MAGNETOTRANSPORT PHENOMENA IN
MODULATION DOPED N-CHANNEL Si/Si$_{0.7}$Ge$_{0.3}$
QUANTUM WELL STRUCTURES

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A Thesis submitted to the University of London
for the Degree of Doctor of Philosophy

UCL
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University College London
1999
Abstract

The work described in this thesis involves magnetotransport measurements performed on n-channel Si_{0.7}Ge_{0.3}/Si modulation doped quantum well structures. These systems are grown by gas source molecular beam epitaxy, and are found to be quasi two-dimensional. Most of the work is carried out at low temperatures between 0.05K and 4.2K using magnetoresistivity measurements on Hall bar devices, in order to understand the physics of the two-dimensional electron gas (2DEG).

Shubnikov-de Haas measurements between 0.05 and 1.6K have been performed on three modulation-doped n-type Si/Si_{0.7}Ge_{0.3} heterostructures, and the results analysed to extract the quantum relaxation time. Use of the conventional Dingle formula resulted in deviations from the expected theoretical behaviour above ~0.3K. The corresponding quantum relaxation time appeared to increase with temperature in the same range. The data are then re-analysed using a modified expression, in which the thermal damping term is neglected. This gave plots with the correct characteristics, and as expected for ionised impurity scattering in a degenerate electron gas, the quantum relaxation time is found to be approximately constant with temperature.

SdH oscillations corresponding to the spin-Landau levels of a 2DEG at 100mK are also studied. The method used to obtain the effective Landé g factor can be described as the method of coincidences, where the tilt angle that causes adjacent SdH minima to be equal, is used to determine the $g^*$ factor. The effective g factor for $\nu = 6\leftrightarrow 8$ is $3.34 \pm 0.05$. The results demonstrated that
the effective g factor oscillates as a function of the Landau level.

The temperature dependence of the $\sigma_{xx}$ in their $\rho_{xx}$ minima in SdH oscillation at integer filling factor is analysed. Above 1K, the resistivity is thermally activated. However, the activation energy $\Delta$ obtained from the experiments is much lower than the cyclotron energy. This has been attributed to the fact that the Landé g factor should be larger than the bulk value of $g=2$ and the Landau levels are broadened due to impurity scattering. The temperature dependence of the resistivity deviated from the expected behaviour below 1K, as also found in measurements on disordered systems, and this is interpreted as being due to hopping conduction.
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Acknowledgement

During the course of the last three years many persons have offered useful advice and suggestions. For this I’m extremely grateful. In particular I wish to thank the following persons:

Firstly and most importantly, I wish to thank my supervisor Dr. J.J. Harris for his effective guidance, the great amount of time and help he offered me and the enormous patience he had with me during the last three years.

I wish to thank Dr. Juan Fernández who grew the set of samples and many thanks go also to Dr. N. Wood and Dr. T. Thornton who helped me with the samples fabrication.

The team involved in the collaborative at the CNRS in Grenoble were pivotal in the completion of this thesis. I like to thank Dr. D.K.Maude for his invaluable low temperature expertise and Prof. J.C. Portal who made the collaboration possible.

Next, I wish to thank my follow PhD. students, C. Becker, Kwon Lee, Jay Ku for their friendship and useful discussion.

I owe my parents a great debt of love and thanks for being very supported of me during this period. A last thank you goes to my wife and my children, who kept me good company and my spirits high.
To my parents, daughter, son, and wife
Chapter 1
Introduction

In the last couple of decades, heterojunction structures have become one of the basic research topics in solid state science. The development of epitaxial growth techniques, such as molecular beam epitaxy and chemical vapour phase epitaxy, have allowed us to control the growth of individual semiconductors layers on an atomic scale. These achievements provide a strong basis for designing semiconductor materials with tailored band structure, and tailored electronic and optical properties. The important parameters in this “band structure engineering” are the microscopic nature of the interfaces: the bandgap discontinuities, the doping profiles and the related space charge layers.

The most important fundamental studies arise from the high electron mobility of two dimensional electron systems which can be achieved at modulation-doped semiconductor heterostructure interfaces (the physical separation of the ionised impurities from the two dimensional electron gas reduces the strength of collision with impurities).

The most widely studied two-dimensional systems are based on III-V compound semiconductors, such as GaAs/Al\textsubscript{x}Ga\textsubscript{1-x}As. The main reason for their popularity comes from the fact that the lattice constants of AlAs and GaAs are nearly equal and epitaxial layers of Al\textsubscript{x}Ga\textsubscript{1-x}As can be grown on GaAs with very high quality interfaces. Furthermore, most of these semiconductors have a direct fundamental energy gap which makes them suitable materials for optical devices and optoelectronics.
Despite these developments, the most important semiconductor is elemental Si. Cheap, and available in large quantities, Si has a very stable native oxide, namely SiO₂, which serves as an insulator and as a protecting layer, and forms the basis of nearly all-existing microelectronic devices on the market. However, a variety of fast-growing market segments, especially in the areas of millimetre-wave and optical communication, appear to be beyond the scope covered by the electronic and optoelectronic properties of Si. On the other hand, the III-V compounds completely lack a natural oxide or other insulator, with the quality and versatility required for VLSI technology, although properly designed III-V heterostructures can excel in almost every category of electronic and optoelectronic applications. These problems certainly act as a major drawback in the development of III-V compound heterostructures. It is therefore a great challenge to be involved in the development of novel devices, related to the Si-based technologies.

In this respect, more suitable heterostructures can be made from Si/Si₁₋ₓGeₓ, which can be considered as a kind of 'natural' choice since: i) the two group-IV elements crystallise in the same diamond lattice, and ii) can form random Si₁₋ₓGeₓ alloys of arbitrary composition. By means of controlling these alloys the bandstructure can be tailored within the relative margin given by the two elemental semiconductors. These obvious advantages of Si-based group IV heterostructures were recognised at an early stage of SiGe research. However, progress fell behind similar investigations on the more popular lattice-matched GaAs/AlGaAs heterostructures, which diverted much of the focus from SiGe growth and characterisation. The reasons can be seen not only in the quite significant lattice mismatch between Si and Ge, but also in several doping problems caused by the strong segregation of most dopants then
available.

In the early 1980’s, it was found [1] that by utilising strained layer epitaxy, defects could be significantly reduced in the silicon-based heterostructures. In these strained alloys of $\text{Si}_{1-x}\text{Ge}_x$, it was found that heterostructure effects are much stronger than expected, (for example, higher carrier mobility etc.), making them much more attractive for device applications.

Though considerable theoretical work has shown the possibility that pseudomorphic epilayers of lattice-mismatched materials can be grown, it has been difficult to produce high quality epilayers of $\text{Si}_{1-x}\text{Ge}_x$ alloys on single crystal silicon or germanium substrates. However, low growth temperatures and an exact composition control, which became available by introducing growth techniques such as MBE or CVD, allowed us to grow lattice-mismatched heterostructures. Bean et al. [2] established that high-quality layers can be grown provided that the growth temperature is kept within a certain range and that the layers do not exceed a critical thickness $h_c$.

Structures based a $\text{Si}_{1-x}\text{Ge}_x$ strained layers play an important role in improving existing devices and in developing new devices. So far, solid source molecular beam epitaxy (SSMBE) has been widely used as a growth method for SiGe/Si modulation doped heterostructures, because of its ultra-high vacuum ambient and the high controllability of layer structure that it provides. However, the SSMBE technique contains many problems. For example, during growth, Si is evaporated at high temperature and consequently introduces the problem of heavy metal incorporation. Therefore, by using low temperature gas source molecular beam epitaxy (GSMBE), such effects can be greatly reduced. Hence, this thesis studies the magnetotransport of the two-dimensional
electron gas in the Si/Si_{0.7}Ge_{0.3} heterostructure system grown by gas source molecular beam epitaxy (GSMBE). A good control over the growth parameter and the individual layer thicknesses in conjunction with the ultra-high vacuum ambient of the molecular beam epitaxy guarantees the production of very high quality heterointerfaces. Being a relatively new growth technique and hence not as perfected, the crucial advantage of gas source over solid source molecular beam epitaxy is its high growth selectively and the reduced incorporation of impurities from the source.

In the following sections the contents of the various chapters which constitute this thesis are previewed.

Chapter 2 describes the theoretical background of the project. First, the physical properties of the Si/SiGe heterostructures are discussed. Afterward, the physics behind two-dimensional system is explained. Two-dimensional transport in a quantising magnetic field applied perpendicular to a system and the associated Shubnikov-de Haas and quantum Hall effect are described in detail. Finally we cover device application of this material system.

Chapter 3 provides a general description of the experimental apparatus and techniques employed in attaining stringent requirements necessary for the study of the magnetotransport.

In chapter 4, the results of the magnetotransport measurements performed are given in order to determine the electrical parameters of modulation doped quantum wells such as mobility, carrier concentration, and quantum relaxation time. From these magnetotransport measurements, a range of mobility values is obtained, which includes the 'best' $\mu \sim 69700\text{cm}^2/\text{Vs}$ at 100mK. Further investigation have demonstrated the high quality of this material, as seen from the presence of dissipationless minima in $\rho_{xx}$ with
corresponding quantised plateaux in $\rho_{xy}$ (both of which are indicative of a well behaved two-dimensional electron gas in a Si channel). Experimental details and the results of the quantum relaxation time analysis using Dingle plots are also given.

Chapter 5 covers the tilted magnetic field measurements. Such experiments employed tilted magnetic fields to independently vary the Landau and spin splittings. This discrimination becomes possible, because the Landau level spacing depends only on the magnetic field component perpendicular to the 2D carriers, whereas the spin splitting is proportional to the total magnetic field. Hence, tilted magnetic fields allow an enhancement of the spin splitting with respect to the Landau splitting, and a coincidence is reached, when both splittings become equal. Such coincidence experiments yield the effective Landé g factor.

The temperature-dependent conductivity $\sigma_{xx}$ is investigated in the range from ~50mK to 4.2K in chapter 6. Above 1K, the $\sigma_{xx}$ follows a thermally activated temperature dependence, while below 1K, hopping conduction is observed.

Finally, chapter 7 presents the general conclusions of the results, and summarises the main achievements of this work.

References
Chapter 2
Theoretical consideration

2.1 Introduction

This chapter covers the general properties and basic underlying physics of the silicon-germanium system and its two-dimensional electron gas (2DEG) which are the strength of this thesis. The following experimental chapters will use much of the theory which is discussed here.

The physics of lattice mismatched epilayer structures, and of the band structure and the band alignment of strained quantum wells, are referred to in the first section. In the second section, we deal with low-field transport in a two dimensional system. The fundamental magnetotransport phenomena, Shubnikov-de Haas effect, and the integer quantum Hall effect are then discussed. Finally, possible device applications are described.

2.2 Physical properties of silicon germanium system

2.2.1 Crystal structure

Both silicon and germanium, which form a continuous series of Si$_{1-x}$Ge$_x$ solid solutions with $x$ ranging from 0 to 1, crystallise into the diamond cubic structure, where the atoms are covalently bonded, as shown in figure 2.1a. The lattice spacing of diamond consists of two interpenetrating face-centred cubic (fcc) sublattices which are displaced by a quarter of the space diagonal (see figure 2.1b). In diamond structures, the cubic unit cell contains eight atoms,
and each atom is bonded with four nearest-neighbours arranged at the corners of a regular tetrahedron, and twelve next nearest neighbours.

So far, the work carried out by Dismuket et al. [1] has been the most precise and comprehensive determination of the bulk lattice parameters across the whole composition of the Si$_{1-x}$Ge$_{x}$ system. Their experimental data reveal a small deviation from Vegard’s law. According to this law, the lattice constant for an alloy should lie on a linear fit when plotted against the alloy concentration, i.e.

\[ a_{\text{alloy}} = xa_A + (1-x)a_B \]  

(2.1)

The deviation \( \Delta \) from this law is given by

\[ \Delta = a_{\text{SiGe}} - [a_{\text{Si}}(1-x) + a_{\text{Ge}}x] \]  

(2.2)

The experimentally determined deviations have also been theoretically confirmed by Monte Carlo simulations on Si$_{1-x}$Ge$_{x}$ alloy [2]. Herzog [3] has derived a parabolic relation for the Si$_{1-x}$Ge$_{x}$ lattice parameter as a function of composition \( x \) by using the values given in [1].

\[ a(x) = 0.5431 + 0.01992x + 0.002733x^2 \]  

(2.3)

This equation approaches the experimental data with a maximum deviation of $10^{-4}$nm. The small quadratic term, which causes a negligible deviation from the linear fit, can be ignored. Hence, the lattice constant of the bulk Si$_{1-x}$Ge$_{x}$ alloy can be written as
\[ a_{\text{Si}, \text{Ge}_x} = a_{\text{Si}} + (a_{\text{Si}} - a_{\text{Ge}})x = a(x) \] (2.4)

(a) Diamond lattice

(b) Sublattice

Figure 2.1 Schematic diagram of the diamond structure.
2.2.2 Lattice mismatch and critical thickness

The stresses and the energies at the interface in a bi-crystal grown as an epilayer on a lattice-mismatched substrate were discussed by Frank et al. [4]. Their work has been extended, compared with experiment, and reviewed by several research groups [5-7]. These studies show that if the lattice mismatch is sufficiently small (<10%) or the thickness of the epilayer is not large, the atoms on both sides of the interface are in perfect register, and the mismatch is accommodated entirely by the strain configuration in the epilayer. However, a heterostructure with different lattice constants is limited to a certain thickness (the so-called critical thickness, \(h_c\)) up to which the mismatch is accommodated by strain. If the thickness of the epilayer increases beyond the \(h_c\), the strain layer will relax by introducing misfit dislocations. An energy \(E_s\) is associated with the strain configuration. If \(E_s\) exceeds the energy required to form misfit dislocations, perfect bonding will collapse, and SiGe alloys on Si substrates will revert to the unstrained structures above a critical thickness \(h_c\).

Figure 2.2 gives a simplified view of the alternatives when two lattice-mismatched semiconductors are combined.

i) If the epitaxial layer is flexible (and much thinner than the substrate), it may distort itself in the plane of growth in order to conform to the substrate spacing, as illustrated by the "strain" configuration at figure 2.2(b).

ii) If both lattices are rigid, they will retain their original crystal structures as shown in figure 2.2(c). Therefore, the interface will contain rows of misbonded atoms. These improperly bonded atoms form misfit dislocation in the form of dangling bonds that can become an unwanted trap site. The case ii) is damaging to device application and therefore the strain configuration (case i)) is preferred.
Figure 2.2 (a) Focusing a mismatched overlayer into interfacial coherence with a substrate forms a strained layer structure. (b) When below the critical thickness for strained layer stability, the resulting coherent structure is shown. (c) If the overlayer is thick enough that the coherent structure is unstable, the mismatch will partially be accommodated by lattice strain and partially by the introduction of misfit dislocation [8].

In the case of a Si$_{1-x}$Ge$_x$ layer on a Si substrate, the value of the misfit associated with both layers is defined as

$$f_m(x) = \frac{a(x) - a_{Si}}{a_{Si}} \approx 0.042x$$  \hspace{1cm} (2.5)

This means the lower the Ge concentration in the Si$_{1-x}$Ge$_x$ layer, the smaller the lattice mismatch between the alloy and the Si substrate. The epitaxial growth of pseudomorphic Si$_{1-x}$Ge$_x$ layers grown on Si substrates have biaxial in-plane compression of the alloy and an extension normal to the interface, because the lattice constants of Si$_{1-x}$Ge$_x$ alloys are larger than that of the Si
substrate. However, when Si layers are grown pseudomorphically on a Si$_{1-x}$Ge$_x$ substrate as depicted in figure 2.3, the reverse case applies, i.e. an in-plane extension and compression normal to the interface.

![Diagram](image)

**Figure 2.3** Pseudomorphic strain produced by epitaxy of an overlayer with a bulk lattice constant (a) larger, or (b) smaller than the substrate. The overlayer must match the in-plane lattice constant of the substrate [8].

The theoretical study on the critical thickness ($h_c$) is discussed by People et al. [9], which states that the $h_c$ is inversely related to the Ge concentration of the Si$_{1-x}$Ge$_x$ alloy. They also found that the theoretical $h_c$ agreed with the experimental data obtained by Bean et al. [10] for a Si$_{1-x}$Ge$_x$ layer strained to a Si substrate with the Ge concentration ranging from $x=0.14$($h_c=7500\text{Å}$) to $x=0.70$($h_c=30\text{Å}$).

A plot of the critical thickness as a function of the Ge concentration [10] is shown in figure 2.4. The plot can be explained by dividing it into three regions.
Region I: This is known as the stable strain region where SiGe alloy layers are strained and free of dislocation.

Region II: SiGe alloy layers in this region is metastable (actually stable up to a critical thickness) due to low growth and processing temperatures. Hence, this is known as the metastable strain region. The metastable critical thickness is temperature dependent and is here drawn for layers grown at 550°C.

Region III: SiGe alloys in this region contains misfit dislocations.

Figure 2.4 Limits of strained layer (defect-free) growth for SiGe layer growth on Si (001)[10].

Top right: layer thickness and/or Ge fraction is too large to be accommodated by strain and misfit dislocations form. Centre: configuration in which strained layer growth can be produced at low temperatures but for which strain is metastable and relaxation may ultimately occur upon extended thermal processing. Bottom left: configurations for which strained layer growth is the lowest energy state and for which dislocations will not form.
2.2.3 Band structure and band alignment of Si$_{1-x}$Ge$_x$

![Figure 2.5 Schematic diagram of the nature of the central cell states in direct and indirect bandgap materials][11].

In order to fully understand the transport properties of the Si/Si$_{1-x}$Ge$_x$ heterostructure, it is useful to discuss the band structure of silicon and germanium. Both semiconductors have an indirect bandgap because their band structures have different $k$-points for the top of the valence band and the bottom of the conduction band as shown figure 2.5. The top of the valence band is located at the $\Gamma$-point in both semiconductors whereas the bottom of the conduction band is at the $L$-point along the $<111>$ axis ($\bar{A}$) in germanium, and close to the $X$-point along the $<100>$ axis ($\bar{A}$) in silicon.
Figure 2.6 The shapes of constant energy surfaces in silicon and germanium. For silicon there are six ellipsoidal, band edge along the <100> axis and for germanium there are eight half-ellipsoids along the <111> axes with the Brillouin zone boundaries being in the middle of the ellipsoids.

Due to the symmetry of the fcc lattice, there are eight degenerate $L$-points and six degenerate $X$-points, consequently leading to eight conduction bandedge valleys in germanium and six conduction bandedge valleys in silicon. Thus the bottom of the conduction band in silicon is close to the $X$-point of the Brillouin zone ($\sim (2\pi/\alpha)(0.85,0,0)$), where $\alpha$ is the lattice constant of Si. Each of the six band has an ellipsoids form [11],

$$E(k) = \frac{\hbar^2 k_x^2}{2m_i} + \frac{\hbar^2 k_y^2}{2m_i}$$

(2.6)

with the longitudinal direction aligned along (100) and the transverse
components perpendicular to this. Two corresponding effective masses are introduced (one along the axes and one perpendicular to them), represented by $m_i^*$ and $m_{i \perp}^*$, respectively. The six constant energy ellipsoids for the Si conduction band along with the eight half-ellipsoids for Ge are shown in figure 2.6. The measured values for the transverse mass $m_i^*$ and the longitudinal mass $m_{i \perp}^*$ are listed in table 2.1[12].

<table>
<thead>
<tr>
<th></th>
<th>Silicon</th>
<th>Germanium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_i$</td>
<td>0.98$m_o$</td>
<td>1.64$m_o$</td>
</tr>
<tr>
<td>$m_{i \perp}$</td>
<td>0.19$m_o$</td>
<td>0.082$m_o$</td>
</tr>
<tr>
<td>$\mu_e [cm^2/Vs]$</td>
<td>1450</td>
<td>3900</td>
</tr>
<tr>
<td>Hole</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_{lh}$</td>
<td>0.16$m_o$</td>
<td>0.044$m_o$</td>
</tr>
<tr>
<td>$m_{hh}$</td>
<td>0.49$m_o$</td>
<td>0.28$m_o$</td>
</tr>
<tr>
<td>$\mu_h [cm^2/Vs]$</td>
<td>505</td>
<td>1900</td>
</tr>
<tr>
<td>Bandgap[eV]</td>
<td>1.12</td>
<td>0.66</td>
</tr>
</tbody>
</table>

**Table 2.1** Effective masses, mobilities and bandgap for Si and Ge at room temperature with $m_o=0.91 \times 10^{-30}kg$.

The composition dependence of the indirect bandgap in Si$_{1-x}$Ge$_x$ bulk alloys was studied by Braunstein et al.[13]. From their experimental results, the conduction band remains Si-like up to the Ge composition of approximately 85%. Si$_{1-x}$Ge$_x$ alloys have smaller fundamental bandgaps compared to Si, principally because of the larger lattice constant, and of the acquired tetragonal distortion (in case of the pseudomorphic SiGe layer grown on Si substrate). Strain in the alloy layer results in modifications of the band structure, i.e. these changes include the splitting of the degenerate valence
band and conduction band. The influence of the strain in both the Si and SiGe layers is depicted in figure 2.7. For compressively strained SiGe on (100) Si the four in-plane conduction band valleys are lowest in energy, while the two valleys normal to the interface are shifted upwards. Under tensile strain (growth on a substrate with a larger lattice constant) the twofold minima are lowered and the fourfold minima are lifted in energy.

Figure 2.7  (a) Schematic picture of the band offsets at the Si/SiGe interface for both material being under strain (b) The effect of strain on hh (heavy hole) and lh (light hole) states in the valence band.
Figure 2.8 Fundamental indirect bandgap of strained Si$_{1-x}$Ge$_x$ alloys in comparison with the bulk alloy at 90K plotted against the Ge content $x$ [14]. The hatched areas between the full lines show the theoretical results of People[14] for transitions involving heavy and light hole.

For an in-plane compression of the strained layer, the upper (lowest energy) valence band states are comprised of the hh states $(3/2, \pm 3/2)$, with the lh states being shifted in energy. However, under biaxial tensile strain, the lowest energy state consists of the lh states $(3/2, \pm 1/2)$, thus altering the character of the valence band and the effective hole mass. A schematic description of the valence band splitting for both cases is illustrated in figure 2.7(b). The difference effective masses of the light and heavy hole bands lead to different behaviour in terms of mobility of carriers.

Figure 2.8[14] shows the lowest energy indirect bandgap as a function of the Ge concentration. The top solid curve represents the bandgap for a bulk alloy layer. Until the alloy composition reaches about 85 atomic percent Ge, the characteristics retain a silicon-like structure. At greater concentrations,
the alloy acquires a germanium-like character. The value of the bandgap for strained Si$_{1-x}$Ge$_x$ on Si substrates falls almost linearly with increasing Ge content. Similarly, the bandgap of strained Si on a relaxed Si$_{1-x}$Ge$_x$ buffer also decreases linearly with increasing Ge concentration. The calculation of the bandgap change for the first case is straightforward, whereas for the second case the bandgap variation of the relaxed Si$_{1-x}$Ge$_x$ buffer with different Ge concentration should be considered as well.

![Diagram](image)

**Figure 2.9** Conduction band and valence band offsets in the Si/SiGe heterostructures. Solid line represent the highest valence band and the lowest conduction band states in each material. Dashed lines indicate the splitting of the bands due to compressive strain in SiGe and tensile strain in Si.

Another decisive parameter for the behaviour of Si/SiGe heterostructures is the band alignments, because an exact knowledge of the band offset would be important in determining whether electron or hole confinement at the interface is possible. It turns out that for pseudomorphic SiGe layers on Si the difference in the energy gap is approximately equal to
the valence band offsets. This band alignment at the heterointerface of $\text{Si}_{1-x}\text{Ge}_x$ and Si is type I. The $\text{Si}_{1-x}\text{Ge}_x$ alloy is compressively strained to the Si substrate which causes the sixfold degenerate conduction band to split into two degenerate sets of two upper and four lower valleys, and the twofold degenerate valence band to split into heavy hole and light hole. There are nearly no offsets between the sixfold degenerate conduction band minima of the unstrained Si and the fourfold degenerate minima, $\Delta(4)$, in the strained SiGe alloys, i.e. the conduction band offset $\Delta E_c$ is less than 20meV [15]. Hence, the type I alignment can be used in devices that require hole confinement.

Due to the small conduction band offset in strained $\text{Si}_{1-x}\text{Ge}_x$ layer on Si, a high mobility electron system cannot be realised. In order to realise a high mobility electron channel in Si/$\text{Si}_{1-x}\text{Ge}_x$ heterostructures, tensile strained Si is required instead of compressively strained $\text{Si}_{1-x}\text{Ge}_x$. It has been demonstrated that the growth of Si on strain-relaxed SiGe buffer layers can result in tensile strain in Si [16]. Tensile strain causes the sixfold degenerate conduction band to split into degenerate sets (neglecting valley splitting and spin degeneracy) of two lower valleys $\Delta(2)$ and four upper valleys $\Delta(4)$, and the twofold degenerate valence band to split into heavy and light hole (type II). Recently, improved concepts on strain relaxation have been demonstrated by using graded SiGe buffer layers [17-19]. In type II, the valence band offset as a function of the composition Ge content is always higher in SiGe than in Si. Type II is shown schematically in figure 2.9. For a 30% Ge concentration in the relaxed $\text{Si}_{1-x}\text{Ge}_x$ buffer layers, doubly degenerate conduction band energy of the strained Si is sufficiently less than the conduction band energy of the $\text{Si}_{1-x}\text{Ge}_x$. The strain-induced conduction band offset of roughly 180meV has been measured[20], creating a large enough potential barrier to confine electrons in the strained Si
layer.

In conclusion, this ability to engineer the band structure is the main reason for employing Si$_{1-x}$Ge$_x$ alloys, which has already led to several new device applications [21] to be discussed in section 2.5.

2.3 Two-dimensional structures

As has already been discussed, a two-dimensional electron gas system can be realised when two semiconductors with different bandgaps are brought into contact with each other.

In the sections that follow, various properties of a two-dimensional heterostructure, in particular modulation-doping techniques in a single heterostructure and quantum well, and the density of states dependence on the dimensionality of a structure are discussed. Then, the electron transport and the effect of various scattering mechanisms are described.

2.3.1 Modulation-doped heterostructure and quantum well

The purpose behind the doping of semiconductors is to intentionally control the free carrier density. This requires the dopant to be ionised and thus positively or negatively charged. This fixed charge centre causes scattering of free electrons (ionised impurity scattering mechanism). Modulation-doping is a doping technique utilised in semiconductor heterostructures which spatially separates mobile carriers from their parent donor or acceptor atoms. The scattering of the electrons is profoundly reduced by this spatial separation, as described by Dingle et al.[22]. A further reduction of the scattering effects can be achieved by introducing an undoped spacer layer, which increases the
distance between the ionised impurities and electrons.

A confined two-dimensional electron system with modulation-doping exists in the form of a single heterostructures or a quantum well. The most commonly employed single heterostructure in two-dimensional transport experiments is composed of the two semiconductors GaAs and Al\textsubscript{x}Ga\textsubscript{1-x}As. This is formed by introducing n-type dopant impurities (e.g. Si) into the wide-bandgap material, Al\textsubscript{x}Ga\textsubscript{1-x}As, usually at a distance away from the interface (the undoped Al\textsubscript{x}Ga\textsubscript{1-x}As spacer layer), while keeping the narrow-bandgap material (GaAs) free from intentional doping. Some of the electrons introduced by Si donors into the conduction band of the Al\textsubscript{x}Ga\textsubscript{1-x}As transfer into the lower-lying conduction band of the GaAs. These electrons are confined in the nearly triangular well established by the conduction band discontinuity and the Coulomb potential of the parent ionised impurities.

Another semiconductor structure in which a quasi two-dimensional electron system can be realised is the square potential well. This consists of a thin layer of semiconductor between two layers of a wider-bandgap material. In a narrow quantum well (<<30nm), the confinement of the carriers between the two band discontinuities causes quantisation in the growth direction.

The generally applied heterostructure in n-type or p-type Si/Si\textsubscript{1-x}Ge\textsubscript{x} systems is a modulation doped quantum well structure. As mentioned in section 2.2.3, pseudomorphic growth of a SiGe layer on a Si substrate does not produce a conduction band offset, i.e. the bandgap difference between Si and SiGe is completely realised by the offset in the valence band (p-type). This is no longer the case if the Si layer is also tensilely strained, when splitting of the sixfold degenerate conduction band occurs.
The required strain adjustment for n-type modulation doped structures is achieved by introducing a relaxed SiGe buffer layer between the Si substrate and the modulation doped structure.

Figure 2.10 shows n- and p-type modulation-doped Si/Si$_{1-x}$Ge$_x$ quantum well. In the case of n-type, it is formed by introducing n-type dopant impurities (e.g. As) into Si$_{1-x}$Ge$_x$ (which has the high conduction band minimum) some distance away from the interface (undoped Si$_{1-x}$Ge$_x$ is called the spacer), while keeping Si (with the low conduction band minimum) free from intentional doping. Some of the extra electrons which are introduced into the conduction band of Si$_{1-x}$Ge$_x$ by the As donors diffuse away and fall into the silicon quantum well which has a lower potential than Si$_{1-x}$Ge$_x$ (type II alignment). These electrons are confined in a nearly triangular well established by the conduction band discontinuity and the Coulomb potential of the parent ionised impurities.

An undoped Si$_{1-x}$Ge$_x$ spacer layer is usually placed between the doped Si$_{1-x}$Ge$_x$ and the very pure Si channel in order to increase the spatial separation of the
ionised donors from the electrons in the well, leading to reduced ionised impurity scattering and thus high mobility in quantum wells at low temperatures. Similarly, a p-type structure is formed by introducing acceptor impurities (e.g. B) into a relaxed Si supply layer which has the low valence band maximum, while keeping the Si$_{1-x}$Ge$_x$ channel with the high valence band maximum free from intentional doping (type I alignment).

2.3.2 Density of states

The concept of the density of states is extremely powerful and versatile, and important physical properties, such as optical absorption, transport, etc., are intimately connected with this concept. The density of states is the number of available electronic states per unit volume per unit energy around energy $E$. The density of states (DOS) of electrons is significantly altered by a reduction in the dimension of the system. In a three-dimensional (3D) system the density of states $D(E)$ scales with the square root of energy, $E$:

$$D(E)dE = \frac{g_s g_v}{4\pi^2} \left( \frac{2m^*}{\hbar^2} \right)^{3/2} E^{1/2} dE \quad \text{for 3-D} \quad (2.7)$$

where $m^*$ is the effective mass of an electron, and $g_s$ and $g_v$ are the spin and valley degeneracy, respectively. The value of spin degeneracy is always two, because of the up and down spin of the electrons. The value of valley degeneracy for the electrons in a strained Si layer is also two, because the lowest conduction band minimum is two-fold in tensile strained Si. The DOS of a two-dimensional (2D) system (i.e. quantum well), when one degree of freedom is eliminated, is given by
Figure 2.11 Density of states versus energy for three-dimensional and two-dimensional systems.

\[ D_{2D}(E) \, dE = \frac{g_s g_v}{4\pi} \left( \frac{2m^*}{\hbar^2} \right) \, dE \quad \text{for 2-D} \quad (2.8) \]

and is therefore independent of energy. This relation is an important feature that will be re-addressed when discussing the SdH effect. When the electrons are restricted to only one-dimensional (1D), the DOS of the 1D system (so-called a quantum wire) is given by

\[ D_{1D}(E) \, dE = \frac{g_s g_v}{4\pi} \left( \frac{2m^*}{\hbar^2} \right)^{1/2} \, E^{-1/2} \, dE \quad \text{for 1-D} \quad (2.9) \]

The behaviour of the 2-dimensional and 3-dimensional DOS are illustrated in figure 2.11. As the dimension of the system changes, the energy dependence of the density of states also changes. The density of states in 3D systems is proportional to \( E^{1/2} \) as shown in figure 2.11. In 2D systems, however,
there is no energy dependence, while 1D systems exhibit an $E^{1/2}$ dependence. These variations are extremely important and are a key driving force in lowering the dimensionality of the semiconductor structures. However, for real 2D semiconductor materials, there exists disorder which creates localised states resulting in a band tail as shown in figure 2.12 rather than sharp edges obtained for the 2D DOS.

In general, conduction takes place only in extended states which exist throughout the crystal, while localised states are bound to a finite region in the crystal and do not contribute to the electron transport. Electrons in localised states can only move by being thermally activated to energy states above the mobility edge or by hopping between localised states at low temperatures, where the mobility edge $E_m$ is defined as the energy which separates the regions of localised and extended states. The concept of the localised and extended states becomes important when explaining the integer quantum Hall effect, and this will be discussed further in section 2.4.2.
2.3.3 Electron transport in two-dimensional structures

The transport properties of semiconductors are profoundly changed when an external electric or magnetic field is applied. In free space, carriers influenced by an electric field are accelerated. However, the motion of the charged carriers in the unconfined direction, i.e. x-y direction, of a two-dimensional structure is not free because of numerous scattering sources. Therefore, the effect of scattering by such sources needs to be taken into account. A first-principles treatment of scattering in a 2-DEG is a complicated task but considerable simplification can be made by adopting a phenomenological approach called the relaxation time approximation [23]. The essential assumptions of this approximation are that the various interactions experienced by the “free” carriers only slightly restrain their motion and there is no qualitative change in their trajectory. Under such assumptions we can make the following modifications. Firstly, the rest mass of the carriers is replaced by an effective mass, $m^*$, to take into account the periodic crystal potential. Secondly, the notion of an average or "drift" velocity is introduced which reflects the random interruption to the motion experienced by the carriers due to scattering involving impurities, lattice imperfections, phonons etc.. The equation of motion for an electron in an electric field $\vec{E}$ is as follows

$$m^* \frac{d}{dt}(\vec{v}) + \frac{m^* \vec{v}}{<\tau>} = -e\vec{E} \quad (2.10)$$

The first term on the left represents the carrier acceleration and the second term represents the scattering experienced by an electron drifting through the semiconductor material, where $<\tau>$ denotes the average relaxation time, and $\vec{v}$ is then the carrier velocity. The steady state solution of equation (2.10) is
then given by

\[ \bar{v} = \bar{v}_d = -\frac{e(\tau)}{m^*} \tilde{E} \]  

(2.11)

where \( \bar{v}_d \) is the electron drift velocity. Equation (2.11) shows that the electron drift velocity is proportional to the applied field. The proportionality factor depends on the average relaxation time and the effective mass. The proportionality factor is known as the electron mobility \( \mu \), usually expressed in units of cm²/V·s,

\[ \mu = \frac{e(\tau)}{m^*} \]  

(2.12)

Thus,

\[ \bar{v}_d = -\mu \tilde{E} \]  

(2.13)

The carrier mobility is an important parameter for carrier transport because it describes all the scattering mechanisms that a carrier experiences, when moving through a semiconductor in the presence of an electric field. The current density \( \bar{J} \) is given by

\[ \bar{J} = -ne\bar{v}_d = \frac{ne^2(\tau)}{m^*} \tilde{E} = \sigma \tilde{E} \]  

(2.14)

where \( \sigma(=1/\rho) \) is the conductivity of the material. This equation relates a macroscopic material property with the microscopic quantities such as the
carrier density, scattering time and electron mass.

2.3.4 Scattering mechanisms in two-dimensional structures

As mentioned previously, the extremely high mobility obtained in modulation-doped heterostructures arises from the separation between the parent ions and the carriers by building a spacer layer between the carrier channel and the donor supply layer.

At the heterojunction, electrons from the dopant transfer across the interface to the lower lying band edge of the neighbouring semiconductor which is spatially separated from the ionised parent donors. The electron mobility in the channel is considerably enhanced when compared to the mobility in a bulk material of equivalent carrier concentration because of the reduction in ionised impurity scattering. Hence, the modulation-doped heterostructure has the greater advantage at lower temperatures where, in regularly-doped semiconductors, carrier scattering by ionised impurities prevails. This advantage is somewhat reduced at higher temperatures due to phonon scattering.

The following discussion will consider various scattering mechanisms present within a bulk non-polar elemental semiconductor such as Si and Ge;

i) Phonon scattering: Lattice vibrations disturb the motion of the electrons. The vibrations may be represented by acoustic and non-polar optical phonons by which electrons are scattered. The phonon population and hence their scattering rate increases with temperature. Therefore, at 0K, no phonons are present. Acoustic phonons become important above about 1.5K, whereas non-polar optical phonons occur above 80K.

ii) Intervalley scattering: Intervalley scattering can occur for electrons, which
are scattered from the vicinity of one conduction band minimum to another minimum.

iii) *Alloy scattering for Si$_{1-x}$Ge$_x$ crystals:* The alloy scattering process has to be taken into account in Si$_{1-x}$Ge$_x$ alloys. The random distribution of Ge atoms in the alloy gives rise to potential fluctuations, causing scattering of the mobile carriers.

No systematic mobility investigations on bulk Si$_x$Ge$_{1-x}$ alloys have been done so far, because all research into Si$_{1-x}$Ge$_x$ alloys is mainly concentrated on strained layers in 2D systems. All the above mentioned scattering mechanisms are not only present in the bulk semiconductor material, but more or less influence the transport behaviour of the carriers in a 2D system, as well. But pseudomorphical growth and strain alter the crystal structure and thus the physical properties of such a system. Consequently, existing scattering mechanisms have to be modified and additional ones have to be taken into account for the calculation of the carrier mobilities for a 2D Si/Si$_{1-x}$Ge$_x$ heterostructures:

iv) *Remote impurity scattering:* Scattering of electrons by the Coulomb potentials of the remote ionised impurities in the Si$_{1-x}$Ge$_x$ supply layer is the dominant source of scattering in modulation-doped structures at low temperature. Usually their influence on the 2DEG is reduced by adding an undoped spacer layer as mentioned earlier. Some general statements can be made about this scattering process: As the Coulomb interaction follows an inverse-square law, the scattering rate would be expected to show a similar dependence on the separation between the well and donor regions, i.e. on the spacer layer thickness. Theoretical studies of the mobility dependence on spacer thickness in modulation-doped Si/Si$_{1-x}$Ge$_x$ have been done by Stern et
al. [20]. They have obtained the following conclusion that it has been reasonable to expect that higher mobilities might be realised by using thicker spacer layers. Don Monroe et al. [24] have estimated the remote impurity scattering limited mobility at 4.2K as 170,000 cm$^2$/Vs for a spacer thickness of 150 Å giving $n_x = 5 \times 10^{11}$ cm$^{-2}$. The other side, its contribution to the so far experimentally realised hole mobilities is negligibly small. Only for very high hole mobilities $>10^6$ cm$^2$/Vs do the remote impurities theoretically become important in the calculations.

v) **Background impurity scattering:** The impurities in the nominally undoped Si channel region of a heterostructure are a source of electron scattering in modulation-doped structures. The purpose of the modulation-doping technique is to separate the carriers in the channel from the dopant in supply layer. Practically, this is not possible due to growth affects such as segregation of the dopant and germanium, and the incorporation of impurities from the growth chamber, leading to unintentional doping in the channel. The resulting background impurity scattering effects the electron mobility already at rather modest densities, while the limit for the hole mobilities again is too high to see its significance in experiments.

vi) **Interface roughness scattering:** One or two monolayers at the heterojunction should be considered as a transition region between the Si and Si$_{1-x}$Ge$_x$ layers. Moreover, inappropriate growth conditions can cause growth by 3D islands, leading to a nonplanar interface. This interface roughness in quantum wells is usually considered as a random modulation of the width of a quantum well. The roughness of the interface between Si and Si$_{1-x}$Ge$_x$ has a great influence on the mobility. However, modern advanced growth techniques such as MBE or MOCVD, make it possible to grow epilayers of high quality.
interfaces with negligible interface roughness scattering effects.

vii) **Threading dislocation scattering:** Threading dislocations in the channel, which are strongly dependent on the density of the misfit dislocations, cause this scattering process. Threading dislocation densities as low as $10^6 \text{cm}^{-2}$ can be realised with present growth techniques, and the limit set by this scattering mechanism exceeds by far the highest electron ($5 \times 10^6 \text{cm}^2\text{V}^{-1}\text{s}^{-1})$[25] and hole mobilities ($2 \times 10^4 \text{cm}^2\text{V}^{-1}\text{s}^{-1})$[26] achieved so far, and thus threading dislocation scattering can be neglected.

A discussion of the factors limiting the mobility in our samples will be given in chapter 4.

### 2.4 Effect of magnetic field on two-dimensional electron gas

The most important parameter when determining the characteristics of 2DEG is the electron mobility. The best method to investigate the electron mobility is to examine the electric field response of the electrons under the influence of magnetic field. The study about the behaviour of the electrons in the presence of magnetic field can be considered by the semi-classical approach or the quantum mechanical approach, i.e. the transport phenomena in weak or in strong magnetic fields.

Experiments such as the Hall effect or the van der Pauw method are carried out in weak magnetic fields. This is the semi-classical approach with the $E$ vs. $k$ relation of the carriers given by the band structure. However, in situations where the free electrons are under the influence of strong magnetic fields, for which the condition $\hbar \omega_c > k_B T$ is fulfilled, one can no longer apply the semi-classical approach since their behaviour is profoundly modified.
Therefore, experiments using strong magnetic fields employ the quantum mechanical approach.

In the first section, a mobility study in weak magnetic fields is examined including Hall effect measurements. In the subsequent sections, two of the main magnetotransport phenomena under strong magnetic fields, the Shubnikov-de Haas and the quantum Hall effect, are dealt with.

2.4.1 Classical magnetotransport phenomena in two-dimensional structures

Magnetotransport measurements carried out on homogeneous samples are usually characterised by a resistivity tensor. For a standard measurement of magnetoresistivity, a so-called Hall bar (figure 2.13) is used [27]. It is essentially a rectangularly shaped sample with current contacts (1 and 5) at the two ends and potential probes (2, 3, 4, 6, 7, and 8) at the sides. Usually, the structure of the Hall bar is etched and the electrical conduction through the 2DEG is possible by alloying a metal into the contact regions.

Because of the quasi 2D nature of our samples, we will only discuss the 2D resistivity tensor ρ, which describes the electrical transport by generalising Ohm's law. If a current is injected in the x-direction and a magnetic field B is applied along the z-direction, this gives rise to an electric field $E_y$ in the y-direction due to the Lorentz force on the charge carriers (Hall effect). The Hall voltage can be measured at the potential probes of the Hall bar. Hence, a current density $\vec{J}$ in the Hall bar shaped sample is related to an electric field $\vec{E}$ from equation (2.14)

$$\vec{E} = \rho \vec{J} \quad \text{or} \quad \vec{E} = \frac{1}{\sigma} \vec{J} \quad \text{(2.15)}$$
Figure 2.13 Hall bar shape. Contacts 1 and 5 are for the current source, whereas the other contacts are for the measurements of the magnetoresistivity and Hall voltages [27].

where the resistivity tensor $\rho$ is given by

$$\rho = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{yy} \end{pmatrix} = \begin{pmatrix} \rho_0 & \frac{B}{n_s e} \\ -\frac{B}{n_s e} & \rho_0 \end{pmatrix}$$

(2.16)

and $\rho_0 = m^* / n_s e^2 \langle \tau \rangle$, where $n_s$ is carrier density in 2DEG. The average relaxation time $\langle \tau \rangle$ is related to mobility $\mu$ by equation (2.12). The inverse of $\rho$ is the conductivity tensor, $\sigma$, which is explicitly described by
\begin{align}
\sigma &= \begin{pmatrix}
\sigma_{xx} & -\sigma_{xy} \\
\sigma_{xy} & \sigma_{xx}
\end{pmatrix} = \frac{n_e e^2}{m^*} \begin{pmatrix}
\frac{\tau}{1 + \omega_c^2 \tau^2} & -\omega_c \tau^2 \\
\omega_c \tau^2 & \frac{\tau^2}{1 + \omega_c^2 \tau^2}
\end{pmatrix} \\
\text{where } \sigma_{xx} &= \frac{n_e e^2 \tau}{m^* (1 + \omega_c^2 \tau^2)}, \quad \sigma_{xy} = \frac{n_e e^2 \tau \omega_c}{m^* (1 + \omega_c^2 \tau^2)} \quad \text{and } \omega_c = eB/m^* \text{ is cyclotron resonance frequency. The components of } \sigma \text{ are related to those of } \rho \text{ by the tensor relations}
\end{align}

\begin{align}
\sigma_{xx} &= \frac{\rho_{xx}}{(\rho_{xx}^2 + \rho_{xy}^2)} \\
\sigma_{xy} &= \frac{\rho_{xy}}{(\rho_{xx}^2 + \rho_{xy}^2)}
\end{align}

where use has been made of the relationships \( \rho_{xx} = \rho_{yy} \) and \( \rho_{xy} = \rho_{yx} \), appropriate for an isotropic material. The form of the conductivity tensor in a magnetic field can be simply calculated if the relaxation time is assumed to be constant.

The semiclassical equation for the motion of an electron in an electromagnetic field \( \vec{E} = (E_x, E_y, 0); \vec{B} = (0, 0, B_z) \) is defined as

\[
m^* \frac{d\vec{v}}{dt} + \frac{m^* \vec{v}}{\tau} = -e(\vec{E} + \vec{v} \times \vec{B})
\]

and by using equation (2.14), the steady state solution can be expressed as

\[
J_x = \frac{n_e e \mu}{1 + \mu^2 B^2} E_x + \frac{n_e e \mu B}{1 + \mu^2 B^2} E_y = \sigma_{xx} E_x + \sigma_{xy} E_y
\]
Because there is no current path in the y-direction for a standard Hall bar shaped sample, the Hall coefficient, $R_H$, is defined as:

$$J_y = \frac{n_e \mu}{1 + \mu^2 B^2} \frac{E_y}{J_x B} = \frac{n_e \mu^2 B}{1 + \mu^2 B^2} E_x - \sigma_{xx} E_y - \sigma_{xy} E_x$$  (2.22)

and at low field the magnetoconductivity becomes

$$\sigma_{xx} = \frac{1}{\rho_{xx}} = \frac{J_x}{E_x} = n_e e \mu$$  (2.24)

This is the classical Hall effect. However, equation (2.24) is not totally accurate since we have assumed that all electrons in the sample move with the same velocity. In facts, this velocity is the average drift velocity superimposed on the random thermal motion. Hence, some electrons move more slowly and some move faster. The Lorentz forces acting on electrons moving with different velocities are different, but the Hall electric field acts with the same force on all electrons, independently of their velocities. A more accurate analysis of this problem shows that equation (2.23) has to be modified as follows ; $R_H = \frac{r_H}{en_e}$

where $r_H$ is called Hall factor. The Hall factor ranges between one and two, depending on the Fermi energy, the strength of the magnetic field and the scattering mechanisms presented in the material, and provided $\mu B << 1$, is given by
where \(<t>\) is the average of all relaxation times. As a result, the Hall mobility \(\mu_H\) can be obtained from

\[
\rho_{xx} = \frac{r_H}{\mu_H n_e e} \Rightarrow \mu_H = \frac{r_H}{n_e e \rho_{xx}}
\]  

(2.26)

The measured Hall mobility \(\mu_H\) is related to the actual drift mobility \(\mu\) via the Hall factor, leading to \(\mu_H = r_H \mu\). In our two-dimensional electron gas Si/Si_{1-x}Ge_x heterostructures at low temperatures the carrier distribution is degenerate, and in this case the energy averages in equation 2.25 are replaced by the value of the relaxation time at the Fermi energy. This results in \(r_H = 1\), i.e. the carrier drift mobility is exactly equal to the measured Hall mobility.

Because of the ratio of the width to the length of the channel between the two adjacent contacts, the resistivity \(\rho_{xx}\) is related to the measured resistance \(R_{xx}\) by

\[
\rho_{xx} = \frac{d_{x,y}}{d_{x,i}} R_{xx}
\]

(2.27)

\[
\rho_{xy} = R_{xy}
\]

(2.28)

with \(d_{i,j}\) being the sample dimensions between the contacts \(i\) and \(j\) of the Hall bar in figure 2.13. Therefore, the carrier density \(n_e\) of the 2DEG is given by the following relation

\[
r_H = \frac{\langle \tau^2 \rangle}{\langle \tau \rangle^2}
\]

(2.25)
where $B$ the magnetic field and $e$ the carrier charge.

2.4.2 Two-dimensional magnetotransport phenomena in quantising magnetic fields

When transport measurements are carried out under strong magnetic fields (in the so-called quantum regime, where $\omega_c \tau \gg 1, \tau$ being the carrier relaxation time and $\omega_c$ being the cyclotron resonance frequency), a vast amount of information can be obtained about the parameters of the 2DEG electron gas. Such measurements also opened up the new area of the quantum Hall effect. In this section a detailed description of these studies is presented.

A strong magnetic field $B$ with a component $B_x$ normal to the interface causes electrons in a two-dimensional layer to move in cyclotron orbits parallel to the surface. As a consequence of the orbital quantisation, the energy levels of the 2DEG are quantised for motion in a direction perpendicular to the applied magnetic field. In order to derive an expression for the quantised energy levels, the Schrödinger equation

$$\hat{H}\psi = E\psi$$

should be solved under the influence of the magnetic field[28]. The general Hamiltonian for an electron in the electric-magnetic field is as follows

$$\hat{H} = \left(\frac{\hat{p} - e\hat{A}}{2m_\theta}\right)^2 + V_c(r)$$

(2.31)
Therefore, the Schrödinger equation is given by

$$\frac{1}{2m^*}(\frac{\hbar}{i} \nabla - eA)^2 \psi = E\psi$$  \hspace{1cm} (2.32)

Here, $\vec{A}$ is the magnetic vector potential, $\hat{p}$ is the momentum operator. If we allow $\vec{A}=(0,Bx,0)$ and the magnetic field $B=(0,0,B)$, and take into account the interaction of the spin of the electron, the above equation's eigenvalues can be described by

$$E_{i,n,s} = E_i + \left( n + \frac{1}{2} \right) \hbar \omega_c + sg\mu_B B$$  \hspace{1cm} (2.33)

where $g$ is the Landé $g$-factor, $s=\pm 1/2$ is spin quantum number, $\omega_c$ is the cyclotron resonance frequency (see page 46) and $\mu_B = e\hbar/2m_e$ is Bohr magneton.

From these equations it can be said that when a magnetic field, $B$, is applied to a two-dimensional electron gas with a component $B_z$ along the growth axis, the electrons are forced to move in cyclotron orbits parallel to the interface and the system becomes fully quantised with energy levels. The orbital motion of the electrons in the magnetic field $B_z$ component is characterised by a classical cyclotron radius of the ground state of the Landau levels, which is independent of material parameters

$$l_B = \left( \frac{\hbar}{eB_z} \right)^{1/2}$$  \hspace{1cm} (2.34)

The $E_i$ of equation (2.33) is energy associated with the $z$ motion of the
carrier. The second term refers to the energy distribution in the plane perpendicular to the magnetic field, i.e. the energy condenses from a continuum of states (zero magnetic field) onto a states of equally spaced Landau energy levels separated in energy by $\hbar \omega_c$. The quantised energy of the cyclotron movement perpendicular to the magnetic field is similar to a classical harmonic oscillator. Although the wave vector components $k_x$ and $k_y$ are not contained in equation (2.33), the second term can be changed to the following relation

$$
\left( n + \frac{1}{2} \right) \hbar \omega_c = \frac{\hbar^2 (k_x^2 + k_y^2)}{2m^*}
$$

(2.35)

![Diagram of Landau levels](image)

**Figure 2.14** Non-degenerate broadened Landau levels, on the right side, and on the left side the degeneracy lifted because each Landau level of Si consists of 2-fold spin splitting and 2-fold valley splitting.
Hence, the second term described the energies corresponding to the degree of freedom for the motion into a direction perpendicular to the magnetic field, i.e. x-y plane. The third term of equation (2.33) represents the energy term introduced as a result of the electron spin. This is called the Zeeman energy and the effect of spin on the electron energy is to split each Landau level into two energy states of opposite spin, this effect is illustrated in figure 2.14.

In the presence of magnetic field the energy independent 2D density of states is formed into a series of delta function positioned at Landau level energies (see figure 2.15 (a))

\[
D_{2D}(E) = \sum_{n} N_L \delta(E - E_n)
\]  

(2.36)

The number of states per Landau level, per spin and per unit area, \( N_L \), is given by

\[
N_L = \left( \frac{m^*}{2\pi \hbar^2} \right) \hbar \omega_c = \frac{eB_z}{\hbar}
\]  

(2.37)

At this stage, the filling factor \( \nu \) is introduced, which defines the relation between the density of electron \( n_s \) and the number of states per Landau level \( N_L \)

\[
\nu = \frac{n_s}{N_L} = \frac{\hbar n_s}{eB_z}
\]  

(2.38)
In real 2-D systems, the density of states is not δ-function like, but rather expected to have a form shown in figure 2.15(b). It is broadened due to scattering of electrons by residual structural disorder present in all samples. The states in the tails of the levels are localised in space and the presence of extended and localised states will be discussed in the next section.

Figure 2.15 Density of states in a 2-D electron gas in a strong magnetic field (a) ideal 2-D structure (b) real 2-D structure, with impurities and imperfections.
A. The Shubnikov-de Haas effect

Figure 2.16 Landau fan showing the energy of consecutive Landau levels as a function of the magnetic field, where $N$ is Landau level index.

The Shubnikov-de Haas effect (SdH) consists of an oscillatory variation of the longitudinal magnetoresistivity in a strong magnetic field ($\omega_c \gg 1$) at low temperatures, due to the oscillation of the density of states in the vicinity of the Fermi level. This effect is caused by the fact that there is a formation of Landau levels when a magnetic field is applied perpendicular to a 2DEG, as already discussed. The Landau levels increase in energy linearly with increasing magnetic field (due to cyclotron frequency $\omega_c = eB/m^*$) and this is illustrated in figure 2.16. Consequently, as the magnetic field is increased, the Landau levels move up and eventually pass through the Fermi energy causing oscillations in the conductivity. Maximum conductivity (or resistivity) occurs whenever the peak of a Landau level passes through the Fermi level, because only the DOS close to the Fermi energy is of interest for it. As the Landau
levels move further upwards, the DOS around the Fermi energy gets smaller and so does the conductivity, until it reaches a minimum when the Fermi energy lies exactly between the two adjacent Landau levels. Not only does the DOS become smaller in this process resulting in fewer carriers contributing to the transport, but also these carriers change from being extended to being localised. With increasing magnetic field the Landau levels separate further in energy leading a lower frequency of the oscillations.

The oscillations of the longitudinal magnetoresistivity have long been a very powerful tool in studying 2D systems regarding various physics effects such as the carriers transport, especially in new heterostructures and alloys, and in determining parameters such as the effective mass, the carrier density, the carrier mobility and the quantum and transport relaxation time.

The conductivity obtained from Shubnikov-de Haas oscillations can be divided into oscillatory and non-oscillatory portions

$$\sigma = \sigma_{\text{oscillatory}} + \sigma_{\text{non-oscillatory}} \quad (2.39)$$

where the oscillatory portion of the conductivity yields the information on effective masses, carrier density, and scattering times.

Full expression for $\sigma_{xx}$ from Isihara et.al. [29] is given by

$$\sigma_{xx} = \frac{n_e e^2 \tau_i}{m^*} \left[ \frac{1}{1 + (\omega_c \tau_q)^2} \sum_{s=\pm1, \sinh s \chi} \exp \left( -\frac{s\pi}{\omega_c \tau_q} \right) \cos \left( \frac{4\pi^2 \hbar n_g}{g_s g_v e B} + s\pi \right) \right] \quad (2.40)$$

with $\chi = 2\pi^2 m^* k_B T / \hbar e B = 2\pi^2 k_B T / \hbar \omega_c$, where $\tau_i$ is the transport relaxation time, $\tau_q$ is the quantum relaxation time and $g_s, g_v$ are spin and valley degeneracy,
respectively (see 2.3.2). It will be seen in chapter 4.4.3 (see figure 4.11) that the typical exponential decay $e^{-s}$ in the field range studied here. Other workers have assumed that the major contribution to this decay is from the fundamental, $s=1$, term [30,31]; this implies that higher harmonic terms, i.e. $s \geq 2$ will decay even faster, and will contribute less than 10% to the measured amplitude at the higher fields used, and much less at lower fields. We shall therefore also only consider the fundamental component, i.e.

$$
\sigma_{xx} = \frac{n_e e^2 \tau_i}{m^*} \frac{1}{1 + (\omega_c \tau_i)^2} \left[ 1 + \frac{4(\omega_c \tau_q)^2}{1 + (\omega_c \tau_q)^2} \cdot \frac{\chi}{\sinh \chi} \exp \left(-\frac{\pi}{\omega_c \tau_q}\right) \cos \left(\frac{4\pi^2 \hbar n_q}{g_s g_v eB} + \pi\right) \right] \quad (2.40a)
$$

Further discussions of this point will be given in section 4.4.3.

The expression given in equation (2.40a) gives conductivity whilst the measured quantity is usually the resistivity. In order to compare theory and experiment, the measured resistivity values need to be converted into conductivity by inverting the $\rho$ tensor

$$
\rho_{xx} = \sigma_{xx} \left( \rho_{xx}^2 + \rho_{xy}^2 \right) \quad (2.41)
$$

The Hall resistivity can now be written by

$$
\rho_{xy} = R_H B = \frac{\mu_H B}{\sigma_{xx}(B = 0)} = \rho_{xx}(B = 0) \mu_H B \quad (2.42)
$$

and by using equation (2.42)
\[ \rho_{xx}^2 + \rho_{xy}^2 = \rho_{xx}^2 + [\rho_{xx}(B=0)\mu_{ii} B^2] = \rho_{xx}(B=0)[1+\frac{\Delta \rho}{\rho_{xx}(B=0)}]^2 + \mu_{ii} B^2 \]  

(2.43)

where \( \Delta \rho \) is \( \rho_{xx}(B) - \rho_{xx}(B=0) \), with the assumptions

\[ \rho_{xx}^2 = [\rho_{xx}(B=0) + \Delta \rho]^2 = \rho_{xx}(B=0)[1+\frac{\Delta \rho}{\rho_{xx}(B=0)}]^2 \]  

(2.44)

and

\[ \mu_{ii} B >> \frac{\rho_{xx} - \rho_{xx}(B=0)}{\rho_{xx}(B=0)} \frac{\Delta \rho}{\rho_{xx}(B=0)} \]  

(2.45)

These are very good approximation in high mobility samples, and in this case equation (2.43) becomes

\[ \rho_{xx}^2 + \rho_{xy}^2 \approx \rho_{xx}(B=0)[1 + \mu_{ii} B^2] \]  

(2.46)

By combining equation (2.40a), (2.41), and equation (2.46), and using the relation \( \omega_c \tau = \mu_{ii} B \), the magnetoresistivity can be obtained by

\[ \rho_{xx} = \rho_{xx}^2(B=0)\frac{e^2 \tau_i}{m^*} \times [1 + \frac{2(\omega_c \tau_q)^2}{1+(\omega_c \tau_q)^2} \cdot \frac{\chi}{\sinh \chi} \exp\left(-\frac{\pi}{\omega_c \tau_q}\right) \cos\left(\frac{4\pi^2 \hbar n}{g_s g_e e B} + \pi\right)] \]  

(2.47)

The most important aspect of this transformation is that the resistivity is directly proportional to the conductivity, arising from the approximation of equation (2.46).

In section 2.3, it was noted that for 2D systems the energy independence of the DOS would be important in the observation of SdH
oscillations. In the absence of magnetic field the Fermi energy is given by

\[ E_F = \frac{2\hbar^2 \pi}{m^* g_s g_v} n_s \]  
\[ (2.48) \]

If the resistivity minimum at \( B_N \) corresponds to \( N \) Landau levels below \( E_F \), then \( E_F \) in presence of magnetic fields becomes

\[ \left( N + \frac{1}{2} \right) \hbar \omega_c = E_F = \frac{2\hbar^2 \pi}{m^* g_s g_v} n_s \]  
\[ (2.49) \]

For successive Landau levels, we can get the following:

\[ \Delta(1/B) = \frac{1}{B_{N+1}} - \frac{1}{B_N} = \frac{e\hbar}{m^* E_F} \]  
\[ (2.50) \]

Using the values for spin and valley degeneracy, the carrier density \( n_s \) becomes \( (2m^*/\pi\hbar^2)E_F \) in a 2D system. Hence, from equation (2.50), one can deduce

\[ \Delta(1/B) = \frac{4e}{\hbar n_s} \]  
\[ (2.51) \]

Thus the carrier density can be obtained from the period of oscillations. The periodicity of the SdH oscillations in the reciprocal of magnetic field is a signature of the 2D system. A plot of the magnetoresistivity minima positions (which correspond to the filling factor number \( v \)) as a function of the reciprocal magnetic field enable the evaluation of the carrier density. This method will be discussed in section 4.4.1.

Observation of the oscillations can be made difficult by two
mechanisms; i) the collision broadening of the Landau levels and ii) the thermal broadening of the Landau levels.

The collision broadening of the Landau levels, $\Gamma$ (the full width at half maximum), should be much less than the separation of the levels in order to resolve the oscillations

$$\Gamma \leq \hbar \omega_c \quad (2.52)$$

Alternatively, $\Gamma$ can be determined according to the principle of uncertainty by

$$\Gamma = \frac{\hbar}{\tau} \quad (2.53)$$

and clearly shows that the broadening of the Landau levels is only dependent on the existing scattering in the system. Substituting equation (2.53) into equation (2.52) results in the condition for oscillations to be observed, which is

$$\omega_c \tau \geq 1 \quad (2.54)$$

or

$$\mu B \geq 1 \quad (2.55)$$

obtained from the relation

$$\omega_c \tau = \frac{eB}{m^*} \tau = \mu B \quad (2.56)$$

A physical interpretation of equation (2.54) is that the higher the cyclotron
frequency, the smaller the electron orbit, and hence the smaller the probability for scattering. The method for the determination of the relaxation time will be discussed in chapter 4.

If the thermal energy exceeds the Landau level spacing, the levels will overlap. This smearing of the Landau levels will diminish the amplitude of the oscillation. Therefore, the thermal broadening of the Landau levels should be small compared to their separation, which from the definition of $\chi$ means that

$$\hbar \omega_c \geq 2\pi^2 k_B T \tag{2.57}$$

Hence, the minimum magnetic field to observe SdH oscillations can be determined from equation (2.57) by the following equation,

$$2\pi^2 k_B T = \hbar \omega_c \tag{2.58}$$

and thus

$$B = \frac{2\pi^2 m^* k_B T}{e\hbar} \tag{2.59}$$

Substituting for our Si/SiGe system where $m^* = 0.19m_0$ and $T \approx 0.1K$, the oscillations should be resolved at about 0.2T. In practice, oscillations begin at fields greater than 0.5T which indicates that it is a combination of thermal broadening of the Landau levels and broadening caused by scattering which limits the onset of the SdH oscillations.

B. The integer quantum Hall effect

The quantisation of the Hall effect observed in 1980 by von Klitzing et al.
in a Si MOSFET [32] is a remarkable macroscopic quantum phenomenon for two-dimensional electron systems at low temperatures and strong perpendicular magnetic fields. Under these conditions, the Hall resistivity deviates from the classical linear relation defined by equation (2.29) to form plateaux in the Hall resistivity

\[ \rho_y = \frac{h}{ye^2} \]  

(2.60)

where \( v \) is the Landau level filling factor given by equation (2.38) and the value of \( h/e^2 \) equals 25.812KΩ. The plateaux of the quantum Hall effect occur at each integer filling factor and at this point, the longitudinal resistivity \( \rho_{xx} \) drops to zero.

The explanation of the integer quantum Hall effect is aided by the existence of extended and localised states, as mentioned in section 2.4.2. The extended states contribute to the carrier transport and the magnetic field can affect the conductivity. On the other hand, the localised states are not responsible for current flow and the conductivity is independent of the magnetic field. As the magnetic field is varied, the Fermi energy will either reside in localised or extended states. When \( E_F \) lies within the localised states, a variation in magnetic field adds carriers to or subtracts carriers from successive localised states. Consequently, the extended states are not altered. Since only the extended states contribute to conduction, the transport properties of the systems remain constant, i.e. \( \rho_y \) shows plateaux in a finite range of \( B \)-field. As the Landau levels move upwards in energy with increasing magnetic field the extended states will eventually pass through the Fermi energy resulting in a reduced total number of occupied extended states which
leads to the expected rise in the Hall resistivity. This explains only the formation of the plateaux seen in the integer quantum Hall effect, but does not explain why the Hall resistivity has the precise values ($\rho_{xy} = h/ev_f^2$) regardless of the investigated material or structure. It is as if all electrons are actually in extended states, independent of the fact that a fraction of them are in localised states, which crucially depends on sample disorder and therefore should vary from sample to sample. Since a given fraction of the carriers are localised, the density of extended states is diminished and the number of carriers to support the current flow in each Landau level deviates considerably from its value given by equation (2.37). Therefore, the Hall resistivity is expected to deviate from the quantised value $\rho_{xy} = h/ev_f^2$. There have been several approaches to understand the origin of this paradox [33-36].

With the Fermi energy located in the localised states region, it might be expected that the value of $\rho_{xy}$ would change significantly (the classical $\rho_{xy}$ depends on the density of mobile carriers) from the quantised value. However, several authors [33,36] have explained the reason why this does not occur as follows: the potential which localises some of the electrons, accelerate the remaining carriers so that the reduction in the number of mobile carriers is exactly compensated by their increased velocity, thereby keeping the total current undiminished.

Hence, the overall gradient of the Hall resistivity can still be used to determine the carrier density and the Hall mobility.

2.4.3 Parallel conduction

In the case of an ideal sample conduction only takes place in 2DEG. It is possible, however, that the Si$_{1-x}$Ge$_x$ supply layer is not totally depleted from
carriers and the Fermi level resides within the conduction band. In that case conduction can also take place in the $\text{Si}_{1-x}\text{Ge}_x$ supply layer, where the mobility is much lower than in the 2DEG. This effect has been considered by many group [37-38], either to enable the carrier densities and mobilities of the two channels to be separated, or to explain the degradation of the quantum Hall effect and Shubnikov-de Haas oscillations in the presence of conducting the $\text{Si}_1\text{Ge}_x$ supply layer.

2.5 Device applications

III-V semiconductor technology has consistently demonstrated superior performance to silicon technology, but yet silicon devices have an overwhelming commercial market share of about 97%. There are many direct and indirect reasons. The most important reason for the dominance of Si devices lies in the combination of Si being an easily available semiconductor and Si having an excellent natural oxide, namely $\text{SiO}_2$, which serves as an insulator and as a protecting layer. The structure $\text{Si/\text{SiO}}_2$ is the basis for the MOSFETs that can be integrated in enormous quantities.

The properties of the $\text{Si/\text{SiO}}_2$ material system make it ideally suited for digital applications. However, the optoelectronic devices are the only ones that cannot be accomplished in a pure Si system, due to its physical limits. Again, the performance of III-V heterostructures in this area is far superior to the Si systems. However, being widely unsuited for large-scale integration and lacking compatibility with Si VLSI (very large scale integration) technology, the potential market segments for III-V heterostructures are quite restricted.

An alternative would be to introduce a heterostructure that allows the
exploitation of band offsets for application-specific tailoring of the electronic and optical material properties without sacrificing the preconditions for future heterointegration, namely the basic compatibility with Si substrates and technology. In this respect, the Si/Si$_{1-x}$Ge$_x$ heterostructure is a much better suited heterostructure.

The most important application of Si/Si$_{1-x}$Ge$_x$ heterostructures is the heterojunction bipolar transistor (HBT). An example of the high level performance reached 1997 in the Si/Si$_{1-x}$Ge$_x$ heterojunction bipolar transistor, which demonstrated cut-off frequencies and maximum oscillations frequencies well beyond 150GHz, is given in reference [39].

Another potential application of Si/SiGe heterostructures is the modulation-doped field effect transistors (MODFET). In contrast to III-V materials, which are mostly used for n-type devices, the Si/SiGe heterostructures can in principal provide both n-and p-type MODFETs with excellent properties.

In the following sections both the pseudomorphic Si/SiGe HBT and the complementary types of MODFET will be discussed.

2.5.1 Heterojunction bipolar transistors

Both npn and pnp heterojunction bipolar transistors (HBT) have been fabricated successfully, the most recent ones exhibiting transit frequency around 150GHz. However, nearly all of the technology and scientific interest is currently directed toward the npn devices. How the use of heterostructures can increase the device performance of the bipolar transistor is best illustrated by looking at the basic device principle which is explained in short.

In all npn bipolar transistors the collector current in all npn bipolar
transistors relies on the injection of electrons from the emitter into the base. Current amplification in an npn bipolar junction transistor is mainly determined by the ratio between the density of electron injected from the emitter into the base, and the density of holes re-injected from the base into the emitter:

\[ \beta = \frac{I_c}{I_b} \]  
\[ (2.61) \]

\[ P = \frac{I_c}{\mu_e v_T} \]  
\[ \mu_e v_T (2.61) \]

\[ \text{drift field through graded doping and graded Ge content in base} \]

\[ \text{E}_c \]
\[ \text{e}^{-} \]

\[ \text{E}_v \]
\[ \text{h}^{+} \]

\[ \text{n}^{+}\text{Si} \text{ emitter} \]
\[ \text{p-SiGe} \text{ base} \]
\[ \text{n}^{+}\text{Si} \text{ collector} \]

\[ \text{Energy} \]

**Figure 2.17** Schematic band diagram Si and SiGe HBT. The valence band offset increases the emitter injection efficiency.

where \(I_c\) and \(I_b\) are the collector and base current, respectively.

The basic principle behind the HBT devices is the introduction of a heterojunction at the emitter/base interface. This leads to an enhancement of \(\beta\) by an exponential factor which depends only on the bandgap difference \(\Delta E_g\) and the thermal energy \(kT\). Therefore,

\[ \beta_{\text{HBT}} = \beta_{\text{BJT}} \cdot \exp\left( \frac{\Delta E_g}{kT} \right) \]
\[ (2.62) \]
The potential variation in a Si/SiGe HBT is schematically shown in figure 2.17. Since the narrow-gap material SiGe is used for the base, the emitter and the collector are made of Si, and thus the Si/SiGe HBT contains two heterointerfaces, with the base/collector junction being of little relevance for device performance. Si/SiGe HBTs have two-principle advantage over their Si counterparts. For a given base-emitter bias the conduction band offset between the base and the emitter is much smaller than the valence band offset. Therefore, the reverse injection of holes into the emitter is reduced and the emitter efficiency is increased. This in itself is of minimal benefit for the performance of most circuits, but the increased gain can be traded for a higher doping level in the base. The base with a higher doping can be made narrower, further reducing the transit time of the electrons and increasing the transit frequency. The other advantage is the possibility of grading the Ge content across the base providing a built-in field, thereby enhancing the speed at which the minority carriers cross the base, improving the frequency response of the device.

A exceedingly impressive performance has been reported at the end of 1994 with $f_T$ values well in excess of 100 GHz [40] and a maximum oscillation (unity power gain) frequency ($f_{max}$) of 120GHz [41]. These data are about twice that of a typical standard Si bipolar transistor [42].

2.5.2 Modulation-doped field effect transistor

A modulation doped FET(MODFET) basically operates on the same principles as the Si-based MOSFET with the exception that the channel is built in the structure in the form of a quantum well and the carriers originate from a modulation doped supply layer.
In Si-based MOSFET structure, a thin layer of oxide is grown on top of the Si to insulate the metal gate from the inversion layer that builds up directly underneath the oxide at the surface of the Si. For n-channel MOSFETs the substrate is lightly p'-doped, whereas the source and drain regions are heavily n'-doped. There are two types of channel which can be formed between the source and drain regions. If at zero gate bias the channel conductance is very low, a positive voltage needs to be applied to the gate to form the n-channel, and hence it is called the normally-off or enhancement mode n-channel MOSFET. In the case where the channel conductance already exists at zero bias, a negative gate voltage will deplete the carriers in the channel to reduce its conductance. This type is called the normally-on or depletion mode n-channel MOSFET. Similarly, there exists the concept of the p-channel enhancement and depletion mode MOSFET. The basic device parameters to describe a MOSFET are the channel length, the channel width, the insulator(oxide) thickness, the junction depth and the substrate doping, while its performance is characterised by the values of the transconductance, the cut-off and maximum operating frequency, and the gate voltage swing(logic swing) which defines the abruptness between the on and off state when used as a switch in digital logic circuits. Other parameters such as the leakage current, subthreshold characteristic or “off” current are also very important and should be considered in order to be able to fully characterise a MOSFET device [8].

The n-channel and p-channel mode described for the MOSFET have been realised in MODFET structures too, and their performance has been studied for many years. A schematic view of the MODFET structure and its band alignment is shown in figure 2.18.
**n-Channel MODFETs;** The first reported Si-channel n-MODFET devices[44] were based on a single step buffer layers that led to quite moderate mobilities. These devices employed a Pt/Ti/Au Schottky gate with a barrier height of around 0.9eV. The well-behaved dc characteristics of the devices have been observed, a maximum extrinsic transconductance $g_m$ value of 50mS mm$^{-1}$ at a gate length of 1.6µm has been reported. Drastic improvements resulted from the introduction of a graded SiGe buffer layer, giving significantly enhanced electron mobilities. Ismail et al. [45] reported on Schottky-gated n-MODFETs with a layer sequence grown in two successive UHV-CVD processes. They fabricated a 0.25µm gate length n-channel MODFET which gave mobilities of $1500\text{cm}^2\text{V}^{-1}\text{s}^{-1}(300\text{K})$ and $9500\text{cm}^2\text{V}^{-1}\text{s}^{-1}(77\text{K})$ with corresponding sheet densities of $2.5\times10^{12}\text{cm}^{-2}$ and $1.5\times10^{12}\text{cm}^{-2}$. In comparison, the room temperature mobility for bulk Si doped to the same level is only one third of the MODFET value, and the 77K mobility for Si MOSFET is approximately one third as well.
Although some parallel conduction was present at room temperature, the depletion-type (normally on) transistors behaved quite well. Impressive maximum extrinsic transconductance values of 330 and 600 mS mm$^{-1}$ have been obtained at 300 and 77K respectively. König et al. [46] reported a similarly successful approach. They employed MBE-grown n-MODFETs. The maximum transconductance obtained from these samples are quoted to be 340 mS mm$^{-1}$ at room temperature and 670 mS mm$^{-1}$ at 77K, and they demonstrated excellent Hall mobilities of 2200 (300K) and 15700 cm$^2$V$^{-1}$s$^{-1}$ (77K).

**p-channel MODFETs;** Pearsall et al. [47] reported the first p-MODFETs, which employed a pseudomorphic Si$_{0.8}$Ge$_{0.2}$ channel. These devices exhibited well-behaved transistor characteristics, but their dc performance were found to be modest because of the low room temperature mobilities of pseudomorphic SiGe channels. Little progress has been made over the years regarding the p-type devices.

Although p-type MODFETs can be implemented with pseudomorphic SiGe channels, the low mobilities caused by alloy scattering in such layers suggested the use of pure Ge as the channel material, which again requires a relaxed SiGe buffer layer.

Mobilities at 300K(77K) of 1300 cm$^2$V$^{-1}$s$^{-1}$ (14000 cm$^2$V$^{-1}$s$^{-1}$) at carrier concentration of $1.5 \times 10^{12}$cm$^{-2}$ ($1.0 \times 10^{12}$cm$^{-2}$) for the Ge channel p-MODFET have been reported [48]. This high mobility at room temperature indicated promising figure of merit regarding the dc performance. Transconductance values for 1.2μm gate-length devices at 300K(77K) have been found to be 125 mS mm$^{-1}$ (290 mS mm$^{-1}$).

While the p-type FETs with a Ge channel are certainly the best...
possible choice in terms of high-speed performance, they are not well suited when combined with n-channel FETs employing a pure Si channel for CMOS. For this reason most research on p-channel MODFETs reverted to the method of using a SiGe alloy channel. In contrast to the aforementioned pseudomorphic SiGe channels, which have shown low hole mobilities, new efforts have been focused on SiGe channel with the composition x significantly larger than 50%. Increased hole mobilities in these p-channel samples have been observed due to the reduction in the effective mass. Arafa et al. [49] investigated Si_{1-x}Ge_{x} channel p-MODFETs with x around 50% used an inverted supply layer. These structures showed outstanding hole mobilities of 800-1000 cm²V⁻¹s⁻¹ at room temperature and of 3300-3500 cm²V⁻¹s⁻¹ at 77K. They obtained remarkable results in both the dc performance and the high-frequency behaviour of the p-MODFETs. A maximum extrinsic transconductance of 230mS mm⁻¹ has been measured at room temperature for a gate length of 0.25μm and a gate/source distance of 0.2μm. This high transconductance value led to a transit frequency of 24 GHz and the maximum oscillation frequency of 37GHz. Preliminary results on 0.1μm devices indicated a transit frequency as high as 70GHz[50].

In conclusion, the performance demonstrated experimentally by SiGe MODFETs is an extremely impressive one and one can safely state that these devices have the ability to compete at least in terms of speed with some of the III-V technologies. However, until high quality, low defect density virtual substrates or bulk SiGe substrates become available, the technology will be confined to the laboratories. Other performance related issues such as noise have yet to be addressed.
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Chapter 3

Experimental procedures

3.1 Introduction

This chapter presents a general description of the experimental techniques and equipment that are employed in obtaining the results of this thesis.

First, a brief description of the growth technique of Gas Source Molecular Beam Epitaxy is given, followed by an account of experimental equipment and techniques for magnetotransport measurements.

The contents of the section regarding experimental equipment are divided into two parts. First, the basic operating principles and design features of the cryogenic systems utilised are discussed, and then details of the experimental techniques performed below 1K are described. In the following sections, the experimental set-up and techniques for the measurements of the SdH effect and the integer quantum Hall effect are discussed. The experiments employed in this thesis invariably required the measurement of the sample resistance as a function of magnetic field and temperature. Typically, this measurement can be carried out by passing a known current through the samples of well-defined geometry (see figure 2.13), while simultaneously measuring the voltages developed across various contact configurations as the magnetic field is swept.

Finally, details of sample preparation before the experimental measurements are given.
3.2 Gas source molecular beam epitaxy

The Si/Si₁₋ₓGeₓ modulation doped quantum well structures investigated in this thesis have been grown by gas source molecular beam epitaxy (GSMBE). The samples are grown at the Interdisciplinary Research Centre (IRC) for Semiconductor Materials, Imperial College, by several co-workers [1]. The purpose of the gas source molecular beam epitaxy system is to overcome some of the problems associated with solid source molecular beam epitaxy (SSMBE) using Si. In GSMBE, a volatile hydride is used instead of the solid sources. Although the advantages of GSMBE over SSMBE are covered in this section, for a more detailed account of this and the growth technique see Ohtani [2].

Figure 3.1 Schematic diagram of GSMBE
In Si-SSMBE, activities on fabrication of Si based bipolar junction transistor (BJT)[3], Si$_{1-x}$Ge$_x$ based heterojunction bipolar transistor (HBT)[4], and modulation doped field effect transistor (FET)[5] fabrication have been reported. However, conventional SSMBE contains several difficulties for device applications.

Firstly, the electron-gun evaporator typically used Si-SSMBE produces a characteristic surface defect, called the spitting defect, arising from Si particles from the Si-source. This spitting defect cause stacking faults, even if the silicon molecular beam is not contaminated. In order to create the silicon molecular beam, the temperature of the Si source, using the electron-beam evaporator, is raised to above 2000°C. This causes radiative heating of the steel walls of the MBE machine and consequently, contamination of the growth occurs by impurities effusing out of the walls. These problems can be overcome by using the GSMBE method. The high temperature Si source is replaced with the volatile hydride gases, which impinge upon the substrate at room temperature.

Secondly, interrupted throughput due to Si or Ge solid source exchange in SS-MBE is also problem. The ultra-high vacuum growth chamber has to be opened for source exchange when the solid Si or Ge source was exhausted. This causes low throughput. However, in GSMBE these problems are eliminated because gas source bottles can be exchanged outside the ultrahigh vacuum chamber.

Thirdly, the steel walls of the main chamber in the SSMBE and GSMBE are coated due to successive growth runs with various contents of the Knudsen cells. Subsequent growth at high temperature in SSMBE can result in the effusion of the impurities from the walls, which can contaminate the
growth process. This phenomenon is called the memory effect, whereby dopants used in the previous layer growth are incorporated into the present growth. However, GSMBE avoids this problem, since the volatile hybrid gases are released at room temperature. This means that the film growth takes place only on the heated substrate and not on the steel walls of the growth chamber [2]. Finally, the lower temperature epitaxial growth in GSMBE allows superior abrupt doping profiles due to the absence of high thermal energy which encourages the dopant species to diffuse [6].

The GSMBE apparatus is shown schematically in figure 3.1. The gas source consist of disilane (Si$_2$H$_6$) and germane (GeH$_4$) for the semiconductor matrix and arsine (AsH$_3$) for the arsenic n-type dopant. Typical growth temperatures are 550-700°C. Gas purifiers are fitted between the gas source bottles and the mass flow controllers. The purpose of these purifiers is to filter out heavy mass molecules such as phosphine that may contaminate the gas sources.

Details of the structure for modulation doped quantum well structures are presented in chapter 4.

3.3 Cryogenic system

This section presents the basic operational principles and design features of the cryogenic systems utilised. The experimental techniques and apparatus that are employed for the experiments performed below 1K are explained.
3.3.1 The principle and design of cryogenic system

The magnetotransport experiments discussed in this thesis can be roughly divided into three stages:

i) Shubnikov-de Haas and Hall effect measurements at 4.2K and 1.7K in a 8T cryomagnetic system constructed by Technology System Limited, at the IRC Characterisation Facility, Imperial College.

ii) Shubnikov-de Haas and Hall effect measurements at 4.2K and 1.5K in an Oxford Instruments superconducting solenoid cryostat at the High-Field Magnetic Facility at the CNRS, Grenoble.

iii) Shubnikov-de Haas and Hall effect measurements below 1K in the dilution fridge constructed by Oxford Instruments at the CNRS, Grenoble.

Each of three cryomagnetic systems that were used for the low temperature measurements of our samples consisted of three major components, a superconducting solenoid, a variable temperature insert (VTI), and a sample chamber. The Oxford Instruments System at Grenoble enables the use of both the $^3$He/$^4$He dilution refrigerator and the VTI.

1) $^4$He cryogenic systems, 4.2K-1.5K

The cryostat system constructed by Technology System and Oxford Instrument is aimed to provide variable temperatures in the range 1.5K-300K. It consisted of a vacuum insulated liquid helium reservoir surrounded by the liquid nitrogen chamber.

The NbTi superconducting solenoid designed by Technology System is capable of achieving a maximum central field of 8T at 4.2K in a 40mm bore, while the magnet constructed by Oxford Instrument is composed of two sets of concentric solenoids with outer sections from NbTi superconducting wire and inner sections from Nb$_3$Sn. These sections are arranged to provide a maximum
vertical magnetic field of 15T at 4.2K in a 52mm bore. Magnetic fields are found to be homogenous to within 1% and 0.1% for the two arrangements respectively, over a cylinder of 10mm length by 10mm in diameter. Both magnets were of excellent quality as judged from the near negligible hysteresis measured. The critical field and current of a superconducting magnetic are a function of the temperature, and magnetic performance is enhanced by reducing the temperature. There are two methods of reducing the temperature of the magnet space. The entire helium bath is pumped, or only the lower portion of the helium bath surrounding the superconducting coil can be cooled. The latter method is achieved by a device known as a λ-point refrigerator which operates by drawing liquid from the main bath into the sample chamber and reducing the vapour pressure by means of a rotary pump.

A schematic diagram of the “Technology Systems” cryomagnetic set-up is shown in figure 3.2. The liquid helium is placed in the liquid helium reservoir. Both the superconducting solenoid is in the 4He reservoir and the sample chamber is in thermal contact for part of its length. The sample chamber itself can be filled with liquid helium through a needle valve, in order to maintain a constant temperature during the measurements. The nitrogen reservoir filled with liquid nitrogen acts as an additional radiation shield for the liquid helium chamber against the high temperatures outside the cryomagnetic system. A sample can be inserted into the system through the sample tube and placed directly in the centre of the solenoid. A copper block is used as the sample holder to keep the temperature stable over a period of time for measurements which are performed at temperatures greater than 4.2K.
To increase the temperature of the samples above 4.2K, the copper block contains a resistive heater and a diode acting as a thermometer. The liquid helium in the sample chamber should be pumped out in order to decrease temperatures below 4.2K. Due to the falling pressure of the liquid helium in the sample chamber the temperature decreases to keep the vapour pressure of the liquid helium constant. Eventually, the temperature in the sample chamber can cool down to 1.7K. There are three main causes for high helium boil off: i) the cooling down of a newly inserted hot sample, which can
be either at room temperature or pre-cooled at 77K, ii) the heating of a sample above 4K and iii) the cooling of the solenoid which is heated each time a magnetic sweep is performed.

A Hall probe is used to measure the magnetic field of the solenoid during a sweep and the resulting voltage is fed via an A/D converter directly into the computer. In order to provide easy optical access to the sample space, a 12mm-spectrosil B quartz window is included in its base plate.

Figure 3.3 shows a schematic diagram of the Oxford Instruments superconducting solenoid cryostat at Grenoble. The sample chamber is thermally insulated from the helium bath by an inner vacuum space, and is surrounded over most of its length by a radiation shield. For rapid cooling and operation below 4.2K the needle valve allows liquid helium to flow dynamically from the main ‘He bath into the sample chamber. Subsequent closure of the needle valve and control of the vapour pressure via a manostat, gives accurate temperature control in the range 1.5~4.2K. The sample chamber temperature can be monitored by means of a calibrated carbon glass resistance temperature sensor mounted close to the sample. Hence, temperature dependence of the magnetotransport measurements between 4.2K and 1.5K were exclusively performed using this system.

Samples for the measurements are introduced into the central magnetic field region attached to the thin walled stainless steel inserts in both cryomagnetic systems. A top-loading system is utilised, where sample rods emerge through O-ring seals at the top of the apparatus. This allowed easy access to replace the samples from the sample space while the cryostat is cold. For the insert used in the 8T magnet by “Technology Systems”, leads for the sample connection are terminated at a 10-pin DIL socket. Samples are
mounted independently on separate 10-pin DIL headers which could then be easily plugged into the insert. In the 15T system (constructed Oxford Instrument) samples can be mounted directly on to the samples stage at the lower end of the insert via an 8-pin DIL socket.

Figure 3.3 Schematic diagram of the Oxford Instruments cryomagnetic system.
2) $^3\text{He}/^4\text{He}$ cryogenic system below 1K

A specialised cryomagnetic system, which can access temperatures below 1K, is needed to observe the various magnetotransport phenomena, such as spin splitting, valley splitting, and the fractional quantum Hall effect. Achieving base temperatures below 1K in a high magnetic field environment is difficult. There are several major problems to be overcome in designing low temperature experiments, such as i) reaching the necessary low temperatures, ii) measuring these temperatures accurately and, iii) maintaining the low temperatures for sufficiently long periods to allow the measurements [7].

Several techniques have been used to reach temperatures significantly below 1K, but most commercial systems use the valuable lighter isotope of helium, $^3\text{He}$. Temperature reduction of the system is accomplished by lowering the vapour pressure of the liquid, using suitable vacuum pumps, and is only effective for temperatures above approximately 0.3K. As is discussed in the previous section, using the heavier isotope, $^4\text{He}$, temperatures down to approximately 1K can be achieved by reducing the vapour pressure with suitable vacuum pumps. Temperatures below 0.3K are obtained by using $^3\text{He}/^4\text{He}$ dilution refrigerators, in which $^3\text{He}$ is diluted by $^4\text{He}$. These machines can maintain temperature as low as the millikelvin range continuously at high magnetic fields. A few larger specialised (non-commercial) refrigerators have achieved temperatures as low as 2mK, and machines reaching 5mK are available commercially.

The low temperatures, necessary for the magnetotransport experiments discussed in this thesis, were carried out using a transportable dilution refrigerator operated by Dr. D.K. Maude and constructed at the low temperature research laboratories of the CNRS. The dilution fridge can reach
base temperatures of ~20mK at zero magnetic field with a stability of the order of 0.01%. Under high magnetic fields, however, the performance of the fridge suffered at low temperatures (below 50mK), due to eddy current heating in the metal of the mixing chamber. The rate of heat production is proportional to the magnet sweep rate. Heating can be minimised by reducing the sweep rates to 0.1-0.2T/min at low temperatures. After a typical sweep of the magnetic field it was found that the temperature of the sample space rose by about 5-10mK above its initial temperature. This gives an estimate of the error involved in temperature measurements in a magnetic field. To minimise this error, temperatures used in the analysis of the results are averaged between the measured values at the start and at the end of the sweep.

![Figure 3.4 Phase diagram of a $^3$He/$^4$He mixture at low temperatures and pressures.](image)

The principle of the dilution refrigerator was originally proposed by Heinz London of Harwell in 1951, but first working systems were not built until more than 10 years later. Since then, systems have been steadily
improved in performance and the physical processes involved, such as the phase separation of the helium isotope mixture, have become much better understood.

The phase separation diagram of a $^3$He/$^4$He mixture is shown in figure 3.4 [8]. Above 0.86K, the two liquids are miscible in all proportions. As the temperature is lowered, the mixture can become superfluid, where the value of the transition temperature is dependent on the $^3$He concentration. When a liquid mixture of the isotopes is further cooled below the tricritical temperature ($\approx$0.86K), it separates into two different phases. The lighter “concentrated phase” is rich in $^3$He and the heavier “dilute phase” is rich in $^4$He. The mixture ratio in the two phases at any temperature is given by the points where a horizontal line corresponding to the temperature of interest crosses the phase separation curve. Of particular interest is the behaviour at very low temperatures. As $T\to 0$K, the concentrated phase is pure $^3$He, as shown the figure 3.4. However, there is a finite solubility of $^3$He in the dilute phase of approximately 6.4% even at the absolute zero temperature. This fact is the basis of the fridge’s operation. The concentrated phase of the mixture may be regarded as liquid $^3$He, and the dilute phase as $^3$He “gas”. At temperatures less than about 0.5K, the $^4$He which forms the majority of the dilute phase is inert, and the $^3$He “gas” moves through the liquid $^4$He without interaction. The properties of the liquids in the dilution refrigerator are described by quantum mechanics; i.e. this fundamentally different behaviour of the two isotopes at low temperatures is explained by the differing quantum mechanical statistics applied to their atoms [7].

At a given temperature, each phase has an equilibrium concentration. $^3$He is pumped deliberately from the dilute phase so that the equilibrium
concentration between the two phases is destroyed. In order to restore equilibrium, $^3\text{He}$ atoms evaporate across the phase boundary from the concentrated phase to the dilute phase. But for an atom to migrate across the boundary, latent heat must be supplied by the sample and components of the dilution unit. This latent heat is responsible for the cooling process. By removing of $^3\text{He}$ atoms from the dilute phase (to prevent saturation) and returning them to the concentrated phase a dynamic equilibrium is established, thus providing continuous cooling.

Figure 3.5 shows a schematic diagram illustrating the principal parts of the dilution fridge. The $^3\text{He}$ is pumped away from the liquid surface in the still, which is kept at typically 0.6-0.8K. At this temperature the vapour pressure at the $^3\text{He}$ chamber is much higher than at the $^4\text{He}$ chamber. Therefore, $^3\text{He}$ evaporates preferentially. A small amount of heat is supplied to the still to promote the required flow. Hence, the concentration of $^3\text{He}$ through the dilute phase in the still becomes lower than it is in the mixing chamber, and there is thus an osmotic pressure difference which drives a flow of $^3\text{He}$ in a series of heat exchangers.

$^3\text{He}$ is removed from the still and compressed to a pressure of a few hundred millibar by the external vacuum pumping system. Then the gas passes through liquid nitrogen cold traps to remove impurities, and returns to the cryostat, where it is pre-cooled in the main helium bath and condensed on the 1K pot. A narrow capillary tube creates a flow impedance to maintain a high enough pressure in the 1K pot region for the gas to condense.

The basic operating procedure of the fridge can be explained briefly as followed. When the refrigerator starts, the $^3\text{He}/^4\text{He}$ mixture is condensed into the dilution unit by the 1K pot. The temperature has to be reduced below
0.86K (the tri-critical point as showing in figure 3.3), to reach phase separation. The necessary cooling is achieved by the still and it is the first part of the fridge to cool below 1.2K. The incoming $^3\text{He}$ gas is then cooled by the still before it enters the heat exchangers and the mixing chamber. Gradually, the rest of the dilution unit is cooled to the onset of phase separation. The correct concentration and volume of the $^3\text{He}/^4\text{He}$ mixture should be chosen carefully so that the phase boundary is inside the mixing chamber, and the liquid surface is in the still. These conditions are calculated from the known volumes of the mixing chamber, the pressure cell utilised, and the heat exchangers. Gas circulation is initiated by a rotary pump after which, at sufficiently low still pressures, a turbomolecular pump is switched on. The various parts of the fridge cool to their equilibrium temperature over a period of about five hours.

A top-loading system allowed easy access to the mixing chamber via a long sample rod that emerged through O-ring seals at the top of the apparatus. Samples can be mounted in the specially designed 8-pin DIL header which is mechanically coupled to the lower end of the sample rod by means of an attachment screw. The samples and resistance thermometers are immersed directly into the dilute phase.
Figure 3.5 The basic components of a dilution refrigerator.
3.4 Electrical Wiring and Measurement Techniques

Figure 3.6 Schematic diagram of set up of electrical equipment.

The different components of the measuring circuitry are depicted in figure 3.6. A Hall probe is used to measure the magnetic field of the solenoid during a sweep, and the resulting voltage is fed via an A/D converter directly into a computer for data acquisition. The sweep of the solenoid itself is controlled by a programmable power supply which acts as a ramped current source.

The transport properties of SiGe/Si samples employed in the longitudinal and transverse resistivity measurements are studied using the lock-in technique to obtain the highest possible signal-to-noise ratio. At the
IRC only one channel could be recorded for measurements at the time since only one lock-in amplifier could be used, while at the Grenoble laboratory two lock-in amplifiers (EG&G models 5207 and 5210) could be used to measure two resistivities simultaneously. Both signals are pre-amplified before being fed into the lock-in amplifier for measurements in the dilution fridge and the VTI. The voltage output of the lock-in amplifier and different resistors are used as a variable current source for the measurements. The amplified dc voltage output from the lock-ins can be monitored by Keithley multimeters which are controlled by computer via IEEE-488 interfaces.

It will be shown in chapter 6 that sample heating effects can be neglecting for deriving currents of $\leq 100\mu A$. For a typical samples resistivity of $154.5\Omega$ at low temperatures, this corresponds to a electric field of $1.9 \, mV/cm$ along the channel. This field is well below the “knee” field of $2 \times 10^3 \, V/cm$ at which velocity saturation starts to occur in Si.

### 3.5 Sample preparation

All samples have been grown by gas source molecular beam epitaxy by Dr. J. M. Fernández et. al.[1] at the IRC. Initial Hall measurements were performed on the Biorad System using the van der Pauw method (see chapter 4). The material used to form the contacts in n-type structures is chosen to be indium, which can simply be soldered close to the edges of the samples. Hence a tunnel contact is formed for all 2DEG structures and within the temperature range used in the Biorad system no negative effects from the contacts could be observed.
Samples which showed reasonably good transport properties at 300K and 77K in the Biorad system were then processed into Hall bars for further low temperature measurements. Most Hall bar structures were prepared in the clean room facilities of Imperial College by Dr. T. J. Thornton and Dr. N. Woods. The fabrication process was performed by means of photolithography employing two different masks, one for etching the mesa and one for defining the ohmic contacts. The mesa pattern for the Hall bars was etched by using reactive ion etching to define the channel geometry (width 80μm×length 400μm). Alloyed AuSb ohmic contacts to the Hall bars are deposited by thermal evaporation.

Finally, each sample was bonded onto a chip carrier using Al wires in the thermo-compression bonding machine at UCL, and then placed on the samples holder of the cryostat.
References


Chapter 4
Low-temperature magnetotransport phenomena

4.1 Introduction

This chapter focuses on the investigation of 2DEG structures with respect to their sheet carrier densities, mobilities, and relaxation times. In order to obtain information about the quality of the samples, low temperature experiments have been adopted. The basic transport properties such as dominant scattering mechanisms (e.g. remote impurity ionised scattering, phonon scattering etc.), parallel conduction, and negative magnetoresistivity which in turn give information about the structural quality of the samples, can be studied by low temperature magnetotransport measurements. These results are vital in providing constructive feedback to the growth process involving the relatively new growth technique of GSMBE, which still need to be optimised. A good performance of the 2DEG from devices fabricated using those samples is also required at room temperature.

Immediately after new samples are grown, crude van der Pauw measurements for obtaining the carrier density and mobility are performed at room temperature. Samples that exhibit good mobilities at room temperature are then processed into Hall bars and measurements are carried out continuously at low temperatures and high magnetic fields in the cryomagnetic system, constructed by “Technology Systems”. Finally, the best samples are taken to the high magnetic field laboratory in Grenoble where the measurements in the dilution refrigerator are carried out.

The remainder of this chapter is organised as follows. In section 4.2, the experimental method is discussed. In section 4.3, the theory and the
experimental results of the van der Pauw method are described to obtain the carrier density and mobility at room temperature and 77K. Various experimental results, for example the determination of the mobility and carrier density from the SdH and the quantum Hall effect, transport relaxation times and quantum relaxation times, and dominant scattering mechanisms, are dealt with in section 4.4. General conclusions are summarised in section 4.5.

4.2 Experimental procedure

![Schematic layer diagram of the n-channel modulation-doped Si/SiGe used in this thesis](image)

**Figure 4.1** Schematic layer diagram of the n-channel modulation-doped Si/SiGe used in this thesis

The n-type modulation-doped Si/Si<sub>1-x</sub>Ge<sub>x</sub> structures discussed in this thesis are grown by Gas Source Molecular Beam Epitaxy at the IRC for the
Semiconductor Materials [1], as described in chapter 3. The gas sources are disilane (Si₂H₆) and germane (GeH₄) for the semiconductor matrix and arsine (AsH₃) for the arsenic doped n-channel supply layers. Growth rates varied between 0.1 and 1Å/s, depending on the alloy composition and the substrate temperature of all samples was kept constant at 520°C.

As discussed in chapter 2, in order to confine the two-dimensional electron gas in a quantum well, a conduction band offset between the Si channel and the surrounding SiₓGeₓ layers is essential. Also, the underlying SiₓGeₓ buffer layer must be completely relaxed. A sketch of the sample cross section is shown in figure 4.1, and the various samples are categorised in table 4.1. After depositing a 0.3μm thick Si buffer layer onto a high resistivity p-type Si(100) substrate, a 1.5-2μm graded SiₓGeₓ layer, in which the Ge content increased linearly from 5 to 35%, is then grown, followed by 0.5-1μm of constant composition (Si₀.₅Ge₀.₅) alloy. These layers produced a strain-relaxed surface with low dislocation densities onto which the tensile-strained Si channel (110Å thick) was deposited, followed by an undoped Si₀.₇Ge₀.₃ spacer layer, a doped Si₀.₇Ge₀.₃ supply layer, and finally undoped Si₀.₇Ge₀.₃ and Si cap layers. The thickness and As doping levels in the supply layer are varied slightly between the samples, as indicated in table 4.1; for BF197 and BF213, this layer was nominally uniformly doped, whereas for BF313 and BF314, the As doping is supplied by a delta- (δ-) doped layer at the start of the supply layer. It should be pointed out here that surface segregation has been thought likely to produce a redistribution of the As through the subsequently-deposited material [2].

The two-dimensional electron gas samples have been mesa-etched by Dr. T.J. Thornton and Dr. N.J. Wood at the Electrical Engineering Department,
Imperial College. AuSb ohmic contacts to the Hall bars have been prepared by thermal evaporation and alloying.

<table>
<thead>
<tr>
<th></th>
<th>BF197</th>
<th>BF213</th>
<th>BF313</th>
<th>BF314</th>
</tr>
</thead>
<tbody>
<tr>
<td>linearly graded SiGe buffer layer (μm)</td>
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<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Uniform Si_{0.7}Ge_{0.3} buffer (μm)</td>
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<td>0.5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Si channel (nm)</td>
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<td>11</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>Si_{0.7}Ge_{0.3} Spacer (nm)</td>
<td>15</td>
<td>15</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>Si_{0.7}Ge_{0.3} supply (nm)</td>
<td>15</td>
<td>30</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Delta Deposition (Torr)</td>
<td>—</td>
<td>—</td>
<td>27'@2</td>
<td>20'@2</td>
</tr>
<tr>
<td>As doping density</td>
<td>~10^{18} cm^{-3}</td>
<td>~10^{18} cm^{-3}</td>
<td>2×10^{12} cm^{-2}</td>
<td>1.5×10^{12} cm^{-2}</td>
</tr>
<tr>
<td>Si_{0.7}Ge_{0.3} cap (nm)</td>
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<td>6</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Si cap (nm)</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 4.1 Layer structure of modulation doped quantum well investigated.

4.3 Van der Pauw measurements

4.3.1 Theory of van der Pauw method

For samples without Hall bar structures, it is still possible to determine the sheet carrier density and mobility by Hall effect measurements, using the van der Pauw four-contact method [3]. The requirements of this
method are as follows; the samples should be flat, homogeneous, and the point contacts should be well-separated and placed as symmetrically as possible near the edge of the sample. The shape of four-fold symmetry is usually employed in practice, because this arrangement produces smaller misalignment voltages. There are various patterns in common use. However, the standard configurations used are the simple square geometry and the clover-leaf geometry, as shown in figure 4.2.

![Simple Square Geometry and Clover-Leaf Geometry](image)

**Figure 4.2** The simple square geometry and clover-leaf geometry for the van der Pauw method.

To measure the resistance, a current is injected using two adjacent contacts and the voltage drop is measured between two opposite contacts as shown in figure 4.2. For example, a current is passed in the x-direction from 1 to 2 (i.e. $I_{12}$), the voltage between 3 and 4 (i.e. $V_{34}$) is measured, and the same measurement can be repeated in the y-direction. $R_1$ and $R_2$ are given by

$$R_1 = \frac{V_{34}}{I_{12}} \quad R_2 = \frac{V_{32}}{I_{44}}$$

(4.1)
These equations are used in an expression for the calculation of the sheet resistivity $\rho_s$ which is given by

$$\rho_s = \frac{\pi}{2 \ln 2} [R_1 + R_2] f(Q) \ \Omega/\text{square} \quad (4.2)$$

where $f$ is a geometry factor. The geometry factor is determined from the transcendental equation

$$\frac{Q - 1}{Q + 1} = f \cosh^{-1} \left[ \frac{1}{2} \exp \left( \frac{\ln 2}{f} \right) \right] \quad (4.3)$$

where $Q$ is the ratio

$$Q = \frac{R_1}{R_2} \quad (4.4)$$

If $Q$ is almost equal to 1, $f$ can be approximated by the formula;

$$f \approx 1 - 0.34657 \left( \frac{Q - 1}{Q + 1} \right)^2 - 0.09236 \left( \frac{Q - 1}{Q + 1} \right)^4 \quad (4.5)$$

The Hall effect can be observed when a magnetic field is applied. The current is injected between any opposite pair of contacts such that the Hall
voltage, $V_H$, can be measured across the current flow, $I$, between the remaining contacts, when the magnetic field is applied perpendicular to the sample. The Hall coefficient is then given by

$$R_H = \frac{V_H}{IB} = -\frac{1}{n_e e} \text{ cm}^2/\text{coulomb} \quad (4.6)$$

where the Hall factor is still assumed to be one as discussed in chapter 2. Ignoring the sign in the Hall coefficient relation, the sheet carrier density and mobility can be calculated according to

$$n_s = \frac{1}{eR_H} \text{ cm}^2 \quad (4.7)$$

$$\mu = \mu_H = \frac{R_H}{\rho_s} \text{ cm}^2/\text{Vs} \quad (4.8)$$

with $\rho_s$ being the sheet resistivity which, as described previously, can also be measured using the van der Pauw method.

4.3.2 Experimental set-up

The van der Pauw measurements are conducted using a modified Biorad HL 5200. An Alomax-III permanent magnet produces a constant magnetic field of 0.32T. The manufacturer has calibrated this value within ±1% over a radius of 12mm from the pole centre. The magnetic field is reversed
by rotating the magnetic assembly. Errors due to the magnet misalignment are expected to be small.

The current source can provide currents of 1-20000μA and a phase-sensitive detector allows AC measurements to be made at 200Hz. Voltages are measured by a voltmeter with an input impedance of ~10GΩ, and adjustable gain from ×1-1000. The maximum measurable voltage is just over 4V, and the maximum resistance between any two contacts must be kept <1MΩ.

To perform the measurements the van der Pauw samples are inserted into the probe gear. The probes consisted of four plated spikes which can be positioned onto contact pads of the samples. After applying the input current to the van der Pauw current contacts the quality of the contacts is monitored. This involved the resistances of all the permutations of the contact pairs being measured at varying fractions of the applied current to determine linearity of the ohmic resistance. Next the resistivity data are used to determine the sample symmetry and the sheet resistivity. Finally a magnetic field is applied to the sample. Then, the Hall resistivity, Hall voltage, degree of misalignment, and hence the carrier density and mobility are determined in both field orientations. These measurements are carried out at 77K temperature using liquid nitrogen, and at room temperature.

4.3.3. Results and discussion

Van der Pauw experiment using the Biorad system are carried out first to determine simply the carrier density and mobility of all samples at 77K and room temperature. Table 4.2 shows the results of van der Pauw measurements on our four samples.
Table 4.2 Results of the van der Pauw measurements of the 2DEG structures at 300K and 77K

<table>
<thead>
<tr>
<th></th>
<th>BF197</th>
<th>BF213</th>
<th>BF313</th>
<th>BF314</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$[K]</td>
<td>300</td>
<td>77</td>
<td>300</td>
<td>77</td>
</tr>
<tr>
<td>$n_H$ [$10^{12}$cm$^{-2}$]</td>
<td>1.30</td>
<td>0.63</td>
<td>1.9</td>
<td>0.86</td>
</tr>
<tr>
<td>$\mu_H$ [cm$^2$/Vs]</td>
<td>1300</td>
<td>15500</td>
<td>1210</td>
<td>16000</td>
</tr>
</tbody>
</table>

The samples BF313 and BF314 which share a similar structure have a remarkably high carrier density at room temperature. These finding suggest the effect of a strong parallel conductive layer (see chapter 2) on the 2DEG structures. However, their mobilities at 300K are not much lower than for samples BF197 and BF213 which have a weaker influence of the parallel conduction layer, and this would be unexpected for a structure with the evidence of strong parallel conduction because of its degrading character to the overall mobility of the structure. However, we could infer a cause from the fact that the parallel conductive layer would most probably be formed in the heavily doped Si$_{0.7}$Ge$_{0.3}$ supply layer. The room temperature mobility for the electrons in Ge is around 1000cm$^2$/Vs compared to the drastically reduced value of 100cm$^2$/Vs in Si at a doping concentration in low $10^{19}$cm$^{-3}$. Assuming a mobility value for the 2DEG which is only slightly higher than 1000cm$^2$/Vs, then that would explain why the overall mobility is not much lower than for BF197 and BF213.

At 77K most of the additional carriers in the supply layer are frozen out, eliminating the parallel conductive layer and with it, the degrading
influence on the overall mobility, as well. Still the carrier density of samples BF313 and BF314 are higher than in BF197 and BF213 due to the narrower spacer layer in BF313 and BF314.

4.4 Shubnikov-de Haas and quantum Hall Measurements

4.4.1 Carrier density determination

![Figure 4.3](image)

**Figure 4.3** Sketch of the order of the minima values versus the inverse magnetic field. A linear regression yields in a straight line whose slope represents the fundamental field and from which the carrier density can be calculated.

The carrier density can be obtained from the magnetotransport data using the periodicity of the SdH oscillations as discussed in section 2.4.2. The cosine term in the Shubnikov-de Haas equation (see equation (2.40)) induced the oscillations to be periodic with the reciprocal magnetic field. The period of
the oscillatory magnetoresistivity in the reciprocal of the magnetic field relates to the carrier density as shown in equation (2.51).

The period of the SdH oscillations has been obtained from the plots of the reciprocal magnetic field ($1/B_L$), at which the $L$th minima occurs, against the minima number ($L$). If the electrons in only one subband participate in the SdH oscillations, the graph of $1/B_L$ versus $L$ gives a straight line (figure 4.3) and the gradient which yields the oscillation period (so-called fundamental fields $B_f$) is given by

$$\text{gradient} = \frac{\hbar n_s}{g_v g_s e}$$  \hspace{1cm} (4.9)

where $g_s$ is the spin-splitting factor ($g_s = 2$), and $g_v$ is the valley-splitting factor ($g_v = 2$). The carrier density from equation (4.9) can be calculated by the equation

$$n_s = \frac{g_v g_s e}{\hbar} \times \text{gradient}$$  \hspace{1cm} (4.9a)

This method can only be applied in the case of a 2DEG system where only one subband exists. If there are more than one occupied subband, this method cannot be used because the $L$ vs. $1/B_L$ graph would not be represented by a straight line any more. It would be impossible to find the gradient, since the oscillations would be the sum of as many sine waves as there are occupied subbands, all with different frequencies.
Figure 4.4 Sketch of fast Fourier transform of the oscillations. The largest peak represents the fundamental field. The carrier density can be calculated from this fundamental field. The side lobes are harmonics due to the finite data set.

An alternative method to determine the carrier density of a multisubband populated system can be employed using the fast Fourier transformation [4]. Since Shubnikov-de Haas oscillations are periodic in reciprocal magnetic field, a Fourier transformation of the data yields directly the fundamental field of any series present. The systems with one subband exhibit one pronounced peak with the maximum being located at the fundamental field as shown in figure 4.4, representing the Fourier transform for an ideal 2D system. For multisubband systems, where the peaks of the Fourier transform will show more than one pronounced peak, each one can be assigned to the fundamental field of the corresponding subband. Thus, the carrier density for each occupied subband can be calculated, and when summed up over all subbands, the total carrier density is obtained.
4.4.2 Determination of transport relaxation time and quantum relaxation time

From transport measurements, two characteristic parameters can be obtained, the transport relaxation time (transport mobility), and the quantum relaxation time (quantum mobility). It is important to distinguish between these two values, which originate from physically different measurement conditions. The transport relaxation time \( \tau_t \) is governed by the Boltzmann equation and Drude relations, while the quantum relaxation time is mainly concerned with the quantum mechanical Landau level broadening.

The mobility and its appropriate relaxation time, \( \tau \), are related directly by

\[
\mu = \frac{e\tau}{m^*} \quad (4.10)
\]

where \( m^* \) and \( e \) are the effective mass and charge of the electron, respectively. Therefore, the mobility and relaxation time in equation (4.10) can be used interchangeably.

A. Transport relaxation time

The transport relaxation time \( \tau_t \), which is directly determined from the measured Hall or drift mobility, can be found by solving the Boltzmann equation in the relaxation time approximation [5-7]

\[
\frac{1}{\tau_t} = \int dk' P(k, k')(1 - \cos \theta) \quad (4.11)
\]
where \( P(\tilde{k}, \tilde{k}') \) is the probability of scattering from state \( \tilde{k} \) to \( \tilde{k}' \) and \( \theta \) is the scattering angle. Transport relaxation time measurements favour large angle scattering over small angle scattering due to the weighting factor of \((1-\cos \theta)\) in equation (4.11) and is related directly to the Drude relation by

\[
\sigma = n_e e \mu_i = \frac{n_e e^2 \tau_t}{m^*} \tag{4.12}
\]

where \( n_e \) is the two dimensional carrier density, \( \sigma \) is the conductivity, and \( \mu_i \) is the transport mobility. Rearranging equation (4.10) shows that the transport relaxation time \( \tau_t \) is related to transport mobility by

\[
\tau_t = \frac{m^* \mu_i}{e} \tag{4.13}
\]

Experimentally, the transport mobility in a system with single subband occupancy such as in our samples can be determined using equation (4.12), from dividing the zero-field conductivity by the sheet carrier density obtained from the SdH effect.

**B. Quantum relaxation time**

The quantum relaxation time \( \tau_q \) (i.e. the lifetime of a given quantum state) becomes finite in the presence of perturbations produced by scattering potentials. The quantum relaxation time essentially measures the total scattering rate due to collisions with lattice defects and impurity atoms, or equivalently measures the broadening of the Landau levels due to these collisions. The quantum relaxation time relates to the half width the broadened Landau level [7].
The quantum relaxation time, $\tau_q$, is also given by the total scattering probability, $P(\vec{k}, \vec{k}')$ [5,6],

$$\frac{I}{\tau_q} = \int d\vec{k}' P(\vec{k}, \vec{k}') \quad (4.14)$$

One can note that this definition differs from the corresponding expression for the transport relaxation time, given in equation (4.11), because of the absence of $(1 - \cos \theta)$ term. The transport relaxation time favours large-angle scattering due to the weighting factor of $(1 - \cos \theta)$, while the quantum relaxation time is sensitive to all scattering events and effectively measures the entire collision cross-section; small and large angle scattering events are counted equally.

C. Dominant scattering mechanism

The measured $\tau_t/\tau_q$ ratio differs for each scattering mechanisms and is a useful indicator of the dominant scattering mechanism in limiting the mobility of the 2DEG[6,8,9]. Large angle scattering is usually associated with short range potentials such as that from the close proximity of ionised impurities and interface roughness, whereas small angle scattering is associated with long range potentials such as that from remote ionised impurities. Das Sarma et al. [9] have pointed out that for a short-range potential the ratio $\tau_t/\tau_q$ is close to unity, whereas for a long-range potential, this ratio is much larger than unity. The latter case is typical for remotely doped GaAs/AlGaAs structures [10], whereas the former has been confirmed for Si-inversion structure [11] which is formed at a Si/SiO$_2$ interface under positive gate bias.

Large variations in the transport to quantum relaxation time ratio have been reported for Si/Si$_{1-x}$Ge$_x$ heterostructures [12-15]. The values range
from ~26 for the highest mobility samples to ~3.6 for the lowest mobility samples.

D. Experimental method for the determination of both relaxation times

Experimentally, the transport and quantum mobility (or relaxation time) can be determined using completely different measurements. The former can be obtained by various methods (such as Shubnikov-de Haas, and Hall measurements and the van der Pauw method), and the latter, through SdH measurements. Having determined the sheet carrier density, the transport mobility is found by

\[ \mu_t = \frac{1}{n_e \epsilon \rho_x (B = 0)} \]  

(4.15)

Subsequently, the calculation of the transport relaxation time becomes straightforward by using equation (4.13).

In contrast to the transport relaxation time, the quantum relaxation time can be extracted by fitting the oscillatory longitudinal resistivity from the SdH measurements to theory (the so-called Dingle plot [16]) or from the width of the peaks resulting from the Fourier analysis of the SdH oscillations [17]. Possible errors associated with these methods will be discussed in section 4.4.3 section.

**Dingle plot:**

The quantum relaxation time at any given temperature, T, is usually determined from so-called Dingle plot [16], i.e. the amplitude of the Shubnikov-de Haas oscillations, \( \Delta \rho_x (B) \), plotted logarithmically as a function
of the reciprocal of magnetic field, \( B \). \( \tau_q \) is then obtained by comparing the result with expression[18]

\[
\Delta \rho_{xx} = 4 \rho_0 X(T) \exp\left( -\pi / \omega_c \tau_q \right)
\]

(4.16)

where \( \rho_0 \) is the zero-field resistivity, \( \omega_c \) is the cyclotron frequency, and \( X(T) \) is a thermal damping factor. This is given by

\[
X(T) = \frac{\left(2\pi^2 k_B T / \hbar \omega_c \right)}{\sinh\left(2\pi^2 k_B T / \hbar \omega_c \right)}
\]

(4.17)

Equation (4.16) shows that the amplitude of the SdH oscillations depends primarily on (i) the collision broadening of Landau levels due to scattering, characterised by the exponential term containing \( \tau_q \), and (ii) the thermal broadening of the levels, entering through the thermal damping term. By rearranging this equation, it can be shown that, if \( \ln[\Delta \rho_{xx}(B)/\rho_0 X(T)] \) is plotted against \( 1/B \), then the slope of the plot gives \((-\pi m^* \tau_q)\) with an intercept (at \( 1/B=0 \)) of 4[18]. From equation (4.17), the thermal damping factor saturates to unity at very low temperatures, but over the regime of interest for the magnetotransport measurements, the deviation from 1 can be significant. The behaviour of the thermal damping term is investigated in detail in section 4.4.3.B.

4.4.3 Results and discussion

The first measurements were carried out at 4.2K and 1.6K, and from 0 to 15 tesla in the High Magnetic Field Laboratory in Grenoble. After investigating the transport properties in this range, sample which
demonstrated good characteristics are taken into the dilution fridge for a measurement at 50mK.

Sample BF197 did exhibit some oscillations but they did not represent typical SdH oscillations and no parameters could be extracted. Even when BF197 was cooled down to 100mK, no improvement could be observed. However, the 2DEG structures of the samples BF213, BF313, and BF314 showed excellent SdH oscillations at 100mK.

Typical graphs obtained from those measurements at 4.2K and 1.66K for BF213, and at 4.2K and 1.6K for BF314, are shown in figure 4.5. At 4.2K, pronounced Shubnikov-de Haas oscillations in $\rho_{xx}$ are observed, for all samples, along with quantum Hall plateaux in $\rho_{xy}$, both of which are indicative of the presence of a well-behaved two-dimensional electron gas (2DEG) in the Si channel. However, only a few minima in the $\rho_{xx}$ and a few plateaux of $\rho_{xy}$ could be resolved. The samples were then cooled down to 1.6K with otherwise unchanged parameters. The significant increase in resolution of the longitudinal resistivity at lower fields is in good agreement with the temperature dependence found in the SdH formula. For filling factors between $\nu=4$ and 8, the spin splitting is clearly resolved in $\rho_{xx}$ for both samples. For high magnetic fields $\rho_{xx}$ exhibits the results of the lifting of the two-fold valley degeneracy at the odd filling factor $\nu=3$ (BF213) or $\nu=5$ (BF314) at around 1.7K. For magnetic fields $B<4T$, the magnetoresistivity oscillation corresponds to filling factors which are multiples of 4, due to the spin and the remaining two-fold valley degeneracies.
Figure 4.5 $\rho_\text{xx}$ and $\rho_\text{xy}$ vs magnetic field B, for two or three different temperatures for BF213 and BF314.
In order to obtain more information, the SdH and Hall effect of the samples were measured at the base temperature in the dilution fridge. The temperature at which the measurements have been performed varied from sample to sample due to the fluctuating nature of the base temperature in the dilution fridge. All measurements discussed here are conducted at the base temperature, i.e. the sweep time lasted 30 min to avoid the heating effect of the varying magnetic field on the sample holder. The Shubnikov-de Haas and Hall effect measurements of sample BF213 at a temperature of 93mK are shown in figure 4.6.

Well-pronounced SdH oscillations and Hall plateaux can be seen in figure 4.6 with clear signs of spin and valley splitting. The large plateau between 12T and 15T and the simultaneous dissipationless minima in $\rho_{xx}$ correspond to a filling factor of $\nu=2$, indicating the spin splitting of the lowest
Landau level, i.e. the first spin level which consists of two valley levels. At around 9.4T, the first valley splitting at $\nu=3$ can be seen. The plateau around $B=7T(\nu=4)$ indicates that the first Landau level is filled. When the magnetic field is decreased, the minima corresponding to valley splittings slowly begin to vanish. The first onset of the Shubnikov-de Haas oscillations is already observed at around $B=0.5T$. Hall plateaux can obviously be seen at the integer filling factors for $\nu=4, 8, 12, 16$. The spin degeneracy begins to lift around $B=2T$ with the first distinct Hall plateau. The onset of valley splitting occurs at around $B=4T$ but no corresponding Hall plateau can be observed. Well-defined quantum Hall plateaux are also observed for the even filling factors $\nu=2, 6, 10, 14$ due to the spin splitting as indicated from the tilted magnetic field experiments. This will be discussed in chapter 5. Hall plateaux due to valley splitting occur for the odd filling factors $\nu=3, 5$ and start to be well resolved at 5.5T.

The graphs of longitudinal and transverse resistivity for samples BF313 and BF314 are shown in figure 4.7. Both samples have nearly the same structures except for the supply doping concentration. The base temperature for the measurement of both samples are identical since they were placed concurrently on the same sample holder, and thus measured within the same run in the dilution fridge. Since the base temperature varies each time that the dilution fridge is cooled down, the magnetotransport measurements have been carried out at a lower temperature of 50mK. The Shubnikov-de Haas oscillations start to appear at around $B=0.95T$ for BF313 and at around $B=0.8T$ for BF314, which are at higher magnetic fields than BF213. This indicates lower mobilities for BF313 and BF314 than BF213 as shown in equation (2.55).
Figure 4.7 SdH oscillations and quantum Hall effect up to B=15T at 50mK for samples (a) BF313 and (b) BF314
The onset of SdH oscillation for BF314 starts slightly earlier in magnetic field than BF313, suggesting a higher mobility as described in section 2.4.2. At 2.8T, splitting of the spin degeneracy for BF313 sets in, while the effect of spin splitting on the SdH oscillations for BF314 can be seen first at 2.6T. However, higher fields are required to observe distinct Hall plateau. These are visible at integer filling factors \( \nu=4,8,12,16,20 \) and at even filling factors \( \nu=6,10,14,18 \), caused by the spin splitting. Only two valley splittings for both samples are resolved at filling factors \( \nu=5,7 \) in \( \rho_{xx} \) but the distinct Hall plateau occurs at a filling factor \( \nu=5 \). A feature of the Hall resistance traces for these samples is the presence of a small peak at each end of the quantum Hall plateau. The coincidence of these peaks with the maxima in the SdH curves suggests that they arise from a small contribution to the Hall voltage from the \( \rho_{xx} \) trace. This is not thought to be due to a misalignment of opposite Hall probes, but more likely to be caused by lateral fluctuations in the barrier composition or dopant distribution.

A. Carrier density and transport mobility

Carrier density values obtained in this thesis have been deduced from the period of the SdH oscillations, and from the slope of the Hall resistance, together with mobility values, \( \mu \), and the corresponding transport relaxation time, \( \tau_p \), from \( \rho_{xx} \). The carrier density can be evaluated from the slope of the Hall resistivity in figure 4.6 and 4.7 before the first plateau appear. To determine the carrier density from the SdH oscillations the oscillations before the onset of the spin splitting can be used. Hence the resolution of this part of graph has to be increased for more accurate calculations. This has been done in the next three figures which show the SdH oscillations between 0 and 2.5T.
(figure 4.8 continued)
Self-consistent calculation on similar structures have shown that the second subband lies ~15meV above the Fermi level in the first band and is there unoccupied at low temperature [14].

To determine the carrier density from the SdH oscillations, the minima number should be plotted against the corresponding inverse magnetic field as discussed section 4.4.1. From the gradient of the resulting straight line the carrier density can be deduced using the equation (4.9a). Also, this data can be analysed by the fast Fourier transform (FFT).
Figure 4.9 The reciprocal magnetic field ($1/B_0$) plotted as a function of the oscillation minima values. The insert shows the fast Fourier transform of the oscillations (Field range up to 25T).
Figure 4.9 presents typical results obtained from both methods. The carrier densities obtained from both methods are of the same value within the limit of experimental error. Transport mobility and relaxation time can be obtained using equation (4.15) and (4.13).

Carrier density values from the period of the SdH oscillations, \( n_s \), and from the Hall resistivity, \( n_H \), together with mobility, \( \mu \), and the corresponding transport relaxation times, \( \tau_t \), are summarised in Table 3.

<table>
<thead>
<tr>
<th>Sample I.D</th>
<th>( T(K) )</th>
<th>( n_s \times 10^{12} \text{cm}^{-2} )</th>
<th>( n_H \times 10^{12} \text{cm}^{-2} )</th>
<th>( \mu \text{cm}^2/\text{V-s} )</th>
<th>( \tau_t \text{ps} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BF213</td>
<td>4.2</td>
<td>0.71</td>
<td>0.7</td>
<td>56460</td>
<td>6.3</td>
</tr>
<tr>
<td></td>
<td>1.60</td>
<td>0.71</td>
<td>0.7</td>
<td>65900</td>
<td>7.12</td>
</tr>
<tr>
<td></td>
<td>0.093</td>
<td>0.70</td>
<td>0.69</td>
<td>69700</td>
<td>7.53</td>
</tr>
<tr>
<td>BF313</td>
<td>4.17</td>
<td>1.29</td>
<td>1.22</td>
<td>29600</td>
<td>3.21</td>
</tr>
<tr>
<td></td>
<td>1.63</td>
<td>1.26</td>
<td>1.22</td>
<td>32600</td>
<td>3.53</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>1.26</td>
<td>1.27</td>
<td>33700</td>
<td>3.64</td>
</tr>
<tr>
<td>BF314</td>
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<td>1.22</td>
<td>1.11</td>
<td>32770</td>
<td>3.55</td>
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</tr>
<tr>
<td></td>
<td>0.05</td>
<td>1.13</td>
<td>1.09</td>
<td>37080</td>
<td>4.03</td>
</tr>
</tbody>
</table>

Table 4.3 Magnetotransport results for samples BF213, BF313, and BF314, between 4.2K and 0.05K.

The close agreement between \( n_s \) and \( n_H \) indicates that parallel conduction in the doped supply layer has been completely frozen out; this is confirmed by the absence of background positive magnetoresistivity in the
curves in figure 4.7 and 4.8. Although BF313 and BF314 have similar structures, sample BF314 has a smaller carrier density than BF313 due to the longer time taken for the delta deposition in BF313. Sample BF213 presents a typical value for the carrier density found in the literature for similar structures [20]. It also contains the largest mobility and relatively small carrier density, whereas the lower mobility together with the highest carrier density is found in BF313. All measured values for the electron mobility of our samples are much lower than the best values which can be found in the literature [21,22] because the level of large-angle background impurity scattering is slightly greater than in our samples. This finding will be discussed further in the next section.

B. Quantum relaxation time and dominant scattering mechanism

The SdH oscillations in the low field regime, i.e. before the spin degeneracy is lifted, yield interesting information regarding the dominant scattering mechanism.

The magnetoresistivity of a 2DEG is not solely governed by equation (4.16), since there may also be contributions from electron-electron interaction [23,24] or parallel conduction [25]. To obtain the purely oscillatory component of magnetoresistivity, the \( \rho \) data need to be filtered to remove the background contribution which, in our samples, is quadratic in \( B \), and is believed to be associated with electron-electron interactions [23,24]. Figure 4.10 shows the purely oscillatory part \( \Delta \rho \) vs the inverse magnetic field. Initially, the quantum relaxation time, \( \tau_q \), can be determined from equation (4.16).
Figure 4.10 The pure oscillations vs the inverse magnetic field for BF213 at 0.093K.

A typical result of the analysis using this approach, for samples BF213 at 93mK, is shown in figure 4.11(a), where $\frac{\rho_{xx}}{\rho_0} X(T)$ is plotted against $1/B$ on the semi-logarithmic scale. Figure 4.11(a) compares well with the results of equation (4.16) at this low temperature. In contrast, a set of data from the same sample at 0.9K is shown in figure 4.11(b)(open circles). Open circles in figure 4.11(b) indicates that the intercept has deviated from the expected value of 4 [18]. The filled circles in figure 4.11(a) and (b) show the effect of neglecting the thermal damping term $X(T)$ in equation (4.16) by assuming that it is unity, i.e. rewriting equation (4.16) as:

$$\Delta \rho_{xx}(B) = 4 \rho_0 \exp\left(-\frac{\pi}{\omega_c \tau_q}\right)$$

(4.18)
Figure 4.11 Dingle plots for BF213 (a) at 0.09K (b) 0.9K. The open circles represents the data with thermal damping included using equation (4.16), closed circles show the data without this term, from equation (4.18)
The results obtained from using equation (4.18) are linear over a wide range of $1/B$, and also give the correct intercept. The thermal damping factor indeed reduces to unity in the limit of extremely low temperatures at which point the amplitude of the oscillations becomes temperature independent. At higher temperatures, however, the theoretical thermal broadening contribution is expected to be significant. A systematic error is observed when equation (4.16) is used to estimate the quantum relaxation time, but this error can be removed by ignoring the thermal damping factor. Figure 4.12 shows that the quantum relaxation time obtained from equation (4.16)(open circles) increases with temperature between 0.05K and 1.6K, for both BF213 and BF313. Thus, the effect of incorporating the thermal damping factor is to reduce the slope of the Dingle plot, giving not only an incorrect intercept (the largest intercept of our samples is 2 above 0.9K, as shown fig. 4.11), but also giving the highly improbable result that $\tau_q$ increases with temperature. The results of analysis without the thermal damping term show that $\tau_q$ is nearly constant, independent of temperature (figure 4.12, filled squares) as expected for ionised impurity scattering in a degenerate electron gas. The fall in $\tau_q$ at the highest temperature point can be ascribed to greater acoustic phonon scattering, where the increased rate at 1.6K corresponds to an acoustic phonon limited quantum relaxation time of 1.4ps and 1.1ps in samples BF213 and BF313, respectively. These relaxation times scale with the carrier density approximately as $n^{-1/3}$, consistent with deformation potential scattering in a 2D system [27]. The reduction in $\tau_q$ with increasing temperature is significantly greater than the corresponding decrease of 5 and 3%, respectively, in $\tau_r$, which can be related to acoustic phonon limited transport relaxation time of 130ps and 120ps at 1.6K. The difference in behaviour can be explained by the greater
Figure 4.12 Dependence of the quantum relaxation time on the temperature for (a) BF213 and (b) BF313.
sensitivity of the quantum relaxation times to small-angle scattering, which will dominate in the present situation where electrons at the Fermi energy of a degenerate distribution are scattered by low temperature phonons.

Recent work [28,29] has also indicated that if equation (4.16) is used at similar temperatures to analyse the data from GaAs/AlGaAs heterojunction, the intercept deviates from 4, and the corresponding value of $\tau_q$ increases with temperature, but that if the thermal damping term is ignored, these anomalies are removed. There is currently no established theoretical explanation for the apparent lack of thermal damping in the SdH oscillations. For GaAs/AlGaAs heterojunctions, deviation from the theoretical model of equation (4.16) has been attributed to a field-dependent effective mass [30], while Coleridge et al. [28] have considered, but rejected, temperature dependent oscillations in the Fermi level, and weak localisation. More recently, Hayne et al. [29] have observed quantum Hall plateaux using the time-dependence of persistent photoconduction in GaAs/AlGaAs 2DEGs, and interpreted their results using a model in which the extended states at the centre of each Landau level occupy a fixed energy range; one consequence of this approach would be that the thermal damping term disappears from the expression for the evaluation of SdH amplitudes, in accordance with their experimental results.

It has been pointed out in section 2.4.2 that the formula used here to describe the oscillations is in fact only the first terms of a harmonic series. If higher frequency terms were not negligible, the amplitude of the fundamental term would decay more slowly with increasing $1/B$, corresponding to an even higher value of $\tau_q$. Thus the omissions of these terms does not explain the experimental observations.
Coleridge et al. [17] and Das Sarma et al. [9] have pointed out that for a short-range range scattering potential the ratio $\tau_i/\tau_q$ is close to unity, whereas for long-range scattering this ratio is much larger as already explained in section 4.4.2.C. Indeed, for modulation doped GaAs/Al$_x$Ga$_{1-x}$As structures, this ratio has been reported to be 10 or greater [10].

<table>
<thead>
<tr>
<th>Samples I.D.</th>
<th>$T$ [K]</th>
<th>$\tau_q$ [ps]</th>
<th>$\tau_i$ [ps]</th>
<th>$\mu_q$ [cm$^2$/Vs]</th>
<th>$\mu_i$ [cm$^2$/Vs]</th>
<th>$\tau_i/\tau_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BF213</td>
<td>0.09</td>
<td>0.81</td>
<td>7.53</td>
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<td>7.6</td>
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</table>

Table 4.4 Transport and quantum relaxation times determined in 2DEG samples.

The ratio, $\tau_i/\tau_q$, for all our samples ranges about 7~9, demonstrating the expected dominance of small-angle remote impurity scattering. However, Stern et al. [31] have calculated $\tau_i/\tau_q$ values for the spacer layer thickness of 15nm. The ratio, $\tau_i/\tau_q$, obtained from their calculation is found to be above 10 for small-angle remote impurity scattering. However, in our samples, where the spacer thickness is also 15nm one cannot draw the conclusion that based on their calculations, the mobility is limited by remote impurity scattering only. Consequently we suggest that the level of large-angle background impurity
scattering is slightly greater in our samples than reported in other literature [21,22].

The nearly constant value of the ratio suggest that the difference in mobility between BF213 and the similar structures BF313 and BF314 (grown later) cannot be ascribed solely to an increase in the background doping level in the latter samples; there must also be an increase in the small-angle scattering rate. A possible interpretation arises from the difference in dopant distribution in the two sets of samples, since in BF313 and BF314, the growth was interrupted briefly at the start of the supply layer, to allow an accumulation of As on the surface which is expected to compensate for the strong tendency of this dopant to surface segregate. Thus the concentration of ionised impurities close to the 2DEG would be greater in the samples than in BF213, where the segregation would introduce a gradual build-up in doping level, as the growth of the supply layer progressed. The transport relaxation time would therefore be shorter in the latter structures, but an increase in background scattering would also have to have occurred, to change the quantum relaxation time by approximately the same factor.

4.5 Conclusion

To summarise, measurements of the oscillation of the transverse magnetoresistivity, i.e. Shubnikov-de Haas effect (SdH) and the quantum Hall effect, have been carried out in the low temperature regime (0.05K to 4.2K). Pronounced Shubnikov-de Haas oscillations in $\rho_{xx}$, with corresponding quantum Hall plateaux in $\rho_{xy}$, which are indicative of a well-behaved two-dimensional electron gas (2DEG) in the Si channel can be seen in all three
samples. To determine the characteristics of the two-dimensional electron gas, the electron density and mobility are calculated from the period of Shubnikov de Haas oscillation and the zero field resistivity. From this method, the electron mobility is found to be 69700 cm$^2$/Vs at carrier density of $\sim 7 \times 10^{11}$ cm$^{-2}$ for BF213, 33700 cm$^2$/Vs with the carrier density $\sim 1.25 \times 10^{12}$ cm$^{-2}$ for BF313, and 35810 cm$^2$/Vs with $n_s \sim 1.13 \times 10^{12}$ cm$^{-2}$ for BF314. All samples have shown that the lifting of four-fold degeneracy for each Landau level (two for spin and valley) is well resolved at relatively low fields, and the lifting of the spin degeneracy can first be observed for the magnetic field at around $B=2T$ for all samples. The valley splitting, characterised by the odd filling factors, can be resolved at about 6T. Vanishing longitudinal resistivity $\rho_x$ can be observed at filling factors $\nu=4, 6, 8$. It has been observed that the carrier density evaluated from the SdH oscillations is very similar to that obtained from the low-field gradient of the Hall resistivity $\rho_{xy}$, suggesting that there is little parallel conduction in the Si$_{0.7}$Ge$_{0.3}$ supply layer at these low temperature. The low temperature Shubnikov-de Haas effect has been analysed to extract the quantum relaxation time. Using conventional Dingle plots based on equation (4.16), the intercept of the fitted curve has been found to fall below the expected value of 4 at temperatures where the thermal damping term, $X(T)$, departed significantly from unity, i.e. above $\sim 0.3$K. The corresponding quantum relaxation time increased with temperature in the range between $\sim 0.3$K and 1.6K. We then analysed the data using a modified expression, equation (4.18), in which the thermal damping term was neglected. This gave plots with the correct intercept, and a quantum relaxation time which was nearly constant with increasing temperature. We interpret these results as showing that, since at very low temperatures the SdH oscillations appear to
behave as expected theoretically, is it the thermal damping term which is anomalous. The decrease in quantum lifetime observed at 1.6K is attributed to acoustic phonons, whose increased contribution to the small-angle scattering has a large effect on $\tau_q$, but only changes $\tau_i$ by a few percent.

The ratio of transport and quantum relaxation time indicate that the dominant scattering mechanism limiting our sample's mobility is small angle remote impurity scattering. However, the level of large-angle back-ground impurity scattering in all samples is slightly greater than samples reported from the other group[21,22].
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Chapter 5
Tilted magnetic field studies of spin splitting of the Landau levels.

5.1 Introduction

In the past three decades there has been considerable interest in the magnetic field dependence of the spin and valley degeneracy. Various groups [1-6] have extensively studied the magnetic field induced lifting of the spin and valley degeneracy in the metal-oxide-silicon (MOS) systems. Such systems exhibit properties that are very different from the bulk material. One such property is that the Landé g-factor is considerably larger than the bulk value of 2 [7], this was discovered using the tilted magnetic field method first developed by Fang and Stiles [1], which increases with decreasing electron concentration. The enhancement of $g$ and its inverse dependence on $n_s$ have been attributed to the exchange interaction as proposed by Janak [8]. Since then, a number of groups [9-12] studied the interesting consequences of $g$-factor enhancement, $g^*$, in a two-dimensional system, in particular Ando and Uemura [10], who first pointed out that the $g$-factor enhancement should be an oscillatory function of the magnetic field. The physical idea behind this periodic $g$-factor enhancement is the following: the Zeeman spin splitting has been shown [8] to be increasing by the exchange interaction, which is greatest where difference between the occupancies of the spin-up and spin-down Landau levels is large. This population difference depends on the position of the Fermi level, which changes relative to the Landau levels as the field is
changed. Resistivity or conductivity minima occurring when the Fermi level lies between two spin split states of the same Landau level will correspond to a maximum enhancement, leading to a larger $g^*$-factor, while for a Fermi level between the spin states originated from different levels the population difference will be much smaller leading to a smaller $g^*$-factor. This gives rise to the observed oscillatory enhancement of the effective $g$-factor.

The most commonly employed method to obtain the magnitude of the $g$-factor involves tilting of the sample relative to the magnetic field direction. The Landau splitting is only determined by the perpendicular component of magnetic field, $B_\perp$, whereas the spin splitting is proportional to the total magnetic field, $B$. Therefore in tilted magnetic fields, the spin splitting can be enlarged in comparison with the Landau level splitting, and can be determined with respect to the Landau splitting, using, for example, measurement of the corresponding change in SdH oscillations.

Another unsolved property of 2DEGs formed on (100) Si is the nature of the two-fold valley degeneracy. Several models [13,14] have been proposed to describe the occurrence of valley splitting, but agreement between theory and experiment remains unsatisfactory. The magnitude of this splitting is expected to increase with the electric field, and thus with the carrier density, within the quantum well [15], although the opposite has been observed experimentally in Si MOSFET structures. Köhler et al.[16-17] tried to determine the magnitude of the valley splitting, $\Delta E_v$, using a slightly different argument of the phase change of the SdH oscillation. They tilted the magnetic field and realised a situation $\hbar \omega_c / 4 \sim \Delta E_v$. From this method, they obtained $\Delta E_v \sim 0.69 \text{meV}$ at $B_\perp = 15.2T$, for an electron concentration of $n_e = 2.4 \times 10^{12} \text{cm}^{-2}$. Most theoretical calculations in the literature have only considered the bare
valley splitting [13,14,18], and predict values for the magnitude of $\Delta E_v$ that are much smaller than that are observed experimentally [16-17]. This situation strongly suggests that the valley splitting is also enhanced by exchange interactions in high magnetic fields as proposed by Ando et. al. [10].

In this chapter, a study of the $g^*$-factor is presented in a Si/Si$_{0.5}$Ge$_{0.5}$ heterostructures by the tilted field method, showing clear evidence for both enhancement and oscillatory behaviour. However, the value of the valley splitting can not be determined due to the limited mobility of our samples.

The remainder of this chapter is organised as follows. In section 5.2 the theoretical background is presented. In section 5.3, a description of the experimental method used to obtain the data is presented and the corresponding results are discussed in section 5.4. The conclusions are summarised in section 5.5.

5.2 Theoretical background

As discussed in previous section, the tilted field method allows separate control of spin and Landau level splitting, and this can be applied to measure the g-factor enhancement. It is also worth discussing the measurement conditions used in some detail, since these can be quite complex for a system with an oscillating $g^*$-factor. In tilted-field, the energy levels are given by

$$E = \left(n + \frac{1}{2}\right)\hbar \omega_c \pm \frac{1}{2}g^*\mu_B B = \left(n + \frac{1}{2}\right)\frac{\hbar e B}{m^*} \cos \theta \pm \frac{1}{2}g^*\mu_B B$$

(5.1)
Figure 5.1 A schematic view of the energy levels for Landau quantum number (N-1) and N.

where $\theta$ is the tilt angle. Making the simplifying assumption of a constant $g^*$-factor will lead to an evenly spaced ladder of levels whenever the condition

$$|g^* \mu_B B| = r \hbar \omega_c = r \frac{\hbar e B}{m} \cos \theta, \quad \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$$

is satisfied, where $r=1/2, 3/2, 5/2$ corresponds to a ladder of alternating spin levels, and $r=1, 2, 3, \ldots$ corresponds to a ladder with coincident spin-up and spin-down levels from different Landau level. These conditions are shown schematically in figure 5.1, where the energy levels are shown for a constant perpendicular field component and increasing total field, and hence spin splitting.

Since the resistivity values of the SdH oscillation minima are
dependent upon the energy spacing of the broadened Landau levels, the level spacing are approximately equal when the resistivity of adjacent minima are equal, assuming that change in Landau level half-width is small over field range involved. At this point equation (5.2) is described by the following formula [19]

\[
\frac{g^* \mu_B B_{\perp(n \pm 2)}}{\cos \theta_{n\pm 2}} = \frac{\hbar e B_{\perp(n)}}{m^*} - \frac{g^* \mu_B B_{\perp(n)}}{\cos \theta_{n\pm 2}}
\]  (5.3)

where \( B_{\perp(n)} \) is the perpendicular component of the magnetic field at the \( n \) SdH minima (Landau level occupancy, i.e. \( n=4, 8, \text{etc.} \)) and \( \theta_{n\pm 2} \) is the tilt angle at which the resistivities of the \( n \) and \( n \pm 2 \) minima are equal. Since, for constant carrier density,

\[
nB_{n=\nu} = (n \pm 2)B_{\nu=n \pm 2}
\]  (5.4)

equation (5.3) can be solved for \( g^* \) and simplified to

\[
g^* = \left( \frac{n \pm 2}{n \pm 1} \right) \left( \frac{m_0}{m^*} \right) \cos \theta_{n\pm n\pm 2}
\]  (5.5)

where the definition \( \mu_B = e\hbar/2m_0 \) has been used as mentioned chapter 2. Therefore, from (5.3), one can obtained the value of \( g^* \) by determining the tilt angle at which the resistivity of adjacent minima are equal [19].
5.3 Experimental Method

The tilted magnetic field experiments have been performed on BF314. The samples were mounted in the dilution refrigerator to perform magnetotransport measurements at temperatures down to 0.1K. At this temperature, the samples show an electron mobility as high as $\mu=35810\text{cm}^2/\text{Vs}$ with the carrier density $n_c=1.13\times10^{12}\text{cm}^{-2}$ as discussed chapter 4. The sample holder allowed the tilt angle $\theta$ between the magnetic field and the sample normal to be changed in the range 0-90°.

The magnetic field dependencies of the longitudinal resistivity $\rho_x$ and Hall resistivity $\rho_y$ were measured for different tilt angles at 0.1K in magnetic fields up to 15T. All measurements are performed using phase-locked measurement techniques, with a driving current of 50nA.

5.4 Results and discussion

In figure 5.2, data from the SdH and Hall measurements at $\theta=0^\circ$ are displayed. The large plateau, visible around $B=15T$, corresponds to a filling factor $\nu=3$, indicating the valley splitting. The plateau around $B=11.5T$ represents the first filled Landau level ($\nu=4$). Between 12T and 6T, $\rho_x$ exhibits three minima. At 9.1T and 6.54T the valley splitting at $\nu=5$ and at $\nu=7$ can be observed. Spin splitting is resolved at around 7.53T($\nu=6$). At lower fields, SdH oscillations can be observed as the spin degenerate Landau levels are depopulated, but higher-index valley degenerate Landau levels can be not resolved due to the limited' mobility in this sample. The spin splitting is observable down to $B=2.6T$, with the SdH oscillations themselves beginning at $B=0.8T$. 

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Figure 5.2 Longitudinal resistivity and Hall resistivity versus magnetic field $B(T=0.1K)$.

Filling factors indicated with arrow and numbers.

Typical $B_z$ dependencies of the longitudinal resistivity at different values of $\theta$ are presented in figure 5.3. For the traces at $\theta=65.1^\circ$, the minima corresponding to the integer filling factor become shallower, while the minima of the spin splitting get deeper. For the traces at $\theta=71.33^\circ$, the minima corresponding to the Landau splitting disappear. Instead, maxima are observable at filling factors $\nu=16, 20, 24\cdots$, due to the coincidence of the spin levels of adjacent Landau levels as shown figure 5.1(condition $r = 1$). As mentioned earlier, since the spin splitting is an isotropic effect, it is determined by the total magnetic field, $B$, while the Landau splitting depends only upon the perpendicular component of the field. The spin splitting thus becomes relatively more pronounced as the sample is rotated toward the higher magnetic field, as can be seen in figure 5.3. This fact may be used to
measure $g^*$, which can be analysed from the relative amplitudes of the oscillations and their angular dependence using equation (5.2) by $r = 1$, as first reported by Fang et al. [1].

Figure 5.3 Longitudinal resistivity $\rho_{xx}$ for different angles ($\theta = 71.33^\circ$, 65.1°, 53.29°, 42.33°, 31.87°, 0°) versus normal component of magnetic field $B_\perp$. The positions of some integer filling factors are indicated by the dotted lines.

As shown the figure 5.3, $\rho_{xx}$ at integer filing factor rises until the coincidence is reached at a certain angle, $\theta$. Since spin-and Landau-splitting have the same size at this angle [$g^* \mu_B B = \hbar \omega_c = \hbar e B \cos \theta / m^*$ from equation (5.2)], it is possible to extract the effective g-factor. Weitz et. al. [20] obtained the results from this condition that the $g^*$ stayed almost constant $28 \geq \nu \geq 16$ in high mobility Si/Si$_{0.75}$Ge$_{0.25}$ heterostructures.
Figure 5.4 The typical results of $\rho_x$ for $\theta=60.72^\circ$ and $50.1^\circ$ versus normal component of magnetic field $B_\perp$ between 3 and 6T. One can see that the resistivities of the adjacent minima $\nu=8 \leftrightarrow 10$ and $10 \leftrightarrow 12$, respectively, are equally resolved.
On the other hand, the relation in equation (5.5) can be used to determine the value of $g^*$, when the resistivity of minima are equal as discussed previous section. We observed equal minima of the adjacent resistivities for followed situation: 42.33°, 60.72°, 50.1°, 58°, 50.7°, and 57.1° for $v=6\leftrightarrow8$, $8\leftrightarrow10$, $10\leftrightarrow12$, $12\leftrightarrow14$, $14\leftrightarrow16$ and $16\leftrightarrow18$, respectively. Typical traces for two of these are shown in figure 5.4.

The effective $g$-factor has been measured as a function of the Landau level index for a fixed electron concentration at $T=0.1$K. As shown in figure 5.4, since $\rho_x$ at $v=10\leftrightarrow12$ are equal at the tilt angle of 50.1°, it is possible to extract the effective $g$ factor from equation (5.5). The effective $g$ factor obtained from such method is $3.07\pm0.01$. The resulting values of $g^*$ determined from equation (5.5), at which equal resistivity minimum for $v=6\leftrightarrow8$, $8\leftrightarrow10$, $10\leftrightarrow12$, $12\leftrightarrow14$, $14\leftrightarrow16$ and $16\leftrightarrow18$, occur, are plotted in figure 5.5.

The $g^*$ from the data is seen to oscillate, with greater variations as $v$ is reduced. The values of $g^*=3.34\pm0.05$ for $v=6\leftrightarrow8$, decreases to $2.86\pm0.06$ for $v=8\leftrightarrow10$, and then increases again to $g^*=3.07\pm0.01$ for $v=10\leftrightarrow12$. The $g^*$ below $v=6\leftrightarrow8$ can not be observed because of the limited field range of magnetic field or lower mobility. It should be the effective $g^*$-factors obtained by our method are average values for the two Landau levels $g^*$-factor. However, it does not contradict the behaviour of an oscillatory $g$-factor because the value of $g_{n=\pm2}^*$ determined by this method is a weighted average of $g^*$-factors for $v=n$ and $v=n\pm2$. For instance, if we assume that each minimum $v=n$ in the SdH characteristic has a certain value $g^*$ associated with it, for example $g^*_8 = g_{v=8}^*, g^*_{10} = g_{v=10}^*$, etc, then the $g$-factor determined from equation (5.5) is equal to [19]
Figure 5.5 Plot of effective g-factor for various Landau level coincidences on BF314.

\[
g^{*}_{n \pm \frac{2}{2}} = \frac{1}{2} \left[ \left( \frac{n \pm 2}{n \pm 1} \right) g^{*}_{n} + \left( \frac{n}{n \pm 1} \right) g^{*}_{n \pm 2} \right]
\]

(5.6)

Now, for example, considering the case \( \nu = 10 \leftrightarrow 12 \) and \( \nu = 12 \leftrightarrow 14 \), the following result can be obtained

\[
g^{*}_{8 \leftrightarrow 10} = \frac{1}{9} \left[ 5g^{*}_{8} + 4g^{*}_{10} \right], \quad g^{*}_{10 \leftrightarrow 12} = \frac{1}{11} \left[ 6g^{*}_{10} + 5g^{*}_{12} \right]
\]

(5.7)

From equation (5.7) it is clear that \( g^{*}_{8 \leftrightarrow 10} \) is weighted toward \( g^{*}_{8} \), while \( g^{*}_{10 \leftrightarrow 12} \) is weighted toward \( g^{*}_{10} \). Since the Fermi level lies between two spin split states of the same Landau level at \( \nu = 10 \), \( g^{*}_{10} \) is expected to be larger than
$g^*_s$ due to the exchange interaction as discussed section 5.1. From the experimental results the above relations are satisfied since $g^*_{10=12}=3.07$, while $g^*_{8=10}=2.86$. From equation (5.6), assuming a value for the minimum $g^*$ at $\nu=4,8,12$ etc., it is possible to evaluate the maximum values for $\nu=6,10,14$ etc.. This is done in table 5.1 for a) the bare $g^*$ value of 2, and b) values estimated from the activation analysis in chapter 6.

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<th>$g^*_{\text{min}=2.8}$</th>
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<tr>
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<td>4.08</td>
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</table>

Table 5.1 Calculated values of the maximum $g^*$-factor under the assumption of the constant minimum $g^*$-factor.

Table 5.1 shows the maximum $g^*$ calculated under the assumption of constant minimum $g^*$-factor. Hence, the oscillatory features observed in this sample agree with the theoretical prediction [10].

If the valley splitting is significant in this field range, one cannot obtain an effective $g$ factor from a study of the angular dependence of $\rho_{xx}$ near coincidence in this magnetic field range. However, level widths of spin splitting levels estimated from equation (5.2) are typically $\sim1.75\text{meV}$ at $B=7.6\text{T}$, which is much larger than the valley splitting calculated theoretically, i.e. Ohkawa et.al.[21] determined values of $\Delta E_v=0.2(n_e/10^{12})\text{meV}$ for Si inversion layer giving $0.2\text{meV}$ for our sample. Thus, it can be expected that the influence of the valley splitting can be neglected.
5.5 Conclusion

In summary, SdH oscillations corresponding to the spin-split Landau levels of a 2DEG in \(	ext{Si}_{0.7}\text{Ge}_{0.3}/\text{Si}\) heterostructure have been studied with magnetic fields of up to 15T. Unfortunately, the information about the valley splitting cannot be obtained by our experiment, because of the limits set the mobility and the magnetic field. The experiment used to obtain the effective \(g\)-factor employed the method of the tilt magnetic field, where a tilt angle that causes adjacent SdH minima to be equal is used to determine the \(g^*\). The effective \(g\) factor for \(\nu=10 \leftrightarrow 12\) is found to be \(3.07 \pm 0.01\). The results indicated that the effective \(g\) factor oscillates as a function of the filling with greater variations as \(\nu\) is reduced. The values of \(g^*\) obtained from this method are weighted average values, and thus the actual individual oscillatory behaviour is likely to be much stronger. The oscillatory behaviour of the \(g^*\) from the data exist throughout the whole range of the filling factor, with greater variations as \(\nu\) is reduced.
References

Chapter 6
Electron conduction within Landau level tails

6.1 Introduction

It was first demonstrated by von Klitzing et al. [1] that a two-dimensional electron gas in a Si MOSFET at low temperatures and in high magnetic fields perpendicular to the plane could exhibit very precise quantisation of the Hall resistivity at integer filling factors as discussed in chapter 2.4.2. The essence of the integer quantum Hall effect (IQHE) is that the quantisation $\rho_{xy} = \frac{h}{ne^2}$ at a filling factor $\nu$ exists for a wide range of the magnetic field, and of the carrier density $n_e$. Accuracy of ~1 part in $10^8$ has been obtained for samples with different electron and impurity concentrations, despite the effect of electron-electron interactions and random impurity potentials. At the same time the longitudinal resistivity $\rho_{xx}$ displayed a sharp minimum. The density of states of the two-dimensional electron gas becomes completely discrete in the presence of a strong magnetic field perpendicular to the surface. Prange [2], Laughlin [3], and Halperin [4] have argued that as long as the Fermi level lies in the region of localised states between two current carrying regions of extended states, the Hall conductivity $\sigma_y$ is quantised and $\sigma_x$ vanished. Under these conditions, the measured value of $\rho_{xy}$ approaches the universal value $\left(\frac{h}{ne^2}\right)$ as the temperature is lowered. One of the mechanisms responsible for the appearance of dissipative longitudinal conductivity in the 2DEG, arising as a result of localisation of electrons, is the thermal activation of electrons into the extended electronic states.
Measurements of the thermal activation dependence of the dissipative conductivity $\sigma_\text{xx}$ are widely employed for determining (a) the difference, $\Delta$, between the mobility edge and the Fermi level, and (b) the density of states of Landau levels [5-7]. Energy gaps, $2\Delta$, between adjacent Landau levels can be observed if the level broadening is smaller than the energy separation. With decreasing temperature, the electrical conductivity of doped semiconductors depends on the density of states close to the Fermi energy and the radius of localisation of the charge carriers bound to impurity states ("Bohr radius"). At very high doping levels the large overlap of the wavefunctions leads to impurity-band conduction and results in a finite conductivity at $T=0\text{K}$: the system has become metallic. Below the metal-insulator-transition, however, the carriers are localised at the doping sites and charge transport takes place by hopping conduction. This is a thermally activated tunnelling process and can occur over a range of distances between doping sites; it is therefore known as "variable-range hopping" (VRH). In this chapter, the temperature-dependent conductivity $\sigma_\text{xx}$ is investigated in the temperature range from $\sim50\text{mK}$ to $4.2\text{K}$. Above $1\text{K}$, $\sigma_\text{xx}$ followed a thermally activated temperature dependence, while below $1\text{K}$, $\sigma_\text{xx}$ can be analysed by hopping conduction theory.

The subsequent sections are organised as follows: the experimental conditions are presented in section 6.2. The theory, the experimental results and discussions of results are divided into three parts. Firstly, the theory and the important results from thermally activated transport measurements are discussed in section 6.3. Secondly, section 6.4 gives a discussion about the density of states between the Landau levels. Thirdly, the theory and the experimental results of the hopping conduction are covered in section 6.5. Finally, conclusions are presented in section 6.6.
6.2. Experiments

Two different samples have been employed in this study. All samples are grown on top of a "relaxed" virtual substrate as discussed in chapter 4 at a temperature of 520°C, and patterned into a Hall bar geometry (400×80μm²). The details of the layers are discussed in chapter 4.2. Sample BF213 had following basic parameters: \( n_s(93\text{mK})=7.02\times10^{11}\text{cm}^{-2}; \mu(93\text{mK})=69700\text{cm}^2/\text{Vs} \). The other sample denoted here as BF314 has been characterised by the values: \( n_s(50\text{mK})=1.13\times10^{12}\text{cm}^{-2}; \mu(50\text{mK})=37080\text{cm}^2/\text{Vs} \).

Both components, \( \rho_{xx}(T) \) and \( \rho_{xy}(T) \), of the resistivity tensor have been simultaneously measured at temperatures between 4.0K and 50mK using standard low frequency ac transport techniques, at a frequency of around 10Hz and in perpendicular fields of up to 15T. \( \rho_{xx}(T) \) is measured within the

\[\begin{align*}
\rho_{xx}[\Omega] \\
0 & 50 \\
100 & 150 \\
200 & 250
\end{align*}\]

\[\begin{align*}
0.95\text{mV/cm} & \quad \text{dotted line} \\
1.9\text{mV/cm} & \quad \text{dash-dotted line} \\
19\text{mV/cm} & \quad \text{dashed line} \\
95\text{mV/cm} & \quad \text{solid line}
\end{align*}\]

**Figure 6.1** SdH oscillations \( \rho_{xx}(B) \) at \( T=0.1\text{K} \) for different bias currents (sample BF314)
temperature range 50mK-1.5K (in a dilution refrigerator) and 1.6-4.2K (in a variable temperature insert cryostat), at the High Magnetic Field facility in Grenoble.

In order to make sure that the temperature-dependent experiments were not influenced by electron heating effects, the measurement of SdH oscillations, by varying the bias current, has been performed at the lowest possible temperature. Figure 5.1 shows such SdH traces of $\rho_{xx}(B)$ for BF314 with current levels $50\text{nA} \leq I \leq 5\mu\text{A}$ at the temperature of 0.1K. A clear decrease in the SdH peak intensity is observed with increasing current. While the oscillations strongly depend on $I$ at high current levels, they saturate below about $I \approx 100\text{nA}$. The conductivity $\sigma_{xx}(T)$ can be calculated by using the expression

$$
\sigma_{xx}(T) = \frac{\rho_{xx}(T)}{\rho_{xx}^2(T) + \rho_{xy}^2(T)}
$$

(6.1)

6.3. Thermal activation energy

6.3.1 Theoretical background

Transport measurements in the quantum Hall regime have been widely used to investigate the fundamental physics of electron conduction in the case of a quantised two-dimensional electron gas in strong perpendicular magnetic fields. It is generally accepted that the integer quantum Hall effect (IQHE) can be understood in terms of localised and extended single electron states as discussed in chapter 2.4.2. While the latter form very narrow energy
bands of width $\Gamma_c$ centred at Landau-level energies $E_N$, the former fills the mobility gap of width $(\hbar \omega_c - \Gamma_c)$ as shown in figure 6.2 and do not contribute to the longitudinal conductivity $\sigma_{xx}$ at least in the limit $T \to 0$. If the energy separation between the Landau levels (LL's), i.e., the cyclotron energy $\hbar \omega_c$, is much larger than the LL linewidth, all electrons within the LL tails are considered to be localised. Such a picture has been supported by the measurement of thermally activated electron transport in two-dimensional electron gas (2DEG) structures in strong magnetic fields, corresponding to an integer filling factor $\nu$, i.e. to the centre of a mobility gap. When the Fermi level lies in the middle between two Landau levels, $\sigma_{xx}$ has the form [8-10], provided the temperature $T$ is not too low

$$\sigma_{xx} = \sigma_0 \exp\left(-\Delta / k_B T\right)$$  \hspace{1cm} (6.2)
with the activation energy $\Delta = h\omega_c/2$ for the ideal case. The prefactor $\sigma_0$ has attracted a great deal of interest since it has been found from the investigation of high-mobility heterostructures in GaAs/AlGaAs [10,11] that $\sigma_0$ is equal to $e^2/h$ independent of the sample and the filling factor. Although good fits to equation (6.2) in a finite temperature interval have been reported [12-15], the universality of the prefactor $\sigma_0$ has been disputed both experimentally [13] and theoretically [14]. Recently, Polyakov and Shklovskii [15] have reported that $\sigma_0$ in equation (6.2) can be derived from percolation theory in the limit of a long-range impurity potential, which is typical for high mobility modulation-doped GaAs/AlGaAs heterostructures, and predicted that it is just in the centre of a mobility gap that this prefactor equals $2e^2/h$, i.e. it is twice as large as that found experimentally by Clark [10,11]. Dorozhkin et al. [7] found experimentally that the prefactor $\sigma_0$ was changed from $2e^2/h$ to $e^2/h$. Dunford et al.[16] have found in an experiment using Al$_{0.2}$Ga$_{0.8}$As/In$_{0.2}$Ga$_{0.8}$/GaAs quantum wells that $\sigma_0$ was increased continuously with decreasing magnetic field.

6.3.2 Results and discussions

Figure 6.3 shows the magneto-conductivity of BF213 as a function of magnetic field at three different temperatures. The oscillation minima are marked with the corresponding filling factor $\nu$. Spin splittings are resolved for $\nu=2$ at temperatures between 1.6K and 4.0K, but for $\nu=6$ they can only be resolved only at 1.6K. The valley splittings, however, are not resolved in this temperature region. In figure 6.4, the graph of $\sigma_{xx}$ measurements for BF314 is shown.
Figure 6.3 Calculated $\sigma_{xx}$ using equation (6.1) from experimental recording of Shubnikov-de Haas oscillation and the quantum Hall effect between 1.6K and 4.0K for BF213.

Figure 6.4 Magneto-conductivity $\sigma_{xx}$ between 1.6K and 4.0K for BF314.
Although the curves for both samples are very similar, BF314 displays $\nu=4$ at a higher field due to its higher carrier concentration than BF213 (see chapter 4.4.3). Spin splittings can be resolved for $\nu=6,10$, but valley splitting at $\nu=5$ is resolved only at 1.6K.

<table>
<thead>
<tr>
<th>Filling factor $\nu$</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B(T)$</td>
<td>7.10</td>
<td>3.58</td>
<td>2.38</td>
<td>1.82</td>
</tr>
<tr>
<td>$\hbar\omega_c/2$ (meV)</td>
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<td>1.10</td>
<td>0.72</td>
<td>0.56</td>
</tr>
<tr>
<td>$\Delta$ (meV)</td>
<td>0.90</td>
<td>0.45</td>
<td>0.20</td>
<td>0.23</td>
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</tbody>
</table>

(a) BF213

<table>
<thead>
<tr>
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<td>$\Delta$ (meV)</td>
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<td>0.65</td>
<td>0.36</td>
<td>0.24</td>
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</tbody>
</table>

(b) BF314

**Table 6.1** Extrapolated values of $\Delta$ from figures 5.3 and 5.4 for BF213 and BF314.

From the temperature dependence of magneto-conductivity $\sigma_{xx}(T)$ obtained by equation (6.1), the activation energy $\Delta$ can be extracted from the linear part of an Arrhenius of the data points. In all cases, $\sigma_{xx}$ has been measured at the symmetry point between adjacent levels where the density of states $D(E)$ is assumed to have a minimum (i.e. where $dD(E)/dE=0$).

Figure 6.5 shows the experimental results of $\sigma_{xx}(T)$ at $\nu=4,8,12,$ and 16 in standard Arrhenius plots for BF213 and BF314. All the plots have substantial straight-line regions, but curve away from the straight-line at low temperature.
Figure 6.5 Detailed temperature dependence of $\sigma_{xx}$ for the integer filling factor between 4 and 16 of BF213 and BF314. The solid lines are fits to data using equation (5.2).
The least-squares fit to the expression $\sigma_{xx}(T) = \sigma_0 \exp(-\Delta/k_BT)$ is carried out between 4.2K and 1.5K, and the expected exponential behaviour has been observed. These $\sigma_{xx}$ values at integer filling factors between Landau levels 1-2, 2-3, 3-4, and 4-5 obtained from figure 6.5 are summarised in Table 6.1. The $\Delta$ is equal to the cyclotron energy, $\hbar\omega_c/2$, for an idealised system. However, it is usually smaller due to spin-splitting, valley-splitting, and a finite linewidth of the extended states. Hence, in general, the condition, $\Delta \sim \frac{1}{2} (\hbar\omega_c - \Gamma_v - g^*\mu_B - \Delta E_u)$ (assuming $\Delta E_u = 0.2(n_e/10^{12})$meV is valley splitting in Si inversion layer[17]), is expected, where $\mu_B$ is the Bohr magneton, $V_N$ is the energy of valley splitting and $\Gamma_v$ is the width of the region of extended states (see figure 6.2). The width of region of extended states at integer filling factor $\nu=8$ and 12 for BF314 are smaller than $\Gamma$ which characterises the total broadening of Landau as shown the table 6.2, using the enhancement $g$-factor which is obtained in chapter 5.

<table>
<thead>
<tr>
<th>Filling factor $\nu$</th>
<th>$\Gamma_v^{(1)}$(meV)</th>
<th>$\Gamma_v^{(2)}$(meV)</th>
<th>$\Gamma_v^{(3)}$(meV)</th>
<th>$\Gamma_v^{(4)}$(meV)</th>
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<tr>
<td>8</td>
<td>0.99</td>
<td>1.29</td>
<td>1.12</td>
<td>1.59</td>
</tr>
<tr>
<td>12</td>
<td>0.71</td>
<td>0.95</td>
<td>1.12</td>
<td>1.07</td>
</tr>
</tbody>
</table>

(1) using the enhancement $g$-factor (see chapter 5): $g^*_{8\nu=10}=2.86$ for $\nu=8$, $g^*_{12\nu=14}=3.00$ for $\nu=12$
(2) using the bare $g$-factor
(3) using the broadening of the Landau levels by the uncertainty principles
(4) using $\Gamma$ by the self-consistent Born approximation

Table 6.2 Comparison with the width of the region of extended states and the Landau level broadening (the full width at half maximum) for BF314.
However, if we used the bare g-factor, we obtained the unreasonable value of the width of extended states at \( \nu=8 \), i.e. it is larger than the width of the broadened Landau level evaluated from uncertainty principle. Therefore, such a reduction in the activation energy has been attributed to the fact that the Landé g-factor should be considerably larger than the bulk value \( g=2 \)[18], and could be originated from the broadened Landau levels.

Calculations using equation (6.2) provides not only the energy gap \( \Delta \) but also the value of the pre-exponential factor \( \sigma_\theta \). The pre-exponential factor obtained from the Arrhenius plots are \( \sigma_\theta=1.2\sim 2e^2/h \) between \( \nu=4 \) and \( \nu=16 \) for BF213 and \( 1.7e^2/h, 1.9e^2/h \) for BF314 at \( \nu=4 \) and \( \nu=8 \). Therefore, the values of prefactor at \( 4 \geq \nu \) for both samples are markedly higher than \( e^2/h \). Other experiments on GaAs/Al\(_x\)Ga\(_{1-x}\)As heterostructures have reported that the values of \( \sigma_\theta \) deduced from Arrhenius plots showed considerable scatter. Usher et al.[12] have obtained larger values of \( \sigma_\theta \) of up to \( 2e^2/h \) at \( \nu=2 \). For a p-type GaAs/In\(_{0.2}\)Ga\(_{0.8}\)As/GaAs quantum well, \( \sigma_\theta \) has been found to be \( \sim 2e^2/h \) for \( \nu=3\)-7[19]. Polyakov et al.[14] have reported theoretical calculations of the pre-exponential factor for activated conductivity as discussed in 6.3.1. These calculations predict that when \( \sigma_\theta \) is \( \sim 2e^2/h \), long-range scattering dominates and short-range scattering dominates when \( \sigma_\theta \) is \( \leq 1 \). Hence, for both samples, one can imply that the values of prefactor at \( \nu\geq4 \) should obviously be higher than \( e^2/h \). Therefore, our results for both samples seem to be better explained by the theory of Polyakov et al. [14,15] based on long-range scattering.

As demonstrated in figure 6.6, our samples show clearly that \( \sigma_\theta \) increases continuously with decreasing magnetic field. A similar result is reported for an Al\(_{0.2}\)Ga\(_{0.8}\)As/In\(_{0.2}\)Ga\(_{0.8}\)As/GaAs quantum well [16].
There is currently no established theoretical explanation for the observed increase of $\sigma_0$ at low magnetic fields. However, Dunford et al. [16] suggested that the results seemed to be consistent with the behaviour that would be expected from self-consistent Born approximation calculations.

### 6.4 Density of states between Landau levels

#### 6.4.1 Theoretical background

A microscopic theory of the quantum Hall effect should give a correct description not only of the quantised resistivity values but also of the transitions between the plateaux and the finite values of $\rho_{xx}$. The origin of
these plateaux is attributed to the position of the Fermi energy within a mobility gap. The density of localised states is unimportant for the discussion of transport data at zero temperature, but becomes significant if variable range hopping (which includes the density of states at the Fermi energy) plays a role.

![Diagram of density of states at two different magnetic fields](image)

**Figure 6.7** Model for density of states at two different magnetic fields used to explain the reconstruction of the activation energies $\Delta$ [23].

Moreover, all transport theories include information about the density of states (DOS) and the microscopic theory of the quantum Hall effect or the longitudinal resistivity should give a correct answer for the density of states. One of the first theories on the density of states (DOS) assumed short-range scattering. According to this theory [20], the energy spectrum of 2D electrons under a strong transverse magnetic field consists of a set of Landau levels of width as shown figure 6.7, which is represented by
This self-consistent Born-approximation (SCBA) result depends on $B$ and $\tau_q$, thus leading to a broadening of the discrete energy spectrum into an elliptic line-shape for the DOS. Therefore, the density of states in the gaps between the levels is found to be extremely low, while within the level, it is determined by the line-width $\Gamma_N$ derived in equation (6.3)[20]

\[ \Gamma_N = \hbar \left( \frac{2 \omega_c}{\pi \tau_q} \right)^{1/2} \]  

(6.3)

\[ D(E) = \frac{eB}{\hbar} \sum \left[ 1 - \left( \frac{E - E_N}{\Gamma_N} \right)^2 \right]^{1/2} \]  

(6.4)

where $E_N = (N + 1/2) \hbar \omega_c$ as discussed in chapter 2.4.2. In the absence of scatterers the density of states has a $\delta$-function peak at the position of each Landau level $E = E_N$.  

In various experiments carried out on GaAs-AlGaAs structures, involving an analysis of the heat capacity [21], the magnetisation [22], and the thermally activated magnetoconductivity [23], a common conclusion can be reached: The density of states of the gaps in the energy spectrum is approximately constant ($D = D_N$), and it amounts to a rather significant fraction of the zero-field density of states of 2D electrons, $D_0 = (g^* m^*/\pi \hbar^2)$ in case of Si ($m^*$ is the effective mass, and $g_*$ is the multiplicity of the valley degeneracy.).

The density of states can be directly determined by measuring the energy of the Fermi level as a function of the filling factor of a Landau level. The filling factor $\nu$ can be changed either by the carrier density or the magnetic field, and the position of the Fermi level can be deduced from an
analysis of the thermally activated conductivity (or resistivity) in the magnetic
field range of the Hall plateaux, i.e. the localised region.

The temperature dependence of \( \rho_{xx}^{\text{min.}} \) for 1~4.2K is usually dominated
by an exponential term corresponding to

\[
\rho_{xx}^{\text{min.}} \propto \sigma_{xx}^{\text{min.}} \propto \exp(-\Delta/kT)
\]  

(6.5)

The data can be analysed by using a model [24] in which the
temperature dependence of the resistivity in the plateau region (not only at
integer filling factor) is described by an expression

\[
\rho_{xx} \propto \exp\left(-\frac{|E_N - E_F|}{kT}\right)
\]  

(6.6)

This forms the basis for the determination of the Fermi level positions as a
function of the filling factor, and for the direct measurement of the density of
localised states.

Changing the position of a Landau level \( E_N \) relative to the Fermi level
\( E_F \) (by changing the magnetic field) results in reduced activation energy
\( \Delta = |E_N - E_F| \). The change in the activation energy, which corresponds to the
change in the filling factors, can be clearly seen from the experimental data
figure 6.8. A shift of the Fermi levels, equivalent to a change in carrier density
\( \Delta n \) plotted in a fixed field, induces a change in the filling factor, from which a
mean value for density of states can be deduced. This better illustrated in
figure 6.7 where a model for the DOS at two different magnetic fields, \( B_2 > B_1 \), is
shown. If the magnetic field is raised from $B_1$ to $B_2$, the Fermi energy is shifted from $E_F(B_1)$ to the lower energy position $E_F(B_2)$ assuming that the carrier density is maintained at a constant value. The variation $\Delta E$ of the Fermi energy corresponds approximately (if the Fermi energy is located in the tails of the Landau levels between $N=0$ and $N=1$) to a change in the carrier density of

$$\Delta n \approx \frac{e g_s g_v}{h} (B_2 - B_1) \quad (6.7)$$

The energy difference between the two Fermi level positions due to a change in the field from $B_1$ to $B_2$ is given by

$$\Delta E = \Delta(B_1) - \Delta(B_2) - \frac{\hbar}{2} (\omega_{c2} - \omega_{c1}) \quad (6.8)$$

where $\Delta(B_1)$ and $\Delta(B_2)$ are the activation energy at magnetic field $B_1$ and $B_2$, respectively. Therefore, the density of states ($D(E) = \Delta n / \Delta E$) can be estimated from

$$D(E) \approx \frac{g_s g_v}{h} \frac{B_1 - B_2}{(\Delta(B_1) - \Delta(B_2)) - (e\hbar/2m^*) (B_2 - B_1)} \quad (6.9)$$

6.4.2 Results and discussion

The results in this section are mainly concerned with BF213. The motion of the Landau levels relative to the Fermi level, and corresponding reduction in the activation energy ($\Delta = |E_N - E_F|$), is clearly visible in figure 6.8, as the filling factor of the Landau level is varied. The activation energy from $B=7.1T$ ($v=4$) to $B=7.2T$ ($v=3.94$) is reduced by about $\Delta E=0.17\text{meV}$.
Figure 6.8 Temperature dependence of the conductivity $\sigma_{xx}$ of a BF213 at different magnetic field values close to a filling factor $v=4$. The insert shows magnetoconductivity at $v=4$ as a function of $B$ at different temperatures.

A filling factor change of 0.06 at this magnetic field represents the carrier density variation of $\Delta n_e=1.13\times10^{10}\text{cm}^{-2}$. As previously mentioned in section 6.4.1, the density of states in the gap between the Landau level can be obtained from $D=\Delta n_e/\Delta E$. Figure 6.9 shows a plot of the density of states versus the energy in the gap between the Landau levels of these 2D electrons.

Although this approach is restricted to the tails of the Landau levels, a
mean density of states can at least be roughly estimated from the activation energy near the middle of the mobility gap. The experimental result gave shown that $D(E_F) \leq 6.3 \times 10^{10}$ meV$^2$cm$^{-2}$, i.e. about one third of the zero field two-dimensional density of state, within the interval of $\Delta E \approx 7$K around the centre of the mobility gap. Similar curves of $D(E_F)$ were found for $v=8$.

Measurements at the maximum of the density of states are not possible because the position of the Fermi level can be determined only if it is located well below the mobility edge and if complication due to spin and valley splitting are absent.

In summary, on the basis of the analysis of the thermally activated magneto-resistivity of 2D electrons, the experimental data have been shown that the density of states is large and almost constant in the gaps between the Landau levels, suited to previous derivation in GaAs/AlGaAs 2DEGs[23].

![Density of states as a function of energy for BF213](image)

**Figure 6.9** Density of states as a function of energy for BF213. $\Delta E=0$ corresponds to the centre ($v=4$) between two Landau levels. Here $E_m$ indicates mobility edges, determined from the activation energy analysis.
6.5 Hopping conduction in the Landau level tails

6.5.1 Theoretical background

With decreasing temperature, deviations from the linear relation of equation (6.2) can occur. In this low temperature regime, an electron conduction mechanism known as variable-range hopping (VRH) becomes the dominant transport process [25]. The theory for the conductivity in a strong magnetic field within the hopping regime is a fairly straightforward extension to Mott’s original derivation [25]. According to the Mott, for two dimensions the conductivity is given by

\[ \sigma_x \propto \exp(-T_0/T)^{1/3} \]  \hspace{1cm} (6.10)

where

\[ T_0 = \left(27d^2\right)/\pi D(E_F)k \]  \hspace{1cm} (6.11)

with \( d \) is derived from the exponentially decaying wave function of the localised electronic states, \( \psi \propto \exp(-dr) \) where \( r \) is the distance from the impurity.

The form assumed for the localised wavefunction strongly influences the predicted behaviour. Ono [26] has calculated the contribution of the hopping conduction in the Landau-level tails under the assumption that the magnetic field causes a Gaussian localisation of the electron wavefunction on a scale given by the magnetic length \( \ell = \sqrt{\frac{h}{eB}} \), where the electronic wavefunction, \( \psi \propto \exp\left(-r^2/2\ell^2\right) \), is localised around a single maximum or minimum of the impurity potential. His calculation results in
\[
\sigma_{xx} = \frac{e^2}{k_B T} \gamma_0 \exp\left[-(T_0 / T)^{1/2}\right]
\]

(6.12)

where \( \gamma_0 \) is a material parameter, which depends on the electron-phonon coupling and \( T_0 \) is a critical temperature given by

\[
T_0 = \frac{c}{k_B D(E_F) e^2}
\]

(6.13)

with \( c \) being the percolation constant of the order of unity. Several research groups [27,28] have experimentally found that although the results fitted from equation (6.12) agreed well at low temperatures, the density of states obtained from the calculated critical temperatures \( T_0 \) on the basis of equation (6.13) became unrealistically high. Briggs et al. [28] reported values of \( D(E_F) \) a factor 70 higher than the zero-field values, \( D_0(E) = m^*/\pi\hbar^2 \), in \( \text{In}_{x}\text{Ga}_{1-x}\text{As/InP} \) heterojunctions. On the other hand, the conductivity \( \sigma_{xx} \propto \exp\left( T_0 / T \right)^{1/2} \) has been found in Si-inversion layers by Nicholas et al.[29], and in \( \text{GaAs/Al}_x\text{Ga}_{1-x}\text{As} \) heterostructures by Störmer et al. [30].

The theory by Ono [26] leading to equation (6.12) assumes a finite density of states at \( E_F \) and unperturbed wavefunctions of isolated impurities in the form \( \psi \propto \exp\left(-r^2 / 2\xi^2\right) \). This has been criticised by Polyakov et al. [31,32] who provide another expression for the hopping conduction which relies on the existence of a Coulomb gap at \( E_F \) predicted by Efros and Shklovskii [33]. They assume that tail of the wavefunction has a simple exponential form \( \psi \propto \exp\left(-r / \xi \right) \) due to multiple scattering of a tunnelling electron [32].

Assuming that the experimentally observed variable-range-hopping (VRH) at integer filling factors and temperatures below 1K remains the
dominant process even in the vicinity of the conductivity peak, they describe
the low-temperature conductivity at the particular filling factors by the law

$$\sigma_{xx} = \sigma_0 \exp\left(-\frac{T_1}{T}\right)^{1/2} \quad (6.14)$$

with a characteristic temperature $T_1$

$$T_1 = C \frac{e^2}{\varepsilon k_B \zeta(\nu)} \quad (6.15)$$

Here $C$ is a numerical constant (in two dimensions $C \approx 6$ [34]), $\varepsilon$ is the dielectric constant, and $\zeta(\nu)$ denotes the localisation length of the states at the Fermi energy for a given $\nu$.

The role of the prefactor $\sigma_0$ in equation (6.14) could not be clearly identified theoretically. However, previous experiments on hopping transport, measuring conductivity minima between Landau levels [27,28,33], found $\sigma_0 \propto 1/T$. Thus equation (6.14) is functionally similar to (6.12), but with a different expression for the characteristic temperature.

The following result provide an experimental comparison with the predictions by Mott et al. [25], Ono [26] and Polyakov et al. [31,32] in Si$_{0.7}$Ge$_{0.3}$/Si modulation doped quantum wells. In particular, the experimental determination of the conductivity $\sigma_{xx}$ in the peak region between adjacent integer quantum Hall plateaux has been carried out, and the results show that $\sigma_{xx}$ is best described by the Polyakov model[equation (6.14)].
6.5.2 Results and discussion

The typical results of the magneto-conductivity measurements on BF213 are shown in figure 6.10 as a function of magnetic field at three different temperatures: 88mK, 0.37K, and 0.9K. Spin splitting is resolved for \( v=2,6,10 \) designated with signs \( \uparrow, \downarrow \), whereas valley splitting is resolved for \( v=3,5 \) below 1K as discussed in chapter 4. A series of broad minima of \( \sigma_\alpha \) tend towards zero as the temperature is lowered.

First of all, in order to check Ono's and Mott's theory in our samples, the conductivity data at filling factors \( \nu=6,8,12 \), corresponding to magnetic fields of 4.37, 3.55, 2.38T for BF213 and 7.68, 5.78, and 3.86T for BF314, are fitted to the equation (6.10) and (6.12).
Figure 6.11 $\sigma_{xx}$ plotted against $T^{1/3}$ and $\sigma_{xx}T$ plotted against $T^{1/2}$ (according to equation (6.4) and (6.6)) for the conductivity minimum at $B=4.75T$ for BF213.

In all cases, $\sigma_{xx}$ has been measured at the conductivity minima point between the adjacent levels. Figure 6.11 shows typical results of the temperature dependence of $\sigma_{xx}$ from the filling factor $\nu=6$ at $B=4.75T$ for BF213, i.e. the spin-splitting oscillation minimum between Landau levels of index $N=2\downarrow$ and $N=2\uparrow$ as described in figure 6.10. The data varied in the temperature range between 88 and 900mK. The open circles in figure 6.11 indicate log $\sigma_{xx}$ against $T^{1/3}$. 
Figure 6.12 Analysis of the temperature-dependent conductivity for BF314 at v=8.

The straight line, expected on the basis of equation (6.10), could not be obtained from these results. On the other hand, the filled circles represent a plot of \(\log \sigma_x T\) against \(T^{1/2}\) for the same data. These values are well fitted by a straight line as expected from equation (6.12). The same analysis has been also applied to BF314, resulting in showed similar behaviour. Figure 6.12 shows the result of BF314.

Figure 6.13 shows the temperature dependence of \(\sigma_x\) at four conductivity minima for BF213, and BF314. It can be see that the data can be well fitted to the equation (6.12) over a wide range of magnetic field.
Figure 6.13 Temperature dependence of $\sigma_{xx}$ at the conductivity minima for $\nu=6,8,10,12$ for (a) BF213 and (b) BF314.
The critical temperatures \( T_0 \) obtained from the experimental results for three samples are summarised in table 6.3. It is apparent from table 6.3 that the sample’s \( T_0 \) values do not follow the magnetic field dependence of equation (6.12), i.e. \( T_0 \propto B \); similar results have been seen for In\(_x\)Ga\(_{1-x}\)As/InP heterojunctions [28].

<table>
<thead>
<tr>
<th>Filling factor [( \nu )]</th>
<th>( B [T] )</th>
<th>( T_0 [K] )</th>
<th>( B [T] )</th>
<th>( T_0 [K] )</th>
</tr>
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<tr>
<td>6</td>
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<tr>
<td>10</td>
<td>2.83</td>
<td>5.90</td>
<td>4.60</td>
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</tr>
<tr>
<td>12</td>
<td>2.38</td>
<td>6.41</td>
<td>3.85</td>
<td>8.46</td>
</tr>
</tbody>
</table>

**Table 6.3** \( T_0 \) values from the experimental results for both samples.

It has been suggested that this discrepancy could be explained by the variation of the density of states \( D(E_F) \) with \( B \) at the \( \sigma_{xx} \) minima. Guldner et al. [36] explained that when some \( \sigma_{xx} \) minima do not drop to zero (see figure 6.1), the overlap between the tails of the adjacent Landau levels becomes significant and, as a consequence, the corresponding \( T_0 \) values become small. From the values of \( T_0 \), the density of states can be deduced using equation (6.13), if the percolation constant \( c \) is known. Assuming the \( c \) to be \( \approx 1 \), the density of states is found to have a value at \( \nu = 8 \) of \( D(E_F) = 7.2 \times 10^{11} \) and \( 1.26 \times 10^{12} \) meV\(^{-1}\)cm\(^2\) for BF213 and BF314, respectively. These values are higher than the density of states without magnetic field, and quite unrealistic. This finding has also been found by other groups [27,28,35,36] in different materials, and thus raises the question of whether the expression for \( T_0 \) in equation (6.13) is correct, or whether other mechanisms influence the conductivity in the Landau level.
minima of 2DEG structures. Ebert et al. [27] have suggested that under low temperature conditions, as in our experiments, Coulomb effects (not included in the Ono theory) may play an important part, and appropriate corrections need to be implemented.

Another approach to describe variable-range hopping (VRH) conduction has been suggested by Polyakov and Shklovskii [31,32] as discussed in section 6.4.1. The equation (6.14) has been derived under the assumption that the experimentally observed VRH at integer filling factors and at temperatures below 1 K [35] remains the dominant transport process even in the vicinity of the conductivity peak. Although the question as to the nature of the localisation still remains unresolved, various computer simulations [37-39] strongly support the existence of localisation, thereby yielding a power-law divergence of the localisation length \( \xi \propto |\nu - \nu_c|^{\gamma} \). Here \( \nu \) is the filling factor and \( \nu_c \) is the critical filling factor. The value of \( \nu_c \) corresponds to the position where \( E_F \) coincides with the Landau level centre and \( \gamma \approx 2.3 \) is a critical exponent [40-43]. In a number of experiments [40, 44-46], a remarkable result has been shown, where \( \gamma \) can be obtained from the half-width \( \Delta B \) of the \( \sigma_{xx} \) peaks. The half-width \( \Delta B \) of the \( \sigma_{xx} \) peaks shrinks with decreasing temperature resulting in a power law relation [31],

\[
\Delta B \propto T^\kappa
\]

(6.18)

with \( \kappa = 1/\gamma \). Sample BF314 has been used to check experimentally the predictions of Polyakov et al.[31], by comparing the half-width analysis and the hopping conduction analysis of the conductivity \( \sigma_{xx} \) determined experimentally from the vicinity of the conductivity peak between adjacent
quantum Hall effect plateaux; similarly to the case using equation (6.12), the data throughout this field can indeed be described by equation (6.14) in a SiGe/Si system, as has been seen in the AlGaAs/GaAs system [47]. The magnetic field dependence of the characteristic temperature $T_i$ is analysed for the peak region between the filling factor $\nu=5$ and $\nu=6$, and followed the power law as expected by theory.

![Graph](image)

**Figure 6.14** Diagonal conductivity data $\sigma_{xx}$ for BF314 as a function of magnetic field $B$ at filling factors between $\nu=5$ and $\nu=6$ for various temperatures.
Figure 6.15 The half-width $\Delta B$ as a function of temperature for BF314 at filling factors between $\nu=5$ and 6.

Experimental results for the measured longitudinal conductivity $\sigma_{xx}$ are presented in figure 6.14 as a function of magnetic field $B$ at various temperatures for filling factors between $\nu=5$ and $\nu=6$. The half-width of the $\sigma_{xx}$ peak increases with increasing temperature. At low temperatures below 0.27K, the half-width saturates. This phenomenon is proposed by Koch et al. [45] that at these low temperatures the influence of the thermal broadening of the Landau level no longer plays a role. The solid line in figure 6.15 is a fit to the higher-temperature data, and corresponds to the temperature exponent $\kappa$ of $1.01 \pm 0.04$. 
Figure 6.16 $\sigma_x T$ as a function of temperature $T^{-1/2}$ according to $\sigma_x T \propto \exp[-(T_0/T)^{1/2}]$ at various magnetic field. 

The data shown in figure 6.16 has been analysed in terms of hopping transport according to equation (6.14). The diagonal conductivity $\sigma_x T$ is plotted as a function of $T^{-1/2}$ for the peak between the filling factors $\nu = 5$ and 6. The results calculated by equation (6.14) as indicated by the full lines compare well with the experimental data over the whole temperature range applied.

The characteristic temperature $T_i$ obtained from figure 6.16 is then plotted in figure 6.17, as a function of filling factor difference $\Delta \nu$ on a double-logarithmic plot, checking the already-cited prediction $T_i \propto (\Delta \nu)^{1/\kappa}$ [31].

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The results obtained from this plot show straight line corresponding to the power law $T_i \propto (\Delta \nu)^\beta$. The exponent $\beta$ found to be 0.9±0.07. The value of $\beta$ coincides with $1/\kappa$ obtained from half-width experiments, within the experimental error as predicted by [31]. However, the value of $\kappa$ is not equal to that given by the theory of Polyakov et al. [31].

They suggested that the temperature exponent $\kappa$ is solely characterised by the localisation length exponent $\gamma$, specifically $\gamma \approx 1/\kappa$. Our $1/\kappa$ values do not satisfy the Polyakov's condition (since $\gamma$ is a universal constant) and such
finding has been also reported by Koch et al.[47] who investigated AlGaAs/GaAs structures. Nevertheless, the essential features of the theory remain unaffected by this. The theory of hopping conduction [31] is applicable to the present sample but because of the difference between $\gamma$ and $\kappa$, it may have to be supplemented to some extent, by taking the actual nature of disorder into account.

6.6 Conclusion

The temperature dependence of the longitudinal conductivity $\sigma_{xx}$ has been studied in samples which show well-developed IQHE plateaux around the centres of the mobility gap and in the vicinity of the peaks between adjacent quantum Hall plateaux. A clear distinction between thermally activated transport and transport by hopping conduction can be made. In the former case, the longitudinal conductivity in our samples agreed well with the thermal activation dependence theory. The activation energies $\Delta$ obtained by fitting the data to equation (6.2) are found to be much smaller than the half width $\hbar \omega_c / 2$ of the interval between adjacent Landau levels. This reduction has been attributed to the fact that the Landé g-factor is expected to be considerably larger than the bulk value $g=2$ and that will also be some broadening of the Landau levels. The prefactor $\sigma_0$ in equation (6.2) should equal $\sim 1e^2/h$ at $\nu=4$ for BF213. However, the results indicate that $\sigma_0 \sim 2e^2/h$ at $\nu=4$ and $\nu=8$ for BF314, and shows clearly that the $\sigma_0$ increases continuously with an increase in the filling factor, similar the results on Al$_{0.2}$Ga$_{0.8}$As/In$_{0.2}$Ga$_{0.8}$As/GaAs obtained by Dunford et al. [16].

From an analysis of the thermally activated resistivity (or conductivity)
as a function of the magnetic field in the quantum Hall regime, the density of states within the mobility gap can be deduced. On the basis of this method, the experimental data have shown that the density of states appears to be large and almost constant in the gaps between the Landau levels.

At low temperatures (<1K), the temperature dependent longitudinal conductivity, \( \sigma_{xx} \), in the Landau level tails of our samples has been analysed according to Ono's and Polyakov's hopping conduction theory. We found that the temperature dependence of the longitudinal conductivity \( \sigma_{xx} \) in the Landau tails of our samples is in good agreement with Ono' theory, i.e. equation (12), which has been deduced of variable hopping on the basis of Gaussian localisation of the electron wavefunction. However, the magnetic field dependence of the characteristic temperature \( T_0 \) cannot be explained quantitatively. To solve this problem, Polyakov et al.[31,32] proposed a new hopping conduction theory. The conductivity \( \sigma_{xx} \) in the vicinity of the peaks between adjacent quantum Hall plateaux has been analysed according to Polyakov's theory, and the results have shown that the data can indeed be described by this hopping conduction theory. The magnetic field dependence of the characteristic temperature \( T_j \) is analysed for BF314, demonstrating the power low relation, \( T_j \propto (\Delta v)^{1/\kappa} \), as expected from theory. The experiment value agreed with that determined from half-width analysis of the \( \sigma_{xx} \) peaks, but was different to that predicted theoretically.
References


Chapter 7
Conclusions

This thesis has presented the results from studies on the electron transport properties of the two-dimensional electron gas in the Si$_{0.7}$Ge$_{0.3}$ modulation doped heterostructures system, grown by gas source molecular beam epitaxy (GSMBE). Magnetotransport measurements with magnetic fields of up to 15T at temperatures down to 0.05K have been performed to investigate the various phenomena in low temperature and high magnetic field.

Shubnikov-de Haas oscillations in the longitudinal resistivity, $\rho_{xx}$ of the samples, have been observed even at 4.2K, indicating the presence of a good quality 2DEG in these samples. These features were dramatically enhanced on cooling down to 100mK and below. All three samples gave rise to clear quantised plateaux in the Hall resistivity, $\rho_{yx}$, and dissipationless minima in $\rho_{xx}$ corresponding to integer, spin-split and valley-split Landau level filling. Close agreement between carrier density values deduced from the fundamental field of the SdH oscillations, $n_e$, and from the slope of the Hall resistivity, $n_{H}$, demonstrated that the presence of parallel conduction in the doped supply layer had completely frozen out at 0.1K.

The low temperature mobility values are not easily related to room temperature transport, since optical phonon scattering becomes significant above ~100K. Nevertheless, the magnitude can be taken into account, indicative of the overall material quality including such factors as the
background doping level in the Si channel, the interface roughness between this channel and the SiGe barriers, and possible Ge and As segregation effects. Although the mobility results obtained in this thesis do not exceed the level of over $10^5 \text{m}^2/\text{Vs}$, as reported by other groups, they are completely acceptable for device applications.

The ratio of transport to quantum relaxation times, $\tau_\text{t}/\tau_\text{q}$ [obtained from equation (4.22)] is found to be close to $\sim 7$-9 for all three samples, showing the expected dominance of small-angle remote impurity scattering. Once again, higher values ($\geq 10$) have been reported, suggesting that the level of large-angle background impurity scattering is slightly greater in these three samples.

To obtain the effective $g$-factor, tilted magnetic field experiments have been performed on BF314. The tilt angle that causes adjacent SdH minima to be equal has been employed to determine $g^*$. The effective $g$-factor for $\nu=6\leftrightarrow 8$ is found to be $3.34\pm0.05$, much higher than of the bulk. The results also have shown that the effective $g$ factor oscillates as a function of the Landau level.

The temperature dependence of the longitudinal $\sigma_{xx}$ has been analysed on samples with well-developed IQHE plateaux around the centres of the mobility gap and in the vicinity of the peaks between adjacent quantum Hall plateaux. A clear distinction between thermally activated transport and transport by hopping conduction has been made in our samples.

At temperature between $1\text{K}\leq T\leq 4.2\text{K}$, the longitudinal conductivity, $\sigma_{xx}$, has been analysed by the thermally activated transport, and the results agreed
well with the Arrhenius law, enabling the determination of the prefactor $\sigma_0$. The activation energy $\Delta$ obtained from experiments is much lower than the cyclotron energy, $\hbar\omega_c$. This reduction has been attributed to the fact that the Landé $g$-factor should be considerably larger than the bulk value of $g=2$, and could also originate from the broadening of the Landau levels. The prefactor $\sigma_0$ obtained from the Arrhenius law has been analysed. The prefactor $\sigma_0$ of $\sim 1e^2/h$ at $\nu=4$ for BF213 compared with the results reported by Clark et al. [1]. However, it is $\sim 2e^2/h$ at $\nu=4$ and $\nu=8$ for BF314 and increases continuously with an increase in the filling factor as do the results obtained by Dunford et al. [2].

From an analysis of the thermally activated conductivity (or resistivity) as function of the magnetic field in the quantum Hall regime, the density of states has been deduced between the mobility gap. Based on this method, the experimental data have shown that the density of states is large and almost constant in the gap between the Landau levels.

Below 1K, the temperature dependence of the longitudinal conductivity, $\sigma_{xx}$, in the Landau level tails, according to Ono's and Polyakov' hopping conduction theory has been investigated, the resulting behaviour agrees well with both theories, and this can be understood by the notion of hopping conduction through the electron wavefunction. However, for Ono's analysis which has been based on Gaussian localisation, there were difficulties in explaining quantitatively the magnitude and magnetic field dependence of the characteristic temperature $T_\theta$ obtained in our samples. Similar difficulties have been experienced by other groups.
Our samples have been studied using another method suggested by Polyakov and Shklovskii, who used exponential wavefunction. They have derived their method under assumption that the experimentally observed hopping conduction at integer filling factors at temperatures below 1K remains the dominant process even in the vicinity of the conductivity peak. Therefore, we have experimentally determined the conductivity \( \sigma_{xx} \) near the peaks between adjacent quantum Hall plateaux according to Polyakov' theory and have shown that our data can indeed be described by their method.

From the experimentally determined activation energy \( \Delta \) and enhancement Landé g-factor information about the linewidth of the extended states of Landau level is obtained. The finite width of the extended state region of each Landau level leads to anomalously low activation energy at whole magnetic field. These results suggest that the extended states occupy a region, of the width of a few Kelvins, at the centre of each Landau level. The observations of the hopping conduction in agreement with Ono's and Polyakov' prediction for the transport mechanism of localised electrons in the tails of Landau levels confirm that localised states in the tails of the broadened Landau levels should be involved to interpret the Hall plateaux as discussed in chapter 2.4.2.
References


Publications

Conference


Journals
