DELINEATION OF GROUNDWATER PROTECTION ZONES FOR FRACTURED AQUIFERS IN THE UK

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Submitted in fulfilment of the degree of
Doctor of Philosophy,
University College London, September 2002
ABSTRACT

Groundwater protection zones (GPZs) are defined as part of the policy for the protection of groundwater for England, Wales, Scotland and Ireland. GPZs are delineated by the relevant UK regulatory authority for boreholes and springs on the basis of horizontal flow time and distance from the source.

The zones are usually determined through reverse particle tracking using 'porous media' numerical models. There are two principal drawbacks with the existing methodology: a) the majority of the major and minor aquifers of the UK are fractured to a certain degree, but are being represented by porous media models, and b) the regulatory authority's existing protection zone methodology only has a limited uncertainty analysis associated with it. This thesis aims to provide alternatives to the sole use of porous media models in fractured rock scenarios to delineate protection zones, as well as a more robust uncertainty analysis.

Having conducted an extensive appraisal of protection zone methodologies world-wide it was found that no country reviewed had an adequate policy for delineating protection zones in fractured rock. Therefore any methodology developed for the UK must be from first principles. Possible modelling approaches were then explored, together with potential methods of providing a rigorous uncertainty analysis, leading to probabilistic protection zones. Since fracture models were found to require large amounts of data, standard and alternative methods of obtaining fracture data were investigated. The characteristics of anisotropic kinematic porosity in fractured rock were also examined. From a series of stochastic fracture modelling studies in 2-D, it was considered necessary to develop a methodology in 3-D. A pilot study was conducted to test a proposed 3-D methodology. Although the confidence in the probabilistic protection zones produced was much greater than in the porous media zones, successful calibration of stochastic fracture models was discovered to be a significant issue.
ACKNOWLEDGEMENTS

My sincere thanks go to John Barker for all his advice, support and discussion over the last few years. My thanks also go to the people who helped me plan and set up the tracer tests at Alton Court, including Rob Ward of the Environment Agency, Ann Williams of BGS, and Dave Headworth, Wayne Davies and Ruth Jones of the Welsh Region Environment Agency. Janet Whittaker was also extremely generous in providing me with information on German groundwater protection zone policies. Enormous thanks go to John Bloomfield and my very good friend Shona who had the dubious pleasure of proof reading this thesis. And last but definitely not least, a very big thank you to my husband, Mark, for all the love and support (and proof reading) throughout the production of this thesis.
For my Dad
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1 INTRODUCTION

This thesis examines the delineation of groundwater protection zones in fractured UK aquifers. The aim of the thesis is principally to provide a practical methodology for the UK regulatory authorities for zone delineation in fractured rock which is robust, transparent and, most importantly, defensible. Whilst the main objective has been to provide a practical methodology the thesis also examines the more fundamental aspects of transport in fractured rock that relate to protection-zone delineation.

The impetus for the thesis originally came from an interest in the work of Bradbury and Muldoon (1993), and an Environment Agency R&D requirement to assess whether a methodology could be developed delineating protection zones in fractured rock. The R&D project was undertaken at UCL by John Barker and myself and can be found in the Environment Agency WD-060 series of reports (Robinson and Barker, 2000a, b). This work has subsequently been expanded to form this thesis.

To the best of the author’s knowledge the delineation of protection zones in the fractured aquifers of the UK has not previously been attempted using the techniques in this thesis. Stochastic fracture modelling techniques are used to obtain the appropriate effective aquifer parameters which are then upscaled and used in stochastic porous media models to produce probabilistic protection zones. The methodology is kept as practical as possible, in particular recognising the scarcity of field data.

It is acknowledged that there is much in the literature on delineating probabilistic protection zones in porous media, for example, Bair et al., 1991, Varljen and Shafer, 1991, Kinzelbach et al., 1996, Franzetti and Guadagnini, 1996, Wheater et al., 2000, van Leeuwen et al., 1998 and 1999, and van Leeuwen 2000. However, the majority of these techniques have onerous data requirements such as knowledge of the transmissivity spatial correlation structure; data which are extremely difficult to obtain given the small amount of transmissivity data available around a typical groundwater source. It is also recognised that there is a vast body of literature on groundwater flow in fractured rock, principally from the nuclear waste repository test sites. Chevalier et al. (2001) is the only recent research found in the literature that does deal with stochastic delineation of
protection zones in fractured rock, although this is based on an analytical approach with very simple hydrogeological scenarios.

1.1 Terminology

It is important before any further discussion on delineation of protection zones in fractured rock that the terminology used throughout this thesis is understood. In much of the literature on groundwater protection zones the words 'protection zone', 'wellhead protection area' and 'capture zone' are used synonymously. However, in this thesis capture zone refers to the entire body of water that will eventually be captured by the source, or in regulatory authority terms the body of water required to support the long term abstraction rate, usually referred to by the term 'total catchment zone'. In much of the literature capture zone refers to a zone around the source defined by any finite travel time, for example 50 days or 400 days. In this thesis these zones are simply referred to as the 50-day or 400-day zones. The only exception to this is in Chapter 4, where protection zone delineation across the world is examined, with some countries having their own particular nomenclature.

A definition of the term 'fracture' must also be given. In the context of this thesis a fracture refers to any discontinuity in rock, be it a fault, fault related fracture, joint or bedding plane fracture. The term fracture does not imply any particular mode of discontinuity formation.

1.2 Structure of the thesis

Chapter 2 outlines the problem with the existing methodology used by the UK regulatory authorities for protection zone delineation in fractured rock, and from this the two main issues to address within the thesis are presented.

Chapter 3 presents the approach adopted in the thesis and the assumptions made in tackling this large area of research.

Chapter 4 examines the delineation of protection zones in fractured rock in other countries around the world to assess whether: a) a methodology already exists and b) whether parts of any methodology found should be incorporated into a UK methodology.
The following four chapters are then presented as if a source were chosen for protection zone delineation. The steps from conceptual model development, to model selection, data collection and collation for a model, through to simulation of probability-based protection zones are given.

Chapter 5 examines the considerations for development of a fractured rock conceptual model for a site and the numerical modelling options for the different conceptual model types, together with the data required for particular models.

Chapter 6 studies how fracture data could be acquired for fractured rock models, to include sources of data that would not be used as standard in order to constrain fracture parameters. The distribution types that the different fracture parameters tend to follow are also investigated.

Chapter 7 looks at ways of characterising protection zone uncertainty and how probability contours could be developed and interpreted.

Chapter 8 presents the simulation of probability-based protection zones in 2-D. Initial investigations were conducted in 2-D to be consistent with the existing regulatory authority methods and the approach taken by Bradbury and Muldoon (1993). Fracture modelling exercises were conducted to assess whether generalisations could be made on the shape and form of protection zones for particular combinations of 2-D fracture parameters. Probabilistic protection zones were then developed for two case study sites. Fundamental problems were found with modelling in 2-D which are discussed in this chapter, resulting in a need for fractured rock protection zone modelling in 3-D, as examined in chapter 9 and 10.

Chapter 9 gives a discussion on the nature of anisotropic kinematic porosity, with examples from 2-D and 3-D fractured systems.

Chapter 10 examines the possibility of developing a methodology in 3-D, since fracture flow is, in the majority of cases, a 3-D problem. A method of upscaling to allow
catchment scale modelling is tested, taking into account the fracture flow properties of
the system. The methodology is tested at a case study site to produce probabilistic
protection zones, with a discussion on the different methods of zone validation.

Chapter 11 concludes the thesis with a summary of the principal results, together with
recommendations for further research.
2 THE PROBLEM OF SOURCE PROTECTION IN FRACTURED AQUIFERS

Groundwater is an extremely valuable resource worldwide. Maintenance of this resource in terms of both quality and quantity is a far cheaper and a more immediate option than the cleanup and restoration of the resource after contamination or over-abstraction. Therefore the protection of groundwater quality and quantity should be recognised as a priority by all countries.

The protection of groundwater quality and quantity is supported by both UK and EU law. The Water Framework Directive is the most recent EU legislation to address both the quality and quantity of groundwater, and in particular recognises the need for groundwater protection zones. The Water Framework Directive (2000) states under Article 7 “Member States shall ensure the necessary protection for the bodies of water identified with the aim of avoiding deterioration in their quality in order to reduce the level of purification treatment required in the production of drinking water. Member States may establish safeguard zones for these bodies of water.”

The protection of groundwater quality in the UK is based on a framework that is comprised of three components, and is implemented by the regulatory body responsible for groundwater protection: in England and Wales this is the Environment Agency, in Scotland this is SEPA (Scottish Environment Protection Agency) and in Northern Ireland this is The Environment and Heritage Agency (EHA). The three components of the framework are:

- groundwater vulnerability;
- definition of source protection zones; and
- statements on groundwater protection policy.

The Environment Agency (the Agency) have chosen to define groundwater vulnerability through a series of 1:100,000 maps which are designed to show the effect on vulnerability of the:
• presence and nature of the overlying soil;
• presence and nature of drift cover;
• nature of the strata; and
• depth to water table/thickness of the unsaturated zone.

These maps are designed for land-use planners in the decision making process of granting land use changes and the granting of planning applications, in order that a reasonably accurate assessment of the risk to the aquifer of that particular activity can be made.

The second component of the framework is the delineation of source protection zones. Within the zones land use is restricted or controlled. For example, close to a well new activities involving petrochemicals or herbicides and pesticides would not be permitted. Exactly which activities are permitted or restricted in which zone is set out in the third component of the framework, which consists of a set of policy statements.

The most technically challenging part of the framework is the second component of policy regarding the delineation of source protection zones. Research into the most appropriate methods for zone delineation forms the basis of the research presented in this thesis.

Source protection zones or ‘groundwater protection zones’ (GPZs) are delineated on the basis of the time of travel of potential pollutants to the source. Three zones are defined:

• inner protection zone defined by the 50-day travel time for contaminants. This zone is designed to protect the source from biological contaminants, and is based around the idea that most bacteria/viruses tend to have a half-life of less than 50 days;
• outer protection zone defined by the 400-day travel time for contaminants. This zone is designed to protect the source from chemical contaminants, and is based around the time it takes for chemicals to attenuate in the aquifer; and

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1 Pekdeger et al. (1985) give elimination rate constants or ‘half-lives’ of bacteria ranging from 0.4 days to 65 days, and for viruses from 25 days to 108 days. These ‘half-lives’ become longer as the pore size of the aquifer increases.
catchment zone defined by the area required to support the long term abstraction rate by the average annual recharge.

Travel-time zones have been developed for over 2000 sources in England and Wales by the Environment Agency, most using computer (numerical) modelling techniques that assume the aquifer is a porous medium and relatively homogeneous in terms of porosity and permeability. The numerical models used to define the existing zones are principally Flowpath (Franz and Guiguer, 1988) and Modflow (McDonald and Harbaugh, 1988). However, most aquifers (Major and Minor) are fractured to varying degrees, and these fractures are known to have a significant effect on groundwater flow patterns and velocities. Fracture flow effects result in many existing protection zones being inaccurate, a fact that is recognised by the Agency. One example of how fracturing affects the direction of groundwater flow is in the Yorkshire Chalk where tracer testing proved flow perpendicular to the regional flow direction (Ward et al., 1997). The inaccuracy in protection zone delineation can be seen through both planned and unplanned (spills) tracer testing. One example of this comes from a source in the Hythe Beds near Maidstone in Kent where conductivity in the source increased immediately after a rainfall event. This sudden rise in conductivity was traced to a storage depot for road salt. The road salt was not adequately protected from rain so when precipitation occurred, runoff from the salt store entered a storm water drain and subsequently into the aquifer. According to the protection zone the storage depot was in the outer protection zone (between 50 and 400 days travel time), however the actual travel time was less than 1 day. The Hythe Beds are faulted and fractured.

There are a variety of mathematical techniques that are used to define travel-time zones:

- the fixed radius method where recharge balances the volume of water required by the abstraction rate, producing zones that are circular;
- an analytical manual method can be used whereby the hydraulic gradient, recharge, kinematic porosity, aquifer thickness, and hydraulic conductivity are also taken into account;
- an analytical modelling method, which is good for simple scenarios and is only used extensively by the US EPA; and
• a numerical modelling method, which attempts to simulate variations in porosity and permeability, different boundary conditions across the area, as well as surface water interactions. This is the method favoured by the Agency.

The numerical modelling techniques require reasonably large amounts of data, many of those data originating from testing in the field or laboratory. All hydrogeological data have an element of uncertainty associated with them. The Agency has attempted to take this into account by performing limited qualitative uncertainty analyses where parameters in the models are varied: permeability, kinematic porosity and recharge, and then combining the resulting zones to produce zones of confidence and uncertainty. However, there are also many other types of uncertainty associated with the modelling process that are not taken into account in the current Agency analysis. These are uncertainties in:

• the conceptual model (for example whether a site should be represented in 2-D or 3-D);
• the spatial distribution of parameters;
• flow equation parameters;
• output parameters/calibration parameters;
• dispersion in the aquifer; and
• numerical dispersion in the model.

In the case of conceptual model uncertainty, the Agency use porous media models, incorrectly in some cases, to represent fractured aquifers. The research presented here aims to produce an alternative modelling technique to be used for fractured rock conceptual models.

Uncertainty is dealt with in a limited qualitative manner by the present Agency methodology. Given the extreme heterogeneity of fractured rocks and the uncertainty in their characterization, the research presented here aims to deal with uncertainty in a more rigorous quantitative manner. There are many aspects to fractured rock uncertainty such as defining the active flowing fractures within the system, the effective aperture of those
fractures, and the variation of connectivity and hydraulic conductivity over space and time.

But there are also those factors not connected with the fracturing of the system that contribute significantly to the uncertainty of protection zones. These factors include boundary conditions, transient effects and river/aquifer interaction. However, perhaps the most important consideration is recharge and recharge is a factor that is relatively unknown. Therefore there may be other factors that are as important or more important than the nature of fracturing in the delineation of protection zones, but we must deal with the different factors in a methodical manner. This research looks solely at the impact of fracturing on protection zones in steady-state, but it is acknowledged that fracturing is not the only major uncertainty in the system.

One approach for incorporating the effects of these uncertainties into groundwater models is by producing probabilistic protection zones (i.e. there is a certain probability of the true protection zone lying inside probabilistic protection zone). Probabilistic zones will allow a much more detailed and robust risk analysis to be performed in order that sources can be more effectively protected.

In summary, any proposed methodology developed to delineate groundwater protection zones in fractured rock should attempt to address the following problems with the existing methodology:

- use of inappropriate models for fractured rock scenarios – defensible alternatives to the sole use of porous media models to represent fractured rock should be found; and
- inadequate uncertainty analysis – conceptual model and parameter uncertainties must be taken into account in a rigorous and quantitative manner.

This thesis aims to tackle both of these issues.

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2 The 'true' protection zone refers to the area from which the abstraction well actually derives its water.
3 THE APPROACH

3.1 The approach

The approach adopted in the initial stages of this work was to review many different areas of fractured rock modelling research and assess whether and how these methods could be applied to groundwater protection zones (GPZs) in the UK.

While different rock types impose different characteristics on the nature of fracturing, it was decided early on to regard zone delineation as two separate issues. The first issue was to identify an appropriate approach to protection-zone delineation in relation to fracture characteristics. Those characteristics were seen as spanning a (multidimensional) spectrum of types. This issue was the primary concern of the research. The secondary issue was to assign rock types into that spectrum, although this secondary issue has not been tackled in this thesis. This approach is summarised in Figure 3.1. One motivation for this two-aspect approach is that it avoids the complication of becoming caught up in arguments over issues such as whether or not the Chalk should be considered karstic.

The first step was to review methodologies for delineating GPZs in fractured rock in countries around the world to assess whether the present Environment Agency, SEPA and EHA policies are in line with current thinking. The review was also to assess whether there were methodologies elsewhere that were applicable to the UK.

The second stage was to conduct an extensive review of fractured rock modelling literature to include subjects such as:

- the different methods of fracture modelling – discrete, stochastic and continuum, for example;
- the different fracture models available, their capabilities and data requirements;
- how the data required for the modelling can be obtained in the field;
- if data are not available, how can structural and other geological theories help predict fracture parameters; and
- fracture parameter distributions.
The third stage was to conduct a literature review on the different methods of characterising uncertainty and hence how to create zones based on probability. Based on these ideas, a code was created that would create these probability-based zones from standard stochastic modelling exercises.

**Different Approaches**

<table>
<thead>
<tr>
<th>GPZS WORLDWIDE</th>
<th>FRACTURE CHARACTERIZATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEOSTATISTICS</td>
<td>MODELLING</td>
</tr>
</tbody>
</table>

Spectrum of fractured rock types based around many factors e.g. hydraulic conductivity, fracture density and orientation, and parameter statistics

**FRACTURED ROCK SPECTRUM**

- LIMESTONE
- METAMORPHICS
- SANDSTONE
- CHALK
- VOLCANICS

**Different Aquifer Types**

Figure 3.1: Division of the problem into two aspects: delineation according to fracture type and assignment of aquifer types.
The next and fourth phase was to examine the possibility of developing a methodology in 2-D. Initial investigations were conducted in 2-D to be consistent with the existing Agency methods. Fracture modelling exercises were conducted to assess whether generalisations could be made on the shape and form of protection zones for particular combinations of 2-D fracture parameters. Tests using 2-D modelling techniques were then completed on a number of case study sites.

The final phase of work examined the possibility of developing a methodology in 3-D, since fracture flow is generally a 3-D problem. A method of upscaling to allow catchment scale modelling was tested, but taking into account the fracture flow properties of the system. The method was applied to a case study site in Ross-on-Wye to produce probabilistic protection zones, with a discussion on the different methods of validating these zones, since the zones must be defensible.

3.2 Constraints on the approach

Any proposed methodology for delineating groundwater protection zones in fractured rock should take into account the needs of the regulatory body responsible for groundwater protection: in the UK this is the Environment Agency, SEPA and the EHA. The methodology should also attempt to deal with any problems with the existing methodologies, these problems being the need to provide a more robust uncertainty analysis and also to provide alternatives to the sole use of porous media models in fractured rock scenarios to delineate travel time zones.

The approach to the problem should, in general, take the following issues into account:

i) **Transparency.** Any proposed methodology should be transparent with the steps within a methodology both well documented and explained;

ii) **Practicality.** The output of any newly proposed methodology should be easily understood by both land-use planners and non-specialists, preferably using techniques with which the end-users are already familiar;

iii) **Limited resources.** Any proposed methodology should be compatible with and enhance the existing methodology since much time, effort and expense has already been spent on delineating the existing zones. Limited resources
would be available for any reassessment of the existing zones by the regulatory authorities;

iv) **Defensibility.** Economic decisions regarding land-use will be made as a result of the location and type of the protection zone in the area. Therefore, any proposed methodology should attempt to be at least as, and preferably more defensible than, any existing methodology;

v) **Few data, especially for fractures.** The proposed methodology must not have onerous data requirements but must rely on data that already exists, or can be extrapolated from elsewhere, or can be collected quickly and simply; and

vi) **Need for a rigorous uncertainty analysis.** As is the case for all aquifer systems, the aquifer parameters can take on a wide range of values both because of the heterogeneity of the system, but also because of the uncertainties in characterising that system. For fractured rock these uncertainties can be extreme. In the current methodology the uncertainty analysis is a deterministic one. Any new methodology should have a more rigorous uncertainty analysis founded on a risk-based approach to produce probabilistic zones using stochastic models.

The approach taken in this research assumes that protection zones will be delineated using travel-time, as is the current practice in the Environment Agency, SEPA and the EHA. Groundwater protection zones could, however, be defined by other criteria, for example:

- Distance from the source.
- Dilution factors. Zones could be based on lines of equal dilution to the source. Zones would be contaminant specific because of different contaminant toxicities. Zones of dilution factors could be presented and for each substance the dilution factor multiplied by the contaminant concentration at source to assess the risk — hence a relatively simple method. The use of dilution factors would therefore be more defensible since they take into account the contaminants’ toxicities whereas travel-time does not.
- Drawdown
- Flow boundaries
• Assimilative capacity

Travel time is, however, the one of the less complex methods and one that is used by many other countries, and hence is favoured by the UK regulatory authorities. If a practical methodology is to be developed for use by these authorities it must be compatible with and enhance the present methodology since much expense and time has already been expended in creating the existing zones. It is for this reason that the research presented here is based solely on travel-time protection zones.

From the work by Pekdeger et al. (1985) it is recognised that the 50-day travel time for the protection of a source in fractured rock from biological pollutants may be inappropriate. The 50-day travel-time zone is based around elimination rate constants or ‘half lives’ of bacteria and viruses in groundwater being, in general, less than 50 days. However, Pekdeger et al. (1985) state that in aquifers with a large porosity, and particularly in fractured aquifers, bacteria and viruses are not subject to significant filtration from the groundwater flow path, and thus have a very long ‘half-life’. They recognise that other protection criteria should be used rather than distance or time in these cases. In fractured rock the 50-day travel-time zone may therefore offer little protection from biological contamination of a source, and may be an inappropriate zone to delineate in fractured rock. However, in order to be consistent with the existing methodology of the UK regulatory authorities the 50-day travel-time has been considered relevant in this thesis.

As discussed in Chapter 2, the approach here has been to concentrate on the effects of fracturing and fracture flow on the delineation of protection zones. Other factors, such as recharge, transient effects and boundary conditions are not discussed here but may be as equally important as fracture effects, since they also have much uncertainty associated with them. Chapter 2 also discussed the need for a more robust uncertainty analysis to be used, which constrains the approach to principally using stochastic techniques in zone delineation.
4 SOURCE PROTECTION METHODOLOGIES WORLD-WIDE: A REVIEW

This chapter reviews the methodologies for groundwater protection used across the world, focussing on methods that are used in fractured rock areas. It is possible that delineation techniques have already been developed in other countries for fractured rock situations, and that they can be adapted for a UK methodology. It is also important to obtain an understanding of the current state of research in this area in order that no research is unnecessarily duplicated here. Methods employed to review worldwide methodologies included querying the web, conducting many literature searches, and requesting information from government departments and universities. If sufficient data were available on a specific country’s methodology, a critique was of that methodology was undertaken. In particular, a critique on the methodologies of South Africa and the United States are presented in this chapter.

4.1 EUROPE

Details of the legislation and the technical aspects of groundwater protection zones in European countries are outlined in alphabetical order below. For some European countries it has proved difficult to find data more recent than 1989 (Lallemand-Barres and Roux, 1989) and therefore it is recognized that some policy changes are likely to have occurred. In general most European countries have developed their protection zone policies within the last ten or twenty years (since 1980). Table 4.1 presents a summary of the criteria for delineating the different zones in the different European countries. Due to lack of information, some of the countries represented in the table are not detailed in the following section and some of the countries in the text are not in the table.

As mentioned in chapter 2, the recent EC Water Framework Directive (2000) states under Article 7 that “…Member States may establish safeguard zones…” for the protection of groundwater quality. This Framework Directive has yet to be interpreted under the laws of the individual European countries. Given that most European countries have an established groundwater protection zone policy, it is not known how this article will be incorporated into national law. Article 7 could be used as an opportunity to rethink groundwater protection zone policy and delineate zones using more robust methods, although for most countries the cost involved may be prohibitive.
4.1.1 BELGIUM

On average 67% of the country’s drinking water is derived from groundwater. This value varies from region to region with the Walloon region obtaining nearly all its drinking water from groundwater sources, the Flanders region having 50% derived from groundwater sources and 75% for Brussels.

The concept of catchment areas and protection zones within catchments was introduced in the law on the Protection of Groundwater (26th March 1971), with the responsibility for defining zones left to individual regions. In Walloon region the groundwater zones have been more precisely defined (Derouane and Dassargues, 1998) as:

- Zone of Water Supply – an area encompassing the water supply installations with an extra distance of 10 m from the water supply area added in all directions;
- Zone of Protection (Type 1) - an area defined by a travel time of 24 hours;
- Zone of Protection (Type2) - an area defined by 50-day travel time to protect against bacteriological contamination and has a minimum radius of 100 m in sandy aquifers, 500 m in gravel aquifers and 1000 m in fissured and karstic aquifers; and
- Zone of Protection (Type 3) – the entire source catchment area.

A very similar designation of zones is found in the Flanders region.
Table 4.1: Comparison of the different groundwater protection zones used in Europe (after Lallemand-Barres and Roux, 1989) – continued over.

<table>
<thead>
<tr>
<th>Prohibition</th>
<th>Austria</th>
<th>Belgium</th>
<th>Canada</th>
<th>Czech Republic</th>
<th>England &amp; Wales</th>
<th>Finland</th>
<th>France</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>Only water supply activities allowed</td>
<td>Immediate protection area</td>
<td>Zone of water supply &gt;10 m</td>
<td>Protection Area 1 100 day or 250 day</td>
<td>First sanitary protection zone 10-50 m</td>
<td>Included in zone I</td>
<td>Intake area 20-25 m</td>
<td>Immediate protection zone 100-1000 m²</td>
<td>Zone I wellfield &gt;10 m</td>
</tr>
<tr>
<td>Restriction on buildings and agriculture</td>
<td>Protection area 60 days</td>
<td>Zone of Protection (Type 1) 24 hours Zone of Protection (Type 2) 50 days</td>
<td>Protection Area 2 5 year</td>
<td>Internal second sanitary protection zone</td>
<td>Zone I 50 days and &gt;50 m</td>
<td>Inner protection zone 60 days</td>
<td>Inner protection zone 100,000 m²</td>
<td>Zone II 50 or 100 days</td>
</tr>
<tr>
<td>Restriction on certain industries, storage and transport of certain chemicals and oils</td>
<td>Zone of partial protection</td>
<td>Zone of Protection (Type 3) Entire catchment area</td>
<td>Protection Area 3 25 years</td>
<td>External second sanitary protection zone</td>
<td>Zone II 400 days and &gt;25% of catchment area</td>
<td>Outer protection zone</td>
<td>Remote protection area</td>
<td>Zone IIIa &lt;2000 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Zone III Catchment area</td>
<td></td>
<td></td>
<td>Zone IIIb Catchment area</td>
</tr>
</tbody>
</table>

Chapter 4 - GPZs worldwide
Table 4.1 (cont.): Comparison of the different groundwater protection zones used in Europe (after Lallemand-Barres and Roux, 1989).

<table>
<thead>
<tr>
<th>Prohibition</th>
<th>Hungary</th>
<th>Ireland</th>
<th>Italy</th>
<th>Netherlands</th>
<th>Spain</th>
<th>Sweden</th>
<th>Switzerland</th>
<th>United States (Ohio)</th>
<th>USSR (as was)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Only water supply activities allowed</td>
<td>Protection zone</td>
<td>Zone I &gt;10 m</td>
<td>Zone I &gt;10 m</td>
<td>Well field</td>
<td>Absolute restriction zone 1 day or &lt;400 m³</td>
<td>Well area Zone I 10-20 m</td>
<td>Zone I 10-20 m</td>
<td>Zone I &lt;400 ft</td>
<td>Zone I 15-50 m</td>
</tr>
<tr>
<td>Restriction on buildings and agriculture</td>
<td>60 days</td>
<td>Inner protection area 100 days 300 m</td>
<td>Zone 2 &gt;200 m</td>
<td>Catchment area 50-60 days &gt;30 m</td>
<td>Maximum Restriction zone 50-60 days</td>
<td>Inner protection zone &gt;60 days &gt;100 m</td>
<td>Zone II 10 days &gt;100 m</td>
<td>Zone 2 100-400 days</td>
<td></td>
</tr>
<tr>
<td>Restriction on certain industries, storage and transport of certain chemicals and oils</td>
<td>Hydro-geological protection zone 25-100 m</td>
<td>Catchment area or 1000 m</td>
<td>Zone 3 Recharge area</td>
<td>Protection area 10 years or 25 years &lt;2000 m</td>
<td>Moderate restriction zone Several years Satellite protection zone</td>
<td>Outer protection zone</td>
<td>Zone III &gt;200 m Interim Wellhead Protection Area Zone A</td>
<td>Zone III time dependent zone</td>
<td></td>
</tr>
<tr>
<td>Regional protection</td>
<td>Regional protection</td>
<td>Far recharge area</td>
<td>Quantity protection zone</td>
<td>Zone B</td>
<td>Entire watershed</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Chapter 4 - GPZs worldwide
4.1.2 DENMARK

Groundwater and spring water provide virtually all potable water in Denmark (99%), with the remainder being abstracted from lakes. The north and southeast regions have a complex geology with clay and glacial till overlying fissured Chalk. In the southwest, sandy soil and gravel overly the clay beds.

Groundwater abstraction usually requires a permit and once it is granted an automatic invocation of Section 12 of the Environmental Protection Act (1/10/74) is applied, and a protection zone is immediately set up around the borehole. The Environmental Protection Act of January 1983 takes this slightly further by defining which activities are prohibited in the protection zone, the size and the shape of the zone being controlled by the individual hydrogeological situation.

In August 2000 the Danish Environment Protection Agency set up guidance for detailed mapping and the setting up of protection zones (Thomsen, 2001). As part of the groundwater protection plans in Denmark an extensive geological and hydrogeological mapping exercise has been and is being undertaken (1998 onwards) in order that all groundwater protection zones are based on well-founded conceptual models.

4.1.3 FRANCE

Groundwater provides approximately 64% of the country's drinking water, although this figure varies from region to region. In the north eastern and central parts of France, Cretaceous Chalk and Jurassic limestone are the main aquifers and, as in East Anglia and the Severn-Trent Regions in the UK, are the main drinking water sources. The granitic and schistose areas have some small, unconfined aquifers. Shallow aquifers are also present in Brittany where the geology is similar to that found in southwest England.

In France groundwater law comes under the general laws of surface water protection. The principal law which prohibits any action that is likely to impair water quality was brought into force in December 1964. The law allows protection zones to be developed and fines to be collected if disposal of polluting substances is found to have occurred. This law has since been reinforced by Article 17 of the Decree concerning Water Intended for Human Consumption (March 1988) and Article L 20 of the Public Health
Code (last revision January 1989). The protection zones are divided into three zones around each source:

- Zone I - This area generally covers an area of 100-1000 m² around the well and in this area no activities other than those related to water abstraction are permitted;
- Zone II - This is the near zone and typically covers 100,000 m² (equivalent to ~300 m radius). The size of the zone depends on the hydrogeological setting; and
- Zone III - This outer protection zone is more extensive and can be established if needed in areas of higher groundwater vulnerability.

In 1990 protection zones had been defined for only 12% of supply wells, although this figure varied from region to region.

### 4.1.4 GERMANY

Groundwater, including spring water, provides over 70% of Germany’s drinking water, with the proportion continuing to increase (Gramel, 2001). This value varies from state to state with 100% in West Berlin, Schleswig Holstein and Bremen, to only 38% in North Rhine-Westphalia. It is recognized that protecting groundwater quantity is much less of a problem than groundwater quality in Germany since the reduction in water consumption by both industrial and private users (Gramel, 2001).

Groundwater legislation is covered by water supply laws, which are enforced by the regional authorities or states. There are numerous laws that help to protect groundwater:

- Law of Public Order;
- Federal Waste Avoidance and Disposal Act 1986;
- Potable Water Regulation; and
- Section 19 of the German Water Act (Wasserhaushaltsgesetz WHG) (http://www.penelope.uab.es/penelope/Library/Libs/dlib/whge). These regulations are given by the Deutscher Verein des Gas- und Wasserfaches (DVGW; German Association for Gas and Water).
first guidelines were published in 1953 and these were updated in 1961, 1975 and 1995.

Section 19 of the Water Act states that “water protection areas may be established where it is necessary, in the interests of the common good:

1. to protect certain waters against detrimental effects in the interests of the existing or any future public water supply, or
2. to replenish the stock of ground water, or
3. to prevent the harmful effects caused by rainwater run-off as well as by erosion and the introduction of soil components, fertilisers, herbicides and pesticides into waters.

Within these water protection areas:

1. certain activities may be prohibited or permitted only to a limited extent; and
2. the owners and persons entitled to use land may be obliged to tolerate certain measures. This also includes measures for observing the water and the soil.”

Three zones are defined:

- Zone I - A minimum of 10 m from the abstraction point to protect the area immediately surrounding the well. The land is owned by the water company. All forms of land use not relating to water supply production are forbidden;
- Zone II - Defined as 50 or 100 day travel time from the well depending on aquifer vulnerability, and protects the groundwater from bacterial pollution. Land restrictions include, amongst others, the construction of sewer pipes, roads and railways, and the use of fertilizers; and
- Zone III - This zone is generally considered as the catchment area. It is split into zone IIIa which is less than 2 km from the abstraction point, and zone IIIb which comprises the remaining catchment area.

The zones are based mainly on hydrogeological modelling, but consultation with local people and organisations may cause the zones to be altered slightly. By 1989 over 13,000 protection zones had been established or were planned out of a total of 26,000
which were considered necessary. The protection zones in Germany are estimated on
the basis of the technical guideline ‘DVGW Regelwerk W 101’ which is a regulation of
the German Association of Water and Gas Specialists. The guideline is only advisory
and does not bind any of the sixteen federal states to observe it. The states all have their
own water acts, and the organisation that carries out the zone delineation varies from
state to state. For example in Baden-Württemberg (SW Germany) the Federal State
Geological Survey determines all GPZs.

Lecture notes from the University of Tübingen (pers. comm. J. Whittaker, 1997) state
that for karstic aquifers and aquifers that do not behave as an equivalent porous medium
the Baden-Württemberg authorities use tracer tests to determine protection zones. The
maximum velocity from the breakthrough curves is used to delimit the zones. It is
recognised that this can lead to unfeasibly large zones due to velocities up to several
hundred metres per hour.

The zones in Baden-Württemberg fractured rock aquifers are then defined as follows:

- Inner zone - 30 m radius;
- Middle zone – from 300 m to 1000 m from the source. Although the middle
  zone is usually defined by a 50-day travel time, it is recognised as not generally
  practical at the flow velocities measured. Any particularly sensitive areas
  outside the 1000 or 300 m zone such as sink holes, quarries, or deeply incised
  dry valleys should also be designated as zone II. This often leads to the
  protection zones assuming a mosaic type pattern; and
- Outer zone – total capture zone.

If the whole well capture zone lies within a 50-day travel time then those areas not
classified as zones I or II are to be classified as zone III. Division into zones IIIa and
IIIb can only take place inside the 50-day travel time if the aquifer is overlain by an
extensive, thick, low permeability layer. Zone IIIb must be at least 1000 m from the
well and overlain by at least 8 m of loamy clayey, silty layers or a layer containing its
own perched aquifer table over a separating layer of at least 5 m in thickness. These
criteria were originally developed by the Federal Geological Society (GLA) for the
Baden-Württemberg state, in particular the region of Rottenburg (Villinger, 1994), but
have now been written into the DVGW Guidelines W101 and apply to all regions.
Lallemand-Barres (1990) lists the same recommendations for Germany for fractured and karstic rock.

The GLA in Baden-Württemberg calculates protection zones for karstic and non EPM areas using aquifer parameters either defined through tracer tests or pump tests, but more frequently through values obtained from the best professional judgement of aquifer conditions. Numerical modelling is sometimes carried out when there are sufficient data, but delineation is completed using Darcy’s Law combined with vulnerability mapping to demarcate the sensitive recharge areas, as described above for Zone II delineation (pers. comm. J. Whittaker, 1997).

4.1.5 IRELAND

Approximately 25% of the total water supply in Ireland comes from groundwater, with most sources being of good quality. Contamination tends not to be widespread and nitrate levels are low, even though the main hazard to groundwater quality is from agricultural sources such as farmyards and septic tank systems. However, they do have karstic aquifers and the potential for rapid migration of pollutants is high.

Responsibility for groundwater protection in Northern Ireland falls to the Environment and Heritage Service, which is part of the Department of the Environment of Northern Ireland. Their policy for groundwater protection mirrors that of Environment Agency for England and Wales. It is stated in the ‘Policy and Practice for the protection of Groundwater in Northern Ireland’, July 2001, that source protection zones may be delineated by any ‘technically valid method’ since they recognize the difficulty of delineating zones in complex hydrogeological environments such as karst.

Protection of groundwater in Eire is included in the laws on water pollution and is primarily controlled by local authorities, for example in the Local Government (Water Pollution) Acts of 1987 and 1990.

The Irish (Eire) programme of pollution prevention is designed as a practical way of complying with the 1987 and 1990 Water Pollution Acts. This has been drawn up by Geological Survey of Ireland (GSI), with influences for the scheme coming from the US, Switzerland and England and Wales. The scheme was formally published in 1999.
but has been in use since the early 1990s (Daly and Misstear, 2001). There are three main components to the proposed scheme (Daly and Deakin, 1996):

- A source protection map, which shows zones requiring different degrees of protection;
- An inventory of potential pollution sources within the source protection area; and
- Control measures (code of practice or management plan) for existing and new potentially polluting activities.

There are two elements to the first component of the programme, which are integrated to give the source protection zones:

- Areas surrounding individual groundwater sources; and
- Division of the source protection areas on the basis of the vulnerability of the underlying groundwater to contamination.

Around each individual source there are three source protection areas: the source site area, the inner protection area and the outer protection area. It is recognised that these areas are only seen as a guide for decision-making due to the heterogeneity of Irish aquifers and the variations in data availability. The GSI uses four methods of protection area delineation:

- Calculated fixed radius;
- Analytical methods;
- Hydrogeological mapping; and
- Numerical modelling, using FLOWPATH.

The source protection areas are defined as follows:

- The source site area includes the source itself and the operational activities associated with water supply. It should be at least 10 m in radius, and all potentially polluting activities are prohibited in this area. The size and shape of this zone does not depend on hydrogeological characteristics;
The inner protection area is defined by 100-day travel time and is designed to prevent microbial pollution. This travel time is greater than in many other countries due to the heterogeneous nature of Irish aquifers and the fact that some bacteria may live longer than 50 days in groundwater. In karstic areas it is recognised that the time-of-travel approach is not applicable as there are large variations in permeability, high flow velocities and a low level of predictability. In some cases it may be necessary to use the arbitrary fixed radius method, and in this case a radius of 300 m is used. For springs a semi-circular area is used. This distance may be increased in karstic areas, and reduced in granular aquifers; and

The outer protection area covers the zone of contribution or catchment area. The catchment area is currently defined using a 50% increase in the present abstraction rate to allow for possible expansion in abstractions and for dry periods. Also, a 20° variation in the direction of the hydraulic gradient is included as a safety margin. If the arbitrary fixed method is used a radius of 1000 m is chosen, with some variations occurring in regions of karst or low yielding wells.

These areas are then combined with vulnerability mapping to take account of relative vertical travel times to the water table from the surface. Four groundwater vulnerability mapping categories are used by the GSI: extreme, high, moderate and low, and these categories depend on the subsoils that overlie the groundwater, the recharge type and the thickness of the unsaturated zone. The source protection zones are formed by superimposing the vulnerability map on the source protection area map to produce a possible 12 types of source protection zones.

Prior to 1996 the GSI had delineated 30 protection zones around public supply wells. They drew several conclusions that are relevant to this project. Around karst springs the only realistic approach is to define the catchment area using hydrogeological mapping, tracing, water balance estimates and by defining sinking stream inputs and other relevant karst features. They also noted that the lack of information on spring outflows and paucity of hydrogeological information around sources hindered their work.

4.1.6 ITALY

Approximately 88% of Italy's drinking water is obtained from groundwater or spring water. There are three main aquifer types in Italy: alluvial, limestone and
arenaceous/volcanic. The karstic and fissured aquifers occur in the Alps and also in the Apennines. It is recognised that groundwater flow is relatively rapid in these areas.

Most of the laws relating to groundwater are detailed in the regional legislation rather than the national legislation, with most of the laws designed to control emissions to waters. For example:

- Law 319 (1976) amended by law 650 (1979) which regulates all kinds of liquid and solid emissions affecting the aquatic environment;
- Law 441 (1978) regulating, by means of Article 5 studies and/or operations dealing with the restoration of contaminated land; and
- Law 175 (1987) implementing Directive 82/501/EEC on major accident hazards of certain industrial activities into national legislation, e.g. petrochemical plants, oil tanks etc.

However, the most important law (to date 1990) was the Presidential Decree 236 of May 1988 which amongst many other regulations detailed the creation of protection zones to safeguard drinking water abstraction points. Three zones were defined:

- Zone 1 - The radius must be not less than 10 m and only activities relating to the abstraction of water are allowed within this area. The size of the area must take into account the vulnerability of the source to contamination;
- Zone 2 - This should have a radius of not less than 200 m with an adjustment made to take into account the vulnerability of the source. Many activities are prohibited in this area; and
- Zone 3 - This is defined as the recharge area, and depending on the risk of contamination to the groundwater, various restrictions may be imposed on land-use.

However, as mentioned earlier, the implementation of the zones and most of the legislation is based at a regional level. For example, in Lombardy the zones are based on travel times with the inner zone being defined by a 60 day time and the outer zone by a 365 day travel time.
4.1.7 NETHERLANDS

Approximately 70% of Dutch drinking water is obtained from groundwater and spring water sources, with the remainder being extracted from surface water sources. The majority of Dutch aquifers are sand and gravel with porous-media type flows. However, in the south-west part of the Netherlands karstic and highly fissured aquifers are present. The Dutch do allow for these fractured rocks within their main groundwater protection policy.

There are many laws in the Netherlands which aid the protection of groundwater:

- Groundwater Act 1982- protection of catchment areas and abstraction;
- Soil Protection Act 1980 and 1986- protection of soil and groundwater from contamination;
- Provisional Soil Decontamination Act 1982- clean-up and restoration of soil and groundwater pollution; and

The groundwater protection zones are based on travel time with the shape and size of the zones being determined by hydrogeological parameters and modelling. Within the zones various land-use restrictions are in force. The zones themselves are defined as follows (Headworth, 1986):

- Zone I (Catchment Area) – based on 50-60 day travel time for granular formations, resulting in a zone of between 30 and 150m radius;
- Zone II - (Protection Area) – based on 10 year and 25 year travel times, which are approximately equivalent to 800 and 1200 m radius respectively, up to a maximum of 2 km radius. This is based on sufficient time allowed to clean up an acute spillage within the catchment area; and
- Zone III - (Remaining Recharge Area)

For fissured and karstic aquifers in the South-West Netherlands the definition for Zone III remains identical, whereas the Zone II is restricted to a radius of 2 km from the abstraction wells. It was recognised that the high flow rates in such areas would lead to protection areas which would be impossible to implement.
4.1.8 SCOTLAND

In 2000 only 3% of Scotland’s water supply came from groundwater. It is, however, a vital water supply for those in more rural areas. The regulatory authority in Scotland, the Scottish Environment Protection Agency (SEPA) produced a Groundwater Protection Policy for Scotland, which was adopted in June 1997. This policy is almost identical to the equivalent document of the Environment Agency of England and Wales, with the three components of the framework being:

- groundwater vulnerability;
- definition of source protection zones; and
- statements on groundwater protection policy.

Three source or groundwater protection zones are defined:

- Inner protection zone, defined by the 50-day travel time for contaminants. This zone is designed to protect the source from biological contaminants, and is based around the idea that most bacteria/viruses tend to have a half-life of less than 50 days;
- Outer protection zone, defined by the 400-day travel time for contaminants. This zone is designed to protect the source from chemical contaminants, and is based around the time it takes for chemicals to attenuate in the aquifer; and
- Catchment zone, defined by the area required to support the long term abstraction rate by the average annual recharge.

Although the policy recognises the different hydraulic conductivities of the aquifers in Scotland, no guidance is given in this document as to how the zones should be defined, and hence no mention of how zones should be defined in fractured rock.

Robins (1999) does, however, present some ideas of how to tackle fractured rock protection zones, particularly those in the Lower Palaeozoic and Precambrian rocks of Scotland, Wales, Cumbria, the Channel Islands and Ireland.

Robins (1999) divides these basement rocks into four categories:

a. karstic limestone;
b. massive rock with few fractures;
c. fractured rock; and
d. fractured rock with regolith.

Robins (1999) recognises that delineation of capture zones by numerical modelling in these weakly permeable fractured aquifers with steep topography is not practical because data are sparse and the since most sources are small it is not economically viable to collect more data or spend time on numerical modelling. Therefore an empirical approach is advocated for capture zone delineation depending on:

i) the specific groundwater setting;
ii) the hydraulic gradient; and
iii) the abstraction rate and recharge.

Six example source capture zones are given for specific groundwater settings which include settings with a dominant fracture set orientation and preferred flow paths for karstic scenarios. In karstic regions it is recognised that any sinkholes and the surface drainage catchment supplying the sinkholes should have a 30 m diameter protection zone around them. The areas of the source capture zones for different yields of borehole given by Robins (1999) are illustrated in Table 4.2.

Table 4.2: Probable source capture zone area (km$^2$) in Lower Palaeozoic and older rocks (after Robins, 1999).

<table>
<thead>
<tr>
<th>Location</th>
<th>Infiltration (mm/annum)</th>
<th>Borehole yielding 1 l/s for 12hr/day</th>
<th>Borehole yielding 0.2 l/s for 12hr/day</th>
<th>Spring of maximum yield 51/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel Islands</td>
<td>100</td>
<td>0.2</td>
<td>0.03</td>
<td>1.6</td>
</tr>
<tr>
<td>East and Central Scotland</td>
<td>200</td>
<td>0.1</td>
<td>0.02</td>
<td>0.8</td>
</tr>
<tr>
<td>Other Upland areas</td>
<td>500</td>
<td>0.3</td>
<td>0.01</td>
<td>0.3</td>
</tr>
</tbody>
</table>

4.1.9 SPAIN

In Spain only 26% of drinking water supplies come from groundwater. Even though groundwater supplies a relatively small proportion of water to the population, the
delineation of wellhead protection areas in urban catchments is mandatory under Spanish water law (Martinez-Navarrete and Garcia-Garcia, 2001). The approval of wellhead protection areas is the responsibility of the Government Board of Water Managers Association. The water laws allow 5 zones to be defined:

- **Absolute restriction zone** – travel time of 1 day or an arbitrary fixed area of between 100 and 400 m²;
- **Maximum restriction zone** – 50-60 days travel-time to protect against microbial contaminants;
- **Moderate restriction zone** – either a travel time of several years or hydrogeological mapping, or both are used to protect against long term persisting pollutants;
- **Satellite protection zone** – this zone is used in karstic and fractured aquifers and allows an extra level of protection to be added. It is used for areas that are outside of the moderate restriction zone but which are still in hydraulic connection with the catchment; and
- **Quantity protection zone** – the objective of this zone is to preserve the volume of water required to supply the population.

Not all of these zones tend to be defined, the number defined depends on the hydrogeological conditions.

The methods of zone delineation include analytical (Wyssling technique – see Appendix A) and numerical techniques as well as hydrogeological/flow and vulnerability mapping.

Although it has been recognized in the zone divisioning that fractured rock situations have more uncertainty associated with them than porous rocks and generally require more aquifer protection, no specific fractured rock methodology is mentioned (Martinez-Navarrete and Garcia-Garcia, 2001).

**4.1.10 SWITZERLAND**

In 1988 approximately 82% of all water used in Switzerland was derived from groundwater: 43% of this water was derived from springs and 39% from aquifers. The
most important aquifers in Switzerland are the glacial sand and gravels aquifers in the valley regions and karstic limestone aquifers elsewhere.

Groundwater legislation was first introduced in 1972 when the federal law of groundwater protection came into force. Article 30 of this law states that the Cantons are responsible for determining groundwater protection zones around spring and borehole sources. The owner of the borehole or spring has to provide the data for delineating the protection zone. The methodology for delineating protection zones and their implementation was eventually produced in 1977 (BUS 1977/82). This was updated in Article 20 of the Swiss Federal law on the Protection of Groundwater (Water Protection Law) of January 24th 1991 requiring the delineation of groundwater protection zones for all public groundwater catchments (springs and wells), as well as artificial recharge areas. Within the zones a ban is placed on industrial development and extractive industries. The Water Protection Ordinance of 28th October 1998 advocates three protection zones: S1, S2 and S3 which relate to land use. These zones are defined as follows (Doefliger and Zwahlen, 1998):

- **S1 Zone** – *this zone must prevent damage to the groundwater catchment installations or artificial recharge facilities as well as prevent pollution in their immediate surroundings;*

- **S2 Zone** – *this zone defines an area suitable to prevent biological contamination to drinking water catchments. It must also prevent drinking water supply from being polluted by excavations and subsurface works, or that the flow of water towards the surface is disturbed by subsurface works; and*

- **S3 Zone** – *this zone must provide sufficient space and time for remediation when accidental pollution threatens a catchment installation."

Blau (1990) states that the zones will be revised every 10 years as new information becomes available. It was recognised before the 1977 methodology was initiated that 50 days should be used for a travel time for biological contaminants, but it was decided that groundwater velocities were too fast for that criteria to be practical, and that 10 days would be sufficient.
A very recent document has been published in March 2002 on delineation of protection zones in fractured rock (Pochon and Zwahlen, 2002). This document details the techniques to be applied in different fractured rock settings depending on their vulnerability. In low vulnerability settings the distance method is used (fixed size zones around the well) with the zones defined as follows:

- **Zone S1**: 10 m around the source, including any known adits and fractures intersecting adits and/or the abstraction well;
- **Zone S2**: a minimum of 100 m in the up-gradient principal flow direction from the source; and
- **Zone S3**: The same distance again as the distance from the edge of zone S1 to the edge of zone S2.

In more vulnerable areas where an equivalent porous medium is present and groundwater flow velocity is of the order of tens of metres per day travel-time zones should be used:

- **Zone S1**: 10 m around the source, including any known adits and fractures intersecting adits and/or the abstraction well;
- **Zone S2**: 10-day travel time around the source, or a minimum of 100 m in the up-gradient principal flow direction from the source; and
- **Zone S3**: The same distance again as the distance from the edge of zone S1 to the edge of zone S2.

In extremely vulnerable areas, where groundwater flow velocities are of the order of hundreds of metres per day, the 'DISCO' (discontinuités - couverture protectrice) method is used to delineate zones. This is a vulnerability classification technique whereby, depending on the total score from several factors, the zones S1, S2 and S3 are defined. The factors used in the DISCO method are:

1. Fractures - their location and changes of density within the catchment and the flow velocities within them;
2. Unsaturated zone (protective cover)- soil type, presence of Quaternary deposits, and pre-Quaternary layers; and
3. Surface water – the nature of surface water flow before infiltration; location and nature of temporary or permanent rivers, slope of the ground.

The resulting zones are very different to standard travel-time zones with many zones of all types juxtaposed, producing a mosaic type pattern. The initial determination of whether to go for the fixed radius zones, isochrones or the DISCO technique depends on fracture pattern observations, the response of the aquifer to heavy rainfall (chemistry, biology and water levels), and tracer testing. Tracer testing is cited as being a very important factor in this decision process.

Karstic environments have also been extensively studied in Switzerland. The EPIK method of karst vulnerability mapping (Doefliger and Zwahlen, 1998) was developed in Switzerland. The four parameters taken into account are:

1. development of Epikarst;
2. effectiveness of Protective cover;
3. conditions of Infiltration; and
4. development of the Karst network.

After mapping these parameters a weighting factor is applied according to the degree of protection against contamination. The protection values are then added together to create the final vulnerability map. Protection zones S1, S2 and S3 can then be defined, as with the DISCO method.

Three cases studies for the COST 620 programme also investigate groundwater risk in karstic environments through vulnerability mapping. In this programme it has also been recognised that computer modelling is almost impossible and hence protecting karstic areas can only be achieved by sophisticated vulnerability mapping.

4.2 SOUTH AFRICA

More than 70% of South Africa’s water users, including more than 100 medium-sized towns, are dependent on groundwater. The maximum quantity of groundwater that can be developed economically is estimated at 5400 million m³ a year. Nearly 90% of the area of South Africa is considered as being under fractured rock conditions and
therefore classic approaches to groundwater protection, as used in Europe and the USA, can rarely be used.

Groundwater research is co-ordinated by the Water Research Commission in Pretoria. It is the Department of Water Affairs and Forestry (DWAF) that is tasked with the integrated management of the national water resources. In 1991 the Committee of Groundwater Quality Management Policies and Strategies for South Africa was established. This committee identified the need for an overall groundwater protection strategy in South Africa, rather than the fragmented and reactive approach previously taken to pollution incidents. Further to this DWAF embarked on a programme of policy and strategy formation for groundwater quality. A white paper was produced by DWAF in 1994 forming the Reconstruction and Development Programme (RDP). Part of this is the Community Water Supply and Sanitation Programme (CWSS) where it was recognised that there was an immediate need for the formulation of a guideline for groundwater protection. A short term objective of the RDP is that the CWSS has to ensure a clean, safe drinking water supply of 25 l/day/capita within a 200 m walking distance.

Government legislation introduced in November 1996 states that groundwater is no longer private property but should be used for the good of the nation. The policy (Principle D7) also states that regulation of land use should be used as an instrument to manage water resources. As part of this a vulnerability map for South Africa has been developed which has been described by Lynch et al. (1994) and is based around the widely used DRASTIC methodology (Aller et al., 1985). The methodology uses a scoring system for each of the hydraulic parameters which are then combined to produce the final DRASTIC index for the area. The new groundwater protection guidelines (Xu and Braune, 1995 and Xu and Reynders, 1995), which emphasise the need for source protection zones can now be implemented more readily with the introduction of the November 1996 law stating that groundwater is no longer private property.

It is recognised in South Africa that there are two aspects to groundwater protection: vulnerability classification and source protection. The South Africans have adopted a three-tier approach to dealing with these two aspects, as well as satisfying the CWSS criteria for drinking water supplies:

Chapter 4 - GPZs worldwide
- Tier 1 - Minimum Protection Standards;
- Tier 2 - Vulnerability Classification; and
- Tier 3 - Designation of Protection Zones.

The third tier involves the designation of protection zones around supply wells. It has been recognised that in some cases protection zone delineation may not be necessary or viable for economic reasons, and to perform the zoning adequately sufficient data must be available. It has also been recognised that methods other than numerical modelling may have to be used to delineate zones, especially in highly fractured and cavernous rocks. The Xu and Braune (1995) document suggests that flow system mapping along with isotopic and hydrogeochemical analyses should be used to support any travel time zones defined. Xu et al. (1997) recognised that defining protection zones in fractured rock should also be supported by tracer testing.

Therefore, although the South Africans recognise the problem of being able to define protection zones in fractured rock, they are only in the first stages of developing a policy for delineating such zones. There have been several programmes to look at the problem, in particular a tracer test programme entitled ‘Utilisation of tracer experiments for the development of rural water supply management strategies for secondary aquifers’ (van Wyk et al., 2001) which has studied the use of natural and artificial tracers together with protection zone delineation. However, this document only studies the impact on zone delineation of single fractures or fracture zones that dominate the flow system, rather than a fracture flow network. The following protection zones are suggested by van Wyk et al. (2001):

- Protection Zone I – the immediate fenced area around the abstraction well, which should be at least a 5 m radius;
- Protection Zone II – this zone is designed to protect the well from biological contaminants and should be taken as a minimum of 50 m radius, but should also encompass the area directly above any large fracture zone passing near or through the well, be it horizontal or vertical; and
- Protection Zone III – the catchment area of the borehole should be defined if there are “persistent hazardous non-degradable elements present”.

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For Protection Zone II the estimation of fracture area is taken from the analysis of the first minute of pumping test data. (In my opinion the drawdown curve at this early time would be purely from borehole storage effects rather than the hydraulic effects of a fracture zone, unless pumping rates were exceptionally high. The hydraulic effect of a large fracture is likely to kick in at times greater than one minute). Fracture transmissivity is estimated by the Logan approximation, and the fracture half-length is from pumping test data analysed through the technique given by Gringarten and Ramey (Krusemann and De Ridder, 1994) for vertical fractures.

The protection zones presented by van Wyk et al. (2001) are not based on travel-times, and therefore in this context the tracer testing is only used for determination of aquifer parameters, rather than for the partial verification of travel-time based zones.

4.3 AUSTRALIA

As part of the national water quality management strategy, a document was produced in September 1995 to help the States, Territories and Commonwealth in Australia to formulate their own specific groundwater protection policies with the same goals and objectives nationwide.

In Australia only 18% of all water used is derived from groundwater. However, for many people in more rural areas and for many agricultural and industrial areas groundwater is the sole water source, and hence very important. For example, almost one million people rely on groundwater for their drinking supply. Also many of the pastoral activities in Queensland and grape growing activities in Southern Australia could not occur if it were not for the availability of groundwater. There are some exceptions to this mainly rural groundwater use; for example, in Perth two thirds of all water used is derived from groundwater.

Control of water resources and water quality in Australia is a State and Territory responsibility with some of the funding being provided by national programmes through the Commonwealth Government. In the past, many States and Territories have developed reactive policies to deal with wilful or negligent acts of pollution, but these have not been devised using any systematic framework. The national guidelines
attempt to produce a more pro-active policy and to provide a systematic framework for groundwater protection policy.

At present in Australia there are a variety of State, regional and local government legislative powers that are being used to develop and implement groundwater protection plans. This legislation can be grouped under three broad headings:

- Groundwater Management;
- Land Use Planning; and
- Environment Protection.

Wellhead protection plans form part of the groundwater management section of legislation. The plan comprises four components: well integrity assurance, wellhead protection zones, monitoring systems and contamination sources/land-use control.

Wellhead protection zones (or Underground Water Pollution Control Area in Western Australia) are aimed at protecting the part of the groundwater flow system that contributes to the discharge of the well. The size and location of these zones needs to be designed on a site specific basis. Three zones are to be defined:

- the inner (zone I) 50m radius zone;
- the middle (zone II) zone defined by a 10 year residence time; and
- the outer (zone III) zone by a greater than 10 years residence time which is usually the catchment area.

Implementation of Policy
Contact has been made with some of the States and Territories, in particular Western Australia, New South Wales and South Australia to assess the extent to which protection zones have been defined and whether separate approaches are adopted for areas with fractured aquifers.

According to the Water and Rivers Commission (WRC), fractured rock sources are only used for very small country towns and populations more towards the centre of the continent (pers. comm. 1997, G. Prince and R. Sheridan, WRC). Protection zones have
been defined for these small sources using very basic approaches and equations. The protection zones are typically very large, reflecting the amount of information available and the fact that there is little competition for land. The zones have not been delineated using computer modelling. In Western Australia most protection zone delineation is done in non-fractured environments using MODFLOW (Dames & Moore, 1996).

4.4 CANADA

Groundwater supplies a large number of communities within Canada with drinking water. The reliance on groundwater varies considerably across Canada with 100% of the population of Prince Edward Island relying on groundwater and only 22% of the population in British Columbia. 63% of the population of New Brunswick rely on groundwater for supply (Craig and Chowdhury, 1995). 90% of rural Canadians rely on groundwater for supply.

Groundwater protection legislation exists at both a national level and also on a provincial level. Responsibilities for groundwater protection lie mainly at the provincial and municipal level. For example, the New Brunswick Clean Water Act states that water suppliers have a responsibility to provide a new or alternative supply should the existing one become contaminated. Section 14 of this Act can be used to prohibit any land use that might pose a threat to the public water supply. The Municipalities Act and the Community Planning Act control land use through zoning. Three zones are used in New Brunswick:

- Protection Area 1 (bacterial potential hazard area) – Either a 100 day travel-time zone in porous media or 250 day travel time in fractured media;
- Protection Area 2 (petroleum potential hazard area) – 5 year travel-time zone; and
- Protection Area 3 (persistent hazardous chemical potential hazard area) – 25 years travel-time zone.

Although bedrock aquifers are not uncommon in New Brunswick most protection areas are delineated using porous media models, such as Flowpath, as it is recognised as both costly and difficult to characterise fractured rocks (pers. comm. Ali Chowdhury, May 1997).
Ontario has extensive bedrock aquifers as well as near surface sand and gravel aquifers. In Ontario there have been many Wellhead Protection Areas designated over the last two years (Neufeld, 2001), for example in Oxford County. There is also a considerable concentration of research in the Municipality of Waterloo, principally because of the location of the University of Waterloo and the Engineering Department’s Water Resources Group at the Municipality Council. Most protection areas have been delineated using porous media approaches but it is recognised that in many bedrock cases this approach may not be valid (pers. comm. Eric Hodgins, May 1997) and fracture modelling techniques should be used.

There has found to be a paucity of information from the other Canadian states. In Nova Scotia no document had been published on delineating protection zones, but some had been delineated on an individual basis (pers. comm. David Briggins, N.S. Dept of the Environment, July 1997). In Newfoundland hydrogeological mapping and vulnerability mapping were used to delineate zones in bedrock aquifers. Typically only 5-50 houses are supported by each bedrock well, so funds were not available for a more advanced technique to be applied.

4.5 UNITED STATES

Groundwater, sourced from both wells and springs, supplies more than half the population of the United States with drinking water. More than 90% of the population in rural areas relies on groundwater for a drinking supply. In 1986 amendments to the Safe Water Drinking Act of 1974 required States to establish wellhead protection programmes to protect groundwater used for public supplies from possible contamination. This wellhead protection (WHP) programme is designed to protect the surface and underground areas through which contaminants could potentially pass before reaching a well. Specifically, the aim of the programme is to delineate wellhead protection areas for each public supply well, identify sources of contamination within these areas and develop management strategies to prevent contamination in wells.

As part of the WHP programme the US Environmental Protection Agency have developed a series of methodologies for delineating groundwater protection zones in different situations. They have developed a general methodology for unconfined porous aquifers, and have modified this to create two further documents, one for confined aquifers and the other for fractured rock aquifers. It is this last document on delineation
of protection zones in fractured rock that is of interest to this project. A critique of this document is presented in the next section along with an outline of their methodologies.

In 1996 the Safe Water Drinking Act was further amended by Congress, and required that States develop Source Water Assessment Plans (SWAPs) to investigate the areas serving as public sources of drinking water, in order to identify potential threats and initiate protection efforts (Farrelly, 2001). The US EPA guidance given on protection zone delineation methodologies does not appear to have changed subsequent to this.

The zones that are delineated vary from state to state, depending on their own interpretation of national guidelines and laws. For example, Ohio has produced guidelines for wellhead protection area delineation techniques (Goncalves, 1994) whereby the zones to be defined are as follows:

- Zone I is defined as the area encompassing a maximum 400-foot radius around the source (assuming a greater than 100,000 gpd withdrawal rate);
- Zone II is defined as the entire extent of the aquifer deposits which could fall within, and are upgradient from, the production well's capture zone based on the predicted drawdown after 180-day drought conditions at the approved pumping rate;
- Interim Wellhead Protection Area (IWPA) is defined as the area encompassing a ½ mile radius around a public supply well that does not have a delineated Zone II; and
- Zone III is defined as the entire watershed upgradient of Zone II.

In general this division of the zones does not vary much between states, for example, in Pennsylvania Zone I is also a radius of 400 feet, Zone II is the total capture zone and Zone III is the total contributing area which includes any waters from other sources, such as losing streams (Barton et al., 1999).

US EPA DOCUMENT - DELINEATION OF WELLHEAD PROTECTION AREAS IN FRACTURED ROCK: A CRITIQUE

The document on the delineation of groundwater protection zones in fractured rock was produced in June 1991 (US EPA, 1991). It is a follow up of the US EPA technical
The methods devised in this document are based around two case study areas, both in densely fractured rocks that are thought to behave as an equivalent uniform, porous media at the scale of the tests. One site is in Sevastopol, Door County and the other in Junction City, both of which are in Wisconsin. Suggestions for approaches to be adopted in fractured rock that does not behave as a porous medium are also made, as outlined below. However, the non-porous assumption methods are not tested.

**The Methodologies**

First of all the document describes the various parameters/criteria which may be considered for use in determining WHPAs. These are:

- distance;
- drawdown;
- time of travel;
- flow system boundaries; and
- assimilative capacity of the subsurface.

The text states that these criteria can be used by themselves or in combination to define the WHPA to define a protection zone in fractured rock. The methodology used will depend on how much data are available, the personnel and financial resources, and the complexity of the fractured rock setting. Six delineation methods were tested by the US EPA in this document based on the above criteria/parameters:

- arbitrary fixed radius;
- calculated fixed radius;
- vulnerability mapping;
- flow system mapping with a) time of travel criterion  b) analytical equations;
- residence time approach; and
- numerical flow/transport models.
**Arbitrary Fixed Radius**
This method defines the WHPA as extending to an arbitrary distance from the well, without regard for groundwater flow direction. As stated in the document this method does not incorporate any hydrogeological knowledge or contaminant transport considerations and is not recommended for use in fractured rock situations.

**Calculated Fixed Radius**
Analytical flow equations are applied to calculate a fixed radius, based on the flow rate of the well, kinematic porosity of the system and the thickness of the aquifer. This is clearly an improvement on the arbitrary fixed radius method as some hydrogeological knowledge is incorporated. However, heterogeneity, anisotropy, recharge and vertical components of flow will not be incorporated at this stage. The document recognises that this “may not give acceptable results in unconfined fractured rock settings”. WHPAs were defined at the two case study sites using the Theis non-equilibrium equation and the protection zones were found to be very large, encompassing an unacceptably large area. Having very large protection areas is not a feasible option in the UK because of high demands for land and the difficulties of implementation.

**Vulnerability Mapping**
Vulnerability mapping uses geological maps, soil maps, water table maps and aerial photographs to identify areas particularly vulnerable to groundwater contamination. These areas may include sink holes, permeable soils, exposed bedrock and a shallow water table. The method does not define protection zones for a given well, but it may be used in combination with other methods such as the method of variable shapes (US EPA, 1987). Mapping of areas vulnerable to groundwater contamination should be undertaken as part of the WHPA plan to collect data for use on other delineation methods. The advantages of the vulnerability mapping method are that it does not require detailed measurements of aquifer parameters or assumptions about the aquifer type, as well as relying on many different data sources. The disadvantages are that no direct protection zone is defined and the results are somewhat subjective.

**Flow System Mapping**
Flow system mapping identifies the hydrological and physical features and boundaries that control the groundwater flow system. Physical boundaries to the system will include the base of the aquifer, structural features such as impermeable faults and...
topographic features that may determine the position of groundwater divides. Rivers, streams and canals may also function as boundaries to the flow system. This mapping method assumes that these boundaries are constant over time. Many groundwater divides and ephemeral stream boundaries may not be permanent and therefore caution should be exercised in boundary definition. Careful checks should be made to see whether they are true system boundaries or whether the protection zone has to be made larger to account for boundary movements.

Once the flow system boundaries have been established, flow lines are drawn away from the well and extend up-gradient towards the groundwater divide/boundary. However, drawing flow lines away from the well assumes an isotropic aquifer, which in a fractured system is unlikely to be the case.

Flow system mapping can be used to define WHPAs in two ways:

a) With Time-of-Travel Calculations
Once the flow system boundaries have been established, the area contributing to the WHPA may have to be limited for practical reasons. This can be done by assuming a suitable time of travel to the well, for example, the time it takes for a particular chemical pollutant to decay by 50%. The velocity of the groundwater is then calculated from the aquifer parameters (hydraulic conductivity, porosity and average hydraulic gradient) and hence the distance to the edge of the protection zone determined. The advantages of this method are that once the flow system boundaries are defined, there is very little further work that needs to be done and the method requires only elementary mathematics. Also the size of the WHPA is limited as some flow system mapping results can be unacceptably large. The disadvantages are that the method assumes that the aquifer is uniform, two dimensional, homogeneous and isotropic. This is unlikely to be the case. Also, if large conductive flow zones are present then large errors could occur in the time of travel calculation.

Both the case study areas were used to test the methodology. One important point that came out of the Sevastopol case study was that the protection zones for the upper and lower aquifers must be combined, and it must be assumed that leakage from the upper aquifer to the lower aquifer can occur anywhere. The work at the Junction City site also emphasises the need for accurate hydraulic conductivity data, otherwise the variation in

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the possible size of zone will be large. Variations in hydraulic gradient, especially in the vicinity of the well, will cause large variations in velocity and hence large errors in the delineation of the time of travel zone.

b) With the Uniform Flow Equation

Instead of using Darcy's law directly to calculate groundwater velocity and hence the distance at which to apply the cut-off to the flow system, the uniform flow equation can be applied (Todd, 1980). However, this equation also assumes a uniform porous medium with an average value of hydraulic conductivity, porosity, hydraulic gradient and aquifer thickness.

The advantages of this method are that it accounts for some of the effects of pumping, and it is simple and requires very limited amounts of data. The disadvantages are that the method ignores aquifer heterogeneities, non-uniform recharge, and many types of hydrological boundary. The method can also produce unacceptably large WHPAs if the well to be protected is far from the groundwater divide. The method assumes a porous medium approach and a two-dimensional aquifer. Errors in the hydraulic conductivity, porosity and the hydraulic gradient can cause large errors in the protection zone delineation.

Residence Time Approach

The residence time approach uses water chemistry and isotope analysis to identify groundwater sources and flow rates. Geochemical parameters such as saturation indices and mineral concentrations can help to identify the source area of the groundwater. Environmental isotopes such as $^{18}$O can be used to establish the approximate age of water in the well. Tritium ($^3$H) can also be used to assess the age of the water, since tritium was released during the 1950s and 1960s nuclear weapons testing programme, and so all water recharging at that time was enriched with $^3$H. The text states that such chemical and isotopic analyses are useful for three reasons:

1. Relative age determinations can provide a check on travel-time estimates obtained by the hydraulic approaches above;
2. In areas where water produced from a well can be proved to be hundreds or thousands of years old, the protection zone may be so large that local wellhead protection zones may not be appropriate;
3. In areas where isotopic and geochemical signatures vary considerably from place to place, areas of low recharge and high recharge can be differentiated.

The advantages of this approach are that the porous media assumption and detailed knowledge of aquifer parameters are not necessary. The method also helps to confirm WHPAs defined by other methods. It gives information about relative groundwater ages, which can be useful in determining whether WHPA delineation is appropriate. The disadvantages of this method are that it requires much skill and experience in geochemical and isotopic interpretation and can be expensive. The results can also be ambiguous, not applicable in all settings, and do not directly produce a delineated protection zone.

Both the Junction City and Sevastopol sets of boreholes were tested for isotopic and geochemical signatures. The text claims that the results prove the sites can be modelled using an equivalent porous medium approach, although my critique presented in the next few sections considers the results to be at best ambiguous and at worst incorrectly interpreted.

**Numerical Flow/Transport Models**

The most time consuming and technically complex of the delineation methodologies suggested is the use of numerical models. These models simulate a groundwater flow system, with the potential to simulate contaminant transport. Modelling of the system involves discretising the area into nodes or cells in either two or three dimensions, with each of these nodes or cells having a particular hydraulic conductivity, porosity, recharge, pumping rate and aquifer thickness assigned to it. In this way the heterogeneous properties of the aquifer can be modelled. However, most of the models used assume the rock to behave as a porous medium, which is not necessarily valid in many fractured rock scenarios.

The advantages of numerical modelling are that groundwater flow paths and travel times can be determined with much greater precision than any of the other methods. Also the models can simulate the heterogeneities of the system, and in some cases can adequately model large, discrete fractures. The disadvantages are that the rock must behave as a porous medium at some scale, the modelling itself can be very expensive and time consuming, and much data are required for calibration and verification.
For the two case studies presented in the US EPA document, numerical modelling is assumed to be the preferred methodology for creating defensible protection zones. The data for the two sites were examined and the type of model that should be used to best represent the flow regime was assessed. The document lists eleven principal criteria (numbered [1] to [11] below) for assessing whether the fractured rock sites should be modelled as equivalent porous media or whether another methodology should be adopted.

The Case Study Criteria

It has been suggested (Long et al., 1982) that there are various theoretical criteria that can be used to determine whether fractured systems behave as porous media. These criteria are:

- high fracture density [1];
- apertures are constant rather than distributed [2];
- orientations are distributed rather than constant [3];
- larger sample sizes are tested [4].

Bradbury, Muldoon, Zaporozec and Levy who wrote this US EPA document came up with suggestions for more criteria that could be used to assess whether the porous media assumption is valid:

- pumping test responses:
  1) drawdown in OBHs increases linearly with increase in discharge at the pumping well [5];
  2) drawdown curves for OBHs in different directions from the ABH should have similar curves and should not have sharp inflections [6];
  3) drawdown cones should be circular or elliptical [7];

- Water table configuration should show a smooth continuous surface with no rapidly changing or anomalous water levels over time and space [8];

- Ratio of fracture scale to problem scale - the minimum dimension of the WHPA should be 100 times the size of the average fracture spacing [4];
- Hydraulic conductivity distribution - according to work completed by Bradbury and Rothschild (1985) hydraulic conductivity as estimated by slug tests and specific capacity data should follow a lognormal distribution [9];

- Variations in water and isotopic chemistry - if fractured systems are to be approximated as porous media systems then variations in water chemistry and isotopic data [10] and [11] should be small, both over time and space.

**Case Study 1**
The first case study area is near the village of Junction City, and lies on a Precambrian shield of metamorphic and igneous rocks in a relatively flat topographic region. Fractures are said to be numerous and concentrated in the upper portion of the aquifer. Surficial materials are present up to 50 feet deep, but are laterally discontinuous. A large quantity of field data was collected for analysis of this site including borehole logs, geophysical logs, pumping test analyses, chemical data, water level data and isotopic data. From all these data the authors conclude that the site can be adequately represented as a porous medium. However, of the eleven different criteria set out above that should be satisfied for the fractured system to be represented as a porous medium, either the data are insufficient for the criteria to be tested (or not given in the document) or the data show that the system should really be modelled using another method.

This thesis examines each criterion in turn for the Junction City data to assess the validity of using a porous medium approach for zone delineation.

1. **High fracture density.** The text states that the area has a high fracture density but does not give a detailed analysis. On page 25 it states that fracture spacings are irregular and range from a few tenths of an inch to several feet. One geological log is presented which indicates that relatively intense fracturing occurs down to 35 feet, however, there is no analysis. There is a temperature log for well JC23 which indicates that fracture density in terms of flowing fractures is very low with just one main water bearing fracture at 160 feet.

2. **Apertures are constant.** There are no aperture data given, although a value of $<0.01 \text{m}$ is mentioned (p. 25).
3. Orientations are distributed. Again there is no detailed fracture analysis and no orientation data. On page 25 it is stated that at depths of 130 to 160 feet below ground level fractures are frequently vertical but gives no other clue as to their orientation.

4. Ratio of fracture scale to problem scale. Since the average fracture spacing is unknown (but thought to be between a few tenths of an inch and several feet) the problem scale cannot be well defined, but may be from several feet to several hundred feet (100 times fracture spacing). This uncertainty in problem scale is further complicated by the fact that fractures in the Junction City area are thought to be typically less than thirty feet long (p.25). The problem scale would have to be at least several hundred feet otherwise a fracture of 30 feet would dominate the flow pattern across the area, and the fracture system could not be represented as a porous media. Fortunately, the length dimension of area modelled is very much larger than the several hundred feet needed to meet the criterion, and is more like several thousand feet.

5. Linear increase in drawdown with increase in Q in an OBH. Although a 24-hour pumping test was performed, the results of the test are not presented. Therefore no judgement can be made as to whether this criterion was met.

6. OBH drawdown curves similar and no sharp inflections. As discussed in point five, although a 24-hour pumping test was performed, the results of the test are not presented. Therefore no judgement can be made as to whether this criterion has been met.

7. Drawdown cones should be circular or elliptical. During the 24-hour pumping test eight observation wells were monitored. However, six of these wells are positioned in approximately the same direction from the pumping well, not giving a good spread of observation wells around the pumped well. The text claims that the drawdown cone is nearly elliptical, but with such a poor spread of observation wells, this is more a case of how the interpolation has been completed. Also in the immediate vicinity of the six OBHs, the cone of depression becomes much more
uneven indicating that the fractures are not evenly distributed enough to be represented by a porous media.

8. *Water table configuration.* The water table contour map presented does appear to be relatively complex for a small area (~5×7 km). For a relatively flat region of country the water table is considered to vary quite considerably in elevation. Also the water table configuration does have a 'stair'-like appearance as presented on page 17, which is characteristic of a fractured rock system rather than an equivalent porous medium. Temporal water table fluctuations are stated as being approximately six feet in most wells. However, the groundwater hydrograph variations are not linked with recharge events. The groundwater hydrographs are not presented and hence an indication as to whether variations in water level are gradual, and therefore more likely to be represented by a porous medium, or rapid, and therefore more likely to represent linkage with the surface via fracturing.

9. *Hydraulic conductivity distribution.* Many hydraulic conductivity measurements are presented in the document as well as a diagram of hydraulic conductivity variation across the area on a logarithmic scale. Although the hydraulic conductivity measurements do vary by several orders of magnitude, there is no evidence presented in the document that the data follow a lognormal distribution.

10. *Water chemistry.* The document indicates that the water quality in the upper fractured bedrock is relatively uniform and can therefore be represented using a porous media approach. However, on closer inspection of the results this view may not be the case. Considering all those boreholes that are screened in the same part of the aquifer, the variation in the standard water parameters such as temperature, conductivity and pH is quite considerable. For example, temperature ranges from 13°C in well JC24 to 7.5°C in wells JC01 and JC11 on the same day. Groundwater temperature variations are usually small within such a restricted area. This would indicate that water is coming from different sources and is not uniform throughout the system. The pH variation is similarly large with a range of 6.88 in well JC11 to 8.92 in well JC02. Although these values are still within the range of natural groundwaters, they are at either ends of the spectrum and this seems unusual for groundwater within such a small area. Dissolved oxygen shows a comparable variation. All these parameters are relatively easy to measure and the data should be
relatively accurate, therefore the variation in parameters is likely to be due to different sources of water. Hence it would be unsuitable to represent the fracture flow system by an equivalent porous media.

11. Isotopic data. The isotopic data for both $^{18}$O and $^3$H indicate that the water in the main part of the aquifer is relatively young (<35 years) and is fairly uniform across the area. Although this data would indicate that the water comes from a source of the same age, the chemical data suggests that different sources are involved. The explanation for this may be due to the prevalence of vertical fractures in the area which allow recharge of the same age into the system and the lack of horizontal connection between fractures. If the horizontal fractures are unconnected, groundwater in different fractures may take on different chemical characteristics.

Therefore since only one of the eleven criteria are directly met and four of the eleven are clearly not met, it should be concluded that the site should not be represented by an equivalent porous media. There does not appear to be sufficient data to pass judgement on the other six criteria.

Case Study 2
The second case study is based on the Door Peninsula near the town of Sevastopol where the aquifer is composed of fractured Silurian dolomite. The dolomite is reported to be exposed at the surface or covered by only a thin layer of soil. The dolomite is densely fractured with horizontal fractures occurring every one to ten feet, and vertical fractures forming a more or less regular pattern. Topography is moderate across the area and relief slopes very gently down in all directions from the site. Data for the Sevastopol site includes water elevations, fracture characterisation, borehole geophysics, pumping test data, water chemistry and isotopic data. From this data the authors conclude that the area can be adequately represented by an equivalent porous media approach.

Again, this thesis examines each criterion in turn for the Sevastopol data to test whether the criteria set out in the start of this document can be met to an acceptable degree for the site to be modelled using a porous media approach.
1. **High fracture density.** Evidence from aerial photographs over the county indicates a fracture spacing of 3-7 m. From the geophysical logs, it would appear that the interval between flowing fractures is considerably more than the 3-7 m measured by aerial photograph, at approximately 17 m.

2. **Apertures are constant.** Fracture apertures are said to range between a fraction of an inch to several inches. However, no tables of data are presented and even without them apertures will vary by more than an order of magnitude. It is also understood that some of the fractures are widened by solution and minor karst features are present in the area, and therefore channelling within the fractures is likely to be significant. Since flow rate is determined by the cubic law in fractured rock, then one order of magnitude difference in aperture results in three orders of magnitude difference in flow rate.

3. **Orientations are distributed.** Principal joint orientations were measured at 25°, 70° and 155°, with major visible fractures orientated at N30°E at the site. Therefore it would appear that there are three principal joint directions across the area which are only 40° - 45° apart. However, no stereograms are presented and therefore it is not possible to assess whether each fracture set has a wide distribution of orientations and hence whether the orientations are well distributed.

4. **Ratio of fracture scale to problem scale.** From the aerial photography it can be seen that the fractures appear to be variably spaced from 3-17 m, and hence the grid of the modelled system should be at least 1700 m in width/length. In fact the actual modelled grid is 50,000 ft (17 km) along its shortest side and so the ratio of fracture scale to problem scale is more like 1000 rather than 100, and hence suitable for modelling a fractured system.

5. **Linear increase in drawdown with increase in Q in an OBH.** Data from the pumping tests is not presented for the observation boreholes and hence no judgement can be made as to the whether this criteria is met.

6. **OBH drawdown curves similar and no sharp inflections.** Data from the pumping test is not presented for the observation boreholes and hence no judgement can be made as to whether this criterion is met.
7. **Drawdown cones should be circular or elliptical.** It is stated in the text that the aquifer is anisotropic both vertically and horizontally. Analysis of the pump test data appears to indicate that the ratio of $T_{xx}$ to $T_{yy}$ is 4.2 for the transmissivity ellipse. The authors do admit, however, that this result comes from the analysis of only two observation wells and therefore this ratio may not represent the true transmissivity ellipse (if it is an ellipse). It may be argued that there is no way of knowing whether the drawdown cones are elliptical or circular with only two observation wells (Kruseman and de Ridder, 1994, present a method from Neuman that allows this ellipse to be calculated but two of the wells must be pumped). Therefore no valid judgement can be made as to whether this criterion is met.

8. **Water table configuration.** The water table configuration for the Sevastopol site is very different in appearance to that of the Junction City site (p. 127). Both the water table in the upper and lower aquifers appear to be smooth with no sharp changes in gradient and no stepped appearance. However, consideration of the number of data points that have been used to form the plot must be taken into account. If there are not as many data points as for the Junction City groundwater table plot, then it is likely that the appearance will be much smoother. There is no information on the number of points used for either of the plots and therefore they must be taken at face value. Groundwater hydrographs for boreholes in the area indicate that there is direct connection between the surface and the groundwater as there is a very rapid response time in the piezometers to snowmelt. This type of response would not be expected in a porous medium. Therefore this criterion has been satisfied over space and partially over time.

9. **Hydraulic conductivity distribution.** The text indicates that the hydraulic conductivity varies by over four orders of magnitude and that the hydraulic conductivity distribution is lognormal. Therefore this criterion is met, although the actual data has not been seen and therefore cannot be critiqued.

10. **Water chemistry.** The document indicates that the water quality in both the upper and lower fractured aquifers is relatively uniform and can therefore be represented using a porous media approach. The variation in the standard water parameters such as temperature, conductivity and pH is quite considerable, although very much less
varied than at the Junction City site. For example, temperature ranges from 10.7°C in well H3A to 6.9°C in well J2C on the same day, however, the number of values at these outer values is small, with most lying in the 7-8°C range. The pH variation is much smaller than that at Junction City with a range of 6.90 in well MW3 to 7.99 in well H1A on the same day. Also the range of the ratio of Ca/Mg is small, from 1.83 in H1A to 2.10 in well MW6B on the same day. The small range in the water parameter values would tend to indicate that the groundwater sources for the boreholes are reasonably well connected and mixed, producing water of a similar type across the area.

11. Isotopic data. The isotopic data for both $^{18}$O and $^3$H indicate that the water in the main part of the aquifer is relatively young (<35 years) and is fairly uniform across the area. As the text indicates, the water is likely to come from the same source and is young due to the rapid recharge. The explanation for this may be due to the prevalence of vertical fractures in the area which allow recharge into the system.

Therefore, since four of the eleven criteria are directly met and four of the eleven are clearly not met, it should be concluded that the site can be represented by an equivalent porous media, although much caution would have to be exercised. There do not appear to be sufficient data to pass judgement on the other three criteria.

**General Methodology Critique**

The US EPA methodology document (op. cit.) only contains a few paragraphs on how to simulate conditions in fractured rocks that do not behave as porous media at any scale. The document suggests that a combination of vulnerability mapping and hydrogeological mapping should be used. The residence time approach is also suggested as being a valuable tool for confirming any results from the vulnerability and hydrogeological mapping.

The document does briefly recognise the fact that there are other ways in which fractured rock that does not behave as an equivalent porous medium can be modelled. At the time the document went to press, stochastic modelling, fractal representations and discrete fracture modelling were considered to be in their infancy. Therefore these type of models were deemed unsuitable for delineation of protection zones.
The document does attempt to bring in the site-specific fracture data by making an assessment as to whether the fracturing is dense and of varied orientations, and whether fracture apertures are of constant width. However, the document does not provide any comparable values for these parameters, for example, the frequency at which the fracturing becomes dense. Surely fracture density taken in isolation is not the key parameter, depending also on the scale of the model. For example, if there are two fractures per metre and the model area is only 2 m by 2 m then the fracturing would be sparse, but if the model area were 500 by 500 m then the fracturing may be considered dense. Also, how many different fracture sets do there have to be, and of what width of distribution, for the fracture distribution to be considered even? When fracture apertures have to be a constant size for the system to be modelled as an equivalent porous media, what is the range over which fracture apertures can be called constant? Variations in fracture aperture have a significant impact on flow because of the cubic law (i.e. the flow is proportional to the cube of the aperture and therefore any increase in aperture causes a large increase in flow).

The other criteria for differentiating between fractured systems that can be modelled using an equivalent porous media technique and other fracture system techniques could also be criticised. For example, the document states that the hydraulic conductivity distribution should be lognormal for a fractured system to be represented by a porous media approach. It has been thought that most natural systems exhibit lognormal behaviour for many of their characteristics. However, it has been pointed out recently that this may not be the case, and it is the way in which the characteristics are measured that produces the lognormal distribution, and that natural distributions should really follow a negative exponential form (Barton & LaPointe, 1995).

State Specific Examples
In areas that approximate to karstic conditions, it may not be appropriate to use travel time criteria for delineating protection zones. The zones produced are likely to be very large, and the degree of uncertainty of the zones may be so great as to make the zones unmanageable. Another method should be found for groundwater protection under these conditions. Hence the suitability of using travel time to delineate zones could be another criteria, although the exact conditions when the criteria would cease to apply would have to be investigated further. For example, the cut-off point could be when the definition of karst applies. Vulnerability mapping techniques may be more suitable for
defining protection zones, such as those used in the Swiss EPIK system (Doefliger and Zwahlen, 1998). Chin (1996) suggests a combination of vulnerability mapping and numerical modelling to delineate protection zones in karstic aquifers in Pennsylvania. Quinlan et al. (1995) suggest that only hydrogeological mapping combined with tracer testing produces more defensible protection zones in karstic aquifers, especially in relation to Kentucky, with the use of time of travel zones being “dopey and unrealistic”.

A US EPA document (US EPA, 1993) on wellhead protection in fractured and karst aquifer settings is less damning of the time of travel method, but states that it should only really be used for EPM situations. In most karstic aquifers a combination of hydrogeological mapping, tracer testing and lineament mapping should be used.

Some of the States of the US are such that the majority of their aquifers are fractured, such as Pennsylvania, Ohio and Kentucky. Pennsylvania has drawn up a strategy for delineating groundwater protection zones in fractured bedrock aquifers (Risser and Barton, 1995). It follows the US EPA document for delineating protection zones in fractured rock closely (US EPA, 1991). The report emphasizes the importance of obtaining the best conceptual model possible for an area before a protection zone delineation technique is chosen. Although the report recognises the many types of fractured rock and the ways in which they can be represented (discrete fracture modelling, stochastic modelling, continuum modelling), these techniques do not appear to have been used in either this report or other Pennsylvanian case study reports (Barton and Risser, 1991, Barton et al., 1999). The favoured modelling techniques are a mixture of flow system mapping with simple analytical time of travel equations, and equivalent porous medium techniques.

Ohio (Goncalves, 1994) recognises the importance of using the correct modelling technique for fractured rock situations, since the majority of Ohio is underlain by fractured sandstones and carbonates. Gonclaves (1994) follows the methods of the US EPA document for delineation of protection zones for porous media. However, delineation of zones in fracture flow settings are discussed. It is recommended that fracture characterisation of the area in question should be carried out to determine the appropriate type of zone delineation method. The use of a discrete fracture approach is considered, but it is rejected, as it is not practical in the majority of cases. It is stated that elongation of the protection area may be necessary in the direction of the dominant fracture set, but no details are given as to the magnitude of this elongation. The details
given on the continuum methodology are derived from the US EPA document on delineation of protection zones in fractured rock (US EPA, 1991). It is also recognised that in many areas hydrogeological mapping may be necessary. Within this technique tracer testing is recognised as being the most accurate way to determine flow paths in fractured rock. However, they are not recommended because of the potential risk of contamination to the community’s drinking water by the tracer.

4.6 CONCLUSIONS

All the countries reviewed have comprehensive groundwater protection zone policies, and are very much in concord with that of the UK regulatory authorities, most protection zones being based on travel-time and most zones are defined using numerical modelling. Also most countries recognise the fact that fractured rock aquifers require different delineation methods to porous media aquifers. However, no country has a methodology that deals specifically and comprehensively with fractured rock protection zones. Ideas from some countries, such as Switzerland, Germany, the US, and South Africa, on protection measures for fractured rock aquifers should be incorporated into any methodology proposed for UK aquifers, as shown in Table 4.3.

Table 4.3: Potential overseas groundwater protection technique to be incorporated into a UK methodology.

<table>
<thead>
<tr>
<th>Potential groundwater protection technique to be incorporated into a methodology for UK aquifers</th>
<th>Originating Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>The importance of tracer testing in validating protection zones</td>
<td>South Africa</td>
</tr>
<tr>
<td>The use of vulnerability classification in delineating protection zones for different fractured rock types, together with the extensive use of tracer testing</td>
<td>Germany and Switzerland</td>
</tr>
<tr>
<td>Criteria for identifying fractured rocks that behave as an EPM</td>
<td>United States</td>
</tr>
</tbody>
</table>

Ultimately, however, a methodology must be developed from first principles for UK aquifers.
5  FRACTURED ROCK MODELLING

Having determined from chapter 4 that no country reviewed has fully devised a methodology for protection zone delineation in fractured rock, we should attempt to develop our own methodology. The first stage of this should be to examine the fracture characteristics of a site and then establish an appropriate conceptual model. A method of numerical modelling should then be chosen that best suits the conceptual model, as well as having the capability to produce protection zones.

The structure of this chapter is based around the need for modelling at a particular site, assuming data for the site has already been collected and collated (see Chapter 6 for more detail on data collection and collation). The first section considers the different influences on the fractured rock system that lead to the determination of the conceptual model. The second section considers the different types of models that can be used to simulate fractured systems. The data requirements for more commonly used models are discussed, and then the final choice of the numerical model(s) to use. Alternatives to the use of fracture models are also presented.

5.1  Determination of the fractured rock conceptual model

Consideration of the rock type, the relative dominance of the fractures within the flow system, the fracture pattern type and the scale at which the fracture rock mass is to be studied should be made in order to build a comprehensive conceptual model of the fractured rock mass under review. If the groundwater flow is predominantly through the fractures then the effects may be seen in a number of standard field measurements – these effects are discussed in some detail, especially with reference to rocks that behave as an equivalent porous medium at some scale.

5.1.1  Rock type

In general, rock types will fall into certain categories that will lend themselves to particular types of models. This idea was presented in chapter 2 where different fractured rocks fall on a (multi-) dimensional spectrum of fracture types. Some rock types have a very low permeability and porosity matrix, so that transport processes within it are negligible, but significant flow can occur in the fractures. The sorts of rock types that would come into this category are, for example, the unweathered volcanics and metamorphics. In other rocks there will be significant flow within the matrix as
well as in the fractures. These types of rocks would include the Permo-Triassic sandstones. Other rock types are such that the matrix permeability is very low and no significant flow occurs within it, but there is significant matrix porosity and fracture flow. This results in some transport processes occurring in the matrix. The Chalk (Barker, 1993), some of the Triassic sandstones, and possibly the Lincolnshire Limestone (Lloyd et al., 1996) come into this category.

5.1.2 Relative dominance of fractures

In some situations the permeability provided by flow through the pores and flow through the fractures may be very different. The hydraulic significance of fractures will depend on aperture, degree of interconnection and on the matrix hydraulic conductivity. This relative dominance of fractures will determine the fracture model selection procedure. The Aquifer Properties Manual (Allen et al., 1997) gives good examples of the relative dominance of fracture and matrix flow in the Permo-Triassic sandstones. Faults within the Permo-Triassic vary from being recharge boundaries, for example the Topcliffe Fault in North Yorkshire, to impermeable barriers to flow, for example the Roaring Meg Fault in Merseyside.

As noted in a BGS report on fracturing and hydrogeology of the Permo-Triassic sandstones in England and Wales, the relative dominance of fractures can also depend on the scale of observation (Allen et al., 1998). In general, it is considered that at smaller scales fracture flow predominates, whereas at larger scales the flow and transport is matrix controlled. However, it has been noted that in some areas of the Permo-Triassic sandstone the intergranular permeability is so low that the transmissivity is derived solely from the fractures, even at large scales of observation. This is known to occur in parts of the St. Bees Sandstone in north Cumbria, as well as in parts of Yorkshire and Teeside. Little work on this subject appears to have been conducted on other aquifers.

Consideration of the form of the fractures themselves should feature in decisions regarding the conceptual model. The dominance of the fractures within the system depends on their connectivity and also on their aperture. If fractures are not connected then flow will be predominantly through the matrix. In many conceptual models fractures are assumed to be a parallel plate for simplicity and for ease of calculation in
modelling. Flow within fractures can be described by the “cubic law”, where the flow is related to the cube of the aperture:

\[ Q = \frac{\rho g b^3 dh}{12 \mu dl} \]

where:
- \( Q \) is the volumetric flow through the fracture per unit width
- \( g \) is the acceleration due to gravity
- \( dh/dl \) is the hydraulic gradient
- \( b \) is the fracture aperture
- \( \mu \) is the fluid viscosity
- \( \rho \) is the fluid density

Therefore, even a small increase in the aperture can increase the flow significantly and hence the dominance of the fractures in the flow system.

However, the cubic law has not been shown to be applicable in many cases, both in laboratory studies (for example, Pyrak Nolte et al., 1987) and from observations in the field (for example, Tsang et al., 1991) where flow is dominantly through only a few channels across the fracture surface. Characterisation of these channels is only possible on very small scales in the laboratory or by extremely detailed tracer testing experiments, such as those at Stripa. Whilst the concept of channelling of fluids within fractures could be included within a conceptual model, the details of these channels and the modelling of these channels would not be practical on a large scale.

The relative dominance of fractures in a system can also change over time. In some cases the length of time required for the fracture aperture system to develop will be great. However, locally, fractures may become enlarged within a relatively short time, for example, in the vicinity of chalk boreholes that have been acidised, or new pumping boreholes in relatively friable sandstones. The modelling of such growth and channelling within the fracture network by solution and mechanical processes has been studied extensively (Daccord, 1987, Liétard and Daccord, 1989). Such fracture aperture development should be taken into account when the conceptual model is produced, and hence in the ultimate model selection. It is recognised that if the ultimate model
selected contained this capability it would be complex, and perhaps too complex and impractical for a transparent, easy-to-use protection zone modelling methodology.

In order to assess the relative dominance of fractures in the system, all possible hydrogeological methods of testing should be used, for example: geophysical logging, hydraulic testing at the scale of interest; tracer testing; and fracture mapping. These techniques are discussed further in chapter 6.

5.1.3 Fracture pattern type
The fracture geometry of an area will also influence the choice of model. The fracture pattern type will depend on the rock itself and how it has responded to the tectonics of the region. At one end of the spectrum large faults may dominate the fracture system, with very little fracturing within the fault-bounded blocks. Or it may be that the faults are the major fractures in the system but that there are also conductive fractures within fault-bounded blocks. One such example is given by Allen et al. (1998) where many Permo-Triassic fracture systems can be characterised by a series of faults with bedding plane fracture intersecting them. Alternatively one fracture set only may dominate, such as the bedding plane fractures. At the other end of the spectrum there may be fracturing at all angles throughout the entire system.

5.1.4 Scale of the fractured rock problem
The modelling of fractured systems for the delineation of protection zones should be on a catchment scale, as a minimum. The relationship between the scale of the fracturing and the problem scale is crucial in building the conceptual model. If the fracturing is dense under the scale of investigation then it is possible that the system has definable average aquifer properties. However, if the fractures are sparse and fracture interconnections are spaced at the same scale as the problem scale it may be that no average aquifer properties can be defined, so each individual fracture should be represented to obtain the flow characteristics of the system. The scale of the problem is dealt with further in section 5.2 on the equivalent porous media (EPM) approach.

5.1.5 Effects on standard field measurements
The rock type, the relative dominance of the fractures within the system and the fracture pattern type all have an influence on standard field observations. The scale at which a field observation is taken also determines to what extent fracture flow is observed.
Consideration of the following standard field observations, taken mostly from the EPA (1991) document on delineation of protection zones in fractured rock, allows an assessment of the relative importance of fracture flow within an aquifer system, and hence a conceptual model to be developed.

- **Aquifer test responses in fractured rock**
  
  If fracture flow predominates the following observations may be seen during aquifer testing:
  
  1. drawdown in observation boreholes may not increase linearly with increase in discharge at the pumping well;
  2. drawdown curves for observation wells in different directions from the abstraction well will tend to be different and may have sharp slope changes/inflections; and
  3. drawdown cones are unlikely to be circular or elliptical.

It should be noted that, even in highly irregularly fractured areas with low connectivity, drawdown curves are often apparently smooth and show few signs of the underlying fracture geometry. Therefore pumping test responses should not be taken in isolation as a basis for the conceptual model. At the sites studied later in Chapter 8, most drawdown curves were smooth, however the areas were known to be highly fractured.

The relative dominance of fracture flow at difference scales of observation may be obtained from the early and late time data from an aquifer test. For example, early time data may show a fractured response as a single fracture dewatered, but at longer time scales a Theis response could be seen because the volume of rock tested will tend to the representative elementary volume (REV – see section 5.2.3 for details), hence giving an EPM response.

- **Water table configuration** may show a discontinuous surface with rapidly changing or anomalous water levels over time and space in fractured rock. This criterion implies that there is sufficiently dense information to assess whether the surface is smooth or not. The effect can often be seen if faulting in the area is acting as a barrier to flow, and the heads either side of the fault will be significantly
different. In most cases the density of information will not be high enough to make an informed decision.

- **Ratio of fracture scale to problem scale** – if the approximate minimum dimension of the capture zone is much less than 100 times the size of the average fracture spacing then fracture flow is likely to be a significant flow mechanism. From semi-analytical models and the existing Flowpath/Modflow Environment Agency zones (providing they are of the correct order of magnitude) a guide to the dimension of the capture zone should be attainable. Fracture spacing can be obtained by many methods (see Chapter 6), but the spacing should be in the direction of the minimum dimension of the capture zone.

- **Hydraulic conductivity distribution** - according to work completed by Bradbury and Rothschild (1985), hydraulic conductivity, as estimated by slug tests and specific capacity data, is likely to follow a bimodal distribution if the aquifer is highly fractured. If the aquifer can be approximated as an EPM then a lognormal distribution of hydraulic conductivity values tends to be found.

- **Variations in water and isotopic chemistry.** If a very dense, highly-connected fracture system is present, variations in water chemistry and isotopic data should be small, both over time and space. Differences may appear in a less densely fractured system because:
  
a) wells sampling different fractures close to each other may not be connected and hence have different sources of water; and

b) some water will travel relatively quickly through the fractures and some will travel relatively slowly through the matrix, hence acquiring differing chemical signatures, being at different degrees of chemical equilibrium with the rock.

- **Geophysical logging responses** – downhole geophysical logging techniques have the capability to detect non-flowing and flowing fractures. Using techniques such as flow meter logging, differential temperature and differential conductivity logging flowing fractures can be identified. Caliper, digital CCTV and acoustic televiewer logging may also allow non-flowing fractures to be detected. The magnitude of the
sonde responses may indicate the relative dominance of fracture and matrix flow within the system.

- **Outcrop analysis** – simple outcrop observations should yield some information on the extent and pattern of fracturing, although it should not be forgotten that weathering of outcrop can change its fracture properties significantly.

5.2 Fracture modelling techniques

After a conceptual model for a fractured rock site has been developed the next decision is on which type of numerical fracture model to use to best represent the conceptual model. There are an enormous variety of numerical fracture model types. Choosing the appropriate numerical model to match the conceptual model requires an understanding of the numerical model types and their capabilities and limitations. This section examines various aspects of numerical modelling in relation to their use in fractured rock situations.

5.2.1 Transfer functions

A transfer function allows the response of a system to any set of input parameters to be predicted. The transfer function is developed from knowledge of which input parameters gives a particular set of output parameters, and so it can then be used to predict output from different input parameters. The function can be said to represent a 'black box' where the function inside the box can transform the input to the output. However, these black box functions do not necessarily have parameters that are linked directly with parameters that have a physical meaning. One of the principal aims of the GPZ modelling methodology to be developed is defensibility. Parameters that have no physical basis would not provide a defensible methodology. This type of modelling is therefore not considered further.

5.2.2 2-D versus a 3-D approach

Depending on the fractured rock conceptual model that has been developed, there may be a decision to be made on the dimension of the model to be used. In most cases a 3-D approach will be more appropriate since fractures tend to connect at different depths and flow paths are not simply horizontal but zigzag in three dimensions. However, in some systems, such as those dominated by horizontal bedding plane fractures that are almost continuous, it may be possible to use a 2-D model to represent the system without
significant alteration to flow characteristics and parameters. Consideration as to how the water table is linked to the surface is also important. If the linkage is provided by fractures at a low angle and the water table is relatively deep then there could be a significant impact on the size and shape of the protection zones (Bradbury and Rayne, 1998), and it becomes even more important that 3-D modelling is undertaken.

5.2.3 Discrete versus a continuum approach

A discrete approach to modelling attempts to represent all of the individual features of the system. In fractured rock modelling this means that all fractures and their individual properties are represented. Discrete models or fracture network models are capable of capturing the extreme heterogeneity of a system, allowing the variability in flow patterns to be simulated. In discrete models such as Fracman (Dershowitz et al., 1998), or SDF (Rouleau, 1988) the fracture networks can be created using a number of assumptions on the fracture shape and the distribution of fractures, for example, the fractures can be represented by polygons, ellipses or circles. Also there are numerous methods for generating different distributions of fractures depending on the conceptual model, for example in Fracman the ‘enhanced Baecher model’ allows terminations of fractures at intersections or the ‘war zone model’ imitates the geometry of shear zones.

If the networks to be simulated are very large (more than a few hundred metres) then the computational effort required to create the matrices of fracture linkages and solve the flow equations tends to become prohibitive. Therefore, to simulate flow in large fractured rock systems, continuum approaches are often used.

A continuum approach to modelling assumes that properties of a medium can be represented by effective or average values at some scale. In fractured rock this approach assumes that the combined hydraulic effect of fracture and matrix can be represented by a continuum. The effective flow parameters can be derived from the concept of either the representative elementary volume or random functions. There are two principal types of continuum models:

1. the equivalent porous media model; and
2. the multi-porosity model.
The concept of a representative elementary volume and a random function is dealt with next and then the two principal types of continuum model are discussed.

**Representative elementary volume or random function?**

There are two ways in which the local properties of a medium can be defined: using a representative elementary volume approach, and using a random function approach.

The representative elementary volume (REV) is a certain volume of material for which there can be a ‘mean’ property defined for the parameter in question. De Marsily (1986) defines the size of the REV as:

- Sufficiently large that the great number of pores or fractures allow us to define a mean global property.
- Sufficiently small so that the parameter variations from one domain to the next may be approximated by continuous functions, in order that infinitesimal calculus can be used, without introducing any error that may be picked up by the measuring instruments at the macroscopic scale.

For a medium where a REV is definable, the variation the average parameter with increase in volume or length scale may approximate that shown in Figure 5.1.

![Figure 5.1: Definition of the REV (after De Marsily, 1986).](image)

The size of an REV always stays arbitrary, as there is nothing to say that the increase in size will ensure the stabilising of a given hydraulic parameter. The size of the REV in a fractured medium may be enormous, or even indefinable. For example, if the fractured system had a fractal structure an REV would be indefinable. Such fractal systems are reported by Gillespie *et al.* (1993) who found that faults tend to have fractal qualities.
and Odling et al. (1999) who report that, in general, fracture sizes are fractal in non-stratabound systems, such as deeper volcanic and metamorphic rocks (see Chapter 6 for more details). Another example is dispersion, which has scale dependent properties, and an underlying fractal geological structure can be invoked to explain this phenomenon (Ross, 1986).

Use of the REV method has drawbacks in the definition of the mean properties of a medium in a given volume, with the REV concept not allowing us to study the structure of the property in space.

A more powerful alternative to the REV approach is the random function (RF) approach, which is a stochastic approach (see section 5.2.4) that deals in terms of ensemble averages rather than the spatial averages of the REV technique.

**Equivalent porous media (EPM) models**

An equivalent porous medium approach assumes that both the fractures and the matrix can be represented by one porosity. If an EPM approach is to be used because the size of the block to be modelled is greater than the REV then the equivalent flow parameters, hydraulic conductivity and porosity, need to be defined. The hydraulic conductivity and porosity of the EPM system will be different to that of the individual fractures and matrix. Long et al. (1982) state that for a heterogeneous system to be replaced by an equivalent homogeneous system for the purposes of analysis the following criteria must be met:

> 1. *There is an insignificant change in the value of the equivalent permeability with a small addition or subtraction to the test volume.*
> 2. *An equivalent symmetric permeability tensor exists which predicts the correct flux when direction of gradient in the REV is changed."

For the equivalent porous medium approach to be applicable the size of the grid cells used in the numerical model should be larger than the REV.

Whether an EPM approach is applicable for modelling purposes can be assessed from standard field observations, as discussed in the section 4.5 critique of the USEPA 1991 document on the delineation of protection zones in fractured rock.
Multiporosity models

Some of the earliest papers on fractured rock modelling were based around double porosity or multiporosity models in the 1960s (Sahimi, 1993). Double porosity models are such that a network of connected fractures (one porosity) is embedded within a porous matrix (the second porosity). Flow occurs only in the fractures and not the matrix (as distinct from double permeability systems where flow occurs in both the fractures and the matrix). Since the 1960s other models have been developed to extend from two to three porosities.

Barker (1985) gave a general mathematical formulation of the double-porosity model. Other numerical double porosity models, such as that of Bibby (1981), which dealt with slab shaped matrix blocks, are numerical formulations of that equation. Groundwater protection zone delineation in double porosity aquifers has been discussed in Robinson and Barker (2000a), but is not discussed further in this thesis.

5.2.4 Deterministic versus a stochastic approach

A deterministic approach to modelling treats the hydraulic and fracture parameters as having exact values. One argument for using this approach could be that all the fractured rock properties do have exact values since rocks tend to be rigid and permanent under the timescales usually considered. The obvious counter-argument to this is that the exact values of the parameters can never be known since exhaustive field-testing and analysis would have to be undertaken. Such intensive field-testing, usually by intrusive methods, will ultimately change the properties of the rock because the number of boreholes drilled may cause flow patterns to change and introduce foreign materials into the ground.

A stochastic approach to modelling treats the hydraulic and fracture parameters as having a distribution of values. The approach recognises that the parameters are highly variable due to the complexity of the structures and sediments involved, as well as there being considerable uncertainty in the characterisation of these complexities.

Gelhar (1993) views the question as to whether the groundwater system is a stochastic or a deterministic one as being a philosophical one, with no real answer. In principle
the world is a deterministic place and with exact knowledge of boundary conditions and
governing equations any action can be accurately predicted, for example the flip of a
coin. But, in practice the world is not like that and we can never measure and know
everything (and if we did measure everything then we would probably change its initial
state anyway) and hence processes are random. However, Gelhar recognises that
whether a stochastic or a deterministic approach is used is essentially a pragmatic one
rather than a philosophical one, and depends on the specific application, the field
conditions, and the quality and quantity of data available. However, the results from
both methods are generally complementary (Gelhar, 1993).

The deterministic and stochastic approach as applied to groundwater protection
zones

Many capture zone delineation exercises have been conducted using a deterministic
approach. To a certain extent, this approach can be used to deal with parameter
uncertainty by varying each of the hydrogeological parameters in turn and assessing the
change in the resulting protection zone. Bhatt (1993) and Evers and Lerner (1995,
1998) use this approach.

However, the deterministic approach does not deal with the fact that much of the
uncertainty arises from an imprecise knowledge of parameter values, and in fractured
rock these uncertainties are normally large. Conventional deterministic methods tend to
show just one capture zone, rather than a range of equally likely capture zones from a
range of equally likely input parameters. To present just one capture zone using the
deterministic approach, or even the combination of a few capture zones from a small
range of input parameters, gives a false impression as to the true uncertainty in capture
zone delineation. A rigorous uncertainty analysis is not possible using this deterministic
technique.

Stochastic approaches to protection zone delineation have been used often in the
literature, for example, Bair et al., 1991, Bradbury and Muldoon, 1993, Kinzelbach et
al., 1998, 1999 and van Leeuwen 2000. Many of these protection zone delineation
studies have relied on spatial correlation data within the stochastic analysis, data that are
only available in extremely rare cases, if at all. We should remember that the aim of
this research is to provide a practical model tool based on data that are commonly
available. One of the other principal aims is also to produce a methodology that deals with uncertainty in the conceptual model and the aquifer parameters in a rigorous manner. A stochastic approach appears to offer the most potential to provide this rigorous uncertainty analysis. These stochastic modelling methods are examined in more detail below.

**Basic stochastic concepts**

As mentioned in section 5.2.3 there are two methods of defining the local properties of a medium:

- Using the representative elementary volume (REV) concept
- Using the random functions method

The REV concept has been discussed in section 5.2.3.

The random functions (RF) method is a powerful tool that allows us to study the local properties of a medium. The concept states that the studied medium is a realization of a random process. The property, \(Z(x, \xi)\), is a random function if it varies both with the spatial coordinate system, \(x\), and with the state variable, \(\xi\), in the ensemble of realizations (\(\xi\) is usually omitted in notation) (De Marsily, 1986). The immense advantage of this stochastic approach is that all statistical properties of the medium can be studied through the ensemble of realizations. For example, the average of a given property over many realizations is known as the ensemble average.

In order to understand further the nature of the random medium approach two concepts should be defined: stationarity and ergodicity. A stationary process is one in which the probabilistic descriptions become independent of the time or space origin (Gelhar, 1993). This means that there is no systematic trend to the variability of a parameter across a region at different times or over space. Thus to obtain an average and standard deviation of the parameter, realizations from different times and positions can be used to compute the values. All moments of the distribution are invariant under translation. In practice it is often just the first two moments that are estimated, and the assumption of stationarity is based only on this. This type of stationarity is termed weak stationarity or second-order stationarity.
A non-stationary process is one in which the probabilistic descriptions are dependent on time or space. To obtain an average for a non-stationary process it would not make sense to use data from different times or locations, since the variability in the parameter changes. The parameter distributions will exhibit drift of some sort or there is some systematic trend in space.

In order to analyse data from a single realization (as is the case with the one realization of reality) it is often assumed that expected values or ensemble averages can be replaced by appropriate time or space averages. This assumption is referred to as the ergodic hypothesis. For example, the statistics of a random process can either be derived from repeated sampling of an ensemble of realizations from statistically equivalent media, or from samples collected at different locations within a single realization. In practice to adopt the ergodic hypothesis is the only option that is open to us since we have to assume there is sufficient information contained in the one realization of the real world to characterize the process statistically.

**Solutions to stochastic equations**

In a stochastic approach aquifer properties are regarded as random variables with known distributions. The outputs of this approach are characterized by a full probability density function. There are, in general, no exact solutions of the stochastic differential equations with random coefficients in groundwater flow problems, because the equations are non-linear. However, there are several methods by which an approximate solution to the equations can be obtained, for example:

- Spectral analysis. If the stochastic process is stationary then it is possible to represent the stochastic differential equation by the analogous Fourier series and Fourier integrals for deterministic functions. The spectral method provides an efficient method to solve the stochastic differential equations, even for multi-dimensional systems. The spectral approach is usually used to treat time variant problems and problems with spatial variability.

- Direct solution approach via a response function. This is more complex than the spectral method but non-stationary situations can be treated, as well as the effect of conditioning.
Solution of ordinary differential equations describing the covariance functions. The properties of the mean square derivative of a random process can be expressed entirely in terms of the second moment properties, that is, by the covariance function. If the mean square derivative and its integral exist then the rules of calculus can be applied for their solution.

Perturbation techniques. The coefficients of the differential equation are random when representing the random input variables. These coefficients can be decomposed into a mean and a perturbation about that mean. There are several approaches for making the solution of these equations simpler, such as when the perturbation is small compared with the mean, and if most of the terms in the equations follow a Gaussian distribution.

Monte Carlo simulations. The Monte Carlo approach does not actually attempt to solve the stochastic differential equations, but computes deterministic solutions for a large number of realisations and then statistically analyses the ensemble of realisations to give means, variances and sometimes probability density functions. There are three drawbacks with the Monte Carlo system, which result in it being inexact. The first is that the statistics derived are based on a finite number of realisations and therefore have an element of uncertainty in them. The second is that the process of generating the random field is inherently an approximate one. Thirdly the flow solution of the random field may only be approximate because of, for example, the large variation in hydraulic conductivity in adjacent nodes or cells.

Although the Monte Carlo technique is not exact in its solution of the stochastic equation, it is a technique with which many hydrogeologists are familiar. Since one of the aims of any new methodology is to be transparent and easy to understand, the use of the Monte Carlo method gives the most promise for GPZ delineation in stochastic systems.

5.3 Alternatives to fracture modelling – ‘correction’ factors

In England and Wales protection zones in fractured rock have already been delineated using numerical models. Due to limitations in software availability, technical expertise and data, there has been infrequent use of fracture flow numerical models. Most protection zones have been modelled using the porous media models Modflow and Flowpath. Since so much time and money has been spent on the existing source.
protection zones, an assessment is made here as to whether it is possible to adjust the existing zones in some way to take account of the fracture properties in that area, or to find a simple, yet improved, method of remodelling the zones. These ideas have not been considered in any previous literature. Two potential approaches are examined in this thesis; these involve the application of what will be referred to as:

1. A 'fuzzy factor'; and
2. An effective dispersion coefficient, respectively.

An effective dispersion coefficient, $\alpha_{\text{eff}}$, would be used in exactly the same way as a dispersion coefficient in the advection-dispersion equation. During reverse particle tracking modelling exercises from an abstraction well this would have the effect of producing a distribution of particles, allowing probabilistic protection zones to be defined. The effective dispersion coefficient would be defined from many particle-tracking exercises in a stochastic fracture model, with the value of effective dispersion coefficient depending on the fracture network characteristics. A brief review of dispersion in fracture networks can be found in section 5.3.2 and a more detailed analysis can be found in section 8.3.1.

The 'fuzzy factor' is envisaged to act like a distribution of distance superimposed onto the existing zone to obtain the different probability contours. One way in which this might work would be to have a different fuzzy factor (or distribution of distance) for each travel-time zone, superimposed on the existing zones. Since fuzzy factors are already based on percentages it would be a simple superposition into the existing zones to obtain the final probability contours.

5.3.1 'Fuzzy factors'

The first few paragraphs of this section describe fuzzy theory (Klir and Yuan, 1995) and its background in groundwater flow modelling. The second part of this section outlines how this theory might be used in the production of probability-based protection zones.

The traditional 'crisp' set theory is based on the Aristotelian two-valued logic where, if A is the set and x is a relevant object, then x is either a member of A or not a member of A. However, the state where “x is a member of A” may not necessarily be true (1) or false (0) and may only be a member of A to some degree (say 0.4, for example).
capability of fuzzy sets to express gradual transitions from membership to non-membership is a powerful tool. It allows vague concepts in natural language to be expressed, as well as measurement of uncertainties. For example, take the graduation in weather between sunny and cloudy. At 100% cloud the weather is obviously cloudy and at 0% it is obviously sunny. To most people a cloud cover of 10 or 20% would still be sunny, but where does one draw the line? If say at 25% cloud cover it is no longer sunny, then what classification does 24% cloud cover come under as it is clearly not significantly different from the 25%? A difference of 1% cannot clearly make the difference between sunny and cloudy. Hence it makes more sense is if we adopt a different approach to the problem which is not based on crisp set theory.

The fuzzy approach says that each individual is assigned a degree or grade of membership to the set. For example 26% cloud cover might have a membership to the set of sunny of 0.5 (arbitrary in this case – it depends on the membership function). Membership functions describe the degree to which a concept or value belongs to a set. These membership functions may take many forms as shown in Figure 5.2.

In groundwater modelling fuzzy theory could be used to describe parameter uncertainty/vagueness in a non-probabilistic framework. Two uses of fuzzy theory are considered here, within the framework of fractured rock groundwater protection zone modelling:

1. Membership functions or ‘fuzzy factors’ could be used to describe the fracture and aquifer parameters, which are then used as input to the finite difference groundwater flow equations. Solution of these equations results in heads which have an associated membership function, which will give fuzzy protection zones; and
2. A membership function or ‘fuzzy factor’ could be applied to an existing protection zone based on the overall expert knowledge of the area.

1. **Use in aquifer parameter description**

Membership functions could be used to describe fracture and aquifer parameters. This would facilitate the use of very uncertain and subjective information, such as the opinion of experts as well as the hard data that already exist. The question arises as to what is the meaning of a membership function for an aquifer/fracture parameter, since it...
does not follow the same argument as the sunny/cloudy example earlier. The membership function for transmissivity, for example, would be based around the set of $T$ values that are near, say, 200 m$^2$/day. Some experts may regard the transmissivity as being between 100 and 300 m$^2$/day but are unwilling to say that it is precisely 200 m$^2$/day. Pumping test analyses may give values of 150 and 250 m$^2$/day. Therefore a membership function could be defined that approximates, for example, the form of $A_1(x)$ in Figure 5.2. At first sight this type of membership function seems little different to a frequency distribution that would be used in a standard probabilistic analysis of groundwater flow. However, these membership functions can be used in a fuzzy solution to the groundwater flow equations, as opposed to the solution of the partial different stochastic equations using perturbation techniques, spectral analysis or Monte Carlo analysis. The advantage of the fuzzy approach is that solution of the flow equations are relatively simple and vast numbers of realisations do not have to be used to obtain an output, as with the Monte Carlo stochastic approach.

![Figure 5.2: Examples of membership functions that may be used in different contexts for characterising fuzzy sets of real numbers close to 2.](image)

Only two papers have been found in the literature that relate to fuzzy theory and groundwater flow modelling. There are several methods of solving the finite difference
set of equations when modelling groundwater flow using fuzzy parameters. The first is by using interval arithmetic (Appendix C in Dou et al., 1995) in an iterative algorithm, and the second by using a groundwater model operator method (Woldt et al., 1996). Dou et al. (1995) indicate that the iterative method is the preferred solution option for membership functions that are relatively narrow, but they do not define narrow.

The groundwater modelling operator method of Woldt et al. (1996) relies on the operations being performed between various ‘α-levels’ or ‘α cuts’ which are represented by intervals of membership, for example, between 25% and 30% ‘sunny’ which would be equivalent to α-levels of 0.25 and 0.30. The linear fuzzy equations are essentially a system of linear interval equations for a specified membership level. At each α-level cut a search is performed to find the solution for the hydraulic heads to find the complete range of heads for each node on the grid and hence the upper and lower bound of the unknown heads in the system. Once the head intervals for all the α-level cuts have been completed they generate a membership function for the head. The resulting width of the head interval at an α-level of 0.0 (the base width) can be viewed as the uncertainty associated with the head as a result of the uncertainty in the input parameters.

Both papers that have been reviewed dealing with fuzzy logic and groundwater modelling have used membership functions that are similar to the form of $A_1(x)$ (Dou et al., 1995 and Woldt et al., 1996). Intuitively, the greater the fuzziness of the input parameters the greater the fuzziness of the output hydraulic head, and this indeed proved to be the case in these papers. The Dou et al. (1995) paper fully examines the effect of different forms of membership function applied to the groundwater parameters on the resulting head variation and its corresponding membership function. It was found that if a non-symmetric transmissivity membership function were used the resulting head variation was symmetrical. Also, if symmetrical and non-symmetrical functions of the same base width were used, the output hydraulic head base width was greater for the symmetrical membership function.

Although consideration as to how to define the aquifer/fracture membership functions for input to groundwater flow equations could be made here, it is considered that this use of fuzzy theory is not suitable for our purposes.
2. Use in ‘fuzzy factor’ description

Membership functions could, however, be used directly as a factor to be combined with existing travel-time and catchment zones. If fuzzy factors were used to produce protection zones by applying a particular membership function to the zone itself, it may be because there is not enough data or knowledge about an area to apply the dispersivity factor option (see section 5.3.2 below).

If a membership function were to be used as a correction factor, the meaning of the membership function should be understood. For the catchment zone, a location could be described as being inside it or outside it, or in an area where there is some doubt as to exactly where the capture zone is positioned, so a membership function could be defined which could be of the form of \( A_4(x) \). The same type of membership function could extend to the 400-day and 50-day travel-time zones.

The membership function would be defined through expert opinion, i.e. whether a given location is considered likely to be inside or outside a particular protection zone. The form of the membership function is likely to reflect the rock type and also the degree and nature of fracturing within an area. For example some double porosity rocks are likely to show less uncertainty than a semi-karstic rock, and therefore each rock type could have a certain base width of membership function. It may be that the shape of the function type (\( A_1 \) to \( A_4 \), for example) would always remain the same, but the base width would change. These ideas are however only hypotheses, and more work would have to be done if this option is to be taken further. Although the use of fuzzy set theory is principally used to make decisions in areas where there is very low-level information, it is possible that there are more defensible options available to us since the membership functions may not be based on physically based parameters but on expert judgement only.

As stated previously, a methodology that is developed for delineating groundwater protection zones in fractured rock should preferably be based around techniques that are more widely used and understood, in order that the methodology is transparent. Fuzzy theory is not widely understood and even in the literature there are few examples of its use in groundwater modelling.
5.3.2 Effective dispersion coefficients

Protection zones could be created by reverse particle tracking within a porous media model using the advection-dispersion equation with effective dispersion coefficients. The use of an effective dispersion coefficient to represent uncertainty in protection zone modelling has been discussed by Neupauer and Wilson (2000, 1999). During reverse particle tracking modelling exercises from an abstraction well this would have the effect of producing a distribution of particles, allowing probabilistic protection zones to be defined. The probability is computed using only one reverse track simulation, using the advection dispersion equation for a conservative contaminant. The computational burden is thus greatly reduced compared with stochastic modelling to obtain probabilities of particle capture in a given time.

The effective dispersion coefficients could be based on the characteristics of the fracture network of the area. A series of effective dispersion coefficients could be developed for different fractured rock areas by undertaking multiple particle tracking exercises, using a stochastic fracture model to determine the dispersion characteristics of the fracture network in each direction. These effective dispersion coefficients could then be used in a porous media model with the advection-dispersion equation to produce probabilistic zones. It is, however, possible that effective dispersion coefficients will only be definable for certain fracture networks of high connectivity and fracture density.

Dispersion in fracture networks has been studied by several authors, including Berkowitz and Braester (1991), Gelhar (1993) and Lee et al. (1994). Berkowitz and Braester (1991) and Lee et al. (1994) examine dispersion in fracture networks at or close to the percolation threshold, and below the REV of the fracture system. Both these papers look at the dispersion characteristics of large numbers of particles travelling through many realisations of stochastically generated fracture networks. They found that the dispersion in these networks was scale dependent, with a power law relationship between the average square of the distance travelled by the particles and the average time taken to reach that location.

Lee et al. (1994) also present histograms of particle locations parallel and perpendicular to the hydraulic gradient for two different fracture networks of differing connectivities. Although qualitative descriptions of these histograms were made, no qualitative analysis
was done to characterise the longitudinal and transverse dispersion. They did note, however, that as the connectivity of the fracture system increased the particle dispersion became more Gaussian in form, whereas at lower connectivities the particles were dispersed in a multi-modal manner because of the limited number of pathways. The work of Lee et al. (1994) is interesting in that it hints at the problems of defining dispersivity for fracture systems of low connectivity.

Gelhar (1993) describes the 1984 PhD work of P. C. Robinson on dispersion in fracture networks. The scale dependency of longitudinal dispersion with distance by the particles in the network was noted. This work has been extended to examine longitudinal and transverse dispersion in fracture networks, and is presented in section 8.3.1.

Because of the possibility of scale dependent dispersion, different effective dispersion coefficients may be required for the different travel-time zones. The effective dispersion coefficient may be difficult to define for catchment zones unless the dispersion coefficients tend to a constant value at large times/distances, as catchment scale fracture models would be required to determine the effective dispersion coefficients. At present catchment scale fracture models are beyond standard computer technology.

In conclusion, it is thought that the determination of effective dispersion coefficients for a fractured system can only be achieved after a full, validated, numerical fracture modelling exercise of the area is undertaken. Once this full modelling exercise has been conducted the protection zones could be obtained straight away and therefore the determination of an effective dispersion coefficient would be futile. However, if after many such modelling exercises were conducted in the same area it was found that a particular value of the effective dispersion coefficient was representative of that area, the value could be used in any subsequent zone remodelling within that area.

5.4 Data requirements

Integral in the choice of model to be used to simulate flow in a fractured system is the amount of data available as compared with the data requirements of the model. Different models require different parameters for fracture flow simulations and also different amounts of data.
For example, 2-D EPM models require aquifer thickness, hydraulic conductivity and kinematic porosity. These values may be obtained from pump tests, tracer tests and geological logs. However, if a fractured rock is being represented through the porous medium assumption, then the permeability that provides the best model calibration will be an effective permeability and not necessarily close to hydraulic conductivity values derived from pumping tests (if the pumping test samples a volume of rock less than the REV then the hydraulic conductivity could be significantly different to that of the REV value).

For double-porosity models, parameters such as the relative porosities of the matrix and fractures, and the dimensions and shapes of the aquifer blocks are required. The porosity of matrix and fractures may be obtained from core analysis, tracer testing and some pumping test analyses, but the accuracy of the values obtained and their true meaning will always be a subject for debate. For double-permeability models, the shape and relative dimensions of the matrix block will also be needed along with the relative permeabilities of the matrix and fractures.

For discrete-fracture models much more information is needed. The orientations, dips, density, aperture and the trace lengths of all the fracture sets are required as well as their exact positions, assuming a deterministic model. If a stochastic model is required then distributions of the fracture parameters will be needed. For protection zone modelling it is more likely that a stochastic discrete fracture model will be used to obtain all the possibilities for particle tracking. These data will be difficult to collect unless a study is performed specifically for that purpose.

The final model choice therefore should depend on the amount of data available. If enough data are available then a fracture model should be chosen. If, however, the amount of data is small and there is no possibility of deriving data from other sources (see Chapter 6) then it is probably more prudent to delineate the protection zones through analytical methods or the flow mapping approach. However, for the purposes of this part of the exercise, let us assume that there are sufficient data available for a fracture model to be used. The question then arises as to which model is most suitable for which situation.
5.5 Model selection

A thorough review of the fracture models available on the market was conducted by the BGS in 1995 (Environment Agency R&D Note 309 and Project Record 292/21/A). There are also many other sources of information regarding available numerical models such as through the USGS and the International GroundWater Modelling Centre (IGWMC – Colorado School of Mines). Appendix B also describes many available fracture models. Suggested below are models that would be suitable for various scenarios of data quality and quantity for fractured rock situations. Table 5.1 indicates the actual data required for the recommended models. Chapter 6 presents a discussion on how to derive the fracture parameter and distribution data required for fracture models.

For large amounts of data:

Double Porosity: SWIFT/486. This 3-D model does do particle tracking, but it would appear that it only works in the forward direction.

Impermeable Matrix: FRACMAN. This 3-D stochastic model has particle tracking capabilities as well as the capability to model matrix diffusion.

EPM: MODFLOW/MODPATH. This 3-D model has particle tracking and stochastic capabilities.

For small amounts of data:

Impermeable Matrix: SDF is a stochastic discrete fracture model. Data required for the model could be derived using the methodologies outlined in Chapter 6 (2-D).

EPM: FLOWPATH. This 2-D model has particle tracking capabilities.

In summary, the final model chosen should be such that:

- The model has a particle tracking capability;
- The model has a stochastic capability;
- The model data requirement matches the data that are available;
- The model can be run on Agency machines and can be understood by Agency personnel; and
- Is capable of representing a fractured rock conceptual model.
Table 5.1: Data Requirements for EPM and fracture flow models.

<table>
<thead>
<tr>
<th>Data/Model</th>
<th>FLOWPATH</th>
<th>MODFLOW/MODPATH/MT3D</th>
<th>SDF</th>
<th>SWIFT</th>
<th>FRACMAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydraulic conductivities/</td>
<td>$K_x, K_y$</td>
<td>$K_{x1}, K_{y1}, K_z$</td>
<td></td>
<td>$K_x, K_y, K_z$ for each layer (global systems)</td>
<td>Average transmissivity for all sets, uncorrelated or correlated to aperture, fracture size or depth</td>
</tr>
<tr>
<td>Transmissivity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Storativities and porosity</td>
<td>$n_e$</td>
<td>$S, S_Y, n_e$</td>
<td></td>
<td>Porosity fraction for each layer (global and local systems)</td>
<td>Average storativity for all sets, uncorrelated or correlated to aperture, fracture size or depth</td>
</tr>
<tr>
<td>Aquifer thickness</td>
<td>$d$</td>
<td>$d$</td>
<td></td>
<td>Layer thicknesses</td>
<td>Block thickness</td>
</tr>
<tr>
<td>Disperivity</td>
<td></td>
<td></td>
<td></td>
<td>Longitudinal and transverse dispersivity</td>
<td></td>
</tr>
<tr>
<td>Diffusion</td>
<td></td>
<td></td>
<td></td>
<td>Molecular diffusivity (global and local systems)</td>
<td></td>
</tr>
<tr>
<td>Spacing/density of fractures</td>
<td></td>
<td></td>
<td></td>
<td>Fracture density for all sets defined</td>
<td>Fracture density and distributions for all sets</td>
</tr>
<tr>
<td>Orientation of fractures</td>
<td></td>
<td></td>
<td></td>
<td>Orientations and distributions of all sets</td>
<td>Orientations and distributions of all sets</td>
</tr>
<tr>
<td>Trace Length of fractures</td>
<td></td>
<td></td>
<td></td>
<td>Trace lengths and distributions of all sets</td>
<td>Trace lengths and distributions of all sets</td>
</tr>
<tr>
<td>Aperture of fractures</td>
<td></td>
<td></td>
<td></td>
<td>Aperture and distributions for all sets</td>
<td>Average aperture for all sets, uncorrelated or correlated to fracture size or depth</td>
</tr>
<tr>
<td>Termination of fractures</td>
<td></td>
<td></td>
<td></td>
<td>Abutting or free</td>
<td>Termination probability</td>
</tr>
</tbody>
</table>
6  FRACTURE CHARACTERIZATION

6.1  Predicting fracture parameter values

As described in Chapter 5, the models that are most likely to be used for modelling fractured rock under non-EPM scenarios will require significant amounts of data. The following sections describe how this data can be obtained, whether it be through field work, through assumptions about the local geological conditions to give fracture data, or various theoretical arguments.

Studies of fracture characterization are rare within hydrogeological literature, and the need for extensive study is great. In order that groundwater flow models can be set up to simulate fracture flow with confidence, details of the fractures are needed. Such details in the form of distributions of fracture parameters can be obtained from either real fracture data or predictions from other available geological data. It can be seen from some of the literature that it may be possible to predict the forms of the fracture distributions from other known geological parameters. This chapter takes each of the fracture parameters: density/spacing, orientation, trace length, aperture, infilling and fracture roughness, and describes possible ways in which these parameters could be predicted using standard geological knowledge. It then goes on to describe the distributions measured across England and Wales, as well as describing the ways in which these parameters can be best measured.

In the context of the thesis the term fracture is taken to include all types of discontinuity in rock, such as joints, fissures, veins, bedding planes and faults. For this chapter it is also useful to distinguish between faults and joints. Faults are fractures with a shear displacement whereas joints are open fractures with negligible shear displacement.

6.1.1  From geological and geotechnical data

Figure 6.1 illustrates some of the nomenclature associated with fractured systems. Systematic fractures are those which have straight traces as seen on the bedding plane surface, and non-systematic fractures are those with curved fracture traces as seen on the bedding plane surface. Figure 6.1 presents some of the joint nomenclature for an H type joint pattern (Hancock, 1985) showing systematic jointing and non-systematic jointing. When systematic joints form parallel to one another they form a set. It is
considered that non-systematic fractures tend to occur mainly in the weathered zone (Price and Cosgrove, 1991), whereas systematic fractures can occur at all depths. All the fractures in the rock mass under consideration may be referred to as the fracture network or fracture system.

Figure 6.1: Nomenclature for joint and fracture patterns (after Gross (1993) and Hancock (1985)).

Density/Spacing

In the literature there are two distinct schools of thought for the way in which fracture spacing develops. One school of thought is generally held by structural geologists, who consider the stress regimes, strains and layer boundaries in their theories of fracture development. The other view is held by engineering geologists, who consider the fracture development process as a random one (a Poisson process), producing a negative exponential distribution in fracture spacing. To an engineering geologist it is less important to study the relative timings of the formation of different fracture sets, but more important to study the overall fracture system. However, for our purposes it is more instructive to consider the structural relationships, as these allow us to predict fracture intensity to a certain degree in some situations. These relationships are discussed further below, but without a detailed discussion of the processes that give rise to these relationships. The processes involved are complex both in time and space and beyond the scope of the work presented here. If predictions can be made of fracture
spacings from known stress regimes and bed thicknesses, some sort of fracture model can be developed in the absence of real fracture characterization data. To characterize fully the fractures at a particular location can be relatively costly and time consuming.

The density and spacing of fractures may at first appear to be completely random, however, there are a few theories that may be able to help us predict patterns once various rock parameters are understood. The processes which produce the spacings of fractures are complex both in 3-D and in time and therefore it is considered possible that the relationships detailed below are not entirely fundamental but may apply in certain situations. When and where these situations are valid must be investigated.

One important point to recognise is that the spacing of fractures varies with depth. Many fractures are associated with stress relief during uplift and stress relief is greater nearer the surface, hence producing lower fracture spacings at shallow depths.

**Relationship between Bed Thickness and Fracture Spacing**

Within the structural geology literature it is widely accepted that there is a linear relationship between fracture spacing and bed thickness in interbedded sediments (Price and Cosgrove, 1991, Ji and Saruwatari, 1998).

Many different mechanical models have been used to simulate this observed relationship. There are three categories of these mechanical models:

i) Shear lag models – this category of model proposes that the linear relationship is controlled by the transfer of stress from adjacent incompetent beds to the competent layer. Hobbs (1967) first proposed this model to the geological community. Ladeira and Price (1981), Narr and Suppe (1991), Huang and Angelier (1989) and Gross (1993) also support this hypothesis.

ii) Stress perturbation models – this category of model considers that joints/fractures are free surfaces and so the normal tensile stress in the vicinity of the fracture is locally reduced with respect to the far field stress. This stress reduction prevents new fractures forming in the vicinity of the old ones, and this is often referred to as the stress reduction shadow (Gross, 1993). The size of the stress reduction shadow controls the joint spacing.
iii) Energy balance models – these set of models look at the energy
dissipation in the elastic deformation and brittle fracture process to form
a relationship between fracture spacing and bed thickness.

There are some important and relevant relationships that come from the agreement of
the models and field data.

Price and Cosgrove (1991) state that there is some physical evidence for a linear
relationship between bed thickness and fracture spacing in sedimentary sequences of
bed thickness less than 1.5 m (OA on Figure 6.2). Ladeira and Price (1981), in their
shear lag model, show that the thickness of the incompetent beds either side of the
competent bed also influences the fracture spacing. As the thickness of the incompetent
beds increase, the fracture spacing increases. The rock formations analysed by Ladeira
and Price (1981) also show that beyond a certain fracture spacing, which depends on the
rock type, the relationship is no longer linear, as seen by AB on Figure 6.2.

\[
\frac{dt}{ds} = \frac{1}{FSI}
\]

Figure 6.2: Variation of bed thickness with fracture spacing for a hypothetical rock (after
Ladeira and Price (1981)).

However, this linear relationship is still useful for bed thicknesses less than 1.5 m, and
providing that adequate geological logs are available, bed thicknesses can be converted
to fracture spacings. A constant of proportionality of between 0.5 and 6 will give good
approximations for most rock types (Price and Cosgrove (1991) and Ladeira and Price
(1981)), although it does vary with rock characteristics such as lithology and degree of
consolidation.

Further rock fracturing and fragmentation may occur through hydraulic fracturing
(when the fluid pressure is greater than the least total stress, \(\sigma_3\)), although in very thin
beds this mechanism is thought to play a secondary role. From hydraulic considerations, the fracture spacing can be linked to the pressure gradient, and hence also the rate of propagation of the fracture and the permeability of the unfractured rock. Hence one would expect fracture spacing to be much smaller in high permeability sandstones than in low permeability mudstones, for example. However, Ladeira and Price (1981) realized that much more work needs to be completed before the mechanisms outlined can be verified.

Gross (1993) describes a stress perturbation model for the relationship between mechanical layer thickness and cross joint spacing seen in the Monterey formation, California, based on the analysis of stress reduction in the vicinity of a newly formed joint. Mechanical layer thickness can be defined as either the layer thickness or the width between fractures in the initial fracture set. The fracture spacing effectively acts as another layer thickness but in a different orientation. The ideas of Gross (1993) follow on from the work completed by Narr and Suppe (1991) who also studied fracture spacing in the Monterey Formation. Gross suggests that it is unlikely that fractures are produced in a mechanical layer simultaneously but are produced by a process of sequential infilling. However, the results presented do not indicate a two-process graph as illustrated in Figure 6.2. The graphs presented indicate a good linear relationship between bed thickness and fracture spacing with a fracture spacing index (FSI) (or inverse of constant of proportionality) of 1.3. Gross suggests that there is no change of FSI over much of the Monterey Formation due to fracture saturation (see next section), and that when an FSI of 1.3 is reached, any extra stress is accommodated by the opening of existing joints rather than the formation of more joints.

**Fracture Saturation**

It has been documented by a variety of sources (Wu and Pollard, 1995 and Narr & Suppe, 1991, for example) that, if a steadily increasing strain is applied to a rock, fractures will form, but there will be a certain fracture density/spacing beyond which no more fractures will be formed, no matter how much the strain is increased. This fracture spacing is termed the fracture saturation. Once the fracture saturation has been reached the stress is accommodated by the opening of fractures rather than by the production of new ones.
The assessment as to whether a fracture set has reached saturation has important implications for the interpretation of data. For sets that have not reached saturation, considerable spread on a plot of bed thickness against fracture spacing may be observed, as minor differences in strain across the bed may produce major differences in fracture spacing. Rives et al. (1994) use the mode/mean ratio from the frequency spacing distribution graphs as an indicator of the fracture pattern development stage. A low value (towards 0) indicates that the set is poorly developed and a high value (towards 1) indicates a nearly saturated set. Wu and Pollard (1995) also developed a method for describing the degree of fracture saturation which is based on assessing the deviation, d, between the normalized individual spacings measured along a scanline and the area spacing obtained from the area method as described in the section on measuring fracture density/spacing. If $d > 0.7$, then the jointing is considered poorly developed and if $d < 0.4$ then the jointing is considered well developed.

As well as the fracture spacing being proportional to bed thickness, it is recognized that the spacing will be influenced by the degree of orogenic deformation. This was confirmed by, among others, Huang and Angelier (1989) who compared two very similar rock types in the same location, but ones that had undergone different amounts of compaction. It was found that for the more compacted rock the fracture spacing was smaller than for the less compacted rock.

**Relationship between RQD and fracture spacing**

In geotechnical investigations the Rock Quality Designation (RQD) is a parameter often measured in order to provide an assessment of the rock mass behaviour and quality. The RQD is defined as the percentage of intact lengths greater than a threshold value which is usually taken as 10 cm, and is given by the following equation:

$$RQD = \frac{100}{L} \sum_{i=1}^{n} x_i$$

(1)

where $x_i$ is the length of the $i$th length $> 0.1$ m, $n$ is the number of intact lengths and $L$ is the total length of the scanline/borehole measured. According to values within the percentage range, the rock quality is classed as excellent (90 – 100%), good (75 – 90%), fair (50 - 75%), poor (25 – 50%) or very poor (<25%). To a certain extent it is possible to use RQD values to obtain a value for fracture spacing to be used in a fracture geometry model of a site.
Priest and Hudson (1976) give a relation between the RQD and fracture frequency based on the assumption that fracture spacings follow a negative exponential distribution:

\[
RQD = 100e^{-\frac{\lambda}{10}} \left( \frac{\lambda}{10} + 1 \right)
\]  

(2)

where \( \lambda \) is the fracture frequency (m\(^{-1}\)) and equal to \( 1/x \) where \( x \) is the fracture spacing (m). This is for a threshold value (t) of 0.1 m, although the equation can be altered simply for other threshold values. Since this equation is derived simply by integrating all spacing values above the threshold value in the spacing distribution (0.1 m in this case), expressions for theoretical RQD can be found in a similar way for other spacing distributions.

Hudson and Priest (1979) stress that the fracture frequency in the above equations will vary with scanline orientation. Therefore if the scanline intersects two orthogonal fracture sets at an angle \( \theta \) then \( \lambda \) becomes \( \lambda_\theta \) and:

\[
\lambda_\theta = \lambda_1 \cos \theta + \lambda_2 \sin \theta
\]  

(3)

where \( \lambda_1 \) is the fracture frequency of set 1 and \( \lambda_2 \) is the fracture frequency of set 2 normal to the strike of the fractures. Hence RQD is a function of scanline/borehole orientation. If \( \lambda \) lies in the range 6-16 per m then use can be made of an approximation produced by Priest and Hudson (1976):

\[
RQD = 110.4 - 3.68\lambda
\]  

(4)

However, there is a caveat when using RQD values to predict fracture spacings. During the drilling and sampling process extra fractures may develop due to the stresses involved and hence when RQD values are measured from the core, calculations of fracture spacings from RQD values may be artificially low.
Measuring Fracture Density/Spacing

Fracture spacing and densities can be measured in several different ways, the most common method being along a scanline (1-D). Fracture spacing can also be measured areally (2-D) and volumetrically (3-D). Ideally the fractures should be classified into sets and then fracture spacing measured for each of the sets as this makes for a simpler analysis. However, methods have been developed by Hudson and Priest (1979) to enable fracture spacings to be calculated for different sets without the need for assigning individual fractures to a set. The method relies on rotating the scan line about the rock surface to get the fracture frequency with orientation. The shape of the resulting curve enables the number of fracture sets and their associated fracture densities to be identified.

Hudson and Priest also set out examples of how to perform block area distributions and frequencies, as well as block volumes. Although an interesting exercise in predicting fracture frequencies, these methods are not wholly practical for our purposes. The areal analysis involves taking photographs of the rock surface, cutting up the area in question into the fracture defined blocks and weighing the pieces of paper. The volume analysis would be almost impossible in practical terms as it is analogous to performing a particle size distribution test on fractured blocks. Therefore the discussion set out here will concentrate on scanline methods and analysis presented by Priest and Hudson (1976).

Hudson and Priest (1979) recommend that to perform a thorough assessment of N fracture sets, there needs to be at least N scanlines. Each scanline should be orientated so that it is perpendicular to the fracture set to be measured. However, they recognize the problem that many fractures do not fall into obvious sets. Therefore they suggest two different methods for establishing scanline positions:

- Orthogonal pairs of scanlines positioned without reference to fracture orientation.
- Pairs of scanlines positioned parallel to two major fracture sets.

Obviously, if there are more than two fracture sets then more scanline orientations will be needed. Hudson and Priest also recommend that the total scanline length should be 50 times the estimated average fracture spacing. This gives a mean fracture frequency within ±20% of the true value at an 80% confidence level. Priest and Hudson (1981) use the central limit theorem to prove that the number of measurements along the
scanline must be larger than 30 for reasonable precision of the average fracture spacing. The central limit theorem states that the means of a random number of samples, n, taken from a population that follows any distribution will tend to be normally distributed. As the distribution tends towards a Gaussian then the true average fracture spacing can be calculated with more precision. This distribution of the means only closely approximates the normal distribution if n>30 (this is also closely related to the t-distribution which approximates a normal distribution when the number of samples or the degrees of freedom are greater than 30).

Kulatilake et al. (1990) also described the process of fracture spacing analysis using a scanline as illustrated in Figure 6.3.

![Flowchart of the process of fracture spacing analysis using a scanline (after Kulatilake et al., 1990).](image)

Ryan et al. (2000) present an alternative approach to quantifying fracture density of 2-D fracture networks (scan planes and outcrops) which gives both fracture density in a
given orientation and the orientation of the principal fracture sets, the results being presented on a diagram they term an 'itogram'. The method gives the dominant fracture spacing rather than the average, although the bias produces compared with a 1-D sampling method is reduced. However, the data that are required for input into fracture flow models are average fracture orientations and spacings within that fracture set, along with their distributions, and not the dominant fracture spacings. Distributions required for stochastic analysis are not given using this method.

Wu and Pollard (1995) suggest that different methods for measuring fracture spacing should be used for different stages of fracture development. They indicate that scanlines should only be used on well-developed fracture patterns, i.e. those patterns where fracture trace length is much greater than the fracture spacing. In a poorly developed fracture pattern (trace lengths are roughly equal to or smaller than fracture spacings) the fracture spacing can vary considerably from one scanline to another. This poorly developed pattern is unlikely in the areas where fracture analysis is required for fractured rock modelling. However, a method for poorly developed fracture systems is presented here for completeness.

Wu and Pollard (1995) present an area method that is much more appropriate for poorly developed fracture patterns. The area method relies on a fractured surface being exposed. A square is marked out on the surface and the lengths of all the fractures of the set under consideration are measured and added together. The number of fractures in the area under consideration is then counted and the average fracture spacing calculated from:

$$S = \frac{A}{I_0 + \sum_{i=1}^{n} l_i} = \frac{A}{I_0 + L}$$

where $I_0$ is the side length of the measuring region and the area of the square is $A = I_0^2$,

$L$ is the total length of fracture $i$ in that region,

$n$ is the number of fractures in the square.

Wu and Pollard go on to recommend that the area method is used in preference to the scanline method since:
a) for well developed fracture patterns, the difference between the two methods is negligible and;
b) for poorly developed patterns, the area method is superior.

It is possible to measure fracture density and spacing by a number of methods. The most practical methods are by scanline analysis in the field, scanline analysis on photographs of the outcrops, and scanline analysis from boreholes. When obtaining fracture spacing data from scanlines (1-D) and trace planes (2-D), care must be taken to compensate for the orientation bias of the sampling plane or line. The most frequent bias compensation used is the Terzaghi correction (see later section on measuring orientations/analysis). Where this is applied there is an increase in the number of records in the dataset for fractures that have normals at a high angle to the direction of the borehole or scanline, by a quantity that is inversely proportional to the probability of intersecting those fractures within the borehole or scanline. Analysing photographs back in the office may seem like a much more pleasant option, but produces even more bias in the data obtained.

However, in more recent years with the advent of GIS systems, fracture spacing analysis has become much more simple. If photographs of fracture systems are taken then these photographs can be processed and analysed by GIS packages to give fracture spacings. This has been done for a study site in the La Rochefoucauld limestone massif, Charente, France by Bodin and Razack, 1999. It is important that bias on the data is still taken into account after the GIS analysis.

Orientation

The orientation in which fractures develop depends on the stress regime at the time of formation. In general the orogenic and tectonic history of England and Wales is well understood, with the stress regimes at different times being known. Hence, from structural considerations, it should be possible to predict the types of fracture patterns seen from the stress history of that particular region.

If the stress history is known for a given region then the theory of the Mohr circle (introduced by Otto Mohr in 1882) can be used to predict approximate fracture orientations. The theory is not presented here as it is assumed that the reader is familiar
with the technique. The theory can be used to predict fracture patterns under different principal stress ($\sigma_1$) orientations as illustrated in Figure 6.4.

![Figure 6.4: Block diagrams indicating dominant conjugate fracture sets under different stress conditions.](image)

In Figure 6.4 the situation most common in the UK is that depicted in diagram b where the principal and minimum stresses are in the horizontal plane producing vertical conjugate sets of fractures. Fractures can also be symmetrically related to folding, although the precise orientations and sets present may depend on particular circumstances. However, general fracture orientations with relation to a fold can be depicted as illustrated in Figure 6.5.

If systematic joints are considered, then it is suggested (e.g. Engelder and Geiser, 1980) that vertical joints propagate normal to the least principal stress and thus follow the trajectory of the stress field present at the time of propagation. Price and Cosgrove (1991) present examples of fracture orientations in relatively unfolded sediments such as those found over much of southern England, and a fracture pattern derived by considering a simple idealized sedimentary basin that undergoes downwarp and subsequent uplift and exhumation. The two patterns are found to match closely. On the downwarp, the fractures would tend to develop parallel and vertical to the long axis of the basin in the first instance, and then vertical and perpendicular to this in the second. For non-parallel uplift (the standard case) then fractures form at an angle to the long axis of the basin with a related conjugate set. This can be seen more clearly in the Figure 6.6.
Conjugate shear fracture sets intersecting fold axis

Tension fractures perpendicular to fold axis

Figure 6.5: Typical fracture orientations on a fold.

Conjugate shear fracture sets formed during uplift
Orthogonal extension fractures formed during downwarp

Figure 6.6: Fracture pattern observed in the generally flat lying sediments of the Cotswold Hills, which is very similar to that predicted by basin analysis (after Price and Cosgrove, 1991).

However, it must be emphasized that this type of faulting pattern and the mechanisms involved work well only for strong rocks (high Youngs modulus and Poissons ratio). For clay-type rocks, it is likely that for a basin undergoing relatively shallow burial only vertical orthogonal joint sets will be formed.

The above examples demonstrate that sensible estimates of fracture orientation can be made from knowledge of the tectonic history of a region, and knowledge of the axes of large folds and faults.
Measuring Orientations/Analysis

Fracture orientation data can be obtained from outcrop analysis or from borehole analysis. When analyzing these data one has to be aware that most data will be biased to a certain degree. For example, data from a vertical borehole will under-represent those fractures that are parallel or sub-parallel to the borehole, especially in the region of $0 - 30^\circ$ from the borehole (Martel, 1999). There are several methods for correcting this bias, with the most straightforward being devised by Terzaghi (1965).

Orientation analysis of rock fractures must first start with identifying the different sets within the fracture pattern along a scanline and assigning each fracture to a particular set. Usually this has been done by the relatively simple method of using orientation densities on stereographic projections, as described by Toynton (1983). Poles to the fracture planes are plotted on the stereogram. The concentration of fracture poles is then measured in terms of the percentage of measured fracture poles within a 1% of the area of the stereogram. There are several computer packages that will perform this density analysis simply, such as Rockworks '99 (Rockware Inc., 1999). Various statistical tests can then be applied to the densities to determine where the bounds of the clusters lie. Bridges (1990) presents an alternative method for determining fracture sets by recognizing different angular relationships between the fractures. Martel (1999) also presents a full analysis of orientation characterization and analysis of bias.

Shanley and Mathab (1976) present a methodology for determining the delineation and analysis of clustering in orientation data. Analysis of the clustering is based around the Bingham distribution, which is an elliptical rather than the more commonly assumed circular distribution. Kulatilake et al. (1990) applied the methods set out by Shanley and Mathab (1976) to the Stripa mine data, which was corrected for sampling bias and also tested for the goodness-of-fit to the Bingham distribution.

Although it is recognized that orientation analysis should be performed rigorously and to a high standard, it may not be appropriate, due to time and technical constraints in fracture characterization, for the method proposed by Bridges to be adopted. However, it is suggested that orientation density plots are used combined with statistical tests to assess the boundaries between fracture sets, and that the Terzaghi correction is applied for those data with a borehole or trace plane bias.
As mentioned in the section on measuring fracture spacing, the advent of GIS systems allows fracture system analysis to be done rather more simply. If photographs of fracture systems are taken then these photographs can be processed and analysed by GIS packages to give orientations. This has been done for a study site in the La Rochefoucauld limestone massif, Charente, France by Bodin and Razack (1999).

**Trace Lengths**

The trace length of a fracture is the length of fracture seen at the exposed surface, such as at outcrop or in cross-section on a geophysical profile. These observed trace lengths are a function of the shape of the fracture. However, there is some debate as to the actual shape of fractures within a rock body. Baecher et al. (1977) assume that fractures are two-dimensional discs with the centre points of the fractures forming a Poisson field. This random disc model allows the shape of the discs to be either circular or elliptical (Baecher, 1983). However, it is unlikely that perfectly circular or elliptical fractures will occur since rocks are generally heterogeneous. Fractures are often represented as polygons of equal area to the ellipse because of the terminations of the fracture at other intersecting fractures (Dershowitz et al., 1998).

The trace length of a fracture depends on the stress regime the rock has encountered and the rock type. There are certain generalities that can be made about trace lengths and the terminations of fractures. Gross (1993) states that fracture terminations can occur in two principal ways: those that terminate at random locations in the rock mass and those that terminate at mechanical layer boundaries (other fractures or bedding planes). The former fracture termination type is observed in units that are relatively homogeneous such as granites and massive thick-bedded carbonates. The latter are observed in well stratified sections where adjacent layers have different mechanical properties such as in interbedded limestones and shales (these data are required by many fracture models such as SDF and Fracman).

Huang and Angelier (1989) reviewed work by Cruden (1977) and Baecher (1983) where trace lengths across an area were measured. The trace length distributions were found to fit a negative exponential in the first case and a lognormal distribution in the second case. However, as Huang and Angelier point out, the distribution obtained will depend
on the scale of observation, i.e. whether large fractures are not taken into account as they are rarely totally exposed and whether very small fractures are ignored.

It has been stated in some texts that there may be a relationship between fracture length and fracture spacing. For example, Gale et al. (1982) found a spacing to length ratio of 1.15. Clemo (1989) developed this idea further using an average of the literature values (2.15) for estimates of fracture length in a dual permeability model. However, no general model has been found and the relationship is therefore not considered to be appropriate for the fracture characterization analysis presented here.

**Measurement of Trace Lengths**

Priest and Hudson (1981) recognize that the problem of trace length estimation lies in both accuracy and precision. Measuring trace lengths can be fraught with problems since fractures often finish out of sight in the rock face or under scree or vegetation. Often discontinuity spacings are measured along a scan line, and the trace lengths of those fractures that cross the scan line are also measured. Unfortunately this produces bias in the data.

Biases in trace length data can occur for the following reasons (Zhang and Einstein, 2000):

i) Orientation bias – the probability of a fracture appearing at outcrop depends on the relative orientation between the outcrop and the fracture.

ii) Size bias – a) large fractures are more likely to be sampled than small ones, and b) the longer the fracture the more likely it is to appear on the outcrop.

iii) Truncation bias – very small fractures are very difficult to measure, therefore trace lengths are often not recorded below some cut-off. This cut-off is usually at 10 mm.

iv) Censoring bias – long trace lengths may extend beyond the visible exposure so that one or both ends of the fracture cannot be seen.

If the discontinuity size distribution is required from the trace length data then corrections need to made for i) and ii)a. Truncation bias can be corrected for by the method of Warburton (1980). Zhang and Einstein (2000) present a method for reducing the bias from ii)b and Priest and Hudson (1981) present a method for reducing the bias
from iv). Mauldon (1998) also presents a method for reducing the bias from iv) and ii)b.

Priest and Hudson (1981) present three scanline sampling scenarios (Figure 6.7) that could potentially be used to assess mean trace lengths and the associated distributions, and the methods available for correcting the censoring bias produced in these scenarios:

- Scan line located randomly to a parallel set of fractures (Figure 6.7a);
- Scan line located randomly to a parallel set of fractures and only measuring the length of traces on one side of the line (e.g. scan line placed at base of rock face) (Figure 6.7b);
- Scan line located randomly to a parallel set of fractures and only measuring the censored length of traces on one side of the line (e.g. scan line placed at base of rock face and cut-off point required at some distance from scan line, possibly due to vegetation restrictions) (Figure 6.7c).

Figure 6.7: Diagrammatic representation of fracture traces intersecting scanlines under different scenarios (after Priest and Hudson, 1981).
The details are not presented here, but can be found in the original paper of Priest and Hudson (1981).

There is also a method for measuring the size/trace length of discontinuities that relies on sampling through a ‘window’. This method is presented by Zhang and Einstein (2000) who measure the intensity of discontinuities in a rock mass. They use a circular window on the rock mass outcrop that has the advantage over scanline sampling since the orientation bias can automatically be eliminated when estimating the trace lengths. However, because measurement of all trace lengths for all fracture sets occurs within the circular window, the trace length distribution for each fracture set required for input into fracture models is not obtained.

As mentioned in the section on measuring fracture spacing and orientation, the advent of GIS systems allows fracture system analysis to be done rather more simply. If photographs of fracture systems are taken then these photographs can be processed and analysed by GIS packages to give trace lengths. This has been done for a study site in the La Rochefoucauld limestone massif, Charente, France by Bodin and Razack, 1999.

**Aperture**

Small-scale aperture characterization is highly technical and complex. Many methods have been used to attempt to characterize apertures of individual fractures, for example with resin casts, metal injection and surface profiling. However, we need to find more practical ways in which to characterize fracture apertures on a larger scale for flow models and for predicting breakthrough curves. If these more complex small-scale 2-D methods are used then a spatial correlation length\(^1\) is required for the extrapolation to 3-D. Pyrak-Nolte *et al.* (1997) suggest that as long as the core sampled is larger than the correlation length, then measurements at the core scale can be scaled up to the field scale.

Hatton *et al.* (1994) studied the relationship between fracture length and fracture width or aperture. They examine the validity of applying a universal scaling law for the relationship between fracture length (L) and fracture width (D):

---

\(^1\) Correlation length is the distance from a point beyond which there is no further correlation of a physical property associated with that point. Values for a given property at distances beyond the correlation length can be considered purely random. The correlation length is also the same as the integral scale in stochastic systems.
by studying tension fractures in Iceland which were all formed within the Holocene in the same environment. However, it was found that with increasing width there was a change in \( n \) from 2 to 1 at a fracture length of approximately 10 times the fracture spacing. Hence the growth rule changes once the fracture reaches a critical length, which may approximate to 10 fracture spacings. Although only tensile fractures were measured in this study, Hatton et al. (1994) are confident that the findings can be applied to shear fractures, as the models used to describe the tensile fractures are independent of the mode of fracture growth.

It is possible that the relations given by Hatton et al. (1994) may be able to provide some outer limits on fracture apertures for various lengths. At 10 times the fracture spacing the relationship should change from \( D \propto L^2 \) below 10 times the fracture spacing to \( D \propto L \) above 10 times the spacing. The constant of proportionality for below 10 times the fracture spacing varies from \( 3.16 \times 10^{-4} \) to \( 3.16 \times 10^{-5} \), and above 10 times the fracture spacing it varies from \( 3.16 \times 10^{-2} \) to \( 3.16 \times 10^{-3} \).

Vermilye and Scholz (1995) studied the relationship between vein length and maximum aperture in a range of rock types and tectonic environments. They found that there was a positive correlation between the length and aperture with the constant of proportionality ranging from \( 2.1 \times 10^{-4} \) to \( 8.2 \times 10^{-3} \), with correlation coefficients ranging from very good to weak. It may be possible, therefore, by using a combination of the relationship of Hatton et al. (1994) and Vermilye and Scholz (1995) to obtain bounds on fracture apertures for various lengths.

It is also important to remember that fracture apertures are, in effect, pressure sensitive so that fracture permeability generally decreases with depth.

Tsang and Tsang (1990) describe a method to characterize the fracture aperture which relies mainly on hydrological measurements rather than more complex methodologies. They use three parameters to characterize the fractures that have been generated statistically: the mean aperture, the standard deviation, and the spatial correlation length (related to the spatial arrangement of the aperture variation). These parameters rely on a tracer test being conducted.
Mean Aperture

Fracture apertures are not usually considered by hydrogeologists in terms of their actual size and size distribution. They are more often considered in terms of hydraulic effect, i.e. the ‘size’ of aperture the water flow ‘sees’. Fractures are often treated as a pair of parallel plates for ease of calculation and understanding. For the parallel-plate fracture model there is no ambiguity as to the fracture aperture, it is always a single value, usually denoted as ‘b’. However for real fractures the fracture aperture varies enormously and can be described by a fracture aperture density distribution and a correlation length. Although the following discussion does not describe relationships between standard geological parameters that can be measured and the true fracture aperture, it is perhaps more relevant to describing the actual parameter required for fracture flow modelling - the hydraulic aperture.

From literature on field studies involving tracer tests and hydraulic tests an ‘equivalent aperture’ value is often derived. However, there appears to be a discrepancy in the literature as to values obtained for the ‘equivalent aperture’ from the different field study types. This has arisen from an inconsistency in the use of the term for equivalent aperture. Tsang (1992) addresses this problem, and proves that the results in the literature are in fact consistent with each other, with the confusion arising due to the terminology used for the fracture aperture.

Tsang (1992) notes that there are three ways in which the ‘equivalent aperture’ can be defined:

a) Mass Balance Aperture

This aperture type derives from tracer test data and the measurement of the volumetric flow rate, $Q$, the mean residence time of tracer transport and the assumption that the areal extent of the single fracture $Length \times Width = A$, in which the transport takes place is known. This gives:

$$Qt_w = A \delta_m$$

and so

$$\delta_m = \frac{Q t_w}{LW}$$

(7)
where the mean residence time is determined from the time moment of the measured breakthrough curve. The determination of the mass balance aperture, $\delta_m$, is related to the pore volume of the fracture and can also be related to the arithmetic mean of all the aperture values in the flow paths.

**b) Frictional Loss Aperture**

The frictional loss aperture is also derived from tracer test data, and involves the mean residence time, $t_m$, for tracer transport from the injection point, $l_1$, to the collection point, $l_2$, in terms of transport velocity. If the velocity is assumed to be constant from $l_1$ to $l_2$ then Darcy’s Law can be used with the hydraulic gradient being known across the distance $L (l_1-l_2)$ and assuming that the real fracture can be represented by a parallel plate one with a frictional loss aperture $\delta_f$. This gives:

$$\delta_f = L \left( \frac{12 \mu}{\gamma \Delta H t_m} \right)^{1/2}$$  \hspace{1cm} (8)

where: $L$ is the distance the tracer travels along a parallel plate aperture of frictional-loss aperture.

$\gamma$ is the weight density of the fluid = $\rho g = 9808.9 \text{ kgm}^2 \text{s}^{-2}$

$\mu$ is the dynamic viscosity of the fluid = $1.3037 \times 10^{-3} \text{ Nsm}^{-2}$

$\Delta H$ is hydraulic head difference over length $L$

$t_m$ is the mean residence time of the tracer

$\delta_f$ is the ‘frictional-loss’ aperture

This equation must be changed for flow in radially converging or diverging situations and injection-withdrawal dipole flow. For radial flow symmetry (8) becomes:

$$\delta_f = L \left[ \frac{6 \mu}{\gamma \Delta H t_m} \ln \left( \frac{r_1}{r_0}\left(\frac{r_1^2 - r_0^2}{r_1^2 r_0^2}\right) \right) \right]^{1/2}$$  \hspace{1cm} (9)

where: $r_0$ is the distance from the abstraction well (the injection well)

$r_f$ is the radius of the abstraction well
c) Cubic Law Aperture

The cubic law fracture aperture is the equivalent parallel plate aperture that would allow a certain flow rate at a given pressure drop. For linear flow between parallel plates with constant aperture $b$, fracture breadth $W$ and flow path length $L$, and applying Darcy's Law:

$$Q = -\frac{\gamma}{12\mu} \frac{\Delta H}{L} \delta_c^3 W$$

(10)

$b$ then may be replaced by $\delta_c$ and solved. For radial flow situations then the above equation can be modified and gives the following expression for the cubic law aperture:

$$\delta_c = \left[ \frac{6\mu Q}{\pi \gamma \Delta H} \ln \left( \frac{r_1}{r_2} \right) \right]^{1/3}$$

(11)

The cubic law aperture is probably the most widely used equivalent aperture in the literature, and is often referred to as the 'hydraulic aperture'.

A relationship can now be found to relate all three types of equivalent aperture:

$$\delta_c^3 = \delta_i^2 \delta_m$$

(12)

Tsang (1992) also show that, for a given fracture, their relative magnitudes may be ranked as follows:

$$\delta_m \geq \delta_c \geq \delta_i$$

(13)

Tsang (1992) expands on this and explains that $\delta_i^2$ is proportional to the equivalent permeability for a homogeneous porous medium, and the equivalent permeability for a random 2-D field has the upper bound of an arithmetic mean and lower bound of a harmonic mean. For a linear flow geometry the permeability is well approximated by the geometric mean. $\delta_m$ is representative of the pore volume of a fracture and is the true arithmetic mean. $\delta_i$ is a multiplicative average of the other two apertures and therefore occupies the middle position in the inequality.

The frictional loss and the cubic law apertures both rely on measurements made under a pressure drop in laminar flow. It is interesting to note that the pressure drop is very sensitive to local heterogeneities, so derived apertures will always be weighted towards the smaller apertures where there is the most resistance to flow and which cause the largest head drop.
Tsang and Tsang (1990) investigated the relationship between residence times of particles flowing through a generated fracture and the ratio of total fracture void volume to the total flowrate through the fracture. They found that the time given by the two methods was very similar and hence the mean aperture can be calculated using just the total flow rate and the mean residence time from tracer breakthrough curves:

\[ t_m = \frac{\bar{b}LW}{Q} = \frac{\bar{b}L^2}{Q} \]  

(14)

where \( t_m \) is the residence time,

- \( L \) is the distance between tracer injection and collection,
- \( W \) is the width of the flow region,
- \( b \) is the average fracture aperture,
- \( Q \) is the total flow rate.

Tsang and Tsang (1990) state that it has been common practice to relate the ratio of flow over hydraulic gradient to some mean aperture of a fracture. They emphasize the fact that this ratio gives an incorrect mean aperture as it gives the effective permeability of the fracture which is the geometric mean of all the permeabilities, rather than the arithmetic mean. For a lognormal distribution (as with all distribution types) the geometric mean is smaller than the arithmetic mean, and as the standard deviation increases the discrepancy between the two means increases.

All these methods of deriving apertures from tracer test data allow bounds to be placed on the aperture for the fracture modelling input.

**Standard Deviation**

Intuitively, the larger the fracture aperture variation the larger the dispersion on a tracer breakthrough curve. One empirical measure of dispersion on a breakthrough curve represented as fraction of particles arrived vs. time may be defined in terms of \((t_{0.9} - t_{0.1})/t_{0.5}\) where \( t_x \) is the time when the fraction of particles has reached that point, as shown in Figure 6.8.
Tsang and Tsang's (1990) statistically generated fracture with variable apertures exhibited a strong positive correlation between \((t_{0.9} - t_{0.1})/t_{0.5}\) and the standard deviation. They present a figure from the experiments which could be used to relate \((t_{0.9} - t_{0.1})/t_{0.5}\) from a tracer test breakthrough curve to the standard deviation for different spatial correlation lengths.

**Spatial Correlation Length**

Through a series of computer modelling experiments conducted by Tsang and Tsang (1990) it has been shown that flow channelling appears to occur when the fracture aperture varies between two planes. The spacing between these flow channels was found to be of the same order as the spatial correlation length. The ‘length’ in the direction of the flow over which the flow appears to coalesce into channels is also the order of one correlation length. Therefore it may be possible to get a rough estimate of the correlation length from observing the spatial pattern of the non-uniform flow rates in a line measurement.

Hence, through the three parameters described above, it may be possible to model at least the morphology of the fractures from values that are relatively simple to obtain. It may not be necessary, therefore, for the roughness coefficients defined in the next section to be used.
Montemagno and Pyrak-Nolte (1999) describe experimental measurements of fracture aperture using X-ray tomography. Spatial correlation data indicate apertures in coal are correlated over distances of 10-20 mm.

**Roughness**

Gentier and Riss (1990) describe the morphology of a fracture surface in terms of heights of the asperities and angularity of the surface profile. Angularity of the profile can be measured by considering the positive and negative inclinations of each part of the profile. A roughness parameter can then be assigned to the profile, which can be defined as:

$$R_L = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{\cos \theta_i}$$  \hspace{1cm} (15)

Where $N$ is the number of definable inclinations and

$\theta_i$ is the inclination angle of each part of the aperture profile

Other ways in which Gentier and Riss suggest fracture morphology can be studied are spectral analysis and fractal analysis. (Both these types of analysis are described later on.) They conclude that the relationship between channelling and angularity must be studied, as it is possible that there is a critical inclination for a given aperture fracture that may lead to flow channelling.

However, it must be stressed that these type of measurements are not practical on the timescales usually associated with protection zone delineation and also on a financial basis.

**Infilling**

The infilling of fractures can be by many materials such as quartz or calcite, and some may exhibit iron staining. The fracture infills are useful for determining the relative ages of the different fracture sets, for example, a fracture set with no infill will be younger than an intersecting set infilled by vein material. The presence of vein material, such as quartz or calcite also tends to suggest fluid flow has occurred at some time in the past. Staining may also indicate preferential flow paths, but probably does not indicate all flowing fractures. Infill in fractures will also substantially reduce their permeability.
Summary

Two methods are used for obtaining fracture parameter data depending on availability of staff time and money. Either scanline analyses or tracer tests can be performed with the associated corrections for biasing to obtain fracture parameters and characteristics.

If it is not possible to do a scanline and tracer studies then estimations of the various parameters can be made by the following methods:

i) fracture spacing
   - from ‘nearby data’ (see below)
   - estimate from bed thickness from lithological logs, and also get cross joints spacing from bed thickness fracture spacing. Use constant of proportionality as ~1
   - from RQD values after estimating distribution
ii) orientation
   - from nearby data
   - if stress regime known use Mohr diagrams
   - is it on a fold, then use Figure 6.5
   - is near a fault, then use Rawnsley et al. (1992)
iii) trace length
   - from nearby data
   - distribution may be fractal
iv) aperture
   - from nearby data
   - from tracer studies
   - from relationship with trace lengths (Hatton et al., 1994)
   - from the cubic law, known transmissivities and flowing fracture spacings
v) roughness
   - from laboratory studies
   - fractal properties

‘Nearby data’ refers to the data being derived from the same structural region as the site, such as from near by quarries, cliffs and other outcrops. In other words there should be no major faulting, folding or change of lithology between the site to be characterized and the site where the data originates.

6.1.2 From geophysical data

Table 6.1 summarises those geophysical methods which can be used to detect certain fracture parameters. Those methods in italic type are those which are more strongly
recommended for the detection of that particular parameter. For all geological environments the TV logs, microresistivity (Formation Microscanner), televiewer, differential temperature and fluid conductivity logs are recommended. For crystalline environments useful information can also be gained from guard logs, sonic and density logs combined, and cross borehole tomography of different kinds.

It is important to note that only one method (GPR) can be used (approximately) to assess the trace length of fractures. The main disadvantage with this is that the results cannot be confirmed by other geophysical methods. The method also only gives very approximate length values, which will probably be on the short side, since it is unlikely that very narrow fractures will reflect much EM radiation. The depth of penetration of GPR depends on the material and the wavelength of the radar. However, the depth of penetration rarely exceeds 10 to 20 m, and therefore cannot detect fractures at larger depths.

Table 6.1: Summary of geophysical methods used for detection of fracture parameters.
(Italics indicate methodology specifically recommended for the particular parameter.)

<table>
<thead>
<tr>
<th>Parameter required</th>
<th>Geophysical method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Fracture Sets</td>
<td>• Televiwer&lt;br&gt;• Formation Microscanner&lt;br&gt;• TV&lt;br&gt;• Azimuthal resistivity and Electromagnetics (EM)</td>
</tr>
<tr>
<td>Average Trace Length and Distribution</td>
<td>• Ground Penetrating Radar (GPR)</td>
</tr>
<tr>
<td>Average Orientation and Distribution</td>
<td>• Televiwer – for all fractures, especially near horizontal ones&lt;br&gt;• Azimuthal resistivity - for vertical sets&lt;br&gt;• Azimuthal electromagnetics - for vertical sets&lt;br&gt;• Magnetics – for large fracture zones only&lt;br&gt;• Gravity - for large fracture zones only&lt;br&gt;• Seismics - for large fracture zones only&lt;br&gt;• Cross borehole tomography using both acoustics and electromagnetics for detecting flowing fracture sets - only useful in crystalline rocks</td>
</tr>
<tr>
<td>Average Density (actual fracture density) and</td>
<td>• Combined P-wave and density logs&lt;br&gt;• Neutron-neutron logs - only in crystalline rocks</td>
</tr>
</tbody>
</table>

Chapter 6 - Fracture characterisation
<table>
<thead>
<tr>
<th>Parameter required</th>
<th>Geophysical method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distribution</td>
<td>• Guard logs, especially in crystalline rocks</td>
</tr>
<tr>
<td></td>
<td>• Caliper logs</td>
</tr>
<tr>
<td></td>
<td>• TV</td>
</tr>
<tr>
<td></td>
<td>• Televiewer</td>
</tr>
<tr>
<td></td>
<td>• Formation microscanner</td>
</tr>
<tr>
<td>Average Density (flowing fracture density) and Distribution</td>
<td>• Temperature</td>
</tr>
<tr>
<td></td>
<td>• Fluid conductivity</td>
</tr>
<tr>
<td></td>
<td>• Cross borehole tomography using both acoustics and electromagnetics - only useful in crystalline rocks</td>
</tr>
<tr>
<td>Average Aperture</td>
<td>• TV</td>
</tr>
<tr>
<td></td>
<td>• Televiewer – not exact</td>
</tr>
</tbody>
</table>

The above review and summary is ideal for choosing geophysical methods for a new study site.

6.1.3 From percolation theory

Flow through a set of fractures is clearly related to more general problems of flow in networks. Such problems arise in many fields from electronics to traffic management. Generic studies of such systems have progressed in recent years mainly through developments in ‘percolation’ theory.

i) Percolation theory

Percolation theory examines flow within a medium consisting of an infinite set of sites which are joined by paths called ‘bonds’ (see Figure 6.9 a)). Each site may have several bonds joining to other sites. The number of bonds leaving each site is known as the coordination number. The percolation process can be considered to occur in one of two ways, either that all bonds are open and a stochastic mechanism blocks sites at random (site percolation) or all sites are open and bonds are blocked randomly (bond percolation). For fluid flow problems, bond percolation considerations are usually used.

If a set of unblocked bonds are connected to one another a cluster will exist (see Figure 6.9 b)). If the cluster is finite then the fluid can only ‘wet’ a finite number of sites. An infinite cluster exists when the critical threshold of the lattice is exceeded and there is connection theoretically between an infinite number of sites. In reality, for example
when computer modelling a finite sized area, the critical percolation threshold is reached when connection between one side of the area to the other is obtained. Hence the bond percolation threshold is defined as the largest fraction of occupied bonds below which there is no infinite connection. Already from this argument it can be seen that percolation properties are scale dependent, and that percolation parameters will be different for infinite networks than for networks used in computer simulations.

The probability of an arbitrary bond being open is defined as \( p \), and \( P(p) \) is the probability of the bond belonging to an infinite network. The percolation threshold of the system is usually denoted as \( p_c \). If it were possible to find the percolation threshold of any given system, then theoretically the lowest network permeability of that system could be found, which would be useful for fracture flow modelling.

![Diagram](image)

**Figure 6.9:** a) A lattice showing bonds and sites which is fully percolating (connected), b) A lattice showing clusters of linked bonds below the percolation threshold.

Close to the percolation threshold it has been hypothesised that permeability can be described by the following expression:

\[
K(p) \sim (p - p_c)^\mu
\]  

(16)
where \( K(p) \) is the intrinsic permeability and \( \mu \) is a constant for the particular network (Lee et al., 1994). The form of the equation above is the same as many others relating to percolation theory and all are of the type:

\[
X \sim (p - p_c)^\nu
\]

(17)

where \( X \) is the observable geometrical or physical quantity and \( \nu \) is an exponent specific to quantity \( X \), sometimes called the universal exponent.

However, it has proven difficult to find the percolation thresholds for even the simplest 2-D lattices such as the honeycomb, the triangular and the grid systems. It took nearly two decades to find the first value for square bond percolation. For 3-D systems such as the body centred cubic and face centred cubic lattices the threshold values have proved almost impossible to calculate. Therefore for randomly fractured systems it would seem unlikely that estimates for the percolation threshold and hence the permeability are attainable.

**ii) Permeability and connectivity estimation**

However, percolation theory can be used in some circumstances to assess the conductivities and connectivities of fracture networks. Berkowitz and Balberg (1993) provide an excellent review on the many applications of percolation theory to hydrology.

Expressions of the form described above in equation (17) are used frequently in the literature. One of the first people to use percolation theory in hydrology/hydrogeology was Robinson (1983, 1984). Robinson (1983) derived the percolation threshold for two sets of equal length fractures at \( \pm 45^\circ \) in two dimensions, which can be related to fracture density and fracture length by:

\[
(N(l)^2)_c = 1.54
\]

(18)

where \( N \) is the fracture density (number of fractures per unit area), \( l \) is the length of the fractures, \( c \) denotes the critical value for percolation and \( <> \) denotes the average value. Robinson (1983) also examines other fracture distribution cases of varying length and orientation in two dimensions:
i) for constant line length and angles distributed uniformly in the range \((-\alpha, \alpha)\) for a range of \(\alpha\)'s. For \(\alpha = 90^\circ\) then \((N\langle l^2 \rangle)_c = 1.45\).

ii) for constant line length and angles either \(\beta\) or \(-\beta\) with a probability of 0.5 for a range of \(\beta\)'s. For \(\beta = 45^\circ\) then \((N\langle l^2 \rangle)_c = 1.54\).

iii) line length uniformly distributed in the range \([l_{av}(1-f), l_{av}(1+f)]\) for a range of \(f\)'s, angles uniformly distributed in the range \([90^\circ, -90^\circ]\). If \(f = 0\) then \((N\langle l^2 \rangle)_c = 1.488\).

iv) line length uniformly distributed as case iii) angles are \([45^\circ, -45^\circ]\).

Although these values relate critical densities, lengths and orientations of fractures to percolation, these relations could be used to provide some bounds on fracture parameters for the models if the types of fracture pattern are generally known. Lengths of fractures are particularly difficult to measure in the field, however, so if we know the system is percolating and we can measure fracture density from boreholes, we can get a lower bound on mean fracture lengths. This is obviously not a precise method nor a technically correct one since the percolation probability will be higher than the critical percolation in the situations observed for pumping wells, but it will allow trace length estimation.

The work of Robinson was extended by Charlaix et al. (1984) who proposed a percolation threshold for a random array of flat discs in 3-D:

\[
(Nr^3)_c \sim 0.15 \text{ to } 0.3
\]  

(19)

where \(N\) is the fracture density (number of fracture centres per unit volume) and \(r\) is the radius of the disc. The radius will also give some estimate of the fracture trace length \((2r)\).

Table 6.2 presents trace lengths for a given fracture density for the different relationships given by Robinson (1983) and Charlaix et al. (1984) for use in obtaining bounds on field data given either the fracture density or the trace length at a site. The definition of fracture density varies between Robinson (1983), Gueguen and Dienes (1989) and Charlaix et al. (1984), and the definition that is likely to come from field investigations (number of flowing fractures in a borehole being 1-D). However, we are only investigating the order of magnitude on the fracture parameter bounds so the
precise definition of fracture density is not considered to be crucial at this stage. Care should be taken if this table is used to ensure that a similar type of fracture system exists at the site as the percolation relationship (e.g. random array of discs or cracks in 3-D if the Charlaix et al. (1984) or Gueguen and Dienes (1989) relationships are to be used, or two sets of approximately orthogonal fractures for the Robinson (1983) relationships to be used).

Table 6.2: Trace lengths predicted by Robinson (1983), Charlaix et al. (1984) and Gueguen and Dienes (1989) for given fracture densities.

<table>
<thead>
<tr>
<th>Fracture Density</th>
<th>Constant trace length and angles between -90° to+90°</th>
<th>Constant trace length and angles either -45° or +45°</th>
<th>Trace length distributed and angles between -90° to+90°</th>
<th>Charlaix et al. relationship = 0.15</th>
<th>Charlaix et al. relationship = 0.3</th>
<th>Gueguen and Dienes (1989) c/A &gt; 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>12.04</td>
<td>12.41</td>
<td>12.20</td>
<td>3.94</td>
<td>6.22</td>
<td>100.00</td>
</tr>
<tr>
<td>0.05</td>
<td>5.39</td>
<td>5.55</td>
<td>5.46</td>
<td>2.88</td>
<td>3.64</td>
<td>20.00</td>
</tr>
<tr>
<td>0.1</td>
<td>3.81</td>
<td>3.92</td>
<td>3.86</td>
<td>2.28</td>
<td>2.88</td>
<td>10.00</td>
</tr>
<tr>
<td>0.2</td>
<td>2.69</td>
<td>2.77</td>
<td>2.73</td>
<td>1.82</td>
<td>2.28</td>
<td>5.00</td>
</tr>
<tr>
<td>0.5</td>
<td>1.70</td>
<td>1.75</td>
<td>1.73</td>
<td>1.34</td>
<td>1.68</td>
<td>2.00</td>
</tr>
<tr>
<td>1</td>
<td>1.20</td>
<td>1.24</td>
<td>1.22</td>
<td>1.06</td>
<td>1.34</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>0.85</td>
<td>0.88</td>
<td>0.86</td>
<td>0.84</td>
<td>1.06</td>
<td>0.50</td>
</tr>
<tr>
<td>5</td>
<td>0.54</td>
<td>0.55</td>
<td>0.55</td>
<td>0.62</td>
<td>0.78</td>
<td>0.20</td>
</tr>
<tr>
<td>10</td>
<td>0.38</td>
<td>0.39</td>
<td>0.39</td>
<td>0.50</td>
<td>0.62</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Robinson (1984) expands these investigations to three dimensions and examines how the critical fracture density varies with increasing system size. He found that for a 3-D system of fixed size orthogonal planes, there would be 2.00 intersections per plane for percolation to occur. In 2-D, 3.11 intersections per line are needed for percolation (an increase of 55%). Therefore if a 3-D fracture system is modelled in 2-D using 3-D fracture statistics percolation is unlikely to occur. This was proved during the work on the case studies where no percolation occurred on modelling in 2-D with the measured 3-D parameters. Englman et al. (1983) also conducted work along a similar line to Robinson (1983, 1984) and Charlaix et al. (1984) where the overall permeability of a percolation network was estimated as functions of fracture area density and the size of the medium.
Long and Witherspoon (1985) also developed a 2-D numerical model of fracture networks, and measured permeability as a function of fracture intersection. They conduct what they term a 'length-density' study. They set out by stating that most fracture parameters are difficult to measure in the field, but two measurements are obtainable from boreholes, namely the fracture frequency down the well and the angle the fractures make to the well. For a given set of fractures it can be shown that (Baecher et al., 1977) fracture frequency can be expressed as:

$$\lambda_L = \lambda_A \bar{l} \cos \theta$$  \hspace{1cm} (20)

where \(\lambda_L\) is the fracture frequency, \(\lambda_A\) is the areal density, \(\bar{l}\) is the mean length, and \(\theta\) are the angles between the sample line and the fracture poles of the given set, and \(\cos \theta\) is the mean of the cosines of the angles. If we know \(\lambda_L\) and \(\theta\) then rearranging the above equation gives:

$$\frac{\lambda_L}{\cos \theta} = \lambda_A \bar{l} = LD$$  \hspace{1cm} (21)

Hence we can estimate the product LD (length multiplied by density), which Long and Witherspoon refer to as the length-density parameter. However, the product LD could be made up of an infinite number of pairs of \(\lambda_A\) and \(l\). Therefore, if two boreholes of the same depth cut through twelve fractures at the same angles to the well, for example, there may be significant differences in the fracture network geometry, and hence in the hydraulic behaviour of the systems. The above equation can be related to percolation theory through an equation describing the 'effective bond occupation probability', \(\zeta\), where:

$$\zeta = \pi \lambda_A \bar{l}^2$$  \hspace{1cm} (22)

As \(\zeta\) increases the permeability of the system increases. Hence if LD is held constant then the permeability or \(\zeta\) increases with \(l\). Long and Witherspoon (1985) modelled different fracture geometries with the same LD and noted the permeability variations. It was found that networks with shorter fracture lengths and higher fracture densities have a lower permeability than systems with a longer fracture length and a lower fracture density, as expected. The behaviour of systems with shorter fracture lengths was found to be less like that of a porous medium than systems with longer fracture lengths. One most interesting result was that if the fracture system did not behave as a porous
medium at one scale it did not necessarily behave as a porous media at larger scales, Hence a REV did not appear to be definable. This has implications for the choice of fracture flow model to be used for a given situation. Also, as fracture lengths increase, permeabilities approach a maximum. Therefore, for systems with fracture lengths longer than a certain value, it may not be necessary to specify the exact length or areal fracture density of the system. The method described by Snow (1969) could be used to predict the maximum permeability of the system assuming infinite length fractures (assuming the cubic law, where the permeability of the fracture is proportional to the square of the aperture).

Gueguen and Dienes (1989) studied two simple models to describe low porosity rocks in terms of their conductivity and permeability. The first model assumes flow through pipes and the second flow through cracks. Conductivity and permeability are then related to pipe/crack parameters such as average length, average radius and average spacing. The relationships produced are valid for only narrow distributions of the crack/pipe parameters. Possibly the most useful relation that is produced is:

\[ P = \frac{\pi^2 c^3}{4 l^3} \]  

where \( P \) is the probability of two cracks intersecting

\( c \) is the average crack radius

\( l \) is the average crack spacing and \( l^3 \) is the fracture density

Assuming a Bethe lattice (a solvable permeability lattice problem) Gueguen and Dienes (1989) show that there is a non-zero permeability for \( p > 1/3 \). A diagram of a Bethe type lattice is shown in Figure 6.10 below. This results in \( c/l > 0.5 \) from the above equation. Therefore, for a system which is just percolating, and where the average crack spacing is known (from borehole logs, for example) then a minimum bound on the crack radius can be found. This could potentially be useful for putting constraints on the bounds for the fracture parameters for use in fracture models.
Hestir and Long (1990) use percolation theory and equivalent media theory to obtain expressions for permeability and the REV as related to percolation parameters for random 2-D fracture networks, as opposed to ordered networks studies by the authors mentioned above.

**Conclusions**

From work on percolation theory and connectivity various theoretical relations have been found that may be able to give a rough guide to some fracture parameters. For example, the work of Robinson (1983, 1984) gives relationships between fracture length and density, and Charlaix et al. (1984) gives a relationship between fracture density and fracture radius under certain conditions. Gueguen and Dienes (1989) also give a similar relationship to that of Charlaix et al. (1984).

6.1.4 Connectivity considerations

As a follow on from the work on percolation theory, which is related to considerations of connectivity, it is useful to look at how connectivity has been defined. This is relevant to work that occurs later in the document (section 8.2).

The fundamental flow properties of a fracture network will be determined by the connectivity of the individual fractures. The connectivity of a rock will depend on a complex function of the discontinuity parameters of orientation, density, and trace length as well as the nature of the fracture fill. The connectivity of a system could potentially lead to fracture parameters such as fracture density and length, but this would be unusual since it is rare that the connectivity of a fracture network is actually defined. However, for completeness the ways in which connectivity may be measured are described here so the fracture parameters could theoretically be derived (providing the method of obtaining the connectivity is known).
Various authors have attempted to define connectivity indices:

- Robinson (1984) used a measure equal to the mean number of intersections per line weighted by line length. He showed that as the size of the domain increases the density of fractures must increase to ensure connectivity. This relates directly to the REV, such that as the density of fractures increases the size of the REV decreases.

- Zhang *et al.* (1992) defined a connectivity ratio, $C$, as:

$$C = \frac{B_c}{B_c + B_o}$$

where $B_c$ is the total number of connected branches (or fractures) and $B_o$ is the total number of non-connected branches in a fracture pattern. However, Zhang *et al.* recognised that $C$ was not unique and the same $C$ might be obtained for different fracture patterns. Therefore a second ratio, $C_k$, is described, which is related to the type of fracture pattern and the number of nodes in the pattern. In a fracture pattern there will be several connected networks of different orders ($k$'s) and some non-connected branches. If the number of branches of the $k$th order network is $B_k$ then $C_k$ is defined as:

$$C_k = \frac{\sum B_k}{B_o + B_c}$$

where $\sum B_k$ is the number of the connected branches of the $k$th order and $C = \sum C_k$ (summed from $k = 0$ to $k = s$, where $s$ is the maximum $k$th order in the system). Connectivity is dependent on fracture density, length and relative orientation between the two sets, as well as the sample area size.

The connectivity of natural and simulated systems were then calculated and compared for the same fracture densities and lengths. The two were found to be closely correlated. It was also found that connectivity varied with sample size, although at a sample size of greater than 10 times the fracture length, the connectivity ratios stabilised. This feature of the analysis would tend to suggest that a minimum domain size for modelling must be $>10$ times the fracture length so that scaling issues in connectivity are not so much of a problem. Zhang *et al.* (1992)
then go on to derive a complex, but more rigorous approach to characterising the connectivity of a fracture system. This approach is then tested using a case study from the Lake District, with the connectivity ratio having a major effect on the flow rate of fluid and the two being strongly correlated.

- Guérin and Billaux (1994) use the mean number of intersections per fracture weighted by fracture diameter.

- Long et al. (1991) use a process called simulated annealing to find the appropriate pattern of connected fractures, given some experimental data from the rock. Simulated annealing can be used to construct a system (of fractures, say) which is functionally equivalent to the observed system: i.e. a model which emulates the same observations as we have. For example, a fracture network model is ‘annealed’ by continually modifying the base or template model such that the modified system behaves more like the observed system. Simulated annealing can be used if:

  i) there is a set of possible configurations of the system, i.e. there is a network of fractures defined, some of which allow flow to take place (fractures are ‘on’) and some which do not (fractures are ‘off’).

  ii) there is the capability within the numerical model for systematically changing the configuration, i.e. there is a probability function for randomly turning fractures on and off and hence creating a new network.

  iii) there is an ‘energy’ function to minimise i.e. taking a configuration from all the sets of fracture configurations and comparing to one configuration in particular. In the case of Long et al. (1991) they compare observed and simulated responses of geophysical measurements, but it could be any other hydrological or geological parameter. The energy function is then the sum of the differences from the observed and simulated measurements to some power. A type of probability distribution is then assumed for the energy function.

  iv) there is an annealing schedule of changing a temperature-like variable, so the system can reach its minimum. As the annealing continues one is less likely to keep those configurations that increase the energy function, and hence the configurations that minimise the difference between the observed and
simulated states are kept. Thus a good approximation to the 'correct'
configuration producing the measurements can be obtained.

- Forde and Wei (1997) produced a document through Golder Associates entitled 'A
discussion document on connectivity definition in fractured rock' which summarises
the many ways in which connectivity can be measured and defined. It is noted that
the problem with attempting to define connectivity is that, firstly, the term is
subjective. It is also problem specific (geological, hydrogeological, thermal,
electrical etc.), scale dependent, lithology dependent and time dependent. They
detail a number of connectivity measures which have been used in the past and
those which are suitable for further characterisation of connectivity. These include
  - Fracture density (de Marsily, 1985)
  - Mean number of fracture intersections (de Marsily, 1985)
  - number of intersections per volume - also used in this document, section 8.2
  - fracture intersection length per volume
  - number of intersections per fracture weighted by fracture diameter/length
    (Robinson, 1984; Guérin and Billaux (1994)
  - Fractal dimension (La Pointe, 1988)
  - Flow dimension
  - Ratio of effective permeability to geometrical mean of fracture permeability for
    an idealised network

The number of intersections per fracture appears to the favoured option as a measure of
connectivity.

6.1.5 From geostatistics

In the previous sections on predicting fracture parameters it has been assumed that
either data has to be collected for the site under consideration or that, in some cases,
data from the same structural provenance can be used at the site. However there is
another method that could be used to extrapolate and interpolate the data from other
locations to the site under consideration. This is through geostatistics, and more
specifically kriging. Kriging provides values for points which are between known data
points by assuming that data is correlated over a certain distance (correlation length).
This would be one way in which fracture parameters could be obtained for areas in
between other data points, and hence could be used for modelling purposes.
However, it is unlikely that fracture parameters can be correlated over large length scales due to the variation in the rock characteristics, structural forces, varying degrees of weathering and/or development of preferential fracture apertures due to hydraulic conditions. Also there are likely to be very few data points on which to perform the kriging and therefore any such analysis is unlikely to be rigorous.

Despite this, some work has been done in this area to study the variance in travel time over areas where the permeability has been defined. Andrews et al. (1987) varied correlation lengths and investigated the effect on kriged permeability fields and the effect on the variance in travel time. It was found that if the permeability field was perfectly defined then the uncertainty in travel-time decreased as the permeability correlation length increased. Furthermore, the uncertainty decreases as the permeability correlation length increases. However, for permeability fields with limited observations or sampling points, increasing the correlation length causes an increase in the uncertainty in the travel-times.

### 6.2 Fracture parameter distributions

Our understanding and quantification of the hydraulic behaviour of fractures requires a knowledge of the statistical distributions such as length and aperture. These distributions are needed explicitly as data input for fracture network models. It is therefore appropriate to review our knowledge of these distributions.

#### 6.2.1 Possible distribution types

Before the actual distributions of fracture parameters found in England and Wales are presented, an understanding of the forms that these distributions could take is valuable. The distributions types presented vary from the simple Gaussian type curve to the more complex gamma distributions. The distributions presented here are only a small fraction of those that have been defined by statisticians. The ones presented here are those that have some physical foundation and theory for their form.

**Gaussian**

The Gaussian or normal distribution was derived by Gauss, the probability density function for the normal distribution of a given population is described by:
\[ P_N = \frac{1}{\sigma \sqrt{2\pi}} \exp\left[ -\frac{(S - \bar{S})^2}{2\sigma^2} \right] \] (26)

where  \( S \) is a value within the population

\( \bar{S} \) is the mean of the population

\( \sigma \) is the standard deviation

It has the approximate form shown in Figure 6.11.

![Figure 6.11: Gaussian distribution.](image)

The Gaussian distribution is convenient and easy to use, however, it allows negative values. Negative values are not possible when dealing with fracture parameters such as length and aperture but it is a feasible distribution for orientations.

**Lognormal**

\[ P_N = \frac{1}{\sigma \sqrt{2\pi}} \exp\left[ -\frac{(\ln S - \ln \bar{S})^2}{2\sigma^2} \right] \] (27)

where  \( S \) is a value within the population

\( \bar{S} \) is the mean of the population (geometric mean, or the arithmetic mean of the log of the values)

\( \sigma \) is the standard deviation of the lognormal distribution (standard deviation of the log values)
If the scale along the S-axis is changed to a logarithmic scale then the function assumes a Gaussian form. One advantage of using the lognormal distribution is that the values are always positive, as shown in Figure 6.12.

**Negative Exponential**

Negative exponential distributions arise through random processes. In particular the random process of fracture or the Poisson process produces a negative exponential distribution. The Poisson process can be envisaged by a scanline along which fractures are located by a purely random process along that line with no interaction between fractures.

\[
P_N = \lambda \exp(-\lambda S)
\]

where \(P_N\) is the probability density function

- \(\lambda\) is the ‘half-life’ of the process and more specifically the inverse of the arithmetic average fracture spacing
- \(S_i\) is a value within the population

Negative exponential functions have the properties of a) their mean and standard deviation being equal (to \(\lambda^{-1}\)) and b) all values being positive, as illustrated in Figure 6.13.
The gamma probability distribution function is described by:

\[
P_N = \frac{1}{\Gamma(\alpha + 1)} \beta^\alpha \exp(-S / \beta)
\]

where \( \Gamma \) is the gamma function.  
\( \alpha \) and \( \beta \) are constants which are related to the arithmetic and geometric means (Huang and Angelier, 1989). \( \beta \) controls the decrease of the function value when \( S = \alpha \beta \). When \( \alpha = 0 \) the gamma function becomes a negative exponential distribution.

An approximation of the forms of the gamma function is illustrated in Figure 6.14.
Weibull

The Weibull probability distribution function is described by:

\[
P_N = \frac{m}{S_0} \left( \frac{S}{S_0} \right)^{m-1} \exp \left[ - \left( \frac{S}{S_0} \right)^m \right]
\]

(30)

where \( S_0 \) is the lowest value of the distribution

\( m \) is the Weibull modulus and a measure of the width of the distribution as well as a measure of the skewness of the distribution

Special cases of the Weibull distribution are the negative exponential distribution (\( m=1 \)) and the normal distribution (\( m = 3.44 \)). Examples of the Weibull distribution for different values of the Weibull modulus are illustrated in Figure 6.15:

![Figure 6.15: Weibull distribution.](image)

The Weibull distribution is often considered suitable for fracture distributions because the distribution types are frequently described as extreme value distributions. However, as is pointed out by Doremus (1983) the Weibull distribution does not have any exact theoretical foundation and he concluded therefore that it should be considered as probable as any other distribution for describing fracture distributions. Only the special cases of the Weibull distribution: the lognormal, and the negative exponential distributions have a physical foundation. The negative exponential distribution is that given when only two facts about the data set are known: the mean and all the values are positive. The Gaussian distribution also requires two sets of information – the mean
and the standard deviation. The lognormal distribution can be formed from three pieces of information: the mean, the variance and positiveness of the values.

Fractal

A fractal distribution is the only distribution that is scale invariant, and at whatever scale the object is examined the properties are identical. Fractal concepts and distributions have been shown to apply to many geological problems. The scale invariance of many geological features is apparent from simple observation, such as the need to include a scale in a geological photograph otherwise the feature may be any size from the centimetre scale to the kilometre scale. It is possible therefore that fractal theory may apply to fractured rock so that aperture, density and orientation distributions can be characterised in a relatively simple way using few parameters.

Fractal concepts can be applied to many different areas such as to a number of objects with a characteristic dimension or to a statistical distribution of objects. This relationship is usually represented in the form:

\[ N(r) = Cr^{-D_f} \]  

where: \( N(r) \) is the number of objects with a characteristic linear dimension \( r \)  
\( C \) is the constant of proportionality  
\( D_f \) is the fractal dimension

For most natural phenomena the fractal dimension will lie between zero and three. The fractal dimension will not necessarily be a whole number but somewhere in between. For example, the implication of a fractal dimension of between 1 and 2 is that the perimeter of the 'object' is no longer a continuous differentiable line. When the fractal dimension is a whole number then the space in which the object can be characterised is Euclidean. Otherwise the space the object/pattern lies in should be considered as "non-overlapping \( d \)-dimensional hyperspheres of Euclidean radius \( r \)" (Sahimi, 1995).

As can be seen from equation (31), there is only one parameter which characterises the self-similar distribution - \( D_f \). To have only one parameter to describe the properties of a fracture aperture distribution, for example, would be extremely useful. Theoretically, it may be possible to have a set of fractal dimensions for the orientation, density and
aperture that could characterise the entire rock of an area. It therefore may be possible to get a set of these dimensions for each tectonically different area of each rock type and hence characterise fractured rock.

It has often been thought that most natural phenomena can be characterised by a lognormal distribution. However, Einstein and Baecher (1983) speculated that the reason for lognormal distributions was due to sampling procedures, and that at very small scales the features to be sampled would be missed, hence producing the tail-off at very small scales. Barton and Zoback (1992) considered that the lognormal distribution should be replaced with a power law distribution since the small-scale features were often undersampled, hence fractal analyses may be appropriate for natural phenomena.

It is possible that whether fracture properties have a fractal structure depends on the type of fracture and the medium in which the fracturing occurs. Gillespie et al. (1993) found that faults tend to have fractal qualities whereas joints tend to be scale dependent. Odling et al. (1999) report that, in general, in strata-bound systems lengths tend to follow lognormal distributions and spacings are regular, whereas in non-strata-bound systems, such as deeper volcanic and metamorphic rocks, fracture sizes tend to follow a fractal distribution.

There are some standard fractal shapes which can be developed easily. These are the Cantor dust, Serpinski Carpet and Menger Sponge which are the 1, 2 and 3 dimensional equivalents of each other. For the Cantor Dust imagine a line, divide it into three and then remove the middle part. Repeat this process with the remaining lines and repeat. If this is repeated many times it can be seen that clustering occurs. This clustering is often recognised in nature in a faulted area where smaller faults often cluster around larger ones, hence producing both densely and sparsely faulted or fractured areas. (The dimension usually assigned to the Cantor Dust is approximately 0.631 (ln2/ln3).)

The above description really applies to fractals that are self-similar i.e. the scale invariance applies over all scales and directions. However, there is another type of fractal called a self-affine fractal. Mountainous terrain, Brownian motion, clouds and fractured rock often come into this category where the self-affine surface z(x,y), isotropic in the x,y plane, is scale invariant under the scale transformation (λx, λy, λ^H z), where H is the roughness exponent. Another parameter is required to characterise self-
affine fractals - the scaling factor in the ‘z’ direction. If one thinks about this in surface roughness terms, it is necessary to describe the way in which the effective ratio of vertical to horizontal magnification must be changed with the scale in order to preserve the self-similarity (Poon et al., 1992). This second parameter has been termed the topothesy.

Using classical (Newtonian laminar flow) theory, the transmissivity of a fracture is proportional to the cube of the fracture aperture. When fluid flow across a rough fracture surface is modelled as a self-affine fractal, for example Zhang et al. (1996), it can be proved using fractal theory that over rough fracture surfaces the permeability of the fracture deviates from the classical law. The permeability measured over a rough surface at large apertures is much greater than that predicted by Darcy’s Law, and the transmissivity of the fracture is approximately proportional to the fourth power of the fracture aperture. The work by Zhang et al. (1996) confirms that as the roughness of the fracture decreases, the permeability increases, so long as Darcy’s Law is observed. However, as roughness decreases the permeability dependence on the aperture changes, with the transmissivity being proportional to the approximately the fifth power of the aperture as roughness becomes very small. Hence there is a large deviation from classical results.

It is also possible to obtain ‘multi-fractal’ distributions. These objects or distributions are self-similar over all scales, but the fractal dimension changes from one scale to another (Sahimi, 1995). Also multi-fractals may exhibit different gradient segments on the log-log plot so that the fractal dimension varies with scale distribution (Boadu and Long, 1994). This is not a continuous variation in fractal dimension, otherwise it would not be a fractal. These type of fractals tend to be found in non-linear dynamical systems, and can be used to characterise, for example, turbulent flow and the flow of tracers through porous media.

Another interesting aspect of fractal theory is that, if it is used to create a fracture network, then the linear scale-dependence of dispersivity emerges as a natural consequence of flow in that system. Hence fractal theory can be used to prove that there is a physical basis for the observed scale dependence of dispersivity, rather than it just being an empirical relation (Ross, 1986). Equivalently, scale dependent dispersivity suggests an underlying fractal structure.
6.2.2 Actual field measurements

Although actual field data measurements from the literature are extremely useful in making generalizations for data distributions in modelling when there are few data, care must be taken to ensure that the distribution assumed applies to the scale of the investigation (Odling et al., 1999). A fracture system may therefore have a hierarchical structure with different distributions dominating at different scales. Also different rock types may have different generalizations in their distributions for the different fracture characteristics. In general, in strata-bound systems lengths tend to follow lognormal distributions and spacings are regular, whereas in non-stratabound systems, such as deeper volcanic and metamorphic rocks, fracture sizes tend to follow a fractal distribution.

Also, if a differentiation is made between joints (opening mode fractures) and faults (shear mode fractures) there are systematic differences in the fracture parameter distributions. In general faults are fractal in character and joints are scale dependent (Gillespie et al., 1993).

Measured Fracture Density Distributions

Fracture spacing (or equivalently fracture frequency) populations have been described by a number of different distribution types. From the available literature it would appear that the negative exponential distribution is probably the most frequently used and measured. Priest and Hudson (1976) measured nearly 5,000 fractures in a Chalk tunnel near Chinnor, Oxfordshire, the distribution of which fitted a negative exponential distribution well. The average fracture spacing was 0.105 m and the standard deviation 0.113 m. Fracture spacing in the Carboniferous Limestone sequence of County Durham and the Lower Chalk in the Channel Tunnel were also measured. Nearly 2,000 measurements for the mudstone in Durham were taken with fracture spacings approximating a negative exponential distribution. Measurements were also taken for the Lower Chalk and a sandstone (although fewer in number than in the other two locations), with the distributions still approximating a negative exponential curve. Hudson and Priest (1979) also measured fracture spacings from 10 different rock types ranging from Jurassic limestone to Pre-Cambrian diorite. All distributions could be approximated as a negative exponential form. Priest and Hudson (1981) performed measurements at two case study sites; one in the Ordovician mudstone and the other in a Cambrian sandstone. Again the data were found to fit a negative exponential
distribution well. It is interesting to note that the negative exponential distribution is a special case of the gamma function. However, as pointed out by Gross (1993) and Bloomfield (1996) the Priest and Hudson data do not appear to differentiate between the different fracture sets for the distribution analysis. The negative exponential forms found therefore represent bulk or global values rather than individual fracture set distributions. It is more than likely that individual fracture sets have different distributions to the bulk data.

Baecher (1983) used 15,000 fracture spacings from seven construction and mining sites of varying geologies. The data would appear to fit negative exponential distributions for each of the sites.

La Pointe and Hudson (1985) present 321 joint spacings from the Niagaran dolomite, Wisconsin. Villaescusa and Brown (1990) show the distribution from 212 joint data points from the Panguna andesite. Kulatilake et al. (1990) investigated joint spacing distributions at the Stripa mine in Sweden, and collected 212 data points, analysing all the different joint sets. In all these cases a negative exponential distribution was found to fit the field data most successfully.

Huang and Angelier (1989) studied tension and shear tectonic joints only, rather than all discontinuities. The examples used come from Gulf of Suez and south-eastern France, and in all cases follow a gamma distribution or the lognormal form well.

Narr and Suppe (1991) have described their 596 data points from the joint sets of the Monterey Formation in California in terms of a lognormal distribution. However, they do not appear to have compared the data with any other distributions. It has been noted by Huang and Angelier (1989) that the lognormal distribution varies only slightly from the skewed gamma function. Therefore it is quite possible that the Narr and Suppe data better follows a gamma function.

Rives et al. (1994) hypothesised that fracture spacing distribution varies with fracture development, or the degree of fracture saturation. Through a series of experiments on polystyrene plates they showed that as the number of fractures increased (i.e. the degree of fracture saturation increased) the spacing distribution changed from being negative exponential to lognormal through to normal. It is interesting to note that this is exactly
how gamma and Weibull distributions evolve with an increase in the $\alpha$ or $m$ coefficient from negative exponential to lognormal type through to normal.

Gross (1993) presents data from cross joints of the clayey-siliceous member of the Monterey Formation, California. 710 measurements were taken and the resulting distribution approximated a gamma distribution most closely of a lognormal type shape.

Boadu and Long (1994) collected fracture spacing data from the gneisses around Lake Strom Thurmond, Georgia. They compared the field data distributions with Schumann, fractal and Weibull distributions. Data from the different fracture sets appeared to fit the fractal and the Weibull distributions more closely. Comparison with the gamma distribution was not made. However, from the section comparing the different distributions, it can be seen that the forms of the gamma and the Weibull distributions are very similar. Also a fractal distribution is very similar in form to the gamma distribution when $\alpha$ is negative and $\beta$ large and positive (see section 4.2.1).

Bloomfield (1996) measured 167 fracture spacings in the Chalk of Play Hatch Quarry, Berkshire. The results were corrected for sampling biases and it was found that bedding plane fractures followed a lognormal distribution and the joint sets perpendicular to this followed a negative exponential distribution. Both these types of distribution are special cases of the Weibull and gamma distributions.

A summary of the distributions found in the literature is presented in Table 6.3.

<table>
<thead>
<tr>
<th>Source</th>
<th>Fracture Spacing Distribution</th>
<th>Number of Measurements</th>
<th>Geology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Priest and Hudson (1976)</td>
<td>Negative exponential</td>
<td>5,000</td>
<td>Lower Chalk</td>
</tr>
<tr>
<td>Priest and Hudson (1976)</td>
<td>Negative exponential</td>
<td>2,000</td>
<td>Carboniferous sst + mudst</td>
</tr>
<tr>
<td>Priest and Hudson (1976)</td>
<td>Negative exponential</td>
<td></td>
<td>Chalk</td>
</tr>
<tr>
<td>Hudson and Priest (1979)</td>
<td>Negative exponential</td>
<td></td>
<td>All types</td>
</tr>
<tr>
<td>Priest and Hudson (1981)</td>
<td>Negative exponential</td>
<td></td>
<td>Ordovician mudstone</td>
</tr>
<tr>
<td>Priest and Hudson (1981)</td>
<td>Negative exponential</td>
<td></td>
<td>Cambrian sandstone</td>
</tr>
</tbody>
</table>

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From the early data (pre 1990) negative exponential distributions appear to fit the data well. However, as Gross (1993) and Bloomfield (1996) stated, these distributions appear to apply to global data rather than to individual fracture sets. It is the spacing distribution for individual fracture sets that would be required for modelling purposes. Many of the later analyses show a lognormal form, although it is recognised that this type of distribution is often a product of sampling biases. Distributions of the lognormal form and negative exponential form are special cases of the Weibull and gamma distributions.

It can be concluded from the literature reviewed that fracture spacing distributions tend to follow gamma or Weibull type distributions. The Weibull and gamma distributions both require two parameters ($S_o$ and $m$, and $\alpha$ and $\beta$, respectively) which are not directly related to easily obtainable parameters such as the mean and standard deviation or rock related properties. In the absence of a method for estimating the parameters $S_o$ and $m$, 

### Table: Fracture Spacing Distribution

<table>
<thead>
<tr>
<th>Source</th>
<th>Fracture Spacing Distribution</th>
<th>Number of Measurements</th>
<th>Geology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baecher (1983)</td>
<td>Negative exponential</td>
<td>15,000</td>
<td>Many types</td>
</tr>
<tr>
<td>La Pointe and Hudson (1985)</td>
<td>Negative exponential</td>
<td>312</td>
<td>Niagaran dolomite</td>
</tr>
<tr>
<td>Villaescusa and Brown (1990)</td>
<td>Negative exponential</td>
<td>212</td>
<td>Panguna andesite</td>
</tr>
<tr>
<td>Kulatilake et al. (1990)</td>
<td>Negative exponential</td>
<td>212</td>
<td>Stripa mine</td>
</tr>
<tr>
<td>Huang and Angelier (1989)</td>
<td>Gamma (lognormal form)</td>
<td>710</td>
<td>Gulf of Suez</td>
</tr>
<tr>
<td>Huang and Angelier (1989)</td>
<td>Gamma (lognormal form)</td>
<td></td>
<td>SE France</td>
</tr>
<tr>
<td>Narr and Suppe (1991)</td>
<td>Lognormal</td>
<td>596</td>
<td>Monterey Formation</td>
</tr>
<tr>
<td>Gross (1993)</td>
<td>Gamma (lognormal form)</td>
<td></td>
<td>Monterey Formation</td>
</tr>
<tr>
<td>Boadu and Long (1994)</td>
<td>Fractal and Weibull</td>
<td></td>
<td>Lake Thurmond gneiss</td>
</tr>
<tr>
<td></td>
<td>(negative exponential form)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bloomfield (1996)</td>
<td>Lognormal</td>
<td>167</td>
<td>Chalk - bedding planes</td>
</tr>
<tr>
<td>Bloomfield (1996)</td>
<td>Negative exponential</td>
<td></td>
<td>Chalk - vertical joints</td>
</tr>
</tbody>
</table>
and $\alpha$ and $\beta$ for the Weibull and gamma distributions respectively, using the lognormal distribution appears to be the best option.

Aside from this, it would be useful to discover whether those examples showing a negative exponential distribution are less fracture saturated than those exhibiting a lognormal distribution, and those in turn by the fractures showing a normal distribution (c.f. Rives et al., 1994). It may therefore be possible to relate the $\alpha$ and $\beta$ parameters of the gamma distribution and/or the $\mu$ and $S_p$ parameters of the Weibull distribution to the fracture saturation. Hence predictions of fracture spacing distributions could possibly be made from knowledge of rock type and fracture saturation, which in turn may come from the orogenic history of the region.

**Measured Fracture Orientation Distributions**

From the literature reviewed, very few orientation distributions have been studied or presented. Most of the data concentrate on assigning particular fractures into fracture sets. There does not appear to be much mention of distributions within these sets. Exceptions to this are Baecher (1983) and Kulatilake et al. (1990) who attempted to assign the distribution within the sets to various distributions. Both Kulatilake (1990) and Baecher (1983) found that the data fitted no distribution adequately, although there was a slight correlation to the Bingham and bivariate Fisher distributions for the Baecher data. Bingham distributions tend to be elliptical under certain conditions. Shanley and Mathab (1976) indicate that distributions about a mean direction are generally elliptical on a stereogram rather than circular, confirming Baecher’s results. It is therefore recommended that an elliptical distribution be used in 2-D, or a gaussian distribution in 1-D.

**Measured Fracture Trace Length Distributions**

Bloomfield (1996) studied the trace length distributions in Chalk at Play Hatch Quarry, Berkshire. It was found that, after correcting for sampling biases as described in Priest and Hudson (1981), the trace populations approximated to a negative exponential distribution with an average length of 0.09 m. When considering the trace lengths from the individual fracture sets, approximation to a lognormal distribution was found.

Priest and Hudson (1981) conducted two case studies, one on an Ordovician mudstone and one on a Cambrian sandstone for trace length distributions. It was found that the...
actual distributions approximated negative exponential distributions, except that at very small lengths a higher frequency of traces occurred than is predicted by the negative exponential pattern. This type of distribution can occur for certain values of $\alpha$ and $\beta$ in a gamma distribution. The mean trace length in the Ordovician mudstone was 0.87 m and the mean trace length in the Cambrian sandstone was 1.22 m. Again the fracture sets have not been differentiated and the values and distributions given are for the bulk or global values.

Baecher (1983) studies the trace length distributions of 15,000 measurements and concluded that the lognormal distribution represented the data most adequately. However, Baecher recognises that lognormal distributions are often a product of sample biasing, and therefore the result may not be 'real', even though the data were corrected for sampling biases as far as possible.

Kulatilake et al. (1990) measured the trace lengths for four different joint sets in part of the Stripa mine, Sweden. It was observed that all four sets of data approximated to gamma distributions of the lognormal type shape.

Even though the lognormal distributions observed may be a product of sampling biases that have not been recognised and corrected for, in the absence of further data and correction of biases, it is recommended that a lognormal distribution is used in the modelling process.

A summary of the distributions found in the literature is presented in Table 6.4:

<table>
<thead>
<tr>
<th>Source</th>
<th>Fracture Trace Length Distribution</th>
<th>Number of Measurements</th>
<th>Geology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Priest and Hudson (1981)</td>
<td>Negative exponential</td>
<td></td>
<td>Ordovician mudstone + Cambrian sandstone</td>
</tr>
<tr>
<td>Baecher (1983)</td>
<td>Lognormal</td>
<td>15,000</td>
<td>Many types</td>
</tr>
<tr>
<td>Kulatilake et al. (1990)</td>
<td>Gamma (lognormal form)</td>
<td></td>
<td>Stripa mine</td>
</tr>
<tr>
<td>Bloomfield (1996)</td>
<td>Lognormal</td>
<td>167</td>
<td>Chalk</td>
</tr>
<tr>
<td>Odling et al. (1999)</td>
<td>Lognormal/fractal</td>
<td>NK</td>
<td>W. Arabian sandstone</td>
</tr>
</tbody>
</table>
Measurements of Fracture Aperture Distributions

Snow (1970) measured aperture distributions in a granite in Colorado using a method involving photography and fluorescent dye penetrants. It was found that the apertures followed a lognormal distribution. Snow also derived aperture distributions from packer testing data, with distributions for granite, gneiss, meta-volcanics, schists, slates and sandstones being calculated. It was found that the apertures varied very little with rock type but varied more with depth. The distributions of apertures were again found to be mainly lognormal.

It is reported by Tsang and Tsang (1990) that aperture distributions in a single fracture in the laboratory usually follow a lognormal distribution. They do not report actual aperture distributions in the field.

Bloomfield (1996) measured apertures along a single fracture at Play Hatch Quarry, Berkshire and found that the field data approximated to a negative exponential distribution. It was considered, however, that two different populations made up the negative exponential curve with those apertures below 7 mm following a negative exponential distribution but those above 7 mm following a lognormal distribution. Those apertures >7 mm had solution-enhanced characteristics.

It could be concluded therefore that it is unwise to measure aperture distributions on weathered surfaces and on rocks that are prone to solution weathering. Fresh surfaces should be used wherever possible, although it is recognised that if quarry faces are used the blasting process may enlarge fracture apertures or create new misleading fractures.
Pyrak-Nolte et al. (1997) measured aperture distributions using a computer tomography (CT) method and found that aperture distributions in various coal series varied between normal and lognormal. It was found that cores which tended to have few long fractures tended towards a lognormal aperture distribution compared with cores with filamentary fractures having normal distributions of apertures.

Johns et al. (1993) used CT methods to measure apertures in crystalline rock and found that the aperture distributions followed a lognormal distribution.

A summary of the distributions found in the literature is presented in Table 6.5.

Table 6.5: Summary of fracture aperture distributions obtained in literature.

<table>
<thead>
<tr>
<th>Source</th>
<th>Fracture Aperture Distribution</th>
<th>Number of Measurements</th>
<th>Geology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snow (1970)</td>
<td>Lognormal</td>
<td>N/K</td>
<td>Colorado Granite</td>
</tr>
<tr>
<td>Gale (1987)</td>
<td>Gaussian</td>
<td>&gt;20000</td>
<td>Granite</td>
</tr>
<tr>
<td>Tsang and Tsang (1987)</td>
<td>Gamma/lognormal</td>
<td>100</td>
<td>N/K</td>
</tr>
<tr>
<td>Johns et al. (1993)</td>
<td>Lognormal</td>
<td>CT scan</td>
<td>Coal series</td>
</tr>
<tr>
<td>Bloomfield (1996)</td>
<td>Lognormal to -ve exponential</td>
<td>~150</td>
<td>Chalk</td>
</tr>
<tr>
<td>Keller (1997)</td>
<td>Lognormal</td>
<td>CT scan</td>
<td>Granite and sandstone</td>
</tr>
<tr>
<td>Pyrak-Nolte et al. (1997)</td>
<td>Lognormal to normal</td>
<td>CT scan</td>
<td>Crystalline rock</td>
</tr>
<tr>
<td>Montemagno and Pyrak-Nolte (1999)</td>
<td>Lognormal and Gaussian</td>
<td>CT scan</td>
<td>Coal</td>
</tr>
</tbody>
</table>

As with trace lengths, even though the lognormal distributions observed may be a product of sampling biases that have not been recognised and corrected for, in the absence of further data and correction of biases, it is recommended that a lognormal distribution is used in the modelling process.
6.3 Summary

Three possible data scenarios are likely to exist when considering a site for fractured rock protection zone delineation:

1. Full fracture characterisation of a site already exists or there is sufficient resources to perform at full fracture characterisation at a site;
2. Only some fracture data for a site exist; and
3. No fracture data exist for a site.

If a stochastic fracture model is to be used to delineate protection zones then the following data are required as a minimum for each fracture set:

- Fracture density values and the associated distribution type;
- Fracture trace length values and the associated distribution type;
- Fracture orientation values and the associated distribution type; and
- Fracture aperture values and the associated distribution type.

Table 6.6 presents some options for obtaining these data, if they do not already exist for a site, based on relationships presented in the reviewed literature. It is possible that all these data can be derived from areas with fracture data within the same structural provenance.
Table 6.6: Summary of methods for obtaining fracture data when collection of more field data is not possible.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Data required</th>
<th>Method for deriving data</th>
</tr>
</thead>
</table>
| Fracture spacing  | Values        | • Estimate from bed thickness from lithological logs, and also get cross joints spacing from bed thickness fracture spacing. Use constant of proportionality as $\sim 1$
|                   |               | • From RQD values after estimating distribution                                             |
|                   |               | • From percolation theory relations (Robinson, 1983 and Charlaix et al., 1984) and knowing trace lengths |
| Distribution type |               | Lognormal assumed in absence of no site data                                              |
| Fracture orientation | Values    | • If stress regime known use Mohr diagrams                                                |
|                   |               | • Is it on a fold, then use figure 6.5                                                    |
|                   |               | • Is near a fault, then use Rawnsley et al. (1992)                                         |
| Distribution type |               | Gaussian in 1-D and elliptical in 2-D assumed in absence of no site data                   |
| Trace length      | Values        | • From relationship with apertures (Hatton et al., 1994)                                  |
|                   |               | • From percolation theory relations (Robinson, 1983 and Charlaix et al., 1984) and knowing fracture densities |
|                   |               | • Ground penetrating radar data                                                           |
| Distribution type |               | Lognormal assumed in absence of no site data                                              |
| Fracture aperture | Values        | • From tracer studies                                                                    |
|                   |               | • From relationship with trace lengths (Hatton et al., 1994)                              |
|                   |               | • From the cubic law, known transmissivities and flowing fracture spacings                |
| Distribution type |               | Lognormal assumed in absence of no site data                                              |
7 CHARACTERISING PROTECTION ZONE UNCERTAINTY

This chapter presents methods by which uncertainty in protection zone modelling can be quantified. Initially, the sources of uncertainty arising in groundwater flow modelling are discussed. The existing approach to characterising uncertainty used by the Environment Agency is presented together with quantitative options available for characterising uncertainty in a more rigorous manner. Several methods are then developed for producing uncertainty contours from particle tracking simulations. The last section presents a discussion on the interpretation of the uncertainty contours.

7.1 Sources of uncertainty

Not only does modelling provide us with groundwater protection zones themselves, but it also furthers our understanding of the nature of groundwater flow, as well as providing us with a tool to make management decisions. As part of this management decision process, risk analysis has to be undertaken with respect to protection zones. If a deterministic approach to the groundwater modelling is taken then only a single representation of the aquifer system is used to produce the protection zones and no form of quantitative risk analysis is undertaken. For an adequate risk analysis the uncertainty associated with both the conceptual model and the input parameters must be taken into consideration. One approach to incorporating the effects of this uncertainty into the groundwater models is by producing probabilistic protection zones.

Protection zone uncertainty may originate from a variety of causes. In particular:

- *Uncertainty in the conceptual model.* For example, the positions of geological boundaries, type of flow (porous medium of fractured), and the interaction between a particular stream and the aquifer may be unknown or unquantifiable.

- *Uncertainty in the spatial distribution and the directional characteristics of parameters.* For example, hydraulic conductivity may vary horizontally and vertically from place to place (a heterogeneous medium), but the variation is unknown in some areas due to the sampling regime. Parameters such as hydraulic conductivity and porosity may also be anisotropic (vary directionally at one place).
Quantification of both the heterogeneity and the anisotropy can be both difficult and time consuming.

- **Uncertainty in flow equation and boundary condition parameters.** For example, aquifer thickness, hydraulic conductivity, storage and flows in and out of the system. There may be inaccuracies in measuring these parameters, or assumptions that are made when analysing for values of hydraulic conductivity and storage that are not strictly valid. An order of magnitude may be obtained for these parameters but the precise values will be unknown. Uncertainty in the boundary conditions for the model may also exist, due to lack of data in the area to be modelled.

- **Uncertainty in calibration parameters.** When attempting to calibrate the groundwater model, observed values of head, velocity or flow may be used in the comparison to the simulated values. These observed values may not cover a full spatial and temporal range, leading to a non-unique solution to flow within the aquifer system and potentially to a misleading representation of the aquifer system. The calibration parameters may also have measurement errors associated with them, for example flow gauging of streams can be highly inaccurate. The output of the model has to be matched to this representation and hence uncertainty may occur in the final selection of calibrated model parameters.

- **Uncertainty due to numerical dispersion in the model.** If the model is not correctly discretised over time and space then more dispersion can occur than is physically correct leading to further model output uncertainty.

### 7.2 Present Environment Agency approach to characterising uncertainty

The Environment Agency takes a ‘qualitative’ deterministic approach to uncertainty analysis in zone delineation (Environment Agency, 1996a). The Environment Agency recognises that in an ideal situation all input parameters to the groundwater flow model should be represented as probability distribution functions, but due to the lack of data for the majority of UK catchments it is not possible to define these distributions.

The Environment Agency ‘qualitative approach’ is such that three different zones are defined by reverse particle tracking through a basic uncertainty analysis. The ‘qualitative approach’ is in fact a semi-quantitative analysis. A ‘best estimate’ model is initially defined through model calibration, with the model parameters being varied only
over physically plausible ranges. A (brief) sensitivity analysis is then performed on three input parameters: hydraulic conductivity, recharge and kinematic porosity\(^1\). These parameters are then each varied within three multipliers of less than one, more than one, and one, from the best estimate model parameter set. The multipliers are developed from a sensitivity analysis where the calibration must remain within pre-defined limits. For capture zones, hydraulic conductivity and recharge are varied, and for the time-of-travel zones the kinematic porosity is also varied to produce a set of different models. This set of plausible models is then combined to define the zone of confidence (ZOC), the zone of uncertainty (ZOU) and the best estimate zone (BEZ). All these types of zones are then superimposed in a CAD environment and a confidence index (CI) is calculated as:

\[
CI(\%) = \frac{\text{Area}(ZOC) \times 100}{\text{Area}(ZOU)}
\]

The methodology recognizes the fact that time-of-travel zones will be more uncertain than catchment zones due to the fact there is an additional variable parameter used to calculate the zones (the kinematic porosity). The methodology also acknowledges the fact that uncertainty in an incorrect conceptual model, incorrect discretisation of the area and incorrect choice of model code are not taken into account, and the correct application of these three uncertain areas lies with professional judgment. Although the confidence index may give an indication as to how representative the zones are for the data available, the methodology does not give a criterion at which the model should be rejected, and a new conceptual model devised, a new model code tried or different hydraulic parameters used. The methodology also does not include variation in boundary conditions, river interaction and aquifer thickness in the sensitivity analysis of the subsequent definition of the BEZ, ZOU and the ZOC: these factors are likely to have a significant impact on zone uncertainty.

### 7.3 Methods of characterising uncertainty

There are various methods by which uncertainty can be taken into account in a model simulation. The following is a list of options that could ultimately lead to a range of protection zones for each travel time, the range of protection zones giving some measure of the uncertainty in the system.

---

\(^1\) See section 9.1 for a definition of kinematic porosity
1. Deterministic approach using a combination of input parameters to produce the mean, maximum and minimum sized protection zones for each travel time (bearing in mind that the maximum sized protection zone will not necessarily be a combination of maximum and minimum input parameters and vice versa).

2. Deterministic approach using the mean and a variance of field-derived input data to produce a mean protection zone for each travel time, and then an inner and an outer protection zone from, for example, two standard deviations from the mean data.

3. Fuzzy approach where the output of the fuzzy modelling is a membership function and which acts essentially as a probability density function. Fuzzy approaches tend to be used where there are very few data available.

4. Stochastic approach where aquifer properties are regarded as random variables with known distributions. The output of this approach is characterized by a full probability density function. There are no general methods of solving stochastic differential equations with random coefficients exactly, because the equations are non-linear. There are several methods by which an approximate solution to the equation can be obtained, for example:
   - Spectral analysis;
   - Direct solution approach via a response function;
   - Solution of differential equations describing the covariance functions;
   - Perturbation techniques; and
   - Monte Carlo simulations.

These solution methodologies were discussed in more detail in section 5.2.4.

Both deterministic methods (points 1 and 2 listed in the previous paragraph) are technically relatively simple to achieve and Evers and Lerner (1998) advocate such an approach whereby the aquifer parameters are varied between known ranges to obtain zones of confidence (ZOC) and uncertainty (ZOU) for groundwater sources. They state that it is inappropriate to attach confidence levels to an estimate of the capture zones because this implies a good knowledge of the uncertainties in the system. Therefore because of the lack of statistical data for each of the aquifer properties they restrict their approach to a qualitative one, incorporating only the range in each of the aquifer parameters to produce the ZOC and ZOU. Whilst it is difficult to argue with this pragmatic approach since for a typical groundwater source there are few data, it has
been shown in Chapter 6 that more can be done for fractured rock to represent the fracture parameters by probability density functions and to give upper and lower bounds to those functions. The fuzzy approach requires few data but still allows the results to be presented in a probabilistic way. Also, in recent years a risk-based approach to groundwater modelling has been advocated by the regulatory authorities (McMahon et al., 2001). To provide results in a risk-based framework a stochastic or fuzzy approach is required.

One of the principal aims of any new protection zone methodology should be to provide an uncertainty analysis that is more rigorous than the existing methodology. Therefore the fuzzy approach or the stochastic approach should be taken. A fuzzy approach is generally only used for situations where there are few data and also it is a technique with which most people are unfamiliar. One other aim of the proposed methodology was that it should be practical and transparent. This leaves us with using standard stochastic techniques through Monte Carlo simulations. The Monte Carlo technique is one with which most people are familiar as well this feature being included in many of the standard groundwater modelling software packages (e.g. Groundwater Vistas).

7.4 Developing methods to create probability contours

Having determined that a Monte Carlo approach to stochastic modelling appears to be the most appropriate way forward, methods now need to be developed to derive probability contours from the multiple reverse particle tracking exercises within the stochastic framework for each of the travel time zones and the capture zone. Two principal methods have been developed within this thesis:

1. a) End-point method based on equal angle sectors;
   b) End-point method based on equal numbers of particles in each sector; and

2. Travel time method.

These methods rely on reverse particle tracking since this is the standard method used by the Environment Agency for producing protection zones. Forward tracking methods have been used in the past to create probability contours, for example Wheater et al. (2000), but since one of the aims of a proposed new methodology is to use techniques with which the end users are already familiar it is more appropriate to use reverse particle tracking to create probability contours.
1. **End-point method**

This method relies on the use of multiple end points of a reverse particle tracking exercise for a given time to determine probability contours. The use of end points in determining regions of confidence or probability contours from stochastic groundwater modelling appears to have been used first by Bair *et al.* (1991) and Varljen and Shafer (1991).

The simplest way to explain the end point method is graphically. Figure 7.1 illustrates a typical set of end points after reverse particle tracking for a given time from a well using a stochastic discrete fracture network model, SDF (see Chapter 8 for more detail in the use of the SDF model).

![Figure 7.1: End points of 2000 reverse tracked particles using 40 realizations from a stochastic discrete fracture network model (see Chapter 8 for explanation of this 'diamond' shape).](image)

In considering the location of the end points the only criteria by which probability contours can be defined is the distance from the well in a given direction (in this case in Figure 7.1 the well is located at $x = 40$ m, $y = 20$ m). For example, to obtain the location of the 95% probability contour the area around the well could be divided up into either equal angle sectors or sectors containing equal number of end points. Within
each sector the location of the 95% contour would be the radius from the well where 95% of the end points would be positioned inside, or nearer the well. Only 5% of the particles would be located outside of this point. The radii or points lying on the 95% contour from all the sectors would then be joined to form the 95% probability contour. The meaning of this probability contour is discussed in section 7.5.

Figure 7.2: 10, 50 and 90% probability contours derived from the equal angle sector method.

Figure 7.3: 10, 50 and 90% probability contours derived from the equal numbers method.
The reasoning behind having not only an equal angle sector method, but one which considers equal numbers of end points within each sector is that the equal angle sector method may not produce satisfactory results for zones that are long and thin. Long and thin zones are likely to have some equal angle sectors that have a high density of particles, and other areas that have a low density within equal angle sectors. Ideally the solution to this problem would be increase the number of realizations used and hence the total number of end points, leading to more end points in the low density sectors. However, it may be more computationally efficient to consider sectors which have equal numbers of particles in them.

Macros have been written for both of these methods using Visual Basic for Applications. The equal angle sector method macro can be found in Appendix C. Examples of the 10, 50 and 90% probability contours from the end points presented in Figure 7.1 for the equal angles and equal numbers method are given in Figures 7.2 and 7.3 respectively. Due to the fact that the end points are relatively evenly dispersed around the well in all directions the equal angle sector and the equal number methods show almost identical probability contours.

Other similar end-point methods could be used to delineate probability contours, such as a moving average method. This type of method would consider a sector of given angle which would then sweep around the well creating the probability contours for every degree. This is likely to produce more even probability contours than the equal number or equal angle sector methods. However, the idea for this method has not been further developed here.

2. **Travel Time Method**

Although it has been shown to be possible to create probability contours using a relatively simple geometrical method, it is unlikely that for 3-D fracture networks this method is sufficiently robust. For example, in a 3-D fracture network it is possible, due to the complex connectivity of the system, the end point at a given time is not the furthest point from the well that the particle has been in its history. A false impression may be given as to the extent of the protection zone if only end points were used in the analysis. It is therefore necessary to record the path the particle travels over all modelled time to obtain an accurate definition of the protection zone. A new method of
combining all of the travel-time and position information to produce probability contours which is computationally efficient, robust and simply understood has been developed, as described below.

A simple and robust method would be to consider the times at which the reverse tracked particles from multiple scenarios and multiple realisations within each scenario pass through a section of the modelled area. Each section or area would then have a distribution of travel times associated with it. To model the probability contours for a particular time, the position of that time on the distribution would be recorded as a percentage of the total times, and these percentages could then be contoured. This method would have the advantage of containing all the particle track data within it, and therefore if contouring of different times was required it could easily be achieved. These ideas are illustrated in Figure 7.4.

Two different techniques for creating the probability contours are suggested: one for delineating the 50-day and 400-day contours and another one for delineating the total capture zone contours. For the 50-day and 400-day contour technique, the times at which particles pass through each element of the grid (e.g. a grid square) are noted. Many particles will pass through a grid element, because of the multiple scenarios and multiple realisations within each scenario that have been reverse tracked, so a distribution of travel times for each element can be determined. For the chosen travel time, i.e. 50 days or 400 days, the probability of particles taking longer than that time to reach the well is determined from examining the distribution. The probability is then contoured across the model domain to obtain the full set of probability contours for each travel time.

The technique for calculating the total capture zone probability contours is slightly different to the other protection zones. For each model realization, a flag is set for each grid element to indicate whether any particle passes through it. This is repeated for all scenarios and multiple realizations within each scenario. The probability of a grid element lying inside the capture zone is calculated as the proportion of realizations for which no particle has entered. The probabilities are then contoured in the same way as with the other protection zones. Thus, higher probabilities occur at greater distances from the well and indicate increasing certainty that the (probability) zones encompass the true capture zone. This method for defining probability contours for capture zones
is similar to that presented by Vassolo et al. (1998) where the probability is given by the fraction of catchments among all realizations which contain the point. This method results in higher probability contours closer to the well and is therefore not suitable for our requirements.

1. Reverse particle tracking

2. Distribution of travel times for each grid point

3. Contour required travel time

Figure 7.4: Technique for creating probability contours for 50-day and 400-day protection zones.

A code has been developed to produce the uncertainty or probability contours using distributions of travel times on grid points superimposed across the model domain from particle tracking. The probability contours produced by this code can be found in Chapters 8 and 10 and the Fortran coding in Appendix D.
The only problems found so far with the code produced are that a large amount of computer memory is required for the input file of particle track data to give a sufficient number of values for the distributions at each data point in the output probability file. The output file can then be contoured using a computer package such as Surfer™, although the choice of contouring method (e.g. kriging or linear regression) can make a significant difference in the form of the contours produced. Many of the methods do not tend to create the contours continuously, but will create a 'bulls-eye' effect or 'islands' with gaps where there is little data, rather than interpolating between points that should lie on the same contour. However, if care is taken with the contouring method, by weighting points over a reasonable area in linear regression or including a nugget effect in kriging, smooth contours result. If the number of data points is sufficiently small it maybe easier to hand-contour some probability zones. An example of the type of contouring obtained with Surfer™ can be seen in Figure 7.5 below.

In order that further confidence may be gained in the code for creating probability contours, a comparison has been made between the equal number end-point method and the travel time method for the same scenario with the same particle tracks. 10-day probability contours have been created for a site in Kent – Coombe Farm (see Chapter 8 for further details) using particle tracking with a 2-D code, SDF. Figure 7.5 illustrates the contours from the travel-time method and Figure 7.6 illustrates the contours from the equal number end-point method.

It can be seen from these diagrams that the size and shape of the probability zones are very similar to each other. This is expected since the same data were used to create the zones. However, on closer examination there are important differences that must be explained. The travel-time method contours are further from the well in all directions compared with the end point method, making the travel-time contouring method more conservative. For example, in the ‘x’ direction for the travel-time 10% contour the zone is 37 m larger, and in the ‘y’ direction it is 18 m larger. The discrepancy is smaller for the 90% contour with the travel-time zone being only 25 m larger in the ‘x’ direction and 15 m larger in the ‘y’ direction. Although these discrepancies are relatively small, the discrepancy is between 5 and 17% of the overall zone length. At travel times of greater than 10 days this discrepancy could become large in terms of area. Therefore the method of defining the probability contours is important, especially in terms of land use planning.
Figure 7.5: 10-day probability contours for Coombe Farm using the travel-time method for contouring.

Figure 7.6: 10-day probability contours for Coombe Farm using the equal numbers endpoint method for contouring.

The reason for the travel time method producing more conservative probability contours is that the accuracy of the different methods and the different grid spacing of the contoured points. Specifically the differences are due to:
i) \text{the way in which the contouring package interprets the data means that a general trend in the data (Surfer) is followed rather than the exact values at each point. The end point method is based on precise values at a point; and}

ii) \text{the difference in the grid spacing of data points for the two methods. For the travel-time method the grid spacing is even over the modelled area, whereas the end point method has a set number of points for each contour (20), as can be seen in Figure 7.6. Because the end-point method is based on a division of the end points into sectors of equal numbers of particles the spacing of the points used to define the contours can be uneven. For the 90\% contour, the spacing varies from approximately 10 m to 80 m.}

The discrepancy of the two methods is of the order of the spacing of the points used to create the contours. From this comparison it is considered, therefore, that much care should be taken in choosing either the grid spacing for the travel-time method or the number of sectors for the end point method. Also the comparison shows that the travel-time method tends to be the more conservative of the two, as well as being the more robust and efficient method for contouring many different travel times. Therefore, the travel-time method is the preferred method.

7.5 Understanding probability contours

Once the probability contours have been produced for a particular travel time, a question then arises as to what these contours mean and how they should be interpreted and used in terms of protection policy. It is easiest to begin to consider these issues and the definition in 1-D.

Let us suppose that probability contours have been produced for the 50-day travel time zone. The 10\% probability contours will be closer to the abstraction well than the 90\% probability contour. This is illustrated more clearly in Figure 7.7 which presents the travel-time distributions as they would appear from the code at two individual points around the well.
Figure 7.7: Diagram comparing the travel-time distributions obtained for a point near the well (a) and further from the well (b).

The probability contours are defined by the proportion of ensemble realisations for which particles released at a point reach the well in more than a given time e.g. 50 days.

The probability contours aim to show the confidence in modelling the system. The 10% probability contour suggests that we are not very confident that the well will be protected by the 50-day travel time contour and that the true 50-day travel time contour is not very likely to lie within this zone. The 90% contour suggests that we are much more confident about the protection the 50-day contour will provide and that it is quite likely that the true 50-day contour will lie within this zone. Therefore the higher the probability, the higher the confidence that the actual protection zone for the well lies within the area.

For the practical purposes of delineating the 50-day contour on a map, the 90% probability contour would provide a much more ‘precautionary’ zone. It is likely to be impractical to use probability contours higher than 90%, since as the percentage becomes very high (or very low) there is a large amount of uncertainty associated with the contours. This is because the number of particles associated with these very high and low contours is very small. If more realizations are included in the analysis then even if just a few particles are added at either extreme it can make a large difference in the contour position. The middle value contours (50%) remain approximately constant in position. The 90% contour does, however, appear to be reasonably stable from studies of the effect of number of particle tracking runs on the form of the probability contours (beyond about 1500 end points for the case of the equal angle sector method).
Figures 7.8 and 7.9 show this effect more clearly using the equal angle sector method with end points from reverse particle tracking from the 2-D fracture model SDF. Figure 7.8 presents results of the 50% contour using 500, 1000, 1500 and 2000 end points. The position of the contour remains almost constant, even comparing the contour using 500 end points, to that using 2000 end points. Figure 7.9 shows the increase in variability of the 90% contour definition with number of end points used compared with the 50% contour. For both the 50% and the 90% contours an infinite set of end points would produce an even diamond shape.

The width of the travel-time distributions at the individual points around the well also give an indication of the overall uncertainty in travel-times for the fracture network over that length of time and distance from the well. As the average travel time or distance to the well increases, the width of the travel-time distributions increases and so the uncertainty increases. This has been verified through a series of trials presented in Chapter 8. The trials also examine the impact of differing connectivities on this width of travel-time distributions.

The width of the travel-time distributions in a direction perpendicular to the contours, for example, along a streamline, can also be related to the distance between the probability contours. If the width of the probability contours (say between 10% and 90%) is large then the uncertainty in travel-times in that fracture network is large. Also vice versa: if the distance between the probability contours is small then there is a smaller amount of uncertainty associated with the system. This can be more clearly seen in Figure 7.10 where, for the same distance from the well, R, the distance between the contours can vary depending on the uncertainty in the system.
Figure 7.8: 50% probability contours derived from the equal angle sector method for 500, 1000, 1500 and 2000 end points.

Figure 7.9: 90% probability contours derived from the equal angle sector method for 500, 1000, 1500 and 2000 end points.
Even though we now have a method to define the probability contours and we are confident that the results produced are meaningful and (potentially) accurate, there is still the question as to whether the probability contours produced are correct. The confidence in the contours produced is only as great as the confidence we have that the model can adequately represent the fracture network underground. For example, there may be many assumptions used in the modelling, such as a uniform aperture distribution, or the model may be 2-D attempting to simulate a 3-D system. Therefore the probability contours will have an uncertainty associated with them due to the uncertainty in the conceptual model of the system, the modelling procedures and the data used (other factors are listed in section 7.1).

It is important therefore that the uncertainty on the probability contours is reduced as far as is practical. Identification of the factors that cause this uncertainty on uncertainty is thus a crucial process. Further to this, the relative degree to which these factors contribute to the uncertainty (on the uncertainty) and the ease with which these uncertainties can be reduced should be assessed. This assessment would allow any further work undertaken on the uncertainty of probability-based protection zones to be conducted in the most efficient manner possible.

If probability contours are to be used by the Agency in the future then comparisons should be made between the old and the new contours. It is possible that for some fractured rock situations the contours of probability may be close to a particular travel-
time contour from the porous media models of the Agency. It may also be possible that, within the subset of fractured rock scenarios where old and new zones are shown to have a relationship, that there is also a correlation with the type of fractured rock (sandstone, limestone, double porosity, double permeability or even fault bounded block). These possibilities need to be fully assessed since any correlations could be exploited for zones that have not had probability contours developed for them.

Having shown that it is possible to create probability contours, the next chapter attempts to delineate probability contours for two case studies in 2-D.
TESTING A 2-D METHODOLOGY

This chapter presents delineation of protection zones in fractured rock using numerical modelling in 2-D. The relevant regulatory authorities across the UK have taken a 2-D approach to protection-zone delineation, so to provide consistency with this approach, initially a 2-D approach is assessed. A 3-D approach to protection zone delineation is presented in Chapter 10.

In this chapter, some numerical models that might be used are discussed and then a fracture model is chosen with which to attempt 2-D protection zone delineation. The model is initially used in a series of studies to assess the impact of fracture geometry on protection zone shape and size. It is also used to assess the possibility of using an effective dispersion coefficient to define protection zones. Protection zones are then developed for two case study sites, going through the stages of data collection and collation, development of an appropriate conceptual model, through to numerical modelling. Finally, the problems of using a 2-D methodology are discussed.

8.1 2-D model selection

A review of the more widely available numerical 2-D fracture models is presented in Appendix B and section 5.5 and, based on these models, a subset has been selected that would be appropriate in a 2-D fractured rock protection zone methodology. The criteria for code selection were the capability for fracture flow and particle tracking, as well as being inexpensive to acquire, easy to use and having a stochastic capability. The only model available that fulfils these criteria appears to be SDF. SDF has also been used in previous fractured rock protection zone studies (Bradbury and Muldoon, 1993) and therefore has proved itself suitable for the types of exercise attempted in this chapter, with the added advantage that it has a stochastic capability.

SDF is a two-dimensional steady-state flow model with fractures in an impermeable matrix (Rouleau, 1988). The fracture input data required are the statistical distributions of fracture densities, lengths, apertures and orientations for each fracture set. Single values and lognormal distributions of the fracture parameters can be simulated. Transport is by advection simulated by stochastic particle tracking in the fracture network. Transmissivities are calculated using the cubic law. Many fracture sets can be simulated. The fracture network generation part of the code uses a Monte Carlo method.
to create a pattern of lines of distributed lengths and orientations. Each line represents
the trace of a fracture that cuts a slice of rock one unit in thickness. The lines are
generated one set at a time using specific distribution types given in the input file for
orientations and lengths. Fracture apertures are also selected from a given distribution
to each fracture trace. The model only has the capability to deal with no-flow, linear
head gradients or constant head boundary conditions (at the model boundaries as well as
at pumping wells) along fixed lengths of the model boundaries. Given this limitation, it
is possible that the model may be inappropriate for some situations. The other
limitations of the model are that it cannot handle dual porosity or dual permeability
systems, and has no transient capability.

Despite these limitations, the model is suitable for fractured rocks with an impermeable
matrix, it is relatively straightforward to use and has a stochastic capability, thus
allowing a rigorous uncertainty analysis. This model would be appropriate for many
aquifers within the UK with an impermeable matrix, such as volcanic and metamorphic
rocks together with some sandstones of low matrix porosity. As described in Chapter 2
there is large spectrum of fractured rock types in the UK and it is possible that different
modelling methods may have to be found for each type, for example double porosity
and double permeability systems. It would also be suitable if particulate transport in
any aquifer were being considered.

8.2 Protection zone geometry

This section initially describes the use of SDF to create probabilistic protection zones
and goes on to present probabilistic zones for a variety of fracture network geometries.
The impact on protection zone form is assessed for these differing fracture networks. It
is instructive to investigate the effects of varying fracture network geometry on
protection zone shape and form so that when protection zones are developed for real
scenarios in fractured rock, potentially using more sophisticated techniques, an
assessment of the feasibility of the size and form of the zones can be made, given the
starting fracture network geometry.

As discussed earlier (chapter 2) it should be recognised that there are many other factors
that will impact the form of a protection zone, such as recharge, boundary conditions
and groundwater-surface water interactions. These factors have not been included in
this study, but it is acknowledged that these may have even more influence on protection zone form than fracture flow characteristics.

The zones presented in Figures 8.1 through to 8.9 have all been developed for a travel-time of one year using reverse tracking of 50 particles for each realization. 40 realisations have been simulated for each scenario resulting in 2000 particles being used to create the probability contours. The pumping well is located in the centre of the model domain (x=30 m, y=30 m) using the same constant head condition in the well for each scenario (as opposed to pumping rate). The 10, 50 and 90% probability contours have been created using the end points of the particle tracks after one year using the equal number sector method, as detailed in section 7.4. Only one variable is changed for each scenario for comparative purposes.

The base case scenario is two orthogonal fracture sets of equal densities (1.0 m$^{-1}$), equal trace lengths (10.0 m) and equal apertures (10 μm), with a single value for each parameter used, as opposed to distributions of fracture parameters. The fracture parameter values are varied, and in some cases a lognormal distribution of the fracture parameters is used. The effect on the size and shape of the probability contours are then observed. The arrows on Figures 8.1 to 8.9 indicate the principal orientation of the fracture sets used in the simulation, as well as the standard deviation used for the case of Figure 8.7.

The connectivity of the fracture systems generated is also investigated for each scenario. Different measures of connectivity are detailed in section 6.1.4, but for this exercise the number of fracture intersections per unit area is used (Forde and Wei, 1997). From the 40 realisations used for each scenario an average number of fracture intersections is calculated. For the base case scenario this average is 3749 fracture intersections for the model domain area, and is the value to which all other scenarios are compared.

8.2.1 Variation in zone form with fracture density

Three different fracture densities were studied: 0.3, 0.6 and 1.0 m$^{-1}$. The resulting shapes of the probability contours are very different as can be seen in Figures 8.1a) and 8.1b).
Figure 8.1: One year probability contours developed using SDF with the equal number sector method for a) density of fracture sets = 1.0 m\(^{-1}\), and b) density of fracture sets = 0.3 m\(^{-1}\).
Figure 8.1a) illustrates the base case scenario of an even diamond shape with the vertices of the rhomboid parallel to the fracture set orientations. The diamond shape arises in this denser scenario because relatively direct paths will be available parallel to the principal fracture directions resulting in a higher effective hydraulic conductivity, allowing particles to travel further from the well in one year. The effective hydraulic conductivity will be the lowest midway between the two principal fracture orientations as there will be no direct path in these directions and hence particles cannot travel as far from the well. Figure 8.1a) also shows the 10% and 90% contours approximately the same distances away from the 50% contour.

Figure 8.1b) shows a very different situation with highly uneven contours. The variation of contours appears to show little correlation with fracture direction and distances between these probability contours vary significantly. The connectivity of this system is only 8% of that of the base-case scenario, showing that the fracture system is very sparsely connected. Relatively direct paths will not occur through the fracture network in particular directions because of the sparsely connected system.

A scenario where the fracture density is 0.6 m\(^{-1}\) has also been modelled. This modelling showed a result similar to Figure 8.1a) except the rhomboid was slightly less well defined. The connectivity of the system was 35% of the base case scenario.

However, the above observations are based around the comparison of contours for a fixed number of realisations and particle end points for different fracture densities. Because of the nature of the particle tracking at low fracture densities, many of the particles end in the same fracture for a given realisation, and therefore for the same number of realisations as the dense fracture systems, the number of particle positions to produce the contours from is limited. Therefore the contours in Figure 8.1b) appear uneven.
Figure 8.1 (cont.): One year probability contours developed using SDF with the equal number sector method for a fracture density of 0.3 m\(^{-1}\) for c) 80 realisations d) 120 realisations e) 160 realisations.
A brief investigation has been made to assess the impact of the number of realisations on the unevenness of the contours at a density of 0.3 m\(^{-1}\). Figures 8.1c), 8.1d) and 8.1e) illustrate the change in the contour unevenness with 80, 120 and 160 realisations respectively. It was considered possible that as the number of realisations increased the evenness of the contours would have increased. Figures 8.1c) through to 8.1e) illustrate that this is not the case for up to 160 realisations, with the contours remaining highly uneven. In Figure 8.1e) the 50% contour is furthest away from the well in the direction of the fracture sets, indicating that if the number realisations were dramatically increased the contours may tend towards the diagonal shape of those presented in Figure 8.1a). The distance between the 10%, 50% and 90% contours would, however, be much greater than that in Figure 8.1a). Figure 8.2 illustrates the fact that the contours would be more of a diamond shape if there were only one particle end point at each location, rather than many because of the limited number of fracture pathways for the particles. Figure 8.2 also illustrates that the distance between the 10% and the 90% contours would be much greater than in Figure 8.1a).

The simulation in the sparsely fractured system does, however, give an indication of the extremely uneven shape that true protection zones would assume in such rock types because of the extreme heterogeneity of the system.

Figure 8.2: Distribution of particle end points for 160 realisations (8000 particles) given a fracture density of 0.3 m\(^{-1}\) after one year.
8.2.2 Variation in zone form with fracture length

Three different fracture length scenarios were modelled: 5 m, 10 m and 20 m. Lognormal distributions were then used with the standard deviation being 10% of the mean. The results for the 5 m and 20 m scenarios are shown in Figures 8.3 a) and 8.3b) (10 m is the base case scenario as presented in Figure 8.1 a).

Figure 8.3a) illustrates the case for a trace length of 5 m. Compared with the base case in Figure 8.1a) the contours appear slightly more uneven and the shape of the contours is more rounded rather than diamond-like. Figure 8.3b) is much more angular and even in its contours, probably due to the improved connectivity between the fracture sets as the fractures get longer, accentuating the distance travelled by the particles in the direction of the fracture sets.

Figure 8.4 presents a scenario where a lognormal distribution has been applied to the trace lengths, with the standard deviation being 10% of the average length. It would appear that Figures 8.4 and 8.3b) are very similar indeed, with no appreciable difference in the area covered by the zones or the shape and evenness of the form of the probability based protection zone. The connectivity measure also indicates very little difference in the fracture systems with a variation of only 3%.
Figure 8.3: One year probability contours developed using SDF with the equal number sector method for a) trace length of fracture sets of 5 m, and b) trace length of fracture sets of 20 m.
Figure 8.4: One year probability contours developed using SDF with the equal number sector method for an average trace length of 20 m and a lognormal distribution.

8.2.3 Variation in zone form with orientation

Three different relative fracture orientations were modelled at 90°, 60° and 30° apart. Three further scenarios were then modelled with lognormal distributions applied to the orientations with a standard deviation of 10 degrees. Figure 8.1a) illustrates the case for fracture sets 90° apart and Figures 8.5a) and b) illustrate the cases for 60° and 30° respectively.

Note the marked elongation parallel to the fracture directions and also that the area covered by the different contours remains almost identical for the three different scenarios. The evenness of the contour shapes also changes with the relative angle with the contours becoming more uneven as the relative angle decreases. This effect is due to the connectivity of the system reducing markedly as the relative angle decreases, from 85% connectivity of the base case scenario at 60°, to 50% at 30°.
Figure 8.5: One year probability contours developed using SDF with the equal number sector method for a) a relative angle of fracture sets of 60° and b) a relative angle of fracture sets of 30°.
Figure 8.6: One year probability contours developed using SDF with the equal number sector method for a) a relative angle of fracture sets of $60^\circ$ and b) a relative angle of fracture sets of $30^\circ$, both with lognormal distributions on the orientations.
Figure 8.6a) and b) show the effect of applying a lognormal distribution to the relative fracture orientations. It would appear that the contours have become much smoother and the vertices of the rhomboid are less well defined since the fracture directions are now less well defined. The increased smoothness of the contours is a result of the increased connectivity compared with the probability zones without a distribution. The area enclosed by the protection zone probability contours also remains approximately the same.

8.2.4 Variation in zone form with number of fracture sets

Three different scenarios were studied, the base case with two orthogonal fracture sets, one with three fracture sets at 60° to each other and the case of four fracture sets at 45° to each other. The results for the 60° and 45° cases are presented in Figure 8.7a) and b) respectively, with the base case presented in Figure 8.1a).
It is noted that there is an increasing roundness of the protection zone shape with the increase in number of fracture sets that are evenly orientated. From the earlier modelling exercises on relative fracture orientations it should be relatively simple to infer the shape of protection zone contours for almost any combination of fracture sets in 2-D, if combined with the results from the effect of the number of fracture sets.

8.2.5 Variation in zone form with aperture

Three different aperture scenarios were modelled, at apertures of 10 \( \mu m \), 20 \( \mu m \) and 5 \( \mu m \). The effects of distributed apertures are not illustrated here since the results were too variable to be presented on the same scale as the other diagrams in order to make a comparison. The results of the modelling for apertures of 20 \( \mu m \) and 5 \( \mu m \) are presented in Figures 8.8a) and b) respectively. Figure 8.1a) illustrates the results for modelling of apertures of 10 \( \mu m \). Unlike in Figure 8.1a) the zones in Figure 8.8a) do not have the same dimensions parallel to each of the fracture directions. The reason for this is likely to be a combination of the variability in end point positions for each of the realisations at high apertures producing an uneven zone and the impact of the particles.
reaching the boundary of the model domain. As expected, in Figure 8.8b) the dimensions of the zones parallel to the fracture directions are the same.

It should be emphasised that Figures 8.8a) and b) are for constant drawdown rather than constant pumping rate. The higher the aperture, the larger the probability zones for a constant drawdown. For a constant pumping rate a larger aperture would produce smaller probability zones.

The size of the protection zone and also the relative spacing of the contours appear to be most closely linked with the aperture of the fractures. It can be concluded that out of the fracture parameters examined, aperture is the most important factor controlling the size of the protection zones. If the shape of the contours can be predicted to a certain degree it is therefore the aperture that controls the probability contours to the greatest extent. Unfortunately aperture is one of the most difficult fracture parameters to measure, and there is also the added complication that aperture will vary from fracture to fracture, and also within fractures (channelling).
10, 50 and 90% Probability contours (equal numbers method) for an aperture of 20 µm

10, 50 and 90% Probability contours (equal numbers method) for an aperture of 5 µm

Figure 8.8: One year probability contours developed using SDF with the equal number sector method for a) a fracture aperture of 20 µm and b) a fracture aperture of 5 µm.
8.2.6 Variation in connectivity with fracture network

For the SDF scenarios presented in section 8.2.5 the number of intersections over the model domain was recorded. The number of intersections per unit area (usually taken as volume but we are working in 2-D) volume gives a measure of connectivity. The results are presented in Table 8.1.

Table 8.1: Variation in connectivity with SDF scenario modelled.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Connectivity (no. of intersections)</th>
<th>% of base case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>3749</td>
<td>100.0</td>
</tr>
<tr>
<td>Density = 0.3 m(^{-1})</td>
<td>301</td>
<td>8.0</td>
</tr>
<tr>
<td>Density = 0.6 m(^{-1})</td>
<td>1341</td>
<td>35.8</td>
</tr>
<tr>
<td>Length = 5 m</td>
<td>3631</td>
<td>96.8</td>
</tr>
<tr>
<td>Length = 20 m</td>
<td>3718</td>
<td>99.2</td>
</tr>
<tr>
<td>Length = 5 m + distribution</td>
<td>3724</td>
<td>99.3</td>
</tr>
<tr>
<td>Length = 10 m + distribution</td>
<td>3743</td>
<td>99.8</td>
</tr>
<tr>
<td>Length = 20 m + distribution</td>
<td>3608</td>
<td>96.2</td>
</tr>
<tr>
<td>2 sets at 60(^\circ)</td>
<td>3208</td>
<td>85.6</td>
</tr>
<tr>
<td>2 sets at 30(^\circ)</td>
<td>1891</td>
<td>50.4</td>
</tr>
<tr>
<td>2 sets at 90(^\circ) + distribution</td>
<td>4289</td>
<td>114.4</td>
</tr>
<tr>
<td>2 sets at 60(^\circ) + distribution</td>
<td>3936</td>
<td>105.0</td>
</tr>
<tr>
<td>2 sets at 30(^\circ) + distribution</td>
<td>2608</td>
<td>69.6</td>
</tr>
</tbody>
</table>

From Table 8.1 it can be seen that there is a large change in connectivity with change in density of the fracture sets. Reducing the fracture spacing by approximately two-thirds reduced the connectivity to 35% of its original value. Reducing the fracture spacing by a factor of three reduced the fracture connectivity to 8% of its original value. The number of intersections is likely to reach zero as the fracture spacing approaches the length of the fractures.

From Table 8.1 it can also be seen that as the two fracture sets approach the same orientation, the connectivity falls dramatically. The function that the connectivity seems to take with relative fracture angle is the sine function (\(\sin 90^\circ = 1.0\), \(\sin 60^\circ = 0.86\), \(\sin 30^\circ = 0.5\)).
From the above table it would appear that the variation in lengths makes little difference to the connectivity of the system. This fact would seem to be counter-intuitive since as the length of a fracture increases the number of intersections per fracture would increase (a measure of connectivity). The small variation in connectivity with fracture length could be due to the way in which the connectivity has been defined for the above comparisons. Zhang et al. (1992) pointed out, the fact that the magnitude of the connectivity does not change does not imply that the pattern/form of the connectivity is the same. Different fracture patterns will often have the same connectivity (depending on how it is defined). Zhang et al. (1992) also note that when the model domain length/width becomes more than 10 times the fracture length the connectivity stabilises. This should surely have implications for the size of a REV in fractured modelling exercises.

From section 8.2 it can be seen that the changes in connectivity are related to the changes in form of the probability zones for a defined number of realisations. The density changes and the relative changes in fracture orientation had the most effect on unevenness and shape of the probability contours, a pattern that is also reflected in the connectivity. These changes in connectivity would reflect the true protection zone unevenness in shape. The probability-based zones, however, are unlikely to exhibit such unevenness if very large numbers particles were reverse tracked for many realisations.

8.3 Case studies
Following on from section 8.2, where it has been shown that probability contours can be created for the 2-D case, two case study sites have been chosen to assess the viability and practicality of a 2-D methodology. The selection of these particular two case study sites came from a group of six sites recommended by the Environment Agency following the completion of a questionnaire sent out to all Regions. The questionnaire, developed as part of this work, requested details of sites known to exhibit fracture flow characteristics that were considered to be of particular interest, and also the quantity of data that were associated with each of the sites. The six sites chosen are detailed in Table 8.2.
Further details of the six case study sites are given in R&D Project Record W6-020-1 (Robinson and Barker, 2000a). Only the two case studies for which probability contours were ultimately produced are presented here, Coombe Farm and Alton Court.

Table 8.2: Summary of case study sites.

<table>
<thead>
<tr>
<th>Case Study Site</th>
<th>Location</th>
<th>Geology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alton Court</td>
<td>Ross-on-Wye</td>
<td>Devonian Old Red Sandstone</td>
</tr>
<tr>
<td>Coombe Farm</td>
<td>Dover</td>
<td>Cretaceous Middle Chalk</td>
</tr>
<tr>
<td>Forstal</td>
<td>nr. Maidstone</td>
<td>Cretaceous Hythe Beds</td>
</tr>
<tr>
<td>Meysey Hampton</td>
<td>Cotswolds</td>
<td>Jurassic Inferior Oolite</td>
</tr>
<tr>
<td>Scales</td>
<td>north Cumbria</td>
<td>Permo-Triassic Sandstone</td>
</tr>
<tr>
<td>Tadcaster</td>
<td>nr. York</td>
<td>Jurassic Limestone</td>
</tr>
</tbody>
</table>

The following discussion of the case studies initially examines the relevant fracture data available at the site and data from literature sources, conceptual models for the sites are then developed, and finally numerical modelling and time-of-travel probability contours are simulated.

8.3.1 Coombe Farm

Coombe Farm public supply borehole is located on the Western outskirts of Dover in the base of a Chalk valley that trends approximately WSW-ENE, as illustrated in Figure 8.9. This dry valley (Coombe Valley) forms one of several valleys trending in this direction, and all are approximately parallel. The floors of these valleys are narrow and flat due to the infill from the valley sides, with the infill up to 4 m thick. These valleys all connect with the Dour valley which trends NW-SE, passing through the centre of Dover and containing the River Dour. The Dour valley is known to follow a fault zone. The Coombe Farm borehole penetrates the Middle and the Lower Chalk and was drilled and acidised in 1974.

The Coombe Farm site lies on the northern limb of the Wealden anticline structure which dips at 1 to 2 degrees to the NNE to NE, with the dip altered locally by folding and faulting. The folds are commonly monoclinal with an axis striking NW, with the northerly limbs being steeper. Faults are generally not of large throws and not usually traceable away from outcrop. There are many dry valleys within the area which trend in two general directions: NW-SE and NNE-SSW. These directions mirror the conjugate joint directions of N60°E and N25°E and it seems likely that they may be fault
controlled such as has been established in the River Dour valley (EA source proforma for Coombe Farm).

In previous porous media modelling, groundwater protection zones around Coombe were simulated using the East Kent Chalk model, which encompassed a large area with the following model boundaries:

- The Great River Stour to the west – approximately 28 km from Coombe Farm;
- The English Channel to the east – approximately 8 km from Coombe Farm;
- The Chalk escarpment forming the highest part of the aquifer extending from near Ashford to Folkestone formed the southern limit of the area - Folkestone being approximately 8 km WSW of Coombe Farm; and
- The Chalk dips gently to the NNE forming the southern limb of the Richborough syncline, which forms the valley in which the River Stour flows eastwards towards the sea at Pegwell Bay. This formed the northern limit of the model approximately 26 km north of Coombe Farm.

Available Data

i) Pumping Test Data

A pumping test was performed on the Coombe Farm borehole in November and December 1979 for 16 days. A representative transmissivity for the Middle Chalk was found to be 500 m²/d and a storage coefficient of 0.4%. Only one monitoring well, CV10, was found to have any drawdown over the pumping test period (2.49 m at the end of the test period). This monitoring well was 180 m west of the abstraction well; all the other monitoring wells were over 1 km from the abstraction well. Prior to the pumping test groundwater flow directions in Coombe valley were towards the WNW at a gradient of approximately 0.008.

ii) Geophysical Logging

Geophysical logs were run in CV10, 180 m west of the abstraction well, in June 1980. The geophysical logs (temperature, fluid conductivity, lateral resistivity, caliper and long normal resistivity) indicate fracturing in the Middle Chalk, but very few fractures in the Lower Chalk. The Lower Chalk boundary occurs about 55 m bgl. There is also evidence of large fissures at the Lower Chalk/Middle Chalk boundary in the Melbourn Rock. Data for fracture density and flowing fracture spacing has been obtained from the geophysical logs. The caliper log indicates that there are at least 24 major fractures
in the saturated part of the Middle Chalk. The temperature and conductivity logs indicate that of those 24 fractures approximately nine of those are hydraulically active, of which the majority are in the Melbourn Rock. Scanline data from the Channel Tunnel and its associated works has also been used for obtaining fracture data.

iii) Channel Tunnel data

Much data are available for the Channel Tunnel and the associated links, with most of the data relevant to the Coombe Farm site coming from the “Engineering Geology of the Channel Tunnel” (Harris, 1996). The larger structural features of the Dover area can be seen in Figure 8.9. The engineering geologists working on the Channel Tunnel divided the area up into different structural sectors - Coombe Farm lies in sector D.

![Figure 8.9: Location and major structural features around Coombe Farm.](image-url)
Table 8.3: Summary of fracture data for Sector D (Coombe Farm).

<table>
<thead>
<tr>
<th>SET</th>
<th>Orientation Range (°)</th>
<th>Dip Range (°)</th>
<th>Mean Dip (°)</th>
<th>Mean Orientation (°)</th>
<th>Approximate Density (m⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>025 - 035</td>
<td>50 - 90</td>
<td>65</td>
<td>030</td>
<td>&lt;0.5</td>
</tr>
<tr>
<td>F2</td>
<td>050 - 080</td>
<td>50 - 90</td>
<td>65</td>
<td>065</td>
<td>&lt;0.5</td>
</tr>
<tr>
<td>F3</td>
<td>085 - 095</td>
<td>50 - 90</td>
<td>70</td>
<td>090</td>
<td>&lt;0.5</td>
</tr>
<tr>
<td>F4</td>
<td>115 - 125</td>
<td>50 - 90</td>
<td>90</td>
<td>120</td>
<td>&lt;0.5</td>
</tr>
<tr>
<td>F5</td>
<td>130 - 140</td>
<td>50 - 90</td>
<td>90</td>
<td>135</td>
<td>~1</td>
</tr>
</tbody>
</table>

Harris (1996), Warren and Harris (1996) and Birch and Warren (1996) provide detail of the structural aspects of the Dover area. The folding in the area is such that anticlinal structures are superimposed on the gentle NNE dip of the northern limb of the Wealden anticline. In the Dover area the anticlines are subparallel and trend WNW-ESE to NW-SE. Minor folds also exist with amplitudes of approximately 1 m and follow the trend of the main Alpine system (WNW-ESE).

Extensive work has been conducted on the joint systems within sector D by Harris, Warren and Birch in their publications. A summary of the fracture sets, strikes and dips can be found in Table 8.3.

Harris (1996) also calculates the percentage of flowing fractures as a percentage of the total fractures detected at around 15 %. Therefore the actual flowing fracture density of sets F1 to F4 is 0.075 m⁻¹, and F5 is 0.15 m⁻¹.

As well as the five fracture sets identified by Harris (1996) there is also likely to be a bedding plane fracture set. The flowing fracture spacing of this set is not known, but from the geophysics data and the Channel Tunnel data an estimate can be made. Given the angle at which the fracture sets F1 to F3 intersect the Coombe Farm borehole (sets F4 and F5 are unlikely to intersect the borehole since they are vertical) and given that the fracture spacing for each set is 13.3 m, only 3 fractures would intersect the Middle Chalk saturated thickness of the borehole. However, nine flowing fractures were detected with the geophysics. Six fractures could therefore be attributed to being from the bedding plane set, at a spacing of approximately 4.2 m (although it is possible that all flowing fractures are from sets F1 to F3, given that density information is only approximate). The spacings and numbers of fractures intersecting the Coombe Farm...
borehole are only approximate since average values have been used, and also the data is not site specific (although data are from within the same structural block).

Due to the nature of the Channel Tunnel and the surrounding cliffs, the scanlines performed by Harris (1996) were generally no longer than 17 m. All fractures were found to be at least this long, so their exact length and distribution of lengths is unknown. Aperture data are also sparse so data must be derived from other sources.

iv) Apertures
The transmissivity data from the pumping test can be converted to hydraulic conductivity values since the effective saturated thickness of the well is known. Then, assuming the cubic law and knowing the number of flowing fractures in each well from a combination of the geophysical data and the Channel Tunnel data, an approximate ('hydraulic') fracture aperture can be obtained. It is recognised that this aperture is probably the lower end of any observed aperture variation since the cubic law does not take into account that fractures are not parallel plates and will not be completely open at all points. The true fracture aperture could potentially be very different from that calculated because of channelling processes, with flow being concentrated in several larger aperture channels rather than narrower parallel plates. Fracture apertures cannot be obtained for each fracture set, but only for all fracture sets since the transmissivity relates to the combination of all the flowing fractures, regardless of set.

From the pumping test data the hydraulic conductivity of the Middle Chalk at Coombe Farm is considered to be in the region of 20 m/d. From the geophysics there are nine flowing fractures, with each fracture therefore having an average hydraulic conductivity of 2.22 m/d. Using the cubic law this gives an average aperture of $6.40 \times 10^{-6}$ m.

v) Trace Lengths
The trace lengths of the fractures are known to be longer than 17 m from the Channel Tunnel scanlines. However, the upper limit on the fracture lengths is unknown. In the discussion on trace lengths in section 6.1.1, Hatton et al. (1994) present a relationship between aperture and length. Since we have an estimate of aperture we can obtain an approximate trace length. From the Hatton et al. (1994) relationship and using an aperture of $6.40 \times 10^{-6}$ m, an upper limit trace length of 0.45 m is calculated.
Robinson (1983, 1984) and Charlaix et al. (1984) also demonstrate relationships between fracture length and density, as described in Section 6.1.3. At least nine fractures are known to be hydraulically active in borehole CV10 over a thickness of 25 m, giving a 1-D fracture density of 0.36 m\(^{-1}\). Xu and Jacobi (1998) developed a relationship between the 1-D fracture density and the 2-D fracture density:

\[ D = 1.3488f + 0.0285 \]

where: 
- \( f \) is the 1-D fracture density (m\(^{-1}\))
- \( D \) is the 2-D fracture density (m\(^2\))

We can use this to convert the 1-D site data to 2-D for use in the Robinson (1983, 1984) relationships. The 2-D fracture density for the site is therefore 0.51 m\(^2\). If the relationships of Robinson (1983, 1984) are used fracture lengths of between 1.68 m and 1.73 m are obtained. If the relationship of Charlaix et al. (1984) is used a range of lengths between 1.66 and 1.32 m is obtained (although the site data has not been corrected to a 3-D fracture density).

These trace lengths are well below the correct order of magnitude, which contradicts what is known about the actual trace length from the Channel Tunnel data. Therefore a trace length must be approximated for the modelling. The low values obtained from percolation theory relations may be partly due to the fact they are the minimum required for percolation, with flow at Coombe Farm being under conditions well above the percolation threshold at longer trace lengths.

**Conceptual Model**

The principal features of the conceptual model at Coombe Farm are as follows:

- Groundwater flow is predominantly through fractures concentrated in the unconfined Middle Chalk, specifically in the Melbourn Rock, with negligible flow through the matrix. Flow is concentrated in the upper 25 m of the saturated thickness, with negligible flow through the Lower Chalk.
- Groundwater levels are approximately 30 m bgl and the hydraulic gradient across site is approximately 0.008, with flow from west to east (data from 24\(^{th}\) October 1979 water levels).
• The transmissivity of the Middle Chalk at the site is 500 m^2/day. The effective saturated thickness is 25 m giving a hydraulic conductivity of 20 m/d.

• Six fracture sets are considered to be present at the site. The orientations and dips of five of these sets are given in Table 8.3. The flowing fracture density of the sets F1-F4 is 0.075 m^-1 and of F5 is 0.15 m^-1. A bedding plane fracture set will also be present at the site which is approximately horizontal, sloping at 1 to 2 degrees to the NNE to NE. The flowing fracture spacing of this bedding plane set is not known, but is considered to be approximately 4 m. The average hydraulic aperture for the fractures is $6.40 \times 10^{-6}$ m. The trace length of the fractures is considered to be in the region of 20 m from Channel Tunnel data.

• Recharge in the vicinity of the site depends on the presence of the low hydraulic conductivity Clay-with-Flints which overlies the Chalk on higher ground. It is estimated by the Environment Agency that recharge through the clay is approximately 36.5 mm/year and that recharge in the vicinity of the Coombe Farm site is 277 mm/year. 1 km to the west of Coombe Farm Clay-with-Flints is present.

• The site is located within a dry valley whose axis slopes downwards towards the ENE. In the immediate vicinity of the site there are no major surface water features.

• The regional flow boundaries for Coombe Farm are considered to be the English Channel to the east and south, the Richborough syncline coinciding with the River Stour to the north, the Great River Stour to the west and the chalk escarpment to the south and west. More locally the groundwater flow system boundaries are the fault/River Dour valley to the northeast of the site, the River Dour to the north of the site, the English Channel to the south of the site and the Chalk escarpment groundwater divide to the west and south of the site.

The Coombe Farm borehole is licensed to abstract 2500 m^3/day. A drawdown of approximately 20 m is achieved at this abstraction rate.
Numerical Modelling

SDF was used in an attempt to model travel-time protection zones at Coombe Farm. Although the Chalk is recognised as a double porosity medium, the first arrival times of a potential dissolved phase contaminant or particle at the well will be governed by fracture flow and therefore the use of SDF in this case is considered acceptable.

The model was set up using the fracture parameters and hydraulic gradient outlined in the conceptual model. A trace length of 40 m had to be used in order to allow the fracture network to percolate fully. The use of a fracture length greater than on-site data suggest is discussed further in section 8.4. Recharge is not considered within the model. A drawdown of 20 m was used at the pumping well.

Fracture aperture was the parameter chosen to calibrate the model since this is the value that is the most uncertain at the site. Aperture was varied to achieve the correct pumping rate for the 20 m of drawdown. The calibrated aperture was $7 \times 10^{-5}$ m. This value is an order of magnitude more than that estimated from the field data, and even though this value is derived from 2-D modelling, it is considered to be more representative of conditions at the site.

It was discovered that because of the number of fracture sets and the density within each set a model domain larger than 275 by 475 m could not be simulated. This was due to restrictions in the fixed array sizes within the SDF Fortran 77 code. Attempts were made to increase the array sizes for number of fractures but it was found that the majority of the other arrays also required resizing as a result. To make the code accept any number of fractures it would require converting from Fortran 77 to Fortran 90, and this was considered to be beyond the scope of this exercise. It is also known that other fracture models (e.g. Fracman) have similar problems, with the maximum model domain size often being small compared with catchment size, because of computer memory limitations. Therefore, a solution to this more fundamental problem must be found other than adjusting array sizes in SDF.

Despite the model domain limitation, protection zone modelling was then attempted. 20 realisations were simulated each with 50 reverse tracked particles. The 1000 reverse tracked particles were then used to create the probability contours using the travel-time
method. Because of the size of the model domain only 10-day probability contours could be created. The contours presented in Figure 8.10 have been created using Surfer™ using the inverse distance to a power method of interpolation (power of 2 and smoothing factor of 5).

It is important to recognise that the accuracy of the probability contours is not known since there are no tracer test data with which to calibrate/validate the model.

Figure 8.10: 10-day probability contours for Coombe Farm derived from 20 realisations using flowing fracture spacings, and using the travel-time method for contouring. Groundwater flows approximately west to east.

The SDF derived 10-day probability contours show an approximately rectangular shape, the long side of which is aligned in an ESE direction. The two diagonal axes of this rectangle are aligned in the direction of:

1. the principal hydraulic gradient from west to east and also parallel to sub-vertical fracture set F3
2. the strikes of the vertical fracture sets F4 and F5 of 120° and 135° respectively.

It is interesting to note that the probability contour shape is governed by the vertical fracture sets and fracture sets aligned in the principal flow direction. The fracture sets that are not aligned with the principal flow direction and are sub-vertical (dip of 65°) do not appear to have as much of an impact on the zone shape.
It is important to recognise that the shape of SDF derived zones are dictated by the orientation of the model boundaries, the direction of the hydraulic gradient and the fracture orientations. Ideally the model domain should be much larger, with the boundaries governed by the River Dour and its valley, the coast, the Great River Stour and the Chalk escarpment as a minimum size, rather than their location being dictated by the SDF array sizes. Therefore the numerical modelling exercise for Coombe Farm should be considered only as a more complex form of the simulations performed in section 8.2 on the effect of fracture orientation and number of fracture sets on protection zone size and shape. The Coombe Farm SDF simulations should not be regarded as a realistic assessment of protection zone size and shape. This is emphasised further in the following paragraphs.

Figure 8.11 gives a comparison of the 10-day probability contours from SDF and the 50-day contour from the porous-media modelling. The porous media model hydraulic conductivity is defined as 60 m/d at the site, but drops to 0.6 m/d beneath the Clay-with-
Flints 1 km to the west of the site. The recharge in the porous media model is defined as 277 mm/year at the site, dropping to 36.5 mm/year 1 km west of the site beneath the Clay-with-Flints. Both these distributions combined have resulted in the triangular shaped 50-day porous media zone, with the abstraction well preferentially drawing water from either side of the Clay-with-Flints area. This effect becomes more pronounced for the 400-day travel-time zone.

SDF does not have the capability to create zones of different average hydraulic conductivity of the fracture system within the model domain. Thus, areas of Clay-with-Flints, for example, cannot be taken into account. It is for this reason that the shapes of the 10-day probability contours and the 50-day travel-time zone are not similar. Therefore, a method must be found to incorporate different average hydraulic conductivity zones within the geometry of the fracture system in order to obtain more accurate protection zones in fractured rock scenarios.

Recharge is also not included in the fracture model. Recharge distribution is an extremely important parameter in determining protection zone size and shape. Any fracture model used must include recharge in order that fractured rock probability contours are at least as accurate as the porous-media travel-time zones. Because there is no recharge included in the fracture model, the fracture probability zones represent the worst-case scenario, i.e. during summer when there is minimal recharge the zones are at their largest.

The area covered by the 10-day probability contours is approximately one fifth of the area of the 50-day travel-time zone. Even though this relationship between the areas of the zones using the two models exists, both the kinematic porosities and the velocity fields in the Flowpath and SDF modelling are different. A kinematic porosity of 1% is used in Flowpath and a kinematic porosity of 1.75% is calculated by SDF. Therefore to compare the two zones would be inappropriate.

It can be seen from the comparison of the SDF fracture modelling and the porous media modelling for Coombe Farm that the impact of recharge, the model boundaries and zoning of hydraulic conductivity appear to be more important factors in governing the shape and size of the zone than the fracture system. Although it is possible that the fractured rock effects for some sites are greater than the more regional factors of
recharge and hydraulic conductivity zoning, for this case fracture flow effects appears to be less important.

8.3.2 Coombe Farm effective dispersion coefficient modelling

The use of effective dispersion coefficients to produce probabilistic protection zones has been discussed in section 5.3.2. Effective dispersion coefficients could potentially be used in remodelling within the area, but would initially have to be defined from many stochastic modelling exercises.

From the fracture modelling for Coombe Farm it has been shown that the probability contours created are almost elliptical in shape. From section 8.2 where the change in probability contour size and shape with connectivity was examined, it was seen that at higher connectivities the zone shape is even and to some extent predictable. Therefore, for these higher connectivity situations is it possible to define an effective dispersion coefficient for the fractured rock system?

In this section a short study is presented to assess whether at certain connectivities an effective dispersion coefficient can be defined for relatively densely fractured systems, and to assess the characteristics of this dispersion. The fractured system used for this study is based around the fracture system data used for the Coombe Farm modelling. Four different connectivities are examined: 130, 670, 1700 and 2650 nodes (or intersections) within the model domain. The connectivity of 2650 nodes refers to the fractured system at Coombe Farm. The densities of the fracture sets were lowered to achieve the other three connectivities. The apertures of the fractures were lowered compared with the Coombe Farm model so that particles took approximately one year to cross the model domain under a hydraulic gradient of 0.008 (as at Coombe Farm) down the length of the model domain. 50 particles were started from the middle of the up-gradient side of the model domain and 10 realisations of the fracture network were used, to give 500 particles for each scenario. The particle distributions were examined every 50 days until 300 days and then at 365 days.

When studying the nature of dispersion in a fracture model it is important to understand how particles behave at the fracture intersections. Particles travel through the fractures at a rate determined by the hydraulic gradient and size of the fracture. At an intersection in a modelled system various options could be programmed to determine
the next fracture that the particle enters depending on the conceptual model of mixing at the intersection. The direction in which the particle will go will depend on the way in which the mixing at the intersection is treated. The fracture modelling of Schwartz et al. (1983) assumed that complete mixing would occur, and this assumption is based on work done by Krizek et al. (1972) in laboratory experiments of a single fracture junction. For complete mixing, transfer of particles at an intersection is treated as a random function weighted by the flow into the exit fractures. However, Hull et al. (1987) point out that complete mixing at a junction will only occur if apertures are small and velocities are relatively low (apertures on the order of microns and velocities of up to several metres per day are quoted). Streamlines have also been used for modelling of tracers across junctions. As the individual particle reaches the junction the streamline on which it is located is identified and the corresponding exit point calculated. However, the streamline approach can underestimate the lateral spread of a plume, with Hull et al. (1987) showing that transfer of particles across streamlines plays a significant role in dispersion of particles in fracture systems. Hull et al. (1987) state that for permeable systems complete mixing at intersection is probably not justified, and it can overestimate the lateral spread at intersection. The conceptual models of mixing at an intersection are presented in Figure 8.12.

![Figure 8.12: Two models for particle behaviour at fracture intersections.](image)

In the study presented here dispersion within individual fractures has not been included. Only dispersion within the overall fracture system has been examined. Within the SDF model complete mixing is assumed at fracture intersections, forming the worst-case scenario in terms of magnitude of dispersion.
Figure 8.13 Particle locations perpendicular or transverse (y direction) and parallel or longitudinal (x direction) to the hydraulic gradient after 200 days for fracture systems with 2650 nodes and 130 nodes.
The results of the dispersion modelling are presented in full in Appendix E, with two examples from a time of 200 days for the greatest and least number of nodes tested (2650 and 130 nodes respectively) shown in Figure 8.13. The locations of the particles perpendicular (y or transverse direction) and parallel (x or longitudinal direction) to the principal flow direction over time were examined initially.

Considering the location of particles in the transverse direction after 200 days, Figure 8.13 shows that with increasing number of nodes and connectivity of the system the spread of the particles is much less. This observation is unsurprising since the distance from one node to the next is relatively small, implying that there will be a fracture parallel to the overall hydraulic gradient along which the particle will preferentially migrate after only travelling a small distance in the transverse direction. From Appendix E it can also be seen that as the time after particle release increases the spread of the particle locations becomes larger. This can be explained by the fact that as time increases the number of nodes that the particle passes through increases, thus increasing the possibility of particles travelling further away from the initial transverse position, as explained in the paragraph on mixing at junctions. In the transverse direction the distribution of particle locations is relatively smooth for all times, with a significant possibility that the distribution observed may closely match a distribution of a standard type, for example, Gaussian, lognormal, or triangular.

Considering the location of particles in the longitudinal direction after 200 days, Figure 8.13 shows that with increasing number of nodes and connectivity the spread of particles becomes narrower. Again, this is because the distance from one node to the next is relatively small, implying that there will be a fracture parallel to the overall hydraulic gradient along which the particle will preferentially migrate after only travelling a small distance in the transverse direction. From Appendix E it can also be seen that as the time after particle release increases the spread of the particle locations becomes larger. The explanation for this has been given in the previous paragraph. Note that even at higher connectivities the distribution of particle locations is not as even as that in the transverse direction. As the connectivity of the system decreases the unevenness in distribution becomes even more apparent, with no apparent recognisable form to the distribution.
One unexpected observation on the distributions of transverse particle locations is that very few particles are located where they initially started off. The reason for this is unclear, but could be an artefact of the way in which the particle locations are divided into ‘bins’ to illustrate the frequency distribution. The bin next to the initial location often shows anomalously high frequencies, especially for times less than 250 days, which combined with the anomalously low frequencies at the initial location would provide a smoother overall distribution.

In the next stage the distribution that the data most closely followed was examined. This was done using the Crystal Ball® 2000 software (Decisioneering Software Inc.). In porous media the concentration distribution from hydrodynamic dispersion is considered to follow a Gaussian distribution (Fetter, 1993), the hydrodynamic dispersion being a combination of mechanical dispersion (pore size differences, path length differences and friction differences across the pore) and molecular diffusion. Although, by eye, at higher connectivities the transverse particle frequency diagrams appear to follow a Gaussian distribution, the longitudinal diagrams (x direction) seem to show more of an extreme value distribution, with many particles having travelled smaller distances than the most frequent particle location. This extreme value distribution fits well with the tailed distribution seen when tracer testing is performed in fractured rock, with many of the particles lagging behind the main peak because of the many different flow paths.

Before making a more formal analysis of the distributions, a check must be made as to the stability of the distributions as the number of particles used changes. The four graphs in Figure 8.14 show how the variance stabilises with number of particles used. It can be seen that for 2650, 1700 and 670 nodes that the variances in the longitudinal and transverse directions stabilise after 250 to 300 particles. However, no stabilisation is observed for 130 nodes. It is possible that this occurs because more particles are required in order to define a stable variance, but is more likely due to the fact that a variance (and hence an effective dispersion coefficient) is not definable for this low connectivity.
Figure 8.14: Stabilisation of particle location variance with number of particles in the longitudinal and transverse directions.
Table 8.4 presents the results of the best-fit match of the distribution of locations to recognised continuous distributions using Crystal Ball®. The chi-squared test has been used to assess the goodness of fit to the most closely matching distribution. From this table it can be seen that the results do not closely match any one type of distribution for either the longitudinal or the transverse results. It can be seen that as time increases the fit to a particular distribution improves with time (lower chi-squared values).

Table 8.5 presents the results of matching the distributions of location data to a Gaussian distribution, with the goodness of fit assessed using the chi-square statistic, and whether the data match to a Gaussian distribution is statistically significant (significance level of more than 10%). For the number of bins of data used (see Appendix E) the number of degrees of freedom varies between 12 and 52 (higher at later times), resulting in a chi-squared statistic of between 3.6 (for 12 degrees of freedom at the 99% significance level) and 65 (52 degrees of freedom at the 10% significance level). From Table 8.5 it can be clearly seen that the match to a Gaussian distribution is very poor at all times for the data in the longitudinal direction. Only at the highest connectivity is there some match to a Gaussian distribution. However, for the transverse data the match to a Gaussian curve is good for times of greater than 150 days. Contrary to observation of the frequency distributions, the fit to a Gaussian curve improves with decreasing connectivity in a transverse direction. The reason for this is likely to be because at shorter times the match to a Gaussian is poorer than expected, as it is actually tends to a more triangular distribution, even though the form of the distribution curve is very symmetrical and even.

From these tables and graphs it can therefore be concluded that the nature of the dispersion in the transverse and longitudinal directions is very different, with the dispersion in the transverse direction tending to follow a Gaussian distribution relatively closely.
Table 8.4: Best fit distributions for transverse and longitudinal particle locations for different connectivity systems.

Transverse dispersion characteristics

<table>
<thead>
<tr>
<th>Nodes</th>
<th>2650</th>
<th>1700</th>
<th>670</th>
<th>130</th>
</tr>
</thead>
<tbody>
<tr>
<td>Days</td>
<td>Z²</td>
<td>Distribution</td>
<td>Mean</td>
<td>St. Dev²</td>
</tr>
<tr>
<td>50</td>
<td>9.36</td>
<td>Normal</td>
<td>0.04</td>
<td>0.26</td>
</tr>
<tr>
<td>100</td>
<td>1.89</td>
<td>Logistic</td>
<td>0.02</td>
<td>0.19</td>
</tr>
<tr>
<td>150</td>
<td>4.8</td>
<td>Normal</td>
<td>-0.01</td>
<td>0.45</td>
</tr>
<tr>
<td>200</td>
<td>2.6</td>
<td>Normal</td>
<td>0.04</td>
<td>0.51</td>
</tr>
<tr>
<td>250</td>
<td>1.9</td>
<td>Normal</td>
<td>-0.08</td>
<td>0.55</td>
</tr>
<tr>
<td>300</td>
<td>1.7</td>
<td>Normal</td>
<td>-0.12</td>
<td>0.56</td>
</tr>
<tr>
<td>365</td>
<td>1.7</td>
<td>Normal</td>
<td>-0.17</td>
<td>0.63</td>
</tr>
</tbody>
</table>

This column contains the mean for the normal distribution, for other distributions it contains the 'first number'

This column contains the standard deviation for the normal distribution, for other distributions it contains the 'second number' and the 'third number'

Extreme distribution - first number is the mode and second is the scaling factor

Logistic distribution - first number is the mean and second is the scaling factor

Weibull distribution - first number is the location, the second is the scaling factor and third is the shape factor

Gamma distribution - first number is the location, the second is the scaling factor and third is the shape factor

Beta distribution - the first number is the uncertain variable, the second two are shape factors which determine the skewness of the distribution

Triangular - the first number is the minimum, the second the mode and the third the maximum

Longitudinal dispersion characteristics

<table>
<thead>
<tr>
<th>Nodes</th>
<th>2650</th>
<th>1700</th>
<th>670</th>
<th>130</th>
</tr>
</thead>
<tbody>
<tr>
<td>Days</td>
<td>Z²</td>
<td>Distribution</td>
<td>Mean</td>
<td>St. Dev²</td>
</tr>
<tr>
<td>50</td>
<td>67</td>
<td>Weibull</td>
<td>-0.16</td>
<td>1.73,2.65</td>
</tr>
<tr>
<td>100</td>
<td>70</td>
<td>Extreme</td>
<td>2.3</td>
<td>0.67</td>
</tr>
<tr>
<td>150</td>
<td>52</td>
<td>Normal</td>
<td>2.55</td>
<td>1.2</td>
</tr>
<tr>
<td>200</td>
<td>58</td>
<td>Normal</td>
<td>3.12</td>
<td>1.41</td>
</tr>
<tr>
<td>250</td>
<td>45</td>
<td>Extreme</td>
<td>4.62</td>
<td>1.29</td>
</tr>
<tr>
<td>300</td>
<td>25</td>
<td>Extreme</td>
<td>5.31</td>
<td>1.61</td>
</tr>
<tr>
<td>365</td>
<td>42</td>
<td>Extreme</td>
<td>6.15</td>
<td>1.78</td>
</tr>
</tbody>
</table>

1 This column contains the mean for the normal distribution, for other distributions it contains the 'first number'

2 This column contains the standard deviation for the normal distribution, for other distributions it contains the 'second number' and the 'third number'

Extreme distribution - first number is the mode and second is the scaling factor

Logistic distribution - first number is the mean and second is the scaling factor

Weibull distribution - first number is the location, the second is the scaling factor and third is the shape factor

Gamma distribution - first number is the location, the second is the scaling factor and third is the shape factor

Beta distribution - the first number is the uncertain variable, the second two are shape factors which determine the skewness of the distribution

Triangular - the first number is the minimum, the second the mode and the third the maximum

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Table 8.5: Goodness of fit to a Gaussian distribution for transverse and longitudinal particle locations for different connectivity systems.

Transverse dispersion characteristics

<table>
<thead>
<tr>
<th>Nodes</th>
<th>2650</th>
<th>1700</th>
<th>670</th>
<th>130</th>
</tr>
</thead>
<tbody>
<tr>
<td>Days</td>
<td>Accept Gaussian match?</td>
<td>$\chi^2$</td>
<td>Mean (m)</td>
<td>St. Dev (m)</td>
</tr>
<tr>
<td>50</td>
<td>✓</td>
<td>936</td>
<td>0.04</td>
<td>0.26</td>
</tr>
<tr>
<td>100</td>
<td>x</td>
<td>290</td>
<td>0.02</td>
<td>0.35</td>
</tr>
<tr>
<td>150</td>
<td>x</td>
<td>48</td>
<td>-0.01</td>
<td>0.45</td>
</tr>
<tr>
<td>200</td>
<td>✓</td>
<td>26</td>
<td>0.04</td>
<td>0.51</td>
</tr>
<tr>
<td>250</td>
<td>✓</td>
<td>19</td>
<td>-0.08</td>
<td>0.55</td>
</tr>
<tr>
<td>300</td>
<td>✓</td>
<td>17</td>
<td>-0.12</td>
<td>0.56</td>
</tr>
<tr>
<td>365</td>
<td>✓</td>
<td>17</td>
<td>-0.17</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Longitudinal dispersion characteristics

<table>
<thead>
<tr>
<th>Nodes</th>
<th>2650</th>
<th>1700</th>
<th>670</th>
<th>130</th>
</tr>
</thead>
<tbody>
<tr>
<td>Days</td>
<td>Accept Gaussian match?</td>
<td>$\chi^2$</td>
<td>Mean (m)</td>
<td>St. Dev (m)</td>
</tr>
<tr>
<td>50</td>
<td>x</td>
<td>103</td>
<td>1.38</td>
<td>0.61</td>
</tr>
<tr>
<td>100</td>
<td>x</td>
<td>112</td>
<td>1.93</td>
<td>0.79</td>
</tr>
<tr>
<td>150</td>
<td>✓</td>
<td>52</td>
<td>2.55</td>
<td>1.2</td>
</tr>
<tr>
<td>200</td>
<td>✓</td>
<td>58</td>
<td>3.12</td>
<td>1.41</td>
</tr>
<tr>
<td>250</td>
<td>✓</td>
<td>60</td>
<td>3.9</td>
<td>1.5</td>
</tr>
<tr>
<td>300</td>
<td>x</td>
<td>65</td>
<td>4.42</td>
<td>1.87</td>
</tr>
<tr>
<td>365</td>
<td>x</td>
<td>62</td>
<td>5.17</td>
<td>2.07</td>
</tr>
</tbody>
</table>
Since a match to the Gaussian distribution has been proved, certainly for the transverse direction, an effective dispersion coefficient can be defined from the variance of the distributions and the mean travel time of the particles (Fetter, 1993) as:

\[ D_L = \frac{\sigma_L^2}{2\langle t \rangle} \]

or alternatively the dispersion coefficient can be related to the distance travelled by the particles since \( D_L = \alpha_L v_L \) and \( D_T = \alpha_T v_T \):

\[ \alpha_L = \frac{\sigma_L^2}{2\langle x \rangle} \]

\[ \alpha_T = \frac{\sigma_T^2}{2\langle x \rangle} \]

where: 
- \( \langle t \rangle \) - mean travel time of the particles
- \( \langle x \rangle \) - mean distance travelled by particles
- \( \sigma_T^2 \) - variance of the transverse spreading of the particles
- \( \sigma_L^2 \) - variance of the longitudinal spreading of the particles
- \( D_L \) - longitudinal dispersion coefficient
- \( D_T \) - transverse dispersion coefficient
- \( \alpha_L \) - longitudinal dynamic dispersivity
- \( \alpha_T \) - transverse dynamic dispersivity

Table 8.6 and Figure 8.15 present the results of these calculations in terms of time and effective dispersion coefficients. From Figure 8.15 it can be seen that the effective transverse dispersion coefficient does not vary significantly over time (and therefore distance) for the 2650, 1700 and 670 node scenarios. With increasing connectivity the effective dispersion coefficient decreases, which is what would be expected. At a connectivity of 130 nodes the effective transverse coefficient does vary with time, but we know that the definition of this coefficient is not stable, as discussed earlier and as seen in Figure 8.14. We can conclude that an effective transverse dispersion coefficient is definable for some relatively densely fractured systems.

The effective longitudinal dispersion coefficient increases with time for the 2650, 1700 and 670 node scenarios. The lines are not as smooth as the transverse ones, which is likely to be as a result of goodness of fit to a Gaussian distribution not being as good. There is also no significant trend with magnitude of dispersion coefficient and connectivity. As with the transverse dispersion, at 130 nodes the longitudinal effective
dispersion coefficient falls with time, which is likely to be due to the instability of the distribution through sparse fracturing.

![Graph showing variation of transverse and longitudinal dispersion coefficients with time and connectivity (represented as number of nodes).](image)

**Figure 8.15:** Variation of a) transverse and b) longitudinal dispersion coefficients with time and connectivity (represented as number of nodes).

Gelhar (1993) describes the 1984 PhD work of P.C. Robinson on the variation of longitudinal dispersion with domain size for 2-D networks of orthogonal fractures for a set fracture density. The domain size varied from 50 to 1000 m for fracture lengths of 50 m at a density of 0.004 m$^{-2}$. The shape of the curve obtained by Robinson was of a logarithmic form, with little increase in longitudinal dispersion at the larger domain sizes. Robinson's curve is effectively equivalent to that in Figure 8.15, although in Figure 8.15 the flattening of the curve is not seen at higher times. This may be because
the time at which the flattening would occur is much greater than those given in Figure 8.15. Gelhar (1993) also shows the variation in longitudinal dispersion with fracture density, with the dispersion decreasing as the density increases. This is definitely observed in the transverse dispersion presented in Figure 8.15, although is not as conclusive for the longitudinal dispersion because of the goodness of fit to a Gaussian distribution. It is not known whether transverse dispersion relations were developed by Robinson in 1984 as they are not shown by Gelhar (1993).

In order to define an effective dispersion coefficient that can be used in the delineation of probabilistic protection zones, delineation of protection zones using stochastic modelling would have to be undertaken many times in a given area of the same structural provenance, fracture type and rock type in order to understand the relationship between the effective dispersion coefficients and the fracture parameters. Because of the scale dependency of dispersion it is possible that different effective dispersion coefficients would be required to model the different zones using the advection-dispersion equation.
Table 8.6: Longitudinal and transverse effective dispersion coefficients for different connectivity systems.

Transverse characteristics

<table>
<thead>
<tr>
<th>Nodes</th>
<th>2650</th>
<th>1700</th>
<th>670</th>
<th>130</th>
</tr>
</thead>
<tbody>
<tr>
<td>Days</td>
<td>St. Dev</td>
<td>Variance</td>
<td>$D_T$ (m$^2$/day)</td>
<td>St. Dev</td>
</tr>
<tr>
<td>50</td>
<td>0.26</td>
<td>0.07</td>
<td>0.0007</td>
<td>0.27</td>
</tr>
<tr>
<td>100</td>
<td>0.35</td>
<td>0.12</td>
<td>0.0006</td>
<td>0.42</td>
</tr>
<tr>
<td>150</td>
<td>0.45</td>
<td>0.20</td>
<td>0.0007</td>
<td>0.48</td>
</tr>
<tr>
<td>200</td>
<td>0.51</td>
<td>0.26</td>
<td>0.0007</td>
<td>0.61</td>
</tr>
<tr>
<td>250</td>
<td>0.55</td>
<td>0.30</td>
<td>0.0006</td>
<td>0.62</td>
</tr>
<tr>
<td>300</td>
<td>0.56</td>
<td>0.31</td>
<td>0.0005</td>
<td>0.67</td>
</tr>
<tr>
<td>365</td>
<td>0.63</td>
<td>0.40</td>
<td>0.0005</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Longitudinal characteristics

<table>
<thead>
<tr>
<th>Nodes</th>
<th>2650</th>
<th>1700</th>
<th>670</th>
<th>130</th>
</tr>
</thead>
<tbody>
<tr>
<td>Days</td>
<td>St. Dev</td>
<td>Variance</td>
<td>$D_L$ (m$^2$/day)</td>
<td>St. Dev</td>
</tr>
<tr>
<td>50</td>
<td>0.61</td>
<td>0.37</td>
<td>0.0037</td>
<td>0.50</td>
</tr>
<tr>
<td>100</td>
<td>0.79</td>
<td>0.62</td>
<td>0.0031</td>
<td>0.86</td>
</tr>
<tr>
<td>150</td>
<td>1.20</td>
<td>1.44</td>
<td>0.0048</td>
<td>1.15</td>
</tr>
<tr>
<td>200</td>
<td>1.41</td>
<td>1.99</td>
<td>0.0050</td>
<td>1.43</td>
</tr>
<tr>
<td>250</td>
<td>1.50</td>
<td>2.25</td>
<td>0.0045</td>
<td>1.75</td>
</tr>
<tr>
<td>300</td>
<td>1.87</td>
<td>3.50</td>
<td>0.0058</td>
<td>1.80</td>
</tr>
<tr>
<td>365</td>
<td>2.07</td>
<td>4.28</td>
<td>0.0059</td>
<td>2.10</td>
</tr>
</tbody>
</table>
8.3.3 Alton Court

The Alton Court public supply well is located on south east side of Ross-on-Wye, as presented in Figure 8.16, and has been abstracting from the Devonian Lower Old Red Sandstone (LORS) since 1887. The borehole is thought to penetrate the Brownstone Group of the LORS, which is a complex sequence of interbedded marls and red sandstones. The ponds 50 m to the north and west of the abstraction well are fed by springs. These springs dry up in the summer months as groundwater levels fall.

Figure 8.16: Location and layout of Alton Court abstraction well and surrounding monitoring wells.
The ponds at the site feed a small stream which eventually discharges into Rudhall Brook approximately 1 km to the north of the site. Rudhall Brook flows westwards where it confluences with the River Wye. The River Wye is located approximately 1.5 km to the west of the site and flow southwards. Elevated topography of Penyard Park/Tudorville exists to the south of the site. Approximately 4 km to the south of the site is a valley containing Coughton Brook, which, again, flows westwards to confluence with the River Wye.

Little groundwater flow modelling has been conducted in the area. The only modelling conducted has been for the simulation of groundwater protection zones around Alton Court, using the porous media model, Flowpath, during 1996. It is stated in the Agency data request proforma that little data exist in the area, with the geology, aquifer properties and boundaries to the system poorly understood. No known groundwater divides exist in the Ross-on-Wye area and the groundwater catchment is considered to be similar to the surface water catchment in the area. The groundwater flow system is considered to be topographically controlled. The following groundwater divides are considered to exist in the vicinity of Alton Court due to the topography:

- An east-west groundwater divide centred over the elevated topography of Penyard Park and Tudorville, approximately 1 km south of Alton Court. This groundwater divide is, however, not considered to be stable;
- An east-west groundwater divide south of Coughton Brook over elevated land approximately 4 km south of Alton Court;
- A NW-SE trending groundwater divide which forms the eastern limit of the Wye and Rudhall Brook catchments; and
- A varying north to south through to northeast-southwest groundwater divide following the high land to the west of the River Wye approximately 5 to 7 km to the west and northwest of Alton Court.

Apart from the probable groundwater divide immediately south of Alton Court around Penyard Park, the other groundwater divides were used to delineate the Flowpath model domain as no flow cells. The remainder of the model boundaries were defined as river cells where streams entered or left the model, and also two constant head cells.
Available Data

During April 1998 three new monitoring boreholes were drilled at the Alton Court site. The locations of these new monitoring boreholes are illustrated in Figure 8.16. The boreholes were all located within the Alton Court supply compound since the surrounding land is owned by the Ministry of Defence. The boreholes were cored, packer tested and geophysically logged. Radially converging tracer testing was then performed in conjunction with a 7-day pumping test. The results that are relevant to fracture flow modelling are discussed below.

i) Pumping Test Data

Water levels throughout the 7-day pumping test, starting on the 15\textsuperscript{th} April 1998, were recorded in the abstraction well, the observation wells and the pond on site. The drawdown data for all wells are presented in Appendix F. The data were analysed using the Cooper-Jacob solution for all wells. A fractured response to pumping the abstraction well was seen in BH2, and using Kazemi’s straight-line method (Kazemi \textit{et al.}, 1969) of analysis a fracture storativity of $6.6 \times 10^{-5}$ and a matrix storativity of $2 \times 10^{-3}$ was derived. A summary of the aquifer parameters obtained is presented in Table 8.7. From the pumping test analyses a large range in hydraulic conductivity was obtained (45 to 555 m/d), as presented in Table 8.8. This analysis was conducted as part of this thesis.

<table>
<thead>
<tr>
<th>Borehole</th>
<th>Early time</th>
<th>Middle time</th>
<th>Late time</th>
<th>Recovery</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T$ (m$^3$/d)</td>
<td>$S$</td>
<td>$T$ (m$^3$/d)</td>
<td>$S$</td>
</tr>
<tr>
<td>ABH</td>
<td>1087</td>
<td>2.71x10$^{-4}$</td>
<td>1115</td>
<td>1.40x10$^{-3}$</td>
</tr>
<tr>
<td>BH1</td>
<td>1115</td>
<td>3532</td>
<td>942</td>
<td>2354</td>
</tr>
<tr>
<td>BH2</td>
<td>3532</td>
<td>6.56x10$^{-5}$</td>
<td>3853</td>
<td>1.24x10$^{-3}$</td>
</tr>
<tr>
<td>BH3</td>
<td>5297</td>
<td>1.37x10$^{-2}$</td>
<td>3260</td>
<td>3260</td>
</tr>
<tr>
<td>POND</td>
<td>7063</td>
<td></td>
<td></td>
<td>4036</td>
</tr>
<tr>
<td>WELL</td>
<td>1324</td>
<td>1.29x10$^{-2}$</td>
<td>2825</td>
<td>1284</td>
</tr>
</tbody>
</table>

Table 8.7: Aquifer property variations from pumping test analyses at Alton Court.

$S_f = 6.56 \times 10^{-5}$, $S_m = 1.17 \times 10^{-3}$ assuming bedding plane fractures and $S_m = 3.52 \times 10^{-3}$ assuming orthogonal fractures

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Table 8.8: Average aquifer hydraulic conductivity values at Alton Court.

<table>
<thead>
<tr>
<th>Borehole</th>
<th>Average T (m²/d)</th>
<th>Borehole depth (m)</th>
<th>Saturated thickness (m)</th>
<th>Average hydraulic conductivity (m/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABH</td>
<td>1028</td>
<td>30</td>
<td>23</td>
<td>44.7</td>
</tr>
<tr>
<td>BH1</td>
<td>921</td>
<td>16.0</td>
<td>13.8</td>
<td>66.7</td>
</tr>
<tr>
<td>BH2</td>
<td>3246</td>
<td>22.0</td>
<td>21.0</td>
<td>154.6</td>
</tr>
<tr>
<td>BH3</td>
<td>3939</td>
<td>36.0</td>
<td>35</td>
<td>112.5</td>
</tr>
<tr>
<td>POND BH</td>
<td>5550</td>
<td>10.7</td>
<td>10</td>
<td>555.0</td>
</tr>
<tr>
<td>WELL BH</td>
<td>1811</td>
<td>9.7</td>
<td>9</td>
<td>201.2</td>
</tr>
</tbody>
</table>

ii) Tracer Test Data

The aim of the tracer testing at Alton Court, undertaken as part of this thesis, was principally to obtain a better understanding of the fracture flow characteristics at the site in order to conduct protection zone modelling with probability contours. The secondary aim of the tracer test was to trial the guidelines being developed for a ‘tracer testing manual’ by the BGS/Environment Agency (Ward et al., 1998) to ensure that, even with limited knowledge of tracer testing, a well planned and conducted test could be performed. I performed several types of tracer test:

- a radially converging flow tracer test using the injection of three different fluorescent dyes into BH1, BH2 and BH3;
- three single borehole dilution tests in BH1, BH2 and BH3 using sodium chloride; and
- one natural gradient tracer test using sodium chloride from BH2 to the pond.

Converging-Flow Tracer Testing

Two weeks and also two hours prior to the pumping and tracer tests, groundwater samples were taken from all the wells on site in order that background levels of fluorescence in the groundwater could be measured at the different dye frequencies. Low concentrations of compounds at the absorbance wavelengths of the dyes were detected in the all the samples. Concentrations were lower in samples taken after Alton Court had been pumping for two hours than two weeks previously. The results of the background fluorescence sampling are shown in Table 8.9.
Table 8.9: Background concentrations of compounds in groundwater at Alton Court with the same absorbance wavelength as the tracer dyes.

<table>
<thead>
<tr>
<th>Borehole</th>
<th>Background concentration (μg/l) at Fluorescein wavelength</th>
<th>Background concentration (μg/l) at Rhodamine wavelength</th>
<th>Background concentration (μg/l) at Photine wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>BH1</td>
<td>48</td>
<td>0.007</td>
<td>1</td>
</tr>
<tr>
<td>BH2</td>
<td>48</td>
<td>0.006</td>
<td>1</td>
</tr>
<tr>
<td>BH3</td>
<td>95</td>
<td>0.012</td>
<td>3</td>
</tr>
<tr>
<td>ABH</td>
<td>24</td>
<td>&lt;0.006</td>
<td>1</td>
</tr>
</tbody>
</table>

After approximately four hours of pumping at the abstraction well at 2.3 Ml/day on 15th April 1998, Fluorescein was injected into BH3 (55 m from the pumping well), Rhodamine WT was injected into BH2 (44 m from the pumping well) and Photine was injected into BH1 (41 m from the pumping well), using the injection guidelines in the tracer test manual.

Samples of the abstracted groundwater were taken manually from the sample tap on the wellhead, and were transferred to brown glass bottles and stored in the dark and at < 4°C until analysis. Samples were taken at 5-minute intervals for the first two hours, at 10-minute intervals for the next hour and at 15-minute intervals for the next four hours. Thereafter samples were taken at approximately hourly intervals. Analyses of samples were performed at UCL on a spectrophotometer (PYE Unicam PU 8600 UV/VIS spectrophotometer).

Figure 8.17: Breakthrough of Fluorescein in the Alton Court abstraction well during the converging flow tracer test.

Chapter 8 – Testing a 2D Methodology
The different breakthrough curves detected in the pumping well are shown in Appendix G. No breakthrough of Rhodamine was observed in the pumping well. Breakthrough was present for Photine, although concentrations were all below two times background levels. First arrivals occurred 25 minutes after injection, with the main peak detected at 135 minutes. A well defined breakthrough curve for Fluorescein was detected at the abstraction well, as presented in Figure 8.17, with concentrations peaking at approximately 265 minutes after tracer injection and first arrivals detected at approximately 220 minutes.

The timing of the main peak of the Photine breakthrough from BH1 to the ABH can be used to calculate the kinematic porosity of the aquifer from BH1 to the ABH:

- Time for peak to reach pumping well = 135 minutes
- Distance from BH1 to ABH = 41 m
- \( \Rightarrow \) mean velocity in aquifer = \( \frac{41 \times 60 \times 24}{135} \) = 437 m/d

From pumping test hydraulic conductivity of BH1 = 67 m/d
Hydraulic gradient in the vicinity of BH1 = 0.049

\[ \Rightarrow \text{kinematic porosity in aquifer} = \frac{K_i}{\nu} = 67 \times 0.049 / 437 \]
\[ = 0.8\% \]

The timing of the main peak of the Fluorescein breakthrough from BH3 to the ABH can be used to calculate the kinematic porosity of the aquifer from BH3 to the ABH:

- Time for peak to reach pumping well = 265 minutes
- Distance from BH3 to ABH = 55 m
- \( \Rightarrow \) mean velocity in aquifer = \( \frac{55 \times 24 \times 60}{265} \) = 299 m/d

From pumping test hydraulic conductivity of BH3 = 112 m/d
Hydraulic gradient in the vicinity of BH3 = 0.075

\[ \Rightarrow \text{kinematic porosity in aquifer} = \frac{K_i}{\nu} = 112 \times 0.075 / 299 \]
\[ = 2.8\% \]
Breakthrough of Fluorescein, Photine and Rhodamine also occurred in the pond to the NNW of the site, down natural hydraulic gradient of the observation wells. Breakthrough of Rhodamine (orange/pink colour in pond) occurred about ten to fifteen minutes after the tracer injection. This gave a maximum flow velocity in the fractured part of this aquifer from BH2 to the pond of approximately 1440 m/day. The tracer could be seen passing through the chamber of the Pond Borehole, on route to the outfall in the pond. From the pumping tests the hydraulic conductivity in the vicinity of BH2 is approximately 155 m/d and the hydraulic gradient is 0.044 in that region. A value for the kinematic porosity for this section of ORS can be calculated as 0.5%.

At approximately 17:30 on the 15/4/98 one of the springs in the pond appeared to turn from a pink/orange colour to a more fluorescent green colour, which was proved later to be the Fluorescein breakthrough. The Fluorescein was injected three hours prior to its appearance in the pond. The Fluorescein breakthrough in the pond gives an approximate maximum flow velocity from BH3 to the pond of 370 m/d. Assuming a hydraulic conductivity of 112 m/d in the vicinity of BH3 (from the recovery test results), a hydraulic gradient of 0.042 to the pond (from the piezometric surface four hours after the start of abstraction), then the kinematic porosity of the aquifer can be estimated at 1.3%.

Breakthrough of the Photine was not observed in the pond since at relatively low concentrations it is colourless, but analysis of pond water samples indicated that Photine did migrate to the pond.

Analysis of the pond water from seven locations in the pond was conducted for the three tracing dyes for four sets of samples taken approximately at 12 hourly intervals, up to 50 hours from Fluorescein injection. From this it was possible to do a mass balance calculation for the dye tracers. It was found that all the Fluorescein could be accounted for by the mass arriving at the abstraction well and the mass in the pond. However, for the Photine and the Rhodamine not all of the mass of tracer introduced to the groundwater system could be accounted for by the mass in the pond and the mass arriving at the abstraction well. An Environment Agency pumping test report at the site suggested the presence of a deeper groundwater system that by-passes the spring-flow
to the pond (Environment Agency, 1996b). It is considered that the remaining mass of tracer travelled off-site to the northwest in this deeper groundwater system.

Because of the short time taken for the tracer to reach the abstraction well and the short distances involved, it has not been possible to assess whether diffusion into the sandstone matrix is a significant transport process. The breakthrough curves observed do not appear to have long tails indicating that diffusion is not a significant process over the few hours and/or the different pathways taken by the tracer are not significantly different in terms of travel-time. The porosities measured at the site are likely to represent the fracture porosity only.

From the rate at which the Fluorescein and Rhodamine were seen to disappear from the pond, a rough estimate of the total spring flow into the pond and flow through the pond can be made. The dimensions of the pond are approximately $65 \times 55 \times 0.4$ m, giving a volume of water of $1144$ m$^3$ assuming 80% of the pond volume is available for flow (much vegetation and trees form islands in the pond). The Rhodamine and Fluorescein took 24 to 36 hours to pass through the pond. Therefore, the flow rate into and out of the pond is between 763 and 1144 m$^3$/d. This is the same order of magnitude as quoted by the BGS memoir on the Alton Court borehole (Environment Agency, 1996) of 2273 m$^3$/d in the spring season. A value less than that quoted by the BGS memoir would be expected under pumping conditions since the pumping well would be drawing water away from the pond.

**Single Borehole Dilution Testing**

Single borehole dilution tests were performed on monitoring wells BH1, BH2 and BH3 to assess the principal flow horizons for tracer migration. These tests were performed on the second and third days of the pumping test. Salt (sodium chloride) was added in the same way as the dye tracers to the column of water in the borehole, in a concentration high enough to ensure values were well above background levels. Background conductivity readings in the observation wells ranged from 675 - 720 $\mu$S/cm. The mass of salt added aimed to give an initial conductivity in the borehole of $\sim 1700$ $\mu$S/cm. The conductivity profile of the borehole with depth was then measured over time. Ideally the conductivity with depth would have been measured with a probe on a cable of sufficient length to reach the base of the well. However, an
instrument of this nature was unavailable at the time, so measurements were made using a portable EC probe placed in samples of the well water taken with a 1 cm diameter double-valved bailer to obtain discrete depth samples. The results of the concentration profiles are shown in Figure 8.18. The following analysis of the results assumes that a uniform injection concentration was achieved.

Figure 8.18 illustrates the variation in conductivity with depth over time in BH1, BH2 and BH3. In the following analysis it is assumed that the initial conductivity was uniform over the saturated thickness of the three boreholes (in reality this is not likely to occur due to difficulty in injecting the salt evenly over the water column). In BH1, assuming the initial conductivity was even over the length of the borehole, it could be seen that the conductivity falls the fastest in the 7-10 m zone and also at the base of the borehole at 15 m. At 11 - 13 m depth the conductivity appeared to rise above the initial values. This may be due to the dense salt solution sinking in the borehole, or could be simply due to the fact that in the first sampling run those depths were not tested. The latter reason would seem more appropriate, and it may be that the zone around 11 - 14 m is less transmissive. The most transmissive zones would appear to at 7 - 10 m depth and at 15 m depth. At all depths in the well the conductivity returned to background levels within 1 hour of the salt injection. On average the borehole would appear to be very transmissive. The conductivity of the springs into the pond was measured at the start and throughout the dilution test, but no elevated conductivities were recorded.

In BH2 it can be seen that the conductivity falls fastest in the 15 - 19 m zone. At the base of the borehole the conductivity appears to remain much higher than elsewhere in the borehole. The base of the borehole would therefore appeared to have a much lower transmissivity. This is confirmed by the fact that a small concentration of Rhodamine was detected at the very base of the well (2 µg/l). The base of the borehole was bailed out at the end of the dilution test to ensure no salt or Rhodamine remained in the well. Again, at all depths the conductivity returned to approximately background levels within one hour of salt injection.
Figure 8.18: Variation of conductivity with depth for BH1, BH2 and BH3 at Alton Court after single well tracer injection.
In BH3 it can be seen that the conductivity falls fastest in the 15 - 17 m and the 25 - 31 m zones. The conductivity drops less rapidly in the 19 - 23 m zone and the upper 5 - 9 m zone. The upper zone seems to be least transmissive of the two. As with BH1 and BH2, at all depths of BH3 the conductivity returned to approximate background concentrations within 45 minutes of the salt injection, indicating a high groundwater velocity.

In summary there appears to be a transmissive zone at around 15 – 19 m bgl in all boreholes. The other conductive zones do not appear to correlate between the boreholes. It was not possible to obtain an estimate of the groundwater velocities from these single borehole dilution tests because of the limited number of conductivity profiles obtained over time for each borehole.

Natural Gradient Tracer Testing
The conductivity of one of the springs into the pond was measured at the start of the dilution test on BH2 and for approximately one hour into the test at regular intervals, as the two were known to be connected. The results are presented in Figure 8.19. The tail of the conductivity breakthrough curve is not well defined since conductivity probes had to be exchanged half way through the test. The results have been analysed as a natural gradient test (BH2 is considered to be outside the capture zone of the pumping well). The peak conductivity reached the outfall within 13 minutes, which was a distance of approximately 10 m from BH2. Therefore the transport velocity can be calculated as approximately 1108 m/d. From the pumping tests the hydraulic conductivity in the vicinity of BH2 is 155 m/d and the hydraulic gradient is 0.044 in that region. A value for the kinematic porosity for this section of ORS can be calculated as 0.6%.

Summary of Tracer Testing
The principal conclusions of the tracer testing can be summarised as follows:

- BH2 is outside the capture zone of the well at an abstraction rate of 2.3 ML/day, with BH2 being only 44 m from the abstraction well;
- BH3 and BH1 are on the edge of the capture zone of the well at 2.3 ML/day since dye went to both the ABH and the pond. These results emphasise the 3.D nature of the capture zone, with the shallower levels being outside of the
capture zone, but the deeper groundwater being inside the capture zone of the Alton Court well;

- In the vicinity of BH2 large aperture fractures exist, with high groundwater flow velocities of up to 1440 m/day;
- Mass budgets for the tracers in BH2 and BH3 suggest that there is a deeper groundwater system present at the site that does not outflow to the pond, but bypasses it, flowing towards the north-west;
- A major flow horizon appears to exist between 15 and 19 m bgl;
- Groundwater flow velocities towards the pond vary between 370 and 1440 m/day;
- Groundwater flow velocities towards the ABH vary between 299 and 437 m/day at an abstraction rate of 2.3 Ml/day; and
- The representative kinematic porosity at the site is considered to be 1%.

![Graph showing conductivity variation at a spring in the pond at Alton Court after tracer injection into BH2.](image-url)

**Figure 8.19:** Variation in conductivity at a spring in the pond at Alton Court after tracer injection into BH2.

On the timescales and distances involved at Alton Court, the groundwater flow in the ORS is considered to be principally by fracture flow, rather than by matrix flow. This is shown in the Fluorescein breakthrough curve at the pumping well where the tail of the breakthrough curve does not appear to be particularly long.
However, care must be taken when using the kinematic porosity values from the tracer testing since the distances involved from the abstraction well are likely to be in non-uniform flow conditions. The calculations used for defining the kinematic porosity are based on Darcian flow conditions.

**iii) Packer Testing Data**

The packer testing on BH2 and BH3 showed very little difference in the hydraulic conductivity with depth, indicating that the average flowing fracture spacing is much less than 3.3 m at all depths. For BH2 it was found that there was average hydraulic conductivity within the 3.3 m packer interval of 1.15 m/d, varying from a maximum of 1.26 m/d to a minimum of 1.07 m/d. For BH3 it was found that there was average hydraulic conductivity within the 3.3 m packer interval of 1.15 m/d, varying from a maximum of 1.35 m/d to a minimum of 0.86 m/d.

These estimates of hydraulic conductivity are low compared with those obtained from the pumping test analyses. The reasons for this are unclear, especially since large transmissive fractures are known to occur in BH2. The packer test results are therefore not considered to be representative of the fracture system.

**iv) Geophysical Logging Data**

Many different geophysical log types were run down the new monitoring wells at Alton Court by Robertson Geologging, these included spontaneous potential (SP), gamma (γ), single point resistance (SPR), short normal resistance (SNR), long normal resistance (LNR), caliper, differential temperature (δT) and differential conductivity (δC) logs. The logs were analysed as part of this thesis.

Since caliper, δT and δC logs give less ambiguous information about the spacing of flowing and non-flowing fractures than the other logs, only these have been used in the fracture assessment. As with most values derived from logs run vertically down the borehole, the fracture spacing is only an apparent one since the methods cannot distinguish between fracture sets and the angles at which they intersect the borehole. Only optical and acoustic televiewer data can distinguish between the sets and give true fracture set spacings.
Figure 8.20 illustrates the depths of these flowing fractures along with the depths of the fractures detected using an acoustic televiewer log (see next section). The aim of this diagram was to assess whether correlations of actively flowing fractured zones exist between boreholes, and whether this could be related to the single borehole dilution testing data. However, there appears to be no obvious correlation between the two sets of results, except perhaps in BH3 where the most actively flowing zones do approximately correspond to the flowing fracture locations as identified by the differential temperature logging.

In BH1 the caliper response indicates an average fracture spacing of 0.67 m. The δT log indicates an average flowing fracture spacing of 0.44 m. Overall the spacings in BH1 show a lognormal distribution.

In BH2 the caliper response indicates an average fracture spacing of 1.32 m. The δT log indicates an average flowing fracture spacing of 4.8 m. There was insufficient data to assess the spacing distribution.

In BH3 the caliper response indicates an average fracture spacing of 1.28 m. The δT log indicates an average flowing fracture spacing of 2.09 m. Overall the spacings in BH3 show a lognormal distribution.
Locations of flowing fractures in BH1, BH2 and BH3 at Alton Court from differential temperature logging

Figure 8.20: Depths of fractures and flowing fractures in BH1, BH2 and BH3 at Alton Court.
Televiewer Data

Acoustic televiewer data was collected for all three boreholes and all dips and strikes of the fractures were plotted on a stereogram. From the stereographic analysis completed as part of this thesis three distinct fracture sets were identified. The data were then divided up into these sets, and histograms of orientations, dips and spacings were plotted. A summary of these data can be found in Table 8.10.

Table 8.10: Summary of fracture data for Alton Court.

<table>
<thead>
<tr>
<th>Fracture Set</th>
<th>Parameter</th>
<th>Average</th>
<th>Standard Deviation</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>SET 1</td>
<td>Orientation</td>
<td>163°</td>
<td>56°</td>
<td>normal</td>
</tr>
<tr>
<td></td>
<td>Dip</td>
<td>23.2°</td>
<td>13.2°</td>
<td>normal</td>
</tr>
<tr>
<td></td>
<td>Spacing (m)</td>
<td>0.11</td>
<td>0.03 – 0.38</td>
<td>lognormal</td>
</tr>
<tr>
<td>SET 2</td>
<td>Orientation</td>
<td>201°</td>
<td>142°</td>
<td>normal</td>
</tr>
<tr>
<td></td>
<td>Dip</td>
<td>58.6°</td>
<td>13.5°</td>
<td>normal</td>
</tr>
<tr>
<td></td>
<td>Spacing (m)</td>
<td>0.24</td>
<td>0.34 – 0.72</td>
<td>lognormal</td>
</tr>
<tr>
<td>SET 3</td>
<td>Orientation</td>
<td>33°</td>
<td>13.6°</td>
<td>assume normal</td>
</tr>
<tr>
<td></td>
<td>Dip</td>
<td>8.2°</td>
<td>2.3°</td>
<td>assume normal</td>
</tr>
<tr>
<td></td>
<td>Spacing (m)</td>
<td>1.02</td>
<td>0.26 – 4.03</td>
<td>assume lognormal</td>
</tr>
</tbody>
</table>

The distributions for set 3 have been assumed because not enough fractures in this set were sampled to make a representative frequency distribution diagram. The distributions for sets 1 and 2 were established using the statistical software package Crystal Ball® to find the best-fit distribution (of approximately fifteen different distribution types) to the data. A full presentation of the acoustic televiewer data can be found in Appendix H.

Having located which fractures were in each set and at what depths, they could then be cross-correlated with the differential temperature logs to find out the flowing fracture spacing for each of the sets. Table 8.11 is a summary of the fracture spacings and the flowing fracture spacings found for each of the sets.

The excellent quality of the televiewer data enables flowing fracture spacings to be determined for each fracture set, as opposed to caliper, differential temperature and differential conductivity logs which can only give the average flowing fracture spacing for all fracture sets in a given borehole.
Table 8.11: Summary of flowing fracture data for Alton Court.

<table>
<thead>
<tr>
<th>Fracture Set</th>
<th>Average spacing (m)</th>
<th>Standard Deviation</th>
<th>Flowing fracture average spacing (m)</th>
<th>Percentage of flowing fractures</th>
</tr>
</thead>
<tbody>
<tr>
<td>SET 1</td>
<td>0.11</td>
<td>0.03 - 0.38</td>
<td>1.98</td>
<td>5.5</td>
</tr>
<tr>
<td>SET 2</td>
<td>0.24</td>
<td>0.34 - 0.72</td>
<td>2.66</td>
<td>9.0</td>
</tr>
<tr>
<td>SET 3</td>
<td>1.02</td>
<td>0.26 - 4.03</td>
<td>6.35</td>
<td>16.1</td>
</tr>
</tbody>
</table>

v) Apertures

The transmissivity data from the pumping tests on boreholes BH1, BH2 and BH3 can be converted to hydraulic conductivity values since the saturated thickness of wells is known. Then, assuming the cubic law and knowing the number of flowing fractures in each well from the televiewer data, an approximate ('hydraulic') fracture aperture can be obtained. It is recognised that this aperture is probably the lower end of the aperture variation since the cubic law does not take into account that fractures are not parallel plates and will not be completely open at all points. The true fracture aperture could potentially be very different from that calculated because of channelling processes, with flow being concentrated in several larger aperture channels. Also the data used to obtain the fracture aperture is for all fracture sets since the transmissivity relates to the full saturated thickness of the borehole, so apertures for individual sets cannot be distinguished.

Even though the transmissivities varied greatly across the boreholes, the apertures calculated were very similar (due to the corresponding large difference in fracture numbers in the boreholes and taking the square root of the hydraulic conductivity). The minimum average aperture calculated was for BH1 at 6.0 × 10⁻³ mm and the maximum average was in BH2 at 2.4 × 10⁻² mm. If the same calculation is performed for the packer test data an average aperture of 4.6 × 10⁻³ mm is obtained for BH2 and 3.3 × 10⁻³ mm for BH3. These aperture data assumed that there are 34 flowing fractures in BH1, 5 in BH2 and 17 in BH3 from the differential temperature logging. These apertures seem extremely small given the very high groundwater velocities across the site observed during the pumping and tracer testing.

Chapter 8 – Testing a 2D Methodology
Apertures can also be calculated from the radially converging tracer test data to give the frictional-loss aperture as opposed to the cubic law hydraulic aperture (see section 6.1.1 and Tsang, 1992). For borehole BH3 where the mean residence time of the Fluorescein was 265 minutes, an aperture of 0.47 mm is obtained. For the breakthrough of Photine in BH1 an aperture of 0.68 mm is obtained using the same method.

The equation for the frictional loss aperture for non-radial symmetry is also presented in section 6.1.1. This equation can be used to calculate the frictional loss aperture for the breakthrough of sodium chloride in the pond as 0.68 mm. These values are at one to two orders of magnitude higher than the cubic law apertures and are considered to be more representative of the larger fractures on site.

vi) Trace Lengths
The only fracture parameter not directly obtainable from the testing on-site is trace length data. In the discussion on trace lengths in section 6.1.1 Hatton et al. (1994) present a relationship between aperture and length. Since we have an estimate of aperture we can obtain the approximate upper and lower limits to the trace lengths. From this relationship and using an aperture of between $6.8 \times 10^{-4}$ m to $3.3 \times 10^{-6}$ m, an upper limit trace length of 4.6 m and a lower limit trace length of 0.0001 m are calculated.

Robinson (1983, 1984) and Charlaix et al. (1984) also demonstrate relationships between fracture length and density, as described in Section 6.1.3. The flowing fracture spacings in BH1, BH2 and BH3 are known to vary between 0.44 m and 4.8 m, which give a range of densities in 1-D of between 2.27 and 0.21 m$^{-1}$. Xu and Jacobi (1998) developed a relationship between the 1-D fracture density and the 2-D fracture density:

$$D = 1.3488f + 0.0285$$

where: $f$ is the 1-D fracture density (m$^{-1}$),
$D$ is the 2-D fracture density (m$^2$)

We can use this equation to convert the 1-D site data to 2-D for use in the Robinson (1983, 1984) relationships. The 2-D fracture density for the site is therefore between 3.09 and 0.31 m$^2$. If the relationships of Robinson (1983, 1984) are used fracture lengths of between 0.68 m and 2.22 m are obtained. If the relationship of Charlaix et al.
(1984) is used a range of lengths between 0.72 and 1.98 m is obtained (although the site data has not been corrected to a 3-D fracture density).

However, we know that these relationships underestimated the trace lengths for Coombe Farm. Therefore, we must be mindful of this when setting up the fracture model. The low values obtained from percolation theory relations may be partly due to the fact they are the minimum required for percolation, with flow at Alton Court being under conditions well above the percolation threshold and hence having longer trace lengths.

**Conceptual Model**

The principal features of the conceptual model at Alton Court are presented in Figures 8.21a) and b) and can be described as follows:

- Groundwater flow is predominantly through fractures in the LORS with negligible intergranular flow. Flow is concentrated in the upper 30 to 50 m of the saturated thickness.

- Groundwater levels are approximately 1 to 2 m bgl and the hydraulic gradient across site is between 0.01 and 0.04, with flow towards the NNW and Rudhall Brook (data from 15th April 1998 water levels, and confirmed by September 1993 data). Groundwater flow is thought to then turn westwards towards the River Wye. The vertical groundwater gradient is upwards during non-pumped conditions and downwards during pumped conditions.

- The transmissivity of the LORS at the site varies from 920 and 5550 m²/day. The saturated thickness values allow hydraulic conductivity values to be calculated between 45 m/d to 555 m/d.

- Three fracture sets are considered to be present at the site. The orientations and dips of these sets are given in Table 8.6. The flowing fracture densities of the sets are given in Table 8.7. A representative aperture for the larger fractures is considered to be $6.0 \times 10^{-4}$ m, whereas a representative aperture for the fractures overall is considered to be derived from the cubic law at $1.0 \times 10^{-5}$ m. The trace length of the fractures is considered to be over 4.6 m.

- Recharge in the vicinity of the site is considered to be 202 mm/year (Mott MacDonald, 1996).
The tracer test results emphasise the 3-D nature of the capture zone. The interbedded nature of the LORS marls and sandstones allows the aquifer to act as a multi-layered leaky system. The upper water-bearing sandstone bands are in hydraulic continuity with the pond but also respond to abstraction, with leakage to the lower layers. The upper layers are likely to be relatively thin since the pond is known to dry up in the summer months. Some of the lower sandstone layers are in hydraulic continuity with the abstraction well whilst some form part of a deep groundwater system that flows towards the NNW, even under pumping conditions. Under non-pumping conditions groundwater in all these deeper sandstone layers would be beneath the pond to the NNW.

The capture zone of the well at 2.3 Ml/day has a limited down-gradient extent of less than 44 m.

Groundwater velocities are high (299 to 1440 m/day) in both the upper and the lower layers of the LORS.

Localised high aperture fractures exist. In the upper layers these high apertures are considered to have grown over time because of the spring outfalls. In the lower layers the high apertures may have developed because of over 110 years of abstraction at the site.

The representative kinematic porosity at the site is considered to be 1%.

The regional flow boundaries for Alton Court are considered to be the east-west groundwater divide approximately 4 km south of the site and south of Coughton Brook, the NW-SE trending groundwater divide which forms the eastern limit of the Wye and Rudhall Brook catchments, and a varying north to south through to northeast-southwest groundwater divide following the high land to the west of the River Wye approximately 5 to 7 km to the west and northwest of Alton Court.

The Alton Court borehole is licensed to abstract 2273 m³/day. A steady-state drawdown of approximately 12 m is achieved at this abstraction rate (Environment Agency, 1996b).

From the conceptual model the Alton Court site could be modelled using a fracture flow model with an impermeable matrix rather than a double porosity model.
Outflow of spring-fed pond to small stream

Abstraction borehole

Upper flow system, \( K > 100 \text{ m/d} \)

Lower flow system, \( K < 50 \text{ m/d} \)

Outflow to River Wye

Figure 8.21: a) Conceptual groundwater flow system at Alton Court (above)

b) Conceptual 3-D capture zone at Alton Court at an abstraction rate of 2.3 ML/day (below).

Numerical Modelling

SDF was used in an attempt to model probabilistic protection zones at Alton Court. The LORS at Alton Court does not appear to have any significant matrix flow and therefore the use of an impermeable matrix fracture flow model, in this case SDF, was considered suitable.

The model was set up using the fracture parameters and hydraulic gradient outlined in the conceptual model. A trace length of 20 m was used in order to allow the fracture network to percolate fully. Recharge is not considered within the model. A drawdown of 6 m was used at the pumping well since this was the drawdown achieved at the end of the April 1998 pumping test.
The model was calibrated using fracture aperture in order to achieve the correct pumping rate for the 6 m of drawdown. Fracture aperture was the parameter chosen to calibrate the model since this is the value that is the most uncertain at the site. This best-fit aperture was $1 \times 10^{-5}$ m. This value matches the average representative cubic law derived value.

It was discovered that because of the number of fracture sets and the density within each set a model domain larger than 500 by 100 m could not be simulated. As detailed in section 8.3.1 on the Coombe Farm modelling, attempts were made resolve this. Given the location of the regional boundaries with respect to Alton Court, it is recognised that a domain 500 by 100 m will impose significant restrictions on the groundwater flow system, and apply boundaries to the flow regime that are indefensible.

Despite the model domain limitation, protection zone modelling was then attempted. 20 realisations were simulated each with 50 reverse tracked particles. The 1000 reverse tracked particles were then used to create the probability contours using the travel-time method. Because of the size of the model domain only 50-day probability contours could be created. The resulting 50-day probability contours are presented in Figure 8.22 and have been created using Surfer™ using the inverse distance to a power method of interpolation (power of 2 and smoothing factor of 5, as for the case of Coombe Farm). The 50-day porous media protection zones have also been presented in Figure 8.23.

From Figures 8.22 and 8.23 it can be seen the distance between the probability contours to the north, east and west is not large between the 10% and the 90% contours. Because of this small distance the confidence in the location of the 50-day contour is high. The interesting feature to note is that the tracer testing broadly agrees with this modelling, with BH2 being completely outside the capture zone and the 50-day zone (groundwater flowing in the opposite direction to the pumping well). 11% of the mass of Fluorescein injected into BH3 ended up in the well, with the rest being discharged to the pond in the opposite direction to the pumping well. This would suggest that BH3 is on the very edge of the 50-day probability contours since the mass that did arrive at the borehole arrived in much less than 50 days (a few hours) and the rest went in the opposite direction. The Photine placed in BH1 appeared to migrate, in the main, away from the pumping well, with only 0.2% arriving at the abstraction well. This would put the well on the 99% probability contour of the capture zone. BH1 lies just outside the 100%
probability contour of the 50-day zone. It is likely for Alton Court the 50-day and the capture probability zones nearly coincide on the north, east and western edges.

Figure 8.22 clearly shows the restrictions placed on the groundwater flow in the fracture model by the size of the model domain, with the position of the cross natural gradient probability contours being restricted by the model boundary shape and location. So, whilst the probability contours do not contradict the tracer testing results, the model domain is not large enough to simulate groundwater flow patterns and magnitudes with any confidence.

The porous media protection zones (Figure 8.23) do not show good agreement with either the tracer testing at the site or the fracture modelled zones. Although the regional groundwater flow direction is towards the southwest and along the River Wye valley, around Alton Court this is not the case with flow towards the NNW. This suggests that the Flowpath model used in the porous-media modelling was poorly calibrated. The area of the porous media modelled best estimate zone and the fracture-modelled zones are very different indeed at 1.125 km² and 0.05 km² respectively. If one does a simple calculation to balance the abstraction rate of the borehole with the volume of aquifer required for a 50-day travel-time, assuming a kinematic porosity of 0.5% and an effective saturated thickness of 20 m, the radius of aquifer required is 601 m, equivalent to 1.14 km², which is very close to the Flowpath modelling result. This simple calculation does not, however, include recharge. The model used to derive the porous media protection zones used a conservative saturated thickness value of 20 m and a kinematic porosity of 0.5%. From the recent field work at Alton Court a more representative kinematic porosity is 1% with an effective thickness of 30 m. Therefore a more representative area of contribution for 50 days is 0.38 km², and this value does not include recharge resulting in the true area required for supporting the abstraction rate for 50 days being less than 0.38 km².

The fracture modelled zone area does appear to be overly small. If this fracture modelled area were correct then this would result in a kinematic porosity of approximately 7%, which is much higher than the highest value calculated for the site of 2.8% in BH3. 7% is an unrealistically high kinematic porosity for the Old Red Sandstone. The reason for this discrepancy is likely to be the fact that the zone has been modelled in 2-D, rather than an appropriate 3-D method to match the conceptual model.
It is also possible that this apparent high kinematic porosity value from the fracture model is partly due to the increased fracture lengths required in order to make the system percolate. This leads us to the fact that fracture lengths could be used as a calibration parameter in the model, as well as aperture. Lengths would have to be long enough for the system to percolate, but also such that the lengths and apertures combined result in a kinematic porosity that agrees with site data. Percolation of the fractured rock system is therefore not a stopping point as fracture lengths are increased; they may have to be lengthened further for the better agreement with site kinematic porosity data.

The other principal reason for the discrepancy in the porous media and the fracture-modelled zones will be due to the restrictions imposed by the size of the model domain on the fracture flow model. The probability contours may be unrealistically small due to the model boundaries perpendicular to the up--gradient hydraulic gradient being no flow boundaries, hence not allowing particles to cross these model boundaries.

Although probability based contours have been produced for the 50-day travel-time zone, the confidence in the zones is, however, fairly small in the up-gradient direction. This is due to the lack of data for validation. Although the tracer testing has been able to constrain the down-gradient side of the zones well, the up-gradient side has not been validated. Further tracer work would have to be performed to validate the up-gradient side of the zones. The good agreement of the fractured media zones with the tracer testing are considered to be fortuitous, rather than by design in using the correct 3-D conceptual model for the site.

Even though the zones are apparently validated to some extent, the model used to simulate flow in the fractures is only 2-D (SDF), and it is known at the site that the flow and transport is very much a 3-D problem from the tracer testing. It would be useful, therefore, to repeat the modelling exercise using a 3-D model such as FRACMAN to reassess the probability zones produced.
Figure 8.22: 50-day probability contours at Alton Court (probability that particles take longer than 50 days to reach the pumping well) compared to the original porous-media protection zone modelling.

Figure 8.23: Comparison of 50-day probability based protection zone contours and the original 50-day protection zones at Alton Court.
8.4 Conclusions

There are several important conclusions to be drawn from the 2-D investigations:

1. Protection zones become elongated in the direction of the principal fracture sets;
2. In high connectivity fractured rock systems it is possible that an effective dispersion coefficient could be used to create probabilistic protection zones through the advection-dispersion equation. However, a significant amount of stochastic fracture modelling would have to be completed for each area of a given fracture type before an effective dispersion coefficient could be defined;
3. Recharge is extremely important in determining protection zone size and shape and must be included in fractured rock protection zone modelling. In fractured rock protection zone modelling a capability is required for zoning of hydraulic conductivity, for example, due to fracture blocks of differing characteristics, or the impact of different surface cover on fracture aperture. The impact of hydraulic conductivity zoning, model boundary location and type, and recharge may be more significant on the protection zone size and shape than fracture flow effects, as shown in the Coombe Farm example;
4. Fractured rock modelling should be done in 3-D in order that the conceptual model is represented as defensibly as possible. This would avoid problems with connectivity (the unjustified increase of trace lengths to get 2-D systems to percolate). It would also allow more than one layer to be simulated in the fractured rock system;
5. A method for fractured rock catchment scale modelling is required. 2-D modelling using a small domain size is inappropriate, and imposes unnatural boundary conditions and restrictions on the system, not allowing the flow to vary to external forces appropriately; and
6. Validation of probabilistic zones is extremely important, and tracer testing offers an important tool. The validation is concerning the uncertainty in our knowledge of the fractured system, or in other words to what extent the system is unknown, rather than assessing the uncertainty in the aquifer parameters.
9 ANISOTROPIC KINEMATIC POROSITY

9.1 Introduction

When making the step from modelling fractured rock at relatively small scales using discrete fracture models, to catchment scale modelling using continuum models, we must ensure that the essential properties of the fractured medium are incorporated into the continuum. The aquifer parameters that are required in continuum modelling derived from the fracture modelling are hydraulic conductivity for the flow simulation and kinematic porosity for the particle tracking simulations, to produce groundwater protection zones. Anisotropic hydraulic conductivity is a concept that is accepted, but little work has been done on the concept of directional kinematic porosity. When tracer tests are performed it may be observed that the resulting breakthrough curves (and hence the kinematic porosity derived) show directional dependence. This effect has been seen at Alton Court where kinematic porosity values over the range 0.5% to 2.8% were derived from the converging flow tracer testing for different tracer input locations. Guérin and Billaux (1994) obtained anisotropic kinematic porosity from their fracture modelling of the 3-D fractured system at Stripa. Because the kinematic porosity was highly anisotropic it was considered that a continuum approximation for the transport was not valid.

Kinematic porosity is defined as ratio of the volume of water able to flow to the total volume of the rock, or as is more often used in this chapter, the ratio of the darcian flux to the bulk average velocity. Dead-end fractures or pores, unconnected fractures or pores, or rock matrix through which there is no significant flow, do not contribute to the kinematic porosity.

McKee and Way (1988) have investigated the concept of directional kinematic porosity in fractured systems and came up with some general expressions for the kinematic porosity variation with direction of the hydraulic gradient and direction of the fracture sets. However, Sánchez-Vila and Carrera (1997) consider that kinematic porosity does not have directional dependence, but it is an artefact of an inappropriate selection of conceptual model in tracer test interpretation. They state that it is the anisotropic hydraulic conductivity that has been incorrectly characterised, producing the effect of apparent directional kinematic porosity. However, Fernandez-Garcia et al. (2002) state
that Sánchez-Vila and Carrera (1997) claim "porosity has directional qualities, and that porosity delineates an ellipse perpendicular to the ellipse of the transmissivity field. Apparent porosity can be represented by a second order symmetric tensor." This statement apparently contradicts the Sánchez-Vila and Carrera (1997) paper even though two authors of the 1997 paper are also involved with the Fernández-García et al. (2002) paper. The conclusions of both Sánchez-Vila and Carrera (1997) and Fernández-García et al. (2002) are examined further in this chapter.

Despite this confusion in the literature, for the case of protection zone delineation we must look at the possibility of directional kinematic porosity on a larger scale than that usually considered during convergent flow tracer tests. Over the next few pages the case for directional kinematic porosity in porous media and fractured rock is examined. The porous media case is examined first.

Consider the anisotropic porous media presented in Figure 9.1. The grains are closely packed in the direction of the short axis of the grain, but packed quite widely in the direction of the long axis of the grains. The hydraulic conductivity of the system will be higher in the direction of the long axis of the grains and lower in the direction of the short axis of the grains. The tortuosity of the two pathways, parallel and perpendicular to the grains, is also completely different. The flow samples the same porosity as it passes through the pores either in a direction parallel or perpendicular to the grains, since the water is essentially travelling through the same pores but just in a different direction.

Figure 9.1: Flowpaths parallel and perpendicular to grains in an anisotropic media.
If the grains were packed in a cubic pattern (almost impossible to occur in reality because it is an inefficient packing system) then the flow may not necessarily pass through the same pore space, as illustrated in the example in Figure 9.2.

![Figure 9.2: Grains packed in a cubic pattern (unlikely in reality).](image)

In the cubic packed example the tortuosity (or lack of it) is the same in both directions. The hydraulic conductivity is higher in a vertical direction than a horizontal one. If one imagines particles passing through the medium in the two different directions, the porosity the particles see is slightly different to the previous example in a more efficiently packed system, with the particles passing through different pore spaces in the cubic packed example.

This type of example can be extended to fractured rock systems. Generally, the different fracture sets in a fracture system will be formed under different stress regimes. The apertures of different fracture sets will then tend to be dissimilar because of the differing stress regimes that created them. Also, some fracture sets may be preferentially enlarged after their creation, by weathering and solution processes. Unlike the anisotropic porous media example above, flow through fractured systems is unlikely to sample the same spaces for different flow directions, but will sample different fracture sets with different apertures. Kinematic porosity is likely to vary with direction if, for example, one fracture set has a larger aperture than the other, or one fracture set is more dense than the other. An illustrative example of this can be seen in Figure 9.3.

### 9.2 Porosity of a simple fractured system

Let us examine the fracture porosity of the simple fractured system given in Figure 9.3.
A block has apertures in the x, y and z directions of size a, b and c respectively, where \( b > c > a \). The spacing of the fractures is p, q and r in the x, y and z directions respectively. The size of the whole block is \( \Delta x, \Delta y \) and \( \Delta z \) in the x, y and z directions.

Therefore:

Volume of the block = \( \Delta x \Delta y \Delta z \) \hspace{1cm} (1)

Number of fractures in the x direction = \( \Delta x/p \) \hspace{1cm} (2)

Number of fractures in the y direction = \( \Delta y/q \) \hspace{1cm} (3)

Number of fractures in the z direction = \( \Delta z/r \) \hspace{1cm} (4)

And:

Volume of individual blocks = \( (p-a)(q-b)(r-c) \) \hspace{1cm} (5)

Figure 9.3: Fracture and aperture spacing notation used for calculating the porosity of a fracture block.

In the x-z plane the area of the fractures = area of each of the sides – area of the blocks

\[
= \Delta x \Delta z - \left[ (p - a)(r - c) \frac{\Delta x \Delta z}{pr} \right]
\]
Similarly, area of fractures in x-y plane = \( \Delta x \Delta y \left[ 1 - \frac{(p - a)(q - b)}{pq} \right] \) (7)

And, area of fractures in z-y plane = \( \Delta y \Delta z \left[ 1 - \frac{(q - b)(r - c)}{qr} \right] \) (8)

Total volume of fractures in block = volume of block - volume of individual blocks

\[
\Delta x \Delta y \Delta z \left[ 1 - \frac{(p - a)(q - b)(r - c)}{pqr} \right]
\]

\[
\Delta x \Delta y \Delta z \left[ 1 - \frac{(p - a)(q - b)(r - c)}{pqr} \right]
\]

\[
\Delta x \Delta y \Delta z \left[ 1 - \frac{(p - a)(q - b)(r - c)}{pqr} \right]
\]

(9)

Therefore the 'total fracture porosity' of the block =

\[
\frac{\Delta x \Delta y \Delta z \left[ 1 - \frac{(p - a)(q - b)(r - c)}{pqr} \right]}{\Delta x \Delta y \Delta z}
\]

\[
\frac{1 - \frac{(p - a)(q - b)(r - c)}{pqr}}{pqr}
\]

(10)

As well as the total porosity of the block we are interested in the porosity of the block that the flow 'sees' when travelling in a particular direction: kinematic porosity.

For example, for flow in the x direction the porosity 'seen' =

\[
1 - \frac{(q - b)(r - c)}{qr}
\]

(11)

Similarly the porosity 'seen' for flow in the y direction =

\[
1 - \frac{(p - a)(r - c)}{pr}
\]

(12)

And for flow in the z direction the porosity 'seen' =

\[
1 - \frac{(p - a)(q - b)}{pq}
\]

(13)

Just from the difference in the three equations above it can be seen that porosity, in this example, has a directional quality. This is a very simplistic view of fracture flow in that it is assumes that the flow at fracture intersections only 'sees' the voids directly in front of it. It does not see the space to either side due to the intersecting fracture. In reality.
the flow will be impacted by this intersection. Figure 9.4 illustrates this more clearly. However, if the path lengths in the fractures are much longer than those across the intersections this effect may become insignificant.

| a) flow does not 'see' intersections | b) flow affected by intersections (not considered here) |

Figure 9.4: Kinematic porosity calculations assume flow does not 'see' intersections as in a) rather than b).

9.3 Kinematic flow parameters for simple fractured systems

9.3.1 Kinematic porosity using the McKee and Way approach

The question arises as to whether we can make more general statements for kinematic porosity for any number of fracture sets with the sets in arbitrary directions. McKee and Way (1988) state that they examined \( n \) fracture sets in 2-D, each set at an angle \( \theta_n \) to the x-axis, under a hydraulic gradient \( i \), at an angle \( \alpha \) to the x-axis, with the hydraulic conductivity of each fracture, \( k_n \). They also state that the total number of fracture sets is \( m \). The notation used in the following analysis is given in Figure 9.5. (The notation has been altered from that used by McKee and Wey (1988) in order that their equations can be compared more easily with those presented later in this chapter and Appendix I.)

Figure 9.5: Notation for 2-D kinematic porosity calculations by McKee and Way (1988).
McKee and Way (1988) give the flow through each fracture per unit area (or Darcian velocity) as:

\[ q_n = k_n \cos(\alpha - \theta_n) \]  

(14)

McKee and Way (1988) do not state how they define \( k_n \), although it is considered that this may be related to fracture aperture through the cubic law.

For particles travelling through the fractures they examine the "mean space coordinates" of \( x \) and \( y \) as \( \langle x \rangle \) and \( \langle y \rangle \), and \( \langle t \rangle \) as the average residence time such that:

\[ \langle x \rangle = \frac{\sum_{n=1}^{m} l_n \cos \theta_n q_n}{\sum_{n=1}^{m} q_n} \]  

(15)

\[ \langle y \rangle = \frac{\sum_{n=1}^{m} l_n \sin \theta_n q_n}{\sum_{n=1}^{m} q_n} \]  

(16)

where \( l_n \) is defined as the 'spacing'. The mathematical definition of these terms is rather confusing on the following points:

1. \( l_n \) is not the spacing perpendicular to fracture set \( n \) (which is defined as \( D_n \) in Figure 9.5). McKee and Way (1988) illustrate the definition of \( l_n \) graphically using two fracture sets such that the following relation between \( D_n \) and \( l_n \) has been deduced:

\[ D_1 = l_2 \sin(\theta_1 - \theta_2) \]  

and

\[ D_2 = l_1 \sin(\theta_1 - \theta_2) \]

where \( l_1 \) is the spacing of set 2 perpendicular to set 1 and vice versa, as shown in Figure 9.6. McKee and Way (1988) appear to have derived equations 15 and 16 from the idea of particles moving through the connected fracture system in a particular way, as shown in Figure 9.6. This definition of spacing results in the spacing of no more than two sets being definable. If three fracture sets were present then there would be a non-unique definition of the spacing. This is a fundamental flaw with the McKee and Way (1988) methodology, since they set out the methodology to have up to \( m \) fracture sets, and they do not state that \( m \) can be no greater than 2. They also make assumptions that the fracture system is
connected as shown in Figure 9.6. This makes their kinematic porosity solution applicable for only two fracture sets.

2. McKee and Way (1988) do not give a conceptual definition of the mean space coordinates. From a diagram they present in their paper it would appear that mean space coordinates refer to the stochastic variables \(x, y\) and \(t\) of average location in time and space of a large number of particles that start at the origin. However, the location of the plume and the plume movement should be related to the bulk flow in the fracture system (which depends on fracture separation of each of the fracture sets and the Darcian velocity in those sets). In equations 15 and 16 the location of the plume is only related to the Darcian velocity in each fracture, and so does not allow the importance/density of the fractures in each of the fracture sets to be taken into account.

\[
\beta = \tan^{-1}\left(\frac{\langle y \rangle}{\langle x \rangle}\right)
\]  

(17)
and the residence time in the fractures is given by equating two different velocity expressions; velocity being the distance travelled over the residence time, and velocity also being flow in a fracture over the fracture’s kinematic porosity, $n$.

Solving for the residence time gives:

$$
\langle t \rangle = \frac{\sum_{n=1}^{m} n \sqrt{(x_n)^2 + (y_n)^2} \cos(\theta_n - \beta)}{\sum_{n=1}^{m} q_n}
$$

(18)

Now calculating the velocities in the $x$ and $y$ directions, $V_x$ and $V_y$, respectively,

$$
\langle V_x \rangle = \frac{\langle x \rangle}{\langle t \rangle}
$$

(19)

$$
\langle V_y \rangle = \frac{\langle y \rangle}{\langle t \rangle}
$$

(20)

and solving for the mean velocity, $\langle V \rangle$:

$$
\langle V \rangle = \sqrt{\langle V_x \rangle^2 + \langle V_y \rangle^2} = \sqrt{\frac{\langle x \rangle^2 + \langle y \rangle^2}{\langle t \rangle}} = \frac{\sum_{n=1}^{m} ik_n \cos(\alpha - \theta_n)}{\sum_{n=1}^{m} n \cos(\theta_n - \beta)}
$$

(21)

Assuming that $k_x$ and $k_y$ are the major and minor hydraulic conductivity, the directional conductivity in the direction $\langle V \rangle$ (McKee and Way have assumed this to be the direction of flow) is $k_f$ (Bear, 1972 for anisotropic porous media):

$$
k_f = \frac{1}{\cos^2 \beta + \cos^2(90^\circ - \beta)}
$$

(22)

and the velocity in the direction of flow, $V_f$ from Darcy’s Law is:

$$
V_f = \frac{k_f}{n_f} \cos(\alpha - \psi)
$$

(23)

which gives the apparent porosity in the direction of flow as:

$$
n_f = \frac{k_f \cos(\alpha - \psi) \sum_{n=1}^{m} n_n \cos(\theta_n - \beta)}{\sum_{n=1}^{m} k_n \cos(\alpha - \theta_n)}
$$

(24)
On attempting to code up equation (24) to verify the graphical variation in effective porosity presented by McKee and Way (1988), it was found that equation 24 did not give the graphically presented variation in kinematic porosity. The actual equation deduced to obtain the figure was very different, and is given in equation 25. This gives us little confidence in the methodology presented by McKee and Way (1988).

\[
    n_f = \frac{\sum D_n k_n \cos(\theta_n - \alpha) \cos(\theta_n - \beta)}{\langle V \rangle}
\]  

(25)

It was also found that the porosity used in the example presented by McKee and Way (1988) did correlate directly to aperture (assuming a fracture spacing of 1), with the individual fracture sets in example they code up having a permeability of 1 and a porosity of 0.01 for the first set, and a permeability of 8 and a porosity of 0.02 for the second set. However, the porosity of a fracture set should depend on the spacing of the fractures as well as the aperture. No such relation is given in McKee and Way's paper.

9.3.2 Corrected approach for kinematic porosity

As presented above, the analysis by McKee and Way (1988) is flawed. Equations (15) and (16) weight the distances by the individual fracture Darcian velocities. But it is the bulk volumetric flux of a particular fracture set that is required for weighting purposes. At no point in the above analysis is the flow per unit area considered, hence bringing in the fracture spacing which is required for the bulk flow. The 'spacing' used by McKee and Way (1988) only allows two fracture sets to be considered, not more, which is
another flaw in the general methodology. They also do not state how the hydraulic conductivity of a fracture set is related to the aperture of the fracture set.

The correct approach to obtain the kinematic porosity in 2-D is now given.

![Diagram](image)

**Figure 9.7:** Notation for 2-D kinematic porosity calculations (showing only one fracture set, n).

Fracture set $n$ has an aperture $a_n$ and a spacing of $D_n$ at an angle of $\theta_n$ to the x-axis. The hydraulic gradient across the fracture system is $i$ at an angle $\alpha$ to the x-axis. The bulk flow through the system, $q_p^b$, is at an angle $\beta$ to the x-axis. This notation is illustrated in Figure 9.7. Note the presumption in the figure that the gradient, flux and velocity are, in general, in different directions.

The flow per unit area (also the Darcian velocity) in a fracture in set $n$ is given by:

$$q_n = \frac{\rho g}{12 \mu} a_n^2 i \cos(\theta_n - \alpha)$$  \hspace{1cm} (26)

Therefore the flux across an area $D_n \times W$ is $W \times a_n \times q_n$, so the bulk flux per unit area in the direction of the fracture, $q_n^b$, is:

$$q_n^b = \frac{a_n q_n}{D_n} = \frac{\rho g}{12 \mu} \frac{a_n^3}{D_n} i \cos(\theta_n - \alpha)$$  \hspace{1cm} (27)

The bulk flux in an arbitrary flow direction, $\psi$, due to set $n$ is:

$$q_{n\psi}^b = q_n^b \cos(\theta_n - \psi)$$  \hspace{1cm} (28)

So the total flux in direction $\psi$ due to all fracture sets is:
\[ q^b_\psi = \sum_n q^n_\psi \cos(\theta_n - \psi) \]  

(29)

Velocity in the direction \( \psi \) for water in fracture set \( n \):

\[ v^b_\psi = q^n \cos(\theta_n - \psi) \]  

(30)

The velocity in direction \( \psi \) must be weighted by the magnitude of the bulk flow in each of the fracture sets to give the mean velocity:

\[ V^B_\psi = \frac{\sum_n |q^n_\psi| q^n \cos(\theta_n - \psi)}{\sum_n |q^n_\psi|} \]  

(31)

Therefore we can define the kinematic porosity, \( n_\psi \) in direction \( \psi \) as:

\[
 n_\psi = \frac{q^b_\psi}{V^B_\psi} = \frac{\sum_n q^n_\psi \cos(\theta_n - \psi) \sum_n |q^n_\psi|}{\sum_n |q^n_\psi| q^n \cos(\theta_n - \psi)} \]

\[
 = \frac{\sum_n \frac{a_n^3 \cos(\theta_n - \alpha) \cos(\theta_n - \psi) \sum_n a_n^3 \cos(\theta_n - \alpha)}{D_n}}{\sum_n a_n \cos(\theta_n - \alpha) \cos(\theta_n - \psi) \cos(\theta_n - \alpha)} \]  

(32)

One of the interesting aspects of the relationship in equation 32 is that the kinematic porosity is proportional to the aperture to the power of six over the aperture to power of five. These exponents are not particularly common in the natural world and perhaps point to the unusual nature of kinematic porosity in fractured systems.

The directions of the average bulk flux and average bulk velocity can be determined from their tangents:

\[
 \tan \beta = \frac{q^b_{90^\circ}}{q^b_\psi} = \frac{\sum q^n_\psi \sin(\theta_n)}{\sum q^n_\psi \cos(\theta_n)} \]  

(33a)

\[
 \tan \gamma = \frac{v^b_{90^\circ}}{v^b_\psi} = \frac{\sum |q^n_\psi| q^n_\psi \sin(\theta_n)}{\sum |q^n_\psi| q^n_\psi \cos(\theta_n)} \]  

(33b)

Note that \( \gamma \) and \( \beta \) are functions of \( \alpha \), through equation 27.
9.3.3 Effective hydraulic conductivity

From the fracture network set up in Figure 9.7 we can also examine the variation in hydraulic conductivity with change in hydraulic gradient for multiple fracture sets of infinite length. Equation 27, 28 and 29 give the bulk flux in fracture set \( n \) in the direction of the fracture, the bulk flux in an arbitrary flow direction and the bulk flux due to all sets in an arbitrary flow direction.

Now if we consider Darcy's Law in an arbitrary direction \( \psi \):

\[
q^b = K^b \cos(\alpha - \psi)
\]

where:

\[
q^b = \sum_n q^b_n \cos(\theta_n - \psi)
\]

giving:

\[
K^b = \frac{q^b}{\cos(\alpha - \psi)}
\]

\[
= \frac{\sum_n q^b_n \cos(\theta_n - \psi)}{\cos(\alpha - \psi)}
\]

\[
= \frac{\sum_n a^3_n \frac{P \cos(\theta_n - \alpha) \cos(\theta_n - \psi)}{12D_n \mu}}{\cos(\alpha - \psi)}
\]

The intrinsic permeability in the bulk flow direction, \( \beta \), would therefore be:

\[
k^b = \frac{\sum_n a^3_n \cos(\theta_n - \alpha) \cos(\theta_n - \beta)}{\cos(\alpha - \beta)}
\]

and in the direction of the gradient the intrinsic permeability would be:

\[
k_\alpha = \frac{\sum_n a^3_n \cos^3(\theta_n - \alpha)}{12D_n}
\]
9.3.4 The tensorial nature of kinematic porosity

The directional nature of kinematic porosity is examined mathematically in Appendix I (pers. comm. J.A. Barker).

It is suggested that a kinematic-porosity matrix (tensor), \( N \), should be defined through:

\[ q = N v \tag{38} \]

where:

- \( q \) - is the darcian flux vector, and
- \( v \) - is the bulk average velocity vector.

In Appendix I the same 2-D model as studied in this chapter (Figure 9.7) is considered. It is shown that for this model:

- the permeability tensor can be defined, but
- it is not possible, in general, to define the porosity tensor.

While these results apply to a specific, simple, two-dimensional model of a fractured medium, this single counter-example it is sufficient to show that it is unlikely that the porosity tensor will exist for any discrete fracture model comprising fractures in more than one direction.

9.3.5 Graphical illustration of change in kinematic porosity and effective permeability with hydraulic gradient

Equations 31, 32 and 37 have been coded up in a spreadsheet format in order to examine the velocity (equation 31), kinematic porosity (equation 32) and permeability variation (equation 37) with change in hydraulic gradient direction. In all cases the value in the direction of the gradient are plotted: \( \psi = \alpha \). These variations have been examined for different numbers of fracture sets and fracture parameters for change in hydraulic gradient direction, as illustrated in Figure 9.8 and 9.9. Also plotted is \( 1/\sqrt{k_a} \) as shown in Appendix I an ellipse should always result for the case of infinite fracture sets as detailed in section 9.3.3. Figures 9.8 and 9.9 are also normalised to a unit average radius in order that the form of the relationships can be shown on the same diagram.
Details of the fracture set and fracture parameter scenarios examined are given in Table 9.1 along with the corresponding figure numbers. The corresponding bulk flow direction, $\beta$, and velocity direction, $\gamma$, are also presented.

Table 9.1: Summary of fracture sets and parameters used to show variation of velocity, kinematic porosity and permeability with change in hydraulic gradient direction.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Fracture Set Number</th>
<th>Aperture (mm)</th>
<th>Fracture Direction ($^\circ$)</th>
</tr>
</thead>
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<tr>
<td>9.8a)</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>9.8b)</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>90</td>
</tr>
<tr>
<td>9.8c)</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10</td>
<td>90</td>
</tr>
<tr>
<td>9.9a)</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
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Where possible the fracture parameter scenarios in Table 9.1 have been examined using the McKee and Way (1988) methodology. Only the scenarios containing two fracture sets at ninety degrees to each other were examined. It was not possible to compare the other scenarios due to the limitations of the McKee and Way (1988) methodology. The McKee and Way fracture parameter scenarios are illustrated in Figure 9.10.
Figure 9.8: Left: polar plots of the variation with direction of the hydraulic-gradient of: the kinematic porosity, velocity, permeability (k) and $1/\sqrt{k_a}$, all measured in the direction the gradient ($\psi = \alpha$). Right: x-y plot of the variations of the directions (in degrees) of the bulk flux (beta), bulk velocity (gamma) with the direction of the gradient (alpha) plotted as the x-axis. First three cases in Table 9.1.

Chapter 9 – Anisotropic kinematic porosity
Figure 9.9: Left: polar plots of the variation with direction of the hydraulic-gradient of: the kinematic porosity, velocity, permeability ($k$) and $1/\sqrt{k}$, all measured in the direction the gradient ($\psi = \alpha$). Right: x-y plot of the variations of the directions (in degrees) of the bulk flux (beta), bulk velocity (gamma) with the direction of the gradient (alpha) plotted as the x-axis. Final three cases in Table 9.1.
Figure 9.10: Left: polar plots of the variation with direction of the hydraulic-gradient of: the kinematic porosity and velocity, all measured in the direction the gradient ($\psi = \alpha$). Right: x-y plot of the variations of the directions (in degrees) of the ‘bulk’ flux (beta), ‘bulk’ velocity (gamma) with the direction of the gradient (alpha) plotted as the x-axis. Cases 9.7b) and 9.7c) in Table 9.1 using the McKee and Way methodology.

In Figure 9.8a) for two identical (parallel) fracture sets it can be seen that permeability forms a dumbbell shape. This shape arises because of the form of equation 36, where $\theta_n$ (angle of the fracture sets) is zero for both fracture sets, and therefore the permeability goes as the cosine squared of the hydraulic gradient. The kinematic porosity is, as expected, shown to be isotropic for this simple case, and this arises because of the form
of equation 31. This result is therefore isotropic and forms a circle in Figure 9.8a). The velocity has an identical form to the permeability and, because of their coincidence, cannot be seen in the figure. Because the fractures are only in one direction in the case in Figure 9.8a) the flow and velocity can only be in one direction as shown on the right.

Figure 9.8b) presents the results for two perpendicular fracture sets, with identical parameters. The kinematic porosity is shown to be anisotropic, with minima in the directions of the fracture sets and maxima midway between the fracture sets. The velocity peaks in the direction of the fracture sets, and goes to minima midway between fracture sets. Permeability is isotropic because equation 36 reduced the sum of the cosine squared and the sine squared of the hydraulic gradient, being unity. The direction of the (bulk average) velocity in this case (γ) does not follow the hydraulic gradient and bulk flow directions (α and β respectively).

Figure 9.10a) presents the results for the same scenario as in Figure 9.8b), but with only the kinematic porosity and the velocity shown. The permeability is not illustrated in this figure because the permeability used by McKee and Way (1988) has only been deduced from the results that they present, and the exact definition of the effective permeability is not known. Figure 9.10a) illustrates that both kinematic porosity and velocity are essentially isotropic, except for the hydraulic gradient at an angle of 315°. At this angle the sum of the flows in the fractures totals zero because McKee and Way (1988) do not state to take the magnitude of the flows only. This highlights yet another error within the McKee and Way (1988) methodology. The issue of the sign of the flows can be seen in the plot of alpha, beta and gamma in Figure 9.10a) where the 'bulk' flux and the 'bulk' velocity are always parallel to the hydraulic gradient, but are often in the opposite direction. The forms of the kinematic porosity shown in Figure 9.8b) and 9.10a) are very different, again indicating the major differences between the two different methodologies.

Figure 9.8c) is the result of two perpendicular fracture sets, with the one at ninety degrees having an aperture an order of magnitude higher than the other. Figure 9.8c) shows that the kinematic porosity is very small in the direction of the narrow aperture fracture set, and away from this direction it tends towards the case shown in Figure 9.8a). This presumably arises because of the dominance of the one large aperture.
fracture set. The permeability and velocity also behave as in Figure 9.8a) because of the dominance of the large aperture set, with the maximum permeability and velocity in the direction of this set. Further testing in the spreadsheet with two fracture sets shows that the form of the velocity, permeability and the kinematic porosity remains almost the same if the aperture of one set is more than two to three times that of the other. The directions of the flow gradient and velocity are then very similar to those in Figure 9.8a).

Figure 9.10b) presents the results for the same scenario as in Figure 9.8c), but with only the kinematic porosity and the velocity shown, and not the permeability as discussed earlier. Figure 9.10b) is very similar in form to that in Figure 9.8c) for both kinematic porosity and velocity. The velocity is shown to be a maximum in the direction of the larger permeability and porosity fracture set, with a minimum in the direction of the smaller permeability and porosity fracture set. The kinematic porosity for most angles of hydraulic gradient is shown to be isotropic, apart from in the direction of the smaller porosity fracture set. If the difference in porosity and permeability of fracture sets is further increased for the McKee and Way (1988) methodology, the form of the velocity and kinematic porosity remains the same. In this example it can be seen that there is close agreement between the two methods of deriving the kinematic porosity, indicating that whilst there are many errors in the McKee and Way (1998) methodology, there are some similar features (which may also be a coincidence). From Figure 9.10b) of the ‘bulk’ flux and ‘bulk’ velocity directions it can be seen that the directions are dominated by the larger permeability set, although the velocity and flux directions are non-coincident and often 180° apart, as also seen in Figure 9.10a).

Figure 9.9a) is derived from four fracture sets at forty-five degrees to each other with identical fracture properties. It can be seen that the kinematic porosity, velocity and permeability are almost isotropic. There are, however, still minima in porosity and maxima of permeability and velocity in the direction of fractures. There is little variation in the bulk flow and velocity angles from the hydraulic gradient.

Figure 9.9b) illustrates the case of four fracture sets at forty-five degrees but with two of the fracture sets having apertures five times of the other two. This case is very similar to that observed in Figure 9.9b) and illustrates the dominance of the larger aperture fracture sets on the characteristics of the system. The minima of kinematic porosity
remain in the directions of the larger aperture fracture sets. Permeability appears to be isotropic and velocity has maxima in the direction of largest aperture sets. The direction of the velocity, \( \gamma \), comes into line with the hydraulic gradient, \( \alpha \), and flow, \( \beta \), as in the case presented in Figure 9.8b).

Figure 9.9c) shows that when the apertures are only two or three times greater than the smallest aperture fracture set, and the fracture sets are not at equal angle spacing, asymmetric shapes are observed. The permeability is still observed to be a dumbbell type shape and \( 1/\sqrt{k_a} \) remains an ellipse. The velocity is similar to the permeability but with small peaks in the direction of fracture set four. The kinematic porosity is a bizarre shape, but with minima again occurring the directions of the greatest aperture set.

The kinematic porosity minima occurring in the direction of the greatest aperture sets is a feature of all the scenarios presented. It must be noted that these kinematic porosity variations have been obtained assuming infinite fracture systems. In reality fractures have a finite length and the nature of the connectivity of a fracture system with finite fractures will be very different to that with infinite fractures, possibly producing very different kinematic porosity variations with direction of hydraulic gradient.

### 9.3.6 2-D numerical modelling exercises

Having shown that theoretically kinematic porosity has an anisotropic nature for infinite length fractures, it would be interesting to discover whether this property can also be demonstrated through numerical modelling exercises with finite lengths of fractures. To investigate the effect of finite fracture lengths the 2-D fracture flow program SDF was used. Two scenarios were investigated:

1. A fracture system comprising of two perpendicular fracture sets with equal apertures of \( 5 \times 10^{-6} \) m.
2. A fracture system comprising of two perpendicular fracture sets with apertures of \( 5 \times 10^{-6} \) m and \( 5 \times 10^{-5} \) m.

The fracture block used was 20 m by 20 m with the fracture lengths for all sets following a lognormal distribution with an average length of 3.0 m, and a fracture
density of 0.8 m$^{-1}$. Single values for orientations and apertures were used; only lengths were varied in the stochastic fracture generation. The direction of the hydraulic gradient across the network was rotated by rotating the fracture set directions rather than rotating the hydraulic gradient: SDF does not allow the hydraulic gradient to be rotated, it can only be perpendicular to the model boundaries (see Appendix J for a fuller explanation of SDF capabilities). The hydraulic gradient was applied using fixed head boundaries on opposite boundaries and no flow boundaries on the other two sides. The fracture sets were rotated in ten-degree intervals. The flow across the block was given in the program output, the hydraulic gradient and the area of the block were known, and therefore the hydraulic conductivity in the direction of the hydraulic gradient could be calculated. Five realisations were simulated for each fracture set direction. A typical fracture network realisation is illustrated in Figure 9.11.

Particle tracking across the block was then simulated using 200 particles for each simulation. The distribution of times of particles arriving at the down gradient boundary always produced a lognormal distribution, as shown in Figure 9.12, and therefore the geometric mean for the average time for particle arrival at the boundary was used in further calculations. From knowledge of the hydraulic conductivity, $K$, hydraulic gradient, $i$, and the average velocity across the block, $v$, a kinematic porosity, $n_e$, could be calculated by $n_e = Ki/v$.

![Figure 9.11: Typical fracture network used to create the kinematic porosity variation with hydraulic gradient direction.](image)
The exercise was attempted with a linear gradient along the side boundaries. For the case of non-equal apertures for the two fracture sets the particles tend to move along the fractures of larger aperture. Therefore at some angles of the fracture sets all the particles pass out of the side boundaries, making definition of an average particle velocity in the direction of the hydraulic gradient impossible to obtain. The partial results of this exercise are therefore not presented.

Figure 9.12: Typical distribution of particle travel times used in calculating the kinematic porosity for a 2-D fracture network using SDF.

Figure 9.13: Variation in kinematic porosity with direction of hydraulic gradient for two vertical fracture sets at 0 and 90 degrees with equal apertures.
The variation of kinematic porosity with direction is presented in Figure 9.13 for the first scenario with equal aperture fractures for both fracture sets. It can be seen that the minimum kinematic porosity is actually in the direction of the two fracture sets.

Figure 9.14 presents the results of the second scenario with the apertures an order of magnitude different (\(5 \times 10^{-6}\) m and \(5 \times 10^{-5}\) m). It can be seen that, again, the minimum kinematic porosities are in the direction of the fracture sets. Whereas the higher kinematic porosities in Figure 9.13 are in all directions except that of the fracture sets, in Figure 9.14 the higher kinematic porosities are midway between the two fracture set directions and are an order of magnitude higher than the values in between. It is also interesting to note that the maximum kinematic porosities midway between the fracture set directions are different, with one being three times the other. Given that the maxima are midway between the fracture sets it is difficult to understand why they should be different. It is possible that it is due to insufficient numbers of realisations to obtain the average kinematic porosity for a particular direction, but this is unlikely since the individual kinematic porosities from the realisations only vary within a few percent.

Figure 9.14: Variation in kinematic porosity with direction for two vertical fracture sets at 0 and 90 degrees and apertures of \(5 \times 10^{-6}\) and \(5 \times 10^{-5}\) m respectively.
Extreme care must be taken when attaching an interpretation to the results of Figures 9.13 and 9.14. Ideally the hydraulic gradient would be rotated around a fixed fracture set and the kinematic porosity plotted for change in gradient direction. However, SDF, the program used to create the fracture network and simulate flow is such that the fracture sets must be rotated and not the hydraulic gradient. Thus, instead of having a fixed fracture network around which the hydraulic gradient is rotated to give a kinematic porosity with direction, the fracture network is different for each different direction of the fracture sets with respect to the hydraulic gradient. Figures 9.13 and 9.14 represent an average variation in kinematic porosity, and not the actual variation in kinematic porosity with direction for a fixed fracture set. One other factor must be born in mind when considering Figures 9.13 and 9.14: two sides of the model are defined as no flow boundaries. Because of the nature of fracture flow, flow would occur across this boundary, which would alter the flows at the down gradient boundary and ultimately affect the kinematic porosity calculation.

The above examples are only for a fracture domain of 20 m by 20 m and therefore it is possible that the fracture network is close to the REV. Given that the REV size can be described as approximately ten times the fracture spacing (see section 4.5), the REV could be of the order of 12 m (given a fracture spacing of 0.8 m\(^{-1}\)), which is close to the fracture domain size of 20 m. It is possible therefore that Figures 9.13 and 9.14 are for scenarios below the REV. Therefore it is important that scenarios that are definitely above the REV scale are examined. This could not be done using SDF because of the limitation on array sizes and compiling errors in the code. The problem of anisotropic kinematic porosity was therefore examined using 3-D techniques.

### 3.3.7 3-D numerical modelling exercises

It has been shown in the previous section that anisotropic kinematic porosity does appear to exist in 2-D fractured systems. However, the nature of this anisotropy remains uncertain, therefore the possibility of anisotropic kinematic porosity in 3-D fractured systems should also be examined. The relationship between the kinematic porosity and effective hydraulic conductivity is also studied. The 3-D fracture model Fracman/Mafic was used to investigate three scenarios:

1. Two vertical perpendicular fracture sets of equal aperture and size, and with their centres located according to the enhanced Baecher mode of generation with
equal densities (see Appendix J for an explanation of Fracman/Mafic terminology);
2. Two perpendicular fracture sets of equal aperture and size, one horizontal and
one vertical and with their centres located according to the enhanced Baecher
mode of generation with equal densities; and
3. Three orthogonal fractures sets of equal aperture and size, and with their centres
located according to the enhanced Baecher mode of generation with equal
densities.

The kinematic porosity was obtained in the same manner as with the SDF scenarios, as
explained in section 9.3.6.

Scenario 1
Figure 9.15 illustrates one realisation of the fracture system from scenario one using
Fracman/Mafic. Figure 9.16 presents the variation of kinematic porosity with hydraulic
gradient direction and Figure 9.17 presents the variation in hydraulic conductivity (as
$1/\sqrt{k_w}$) with hydraulic gradient direction for the same system.

Figure 9.15: One realisation of the two vertical fracture sets used to examine kinematic
porosity in 3-D systems using Fracman.
Figure 9.16: Variation in kinematic porosity in the horizontal plane with gradient direction for two vertical fracture sets using Fracman/Mafic, as illustrated in Figure 9.15.

Figure 9.17: Variation in $1/\sqrt{K_p}$ in the horizontal plane for two vertical fracture sets using Fracman/Mafic, as compared to the isotropic case, for a fracture system similar to that illustrated in Figure 9.15.
From Figure 9.16 a clear variation in the kinematic porosity can be seen with hydraulic gradient direction, with the highest kinematic porosities in the direction of the fracture sets, dropping to lowest values exactly midway between the fracture sets. Figure 9.17 shows that the hydraulic conductivity for this system is essentially isotropic, since it almost coincides with the dotted line which represents a purely isotropic system. Figure 9.18 shows that there is negligible correlation between the hydraulic conductivity and the kinematic porosity, with an R² value of 0.018.

**Scenario 2**

Since most sedimentary fractured systems in the UK have a bedding plane fracture set that is almost horizontal it is important to understand the influence of this set on directional kinematic porosity and hydraulic conductivity. Scenario two examines the case of one vertical and one horizontal fracture set of the same characteristics.

Figure 9.19 illustrates one realisation of the fracture system from the second scenario using Fracman/Mafic. Figure 9.20 presents the variation of kinematic porosity with hydraulic gradient direction and Figure 9.21 presents the variation in hydraulic conductivity (as $1/\sqrt{K_w}$) with hydraulic gradient direction for the same system. Figure 9.22 shows the correlation between the hydraulic conductivity and the kinematic porosity for this system.
Figure 9.19: One realisation of the two fracture sets, one vertical and one horizontal at ninety degrees, used to examine kinematic porosity in 3-D systems with Fracman.

Figure 9.20: Variation in kinematic porosity in the horizontal plane with gradient direction for two fracture sets, one vertical and one horizontal with Fracman/Mafic, as illustrated in Figure 9.19.
From Figure 9.20 it can be seen that the maximum kinematic porosity is parallel to the direction of the vertical fracture set. The minimum value is perpendicular to this vertical fracture set, at approximately half the maximum value. Figure 9.21 illustrates the variation in hydraulic conductivity with hydraulic gradient direction, with the maximum of the ellipse in the direction of the vertical fracture set and the minimum perpendicular to this fracture set. However, it must not be forgotten that the hydraulic conductivity has been plotted as $1/\sqrt{K}$ and therefore the maximum hydraulic conductivity is actually perpendicular to the direction of the vertical fracture set.

From Figures 9.20 and 9.21 it can be seen that the direction of maximum hydraulic conductivity is in fact perpendicular to the maximum of the kinematic porosity 'ellipse'. Fernandez-Garcia et al. (2002) state that the kinematic porosity delineates an ellipse perpendicular to the ellipse of the transmissivity field, which for this case does appear to be correct.

Figure 9.21: Variation in $1/\sqrt{K}$ in the horizontal plane for two fracture sets, one vertical and one horizontal with Fracman/Mafic, as compared to the isotropic case, for a fracture system similar to that illustrated in Figure 9.19.
Figure 9.22 illustrates that there is a correlation between the kinematic porosity and hydraulic conductivity for this fractured system. This points to the possibility that joint probability density functions could be defined for some fractured systems.

![Graph showing correlation](image)

**Figure 9.22:** Correlation between effective hydraulic conductivity and kinematic porosity for two fracture sets, one vertical and one horizontal.

**Scenario 3**

Having examined the case for one vertical and one horizontal fracture set, it is instructive to look at the case of three orthogonal fracture sets since many sedimentary fractured systems in the UK have a bedding plane fracture set that is almost horizontal and two conjugate vertical fracture sets (for example the Cotswolds – see section 6.1 for more details). Scenario three examines the case of three orthogonal fracture sets of the same characteristics.

Figure 9.23 illustrates one realisation of fracture system using Fracman/Mafic for the third scenario. Figure 9.24 presents the variation of kinematic porosity with hydraulic gradient direction and Figure 9.25 presents the variation in hydraulic conductivity (as $1/\sqrt{K_e}$) with hydraulic gradient direction for the same system. Figure 9.26 shows the correlation between the kinematic porosity and the hydraulic conductivity for the system of three orthogonal sets.
Figure 9.23: One realisation of the three orthogonal fracture sets used to examine kinematic porosity in 3-D systems with Fracman.

Figure 9.24: Variation in kinematic porosity in the horizontal plane with gradient direction for three orthogonal fracture sets using Fracman/Mafic, as illustrated in Figure 9.23.

Chapter 9 – Anisotropic kinematic porosity
Figure 9.25: Variation in $1/\sqrt{K_{\alpha}}$ in the horizontal plane for three orthogonal fracture sets using Fracman/Mafic, as compared to the isotropic case for a fracture system similar to that illustrated in Figure 9.23.

Figure 9.26: Correlation between effective hydraulic conductivity and kinematic porosity for three orthogonal fracture sets.
As can be seen in Figure 9.26 there appears to be little correlation between the kinematic porosity and the hydraulic conductivity for the fractured system with three orthogonal sets.

As with the case for two vertical fracture sets only, the hydraulic conductivity for three orthogonal sets is isotropic. The kinematic porosity is, however, anisotropic, although the difference in kinematic porosity with direction is not as pronounced as when the horizontal set was not present (scenario one). All three scenarios have assumed that the fracture set dip and strike directions do not vary within a set. However, within natural fracture systems there will be some variation in direction of a set. It is therefore likely that for such a system with three fracture sets, one of which is horizontal, the kinematic porosity will be isotropic, or extremely close to isotropic.

In summary it is considered that the kinematic porosity for real fractured system may be isotropic for cases with a horizontal fracture set and several sub vertical fracture sets with varying orientations within a set. However, for cases where a horizontal set is not present and where the fracture set directions vary only minimally, kinematic porosity is likely to be anisotropic and must be included in an analysis for kinematic flow parameters.

Extreme care must be taken when attaching an interpretation to the results of Figures 9.16, 9.20 and 9.24. Ideally the hydraulic gradient would be rotated around a fixed fracture set and the kinematic porosity plotted for change in gradient direction. However, the program Fracman/Mafic is such that the fracture sets must be rotated and not the hydraulic gradient. Thus, instead of having a fixed fracture network around which the hydraulic gradient is rotated to give a kinematic porosity with direction, the fracture network is different for each different direction of the fracture sets with respect to the hydraulic gradient. Figures 9.16, 9.20 and 9.24 represent an average variation in kinematic porosity, and not the actual variation in kinematic porosity with direction for a fixed fracture set. One other factor must be born in mind when considering Figures 9.16, 9.20 and 9.24: four sides of the model are defined as no flow boundaries. Because of the nature of fracture flow it is considered more than likely that flow would occur across this boundary, which would alter the flows at the down gradient boundary and ultimately affect the kinematic porosity calculation.
The above analysis has assumed that fractures are effectively parallel plates. However, we know that this is not generally considered to be the case, and that fractures are more accurately represented as channels (Pyrak Nolte et al., 1987, Tsang et al., 1991). Because of the channelling effect, when flow occurs across the network in different directions entirely different channels may be sampled by the flow, the channels being independent of one another. The channels are likely to have different kinematic porosities. Therefore there is a case for directional variation of kinematic porosity in real channelled systems being even stronger than in parallel plate systems.

This channelling effect could, however, work in one of two ways depending on the density of channels in the system. If the density of channels is high there will be many channels within a single fracture in many different directions. Because of the large number of directions it is likely that the kinematic porosity will tend to be isotropic. However, if the density of channels is low there may be just a few distinct channels in preferential flow directions resulting in anisotropic kinematic porosity.

9.4 Applying anisotropic kinematic porosity to Modpath

Kinematic porosity is required when calculating the velocity of the particles travelling in a fractured system, and is used in the particle tracking exercises for protection zone delineation. Velocity is a vector quantity, as well as the flow in a system being a vector quantity. The two are related by the following relationship:

\[ n v = q \]

Therefore the kinematic porosity must either be a scalar or tensor quantity, so that both \( q \) and \( v \) remain vectors. We know from the examples in section 9.2 and 9.3 that the kinematic porosity can vary with direction and, according to Fernandez-Garcia et al. (2002) it should be a second order symmetric tensor. However, from the analysis in Appendix I, defining the kinematic porosity as either a scalar or a tensor is not straightforward.

The nature (scalar or tensor) of kinematic porosity will affect the particle tracking equations used for delineation of protection zones. However, since we do not know the exact nature of kinematic porosity it has not been considered prudent to make code changes within particle tracking programs to take account of the possible tensorial
nature of kinematic porosity. It is however, useful to examine how particle tracking routines should be changed if kinematic porosity were found to be a tensor.

Pollock (1988 and 1989) developed a particle tracking program based on linear interpolation of velocities. This interpolation technique has shown to be one of the most robust methods for particle tracking (Semra et al., 1994). The brief analysis presented here is therefore based on the method of Pollock (1988 and 1989).

The partial differential equation describing conservation of mass in a steady-state 3-D groundwater flow system can be expressed as:

\[
\frac{\partial(nv_x)}{\partial x} + \frac{\partial(nv_y)}{\partial y} + \frac{\partial(nv_z)}{\partial z} = W
\]  

(1)

where \( v_x, v_y \) and \( v_z \) are the principal components of the average linear groundwater velocity vector

\( n \) is the porosity of the system (isotropic)

\( W \) is the volume rate of water that is created or consumed at the internal sources or sinks per volume of aquifer.

Now, if we consider that kinematic porosity is directional (more specifically, a tensor with principal directions in the \( x, y \) and \( z \) directions) equation (1) becomes:

\[
\frac{\partial(n_xv_x)}{\partial x} + \frac{\partial(n_yv_y)}{\partial y} + \frac{\partial(n_zv_z)}{\partial z} = W
\]  

(2)
Consider a cube of dimension $Ax$ by $Ay$ by $Az$ with flows, $Q$, either in or out of each of the sides in the $x$, $y$ and $z$ directions as shown in Figure 9.27.

![Figure 9.27: Unit cell used in the consideration of flow velocities for anisotropic kinematic porosity.](image)

We can then define the velocity across each of these faces:

$$v_{x1} = \frac{Q_{x1}}{n_x AxAy}$$

$$v_{x2} = \frac{Q_{x2}}{n_x AyAz}$$

$$v_{y1} = \frac{Q_{y1}}{n_y AxAz}$$

$$v_{y2} = \frac{Q_{y2}}{n_y AxAy}$$

$$v_{z1} = \frac{Q_{z1}}{n_z AxAx}$$

$$v_{z2} = \frac{Q_{z2}}{n_z AxAx}$$

Then the mass balance for the cell is given by the following expression, assuming that $Q_s$ accounts for the internal sources and sinks within the cell:

$$\frac{n_x (v_{x2} - v_{x1})}{Ax} + \frac{n_y (v_{y2} - v_{y1})}{Ay} + \frac{n_z (v_{z2} - v_{z1})}{Az} = \frac{Q_s}{AxAyAz}$$

Following the logic of Pollock (1988) we only have to make changes to the code at the beginning of the velocity calculation and then the code can carry through the linear interpolation method as normal.
9.5 Summary

In summary, it has been shown that:

- in general, from a model of sets of infinite fractures and numerical modelling of finite length fractures in 2-D the lowest values of kinematic porosity appear to be in the direction of the fracture sets;
- from more realistic numerical simulations in 3-D of finite length fractures, kinematic porosity is highest in the direction of the fracture sets;
- the existence of a horizontal or bedding plane fracture set in 3-D numerical simulations reduces the degree of anisotropy in kinematic porosity. However, the presence of a bedding plane set may not necessarily result in isotropic kinematic porosity;
- in general, anisotropic kinematic porosity cannot be expressed as a tensor. Therefore it is difficult to define an equivalent kinematic porosity for use in continuum models. Further work on this aspect, particularly averaging effects, is recommended so that the kinematic porosity used in particle tracking for zone delineation can be more accurately represented.
10 TESTING A 3-D METHODOLOGY

As shown in Chapter 8, problems were discovered with attempting to delineate probabilistic protection zones using 2-D fracture modelling methods. For example, there was the problem of unjustified increase in fracture lengths for the 2-D fracture system to percolate. This increase in fracture length is not defensible. Therefore, since fracture connectivity appears to be very much a 3-D problem, the possibility of developing a methodology in 3-D should be investigated. The first section of this chapter presents an outline 3-D methodology that attempts to resolve the issues discussed in Chapter 8. The second section uses Alton Court as a pilot study for this methodology. The last two sections of the chapter go on to examine the problem of probabilistic protection zone validation, and how the probabilistic protection zones should be used and interpreted by the regulator and land-use planner.

10.1 Developing protection zones in 3-D

As outlined in the previous chapters, to perform zone delineation for a source in fractured rock and to obtain probability contours with confidence, considerable volumes of data are required along with fracture flow models requiring not insignificant amounts of computer power (speed and memory). At the present time, standard computer power is not great enough to be able to model groundwater flow on a catchment scale using current fracture flow models, which are required for zone delineation. Catchment scale modelling can only be done using porous media flow models. Since one of the principal aims of the methodology was to be practical and use methods that are understood and easily available, a tool has to be found that provides an intermediate measure between porous media catchment models and fracture flow catchment models until standard PCs have the power to deal with catchment scale fracture models.

This intermediate measure should incorporate the heterogeneity of fracture flow, with porous media models permitting catchment-scale stochastic modelling, leading to probability contours. This intermediate measure should be 3-D to simulate the connectivity and heterogeneity of the fracture systems with confidence. This can be achieved by combining the flow characteristics from a stochastic 3-D fracture flow model, such as Fracman, and a 3-D porous media model, such as Modflow/Modpath to achieve the catchment scale modelling. This intermediate methodology should have the capability to:
• simulate 50-day, 400-day and catchments zones;
• allow zoning of aquifer properties;
• have a stochastic capability; and
• allow anisotropy of hydraulic conductivity and kinematic porosity.

The 3-D modelling technique suggested here works up through several levels (see Figure 10.1 and Table 10.1). The lowest level involves the collation and collection of distributions of fracture parameters such as trace lengths, apertures, spacings and orientations. These distributions then feed into the second level of stochastic fracture modelling, at a scale that is greater than the REV for that fractured rock type (if there is a definable REV). The equivalent directional hydraulic conductivities and kinematic porosities are calculated for the fracture block i.e. \(K_x, K_y, K_z, n_x, n_y\) and \(n_z\). These equivalent parameters are then calculated many times for many different realisations of the fracture block with the same fracture statistics to obtain distributions of the equivalent parameters. The distributions of \(K_x, K_y, K_z, n_x, n_y\) and \(n_z\) are then fed into the catchment-sized Modflow type model, with a cell size at least as large as the REV for the fractured system. The porous media model would have a stochastic capability to incorporate the \(K\) and \(n\) distributions. Thus, the catchment-sized model contains the fracture flow heterogeneities provided by the Fracman model while the Modflow model provides the ability to represent zoned aquifer parameters, boundary conditions, surface water features and major structural features. Particle tracking is then performed across the model for many realisations of the model to obtain enough data to define the probability contours for a given time. These last two stages, involving the hydraulic conductivity and kinematic porosity distributions and the Modflow modelling, form the top level of the technique.

The method has several assumptions that should not be overlooked:

• Using a Fracman block size that is an optimal size for the cells in the Modflow model assumes that the Fracman block is larger than the REV for the fracture system. Therefore the method would not be suitable for very sparse fracture systems, especially karstic ones where a REV is not definable.
• The $K_x$, $K_y$, $K_z$, $n_x$, $n_y$ and $n_z$ parameters for input into Modflow can be defined and represent the heterogeneity adequately.

• The data collected and used as the initial input for the Fracman model are sufficiently accurate to represent the fracture system and fracture flow characteristics.

• First arrivals are the most important factor in defining the protection zones. Fracman assumes an impermeable matrix, and hence for aquifers of the double-porosity type the protection zones would be much larger than otherwise predicted through double porosity modelling. First arrivals are often at very low concentrations in a double porosity aquifer and could be considered to be insignificant in terms of contamination.

The aim of this style of modelling is ultimately to give a set of $K_x$, $K_y$, $K_z$, $n_x$, $n_y$ and $n_z$ distributions for the various fractured aquifer types (across the spectrum of fractured rock) which could then be used in the stochastic Modflow modelling. The sets of hydraulic conductivity and kinematic porosity distributions would be produced using Fracman. A library of parameters (for example the mean and standard deviation of $K_x$) would then exist either at the Fracman or the Modflow level that could be used for any site considered to have similar characteristics. Hence, if the Fracman process has been undertaken for one site, then it may not be necessary to perform the process for another site of the same fracture provenance, unless there were more fracture parameter data that could be used to constrain the old $K$ and $n_e$ distributions. Therefore, after some period of application the Fracman section of the modelling exercise could become unnecessary. It would then only be the Modflow modelling that would have to be undertaken for each source.

The probability zones, the sets of $K$ and $n_e$ parameters and the initial fracture parameter data must go through a process of continual updating as more data are collected. The data used to develop the $K$ and $n_e$ parameters will not, initially, be well constrained. The aim, however, is that over time more fracture data are collected and that the fracture parameters gradually become further constrained. The confidence in the $K$ and $n_e$ directional parameters should then increase with time. The $K$ and $n_e$ directional parameters would be further constrained throughout the updating exercise using data from packer testing, pumping tests and tracer testing so that the distributions can be
extended or truncated depending on existing site data (i.e. removing options of $K$ and $n_e$ that are deemed to be unrealistically high or low depending on the actual site data, or extending the distributions in the light of the site data). The constrained $K$ and $n_e$ values would then be fed into Modflow and the probability contours reassessed.

As pointed out in Chapter 7 it is also important that, within the methodology, different conceptual model scenarios are included in the modelling since different conceptual models may produce equally acceptable calibrations, for example alternative boundary conditions or fracture generation methods. Inclusion of these different conceptual model scenarios will allow a rigorous uncertainty analysis to be performed, so that not only data uncertainty is included but also conceptual model uncertainty. The importance of including both these types of uncertainty in protection zone delineation is emphasised in Evers and Lerner (1998).

Figure 10.1 summarises how the process of obtaining the probability contours using Fracman and Modflow would work. Figure 10.2 shows what considerations must be taken into account when attempting to model a site.
**Top level:** Probability contours created via particle tracking using Modpath through a series of realisations of the blocks of hydraulic conductivity and porosity in Modflow.

Each cell has a $K_x$, $K_y$, $K_z$, $H_x$, $n_x$ from the distributions below.

**Middle Level:** Sets of permeability and porosity distributions for different fracture types in different areas for input to Modflow, constrained by pump test results, packer tests and tracer tests (within a fracture type the parameters should be correlated).

**Bottom Level:** Use Fracman stochastically to obtain distributions of effective permeabilities and porosities for each fracture system type through a series of realisations to provide the above sets. The number of sets of $K$ and $n_e$ will increase over time as the amount of fracture data in an area increases. Fracman models will be calibrated as far as possible.

Fracture pattern created via Fracman using statistics for fracture aperture, density, length and orientations obtained either at the site or from nearby.

---

Figure 10.1: Summary of the method used to obtain probability contours from the combined use of Modflow and Fracman.
Figure 10.2: Diagrammatic representation of a methodology for creating probability contours under different fractured rock conditions.
Two situations are envisaged in the UK: sites with adequate fracture data and those without. In the former case the site data are used to run the Fracman code and generate site-specific K and $n_e$ distributions. In the latter case the site will have to be associated with one or more standard 'types' of fractured rock in that aquifer and a weighted average of the statistics of the types taken to represent the site. As the methodology becomes much more used then it should be possible for the former case to become almost redundant, and only applied in cases of fracture data updates and addition of more data. There should be a sufficient number of distributions available for most sites to be modelled without using Fracman.

In order to implement this intermediate methodology there are a number of stages that must be undertaken, which are outlined in Tables 10.1 and 10.2. These tables are a simple version of the explanations presented in the previous paragraphs.

Table 10.1: Stages in the proposed modelling technique for producing probability contours from 3-D models for sites with little adequate data.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Description of work to be completed for sites with little adequate data</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Find the K and $n_e$ distributions which represent the site’s fracture system adequately (if necessary taking a weighted average of the possible statistics) and constrain the K and n values so that they agree with the available field data and give more realistic distributions.</td>
</tr>
<tr>
<td>2</td>
<td>Set up a stochastic version of Modflow with grid directions in direction of principal axes of anisotropy, appropriate boundaries, drains, streams, and other features.</td>
</tr>
<tr>
<td>3</td>
<td>Calibrate the model to any pumping test data, head distributions, velocities or flow data. Perform rigorous sensitivity analysis to obtain ranges of model parameters to include in the stochastic analysis. K and $n_e$ distributions should be of the type derived from Fracman modelling.</td>
</tr>
<tr>
<td>4</td>
<td>Perform Modpath simulation with a large number of particles for just greater than the time zone required.</td>
</tr>
<tr>
<td>5</td>
<td>Save all particle tracking positions and times.</td>
</tr>
<tr>
<td>6</td>
<td>Repeat stochastically for Modflow selection of all stochastic parameters; perform particle tracking again with Modpath and save all particle positions and times.</td>
</tr>
<tr>
<td>7</td>
<td>Create the 3-D probability contours.</td>
</tr>
</tbody>
</table>
Table 10.2: Stages in the proposed modelling technique for producing probability contours from 3-D models for sites with adequate data.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Description of work to be completed for sites with adequate data</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Find fracture parameter distributions (lengths, apertures, densities and orientations).</td>
</tr>
<tr>
<td>2</td>
<td>Input data into Fracman and model flow. Assess whether a REV is definable. Calibrate to any head/velocity data existing for the site.</td>
</tr>
<tr>
<td>3</td>
<td>Obtain $K_x$, $K_y$, $K_z$, $n_x$, $n_y$ and $n_z$ values by particle tracking and varying the hydraulic gradient direction across the fracture block.</td>
</tr>
<tr>
<td>4</td>
<td>Perform multiple realisations of Fracman to obtain distributions of $K_x$, $K_y$, $K_z$, $n_x$, $n_y$ and $n_z$ for a fracture block size greater than the REV.</td>
</tr>
<tr>
<td>5</td>
<td>Constrain the $K$ and $n$ values so that they agree with the available field data and give more realistic distributions.</td>
</tr>
<tr>
<td>6</td>
<td>Set up a stochastic version of Modflow with grid directions in direction of principal axes of anisotropy, appropriate boundaries, drains, streams, and other features.</td>
</tr>
<tr>
<td>7</td>
<td>Calibrate the model to any pumping test data, head distributions, velocities or flow data. Perform rigorous sensitivity analysis to obtain ranges of model parameters to include in the stochastic analysis. $K$ and $n_e$ distributions should be of the type derived from Fracman modelling.</td>
</tr>
<tr>
<td>8</td>
<td>Perform Modpath simulation with a large number of particles for just greater than the time zone required.</td>
</tr>
<tr>
<td>9</td>
<td>Save all particle tracking positions and times</td>
</tr>
<tr>
<td>10</td>
<td>Repeat stochastically for Modflow selection of all stochastic parameters; perform particle tracking again with Modpath and save all particle positions and times.</td>
</tr>
<tr>
<td>11</td>
<td>Create the 3-D probability contours</td>
</tr>
</tbody>
</table>

10.2 Case study

Having presented the proposed methodology it is important to test its viability and assess whether amendments are required. This section should therefore be treated as an initial exploration of the issues associated with the proposed methodology and the possible ways forward. As in Chapter 8, Alton Court has been chosen as the case study site because of the quality and quantity of data available. The data have been discussed at some length in Chapter 8 and that discussion is therefore not repeated here. This section describes the eleven stages in Table 10.2 for Alton Court from data input to the fracture model in stage 1 to the production of probabilistic protection zones in stage 11.
10.2.1 Data input – Stage 1

The data initially used as input for Fracman to create the fracture network model is presented in Table 10.3. In addition to these data, storativity data in the range of $6.56 \times 10^{-5}$ (BH2 early time data) to $3.37 \times 10^{-2}$ (BH3 late time data) can be used in the calibration process. Hydraulic conductivity data should be in the range of 44.7 m/day (abstraction well) to 555 m/day (Pond Borehole) for calibration of the fracture model.

Table 10.3: Initial fracture parameters for input into Fracman.

<table>
<thead>
<tr>
<th>Fracture Set</th>
<th>Parameter</th>
<th>Average</th>
<th>Standard Deviation</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>SET 1</td>
<td>Orientation</td>
<td>163°</td>
<td>56°</td>
<td>normal</td>
</tr>
<tr>
<td></td>
<td>Dip</td>
<td>23.2°</td>
<td>13.2°</td>
<td>normal</td>
</tr>
<tr>
<td></td>
<td>Aperture (m)</td>
<td>$6.8 \times 10^{-4}$ to $3.3 \times 10^{-6}$</td>
<td>assume lognormal</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Flowing Fracture</td>
<td>1.98</td>
<td></td>
<td>lognormal</td>
</tr>
<tr>
<td></td>
<td>Spacing (m)</td>
<td>&gt;4.6</td>
<td></td>
<td>assume lognormal</td>
</tr>
<tr>
<td></td>
<td>Fracture length (m)</td>
<td>&gt;4.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SET 2</td>
<td>Orientation</td>
<td>201°</td>
<td>142°</td>
<td>normal</td>
</tr>
<tr>
<td></td>
<td>Dip</td>
<td>58.6°</td>
<td>13.5°</td>
<td>normal</td>
</tr>
<tr>
<td></td>
<td>Aperture (m)</td>
<td>$6.8 \times 10^{-4}$ to $3.3 \times 10^{-6}$</td>
<td>assume lognormal</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Flowing Fracture</td>
<td>2.66</td>
<td></td>
<td>lognormal</td>
</tr>
<tr>
<td></td>
<td>Spacing (m)</td>
<td>&gt;4.6</td>
<td></td>
<td>assume lognormal</td>
</tr>
<tr>
<td></td>
<td>Fracture length (m)</td>
<td>&gt;4.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SET 3</td>
<td>Orientation</td>
<td>33°</td>
<td>13.6°</td>
<td>assume normal</td>
</tr>
<tr>
<td></td>
<td>Dip</td>
<td>8.3°</td>
<td>2.3°</td>
<td>assume normal</td>
</tr>
<tr>
<td></td>
<td>Aperture (m)</td>
<td>$6.8 \times 10^{-4}$ to $3.3 \times 10^{-6}$</td>
<td>assume lognormal</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Flowing Fracture</td>
<td>6.35</td>
<td></td>
<td>assume lognormal</td>
</tr>
<tr>
<td></td>
<td>Spacing (m)</td>
<td>&gt;4.6</td>
<td></td>
<td>assume lognormal</td>
</tr>
<tr>
<td></td>
<td>Fracture length (m)</td>
<td>&gt;4.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Some distributions are assumed because of insufficient site data. The type of distribution selected in the case of insufficient data was on the basis of chapter 6 conclusions. The known distributions have been proven statistically as the best fit to field data – see section 8.3.3.

A fracture block $50 \, m \times 50 \, m \times 50 \, m$ was used in the simulations. This block size was the largest for which groundwater flow could be simulated given the Alton Court fracture parameters using Fracman/Mafic. Fracworks, which is part of the Fracman
suite of programs, was used to create the fractures within the 50 m × 50 m × 50 m block. Figure 10.3 illustrates a typical fracture block created through Fracworks for Alton Court.

However, the Fracman model requires data over and above that presented in Table 10.3. A more detailed explanation of the Fracman input parameters is given in Appendix J. Various assumptions have therefore been made within this 3-D modelling exercise about the nature of the fractures:

1) Fracture network generation model type: Fracman gives a choice of Enhanced Baecher, Poisson rectangle, Levy-Lee, nearest neighbour, probabilistic war zone, BART, nonplanar zone, fractal, Fractal POCS or a geostatistical model for locating fracture centres. The enhanced Baecher model was chosen as fracture centres are assumed to be distributed in space by a random Poisson process. Since there is no evidence on the nature of the fracture centre distributions a random process must be assumed. The fractures are assumed to be elliptical rather than polygonal since we do not know whether they terminate at fracture intersections.

2) Fracture density distribution: Fracman gives a choice of distributions of constant, gamma, and correlated to depth. Gamma was chosen since with a carefully chosen gamma coefficient the gamma distribution can approximate the form of a lognormal distribution. It is also known that the fracture density is not correlated with depth, as illustrated in Figure 8.19.

3) Aspect ratio – this parameter is required for the enhanced Baecher model and determines the ‘roundness’ of the fracture. It has been assumed to be 1 (round) since we have no site-specific information on this.

4) Termination percentage. This parameter determines whether fractures terminate against each other or end freely within the rock mass. It is assumed to be 0% since we have no evidence that fractures terminate against each other.

5) Transmissivity – Fracman gives a choice of transmissivity being correlated with size, width or depth, or uncorrelated. If the transmissivity is correlated then a correlation exponent, correlation factor and deviation factor must be defined. This selection was determined through the model calibration.

6) Storativity – Fracman gives a choice of storativity being correlated with size, width or depth, or uncorrelated. If the storativity is correlated then a correlation...
exponent, correlation factor and deviation factor must be defined. This selection was determined through the model calibration.

7) Fracture thickness (aperture) - Fracman gives a choice of aperture being correlated with size, width or depth, or uncorrelated. If the aperture is correlated then a correlation exponent, correlation factor and deviation factor must be defined. This selection was also determined through the model calibration.

Figure 10.3: Typical fracture block produced by Fracman using Alton Court fracture statistics.

10.2.2 Fracture model calibration – Stage 2
The initial fracture statistics were input into Fracman and the fracture network model was calibrated so that good agreement was obtained between:

a) simulated and observed fracture network parameters; and
b) simulated and observed heads for the 1998 pumping test.
The fracture network created depends on the seed used in sampling the fracture parameter distributions. If the same seed is used for the same input parameters the same fracture network is generated.

**Fracture network calibration**

Once the fracture network was created using Fracworks (see Appendix J for a more detailed explanation of the Fracworks part of Fracman), the fracture block was tested using the statistical part of that program. A 30 m vertical test borehole was located in the centre of the block against which to calibrate the fracture parameters. The field data used in calibration were:

- flowing fracture spacing within a 30 m borehole to be between 0.44 m (BH1) and 4.8 m (BH2), with an average of 2.44 m;
- fracture spacing of individual fracture sets within a vertical 30 m borehole. Set 1 at 2.15 m spacing, set 2 at 5.10 m spacing and set 3 at 6.42 m spacing; and
- numbers of fractures intersecting a 30 m borehole between 64 and 6.3 fractures, with an average of 28 fractures.

From the geophysical analysis there appeared to be no correlation of fracture spacing with depth (Figure 8.19) and the spacings were definitely not constant, therefore a gamma distribution had to be used (the only other option given by Fracman). It was found, by trial and error, that in order to get a) the number of fractures in a 30 m borehole, b) the overall fracture spacings and c) the spacings of the individual sets correct, the gamma distribution had to be used with coefficients for sets 1, 2 and 3 being 0.05, 0.01 and 0.001 respectively.

**Fracture flow calibration**

Having calibrated the fracture network model as far as possible, an attempt to calibrate the flow model was made. Calibration was performed manually. An abstraction well pumping at 2273 m³/day was simulated, with four monitoring wells in the same relative positions to the abstraction well as BH1, BH3, Pond and Well boreholes. Transmissivity, storativity and aperture and their associated correlation parameters were used as the calibration parameters. The calibration parameters were varied only within
the data ranges obtained on site in order to attempt to match the observed and simulated heads for the pumping test of April 1998. The field data used in the calibration were:

- a storativity range of between $6.56 \times 10^{-5}$ (BH2 early time data) and $3.37 \times 10^{-2}$ (BH3 late time data);
- transmissivity of a 30 m borehole to range between $1.55 \times 10^{-2}$ m²/s (K of 44.7 m/day in the abstraction well) to $1.93 \times 10^{-2}$ m²/s (K of 555 m/day in the Pond Borehole); and
- aperture sizes between $6.8 \times 10^{-4}$ m to $3.3 \times 10^{-6}$ m.

Figure 10.4: Comparison of simulated and observed heads in BH1 for the April 1998 pumping test using Fracman.

Figure 10.5: Comparison of simulated and observed heads in BH3 for the April 1998 pumping test using Fracman.
Thirty different calibration attempts were made in order to find the best fit to the 1998 pumping test drawdown data. The best match was found with both transmissivity and storativity correlated to the size of the fracture, and aperture being uncorrelated. The correlation exponent in both cases was 0.3 with correlation coefficients of $2.57 \times 10^{-6}$ for transmissivity and $2.5 \times 10^{-6}$ for storativity. These correlation factors are explained further in Appendix J.

Figure 10.6: Comparison of simulated and observed heads in the Well Borehole for the April 1998 pumping test using Fracman.

Figure 10.7: Comparison of simulated and observed heads in the Pond Borehole for the April 1998 pumping test using Fracman.
Figure 10.8: Comparison of simulated and observed heads in the abstraction well for the April 1998 pumping test using Fracman.

The degree of match between the simulated and observed values was determined by the chi-squared value. For a 95% significance that the simulated and observed data agree the chi-square statistic needs to be less than 1.635. For the Pond borehole, BH1 and BH3 this criterion was met, with best-fit chi-squared values of 1.48, 0.45 and 1.42 respectively. Figure 10.4 through to Figure 10.8 present the simulated and observed heads for the best-fit case from the same realisation of the fracture network. Whilst it can be seen that the fit for the abstraction well and the Well borehole are, quite frankly, dreadful, this appears to be the best that can be done within the limits of the site data constraints. The fracture parameters used to obtain the best calibration are presented in Table 10.4.

One of the major difficulties in calibration of the fracture model is that different seeds in the model can generate very different connections within the fracture network for the same network statistics. This results in the monitoring well locations having very different degrees of connection with the abstraction well, resulting in a large variation in drawdown at a monitoring well from one realisation to the next. An example of this is shown in Figure 10.9 for the Well borehole. The fracture input statistics are exactly the same as those used to obtain Figure 10.6, but a different seed has been used. Figure 10.9 shows an order of magnitude decrease in drawdown compared to that presented in Figure 10.6.
Despite the general poor calibration it should be noted that there is little consistency between the simulated and the observed results, i.e. the simulated results are not always higher than the observed results, but a mixture of the two cases. Therefore, on average it may be possible to say that the calibration is reasonable. The question is whether this degree of calibration is the best that can be expected for a stochastic model. There certainly appears to be no straightforward way of improving the calibration.

The other major difficulty with calibration is seen in Figures 10.4 to 10.9 after a time of 100 minutes. After this time drawdown no longer increases with time, indicating that the groundwater is supplied through the boundaries to the abstraction well. Since the model domain is only 50 by 50 by 50 m the boundary conditions are going to have a significant impact on the flow within the block. Therefore, in the calibration process only the early time data can be used for calibration of the model (the mid time data usually being used as it is more reliable to assess aquifer properties than the early or late time data).

**Figure 10.9: Comparison of simulated and observed heads in the Well borehole for the April 1998 pumping test using Fracman (different fracture block realisation).**

Other difficulties with fracture model calibration included the time taken to create the fracture network, determine the fracture connection matrix and then conduct the flow modelling. Once the fracture network is created, the fracture array has to be interrogated to find which fractures pass through the monitoring well used in the calibration. The flow simulation then has to be altered to include the correct fracture to
monitor during the flow simulation. The fracture or fractures to monitor change with each realisation and therefore this process must be completed for each trial during the calibration. Compared to an equivalent process with a standard porous media flow package such as Modflow within GW Vistas, the fracture flow modelling takes a much greater length of time and is not as straightforward.

Table 10.4: Fracture parameters for Alton Court after calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fracture Set Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Set 1</td>
</tr>
<tr>
<td>Strike (degrees)</td>
<td>144.0</td>
</tr>
<tr>
<td>Dip (degrees)</td>
<td>24.6</td>
</tr>
<tr>
<td>Distribution</td>
<td>Fisher</td>
</tr>
<tr>
<td>Distribution coefficient – dispersion$^1$</td>
<td>10</td>
</tr>
<tr>
<td>Size (m)</td>
<td>10</td>
</tr>
<tr>
<td>Size Distribution</td>
<td>Lognormal</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>5</td>
</tr>
<tr>
<td>Direction of Elongation (degrees)</td>
<td>0</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>1</td>
</tr>
<tr>
<td>Termination %</td>
<td>0</td>
</tr>
<tr>
<td>Intensity (m$^{-1}$)</td>
<td>0.505</td>
</tr>
<tr>
<td>Intensity Distribution</td>
<td>Gamma</td>
</tr>
<tr>
<td>Distribution coefficient$^1$</td>
<td>0.05</td>
</tr>
<tr>
<td>Transmissivity (T) correlation$^1$</td>
<td>With size</td>
</tr>
<tr>
<td>T Correlation exponent$^1$</td>
<td>0.3</td>
</tr>
<tr>
<td>T Correlation factor$^1$</td>
<td>$2.57 \times 10^6$</td>
</tr>
<tr>
<td>T Deviation factor$^1$</td>
<td>1</td>
</tr>
<tr>
<td>Storativity (S) correlation$^1$</td>
<td>With size</td>
</tr>
<tr>
<td>S Correlation exponent$^1$</td>
<td>0.3</td>
</tr>
<tr>
<td>S Correlation factor$^1$</td>
<td>$5 \times 10^6$</td>
</tr>
<tr>
<td>S Deviation factor$^1$</td>
<td>1</td>
</tr>
<tr>
<td>Fracture thickness correlation$^1$</td>
<td>Uncorrelated</td>
</tr>
<tr>
<td>Fracture thickness (m)</td>
<td>$10^4$</td>
</tr>
</tbody>
</table>

Note: $^1$Parameters as defined in the Fracman model (Dershowitz et al., 1998) and detailed further in Appendix J.
10.2.3 Defining directional parameters – Stages 3, 4 and 5

Having calibrated the fracture network and the flow parameters as far as possible to the field data, the next stage was to define the effective flow parameters for input into the equivalent porous media model.

At this stage ideally one would take a fracture block created by Fracman for a fixed seed and then apply a hydraulic gradient at different angles across the fracture block to obtain the effective hydraulic conductivity of the block in the direction of the gradient, since the cross sectional area in the direction of flow and the hydraulic gradient are known. An effective hydraulic conductivity of the block can then be obtained using Darcy’s Law for each different direction.

However, Fracman does not allow the hydraulic gradient to be applied in this way (it can only be applied perpendicular to the sides of the block). Therefore, in order to obtain the effective parameters the orientation of the fractures sets have to be rotated, keeping the hydraulic gradient in a constant direction. It must be recognised that in rotating the fracture sets in this way the fracture blocks tested in the different directions are only statistically identical, but are not the same fracture block realisation. 10 different fracture blocks have been created for every 10 degrees and the effective parameters derived for each block to obtain an average for each direction.

Particle tracking was also performed at the same time across each of the blocks to determine the distribution of times for the particles to cross the block. At least 1000 particles were tracked across the block in each simulation (particle numbers are not constant because of the method used by Fracman/Mafic to create particles by setting a particle mass and a total concentration of particles required. See Appendix J for a more detailed explanation). The distribution of effective velocities was then obtained since the distance across the block was known. The effective hydraulic conductivity in that direction was also known from the flow across the block, and hence a kinematic porosity in that direction could be derived.

A typical distribution of travel times arriving at the down gradient side of the fracture block is illustrated in Figure 10.10. Note the long tail of the distribution, with a few particles arriving up to four times later than the main peak of the particles. The tail is
long because some particles will pass down paths that have very low velocities. This arises because of the way in which groups of particles are split at fracture intersections, with the number of particles travelling down a fracture proportional to the flow in each fracture. Therefore a small proportion of the particles will end up travelling paths of low flows.

The average effective parameters were then plotted to assess whether an ellipse was obtained. If this was the case then the size of the fracture block used in the simulations was considered to be at or above the representative elementary volume for that fracture system. The variations of effective hydraulic conductivity and kinematic porosity with hydraulic gradient direction are illustrated in Figures 10.11 and 10.12 respectively, for a fracture block of 50 x 50 x 50 m. The full set of data used to derive these figures is presented in Appendix K.

![Typical distribution of travel times across a fracture block (set 1 at orientation of 274 degrees - 7th realisation).](image)

Figure 10.10: Typical distribution of travel times across a fracture block (set 1 at orientation of 274 degrees - 7th realisation).

It can be seen from Figure 10.11 that the hydraulic conductivity is anisotropic with a major to minor axis ratio (eccentricity) of the hydraulic conductivity ellipse of 1.13. Since Figure 10.11 represents the hydraulic conductivity as $1/\sqrt{K}$, the maximum hydraulic conductivity is in the direction 130° - 310°, and the minimum is perpendicular to this at 40° - 220°. Therefore, in order to obtain the correct anisotropy in the Modflow modelling the domain grid must be orientated parallel or perpendicular to the major and minor axes of this ellipse. The variation in hydraulic conductivity across the site determined from the pumping test (Table 8.8) indicates that the hydraulic conductivity...
at Alton Court is anisotropic with the highest values to the west of the abstraction well. The Fracman/Mafic modelling indicates that the highest hydraulic conductivities are in a northwest and a southeast direction, rather than a westerly direction. Data on site do not exist to the east or the southeast of the abstraction well and therefore it is not possible to assess the full extent of anisotropy at the site and its agreement or otherwise with the fracture modelling.

Figure 10.11 also indicates that the hydraulic conductivity of the fracture block is of the order of 1 m/d. This is significantly less than the values obtained on site (45 to 555 m/d). It is considered that the principal reason for this discrepancy is due to the impact of the boundary conditions on the fracture flow system. This effect is discussed further in sections 9.3.6 and 9.3.7.

Figure 10.12 shows that the kinematic porosity from the Fracman/Mafic modelling is isotropic with a value of 0.0007. From the on-site tracer testing the kinematic porosity calculated was of the order of 0.01. The value from the tracer testing is considered to be more representative, especially since the calibration of the Fracman/Mafic modelling is not as good as it could be (although as good as it was possible to make it).

Figure 10.11: Variation of $1/\sqrt{K}$ with direction of hydraulic gradient across the fracture block in the horizontal plane, as compared to the isotropic case (dotted line).
The discrepancy between the simulated kinematic porosity data and the field data may be again attributed to the impact of the boundary conditions on the flow system, as explained in sections 9.3.6 and 9.3.7, with limitations imposed on the flow system by rotating the fractures and not the hydraulic gradient.

It is possible, as shown in section 9.3.6, that the kinematic porosity and effective hydraulic conductivity are correlated. If there is a correlation then it may be possible to define joint probability distribution functions for a particular fractured rock type, and thus build up a library of values that can be used within the later stages of the modelling methodology. Figure 10.13 presents the correlation between kinematic porosity and effective hydraulic conductivity. However, from this figure it would appear that no such correlation exists for Alton Court. Therefore for the purposes of the pilot study it was decided not to add the complication of considering joint probability distribution functions further.

Figure 10.12: Variation of kinematic porosity with direction of hydraulic gradient across the fracture block in the horizontal plane, as compared to the isotropic case (dotted line).
Figure 10.13: Correlation between kinematic porosity and effective hydraulic conductivity at Alton Court.

Figure 10.14 presents the distributions of kinematic porosity and hydraulic conductivity obtained with the hydraulic gradient in the direction of the principal axes of the hydraulic conductivity ellipse (130 and 40 degrees). All four distributions crudely approximate a normal or a triangular distribution. In order to define these distributions more accurately more realisations would have to be performed. In theory these distribution types then go on to be used as the distribution types for the kinematic porosity and hydraulic conductivity in the stochastic porous media flow model. However, as will be seen in section 10.2.5 calibration of the porous media flow model reveals that the hydraulic conductivity and kinematic porosities derived here are inappropriate for a steady-state porous media flow model. Therefore it is possible that the distribution types derived here also cannot be applied.
Figure 10.14: Frequency distributions for kinematic porosity and hydraulic conductivity in the directions of the principal axes of the hydraulic conductivity ellipse.

Chapter 10 - Testing a 3-D methodology
10.2.4 Stochastic Modflow set up – Stage 6

Having obtained the effective flow parameters for Alton Court, a 3-D porous media model of the system had to be developed in order to obtain the probabilistic zones. Few data are present for the Old Red Sandstone aquifer, apart from the area immediately surrounding Alton Court. The aquifer parameters used for the original Flowpath modelling were therefore used as an initial guide. The model cells were 200 m by 200 m, with the grid being aligned with the major and minor axes of the hydraulic conductivity ellipse obtained in the Fracman modelling.

The system was represented by a one-layer unconfined aquifer using GW Vistas version 3.08 (which includes the Modflow and Modpath programs). The thickness of this layer was initially set at 50 m across the entire model, but because of the steep topography across the area the thickness had to be increased to 100 m beneath the elevated areas to prevent the drying of cells during simulations. The topography was obtained by digitising the 1:50,000 OS map of the Gloucester and Forest of Dean area (162), and then interpolating between points using GW Vistas own interpolation package. The topography of the area is illustrated in Figure 10.15. The location and elevations of the streams and rivers were also obtained from the OS map. Streams were used as opposed to rivers to allow full surface water-groundwater interaction (see Appendix J for further explanation of the river versus stream formulation). The base of the aquifer was obtained from the surface topography data less 50 m, with additional editing of the elevated areas to give 100 m thickness.

The boundary conditions of a regional model for the area should be similar to that described in section 8.3.3 with an east-west groundwater divide approximately 4 km south of the site and south of Coughton Brook, a NW-SE trending groundwater divide which forms the eastern limit of the Wye and Rudhall Brook catchments, and a varying north to south through to northeast-southwest groundwater divide following the high land to the west of the River Wye approximately 5 to 7 km to the west and northwest of Alton Court. These boundaries were used in the Flowpath model, with groundwater flow within the Flowpath model being controlled by topography. For the majority of the model a grid spacing of 250 m was used. The flow balance within the Flowpath model was considered to be good (<2% difference between inflows and outflows) and
also the match of the simulated and observed regional piezometry was good, although locally the match to the known piezometry was poor.

Figure 10.15: Topography of the Alton Court Modflow model.

Figure 10.16: Boundary conditions of the Alton Court Modflow model.

Since the regional piezometry was considered to be well defined in the Flowpath model and focus was only required on the Alton Court abstraction, it was considered that a
smaller scale, nested 3-D model could be used to simulate groundwater flow within the area. The boundary conditions were derived from the regional Flowpath model such that the southwestern boundary was defined by the surface water catchment boundary (no flow boundary) the remaining boundaries were defined as general head or stream boundaries with the heads derived from the calibrated Flowpath model. The boundary types are illustrated in Figure 10.16. General head boundaries were used in the eastern part of the model as opposed to constant head boundaries, since constant head boundaries often allow either too much flow into a model or too little, as the heads remains constant at that location (see Appendix J for further explanation of general head boundaries).

Recharge was set at 81 mm/year across the entire model, as was used in the Environment Agency Flowpath model for Alton Court zone delineation. The hydraulic conductivity of the elevated areas of the model had to be much lower than that determined from the Fracman modelling and the lower lying areas of the model required a higher hydraulic conductivity of between 3 m/day and 13 m/day, depending on the direction of anisotropy, in order for the model not to dry out. This pattern of hydraulic conductivity required, with higher hydraulic conductivity in the river valleys and lower in the interfluves is one that is frequently observed in the Chalk (Allen et al., 1997).

10.2.5 Stochastic Modflow Calibration – Stage 7
Calibration of the model was extremely difficult given that there are no groundwater hydrographs for the area. It was also not possible to calibrate the model to the pumping test at Alton Court since all the monitoring wells are located in the same grid square as the abstraction well.

The calibration of the model has therefore been performed using the available water level information presented in Table 10.5. It is known that under non-pumping conditions groundwater flow is towards the north-north-west, with flow further down hydraulic gradient towards the west and the River Wye. Under non-pumping conditions the model does simulate this pattern as presented in Figure 10.17.
Table 10.5: Water level data used for calibration of the Alton Court Modflow model.

<table>
<thead>
<tr>
<th>Borehole Name</th>
<th>Non-pumping conditions</th>
<th>Pumping conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water Level (m AOD)</td>
<td>Date</td>
</tr>
<tr>
<td>Alton Court ABH</td>
<td>42.31</td>
<td>19/7/96</td>
</tr>
<tr>
<td></td>
<td>44.05</td>
<td>15/4/98</td>
</tr>
<tr>
<td>Alton Court Well OBH</td>
<td>41.83</td>
<td>19/7/96</td>
</tr>
<tr>
<td></td>
<td>42.24</td>
<td>15/4/98</td>
</tr>
<tr>
<td>Alton Court Pond OBH</td>
<td>41.28</td>
<td>19/7/96</td>
</tr>
<tr>
<td>Alton Court BH1</td>
<td>41.83</td>
<td>15/4/98</td>
</tr>
<tr>
<td>Alton Court BH2</td>
<td>41.95</td>
<td>15/4/98</td>
</tr>
<tr>
<td>Alton Court BH3</td>
<td>43.07</td>
<td>15/4/98</td>
</tr>
<tr>
<td>Hildersley Farm</td>
<td>40.76</td>
<td>N/K</td>
</tr>
<tr>
<td>John Crane Ltd.</td>
<td>38.88</td>
<td>N/K</td>
</tr>
</tbody>
</table>

Figure 10.17: Simulated groundwater elevations (m AOD) during non-pumping conditions at Alton Court.

Hydraulic conductivity has been used to calibrate the model, and the range of possible hydraulic conductivity values has been obtained for the observed range in heads within the Alton Court grid cell for both non-pumping and pumping conditions. Aquifer thickness, stream parameters and recharge were not used within the calibration exercise.
since it was found that for model convergence and for areas close to rivers not to flood, these values had to be within a relatively narrow range. Therefore hydraulic conductivity was the principal aquifer parameter that was used for model calibration. It was found that the hydraulic conductivity could vary between 3 and 13 m/day in order to match the observed range of heads at Alton Court.

In order to then find the range of aquifer parameters to be carried forward into the stochastic analysis a full sensitivity analysis of the model should be conducted. However, this was not considered appropriate because of the restriction on aquifer parameters for convergence and non-flooding of the model. Therefore just hydraulic conductivity has been carried forward to the stochastic flow modelling stage with a range between 3 and 13 m/day.

However, at this stage of the proposed methodology, the aspect of most concern is the fact that the hydraulic conductivity obtained through the fracture flow modelling has been found to be inappropriate for the porous media flow modelling. Because the hydraulic conductivity values are different from the fracture flow modelling to that required for the porous media flow modelling, it is difficult to justify using the distributions derived from the fracture modelling for the stochastic porous media flow modelling. Ideally the methodology should be taking the fracture flow derived hydraulic conductivity, along with its associated distribution, as illustrated in Figure 10.14, and then apply them directly to the porous media flow modelling. The reason for this discrepancy at Alton Court could be that Modflow is being used in steady state, and so the fracture flow model derived values are inappropriate for this usage. The hydraulic conductivity required to calibrate a transient catchment model at Alton Court may be very different to the steady state calibrated value. The most likely explanation is that the Fracman derived hydraulic conductivities are incorrect because of the boundary conditions required to obtain the values in the first place.

Having discovered that the fracture model hydraulic conductivities are inappropriate for use in the steady state porous media flow model, kinematic porosities derived from the fracture model could also likely to be inappropriate for the porous media flow model. The difference in kinematic porosity determined from the Fracman/Mafic modelling and from the tracer testing data has already been discussed in section 10.2.3.
10.2.6 Particle tracking – Stage 8, 9 and 10

Having calibrated the porous media flow model (Modflow) as far as possible, particle tracking was initially conducted for the deterministic case with a flow rate of 2273 m$^3$/day at Alton Court. Two other abstraction wells were also included in the model, one at Hildersley Farm to the east of Alton Court and one at John Crane Ltd to the north of Alton Court pumping at 99.6 m$^3$/day and 140.9 m$^3$/day respectively. Since the kinematic porosity from the fracture modelling was shown to be isotropic, there was no need to enter a directional kinematic porosity. The kinematic porosity used was 0.01 which is based on the tracer testing results (see section 8.3.3 on the forced-gradient tracer testing) rather than the Fracman/Mafic modelling results which gave a value of 0.0007, but with poor model calibration. Particles had to be released from the base of the model in order for the reverse particle tracking to be successful.

Figures 10.18 and 10.19 show the 400-day and the total catchment particle tracks, respectively, for the deterministic porous media model. The 50-day particle tracking is not illustrated since the particles have not travelled beyond the grid cell containing the Alton Court abstraction well. Figure 10.18 illustrates that in 400 days the particles only (reverse) track approximately 200 m in easterly and northerly directions, only just making beyond the grid square containing the Alton Court abstraction well. Figure 10.19 illustrates the particle tracks for the capture zone of the well extending towards the east, with contribution also from the area south of Alton Court. The area covered by the capture zone appears to agree much better with the overall groundwater hydraulic gradient in the area and also the tracer testing at the site than the Flowpath model zones. The original 2-D Flowpath modelling showed the travel time and capture zones extending towards the north which is known not to be the case, both from the tracer testing and the hydraulic gradients in the area.

However, the tail of the capture zone extends out of the model domain to the east suggesting that the general head boundary is providing a source of water for the abstraction well. A catchment (no flow) boundary is considered to be located approximately 3 km further to the east of the model domain boundary. Ideally, therefore, the model should be extended further to the east, with the boundary represented by no flow cells in order that the model gives a better representation of the system. If this were done it is likely that the tail of the capture zone would become
wider, allowing an improved balance between the recharge from the capture zone area and the abstraction rate.

Figure 10.18: Particle tracking for 400 days using the best estimate deterministic porous media model for Alton Court.

Figure 10.19: Particle tracking for the total catchment zone using the best estimate deterministic porous media model for Alton Court.
Despite the limitations, the confidence in these modelling results is much higher than those from the original Flowpath modelling, and allows us to move to the next stage of stochastic porous media modelling. However, the limitations of the modelling should in no way be overlooked.

Stochastic particle tracking was then performed using the array of flow fields obtained when the hydraulic conductivity was varied between 3 and 13 m/d with a normal distribution (only one hydraulic conductivity applies to the zone, rather than different cells sampling a normal distribution between 3 and 13 m/d for a single realisation). The normal distribution was chosen on the basis of the distribution type found from the Fracman/Mafic modelling of the effective hydraulic conductivity for the fracture block.

The details of the particle tracks over time were saved for each realisation for input into the program travtim2, described in Appendix C, to create the probability contours for 50-day, 400-day and total catchment zones.

10.2.7 Probability based protection zones – Stage 11

The probability-based protection zones for 50-day 400-day and catchment zones are presented in Figure 10.20, 10.21 and 10.22 respectively.

Figure 10.20 compares the 50-day probability contours derived from the 3-D methodology, the 50-day probability contours from the 2-D SDF fracture modelling and the 50-day zones from the Flowpath modelling. The 3-D contours are elongated in the direction of the highest hydraulic conductivity, as obtained from the Fracman modelling. This direction is about thirty degrees from the direction of the hydraulic gradient. The 2-D probability contours align themselves with the hydraulic gradient direction, but this is because the model size is such that it constrains the contours in this direction. Although the 3-D and the 2-D zones are not identical they are elongated in similar directions and are similar in shape. The maximum extent of the 3-D zone covers approximately the same area as the zone of confidence from the Flowpath modelling, which are both approximately three times the area of the 2-D derived zones. Both the 3-D derived zones and the 2-D derived zones are much smaller than the best estimate zone from the Flowpath modelling. The fact that the 2-D fracture flow and the 3-D zones are very similar puts more confidence in the fracture-based modelling results (2-D and 3-D) than the Flowpath results.
Figure 10.21 presents the 400-day probability contours derived from Modflow and the 400-day Flowpath zones. The Modflow based zone derives most its water from the hills immediately to the southeast of the site. The contours immediately adjacent to the abstraction well appear slightly strange with two separate areas that have a probability of less than 5%. This is an artefact of the contouring package Surfer™. These two zones should in fact be joined. The probability contours in Figure 10.21 appear to be much more extensive than the pathlines presented in Figure 10.18, this feature is a product of the many hydraulic conductivities used to derive the probability contours, with the lower hydraulic conductivities producing zones that extend further towards the south-east.

Figure 10.22 presents the catchment zone for Alton Court derived from both the Modflow and the Flowpath modelling. A significant difference between the two sets of zones can be seen, with the Modflow catchment deriving most its water from the hills to the south east of the site and also from valley to the west containing Rudhall Brook (see Figure 8.16). From what is known of the hydraulic gradient direction in the area this Modflow catchment zone is much more plausible than the Flowpath zone. The Flowpath zone can only be obtained if the hydraulic gradient is principally towards the southwest.

The maximum area of the Modflow catchment zone is approximately two times smaller than the best estimate Flowpath zone. The size of the catchment zone should be a reflection of the balance of two factors only: the recharge and the abstraction rate. From a simple mass balance calculation, the area of the catchment zone should be the equivalent of a circular catchment zone of 2 km radius \((1 \times 10^7 \, m^2)\). It would appear that the Modflow/Modpath 95% probability zone is approximately \(3.5 \times 10^6 \, m^2\), which is approximately one third of the size that it should be for a recharge/abstraction water balance. The simulated abstraction is therefore deriving much of its water from the eastern general head boundary. The Flowpath best estimate zone has an area of \(8.2 \times 10^6 \, m^2\) which is much closer to the recharge/abstraction water balance figure.

In summary, whilst the shape of the 50-day protection zone appears to be governed mainly by the fracture sets and fracture flow, the shape of the 400-day and total
catchment zones appears to be governed by the regional flow system rather than fracture flow effects.

Figure 10.20: Alton Court probability contours for the 50-day travel-time zone using the 3D methodology compared with the 50-day Flowpath zones and the 50-day 2-D probability contours.
Figure 10.21: Alton Court probability contours for the 400-day travel-time zone using the 3D methodology compared with the 400-day zones created using Flowpath.

Figure 10.22: Alton Court probability contours for the catchment zone using the 3D methodology compared with the catchment zones created using Flowpath.
The tracer testing results discussed in section 8.3.3 allow reasonable confidence to be placed in the down gradient side of the 3-D and 2-D probability contours at Alton Court. The zones for the 50-day, 400-day and catchment zones do not extend more than 150 to 200 m to the northwest of the abstraction well. From the tracer testing it is known that the edge of the actual capture zone is approximately 50 m to northwest of the abstraction well. Considering that the cell size in Modflow is 200 m by 200 m, the zones are about as close as it is possible to get to the actual data (the tracer testing data were not used to constrain the 3-D zones).

10.3 Validation of probability based protection zones

The last major section of this chapter has shown that it is possible to create probability-based protection zones in fractured rock. Probability-based protection zones must be made as defensible as possible, initially through good science and then through validation. If the zones can be validated then their defensibility improves. The question then immediately arises as to the definition of validation.

Care must be taken with the term validation, as three terms: validation, verification and calibration are often confused in the literature (Environment Agency, 1995). In the discussion here it is assumed that verification refers to the process of ensuring that the model code accurately solves the governing equations and that the code is fully operational. Calibration involves the determination of model parameters, such as aquifer parameters and boundary conditions, within the bounds of field observation, so that the model produces results that agree with observations to an acceptable degree. Validation is described in section 10.3.1 below.

10.3.1 Definition of validation

There have been a number of definitions of model validation. The International Atomic Energy Authority (IAEA, 1982) definition of validation is: “A conceptual model and the computer code derived from it are validated when it is confirmed that the conceptual model and the derived computer code provide a good representation of the actual processes occurring in the real system”. Of course, it can be debated as to the definition of ‘good representation’. Schlesinger (1979) defined validation as “substantiation that a computerized model within its domain of applicability possesses a satisfactory range of accuracy consistent with the intended application of the model”.

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However these definitions do not distinguish between validation of generic concepts/processes simulated by the model (e.g. matrix diffusion and sorption etc.) and validation of site-specific aspects of the model. Both issues require validation.

For validation of a process (e.g. buoyancy convective flow) it first has to be conceptualised (e.g. as temperature-dependent density and viscosity) and coding is performed on this conceptualisation. The model is then applied to a buoyancy experiment and the results compared with the measurements. If the agreement is satisfactory then the model can be said to be validated with respect to this process. Tsang (1991) states that the most common approach to the site-specific aspect of validation is to first select a field or a laboratory experiment. The experimental conditions are specified in terms of boundaries and initial conditions. Model computations are made and then the predictions are checked against the field or laboratory data.

However, there are several more specific validation methods that can be used (Tsang 1991):

- **Event Validity**: an initial validation test of a qualitative nature, in which simulation of an event in the model are compared with those of the real system e.g. a large rainstorm event (this validation step is usually considered to be part of the calibration exercise);
- **Face validity**: knowledgeable people are asked whether they consider the model and the model input-output to be reasonable;
- **Traces**: the behaviour of different elements or entities of a model are traced or followed through the numerical model code to determine if the logic and the program are correct, and to assess whether the necessary accuracy is maintained (alternatively this method could be termed validation of the program logic). This is usually termed part of verification rather than validation;
- **Historical methods**: this typically consist of three steps: a) examining the model’s theoretical assumptions, b) investigating each of the model’s assumptions, where possible by empirically testing them, and c) comparing the input-output relationship
of the model to field behaviour (alternatively this method could be termed validation of the processes);

- **Internal validity**: Several realisations are used to determine the amount of stochastic variability in the model. A high degree of variability may cause the model’s result to be questionable and may require a redefinition of the parameter of interest. This is particularly important for the validation of stochastic models;

- **Historical data validation**: part of the data on site is used to construct and calibrate the model and the other part is used to validate the model (this is the type of validation which is most commonly used);

- **Predictive validation**: the model is used to make predictions for a given field or lab test and further measurements are made to test these predictions; and

- **Turing tests**: knowledgeable people are asked whether they can distinguish between model results and field observations. This is particularly important for stochastic models.

Validation is typically attempted through the same process that is identified as calibration (event validity and historical data validation). However, the non-uniqueness of a model solution means that a good comparison can be achieved with an inadequate or an erroneous model. Also, because the definition of ‘good’ in terms of the IAEAs statement ‘...provide a good representation of the actual processes occurring in the real system’ is subjective, one scientist may consider the model validated whereas another may use the same data to declare the model invalid.

Konikow and Bredehoft (1992) believe that the terms validation and verification should not be used in groundwater modelling, rather that the terms model testing, model evaluation, model calibration, sensitivity testing, benchmarking, history matching and parameter estimation should be used. They believe that it is naive to think that a model can be validated so that it will make predictions far into the future, as is the case for waste repository studies. They also state that, while models cannot be validated, they still provide an extremely valuable tool for critical analysis.

Leijnse and Hassanizadeh (1994) consider validation in the ‘weak’ sense and the ‘strong’ sense. Validation in the weak sense refers to the validity of the ‘conceptual part of the model’ – that is, the dominant processes are determined and the range of
applicability of the basic equations. They believe that this type of validation is feasible. This type of validation often occurs on ‘analysis’ type models where an understanding of system behaviour is of interest e.g. in the analysis of pumping test data. Validation in the ‘strong’ sense refers to the validity of the site-specific model. Leijnse and Hassanizadeh (1994) believe that this type of validation is not truly possible, and in this sense agree with Konikow and Bredehoft.

10.3.2 Philosophy of validation

Konikow and Bredehoft (1992) state that there are two principal schools of thought on validation in science. One school, called positivism holds that ‘...theories are confirmed or refuted on the basis of critical experiments designed to verify the consequences of the theories’. A second school argues that ‘...as scientists we can never validate a hypothesis, only invalidate it’. There is clearly an incompatibility between these two schools. As noted by Stephen Hawking in the recent book ‘A Brief History of Time’:

‘Any physical theory is always provisional, in the sense that it is only a hypothesis: You can never prove it. No matter how many times the results of experiments agree with some theory, you can never be sure that the next time the result will not contradict the theory. On the other hand, you can disprove a theory by finding even a single observation that disagrees with the predictions of the theory. As philosopher of science Karl Popper has emphasized, a good theory is characterized by the fact that it makes a number of predictions that could in principle be disproved or falsified by observation. Each time new experiments are observed to agree with the predictions the theory survives, and our confidence in it is increased; but if ever a new observation is found to disagree, we have to abandon or modify the theory.’

Site-specific groundwater models are an agglomeration of multiple hydrogeologic theories. They are subject to improvement via invalidation, but cannot be proven valid. Validation cannot add to the fund of knowledge.

One can also ask whether there is evidence that the current practice of calibration and verification lead to a reliable predictive capability. Several authors have examined this
question and have concluded that there is little evidence to support high confidence in long-term model predictions e.g. Konikow, 1986.

10.3.3 Validation of probabilistic protection zone modelling

From the above discussion it is considered that there are two separate validation issues. There is the validation of the flow model used to produce the probability based protection zones and then there is the validation of the zones themselves. One is a validation of the flows involved, the heads, and the velocities and the other is a validation of the probabilistic protection zones.

The flow model can be validated using all the methods outlined by Tsang (1991), although since the methodology relies on standard, previously validated codes such as Modflow, Modpath and Fracman/Mafic it is not considered necessary to include the code validation (traces) and process validation (historical methods).

The validation of non-probabilistic protection zones is such that, if one takes the view of Konikow and Bredehoft (1992), ultimately it is impossible to totally validate the zones unless we can measure the groundwater velocity at all points in the aquifer. Clearly this is not a viable option since the groundwater flow would be disrupted by the number of boreholes required to measure the velocity at all points (unless a remote method is devised).

The question then arises as to how we validate the probabilistic protection zones. There are quite a number of points that must be considered before such a discussion is considered:

1. The probability-based protection zones are defined through an ensemble of realizations which are based around distributions of aquifer parameters.
2. The actual geometry, hydraulic properties and flow at a site represent a single realization (reality), not an ensemble of realizations. Therefore in using stochastic based protection zones we are in fact attempting to say to what degree our simulation of reality matches the true protection zone (one realisation: reality). In validating these probability based protection zones using field data we are trying to compare reality (one realisation) with an ensemble of realisations.
3. At a site the hydraulic properties will change over time. This will either be because of erosion of the fractures (over very long time scales) or over seasons where the water level changes over time. The changes in water level will introduce or remove various fracture connections and systems near the surface of the groundwater system. Therefore the true protection zone will also change over time. The true protection zone of a well will hence be an ensemble of protection zones over time. This ensemble of zones is not the same as the ensemble of realizations created in probability-based protection zone modelling. The difference lies in the fact the true time-based ensemble of zones will have different aquifer parameter distributions to those used for creating the probability based protection zones (because of the fractures included or not in the zone of water table fluctuation).

4. This leads to the idea that probability-based protection zones should not be defined on a steady state basis but on a transient basis\(^1\). However, if we did decide to create the protection zones on a transient basis then this does not ultimately solve the problem of how to validate the zones, as the true protection zone, represented by an ensemble of zones over time, will still not be the same as the ensemble of realizations from distributions of hydraulic conductivity and porosity over time.

5. There is however, some chance of validating deterministic transient based zones to a certain extent. This might be achieved by performing tracer tests at different times of the year in different parts of the catchment.

In order to examine validation of simulated travel-time probabilities we must look at the type of field tests that yield travel times and distributions of travel times and compare the two sets of results. The field test that reveals this type of travel-time data is tracer testing. Most tracer testing results in a curve of tracer concentrations over time at a monitoring point, often termed the breakthrough curve. This distribution of travel times arises from the different flow paths and velocities encountered by the tracer on its way to the monitoring point.

The distribution of travel times from a tracer breakthrough curve cannot, however, be equated with the distribution of travel times at a point obtained from the stochastic

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\(^1\) Protection zones should be defined within a transient modelling framework anyway, since groundwater catchments are rarely in steady state.
particle tracking exercises. So the question arises as to how the two distributions are related and whether the tracer breakthrough curves could be used in any way to validate the simulated travel-time distributions. The probabilistic zones can be validated to some extent if the tracer breakthrough curve falls entirely inside the simulated distribution of travel-times. By this, it is meant that reality must lie within the simulated distribution of travel-times. If tracer tests were conducted at different times of the year these must all lie within the simulated distribution. In this way we can ensure that the model is not invalidated (although not validated). However, it is not considered meaningful to attach a degree to which the two distributions match.

By considering the methods of validation presented by Tsang (1991) we can also look at other ways of partially validating (or not invalidating) the probabilistic zones:

- **Event Validity.** This sort of validation is not likely to be possible for the probabilistic protection zones unless there is a local source (e.g. a spill or a tracer test);

- **Face validity.** This is achievable for the stochastic model output, but does not ultimately validate the probability-based zones in a defensible manner;

- **Internal validity.** Tsang (1991) states that this method is particularly important for the validation of stochastic models. Several realisations are used to determine the amount of stochastic variability in the model. A high degree of variability may cause the model’s result to be questionable and may require a redefinition of appropriate quantity of interest. However, at many of the sites where the methodology would be applied, the extreme heterogeneity of the system would manifest itself as a high degree of variability in the model output. In the case of the extreme heterogeneity of fractured rock this high variability in the model output is probably appropriate, and therefore this method of validation may not be applicable;

- **Historical data validation.** This is standard practice for flow model calibration and validation and so is perfectly reasonable. If sufficient data from tracer testing were available then some could to be kept back from model calibration and used in a partial validation of the model;

- **Predictive validation.** The model could be used to predict the envelope in which the breakthrough curve of a tracer test would occur and then tests could be carried out in
the field to assess the similarity (bearing in mind that tracer breakthrough curves and simulated travel-time distributions cannot be equated); and

- *Turing tests.* This is particularly important for stochastic models, however, in the case of probability-based protection zones there will be few field observations with which to compare the model results since the field observations would not be in a probabilistic form.

### 10.3.4 Tracer Testing

As tracer tests may lead to partially validated probabilistic protection zones, consideration as to how many tracer tests and of what type should be performed must be given. However, within this discussion it must not be forgotten that permission to perform tracer testing may be difficult to acquire close to public supply wells.

The following discussion is based on validation of transient probabilistic protection zones derived from distributions of aquifer parameters.

**Type of tracer test**

There are many different types of tracer test, for example forced gradient, natural gradient, single borehole tests and multiple borehole tests (Ward et al., 1998). The most appropriate test to validate, to some extent, probability-based protection zones is a forced gradient tracer test. Different tracers would be injected into multiple observation wells which would then migrate to the supply well which would be pumping at the standard supply rate. This type of test would yield breakthrough curves for each of the different tracers at the well, acting in a very similar manner to forward tracking of particles in a modelling exercise. Single borehole dilution tests also yield groundwater flow velocities (if a kinematic porosity is known) which would also be extremely useful in partially validating the zones as these velocities could be converted to travel times. The single borehole dilution test has the advantage of requiring less mass of tracer than a forced gradient test and also does not require the use of the supply well, only monitoring wells at distance from the supply well.

**Data used from tracer testing**

How the data from tracer testing is used is one of the more challenging aspects of tracer testing in relation to zone validation. As stated earlier the question arises as
to how one takes the single realization of the tracer test results to validate an ensemble of realizations from the modelling? Initially the actual breakthrough curve from a point injection of tracer can be visually compared with the probability distribution of travel times at the injection point in the model. Extreme care must be taken at this point since the two distributions are not the same thing and should not be equated. However, the two distributions could be examined in a qualitative way to assess whether the actual distribution of times lies within the model distribution. If this is the case then the model cannot be invalidated.

**Number of tracer tests**

In order to constrain a protection zone, one would expect that a minimum of four tracer tests would have to be conducted, one in each quadrant of the protection zone. This is clearly the absolute minimum. If the zone is highly irregular, as found in areas with significant river/aquifer interaction, fault bounded aquifer blocks, or geology that produces long tails on protection zones then many more tracer tests would have to be undertaken. However, there will be economic and time restraints on the number of tracer tests that can be performed (if they are not prohibited by their very suggestion).

**Location of tracer tests**

If at least four tracer tests are to be undertaken then they could be positioned such that there is one directly upstream of the well, one directly down natural hydraulic gradient and then a third and fourth parallel to the natural gradient contours. However, it should be borne in mind that in a highly heterogeneous aquifer the protection zone is unlikely to be elliptical. Therefore more locations may have to be chosen in keeping with the estimated extent of probability-based protection zones in highly heterogeneous environments.

If the number of tracer tests is to be restricted then it is considered more important to focus on validating the 50-day probability contours since if these are not reasonably accurate there is little time in which to rectify mistakes made by spillages. Hence the location of the sites for tracer testing would preferably be closer to the well.
Interpretation of results from tracer testing

In many cases the results of a tracer test will be negative, in that the tracer has either come through in concentrations that are below method detection limits, or that the estimation of the location where the tracer was anticipated to arrive was incorrect, or even that the time over which the samples were taken was incorrectly estimated. The interpretation of a negative result has to be taken with extreme caution in that it does not mean that the location of the tracer test is not in the capture zone. The interpretation should also be treated with caution in that it may mean that the conceptual model is incorrect for the site and should be rethought, as in the example of tracer testing at Alton Court.

If a positive result is obtained in that tracer concentrations are recorded over time then analysis can proceed as normal.

10.4 Interpretation and use by the land use planner and regulator

Having developed a methodology whereby protection zones can be described in a risk-based framework, consideration must be given to the following issues if they are to be used by the land-use planner and the regulator:

a. Which probability contour should be used for making land-use decisions;

b. What decisions should be made if the new probability-based contours are impractically large?

c. How the probability-based zones should be represented in map format;

d. If the protection zones change dramatically in size and shape, some landowners may find their land, which has previously been in a protection zone, no longer being located within a zone, and vice versa.

If we consider the first issue, in order to protect the groundwater source to a high degree, the high probability contours should be used, for example the 90% or 95% contour, since these contours would encompass the larger area around the groundwater source. However, in practice the probability-based protection zone is likely to become extremely large, especially in areas of sparsely fractured rock, and therefore unworkable. In this instance, can the 50% probability contour be used, for example? If this is done, then all zones might have to be set at the 50% contour for consistency. The regulator would have to decide which probability would represent the protection zone,
and it is likely that this decision could not be made until many case studies had been completed to obtain the full range of protection zone sizes and shapes.

Another option could be to follow the example of other probabilistic frameworks, for example the CLEA human health framework (DEFRA/EA, 2002 – CLR7), for guidance on the appropriate probability level to use. The CLEA framework uses the 95\textsuperscript{th} percentile for decision making on whether a particular contaminated site poses a risk to human health (a decision made by the Department of Health). Therefore, use of the 95\% probability contour would provide consistency with other probabilistic tools. Although probability based protection zones are designed to protect groundwater resources, if an incident did occur within the capture zone that went undetected it could be human health that is impacted in the short term. As stated in Chapter 3, Pekdeger et al. (1985) recognise that in fractured aquifers bacteria and viruses have a ‘half-life’ much greater than the 50 days allowed for by Zone I delineation. Therefore, even if the 95\% probability contour were to be used, contamination of water supplies by bacteria, which could adversely affect human health, could not be ruled out.

Another alternative to this approach would be to have different probabilities applying to different land uses. For example, activities such as sheep dipping and the new location of petrol stations that put groundwater at a higher risk of contamination should not be permitted within, for example, the 90\% contour. Consideration of the probability contours in this context would require the set of policy statements that support the protection zones to be revised.

The problem with having previously defined protection zones for fractured rock aquifers is that, if they are redefined using technically more defensible techniques, the likelihood is that the areas of the zones will increase. This is a problem that has been noted elsewhere in the world (US EPA, 1991) where it has been found to be more advantageous to over-protect groundwater initially than to under protect. This is because politically it is simpler to reduce the size of zones that to increase them.

In considering the third issue, mapping, the most practical option would be to illustrate the probabilistic 50-day, 400-day and capture zones on separate maps for clarity. It is possible that in some instances the low-probability contours of the 400-day zone could overlap with the high probability contours of the 50-day zone. Therefore, to avoid
misinterpretation the different sets of probability contours should be illustrated separately.

If landowners find that their land has moved from being within a restricted area to being outside, and that land-use restrictions have been applied in the past but are then lifted, does this entitle them to compensation? It must remembered that dramatic changes in the extent and shape of the zones will not be well received by those whose land alters in the restrictions applied. However, it should be remembered that when working with probabilities, a point outside a zone may be within the 'true' protection zone, and therefore from a technical point of view it is unlikely that compensation could be claimed. This point does emphasise the fact that any modelling methodology must be defensible.

10.5 Summary

A methodology has been proposed for delineating protection zones in fractured rock for scenarios where a REV is definable. The methodology was tested using a pilot study, and whilst probabilistic protection zones were delineated that were an improvement to the existing porous media derived zones close to the well, as indicated by tracer testing, there are several significant issues that would have to be resolved before the methodology is both defensible and practical. These issues are:

1. Calibration of the fracture model is difficult with many fracture parameters to define. There is also the problem of different realisations of the same statistics producing very different calibrations, some reasonable and some not. It is difficult to envisage a way round this problem. Calibration could only really be performed in an average sense by averaging the results of many realisations using a range of fracture parameter values as input.

2. Calibration of the flows in a fracture block by comparing to pumping test data should also be undertaken with caution. After only a short time, in the pilot study example 100 minutes, the abstraction well draws its water from the boundaries, and therefore simulated and observed late time data can not be compared.

3. The effective flow parameters defined in this chapter have been obtained by rotating the fracture sets within a fixed direction hydraulic gradient, because of software limitations. This method results in different fracture blocks being
tested at different angles of fracture set rotation. Preferably, to obtain the effective flow parameters, the hydraulic gradient would be rotated around a fixed fracture block, rather than the fracture sets rotating around the hydraulic gradient. This would be done for many realisations of the fracture block. In the situation presented in this chapter the effective flow parameters are a statistical average of many fracture blocks at each angle. The two cases are not the same and it is possible that they may produce different results.

4. The boundary conditions on the fracture blocks tested for the effective flow parameters were such that two were fixed head to apply a gradient across the block and the other four were no flow boundaries. The application of no flow boundaries was the only practical solution to the problem, but this imposes restrictions on the fracture flow, as flow should be allowed in and out of the side boundaries. This will impact the flow observed at the down gradient boundary, which has knock-on effects on the hydraulic conductivity and the kinematic porosity. Changes to the Fracman/Mafic code would have to occur to make it simple to apply a linear gradient along boundaries.

5. It has been found from the Alton Court case study that the hydraulic conductivity and kinematic porosity values and distributions derived from the fracture flow modelling were inappropriate for calibration of the steady-state porous media model. Aquifer parameter distributions and values for the stochastic porous media model then had to be derived from a sensitivity analysis and field data, rather than from the fracture model. Only the principal axes directions for the anisotropic hydraulic conductivity obtained from the fracture model were used in the porous media model. It is likely that if the fracture model calibration were successful and also if the hydraulic gradient could be rotated rather than the fracture sets to obtain the effective hydraulic parameters, more data from the fracture modelling could be used in the porous media model.

6. In the case of Alton Court, the shape of the 50-day protection zone appears to be governed mainly by the fracture sets and fracture flow, whereas the shape of the 400-day and total catchment zones appears to be governed by the regional flow system rather than fracture flow effects. It is considered likely that for those fracture systems that are not governed by a single major fault or fracture, this result may apply to many fractured rock scenarios.
7. One limitation with the porous media flow and transport software, GW Vistas, is that it cannot handle anisotropic hydraulic conductivity or anisotropic kinematic porosity within its stochastic capability, hence stochastic simulations have to be performed outside of the main Vistas shell. Whilst the limitation can be overcome it does not make the simulation of probabilistic zones straightforward (one of the main aims of the methodology being transparency and ease of use).

8. It was fortunate that the kinematic porosity for the case study was found to be isotropic since at present Modpath does not have the capability to deal with anisotropic kinematic porosity. Although the Modpath code can be altered, further work needs to be completed on the directional nature of kinematic porosity before this step is taken (a significant start has been made in section 9.3.2 and Appendix I).

9. The entire exercise is computationally intensive.

10. Tracer testing appears to be the principal method by which a partial validation of the probabilistic protection zones can be obtained.

11. If regulatory authorities decide to adopt the idea of probabilistic protection zones then they will have to make many practical decisions regarding the use and application of the type of zones.
1 SUMMARY AND RECOMMENDATIONS

1.1 Summary

This thesis outlines a methodology for delineating groundwater protection zones in fractured UK aquifers. The pilot study used to test this methodology shows that it is not yet possible to delineate protection zones for fractured rock using the 2-D or 3-D fracture flow software (SDF or Fracman/Mafic) in a defensible or practical manner. With further work on the calibration of stochastic fracture models under appropriate boundary conditions and using different fracture flow software (e.g. Napsac) a slightly modified methodology could be developed that would be defensible. However, at present it would appear that a defensible methodology does not exist.

From the individual chapters there are several more specific conclusions that can be made:

- Of the protection zone delineation techniques examined for different countries, a numerical fracture modelling technique has not previously been incorporated into groundwater protection policy. However, the Swiss have successfully implemented a sophisticated vulnerability mapping technique for fractured rock groundwater protection that appears to require much less time, effort and data per source that the methodology presented in this thesis. Techniques akin to this vulnerability mapping offer another potential solution for defensible protection zones in fractured UK aquifers.

- Although the methodology devised in this thesis does have considerable data requirements, fracture parameters can be constrained using standard field data to a greater degree than previously considered at the beginning of this exercise. Upper and lower limits can be placed on most fracture parameters and their probability density functions constrained.

- In well connected fracture systems, given known numbers of fracture sets, orientation of those sets and the distribution of orientations in a fracture set, the shape of protection zones can, to some extent, be predicted. The size of protection zones is governed principally by the fracture aperture. Aperture is the most difficult fracture parameter to characterise and therefore the size of protection zones is subject to considerable uncertainty.
• Uncertainty in protection zones can be successfully quantitatively characterised for 50-day, 400-day and the catchment zones.

• 2-D modelling of protection zones in fractured rock is inappropriate since the connectivity of the real system in 3-D cannot be represented in 2-D without 'adjusting' fracture parameters (length) in an indefensible manner.

• For densely fractured systems an effective dispersion coefficient could be used to create probabilistic protection zones. This method of delineation is, however, for the future since significant numbers of fracture model simulations would have to be conducted for an area of a particular fracture pattern type to determine the effective dispersion coefficient adequately.

• Preferably a 3-D methodology for delineating protection zones in fractured rock would use the same appropriate stochastic fracture model to simulate all of the protection zones. However, until computer technology becomes more advanced this is not possible, so an intermediate methodology is proposed whereby a fracture model is used to derive effective aquifer parameters (K and n_e). These parameters are then used in a stochastic porous media model to enable catchment scale modelling. In the long term it is hoped that a library of effective aquifer parameters can be built up in order that the fracture modelling stages do not have to be conducted. However, the pilot study used to test the methodology has shown that many aspects of the methodology need to be improved before it can be shown to be both defensible and practical. If even one of the problems with the methodology remains, then it cannot be used, as the different stages are interlinked in such a way that if there is no confidence in one stage then the whole methodology becomes invalid.

• In fractured rock, anisotropic kinematic porosity can have a significant impact on particle tracking for protection zone modelling, and should be included in all fractured rock effective parameter analyses if using the 3-D methodology presented in this thesis. This effect is particularly important in rocks with no horizontal/sub-horizontal fracture set.

• Validation of probabilistic protection zones can be partially achieved through tracer testing.
1.2 Recommendations

1. During the course of this thesis the paucity of fracture parameter data has become apparent. Therefore a concerted effort should be made by the regulatory authorities, the BGS, water companies and consultancies to collect fracture parameter data, especially through the use of geophysics during the drilling of new boreholes. This would allow a fracture parameter database to be set up and improve the conceptual understanding of fractured rock sites.

2. The fracture parameter that has the most impact on the size and uncertainty of protection zones is aperture. In order to obtain a better understanding of the range of hydraulic apertures at a site the use of tracer tests must be encouraged. It is recognised that it is often difficult to gain permission to for tracer injection, especially near public supply wells. Therefore it would be advisable to set up a tracer test database that can then be correlated with transmissivity and fracture density. If sufficient tracer tests were done then further tracer tests would be unnecessary if transmissivity and fracture density data were available. Apertures can also be obtained from comparison of geophysical data on flowing fractures and pumping test data – another reason for encouraging the use of geophysics where possible.

3. Further work should be conducted on the nature of anisotropic kinematic porosity to assess whether it needs to be incorporated into particle tracking codes, such as Modpath, for general use. The anisotropic nature of hydraulic conductivity (and perhaps kinematic porosity) also needs to be incorporated into stochastic porous media models, such as GW Vistas, for computational efficiency and transparency of the 3-D methodology (anisotropy of hydraulic conductivity is only incorporated in the deterministic part of the code).

4. It is important for the regulatory authorities to revisit the problem of protection zone delineation in fractured rock on a regular basis. Advances in computer technology occur at a rapid pace, and therefore it may become possible in a few years to delineate catchment zones using 3-D stochastic fracture models only.

5. In the meantime, if the interpretation of the Water Framework Directive in UK law results in further action being required on protection zone delineation, it may be appropriate for regulatory authorities to investigate other methods of delineating protection zones in fractured rock. Methods akin to the Swiss technique of sophisticated vulnerability mapping (DISCO method) could be used, for example.
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Appendix A - The Wyssling technique

The Wyssling technique is an analytical technique for delineating capture zones and protection zones in porous, isotropic, homogeneous aquifers. It is one of three analytical methods described in Lallemand-Barres and Roux (1989) for protection zone delineation.

The following notation and geometry of the protection zone is assumed:

![Diagram showing capture zone and protection zone with notation](image)

The radius of capture is defined as:

\[ x_0 = \frac{Q}{2\pi K b i} \]

where:
- \( Q \) – abstraction rate
- \( i \) – hydraulic gradient
- \( K \) – hydraulic conductivity
- \( b \) – saturated thickness of the aquifer
The width of the front of capture is defined as:

\[ B' = \frac{B}{2} = \frac{Q}{2Kbi} \]

where \( B \) is the width of the capture zone.

The effective velocity of the groundwater is such that:

\[ v = \frac{Ki}{n_e} \]

where \( n_e \) is the effective porosity of the aquifer.

The next stage is calculate the distance travelled by a conservative particle in groundwater for a given time, e.g. 50 days, by \( l = vt \). The distances immediately up and down gradient of the well for this time are then calculated by:

\[ S_o = \frac{l + \sqrt{l(l + 8x_o)}}{2} \quad \text{and} \quad S_u = \frac{l + \sqrt{l(l + 8x_o)}}{2} \]

Lallemand-Barres and Roux (1989) do not state how these equations have been derived.

This technique is an extreme simplification of the aquifer system, with no other external stresses, heterogeneity or recharge taken into account. The technique has been used in Spain (Martinez-Navarrete, C. and Garcia-Garcia, A., 2001) for delineating wellhead protection areas in limestone and detritic deposits.
Appendix B – Review of fracture models

The appendix outlines the main features of a selection of some fracture flow models. The details of the models have been found from web searches and literature searches, specifically the web pages of the International Groundwater Modeling Centre, the Hydrogeologists Home Page and the Energy Science and Technology Software Centre (US Department of Energy).

AVAILABLE 2-D FRACTURE FLOW AND TRANSPORT MODELS:

1. **BIM/BIM2D/BIM3D/FRACGEN.** Authors: Rasmussen, T.C., and D.D. Evans - University of Arizona. This is two- or three-dimensional steady-state saturated, unsaturated, or multiphase flow model in fractured, porous or non-porous media. Solute transport is by advection and diffusion.

The BIM/FRACGEN package simulates flow and (advective) solute transport in unsaturated, fractured, porous or non-porous media. It solves the boundary value problem within intersecting fracture planes using the boundary integral method applied to two-dimensional (BIM2D) and three-dimensional (BIM3D) formulations for flow using a constant capillary head within individual fractures. The transport problem is solved through calculating travel times and breakthrough curves by integrating the inverse velocity over a streamline, and then summing over all streamlines. The transport equation includes linear equilibrium reversible sorption (retardation) and diffusion from fractures into the rock matrix. FRACGEN generates synthetic fracture networks for sensitivity analysis with respect to fracture network parameters. The program BIM provides estimates of steady flow rates, hydraulic head distributions, travel times and breakthrough curves for discrete fracture networks, incorporating both saturated and variably saturated flow. The program BIM2D is used to discretize the fracture network and then solve for steady fluid flow and transport using the boundary integral method. It is limited to applications involving non-porous rock. The program BIM3D is used to study coupled flow through a fracture network embedded within a permeable matrix. The program FRACGEN is used to determine the global hydraulic conductivity of a fractured rock mass by generating finite fractures within a specified rock volume and then solving the finite lines of intersections between fractures and between fractures and the rock.
volume boundary. It uses site-specific geometric data, or generates synthetic fractures.

2. **FRACFLO.** Author Gureghian, A.B. Two-dimensional steady-state fracture network flow model. Radionuclide transport by advection, dispersion, diffusion into the rock matrix with decay. Source configurations can be variable in space and time.


4. **FRACTRAN.** Authors: Sudicky, E.A., and R.G. McLaren. A two-dimensional steady-state flow in fractured or unfractured porous media. Solute and radionuclide transport by advection, dispersion, diffusion, retardation, and first-order decay. It is a finite element model for simulating steady state groundwater flow and transient contaminant transport in a discretely fractured porous media. The porous media is represented by block elements and fractures are represented by line elements. The model accounts for flow and transport in both the discrete fractures and the matrix block.

5. **FTRANS.** Author: Huyakorn, P.S. Two-dimensional, transient, density-dependent flow model in fractured or unfractured anisotropic, heterogeneous, porous media. Solute transport, heat, and radionuclide transport by advection, conduction, dispersion, diffusion, sorption, and first-order decay with decay chains capability.

6. **MAGNUM2D.** Authors: England, R.L., N.W. Kline, K.J. Ekblad, and R.G. Baca, The Nuclear Energy Agency (NEA), Paris, France. MAGNUM2D is a two-dimensional finite element numerical model for transient or steady state analysis of coupled processes of advective and conductive heat transfers in a porous continuum and buoyancy driven groundwater flow in a saturated, fractured, porous medium. The simulation domain may be modelled in two Cartesian dimensions or in a radial coordinate system that is symmetric around the vertical axis. Flow Model Two-dimensional, transient or steady-state, flow in fractured anisotropic, heterogeneous, porous media.
7. **SDF** Author: Rouleau, A. - University of Quebec. SDF is a Stochastic Discrete Fracture model. A two-dimensional steady-state flow model for fractures in impermeable non-porous media. Transport is by advection simulated by stochastic particle tracking in virtual network based on selected directional parameters: relative flow rate, mean flow velocity, and mean length of fracture segment. It is composed of 4 modules: NETWRK, NETFLO, NETTRANS and APEGEN. NETWRK generates a 2D fracture network using a Monte Carlo method based on the statistics of field data on fracture geometry i.e. fracture orientation, trace length, aperture and density. The program APEGEN can be used to generate supplementary aperture distributions for the fracture network generated by NETWRK. The fractures are simulated as parallel plates and there is no matrix storage in the blocks. NETFLO then computes the steady state fluid flow through the network created in the previous two modules (see above). NETFLO also computes the statistics of selected parameters in every 10-degree range of direction, including the total length of fracture segments, total flow velocity and the total flow rate. NETTRANS then uses the latter directional parameters to calculate the travel time of particles over an arbitrary distance using a second-level stochastic process.

The data required for the modules are the statistical distributions of fracture lengths, apertures and orientations. Transmissivities are calculated using the cubic law.

8. **TRAFRAP-WT.** Authors: Huyakorn, P.S., H.O. White, and T.D. Wadsworth. TRAnsport in FRActured Porous media with Water Table boundary conditions is a two-dimensional finite element code designed to simulate ground-water flow and solute transport in fractured or granular aquifers, and is capable of treating both (leaky-)confined and water table systems. Fractured porous media are represented by either a dual porosity or a discrete fracture approach or a combination of both. The code takes account of fluid interactions between fractures and porous matrix blocks, the advective-dispersive transport in the fractures and diffusion in the porous matrix blocks and fracture skin, as well as chain reactions of radionuclide components. The matrix blocks can have varying permeability. In addition, for the fractured system the model distinguishes between cases in which the matrix blocks have low permeability and those in which the matrix blocks have substantial permeability. The model can handle various model geometries and element shapes (rectangular, triangular and linear).

10. STAFF2D. Author: Huyakorn, P.S. A two-dimensional, steady-state or transient, flow model in fractured or unfractured porous media. Real, cross-sectional, or radial-grid orientations. Solute and radionuclide transport by advection, dispersion, sorption, and first-order degradation, including chain decay of multiple species.
AVAILABLE 3-D FRACTURE FLOW AND TRANSPORT MODELS

1. BIM/BIM2D/BIM3D/FRACGEN.
   Authors: Rasmussen, T.C., and D.D. Evans - University of Arizona. See previous section for details.

2. FRACAS
   FRACAS is a 3D stochastic code for modelling fluid flow and heat transfer through a fractured rock mass. The fractured network is represented by a set of intersecting discs with a distribution of radii and directions inferred from statistical analysis of the data. The available flow paths are simulated by a series of 'flow lobes', or channels linking the centres of the discs, assuring that they intersect, and passing through the middle of each intersection. Hydraulic conductivities are assigned to the discs as the network is generated. Mass transport is modelled using an advective biased random walk technique which allows consideration of channel shape, complexity and local dispersion due to irregularities in the walls of the natural fractures. For purposes of considering heat transfer, each disc is considered as a simple heat exchanger: heat transfer by conduction in the rock matrix and by convection in the fractures. Conduction is assumed to be 1D and in a direction perpendicular to the fracture channel. Thermal perturbations exist only up to a certain distance where a condition of zero heat flow is imposed. FRACAS is a modular code available in Fortran 77, Pascal and C. A dynamic memory allocation facility is incorporated which enables its use on machines of variable capacity. Transient flow, hydromechanics and thermochemistry are options which are currently being developed.

3. FRACMAN/MAFIC
   This is a 3D finite element model for simulating flow and transport in discrete fractures. The code is made up of two parts: FRACMAN and MAFIC. The FRACMAN part of the code allows fracture patterns to be simulated using a stochastic approach to generate theoretical fracture patterns form the different properties of the field data.
Data input requirements are large but there are many options for the type of fracture system required. For example, whether a fracture network is required or a discrete fracture, fracture roughness type, rock type, whether the fractures are fresh or sealed and if fractures terminate against each other or if they are free etc., etc.

From the data input the simulation of the fracture sets is then completed. There is a choice of 9 different approaches to the geometrical arrangement of the fractures if a network model is required. For example, a fractal box model, a geostatistical approach, a war zone model or a Poisson rectangle model. After the geometric model has been chosen the grid or the centres of the fracture ellipses have to be specified.

The fracture set characteristics and grid are then fed to the MAFIC routine for steady state or transient flow and particle tracking runs.

The output from FRACMAN can be in several forms: a head contour map, a head histogram, head variations over time and particle distributions.

4. MODFLOW-SURFACT

MODFLOW-SURFACT is an updated version of MODFLOW with added features. It now has more robust and efficient solution schemes, rigorous treatment of unconfined conditions, treatment of non-ponding and ponding recharge and a fracture well package. The package claims to “provide a rigorous treatment of wells by superposing tabular fracture elements to the discretised system. This package automatically determines the contributions from various nodal layers to a total pumping rate, adjusts these contributions based on aquifer conditions, adjusts the total pumping rate of the system when demand exceeds supply, and includes wellbore storage effects.”

5. NAPSAC

NAPSAC is a Fortran computer program developed by AEA Technology at Harwell Laboratory, UK to calculate groundwater flow and radionuclide transport in fractured rocks using stochastic fracture network models. The current release supersedes the earlier NAMNET and DISPER programs. NAPSAC represents the flow system as a 3D network of planar fractures, which is defined by the statistical distributions for
fracture size, aperture and orientation. The steady-state flow through the network is calculated using a finite element idealisation for each fracture. Transport in the resulting flow field is calculated by tracking a large number of particles to determine breakthrough curves.

6. NEFTRAN/NWFT/DVM

NEFTRAN (NEtwork Flow and TRANsport) is a discrete finite difference model for groundwater flow and radionuclide transport in high-level radioactive waste repositories in deep saturated and fractured basalt formations. It handles a generalised flow network, matrix diffusion, mixing cell and multiple radionuclide decay chains. The underlying assumption is that all significant flow and radionuclide transport takes place along 1D discrete legs or paths. These legs are assembled into a multi-dimensional network. A particle tracking model is used to define the trajectory a particle follows from a given point until it crosses a boundary. The resulting information is used to construct the network and define the boundary conditions.

7. NETFLO

This is a model which simulates steady state 3D groundwater flow in a heterogeneous medium using an equivalent network of series and parallel flow members. Where the stratigraphy does not suggest a particular network configuration, a network may be specified along principal stream tubes using a more general 3D groundwater flow code.

The solution of the code is based around Darcian flow occurring in all of the flow branches and the conservation of mass at each node.

The output of the system is in the form of pressure at all nodes and velocities and fluxes in all the branches in a text file. The code also determines all possible flow paths from the input to the output point, and the mean flow, time of travel and length for each path.

8. FRAC3DVS

This is a 3D variably saturated groundwater flow and contaminant transport model. It is a finite element model for simulating steady state or transient flow and advective-dispersive solute transport in porous or discretely fractured media.
Fractures are represented by triangular plane elements, and block elements can be subdivided into tetrahedra to produce non-orthogonal block elements (and hence fractures at various orientations).

Transport processes include advection, hydrodynamic dispersion, sorption according to the Freundlich isotherm and multi-species transport of straight or branching decay chains.

The solutions techniques can be changed from control-volume finite element, Galerkin finite element of finite difference method.

2D slices are produced of the 3D domain to show hydraulic head, saturation or concentration outputs. Flow velocity vectors can also be produced.

9. MOTIF
Authors: Guvanasen, V., and T. Chan, Atomic Energy Canada, Ltd. A one-, two-, or three-dimensional, variably saturated flow model in fractured, deformable, porous media. Solute, heat, and single-species radionuclide transport by convection, advection, dispersion, diffusion, adsorption, decay.

10. PORFLOW
Author: Runchal, A.K. Two- or three-dimensional, cartesian or radial, steady-state or transient, variably saturated or multiphase flow in fractured or unfractured, anisotropic, heterogeneous porous media. Supports freezing/thawing and evaporation/condensation. Solute, heat, or radionuclide transport by advection, dispersion, diffusion, sorption, retardation, convection, conduction, dispersion, decay. Highly variable source configuration.

11. STAFF3D
Solute Transport and Fracture Flow in 3D is a finite element model that simulates groundwater flow and transport of conservative/reactive solutes in fractured or granular porous media. STAFF3D can handle various fractured systems including those containing an intricate network of fractures and/or several discrete fractures.

The code may perform steady-state and transient simulations in a cross-section, an
areal plane, an axisymmetric configuration, or a fully 3D configuration. A wide range of boundary conditions can be treated including those involving water table conditions, infiltration, aquitard leakage and pumping and injection wells. The contaminant transport module may account for advection, hydrodynamic dispersion, linear equilibrium sorption and first-order degradation. Transport of a single species or decay chain may be handled.

Fractures in porous media are represented using discrete fracture and dual-porosity approaches and allow for fluid interactions between the matrix blocks and the fractures. 1D elements are used to discretise porous matrix blocks of a dual-porosity medium, or to discretise discrete fractures.

12. SWIFT/486

SWIFT (Sandia Waste-Isolation Flow and Transport) is a transient, 3D finite difference model which simulates the flow and transport of fluid, heat, brine and radionuclide chains in porous and fractured media. The model supports both dual-porosity and discrete fracture network simulations. Migration within the rock matrix is characterised as a 1D process. Aquifer hydraulic conditions may be heterogeneous and anisotropic under confined or unconfined conditions. The model also has the capability for diffusion, adsorption and dispersion.

Solution of the flow and transport equations are by a finite difference technique using centred or backward weighting in the time and space domains. Gaussian elimination and two-line successive over-relaxation techniques are used to solve the matrices.

Output of SWIFT can be interfaced with the SURFER and GW Vistas software packages.

11. TOUGH2

Author: Pruess, K. TOUGH2 is a general-purpose numerical simulation program for multi-phase fluid and heat flow in porous and fractured media. It belongs to the MULKOM family of codes, developed at the Lawrence Berkeley National Laboratory for applications in geothermal reservoir engineering, nuclear waste disposal, and unsaturated zone hydrology. It is a multi-dimensional numerical model for simulating the coupled transport of water, vapour, non-condensible gas, and heat.
in porous and fractured media. TOUGH2 offers added capabilities and user features, including the flexibility to handle different fluid mixtures (water, water with tracer; water, CO2; water, air; water, air, with vapour pressure lowering, and water, hydrogen).

**12.TRACR3D.**

**Author:** Travis, B.J. A one-, two-, or three-dimensional, steady-state or transient, flow model in fractured or unfractured, deformable, anisotropic, heterogeneous porous media. Flow options include single-phase saturated, single-phase unsaturated, two-phase immiscible, and others. Multicomponent transport model in air and/or water phases by advection, dispersion, diffusion, equilibrium or kinetic adsorption/desorption, up to n chains of radioactive decay, and biological transformation.

**13.TRUMP**

**Authors:** Edwards, A.L., A. Rasmuson, I. Neretneiks, and T.N. Narasimhan. A one-, two-, or three-dimensional, steady-state or transient, flow in fractured heterogeneous porous media. Solute or heat transport by advection, dispersion, diffusion, conduction.

**14.TRUST84**

**Author:** Narasimhan, T.N. A one-, two-, or three-dimensional, steady-state or transient, variably saturated flow in fractured or unfractured, anisotropic, heterogeneous deformable porous media model.

**OTHER DUAL-POROSITY FLOW ONLY MODELS**

- BACRACK
- DCM3D
- FEFLOW(TIHAVASSALO)
- FRACTEST
- GMF
- MLU
- ROCMAS-H
OTHER DUAL-POROSITY FLOW AND TRANSPORT MODELS

3D FE DUAL POROSITY FLOW AND TRANSPORT MODEL
CHAINT
CRACK
FEFLOW(DIERSCH)
FRACPORT
FRACSL
FRACT
FTRANS
MULKOM
PORFLO-3
Appendix C – End-point macro

This macro is referred to in section 7.4 as one of the methods by which probability contours can be created; the equal angle sector method. The macro used to create probability contours using the equal numbers sector method is not presented here as the end points simply have to be ordered in terms of angle around the well and then divided into groups of the same number to be averaged.

Equal Angle Sector Method

Sub contour()

' This section copies the r and theta values to columns for sorting

    Sheets("Sheet2").Select
    Columns("D:D").Select
    Selection.Copy
    Columns("K:K").Select
    Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, _
        SkipBlanks:=False, Transpose:=False
    Columns("I:I").Select
    Application.CutCopyMode = False
    Selection.Copy
    Columns("J:J").Select
    Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, _
        SkipBlanks:=False, Transpose:=False

' This section sorts column J on theta

    Range("J4:K2003").Select
    Application.CutCopyMode = False
    Selection.Sort Key1:=Range("J4"), Order1:=xlAscending, Header:=_
        xlGuess, OrderCustom:=1, MatchCase:=False, Orientation:= _
        xlTopToBottom

' This section loops through the rows and sorts r on 10 degree sectors

    counter = 0
    count = 4
    n = 0
For count = 1 To 40
    ActiveSheet.Range(Cells((50 * count) - 46, 10), Cells((50 * count) + 3, 11)).Select
    thetamin = Cells((50 * count) - 46, 10).Value
    thetamax = Cells((50 * count) + 3, 10).Value
    Selection.Sort Key1:=Range(Cells((50 * count) - 46, 11),
    Cells((50 * count) + 3, 11)), _
    Order1:=xlAscending, Header:=xlGuess, OrderCustom:=1,
    MatchCase:=False, Orientation:= _
    xlTopToBottom

'Having sorted a segment, find the radii for the percentiles and copy
to elsewhere
'in the spreadsheet

roundcent = 5
    ActiveSheet.Cells(1, 12 + count).Value = count
    ActiveSheet.Cells(2, 12 + count).Value = thetamin
    ActiveSheet.Cells(3, 12 + count).Value = thetamax
    ActiveSheet.Cells(4, 12 + count).Value = Cells((50 * count) -
        46, 11)
    ActiveSheet.Cells(14, 12 + count).Value = Cells((50 * count) +
        3, 11)

For n = 1 To 9
    ActiveSheet.Cells(((50 * count) - 46) + (roundcent * n),
        11).Select
    Selection.Copy
    ActiveSheet.Cells(4 + n, 12 + count).PasteSpecial
Next n
10 Next count

' This section then reconverts the r,theta values back to (x,y)
coordinates
' for plotting

For m = 13 To 53
    Cells(1, m).Select
    Selection.Copy
    Cells(17, ((2 * m) - 13)).PasteSpecial
For p = 4 To 14
xcoord = (Cells(p, m).Value * Cos(((Cells(2, m).Value + Cells(3, m) _ .Value) * 0.5) * 3.14159 / 180)) + 20
ycoord = (Cells(p, m).Value * Sin(((Cells(2, m).Value + Cells(3, m) .Value) * 0.5) * 3.14159 / 180)) + 40
Cells(15 + p, (2 * m) - 12).Value = xcoord
Cells(15 + p, (2 * m) - 13).Value = ycoord
Next p
Next m
' This section copies the results to a new sheet.
Sheets("Sheet2").Select
Range("M17:Cn29").Select
Selection.Copy
Sheets("Sheet3").Select
Range("A1").Select
ActiveSheet.Paste
Sheets("Sheet2").Select
Range("m17:n29").Select
Selection.Copy
Sheets("Sheet3").Select
Range("cc1:cd13").PasteSpecial
' This section rearranges the data to enable it to be plotted
For r = 3 To 13
    For s = 1 To 41
        Range(Cells(r, (2 * s) - 1), Cells(r, 2 * s)).Select
        Selection.Copy
        Range(Cells(s + 15, (2 * r) - 5), Cells(s + 15, (2 * r) - 4)).PasteSpecial
    Next s
Next r
End Sub

Appendix C – End-point macro
Appendix D - Contouring program for probability contours

This program is first described and used in section 7.4 on developing methods to create probability contours. It is also used in section 8.3 for the 2-D Alton Court and Coombe Farm case studies, and again in section 10.2 for the 3-D Alton Court case study.

PROGRAM TRAVTIM2

! ***********************************************************************
! This program creates a distribution of travel times for each point on a grid across the model domain, and then allows contours of probability to be defined by contouring these grid points. This program is the second version based on distances from points to the segments for a radius about the grid points.
! ***********************************************************************

IMPLICIT NONE
CHARACTER*12 ONIT(46)
CHARACTER*80 PARTICLE
REAL :iXGRDSP,XDIFF,YDIFF,XPOINT,YPOINT,XP,YP
REAL :  :K,P,XMINO,YMINO,XMAXO,YMAXO
REAL :  :DMXCOORD,DUMYCOORD,DUMTIM,RANGE2
REAL :SUMTT, AVERAGETT, SUMVAR, STDEV, MAXTT, MINTT,
SUMSWK, SUMKURT
REAL :SKEW, KURT,CNT, TTREQ, PERCENT, RNO, TTMAXIN
INTEGER : M, I, M, J, NOTTPTS, NOGDPPTS, UNITNUM, UNITEND, NOSEG2, RN,
NOSEG, NO
INTEGER : TOTALNUM, ERROR, ERROR_TT, NOPROTS, DUMNUM, DUMNUMELE
INTEGER : ERROR_ST, YGRDSPNO, ERROR_XG, ERROR_YG, INTERVAL
INTEGER, DIMENSION(55970) :: NUMP, NUMELT
REAL, DIMENSION(55970) :: XCOOR, YCOORD, TIM
LOGICAL, DIMENSION(55970) :: DORUN
REAL, ALLOCATABLE, DIMENSION(:,:) :: TT, STATS
INTEGER, ALLOCATABLE, DIMENSION(:,:) :: NOTT, XGRID, YGRID

N=0
M=0
I=0

! Open and read in data from various SDF output files
!
WRITE(*,*)'OPENING FILES'
OPEN(90, FILE='files.txt', STATUS='OLD')
UNITEND=0
DO WHILE (UNITEND .EQ. 0)
 READ(90, '(12,IX,A)', IOSTAT=UNITEND)UNITNUM, UNIT(UNITNUM)
END DO
CLOSE(90, STATUS='KEEP')
OPEN(25, FILE=UNIT(25), STATUS='OLD')
OPEN(44, FILE=UNIT(44), STATUS='NEW')
OPEN(45, FILE=UNIT(45), STATUS='NEW')
OPEN(46, FILE=UNIT(46), STATUS='OLD')
REWIND(25)
REWIND(44)
REWIND(45)
REWIND(46)

! Start reading in the data from PARTRACK.OUT

! WRITE(*,*)'STARTING TO READ IN DATA'
READ(25,1) PARTICLE
1 FORMAT(A80)
NOSEGS=0
TTMAXIN=0
DO 2 I=1,35500
   READ(25,*) DUMNUMP, DUMNUMELT, DUMXCOORD, DUMYCOORD,
           DUMTIM
   NUMP(I)=DUMNUMP
   IF (NUMP(I)==-999) THEN
      WRITE(*,*)'END OF FILE'
      GO TO 3
   END IF
   NUMELT(I)=DUMNUMELT
   XCOORD(I)=DUMXCOORD
   YCOORD(I)=DUMYCOORD
   TIM(I)=DUMTIM
   TTMAXIN=MAX(TTMAXIN,DUMTIM)
   WRITE(*,*) TTMAXIN, NOSEGS
   NOSEGS=NOSEGS+1
2 CONTINUE

! Loop through all the particle tracks to find where end point is
! so as not to calculate times between start and end of the different
! particle tracks
!
3 NOSEGS2RN=0
TOTALNUM=0
DO 4 I=1,NOSEGS-1
   IF (NUMP(I)==NUMP(I+1)) THEN
      DORUN(I)=.TRUE.
      NOSEGS2RN=NOSEGS2RN+1
   ELSE
      DORUN(I)=.FALSE.
   END IF
4 CONTINUE
TOTALNUM=NOSEGS2RN

! Read in the data required on the grid size and the time for which the
! probability contours are to be created (CONT) and also the number of

Appendix D - Fortran coding
divisions required along each side of the grid (INTERVAL) and hence the
number of points required to define the probability contours

    READ(46,*)XMAXO,YMAXO,CONT,INTERVAL
    XMINO=3000.00
    YMINO=1500.00

    NOPOINTS=0
    XGRDSP=(XMAXO-XMINO)/INTERVAL
    YGRDSPNO=INT((YMAXO-YMINO)/XGRDSP)
    NOPOINTS=(INTERVAL+1)*(YGRDSPNO+1)
    PRINT *,NOPOINTS,TOTALNUM

Allocate the arrays needed for x,y coords of grid positions (XGRID,YGRID),
the travel times for each grid point (TT) and the number of travel times
associated with each grid point.

    ERROR=0
    ERROR_TT=0
    ERROR_XG=0
    ERROR_YG=0

    ALLOCATE(NOTT(NOPOINTS),STAT=ERROR)
    IF (ERROR/=0) THEN
        PRINT *,"Nott array not allocated - more space needed"
        STOP
    END IF

    ALLOCATE(TT(TOTALNUM,NOPOINTS),STAT=ERROR_TT)
    IF (ERROR_TT/=0) THEN
        PRINT *,"tt array not allocated - more space needed"
        STOP
    END IF

    ALLOCATE(XGRID(NOPOINTS),STAT=ERROR_XG)
    IF (ERROR_XG/=0) THEN
        PRINT *,"X points array not allocated - more space needed"
        STOP
    END IF

    ALLOCATE(YGRID(NOPOINTS),STAT=ERROR_YG)
    IF (ERROR_YG/=0) THEN
        PRINT *,"Y points array not allocated - more space needed"
        STOP
    END IF

RANGE2 is the radius of the circle that encompasses the points going towards
the distribution of times at a grid point

    RANGE2=0.5*(XGRDSP**2)

Start calculating the grid coordinates for the times to be superimposed on.
WRITE(*,*)'START MAKING GRID'
NOGRDPTS=0
DORUN=.TRUE.
DO 5 N=1,INTERVAL
   DO 7 M=1,YGRDSPNO
      NOGRDPTS=NOGRDPTS+1
      XGRID(NOGRDPTS)=XGRDSP*(N-1)
      YGRID(NOGRDPTS)=XGRDSP*(M-1)
      NOTT(NOGRDPTS)=0
   PRINT *,NOGRDPTS
5 CONTINUE
7 CONTINUE
WRITE(*,*)'FINISH MAKING GRID AND EVERYTHING ELSE'

! Loop through all segments of particle tracks and calculate the shortest
! distance from the segment to the nearest grid point, and then calculate
! the travel-time at that closest point and store in the TT array.
!
NOTTPTS=0
DO 20 I=1,NSEGS-1
   IF (DORUN(I)) THEN
      XDIFF=XCOORD(I+1)-XCOORD(I)
      YDIFF=YCOORD(I+1)-YCOORD(I)
      XPOINT=XGRID(NOGRDPTS)
      YPOINT=YGRID(NOGRDPTS)
      X=P=XCOORD(I)-XPOINT
      Y=YCOORD(I)-YPOINT
      K=-((XP*XDIFF)+(YP*YDIFF))/((XDIFF**2)+(YDIFF**2))
      IF (K>=0.00 .AND. K<1.00) THEN
         P=(XP**2)+(YP**2)+((K**2)*(XDIFF**2+YDIFF**2))
         IF (P>RANGE2) THEN
            GO TO 20
         ELSE
            NOTTPTS=NOTTPTS+1
            TT(NOTTPTS,NOGRDPTS)=TIM(I)+(K*(TIM(I+1)-TIM(I)))
            IF MAX(TT(NOTTPTS,NOGRDPTS),TIM(I),TIM(I+1))>TTMAXIN) THEN
               PRINT *,N,I,K,TT(NOTTPTS,NOGRDPTS),TIM(I),TIM(I+1)
               STOP
         END IF
      END IF
   ELSE
      GO TO 20
   END IF
20 CONTINUE
! ERROR_ST=0

ALLOCATE(STATS(7,NOGRDPTS),STAT=ERROR_ST)
IF (ERROR_ST/=0) THEN
  PRINT *,"Stats array not allocated - more space needed"
  STOP
END IF

DO 60 N=1,NOGRDPTS
NO=NOTT(N)
IF (NO>0) THEN
  SUMTT=0.0
  MAXTT=0.0
  MINTT=1.0
  AVERAGE TT=0.0
  DO 70 J=1,NO
    SUMTT=TT(J,N)+SUMTT

  ! Calculate max and mins in t-t data and average
  !
  IF (TT(J,N)>MAXTT) THEN
    MAXTT=TT(J,N)
  ELSE IF (TT(J,N)<MINTT) THEN
    MINTT=TT(J,N)
  END IF
  70 CONTINUE
  AVERAGE TT=SUMTT/NO
  IF (MINTT==1.0) THEN
    MINTT=0.0
  END IF
  STATS(1,N)=AVERAGE TT
  STATS(2,N)=MINTT
  STATS(3,N)=MAXTT

  ! Calculate standard deviation
  !
  IF (NO>5) THEN
    WRITE(*,*)'START STANDARD DEVIATION CALC'
    SUMVAR=0.0
    STDEV=0.0
    DO 80 J=1,NO
      SUMVAR=SUMVAR+(TT(J,N)-AVERAGE TT)**2
    80 CONTINUE
    RNO=REAL(NO)
    STDEV=SQRT((1/(RNO-1))*SUMVAR)
    WRITE(*,*)RNO,N,AVERAGE TT,STDEV,NO
    IF (STDEV==0.0) THEN
      STATS(4,N)=0.0    !STDEV
      STATS(5,N)=0.0    !SKEW
      STATS(6,N)=0.0    !KURT
  END IF
GO TO 60
END IF

! Calculate skewness and kurtosis of distribution
!
SUMSKEW=0.0
SUMKURT=0.0
SKEW=0.0
KURT=0.0
DO 90 J=1,NO
    SUMSKEW=SUMSKEW+(((TT(J,N)-AVERaget)/STDEV)**3)
    SUMKURT=SUMKURT+(((TT(J,N)-AVERaget)/STDEV)**4)
90 CONTINUE
SKEW=SUMSKEW/RNO
KURT=SUMKURT/RNO
WRITE(*,*),SKEW,KURT,AVERaget,STDEV,NO,N
STATS(4,N)=STDEV
STATS(5,N)=SKEW
STATS(6,N)=KURT
ELSE
    GO TO 60
END IF
ELSE
    GO TO 60
END IF
60 CONTINUE

! Calculate percentages for the given travel-time for each grid position
!
TTREQ=0.0
PERCENT=0.0
DO 55 N=1,NOGRDPTS
    TTREQ=0.0
    STATS(7,N)=0.0
    MAXTT=STATS(3,N)
    NO=NOTT(N)
    IF (NO<1) THEN
        STATS(7,N)=100.0
        GO TO 55
    END IF
    IF (NO>0) THEN
        DO 57 J=1,NO
            IF (TT(J,N)>CONT) THEN
                TTREQ=TTREQ+1
            ELSE
                GO TO 57
            END IF
57 CONTINUE
    PERCENT=(TTREQ/NO)*100
    STATS(7,N)=PERCENT

Appendix D - Fortran coding
END IF

55 CONTINUE

WRITE all relevant data to output file

WRITE (*,*)'START WRITING X,Y DATA TO FILE'
DO 100 N=1,NOPOINTS
   WRITE(44,*),XGRID(N),YGRID(N),NOTT(N)
100 CONTINUE

WRITE (*,*)'START WRITING STATS DATA TO FILE'
DO 200 N=1,NOGRDPTS
   WRITE(45,*),STATS(7,N)
   WRITE(45,*),STATS(1,N),STATS(2,N),STATS(3,N),STATS(7,N)
   & STATS(5,N),STATS(6,N),STATS(4,N)
200 CONTINUE

DEALLOCATE(TT)
DEALLOCATE(XGRID)
DEALLOCATE(YGRID)
DEALLOCATE(NOTT)
DEALLOCATE(STATS)

CLOSE(45,STATUS='KEEP')
CLOSE(44,STATUS='KEEP')
CLOSE(25,STATUS='KEEP')
CLOSE(46,STATUS='KEEP')
END PROGRAM TRAVTIM2
Appendix E – Results of the dispersion modelling

The following four pages present the results of the dispersion modelling discussed in section 8.3.2. Each page illustrates the results of the modelling at different fracture system connectivities. Particles have been tracked through a fracture network under a hydraulic gradient using the discrete fracture model SDF. The fracture system is based on that at Coombe Farm.

The two columns of graphs on each page show histograms of the particle locations in the longitudinal direction (parallel to the hydraulic gradient) on the left hand side of the page, and the transverse direction (perpendicular to the hydraulic gradient) on the right hand side of the page. Histograms are shown for seven different times up to a year, with the time steps in approximately 50-day intervals, and with the time increasing down the page.

The results are discussed in full in section 8.3.2, but in general it can be seen that as the time increases the dispersion of the particles increases, and as the connectivity decreases the dispersion increases. From these results it has been possible to determine effective dispersion coefficients for this fracture system.
Appendix E - Results of the dispersivity modelling:

Distribution of particles at different times parallel and perpendicular to the hydraulic gradient for 2650 nodes.

Increasing time
Appendix E - Results of the dispersivity modelling -
Distribution of particles at different times parallel and perpendicular to the hydraulic gradient for 1700 nodes

Particle Distributions Parallel to the Hydraulic Gradient

Particle Distributions Perpendicular to the Hydraulic Gradient

Increasing time
Appendix E - Results of the dispersivity modelling -
Distribution of particles at different times parallel and perpendicular to the hydraulic gradient for 670 Nodes

**Particle Distributions Parallel to the Hydraulic Gradient**

1. Distribution of particle locations after 50 days from the 'y' starting position (0 m)
2. Distribution of particle locations after 100 days from the 'y' starting position (0 m)
3. Distribution of particle locations after 150 days from the 'y' starting position (0 m)
4. Distribution of particle locations after 200 days from the 'y' starting position (0 m)
5. Distribution of particle locations after 250 days from the 'y' starting position (0 m)
6. Distribution of particle locations after 300 days from the 'y' starting position (0 m)
7. Distribution of particle locations after 365 days from the 'y' starting position (0 m)

**Particle Distributions Perpendicular to the Hydraulic Gradient**

1. Distribution of particle locations after 50 days from the 'y' starting position (0 m)
2. Distribution of particle locations after 100 days from the 'y' starting position (0 m)
3. Distribution of particle locations after 150 days from the 'y' starting position (0 m)
4. Distribution of particle locations after 200 days from the 'y' starting position (0 m)
5. Distribution of particle locations after 250 days from the 'y' starting position (0 m)
6. Distribution of particle locations after 300 days from the 'y' starting position (0 m)
7. Distribution of particle locations after 365 days from the 'y' starting position (0 m)
Appendix E - Results of the dispersivity modelling -
Distribution of particles at different times parallel and perpendicular to the hydraulic gradient
for 130 Nodes
Appendix F – 1998 Alton Court pumping test results

Figure F.1: Water Levels in Production Well 15 - 21/4/98 at Alton Court
Time (mins)

Figure F.2: Water Levels in BH1 15 - 21/4/98 at Alton Court
Time (mins)

Figure F.3: Water Levels in BH2 15 - 21/4/98 at Alton Court
Time (mins)
Figure F.4: Water Levels in BH3 15 - 21/4/98 at Alton Court

Figure F.5: Water Levels in Well Borehole 15 - 21/4/98 at Alton Court

Figure F.6: Water Levels in Pond Borehole 15 - 21/4/98 at Alton Court

Appendix F - Alton Court pumping test results
Figure F.7: Water Levels at Pond Stage 15 - 21/4/98 at Alton Court

Figure F.8: Recovery in Production Well 21/4/98 at Alton Court

Figure F.9: Recovery in BH1 21/4/98 at Alton Court

Appendix F - Alton Court pumping test results
Figure F.10: Recovery in BH2 21/4/98 at Alton Court

Figure F.11: Recovery in BH3 21/4/98 at Alton Court

Figure F.12: Recovery in Well Borehole 21/4/98 at Alton Court

Appendix F - Alton Court pumping test results
Figure F.13: Recovery in Pond Borehole 21/4/98 at Alton Court

Appendix F - Alton Court pumping test results
Appendix G – Tracer test breakthrough curves

Figure G.1: Fluorescein concentration over time in the abstraction well at Alton Court - 15th to 16th April 1998

Figure G.2: Rhodamine concentration over time in the abstraction well at Alton Court - 15th to 16th April 1998

Figure G.3: Photine concentration over time in the abstraction well at Alton Court - 15th to 16th April 1998
Appendix H: Alton Court televiewer data

**Frequency distribution of spacings for Set 1, Alton Court**

- Best fit to log normal distribution
- to > 95% significance ($\chi^2 = 9.13$)

**Frequency distribution of dips for Set 1, Alton Court**

- Best fit to normal distribution
- to only 2% significance ($\chi^2 = 31.12$)

**Frequency distribution of orientations for Set 1, Alton Court**

- Best fit to normal distribution
- to 63% significance ($\chi^2 = 31.54$)
Appendix H: Alton Court televiewer data

**Frequency distribution of spacings for Set 2, Alton Court**

Best fit to log normal distribution of >95% significance ($\chi^2 = 13.34$)

**Frequency distributions of dips for Set 2, Alton Court**

Best fit to normal distribution with a significance of 56% ($\chi^2 = 15.48$)

**Frequency distribution of orientations for Set 2, Alton Court**

Best fit to normal distribution with >95% significance ($\chi^2 = 11.32$)
Appendix H: Alton Court televiewer data

Frequency distribution of spacings for Set 3, Alton Court

Frequency distribution of dips for Set 3, Alton Court

Frequency distribution for orientations of Set 3, Alton Court
Appendix I – Characteristics of anisotropic kinematic porosity

(Provided by J.A. Barker)

This appendix was provided in support of the computational investigation of directional properties of a fractured system. In essence, it shows how a porosity tensor could be formulated and then goes on to show that, using the simple model introduced in Chapter 9, this tensor will not in general exist.

A candidate kinematic-porosity matrix

Let us consider a homogeneous but anisotropic flow system. A given head gradient will give rise to a darcian flux, as described by Darcy’s law:

\[ q = K_i \]  

(1)

where:

- \( \mathbf{i} \) is a vector in the direction of the decreasing head gradient of magnitude equal to the head gradient,
- \( \mathbf{q} \) is the darcian flux vector, and
- \( K \) is the hydraulic conductivity matrix, which we will take to be symmetric.

In general we do not expect the direction of the (mean) velocity to be in the same direction as the darcian flux. In Figure I.1, identical bulk fluxes occur in the horizontal and vertical directions. However, the velocity is not at 45 degrees to the axes as the porosity is different in the different directions.
Figure I.1: Demonstration that the mean velocity is not necessarily in the same direction as the flux, which is at 45 degrees to horizontal for this case where the flux is the same in both the x and y directions.

So, let us assume for now that, in general, the velocity can be related to the head gradient through an equation of the form:

\[ \mathbf{v} = \mathbf{C} \mathbf{i} \]  \hspace{1cm} (2)

where:
\( \mathbf{v} \) - is the bulk average velocity vector, and
\( \mathbf{C} \) - is a matrix which we will call the ‘velocity-conductivity’ matrix. (This does not seem to have been studied previously in the literature.)

We can now combine the above two equations to give

\[ \mathbf{q} = \mathbf{K} \mathbf{C}^{-1} \mathbf{v} \]  \hspace{1cm} (3)

Let us introduce a new matrix defined by

\[ \mathbf{N} = \mathbf{K} \mathbf{C}^{-1} \]  \hspace{1cm} (4)

so
From this equation is it seen that the matrix \( N \) should be a natural candidate for a 'kinematic-porosity matrix (tensor)'. However, it cannot be assumed that such a matrix exists and, indeed, we will demonstrate that, in general, it does not!

**Proof of the non-existence of the kinematic-porosity matrix, \( N \).**

From Equation (4) we see that the existence of \( N \) depends on the existence of \( K \) and \( C \). We consider the case of the model introduced in Chapter 9, comprising several sets of fractures where each set comprises an infinite number of infinitely long parallel fractures, which are all of the same aperture and separation (see Figure 9.6).

**The hydraulic conductivity matrix, \( K \)**

The formula obtained for permeability in any given direction \( \psi \) is

\[
k_\psi = \sum_n \frac{a_n^3}{12D_n} \cos(\theta_n - \alpha) \cos(\theta_n - \psi) \frac{\cos(\alpha - \psi)}{\cos(\alpha - \psi)}
\]

So in the direction of the gradient, taken to be angle \( \alpha \),

\[
k_\alpha = \sum_n \frac{a_n^3}{12D_n} \cos^2(\theta_n - \alpha) = \sum_n \frac{a_n^3}{12D_n} (\cos \theta_n \cos \alpha + \sin \theta_n \sin \alpha)^2
\]

or

\[
k_\alpha = \sum_n \frac{a_n^3}{12D_n} (\cos^2 \theta_n \cos^2 \alpha + 2 \cos \theta_n \cos \alpha \sin \theta_n \sin \alpha + \sin^2 \theta_n \sin^2 \alpha)
\]

The permeability and hydraulic conductivity are related by a constant (including the density and viscosity of water). So we can write the hydraulic conductivity in the direction of the gradient as

\[
K_\alpha = E \cos^2 \alpha + F \cos \alpha \sin \alpha + G \sin^2 \alpha
\]
where E, F and G represent summations of terms containing the fracture set parameters. (Incidentally, by writing \(x(\alpha) = \cos \alpha / \sqrt{K_\alpha}\) and \(y(\alpha) = \sin \alpha / \sqrt{K_\alpha}\), it is readily shown from Equation (9) that \(1/\sqrt{K_\alpha}\) will plot as an ellipse in a radial plot.)

Let us now return to the matrix formulation and obtain the directional hydraulic conductivity as the projection of the flux in the direction of the gradient divided by the gradient:

\[
K_\alpha = \frac{q_i / \|i\|}{\|i\|^2} = \frac{i.K_i}{\|i\|^2}
\]

(10)

Since the components of the gradient in the \(x\) and \(y\) directions are simply given by the magnitude of the gradient times the cosine and sine of the angle \(\alpha\)

\[
K_\alpha = (\cos \alpha \sin \alpha \begin{pmatrix} K_{xx} & K_{xy} \\ K_{xy} & K_{yy} \end{pmatrix} \begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix}) = K_{xx} \cos^2 \alpha + K_{xy} 2 \sin \alpha \cos \alpha + K_{yy} \sin^2 \alpha
\]

(11)

where the hydraulic conductivity matrix has been assumed symmetric.

Comparing Equations (9) and (11) we see that the two forms are identical and if we wished we could directly express the matrix components in terms of the fracture parameters.

Most importantly, since the form is the same, we have shown that for this model the hydraulic conductivity matrix can be defined.

The velocity-conductivity matrix, \(C\)

Now let us turn our attention to the newly-introduced velocity-conductivity matrix, which characterises the velocity in terms of the gradient. For the simple model being considered, the average bulk velocity in any given direction \(\psi\) was found to be (Chapter 9, Equation (30)):

Appendix I – Characteristics of anisotropic kinematic porosity
\[ V^B_{\psi} = \frac{\sum_n |q_n^B| q_n \cos(\theta_n - \psi)}{\sum_n |q_n^B|} \tag{12} \]

Since
\[ q_n = \frac{PG}{12\mu} a_n^2 i \cos(\theta_n - \alpha) \tag{13} \]

and
\[ q_n^B = \frac{PG}{12\mu D_n} a_n^3 i \cos(\theta_n - \alpha) \tag{14} \]

we have, after some simplification:
\[ V^B_{\alpha} = \frac{i \sum_n \left| \frac{PG}{12\mu D_n} a_n^5 \cos(\theta_n - \alpha) \right|}{\sum_n \left| \frac{a_n^3}{D_n} \cos(\theta_n - \alpha) \right|} \tag{15} \]

Now, if we follow the same development as in going from Equation (7) to (9), we obtain an expression of the form:
\[ C_\alpha = \frac{V^B_{\alpha}}{i} = \frac{P \cos^3 \alpha + Q \cos^2 \alpha \sin \alpha + R \cos \alpha \sin^2 \alpha + S \sin^3 \alpha}{T \cos \alpha + U \sin \alpha} \tag{16} \]

The coefficients P to U are distinct and the trigonometric functions of \( \alpha \) are linearly independent (since their Wronskian is non zero).

The same quantity obtained from the velocity conductivity matrix (if it exists) would be given (as with hydraulic conductivity) by
\[ C_\alpha = (\cos \alpha \sin \alpha) \left( \begin{array}{cc} C_{xx} & C_{xy} \\ C_{xy} & C_{yy} \end{array} \right) \left( \begin{array}{c} \cos \alpha \\ \sin \alpha \end{array} \right) = C_{xx} \cos^2 \alpha + C_{xy} 2 \sin \alpha \cos \alpha + C_{yy} \sin^2 \alpha \tag{17} \]

We can see that this is only characterised by three parameters (four if we make C non-symmetric) while the equivalent expression in Equation (16) requires 5 parameters (not...
six, since we can divide through, top and bottom, by any one of the parameters P to U without changing the value). So (16) and (17) are, in general, incompatible.

**Summary**

The existence of a kinematic-porosity matrix has been shown to depend on the existence of both a hydraulic conductivity matrix and what has here been referred to as a velocity-conductivity matrix.

By considering a simple two-dimensional model, it has been shown that, while the hydraulic conductivity matrix can be defined, a velocity conductivity matrix is not capable of representing the angular dependence of velocity. This counter-example therefore proves that, in general, it is not possible to construct a kinematic-porosity matrix for homogeneous anisotropic fractured media.

**The nature of porosity variation for the two-dimensional model**

For completeness rather than as part of the logical development, it is of interest to consider the actual form of the porosity variation with direction for the model being considered:

Dividing the results given in Equations (9) and (16) we can obtain an expression for the porosity in the direction of the gradient as the ratio of the darcian flux to the bulk velocity:

\[
n_a = \frac{K_a i}{V^B_a} = \frac{K_a}{V^B_a / i} = \left(\frac{E \cos^2 \alpha + F 2 \cos \alpha \sin \alpha + G \sin^2 \alpha (T \cos \alpha + U \sin \alpha)}{P \cos^3 \alpha + Q \cos^2 \alpha \sin \alpha + R \cos \alpha \sin^2 \alpha + S \sin^3 \alpha}\right)
\]

We see that this expression is a rational polynomial, of order three, in terms of the sine and cosine of the angle. This can be a very complex function and no obvious manner of plotting this porosity seems likely to give a familiar form (unlike the conductivity, the inverse square root of which can be plotted as an ellipse).
Appendix J – Brief explanation of the use of Fracman/Mafic, Modflow and SDF in this thesis

FRACMAN/MAFIC

For the full explanation of the Fracman/Mafic suite of programs the reader is referred to the user manuals (Dershowitz et al., 1988). However, it is useful to give a summary of how Fracman/Mafic has been used in this thesis and a brief explanation of the terms used in Fracman/Mafic. The general structure of the Fracman/Mafic suite of programs is as follows:

\[
\begin{align*}
\text{Fracman} & : \begin{cases} 
\text{Fracs} & \text{analysis of orientation, size, transmissivity and intensity data} \\
\text{Fracworks} & \text{stochastic fracture generation and sampling} \\
\text{Meshmaster} & \text{mesh generation and boundary condition assignment} \\
\text{Mafic} & : \begin{cases} 
\text{Mafic} & \text{finite element flow and particle tracking/solute transport in steady state or transient} 
\end{cases}
\end{cases}
\end{align*}
\]

Fracs was used to confirm the results of the stereographic analysis for Alton Court by plotting the televiewer data and analysing the fracture set statistics.

Fracworks was used to generate blocks of discrete fractures from the stochastic input parameter descriptions. There are many parameters that must be defined:

**Seed:** This is used to initialise the random number generator, with identical seeds producing identical fracture networks.

**Model:** This is the fracture network generation model type. Fracman gives a choice of Enhanced Baecher, Poisson rectangle, Levy-Lee, nearest neighbour, probabilistic war zone, BART, nonplanar frac zone, fractal, Fractal POCs or a geostatistical model for locating fracture centres. The enhanced Baecher model was chosen for all the generated networks in the thesis, as fracture centres are assumed to be distributed in space by a random Poisson process. Since there is no evidence at Alton Court on the nature of the
fracture centre distributions a random process must be assumed. The fractures are assumed to be elliptical rather than polygonal since we do not know whether they terminate at fracture intersections.

**Strike** – average strike of the fracture set

**Dip** – average dip of the fracture set

**Orientation distribution** – This can defined as either Fisher, bivariate normal, bivariate Fisher, Bivariate Bingham, Bootstrap or Multibootstrap. The Fisher distribution was chosen as it produced the closest match to the field observations in a test block.

**Distribution coefficient** – For the Fisher distribution this refers to the Fisher dispersion coefficient and defines the amount of variability of the distribution. 1-5 represents high variability and values of between 20 and 50 represent low variability.

**Size** – this is the mean equivalent radius of the fractures. Since fractures are not always generated as circular disks, the equivalent radius is the radius of a circular disk with the same area as the generated fracture.

**Size distribution** – The choice of distribution is between exponential, lognormal, normal, truncated exponential, truncated normal, truncated lognormal, power law and uniform. Lognormal was chosen for the Alton Court site from the literature review results in Chapter 6.

**Standard deviation** – this is entered as a value in true space as opposed to log space, even for the lognormal distribution.

**Direction of elongation** – If elliptical fractures are required then this allows the trend and plunge of the major axis of the ellipse to be defined. If circular fractures are modelled this direction is irrelevant (as in the case used in this thesis).

**Aspect ratio** – this parameter is required for the enhanced Baecher model and determines the ‘roundness’ of the fracture. It has been assumed to be 1 (circular fractures) since we have no information on this from field data.

Appendix J – Brief explanation of the use of Fracman/Mafic, Modflow and SDF
**Termination %** - This parameter determines whether fractures terminate against each other or end freely within the rock mass. It is assumed to be 0% since we have no evidence that fractures terminate against each other.

**Intensity**: (or fracture set density) entered as area/vol, vol/vol or number of fractures, depending on field data type.

**Intensity distribution** - Fracman gives a choice of distributions of constant, gamma, and correlated to depth. Gamma was chosen for the Alton Court modelling since with a carefully chosen gamma coefficient, the gamma distribution can approximate the form of a lognormal distribution (field data at Alton Court shows a lognormal distribution of fracture spacings).

**Distribution coefficient** – this coefficient is required for the gamma distribution and relates to the mean and standard deviation of the distribution. This coefficient was chosen by trial and error until the statistics of the distributions of the fracture network matched those in the field as closely as possible.

**Transmissivity/Storativity/Fracture thickness (aperture) correlation** - Fracman gives a choice of transmissivity, storativity and fracture thickness (aperture) being correlated with size, width or depth, or uncorrelated. If the parameters are correlated then a correlation exponent ($b$s), correlation factor ($S_0$) and deviation factor ($a_s$) must be defined.

These factors are related by the following equation:

$$\log_{10} S_i = \log_{10} S_0 + b_s \log_{10} X_i + e_s$$

where

- $S_0$ is the correlation factor
- $b_s$ is the correlation exponent – a value of one will result in a linear correlation with fracture size
- $X_i$ is the area or width of the fracture
- $e_s$ is a random deviate, uniform of the interval $[-\log_{10}a_s, \log_{10}a_s]$
is the deviation factor

Once the fracture block had been simulated the initial stages of calibration involved adding boreholes into the block to obtain the statistics of the fractures sampled. This was done within the Fracworks part of the program, under the ‘Sample’ option. During this stage the distribution coefficient for the gamma distribution was obtained so that approximately the correct number of fractures of each set were sampled in a typical borehole, and the spacing was correct.

Once this had been done the boundary conditions were set up so that the Alton Court pumping test could be simulated. Boreholes were added into the block in the relative orientations and spacings of those at the Alton Court site. The transmissivity, storativity and fracture thickness parameters were then used to calibrate the model as far as possible.

The fracture statistics obtained were then used to create fracture blocks across which a hydraulic gradient was applied and particles were tracked to obtain the effective hydraulic conductivity and the kinematic porosity of the block. The mass of the particles must be set in the input file, with a smaller mass producing more particles. A concentration has to be assigned to the ‘solute’ in order to obtain the number of particles. Average concentrations are determined at the end of each time step by summing the particle mass within each fracture element and dividing by the mass of fluid within each element. The equation for determining the particle concentrations within a specific element is:

\[ c = \frac{M_p N_p}{P_w A_e b_e} \]

where:
- \( c \) is the concentration
- \( M_p \) is the particle mass
- \( N_p \) is the number of particles in the element
- \( P_w \) is the fluid density
- \( A_e \) is the fracture element area
- \( b_e \) is the fracture aperture
Therefore since the average volume of water within all the elements will change with
each realisation the number of particles that are generated will also change.

MODFLOW WITHIN GROUNDWATER VISTAS

This section only describes the possible boundary conditions types that can be used
within Groundwater Vistas (GV) relating to the simulation of the groundwater system
around Alton Court described in chapter 10, and the principal differences between the
boundary types.

Constant head boundary conditions. If a constant head and/or concentration is
assigned to a boundary cell, the head or concentration does not vary throughout a given
stress period within the simulation. Different values of the head can be assigned to
different stress periods for the same cell, to allow representation of tidal influences, for
example. GV allows you to specify whether a constant head cell refers to head,
concentration, or both.

Constant flux boundary conditions. These are called wells in GV. A constant flux is
specified in a cell by entering the volumetric flow rate (e.g. m^3/d) that the model (e.g.
MODFLOW) will extract or inject into that cell. The sign of the flow rate (positive or
negative) depends upon the model. For example, MODFLOW assumes that negative
flow rates indicate pumping and positive refers to injection. Recharge is a form of
constant flux boundary condition; however, it is normally distributed over large areas of
the model and is thus categorized as a parameter in GV.

No-Flow boundary conditions. This is a form of constant flux boundary and is applied
to cells that are outside the computational domain of the model. These are termed
inactive cells in MODFLOW (IBOUND = 0). Head and concentration are not
computed in cells designated as no-flow.

Head dependent flux boundary condition. GV supports the use of four types of
mixed-type or head-dependent flux boundary conditions, including drain, river, general-
head, and stream. Evapotranspiration is another form of head-dependent flux boundary
condition, but it is treated like recharge, as a property zone. The general-head boundary
condition is the most generic of the four types. In all four head-dependent boundary
types, a boundary head and a conductance term is specified as a minimum. In most models, the flux of water into or out of the cell is then computed as follows:

\[ Q = C(H_b - H_m) \]

where:
- \( Q \) = flux into or out of boundary cell (\( \text{L}^3/\text{T} \)),
- \( H_b \) = boundary head (\( \text{L} \)),
- \( H_m \) = head computed by model (\( \text{L} \)), and
- \( C \) = boundary conductance (\( \text{L}^2/\text{T} \)).

The conductance term is a coefficient that is usually computed using an equation similar to the following:

\[ C = \frac{K_b A}{B} \]

where:
- \( K_b \) = hydraulic conductivity of the boundary material (\( \text{L}/\text{T} \)),
- \( A \) = area of the boundary (\( \text{L}^2 \)), and
- \( B \) = thickness or width of boundary (\( \text{L} \)).

For example, the conductance term for the MODFLOW river boundary type is computed using the hydraulic conductivity of the riverbed material, the area of the river bottom within the finite-difference cell, and the thickness of the river bottom. Because little is known about the conductance of a river or a stream bed, this parameter is often used as one of the main calibration parameters.

The generic form of the head-dependent flux boundary condition (general-head boundary in GV and MODFLOW) computes the flux of water into or out of the model and assigns that flux to the cell. The other types of head-dependent boundary conditions (drains, rivers, and streams) modify this flux term depending upon the relationship of boundary head to model-computed head in the cell. The drain boundary condition will only allow water to be removed from the system; if the head computed by the model is less than the head in the boundary (drain), the boundary condition is turned off. The river boundary condition also limits the amount of water injected into the aquifer if the aquifer head drops below the bottom of the river (McDonald and Harbaugh 1988). The stream boundary is a special case of the river boundary in which the amount of water injected into the aquifer is further limited by the available water flow in the stream. The stream boundary is therefore more suitable for conditions
where there is significant groundwater-surface water interaction, such as in the Alton Court area.

SUMMARY OF SDF PROGRAMS

The SDF suite of programs has the capability to simulate flow through a 2-D fracture network with an impermeable matrix, with or without a well, to include particle tracking if required. Four main programs are required to simulate flow and particle tracking: NETWRK, NETFLO or WELFLO, and PARTRACK. WELFLO is used if an abstraction or injection well is to be included, and NETFLO is used for simulations containing no well.

NETWRK

This is essentially a 2-D line network generation code which uses a Monte Carlo method to generate a pattern of lines of distributed lengths and orientations. Many fracture sets can be simulated with a given density for each set. Each line represents the trace of a fracture that cuts a slice of rock one unit in thickness. The lines are generated one set at a time using specific distribution types given in the input file (single or lognormal) for orientations and lengths. Fracture apertures are also selected from a given distribution (single or lognormal) for each fracture trace.

NETWRK can handle two types of fracture domain geometry - rectangular and circular. In the exercises in the thesis the rectangular option has been used. The model has both inner and outer boundaries in order to minimise edge effects. In generated line networks fractures can be systematically less dense near the edges.

NETWRK handles the input fracture parameters in the following way:

- **Fracture Density**
  
  To get the fracture density specified in the input files, the area inside the outer boundary is calculated and the estimated number of fractures needed for that area calculated.

- **Orientations and Lengths**
  
  With the estimated number of fractures needed, random points are generated within the outer boundary, each point corresponding to the centre of the

Appendix J - Brief explanation of the use of Fracman/Mafic, Modflow and SDF 397
fracture trace. The distribution of trace lengths (lognormal or single) and orientations (lognormal or single) is then applied.

- **Abutting Fractures**
  There is an option to abut the fractures of the set being generated against those of the previously generated set, as often occurs in natural systems. Only one end of the fracture is allowed to abut. The length of fracture that would have traced past the existing fracture is taken and put on the opposite end of the fracture to keep the total fracture length the same.

- **Scanlines**
  There is an option to place a scanline perpendicular to a fracture set and measure the spacings between the fractures of that set. Therefore the model fracture spacings can be compared with any actual fracture spacing measurements from the field.

- **Fracture Intersections**
  The program then calculates where fractures intersect, the positions of which are known as nodes. An attempt is then made to find all the effective fracture intersections i.e. all those junctions located on a continuous flow path between two fixed head boundaries. Where there are only two fractures involved in an intersection, so acting as a ‘continuous flowpath’, the junction is marked as a non-effective intersection.

- **Apertures**
  Apertures are then assigned to the fractures according to a lognormal or single distribution, depending on the fracture set, as specified in the input files.

**NETFLO/WELFLO**

The fracture network defined in NETWRK is then taken by either NETFLO or WELFLO to perform the fluid flow simulation. NETFLO and WELFLO are steady state programs, with NETFLO simulating the flow without any abstractions wells and WELFLO solving the flow equations for scenarios containing wells.
For the fluid flow simulation, the rock matrix is assumed to be impermeable and the fractures are assumed to consist of parallel plates in which the cubic law applies. Boundaries to the system are assumed to be no flow, linearly varying constant head or one value of constant head. Each boundary has two sections, as seen in the diagram below. This could cause restrictions on the different scenarios that could be simulated.

Heads at the fracture intersections and within the fractures are calculated using the Choleski algorithm. The flow rates, flow velocity and Reynolds number are then calculated for each of the segments. Hence the cumulative flow rate through the system can be calculated.

Abstraction/injection is simulated by specifying the head in the well, rather than a pumping rate, presumably to make calculations easier and to minimize memory usage. After the flow calculations are complete, the program calculates the discharge rate from the well. Up to 50 wells can be used across the model domain. Hence to achieve a precise flow rate from a well several iterations will presumably be required.

PARTRACK

The PARTRACK program then takes the flow regime calculated in WELFLO/NETFLO and applies particle tracking. Particles can be tracked in the forward or the reverse direction, and at present the limit on the number of particles is 200. When a particle reaches an intersection, they are moved through to the adjoining fractures on the basis of flow rate through the fracture and a random number generated internally.