FREQUENCY DOMAIN FATIGUE ANALYSIS OF DYNAMICALLY SENSITIVE STRUCTURES

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Abstract

This thesis presents several developments in frequency domain fatigue analysis for dynamically sensitive structures. A sophisticated study is carried out for various factors which affect the prediction of fatigue damage using different spectral fatigue analysis tools. Applications were made to data from wind turbine blade WEG MS1, Howden HWP330 and other data to predict fatigue damage of the structure/component under different loading cases. A new frequency domain based theoretical approach for fatigue prediction has been developed which is applicable for general random, stationary signals. This overcomes the difficulties of assuming Gaussian, narrow band loading or applying correction factors in the existing spectral fatigue analysis tools. Effects of deterministic component (including harmonic and equally spaced spikes) and non-Gaussianity, which are ignored in existing frequency domain fatigue analysis tools, are studied. Extensive computer modeling and Artificial Neural Networks (ANN) have been applied to study the effect of non-Gaussianity. Analysis of WEG, HWP and other data suggested kurtosis as the best description of the degree of the departure from a Gaussian distribution. Several parameters were related to the rainflow cycle range p.d.f. of a non-Gaussian stress signal, these being the root mean square of the signal, the irregularity factor, mean frequency and kurtosis. Among various developed ANN systems, back-propagation, a method of training a neural network to approximate any function, including arbitrarily complex nonlinear functions, was chosen to study the effect of non-Gaussianity. Neural networks were trained to provide a fast, efficient toolbox for practical engineering applications.
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NOMENCLATURE

General

PSD \hspace{1em} \text{Power Spectral Density}
PDF, p.d.f., pdf \hspace{1em} \text{Probability Density Function}
DFT \hspace{1em} \text{Discrete Fourier Transform}
FFT \hspace{1em} \text{Fast Fourier Transform}
RAT \hspace{1em} \text{Reverse Arrangement Test}
FEA \hspace{1em} \text{Finite Element Analysis}
ANN \hspace{1em} \text{Artificial Neural Networks}
RMS, rms \hspace{1em} \text{root mean square}

\( S \) \hspace{1em} \text{stress cycle range}
\( S_0, S_c \) \hspace{1em} \text{transition stresses}
\( N \) \hspace{1em} \text{cycle number}
\( b, b_1, b_2 \) \hspace{1em} \text{inverse slopes of S-N curve}
\( c, c_1, c_2 \) \hspace{1em} \text{S-N curve intercepts}
\( \forall \) \hspace{1em} \text{for all}

Chapter 2

\( \Delta D \) \hspace{1em} \text{damage ratio}
\( D \) \hspace{1em} \text{accumulated fatigue damage}
\( E[P] \) \hspace{1em} \text{expected peak number in unit time (peak frequency)}
\( E[0] \) \hspace{1em} \text{expected zero-crossing number in unit time}
\( p(S) \) \hspace{1em} \text{probability density function of cycle ranges}
\( S_h \) \hspace{1em} \text{Equivalent Stress Parameter}
\( \omega \) \hspace{1em} \text{radius frequency}
\( S(\omega) \) \hspace{1em} \text{double-side (radius frequency) PSD}
\( G(f) \) \hspace{1em} \text{single-side Hertz PSD}
\( m_n \ (n=0,1,2...) \) \hspace{1em} \text{n-th moment of the one-sided Hertz PSD}
\( M_n \ (n=0,1,2...) \) \hspace{1em} \text{n-th moment of the double-sided radius PSD}
\( a \) \hspace{1em} \text{crack size}
\( a_0 \) \hspace{1em} \text{initial crack size}
\( C, m \) \hspace{1em} \text{crack growth constants}
\( \Delta K \) \hspace{1em} \text{stress intensity factor range}
\( F(a) \) \hspace{1em} \text{a geometry function}
\( X(t), y(t), z_a(t) \) \hspace{1em} \text{random processes}
### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$\tau$</td>
<td>time lag</td>
</tr>
<tr>
<td>$T$</td>
<td>time duration</td>
</tr>
<tr>
<td>$P(x)$</td>
<td>probability distribution of $x$</td>
</tr>
<tr>
<td>$p(x)$</td>
<td>probability density function of $x$</td>
</tr>
<tr>
<td>$\alpha_n$</td>
<td>$n$th order moment</td>
</tr>
<tr>
<td>$\mu_n$</td>
<td>$n$th order central moment</td>
</tr>
<tr>
<td>$\bar{x}$</td>
<td>mean value of random process</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>root mean square</td>
</tr>
<tr>
<td>$E{\cdot}$</td>
<td>mathematical expectation</td>
</tr>
<tr>
<td>$\phi(u)$</td>
<td>characteristic function</td>
</tr>
<tr>
<td>$R_{xy}$</td>
<td>cross-correlation function</td>
</tr>
<tr>
<td>$R_x$</td>
<td>autocorrelation function</td>
</tr>
<tr>
<td>$R_{xx}$</td>
<td>autocorrelation function</td>
</tr>
<tr>
<td>$a_n, b_n$</td>
<td>Fourier constants</td>
</tr>
<tr>
<td>$X(\omega)$</td>
<td>Fourier integral</td>
</tr>
<tr>
<td>$A(\omega)$</td>
<td>real part of $X(\omega)$</td>
</tr>
<tr>
<td>$B(\omega)$</td>
<td>imaginary part of $X(\omega)$</td>
</tr>
<tr>
<td>$X$</td>
<td>stochastic process</td>
</tr>
<tr>
<td>$\dot{X}$</td>
<td>first order differential process</td>
</tr>
<tr>
<td>$\ddot{X}$</td>
<td>second order differential process</td>
</tr>
<tr>
<td>$G_k(f)$</td>
<td>power spectral density function</td>
</tr>
<tr>
<td>$L_s$</td>
<td>duration of signal</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>time interval width of signal</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>irregularity factor</td>
</tr>
<tr>
<td>$f_m, x_m$</td>
<td>mean frequency</td>
</tr>
<tr>
<td>$M$</td>
<td>mass matrix</td>
</tr>
<tr>
<td>$C$</td>
<td>damping matrix</td>
</tr>
<tr>
<td>$K$</td>
<td>stiffness matrix</td>
</tr>
<tr>
<td>$V$</td>
<td>deformation</td>
</tr>
<tr>
<td>$p(\tau)$</td>
<td>loading</td>
</tr>
<tr>
<td>$T_p$</td>
<td>period of load</td>
</tr>
<tr>
<td>$h(\cdot)$</td>
<td>unit impulse response function</td>
</tr>
<tr>
<td>$P(i\omega)$</td>
<td>Fourier transform of load $p(t)$</td>
</tr>
<tr>
<td>$v(t)$</td>
<td>time domain response</td>
</tr>
<tr>
<td>$V(i\omega)$</td>
<td>frequency domain response</td>
</tr>
<tr>
<td>$H(\omega)$</td>
<td>frequency response function</td>
</tr>
<tr>
<td>$S(i\omega)$</td>
<td>response spectrum</td>
</tr>
<tr>
<td>$u[\cdot]$</td>
<td>unit step function</td>
</tr>
<tr>
<td>$\delta(t)$</td>
<td>Dirac delta function</td>
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<tr>
<td>$\text{sign}(\cdot)$</td>
<td>sign function</td>
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<tr>
<td>$\nu_a(t)$</td>
<td>number of level $a$ crossing in unit time</td>
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<tr>
<td>$\nu_a^{-}(t)$</td>
<td>number of down level $a$ crossing in unit time</td>
</tr>
<tr>
<td>$\nu_a^{+}(t)$</td>
<td>number of upper level $a$ crossing in unit time</td>
</tr>
</tbody>
</table>
Chapter 3

\[ \sigma = \sqrt{m_0} \text{ root mean square} \]
\[ E[D] \text{ expected damage} \]
\[ b, c \text{ material (S-N curve) parameters} \]
\[ S_h \text{ equivalent stress parameter} \]
\[ \gamma \text{ irregularity factor} \]
\[ E[P] \text{ expected peak number in unit time (peak frequency)} \]
\[ E[0] \text{ expected zero-crossing number in unit time} \]
\[ \Gamma(.) \text{ Gamma function} \]
\[ E[D]_{NB} \text{ expected damage by narrow band solution} \]
\[ Q(x^2|\nu) \text{ chi-square probability function} \]
\[ \rho \text{ correction factor for the fatigue damage rate} \]
\[ erf(.) \text{ error function} \]
\[ z \text{ normalised cycle range (} = S/2\sqrt{m_0}) \]
\[ p_{RBR}(z) \text{ rainfall cycle range PDF} \]
\[ C_1 \text{ coefficients in Dirlik’s formula} \]
\[ C_2 \text{ coefficients in Dirlik’s formula} \]
\[ C_3 \text{ coefficients in Dirlik’s formula} \]
\[ \tau \text{ factor of exponential distribution} \]
\[ \alpha \text{ factor of Rayleigh distribution} \]
\[ Y_1 \text{ first event in rainfall cycle} \]
\[ Y_2 \text{ second event in rainfall cycle} \]
\[ Y_3 \text{ third event in rainfall cycle} \]
\[ Y_1(.) \text{ probabilities of } Y_1 \]
\[ Y_2(.) \text{ probabilities of } Y_2 \]
\[ Y_3(.) \text{ probabilities of } Y_3 \]
\[ ip \text{ peak level} \]
\[ kp \text{ trough level} \]
\[ dh \text{ level interval width} \]

Chapter 4

\[ N \text{ number of signal blocks} \]
\[ \text{RAT reverse arrangement test} \]
\[ \mu \text{ mean value} \]
\[ \sigma^2 \text{ variance} \]
\[ S.D. \text{ standard deviation} \]
\[ RMS \text{ root mean square} \]
\[ \alpha \text{ significant level} \]
\[ A \text{ number of reverse arrangements} \]
\[ \chi^2 \text{ chi-square distribution} \]
\[ \kappa \text{ kurtosis} \]
Nomenclature

Cal
clipped factor

$\rho$
clipping ratio

$X_{\text{max}}$
assumed maximum value of random process

Chapter 5

$X(t)$
stochastic response caused by the wind turbulence

$Y(t)$
gravity induced response

$Z(t)$
combined response

$A$
amplitude of the harmonic component

$D$
global mean value of the signal

$\phi = \omega t$
azimuth

$\xi$
a suitable initial phase

$\sigma_x = \sqrt{\mu_0}$
Root Mean Square

$f_m = \frac{m_1}{m_0} \sqrt{\frac{m_2}{m_4}}$
mean frequency

$A$
amplitude

$f$
frequency

Chapter 6

$y(t)$
loading history

$\{y(i), i = 1, 2, ..., N\}$
sequence of extrema

$M$
number of levels

$s_1, s_2, ..., s_M$
values of levels

$\Delta$
interval

$p(.)$
probability density function

$p_k(v_j)$
probability of peak (trough) falling in interval k (j)

$P_k(V_j)$
probability of peak (trough) falling above (below) interval k (j)

$t_{jk}, u_{jk}, d_{jk}$
transition probabilities

$T, U, D$
transition probability matrices

$m, k, i, j, l$
indices for extrema

$q_{ki}^{(n)}$
probability of a transition from a peak $s_k$ to another peak $s_l$ with $n$ troughs inbetween

$I_j, G_k, H_k$
special matrices

$r_k^j$
rightwards PDF

$l_k^j$
leftwards PDF

$p_{R}(k)$
rainflow cycle range PDF with range $k\Delta$

$E[P] = \sqrt{\frac{m_4}{m_2}}$
expected value of the number of peaks per unit time

(peak frequency)

$E[PI] = \sqrt{\frac{m_4}{m_6}}$
expected value of the number of inflection points per unit time (inflection point frequency)

$p_{\text{m,amp}}(\alpha_m, \alpha_a)$
probability density function (p.d.f.) of the amplitudes
Nomenclature

- $N_0$: zero crossing frequency
- $N_1$: peak frequency
- $c_1, c_2$: parameters
- $\gamma$: irregularity factor
- $\alpha_m$: mean value
- $\alpha_a$: amplitude
- $\alpha_1$: trough value
- $\alpha_2$: peak value
- $J$: Jacobian
- $\text{sign}(\cdot)$: sign function
- $a = \frac{\alpha}{\sqrt{m_0}}$: normalised peak value
- $erf(\cdot)$: error function

Chapter 7

- $x(t)$: random process
- $p(x)$: joint probability density function (p.d.f.)
- $x_0$: mean value vector
- $R$: Covariance Matrix
- $\gamma_1 = \frac{m_3}{\sigma^3}$: skewness
- $\gamma_2 = \frac{m_4}{\sigma^4} - 3$: kurtosis
- $F(x)$: distribution function
- $\varphi(t)$: characteristic function
- $\chi_\nu (\nu = 1, 2, \ldots)$: cumulants (semi-invariants)
- $\phi(x)$: Gaussian probability density function
- $C_k (k=3, \ldots n)$: constants
- $H_k(x) (k=3, \ldots n)$: Hermite polynomials
- $\delta_{mn}$: unit function
- $\xi, \xi_1$: stress process and its derivative
- $b, c$: material constants
- $k_{mn}$: cumulants
- $\lambda_{mn}$: "normalized" cumulants
- $\lambda_{40}, \lambda_{04}$: kurtoses of $\xi$ and $\xi_1$
- $\Gamma(\cdot)$: Gamma function
- $\chi_k (k=3, \ldots n)$: cumulants
- $H_p$: entropy
- $G[\cdot]$: non-linear function
- $\omega$: a parameter
- $R_{xx}(t_1, t_2)$: autocorrelation function
- $R_{xxx}(t_1, t_2, t_3)$: 3rd order correlation (Bi-correlation)
- $R_{xxxx}(t_1, t_2, t_3, t_4)$: 4th order correlation (Tri-correlation)
- $E[\cdot]$: ensemble average
- $t, t_1, t_2, t_3, t_4$: time
- $\tau, \tau_1, \tau_2, \tau_3$: time lag
**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$S_{xxx}(\omega_1, \omega_2)$</td>
<td>bi-spectrum</td>
</tr>
<tr>
<td>$S_{xxxx}(\omega_1, \omega_2, \omega_3)$</td>
<td>tri-spectrum</td>
</tr>
<tr>
<td>$U(t)$</td>
<td>standard normal process</td>
</tr>
<tr>
<td>$g(\cdot)$</td>
<td>monotonic transfer function</td>
</tr>
<tr>
<td>$\bar{x}, m_x$</td>
<td>mean value of $x$</td>
</tr>
<tr>
<td>$\alpha_n, (n=0,1,\ldots)$</td>
<td>parameters</td>
</tr>
<tr>
<td>$\eta_D$</td>
<td>mean damage accumulation rate</td>
</tr>
<tr>
<td>$\alpha_1, \alpha_2$</td>
<td>trough, peak value</td>
</tr>
<tr>
<td>$p_{\text{max}, \text{min}}(\alpha_1, \alpha_2)$</td>
<td>joint PDF of adjacent peaks and troughs</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>irregularity factor</td>
</tr>
<tr>
<td>$m_0$</td>
<td>0-th moment of PSD (variance)</td>
</tr>
</tbody>
</table>

**Chapter 8**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>root mean square</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>irregularity factor</td>
</tr>
<tr>
<td>$x_m$</td>
<td>mean frequency</td>
</tr>
<tr>
<td>$S$</td>
<td>cycle range</td>
</tr>
<tr>
<td>$G(f)$</td>
<td>single-side Hertz PSD</td>
</tr>
<tr>
<td>$A_i, f_i, Q_i, (i=1,2)$</td>
<td>parameters</td>
</tr>
<tr>
<td>$f_c$</td>
<td>cutoff frequency</td>
</tr>
<tr>
<td>$net_{p,j}$</td>
<td>input to neuron $j$ from system input $p$</td>
</tr>
<tr>
<td>$w_{ij}$</td>
<td>weight factor of neuron $i$ to neuron $j$</td>
</tr>
<tr>
<td>$o_{pi}$</td>
<td>output of neuron $i$ for system input $p$</td>
</tr>
<tr>
<td>$\delta_{pk}$</td>
<td>error at neuron $k$ for system input $p$</td>
</tr>
<tr>
<td>$E_p$</td>
<td>total error of output layer</td>
</tr>
<tr>
<td>$\mu$</td>
<td>coefficient of sigmoid function</td>
</tr>
<tr>
<td>$\delta_{pk}^\circ$</td>
<td>intermediate quantity</td>
</tr>
<tr>
<td>$\eta$</td>
<td>upgrading step size</td>
</tr>
</tbody>
</table>
Acknowledgements

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I am also indebted to Stuward Kerr and Andrew Halfpenny who have read and commented on part of the dissertation.

I would like to thank my parents, my wife Zhinong and my daughter Cherry for their love and support. Without their support, this thesis could never have been finished.

Ruhuai Wang
Declaration

This dissertation is submitted in support of an application for the Degree of Doctor of Philosophy in Engineering Science, from University College London, University of London.

No part of the work contained in the thesis has been submitted for any other Degree or Diploma from this University or any other Institution.

All the computation work in this thesis was performed by programs coded and developed by me unless otherwise stated in the text.

I hereby declare that this declaration is true in every respect.

Ruhuai Wang
Chapter 1

INTRODUCTION

1.1 Introduction

The aim of using stochastic process theory for structural analysis is to judge the reliability of a structure which has been designed to withstand random loading. The most common structural failures are first-excursion failures and fatigue failures, particularly the latter which account for the majority of all service failures due to mechanical causes (in metallic structures, it has been claimed that some 80-90% of failures are related to fatigue and fracture[22]). Traditionally, fatigue damage prediction is carried out in the time domain, that is, using time histories of response (stress or strain) to estimate fatigue life. In recent years, several frequency domain techniques[14, 17, 18, 19, 94, 93] have been developed to predict fatigue life. Rather than directly using the measured stress signal, these methods predict fatigue life from the Power Spectral Density (PSD), or the relevant moments, of the stress signal. In many engineering fields this is sometimes the only loading information which is available, particularly at the design stage. Also, in some engineering applications, e.g., the aircraft industry, standard loading spectra (PSD, etc.) have been developed, which precisely simulate the service environment. The frequency domain approaches (spectral fatigue methods) can then be applied to produce more reliable designs.

Fatigue is defined as the process of progressive localized permanent structural change occurring in a material subjected to conditions which produce fluctuating stresses and strains at some point or points and which may culminate in cracks or complete fracture after a sufficient number of fluctuations [7][12]. Fatigue of metals
has been studied for over 150 years[34, 8, 35]. The first fatigue investigation seem to have been reported by a German mining engineer, W.A.S. Albert, who in 1829 performed some repeated loading tests on iron chain[26]. During the period from 1852 to 1870, a German railway engineer, August Wöhler, set up and conducted the first systematic fatigue investigation. These experiments were conducted to establish a safe alternating stress below which failure would not occur. Full scale axles as well as smaller laboratory specimens were employed to establish the endurance limit concept for design. Many important aspects of fatigue behaviour were pointed out. The most important one being that fatigue depends more on the range of stress than the maximum stress and the life of specimens reduces when the amplitudes of repeated loading increases. Test data were plotted on what has become known as the $S$-$N$ diagram, and fatigue data today are often presented in much the same way.

After the initial research from 1850 to 1875, more experimental work was conducted to establish a clearer understanding of the fatigue phenomena[8], i.e., the process of crack propagation under cyclic loading. The importance of cyclic deformation was clearly established in 1930’s. Research in fatigue during the 1930’s and 1940’s was largely devoted to experimentally establishing the effects of the many factors that influence the long-life fatigue strength. Tests were usually conducted in rotating bending and the life range of interest was about $10^8$ cycles and greater.

During the 1950’s the quantitative relationships between plastic strain and fatigue life was established. Many significant contributions were made during the 1960’s. Fracture mechanics was developed as a practical engineering tool for fatigue analysis. Paris[76] quantified the relationships for fatigue crack propagation. By the 1970’s fatigue analysis became an established engineering tool in many industrial applications.

There are three major approaches to analysing and designing against fatigue failures[31][86]. They include:

(i) **The nominal stress approach.** The amplitude of some representative stress
in the component is used to predict its life. The stress is often a nominal stress and local features such as holes and notches are dealt with by introducing stress concentration factors. Failure may be taken as the appearance of a crack, a specific length of crack, or total failure depending on the test data available.

(ii) **The fracture mechanics approach** [83]. Crack propagation is assumed to depend on a fracture mechanics parameter, usually the range of crack tip stress intensity factor $\Delta K$. Life is then calculated by assuming an initial crack length and finding how many cycles are needed to make this crack grow to an unacceptable size.

(iii) **The local stress-strain or critical location approach** [84] [103]. The strain history of some critical location is estimated from the nominal loading history, including plasticity effects. Life is then estimated from test data taken under strain controlled conditions.

The nominal stress approach is used in this thesis. It is chosen because methods such as the other ones described above, either have no relevant influence on the focus of the present study, or are unsuitable for dealing with the loading problem investigated because there is a need to define a stress (or strain) “cycle” for the loading conditions which are more complex than constant amplitude. The nominal stress approach is best suited to so-called long life (low stress) situations where plasticity effects are small. Fortunately for dynamic or vibration problems, where frequency based fatigue tools are particularly relevant, this is usually the case.

The $S-N$ curve can be reliably used to predict fatigue life in the case of a constant amplitude loading history. However, when a structure or component is subjected to normal service loadings this approach has to be adapted to account for the fact that the loadings will not be of constant amplitude. For such situations, firstly there must be a way to count the accumulation of fatigue damage, and secondly a method must be used to extract the “cycles” which contribute to such damage.
from the loading time history. For the first problem, Miner's rule[71] is generally adopted. This rule assumes a linear fatigue accumulation and ignores the order of cycles of different range and their interactive effects [82]. For the second problem, many methods of defining or counting a “cycle” have been proposed. Among them, the rainflow cycle counting method[72] is generally used because it is believed that this method gives the best correlation with test results. The original definition of Rainflow Cycle Counting is complicated, Rychlik[79] gave an alternative definition and proved it is equivalent to the original one. The new definition enables easier application.

In general, stochastic loading is hard to express using a conventional mathematical formula. A probabilistic description is usually inevitable. A common way is to express the loading in the frequency domain as a PSD, as with, for instance, wind loading, sea wave loading, etc. In most cases the structure and/or component is assumed to be linear, i.e., the response spectrum can be linearly linked to the loading spectrum by the frequency response function. This analysis technique has many advantages. The most important one is that, the tedious and time consuming computing work in the time domain can be avoided and the response spectrum can be obtained without knowing the time history of the loading (actually it is very difficult to know). With most Finite Element packages used for structural analysis, such spectra can be obtained directly.

It is for this reason that considerable attention has focused on the spectral fatigue analysis approach for structures and/or components subjected to stochastic loadings [12]. This approach uses frequency domain information describing structural response to predict the fatigue damage, rather than relying on the more traditional deterministic or time domain solutions.

Frequency domain fatigue analysis is strongly related to the moments of the loading stress PSD. Peak value distribution and level crossing frequency are desired in the analysis. Rice [78] and then Bendat [10] worked out relationships for calculat-
ing the number of peaks and zero crossings per unit time from the joint probability
density function of the process and its first and second order differential processes.
For a Gaussian signal, this joint probability density function can be determined from
the frequency domain representations of the loading. This relationship provides the
basic foundation for spectral fatigue analysis.

The simplest fatigue damage estimation method is the so-called “Three-Band
Technique” [88], which can be regarded as either a time domain method or frequency
domain method. It can only give a very rough estimation.

The first frequency domain approach was the so-called “Narrow Band Solution”
[70] which assumes that the response has a narrow frequency band of one predom­
inant frequency. However, this is not always the situation, especially when taking
account of dynamic behaviour or possible structural nonlinearities. It is conserva­
tive and usually underestimates fatigue life. Other methods were developed to
modify it to deal with more general loading situations and to use the rainflow cy­
cle definition. Some methods have also been developed to calculate the rainflow
cycle probability density function directly, either using numerical simulation [28]
or Markov chain theory [12]. Existing frequency domain methods can be classified
into four categories: Closed Form Analytical solutions, Correction Factor solutions,
Dirlik’s Empirical solution and Theoretical solutions. Among these approaches, Dir­
lik’s empirical solution [28] and Bishop’s theoretical solution [13] [12] are the most
promising methods. All of the Closed Form Analytical Solutions and Correction
Factor Solutions are either derived by making some simplified assumptions or using
computer modelling, and curve fitting, and have no significant theoretical backing.
Therefore, although useful for certain cases, these results alone were not substantial
enough to influence the design practices of structures such as offshore oil platforms
or wind turbine blades. Before such a change in design practices could take place
more substantial theoretical backing was needed. As a means of widening the appli­
cability of frequency domain methods, Bishop’s Markov Chain model is promising,
but the calculation procedure is complicated and susceptible to computational instabilities. For this purpose, a new theoretical solution is derived in this thesis and presented in chapter 6. Furthermore, all existing approaches require the stress signal to be stationary, Gaussian and random, i.e. without any significant deterministic component. Perhaps there are two reasons for this assumption. The first is that, according to the central limit theorem, if there is no significant deterministic component in the loading, the structural responses should be Gaussian. The second is that, conventional one-dimensional Power Spectral Density functions can only provide enough information about the distribution of Gaussian processes. Recent studies[17, 18] show that the effect of non-Gaussianity cannot be ignored, at least for the Howden HWP330 wind turbine blade data. In some cases the effect is significant. Experience suggests kurtosis is the best and simplest parameter for the description of the departure from the Gaussian distribution. It is known that the distribution of the process and its first and second order differential processes is essential for such analysis. For non-Gaussian responses, there is currently no efficient way to perform the fatigue analysis using frequency domain information. Actually, non-Gaussian processes are too wide a class of distributions to deal with as a whole. In this thesis, efforts are made to study the effect of kurtosis-described non-Gaussian signals. This is achieved by applying extensive simulation and artificial neural networks.

The first large scale application of frequency domain fatigue analysis was for offshore engineering. In fact, some methods were purposely developed for offshore engineering. Much material has been published on the spectral fatigue design of offshore platforms. This technique has also been applied for railway engineering design [91] and wind turbine blade design. However, in some cases the loading on wind turbine blades does not satisfy the Gaussian assumption since the gravity component in the edgewise direction sometimes becomes dominant. This deterministic (gravity) component is applied predominantly in the blades edgewise direction although there is some coupling into the flapwise direction. In all but the purely flapwise direction
there is therefore a combined stochastic (wind loading) and deterministic (gravity) mixed signal. Such a deterministic component makes the response a mixture of a random process with a strong deterministic component. Effects of mixing a stationary signal with various deterministic components can be found in [19] and chapter 5.

In this thesis, the problems of non-Gaussianity have been studied. For overcoming the difficulties of assuming Gaussian, narrow band loading, or applying correction factors, a new frequency domain based approach has been developed[94]. This approach itself doesn’t require the signal to be Gaussian. The PSD of the stress signal is used to predict the probability density function of rainflow cycle ranges (for a non-Gaussian signal, the tri-spectrum is also required). Calculations confirm it is efficient and easy to apply. For Gaussian signals, Kowalewski’s[60] joint PDF of adjacent peaks and troughs is applied. For non-Gaussian signals, an alternative expression for the joint PDF of adjacent peaks and troughs is required. Unfortunately, due to the mathematical difficulty of the problem, there is no closed form theoretical expression available. Extensive simulation and modeling was carried out and an Artificial Neural Network (ANN) has been applied to study the effect of non-Gaussianity. A trained network is provided as a toolbox for frequency domain fatigue analysis of general stationary signal (Gaussian or kurtosis-described non-Gaussian).

It should be realised that for non-Gaussian processes it is necessary to know more than the one-dimensional probability distribution in order to find an adequate description of the statistical properties of the process. For example, calculation of the mean squared value (second moment) of structural response requires prior knowledge of the autocorrelation function (or power spectral density). Analogous to the calculation of second moment, the calculation of kurtosis (or coefficient of excess) requires another operation involving the 4th order moment functions. In other words, the 4th order cumulant function (in the time domain) or the 3-dimensional
spectral density, tri-spectrum, (in the frequency domain) is required.

1.2 Outline of the Thesis

An outline of the thesis is given in Fig. 1.1. The main achievements are: (i) development of a new theoretical approach (Fig. 1.2) which can be applied to a wide range of random stationary stress loadings; (ii) study of the effect of non-Gaussianality (Fig. 1.3) with Monte-Carlo simulation and artificial neural networks, neural networks are trained to provide a fast, efficient toolbox for practical engineering applications; (iii) a sophisticated study of the factors affecting the accuracy of spectral fatigue analysis; (iv) development of a software package which applies some of the popular frequency domain methods and the new theoretical method. Part of the work has been incorporated into the software product (NSOFT FATIMAS-SPECTRAL) of nCode International.

1.3 New Developments

1.3.1 A new frequency domain theoretical approach

A new frequency domain theoretical approach [94] has been developed to calculate the pdf of rainflow cycle ranges and hence the fatigue life. It is based on Rychlik's alternative definition [79] of rainflow cycle ranges. The method itself doesn't require the signal to be Gaussian. For a Gaussian signal, Kowalewski's joint PDF of adjacent peaks and troughs can be applied. The forward and backward peak-peak transition (with any number of intermediate troughs in between) probability matrices are calculated. Two cases have been considered. Firstly peak-peak transitions with the lowest trough to the right and secondly with the lowest trough to the left. Rainflow transition probability and therefore the p.d.f. of rainflow cycle ranges has been calculated. Calculations confirm it is efficient and easy to apply. Work has

\footnote{nCode International Ltd., 230 Woodbourn Road, Sheffield S9 3LQ, UK}
chapter 1

Fatigue Analysis: Frequency Domain Approach

A New Theoretical Approach

Effect of non-Gaussianity

Artificial Neural Networks (ANN)

Case Studies

Joint PDF of Adjacent Peaks and Troughs

Peak Value PDF

Monte-Carlo Simulations

Non-linear Transform of Gaussian Process

Wind Energy Group MS-1 Wind Turbine

Howden HWP 330 Wind Turbine Blades

Other Data

Figure 1.1: Outline of the thesis
Flow Chart of the New Method

Fatigue Life

Fatigue Damage Rate

Stress/Strain vs Life (S-N) Curve

Rainflow Cycle Range p.d.f.

Forward & Backward Transition Probability

Rychlik's Rainflow Cycle Counting Definition

Joint p.d.f. of Adjacent Peaks & troughs

Peak Value p.d.f.

PSD of Stress/Strain Signal

Figure 1.2: A new theoretical approach
Effect of Non-Gaussianity

Figure 1.3: Study of the effect of non-Gaussianity
been carried out to validate the method for various engineering applications, including WEG (Wind Energy Group) MS1 and Howden HWP330 wind turbine blade data.

1.3.2 Effect of non-Gaussianality

Efforts have been made to get the joint probability density function of adjacent peaks and troughs for general stationary (not necessary Gaussian) signals. Due to mathematical difficulties, no closed form theoretical results have been achieved. Extensive computer modelling[93] (using various methods, including use of the Monte-Carlo method) and Artificial Neural Network (ANN) have been applied to study the effect of non-Gaussianality. For non-Gaussian stress signals, the rainflow cycle range p.d.f. appears to be related to several parameters, these being the root mean square of the signal, the irregularity factor, and kurtosis, a parameter representing the non-Gaussianality of the signal. Analysis of WEG, HWP data and simulation suggests kurtosis as the best description of the degree of the departure from Gaussian distribution. Among various developed Artificial Neural Network systems, backpropagation, a method of training a neural network to approximate any function, including arbitrarily complex nonlinear functions, is the only neural network technique to produce any number of fielded applications. Even though backpropagation has played a key role in the development of artificial neural networks, it is really a statistical modeling technique. More specifically, it is a non-parametric modeling method: one in which the shape of the relationship between inputs and outputs is decided by the data rather than predetermined by the tool. The simulation procedure is as follows:

- Generate Gaussian signals with various $\gamma$ (irregularity factor) and mean frequency; seventy PSDs were used.

- Apply a nonlinear function transform and thereby generate non-Gaussian signals with various $\kappa$ (kurtosis). Twenty five kurtosis values were chosen there-
fore totally 1750 non-Gaussian signals were simulated.

- Using the generated non-Gaussian signals, calculate the sample p.d.f. for rainflow cycle ranges;

- Study the sample p.d.f. of rainflow cycle ranges for the non-Gaussian signal and apply an Artificial Neural Network (ANN) to establish a neural network to model the effect of non-Gaussianity.

Software has been developed. The trained neural network provides a fast, efficient toolbox for practical engineering applications.

1.4 Applications

Sophisticated studies have been carried out for all the existing and newly developed frequency domain fatigue analysis methods. Various factors affecting the prediction have been considered. The methods have been applied to both simulated data and measured data. As a result of the invaluable collaboration with industry, WEG Ltd and Garrad Hassan Ltd provided MSI and Howden HWP330 wind turbine data. Extensive studies have been carried out on the data. Some results and details are published in [17, 18]. Work on the data includes

- Data logging, sorting and classification;

- Statistical Tests: Calculation of various statistics (mean, rms, zero crossing frequency, peak frequency, irregularity factor, skewness and kurtosis); Stationarity and Trend tests; Gaussianity tests.

- Calculation of Power Spectral Density and first 4 (0th, 1st, 2nd, 4th) moments.

- Application of various spectral fatigue methods, i.e., Dirlik's, Narrow Band Solution, Wirsching's solution, Hancock's solution, Kam & Dover's solution, Steiberg's solution and the new theoretical solution to calculate the probability
density function of rainflow ranges (thereby fatigue damage rate). Comparison of the results with the time domain method.

Analysis of the data has been used mainly to establish the sensitivity of frequency domain techniques to the assumption that the data be Gaussian and stationary. The WEG and Howden data differs in two important ways. Firstly the WEG data is for flapwise blade data and so only the stochastic wind induced loading is presented. The Howden data, however, contains edgewise data which includes a significant (non-random) gravity component. In addition, the Howden data comes from machines in complicated terrain and with a complicated control system which causes the data to be less ideal. Some statistical analysis results and fatigue results are given in [18].

It is clear that the deterministic gravity component causes all the edgewise results to become very conservative. A study into the effects of combining stationary signals and a deterministic component is given in chapter 5.

Furthermore, the presence of non-Gaussian and non-stationary trends in the flapwise signal also cause problems. The new methods described in section 1.3.2 for non-Gaussian signals are applied here and also presented in [93].

1.5 Arrangement of the Thesis

Chapter 2 presents a brief summary of some of the statistical techniques and aspects of fatigue analysis which are encountered throughout the thesis.

Chapter 3 reviews existing frequency domain fatigue analysis methods. Comparison were made for different methods.

Chapter 4 presents the results of the analysis of two types of wind turbine blade data: WEG MS1 and Howden HWP330 data. Statistical analysis was undertaken for these data. Results show the WEG and Howden flapwise data can, in general, be accepted as stationary Gaussian signals, whilst the Howden edgewise data has strong deterministic components. Various spectral fatigue analysis methods were applied
to the data. Effects of various factors on the fatigue prediction were studied.

Chapters 5-8 form the major part of the original work presented in this thesis, chapters 6-8 being the most important. Chapter 5 studies the effect of deterministic components in the spectral fatigue analysis. Two cases were considered: stationary signals mixed with sine waves; stationary signals mixed with equal-spaced spikes.

Chapter 6 presents a new theoretical frequency domain fatigue analysis method. In theory, the method can be applied to any stationary signal. The method itself doesn't require the signal to be Gaussian, though a closed form theoretical expression for the joint distribution of adjacent peaks and troughs for non-Gaussian signals is required. Efforts have been made to get the joint probability density function of adjacent peaks and troughs for general stationary (not necessarily Gaussian) signals. However, due to mathematical difficulties, no closed form theoretical results have been achieved. A trained neural network can be used to provide numerical solution for the joint PDF of adjacent peaks and troughs for non-Gaussian signals. Examples show the new method is easy to use and agrees well with time domain results.

Chapters 7 and 8 study the effect of non-Gaussianity. Extensive computer modelling and Artificial Neural Networks (ANN) have been applied to study the effect of non-Gaussianity. Analysis of WEG, HWP and other data suggests kurtosis as the best description of the degree of the departure from a Gaussian distribution. Several parameters appear to relate to the rainflow cycle range p.d.f. of non-Gaussian stress signals, these being the root mean square of the signal, the irregularity factor, and kurtosis. Among various developed Artificial Neural Network systems, back-propagation, a method of training a neural network to approximate any function, including arbitrarily complex nonlinear functions, is chosen to study the effect of non-Gaussianity. Neural networks are trained to provide a fast, efficient toolbox for practical engineering applications.

Chapter 9 gives a summary of the conclusions from each chapter, an overall discussion of the work and future directions the research can take.
Chapter 2

THEORETICAL BACKGROUND

This chapter presents some background theory which is used in later chapters. It is given mainly for two reasons: firstly some work in later chapters is based on the background theory given here, and secondly to avoid the need to consult texts. If a more detailed treatment of any topic is required readers can refer to references indicated in appropriate places.

2.1 Fatigue Failure and Palmgren-Miner Cumulative Damage Hypothesis (Linear Damage Rule)

A fatigue failure has been described as the result of cumulative damage that arises when the response of a structure, to external excitations, fluctuates at small or moderate excursions. Though the exact nature of fatigue failures has not been fully understood, many experiments confirm the logarithm-logarithm relationship between the response stress amplitude and the number of cycles to fatigue failure (S-N curve).

\[ NS^b = c \]  

where \( N(S) \) is the number of cycles with constant stress range \( S \) to cause fatigue failure or simply the fatigue life at stress range \( S \), \( b \) and \( c \) are positive constants depend only on material strength. Values of \( b \) and \( c \) for typical materials can be found in, e.g., [31]. A generalised form of S-N curve has three staged log-log straight
lines with different slopes (Fig. 2.1)

\[
N = \begin{cases} 
  c_1 S^{-b_1} & \forall S \geq S_0 \\
  c_2 S^{-b_2} & \forall S_0 \geq S \geq S_c \\
  \infty & \forall S \leq S_c
\end{cases}
\]  

\[ (2.2) \]

Figure 2.1: Generalised S-N curve

When the stress amplitude is not constant, the above equation cannot be used without additional assumptions. Many hypotheses have been proposed as means of assessing the fatigue damage due to different levels of stress range. The simplest theory is the Palmgren-Miner hypothesis. It states that the percentage of damage (or damage ratio) \( \Delta D_i \) due to loading with \( n_i \) cycles of range \( S_i \) is accumulated linearly. That is

\[
\Delta D_i = \frac{n_i}{N(S_i)}
\]

\[ (2.3) \]

where \( N(S_i) \) is the fatigue life (number of cycles) with cycle load of constant stress range \( S_i \).

The Palmgren-Miner hypothesis has the advantage that it can easily be adapted to the case of continuous variable and random stresses, i.e.,

\[
dD(S) = \frac{dn(S)}{N(S)} = dn(S) \frac{S^b}{c}
\]
Note that the Palmgren-Miner theory implies that the order of application of the different stress levels has no effect on the resulting total damage accumulated, a very desirable property when the theory is used in the case of stationary random stresses.

Consider now that the cyclic loading is a random load, $S - N$ curve has a single slope $b$, the accumulated damage due to random loading is

$$D = \int_0^{D(M)} dD = \frac{1}{c} \int_0^M S^b dn \approx \frac{1}{c} \sum_{i=1}^M S_i^b \approx \frac{1}{c} M E[S^b] = E[P] \frac{T}{c} \int_{S=0}^{\infty} S^b p(S) dS$$

(2.5)

where $M$ is the number of cycles up to the accumulated damage $D(M)$, $E[P]$ is the expected number of peaks per unit time (peak frequency), $p(S)$ is the probability density function of the stress cycle range. This is the damage accumulated by $M$ cycles. For one cycle, it should be

$$D_1 = \frac{1}{c} E[S^b] = \frac{1}{c} \int_{S=0}^{\infty} S^b p(S) dS$$

Equation 2.5 can be written as

$$D = E[P] \frac{T}{c} S_h^b$$

(2.6)

with

$$S_h = \left[ \int_{S=0}^{\infty} S^b p(S) dS \right]^{\frac{1}{b}}$$

(2.7)

$S_h$ is called "Equivalent Stress Parameter".

When $D$ approaches 1, fatigue failure occurs. From equation 2.5, the fatigue life is

$$T = \frac{c}{E[P] \cdot \int_{S=0}^{\infty} S^b p(S) dS}$$

(2.8)
From this equation, once $E[P]$ is available, the only unsolved is $E[S^k]$ or $p(S)$, the probability density function of the stress cycle range. In fact, for stationary random process $x(t)$,

$$E[P] = \int_{-\infty}^{\infty} dx \int_{-\infty}^{0} |x_2| p(x, x_1, x_2) dx_2$$  \hspace{1cm} (2.9)

where $p(x, x_1, x_2)$ is the joint PDF of $x, x_1, x_2 (x_1 = \frac{dx}{dt}, x_2 = \frac{d^2x}{dt^2})$. Furthermore, if $x(t)$ is Gaussian,

$$E[P] = \left[ \frac{m_4}{m_2} \right]^{\frac{1}{2}}$$  \hspace{1cm} (2.10)

where

$$m_n = \int_0^\infty f^n G(f) df$$  \hspace{1cm} (2.11)

is the n-th moment of the one-sided Power Spectral Density function $G(f) = 4\pi S(\omega = 2\pi f)$, with $f$ in units of Hertz.

As an alternative to the calculation of fatigue damage, fracture mechanics may also be used to calculate the fatigue crack growth. The Paris equation[76] may then be applied:

$$\frac{da}{dN} = C(\Delta K)^m$$  \hspace{1cm} (2.12)

where $a$ is the crack size, $N$ is the number of stress cycles, $C$ and $m$ are crack growth constants, and the stress intensity factor range is given by

$$\Delta K = F(a)S\sqrt{\pi a}$$  \hspace{1cm} (2.13)

in which $F(a)$ is the geometry function.

Integrating equation 2.12, the crack size $a(T)$ at time $T$ is then obtained as
if a so-called 'mean crack driving stress' is defined as the expected value of the stress range to the power of crack growth exponent, i.e.

$$
E[S^m] = \frac{1}{N(T)} \sum_{i=1}^{N(T)} S_i^m
$$

Then, from equations 2.8 and 2.14, it is clear that a major task for the prediction of fatigue in terms of either damage or crack growth is the calculation of 'mean crack driving stress' or cycle range probability density function $p(S)$. As Rain-Flow-Cycle counting gives the best prediction of fatigue damage, obtaining the Rain-Flow-Cycle Range p.d.f. $p_{RR}(S)$ is therefore our major task.

### 2.2 Random Process

#### 2.2.1 Basic concept

A random or stochastic process is defined as a parametered family of random variables $X$ with the parameter $t$. For any fixed time $t_i$, the corresponding value $X(t_i)$ is a random variable. Theoretically a random process is completely described by all the joint probability density functions (PDF's) of the random variables at all times. If all the PDF's remain unchanged when the time scale is shifted by an arbitrary value $\tau$, the process is called strongly stationary. If only the first and second order PDF's remain unchanged while the time scale is shifted by an arbitrary value $\tau$, the process is called weakly stationary. Physically a stationary random process is one where the statistical properties measured across a set of records, or ensemble, at a particular time, are identical with the statistics measured across the ensemble at any other time. Since the mean values and covariance functions are consequences only of the first- and second-order probability distributions, it follows that the class of strongly stationary processes is a subclass of the class of weakly stationary ran-
dom processes. For Gaussian random processes, however, weak stationarity implies strong stationarity since all possible probability distributions may be derived from the mean values and covariance functions. Thus, for Gaussian random processes, these two stationary concepts coincide.

In general, a particular sample function would not be suitable for representing the entire random process to which it belongs. However, under certain conditions, it turns out that for the special class of ergodic random processes, it is possible to derive desired statistical information about the entire random process from appropriate analysis of a single arbitrary sample function. It is much more convenient for statistical computation if the process has such a property because the statistics can then be obtained from one sample.

In this thesis, all stationary processes are weakly stationary and ergodic unless otherwise indicated.

2.2.2 Moments and characteristics function

The best description for a stationary process is its probability density function (PDF) \( p(x) \) or probability distribution function \( P(x) \), which are independent of time \( t \). The moments of the process are defined by its probability density function \( p(x) \) as:

\[
\alpha_n = E\{X^n\} = \int_{-\infty}^{\infty} x^n p(x) dx
\]

The central moments are similarly defined as:

\[
\mu_n = E\{(X - \bar{x})^n\} = \int_{-\infty}^{\infty} (x - \bar{x})^n p(x) dx
\]

where \( \bar{x} \) denotes the global mean value of the process. The variance of the process is then given by the second order central moment, and its square root gives the standard deviation (or root mean square - RMS).
The characteristic function of the process is defined as

$$\phi(u) = E\{e^{itu}\} = \int_{-\infty}^{\infty} e^{itu} p(x) dx \quad (2.16)$$

Thus, the PDF is obtained by applying a Fourier transformation to the characteristic function.

$$p(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itu} \phi(u) du \quad (2.17)$$

The characteristic function can be expanded as a MacLaurin series as follows

$$\phi(u) = \phi(0) + \phi'(0)u + \frac{\phi''(0)}{2} u^2 + \ldots = \sum_{j=0}^{n} \frac{\alpha_j}{j!} (iu)^j + O(u^n) \quad (2.18)$$

From Equation 2.16,

$$\phi^{(n)}(0) = i^n \int_{-\infty}^{\infty} x^n p(x) dx = i^n \alpha_n$$

For the situation of more than one random variable, the parameters are defined similarly.

It is obvious that for a general stochastic process, the probability distribution of the process should be described by all the moments of the process. In other words, finite order moments of the process are not enough to fully describe the process. Any truncation causes errors unless the higher order moments can be expressed as functions of lower order moments. The widely used Gaussian distribution with the PDF expressed as

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

is a good example of where all higher order central moments can be calculated from the lower order moments as:

$$\mu_n = (n - 1)\sigma^2 \mu_{n-2} \quad n = 2, 4, 6, \ldots$$

while the odd moments vanish.
2.2.3 Correlation (covariance) function

The autocorrelation function gives a measurement of the amount by which a signal is correlated with itself. It is defined as the average value of the product $x(t)x(t+\tau)$. For a stationary process, the value of $E[x(t)x(t+\tau)]$ is independent of time $t$ and depends only on the time separation $\tau$:

$$R_{xx}(\tau) = E[x(t)x(t+\tau)]$$

or, alternatively

$$R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)x(t+\tau)dt = R(\tau) \quad (2.19)$$

The autocorrelation function $R_{xx}(\tau)$ for a stationary process is always an even function of $\tau$ and has the maximum value at $\tau = 0$. Provided the mean value of a process is adjusted to zero and the process has no periodic components, the autocorrelation function does satisfy:

$$R(\tau \to \infty) = 0$$

and then the condition

$$\int_{-\infty}^{\infty} |R(\tau)|d\tau < \infty$$

is satisfied.

The cross-correlation (covariance) function gives a measurement of the amount by which two functions are related to each other. For two random variables $x(t)$ and $y(t)$, their cross-correlation function is given by:

$$R_{xy} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)y(t+\tau)dt \quad (2.20)$$

2.2.4 Power spectral density (PSD)

While the realisation of any process (samples) can be described in the time domain, the process can be described by its Fourier components in the frequency domain [73, 41]. For a stationary process, $x(t)$, the concept of the Power Spectral Density
(PSD), denoted \( S_x(\omega) \), is particularly useful. \( S_x(\omega) \) may be defined as the Fourier transform of the autocorrelation function, \( R(\tau) \):

\[
S_x(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\tau) e^{-i\omega\tau} d\tau
\]  \hspace{1cm} (2.21)

The inverse transform is:

\[
R(\tau) = \int_{-\infty}^{\infty} S(\omega) e^{i\omega\tau} d\omega
\]  \hspace{1cm} (2.22)

The spectral density function defined in this way is known as the two-sided (radians frequency \( \omega \)) spectral density function \( S(\omega) \). It gives a "negative frequency" which only makes sense mathematically. In practical applications, the one-sided (Hertz) spectral density function \( G(f) \) is defined to give only positive frequency components and can still give the same mean square value of the process. If the frequency \( f \) is defined in Hz, it is related to the two-sided spectral density function (in radians \( \omega \)) as:

\[
G(f) = 2S(f) = 4\pi S(\omega = 2\pi f)
\]

Physically the PSD \( S(\omega) \) can be interpreted as a spectral decomposition of the mean square, \( E[x^2] = R_x(0) \). Thus, if a frequency band \( (\omega, \omega + d\omega) \) is considered, then the contribution to \( E[x^2] \) from these harmonic components in the process which lie in this frequency band is \( S(\omega)\Delta\omega \), if \( \Delta\omega \) is small.

The spectra of the stochastic process \( X \) and its derivative \( \dot{X} \) are connected by

\[
S_{\dot{X}}(\omega) = \omega^2 S_X(\omega)
\]  \hspace{1cm} (2.23)

Similarly,

\[
S_{\ddot{X}}(\omega) = \omega^4 S_X(\omega)
\]  \hspace{1cm} (2.24)

2.2.5 Fast Fourier transform (FFT)

In practical calculations, the transform is generally performed on the discrete time series \( \{x_r\} \) as:

\[
X_k = \frac{1}{N} \sum_{r=0}^{N-1} x_r e^{-i2\pi \frac{kr}{N}}, \hspace{1cm} k = 0, 1, 2, ..N - 1
\]  \hspace{1cm} (2.25)
The one sided Power Spectral Density (PSD) is given by:

$$G_k(f) = 2L_s||X_k||^2$$  \hspace{1cm} (2.26)

where, \(L_s = (N \cdot \Delta t)\) and \(\Delta t\) is the time interval between each time point in \(\{x_r\}\). The PSD defined in this way takes the energy information from the time series but discards the phase information.

The computation of the discrete Fourier transform of Equation 2.25 is time consuming, especially when \(N\) is big. The Fast Fourier Transform (FFT) is therefore generally adopted for this computation. The methodology is that the work can be performed by partitioning the whole sequence \(\{x_r\}\) into a number of shorter sequences. Then, combination of these subsequences together will yield the full DFT of the original sequence.

Suppose that \(\{x_r\}, r = 0,1,2,\cdots (N-1)\) is the sequence where \(N\) is an even number and that this is partitioned into two shorter sequences \(\{y_r\}\) and \(\{z_r\}\) where\[\begin{align*}
  y_r &= x_{2r} \\
  z_r &= x_{2r+1}
\end{align*}\]

The DFT's of these two short sequences are \(Y_k\) and \(Z_k\) given as:

\[
\begin{align*}
  Y_k &= \frac{1}{N/2} \sum_{r=0}^{N/2-1} y_r e^{-i2\pi\left(\frac{kr}{N}\right)} \\
  Z_k &= \frac{1}{N/2} \sum_{r=0}^{N/2-1} z_r e^{-i2\pi\left(\frac{kr}{N}\right)}
\end{align*}\]

(2.27)

Recombination of Equation 2.25 would give:

\[
X_k = \frac{1}{N} \sum_{r=0}^{N-1} x_r e^{-i2\pi\left(\frac{kr}{N}\right)} = \frac{1}{N} \left\{ \sum_{r=0}^{N/2-1} x_{2r} e^{-i2\pi\left(\frac{kr}{N}\right)} + \sum_{r=0}^{N/2-1} x_{2r+1} e^{-i2\pi\left(\frac{(k+1)r}{N}\right)} \right\}
\]

(2.28)

It is found from Equation 2.27 and 2.28 that

\[
X_k = \frac{1}{2} \{Y_k + e^{-i(2\pi k/N)}Z_k\} \hspace{1cm} k = 0,1,2,\cdots,(N/2 - 1)
\]

(2.29)
The DFT of the original sequence can therefore be obtained directly from the DFT's of the two half-sequences $Y_k$ and $Z_k$ according to Equation 2.29. If the original length $N$ of sequence $\{x_r\}$ is a power of 2, then the half-sequences $\{y_r\}$ and $\{z_r\}$ may themselves be partitioned into quarter-sequences, and so on, until eventually the last sub-sequences have only one term each. As $Y_k$ and $Z_k$ are periodic in $k$ with period $N/2$, the full computation is[73]:

$$X_k = \frac{1}{2}\{Y_k + W^kZ_k\}$$

$$X_{k+N/2} = \frac{1}{2}\{Y_k - W^kZ_k\}$$

in which $W = e^{-i(2\pi/N)}$

2.2.6 Statistics in the frequency domain

For the purpose of this thesis the spectrum is characterised by its moments as shown in Figure 2.2. Define the n-th moment of the PSD of process $y(t)$

$$M_n = \int_{-\infty}^{\infty} \omega^n S_y(\omega) d\omega$$

$$m_n = \int_{0}^{\infty} f^n G(f) df = \sum_{k=1}^{M} f_k^n G_k(f_k) \Delta f$$

where $\omega$ (radian frequency) and $f$ (in Hz) are frequencies, $S(\omega)$ and $G(f)$ are double and single-sided PSDs, $M_n$ and $m_n$ ($n=0,1,\ldots$) are nth moments of the PSD, $M$ is the total number of components in the discretised PSD, $\Delta f$ is the frequency interval. $M_n$ and $m_n$ ($n=0,1,\ldots$) are related by

$$M_n = (2\pi)^n m_n$$

The even order moments can be calculated from both one-sided and two-sided PSD's. In theory, the odd order moments for double-side PSD are usually zero because of the symmetry of double-side PSD. In practice, the odd order moments are usually defined for one-sided PSD's.
Figure 2.2: PSD moments calculation
From equations 2.23 and 2.32, the zeroth order moment from the spectrum gives
the standard deviation of the original process. The 2\textsuperscript{nd} order moment gives the
standard deviation of the first order differential process.

Rice [78] and Bendat [10] worked out statistics of the spectrum. Some details
related to the calculation of these statistics are shown later in this chapter. If the
signal is stationary, ergodic and Gaussian, results were produced for the number of
zero crossings and number of peaks per unit time.

The number of zero crossings and number of peaks per unit time (peak rate) are
given as:

\[ E[0] = \sqrt{m_2/m_0} \]  \hspace{1cm} (2.34)
\[ E[P] = \sqrt{m_4/m_2} \]  \hspace{1cm} (2.35)

Using the number of zero crossings and number of peaks, the \textit{irregularity factor}
is defined as:

\[ \gamma = E[0]/E[P] = m_2/\sqrt{m_0 m_4} \]  \hspace{1cm} (2.36)

The irregularity factor is generally taken as an indication of the frequency band
width of the signal and its spectrum. It can take any value between 0.0 and 1.0.
When \( \gamma \) approaches 1.0, the signal becomes more like a regular sine wave. In this
limiting case the signal is said to be \textbf{narrow band} and its probability density
function of peaks becomes Rayleigh; cycle counting in this case is relatively easy.
As the irregularity factor approaches 0.0 the signal becomes more like white noise. In
this limiting case the signal is said to be completely \textbf{wide band}, and its probability
density function of peaks becomes Gaussian. In practice the response is rarely
narrow nor completely wide band but somewhere between.

In some circumstances, the centroid of the spectrum is taken as a measurement
of the frequency level of the spectrum and is defined as “mean frequency” and is
made dimensionless by normalisation using the peak rate [28].

\[ f_m = \frac{m_1}{m_0 \cdot E[P]} \]
2.3 Response Spectrum of Dynamic System

From the theory of linear vibrations, the motion equations of a linear structure subjected to dynamic loading can be written as [25]:

\[ M\ddot{v} + C\dot{v} + Kv = F(\Omega, t) \]  (2.37)

where \( M, C, \) and \( K \) are mass, damping, and stiffness matrices respectively, \( v \) is the structural response, \( \Omega \) is the space in which the structure is defined. Under the circumstances that the structure is under the action of deterministic dynamic loading, structural analysis can be performed in the time domain using suitable numerical integration methods, such as Newmark method, Wilson-\( \theta \) method, etc.

For the case of arbitrary dynamic loading, Duhamel’s integral equation can be taken as the general solution for structural response in time domain. For single degree of freedom systems, this integral can be expressed as:

\[ v(t) = \int_{0}^{t} p(\tau)h(t - \tau)d\tau \]  (2.38)

where, \( h(t - \tau) \) is the unit-impulse response function and is expressed as:

\[ h(t - \tau) = \frac{1}{m\omega} \sin \omega(t - \tau) \]  (2.39)

for undamped single degree of freedom systems.

Most systems are of multi-degree of freedom. If the loading in coordinate \( j \) is a general time varying load \( p_{j}(t) \), the dynamic response in coordinate \( i \) could be obtained by superposing the effects of a succession of impulses as specified using Duhamel’s integral, assuming zero initial conditions. The generalised expression for the response in coordinate \( i \) to the load at \( j \) is the integral as follows:

\[ v_{ij}(t) = \int_{0}^{t} p_{j}(\tau)h_{ij}(t - \tau)d\tau \quad i = 1, 2, \cdots, N \]  (2.40)

where \( N \) is the total number of degrees of freedom. \( h_{ij}(t) \) denotes the response at coordinate \( i \) to the unit-impulse loading in coordinate \( j \). The total response in
coordinate $i$ produced by a general loading involving all components of the load vector $p(t)$ is obtained by summing the contributions from all load components:

$$v_i(t) = \sum_{j=1}^{N} \left[ \int_{0}^{t} p_j(\tau) h_{ij}(t-\tau) d\tau \right] \quad i = 1, 2, \ldots, N \quad (2.41)$$

This type of time domain analysis is complex and time consuming but is possible. However, if the loading is stochastic instead of deterministic in type, this analysis method is generally invalid because the history of the loading is unknown except for some statistical characteristics. Frequency domain analysis then becomes quite useful.

To perform a frequency domain analysis, the variables (load and response) must be expressed as Fourier series. For the periodic loading $p(t)$ with period $T$, the Fourier series form is given by Equations 2.42 and 2.43.

$$p(t) = \sum_{n=-\infty}^{\infty} P(i\omega_n) e^{i\omega_n t} \quad (2.42)$$

where the complex amplitude coefficients are given by

$$P(i\omega_n) = \frac{1}{T} \int_{-T/2}^{T/2} p(t) e^{-i\omega_n t} dt \quad (2.43)$$

with $n\omega_1 = \omega_n = n \cdot 2\pi / T$.

When $p(t)$ is an arbitrary nonperiodic loading, the above equations can still be used by letting $T \to \infty$, as in integral form

$$\begin{cases} p(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(i\omega) e^{i\omega t} d\omega \\ P(i\omega) = \int_{-\infty}^{\infty} p(t) e^{-i\omega t} dt \end{cases} \quad (2.44)$$

which is the Fourier transform pair.

The Fourier series expression of the response can then be connected with the Fourier series of the loading by the complex frequency response function $H(i\omega)$, which is the inverse Fourier transform of $h(t)$, as

$$V(i\omega) = \int_{-\infty}^{\infty} v(t) e^{-i\omega t} dt = H(i\omega) P(i\omega)$$
When the case of multi-degree of freedom is considered, the loading and its Fourier series become a vector \( p(t) \) and \( P(\omega) \), and the frequency response function becomes a matrix \( H(\omega) \). Their components in coordinate \( j \) are:

\[
P_j(\omega) = \int_{-\infty}^{\infty} p_j(t)e^{-i\omega t} dt
\]

and

\[
V_{ij}(\omega) = \int_{-\infty}^{\infty} v(t)e^{-i\omega t} dt = H_{ij}(\omega)P_j(\omega)
\]  \hspace{1cm} (2.45)

The response in coordinate \( i \) can be obtained as

\[
v_i(t) = \sum_{j=1}^{N} v_{ij}(t) = \sum_{j=1}^{N} \left[ \int_{0}^{t} p_j(\tau)h_{ij}(t-\tau)d\tau \right]
\]  \hspace{1cm} (2.46)

The unit impulse and the complex frequency response functions are two transfer function in time and frequency domains respectively. They are related as a Fourier transform pair:

\[
\begin{align*}
H(\omega) &= \int_{-\infty}^{\infty} h(t)e^{-i\omega t} dt \\
h(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{H(i\omega)}{i\omega} e^{i\omega t} d\omega
\end{align*}
\]  \hspace{1cm} (2.47)

In multi-degree of freedom cases, this relationship becomes:

\[
\begin{align*}
H_{ij}(\omega) &= \int_{-\infty}^{\infty} h_{ij}(t)e^{-i\omega t} dt \\
h_{ij}(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{H_{ij}(i\omega)}{i\omega} e^{i\omega t} d\omega
\end{align*}
\]  \hspace{1cm} (2.48)

From the point of view of the spectrum, the input and output spectrum are connected by the relationship

\[
S_u(\omega) = H(-i\omega)H(i\omega)S_p(\omega) = |H(\omega)|^2S_p(\omega)
\]  \hspace{1cm} (2.49)

### 2.4 Level Crossing and Distribution of Extrema

The expected number of zero crossings of a stochastic process \( X(t) \) within a given interval \([t_1, t_2]\) or, more generally, the expected number of crossing of \( X(t) \) at some arbitrary level \( x_0 \) in \([t_1, t_2]\) is of considerable importance in spectral fatigue analysis.
When Miner's rule is used for random fatigue analysis, the expected number of peaks in unit time is also an important parameter for the damage accumulation computation. Together with the zero crossing rate, it should be calculated from the spectrum[87].

2.4.1 Level crossing

Suppose there is a random process \( y(t) = \{Y(\xi, t), \xi \in S, t \in T \} \), we are seeking the number of \( y(t) = a \) within \([t_1, t_2] \subset T\), denoted as \( n_a(t_1, t_2) \).

Since \( y(t) \) is random, \( n_a \) is random as well. For any determined \( a, t_1 \) and \( t_2 \), \( n_a \) is a random variable. We need to calculate its statistical values, i.e., mean value, deviation, etc.

A sample record is shown in Fig.2.3(a). Introduce a random 0-1 process \( z_a(t) \), corresponding to \( y(t) \geq a \) (Fig.2.3(b)):

\[
z_a(t) = u[y(t) - a]
\]  \hspace{1cm} (2.50)

where \( u(y) \) is unit step function.

Derivative \( z_a(t) \) with respect to \( t \), we get a series unit impulse function (Dirac delta function) (Fig.2.3(c)) \( \delta(t - t_j) \), where \( t_j \) \((j=1,2,...)\) are the crossing times.

\[
\dot{z}_a(t) = \left\{ \frac{d}{dy}u[y(t) - a] \right\} \cdot \frac{d}{dt}[y(t) - a] = \dot{y}(t)\delta[y(t) - a] 
\] \hspace{1cm} (2.51)

From the properties of Delta functions, \( \delta(at) = \frac{1}{|a|}\delta(t) \) \((a \text{ is a non-zero constant})\), the above equation becomes

\[
\dot{z}_a(t) = \sum_j \text{sign}[\dot{y}(t_j)]\delta(t - t_j) 
\] \hspace{1cm} (2.52)

where \( \dot{y}_j > 0 \) corresponds to upper-crossing, \( \dot{y}_j < 0 \) corresponds to down-crossing, \( \text{sign}(\cdot) \) is a sign function.
The total number of level crossings \((y = a)\) within \([t_1, t_2]\) is

\[
n_a(t_1, t_2) = \int_{t_1}^{t_2} |\dot{y}(t)| \delta[y(t) - a]dt \tag{2.53}
\]

The ensemble average of \(n_a(t_1, t_2)\) is

\[
N_a(t_1, t_2) = E[n_a(t_1, t_2)] = \int_{t_1}^{t_2} dt \int_{-\infty}^{+\infty} |\dot{y}(t)| \delta[y(t) - a]p(y, \dot{y}, t)dyd\dot{y} = \int_{t_1}^{t_2} dt \int_{-\infty}^{+\infty} |\dot{y}(t)|p(y = a, \dot{y}, t)d\dot{y} \tag{2.54}
\]
where \( p(y, \dot{y}, t) \) is the joint probability density function of \( y, \dot{y} \) at time \( t \). For a stationary process, \( p(y, \dot{y}, t) \) is independent of \( t \).

Denote

\[
\nu_a(t) = \int_{-\infty}^{+\infty} |\dot{y}(t)| p(y = a, \dot{y}, t) d\dot{y}
\]  

(2.55)

This is the ensemble average level crossing frequency (times/unit time) at time \( t \). In general it is the function of \( t \).

For an upper crossing,

\[
\nu_a^+(t) = \int_{0}^{+\infty} \dot{y}(t) p(y = a, \dot{y}, t) d\dot{y}
\]  

(2.56)

For a down crossing,

\[
\nu_a^-(t) = \int_{-\infty}^{0} (-\dot{y}(t)) p(y = a, \dot{y}, t) d\dot{y}
\]  

(2.57)

In particular, for a stationary process,

\[
\nu_a^+(t) = \int_{0}^{+\infty} \dot{y}(t) p(y = a, \dot{y}) d\dot{y}
\]  

(2.58)

\[
\nu_a^-(t) = \int_{-\infty}^{0} (-\dot{y}(t)) p(y = a, \dot{y}) d\dot{y}
\]  

(2.59)

As \( y(t) \) is continuous, the difference between \( \nu_a^+ \) and \( \nu_a^- \) is less than \( \frac{1}{t_2 - t_1} \). They can, in general, be regarded as equal.

Furthermore, for a Gaussian process, \( y \) and \( \dot{y} \) are independent and level crossing frequency is independent of time \( t \),

\[
\nu_a^+ = \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{1}{2}(\frac{a}{\sigma_y})^2} \int_{0}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_y} \dot{y} e^{-\frac{1}{2}(\frac{\dot{y}}{\sigma_y})^2} d\dot{y} = \frac{1}{2\pi\sigma_y} e^{-\frac{1}{2}(\frac{a}{\sigma_y})^2}
\]  

(2.60)
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For zero-crossing, \( a = 0 \),

\[
E[0] = \nu_0^+ = \frac{1}{2\pi} \frac{\sigma_y^2}{\sigma_y} \quad (2.61)
\]

This is sometimes called \textit{Statistical Average Frequency}.

For narrow band Gaussian process with PSD

\[
S(\omega) = \begin{cases} 
S_0 & \forall \omega_1 \leq \omega \leq \omega_2 \\
0 & \text{Others}
\end{cases}
\]

\[
\sigma_y^2 = \frac{2}{3}(\omega_2^3 - \omega_1^3) \approx 2S_0\omega_0^2\Delta_0 \quad (2.62)
\]

\[
\sigma_y^2 \approx 2S_0(\omega_2 - \omega_1) \quad (2.63)
\]

therefore,

\[
\nu_0^+ = \frac{1}{2\pi} \sqrt{\frac{\omega_2^2 - \omega_1^2}{3(\omega_2 - \omega_1)}} \approx \frac{\omega_0}{2\pi} \quad (2.64)
\]

Similarly, the variance of \( n_a(t_1, t_2) \) is

\[
E[(n_a(t_1, t_2))^2] = \int_{t_1}^{t_2} dr_1 \int_{t_1}^{t_2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\hat{y}(\tau_1)| \cdot |\hat{y}(\tau_2)| \\
\cdot \ p(y(\tau_1), \hat{y}(\tau_1), \tau_1; y(\tau_2), \hat{y}(\tau_2), \tau_2)dy(\tau_1)dy(\tau_2) \quad (2.65)
\]

where \( p(y(\tau_1), \hat{y}(\tau_1), \tau_1; y(\tau_2), \hat{y}(\tau_2), \tau_2) \) is the joint probability density function of \( y(\tau_1), \hat{y}(\tau_1), \tau_1; y(\tau_2), \hat{y}(\tau_2), \tau_2 \). If it is stationary, \( p(\cdot) \) will be independent of \( \tau_1, \tau_2 \).

A less mathematical derivation can be found in [74].

\subsection*{2.4.2 Zero crossings and peak frequency}

Results for zero crossing can be derived by simply taking it as a special case of level crossing, i.e., set \( a = 0 \). The positive zero crossing rate for a stationary signal is

\[
E[0] = \nu_0^+ = \frac{1}{2\pi} \frac{\sigma_y^2}{\sigma_y} \quad (2.66)
\]
For narrow band Gaussian process,

\[
E[0] = \nu_0^+ \approx \frac{\omega_0}{2\pi} \quad (2.67)
\]

Notice that for every peak value for \( y(t) \), there is a negative zero-crossing for \( \dot{y}(t) \), therefore the number of peaks in unit time (peak frequency) is

\[
E[P] = \frac{1}{2\pi} \frac{\sigma_{\dot{y}}}{\sigma_\dot{y}} \quad (2.68)
\]

Notice that

\[
\sigma_y^2 = \int_{-\infty}^{\infty} S_y(\omega) d\omega = M_0 = m_0
\]

\[
\sigma_{\dot{y}}^2 = \int_{-\infty}^{\infty} \dot{S}_y(\omega) d\omega = \int_{-\infty}^{\infty} \omega^2 S_y(\omega) d\omega = M_2 = (2\pi)^2 m_2
\]

\[
\sigma_{\ddot{y}}^2 = \int_{-\infty}^{\infty} \ddot{S}_y(\omega) d\omega = \int_{-\infty}^{\infty} \omega^4 S_y(\omega) d\omega = M_4 = (2\pi)^4 m_4
\]

therefore the zero-crossing and peak frequencies are

\[
E[0] = \frac{1}{2\pi} \sqrt{\frac{M_2}{M_0}} = \sqrt{\frac{m_2}{m_0}} \quad (2.69)
\]

\[
E[P] = \frac{1}{2\pi} \sqrt{\frac{M_4}{M_2}} = \sqrt{\frac{m_4}{m_2}} \quad (2.70)
\]

Define irregularity factor

\[
\gamma = \frac{E[0]}{E[P]} \quad (2.71)
\]

therefore

\[
\gamma = \frac{m_2}{\sqrt{m_0 m_4}} = \frac{M_2}{\sqrt{M_0 M_4}} \quad (2.72)
\]

\( \gamma \) is a parameter describing the width distribution of the process. Notice that \( E[0] > 0, E[P] > 0 \) and \( E[0] \leq E[P] \), the value of \( \gamma \) is always in the range of \((0,1)\). When \( \gamma \) approaches 1, the process is narrow-banded. When \( \gamma \) approaches 0, the process is wide-banded.
2.4.3 Distribution of extrema

For stationary, Gaussian, zero-mean-value random processes, the p.d.f. of its peak value\[62, eq.(9-36a)\] \(p(s)\) (i.e., the probability that its peak value falls in \([s, s + ds]\)

\[
p(s) = \frac{1}{\sqrt{2\pi m_0}} \exp\left\{-\frac{s^2}{2m_0(1 - \gamma^2)}\right\} + \frac{\gamma s}{2m_0} \left\{1 + \text{sign}(s) \cdot \text{erf}\left[\frac{\gamma s}{\sqrt{2m_0(1 - \gamma^2)}}\right]\right\} \exp\left(-\frac{s^2}{2m_0}\right) \tag{2.73}
\]

where \(\gamma = \frac{m_0}{\sqrt{m_0 m_4}} \leq 1\) is the irregularity factor; \(m_0 = \sigma_x^2\) is the variance (0-th moment of the Power Spectral Density); \(\text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-u^2} du\) is the error function; \(\text{sign}(s)\) is sign function:

\[
\text{sign}(s) = \begin{cases} 
1 & \text{if } s > 0 \\
0 & \text{if } s = 0 \\
-1 & \text{if } s < 0
\end{cases}
\]

Furthermore, if the process is narrow-banded, i.e., \(\gamma = \frac{m_0}{\sqrt{m_0 m_4}} \approx 1\), then from eq.(6.43),

\[
p(s) \approx \frac{s}{m_0} e^{-\frac{s^2}{2m_0}} \tag{2.74}
\]

This is Rayleigh distribution with mean value \(\sqrt{\pi m_0}/2\) and variance \(\frac{4 - \pi}{2} m_0\).

2.5 Stress (Strain) cycle counting

The definition of stress cycle is the first problem encountered in random fatigue analysis. The cycle counting process is actually trying to find a set of sinusoidal cycles which has the same fatigue damage as the original stochastic sequence. Up to now, there are more than ten types of counting methods which have been reported in the literatures \[63][37][80][12\]. Some of them are listed below.
(i) **Peak count method.** The number of peaks and/or troughs at particular levels are counted.

(ii) **Mean-crossing peak count method.** As (i) above except that only the maximum peak or minimum trough is counted between zero crossings.

(iii) **Ordinary range count.** The height of ranges between adjacent peaks and troughs is counted. From this a probability density of ordinary ranges can be calculated.

(iv) **Range-mean Count.** This method is identical to (iii), except that the mean value of each ordinary range is also counted.

(v) **Level crossing count.** The number of upwards (or downwards) crossings of particular levels are counted.

(vi) **Fatiguemeter count.** A technique developed in the aeronautics industry to measure variations of acceleration. This is a similar technique to (v) except that small variations in the signal, such as noise, are removed by using a gate or trigger level. Signal excursions from the previous recorded level are only recorded if the trigger level is exceeded [89].

(vii) **Range-pair count** [63].

(viii) **Wetzel’s method** [95].

(ix) **Rainflow method** [68][33].

Among all of them, the last three have generally been accepted as better methods of calculating fatigue damage from random signals [30]. Among the last three, the rainflow counting method is now widely accepted as the one which gives the most consistent prediction compared to the actual life result [30][79]. It is for this reason that, rainflow cycle counting is accepted as the default counting method in the whole of this thesis.
2.6 Rainflow Cycle Counting

The original Rain-Flow-Cycle definition (see below) is rather complicated and difficult to analyse, some alternative Rain-Flow-Cycle counting algorithms (which are equivalent to the original definition), such as [32], have been proposed. However, these procedures have a complicated “sequential” structure, which makes them difficult to apply when their statistical properties are to be studied.

2.6.1 Original definition

Definition The Rain-Flow-Cycle counting method is illustrated in Fig. 2.4. The strain/stress-time history is plotted so that the time axis is vertically downward, and the lines connecting the strain peaks are imagined to be a series of pagoda roofs. Several rules are imposed on rain dripping down these roofs so that cycles and half cycles are defined. Rain flow initiated at a peak is allowed to drip down and continue except that, if it is initiated at a minimum, it must stop when it comes opposite a minimum more negative than the minimum from which it was initiated. Similarly, if the rain flow initiates at a maximum, it must stop when it comes opposite a maximum more positive than the maximum from which it was initiated. A rain flow must also stop if it meets rain from a roof above. Finally, one pairs a maximum-originating rainfall with a minimum-originating rainfall, of the same amplitude, to form a full Rain-Flow-Cycle. The strain ranges which cannot be paired in that way are regarded as half cycles.

2.6.2 Alternative definitions

Rychlik [79] proposed an alternative definition of Rain-Flow-Cycle counting method and proved that it is equivalent to the original definition (Fig.2.5). The new definition can be stated as
**Definition 1** Let \( y(s), -T < s < T \), be the load function, and suppose it has a local maximum at time \( t \). Let \( t^+ \) be the time of the first upcrossing after \( t \) of the level \( y(t) \), (or \( t^+ = T \) if no such upcrossing exists for \( t < s < T \)), and Let \( t^- \) be the time of the last downcrossing before \( t \) of the level \( y(t) \), (or \( t^- = -T \) if no such downcrossing exists for \( -T < s < t \)). Then the Rain-Flow-Cycle amplitude originating at \((t, y(t))\) is defined by

\[
S(t) = \min \left( \max_{t^- < s < t} (y(t) - y(s)), \max_{t < s < t^+} (y(t) - y(s)) \right)
\]

If the load history is a stationary ergodic time signal, a symmetry about \( t = 0 \) exists. For this reason, another restriction of \( S^- \geq S^+ \) can be applied to the definition. Every cycle counted then should be considered as two cycles with the same amplitude. This modified the definition as [12]:

**Definition 2** For a rainflow cycle valued \( S \) to exist at a current peak, the signal must have the following configuration as in Figure 2.6:

i). takes the signal forwards (+ve time) from point 1 to point 2, a distance \( S \) below it.
ii). takes the signal forwards from point 2 to point 3, some level at or above point 1.

iii). takes the signal backwards (-ve time) from point 1 to point 4, some level at or below point 2.

iv). takes the signal backwards from point 4 to point 5, some level at or above point 1.

However, for stationary ergodic signals, when considering the long term distribution of the signal, event iv) of this definition is redundant. Because if the signal comes from below the level of point 2 there is a probability of 1.0 that it originally
came from a level above point 1 prior to this (given that it could go to any level below point 2 during this process).

Based on the above analysis, a new definition was made by Bishop[13] and is given as below (Figure 2.7):

**Definition 3.** For a rainflow cycle valued $S$ to be defined from a particular peak the following events must happen:

i). $Y_1$ The signal must have come from a level at least $S$ below the level of point 1 without at any time going above the level of point 1 (with any number of extreme points in between).

ii). $Y_2$ The signal must then go from the level of point 1 to some level a distance $S$ below without at any time between going back to the level of point 1 or below the level of point 2 (with any number of extreme points in between).

iii). $Y_3$ The signal must then go from the level of point 2 to some level at or above point 1 without at any time going back to the level of point 2 (with any number of extreme points in between).

The essential idea of rainflow cycle counting is to characterise the stress (or strain) history over a long time period. That is, to allow the hysteresis cycle to be closed after a long time interval by using the stress-strain “memory” information. The transitions in-between can then be processed separately. The advantage is, therefore, that large cycles which can be missed very easily by ordinary counting methods are included by this method. Figure 2.8 shows such an example. Figure 2.8(a) shows a typical strain history. The initial strain excursion, from 0 to A, uses the cyclic stress-strain curve. The strain range 0 to A is plotted on the strain axis, and the stress at point A is calculated from the equation for the cyclic stress-strain curve. Point A is then taken as an origin, the stress range from A to B is then calculated from the hysteresis curve. The actual stress at B then is obtained by
subtracting the stress range A to B from the value at point A. By continuing this plotting process until the end of the local strain history, the stress-strain hysteresis history can be derived as shown in Figure 2.8(b).

As seen from this stress-strain history, apart from two closed hysteresis loops ranged as E-F and B-C, there exists a large cycle which has the range A-D, which could easily be missed by other counting methods.
(a). A typical local train history

(b). The stress-strain hysteresis loop for strain history (a)

Figure 2.8: An example of rainflow cycle counting
Chapter 3

EXISTING FREQUENCY DOMAIN METHODS

There are several existing frequency domain fatigue damage methods. Analytical results are available for narrow band and some special wide-band stationary Gaussian processes. Empirical formulae (mostly lead to the correction factors) and theoretical method also exist for general stationary processes. This chapter presents the existing methods. Some results have been extended by the author.

3.1 Analytical Solutions

3.1.1 Steinberg’s three-band technique

As a rough estimation for the fatigue damage, Steinberg proposed a three-band technique[88]. The basis for this method is the Gaussian distribution. The instantaneous stresses (or accelerations) between $+1\sigma$ ($\sigma$ is the root mean square) and $-1\sigma$ are assumed to act at the $1\sigma$ level 68.3% of the time. The instantaneous stresses (or accelerations) between $+2\sigma$ and $-2\sigma$ are assumed to act at the $2\sigma$ level 95.4-68.3, or 27.1% of the time. The instantaneous stresses (or accelerations) between $+3\sigma$ and $-3\sigma$ are assumed to act at the $3\sigma$ level 99.73-95.4, or 4.33% of the time. Stresses larger than $+3\sigma$ or smaller than $-3\sigma$ are ignored.

The fatigue damage is thus estimated according to the $S - N$ curve. The corresponding accumulated fatigue damage is
\[ E[D] = E[P] \frac{T}{c} S_h^b \]  
(3.1)

where the equivalent stress parameter is

\[ S_h = 2\sqrt{m_0}[0.683 + 0.271 \cdot 2^b + 0.043 \cdot 3^b]^\frac{1}{b} \]  
(3.2)

This method is easy to use. However, it can only roughly estimate the fatigue life as stresses with amplitude less than \(3\sigma\) are enlarged, while stresses with amplitude larger than \(+3\sigma\) are ignored. This is inaccurate in terms of fatigue estimation.

### 3.1.2 Narrow band solution

For stationary, Gaussian, zero-mean-value random processes, the p.d.f. of its peak value[62, eq.(9-36a)] \(p(s)\) (i.e., the probability that its peak value falls in \([s, s + ds]\) is \(p(s)\)\(ds\))

\[ p(s) = \sqrt{\frac{1 - \gamma^2}{2\pi m_0}} \exp \left\{-\frac{s^2}{2m_0(1 - \gamma^2)} \right\} \]

\[ + \frac{\gamma s}{2m_0} \left\{ 1 + \text{sign}(s)erf \left( \frac{\gamma s}{\sqrt{2m_0(1 - \gamma^2)}} \right) \right\} \exp \left\{-\frac{s^2}{2m_0} \right\} \]  
(3.3)

where \(\gamma = \frac{m_0}{\sqrt{m_0 m_4}} \leq 1\) is the irregularity factor; \(m_0 = \sigma^2\) is the variance (0-th moment of the Power Spectral Density); \(erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-u^2} du\) is the error function; \(\text{sign}(\cdot)\) is sign function.

For the ‘normalised’ peak value \(a = \frac{s}{\sqrt{m_0}}\), notice the Jacobian is \(J = \sqrt{m_0}\), the probability density function is

\[ p(a) = \sqrt{\frac{1 - \gamma^2}{2\pi}} \exp \left\{-\frac{a^2}{2(1 - \gamma^2)} \right\} \]

\[ + \frac{\gamma a}{2} \left\{ 1 + \text{erf} \left( \frac{\gamma a}{\sqrt{2(1 - \gamma^2)}} \right) \right\} \exp \left(-\frac{a^2}{2} \right) \]  
(3.4)

Furthermore, if the process is narrow-banded, i.e., \(\gamma = \frac{m_0}{\sqrt{m_0 m_4}} \approx 1\), then from equation 3.3,
This is Rayleigh distribution. $E[P]$ (the expected number of peaks per unit time) is approximately equal to $E[0]$, the expected number of (positive) zero crossings per unit time. (Plottings of the distribution of $p(s)$ can be found in chapter 6, Fig.6.6). Notice that for narrow band processes, the stress range $S = 2 \cdot s$, Jacobian $J = \frac{ds}{dS} = \frac{1}{2}$, then

$$p(S) = \frac{S}{4m_0} e^{-\frac{S^2}{8m_0}}$$

therefore $E[D]$ reduces to

$$E[D]_{NB} = E[0] \frac{T}{c} \int_0^{\infty} S^b \left[ \frac{S}{4m_0} e^{-\frac{S^2}{8m_0}} \right] dS = E[0] \frac{T}{c} (2\sqrt{2m_0})^b \Gamma(1 + \frac{b}{2})$$

where $\Gamma(z) = \int_0^\infty u^{z-1} e^{-u} du = \int_0^1 (-\ln u)^{z-1} du$ is Gamma function.

This result was first derived by Miles[70].

When a Gaussian process is wide-banded, however, the Rayleigh stress range distribution assigns larger probabilities to larger stress ranges. When $b \geq 1$, the above equation predicts a conservative estimation of the mean damage[99]. Some analytical solutions have been derived for special cases of wide band random processes.

For the generalised form of $S-N$ curve, i.e, it has three staged log-log straight lines with different slopes (Fig. 3.1),

$$N = \begin{cases} \frac{c_1S^{-b_1}}{} & \text{if } S \geq S_0 \\ \frac{c_2S^{-b_2}}{} & \text{if } S_0 \geq S \geq S_c \\ \infty & \text{if } S \leq S_c \end{cases}$$

the cumulated fatigue damage can be expressed in terms of Chi-square probability function [90, 91].
\[ E[D]_{NB} = E[P] \cdot T. \]
\[
\left\{ \frac{3b_1}{c_1} \Gamma \left( \frac{b_1}{2} + 1 \right) Q\left( \chi_0^2 | b_1 + 2 \right) + \frac{3b_2}{c_2} \Gamma \left( \frac{b_2}{2} + 1 \right) \left[ Q\left( \chi_0^2 | b_2 + 2 \right) - Q\left( \chi_0^2 | b_2 + 2 \right) \right] \right\}
\]
where \( \chi_0 = \frac{S_0}{2\sqrt{m_0}} \), \( \chi_c = \frac{S_c}{2\sqrt{m_0}} \); \( Q(\chi^2 | \nu) \) is chi-square probability function[6]:
\[
Q(\chi^2 | \nu) = \frac{1}{2^{\nu/2} \Gamma(\nu/2)} \int_{\chi^2}^{\infty} t^{\nu/2 - 1} e^{-t/2} dt = \frac{1}{\Gamma(\nu/2)} \int_{\chi^2}^{\infty} t^{\nu/2 - 1} e^{-t} dt
\]
(3.10)
Notice that \( Q(0 | \nu) = 1 \) and \( Q(\infty | \nu) = 0 \) and for narrow band process, \( E[P] \approx E[0] \), when \( b_1 = b_2 = b \) (hence \( c_1 = c_2 = c \) and \( S_c = 0 \), eq. 3.9 converges to eq. 3.7.

![Generalised S-N curve](image)

**Figure 3.1: Generalised S-N curve**

### 3.1.3 Wide band solution

Jiao and Moan[50] derived the analytical solution for the fatigue damage under stationary Gaussian processes with well-separated bimodal spectral density functions, which are the combinations of a low frequency Gaussian component and a high frequency one. The result leads to a correction factor for the fatigue damage rate:
\[ \rho = \frac{D_{WB}}{D_{NB}} = \frac{E_p[0]}{E_z[0]} \left[ a_1 \frac{b}{2} + (1 - \sqrt{\frac{a_2}{a_1}}) + \sqrt{\pi a_1 a_2} \frac{b \Gamma(\frac{1+b}{2})}{\Gamma(1 + \frac{b}{2})} \right] + \frac{E_h[0]}{E_z[0]} a_2^\frac{b}{2} \] (3.11)

where \(E_z[0]\) is the zero crossing frequency of the process; \(E_p[0]\) is the zero crossing frequency of the envelope component; \(E_h[0]\) is a parameter related to the zero crossing frequency of the high frequency component, \(a_1\) and \(a_2\) are the "normalised" variance of the low- and high-frequency component respectively. The expressions of these parameters are listed below:

\[
E_p[0] = a_1 \nu_1 \sqrt{1 + \frac{a_2}{a_1} (\frac{\nu_2}{\nu_1})^2} \\
E_z[0] = \sqrt{a_1 \nu_1^2 + a_2 \nu_2^2} \\
a_1 = \frac{m_{0,1}}{m_{0,1} + m_{0,2}} \\
a_2 = \frac{m_{0,2}}{m_{0,1} + m_{0,2}} \\
\nu_1 = \frac{m_{2,1}}{m_{0,1}} \\
\nu_2 = \frac{m_{2,2}}{m_{0,2}} \\
\delta_2 = \sqrt{1 - \frac{m_{0,2}^2}{m_{0,2}/m_{2,2}}}
\]

\(m_{i,j}, \ i = 0,1,2, \ j = 1,2\) are the spectral moments in the two spectral bands (j=1,2).

Yang[102] generalized Miles’ result. He introduced a correlation parameter \(k\), when \(k\) approaches to 1, his result converges to Miles’.

Another similar derivation[20] is based on the definition of a so-called double envelope process. The result leads to the correction factor:

\[ \rho = \gamma^{b-1} \] (3.12)
Obviously, when $\gamma \rightarrow 1$ or $b \rightarrow 1$, then $\rho \rightarrow 1$. When $\gamma$ is small and $b$ is large, this will be very unconservative, leads to a much longer fatigue life than narrow band solution. For example, $\gamma = 0.35$, $b = 9$, $\rho = \frac{1}{441}$. This is a dangerous prediction for a design engineer.

### 3.1.4 Tunna’s formula

Tunna[90] [91] derived the analytical expression of damage rate for narrow band Gaussian loads situations while the S-N curve is made up from 3 staged log-log straight lines with different slopes,

$$N = \begin{cases} 
c_2 S^{-b_2} & \text{if } S \geq S_0 \\
c_3 S^{-b_3} & \text{if } S_0 \geq S \geq S_c \\
\infty & \text{if } S \leq S_c 
\end{cases} \quad (3.13)$$

the cumulated fatigue damage can be expressed in terms of Chi-square probability function:

$$E[D]_{NB} = E[P] \cdot T \cdot \left\{ \frac{3b_1}{c_1} \Gamma\left(\frac{b_1}{2} + 1\right)Q(\chi_0^2|b_1 + 2) + \frac{3b_2}{c_2} \Gamma\left(\frac{b_2}{2} + 1\right)\left[ Q(\chi_c^2|b_2 + 2) - Q(\chi_0^2|b_2 + 2) \right] \right\}$$

where $\chi_0 = \frac{S_0}{2\sqrt{m_0}}$, $\chi_c = \frac{S_c}{2\sqrt{m_0}}$; $Q(\chi^2|\nu)$ is chi-square probability function[6]:

$$Q(\chi^2|\nu) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} \int_{\chi^2}^{\infty} t^{\nu/2-1} e^{-\frac{1}{2}t} dt = \frac{1}{\Gamma(\nu)} \int_{\chi^2}^{\infty} t^{\nu/2-1} e^{-t} dt \quad (3.15)$$

A special case is the single slope (S-N curve) narrow band solution (eq. 3.7).

For wide band signal, Tunna applied the range-mean cycle counting and derived an improved Cycle Range PDF

$$p_{\text{tunna}}(S) = \frac{S}{4\gamma^2\sigma^2} \exp -\frac{S^2}{8\gamma^2\sigma^2} \quad (3.16)$$
where $S$ is cycle range, $\gamma$ is the irregularity factor. The damage rate can be similarly expressed as

$$E[D]_{\text{Tunna}} = E[P] \cdot T \cdot \frac{1}{c_1} \int_{S_0}^{\infty} S^{b_1} p(S) dS + \frac{1}{c_2} \int_{S_c}^{S_0} S^{b_2} p(S) dS$$

$$= E[P] \cdot T \cdot \left\{ \frac{2.5b b_1 \gamma^{b_1}}{c_1} \Gamma \left( \frac{b_1}{2} + 1 \right) Q(x_0^2 | b_1 + 2) + \frac{2.5b b_2 \gamma^{b_2}}{c_2} \Gamma \left( \frac{b_2}{2} + 1 \right) \left[ Q(x_0^2 | b_2 + 2) - Q(x_0^2 | b_2 + 2) \right] \right\}$$

where $x_0 = \frac{2S_0}{2\sqrt{m_0}}, \ x_c = \frac{2S_c}{2\sqrt{m_0}}.$ (3.17)

This formula actually contains two special cases: one is the narrow band solution as $\gamma \approx 1$ for narrow band signals. The other case is the single slope $S-N$ curve: $c_1 = c_2 = c, \ b_1 = b_2 = b, \ S_c = 0$. Notice the property of Chi-square probability function: $Q(0|\nu) \equiv 1$, $Q(\infty|\nu) \equiv 0$, eq.(3.17) reduces to

$$E[D]_{\text{Tunna}} = \frac{E[P] \cdot T}{c} (\sqrt{S} \gamma)^b \Gamma \left( \frac{b}{2} + 1 \right)$$

(3.18)

If we compare this with the Narrow Band solution, it is actually a solution with correction factor $\rho = \frac{E[D]_{\text{Tunna}}}{E[D]_{\text{NB}}}$ = $\gamma^b$. Take an example, for $\gamma = 0.75, \ b = 5$, the predicted fatigue life by Tunna’s method will be 4.214 times of Narrow Band solution. This method does not work well when $\gamma$ is small and $b$ is large. For $\gamma = 0.4, \ b = 10$, the predicted fatigue life will be 9537 times of Narrow Band solution, again a dangerous prediction.

### 3.2 Correction Factor Solutions—Empirical Formulæ

Several empirical formulæ[20, 17, 61, 18] have been proposed for the prediction of the Rain-Flow-Cycle range probability density function by introducing “Correction factors”, $\rho$, which is defined as $\rho = \frac{D_{WB}}{D_{NB}}$, i.e. the wide band damage divided by the damage obtained after assuming the process is narrow banded.
3.2.1 Wirsching’s formula

Wirsching et al. [99] derived an empirical expression of $E[D]$ for the wideband situation. The correction factor is,

$$\rho = \frac{E[D]}{E[D]_{NB}} = a(b) + [1 - a(b)](1 - \epsilon)^{\sigma(b)}$$

(3.19)

where $a(b) = 0.926 - 0.033b; c(b) = 1.587b - 2.323; \epsilon = \sqrt{1 - \gamma^2}$. $\gamma$ is the irregularity factor; $b$ is material constant.

3.2.2 Kam, Chaudhury and Dover

Their result [51, 53, 52, 24] leads to the equivalent stress range parameter, which is defined as $S_h = \left[\int_0^\infty S^b p(S) dS\right]^{\frac{1}{b}}$.

$$E[D] = E[P] \frac{T}{c} S_h^b$$

$$S_h = \left(2\sqrt{2m_0}\right) \left[\frac{\epsilon^{b+2}}{2\sqrt{\pi}} \Gamma\left(\frac{1 + b}{2}\right) + \frac{1 + erf(\gamma)}{2} \gamma \Gamma(1 + \frac{b}{2})\right] \frac{1}{b}$$

(3.20)

where the error function is approximated by $erf(\gamma) \approx 0.3012\gamma + 0.4915\gamma^2 + 0.9181\gamma^3 - 2.3584\gamma^4 - 3.3307\gamma^5 + 15.6524\gamma^6 - 10.7846\gamma^7$, for $0.1 < \gamma < 0.96$; $erf(\gamma) = 1$, for $\gamma \geq 0.96$; $\epsilon = \sqrt{1 - \gamma^2}$ is a parameter to describe the bandwidth.

3.2.3 Hancock

Two equations, developed by Hancock et al. [40], incorporate curve fitting parameters into the Weibull distribution, and thereby obtained the expression for equivalent stress are given below.

**Handcock A** :

$$S_h = 2\sqrt{2m_0}[\gamma \Gamma(\frac{b}{2} + 1)]^{\frac{1}{b}}$$

(3.21)
Handcock B:

\[ S_h = \gamma \sqrt{2} \frac{m_0}{(2 - \xi^2) \Gamma\left(\frac{b}{2} - \frac{b}{\xi^2} + 1\right)^\frac{1}{b}} \] (3.22)

in which \( \xi \) is the damping factor.

### 3.2.4 Madsen

The work of Madsen et al. [67] was derived specifically for application to wind turbines. It deals with signals which combine stochastic and deterministic loads. These two components of a signal are treated separately. For a random stationary process, damage is assumed to be a constant function of time, the value of the constant being dependent on the signal characteristics. For a zero-mean, Gaussian, narrow band stochastic process, a Rayleigh distribution of peaks applies and each range is taken as a half cycle as before. For this idealised signal, Madsen proposes a dimensionless damage parameter, which is equal to the actual damage of the signal, normalised by dividing by the damage due to a constant amplitude signal having the same rms. This was shown to be a function of the slope of \( S-N \) curve.

For more complex and realistic wide band processes, Madsen assumed a Gaussian distribution, and hence a solution for the distribution of the peaks. Refering to Wirsching [99], he uses the assumption that each peak will eventually pair with an equal and opposite trough to form a closed cycle. This is exceptionally conservative, and so a ‘bandwidth correction factor’ was proposed. Madsen proposed a formula, independent of \( S-N \) curve slope, and supported it by reference to published fatigue test data. His basis for the formula is a comparison of fatigue lives, rather than the signal statistics themselves.

For the deterministic component of load, the simple case of two superimposed sine waves is considered, and a constant amplitude sine wave of equivalent damage was proposed, as a function of \( S-N \) curve slope. Making the assumption of uniform distribution of phase between the two original sine waves, he fitted an empirical
relationship to calculate the equivalent constant amplitude stress range as a function of irregularity factor and $S-N$ curve slope.

For the general case of a sum of periodic and stochastic terms, Madsen built up from the special case of a sinusoid plus a narrow band stochastic process. He proposes to view the confluent hypergeometric function as an interpolating function between the purely periodic signal and the Gaussian stochastic signal. To allow for wide band processes, the periodic rms and stochastic rms were each corrected for irregularity.

theory

Throughout the theory of Madsen, the expected damage rate is defined as

$$E[D] = E[0](\frac{S_h}{k})^b$$

(3.23)

where $E[0]$ is the zero-crossing rate with positive slope, $k$ the stress intercept and $b$ the fixed inverse slope of $S-N$ curve, and $S_h$ an equivalent constant amplitude stress range calculated for the signal.

stochastic loading

For a purely stochastic signal, $X(t)$,

$$S_h = 2\sqrt{2} g_x(\gamma_x) \sigma_x [\Gamma(1 + \frac{b}{2})]^{1/b}$$

(3.24)

where the bandwidth correction term is defined as

$$g_x(\gamma) = 0.93 + 0.07\gamma^5$$

deterministic loading

The equivalent stress range for the deterministic component, $Z(t)$ is

$$S_h = 2\sqrt{2} g_x(\gamma_x) \sigma_x$$

(3.25)
with bandwidth correction factor

$$g_z(\gamma) = 1.24 - (0.325 - 0.025b)(2.2\gamma - \gamma^2)$$

The bandwidth parameter (irregularity factor) is defined as $\gamma = E[0]/E[P]$, where $E[0]$ is the number of mean up-crossing, and $E[P]$ the number of peaks, in unit time ($T_0 = 2\pi/\omega_0$). By defining $\gamma$ in this form, Madsen believed that some information on the relative phase between the Fourier components is retained, whereas all such information is lost in the spectral moment parameters.

The standard deviation is calculated as

$$\sigma_Z = \left( \frac{1}{2} \sum_{k=1}^{K} c_k^2 \right)^{1/2}$$

where $c_k$ is the amplitude of the sine waves.

**combined loading**

For the combined time history, $Y(t) = X(t) + Z(t)$,

$$S_h = 2\sqrt{2g_z(\gamma_y)\sigma_z}[\Gamma(1 + \frac{b}{2})M(-\frac{b}{2}, 1, -\beta^2)]^{1/b} \quad (3.26)$$

with $\beta = \frac{g_z(\gamma_y)\sigma_z}{g_z(\gamma_y)\sigma_z}$

$M(\cdot, \cdot, \cdot)$ is a confluent hypergeometric function satisfying

$$M(-\frac{b}{2}, 1, -\beta^2)|_{\beta=0} = 1$$

and

$$M(-\frac{b}{2}, 1, -\beta^2)|_{\beta=\infty} = \frac{\beta^b}{\Gamma(1 + \frac{b}{2})}$$

The combined model therefore also includes the previous models as special cases.

The correction for irregularity is applied using the bandwidth parameter,

$$\gamma_y = E[0]/E[P]$$
defined for the combined signal from

\[ \nu^y_0 = E[0]_x \cdot \frac{1}{T} \int_0^T \phi\left(\frac{r}{\sigma_x}\right) \xi\left(\frac{\dot{r}}{\sigma_z}\right) dt \]

and

\[ \nu^y_m = E[P]_x \cdot \frac{1}{T} \int_0^T \phi\left(\frac{\dot{r}}{\sigma_z}\right) \xi\left(\frac{\dot{r}}{\sigma_z}\right) dt \]

where

\[
\begin{cases}
\phi(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2} \\
\xi(t) = \phi(t) - t\psi(t) \\
\psi(t) = \int_{-\infty}^t \phi(p) dp = \text{erf}(t)
\end{cases}
\]

with \(\sigma_z = \sqrt{m_2}\) and \(\sigma_z = \sqrt{m_4}\)

The function \(r(\cdot)\) is derived by subtracting the deterministic time-history from the mean of the combined signal, \(\mu_y = c_0\), so that

\[ r(t) = \mu_y - Z(t) = -\sum_{k=1}^K c_k \sin(k\omega_0 t + \theta_k) \]

### 3.2.5 Ortiz/Chen

Their result leads to a correction factor which is the function of \(m_0, m_2, m_4\), together with two extra moments \(m_b\) and \(m_{2+b}\) (\(b\) is the inverse \(S-N\) curve slope) [20, ref(26)]

\[ \rho = \frac{1}{\gamma} \sqrt{\frac{m_2 m_b}{m_0 m_{2+b}}} \quad (3.27) \]

### 3.2.6 Larsen/Lutes

Larsen & Lutes\[65, 61\] proposed the single moment approach to the estimation of fatigue damage.

\[ \rho = m_b^{b/2} \quad (3.28) \]

Obviously, the special order of moment \(m_b\) must be first calculated.
3.2.7 Dirlik

Dirlik [28] conducted extensive computer simulation on the rainflow cycle distribution of random Gaussian stress histories. He studied a wide range of spectra of various shapes, with \( \gamma \) in the range 0.160 to 0.988 and 'mean frequency' from 0.103 to 0.907. Seventy Gaussian stationary stress histories were simulated from these spectra. Both countings for ordinary range and rainflow range cycles were performed on the simulated signals. The rainflow cycle PDF was modelled by using the following expression:

\[
q_{RR}(z) = C_1 e^{-\frac{z}{2}} + C_2 \frac{z}{\alpha^2} e^{-\frac{z^2}{2\alpha^2}} + C_3 z e^{-\frac{z^2}{2}}
\]  (3.29)

where \( z \) is a normalised cycle range parameter: \( z = \frac{S}{2\sqrt{m_0}} \), \( S \) is cycle range, \( \sqrt{m_0} \) is the RMS of the signal. \( q_{RR}(z) \) and \( q_{RR}(S) \) are related by

\[
q_{RR}(S) = \frac{q_{RR}(z = \frac{S}{\sqrt{m_0}})}{2\sqrt{m_0}}
\]  (3.30)

the parameters in the formula are all related to the first 4 moments \((m_0, m_1, m_2, m_4)\) of the PSD, they are

\[
\begin{align*}
\gamma &= \frac{m_2}{\sqrt{m_0 m_4}} \\
f_m &= \frac{m_1 \sqrt{m_2}}{m_0 m_4} \\
C_1 &= \frac{2\left(f_m - \gamma^2\right)}{1 + \gamma^2} \\
C_2 &= \frac{1 - \gamma - C_1 + C_1^2}{1 - \alpha} \\
C_3 &= 1 - C_1 - C_2 \\
\alpha &= \frac{\gamma - f_m - C_1^2}{1 - \gamma - C_1 + C_1^2} \\
\tau &= \frac{1.25(\gamma - C_3 - C_2\alpha)}{C_1}
\end{align*}
\]

This model used three combined PDF's, exponential, Rayleigh and standard Rayleigh distributions, to fit the probability density of low, middle and high range cycles respectively. It was achieved by minimising the cost function (mean square
error of the model equation with the time domain counting result) and regression
analysis.

This important empirical formula was the first to directly link the rainflow cycle
distribution with the spectrum moments. Because of the wide range covered by
the selected irregularity factor and mean frequency, the formula works very well for
Gaussian stress/strain histories [16][15][17]. The simulation process here seems to
have worked quite successfully. The phases used for simulating the time histories
from the spectra are assumed to have a uniform distribution in \((0, 2\pi)\), the Gaussian
assumption was therefore implicitly adopted.

The corresponding equivalent stress range parameter is

\[
S_h = \left[ \frac{C_1 \Gamma(1 + b)(2|\tau|)^b + \Gamma(1 + \frac{b}{2})(2\sqrt{2})^b(C_2|\alpha|^b + C_3)}{2\sqrt{m_0}} \right]^{\frac{1}{b}}
\]

\[ (3.31) \]

3.3 Bishop's Theoretical Approach

Bishop[13] carried out theoretical studies on the connection between the spectrum
and rainflow cycle distribution[12][23]. For a rainflow cycle as defined in Definition
3 in last chapter, the three events which constitute a rainflow cycle can be separated
and considered as three single events. The probability of a rainflow cycle existing was
therefore considered to be equal to the probability of these three events occurring
together. This consideration significantly simplified the computation of rainflow
cycles because each event was dealt with as a series of transitions of the signal
from a peak to a trough or from a trough to a peak rather than the complicated
configuration associated with the original definition. The rainflow cycle PDF can
then be expressed as:

\[
p_{RR}(h) = \frac{2.0}{dh} \sum_{i_p = h+1}^{\infty} Y_1(i_p, i_p - h)Y_2(i_p, i_p - h)Y_3(i_p, i_p - h)p(ip)
\]

\[ (3.32) \]

where, \( Y_1(i_p, i_p - h) \), \( Y_2(i_p, i_p - h) \), and \( Y_3(i_p, i_p - h) \) are the probabilities of events
\( Y_1, Y_2 \) and \( Y_3 \), respectively and can be calculated by Markov process theory. \( dh \) is
the interval width used to divide the signal and \( p(ip) \) is the probability of the signal being at peak \( ip \). The coefficient 2.0 occurs because there exists a symmetry about \( t=0 \) for stationarity.

To calculate the probability of the signal transition from peak to trough and from trough to peak, the peak-trough series can be assumed as a Markov chain. The probability of the above three events can then be calculated using Markov stochastic process theory. The complete PDF of rainflow ranges can therefore be obtained.

Theoretically, this method can be taken as a universal method and is suitable for any type of signals either Gaussian or non-Gaussian. However, this method needs a one step signal transition matrix to set up the Markov model matrix. This transition matrix is generally very difficult to derive except for a Gaussian distribution. So, this method is restricted mainly to the Gaussian signals. Furthermore, the computation procedure is complicated and susceptible to computational instabilities. However, due to its theoretical background, this method can be used for some special non-Gaussian situations once the heavy mathematical task is achieved to produce the required one step transition matrix. Additionally, this is the only existing method which offers the possibility of retaining the information about the relative mean of the rainflow cycles. This is obviously needed if the influence of mean stress is to be considered.

### 3.4 Discussion

Existing frequency domain methods for fatigue prediction are reviewed in this chapter. Steinberg's 3-band solution and Narrow band solution are easy to use, but conservative and can only give a very rough estimation. Most correction factor solutions work well only for some special situations. The computation procedure for Bishop's theoretical solution is complicated and susceptible to computational instabilities. Dirlik's empirical formula is the most promising one since it is derived
from extensive modelling of Gaussian signals. None of these methods can cope with non-Gaussian signals. A practical method, preferably with theoretical backing, is therefore desired for general stationary (Gaussian and non-Gaussian) signals.
Chapter 4

APPLICATIONS TO WEG AND HOWDEN WIND TURBINE DATA

4.1 Introduction

The development of various frequency domain methods dealing with fatigue damage prediction has gradually gained acceptance in many engineering applications. For any method developed, the best assessment of how well it performs is to apply it to the analysis of monitored structural response histories. Power Spectral Density (PSD) of the monitored times series can be calculated, hence the cycle range Probability Density Function (PDF) and fatigue life (or damage) by various frequency domain solutions. The monitored time sequences would be able to provide a time domain fatigue life prediction which can be taken as a reference solution with which to compare the frequency domain predictions. The work presented in this chapter performs this assessment.

As a result of the invaluable collaboration with industry, two sets of data from WEG MS-1 and Howden HWP330 wind turbine machines were provided by Wind Energy Group (WEG) Ltd and Garrad Hassan Ltd. These data sets are monitored response histories of the wind turbine blades during operation. Since most of the methods developed so far are for fatigue analysis of offshore structures other than wind turbine blades, it is useful to perform fatigue analysis on these monitored turbine responses in order to assess the validity of applying these methods to wind turbine blade fatigue analysis.

The results from such an analysis are presented in this chapter. Statistical anal-
ysis and fatigue life predictions using frequency domain methods and directly from
time series were carried out. Programs were developed to carry out the calculation.
Factors affecting calculation accuracy and problems revealed by the analysis and
some possible solutions are presented.

4.2 The WEG and Howden Wind Turbine Data

Extensive studies have been carried out on the WEG Ltd and Howden wind turbine
data. Some results and details are published in [17, 18, 19].

4.2.1 WEG MS-1 data

The WEG MS-1 data is made up of bending moments measured from a Wind Energy
Group MS1 wind turbine blade, corresponding to the flapwise load cases at 1.35m,
3.28m, 4.94m and 7.24m along the blade respectively. They were measured at six
different time periods (A,B,C,D,E,F) with different wind speed, yaw, turbulence
intensity and hub configuration. The sampling rate was 125 Hz (125 points/sec)
which corresponds to a 62.5 Hz Nyquist frequency. A description of the WEG MS1
wind turbine data is given in Table 4.1.

The original measured data was the flapwise moments at each section, rather
than the desired stress time histories. However, from beam theory, stresses can
easily be derived by dividing the bending moment history by the sectional bending
resistance module. But this transformation can be avoided since we are most con­
cerned about the PDF of cycle ranges. The transformation from bending moment
cycle range into stress cycle range has no effect on the probability density function
of cycle ranges. The fatigue damage (hence life), of course, has changed. However,
since we are trying to compare the damage (hence life) with the time domain re­
result, the ratio will remain unchanged. Thus, all the work here is performed directly
on the bending moment histories. Normalised fatigue damage rates (or lives) give
a comparison between various frequency domain methods and the result directly calculated from the time signal.

Table 4.1: WEG MS-1 Wind Turbine Blade load cases

<table>
<thead>
<tr>
<th>Case</th>
<th>wind speed (m/s)</th>
<th>Yaw (deg.)</th>
<th>Turb. Int. (%)</th>
<th>Hub Config.</th>
<th>Duration (secs)</th>
<th>Length (points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>18.4</td>
<td>7.1</td>
<td>9.7</td>
<td>Fixed</td>
<td>300</td>
<td>37500</td>
</tr>
<tr>
<td>B</td>
<td>23.7</td>
<td>3.0</td>
<td>11.1</td>
<td>Fixed</td>
<td>240</td>
<td>30028</td>
</tr>
<tr>
<td>C</td>
<td>11.1</td>
<td>-12.5</td>
<td>8.9</td>
<td>Fixed</td>
<td>300</td>
<td>37500</td>
</tr>
<tr>
<td>D</td>
<td>16.5</td>
<td>2.5</td>
<td>10.1</td>
<td>Fixed</td>
<td>300</td>
<td>37500</td>
</tr>
<tr>
<td>E</td>
<td>15.6</td>
<td>-3.1</td>
<td>6.6</td>
<td>Tethered</td>
<td>102</td>
<td>12823</td>
</tr>
<tr>
<td>F</td>
<td>11.3</td>
<td>12.3</td>
<td>15.0</td>
<td>Tethered</td>
<td>300</td>
<td>37500</td>
</tr>
</tbody>
</table>

4.2.2 Howden HWP 330 data

During the mid-1980’s, Howden Wind Turbines Ltd recorded a considerable volume of measured data from two comprehensively instrumented 3 bladed machines in California. The Howden HWP330 data used in this thesis is the measured data from one of the two instrumented machines, a 33m diameter HWP330 located within a large windfarm in the complex terrain of Altamont Pass. Four monitored data files were given, each containing three flapwise and three edgewise bending moment histories at 3.00m, 8.09m and 13.04m radius locations. The sampling rate was 40 Hz. A summary of the four basic load cases is given in Table 4.2.

Table 4.2: Howden HWP330 data load cases

<table>
<thead>
<tr>
<th>Tape</th>
<th>Windspeed (m/s)</th>
<th>Turbulence Intensity(%)</th>
<th>Mean yaw (deg)</th>
<th>Duration (s)</th>
<th>Length (points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>10.68</td>
<td>19.6</td>
<td>-11.7</td>
<td>2560</td>
<td>102000</td>
</tr>
<tr>
<td>26</td>
<td>14.07</td>
<td>9.2</td>
<td>-6.5</td>
<td>3260</td>
<td>130400</td>
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<tr>
<td>27</td>
<td>16.86</td>
<td>10.7</td>
<td>-11.8</td>
<td>3863</td>
<td>154518</td>
</tr>
<tr>
<td>30</td>
<td>8.51</td>
<td>15.3</td>
<td>2.4</td>
<td>3512</td>
<td>140398</td>
</tr>
</tbody>
</table>

Examples of WEG and Howden data are given in Fig.4.1 and 4.2 respectively. A brief inspection of the signals shows there is strong deterministic component in the
Howden edgewise signals. A short time sample from the tape 26 edgewise signals is given in Fig.4.3.

### 4.3 Analysis Program

A program was developed to perform statistical analysis and fatigue analysis calculations in both time and frequency domains. The program consists of more than 40 subroutines and is about 3500 lines in Fortran 77 code. The flow chart for fatigue analysis is shown in Appendix A. An earlier version of part of the program has been incorporated into \textit{nCode International} software product (\textit{NSOFT FATIMASPECTRAL}) which has already found wide applications in many engineering fields.

In the time domain analysis, extreme points (peaks and troughs) are first extracted, rainflow cycle counting is then performed, together with material/component properties (S-N curve), to obtain the time domain fatigue life estimation. This estimation is then taken as the reference solution. This is because it is the result that all the frequency domain approaches are trying to emulate. When working in the frequency domain, the PSD is computed from the given time series or input from a data file which stores the spectrum. If the latter is the case, there would be no reference solution with which to compare the frequency domain solutions.

Most of the methods described in the previous chapter are implemented in the program. They are:

- Dirlik’s empirical formula
- Narrow band solution
- Tunna’s solution
- Wirsching’s modification
- Hancock’s modification
- Kam and Dover’s modification

\footnote{\textit{nCode International Ltd., 230 Woodbourn Road, Sheffield S9 3LQ, UK}
Figure 4.1: WEG MS-1 wind turbine blade data: y12c, y19f, y27a, y35d
Figure 4.2: Howden data: tape 26
Figure 4.3: Howden data: Edgewise: tape26-ch6,8,10, Zoomed

- Steinberg's three-band solution

A new theoretical solution developed by the author is added later.

Statistical analysis (see next section for details) was performed to check if the signals were stationary and/or Gaussian. A brief inspection, and statistical analysis, show the WEG MS-1 data can be accepted as stationary, Gaussian signals. This is because they are all flapwise signals which contain no strong deterministic component. However, the Howden HWP330 data consists of flapwise and edgewise data, the latter are combined deterministic and stochastic response histories which are a class of special non-Gaussian signals.

When appropriate, the rainflow cycle range PDFs predicted by the frequency domain methods are calculated first. The estimated fatigue livies derived from these rainflow cycle PDFs are then compared with the ones from the time domain analysis. The ratio of the fatigue lives predicted using the frequency domain methods to those directly from time domain solution is defined as the normalised life in this chapter.
(the fatigue damage rate (or ratio) is similarly defined and is the inverse of fatigue life ratio). A value less than 1.0 implies that the frequency domain result is conservative, i.e., gives a life value less than the time domain solution. Conversely, a value greater than 1.0 implies that the frequency domain result is unconservative.

As part of the initial development the program was first used for the analysis of two sets of simulated data. The first set of data is created by filtering white noise, as produced by a Brüel & Kjaer signal generator[12]. These data sets are denoted as nbdata, nbdatb, nbdatc and nbdatd. For a typical data set, nbdatc, the PSD function is shown in Figure 4.4.

The rainflow cycle PDF's counted from the time series and predicted by the frequency domain methods mentioned above are shown in Figure 4.5. Since the signal is a simulated Gaussian time history, it was found that most of the methods agree well with the time domain analysis results. The narrow band solution, as expected, is conservative as the middle and high range part of the probability density is over predicted. This part of the PDF contributes most to the total fatigue damage because of the nonlinear $S - N$ equation.

The program is also applied to another set of computer simulated Gaussian data corresponding to PSDs with different irregularity factors and other parameters (see Chapters 7 and 8 for details of the simulated data). Results are given in Table 4.3 and 4.4. As expected, Dirlik's method gives the best agreement with the time domain results, while the Narrow Band, Steinberg and Wirsching's formulaes provide conservative predictions.

### 4.4 Statistical Analysis

Statistical analysis was carried out for all the data. For convenience of analysis, each signal was broken up into a number of blocks (a special case is using the whole signal as one block) which were then analysed as independent records. Arithmetic
Figure 4.4: Example of first set of simulated data: *nbdata*
Table 4.3: Comparison of Fatigue damage prediction by different methods: Simulated data: Part 1

<table>
<thead>
<tr>
<th>Sig</th>
<th>Gamma</th>
<th>b</th>
<th>NB</th>
<th>Dirlik</th>
<th>Wirsch</th>
<th>KamDov</th>
<th>Steinberg</th>
</tr>
</thead>
<tbody>
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<td>0.98</td>
<td>2</td>
<td>0.917</td>
<td>0.998</td>
<td>0.985</td>
<td>0.985</td>
<td>0.886</td>
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<tr>
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<td>3</td>
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<td>1.031</td>
<td>0.972</td>
<td>0.881</td>
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<td>0.927</td>
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<td>1.077</td>
<td>0.956</td>
<td>0.869</td>
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<tr>
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<td>5</td>
<td>0.907</td>
<td>1.027</td>
<td>1.120</td>
<td>0.936</td>
<td>0.858</td>
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<tr>
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<td>6</td>
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<td>1.070</td>
<td>1.156</td>
<td>0.911</td>
<td>0.854</td>
</tr>
<tr>
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<td>1.006</td>
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<td>0.990</td>
<td>0.939</td>
<td>0.817</td>
</tr>
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<td>0.916</td>
<td>0.798</td>
</tr>
<tr>
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Table 4.4: Comparison of Fatigue damage prediction by different methods: Simulated data: Part 2

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</table>
averages were then used to compute a single value for the particular variable of interest.

4.4.1 Pre-processing of the measured data

It is noted that some data file(s) contain spurious and/or degraded components that probably have resulted from acquisition and recording problems such as excessive noise, signal dropouts and loss of signal because of transducer malfunction. For example, for all Howden data, there's a big spike in around 1550 sec. This might be caused by an excessive recording noise or others. These kind of 'wild points' have been removed before detailed processing.
4.4.2 Basic statistics

The Mean, Root Mean Square (RMS) and Irregularity Factor were calculated directly from the appropriate number of blocks of signal and then summed to get the arithmetic average.

4.4.3 Stationarity and trend tests

For Stationarity and Trend Tests, since the sampling distributions of the data parameters were not known, a nonparametric approach was desirable. Since Reverse Arrangement Test (RAT) is powerful for detecting monotonic trends in a sequence of observations, it was chosen for Stationarity and Trend Tests. A brief description of the method follows. After calculating the mean/rms of every block, a sequence of $N$ (number of blocks) observations (denoted as $x_i = 1, 2, \ldots N$) of a random variable $x$ is obtained. The number of times that $x_i > x_j$ for $i < j$ is then counted (this is called the number of reverse arrangements). If the sequence of $N$ observations are independent observations of the same random variable, then the number of reverse arrangements is a random variable $A$, with the mean and variance as follows:

$$
\mu_A = \frac{N(N - 1)}{4}
$$

$$
\sigma_A^2 = \frac{N(N - 1)(2N + 5)}{72}
$$

For the significant level $\alpha = 0.05$, the accepted region (upper and lower bound values) for the hypothesis that the observations are independent observations of a random variable are given in Table 4.5 (for different block numbers). When block number $N = 6$, the values are not listed as the number of observation is too small for there to be any practical significance).
Table 4.5: 95% Confidence Region of Reverse Arrangement Test

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4.4.4 Testing for normality (Gaussianity)

The most obvious way of testing the normality of samples of stationary random data is to calculate the probability density function of the data and compare it to the theoretical normal distribution. If the sample record is sufficiently long to permit a measurement with small error compared to the deviations from normality, the lack of normality will be obvious. If the sampling distribution of the probability density estimate is known, various statistical tests for normality can be performed even when the random error is large. However, a knowledge of the sampling distribution of probability density measurements requires frequency information for the data which may be difficult to obtain in practical cases. Hence a nonparametric test is desirable. In this thesis, a normality test is carried out using two approaches, the $\chi^2$ (Chi-square) Goodness-of-Fit Test and the Kurtosis Test. The $\chi^2$ Goodness-of-fit Test was performed using K.Pearson's statistics[9],

$$\chi^2 = \sum_{i=1}^{k} \frac{(f_i - F_i)^2}{F_i}$$

where $f_i$ is the observed frequency, i.e., the number of observations falling within the $i$th class interval; $F_i$ is the expected frequency, i.e., the number of observations that would be expected to fall within the $i$th class interval if the true probability
density function of $x$ is Gaussian. According to the Pearson Theorem, $\chi^2$ obeys the $\chi^2$ distribution of degree of freedom $k - 1$. For practical problems, the statistical parameters (i.e., mean and root mean square) are not known, and have to be estimated. The actual Degree of Freedom is then $k - 3$. The results for $k=43$ (i.e., D.O.F=40) are listed. For reducing error, the intervals are chosen so that all $f_i > 3$. For the significant level of $0.01$, the upper bound of the accepted regions for $\chi^2(40)$ is 63.69.

Without losing generality, we assume the signal is stationary with a zero mean value, when carrying out the Kurtosis (Coefficient of Excess) Test [27][11] (significant underlying trend is removed before carrying out the statistical test). Denote $\mu_i$ ($i=1,2,4$) as the $i$th central moment, $\sigma^2$ as the variance. Kurtosis (or Coefficient of Excess) is then defined as

$$\kappa = \frac{\mu_4}{\sigma^4} - 3$$

$\kappa$ can be used as a measure of the degree of flattening of a frequency curve near its centre. It gives an indication of the drift of the signal from a Gaussian distribution. The minimum value of $\kappa$ is -2, and this occurs only when $x$ is a symmetric binary random variable ($|x| = constant$). At the other extreme, $\kappa$ may be infinite for a probability density function with slowly decaying tails. For a Gaussian distribution, since $E[x^4] = 3\sigma^4$ [69], then $\kappa = 0$.

For a real signal, it is transformed to a zero-mean random variable. $\sigma^2$ and $E[x^4]$ are then estimated and used to get an estimation of kurtosis. If kurtosis is around zero, the signal can be regarded as Gaussian. When taking an arithmetic average, some blocks of the signal give a positive kurtosis and some give a negative kurtosis. A very small kurtosis can then be obtained. An improvement of this may be obtained by averaging the absolute value of kurtosises for all blocks. $\kappa_1$ is therefore the average of the absolute value and $\kappa_2$ is the arithmetic average.
4.4.5 Results

The statistical test results for the WEG data are listed in Table 4.6. The following points should be noticed:

- The whole signal is divided into appropriate number of segments with 2048 points in each block. The final result is calculated by averaging the results from all the blocks.

- Stationarity and Trend Tests were carried out by using the Reverse Arrangement Test (rat). The numbers of reverse arrangements for mean value and root mean square are listed as 'ratm' and 'ratr'.

- Normality Tests were carried out by using a $\chi^2$ Goodness-of-Fit Test and a Kurtosis Test respectively. Results are listed as $\chi^2$ (Degree of Freedom: 40) and kurtosis: $\kappa_1$ and $\kappa_2$. $\kappa_1$ is obtained by averaging the results of the absolute value from all blocks, while $\kappa_2$ is the arithmetic average from all blocks.

- Bandwidth characteristics are specified using the Irregularity Factor $\gamma$.

The results of statistical tests on the Howden data are given in Table 4.7. The Chi-square test and Kurtosis values clearly shows that all edgewise data sets are strongly non-Gaussian. In fact, all of the edgewise data files have strong deterministic components.

The amplitude distribution of the signals was also computed. Examples of the amplitude distribution for the WEG data are shown in Fig. 4.6, 4.7 and 4.8. These distributions show that the WEG and Howden flapwise signals can reasonably be accepted as Gaussian signals, but that the Howden edgewise signals are non-Gaussian.
Figure 4.6 Amplitude distribution of WEG Y12a data

Figure 4.7: Amplitude distribution of Howden H2605 Flapwise data
### Table 4.6: Results of Statistical Tests on WEG Data

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<th>rms</th>
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### 4.5 PSDs of WEG and Howden Data

PSDs were calculated by applying the FFT (See [9] 11.5.3 for details of procedures).

The FFT window size was chosen as 2048. A Hanning taper was also applied to suppress side-lobe leakage. Examples of the PSD’s for the WEG data are given in Fig.4.9 and 4.10.

Examples of the PSD’s for the Howden data are shown in Fig. 4.11 and 4.12.
Figure 4.8: Amplitude distribution of Howden H2606 Edgewise data

Figure 4.9: PSD of WEG data: y27a – linear scale
Figure 4.10: PSD of WEG data: y27a – logarithm scale

Figure 4.11: PSD of Howden data: Tape26-ch5
Figure 4.12: PSD of HWP26 (logarithm scale): Tape26-ch6
### Table 4.7: Statistics of Howden HWP 330 data

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<th>RMS</th>
<th>Ratr</th>
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<th>$\gamma$</th>
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#### 4.6 Fatigue Life Prediction Results

Ignoring the influence of mean stress, several methods (Dirlik, Narrow Band, Tunna, Wirsching, Hancock, Kam & Dover and Steinberg) were applied to the WEG and Howden data. Because these spectral methods are very sensitive to the moment calculation procedure, and this is influenced by parameters such as the FFT window size and the cutoff frequency of the PSD, predefined techniques were used to set these variables. Effects of various parameters were studied later. For calculation of the PSD, the window size of the FFT was set at 2048 data points (in the time domain). When calculating the moments of the PSD, the frequency cutoff point was set at the frequency corresponding to 99.95% of the zeroth moment obtained by integrating
the PSD up to the maximum frequency limit. As a comparison, the fatigue life was calculated directly from the time domain using rainflow cycle counting. The clipping ratio was set at 4.5 (i.e., the maximum range of cycles is 9 times the rms value). Different $S - N$ curve slopes (3, 5, 8, 10) were chosen for comparison. Results for the WEG data are given in the Tables 4.8 to 4.11. All numbers are fatigue lives normalised with the result from time domain. Fatigue damage is the inverse of the life.

Results show that the Narrow Band, Wirsching, Hancock and Steinberg's methods are in general conservative. For WEG data, the predicted fatigue life is up to 20 times shorter than from the time domain. For the Howden data, it is up to 100 times shorter. Dirlik’s and Kam & Dover’s formula in general give good prediction. As indicated in the last chapter, Tunna’s formula is unconservative, particularly when the signal is wide banded (with small $\gamma$) and the inverse S-N curve slope is large, it gives far too high fatigue lives. This is dangerous for a designer and so this method is not recommended.

Examples of rainflow cycle range PDF’s for the $y12a,b,c$ and $f$ data sets, calculated from both time and frequency domain analysis, are presented in Figure 4.13. It is clear that the narrow band solution over estimates the middle and high range cycles. However, Dirlik’s formula gives a reasonably good prediction for the cycles in these ranges. In terms of fatigue, the middle and high range cycles always contribute most to the fatigue damage.

Examples of rainflow cycle range PDF’s for the Howden data are given in Figure 4.14 and 4.15. For edgewise signals, due to the strong deterministic component, there is an enormous difference between the PDF’s predicted by any frequency domain method and the actual (time domain) cycle range distributions. In fact, the strong deterministic component is nearly a harmonic (sine or cosine) wave, therefore all cycles have a similar range and this is indicated by the big spike in the cycle range PDF. None of the existing methods can predict this spike in the cycle range PDF.
Figure 4.13: Cycle range probability density functions for WEG MS-1 data \( y12a, b, c, f \)
Actually, the harmonic component ends up as a delta function in the PSD. This makes the calculation of moments very difficult and sensitive to computation errors.

Table 4.8: Comparison of Fatigue Life results using the different methods: WEG data, Part 1

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<th>Wirsch</th>
<th>Hancock</th>
<th>KamDov</th>
<th>Stnberg</th>
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4.7 Practical Considerations in the Calculation

Previous sections have provided an overall assessment of the existing frequency domain methods used for fatigue life estimation of the WEG MS1, Howden HWP330 and other data. Calculations show that the accuracy of the fatigue life prediction is affected by many factors which include the cutoff frequency, the clipping ratio of the probability density function, $S - N$ curve slope etc. In practical calculation these parameters should be selected carefully, otherwise an enormous error might occur in
Figure 4.14: Rainflow cycle range probability density functions for Howden flapwise data Tape 27, channel 5
Figure 4.15: Rainflow cycle range probability density functions for Howden edgewises data Tape 27, channel 6.
Table 4.9: Comparison of Fatigue Life results using the different methods: WEG data, Part 2

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Table 4.10: Comparison of Fatigue Life results using the different methods: WEG data, Part 3

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Table 4.11: Comparison of Fatigue Life results using the different methods: WEG data, Part 4

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Table 4.12: Comparison of Fatigue Life results using the different methods: Howden data, Part 1 (ch5, ch7 and ch9 are flapwise, ch6, ch8 and ch10 are edgewise)

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Table 4.13: Comparison of Fatigue Life results using the different methods: Howden data, Part 2 (ch5, ch7 and ch9 are flapwise, ch6, ch8 and ch10 are edgewise)

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Table 4.14: Comparison of Fatigue Life results using the different methods: Howden data, Part 3 (ch5, ch7 and ch9 are flapwise, ch6, ch8 and ch10 are edgewise)

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the fatigue life estimation. This section presents the results from an investigation of these parameters. Some are related to the assessment of frequency domain analysis tools and some are related to the use of these tools in design. Suggestions are made for choosing appropriate values for these parameters.

### 4.7.1 Choice of cutoff frequency

All frequency domain fatigue analysis methods developed so far are directly related to the first few moments of the PSD, these being \( m_0, m_1, m_2, m_4 \) and others. For practical calculation, accuracy of the calculation of the moments, hence the fatigue lives, is related to the choice of the cutoff frequency of a spectrum. For measured stress or strain signals, the sampling rate should be chosen so that it is at least two times the maximum frequency of interest (Nyquist frequency). This implies, in theory, the cutoff frequency of a spectrum should be half of the time series sampling frequency to take account of all frequency components. However, this cutoff frequency is decoupled from the process of calculating the moments of the spectrum when performing a fatigue analysis. The reason for this is that high frequency components are often just acquisition noise, rather than the structural responses. This may cause serious problems in the calculation of the moments.

Figure 4.16 shows examples of effects of cutoff frequency from the WEG MS-1 data y12a. Fig. 4.17 and 4.18 show results for Howden HWP330 data. In Figure 4.16, the PSD is plotted up to 62.5Hz, the Nyquist frequency. For a clear view of the tails, the PSD's are plotted in logarithm scale. The moments and irregularity factor calculated from the spectrum are also plotted (they are re-scaled so that they can be plotted in the same graph). It can be seen that the high frequency components have a very big influence on the higher order moments and finally on the irregularity factor, as expected. The damage rates (normalised by the time domain fatigue result) of Dirlik’s method, the best existing frequency domain method, are also plotted. It is not stable until after approximately 15Hz. Actually this is the point to the left of
which most of the power is included in the PSD.

The question which arises is, for the monitored time history or the PSD, how should the cutoff frequency be determined? If the high frequency components simply represent noise produced in data acquisition process, how should the boundary between structural response and noise be set? In another words, which part of the signal should be taken as structural response and which part as acquisition error?

The answer to this question depends largely on the bit number used in the data acquisition system. (see [77] for detail). Increasing the bit number in the data acquisition process would greatly reduce the error produced. Of course, the memory requirement would increase as well.

For WEG MS-1 and Howden data, the cutoff frequency was set at the point at which the area of the spectrum reached $99.95\%$ of its whole area. This is attempting to include all the structural response data in the spectra whilst excluding the noise which is either caused by acquisition error or by electronic noise during measurement.

Two typical results are plotted for the Howden data tape 26. Figure 4.17 shows the result for 5 meters flapwise and 4.18 for 5 meters edgewise. As indicated before, the PSD's are plotted with a log scale so that the high frequency components can be observed more clearly. It seems that the influence of these high frequency components is more serious here than with the WEG MS-1 data as seen from Figure 4.17 and 4.18. The high order moments and irregularity factor change more rapidly. Again, only the damage rates from Dirlik's formula is plotted here for simplicity.

It is clear that the high frequency components have a big influence on the higher order moments. This increases the expected number of peaks per unit time. It would be encouraging if this increase kept the total number of higher range cycles unchanged. Since the small range cycles contributes little to the total damage, keeping the number of big cycles constant would mean the value of fatigue damage could remain the same. Unfortunately, this is not the case as shown in Figure 4.17.
The cutoff frequency issue is mainly a problem for the frequency domain tools when they are used to analyse monitored data. There is not a high frequency noise problem when a theoretical spectrum is used for structural analysis at the design stage but the spectrum truncation problem still exists. Loading spectra for some structures are provided as theoretical, empirical, or semi-theoretical formula. The sea wave load spectrum for an offshore platform, for example, is expressed as a semi-theoretical formula which is a function of the significant wave height and the wave dominant period [54]. No matter where the spectrum comes from, the cutoff frequency has to be set by the user. This truncation problem is then the same as the cutoff frequency problem. For practical application, similar to the analysis of WEG MS-1 and Howden data, the cutoff frequency can be set at the point at which the area of the spectrum reaches 99.95% of its whole area.

4.7.2 Effect of S-N curve slope

Most existing frequency domain fatigue methods produce cycle range PDF’s, then fatigue life. Generally speaking, there is always a difference between the calculated cycle range PDFs from different methods (see Fig. 4.19, for example). The closer the two rainflow cycle pdf’s are, the closer the damages derived from these two pdf’s will be. The difference between these PDFs will then produce different damage ratios for different S-N curve slope $b$ value. The term damage ratio is used here again to represent one damage value divided by the other.

Figures 4.19 and 4.20 highlight this variance for two typical WEG MS-1 data files. They were simply derived by fixing the rainflow cycle PDF’s and changing the value of $b$ in the damage calculation. It is interesting to see from Figures 4.19 and 4.20 that, both over-prediction and under prediction of the fatigue damage is possible. This is obviously caused by the weighted integral in the damage calculation. The importance of the difference between the PDF’s from the frequency domain analysis compared with the one from the time domain analysis is actually changing with $b$. 
Figure 4.16: Influence of cutoff frequency of WEG MS-1 data y12a
Figure 4.17: Influence of cutoff frequency of Howden data, tape 26-5m flapwise
Figure 4.18: Influence of cutoff frequency of Howden data, tape 26-5m edgewise
Figure 4.19: Effect of S-N curve slope: WEG MS-1 data y35d
Figure 4.20: Effect of S-N curve slope. WE1 MS-1 data.
A similar investigation on the Howden data has also been performed. Figure 4.21 and 4.22 show the results for data tape 27, 3 meter flapwise and 3m edgewise respectively. The result for the 3m flapwise signal shows a underprediction of fatigue damage. The result for 3m edgewise signal shows a consistent upward tendency. Once again, this is caused by the dominant deterministic component in the edgewise signal. The deterministic component results in cycles being concentrated in a certain range and the weighted sum is dominated by these cycles.

Figs. 4.23 and 4.24 show the effect of $S - N$ curve slope on fatigue life (normalised) for simulated signals (see chapters 7 and 8) by Wirsching and Steinberg’s methods and they don’t produce cycle range PDFs directly. It can be seen both methods work well for narrow band signals. For wide band signal, they are in general conservative.

This phenomena suggests that some methods which work well for one kind of material may give very poor estimation results for other kinds. The accurate prediction of both medium and higher range cycles is very important if a method is to be used for different kind of materials. It should also be noted that the term of “equivalent stress” should be understood as valid only for the specified value of $b$ which is used in its derivation.

### 4.7.3 Effects of bandwidth

A simple definition of the bandwidth of the PSD is the irregularity factor. Each of the developed frequency domain methods (other than Dirlik) work well only for a certain cases. Obviously, the Narrow Band solution only works well for narrow band signals. Correction factors also only work well for some specific cases. Dirlik’s empirical formula is based on an extensive simulation of a wide range of stationary signals. Since neither the WEG nor HOWDEN data can provide a wide range of signals with different bandwidths, here we have used simulated signals to evaluate the approach (see chapters 7 and 8 for details). Figs. 4.25 to 4.28 show the effect of
Figure 4.21: Effect of S-N curve slope: Howden data tape 27 3m flapwise
Figure 4.22: Effect of S-N curve slope: Howden data tape 27 3m edgewise
Figure 4.23: Effect of the S-N curve slope: Wirsching Solution
Figure 4.24: Effect of the S-N curve slope: Steinberg Solution
irregularity factor on normalised life for different methods (Narrow Band, Wirsching, Steinberg and Dirlik). Only Dirlik's method can give a consistently stable prediction for both wide band and narrow band signals. All the other methods give conservative predictions for wide band signals (except Tunna's as explained in the last chapter).

![Fatigue Life Ratio (Narrow Band)](image)

Figure 4.25: Effect of the Bandwidth: Narrow Band Solution
Figure 4.26: Effect of the Bandwidth: Wirsching Solution
Figure 4.27: Effect of the Bandwidth: Steinberg Solution
Figure 4.28: Effect of the Bandwidth for Fatigue life: Dirlik Solution
When carrying out rainflow cycle counting for a stress time series, the value of the maximum cycle can easily be determined using the difference between the highest peak and the lowest trough in the time signal. However, when a frequency domain analysis is performed, the sample stress signal corresponding to the PSD could, in theory, range from $-\infty$ to $\infty$. Hence the distribution of the rainflow cycle ranges extends from 0 to $\infty$. A typical Gaussian distribution of stress amplitude is shown in Fig. 4.29. If the time series from which the spectrum is calculated is available, it is possible to derive the maximum range by referring to the time domain analysis. However this only makes sense in a research environment. A practical computation is unable to deal with this infinite range. Thus, a clipping point must be selected to set a finite maximum range other than infinity. This point is described as the so-called “clipping ratio”, which is defined as the ratio of the maximum value divided by the root mean square. That is,

$$\rho = \frac{X_{\max}}{\sigma}$$

The maximum range is $2\rho \sigma$ because of symmetry (the mean value is assumed as zero).

The principle behind the selection of clipping ratio is that it should include most of the probability inside the range determined. However, the important question is what level of probability can be truncated. Some methods used for fatigue estimation, such as Dirlik’s empirical formula, can theoretically extend to infinity. A problem for practical computation is therefore to select a suitable cutoff point.

The relationship between damage rate and clipping ratio can be examined to answer this question. Figure 4.30, 4.31 and 4.32 show the typical results of this relationships when Dirlik’s formula is used. The examination of the WEG MS-1 and the Howden data shows that the fatigue damage rate generally converges to the stable value for a clipping ratio range between 4.0 and 6.0. So, selecting a clipping
ratio of 6.0 would be enough from the point of view of fatigue estimation.

4.8 Discussion

This chapter presents results which were obtained by applying existing frequency domain methods to blade data from two wind turbines (WEG MS1 and Howden HWP330). Various parameters associated with the use of the correct methods have been investigated. This study provides useful guidance for the application of current methods in fatigue damage estimations.

The effect of cutoff frequency, S-N curve slope, bandwidth of the PSD and clipping ratio were investigated. This study will help to make existing frequency domain tools practically useful in engineering design.

The accuracy of most existing frequency domain methods is strongly influenced by the calculation of the first 4 moments of the PSD. Improvement for this can be achieved by setting a threshold for filtering out acquisition noise at the stage of
Figure 4.30: Choice of clipping ratio: WEG data y27d

Figure 4.31: Choice of clipping ratio: Howden data tape 26 3m flapwise
calculating the PSD. Clipping ratio can in general be set at around 6. Most methods work well for narrow band Gaussian signals, while Dirlik’s method can cope with both narrow band and wide band signals. As expected, none of the methods work well on signals with strong deterministic component.
Chapter 5

EFFECT OF DETERMINISTIC COMPONENT

5.1 Introduction

All frequency domain fatigue analysis methods developed so far are based on the assumption that the stress signals are stationary, Gaussian, and without strong deterministic components. As seen from the analysis of the Howden data in the previous chapter, the existence of a deterministic component in the stress time history represents an important problem for fatigue life estimation in the frequency domain. In fact, when the deterministic component is (or nearly) a harmonic (sine or cosine) wave, it produces a large spike in the cycle range PDF at a range of two times the harmonic component amplitude. If the deterministic component is strong (with higher amplitude), it produces a spike in the higher range part of the PDF. The Howden edgewise data is of the type. When the dynamic loading has a strong deterministic component in it, it is no longer stationary or Gaussian. The big spike in the cycle range PDF can not be predicted by frequency domain methods which are related to the loading PSDs.

For wind turbines stochastic loadings are usually caused by wind turbulence. Deterministic loads, on the other hand, generally exist in the following cases:

- wind shear.

- skew wind.

- tower interference.
Among all the deterministic components, the most important is undoubtably the one caused by gravity. Its value is usually so big that all the others can be classified into the stochastic part. In most cases there is only one strong deterministic component, though there may be some situations where more than one deterministic component is important.

A question related to the practical application of frequency domain methods will be raised: how seriously will the deterministic component affects the prediction accuracy? Or, at what magnitude will the deterministic component significantly affect the prediction accuracy of the frequency domain methods? Obviously, when the stress signal has a strong harmonic component, it is inappropriate to represent it by its PSD. However, for a design engineer, it is desirable to have a criteria to judge under what circumstances (how strong is the deterministic component) the frequency domain fatigue method still works. This chapter will try to address these issues, and presents results from a study which investigated the effect of a combination of a stationary signal mixed with various deterministic components. A combination of stationary signals mixed with sine waves and with equally spaced spikes were considered. The aim of the study was to try to understand in what cases does the deterministic component start to cause serious fatigue prediction inaccuracy and how serious is the effect. This provides a guide for engineering application of frequency domain fatigue analysis.
5.2 Separation of a Deterministic Component From Mixed Signal

As seen from the previous chapter, Howden edgewise signals can be regarded as a mixture of a stationary random signal with a harmonic (sine or cosine) component (gravity component). Let \( X(t) \) be the stochastic response caused by the wind turbulence, and \( Y(t) \) be the one caused by the gravity of the blade. The monitored signal is then \( Z(t) = X(t) + Y(t) \). The gravity induced response takes the following form:

\[
Y(t) = A \sin(\phi + \xi) + D \tag{5.1}
\]

where \( A \) is the amplitude of the harmonic component, \( D \) is the global mean value of the signal, \( \phi = \omega t \) is the azimuth, and \( \xi \) is a suitable initial phase. For Howden data, channel 4 for all tapes stores azimuth information.

5.2.1 Band pass filter

In theory, the deterministic component in a mixed signal can be extracted by filtering the mixed signal through a band pass filter which only allows the single sine wave (or a very narrow band of energy around it) to pass. In practice, this method did not work well because of the leakage problem associated with FFT calculations. This is inevitable in discrete FFT computation, regardless of the type of window used. The sine wave becomes a narrow band signal because of this leakage. Also, there is no guarantee that the blade is always rotating at a constant frequency during the time period in which the data is acquired. This ruled out the possibility of using a band pass filter.

5.2.2 Least square sine wave fitting

Without losing generality we can assume the mean value of \( Z(t) \) is zero. The stochastic response \( X(t) \) is also a process with zero mean. The gravity induced response can be extracted by least square error fitting to calculate the parameters:
A better strategy for calculating the parameters of the sine wave is to apply a band pass filter to the original signal and then apply a least square error fitting to the filtered signal.

5.3 The Program for Carrying Out the Study

This study was carried out by running the nCode software product which included the module, NSOFT FATIMAS-SPECTRAL, developed as part of this thesis. Two cases were considered:

- The mixture of stationary signals with sine waves.
- The mixture of stationary signals with equally-spaced spikes.

Three stationary signals were chosen: XFILE is a rescaled WEG data file (y12a), YFILE is a simulated signal created by filtering the white noise generated from a Brüel & Kjaer signal generator[12]; ZFILE is a simulated non-Gaussian signal (see chapters 7 and 8). PSDs of the mixed signals were calculated. Initially, when calculating PSDs, the default threshold value of the noise floor $-72dB$ was chosen to reduce all PSD values smaller than $\frac{1}{3981}$ of the maximum PSD value to zero. However, this causes inaccuracy in the calculation of higher moments. When this noise floor was changed to $-90dB$, $-105dB$, $-120dB$, all results were stable. In the whole calculation, this noise floor was then set at $-120dB$, i.e., set all PSD values less than $\frac{1}{10^9}$ of the maximum PSD value to zero. Fatigue lives by frequency domain methods and time sequence are then calculated and compared.
5.4 Mixture of Stationary Signals with Sine Waves

A random signal with a deterministic component can be obtained by simply superimposing the deterministic component on to the stationary signal. One problem involved in mixing these two signals could be deciding on a suitable choice of the sine wave initial phase. However, it was found that only when trying to mix more than one sine wave to a stationary signal was the phase difference between the sine waves important from the point of view of fatigue damage. Since the phase information in the PSD is lost, the initial phase of a single sine wave is not important relative to the spectrum. Therefore the effect of an initial phase of the sine wave was ignored. Initial phase of the added sine wave was therefore chosen at random in $(0, 2\pi)$.

Without losing generality, the stationary signal is assumed to have a zero mean value. Assume the stationary signal $X(t)$ has Root Mean Square $\sigma_x = \sqrt{m_0}$ and mean frequency $f_m = \frac{m_1}{m_0} \sqrt{\frac{m_2}{m_1}}$. This stationary signal is mixed with 400 sine waves $Y(t)$ with 20 amplitudes $A_y = \frac{1}{20} 5\sigma_x, \frac{2}{20} 5\sigma_x, \frac{3}{20} 5\sigma_x, \ldots, \frac{20}{20} 5\sigma_x$ and 20 frequencies $f_y = \frac{1}{20} f_m, \frac{2}{20} f_m, \frac{3}{20} f_m, \ldots, \frac{20}{20} f_m$. Fig. 5.1 shows an example of the original stationary signal (XFILE), the added sine wave and the mixed signal.

PSD's of the mixed signals were calculated and fatigue lives are calculated by frequency domain methods (here we apply Dirlik's method) and directly from the time sequence. Fig.5.2 shows an example of PSD's of XFILE, the sine wave and the mixed signal.

Figs 5.3 and 5.4 show the histogram of rainflow cycle range-mean distribution of XFILE and the mixed time sequence. Since the amplitude of the added sine wave is $\frac{3}{20} 5\sigma_x$, there is a big increase, as expected, in the low range part of the histogram for the mixed signal.

Figs. 5.5 to 5.7 show the effect on fatigue life (S-N curve slopes $b=4,7,10$) for XFILE. For the convenience of comparison, they are plotted as the percentage error of the equivalent stress parameter (eq.(2.7)) estimated from the time domain method.
Figure 3.1: XFILE, the sine wave and the mixed signal

Original Signal: XFILE

RMS: 20.419
Mean: 0.021
Sample Rate: 125
Amplitude: 13.515
Time: 300

Resultant Signal: SIN0003

RMS: 18.024
Mean: 0.024
Sample Rate: 125
Amplitude: 6.327
Time: 299.992

Offset Signal: SIN0003

RMS: 9.555
Mean: 0.021
Sample Rate: 125
Amplitude: 13.515
Time: 300
Figure 5.2: An example of PSDs of XFILE, the sine wave and the mixed signal
Figure 5.3: Histogram of rainflow cycle range-mean distribution of XFILE
Figure 5.4: Histogram of rainflow cycle range-mean distribution of a mixed file
divided by the result obtained by using Dirlik's method. This is proportional to the $b$-th root of the percentage error of fatigue life obtained using Dirlik's method divided by the time domain result.

When mixing a sine wave with a stationary signal, both the amplitude and frequency of the sine wave have an important effect on the fatigue life. In Fig. fig:xsine2 (b=7), when $A \leq 1$, the maximum percentage error of equivalent stress parameter is 7.9% (at $f_m = 0.4$), this corresponds to a fatigue damage ratio (frequency domain/time domain, see section 4.3) of 1.70. When $A \leq 2$, the maximum percentage error of the equivalent stress parameter is 17.1% (at $f_m = 0.3$), this corresponds to a fatigue damage ratio of 3.02, which is too conservative. When $A > 2$, the error is in general bigger, fatigue prediction becomes unconservative. This is not acceptable for practical applications. Actually, this is expected, as the sine wave is becoming dominant.

Figs. 5.5 and 5.7 show a similar trend, though Fig. 5.5 shows smaller error. Therefore, when the amplitude of the added sine wave is within the value of $\sigma_z$, results calculated by the frequency domain method and the time domain method are in good agreement.

5.5 Mixture of Stationary Signals with Equally-spaced Spikes

The spikes were chosen to have a very narrow band (1/1000 of the time interval) with various amplitude and frequency values. Similar as in previous section, 400 spikes $Y(t)$ with 20 amplitudes $A_y = \frac{1}{20}5\sigma_z, \frac{2}{20}5\sigma_z, \frac{3}{20}5\sigma_z, \ldots, \frac{20}{20}5\sigma_z$ and 20 frequencies $f_y = \frac{1}{20}f_m, \frac{2}{20}f_m, \frac{3}{20}f_m, \ldots, \frac{20}{20}f_m$ were generated and mixed with stationary signal $X(t)$. Fig 5.8 to 5.10 show the effect on fatigue life (S-N curve slopes $b = 4, 7, 10$) for XFILE. Again, they are plotted as the percentage error of the equivalent stress parameter estimated from the time domain method divided by the result obtained by using Dirlik’s method. This is proportional to the $b$-th root of the percentage error.
Effect on fatigue life (S-N slope of 4) of adding sine waves to a stationary Gaussian data file (XFILE) with amplitude $A\cdot\text{rms}$ and frequency $B\cdot\text{mean frequency}$.

Figure 5.5: Effect of mixing sine wave with XFILE. S-N slope $b=4$
Effect on fatigue life (S-N slope of 7) of adding sine waves to a stationary Gaussian data file (XFILE) with amplitude $A^\text{rms}$ and frequency $B^\text{mean frequency}$.

Figure 5.6: Effect of mixing sine wave with XFILE, S-N slope $b=7$
Effect on fatigue life (S-N slope of 10) of adding sine waves to a stationary Gaussian data file (XFILE) with amplitude $A^\text{rms}$ and frequency $B^*\text{mean frequency}$.

Figure 5.7: Effect of mixing sine wave with XFILE, S-N slope b=10
of fatigue life obtained using Dirlik's method divided by the time domain result.

Unlike in the mixing of random signal with sine waves, here as $A_y$ increases, the frequency domain method is always conservative. In general, the frequency of the spikes has much less important effect than the amplitude has on fatigue life. However, when the amplitude of the added spikes is within the value of $\sigma_x$, results calculated by the frequency domain method and time domain method are in good agreement. This is the same as in the previous section.

5.6 Conclusion

When mixing a sine wave with a stationary signal, both the amplitude and frequency of the sine wave have an important effect on the fatigue life. However, when mixing equally-spaced spikes with a stationary signal, the frequency (or time interval) of the spikes has a much less important effect than the amplitude has on the fatigue life. The frequency domain fatigue method still works well when a stress/strain signal has harmonic or equally spaced spikes (deterministic) components whose amplitude is within the value of the root mean square of the stationary component to whom the deterministic component is mixed with, no matter what the frequency of the sine wave or equally-spaced spikes is.
The effect on fatigue life (S-N slope of 4) of adding spikes to a stationary Gaussian data file (XFILE) with amplitude \( A \times \text{rms} \) and frequency \( B \times \text{mean frequency} \).

Figure 5.8: Effect of mixing spikes with XFILE, S-N slope \( b=4 \)
The effect on fatigue life (S-N slope of 7) of adding spikes to a stationary Gaussian data file (XFILE) with amplitude $A\cdot\text{rms}$ and frequency $B\cdot\text{mean frequency}$.

Figure 5.9: Effect of mixing spikes with XFILE, S-N slope $b=7$
The effect on fatigue life (S-N slope of 10) of adding spikes to a stationary Gaussian data file (XFILE) with amplitude $A\cdot \text{rms}$ and frequency $B\cdot \text{mean frequency}$.

Figure 5.10: Effect of mixing spikes with XFILE, S-N slope $b=10$
Chapter 6

A NEW THEORETICAL APPROACH

6.1 Introduction

Existing frequency domain methods described in the previous chapters either assume the loading signal is Gaussian and narrow-banded, or they use various correction factors to extend the Narrow Band solution. Some were achieved by applying further restrictions, some were produced using simulations. In this chapter, a new frequency domain based theoretical approach is presented. This approach itself doesn't require the signal to be Gaussian. The PSD of the stress signal is used to predict the probability density function of rainflow cycle ranges (for a non-Gaussian signal, the Tri-spectrum is also required). Calculations confirm it is efficient and easy to apply.

6.2 A New Approach

6.2.1 Definitions

Suppose the loading history \( y(t) \) has been reduced to a sequence of extrema \( \{ y(i), i = 1, 2, \ldots, N \} \). This sequence of extrema is then discretized into a finite number \( M \) of levels, with \( \max\{ y(i), i = 1, \ldots, N \} = s_M \), \( \min\{ y(i), i = 1, \ldots, N \} = s_1 \), \( \Delta = (s_M - s_1)/(M - 1) \). Denote \( p_y \) as the peak value, \( p_v \) as trough value.

Let

\[
p_k = \text{Prob}\{ p_y = s_k \} \approx p(s_k) \cdot \Delta
\]  

(6.1)
Define the following transition probability from a peak (trough) to the following (adjacent) trough (peak)

\[ t_{jk} = \text{Prob}\{s_j \text{ to } s_k\} \]  \hspace{1cm} (6.5)
\[ u_{jk} = \text{Prob}\{v_{s_j} \text{ to } p_{s_k}\} \]  \hspace{1cm} (6.6)
\[ d_{jk} = \text{Prob}\{p_{s_j} \text{ to } v_{s_k}\} \]  \hspace{1cm} (6.7)

\[ (k, j = 1, \ldots M) \]

Obviously, the transition probability matrix \( T \) is

\[ T = [t_{jk}] = [u_{jk} + d_{jk}] = U + D \]  \hspace{1cm} (6.8)

\( U \) and \( D \) are upper- and lower triangle matrices respectively. The calculation of the adjacent extremum transition probability matrix is discussed in a later section.
6.2.2 Peak-peak transition probability

One intermediate trough

Let \( q_{kl}^{(1)} \) be the probability of a transition from a peak \( s_k \) to another peak \( s_l \), with a single trough in between which is larger than \( s_j \). Then

\[
q_{kl}^{(1)} = \sum_{m=j}^{\min(k,l)-1} d_{km} u_{km}
\]

\((k, l, j = 1, \ldots M)\)

Since \( \forall m \geq k, d_{km} = 0 \); and \( \forall m \geq l, u_{ml} = 0 \), then

\[
q_{kl}^{(1)} = \sum_{m=j}^{M} d_{km} u_{ml}
\]

\((k, l, j = 1, \ldots M)\)

Define matrix

\[
I_j = \begin{pmatrix}
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & \cdots & 1
\end{pmatrix}
\]
chapter 6

Define vectors

\[
G_k = \begin{bmatrix}
0 \\
0 \\
\cdot \\
1 \\
\cdot \\
0
\end{bmatrix}
\]

\[
H_k = \begin{bmatrix}
0 \\
0 \\
\cdot \\
1 \\
\cdot \\
1
\end{bmatrix}
\]

Obviously

\[
H_k = \sum_{i=k}^{M} G_i
\]  \hspace{1cm} (6.11)

Then \( q_{kl}^{(1)} \) can be written as

\[
q_{kl}^{(1)} = G_k^T DI_j U G_l
\]  \hspace{1cm} (6.12)

and

\[
Q^{(1)} = [q_{kl}^{(1)}] = DI_j U
\]  \hspace{1cm} (6.13)

Notice that \( q_{kl}^{(1)} = 0 \), \( \forall k, l \leq j \) (Fig. 6.3). This is obvious: the value of the trough in between can never exceeds the value of either of the peaks.
two intermediate troughs

The probability of a transition from a peak $s_k$ to another peak $s_l$, with two troughs inbetween (and one peak inbetween, of course), neither trough is lower than $s_j$, the intermediate peak less than $s_j$:  
\[
q_{kl}^{(2)} = \sum_{m=j}^{k-1} q_{km}^{(1)} q_{ml}^{(1)} = G_k^T D I_j U (I - I_k) D I_j U G_l
\]  
(6.14)

n intermediate troughs

Similarly, the transition probability from a peak $s_k$ to another peak $s_l$, with $n$ intermediate troughs in between, none of the troughs is lower than $s_j$, the intermediate peak less than $s_k$:  
\[
q_{kl}^{(n)} = G_k^T D I_j U [(I - I_k) D I_j U]^{n-1} G_l
\]  
(6.15)
6.2.3 Peak-peak transition with lowest trough to the right

The probability that the n-th peak is equal to or higher than the starting one, and that nothing inbetween is less than \( s_j \):

\[
a_k^{(n)j} = \sum_{i=k}^{M} q_{ki}^{(n)j} = G_k^T D I_j U [(I - I_k) D I_j U]^{n-1} H_k
\]

(6.16)

Obviously, when \( n \) approaches infinite, \( a_k^{(n)j} \) approaches 0, since one can always expect reaching level \( s_k \) again. That is,

\[
\lim_{n \to \infty} [(I - I_k) D I_j U]^n = 0
\]

The probability that the deepest trough to the right of a level \( s_k \) peak, before level \( s_k \) is reached again, is greater than or equal to \( s_j \) is

\[
R_k^i = \sum_{n=1}^{\infty} a_k^{(n)j} = \sum_{n=1}^{\infty} G_k^T D I_j U [(I - I_k) D I_j U]^{n-1} H_k
\]

(6.17)

notice that for any matrix \( A \), if \( \lim_{n \to \infty} A^n = 0 \), then \( \sum_{n=0}^{\infty} A^n = (I - A)^{-1} \), then

\[
R_k^i = G_k^T D I_j U [I - (I - I_k) D I_j U]^{-1} H_k
\]

(6.18)

\[\text{let } S = \sum_{n=0}^{\infty} A^n, \text{ notice that } \lim_{n \to \infty} A^n = 0, \text{ since } (I - A) \cdot S = S \cdot (I - A) = I, \text{ therefore } S = (I - A)^{-1}\]
The probability that the lowest trough to the right of a level $s_k$ peak, before level $s_k$ is reached again, falls into $s_j$, is then

$$r^i_k = R^i_k - R^{i+1}_k$$  \hspace{1cm} (6.19)$$

i.e.,

$$r^i_k = G^T_k (D I_j U [(I - (I - I_k) D I_j U)^{-1} - D I_{j+1} U [(I - (I - I_k) D I_{j+1} U)^{-1}]) H_k$$  \hspace{1cm} (6.20)$$

### 6.2.4 Peak-peak transition with lowest trough to the left

Applying the same method to the peak-peak transition with the lowest trough to the left. The probability that the current peak is at level $s_k$, when the $n$-th previous one is higher, and there is neither a peak higher than $s_k$, nor a trough inbetween lower than $s_j$, is

$$b_k^{(n)} = G^T_k D_b I_j U_b [(I - I_{k+1}) D_b I_j U_b]^{n-1} H_{k+1}$$  \hspace{1cm} (6.21)$$

where $D_b$ and $U_b$ are the backwards transition matrices. Notice that

$$d_{ji} = \text{Prob}(s_j - s_i)/p_j$$  \hspace{1cm} (6.22)$$

and
\[ u_{bij} = \frac{\text{Prob}(s_j - s_i)}{v_i} \] (6.23)

then

\[ u_{bij} = \frac{d_{ji}p_j}{v_i} \] (6.24)

Similarly,

\[ d_{bij} = \frac{u_{ji}v_j}{p_i} \] (6.25)

here \( t_{(n+1)}^{(n)} \) is not defined. Obviously \( b_k^{(n+1)} \) approaches 0 when \( n \) approaches \( \infty \), since one can always expect to reach a level higher than \( s_k \).

The probability that the deepest trough to the left of a level \( k \) peak, after a higher level than \( S_k \) is reached for the last time is not less than \( s_j \) is

\[ L_k^j = G_k^{T} D_b I_j U_b [I - (I - I_{k+1})D_b I_j U_b]^{-1} H_{k+1} \] (6.26)

recalling the physical meaning of \( L_M^j \), the probability that the overall minimum to the left of a given peak falls into \( s_j \):

\[ L_M^j = 1, \quad \forall j = 1 \]
\[ = 0, \quad \forall j = 2, \ldots, M \] (6.27)

Therefore, the probability that the deepest trough to the left of a level \( s_k \) peak, after a higher level than \( s_k \) is reached for the last time, falls into \( s_j \), is

\[ l_k^j = L_k^j - L_k^{j+1} \] (6.28)

### 6.2.5 Rainflow transition probability

The probability of a rainflow cycle \((s_j, s_k)\) is
\[ p_{kj} = p_k \left\{ [1 - L_k^{j+1}] r_k^j + [1 - R_k^j] r_k^j \right\} \quad (j, k = 1, \ldots M) \quad (6.29) \]

### 6.2.6 The probability density function of rainflow cycle ranges

The probability of rainflow cycle range \( k\Delta \) is

\[ P_R(k) = \sum_{i=1}^{M-k} p_{i,i+k} + \sum_{i=k+1}^{M} p_{i,i-k} \quad (k = 1, 2, \ldots M - 1) \quad (6.30) \]

i.e.,

\[ P_R(1) = \sum_{i=1}^{M-1} p_{i,i+1} + \sum_{i=2}^{M} p_{i,i-1} \]

\[ \ldots \]

\[ P_R(0) = 0 \]

The corresponding probability density function of rainflow cycle range \( k\Delta \) is therefore

\[ p_R(S_k = k\Delta) = \left( \sum_{i=1}^{M-k} p_{i,i+k} + \sum_{i=k+1}^{M} p_{i,i-k} \right) / \Delta \quad (k = 1, 2, \ldots M - 1) \quad (6.32) \]

### 6.3 Flow chart of calculation

As indicated in the previous sections, the whole calculation procedure for this method is based on the transition probabilistic matrix of adjacent peaks and troughs of the random process, peak value distribution of the process. The transition probabilistic matrix and the peak value probability distribution for general stationary signals are therefore required.

Efforts have been made to derive the joint probability density function of adjacent peaks and troughs for general stationary (not necessary Gaussian) signals. Due to mathematical difficulties, no closed form theoretical results have been achieved.
For Gaussian signals, Kowalewski’s joint probability density function for adjacent peaks and troughs is applied. This and the peak value probability density function for Gaussian signal are given in the next two sections. For general non-Gaussian signals, extensive simulation and analysis has been carried out and this is described in the next two chapters. Kurtosis is chosen to describe the departure of a non-Gaussian signal from a Gaussian distribution. By applying the rules of Artificial Neural Networks, the joint probabilistic distribution of adjacent peaks and troughs can be simulated. It is given by a trained neural network. This can then be applied to the theoretical solution. Procedures for this calculation are listed in the flow chart in Fig. 6.4.

6.4 Kowalewski’s Joint p.d.f. of Adjacent Peaks & Troughs

Applying the following assumptions:

- The process is a Gaussian, stationary random process;
- The distributions of means and amplitudes are independent (which is valid only for "sufficiently" small time interval $\tau$ between adjacent peaks and troughs);
- The expected value of the number of peaks per unit time (peak frequency) is approximately equal to the expected value of the number of inflection points per unit time (inflection point frequency), i.e.

$$E[P] = \sqrt{\frac{m_4}{m_2}} \approx E[PI] = \sqrt{\frac{m_6}{m_4}}$$

Then for "sufficiently" small time intervals $\tau$ between adjacent peaks and troughs, Kowalewski derived the probability density function (p.d.f.) of the amplitudes and mean values of adjacent extremes[60, eq.(10.46)]
Figure 6.4: flow chart of the calculation
\[ p_{m,\text{amp}}(\alpha_m, \alpha_a) = \begin{cases} c_1 \alpha_a \exp \{-c_2 [\alpha_a^2 (1 - \gamma^2) + \alpha_m^2 \gamma^2]\} & \forall \alpha_a \geq 0, -\infty < \alpha_m < +\infty \\ 0 & \forall \alpha_a < 0 \end{cases} \tag{6.33} \]

where

\[
\begin{align*}
    c_1 &= \frac{1}{m_0 \gamma^2 \sqrt{2\pi m_0 (1 - \gamma^2)}} \\
    c_2 &= \frac{1}{2m_0 \gamma^2 (1 - \gamma^2)} \\
    \gamma^2 &= \left( \frac{N_0}{N_1} \right)^2
\end{align*} \tag{6.34} \]

\( N_0 \) and \( N_1 \) are the zero crossing frequency and peak frequency respectively; \( m_0 \) is the variance (0-th moment of the PSD).

Equation 6.33 satisfies the necessary conditions of a p.d.f., i.e.

\[
\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p_{m,\text{amp}}((\alpha_m, \alpha_a)) d\alpha_m d\alpha_a = 1 \tag{6.35} \]

Notice the definition of peak and trough height \( \alpha_1, \alpha_2 \)

\[
\alpha_1 = \alpha_m - \alpha_a, \quad \alpha_2 = \alpha_m + \alpha_a \]

\[
\alpha_m = \frac{\alpha_2 + \alpha_1}{2}, \quad \alpha_a = \frac{\alpha_2 - \alpha_1}{2} \tag{6.36} \]

Substitute into eq 6.33 and notice that the joint p.d.f. of new variables after transformation is the original p.d.f. times the Jacobian (see, for example, [75, eq.(8-6~9)]). Here the Jacobian is \( J = \frac{\partial(\alpha_m, \alpha_a)}{\partial(\alpha_1, \alpha_2)} = \frac{1}{2} \), we get the joint p.d.f. of adjacent extreme values:
As linear transformation doesn’t affect the “normality” of p.d.f., we have

$$\int \int_{-\infty}^{+\infty} p_{\text{max, min}}((\alpha_1, \alpha_2)) d\alpha_1 d\alpha_2 = 1$$

let \( m_0 = \sigma^2, \alpha_1 = \alpha_1/\sigma, \alpha_2 = \alpha_2/\sigma \). Notice the transform Jacobian is \( J = \sigma^2 \),

then equation 6.37 becomes

$$\left\{ \begin{array}{ll}
\frac{\alpha_2 - \alpha_1}{4m_0\gamma \sqrt{2\pi m_0(1-\gamma^2)}} exp\left\{-\frac{1}{2m_0(1-\gamma^2)} \left[ \alpha_1^2 + \alpha_2^2 + 2\alpha_1 \alpha_2 (2\gamma^2 - 1) \right]\right\} & \forall \alpha_1 < \alpha_2 \\
0, & \forall \alpha_1 \geq \alpha_2
\end{array} \right.$$  

(6.38)

This equation shows that the joint p.d.f. of normalised adjacent peaks and troughs of Gaussians signals is governed by the irregularity factor \( \gamma \). For general non-Gaussian signals, there is no similar closed form expression available. From the next two chapters, it can be seen, by extensive simulation and analysis, a neural network can be trained to express the joint probabilistic distribution of adjacent peaks and troughs with kurtosis as the description of the departure of a non-Gaussian signal from a Gaussian distribution.

Eq. 6.37 can also be expressed in terms of peak value \( \alpha_2 \) and range value \( \alpha_{\text{rng}} = \alpha_2 - \alpha_1 = 2\alpha_a \):

$$p_{\text{max, rng}}(\alpha_2, \alpha_{\text{rng}}) = \left\{ \begin{array}{ll}
\frac{\alpha_{\text{rng}}}{4m_0\gamma \sqrt{2\pi m_0(1-\gamma^2)}} exp\left\{-\frac{1}{2m_0(1-\gamma^2)} \left[ \alpha_2^2 + \frac{\alpha_{\text{rng}}^2}{4\gamma^2} - \alpha_2 \alpha_{\text{rng}} \right]\right\} & \forall \alpha_{\text{rng}} \geq 0 \\
0, & \forall \alpha_{\text{rng}} < 0
\end{array} \right.$$  

(6.39)

Similarly, the joint p.d.f. of peak value \( \alpha_2 \) and amplitude value \( \alpha_a = \frac{\alpha_{\text{rng}}}{2} \) is
The marginal distribution p.d.f. of range is

\[
\begin{align*}
    p_{\text{rng}}(\alpha_{\text{rng}}) &= \int_{-\infty}^{+\infty} p_{\text{max, rng}}(\alpha_2, \alpha_{\text{rng}}) d\alpha_2 \\
    &= \frac{\alpha_{\text{rng}}}{4m_0\gamma^2} e^{\frac{\alpha_{\text{rng}}^2}{4m_0\gamma^2}}
\end{align*}
\]

(6.41)

This is a Rayleigh Distribution.

Similarly, the joint p.d.f. of trough value \( \alpha_1 \) and Range value \( \alpha_{\text{rng}} \) is

\[
\begin{align*}
    p_{\text{min, rng}}(\alpha_1, \alpha_{\text{rng}}) &= \left\{ \frac{\alpha_{\text{rng}}}{4m_0\gamma^2\sqrt{2\pi m_0(1-\gamma^2)}} e^{\frac{-\alpha_1^2}{2m_0(1-\gamma^2)} + \frac{\alpha_{\text{rng}}^2}{4m_0\gamma^2}} \right\} \\
    &\quad \forall \alpha_{\text{rng}} \geq 0 \\
    &\quad \forall \alpha_{\text{rng}} < 0
\end{align*}
\]

(6.42)

Fig. 6.5 shows the joint p.d.f. of adjacent peaks and troughs of a Gaussian signal for \( \gamma = 0.1, 0.4, 0.7, 0.9 \).

6.5 Peak Value Probability Density Function

For stationary, Gaussian, zero-mean-valued random processes, the p.d.f. of its peak value[62, eq.(9-36a)] \( p(s) \) (i.e., the probability that its peak value falls in \([s, s + ds]\)) is \( p(s)ds \)

\[
\begin{align*}
    p(s) &= \sqrt{\frac{1-\gamma^2}{2\pi m_0}} e^{\frac{-s^2}{2m_0(1-\gamma^2)}} \\
    &\quad + \frac{\gamma s}{2m_0} \left\{ 1 + \text{sign}(s) \cdot \text{erf} \left[ \frac{\gamma s}{\sqrt{2m_0(1-\gamma^2)}} \right] \right\} e^{\frac{-s^2}{2m_0}}
\end{align*}
\]

(6.43)

where \( \gamma = \frac{m_2}{\sqrt{m_0 m_4}} \leq 1 \) is the irregularity factor; \( m_0 = \sigma_z^2 \) is the variance (0-th moment of the Power Spectral Density); \( \text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-u^2} du \) is the error function;
Figure 6.2: Joint p.d.f. of adjacent peaks and troughs of a Gaussian Signal
sign(s) is sign function:

\[ \text{sign}(s) = \begin{cases} 
1 & \text{if } s > 0 \\
0 & \text{if } s = 0 \\
-1 & \text{if } s < 0 
\end{cases} \]

For the normalised peak value \( a = \frac{s}{\sqrt{m_0}} \), Jacobian \( J = \sqrt{m_0} \), then

\[
p(a) = \sqrt{\frac{1 - \gamma^2}{2\pi}} \exp \left\{ -\frac{a^2}{2(1 - \gamma^2)} \right\} \\
+ \frac{\gamma a}{2} \left\{ 1 + \text{sign}(a) \cdot \text{erf} \left[ \frac{\gamma a}{\sqrt{2(1 - \gamma^2)}} \right] \right\} \exp \left( -\frac{a^2}{2} \right) \tag{6.44}
\]

Fig. 6.6 shows the distribution.

Furthermore, if the process is narrow-banded, i.e., \( \gamma = \frac{m_2}{\sqrt{m_0 m_4}} \approx 1 \), then from eq. (6.43),

\[
p(s) \approx s e^{-\frac{s^2}{2m_0}} \tag{6.45}
\]

This is a Rayleigh distribution with mean value \( \sqrt{\pi m_0/2} \) and variance \( \frac{\pi m_0}{2} \).

### 6.6 Examples of the Applications

The new method has been applied to a number of simulated Gaussian signals as well as some measured data. For Gaussian signals, Kowalewski’s joint PDF for adjacent peaks and troughs is used. For non-Gaussian signals, a trained neural network (see chapter 8) is applied. Examples for WEG data are given in Fig. 6.7 to Fig. 6.10. It can be seen that the new developed method produces a closer cycle range PDF to the time domain solution than other methods such as the Narrow Band approach and it is at least as good as Dirlik’s. This method is also applied to Howden data. Fig. 6.11 shows the calculated PDF for H26, 5m flapwise and edgewise. Since there is a strong deterministic component in the edgewise signal, none of the methods can cope with it.
Figure 6.6: Peak Value p.d.f. of a Gaussian Signal: Gamma = 0.01, 0.255, 0.5, 0.745, 0.99
Figure 6.7: Cycle Range PDF for WEG data: y12a & b
Figure 6.8: Cycle Range PDF for WEG data: y27a & b
Figure 6.9: Cycle Range PDF for WEG data: y27c & d
Figure 6.10: Cycle Range PDF for WEG data: y27e & f
Figure 6.11: Cycle Range PDF for Howden data: H26: 5m Flapwise & Edgewise
The corresponding fatigue damage rates (section 4.3) are given in Table 6.1 (assume S-N curve slope $b = 5$). Except for the WEG-y27a data case, the results for all other data sets have been improved.

Table 6.1: Fatigue Damage rates calculated by the new method

<table>
<thead>
<tr>
<th>Signal</th>
<th>Narrow Band</th>
<th>Dirlik</th>
<th>Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>WEG-y12a</td>
<td>2.79</td>
<td>1.12</td>
<td>1.06</td>
</tr>
<tr>
<td>WEG-y27a</td>
<td>4.39</td>
<td>1.16</td>
<td>1.19</td>
</tr>
<tr>
<td>WEG-y27b</td>
<td>2.56</td>
<td>1.19</td>
<td>1.09</td>
</tr>
<tr>
<td>WEG-y27c</td>
<td>1.85</td>
<td>0.779</td>
<td>0.966</td>
</tr>
<tr>
<td>WEG-y27d</td>
<td>2.72</td>
<td>0.928</td>
<td>0.958</td>
</tr>
<tr>
<td>WEG-y27e</td>
<td>1.89</td>
<td>0.917</td>
<td>0.921</td>
</tr>
<tr>
<td>WEG-y27f</td>
<td>2.15</td>
<td>1.08</td>
<td>1.06</td>
</tr>
<tr>
<td>H26 5m flapwise</td>
<td>3.34</td>
<td>0.521</td>
<td>0.752</td>
</tr>
<tr>
<td>H26 5m edgewise</td>
<td>2.94</td>
<td>0.884</td>
<td>1.032</td>
</tr>
</tbody>
</table>
Chapter 7

LINK BETWEEN NON-GAUSSIANALITY AND FATIGUE LIFE

7.1 Introduction

In most structural dynamics analyses, the loading process and the structural response are assumed to be normal (Gaussian), an assumption many structural engineers would be comfortable with. The main reasons are perhaps that it can enable a more elegant analysis and that most loading processes due to natural phenomena are thought to be Gaussian. If one assumes that the structure is linear, the structural response will be Gaussian as long as the dynamic loading is. However, in many cases the loading is non-Gaussian (e.g. dynamic/viscous effects of wave loading) and furthermore, if structural nonlinearity (e.g. geometric or material) is an issue, the structural response will be non-Gaussian even if the loading is Gaussian. Some studies[36, 48, 96, 56, 98, 45, 43, 59, 97, 57] have indicated the effect of non-Gaussianality cannot be ignored. Hu and Lutes[64, 47] carried out work into the understanding of the nature of the non-Gaussianity of the response of structural systems subjected to random loading. Hooper and Swannell[44] carried out an extensive numerical study of the response of compliant offshore structures subjected to Gaussian waves. They showed that, depending on the structural parameters, the response can be highly non-Gaussian. Lutes et al[66] carried out an investigation into the effect of the non-Gaussianity of the loading process on the rate of fatigue damage accumulation. Using limited simulation results, they concluded that under
certain circumstances non-Gaussianity can significantly alter the rate of fatigue damage accumulation. This non-Gaussianity is not readily evident if the measurement is quantified by a PSD curve and is often overlooked (see the following sections). It is therefore important to investigate the consequences on fatigue damage estimates of assuming that a non-Gaussian process is Gaussian when in reality it is not.

Most existing frequency domain fatigue approaches are based on the assumption that the loading stress is Gaussian. This, in some cases, leads to a significant error for fatigue damage estimation, e.g. as for the Howden wind turbine blade fatigue analysis.

In this chapter the mathematical description of a non-Gaussian process is firstly given. Existing approaches dealing with non-Gaussianity are then reviewed. A study of the link between non-Gaussianity and fatigue life is then carried out by using extensive simulations and an Artificial Neural Network. Details about applying artificial neural networks to establish a toolbox are given in the next chapter.

7.2 Mathematical Description of Non-Gaussian Random Processes

For reference, this section briefly describes the non-Gaussian random process and shows a few approaches dealing with the effect of non-Gaussianity.

7.2.1 Basic concepts

• Gaussian Distribution

A process $x(t)$ ($x(t) = (x_1, x_2, ..., x_n)^T$) is called Gaussian if the joint probability density function (p.d.f.) of its components is
\[
p(x) = \frac{1}{(2\pi)^{3/2}\sqrt{\det{R}}} \exp\left\{ -\frac{1}{2}(x - x_0)^T R^{-1}(x - x_0) \right\}
\]  
(7.1)

where \(x_0\) is the mean value vector, \(R\) is the Covariance Matrix.

The p.d.f. is determined by its first two moments and is symmetric to the mean value. Any higher moment can be represented by the first two moments. This is one of the most important properties of a Gaussian process.

- **Skewness (or Coefficient of Skewness)**— This is a measure of asymmetry of the distribution.

\[
\gamma_1 = \frac{\mu_3}{\sigma^3}
\]

(7.2)

where \(\mu_3\) is the 3rd central moment, \(\sigma\) is the RMS (Root Mean Square).

- **Kurtosis (or Coefficient of Excess)**— A measure of how far the process departs from a Gaussian distribution

\[
\gamma_2 = \frac{\mu_4}{\sigma^4} - 3
\]

(7.3)

where \(\mu_4\) is the 4th central moment.

Kurtosis is actually a measure of the peakedness of the probability density function (pdf) of the process. For Gaussian processes, the value of kurtosis is exactly equal to zero. Any deviation from this value indicates that the process is non-Gaussian. The minimum value of kurtosis is \(-2\) for a symmetric binomial process; the maximum value of kurtosis may be infinite, as in the case of a probability density function with a very slowly decaying tail. A kurtosis value larger than zero indicates a more highly peaked distribution (leptokurtic) having more probability mass near the mean value and in the tails of the distribution than in a Gaussian distribution having the same standard deviation. The converse is true for a process with a kurtosis value less than
zero (platykurtic) having less probability mass near its mean value and in the tails of the distribution than a Gaussian distribution having the same standard deviation. This parameter, kurtosis, is used in this thesis to characterize the degree of non-Gaussianity of a loading process.

- Cumulants (or Semi-invariant)

Denote the characteristic function

$$\varphi(t) = \int_{-\infty}^{\infty} e^{itx} dF(x)$$  \hspace{1cm} (7.4)

Expand

$$\log \varphi(t) = \sum_{\nu=1}^{k} \frac{\chi^{\nu}}{\nu!} (it)^\nu + O(t^k)$$  \hspace{1cm} (7.5)

where $F(x)$ is the distribution function, $\chi^{\nu}$ ($\nu = 1, 2, ...$) are called the Cumulants (or Semi-invariants of the distribution).

Skewness and kurtosis can be expressed in terms of cumulants

$$\gamma_1 = \frac{\chi_3}{\chi_2^{3/2}}$$  \hspace{1cm} (7.6)

$$\gamma_2 = \frac{\chi_4}{\chi_2^2}$$  \hspace{1cm} (7.7)

7.2.2 Mathematical description of non-gaussian random variables

The most sophisticated description of a random variable is the p.d.f. However, for variables derived from experimental data, this type of description is usually not available. In practical cases, only those statistical quantities, such as mean, variance, skewness, kurtosis, etc. can be estimated. If one assumes the signal is Gaussian, then the mean value and variance provide sufficient information to describe the p.d.f. For non-Gaussian signals, some higher moments need to be introduced to approximately describe it.
Gram-Charlier expansion

This is to expand the p.d.f as

\[ p(x) = \varphi(x) \left[ 1 + \sum_{k=3}^{n} \frac{C_k}{k!} H_k(x) \right] \quad (7.8) \]

where \( \varphi(x) \) is the Gaussian distribution with given mean value and variance; \( C_k \) \( (k=3,...n) \) are constants, which to be determined from higher statistical moments; \( H_k(x) \) \( (k=3,...n) \) are Hermite Polynomials of order \( k \):

\[ H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2} \quad (7.9) \]

i.e. \( H_0(x) = 1, H_1(x) = 2x, H_2(x) = 4x^2 - 2, H_3(x) = 8x^3 - 12x, H_4(x) = 16x^4 - 48x^2 + 12, \ldots \)

The most important property of Hermite Polynomials is orthogonality:

\[ \int_{-\infty}^{\infty} H_m(x)H_n(x)e^{-x^2}dx = 2^n n! \sqrt{\pi} \delta_{mn} \quad (7.10) \]

\( (m, n = 0, 1, 2, \ldots) \)

This method actually introduces a "correction factor" to the Gaussian p.d.f. The correction factor relates only to the higher moments. The drawback of this method is that the derived p.d.f. could be negative.

In [48], Jenson applied this method and derived the expected value of the cumulative damage \( \Delta D \) per cycle

\[ \Delta D = \frac{\Gamma(1 - \frac{b}{2})}{c(2\sqrt{2}\sigma_{\xi})^b} \frac{1 + \frac{1}{24}(b^2 + 4b + 3)\lambda_{40} - \frac{1}{4}(m + 1)\lambda_{22} - \frac{1}{24}\lambda_{04}}{1 + \frac{1}{8}\lambda_{40} - \frac{1}{4}\lambda_{22} - \frac{1}{24}\lambda_{04}} \quad (7.11) \]

where \( b, c \) are material constants; \( \lambda_{mn} = \frac{K_{mn}}{\sigma_{\xi}^m \sigma_{\xi}^n} \) are "normalized" cumulants (where \( K_{mn} \) are cumulants; \( \xi_1 = \frac{d\xi}{dt} \)); \( \lambda_{40} \) and \( \lambda_{04} \) are Kurtosises of \( \xi \) and \( \xi_1 \) respectively.
If \( \lambda_{40}, \lambda_{04}, \lambda_{22} \ll 1 \), then
\[
\Delta D = \frac{\Gamma(1 - \frac{b}{2})}{c(2\sqrt{2}\sigma_q)^b} \left\{ 1 + \frac{1}{24} \left[ b^2 \lambda_{40} + b(4\lambda_{40} - 6\lambda_{22}) \right] \right\} \tag{7.12}
\]

The interesting fact is the correction factor is independent of \( \lambda_{04} \).

Cumulant neglect closure method

The central idea of this method is:

- assume the cumulants higher than a certain order vanish.
- construct a characteristic function
  \[
  \varphi(t) = \exp \left[ \sum_{k=3}^{n} \frac{\chi_k}{k!} (it)^k \right] \tag{7.13}
  \]
  where \( \chi_k \) (k=3,...,n) are the cumulants calculated from, say, original data.
- carry out the inverse Fourier transformation of \( \varphi(t) \) and get the p.d.f.

This method could also lead to a negative p.d.f.

Maximum entropy method (MEM)

This is a variational problem[9]: Maximizing the “entropy” \( H_p \) of a p.d.f. \( p(x) \)
\[
H_p = -\int_{-\infty}^{\infty} p(x) \ln p(x) dx \tag{7.14}
\]
subject to the constraints:
\[
E[x^k] = \int_{-\infty}^{\infty} x^k p(x) dx \tag{7.15}
\]
This problem has a solution in the form
\[ p(x) = \exp\left[ \sum_{k=0}^{n} C_k x^k \right] \]  
(7.16)

where coefficients \( C_k \) are determined from the statistical moments, i.e. to satisfy the constraints.

A drawback of this method is that for \( n \) odd the p.d.f. may become unbounded at either \( x \to \infty \) or \( x \to -\infty \).

**Nonlinear functional transform**

Taking a nonlinear functional transform\([36, 96, 47]\)

\[ x(t) = G[u(t)] \]  
(7.17)

where \( x(t) \) is the Non-Gaussian process, \( u(t) \) is a Gaussian process; \( G[.] \) is a monotonically increasing odd nonlinear function.

The principle for choosing \( G[.] \) is make \( x(t) \) and \( u(t) \) "equivalent" in the sense of variance, zero-crossing rate and irregularity factor. An example in \([47]\) is

\[ G(u) = a \left( u + \frac{u^3}{\sigma_u^2} \right) \]

with

\[ a = \frac{1}{\sqrt{1 + 6\omega + 15\omega^2}} \]

**7.3 Statistical Description of Non-Gaussian Process: Time Domain and Frequency Domain**

As is the case for random variables, the optimal description of a random process is given in terms of all possible joint probability density functions (pdf) at different times. Of course, there are an infinite number for continuous time processes. Even under conditions of stationarity and ergodicity it appears to be impossible to obtain
the required statistical information from the analysis of experimental data. On the other hand, it is relatively easy to determine moment functions such as mean, root mean square, autocorrelation, 3rd order correlation up to some order with sufficient accuracy and confidence. These higher order moment functions can be transformed into the frequency domain—provided stationarity exists—yielding higher order spectra.

7.3.1 Time domain

Quite analogously to the well-known autocorrelation

\[ R_{xx}(t_1, t_2) = E[x(t_1)x(t_2)] \]  

(7.18)

higher order moment functions can be defined. For example, 3rd order correlation (Bi-correlation)

\[ R_{xxx}(t_1, t_2, t_3) = E[x(t_1)x(t_2)x(t_3)] \]  

(7.19)

and 4th order correlation (Tri-correlation)

\[ R_{xxxx}(t_1, t_2, t_3, t_4) = E[x(t_1)x(t_2)x(t_3)x(t_4)] \]  

(7.20)

In the above equations, \( E[.] \) denotes the ensemble average, \( t_k (k=1,2,3,4) \) denote time arguments and \( x \) is a process. If \( x(t) \) is stationary then the number of time variables is reduced by one, i.e. only the time lags are important, so that

\[ R_{xxx}(\tau_1, \tau_2) = E[x(t)x(t+\tau_1)x(t+\tau_2)] \]  

(7.21)

\[ R_{xxxx}(\tau_1, \tau_2, \tau_3) = E[x(t)x(t+\tau_1)x(t+\tau_2)x(t+\tau_3)] \]  

(7.22)

Without losing generality, we assume \( x(t) \) is zero mean process. It should be noted that in the case of ergodic \( x(t) \) the ensemble averages may be replaced by time averages.

For zero time lags, the correlation function yield the one time central statistical moments of the process \( x(t) \):

\[ \mu_{3,x} = R_{xxx}(0,0) = E[x(t)^3] \]  

(7.23)
\[ \mu_{4,x} = R_{xxxx}(0, 0, 0) = E[x(t)^4] \]  

while not giving complete information, these two quantities still provide some measure for non-Gaussian properties of the process \( x(t) \). Kurtosis can then easily be derived.

### 7.3.2 Frequency domain

Analogous to the well known Wiener-Khintchine relationship, multiple Fourier transforms can be applied to higher order spectra. The Bi-spectrum and Tri-spectrum are defined by

\[
S_{xxx}(\omega_1, \omega_2) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{xxx}(\tau_1, \tau_2) e^{-i\omega_1 \tau_1} e^{-i\omega_2 \tau_2} d\tau_1 d\tau_2
\]

\[
S_{xxxx}(\omega_1, \omega_2, \omega_3) = \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{xxxx}(\tau_1, \tau_2, \tau_3) e^{-i\omega_1 \tau_1} e^{-i\omega_2 \tau_2} e^{-i\omega_3 \tau_3} d\tau_1 d\tau_2 d\tau_3
\]

The inverse relations are given by:

\[
R_{xxx}(\tau_1, \tau_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{xxx}(\omega_1, \omega_2) e^{i\omega_1 \tau_1} e^{i\omega_2 \tau_2} d\omega_1 d\omega_2
\]

\[
R_{xxxx}(\tau_1, \tau_2, \tau_3) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{xxxx}(\omega_1, \omega_2, \omega_3) e^{i\omega_1 \tau_1} e^{i\omega_2 \tau_2} e^{i\omega_3 \tau_3} d\omega_1 d\omega_2 d\omega_3
\]

Introducing zero time lags in above equations:

\[
\mu_{3,x} = R_{xxx}(0, 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{xxx}(\omega_1, \omega_2) d\omega_1 d\omega_2
\]

\[
\mu_{4,x} = R_{xxxx}(0, 0, 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{xxxx}(\omega_1, \omega_2, \omega_3) d\omega_1 d\omega_2 d\omega_3
\]

skewness and kurtosis can then be derived from eqs7.2 and 7.3.

### 7.4 Methods Dealing With Non-Gaussianity in Random Fatigue Analysis

In this section, we'll briefly review the existing methods dealing with non-Gaussianity in random fatigue analysis.
7.4.1 Correction factor approach

Applying the methods presented in the previous section usually leads to a correction factor for the fatigue damage rate ($\Delta D$). This correction factor is usually the function of some higher moments (say, skewness, kurtosis or higher-order cumulants). In this case the effect of Non-Gaussianity is taken into account in the correction factor.

transformation method

Suppose the non-Gaussian response of a structure is denoted as $X(t)$. It can be the result of applying a monotonic transfer function, $g$, to a standard normal process, $U(t)$: [4] [5] [55]

$$X(t) = g[U(t)]$$

in which $g(u) = F_x^{-1}[\Phi(u)]$ in terms of the cumulative distribution functions, $F_x$ and $\Phi$, of $X(t)$ and $U(t)$.

The difficulty in using this equation is that, the transfer function $g$, in general, must be determined numerically, complicating the subsequent fatigue analysis. And, it is not clear how $g$ should be chosen if only certain response moments are available. To overcome these difficulties, a Hermite series approximation to $g$ is constructed based on the known response moments.

$$X(t) = \sum_{n \geq 0} \alpha_n H_n[U(t)]$$

Since $\bar{x} = \alpha_0$, $\sigma_x = \alpha_1$, the above series can be rearranged into a standardised form:

$$X_0(t) \equiv \frac{X(t) - \bar{x}}{\sigma_x} = U(t) + \sum_{n \geq 2} \epsilon_{n+1} H_n[U(t)]$$

in which it assumed that $\epsilon_{n+1} \equiv \alpha_n/\sigma_x \ll 1(n > 2)$, so that the $O(\epsilon_n \epsilon_m)$ terms are negligible. Depending on the known moments, a suitable truncated order can be selected as an approximation of $g$. 
A fatigue analysis can then follow this approximation. If $X(t)$ is narrow-band, the peak distribution of $U(t)$ would have a Rayleigh distribution. If $U(t)$ has a peak at level $S$, $X(t)$ would have a peak at level $g(S)$ and there would exist a cycle $g(S)-g(-S)$. The fatigue damage is then determined by using the moments of the signal.

In [96], Winterstein applied the nonlinear functional transform

$$\frac{X(t) - m_x}{\sigma_x} = U(t) + \frac{\alpha_3}{6}[U^2(t) - 1] + \frac{\alpha_4 - 3}{24}[U^3(t) - 3U(t)]$$

and derived the mean damage accumulation rate

$$\eta_D = cE[P](2\sqrt{2}\sigma_x)^b \left( \frac{b}{2} \right)! \left[ 1 + \frac{b(b-1)(\alpha_4 - 3)}{24} \right]$$

weakly non-Gaussian approximation

The Gram-Charlier approximation of PDF described in the previous section can be developed into another expression for calculating the distribution of signal peaks. Firstly, the joint probability density function $p(\xi, \dot{\xi})$ of a random variable $\xi$ can be calculated. Then, if the response is narrow-banded, the probability distribution $F(\dot{\xi})$ for the peak values of $\xi$ is approximated by [3]:

$$F(\dot{\xi}) = 1 - \frac{N(\dot{\xi})}{N(0)}$$

where $N(\dot{\xi})$ is the average number of crossing of level $\xi = \dot{\xi}$ per unit time

$$N(\dot{\xi}) = \int_{-\infty}^{\infty} p(\xi, \dot{\xi})|\dot{\xi}|d\xi$$

This approximation assumes that the most-often crossed level is $\dot{\xi} = 0$. This holds for a symmetric response, but is just an approximation for an asymmetric response, even with zero mean.

Using this approximation, the probability distribution for the peak values is obtained. The peak range distribution is then found to be

$$F_a(\dot{\xi}) = \frac{1}{2}[F(\dot{\xi}) + F(-\dot{\xi})]$$
because of the narrow-band assumption. The fatigue damage is then evaluated from
the cycle probability density function as before.

7.4.2 Koliopulos's "separability" approach

In [59] Koliopulos proposed a "separability" approach for linear structural systems,
but has not been widely applied in engineering applications.

7.5 Peak-trough Series Regeneration

Because of the difficulties arising in dealing with non-Gaussian response histories,
signal simulation or peak-trough regeneration could be a better choice. Extensive non-
Gaussian signal simulation and analysis reveals the link between non-Gaussianality
and fatigue damage. The methodology used for Standardised Load Sequence regen­
eration is examined here as one possible approach for non-Gaussian fatigue analysis
[29].

7.5.1 Transition matrix

Modern structural fatigue laboratories are usually equipped with computer-controlled
servohydraulic machines. A variable amplitude fatigue experiment can then easily
be performed using complicated loading histories. One problem which arises with
such tests is, if several experiments are conducted with different loading histories
in different laboratories, the results are difficult to compare. To overcome this dif­
ficulty, standardised load sequences have been developed, such as FALSTAFF for
aircraft [46] [92], WASH for offshore structures [81], etc. Generally, the load se­
quencies are stored as a Markov matrix which denotes the transition probabilities
from peak to trough and from trough to peak [38].

The requirements for a standardised load sequence are:

- the choice of the essential parameters of the sequence must be well founded;
- the sequence must be realisable in a practicable manner on the test equipment.

The basis of a meaningful standardised load history is either strain or load measurements in service, preferably from a number of similar structures. From these many measurements, common features can be extracted; that is, their spectrum shapes must be similar. Based on these strain measurements in service, an "average" spectrum can then be selected.

### 7.5.2 Load sequence generation

The procedure for generating a random sequence of peaks and troughs from the transition matrix is explained in Fig. 7.1. Transitions from troughs to peaks (the corresponding transition frequencies are found right above the diagonal elements) is distinct from transitions from peaks to troughs (the corresponding transition frequencies are found left below the diagonal elements). All transitions from an interval \( i \) to an interval \( j \) will be simulated by a transition from a definite level \( i \) to a definite level \( j \), each of these levels being defined by the centre of the particular interval.

The routine for load sequence generation is as follows:

1). Determine the cumulative distribution for each row of the transition matrix.

2). Generate a random number between 1 and the maximum level of signal as the start point. Nearly all computers nowadays provide an intrinsic function as a pseudo random number generator which has a uniform distribution in a given area.

3). Generate another random number inside the range of the cumulative distribution of the starting row. Take the hit column number as the trajectory turning point (next peak or trough).

4). Repeat procedure 3. until the return period length is reached.

This method has been applied to the WEG MS-1 data with the transition matrices obtained directly from the time histories. Figure 7.2 shows the rainflow cycle
PDF's for such a regenerated load sequence, together with the original time series from which the transition matrix is extracted. This time series was simulated using the turning point matrix of the of WEG data $y_{35d}$ time history after the noise above $15.625\text{Hz}$ was filtered out. The results here show a good consistency.

The random number generator plays a very important role in the peak-trough regeneration. An ordinary random number generator guarantees the generated sequence is statistically equivalent to the original sequence. It does not necessarily give the same rainflow cycle range distributions. A carefully chosen pseudo random number generator can guarantee the generated sequence has identical rainflow cycle range distribution as the original sequence. (Details can be found in [39] and [58].)

### 7.6 Modelling the Effect of Non-Gaussianity

A simple and natural way to include non-Gaussianity is through consideration of moments higher than the second. In particular the 4th moment is important for characterising non-Gaussianity, especially if the random variable is symmetric about its mean values so that the 3rd moment gives no new information.
Figure 7.2: PDF's from regenerated load sequence: WEG y35d
experience indicates that kurtosis, proportional to the 4th moment, is the simplest and best description of non-Gaussianity for practical usage. Kurtosis is therefore taken as the non-Gaussian parameter in the modelling outlined below.

7.6.1 Outline of the modelling

As indicated in last chapter and in [94], an improved frequency domain method for calculating rainflow cycle range PDF's is proposed. While dealing with Gaussian distributed signals, Kowalewski's joint PDF for adjacent peaks and troughs is applied. An expression of the joint probability density function of adjacent peaks and troughs for non-Gaussian signal is also desired. Unfortunately, due to the mathematical difficulty, there is no theoretical expression available. However, numerical simulation and Artificial Neural Network training have been carried out to provide a practical toolbox for frequency domain fatigue analysis for non-Gaussian signals.

Kowalewski derived the expression

\[
\begin{align*}
p_{\text{max, min}}(\alpha_1, \alpha_2) &= \\
&= \frac{\alpha_2 - \alpha_1}{4m_0\gamma^2 \sqrt{2\pi m_0(1 - \gamma^2)}} \exp \left\{ -\frac{1}{8m_0\gamma^2(1 - \gamma^2)} \left[ \alpha_1^4 + \alpha_2^4 + 2\alpha_1 \alpha_2(2\gamma^2 - 1) \right] \right\} \quad (7.33) \\
&\forall \alpha_1 < \alpha_2, -\infty < \alpha_2 < +\infty \\
&0, \quad \forall \alpha_1 \geq \alpha_2
\end{align*}
\]

This expression is based upon the assumptions

- the process is a Gaussian, stationary random process;

- the distributions of means and amplitudes are independent (which is valid only for "sufficiently" small time interval \( \tau \) between adjacent peaks and troughs);

- the expected value of the number of peaks per unit time (peak frequency) is approximately equal to the expected value of the number of inflection points
per unit time (inflection point frequency), i.e.

\[ E[P] = \sqrt{\frac{m_4}{m_2}} \approx E[PI] = \sqrt{\frac{m_6}{m_4}} \]

Obviously, in Kowalewski’s formula, only \( m_0 \) (variance) and \( \gamma \) (irregularity factor) are related to the signal. This expression needs to be extended for non-Gaussian signals. For this purpose, a non-Gaussian parameter, kurtosis, will be introduced.

For the purpose stated above, Monte-Carlo simulation is applied. Without losing generality, we assume the signal is with mean value 0.

Simulation procedure is as following:

- Generate Gaussian signals with various \( \gamma \) (irregularity factor) and mean frequency; Seventy PSDs are chosen which represent a wide range of PSDs with \( \gamma \) from 0.167 to 0.989. (see next chapter).

- Applying nonlinear function transform and iteration, generate non-Gaussian signals with various \( \kappa \) (kurtosis); 25 kurtosis values were chosen ranging from 0.0 to 12.0, in steps of 0.5;

- Using the generated non-Gaussian signals, calculate the sample p.d.f. for Rain-Flow-Cycle ranges;

- Study the sample p.d.f. of Rain-Flow-Cycle ranges of the non-Gaussian signals, and then apply Artificial Neural Network (ANN) to establish a neural network to model the effect of non-Gaussianity (this is presented in Chapter 8).

In the following subsections, we briefly describe the simulation of Gaussian and non-Gaussian signals.

### 7.6.2 Simulation of stationary Gaussian process

A detailed description concerning the simulation of stationary Gaussian processes is given in [101].
The Fourier transform of a time series is the frequency domain representation of the time series information. No information has been lost up to this stage and in fact it is quite straightforward to transform back to the original time signal. When the power spectral density function is formed from the transformed data, however, the phase information is lost. One spectrum can therefore be theoretically derived from an infinite number of time signals in so far as they contain the same amplitude constitution but different phase information. In other words, only one spectrum can be derived from a time series but a lot of time series can be derived from one spectrum given that the phase information is unknown. Since the PSD does not contain all the information from the original signals, it is necessary to assume the probability distribution of the time series. This assumption generally makes it possible to derive a sample time series from a given spectrum.

To simulate a stationary Gaussian process for a given power spectral density function, consider a stationary Gaussian process $x_0(t)$ with zero mean and power spectral density $S(\omega)$. The process $x_0(t)$ can be expressed as

$$x_0(t) = \int_{-\infty}^{\infty} e^{i\omega t} dX_0(\omega)$$

(7.34)

Where $X_0(\omega)$ is an orthogonal random process with zero mean and

$$E[dX_0(\omega_1)dX_0^*(\omega_2)] = 0$$

(7.35)

$$E[|dX_0(\omega_1)|^2] = S(\omega) d\omega$$

where the asterisk denotes the complex conjugate. Using the above orthogonal condition, one can show that the autocorrelation function $R_{x_0}(\tau)$ of $x_0(t)$ and $x_0(t+\tau)$ is related to the spectral density as follows:

$$R_{x_0}(\tau) = \int_{0}^{\infty} G(\omega) \cos \omega \tau d\omega$$

(7.36)

Where $G(\omega) = 2S(\omega)$ for $\omega \geq 0$ is the one-sided spectral density function. The above equation is actually the Wiener-Khintchine relationship. Since $x(t)$ is a real
process, equation 7.34 can be written as[101]

\[ X(t) = \int_0^{\infty} \cos \omega t dU(\omega) + \sin \omega t dV(\omega) \] (7.37)

where

\[ dU(\omega_k) = [2G(\omega_k) \Delta \omega_k]^{1/2} \cos \psi_k \] (7.38)

\[ dV(\omega_k) = -[2G(\omega_k) \Delta \omega_k]^{1/2} \sin \psi_k \]

In which, \( \psi_k (k = 1, 2, \cdots) \) are independent and all uniformly distributed in \((0, 2\pi]\).

Substitute eq.(7.38) into (7.37) and approximate the integral by summation, the random process can be simulated by:

\[ X(t) = \sum_{k=1}^{N} \sqrt{2G(\omega_k) \Delta \omega_k} \cos(\omega_k t + \psi_k) \] (7.39)

Take the complex Fourier transform of \( 2G(\omega)e^{i\psi_k} \), a complex random process is expressed as:

\[ Y(t) = \sum_{k=1}^{N} \sqrt{2G(\omega_k) \Delta \omega_k} e^{i\omega_k t} \] (7.40)

The random process described by eq.(7.39) then become the real part of \( Y(t) \):

\[ x(t) = \sqrt{\Delta \omega} \text{Re}(Y(t)) \] (7.41)

From the theory described above, a sample random process for a given power spectral density can be simulated by FFT. Due to the central limit theorem, the process obtained this way is Gaussian distributed given phase \( \psi_k \) has a uniform distribution in \((0, 2\pi]\). It can also shown that such a simulated process is ergodic irrespective of \( N \). Thus, a relatively long process can be obtained by putting different simulated samples together.

### 7.6.3 Regeneration of Non-Gaussian signal

The flow chart of the modelling for non-Gaussian signals is given in Fig. 7.3. This is a modified version of the simulation of non-Gaussian signals described in [100].
Simulation of a non-Gaussian signal has been achieved by taking a nonlinear transformation of a Gaussian signal. That is, let \( x(t) \) be a time history from a Gaussian process, and let

\[
y(t) = G[x(t)]
\]

(7.42)
in which \( G(x) \) is a monotonically increasing odd nonlinear function. Then \( y(t) \) is non-Gaussian. The \( G(x) \) here are chosen simple so that \( \sigma_y^2 \) and \( E[y^4] \) can be evaluated analytically in terms of \( \sigma_x \), therefore the kurtosis can easily be calculated. The nonlinear functions have been scaled such that

\[
\sigma_y = \sigma_x
\]

(7.43)

Also, \( y(t) \) has zero crossings and peaks at precisely those times when \( x(t) \) has them. Thus the \( y \) and \( x \) processes are equivalent in terms of \( \text{rms} \) value, zero crossing rate and irregularity factor— the three parameters most commonly used to describe a fatigue process. The most obvious difference between the two processes is the non-Gaussianity of \( y \), which is measured by the parameter \( \kappa \).

### 7.6.4 The nonlinear function transform

The transformation is chosen so that the non-Gaussian process has the same \( \text{rms} \) as the Gaussian process. The following transform function is used

\[
y = \frac{1}{\sqrt{1 + 4 \cdot 0.919 \sqrt{\frac{2}{\pi}} \omega + \sqrt{\frac{8}{\pi}} \omega^2}} \cdot x[1 + \omega \sqrt{|x|}] 
\]

(7.44)

where \( \omega \) is a parameter.

The corresponding kurtosis is

\[
\kappa = 3 + \frac{a_1 \omega + a_2 \omega^2 + a_3 \omega^3 + 15 \omega^4}{(1 + b_1 \omega + b_2 \omega^2)^2} - 3
\]

(7.45)

where \( a_1 = 64 \sqrt{\frac{2}{\pi}} \Gamma(\frac{11}{4}), \ a_2 = 48 \sqrt{\frac{2}{\pi}}, \ a_3 = 128 \sqrt{\frac{2}{\pi}} \Gamma(\frac{13}{4}), \ b_1 = 4 \sqrt{\frac{2}{\pi}} \Gamma(\frac{7}{4}), \ b_2 = 2 \sqrt{\frac{2}{\pi}} \).
The relationship of parameter $\omega$ and $\kappa$ is shown in Fig. 7.4.

Fig. 7.5 shows the nonlinear transform when kurtosis are $0$, $0.5$, $2$, $5$, $10$ (corresponding to parameter values $0$, $0.0096$, $0.045$, $0.1779$, $0.8897$).

Examples of the generated non-Gaussian signals are shown in Fig. 7.6 - 7.9.
Figure 7.3: Flow chart of digital generation of Non-Gaussian Stochastic Signal
Figure 7.4: The relationship of parameter and kurtosis
Figure 7.5: Nonlinear transform
Figure 7.6: Simulated non-Gaussian Signal: example 1
Figure 7.7: Simulated non-Gaussian Signal: example 2
Figure 7.8: Simulated non-Gaussian Signal: example 3
Figure 7.9: Simulated non-Gaussian Signal: example 4
Chapter 8

MODELING THE EFFECT OF NON-GAUSSIANALITY

As indicated in the previous chapter, the rainflow cycle range p.d.f. for non-Gaussian stress signals is related to several parameters: root mean square of the signal; irregularity factor; mean frequency; parameter (e.g., kurtosis) representing the non-Gaussianality etc. This chapter presents details of a simulation procedure and applications of an Artificial Neural Network (ANN) for studying the effect of non-Gaussianality. A neural network is trained to link the rainflow cycle range PDF to these parameters. Though not expressed explicitly, this trained network is equivalent to a close form formula which represents rainflow cycle range PDF in terms of root mean square of the signal ($\sigma$), irregularity factor ($\gamma$), mean frequency ($x_m$) and rainflow cycle range ($S$). In this chapter, details of the simulation are first presented, followed by some basic concepts about artificial neural networks and the procedure for applying ANNs. Software is developed to provide a practical toolbox for frequency domain fatigue analysis for general stationary (non-Gaussian) signals. Results and discussions are finally given.

8.1 Procedure of Simulation

The simulation procedure is as following:

- Generate Gaussian signal with various $\gamma$ (irregularity factor) and $x_m$ (mean frequency, see eq.(8.2)); Seventy PSDs are chosen to cover a wide range of $\gamma$ (0.167 - 0.989) and $x_m$ (0.103 - 0.982).
• Applying nonlinear function transforms, generate non-Gaussian signals with
various $\kappa$ (kurtosis); 25 $\kappa$ values (0.0-12.0, step 0.5) were used.

• Using the generated non-Gaussian signals, count the sample p.d.f. (histogram)
for Rain-Flow-Cycle ranges (as an application of the simulation procedure,
transition PDF (histograms) of adjacent peaks and troughs were first counted);

• Study the sample p.d.f. of Rain-Flow-Cycle ranges of non-Gaussian signal (and
the transition PDFs of adjacent peaks and troughs), applying an Artificial
Neural Network (ANN) to establish a neural network to model the effect of
non-Gaussianity.

A total of 70 different spectral densities were used. These PSD's were carefully
chosen to cover a wide range of spectral types. For the ease of comparison, the
densities are shaped in such a way that they all have the same root mean squares
and the same peak frequency (expected rate of peaks). The two types of spectra used
in this study are smooth and rectangular bimodal spectra (Fig.8.1). The smooth
spectrum is a combination of two band pass filters (truncated at a certain frequency
$f_c$), each having the analytical form

$$G_i(f) = \frac{A_i^2}{1 + \left(\frac{f - f_c}{Q_i}\right)^2}$$  \hspace{1cm} (8.1)

The rectangular bimodal spectrum is used because of its simplicity. It provides
the PSDs with a wide range of values of the irregularity factor, $\gamma$, having the same
RMS, and the same peak frequency $E[P]$, by adjusting the amplitudes and the fre­
quency boundaries. As Dirlik[28] indicated, though the first moment of any (double
side) spectral density of a real process is zero, an artificially defined single-sided first
moment (see below) plays an important role in predicting the rainflow cycle range
PDF:

$$m_x = \frac{1}{\pi} \int_0^\infty fG(f)df$$
then the quantity $\frac{m_x}{m_0}$ could be considered as the *mean frequency*.

For convenience of use, the *mean frequency* is described by a normalised dimensionless quantity, $x_m$

$$x_m = \frac{m_x}{m_0E[P]}$$

(8.2)

where $E[P]$ is the peak frequency (expected rate of peaks).

The spectral parameters described above are listed in Table 8.1 and 8.2. PSDs 1-14 are smooth spectra, while all the others are rectangular spectra. Among the rectangular spectra, six groups (PSDs 29-70) were chosen so that each having similar irregularity factor but different mean frequency.

For each spectrum, 25 kurtosis values were chosen from 0.0 to 12.0, stepped by 0.5, to produce non-Gaussian signals. A total of 1750 input-output trials were therefore obtained. An Artificial Neural Network was then applied to link the rainflow cycle range PDF's to various parameters including the irregularity factor, kurtosis and mean frequency for the spectrum. In this way, the trained neural network toolbox can be envisaged as a sophisticated “transfer function” which does not have any predefined limitations, such as linearity or predefined function format.

Figure 8.1: Power Spectral Density shapes
Table 8.1: 70 PSD’s used in stress history simulation (1)

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<th>PSD No.</th>
<th>$A_1$</th>
<th>$f_1$</th>
<th>$Q_1$</th>
<th>$A_2$</th>
<th>$f_2$</th>
<th>$Q_2$</th>
<th>$f_c$</th>
<th>$\gamma$</th>
<th>$x_m$</th>
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<td>90</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>210</td>
<td>0.8684</td>
<td>0.8380</td>
</tr>
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<td>82</td>
<td>11</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>226</td>
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<td>0.7739</td>
</tr>
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<td>12</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>233</td>
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<td>0.7092</td>
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Examples of simulated non-Gaussian signals are given in the last chapter. Examples of rainflow cycle range PDFs calculated from the non-Gaussian signals are given in Fig. 8.2. It shows there must be some ‘inherent’ link between rainflow cycle range PDFs and the root mean square of the signal, irregularity factor, mean frequency and kurtosis. It is desirable to find this ‘link’. An explicit expression would be easy to use. Dirlik’s formula is a solution for a special case: when kurtosis is zero. However, in deriving such a formula, a sacrifice must be made to a pre-defined formula. The development of ANN provides a solution for this problem. A trained neural network therefore replaces the pre-defined formula. By finding the minimum in the error surface of the neural network (see next two sections), accuracy is therefore improved for the non-Gaussian case. The following section presents some basic concepts about artificial neural networks.

![Rainflow Range p.d.f with different kurtosis](image)

Figure 8.2: An example of the Rainflow Range p.d.f with different kurtosis
8.2 Artificial Neural Network (ANN) (Backpropagation)

8.2.1 The concept

Artificial Neural Networks (ANN) represent a new computer based approach which has been applied to many different fields since the early 1980s. This technique is fundamentally different from the well known traditional programmed computing approach. Programmed computing has been used in all types of information processing applications. Solving a problem using conventional programmed computing involves devising an algorithm and a set of rules for solving the problem and then correctly coding these in software. Obviously, programmed computing can only be used for situations where the processing to be accomplished can be described in terms of a known procedure or a known set of rules. If the required algorithm procedure or set of rules are not available then the traditional programmed computing method cannot be used. If the algorithm required is not simple (which is frequently the case with the most desirable capabilities), the development process may have to await a flash of insight. Under these circumstances Artificial Neural Networks appear to be very powerful.

The field of neural networks has a history of some five decades but has found solid application only in the past eight to ten years. Neural Networks have been trained to perform complex functions in various fields of application.

The initial concept of neural networks was presented as early as in the 1940s. Later Wiener (1951) and von Newmann (1953) made the suggestion that research into the design of brain-like or brain-inspired computers might be interesting. Unfortunately no substantial progress was made until the first successful neurocomputer was developed during 1957 and 1958 by Rosenblatt, Wightman (1958) and others. It is widely accepted that Rosenblatt is the founder of neurocomputing as we know it today. Besides Rosenblatt, Wightman and Widrow, there were a number of other
people during the late 1950s and early 1960s who contributed to the development of neural network architecture and implementation concepts. However, in the following years this field suffered from overambitious claims made by some researchers. The leading neural network, the perceptron, was incapable of learning to distinguish classes of patterns that were not linearly separable. Furthermore, research in neural computing had run out of good ideas in the mid and late 1960s. These problems led to there being a lack of rigor in the field and research in the area reduced to a low level.

The solving of the "non-linear separable" problem in the mid-1980s sparked renewed interest in neural networks. In the years 1983 and 1986 Hopfield, an established physicist of worldwide reputation, made great progress. He produced two highly readable papers (1982, 1984) on ANN. He also delivered many lectures all over the world and persuaded many highly qualified scientists to join this new research field. Now an ever increasing number of difficult problems which could not be satisfactorily dealt with using traditional programmed computing techniques were being solved by using ANN.

8.2.2 Neural networks

Neural Networks are composed of many simple elements operating in parallel. These elements are inspired by biological nervous systems. The network function is determined largely by the connections between elements.

As indicated in[42], a Neural Network is a parallel, distributed information processing structure consisting of processing elements (which can possess a local memory and can carry out localized information processing operations) interconnected via unidirectional signal channels called connections. Each processing element has a single output connection that branches ("fans out") into as many collateral connections as desired; each carries the same signal— the processing element output signal. The processing element output signal can be of any mathematical
type desired. The information processing that goes on within each processing element can be defined arbitrarily with the restriction that it must be completely local; that is, it must depend only on the current values of the input signals arriving at the processing element via impinging connections and on values stored in the processing element’s local memory.

A directed graph is a geometrical object consisting of a set of points (called nodes) along with a set of directed lines segments (called links) between them. A neural network is a parallel distributed information processing structure in the form of a directed graph, with the following sub-definitions and restrictions:

- The nodes of the graph are called **processing elements**.
- The links of the graph are called **connections**. Each connection functions as an instantaneous unidirectional signal-conduction path.
- Each processing element can receive any number of incoming connections (also called **input connections**).
- Each processing element can have any number of outgoing connections, but the signals in all of these must be the same. In effect, each processing element has a single output connection that can branch or fan out into copies to form multiple output connections, each of which carries the same identical signal (the processing element’s output signal).
- Processing elements can have local memory.
- Each processing element possesses a transfer function which can use (and alter) local memory, can use input signals, and which produces the processing elements’ output signal. In other words, the only inputs allowed to the transfer function are the values stored in the processing element’s local memory and the current values of the input signals in the connections received by the processing element. The only outputs allowed from the transfer function are values to
be stored in the processing element’s local memory and processing element’s output signal. Transfer functions can operate continuously or episodically. If they operate episodically, there must be an input called “active” that causes the processing element’s transfer function to operate on the current input signals and local memory values and to produce an updated output signal (and possibly to modify local memory values). Continuous processing elements are always operating. The “active” input arrives via a connection from a scheduling processing element that is part of the network.

- Input signals to a neural network from outside the network arrive via connections that originate in the outside world. Outputs from the network to the outside world are connections that leave the network.

Figure 8.3 shows a typical neural network architecture

![Figure 8.3: A typical neural network architecture](image)

8.2.3 The processing elements of a neural network

As mentioned above, processing element consists of input and output connections, transfer function and local memory. A typical structure of a processing element is
shown in Fig. 8.4.

![Figure 8.4: An illustration of a processing element](image)

### 8.2.4 Transfer functions and local memories

The transfer function receives values from the incoming connections and from local memory. It produces outputs from the processing element and values from storage in the local memory. Each time an active signal is sent, these operations are performed. If the neural network is not continuously running, the processing element ceases to function after each operation is performed.

A typical example of such a transfer function is a case where the input signal (assumed to be a floating point number) of the connections are combined to form a weighted sum of the form $I_k = w_k \cdot x_k$ for input class $k$ with $x_k$ as the input vector and $w_k$ as the weight. A weight is a local memory variable of a specified data type assigned to input connections. A vector which has weights as components is known as a weight vector.
8.2.5 Training of a neural network and learning laws

Weight plays a very important role in most neural networks that have learning capabilities. The changing of processing element weights so that the mean square error reduces is called learning. Any method for determining the amount of change is therefore called a learning law. There are many kinds of training methods for different types of networks which depend on different learning laws. They can be divided into supervised training, graded training, and self-organised training at the fundamental level.

Supervised training is used in this thesis. This type of training is generally used for the situation where the network is functioning as an input/output system. In other words, the network receives an input vector \( \mathbf{x} \) and emits a vector \( \mathbf{y} \). Supervised training for such a system implies a regime in which the network is supplied with a sequence of examples \((x_1, y_1), (x_2, y_2), \ldots, (x_k, y_k), \ldots\) of “desirable” or “correct” input/output pairs. As each input \( x_k \) is entered into the neural network, the “correct output” \( y_k \) is also supplied to the network. The network is thus told precisely what it should be emitting as its output. The actual output is then taken as an estimation of the correct output.

In many supervised training situations the \((x_k, y_k)\) pairs used during training are assumed to be examples of a fixed function \( f \). The neural network is then used to identify the system. It is generally of use for situations where examples can be obtained but where the function is difficult to establish using traditional regression methods.

There are many learning laws available for training. Steepest Descent is the only learning law that has been mathematically proven to converge on the set of weights producing minimum error. However, this is a slow technique. Adaptive Learning Rates, on the other hand, enjoys several important advantages over steepest descent, though it does not enjoy the benefit of guaranteeing a minimum error.
8.2.6 Artificial neural networks (backpropagation)

Among various developed Artificial Neural Network systems, backpropagation, a method of training a neural network to approximate any function, including arbitrarily complex nonlinear functions, is the "only form of Neural Network that has yet produced an appreciable number of commercial applications" [85]. It is also "the only neural network technique to produce any number of fielded applications" [85].

Even though backpropagation has played a key role in ANN, it is really a statistical modelling technique. More specifically, it is a non-parametric modelling method: one in which the shape of the relationship between inputs and outputs is decided by the data rather than predetermined by the tool.

The structure of a back propagation network

The typical structure of a back propagation network is shown in Figure 8.5 [1]. Basically, this type of neural network has a fully interconnected multi-layer structure. It uses supervised training.

![Figure 8.5: Layout of back propagation network](image)

Along with the arrow direction of the connections, the first layer is the input layer which reads in the inputs $x_p$ from the input/output pairs $(x_1,y_1)$, $(x_2,y_2)$, $..., (x_k,y_k)$. The last layer, the output layer, gives the estimation results of $y_p$. The layers in between are the so-called hidden layers. The size of the input and output layers is determined by the function which the network is modelling. The
number of hidden layers and the sizes for each layer, however, can in theory be arbitrary. According to the Kolmogolov's mapping neural network existence theorem [2] (p122), any continuous function of \( n \) variables can be implemented exactly by a three layer network with \((2n+1)\) processing elements in the hidden layer. This theorem provides some guidance for the selection of the hidden layer size.

The work of a back propagation network can be divided into a forward pass and a backward pass. The backward pass only occurs on training trials. This forward and backward pass form a loop during the network training which is used to search for a set of weights which gives the network its optimum performance. Mathematically, the optimum performance means the best estimation of the desired output (or least error between them). After the training process, this set of weights can be regarded as one special regression of the function relationship between the input and output. The trained network can then be used for further computation using an input for which the desired output is not known.

**Forward pass**

The inputs for the processing elements in the input layer are determined by the system input. For any processing elements in the hidden layer and output layer, the input from the connections are taken as the weighted sum, that is,

\[
net_{pj} = \sum_i w_{ji}o_{pi}
\]  

(8.3)

The output from each processing element \( j \) can be assumed as any differentiable monotonic function,

\[
o_{pj} = f_j(net_{pj})
\]  

(8.4)

From the output layer, the estimation for function value \( y_p \) can then be obtained:

\[
net^o_{pk} = \sum_j w^o_{kj}o_{pj}
\]  

(8.5)

\[
o_{pk} = f_k^o(net^o_{pk})
\]  

(8.6)
The superscript "o" here refers to the quantities on the output layer.

**Backward propagation**

During the network training, the mean square error of the output vector is taken as an objective function which need to be minimised. The error can be defined as \( \delta_{pk} = (y_{pk} - o_{pk}) \). The total error of the output layer is

\[
E_p = \frac{1}{2} \sum_k \delta_{pk}^2 \quad (8.7)
\]

The value \( \frac{1}{2} \) here is used to make the calculation process easier. The values of the weights can be adjusted such that the total error reduces. This can be done by using a gradient direction search, such as

\[
\frac{\partial E_p}{\partial w_{kj}^o} = -(y_{pk} - o_{pk}) \frac{\partial f'_k}{\partial net_{pk}^o} \frac{\partial net_{pk}^o}{\partial w_{kj}^o} \quad (8.8)
\]

As \( \frac{\partial net_{pk}^o}{\partial w_{kj}^o} = o_{pj} \), the negative gradient direction is:

\[
- \frac{\partial E_p}{\partial w_{kj}^o} = (y_{pk} - o_{pk}) f'_k(\text{net}_{pk}^o) o_{pj} \quad (8.9)
\]

If the sigmoid function is used, then

\[
f(\text{net}_{jk}^o) = \frac{1}{1 + e^{-\mu \cdot \text{net}_{jk}^o}} \quad (8.10)
\]

and

\[
f'(x) = \mu f_k^o (1 - f_k^o) = \mu o_{pk} (1 - o_{pk}) \quad (8.11)
\]

where \( \mu \) is a constant.

By defining a step size \( \eta \) and a quantity as

\[
\delta_{pk}^o = (y_{pk} - o_{pk}) f'_k(\text{net}_{pk}^o) \quad (8.12)
\]

The weights can be upgraded as

\[
w_{kj}^o \leftarrow w_{kj}^o + \eta_{pk}^o \delta_{pk}^o o_{pj} \quad (8.13)
\]
Following on further, the weights of the hidden layers can be upgraded by changing $\delta_{pk}$ to

$$\delta^h_{pj} = \sum_k \delta^o_{pk} w_{kj}$$

(8.14)

### 8.3 ANN to Model Rainflow Range p.d.f. for Non-Gaussian Signals

#### 8.3.1 Program and flow chart

Backpropagation provides a way of using examples of a target function to find the coefficients that make a certain mapping function approximate the target function as closely as possible. Details of applying backpropagation can be found in, say, [85]. A flow chart is given in Fig.8.6. The program is given in Appendix B.

#### 8.3.2 Choice of neural network structure and parameters

The backpropagation neural network model was chosen to have one hidden layer. The number of neurons in the hidden layer is more than twice the number of inputs. A typical example is as following: four inputs (irregularity factor; kurtosis; mean frequency for the spectrum and rainflow cycle range (normalised by RMS)), one output (the rainflow cycle range p.d.f.); 10 hidden layer neurons; using adaptive learning laws (so that the error is decreasing in the deepest direction), with learning parameters 0.1, 0.5, 0.7 and 0.9 respectively; Error goal is set at 0.00001. Training stops if it fails to reach the error goal after 10000 epoches.

**Considerations in network training**

The back propagation algorithm described previously provides a way of network training. Implementation of the algorithm is, in many ways, a more difficult problem. Since the weights are generally assigned as random initial values at the beginning of the training loop, the problem which arises is how can the network find a global
minimum in the error space. As shown in Figure 8.7, it is quite possible for a network to cease its training loop at a local stationery point. This is a general problem for nonlinear programming and there is no universal solution. Four steps have been taken to avoid such problems arising.

1). Different initial values for the weight and bias were used;

2). Different sizes of network were tested;

3). The iteration was allowed to jump out from a stationary point and search in a wider area;

4). An "eye test" (plot checking) was used to compare the results with the time domain analysis results.

8.3.3 Results

Examples of the predicted PDFs from the toolbox compared with the original PDFs for the simulated signals are given in Fig. 8.8 and 8.9. They show good agreement.

8.3.4 Application of the toolbox to WEG and Howden data

The trained toolbox was applied to the WEG and HOWDEN data. Figs. 8.10 and 8.11 show examples of rainflow cycle range PDFs predicted by the toolbox, Dirlik’s formula and directly from time signals. Result from the trained toolbox shows good agreement for WEG and Howden flapwise data. It is an improvement over Dirlik's empirical formula. As expected, it is impossible to predict the large spike in the rainflow cycle range PDF for Howden edgewise data since this kind of special signal with strong deterministic component can hardly be classified as a kurtosis-described non-Gaussian signals. In fact, this kind of signals is not even stationary. Frequency domain fatigue analysis can not be applied to it.
8.4 Summary and Conclusions

While spectral fatigue analysis could provide a fast, efficient tool to the capability designers for design optimization of wind turbines or other dynamically sensitive structures, it is desirable to extend it to non-Gaussian signals. This chapter has presented a trained neural network which is applicable for kurtosis-described non-Gaussian signals. This provides a practical toolbox for designers.
Read from a file the dimensions and data of the training examples

Read from a file the architecture of the network and the parameters controlling learning:
  numbers of inputs, hiddens & outputs
  learning parameters
  error requirement

Assigns initial values and learning rates to weights

For all examples:
  Forward pass through the network
  Backward: calculates derivatives of error for each weight

Change weights

< required error?

yes

no

End

Figure 8.6: Flow chart of backpropagation programming
Figure 8.7: A typical error surface
Figure 8.8: Comparison of PDFs by trained neural network: example 1: sp7, kurtosis=1 & 4
Figure 8.9: Comparison of PDFs by trained neural network: example 2: sp27, kurtosis=0.5 & 3
Figure 8.10: Example of PDFs predicted by the toolbox: WEG data
Figure 8.11: Example of PDFs predicted by the toolbox: HOWDEN data
Chapter 9

CONCLUSIONS AND FURTHER WORK

9.1 Summary and Conclusions

This thesis has achieved its objectives. The frequency domain fatigue analysis approach has many advantages for dynamically sensitive structures, particularly at the design stage. Since the rainflow cycle counting method has widely been accepted as the best method of estimating the fatigue damage caused by random fluctuating loading conditions, it is desirable to link the frequency domain representation of a stress signal to the rainflow cycle range distribution, and hence predict the fatigue life of the structure/component being analysed. A new theoretical approach has been developed which is applicable for general stationary signals. Since the original rainflow cycle counting method is complicated and difficult to apply, Rychlik's alternative rainflow cycle counting definition is adopted. This overcomes the Gaussian assumption and other inherent limitations of existing frequency domain fatigue methods, although a closed form expression of the joint PDF of adjacent peaks and troughs is desired. For Gaussian signals, Kowalewski's formula for the joint PDF of adjacent peaks and troughs is applied. For non-Gaussian signals, a trained neural network provides the joint probability distribution. Details of the new theoretical method are given in Chapter 6. Based on extensive simulation and artificial neural network training, a new toolbox has been developed for general stationary stress signals. Kurtosis was introduced as the description of the signal departure from a Gaussian distribution. Details of the simulation and neural network training were
given in Chapters 7 and 8. Chapters 6, 7 and 8 form the major part of the original work presented in this thesis although some original contributions are also made in chapters 3, 4 and 5.

Frequency domain and time domain fatigue analysis were applied to wind turbine blade data WEG MS1 and HOWDEN HWP330 as well as other data. A sophisticated investigation was carried out to study the effects of various parameters affecting the fatigue prediction accuracy. The effect of stress signals with strong deterministic component was also studied. Software was developed to apply the frequency domain fatigue analysis methods, particularly the commercialised nCode product (FATIMAS-SPECTRAL) which has already found wide applications.

Chapter 3 reviews the existing frequency domain fatigue methods. Some results have been extended by the author. Most methods relate the cycle range PDF or equivalent stress factor to the first 4 moments ($m_0, m_1, m_2, m_4$) of the stress PSD. Advantages and disadvantages of each method were discussed. While Dirlik's solution was for rainflow cycle range PDFs, most other methods give equivalent stress values directly. A program was developed to implement these methods. Part of the program forms the commercialised nCode product (FATIMAS-SPECTRAL) which has already found wide applications.

Chapter 4 presents results of application to two sets of wind turbine blade data (WEG MS1 and HOWDEN HWP330) as well as some other simulated data. Results and problems were shown and discussed. It was found that all Howden HWP330 edgewise data had a strong deterministic component which is caused by the blade gravity. This deterministic component produces a large spike in the rainflow cycle range PDF, which cannot be predicted by any frequency domain methods. In fact, this deterministic component is nearly a harmonic wave (sine or cosine), which can be mirrored as a Dirac delta function (or a very large spike) in the PSD. Calculation of the moments of the PSD is very inaccurate. Generally speaking, this kind of signal cannot even be classified as a stationary signal, therefore the term PSD has
little meaning for it. A detailed study was carried out by mixing stationary signals with various deterministic components and then calculating fatigue lives. This is presented in Chapter 5. It is found when the amplitude of the mixed sine wave or equally-spaced spikes is within the value of the root mean square of the mixed random stationary signal, frequency domain fatigue method still works well. Various factors affecting frequency domain fatigue analysis have been studied. As most frequency domain methods use the first four moments of the PSD, the accuracy of calculating the moments is very important for the prediction. This is affected by the calculation of the PSD and the choice of cutoff frequency of the PSD. If PSDs are calculated from the time signals, a threshold is recommended to filter out measurement and other noises. This noise floor can be set at $-120dB$ of the largest PSD value, i.e., $\frac{1}{10^6}$ of the largest PSD value. The choice of cutoff frequency for calculating the moments mainly affects the higher order moments. The cutoff frequency can be set at the frequency corresponding to 99.95% of the total area under the PSD. Since the $S-N$ curve slope affects the weighted integral of cycle range PDF and hence fatigue damage, the effect of $S-N$ curve slope was studied. The application of frequency domain methods to stationary signals with various bandwidth values (from narrow band to wide band) was also studied. Dirlik’s solution shows consistency for narrow band and wide band signals. This is expected because Dirlik’s formula was derived from extensive simulations of a wide range of signals. The effect of clipping ratio was also studied. A value of 6 for clipping ratio is recommended.

Chapter 5 is concerned with the effect of mixing a stationary signal with a deterministic component. The aim of this study was to provide a guide for engineers applying frequency domain fatigue analysis techniques. Two cases for the deterministic component were considered: sine waves with various amplitude and frequency, equally-spaced square spikes with various amplitude and frequency (interval).

Chapters 7 and 8 show details of simulation and artificial neural network training for non-Gaussian signals. A program was developed for the simulation
and artificial neural network training. A neural network was trained, applicable for
general kurtosis-described non-Gaussian signals. This provides a practical frequency
domain fatigue analysis toolbox for stationary signals.

9.2 Recommended Future Work

Despite the success of implementing frequency domain fatigue analysis techniques,
several fundamental areas of research remain to be undertaken. A number of industry specific problems remain to be addressed such as the kerb strike problem in
automotive design where a transient load is superimposed onto a stochastic signal.
(In this case, the mixed signal cannot be represented by its PSD, but it remains
a problem to use the PSD of the stationary signal, together with properties of the
transient load, to predict fatigue life). Furthermore, a number of general fundamental problems require attention such as non-stationary signals, short samples and
multiaxial fatigue assessment. The topics which require further work are therefore,

- Industry specific Transient and stochastic load combinations- the kerb strike
problem, here the transient load is not necessarily harmonic (this will not be a
pure frequency domain problem); Inputs of more than one deterministic com-
ponent, in this case the phase difference between the deterministic components
must be taken into account; Practical computation problems such as the 4th
moment problem in wave and wind spectral descriptions.

- General Non-stationary signals. Conventional PSD's cannot be applied to
describe a nonstationary signal. An extended description like evolutionary
PSD’s must be introduced. It is desirable to extend current frequency domain
methods to the new PSDs.; Short samples. For practical applications, it
is desirable to overcome the effects of short samples. Multiaxial fatigue.
In theory, frequency domain fatigue methods can be extended to multiaxial
fatigue. Particularly for those multiaxial criterias involving linear transform
like *Maximum Normal Stress, Maximum Strain* or *Maximum Shear Stress*, an equivalent stress can be expressed as a linear combination of the stress state tensor components. The PSD's of the equivalent stress can be expressed as linear combinations of the PSD's of the stress state tensor components. The existing frequency domain methods can then be applied to the equivalent stress. This strategy might not work for those nonlinear criteria such as the *Distortion Strain Energy (von-Mises)* or *Total Strain Energy (Beltrami)* as it causes difficulties in relating the PSDs of equivalent stress to the stress state tensor components.
Bibliography


[37] Haas, T. Loading statistics as a basis of structural and mechanical design. Engineers Digest, 23(3,4 and 5), 1962.


Appendix A

SPECTRAL FATIGUE ANALYSIS PROGRAM

A program was developed for the fatigue calculation. The program consists of more than 40 subroutines and is about 3500 lines in Fortran 77. The flow chart is shown in Figure A.1.
Figure A.1: Flowchart of the program for random fatigue analysis
A Fortran 77 program for the study of the effect of non-Gaussianity by using Artificial Neural Network is developed. The program is listed below.

```fortran
PROGRAM HEUH
     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
     C ARTIFICIAL NEURAL NETWORK (BACKPROPAGATION)
     C----------------------------------------------------------
     C USAGE: HEUH <PARAMIN> <PDFFILE> <RSLTFILE>
     C WHERE <PARAMIN> IS THE INPUT PARAMETER FILE NAME;
     C   User provides this file
     C <PDFFILE> IS EXAMPLE PDF FILE;
     C   User provides this file
     C <RSLTFILE> IS THE RESULT FILE NAME (weights).
     C
     INS---- NUMBER OF INPUTS
     HIDS---- NUMBER OF HIDDENS
     OUTS---- NUMBER OF OUTPUTS
     I-------- INDEX OF INPUTS
     J-------- INDEX OF HIDDENS
     K-------- INDEX OF OUTPUTS
     M-------- INDEX OF EPOCHS
     N-------- INDEX OF EXAMPLES
     EXAMPLES---- NUMBER OF EXAMPLES
     SUMSQERR---- ACCUMULATING SQUARE ERROR
     ERRGOAL---- ERROR GOAL: WHEN <MEAN SQERR < ERRGOAL>, STOP
     MEAN SQERR---- MEAN SQUARED ERROR
     KAPPA, PHI, THETA, MU---- PARAMETERS
     RECOMMENDED: KAPPA=0.1; PHI=0.5; THETA=0.7; MU=0.9
     PARAMIN---- A CHARACTER FILE WHICH STORES THE INPUT
     PARAMETERS
     RSLTFILE---- A CHARACTER FILE WHICH STORES THE RESULTS
     A---- HIDDEN WEIGHTS
     B---- OUTPUT WEIGHTS
     CHID & COUT---- WEIGHT CHANGES
     DHID & DOUT---- DERIVATIVES
     EHID & EOUT---- ADAPTIVE LEARNING RATES
     FHID & FOUT---- RECENT AVERAGE OF DERIVATIVES
     U---- WEIGHTED SUM FOR HIDDEN NODE
     Y---- HIDDEN NODE OUTPUTS
     V---- WEIGHTED SUM FOR OUTPUT NODE
     Z---- OUTPUT NODE OUTPUTS
     P---- DE/DV
     Q---- DE/DU
     ******************************************************************************************
     IMPLICIT REAL*8 (A-H,O-Z)
     INTEGER INS,HIDS,OUTS,I,J,K,M,N
     224
```
APPENDIX B. ARTIFICIAL NEURAL NETWORK PROGRAM

REAL*8 SUMSQERR, MEANSQERR, KAPPA, PHI, THETA, MU
REAL*8 A, B, CHID, COUT, DHI, DOUT, EHI, EOUT, FHI, FOUT
REAL*8 U, V, W, X, Y, Z, P, Q, T
REAL*8 LOGISTIC, PI
REAL*8 TMP1, TMP2, PDFMAX, TINY, HUGE
CHARACTER PARAHIH*15, RSLTFILE*15, PDFFILE*15
INTEGER INSUP, HIDSUP, OUTSUP, NFPTS, IRET
INTEGER EXAMPLUP
PARAMETER (IBSUP=6, HIDSUP=52, OUTSUP=2, EXAMPLUP=200001)
PARAMETER (PI=3.1415926535, HFNTS=50, TINY=1.E-40, HUGE=1.E60)
DIMENSION A(0:INSUP, 1:HIDSUP), B(0:HIDSUP, 1:OUTSUP)
DIMENSION CHID(0:INSUP, 1:HIDSUP), COUT(0:HIDSUP, 1:OUTSUP)
DIMENSION DHI(0:INSUP, 1:HIDSUP), DOUT(0:HIDSUP, 1:OUTSUP)
DIMENSION EHI(0:INSUP, 1:HIDSUP), EOUT(0:HIDSUP, 1:OUTSUP)
DIMENSION FHI(0:INSUP, 1:HIDSUP), FOUT(0:HIDSUP, 1:OUTSUP)
DIMENSION Y(0:HIDSUP), Z(0:OUTSUP), P(0:OUTSUP)
COMMON KAPPA, PHI, THETA, MU
IRET=0
PDFMAX=0.0
CALL GETARG(1, PARAMIN)
CALL GETARG(2, PDFFILE)
CALL GETARG(3, RSLTFILE)
OPEN(7, FILE=PARAMIN)
READ(7, *) INSUP, HIDSUP, OUTSUP, EXAMPLUP, KAPPA, PHI, THETA, MU, ERRGOAL, MAXEPOCH
CLOSE(7)
OPEN(20, FILE=RSLTFILE)
OPEN(8, FILE=PDFFILE)
DO N=1, EXAMPLES
READ(8, *) N, X(N, 1), X(N, 2), X(N, 3), X(N, 4), T(N, 1)
IF (T(N, 1).LT.0.) THEN
WRITE(*,*) "PDF must not be negative! program halted!"
WRITE(*,*) "no, gamma, kurtosis, meanfreq, range: pdf"
WRITE(*,*) N, X(N, 1), X(N, 2), X(N, 3), X(N, 4), T(N, 1)
IRET=-10
GOTO 999
ENDIF
PDFMAX=AMAX1(PDFMAX, T(N, 1))
ENDDO
CLOSE(8)
IF (PDFMAX .LT. TINY) THEN
WRITE(*,*) "all pdf values are zero! program halted!"
IRET=-10
GOTO 999
ENDIF
C***** RE-SCALES INPUT DATA VALUES SO THEY ALL FALL IN (0.1,0.9)***
WRITE(*, '(/)')
WRITE(*,*) "maximum pdf value: ", PDFMAX
WRITE(*, '(/)')
WRITE(20,510) PDFMAX
DO N=1, EXAMPLES
T(N, 1)=0.8*T(N, 1)/PDFMAX+0.1
ENDDO
C***** ASSIGN INITIAL VALUES AND LEARNING RATES TO WEIGHTS***
ISEED=738451
DO J=1, HIDSUP
I=0, INS
A(I, J)=0.2*(RAN(ISEED)-0.5)
EHI(I, J)=KAPPA
ENDDO
ENDDO
DO K=1, OUTSUP
DO J=0, HIDSUP
IF (MOD(J, 2).EQ.0) THEN
APPENDIX B. ARTIFICIAL NEURAL NETWORK PROGRAM

\[ B(J,K) = 1. \]
\[ \text{ELSE} \]
\[ B(J,K) = -1. \]
\[ \text{ENDIF} \]
\[ \begin{align*}
E_{OUT}(J,K) &= \kappa \\
\end{align*} \]
\[ \text{ENDDO} \]
\[ \text{ENDDO} \]

C*****THE MAIN PROGRAM***************

M = 0

10 M = M + 1

SUMSQERR = 0.

DO \( M = 1, \text{EXAMPLES} \)

C*****THE FORWARD PASS THROUGH THE NETWORK**

DO \( J = 1, \text{HIDS} \)

U = A(0, J)

DO 1 = 1, INS

U = U + (A(I, J) \times X(I,N))

ENDDO

Y(J) = \text{LOGISTIC}(U)

ENDDO

DO K = 1, OUTS

V = B(0, K)

DO J = 1, HIDS

V = V + (B(J,K) \times Y(J))

ENDDO

Z(K) = \text{LOGISTIC}(V)

ENDDO

C*******BACKWARDS, CALCULATES DERIVATIVES OF ERROR FOR EACH WEIGHT

DO \( J = 0, \text{HIDS} \)

Q(J) = 0.

ENDDO

DO K = 1, OUTS

SUMSQERR = SUMSQERR + (Z(K) - T(K,N)) \times (Z(K) - T(K,N))

P(K) = (Z(K) - T(K,N)) \times Z(K) \times (1 - Z(K))

DOUT(0, K) = DOUT(0, K) + P(K)

DO \( J = 1, \text{HIDS} \)

DOUT(J, K) = DOUT(J, K) + P(K) \times Y(J)

Q(J) = Q(J) + P(K) \times B(J,K)

ENDDO

ENDDO

C WRITE(*,*) M, N, SUMSQERR

DO \( J = 1, \text{HIDS} \)

Q(J) = Q(J) \times Y(J) \times (1 - Y(J))

DHID(0, J) = DHID(0, J) + Q(J)

DO I = 1, INS

DHID(I, J) = DHID(I, J) + Q(J) \times X(I,N)

ENDDO

ENDDO

C*****CHANGE WEIGHTS*********************

DO \( J = 1, \text{HIDS} \)

DO I = 0, INS

CALL CHANGE(A(I, J), DHID(I, J), FHID(I, J))

& , EHID(I, J), CHID(I, J))

ENDDO

ENDDO

DO \( K = 1, \text{OUTS} \)

DO \( J = 0, \text{HIDS} \)

CALL CHANGE(B(J,K), DOUT(J,K), FOUT(J,K))

& , EOUT(J,K), COUT(J,K))

ENDDO

ENDDO

DO I = 0, INS

DO \( J = 1, \text{HIDS} \)

DHI(1, J) = 0.

ENDDO

ENDDO

DO \( I = 0, \text{HIDS} \)
DO J=1,OUTS
   DGUT(I,J)=0.
ENDDO
ENDDO
MEAN SQ ERR=SQRT(SUM SQ ERR)/REAL(EXAMPLES)/REAL(OUTS)
IF(M.LE.10)THEN
   WRITE(20,*)M,' ',MEAN SQ ERR
ENDIF
IF(MOD(M,1).EQ.0)THEN
   WRITE(*,200)H,' > ',MEAN SQ ERR
ENDIF
IF(MEAN SQ ERR.LE.ERR)GOTO 20
IF(M.GE.MAX EPOCH)GOTO 20
GOTO 10
20 CONTINUE
WRITE(*,*)' NO OF EPOCHES: ',M
WRITE(*,100)' mean sq error: ',MEAN SQ ERR
C
DO I=0,INS
   DO J=1,HIDS
      THP1=A(I,J)
      WRITE(20,520)I,J,THP1
   ENDDO
ENDDO
C
DO J=0,HIDS
   DO K=1,OUTS
      THP2=B(J,K)
      WRITE(20,S20)J,K,THP2
   ENDDO
ENDDO
CLOSE(20)
C
FUNCTION LOGISTIC(UV)
REAL*8 LOGISTIC,UV
C
DO=(F.GT.0.)THEN
   E=E*KAPPA
ELSE
   E=E*PHI
ENDIF
F=(1.-THETA)*D+THETA*F
C=(1.-MU)*(1.)*E+D+MU*C
W=W+C
RETURN
END
SUBROUTINE CHANGE(W,D,F,E,C)
C
REAL*8 PH remembered

C
IF(D*F.GT.0.)THEN
   E=E*KAPPA
ELSE
   E=E*PHI
ENDIF
F=(1.-THETA)*D+THETA*F
C=(1.-MU)*(1.)*E+D+MU*C
W=W+C
RETURN
END
IF(UV.LT.(-88.)) THEN
  LOGISTIC=0.
ELSEIF(UV.GT.88.) THEN
  LOGISTIC=1.
ELSE
  LOGISTIC=1./(1.+DEXP(-UV))
ENDIF
RETURN
END