

# Is Gravity Actually the Curvature of Spacetime?

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## Abstract

The Einstein equations, apart from being the classical field equations of General Relativity, are also the classical field equations of two other theories of gravity. As the experimental tests of General Relativity are done using the Einstein equations, we do not really know, if gravity is the curvature of a torsionless spacetime, or torsion of a curvatureless spacetime, or if it occurs due to the non-metricity of a curvatureless and torsionless spacetime. However, as the classical actions of all these theories differ from each other by boundary terms, and the Casimir effect is a boundary effect, we propose that a novel gravitational Casimir effect between superconductors can be used to test which of these theories actually describe gravity.

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*General Relativity* (GR) is one of the most well tested theories in Nature, but in all those tests, what is actually tested are the predictions made by the Einstein equations [1]. It is possible to construct two other geometrical theories describing gravity, which are fundamentally different from GR, but whose classical field equations are the Einstein equations. To understand these theories, we first note that the spacetime has to be described by a differential manifold in any geometrical theory of gravity. Now a general affine connection  $\Gamma^\alpha_{\mu\nu}$ , on such a manifold, can be decomposed into three pieces [2, 3]

$$\Gamma^\alpha_{\mu\nu} = \left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\} + K^\alpha_{\mu\nu} + L^\alpha_{\mu\nu}. \quad (1)$$

The first term  $\left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\}$  is the standard *Levi-Civita connection*, which is obtained from the metric. The second term  $K^\alpha_{\mu\nu}$  is the *contortion tensor*, which is obtained from the torsion tensor  $T^\alpha_{\mu\nu}$  as  $K^\alpha_{\mu\nu} \equiv (1/2)T^\alpha_{\mu\nu} + T_{(\mu}{}^\alpha{}_{\nu)}$ . The last term  $L^\alpha_{\mu\nu}$  is the *disformation tensor*, which is constructed from the non-metricity tensor  $Q_{\alpha\mu\nu} \equiv \nabla_\alpha g_{\mu\nu}$  as  $L^\alpha_{\mu\nu} \equiv (1/2)Q^\alpha_{\mu\nu} - Q_{(\mu}{}^\alpha{}_{\nu)}$ .

GR is described by a torsionless spacetime ( $T^\alpha_{\mu\nu} = 0$ ), which satisfies the metric compatibility condition  $\nabla_\alpha g_{\mu\nu} = 0$  ( $Q_{\alpha\mu\nu} = 0$ ). So, as  $K^\alpha_{\mu\nu} = L^\alpha_{\mu\nu} = 0$  in GR, the affine connection of Eq. (1) can be written in terms of the Levi-Civita connection as  $\Gamma^\alpha_{\mu\nu} = \left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\}$ . The curvature tensor constructed from this Levi-Civita connection  $\bar{R}^\alpha_{\beta\mu\nu}$  is used to obtain the Einstein-Hilbert action

$$\mathcal{S}_G = \frac{1}{16\pi G} \int \sqrt{-g} \bar{R}, \quad (2)$$

where  $g$  is the determinant of the metric  $g_{\mu\nu}$  and  $\bar{R} \equiv g^{\beta\nu} \bar{R}^\alpha_{\beta\alpha\nu}$  is the scalar curvature obtained from  $\bar{R}^\alpha_{\beta\mu\nu}$ . Einstein equations are the classical field equations obtained from this action.

*Teleparallel Gravity* (TG) is another geometrical theory of gravity, whose classical field equations are the Einstein equations. In this theory, the general connection of Eq. (1) is equated to the *Weitzenböck connection*, and so the curvature of spacetime vanishes. Thus, TG is constructed using such a curvatureless spacetime, which satisfies the metric compatibility condition  $Q_{\alpha\mu\nu} = 0$  ( $L^\alpha_{\mu\nu} = 0$ ) [4–9]. This theory is constructed from the torsion tensor in the tetrad formalism, and its action is given by

$$\mathcal{S}_T = -\frac{1}{16\pi G} \int e T, \quad (3)$$

where  $e = \sqrt{-g}$  is the determinant of the tetrad and  $T$  is the scalar torsion (which is constructed from contractions of the torsion tensor).

It may be noted that in curvatureless spacetime of TG, the curvature tensor ( $R^\alpha{}_{\beta\mu\nu}$ ) obtained from the general affine connection of Eq. (1) vanishes, but the curvature tensor constructed using the Levi-Civita connection ( $\bar{R}^\alpha{}_{\beta\mu\nu}$ ) does not vanish. In TG, the scalar curvature  $R$  obtained from  $R^\alpha{}_{\beta\mu\nu}$  is related to the scalar curvature  $\bar{R}$  obtained from  $\bar{R}^\alpha{}_{\beta\mu\nu}$ , as  $R = \bar{R} + T - (2/e)\partial_\mu(eT^\lambda{}_\chi{}^\mu) = 0$ , so we can write

$$\bar{R} = -T + B_T, \quad (4)$$

where  $B_T = (2/e)\partial_\mu(eT^\lambda{}_\chi{}^\mu)$  is a boundary term. Thus, the action for GR given by Eq. (2) and the action for TG given by Eq. (3), differ from each other by the boundary term  $B_T$ .

It is also possible to formulate a *Theory of Non-Metricity* (TNM) to describe gravity [10–14]. This theory is also called Coincident General Relativity or Symmetric Teleparallel Gravity, as it has certain features which resemble both GR and TG, but we shall call it as TNM, as the theory is based on the concept of non-metricity. In this theory, both the torsion tensor and  $R^\lambda{}_{\mu\nu\beta}$  vanish, and gravity is produced because of the non-vanishing non-metricity tensor,  $\nabla_\alpha g_{\mu\nu} = Q_{\alpha\mu\nu} \neq 0$  ( $L_{\alpha\mu\nu} \neq 0$ ). The action for this theory is constructed using the non-metricity scalar  $Q$  (which is obtained from the non-metricity tensor  $Q_{\alpha\mu\nu}$ ) as

$$\mathcal{S}_N = -\frac{1}{16\pi G} \int \sqrt{-g} Q. \quad (5)$$

As TNM is described by a torsionless and curvatureless spacetime,  $Q$  can be related to  $\bar{R}$  (curvature obtained from the Levi-Civita connection) as

$$\bar{R} = -Q + B_N, \quad (6)$$

where  $B_N = (1/\sqrt{-g})\partial_\alpha(Q^\alpha{}_\lambda{}^\lambda - Q^\lambda{}_\lambda{}^\alpha)$  is again a boundary term (different from the boundary term obtained in TG). So, the action for GR given by Eq. (2) and the action for TNM given by Eq. (5) differ from each other by the boundary term  $B_N$ .

Even though the actions of GR, TG and TNM differ from each other by boundary terms, they have the same classical field equations (Einstein equations), so they cannot be classically distinguish from each other. The only reason for the preferential attention given to GR (over the other two geometrical theories) is historical and not scientific. However, they can be differentiated using quantum effects because these theories are fundamentally different from each other and will produce different quantum corrections. We do not have a full theory of quantum gravity, but it is possible to get an estimate of perturbative quantum gravitational

effects, using the formalism of effective field theories [15–17]. Thus, the classical actions for GR ( $\mathcal{S}_G$ ), TG ( $\mathcal{S}_T$ ) and TNM ( $\mathcal{S}_N$ ) get corrected by quantum corrections  $\mathcal{S}_{QG}$ ,  $\mathcal{S}_{QT}$  and  $\mathcal{S}_{QN}$ , such that

$$\mathcal{S}_1 = \mathcal{S}_G + \mathcal{S}_{QG}, \quad \mathcal{S}_2 = \mathcal{S}_T + \mathcal{S}_{QT}, \quad \mathcal{S}_3 = \mathcal{S}_N + \mathcal{S}_{QN}. \quad (7)$$

It is not possible to use cosmological and astrophysical observations to differentiate between  $\mathcal{S}_G$ ,  $\mathcal{S}_T$  and  $\mathcal{S}_N$ , however, such observations can differentiate between  $\mathcal{S}_1$ ,  $\mathcal{S}_2$  and  $\mathcal{S}_3$ . It has been demonstrated that the quantum corrected GR [16] and quantum corrected TG [17] are both consistent with the cosmological data obtained from SNe Ia + BAO + CC +  $H_0$  [18–21], and so at present, quantum corrections cannot rule out either of them. However, it is still possible that future cosmological observations may rule out one of these theories.

Even though, at present, we are not able to use quantum corrections to differentiate between these theories, it is still possible to use a combination of quantum effects and boundary effects to distinguish them from each other. As the actions of GR, TG, TNM differ from each other by boundary terms, and the Casimir effect is a quantum mechanical boundary effect, a gravitational Casimir effect can be used to distinguish them from each other. The reflection of gravitational waves in the microwave regime by quantum properties of superconductors (Heisenberg-Coulomb effect) [22–25] can produce a novel measurable gravitational Casimir effect [25–29]. In ordinary metal plates, the lattice of ions and electrons move along the same geodesic, in the presence of gravitational waves. However, when Cooper pairs form below the superconducting transition, they move along a non-geodesic path due to their quantum non-localizability. It has been demonstrated that this produces a large mass conductivity due to an enhanced mass current [25–29]. As the electromagnetic waves are reflected due to the electrical conductivity, this mass conductivity reflects gravitational waves [22–25]. Thus, for such systems, a gravitational Casimir effect can be produced [25–29], in analogy with the conventional electromagnetic Casimir effect [30–33].

As the actions for GR, TG and TNM are related to each other by boundary terms, we can relate the gravitational Casimir energy in GR ( $\langle E \rangle_G$ ) [25–29] to the gravitational Casimir energies in TG ( $\langle E \rangle_T$ ) and TNM ( $\langle E \rangle_N$ ) as

$$\langle E \rangle_T = \langle E \rangle_G + \langle E \rangle_{B_T}, \quad \langle E \rangle_N = \langle E \rangle_G + \langle E \rangle_{B_N}, \quad (8)$$

where  $\langle E \rangle_{B_T}$  is the contribution from the boundary term  $B_T$ , and  $\langle E \rangle_{B_N}$  is the contribution from boundary term  $B_N$ . Since the boundary action for these theories is different, so

$\langle E \rangle_{B_T} \neq \langle E \rangle_{B_N} \neq 0$ , thus we obtain  $\langle E \rangle_G \neq \langle E \rangle_T \neq \langle E \rangle_{B_N}$ . So, these theories will produce different gravitational Casimir effects, and such effects can be used to test which of these theories is actually the geometrical theory of gravity. It may be noted that the Casimir force between superconductors has been recently experimentally measured [25, 34–38]. Thus, it is possible to measure the novel gravitational Casimir effect due to the onset of superconductivity between two aluminum nanostrings. With an optomechanical cavity readout, these experiments could detect 6 *mPa* differences in the Casimir force between such nanostrings [25, 34–38]. The magnitude of a gravitational Casimir effect depends on the difference between the change in momentum of the Cooper pair and change in the momentum of the ion core [25–29]. Even if a more detailed analysis, reduced the magnitude of this novel gravitational Casimir effect by ten orders of magnitude, it would still remain a measurable effect, using the currently available technology. It is important to achieve sufficiently accurate parallelism between two superconductors at low temperatures to produce this novel gravitational Casimir effect. The technology needed to obtain such an accurate parallelism has already been used in resonator platforms for superconducting circuits [39, 40]. So, such an experiment can be performed using the currently available technology, and we can know which theory actually describes gravity in our Universe.

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