissue birefringence is lost. It was found that tissue birefringence was a more sensitive indicator of tissue thermal damage than were changes in tissue scattering properties.

A longer discussion of the medical uses of polarised light is given in Appendix 4A (s.4A.3).
Chapter 5 Experimental work on the polarisation filter

5.1 Introduction and aims

In order to use the LCTVD as a polarisation filter it was necessary to fully characterise its polarisation modulation capabilities. As mentioned in Chapter 3 (s.3.4.5), a TNLCD can be modelled as a Jones matrix. A Jones matrix model of the commercially produced LCTVD used in this thesis was computed by researchers at UCL [Kilpatrick et al., 1998]. The Jones matrix model was constructed using theoretical predictions of the director profile (see s.3.4.5), ignoring temporal variations such as those caused by the switching behaviour of the LC. The aim of this chapter was to investigate the actual behaviour of the LCTVD, and determine whether a more accurate model could be determined based on experimental observations.

5.2 Design of experiment

5.2.1 Single pass

This experiment consisted of passing linearly polarised light through a LCTVD and analysing the polarisation of the output light for various grey levels (GLs) of the LCTVD. The experimental layout is shown in Figure 5-1.
Chapter 5 Experimental work on polarisation filter

5.2.2 Double pass

The experiment was repeated in double pass, with an aluminium mirror placed immediately behind the display. The details of this are covered in [Blakeney et al., 1999], a copy of which is attached at the end of this thesis.

5.3 Equipment used

5.3.1 Laser

The requirements of the laser were:

- stable polarisation
- stable intensity
- enough power for the beam to be expanded if necessary
- visible wavelength

The laser chosen was the class IIIb Ealing-Coherent 10mW HeNe laser. It has a polarisation ratio of 500:1, stated intensity variation of 3% after 20 minutes warm up, beam diameter quoted of 0.68mm, Gaussian profile, wavelength of 632.8nm.

The stability of the polarisation was checked over time and it was found that it required to be switched on for 90 minutes for the intensity through a polariser to stabilise. After 90 minutes the measured variation in the laser intensity through a polariser was 0.4%.
5.3.2 ND filter

The laser was chosen to be powerful enough for the beam to be expanded (if necessary) in future experiments. When it was initially used, as an unexpanded beam, it was necessary to reduce the power so that it did not damage the LCTVD. The maximum quoted power which should be incident on the LCTVD is 2.5W per device. This corresponds to 0.377mW over the beam diameter of 0.68mm.

The combination of polariser and 1.2 ND filter reduced the power incident on the LCTVD to 0.3155mW, which is less than the threshold of 377μW. A 1.2 ND filter was most suitable for the single pass experiment, as this gave an acceptable variation of readings on the detector.

5.3.3 Quarter wave plate

This was used to produce circularly (or nearly circular) polarised light so that when the polariser was rotated, some light would be transmitted at all polariser orientations. Perfect circularity was unobtainable in practice, so some degree of ellipticity was present. In practice, the polarisation of the light falling on the polariser had an azimuth angle of 47.5°, and ellipticity of 1.036. This was determined using readings taken with the analyser rotated between 0° and 180° (see s. 5.3.4. below).

5.3.4 Polariser

The polarisers chosen were the Coherent-Ealing HN32 precision sheet polarisers. These have a vernier scale allowing the scale to be read to 1/10 degree. The quoted transmission when crossed is 0.05% 1.

It was found that there were slight irregularities in the polarisers which caused the intensity transmitted when the analyser was at θ° to differ from that transmitted at θ+180°. This can be demonstrated from the readings obtained when calibrating the circularity of the light incident on the first polariser (no LC present) - Table 5-1. The maximum difference between intensity readings of θ° and θ+180° was 1.26%. The ‘quality’ of the fit is quantified using the correlation coefficient, R² (as in s.2.6.3.a).

---

1 For the intensities used in this work, the intensity transmitted between crossed polarisers was within the noise level of the detection system.
Table 5-1 Variation in values of α and e for ‘circularly’ polarised light falling on first polariser, depending on how many analyser positions were used to measure intensity (using linear regression method described in s.2.6.3)

<table>
<thead>
<tr>
<th>Range of analyser positions used (10° steps)</th>
<th>Measured α</th>
<th>Measured e</th>
<th>Correlation coefficient, $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°-360°</td>
<td>44°</td>
<td>1.035</td>
<td>0.8706</td>
</tr>
<tr>
<td>0°-180°</td>
<td>47.5°</td>
<td>1.035</td>
<td>0.971</td>
</tr>
<tr>
<td>180°-360°</td>
<td>40.5°</td>
<td>1.03</td>
<td>0.9672</td>
</tr>
</tbody>
</table>

5.3.5 LCTVD

The unit used was the Philips LDK036T-20 liquid crystal projection TV (as detailed by [Gardner et al., 1998]).

5.3.5.a Thickness

This has been discussed in s.3.4.4. The retardance of the display ($\Delta nd$) is 460.1 nm (physical thickness 4.3 µm).

5.3.5.b Pixel pitch

The pixel size is 83 x 69 µm, with a covering mask (pitch) of 88.5 µm square (Figure 5-2).

When illuminated by the laser beam the pixels of the display caused a diffraction pattern to be formed as shown below (Figure 5-3). This figure only shows the brightest part of the
diffraction pattern, because of the limited dynamic range of the CCD detector used to image the pattern. The pitch of the diffraction pattern was consistent with that of the pixels. The quoted brushing directions are $+/47^\circ$, and the pre-tilt is approximately $3^\circ$. To confirm these the LCTVD was switched off and the transmitted intensity was measured with the polariser and analyser parallel. The polariser and analyser were then rotated at $10^\circ$ intervals (still parallel) and the intensity was again recorded. The results are shown in Figure 5-4 below.

5.3.5.c Brushing direction

The quoted brushing directions are $+/47^\circ$, and the pre-tilt is approximately $3^\circ$. To confirm these the LCTVD was switched off and the transmitted intensity was measured with the polariser and analyser parallel. The polariser and analyser were then rotated at $10^\circ$ intervals (still parallel) and the intensity was again recorded. The results are shown in Figure 5-4 below.

When the diffraction pattern was projected onto a screen 230mm from the LCTVD, the distance between the central spots was 1.6mm. The diffraction equation is $A \sin \theta = m \lambda$ for $\lambda=633\text{nm}$, this equates to $A=90\mu\text{m}$. 

Figure 5-3 Central portion of diffraction pattern produced by LCTVD pixel structure
Chapter 5 Experimental work on polarisation filter

LCTVD off, intensity measured through parallel polarisers vs. pol & analyser position.

Figure 5-4 Intensity transmitted through unswitched LCTVD through parallel polarisers

This figure can be compared with one calculated using Raynes’ formula for transmission through a TNLC with a twist angle of greater than 90° [Raynes, 1998]. How the intensity transmitted through parallel polarisers varies with polariser rotation, for the parameters of the LCTVD, is shown in Figure 5-5. For a twist angle of 90°, the minimum transmission is obtained when the polarisers are parallel with the input brushing direction. However, for the actual twist angle of 94°, the minimum intensity transmitted occurs when the polariser is rotated +2° (or 92°) from the input director.

Figure 5-5 Change in calculated transmission obtained through parallel polarisers through unswitched LCTVD. Rotation of polarisers through 180°.
From the experimental measurements (Figure 5-4), it can be seen that there is a minimum at approximately 46° and 137.5°. Comparing Figure 5-4 and Figure 5-5 enables the true brushing directions to be estimated at 44° and 135.5° which compare with the quoted values of 47° and 133°. It is possible that the LCTVD was tilted slightly in its mount, which would explain this discrepancy.

5.3.5.4 \textit{Driving signal}

The LCTVD was powered with a 5V supply, and images were transferred to it from a VGA monitor running Paint Shop Pro in windows 3.1 via a video splitter. The signal from the computer was sent to the splitter and one colour signal (red, green or blue) was taken from this and wired into the LCTVD. The picture displayed on the LCTVD was therefore a monochrome version of that displayed on the computer screen. The computer showed grey levels from 0 (black) to 255 (white). To find the relationship between GLs written to the LCTVD and the voltage applied to it, data was obtained from the manufacturer relating voltage applied to intensity transmitted [van den Eerenbeem 1997]. This was compared with an experimental curve of GL vs. intensity. A calibration curve relating GL to voltage could then be drawn (see appendix 5A).

The LCTVD could be driven in either field inversion, or single row inversion mode. Modern displays are normally driven in single (or double) row inversion because of the advantages discussed in s.3.4.6.d. The display was therefore always run in this mode.

5.3.5.e \textit{Calibration of contrast and brightness}

The contrast and brightness controls on the LCTVD were adjusted so that the largest change in intensity was achieved. A plot of intensity through crossed polarisers against GL is shown in Figure 5-6. The LCTVD was always used with the contrast setting at maximum and the brightness at minimum.
5.3.5.5 Stability of reading over time

The LCTVD was switched off and then on again, to determine its stability. When first switched on, the intensity transmitted varied by +/- 4%, but after one hour the variation dropped to around 0.5%. As remarked in Chapter 3 (s.3.3), the behaviour of a LCTVD is temperature sensitive, so it is probable that the electronics on the display warm up the pixels and affects their behaviour until equilibrium is reached. The LCTVD was therefore switched on at least 90 minutes prior to an experiment.

5.3.6 Detector

A Centronics silicon photodiode with a 15mm² sensitive area was zero biased to give a linear response, with minimum noise. The linearity was verified using neutral density filters and by checking that the intensity transmitted by a rotating analyser gave the cosine squared curve expected (as in s.2.5.2). The linearity of the photodiode will be compared with that of a CCD camera in Chapter 8 (s.8.2.4). The photodiode was connected to wide bandwidth amplifier with a range of 200nA to 2mA. The photodiode was not polarisation sensitive. This was checked by measuring the intensity recorded (no LCTVD present), for an input polarisation of 0°, whilst the photodiode was rotated by 110° (this was the maximum the diode could be rotated without the wires becoming crossed). The maximum variation in intensity was 0.8%. A mask was put over the photodiode to ensure that only the central order of the diffraction pattern was measured.
5.4 Method

Using the layout shown in Figure 5-1 linearly polarised light was passed through the LCTVD to the photodetector. The polarisation of the light emerging from the LCTVD was measured using the \( I_{\text{max}} \) and \( I_{\text{min}} \) method described in s.2.5.4, and quantified in terms of \( \alpha \) and \( e \). (To check the reliability of the readings, this was later repeated for certain incident polarisations using the linear regression method). It was repeated for different GLs of the LCTVD starting from GL255 (white, low voltage) to GL0 (black, high voltage). The pattern written to the LCTVD was a single screen of the same GL. This was then repeated for different input polarisations.

Detector readings were generally taken to 3 significant figures, unless the readings were varying so much as to make this impractical, in which case readings to 2 significant figures were used.

5.5 Results

The ellipse of polarisation measured, together with that predicted by the Jones matrix model is shown in Figure 5-7. It can be seen that although the model predicts the trend of polarisation change, the exact values for \( \alpha \) and \( e \) which are measured are slightly different. This difference was maintained when the polarisation was measured using each of the three methods described in 2.6.1-2.6.3. The experimental results deviated from the theoretical model particularly when the input polarisation was \(-15^\circ\). For this input polarisation, the model predicted that the output polarisation would have a high ellipticity around the middle of the voltage range. This would be particularly difficult to measure accurately, and errors would occur if the actual polarisation fluctuated. For this reason this particular set of results is discussed further in s.5.6.

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\[ \text{iii Where a reading was regularly alternating between two readings, the mean was recorded.} \]
Figure 5-7 Single pass results for LCTVD. Experimental ellipse of polarisation in black, theoretical model in red.
5.6 Errors between model and experimental results

It can be seen that the experimental behaviour does not exactly match the Jones matrix model. There were several possible causes of this:

5.6.1 Experimental error

5.6.1.a Detector noise & unpolarised light

The difficulties in accurately measuring high ellipticities of polarisation have been discussed in s.2.5 and 2.6 and are discussed more fully in [Liang et al., 1996]. Since polarisation properties are calculated on the basis of intensity ratios, a fixed noise level on the smallest intensity can have a devastating effect. It was possible that lower ellipticities than predicted were caused by a small proportion of unpolarised light being present in the system. This may be caused by the director fluctuating during the frame time, which would cause the polarisation to fluctuate temporally, as in the definition of unpolarised light (s.2.1.2). The temporal fluctuation in intensity will be discussed more fully in s.5.6.5. If present, unpolarised light would add to both the maximum and minimum intensities recorded through the analyser, and decrease the recorded ellipticity. To measure the unpolarised light present, a QWP was inserted into the path of the beam, and rotated to see if complete extinction could be achieved through the analyser [Mansuripur, 2000]. It was very difficult to determine small amounts of unpolarised light because of the detection limit of the photodetector, which meant that the reading fluctuated even in a dark field.

To investigate the effect of unpolarised light and noise, polarisation was measured using the linear regression method (s.2.6.3). The values of \( I_{\text{max}} \) and \( I_{\text{min}} \) were calculated to give the 'original' value for \( \varepsilon \). Then, a QWP was inserted in front of the analyser and it and the analyser were rotated until the position of minimum intensity was found, \( I_{\text{min(QWP)}} \). The residual intensity, \( I_{\text{min(QWP)}} \) was caused by the noise of the photodetection system and the unpolarised light, so:

\[
I_{\text{min(QWP)}} = I_{\text{unpol}} + I_{\text{noise}}
\]

Equation 5-1

\( I_{\text{noise}} \) was measured by recording how much the detector reading reduced when the light source was covered. This reduction in intensity was \( I_{\text{unpol}} \). The 'corrected' value for \( \varepsilon \) was found using Equation 5-2.

\[
\varepsilon = \frac{I_{\text{max}} - I_{\text{unpol}}}{I_{\text{min}} - I_{\text{unpol}}}
\]

Equation 5-2
This was measured for several GLs of the LCTVD, and the effect on the experimental ellipticity values for an input polarisation of -15° (where the theoretical ellipticity went to infinity at a voltage of 2.5V (GL 120)) is shown in Figure 5-8.

Whereas unpolarised light will always act to reduce the apparent value of ellipticity measured, the detector noise may reduce or increase it. The upper error bar in Figure 5-8 was the maximum value for $e$ that would be found using Equation 5-2, and the lower error bar represents the minimum ellipticity possible taking the noise into account. It was anticipated that using the linear regression method to measure polarisation would 'even out' the effect of noise (which is fairly random). This should not affect the values of $\alpha$ and $e$ that are obtained, but will affect the correlation coefficient, $R^2$. This is one reason for $R^2$ being less than 1 (s.2.6.3). It can be seen that the unpolarised light does not fully explain the difference between the experimental and theoretical values for ellipticity. The temporal fluctuation in intensity throughout the frame time of the LCTVD will be discussed further in s.5.6.5.

Whilst the relationship between unpolarised light and ellipticity may be easily understood, it is more difficult to determine whether or not this would cause the difference between the experimental and predicted azimuth angle. Whether or not the azimuth angle is affected by the presence of unpolarised light will depend on how the polarisation fluctuates. If the polarisation fluctuates between states A, B, and C, the azimuth angles of these three states may all be the same, or they may differ. If they differ the azimuth angle that is measured will
be determined by the combination of all three azimuth angles, taking into account the period of each polarisation state.

5.6.1.b Azimuth angle variation

It was found that the azimuth angle which was measured varied from one set of readings to the next. The readings were taken on different days, and were rechecked over a period of some months. The fluctuations in ellipticity and azimuth angle would not affect the performance of the LCTVD as a display, as the relative intensity transmitted by the analyser would always be greater for GL 255 than GL 200 and so on. Only the absolute intensity would vary. The variation in $\alpha$ (and $e$) are shown, in the complex polarisation plane, in Figure 5-9.

**Variation of polarisation of output through LCTVD for GLs 255, 140 and 0. Inputs -15 & +45 deg.**

![Figure 5-9 Fluctuation of resultant polarisation for input polarisations of -15° and +45° passing through LCTVD. GLs 255, 140 and 0.](image)

5.6.1.c Positioning of analyser

To determine how accurately the analyser could be positioned, with the LCTVD off, repeat intensity readings were taken with the analyser rotated, and then returned to the same
position (40°). The mean photodetector reading was 0.256553 (μA) with a standard deviation of 0.001087.

### 5.6.2 Temperature fluctuations

The room temperature varied between 21 and 28°C. The variation of birefringence with temperature is shown in Figure 5-10, which is plotted assuming the birefringence varies linearly with the order parameter.

The variation in $\Delta n$ between 20 and 30°C is only 0.01161μm, (i.e. $2\pi\Delta n/\lambda = 0.11524$). This difference in birefringence would change linearly polarised light ($\alpha = 30°$) to elliptically polarised light with $\varepsilon = 401.379$, $\alpha = 29.917°$, or linearly polarised light ($\alpha = 15°$) to light with $\varepsilon = 1208.14$ and $\alpha = 14.918°$. The maximum change in ellipticity is when the input is along 45°, in which case, although $\alpha$ will not change, $\varepsilon$ becomes 300.533. In practice, this will have some effect, particularly on $\varepsilon$.

The Freedericks threshold is inversely proportional to $\Delta e$, so as temperature increases the threshold voltage will increase, so the exact response of the LC to a particular voltage will also change.

### 5.6.3 Diffraction pattern

It has been seen in 5.3.5.b above that a diffraction pattern was formed by the LCTVD. When intensity measurements were taken, the photodiode was masked so that only the central diffraction spot was measured. However, it has been discovered that the intensity change caused by a LCTVD may not be uniform across the diffracted orders, particularly in the mid-voltage range [Davis et al., 1999]. Davis et al. investigated the transmission of
polarised light through a parallel aligned nematic pixellated LC spatial light modulator (SLM). They found that when the incident light was polarised parallel to the extraordinary axis of the SLM the intensity of the central diffracted spot varied. They also noticed a change in the diffraction pattern with LC voltage with extra diffracted orders appearing at some LC voltages. The maximum number of diffracted orders was obtained near a GL of 104 (i.e. near the middle of the voltage range). The change in the intensity of the central order was postulated to be caused by interference and diffraction effects. The effect of interference upon transmission through, and reflection from, a LCD has been noticed in the work done in this thesis and is considered in the next chapter (s.6.4.2). A change in the diffraction pattern would affect the intensity of the central order because this would decrease as more peripheral orders appeared. Any factor which affects the intensity of one orthogonal component of light differently to the other component will affect the polarisation (s.2.4.1).

It is suggested that the change in the diffraction pattern may occur because of a non-uniform electric field across each pixel that may occur because of capacitative edge effects. This would cause a gradient in the tilt angle of the director across the pixel, and lead to variation in the effective optical thickness of the pixel from one side to the other. This would change the intensity distribution in the various diffracted orders. If there was no phase gradient across each pixel the diffraction pattern would be centred about the zero order. Although the absolute intensity is not changed by a phase gradient, a phase gradient causes the diffraction pattern to be displaced so that it is no longer symmetrical about the zero order. The intensities of the +1 and −1 orders (and other peripheral orders) are no longer equal. The asymmetry between the +1 and −1 intensities, as well as the intensity of the zero order will depend on the size of the phase gradient across each pixel. This will vary with the voltage applied to the LC. The maximum phase shift was found to be at LC GL 104 (the GLs in Davis’ paper were the opposite to those used in this thesis, with a GL of 0 representing low voltage). The effect of this would be that the intensity of the central order would be highest when the diffraction pattern was symmetrical (i.e. no voltage applied), and lowest at a GL in the middle of the range (equivalent to Davis’ 104).

These effects have also been observed in TN LCSLMs, but because of the polarisation effects induced by the twist it is more difficult to separate the effects of the twist from that of phase non-uniformities across each pixel [Davis et al., 1999]. To the author’s knowledge, the effects using TN LCSLMs have not been quantified.

5.6.4 Reflection

It was found that the intensity of the light reflected from the display (towards the laser) varied considerably according to the voltage applied to the display, the incident polarisation,
and the vertical and horizontal position of the display. This result will be discussed more fully in Chapter 6.

5.6.5 Driving signal

5.6.5. a Connection of oscilloscope

It has been shown (s.5.6.1.a) that one cause of the underestimation of the ellipticity values is that some unpolarised light is being transmitted by the LCTVD. This could be caused by the complex video signal, which was driving the LCTVD, causing a temporal fluctuation in the voltage applied to each pixel. In addition, the switching time of the LC is not zero. To investigate this further an oscilloscope was connected to the amplifier (as shown in Figure 5-11), and the temporal intensity variation per frame was examined (quoted photodiode response time 4ns from 10-90%). The oscilloscope was triggered from the video signal that was sent to the monitor and LCTVD.

![Diagram of connection of oscilloscope to photodetector](image)

Figure 5-11 Connection of oscilloscope to photodetector

5.6.5.b Oscilloscope plots

An example of the type of intensity variation during the frame time that was seen using the oscilloscope is shown in Figure 5-12. The upper plot shows the temporal response of the photodetector, and the lower is the video signal sent from the computer monitor to the LCTVD. In the lower plot, the frame time is marked by a large pulse, followed by the pulses for addressing each row of the display. Figure 5-12 shows the variation at grey level 120 when the input polarisation was -15°, and the analyser was set to give maximum intensity (23°).
Figure 5-12 Variation of photodetector response during one frame time. GL120, input polarisation -15°. Analyser at 23°. Horizontal scale 2ms/square, vertical scale 1mV/square. The actual voltages of the upper trace recorded by the oscilloscope are shown in the figure. The average value is 550.28mV +/-0.28%.

The background noise is shown in Figure 5-13. Here, the average voltage displayed in the top trace is only 330μV, but varies a lot more, at +42% and -58% (value varies from 137-469μV). It should be mentioned that when an average trace was being recorded, the numerical values for the average, minimum and maximum voltages on the oscilloscope figures do not necessarily represent the exact trace which is shown in the figure: they represent the voltages which were obtained as a moving average since the ‘clear’ button was pressed on the oscilloscope.
The oscilloscope traces obtained when the light source was not covered are the superposition of the noise trace shown in Figure 5-13 and that produced by the behaviour of the LCTVD.

The readings from the oscilloscope were used to investigate what effect this temporal intensity variation would have upon ellipticity, and whether this could explain the discrepancies between the theoretical model and the experimental results.

When the analyser had been positioned so as to give a maximum (or minimum) reading using the photodiode (i.e. at α or (α+90°), the image on the oscilloscope was frozen and the average voltage reading \( V_{\text{max}}(\text{ave}) \) and \( V_{\text{min}}(\text{ave}) \), together with the maximum \( V_{\text{max(max)}} \) and \( V_{\text{min(min)}} \) and minimum \( V_{\text{max(min)}} \) and \( V_{\text{min(max)}} \) voltage readings on the oscilloscope were recorded. This was done when just one frame time was displayed. This was then repeated, using markers to isolate a specific portion of the oscilloscope trace, so that the voltages were recorded for the addressing of a single row only. The row chosen was that just before the frame pulse, so it was always in the same part of the addressing cycle.

From these readings, various values of ellipticity could be calculated. These were the ellipticity using the average voltages over the frame time \( (V_{\text{max(ave)}}/V_{\text{min(ave)}}) \), maximum ellipticity over the frame time \( (V_{\text{max(max)}}/V_{\text{min(min)}}) \) and minimum ellipticity over the frame time \( (V_{\text{max(min)}}/V_{\text{min(max)}}) \). Similar values could be calculated using the voltages displayed for a single row. The variation in ellipticity values arrived at using these readings is presented in Figure 5-14.
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Input -15 deg. ellipticity vs. LCTVD voltage. Oscilloscope measurements.

Figure 5-14 Changes in output ellipticity values with specific values of intensity taken from the frame time. Input -15°.

It can be seen that for lower values of ellipticity the difference between the average readings measured over a single row and those measured over (just more than) a frame is not significant. However, when the ellipticity becomes large, the value of the measurement is very sensitive to the method used. This figure can be compared with Figure 5-8, and it can be seen that the theoretical values for ellipticity are not always within the extremes calculated using the intensities recorded by the oscilloscope, particularly for the mid-voltages (cf. [Davis et al. 1999]).

A similar graph for a different input polarisation, where the ellipticity values are lower is shown in Figure 5-15. Again, it can be seen that the behaviour of the LCTVD at mid-voltages is not as expected.
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Input -75 deg, ellipticity vs voltage. Oscilloscope measurements.

Theoretical ellipticity

Frame time ellipticity
Max
Min
Average

Experimental e

LCTV voltage (V)

Figure 5-15 Changes in output ellipticity values with specific values of intensity taken from the frame time. Input -75°.

Figure 5-14 and Figure 5-15 indicate that it is not only the switching behaviour of the LCTVD which is causing the discrepancy with the theoretical model.

5.6.5. c Discussion of oscilloscope traces

Using the oscilloscope, it could be seen that the signal from the photodiode was not constant, but varied in time. The magnitude of the temporal intensity variation depended on the input polarisation, the analyser position, and the particular voltage applied to the LCTVD. This is to be expected, because if the voltage of the LCTVD was varying by a fixed amount around that which was expected, the intensity variation this would produce through the analyser would depend on the voltage/intensity curve for that particular input polarisation and analyser position. This is shown in Figure 5-16, and it can be seen that for some analyser positions there is very little intensity variation between some voltages.
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Input -15 deg. Theoretical voltage/intensity curves for LCTVD. Different analyser positions

![Graph showing theoretical voltage/intensity curves for LCTVD with different analyser positions.]

Figure 5-16 Voltage intensity curve of LCTVD. Different analyser positions. Theoretical model. Maximum intensity 1.

The voltage/intensity curves above are for a steady voltage applied to the liquid crystal. The driving signal to the LCTVD is not a steady state, so as the liquid crystal takes a finite time to switch it is likely that the liquid crystal is in a state of flux throughout the frame time. Figure 5-14 and Figure 5-15 should be interpreted to show the range of variation in $\epsilon$ that can occur, rather than the absolute value. The absolute value can vary considerably due to interference effects which are discussed in 5.6.6. below.

It appears that this fluctuation in intensity goes some way towards explaining the difference between the ellipticity measurements taken experimentally and those predicted theoretically. However, there still appears to be some unexplained discrepancy between the theory and results.

5.6.6 Interference and thickness variations of LC layer

5.6.6.a Spatial fluctuation of ellipticity measurements

It was found that the values of $I_{\text{max}}$ and $I_{\text{min}}$ that were measured depended on which part of the LCTVD the laser passed through. In order to quantify this, the LCTVD was mounted on a translation stage, and a graph of variation in $I_{\text{max}}$, $I_{\text{min}}$, and resultant ellipticity with horizontal vernier position was plotted. The plot was similar for all input polarisations. Examples are shown in Figure 5-17 and Figure 5-18.
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Input -15 deg. Analyser at 55/145 deg. Intensity of central diffraction spot only vs. horizontal vernier position. LCTVD off.

Figure 5-17 Translation of LCTVD horizontally: maximum and minimum intensities measured with vernier position.

Figure 5-18 Translation of LCTVD horizontally: resultant ellipticities from intensities shown in Figure 5-17. Theoretical $\epsilon$=20.11.

The variation in ellipticity with vernier position for several GLs of the LCTVD is shown in Table 5-2 below.
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<table>
<thead>
<tr>
<th></th>
<th>LCTVD off</th>
<th>GL255</th>
<th>GL0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum $\epsilon$</td>
<td>23.16</td>
<td>27.97</td>
<td>122.67</td>
</tr>
<tr>
<td>Minimum $\epsilon$</td>
<td>18.18</td>
<td>20.83</td>
<td>92.62</td>
</tr>
<tr>
<td>Variation: $\frac{\epsilon_{\text{max}} - \left( \frac{\epsilon_{\text{max}} + \epsilon_{\text{min}}}{2} \right)}{\left( \frac{\epsilon_{\text{max}} + \epsilon_{\text{min}}}{2} \right)} \times 100%$</td>
<td>+/-12.04%</td>
<td>+/-14.63%</td>
<td>+/-13.96%</td>
</tr>
</tbody>
</table>

Table 5-2 Variation in ellipticity values obtained on translating the LCTVD by 14mm horizontally. Input polarisation -15°.

It is likely that this variation in ellipticity is not caused by variations in the LC layer. This is for two reasons: firstly, there was no significant change in the azimuth angle of the polarisation ellipse as the LCTVD was translated. If the variation in intensity recorded was due to changes in the LC layer, it would be expected to cause a change in polarisation (e.g. if the thickness of the LC layer changed – see s.5.6.6.b). This would cause a change in the azimuth angle of the polarisation ellipse. Secondly, a similar variation in ellipticity occurs when the LCTVD is fully switched and when it is at its minimum voltage (GL255). When the LCTVD is fully switched, it has minimal birefringence ($n_e=n_o$), but when it is at GL255 it has maximum birefringence. Any effect caused by the LC layer would therefore be different at the two GLs (0 and 255), and this is not the case in Table 5-2. This variation in ellipticity should be thought of as being superimposed on the variations shown in Figure 5-14 and Figure 5-15, which were obtained without moving the LCTVD. The intensity variation did not occur when the display was illuminated using non-coherent (polarised) light from an LED ($\lambda=660\text{nm}$, described in Chapter 8), so it was most likely that the fluctuation shown in Figure 5-17 was due to interference effects caused by reflection from the surfaces of the glass or ITO. Changes in thickness of the glass will cause such an effect, as the display is moved. The thickness of the glass is not strictly controlled during the manufacturing processiv, as this is not important when using the display under normal operating conditions. A more comprehensive review of the problems caused by reflection is given in Chapter 6, s.6.4.

The measurements of polarisation were normally only taken through one particular part of the LCTVD, so the ellipticity that was measured could fall anywhere within the range shown in Figure 5-18 and Table 5-2. This fluctuation was not due to the switching behaviour of the LCTVD, as it was present even when there was no power supplied to the display. It also occurred when the LCTVD was replaced by a single pixel LC cell. This indicates that it is not caused by the pixellated structure of the display.

iv Pearson, Andrew, Philips Research Laboratories, Redhill, Surrey, Personal communication
5.6.6.b Thickness variation of LC layer

Discrepancies between the behaviour of the theoretical model and the experimental results when the LCTVD is not fully switched could occur if the thickness of the LC layer was not the quoted 4.3μm. Thickness variations in the LC layer could occur due to the presence of the electronics (diodes) on the pixels - if a spacer gets underneath a diode, the thickness of the display will be greater in this region than where the spacers are not on a diode. The thickness of a LC layer can be calculated by illuminating the unswitched (i.e. no power applied) display with light of a known polarisation and measuring the polarisation of the emergent light. The values of $\alpha$ and $\epsilon$ which can be expected for different LC thicknesses can be calculated using the formulae proposed by Gooch & Tarry ([1974], [Gooch & Tarry, 1975]) (for light incident along the director) and Raynes ([Raynes & Tough, 1985], [Raynes, 1987]) (for light incident at any orientation). As an example, for an input polarisation of $+45^\circ$ (along the director), the effect of different LC thicknesses on $\alpha$ and $\epsilon$ are shown in Figure 5-19 and Figure 5-20. The Gooch & Tarry curves are based on a twist angle of 90°, whereas the Raynes curves are based on the true twist angle of 94°. The theoretical model is based upon modelled director profiles.

![LCTVD OFF. Variation in azimuth angle with cell thickness. Input polarisation $+45$ deg (along director).](image)

Figure 5-19 Variation in values for $\alpha$ for light emerging from LCTVD (unswitched). Input polarisation $+45^\circ$. Various theoretical models.
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LCTVD OFF. Change in ellipticity with cell thickness. Input polarisation +45 deg (along director).

Figure 5-20 Variation in values for $\varepsilon$ for light emerging from LCTVD (unswitched). Input polarisation +45°. Various theoretical models.

Using the experimentally determined value for $\alpha$ of 118°, the thickness of the LCTVD can be calculated as approximately 4.1 µm. However, on translation of the LCTVD, it was found that the average ellipticity fluctuated between approximately 62 and 78. The theoretical Jones matrix model suggests this would equate to a thickness of approximately 4.2 µm, and the Raynes model to a thickness of 4.1-4.15 µm. The quoted value for the thickness of the display is 4.3 µm. It can be seen that changes in LC thickness affect the value of $\alpha$ of the resultant polarisation. The fact that this did not change with translation of the LCTVD indicates that the change in intensity measured with translation is not caused by changes in the thickness of the LC layer.

5.6.6.c The effect of interference, thickness variations, and driving signal

In summary the effect of interference variations is to vary the actual value of $\varepsilon$ that is measured (by an unknown amount, because it is not known where in the interference sinusoid the 'correct' value is), but this does not significantly affect the azimuth angle which is measured. The effect of the driving signal is to add an uncertainty to that value of $\varepsilon$ (and probably $\alpha$, but this is more difficult to investigate) as shown by and Figure 5-14 and Figure 5-15. Any variations in the thickness of the LC will cause a change in $\varepsilon$ as well as $\alpha$, as shown

\[\varepsilon\] Using an input of -15°, the experimental value of $\alpha$ (55°) is consistent with a thickness of 4.20-4.25 using the theoretical model, and 4.15-4.2 using the Raynes model, but the value of ellipticity does not vary significantly over this range.
by Figure 5-19, but \( \alpha \) will only change by about 2° (for a thickness change of 100nm). LC thickness changes will only affect polarisation when the LC is not fully switched, and will have less effect the nearer it is to being fully switched.

5.6.6.d Coherent and incoherent light

The intensity variation seen when the LCTVD was translated did not occur when it was illuminated with incoherent light. An advantage of a polarisation-based system rather than a phase-based system is that polarisation depends upon relative phase (between the \( x \) and \( y \) components), rather than absolute phase. This can be detected using incoherent light, but in order to detect changes in absolute phase, the system must be illuminated with coherent light. For a phase-based system, therefore, the thickness of all optical components must be very carefully controlled, as any variation can lead to errors. For a polarisation-based system, it is only necessary to control the thickness of birefringent layers. This is an important distinction between the polarisation varying security marking and the phase masks discussed in Chapter 7 (s.7.5).

5.7 Optimisation

In order to try and obtain a more accurate model of the experimental behaviour of the LCTVD, a Jones matrix model was constructed using the experimental results. The technique of singular value decomposition (SVD) can be used to calculate optimum values [Sabatke, 2000].

5.7.1 The problem

A method was needed to find the Jones matrix for each voltage applied to the LCTV which best described its behaviour in the light of the experimental results. The mathematical problem can be described as follows.

The complex Jones matrix of the device (to be determined) can be represented as

\[
\begin{bmatrix}
    a + ib & c + id \\
    e + if & g + ih
\end{bmatrix}
\] (where \( i = \sqrt{-1} \)).

The polarisation of the incident light is linear, and is represented by the Jones vector

\[
\begin{bmatrix}
    \cos \theta \\
    \sin \theta
\end{bmatrix}
\]

The polarisation of the emergent light, having passed through the display is represented by the
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Jones vector \[
\begin{bmatrix}
A \\
C + iD
\end{bmatrix} e^{i\phi}.
\]
The absolute phase of the x component, \(e^{i\phi}\), is not known, as only the polarisation has been measured.

The action of the display can therefore be represented by equation 5-3, where, for simplicity of calculation the phase of the x component has been taken out of the vector\(^{ii}\).

\[
\begin{bmatrix}
a + ib \\
e + if
\end{bmatrix}
\begin{bmatrix}
cos \theta \\
\sin \theta
\end{bmatrix} = \begin{bmatrix} A \\
C + iD \end{bmatrix} e^{i\phi}
\]
equation 5-3

Multiplying out, we have:

\[(a + ib) \cos \theta + (c + id) \sin \theta = Ae^{i\phi}\]
equation 5-4

\[(e + if) \cos \theta + (g + ih) \sin \theta = (C + iD) e^{i\phi}\]
equation 5-5

To eliminate the absolute phase, \(e^{i\phi}\) from the equations, dividing equation 5-4 by equation 5-5 gives

\[
\{(a + ib) \cos \theta + (c + id) \sin \theta\} (C + iD) = \{(e + if) \cos \theta + (g + ih) \sin \theta\} A
\]
equation 5-6

Grouping real and imaginary parts gives

\[
Ca \cos \theta +Cc \sin \theta - Db \cos \theta - dD \sin \theta + i(Cb \cos \theta + Cd \sin \theta + Da \cos \theta + cD \sin \theta)
= Ae \cos \theta + Ag \sin \theta + i(Af \cos \theta + Ah \sin \theta)
\]
equation 5-7

Equating real and imaginary parts of leads to:

\[
Ca \cos \theta - Db \cos \theta + Cc \sin \theta - dD \sin \theta - Ae \cos \theta - Ag \sin \theta = 0
\]
equation 5-8

\[
Da \cos \theta + Cb \cos \theta + Dc \sin \theta + Cd \sin \theta - Af \cos \theta - Ah \sin \theta = 0
\]
equation 5-9

If measurements of output polarisation, \[
\begin{bmatrix}
A_n \\
C_n + iD_n
\end{bmatrix},
\]
are taken for a series of linear input polarisations, each at angle \(\theta\), then equation 5-8 and equation 5-9 can be written as a matrix, separating out the Jones matrix components (a to h) we wish to find\(^{iii}\):

---

\(^{ii}\) To convert a Jones vector to one representing the same polarisation with a zero x phase, the vector is simply multiplied by the complex conjugate of the x component, hence

\[
\begin{bmatrix}
A + iB \\
C + iD
\end{bmatrix}
\begin{bmatrix}
A - iB \\
A - iB
\end{bmatrix} = \begin{bmatrix} A^2 + B^2 \\
CA + BD + i(DA - BC) \end{bmatrix}
\]

\(^{iii}\) equation 5-10 shows an 8 x 8 matrix on the left hand side, corresponding to 4 sets of experimental measurements, but it can be extended to any \(n \times n\) matrix, where there are \((n/2)\) sets of experimental results from output polarisation measurements.
5.7.2 SVD to find least squares fit

It can be seen that in an ideal situation, equation 5-10 should be singular, as the LCTVD should behave as a perfect Jones matrix for any input polarisation. However, in reality, the right hand side of equation 5-10 is not zero. To minimise the errors, the desired values of \([a, b, \ldots f]\) are those which result in the smallest values on the right hand side of equation 5-10. Singular value decomposition can be used for this purpose [Golub & van Loan, 1996 pp. 70 et seq.].

SVD is used to decompose the matrix on the left hand side of equation 5-10 into 3 components so that:

\[
M = U \cdot S \cdot V^T
\]

where \(M\) is the matrix to be decomposed. The matrices \(U\) and \(V\) have the properties that when multiplied by their transposes they each give the identity matrix. The diagonal matrix, \(S\), gives the singular values of the matrix, \(M\).

To find the optimum solution for equation 5-10, the values from the \(V\) matrix which correspond to the smallest values of the diagonal matrix in equation 5-11 are needed. A proprietary mathematical package was used to obtain both the diagonal matrix (which gave an estimation of how singular the matrix was) and the \([V]\) matrix. The solution to equation 5-10 was given by those columns of the \([V]\) matrix which corresponded to the smallest values.

\[\begin{bmatrix}
C, \cos \theta_1 & -D, \cos \theta_1 & C, \sin \theta_1 & -D, \sin \theta_1 & -A, \cos \theta_1 & 0 & -A, \sin \theta_1 & 0 & \text{a} \\
D, \cos \theta_1 & C, \cos \theta_1 & D, \sin \theta_1 & C, \sin \theta_1 & 0 & -A, \cos \theta_1 & 0 & -A, \sin \theta_1 & \text{b} \\
C, \cos \theta_1 & -D, \cos \theta_1 & C, \sin \theta_1 & -D, \sin \theta_1 & -A, \cos \theta_1 & 0 & -A, \sin \theta_1 & 0 & \text{c} \\
D, \cos \theta_1 & C, \cos \theta_1 & D, \sin \theta_1 & C, \sin \theta_1 & 0 & -A, \cos \theta_1 & 0 & -A, \sin \theta_1 & \text{d} \\
C, \cos \theta_1 & -D, \cos \theta_1 & C, \sin \theta_1 & -D, \sin \theta_1 & -A, \cos \theta_1 & 0 & -A, \sin \theta_1 & 0 & \text{e} \\
D, \cos \theta_1 & C, \cos \theta_1 & D, \sin \theta_1 & C, \sin \theta_1 & 0 & -A, \cos \theta_1 & 0 & -A, \sin \theta_1 & \text{f} \\
C, \cos \theta_1 & -D, \cos \theta_1 & C, \sin \theta_1 & -D, \sin \theta_1 & -A, \cos \theta_1 & 0 & -A, \sin \theta_1 & 0 & \text{g} \\
D, \cos \theta_1 & C, \cos \theta_1 & D, \sin \theta_1 & C, \sin \theta_1 & 0 & -A, \cos \theta_1 & 0 & -A, \sin \theta_1 & \text{h}
\end{bmatrix}
\]

\[\text{equation 5-10}\]

\[\text{equation 5-11}\]
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To make this data easier to see, the matrix \([V]\) was transposed, so that the final two rows of \([V]^T\) gave the required values for \(a, b, c, d, e, f, g\) and \(h\). This is described in more detail in Appendix 5B. These values for \(a, b, c, d, e, f, g\) and \(h\) were used to form a Jones matrix model of the LCTVD which was based on experimental results (the 'experimentally based Jones matrix model').

The requirement for absolute phase to be measured is eliminated in equation 5-10 but a consequence of this is that there is no unique solution. This means that the matrix obtained using the SVD method can be multiplied by any complex number, and it will still be a solution to equation 5-10.

5.7.3 Results of experimentally based Jones matrix model compared with the theoretical model

A Jones matrix model was calculated for each voltage of the LCTV using the results obtained using eight linear input polarisations. This model predicted the azimuth angle of the output polarisation ellipse considerably more accurately than the UCL Jones matrix model.

This experimentally based model had been obtained using only linearly polarised light as an input. With the phase difference of linearly polarised light being zero, it was possible that the phase component of the model would have been incorrect. This would only show up if elliptically, or circularly polarised light was used as an input through the model. As a test of the accuracy of the model, circularly polarised light was used as an input through the LCTVD, and the results of this are plotted in the circular complex polarisation plane (explained in §2.3.5) in Figure 5-21. The equal \((x\) and \(y)\) amplitudes of circularly polarised light should show up any discrepancies in either the \(x\) or \(y\) components of the model.
Figure 5-21 Experimental polarisation change produced by LCTVD compared with that predicted by experimentally based and theoretical Jones matrix models. Input polarisation circular. GLs 255 (orange), 180 (blue), 140 (pink), 100 (black), 40 (brown) and 0 (green).

Each coloured point represents a particular GL. Each symbol represents a model used.

Each point in the figure represents the mean of a series of repeat measurements. For each value of $\alpha$ and $e$, the standard deviation (SD) was calculated and added or subtracted from the values of $\alpha$ and $e$ to give maximum and minimum $\alpha$ and $e$ values. The cluster of complex polarisation variables which would represent this error was then the value of $\chi$ for the $[\text{minimum } \alpha + \text{maximum } e]$, $[\text{minimum } \alpha + \text{minimum } e]$, $[\text{maximum } \alpha + \text{maximum } e]$, and $[\text{maximum } \alpha + \text{maximum } e]$. Each mean value therefore had four values which represented the errors. These are plotted as open circles on Figure 5-22.
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Circular complex polarisation variable variation for LCTVD measurements. Input R circular. Expanded to show experimental errors.

Figure 5-22 An expansion of Figure 5-21 to show experimental errors (alpha and ellipticity calculated +/- 1 SD). The experimental values are represented using open squares, so that all error points can be seen.

It can be seen that there is still not perfect agreement between the experimental results and the model, but it should be noted that the output ellipticities are fairly low, so it is expected that the azimuth angle would not be measured as accurately as at higher ellipticities. It was also noticed that if measurements were repeated on different occasions, they differed in azimuth angle (s.5.6.1.b), though they did not dramatically differ in ellipticity. The measurements which lead to the experimentally based model were taken over a period of several days (but through the same part of the LCTVD) and so the variation in polarisation which was measured from one day to the next will lead to imperfections in the model. The measurements shown in Figure 5-21 were all taken on one day.

5.7.4 Discussion

The experimentally based model generally characterises the actual behaviour of the LCTVD more accurately than the theoretical model. Although it may seem obvious that a model calculated from experimental results would give a better prediction of those results this would not necessarily be the case if the LCTVD was behaving inconsistently, or if there were
large errors in the experimental results. When the LCTVD was used as a Jones polarimeter, it was necessary to have an accurate model of the LCTVD. For this purpose the experimentally based model was used, but it was found that this model was not accurate enough given the sensitivity of the method. Therefore, a method of quickly calibrating the LCTVD was needed. This was used when the LCTVD was used as part of a complete Stokes polarimeter. The use of the LCTVD to measure polarisation is discussed in Chapter 6.
Chapter 6

Algorithm to determine unknown input polarisation

6.1 Introduction & aims

The aim of this chapter was to see if the LCTVD could be used as part of a full polarimeter with no moving parts. It was anticipated that polarisation of light incident on the LCTVD could be determined simply by applying known voltage signals to the display, and measuring the intensity transmitted by an analyser. The LCTVD would induce a phase difference between the orthogonal components of the incident light, and this would enable all four Stokes vectors to be determined. The sense of rotation of the polarisation ellipse would also be known. As mentioned in Chapter 2, polarisation can be quantified using Jones or Stokes vectors, and each method has advantages and disadvantages. Because the LCTVD had been modelled as a Jones matrix, an algorithm was first developed using Jones calculus. Then a method is described which uses Stokes vectors and a partial Mueller matrix.

This chapter will begin by analysing both methods, and presenting the results obtained using them. It will then consider the effect errors have on each method, and the sources of those errors. The advantages and disadvantages of the LCTVD polarimeter over polarimeters proposed by previous researchers will then be discussed, and the effect of polychromatic sources and oblique angles of incidence considered. The chapter will conclude with two specific applications of the device.
Chapter 6 Algorithm to determine unknown input polarisation

6.2 Jones calculus method

6.2.1 Theory

If the LCTVD is represented by a (known) Jones matrix,

\[
\begin{bmatrix}
R_1 + iM_1 & R_2 + iM_2 \\
R_{y1} + iM_{y1} & R_{y2} + iM_{y2}
\end{bmatrix},
\]

its effect on polarised light can be written as:

\[
\begin{bmatrix}
R_1 + iM_1 & R_2 + iM_2 \\
R_{y1} + iM_{y1} & R_{y2} + iM_{y2}
\end{bmatrix}\begin{bmatrix}
J_1 + iJ_2 \\
J_3 + iJ_4
\end{bmatrix} = \begin{bmatrix}
L + iM \\
N + iP
\end{bmatrix}
\]
equation 6-1

where the incident light polarisation is represented by the vector \(\begin{bmatrix} J_1 + iJ_2 \\ J_3 + iJ_4 \end{bmatrix}\), the components of which are to be determined, and the output light is fully polarised with Jones vector components, \(L, M, N\) and \(P\). If the analyser is fixed so that its transmission is along the \(x\)-axis, then, from equation 6-1, the intensity measured is equal to \(|L|^2 + |M|^2\), and the \(y\) component can be ignored. Multiplying out the left-hand side of equation 6-1, separating into real and imaginary components and squaring\(^1\) leads to equation 6-2. In this equation the \(K_n\) values are constants derived from the real and imaginary components of the Jones matrix for each grey level used.

\[I_{GL} = K_1(J_1^2 + J_2^2) + K_2(J_3^2 + J_4^2) + K_3(J_1J_3 + J_2J_4) + K_4(J_1J_4 - J_2J_3)\] equation 6-2

In order to determine the 4 unknown components \((J_1, J_2, J_3, J_4)\), four GLs were applied (in sequence) to the LCTVD. The intensity transmitted by the analyser, \(I_{GL}\) (where GL =255, 140, 60 and 0) was measured for each GL. This gave 4 simultaneous equations that could be solved for \(J_1, J_2\) and \(J_4\) (equation 6-3). As polarisation rather than absolute phase is being determined, \(J_2\) can be taken to be zero. \(J_4\) therefore represents the relative phase of the \(y\) component of the Jones vector of the input light. equation 6-3 enables the Jones vector of the input light, and therefore the polarisation, to be calculated.

\[
\begin{bmatrix}
K_1 & K_2 & K_3 & K_4 \\
K_5 & K_6 & K_7 & K_8 \\
K_9 & K_{10} & K_{11} & K_{12} \\
K_{13} & K_{14} & K_{15} & K_{16}
\end{bmatrix}^{-1}\begin{bmatrix}
I_{255} \\
I_{140} \\
I_{60} \\
I_0
\end{bmatrix} = \begin{bmatrix}
J_1^2 + J_2^2 \\
J_3^2 + J_4^2 \\
J_1J_3 + J_2J_4 \\
J_1J_4 - J_2J_3
\end{bmatrix} = \begin{bmatrix}
E \\
F \\
G \\
H
\end{bmatrix}
\]
equation 6-3

\(^1\)i.e. multiplying by the complex conjugate
Measuring intensity only (i.e. no phase information) lead to a sign ambiguity because $J_1$ was determined as the square root of $E$. If $J_1$ was negative, the phase of the $x$ component would be different by 180°. This would have the effect that the ellipticity of the light would remain unchanged, (although its sense of rotation would be reversed), but the azimuth angle would be reflected about the co-ordinate system. It was anticipated that this ambiguity could be resolved using some other knowledge of the unknown polarisation.

6.2.2 Results

The Jones matrix algorithm worked well in theory [Blakeney et al., 1999], but was very sensitive to error. Neither the theoretical model, nor the SVD model of the LCTVD predicted the behaviour of the LCTVD sufficiently accurately for this purpose. An example of the sensitivity of the method is shown in Table 6-1 (for an input polarisation at +30° (linear)).

<table>
<thead>
<tr>
<th>Error induced into modelled intensity values ($I_{255}, I_{140}, I_{60}, I_0$)</th>
<th>$\alpha$ (should be 30°)</th>
<th>$e$ (should be infinity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All +10%</td>
<td>29.606°</td>
<td>5839</td>
</tr>
<tr>
<td>Mixed +/−2% ii</td>
<td>43.63°</td>
<td>10.24</td>
</tr>
</tbody>
</table>

Table 6-1 Values for $\alpha$ and $e$ for input polarisation of +30° (linear) if theoretical intensities are corrupted by error.

These values were obtained using the theoretical model of the LCTVD to calculate the intensities that would be measured for this particular input polarisation. These values were then corrupted by various errors. If the four intensities were all increased or decreased, even a comparatively large error in intensity (10%) had little effect on the polarisation which would have been calculated using these values. However, if some intensities were increased and some were decreased, even as small an error as +/−2% caused an unacceptable value of $\alpha$ and $e$ to be obtained. The SVD model of the LCTVD, although it matched the experimental results more closely than the theoretical model, was not accurate enough for this method to give good results.

The poor experimental results caused by the sensitivity of the method, together with the inherent sign ambiguity, and the limitation of only completely polarised light being measured, lead to an alternative method being developed. This was based on Stokes vectors and Mueller calculus.

ii $I_{255}$ and $I_{140} +$2%; $I_{60}$ and $I_0 -$2%
6.3 Stokes vector method

Stokes vectors are intensity based, as opposed to the phase based Jones vectors. They are derived from experimental measurements of intensity (s.2.3.7) and so lend themselves to situations where intensity (rather than phase) is measured.

6.3.1 Theory

The action of the LCTVD upon incident light can be written as:

\[
\begin{bmatrix}
    m_1 & m_2 & m_3 & m_4 \\
    m_5 & m_6 & m_7 & m_8 \\
    m_9 & m_{10} & m_{11} & m_{12} \\
    m_{13} & m_{14} & m_{15} & m_{16}
\end{bmatrix}
\begin{bmatrix}
    S_0 \\
    S_1 \\
    S_2 \\
    S_3
\end{bmatrix}
= \begin{bmatrix}
    S'_{0} \\
    S'_{1} \\
    S'_{2} \\
    S'_{3}
\end{bmatrix}
\]

\text{equation 6-4}

Where \( [S_0, S_1, S_2, S_3]^T \) are the Stokes components of the input light (unknown), and \( [S'_{0}, S'_{1}, S'_{2}, S'_{3}]^T \) are the Stokes components of the output light. The matrix, \( m \) is the Mueller matrix of the LCTVD. Assuming the LCTVD is not a depolarising optical element, a Jones matrix can be converted to a Mueller matrix [Lu & Chipman, 1998], using the procedure laid out by Azzam [1979]. From the definitions of the Stokes vectors laid out in s.2.3.7, the intensity transmitted by a polariser with its transmission along the \( x \) axis, is:

\[
I_x = \frac{S'_{0} + S'_{1}}{2}
\]

\text{equation 6-5}

If the intensities measured, using the photodiode, through the polariser (along the \( x \) axis) for each of the four LCTVD voltages are denoted by \( I_{GL} \) (where \( GL = 255, 140, 60 \) and 0), and the Mueller matrix components for each \( GL \) are denoted by \( m_{(GL)} \) then combining equation 6-4 and equation 6-5:

\[
m_{5(GL)}S_0 + m_{6(GL)}S_1 + m_{7(GL)}S_2 + m_{8(GL)}S_3 = 2I_{(GL)} - S_0
\]

\text{equation 6-6}

Equation 6-6 assumes that the total intensity remains the same before and after the light has passed through the LCTVD, so \( S'_{0} \) of the output is the same as \( S_{0} \) of the input.
6.3.2 Calibration of LCTVD

The LCTVD can be quickly and easily calibrated by measuring the intensities transmitted by the analyser with its transmission along the x- and y-axes \( I_x \) and \( I_y \) for four independent input polarisations [Schirmer et al., 1998]. These polarisations are chosen, for convenience, to be those with Stokes vectors \([1, 1, 0, 0]^T\) (0°), \([1, -1, 0, 0]^T\) (90°), \([1, 0, 1, 0]^T\) (45°), and \([1, 0, 0, 1]^T\) (R circular). The components of the matrix are found using:

For input 0°: \[ m_5 + m_6 = I_{x(0)} - I_{y(0)} \]
For input 45°: \[ m_3 + m_4 = I_{x(45)} - I_{y(45)} \]
For input 90°: \[ m_5 - m_6 = I_{x(90)} - I_{y(90)} \]
For input R circular: \[ m_3 + m_5 = I_{x(r)} - I_{y(r)} \]

Equations 6-7

Therefore, manipulation of equations 6-7 gives values for \( m_n \) so that:

\[ m_5 = I_{x(0)} + I_{x(90)} - 1 \] (0°)
\[ m_6 = I_{x(0)} - I_{x(90)} \]
\[ m_3 + m_4 = 2I_{x(45)} - 1 \]
\[ m_3 - m_4 = 2I_{x(45)} - 1 \]

If the LCTVD is not anisoabsorbic (i.e. does not absorb light of different polarisations differently) then \( I_{x(0)} + I_{x(90)} \) is constant (equal to 1 in the equations above), and \( m_5 \) is always zero.

There is some duplication when measuring \( I_x \) and \( I_y \), however this was carried out as a check that the total intensity was not, in fact, varying. It is also not necessary to use an input of 0° as well as 90° because (from equation 6-6)

\[ m_3 + m_6 = 2I_{r(0)} - 1, \]
and \[ m_3 - m_6 = 2I_{r(0)} - 1 = -2I_{r(90)} \] [because \( I_{x(90)} + I_{y(90)} = 1 \)]

Therefore, adding:

\[ m_3 + m_6 + m_3 - m_6 = 2I_{r(0)} - 1 + 1 - 2I_{r(90)} \]

Therefore \( I_{x(0)} = I_{y(r)} \)

6.3.3 Calculation of unknown input Stokes vector

Using the calibration procedure in s.6.3.2 the components of the partial Mueller matrix are known. With the total intensity, \( S_0 \) normalised to 1, the final partial Mueller matrix equation is:
Chapter 6 Algorithm to determine unknown polarisation

\[
\begin{bmatrix}
1 & m_{6(255)} & m_{7(255)} & m_{8(255)} \\
1 & m_{6(140)} & m_{7(140)} & m_{8(140)} \\
1 & m_{6(60)} & m_{7(60)} & m_{8(60)} \\
1 & m_{6(0)} & m_{7(0)} & m_{8(0)}
\end{bmatrix}
\begin{bmatrix}
S_0 \\
S_1 \\
S_2 \\
S_3
\end{bmatrix}
= \begin{bmatrix}
I_{(255)} \\
I_{(140)} \\
I_{(60)} \\
I_{(0)}
\end{bmatrix}
\]
equation 6-8

The polarisation is represented by the ratio of the components of the Stokes vector, so the constant factor of 2 (in equation 6-6) can be ignored. If the absolute intensity is required, this factor should be reinserted.

When the components of the partial Mueller matrix represented on the left hand side of equation 6-8 are known, SVD is used to find the best solution for the unknown polarisation vector \([S_0, S_1, S_2, S_3]^T\). SVD can be used to find the best solution to the problem \([M] \cdot [x] = [A]\). The solution is given by:

\[
[x] = [U]^{-1}
\begin{bmatrix}
1/s_1 & 0 & 0 & 0 \\
0 & 1/s_2 & 0 & 0 \\
0 & 0 & 1/s_3 & 0 \\
0 & 0 & 0 & 1/s_4
\end{bmatrix}
[U] \cdot [A]
\]
equation 6-9

where \(s_n\) is the singular value of the matrix, \(M\). The vector, \(x\) in equation 6-9 is the Stokes vector of the incident light.

6.3.4 Results

6.3.4.a Using LCTVD

Various input polarisations were used to test the system: linear polarisation, both fully and partially polarised, and elliptically polarised light of both senses (again fully and partially polarised). The degree of polarisation, \(P\) is defined as the ratio of the polarised intensity to the total intensity, so that \(P = I_{pol}/I_{total}\). Therefore for linearly polarised light, \(P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}\) (equation 2-27). For all polarisations, this was related to the Stokes parameters using

\[
P = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \quad [Azzam & Bashara, 1979, p. 59].
\]

For fully polarised light, \(P = 1\). Some of the results are plotted in Figure 6-1.
Figure 6-1 Use of LCTVD as Stokes polarimeter. Polarimeter-measured and actual input polarisations. Each colour represents a pair of points (actual and experimental). Triangles represent experimental values, squares actual inputs.

Each pair of points (experimental and actual polarisations) are represented by the same colour. For clarity, $P$ is not shown on the figure, but is shown separately in Figure 6-2.
Chapter 6 Algorithm to determine unknown input polarisation

Experimental values for \( P \) vs. values obtained using LCTVD as Stokes polarimeter

Figure 6-2 Comparison of actual values for the degree of polarisation, \( P \), with values obtained using LCTVD Stokes polarimeter. If no errors were present all the points would lie on the dotted line.

6.3.4.b Accuracy of measurement of input polarisation using LCTVD polarimeter

The accuracy of the determination of azimuth angle and ellipticity depend upon the actual values of the input polarisation. As expected, the azimuth angle is determined most accurately when the input polarisation is linearly polarised. The results obtained using the LCTVD polarimeter compare well with those published using the LC polarisation camera: it is reported that using the LC polarisation camera the azimuth angle can be determined to just over +/-1° (completely linearly polarised input) [Wolff & Andreou, 1995]. Using monochromatic light, the LCTVD polarimeter has a similar degree of accuracy: linearly polarised light is normally identified to within +/-1°, and the azimuth angle of elliptically polarised light can also be determined with this degree of accuracy. It can be seen from Figure 6-1 that the polarimeter also delivers the ellipticity of elliptically polarised light very well: for the four ellipticities tested (range 2.8-7.04 \(^{iii}\)), the largest error in ellipticity was 0.37 (when \( e=2.925 \)). When the light was linearly polarised, the lowest value for ellipticity obtained was 683. The sense of rotation was always correctly identified. The largest error was in the determination of the degree of polarisation, \( P \), and this is again demonstrated in the consideration of a polychromatic source discussed in s.6.5.2.
The accuracy of the LCTVD Stokes polarimeter in determining the degree of polarisation is not as good as that of the LC polarisation camera, where the partial polarisation of a linearly polarised source varied between 0.99 and 1 [Wolff et al., 1997]. Figure 6-2 shows that this was not the case for the LCTVD Stokes polarimeter. The probable reason for this is that in the LC polarisation camera the 2 LC cells are either unswitched or fully switched. Both of these states should be stable because either there is no voltage applied to the cells (and therefore this cannot vary) or the voltage is so strong (+/-9V) that the elastic forces which hold the LC director in its twisted configuration are completely overcome by the strength of the electric field applied. In addition to the fully switched and unswitched states, the LCTVD polarimeter uses intermediate voltages. The response of the LC director to these intermediate voltages is more subtle than that to a high voltage, and it is likely that this response will be more significantly affected by factors such as environmental changes and fluctuation of the actual addressing voltage. As an example, if the addressing voltage fluctuates by +/-10%, if the mean voltage is 9V +/-10%, the director will always be fully switched. However, if the voltage is in the middle of the switching curve (figure 3-7) a small fluctuation will lead to a change in director orientation, which will cause the transmitted polarisation to vary. This will lead to the actual intensities being transmitted through the analyser not being precisely constant. It would be expected that the results of a polarimeter using a single pixel TNLC cell instead of the LCTVD may be more accurate than using the LCTVD because the single LC cell could be driven with a constant directly applied voltage. This would eliminate the director fluctuations that may occur in the LCTVD as a consequence of its complex driving signal. However, it was found that different errors occurred when using a single pixel LC cell. These are discussed in s.6.4.

The results reported by Schirmer et al. [1998] for a broad-band LC Stokes-meter are not as accurate as the +/-1° in azimuth angle determination reported by Wolff. However, this polarimeter is designed to be used over a broad spectral range. The birefringence of a material varies with the wavelength of light passing through it, and so it would be expected that the results obtained using a variety of incident wavelengths would be less accurate than when using a monochromatic source. Schirmer’s polarimeter is designed to reduce this effect, which will be further discussed below (s.6.5.2). Wolff does not discuss how polychromatic sources affect his polarisation camera, but how his camera would be affected by a change in wavelength of the incident light is also discussed in s.6.5.2.

iii For clarity, the results for ellipticity of 6.78R are not shown in the figure.
6.3.4.3  Effect of errors

It can be seen that the LCTVD polarimeter works well in practice. It can distinguish between partially polarised and fully polarised light, regardless of whether the light is linearly or elliptically polarised. It is accurate for both senses of elliptical polarisation.

However, as for the Jones matrix polarimeter, the method is very sensitive to error. As a comparison with the Jones method, the intensities predicted by the theoretical model were corrupted by a certain amount, and then put into the Stokes algorithm above (as in Table 6-1). The effect of this is shown in Table 6-2, for a linear input polarisation of 30°, fully polarised (P=1).

<table>
<thead>
<tr>
<th>Error induced into modelled intensity values</th>
<th>$\alpha$ (should be 30)</th>
<th>$e$ (should be $=1$)</th>
<th>$P$ (should be 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All +10%</td>
<td>30.00</td>
<td>7.73x10$^{-6}$</td>
<td>0.996</td>
</tr>
<tr>
<td>Mixed +/-2%  ($I_{333}$ and $I_{440}$ +2%; $I_{600}$ and $I_0$ -2%)</td>
<td>28.71</td>
<td>106.47</td>
<td>1.21</td>
</tr>
<tr>
<td>Mixed +/-2%  ($I_{440}$ and $I_0$ +2%; $I_{333}$ and $I_{600}$ -2%)</td>
<td>27.76</td>
<td>12.31</td>
<td>2.65</td>
</tr>
</tbody>
</table>

Table 6-2  Effect of error on LCTVD Stokes polarimeter. Theoretical intensities corrupted by 10 or 2%. Input polarisation 30° linear.

The exact effect upon the results depended on which particular intensities were ‘corrupted’. The different values for ‘mixed +/-2%’ in Table 6-2 shows the different effect obtained when different intensities were increased and decreased by 2%. The exact figures in Table 6-1 and Table 6-2 are not important as they are only shown to demonstrate that slight errors can destroy the accuracy of the results. The exact effect depends upon the input polarisation and which intensities are corrupted. The effect on the same intensities but corrupted by only 1% is shown in Table 6-3. It can be seen that even this small error degrades the results. Similar results were obtained using other input polarisations. Calibration of the device before each set of readings reduces the effect of this.

<table>
<thead>
<tr>
<th>Error induced into modelled intensity values ($I_{333}$, $I_{440}$, $I_{600}$, $I_0$)</th>
<th>$\alpha$ (should be 30)</th>
<th>$e$ (should be $=1$)</th>
<th>$P$ (should be 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixed +/-2%  ($I_{333}$ and $I_{440}$ +1%; $I_{600}$ and $I_0$ -1%)</td>
<td>29.33</td>
<td>394.58</td>
<td>1.42</td>
</tr>
<tr>
<td>Mixed +/-2%  ($I_{440}$ and $I_0$ +1%; $I_{333}$ and $I_{600}$ -1%)</td>
<td>28.64</td>
<td>30.14</td>
<td>1.61</td>
</tr>
</tbody>
</table>

Table 6-3  Intensities as in Table 6-2, but corrupted by +/-1%. Input polarisation 30° linear.

iv In practice it was only re-calibrated every few days, unless something had changed (such as LCTVD position).
6.3.4.d  Repeatability

For a linearly polarised input of 60°, repeat readings of intensities were taken, and the polarisation, using the Stokes polarimeter was calculated. This is shown in Figure 6-3 below.

Reproducibility of LCTVD Stokes polarimeter for linear input polarisation along 60 deg.

These measurements were taken on a single day. The mean and standard deviation values are shown in Table 6-4. Sample size: 16.

<table>
<thead>
<tr>
<th></th>
<th>α</th>
<th>e</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal value</td>
<td>60</td>
<td>Infinity</td>
<td>1</td>
</tr>
<tr>
<td>Mean</td>
<td>60.9177</td>
<td>10786.52</td>
<td>0.9972</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.2855</td>
<td>13598.28</td>
<td>0.0095</td>
</tr>
</tbody>
</table>

Table 6-4 Mean and standard deviation values for α, e and P for the polarisations shown in Figure 6-3. Repeat readings for input linear polarisation of 60°.

It can be seen that there is little variation in values for α and P. The large standard deviation in the value for ellipticity is a result of the small denominator. The practical effect of this variation is best understood by referring to Figure 6-3. It can be seen that the points
closely grouped, and hence represent similar polarisations. The actual values for ellipticity varied from 40852 to 693.

6.3.4.e Using single pixel LC cell

Following on from the success of the method using the LCTVD, the method was repeated using a single pixel TNLC cell. This cell had a similar LC material and thickness to the LCTVD, and had the advantage that it was cheaper and simpler to produce than the LCTVD. It also did not produce the diffraction and drive signal effects discussed in Chapter 5. It was driven with a 50Hz square wave from a calibrated function generator.

The results using the single pixel cell were not as good as using the LCTVD. This was because the total intensity transmitted by the cell (without an analyser present) varied with voltage, and the degree of this variation depended on the polarisation of the incident light. To investigate this variation in intensity transmission polarised light was passed through the LC cell, and the intensity recorded by the photodiode, without an analyser present, was recorded. The voltage of the LC cell was varied. The results are shown in Figure 6-4, which shows the change in this total intensity with LC voltage for several incident polarisations. This variation in total intensity transmitted by the LC invalidates the assumption made in Equation 6-6 and this cannot be corrected without leaving more unknowns than equations.

![Variation of total intensity reading with single pixel LC voltage for different incident polarisations. No analyser present.](image)

Figure 6-4 Variation in total intensity transmitted by single pixel LC cell

Five different input polarisations are shown in Figure 6-4. The maximum degree of intensity change is 11.04% (of the maximum), when the polarisation of the incident light is at 45°, and
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the minimum variation is 5.05% when the polarisation is 90°. The reasons for this variation will be discussed in the next section.

6.4 Reflection

Whenever there is a boundary between two layers of different refractive index some light will be transmitted, and some will be reflected. It has recently been reported that the total intensity transmitted by a pixellated parallel-aligned nematic LC SLM varies with LC voltage, even when the incident light is polarised along the extraordinary axis of the SLM. This variation can be nearly 25%, and is suggested as being caused by diffraction and interference effects. Similar effects have been observed with TNLCDs [Davis et al., 1999]. The single LC cell used in this thesis did not produce a diffraction pattern, so only the effect of interference upon reflection from the LC/ITO boundary is considered in this section.

Examination of the reflections produced from the single LC cell showed that one (or more) of the reflections was varying with cell voltage, and the degree of this variation depended on the input polarisation. To prove it was the reflection that was causing the change in transmitted intensity that is shown in Figure 6-4, an extra photodiode was inserted to intercept this reflection. As expected, it was found that the variation in transmission was opposite to the variation in reflection. The variation in total intensity (reflected + transmitted) was minimal: the same polarisations were tested as in Figure 6-4 and the maximum variation in the (maximum) intensity was only 1.26%.
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6.4.1 Reflecting layers

Figure 6-5 shows a diagram of the different boundaries that are present in a LC cell (or LCTVD).

![Diagram of LC cell boundaries](image)

The main reflections from the LC cell will occur where the change in refractive index is greatest. This will occur at the glass/air interface, and the ITO/LC interface. The polyimide has a refractive index of 1.58-1.60 and is approximately 100nm thick [Pearson, 2000], so the intensity of the reflection between this, and the LC material is not as great as that between the ITO ($n=1.95$) and the polyimide or the LC. For simplicity the polyimide layer will be ignored, and only the ITO/LC interface will be considered, except in the case of thin film interference (s.6.1.3.b below). The reflection from the ITO/glass boundaries will not change with LC voltage, so this too will be ignored. The ITO is sputtered onto the glass. ITO, being a conductor, has a complex refractive index (defined as $\tilde{n} = n + i\kappa$) but the extinction coefficient, $\kappa$ is very low for the visible spectrum, being in the region of 0.05 [Hultaker, 2000]. The extinction coefficient indicates how far the incident wave penetrates into the refracting material before it is reflected. The energy density falls to 32%$^\dagger$ of its value after the wave has advanced a distance $d$, where $d = \frac{2n}{4\pi\kappa}$ [Born & Wolf, 1999, p.738 vii]. For most metals (which are good reflectors), this distance is a fraction of a wavelength, but for an ITO layer with $\kappa=0.05$, $d=1.96\mu$m. The thickness of an ITO film on a glass plate, such as found in a LC cell has been reported as 112nm [Yang & Sambles, 1997] (i.e. 17.5$d$). So, most of the incident light will not be reflected, and the ITO appears transparent. The reflectivity (and transmissivity) of a thin metallic film vary with film thickness and the refractive indices of the

$^\dagger$ i.e. $1/\exp$
surrounding layers [Born & Wolf, 1999, p.752 et seq.]. However, the comparatively small difference in refractive index between the LC layer and the ITO (compared with, say, an ITO/air interface) mean this will not have a significant effect, and so this will not be considered here. The low value of $\kappa$ does not significantly affect the reflectance (for a constant film thickness), as shown in Figure 6-6 below. Figure 6-6 was drawn using the formula for reflectance:

$$ R = \frac{(n_r - n_i)^2 + \kappa_r^2}{(n_r + n_i)^2 + \kappa_r^2} $$

Equation 6-10

(From [Kaye & Laby, 1986])

Figure 6-6 Variation of reflectance (from ITO/LC interface) with absorption coefficient of ITO (normal incidence)

It was noticed that the position of the reflected spot varied depending on the horizontal translation of the LC cell. The LC cell was always positioned so that the incident light was normal. This was determined by tilting the cell so that its primary reflection (from the front surface of the glass) was directed back along the path of the laser beam. The fact that the secondary reflection (which varied with LC cell voltage) was not coincident with this beam indicated that the surface from which it was reflected was not parallel to that producing the primary reflection. According to the Fresnel equations (s.2.4.1.b), the polarisation of light which is reflected at non-normal incidence will be changed. However, for small angles of incidence this is not significant. The angle of incidence in this experiment was always less

---

vi Born & Wolf define $\tilde{n} = n(1+i\kappa)$ [Born & Wolf, 1999, p.737], hence the extra factor of $n$ in the numerator.
than $10^\circ$. For an angle of incidence of $10^\circ$ the Fresnel reflection coefficients (squared) for the ITO/LC boundary were $R_p=0.01349$ and $R_s=0.01584$ (taking the average value of the refractive index of the LC). There is therefore only a 0.2% difference in reflected intensity between the parallel and perpendicular components, and even if this is doubled (to take into account the second LC/ITO interface), this is not sufficient to explain the 11% variation in transmitted intensity shown in Figure 6-4.

6.4.2 Thin film interference

Interference effects can increase or decrease the intensity of reflected light at a given point. If the intensity of light incident on a plane parallel transparent material is $I$, then the maximum intensity that can be reflected back is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

where $I_1$ and $I_2$ are the intensities of the two waves being interfered (i.e. one from the front surface and one from the back surface of the boundary), and $\delta$ is the phase difference between them. If $I_1$ and $I_2$ are equal, then the maximum reflected intensity is $4I$ and the minimum is 0 [Born & Wolf, 1999, p.289]. These maxima and minima occur when the optical path traversed through the film ($\delta$) is equal to $\lambda$ or $\lambda/2$ respectively. As shown in Figure 6-5 there are several parts of the LC cell which could cause this phenomenon, and these will be considered in turn.

6.4.2.a Polyimide layer:

An antireflection film works by reflecting light so that it destructively interferes with itself. For complete elimination of the reflected beam the refractive index of a single film should equal $\sqrt{n_1 n_{1'}}$ [Kaye & Laby, 1986], and the optical thickness, $\delta$ should equal $\lambda/4$. The optical thickness of the polyimide layer is 158-160nm. This means it can act as an antireflection layer between the ITO and the LC layers. $\sqrt{n_1 n_{1'}} = 1.70$ for $n_o$ of the LC, and 1.76 for $n_e$. The reflected beam will therefore not be completely eliminated by the polyimide layer, but will have a residual reflectance ($R$) of 0.00379 for $n_o$ and 0.00915 for $n_e$. This is very small, and unlikely to be significant.

6.4.2.b ITO thin film

The two ITO layers could also act as thin films. It is possible that the thickness of the ITO layers is near the 162nm required for the optical thickness to be $\lambda/2$. If this were the case, the reflection (intensity) coefficient would increase from 0.0103 and 0.0183 for $n_o$ and $n_e$ of the...
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LC to 0.05045 and 0.06664 (using equation 6-11) from each ITO boundary. If the refractive index of the ITO is \( n_r \), that of the incident medium is \( n_i \), and the thickness of the film is \( d \) (Figure 6-7) then \( \delta = 2n_r d \cos \theta' \pm \frac{\lambda_0}{2} \). \( \lambda_0 \) is the wavelength of the light in a vacuum.

For the ITO film on the front surface of the LC cell, for a fixed value of ITO film thickness, refractive index, and angle of incidence, \( \delta \) will stay constant. However, the intensity of the reflection at the ITO/LC interface will change with LC voltage, as the refractive index of the LC changes.

For the ITO film on the back surface of the LC cell, the situation is a little different, because the refractive index of the incident medium will change with LC voltage. This will cause a change in the incident angle, \( \theta \), which will change the value of \( \delta \). However, the effect of this is very small, as for an incident angle of 10°, \( \delta \) only changes from 942.65nm (i.e. 1.4897) for \( n_o \) to 941.83nm (1.4884) for \( n_r \). Using equation 6-11, if \( I_0=I_f=0.5 \), then this change in \( \delta \) would cause a change in \( I \) from 1.99788 (\( n_o \)) to 1.99732 (\( n_r \)), which is negligible.

6.4.2.c  LC cell as reflecting cavity

The interference effects caused by the LC cell can be modelled by two mirrors (equivalent to the ITO layers) separated by the LC medium i.e. the LC cell can act as the plane parallel layer in Figure 6-7, between two layers of ITO [Davis et al., 1999]. As the LC switches, its effective refractive index, \( n_e \), changes. Considering the simpler case of a parallel aligned nematic LC, the effect of the change in refractive index upon transmitted intensity is shown in Figure 6-8 and Figure 6-9. These graphs show how multiple beam reflection (not the simpler two beam interference case considered in [Davis et al., 1999]) change the intensity of reflected light with both refractive index (\( n_o \) of equation 2-21) and director orientation (from 0 to 90°). These figures were plotted using Airy’s formula for a plane parallel plate which is:
Chapter 6 Algorithm to determine unknown polarisation

\[ I_r = \frac{4R \sin^2 \frac{\delta}{2}}{(1-R)^2 + 4R \sin^2 \frac{\delta}{2}} \]

Equation 6-12
(from [Born & Wolf, 1999, p.361])

Where \( \delta = \frac{4\pi l d \cos \theta'}{\lambda_0} \) and \( R \) is given by Equation 6-10.

It can be seen that for normal incidence, the reflected intensity varies between 7% and zero (6.2% and zero if only the two beam interference case is considered), depending on \( n_c \).

Figure 6-8 and Figure 6-9 ignore the effect of polarisation. The variation in reflection will depend on the proportions of the incident vector which lie along the \( n_g \) and \( n_o \) directions, and this, of course, depends upon the polarisation of the incident light.

6.4.2.4 Electric field effects/variation in LC cell thickness

When a voltage is applied across a LC cell, the opposite sides of the cell are oppositely charged. The walls of the cell will be attracted to each other, and this may cause the thickness of the cell to change. Figure 6-10 shows the effect a change in LC cell thickness would have upon the reflected intensity. The maximum reflected intensity for \( n_o \) is 7.2%. It can be seen that the reflected intensity is very sensitive to changes in LC thickness.
The effects shown discussed in sections 6.4.2.c. and 6.4.2.d are not additive, as the maximum reflected intensity occurs when the optical path lengths between the two interfering beams are a multiple of $\lambda$. The maximum reflected intensity in each case (LC thickness or refractive index variation) is in the region of 7%. Although these effects do not fully explain the variation shown in Figure 6-4, they do explain why some variation in reflected intensity would occur as the LC voltage changes. The structure of the TNLC is far more complicated than assumed in Figure 6-8 and Figure 6-9 and so it is likely that this does not fully represent the actual reflection that is occurring from the LC. The reason why these effects were not so marked (a slight change in reflection was noted) with the LCTVD is not clear. It is possible that the diffraction pattern masked some of the effects: only the central order was measured, and it is possible that, at the point where this should have been reducing in intensity due to the interference effects discussed above, some of the light from a peripheral order was being redirected into the central order. The unexpected effect of the diffraction pattern on intensity of the diffracted orders is discussed in s.5.6.3.
6.5 Discussion

6.5.1 Advantages of the LCTVD polarimeter

The results show that the commercially produced LCTVD can be used as part of a complete polarimeter. The Stokes based method used above is superior to the Jones based method: it does not have the sign ambiguity caused by taking the square root, can distinguish partially linearly polarised from elliptically polarised light, and gives far better experimental results. The better experimental results occur because, using the Stokes method, the LCTVD can be easily calibrated before each set of readings. This is important because both Jones and Stokes methods are very sensitive to errors. The Jones matrix method used a model of the LCTVD that was calculated using polarisation measurements taken over several days. The LCTVD did not cause exactly the same polarisation change from one day to the next, and so the SVD model was not accurate enough for the Jones matrix method.

The fact that the Stokes polarimeter can be easily calibrated means that it can be used for a variety of wavelengths, and can be used at different temperatures. The polarimeter has no moving parts (except during the calibration procedure), and can be assembled from commercially available components with minimum set-up. Unlike the multiple cell device described by Schirmer [Schirmer et al., 1998], and Wolff [Wolff & Mancini, 1992] it consists of a single LC device, and therefore has fewer reflecting surfaces. The simplicity of the design means it could be assembled into a portable unit, as can the polarimeters described by Schirmer and Wolff. The LCTVD polarimeter could be used in the applications discussed in s.4.6.

The polarimeter does not have the complex optical arrangement of some other polarimeters, which split the beam into four separate components. The use of the same optics to measure each Stokes parameter in turn slows down the measurement process. However, as discussed in s.4.5, it should still be possible to take six sets of measurements per second. If it is known that the light has no circular component, the polarimeter can also be easily adapted to measure only the first three Stokes components. This would be done by assuming component $S_3$ in equation 6-4 was zero, and then just taking three intensity measurements to find $S_0$, $S_1$ and $S_2$. This would decrease the time needed for each set of readings to be taken (and subsequently processed).
6.5.2 Polychromatic sources

6.5.2.a The problem

As mentioned above, the LCTVD polarimeter can easily be calibrated for different wavelengths of incident light. However, the light source may be polychromatic, so it is informative to examine the effect of wavelength upon the results obtained using the polarimeter algorithm. The wavelength of the incident light will affect the polarisation of light emerging from the LCTVD in two ways: the state of polarisation depends (partly) upon the phase difference, $\phi$, between the $x$ and $y$ components of the incident light. This varies according to $\Delta \varphi / \lambda$ (s.2.4.2), so if $\lambda$ changes, so will $\phi$. In addition to this, refractive index varies with wavelength, [Born & Wolf, 1999, p.95], and this phenomenon is called dispersion. This can affect $\Delta n$, which will again change $\phi$. When there is no birefringence (i.e. when the LC is fully switched), $\Delta n=0$, so wavelength will not affect the polarisation. (Although, because dispersion affects the absolute value of $n$, this will change the absolute phase of the wave passing through the LC. This thesis is not concerned with that effect, as it does not affect polarisation). When the LC is unswitched, or partially switched, its birefringence will change $\phi$, and so will be affected by wavelength as discussed above.

6.5.2.b How this affects the LCTVD polarimeter

The LCTVD polarimeter relies upon taking intensity readings at four LCTVD voltages. For at least three of these four readings the birefringence of the LCTVD will be affected by the factors discussed in s.6.5.2.a. The change in refractive index of the LCTVD ($n_e$ and $n_o$) with wavelength is shown in Figure 6-11.
The Jones matrix of the LCTVD was calculated using the program developed at UCL [Kilpatrick et al., 1998], for several wavelengths and associated refractive indices of the LCTVD (from Figure 6-11). This enabled the effect of the LCTVD upon light of different wavelengths to be modelled. The intensity that would be transmitted by an analyser (assumed to absorb equally at all wavelengths) placed with its transmission axis along the x axis was calculated, and this value was then put into the algorithm described above. The algorithm was 'calibrated' using the wavelength 550nm, as this is near the peak sensitivity of the human photopic sensitivity curve [Fletcher & Voke, 1985] and the middle of the visible spectrum. The results are shown in Figure 6-12. Wavelengths modelled were 400, 450, 500, 525, 550, 575, 600, 633, 640, 650 and 700nm. For clarity, only L elliptically polarised light is shown on the figure, but similar results were obtained with R elliptical light.
Chapter 6 Algorithm to determine unknown polarisation

LCTVD as Stokes polarimeter. Variation with wavelength.

- Linear input 30 deg
- Input L circular
- Alpha 120 deg, e=6
- Alpha 120 deg, e=20

Figure 6-12 Variation of results obtained (using theoretical model including dispersion) for different wavelengths (400-700nm) using Mueller matrix model for 550nm. 400nm shown in blue, and 700nm in red. Results for 550nm shown with solid figures.

The range of variation with wavelength varies according to the particular input polarisation. Considering certain input polarisations: when the input polarisation is 30° (linear) the predicted input varies from an azimuth angle of 47.7° with an ellipticity of 32.2 for a wavelength of 400nm, to an azimuth angle of 20.8° and ellipticity of 20.2 for a wavelength of 700nm. Restricting the range of wavelengths from 525-575nm results in the azimuth angle being accurately predicted to +/-3°, and the minimum ellipticity predicted being 277. As a contrast, if the input polarisation is linear with an azimuth angle of 75° (not shown in Figure 6-12) merely changing the wavelength by 25nm, to 525nm results in an azimuth angle of 73.4°, and ellipticity of only 77.5. When the wavelength drops to 500nm, the azimuth angle reduces to 69.1°, and ellipticity to only 8.1 (compared with $\alpha$ of 37.3° and $e$ of 77.3 for 500nm with an input of 30° linear).

If the input polarisation is left circularly polarised, the range of ellipticities which are predicted vary from 5 ($\alpha=145.1°$) at 400nm to 251 ($\alpha=92.8°$) at 700nm, but only from 1.6 ($\alpha=176.3°$)-2.2 ($\alpha=91.1°$) if the range is restricted to 500-600nm. For some elliptical polarisations changing the wavelength can affect the sense of rotation which is predicted: for
an input polarisation of $\alpha=120^\circ$, $e=6$ (left), the ellipticity is again significantly affected by a change of wavelength. Even restricting the wavelength range to +/-50nm produces a range of polarisations predicted from $\alpha=136.3^\circ$, $e=3.6$ (left) to $\alpha=112.1^\circ$, with $e=113.1$ (right). The degree of polarisation also varies from 5.9-0.5 respectively.

For all the input polarisations that were tested, it was found that the values obtained for degree of polarisation, $P$, varied considerably with wavelength. The minimum value was 0.08 for an input polarisation of left circular light, wavelength 700nm, and the maximum value was 2024.2 (sic) for an input polarisation of $\alpha=120^\circ$, $e=10$ left, wavelength 400nm. Even if the wavelength range was restricted from 500-600nm, $P$ varied from 5.6-0.2 (both of these values were for left circularly polarised input). It is important to remember that whereas a value for $P$ of greater than 1 indicates some inaccuracy in the measurement, a value of less than 1 does not, and could lead to the state of polarisation being misinterpreted. (For all the examples quoted in this subsection $P$ should equal 1).

It can therefore be seen that the effect a polychromatic source will have on the results obtained using the LCTVD Stokes polarimeter will vary according to the particular input polarisation. This makes it difficult to adapt the polarimeter to take a polychromatic source into account. The polarimeter works best with a monochromatic source.

6.5.2. c How other researchers have approached this problem

The LC polarimeter proposed by Schirmer relies on taking intensity measurements at four different birefringences. As for the LCTVD polarimeter, these readings would normally be affected by wavelength. To reduce this effect, several (parallel aligned) LC cells of two differing thicknesses (4 and 10$\mu$m) are arranged in series. The LC materials are chosen so that the thinner cells are filled with a LC of a high dispersion and the thicker cells with a LC of a low dispersion. By crossing the fast axes of the thin LC cells with that of the thicker LC cells in this layered structure, the retardance can be made insensitive to wavelength within the specified spectral interval [Schirmer et al., 1998]. To create the four retardation states necessary to measure the Stokes components, the LC cells are alternately switched (fully) on and off, in the same way as those in Wolff's LC polarisation camera. (The LC response to an intermediate voltage is not used). The results Schirmer obtained with this layered structure varied across the spectral range (500-800nm), with the azimuth angle being predicted to within +/-2.95$^\circ$, and the lowest value of $e$ being 148.7 for linearly polarised input light. The achromatism could be improved by combining more than two LC layers, but this increases the loss of light from reflection at the ITO interface (see s.6.4 above).

The LC polarisation camera [Wolff et al., 1997] also only uses LC cells that are switched fully on or off (although these are TN rather than the parallel aligned ones used by
Schirmer). When the cells are fully switched, their birefringence is minimal, so wavelength should not affect the polarisation. However, the behaviour of the TNLC cells when they are unswitched will depend upon the wavelength of the incident light, and for the full 'guiding effect' to occur the Mauguin limit should be satisfied for all wavelengths (s.3.4.4). If the cells were of a similar thickness to the LCTVD used in this thesis the intensity transmitted through an analyser, when the LC cells were unswitched would vary considerably with wavelength. An example of this is shown in Figure 6-13 below, which shows how intensity transmitted through an analyser at 0° would vary with wavelength for an input polarisation of 0° and 45°. This is calculated using the Jones matrix model of the LCTVD described above, and it can be seen that for the fully switched state, the intensity transmitted does not vary significantly with wavelength. (Some slight variation occurs because the actual birefringence in the fully switched state is not zero. This is because the director layers near the LC cell boundary are fixed in place by the alignment layer and do not reorient along the applied electric field. vii)

![Theoretical change in intensity transmitted by analyser along 0 deg with wavelength for unswitched and fully switched LCTVD.](image)

Figure 6-13 Change in intensity transmitted by analyser with wavelength for fully switched and unswitched states of LCTVD.

vii The residual birefringence in a fully switched TNLC cell is less than that in a fully switched parallel aligned cell because the alignment layers are crossed with respect to each other. Any residual birefringence caused by the boundary layers remaining fixed on one side of the cell is therefore cancelled out by that on the opposite side of the cell. In a parallel aligned cell, the boundary layers are parallel to each other, and so the residual birefringence will be a result of the addition of the two layers [Pain et al., 1997]. In its fully switched state, therefore, a parallel aligned cell will be affected by a change in wavelength more than a TNLC cell will be. A slight residual birefringence will occur in the TNLC however, because of the pre-tilt of the LC molecules near the alignment layers (see s.3.4.1). This will not cancel itself out.
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The author assumes that the LC cells used in the LC polarisation camera are considerably thicker than the LCTVD used in this thesis, however, there may still be some variation in transmitted intensity with wavelength for the unswitched states.

6.5.3 Oblique transmission

The experiments described in Chapter 5 were all conducted using normally incident light from a laser beam. If the light is not normally incident on the LC surface the situation is complicated by two main factors: a) changes caused by reflection, and b) the fact that the birefringence of the LC varies with angle of incidence. These will be considered in turn.

6.5.3.a Off axis reflection

The effect reflection has upon normally incident light has been considered above (s.6.4). If light is obliquely incident upon the reflecting layers two differences will occur. Firstly, the polarisation of light will be changed upon reflection according to the Fresnel equations (s.2.4.1.b), although this is not significant for angles of incidence of less than 10° (s.6.4.1). Secondly, the amplitude of the reflection will change as shown in Figure 6-8 and Figure 6-9, because the optical path length depends upon the angle of incidence (partly because the distance through the sample is greater if it is traversed obliquely, and partly because of the change in refractive index of the LC with angle of incidence).

6.5.3.b Off axis behaviour of LCD

The polarisation change produced by a LCD depends upon the angle the incident light makes with the director as the light passes through the display (see Figure 2-6). Therefore, the polarisation change will be different for oblique angles of incidence than for normal incidence. This can be understood by considering the tilt of the director in the centre of the cell, and relating this to the angle of incidence of the light passing through the cell (using Figure 2-4). (This is a very simplified model - how the tilt angle of the director varies throughout the cell with the application of voltage can be found in liquid crystal textbooks (eg. [Yeh & Gu, 1999, p.361]) and will not be reproduced here.)

This effect is further complicated if the pre-tilt of the director is taken into account: to avoid an ambiguity of twist direction when the LC relaxes, most TNLCDs are manufactured so that the director adjacent to the alignment ('rubbing') layer has a small amount of pre-tilt (see s.3.4.1). This has the effect that if the TNLCD is illuminated with a non-collimated beam of light, the optical path length of light which passes through the superior portion of the LCD (at
an angle of $+\theta$ to the $z$ axis) will not be the same as that passing through the inferior portion (at an angle of incidence of $-\theta$ to the $z$ axis). This is demonstrated in Figure 6-14 which shows the difference between two vertical viewing positions, A and B. At position A, there is low birefringence, so the polarisation will not be changed much by the LC cell. The contrast will therefore be high (this is near the 'on' state). At position B, there is more birefringence (refer to figure 2-6), so the polarisation state will be changed by the LC cell, and some light will pass through the analyser leading to a loss of contrast in the 'on' state. In practice this has the effect of limiting the viewing angle of the LCD, i.e. the range of incident angles over which the contrast of the display is acceptable [Stallinga et al., 1998].

A result of this is that even if the polarisation is uniform across the incident beam, the polarisation of the emergent beam will not be uniform.

The LCTVD polarimeter relies on luminance measurements, and so the effect of oblique incidence can be gauged by considering the change in luminance with viewing angle. The variation in the calculated luminance transmitted by a (normally white) TNLC cell depends upon the voltage applied, but is reasonably constant (to within approximately 5% of the maximum luminance) for horizontal angles of incidence of +/-15°. However, there is no such constancy for vertically oblique angles of incidence, with even a small change in angle of incidence having a large effect upon luminance for intermediate grey levels [Stallinga et al., 1998, Yeh & Gu, 1999 p.358 et seq.]. The variation in transmission with oblique horizontal and vertical viewing angles is shown in Figure 6-15 and Figure 6-16.

Figure 6-14 Effect of oblique viewing angles: simplified structure of TNLC cell showing orientation of mid-plane director for mid-high voltage. Vertical viewing position A has lower birefringence than viewing position B. From [Stallinga et al., 1998].
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Figure 6-15 Calculated variation in transmission, $T$, for a normally white TNLC cell with horizontal viewing angle for various applied voltages (from [Yeh & Gu, 1999, p.363]). Maximum transmission 0.5 as 50% of incident light is lost through the polarisers.

It has been stated that, in practice, the polarising properties of TNLCDs are fairly constant across a $+/−30^\circ$ conical field of view relative to the normal face of the LC [Wolff & Andreou, 1995; Wolff et al., 1997]. Figure 6-15 and Figure 6-16 show that a variation in horizontal angle of incidence of $+/−30^\circ$ can cause a variation of approximately 10% of the maximum luminance, and a variation of similar dimensions vertically can cause a 90% variation. The sensitivity of the LCTVD polarimeter to slight errors in intensity measurements (s.6.3.2.c) means it is going to produce best results when the incident light is collimated, or of near normal incidence (within $+/−5^\circ$ horizontally and nearer $+/−1^\circ$ vertically). If it is important to maintain the accuracy of the LCTVD polarimeter over a range of horizontal angles of incidence, one or more birefringent compensating films can be sandwiched between the LC cell and the analyser. These ‘neutralise’ the unwanted off axis birefringence and can
significantly decrease the variation of transmission which occurs with viewing angle. Use of appropriate compensation films could enable the polarimeter to be used over a horizontal viewing angle of +/-30°. However, the transmission still varies considerably with a small change in vertical viewing angle [Yeh & Gu, 1999, p.375]. (The polarimeter would be calibrated with the birefringent film in situ).

With a vertical acceptance angle of 8° (-5° to +3°) the parallel aligned LC polarimeter has been found to be accurate in predicting α +/-1.65°, and e of 369-3906 (linearly polarised input) [Schirmer et al., 1998]. The use of compensating films was not discussed.

6.6 Non-polarimeter applications

6.6.1 Reducing reflected highlights

The pixellation of the LCTVD has the advantage that the device could be used to reduce unwanted reflections from a scene selectively, as discussed in s.4.6.2.a. The LCTVD could first be used as a polarimeter, to determine the polarisation of light passing through it. If the scene was imaged on a pixellated detector, this information could then be fed back to the LCTVD. The LCTVD could then be driven to change the polarisation of the light coming from the scene so it is optimally absorbed (or transmitted) by the same analyser (subject to the constraints imposed by the thinness of the display – s.3.4.4). This has the obvious advantage that if the polarisation of light from one portion of the scene is such that it is already optimally absorbed (or transmitted) by the analyser, this need not be changed by the LCTVD.

6.6.2 Security mark reader

A spatially varying pattern of polarisation is produced by the security marks which are discussed in the following two chapters. The LCTVD and single LC cell can be used to read this pattern of polarisation using a similar method to that discussed in the Stokes polarimeter above. The problems caused by reflections when using the single LC cell are not important when using the LC cell as a security mark reader, as this is simply authenticating a known polarisation pattern, rather than trying to identify the exact polarisation state of light incident on the cell.

6.6.3 Polarisation encoding

Recent papers ([Davis et al., 2000; Eriksen et al., 2001]) have demonstrated a system for two dimensional polarisation encoding. Any desired state of polarisation can be generated
using a polariser, two SLMs and two quarter wave plates. Both SLMs have their extraordinary axes at 45° to the input polarisation and are arranged so that the second SLM is between two crossed quarter wave plates (with extraordinary axes parallel and perpendicular to the input polarisation) -see Figure 6-17.

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Figure 6-17 An optical system for converting incident polarised light to any state of polarisation. The lines denote the extraordinary axes of the SLMs and quarter wave plates.

After [Eriksen et al., 2001].

The first part of the system generates any ellipticity of polarisation, and the second part rotates this to the desired angle, the angle of rotation being given by \( \delta_2/2 \) where \( \delta_2 \) is the retardation of the second SLM. The rotation of polarisation comes from the Jones matrix calculation (see equation 2-23):

\[
Rotation = \begin{bmatrix}
1 & 0 \\
0 & -i
\end{bmatrix}
\begin{bmatrix}
\cos^2 \theta \exp\left(i \frac{\delta_1}{2}\right) + \sin^2 \theta \exp\left(-i \frac{\delta_1}{2}\right) & i \sin \left(\frac{\delta_1}{2}\right) \sin 2\theta \\
0 & \cos^2 \theta \exp\left(i \frac{\delta_1}{2}\right) + \sin^2 \theta \exp\left(-i \frac{\delta_1}{2}\right)
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & i
\end{bmatrix}
\]

When \( \theta \) is 45°, this simplifies to

\[
Rotation = \begin{bmatrix}
\cos \left(\frac{\delta_1}{2}\right) & -\sin \left(\frac{\delta_1}{2}\right) \\
\sin \left(\frac{\delta_1}{2}\right) & \cos \left(\frac{\delta_1}{2}\right)
\end{bmatrix}
\]

Equation 6-13

Equation 6-13 is a pure rotation matrix with a rotation angle of \( \delta_2/2 \).

The pixellation of the (LC)SLMs enables a spatial pattern of polarisation to be produced. This could then be identified/decoded using the LCTVD Stokes polarimeter arrangement discussed in this part of the thesis.
Chapter 7

Security device: introduction and literature survey

7.1 Introduction

As a precaution against counterfeiting it may be desirable to attach a mark, which is difficult to reproduce, to an article that indicates its authenticity. In the same way as official scripts are stamped or typed on watermarked papers for authenticity proof, this chapter will introduce the concept of a polarisation varying security marking which could be applied to items such as high value goods, tickets, credit cards, passports etc. The advantages of a polarisation varying security mark will be discussed and security marks already patented will be covered. The chapter will end with a discussion on an alternative method for protecting data, that of the phase mask. The differences between this concept and that of polarisation variation will be explained.

A liquid crystal reader that would automatically verify or reject a polarisation varying security marking will be discussed more fully in Chapters 8 and 9.

\[\text{Digital watermarking (either visible or invisible) techniques have been proposed to protect digital products. Invisible watermarks, which are defined as small alternations of the image data, aim at protecting either authenticity or copyright. For a full discussion of these, and other techniques for protecting information see [Voyatzis & Pitas, 2000].}\]
7.1.1 Proposal for a polarisation varying security mark and automatic reader

A security mark will only be effective against counterfeiting if it is sufficiently difficult to replicate by an unauthorised party. One way to do this is to make the mark difficult to see without special equipment. If the counterfeiters cannot see the mark, it will be more difficult for them to copy it. If the security mark contains a spatially varying pattern of polarisation states, this pattern will be impossible to see without special equipment (at least a pair of polarisers). Even with this special equipment, the counterfeiters may not be able to discern the full polarisation state. One other advantage of using a polarisation varying security mark is that slight changes in the properties of the material of which the mark is made can have considerable effects on the final polarisation state emitted. Only authorised manufacturers of the mark would know the exact parameters required to replicate the mark.

Several patents have been published that detail security marks which, when illuminated by polarised light, emit a spatially varying pattern of polarisation. No automatic reader has so far been patented for such marks. The pattern emitted by the marks patented generally consists of linear polarisation states. It has been seen in Chapter 2 that linear polarisation occurs when the phase difference, $\phi$, between the $x$ and $y$ electric field components of the light is an exact multiple of $\pi$. Any other phase difference leads to the polarisation being elliptically polarised. The marks patented are currently verified by examining or identifying an intensity pattern visible through polarisers. It will be shown in section 7.3.2 that an infinite number of elliptical polarisation states can lead to the same intensity being transmitted by an analyser. To enhance the accuracy of a security mark detection system it is desirable to be able to distinguish between these elliptical states. This work will describe a reader which is sensitive to the full polarisation state emitted by a polarisation varying security mark. The reader is not ‘fooled’ by a counterfeit mark that is made to transmit the same intensity through crossed polarisers as the genuine mark. It is anticipated that the reader could be used in transmission mode, as in Figure 7-1 or in reflection mode, as in Figure 7-2.
Chapter 7. Security device: introduction and literature survey

Figure 7-1 Security mark reader in transmission mode

P=Polariser (fixed), A=Analyser (fixed), LC=Liquid crystal display or single cell, M=polarisation varying security mark, CCD=CCD camera with frame grabber.

Figure 7-2 Security mark reader in reflection mode

BS=non-polarising beamsplitter

The main advantage of using transmission mode is that it requires a less complicated experimental set up to test it. The security marks used in this thesis were transmitting rather than reflecting. However, in real life situations such a mark would have to be inset into a transparent part of the object to be authenticated. This is not as versatile as a mark that is reflective and could be attached to any piece of the article to be protected, such as a passport or a ticket.

7.1.2 Manufacturing methods

It has been seen in Chapter 2 (s.2.4.2) that the state of polarisation can be changed by passing polarised light through a birefringent material. The resultant polarisation depends on the thickness of the material \(d\), the birefringence \(\Delta n\), and the orientation \(\theta\) of the optic axis of the birefringent material.
7.1.2. Changing thickness

A mark made of birefringent material (such as a film made of a stretched polymer) can be made to transmit a pattern of polarisation states by varying the thickness of the material, \( d \), across the mark [Thomas et al., 1997]. This varies \( \Delta n d \), which affects the phase difference induced between the orthogonal components of the electric field, and hence the polarisation. For the same input polarisation, each thickness produces a different output polarisation.

7.1.2.b Changing orientation

Alternatively, some birefringent polymers can be polymerised (for example by curing with polarised UV light) in such a way to make the birefringence vary spatially [Schadt et al., 1992]. The orientation of the optic axis of the polymer layer can be varied across the sample (by the method described below). The birefringence of this material can then be increased by using the polymer layer as an alignment layer for nematic liquid crystals [Schadt et al., 1992]. In this case, \( \Delta n d \) does not change, but \( \theta \) does. As the optic axis rotates, the amplitudes of the components of the incident electric field which are parallel and perpendicular to the optic axis of the birefringent material vary. This leads to a spatial variation of polarisation, when the material is illuminated with polarised light. This is shown schematically in Figure 7-3, which shows how the optic axis of such a material could be patterned.

![Figure 7-3 Spatial variation of birefringence.](image)

Each circle represents an area with a different optic axis. The arrows represent the optic axis. The resultant polarisation pattern is represented by the colours, with each optic axis direction producing a different polarisation.

The security marks used in this work are made using this method. The method of manufacture is described in s.8.4.1 and s.8.5.1. The result is a ‘security mark’ which produces a pattern of polarisation that is determined by the orientation of the photo-alignment layer and the thickness of the nematic LC layer aligned by it. This technique has been patented by Rolic

\(^{ii}\) (provided the input polarisation is not aligned with or perpendicular to the optic axis of the material)
Chapter 7. Security device: introduction and literature survey

Research Ltd. of Switzerland (www.Rolic.com) [Schadt et al., 1992; 1995a; 1995b], and can be used to produce photographic quality images. These images are not visible unless the image is placed between crossed polarisers. The image appears negative when viewed between parallel polarisers.

7.1.3 Verification

Any polarisation varying security mark could simply be examined by viewing it through a sheet polariser. This would reveal an intensity pattern similar to that shown in Figure 7-3. Alternatively, the polariser could be rotated to see if the pattern produced by the mark 'flashes' (i.e. the intensity changes as the polariser is rotated). A 'flashing' would indicate that the pattern produced is (partially or fully) linearly polarised [Sage, 1999]. However, examination of the mark through a linear polariser alone would only reveal the intensity information contained in the birefringent mark. This is equivalent to only measuring the first three Stokes vectors as discussed in Chapter 2. It is clear from Chapter 2 that without a retarding element in the reading system, it is not possible to distinguish partially linearly polarised light from elliptically polarised light as both may transmit the same intensity through an analyser. There are an infinite number of elliptical polarisations that transmit the same intensity through an analyser (s.7.3.2). A reader without a retarding element (the LC in this case) would be unable to distinguish between them, and may falsely verify a polarisation pattern which has an identical intensity 'signature'. This chapter shows how a liquid crystal device can be used to authenticate the full (phase and intensity) polarisation signature of a polarisation varying security mark. An added advantage to this system is that it has no mechanically moving parts.

7.2 Requirements for a security mark

Marks applied to articles may be either covert or overt. The mark proposed in this section is overt in that its presence can be seen, but it has covert features in that its detail cannot be distinguished without the use of verification equipment. The requirements that must be fulfilled for a security mark to be useful are detailed in the following sections.

7.2.1 Difficult to replicate without authorisation

The availability of the material from which the mark is made can be restricted. However, if this was the only defence against the mark being counterfeited, and the
counterfeiters illegally obtained the material, the system would be open to abuse. The marks used in this work do not rely on this factor, as the manufacturing process itself is complex. Even if the counterfeiter were to obtain the photo-alignment layer, the layer would have to be patterned correctly, and then a LC layer of exactly the correct birefringence and thickness would have to be spun on the mark.

In addition to being made from special paper, banknotes include complex patterns that are designed to be difficult to counterfeit. However, with modern reprographic techniques, a printed pattern (i.e. consisting of variations in the intensity domain) is no longer difficult to copy. Even embossed holograms can now be copied [Volodin et al., 1996]. A pattern that is difficult to view would be difficult to counterfeit because it would be difficult to determine if the pattern had been copied correctly. Merely producing a pattern that has the same intensity signature when viewed through linear polarisers would lead to the mark being rejected by the reader.

One other advantage of a polarisation varying security marking is that polarisation is very sensitive to changes in the birefringent layer producing it. The LC polymer used in creating the mark described in the next chapter has $\Delta n = 0.137$ and $d = 2.74 \mu m$. The variation in resultant polarisation with differing film thicknesses is shown in Figure 7-4. It can be seen that a small change in film thickness causes a significant change in azimuth angle and ellipticity. This leads to the actual thickness of the film being critical to produce the desired polarisation pattern. A counterfeiter would have to be able to match the birefringence exactly to replicate the mark. The values for azimuth angle and ellipticity were calculated using equations 2.4-2.6.
Variation of output polarisation with birefringent film thickness. 
Input polarisation 30 deg.

![Graph showing variation of azimuth angle and ellipticity vs. birefringent film thickness. Input polarisation 30°.](image)

Figure 7-4 Variation in azimuth angle and ellipticity vs. birefringent film thickness. Input polarisation 30°.

The sensitivity of polarisation to birefringent film thickness means that the marks have to be manufactured to a high degree of tolerance. The variation in the photographic marks supplied by Rolic is discussed in Chapter 8 (s.8.5.2).

### 7.2.2 Cost effective

The high degree of tolerance required for the security marks may make them expensive to produce. The cost of the mark should be cheap relative to the product it is authenticating, and this may be the downfall of polarisation varying security marks. The optical methods of producing birefringent security marks (see s.7.1.2.b and s.8.5) enable marks to be mass produced, but the exact details of production and repeatability are commercially sensitive.

### 7.2.3 Easy to verify

Ideally, the mark should be able to be verified quickly and automatically to avoid human error. The verification device that is proposed in this work will be able to automatically distinguish between true and false marks. This avoids the necessity for a human operator to examine the mark and verify a pattern (as in the patent published by [Sage, 1999]).
7.2.4 Easy to duplicate by an authorised party

The photo-alignment method described above uses purely optical methods (the polarised UV laser) to align the director of the LC polymer. The lack of any mechanical contact with the alignment layer enables good reproduction of the samples, if the exposure time and polarisation of the polymerising laser are controlled [Jackson et al., 1999]. Some manufacturing tolerance is to be expected from variation in the deposition of the LC polymer layer onto the photo-aligned substrate. Assuming any inaccuracies in the thickness of the LC polymer are uniformly distributed across the security mark, the image will still be visible when it is viewed through polarisers. However, the exact polarisation produced by each part of the mark will be different to that expected if the thickness of the LC polymer layer is not well controlled. This can be an advantage, in that it will be difficult for counterfeiters to duplicate the mark, but it can also be a disadvantage, in that the manufacturing process has to be tightly controlled.

7.3 Liquid Crystal security mark reader: the concept

7.3.1 Original concept

It was initially envisaged that the mark would be illuminated with polarised light as shown in Figure 7-1 and two spatially varying voltage patterns would be written to the LCTVD. These voltage patterns would match the security mark and would change the phase difference of the x and y electric field components of the incident light by two different amounts. It was hoped that these voltages would have the effect so that for one voltage the azimuth angle of the resultant polarisation ellipse would be parallel to, and for the other voltage the azimuth angle would be perpendicular to, a fixed analyser, as shown in Figure 7-5.
These voltage patterns would be unique to the particular security mark, and would give a resultant maximum and minimum intensity transmitted by the analyser respectively. This would only happen for the true mark, and would allow it to be automatically verified or rejected. Figure 7-5 shows how such a security mark reader could be used.

Figure 7-5 Proposed use of LCD to verify birefringent security mark

It has been seen in Chapter 3 (s.3.4.4) that the LCTVD is too thin to achieve the full range of phase change for a wavelength of 632.8nm. However, even if a full range of phase change was available it would not be possible to change any input polarisation to linear polarisation at a specified angle purely by changing the birefringence of the LCTVD (see s.7.3.2 below). To do this the input polarisation (but not the mark or the LC) would have to be rotated. One of the advantages of using a liquid crystal device as a variable retarder is the lack of mechanically moving parts, so it is not desirable to have to rotate the input.

### 7.3.2 Flaws in original concept

Examination of equations 2-4 – 2-6 show that to obtain complete control of the azimuth angle of the polarisation ellipse and its ellipticity both the phase difference (between the x and y electric field components) $\phi$, and the magnitudes of those components ($a$ and $b$), must be controlled. If the input polarisation is fixed, $a$ and $b$ cannot be changed. For the light to be linearly polarised, $\phi$ must be a multiple of $\pi$. If $\cos\phi=1$, equation 2-4 shows that the value of $\alpha$ is determined by the particular values of $a$ and $b$. Without changing $a$ and $b$, the best that can be hoped for is to induce an appropriate $\phi$ so that the azimuth angles of the output polarisation are aligned with (or orthogonal to) the chosen analyser angle. The ellipticity cannot be controlled as well, so (because the light is not linearly polarised) any minimum intensity measured will not be zero. The effect of changing the birefringence of a material on the polarisation of light passing through it, when the input polarisation state is constant, is shown in Figure 7-4.
If the light passing through the LCTVD cannot be converted to linear polarisation aligned with the analyser, the resultant intensity transmitted will not be the maximum possible. When the intensity is not at a maximum (or a minimum) there are many different polarisations which can lead to the same intensity being transmitted by the analyser. The different values of azimuth angle and ellipticity of polarisation that would transmit the same intensity through a fixed analyser can be found from equation 2-25: the intensities are normalised so that \( a^2 + b^2 = 1 \), so the relationship between \( b^2 \) and \( \gamma \) becomes

\[
\frac{I - b^2}{1 - 2b^2} = \cos^2 \gamma
\]

The values of ellipticity and \( \gamma \) required to produce a constant intensity through an analyser can be found from equation 7-1.

\[
\theta = \angle \text{analyser relative to } x \text{ axis} \quad \text{and} \quad \alpha = \angle \text{azimuth angle of polarisation ellipse}
\]

Figure 7-6 shows the graph of ellipticity against \( \gamma \) for a various values of intensity (relative to input intensity of 1). It can be seen that the range of values for the ellipticity and azimuth angle of the polarisation ellipse, which can lead to the same intensity being transmitted by the analyser, depends on the particular transmitted intensity chosen. For example, if the desired intensity is 1, the only output polarisation which will achieve this is linear polarisation with the azimuth angle parallel with the analyser. If the azimuth angle or ellipticity changes from this, the intensity measured through the analyser will decrease (as...
(equation 7-1 confirms this because when \( I = 1 \), the only real solution for \( b_1^2 \) is zero. This occurs when \( \cos^2 \gamma = 1 \). A smaller value for \( \cos^2 \gamma \) gives a negative value for \( b_1^2 \).)

When \( I < 1 \), there are several values of \( b_1^2 \) and \( \gamma \) which give solutions. An interesting case occurs when \( I = 0.5 \). At this position the solution for \( \gamma \) is \( 45^\circ \), regardless of the ellipticity of the polarisation passing through the analyser. This occurs because, when the ellipse is oriented at \( 45^\circ \) to the analyser, the analyser transmits equal components of the major and minor axes of the ellipse. The normalisation of the sum of the squares of the major and minor axes leads to the transmitted intensity remaining constant regardless of the ellipticity.

Figure 7-6 shows lines of constant intensity that would be transmitted through an analyser. The range of ellipticities and azimuth angles that this represents can be read from the graph.

If the reading device could be arranged so that a near minimum or near maximum intensity was transmitted through the analyser, the range of polarisations emitted by the security mark that would cause this would be very restricted. This would mean that the reader could distinguish between a false and a true mark by simply registering this one intensity (as no other polarisation pattern would produce this result). However, as has been shown, this is not possible. Consequently, a false mark may transmit the same intensity as the true mark even though its polarisation pattern was quite different. This could lead to a false positive result.

The security mark would need to be verified using at least 2 LCTVD settings. This is to avoid having to measure the absolute intensity of the source (as one reading is divided by the other to normalise it), and to avoid a false mark being made using simply a linear polariser oriented so as to give the required intensity. In fact, if this were not done, an opaque object could be used to give a minimum intensity.

### 7.3.3 Improved concept

As previously discussed, in order to fully distinguish between different states of polarisation which may be emitted by the security mark it is necessary to do more than just measure the intensity which is transmitted by an analyser. It was felt that, in order to analyse the transmission of each part of the security mark individually, a pixellated detector should be

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From equation 2-25: \( I(\gamma) = a_1^2 \cos^2 \gamma + b_1^2 \sin^2 \gamma \). If \( \gamma = 45^\circ \) then \( \cos^2 \gamma = \sin^2 \gamma = 0.5 \). Therefore \( I(\gamma) = 0.5(a_1^2 + b_1^2) \). As \( a_1^2 + b_1^2 \) is constant, the intensity transmitted will be constant regardless of the individual values of \( a_1^2 \) and \( b_1^2 \).
used. This also had the advantage of enabling an image to be viewed and saved. It has already been shown that the Stokes vector of an incident beam can be fully characterised by passing the beam through the LCTVD at four separate grey levels, and comparing the four intensities transmitted by an analyser. It was felt likely that a similar method would work for verifying the polarisation pattern produced by the security mark. As uniform voltages were applied to the LCD, a pixellated LCD was not necessary, so the reader was designed using the single LC cell rather than the LCTVD (although comparison with the LCTVD was made). A single LC cell is considerably cheaper to produce than the LCTVD, and requires simpler drive electronics. It will be shown in the next chapter that it was possible to verify even the complex photographic slides by passing linearly polarised light through the security mark, LC device and analyser to the CCD detector (as shown in Figure 7-1). The mark was verified by analysing the resultant intensity pattern produced for each of four voltages of the LC device. The method is automatic, quick and simple, and the components of the reader can be obtained for modest cost. The reader was robust to image distortion such as rotation, and sensitive enough to distinguish between slides whose optical path length, $\Delta n d$, differed by only 25nm.

7.4 Relevant patents published

Published patents fall into two categories: those that describe a security mark, and those that describe a reading system. There are many patents relating to security marks, but very few which describe an automatic reading system.

7.4.1 Security marks:

Several techniques have been patented for the fabrication of security marks. Two of these utilise (chiral) liquid crystals as part of a security system. Chiral LCs can be used to change the colour of reflected light so that a document cannot be replicated by colour photocopying [Mueller-Rees et al., 1999]. Alternatively, layers or particles of chiral LC can be used to pattern items with unique reflection patterns [Shanks & Dobrusskin, 1994; Dobrusskin, 1995]. These could be used in inks for security purposes. The polarising properties of the chiral liquid crystals mean that these patterns cannot be reproduced photographically. Several patents detail the use of birefringent materials to produce a polarisation pattern [Shanks & Dobrusskin, 1994; Lynch, 1995; Kanti, 1994; and Sage, 1999], but these rely on producing a pattern of linearly polarised light which is usually verified by human inspection. Other techniques use retarding elements but these are only used as a means
Chapter 7. Security device: introduction and literature survey

to an end to manipulate linearly polarised light, rather than to produce an end form of elliptical polarisation. A retarder may be used to convert linearly polarised light to circular prior to reflection, so that when the light is reflected it remains linearly polarised, but rotated [Lynch, 1995].

Lynch, [1995] describes a method of passing light of a predetermined polarisation through a layer or series of layers of a birefringent material attached to the item to be authenticated. The emergent light is then directed through a fixed second polariser which enables a particular pattern (such as a number) to be revealed. This mark can be made from cellulose derivatives and polyesters, and the patent acknowledges that repetition of an ‘identical’ manufacturing process will result in a marginally different light rotation pattern, even where the same materials are used.

A US patent filed by Kanti, [1994] describes a polarisation-altering overlay sealed to a base print of data (such as a photograph). The overlay is encoded with information that is only readable with a polarising viewer. It describes tamper-resistant and tamper-evident features. After tampering, the overlay provides both evidence of tampering and a view of otherwise non-visible security data. The patent describes the use of polarising material and a quarter wave film overlaid upon the photograph (or document) to be verified. The polarisation altering materials will reveal a pattern when viewed through an analyser. Without the analyser, the marking appears fairly transparent. There are several methods of producing the marking, one of which includes mechanically stretching polymers, to make the film become a polariser. The polymer is then doped with dyes. These dyes can be bleached by exposing them to intense UV, visible or IR radiation which can be used to pattern the films. The authors anticipate generating a pattern of linearly polarised light, rather than the pattern of elliptical polarisations described in s.7.1.2.b above. However, a marking that is made using this method could be read by the reader proposed in this work.

A fluorescent or phosphorescent mark is described by Sage [1999]. This has a unidirectionally aligned structure and when illuminated with UV light emits characteristic polarised fluorescent or phosphorescent radiation. This is detected by rotating an analyser to give a ‘characteristic flashing effect’, which may be an intensity or colour variation. The mark may comprise several sections, each of which emits polarised light along a different orientation, and it may be used in transmission or reflection mode.

Lynch [1998] documents a security marking which either rotates the plane of linear input polarisation by 90° or does not affect it at all. The marking consists of a pattern of blocks of material that either rotate or do not affect the plane of polarisation. The marking therefore has only two states. The binary number of states means that the marking would be
easier to copy fraudulently than would a more complex mark, and would have a reduced
number of possible combinations. However the patent does admit that, for security reasons,
the number of patterns in a system should not be too small. Once again, this patent relies on
the human eye as a detection system, and has no automated method for verifying authenticity.

Finally, the patents filed by Thomas [et al., 1997] and Ames & Thomas [1999] detail
security devices composed from a birefringent film. Thomas’ device consists of a birefringent
film of varying thickness, such that when the material is viewed through a polariser a
characteristic pattern is exhibited. The thickness of the film is varied in production using a
pulsed laser beam, which removes a certain thickness of the material. In addition, the
birefringent property can be removed using a CO\textsubscript{2} laser so that security indicia can be
patterned on it. This patent differs from the others discussed in that it does not consist of
several components being stuck together, as the film is continuous with just its thickness
varying. It can be viewed in transmission, or in reflection, with the addition of a reflective
layer. Ames & Thomas’ patent is designed to be used in reflection mode and consists of a
layered structure with birefringent material between the layers. The thickness of the
birefringent material can be varied to alter the polarising characteristics. No automatic reading
system is detailed for either of these patents.

It should be possible for the polarisation pattern emitted by any of these patents to be
read by the LC reader proposed in this work.

### 7.4.2 Reading systems

Most of the patents for ‘security readers’ relate to readers for smart cards (i.e.
electronic methods, not optical). One patent that uses a liquid crystal display is by Hobbs
[1999]. This patent describes a method of driving and backlighting a LCD to act as a smart
card reader. The LCD is used as a conventional display, to show information obtained from
reading an identity card or driving licence (etc). The pattern displayed is used to verify the
information on the card. This system does not use the LCD to directly authenticate the card –
a human operator has to view the information display on the screen.
Kanti [1994] mentions using a twisted nematic liquid crystal cell in front of a fixed analyser as
part of a verification system, but this is only used to produce a ‘flashing effect’ equivalent to
rotating the analyser. In Lynch’s patent [1995] the light pattern which is revealed ‘can then be
analysed by suitable electro-optic methods to produce a machine readable identification of the
light pattern’. Unfortunately, these methods are not described.
The security mark reader proposed in this thesis does not rely on producing linearly polarised light, but can cope with all forms of elliptically polarised light too. It does not rely on human judgement of whether or not the mark is genuine.

7.5 Phase masks

Javidi and colleagues ([Javidi, 1998; Javidi, Sergent & Ahouzi, 1998; Javidi & Sergent, 1997]) have published work using a phase mask as part of a security marking system. A phase mask works by altering the (absolute) phase of light. This is concerned with the phase of light in one spatial area compared with that in another, rather than the phase of components of polarised light along the \(x\) and \(y\) axes which affect polarisation. When light passes through any transparent material that has a different refractive index to the incident medium, the phase of the emergent light is different to that of the incident light. This is changed by the optical path length of the material. If the material is patterned so that the path length, \(nd\) is varied spatially (in a similar way to the way birefringence is patterned in Figure 7-3) then the resultant wavefront will have a phase pattern which also varies spatially. The polarisation, however, will not be altered unless the material is also birefringent.

Phase information, like polarisation information, cannot be detected by the naked eye, so this is recovered using interferometric techniques. For the phase change induced by the phase mask to be recovered, the phase mask must be illuminated by coherent light.

The phase mask can be a thin sheet of transparent plastic [Javidi, 1998], which can be made from an embossed plastic film (to vary its thickness, \(d\)), bleached photographic film, or etched onto a glass or metallic reflector [Javidi & Horner, 1994]. The mask is bonded to the image to be protected (for example a photograph or a fingerprint). This encodes the image. The verification procedure involves illuminating the phase mask and image with coherent light, and interfering this with a reference image in the Fourier plane. The verification procedure is in effect a two dimensional correlation between the pattern and a reference pattern in the phase domain. The correlation can be performed either using one or two spatial light modulators (SLMs) [Javidi & Horner, 1989] or using a photorefractive polymer [Volodin et al., 1996]. When only one SLM is used to verify the phase mask, the SLM first displays the reference and input signals. The interference between the Fourier transforms of the input signals is obtained optically and collected by a charge-coupled device (CCD). This interference intensity array is then stored in a frame buffer. This array is then directed to the SLM and the Fourier transform of this pattern (illuminated with the same coherent light) is...
used to correlate the image. The same SLM is therefore used in the correlation as well as the acquisition procedures.

### 7.5.1 Advantages

1. The phase mask system is invisible to the naked eye and this means it is more difficult to copy than an intensity based system.
2. The correlation system is also resilient to noise [Volodin et al., 1996] and bending of the mask [Javidi et al., 1996]
3. The mask can be used in transmission or reflection.
4. The mask can also be encoded with biometric data in addition to the data on the intensity image it protects [Javidi & Horner, 1994].

### 7.5.2 Disadvantages and comparison

1. As polarisation is sensitive to relative phase changes, absolute phase change, as produced by the phase mask is likely to suffer from problems with reproducibility. The method developed by Rolic Ltd. and at Hull University (see Chapter 8) means that birefringent layers can be optically patterned. It is hoped that this will be a more reproducible manufacturing technique than mechanical methods. Unfortunately, the manufacturing process of the phase masks ([Javidi, 1998; Javidi & Homer, 1994] is not detailed, but it is presumed that the phase masks, like the polarisation marks presented here, are very sensitive to small changes in the security features produced. As mentioned above, this can be both an advantage, in that counterfeiting is made more difficult, and a disadvantage, in that manufacturing processes have to be very tightly controlled.
2. It has been seen that when it was illuminated by coherent light, interference caused the intensity transmitted by the LCTVD to vary as it was moved horizontally and vertically. This means that the thickness of all optical components must be very tightly controlled, as variation in these components can lead to errors (s.5.6.6). Polarisation change can be detected if the system is illuminated with non-coherent light (e.g. the LED used in Chapter 8), so the thickness of the optical components is not so crucial (although the thickness of birefringent components must still be controlled).
3. The system requires that, usually, an additional phase mask is held by the authenticator to interfere with the pattern coming from the encrypted image. This may not be a problem if the mask is only to be verified in one place, but if the mask was to be verified at several places (such as airports if it was attached to a passport) every entry point would need a
genuine mask to compare the tendered version with. Using the polarisation varying mark, all that is required is knowledge of the intensity pattern transmitted through the liquid crystal layer. This means that there can be many 'stations' at which the mark can be verified, which is essential if the mark is to be applied to items such as banknotes which can be tendered at retail outlets. A method is described whereby the authentication code is written onto a SLM in the input plane, and presumably this could be done remotely and distributed to authorised users. However, it relies on each user having an identical SLM for the system to be effective.

4. The correlation procedure is far more complex than the technique proposed in this thesis. At least one SLM is usually required, together with Fourier optics and a coherent light source. This adds to the cost of the equipment (see s.1.2). The SLM needs to be flatter than the LC device in the polarisation system because phase variations in the SLM will add to the variations in the phase mask. The method proposed in this work does not need a coherent light source, and only requires lenses to ensure that the whole polarisation mark is illuminated and imaged onto a CCD detector. The only other equipment required is the liquid crystal device, a pair of linear polarisers and the frame grabbing and processing equipment to convert the CCD image to intensity values.
Chapter 8. Security device

Chapter 8

Security device

8.1 Introduction

The background of optical security markings has been covered in the preceding chapter, and it has been shown that some security marks are produced which emit a spatially varying polarisation pattern. The aim of this chapter is to design a system that can distinguish between security marks that emit different patterns of polarisation, and to explore the limitations of such a system. The system was initially tested with very simple marks consisting of four areas of different birefringence [Blakeney et al., 2000], then with more complex ('Hull') marks consisting of 9 independent areas of birefringence. Finally, the LC security mark reader was tested using photographic quality ('Rolic') marks that emit a very complex pattern of polarisation which is not discernible to the unaided eye. The results show that the reader is reliable in distinguishing between complex polarisation varying security marks and is sensitive to slight rotations of the marks. The chance of a false positive result is very low. The sensitivity of the system can be customised to meet the requirements of the end user (this will be discussed in Chapter 9).

This chapter will begin by describing the method and experimental set-up used to test the system, it will then detail the two types of security marks that were used. The results for both the 'Hull' security marks, and the complex 'Rolic' marks will then be presented. The chapter ends with a critical analysis of the results and discussion.
8.2 Experimental set-up used to measure and verify the security marks

8.2.1 Overview

Figure 8-1 shows the experimental arrangement used to measure the polarisation coming from each piece of the mark. The initial tests on the system were done using the HeNe laser and the photodiode described in Chapter 5. These were used because their characteristics were well known and it was considered prudent not to introduce too many unknown factors into the experiment initially. These results were published [Blakeney et al., 2000] and will not be considered further. The system was then redesigned to enable it to cope with more complex birefringent security marks.

The following sections describe the apparatus used to test the LCTVD, and the single LC cell described in Chapter 6, as part of a reader to automatically read polarisation patterns. The principle used was similar to that of the Stokes polarimeter (Chapter 6) in that polarised light was passed through the security mark, the LC, and a fixed analyser to the detector. The particular pattern of polarisation, which is emitted by each security mark, is changed by the LC, and the amount of that change depends on the voltage applied to it. This, in turn affects the intensity that is transmitted by the analyser to the detector. The intensity recorded by the detector is measured for four different voltages applied to the LC and this enables the polarisation 'signature' emitted by the security mark to be verified. When the LC is used as a security mark reader, it is not necessary to determine the Stokes vectors of the light arriving at the LC. All that is required is to determine whether the polarisation signature is that of the genuine mark or not.
8.2.2 Light source

It was felt that it was unnecessary (and undesirable because of the interference problems) to have a coherent light source, and it was desirable if the security mark reader could be designed using a portable, cheap, low power light source instead of the laser. For this purpose, the laser was replaced with a LED (Kingbright 5mm round superbright LED). This had a wavelength of 660nm, and spectral line half width of 20nm. The light emitted was unpolarised (checked by rotating an analyser \(^1\)). The stability of the LED over time was checked, and is shown in Figure 8-2. It can be seen that although the total intensity drops by 3% from 10 mins to 160 mins, after an initial warm up period of approximately 15 mins, the intensity is stable for a period of at least 8 minutes. All the measurements were conducted within this period, so the probability of intensity variations during the time interval in which measurements were taken is small. The security mark was verified using the ratio of the intensities recorded by the detector for the four voltages applied to the LC. This meant that the absolute intensity was not important, so long as it didn’t vary whilst each set of four readings was being taken.

\(^1\) with and without a QWP in place
Chapter 8. Security device

LED calibration: variation of intensity through analyser at 0 deg with time

Figure 8-2 LED stability with time

The divergence of the light from the LED meant that the beam covered the whole security marking. To ensure normal incidence across the LCD (see the discussion on oblique incidence in s.6.5.3), the light was collimated before it passed through the security mark. A disadvantage of the LED is that it does not produce a spatially uniform beam. Therefore it was important that the security mark was not allowed to move between intensity readings, and the same area of the image was always compared with itself.

8.2.3 Liquid crystal display

It was hoped that it would be possible to use the arrangement of the Stokes polarimeter to determine the Stokes vectors of the light coming from the security mark. However, it was found that the dynamic range of the CCD (see next section) was not wide enough for this to be done. The experiments done using the Hull security marks (s.8.4) were performed using both the LCTVD and the single LC cell. It was found that the LC cell gave a more definite cut off between the ‘true’ and ‘false’ slides and so subsequent experiments were done using the single LC cell. The single LC cell has cost and simplicity advantages over the LCTVD.
8.2.4 Detection system

8.2.4.a Advantages and disadvantages of CCD system over photodiode

When the laser was used, the photodiode described in Chapter 5 was used as a detector. However, it was considered advantageous to be able to take measurements from the whole area of the mark simultaneously. In order to do this, a CCD camera was used. The camera used was a Pulnix TM-6CN camera, with a Pentax 25mm f1.4 TV lens. The intensity of the captured image was then digitised using a Matrox Meteor frame grabber and image processing software (Image Pro Plus). An advantage of the CCD system was that the whole image could be viewed and saved, and each area could then be analysed separately. A disadvantage was its limited dynamic range compared with the photodiode and amplifier. When using the CCD, to achieve as near linear response to intensity as possible, the gamma setting was set to 1, and the automatic gain control was disabled [Liang et al., 1996]. The brightness and contrast settings were set on the frame grabber to give the largest dynamic range of the CCD (see s. 8.2.4.a). However, this still did not give as large a dynamic range as when using the photodiode. This is because when the photodiode was used, it was connected to an amplifier, the sensitivity of which could be changed. This meant that when the reading saturated one range, the sensitivity could be reduced by a factor of 10. The power that was incident on the photodiode was limited by the maximum power that could be incident on the LCTVD (s.5.3.2). This intensity did not saturate the photodiode.

To avoid the image saturating the CCD unnecessarily, an extra polariser was placed between the LED and the ‘initial’ polariser. Rotation of this extra polariser enabled the total light intensity to be varied without affecting the polarisation.

8.2.4.b Settings used on CCD system/frame grabber and linearity of the system

The linearity of the CCD detection system was checked by recording the intensity of an area of the image (of the LED – no LC present) through an analyser as it was rotated. This was repeated for different contrast and brightness settings on the frame grabber, and compared with readings taken using the photodiode. The results are shown in Figure 8-3. To compare the response of the CCD with the photodiode, the photodiode has been adjusted to GLs by

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ii The same effect could be achieved by changing the voltage on the LED, but this would not give as great a dynamic range.
normalising it and multiplying by 254 (which was the maximum GL reading in the grabbed images).

If the response of the CCD is linearly proportional to the intensity of light falling on it the GL measured should vary with \( \cos^2 \gamma \), where \( \gamma \) is the angle of the analyser relative to the azimuth angle of the ellipse of polarisation (equation 2-25). It can be seen from Figure 8-3 that the photodiode response very nearly fulfils this criterion. The response of the CCD is not exactly linear, but the most linear response with the greatest dynamic range was achieved with the contrast set to 255 and the brightness to 230.

![Average GL recorded vs analyser position. Frame grabber contrast set at 255, different brightness settings. Comparison with photodiode response.](image)

Figure 8-3 Variation of intensity with analyser position. Contrast constant at 255, different brightness settings. Comparison with photodiode response and \( \cos^2 \) curve.

### 8.3 Reading technique

#### 8.3.1 Method

To test the system it was necessary to fix the input polarisation and the analyser and capture images of the security slides with the LC (LCTVD or single LC cell) at four different voltages. Using the experimental arrangement as shown in Figure 8-1, the (second) polariser and analyser were set with their transmission axes along the x axis (0°). Each test slide was mounted in front of the LC, and the lens on the CCD camera was focussed on the slide. The image of the slide, through the LC and analyser was grabbed and stored. This was then repeated with a different voltage applied to the LC. The experiment was performed first using
the LCTVD, displaying a uniform GL for all pixels (as in Chapter 5), and then the LCTVD was replaced by the single LC cell.

When the initial experiments were conducted [Blakeney et al., 2000] it was found that the accuracy of the system did not increase if more than 4 LCTVD voltages were used. Interestingly, this is the same number of voltages that were necessary to determine the Stokes parameters of light passing through the LCTVD. The LC voltages that were used were evenly spaced across the voltage range that could be applied to the LC. For the LCTVD the GLs chosen were 255, 140, 60, and 0. When the single LC cell was used the r.m.s. voltages were 0V, 1.85V, 2.56V and 3.86V.

For each voltage of the LC, the grabbed image of the slide was divided up into several areas of interest (AOIs) and then analysed using image processing software. The average intensity of each AOI was recorded. The change in intensity of each AOI with LC voltage was unique to each security mark, and it was this which was used to authenticate the mark (see s. 8.3.4).

8.3.2 Registration of slides

It was envisaged that in a practical security mark reader, each slide would have a marking, notch, or similar so that when it is inserted into the reader its position would be registered correctly. This was not present in the prototype slides, so a convenient registration point was input, by eye. The AOIs were a fixed distance from this point (see Appendix 8C). (This point was the upper left-hand corner of the Hull slides, and the seam in the lapel in the Rolic slides). Each slide was mounted in a slightly different position on the microscope slide, so a different start pixel co-ordinate was input for each slide. When the measurements were taken on different days, the slide may have moved slightly, so, again the starting pixel co-ordinate could be slightly different.

8.3.3 Limitations of detector

The range of GLs on the captured images was 1-254 inclusive. It was anticipated that if the image of the slide was so bright (or dark) that the CCD was saturated (or clipped), the accuracy of the system as a security mark reader would be reduced. This would occur because if the image from the true slide was saturated, the GL of the captured image would be 254 in all areas which were measured. A false slide which also saturated the detector would give the

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iii These corresponded to approximately evenly spaced voltages rather than GLs, as the relationship between GL and voltage was not linear (see Appendix 5A, figure 5A-2).
iv In a similar way that a floppy disk on a computer can only be inserted in one position.
same intensity pattern. This could happen for one or more of the voltages applied to the LC. To investigate the effect of this absolute intensity variation, the images of each slide were grabbed with the first polariser at different orientations. The effect of this was that each set of four images (one for each LC voltage) had a different intensity, and so more or less points in the image which clipped or saturated the detector.

In practice, it was found that the limited dynamic range of the CCD did lead to some of the intensities which were recorded from parts of the slides being either saturated or clipped. It can be seen from Figure 8-3 that a reasonable linearity of the CCD is obtained for GLs > 40. Below this GL, it was considered that the intensity could be clipped, and therefore not reliable for the purposes of the security mark verification. To set the higher limit of linearity, it was felt that any slight imperfections, such as particles of dust, on the AOI would reduce the average intensity recorded. The theoretical maximum GL that could be recorded was 254, so an average GL of 210 was taken as the maximum intensity that could be recorded before the detector was regarded as being saturated. Therefore, if the average intensity of an area was > 210 or < 40, it was disregarded completely. This is discussed in s.8.3.5.

8.3.4 Error calculation

The image of each slide was captured four times (one for each LC voltage). Then each slide was divided up into separate AOIs. There were 9 AOIs for the Hull slides (see s.8.4.2) and 99 for the Rolic slides (see s.8.4.2). The average intensity over each AOI was obtained using image processing software, so this gave a series of four intensities for each AOI of each slide: \( I_{y1}, I_{y2}, I_{y3}, I_{y4} \). It was these intensities which were used to determine whether or not the ‘test’ slide was identical to the ‘master’ slide with which it was being compared. Each area of each slide was considered separately, and an error score for each area was calculated as follows:

1) The mean for each area was calculated \{i.e. \((I_{y1}+I_{y2}+I_{y3}+I_{y4})/4\)\} giving, \( I_{\text{ave(test)}} \) and \( I_{\text{ave(master)}} \). NB only those points which were being compared were used in calculating this average. If, say, \( I_{y2} \) had intensity of GL10, the average would be \((I_{y1}+I_{y2}+I_{y4})/3\)

2) For each area of the test slide, each intensity was multiplied by \( I_{\text{ave(master)}}/I_{\text{ave(test)}} \) to give a corrected intensity \( I_{y1(\text{corrected test})}, I_{y2(\text{corrected test})}, I_{y3(\text{corrected test})}, I_{y4(\text{corrected test})} \). This reduced the effect of absolute intensity fluctuations between the ‘test’ image grab and the ‘master’ image grab.

3) For each voltage, this corrected intensity was divided by the master intensity for that voltage to give an error score for that voltage so that \( E_{y1} = I_{y1(\text{corrected test})}/I_{y1(\text{master})}; E_{y2} = I_{y2(\text{corrected test})}/I_{y2(\text{master})}; \) and so on.

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4) The total error score, $E_{AOI}$, for each AOI was found using
\[ E_{AOI} = |1 - E_{\nu_1}| + |1 - E_{\nu_2}| + |1 - E_{\nu_3}| + |1 - E_{\nu_4}| \]

5) The total error score, $E$, for the slide was calculated using
\[ E = \frac{\sum_{i=1}^{m} E_{AOI}}{m} \]

where $m$ is the number of slide areas used (i.e. 9 or 99 for the above examples). A perfect match would have a total error score of zero.

Several 'correlation-like' algorithms have been used in the image processing literature to quantify image similarity (see [Trucco & Verri, 1998]), but the method above was simple and appropriate and there was no time to explore this aspect further.

### 8.3.5 Comparison points

It has been seen (s.8.3.3) that only AOIs with an average intensity of 40 to 210 were considered to be reliable, and intensities outside this range were disregarded. If an AOI had an average intensity of between 40 and 210 it was called a comparison point, as it was used in the verification of the slide. It was found that if the range of comparison GLs was expanded, the sensitivity of the system reduced, because the detector was saturated or clipped. If the range of comparison GLs was restricted, less comparison points would be obtained for each slide, and this would again reduce sensitivity. This can be easily understood by considering what would happen if only (say) one AOI at (say) two LC voltages were compared. This would give only two comparison points and it is possible that a counterfeit mark may match the genuine mark for these two points even if the counterfeit mark was totally different in every other respect to the genuine mark. It can therefore be seen that the more the number of areas and voltages of the LC which are compared between true and false slides the less likely it is that a counterfeit mark would be authenticated.

As the slide was verified by comparing at least 2 out of 4 intensities for each area, if there was only one intensity reading for that particular area which was within the range of GL 40-210, then that whole area was disregarded. The effect of the range of GL values used to compare each AOI, together with the total possible error scores are discussed in s.9.4.2.b.

From the above discussion it can be seen that after examination, each slide had a figure for error score, $E$, and the number of points which were compared. These will be called comparison points, and denoted by $C$. 

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8.4 Hull security marks

8.4.1 Manufacture and characteristics

The theory behind the manufacture of the security marks has been covered in Chapter 7 (s.7.1.2.b). Some birefringent security marks were made during a visit to Hull University. Each 'Hull' mark consisted of a 3 x 3 matrix of 5.5mm diameter circles on a 1” square glass slide. Each mark was made by covering the glass slide with a photo-alignment layer. This was then polymerised so that the molecular orientation in each circle was different from its neighbour (rotated in the xy plane) [Jackson et al., 1999]. The exact orientation of the slides in front of the laser is presented in Appendix 8A. The polymerisation was achieved using a 10mW Argon-Ion laser (301nm) for 15s. In order to render this layer sufficiently birefringent to change light polarisation significantly, this layer was then coated with a LC polymer. The polymer used was Merck RMM17 dissolved 50 wt% in xylene. This was heated to 70°C (to reach its isotropic phase), filtered through 0.2 micron PTFE tape, and then spun onto the substrate at 3000 RPM for 30s. This gave a film thickness of 2.74 microns giving an approximate $\Delta n d$ value of 375nm. The LC layer was then polymerised by exposing it to an unpolarised UV source (400nm) at 9mW/cm² for 10 minutes. A microscope slide was then attached to the 1” square, face down, to protect the LC polymer from dust etc. The end result was a slide where each circle on the slide represented a portion of birefringence oriented at a different angle to the x axis in the xy plane. Figure 8-4 shows an image of slide 3 captured by the CCD. The intermediate regions were non-aligned, so scattered light, can be seen between the circles of aligned LC polymer. 10 different slides were produced.

* Richard Harding, Merck UK, Personal communication
**8.4.2 AOI selection used to verify Hull slides**

It can be seen from Figure 8-4 that each slide contained some imperfections (partly from the manufacturing process, and partly from sticking the slides to the microscope slide). It was desirable to concentrate on looking at only the polarisation varying properties of each slide, and this was done by only analysing the portions of each slide in the centre of each circle. Therefore, for each slide, the AOIs were selected to only encompass the 9 specific areas shown in Figure 8-5. The image of each slide was grabbed four times (one for each LC voltage), so there were 36 AOIs in total to be compared. These were used to authenticate the slide’s polarisation signature. Each AOI was 52 x 65 pixels in size (rectangular, for ease of measurement). For each AOI, the average intensity was recorded, and saved.

**8.4.3 Experimental details**

Ten Hull slides were made, and these were measured both upright and upside down, to give 20 different slides that could be compared with each other. An error score was calculated for each of the 20 Hull slides as master compared with the other 19 slides as ‘tests’. The image of each slide was grabbed at least three times (at different intensities), so each master was compared with at least 57 test slides. To determine the reliability of the system in
identifying true slides (i.e. the same slide at different intensities) some slides were grabbed many times and each of these grabs was compared with the other grabs of the same slide. In order to analyse the effect of the number of comparison points, some of the images were grabbed when parts of the slide were quite clearly saturating or clipping the detector. The results were then separated into true and false slides and a histogram of the error scores was plotted.

8.4.4 Results for Hull slides

The error score histograms are shown below. It was found that when the number of comparison points, $C$, was low (the maximum number of comparison points was $4 \times$ the number of areas compared), any overlap between true slides and false slides increased. The results presented below show both the full results, and those obtained when the error scores were ignored if $C$ fell below a certain threshold. The scales on the histograms of true and false slides are different due to there being many more false than true slides.
8.4.4.a Using LCTVD

Figure 8-6 and Figure 8-7 show the histograms for the error scores obtained using the LCTVD as a reader. In these figures, all points were counted, regardless of the value of C.

![Histogram of error scores. True slides only. C>0. LCTVD as reader. Mean error score=0.0187 Standard deviation=0.0121](image1)

Figure 8-6 Histogram of error scores. True slides only. C>0. LCTVD as reader.

Mean error score=0.0187 Standard deviation=0.0121

![Histogram of error scores. False slides only. C>0. LCTVD as reader. Mean error score=0.3446 Standard deviation=0.1262](image2)

Figure 8-7 Histogram of error scores. False slides only. C>0. LCTVD as reader.

Mean error score=0.3446 Standard deviation=0.1262

Figure 8-8 and Figure 8-9 show the corresponding histograms for points where C≥10 (max value 36), using the LCTVD as a reader. It can clearly be seen that the area of overlap between the true and the false slides is reduced.
Only including data where the number of comparison points is greater than 9 not only reduces the area of overlap between the true and the false slides, but also reduces the standard deviation of the errors obtained for the true slides much more than it does for the false slides. The standard deviation for the true slides has reduced by 11.5% when C>9, whereas for the false slides it has only reduced by 4.2%. This indicates that increasing the number of comparison points decreases the spread of error scores obtained for true slides (as would be expected).
8.4.4.b Using single LC cell

Similarly, Figure 8-10 and Figure 8-11 show the error histograms for all true and false slides with the single LC cell as the reader.

![Histogram for true slides](image1.png)

Figure 8-10 Histogram of error scores. True slides only. $C \geq 0$. LC cell used as reader.

Mean error score=0.0153  Standard deviation=0.0097

![Histogram for false slides](image2.png)

Figure 8-11 Histogram of error scores. False slides only. $C \geq 0$. LC cell used as reader.

Mean error score=0.4309  Standard deviation=0.1869
Figure 8-12 and Figure 8-13 show the histograms when the number of correlating points is restricted to 10 or more.

**Figure 8-12** Histogram of error scores. True slides only. \( C \geq 10 \). LC cell as reader.

Mean error score = 0.0136
Standard deviation = 0.0076

**Figure 8-13** Histogram of error scores. False slides only. \( C \geq 10 \). LC cell as reader.

Mean error score = 0.4395
Standard deviation = 0.1562

As for the LCTVD reader, the standard deviation of the true slides reduces by more for the true slides (21.7%) than it does for the false slides (16.4%) when the number of comparison points is restricted to 10 or more.

Looking at these histograms, when \( C \geq 10 \), using the LCTVD as reader, the minimum value of \( E \) for a false slide is 0.0347, whilst the maximum value for a true slide is 0.055. There
is therefore some overlap between the error scores obtained for true and false slides. However, when the single LC cell is used as a reader, when \( C \geq 10 \), the comparable values are 0.0786 and 0.0324. There is therefore no overlap between true and false slides. Therefore, the single LC cell, appears to give a better cut off between true and false slides. This is probably because of temporal fluctuations in the director configuration caused by the drive electronics. Although these histograms indicate that the LCTVD would make a fairly successful security mark reader, it was felt that for reasons of cost and simplicity, together with a better cut off between true and false slides, the single pixel LC would be more effective in an optimised system. For the rest of the experiments therefore, the single LC cell was used instead of the LCTVD. It should be noted that just because there is no overlap between the true and the false slides measured does not necessarily mean that the reader is 100\% accurate at distinguishing between them. The statistical analysis of these histograms will be discussed in Chapter 9.

8.5 Photographic quality security marks from Rolic Research Ltd.

8.5.1 Manufacture and characteristics

Some photographic polarisation marks were kindly donated by Rolic Research Ltd. of Switzerland. These are made by patterning a photo-alignment layer onto a substrate (in this case plastic) and the coating the substrate with LC polymer. The image used is shown in Figure 8-14. The marks were supplied on plastic film, but were then mounted on glass microscope slides, for ease of handling. The image in Figure 8-14 is obtained using the experimental arrangement shown in Figure 8-1.
8.5.2 Verification of photographic slide parameters

Three slides were supplied by Rolic (numbered 20, 22 and 24), with quoted \( \Delta nd \) values of approximately 235, 210 and 280nm respectively (wavelength unknown). It was decided to verify these values experimentally. To do this, the apparatus was arranged as in Figure 8-15. The laser was passed through two separate areas (areas A and B in Figure 8-14). The diameter of the laser beam was approximately 1mm. To see what area of the slide this represented a transparent ruler was imaged by the CCD. It could then be seen how many image pixels represented a circle of 1mm diameter.

Areas A and B were chosen because they had different intensities when viewed through crossed (or parallel) polarisers, and appeared relatively uniform over the area of interest (AOI). For each area, polarised light was passed through the slide, and the resultant polarisation was measured. This was repeated for several input polarisations. Equation 2-23 allows the polarisation of light emerging from a birefringent sheet with birefringence \( \Delta n \), thickness \( d \), and optic axis orientation \( \theta \) to be determined. This was calculated for each input polarisation used and the results were compared with the experimental results to give an optimum value for \( \Delta n \) and \( \theta \). How \( \alpha \) and \( \epsilon \) change with various values of \( \Delta nd \) is shown in Figure 8-16 and Figure 8-17 for input polarisations of 0° and 60°.
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Figure 8-16 Change in ellipticity and azimuth angle of polarisation with $\Delta nd$ and $\theta$ of birefringent layer. Input polarisation $0^\circ$.

It can be seen from Figure 8-16 that for an input polarisation of $0^\circ$, when $210 > \Delta nd > 310$ nm, there is very little change in azimuth angle though there is a significant change in ellipticity. Considering the difficulties discussed in s.2.5 and s.2.6 in accurately measuring
ellipticity, using this input polarisation alone does not allow an exact determination of $\Delta n$ and $\theta$. It was therefore necessary to use more than one input polarisation. Looking at Figure 8-17 it can be seen that for an input polarisation of 60° there is a significant change in $\alpha$ with $\Delta n$ and $\theta$ for $\Delta n$ values of 150-270nm. It was considered that the best estimation of $\Delta n$ and $\theta$ could therefore be made if the results for a selection of input polarisations were compared.

Four input polarisations were used and the closest values of $\Delta n$ and $\theta$ which would give the experimentally measured output polarisation are shown in Figure 8-18. The full results of the measurements are shown in Appendix 8B.

![Closest match for values of theta and $\Delta n$ for Rolic slides. Areas of bubbles represent number of input polarisations used.](image)

Figure 8-18 Bubble plot of optimum values of $\Delta n$ and $\theta$ for Rolic slides areas A and B. Four different input polarisations.

There is no exact match for the polarisation parameters produced by a particular value of $\Delta n$ and $\theta$. This could be caused by experimental error (of measuring azimuth angle and ellipticity), but it could also be due to the AOI of the slide emitting more than one polarisation. The minimum intensity which was transmitted with a QWP inserted into the path of the beam was measured, and a corrected value for ellipticity was calculated as described in s.5.6.1.a. $I_{\text{min(QWP)}}$ was fairly constant for all three slides for each area, but was significantly less (approximately 1/3), for all three slides, at point B, than at point A. Inspection of the slides did not reveal any ‘cloudy’ or ‘hazy’ areas which would indicate that the slide was scattering some light, so it was concluded that a mixture of polarisations was in fact being emitted by each of the two AOIs, particularly at point A.
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It can be seen that the quoted values for $\Delta n$ do not exactly match those found experimentally, and the actual values measured depend on the particular area of the slide which is used. If it is assumed that the slides are manufactured from a LC polymer with $\Delta n$ of approximately 0.13 (as for the RMM 17 described in s.8.4.1 above) and the layer is assumed to have a thickness of approximately 2 microns, a tolerance of approximately 0.1 micron will cause a change in $\Delta n$ of 13nm. A variation of $\Delta n$ of 25nm is therefore not unreasonable. It is also likely that errors in the measurement of the output polarisation (above) has lead to inaccuracy in the calculation of $\Delta n$. Interestingly, the values calculated for $\Delta n$ using area B are nearer to the quoted values than those calculated using area A. This is almost certainly linked with the higher degree of partial polarisation found with area A (discussed above). The experimental order of the $\Delta n$'s of the slides (i.e. $\Delta n$ of slide 22<slide 20<slide 24) is the same as the quoted order.

8.5.3 AOI selection for Rolic slides

These slides had fewer imperfections than the Hull slides, and the whole area of the slide affected polarisation. Therefore as much of the slide as possible was used. The slide was divided into 99 areas of 27 x 35 pixels. A discussion of the effect of the size of the AOI is given in s.8.7.5. The Image Pro Plus macros to determine the AOIs and calculate the average intensities are shown in Appendix 8C. A series of the Rolic slide images, together with the AOIs used is shown in Figure 8-19 to Figure 8-22 below:

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vi The exact variation in a spun layer will depend on the spinning conditions, and the area over which the spun material is deposited [Law, V., UCL, personal communication].
8.5.4 Experimental details

As there were only three Rolic slides to use, and they were not symmetrical enough to enable them to be used upside down, extra 'false' slides were simulated by putting the input polariser at 10° and 20°, instead of the correct 0°. To further investigate the sensitivity of the system the slides were also rotated in a rotatable mount. After the image had been grabbed, an analysis was also performed of the effect of translation (vertical and horizontal) of the image, and rotation of the (grabbed) image. The effect of image rotation is shown in Figure 8-23 and Figure 8-24, and it can be seen that some of the AOIs did not fall on the slide.
8.5.5 Results for Rolic photographic slides

Following on from s.8.4.4 the Rolic slides were only analysed using the single LC cell. Figure 8-25 and Figure 8-26 show the histograms for the true and false slides when all points were considered.

![Histogram](image)

Figure 8-25 True Rolic slides. $C > 0$.

Mean error score = 0.0150 Standard deviation = 0.0010
Figure 8-26 False Rolic slides. C ≥ 0.
Mean error score = 0.2041 Standard deviation = 0.0964

Figure 8-27 and Figure 8-28 show the corresponding histograms when C ≥ 100 (maximum value of C = 396). It can be seen that the cut off between the true and false slides is more distinct when the number of comparison points is limited.

Figure 8-27 True Rolic slides. C ≥ 100.
Mean error score = 0.0136 Standard deviation = 0.0090
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8.5.6 Effect of polariser rotation

The histograms above include ‘false’ slides where the input polariser was set at 10° or 20° instead of the expected 0°. This was done to simulate a pattern of polarisation which was fairly similar to that expected, but not identical. Figure 8-29 shows an analysis of the error scores for slide 22 with those obtained when the polariser was rotated by 10° and 20°. It can be seen that even a 10° rotation is distinguishable from the ‘true’ slide pattern.
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Rolic slide 22. Error score for true slide compared with that with polariser rotated 10 and 20 deg. \( C > 0 \).

![Error score comparison graph](image)

Figure 8-29 Rolic slide 22 only. Comparison of error score with polariser at 0° ('true'), 10° and 20°. All points considered (\( C > 0 \)).

The means and standard deviations of the error scores shown in Figure 8-29 are shown in Table 8-1 below.

<table>
<thead>
<tr>
<th></th>
<th>Slide 22 no rotation</th>
<th>Slide 22 polariser at 10°</th>
<th>Slide 22 polariser at 20°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0110</td>
<td>0.0735</td>
<td>0.1861</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.0053</td>
<td>0.0240</td>
<td>0.0459</td>
</tr>
</tbody>
</table>

Table 8-1 Mean and standard deviation values for the error score histogram shown in Figure 8-29. (\( C > 0 \)).

The overlap between the 'true' slide and the measurements taken when the polariser is at 10° disappears when \( C \) is restricted to \( \geq 100 \) as previously. This is seen in Figure 8-30 below.
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Rolic slide 22. \( C \geq 100 \). Comparison of true slide with polariser rotated 10 and 20 deg.

![Graph showing error score comparison](image)

Figure 8-30 Rolic slide 22 only. Comparison of error score with polariser at 0° (‘true’), 10° and 20°. \( C \geq 100 \).

The means and standard deviations of the error scores shown in Figure 8-30 are shown in Table 8-2 below.

<table>
<thead>
<tr>
<th></th>
<th>Slide 22 no rotation</th>
<th>Slide 22 polariser at 10°</th>
<th>Slide 22 polariser at 20°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0090</td>
<td>0.0855</td>
<td>0.1899</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.0028</td>
<td>0.0101</td>
<td>0.0414</td>
</tr>
</tbody>
</table>

Table 8-2 Mean and standard deviation values for the error score histogram shown in Figure 8-30. \( C \geq 100 \)

Similar results were obtained for the other two slides. These are presented in Appendix 8D. In the cases tested there was no overlap between the scores for the slide when the polariser is not rotated and the scores for when the polariser is rotated, provided the number of comparison points is limited to 100 or more.

The reader is therefore sensitive to slight variations in the polarisation pattern emitted by the security mark.

8.5.7 Effect of slide rotation

To assess how sensitive the reading system was to rotation of the slide, the Rolic slides were placed in a rotatable mount and rotated in front of the LC reader. The input polarisation
remained at 0°, and the analyser was also fixed at 0°. The slide was rotated by up to ±5°. The
effect this has on the error score histogram for one particular slide (number 22) is shown in
Figure 8-31. In this figure the negative rotations of the slide are shown with a pattern filling
the bars, and the positive rotations are shown with the same colour, but with a solid fill.

![Rolic slide 22. Change in error score with slide rotation.](image)

Figure 8-31 Rolic slide 22, effect of slide rotation on error score

<table>
<thead>
<tr>
<th>Error score</th>
<th>0°</th>
<th>0.5°</th>
<th>1°</th>
<th>1.5°</th>
<th>2°</th>
<th>2.5°</th>
<th>3°</th>
<th>4°</th>
<th>5°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0069</td>
<td>0.0187</td>
<td>0.0363</td>
<td>0.0499</td>
<td>0.0642</td>
<td>0.0734</td>
<td>0.0896</td>
<td>0.1040</td>
<td>0.1122</td>
</tr>
<tr>
<td>SD</td>
<td>0.0019</td>
<td>0.0025</td>
<td>0.0030</td>
<td>0.0052</td>
<td>0.0073</td>
<td>0.0083</td>
<td>0.0115</td>
<td>0.0151</td>
<td>0.0155</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Error score</th>
<th>0°</th>
<th>-0.5°</th>
<th>-1°</th>
<th>-1.5°</th>
<th>-2°</th>
<th>-2.5°</th>
<th>-3°</th>
<th>-4°</th>
<th>-5°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0069</td>
<td>0.0245</td>
<td>0.0254</td>
<td>0.0352</td>
<td>0.0434</td>
<td>0.0596</td>
<td>0.0810</td>
<td>0.1097</td>
<td>0.1387</td>
</tr>
<tr>
<td>SD</td>
<td>0.0019</td>
<td>0.0025</td>
<td>0.0022</td>
<td>0.0029</td>
<td>0.0030</td>
<td>0.0039</td>
<td>0.0052</td>
<td>0.0049</td>
<td>0.0068</td>
</tr>
</tbody>
</table>

Table 8-3 Mean and standard deviation values for the error scores shown in Figure 8-31.

If the borderline between true and false slides is taken as \( E = 0.055 \) (see Figure 8-27 and
Figure 8-28) then it can be seen from Figure 8-31 that for a rotation of -2° to +1.5° the error
score is such that the slide is likely to be authenticated. When the slide is rotated by +1.5°, \( E \) is
between 0.042 and 0.059, which is likely to be borderline. The reader is therefore very
sensitive to inaccuracies in the rotational position of the security mark. If this degree of
accuracy is to be maintained, the registration of the slide must be accurate to within one degree. A similar result was found for slide 24, which is presented in Appendix 8D.

### 8.5.8 Effect of image rotation

The effect of rotation of the *slide* with rotation of the *image* of the slide was then compared: after the image had been grabbed, it was rotated by ±1, 2, 3, 4, 5, 6 and 10°. The AOIs were then measured (from the same start point), as if there had been no rotation. This is different from rotating the slide (or simply rotating the input polarisation) because when the image was rotated the slide was always in the same place relative to the input polariser, LC, and analyser (as the image contained all of these elements). When the slide itself was physically rotated these other elements remained fixed. The results for one slide are shown in Figure 8-32. The positive rotations are shown with solid fills and the negative ones with patterned fills.

![Rolic slide 22. Effect on error score of rotation of image using image analysis software.](image)

**Figure 8-32** Rolic slide 22. Variation in error score with rotation of image in image processing program.
Again, if the borderline between true and false slides is taken as being an error score of 0.055 a rotation of ±1° has a minor effect on the error score, ±2° moves the error score up to the borderline between true and false slides. A rotation of >±3° would lead to the slide being rejected. If rotation is likely to be a problem, it may be necessary to move the cut off point at which the slide is rejected. This would have the effect of increasing the numbers of false positive slides accepted, but depending on the application this may be less of a problem than falsely identifying slides that may have been inadvertently rotated.

8.5.9 Effect of image translation

To give an indication of the effect image translation would have on the system, the starting point for the scanning of the AOIs was altered by various amounts both horizontally and vertically. It was found that the error score was far less affected by this translation than by rotation. The histogram for the change in error score with translation is shown in Figure 8-33.
Figure 8-33 Rolic slide 22. Effect of horizontal and vertical translation on error score. Image translated horizontally by up to +20 HP (horizontal pixels) and −13 HP, and vertically by +20 to −15 VP (vertical pixels). Horizontal translation represented by solid fills, vertical by patterned fills.

<table>
<thead>
<tr>
<th>Error Score</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0110</td>
<td>0.0053</td>
</tr>
<tr>
<td>+2HP</td>
<td>0.0146</td>
<td>0.0040</td>
</tr>
<tr>
<td>+4HP</td>
<td>0.0229</td>
<td>0.0041</td>
</tr>
<tr>
<td>+6HP</td>
<td>0.0318</td>
<td>0.0049</td>
</tr>
<tr>
<td>+8HP</td>
<td>0.0410</td>
<td>0.0074</td>
</tr>
<tr>
<td>+10HP</td>
<td>0.0495</td>
<td>0.0075</td>
</tr>
<tr>
<td>+13HP</td>
<td>0.0633</td>
<td>0.0077</td>
</tr>
<tr>
<td>-13HP</td>
<td>0.0558</td>
<td>0.0058</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Error Score</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0110</td>
<td>0.0053</td>
</tr>
<tr>
<td>+15HP</td>
<td>0.0728</td>
<td>0.0076</td>
</tr>
<tr>
<td>+20HP</td>
<td>0.0922</td>
<td>0.0099</td>
</tr>
<tr>
<td>+5VP</td>
<td>0.0286</td>
<td>0.0087</td>
</tr>
<tr>
<td>+10VP</td>
<td>0.0506</td>
<td>0.0133</td>
</tr>
<tr>
<td>+15VP</td>
<td>0.0669</td>
<td>0.0133</td>
</tr>
<tr>
<td>+20VP</td>
<td>0.0797</td>
<td>0.0116</td>
</tr>
<tr>
<td>-15VP</td>
<td>0.0739</td>
<td>0.0141</td>
</tr>
</tbody>
</table>

Table 8-5 Mean and standard deviation values for the error scores shown in Figure 8-33.

It can be seen that the error score does not approach 0.055 until the image is translated by at least ±10 pixels horizontally or vertically. How much image translation will affect the accuracy of the reading system will depend on the size of the AOIs, and the density of the spatial variation of the security mark. The AOIs in this case were 27 x 55 pixels in dimension. The image could therefore be translated by ±18% horizontally and ±37% vertically without a false negative identification of the Rolic slide.
8.6 Reproducibility of polarisation change caused by single LC cell over time

In a perfect world, the error score for a true slide would always equal zero. Any variability in the behaviour of the LC cell was investigated by taking repeat measurements of output polarisation through the test cell on several occasions. The input polarisation was 0°, and the test cell voltages were set at: test cell off, 1.85V, 2.56V and 3.86V. The results were plotted on the complex polarisation plane, as described in Chapter 2 (s.2.3.5). The results are shown in Figure 8-34.

Figure 8-34 Variation of polarisation produced by LC test cell with time. Various displayed voltages.

It can be seen that there is a considerable variation in the behaviour of the test cell at each voltage. As would be expected there was less variability in the behaviour of the cell when it is either switched off, or fully switched. Intermediate voltages cause some variation, and it appears that the largest variability was when the voltage was 0.78V. For this voltage, $\alpha$ varied between 72° and 76.36°, and $\epsilon$ between 19.44 and 120. This is because this voltage is close to the Freedericksz threshold (see Chapter 3 s.3.4.3), and so the behaviour of the liquid crystal is very temperature dependent around this point (the environmental temperature was...
not controlled – maximum room temperature was 28°C). The voltages that were chosen for the security mark reader were not around this threshold. The variation when the LC voltage was 1.85V was \( \alpha = 56°-61.42° \) and \( e = 7.13-17.79 \), and when the voltage was 2.56V \( \alpha \) varied between 22° and 28.28° and \( e \) between 5.29 and 11.75.

It was not expected that a dynamic system such as a liquid crystal would give exactly the same results from one day to the next, so the system was designed to expect some error score.

8.7 Discussion

8.7.1 Accuracy

It can be seen from Figure 8-27 and Figure 8-28 that the liquid crystal security mark reader can be used to distinguish true from false security marks. The error score above which a slide will be rejected can be chosen by the user depending on how important it is to avoid false negatives (i.e. a true slide being identified as false), and the repeatability of the security mark manufacturing process. The manipulation of this cut off point will be discussed more fully in Chapter 9.

The security mark reader is very sensitive to changes in polarisation, which is how it can be used to distinguish between photographic quality images where the only difference is an optical phase difference of approximately 25nm. This is an accuracy of \( 1/25^{th} \) of the wavelength of light used. This sensitivity has the disadvantage of making the reader susceptible to rejecting slides that are inadvertently rotated. It is envisaged that if the reader were to be made into a commercial device, the holder into which the security mark was to be inserted for verification would ensure that the mark was accurately lined up in the reader. This could be accomplished by carving a notch or hole into the security mark (if the mark was made into a ‘tag’ or similar) that could be lined up with a corresponding peg in the reader.

It should be pointed out that all of the slides used in this chapter had an identical image on them. This was meant to test the ability of the system to distinguish between slides that were nearly identical in the polarisation pattern they emitted. Some ‘fakes’ were simulated by illuminating the ‘true’ slides with circularly polarised light, instead of linearly polarised. It was found that the error scores for these images were so high that they were discounted as being ‘too easy’. In practice it is considered unlikely that such good fakes will be encountered. This will mean that the error scores for counterfeit slides will be considerably higher than those presented in this chapter. This will have the effect that the cut off point between true and false slides will not be so critical. This will lead to true security marks that are rotated by more than \( \pm 2° \) being verified.
8.7.2 Twisted nematic vs. parallel aligned nematic

Following on from Part I of the thesis, the security mark reader was originally designed using the twisted nematic LCTVD. However, it was found that better distinction between the true and false slides was achieved using a single pixel LC cell. The single LC cell has cost and simplicity advantages over the LCTVD: it is cheaper and easier to make and more easily driven than the LCTVD. This cell was twisted nematic, with similar properties to that of the LCTVD. However, it would be possible to use a parallel aligned LC cell if required. If the pixellation of the LCD was required, the mass production of LC displays means that there are considerable cost advantages in using a twisted nematic device, but this does not apply if a single LC cell is required instead. The effect of a parallel aligned nematic LC device has been discussed in s.3.5. It has been seen that the intensity transmitted by the TNLC cell (through an analyser) always varies with LC voltage. However, there are certain situations where the intensity transmitted through a PALC cell will not change with applied voltage. These are:

1. if the input polarisation is linear, and along or perpendicular to the optic axis of the PALC cell (regardless of the orientation of the analyser) or,
2. the analyser and optic axis are parallel or perpendicular to each other (regardless of the particular input polarisation). This can be avoided by careful experimental set-up.

Therefore the security mark reader will be able to distinguish between a larger range of input polarisations if a TNLC cell is used rather than a PALC cell.

8.7.3 Environmental changes

As discussed in Chapter 3 the response of a LC to voltage varies with temperature, however it is anticipated that the reader could be re-calibrated by inserting a genuine slide if the operating temperature of the device should vary (such as could happen if it is used outdoors). This would be similar to the calibration of the LC Stokes polarimeter discussed in Chapter 6.

8.7.4 Wavelength

The reader could be used with any wavelength which is not absorbed by the optical components. A shorter wavelength would cause the effective birefringence ($\Delta n d / \lambda$) of the security mark to increase, and hence cause more of a polarisation change. This should enhance the distinction between slides of similar $\Delta n d$ values. However, this is not necessarily an advantage, because a fake slide could be made (in theory) with any thickness of birefringent...
material as long as the $\Delta$nd value is an exact multiple of $\lambda$. This would occur whatever the wavelength is, and it may not be more difficult to make an exact number of 430nm (say), than 660nm. Now that coloured, or white, LEDs are readily available, it is possible that several coloured sources could be used to illuminate the mark. The LC cell could then be replaced by a colour LCTVD. Each colour could have a different voltage written on to the LCTVD. This would avoid the necessity for 4 separate voltages to be written to the device (although with only 3 colour filters on the colour LCTVD, it would have to be addressed at least twice). However, it would also be necessary to have a colour CCD, which would increase the cost of the device.

8.7.5 Density of information

For the Hull slides, each AOI was chosen so as to encompass as much as possible of the polarisation varying portion of the slides whilst avoiding the edges of the circular birefringent areas. For the Rolic slides, it was considered desirable that as much as possible of the patterned area of the slide was covered. Each AOI had to be small enough to avoid the problems of summing a large area discussed in s.7.3.2., but large enough to be processed in a reasonable time. This is particularly important if the reader is to be incorporated into something like a turnstile to read tickets on entry to an event.

It is anticipated that, following the discussion in s.7.3.2., the larger each AOI is, the less sensitive the LC reader will be at distinguishing a true from a false slide. However, it is also anticipated that the larger each AOI is, the less sensitive the reader will be to translation or rotation of the slide, as a larger proportion of the AOI will remain unchanged when the slide is translated or rotated. The size of the AOI that is chosen will depend on the amount of detail which is in the slide (the more detail, the smaller the AOIs which will be needed to resolve this), the degree of accuracy which is required, and the accuracy of the positioning system for registering the security mark up with the reader.

8.7.6 Switching time

The time taken to read the security mark will depend upon the switching time of the LC. As discussed in s.4.5, it should be possible to apply six sets of four voltages to the LC per second. Assuming they can be placed in the reader this quickly, this will result in 6 marks being read per second.
Chapter 9

Discussion of LC security mark reader

9.1 Introduction

It has been seen from Chapter 8 that the LCTVD or the single LC cell can be used as part of a system to read polarisation varying security markings. The method has similarities to that of the Stokes polarimeter discussed in Chapter 6, and it is interesting that both methods require the application of four separate voltages to the LC to obtain an accurate result. This chapter will begin by analysing the problem of distinguishing between true and false slides using a statistical approach. This will show how the reader can be customised to meet the needs of the user. The chapter will conclude with a discussion of the advantages and disadvantages of the LC security mark reader.

9.2 Statistical analysis of results from security mark reader

The following discussion will be confined to the results obtained using the Rolic slides, as they are more complex, and hence more difficult to counterfeit, than the Hull slides. Only the data where the number of comparison points, C, exceeds 99 will be considered, as it has been shown in Chapter 8 that this gives a more definite distinction between true and false slides.
9.2.1 The problem and statistical solution

It has been seen (s.8.5.5) that there is hardly any overlap between the error scores for the true and false slides (figures 8-27 and 8-28). However these error scores do not represent all possibilities of true and false slides. The true slides belong to a closed class (there is a finite number of slides that can claim to be ‘true’), but these were not measured for all situations. Only a certain number of measurements were taken, at a finite number of incident light intensities, to simulate differing conditions. The number of false slides is infinite.

In order to predict the behaviour of the system it was necessary to know what distribution the true and false slides follow. To get an idea for possible distributions, particularly in the presence of a priori expectations, curves were fitted to the histogram data for the true and false slides (separately). This enabled the data to be extrapolated to give an idea of situations that were not measured, although proper statistical methods for estimating the probability that a set of observations follows a particular distribution [Lapin, 1990] were not considered.

The total area under the ‘true’ or ‘false’ histogram represented all the possible error scores for the true (or false) slides. To estimate how many slides would be classified as true or false for a particular error score $E$, all that had to be done was to determine the area under the curve(s) between $E$ and zero for classifications as ‘true’, and between that error score and infinity (practically taken as 0.1 in the examples used) for a ‘false’ classification. The relationship of this integral to the whole area (multiplied by 100) gave the percentage of slides that would be classified as true (or false) for this value of $E$. The intersection of the curves represents the error score at which the minimum number of misclassifications (i.e. false positives and false negatives) would occur. The areas under each curve, between the intersection and zero (for the false slides), and between the intersection and the upper limit of the error score (for the true slides) represent the probability of a false positive and false negative result respectively. The total of these areas (which represents the total misclassifications) is minimised at the intersection of the two curves. This is demonstrated in Figure 9-1.
Chapter 9. Discussion of LC security mark reader

Curves representing true and false slides

Figure 9-1
Demonstration of false negative and false positive results around intersection of error score curves

As an example, say the area under the 'true' curve is $T$ and that under the false curve is $F$, and the cut-off point is taken as the intersection of the two curves. If the area under the 'true' curve between the intersection and zero is $0.8T$ then 80% of the true slides will be correctly identified. However, if the area under the 'false' curve between the intersection and zero is $0.1F$, then as well as 80% of the true slides being correctly identified, 10% of the false slides will be falsely authenticated. Manipulation of the cut-off point can be used to vary these percentages, and this will be discussed next.

9.2.2 Customisation of cut-off point

When computing whether or not the mark is genuine, it is possible to adjust the weighting of the data to take into account how likely the slide is to be true or false. The area under each curve represents the probability of that event occurring. If it is equally likely that the mark will be true or false, the areas under each curve should be equal. However, if it is 9 times more likely that the mark would be false than true, the area under the 'false' curve could be adjusted to equal 0.9, and the 'true' curve area would be 0.1. This is demonstrated in Figure 9-2 which shows the same graphs as shown in Figure 9-1, but the area under the 'false' curve has been increased by 5 times. It can be seen that the position of the intersection has moved to the left (same scale as in Figure 9-1).
Chapter 9  Discussion of LC security mark reader

Figure 9-2 Same graphs as in Figure 9-1, but with area under false slide curve increased by 5

The change in the cut-off point means that the chance of identifying a true slide is reduced, to, say, 0.77, but fewer false positives would occur (relative to the area under the 'false' curve, say 0.08F). It is also inevitable that more false negatives will occur (relative to the area under the 'true' curve ~ 0.37) but this is desirable to maintain the accuracy of the reader if there are far more false slides than true slides. The reader would be very good at rejecting the large number of false slides at the expense of erroneously rejecting a few more true slides.

If it is expected that it will be more likely that a true mark is encountered (as would be the case, for example, when checking tickets), then the area under the 'true' curve would be increased. This would lead to true marks being more easily verified, as the cut-off point would move to the right. Fewer false negatives (relative to the area under the ‘true’ curve) would occur. The number of false positives would necessarily increase, but if the likelihood of encountering a false mark was so remote, this would be a price worth paying.

9.3 Curve fitting: results

In order to use this technique, it was necessary to fit a curve to the error histograms shown in Chapter 8. The aim was to minimise the least squares function between the (experimentally determined) histograms, and the curve. This was done using a program that utilises the Levenberg-Marquardt algorithm, which has become the standard of least-squares routines to solve non linear equations [Press et al., 1995 p.683]. This algorithm minimizes $\chi^2$ using two insights: a) near the actual minimum of the error function, the error function should be very nearly parabolic, and thus quadratic methods will be most efficient, and b) a long way from the actual minimum quadratic methods take too small steps, and steepest descent methods using the gradient are more efficient. The quadratic approximation uses a
Taylor series expansion, and the steepest descent method takes a step \( \Delta x \) along the direction of the negative gradient. Using a combination of these processes the iteration proceeds until a minimum error is found. The details of the method are described in mathematical textbooks (such as [Borse, 1997, p.344 et seq.]).

To judge the accuracy of the security mark verification system, it was considered equally likely that a mark was true or false. The curves were adjusted so that the area under each of the true and false curves was equal. This was done after the curve had been fitted to the data.

9.3.1 True slides

The fit of several non-linear equations to the experimental data was tested by quantifying the correlation coefficient, \( R^2 \), as described in Chapter 2 (s.2.6.3.a, and equation 2-29, which is reproduced here):

\[
R^2 = \frac{S_y - S_r}{S_y}
\]

The values for \( \bar{y} \) and \( S_y \) (defined in s.2.6.3.a) should not be confused with the mean and standard deviation of the original data. Here, \( \bar{y} \) and \( S_y \) are concerned with the \( y \) values on the histogram, not the \( x \) values, and are used only in assessing the quality of the fit of the regression curve to the data histogram. It is the \( x \) values that indicate the mean and standard deviation of the original data, and these are given below the relevant histograms in Chapter 8.

Two forms of equations gave curves where \( R^2 \) was greater than 0.81 (i.e. \( R \) greater than 0.9). These will be called a rational fit and a gaussian type fit. The rational fit had the form \( y = \frac{a + bx}{1 + cx + dx^2} \) and the 'gaussian type' was \( y = ax \exp \left( \frac{(x - b)^2}{c} \right) \) where \( a, b, c, \) and \( d \) are the fitting parameters. It was considered impossible in practice that the error score obtained from a true slide would exceed 0.1, so this was taken as the upper limit for the curve fitting. The curve fits that gave the highest correlation coefficients are shown in Figure 9-3. The data points for both curves have been scaled so that the area under the curve is 0.5 (\( \int_{-1}^{1} f(x) \, dx = 0.5 \)).
Chapter 9  Discussion of LC security mark reader

True slides: best curve fits. Area under each curve equal to 0.5.

Figure 9-3 Best curve fits obtained for true Rolic slides histogram

The correlation coefficients and standard errors for these curves are shown in Table 9-1. The scaling of the data affected the value obtained for standard error (but not the correlation coefficient as this is normalised \{s.2.6.3.a\}), so a better comparison between the standard errors was obtained if it was divided by the mean (of the histogram).

<table>
<thead>
<tr>
<th>Standard error</th>
<th>Standard error/mean</th>
<th>Correlation coefficient, $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rational fit</td>
<td>2.0712</td>
<td>0.4726</td>
</tr>
<tr>
<td>Gaussian type fit</td>
<td>3.5137</td>
<td>0.5964</td>
</tr>
</tbody>
</table>

Table 9-1 Standard errors and correlation coefficients for curves fitted to true slide data

The standard error of the estimate, $S$, is defined as $S = \sqrt{\frac{1}{m_{\text{points}} - m_{\text{param}}} \sum_{i=1}^{m_{\text{points}}} (y_i - f(x_i))^2}$, and quantifies the spread of data points around the regression curve. It can be seen that this is very similar to the definition of $S$ in s.2.6.3.a. The denominator is the number of degrees of freedom: $m_{\text{points}}$ is the number of data points used, and $m_{\text{param}}$ is the number of variables in the regression curve (i.e. 3 for the Gaussian type fit and 4 for the rational fit). As the quality of the data model increases, $S$ approaches zero.
9.3.2 False slides

Fitting of a curve to the data for the false curves was more problematic as the selection of slides to use as false slides was quite arbitrary. As mentioned previously (s. 8.7.1.), when ‘false’ slides were simulated by illuminating the true slides with circularly polarised light rather than linearly polarised light, the error score which was obtained was so high it was considered too simple to distinguish. The maximum error score which could be obtained for the Rolic slides was 2.45. This would occur when the intensity of the fake slide reads 41 when the true slide reads 209 (GLs >209 or <41 are ignored from the definition of C) and vice versa. From s.8.3.4., the error score for each point $E_n$ is therefore 209/41 and 41/209. This gives a total error score, $E$ of 2.45. The error scores that are plotted for the false slides in figure 8-28 are far less that this figure, and this shows just how good the ‘fakes’ are. It should be noted that the values that are chosen for the limits of C have a dramatic effect on the maximum possible error score. If all points are used (GLs 254-1) the maximum possible error score is 127.

As for the true slides, the rational and gaussian types of functions were found to fit the data so that the correlation coefficient exceeded 0.81. The best fits are shown in Figure 9-4. The data points were scaled so that $\int_0^1 f(x) \, dx = 0.5$.

![False slides: best curve fits.](image)

**Figure 9-4** Best curve fits obtained for false Rolic slides histogram

The correlation coefficients and standard errors for these curves are shown in Table 9-2.
Chapter 9. Discussion of LC security mark reader

<table>
<thead>
<tr>
<th></th>
<th>Standard error</th>
<th>Standard error/mean</th>
<th>Correlation coefficient, $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rational fit</td>
<td>0.2854</td>
<td>0.4309</td>
<td>0.8435</td>
</tr>
<tr>
<td>Gaussian type fit</td>
<td>0.3636</td>
<td>0.4376</td>
<td>0.8358</td>
</tr>
</tbody>
</table>

Table 9-2 Standard errors and correlation coefficients for curves fitted to false slide data

9.3.3 Summary

Looking at Figure 9-3 and Figure 9-4 it appears that the rational function is a closer approximation to the true and false data than the gaussian type function. However, this is slightly misleading for the true slides because although the highest error score for any true slide was less than 0.0475 the rational function does not decay to zero like the gaussian type function does. This means that if the rational function was assumed to represent the distribution of the true slides a deceptively high number of false negatives may be predicted. The fit of both functions to the false slide data was similar.

9.4 Distinguishing between true and false slides

The curves shown in Figure 9-3 and Figure 9-4 are superimposed and enlarged in Figure 9-5 below.

Figure 9-5 Best curve fits to true and false Rolic slides. Graph enlarged to show intersection.
Chapter 9. Discussion of LC security mark reader

The areas under the curves between the intersection and 0.1 for the 'true' curves, and between the intersection and zero for the 'false curves' was then calculated to give the areas shown in Figure 9-1. The effect of using the four possible intersections is shown in Table 9-3.

<table>
<thead>
<tr>
<th>False positives %</th>
<th>False negatives %</th>
</tr>
</thead>
<tbody>
<tr>
<td>True gaussian/false rational</td>
<td>0.004</td>
</tr>
<tr>
<td>True gaussian/false gaussian</td>
<td>0.077</td>
</tr>
<tr>
<td>True rational/false rational</td>
<td>2.626</td>
</tr>
<tr>
<td>True rational/false gaussian</td>
<td>4.689</td>
</tr>
</tbody>
</table>

Table 9-3 Effect of the different curve fits shown in Figure 9-5 on misclassification of slides

It can be seen from Figure 9-3 and Figure 9-4 that the fit of all four curves to the experimental data is acceptable. If the correlation coefficient is used as an indicator of the best fit, then the intersection which should be used is that between the rational fit for both the true and false slides. This gives an indication that 2.626% of the slides may be misclassified as false positives, and 3.012% of them may be false negatives.

However, as mentioned in 9.3.3, it can be seen from the histograms that although the rational curve fits the data for the true slides well for the points where the error score is below 0.05, it does not reduce to zero when the error score exceeds this. Fitting the curves to the data is only used to give an indication of the possible accuracy of the security mark reader. In practice, for the situations used in the experiment, no true slide had an error score of greater than 0.05. Consequently if the cut-off point was taken as 0.06, no false negatives would have arisen, despite the figures given in Table 9-3. It is also anticipated that in practice, very few (if any) fake slides will have error scores as low as those used in these experiments (see 8.7.1), which will reduce the chance of any false positive classifications.

9.5 Discussion

9.5.1 Sensitivity of reader

As has been shown in Chapter 8, the security mark reader is very sensitive to changes in polarisation. This has advantages and disadvantages and is fully discussed in 8.7.1. The Rolic security marks are designed to reveal hidden detail when viewed through crossed polarisers. This relies on a purely relative change in polarisation across the mark (which causes the intensity pattern through crossed polarisers). For the image on the mark to be identified all that is necessary is that the intensity of one area of the image is more or less dark than that of another area of the image. The actual optical thickness of each area of the mark is
not critical, providing that the pattern of polarisation change from one area to the next is constant. However, the security mark reader detects absolute values of polarisation and will reject marks with different optical thicknesses, even if the pattern of polarisation change is the same (the optical thickness was the only difference between the marks used for the tests in Chapter 8). The author feels that this may be a weakness in the system, with it being difficult or uneconomic in practice, to produce large numbers of identical marks. The sensitivity of the reader could be reduced to cope with this, but until a large number of 'true' marks are available, it is unclear how much the sensitivity would have to be reduced by, and the implications this would have for the number of false positives.

9.5.2 Customisation

The method described in s.9.2.2 shows how the reader can be customised to meet the requirements of the end user. The cut-off point can be moved depending on how likely it is that a true slide or a false slide would be encountered, and how important it is to avoid false (positive or negative) results. If the reader is used as part of a security system on, say, a visa, it may be decided that it is very important that false positives are avoided. The cut-off point for a 'true' identification could then be put to a very low error score. However, if the security marks may become slightly mutilated in use (such as may happen if they were put on concert tickets), the cut-off point could be increased to reduce the sensitivity of the reader to rejecting damaged (but true) marks.

9.5.3 Temperature changes

As discussed in Chapter 3, LCs are very temperature dependent. The temperature of the environment in which the reader is used may be fairly constant (such as the entry point in a concert venue where tickets are being checked). In this case the accuracy of the reader should not be significantly affected. However, if the reader is used in environments where the temperature may be expected to change significantly (such as outdoors), it would have to be recalibrated for each change in environment. This could be done by inserting a 'true' security mark into the reader and recording the intensity pattern transmitted at the four voltages of the LC. Although this is quite feasible, it is likely to be inconvenient in practice, as it necessitates each reader being supplied with the number of true marks that are to be identified. If the mark changes from one day to the next (which may happen if it was put on an item such as a rail ticket – each day may have a differently patterned mark) this may not be practical.
Chapter 9. Discussion of LC security mark reader

9.6 Summary

Chapters 8 and 9 describe a LC security mark reader that can accurately authenticate polarisation varying security marks. The security marks are verified by comparing the intensity pattern produced by the slide under test, with that of a master slide. If required, this 'master' intensity pattern can be transmitted remotely to the reader which could be at, say, an airport if visas are required to be checked.

The exact operation of the reader can be customised depending on the requirements of the end user: the complexity of the slides will determine the size of the areas of interest used to examine the slides. The greater the complexity of the slide the more time will be taken to authenticate the slide, but the slide will be more difficult to counterfeit. The reading process (as opposed to the authenticating process, when the grabbed images are processed) can be done at a rate of 6 slides per second. The cut-off point between where a slide is authenticated or not can be moved depending on the likelihood of a false or true slide being encountered, and the consequences of a misclassification. The reader is sensitive to slight changes in polarisation, but this sensitivity can be reduced if necessary.

The reader has been designed using a cheap and portable monochromatic light source, a single LC cell, and a monochrome CCD detector with image processing software. The reader can be easily recalibrated to counteract changes in environmental conditions by inserting a 'true' slide with which the test slides will be compared. If the environmental conditions in which the reader is used are not expected to change significantly then the 'true' slide pattern can be transmitted remotely to the reader.

This part of the thesis describes a technique that can be used for automatic verification of complex polarisation varying security marks. These marks can be attached to a variety of items, and because of the sensitivity of polarisation to changes in birefringence (and orientation) are very difficult to replicate without specialised knowledge and equipment. The cost of replicating the marks, and the repeatability of the process are commercially sensitive. The maximum cost allowable for each mark will depend upon the value of the item to be protected. This could be a relatively low value item, such as a ticket, or something of higher value such as a visa or passport.
Chapter 10

Conclusions and suggestions for future work

10.1 Introduction

Two possible applications have been presented for a commercially produced LCTVD: a complete polarimeter and an automatic security mark reader. The main motive for this work was the considerable difference in cost between a commercially produced TNLCD and a parallel aligned nematic spatial light modulator (SLM) (i.e. a NLC cell with 0° twist — see s.3.5). The huge market for TNLCDs enables them to be produced relatively cheaply (from a few pounds, but the exact price depends upon the specifications required, such as number of pixels and mode of addressing). However a LC SLM can cost many thousands of pounds. The Hamamatsu VGA LC SLM with a similar specification (VGA) to the LCTVD used in this thesis costs over £9000 [Hamamatsu, 2000].

This chapter will summarise the two applications proposed and conclude with some suggestions for future work.

10.2 Comparison of TNLCD performance with that of a parallel aligned nematic device

A discussion of the differences between the operation of a TNLC device and a PALC was given in s.3.5 and summarised in s.8.7.2. Both the polarimeter and the security mark reader rely on detecting changes in intensity of light transmitted through the analyser and there
are situations where this does not happen when a PALC device is used. The security mark reader is designed to distinguish between different spatially varying patterns of polarisation. It is possible that at least part of these patterns will consist of polarisations where there would be no intensity change with voltage using a PALC device. Therefore a wider range of polarisation patterns will be distinguishable if a TNLC device is used.

10.3 LC polarimeter

The LC polarimeter described in this thesis has several advantages over polarimeters proposed by previous researchers: it is made from a commercially produced LCD which is easy to obtain and relatively cheap. As the device is calibrated before each (set of) reading(s), any TNLCD can be used. This calibration also enables the device to be used for any wavelength (to which the optics is transparent) and in any environment in which the LC is switchable. The device uses only one LC layer, and so suffers from fewer reflections than polarimeters using multiple layers [Schirmer et al., 1998].

The LC polarisation camera described by Wolff uses two TNLC cells in series to rotate the plane of polarisation by 0°, 45° and 90° in turn. To fully rotate the plane of polarisation these cells must be thick enough to fulfil the Mauguin limit (s.3.4.4) and operate beyond the oscillations in the Gooch and Tarry curve (Figure 3-8). The exact thickness of the LC cells used is not mentioned, but if they are thin enough to switch quickly, they may not fulfil the Mauguin limit and hence they may induce some circularity into the polarisation. The assertion that the LC cells purely rotate the plane of polarisation may therefore not be valid for some wavelengths. The assumption that the cells are thin is deduced from the quoted switching time of 16ms [Shashar et al., 1994]. This is comparable to that of the thin LCTVD used in this thesis. It is possible that the LC cells are thicker than would otherwise be expected, because they are driven with a relatively high a.c. voltage (+/-9V) [Wolff & Mancini, 1992; Wolff, 1997]. This is higher than the voltages used in this thesis, and would lead to a faster switching time than if they were driven with a lower voltage. The algorithm proposed in Chapter 6 of thesis takes into account the actual polarisation change caused by the LCTVD, and so is not restricted to using thick LC cells. It can be used with any LCD that generates four separate polarisation states.

The LC polarimeter proposed in this thesis is simple to set up. It does not require the incident wavefront to be split, or several detectors to be used, to identify each component of the Stokes vector. If the LCTVD driving system can be simplified, the LCTVD polarimeter could be assembled into a portable unit, as are the LC polarimeters proposed by Wolff and Schirmer.
Unlike the polarisation camera developed by Wolff, the LCTVD polarimeter measures all four Stokes vector components. This raises the possibility of near real time measurements of the full state of polarisation, at a similar frequency to that proposed by Wolff [Wolff, 1995; Wolff, 1994]. Wolff's LC polarisation camera was designed to only measure the first three Stokes vector components because almost all naturally occurring light outdoors, as well as light in most indoor environments is partially linearly polarised [Wolff, 1997]. However, the ability of the LCTVD polarimeter to distinguish partially linearly polarised light from elliptically polarised light leads to it being suitable for accurately distinguishing between different polarisation patterns such as those produced by birefringent security marks (described in Part III of this thesis). If a faster reading time is required, and the fourth Stokes component is not needed, the LCTVD polarimeter can be used to measure the first three Stokes vectors only. This would be done by simply taking intensity readings at three, rather than four LCTVD voltages (s.6.3.3).

Although in this work the polarimeter was demonstrated using the LCTVD, the LC polarimeter does not require a pixellated device. However, after the polarisation of light passing through the LCTVD has been determined, the pixellation of the LCTVD could be used to reduce unwanted reflections from a scene in a more selective way than that proposed by Fujikake [et al., 1998]. It is anticipated that an appropriate algorithm could be developed so that a voltage pattern could be written to the LCTVD which resulted in the azimuth angles of the polarised reflections (from a scene) being as near as possible to orthogonal to an analyser. The analyser would then block some of this reflected light and increase the contrast of the viewed image.

The polarimeter described in this thesis is a viable, cheap (if designed using a single pixel LC cell instead of the LCTVD), near real time polarimeter, and could be used for applications such as material classification (distinguishing conducting from non-conducting material) and determining shape from polarisation, as in [Wallace et al., 1999].

10.4 Security mark reader

The LC security mark reader has been tested for a limited range of polarisation varying security markings in transmission mode. It has been shown to be accurate and reliable in distinguishing complex patterns of polarisation. The sensitivity of the reader can be customised to meet the needs of the particular application. To the author's knowledge, no other automatic reading device for authenticating polarisation-varying security marks is currently available. It is anticipated that the reader will be able to be used with markings made using different methods (as described in Chapter 7), and in reflection mode.
10.5 Future work

10.5.1 Polarimeter

10.5.1.a Reflections

1. The LCTVD polarimeter could be developed further to see if an algorithm could be designed for reducing reflections from scenes.

2. The variation in intensity which is transmitted (or reflected) by the LC cell with voltage (s.6.4) could be investigated by accurately modelling the reflection which occurs from the TN structure. The variation of the total transmitted intensity from the single LC cell with voltage has not been completely explained by modelling the cell as a PALC cell (s.6.4.2.c), and so there is scope for further investigation using a model of the twisted structure. This could explain the relationship between the degree of variation in the reflected beam with voltage and the polarisation of the incident beam. Elimination of this reflection variation would mean the polarimeter could be designed using a single LC cell. Enhancement of this reflection variation (e.g. by a suitable choice of ITO layer parameters, LC materials and display thickness), could lead to a reflective TN LCD being developed which could give a visible intensity variation without the need for a second polariser. It is desirable to reduce the number of polarisers in a LCD system because of the loss of light through them.

10.5.1.b Polychromatic sources

The problems caused by the monochromatic LCTVD being used with polychromatic sources were discussed in Chapter 6 (s.6.5.2). If the polarisation of each wavelength of a polychromatic source was to be measured the model could be adapted to cope with this. The light of each colour would need to be separated, so it could be treated individually. This could be done using a colour LCTVD and colour CCD. Instead of addressing the LCTVD with four separate monochromatic voltages, the LCTVD would be addressed with four separate coloured signals. The colour LCTVD and CCD could separate the intensities of each colour, to give a Stokes vector for each wavelength (corresponding to the colour filters used). This would complicate the algorithm described in Chapter 6, but it would enable the polarimeter to be used more in the ‘real’ world, where monochromatic light is uncommon. This procedure would also lead to the polarimeter being more versatile, as it would not have to be separately calibrated for each wavelength of light passing through it: e.g. if the polarimeter had been calibrated for ‘red’ (monochromatic) light, and was then needed to analyse a ‘green’ object. This could be done using the monochromatic polarimeter, but the device would have to be re-
calibrated for the different wavelength. If it had been calibrated for a ‘white’ source, this
would simultaneously calibrate it for the red, green, and blue LCTVD filters.

10.5.1.c Oblique incidence

The polarimeter has been designed for use with normally incident light. However, its
accuracy at oblique incidence could be investigated, together with how its off axis performance
could be improved with the addition of optical compensating films as discussed in s.6.5.3.

10.5.1.d Machine vision applications

The polarimeter described in this thesis is a viable, cheap and near real-time
polarimeter which could be used for the material classification and shape from polarisation
applications discussed in [Wallace et al., 1999]. The ability of the polarimeter to measure all
four Stokes parameters means it is particularly suitable for these tasks, as it has been shown
that the value obtained for $S_j$ as well as the degree of polarisation, $P$ can be used to gain
additional information about a scene ([Wallace et al., 1999; Liang, et al., 1999; Clark et al.,
1997]).

10.5.2 Security mark reader

It is anticipated that the security mark reader will be developed into a portable device,
and used with different types of security marking. It could either be used at close range (so
that the security mark is inserted into it, like a floppy disc is inserted into a computer, or
passed across it, like a barcode reader.) Alternatively, it could be used at long range, where the
security mark is viewed through the reader at a distance of 1m or more. The advantage of
close range is that the position of the mark, and its illumination can be more closely controlled
(although if it was used indoors at long range, the illumination could be controlled in certain
situations – such as on a production line in a factory). The position of the mark at long range
could be judged by inserting an identifying feature, such as a mark at each corner, which was
to be aligned with the viewing system. This could be done automatically.

Initial tests using commercially produced reflective polarisation varying security
marks have been conducted. These have shown that different marks can be distinguished from
each other.

Use of the security mark reader in a commercial environment would necessitate the
reader being tested with a large number of samples, and being tested using mutilated and dirty
samples. The testing described in Chapter 8 shows that the method can work in laboratory
conditions, with a limited range of samples. Only one sample of each security mark was available, so the variation in a large sample of 'true' marks could not be tested. The anticipated degree of variation that could be expected from one true mark to the next was not available. This would depend upon the stringency of the manufacturing process, which will in turn depend upon the cost constraints on producing the marks.

10.6 Summary

This thesis has investigated the use of a commercially produced LCTVD as a polarisation filter, and how it can be used in a device for automatically reading polarisation varying security marks. The motivation for this work was the mass availability of TNLCDs, and their low cost when compared with PALC SLMs. However, it has been shown above that the TNLCD is more suitable for the applications discussed in this thesis than a PALC SLM would be. The LCTVD Stokes polarimeter has advantages over polarimeters proposed by other researchers, and is versatile enough to be used in other applications. The security mark reader is, to the author's knowledge, the only automatic reader currently available to read polarisation-varying security marks. Polarisation-varying security features are already produced by a number of companies (Chapter 7), so there is a need for an automatic verification system for these marks.
Appendix 2A

Sign convention and sense of rotation

2A.1 Definition of phase with respect to sense of rotation of light

We can define the $x$ and $y$ components of the light as being represented by

\[ E_x = a \cos(\omega t - kz + \phi_x) \]  
\[ E_y = b \cos(\omega t - kz + \phi_y) \]

Equation 2A-1
Equation 2A-2

We can also define the phase difference between them, $\phi$ as $\phi_x - \phi_y$. Hence, if $E_x$ is taken as having zero phase, the relative phase of $E_y$ is $-\phi$. This is shown in Figure 2A-1.

![Figure 2A-1 Relative phase of x and y components of electric field](image)

Adding $\phi/2$ to each equation means that the $x$ and $y$ components can also be represented by equation 2A-3 and equation 2A-4.

\[ E_x = a \cos(\omega t - kz + \frac{\phi}{2}) \]  
\[ E_y = b \cos(\omega t - kz - \frac{\phi}{2}) \]

equation 2A-3

equation 2A-4

Assuming $\phi$ is $-90^\circ$, then at a fixed point in space (so $kz$ is constant) when $\alpha x=0$, ignoring the $(kz)$ term, $x$ becomes $a\cos(-45^\circ)$ and $y$ becomes $b\cos(+45^\circ)$.

If, for simplicity, $a=b$, then at $\alpha x=0$, $x=0.707a$, as does $y$.

When $\alpha x=45$, then $x=a \cos(45-45)^\circ = a$; and $y=a \cos(45+45) = 0$.

When $\alpha x=90$, then $x=a \cos(90-45)^\circ = 0.707a$, and $y=-0.707a$ and so on.
Appendix 2A  Sign convention

This is represented by Figure 2A-2 which shows that when the phase difference is -90°, as demonstrated above, the tip of the electric vector rotates in an anticlockwise fashion looking away from the source. This represents right handed light (looking away from the source), and occurs when \( \sin \phi \) is negative. (However, if \( \omega x \) is taken as negative, the situation is reversed).

Direction of rotation of tip of electric vector

\[
\begin{align*}
\omega t=90 & \\
\omega t=45 & \\
\omega t=0 & \\
0.707a & \\
0.707a & \\
y & x
\end{align*}
\]

Figure 2A-2  Sign convention for phase difference when sense of rotation is right handed (looking away from source)

In this work the sign convention of \( E = \text{amplitude} \cdot \cos(\omega t-kz) \) is adopted because the observer is considered as looking with the source behind them. The signs of \( t \) and \( z \) need to be opposite because, as the wave is travelling in space, the value of \( k \) must increase as \( t \) decreases. This can be explained with reference to Figure 2A-3 which represents the variation of the wave in space (either \( x \) or \( y \) component) at a fixed point in time. It can be seen that as \( kz \) increases, the value of the wave (on the \( E \) axis) increases (in a sinusoidal fashion).

\[
\begin{align*}
kz=1 & \\
kz=2 & \\
\text{Distance} &
\end{align*}
\]

Figure 2A-3  Vibration of electric vector

However, if we consider the same wave at a later point in time, the wave has moved along, and is represented by Figure 2A-4.
Looking at Figure 2A-4 and considering just the point $kz=2$, it can be seen that at the later point in time, the value of $E$ has decreased from the value it had at the same point in space in Figure 2A-3. If both $kz$ and $at$ were positive, the wave would be travelling backwards in space.
Appendix 2B

Calculation of Jones vector components from ellipticity and azimuth angle

It can be seen from Figure 2.2 that \( a' \) and \( b' \) correspond to the major and minor axes of an ellipse oriented along the \( X \) and \( Y \) axes. To deduce the Jones vector components it is necessary to calculate the amplitudes and phases of the components along the \( x \) and \( y \)-axes. To rotate clockwise (i.e. through a positive angle \( \alpha \)), the rotation matrix

\[
\begin{bmatrix}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{bmatrix}
\]

is used. This transforms the ellipse along the \( X \) and \( Y \) co-ordinates to the \( x \) and \( y \) system. Multiplying \( x \) and \( y \) by the rotation matrix gives:

\[
X = x \cos \alpha + y \sin \alpha \quad \text{equation 2B-1}
\]

\[
Y = -x \sin \alpha + y \cos \alpha \quad \text{equation 2B-2}
\]

Where, for brevity, \( x \) and \( y \) have been written instead of \( E_x \) and \( E_y \) (in equation 2.3). The equation of an ellipse along the \( X \), \( Y \) co-ordinates is

\[
\frac{X^2}{(a')^2} + \frac{Y^2}{(b')^2} = 1 \quad \text{equation 2B-3}
\]

substituting for \( X \) and \( Y \) gives:

\[
\frac{x^2 \cos^2 \alpha + y^2 \sin^2 \alpha + 2xy \sin \alpha \cos \alpha}{(a')^2} +
\frac{x^2 \sin^2 \alpha + y^2 \cos^2 \alpha - 2xy \sin \alpha \cos \alpha}{(b')^2} = 1
\quad \text{equation 2B-4}
\]

Equating coefficients of \( x^2 \), \( y^2 \) and \( xy \) in equation 2B-4 and equation 2-3 we have:

\[
x^2 \left( \frac{\cos^2 \alpha}{a'^2} + \frac{\sin^2 \alpha}{b'^2} \right) +
y^2 \left( \frac{\sin^2 \alpha}{a'^2} + \frac{\cos^2 \alpha}{b'^2} \right) +
xy \cos \alpha \sin \alpha \left( \frac{1}{a'^2} - \frac{1}{b'^2} \right) = 1
\quad \text{equation 2B-5}
\]

and
Appendix 2B Calculation of Jones vector components

\[
x^2 \left( \frac{1}{a^2 \sin^2 \phi} \right) + y^2 \left( \frac{1}{b^2 \sin^2 \phi} \right) - 2xy \left( \frac{\cos \phi}{ab \sin^2 \phi} \right) = 1
\]
equation 2B-6

This leads to three equations that enable \( a, b \) and \( \phi \) to be calculated. These equations are:

\[
\frac{1}{a^2 \sin^2 \phi} = \frac{b^4 \cos^2 \alpha + a^2 \sin^2 \alpha}{a^2 b^2}
equation 2B-7
\]

\[
\frac{1}{b^2 \sin^2 \phi} = \frac{b^4 \sin^2 \alpha + a^2 \cos^2 \alpha}{a^2 b^2}
equation 2B-8
\]

and

\[
-\frac{\cos \phi}{ab \sin^2 \phi} = \cos \alpha \sin \alpha \frac{(b^2 - a^2)}{a^2 b^2}
equation 2B-9
\]

Dividing equation 2B-8 by equation 2B-7 gives an expression for \( a'/b' \):

\[
\frac{a^2}{b^2} = \frac{b^2 \sin^2 \alpha + a^2 \cos^2 \alpha}{b^2 \cos^2 \alpha + a^2 \sin^2 \alpha}
equation 2B-10
\]

This enables \( a'/b' \) to be calculated. To calculate the phase difference, \( \phi \), equation 2.4 can be written:

\[
\tan 2\alpha = \frac{2 - \cos \phi}{1 - \frac{b^2}{a^2}} + m\pi
\]
equation 2B-11

Therefore,

\[
\cos \phi = \tan 2\alpha \left( \frac{1 - \frac{b^2}{a^2}}{2 - \frac{b}{a}} \right)
equation 2B-12
\]

It should be noted that equation 2B-12 contains the ratio \( a/b \), which can only be calculated by taking the square root of \( a'^2/b'^2 \). Dividing equation 2B-9 by equation 2B-8 gives:

\[
-\frac{b \cos \phi}{a} = \cos \alpha \sin \alpha \frac{(b^2 - a^2)}{a^2 \cos^2 \alpha + b^2 \sin^2 \alpha}
equation 2B-13
\]

The sign ambiguity can now be resolved: the sign of \( \cos \phi \) is known, because the sense of rotation of the light is measured so the sign of \( a/b \) can be deduced.
Appendix 2C

The relationship of the Stokes parameters to the Cartesian co-ordinate system and Poincaré sphere

On the Poincaré sphere the Stokes parameters are related to the ellipticity of the polarised light by the equations

\[ S_1 = \frac{1}{2} \cos 2\varepsilon \cos 2\alpha \]
\[ S_2 = \frac{1}{2} \cos 2\varepsilon \sin 2\alpha \]
\[ S_3 = \frac{1}{2} \sin 2\varepsilon \]

when the radius of the sphere is normalised to unity.

The mathematical description of the parameters as related to the cartesian components is as follows:

\[ S_0, \text{ which represents intensity } = <E_{0x}^2(t)> + <E_{0y}^2(t)> \]
\[ S_1 = <E_{0x}^2(t)> - <E_{0y}^2(t)> \]
\[ S_2 = 2<E_{0x}(t)E_{0y}(t) \cos (\phi_y(t) - \phi_x(t))> \]
\[ S_3 = 2<E_{0x}(t)E_{0y}(t) \sin (\phi_y(t) - \phi_x(t))> \]

To show that \( S_2 \) and \( S_3 \) represent the preference for polarisation at 45° (for \( S_2 \)) or R or L circular polarisations (\( S_3 \)), the general expression for the Jones matrix \( E_{x,y}(t) \) is multiplied by the counter rotation matrix, \( R(-\theta) \), which rotates the matrix clockwise by an angle \( \theta \).

\[ R(-\frac{\pi}{4})E_{xy}(t) = \begin{bmatrix} E_{x,y}(t) e^{i\phi_x(t)} \\ E_{0y}(t) e^{i\phi_y(t)} \end{bmatrix} \]

From Equation 2C-1 to Equation 2C-3 it can be seen that the difference between the intensities of the new x and y-axes (now -\( \frac{\pi}{4} \) and \( \frac{\pi}{4} \)) is

\[ 2 <Re \left( E_x^*E_y \right)> = 2<E_{0x}(t)E_{0y}(t) \cos (\phi_y(t) - \phi_x(t))> = S_2 \]
Appendix 2C Relationship of Stokes parameters to the Poincaré sphere

Similarly, for \( S_3 \), multiplying the expression for the Jones matrix, by the inverse transformation matrix, \( F^{-1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix} \) gives \( \begin{bmatrix} E_l \\ E_r \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} E_x + iE_y \\ E_x - iE_y \end{bmatrix} \). The differences between the intensities of the \( R \) handed and \( L \) handed circularly polarised components is therefore

\[
\frac{1}{2} \langle (E_x - iE_y)(E_x - iE_y)^* \rangle - \frac{1}{2} \langle (E_x + iE_y)(E_x + iE_y)^* \rangle = \Re \langle -iE_x^*E_y \rangle.
\]

This equals:

\[
2 \langle E_{0x}(t) E_{0y}(t) \sin[\phi_y(t) - \phi_x(t)] \rangle = S_3. \tag{2C-5}
\]

Equation 2C-5

Stokes parameters can also be used to describe partially polarised light: if the light is unpolarised, the point representing the light will be at the centre of the sphere, if it is totally polarised, it will be on the surface, and if it is partially polarised the point representing the polarisation of the light will be somewhere within the body of the sphere.
Appendix 2D Polarisation by reflection

Appendix 2D

Change of polarisation on reflection from a dielectric: the Fresnel coefficients

The polarisation of light reflected from a dielectric at non normal incidence changes. This occurs because light polarised perpendicular to the plane of incidence is always partially reflected and partially refracted, for all angles of incidence. However, light polarised in the plane of incidence is totally refracted at one particular angle of incidence (the Brewster angle \(=\tan^{-1} n\) where \(n\) is the refractive index of the reflector. The proportions of reflected and refracted light for vibrations in and perpendicular to the plane of incidence were calculated from Maxwell’s equations by Fresnel. The Fresnel equations are covered in most optical textbooks (such as [Hecht, 1988, pp.112-113]). If \(\rho_\perp\) and \(\rho_\parallel\) are the reflection coefficients for light polarised perpendicular and parallel to the plane of incidence respectively then [Shurcliff, 1962, p.79]:

\[
\rho_\perp = \frac{\sin^2(i-r)}{\sin^2(i+r)} \quad \text{Equation 2D-1}
\]

and

\[
\rho_\parallel = \frac{\tan^2(i-r)}{\tan^2(i+r)} \quad \text{Equation 2D-2}
\]

where \(i\) and \(r\) are the angles of incidence and refraction respectively. \(n\) is the refractive index of the dielectric.

It can be seen that when \((\tan (i+r))\) becomes infinite, at the Brewster angle, the proportion of light which is reflected polarised parallel to the plane of incidence drops to zero. All the reflected light is therefore polarised perpendicular to the plane of incidence. This is shown graphically in Figure 2D-1 where the Brewster angle can clearly be seen.

\[1\] These coefficients are squared because they concern intensities, not amplitudes.
Appendix 2D Polarisation by reflection

Figure 2D-1 Reflection coefficient of a single surface of a dielectric body in a vacuum.
Refractive index, \( n \), is 1.5 (from [Shurcliff, 1962])
Appendix 2E

Effect of birefringent layer using Huygen’s wavelet theory and refractive index indicatrix

2E.1 Crystal plate cut with principal plane parallel to incident face of the crystal

Recapping from s.2.4.2.a, it has been seen that the index indicatrix enables a visualisation of the how the refractive indices of a birefringent material vary with angle of incidence. When the crystal is cut so that the principal plane (defined as a plane containing the optic axis) lies perpendicular to the plane of incidence (as is the case in crystal retarders), the indicatrix shows that the refractive indices encountered by the orthogonal components of the incident light are simply $n_e$ and $n_o$. These orthogonal components vibrate along and perpendicular to the optic axis as they travel through the crystal. Using Huygens’ wavelet theory, the wavelets radiating from the refraction parallel to the optic axis and those radiating from the refraction perpendicular to the optic axis are superimposed, and therefore the light passes through the crystal undeviated, with only a change in phase between the orthogonal components. This is shown in Figure 2E-1.

![Diagram](image-url)

Figure 2E-1 Ordinary and extraordinary wavefronts and rays in a uniaxial crystal when a plane wave is incident normally to the surface and to the optic axis (from [Clarke &Grainger, 1971, p.77])
2E.2 Crystal plate cut with principal plane not parallel to incident face of the crystal

The behaviour of the crystal when the principal plane is not perpendicular to the plane of incidence is quite different.

Considering the uniaxial crystal Calcite: When the vibration of the $E$ field is perpendicular to the principal plane, which is in the plane of the page (as in Figure 2E-2) the vibration is always perpendicular to the optic axis. Consequently, the wavelets extend into the medium in all directions with a uniform speed. The ray then passes through the crystal undeviated (Figure 2E-2).

However, Figure 2E-3 shows an incident wave where the $E$ field is parallel to the principal section (linearly polarised). In this case $E$ has a component normal to the optic axis as well as one parallel to it. The birefringence of the material means that light polarised (i.e. with vibrations) parallel to the optic axis propagates with a different speed to that polarised perpendicular to it. This leads to the wavelets becoming elongated in one direction and hence they appear elliptical. This causes the beam to be deviated from the ordinary ray shown in Figure 2E-2 as it traverses the crystal. If the incident light contains a mixture of the two states shown in Figure 2E-2 and Figure 2E-3 (such as when the light is unpolarised) then both situations will occur simultaneously and two refracted beams will be evident, one polarised orthogonally to the other [Hecht, 1998, p.335]. A more detailed explanation can be found in [Collett, 1993, pp.444 et seq.].
Appendix 2F

Variation of polarisation ellipse calculation with number of intensity readings taken at differing analyser positions

2F.1 No liquid crystal present in optical system

Results using an input polarisation of $0^\circ$ have been presented in s.2.3.7. The method was repeated for an additional input linear polarisation:

2F.1.1 Input polarisation $-30^\circ$

![Intensity vs cos$^2 \gamma$. Input polarisation linearly polarised (-30 deg). No liquid crystal.](image)

$$y = 1.8525x - 0.0057$$

$$R^2 = 0.9988$$

Figure 2F- 1 Experimental intensity measured through analyser vs. cos$^2 \gamma$. Input polarisation $-30^\circ$ (linear).
Appendix 2F Calculation of polarisation ellipse

The results obtained for the whole range of analyser positions, input -30° are shown in Figure 2F-1. The effect differing number of intensity readings have on the values found for ellipticity and azimuth angle is shown in Table 2F-1.

<table>
<thead>
<tr>
<th>No. of int. readings</th>
<th>Analyser positions</th>
<th>Eqn. ( y = )</th>
<th>( R^2 )</th>
<th>( \alpha )</th>
<th>( \varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0-180°, 5° or 10° steps</td>
<td>1.8525x-0.0057</td>
<td>0.9988</td>
<td>-31.75°</td>
<td>(-)324</td>
</tr>
<tr>
<td>11</td>
<td>0-90°, 5° or 10° steps</td>
<td>1.8558x-0.0058</td>
<td>0.998</td>
<td>-32°</td>
<td>(-)318.9655</td>
</tr>
<tr>
<td>10</td>
<td>90°-180° in 5° or 10° steps</td>
<td>1.8105x+0.0336</td>
<td>0.9973</td>
<td>-31°</td>
<td>54.8839</td>
</tr>
<tr>
<td>7</td>
<td>0,30,65,90,120,150 &amp; 180°</td>
<td>1.8545x-0.0033</td>
<td>0.9991</td>
<td>-31°</td>
<td>(-)560.9697</td>
</tr>
<tr>
<td>6</td>
<td>0,30,65,90,120 &amp; 150°</td>
<td>1.8503x-0.0049</td>
<td>0.9993</td>
<td>-31.5°</td>
<td>(-)376.6122</td>
</tr>
<tr>
<td>5</td>
<td>0,30,65,120 &amp; 150°</td>
<td>1.8581x-0.0134</td>
<td>0.9995</td>
<td>-31°</td>
<td>(-)137.6642</td>
</tr>
<tr>
<td>4</td>
<td>0,65,120 &amp; 150°</td>
<td>1.8389x+0.0048</td>
<td>0.9998</td>
<td>-30.5°</td>
<td>384.1042</td>
</tr>
<tr>
<td>3</td>
<td>0, 65 &amp; 120°</td>
<td>1.8219x+0.0092</td>
<td>1</td>
<td>-30.5°</td>
<td>199.0326</td>
</tr>
<tr>
<td>3</td>
<td>0, 45 &amp; 90°</td>
<td>1.8928x-0.0119</td>
<td>1</td>
<td>-31.75</td>
<td>158.0588</td>
</tr>
</tbody>
</table>

Table 2F-1 Comparison of values for \( \alpha \) and \( \varepsilon \) with number of intensity readings taken.

As for the results presented in s.2.3.7 it can be seen that there is a considerable variation in the values obtained for ellipticity depending on the number of intensity readings taken, and the particular positions of the analyser which are chosen. There is a marked difference between the parameters found for the polarisation ellipse when the quadrant 0-90° is used as opposed to that from 90-180°.
2F.2 Liquid crystal test cell present in optical system

2F2.1 Voltage 0.784V (displayed voltage 1.99V)

2F.2.1.a Input polarisation 0 deg

Figure 2F-2 Experimental vs. theoretical intensity. Various analyser positions. Input 0°, LC cell voltage 0.784V

<table>
<thead>
<tr>
<th>No. of int. readings</th>
<th>Analyser positions</th>
<th>Eqn. y=</th>
<th>R²</th>
<th>α</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>0-180°, 5° or 10° steps</td>
<td>1.664x+0.0236</td>
<td>0.9999</td>
<td>72.25°</td>
<td>71.5085</td>
</tr>
<tr>
<td>11</td>
<td>0-90°, 5° or 10° steps</td>
<td>1.6814x+0.009</td>
<td>0.9999</td>
<td>72</td>
<td>187.8222</td>
</tr>
<tr>
<td>12</td>
<td>90°-180° in 5° or 10° steps</td>
<td>1.6493x+0.0261</td>
<td>0.9999</td>
<td>72.5</td>
<td>64.1916</td>
</tr>
<tr>
<td>7</td>
<td>0,30,65,90,120,150 &amp; 180°</td>
<td>1.6648x+0.023</td>
<td>0.9999</td>
<td>72.25</td>
<td>73.3826</td>
</tr>
<tr>
<td>6</td>
<td>0,30,65,90,120 &amp; 150°</td>
<td>1.6646x+0.0236</td>
<td>0.9998</td>
<td>72</td>
<td>71.5339</td>
</tr>
<tr>
<td>5</td>
<td>0,30,65,120 &amp; 150°</td>
<td>1.6718x+0.0223</td>
<td>0.9999</td>
<td>72.25</td>
<td>75.9686</td>
</tr>
<tr>
<td>4</td>
<td>0,65,120 &amp; 150°</td>
<td>1.6721x+0.0242</td>
<td>0.9999</td>
<td>72</td>
<td>70.0950</td>
</tr>
<tr>
<td>3</td>
<td>0, 65 &amp; 120°</td>
<td>1.6832x+0.0153</td>
<td>1</td>
<td>72</td>
<td>111.0131</td>
</tr>
<tr>
<td>3</td>
<td>0, 45 &amp; 90°</td>
<td>1.6838x+0.006</td>
<td>1</td>
<td>71.5</td>
<td>281.6333</td>
</tr>
</tbody>
</table>

Table 2F-2 Input 0°, LC cell voltage 0.784V
## Appendix 2F Calculation of polarisation ellipse

### 2F.2.1.b Input polarisation -30°

Experimental intensity vs predicted intensity. Input -30 deg. Single pixel LC cell 0.784V.

![Graph showing experimental vs. theoretical intensity]

\[ y = 1.5364x + 0.1194 \]
\[ R^2 = 0.9998 \]

Figure 2F-3 Experimental vs. theoretical intensity. Various analyser positions. Input -30°, LC cell voltage 0.784V

<table>
<thead>
<tr>
<th>No. of int. readings</th>
<th>Analyser positions</th>
<th>Eqn. ( y = )</th>
<th>( R^2 )</th>
<th>( \alpha )</th>
<th>( \epsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>0-180°, 5° or 10° steps</td>
<td>1.5357x+0.1201</td>
<td>0.9998</td>
<td>40.5°</td>
<td>13.7868</td>
</tr>
<tr>
<td>11</td>
<td>0-90°, 5° or 10° steps</td>
<td>1.5755x+0.0878</td>
<td>0.9997</td>
<td>40.5</td>
<td>18.9442</td>
</tr>
<tr>
<td>12</td>
<td>90°-180° in 5° or 10° steps</td>
<td>1.5098x+0.125</td>
<td>0.9998</td>
<td>40.5</td>
<td>13.0784</td>
</tr>
<tr>
<td>7</td>
<td>0,30,65,90,120,150 &amp; 180°</td>
<td>1.5355x+0.1194</td>
<td>0.9998</td>
<td>40.5</td>
<td>13.8601</td>
</tr>
<tr>
<td>6</td>
<td>0,30,65,90,120 &amp; 150°</td>
<td>1.5365x+0.1202</td>
<td>0.9999</td>
<td>40.5</td>
<td>13.7829</td>
</tr>
<tr>
<td>5</td>
<td>0,30,65,120 &amp; 150°</td>
<td>1.535x+0.1235</td>
<td>1</td>
<td>40.5</td>
<td>13.4291</td>
</tr>
<tr>
<td>4</td>
<td>0,65,120 &amp; 150°</td>
<td>1.5289x+0.1247</td>
<td>1</td>
<td>40.5</td>
<td>13.2606</td>
</tr>
<tr>
<td>3</td>
<td>0, 65 &amp; 120°</td>
<td>1.5288x+0.1248</td>
<td>1</td>
<td>40.5</td>
<td>13.25</td>
</tr>
<tr>
<td>3</td>
<td>0, 45 &amp; 90°</td>
<td>1.5667x+0.1</td>
<td>1</td>
<td>40.25</td>
<td>16.667</td>
</tr>
</tbody>
</table>

Table 2F-3 Input -30°. LC cell voltage 0.784V
Appendix 2F Calculation of polarisation ellipse

2F.2.1.c Input polarisation -45°

Experimental intensity vs. predicted intensity. Input -45 deg. Single pixel LC cell 0.784V.

\[ y = 1.5726x + 0.0863 \]
\[ R^2 = 0.9994 \]

Figure 2F-4 Results using LC cell at 0.7834V voltage. Input polarisation -45°.

The change in values for \( \alpha \) and \( e \) are shown in Table 2F-4D:

<table>
<thead>
<tr>
<th>No. of int. readings</th>
<th>Analyser positions</th>
<th>Eqn. ( y )</th>
<th>( R^2 )</th>
<th>( \alpha )</th>
<th>( e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>0-180°, 5° or 10° steps</td>
<td>1.5726x+0.0868</td>
<td>0.9994</td>
<td>24.25°</td>
<td>19.1081</td>
</tr>
<tr>
<td>11</td>
<td>0-90°, 5° or 10° steps</td>
<td>1.6048x+0.0652</td>
<td>0.9999</td>
<td>24.25°</td>
<td>25.6135</td>
</tr>
<tr>
<td>12</td>
<td>90°-180° in 5° or 10° steps</td>
<td>1.5326x+0.0958</td>
<td>0.9994</td>
<td>24°</td>
<td>16.9979</td>
</tr>
<tr>
<td>7</td>
<td>0,30,65,90,120,150 &amp; 180°</td>
<td>1.574x+0.0865</td>
<td>0.9999</td>
<td>24°</td>
<td>19.1965</td>
</tr>
<tr>
<td>6</td>
<td>0,30,65,90,120 &amp; 150°</td>
<td>1.581x+0.0855</td>
<td>0.9999</td>
<td>24°</td>
<td>19.4912</td>
</tr>
<tr>
<td>5</td>
<td>0,30,65,120 &amp; 150°</td>
<td>1.5775x+0.0886</td>
<td>0.9999</td>
<td>24°</td>
<td>18.8047</td>
</tr>
<tr>
<td>4</td>
<td>0,65,120 &amp; 150°</td>
<td>1.569x+0.0909</td>
<td>0.9999</td>
<td>24°</td>
<td>16.2607</td>
</tr>
<tr>
<td>3</td>
<td>0, 65 &amp; 120°</td>
<td>1.5672x+0.0934</td>
<td>0.9999</td>
<td>24°</td>
<td>17.7794</td>
</tr>
<tr>
<td>3</td>
<td>0, 45 &amp; 90°</td>
<td>1.6055x+0.0707</td>
<td>1</td>
<td>24.25°</td>
<td>23.7086</td>
</tr>
</tbody>
</table>

Table 2F-4 Changes in values for \( \alpha \) and \( e \) with varying number of intensity readings taken.

Input polarisation -45°
Appendix 2F Calculation of polarisation ellipse

2F2.2 LC voltage 1.8478V (displayed voltage 4.1V)

2F2.2.a Input polarisation -30°

Experimental vs. theoretical intensity. Input -30 deg. Single pixel LC cell 1.8478V.

\[ y = 0.9599x + 0.2561 \]
\[ R^2 = 0.9951 \]

Figure 2F-5 Results using LC test cell at 1.8478V voltage. Input polarisation -30°.

<table>
<thead>
<tr>
<th>No. of int. readings</th>
<th>Analyser positions</th>
<th>Eqn. ( y = \alpha x + e )</th>
<th>( R^2 )</th>
<th>( \alpha )</th>
<th>( e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>0-180°, 5° or 10°</td>
<td>0.9593x+0.2565</td>
<td>0.9954</td>
<td>20.5°</td>
<td>4.7399</td>
</tr>
<tr>
<td>11</td>
<td>0-90°, 5° or 10°</td>
<td>0.9629x+0.2537</td>
<td>0.9924</td>
<td>20°</td>
<td>4.7954</td>
</tr>
<tr>
<td>12</td>
<td>90°-180° in 5° or 10° steps</td>
<td>0.9582x+0.2567</td>
<td>0.9964</td>
<td>20°</td>
<td>4.7328</td>
</tr>
<tr>
<td>7</td>
<td>0,30,65,90,120,150 &amp; 180°</td>
<td>0.9592x+0.2566</td>
<td>0.9977</td>
<td>19°</td>
<td>4.7381</td>
</tr>
<tr>
<td>5</td>
<td>0,65,90,120 &amp;150°</td>
<td>0.9863x+0.2496</td>
<td>0.9999</td>
<td>19.75°</td>
<td>4.9515</td>
</tr>
<tr>
<td>3</td>
<td>0, 45 &amp; 90°</td>
<td>0.9795x+0.2566</td>
<td>1</td>
<td>19.25°</td>
<td>4.8172</td>
</tr>
<tr>
<td>3</td>
<td>0, 65 &amp; 120°</td>
<td>0.9855x+0.2511</td>
<td>1</td>
<td>19.5°</td>
<td>4.9247</td>
</tr>
</tbody>
</table>

Table 2F-5 Changes in values for \( \alpha \) and \( e \) with varying number of intensity readings taken.

Input polarisation -30°. Test cell voltage 1.8478V.

Table 2F-5 shows that when the ellipticity is low, the number of intensity readings taken does not significantly effect the values calculated. This is not surprising because the graph in Figure 2F-5 has a shallower gradient than when the ellipticity is higher. Consequently, slight inaccuracies in the analyser position will not dramatically affect the intensity measured at that point.
Appendix 2F Calculation of polarisation ellipse

2F2.3 LC voltage 3.8649 (displayed voltage 8.1V)

2F.2.3.a Input polarisation -30°

Experimental vs. theoretical intensity. Input -30deg. Single pixel LC cell 3.865V.

\[ y = 1.2702x + 0.0019 \]
\[ R^2 = 0.9991 \]

![Graph showing experimental vs. theoretical intensity](image)

Figure 2F-6 Results using LC cell at 3.8649V. Input polarisation -30°.

<table>
<thead>
<tr>
<th>No. of int. readings</th>
<th>Analyser positions</th>
<th>Eqn. y =</th>
<th>( R^2 )</th>
<th>( \alpha )</th>
<th>( e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>0-180°, 5° or 10° steps</td>
<td>1.2707x+0.0019</td>
<td>0.9991</td>
<td>150.25°</td>
<td>681.0303</td>
</tr>
<tr>
<td>11</td>
<td>0-90°, 5° or 10° steps</td>
<td>1.2827x-0.0005</td>
<td>0.9999</td>
<td>150.5°</td>
<td>(-)2564.4</td>
</tr>
<tr>
<td>12</td>
<td>90°-180° in 5° or 10° steps</td>
<td>1.2811x-0.0102</td>
<td>0.9966</td>
<td>150.5°</td>
<td>(-)124.598</td>
</tr>
<tr>
<td>10</td>
<td>0-180° in 20° steps</td>
<td>1.292x-0.0013</td>
<td>0.9998</td>
<td>150.25°</td>
<td>(-)992.8461</td>
</tr>
<tr>
<td>5</td>
<td>0, 65, 90, 120 &amp; 150°</td>
<td>1.2588x+0.0065</td>
<td>0.9994</td>
<td>150.5°</td>
<td>194.6615</td>
</tr>
<tr>
<td>3</td>
<td>0, 45 &amp; 90°</td>
<td>1.2932x-0.005</td>
<td>1</td>
<td>150.5°</td>
<td>(-)257.64</td>
</tr>
<tr>
<td>3</td>
<td>0, 65 &amp; 120°</td>
<td>1.2813x+0.0021</td>
<td>1</td>
<td>150.5°</td>
<td>611.1429</td>
</tr>
<tr>
<td>3</td>
<td>20, 65 &amp; 110°</td>
<td>1.2685x+0.0026</td>
<td>1</td>
<td>150.5°</td>
<td>488.8846</td>
</tr>
</tbody>
</table>

Table 2F-6 Changes in values for \( \alpha \) and \( e \) with varying number of intensity readings taken.

Input polarisation -30°. LC cell voltage 3.8649V.

Once again, it can be seen that a good approximation to the state of polarisation can be made using intensity measurements from 0-180° in 20° steps.
Appendix 3A

Multiplexing of liquid crystal displays

3A.1 Multiplexing

The two electrodes (one on each surface of the display) are formed to make a number of parallel conductive lines so that there are $N$ row electrodes and $M$ column electrodes (Figure 3A-1). The rows are on one side of the LCD and the columns are on the other.

![Figure 3A-1 Multiplexing of LCD pixels](image)

The number of pixels, $P$, can be addressed by $P/N + N$ individual connections. The voltage is applied in pulses to the rows and columns and is such that the pixel only switches on when it has a voltage from both the row and the column simultaneously. The $N$ rows are sequentially addressed by voltage pulses of strength $S$ during the selection interval $\Delta t$ (the frame time/$N$). The columns are addressed by voltage pulses of strength and polarity $+D$ or $-D$. The voltage for one pixel therefore consists of $(S+D)$ or $(S-D)$ for the selection interval, $\Delta t$, and $+/-D$ for the rest of the frame time $(N-1)\Delta t$.

The LC responds to the rms voltage across the pixel, which is given by:

$$V_{\text{rms}} = \sqrt{\frac{(S+D)^2 + (N-1)D^2}{N}}$$

and

$$V_{\text{off}} = \sqrt{\frac{(S-D)^2 + (N-1)D^2}{N}}$$

equation 3A-1
Appendix 3A Multiplexing of LCDs

It can be seen that when the cell is off, the applied voltage is not zero, as it is for direct addressing. This means that the LC must not switch at the $V_{off}$ voltage or else the contrast will be reduced (see Figure 3A-2).

The number of multiplexing connections (i.e. rows) is governed by the electro-optic properties of the display:

$$\frac{V_{on}}{V_{off}} = \sqrt{\frac{(S + D)^2 + (N - 1)D^2}{(S - D)^2 + (N - 1)D^2}}$$  \hspace{1cm} \text{Equation 3A-2}$$

In order to maximise this, differentiating this with respect to $S/D$ and setting the equation equal to zero, gives $\frac{S}{D} = \sqrt{N}$. This then leads to the ratio of $\frac{V_{on}}{V_{off}} = \sqrt{\frac{N + 1}{N - 1}}$ which is the Alt-Pleshko law of multiplexing which relates the number of matrix rows, $N$, to the required selection ratio, $V_{on}/V_{off}$, to achieve an optimum contrast ratio [Demus et al., 1998, p.205].

For an infinite number of $N$, $V_{on}/V_{off}$ tends to 1. The smaller this ratio becomes, the steeper the electro-optical curve needed (Figure 3A-2). However, this means that a very accurate control of voltage is required to get accurate grey levels. Another disadvantage of multiplexing is the 'crosstalk' which occurs because the voltage at one pixel can influence other pixels.

![Figure 3A-2 Change in brightness of LC display with increasing voltage](image)

**3A.2 Active matrix operation**

**3A.2.1 The benefits of active matrix addressing**

In a passive matrix display the voltage across the LC pixel is determined by the potential difference between the voltages applied to the overlapping row and column electrodes, as shown in Figure 3A-1. The TNLCD is not bistable, and this means that as soon
as the driving electric field is switched off, or reduced, the structure will relax to its off state. If a large number of pixels are being addressed it has been seen in s. 3.4.6.b that multiplexing has some disadvantages. In addition to this, as the driving waveform is applied to the display, by the time the whole display has been addressed, the voltage required to switch the beginning of the display on will have reduced. If there are \( N \) rows, and the frame time is \( T \), then the voltage pulse is only applied to each row for a duration of \( T/N \). By the time the display is refreshed, the LC will have begun to relax to its off state.

Some of these difficulties are overcome by actively addressing the pixels using transistors or diodes to produce sharper threshold characteristics. The display used in this project uses diodes to address the pixels. Unlike in a multiplexed passive display, the row and column electrodes are patterned onto one side of the glass. They drive the diode that is on each pixel, and this charges the LC capacitor. The other surface of the LCD contains the common ITO electrode.

Each row is selected only for a time \( T/N \) but the diode helps to keep the LC pixel in its addressed state until the next pulse of voltage is delivered in the subsequent frame. Each pixel is isolated from other pixels by the diodes and so the voltages remain constant while other pixels are being electrically addressed.

3A.2.2 Field and row inversion addressing

Addressing a TNLCD with direct current induces permanent dipoles on the molecules, which degrades the performance of the LCD. Using alternating current (a.c.) to drive the display eliminates this. The simplest way to ensure that the LCD is addressed with a.c. is to invert the polarity of the LC drive signal every time a pixel is addressed. This can be achieved by inverting the drive signal polarity once every field, and this is known as field inversion. However it is very difficult to ensure complete symmetry in the drive voltages of opposite polarities. This can lead to a 25Hz flicker being seen in the image (if the display is addressed at 50Hz). The flicker can be prevented by ensuring that the drive signal has an opposite phase on alternating rows of the display. The eye only perceives the combined effect of flicker on several rows, and because the flicker on alternate rows has opposite polarities the effect cancels out [Knapp, 1999]. The drive signal for field inversion, single, and double row inversion is shown in Figure 3A-3.
### Appendix 3A Multiplexing of LCDs

<table>
<thead>
<tr>
<th>Timing</th>
<th>Field inversion</th>
<th>Single row inversion</th>
<th>Double row inversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 ms</td>
<td>Pixel polarity in odd fields</td>
<td>Pixel polarity in even fields</td>
<td></td>
</tr>
<tr>
<td>32 μs</td>
<td>+ + + + +</td>
<td>- - - - -</td>
<td></td>
</tr>
<tr>
<td>64 μs</td>
<td>+ + + + +</td>
<td>- - - - -</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ + + + +</td>
<td>+ + + + +</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ + + + +</td>
<td>- - - - -</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ + + + +</td>
<td>+ + + + +</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3A-3 Methods of a.c. addressing of LCTVDs. Addressing rate 50Hz, 625 rows [Edwards, 1998].

The TNLCD used in this thesis has a switch which enables it to be addressed either using field inversion, or single row inversion. It was always used in single row inversion mode.
Appendix 4A

Applications for polarisation measurement

4A.1 Determining conductivity of a reflecting surface

4A.1.1 Intensity based method

Wolff [Wolff, 1990] has validated the theory that can determine the relative conductivity of materials (with metal and dielectrics being extreme cases). He found that if the specular angle of incidence is between 30° and 80°, and the specular component is strong relative to the diffuse component, the quantity $I_{\text{max}}/I_{\text{min}}$ derived from transmitted radiance parameters approximates the Fresnel ratio of $F_{\perp}/F_{\parallel}$ and is very reliable for distinguishing varying levels of electrical conductivity. This ratio for most metals varies between 1 and 2, whereas for dielectrics this ratio is above 3 [Wolff & Andreou, 1995].

The ratio of the maximum to minimum intensity ratio ($I_{\text{max}}/I_{\text{min}}$) always theoretically underestimates the Fresnel ratio of $F_{\perp}/F_{\parallel}$ as the true equation is [Wolff & Boul, 1991]:

$$\frac{F_{\perp}}{F_{\parallel}} = \frac{I_{\text{max}} - (\frac{1}{2} I_d)}{I_{\text{min}} - (\frac{1}{2} I_d)}$$

(Equation 4A-1)

($I_d$ is the magnitude of the diffuse component of the reflection and $I_s$ the component specularly reflected. $I_s$ has been eliminated from the equation.)

4A.1.2 Phase based method

Clark [Clark et al., 1997] noted that linearly polarised light becomes elliptically polarised upon reflection from a metal, and the orientation and the ellipticity of the ellipse depended on the phase shift between the parallel and perpendicular components of the electric fields. To simplify matters, linearly polarised incident light oriented at 45° with respect to the plane of incidence was used so that the orientation of the ellipse was independent of the phase difference. This meant that only the ellipticity varied with phase difference.

Following on from Clark et al., Chen & Wolff [1997] have recently developed a method for distinguishing between conducting and non-conducting materials using a phase-based method. This method relies upon the fact that when light is reflected from a metal, there is a phase change induced between the components parallel and perpendicular to the plane of incidence, for almost all angles of specular reflection. For a dielectric the phase change is
either $\pi$ (for incident angles up to Brewster's angle) or zero degrees, for angles over Brewster’s angle (Hecht 1998, p.116). Wolff’s Fresnel reflectance model becomes inaccurate when the diffuse component of reflection is not small compared with the specular component (for incident angles of less than 30° and greater than 80°), and will misclassify highly diffuse white paper as metal (as white paper has a high diffuse component of reflection).

The phase based method can be utilised to distinguish between metal and dielectric objects illuminated outdoors by a clear or partly cloudy sky, as well as being useful even where the specular reflection from a conductive surface is accompanied by a much larger diffuse reflection component. The measurement of the phase retardance is limited only by the signal-to-noise ratio of the camera sensor being larger than the ratio of diffuse to specular reflection. Wolff modifies his LC camera to detect phase retardance, by placing a quarter wave plate in front of it to change the cosine in his calculations to sine. He measures whether partially or fully linearly polarised light remains linearly polarised, or becomes elliptically polarised. If light is specularly reflected from a dielectric, the phase change between the components of the electric vector parallel and perpendicular to the plane of incidence is $0^\circ$ or $\pi$, and therefore incident linear light remains linearly polarised. If linearly polarised light is specularly reflected from a metal, the phase change is between $0^\circ$ and $\pi$ (unless it is incident at normal incidence, when the metal will not cause a relative phase change), and therefore the linear incident light will become elliptically polarised on reflection. Measurements of the angle of the maximum intensity enable calculation of the phase difference between the orthogonal components, and materials can be classified depending on this angle. The phase-based method relies on the change in polarisation of linearly polarised incident light. Therefore the incident light must have some linear polarisation. This method therefore neatly complements Wolff’s Fresnel reflectance model that is used when the incident light is unpolarised.

Nayar et al., [1997] identified two problems with Wolff’s approach: firstly that except for very smooth surfaces the diffuse component cannot be assumed to be constant; and secondly a constant Fresnel ratio assumption was used. The Fresnel ratio is sensitive to angle of incidence, and therefore except for very smooth surfaces, highlights will include points with different surface normals, and therefore different incident angles. To reduce these problems an algorithm was developed that uses colour and polarisation simultaneously. This can estimate specular components that result not only from direct source illumination but also inter-reflections between points in the scene. However, it does assume that the specular component is polarised, whereas the diffuse component is not. It has been seen above, that the diffuse component is significantly polarised for oblique angles of incidence, so this is a failing in Nayar’s model.
4A.2 Highlight removal

Nayar et al. [1997] made several assumptions in their colour and polarisation based model:

1. The scene consisted of dielectric objects (therefore specular reflections were polarised, while diffuse reflections were not).
2. Specular inter-reflections arose from either a diffuse-specular mechanism, in which the incident light was unpolarised, or from a specular-specular mechanism, in which the incident light was partially polarised (from the first specular reflection), but not both.
3. The Fresnel coefficients are independent of the wavelength of light. Although the Fresnel coefficients depend on refractive index (which is wavelength dependent), the visible light spectrum is reasonably narrow, and the coefficients do not vary greatly across this frequency band.

It should be noted that Nayar’s dichromatic model is not generally suitable for metals, as they rarely produce diffuse reflections with spectral distributions that differ from that of the illumination.

4A.3 Medical uses

Demos et al., [1997] have demonstrated that polarisation can be used in biomedical imaging. One of the major obstacles in optical tomography is the multiple scattering which takes place during light propagation through the tissue. This degrades the image information. Polarisation may be applied in the imaging of biomedical media, as polarised light propagating in normal human breast tissue depolarises less than in malignant breast tissue. The polarisation is preserved to different degrees depending on the type of tissue and the wavelength of the propagating light. Experiments showed that in transmitting 532nm laser light through a 3mm thick tissue sample, the part of the sample containing the cancer tumour depolarised the beam almost completely, while the light propagating through the normal tissue was partially polarised. (Shorter wavelengths provided a higher contrast, but had a greater attenuation of light during propagation due to increased absorption and scattering). The experimental arrangement involved a polariser and analyser together with a light collecting fibre to separate the two polarisation components of the transmitted light parallel and perpendicular to the polarisation of the incident light.

Polarisation-sensitive optical coherence tomography has been used to measure thermal damage to histological samples of medical tissue [Schoenenberger et al., 1998]. Here,
circularly polarised light was incident on a tissue sample, and the birefringence of the tissue was measured using the phase delay of the light returning (through a beamsplitter) to the detector. Tissue has inhomogeneities, which cause linear and circular anisotropy because of the proteins, such as collagen, built up in an extra-cellular matrix. When the temperature of the tissue is raised above a melting threshold, the collagen molecules unfold, and some of the tissue birefringence is lost. The scattering by the tissue, which causes partial depolarisation, was overcome by selectively gating out the multiply scattered component of the detected light by low-coherence interferometry. It was found that tissue birefringence was a more sensitive indicator of tissue thermal damage than were changes in tissue scattering properties.

Demos and Alfano [1997] have also used polarisation difference imaging to look at subsurface structures of the human body. It was found that reflected light which was parallel to the input polarisation imaged surface structures, and reflected light which was perpendicular to the input polarisation, contained information about subsurface structures (as it was backscattered). By temporally separating these two components, an image was formed from different depths, as well as at the surface. It was found that if there was a structure that scattered light more, the partially polarised light reaching the structure would depolarise faster, contributing to increased intensity in the perpendicular image. This leads to the structure appearing brighter in the polarisation-difference image. It was suggested that by gradually changing wavelengths, structures at different depths might be displayed.
Appendix 5A

Calibration of GL to voltage applied to LCTVD

Figure 5A-1 shows the relationship between transmission through crossed polarisers and voltage applied to the TNLCD. This data was provided by the display manufacturer.

A similar curve was then plotted for GL against intensity. These two graphs enabled the voltage to be related to the intensity applied to the LCTVD. This relationship is plotted in figure 5A-2 below.
Appendix 5B

The use of singular value decomposition in finding the least squares solution

From Chapter 5, equation 5-10, which is reproduced here, represents the ideal behaviour of the liquid crystal television.

\[
\begin{bmatrix}
C_1 \cos \theta_1 & -D_1 \cos \theta_1 & C_1 \sin \theta_1 & -D_1 \sin \theta_1 & -A_i \cos \theta_i & 0 & -A_i \sin \theta_i & 0 & a \\
D_1 \cos \theta_1 & C_1 \cos \theta_1 & D_1 \sin \theta_1 & C_1 \sin \theta_1 & 0 & -A_i \cos \theta_i & 0 & -A_i \sin \theta_i & b \\
C_2 \cos \theta_2 & -D_2 \cos \theta_2 & C_2 \sin \theta_2 & -D_2 \sin \theta_2 & -A_i \cos \theta_i & 0 & -A_i \sin \theta_i & 0 & c \\
D_2 \cos \theta_2 & C_2 \cos \theta_2 & D_2 \sin \theta_2 & C_2 \sin \theta_2 & 0 & -A_i \cos \theta_i & 0 & -A_i \sin \theta_i & d \\
C_3 \cos \theta_3 & -D_3 \cos \theta_3 & C_3 \sin \theta_3 & -D_3 \sin \theta_3 & -A_i \cos \theta_i & 0 & -A_i \sin \theta_i & 0 & e \\
D_3 \cos \theta_3 & C_3 \cos \theta_3 & D_3 \sin \theta_3 & C_3 \sin \theta_3 & 0 & -A_i \cos \theta_i & 0 & -A_i \sin \theta_i & f \\
C_4 \cos \theta_4 & -D_4 \cos \theta_4 & C_4 \sin \theta_4 & -D_4 \sin \theta_4 & -A_i \cos \theta_i & 0 & -A_i \sin \theta_i & 0 & g \\
D_4 \cos \theta_4 & C_4 \cos \theta_4 & D_4 \sin \theta_4 & C_4 \sin \theta_4 & 0 & -A_i \cos \theta_i & 0 & -A_i \sin \theta_i & h \\
\end{bmatrix} = 0
\]

\[Mx = 0\]

where the vector \(x\) represents the parameters of the required Jones matrix of the LCTV.

5B.1 Singular value decomposition method

To find the desired values for \(x\), singular value decomposition (SVD) is used to break down the matrix \(M\), and the required value for \(x\) is that which corresponds to the smallest singular value for the matrix, \(M\).

SVD breaks down a matrix into 3 components, which have particular properties. Any matrix, \(M^1\), can be represented as:

\[M^1 = U \Sigma V^*\]

Here, the SVD solution for an 8 x 8 matrix is presented.
The matrices $U$ and $V^T$ have the properties that $U^TU=I$ and $V^TV=I$, where $I$ is the identity matrix. $s_1$ through to $s_8$ are the singular values of the matrix, $M$. Any column of $V$ corresponding to a zero singular value is a solution of $My=0$. Where there is no zero singular value, the smallest singular value gives the nearest solution to $My=0$. The reason for this is as follows:

Using SVD on equation 5B-1 gives:

$$
M = U \begin{bmatrix}
    s_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & s_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & s_3 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & s_4 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & s_5 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & s_6 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & s_7 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & s_8
\end{bmatrix} V^T
$$

where $v_1^T ... v_8^T$ are the columns of the $V$ matrix.

Multiplying out gives:

$$
\begin{bmatrix}
    u_{11} & u_{12} & u_{13} & u_{14} & u_{15} & u_{16} & u_{17} & u_{18} \\
    u_{21} & u_{22} & u_{23} & u_{24} & u_{25} & u_{26} & u_{27} & u_{28} \\
    u_{31} & u_{32} & u_{33} & u_{34} & u_{35} & u_{36} & u_{37} & u_{38} \\
    u_{41} & u_{42} & u_{43} & u_{44} & u_{45} & u_{46} & u_{47} & u_{48} \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{bmatrix}
= \begin{bmatrix}
    s_1 v_1^T x \\
    s_2 v_2^T x \\
    s_3 v_3^T x \\
    s_4 v_4^T x \\
    s_5 v_5^T x \\
    s_6 v_6^T x \\
    s_7 v_7^T x \\
    s_8 v_8^T x
\end{bmatrix}
$$

Remembering that $V^TV=I$, then $v_i^T y_j = 1$ when $i=j$, and 0 when $i \neq j$ it can be seen that, in equation 5B-3, if $x$ is set to equal $v_8$ then the equation becomes:

\[^{ii}\text{Sorted out into numerical order, where } s_1 > s_2 > s_3 \ldots > s_8^{iii}\text{ Where } v \text{ is a column of the matrix, } V\]
Appendix 5B Singular value decomposition

\[
\begin{bmatrix}
  u_{11} & u_{12} & u_{13} & u_{14} & u_{15} & u_{16} & u_{17} & u_{18} \\
  u_{21} & u_{22} & u_{23} & u_{24} & u_{25} & u_{26} & u_{27} & u_{28} \\
  u_{31} & u_{32} & u_{33} & u_{34} & u_{35} & u_{36} & u_{37} & u_{38} \\
  u_{41} & u_{42} & u_{43} & u_{44} & u_{45} & u_{46} & u_{47} & u_{48}
\end{bmatrix}
\begin{bmatrix}
  0 \\
  0 \\
  0 \\
  0 \\
\end{bmatrix}
\begin{bmatrix}
  s_1 \\
  s_2 \\
  s_3 \\
  s_4 \\
  s_5 \\
  s_6 \\
  s_7 \\
  s_8
\end{bmatrix}
\]
equation 5B-4

If \( s_8 \) is zero, then the vector on the right hand side will be zero regardless of the values of \( U \). Hence, the column of \( V \) corresponding to a zero singular value of \( X \) is a solution of \( M \bar{y} = 0 \).

However, in the case of the experimental results, it is unlikely that there will be a zero singular value of the matrix \( M \), so the value of \( s_8 \) (assuming that the computer sorts the diagonal matrix so that \( s_1 > s_2 > s_3 \ldots > s_8 \)) is the nearest solution to equation 5.8 based on the experimental measurements. Why this corresponds to the least squares solution will be discussed in the following section.

5B.2 SVD and the least squares solution

In practice, the right hand side of equation 5B-1 is not zero, due to experimental errors. So, the equation can be written more accurately as

\[
Mx = \bar{y} + e
\]
equation 5B-5

where \( e \) is the experimental error. The desired solution is the values for \( x \) which minimise the total error. To avoid the problems with positive and negative errors appearing to cancel out and reduce the total error, the sum of the squares of the errors is taken as the total error, i.e.

\[
e^T e = E,
\]

where \( E \) is the total error function. equation 5B-5 then becomes:

\[
(Mx)^T (Mx) = E
\]
equation 5B-6

So, \( x^T M^T M x = E \).

From SVD, \( M = U S V^T \), so \( M^T = V S^T U^T \). \( S^T = S \) (because it is a diagonal matrix), so equation 5B-6 becomes:

\[
x^T V S T U^T U S V^T x = E
\]

so, \( x^T V S^2 V^T x = E \)
equation 5B-7

Any vector in eight-dimensional space can be expressed as a linear combination of eight orthogonal vectors, so
Substituting for $x$ in equation 5B-7 gives:

$$
(k_1 v_1^2 + k_2 v_2^2 + k_3 v_3^2 + \ldots + k_g v_g^2) = E
$$

Because $V^TV=I$, then $v_i^Tv_j=1$ when $i=j$, and 0 when $i \neq j$ this simplifies to:

$$
E = \begin{bmatrix}
  k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 & k_8 \\
  k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 & k_8 \\
  k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 & k_8 \\
  k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 & k_8 \\
  k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 & k_8 \\
  k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 & k_8 \\
  k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 & k_8 \\
  k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 & k_8 \\
\end{bmatrix}
$$

and then

$$
E = k_1^2 s_1^2 + k_2^2 s_2^2 + k_3^2 s_3^2 + k_4^2 s_4^2 + \ldots + k_g^2 s_g^2
$$

To minimise the value of $E$ it is, of course, possible to make $E$ zero by setting $x$ as zero, but this would not give a sensible result, so the value of $x$ is set at, say, 1. This implies that, from equation 5B-8, that $1 = k_1^2 + k_2^2 + k_3^2 + k_4^2 + \ldots + k_g^2$ (as $v_i^Tv_j=1$ when $i=j$).

By inspection of equation 5B-9 it can be seen that, in order for the sum of the $k_n$ components to equal 1, the smallest value of $E$ will occur when the largest value of $k_n$ is multiplied by the smallest value of $s_n$. So, for minimum error, the smallest singular value ($s_g$) should correspond with the largest value of $k_n$ (i.e. when $k_g=1$).

Going back to equation 5B-8 it can be seen that when $k_g=1$, $x = v_g$

Therefore, the solution for $x$ which minimises the error, $E$, is the column of the $V$ vector which corresponds with the smallest singular value, $s_g$. If there are two equally small singular values, there are two solutions for $x$, one for each small singular value. This method enables the least squares solution to be found (even) when the data is over constrained. In practice,

---

iv Where $v_n$ is a column vector of the matrix $V$ used in singular value decomposition. The columns of $V$ are orthogonal to each other because $V^TV=I$, so $v_i^Tv_j=0$ when $i \neq j$ (therefore, when $i \neq j$ the vectors are orthogonal, as orthogonal vectors have a zero dot product). Hence, the vectors $v_n$ can be used to represent any $n$ dimensional vector by multiplying them by the appropriate constants.
various numbers of experimental measurements were taken, the most common number of results taken was 8 input polarisations. Each input polarisation gave rise to two rows of the matrix, \( M \), in equation 5.8.

### 5B.3 Multiple solutions

A consequence of there being a random phase element in equation 5.8 is that there are multiple solutions. The singular values identified by the SVD method described above were in pairs, with \( s_1 = s_2 \), \( s_3 = s_4 \), and so on. The solution therefore gives two values for the \( y \) vectors which correspond to the equally small \( s_7 \) and \( s_8 \). These two vectors, \( y_7 \) and \( y_8 \), when put into the Jones matrix, give identical resultant polarisations, so each is a valid solution to the problem. The only difference between them is that the absolute phase is different.

The reason for there being two solutions is as follows: In equation 5.3, there is an arbitrary phase term added to the right hand side \( e^{i\phi} \). There is also an arbitrary phase term on the left hand side (which, for simplicity, has not been written in equation 5.1). Both of these absolute phase terms are eliminated by the division of equation 5.4 by equation 5.5. The effect of the unknown phase term on equation 5.10 is as follows:

If the unknown phase component on the left hand side is \( e^{i\phi} \), equation 5.3 becomes:

\[
(\cos \varphi + i \sin \varphi) \begin{bmatrix} a + ib & c + id \\ e + if & g + ih \end{bmatrix} \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} = \begin{bmatrix} A \\ C + iD \end{bmatrix} e^{i\phi}
\]

Equations 5.4 and 5.5 become:

\[
(\cos \varphi - b \sin \varphi + i \cos \varphi + i \sin \varphi) \cos \theta + (e \cos \varphi - d \sin \varphi + i \cos \varphi + i \sin \varphi) \sin \theta = Ae^{i\phi}
\]

and

\[
(e \cos \varphi - f \sin \varphi + i \cos \varphi + i \sin \varphi) \cos \theta + (g \cos \varphi - h \sin \varphi + i \cos \varphi + i \sin \varphi) \sin \theta = (C + iD) e^{i\phi}
\]

Dividing, and equating real and imaginary parts gives:

**Real part:**

\[
Ca \cos \varphi \cos \theta - Cb \sin \varphi \cos \theta - Db \cos \varphi \cos \theta - Da \sin \varphi \cos \theta + Cc \cos \varphi \sin \theta - Cd \sin \varphi \sin \theta - \\
Dd \cos \varphi \sin \theta - Dc \sin \varphi \sin \theta - Ae \cos \varphi \cos \theta + Af \sin \varphi \cos \theta - Ag \cos \varphi \sin \theta + Ah \sin \varphi \sin \theta = 0
\]

equation 5B-10

**Imaginary part:**

\[
Da \cos \varphi \cos \theta - Db \sin \varphi \cos \theta - Cb \cos \varphi \cos \theta + Cas \sin \varphi \cos \theta + Dc \cos \varphi \sin \theta - Dd \sin \varphi \sin \theta + \\
Cd \cos \varphi \sin \theta + Cc \sin \varphi \sin \theta - Af \cos \varphi \cos \theta - Ae \sin \varphi \cos \theta - Ah \cos \varphi \sin \theta - Ag \sin \varphi \sin \theta = 0
\]

equation 5B-11
Appendix 5B Singular value decomposition

This leads to equation 5.8 being written as:

\[
\begin{bmatrix}
C_1 \cos \theta_1 & -D_1 \cos \theta_1 & C_1 \sin \theta_1 & -D_1 \sin \theta_1 & -A_1 \cos \theta_1 & 0 & -A_1 \sin \theta_1 & 0 \\
D_1 \cos \theta_1 & C_1 \cos \theta_1 & D_1 \sin \theta_1 & C_1 \sin \theta_1 & 0 & -A_1 \cos \theta_1 & 0 & -A_1 \sin \theta_1 \\
C_2 \cos \theta_2 & -D_2 \cos \theta_2 & C_2 \sin \theta_2 & -D_2 \sin \theta_2 & -A_2 \cos \theta_2 & 0 & -A_2 \sin \theta_2 & 0 \\
D_2 \cos \theta_2 & C_2 \cos \theta_2 & D_2 \sin \theta_2 & C_2 \sin \theta_2 & 0 & -A_2 \cos \theta_2 & 0 & -A_2 \sin \theta_2 \\
C_3 \cos \theta_3 & -D_3 \cos \theta_3 & C_3 \sin \theta_3 & -D_3 \sin \theta_3 & -A_3 \cos \theta_3 & 0 & -A_3 \sin \theta_3 & 0 \\
D_3 \cos \theta_3 & C_3 \cos \theta_3 & D_3 \sin \theta_3 & C_3 \sin \theta_3 & 0 & -A_3 \cos \theta_3 & 0 & -A_3 \sin \theta_3 \\
C_4 \cos \theta_4 & -D_4 \cos \theta_4 & C_4 \sin \theta_4 & -D_4 \sin \theta_4 & -A_4 \cos \theta_4 & 0 & -A_4 \sin \theta_4 & 0 \\
D_4 \cos \theta_4 & C_4 \cos \theta_4 & D_4 \sin \theta_4 & C_4 \sin \theta_4 & 0 & -A_4 \cos \theta_4 & 0 & -A_4 \sin \theta_4
\end{bmatrix}
\]

Splitting \( x \) into its sine and cosine components,

\[
\begin{bmatrix}
a \\
b \\
c \\
d \\
e \\
f \\
g \\
h
\end{bmatrix}
= \begin{bmatrix}
\cos \varphi - b \sin \varphi \\
b \cos \varphi + a \sin \varphi \\
c \cos \varphi - d \sin \varphi \\
d \cos \varphi + c \sin \varphi \\
e \cos \varphi - f \sin \varphi \\
f \cos \varphi + e \sin \varphi \\
g \cos \varphi - h \sin \varphi \\
h \cos \varphi + g \sin \varphi
\end{bmatrix}
\]

\[
x = \cos \varphi + \sin \varphi
\]

the solutions for \( x \) therefore represent the two orthogonal vectors shown in equation 5B-12. It is also worth noting, that the solution for \( x \) can be multiplied by any complex number, and still be valid.
After being coated with the alignment layer polymer, each Hull slide was rotated in the $xy$ plane relative to the Argon-Ion laser. The molecular orientation of the alignment layer was determined by the plane of polarisation of the laser. The director of the LC layer which was then spun onto this polymerised layer aligned according to this molecular orientation. The end result was a birefringent slide where the optic axis of each area was determined by the original polarisation of the Argon-Ion laser, as illustrated in Figure 8.5. The particular orientation of each area of the slide is shown in Table 8A-1.

<table>
<thead>
<tr>
<th>Slide no.</th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
<th>Area 4</th>
<th>Area 5</th>
<th>Area 6</th>
<th>Area 7</th>
<th>Area 8</th>
<th>Area 9</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0</td>
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<td>0</td>
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<td>0</td>
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<td>90</td>
<td>45</td>
<td>0</td>
<td>0</td>
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<td>60</td>
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<td>30</td>
<td>60</td>
<td>90</td>
<td>0</td>
</tr>
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<td>45</td>
<td>90</td>
<td>90</td>
<td>0</td>
<td>0</td>
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<td>100</td>
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<td>20</td>
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<td>100</td>
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<td>80</td>
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<td>40</td>
<td>20</td>
<td>0</td>
<td>160</td>
<td>140</td>
<td>120</td>
</tr>
</tbody>
</table>

Table 8A-1 Orientation of photo-alignment layer for each area of each Hull slide
Appendix 8B

Calculations of parameters of birefringent Rolic slides areas a and b

As described in s.8.5.2, the polarisation of light emerging from two areas of the Rolic slides was measured for several input polarisations. The experimentally measured values for $\alpha$ and $\varepsilon$ for each input polarisation are presented below, together with the values for $\Delta\nu d$ and $\theta$ which give the nearest match to these experimentally measured values. It can be seen that for each slide the values for $\Delta\nu d$ and $\theta$ are similar for the two areas A and B.
### Table 8B-1 Birefringence calculations for Rolic slides 20 and 22

<table>
<thead>
<tr>
<th>Area A</th>
<th>Output polarisation</th>
<th>Input polarisation (deg)</th>
<th>Area B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>60</td>
<td>75</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>81</td>
<td>31.5</td>
<td>16.75</td>
</tr>
<tr>
<td>Corrected (e)</td>
<td>3.876</td>
<td>13.32</td>
<td>4.83</td>
</tr>
<tr>
<td>(R^2)</td>
<td>1</td>
<td>0.9998</td>
<td>0.9999</td>
</tr>
<tr>
<td>Average (\Delta nd) (nm)</td>
<td>220</td>
<td>215</td>
<td>215</td>
</tr>
<tr>
<td>Average (\theta) (deg)</td>
<td>42</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>81.83</td>
<td>31.39</td>
<td>16.89</td>
</tr>
<tr>
<td>(e)</td>
<td>3.78</td>
<td>14.14</td>
<td>4.49</td>
</tr>
<tr>
<td></td>
<td>7.5</td>
<td>124.5</td>
<td>108.5</td>
</tr>
<tr>
<td>Corrected (e)</td>
<td>153.2</td>
<td>5.75</td>
<td>14.51</td>
</tr>
<tr>
<td>(R^2)</td>
<td>1</td>
<td>0.9998</td>
<td>0.9999</td>
</tr>
<tr>
<td>Average (\Delta nd) (nm)</td>
<td>210</td>
<td>225</td>
<td>225</td>
</tr>
<tr>
<td>Average (\theta) (deg)</td>
<td>95</td>
<td>95</td>
<td>95</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>7.48</td>
<td>124.71</td>
<td>108.66</td>
</tr>
<tr>
<td>(e)</td>
<td>173</td>
<td>5.1</td>
<td>13.52</td>
</tr>
</tbody>
</table>

*Table 8B-1 Birefringence calculations for Rolic slides 20 and 22*
Appendix 8B \textit{And} calculations for Rolic slides

Slide 24 (quoted $\Delta n d$ 280nm)

<table>
<thead>
<tr>
<th>Output polarisation</th>
<th>Input polarisation (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area A</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>83</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>8.88</td>
</tr>
<tr>
<td>Corrected $\epsilon$</td>
<td>0.9999</td>
</tr>
<tr>
<td>$R^2$</td>
<td>250</td>
</tr>
<tr>
<td>Average $\Delta n d$ (nm)</td>
<td>42</td>
</tr>
<tr>
<td>Average $\theta$ (deg)</td>
<td>83.21</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>8.66</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>11</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>321.78</td>
</tr>
<tr>
<td>Corrected $\epsilon$</td>
<td>0.9999</td>
</tr>
<tr>
<td>$R^2$</td>
<td>260</td>
</tr>
<tr>
<td>Average $\Delta n d$ (nm)</td>
<td>96</td>
</tr>
<tr>
<td>Average $\theta$ (deg)</td>
<td>11.1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>325.94</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>257</td>
</tr>
</tbody>
</table>

Table 8B-2 Birefringence calculations for Rolic slide 2
Appendix 8C Image Pro Plus macros

Image pro plus macros to analyse security slide images

8C.1 Macro used for Hull slides (dividing each slide into 9 areas)

Sub Sue9()

Dim S as integer
Dim Iname as string * 255
ret=IpStGetInt("Please enter slide number",S,1,-32000,32000)

Dim Jname As String * 255
ret=IpStAutoName("d:\sms\TC\H#\GLOFF.tif", S, Jname)
ret=IpWsLoad(Jname, "tif")

Dim Xstart as integer
Dim Ystart as integer
ret=IpStGetInt("Please enter start x-co-ord (top LHS)",Xstart,1,1,512)
ret=IpStGetInt("Please enter start y-co-ord (top LHS)",Ystart,1,1,512)

ipRect.left = Xstart+44
ipRect.top = Ystart+65
ipRect.right = Xstart+44+52
ipRect.bottom = Ystart+47+65
ret = IpAoiCreateBox(ipRect)
ret = IpHstCreate()
ret = IpHstUpdate()
ret = IpStAutoName("c:\asue\TCsecurity images\9Hull#.hst", S, Iname)
ret = IpHstSave(Iname, S_STATS)

ipRect.left = Xstart+44
ipRect.top = Ystart+65
ipRect.right = Xstart+44+52
ipRect.bottom = Ystart+47+65
ret = IpAoiCreateBox(ipRect)
ret = IpHstCreate()
ret = IpHstUpdate()
ret = IpStAutoName("c:\asue\TCsecurity images\9Hull#.hst", S, Iname)
ret = IpHstSave(Iname, S_STATS)

ipRect.left = ipRect.left+113
ipRect.top = ipRect.top+3
ipRect.right = ipRect.right+113
ipRect.bottom = ipRect.bottom+3
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()

ipRect.left = ipRect.left-226
ipRect.top = ipRect.top+130
ipRect.right = ipRect.right-226
ipRect.bottom = ipRect.bottom+130
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

8C.2 Macro used for Hull slides (dividing each slide into 9 areas)

Sub Sue9()

Dim S as integer
Dim Iname as string * 255
ret=IpStGetInt("Please enter slide number",S,1,-32000,32000)

Dim Jname As String * 255
ret=IpStAutoName("d:\sms\TC\H#\GLOFF.tif", S, Jname)
ret=IpWsLoad(Jname, "tif")

Dim Xstart as integer
Dim Ystart as integer
ret=IpStGetInt("Please enter start x-co-ord (top LHS)",Xstart,1,1,512)
ret=IpStGetInt("Please enter start y-co-ord (top LHS)",Ystart,1,1,512)

ipRect.left = Xstart+44
ipRect.top = Ystart+65
ipRect.right = Xstart+44+52
ipRect.bottom = Ystart+47+65
ret = IpAoiCreateBox(ipRect)
ret = IpHstCreate()
ret = IpHstUpdate()
ret = IpStAutoName("c:\asue\TCsecurity images\9Hull#.hst", S, Iname)
ret = IpHstSave(Iname, S_STATS)

ipRect.left = ipRect.left+113
ipRect.top = ipRect.top+3
ipRect.right = ipRect.right+113
ipRect.bottom = ipRect.bottom+3
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
Appendix 8C Image Pro Plus macros

```plaintext
ret = IpHstSave(Iname, S_STATS+S_APPEND)
ipRect.left = ipRect.left+114
ipRect.top = ipRect.top
ipRect.right = ipRect.right+114
ipRect.bottom = ipRect.bottom
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

Dim Mname As String * 255
ret=IpStAutoName("d:\sms\TC\H#\GL41.tif", S, Mname)
ret=IpWsLoad(Mname, "tif")

ipRect.left = Xstart+44
ipRect.top = Ystart+65
ipRect.right = Xstart+44+52
ipRect.bottom = Ystart+47+65
ret = IpAoiCreateBox(ipRect)
ret = IpHstCreate()
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

ipRect.left = ipRect.left+113
ipRect.top = ipRect.top+3
ipRect.right = ipRect.right+113
ipRect.bottom = ipRect.bottom+3
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

ipRect.left = ipRect.left+114
ipRect.top = ipRect.top+1
ipRect.right = ipRect.right+114
ipRect.bottom = ipRect.bottom+1
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

ipRect.left = ipRect.left-228
ipRect.top = ipRect.top+127
ipRect.right = ipRect.right-228
ipRect.bottom = ipRect.bottom+127
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

Dim Kname As String * 255
ret=IpStAutoName("d:\sma\TC\H#\GL551.tif", S, Kname)
ret=IpWsLoad(Kname, "tif")

ipRect.left = ipRect.left+110
ipRect.top = ipRect.top+3
ipRect.right = ipRect.right+110
ipRect.bottom = ipRect.bottom+3
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

ipRect.left = ipRect.left+114
ipRect.top = ipRect.top
ipRect.right = ipRect.right+114
ipRect.bottom = ipRect.bottom
ret = IpAoiCreateBox(ipRect)
ret = IpHstCreate()
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

Dim Kname As String * 255
ret=IpStAutoName("d:\sma\TC\H#\GL551.tif", S, Kname)
ret=IpWsLoad(Kname, "tif")

ipRect.left = Xstart+44
ipRect.top = Ystart+65
ipRect.right = Xstart+44+52
ipRect.bottom = Ystart+47+65
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
```

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Appendix 8C Image Pro Plus macros

ret = IpHstSave(Iname, S_STATS+S_APPEND)

ipRect.left = ipRect.left+113
ipRect.top = ipRect.top+3
ipRect.right = ipRect.right+113
ipRect.bottom = ipRect.bottom+3
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

ipRect.left = ipRect.left+114
ipRect.top = ipRect.top+1
ipRect.right = ipRect.right+114
ipRect.bottom = ipRect.bottom+1
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

Dim Lname As String * 255
ret=lpStAutoName("d:\sms\TC\#GL81.tif", S, Lname)
ret=IpWsLoad(Lname, "tif")
ipRect.left = ipRect.left+110
ipRect.top = ipRect.top+5
ipRect.right = ipRect.right+110
ipRect.bottom = ipRect.bottom+5
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

ipRect.left = ipRect.left+112
ipRect.top = ipRect.top+5
ipRect.right = ipRect.right+112
ipRect.bottom = ipRect.bottom+5
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

ipRect.left = ipRect.left+113
ipRect.top = ipRect.top+2
ipRect.right = ipRect.right+113
ipRect.bottom = ipRect.bottom+2
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

ipRect.left = ipRect.left+114
ipRect.top = ipRect.top+1
ipRect.right = ipRect.right+114
ipRect.bottom = ipRect.bottom+1
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

ipRect.left = ipRect.left-228
ipRect.top = ipRect.top+127
ipRect.right = ipRect.right-228
ipRect.bottom = ipRect.bottom+127
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

ipRect.left = ipRect.left+110
ipRect.top = ipRect.top+3
ipRect.right = ipRect.right+110
ipRect.bottom =ipRect.bottom+3
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

ipRect.left = ipRect.left+114
ipRect.top = ipRect.top
ipRect.right = ipRect.right+114
ipRect.bottom =ipRect.bottom
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

ipRect.right = ipRect.right+113
ipRect.top = ipRect.top+3
ipRect.right = ipRect.right+113
ipRect.bottom =ipRect.bottom+3
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

ipRect.left = ipRect.left+112
ipRect.top = ipRect.top+5
ipRect.right = ipRect.right+112
ipRect.bottom =ipRect.bottom+5
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

ipRect.left = ipRect.left+113
ipRect.top = ipRect.top+2
ipRect.right = ipRect.right+113
ipRect.bottom =ipRect.bottom+2
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)

ipRect.left = ipRect.left-228
ipRect.top = ipRect.top+127
ipRect.right = ipRect.right-228
ipRect.bottom =ipRect.bottom+127
ret = IpAoiCreateBox(ipRect)
ret = IpHstUpdate()
ret = IpHstSave(Iname, S_STATS+S_APPEND)
Sub Suel10O()
Dim S As Integer
Dim Iname As String * 255
ret = IpStGetInt("Please enter slide number", S, 1, -32000, 32000)
Dim R As Integer
ret = IpStGetInt("Please enter rotation", R, 1, -180, 180)
Dim Jname As String * 255
ret = IpStAutoName("d:sma\TC\Rolic\H#\GLO FF.tif", S, Iname)
ret = IpWsLoad(Jname, "tif")
ret = IpWsRotate(R, RA_CENTER)
Dim Xstart as integer
Dim Ystart as integer
ret = IpStGetInt("Please enter start x-co-ord (top LHS)", Xstart, 1, 1, 512)
ret = IpStGetInt("Please enter start y-co-ord (top LHS)", Ystart, 1, 1, 512)
ret = IpStAutoName("c:sau\TCsecurity images\100RSslide#.hst", S, Iname)
For M = 0 to 10
F = 27*M
For N = 0 to 8
C = 36*N
ipRect.left = Xstart + 25 + C
ipRect.top = Ystart + F + 10
ipRect.right = Xstart + 35 + 25 + C
ipRect.bottom = Ystart + 27 + F + 10
ret = IpAoiCreateBox(ipRect)
ret = IpHstCreate()
ret = IpHstUpdate() 
ret = IpHstSave(Iname, S_STATS+S_APPEND)
eend sub

8C.2 Macro used for Rolic photographic slides

Next N
Next M
Appendix 8C Image Pro Plus macros

Dim Kname as string * 255
ret=IpStAutoName("d:\sms\TC\Rolic\H#\GL41.tif", S, Kname)
ret=IpWsLoad(Kname, "tif")
ret=IpWsRotate(R, RA_CENTER)

Dim G as integer
Dim H as integer
Dim J as integer
Dim K as integer
For J = 0 to 10
K=27*J
For G = 0 to 8
H=36*G
ipRect.left = Xstart+25+H
ipRect.top = Ystart+K+10
ipRect.right = Xstart+35+25+H
ipRect.bottom = Ystart+27+K+10
ret = IpAoiCreateBox(ipRect)
ret = IpHstCreate()
ret = IpHstUpdate()
ret = IpHstSave(Iname,
S_STATS+S_APPEND)
Next G
Next J

Dim Lname as string * 255
ret=IpStAutoName("d:\sms\TC\Rolic\H#\GL55.tif", S, Lname)
ret=IpWsLoad(Lname, "tif")
ret=IpWsRotate(R, RA_CENTER)

Dim Mname as string * 255
ret=IpStAutoName("d:\sms\TC\Rolic\H#\GL81.tif", S, Mname)
ret=IpWsLoad(Mname, "tif")
ret=IpWsRotate(R, RA_CENTER)

Dim B as integer
Dim E as integer
Dim W as integer
Dim Z as integer
For W = 0 to 10
Z=27*W
For B = 0 to 8
E=36*B
ipRect.left = Xstart+25+E
ipRect.top = Ystart+Z+10
ipRect.right = Xstart+35+25+E
ipRect.bottom = Ystart+27+Z+10
ret = IpAoiCreateBox(ipRect)
ret = IpHstCreate()
ret = IpHstUpdate()
ret = IpHstSave(Iname,
S_STATS+S_APPEND)
Next B
Next W

Dim Q as integer
Dim A as integer
Dim U as integer
Dim V as integer
For U = 0 to 10
V=27*U
For Q = 0 to 8
A=36*Q
ipRect.left = Xstart+25+A
ipRect.top = Ystart+V+10
ipRect.right = Xstart+35+25+A
ipRect.bottom = Ystart+27+V+10
ret = IpAoiCreateBox(ipRect)
ret = IpHstCreate()
ret = IpHstUpdate()
ret = IpHstSave(Iname,
S_STATS+S_APPEND)
Next Q
Next U

end sub
Appendix 8D

Additional error scores for Rolic security marks

8D.1 Error scores for Rolic slides 20 and 24 when polariser rotated 10 & 20 deg.

Figure 8D-1 and Figure 8D-2 show the error score histograms for Rolic slides 20 and 24, as the polariser is rotated to simulate extra ‘false slides’. These figures can be compared with the similar figure 8-31 in Chapter 8 for slide 22. As for slide 22, the error score is very sensitive to slight degrees of polariser rotation, and the ‘true’ slide (i.e. that when the polariser is at 10°) can easily be distinguished from the ‘false’ slides (when the polariser is at 10° or 20°). There is no overlap in error score between these ‘true’ and ‘false’ slides.

Rolic slide 20. C>=100. Comparison of true slide with polariser rotated 10 and 20 deg.

![Figure 8D-1](image)

Figure 8D-1 Effect of polariser rotation on verification of Rolic slide 20.
Appendix 8D Additional error scores for Rolic security marks

Rolic slide 24. C >=100. Comparison of true slide with polariser rotated 10 and 20 deg.

Figure 8D-2 Effect of polariser rotation on verification of Rolic slide 24.

8D.2 Error scores for Rolic slide 24 when slide is rotated

Figure 8D-3 shows the change in error score with slide rotation for slide 24. When the slide was rotated by more than ±4°, the error score was more than 0.15, and when it was rotated by ±5°, the error score increased to approximately 0.2.

Figure 8D-3 Error score histogram for slide 24. Change in error score with slide rotation.
Use of a commercially produced liquid crystal display as a polarisation filter

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ABSTRACT

This paper describes the characterisation of the polarisation modulation produced by a commercially manufactured liquid crystal television from which the polarisers have been removed. Experimental results are compared with a Jones matrix model of the display which has been developed by researchers at UCL.10,12 Experimental analysis shows that the behaviour of the device agrees with prediction, but deviates quantitatively for certain input polarisations. An algorithm has been developed to determine an unknown input polarisation from intensity measurements taken through a fixed analyser, with the display at several different applied voltages. The commercially produced display, apart from its potentially lower cost, due to mass production, has the advantage that it has a large number of pixels. This allows selective control or measurement of the polarisation across the two dimensional input field of view.

Keywords: polarisation, polarization, liquid crystal display, Jones matrix, light, surface topology, refractive index.

1. INTRODUCTION

Twisted nematic liquid crystal (TNLC) devices are now widely used in display technology. The displays modulate light by altering the polarisation of light. This occurs because the LC molecules rotate the plane of polarisation of incident light, by altering the phase difference between the two orthogonal components of the light vector. In displays, in order to modulate the intensity, TNLC cells are usually mounted between crossed polarisers, with the transmission axis of the polariser being parallel to the brushing direction of the liquid crystal (fig. 1). When the TNLC is in its relaxed (off) state, the average direction of the LC molecules (the director) rotates the plane of polarisation, so that light is transmitted by the crossed polariser and the display appears bright. When a voltage is applied to the LC, the molecules tilt, and therefore do not rotate the plane of polarisation. Hence, light is not transmitted by the crossed polariser, and the display appears dark. The brushing layer holds the surface molecules in place and this provides the resorting force when the voltage is removed.

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Measurement of the polarisation of a light beam is conventionally performed using a linear analyser. This is rotated, and the polarisation can be recorded by measuring the intensity of light transmitted at various positions of the analyser. An alternative method, which avoids the need for a mechanically rotating polariser, is to use TNLC cells to rotate the plane of polarisation, using a fixed analyser. A method for measuring polarisation using two TNLC cells and a camera has been developed by Wolff et al. The TNLC cells are specially made, and have twists of 90° and 45°, so that by alternately switching the cells on and off, light is rotated by either 0°, 45° or 90° as required, and the intensity transmitted by a (fixed) linear analyser is measured.

Light specularly reflected from a dielectric is generally partially polarised. The degree of partial polarisation depends on the refractive index of the reflecting surface, and the angle of incidence. Therefore by measuring the change in polarisation of specularly reflected light it is possible to gauge the refractive index or angle of incidence of the reflecting surface (if the other parameter is known). Much research has already been based on this phenomenon. This principle is used in ellipsometry for determining refractive index.

Two methods, based on Fresnel equations, for gauging refractive index from polarisation have been reported. The Fresnel reflectance model assumes that all partial polarisation is produced by specular reflection, and measures the ratio of the Fresnel reflection coefficients \( R_1/R_0 \) using unpolarised incident light. \( R_1 \) and \( R_0 \) are the reflection coefficients for components perpendicular and parallel to the plane of incidence respectively. For specular reflection, \( R_1 \) is greater than \( R_0 \), so the magnitude of the perpendicular polarisation component is attenuated less than the parallel component, i.e. more perpendicular than parallel light is reflected. The result of this is that incident unpolarised light becomes partially polarised perpendicular to the plane of incidence. This method can only be used for angles of incidence between 20° and 60°, as for incident angles less than 20° the Fresnel ratio approaches unity so the incident light is no longer partially polarised. At greater than 60° the diffuse
component of reflection is not small compared with the specular component and this invalidates the assumptions made in Wolff's equations. When the incident light is partially polarised (as in some natural skylight), Chen & Wolff use a phase-based method to distinguish dielectrics from metal. This uses the fact that linearly polarised light will become elliptically polarised when reflected from a metal, but will remain linearly polarised when reflected from a dielectric. (For a dielectric the phase change on reflection is either 0 or π, but for metals it is somewhere in between.)

In order to fully rotate the plane of polarisation, the TNLC layer must be thick enough to fulfil the Mauguin limit (where, for single pass, \( d\Delta n > \lambda / 2 \) \(^i\)). The methods already reported, have used cells which were thick enough to fulfil this criterion. The work reported here differs from previous approaches in that it uses a thin (<5 μm), pixellated display, rather than a single cell. As the LCD is thin, the Mauguin limit is not fulfilled, and the phase difference introduced between x and y components is always less than 180°. As a result, the polarisation change produced by the display is more complex than merely rotation, and so linearly polarised incident light becomes elliptical after passing through the display in its off state.

2. MATERIALS AND METHODS
Initial experiments were designed to compare the input and output polarisations of a LCD at various voltages. The simplest way to do this was to use linearly polarised incident light, and to measure the polarisation of light emergent from the LCD.

The experimental layout is as in figure 2. The light source was a linearly polarised HeNe laser, (quoted ratio 500:1). A neutral density filter reduced the intensity of the laser to 0.869 mW/mm\(^2\) (0.3155 mW in the beam of diameter of 0.68 mm) to avoid damaging the LCD (maximum intensity threshold 2.5 W per device, area 56.6 x 42.5 mm). The beam was passed through a quarter wave plate to produce circularly polarised light. This was done so that when the linear polariser was rotated to produce different orientations of linearly polarised light, there would be approximately equal intensities of linearly polarised light for each orientation. Linearly polarised light was produced by a sheet polariser. The light was then incident on the LCD and analysed by a sheet polariser in front of a zero-biased silicon photodiode, which had been calibrated with neutral density filters.

The LCD used in this work is a Philips LDK036T-20 liquid crystal projection TV, as detailed by Gardner et al.\(^i\). The pixels are covered by a mask of 88.5 μm square, and the pixels themselves are of dimensions 83 x 69 μm. The experimental layout is shown in figure 2.

---

\(^i\) Where \( d \) is the thickness of the cell
The LCD was powered by a 5V supply. A display signal was generated on the LCD by splitting the video signal from a computer to a VGA display, so that part of the signal went to the LCD, and part to the computer monitor. Consequently, any pattern produced on the computer was replicated on the LCD. A homogeneous full screen pattern was produced on the LCD. The LCD was monochrome, and grey levels from 0 (black) to 255 (white) could be produced. The grey level displayed on the computer screen was related to the voltage applied to the LCD by using a calibration curve of voltage applied to a LC test cell versus transmitted intensity between crossed polarisers. This enabled grey level against intensity to be compared with voltage against intensity which allowed a relationship between grey level and voltage to be established.

Input polarisation was varied by rotating the linear polariser. The following sign convention was adopted: zero degrees was defined as vertical alignment of the transmission axis of the polariser (or analyser), and this was taken as the x axis (a positive direction being vertical). The y axis was horizontal, with the positive direction being to the right looking with the source behind the observer. Light was propagating along the z axis. An anticlockwise rotation was treated as negative, and clockwise as positive. For each input polarisation, measurements of output polarisation were taken for grey levels 0-255 (applied voltages of 4.1V - 1.91V). At least 4 readings of azimuth angle, α (figure 3) and maximum/minimum intensities were recorded for each grey level. This was carried out by rotating the analyser until the maximum (or minimum) readings were noted on the detector. The analyser position and detector reading were then read. It was ensured that the analyser was rotated an equal number of times in each direction (clockwise and anticlockwise) so that where the maximum or minimum reading persisted for a large portion of analyser rotation (such as where the light was nearly circularly polarised) the midpoint was estimated, this range was used to calculate the standard error for the error bar.

The experiment was then repeated in double pass, as shown in figure 4. After traversing the LCD once, light was reflected by a dielectric coated aluminium mirror (mounted so as to nearly touch the LCD). The equipment was positioned so that the light was reflected at as near normal incidence as the layout of the optical bench would allow (approximately 5°. This did not change the polarisation significantly 6). Perfectly normal incidence was not possible in double pass, as this would necessitate the inclusion of a
beam splitter. This would alter the polarisation of the beam according to the Fresnel equations. (A polarising beam splitter was not available.) As the angle of incidence was not completely normal, the beam did not pass through the same pixels in each direction of travel through the LCD, and some of the beam was blocked (or diffracted) by the mask covering the pixels. It was therefore ensured that the same grey level was applied to every pixel of the LCD.

An algorithm was then developed to determine the polarisation of light incident on the LCD from intensity measurements (taken through a linear polariser) at several grey levels (voltages).

3. RESULTS

The polarisation of the output light can be represented by the azimuth angle of the polarisation ellipse, and ellipticity. Here, we define ellipticity as the major axis divided by the minor axis of the ellipse (unlike the definition in Azzam & Bashara 7). Output polarisations for various inputs are depicted below. A Jones matrix model for the LCD has been developed by researchers at UCL.12 The experimental results presented here are compared with the model. The following figures show plots of cell voltage versus azimuth angle, and cell voltage versus ellipticity. It is the combination of azimuth angle of the ellipse in addition to ellipticity (and direction of rotation) that describes the polarisation of the light, so the graphs should be viewed in pairs. It can be seen that the size of the error bars on the azimuth angle graphs depends on the ellipticity of the light in question: i.e. when the ellipticity is low (approximately <5) the error bars on the azimuth angle become larger (figure 9a) because the light is nearly circular (and the azimuth angle is, therefore, less well defined). When the ellipticity becomes large, the errors in the ellipticity measurement increase (although the azimuth angle is more well defined - see discussion).

It should be noted, when looking at the following figures of cell voltage versus ellipticity, that the scale has been varied from one graph to another in order to show the results in greater detail.
3.1 Single Pass

Figure 5a) Input +45°

Figure 5b) Input +45°

Figure 6a) Input -60°

Figure 6b) Input -60°

Figure 7a) Input 0°

Figure 7b) Input 0°
3.2 Double Pass

**Figure 8a) Input +45°**

Double pass, cell voltage versus azimuth angle, input polarization +45° deg

**Figure 8b) Input +45°**

Double pass, cell voltage versus ellipticity, input polarization +45° deg

**Figure 9a) Input -60°**

Double pass, cell voltage versus azimuth angle, input polarization -60° deg

**Figure 9b) Input -60°**

Double pass, cell voltage versus ellipticity, input polarization -60° deg

**Figure 10a) Input 0°**

Double pass, cell voltage versus azimuth angle, input polarization 0 deg

**Figure 10b) Input 0°**

Double pass, cell voltage versus ellipticity, input polarization 0 deg
A pictorial representation of the results for a single pass experiment is given in figures 11 and 12, and that for double pass in figures 13 and 14. In these figures the polarisation ellipse is plotted on the figure for differing voltages and linear input polarisations. The voltages across the figures are as nearly the same as it was possible to make them easily: the model was developed using 0.05V steps, and these did not always exactly correspond with those equivalent to the grey levels used; measurements were taken at steps of 10 grey levels. Where no voltage is specified along the top of the figure, that column is a duplicate of that to its left.

![Pictorial representation](image)

**Figure 11:** LCD experimental results ~ Output polarisation vs. LCD voltage for various linear input polarisations.  
*Single pass.*
Figure 12: LCD model ~ Output polarisation vs. LCD voltage for various linear input polarisations. Single pass.

Figure 13: LCD experimental results ~ Output polarisation versus voltage for various linear input polarisations. Double pass.
Figure 14: LCD model - Output polarisation vs. LCD voltage for various linear input polarisations. Double pass.

4. DISCUSSION

4.1 Analysis of results

It can be seen from figures 5-14 that the azimuth angle of the ellipse is generally predicted well by the model, although the plane of polarisation is rotated more than the model predicts. By looking at figures 11 & 12, and 13 & 14 it is evident that the amount of agreement between the experimental results and the Jones matrix model depends on the ellipticity being measured. In single pass it is expected that the maximum change in polarisation would be obtained at low voltages, and the minimum change at high voltages, when the LC is fully switched. It can be seen from figures 11 and 12 that, for low voltages, the LCD does not change the phase difference between the $x$ and $y$ components of the light by a full $180^\circ$, and so linearly polarised input light does not always stay linearly polarised. The model and experiment agree well in this respect. For middle voltages (about 2.32-3.33 V) the agreement is good for most inputs except $+165^\circ$ where the model predicts more linearity than is measured. For high voltages (above 3.33 V) the model and experiment agree for most input polarisations, but there is a general trend, again, for the model to predict a higher degree of linearity than is found experimentally. This trend is not universal, as can be seen from figures 5-10, which represent selected input polarisations. The whole range of linear input polarisations is shown in figures 11-14. It is difficult to accurately measure ellipticity, when the polarisation of the light is approaching linearity, as whichever method is used to quantify elliptical polarisation, larger errors are necessarily introduced when the denominator is small. The trend appears to be that the agreement between the model and experiment is greatest where the output light was least linear (i.e. most circular), and the voltage at which this occurs depends on the polarisation of the input light. For all cases, the shape of the experimental curves when plotting cell voltage against ellipticity was the same as predicted by the model, but the numerical agreement varied depending on input.
It can be seen from figures 8-10, in double pass, when plotting cell voltage versus azimuth angle, there is a degeneracy (one azimuth angle can occur for more than one LCD voltage) at every angle of linear input except at -60°, where there is a large change in azimuth angle with voltage, (although the ellipticity is low)

4.2 Application
A method is presented for determining the polarisation of light incident on the LCD. Intensity measurements are taken through a fixed analyser, oriented along the \( x \) axis, at 4 different LCD grey levels. It is assumed that the input light has a polarisation represented by a Jones vector with components \( a+ib \), and \( c+id \), and the output light has a polarisation represented by a Jones vector with components \( J+iK \), and \( L+iM \). If the LCD behaves as a single Jones matrix for each grey level, then the polarisation of the emergent light is given by equation 1.

\[
\text{Jones matrix model of LCD} \quad \text{Input Jones vector} \quad \text{Output Jones vector} \\
\begin{bmatrix}
R_{1} + iM_{1} \\
R_{2} + iM_{2} \\
\end{bmatrix} \begin{bmatrix}
a + ib \\
c + id \\
\end{bmatrix} = \begin{bmatrix}
J + iK \\
L + iM \\
\end{bmatrix} \\
\text{(Equation 1)}
\]

This is simply the multiplication of a two dimensional complex matrix with a one dimensional complex vector to give a one dimensional complex vector. Intensity is measured with the analyser along the \( x \) axis, so the \( y \) component can be ignored. The intensity measured is the sum of the squares of the real and imaginary parts of the output Jones vector, i.e: \[
\text{Intensity} \quad A = |J|^2 + |K|^2 \\
\text{(Equation 2)}
\]

Multiplying out equation 1 and separating the real and imaginary components leads to equation 3. In equation 3 \( K_{n} \) are coefficients derived from the real and imaginary components of the Jones matrix for each grey level used.

\[
A = K_{1}(a^2 + b^2) + K_{2}(c^2 + d^2) + K_{4}(ad - bc) + K_{3}(ac + bd) \\
\text{(Equation 3)}
\]

Taking intensity measurements at 4 different grey levels (to give \( A, B, C \) and \( D \)), this gives 4 simultaneous equations which can be solved for \( a, c \) and \( d \) (equation 4) \( b \) is taken to be zero, so \( d \) gives the relative phase of the \( y \) component of the Jones vector of the input light.

\[
\begin{bmatrix}
K_{1} & K_{2} & K_{3} & K_{4} \\
K_{5} & K_{6} & K_{7} & K_{8} \\
K_{9} & K_{10} & K_{11} & K_{12} \\
K_{13} & K_{14} & K_{15} & K_{16} \\
\end{bmatrix}^{-1} \begin{bmatrix}
a + b \\
b^2 + c^2 \\
a + c \\
ad - bc \\
\end{bmatrix} = \begin{bmatrix}
E \\
F \\
G \\
H \\
\end{bmatrix} \\
\text{(Equation 4)}
\]

Measuring intensity only (i.e. no phase information) leads to one ambiguity. This was a result of assuming that \( a \) was positive. If \( a \) was in fact negative, the phase of the \( x \) component would by 180° different, so the ellipticity of the light would remain unchanged, (although its sense of rotation would be reversed), but the azimuth angle would be reflected about the co-ordinate system.

This algorithm was used for the modelled data, and was found to be very error dependent. The coefficients, \( E, F, G \) and \( H \) were obtained from intensity measurements, and then \( a \) was calculated as the square root of \( E \). From that \( G \) and \( H \) enabled values of \( c \) and \( d \) to be found easily by dividing by \( a \). Because these three equations were solved simultaneously, any error in one component increased the error in the others.
The results of the algorithm in theory and practice are presented in table 1. The modelled results were obtained by using a known input Jones vector, then multiplying it by the Jones matrix model representing the LCD. This gave a known output polarisation. The intensity of the x component was then calculated and it was this value which was put into equation 4 (at 4 different grey levels). This then gave values for $E$, $F$, $G$ and $H$, which enabled the input Jones vector to be determined.

<table>
<thead>
<tr>
<th>Input $\alpha$ (°)</th>
<th>Input ellipticity</th>
<th>Predicted input $\alpha$</th>
<th>Predicted input ellipticity</th>
<th>Experimental results</th>
<th>Predicted input $\alpha$</th>
<th>Predicted input ellipticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>-45.00</td>
<td>Linear</td>
<td>-43.94°</td>
<td>4676</td>
<td>Linear</td>
<td>-45.32°</td>
<td>95.80</td>
</tr>
<tr>
<td>-30.00</td>
<td>Linear</td>
<td>-30.46°</td>
<td>3663</td>
<td>Linear</td>
<td>-33.00°</td>
<td>55.93</td>
</tr>
</tbody>
</table>

Table 1 Comparison of experimental and modelled results for algorithm for determining unknown polarisation using Jones calculus

It can be seen from the above results, that the algorithm works well in theory, predicting light with a linearity of about 4000 instead of the theoretical infinity. This discrepancy is due to the limitations of computational accuracy. The azimuth angle is predicted to about 1 degree (this degree of accuracy also occurs for any other values of ellipticity). Applying experimental results to the algorithm, also gives fairly good results in predicting input polarisation; however, the accuracy of these is limited by the quantitative variations between the model (which is used in the algorithm), and the experimental results presented in this paper. Although an ellipticity of nearly 56 may not appear to be near infinity, the light would appear to be very nearly linear to an observer, and so this is considered to be an acceptable result. It is proposed to develop a model based on experimental results to enable the accuracy of the algorithm to be improved.

5. CONCLUSION

The behaviour of the commercially produced liquid crystal television has been modelled as a Jones matrix. If the polarisation of the incident polarised light is known, this model can be used to predict the polarisation of light having passed through the display. Alternatively, if the polarisation of the incident light is not known, by taking intensity measurements through a fixed analyser at 4 different grey levels (voltages applied to the LCD), the model can be used as part of an algorithm to determine the polarisation of light incident on the display. This can then be used in applications to determine surface orientation, for example, using the Fresnel equations. Measuring intensity leads to an ambiguity in the predicted input, but it is anticipated that this can be solved with some existing knowledge of the object being viewed. One advantage of this method is that the display is commercially produced; another is that the pixellation of the display enables the polarisation of different parts of the scene to be individually controlled or measured.

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7. REFERENCES

Use of a commercially produced liquid crystal television as a security mark reader

Susan L. Blakeney¹, Sally, E. Day¹, J. Neil Stewart "

Abstract
A commercially produced liquid crystal television (LCTV) is presented as a device for authenticating security markings made from a birefringent material. The security marks could be applied to tickets, visas or other articles where authenticity is important. Several patents have been published detailing the use of a birefringent material as a security marking. These markings produce a unique pattern of polarisation, which then needs to be authenticated. This usually relies on the judgement of the authenticator. The work presented here is original in that it details an automatic way of doing this.

Introduction
As a precaution against counterfeiting, it may be desirable to apply a marking to an article indicating its authenticity. This marking will only be effective against counterfeiting if it is sufficiently difficult to replicate by an unauthorised party. This can be achieved by either limiting the supply of materials from which the marking is manufactured, and/or by making it difficult to see whether or not it is genuine with the unaided eye. If the marking was difficult to view or distinguish, the counterfeiter would not be able to see whether or not their marking would be authenticated, and this in itself will make it difficult to duplicate.

As the human eye is insensitive to polarisation, one type of marking which would fulfil such a criterion is one which could be made to transmit a pattern of polarisation states. This could be made by patterning a birefringent material, so as to vary its thickness¹, or by using a polymer material which is polymerised in such a way so that its birefringence varies spatially. In both cases, a portion of the material, when illuminated (in reflection or transmission) with light of a known polarisation, will produce a particular pattern of polarisation.

The marking can be verified by examining it through a sheet polariser, and either identifying a known pattern, or rotating the polariser to see if the pattern produced by the marking 'flashes' indicating that it is (partially or fully) linearly polarised².

The use of a commercially produced twisted nematic LCTV as a polarisation filter has been documented in Blakeney et al 1999³. Using this display it is possible to alter the polarisation of incoming light to a limited extent, and the pixellation of the display

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means that polarisation can be controlled pixel to pixel. This paper presents a way of using such a LCTV to authenticate such a security marking automatically.

The concept
It was envisaged that a birefringent material could be patterned in such a way as to produce a complex pattern of polarisation when illuminated with light of a known polarisation. This pattern of polarisation could then be ‘corrected’ by applying a known voltage pattern to the LCTV so that the output polarisations would all have the same azimuth angle. This would then give a maximum intensity when passed through an analyser oriented along that azimuth angle. A second voltage pattern could then be written to the LCTV which would rotate the polarisation through 90° to give a minimum intensity through the same analyser (see figure 1).

![Figure 1: Birefringent security marking concept.](image_url)

If the polarisation pattern/marking was not genuine, the pattern of polarisation emerging from the LCTV would not be as expected, and this would not give the desired intensity readings. This would be a quick and automatic method to authenticate the marking.

Method
Light from the marking was passed through the LCTV. Two (or more) known voltages were sequentially applied to various pixels of the television so that the polarisation from the marking was converted to different (known) polarisations. The LCTV was driven with a video signal from a computer monitor, so voltage was applied by changing the grey level (GL) on the monitor. The range of GLs was 0-255 where GL 255 represents the lowest voltage, and GL 0 the highest voltage. A fixed analyser was placed in front of the photodetector, and the marking was verified by measuring the intensity of light which was recorded by the photodetector for each voltage of the LCTV. The experimental setup was as shown in figure 2.
Circularly polarised light \(^{iii}\) from a HeNe laser + λ/4 plate (or, later, unpolarised light from a red LED, λ 660nm) was passed through a sheet polariser. This was then transmitted through a ‘security feature’, the LCTV, and an analyser before reaching a detector (photodiode or CCD).

A test ‘security feature’ was made by attaching 4 separate pieces of self-adhesive birefringent film (quarter wave film - retardation 155nm) to a glass microscope slide. This produced a unique pattern of polarisation because the pieces of film were oriented at various angles, so that when linearly polarised light passed through them, it emerged from each piece of film with a different polarisation \(^{iv}\). The polarisation from each piece of film (1, 2, 3 and 4) was measured using the setup in Figure 2 but with the LCTV removed\(^{v}\). The security marking was covered by a mask with one 2mm diameter hole in it for each piece of the marking \(^{vi}\).

Birefringent materials have different refractive indices along different directions within the material \(^{iv}\). When a polarised beam of light passes through a birefringent material, it is split into its component \((x \text{ and } y^{vi})\) parts. The resultant polarisation depends upon the phase difference induced between these components, which in turn depends on both the birefringence of the material and its thickness. The phase difference varies with the angle between the incident light and the optic axis of the birefringent material \(^{v}\). Twisted nematic liquid crystal displays have an optic axis (director) which twists

\(^{iii}\) A circular polariser was used in front of the linear polariser, so that rotating the polariser would give equal light intensities in each direction.

\(^{iv}\) The more complex the marking, the more unique the solution, and the more difficult the marking is to counterfeit.

\(^{v}\) To measure the polarisation, the analyser was rotated so that the maximum and minimum intensities were recorded, together with the analyser position at the maximum intensity (azimuth angle). The sense of rotation of polarisation was measured using a quarter wave retarder oriented along the azimuth angle of the ellipse of polarisation

\(^{vi}\) The hole was to ensure that when the marking was illuminated, the laser beam (which was approximately 1mm in diameter) always passed through the same area of the marking. The slide was attached to the LCTV using two spring clips attached to the housing of the LCTV and it was ensured that it was always aligned with the same portion of the LCTV. In order to pass the laser beam through each piece of the marking individually, the whole device (LCTV with marking attached to it) was moved along the \(x\) and \(y\) axes, on a vernier mount.

\(^{vi}\) or right and left
through 90° from one surface of the display to the other, so the optic axis for the whole display is not in a single direction.

The LCTV works as a display because its optic axis can be varied with applied voltage. This enables the polarisation of light emerging from the LCTV to be varied by applying voltage to the LCTV. The polarisation state of the light emerging from the 4 sections of film was calculated for each voltage that could be applied to the LCTV, using a computer simulation. This gave a series of ellipses of polarisation versus LCTV voltage for each piece of the marking.

The polarisation change which can be induced by the change in birefringence of the LCTV depends upon the thickness of the LCTV. To achieve fast switching rates, modern displays are made too thin to fulfil the Mauguin limit of \( d\Delta n \gg \frac{\lambda}{2} \) where \( d \) is the thickness of the display, \( \Delta n \) is the birefringence, and \( \lambda \) is the wavelength of the light. If the display does not operate within the Mauguin limit the display does not purely rotate the plane of linearly polarised light, but induces some ellipticity into the polarisation. This occurs with the display used in this work, and so linearly polarised light passing through the display becomes elliptically polarised unless the display is fully switched (when the birefringence is very low, and the change in polarisation is negligible).

It is not possible to use the LCTV to change any input polarisation to a linear polarisation at a fixed angle to be absorbed by an analyser. This would only be achievable if the full range of phase change were available (i.e. if the LCTV was thicker) and the input could be rotated. It is therefore necessary to choose a suitable orientation for the analyser, and to be able to calculate the intensities of the light from the marking which will pass through it.

For simplicity, each piece of the marking was considered separately. The intensity, \( I \), transmitted by an analyser at an angle \( \theta \) to the major axis of each ellipse of polarisation was calculated using equation 1, where \( a' \) and \( b' \) are the major and minor axes respectively of the ellipse of polarisation.

\[
I = a'^2 \cos^2 \theta + b'^2 \sin^2 \theta
\]  

This gave a series of intensities versus grey level (voltage) for each analyser position (0°-180° in 5° steps). For each analyser position the maximum and minimum intensities were noted, as were the grey levels which had to be applied to the LCTV to produce these intensities. For the experimental measurements, an analyser position was chosen which gave a range of maximum/minimum intensities (from each of the separate pieces), and the measured results were compared with the calculated ones. For comparison, measurements were repeated using several analyser positions.

The experiment was then repeated using an LED as a source, and a CCD to record intensities. The image of the marking was captured for each grey level applied to the LCD, and the intensity of an area of pixels corresponding to each piece of the marking was recorded using proprietary image processing software.
Results
The intensities calculated using equation 1 were normalised, but the intensities which were measured were not. In order to compare the calculated value with the measured value, it was necessary to take at least two separate intensity measurements (at different LCTV voltages) and divide one intensity by another. The two ratios, could then be compared. However, this division accentuated the errors.

a) Calculation
![Figure 3 Plot of the calculated maximum (minimum/maximum) intensities which can be achieved for each position of the analyser. Slide A.](image)

The UCL Jones matrix model was used to calculate the maximum ratios of [maximum/minimum] intensities which were achievable by changing the LCTV GL. These ratios, for each analyser position, are shown in figure 3. In this work the [maximum/minimum] intensity will be called the contrast ratio. It can be seen that there is a considerable variation in the contrast ratio which can be achieved for each piece of the marking. When the minimum intensity is very low (or the maximum is very high), it indicates that light at that point is linearly, or nearly linearly, polarised.

Looking at figure 3, suppose the analyser position is chosen as 170°. The predicted contrast ratio for piece 3, is nearly 24,000, but at that point the maximum contrast ratios which can be achieved with the other parts of the markings are 4.8, 5.2 and 1.2 for pieces 1, 2 and 4 respectively. It can also be seen that the light from piece 4 never has a large change in contrast ratio, with the maximum contrast ratio being only about 4 for any analyser position.

Having a very small denominator means that very small errors translate to large errors in the total fraction. In order to reduce these errors, when comparing theoretical with experimental results, the ratio of maximum and minimum intensities was expressed in terms of visibility where
\[
\text{Visibility} = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \tag{2}
\]

b) Experimental
Results are presented for HeNe laser + photodiode and LED + CCD separately. Three slides were used (A, B and C), each containing 4 pieces of film oriented differently. For slide A, using the laser + photodiode experimental comparison with the results in
Figure 1 is shown in Table 1. This shows positions of the analyser at 55° and 170° (input polarisation 45°).

<table>
<thead>
<tr>
<th>Piece</th>
<th>GLs used</th>
<th>Predict. GLs used</th>
<th>Expt. GLs used</th>
<th>Predict visibility</th>
<th>Expt. visibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0/255</td>
<td>0.8975</td>
<td>0.9035</td>
<td>255/0</td>
<td>0.6593</td>
</tr>
<tr>
<td>2</td>
<td>100/255</td>
<td>0.17012</td>
<td>0.0826</td>
<td>0/150</td>
<td>0.6785</td>
</tr>
<tr>
<td>3</td>
<td>100/255</td>
<td>0.2095</td>
<td>0.0991</td>
<td>0/160</td>
<td>0.9999</td>
</tr>
<tr>
<td>4</td>
<td>10/255</td>
<td>0.3377</td>
<td>0.2424</td>
<td>150/80</td>
<td>0.0909</td>
</tr>
</tbody>
</table>

Table 1 Comparison of intensity calculations with experimental results for Slide A (Measurements taken using laser and photodiode). Input +45°.

Although the results presented in table 1 show a good agreement between calculated and experimental results, when measurements were taken using other slides the agreement was not so encouraging (such as when the analyser was at 125°). The reason for this can be seen by analysing the voltage/intensity curves for each piece of the marking, as shown in figure 4.

Figure 4 shows the change in intensity measured with the photodetector (analyser at 125°) as a function of GL applied to the LCTV. These voltage/intensity curves can be related to the contrast ratios shown in figure 3. The method outlined above compares the theoretical and experimental curves at only two points, and it can therefore be expected that any discrepancy in either of these points will adversely affect this method. It can be seen from figure 4 that the contrast ratio for piece 3 will be measured as higher than predicted, as the experimental maximum is higher and the minimum is lower than that calculated using the model.
To increase the accuracy of the verification process, an alternative verification method was used. This was to compare the theoretical and experimental voltage/intensity curves at several points using a least squares method. The error score obtained was calculated for various numbers of GLs displayed on the LCTV. The results are shown in figure 5. It can be seen that the error score does not increase significantly when more than 4 GLs are used.

The error score was therefore calculated applying 4 (evenly spaced) GLs (255, 160, 80 and 0) to slides A, B, and C, and various fakes. The fakes consisted of a clear slide, quarter wave film, and the slides crossed referenced with each other, and upside down. These results are shown in figures 6, 7, and 8.

Discussion
The pixellation of the LCTV enables complex polarisation changes to be made across the display, with each pixel able to change polarisation individually. It was initially

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*The voltage applied to one column of pixels does have some influence on its neighbours, so each pixel is not completely independent. This has been documented in Mark C. Gardner’s PhD Thesis (UCL, in preparation).*
anticipated that the application of two voltage patterns to the LCTV - unique to the marking under test - would enable the marking to be verified. These voltage patterns would convert the polarisation from the marking to a maximum and then to a minimum intensity, and this would not occur if the marking was not genuine.

However, this method has several flaws: firstly it is particularly sensitive to error, as only two measurements are taken, and secondly taking a total intensity reading across the whole marking means that any areas of the marking which emit linearly polarised light will 'swamp' those which don't. This will occur because the maximum intensities will be highest and the minimum intensities will be lowest from linear polarisations. This latter factor will impair the sensitivity of the system to non-linear polarisation emitting markings.

An alternative method of verification has been presented, which appears to be robust. This method does not rely on the pixellation of the LCTV, and so could be used with a single liquid crystal cell, which will considerably reduce the cost and complexity of such a device. The marking can be verified with the application of only 4 voltages to the liquid crystal television or cell.

More complex markings will soon be tested. These will emit complicated patterns of polarisation, which will make them very difficult to counterfeit. They will still be able to be verified using the LCTV (or a single liquid crystal cell) and a CCD arrangement.

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See patent number GB2328180 (1997). Authors David William Thomas, Anthony David Harman and Timothy Andrew Large


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*It should be noted that the intensity of the incident light must be carefully controlled to give an adequate range of output intensities, as the error score of 26 for the genuine slide A (figure 4) occurred when the input intensity was too low (and so the darkest intensities were not distinguished).
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