Stochastic Modelling for High-Fidelity Differential GPS Quality Measures

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ABSTRACT

The offshore energy industries use differential Global Positioning System (DGPS) methods extensively to provide positioning for activities such as seismic exploration and pipeline inspection surveys. It has long been a criticism of DGPS that the quality measures delivered by most processing algorithms do not truly reflect the real quality of the derived positions and underestimate the likely sizes of the errors. In practice, this problem is often resolved by quoting extremely conservative upper error bounds so that users can be sure that real positioning errors are always within these. Significant improvements have been seen in both solutions and quality measures for post-processed applications with the use of reverse-engineered stochastic models. A common goal for researchers involved with single-frequency DGPS positioning is to identify a general case stochastic algorithm that can be relied upon completely to describe correctly, in real-time, the quality of position solutions. Many groups have found that the use of an elevation-weighting function is preferable over a model that incorrectly assumes all code pseudoranges are of equal precision and uncorrelated. However, such functions cannot accurately represent the high-frequency variations within DGPS measurements.

This research has investigated the improvement in the fidelity of DGPS quality measures when developing and applying stochastic functions that represent the errors in the basic DGPS measurements, i.e. undifferenced phase-filtered C/A-code pseudoranges. These analytical functions take into account the impact of distance-independent errors such as residual multipath and Selective Availability, and the distance-dependent errors within satellite orbits, the ionosphere and the troposphere. A number of processing strategies, incorporating various combinations of the candidate functions, have been afforded to the least squares stochastic model for testing on single epoch DGPS solutions. A data collection campaign was organised that provided high-rate GPS data for seven static sites around the North Sea. With the benefit of truth positions from this campaign, the performance of each processing strategy has been investigated and assessed in terms of accuracy and precision estimates. By computing the fidelity of these quality measures, it was possible to quantify the percentage of position estimates whose quality measures had been wrongly assigned relative to the truth positions.

The results of a large number of post-processing tests on real GPS data have shown that it is extremely difficult to design an a-priori stochastic algorithm containing functions that can correctly reflect all variations in the quality of phase-filtered C/A-code
pseudoranges. The candidate functions have been unable to reduce the impact of unmodelled systematic errors on the final position solutions and quality measures. This has led to their failure in significantly, and consistently, improving the fidelity of quality measures associated with differential code positioning. In general, the research has shown that the use of an elevation-weighting function can provide, for single-baseline datasets, consistent time series with minimal step functions even over periods of constellation change. For multiple-baseline datasets, a full variance-covariance matrix containing an adaptive multipath variance estimation routine generally afforded more robust time series of true and formal errors than other candidate model types. The acknowledgement of mutual spatial correlations between the observations at each receiver was seen to improve the quality measure fidelities for single-baseline datasets only, and not for networks. It has been shown that the effective omission of a stochastic model, i.e. the use of a unit weight matrix, can sometimes yield higher fidelity quality measures than an incorrectly assigned full stochastic model with spatial correlation terms. Clearly the nature of the relationships between the variances and covariances of phase-filtered differentially corrected pseudoranges are more complex than any of the models tested here. In some cases however, significant improvements on current practice have been achieved.
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Finally, I’d like to dedicate this thesis to Mum and Dad for their loving support during my time in academia, not only financially and motivationally, but also in believing that I could do it.
Chapter 1 - Introduction

1.1 PREAMBLE

The offshore energy industries require, for a wide variety of engineering surveys, positioning accuracies ranging from better than 10 metres for preliminary site investigations, seismic exploration and pipeline inspections, to decimetres for specialised marine engineering projects. Differential code GPS (DGPS) can provide the 5-10 metre level positioning satisfactorily; however differential carrier-phase DGPS, or Real-time Kinematic GPS (RTK-GPS), is needed to obtain decimetric accuracy. In large-area marine surveys, the lower accuracy DGPS technique is used over much longer distances than RTK-GPS as any spatial errors present are generally smaller than the background accuracy of DGPS.

It has long been a criticism of differential GPS positioning that the quality measures, both for precision and reliability, delivered by the processing algorithms are very simple and do not truly reflect the rapid variations in the real quality of the estimated positions [Cross, (2000)]. Most software packages make simple assumptions about, or completely ignore, the correlations within the GPS data. This invariably leads to the provision of over-optimistic quality measures, based largely on internal consistency, that underestimate the likely sizes of the errors.

In practice, service and software providers often resolve this problem by quoting extremely conservative general upper error bounds so that users can be sure that real positioning errors are always within these [Roberts et al, (1997)]. This may result in particular GPS methods not being used when they are in fact perfectly adequate. Guidelines accredited by the United Kingdom Offshore Operators Association (UKOOA) recommend specific statistics that should be used and suggest that all formal errors associated with offshore positioning activities be quoted at a 95% confidence level. The incorporation of stochastic functions that correctly acknowledge the real-time variations in, and correlations between, GPS observations can yield improved solutions and quality measures.
1.2 THE OFFSHORE ENERGY INDUSTRY

**Exploration and Production**

The offshore industry relies upon its exploration contractors to discover hydrocarbon resources worldwide. Shell Exploration and Production companies, for example, seek and produce oil and gas from natural deposits in over 45 countries around the world. In total, upstream operations by Shell companies bring over 4 million barrels of oil and 400 million cubic metres of natural gas to the surface every day. Advances in technology mean that the extraction of fossil fuel resources can take place in ever more hostile locations, with exploratory drilling now carried out regularly in waters up to 1,700 metres in depth [Hart’s E&P, (2000a)]. The more advanced tension-leg fixed platforms are now producing oil and gas from deep-water sites, where operating depths exceed 330 metres [Hart’s E&P, (2000a); ExxonMobil, (2000)].

In the week beginning 7th December 1998, the price of Brent crude oil fell to below $10 per barrel for the first time since 1986. This was a direct result of several factors: the mild winter world-wide, and the declining demand for petrochemicals providing oversupply to buoyant markets [Parsley, (1998); Smith, (2000)]. Several national recessions, especially in Asia, aided the latter point of oversupply.

City analysts believed that Shell would do best by cutting down on its capital spending. "With a return well below those of its peers, but a colossal asset base and $80 billion of capital employed, the best method of advancement for the group is to make more of those assets and the capital", - Marshall, [1998]. Shell responded by scrapping business committees and appointing chief executives to run the oil products and Exploration and Production (E&P) divisions under the belief that this approach would permit more aggressive management, not only for rapid decision making, but also forcing new corporate changes through into all sub-divisions.

In the past two years since then, the price of crude oil has fluctuated greatly and is currently, at time of writing, loitering around $25-30 per barrel, higher than in December 1998 [The Sunday Times, (2000)]. As it is driven by supply-and-demand economics, the necessity for hydrocarbon exploration is subject to renewed interest by many corporations and investors. However these parties are keen to maximise returns such that the most current, efficient exploration and production techniques must be implemented. Considering the average daily operating costs of a seismic survey vessel are around £100K
[Jensen, (1998)], the use of navigation systems using algorithms that afford increased levels of positioning reliability, are not to be considered lightly.

**New Prospects**

It is therefore only natural to comprehend that exploration projects should evolve from close-range shallow-water sites, such as the United Kingdom’s continental shelf (UKCS), to deep-water sites further from shore. Examples of these include the UK’s Atlantic Margin, the Gulf of Mexico, Brazil, western Africa and the southern Atlantic Ocean east of the Falkland Islands [Exxon, (1998)].

Once the deep-water resources have been discovered and an Appraisal / Development study confirms that the prospect warrants manageable exploitation (both in financial and technological terms), operations can begin in earnest (cf. Table 1-1). All of the major offshore oil companies have extensive rights for exploration and production on six of the seven continents and are pursuing deep-water acreage with the aim of finding new fields and developing them quickly. The deep-water sites are seen as the boom sites of this century with reserves of approximately 30 years; ExxonMobil predicts in [2000] that deep-water oil and gas wells will account for more than 20% of their global hydrocarbon production by 2010. The transition into deep-water will mean that long-range high-precision positioning will be even more critical for activities featured later in Table 1-1 that were taken for granted in the shallow-water sites.

The level of dependency on satellite-based navigation systems will increase tremendously with a heavy emphasis on the reliability issues of such systems. Drilling costs are correlated to water depths as are data collection costs so greater depths will demand a greater outlay of capital and materials for the majority of upstream activities. Providing some examples, seismic surveys require longer streamers to accommodate the seismic signal’s increased travel time, and the use of swathe-scanning sensors in deeper waters will make greater demands on GPS-based attitude determination systems. Lott [1999] believes that deep-water activities will be the catalyst for data collection and subsea engineering using automated underwater vehicles (AUVs), however these again require accurate absolute positioning to locate the prospect potentially several hundred kilometres offshore. Westwood [1999] supports this with a prediction of a boom in subsea production of $40 billion for the 5 years ending in 2004.

Deep-water operations are becoming commonplace for a number of reasons. As the conventional basins are maturing, i.e. reserves in shallow-water sites are nearly exhausted,
the exploration industry requires another growth engine. There is a substantial volume potential in deep-waters projected, by ExxonMobil [2000], at \(~\text{140 billion barrels}\). Given strong advances in engineering technology [Kvinnsland, (2000)], deep-water development is now technically feasible - some appraisal wells are being drilled in ultra deep-water sites to depths of 2,400 metres in Australian waters [Hart’s E&P, (2000b)].

1.2.1 Navigation and Positioning in the Offshore Industry

The precise navigation of offshore vessels is essential for financial feasibility in exploiting these hydrocarbon resources, not only for exploration tasks, but also for all tasks associated with development, production and decommissioning phases throughout the installation’s working life. Table 1-1 lists the principal categories of offshore activities where high-accuracy positioning is critical to the safety and effective completion of the operation [Edge, (1998)].

<table>
<thead>
<tr>
<th>LIFE-PHASE OF CH4 FIELD</th>
<th>ACTIVITY REQUIRING SURVEYING AND POSITIONING</th>
<th>REQUIRED ACCURACY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exploration and Appraisal</td>
<td>- Environment Impact Assessment (EIA) studies</td>
<td>10 m</td>
</tr>
<tr>
<td></td>
<td>- Surveys: seismic and seabed</td>
<td>&lt;10 m</td>
</tr>
<tr>
<td></td>
<td>- Dynamic Positioning (DP) of vessels</td>
<td>5 m</td>
</tr>
<tr>
<td></td>
<td>- Drilling rig positioning</td>
<td>10 m</td>
</tr>
<tr>
<td>Development</td>
<td>- Positioning of wellheads</td>
<td>&lt;5 m</td>
</tr>
<tr>
<td></td>
<td>- Positioning of jack-up rigs adjacent to platform</td>
<td>&lt;1 m</td>
</tr>
<tr>
<td></td>
<td>- Pipeline positioning</td>
<td>2 m</td>
</tr>
<tr>
<td>Production</td>
<td>- Subsidence monitoring of platform</td>
<td>0.1 m</td>
</tr>
<tr>
<td></td>
<td>- Positioning of FPSO vessels</td>
<td>&lt;5 m</td>
</tr>
<tr>
<td></td>
<td>- Pipeline inspection surveys</td>
<td>5 m</td>
</tr>
<tr>
<td>Decommissioning</td>
<td>- Positioning for platform decommissioning equipment</td>
<td>&lt;10 m</td>
</tr>
<tr>
<td></td>
<td>- Environmental and seabed surveys of abandoned locations</td>
<td>&lt;10 m</td>
</tr>
</tbody>
</table>

Table 1-1 Principle Categories of Offshore Activities requiring High Accuracy Surveying and Positioning Support [after Edge, (1998)]
The Global Positioning System (GPS) has long been established as a tool within the research arena of global geodesy and geodynamics [Beutler, (1996); IGS, (1997)] yet only in the last decade has its potential for engineering applications been exploited. Satellite-based positioning techniques, such as Transit Doppler, have been used in marine engineering tasks for many years [Ackroyd and Lorimer, (1990)] however, GPS is now commonly regarded as the main positioning technique.

Following are examples of the GPS positioning methodologies used, in order of increasing accuracy and precision, to achieve the specifications required in these applications. Indeed, the methods are equally applicable to onshore surveying tasks.

**Point Positioning**

Up until 2

\(^{nd}\) May 2000, civilian users were able to obtain their instantaneous position with one receiver to 100 metres in plan and 156 metres in height (95% of the time) making use of the unencrypted C/A-code available from the Standard Positioning Service (SPS) [US DoT and DoD, (1999)]. Members of the US Armed Forces and their allies, equipped with military-specification equipment, enjoy exclusive access to the Precise Positioning Service (PPS) and its encrypted P-code measurements. Use of these more precise ranging signals allows authorized users to obtain their absolute point position to within 22 metres in plan and 28 metres in height (95% of the time) [US DoT and DoD, (1999)]. Regardless of one’s friends however, the accuracy of an instantaneous point position is limited by satellite, orbital, atmospheric, receiver and multipath errors meaning that sometimes better accuracies can be achieved as will be discussed later in §2.1.

On May 2

\(^{nd}\) 2000, the US military terminated Selective Availability on the GPS signals which in turn afforded positions to civilian SPS users with a five-fold increase in accuracy compared to the 1999 SPS [US DoT and DoD, (1999)]. The termination of SA is a noteworthy point, as the data used in this research was collected prior to May 2000. The particulars surrounding this event are discussed further in §2.1.1.1.

The absolute point position of a single receiver can be solved using a surveying technique called resection. In this, a minimum of four pseudoranges are needed to solve for the four receiver unknowns, i.e. three position components and the receiver clock offset. After the pseudorange observation equations have been linearised, an iterative least squares adjustment is used to solve for the receiver’s point position. The normal cofactor matrix derived from this adjustment can be used to assess the relative strength of the observed satellite geometry and yield an estimated position quality statistic. This Dilution
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of Precision (DOP) statistic quantifies how the level of errors in the measurements can be related to the expected quality of the GPS point position solution.

The DOP statistic is a useful value output by all GPS receiver software that can be expressed in different terms relating to the propagation of the satellite geometrical configuration into the constituent components of the position fix, i.e. one, two or three dimensions. VDOP and HDOP are the DOP statistics corresponding to the vertical and horizontal components respectively, and PDOP refers to the 3-D position vector. PDOP is a mathematical function computed using the inverse of the normal equation matrix, which contains information corresponding to the relative co-ordinates of the receiver and those satellites observed within a particular constellation arrangement. The greater the spread of the satellites relative to the user, then the better (smaller) the resultant PDOP value.

When the PDOP statistic is multiplied by the RMS user equivalent range error (UERE), it yields an a-priori RMS three-dimensional position error. The total UERE is defined as the root sum square (rss) of the individual error sources contributing to a range measurement [Langley, (1997)] as discussed in §2.1.6. This range error is mapped into the relevant DOP statistic depending on the application and user requirements. However as this computation is derived from the normal matrix, it should not be regarded as the definitive accuracy of a GPS position because it assumes that the RMS UERE is identical on each satellite contributing to the position fix. This is not always the case, as lower elevation satellites will be subject to greater atmospheric refraction [Langley, (1997)] and multipath effects than those at the receiver's zenith [Leick, (1995)]. Unrealistic assumptions and estimates about the precisions of GPS observations will usually lead to incorrect quality measures although the application of correct stochastic functions can remedy this. Such concepts form the bulk of this research and are discussed in later chapters.

Absolute point positioning is mainly used within the offshore industry for static monitoring applications. One such example is of 'Air Gap Surveys' (cf. Table 1-1) which correspond to vertical deformations of the offshore platform [Svitzer Limited, (1997)]. Safety legislation requires that there be a minimum level of clearance between the sea surface (in a highly excited state) and the lowest manned section of the offshore platform. The air gap clearance is related to reservoir management activities, in that the magnitude of

---

1 RMS (root-mean-square) is defined as the square root of the average of the squared errors [van Diggelen, (1998)]
vertical deformation is directly proportional to the change in volume of the oil or gas reservoir beneath the platforms.

**Differential GPS Positioning**

In the case of differential positioning whereby two receivers make simultaneous observations to the same satellite, error sources that are common to both receivers, such as satellite orbits and atmospheric delays, will largely cancel. The extent to which they will cancel depends on how these errors affect each receiver and is thus governed by the separation distance, or baseline length, between the two receivers. This unknown mobile receiver can be land, marine, air-based or even stationary.

A differential GPS technique (DGPS) has been developed that enhances the performance of GPS positioning through the application of differential corrections, to either the basic pseudorange measurements or derived positions, and permits higher positioning accuracies in navigation and surveying tasks. Two conditions have to be assumed before this technique can be applied successfully. The first is that the user possesses accurate co-ordinates for the satellites and at least one of the GPS reference stations. The second is that with short distances between both receivers and given the high altitude of the satellites, any errors in the satellite-to-receiver ranges will be present in the ranges to the rover receiver. It is assumed that the transmitted signals have followed such a similar path to both receivers that any atmospheric propagation delays have been effected to the same magnitude on each signal.

In the example of DGPS positioning with pseudorange corrections, it is possible to calculate the geometric range between the satellites and the reference station as these quantities are known. By comparing the observed range to the calculated geometric range, the user can then estimate the range errors as seen by the reference station. These range errors are then sent in the form of corrections to the rover receiver where they are applied enabling the determination of instantaneous position relative to the reference with an accuracy of approximately 2-5 metres, sufficient for many navigation tasks [Wells, (1987)].

For engineering surveying tasks requiring higher accuracies such as positioning of jack-up rigs (cf. Table 1-1), techniques have been developed, similar to those in DGPS that make use of simultaneous measurements of the carrier-phase observables. Achievable accuracies using carrier-phase differential positioning, or RTK GPS, are 1 and 2 centimetres in plan and height respectively [Langley, (1998)].
Introduction

Both of the differential positioning techniques described here assume that the signals transmitted from a particular GPS satellite to the observing receivers have travelled through similar conditions along similar paths affecting the signals to similar amounts. For code pseudorange positioning, this assumption is generally believed to hold true for inter-receiver distances up to about 1000 kilometres before the distance-dependent errors overwhelm the distance-independent receiver errors, e.g. code multipath, and significantly affect user positions [Tiberius and de Jonge, (1995)]. With carrier-phase positioning techniques, this threshold distance is much shorter, at approximately 20 kilometres, as the distance-independent errors are much smaller, e.g. by two orders of magnitude for phase multipath (cf. Table 2-4). Considering the accuracy achievable with carrier-phases, during periods of high solar activity, this threshold will likely drop to around 7-10 kilometres as a result of high ionospheric excitation effecting larger ionospheric delays [Langley, (1998)].

As the inter-receiver distances increase, there is a rapid decorrelation in the satellite-to-receiver paths that the signals follow, namely they travel along paths that become less similar. The effects of the upper atmosphere and errors in the broadcast GPS satellite positions also greatly reduce the accuracy of positions potentially attainable. A relative PDOP statistic can be calculated (akin to the absolute PDOP statistic in point positioning) that represents the quality of the baseline components according to its relative satellite geometry.

Following extensive research by many groups, the technique of differential GPS has improved considerably since its inception in the mid-1980s [Langley, (1998)]. Examples of these improvements follow.

- The combination of carrier-phase measurements with code pseudoranges has been shown to yield more precise positions due to a reduction in signal noise (mainly multipath and receiver noise) by 70% [Hofmann-Wellenhof et al, (1994)].
- The determination of differential corrections from multiple reference stations has reduced the magnitude of systematic errors, improved reliability and increased positional accuracies, in some instances, below 2 metres for baselines of 1000 kilometres [Thomson, (1996)].
- The means of assessing DGPS reliability has been greatly improved with the use of improved outlier detection routines and statistical analysis techniques [Cross, (1983); Roberts et al, (1997)].
1.3 MOTIVATION FOR THIS RESEARCH

There is currently a considerable need for real-time dynamic positioning (DP) of vessels in offshore operations such as geotechnical investigations, deep-water drilling, pipelaying and diving support. This need will be further increased with the projected usage of FPSOs\(^2\) in areas of deep-water operations by the major hydrocarbon exploiters [Hart's E&P, (2000b)]. The critical point here is that the reliability of the position reference system is more important than the absolute positional accuracy [Edge, (1998)].

It has long been a criticism of differential GPS positioning that the quality measures, both for precision and reliability, delivered by the processing algorithms do not truly reflect the rapid variations in the real quality of the estimated positions [Cross, (2000)]. Most software packages either make very simple assumptions about the quality of the raw data they process or completely ignore the spatial, temporal and inter-frequency correlations within the data. In particular, the correlations in space, in time and between frequencies are almost completely ignored. This invariably leads to over-optimistic quality measures based largely on internal consistency that flatter the real quality, that is they underestimate the likely sizes of the errors.

In practice, service and software providers often resolve this problem by quoting extremely conservative general upper error bounds so that users can be sure that real positioning errors are always within these [Roberts et al, (1997)]. This may result in particular GPS methods not being used when they are in fact perfectly adequate. Guidelines accredited by the United Kingdom Offshore Operators Association (UKOOA) recommend specific statistics that should be used and suggest that all formal errors associated with offshore positioning activities be quoted at a 95\% confidence level.

Measurement errors can be modelled in one of two ways: firstly by including additional corrections within the functional model and removing the errors directly, or secondly by properly describing the mismatch of the model and measurements within the stochastic model. Both approaches are essentially equivalent and it is important to note that the choice of stochastic model is dependent on choice of the functional model. Any errors not included in the functional model should then be described stochastically [Blewitt, (1997)], and vice-versa.

\(^2\) Large ocean-going vessels that can provide Floating Production, Storage and Off-take facilities once positioned over the operational wellhead(s). Typically, they are refitted crude oil tankers.
It has been shown in [Roberts et al, (1997); Barnes et al, (1998)] that the application of a full and correct stochastic model can yield improved solutions and quality measures. To give an example for short baseline positioning using single-epoch fixed ambiguities, the stochastic modelling of L1 multipath improves the accuracy of static positioning by almost 60% [Barnes et al, (1998)]. However it must be noted, that this was determined in post-processing with the benefit of a reverse-engineered stochastic model comprising the true rover and satellite positions, and thus the true errors in the observations.

Given the potential improvement in solutions and quality measures as realised in post-processed applications with reverse-engineered stochastic models, the 'Holy Grail' in DGPS positioning is to discover a general algorithm that can be completely relied upon to describe correctly, in real-time, the quality of receiver positions. This algorithm would clearly need to involve proper stochastic modelling of the basic DGPS observables, i.e. phase-filtered C/A-code pseudoranges. Once this has been done, transformation of the statistics through the estimation process is extremely straightforward and well understood.

1.4 RESEARCH OBJECTIVE AND METHODOLOGY

GPS code double-difference observations are riddled with several kinds of errors and biases including residual orbit errors, ionospheric and tropospheric biases, multipath and system noises. Generally these biases are very complex in nature and can not be modelled functionally with any great success [Han, (1997)]. Therefore the remaining error components must be modelled stochastically, whereby the characteristics of these errors are included in the least squares estimation routine as additional terms within the covariance matrix of observations.

In this a-priori matrix, the variance and covariance terms describe the measurement precisions, and the covariances also describe the correlation between the measurements. In practice, the determination of these quantities can be very difficult. A combination of new empirical routines and previous research findings will be the motivations for the determination of a-priori variances and covariances.

The methods available for, and problems associated with, this modelling are reviewed in detail with emphasis on the importance of understanding the spatial and receiver-generated correlations that occur within DGPS. For example, the effects of satellite orbit errors and atmospheric delays have an assumed correlation with the
separation distance between receivers. Also the effects of multipath interference and receiver noises are uniquely correlated to each receiver's location and therefore must be treated individually at each receiver. These correlations must be correctly understood as part of a successful stochastic modelling routine.

The fundamental research objective is to improve the fidelity of achievable quality measures for single-frequency DGPS positioning through more rigorous stochastic modelling, in particular through the better understanding of the correlations between errors within DGPS observations. In order to achieve this objective, it will be necessary to undertake the following tasks.

- Investigate the factors contributing to GPS pseudorange errors in order to gain a better understanding of the correlations afforded to DGPS code measurements: spatially (distance-dependent) and receiver-location specific (distance-independent).
- Investigate the performance benefits of differential positioning using carrier-phase filtered code pseudoranges to reduce the effects of code multipath errors and receiver noise.
- Following an investigation into current stochastic modelling routines for GPS positioning, design a processing package that contains a comprehensive suite of stochastic modelling algorithms to reflect the findings of the correlation studies and that will assist in achieving the research objective. These modelling algorithms should be compatible with a double-difference functional model and be able to use carrier-phase filtered code pseudoranges for baselines of varying length as well as multiple-baseline networks. They must incorporate routines for the real-time estimation of the biases and correlations present in GPS measurements. These will include terms for the decorrelation of orbital, ionospheric and tropospheric delays, and for code multipath and receiver noises.
- Organise and participate in a data collection campaign that will afford, to the research software, a real dataset for a survey vessel operating under typical conditions offshore. The obtained dataset should reflect actual operating conditions and be of sufficient duration, i.e. a minimum of 1 hour, to cover a change in satellite geometry, and contain at least single-frequency GPS code and carrier-phase data at a high data rate of 1 second. A number of different baselines and reference station combinations will be devised for subsequent processing activities.
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- Evaluate the performance of the designed stochastic models with this data, post-processed as though it were collected in real-time, i.e. only data up to, and including the current epoch will be used in the processing and modelling routines.
- The final positions may be described in terms of quality of horizontal, vertical or vector positions. Routines must be created to afford statistics which quantify the quality of the resultant precision estimates in terms of their accuracy with respect to the true positioning errors at each epoch and also for the entire dataset.

An overview of the research activities is provided in the flowchart Figure 1-1.

![Flowchart](image)

**Figure 1-1** Flowchart Overview of GPS Processing Strategy: Functional and Stochastic Modelling

The outcome will include recommendations of the most accurate stochastic modelling techniques for relative code positioning, along with quantitative assessments of the quality of those positions. It is believed that this project’s findings, as well as providing a useful comparative reference, may also identify points suited to further investigation pertaining to the use of code and phase combinations in dynamic GPS positioning and the stochastic modelling of such measurements.
1.5 THESIS OUTLINE

This chapter has detailed the positioning demands of offshore positioning activities along with some discussion on the formative deepwater industry, an area that will rely heavily on high-precision long-range satellite positioning techniques. A brief description has been provided of the methods used to achieve these requirements. The motivation for this research was discussed and followed by the research objectives, namely to improve the accuracy of quality measures in DGPS positioning. Finally there is some background on the Global Positioning System in terms of the signals available to civilian users and means of modelling them.

Chapter 2 begins with background to the major error sources within GPS observations and summarises several current methodologies for differential GPS positioning along with examples of their application.

Tasks relating to the pre-processing of GPS observables, namely phase-filtering and atmospheric delay corrections, are discussed in Chapter 3 as is the double-difference processing method that will be used in this research. The important issues relating to quality measures within DGPS positioning are introduced and discussed.

The concept of stochastic modelling is introduced in Chapter 4 along with a review of a number of current stochastic modelling algorithms. The design and reasoning behind the candidate algorithms are discussed along with the quality measures that will be introduced as part of the processing software.

Chapter 5 contains an overview of the pre-processing, functional and stochastic functions present within the research software. It contains discussion as to how particular modelling problems were overcome when coding all the routines described above.

A summary of the data collection campaign that afforded a real dataset to this research is provided in Chapter 6. There is some discussion as to problems encountered and details surrounding a shift in research focus. In this section, the background pre-processing and functional models are summarized along with the full complement of candidate stochastic models to be tested on the study datasets.

The results of the long-range positioning studies are analysed in Chapter 7 in terms of accuracy, integrity and the fidelity of the quality measures. Comments are made on the performance of the designed algorithms.

Chapter 8 is a summary of the main conclusions reached from this research. A section detailing recommendations for further research investigations, as based on the
interim findings is included. All references made within the body of this text have been listed within Chapter 9.

The eleven appendices contain additional information, basic background on GPS, results, graphs and tables, reference to which will be made when applicable, as evidence of particular and significant processing results, mainly those from the long-range positioning studies.
Chapter 2 - GPS and Offshore Positioning

This chapter discusses the different errors effecting GPS observations and their sources along with some discussion as to the levels of correlation therein. It also introduces a number of differential positioning methodologies currently available to users from commercial providers along with techniques from some recent research projects with discussion as to how their use can mitigate GPS errors. A brief overview of the quality measures used within differential positioning is given with reference to guidelines recommended by UKOOA, i.e. P2/94.

2.1 ERROR SOURCES WITHIN DIFFERENTIAL GPS POSITIONING

As discussed in §1.3, the GPS signals are affected by a number of error biases in their journey from satellite to receiver, namely relating to different phenomena including orbital and atmospheric propagation errors, receiver and multipath errors. The extents to which the signals are affected by each bias are discussed along with methods for their mitigation. A comprehensive understanding of these error sources and their characteristics is a necessary pre-requisite for the logic and reasoning used within the algorithm design process to be discussed in Chapter 4.

2.1.1 Denial of Accuracy and Access

Historically, the largest source of error within GPS arose as a result of the DoD’s Civilian Denial of Accuracy and Access policy, and comprised of two elements respectively:

- Selective Availability (SA)
- Anti-Spoofing (A-S)

2.1.1.1 Selective Availability

Up until its termination on 2nd May 2000 (Day of Year 123, 0407 UTC), the dominant error on those GPS observables available to civilian users was an error source
known as Selective Availability [White House, (2000)]. In this thesis, the phenomena of SA will be discussed in the present tense as it was active during all data collection campaigns associated with this research.

SA is the product of a denial of accuracy initiative implemented through the intentional imposition of error biases onto the broadcast clocks and positions of the GPS satellites. With SA activated, the civilian user is afforded a stand-alone position error of not more than 100 metres horizontally and 156 metres vertically (95% of the time) in the WGS-84 reference frame [US DoT and DoD, (1999)]. This phenomenon consists of two types of error.

i. Dither, $\delta$; a series of rapid and slow desynchronisations or variations in the satellite's fundamental clock frequency, 10.23 MHz. The net effect of these variations is very similar to a satellite clock error propagating directly into pseudorange errors. Magnitude of delta-SA errors range between 50 and 150 metres with periods of several minutes [Hofmann-Wellenhof et al, (1994)].

ii. Epsilon, $\varepsilon$; implemented by introducing additional errors into the predicted satellite position encoded within the almanac subframes of the broadcast GPS Navigation Message (cf. Table A-2). This erroneous data has the effect of suggesting the satellite is at a position differing from that occupied at the time of signal transmission although SA-epsilon is not believed to have been implemented.

According to Casswell [2000], the definitive statement for the accuracy of SPS GPS since the termination of SA, will be included in the next SPS Signal Specification document which was due for release in November 2000.

Since the termination of SA, a number of unofficial statements as to the post-SA accuracy for SPS have been published on numerous web sites concerning GPS, for example [USCG, (2000); NGS, (2000)] and electronic distribution lists such as for [Bartlett, (2000); Moore, (2000)]. The wide spread of these claims is due to the range of receivers and processing methodologies available to civilian users. This is due to a number of factors including: receiver type, whether single-frequency or dual-frequency observables were used, and how the ionospheric and tropospheric delays were modelled (if at all). The influence of multipath interference on the final accuracy of the single point position is minimal given that any multipath effects would most likely average out over the lengthy
Error Sources within Differential GPS Positioning

datasets obtained in these references. However the presence of multipath will be noticeable in the precision figures, i.e. by larger standard deviations.

Figure 2-1 shows SPP scatter plots obtained with a high-quality geodetic receiver with SA and without SA respectively. Both data sets were taken at 30 second intervals with Ashtech Z-XII geodetic receivers. The GPS data recorded were dual-frequency pseudoranges (both L1 and L2) incorporating ionospheric corrections and broadcast WGS-84 orbits. The plan axes for this figure are ±100 metres around the true antenna position. With SA activated, as in Figure 2-1(a), its impact on GPS measurements causes 95% of the resultant positions to fall within a radius of 45.0 metres; SA overwhelms any advantage of using a geodetic quality receiver for single point positioning rather than a hand-held receiver. Without SA, 95% of the points fall within a radius of 6.3 metres [Milbert, (2000)] (cf. Figure 2-1 (b)); this shows that the termination of SA has removed a very significant error source from absolute positioning.

Figure 2-1 Scatter Plots illustrating the Achievable Single Point Positions (a) before and (b) after the termination of Selective Availability [after IGEB, (2000)]

Hill and Moore [2000] observed 24 hours of static positioning with a handheld receiver recording at 30 second intervals. For the 2880 independent position fixes in this period, 95% of the horizontal position fixes were within 8 metres of the mean which in turn was only 40 centimetres from the true WGS-84 position. For vertical positions, 95% of the stand-alone height solutions were within 26 metres of the truth, and the mean height
was biased by 13 metres from the truth. This significant bias is most likely due to atmospheric signal refraction. Single-frequency receivers capable of observing C/A-code measurements (on L1 only) are subject to both ionospheric and tropospheric path delays making them dominant error sources on GPS observations now that SA is no more.

SA is implemented on each satellite clock independently - had it been a bias common in magnitude to all satellites, then it could have been eliminated easily through differencing although it would be indistinguishable from a receiver clock error. For civilian SPP equipment, the user is shown only the final navigated position that includes a combination of SA errors from each of the satellites used in the position fix.

However using the technique of differential GPS (DGPS), can eliminate the effects of SA-dither from the range measurements. By occupying a known co-ordinated reference station and making use of the known satellite positions, it is possible to determine the errors on each individual range and therefore compute appropriate range corrections. These are then transmitted to the roving receiver where they are applied to obtain a position nominally free of SA. There are still considerable errors affecting these differential positions including the latency time taken to determine, transmit and apply the differential corrections. The concept of DGPS is discussed in detail later in this chapter, §2.2.

Note that all data used in this research was collected before 1st May 2000, and was therefore subject to SA-dither. However as a differential positioning methodology was used, the effects of SA were eliminated and did not effect any conclusions drawn in this research; in other words, identical results could be expected with data subjected to SA.

2.1.1.2 Anti-Spoofing (A-S)

Anti-spoofing involves the encryption of the P-code into the classified Y-code for the purpose of preventing ‘an enemy’ of the US from imitating a GPS signal – Denial of Access. The resulting Y-code is the modulo-two sum product of the P-code and the lower rate encrypting W-code [Hofrnann-Wellenhof et al, (1994)]. The encryption afforded means that civilian receivers find it impossible to correctly reconstruct the P-code pseudoranges and L2 carrier-phase measurements.

The precise user is not necessarily curtailed by the presence of A-S since modern receivers now possess complex algorithms and hardware specifically for the reconstruction of the precise P-code pseudorange and L2 carrier-phase information from the civilian GPS signals [Hofmann-Wellenhof et al, (1994); Langley, (1996)]. However, the quality of the
L2 phase available to civilians is reduced relative to decrypted P-code as used by the military.

### 2.1.2 Satellite Orbits and Clocks

Amongst information found in the broadcast navigation message, (cf. §1.2.3), are terms describing predicted orbital parameters and satellite clock corrections for each GPS satellite [Beutler, (1996)]. Any satellite orbit errors will result from that difference between the satellite's actual position and that predicted by the Keplerian elements transmitted within the navigation message. A similar concept exists for the satellite clock errors from the difference between the satellite clock time and the transmitted clock coefficients [Langley, (1991)].

As it is generally regarded that the epsilon component of SA had never been activated [Langley, (1996); Lachapelle et al, (1996a)] and as differential techniques of DGPS remove all clock dither on observed ranges, it can be said that DGPS mitigates completely the effects of SA. Wording in the Presidential statement concerning the termination of SA [White House, (2000)] supports the assumption in [Langley, (1996)] that the dominant error source was the deliberate degradation of the satellite clocks, known as SA.

Jefferson and Bar-Sever [2000] state that the accuracy of the broadcast ephemerides is typically around 5.0 metres with reference to precise JPL hi-rate orbits (as described later in this section) although this is for a probability distribution of 80%. For a 95% probability distribution, broadcast ephemeris accuracy is 8.6 metres, and 14.1 metres for 99% distribution. In conclusion, Jefferson and Bar-Sever [2000] state that the broadcast orbit accuracy (RMS) as compared to the truth orbits is 4.9 metres in three-dimensions.

Some groups prefer to determine the satellite orbits during post-processing themselves as part of their own research software rather than make use of the broadcast navigation messages. Examples shall be given later in this chapter.

The use of precise satellite ephemerides and clock corrections, instead of their broadcast counterparts, yields higher accuracies for both absolute and differential positioning applications respectively [Lachapelle et al, (1996a); Cruddace, (1999)]. However as these products cannot be determined in real-time, the position results must be post-processed; currency is sacrificed for accuracy. A description of the standard format
for precise ephemerides and clock corrections, SP3, can be found in Spofford and Remondi [1993].

The presence of the Internet has greatly increased the ease of accessibility of such precise GPS products to civilian users with several institutions providing them free of charge. One such provider is the International GPS Service for Geodynamics (IGS). The IGS was established formally by the International Association for Geodesy (IAG) in 1993 to provide high-quality geodetic data products, in near real-time, to meet the objectives of a wide range of scientific and engineering applications and studies [IGS, (1997)]. Amongst other data products, the IGS and its members generate high-accuracy GPS satellite ephemerides and GPS satellite clock information. It also provides ionospheric models in real-time and tropospheric path delays for all of, at the time of writing, its 237 permanent reference stations operating around the globe [IGS, (2000)].

The improved and unified IGS orbit (prefixed IGS), is the product of those final orbits as independently predicted by each of the seven IGS Analysis Centres (AC). Quality control is assured through the comparison and combination of results between these analysis centres. The rapid IGS orbits (prefixed IGR) are available within 24 hours and agree with the final unified orbit to about 20 centimetres RMS. The predicted 3 day orbits (prefixed IGP) agree to the final orbit with an RMS of 0.5-0.8 metres after 24 hours, deteriorating to 1-2 metres after 48 hours [IGS, (1997)], but are still suitable for real-time applications.

At the time of writing, precise satellite ephemerides and clock corrections are available to civilian researchers in five versions: predicted, rapid, final [IGS, (1997)], high-rate [Zumberge, (1997)] and, most recently, ultra-rapid [(Martin-Mur et al, (2000))].
Error Sources within Differential GPS Positioning

<table>
<thead>
<tr>
<th>ORBIT TYPE</th>
<th>AVAILABILITY</th>
<th>RMS ACCURACY OF:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>ORBITS (m)</td>
</tr>
<tr>
<td>Broadcast (BRDC)</td>
<td>Real-time</td>
<td>5-8</td>
</tr>
<tr>
<td>IGS Predicted (IGP)</td>
<td>0.5h before start of day</td>
<td>0.5</td>
</tr>
<tr>
<td>IGS Rapid (IGR)</td>
<td>22 hours after observations</td>
<td>0.1</td>
</tr>
<tr>
<td>IGS Final (IGS)</td>
<td>8-10 days after observations</td>
<td>0.05</td>
</tr>
<tr>
<td>JPL Hi-rate (JPL)</td>
<td>~ 12 days after observations</td>
<td>0.05</td>
</tr>
<tr>
<td>IGS Ultra-rapid (IGU)</td>
<td>0.5 - 2 hours before start of day - various</td>
<td>Pred: 0.5 - 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fit: 0.2 - 0.4</td>
</tr>
</tbody>
</table>

**Table 2-1** Availability and Relative Accuracies of Satellite Ephemerides and Clock Correction Products Available to Civilian Users

[after IGS, (1997); Zumberge, (1997); Martin-Mur et al, (2000); AIUB, (2000a)]

As displayed in Table 2-2, the first four IGS products are listed at 15 minute intervals and available over the Internet, mainly for use in geodynamic scenarios where this low observation rate is sufficient for static applications. However, when precise products are applied to kinematic applications where a high one-second data rate is commonplace, the influence of SA becomes significant as its period of a few minutes (cf. §2.1.1) is less than the 15 minute interval. The increase in kinematic applications requiring positions with centimetric precisions was the major motivation for the determination of high-rate satellite clock estimates.

Ultra-rapid products containing the precise satellite orbit and clock estimates are computed by several parties most of whom are Analysis Centres for the IGS [IGS, (2000)]. These groups include NASA’s Jet Propulsion Laboratory (JPL) [Zumberge, (1997); Kouba, (2000)] and Natural Resources Canada (NRCan) [Lachapelle et al, (1996a)]. Researchers at JPL have determined ultra-rapid product files containing post-processed estimates of satellite positions and clock estimates at 30 second intervals, a rate higher than...
GPS and Offshore Positioning

the suspected 106 second autocorrelation[^1] time of SA [Zumberge et al, (1998); Kouba, (2000)]. When applied to code pseudorange measurements from a single receiver, use of these ultra-rapid products allows kinematic post-processing of single-receiver high-rate GPS data with decimetre-level accuracy or better. This is because of the information they contain about the high-frequency variations in the satellite clocks due to SA-dither, the exact level of improvement is dependent on the GPS receiver quality. Zumberge [1997] states that at typical levels of SA in 1997, these 30 second GPS clocks could be interpolated to intermediate times with about 7 centimetre RMS interpolation error.

These ultra-rapid products contain two orbit models; the first is a post-processed orbit fit to the actual observed satellite positions, and the second is a predicted orbit that contains extrapolated orbit and satellite clock estimates [Ray, (2000)]. The JPL ultra-rapid orbits, available at 0200 and 1400 hours every day, contain a 24 hour orbit fit and a 24 hour orbit prediction [Martin-Mur et al, (2000)]. The GeoForschungZentrum (GFZ) Analysis Centre provides similar coverage ultra-rapid orbits at similar timings [Gendt, (1999)] whilst Scripps Oceanography Institute (SIO) provide orbits incorporating 24 hour fit and 12 hour predicted orbits at ~0050 and ~1250 GMT every day [Fang et al, (1999)]. An example of the ultra-rapid clock accuracy is given by CODE in their ‘Statistics for the IGS Ultra Rapid Orbit Combination’ transcript published daily on their website [AIUB-CODE, (2000a)]. For the 11th November 2000, they state that the range of standard deviations for the clock elements as calculated by three contributing IGS AC estimates is between 9 and 27 nanoseconds RMS [AIUB-CODE, (2000a)].

The removal of SA is expected to have a significant impact on precise point positioning with fixed IGS orbits and clocks, as well for satellite clock and data interpolation activities. With no SA, it should now be possible to interpolate the IGS satellite clocks, currently sampled at 15 minute intervals, at or below the 20 centimetre precision level. Consequently, this would allow precise navigation at any instant at this (20 centimetre) precision level with IGS orbits/clocks fixed [Kouba, (2000)]. Kouba [2000] also states that the navigation precision of better than 10 centimetres with fixed IGS orbits and clocks is being demonstrated daily and weekly in the summaries of the IGS Rapid (IGR) and Final (IGS) orbit combinations, respectively (at the 15 min sampling intervals of SP3 orbit files). These ultra rapid products (IGUs) became official IGS products on 6th November 2000 (Day of Year 311) according to [Springer, (2000)].

[^1]: Autocorrelation is defined as the correlation of a variable with itself over successive time intervals.
The main assumption of differential positioning is that the errors on a GPS satellite measurement as seen by two nearby receivers are almost identical when considering the approximate 20,000 kilometres altitude of the satellite above the receivers. An error in the broadcast satellite position will have very little effect on the relative co-ordinates of a mobile receiver. In the worst case, a 20 metre satellite position error will induce a 1 centimetre error in a baseline of 20 kilometres [Tiberius and de Jonge, (1995)] and a 5 metre satellite position error would therefore induce an error of 0.25 centimetres in the same baseline. Extrapolating this orbit error of 5 metres over 1,000 kilometres would yield a baseline error of 15 centimetres, well within the noise levels associated with code differential positioning.

**Discussion: Is DGPS still needed now that SA is gone?**

This research was started at a time when SA was applied to measurements from GPS satellites and the use of a differential positioning technique was the only means of mitigating this large error from the measurements and ultimately, the position solutions. The focus of the research in this thesis is directed towards assessing and improving the quality of the position results from a differential GPS positioning approach, however the question is raised as to whether differential positioning methods are needed now that SA has been removed.

Some users reliant upon differential GPS for navigation and positioning applications at sea or on land, may believe that with the termination of SA, there is no longer any need to subscribe to a service provider for their differential corrections. They may have even believed that SA was the only considerable error associated with GPS (apart from ill-informed users) and that its termination means GPS is now ‘error-free’. In reality, the removal of SA has only served to promote the second largest error source within GPS, that of ionospheric delay, to the primary incumbent error source. Identical position solutions could be expected with data not subject to SA.

The answer to the discussion question is dependent on the accuracy requirement specific to each individual user. If a critical safety-of-life application is being executed, then some form of differential GPS service is required to ensure that the higher accuracies (1-3 metres) are obtained with the commensurate level of integrity and reliability. Those requiring sub-metre level positioning need some form of carrier-phase aided DGPS, to achieve that level of positioning with confidence. The asset / facility management community will now benefit from the ~20 metres plan accuracy that the civilian C/A-code
can afford them without the need for costly augmentation services. In fact, the performance level of SPS should now be comparable to PPS excluding the military benefits of decrypted P-code measurements (cf. §1.6 and 2.1.1.2).

The transmission of corrections to the user in a manner that ensured that the Age of Correction did not exceed 10 seconds (cf. §2.2.1) meant that there was a considerable workload put on the communications channels. Now that SA is gone, the remaining errors, mostly atmosphere and satellite orbits, change much more slowly so that a range rate correction will not 'age' for a few minutes. Because this means that the validity of a differential correction will not deteriorate as rapidly, the latency of the corrections can increase. This in turn will lead to a relaxation on the packet size that the communication channels have to process, thus leading to smaller bandwidth requirements and ultimately lower costs to the subscribers. Multipath interference does not feature within these circumstances because of its distance-independent status.

Users of real-time phase differential systems will be affected in that they may notice a slight improvement in positioning repeatability. Real-time positioning usually involves a prediction of the reference station data and the rate of change of its measurements over the lag time taken for the creation and transmission of the differential corrections. As with DGPS, there is a time at which the predictions are no longer valid in terms of their accuracy with respect to the real-time corrections (cf. §2.2.1). Hill and Moore [2000] believe that a few centimetres of position errors could be attributed to such prediction errors, so now with SA gone, this could be realised as an improvement of 1-2 centimetres in position. This effect would not be noticeable in post-processing activities.

Given the free bandwidth now associated with differential corrections, it may be feasible that new applications could arise for differential positioning that can withstand the lower rate of corrections. It should then be equally feasible that the application's relevant data streams occupy the spare bandwidth especially given lower communication charges. Examples of these new applications and their data streams include RTK-style ionospheric corrections, dual-frequency observations and regional ionospheric models [Patrick and Webb, (2000)].

At the time of writing, it is too soon to quantify the full impact of the termination of SA on a single receiver user in terms of position, velocity and timing applications. It is hoped that an objective viewpoint will be provided within the SPS accuracy statements in the next Federal Radionavigation Plan (FRP), Version 3, due for release in November 2000 [Casswell, (2000)].
2.1.3 Signal Propagation due to Atmospheric Effects

For most of its journey, the GPS signal travels in (essentially) a vacuum at the speed of light. However as the radio signal traverses two particular bands of media comprising the Earth’s atmosphere, the ionosphere and troposphere, it is subjected to both refraction and changes of speed along its propagation path. The combination of these two effects introduces uncertainties in the signal’s time of arrival at the receiver.

Estimates of the ionosphere's thickness span a wide range of values according to different references depending on starting values as shown in Table 2-3. Such a varied range of values exists between references due to the difficulty in defining the ionosphere’s upper boundary, as the electron distribution thins with increasing altitude.

<table>
<thead>
<tr>
<th>REFERENCE:</th>
<th>THICKNESS (km)</th>
<th>ALTITUDE (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Klobuchar [1986]</td>
<td>350</td>
<td>50 – 400</td>
</tr>
<tr>
<td>Leick [1995]</td>
<td>1450</td>
<td>50 – 1500</td>
</tr>
<tr>
<td>Langley [1996]</td>
<td>950 +</td>
<td>50 – 1000 +</td>
</tr>
<tr>
<td>Johansson [1997]</td>
<td>950</td>
<td>50 – 1000</td>
</tr>
</tbody>
</table>

Table 2-2 Estimates of the Ionosphere’s thickness and range from different References

The acknowledged location of the troposphere within adjacent layers of the atmosphere, as Figure 2-2, is also subject to different interpretation between references.
2.1.3.1 The Ionosphere

This is a band of charged particles spanning the atmosphere between 80 and 1,000 kilometres above the Earth’s surface [Langley, (1996)]. Free electrons released as a result of ionising radiation, primarily solar radiation and x-ray emissions, create a dispersive medium with respect to the GPS signal. Consequently the GPS code measurements are delayed whilst the carrier-phase measurements are advanced (cf. §1.6.2). This delay ultimately results in code pseudoranges being too long and phase pseudoranges too short compared to the true geometric range between the satellite and receiver. For both observable types, the deviation from the true range is of the same magnitude.

The magnitude of the ionospheric delay, $l$, is dependent on the signals’ frequency and its path length through the ionosphere, which in turn, is proportional to the total electron count (TEC) along the signal path. TEC values are dependent on several periodic factors such as the time of day, solar activity, the zenith angle, the magnetic latitude and the season [Johansson, (1997)]. TEC is highly variable, both spatially and temporally whereby the dominant variability is diurnal peaking at around 1400 local time when the maximum ionisation due to incident solar radiation [Langley, (2000)].

Typically values of vertical TEC (vTEC) for mid-latitude sites during day and night are $10^{18}$ m$^{-2}$ and $10^{17}$ m$^{-2}$ respectively. However these daytime values can be exceeded by a factor of 2 or more, especially in the equatorial regions. TEC also varies seasonally with higher values during the summer [Langley, (2000)]. For both code and phase observations,
the outcome of differential TEC estimate errors described above, is a shortening of the receiver pair baseline proportional to both the TEC and baseline length [Langley, (2000)]. When the satellite is in the local zenith, the path length is shortest and therefore so is the ionospheric delay that can range from decimetres up to 10-20 metres. Conversely, for a longer path length such as when the satellite is near the horizon, the signal is being advanced more and the delay can reach 60-70 metres [Langley, (1996)].

2.1.3.2 Modelling the Ionospheric Delay

There are several methods with which these ionospheric delays can be modelled. With dual-frequency data, the first-order ionospheric corrections can be computed and then applied independently given the equal magnitude delay and advance affected by the ionospheric refraction on the code and carrier-phase respectively [Grejner-Brzezinka et al, (1998)].

When measurements are made on only one carrier-frequency, alternate means of correcting the ionospheric bias must be considered. One approach is to simply ignore the effect, as is done for short baseline differential positioning whereby double-differencing removes that component of ionospheric range error common to measurements at both stations. For baselines up to 10 kilometres, the residual effects are generally at or below the millimetre level during times of benign ionospheric activity [Tiberius and de Jonge, (1995)]. However around the solar cycle peak every 11 years, increased and erratic ionospheric activity can greatly increase these residual errors [Langley, (2000)]. Christie et al [1999] cite an example where a differential ionospheric error of 0.5 metres was seen over a 9 kilometre baseline.

The use of carrier-phase filtering techniques with single-frequency measurements can also afford an estimate of the single-frequency ionospheric delay [Lachapelle et al, (1989); Hwang et al, (1999)]. After taking the difference between the code range and carrier-phase range at an epoch, the remainder contains, along with some multipath terms, twice the ionospheric delay (cf. Equations 1-3 and 1-4).

Another approach to the correction of ionospheric biases is with the use of empirical (predictive) models. The broadcast navigation message (cf. §1.2.3), for example, contains the parameters of a simple prediction model [Klobuchar, (1986)]. This algorithm attempts to approximate the entire vertical ionospheric refraction to yield the vertical time delay for the code measurements. These delays are then transformed into the actual geometric path travelled using a mapping function such as the basic cosecant function [Hofmann-
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Langley states in [2000] that the broadcast model performs as well as some of the more sophisticated models in part because the upper portion of the atmosphere is inaccurately represented. All empirical ionospheric models are driven by estimates of previous solar activity. The broadcast model parameters are based on a running average of the observed solar flux values over the previous five days. However it must also be noted that the diurnal variability of the ionosphere is not well correlated to flux variations [Langley, (2000)].

Ashkenazi et al [1997] suggested two further options for single-frequency users. The first is a simple method which bundles together the ionospheric and tropospheric delays as one single atmospheric error, which can then be estimated as part of the orbit determination process. Alternatively, the more rigorous Single Frequency Ionospheric Estimation Approach (SFIEA) can be taken in which a regional ionospheric delay model is derived from the reference station network and optimized Klobuchar parameters.

The latter method is of primary benefit to single-frequency users who, as they are unable to compute and apply the necessary corrections, require some form of ionospheric delay correction model relayed from the reference stations [(Ashkenazi et al, (1993)]. Results for these studies showed that the ionospheric delays computed from the broadcast Klobuchar parameters were a poor fit against the truth (derived using dual-frequency carrier-phase measurements) and not surprisingly, the dual-frequency approach using the optimised Klobuchar-like parameters yielded the best fit. The RMS of the recovered ionospheric delays were 0.2, 0.8 and 4.2 metres for the dual-frequency, refined single-frequency approach and the broadcast Klobuchar parameters respectively. It is stated that this refined single-frequency approach is almost equivalent to dual-frequency estimation when applied in real-time positioning. Overall, the single-frequency WADGPS algorithm used by Ashkenazi et al [1997] yielded plan and height accuracies (95%) of 5.0 and 6.1 metres respectively.

2.1.3.3 The Troposphere

The troposphere is the band of neutral medium considered to extend from the ground surface to an altitude of not less than 9 kilometres over the poles to 16 kilometres over the Equator [Langley, (1996)]. While passing through this non-dispersive (for radio
frequencies) band of air and water vapour, the GPS signal is slowed down to a varying extent depending on satellite elevation angle and the troposphere's temperature and relative humidity. The magnitude of the tropospheric delay also varies with the height of the user and the type of terrain below the signal path [Leick, (1995)].

### 2.1.3.4 Estimation of Tropospheric Errors

In the zenith direction, the tropospheric time delay $Z$ results in an increase in measured range of about 2.4 metres. The delay grows with decreasing elevation angle (from the zenith) such that it reaches about 9.3 metres for a satellite with elevation angle of $15^\circ$ [Johansson, (1997)] and 20-28 metres at $5^\circ$ [Leick, (1995)]. Johansson [1997] cites a useful rule of thumb from Brunner and Welsch [1993] that for the effect of mismodelling the tropospheric zenith delay by 1 centimetre can result in a vertical position error of 3 centimetres. Consequently, the effects of tropospheric propagation must be taken seriously when performing high accuracy GPS positioning, in particular for the vertical component.

Hopfield [1969] separated the refractivity $N^{\text{trop}}$ into two components, a dry and a wet component:

$$N^{\text{trop}} = N^{\text{trop}}_d + N^{\text{trop}}_w$$  \hspace{1cm} \text{Equation 2-1}

The dry, or hydrostatic, component is caused by induced dipoles in the molecules, and the wet component is caused by the permanent dipole moment of water vapour [Johansson, (1997)]. The dry component is normally about 2.3 metres in the zenith direction at sea level and can be estimated accurately using surface pressure measurements at the ground point. The wet component is smaller with a range in the zenith direction of 1 to 40 centimetres though is largely dependent on the whole troposphere in the region of the antenna.

Surface meteorological data can be recorded at the observation site to permit the modelling of the refractivities of both components with respect to altitude [Hopfield, (1969); Lanyi, (1984)]. Dry component models require atmospheric pressure and temperature whereas for the wet component, temperature and the partial pressure of water vapour are required. The dry component is generally deemed to invoke 90% of tropospheric refraction with the remaining 10% afforded by the wet [Langley, (1996)].
Modelling of the wet component is much more difficult because the distribution of water vapour cannot be predicted accurately given its high spatial and temporal variability [Langley, (1996)]. Hopfield assumed the same functional model for both the wet and dry components where the mean value of $h_w$ was 11 kilometres. Studies have yielded a range of values for $h_d$ and $h_w$ – unique values cannot be given because of the troposphere’s behavioural dependence on temperature and location, namely height, [Fell, (1980); Kanuiith, (1986)]. Kanuith [1986] suggests an effective height for the troposphere’s components as 40–45 kilometres for the dry component $h_d$, and 10-13 kilometres for the wet component $h_w$.

Several modifications have been made to these Hopfield modules including terms to apply a function to account for the line of sight from the satellite. The most basic mapping function is the cosecant function:

$$\frac{1}{\cos \theta} = \frac{1}{\cos (90 - z)}$$  \hspace{1cm} \text{Equation 2-2}$$

where $\theta$ and $z$ are the elevation and zenith angles respectively of the satellite relative to the observer’s horizon. Seeber [1993] presented formulae including an arbitrary elevation angle, and individual constants for the dry and wet components.

Another approach taken by Saastamoinen [1973] in which the refractivity of the dry and wet components are deduced from the standard gas laws along with a function of zenith angle, atmospheric pressure, temperature and partial pressure of water vapour (PWVP). The tropospheric components are usually written as the product of a zenith delay term (approximating the integral of the refractive index profile) in the vertical direction, and a mapping function that maps the increase in delay with decreasing elevation. Unlike the dry component, the wet component has high spatial and temporal variability [Langley, (1996)] which is being modelled, at least spatially, within the stochastic function detailed in §4.3.2.

The reason for estimating the tropospheric delay is that it is rarely predicted correctly, even with accurate surface measurements of pressure, temperature and relative humidity. A variety of tropospheric model profiles and mapping functions have been designed with the relation of the delay experienced by the signals propagating through the troposphere at arbitrary elevation angles. The most well-known are probably those contributions from Hopfield [1969] and Saastamoinen [1973] which have amassed a
considerable number of users since their inception. These empirical models show that, with surface measurements, the wet delay can be modelled to an accuracy of about 2-5 centimetres RMS in the zenith direction. Using the rule of thumb mentioned earlier by Johansson [1997], 5 centimetres of error in the zenith direction would transform into 15 centimetres of vertical position error.

Many different approaches have been pursued in modelling the tropospheric refraction caused by the wet component mainly due to the difficulty in modelling the water vapour at an observing site using standard surface meteorological instruments. Water vapour radiometers (WVR) calculate the wet path delay by measuring the sky brightness temperature using radiometric microwave observations along the signal path [Ruffini et al, (1999)]. Their incorporation into a reference station configuration can improve accuracy of the tropospheric path delays at that site although they are costly and can experience problems with low elevation angle satellites since the tropospheric path delay at the zenith is amplified by the mapping function [Lanyi, (1984)].

According to Johansson [1997], the measurement approach that will yield the most accurate estimate of the tropospheric zenith delay is a combination of surface meteorology, the use of water vapour radiometry equipment and finally, radiosonde techniques using weather balloons. These techniques can provide an estimate of the meteorology along the signal path. The collection of surface meteorology, in differential positioning tasks, is generally deemed unnecessary for the potential increase in accuracy it can afford. Residual tropospheric delays can also be modelled using sophisticated stochastic functions such as the random walk models and fractals discussed in Gregorius [1998].

**Mapping Functions**

The basic cosecant mapping function (cf. Equation 2-7) is based on the assumption that spherical constant-height surfaces\(^4\) can be approximated to plane surfaces. Of course it is known that the atmosphere is not of constant thickness, especially for lower elevation angles, and should not generally be classed as a plane surface. However, this constant-height approximation forms the basis of most tropospheric delay models, as it is reasonably accurate for high elevation angles and instances where there is a small degree of bending [Langley, (1997)]. The application of mapping functions which rely on the geometric

\(^4\) Also termed ‘laterally homogenous’ [Langley, (1996)]
strength of the observed GPS data, remains as one of the few means of accurately estimating the tropospheric delays [Collins, (1999)].

Janes et al found in their studies [Janes et al, (1991)] that one particular combination of algorithms would provide superior performance to other tropospheric delay models under most conditions. The hybrid algorithm comprised the explicit form of the Saastamoinen zenith delay expression with the Davis hydrostatic (dry), and the Goad and Goodman water vapour (wet) mapping functions, [Davis, (1986); Goad and Goodman, (1974)]. Estefan and Sovers later tested the 'best' empirical mapping functions; Lanyi, Davis et al, Ifadis, Herring and Neill [1996] mapping functions against a look-up table for the Chao model [Estefan and Sover, (1994), and references therein]. They concluded that no one 'best' tropospheric mapping function exists for every application and all ranges of elevation angles; however, the Lanyi and Neill mapping functions were then implemented in GPS processing routines at JPL [Janes et al, (1991)].

In the actual atmosphere, the decorrelation of the signal paths for a network of stations is governed by lateral gradients in atmospheric pressure, temperature and PWVP, such as effected by differences in station elevation [Langley, (1996)]. This is an important point given that the thickness of the troposphere is not fully definable. Beutler et al [1988] showed that, for local GPS networks, the effect of the differential troposphere invokes a relative height error between stations. Johnston and Toor [2000] suggest a similar bias effected by differential ionosphere errors (sometimes termed differential ionospheric gradients).

2.1.4 Multipath Interference

The phenomenon of multipath interference is encountered when transmitted GPS signals are reflected by objects or surfaces, near to the receiver, before entering the receiver (cf. Figure 2-3). As a result of interference with these reflections, the received signals have relative phase offsets. These additional phase offsets are dependent on the path lengths of the reflected signals and can affect both code pseudorange and carrier-phase measurements.
The impact of multipath interference is noticeable on carrier-phases albeit to a lesser extent than with the code measurements. The maximum theoretical multipath on the carrier frequencies L1 and L2 is equal to one-quarter of its wavelength, 90° [Seeber, (1993)]. The binary chipping rate of the code observable dictates the magnitude of the noise associated with it as seen with the reduced levels of noise and hence superior measurement accuracy of the precise P-code over the C/A-code. The chipping rate (or length of the code chip) limits the effective geometric range between the antenna and the reflecting surface.

Langley [1997] refers to work presented to the GPS Joint Program Office [JPO]:

- Multipath can cause both increases and decreases in measured pseudoranges.
- The theoretical maximum pseudorange error for P(Y)-code is about 15 metres, and due to its lower chipping factor (cf. §1.2), 150 metres for the C/A-code.
- Due to the GPS signal's coded-pulse nature, P(Y)-code receivers can discriminate against multipath signals delayed by more than 150 nanoseconds, a range of 45 metres, and 450 metres for C/A-code pseudoranges.
- Typical pseudorange errors show sinusoidal oscillations with periods of 6-10 minutes.

The occurrence of such extreme multipath errors on C/A-code is very rare. Rodgers [1992] noted that multipath could cause errors in the measurements as high as 15 centimetres for the L1 carrier and the order of 15-20 metres for the pseudoranges. These values correspond to the findings of Hill et al [1995] in that the pseudorange
multipath effects are approximately two orders of magnitude greater than those with carrier-phases. This makes the difference between the carrier-phase derived range and its commensurate code range useful for identifying and detecting multipath (as will be critical in the phase-filtering routines discussed in §3.3).

Studies by Martin [1978, 1980] and cited in Langley, [1996] assumed an error budget allocation for multipath of 1-3 metres for P-code measurements and 10-30 metres for C/A-code measurements; the order of magnitude difference accounts for the difference in chipping rate between these two measurement types. Langley [1997] states that multipath interference on a C/A-code pseudorange will generally be at the half-metre level in a benign environment, and 4-5 metres in a highly reflective environment. These findings culminate in a suggested multipath RMS range error budget of 1.2 metres [Langley, (1997)]. The noted difference between these two references can be attributed to the improvement in receiver hardware and signal processing techniques as discussed later.

For both observable types, the multipath interference is cyclic in behaviour and takes the form of a sine wave, whilst its size is dependent on the distance to the reflecting object. As there is a greater demand for high-accuracy solutions within static positioning applications, more resources are directed towards the reduction of multipath at reference stations [Weill, (1997)].

**Multipath in a Kinematic Environment**

Given the immediate surroundings and dynamics of a GPS antenna on a survey vessel, the presence of multipath is inevitable and the prime objective should be its elimination, or at least its mitigation. The dynamics of an operating survey vessel, in particular its change in attitude and orientation, are essentially the same as a change in the satellite constellation relative to the user. This means that the multipath distances have the potential to vary considerably over a short time span, a concept acknowledged within this research. The resultant degradation from these reflected multipath signals depends on a number of factors including antenna design, receiver bandwidth and the type of correlator technology used in the receiver tracking loop [Cox et al, (2000)]. Accordingly, multipath interference can be a major limiting factor when trying to obtain high precision results with GPS. A number of multipath mitigation measures, suggested by researchers, can be

---

5 That difference in distance between the direct path and the reflected multipath signal.
Error Sources within Differential GPS Positioning

Error Sources within Differential GPS Positioning

considered in four categories; i) site location, ii) antenna choice, iii) hardware and iv) software.

Unfortunately it is not possible to implement all these factors within every GPS navigation system as nearly every vessel and reference station is unique. Costwise, the choice of antenna and its final operational location, is dictated by the positioning activity and operational resources. For example, the highest point on the superstructure of an offshore survey vessel would very likely already be home to other types of essential equipment including radio whip antenna, navigation lights and beacon equipment. The use of large choke-ring antennas are preferable for high-precision offshore positioning although their large bulky profiles means that they require secure fixing. The highest free space for mounting a GPS antenna may not be ideal in terms of affording it low multipath signatures as other operational priorities usually prevail.

The potential of radio interference from neighbouring instrumentation is another major issue as is the volume of available space in which a GPS antenna could be located. Ward and Johannessen [1996] found that with 'judicious installation of the GPS antenna relative to other system antenna, the likelihood of mutual interference can be minimised'.

Most GPS manufacturers offer a range of marine antennas designed specifically for offshore surveying use. These lightweight antennas are cheap compared to choke-ring and geodetic antenna types, and have small profiles although they generally lack an external groundplane. The lesser quality of their performance, given the vehicle dynamics, is most likely absorbed into the noise of the code pseudoranges however multipath can still be significant. Use of these marine antennas warrants the implementation of multipath signal processing techniques which would be more practical and perhaps more efficient than an antenna model upgrade.

Distant reflectors typically cause high-frequency multipath at a receiver although some of this can be mitigated by special correlator technologies as referred to by Ray et al [1999b]. In cases where the antenna is mounted less than 2 metres above the ground, typical of many static geodetic and RTK applications, any nearby reflectors can instigate low-frequency multipath. The multipath distance here is at most 4 metres and currently cannot be mitigated successfully by signal processing techniques, receiver correlator technologies or carrier-phase filtering [Ray et al, (1999b)]. Signal processing techniques can mitigate the effect of multipath provided the multipath distance is more than 10 metres [Javad, (1999)]. Any multipath reflected from the ground or objects beneath the antenna can be mitigated by specialised choke-ring antenna. Unfortunately, these antennas cannot
successfully mitigate those multipath signals from high-elevation satellites, or those reflected off nearby tall obstructions, from entering the antenna. Therefore the consideration of the antenna site location, with regards to nearby obstructions, is of major importance.

Smooth reflective surfaces such as water, sheet-metal and tarmac are generally regarded as being especially conducive to multipath propagation [Lachapelle et al, (1989)]. Such surfaces are commonplace on offshore survey vessels, in particular around the aft-deck areas.

**Multipath Templates - Long-term Multipath Signal Observation**

Some groups have investigated the use of multipath templates as means of characterising the long-term effects of significant multipath interference at individual antenna locations [Ray et al, (1999b); Han and Rizos, (2000)]. This method relies upon the assumption that the GPS satellites are in a deep 2:1 resonance with the rotation of the Earth with respect to inertial space [Beutler, (1996)]. That is, the geometry of the satellites is such that they pass over the same portion of the sky each day albeit earlier by 4 minutes.

By analysing the pseudorange and/or carrier-phase measurements for GPS satellites over several consecutive days, it is possible to create, for a specific location, a model of multipath activity identifying both the direct and reflected signals [Ray et al, (1999b)]. Using these template coefficients, it is then possible to remove the effects of the reflected signals from the observed measurements [De Jong, (1999a); Han and Rizos, (2000)]. Because of the long-term observations required by this specialised technique, the execution of multipath audits, and subsequently the application of multipath templates, is limited to important reference sites, e.g., Racal Skyfix DGPS base stations [Racal Survey, (1999)].

The sizes of the multipath errors are a function of the level and phase of the reflected multipath signal relative to the direct signal. Stronger reflections result in greater distortion of the correlator function and increased errors, namely a lower C/N₀ value. Code pseudoranges, carrier-phase and signal-to-noise (C/N₀) measurements are all affected by multipath interference and exhibit high spatial correlation over a small area. [Ray et al, (1999b)]. Breivik et al [1997] were able to exploit the strong correlations between C/N₀ measurements and multipath errors to reduce both the standard deviations of errors and maximum errors in DGPS positioning by 40–50%. Permanent reference stations often show multipath patterns with very long wavelengths from nearby reflectors [Euler and Ziegler, (2000)].
Advances in antenna design have led to, amongst other features, the design of special ground-planes that prevent reflected signals entering the underside of the antenna. Groundplanes of radio-frequency (RF) absorbing foams have also been successfully tested as means of external multipath countermeasures at the reference receiver [Lachapelle et al, (1989); Jaldehag et al, (1996); Zhdanov et al, (1999)]. Jaldehag et al [1996] confirmed that the size of vertical error components for baseline solutions depend strongly on the minimum elevation angle. Errors yielded as a result of lower cut-off angles are due, in significant part, to signal scattering at the baseline antenna. Subsequent studies found that this scattering could be reduced, and thus the vertical components improved, through the positioning of microwave-absorbing materials in the void beneath the antenna and above its mount – tripod, pillar plate or such-like.


**Hardware and Signal Processing Techniques**

Receiver hardware designs have evolved to include several new multipath mitigation technologies. One example is the use of narrow correlator cards that estimate the distortion on a signal due to multipath and then attempt to remove it from the correlation function. This function allows the correlation of a single tracked PRN signal to be measured simultaneously at a number of points, from which the magnitude of the deviation between the observed signal and a receiver-generated signal is indicative of the prevalent multipath delay [Cox et al, (2000)].

Studies by Cox et al [2000] with test data run through a narrow correlator simulation, show that multipath-induced $C/N_0$ changes are correlated with both the multipath pseudorange and carrier-phase errors. Subsequently, a very similar correlation trend was shown between the $C/N_0$ and the calculated pseudorange multipath error for a WAAS\(^6\) receiver under real operating conditions. It is these relationships that have led to the development of receiver processing algorithms which exploit this correlation between multipath error and $C/N_0$ in an attempt to model, and correct for, multipath induced range.

\(^6\) Wide-Area Augmentation System – cf. §2.2.3.2
and phase errors [Comp and Axelrad, (1996)]. Understandably smaller than code errors because of their shorter wavelength, these carrier-phase errors were still deemed significant enough to invoke phase ambiguity resolution problems for kinematic applications.

Solutions for this are to use advanced delay locking loop (DLL) technologies [Lyusin, (1999)] such as the multipath estimating DLL [Cox et al, (2000)] which permits more accurate determination of the correlation function peak caused by multipath as well as improving levels of receiver measurement noise. In tests of their prototype synthesised multipath strobe DLL correlator, Zhdanov et al [1999] used a moving average of 50 seconds to smooth C/A-code pseudoranges and gained a considerable improvement in receiver performance. On static DGPS tests, the improvement in accuracy was noted as over a factor of 2.

Receiver manufacturers are combining hardware technologies with advanced signal processing algorithms to improve the reduction of multipath and receiver noise biases. Two commercial examples of such fusion technologies can be found within Trimble’s Everest™ Multipath Rejection Technology [Trimble, (1996)], and Leica’s ClearTrak™ GPS Receiver Technology [Hatch et al, (1997); Stansell and Maenpa, (1999)].

Generally multipath is very difficult to model as it is both site and frequency dependent, however its influence can be estimated through a combination of L1 and L2 code, and carrier-phase measurements [Langley, (1993); Blewitt, (1998)]. The maximum effect of multipath on phase measurements occurs for a quarter-wavelength (90°) equating to about 5 centimetres on L1 [Seeber, (1993)]. If linear phase combinations are used, then this value will be raised.

<table>
<thead>
<tr>
<th>OBSERVABLE</th>
<th>MAXIMUM THEORETICAL RANGE ERROR DUE TO MULTIPATH (m)</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>C/A-code on L1</td>
<td>150</td>
<td>Equal to ½ wavelength of L1</td>
</tr>
<tr>
<td>P-code on L1</td>
<td>15</td>
<td>Equal to ½ wavelength of L1</td>
</tr>
<tr>
<td>P-code on L2</td>
<td>19</td>
<td>Equal to ½ wavelength of L2</td>
</tr>
<tr>
<td>Phase on L1</td>
<td>0.05</td>
<td>¼ wavelength of L1 (0.19 metres)</td>
</tr>
<tr>
<td>Phase on L2</td>
<td>0.06</td>
<td>¼ wavelength of L2 (0.24 metres)</td>
</tr>
</tbody>
</table>

Table 2-3 Maximum Theoretical Magnitudes of Multipath Interference on GPS Observables [after Seeber, (1993); Langley, (1996)]
A less complex means of reducing the effects of multipath is through the application of carrier-phase filtered code pseudoranges. Given the negligible multipath effects on phases compared to code pseudoranges (cf. Table 2-4), the rate of change of range on the more precise phase pseudoranges can be used to filter any major multipath on the codes. This method is suitable for instances where dual-frequency code and phase data is not available, either due to recording problems or the availability of single-frequency equipment only. Given that this research is aimed primarily at single-frequency applications, this technique will be considered as the means of reducing the code multipath and is discussed further in Chapter 3.

2.1.5 Receiver Clocks and Noise

In general, it is assumed that the differencing process eliminates any receiver clock error along with any satellite clock errors present on the observed pseudoranges. The cheaper oscillators used within GPS receivers are less stable than those on-board the GPS satellites. Most GPS receivers regularly adjust their internal clocks using the code observations (supposedly during the correlation process) and can thus keep the clock error smaller than an arbitrary threshold [Gurtner, (2000)]. In this way, the receiver clock is approximately synchronized to the GPS system time [Misra, (1996)]. The receiver clock’s offset from GPS time, $t_A$, (cf. Equation A-3) will be calculated during the receiver’s single point position computation carried out at each epoch as part of the pre-processing activities.

Receiver noise, or jitter, is the term used to cover any measurement process error occurring within the receiver. These errors are dependent on a number of factors: antenna design, the method used for analogue to digital conversion, the signal correlation processes, and the code tracking loops and bandwidths [Pratt, (1992)].

Receiver measurement noise is proportional to the square root of the correlator spacing in chips yet inversely proportional to the square of the signal-to-noise ($C/N_0$) ratio [Cox et al, (2000)]. A high $C/N_0$ value on a GPS signal is a desirable quality, the higher the ratio, the better the receiver will perform in rapid static and kinematic applications [Leick, (1995)]. $C/N_0$ has a direct bearing on the precision of a receiver’s pseudorange and carrier-phase observations as mentioned in §2.1.4. Receiver noise levels can be quantified as follows [Leick, (1995)].
GPS and Offshore Positioning

\[ \sigma_{\text{receiver - noise}} = \frac{\sqrt{\text{correlator spacing}}}{\sqrt{\text{signal - to - noise ratio}}} \quad \text{Equation 2-3} \]

With the implementation of carrier-phase filtering routines (cf. §3.3), then the pseudorange noise can be reduced, although Langley [1997] assigns an RMS value of 1.5 metres to the measurement noise on an average C/A-code receiver. However it must be noted that most receivers do not usually provide a record of the observed signal-to-noise ratio values for each measurement. Indeed there is provision, albeit minimal, within the RINEX standard [Gurtner, (2000)] for the signal-strength indicators in the form of low-resolution flags where 1 is the weakest and 9 corresponds to the maximum signal-strength.

**Receiver Clock Jumps**

At this point, the concept of receiver clock jumps must be mentioned. Some receivers are subject to deliberate millisecond jumps on their internal receiver clock oscillators which reflect the receiver’s attempts to keep the true instant of observation within ±1 millisecond of GPS time [Gurtner, (1999a)]. Ashtech’s Z-XII series are one example of geodetic receivers that enforce receiver clock jumps. The rate at which these jumps occur is largely dependent on their internal temperature and the stability of the clock oscillator. This has resulted in some parties incorporating external clock oscillators at reference sites to steer the receiver clocks [Chin et al, (1998); Merry, (1998)]. Of course if carrier-phase filtering or receivers that suffer from clock jumps are not used, then the likelihood of the phase-filter being reset is significantly reduced.

Although these clock jumps of 1 millisecond are actually clock errors, when working with combinations of the code and carrier-phase observables, they look like cycle slips. Because the receiver clock is needed to calculate the pseudoranges from the travel time value \( dT^r \), any error in this clock will propagate into a range error (cf. Equation A-3). If \( dT^r \) is in error by a millisecond then all ranges calculated by that receiver will be in error by a nominal 3000 metres (as calculated from the Time-Light equation). Subsequently the receiver and pre-processing software will map that error into its own estimate of the receiver clock error as seen in RINEX files.

The receiver's estimate of the carrier beat phase observable will not be affected as such because it has not 'lost lock' during this time period meaning that carrier-phase measurements are not affected by this phenomenon. On an epoch by epoch basis, these clock interruptions are not noticed within differential positioning activities, however they
become visible over consecutive epochs. Accordingly, routines involving the calculation of pseudorange and range rate corrections (cf. §5.2), and carrier-phase filtered pseudoranges are affected by these step functions.

2.1.6 Summary of GPS Errors

The errors present in GPS code measurements are now summarised in preparation for subsequent chapters, in which these findings will be incorporated within the proposed stochastic modelling algorithms.

All errors in GPS surveying are a combination of noise, bias and blunders. Noise and bias errors combine, resulting in typical RMS ranging errors of around 25 metres for each satellite using Langley's error budget values [1997] in the position solution, as shown in Table 2-4.

<table>
<thead>
<tr>
<th>TYPE</th>
<th>ERROR</th>
<th>TYPICAL RMS RANGE ERROR (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise</td>
<td>Pseudorandom code noise</td>
<td>≈ 1</td>
</tr>
<tr>
<td></td>
<td>Receiver noise</td>
<td>≈ 1.5</td>
</tr>
<tr>
<td>Biases</td>
<td>Selective Availability</td>
<td>≈ 24</td>
</tr>
<tr>
<td></td>
<td>Satellite clock errors</td>
<td>≈ 1</td>
</tr>
<tr>
<td></td>
<td>Ephemeris data</td>
<td>≈ 1 ~ 5</td>
</tr>
<tr>
<td></td>
<td>Unmodelled Ionospheric delays, $I$</td>
<td>≈ 7</td>
</tr>
<tr>
<td></td>
<td>Tropospheric delays, $Z$</td>
<td>≈ 1</td>
</tr>
<tr>
<td></td>
<td>Multipath (code pseudorange), $MP$</td>
<td>≈ 1.2</td>
</tr>
<tr>
<td>Blunders</td>
<td>GPS Control segment</td>
<td>Any size</td>
</tr>
<tr>
<td></td>
<td>Operator segment</td>
<td>Any size</td>
</tr>
<tr>
<td></td>
<td>Receiver errors</td>
<td>Any size</td>
</tr>
<tr>
<td>TOTAL RMS UERE (m)</td>
<td></td>
<td>25.3</td>
</tr>
</tbody>
</table>

Table 2-4 RMS UERE Budget Table for Noise, Biases and Blunders for a Stand-alone GPS Receiver [after Langley, (1997)]

Control segment blunders can include satellite clock errors that have been uncorrected. As far as a civilian user is concerned, these would most likely be absorbed
into the SA-dither component. Examples of control segment operator error could be the input of an incorrect antenna height or reference datum. Due to the computation of the receiver clock offset during GPS pre-processing tasks, no receiver clock component is included within the GPS Error Budget [Langley, (1997)].

Consider a single receiver observing four C/A-code pseudoranges whose RMS UEREs are 25.3 metres. Their orbital configuration yields an HDOP statistic of 2.0. The user can calculate the RMS horizontal positioning accuracy as 50.6 metres, or 101.2 metres (2drms\(^7\)), which is in close agreement to the 100 metres (2drms) figure within the published SPS specification [US DoT and DoD, (1999)] for PDOP = 6.

Values in the conservative error budget suggested by Van Dyke [2000] for C/A-code users given the termination of SA, as shown in Table 2-5, culminate in a system UERE of 12.5 metres.

<table>
<thead>
<tr>
<th>ERROR SOURCE</th>
<th>RMS ERROR (m) WITHOUT SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ionospheric Delay</td>
<td>10</td>
</tr>
<tr>
<td>Tropospheric Delay</td>
<td>2</td>
</tr>
<tr>
<td>Receiver Noise and Resolution</td>
<td>4.8</td>
</tr>
<tr>
<td>Multipath</td>
<td>1.2</td>
</tr>
<tr>
<td>Other (interchannel biases etc)</td>
<td>0.5</td>
</tr>
<tr>
<td>System UERE (rss)</td>
<td>12.5</td>
</tr>
</tbody>
</table>

Table 2-5 Conservative C/A-code User Pseudorange Error Budget [after Van Dyke, (2000)]

The values listed in Tables 2-4 and 2-5 should by no means be taken as correct for every GPS observation made. When assigning precisions to GPS observations, the sizes and natures of GPS errors depend on a number of factors relating to each error source, and to what extent they have been modelled to, if any. Examples of factors that can subjectively affect the final UERE estimates include: different receiver types being capable of recording observations to different levels of precision; the multipath conditions

\(^7\) 2drms (twice distance RMS) is defined as 'twice the RMS of the horizontal errors' [van Diggelen, (1998)]. In practice, any position fix obtained using the given system has a 95% probability of having a radial error equal to or less than the 2drms value expressed [US DoD and DoT, (1999)].
encountered at different antenna locations vary greatly; the antenna types used, and so on without even considering atmospheric delays.

For this reason, the errors are classified into like-groups based on their causes, effects and characteristics. Some groups, in their classifications, use different prefixes for these classes; Ashkenazi et al [1997] use position whilst Wanninger [1999] uses station. As there is some potential for confusion with this terminology, the prefix distance is used within this research to afford unambiguous definition of the errors.

**Distance-Independent Errors**

A distance-independent error is defined as one whose effect at one station will bear no (deliberate) resemblance to the magnitude of the same error type at a second observing station, as long as all observations to satellites are carried out simultaneously.

Sources of distance-independent errors are SA-dither, receiver noise and multipath (both code and carrier-phase). SA-dither affects all receivers to a similar extent and can be classed as a fast error in that it varies in the order of minutes. It is assumed that use of differential positioning techniques, for example the differencing concept as described in §3.1.2, eliminates the effects of SA-dither. Errors due to receiver noise are dictated by the quality of the observables and by thermal noises within the receiver.

Site conditions and obstructions near to an antenna can dictate the magnitude of multipath interference encountered. Multipath is fundamentally driven by satellite and receiver dynamics. Changes in these parameters correspond directly to multipath variations. If one considers a static receiver, the only dynamics effecting multipath are for satellite geometries that are slow relative to the user. Low receiver dynamics will infer some increase in multipath variations and obviously high receiver dynamics afford high-frequency multipath variations to a moving receiver. Therefore relatively speaking, multipath on static receivers is slow compared to dynamic receivers. There is no correlation between the multipath effects on an antenna pair unless they are located very close to each other [Ray et al, (1999b)].

These distance-independent errors, in general, are subject to high-frequency variations changing significantly in a short time. They can be said to exhibit high-temporal correlation, although not necessarily with one other. For example, as the effects of distance-independent errors on two receivers at opposite ends of a baseline 10 kilometres long are uncorrelated, then there will be no deliberate similarity between SA-dither, receiver noise, and multipath they experience.
Distance-Dependent Errors

These are defined as errors that increase in proportion to the separation distance between a pair of observing receivers. Within differential GPS positioning, the sources of distance-dependent errors are the satellite orbits, the ionosphere and troposphere [Wanninger, (1999)]. For receivers very close together, say less than 20 kilometres, the cumulative effect of these errors affects both receivers to similar amounts and there is said to be a very high level of correlation between the satellite common errors (orbits, ionosphere and troposphere). This assumption forms the conceptual basis of differential positioning.

It is worth noting that once a distance-dependent error source affecting two receivers becomes completely decorrelated, it can be said that the error source is now distance-independent and any change in the error would not affect another observing receiver. Examples of this include ionospheric and tropospheric path delays over longer distances. Generally, the troposphere becomes decorrelated over shorter distances than the ionosphere as the former is thinner, although the rate at which they decorrelate is dependent on the spatial variation within these bands of atmosphere. The actual thresholds at which these entities are completely decorrelated are not definitive, any values chosen are generally subjective to many factors as described earlier. The ionosphere is highly dependent on solar activity - time of day, season and geomagnetic latitude, whereas the troposphere depends on atmospheric conditions – temperature, pressure and relative humidity. Their residual biases need to be modelled stochastically, for example, by means of spatial decorrelation functions.

Satellite orbits are not included in this dual-category, in which a distance-dependent error can be reclassified as distance-independent. This is because the likelihood of their complete decorrelation during long-range kinematic positioning is impossible due to a lack of common in-view satellites over such an extreme baseline length. Conversely, when considering single receiver positioning, the concept of distance-dependent errors no longer exists, and the orbits and atmosphere are deemed as distance-independent errors. Their decorrelation spatially cannot be an issue whilst there is no second receiver and therefore no means of differential positioning.

As discussed earlier, these distance-dependent errors do change over time – the ionosphere peaks daily at 1400 LT, the troposphere has some inherent trends from seasons and weather fronts, and the satellite orbits contain prediction errors. All in all, the variations of distance-dependent errors are much slower compared to the distance-
independent errors of clocks, noise and multipath. All GPS error sources discussed here and methods for their mitigation are summarized in Table 2-6.

<table>
<thead>
<tr>
<th>ERROR CLASS</th>
<th>ERROR SOURCE</th>
<th>MITIGATION METHODS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance-Independent</td>
<td>Satellite Clocks (inc. SA-d)</td>
<td>Differencing Techniques</td>
</tr>
<tr>
<td></td>
<td>Multipath</td>
<td>Phase-filtering and Modelling</td>
</tr>
<tr>
<td></td>
<td>Receiver Noise</td>
<td>Modelling</td>
</tr>
<tr>
<td>Distance-Dependent</td>
<td>Satellite Orbits (inc. SA-e)</td>
<td>Differencing Techniques</td>
</tr>
<tr>
<td></td>
<td>Ionospheric Delays</td>
<td>Modelling and Mapping Function</td>
</tr>
<tr>
<td></td>
<td>Tropospheric Delays</td>
<td>Modelling and Mapping Function</td>
</tr>
<tr>
<td>Time-Dependent</td>
<td>All the above errors (to varying extents)</td>
<td>Various Modelling Techniques</td>
</tr>
</tbody>
</table>

Table 2-6 Categorisation of the Error Sources Affecting GPS Observations

2.2 OVERVIEW OF DIFFERENTIAL CODE POSITIONING METHODOLOGIES

The basic form of GPS using SPS (even with SA terminated) is not sufficient to meet the accuracy and reliability requirements of many positioning applications (cf. Table 1-2), and therefore some method of augmentation is needed to satisfy these requirements. Differential GPS positioning is one such method that can do this by reducing the common errors in GPS-derived positions by using additional data from a reference GPS receiver located at a known position (cf. §1.2).

Differential GPS enhances positioning through the use of differential corrections to the basic satellite measurements and is based upon accurate knowledge of the geographic location of one or more reference stations. The reference station tracks all satellites in view and computes corrections based on the combined effects of the errors affecting the measurements at its position. These differential corrections are then broadcast in near real-time to GPS users who subsequently incorporate them to improve their navigation solution.

This is the underlying concept for all differential positioning methodologies discussed in this section. Various differential GPS techniques are employed in applications depending on the accuracy required. The accuracy requirements of the application usually dictate which measurements are used and what algorithms are
employed, where the data processing is to be performed and whether real-time results are required. If they are, then a data link is also required. For applications without a real-time requirement, the data can be collected and processed later, a technique known as post-processing.

The usual means for transmission of these differential range corrections over a local area, are terrestrial radio systems operating on medium, high or very high frequencies depending on the particular system configuration and application. As the receiver separations increase, the means required to transmit these differential corrections becomes more critical (and thus expensive) and will usually involve satellite-based communications [Fugro, (2000); Racal Survey, (2000b)]. Figure 2-4 illustrates the principle of differential GPS along with two methods of transmission for the corrections.

![Figure 2-4 Principle of Differential GPS and Standard Methods of Correction Transmission - Satellite and Terrestrial Means](image)

### 2.2.1 Local-Area Differential GPS (LADGPS)

This is a form of differential positioning in which the user's equipment receives real-time pseudorange, and possibly carrier-phase, corrections from a reference receiver generally located within line of sight, hence the term *local-area* differential GPS. The corrections account for the combined effects of satellite ephemeris and clock errors (including the effects of SA), and depending on the models selected, possibly atmospheric propagation delay errors at the reference station. Assuming that these errors are also
Overview of Differential Code Positioning Methodologies

common to the measurements made by the user's receiver, the application of these scalar corrections to the users pseudoranges will result in more accurate position co-ordinates. This form of DGPS is referred to as conventional DGPS.

LADGPS essentially groups all GPS error sources into one bundled correction comprising the following error sources: satellite orbit errors, atmospheric delays based on an optimized model for the local area, and remaining errors, assumed to be clock-related for all satellites in view of the reference station. For a civil user of SPS GPS, differential corrections can improve navigation accuracy from 100 metres (2drms) to better than 7 metres (2drms) [US DoT and DoD, (1999)].

There are two approaches to the calculation of these corrections; one involves range corrections and the second involves position corrections. It should be noted that both approaches yield identical results only if the same satellites are observed at the reference and rover stations.

Range Correction Method

The most commonly used method is an all-in-view receiver at the reference site that receives signals from all visible satellites and corrects the pseudorange to each. Given that the reference receiver and satellite position are known, it is possible to calculate the geometric range to each satellite. By comparing the observed pseudorange and the calculated range, a correction term for each satellite can be determined. These corrections are then broadcast in near real-time and applied to the user's pseudorange measurements before the GPS solution is calculated by the receiver, resulting in a highly accurate differentially corrected navigation solution.

Position Correction Method

In this method, the reference receiver calculates its own navigation position using all satellites in view. The difference between this navigation position and known position is then transmitted to the user who applies this positional correction to their observed position to obtain an improved navigational solution. This method is also known as the ‘Block Shift’ method [US DoT and DoD, (1999)].

The robustness of the Position Correction method can deteriorate very quickly as the receiver separation increases. This is because the receivers may not be able to use all observed satellites to determine their position due to obstructions at each reference site location.
Comments on the Preferred Method of Differential Corrections

In the range correction method, the reference receiver calculates a code pseudorange correction (PRC) for each satellite as observed at the reference station and also that range correction’s rate of change (RRC). This latter term is used to model the time-varying characteristics of the range corrections over the Age of Correction period (AoC) as the correction is being transmitted, i.e. the latency introduced by the DGPS data link [Roberts and Cross, (1993)]. The code pseudorange correction for a satellite $i$ as seen by reference receiver $A$ at an arbitrary epoch $t$ is found by:

$$PRC_A^i = R_A^i - \rho_A^i + c\tau^i - c\tau_A$$

Equation 2-4

where

$PRC_A^i$: pseudorange correction to satellite $i$

$R_A^i$: geometric (true) distance between satellite $i$ and station $A$

$\rho_A^i$: observed pseudorange between satellite $i$ and station $A$

$c\tau^i$: clock correction for satellite $i$

$c\tau_A$: receiver clock correction at the reference station $A$

The range rate correction, RRC, can be evaluated from a time-series of range corrections by linear interpolation. For example, the range rate correction between the two pseudorange corrections at time $t_0$ and $t_1$ (cf. Equation 2-2) would be:

$$RRC^i = \frac{PRC^i(t_1) - PRC^i(t_0)}{t_1 - t_0}$$

Equation 2-5

where

$RRC^i$: range rate correction for satellite $i$

$\Delta t$: difference in time between the two epochs, $(t_1 - t_0)$

$PRC^i(t_1)$: pseudorange correction for satellite $i$ calculated at time $t_1$

$PRC^i(t_0)$: pseudorange correction for satellite $i$ calculated at time $t_0$

Once calculated, the pseudorange and range rate corrections are combined into one correction which is then added to the observed pseudorange at the mobile station. The total correction as determined at reference station $A$ to be applied at mobile station $B$ is:

$$P_B^s(t_1)_{corr} = P_B^s(t_1)_{raw} + [PRC_A^i(t_1) + RRC^i(\Delta t)]$$

Equation 2-6
Overview of Differential Code Positioning Methodologies

where \( P^i_{B}(t_1)_{corr} \): differentially corrected pseudorange determined at mobile station \( B \)

\( P^i_{B}(t_1)_{raw} \): raw observed pseudorange at mobile station \( B \) at epoch \( t_1 \)

\( PRC_A^i(t_1) \): pseudorange correction determined at reference station \( A \) at epoch \( t_1 \)

\( RRC^i \): range rate correction determined for satellite \( i \)

\( \Delta t \): difference in time between the two epochs, \( (t_1 - t_0) \)

The position correction technique is easier to execute but requires careful satellite selection whereas pseudorange corrections give higher precision at the expense of processing time [Hofmann-Wellenhof et al, (1994)]. Applying a position correction does not necessarily afford a better final solution as it is unlikely that the same satellites and GDOP statistics were used for the individual receivers' navigation solutions. The pseudorange and range rate correction method is preferred as it does not suffer from satellite visibility problems, as the position correction method inherently does. It is currently implemented in the majority of established differential services as detailed later in this chapter.

**Discussion: Considering the Age of Differential Corrections**

As there are some problems associated with transmitting all required information to the user within the limited radio bandwidths at the required rates [Roberts, (1996)], there is a time difference between the generation of a pseudorange correction and its application at the rover station. The greater this time difference, or latency, the less applicable are the differential corrections. Under levels of SA in 1995, use of AoCs of 9 seconds would yield no significant deterioration of position (at the R95\(^5\) level) [Roberts and Cross, (1993)]. This is comparable to the US DoT's [1999] study which showed that the differential pseudorange corrections accumulate an error of about 1 metre after 10 seconds [Hofmann-Wellenhof et al, (1994)]. It is suggested that the differential corrections should be applied at a rate usually less than 20 seconds [US DoT and DoD, (1999)] to prevent the introduction of significant latency-driven positioning errors.
Benefits of Local Area DGPS

With SA activated, achievable accuracies with DGPS are independent of whether SPS or PPS is used as the technique effectively eliminates satellite and receiver clock errors. Because a unique form of SA-dither is implemented onto each satellite clock, as two different receivers record a measurement from one particular satellite, the SA errors from that satellite's clock are effected equally onto both ranges and can be eliminated by cancelling or, differencing techniques (cf. §3.2). In fact, both the pseudoranges and carrier-phases are affected by SA in the same manner because the satellite's atomic clock controls the timing and frequency for the two codes and the carriers [Georgiadou and Doucet, (1990)]. DGPS pseudoranges are correlated because almost the same conditions affect the signals from two satellites to one receiver, i.e. ionosphere, troposphere and SA clock dither.

It must be noted however that the provision interval of differential corrections for real-time DGPS under PPS is longer than when using DGPS under SPS. This is because the rate of change of the nominal system errors (satellite geometry, ionospheric and tropospheric path delays) are slower than the rate of change of SA-dither, commonly regarded as a few minutes [Hofmann-Wellenhof et al, (1994)]. Multipath interference does not affect the rate of differential corrections as it is not eliminated by differential techniques, however its magnitude is increased by differential correction methods. Consequently, other means must be considered to reduce its influence on differential positioning.

GPS positioning using raw C/A-code measurements can yield accuracies at the mobile receiver of around 4-8 metres even when SA is activated. The use of carrier filtering techniques of C/A-code pseudoranges can further improve this figure to the 1-2 metre levels [Hofmann-Wellenhof et al, (1994)]. Use of the P-code observable as found on the L1 frequency can afford sub-metre accuracy. These figures are summarized in Table 2-7.

8 The R95 statistic is the radius of the circle which, when centred on the true position, will contain 95% of the computed positions [Van Diggelen, (1998)].
Overview of Differential Code Positioning Methodologies

<table>
<thead>
<tr>
<th>OBSERVABLE</th>
<th>STATION SEPARATION (km)</th>
<th>POSITIONAL ACCURACY (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw</td>
<td>10</td>
<td>4 - 8</td>
</tr>
<tr>
<td>Code Pseudorange</td>
<td>500</td>
<td>5 - 10</td>
</tr>
<tr>
<td>Carrier-filtered</td>
<td>10</td>
<td>0.3 - 3</td>
</tr>
<tr>
<td>Code Pseudoranges</td>
<td>500</td>
<td>4 - 7</td>
</tr>
<tr>
<td>Carrier-Phases</td>
<td>10</td>
<td>0.03 - 0.20</td>
</tr>
</tbody>
</table>

Table 2-7 Accuracies Achievable with Differential GPS  
[after Hofmann-Wellenhof et al, (1994)]

2.2.2 Network DGPS

With the inclusion of multiple reference stations, the technique of conventional DGPS was enhanced further in that it has been afforded greater coverage, integrity, reliability and increased modelling capabilities. Analysis into the user of multiple reference stations by Roberts and Cross [1993] showed that combining data from two or more stations could reduce the larger DGPS positioning errors and their standard deviations; however the results were not always better than those only from the best (usually nearest) reference station.

The majority of offshore and onshore service providers, have networks of reference stations located around operational areas. For example, Fugro’s Starfix system [Fugro, (2000)] comprises of 10 stations within Northern Europe providing a user with an acceptable level of redundancy should one or more stations cease operating. Not only does the network provider constantly monitor the corrections and their integrity but the user also possesses some freedom to use their own judgement and choose stations best suited to their application.

Two further examples of network DGPS schemes are the United States' Maritime and National DGPS Service Networks (MDGPS and NDGPS respectively). These two complementing services operated by the United States Coast Guard (USCG) are medium-frequency beacon-based augmentations to GPS. The National service is based on the architecture of the maritime system with the combined system consisting of two control stations and more than 120 remote broadcast sites [US DoT and DoD, (1999)]. Pseudorange and range-rate corrections are broadcast to land and maritime users on marine radiobeacon frequencies to improve accuracy and integrity of the standard GPS service.
The 1999 FRP [US DoT and DoD, (1999)] states that the predictable accuracy of this service within all established coverage areas is better than 10 metres (2-drms), and that the achievable accuracy degrades at an approximate rate of 1 metre per 150 kilometres distance from the broadcast site. These two systems are due to be combined into one NDGPS network with full single-frequency coverage that should be completed by the end of 2002. Total dual-frequency coverage is expected for this network over the continental United States, Alaska and Hawaii by the end of 2003 [Canny, (2000)].

### 2.2.3 Augmented Differential GPS Services

The main drawback of conventional local-area DGPS (cf. §2.2.1) with regards to long-distance high-accuracy positioning is that the positional accuracy of a user degrades as the reference-to-user separation increases and is therefore not suitable for longer distances. This spatial decorrelation of the errors with regards to distance is seen as a failing of the fundamental assumption behind differential positioning (cf. §2.2).

#### 2.2.3.1 Wide-Area Differential GPS (WADGPS)

Several groups have conducted research into wide-area augmentation systems which assume that the affecting error sources can be broken down into individual error components and modelled separately. The models differ in how they define their error component modelling routines. The fundamental difference between conventional and wide-area DGPS is that the latter involves the categorisation and modelling of the individual error sources present within the GPS observations.

Data is used from several reference stations to model the separate error components (ephemeris, clock and ionospheric delays) at a master control station (MCS) for application over a wide area. These error models are uploaded to the communications satellite by a near land earth station (NLES) for dissemination to the users [Ochieng et al, (1999)]. Once the observations have been evaluated with reference to selected error models, the user can form a vector of corrections for each error source that can then be applied to each range measurement used. The advantage of transmitting the error models as used to determine the vector correction, is its improved ability in capturing the spatial correlation of the error sources [Hansen, (1998)].

A well-established approach to overcome the errors associated with the broadcast ephemeris is to re-determine the satellite orbits by using a technique that involves the
Overview of Differential Code Positioning Methodologies


Taking into account the positioning accuracy requirements and financial constraints, users will possess either single- or dual-frequency receivers so the system must be designed to serve users on both frequencies. In particular, the number of frequencies available to a user's receiver significantly affects the validity of ionospheric modelling available to their navigation solution (cf. §2.1.3).

The code pseudorange measurements are generally used as the only observable in wide-area systems, as the problem of ambiguity resolution and cycle slips inherent with carrier-phase measurements make them unattractive for long-range real-time applications.

2.2.3.2 Wide-Area Augmentation System (WAAS)

WAAS is being developed by a group led by the US Federal Aviation Authority (FAA) to provide a safety-critical system to enhance the GPS-SPS over a wide geographical area spanning the 48 contiguous states, Hawaii, Puerto Rico and Alaska. This wide-area service is similar to WADGPS in that it provides WADGPS-style corrections for this area however the WAAS system also supports the transmission of integrity measurements. To achieve this, WAAS uses a number of geostationary (GEO) satellites to transmit additional ranging signals along with the necessary integrity data for the constellation’s GPS and GEO satellites [US DoT and DoD, (1999)]. WAAS is expected to be declared fully operational in late 2003.

Figure 2-5 provides a schematic of the WAAS system illustrating its main components [after JPL, (1998)].
In WAAS, a network of 29 precisely located GPS reference stations collect dual-frequency measurements of GPS pseudorange and pseudorange rate for all satellites in view, along with information pertaining to local meteorological conditions. This data is then processed to yield highly accurate ephemeris, ionospheric and tropospheric calibration models, and DGPS corrections for the broadcast satellite ephemeris and clock offsets (including the effects of SA). In WAAS, the GPS corrections and system integrity messages for both GPS and GEO satellites are relayed to civil users via a dedicated package on geostationary satellites. This relay technique also supports the delivery of an additional ranging signal, thereby increasing overall navigation system availability [US DoT and DoD, (1999)].

Most augmented DGPS networks acknowledge the two tiers of receiver capability, that is single or dual-frequency usage, and to cover this have devised algorithms that cover both user scenarios. Assuming the most basic network scenario of single-frequency receivers only - the key issue would be the modelling of ionospheric delays by use of code pseudoranges exclusively. Considering this L1-only scenario does not affect the routines for estimation of the ephemeris and clock-related errors with regards to dual-frequency observation approach. However the issues of atmospheric delay and its direct estimation are more critical given the absence of L2 observations. Research groups have come up with several different approaches to this single-frequency modelling problem.
Overview of Differential Code Positioning Methodologies

In the conventional implementation of wide-area systems, the assumption is made that the ionosphere can be approximated to a thin shell surrounding the Earth and subsequently represented by a rectangular grid in latitude and longitude. Using real-time data from the wide-area network, the respective algorithm estimates the equivalent vertical delay induced on a code pseudorange measurement due to the ionosphere for each grid point, along with a grid ionosphere vertical error (GIVE) relating to the confidence level of the estimated delays. The vertical error and confidence interval effectively summarise the ionospheric model error.

*Clock-related Errors*

Most wide-area research groups assume that if atmospheric delays have been modelled correctly, their effects will have been eliminated (or at least significantly mitigated) and any residual errors can be attributed to the satellite and receiver clocks [Ashkenazi et al., (1997)]. The main assumption of this being that any residual atmospheric and ephemeris errors will be very small compared with the true clock-related errors and can thus be absorbed into the clock biases without significant differential error [Petovello and Lachapelle, (2000)]. The total clock-related error within the GPS observations is distance-independent and consists of SA-dither, satellite clock errors and receiver clock errors. As an estimate of the receiver clock error is determined as part of the single point position computation, its effect is eliminated.

For conventional DGPS, all error corrections are grouped into one differential correction per satellite from a single reference station. For wide-area methodologies, e.g. WADGPS and its network of reference stations, this is not the case. To compute a single differential correction for each satellite, the observations from the different reference stations have to be grouped. To compute a single-difference receiver clock correction within a wide-area network, the observations for all reference stations have to be combined.

One reference station is assigned to act as a master clock and to provide the absolute reference for all remaining receiver clock errors that have been propagated through common view satellites around the network. The resulting differential corrections consist then of one benchmark receiver clock error, and an averaged satellite clock error for each satellite observed within the WADGPS network. This combined satellite clock correction is then broadcast to the user who corrects the pseudoranges accordingly before using them to carry out an independent point position computation to yield the user's own receiver
GPS and Offshore Positioning

clock error estimate. An important conceptual point is that the resulting clock error does not correspond to the user's own receiver clock error, but the sum of the reference station clock errors and the user clock error. As a result, less accurate and therefore inexpensive clocks can be used as reference station clocks without degrading the navigation accuracy [Ashkenazi et al, (1997)].

The algorithms for WADGPS and WAAS systems usually incorporate orbit determination routines permitting them to determine a more precise estimate of the satellite space vector (position and velocity) in place of the broadcast ephemeris. Using data from a fiducial network of reference stations (usually those in the WADGPS network), the orbit algorithm can compute a satellite force model for the entire constellation, consisting of all the forces acting on each individual GPS satellite. That is then used to provide a frequently updated satellite state vector to the users.

The accelerations yielded from these force models are then integrated as a function of time to give high-precision position and velocity estimates of the satellite in near real-time. Using their full dynamic orbit determination process, along with double-differenced raw L1 pseudoranges, Ashkenazi et al [1997] were able to recover satellite orbits to an accuracy, as truthed by an IGS final ephemeris, of 10 metres with the dual-frequency approach, and 12 metres by their refined single-frequency approach.

To benefit from the WAAS service, a user must possess a receiver capable of receiving both GPS and WAAS signals, the latter as transmitted from the geostationary satellites such as INMARSAT. The user's WAAS receiver is required to process:

- integrity data to ensure that the GPS and GEO satellites being used are providing in-tolerance navigation data,
- the differential correction and ionospheric information data to improve the accuracy of the user's position solution, and
- the ranging data from one or more of the GEO satellites for position determination to improve availability and continuity.

As mentioned earlier, one of the main driving forces behind WAAS was the FAA who recognised the benefits that GPS could have for aircraft navigation in the restricted airspace corridors. Originally GPS positioning was sought for the en-route and terminal

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9 A fiducial network consists of a number of geodetic stations whose relative co-ordinates are known to a very high order of accuracy.
Overview of Differential Code Positioning Methodologies

phases of flight but it is soon to become a core component of both non-precision and precision aircraft landing systems [Keedy, (1999)]. The limitations of GPS in terms of coverage and reliability for high-risk aircraft guidance activities have been appreciated, and for that reason alone, have included the GEO satellite constellation to provide additional independent ranging signals to the GPS systems. This will afford much higher levels of navigation reliability, metre-level vertical accuracies and safety-of-life integrity than WADGPS alone [Ceva et al, (1997) and Hansen, (1998)]. Accuracies for the WAAS are currently based on aviation requirements, for the Category I precision approach of flight, horizontal accuracies are guaranteed at 7.6 metres 95% of the time [US DoT and DoD, (1999)]. This is done with an overall reliability approaching 100% with no single point of failure.

2.2.3.3 Research into other Satellite-based Augmentation Systems (SBAS)

Similar research is being conducted into the European Geostationary Navigation Overlay Service (EGNOS), and Japan's Multi-functional Transport Satellite (MTSAT) Satellite-based Augmentation System (MSAS).

EGNOS is being developed to provide regional satellite-based augmentation services to air, marine and land users in Europe. Its goal is to enhance the performance of GPS and GLONASS in terms of accuracy and integrity, in support of multimodal transport applications [ESA, (2001)]. Wide-area differential correction and integrity messages are generated from the data observed at the 34 Reference and Integrity Monitoring Stations (RIMS). These messages are then uploaded to three geostationary satellites in the EGNOS space segment, for onward broadcast to users. Those users in Europe should be able to track at least two geostationary satellites and benefit from a fully operational EGNOS in 2004.

The MSAS augmentation system is being developed by the Japan Civil Aviation Bureau (JCAB) for civil aviation [FAA, (2000)]. Beginning in 2001, this system will provide en-route through to precision approach navigation services for all aircraft within Japanese airspace with full-operational capability by 2005. According to [NGS, (2000)], 25 WAAS sites were incorporated into the National Geodetic Survey's (NGS) National CORS Network in July 2000; the impression being that both systems are evolving in parallel [Leick, (1994)].
There is also considerable research for the development of more efficient and robust wide-area ionospheric models [Blitza, (1999)] and characterisation of local anomalies such as TIDs (travelling ionospheric disturbances) [Christie et al, (1999)].

The Local-Area Augmentation System (LAAS) is another safety-critical precision navigation system designed specifically for supporting terminal area navigation through Category III precision approaches [US DoT, (1999)]. It provides equipped users with:

- local-area differential corrections for GPS satellites, WAAS/Space-based Augmentation System (SBAS), GEOs and Airport Pseudolites (APLs) [Swider et al, (1999)],
- integrity parameters associated with these transmission systems, and
- co-ordinated points describing the final precision approach trajectory.

LAAS uses multiple GPS reference receivers, all located within the bounds of the airport, to receiver and decode the GPS, WAAS GEO and APL range measurements and navigation data. This data is run through quality monitoring algorithms that check various criteria; signal, navigation data, measurement and integrity, before an averaging technique is used to provide optimal differential range corrections for each measurement. These corrections then possess the required level of safety to meet navigation accuracy, integrity, continuity of service and availability criteria. Airborne LAAS receivers receive and apply the differential corrections to their own satellite and pseudolite pseudorange measurements and then assess the error parameters against maximum allowable error bounds corresponding to the category of approach being performed.

An example of the required accuracies for LAAS based on a Category I precision approach, is 16 metres laterally and 4.0 metres in the vertical component 95% of the time [US DoT and DoD, (1999)].

### 2.2.4 Commercial DGPS Service Providers

The continued use of the various GPS positioning techniques within hydrographic and offshore engineering applications is highly dependent on the successful transmission, receipt and application of differential corrections by the mobile receiver.
2.2.4.1 Communication of Data

To ensure compatibility between DGPS users and service providers, it is useful to have standards for the use of differential corrections and a defined data format in which to report them. The *de facto* standard for broadcast DGPS corrections has been developed by the Radio Technical Commission for Maritime Services (RTCM) Special Committee 104 and is duly designated RTCM SC-104 [RTCM, (1994)]. This format contains 64 different message types that reflect information relating to marine GPS positioning and needs to be transmitted in an internationally recognised format. Providing examples, message Types 1 and 9 provide information including pseudorange correction values, range rate corrections and satellite health parameters. Message Types 18 through 21 provide data relevant to carrier-phases, in particular differential phase measurements.

A second standard, NMEA-0183, has been defined by the National Marine Electronics Association to facilitate the electrical interface and data protocol for communications between marine instrumentation [NMEA, (2000)]. Under this standard, all characters used in the data are printable ASCII text and are transmitted in sentences. For example, the GGA string contains data relating to a Global Positioning System position fix.

The majority of commercial vendors provide correction services by two methods of communication, land-based or satellite-based. Satellite-based communications are rapidly becoming the primary means for the dissemination of differential corrections because it affords greater coverage, increased reliability and thus integrity to their final user. The steady drop in the costs of the rapidly expanding global satellite communications infrastructure has greatly assisted this move from terrestrial to satellite-based systems as the most prevalent means of differential correction transmission.

The series of satellites operated by International Maritime Satellite Organisation (INMARSAT) have provided the underlying means of communication for many DGPS correction networks including those operated by Racal Survey [2000] and Veripos [1998]. The INMARSAT satellite communications system consists of four satellites with an orbital coverage such that it affords them continuous seamless coverage for users up to 70 degrees both north and south of the Equator [Ackroyd and Lorimer, (1990)]. There are several types of INMARSAT antenna; Standard-A is transmit and receive whilst Standard-M is receive-only [INMARSAT, (2000)].

User software as developed by the service providers and receiver manufacturers incorporates all relevant corrections into the position fix computation usually along with
some functionality dependent on the differential correction network. There are, in general, several factors common to all products from the service providers.

- The inherent characteristics of these correction products are very similar.
- The broadcast corrections are monitored continuously in real-time for both age and continuity using a static downlink from the communications satellites. INMARSAT services for example, afford very high link reliability and simultaneous data with low latency (AoC) for a large number of reference stations [INMARSAT, (2000)]. Even so, some terrestrial communications are maintained that are suitable for harbour approaches and near-shore activities.
- Subscribers are normally provided with a suite of software packages that contain QC monitoring packages for the analysis of navigated positions in both real-time and post-processing. Commercial service providers log all correction data for post-processing at a later time.

In navigation terms, having a single baseline would yield a unique rover position although most software packages afford some functionality as to how they arrive at their final position solutions when using multiple baselines. They may display the positions as computed from the overall mean or using each one of the multiple baselines or any combination therein. A token averaging or distance-weighting algorithm is applied in all but the most basic commercial packages.

**Modelling Routines with Commercial Software**

As far as openly available literature is concerned, there is very little technical information as to the weighting procedures implemented within these packages. Given that most filter the code pseudoranges to some extent with carrier-phase or Doppler measurements, it is likely that they will make use of statistical information from this process; perhaps the maximum filter length or duration of the filter so far.

Other potential weighting routines could be derived from the DOP statistics or the Age of Correction (AOC) parameters transmitted along with the differential corrections. A study by Roberts et al [1997] showed that the deterioration of differential corrections is caused when SA is activated. If the high-frequency noises imparted by SA were no longer present, then SA would no longer be dominant in terms of contaminating the differential corrections computed at or within a reference station network as is seen in the lower update rate for differential GLONASS corrections [Orpen and Hirsch, (1997)]. Given the
relatively smooth (predictable) motions of the satellites as seen by a static user, then the rate of change of ranges is not as large as when SA was activated. The weighting routines therefore would be amended to acknowledge this.

2.2.4.2 Racal's LandStar and Skyfix positioning services

Of these two services, LandStar is for land-based fleet and agricultural positioning whereas Skyfix is for marine offshore positioning. The operating concept behind each service is very similar and can be summarised as follows. A network of GPS reference stations is located such that they afford optimum coverage to potential users. Differential corrections are then sent via dedicated landlines to the LandStar hub and control centre where the data is checked for quality. The corrections are then formatted into a continuous data stream that is then uploaded to the communications satellite for dissemination to the users within its footprint. These services use L-band spot beam technology for the transmission of their corrections thereby reducing the size of user receivers and antennas (cf. §2.1.4). Given the size of the network for which these corrections have been gathered within, the resultant corrections possess robustness yielding a highly redundant service. Offshore users can subscribe to the Multifix 3 package that allows mobile positioning using corrections from multiple reference stations. Overall positioning accuracies in real-time are quoted to be 1 metre or less [Racal Survey, (1998, 2000); Ochieng et al, (1999)].

When Fugro's Starfix and OmniSTAR services make use of multiple reference stations, they apply an inverse distance-weighted least squares solution to determine the final rover position [Fugro, (2000)]. Technical literature available for Racal's LandStar and Skyfix services do not contain any implicit information about the weighting routines used in multiple reference stations scenarios [Racal Survey, (1998, 1999, 2000b)] although a similar routine is highly likely.

Fugro Starfix is a commercial service providing differential GPS code corrections over a dedicated satellite link to all Fugro vessels and subscribed users in the major offshore exploration areas. Accuracies achievable are 2 metres (95% of the time) providing the transmitted corrections can be received. This positional accuracy is possible as a result of up to 10 Fugro reference stations around Europe determining code corrections for application at the subscriber's receiver [Fugro, (2000)]. Combining the differential corrections as determined from multiple reference stations equipped with Fugro’s Multiple Reference Station DGPS algorithm (MRDGPS), or indeed additional satellites, affords more accurate positioning and quality measures as a result of more error checking and
greater integrity afforded to the final position. The procedures and achievable accuracies mentioned above are typical for other commercial differential code positioning systems.

Fugro currently have two reference stations in Europe monitoring measurements from Russian Federation’s satellite positioning system, GLONASS [Fugro, (2000)]. The system architecture of GLONASS is very similar in concept to GPS as well as its military beginnings [Forsell, (1997); Russian Space Forces, (1998)]. The differential GLONASS (DGLONASS) corrections determined are transmitted over the same L-band satellite link as for DGPS [Chistyakov et al, (1996)]. The update rates required for DGLONASS corrections are around the 60 second level as there are no ‘selective availability’ issues with the GLONASS system [Orpen and Hinsch, (1997)].

2.2.4.3 Virtual Reference Stations (VRS)

The concept of using large-scale networks to determine regional error models is now exploited by many of the major service providers and software manufacturers (cf. §2.2.2). In order to use fast static or RTK centimetric positioning over large networks of active GPS reference receivers, complex algorithms have been developed to model the significant effects of distance-dependent errors. The underlying concept of the algorithms relies upon the assumption that system errors at the mobile’s approximate location can be interpolated linearly between those errors determined from the bounding network of reference stations. An approximate position for the mobile receiver is used along with the error models transmitted from the surrounding network of reference stations. Providing the estimated position is good to less than a kilometre, then the measurements reconstructed for that position are of sufficient quality to be regarded as a virtual reference station. RTK positioning can then be performed with reference to the virtual reference station and a defined co-ordinate system [Spectra Precision Terrasat, (1999)].

Racal’s Skyfix service contains a similar routine in its Virtual Base Station (VBS) module albeit for code DGPS users requiring a multiple-reference station solution [Racal Survey, (1999)]. Similar functionality is offered within the Virtual Base Station Solution (VBSS) options within Fugro’s OmniSTAR system [Fugro, (2000)]. The virtual reference stations in both systems transmit standard RTCM correction messages optimized for the user’s approximate location.
2.2.5 Carrier-phase Positioning Methodologies

Real-time carrier-phase positioning is becoming increasingly employed by non-navigation users. Currently this requires a GPS reference station within a few tens of kilometres of a user's location and involves the transmission of carrier-phase differential corrections. Many users establish their own reference stations that are operated only for the duration of their project.

The greatest problem with real-time carrier-phase differential positioning techniques relates to the resolution of carrier-phase ambiguities. There are many different methodologies to do this including those which search for the correct integer ambiguities in the measurement, position and ambiguity domains [Hatch and Euler, (1994); Han and Rizos, (1997)]. When working over longer distances, the effects of ionospheric refraction introduce more noise to the system making it difficult to successfully determine the correct integer ambiguity [Abidin, (1994)].

A positioning solution called Enhanced DGPS (EDGPS) has been developed by Dassault Sercel in which they combine carrier-phase and code measurements within their Kinematic Ambiguity Resolution Technique (KART) in order to resolve ambiguities [Barboux, (1997)]. KART performs a recursive computation of the approximate position, which it refines iteratively and determines from this position, the most likely 'fixed ambiguities' solution. This method has two advantages: firstly, the computational load is kept at a reasonable level, and secondly, because the method is generic, it is compatible with L1-only data and standard correlation-type receivers providing very low noise measurements. This approximate position as determined with KART, makes use of all available corrected phase-filtered ranges and carrier-phase triple differences. It provides the basis of the EDGPS position at decimetre level accuracy in real-time after tracking only a few minutes of C/A-code measurements. This positioning technique can provide accuracies between, at worst, a standard DGPS solution and a ‘float’ integers pure phase solution depending on the period of tracking. The more accurate the approximate position, the faster the kinematic solution is found.

Considerable research into long-range carrier-phase differential GPS has also been carried out by many parties including Colombo [1997] and Han [1997a] although their efforts have been focussed mainly on post-processing applications. Even with the more precise carrier-phase observables, the phenomena of phase multipath still exerts a considerable influence on RTK-GPS, for example in harbour environments [Spaans, (2000)].
**Racal’s LRTK Positioning Service - Genesis**

The ‘Genesis’ service offered by Racal Survey is a Long-range Real-Time Kinematic (LRTK) service capable of providing sub-20 centimetre accuracies in both horizontal position and height throughout Europe [Racal Survey, (2000a)]. This accuracy is achieved through solution of the integer wide-lane carrier-phase ambiguities in a kinematic environment up to 800 kilometres from a reference station – 20 times greater than other commercially available RTK solutions (cf. §2.2.5). Deep-water exploration and development applications requiring real-time, very high accuracy co-ordinates, in particular heights, would benefit from this LRTK service. As with most long-range positioning services, the determination of user positions from multiple-station networks ensures that a robust solution is maintained throughout periods of poor satellite geometry and availability problems at individual reference stations. As of April 2000, four stations have been installed to optimise coverage in the North Sea and Norwegian Continental Shelf areas and the satellite-based transfer system [Oceanspace, (2000)].

**Global Arrays of GPS Reference Stations**

Applications involving geodetic reference frame definitions, geophysical and meteorological tasks require carrier-phase observations to achieve centimetre level accuracy on a regional-to-global basis requiring large networks of GPS reference stations. At the time of writing, the world-wide tracking network operated by the IGS provides the required accuracy reference frame and sub-decimetre orbits but only supports post-processing applications. However the IGS is moving toward near real-time to real-time provision of information to users via telephones, Internet and broadcast communication means. This information would support such applications as short-term weather prediction and seismic monitoring [IGS, (2000)].

Several large-scale reference station networks have been established by several national mapping agencies (NMA) in order to evaluate the levels of noise induced on dual-frequency code and carrier-phase observations by greater receiver-user separations. Examples of such networks, other than those mentioned in §2.2.2 include the Active GPS Network of Great Britain’s Ordnance Survey (OS) [Ordnance Survey, (2000)] and the US Continuous Operating Reference Station (CORS) [NGS, (2000)].

Taking the UK’s National GPS Network as an example, its presence will allow GPS surveyors to determine high-accuracy GPS co-ordinates in the standard ETRS89 coordinate system and transform these co-ordinates to high-accuracy National Grid co-
ordinates and Ordnance Datum Newlyn (ODN) heights. All GPS data is archived and available to users via the Internet. This should allow the post-processing (subject to a 2 hour latency) of dual-frequency data from the national network to achieve accuracies in the range of 1-10 metres depending on the observing period, receiver and antenna hardware and processing software used [Ordnance Survey, (2000)].

### 2.3 QUALITY MEASURES ASSOCIATED WITH DGPS

The performance of a navigation system can be measured with four quantities; accuracy, integrity, continuity of service and availability of service, which together are known as the Required Navigation Performance parameters (RNP) [Ochieng et al, (1999); Ochieng et al, (2001)]. Historically, several of these quantities have their origins in aviation-navigation although when combined, define the level of safety required of any navigation system. The following bullet points contain a global interpretation of these four parameters.

- **Accuracy** - the degree of conformance of an estimated or measured position at a given time, from the truth.

- **Integrity** - relates to the trust that can be placed in the correctness of information supplied by the navigation system. It includes the ability of a navigation system to provide timely warnings to users when the system is not to be used for navigation / positioning. Specifically, a navigation system is required to deliver a warning (i.e. an alarm) of any malfunction (as a result of a set alarm limit being exceeded) to users within a given period of time (i.e. time-to-alarm) and with a given probability (integrity risk).

- **Continuity of service** - defined as the ability of the total system to perform its function without interruption during an intended period of operation. Continuity risk is the probability that the system will be interrupted and not provide guidance information for the intended period of operation. This risk is a measure of system unreliability.

- **Availability** - defined as the percentage of time during which the service is available for use taking into account all the outages whatever their origins. The service is available if accuracy, integrity and continuity requirements are satisfied.
The representative organisation for the British offshore oil and gas industry is the United Kingdom Offshore Operators Association (UKOOA). Its members are the companies licensed by Her Majesty’s Government to explore for and produce, oil and gas in UK waters [UKOOA, (2000)]. Their responsibilities cover all topics relating to industrial offshore activities (cf. Table 1-2) from environmental aspects and public relations to technical advice, especially with regards to safety matters.

In an attempt to introduce a common standard for the numerous differential service providers operating within UK waters, UKOOA’s Surveying and Positioning Committee commissioned the document ‘Guidelines for the use of Differential GPS in Offshore Surveying – P2/94’ [UKOOA, (1994)]. This document recommends measures that should be used to describe and assess the quality of a DGPS position fix along with test statistics to verify the correctness of the data. These guidelines were later verified by Roberts et al [1997], and will be discussed in more depth in Chapters 3 and 4.

2.4 CONCLUDING REMARKS

- SA was the largest source of error on individual GPS observations at time of data collection. However the implementation of correctly designed differential positioning methodology effectively eliminates all satellite-common errors from the GPS measurements. The position solutions afforded by differential positioning are not affected by the presence, or absence, of SA when assuming correction latency times of zero.

- At the current time, the errors within the broadcast satellite orbits and clock corrections are at their smallest level, and exert a minimal effect on those positions derived using a proven differential positioning approach.

- The effects of atmospheric delays can introduce some significant biases into the position computations especially from low elevation satellites. Means must be taken to reduce the impact of such low quality measurements.

- Multipath interference can exert a considerable bias on GPS code measurements and the positions computed from them. Therefore these adverse effects should be mitigated by some means. The effects of multipath on carrier-phase measurements are assumed to be negligible compared to its impact on code pseudoranges.
Concluding Remarks

- Receiver noise is unique to each receiver yet at an insignificant magnitude in relation to other sources of error on the code pseudorange observables.
- Terms must be included to model the residual effects of these biases and will be done so in the stochastic modelling algorithms in Chapter 4.
- Differential positioning techniques can greatly improve the positioning accuracy of GPS, even more so given the use of multiple reference stations.
Chapter 3 - DGPS Mathematical Models

This chapter provides a background to the approach of differential code pseudorange positioning with the double-difference functional model. The correlation characteristics of the error types on GPS code pseudorange observations are discussed. The concept of carrier-phase filtering and the steps required to derive phase-filtered pseudoranges are also discussed. The double-difference approach is described here along with some background theory on adjustment using least squares techniques.

3.1 CONCEPT OF THE DOUBLE-DIFFERENCING POSITIONING APPROACH

The fundamental objective of differential positioning is to determine the co-ordinates of an unknown point relative to a stationary point of known co-ordinates, in effect, the baseline vector between the two points as described in §1.2. In differential positioning, two receivers simultaneously make observations to the same satellites so that the observation time tags are synchronised for both stations, that is, the observations are recorded nominally at the same time. When linearised and used in a least squares algorithm, estimates for the unknown parameters, i.e. the baseline components, can be computed.

Differential positioning can be carried out with code pseudorange or carrier-phase observables, or perhaps a combination of the two by means of differencing techniques. Linear combinations of these observations can be made so that some parameters are eliminated, e.g. the ionospheric delay and fewer unknowns have to be estimated.

3.1.1 Differencing Techniques

The successful implementation of differential positioning relies on the assumption that satellite-based errors common between two receivers and one satellite can be cancelled or differenced out. These differencing techniques are processes whereby differencing the synchronised code or phase GPS measurements, from different receivers and satellites, can reduce or even eliminate many satellite-common errors such as orbit and clock biases, and
atmospheric effects. The main difference between code and carrier-phase differencing relates to the presence of the integer carrier-phase ambiguity $N$ in the latter.

3.1.1.1 Single-differencing

A single-difference is the difference of the observations made by two receivers to the same satellite at the same epoch as illustrated in Figure 3-1. The satellite clock error bias is present in both observables to the same extent and can therefore be eliminated. Consider the observation equations for two receivers, $A$ and $B$, observing the same satellite, $i$, as in Figure 3-1:

$$P_A' = \rho_A' + c\tau_A - c\tau' + I_A' + Z_A'$$
$$P_B' = \rho_B' + c\tau_B - c\tau' + I_B' + Z_B'$$

Equations 3-1(a) and (b)

The single-difference code pseudorange between these two is defined as:

$$\Delta P_{AB} = P_A' - P_B' = (\rho_A' + c\tau_A - c\tau' + I_A' + Z_A') - (\rho_B' + c\tau_B - c\tau' + I_B' + Z_B')$$

Equation 3-2

$$= \Delta\rho_{AB} + \Delta c\tau_{AB} + \Delta I_{AB} + \Delta Z_{AB}$$

Note that, within the final line of Equation 3-2, the satellite clock bias term $ct_i$ has been eliminated, and the atmospheric terms are reduced also.
3.1.1.2 Double-differencing

This is the difference between two simultaneous single-differences, and its purpose is to eliminate satellite and receiver clock biases. The main disadvantage of double-differencing is that the effects of unmodelled atmospheric errors are increased slightly, by ~40%, compared to single-differencing [Blewitt, (1997a)]. Even so for both single- and double-differences, the carrier-phase ambiguity $N$ is an integer (cf. §Appendix A). This is because the phase offsets of the receiver and satellite oscillator have cancelled in the double-difference combination. Consider the single-differenced observations equations for two receivers $A$ and $B$ observing satellites $i$ and $j$, as in Figure 3-2:

$$\Delta P'_{AB} = \Delta \rho'_{AB} + \Delta c \tau_{AB} + \Delta \lambda'_{AB} + \Delta Z'_{AB}$$

$$\Delta P''_{AB} = \Delta \rho''_{AB} + \Delta c \tau_{AB} + \Delta \lambda''_{AB} + \Delta Z''_{AB}$$

Equation 3-3

The double-difference code pseudorange is defined as the difference between these two single-differences:

$$\nabla \Delta P''_{AB} = \Delta P''_{AB} - \Delta P''_{AB}$$

$$= (\Delta \rho''_{AB} + \Delta c \tau_{AB} + \Delta \lambda''_{AB} + \Delta Z''_{AB}) - (\Delta \rho'_{AB} + \Delta c \tau_{AB} + \Delta \lambda'_{AB} + \Delta Z'_{AB})$$

$$= \nabla \Delta \rho''_{AB} + \nabla \Delta \lambda''_{AB} + \nabla \Delta Z''_{AB}$$

Equation 3-4
A disadvantage of double-differencing and also conventional DGPS, is that the differenced observations are correlated, however there are means of modelling these correlations as discussed later in §4.1. For RTK applications using double-differences, the double-difference phase observation equation takes a similar form to that for the code (cf. Equation 3-7), albeit with an additional term for the double-difference integer ambiguity:

\[
\nabla \Delta l_{AB}^y = \nabla \Delta \rho_{AB}^y - \nabla \Delta l_{AB}^y + \nabla \Delta Z_{AB}^y - \lambda_0 \nabla \Delta N_{AB}^{jk}
\]

Equation 3-5

\[
N_{AB}^y = (N_B^y - N_A^y) - (N_B^j - N_A^j)
\]

Equation 3-6

3.1.1.3 Triple-differencing

A triple-difference is determined from the difference between two double-differences, i.e. over two epochs, as in Figure 3-3. From these two consecutive epochs, it is possible to eliminate the carrier-phase integer ambiguity \( N \) provided the ambiguity has not changed between the two double-differences. This differencing process can help to identify, as outliers, any cycle slips that have occurred during the data set so they can be removed. A disadvantage of triple-differencing is that it propagates the double-difference correlations between observations in time thereby decreasing the data weights. Code pseudorange measurements are not subject to triple-differencing as there are no code ambiguity terms to eliminate.

![Triple-differencing Geometry](image)

Figure 3-3 Triple-differencing Geometry
Considering two double-difference phase observation equations taken over two successive epochs $t_1$ and $t_2$ (cf. Equation 3-5), the triple-difference phase observation equation can be found from:

$$\delta(t_1, t_2)\Delta L_{ab}^\theta = \delta(t_1, t_2)\Delta \rho_{ab}^\theta(t_1) - \delta(t_1, t_2)\Delta I_{ab}^\theta(t_1) + \delta(t_1, t_2)\Delta Z_{ab}^\theta(t_1)$$

Equation 3-7

For further information on the differencing techniques corresponding to GPS measurements, in particular carrier-phases, refer to the comprehensive proofs and discussions in [Hofmann-Wellenhof et al, (1994); Leick, (1995); Teunissen and Kleusberg, (1996); Blewitt, (1997)].

### 3.1.2 Differential Positioning using Double-differencing

Given the advantages of double-differencing discussed above, namely in eliminating the satellite-common biases, its involvement in least squares positioning using code pseudoranges is now covered. A differential positioning example is given in which two receivers observe four satellites as shown in Figure 3-4. Processing the double-differenced data from these two receivers, with Station $A$ as reference, will yield a baseline solution, i.e. the co-ordinate triplet of the baseline’s components, $\{x_B, y_B, z_B\}$. These three components are the unknown parameters to be solved for and effectively represent the co-ordinates of Station $B$.

![Figure 3-4](image)

**Figure 3-4** Simple Example of Double-differencing Geometry and Observations for Differential Positioning
At epoch $t$, these two receivers using satellite $i$ as the reference satellite, will yield three double-difference observations:

$$P^u_{A_i}; P^u_{B_i}; P^u_{A_{ref}}$$

Equation 3-8

**Obtaining Linearly Independent Datasets**

When working with a number of satellite measurements at each epoch, the creation of valid and unique double-differences can be a problem. However, the undifferenced observations can be rearranged to form a linear combination of the others.

Blewitt [1997] provides details of two satellite ordering approaches with which the linearly independent data sets necessary for least squares can be obtained. The first is the Reference Satellite concept which involves using a linearly independent set of double-differenced observations that all make use of one particular satellite. As long as the observation covariance matrix for the reference satellite is constructed properly, any data set is equally valid and will produce identical solutions. The reference satellite must have data for every epoch usually being that one in view for the longest period. Some commercial processing packages, such as Ashtech’s PRISM processing software, possess more complex algorithms that select the reference satellite epoch by epoch [Ashtech, (1995)]. If there are more than two receivers in an application, then a Reference Station approach can be used whereby the set of double-differenced observations all include a common reference station to guarantee linear independence. It must be noted that the order of the visible satellites within the least squares matrices will not affect the final position solution.

**Least Squares Computations**

The observation equations for the baseline solution, as linearised using Taylor’s theorem, can be written as:

$$b = Ax + v$$

Equation 3-9

where $b$: column vector of the observed-minus-computed values of the residual observations; dimensions $[d \times 1]$ where $d$ is equal to the number of linearly independent double-differenced data,
DGPS Mathematical Models

\( A \): partial derivative matrix of the parameters (design matrix); size \([d \times p]\)
where \( p \) are the number of unknown parameters,

\( x \): column vector of parameter corrections, dimensions \([p \times 1]\),

\( v \): column vector of post-fit residuals (observation errors); same dimensions as \( b \), that is \([d \times 1]\).

This set of observation equations represent the parametric case of linear least squares
with one equation per observation. The column vector \( b \) contains the double-difference
code pseudorange observations as calculated from the observed double-differences, \( \rho \),
minus the computed double-differences, \( P \).

\[
\begin{align*}
   b_{AB}^g &= \left( obs_{AB}^g - comp_{AB}^g \right) \\
   &= \left( \rho_B^g - \rho_A^g - \rho_B^g + \rho_A^g \right) - \left( P_B^g - P_A^g - P_B^g + P_A^g \right)
\end{align*}
\]

Equation 3-10

The double-differenced code ranges are incorporated within the design matrix of
partial derivatives, \( A \). Note that \( A \) has the same number of rows as there are data, \( d = 4 \)
and the same number of columns as there are unknown parameters, \( p = 3 \). In the case of
differential positioning using code pseudoranges, the parameters correspond to the co­
ordinate triplet of the baseline components. An example of the coefficients of the design
matrix \( A \) are given by a single row corresponding to the double-difference observation \( P_{AB}^{ik} \)
at epoch \( t \):

\[
\begin{align*}
   A_{AB}^{ij} &= \begin{bmatrix}
   \frac{\partial P_{AB}^{ij}}{\partial x_B} & \frac{\partial P_{AB}^{ij}}{\partial y_B} & \frac{\partial P_{AB}^{ij}}{\partial z_B}
   \end{bmatrix} \\
   &= \begin{bmatrix}
   \frac{\partial P_{AB}^{ij}}{\partial x_B} & \frac{\partial P_{AB}^{ij}}{\partial y_B} & \frac{\partial P_{AB}^{ij}}{\partial z_B}
   \end{bmatrix}
\end{align*}
\]

Equation 3-11

These elements are found by partial differentiation of the double-difference
observation equations and produce the following expression:
The vector of unknown parameters, \( x \), for code positioning, are defined by:

\[
x^T = [\Delta X_B \quad \Delta Y_B \quad \Delta Z_B]
\]  

Equation 3-13

The solution to the normal equations provides corrections to the initial parameters as follows:

\[
\hat{x} = (A^T A)^{-1} A^T b
\]  

Equation 3-14

For a unique baseline solution between two receivers using code double-differences with four satellites, at least two epochs of data are required. Ideally, it is best to have data from different satellite geometries, although with high-rate kinematic positioning, a significant geometry change is not normally possible. If further parameters are to be estimated, such as troposphere and ionosphere, then more satellites will be needed.

**Stochastic Model**

As double-differences are dependent on a common satellite or receiver, the observations are also statistically dependent. For example, at a given epoch, the double-differences

\[
P_{41}^{AB}, P_{42}^{AB}, P_{43}^{AB}
\]  

Equation 3-15

are correlated due to the common single-differenced data relating to satellite number 4, \( P_{4B}^{AB} \). Any measurement in this single-difference will contribute exactly the same error to each of the double-differences. However because of the induced correlations present
within double-difference data, a weighted least squares approach is normally taken whereby the statistical nature of the data is described before being used to determine the least squares vector of corrections $\hat{x}$. An appropriate data weight matrix, $W$, can be created from the inverse of the covariance matrix of the double-difference observational errors:

$$W = C_{\hat{v}\hat{v}}^{-1} \quad \text{Equation 3-16}$$

Following the population of the weight matrix, $W$, the estimates of the corrections to the initial parameter values are then given by:

$$\hat{x} = (A^T W A)^{-1} A^T W b \quad \text{Equation 3-17}$$

The correct population of the weight matrix $W$ is the main objective of stochastic modelling. Not only should it contain statistical descriptions of the precisions of the measurements but also the levels of correlation in spatial, temporal and receiver-dependent terms. Means of populating this weight matrix $W$ are discussed in Chapter 4.

**Baseline Vector Solutions**

The double-differenced observables can then be used to estimate the relative co-ordinates between two receivers, usually in the form of a baseline vector. The observation equation for code pseudoranges is as follows:

$$\nabla \Delta P_{AB}^\| = \rho_{AB}^\| + I_{AB}^\| + Z_{AB}^\| \quad \text{Equation 3-18}$$

For carrier-phase applications, an additional term, $-\lambda \omega N_{AB}^\|$, is included in Equation 3-18 to account for the carrier-phase ambiguity. If the receivers are within 20 kilometres of each other, the differential atmospheric delay terms, $I$ and $Z$, are often assumed to be negligible and ignored (cf. §2.1.3). Conversely, these delay terms can be modelled stochastically, in a manner for example, as Euler and Ziegler [2000] model the residual ionospheric delays using Equation 4-18.
Concept of The Double-differencing Positioning Approach

Consider two GPS receivers at stations A and B. If Station A is held fixed at known, or at least nominal, co-ordinates, then estimating the baseline vector $AB$ is equivalent to estimating the co-ordinates of station B. If in a GPS survey, there were 4 satellites for every epoch ($t$), and satellite 3 was used as the reference, a set of three dependent double-differences $\Lambda$, could be formed (the double-difference notation $\nabla \Delta$ has been dropped for simplicity):

$$\Lambda^3(t) = L_{31}^{AB}(t), L_{32}^{AB}(t), L_{34}^{AB}(t)$$  \hspace{1cm} Equation 3-19

Code double-difference positioning does not require the ambiguity terms to be solved for the co-ordinate triplet. The parameter set to solve for during least squares for carrier-phase positioning would be the following six parameters.

$$x_B, y_B, z_B, N_{31}^{AB}, N_{32}^{AB}, N_{34}^{AB}$$  \hspace{1cm} Equation 3-20

Note that this example does not contain any cycle slips; if it did, they would either be repaired or included in the parameter set as further ambiguity parameters. Depending on the survey specifications, atmospheric terms could also be calculated.

**Ambiguity Resolution using Double-differencing**

Accurate positioning with RTK-GPS at the centimetre level requires that the double-difference ambiguities have been resolved. As mentioned earlier, the double-difference ambiguity is, theoretically, an unknown integer number. However the least squares estimates of the double-difference ambiguities are, in general, real numbers and the coordinate solutions assigned these real numbers are called double-difference float solutions. Affecting these float solutions are unmodelled effects such as residual ionosphere, troposphere, multipath and phase centre variations. The double-difference fixed ambiguity solution is obtained if the integer value of all ambiguities can be uniquely identified, despite the presence of these residual unmodelled errors.

For the first position fix and for single epoch positioning methodologies [Corbett, (1994)], the integer ambiguity needs to be solved for each contributing measurement. Once determined, and by keeping these estimates fixed (known), a fixed double-difference coordinate solution is possible with fewer satellites over short intervals providing cycle slips do not occur.
Table 3-1 summarizes the most important features of the single, double and triple-difference observations when dealing with pseudoranges and carrier-phases.

<table>
<thead>
<tr>
<th>OBSERVATION</th>
<th>EFFECTS ELIMINATED</th>
<th>EFFECTS REDUCED</th>
<th>OPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-difference</td>
<td>Satellite clock bias</td>
<td>orbit errors</td>
<td>constrain</td>
</tr>
<tr>
<td>$\Delta$</td>
<td></td>
<td></td>
<td>ambiguity</td>
</tr>
<tr>
<td>Double-difference</td>
<td>Satellite and receiver clock biases</td>
<td>ionosphere</td>
<td>constrain</td>
</tr>
<tr>
<td>$\nabla \Delta$</td>
<td></td>
<td></td>
<td>ambiguity</td>
</tr>
<tr>
<td>Triple-difference</td>
<td>Satellite and receiver clock biases</td>
<td>troposphere</td>
<td>--</td>
</tr>
<tr>
<td>$\delta \nabla \Delta$</td>
<td>Ambiguity eliminated</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3-1 Summary of Differencing Techniques detailing Common-mode Cancellations

3.2 QUALITY MEASURES AS DERIVED FROM DOUBLE-DIFFERENCE OBSERVATIONS

The least squares algorithm computes parameter estimates and residuals such that $\hat{v}^{T} W \hat{v}$, is minimised, hence the name 'least squares'. Following the successful and swift convergence of the least squares estimates of the correction to the estimated parameters, $\hat{x}$, an operator can use the covariance matrices, from either standard or weighted least squares to compute error information from the adjustment.

**Vector of Post-fit Residuals**

The least squares estimate of the vector of code pseudorange residuals can be computed from:

$$\hat{v} = A \hat{x} - b$$  \hspace{1cm} \text{Equation 3-21}

The elements within this column vector is the difference between the actual observations and the new, estimated model for the observations, namely the amount that each observation has been adjusted by to generate a least squares position fix.
Ideally, the size of the residuals $v_i$ for a particular satellite would reflect the amount of noise on the system and therefore be the same size as the a-priori estimate of this noise as represented by $\sigma_i$ [Miller et al., (1997)]. However if there are sources of error on these observations, then steps need to be taken to model them stochastically within the least squares mathematical model.

**Covariance Matrix of Adjusted Parameters**

A covariance matrix for the estimated parameters $\hat{C}$ can be constructed and used to evaluate the estimated precision of a position fix. Cross [1983] details a number of different precision measures that can be calculated – positional standard errors, error ellipses, standard errors of derived quantities and the single number measures of precision. The first parameter is used mainly in this research to quantify the standard error associated with the co-ordinates. These parameters are transformed from geocentric to topocentric co-ordinates to permit easier analysis. The covariance matrix of adjusted parameters $\hat{C}$ is calculated from:

$$
\hat{C} = \left( A^T (R \hat{C} R^T)^{-1} A \right)^{-1} = (A^T W A)^{-1}
$$

Equation 3-22

where a diagonal matrix is used for the undifferenced data covariance matrix $C_i$ with an a-priori standard deviation of an observation typically defined by the user. $\hat{C}$ is the inverse of the left-hand side of the normal equations. By taking the square roots of the relevant diagonal elements in $\hat{C}$, it is possible to determine the standard deviation of the co-ordinate corrections in plan, height and vector components. These formal errors can be quoted with each estimated co-ordinate.

**Unit Variance**

Once the vector of post-fit residuals has been generated as in Equation 3-22, it can then be used to compute the Unit Variance, also referred to as the standard error of an observation of unit weight. The unit variance statistic $\sigma_0^2$ expresses the correctness of the a-priori standard deviations in describing the quality of the observations:
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\[
\sigma_0^2 = \frac{W^TW}{d-p} \quad \text{Equation 3-23}
\]

where \( d \) and \( p \) are the number of data (double-differences) and unknown parameters respectively. For code positioning, \( p \) corresponds to the co-ordinate triplet only; phase positioning would require the integer ambiguity values also. The unit variance is a measure of the overall fit of the observations to the functional and stochastic models, and its expected value is \( E(\sigma^2) = 1 \), [Cross, (1983)].

**Covariance Matrix of Residuals**

The covariance matrix of post-fit residuals, \( C_v \), is required in some of the later Quality Assessment routines and can be computed from

\[
C_v = C_i - A C_\chi A^T \quad \text{Equation 3-24}
\]

**Quality Assessment**

As part of the quality assessment procedure, statistical testing is usually carried out to determine whether or not the assumptions made in the position calculation and quality measurement process, i.e. the functional and stochastic models, are correct. The Guidelines for DGPS usage in offshore surveying, as provided by UKOOA [1994], recommend a number of tests be applied and measures calculated when assessing the quality of a DGPS position fix. In terms of Statistical Testing, the \( w \)-test should be applied to individual measurement residuals for the purpose of outlier detection, and the \( F \)-test on the unit variance to assess the overall fit of the observations to the functional and stochastic model. The Positional Quality Measures that are recommended are the a-posteriori horizontal error ellipse at the 95% confidence level, and the largest horizontal position vector resulting from a marginally detectable error (MDE) as a measure of the external reliability.

**Detection of Outliers using the \( w \)-test**

Also known as the slippage test, this recursive test looks for a specific outlying error in the functional model that can be addressed by one additional unknown [Tiberius,
Quality Measures as Derived from Double-difference Observations

(1998)]. The w-test statistic is computed from the normalised residual of each satellite used in the double-difference solution at that epoch where $e_i$ is a null vector but for the $i$th element which is unity [Cross, (1983)].

$$\hat{w}_i = \frac{e_i^TW\hat{v}_i}{\sqrt{e_i^TW\Gamma\hat{v}_i}}$$  \hspace{1cm} \text{Equation 3-25}

The $w$-test statistic is then tested against a threshold statistic for the required confidence level as determined for a dataset with normal distribution. For example, at a 99% confidence level, the $w$-test statistic must be less than the threshold of 2.576 else that observation will be deemed, with 99% confidence, an outlying measurement. If the $w$-test statistic does exceed the threshold, this should be rejected from that epoch and the least squares computation repeated. Should more than one measurement fail the $w$-test at a particular epoch, then that one with the largest value should be rejected prior to the $w$-test statistic being recomputed.

**Test of the Unit Variance (F-test)**

Individual values for the unit variance may be very large or very small within a dataset, but the statistical expectation of the overall unit variances are, on average, unity. Occasionally if very large values are obtained that fail the test, the $w$-test should be performed (which would have been done as matter of course if working to the UKOOA guidelines) and this may indicate outliers in the data. Cross et al [1993] state that very small occasional values of the unit variance will be encountered if the degrees of freedom for the epoch in question are small. Because of this limited redundancy, it may appear as though the measurements happen to fit together perfectly. Should this be the case and $\sigma_0^2$ is used as a scaling factor, then the results of the precision and reliability assessment cannot be believed. This is perhaps a superfluous comment when considering that model errors are typically responsible for long-term unit variance averages being significantly greater or less than unity.

If the $F$-test suggests that the unit variance is significantly different from unity, then the observations do not fit well with the mathematical positioning model. Consequently the covariance matrices must be multiplied by the unit variance statistic as the weight matrices used to calculate the unit variance have been incorrect (on average) by the reciprocal of the unit variance [Cross, (1983)].

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\[ C_{rx} = \sigma_0^2 \left( A^T W A \right)^{-1} \]  

Equation 3-26

The inclusion of this scaling factor in Equation 3-26 does not affect the final position solutions at all, only the final quality measures.

**Precision Measures**

Standard deviations are figures associated with individual scalar variables such as pseudorange and height. To depict standard deviations in two dimensions, horizontal error ellipses are used [Cross, (1983)]. The direction of the lowest precision (highest standard deviation) lies along the major axis of this ellipse, and conversely the minor axis shows the direction in which the position is most precise. UKOOA recommends that the precision measure used for the horizontal position be the 95% confidence level ellipse.

**Dilution of Relative Precision Statistics**

A relative Dilution of Precision (DOP) statistic that quantifies how the level of errors in the measurements can be related to the expected quality of the GPS baseline solution. Relative PDOP is a mathematical function computed using the inverse of the normal equation matrix of double-differences and can be used to assess the relative strength of the observed satellite geometry between the observing receivers and yield an estimated position quality statistic of that baseline. The greater the spread of the satellites relative to the user baseline, then the smaller the relative PDOP statistic.

**Reliability Measures**

Reliability describes the performance of the testing and can be classified by two components. Internal reliability concerns the observations themselves and expresses the relationships between the observations, i.e. how much they control each other. External reliability quantifies the resistance of the positioning model to possible anomalous observations (outliers) [Tiberius, (1998)].

Reliability is most usefully measured by means of Marginally Detectable Errors (MDE), or Biases (MDB). For this, routines are designed to detect outlying observations at a certain probability within the outlier test. It is common to assume that only one observation is corrupted by a bias at any one epoch and that the rest of the observations are unbiased. The concept of external reliability is a more practical measure than the internal
reliability [Cross et al, (1993)]. External reliability is assumed by the largest horizontal position MDE, that is the largest effect on the computed position of an observational bias with a magnitude that corresponds to the size of the MDE of that observation.

3.3 TECHNIQUES FOR FILTERING CODE PSEUDORANGES

Although the technique of differential positioning can mitigate the distance-independent errors due to orbital errors and atmospheric delays, it cannot mitigate the biases of multipath interference and receiver noises. It is therefore necessary to consider these error sources individually at each receiver. As mentioned in §2.1.4, the multipath variations afforded to C/A-code pseudoranges are generally regarded as ‘fast’ errors, in that they can exhibit short-term sinusoidal oscillations around the order of 6-10 minutes [Langley, (1997)]. It is beneficial to reduce the influence of multipath on these code pseudoranges, and ultimately, on the final positions yielded from them.

It is possible to mitigate the effects of these fast distance-independent errors with use of a phase-filtering routine that reduces the noises by forming their averages over a predetermined time period. The term phase-filtering refers to the combination of carrier-phase measurements with code ranges in real-time [Euler and Goad, (1991)]. Figure 3-5 gives an example of the noises inherent to the differential positioning solutions determined using raw (unfiltered) C/A-code pseudoranges. This particular baseline is between the two reference stations BEDS and ONSA (cf. §3.3.4 and §6.2.2).
A considerable number of artefacts can be seen within this time series as illustrated by the low-frequency noises that oscillate by up to almost 10 metres. These can be mainly attributed to multipath interference affecting either or both of the observing receivers.

### 3.3.1 Theory behind Carrier-phase Filtering

Although Hatch’s filtering methodology refers to the use of Doppler measurements, it is equally applicable to carrier-phase measurements. This is because, like the Doppler measurements, the carrier-phase observations are considered as time-differenced pseudoranges with a much higher precision level than the code pseudoranges. So much so that when the phase pseudorange’s more precise rate of change is combined with the code pseudoranges, it will yield a series of code pseudoranges containing reduced levels of code multipath. The presence of the carrier-phase integer ambiguity does not affect the filtering process so long as there are no interruptions within the phase measurements. The fundamental assumption behind carrier-phase filtering is that the influence of multipath interference on carrier-phase measurements is considerably smaller than that on code pseudorange measurements (cf. Table 2-4).
Techniques for Filtering Code Pseudoranges

*Cycle Slips*

Depending on the application, an obstruction to the GPS signal or excessive signal noise can cause a receiver to lose count of the number of carrier-phase cycles. This integer discontinuity in the phase data is called a cycle slip and should one occur, it will introduce a step function within the phase pseudorange and consequently the estimate of the phase-filtered pseudorange. Although the code and carrier-phase observations here are both in the form of pseudoranges, they differ nominally by the carrier-phase ambiguity that will remain as an integer until a cycle slip occurs.

*Basic Phase-filtering Methodology*

The smoothing of code pseudoranges by means of phase pseudoranges is an important aspect of real-time trajectory determination and is practised in most commercial DGPS software. A basic example of phase-filtering follows. Dual-frequency instruments yield code and carrier-phase pseudoranges respectively on both frequencies as follows:

\[ P_{L1}, P_{L2}, \Phi_{L1} \text{ and } \Phi_{L2} \]

Assume that the code pseudoranges are scaled to cycles by dividing them by the corresponding wavelength. Using the two frequencies \( f_{L1} \) and \( f_{L2} \), the combination:

\[
P(t_i) = \frac{f_{L1}P_{L1}(t_i) + f_{L2}P_{L2}(t_i)}{f_{L1} + f_{L2}}
\]

Equation 3-27

is formed for the code pseudoranges, and the wide-lane observable:

\[
\Phi(t_i) = \Phi_{L1}(t_i) - \Phi_{L2}(t_i)
\]

Equation 3-28

is formed for the carrier-phase pseudoranges. Combinations of Equations 3-27 and 3-28 are formed for each epoch, and for all epochs \( t_i \) after \( t_1 \), extrapolated values of the code pseudorange \( P(t_i)_{ex} \) can be calculated from:

\[
P(t_i)_{ex} = P(t_i) + (\Phi(t_i) - \Phi(t_1))
\]

Equation 3-29
The smoothed value $P(t_i)_{sm}$ is finally obtained by the arithmetic mean:

$$P(t_i)_{sm} = \frac{1}{2} (P(t_i) + P(t_i)_{ex})$$  \hspace{1cm} \text{Equation 3-30}$$

Generalising the above formulae for an arbitrary epoch $t_i$ (with the preceding $(t_{i-1})$), a recursive algorithm is given by:

$$P(t_i) = \frac{f_{L1} P_{L1}(t_i) - f_{L2} P_{L2}(t_i)}{f_{L1} + f_{L2}}$$  \hspace{1cm} \text{Equation 3-31}$$

The wide-lane signal (difference between L1 and L2 phase) is calculated from

$$\Phi(t_i) = \Phi_{L1}(t_i) - \Phi_{L2}(t_i)$$  \hspace{1cm} \text{Equation 3-32}$$

and is then used to assist in calculating the extrapolated pseudorange:

$$P(t_i)_{ex} = P(t_{i-1})_{sm} + \left(\Phi(t_i) - \Phi(t_{i-1})\right)$$  \hspace{1cm} \text{Equation 3-33}$$

Finally the smoothed pseudorange is calculated from:

$$P(t_i)_{sm} = \frac{1}{2} (P(t_i) + P(t_i)_{ex})$$  \hspace{1cm} \text{Equation 3-34}$$

which works under the initial condition:

$$P(t_1) = P(t_1)_{ex} = P(t_1)_{sm} \text{ for all } i > 1$$  \hspace{1cm} \text{Equation 3-35}$$

### 3.3.2 Modifications to Hatch's Carrier-phase Filtering Technique

There have since been several modifications to Hatch's initial dual-frequency filtering algorithm, primarily because of its idealistic assumption that the data is free of gross errors and cycle slips. Lachapelle et al [1986] countered the problem of data interruptions unsteadying the smoothing process by introducing a time dependent weight.
Techniques for Filtering Code Pseudoranges

factor, $w$, to the recursive algorithm. The filtered code pseudorange for an epoch $t_i$ is found by:

$$P_{(t_i)} = w \cdot P(t_i) + (1 - w) \cdot \left[ P_{(t_{i-1})} + \Phi(t_i) - \Phi(t_{i-1}) \right]$$  \text{Equation 3-36}$$

or alternatively:

$$P_{(t_i)}^\text{fl} = w_{P_{w}} \cdot P_{(t_i)}^\text{raw} + w_{P_{w}} \left[ P_{(t_i)}^\text{fl} + (\Phi(t_i) - \Phi(t_{i-1}) \right]$$  \text{Equation 3-37}$$

where $w_{P_{w}} = 1 - w_{P_{w}}$. For the first epoch $i=1$, the weight factor $w$ is set to 1, thus placing the full weight on the measured code pseudorange. For consecutive epochs, the weight on the code is reduced and progressively increased on the carrier-phase measurements. The time taken for the phase-filter to become stable and impart the influence of phase measurements is known as the convergence time and is defined by the operator as $N_{\text{max}}$.  

Because of its increasing weight on the phase measurements, it is preferable that the carrier-phase data should be free of as many irregularities as possible. Cycle slips may be detected by a simple check of the carrier-phase differences for two consecutive epochs, for example by the multiplication of the Doppler shift by the time interval. As the wavelengths of L1 and L2 phase measurements are small, 0.19 and 0.24 metres respectively, it can be very difficult to identify cycle slips when the reference measurements are code pseudoranges. The noise (precision) associated with these ranges can be several metres in size due to residual errors meaning that several phase cycle slips can be overlooked. Consequently only large numbers of cycle slips can be detected using code pseudoranges.

Should a slip occur, the weight is reset to $w=1$ placing full confidence in the code pseudorange whilst eliminating the influence of the erroneous carrier-phase data. Therefore, in this method, cycle slips must be detected but do not need to be corrected. Signal interruptions will cause discontinuities if less than 4 satellites remain on lock as there is insufficient information with which to determine a unique solution. Filtering algorithms are also applicable to single-frequency data sets and will be discussed in this research.
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Given the opposite signage on the code and carrier-phase ionospheric delays (cf. Equations A-3 and A-4), a single-frequency filtering process could not continue endlessly without regulation. Ashajee et al [1989] suggest that only 2 hours of continuous tracking could result in a few metres of ionospheric bias within the filtered pseudoranges. Their solution was to design ramping techniques in which a number of parallel filters, dual or multiple, were run through the dataset. A moving-window concept was used to record the last \( N \) measurements and provide an average measurement at the leading epoch. Using the dual-ramp example, two ramps (moving-windows) would be started \( N/2 \) measurements apart, each containing the results of the phase-smoothing using Hatch's original algorithm at a 1 Hz rate for \( N \) equal to 200 measurements. Each ramp resets when it reaches \( N_{\text{max}} \) and the weighting emphasis is switched to the other ramp that is at 100 measurements of \( N \). These resets, at intervals of 100 seconds, caused jumps of \(~10\) centimetres in position for Ashajee et al [1989] but are inherently better than dropping back 200 seconds to \( N=0 \) measurements where there is no benefit from phase weighting. Multiple ramps obviously further reduce the magnitude of the drop in smoothing measurements and positional accuracy.

Lachapelle et al [1989] suggested a similar routine in which two parallel filters are run alongside a main filter. The main filter is started at the beginning of the dataset and operated as normal according to its time smoothing constant. The first parallel filter is started at the same epoch as the main filter but resets each time a user-specified maximum count or period occurs, typically a few minutes. The second parallel filter is re-initiated at half the period of the first. Just before one of the parallel filters is reset, its phase-smoothed pseudorange is compared to that of the main filter. If the difference exceeds some user-defined value, say 2-3 metres, the main filter is reset by averaging its values with that of the parallel filter. Over a test dataset of 30 minutes, they found that with a 120 second filter length, the code multipath exhibited peaks of 20 metres and periods of 2-3 minutes. Discontinuities at the level of several metres were noticed immediately after the reset times when code multipath significantly affected the now unweighted pseudoranges.

**Choice of Values for \( N_{\text{max}} \)**

In his method, Hatch [1982] does not provide any specific values for the parameter \( N_{\text{max}} \), although found that his Doppler-filtering method performed acceptably with smoothing time constants of up to 80 seconds. In Ashajee [1989], the \( N_{\text{max}} \) values were
Techniques for Filtering Code Pseudoranges

limited at 200 seconds to prevent excessive ionospheric divergence due to the phase advance and group code delay caused by signal travel through the ionosphere. Lachapelle et al [1989] used a maximum count of 120 seconds. Recommendations made for the LAAS Minimum Aviation System Performance Standards (MASPS) state that receivers should use carrier-smoothing time constants of 100 seconds [McGraw et al, (2000)].

Given that the major concern within such filtering techniques is that of excessive ionospheric divergence, some groups have implemented ionospheric path delay correction routines within the single-frequency filtering routine [Hwang et al, (1999)]. By incorporating the broadcast ionosphere model, around 50% of the ionospheric delay on the L1 code pseudorange [Langley, (1997)] can be mitigated on a day with average ionospheric activity. Presumably there is still some divergence associated with the remaining ionospheric delay that has not or cannot be modelled explicitly. The problem of excessive ionospheric divergence can be overcome by using dual-frequency phase-filtering [Hatch, (1982); Hwang et al, (1999)]. The dual-frequency observables are combined linearly to represent the divergence of the ionosphere between code and carrier-phase measurements, as will be detailed later in this section. Muellerschoen et al [1999] assume that the multipath component on code pseudoranges can be calculated from the standard deviation of the difference between the ionosphere-free linear combinations of code and carrier-phase data.

For Doppler-aided smoothing of code ranges as originally researched by Hatch [1982], Cheng [1999] has reconsidered the validity of Doppler measurements for cycle slip occurrences and single-frequency phase-filter resets. Cheng's research is dependent upon the condition that ‘the accuracy of the Doppler-smoothed code range is better than the accuracy of the raw code pseudorange’ - Cheng [1999], and that the Doppler frequency measurements are immune from cycle slips. The accuracy of a delta-range\(^{10}\) calculated from Doppler is related to both the accuracy of Doppler frequency observables and the epoch interval. The fact that the accuracy of the delta-range is higher for a smaller epoch interval, makes it possible to use Doppler data for continuous smoothing.

Cheng [1999] suggests that Doppler-aided smoothing be used to complement carrier-aided smoothing in times of cycle slips. This is done by implementing a second ramp, containing Doppler measurements, alongside the carrier-phase ramp. At epoch \(t\), when a

\(^{10}\) The delta-range of an observable is equal to the rate of change of that observable between two epochs [Cheng, (1999)].
cycle slip occurs and disrupts the phase ramp, delta-range measurements from the immune Doppler ramp can be used within the filtering routine. For the next epoch \(t+1\), providing no subsequent cycle slip has occurred, the phase delta-range will be used within the filtering routine. This concept is very attractive in terms of continuous uninterrupted single-frequency phase-filtering, however the availability of Doppler measurements within receiver output files, either proprietary or industry standard such as RINEX, is becoming increasingly less commonplace.

3.3.3 Design of Carrier-Phase Filtering Algorithms

A carrier-phase filtering routine was devised for this research software based on the concepts described by Hatch and Lachapelle earlier. A number of additional variables are required to store the respective phase weights for each satellite processed and details of any interruptions that may occur on each satellite. The number of successful uninterrupted filtering observations for each satellite, \(no\_filt\_obs\), must be noted along with a statement as to whether a loss of lock has been detected at that epoch.

In order to make the quantities equal and permit easier visualisation, it is necessary to convert the carrier-phase measurements from units of cycles into units of range by multiplying by the relevant wavelength.

\[
\phi_i(\text{metres}) = \phi_i(\text{cycles}) \cdot \frac{c}{fL_i}
\]

Equation 3-38

It is necessary to check, for each satellite, the number of uncorrupted phase pseudoranges used in the filter so far, to determine whether \(N_{max}\) has been reached by the filter. If so, the value for \(no\_filt\_obs\) has to be redefined as \(N_{max}\) whose value is specified by the user.

**Cycle Slip Detection Routine**

As discussed earlier, it is critical to detect if a cycle slip has occurred on the L1 phase frequency (and on L2 if the dual-frequency algorithm is used). Within the slip detection routine, a conditional algorithm must be satisfied in terms of two components and takes the form:
Techniques for Filtering Code Pseudoranges

\[
\text{if (no\_filt\_obs} \geq 2) \quad \text{and } (\rho(t_i) - \rho(t_{i-1}) - \phi(t_i) - \phi(t_{i-1})) > \text{slip}
\]

Equations 3-39(a) and (b)

For the first condition, Equation 3-41(a), the number of filtering observations recorded on that particular satellite since the phase-filter began, or since the last cycle slip, must be equal to or greater than two epochs. Were it only equal to one epoch, then it would indicate that no previous filtering information was available so it could not be known whether a slip had occurred. Accordingly the code and phase measurements are then afforded equal weighting. The second component contains the actual detection routine as based on the actual delta-range values for two consecutive epochs, \((\rho_{t_i} - \rho_{t_{i-1}})\). Should the difference in code and phase ranges between the current and previous epoch be greater than the slip threshold, then a cycle slip is believed to have occurred and the phase filter for that satellite must be reset.

The next step is to check consecutive carrier-phase measurements for the presence of cycle slips. This is done by comparing the rate of change of phase pseudoranges for the current epoch and its two preceding epochs. For example, given three consecutive phase pseudoranges, \(\phi_{t_i}, \phi_{t_{i-1}}\) and \(\phi_{t_{i-2}}\), it is possible to calculate two independent phase delta-range values, \(\Delta \phi_{12}\) and \(\Delta \phi_{23}\). A similar series of delta-ranges are calculated for the corresponding code pseudoranges. By comparing these consecutive parameters, it is possible to detect whether a cycle slip has occurred; if their difference is not zero, then a cycle slip has occurred. However because the measurements, in particular the code pseudoranges, are influenced by other errors apart from multipath, a buffer threshold was used to overcome their influence. If the combined difference between consecutive code and phase delta-ranges exceeds this buffer, then it is assumed that a cycle slip has occurred and the weighting routine is reset for that particular satellite.

For a phase-filtering scenario whereby dual-frequency observations are used, a slightly more complex cycle slip detection routine must be applied to both frequencies. If a cycle slip is found on only one frequency, then the phase-filter obviously has to be reset for that particular satellite and the pseudorange at that epoch will subsequently be filtered using the remaining single-frequency. The weight afforded to that filtered pseudorange will not be as high as if both frequencies had contributed but obviously much higher that a raw unfiltered code pseudorange. A flowchart schematic illustrating the dual-frequency cycle slip detection routine can be seen as Figure 5-4.
Criteria for the $N_{\text{max}}$ values dictated how many successful filtering observations could be used before the effects of ionospheric divergence became too significant and affect the processed datasets. Obviously if $\text{no}_{\text{filt}}_{\text{obs}}$ were equal to a single observation, then there will have been no previous information with which to calculate delta-ranges. In this case, the code and phase ranges measurements will be afforded equal weight within the phase-filter and the original raw, i.e. unfiltered code pseudorange measurement must be used.

### 3.3.3.1 Single-frequency Model

The following algorithm was given by Hatch [1982] for the filtering of L1 code pseudoranges with L1 carrier-phase measurements at epoch $i$:

$$P_i^{\text{fil}} = \frac{P_i^{\text{raw}}}{N_{\text{max}}} + \frac{(P_{i-1}^{\text{fil}} + \phi_i - \phi_{i-1}) \cdot (N_{\text{max}} - 1)}{N_{\text{max}}}$$

Equation 3-40

where:

- $N$: number of consecutive uninterrupted phase pseudorange observation used to calculate the phase delta-range
- $N_{\text{max}}$: maximum number for $N$ such that the significant effects of ionospheric divergence between the code and phase measurements are limited
- $\phi_i - \phi_{i-1}$: phase delta-range between epochs $i$ and $i-1$

### 3.3.3.2 Dual-frequency Model

The format for the dual-frequency filtering algorithm differs given the presence of the second carrier-phase frequency. This is included in an attempt to restrict the ionospheric divergence between the code and carrier-phase observations. The two additional parameters, $M_i^1$ and $M_i^2$, [after Hatch, (1982)], relate to the dual-frequency algorithm and contain terms to calculate the delta-ranges on both L1 and L2. These ionospherically modified phase delta-ranges reflect the ionospheric effects on the code pseudoranges between epochs $i$ and $i-I$, can be calculated for L1 observations from:

$$M_i^1 = \left[ \left( \frac{L_1^2 + L_2^2}{L_1^2 - L_2^2} \cdot (\phi_i - \phi_{i-1}) \right) - \left( \frac{2L_2^2}{L_1^2 - L_2^2} \cdot (\phi_i^2 - \phi_{i-1}^2) \right) \right] \equiv (P_i^1 - P_{i-1}^1)$$

Equation 3-41
Techniques for Filtering Code Pseudoranges

where $L_1$ and $L_2$: frequencies of the L1 and L2 carriers respectively (cf. Figure 1-1)

$\phi_{i}^{1} - \phi_{i-1}^{1}$: delta-range for L1 phase pseudoranges

$\phi_{i}^{2} - \phi_{i-1}^{2}$: delta-range for L2 phase pseudoranges

$P_{i}^{1} - P_{i-1}^{1}$: delta-range for L1 code pseudoranges

Accordingly, when considering the P-code observables on the L2 frequency, the phase delta-range term $M^2$ takes a different form:

$$M_i^2 = \left[ \frac{2L_1^2}{L_1^2 - L_2^2} \cdot (\phi_i^1 - \phi_{i-1}^1) \right] - \left[ \frac{L_1^2 + L_2^2}{L_1^2 - L_2^2} \cdot (\phi_i^2 - \phi_{i-1}^2) \right] \equiv (P_i^2 - P_{i-1}^2)$$

Equation 3-42

The frequency terms present in the constants $M^f$ and $M^2$ correspond to the inter-frequency relationships between L1 and L2 as shown in Table 3-2:

<table>
<thead>
<tr>
<th>FREQUENCY COMBINATION</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{L_1^2 + L_2^2}{L_1^2 - L_2^2}$</td>
<td>4.091456</td>
</tr>
<tr>
<td>$\frac{2L_1^2}{L_1^2 - L_2^2}$</td>
<td>5.091456</td>
</tr>
<tr>
<td>$\frac{2L_1^2}{L_1^2 - L_2^2}$</td>
<td>3.091456</td>
</tr>
</tbody>
</table>

Table 3-2 L1 and L2 GPS Frequency Constants required when applying Dual-frequency Carrier-phase Filtering (cf. Figure 1-1)

By including these dual-frequency delta-range terms, $M^f$ and $M^2$, within the single-frequency algorithm as Equation 3-42, a filtered L1 pseudorange, i.e. either C1 or P1, can be calculated from:

$$P_i^{f1} = \frac{N-1}{N_{\max}} (P_i^{f1} + M_i^1) + \frac{1}{N_{\max}} P_i^{new}$$

Equation 3-43
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which simplifies to:

$$P_i^{\beta j} = \frac{P_{i-1}^{\beta j} + \left(P_{i}^{\beta j} + M_i^j\right) \cdot (N - 1)}{N_{\text{max}}}$$  \hspace{1cm} \text{Equation 3-44}$$

The final algorithm to be used for the phase-filtering of an L1 pseudorange using
dual-frequency carrier-phases takes the form:

$$P_i^{\beta j} = \left(\frac{P_i^{\text{raw}}}{N_{\text{max}}} \right) + \left(\frac{\left(\frac{L_1^2 + L_2^2}{L_1^2 - L_2^2} \left(\phi_i^1 - \phi_{i-1}^1\right)\right)}{N_{\text{max}}} \right) \cdot \left(\frac{2L_2^2}{L_1^2 - L_2^2} \left(\phi_i^2 - \phi_{i-1}^2\right)\right) \cdot (N - 1)$$  \hspace{1cm} \text{Equation 3-45}$$

Note that the term $N_{\text{max}}$ has been included in the dual-frequency algorithm. Even
though this is meant to prevent excessive ionospheric divergence, some limit needs to be
applied to the number of successive observations that should be used within the filtering
procedure.

Once the filtered pseudorange has been calculated at epoch $(t_i)$ from whichever
algorithm, it will then be assigned within the research software structure as the previous
filtered range for epoch $(t_{i+1})$. These assignments are required for the ongoing cycle slip
detection routines as well as the filtering algorithm. If the phase-filtering algorithm has
been successful, i.e. uninterrupted, then the routine will return the filtered pseudorange
$P_i^{\beta j}$ for use in the least squares computation as discussed later in this section.

As the phase-filtering routine is carried out prior to the adjustment routine, its
execution bears no impact on the final choice of functional model used, be it single point
positioning, or differential positioning using conventional or double-difference DGPS.
3.3.4 Study into Phase-filtering Constants

The previous section has covered the format of the different filtering algorithms based on the L1 frequency with the additional presence of L2 measurements. The following sections assess the differences between these different carrier-phase filtering techniques through a quantitative assessment of their relative performances with some discussion as to the optimal value of the filtering time constant. The size of the threshold for the cycle slip detection routines will, of course, affect the number of observations that are detected as being subject to cycle slips. For the purposes of this research, the slip threshold has been set at 15 metres.

In both studies, the same parameters have been used for pre-processing and processing tasks. The pre-processing tasks include calculation of ionospheric and tropospheric path delays using the broadcast ionosphere and refined Saastamoinen troposphere models respectively (cf. §5.2). The functional model consists of C/A-code and P-code pseudorange on the L1 and L2 frequencies respectively depending on the studies. These observables were double-differenced every 1 second with respect to broadcast satellite ephemerides. The stochastic model for these tests was set at the simplest level - a unit weight matrix whereby the observations were assumed to have equal precision and be independent of each other (cf. §4.1). By doing this, the final performance indicators for these studies would only be affected by the deliberate variations within the phase-filtering routines.

3.3.4.1 Magnitude of the Filtering Time Constant $N_{max}$

The value for the filtering time constant, $N_{max}$, is important for the phase-filtering routine. In the single-frequency algorithm, this value can not be too high otherwise the effects of ionospheric divergence between the code and carrier-phase measurements will become significant and bias the quality of the filtered pseudoranges. Ashajee et al [1989] stated that smoothing the code with integrated Doppler reduces the code phase noise by the "square of $N" where $N$ is the number of continuous smoothing epochs since initialisation or the last loss of lock. With an Ashtech M-XII receiver (predecessor to the popular Z-XII receiver) Ashajee et al [1989] found that the nominal 1 metre of code noise reduced to 10 centimetres after 2 minutes of tracking. As mentioned in §2.1.4, Zhdanov et al [1999] found in their studies that a filtering time constant of 50 seconds was optimal and yielded a factor of 2 improvement in receiver performance accuracy.
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Of the noted references that discuss the settings for the generic carrier-phase filtering (this includes filtering with pure Doppler count measurements), a range of values have been used for the filtering time constant as in Table 3-3.

<table>
<thead>
<tr>
<th>REFERENCE</th>
<th>VALUE FOR N_{\text{max}} (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hatch, [1982]</td>
<td>80</td>
</tr>
<tr>
<td>Ashajee et al, [1989]</td>
<td>200</td>
</tr>
<tr>
<td>Lachapelle et al, [1989]</td>
<td>120</td>
</tr>
<tr>
<td>Zhdanov et al, [1999]</td>
<td>50</td>
</tr>
<tr>
<td>McGraw et al, [2000]</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 3-3 Examples of the Range of Filtering Time Constants used within Quoted References

In general, the filtering time constant is in the 50-200 second range and should, for single-frequency applications, be chosen subjectively with regards to the excessive divergence of ionospheric delays (cf. §2.1.3).

3.3.4.2 Comments on the Phase-filtering Algorithms and Frequencies

To assess the relative benefits of each method, the optimal filtering time constant for both single and dual-frequency filtering methodologies must be determined. For comparative purposes both frequency combinations have been implemented and studies made into the performance of the single-frequency and dual-frequency filtering models. A dual-frequency dataset was collected at a 1Hz rate for a static baseline between the stations BEDS in Newcastle, England, and ONSA in Onsala, Sweden (cf. Figure 6–1) comprising Ashtech Z-XII receivers. The dataset subset used was clean and nominally 3250 epochs long. The baseline length of 820 kilometres has no bearing on the results afforded by the phase-filtering routines, as any distance dependent biases will be constant for all tests.

A number of tests were applied to this dataset, the first series of which were to determine the optimal value for the value \( N_{\text{max}} \) for both models. The second series of tests considered the performance of the single and dual-frequency methods against one another at each \( N_{\text{max}} \) value to illustrate the possible effects of ionospheric divergence.
Techniques for Filtering Code Pseudoranges

**Determination of Optimal $N_{\text{max}}$ values**

Six values for $N_{\text{max}}$ were selected as follows: 0, 10, 50, 100, 200 and 1000 seconds. The zero value provided a benchmark (unfiltered) dataset.

**Single-frequency Filtering**

When the dataset was processed with the single-frequency filtering algorithm featured in Equation 3–42, the following results were obtained as shown in Table 3–4. The accuracies of the different filter settings are shown as RMS errors and their precisions as standard deviations.

<table>
<thead>
<tr>
<th>$N_{\text{max}}$ (s)</th>
<th>RMS ERROR (m)</th>
<th>STANDARD DEVIATION (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lat</td>
<td>Lon</td>
</tr>
<tr>
<td>0</td>
<td>0.145</td>
<td>0.310</td>
</tr>
<tr>
<td>10</td>
<td>0.144</td>
<td>0.309</td>
</tr>
<tr>
<td>50</td>
<td>0.089</td>
<td>0.314</td>
</tr>
<tr>
<td>100</td>
<td>0.100</td>
<td>0.306</td>
</tr>
<tr>
<td>200</td>
<td>0.180</td>
<td>0.340</td>
</tr>
<tr>
<td>1000</td>
<td>0.124</td>
<td>0.393</td>
</tr>
</tbody>
</table>

Table 3–4 Performance of Single-Frequency Phase-filtering Algorithm using Candidate $N_{\text{max}}$ Values

As can be seen from the above results for the single-frequency filter, an increased $N_{\text{max}}$ value increases the precision of the filtered time series. In terms of overall accuracy, the filtering time constant values of 50 and 100 seconds perform best for this dataset but in truth, the spread of the overall position errors is not significantly different between the six settings, only around 17%. To all intents and purposes, the accuracy achievable with these different parameters settings is the same for single-frequency filtering. Note that although the mean accuracy decreases as the value for $N_{\text{max}}$ increases, the precision steadily increases, yet it is difficult to say why this is so.

When applying filtering processes, it can be helpful to consider the actual epoch-by-epoch values as seen in time series plots. Figure 3–6 illustrates such time series of position error vectors as related to the true position of ONSA using single-frequency filtered pseudoranges. It can be seen that the resetting of $N_{\text{max}}$ for the 200 and 1000 second
windows has effected the lower accuracies, and the larger window lengths have effected the higher precision of the errors as seen by a smoother time series.

Figure 3-6 Time Series of Position Error Vectors as determined using the Candidate $N_{max}$ Values within the Single-frequency Phase-filtering Algorithm
**Dual-frequency Filtering**

The same \( N_{\text{max}} \) values were used within the dual-frequency filtering algorithm (cf. Equation 3–47) and yielded the results as seen in Table 3-5. Again RMS errors and standard deviations are used to describe the accuracy and precisions of the results.

<table>
<thead>
<tr>
<th>( N_{\text{max}} ) (s)</th>
<th>RMS ERROR (m)</th>
<th>STANDARD DEVIATION (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lat</td>
<td>Lon</td>
</tr>
<tr>
<td>0</td>
<td>0.145</td>
<td>0.310</td>
</tr>
<tr>
<td>10</td>
<td>0.153</td>
<td>0.293</td>
</tr>
<tr>
<td>50</td>
<td>0.135</td>
<td>0.300</td>
</tr>
<tr>
<td>100</td>
<td>0.127</td>
<td>0.308</td>
</tr>
<tr>
<td>200</td>
<td>0.125</td>
<td>0.325</td>
</tr>
<tr>
<td>1000</td>
<td>0.153</td>
<td>0.365</td>
</tr>
</tbody>
</table>

*Table 3-5 Performance of Dual-Frequency Phase-filtering Algorithm using Candidate \( N_{\text{max}} \) Values*

Again the choice of \( N_{\text{max}} \) has a significant impact on the time series’ overall precision. Figure 3-7 illustrates error vector time series as deduced from data passed through the dual-frequency phase-filtering algorithm.
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Figure 3-7 Time Series of Position Error Vectors as determined using the Candidate $N_{\text{max}}$ Values within the Dual-frequency Phase-filtering Algorithm

The benefit of using the dual-frequency algorithm over that of the single-frequency method, is in the protection it affords from excessive ionospheric divergence between the code and carrier-phase measurements as noted in Hwang et al [1999]. This can be especially noticeable over longer time spans and periods of increased ionospheric activity. Consider the differences in position afforded by the solutions determined using single and dual-frequency filtering. Figure 3-8 shows the differences between single and dual-frequency filtered positions and emphasises the ionospheric divergence between the code and carrier-phase measurements, especially using an extreme $N_{\text{max}}$ value of 1000 seconds.
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Figure 3-8 Time Series of Vector Differences between Position Solutions computed with the Single- and Dual-Frequency Carrier-Phase Filtering Algorithms using the Candidate $N_{\text{max}}$ values

These graphs illustrate the differences in position between the single- and dual-frequency filtering algorithms for each $N_{\text{max}}$ candidate within these studies. The most noticeable features are the step functions as can be seen within Figure 3-8, in particular for $N_{\text{max}} = 1000$ seconds, and also a slight but steady divergence seen in the time series attributed to the effects of ionospheric divergence. The mean differences between the single- and dual-frequency filtering methodologies respectively using the range of $N_{\text{max}}$ values are listed in Table 3-6.
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<table>
<thead>
<tr>
<th>$N_{\text{max}}$ (s)</th>
<th>DIFFERENCE IN RMS ERROR (m)</th>
<th>DIFFERENCE IN STANDARD DEVIATION (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>d Lat</td>
<td>d Lon</td>
</tr>
<tr>
<td>10</td>
<td>0.009</td>
<td>0.603</td>
</tr>
<tr>
<td>50</td>
<td>0.046</td>
<td>0.614</td>
</tr>
<tr>
<td>100</td>
<td>0.028</td>
<td>0.614</td>
</tr>
<tr>
<td>200</td>
<td>-0.055</td>
<td>0.665</td>
</tr>
<tr>
<td>1000</td>
<td>0.029</td>
<td>0.758</td>
</tr>
</tbody>
</table>

Table 3-6 Differences of the Mean Receiver Position yielded by the Single-frequency and Dual-frequency Phase-filtering Algorithms under the Candidate $N_{\text{max}}$ Constants

Care must be taken when assessing time averaging routines, as there are no official thresholds for such an activity. Looking at the last ten plots covering the single-frequency and dual-frequency results, a considerable amount of high-frequency information has been eliminated as the filtering time constant increases. Comparing the null benchmark for single-frequency filtering (cf. Figure 3-6) to the extremes of 10 and 1000 seconds, it is difficult to identify the filter resets for the former, although the latter contains three clearly visible step functions as the 1000 second filter resets (cf. Figure 3-8).

3.3.5 Concluding Remarks on Carrier-phase Filtering Routine

The studies have shown that the inclusion of carrier-phase filtering, whether single- or dual-frequency, affords significant improvement in code multipath and receiver noises. While phase-filtering reduces the level of noise on a pseudorange, the receiver may lose lock due to ionospheric disturbances (single-frequency only) or phase losses of lock (both frequencies) which will affect both the continuity of the phase-filter and the quality of the filtered ranges.

For both frequency combinations, the overall accuracies are very similar as are the overall position solutions. This is because the multipath effects average out over time. If there is any difference, it is that the dual-frequency algorithm yields very slightly more accurate positions.

The analysis and subsequent selection of the ‘optimal’ averaging parameter is highly subjective on two counts: maintaining a longer averaging value removes more high-frequency noise albeit at the cost of removing important ephemeral information. With
Techniques for Filtering Code Pseudoranges

consideration of the above time series plots, the filtering time constant for both filtering algorithm formats has been selected as 100 seconds. As can be seen from the studies carried out earlier, this value can reduce a significant amount of high-frequency noises within the code pseudoranges however it also leaves some information about the time-varying quantities that can be used in later modelling procedures (cf. §4.4).

The reason why Doppler values have not been used within these filtering routines is that although most geodetic quality receivers use them internally, they are not usually made available to users in their output files. This can be seen by their absence in most RINEX files as created from permanent reference stations.

It must be noted that for single point positioning, the use of carrier-phase, or even Doppler-based filtering will not improve the accuracy of the position significantly. This is because the information needed to obtain the absolute position is contained within the ambiguity-free code measurements and they are subject to greater magnitudes of noise than the carrier-phase measurements. The errors induced by SA are far larger than any achievable reduction in code multipath and noise. The more precise carrier-phase measurements have their inherent integer cycle problem (cf. §1.3.2) [Lachapelle et al, (1996)]. However the use of filtered pseudoranges in DGPS applications will and does improve the precision measures due to the noise reductions they afford. Even following the termination of SA, the combined effect of ionospheric and tropospheric delays will typically be greater than the phase-filter corrections meaning that again little improvement in accuracy would be noticed. The precision estimates would be improved though as high-frequency trends and spikes due to multipath would be mitigated.

The following example illustrates how the use of carrier-phase filtering can improve positioning performance in terms of both precision and reduced noise levels within the final position solutions. For comparative purposes, the noisier time series shown in Figure 3-9, corresponds to the raw (unfiltered) code solution as featured earlier in Figure 3-5.
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Figure 3-9 Time Series Plot showing the Absolute Height of ONSA Station using Raw and Dual-frequency Carrier-phase Filtered C/A-code Pseudoranges (cf. Figure 3-5)

Note that for the filtered height time series displayed in Figure 3-9, a reduction in the noises can be seen within the first 80 epochs or so as the solution converges towards the true height of ONSA – 46.565 metres. This lag corresponds to the initialisation time of the phase-filter and the increasing weight afforded to it. The lag’s size is entirely dependent on when the filter is introduced into the height time series and how far the estimated position is from the true position. Note that the means of the two time series are very similar which can be attributed to the multipath effects averaging out over the dataset of 3000 seconds. An artefact can be seen residing within the filtered height time series with a period of around 2.5 minutes and 1 metre amplitude. This is attributed to low-frequency residual multipath.

From Table 3-7, it can be seen that the application of the phase-filtering routine reduces the range of high-frequency noises within the pseudoranges by 77%, yielding an improvement of 76% in the precision of the height estimates. Note that the first 100 seconds of both series has been omitted from these statistics to exclude transient time as the filter populates and converges. Most commercial positioning packages impose similar restrictions on the solutions outputs until their navigation filters have stabilised to acceptable levels.
Techniques for Filtering Code Pseudoranges

<table>
<thead>
<tr>
<th>CODE RANGES</th>
<th>MEAN (m)</th>
<th>MINIMUM (m)</th>
<th>MAXIMUM (m)</th>
<th>RANGE (m)</th>
<th>STD DEV (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw</td>
<td>46.55</td>
<td>41.19</td>
<td>51.87</td>
<td>10.68</td>
<td>1.77</td>
</tr>
<tr>
<td>Phase-filtered</td>
<td>46.56</td>
<td>45.31</td>
<td>47.72</td>
<td>2.41</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Table 3-7 Performance of Carrier-phase Filtering on C/A-code Pseudoranges from the Height Time Series illustrated in Figure 3-9 for epochs 100 to 3000

Figures 3-10 and 3-11 show the elevation angles for two satellites in the BEDS-ONSA dataset; SV10 and SV24. SV10 begins at just over 50° elevation and is rising, whereas SV24 is a low elevation satellite falling over 20° to finish the time series at approximately 15°. Also included are time series of the range corrections applied by the dual-frequency carrier-phase filter where $N_{max}$ equals 100 seconds. Because of the research assumption that any noises reduced by the carrier-phase filter are multipath and receiver noises combined, the corrections applied can be deemed to be equal to the code multipath incident on the observed code pseudoranges.

Figure 3-10 Time Series Plot showing the Elevation Angle and Phase-filter Corrections for SV10
From these two time series plots, it can be seen that the assumed multipath on the code pseudoranges can reach around 4 metres. The assumption about an elevation angle dependency can be confirmed by interpreting the range corrections and satellite elevation angle; there is an increase in size of the range corrections as the elevation angle of the satellite decreases.

3.4 CONCLUDING REMARKS

The concept of least squares estimation has been introduced in terms of the double-difference functional model with particular emphasis on the determination of baseline vectors for differential positioning. These functions form the basis of the research software as summarised in Chapter 5.

Once the pre-processing tasks have been carried out, the corrected and filtered pseudoranges will then be passed through the double-difference functional model with broadcast satellite ephemerides. Even though double-differencing GPS observations nominally eliminates the effects of satellite-common errors, the resultant position solutions are still contaminated by several residual biases due to orbital errors, ionospheric and tropospheric delays, multipath errors and system noise.
It has been shown that code multipath can be reduced considerably by over 75% with the application of carrier-aided phase-filtering techniques. It has also been shown that there is a high degree of correlation between code multipath and the transmitting satellite’s elevation angle. Modelling these remaining errors functionally can be very difficult and therefore they must be modelled stochastically. The next step is to determine the stochastic functions to do just this.
Chapter 4 - Stochastic Modelling for Differential Positioning

This chapter provides background to the concept of stochastic modelling along with discussion on the structure and population of the covariance matrix. There is a review of some current stochastic modelling algorithms for differential positioning with code and carrier-phase measurements. This is followed with background on the design of candidate algorithms by the author to model the precisions of carrier-phase filtered code pseudoranges for use within this research. Discussion is also made into the use of unit variance scaling factors, as derived from the moving window of unit variance.

4.1 CONCEPTS OF STOCHASTIC MODELLING

In standard least squares theory, two models are required to represent the observations in the computation procedure: - firstly, a functional model is needed to determine the adjustment parameters from observations, essentially how to compute the observations. Secondly, a stochastic model is required to describe the quality of the fit of the observations to the functional model.

The errors associated with these observations can be modelled in one of two ways, firstly by including additional corrections within the functional model and removing the errors there, or by including details within the stochastic model of how these errors would affect the observations. In principle, as both approaches are essentially equivalent [Blewitt, (1997); Gregorius, (1998)], the choice of stochastic model is dictated by the functional model applied. In practice, it can be very difficult to model the pseudorange measurement errors rigorously within the functional model [Han, (1997)] and therefore they must be modelled stochastically through the least squares covariance matrix of observations, $C_i$.

Interpretation of the Covariance Matrix

Once the covariance matrix of undifferenced observations $C_i$, has been created, two further covariance matrices can be computed. The first is $C_x$, the covariance matrix of adjusted parameters, and the second is $C_v$, corresponding to the post-fit residuals. The
covariance matrix is a symmetrical positive-definite matrix that has dimensions equal to the number of observations within the functional model. Within this matrix representing the stochastic model, the variances describe the quality of the observations and the covariances describe the correlations between, as well as the quality of, the observations.

A covariance matrix populated with undifferenced observations is shown below:

$$C_i = \begin{bmatrix}
\sigma_a^2 & \sigma_a \sigma_b & \cdots & \cdots \\
\sigma_a \sigma_b & \sigma_b^2 & \cdots & \cdots \\
\vdots & \vdots & \ddots & \cdots \\
\sigma_a \sigma_n & \sigma_b \sigma_n & \cdots & \sigma_n^2
\end{bmatrix}$$

Equation 4-1

It is the population of this matrix with terms that describe the mismatch of the GPS data with the functional model that is of prime interest in this research. Two types of correlation exist within double-difference code pseudorange measurements, physical and mathematical, both of which need to be represented correctly within the stochastic model.

**4.1.1 Physical Correlation**

Because the measurement environment encountered by the GPS observations is assumed to be similar, it is then fair to assume that the observation errors will exhibit some spatial and temporal correlation (cf. Figure 4-1). The correct understanding of these physical correlations is essential for deriving a correct stochastic model.

![Figure 4-1 Physical Correlations in GPS Double-differencing](image)
As discussed in §2.1, the spatially correlated errors are those which vary from point to point in space at a given time. Terms will be created to represent the residual errors associated with these error sources, namely the satellite orbits, the ionosphere and the troposphere.

If the sizes of certain errors at an epoch are similar in size to those same errors some time later, these quantities are said to possess temporal correlation. Examples of such quantities are the errors within satellite clocks (with SA being the largest component by far), delays due to the ionosphere and troposphere, and code signal multipath. SA and multipath are, in general, faster changing error sources than the others, with periods of less than 10 minutes (cf. §2.1) and must be considered in temporal correlation terms. SA is essentially removed through the double-difference process, and code multipath mitigated through the carrier-phase filtering routine. Routines developed to account for variations in residual multipath as will be discussed later in this chapter.

As discussed in §3.1.2, data weights can be introduced into a standard least squares adjustment procedure via the inclusion of the weight matrix $W$ as defined from the covariance matrix of data observations $C_i$ (cf. Equation 4-1):

$$W = C_i^{-1}$$  \hspace{1cm} \text{Equation 4-2}

$C_i$ is populated with the observational variances relating to the code pseudoranges, carrier-phase observations, or as in this research, carrier-phase filtered pseudoranges. Consider the code pseudorange observable as detailed in Equation A-3 and restated as Equation 4-3:

$$P_A^i = \rho^i_A + d\rho_A^i + c\tau_A - c\tau^i + I_A^i + Z_A^i + mp_A^i + e_A^i$$  \hspace{1cm} \text{Equation 4-3}

It is possible to state its overall precision as a function of its component error sources, assuming all are uncorrelated:

$$\sigma_p^2 = \sigma_{\text{orb}}^2 + \sigma_{\text{clocks}}^2 + \sigma_{\text{ino}}^2 + \sigma_{\text{trop}}^2 + \sigma_{\text{mpath}}^2 + \sigma_{\text{rec noise}}^2$$  \hspace{1cm} \text{Equation 4-4}

The determination of these correlations and quantities can be, in practice, very difficult. A combination of new empirical routines and previous research, are the main
motivation for the determination of a-priori variances and covariances. In this research, and in keeping with the objective of creating a general case algorithm, attempts are made to determine the stochastic model from within those measurements as observed by each available GPS receiver.

Because the concept of differential GPS positioning involves two or more receivers observing a number of common satellites, there are physical correlations between the observations in terms of satellite elevation angle $\theta$, and the separation distance $d_{AB}$ between two receivers $A$ and $B$. The following algorithmic functions will be used in the determination of the variances of, and covariances between, the phase-filtered code pseudoranges.

\[
\begin{align*}
\sigma_i^2 &= a_i^2 + (b_i^2 \cdot \csc^2(\theta_i)) \\
\sigma_i \sigma_j &= \sqrt{a_i^2 + (b_i^2 \cdot \csc^2(\theta_i))} \cdot d_{ij} \sqrt{a_j^2 + (b_j^2 \cdot \csc^2(\theta_j))} \cdot d_{ij}
\end{align*}
\]

Equations 4-5 (a) and (b)

The lowercase parameter $a_i$ represents the variances of the constant errors that affect all GPS observations irrespective of receiver location; examples include satellite clock errors under SA and receiver noises. Parameter $b_i$ relates to the variances of errors that depend on receiver location - sources of which are satellite orbits, ionospheric and tropospheric delays. Code multipath is also considered within parameter $b_i$.

For the variances equation, Equation 4-5(a), the spatial correlations within differential positioning are realised according to the elevation angle $\theta$, of an observed satellite relative to the mobile receiver; the smaller the elevation of the satellite, the greater the effects of atmospheric refraction and code multipath effects (cf. §1.2). When considering the covariance between two measurements recorded by two receivers from one common satellite, the precisions of the two measurements must be considered as in the variances algorithm but also the level of correlation in spatial terms. This is done by means of a distance-based factor that represents the decorrelation of distance-dependent errors as the receiver-separation $d_{AB}$ increases.
Temporal Correlations

There are some temporal correlations within the phase-filtered pseudorange datasets related for example, to the rate of change of atmospheric delays on the GPS signals, however the concept of temporal correlation has been ignored in this research.

4.1.2 Mathematical Correlation

The technique of differencing the code pseudorange observations introduces some mathematical correlation into the system, although this is simple to represent with correct modelling. The double-difference code pseudorange observation can be written for one epoch as [Hofmann-Wellenhof et al, (1994)]:

$$\Delta \nabla \rho = R \rho$$  

Equation 4-6

where $\Delta \nabla$ represents the double-difference operator, $R$ is the differencing operator matrix and $\rho$ is the vector of undifferenced code pseudorange observations. The matrix $R$ is one that aids the transformation of a column vector of recorded (undifferenced) data into a column vector of double-differenced data. It is a rectangular matrix with rows equal to the number of linearly independent double-differenced data and columns equal to the number of recorded data. $R$ is also known as the differencing operator. Values of the elements in $R$ must be either +1, -1 or 0 arranged such that they are a linearly independent set of double-differences (cf. Equation 3-9).

To model the mathematical correlations, Gauss' error propagation law is applied to Equation 4-6 and the resultant covariance matrix of the code pseudorange double-difference observations $C_x$, obtained is:

$$C_x = R C_r R^T$$  

Equation 4-7

where $C_r$ is the covariance matrix of the undifferenced code pseudorange errors. The concept of stochastic modelling as followed in this research involves populating the matrix $C_r$ with information pertaining to the precision of the undifferenced observations whilst acknowledging those methods used to determine the estimated parameters within the functional model. This acknowledgement is made through the creation and subsequent population of the differencing operator matrix $R$ (cf. Equation 4-7).
4.2 CURRENT STOCHASTIC MODELLING ACTIVITIES

This section reviews some current stochastic modelling routines used in post-processing and real-time applications. The discussion afforded here covers modelling methods for both code and carrier-phase positioning, as many assumptions about error sources are similar for both observables.

In processing terms, it is generally believed that the GPS observation equations, relating to the functional model, are sufficiently well known and documented, and to all intents and purposes, cannot be advanced significantly further [Han, (1997a); Roberts and Cross [1993]; Barnes et al [1998]; Tiberius, (1999)]. Subsequently this point, along with the requirement for more accurate quality measures, has prompted the increasing interest in the correct population of the variance-covariance matrix. Comparatively speaking, there is limited discussion on the topic of stochastic modelling, and of that, generally most is associated with carrier-phase positioning applications.

4.2.1 Post-processed Applications

In general, most published stochastic modelling routines concentrate on one particular error source within code or carrier-phase measurements, usually that one deemed largest or most influential.

Elevation-Dependent Errors

In his discussion on stochastic modelling for RTK positioning applications, Han [1997b] acknowledges an empirical stochastic model first derived by Jin [1995], in which the precision of code pseudoranges was assumed to be a function of the satellite elevation (cf. §3.3.5). The core stochastic algorithm as derived by Jin [1996] for one-way L1 observations reflects the stochastic characteristics of the observation noise but also residual double-difference biases. The algorithm is as follows:

\[ \sigma = s \left( a_0 + a_1 \cdot \frac{E}{E_0} \right) \]  

Equation 4-8
where $\sigma$ is the standard deviation of L1 observations, and $a_0$, $a_1$ and $E_0$ are approximated by constants, which have been experimentally determined for both code and carrier-phase observations from different kinds of GPS receivers. Typical values for $a_0$ and $a_1$ on code pseudoranges are 7.0 and 60.0 centimetres, and 0.3 and 2.6 centimetres for carrier-phase observations. $E$ corresponds to the elevation of the observed satellite in degrees and $E_0$ is equal to $20^\circ$ [Han, (1997a)]. The scale factor $s$ is determined from analysis of real data over short data sections where it is assumed that the scale factor is constant, i.e. the errors are correlated for longer than the period of the short data sections (cf. §2.1.6). No duration is mentioned for this factor possibly because of this algorithm’s status as a post-processing model.

An elevation angle cut-off of $20^\circ$, as seen in the value of $E_0$, would be acceptable for short-baseline RTK applications. However, in terms of code DGPS, which is generally used over much longer distances, a lower cut-off would be required. Springer [1997] states that with the change of cut-off angle, initially from $20^\circ$ to $10^\circ$, significantly improved the repeatability of the estimate station co-ordinates within the EUREF network as processed by the IGS’ CODE Analysis Centre. In general, the height component improved significantly, whereas the horizontal components worsened - most likely due to satellite geometry [Springer et al, (1997)]. In fact, De Jong [1999b] believes that the application of a basic elevation-weighting routine introduces a time-dependent factor to the stochastic model as a function of elevation over time.

**Code Pseudorange Observables**

RINEXPOS, a GPS and DGPS positioning tool designed by Roberts and Cross [1993], used a simplified stochastic model to obtain a weighted least squares solution from code pseudoranges. Three separate variance components were used to represent the orbital errors, the combined atmospheric errors and a noise figure for the mobile station, as experienced on the undifferenced code pseudoranges:

$$
\sigma_{\text{orbit}} = 2 \sec(Z_M) \\
\sigma_{\text{atmos}} = 20 \left( \sec(Z_R) - \sec(Z_M) \right) \quad \text{Equations 4-9(a), (b) and (c)} \\
\sigma_{\text{noise}} = \frac{5}{\sqrt{sc}}$

where sc is an integer count of the number of consecutive epochs, for each particular satellite, that have been Doppler aided at the mobile station, and \( z \) is the satellite's zenith angle. Roberts and Cross [1993] also carried out positioning tests to investigate the effect of elevation dependency on the final position solutions. Their findings stated that no significant deterioration in positioning accuracy occurs when the elevation mask is reduced to below 5° so long as a weighting strategy, based largely on elevation angle, is employed.

Further post-processing studies by Roberts et al [1997] found that the computed variances within a DGPS error time series did not appear dependent on satellite elevation angle, as is commonly presumed within many commercial packages. The post-processed computation of the covariances between each satellite pair showed that the DGPS pseudoranges were highly correlated due to the influences of atmospheric conditions, receiver environment (multipath effects) and receiver noise. This necessitates the stochastic modelling of these residual biases (cf. §4.1). Their studies used average AoC values of 5-7 seconds for all processing runs.

**Carrier-phase Observables**

Barnes et al [1998] conducted tests whereby L1 carrier-phase positioning was used for a 10 metre baseline dataset heavily contaminated with multipath. A standard double-difference phase functional model was used along with these three stochastic model types:

- **Identity weight matrix** - scaled such that all carrier-phases had a standard deviation of 1.7 millimetres corresponding to the user’s estimates of the receiver’s carrier-phase resolution.
- **Elevation-dependent diagonal covariance matrix of observations** - using a cosine mapping function where the zenith observations were assigned a standard deviation of 1.0 millimetres.
- **Full variance-covariance matrix** - where its contents correctly accounted for the spatial correlation of the measurements - populated from the reverse-engineered variance-covariance analysis of the true double-differences to thus yield true errors.

As the test was conducted over a baseline of only 10 metres using double-differenced phase observables, then the effects of satellite orbital errors, differential ionospheric propagation and differential tropospheric path delay, were assumed to be negligible. The impacts and relative improvements, yielded with the consequential application of these
candidate stochastic models, were analysed in terms of quality of the position solution and the associated quality measures, and the results are summarised in Table 4-1.

<table>
<thead>
<tr>
<th>MODEL</th>
<th>OVERALL HEIGHT RMS (cm)</th>
<th>MAXIMUM HEIGHT ERROR (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variances of Unit Weight</td>
<td>0.59</td>
<td>2.0</td>
</tr>
<tr>
<td>Elevation-dependent</td>
<td>0.43</td>
<td>1.4</td>
</tr>
<tr>
<td>Full variance-covariance</td>
<td>0.22</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Table 4-1 Improvement in Height Accuracy through the Application of a Reverse-Engineered Stochastic Model in a Study by Barnes et al [1998]

The fully correct stochastic model shows a 63% improvement in RMS over the Identity weight matrix, and its true and formal error time series now mainly exhibit random behaviour with occasional increases in noise and no low-frequency multipath fluctuations.

It must be noted that for these results obtained by Roberts and Cross [1993] and Barnes et al [1998], the full variance-covariance matrix has been determined through post-processing and a centred averaging window. Obviously such a routine cannot be used for practical real-time applications. However these results have illustrated the full potential of correctly defined stochastic models.

**General Pseudorange Error Model**

Dixon [1991] used a simple error model:

\[
\sigma = f \sqrt{(a^2, b^2, c^2)}
\]

Equation 4-10

where \( a \) is the receiver and set-up error, \( b \) corresponds to the contribution of the tropospheric delay to the baseline error, and \( c \) is an orbit-related error. Even though this error model is applicable to long-range geodynamic modelling using dual-frequency code and phase observables, no consideration is made for residual ionospheric biases as can be seen by the absence of ionosphere terms within the distance-dependent terms \( b \) and \( c \).
**Ionospheric Modelling**

Many groups have considered stochastically modelling the ionospheric delays to aid faster ambiguity resolution. Amongst those are Odijk [1999, 2000], and Euler and Ziegler [2000] who have used the aforementioned synergy between functional and stochastic modelling (cf. §1.3) to model the residual ionospheric biases through stochastic quantities as opposed to deterministic corrections.

In his ionosphere-weighted model, Odijk [1999] adds stochastic ionospheric corrections to the GPS model as based on dual-frequency phase observations. These corrections can be regarded as pseudo-observations possessing an appropriate variance-covariance matrix to model the ionospheric weights. The advantage of this so-called ionosphere-weighted model, first proposed by Bock [1986] (as cited in Odijk [1999]), is that the benefits of the ionosphere-fixed model (relatively short observation time spans) are realised, as well as those of the ionosphere-float model (relatively long baseline lengths). However as Odijk’s model stands in [Odijk, (2000)], all double-difference ionospheric observations are assumed to have the same precision – at present there are no dependencies assumed between observations in terms of their elevation angle.

Studies by Johnston and Toor [2000], indicate that, although use of the broadcast ionospheric model improves differential positioning accuracies, the Klobuchar model does not accurately reflect the spatial decorrelation of the ionosphere over longer distances. Consequently, attempts have been made to use dual-frequency models to reduce the ionospheric delays, and also refined single-frequency models. This is typical of the approach of many research groups with an interest in modelling residual ionospheric delays. A dual-frequency model was shown to reduce the effects of differential ionospheric delay errors from 10-15 metres to 3-4 metres over a 1500 kilometre baseline [Johnston and Toor, (2000)].

**4.2.2 Real-time Stochastic Modelling**

The implications of applying correct stochastic modelling routines in real-time have been duly noted by research groups investigating the resolution of carrier-phase ambiguities for RTK applications. The reliability of this task is of particular interest. As with post-processing, most groups creating real-time stochastic models concentrate on individual error sources whereas fewer determine general case models for all errors and biases. Typically, a basic elevation-weighting function would be applied to those code
pseudoranges used to derive initial position approximations prior to the carrier-phase
ambiguity resolution procedures.

**Clock Errors**

The satellite clocks are generally regarded as being very stable, varying at only
1-2 parts in $10^{13}$ seconds per day [Langley, (1997)], although the receiver clocks are less
stable (cf. §2.1.2). Muellerschoen et al [1999] demonstrated that by augmenting the
reference and mobile receivers with stable rubidium oscillators, they were able to model
the receiver clocks with some degree of prediction rather than assuming they were subject
to random errors (white noise). Results obtained with dual-frequency measurements
demonstrated that this technique was capable of better than 40 centimetres in the horizontal
and 60 centimetres in the vertical. The accuracy of this system has been verified, through
post-processing, to an accuracy of better than 10 centimetres RMS in all components;
although its general implementation is less than practicable in most navigation systems.

In general, the use of multiple reference stations does not raise many clock-related
problems if considering double-difference positioning, as the clock errors are eliminated.
However if conventional DGPS is used in a WAAS mode, the receiver clock errors can be
estimated in an approach as described in §2.2.3.2. In practice, the technique of double-
differencing removes the effect of satellite and receiver clock biases.

**Multipath Modelling Routines**

The studies conducted by Barnes et al [1998] into carrier-phase multipath modelling
through post-processing (cf. §4.2.1.2) were also applied to real-time datasets through the
implementation of an adaptive carrier-phase multipath error estimation routine as
developed by Comp and Axelrad [1996]. This C/No-based technique (cf. §2.1.4) was
applied in real-time to calculate the double-difference carrier-phase multipath error for
each double-difference phase observation. When applied to the same 10 metre baseline as
for their reverse-engineered studies, this method yielded a 44% improvement in RMS
height errors over a default Identity matrix with no (functional) multipath modelling. It
then yielded a 28% improvement over an elevation-dependent stochastic model, again with
no functional multipath modelling (cf. Table 4-1).

This signal-to-noise ratio (SNR) method also removed a considerable proportion of
low-frequency multipath signals, which is an important benefit as low-frequency multipath
does not generally average out over short periods [Barnes et al, (1998)]. Even so, use of
this estimation method could not eliminate the medium to high-frequency multipath and thus failed to follow the short-term trends in the height error time series.

It must be noted that for the references described here, involving RTK carrier-phase positioning, (Han [1997b]; Teunissen [1997]; Barnes et al, [1998]; Dai, [1999]; Raquet and Lachapelle [2000]; Odijk [2000]) all make use of raw measurements. Even the pseudorange measurements, used to derive the approximate code-based positions, have not been subject to carrier-phase filtering activities. Little mention is made of modelling code and phase multipath, other than assuming constant variances, although it would be assumed that high-frequency errors would be estimated within the measurement noise covariance matrix at each epoch.

**Carrier-phase Integer Ambiguity Resolution**

Reliable ambiguity resolution is highly dependent on applying correct stochastic models. However, for real-time applications without the benefit of past information, it is very difficult to do. Teunissen [1999] showed that the choice of the proper data weight matrix is of significant importance for ambiguity resolution; their weights are optimal when the weight matrix equals the inverse of the ambiguity variance-covariance matrix. Too optimistic or pessimistic a precision description will result in a less than optimal ambiguity success rate.

Wang [2000] found that by directly estimating the variance and covariance components for the differenced measurements from post-fit residuals, success rates for ambiguity resolution and the accuracy of positioning results could be significantly improved. Using the initial preset standard deviations in the covariance matrix of observations, the success rates of single-epoch ambiguity resolution were 49% and 21% for the tested GPS dual-frequency, and combined GPS/GLONASS single-frequency datasets respectively. In contrast, the success rates in three tests of single-epoch ambiguity resolution, using the experimental covariance matrix populated with post-fit residuals, were 87, 92 and 100%, an average of 92%. In this routine, a moving window was used to group the previous section's residuals and then populate the covariance matrix. The optimal size for this window, in terms of highest ambiguity resolution success rates, was in the 7-15 second range.

Han [1997c] has taken Jin's approximate stochastic model [1996] as described in Equation 4-10 and adapted it for the real-time variance estimation of code or carrier-phase observations in RTK applications. In particular, efforts were made to estimate the scale
factor $s$ on a regular basis to reflect the ephemeral variations in the real observational pseudorange data, similar in concept to Wang [2000]. The first data section of course, has no previous information and so is represented by a default algorithm in which the scale factor, $s$ is equal to 1. For the $n^{th}$ section, $s$ is estimated using the $n-1^{th}$ section of data and an algorithm containing terms to summate the number of pseudorange observations, the number of real-valued parameters and the quadratic form of the residuals for the ambiguity-float solution at epoch $k$. Typically the data sections were 2-5 minutes in length. A similar format algorithm was applied to the carrier-phase measurements. When these two models were combined in RTK positioning, the success rate of ambiguity resolution increased from 68% to just over 94% [Han, (1997c)].

Two examples of studies have been discussed that have made use of a similar post-fit variance scaling function for RTK positioning tasks, with similar levels of success. Wang's studies [2000] also included a satellite-based elevation angle stochastic model as suggested by Jin [1996] (cf. Equation 4-8), which yielded an ambiguity resolution success rate of 68%, higher than the results afforded by the preset covariance matrix of 49%. The standard elevation-based stochastic model did not yield any significantly improved results here which is in agreement with the findings of Roberts et al [1997].

It is difficult to attribute the success, or conversely, the failure of this particular format to any one factor. Rather it is more likely the product of a number of factors. The pre-determined constants as used in Equation 4-8 were derived from studies using unique receiver types and therefore a receiver type not featured in these studies may have considerably different noise characteristics [Jin, (1996)]. The overall quality of the GPS data will likely be different and of course there will be unique values for the distance-independent errors according to the antenna and receiver combinations used.

However constant coefficients can only reflect error characteristics of the GPS receiver rather than the unmodelled residual biases that are most probably related to the observing environment [Dai, (1999); Dai et al, (1999)].

**Spatially Decorrelating Errors**

It has long been recognised that using multiple reference stations with code (or carrier-filtered code) observables, typically improves differential positioning, particularly in the context of WADGPS (cf. §2.3.2). Wübben et al [1996] proposed an approach to reducing the distance-dependence of carrier-phase positioning errors by generating a geometric model for phase corrections using the horizontal co-ordinates as parameters.
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Coefficients for the geometric model were generated from a least squares adjustment of a reference station network and were then used to interpolate differential carrier-phase corrections for specific locations within that network's coverage.

A disadvantage of this single geometric model is that the uncorrelated errors (such as multipath) cannot be individually accounted for, a point picked up by Varner [1997] in his partial derivative correction model. In this model, differential errors between a base reference receiver and each of the other reference receivers are used as measurements to estimate two error sources. The first is the multipath at each site and the second is the differential carrier-phase error that is estimated as a function of position by fitting it to a polynomial of position variables, presumably the spatially correlated errors. Once this polynomial is calculated, the user applies it along with the estimated multipath error to calculate the estimated differential error at their location, an approach very similar to that suggested by Han [1997a].

Taking another approach albeit more akin to WADGPS in which error sources are modelled individually, Wanninger [1995] used a linear interpolation algorithm to explicitly model the differential ionosphere delays. With the calculation of the differential ionospheric delays between a network of three reference stations surrounding the mobile receiver, the differential ionosphere can then be linearly interpolated to the user position, a concept used heavily in virtual reference station applications (cf. §2.2.4.3) [Spectra Precision Terrasat, (1999)].

General Case Variance Algorithm used within WAAS Systems

Following their studies into WAAS, Walter et al [1997] propose an equation that attempts to satisfy the integrity requirements of satellite-based augmentation systems with particular reference to aircraft navigation. The algorithm specifically generates bounding variance estimates for the mobile receiver, as based on the maximum deviations of errors observed by the master (reference) station. These bounding estimates are transmitted to the mobile user within the UDRE and GIVE values. The format of the algorithm applied at the mobile receiver to calculate its pseudorange variances is as follows:

\[
\sigma_i^2 = \sigma_{\text{UDRE}i}^2 + F^2(\theta_i)\sigma_{\text{UPE}i}^2 + \sigma_{\text{SNRI}}^2 + \frac{\sigma_{\text{map}45}^2}{\tan^2 \theta_i} + \frac{\sigma_{\text{vd}}^2}{\sin^2 \theta_i}
\]

Equation 4-11

where \( F(\theta)_i \): the ionospheric obliquity factor (mapping function)
Stochastic Modelling for Differential Positioning

\[ \sigma^2_{\text{UVE}} : \text{user-determined ionospheric vertical error} \]

\[ \sigma^2_{\text{SNR}} : \text{variance of signal-to-noise ratio at the mobile receiver} \]

\[ \sigma^2_{\text{mpath} \ 45} : \text{multipath variance at } 45^\circ \]

\[ \sigma^2_{\text{vid}} : \text{variance of the vertical tropospheric delay} \]

It is interesting to note that the multipath modelling function used within this WAAS variance algorithm system is based on the product of an estimated multipath variance at 45° subject to an elevation mapping function. Although this function models the assumed increase in code multipath as the elevation angle decreases, it cannot model the high-frequency variations associated with code multipath (as seen in Figures 3-10 and 3-11).

Walter et al [1997] state that problems can arise with this algorithm when estimating the bounds with which to cover the actual errors at the required level of confidence. Again, this is a subjective task where operators do not wish to infer an over-pessimistic system.

4.2.3 Case Study: A General Case Modelling Algorithm for RTK Positioning - The NetAdjust Algorithm

A novel approach as suggested by Raquet and Lachapelle [2000], is their NetAdjust algorithm that purports to centimetre-level kinematic positioning using a network of reference receivers. In this method, measurements for the reference receivers are corrected as opposed to the conventional method of providing differential range corrections that are then applied to the rover's measurements. Basically, the NetAdjust algorithm is the best linear minimum-variance-of-error estimator that minimises (squared) double-difference errors, and it assumes that the differential errors are well described as zero-mean Gaussian random errors [Raquet and Lachapelle, (2000)].

This algorithm encapsulates all of the information from the reference network into the measurements from a single reference receiver, essentially transforming the multiple reference station network into a conventional single baseline system, albeit with an adjusted reference station. Once the network adjustment has been carried out, the corrected measurements are "essentially more accurate versions of the original raw measurements (with the errors reduced by the network)", - Raquet and Lachapelle [2000].
"The goal of NetAdjust is to determine a set of corrections that, when applied to reference receiver measurements, will minimise the error variance of the double-differenced measurement errors between the corrected reference receiver measurements and measurements obtained from a mobile receiver located at the computation point." - Raquet and Lachapelle [2000].

The major advantages of this approach are quoted as follows.

- It is performed epoch-by-epoch in real-time as long as the carrier-phase ambiguities can be calculated, therefore no assumptions need to be made about error dynamics.
- As it uses one adjusted reference station, only one set of integer ambiguities must be calculated (as opposed to a set for each reference station in conventional multiple reference station networks).
- Errors (including multipath) are isolated and assigned to specific measurements - providing insight into the cause of the multipath at a reference receiver site.
- It can easily be applied to both code and carrier-phase raw measurements.

An unusual aspect of this approach is that the code observations are raw and uncorrected, that is, unfiltered with no explicit ionospheric or tropospheric path delay corrections. Instead these errors are implied within the vector of differential errors, purely as a function of position. Any temporal correlations will be seen, presumably, as a change in estimated error at the reference stations. They are the errors between the location at which the corresponding measurements are taken and an arbitrary point $p_0$ that is normally selected near the centroid of the reference receiver network. Multipath and receiver noise variances are inferred from a table of coefficients computed during previous data processing campaigns.

The differential error vector, $derror\_vector$, describes the measurement errors that are not completely cancelled out by double-differencing techniques.

$$derror\_vector = \partial \rho_{SV} + \partial I + \partial Z + mpath + \nu$$

Equation 4-12

It is these residual errors that propagate into positioning errors although the differential error vector is never calculated explicitly; rather its covariance is calculated and used in the NetAdjust algorithm. The covariance matrix of the differential error vector
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$C_x$ is made up of individual elements $c_{ab}^{xy}$ that correspond to the residual biases that remain once the double-difference operator has been applied (cf. §3.1.3).

**Variance of Double-differenced Observations**

Within the variance $C_x^r$, Raquet and Lachapelle [2000] incorporate estimates for the distance-dependent errors, and a term for the expected levels of distance-independent errors (multipath and receiver noise). These terms are then subject to a mapping function.

$$C_x^r = \text{mapping}_r \times [\sigma_{\text{distance-dependent}} + \sigma_{\text{receiver-independent}}] \quad \text{Equation 4-13}$$

**Covariances of Double-differenced Observations**

For the covariance of two measurements that correspond to the same satellite ($x = y$) albeit as seen from different receivers ($a \neq b$), Raquet and Lachapelle [2000] calculate:

$$C_{ab}^x = \text{mapping}_x \times \sigma_{\text{distance-dependent}} \quad \text{Equation 4-14}$$

Note that the variance component corresponding to distance-independent errors $\sigma_{\text{receiver-independent}}$ has been excluded, as it is assumed that the multipath and receiver noise errors are uncorrelated between measurements from different receivers (cf. §2.1.4).

When considering measurements at one receiver from different satellites, there are elements of mutual correlation introduced between these measurements by the receiver hardware or the processing algorithms used. Raquet and Lachapelle [2000] however, assume that there is no correlation between these measurements and define the corresponding covariance elements as $C_{xy}^x = 0$. These mutual correlations are sometimes known as spatial correlations [Tiberius, (1999); Tiberius and Kenselaar, (2000)], and will be discussed later within this chapter, §4.4.3.

**Error Model Function within NetAdjust**

To calculate the distance-dependent errors, Raquet and Lachapelle [2000] have created a Correlated Variance Function (CVF) which forms the core of their algorithms. This differential variance factor is a function of the products of the distance between the
two points and the predetermined fit coefficients relative to each receiver (based on the separations between the mobile and individual reference stations), and is given by:

\[ \sigma_{c_i}^2(p_m, p_n) = c_1d_1 + c_2d_2 \]  

Equation 4-15

For a network containing three reference receivers, \(a\), \(b\) and \(c\), the CVF for the three baselines with respect to the mobile receiver at point \(p_0\) can be found from:

\[
f_r(p_a, p_b, p_c) = \frac{\sigma_{c_i}^2(p_a, p_0) + \sigma_{c_i}^2(p_b, p_0) + \sigma_{c_i}^2(p_c, p_0) + \sigma_{c_i}^2(p_a, p_b) + \sigma_{c_i}^2(p_a, p_c) + \sigma_{c_i}^2(p_b, p_c)}{2}
\]

Equation 4-16

The mapping function \(\mu(\varepsilon)\) used within NetAdjust is calculated from:

\[
\mu(\varepsilon) = \frac{1}{\sin(\varepsilon)} + c \left( \frac{0.53 - \varepsilon}{180\text{deg}} \right)^3
\]

Equation 4-17

where \(c_s\) is a fit coefficient derived for each observable type based on previous observational data from each receiver used in the network.

According to Raquet and Lachapelle [2000], the NetAdjust algorithm has one major advantage over other modelling algorithms in that it can calculate, using its covariance function, a-priori corrections using only the receiver position, measurement types available and predicted satellite elevations. If the network knows the users’ computation point, then the algorithm can calculate the approximate corrections that should be applied to the mobile’s measurements, and also the grouped corrections based on the reference network. From this, NetAdjust can compute these differential vectors of corrections and transmit the reduced size packet of correction information. Under this approach, only corrected reference receiver measurements are required to minimise the (residual) double-difference errors when using uncorrected measurements from a receiver at the computation point.

**Performance of the NetAdjust Algorithm**

Within a number of test networks, the double-difference L1 code measurement noises were, in general, reduced. Improvements of between 4 and 46% were achieved
between using conventional raw L1 single baselines and raw L1 codes as corrected from the reference station network. There was no trend of improvement noticed with respect to baseline length because the primary errors in the code measurements were multipath and noise, i.e. non-distance dependent errors. However for pure phase measurements, an improvement was noticed with increasing receiver separation because the dominant errors on phase measurement are the distance-dependent satellite orbit and atmospheric errors.

Raquet and Lachapelle [2000] hypothesise that if the tests were carried out during times of high solar activity, the influence of large ionospheric errors would likely force the code measurement errors to become correlated with distance. RMS values of the double-difference errors derived from truth positions, ranged from 0.92 metres to 1.96 metres for a 242 kilometre baseline. For carrier-phase ambiguity resolution, NetAdjust was able to reduce errors on longer baselines (~200 kilometres) by up to 50%, however this was not sufficient to enable effective L1 ambiguity resolution.

Although the NetAdjust algorithm is promoted as a potential modelling system within RTK positioning, the use of predetermined coefficients, as required for the core variance and mapping functions, suggest that the algorithm is highly dependent on previous information with which these parameters are calculated. Such a factor could be said to reduce its attraction as a general-case modelling algorithm.

Instead of using coefficients which, amongst other parameters presume the code multipath noises, perhaps the use of carrier-phase filtering routines to reduce the spurious multipath and receiver noise peaks should be incorporated. Alternatively, an adaptive multipath modelling routine could be used to model the high-frequency terms associated with code multipath.

4.2.4 Modelling Routines in Third-party Software

Research Institute Software

The estimation strategy within JPL’s GIPSY-OASIS II software uses the undifferenced carrier-phases along with smoothed code pseudorange data. Parameters for the Earth’s orientation, satellite states, station and satellite clocks, as well as station coordinates and tropospheric delay, are estimated daily for 24 hour arcs (or however much data is available). The underlying stochastic function within GIPSY is a square root information filter (SRIF) that complements undifferenced data such as an observed
Current Stochastic Modelling Activities

pseudorange [Blewitt, (1998)]. This is a complex algorithm that exploits the equivalence of functional and stochastic modelling to produce a set of statistically uncorrelated parameter set as a function of linear combinations of data and is said to make the development of additional algorithms extremely easy [Blewitt, (1998); Gregorius, (1998)].

The BERNESE program, as developed at the University of Berne [AIUB, (1999)], uses double-differenced data and is probably the most widely used GPS processing package for high precision applications. MIT’s GAMIT program also uses double-difference data along with a Kalman Filter for the stochastic estimation of orbital parameters [Blewitt, (1998)]. The seven Analysis Centres contributing to the IGS product line-up (cf. §2.1.2) typically make use of these double-difference or undifferenced processing methodologies, depending on their personal experience. For these seven centres, comments in [IGS, (2000)] suggest BERNESE is the preferred package.

Considering commercial GPS processing packages, it seems that most typically incorporate elevation-dependent weighting functions, for example Ashtech’s PRISM software [Ashtech, (1995)]. The latest version of Leica’s SKI-Pro program, version 2.1, applies a stochastic function to the ionospheric delays in an attempt to extend the baseline lengths that can be successfully processed. This methodology, when applied to carrier-phase processing, has shown improved position qualities yielded by ionosphere-free fixed ambiguity solutions over the comparable float solutions [Euler and Ziegler, (2000)]. In this routine, ionosphere-free linear combination of dual-frequency carrier-phases are used to eliminate (nominally) the effects of ionospheric delay. The standard carrier-phase observation equation for each satellite (cf. Equation A-4) is then afforded an additional term with which to represent the residual ionospheric effects within the least squares adjustment:

\[
\frac{\Delta \ell}{f_{\ell}^2 \cos(z)}
\]

Equation 4-18

where \(\cos(z)\) is a scalar function to transform the vertical residual ionospheric delay to the actual path delay of each satellite. Use of this stochastic ionospheric modelling function provided a 100% success rate for ambiguity resolution up to 26 kilometres, as opposed to only a 79% success rate as yielded by a standard stochastic model with unit weight. Further tests referenced in Euler and Ziegler [2000] state that a similar stochastic function has been 100% successful when applied in a virtual reference station network scenario.
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**Problematic Ionospheric Decorrelations**

The majority of stochastic modelling references seem to involve studies of short distance for baseline and network RTK applications; generally less than 200 kilometres in distance whereby correlations due to atmospheric errors are assumed to be high and therefore differential errors small. As DGPS is mainly used over longer distances where the operating range for users of satellite-based corrections can extend up to, for example, at least 2,000 kilometres [Racal Survey, (2000)], high atmospheric correlations are not generally the case. It is more likely, therefore, that the atmosphere will be very close to, or have reached, maximum decorrelation. Users would then be forced to include, in the model, estimates of the atmospheric delays without any information on their levels of uncertainty.

Even so, with medium range baselines, a user or software developer must be prepared to decide upon realistic a-priori variances for the ionospheric and tropospheric delays [Teunissen, (1997)]. Accordingly, the WAAS concept described in §2.2.3.2 includes a number of functional models and stochastic functions to model the impact of spatially correlated errors. The use of decorrelation rates for the ionosphere and troposphere has become well-established in network applications. Accuracy models determined by McGraw et al [2000] for LAAS receivers include spatial decorrelation rates for any residual atmospheric biases and elevation-dependent multipath variance estimation routines.

**Use of Kalman Filters within Stochastic Modelling**

For positioning applications whereby the parameters estimated by the least squares process vary with time, there are a number of techniques that can be used to estimate this behaviour. The most popular are the filter equations after Kalman [1960] (as cited in Cross [1983]). The Kalman filter combines a measurement model and a dynamic model in order to compute both the components of a state vector, and their precisions, at particular epochs. These equations include terms for estimating these varying parameters: after the event (smoothing), at the current epoch (filtering) and in the future (predicting).

A number of the mentioned references, including Roberts and Cross [1993], BERNESE [AIUB, (1999)], Wang [2000], Raquet and Lachapelle [2000] make use of these filtering techniques mainly to estimate the change in atmospheric parameters over time for longer baselines. Wang [2000] uses Kalman filter techniques to obtain a more realistic estimate of the measurement noise level and thus its covariance matrix. However,
Wang found that further modelling of the temporal correlations would be required to afford successful processing.

The use of this particular type of filtering algorithm has not been considered within this research for two reasons. The first being that there are already a number of time-averaging filters present within this epoch-by-epoch software, and that the inclusion of further temporal functions and information could result in an unbalanced and biased model. The phase-filtering routine, for example, makes use of information from previous epochs up to, and including, the current epoch. A similar routine is used in two other significant moving average routines; the suite of adaptive multipath variance estimation routines and the unit variance scaling factor. Both of these routines are discussed in detail in §4.3. Secondly, and on a lesser scale, the implementation of Kalman filtering routines would require a considerable amount of further coding within the modelling software. It is appreciated however, that the use of Kalman filters should be seriously considered within any future work, in particular, estimating the temporal decorrelation of the distance-dependent errors over time (cf. §8.2).

4.3 ALGORITHM DESIGN WITHIN THIS RESEARCH

The objective is the determination of a conceptual stochastic model applicable as 'general case' to differential positioning models for kinematic positioning using single and multiple reference stations. To do this, a number of different stochastic model algorithms have been created with functions to estimate, in real-time, the impact of different error sources on GPS code pseudoranges whilst acknowledging some of the observational correlations. The underlying concept is that the covariance matrix of undifferenced observations $C_i$ will then be populated with the real-time precision estimates of the phase-filtered code pseudoranges.

These algorithms have been designed with the intention of providing more accurate quality measures through improved (more sophisticated) stochastic modelling routines. This section chronicles the design of the stochastic functions to be included within the research software. The a-priori weights used in these models cannot be easily computed mathematically in real-time and thus they are derived from previous experience or obtained through an 'educated guess', or at worst, 'trial and error'. If the weight matrix $W$ were omitted, it would imply that all observations have the same precision, an unlikely situation.
4.3.1 Determination of User Variances

In this section, methods for determining the precision of the pseudoranges as observed at each receiver are discussed. To do this, all error sources affecting the code pseudorange observable are considered individually (cf. §1.5.1).

Clock Errors

Receiver clock errors will contribute the same bias to all pseudoranges made at a particular epoch and can be solved for, along with the receiver co-ordinates, during a single point position calculation. The remaining clock uncertainty will be attributed to the satellite clock error on an undifferenced pseudorange with SA active:

\[
\sigma_{\text{clock}}^2 = \sigma_{\text{clock}}^* \sigma_{\text{clock}}^*
\]

Equation 4-19

Atmospheric Terms – Ionosphere and Troposphere

The biases induced on GPS signals by ionospheric and tropospheric delays are proportional to the zenith angle of the satellite and to the additional distance travelled by the signals in the respective media. Consequently a mapping function can be applied that attempts to model the increase in signal path length as a function of the satellite’s elevation angle (cf. §2.1.3).

The most basic mapping function is the cosecant function, \((\sin \theta)^{-1}\), where \(\theta\) is the elevation angle of the observed satellite relative to the user’s antenna location. Although more complex models have been designed to complement tropospheric correction models such as in [Neill, (1996)], they are sometimes dependent on predetermined network-dependent coefficients as in [Jin, (1996); Raquet and Lachapelle, (2000)] (cf. Equation 4-17). In turn this reduces their validity within a general case algorithm. Springer et al, [1997] state that the use of a cosecant-squared mapping function showed a considerable improvement in the height component over the cosecant function albeit at the cost of lesser precision within the horizontal components. Springer et al [1997] believe that this was a function of the satellite geometry at the time of their study.

For the purposes of this research, the simple yet acknowledged cosecant mapping function \(\sin (\theta)^{-1}\) is used in all mapping function requirements to model the elevation-dependent precisions of code pseudoranges:
Algorithm Design within this Research

(mapping function = \frac{1}{\sin(\theta)} = \csc(\theta)) \hspace{1cm} \text{Equation 4-20}

During the pre-processing stages, the acknowledged broadcast GPS ionosphere and Saastamoinen’s refined tropospheric delay correction models have been used to calculate the estimated corrections required on the observed code pseudoranges (cf. §5.2). The ionospheric and tropospheric variances are represented by separate vertical delays, \( v_{id} \) and \( v_{td} \) respectively, that are then subject to the cosecant mapping function to obtain the corresponding delay on the observed signal path. These described functions are as follows:

\[
\sigma_{\text{iono}}^2 = \sigma_{v_{id}}^2 \cdot \csc^2(\theta) \quad \text{Equations 4-21(a) and (b)}
\]

\[
\sigma_{\text{rop}}^2 = \sigma_{v_{td}}^2 \cdot \csc^2(\theta)
\]

**Code Multipath Modelling**

When a pseudorange measurement is suspected as being contaminated by multipath, there are three options available to the navigation software. Firstly, corrections could be made or models applied to the measurement, secondly that the measurement could be significantly downweighted or finally that the measurement could be removed from the position computation altogether. As the latter option is undesirable, alternative means of modelling and / or downweighting the observations must be produced. This is especially so given the limited number of satellites visible to a pair, or network, of receivers considering the elevation cut-off angle or the separation distance between them.

**Discussion: Defining a Constant Multipath Variance**

The simplest option here would be to downweight the measurement, namely by affording a larger value to the multipath variance term \( s_{\text{opath}} \), based say, on a large constant variance. A simple modelling function would be to assume that the precisions of code pseudoranges, subject to code multipath, decreased with increasing zenith angle (cf. Figures 3-10 and 3-11).

Figure 4-2 displays the resultant positional standard deviations calculated using two basic multipath variance functions. The first function (uCMV) assumes that the error on each satellite due to multipath is constant regardless of its elevation, whereas the second assumes that the code multipath precision is related to elevation angle. In this second function (CMV), a constant multipath variance is subject to the basic cosecant mapping
function described earlier. An hour long dataset was processed with two stochastic functions described above for the purposes of assessing the validity of a constant multipath variance. The relative PDOP statistic, as calculated at every epoch, is also included in this plot for purposes of comparison.

Figure 4-2 Time Series showing the A-posteriori Standard Deviations yielded by Two Variants of the Constant Multipath Variance Routine: unweighted (uCMV) and Satellite Elevation-angle weighted (CMV) and the relative PDOP Statistic

Note how the unweighted CMV time series is extremely similar in shape to that of the relative PDOP series, albeit offset by a ratio of the PDOP statistic and the multipath variance – in this case (5 m)². The elevation-weighted CMV routine function (CMV) does afford some slight deviations from the standard deviations of the basic unweighted variance algorithm. This is entirely resultant on the influence of the elevation mapping function as the satellite configuration changes relative to the user. This processing example shows that it is not really feasible to base the multipath variance on satellite geometry or a pure elevation angle weighting function. This is supported by the fact that multipath error in a kinematic environment can change rapidly (cf. §2.1.4).

McGraw et al [2000] found that the method of using code-carrier differences to model airborne multipath was insufficient and that the correlation of code multipath against satellite elevation angle would be more applicable. Their function is given by:
where a $10^\circ$ cutoff angle was specified for the mobile receivers. Estimates of the multipath variances for the ground-based reference receivers are calculated from long-term multipath models similar to those described in §2.1.4, and satellite elevation angles.

These are somewhat contradictory reasons. The use of multipath templates is very rigorous and must be based on previous information; the estimation of code multipath based on the elevation angle exclusively is not necessarily very accurate, and the use of code-carrier differences is one of the few methods that can infer real-time variations. The plots in Figure 4-3 show a time series of phase-filter corrections against the elevation angle for five satellites as seen at a reference station. Note that it has been assumed that the correction applied within the carrier-phase phase-filter is equal to the multipath interference effect onto the code pseudoranges (cf. §3.3.1), hence the phase-filter correction equals the code multipath.
Figure 4-3 Time Series of Dual-frequency Phase-filter Corrections against Satellite Elevation Angle showing the Trends of Code Multipath

Note that for this dataset, although the overall amount of code multipath increases as the elevation decreases, the magnitude of that multipath cannot be linked to the elevation angle at each particular epoch. In this dataset, the incident code multipath can be seen to vary at a high-rate with, for example on SV24, amplitudes of over 3.5 metres were seen. An elevation-weighting function alone cannot truly reflect the high-frequency variations due to code multipath.
Algorithm Design within this Research

**Routines for the Estimation of Multipath Variances**

It is known that multipath instigates a rapid variation within the pseudoranges (cf. §2.1.4 and Figure 4-3), and that the high-frequency terms can be reduced using carrier-phase filtering as shown in §3.3.5. Even so, an amount of multipath interference may remain, albeit at lower frequencies and magnitudes. Therefore, it follows that an important feature of any routines for estimating the residual multipath interference must possess the ability to adapt to changes in this parameter, be they caused by the environment (in static positioning) or especially the dynamics of a kinematic antenna.

Accordingly, the approach taken was to calculate an adaptive variance for the residual code multipath bias as based on a real-time estimate taken from the magnitude of the standard deviation of the range correction applied during the phase-filtering routine. The term adaptive is used as the multipath variances are determined from a time dependent moving average. It has been seen that there are increased noise and multipath trends associated with a satellite decreasing in elevation, and conversely the reduction in noise for rising satellites. It is hoped that variations caused by residual multipath can be approximately isolated from other noises on the pseudoranges as in Lachapelle et al [1996].

At each epoch for each satellite observed, the phase-filter, as described in §3.3, outputs a phase-filtered correction that is applied to the initial raw pseudorange. On the assumption that the phase-filter has removed the significant multipath trends to the best of its ability (under the functional parameters assigned to the filter), it can then be assumed that the residual multipath on the filtered code pseudoranges will possess similar trends. The initial corrections derived by the phase-filter, exhibit some high-frequency trends and provide information about the multipath interference – its magnitude and frequency. From this, a standard deviation can be derived for the corrections as applied to each observed pseudorange by means of a moving average over a predetermined time scale.

A number of algorithms have been created which attempt to describe the higher frequency variations attributable to code multipath using a moving average of previous observations. The estimation routines here are based on the assumptions applicable to the carrier-phase filtering technique, as described in §3.3, in that the influence of reflected multipath signals is considerably less on carrier-phase measurements than on code pseudoranges, at least two orders of magnitude.

Information describing the effects of code multipath on the observations can be obtained as by-products of the phase-filtering routine. The stochastic routines to model the
residual multipath will be taken from this information and will be estimated as a function comprising of two factors:

- the magnitude of phase-filter correction applied at each epoch, and
- a temporal factor based on the length of time that the phase-filter has been successfully implemented (as dictated by the number of consecutive filtering observations used for each satellite).

**Routine for Moving Averages of Carrier-phase Filter Corrections**

The calculation for this parameter commences at the first epoch of the dataset at which time there is no phase-filter correction information available. Consequently, the a-priori multipath variance, as specified in the initialization file, is used for the computations. As the number of epochs increases and the phase-filter begins to converge towards a steady state, a variance statistic is calculated from the standard deviation of those corrections applied so far [Deakin and Kildea, (1999)]. It is likely that these moving variances will be high because they include the user-defined variance at the first epoch and some large correction values. The exact means of defining the moving average variance algorithm to cope with the absence of a correction at the first epoch is subjective and should be decided upon by the user.

Once the multipath variance moving window has been fully populated, the window moves through the dataset with the processed epochs as long as there are no interruptions to that satellite’s signal. The AMV variance is calculated from the standard deviation of these phase-filter corrections within the moving window as follows:

\[
\sigma_{\text{mpath, AMV}}^i = \left( \frac{\sum_{n=1}^{n(a)} (P_{\text{fil}}^i - P_{\text{raw}}^i)^2}{n} \right)^{1/2}
\]

Equation 4-23

where \(\sigma_{\text{mpath, AMV}}^i\) : standard deviation forming the basis of the adaptive multipath variance for satellite \(i\)

\(P_{\text{fil}}^i\) : filtered pseudorange for satellite \(i\)

\(P_{\text{raw}}^i\) : raw pseudorange for satellite \(i\)
Algorithm Design within this Research

$n$: sample size, in epochs, for moving average of carrier-phase filter corrections

Comments on the AMV Routine

The concept of this magnitude-driven multipath variance routine may not be correct as the filtered corrections are dependent on the number of filtered observations since the last cycle slip (cf. §3.3). As the filter operates, there is a progressive increase in the relative weights of the filtered pseudoranges to the raw code pseudoranges. At epoch 1, the raw and filtered code pseudoranges are assigned equal weight because there is no previous phase information with which to reduce any extreme noises. However when the filtering time constant $N_{\text{max}}$ has reached its maximum, in this research 100 seconds (cf. §3.3.4), then a weighting factor of 0.99 is assigned to the filtered pseudorange. No consideration is made for this temporal trend in the AMV routine.

Temporal Multipath Variance Estimation Routine (TMV)

Accordingly, an alternative means of estimating the multipath variance is to acknowledge the duration of a period of successful (i.e. uninterrupted) phase-filtering. So long as the phase filter has been completely populated without a cycle slip or interruption on that satellite's ranges, there will be significant weighting on the phase-filtered pseudoranges (cf. §3.3.2). The TMV routine attempts to relate the increase in the length of the successful filtering pass to an increase in precision of the phase-filtered pseudoranges.

From epoch 1 to $N_{\text{max}}$ epochs, the temporal multipath variance factor increases linearly between a pair of preset values corresponding to the expected upper and lower multipath variances respectively. Once $N_{\text{max}}$ has been reached, the parameter $\text{number\_filtering\_obs}$ is maintained at $N_{\text{max}}$ and the user suggested minimal multipath variance factor is specified until there is another cycle slip and the filtering routine must be restarted. The proposed algorithm format for this routine is:

$$
\sigma_{\text{mpath\_TMV}}^2 = \left( \sigma_{\text{mpath}}^{\text{max}} - \frac{(\sigma_{\text{max}} - \sigma_{\text{min}}) \cdot \text{number\_filtering\_obs}}{N_{\text{max}}} \right)^2
$$

Equation 4-24

where $\sigma_{\text{mpath\_TMV}}^2$: final multipath variance estimate for satellite $i$
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\[ \sigma_{\text{max, multipath}}^{\text{TMV}} : \] maximum a-priori multipath standard deviation defined in initialization file for all satellites

\[ \sigma_{\text{min, multipath}}^{\text{TMV}} : \] minimum a-priori multipath standard deviation

\[ \sigma_{\text{TMV}}^{\text{filtering Obs}} : \] number of successful filtering observations on satellite \( i \).

**Comments on the TMV Routine**

As can be inferred from Equation 4-24, once \( N_{\text{max}} \) is reached, then a constant multipath variance is assumed. This is obviously not the case as any residual multipath will still be causing some variations in the filtered pseudoranges albeit no longer picked up by the TMV routine. For this reason, it has been proposed that the innovations of the AMV and TMV algorithms should be integrated, so that the high-frequency variations in the filtered pseudoranges are picked up as well as the increased precision afforded by a larger number of successful acknowledged filtering observations.

**Adaptive Temporal Multipath Variance Estimation Routine (ATMV)**

This algorithm incorporates the magnitude of the phase-filtering corrections along with their currency as based on the duration of the phase-filter correction. This action refines the TMV method suggested above, by replacing the predefined upper and lower bounds with values derived from within the moving window average of phase-filter corrections. The proposed ATMV algorithm is as follows:

\[
\sigma_{\text{multipath}}^{\text{ATMV}} = \left( \sigma_{\text{max, AMV}}^i - \sigma_{\text{min, AMV}}^i \right) \frac{n_{\text{filtering Obs}}^i}{N_{\max}} \]  

Equation 4-25

where \( \sigma_{\text{max, AMV}}^i \) : maximum phase-filter correction standard deviation within the moving window for satellite \( i \)

\( \sigma_{\text{min, AMV}}^i \) : minimum phase-filter correction standard deviation within the moving window for satellite \( i \)

\( n_{\text{filtering Obs}}^i \) : number of successful phase-filtering observations on satellite \( i \)

\( N_{\max} \) : carrier-phase filtering time constant
Algorithm Design within this Research

Figure 4-4 shows the precision estimates as yielded from the least squares position computations when applying the three adaptive multipath variance estimation routines described in this sub-section - the AMV, TMV and ATMV routines. Note that these standard deviations have been determined directly from the covariance matrix of adjusted parameters $C_x$ and have not been subject to any unit variance scaling factors. Values used for $N_{\text{max}}$ and the phase-filter correction moving windows ($m\text{wrf}$) were 100 and 30 seconds respectively.

**Figure 4-4** Time Series showing the A-posteriori Standard Deviations yielded by the Multipath Variance Estimation Routines (AMV, TMV and ATMV - where $N_{\text{max}} = 100$ seconds and $m\text{wrf} = 30$ seconds) and the PDOP Statistic

Results for the four proposed multipath variance estimation routines within this research can be seen in Figure 4-5 and are plotted against the relative PDOP statistic.
Figure 4-5 Time Series of A-posteriori Standard Deviations yielded by the Four Multipath Variance Estimation Routines proposed in this Research

For reasons of legibility, an offset was applied to the CMV time series so that all output time series could be easily viewed in one plot. The absolute scale of the standard deviations in these time series (as seen by the truncated abscissa for the standard deviations), is not critical but rather the relative variations within them. The assignment of the a-priori multipath standard deviations at the initial epoch affords a large multipath variance to the adaptive algorithms, i.e. AMV and ATMV. Even so, the incorporation of the variance terms for the remaining error terms will affect the overall size and characteristics of these time series. The subsequent application of the unit variance scaling factor will likely change the overall appearance and absolute magnitude of these values (cf. §4.3.5).

The overall similarity between the results from the proposed models and the PDOP statistic is that they are both calculated from the same mathematical component, the design matrix of double-differenced observations $A$. Note that the PDOP is calculated from $\left( A' A \right)^{-1}$ (cf. Equation 3-17). The covariance matrix of adjusted parameters and thus the standard deviations are calculated from $\left( A' W A \right)^{-1}$ (cf. Equation 3-27). The variations seen in Figures 4-4 and 4-5 are thus fully attributable to the inclusion of the weight matrix $W$. In this example, the weight matrix has been populated with the estimated multipath variance values that have essentially taken the form of UERE values (cf. §1.2.1).
time series also illustrate that the approach of using the product of PDOP and RMS UERE values yields neither consistent nor accurate position error statements.

**Receiver Noise**

With the availability of $C/N_0$ ratio values for all GPS observations, it would be possible to afford variance estimates of the receiver noise to the covariance matrix. Given that they are not generally included in RINEX files, a constant variance term for receiver noise was used within the stochastic model of this research:

$$\sigma_{\text{rec\_noise}}^2 = (\sigma_{\text{rec\_noise}})^2$$  \hspace{1cm} \text{Equation 4-26}

### 4.3.2 Determination of User Covariances

**Overall Design of the Covariance Function**

The covariance term describes the correlation between two observations, in this research, between two carrier-phase filtered code pseudoranges. The design of the covariance algorithm (cf. Equation 4-5(b)), is very similar to the variances algorithm as featured as Equation 4-5(a), but is modified to reflect the spatial correlations between same-satellite observations as recorded by different receivers. The overall design of the covariance function is as follows, and will be complemented by the algorithm modules in this section.

$$\sigma_{ab}^2 \text{ (covariance)} = \sigma_{\text{clocks}} + f(\sigma_{\text{orbis}}, \sigma_{\text{iono}}, \sigma_{\text{trop}})$$  \hspace{1cm} \text{Equation 4-27}

Note that there are no multipath or receiver noise terms within the covariance equation. This is because these distance-independent errors exhibit no correlation on the measurements from different receivers a considerable distance apart (cf. §2.1) [Raquet and Lachapelle, (2000)].
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**Satellite Clocks**

When SA is active on satellite clocks, it is the largest error source on phase-filtered code pseudoranges and affects the precision of all observations made to these satellites. Accordingly, the clock covariance is related to the estimate of SA on the satellite clocks:

\[ \sigma_{ij}^{\text{clocks}} = \sqrt{\sigma_i^{SA^2} + \sigma_j^{SA^2}} \]  

Equation 4-28

**4.3.2.1 Spatial Correlations**

Consideration must now be made of those distance-dependent error sources that possess spatial correlation characteristics, i.e. their effect on the measurements made by two receivers depends on the separation distance between them. The biases included in this category include satellite orbits, the ionosphere and the troposphere, and shall be dealt with in that order.

**Satellite Orbits**

An approximate calculation for the propagation of orbital errors into the errors in relative co-ordinates over varying length baselines given in [Bauersima, (1984)] and mentioned in several references as a rule of thumb, for example, in Barnes et al [1998]. This algorithm states that the error in relative co-ordinates \( e_R \), between two receivers separated by distance \( L \) can be expressed as:

\[ e_R = \frac{\kappa e_S}{L} \]

where \( e_S \): error in the GPS satellite broadcast orbits  
\( A \): design altitude of a GPS satellite orbit  
\( \kappa \): a number between 0.2 and 1 (dependent on DOP, estimation strategy and orbit improvement)

The determination of the empirical factor \( \kappa \) can be deemed somewhat subjective as it is dependent on user experience. The approximate parts per million (ppm) error in the baseline distance \( L \) for the broadcast and precise ephemerides is given in Table 4-2:
Algorithm Design within this Research

<table>
<thead>
<tr>
<th>BASELINE (km)</th>
<th>ORBIT ERROR (m)</th>
<th>BASELINE ERROR (cm)</th>
<th>ERROR RATIO (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>0.025</td>
<td>0.05</td>
</tr>
<tr>
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<td>1</td>
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<td>0.01</td>
</tr>
<tr>
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<td>0.25</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td>100</td>
<td>5</td>
<td>2.5</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.5</td>
<td>0.01</td>
</tr>
<tr>
<td>1000</td>
<td>5</td>
<td>25</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>5</td>
<td>0.01</td>
</tr>
</tbody>
</table>

**Table 4-2** Approximate Errors in Relative Station Co-ordinates as determined over Varying Baseline Lengths using Broadcast and Precise GPS Ephemerides

For this research and the algorithms therein, the rate for the orbital decorrelation is based on the satellite altitudes and the quality of the satellite ephemerides (cf. Table 2-4). A 5 metre broadcast orbit error would effect a baseline error of 0.13 metres over 500 kilometres, well within the noise of code DGPS positioning.

**Modelling of the Residual Ionospheric and Tropospheric Delays**

Corrections have been determined for the ionospheric and tropospheric delays using the broadcast ionosphere and refined Saastamoinen models as part of the pre-processing tasks. It has been assumed that any remaining errors attributed to these sources are differential errors that cannot be mitigated by the double-difference functional model. In this research, the limitations of both atmospheric correction models are acknowledged from references and thus empirical terms for these decorrelations have been introduced as based on a linear interpolation of the deteriorating errors.

The troposphere, as the second physical media affecting GPS observations, also decorrelates over distance. Large height differences between two stations above mean sea level (MSL) can introduce biases into the vertical tropospheric delay estimates, although there are correction models, such as the refined Saastamoinen model, which attempt to model these differences (cf. Equation 5-3).
Spatial Decorrelation of Errors

The variability within the ionospheric and tropospheric errors is such that over very long baselines, typically hundreds to thousands of kilometres, there is no correlation between the atmospheric errors encountered at the two observing receivers. Once fully decorrelated in spatial terms, these errors may be considered as reference station unique, that is, a distance-independent error source, and can thus be modelled empirically for the receivers used.

By setting the standard deviation \( \sigma \) as equal to zero (or infinite weight), it is then assumed that the a-priori information is completely known for that variable. Conversely, if the standard deviation for the same variable is specified as \( \sigma = \infty \) (or zero weight), then the a-priori information of the variable does not contribute at all to the solution of the model. An example given by Odijk [2000] and discussed earlier, is the use of ionospheric pseudo-observations; an infinite weight yields an ionosphere-fixed solution whereas a zero weight yields an ionosphere-free position solution.

Values of distance that signify the complete decorrelation of the ionosphere and troposphere are difficult to quantify, and many groups have tried to define rigid boundaries. Roberts and Cross [1993], for example, have suggested decorrelation distances for the ionosphere and troposphere of 2000-3000 kilometres and 50-100 kilometres respectively. The ionospheric decorrelation distance will depend on the calendar’s position within the acting solar cycle (cf. §2.1.3.2).

An alternative means of modelling these decorrelations is to use predetermined rates for the estimated decorrelation of the atmospheric parameters. Their use overcomes the threshold problem by assuming that the parameters do not fully decorrelate, which is obviously not correct over longer distances especially for the ionosphere during times of high ionospheric disturbance. For example, the US DoT [1999] believe that the decorrelation of distance-dependent errors (orbits, ionosphere and troposphere) effects a positional error of 1 metre for every 150 kilometres, almost 7 mm/km, from a Marine or National DGPS Transmitting Station. McGraw et al [2000] suggest a range of values between 2 and 8 mm/km for the decorrelation of the virtual ionospheric gradient (vig) between LAAS stations after which this vertical delay, (now extrapolated for distance), is mapped into the elevation angle of the observed satellites. Odijk [1999] correctly states that the spatial decorrelation of the ionosphere depends on the baseline length and suggests a decorrelation rate of 4 mm/km. Odijk also suggests that one approach not yet studied could be the possible elevation-dependency of the (functional) ionospheric observables.
The decorrelation distance for the satellite orbits is generally regarded as constant in that the satellite geometry, and thus the quality of their positions, does not change very quickly, and will generally be greater than any baseline able to maintain at least four common-view satellites.

Modelling the Distance-dependent Errors using Spatial Decorrelation Functions

The use of linear decorrelation rates appears to be more practical than the use of infinite weights. Any spatial decorrelation function (SDF) needs to increase the uncertainty of the spatial variable, e.g. the satellite orbits, as the receiver separation increases. With zero separation, there is assumed to be no differential orbit error, i.e. the orbital variance is assumed to be known. With increasing distance, the uncertainty in the variable increases, for example, the errors in a baseline vector increase due to errors within the satellite ephemerides (cf. Table 4-3). As with decorrelation thresholds, the use of decorrelation rates is subjective and reliant on previous information on the distance-dependent errors, e.g. the density of the observing network and the atmospheric activity at the time of observation.

A spatial decorrelation algorithm therefore needs to represent the spatial decorrelation of the distance-dependent errors as a function of the separation between the observing receivers. The simplest means of doing this is to assume that the errors decorrelate in a linear fashion as the receiver separation increases. The SDF applied to the distance-dependent errors using the satellite orbits as an example follows:

\[ \sigma_{AB}^{\text{orbits}} = \sqrt{\sigma_A^{\text{orbits}}^2 + \sigma_B^{\text{orbits}}^2} \cdot \frac{L_{AB}}{S_{\text{orbits}}} \]

Equation 4-30

where \( \sigma_{AB}^{\text{orbits}} \): covariance of satellite orbits between receivers A and B

\( \sigma_{\text{orbits}} \): a-priori standard deviation for satellite orbits

\( L_{AB} \): baseline length between receivers A and B (in kilometres)

\( S_{\text{orbits}} \): decorrelation distance threshold for satellite orbits

As the covariances for the ionospheric and tropospheric delays are spatially correlated through the satellite's elevation angles, they are subject to the cosecant mapping function (cf. §4.3.1). This function requires the mean elevation angle \( \theta_{\text{mean}} \) for the
Stochastic Modelling for Differential Positioning

particular satellite as seen by the two observing receivers. The ionospheric component of the covariance algorithm is calculated as follows:

$$\sigma_{AB}^{\text{iono}} = \sqrt{\sigma_A^{\text{iono}}^2 + \sigma_B^{\text{iono}}^2} \cdot \frac{L_{AB}}{S^{\text{iono}}} \cdot \text{cosec}\theta_{AB}^{\text{mean}}$$  \hspace{1cm} \text{Equation 4-31}

A similar procedure is applied when calculating the tropospheric component of the covariance algorithm:

$$\sigma_{AB}^{\text{trop}} = \sqrt{\sigma_A^{\text{trop}}^2 + \sigma_B^{\text{trop}}^2} \cdot \frac{L_{AB}}{S^{\text{trop}}} \cdot \text{cosec}\theta_{AB}^{\text{mean}}$$  \hspace{1cm} \text{Equation 4-32}

A significant downside of this concept and one that has not been investigated, merely acknowledged, is how the ionospheric and tropospheric variances are represented once they have become totally decorrelated. Clearly one cannot extrapolate beyond an infinite weight if a user assumes that the atmospheric term becomes completely decorrelated at a threshold distance. Accordingly in this covariance function, once the decorrelation distance has been exceeded, the decorrelation function continues to extrapolate the covariance according to the actual baseline vector length.

Weighting Routines for Multiple Reference Stations

The spatial weighting routine for multiple-baseline scenarios is based on the differences in covariances determined between all receivers from the stochastic spatial functions described in §4.3.2. This way, there is more correlation between the observations from those receivers closer to the mobile receiver.

When determining the covariances for network scenarios, an estimate of the correlation between the reference stations due to satellite orbits and atmospheric errors is given by including the baseline distances between them. The development of an additional horizontal geometry-based correction model for the multiple reference stations was not considered as it was felt that the proposed SDF routines were sufficient.
4.3.3 Determination of Mutual At-receiver Covariances

**Concept of Mutual Correlations between Observations made by each Receiver**

Consider two satellites \(i\) and \(j\) being tracked by receiver \(A\) as in Figure 4-6; there are correlations between the observations as a result of biases present in that receiver due to:

- atmospheric delays
- mapping function errors
- biases relating to the satellite-tracking channels (inter-channel biases) within the GPS antenna
- multipath effects at the observing receiver
- thermal noises and jitter associated with the receiver electronics

As these terms represent the correlation between two observations made by the same receiver and to identify their distance-independent nature, i.e. their uniqueness to each individual operating GPS receiver, they shall henceforth be referred to as the mutual at-receiver correlations between observations. It is assumed that these errors are smaller in magnitude for the higher elevation satellite \(i\) than for satellite \(k\).

![Figure 4-6](graphic.png)  
*Figure 4-6 Graphic Illustrating the Concept of Mutual At-receiver Correlations*
The main proponents of these correlations are assumed to be associated in the determination of the ionospheric and tropospheric delays along the geometric path, and of errors within these mapping functions.

Tiberius [1999] cites, amongst others, Roberts et al [1997] and Barnes et al [1998] as acknowledging this phenomenon as a spatial correlation and terming it so, yet foregoing details of how it is, or could be, modelled. As seen earlier in Figures 3-11 and 3-12, signals from the higher elevation satellites have less noise than those nearer the horizon. The presence of this characteristic deemed it applicable to design an algorithm that introduces some additional stochastic terms for these correlations as a function of the elevation angle of the individual satellites with respect to the user’s location. The algorithm’s core is based on a function of the coefficient of correlation, the ionospheric and tropospheric zenith delays and the satellite elevation angles:

$$\sigma_{xy} = f(\rho_{xy}, l, Z, \theta)$$  \hspace{1cm} \text{Equation 4-33}$$

The general design of the Mutual At-receiver Covariance Algorithm is given in Equation 4-34:

$$\sigma_i^j \sigma_A^j = \rho_A^{ij} \left[ \frac{\sqrt{\sigma_{\text{iono}}^2 + \sigma_{\text{trop}}^2}}{\sin(\theta_A^i)} \right] \cdot \left[ \frac{\sqrt{\sigma_{\text{iono}}^2 + \sigma_{\text{trop}}^2}}{\sin(\theta_A^j)} \right]$$  \hspace{1cm} \text{Equation 4-34}$$

where \(i\) and \(j\) are two satellites seen by receiver \(A\) (cf. Figure 4-6). As many of the correction terms, within this software and that of other groups, are generally driven by satellite elevation angles, it is feasible that differential errors within these models could contaminate the final solution. This is especially relevant considering the use of correction models for the atmospheric delays.

The correlation coefficient \(\rho_{xy}\) relates the covariance of two data sets divided by the product of their standard deviations.

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \cdot \sigma_y}$$  \hspace{1cm} \text{Equation 4-35}$$
Algorithm Design within this Research

This dimensionless coefficient can range from +1 to -1 and is usually determined empirically. The parameter to be determined is whether positive values of one set are associated with positive values of the other (positive correlation), whether positive values of one set are associated with negative values of the other (negative correlation), or whether the values in both sets are unrelated (correlation near zero).

The errors associated with mapping functions are generally positively correlated in that the atmospheric and multipath errors increase as the zenith angle increases. Some groups have found negative correlations between observations [Roberts and Cross, (1993); Wang, (2000)] through post-processing. Wang [2000] stated that the use of post-fit filtered residuals can yield estimates of the measurement covariances which are negative. Conversely, some groups for example, Raquet and Lachapelle [2000], assume that there is no correlation between measurements from different satellites at different stations, hence $C_{iab}=0$ (cf. §4.2.3).

**Cross-correlations between GPS Observables**

Studies into the temporal and cross-correlations present in both the code and carrier-phase measurements by Tiberius [1999] and Tiberius and Kenselaar [1999], found that the modelling of these terms can lead to an increased ambiguity resolution success rate. However both researchers stated that difficulties were encountered in determining the exact parameters to represent the correlations and that, overall, these activities increased the computational load within the processing software.

It must be noted that no consideration was made, in this research, for the possible correlations between the code and phase measurements. Given the benefits of the carrier-phase filtering routines on the original raw measurements, an investigative study should be made into the impact of inter-observable correlations, especially for those occasions when dual-frequency observables are used, for example, phase-filtering and linear combinations. Tiberius and Kenselaar [1999] believe, however, that there is no direct correlation between the L1 pseudoranges and carrier-phase observables on L1, with a similar hypothesis for the same observables on the L2 frequency.

**Empirical Variances**

The a-priori values for the variances featured in the stochastic models proposed in this research, along with parameters for the spatial decorrelation functions, are discussed further in Chapter 5 and then specified for the processing tasks within Chapter 6.
4.3.4 Unit Variance Factor Scaling Routine

The least squares adjustment results in the computation of a unitless parameter called the a-posteriori variance factor, $\sigma_0^2$, which is suitable for generalised quality control. It is computed from the standard calculation (cf. Equation 3-29):

$$\sigma_0^2 = \frac{\hat{v}^T W \hat{v}}{(d - p)} \tag{Equation 4-36}$$

where $\hat{v}$: the vector of post-fit double-difference code residuals

$d - p$: number of known parameters minus number of unknown parameters (also known as degrees of freedom)

$W$: the weight matrix of the observation data

If proper weighting has been assigned to the observations, the unit variance factor is an indicator of the overall fit of the observations to the mathematical model. Working on the assumption that both models are correct, the unit variance factor is then subject to the influence of two parameters; firstly random errors in the observations and secondly, any biases which may be systematic, periodic or incidental.

Several concerns were raised by users with regards to the applicability of UKOOA’s Differential GPS Guidelines [UKOOA, (1994)]. In particular, users were concerned at the very small unit variance values output by operational systems and incorrectly attributed to the limited redundancy within the DGPS position fix [Roberts et al, (1997)].

The unit variance is an overall model test which has an expectation of unity. Values in this model are tested against a central Fisher distribution [Cross, (1983)]. Assuming there are no observational blunders in the system, and a particular unit variance fails the test, then there are two possible causes [Cross, (1983)]: the functional model is incorrect or the stochastic model is incorrect. Under the assumption that the functional model is correct, then it is usual to expect that the a-priori variances have been underestimated, on average through the dataset, by the reciprocal of the unit variance value. As the unit variance describes the fit between the data and the mathematical positioning model, unit variances analysed on an individual epoch-by-epoch basis can sometimes contain large systematic biases. Consequently, the unit variance scaling factor used is taken from the moving average over a user-defined number of previous unit variance values.
To illustrate the problems that necessitate this routine, a dataset has been processed with similar parameters to those tests by Roberts et al [1997], that is double-differencing with a simple stochastic model. The 2 hour dataset was subject to C/A-code double-differencing at a 1 Hz interval using a diagonal elevation-angle driven stochastic model typical of those used by operational navigation packages that incorrectly assume the observations are uncorrelated. The results of these studies are shown in the form of time series plots as follows. Figure 4-7 provides details of the satellite metadata, namely the number of satellites over 10°, the PDOP statistic and the unit variance statistic at every epoch.

![Time Series of Number of Double-differences, PDOP and Unit Variance Statistics](image)

**Figure 4-7** Time Series for the Unit Variance Scaling Factor Tests – Number of Satellites and the PDOP Statistic

Figure 4-8 shows the true plan errors against the a-posteriori 95% plan error in two forms. The first form corresponds to those unscaled errors direct from the covariance matrix of adjusted parameters $C_{\alpha \alpha}$, and the second are these same quantities albeit scaled with a unit variance scaling factor (as based on the average unit variance over the last 30 seconds).
Figure 4-8 Time Series of Plan Position Errors: True, Formal Unscaled, and Formal Scaled with Moving Average of Unit Variance

Figure 4-9 has been created in a similar vein to Figure 4-8 but corresponds instead to the 3-D vectors of position.

Figure 4-9 Time Series of Position Error Vectors: True, Formal Unscaled, and Formal Scaled with Moving Average of Unit Variance
Concluding Remarks

It can be seen that the use of this scaling factor can help to redress the differences between the observed data and the mathematical model by reflecting some of the variations in the real pseudorange data. The systematic trends present in these figures are typical of those yielded in relative code positioning with an uncorrelated stochastic function. Note that the unscaled a-posteriori error ellipse time series closely mimics that of the PDOP statistics and change slowly as a function of satellite geometry.

Now note that the variance scaled time series contains very little, if any, deliberate similarity with the PDOP statistics (and therefore satellite geometry). It is almost completely dominated by trends from the variance factor (cf. Figure 4-7). In this research, the unit variance scaling factor is the mean of the last 30 unit variance values. An additional benefit of the unit variance scaling factor is that its use can reduce the initial effects of receiver clock jumps (cf. §3.1.1) and their effects on the quality measures. However, the presence of systematic biases within the numerator of the unit variance statistic, namely $v^T \hat{W} \hat{v}$, (cf. Equation 4-38), could effect a considerable influence within the moving average.

The concept of unit variance scaling factors is very similar to that of post-fit residuals as used by Han [1997b]; Wang [2000]. The outcomes of both methods are dependent essentially on the residuals that have gone before. Han et al [1999] state that their post-fit residuals scaling factor can introduce high temporal correlations within consecutive observations.

4.4 CONCLUDING REMARKS

The concept of stochastic modelling has been introduced with respect to both post-processed and real-time applications. It can be seen that with the benefit of truth data enabling the calculation of true errors, the application of stochastic modelling functions in post-processing leads to a considerable improvement in overall performance in terms of accuracy, precision and quality measures. Clearly this is not possible with dynamic real-time positioning applications.

Proposed Stochastic Modelling Routines

As with all empirical models, the activity of designing a stochastic algorithm to describe the precision of pseudoranges from numerous datasets is highly subjective.
Consequently, different conclusions can be made following the analysis of results. This has been seen in a number of models reviewed earlier where, for example, some datasets have been well represented by elevation dependent stochastic models [Jin, (1996); Han, (1997b)] whereas others have not [Roberts et al, (1997); Wang, (2000)].

**Figure 4-10** The Four Candidate Approaches to Populating the Covariance Matrix of Undifferenced Observations (C_ij) applied within these Long-Range Positioning Studies

**Elevation-Dependent Weighting Routines**

The elevation-dependent weighting routines form the basis of many stochastic modelling routines, even though a number of groups have shown that for their datasets, this is generally not the case [Roberts et al, (1997)]. However, because no better performing algorithm has been found, this elevation function is still used [Miller et al, (1997)]. The cosecant mapping function is used for the pure elevation-weighting stochastic function as well as within a number of components of the variance, covariance and mutual covariance modules. Its use allows the realisation of the increased noises associated with measurements from lower elevation satellites that are becoming increasingly beneficial to long-range positioning applications. However, it cannot infer the real-time variations attributed to fast changing errors such as code multipath and receiver noise.

**User-Defined Variances**

The magnitudes of the variances to be used within the stochastic modelling routines have been influenced somewhat by the references as given in Chapter 2 when covering the
major sources of error and bias within GPS observations. It must be noted that these values are therefore subjective user-defined values.

The empirical variance algorithms proposed in this research are very similar in concept to those in the NetAdjust algorithm (cf. §4.2.3) with the exception that here, the mapping function is applied to the relevant terms as and when needed, as opposed to in the NetAdjust algorithm where it scales all variance estimates. This is because Raquet and Lachapelle [2000] assume that the main source of the receiver-based errors, code multipath, is dependent on elevation angle.

For the algorithm in this research, the mapping function is applied to the distance-dependent errors of the ionospheric and tropospheric delays, but not the code multipath variance. It is assumed that the proposed multipath modelling routines reflect the spatial (elevation-dependent) correlations of multipath as well as the larger periodic variations of multipath. As no C/N\textsubscript{0} values are available within the RINEX data files, the receiver noise variances are assumed to be constant.

**Multipath Variance Estimation Routines**

In terms of multipath modelling, a number of algorithms have been created with which to estimate the multipath variances over the data series. These include an adaptive routine that uses the magnitude of the phase-filter corrections applied within short data sections and a temporal routine that estimates a variance as a function of the duration of successful filtering. A third routine was created that intuitively combines the first two routines and their positive qualities into an adaptive and temporal multipath variance estimation routine.

The phase-filter corrections have been calculated on an epoch-by-epoch basis and their standard deviation will be calculated over a longer timescale (cf. §5.2.1 and Table 5-2). An optimal moving window length will be selected for these quantities that will afford valuable information on the time-varying behaviour of the multipath estimates. This can be especially critical should the phase-filter be reset and the subsidiary moving average routine used to calculate the standard deviation of these phase-filter corrections.

**Spatial Decorrelation Functions**

These spatial functions, when incorporated within the covariance terms, assume that the residual delays on the distance-dependent errors, that is orbits, ionosphere and troposphere, decorrelate linearly as the receiver separation increases. The magnitude of
uncertainty is related to the a-priori standard deviation of the satellite orbits and the ionospheric and tropospheric vertical delays. Once the latter pair of delays are factored spatially, they are then subject to a mapping function.

**Mutual At-receiver Correlations**

These algorithms attempt to model the correlations between measurements at each receiver that are induced by atmospheric errors, errors within the mapping functions, interchannel biases and antenna errors. Any level of correlation can be applied within the mutual covariance algorithm.

**Unit Variance Scaling Factor**

This routine is necessary for obtaining more realistic formal precision estimates by introducing some of the high-frequency variations from the actual adjustment into the model. It can be seen that trying to instigate real variations within the formal errors would require some considerable stochastic functions. The current default for the scaling factor sample size is set at the previous 30 unit variance values. Studies should be made to see whether a sensitivity analysis could aid in the more optimal selection of the user-defined variances.

**Ambiguity Resolution**

The use of refined stochastic modelling within carrier-phase ambiguity resolution has led to much improved results, namely in terms of accuracy and quicker resolution [Han, (1997b); Teunissen [1997]; Odijk [1999]; Raquet and Lachapelle [2000]). The results presented in Euler and Ziegler [2000] are a particularly good example; in summary, with the improved quality positions derived from ionosphere-free fixed solutions in which the residual ionospheric biases were modelled stochastically, Euler and Ziegler [2000] achieved 100% success rates for fixing integer ambiguities.
Chapter 5 - Design of Research Software System

This chapter chronicles the key issues relating to the design, creation and application of the GPS processing and modelling algorithms within this research software. The pre-processing tasks and routines within the functional double-differencing model, as detailed in Chapter 3, are summarised with regards to the stochastic modelling methodologies as in Chapter 4. Discussion is given as to how the algorithms are incorporated into the research software and the measures implemented to assess their performance.

5.1 OVERVIEW OF PROCESSING SOFTWARE DEVELOPED FOR RESEARCH PURPOSES

This section provides an overview of the processing software developed for this research by summarising the methodologies described in earlier chapters, and providing discussion on the selection of certain parameters and routines where problems could potentially arise. Beginning with a summary of the pre-processing activities, namely the carrier-phase filtering and correction models for the ionospheric and tropospheric delays, it then covers reasoning behind the population of both the functional and stochastic models and their components. The presentation of the final output solutions and the terms that will be used as quality measures, are also detailed. Figure 5-1 shows in flowchart form, the process of determining positions from GPS datasets.
5.2 PRE-PROCESSING TASKS

This section begins with details of the pre-processing tasks necessary to obtain data that is compatible with the software used in this research. The flowchart as Figure 5-2 illustrates the main processes included within the pre-processing activities. All of these steps will be summarized in this chapter and cross-references made, if they have been discussed earlier in this thesis.
Pre-processing Tasks

**Figure 5-2** Flowchart of Pre-processing Activities Instigated within this Research Software

**GPS Data Formats**

The raw data acquired by a GPS antenna-receiver system usually consists of phase and code observables, the broadcast ephemeris data and individual receiver dependent information. Typically it is recorded in a binary format, dependent on the receiver type, with consideration for the processing software. However, given the popularity in GPS surveying and the increased interchange of data between users of different GPS systems, and hence data formats, a common data exchange format was suggested to overcome such problematic incompatibility.

The Receiver INdependent EXchange format was defined as the international data exchange format [Gurtner, (1989)] that would provide a suitable means of ASCII data exchange prior to data processing. All GPS manufacturers provide conversion software to translate their binary format data into the RINEX format, currently at version 2.10 [Gurtner, (2000)]. RINEX stores GPS data in the receiver time frame, such that the time of the received signal is identical for both observed code and carrier-phase measurements for all satellites. Three ASCII files can be created for observation data, navigation message and meteorological data. These files can be identified by the respective designation of the
character o, n or m as the last character of the file’s extension type. Further details of the naming convention and most recent RINEX structures can be found in Gurtner [2000].

Once the raw GPS data files have been obtained, they must be pre-processed and converted into a format compatible with the processing software. The RINEX format as described earlier contains the basic GPS observables and information about the satellite orbits. The Newcastle Exchange Format (NXF) was developed at the University of Newcastle upon Tyne as a research tool to aid in GPS analysis whilst making use of the GASP software [Corbett, (1994), (1995)]. Its content and layout is very similar to RINEX, although it contains some additional parameters relevant to GPS processing.

The RINTONXF executable is used to convert RINEX files (version 2) to NXF format in which many of the pre-processing computations have already been undertaken, i.e. determination of satellite and receiver co-ordinates, receiver clock corrections and rate of change of satellite range. RINTONXF can use the broadcast satellite ephemerides as well as precise orbits as provided in SP3 format by groups including the IGS (cf. §2.1.2). The receiver clock’s offset from GPS time, $t_A$, (cf. Equation A-3) is calculated during the receiver’s single point position computation carried out at each epoch. An approximate receiver position is a prerequisite of any GPS positioning process.

**Range Rate Terms**

An additional term must be calculated at this point to express the receiver clock terms within the double-difference equation. The rate of change of range is defined as the change of geometric range from satellite to receiver and it allows users to determine by how much a satellite range has changed as a function of the receiver clock error. This range rate term may be calculated from:

$$\frac{\delta \rho_A^i(t_2)}{\delta t} = \frac{\rho_A^i(t_2) - \rho_A^i(t_1)}{t_2 - t_1}$$

Equation 5-1

where $\rho_A^i(t_2)$: the geometric range from satellite $i$ to receiver $A$ at time $t_2$

$\rho_A^i(t_2) - \rho_A^i(t_1)$: change of geometric range between epochs $t_1$ and $t_2$
Pre-processing Tasks

The radial velocity of the satellite and is positive or negative depending on whether the satellite is approaching or leaving its perigee\(^1\) with respect to the ground receiver. The maximum radial velocity appears as a satellite crosses the horizon and amounts to 0.9 kilometres per second [Hofmann-Wellenhof et al. (1994)]. The potential change in range is the product of the radial velocity, or range rate, and the time span of the receiver time-tag bias. Here an offset of one millisecond would correspond to a change in range of 90 centimetres.

Receiver Time-tag Biases

When the receiver obtains the actual transmitted GPS signal, it then compares this to its own generated replica of the GPS signal. The degree of offset between the replica and actual signals represents the time taken for the signal to pass from the satellite to the receiver. This corresponds to about 0.07 seconds at the speed of light. However this value also includes any error present in the receiver clock and therefore this time delay is related to the satellite range. As the true time of signal reception \(t_A\) is not known, it is necessary to calculate the satellite-receiver range term \(\rho_A^i(t_A, t^i)\) from:

\[
t_A = (T_A - t_A)
\]

Equation 5-2

where the epoch \(T_A\) is known exactly, as it is the receiver clock time recorded in the (RINEX) observations file (and hence called the 'time-tag'). A problem arises that due to satellite motion and Earth rotation, the range will change by several metres over the period of a few milliseconds. Blewitt [1997] suggests a number of methods which, when combined, can deal with this range-rate term. The most popular is to include additional terms within the pseudorange observation equation as follows:

\[
P_A^i(T_A) = \rho_A^i(T_A, t^i) + \left( c - \rho_A^i \right) \tau_A - c \tau^i + I_A^i + Z_A^i
\]

Equation 5-3

where \(t^i\): transmit time computed from the nominal receiver time \(T_A\)

\(\rho_A^i\): partial derivative of geometric range with respect to receiver clock \(A\)

---

\(^1\) The perigee is the closest point in a satellite's orbit to the Earth.
5.2.1 Application of the Phase-Filtering Techniques

As the source code behind vendors' programs is not generally available to the public, there is potential for incorrect interpretation of their methodologies. Trimble's DAT2RIN converter contains the option to smooth the recorded code pseudoranges with Doppler measurements. Ashtech’s ASHTORIN software also uses the Doppler measurements to smooth the ranges by default, however use of the ASH-RIN executable written at the University of Newcastle could convert the Ashtech binary measurements into RINEX without this Doppler-aiding smoothing. Leica's L2R2 converter (version 5.15) does not support any phase-filtering activities, neither carrier-phase nor Doppler. Because they can differ somewhat in terms of methodology (single or dual-frequency), observables used (carrier-phase or Doppler), filtering time constant and the cycle slip detection settings, their use would not have afforded consistency to the final position solutions.

Following a blanket request [Keenan, (1999)] to the CANSPACe GPS discussion group [CANSPACE, (2000)], it was deemed that there were no carrier-phase filtering executables available freely within the commercial or public domain. The decision was then taken to incorporate such a function within the research software as part of the pre-processing activities (cf. §3.3.3) for two reasons. Firstly, with the application of a unique phase-filter to these unfiltered datasets, the consistency of the phase-filtering routine could be ensured in terms of methodology and parameters used. Secondly and most importantly, having this full control over the filtering process would allow the direct transfer of information describing the sizes of code multipath into the stochastic modelling process, namely the adaptive multipath variance estimation routines (cf. §4.3.1).

The phase-filtering routines were incorporated into the research software as based on the framework of the GPS Ambiguity Searching Program (GASP) developed at the University of Newcastle upon Tyne [Corbett, (1994); Al-Haifi et al, (1997)]. This program, written in ANSI-C, contained routines for single-epoch carrier-phase ambiguity resolution over short distances, typically less than 10 kilometres. As a point of interest, GASP only contains a standard unit weighting routine only.
An overview of the phase-filtering routine is given in Figure 5-3:

![Flowchart](image)

**Figure 5-3** Overview of Phase-filtering Routine applied within this Research Software

The phase-filter approach selected for use in this research software, was the dual-frequency algorithm (cf. Equation 3-53). Modifications for the weighting routine, as detailed in Lachapelle et al [1986], were also included. A filtering time constant $N_{max}$ of 100 seconds was used in this technique and is assumed to have reduced the majority of high-frequency multipath as reflected by surfaces around the GPS antennas.
The decision to use the dual-frequency algorithm was influenced by the stability of the filtered pseudoranges it would yield (cf. Figure 3-6). With the objective of identifying a correct general case stochastic algorithm, it is preferable to use data that is as free of systematic biases as possible so that the final position solutions and the conclusions subsequently drawn are not overwhelmed by the biases. The restricted filtering time constants associated with single-frequency methods can effect sawtooth trends within the time series of filtered pseudoranges; this trend duly impacts both the position solutions and the performance of any adaptive multipath variance estimation routine.

**Cycle Slip Detection Routine**

In order to provide successful and correct multipath reduction routines by means of phase-filtering, the system must be able to monitor any changes in the filter inputs (both code and phase). As the concept of phase filtering is to infer the precision information from the phases onto the code measurements at the same time-tag, it is critical that the two observables are being compared at the same moment in time.

A routine has been implemented to perform the cycle slip detection routine as discussed in §3.3.3. Within this software, there are two separate filters running through the measurements checking how they change with time, looking out in particular for jumps characteristic of deliberate receiver clock offsets. If there is an error on either one, then the notion that they represent the same rate of change of the satellite is invalid.

In the process where the code and carrier-phase delta-ranges are calculated and differenced (cf. Equation 3-47b), a buffer of 15 metres was specified to accommodate a number of phase cycle slips over the two consecutive epochs. As discussed in §3.3.2, detecting a single cycle slip is very difficult when working with noisy code pseudoranges. The logical process behind the detection routine can be seen in the flowchart as Figure 5-4.
Pre-processing Tasks

![Diagram showing cycle slip detection routine](image)

**Figure 5-4** Schematic of the Cycle Slip Detection Routine within the Carrier-phase Filtering Routine

Note that because the notion of real-time positioning is being investigated, the main objective here is the detection of the occurrence of a cycle slip rather than its subsequent identification and repair [Han, (1997); Teunissen, (1998)]. The result of a cycle slip, in the context of phase-filtering, is to lose all information regarding the previous behaviour of that phase measurement. It will be suggested, as part of the further work, that autonomous cycle slip detection and repair routines be considered for inclusion in the pre-processing tasks.

One problem associated with the satellite ordering routine when considering multiple reference stations, is the loss of lock on a common view satellite. Such an interruption results in a very low weighting for the common satellite when required, as compared to one on which successful lock has been maintained for a time greater than the phase-filtering time constant. Not only is a step function seen in the elements of the matrix \((A^TA)^{-1}\) (cf. Equation 3-17), but also in the weight matrix, albeit to different extents depending on the incumbent stochastic model. Within the stochastic models designed, routines were incorporated to minimise the effects of such interruptions and the subsequent time taken for the weight matrix to return to a steady state.
Applications of Moving Window Routines – Phase-filtering and Unit Variance

These routines primarily made use of running average functions to derive standard deviation statistics as soon as possible after the problematic satellite was reacquired. The modelling routines afforded these running average processes include two of the multipath variance estimation routines, the AMV and ATMV algorithms and the unit variance scaling factor routine.

As seen in §4.3.1, the multipath variance routines estimate a standard deviation of the code multipath over a preset time period of m\text{wrf} epochs, but also populate a moving average window when a satellite is (re)acquired. At the initial epoch, \( t = 1 \), when no previous data is available for the real-time estimate of the multipath variance, then the user-defined a-priori multipath variance must be used in the moving window. As subsequent phase-filter corrections are determined, they are included within the running average calculation as featured in Equation 5-4:

\[
\sigma_{AMV}^i = \sqrt{\frac{\sum_{i=1}^{n} \left( P_{fl}^i - P_{raw}^i \right)^2}{n}}
\]  

Equation 5-4

The use of a relatively short moving window size (m\text{wrf} epochs) is necessary given the use of the a-priori multipath variance as the initial starting value for the moving average. An excessively large a-priori variance for the code multipath can make its presence felt throughout the \( n \) epochs of the running average. However when the predefined m\text{wrf} window size is exceeded and the averaging routine begins to move through the dataset, the estimated multipath variance becomes more realistic in representing the true variations within the data. In particular, the AMV and ATMV routines using Equation 4-25, calculate moving averages of the phase-filter correction values.

The time span of this moving average should be less than the average period of code multipath which, according to the JPO (cf. §2.1.4), is regarded as being 6-10 minutes. The optimal size for this window was investigated through a series of tests not dissimilar in procedure to those used to determine the optimal value of the phase-filtering time constant \( N_{max} \) (cf. §3.3.4).

The statistics analysed from this investigation were the accuracy and precision of the mean error vector, and the percentage of error vectors which exceeded the estimated error
Pre-processing Tasks

vector lengths (at 95% confidence) (cf. §5.5). As the overall statistics did not change significantly over the candidate window sizes of 0, 10, 30, 50, 100 and 500 seconds as seen in Table 5-1, a visual analysis of the resultant time series was necessary.

<table>
<thead>
<tr>
<th>WINDOW SIZE (s)</th>
<th>MEAN RMS ERROR VECTOR (m)</th>
<th>SD OF MEAN RMS ERROR VECTOR (m)</th>
<th>% WRONGLY ASSIGNED VECTORS AT 95% CONFIDENCE</th>
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</thead>
<tbody>
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<td>1.579</td>
<td>3.8</td>
</tr>
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<td>0.555</td>
<td>1.582</td>
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<td>3.7</td>
</tr>
<tr>
<td>50</td>
<td>0.555</td>
<td>1.582</td>
<td>3.7</td>
</tr>
<tr>
<td>100</td>
<td>0.555</td>
<td>1.582</td>
<td>3.7</td>
</tr>
<tr>
<td>200</td>
<td>0.555</td>
<td>1.582</td>
<td>3.8</td>
</tr>
<tr>
<td>1000</td>
<td>0.556</td>
<td>1.584</td>
<td>3.8</td>
</tr>
</tbody>
</table>

Table 5-1 Results of Performance Studies into the Optimal Window Size \( mwrf \) used to calculate the Estimated Multipath Standard Deviation

Again, as in the studies for the determination of \( N_{max} \), the baseline separation between the test sites and the magnitude of the final accuracy statements was irrelevant. The optimal value would be the one that filtered, from the time series, the extreme corrections but still afforded some high-frequency variation commensurate with residual code multipath. Rather the interest was in the sensitivity of each individual receiver to the moving window parameters. The final value of this moving window constant was chosen, subjectively by the author, at 30 seconds as it is not critical in terms of accuracy or precision (cf. Table 5-1). A similar running average procedure was implemented for the unit variance scaling factor. For the first \( n \) epochs until the unit variance averaging filter was fully populated \( mwuv \), the scaling factor would be equal to the sum of the unit variance values divided by the number of values thus far.

**Operation of Phase-Filter Routine**

Because multipath interference affects satellite signals to varying extents, and the multipath effects have been shown to be, in general, greatest on low elevation signals, it is critical that the phase-filtering routine is implemented on all satellites with data regardless of their position above or below the defined elevation cut-off angle.
Design of Research Software System

For example, the phase-filter begins to operate as soon as a satellite becomes visible to a receiver and will continue filtering until signal lock is lost or the satellite is no longer visible by the receiver. This ‘whenever-in-view’ filtering means that when the satellite finally attains the cut-off angle relative to both the mobile and reference receiver, it becomes a common satellite and is duly featured in the double-difference matrix. The position computation is then afforded pseudoranges that have been filtered for some period of time and effect a smaller step function within the position solution than for a raw (unfiltered) pseudorange. In the absence of signal interruptions, this continuous phase-filtering routine will maintain output of more precise phase-filtered measurements than the raw unfiltered code pseudoranges alone. Not only will the phase-filter corrections still be computed, but also the standard deviation of these corrections as required by some of the more rigorous stochastic modelling routines (cf. §4.3.1). This functionality is expected to reduce the number of position and time series spikes experienced at the mobile receiver whenever a new satellite comes into view over the cut-off angle and its signals are subject to considerable multipath noises. Such a routine cannot fail to improve the position quality, given the functional and stochastic models used in this research.

The additional CPU time and dynamic memory overheads required to phase-filter all visible satellites above the horizon are undoubtedly greater than, for example, a 10 degrees cut-off. In this operational post-processing application, system resources are not critical. In a real-time operational version however, this would be a significant issue warranting further source code optimisation, as would the means for transmission of these corrections and the corresponding statements regarding the quality of correction parameters. Currently this research software does not introduce any latency terms into the position computation (cf. §2.2.1).

Compression formats should be considered for these correction parameters, although there may be free slots within the RTCM message structure (cf. §2.2.4.1). If stochastic model terms, such as the variances and covariances, were to be transmitted from the reference stations to the mobile station, then for certain ‘faster’ error sources, investigations should be made into the acceptable ages of the variances and covariances. For example, the rate of change of the standard deviations of the phase-filter corrections is purely dependent on the window size used for the calculation. If a user defined a longer averaging window, i.e. a larger $m_{wrf}$ size, then this will reduce the epoch-to-epoch variation of the estimated multipath variances.
5.2.2 Ionospheric Delay Corrections

The research software made use of the single-frequency broadcast ionospheric model, after Klobuchar [1986], that permits a user to estimate the ionospheric delays on C/A-code pseudoranges. As the C/A-code observable has been defined as the base observable within this research, all pre-processing and processing activities make use of the C/A-code pseudoranges on the L1 frequency.

This broadcast model is basically a culmination of several lesser deterministic functions as detailed in [Leick, (1995)] including the ionospheric plate model and the daily cosine model. The former assumes that the ionosphere can be approximated to a plane surface, and the latter takes into consideration the rotation of the Earth and the daily motion of the Sun with respect to the user. The combination of these factors affords the peak vertical ionospheric delay at around 1400 hours local time (cf. 2.1.3.2). A further refinement was made with the introduction of the ionospheric pierce point model that recognises the ionosphere does not begin at the Earth’s surface, rather at an altitude of approximately 50 kilometres (cf. Table 2-3) and includes a slant factor term for cope for this. The final function within the broadcast model, is one that also includes a generalisation for the azimuth and altitude dependency of the vertical group delay on the observed satellites.

This algorithm was coded into the research software with reference to Leick [1995] and ARINC [1991]. The satellite message contains eight coefficients that are used within the algorithm for computing the group delay, as based on the levels of solar flux recorded over the previous five days [Leick, (1995)].

Were dual-frequency measurements being considered explicitly, then the use of linear combinations such as the ionosphere-free pseudorange would warrant use of this broadcast model unnecessary. Langley [1996] mentions in his discussion of ionospheric models that the empirical broadcast model agrees very well against a limited set of dual-frequency measurements and is generally regarded as being a compromise between accuracy and computational efficiency. Newby and Langley [1992] showed that the broadcast model accounted for approximately 70 to 90% of the day-time ionospheric delay and 60-70% of the night-time delay at a mid-latitude site during a time of high solar activity. Their results indicated that the broadcast model can, at times, remove more than the 50-60% of the ionosphere's effect, which is similar to the model’s acknowledged performance level [Klobuchar, (1986)]. Three more sophisticated models were shown not
to perform significantly better than the broadcast model, with one actually performing somewhat worse [Newby & Langley, (1992)].

Some investigations have been made into the usefulness of ionospheric delay prediction models as predictors of TEC count for the region around a receiver, although most models have difficulty accurately representing the upper part of the ionosphere due to a lack of physical measurements [Langley, (1996)]. The date within the current sunspot cycle, number 23, is also effecting a considerable number of problems on GPS positioning [Langley, (2000)]. The solar-cycle and seasonal variations with commonly-noted periods of 20 to over 100 minutes, although these are hard to model [Langley, (1996)].

5.2.3 Tropospheric Delay Corrections

The tropospheric path delay defines the effect of the neutral atmosphere on the GPS signal. The propagation of radiowaves in this non-ionized and non-dispersive medium is independent of frequency affecting, to equal extents, both code ranges and carrier-phases derived from the L1 and L2 carriers. This renders dual-frequency elimination techniques worthless.

**Modelling the Tropospheric Delay within this Research Software**

This software makes use of the tropospheric correction models derived by Saastamoinen as discussed in §2.1.3.4. The initial Saastamoinen algorithm models the tropospheric delay, expressed in metres as:

\[
d_{\text{map}} = \frac{0.002277}{\cos z_i} \left[ p + \left( \frac{1255}{T} + 0.05 \right) e - \tan^2 z_i \right]
\]

Equation 5-5

where
- \( z_i \): zenith angle of satellite \( i \)
- \( p \): atmospheric pressure in millibars
- \( T \): temperature in Kelvins
- \( e \): partial pressure of water vapour in millibars

This model was further refined by Saastamoinen to incorporate two additional correction terms, \( dR \) and \( B \), the former being dependent on the height of the observing site,
and the latter on the height and the zenith angle. Hofmann-Wellenhof et al [1993] gives the refined model as:

\[ d_{\text{rop}} = \frac{0.002277}{\cos z_i} \left[ p + \left( \frac{1255}{T} + 0.05 \right) e - B \tan^2 z_i \right] + \delta R \quad \text{Equation 5-6} \]

Coefficients for these terms can be interpolated from data tables such as those featured in Hofmann-Wellenhof et al [1993]. Using the refined Saastamoinen model and parameters for the US Standard Atmosphere as in Table 5-2, the vertical tropospheric delay at sea level is about 2.3 metres.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>18.0° Celsius (288.15 Kelvins)</td>
</tr>
<tr>
<td>Total Atmospheric Pressure</td>
<td>1013.25 millibars</td>
</tr>
<tr>
<td>Relative Humidity</td>
<td>50.0 %</td>
</tr>
</tbody>
</table>

Table 5-2 Parameters for the US Standard Atmosphere 
[after Collins and Langley, (1999)]

Given the performance and refinements afforded to Equation 5-3 in terms of above sea level heights and zenith angle coefficients, it was decided that the refined Saastamoinen model would be implemented within this research software. It was coded with reference to the algorithm as featured in [Hofmann-Wellenhof et al, (1994)]. The mode of operation is such that the a-priori tropospheric delays are estimated using Saastamoinen’s refined model for all satellite observations as seen by all receivers. These estimates are then stored within the satellite data structures, to be used during the population of the \( b \) vector as in §5.3.

One of the main benefits of empirical tropospheric models, in particular the refined Saastamoinen model, is its functionality in assigning empirical meteorological parameters to the algorithm. This is especially useful given the difficulties of obtaining meteorological measurements in the vicinity of the antenna in an offshore environment (cf. §2.1.4). Accordingly, the use of an empirical tropospheric delay model is deemed a satisfactory alternative, considering economy of accuracy and resources. The lesser performance of the refined Saastamoinen model, as detailed here with that combination suggested in Janes et al [1991] (cf. §2.1.3.4), would afford little difference when
considering the half-metre accuracy positioning achievable with carrier-filtered code DGPS (cf. Table 2-9) [Hofmann-Wellenhof et al, (1994)].

As with the ionospheric delays, the effects of tropospheric delays are greatest on lower elevation satellites and therefore contribute to larger errors in the final position solution, particularly for height. In general, it is believed that the multipathing effects on these lower elevation satellites overwhelms any accuracy improvements yielded by more rigorous atmospheric modelling [Brunner and Welsch, (1993)], although the carrier-phase filtering routine will significantly mitigate the multipath effects. Once an individual value for the tropospheric zenith delay has been estimated at each epoch for each antenna location, it is then transformed via the cosecant-squared mapping function into the actual geometric path of each observed satellite (cf. §4.3.1).

As the nature of the troposphere is similar to that of the ionosphere in regards to its variation, i.e. assumed linear decorrelation and elevation-dependence for both parameters, similar assumptions are made as to how much the zenith delay changes over time. Some processing packages, such as Trimble’s GPSurvey program, can estimate the tropospheric zenith delay parameters at intervals defined by the user.

5.2.4 Performance of the Atmospheric Correction Models

An investigation was made into the performance of the candidate correction models, namely whether code positioning activities benefited from their inclusion through improved position solutions. A standard phase-filtered C/A-code dataset between the reference stations BEDS and LOND (cf. Figure 6-1), was processed using the double-difference approach and the combination of correction models featured in Table 5-3:

<table>
<thead>
<tr>
<th>TEST NUMBER</th>
<th>CORRECTION MODEL APPLIED FOR:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IONOSPHERE</td>
<td>TROPOSPHERE</td>
</tr>
<tr>
<td>1</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>2</td>
<td>Broadcast Model</td>
<td>None</td>
</tr>
<tr>
<td>3</td>
<td>None</td>
<td>Refined Saastamoinen</td>
</tr>
<tr>
<td>4</td>
<td>Broadcast Model</td>
<td>Refined Saastamoinen</td>
</tr>
</tbody>
</table>

Table 5-3 Combinations of Atmospheric Correction Models as applied to a Test Dataset to illustrate their Benefits
Pre-processing Tasks

A weight matrix comprising variances of unity was used in these tests so that the performance of the correction models could be assessed without any external (modelling) influences. Figure 5-5 summarizes the quality of the satellite configuration available to the BEDS-LOND baseline during processing.

**Figure 5-5** Time Series showing Number of Satellites and Relative PDOP Statistic for the Baseline BEDS-LOND

**Figure 5-6** Time Series of True and Formal Error Vectors for a Dataset with No Ionospheric or Tropospheric Path Delay Corrections Applied
Figure 5-7 Time Series of True and Formal Error Vectors for a Dataset with Both Ionospheric and Tropospheric Path Delay Corrections Applied

Note that the overall accuracy for the uncorrected model is about 2 metres, and for the vectors derived from data corrected for both ionospheric and tropospheric delays, about 1 metre. It can be seen from the results in Table 5-4, that the progressive inclusion of these two correction models increases the actual accuracy of the mobile position, as well as the estimated system accuracy. The improvement is most noticeable with the application of the tropospheric delay model, especially around epoch 3600, and also from 5900 onwards as new low elevation satellites come into view (indicated by the high-frequency step functions in the number of satellites in Figure 5-5).

<table>
<thead>
<tr>
<th>TEST NUMBER</th>
<th>RMS ERROR VECTOR (m)</th>
<th>FORMAL ERROR VECTOR (m)</th>
<th>WRONGLY ASSIGNED VECTORS (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.06</td>
<td>2.21</td>
<td>41</td>
</tr>
<tr>
<td>2</td>
<td>1.85</td>
<td>2.17</td>
<td>29</td>
</tr>
<tr>
<td>3</td>
<td>1.22</td>
<td>1.69</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>1.08</td>
<td>1.68</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 5-4 Results of the Studies into the Benefits of Incorporating Two Acknowledged Empirical Atmospheric Correction Models: The Broadcast Klobuchar Ionospheric and refined Saastamoinen Tropospheric Models
When no correction models are applied, the mean true and estimated accuracies are very similar and 41% of the quality measures are wrongly assigned (cf. Table 5-1 and §5.5). The improvement noted with the application of ionospheric corrections only was less significant that with the inclusion of tropospheric corrections only and may be due to the actual ionospheric conditions being considerably different to those modelled within the broadcast ionosphere message. With the application of ionospheric and tropospheric corrections, only 16% of the position solutions are wrongly assigned. These terms are explained in more depth in §5.5.

5.3 FUNCTIONAL MODEL COMPONENTS

The C/A-code pseudorange observables to be used in the least squares adjustment have been subject to carrier-phase filtering and used to determine both ionospheric and tropospheric path corrections, as described in the pre-processing tasks (cf. §5.2). Broadcast satellite ephemerides have been used to calculate the satellite positions at every epoch. A number of dynamic matrix allocation routines are present in the software that can cope with a large number of visible satellites from a number of reference stations. These undifferenced observations must then be ordered within the computation matrices of the differential positioning functional model, as described in §3.1.3, in a structured and consistent manner. This structure is defined through the ordering of the satellites and their observations as follows.

5.3.1 Satellite Ordering Routines

The ordering of undifferenced observations is a critical procedure and impacts the design of all modelling and processing routines within the research software. The correct population of both the covariance matrix of observations and the differencing operator is fundamental to ensuring the correct representation of correlations throughout the stochastic modelling process, and thus into the final results.

Selection of the Double-differencing Reference Satellite

Theoretically the order in which the satellites are ordered and the double-differences created, has no impact on the final results of the position computation although the
presence of errors within the satellite measurements can effect the measurement residuals. In an approach that differences sequentially through the visible satellites, the impact of an erroneous measurement would only be noticed as that satellite was differenced with. Assuming that there was only one measurement in significant error, then the residuals for this computation would then contain one large residual and the remainder small. Outlier detection routines would be able to identify the erroneous observation. Conversely if one satellite was designated as reference satellite and it contained an outlying measurement, then this error would impact all differences referenced to it. This would mean all residuals would be contaminated by the reference satellite error. Any subsequent outlier detection routines would find it difficult to detect and identify the outlying observation.

The satellite ordering routines within this software are based on a combination of the reference station and reference satellite concepts (cf. §3.1.3). For single-baseline processing, the reference satellite concept is used whereby, on an epoch-by-epoch basis, the highest elevation satellite as seen by the mobile receiver, is selected. For multiple-station processing, a hybrid concept is used in which the highest elevation satellite seen by the mobile receiver is again the reference satellite. To make the assignment of receiver order within the least squares matrices more robust, the mobile receiver has been assigned as the reference station for the double-difference computation at each epoch as this station is common to all baselines of interest.

The highest satellite visible to the mobile receiver was selected, as it is more reasonable to suggest that this satellite will be seen by the reference stations for the longest period, thus affording common viewing with the reference stations. As all satellite ranges have the potential to contain atmospheric induced delays, the use of the highest elevation satellite as the differencing reference is preferable, as there will be very little mapping function error afforded to the ionospheric or tropospheric delays present in its measurements. It is also assumed that the code multipath errors on the higher elevation satellite will be smaller than on lower elevation satellites (cf. §3.3.5). Figure 5-8 illustrates the structure of the covariance matrix of undifferenced observations for a mobile receiver and \( n \) reference stations. Note that the mobile receiver is assigned to the first sub-matrix \( C_{\text{mobile}} \).
The reference receivers \((\text{fixed}_A, B \ldots n)\) are placed into sub-matrices according to their ranking within the software initialisation file. Within these sub-matrices, the satellites are ordered in decreasing elevation according to those seen above the cut-off angle by the mobile receiver. It can be seen that this methodology combines the reference station and reference satellite ordering concepts as discussed in §3.1.3.

**Problems with Satellite Ordering Routines**

Owing to the larger receiver separations associated with differential GPS applications, the elevation of an observed satellite relative to a reference and mobile receiver, and indeed relative to the reference stations, could differ significantly. In some instances, a satellite may be visible to the mobile station and reference station \(A\) yet below the cut-off mask of reference station \(B\). This causes some inconvenience in that this satellite can only be used to create one double-difference equation at this epoch rather than two. If possible, it is preferable to include this observation and the vital positioning information it possesses.

The satellite ordering routine corresponding to multiple reference stations has been modified so that it incorporates all unique double-differences visible at each epoch. This required a considerable amount of logical coding within the ordering algorithm, in particular for instances where satellites may be seen in differing orders of elevation angle at different reference stations.

For reference purposes, the dimensions of the matrices required for the least squares computation of the corrections to the estimated parameters, \(\hat{x} = (A^TW^t)^{-1}A^TWb\) (cf. Equation 3-18) are detailed in Table 5-5. In this notation, \(o\) relates to the number of undifferenced code pseudoranges, \(d\) to the number of data (i.e. double-differenced pseudoranges), and \(p\) to the number of parameters.
### Design of Research Software System

#### 5.3.2 Double-difference Pseudorange Functional Model

Once the pre-processing tasks have been completed, the functional components of the least squares model can be populated, namely the matrices $A$ and $b$ as described in §3.1.3, and listed in Table 5-7. The measurements within the design matrix $A$ and vector of observed-minus-computed double-difference values $b$, have been ordered according to the hybrid satellite ordering routines described in §5.3.1. As the $b$ vector is populated, corrections for the ionospheric and tropospheric delays are subtracted from the observed (filtered) pseudoranges, as are the range rate terms as calculated for each pseudorange (cf. §3.1.1).

The highest elevation satellite, as seen by the mobile receiver, is used as the reference satellite for double-differencing purposes by default. The covariance matrix of the undifferenced observations $C_l$ is also ordered this way to ensure consistency. When considering multiple reference stations, the dimensions of these matrices are merely extended to cope with the additional observations.

The initial parameters for the column vector $x$ are based on the mobile receiver’s single point position for that epoch as included in the NXF dataset. It must be noted that this software calculates the rover position on an epoch-by-epoch basis.

---

<table>
<thead>
<tr>
<th>MATRIX</th>
<th>DESCRIPTION</th>
<th>DIMENSIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Design matrix of double-differences</td>
<td>$[d*p]$</td>
</tr>
<tr>
<td>$W$</td>
<td>Weight matrix</td>
<td>$[d*d]$</td>
</tr>
<tr>
<td>$b$</td>
<td>Column vector of (observed-computed) double-differences</td>
<td>$[d*l]$</td>
</tr>
<tr>
<td>$x$</td>
<td>Column vector of corrections to estimated parameters</td>
<td>$[p*l]$</td>
</tr>
<tr>
<td>$R$</td>
<td>Single-Differencing Operator</td>
<td>$[d*o]$</td>
</tr>
<tr>
<td>$C_l$</td>
<td>Covariance Matrix of Undifferenced Observations</td>
<td>$[o*o]$</td>
</tr>
</tbody>
</table>

Table 5-5 Dimensions of the Least Squares Matrices required for the Computation of Receiver Position from Code Double-differences
5.4 STOCHASTIC MODEL COMPONENTS

One of the main advantages of the double-difference technique for differential positioning over that of conventional DGPS is the ease in which the covariance matrix of undifferenced observations $C_I$ can be populated. Once this has been done, the subsequent application of the differencing operator $R$ to create the matrix product $RC_I R^T$ (cf. Equation 3-21), will transform the undifferenced pseudorange observations into double-differenced observations and deal with the mathematical correlations.

This section briefly features on the a-priori parameters, as defined by the user in the initialisation file, for use within the covariance matrix of undifferenced observations $C_I$ (cf. §3.1.3). Figure 5-9 summarises the candidate options available within the stochastic modelling routine.

Figure 5-9 Schematic summarising the Stochastic Functions available during the Population of the Covariance Matrix of Undifferenced Observations $C_I$
5.4.1 Population of the Covariance Matrix of Undifferenced Observations

The elements of the covariance matrix are ordered using the hybrid satellite ordering routine as described in §5.3.1. The a-priori variances and covariances are created depending on the particular stochastic model selected. In order for the quality procedure to produce meaningful results, the values for the standard deviations of the parameters should be as realistic as possible.

5.4.1.1 Variance of Unit Weight Model

In this model, the precision of each code pseudorange is assumed to be independent of all others taking the form of an unweighted least squares solution, i.e. an Identity matrix (cf. Equation 5-7). This model is used as the benchmark model in these studies and is referred to as the Identity matrix or the conventional stochastic model.

\[ C_{i_{AB}} = [I] \]  

Equation 5-7

5.4.1.2 Elevation-dependent Variance Model

This algorithm assumes that the precision of the carrier-phase filtered pseudorange measurements is purely a function of satellite elevation angle. When the cosecant mapping function, \( cosec(\theta) \), is applied to its leading diagonal, the covariance matrix of undifferenced observations is as Equation 5-8:

\[
C_{i_{AB}} = \begin{bmatrix}
\frac{1}{\sin^2(\theta^i)} & SYM. \\
\frac{1}{\sin^2(\theta^j)} & \\
\frac{1}{\sin^2(\theta^k)} & \\
0 & \\
\end{bmatrix}
\]  

Equation 5-8

For software testing purposes and to illustrate the fact that the absolute stochastic modelling of the a-priori precision estimate has no influence on the final position solutions or quality measures, an additional scaling factor was applied in the two most basic stochastic model algorithms. In the unit weight and elevation-weighting models, it was
assumed that the standard deviation of all code pseudoranges was nominally three metres. These models are identified with the tags of *scaled weight matrix* and *scaled elevation-weight matrix*.

### 5.4.1.3 User-defined Variance Models

In this algorithm type, each individual precision term along the leading diagonal is based either on a user-defined constant variance, or the variance output from an empirical stochastic function - both are described later in §5.4.1.5.

### 5.4.1.4 Variance-Covariance Models

This algorithm type is a refinement of the user-defined variance model and contains additional terms to represent the spatial decorrelation of the distance-dependent errors. The decorrelation of the distance-dependent errors, i.e. satellite orbits, ionosphere and troposphere, is implied within specific spatial functions as detailed in §4.3.2. The spatial correlation terms here reflect the elevation of the observed satellite with respect to the separation of the two observing receivers and the mean observed elevation angle for each satellite, as seen by each receiver. The Spatial Decorrelation Functions also use a-priori estimates of the threshold distances at which it is assumed that the distance-dependent errors have become completely decorrelated. This function is essentially a linear decorrelation rate whereby the rates are determined empirically using previous experience of the operations according to the region, time of year and the equipment used. The spatial decorrelation functions are also applied for the covariances between multiple reference stations (cf. Figure 5-8). Mapping functions are then applied to the vertical ionospheric and tropospheric delays terms, as based on the mean elevation angle of the satellite as seen by the relevant GPS receivers.

### 5.4.1.5 Empirical Variances and Decorrelation Distances

As the standard deviations defined by the user represent the estimates of the error sources affecting the undifferenced code pseudoranges, their magnitudes are similar to values suggested by Langley [1997] in his GPS Error Budget (cf. Table 2-5). The use of a large variance estimate for the satellite clocks, to include SA, will affect all observations but will be removed by the differencing process. Here all results of this work are independent of the presence of SA (cf. §1.2.1). The a-priori standard deviation for the
broadcast satellite orbits was set as 5 metres with an estimated decorrelation distance of 20,000 kilometres (equivalent to their altitude).

The format of the atmospheric variances (corresponding to the ionospheric and tropospheric delays), differs depending on whether they are to be included in the user-defined variances or variance-covariance models. As the former does not include any spatial correlation terms, i.e. no covariances, the ionosphere and troposphere terms can be grouped into one atmospheric delay variance, $s_{\text{atmos}}^2$. However, when determining the covariance of a pair of observations, the ionosphere and troposphere must be considered separately through their own individual spatial decorrelation functions.

The standard deviations of the vertical ionospheric and tropospheric delays were 5 and 0.2 metres respectively, to correspond to the estimated residual delays once the Klobuchar and refined Saastamoinen models had been applied. These delays are then mapped into the observed path delay using the cosecant function.

As detailed earlier in §2.1.2, the use of the broadcast ionosphere model after Klobuchar [1986], has been shown to reduce approximately 50% of the ionospheric delay on the L1 code pseudoranges. A larger variance is defined for the vertical ionospheric delay to correspond to this residual ionospheric delay. Saastamoinen’s models [1973] show that, with surface measurements, the troposphere’s wet delay can be modelled to an accuracy of about 2-5 centimetres RMS in the zenith direction. Using the rule of thumb mentioned earlier in §2.1.3.3, a 5 centimetre error in the zenith direction would map into a 15 centimetres worth of vertical position error. As no surface measurements were made, a larger zenith delay error has been assumed.

Decorrelation distances for the distance-dependent errors due to ionosphere and troposphere were 1000 and 200 kilometres respectively, making them equivalent to decorrelation rates of 5 mm/km and 1 mm/km.

The use of the multipath variance estimation modules is limited to the user-defined variances and the variance-covariance models (cf. §4.3.1). The multipath variance term, $S_{\text{multipath}}$, can be estimated using one of the four following algorithms depending on the user’s specified modelling strategy:

- **Constant Multipath Variance (CMV)** - based on an a-priori value specified by the user and does not require any further computations.
- **Adaptive Multipath Variance (AMV)** - calculates the standard deviations of the phase-filter corrections within a moving average window of 30 seconds.
Stochastic Model Components

- Temporal Multipath Variance (TMV) - calculates the standard deviation of the phase-filter correction as a function of consecutive uninterrupted filtering measurements. Maxima and minima values are used to define the bounds of these standard deviations. Values of 5.0 and 0.5 metres respectively were specified although these values could be optimised with further consideration of different filtering techniques and settings, receiver type and the observable type used.

- Adaptive and Temporal Multipath Variance (ATMV) - combines the adaptive and temporal routines such that weighting is applied based on the largest and smallest multipath variance estimates within the 30 second window, or from a running average routine.

The user can also specify the length of the moving average to calculate the standard deviations of the phase-filter corrections as required by models 2 and 4. In general, its length should be less than the cycle rate of code multipath, commonly regarded to be around 2-5 minutes depending on its frequency (cf. §2.1.4). A window length of 30 seconds has been selected as default for this research (cf. Table 5-1). At the first phase-filtering epoch, there are no filter corrections so the AMV and ATMV routines are assigned the a-priori multipath variance, as defined in the initialisation file. As the window populates, these two routines make use of running averages where the variance is based on the standard deviation of the filter corrections yielded thus far. Once the window size has been reached, a standard operating procedure is followed for the averaging routine (cf. Equation 4-23).

With regards to code multipath, the mitigation methods must be able to cope with all levels of residual multipath interference regardless of antenna hardware performance and receiver dynamics. The a-priori standard deviation for receiver noise was defined as a constant value of 0.1 metres under the assumption that all receivers were modern, of geodetic quality and capable of internal integrity monitoring.

**Mutual At-receiver Correlations within each Receiver**

For the purposes of these studies, the incorporation of additional terms to reflect the correlations affecting all mutual observations at individual receivers is an option for the user-defined variances and variance-covariance model types only. It is felt that the mutual correlation function is not suitable (i.e. too sophisticated) for use within the elevation
dependent weight matrix and of course, the unit weight matrix is the benchmark model, i.e. there is no point in considering 'at-receiver' correlations if 'between-receiver' (spatial) correlation is ignored.

When the mutual correlation algorithm is applied to these models, it introduces off-diagonal elements to the covariance sub-matrix relating to each observing receiver. These particular covariance terms are determined using a-priori variances that represent the ionospheric and tropospheric delays.

As described in §4.3.3, the mutual at-receiver covariance function attempts to model the systematic mapping function errors affecting all measurements made at each observing receiver by assigning a larger uncertainty parameter (weight), according to the differential elevation angle. Correlation coefficients can be applied at any level to reflect the mutual correlation between the observations made at each receiver as a function of their differential elevation angles. Measurements from satellites with large zenith angles are highly unlikely to possess large multipath errors and induced atmospheric errors (cf. §3.3.5). For these specific reasons, it has been assumed that any mutual at-receiver correlations will be positive due mainly to errors within the mapping function increasing as the zenith angle increases. The extent to which the mutual correlations are positive is not exactly known, hence a range of a-priori correlation coefficients have been suggested at four levels as follows: 0.0, 0.1, 0.5 and 0.9. The zero correlation coefficient value corresponds to measurements that are uncorrelated with one another (cf. §4.3.3).

5.4.1.6 Inversion of the Covariance Matrix \( C_l \)

The final step in the creation of the data weight matrix is its inversion from \( RC_l R^T \) to \( W \). This computation is not usually a problem using a standard matrix inversion routine, such as that based on Cholesky decomposition and summarized in Leick [1995]. However it must be noted that, with use of the mutual correlation algorithm and the additional correlations it affords, the covariance matrix can become numerically unstable. This is primarily because with very high levels of correlation terms between the observations at one receiver introduces very high linear dependence within the data and consequently the mathematical model. More advanced matrix inversion routines could be used to overcome these problems, although this would require some considerable coding amendments.

The software has been designed such that information from all steps of the pre-processing and least squares computation, for each epoch, can be output in the form of ASCII logfiles for analysis tasks.
5.5 QUALITY ASSURANCE ROUTINES AND STATISTICS USED

This section details the routines used to assess the output of the processing software described in Chapters 3, 4 and 5, and the comments arising from the approaches followed within the quality assessment procedures.

5.5.1 Performance Assessment

In theory, the use of stochastic modelling should enhance the quality measurement process by better modelling residual differential errors and yielding more realistic quality measures. In kinematic positioning applications, there is no truth reference and consequently it is not usually possible to quantify the true accuracy of the vehicle’s trajectory. Therefore, in order to maintain the integrity of the positioning system and thus the vessel safety, the system developer needs to ensure that the quality indicator correctly describes the positioning performance of the system at all times.

With the benefit of a truthing system, the true navigation errors can be calculated, i.e. the difference between the truth and the navigated solution. The size of these errors, when compared to the formal estimated errors, can indicate the integrity of the positioning system.

The goal of this quality routine then, is not primarily to establish absolute performance levels when using more sophisticated (and hopefully more accurate) stochastic modelling, but rather to show the relative improvement in performance through the precision estimates as resulting from their application.

One approach concerned with the WAAS metrics, i.e. the accuracy, integrity and availability of the WAAS system for all potential users, is the National Satellite Test Bed (NSTB), a WAAS prototype [Hansen, (1998)]. Here the accuracy is determined simply from the comparison of a navigated position to the known truth position. The integrity of a system is quantified as the ratio of the true error to that formal error predicted by the system. A histogram of this ratio, as calculated at every epoch, should portray Gaussian behaviour and fall within a parabola whose equation is based on the size of the sample set [Hansen, (1998)]. Hill et al [1995] used a similar procedure during the Benchmarking of the UKOOA Guidelines and also termed it as the integrity ratio. Availability, in this example, relates exclusively to the denominator of the integrity studies - the position precision estimator - otherwise known as the formal error.
The assessment of positioning accuracy has been carried out with analysis routines on two levels similar to that discussed by Hill et al [1995], Hansen [1998] and Ochieng et al [1999]. The first is to compare the estimated position with the true position and the second relates to the accuracy of the position estimator precision. This is determined from propagation of the assumed observation errors through the data processing model when compared with the true error in position. This figure is the only one that would be yielded by an operating system in real-time and will be taken as the 'system accuracy' in an operating scenario. For the purposes of this research, the studies detailed in Chapter 6 have been carried out on static baselines between known co-ordinated points to afford estimates of the true positioning error.

Outlier and Blunder Detection

For the purposes of this research, no outlier detection routines recommended by UKOOA (cf. §3.2) were implemented as they do not describe the overall quality of the resulting position and are therefore regarded as a background process within the position estimation routines [QUEST, (1996)]. Using such routines, a good (correct) measurement could still be excluded from the least squares computation as an outlier which would then lead to a reduction in the positional and vertical accuracy of a position solution.

Blunder detection is necessary for quality assurance purposes and to ensure that reliable positions are afforded to the positioning application. Such an error trapping process is used here in conjunction with the position confidence analysis (PCA) routine as described later in this section. For example, if the UKOOA recommended $w$-test has to be repeated a number of times, then the amount of redundancy within the adjustment will be reduced (cf. §3.2). Should only one redundant pseudorange remain, then the normalised residuals will come out equal and no outlier detection can be performed. This position would then be flagged as bad - as would any epoch where all of the normalised residuals exceed the test tolerance. The use of a 1% significance level makes the presumption that any residual larger than the tolerance is an outlier.

The incorporation of outlier detection routines applying a recursive exclusion process in measurement space would have demanded considerably more coding within the research software. Ideally, it would be more applicable to include the outlying observation and down-weight its influence within the least squares adjustment; this would result in greater positional (and vertical) repeatability than with one degree of freedom less. This point is
Quality Assurance Routines and Statistics Used

especially relevant given the satellite visibility issues raised from long-range positioning activities as discussed in §3.1.3.

Because this post-processed research makes use of large datasets and truth data, it is possible to determine the blunders in co-ordinate space rather than measurement space as recommended by UKOOA. By comparing the difference in two consecutive position solutions against a pre-defined threshold, it is possible to identify epochs where blunders have occurred, i.e. the measurement errors were not acknowledged by the various modelling processes. From this, the assessment of the quality of the position solutions is free of systematic blunders. This method may be regarded as simplistic in the light of more rigorous UKOOA recommended statistical tests however it has proved to be effective in the analysis of large post-processed datasets.

**Position Confidence Analysis Routine (PCA)**

Although the relative DOP values should be used to obtain an indication of when GPS is not likely to produce good positioning results; they should not really be used for quantifying the quality of a position fix for a number of reasons.

1. The relative DOP statistic does not detect any outliers present in the pseudorange measurements that will ultimately lead to a poor position fix.
2. The use of low elevation satellites can improve the geometrical configuration and hence the GDOP statistic, however their ranges are subject to large atmospheric errors and other biases that will again afford a poor position fix.
3. Relative DOP values give no indication of the magnitude of SA and multipath errors present on all pseudorange observations.

These three reasons illustrate why the relative DOP values, as based on differential satellite geometry, should not be used as precision indicators and why more rigorous a-posteriori means should be taken to indicate the estimated system accuracy.

As can be seen from Equation 3-22, the choice of weight matrix has the potential to significantly affect the size and characteristics of the quality measures yielded at each epoch. Consequently, the reliability of the position solutions is not being considered formally in the sense of external reliability statistics, rather the quality of the solutions in measurement precision terms only. The precision of resulting positions can be assessed from the product of the unit variances and the formal errors yielded by the least squares adjustment. The unit variance value is assessed at each epoch and its value considered
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within a moving average routine. The output of these averages can be used to scale the formal errors, a procedure that is covered in §5.6.2.

For this research, a routine was created to assess the performance of the system in terms of accuracy and integrity (cf. §2.3) by comparing the system accuracy with the true accuracy as determined from the occupation of known co-ordinated locations. The ratio is identical in format to Hansen's integrity ratio [1998], yet renamed here as fidelity ratio which corresponds to the accuracy of the quality measures without any timely alarm warnings. It can be calculated from Equation 5-9:

\[
\text{fidelity ratio component} = \frac{\text{true error component}}{\text{formal error component (at preset confidence level)}}
\]

Equation 5-9

At each epoch, the fidelity ratio for the three components of latitude, longitude and height can be calculated as well as the plan and 3-D position vectors. By comparing the true and estimated errors on each component and categorising the results into quality bins, according to the overall ratio, the performance of the positioning system can then be assessed over a dataset. If the fidelity ratio is at, or below, one for 95% of the time, then the system yielding that figure can be said to possess high-fidelity formal precision estimators – this is what is required for a dynamic real-time system.

The PCA routine at each epoch requires data structures containing the true position errors as well as the raw a-posteriori standard deviations direct from the covariance matrix of adjusted parameters. The term raw means that it is at 1-sigma confidence level, and unscaled by any unit variances. The final required input is the current epoch's unit variance scaling factor.

Modules within this research software can provide these accuracy statements for each epoch or an entire dataset. Values provided on an epoch-by-epoch basis are ideal for dynamic positioning and navigation purposes, however the distribution of the errors can be seen best over a longer period. An error trapping routine is necessary within the position confidence routine to compare the true errors over two consecutive epochs. If they differ by more than a user-specified amount, then it is believed that an outlying observation or interruption has occurred at the current epoch. Epochs containing such interruptions were flagged and not included within the binned values. The default threshold used within this error trapping routine is 0.5 metres over 1 second.
Once an epoch is deemed interruption-free, its unscaled formal errors are multiplied by the unit variance scaling factor obtained up to that particular epoch. Following this, the true error component is divided by the scaled formal error component to yield the fidelity ratio for that component (cf. Equation 5-9). If the ratio is less than one, then the model is correctly estimating the errors within the system at that epoch albeit pessimistically. If the fidelity ratio is greater than one, then those output formal errors indicate a positioning quality better than is actually given and the system is deemed optimistic. This latter scenario is deemed unsuitable for DGPS as the system is optimistic and the quality of positioning is not as high as indicated by the precision measures – the ideal factor is one [Hill et al, (1995)].

Table 5-6 summarizes the classifications afforded by the Fidelity Ratio statistic.

<table>
<thead>
<tr>
<th>FIDELITY RATIO VALUE</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRUE ERROR : FORMAL ERROR</td>
<td>OK – system is overestimating the true errors [PESSIMISTIC]</td>
</tr>
<tr>
<td>&lt; 1</td>
<td></td>
</tr>
<tr>
<td>= 1</td>
<td>DESIRED – the system is functioning correctly [IDEAL]</td>
</tr>
<tr>
<td>&gt; 1</td>
<td>BAD – the system thinks it is performing better than it actually is [OPTIMISTIC]</td>
</tr>
</tbody>
</table>

Table 5-6 Classification of Fidelity Ratios in this Research, and their Definitions

The first two options described in Table 5-6 are desirable in that an operator would rather have overestimation where the system was, pessimistically, suggesting higher errors. Groups conducting research into WAAS believe that if the integrity ratio (akin to fidelity ratio) is less than one, then the positional integrity has been satisfied at the tested confidence level. Should the ratio exceed one, then the model has failed to correctly estimate the magnitude of the errors on the navigation solution and has thus resulted in a breach of system integrity. It is doubtful that any vessel navigators would want to have optimistic quality measures undercutting the actual errors within the system.

Once the entire dataset has been analysed, the statistics can be assessed in terms of overall performance. When considering a normal distribution at a 68% confidence level, it is assumed that the mathematical model would correctly estimate the errors within the system and yield a fidelity ratio of less than one for 68% of the dataset. Conversely, it
could be assumed that 32% of the same dataset would yield fidelity ratio values exceeding one. A similar binning process could also be carried out simultaneously at the 50, 95 and 99% confidence levels whereby sigma-scaling factors of 0.6745, 1.96 and 2.447 respectively should be used.

The ideal positioning system would be one balanced in terms of functional and stochastic models, and would yield a fidelity ratio of one. Ultimately, this would mean that the contents of the aforementioned quality bins would be zero. Discarding this theoretical utopia and considering the real-life PCA routine, any solutions falling above or below the optimal threshold of one are flagged as incorrect; that is, the stochastic model and thus their quality measures, have been wrongly assigned. The final component of this routine classifies the percentage of wrongly assigned epochs according to a specific confidence level, in this case 95% to correspond with UKOOA recommendations. An example of the listings output by the PCA routine is given in Table 5-7, and corresponds to the baseline BEDS-ONSA as featured in §3.3.

<table>
<thead>
<tr>
<th>CONFIDENCE LEVEL (%)</th>
<th>SCALE vs 1s</th>
<th>PERCENTAGE OF WRONGLY ASSIGNED POSITION SOLUTIONS IN:</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.6745</td>
<td>HEIGHT: 18.7, PLAN: 3.7, VECTOR: 10.4</td>
</tr>
<tr>
<td>68</td>
<td>1</td>
<td>HEIGHT: 18.5, PLAN: 4.9, VECTOR: 16.2</td>
</tr>
<tr>
<td>95</td>
<td>1.96</td>
<td>HEIGHT: 4.0, PLAN: 3.9, VECTOR: 4.4</td>
</tr>
<tr>
<td>99</td>
<td>2.447</td>
<td>HEIGHT: 1.0, PLAN: 1.0, VECTOR: 1.0</td>
</tr>
</tbody>
</table>

Table 5-7 Matrix of Results from the Position Confidence Analysis Routine - Percentage of ‘Wrongly Assigned’ Position Solutions at Different Confidence Levels

As can be seen from Table 5-7, raising the confidence level by increasing the confidence interval, improves the fidelity of the final positions. To provide an example of usage, for the vector estimated at 68% confidence, 16% of the epochs have been incorrectly assigned in terms of their fidelity ratio according to a normal distribution. For the 95% confidence level, 4.4% of the position solutions have been wrongly assigned as compared to the true error in position.

Comments on these Quality Assurance Routines

According to Hansen [1998], the confidence interval with which the formal errors are scaled from 1-sigma (68% confidence) to the required interval, is the one and only number
that in-flight avionics will receive from WAAS. For satellite navigation activities, this confidence interval will be time-dependent. It is also noted in Hansen [1998] that system integrity can always be simply protected by increasing the confidence interval denominator in the integrity ratio, as seen in Table 5-7. Changing the criteria by increasing the size of the formal error is not correct and should not be practised as its use can be deemed too subjective. However, for avionics systems, too large a confidence interval could result in the activation of alarm-based failures for precision landings which is also clearly not desired.

It is interesting to note that although the team verifying the UKOOA Guidelines, in Roberts et al [1997], acknowledged the possible outcomes of the integrity ratio, they made no objective statements about departures from the optimal ratio value of 1 (cf. Hill et al, [1995]). The question is, can a solution possessing a fidelity ratio of 1.05 be of equal fidelity (or integrity), as one with a fidelity ratio of 0.95? Although these position solutions are pessimistic and optimistic respectively by 0.05 units of fidelity ratio, are they close enough to be acceptable for positioning activities under UKOOA guidelines?

This concept of subjectivity within the quality measures is where problems always arise and will likely be commonplace for the application of high fidelity differential offshore positioning.

To summarize, the quality measures in this research relate directly to the accuracy and integrity associated with a navigation system (cf. §2.3). As no time of alarm waning has been considered in this research, the term fidelity will be used in place of 'integrity'.

5.5.2 Unit Variance Scaling Factor

The unit variance scaling factor is required to scale the standard deviations for the final precision indicators (cf. §4.3.4) by the reciprocal of the unit variance statistic. The basic arithmetic behind this scaling routine was discussed in §4.3.5 and is not dissimilar to that for the moving window of phase-filter corrections (cf. Equation 4-23). Scaling the entire weight matrix by this factor does not change the final value of the corrections, only changing individual elements of the covariance matrix can do this.

Once the data processing begins, a moving window structure is populated with the real-time unit variance values as calculated at each epoch (cf. §4.3.5). As this window is being populated initially, that is, the total number of epochs processed are still less than the default unit variance scaling window size \( mwwv \), a running average is calculated of the
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unit variances yielded so far. This average is then used to scale the a-posteriori standard deviations at the current epoch. During this time, the carrier-phase filter routine is also converging towards more precise filtered pseudoranges (cf. §3.3).

Once the moving window is fully populated, then the averaging routine begins in earnest and travels in real-time through the time series as long as it encounters no interruptions within the dataset. The scaling factor at each epoch contains information from a previous number of unit variance values (default value for mwuv was 30 seconds). This represents the quality of the adjustment over the last 30 epochs and reduces spurious high-frequency terms from propagating through into the formal precision estimates.

It must be noted that this routine, and indeed any using a moving average of the unit variance values, would afford no error checking capabilities were only four satellites visible. This scenario would afford only three available double-differences and no redundancy meaning that the unit variance value would be one.

More critical in terms of both accuracy and quality measures, is when a problem occurs within the receiver, for example a receiver clock jump. When such an interruption occurs on at least one of the receivers used in the positioning computation, it is said that a receiver cycle slip has affected that receiver (cf. §3.3.1). New receiver clock and approximate position estimates will be computed by the pre-processing algorithm, in this research, RINTONXF, which will significantly affect both functional and stochastic routines. In functional terms, the phase-filtering procedure will need to be restarted for all satellites observed so spikes will be encountered in the position solution and PCA time series. In stochastic terms, the adaptive multipath variance estimation processes must also be restarted along with the unit variance scaling factor. Step functions will be seen in the phase-filtered pseudoranges and the output of the position confidence analysis routines.

Investigations were carried out to determine the optimal value for the unit variance scaling factor with similar findings to those of the multipath moving window (cf. §5.2.1). A window size of 30 seconds would be used to scale those standard deviations yielded from the relevant elements of the covariance matrix of adjusted parameters. It was hoped that this averaging procedure would mitigate the influence of any spurious or extreme unit variance values at individual epochs.
5.5.3 Discussion behind QA Processes and Statements Used

These quality assurance routines are needed in order to quantify the quality of an epoch’s position solution, or a series of solutions. Due to the considerable amount of data that can be generated from GPS processing packages, including formal errors, true errors, satellite details, carrier-phase filter results and least squares adjustment results, obtaining one statistic that can afford meaningful assessments of system performance is a popular goal in GPS positioning. Driven by the research objective of improving the quality measures for differential positioning, the fidelity ratio values at 95% confidence were subject to a ‘Goodness of Fit’ test so as to see the levels of agreement throughout the time series.

A similar quality binning procedure to that in Roberts et al [1997] and Hansen [1998] was created within this research software. It comprised of bins with half standard deviation widths [Hansen, (1998)] as opposed to the quantitative measures of 0.5, 1, 3, 5 and 10 metres as featured in Roberts et al [1997].

**Goodness of Fit (Chi-squared) Test**

The current PCA procedure of displaying the ratio of wrongly assigned position solutions against the total number of processed epochs, requires that values be displayed at the relevant confidence levels. The grouping of these numbers, along with the true and estimated errors, inevitably leads to a surfeit of information to be analysed, some of it ambiguous. Ideally, it would be advantageous to have one statistic that describes exclusively the quality of the formal errors at the 95% confidence level, and can be described as the fidelity of the quality measures.

Using the output of the PCA routine leads nicely to the notion of rigorously analysing the distribution of the processed results. The approach followed was to derive a statistic that reflects the magnitude of similarity between the observed dataset and an expected dataset portraying a normal distribution. This is known as the ‘Goodness of Fit’ test and also as the $\chi^2$ (chi-squared) test.

The chi-squared test statistic, $\chi^2$, is the weighted sum of the deviations of the observed and expected values. The variance of the sampling distribution of the sample means (the standard error$^2$) was taken as being equal to $\frac{\sigma^2}{n}$, or that 95% of the normally distributed population will fall within a range $\mu$ of $\pm 1.96\sigma$ [Kanji, (1994)]
The objective of this routine is to investigate the significance of the differences between observed data arranged in \( k \) classes, and the theoretically expected frequencies in those \( K \) classes. The proposed algorithm will be used to compare observed frequencies against those obtained under assumptions about the parent populations and should permit the classification of the estimated (formal) error according to the magnitude of the true error. This routine has therefore become adaptive in recognising an increase or decrease in the epoch-to-epoch quality of the least squares adjustment as deemed necessary for the successful implementation of such a quality assessment procedure.

Using half-standard deviations, the classifications for the expected distribution of formal errors up to \( 3\sigma \) contain 7 classes either side of the true value (zero) as follows:

<table>
<thead>
<tr>
<th>CLASS NUMBER (FOR TWO-TAILED DISTRIBUTION)</th>
<th>RANGE IN STANDARD DEVIATION (( \sigma ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 / 8</td>
<td>0 – 0.5</td>
</tr>
<tr>
<td>6 / 9</td>
<td>0.5 – 1.0</td>
</tr>
<tr>
<td>5 / 10</td>
<td>1.0 – 1.5</td>
</tr>
<tr>
<td>4 / 11</td>
<td>1.5 – 2.0</td>
</tr>
<tr>
<td>3 / 12</td>
<td>2.0 – 2.5</td>
</tr>
<tr>
<td>2 / 13</td>
<td>2.5 – 3.0</td>
</tr>
<tr>
<td>1 / 14</td>
<td>3.0 – ( \infty )</td>
</tr>
</tbody>
</table>

*Table 5-8 Classification of Expected Distributions for Goodness of Fit Tests*

Considering both the negative and positive tails around the central point (median – NULL error ratio value), this tallies to 14 classes. The outermost class in each tail, numbers 1 and 14, span from negative infinity to negative \( 3\sigma \), and positive \( 3\sigma \) to positive infinity respectively. A one-tailed distribution contains 7 classes spanning from zero to positive infinity.

The formal error component is used to size the \( k \) classes by forming the approximate width of the formal component at the 95% confidence level as scaled by the unit variance scaling factor. The true error component is then compared with the bin widths as part of the sorting procedure. If the true error component falls within the bounds of a bin, a frequency counter is raised on that particular bin and the routine then moves on to assess the next epoch.
Quality Assurance Routines and Statistics Used

Format of 'Goodness of Fit' Algorithm

The chi-squared test statistic for each component is calculated by:

\[ \chi^2 = \sum_{i=1}^{\kappa} \frac{(O_i - E_i)^2}{E_i} \]  

where \( O_i \) and \( E_i \) represent the corresponding frequencies for each of the \( \kappa \) classes. This statistic is compared with a value obtained from the \( \chi^2 \) tables with \( v \) degrees of freedom, in general where \( v = \kappa - 1 \). If the test statistic is greater than the critical value, the null hypothesis that the observed and theoretical distributions agree is rejected. Values can be selected for the left and right-side critical values that represent the lower and upper boundaries of the \( \chi^2 \) distribution, with reference to Table 5 in Kanji [1994].

Expected Distribution Characteristics

When considering the one-dimensional quantities of north, east and height components, their distributions are assumed to follow a two-tailed distribution because the ratio of the height errors to the model errors could be either positive or negative. If the true (real numbered) error is less than the model error, the ratio will be negative and contained within the left-hand tail. However, if the true error is positive, it will then fall within the positive (right-hand) side of the expected distribution plot.

Conversely, as the parameters of plan and vector are derived from two or three position components, they are subject to some squaring and therefore all components are positive. This means that the expected distribution will now fall entirely within the positive tail only. Consideration has been given to this consequence as will be seen later during the comparison of the chi-squared test statistic with the critical value for the plan and vector components.

For the initial purposes of analysis, five chi-squared test statistics were determined.

- A triplet of two-tailed distributions for the position components of north, east and height.
- Two one-tailed distributions for the plan and vector components.
Kanji [1994] states that there are a number of limitations affecting the goodness of fit test, in particular, problems will arise when using a very small dataset considering that the minimum class size should contain at least 5 values. The actual distribution of the position results will be very biased towards one area of the two-tailed distribution and thus show a considerable departure from the expected normal distribution. Calculation of the chi-squared statistic will be impossible with the statistic possibly being a value of infinity. This will be because the products of \((O_i - E_i)^2\) (cf. Equation 5-5) are very large due to the lack of observed values in particular classes. In order for the test statistic to be unbiased, there must be at least 5 occurrences in each of the user-defined classes. As this research assigned, in total, 14 bins of 0.5σ width (cf. Table 5-10), for the outermost bins to contain 5 values, under the premise of a normal distribution, would require a minimum sample size of 3847 values.

**Study of Chi-squared Test Routine**

An investigative study has been carried out into the performance of this chi-squared algorithm, in particular, for the purposes of testing the distribution of the final output data. A baseline has been processed between the BEDS and ONSA stations (cf. §3.3.4.2). The pre-processing settings and functional model used are as described earlier in this chapter with a standard elevation dependent weighting function (cf. §4.3.1). The dataset, recorded at 1 Hertz, is nominally 6120 seconds in length.
<table>
<thead>
<tr>
<th>CLASS</th>
<th>EXPECTED OCCURRENCES</th>
<th>ACTUAL OBSERVED OCCURRENCES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>NORTH</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>773</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>170</td>
</tr>
<tr>
<td>3</td>
<td>101</td>
<td>104</td>
</tr>
<tr>
<td>4</td>
<td>268</td>
<td>134</td>
</tr>
<tr>
<td>5</td>
<td>560</td>
<td>131</td>
</tr>
<tr>
<td>6</td>
<td>912</td>
<td>245</td>
</tr>
<tr>
<td>7</td>
<td>1166</td>
<td>710</td>
</tr>
<tr>
<td>8</td>
<td>1166</td>
<td>694</td>
</tr>
<tr>
<td>9</td>
<td>912</td>
<td>521</td>
</tr>
<tr>
<td>10</td>
<td>560</td>
<td>1018</td>
</tr>
<tr>
<td>11</td>
<td>268</td>
<td>303</td>
</tr>
<tr>
<td>12</td>
<td>101</td>
<td>367</td>
</tr>
<tr>
<td>13</td>
<td>30</td>
<td>377</td>
</tr>
<tr>
<td>14</td>
<td>8</td>
<td>544</td>
</tr>
</tbody>
</table>

Table 5-9 Table showing Summary of Results from Chi-squared Test Study

Although some trends are easily seen in this table, the results for each component can be better appreciated as the time series in Figure 5-10 which shows the expected and observed frequencies within each quality bin. The expected number of values are shown as the lighter quantity and the observed number as the darker quantity.
Figure 5-10 Histogram of Results for the Chi-squared Test Studies showing the Distribution of Observed and Expected Frequencies - North, East, Height, Plan and Vector
According to Hansen [1998], the distribution of the integrity ratio and its tails should be within the parabola of a Gaussian distribution curve to preserve integrity. Clearly, this is not the case as can be seen from Table 5-9 and the five histograms in Figure 5-10. There are some significant systematic biases within the observed frequency population especially within the outer tails of the north and east components. To provide some background to the results this study, the following time series, as Figure 5-11, shows the true and formal errors for the plan and vector positions respectively.

![Figure 5-11 Time Series of True and Formal Position Errors (Plan and Vector) for the Baseline BEDS-ONSA as processed within the Chi-squared Test Study](image)

If the chi-squared test statistic, as calculated from Equation 5-5, is less than the critical value, then it can be said that there are no indications that the distribution is not Gaussian. That is, there are no signs of systematic biases within the output population. However, if the test statistic does exceed the critical value, the hypothesis that the two
distributions are from similar parent populations, is rejected. It can then be said that there are indications that the distribution is not Gaussian.

It must be noted that the critical values for these statistics will differ as they are represented by one and two-tailed distributions. For the two-tailed distributions, i.e. north, east and height, there were 14 classes and therefore 13 degrees of freedom. With a significance level of 5%, the critical test statistics for the two-tailed distribution was calculated to be:

\[ \chi^2_{13} (0.05) = 24.74 \]  

Equation 5-11

With 6 degrees of freedom, the critical value for the one-tailed distributions corresponding to the plan and vector components was:

\[ \chi^2_{6} (0.05) = 12.59 \]  

Equation 5-12

Considering the test dataset for the baseline BEDS-ONSA, the test statistics for the two-tailed and one-tailed distributions were as follows:

<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>CALCULATED $\chi^2$ STATISTIC</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRITICAL VALUE</td>
<td>24.7</td>
<td>CRITICAL VALUE</td>
</tr>
<tr>
<td>North</td>
<td>116232.4</td>
<td>There are indications that the distribution is not Gaussian</td>
</tr>
<tr>
<td>East</td>
<td>232870.2</td>
<td>There are indications that the distribution is not Gaussian</td>
</tr>
<tr>
<td>Height</td>
<td>9767.3</td>
<td>There are indications that the distribution is not Gaussian</td>
</tr>
</tbody>
</table>

Table 5-10 Breakdown of Chi-Squared Test Study Results corresponding to the Two-tailed Distributions of the North, East and Height Components of the BEDS-ONSA Baseline
### Quality Assurance Routines and Statistics Used

<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>CALCULATED χ² STATISTIC</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRITICAL VALUE</td>
<td>12.6</td>
<td>CRITICAL VALUE</td>
</tr>
<tr>
<td>Plan</td>
<td>103428.684</td>
<td>There are indications that the distribution is not Gaussian</td>
</tr>
<tr>
<td>Vector</td>
<td>5218.754</td>
<td>There are indications that the distribution is not Gaussian</td>
</tr>
</tbody>
</table>

*Table 5-11* Breakdown of Chi-Squared Test Study Results corresponding to the One-tailed Distribution of the Plan and Vector Components of the BEDS-ONSA Baseline

**Results of Chi-squared Test Study**

It can be seen from both Table 5-12 and Table 5-13 that the chi-squared test statistics obtained in this study were considerably larger than the critical values calculated earlier. This indicates that the observed distributions are biased and not commensurate with a normal distribution. When one also considers the time series in Figure 5-11, it can clearly be seen that there are a number of systematic errors within the time series where the model is not correctly estimating the real errors in the dataset. For some periods, the true and formal error series are almost coincident and at others, the formal series is considerably greater than that of the true series. The fidelity ratio will be around one, and then greater than 1, for these respective periods. It is these deviations that represent the biases at the tails of the distributions as seen in Figure 5-11. These biases, clearly apparent in the observed distributions, could be a result of incorrect reference station co-ordinates contaminating the fidelity ratios of the five component types although this is unlikely given the status of most sites. There also appeared to be a very large number of epochs falling into the outermost classes of the expected distributions, namely, around the tails of the distribution in the classes corresponding to ±3σ ±∞.
5.6 CONCLUDING REMARKS ON SOFTWARE DESIGN AND TESTING

This chapter has detailed the functionality of the designed research software along with some of the problems encountered with its component functions. A comprehensive piece of research software has been created, as required, to achieve the research objectives, comprising of some existing data conversion and processing routines as well as a number of new routines for pre-processing, modelling and quality assessment tasks. A text-based initialisation file allows users to set specific parameters for the functional and stochastic models as well as for the pre-processing tasks. A number of pre-processing modules have been created that will aid in the reduction or elimination of a number of error sources affecting GPS code pseudoranges. The inclusion of a carrier-phase filtering routine and models from which code pseudorange corrections for the ionospheric and tropospheric delays can be determined, are necessary to reduce the effects of code multipath and atmospheric delays on the undifferenced GPS observables.

The double-difference functional model is capable of processing raw and phase-filtered pseudoranges within single and multiple-baseline networks. Four types of stochastic model have been created to reflect the estimated precision of phase-filtered undifferenced code pseudoranges. Also included are modules capable of real-time multipath variance estimation and the modelling of mutual at-receiver correlations.

Discussion has also been included on the phase-filtering techniques under the conditions of reduced observations and cycle slips, the concepts and potential problems of the moving window routines for the multipath variances and unit variance scaling factor and how they should be calculated in certain scenarios. An analysis routine has also been created that categorises the epoch-by-epoch system accuracy estimates with respect to the true errors, allowing the performance of these models to be assessed in terms of the fidelity of the estimated quality measures.

Summary of Final Processing Options

The following sub-section details the overall processing options. The pre-processing and processing routines include:

- Double-differencing of pseudorange observations – dealing with the induced mathematical correlations with terms based on Gauss’s law of error propagation
Concluding Remarks on Software Design and Testing

- Carrier-phase filtering of code observations using dual-frequency Hatch-based system with a filtering time constant of 100 seconds
- Corrections for the L1 ionospheric pseudorange delay using the single-frequency ionospheric model after Klobuchar as broadcast in the GPS navigation message
- Corrections for the tropospheric delay using the refined Saastamoinen model
- Receiver clock corrections using range-rate terms

These pre-processing and functional routines are constant for all tests detailed in the long-range studies. This research software attempts to eliminate, or at least mitigate, the errors affecting code pseudoranges using the following functional and stochastic routines described in Table 5-12.

<table>
<thead>
<tr>
<th>ERROR</th>
<th>FUNCTIONAL MODEL</th>
<th>STOCHASTIC MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbital Errors</td>
<td>Double-differencing</td>
<td>Spatial Decorrelation Function</td>
</tr>
<tr>
<td>Satellite Clocks</td>
<td>Double-differencing</td>
<td>Constant Variance</td>
</tr>
<tr>
<td>Ionosphere</td>
<td>Double-differencing</td>
<td>Spatial Decorrelation Function</td>
</tr>
<tr>
<td>Troposphere</td>
<td>Double-differencing</td>
<td>Spatial Decorrelation Function</td>
</tr>
<tr>
<td>Multipath</td>
<td>Carrier-phase Filtering</td>
<td>Various Adaptive Multipath Variance Routines</td>
</tr>
<tr>
<td>Receiver Noise</td>
<td>Carrier-phase Filtering</td>
<td>Constant Variance</td>
</tr>
</tbody>
</table>

Table 5-12 Summary of Functional and Stochastic Routines applied within the Research Software to Mitigate and Model Errors on GPS Pseudoranges
Chapter 6 - Long-Range Positioning Studies

This chapter describes the preparation and execution of a data collection campaign that will provide sufficient data against which the designed stochastic models can be assessed. Also detailed are the tasks necessary for the pre-processing of this data prior to its application through the research modelling and processing software summarised in Chapter 5. A number of large GPS datasets are subjected to a broad range of stochastic models covering a range of modelling parameter options. Discussions as to the choice of processing options and stochastic parameters are also given with reference to the research objectives.

6.1 BACKGROUND TO NORTH SEA GPS DATA COLLECTION CAMPAIGN

6.1.1 Campaign Brief and Objectives

For meaningful comparisons and recommendations to be drawn based on the research studies, it was proposed that a real data set be obtained from an operational vessel performing standard survey tasks in actual offshore conditions. The North Sea GPS data collection campaign was co-ordinated by UCL and Shell UK Exploration and Production (SUKEP), and was executed in September 1998. The main objective was to provide a real dataset with which to test and evaluate the designed algorithms and models for the sub-decimetre positioning of offshore vehicles every second, in support of long-range exploration and production activities.

The vessel used within these studies, the MV Tridens 1, was operated by Britsurvey, a subsidiary of the Svitzer Group [Svitzer Limited, (2000)]. MV Tridens 1 set sail from Great Yarmouth (GYAR) for a prospect in Shell’s Indefatigable gas-field, (INDE). Several GPS receivers at varying distances from the prospect were also recording dual-frequency GPS data at specified times during the project. The datasets for these receivers would represent DGPS reference stations during later post-processing. The observing network of six reference stations afforded baseline vectors up to 1160 kilometres to the navigation test
area. The locations of these reference stations, the Shell prospect and the TEST area, can be seen in the map of Western Europe in Figure 6-1.

![Figure 6-1 Plan showing the Location of the Shell Prospect 'INDE', the Sea Trials Area 'TEST' and all GPS Reference Stations Participating in the North Sea Campaign](image)

In order to determine the true errors in the DGPS observations and then determine the correct stochastic model for the position fix, it was necessary to know the true trajectory of the vessel at any precise moment in time. There are essentially two approaches to collecting truth data corresponding to known locations; the first is by using a static point of known co-ordinates, and the other by using a vehicle whose track is being positioned by some other higher accuracy means. Considering the application relates to a dynamic vessel, it was decided to implement the former using the technique of relative carrier-phase positioning (RTK).

An RTK-GPS reference station at GYAR recorded dual-frequency data on both code and carrier-phase for the entire duration of the project from which truth trajectory information could be later calculated. On the assumption that the fixed antenna's co-ordinates are well-known and post-processed using GPS carrier-phase observations, the position of the mobile receiver would then be known to approximately 1 centimetre in the
global reference frame and therefore provide a truth trajectory for the subsequent long-range processing experiments. From these truth positions, it would then be possible to determine the true errors in these position yielded by the candidate models.

So that the highest accuracy co-ordinates could be obtained for the RTK truth trajectory, an offshore test site within about 20 kilometres of Svitzer’s land-based RTK reference station, GYAR, was sought away from the main shipping channels and shallow waters around the Norfolk coast. At this site, a number of set manoeuvres were performed, mainly following standard operating procedures. With the means of truthing these trajectories, using GYAR as an RTK base station, it would be possible to evaluate the accuracy of each proposed functional and stochastic model combination. The point TEST, as seen in Figure 6-1, represents that chosen area whose centroid is a distance of 18.5 kilometres from GYAR.

The Importance of Accurate Reference Station Co-ordinates

Because any biases on the measurements at the reference station will be present to some extent in the differential corrections determined by it, once applied at the roving receiver, the accuracy of their derived final position will be similarly biased. It is generally regarded that reference stations within differential networks should be afforded the most advanced receiver and antenna components along with advanced techniques of mitigating and modelling multipath at these sites. The reduction of errors in the ranges computed at the reference station will duly be noted in the accuracy of the relative position of the mobile. Cox et al [2000] state that a number of receiver and antenna mitigation technologies mentioned, have been combined at WAAS reference sites to help keep overall position errors at, or below, the 1-2 metre level.

6.1.2 Equipment Used

The campaign brief requested that participants used equipment and sites capable of being reference stations in a high-precision geodetic campaign [Keenan, (1998)], typically describing geodetic dual-frequency receivers with high quality antenna at stable sites subject to minimal multipath. As most of stations were permanent reference stations, including participants of the IGS tracking network (cf. §2.1.2), the location of the antenna away from multipath reflecting surfaces should have been near optimal. These stations typically operated at a relatively low data rate recording measurements every 30 seconds,
Background to North Sea GPS Data Collection Campaign

sufficient for static applications. Navigation activities require higher data recording rates, typically less than every 5 seconds.

Each reference station used antenna that possessed particular multipath rejection characteristics, for example, being able to reject those signals reflected from beneath the antenna, typically the concrete pillar head. This criteria was essential as such reflected signals can introduce, in high precision applications, many metres and centimetres of range error when using code pseudoranges and carrier-phase measurements respectively [Javad, (2000)]. All sites used geodetic quality receivers capable of tracking 12 satellites, possessing internal signal processing techniques, and high-quality antenna (either geodetic antenna with attached ground-planes or a choke-ring antenna).

Table 6-1 summarises the receiver and antenna combinations at the participating stations for this data collection campaign. Details for Svitzer’s Trimble receiver TRID on MV Tridens 1 have also been included.

<table>
<thead>
<tr>
<th>SITE ID</th>
<th>RECEIVER</th>
<th>ANTENNA TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRID</td>
<td>Leica MC1000</td>
<td>Leica Marine AT502</td>
</tr>
<tr>
<td></td>
<td>Trimble 4000SSe</td>
<td>Trimble Compact L1/L2</td>
</tr>
<tr>
<td>BEDS</td>
<td>Ashtech Z-FX</td>
<td>DM Choke-ring – Ashtech</td>
</tr>
<tr>
<td>DELF</td>
<td>Trimble 4000SSe</td>
<td>Trimble Compact L1/L2 with groundplane</td>
</tr>
<tr>
<td>GYAR</td>
<td>Trimble 4000SSe</td>
<td>DM Choke-ring – Trimble</td>
</tr>
<tr>
<td>HANN</td>
<td>Ashtech Z-XII</td>
<td>Ashtech Geodetic L1/L2</td>
</tr>
<tr>
<td>LOND</td>
<td>Leica MC1000</td>
<td>DM Choke-ring – Leica</td>
</tr>
<tr>
<td>ONSA</td>
<td>Ashtech Z-XII</td>
<td>DM Choke-ring – B</td>
</tr>
<tr>
<td>PENC</td>
<td>Trimble 4000SSe</td>
<td>Trimble 4000ST L1/L2</td>
</tr>
</tbody>
</table>

Table 6-1 Inventory of GPS Systems for Participants of the North Sea Campaign

6.1.3 GPS Data Archives

GPS observation campaigns are notorious for the amount of data they generate and the North Sea campaign was no exception. This is especially so when the campaign involves navigation activities for research purposes that requires dual-frequency GPS data to be recorded every second. Dual-frequency data comprises both timing codes and both carrier-phase measurements (four GPS observables in total). For the high-accuracy
geodetic receivers used in this campaign, this equated to approximately 2 MB of data per
hour per receiver.

With those receivers at BEDS, GYAR and TRID recording dual-frequency data at 1
Hz rate for 13 days, and up to six European reference stations each recording data at a
similar rate for a total of 66 hours over 3 sessions, almost 2.25 GB of data was generated.
With the exception of the three receivers mentioned above, the majority of reference
station data was retrieved from the participating institutions using the internet, namely file
transfer protocols (ftp). Once obtained, each data set was archived in its proprietary binary
format before being checked, chronicled and converted to the RINEX format. Following
this, the RINEX data files for all receivers were put through a pre-processing package to
calculate further information required by the research software routines (cf. §5.2).

### 6.1.4 Summary of the North Sea Campaign

A second research project was also being conducted on the MV Tridens 1 over the
period of this campaign - a gravimetric survey which required dual-frequency phase data
be recorded at both TRID and GYAR at a 10 Hz rate. Economy of resources on behalf of
the vessel operators demanded that the local reference station data for both projects,
GYAR, was collected concurrently and stored at regular intervals in Trimble’s PMEAS
ASCII format. Although the gravimetric project required data collection for the entire
duration of the project, the intention was to decimate this file to 1 Hz for use in the
author’s research project.

Unbeknownst to the author, and the Svitzer technicians responsible for the logging of
RTK data, the PMEAS format was not supported by Trimble so it was not possible to
obtain code and phase data from these datasets successfully using Trimble software
[Corbett, (1998a)]. Efforts were made by all involved groups to decipher this format into
one readable by standard software [Corbett, (1998b)], however these were met with no
success.

As a consequence of this decoding problem, the original sea trials’ objective of
obtaining a truth trajectory accurate to a few centimetres for the vessel was not fulfilled.
The data collected by the Leica MC1000 receiver on the vessel was not used in subsequent
research for two reasons, the first being that the post-processing softwares available to the
author (at that time) were not capable of yielding centimetre accuracy trajectories for large
receiver separations. This was especially so given the considerable distance of
164 kilometres between the centroid of the test area ‘TEST’ and the nearest campaign reference station DELF. Secondly, the resources were not available within this research project to design and implement software capable of processing this long-baseline RTK data. Therefore the decision was made to amend the focus of this research towards the investigation of a general case stochastic model for differential code positioning and the fidelity of the quality measures such a model would yield. The datasets collected from the reference stations in this campaign were ideal in affording a truth reference network to enable the quantification of differential GPS position errors.

6.2 RESEARCH PROCESSING STRATEGY

6.2.1 Overview of Processing Tasks

The full suite of tasks afforded to the research software have been discussed in the previous three chapters and are briefly summarised here to remind the reader of their background prior to the implementation of the candidate stochastic models. These tasks covered pre-processing, functional modelling, creation of the stochastic models, the least squares computations and the a-posteriori quality assessment and analysis tasks as illustrated in Figure 5-1.

6.2.2 Details of Processed Baselines

In this research, two scenarios were tested relating to the positioning of a mobile receiver: the first was using a series of single baselines, and the second was using a number of reference receivers. From the full complement of six recording stations, five single-baselines were chosen that spanned a wide range of separation distances as listed in Table 6-2 and illustrated in Figure 6-2:
Two multiple-baseline networks were also created whereby each network contained four stations: three fixed reference stations and a mobile receiver location. The fixed reference station measurements were combined within the functional model to position the mobile receiver as located near the centroid of the network. Table 6-3 summarises the multiple-baselines used in these studies where the network comprising BEDS, LOND, HANN and DELF (BLH-DELF) offers a mean baseline length of approximately 401 kilometres. A larger network affording a mean baseline length of approximately 226 kilometres.
704 kilometres was created from the stations ONSA, LOND, PENC and HANN (OLP-HANN).

<table>
<thead>
<tr>
<th>NETWORK</th>
<th>RECEIVER TYPES</th>
<th>BASELINE LENGTHS (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLH - DELF</td>
<td>Ashtech Z-FX + Leica MC1000 + Ashtech Z-12 → Trimble 4000</td>
<td>519 + 316 + 397</td>
</tr>
<tr>
<td></td>
<td></td>
<td>mean ~ 401</td>
</tr>
<tr>
<td>OLP – HANN</td>
<td>Ashtech Z-12 + Leica MC1000 + Trimble 4000 → Ashtech Z-12</td>
<td>575 + 683 + 853</td>
</tr>
<tr>
<td></td>
<td></td>
<td>mean ~ 704</td>
</tr>
</tbody>
</table>

Table 6-3 Network Baseline Studies: Receiver Types and Baseline Lengths

The configuration of these two networks can be seen in Figure 6-3.

Figure 6-3(a) and (b) Maps showing the Multiple-Baseline Networks Processed in the Positioning Studies (a) BEDS, LOND, HANN and DELF (BLH-DELF), and (b) ONSA, LOND, PENC and HANN (OLP-HANN).

The final selection of single and multiple-baselines were dictated by the length of baseline that could be afforded to the study rather than the receiver pair combination. In an operational situation, any baselines possible from the available datasets would be considered for positioning applications.
6.2.3 Data Conversion and Pre-processing

Data Conversion

The original RINEX datasets, comprising unfiltered code pseudoranges, were pre-processed with the RINTONXF executable to convert them into the NXF data format ensuring compatibility with the research processing software (cf. §5.2). During this conversion process, the contents of the RINEX navigation file were used to determine co-ordinates and range rates for all satellites at each observation epoch.

It was decided that for file management purposes, it would be easier to use a global broadcast ephemeris file for the position calculation of those satellites used in the campaign. This broadcast ephemeris file (BRDC) is the daily product of the individual IGS RINEX navigation files when merged into a single non-redundant navigation file [Gurtner, (1999b)]. These orbit files were retrieved from the anonymous ftp-area accessible within the Crustal Dynamics Data Information Service (CDDIS) website [Noll, (1997)]. Use of these non-redundant BRDC files also eliminated formatting issues arising from the use of problematic conversion executables as provided by system manufacturers. For instance, some older versions of Leica’s L2R2 executable have been prone to incorrect RINEX formatting problems.

Once the necessary parameters had been calculated, all information was sorted into the NXF format along with the original RINEX observations and time-tag information.

Data Pre-processing

Following the data conversion routines, the pre-processing functions were applied beginning with the carrier-phase filtering routine; this was applied to reduce the effects of high-frequency code multipath and receiver noise on the code pseudoranges. As this research was concerned with the stochastic modelling of C/A-code pseudoranges available on the L1 frequency, the size of the multipath and receiver noise on these code pseudoranges could be quite considerable (cf. Figures 3-11 and 3-12), and should be mitigated. In particular, the effects of ionospheric divergence and intermittent code multipath due to resetting of the single-frequency phase-filter once $N_{\text{max}}$ is reached. For these reasons, the dual-frequency algorithm was used to filter the raw code pseudoranges. A filtering time constant of 100 seconds was deemed most suitable, not only in terms of its filtering ability and also in retaining information without totally overwhelming the data stream (time series).
Stochastic Algorithms selected for Evaluation

The broadcast ionosphere model, after Klobuchar, and the refined Saastamoinen troposphere model were then used to respectively derive ionospheric and tropospheric path delay corrections on the L1 C/A-code pseudoranges (cf. §5.1.1.2).

6.2.4 Implementation Aspects and Choice of Models

Functional Model Components

For the purposes of analysing the candidate stochastic models on their individual merits alone, the same pre-processing and functional models, as described above, were used for all candidate models within this research. It must be stressed that this research was not attempting to explicitly improve the accuracy achievable with differential code positioning but rather the accuracy of the quality of the position estimates - the fidelity of the quality measures.

Elevation Cut-off Angle

In the majority of typical GPS processing tasks, an elevation cut-off angle of 15 degrees is specified with the intention of minimising the errors induced by mapping fixed zenith delay errors to lower elevation angles (cf. §2.1.3.3), and subsequently the impact of noisy data on the final position solution. It was decided that as some considerable receiver separations were present in the network, (in excess of 1100 kilometres), the elevation angle cut-off should be reduced to 5° in order to permit longer continuous tracking of common satellites. Although this may be regarded as excessive when considering the North Sea arena, the use of lower elevation angle thresholds is a critical aspect of positioning operations and will be more so, given the increase in those long-range surveys associated with deepwater sites.

6.3 STOCHASTIC ALGORITHMS SELECTED FOR EVALUATION

This section briefly summarises the candidate stochastic models designed for the processing tests. A series of initialization files were created to represent these stochastic models and the relevant pre-processing and functional model parameters, and could be amended for each candidate dataset, either single or multiple-baseline scenarios. In theory,
any procedure that attempts to model the residual errors through stochastic means should aid the quality assessment process by yielding more correct measures. To illustrate this improvement, the precision statements for an incorrect matrix of unit weight were determined and output.

Four types of model were used to estimate the precisions of the phase-filtered pseudoranges within the covariance matrix summarised in §5.4.1 and Figure 6-5.

![Figure 6-4 Schematic showing the Four Approaches to Populating the Covariance Matrix of Undifferenced Observations (C_i) applied within these Studies](image)

The first is based on the concept of unit weight for all observations and is used as the benchmark model in these studies, presuming that the observations are independent of one another, i.e. uncorrelated. The second model is based on the assumption that the accuracy of code pseudorange measurements is a function of satellite elevation angle. In this model, variances along the leading diagonal of the covariance matrix are populated with terms that attempt to map this precision deterioration via the elevation angles of the observing satellites. The third model is also based on the concept of uncorrelated variances whereby the diagonal elements are populated with user-defined variances, and from empirical stochastic functions relating to undifferenced pseudorange precision. The fourth model type includes off-diagonal covariance terms that represent the correlations between receivers, and between observations, in particular, the spatial correlations between observations.
The adaptive multipath variance estimation and mutual at-receiver correlation routines were only applied to the user-defined variance and full variance-covariance models. Figure 6-5 illustrates the complete suite of stochastic models applied to each dataset within these long-range positioning studies.

![Figure 6-5 Schematic showing All Options afforded to the Four Candidate Stochastic Model Types and Sub-routines Applied in these Long-Range Positioning Studies](image)

The four multipath variance estimation methods were applied to the long-range datasets to provide the basis for a comparative analysis of each routine’s performance. With the application of four positive levels of mutual at-receiver correlation to each multipath variance estimation algorithm, a total of thirty-six stochastic models were available for testing.

Measurements from satellites with large zenith angles are unlikely to possess large multipath errors and induced atmospheric errors (cf. §3.3.5). For these specific reasons, it has been assumed that any mutual at-receiver correlations will be positive and due mainly to errors within the mapping function increasing as the zenith angle increases. Four predetermined levels of positive correlation coefficient were selected to portray a range of values and were as follows: 0.0, 0.1, 0.5 and 0.9; the null coefficient value corresponds to measurements uncorrelated with one another (cf. §4.3.3).
6.4 SUMMARY OF PROCESSED DATASETS

Seven long-range datasets were subject to the pre-processing, functional and stochastic modelling options as described earlier in this chapter, and the processed datasets are summarised as follows.

- C/A-code pseudoranges and dual-frequency carrier-phase observables.
- Observation time span of 102 minutes for the period 19:43:00-21:24:59 UTC on September 2\textsuperscript{nd} 1998 [GPS Time-frame Identifier: 0973/4]. With a sampling rate of 1 Hz, a nominal 6120 epochs were obtained.
- Broadcast GPS ephemerides were used to calculate the satellite positions.
- Variable numbers of satellites were tracked by the receivers for each dataset as can be seen in Figure 6-6 and Figure 6-7.
- A-priori ionospheric corrections applied using the broadcast ionosphere model.
- A-priori tropospheric corrections applied using the refined Saastamoinen model.

Summary of User-defined Precisions

The a-priori standard deviations defined by the user for processing are detailed in Table 6-4. These values are intended to be indicative of the precisions of undifferenced C/A-code pseudoranges as collected by the geodetic quality receivers in this campaign.

<table>
<thead>
<tr>
<th>ERROR TERM</th>
<th>STANDARD DEVIATION (m)</th>
<th>DECORRELATION DISTANCE (km)</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Satellite Orbits</td>
<td>5</td>
<td>20,000</td>
<td>Broadcast Orbits</td>
</tr>
<tr>
<td>Satellite Clocks</td>
<td>30</td>
<td>--</td>
<td>SA-active</td>
</tr>
<tr>
<td>Ionosphere</td>
<td>5</td>
<td>1,000</td>
<td>Start of Cycle 23</td>
</tr>
<tr>
<td>Troposphere</td>
<td>0.2</td>
<td>200</td>
<td>--</td>
</tr>
<tr>
<td>Code Multipath</td>
<td>5</td>
<td>0</td>
<td>\textit{Bounds} used in TMV Model</td>
</tr>
</tbody>
</table>
\textit{Max: 5; Min: 0.5} & 0                        |                             |
| Receiver Noise        | 0.1                    | 0                           | --                        |
| Code Pseudorange      | 3.0                    | --                          | Scaling Factor            |
| Precision             |                        |                             |                           |

Table 6-4 A-priori User-Defined Parameters for use in these Stochastic Modelling Studies (including Variances and Decorrelation Distances)
Summary of Processed Datasets

Prior to the analysis of results in Chapter 7, a series of time series plots are provided in Figure 6-6 to illustrate the satellite geometry seen for each dataset. These plots, containing the number of common satellites visible over the 5° cut-off angle and the relative PDOP statistic at each epoch, also illustrate the unit variance statistic as yielded by a covariance matrix of unit weight.
Long-Range Positioning Studies

Figure 6-6 Time Series of Number of SV, Relative PDOP Statistic and Unit Variance Statistic (derived from a matrix of unit weight) for the Five Single-Baselines used within the Long-Range Positioning Studies

It can be seen that changes in satellite geometry caused some considerable spikes within the unit variance time series. There is a slight trend that the size of these spikes typically increases in direct proportion to the receiver separation, in particular for the longest single-baseline DELF-PENC. Here the unit variance value peaks at almost 300, hence the truncated abscissa for the last plot in Figure 6-6. Note that for all five plots, the mean unit variance values are considerably less than one, in fact closer to zero. In general, the relative PDOP statistics show very similar trends and magnitudes with a slightly larger value for the longer DELF-PENC baseline. This is likely a function of the differential satellite geometry between these two sites.

Similar plots are provided, as Figure 6-7, for the multiple-baseline networks, BLH-DLF and OLP-HANN. Here the number of satellites refers to the number of observations to common satellites within the four-receiver network (including the mobile receiver), and corresponds to the dimensions of the covariance matrix of undifferenced observations for each network dataset.
Summary of Processed Datasets

As the same functional model was used on all datasets, it was expected that the continuity of results for each dataset would be, in general, very similar with any differences arising from problematic GPS data and hardware, and satellite visibility problems. The impacts of such events can be explicitly seen for the baseline DELF-PENC and the larger network OLP-HANN. Estimates of the receiver clock offsets, as computed from single point position computations, can be seen for each station in Appendix B. Note in Figure B-3, that the Station HANN (comprising of an Ashtech Z-XII receiver) clearly suffers a receiver clock jump around epoch 1710. The single-baseline BEDS-LOND, suffered from an intermittent low elevation satellite as seen by the noise on all three series plotted in Figure 6-7. For the network datasets, there is a trend that the mean unit variance statistic increases in proportion to the increased number of observations (as seen by the increased ordinate values).
Correlation Coefficients within the Mutual At-receiver Correlation Routines

As the commentary in §7.4 and the seven tables in Appendix C state, the linear independence of the datasets is reduced (cf. §5.4.1.6) when higher levels of correlation are applied. Such is this reduction that when considering the full variance-covariance models with a correlation coefficient of 90%, the matrix inversion routines were unable to invert the covariance matrix of double-differenced observations using the standard Choleski inversion method (cf. §5.4.1). In some cases, in particular, for the baselines LOND-DELF and DELF-PENC, a correlation coefficient equal to 50% could incite these matrix inversion problems (cf. Appendices C and G).

6.5 CONCLUDING REMARKS

A considerable GPS dataset has been collected from a survey vessel under real operating conditions, as well as from a number of terrestrial reference stations at varying distances from the survey vessel. Unfortunately due to unforeseen data storage and conversion difficulties, the objective of truthing the vessel’s trajectory using RTK measurements was not achieved. Consequently, the static reference station data was used in these research studies according to the revised objectives (cf. §1.7). The developed research routines were then applied to seven particular subsets of the collected data:

- Five single-baselines ranging from 318 to 1164 kilometres,
- Two multiple-baseline networks each containing three reference stations, where the mean baseline lengths were 401 and 704 kilometres respectively.

A comprehensive processing campaign was then undertaken with all of these datasets using the four candidate stochastic model types as detailed in Figure 6-6. These models were applied in post-processing, albeit in such a manner that inferred real-time processing.

Thirty-six stochastic models, with differing stochastic functions and parameters, were applied to each of the seven datasets yielding, in total, 252 sets of results. However, matrix inversion problems within some models, particularly the variance-covariance models, afforded non-sensical output to a total of 43 sets. These problems were effected primarily by the application of high correlation coefficients within the mutual at-receiver covariance modules that inferred too strong a linear dependence within the observed data.
These instances have been duly excluded from the table of results in Chapter 7 and Appendices C through to K.

Given the benefit of known co-ordinates for the reference stations in the long-range studies, the true position errors were available for quality assessment purposes. Obviously this is not the case in standard real-time positioning but, in this research, is fundamental to assess the performance of the positioning system. In all instances where the true errors had to be compared with system estimates, the modulus value of the true error vectors were taken so that they were always positive.
Chapter 7 - Analysis of Results

This chapter introduces the results of the data processing tasks performed in Chapter 6, for the testing of the stochastic models designed in Chapters 4 and 5. Discussion has been made with regards to the true and estimated errors as yielded by each candidate model as well as the fidelity of the output quality measures.

7.1 PERFORMANCE EVALUATION OF CANDIDATE MODELS

7.1.1 Summary of Processed Datasets

Using data collected in the North Sea GPS campaign (cf. §6.1), studies have been carried out on static GPS positions to evaluate the performance of the candidate stochastic models when applied to real-time GPS data. Dual-frequency data was collected at 1Hz rate for 102 minutes using various geodetic quality receivers (cf. §6.1.2). These long-range datasets were processed with the candidate stochastic models as detailed in Figure 6-6. The permutations for the five single baselines and two multiple-baseline networks are again summarised in Table 7-1:

<table>
<thead>
<tr>
<th>BASELINE / NETWORK</th>
<th>BASELINE LENGTH (km)</th>
<th>MEAN BASELINE LENGTH (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOND-DELF</td>
<td>316</td>
<td>--</td>
</tr>
<tr>
<td>BEDS-LOND</td>
<td>397</td>
<td>--</td>
</tr>
<tr>
<td>ONSA-HANN</td>
<td>575</td>
<td>--</td>
</tr>
<tr>
<td>BEDS-ONSA</td>
<td>881</td>
<td>--</td>
</tr>
<tr>
<td>DELF-PENC</td>
<td>1164</td>
<td>--</td>
</tr>
<tr>
<td>BLH-DELF</td>
<td>519 + 316 + 397</td>
<td>401</td>
</tr>
<tr>
<td>OLP-HANN</td>
<td>575 + 683 + 853</td>
<td>704</td>
</tr>
</tbody>
</table>

Table 7-1 Lengths (in kilometres) of the Processed Baselines and Networks

As discussed in §5.6, the time series of results displayed in this chapter, and the corresponding Appendices, consist of the following statistics.
• True error components as calculated against the true mobile receiver co-ordinates, which in turn reflects positional accuracy.

• Mean Unit Variance statistic and a unit variance scaling factor as based on the previous 30 unit variance values.

• Formal position error components at 95% confidence level (as calculated from the a-posteriori standard deviations and scaled by the unit variance factor) reflecting precision.

• Mean statistics derived once the moving window of the unit variance scaling factor has been successfully populated. These statistics are prefixed with ‘mw’ to signify that they have been subject to the unit variance scaling factor. This moving window routine also discards the first 30 seconds of noisy data associated with the initialising phase-filter.

• The fidelity of the quality measures (at the 95% confidence level) is represented by the values output from the PCA routines. These indicate that percentage of the total dataset that were wrongly assigned (once the unit variance scaling factor has been populated).

Due to the considerable number of results as generated by the 200 or so candidate models applied, the analysis concentrates on the vector results in the formats mentioned above. For discussion purposes, summaries of the important results are provided for each of the baseline datasets processed in the long-range positioning studies. The accuracy and integrity of the formal errors associated with the position estimates as yielded by different stochastic models that are of prime interest. Direct reference will be made to the values of accuracy, precision and fidelity ratio during this analysis. These results tables contain the mean statistics for the entire dataset in terms of vector positions, i.e. those performing best by yielding the smallest mean percentage of wrongly assigned position vector solutions. Results for the two most basic stochastic model types, the unit weight and elevation-weighting models, are also included in these tables for benchmarking purposes.

Although the statistical summaries provide useful information about the performance of each candidate stochastic model (cf. Appendix C), they do not detail any interesting time-dependent trends. To permit the visual interpretation of such trends, the results are shown as time series plots that best illustrate the real-time variation in noise for each baseline (cf. Appendix D through J). Some groups specifically provide plots for the height component and state that the plan errors can be regarded as being half those of height.
Analysis of Results

positions. This is because height is inherently more difficult to determine as a result of the hemispherical configuration of the GPS constellation.

The results of the stochastic model performance tests, as referred to in the latter sections of this chapter and the nine Appendices C through K, correspond mainly to three-dimensional position vectors. The values relating to the complete component set of north, east, height, plan and vector can be found in the Appendices section. A similar format of results table for each baseline and network processed is provided in Appendix C listing the mean true, and mean formal error vectors over the entire dataset albeit excluding the first 30 seconds as the unit variance scaling factor populated. Note each dataset was nominally 6120 epochs (~102 minutes) in length.

Firstly, an identifier is given corresponding to the candidate stochastic model applied; these follow the order of the models as detailed in the schematic of the overall processing strategy as in Figure 6-6. Secondly, is the mean true accuracy of the position estimates over the dataset. The next figure corresponds to the mean estimate of the system accuracy at the 95% confidence level. Finally, there is a mean unit variance value for the entire dataset with the exception of the first 30 epochs as the initial moving window populates. Also included at the 95% confidence level, are those statistics output by the PCA routines corresponding to the overall accuracy of the precision estimators for plan, height and vector components.

It is expected that each candidate algorithm would perform to the same level regardless of whether truthed data was available or not. A chi-squared test is applied to all results to assess whether the distribution of the final results is considered Gaussian. The characteristics of each stochastic model and module type were mentioned during the design stages in Chapter 4, and are again discussed following the analysis of the results for each processed baseline and network. In general, it is expected that the best quality measure fidelity would be seen when applying the more sophisticated stochastic models.

Remarks concerning Data Quality

It must be noted that as real data has been used within these studies, there are disturbances within the data as caused by receiver clock jumps and complete losses of lock. Epochs corresponding to a partial loss of signal lock would mean that the approximate mobile position at that epoch would be of a lesser quality than at the previous epoch and that the carrier-phase filter for that satellite would be reset as would any running multipath variance estimation routines. The performance of a positioning system under
adverse conditions is of as much importance, if not more, than under optimal observing conditions.

### 7.2 SINGLE-BASELINE STUDIES

The results of the studies performed on the five single-baselines detailed in Table 6-3 are discussed here on an individual basis. They are analysed in order of increasing length. Note that the Formal Error Vector and PCA Vector values are at a 95% confidence level, and values for the *Best Performing Model* are highlighted in bold and italics.

#### 7.2.1 LOND-DELF (316 kilometres)

The unit weight model yields the best mean accuracy for the LOND-DELF baseline followed, in decreasing order, by the user-defined variance and elevation-weighting models. The variance-covariance model generally yields the least accurate position solutions, however only some 10 centimetres worse than the unit weight model. In terms of formal errors, the elevation-weighting and variance-covariance models yield the best, and very similar, precisions. The unit weight and user-defined variance models also yield very similar size precisions albeit around 0.4 metres larger than the aforementioned models. The addition of mutual at-receiver correlations increases both the accuracy and precision of the results for the variance-covariance models whilst significantly lowering the overall percentages of wrongly assigned quality measures. Little impact is noted with their addition to the user-defined variance models. These additions also effect an increase on the mean unit variance statistic - however on the variance-covariance models only. This effect is noticeable because there are already a number of spatial correlations inferred within the variance-covariance models.

The unit weight and variances models generally yield the lowest percentages of wrongly assigned solutions as output from the fidelity routines. The elevation and variance-covariance models yield similar PCA values for zero mutual correlations; the increase in the mutual correlation coefficient reduces the PCA values considerably to less than those yielded by the unit weight and variances models. In terms of quality measure fidelity, that stochastic model performing to the highest level was the v-cv model using the Adaptive Multipath Variance (AMV) estimation routine, and a correlation coefficient of
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10%, albeit at the slight expense of accuracy. However this model - \( VCV_{AMV\_corr\_0.1} \) - did not yield much improvement over the benchmark unit weight model.

Table 7-2 contains a selection of results for the baseline LOND-DELF. Note that a full summary of the results, as obtained using all thirty-six candidate models, can be seen in Appendix C.

<table>
<thead>
<tr>
<th>CANDIDATE MODEL</th>
<th>TRUE ERROR VECTOR (m)</th>
<th>FORMAL ERROR VECTOR (m)</th>
<th>WRONGLY ASSIGNED VECTOR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Weight</td>
<td>1.108</td>
<td>1.943</td>
<td>6.1</td>
</tr>
<tr>
<td>Elevation-weight</td>
<td>1.177</td>
<td>1.543</td>
<td>18.2</td>
</tr>
<tr>
<td>Vars_AMV_no-corr</td>
<td>1.127</td>
<td>1.925</td>
<td>6.8</td>
</tr>
<tr>
<td>Vars_AMV_corr-0.9</td>
<td>1.122</td>
<td>1.964</td>
<td>5.6</td>
</tr>
<tr>
<td>VCV_AMV_no-corr</td>
<td>1.205</td>
<td>1.609</td>
<td>14.0</td>
</tr>
<tr>
<td>( VCV_{AMV_corr_0.1} )</td>
<td>1.252</td>
<td>1.988</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Table 7-2 Sample of Results for the Baseline LOND-DELF (316 kilometres)

Referring to the time series of true and formal error vectors for this baseline can provide further information as to how the positioning model responds positively, or negatively, to the GPS dataset used in the baseline LOND-DELF. Appendix D contains plots of the true and formal errors time series as yielded successfully from the processing tests.
Figure 7-1 Time Series Plots of the True and Formal Error Vectors as calculated using Stochastic Models of Unit Weight, Elevation-weight and Variance-Covariances with AMV and 10% Mutual At-receiver Correlations for the Baseline LOND-DELF

A trend at the start of these time series indicates that the true and formal errors are fairly correlated in their behaviour. However following this around epoch 750, the accuracy improves although none of the four models acknowledges this. There is a considerable period of disagreement for the next 1200 seconds until the accuracy steadily deteriorates to become almost coincident with the formal error series around the 1.5 metre mark. The two time series maintain this consistency for the remainder of the dataset, however the shorter term trends, with periods of approximately 700 seconds, continue to remain unmodelled within the true error series.

The unit weight solutions increase in accuracy around epoch 4200 as a new satellite is reacquired and yields the highest mean accuracy for the candidate models on this baseline.
7.2.2 BEDS-LOND (397 kilometres)

For the BEDS-LOND baseline, both the elevation-weighting function and variance-covariance models yielded the best mean accuracy in terms of position vector. The order is then the user-defined variance model followed by the unit weight model for which there is a range in accuracy of 10 centimetres between the four model types. In terms of formal errors, the elevation-weighting and v-cv models again yield the best, and very similar, precisions. The unit weight and user-defined variance models also yield very similar size precisions albeit around 0.3 metres larger than the aforementioned models.

The addition of mutual at-receiver correlations typically has little effect on the accuracies of both the user-defined variance and v-cv models. In terms of quality measure fidelity, the user-defined variances model does not show much improvement although those variance-covariance models featuring the three adaptive multipath routines do see an improvement, in particular at the 50% correlation level. These settings yield a maximum of only 2% wrongly assigned epochs down from around 12.5% when assuming no mutual correlations. Only for the v-cv models do the mean unit variance values show an increase with the addition of mutual correlations. Matrix inversion problems were encountered with 90% correlation levels within the v-cv models.

The elevation-weighting and v-cv models yield the lowest percentages of wrongly assigned solutions from the PCA routines. These models again yield similar PCA outputs under zero mutual correlation; an increase in mutual correlation coefficient reduces the PCA values considerably to less than those for the unit weight and user-defined variance models (cf. Appendix E).

For this baseline, the v-cv model when implemented with an adaptive multipath variance estimation routine and an assumed 50% coefficient of correlation (VCV_AMV_corr-0.5), yielded the highest fidelity quality measures for the vectors at 1.4%. This model yielded a significant improvement over the benchmark stochastic model as seen from Table 7-3 which affords a selection of interesting results for the baseline BEDS-LOND. A full summary of the results as obtained using the candidate models can be seen in Appendix C as can the output time series in Appendix E.
### Table 7-3 Sample of Results for the Baseline BEDS-LOND (397 kilometres)

<table>
<thead>
<tr>
<th>CANDIDATE MODEL</th>
<th>TRUE ERROR VECTOR (m)</th>
<th>FORMAL ERROR VECTOR (m)</th>
<th>WRONGLY ASSIGNED VECTOR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Weight</td>
<td>1.083</td>
<td>1.679</td>
<td>16.5</td>
</tr>
<tr>
<td>Elevation-weight</td>
<td>0.994</td>
<td>1.396</td>
<td>12.3</td>
</tr>
<tr>
<td>Vars_CMV_no-corr</td>
<td>1.057</td>
<td>1.671</td>
<td>14.7</td>
</tr>
<tr>
<td>Vars_CMV_corr-0.9</td>
<td>1.057</td>
<td>1.671</td>
<td>14.7</td>
</tr>
<tr>
<td>VCV_AMV_no-corr</td>
<td>1.003</td>
<td>1.414</td>
<td>12.6</td>
</tr>
<tr>
<td><strong>VCV_AMV_corr-0.5</strong></td>
<td><strong>1.015</strong></td>
<td><strong>1.978</strong></td>
<td><strong>1.4</strong></td>
</tr>
</tbody>
</table>

**BEDS-LOND: True and Formal Error Vectors afforded by a Matrix of Unit Weight**

**BEDS-LOND: True and Formal Error Vectors afforded by an Elevation-weighting Function**
Analysis of Results

Figure 7-2 Time Series Plots of the True and Formal Error Vectors as calculated using Stochastic Models of Unit Weight, Elevation-weight and Variance-Covariances with AMV and 50% Mutual At-receiver Correlations for the Baseline BEDS-LOND

It can be seen that the time series corresponding to the unit weight and user-defined variance models are both very similar in overall trends and also the best model - VCy AMV corr-0.5. Time series artefacts for the elevation-weighting model are very similar to those seen for the variance-covariance models. There is a range in the sizes of the time series and it must be noted that the best model has a similar pattern to the Identity matrix time series; however, there is a larger offset between its true and formal error series. This ‘increased pessimism’ is why it performs best in terms of quality measure fidelity. There are many underlying trends and similarities in the time series as yielded by all four stochastic model types with very little difference noted from a more rigorous stochastic model. For this dataset however, it appears that the elevation-weighting and v-cv models can cope with the presence of minor data interruptions, such as changes in satellite geometry, as seen by the spikes around epochs 3650 and 5900 for the unit weight time series in Figure 7-2.

7.2.3 ONSA-HANN (575 kilometres)

The variance-covariance models yield the best mean accuracy for the ONSA-HANN baseline along with the elevation-weighting model, followed by the user-defined variance and unit weight models. A range of 0.3 metres separates the most and least accurate results. In terms of formal errors, the elevation-weighting and v-cv models yield the best, and very similar, precisions. The unit weight and user-defined variance models also yield very similar size precisions to one another although around one metre larger than those for the elevation-weight and v-cv models.
Single-baseline Studies

The addition of mutual at-receiver correlations has little effect either on the accuracy, precision or PCA values as yielded by the user-defined variance models. Conversely, when the mutual correlations are applied to the variance-covariance models, the results show an increase in accuracy and a drop in precision that significantly lowers the overall percentage of wrongly assigned vector solutions from 20% to 8%. These additional correlations also effect an increase on the mean unit variance statistic mainly so for the variance-covariance models.

Overall, the unit weight model yields the lowest percentages of wrongly assigned solutions, as output from the PCA routines, until a correlation coefficient of 50% is considered within the v-cv models. The variance only and full v-cv models yield similar PCA values for zero mutual correlations. An increase in the mutual correlation coefficient only reduces the PCA values for the user-defined variance models by 2% from 15% to 13%. The model yielding the smallest percentage of wrongly assigned vector solutions for this baseline was the v-cv model with ATMV estimation and 50% mutual correlation between the observations at each receiver - $VCV_{ATMV\_corr-0.5}$.

A selection of results for the baseline ONSA-HANN is shown in Table 7-4. A full summary of the results, as obtained using all thirty-six candidate models, is given in Appendix C. Time series plots for the true and formal error vectors are also provided in Appendix F for all successfully processed models.

<table>
<thead>
<tr>
<th>CANDIDATE MODEL</th>
<th>TRUE ERROR VECTOR (m)</th>
<th>FORMAL ERROR VECTOR (m)</th>
<th>WRONGLY ASSIGNED VECTOR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Weight</td>
<td>1.764</td>
<td>2.847</td>
<td>13.9</td>
</tr>
<tr>
<td>Elevation-weight</td>
<td>1.433</td>
<td>1.874</td>
<td>19.7</td>
</tr>
<tr>
<td>Vars_ATMV_no-corr</td>
<td>1.748</td>
<td>2.742</td>
<td>15.1</td>
</tr>
<tr>
<td>Vars_ATMV_corr-0.9</td>
<td>1.749</td>
<td>2.894</td>
<td>13.4</td>
</tr>
<tr>
<td>VCV_ATMV_no-corr</td>
<td>1.445</td>
<td>1.897</td>
<td>19.7</td>
</tr>
<tr>
<td>$VCV_{ATMV_corr-0.5}$</td>
<td>1.275</td>
<td>2.654</td>
<td>7.6</td>
</tr>
</tbody>
</table>

Table 7-4 Sample of Results for the Baseline ONSA-HANN (575 kilometres)

Time series plots are provided for the unit weight and elevation-weighting results for the baseline ONSA-HANN, as well as the 'best performing' model - $VCV_{ATMV\_corr-0.5}$.
Analysis of Results

![Graphs of True and Formal Error Vectors](image)

Figure 7-3 Time Series Plots of the True and Formal Error Vectors as calculated using Stochastic Models of Unit Weight, Elevation-weight and Variance-Covariances with ATMV and 50% Mutual At-receiver Correlation for the Baseline ONSA-HANN

Again there are periods of agreement and disagreement in these time series regardless of which stochastic model type is used. Considering the time series yielded by the elevation-weighting model as in Figure 7-3, there are number of instances where the formal error is smaller than that of the true error vector as determined from the known position of the station HANN. These features sometimes correspond with smaller features in the formal error series but in general it appears as though any trends between them are
totally uncorrelated. Overall, the elevation-weighting and full variance-covariance models show more continuous time series than those yielded by the unit weight and user-defined variances models. The latter are more erratic around events of phase-filter resets and satellite geometry changes, e.g. around epochs 3430 and 5300.

7.2.4 BEDS-ONSA (881 kilometres)

For the BEDS-ONSA baseline, the elevation-weight and variance-covariance models yield the best mean accuracy followed by the user-defined variance models. On this baseline, the unit weight model generally yields the least accurate position solutions, around 18 centimetres greater than the most accurate. In terms of formal errors, the elevation-weighting and v-cv models yield the best, and very similar, precisions. The unit weight and user-defined variance models also yield very similar size precisions albeit around 0.7 metres larger than the aforementioned models.

The addition of mutual at-receiver correlations reduces both the accuracy and precision of the results for the v-cv models whilst affording a very slight increase in the overall quality measure fidelity values. These additions also effect an increase on the mean unit variance statistic; however only for the three variance-covariance models with the three adaptive multipath routines. These three particular routines also see their mean precisions increase by approximately 50% when considering the transition from the zero to 50% mutual correlation levels (cf. Table 7-5). The user-defined variance models see a slight drop in accuracy, precision and PCA values with the inclusion of mutual at-receiver correlations.

It is noted that the largest PCA value yielded by any stochastic model applied to this baseline is 5% and that the range of PCA values across all candidate models is a mere 0.6% (cf. Appendix C). The use of mutual correlations has very little effect on the output PCA values save from raising them by a few tenths of one percent. As this range of quality measures is only 0.6%, it is quite difficult to state conclusively which is the best performing stochastic model. The elevation-weighting model performs best albeit only by a tenth of a percent over a number of other models.

Table 7-5 affords a selection of results for the baseline BEDS-ONSA. A full summary of the results as obtained using the candidate models can be seen in Appendix C, and the corresponding time series in Appendix G.
Analysis of Results

<table>
<thead>
<tr>
<th>CANDIDATE MODEL</th>
<th>TRUE ERROR VECTOR (m)</th>
<th>FORMAL ERROR VECTOR (m)</th>
<th>WRONGLY ASSIGNED VECTOR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Weight</td>
<td>0.735</td>
<td>2.856</td>
<td>4.9</td>
</tr>
<tr>
<td>Elevation-weight</td>
<td><strong>0.554</strong></td>
<td><strong>2.092</strong></td>
<td><strong>4.4</strong></td>
</tr>
<tr>
<td>Vars_CMV_no-corr</td>
<td>0.693</td>
<td>2.699</td>
<td>5.0</td>
</tr>
<tr>
<td>Vars_CMV_corr-0.9</td>
<td>0.695</td>
<td>2.762</td>
<td>5.0</td>
</tr>
<tr>
<td>VCV_CMV_no-corr</td>
<td>0.560</td>
<td>2.121</td>
<td>4.5</td>
</tr>
<tr>
<td>VCV_CMV_corr-0.5</td>
<td>0.587</td>
<td>3.203</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Table 7-5 Sample of Results for the Baseline BEDS-ONSA (881 kilometres)

Figure 7-4 shows true and formal error time series as determined using the unit weight and elevation-weighting functions applied to the BEDS-ONSA datasets. A plot is also included for those results yielded by the \( VCV\_CMV\_no\text{-}corr \) model.
In the illustrated plots, the formal error series begins at around the 5 metre level and decreases to become coincident with the true series. There appears to be very little correlation between trends in the true and formal error series. A burst of high-frequency noise in the GPS dataset around epochs 4600 to 4800 caused a delayed trough and peak within the formal error series, most noticeable in the unit weight results although it is not known how these high-frequency biases were caused. There were only two short instances when the true errors were greater than those estimated by the system. It can be seen that the elevation-weighting and vcv_CMV models produce extremely similar time series, both in terms of trends and magnitudes suggesting that the spatial decorrelation functions present in the latter model have had little positive effect.

7.2.5 DELF-PENC (1164 kilometres)

For the DELF-PENC baseline, the longest in the positioning studies, the highest accuracies are yielded by those variance-covariance models subject to the AMV and ATMV modules. They are then followed by the user-defined variance models, the unit weight and finally the elevation-weighting function. In terms of formal errors, the elevation-weighting and variance-covariance models yield the best, and very similar, precisions. The unit weight and user-defined variance models also yield very similar size precisions albeit by around 1.1 metres larger than the aforementioned models.

The addition of mutual at-receiver correlations to the variance-covariance models increases the accuracy and decreases precision of the results whilst only lowering the
percentage of wrongly assigned values by approximately 1%. These additions also effect a small increase of the mean unit variance statistic for the variance-covariance models only.

The unit weight and user-defined variance models yield the lowest percentages of wrongly assigned solutions from the PCA routines. The elevation and variance-covariance models yield similar PCA values for zero mutual correlations. An increase in the mutual correlation coefficient reduces the PCA values only very slightly by 0.5%. Only by a very small margin did the user-defined variance models with mutual correlations yielded better fidelity quality measures than the unit weight model.

Table 7-6 affords a selection of interesting results for the baseline DELF-PENC. Note how the elevation-weighting and full variance-covariance models yield similar results that are considerably worse than the identity and user-defined variance models. This suggests that the spatial correlations terms inferred by the first two models are having an adverse effect on the overall quality of the results - but not their accuracy - as seen from the size of the true error vectors. A full summary of the results as obtained using the candidate models can be seen in Appendix C, and corresponding time series plots in Appendix H.

<table>
<thead>
<tr>
<th>CANDIDATE MODEL</th>
<th>TRUE ERROR VECTOR (m)</th>
<th>FORMAL ERROR VECTOR (m)</th>
<th>WRONGLY ASSIGNED VECTOR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Weight</td>
<td>1.401</td>
<td>3.365</td>
<td>10.2</td>
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<tr>
<td>Elevation-weight</td>
<td>1.423</td>
<td>2.267</td>
<td>24.6</td>
</tr>
<tr>
<td>Vars_ATMV_no-corr</td>
<td>1.637</td>
<td>3.265</td>
<td>10.4</td>
</tr>
<tr>
<td>Vars_ATMV_corr-0.9</td>
<td>1.365</td>
<td>3.354</td>
<td>10.0</td>
</tr>
<tr>
<td>VCV_AMV_no-corr</td>
<td>1.233</td>
<td>2.023</td>
<td>23.9</td>
</tr>
<tr>
<td>VCV_AMV_corr-0.1</td>
<td>1.218</td>
<td>2.107</td>
<td>22.8</td>
</tr>
</tbody>
</table>

**Table 7-6** Sample of Results for the Baseline DELF-PENC (1164 kilometres)

The three time series in Figure 7-5 provide examples of the true and formal error time series as determined using the unit weight and elevation-weighting stochastic functions on the DELF-PENC datasets, as well as the user-defined variances model with the full ATMV estimation routine and mutual correlations at the 90% level.
Figure 7-5 Time Series Plots for the True and Formal Error Vectors as calculated using Stochastic Models of Unit Weight, Elevation-weight and User-defined Variances with ATMV and 90% Mutual At-receiver Correlations for the Baseline DELF-PENC

The error vector scale has been truncated to 20 metres so as to afford some idea of the general behaviour of the time series. As with some of the other baseline results, there is very little agreement between the true and formal errors with some periods of intense disagreement. Although the formal errors rose to accommodate the increase in the true errors, once the interruption had passed, the formal error series assumed a lower value than the actual true error. At the time of the interruption, the true error vector reached almost...
Analysis of Results

24 metres in size, whilst the formal 3-D error vector achieved almost 48 metres in length. These interruptions have had an adverse effect on the overall mean statistics.

The elevation-weighting model has managed to reduce the effects of anomalies as seen by the smaller maxima achieved during interruptions. The full v-cv model with 90% mutual correlation tends towards the magnitudes of the unit weight matrix, e.g. around epoch 3000. Some large multipath interference was encountered at station PENC on the signals from SV21, as seen in Figure 7-6, which has affected the results of all studies involving PENC.

![Figure 7-6 Time Series Plot showing the Extreme Phase-filter Corrections seen on SV21 by the PENC Receiver, and the derived Multipath Standard Deviation Estimates](image)

### 7.2.6 Comments on Single-Baseline Studies

For all single-baseline datasets, a number of artefacts can be seen where the model has either failed to compensate for the errors within the data, or conversely where the model is in agreement with the GPS data. Generally, the former was more prevalent in the test datasets. Example artefacts corresponding to the baseline LOND-DELF as subject to an elevation-weight model, are listed in Table 7-7 and shown in Figure 7-7.
Single-baseline Studies

<table>
<thead>
<tr>
<th>EPOCHS</th>
<th>STATUS OF TRUE AND FORMAL ERRORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 190</td>
<td>AGREEMENT</td>
</tr>
<tr>
<td>500 – 800</td>
<td>AGREEMENT</td>
</tr>
<tr>
<td>800 – 1900</td>
<td>DISAGREEMENT</td>
</tr>
<tr>
<td>3200 – 4700</td>
<td>DISAGREEMENT</td>
</tr>
<tr>
<td>4700 – 5000</td>
<td>AGREEMENT</td>
</tr>
</tbody>
</table>

Table 7-7 Sample Periods of Agreement and Disagreement between the True and Formal Error Series when applying an Elevation-weight Stochastic Model to the Single-Baseline LOND-DELF

![LOND-DELF: True and Formal Error Vectors afforded by an Elevation-weighting Function](image)

Figure 7-7 Time Series of True and Formal Errors for the LOND-DELF Baseline processed using an Elevation-weighting Stochastic Model

As can be seen in the Appendices for the Single-Baseline results, (D through H), there are similar zones of agreement and disagreement present in the integrity of the true and formal errors series for all of the single-baseline results albeit to differing extents. These do indeed illustrate that the model is failing to correctly acknowledge the stochastic variations within the GPS measurements.

**Overall Trends within the Single-Baseline Studies**

The overall results of all four types of stochastic model, when applied to the five candidate single-baselines, can be grouped into two approximate trends.
Analysis of Results

1. The first trend contains the results of both the unit weight matrix and the user-defined variance models.

2. The second trend comprises the results of the elevation weighting and variance-covariance models as well as their additional correlations.

This grouping can be attributed to the influence of the elevation-weighting mapping function [Keenan and Cross, (2001)]. When considering the first trend, the large satellite clock variance terms within the user-defined variance models are essentially scaling factors that overwhelm any benefits inferred by the mapping function and the adaptive multipath variances (cf. §4.3.1). Conversely, the presence of the covariance terms in the full variance-covariance algorithm eliminates these larger scaling factors leading to the grouping seen in the elevation-weight and variance-covariance results. There appear to be no direct relationships between the accuracy, precision or quality measure fidelity and the size of the receiver separations. Typically, this would be expected as the dominant measurement errors on the code measurements are the distance-independent errors of code multipath and receiver noise rather than those that are distance-dependent i.e. differential satellite orbits and atmospheric errors. Even so, a comprehensive carrier-phase filtering routine has been used to mitigate code multipath and then the multipath variance estimation routines used to model the remaining multipath on each satellite pseudorange.

With the absence of SA on the undifferenced code pseudoranges, a smaller satellite clock variance would be used affording a lesser scaling effect within the covariance matrix. Accordingly the benefits of the elevation-weighting function would become more noticeable and the user-defined variance model would tend towards the elevation-weighting and variance-covariance trends.

For three of the five baselines, the best results in terms of quality measure fidelities, have been yielded by those stochastic models based on an elevation-weight trend, in particular the full variance-covariance matrix with some level of mutual at-receiver correlation applied. There appears to be no optimal multipath variance estimation routine. Figure 7-8 illustrates the mean vector accuracies achievable with the unit weight and elevation-weighting models for the five single baselines.
Multiple-baseline Network Studies

![Graph showing Mean Vector Accuracies Achieved for all Single-Baselines using Unit Weight and Elevation-weighting Stochastic Functions](image)

**Figure 7-8** Plot showing Mean Vector Accuracies Afforded by Unit Weight and Elevation-weighting Stochastic Models for Each Single-Baseline Datasets

**Comments on the Magnitude of Unit Variance and Chi-squared Statistics**

It can be seen from the results of the five single-baselines (cf. Appendix K), that even when the more sophisticated stochastic models have been applied, the mean unit variance values yielded are still very small, nearer to zero than to unity. This is in agreement with the concerns raised of the initial UKOOA guidelines as noted in Hill et al [1995], and subsequently confirmed by Roberts et al [1997]. Therefore the formal errors have been scaled with a moving average of the reciprocal of the unit variance over the previous 30 seconds. It is reassuring to see the high-frequency information afforded to the formal error estimates by the unit variance scaling factor although slightly disappointing that the true errors are not as well mimicked by the overall positioning model.

The chi-squared test statistics for these single-baselines, as yielded by each stochastic model, are included in Appendix K. In general, the test statistics greatly exceed the threshold statistics as determined in §5.5.3. These results will be discussed in more depth in §7.4.

### 7.3 MULTIPLE-BASELINE NETWORK STUDIES

The results described here correspond to the multiple-baseline networks detailed in §6.2.2 and Table 7-1. The full summary tables and corresponding time series plots of true
and formal errors from each candidate model can be seen for both networks as Appendix C, and Appendices I and J respectively.

7.3.1 The Small Multiple-Baseline Network (BLH-DELF)

For this, the smaller multiple-baseline network comprising of BEDS, LOND and HANN as reference stations and DELF as the mobile receiver, the mean baseline length was approximately 401 kilometres. The unit weight model yields the best accuracy followed by the user-defined variance models. The variance-covariance model as using the CMV routine yields the next best accuracy, better than the elevation-weighting models. Yielding the least accurate position solutions are the variance-covariance models that use the three adaptive multipath variance estimation routines, AMV, TMV and ATMV, (cf. Table 7-8 and Appendix C).

The formal errors yielded by the four stochastic model types do not display such definite trends as in the single-baseline results. In particular, when considering the user-defined variance models, the application of certain adaptive multipath variance estimation routines yield much greater precisions than the CMV routine. The same multipath routines applied in the variance-covariance models yield smaller precisions than those from the CMV routine as applied to the variance-covariance models. These results can be best seen in the summary tables within Appendix C.

The addition of mutual at-receiver correlations increases both the accuracy and precision of the results for the variance-covariance models whilst significantly lowering the overall percentage of wrongly assigned quality measures. These additions again have very little overall effect on the results afforded by the user-defined variance models in terms of accuracy, precision, quality measure fidelity or overall mean unit variance value. However, the v-cv models respond negatively to these correlations with a decrease in accuracy, precision and quality measure fidelity. The mean unit variance statistic also tends to increase by about 50%.

Typically the results for all models, with the exception of those v-cv models using the AMV, TMV and ATMV routines, yield maximum fidelity values of 5% for the position vector estimates. The latter three models yield, under zero mutual correlation, a minimum of 14% wrongly assigned quality measures rising to approximately 21% under a 10% correlation coefficient. Mutual correlations at the 50 and 90% levels resulted in matrix inversion problems (§6.4).
Multiple-baseline Network Studies

The elevation-weighting model yielded the lowest percentage of wrongly assigned solutions as output by the PCA routine. The performance of the adaptive multipath variances, especially with the full v-cv models, is exceptionally poor with at least an extra 9% of solutions being assigned incorrect quality measures.

Table 7-8 affords a selection of results for the smaller multiple-baseline network BLH-DELF, which comprises of three baselines of length 519, 316 and 376 kilometres. Appendix C contains the summarised results for the candidate stochastic models.

<table>
<thead>
<tr>
<th>CANDIDATE MODEL</th>
<th>TRUE ERROR VECTOR (m)</th>
<th>FORMAL ERROR VECTOR (m)</th>
<th>WRONGLY ASSIGNED VECTOR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Weight</td>
<td>0.557</td>
<td>1.811</td>
<td>5.0</td>
</tr>
<tr>
<td>Elevation-weight</td>
<td>0.635</td>
<td>2.345</td>
<td>4.6</td>
</tr>
<tr>
<td>Vars_CMV_no-corr</td>
<td>0.570</td>
<td>1.736</td>
<td>5.0</td>
</tr>
<tr>
<td>VCV_CMV_no-corr</td>
<td>0.600</td>
<td>1.434</td>
<td>5.0</td>
</tr>
<tr>
<td>VCV_AMV_no-corr</td>
<td>0.772</td>
<td>1.072</td>
<td>14.0</td>
</tr>
<tr>
<td>VCV_AMV_corr-0.1</td>
<td>0.855</td>
<td>1.194</td>
<td>19.8</td>
</tr>
</tbody>
</table>

Table 7-8 Sample of Results for the Small Network BEDS+LOND+HANN-DELF (BLH-DELF; average baseline length of 401 kilometres)

Plots of the true and formal error time series as calculated using the unit weight and elevation-weighting stochastic models are shown in the following figures for analysis purposes. For comparative purposes, the results for the variance-covariance model with AMV and 10% mutual correlations are also shown. Time series for all BLH-DELF studies can be seen in Appendix 1.
Analysis of Results

There are two significant interruptions visible within these time series around epochs 3100 and 4200, in particular the true error series. This suggests that the positioning model is not acknowledging the errors within the GPS datasets. These interruptions correspond with new satellites rising above the cut-off angle and a loss of lock on four satellites within the network (cf. Figure 6-7). Again there are periods of agreement and disagreement between actual and estimated error series. It can be seen that the relative scale of the two series to one another is fairly consistent over the entire dataset, in particular, their slight yet steady rise up to, and including, the first interruption. The period between the two interruptions and afterwards is also mimicked well. The larger trends, as clearly present in both series, generally have periods of 100-200 seconds although not necessarily in synchronization between both series. It is uncertain what has effected these features although they possess the characteristics of low-frequency multipath - generally attributed to signal reflections from a nearby surface, or surfaces.

Figure 7-9 Time Series Plots of the True and Formal Error Vectors as calculated using Stochastic Models of Unit Weight, Elevation-weight, and Variance-Covariances with AMV and 10% Mutual At-receiver Correlation for the Network BEDS+LOND+HANN-DEL (BLH-DELF)
The variance-covariance model results afforded in Figure 7-9 do not appear to have been effected by the aforementioned interruptions within their time series; however, the close proximity of the formal error estimates to the actual true errors is concerning. Those time series determined from the more rigorous models contain less erratic high-frequency trends; note for Figure 7-9 in particular, the larger artefacts for the elevation-weighting model compared to the others. The v-cv model has yielded a relatively benign time series albeit close to the true errors.

7.3.2 The Large Multiple-Baseline Network (OLP-HANN)

This section summarizes the results for the larger multiple-baseline network comprising of ONSA, LOND, PENC and HANN. This OLP-HANN network comprises of three baselines of length 575, 683 and 853 kilometres, making an average baseline length of 704 kilometres.

The user-defined variance models yields the most accurate positions, followed by the unit weight and v-cv models, where the elevation-weighting model generally yields the least accuracy. In terms of formal errors, the v-cv models all yield vector precisions around the 2.3 metre level, the lowest from all candidate models. The next level of precision corresponds to the unit weight matrix, and then the elevation-weighting function. The formal errors yielded by the user-defined variance and variance-covariance models are varied depending on the multipath variance estimation routine selected. Of these models, those subject to the AMV routine yield the best precision followed by the CMV solutions, and the TMV and ATMV routines vie for the honour of providing the least precise solutions.

The addition of mutual at-receiver correlations is noticed only in the variance-covariance models by a decrease in the yielded accuracies and precisions. An increase is seen in the mean unit variance values and the percentage of wrongly assigned vector solutions. All candidate models processed successfully yielded quality measure fidelities of between 3.6 and 5.0%. Correlation coefficients at the 50% and 90% levels resulted in matrix inversion problems within the v-cv models.

A selection of results, as obtained using the candidate models, can be seen in Table 7-9, and a full summary of results in Appendix C.
### Analysis of Results

<table>
<thead>
<tr>
<th>CANDIDATE MODEL</th>
<th>TRUE ERROR VECTOR (m)</th>
<th>FORMAL ERROR VECTOR (m)</th>
<th>WRONGLY ASSIGNED VECTOR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Weight</td>
<td>1.069</td>
<td>3.550</td>
<td>4.3</td>
</tr>
<tr>
<td>Elevation-weight</td>
<td>1.395</td>
<td>4.940</td>
<td>4.1</td>
</tr>
<tr>
<td>Vars_AMV_no-corr</td>
<td>0.884</td>
<td>2.353</td>
<td>5.0</td>
</tr>
<tr>
<td>Vars_AMV_corr-0.9</td>
<td>0.883</td>
<td>2.358</td>
<td>5.0</td>
</tr>
<tr>
<td>VCV_ATMV_no-corr</td>
<td>1.21</td>
<td>2.259</td>
<td>3.6</td>
</tr>
<tr>
<td>V-CV_ATMV_corr-0.1</td>
<td>1.257</td>
<td>2.450</td>
<td>4.9</td>
</tr>
</tbody>
</table>

**Table 7-9** Sample of Results for the Large Network ONSA+LOND+PENC-HANN (OLP-HANN; average baseline length of 704 kilometres)

Plots of the true and formal error time series as calculated using three sample models of unit weight, elevation-weight, and full v-cv with ATMV (no mutual correlations - $VCV_{ATMV\_no-corr}$), are shown in Figure 7-10 for analysis purposes.
The multipath affecting SV21 as seen by PENC (cf. Figure 7-6), can clearly be seen to exert an influence on all time series involving PENC albeit to differing extents. An artefact situated around epoch 2000 shows an increase in true error but also an increase in the estimated precision. This is obviously incorrect and, as seen in the unit weight plot (cf. Figure 7-9), suggests that the pre-processing and/or functional tasks are failing to cope with systematic errors. Note that, for the largest (multipath) interruption around epoch 3800, a maximum true error vector of 12 metres is seen whereas the formal error vector leaps up to almost 70 metres. Conversely, a later interruption around epoch 5250 yields a true error vector of 10 metres whilst the system does not acknowledge the error at all within the formal error time series.

The elevation-weighting model has again performed poorly and contains larger artefacts in its time series of solutions, larger even than those yielded by the conventional stochastic model. For this network example, the v-cv model has coped best with the extreme multipath interruption as seen by a true error vector that has not jumped wildly. This would suggest that the more sophisticated models here have been able to respond quickly and correctly to real-time variations within the dataset.

Time series plots of the true and formal errors, as yielded by all successful stochastic models, can be seen in Appendix J.
7.3.3 Comments on Multiple-Baseline Networks

The Summary Tables in Appendix C contain the results of the long-range positioning studies when considering the two multiple reference station networks. Although the single-baseline results could be grouped essentially into two trends, those similar to the unit weight results and those similar to the elevation-weighting results, there were no trends noted within the network results.

Accuracy and Quality of Formal Errors

One significant indicator of improvement from the network solutions is the reduction in the double-difference measurement errors and the sub-metre accuracies achieved. There is a trend in that the 3D RMS error vector increases as the mean baseline length increases, however with a sample set of only two networks, this trend is far from conclusive. Had there been no drop in accuracy over distance, it would have suggested that the majority of the residual errors were distance-independent (code multipath and receiver noises), and not the distance-dependent errors of satellite orbits and atmospherics.

This trend of decreasing accuracy over distance may suggest that the spatial decorrelation functions incorporated within these variance-covariance models do not reflect the atmospheric variations within each particular network. It could be the case for the candidate datasets that the components of ionosphere and troposphere did not possess significant levels of correlation, meaning that the identity weight matrix, with its postulation that all observations are independent of one another, may be valid for longer baselines.

In general, the quality measure fidelity did not improve with the application of the stochastic functions. The greatest percentage of wrongly assigned vector solutions were noted with the presence of the adaptive multipath variance routines. From these network studies, the mean unit variance values over the 100 minute datasets have a greater range for the network solutions than those for the single-baseline results. Consequently, the range of formal errors yielded by the different stochastic model types in the network tests is also greater than for the single-baseline studies.

Comments on the Fidelity of the Quality Measures (Fidelity Ratios)

For the smaller network BLH-DELF, there are, in general, longer periods of agreement between the true and estimated errors, interspersed with periods of disagreement where the actual errors are increasing whilst the system believes they are decreasing; in
particular between epochs 5600 and 6120. These artefacts are listed in Table 7-10 and can be clearly seen within the true and formal error series illustrated in Figure 7-11.

<table>
<thead>
<tr>
<th>EPOCHS</th>
<th>STATUS OF TRUE AND FORMAL ERRORS</th>
</tr>
</thead>
<tbody>
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<td>400 – 750</td>
<td>DISAGREEMENT</td>
</tr>
<tr>
<td>3100 – 4450</td>
<td>AGREEMENT</td>
</tr>
<tr>
<td>5300 – 5600</td>
<td>AGREEMENT</td>
</tr>
<tr>
<td>5600 – 6120</td>
<td>DISAGREEMENT</td>
</tr>
</tbody>
</table>

Table 7-10 Sample of Periods of Agreement and Disagreement between the True and Formal Error Series when applying an Elevation-weighting Stochastic Function to the Small Network BLH-DELF (cf. Figure 7-11)

Figure 7-11 True and Formal Error Time Series for the BLH-DELF Network as processed using an Elevation-weighting Stochastic Model

A time series for the OLP-HANN network, (as in Figure 7-12), also contains many periods where the formal error time series does not appear to be able to follow the general trend in the true error series (cf. Appendix J). At times, the true error and formal error time series are almost coincident and then at other times differ by over a metre. Consequently, the stochastic model has failed to acknowledge the real-time variations in the GPS data. The significant trends within the true errors are not reliably mimicked by the system’s formal estimates. However the extreme multipath relating to SV21, as seen by PENC around epoch 3730 does effect a significant loss of accuracy within the network solution,
almost 12 metres in vector length. At this time, the formal errors jump to almost 70 metres and take some time to return to their mean precision of 3 metres.

![OLP-HANN: True and Formal Error Vectors afforded by an Elevation-weighting Function](image)

**Figure 7-12** Time Series of True and Formal Error Time Series for the Large OLP-HANN Network as processed using an Elevation-weighting Stochastic Model

Note that whilst both the true and formal error series do fluctuate in the event of signal interruptions, the formal error series jump significantly more so. This is because one particular phase-filter reset has affected all satellite measurements as seen by one receiver. The fact that the remaining reference stations may have had a successful uninterrupted period of filtering only serves to magnify any interruption on another receiver. This in turn has the potential to cause disruption within the previously steady time series. Although the elevation-weighting model infers limited spatial information, it can be seen from the results in Appendices C, I and J, that its application to the network datasets did not always yield position solutions with higher accuracy or quality measure fidelity (cf. Figures 7-11 and 7-12).

When applied to the multiple-baseline datasets, the chi-squared tests yielded statistics that significantly exceeded the threshold statistics signifying non-Gaussian distribution.
7.4 ANALYSIS AND COMMENTS

Comments on the Overall Accuracy of the Results

When considering datasets spanning longer baselines, it becomes more difficult to maintain a high number of common satellites between the mobile and reference receivers. As a result, an elevation angle cut-off of only 5° was used. For those models requiring some elevation-weighting functionality, the basic cosec mapping function was used to represent the reduced precision of pseudoranges from lower elevation satellites. This model was expected to perform better than a model of unit weight. However this postulation was not the case as seen in the results of the three single-baselines LOND-DELF, ONSA-HANN and DELF-PENC (cf. Appendix C).

The results from the unit and elevation-weight models, as applied to all datasets, are displayed in Figure 7-13.

![Figure 7-13 Mean Vector Accuracies Afforded by Unit Weight and Elevation-weighting Stochastic Models for All Study Datasets](image)

Overall, there are no significant distance-dependent trends in these results, i.e. no fall in accuracy with increasing receiver separation. The differences in mean error between the unit weight and elevation-weighted solutions are, in general, less than 10 centimetres. Exceptions to this trend are the two medium length single-baselines ONSA-HANN and BEDS-ONSA; 575 and 881 kilometres respectively. When determined using an elevation-weight model following one of unit weights, these baselines were more accurate to the tune...
of 33 and 18 centimetres respectively. As both baselines were observed using Ashtech Z-series receivers with geodetic and choke-ring antennas, it was assumed that receiver hardware characteristics were comparable.

The results grouping noted for single-baselines (cf. §7.2.6) can not be seen in the network results as there were only two networks under analysis, although there appears to be some distance-dependency trend in the networks results (cf. Figure 7-13). In both networks, the unit weight matrix yielded slightly higher positioning accuracies than the elevation-weighting model, and the mean accuracy for the larger network was almost twice that of the smaller network. For the smaller BLH-DELF network, with an average baseline of 401 kilometres, the mean vector accuracy was around 0.65 metres. For the larger OLP-HANN network, which comprised of an average of 704 kilometre baselines, the mean vector accuracy was around 1.15 metres – almost a factor of 2 greater in both mean baseline length and vector accuracy.

The fact that a matrix of unit weight could sometimes yield a more accurate mean position solution than an elevation-weighting model is disconcerting, although it may be due to the possibility that both the ionosphere and troposphere have become completely decorrelated over these distances. Consequently a stochastic model assuming no spatial correlation between the atmospheric errors could have been valid for the datasets used.

As the results for the multiple-baseline networks are more varied than from the single-baseline studies, this has led to fewer trends being visible within the results. For the large OLP-HANN network, a number of losses of lock encountered at the PENC receiver along with some extreme multipath on SV21, has disrupted the time series, in particular around epoch 3730 (cf. Figure 7-6). Further work is needed into creating additional multiple-baseline networks from the North Sea dataset.

Comments on the Overall Precision of Results

The overall precisions as afforded by the unit weight and elevation-weighting models are now considered for each baseline. Figure 7-14 and Figure 7-15 show graphs of the vector component precisions estimated by both models for the processed datasets; firstly for the single-baselines, and secondly, for all seven baselines including the two networks.
For all single-baselines processed, the position estimator precisions are smaller for the elevation-weighted results than for the unit weight results. Although the absolute precision values are not definitive (as they are dependent on a-priori variances), the fact that, in Figure 7-15, the elevation-weighting values are consistently smaller than those from the conventional stochastic model illustrates that the spatial correlation inferred by the elevation-weighting function was acknowledged within the mathematical model.
For the small multiple-baseline network BLH-DELF, the mean vector precisions range from 1.0 to 3.1 metres whereas they are almost twice as large for the OLP-HANN network, from 2.3 to 6.3 metres. As noted earlier, the general overall accuracies of the network results differed by a factor of around two, although this may be coincidence.

For both networks, the mean precisions yielded by the two temporal multipath variance routines (TMV and ATMV), as applied to the user-defined variance model, are amongst the smallest formal error estimates in the network studies. For the variance-covariance models, the three adaptive multipath variance routines fare poorly in terms of quality measure fidelity with at least 14% of the solutions being wrongly assigned at the 95% confidence level. The conventional stochastic model, when applied to the small and large networks respectively, assigns only 5.0 and 4.3% of its solutions wrongly. This suggests that the inclusion of rigorous stochastic functions can reduce the performance of the overall positioning model and fail to correctly represent the GPS biases.

The chart in Figure 7-16 combines the mean accuracy and precision values from the unit weight and elevation-weighting models for each GPS dataset, and plots them against baseline length.

**Figure 7-16** Mean Vector Accuracies and Precisions Afforded by Unit Weight and Elevation-weighting Stochastic Models for Each GPS Dataset
In general, the elevation-weighting function yields mean overall positions with a slightly higher accuracy than the unit weight model. The gradients of the accuracy time series are very similar for the five single-baselines, however, there are fewer similarities within the formal error time series. Precision estimates for the unit weight matrix do increase in proportion to receiver separation although this is likely inferred by the larger direction cosines associated within the design matrix $A$ (cf. Equation 3-15).

Those time series corresponding to the unit weight model are fairly matched in terms of overall trends and gradients for at least six of the seven baselines, however the accuracies and precisions of the elevation-weighting results do not correspond as much. Note that the largest precisions correspond to the ~700 kilometre baseline, (i.e. the large network OLP-HANN), which suffered from extreme multipath at PENC. The smaller network BLH-DELF also affords increased accuracy along with larger precision estimates according to these results. An unusual result was noted in that the mean formal errors for the 575 and 881 kilometre baselines (ONSA-HANN and BEDS-ONSA) are very similar at approximately 2.85 metres. Why the formal error estimate maintained the same level whilst the accuracy improved is unknown, although it may be coincidence. Both the unit weight and elevation-weight models failed to acknowledge, in their quality measures, the improved data quality for these baselines as seen by the improved accuracy statements.
Analysis of Results

Comments on the A-priori User-defined Variance Estimates

The final a-priori variances specified in the research software’s initialisation files are similar to standard operational parameters, and were chosen subjectively given the author’s knowledge of receiver and antenna type, observing conditions and settings within the research software.

The type of stochastic model applied contributes to the quality measures output by the system. Note that all the tests were carried out using data collected before 1st May 2000, and therefore the precisions of the undifferenced code pseudoranges included, within the clock standard deviation, an estimate of the range errors due to SA. The differencing process adopted in this research eliminated the effects of SA and therefore SA does not impact on any conclusions that are drawn. In other words, identical position results could be expected with data not subjected to SA [Keenan and Cross, (2001)].

In the case of the user-defined variance models, the large a-priori satellite clock variance of 900 metres\(^2\) overwhelmed the changes seen in the relatively smaller variances corresponding to the remaining components of the error budget corresponding to the ionospheric and tropospheric delays and multipath variances. With the absence of the large clock variance, it would have been these smaller terms that would have effected any variations within the stochastic model. The application of a full variance-covariance model is more correct than the user-defined variances model as the former acknowledges the correlations between the raw measurements, in particular the correlations due to SA.

It is acknowledged that the final parameters decided upon for use within all of the research routines, including pre-processing, phase-filtering, functional and stochastic modelling, could be subjectively contended against. Although it would illustrate the subjectivity of the precision statements rather than the correctness of the algorithms themselves, the sensitivity of the user-defined variances should be studied further.

Performance of the Stochastic Multipath Modelling Routines

The use of carrier-phase filtering techniques has considerably mitigated the effects of code multipath, in particular that high-frequency multipath typical of being reflected by distant sources. The dual-frequency filtering routine has been shown to improve the repeatability of position estimates by 77% (cf. §3.3). As a means of providing a cleaner dataset for the evaluation of different stochastic models, this method has been invaluable.

It was expected that the more sophisticated multipath modelling routines would be able to model any residual code multipath within the phase-filtered pseudoranges.
Analysis and Comments

(effectively the phase multipath). However for the datasets processed in this research, these routines afforded no significant improvement in the fidelity of the quality measures determined from either the user-defined variance or variance-covariance models.

For single-baselines, the inclusion of an adaptive multipath variance estimation routine within the user-defined variances model yielded, in general, only very slightly higher fidelity quality measures than with a constant multipath variance (CMV).

When considering the multiple-baseline network datasets as processed with the variance-covariance models, the fidelity of the quality measures were again very similar, regardless of the multipath routine used. Similar results were noticed with the application of CMV within the user-defined variance models. However as detailed earlier, the results from the adaptive multipath routines were significantly different for the small network BLH-DELF in which the precision values were reduced to around 1.05 metres (cf. Table C-6 in Appendix C). Consequently this raised the fidelity ratios for these results and effected a larger percentage of wrongly quality measures. It was unknown why this is the case.

In general, although the implementation of the adaptive multipath variance estimation routines, i.e. not the CMV routine, introduced some high-frequency variation to the quality measure time series, they afforded very little difference to the overall quality measure fidelities. The quality measure time series for the v-cv models showed more variations due to the multipath variance estimation routines than the user-defined variance models. This was because in the latter, the large satellite clock variances overwhelmed these variations.

It was expected that the application of these adaptive routines would have improved the repeatability of the v-cv model results, i.e. increased the precision measures. Theoretically the ATMV algorithm (cf. §4.3.1 and Equation 4-27), was expected to be the most realistic multipath variance estimation routine as it utilised the size and period of the multipath corrections applied to each satellite range during the carrier-phase filtering routine. This model would have been expected to perform better than one incorrectly assuming a constant multipath variance on all pseudoranges (cf. Figure 3-12) - however this was not the case. In general, the ATMV routine did not perform significantly better than the less sophisticated multipath variance estimation models. This has been attributed to the fact that the sizes of, and variations in, the residual code multipath are much smaller than the remaining pseudorange errors. That is, the residual code multipath is within the noise associated with undifferenced phase-filtered pseudoranges.
Analysis of Results

Comments on the Mean Unit Variance Values

Generally, the mean unit variance values yielded by the more sophisticated user-defined variance and v-cv stochastic models, were very small and far from unity. In fact, they were very close to zero and did not respond significantly to the introduction of mutual covariance terms. This was again because the larger variance terms, namely the satellite clock variances including SA, overwhelmed the smaller terms afforded by the mutual at-receiver covariances. The less sophisticated unit weight and elevation-weight models yielded larger mean unit variance values, with some statistics for the former greatly exceeding unity.

The variance-covariance models however, responded vigorously to these correlations, at all four levels, by showing an increase in the mean unit variance value towards unity. The three adaptive multipath variance estimation routines yielded similar mean unit variance values albeit slightly larger than by the CMV routine. This suggests that the a-priori variances going into the stochastic model were too pessimistic and that the corrected pseudoranges could not possibly be better than those described by the variances.

The unit variance scaling factor applied to all computed quality measures (cf. §4.3.5), has been previously shown to be beneficial for yielding correct quality measures, albeit under ‘reverse-engineered’ conditions [Roberts and Cross, (1993) and Barnes et al, (1998)]. However, as discussed in earlier time series analyses (cf. §7.2.6 and 7.3.3), there are a number of residual variations within the estimated precision time series which are sometimes completely uncorrelated with the variations of the true position errors. It is believed that these discrepancies and the small unit variance value are highlighting errors within the stochastic model, most likely due to incorrect modelling of the additional variance and covariance terms. This highlights the need for further studies into optimal a-priori variances for the GPS pseudorange error components.

Comments on Moving Average Window Lengths

Three moving average routines have been implemented within the research software: i) the carrier-phase filter, ii) routines to calculate the moving standard deviation of the corrections, and iii) the unit variance scaling factor routine, and studies carried out to determine the optimal window length for them. The optimal value for the phase-filtering time constant was chosen as 100 seconds from studies in §3.3.4.2.

Similar studies were carried out to determine optimal window sizes for the multipath standard deviation and unit variance scaling factor statistics whereby such averaging
Analysis and Comments

procedures were expected to reduce the effects of spike values in the multipath and unit variance times series. This is primarily why larger step functions were seen in the true error time series using the unit weight model (cf. Figures 7-1 through 7-4). There was no stochastic multipath modelling implemented to represent the effects of large code multipath as satellites were reacquired.

The window lengths of 30 seconds used in these studies were chosen subjectively by the author; although their size was insignificant on the mean statistics for the test datasets, they afforded smoother time series (cf. §5.2.1 and §5.5.2). It could be the case for some of the candidate datasets however, that the use of different length windows could have been advantageous.

Comments on the Mutual At-receiver Covariance Algorithm

The concepts of differential atmospheric biases and receiver-generated biases have been considered through the mutual correlation module where additional covariance terms were included for all observations made at each receiver (cf. §4.3.3). Here an assumption has been made that the residual errors within each observing receiver are correlated primarily through atmospheric corrections and mapping function errors, and consequently related to the elevation angle of the observed satellites.

In general, for all of the single-baseline datasets, modelling the mutual correlations between observations at each receiver improved the fidelity of the quality measures. As seen in the tables in Appendix C, there was very little change noticed when these correlations were applied to the user-defined variance models although generally positive and of a slight improvement. This is because there were no explicit functions implemented to model the spatial correlations present in the GPS datasets. Accordingly, more improvement was noted within the full v-cv models, in particular for the shorter baselines. The examples in Table 7-11 correspond to all seven study datasets processed using the full variance-covariance model, the ATMV routine and four levels of mutual at-receiver correlation. The application of higher levels of mutual correlation, i.e. 90% and 50%, caused matrix inversion problems - hence the lack of results for some options.
Analysis of Results

<table>
<thead>
<tr>
<th>BASELINE / NETWORK</th>
<th>LENGTH (km)</th>
<th>% WRONGLY ASSIGNED VECTORS UNDER MUTUAL AT-RECEIVER CORRELATIONS OF:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>ZERO</td>
</tr>
<tr>
<td>LOND-DELF</td>
<td>316</td>
<td>15.4</td>
</tr>
<tr>
<td>BEDS-LOND</td>
<td>397</td>
<td>12.8</td>
</tr>
<tr>
<td>ONSA-HANN</td>
<td>575</td>
<td>19.7</td>
</tr>
<tr>
<td>BEDS-ONSA</td>
<td>881</td>
<td>4.5</td>
</tr>
<tr>
<td>DELF-PENC</td>
<td>1164</td>
<td>26.1</td>
</tr>
<tr>
<td>BLH-DELF</td>
<td>401</td>
<td>14.3</td>
</tr>
<tr>
<td>OLP-HANN</td>
<td>704</td>
<td>3.6</td>
</tr>
</tbody>
</table>

Table 7-11 Improvement in Quality Measures noted from the Progressive Inclusion of Mutual At-receiver Correlations with a Full V-CV Model with the ATMV Routines

A general trend could be seen in that the improvements afforded by the mutual covariances were greatest for the shorter baselines. This trend was expected as the correlation between observations at each receiver would be higher for two receivers close together, i.e. it is more likely that the signal follows the same path to each receiver.

Figure 7-18 through Figure 7-21 illustrate a sample of quality measure fidelities afforded by the full variance-covariance model, two multipath variance estimation routines and variable levels of mutual correlation for each dataset. Figure 7-18 corresponds to the percentage of wrongly assigned quality measures when mutual at-receiver correlation coefficients of 0% and 10% were applied to the variance-covariance model using the CMV estimation routine. Note that the PCA values for the 50% and 90% correlation coefficients are not always available for each baseline. However zero and 10% duly represent the default parameters within this sample.
For the single-baseline datasets, the difference in quality measure fidelity afforded by mutual correlations of 10% are seen to drop off with increasing receiver separation, i.e. the correlation between observations decreases as the satellite geometry relative to the two receivers changes. Introducing the network results into this plot disrupts the trend seen for single-baselines most likely because the spatial decorrelation functions are too simplistic.

Figure 7-18 Percentage of Wrongly Assigned Epochs yielded on each Single-Baseline when applying Mutual At-receiver Correlations of Zero and 10% to the V-CV Stochastic Model featuring the CMV Module

Figure 7-19 Percentages of Wrongly Assigned Epochs yielded on each Study Dataset when applying Mutual At-receiver Correlations of Zero and 10% to the V-CV Stochastic Model featuring the CMV Module
Figure 7-20 shows the outcome of a similar study in which the ATMV routine was incorporated within the full variance-covariance model for single-baselines.

**Figure 7-20** Percentages of Wrongly Assigned Epochs yielded for each Single-Baseline when applying Mutual At-receiver Correlations of Zero and 10% to the Full V-CV Stochastic Model featuring the ATMV Module

With reference to the earlier CMV results (cf Figure 7-18), there is a greater divergence of quality measure fidelities, as seen in Figure 7-20, as a result of applying the more sophisticated ATMV multipath model. This may indicate that this algorithm can aid in representing the (multipath) correlations between observations at individual receivers. The network results are added to the $VCV_{ATMV}$ plot to create Figure 7-21.
Analysis and Comments

Figure 7-21 Percentage of Wrongly Assigned Epochs yielded for each Study Dataset when applying Mutual At-receiver Correlations of Zero and 10% to the Full V-CV Stochastic Model featuring the ATMV Module

Note that for the ATMV routine, the differences in quality measure fidelities between correlation coefficients of 0 and 10% decrease as the receiver separation increases. In general, for the two network datasets, a maximum of 5% of the quality measures were wrongly assigned. Exceptions were noted for the three adaptive multipath routines when applied as part of the variance-covariance model for the small BLH-DELF network.

It must be noted when applying the v-cv models and the three adaptive multipath routines to the network datasets, as the proportion of mutual correlation was raised, the fidelity values increased more significantly for the smaller network - BLH-DELF. As seen in Appendix C, the PCA values output by these models increase from 14.0 to 22.4% as the mutual correlation was raised from 0% to 10%. This is in direct opposition to the trend noticed for the single-baseline results whereby increased mutual correlations yielded a smaller percentage of wrongly assigned quality measures. It is not however known why this was the case for the small network BLH-DELF and not the large OLP-HANN network.

Even though the mutual correlation terms improved the fidelity of the quality measures to differing extents as described earlier, their inclusion was unable to raise the epoch-by-epoch unit variance value by significant amounts towards unity as had been hoped. The mean unit variance values for the entire dataset can be seen to be very small, tending to zero compared to the matrix of unit weight.
Analysis of Results

There are potentially two reasons for this; the first is that the subjective a-priori variances used within the stochastic models, including the terms used in the mutual at-receiver covariance algorithm, were too small. The second is believed to be the lack of any ‘fast’ error terms within the covariances, which are currently driven by constant a-priori variances and a slow elevation-angle function (cf. Equation 4-37). The assumption that any mutual at-receiver correlation between observations would be exclusively positive, was perhaps incorrect and too simplistic given the findings of Roberts and Cross [1993], and Barnes et al [1998].

The inherent lack of fast terms within each receiver’s block of the covariance matrix is duly worthy of further investigation, as is the determination of dynamic correlation coefficient values. The concept of negative correlation should also be considered during subsequent research (cf. §8.2.2).

Comments on the Results of the Chi-squared Tests

As can be seen from a sample of computed chi-squared statistics in Table 7-12, and Appendix K, most of the candidate stochastic models do not show any significant trends even though a considerable number of tests have been performed. Analysing results of the BEDS-ONSA and OLP-HANN datasets, the elevation-weight and VCV_ATMV_no-corr models were deemed the ‘best’ stochastic model in terms of quality measure fidelity (cf. Tables 7-5 and 7-9).

<table>
<thead>
<tr>
<th>CANDIDATE MODEL</th>
<th>BASELINE: BEDS-ONSA</th>
<th>NETWORK: OLP-HANN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plan</td>
<td>Height</td>
<td>Plan</td>
</tr>
<tr>
<td>12.6</td>
<td>24.7</td>
<td>12.6</td>
</tr>
<tr>
<td>Unit Weight</td>
<td>20297</td>
<td>3370</td>
</tr>
<tr>
<td>Elevation-weight</td>
<td>103428</td>
<td>9767</td>
</tr>
<tr>
<td>Vars_CMV_no-corr</td>
<td>22622</td>
<td>3233</td>
</tr>
<tr>
<td>Vars_ATMV_no-corr</td>
<td>21312</td>
<td>3348</td>
</tr>
<tr>
<td>VCV_CMV_no-corr</td>
<td>102793</td>
<td>9600</td>
</tr>
<tr>
<td>VCV_CMV_corr-0.1</td>
<td>102565</td>
<td>9233</td>
</tr>
<tr>
<td>VCV_ATMV_no-corr</td>
<td>101325</td>
<td>8532</td>
</tr>
<tr>
<td>VCV_ATMV_corr-0.1</td>
<td>98388</td>
<td>5538</td>
</tr>
</tbody>
</table>

Table 7-12 Chi-squared Test Statistics yielded by a Selection of Stochastic Models as applied to the Single-Baseline, BEDS-ONSA, and the Large Network, OLP-HANN
Analysis and Comments

It is clear to see that the chi-squared test statistics for these candidate stochastic models were very large and clearly exceeded the threshold statistics of 24.7 and 12.6 for one and two-tailed distributions respectively (cf. Equations 5-6 and 5-7). Such large test statistics are caused by large discrepancies between the observed occurrences and the expected number of occurrences as based on a normal distribution (cf. §5.6.3) and are indicative of non-Gaussian distributions. With consideration of the chi-squared statistics as yielded for all seven datasets (cf. Appendix K), there do not appear to be any trends, in terms of absolute or relative sizes, within the computed values.

Problems with Correlated Data – Linear Dependence of Data

A major disadvantage of acknowledging mutual correlations between observations is that it can exclude successful solutions. For example, when processing single-baselines with a variance-covariance function stochastic model, the inclusion of mutual correlations at the 90% level, and sometimes the 50% level, led to matrix inversion problems. This was also the case in multiple-baseline networks when applied at the 90% and 50% levels of correlation.

Comments on the Multipath Variance Estimation Routines

When considering the performance of the multipath variance estimation routines with additional terms representing mutual at-receiver correlations, there is very little difference between these algorithms. For example, the AMV method typically yielded, an insignificant 0.5% of epochs with wrongly assigned epochs compared to the other three multipath modelling methods.

From the design process, the AMV and ATMV routines were anticipated as being best suited for the estimation of residual code multipath variances for the North Sea datasets. Although the adaptive multipath modelling routines described in §4.3.1 did infer some high-frequency variations into the stochastic model to counter residual multipath effects, their benefit was not significant in improving the repeatability of the position solutions. This has been clearly seen from the tables in Appendix C and the stalwart presence of large artefacts in the true and formal error series regardless of the stochastic model used (cf. Appendices D through J).
Analysis of Results

Comments on the Overall Fidelity of the Quality Measures for Single-Baselines

Using the grouping trends identified in earlier results and specifying both the unit weight and elevation-weighting results, the quality measure fidelities for the study datasets are illustrated in Figure 7-22. There are no distance-dependent trends visible within this fidelity plot. However for the models used, it is expected that there should be some distance-dependency visible in the fidelity measures yielded by the unit weight results.

![Figure 7-22](image)

**Figure 7-22** Percentages of Wrongly Assigned Epochs yielded when applying Unit Weight and Elevation-weighting Stochastic Functions to all Study Datasets

Quality Assessment Routines

The use of the PCA routines (cf. §5.5.1) have been successful in terms of ranking the performance of the candidate stochastic models although concerns have been raised as to the final positional accuracies afforded by some models. For example, when applying an elevation-weighting model to the large network OLP-HANN, only 4.1% of the position solutions had been wrongly assigned in terms of quality measures yet afforded a mean positional accuracy of 1.4 metres (cf. Table 7-6). Applying a variance model with an adaptive multipath variance routine afforded an additional 0.9% wrongly assigned solutions (5.0% in total) but possessed a mean positional accuracy of just 0.9 metres – a 50% improvement in accuracy over the elevation-weighting model. The considerable discrepancy seen in this example gives serious cause for concern as to the validity of identifying the best model as that one with the highest fidelity quality measure. In this respect, the PCA routines will not always identify the more accurate model.
7.5 CONCLUDING REMARKS ON ANALYSIS OF RESULTS

A number of stochastic models have been designed, applied and evaluated for varying length baselines in single and multiple-baseline scenarios.

The use of a cosecant weighting function can represent the general increase in elevation-dependent noises associated with the errors of ionospheric and tropospheric delays and code multipath, although it cannot model the high-frequency variations within the error sources and dataset. Even so, for several datasets, a conventional unit weight model has outperformed the three more sophisticated stochastic models in terms of both accuracy and quality measure fidelity.

A suite of adaptive multipath variance estimation routines was developed to model the residual code multipath errors present in the phase-filtered code observations. Although in theory, the designed ATMV routine was believed to be most realistic in representing residual multipath within the GPS datasets, the results showed that the more sophisticated multipath models did not always perform as well as an incorrect constant multipath variance routine. Generally, the noted benefits of the adaptive multipath variance estimation routines were minimal considering that some larger errors remained within the datasets.

Stochastic functions were incorporated in the covariance terms to model linearly the spatial decorrelation of distance-dependent errors up to a-priori decorrelation distance thresholds. As there did not appear to be any distance-dependent trends within the results afforded by a conventional weight matrix. This suggests that the atmosphere may have already decorrelated for the study datasets and applying more sophisticated stochastic modelling functions may be unnecessary.

The incorporation of mutual at-receiver covariances to both the variances only and the full variance-covariance models has improved the quality measure fidelities for all single-baselines, however an improvement was not noted within the network solutions. Typically the greatest improvement in quality measure fidelity was noted for the shorter baselines processed with a full v-cv model, reducing as the receiver separation increased. This is because the satellite geometry relative to a pair of observing receivers is more correlated for a short baseline than for a longer baseline. Although there was a distinct grouping noted in the single-baseline study results, no definitive trends were seen when the four stochastic model types were applied to the two network datasets.
The use of the unit variance scaling factor has brought the quality measures down to more ‘realistic’ values and introduced an element of real-time variation to the final quality measures.

However for all of the results in these studies, a number of trends remained within the time series of true and formal errors, and unit variance statistics even though corrections had been applied for atmospheric and multipath delays in pre-processing. These trends, attributed to unmodelled systematic errors, contain some low-frequency artefacts of around 100-200 seconds duration as well as some high-frequency noise that has even remained after the application of the carrier-phase filter and multipath variance estimation routines.

One noteworthy concern relates to the quality assessment routines, in particular the absolute accuracy of the solutions afforded by different stochastic models. The quality assessment routine quantified the performance of candidate models according to the percentage of wrongly assigned quality measures. This process however did not always correspond with the best model in terms of positioning accuracy. When applied in real-time situations as intended with no means of truthing, the quality measure fidelity is dependent on the true error at each epoch which, in turn, depends on the success of the pre-processing and functional routines. Considering the table of results for the large network OLP-HANN as in Table 7-9, and Appendices C and J. The VCV_ATMV model yielded the lowest percentage of wrongly assigned quality measures at 3.6% with a mean vector accuracy of 1.21 metres, whilst the Vars_AMV model afforded a mean positional accuracy of 0.88 metres with only 5% of its quality measures wrongly assigned. Assessing the performance of the candidate models using the fidelity ratio does not always identify the best model in terms of positioning accuracy.

When considering the results visually using the time series plots, the elevation-weight and v-cv models performed consistently well for the single-baseline datasets and managed to mitigate the effects of most data interruptions. For the network datasets however, these elevation-models did not perform as well, in fact, sometimes yielding very erratic time series of results. Fortunately, the full variance-covariance models typically yielded more benign time series with less high-frequency noise and could cope with interruptions such as phase-filter resets and extreme multipath - an obvious benefit of their ability to introduce real-time variations to the mathematical positioning model and relate pseudorange precisions to satellite elevation angle.
Concluding Remarks on Analysis of Results

These studies unfortunately have not afforded an optimal general case stochastic algorithm for long-range positioning using phase-filtered differentially corrected code pseudoranges. It is interesting to note that the overall performance of unit weight matrices was, in some cases, better than the more sophisticated models not only in terms of quality measure fidelity, but also absolute accuracy. Empirical modelling will never completely eliminate all biases and noises from GPS datasets although creating real-time algorithms able to cope with every possible scenario and error type, adverse or otherwise, would demand considerable resources.

Ultimately this suggests that the nature of these biases and their relationships to the variances and covariances are clearly more complex than any of the stochastic functions applied in these studies. The designed empirical modelling routines are too simplistic and have failed to acknowledge those systematic errors residing within the GPS dataset, and that there may be a problem, or number of problems, within the pre-processing and functional model.
Chapter 8 - Conclusions and Recommendations for Further Work

This chapter provides an executive summary of the conclusions of this research in terms of the pre-processing, stochastic modelling, testing and position confidence analysis routines. Following this are a number of recommendations for further research relating to aspects of the stochastic modelling of phase-filtered pseudoranges.

8.1 SUMMARY OF MAIN RESEARCH CONCLUSIONS

Objective and Methodologies

This research has investigated the improvement in the fidelity of DGPS quality measures when applying more sophisticated stochastic functions to represent the errors in carrier-phase filtered code pseudoranges. This involved the development of software and algorithms necessary to perform the stochastic modelling of phase-filtered code pseudorange measurements as used in most current DGPS services. The fundamental research objective was to improve the fidelity of the quality measures over those afforded by a basic stochastic model, i.e. a matrix of unit weight. Following the comparison of the accuracies and precisions achievable with a large number of candidate models, the outcome of this research was hoped to be the identification of the best performing (most accurate) stochastic model for high-fidelity relative code positioning.

To do this, an extensive suite of software routines was developed containing existing and new stochastic algorithms for high-precision positioning using carrier-phase filtered code measurements. Through the North Sea data collection campaign, as organised by the author, a large dataset was obtained for six geodetic quality GPS receivers recording dual-frequency GPS data at accurately co-ordinated locations. For purpose of evaluating the candidate stochastic models, five single-baselines, ranging between 316 and 1164 kilometres in length, were selected for the research studies, as well as two multiple-baseline networks each comprising of three baselines whose mean lengths were 401 and 704 kilometres. Although primarily focussed towards marine navigation, these findings could easily be implemented to onshore positioning applications.
Summary of Main Research Conclusions

Pre-processing and Functional Modelling

A number of pre-processing activities were carried out to reduce the effects of biases on the raw code pseudoranges. The Klobuchar and Saastamoinen models were used to determine pseudorange corrections for the ionospheric and tropospheric delays respectively, as they can be found in most GPS processing packages and their algorithms are conveniently accessible. The application of these corrections to a 397 kilometre study baseline between BEDS and LOND was shown to:

- improve the mean vector accuracy, by 48%, from 2.1 to 1.1 metres, and
- reduce the percentage of wrongly assigned quality measures from 41% to 16% when processed using a conventional weight matrix.

The combination of GPS code and phase observables in a carrier-phase filter routine aided in the reduction of high-frequency code multipath, a major error source in differential GPS positioning. A phase-filtering algorithm, implemented in dual-frequency form, was successful in improving the precision (in standard deviations) of double-difference code positions by 76%, and the spread of position solutions by 77%, as compared to positions obtained using raw unfiltered code ranges.

With the objective of identifying a general case stochastic algorithm for single-frequency code positioning, it was preferable to use data as free of as many systematic biases as possible so that the final position solutions and conclusions derived from this data would not be overwhelmed by such biases. The dual-frequency filtering method was used as it was more robust than a conventional single-frequency filter in the event of a single-frequency cycle slip, and does not depend on the size of the filtering time constant ($N_{max}$) as the ionospheric divergence is eliminated through dual-frequency filtering. A value of 100 seconds was used for $N_{max}$ and provided a suitable filtering influence over the time series whilst maintaining the majority of overall trends within the filtered pseudoranges. With the application of this algorithm to the raw pseudoranges, it was possible to maintain consistency in, and full control over, the filtering process in these studies and provide cleaner datasets for the evaluation activities. The stochastic functions designed to represent the precisions of these undifferenced code pseudorange measurements have not been affected by the choice of filtering algorithm.

For the actual processing of the phase-filtered code pseudoranges, a double-difference functional model was used instead of the conventional DGPS approach. Although both methodologies reduce the effects of satellite-common errors and yield
identical position solutions, mathematical correlations between observations can be modelled more conveniently within the double-difference functional model. Means of porting the designed stochastic algorithms into conventional DGPS approaches need to be considered.

**Candidate Stochastic Models**

A comprehensive suite of stochastic algorithms were created that incorporated functions to determine the precision of phase-filtered code pseudoranges by acknowledging real-time varying quantities and correlations present within the datasets. The integration of these stochastic initiatives into the covariance matrix of undifferenced observations would help to determine a more correct weight matrix for the datasets and in turn yield improved quality measures. By implementing these four algorithm types on the test datasets and post-processing them as though they were collected in real-time, it was expected that the effects of residual systematic biases would be mitigated, to varying extents, from the final quality measures and solutions.

- The first stochastic model assumed all observations were of equal precision and independent of one another – a matrix of unit weight.
- The second, an elevation-weighting model, assumed the precision of phase-filtered code pseudoranges decreased as the elevation angle of the observed satellite decreased, and was represented by a basic cosecant mapping function.
- The third stochastic model type, a user-defined variance model, contained terms to describe the precision of the phase-filtered pseudoranges as observed by each receiver along with use of the basic cosecant mapping function. This model had not been expected to perform as well as the full v-cv model because it did not contain covariance terms reflecting the precisions and correlations between observations from common satellites.
- Finally, application of the fourth candidate model, a full variance-covariance model, afforded measurement precision estimates to be made using the user-defined variance algorithm as well as covariance estimates of the spatial correlations between two common satellites or from common receivers. Spatial decorrelation functions were included that assumed linear reduction of the distance-dependent errors of satellite orbits, ionosphere and troposphere with respect to a-priori decorrelation thresholds.
Summary of Main Research Conclusions

The unit weight and elevation-weighting algorithms were included as baseline models for reference purposes, i.e. as baseline models.

A suite of three modelling functions was incorporated in the two more sophisticated models to adaptively estimate the residual code multipath using information from the pre-processing phase-filter, namely the standard deviations (from a 30 second moving average window) and/or currency of the phase-filter corrections. A fourth function assuming residual multipath was identical on all observed pseudoranges included as a benchmark model.

A stochastic function was developed to model the mutual correlations existing between the observations made at each individual receiver. These correlations affected by errors in the atmospheric correction models and the mapping functions, were modelled as a function of a-priori atmospheric delays and the elevation angles of the observed satellites.

Quality Assessment Routines

Procedures were developed to evaluate the performance of the designed algorithms at each epoch and over the entire dataset. Using the Position Confidence Analysis (PCA) routine and a truthed solution for each baseline, a quality measure fidelity (QMF) statistic was determined, at each epoch, from the ratio of the true error to the formal 95% percentile precision estimator. The performance of each candidate model was then quantified using the QMF statistic over the entire dataset, i.e. the percentage of position quality measures wrongly assigned. A QMF value of zero was expected for a GPS dataset containing only random errors with normal distribution (at a 95% confidence level) - any deviation from this suggested that the stochastic model was failing to correctly acknowledge systematic errors within the GPS measurements.

The determination of absolute accuracy was not an objective in this research although the maintenance of positional accuracy was always considered. Even though the UKOOA-recommended guidelines for quality control, i.e. sophisticated blunder detection and reliability testing, did not feature in this research, a simple blunder detection procedure was implemented that excluded, from the quality assessment routines, those position solutions changing by more than 0.5 metres between two consecutive epochs.

A scaling factor determined from the mean of the previous 30 unit variance values, was applied to the quality measures prior to the quality assessment routines and afforded them information on the relationship between the mathematical model and the GPS data. A chi-squared test statistic was calculated for every successful model to ascertain the
distribution of the estimated and true position errors with reference to a Gaussian distribution.

**Analysis of Study Results**

A comprehensive post-processing campaign was undertaken using the four stochastic model types, and the designed stochastic functions. Each of the seven candidate datasets were post-processed as though they were collected in real-time, with the suite of thirty-six stochastic models, and an elevation angle cut-off of only 5 degrees was used to afford longer continuity of satellite visibility when processing the longer baselines. A number of conclusions have been afforded by the results processed according to the candidate stochastic models.

In the single-baseline studies, two general trends were seen in the results afforded by the four model types. Results from the unit weight and user-defined variance models were similar in terms of accuracy and fidelity, as were the results afforded by the elevation-weight and variance-covariance models. Essentially these trends were caused through the dominating effects of large satellite clock variances within the study data. The inclusion of a satellite clock precision in the covariance terms has eliminated this scaling effect and allowed the elevation-weighting functions to be more influential within the single-baseline datasets as seen by the smaller step functions. The largest mean 3-D vector accuracy achieved when processing the single-baseline datasets with the suite of stochastic models did not exceed 1.8 metres.

No similar groupings were seen within the network studies although the use of multiple-baseline datasets provided higher accuracy positioning than conventional single-baseline datasets. In both networks, the lowest mean 3-D vector accuracy achievable did not exceed 1.4 metres. As expected, there was little change noticed in the performance of the user-defined variance models for the network datasets regardless of which multipath variance estimation routine or level of mutual correlation was applied. The response from the full variance-covariance models was more significant although negative, as an increased number of solutions were assigned incorrect quality measures as increasing levels of mutual correlation were applied.

The findings of this research have shown that the elevation-weighting function generally performed well in the single-baseline studies, not only in terms of quality measure fidelity and accuracy, but also in mitigating those errors associated with low elevation satellites as seen in the error time series. Although conceptually incorrect, the
unit weight model, in some cases, performed statistically better than an elevation-weighting model, for example, for the 316 kilometre LOND-DELF baseline. Generally, the continuity of the quality measure time series yielded by an elevation-weight function was less erratic during periods of satellite geometry changes and phase-filter resets. However, the elevation-model did not perform as expected for the multiple-baseline networks and yielded very erratic time series and lower accuracies, even more than a conventional stochastic model. Overall though, this function could not represent the real-time errors in GPS observational data at all times, as seen by the high-frequency variations remaining in all results time series.

In the user-defined variance and full variance-covariance algorithms, a large satellite clock variance (representing SA-dither) was applied to the undifferenced code pseudoranges at each epoch. When using the user-defined variance model in the single and multiple-baseline studies, the choice of multipath variance estimation algorithm has little effect on the overall results as very little variation in the quality measure fidelities was noticed relative to this large satellite clock variance.

It was expected, and seen, that the three adaptive multipath variance estimation routines introduced more variations within the stochastic model than a constant multipath variance estimate and reduced the high-frequency variations within the network datasets. However no-one adaptive algorithm stood out as the best algorithm for either the single or multiple-baseline datasets, i.e. it did not afford significant improvements to the quality measure fidelity over the benchmark CMV algorithm, as there were still large residual unmodelled errors present within the phase-filtered observations.

**Mutual At-receiver Correlation Algorithm**

The acknowledgement of mutual correlations between the observations in single-baseline datasets, even at a level of only 10%, impacted on the final results in terms of accuracy, formal errors and quality measure fidelities by effecting an increase in both the true and formal error vector sizes. The greatest improvement in quality measure fidelity was noted for the shorter single-baselines processed using adaptive multipath variance functions, especially those baselines between similar receiver types. The improvements were more significant for the full variance-covariance model suggesting that the correlations between observations were being modelled correctly, however these improvements reduced as the receiver separations increased and the relative satellite geometries became less correlated.
Conclusions and Recommendations for Further Work

There was little positive benefit noted from applying mutual covariances to the network datasets as the QMF values deteriorated; this was particularly noticeable for results processed with the full variance-covariance models. It is unknown why the incorporation of mutual covariances within the multiple-baseline datasets was not successful in improving the quality measure fidelity.

These studies have not identified the optimum level of mutual correlation as seen from the variation of the results afforded using different coefficients of correlation. In fact, given the greater percentage of position solutions that were wrongly assigned from the network studies, it may have been wrong to assume positive correlations exclusively between those observations at each receiver. The assignment of large coefficients of correlation (typically 50% and 90%) within the mutual covariance algorithm caused problems when inverting the covariance matrix of double-differenced observations into the weight matrix because they inferred considerable linear dependence within the observed data. This was especially prevalent for the networks' larger covariance matrices.

The resultant chi-squared test statistics were considerably larger than the threshold criteria demonstrating that the candidate models did not yield position solutions and quality measures commensurate with a Gaussian distribution, all of which has been attributed to the presence of unmodelled systematic errors within the datasets and functional models.

Using QMF values alone for position quality assessment was insufficient especially considering that the best model, in QMF terms, could be considerably less accurate than other less sophisticated models; for some examples by almost 50%. Excluding the unit variance scaling factor would have afforded quality measures driven solely by the PDOP statistic and the a-posteriori cofactor matrix, both of which lack high-frequency variations.

Overall, this suggests that there are a number of flaws within the candidate modelling functions, essentially in that they fail to correctly acknowledge and model the real variations and correlations for the observations used in differential positioning applications. A prime example of this is the failure of the user-defined variance models to acknowledge spatial correlations and the cancelling of the satellite clock errors (including SA-dither) of common satellites.

Optimal General Case Stochastic Model

A favourable and consistent performance across all seven study datasets would have indicated a potential general case stochastic algorithm, unfortunately, no-one single model performed in this way. Regardless of the stochastic model applied here, many high-
Recommendations for Further Research

frequency artefacts remained within the true and formal error time series, whereas previous research has demonstrated that highly accurate quality measures can be achieved with a correct stochastic model. Therefore it is suggested that, until a correct general case algorithm is discovered and validated across a range of datasets, an elevation-weighting function should be used for single-baseline datasets. From these findings, it has been seen to provide consistent time series with minimal step functions over periods of constellation change and interruptions, such as phase-filter resets and extreme multipath. For multiple-baseline networks, a full variance-covariance matrix populated with an adaptive multipath variance routine will afford more benign time series than any of the other candidate model types tested. These will likely contain smaller high-frequency noises and smaller step functions.

Overall, this research project has shown that the progressive inclusion of sophisticated stochastic terms describing the precisions of, and spatial correlations between, carrier-phase filtered pseudoranges have failed to significantly, and consistently, improve the fidelity of quality measures compared to those afforded by a conventional stochastic model. The designed stochastic functions have failed to eliminate, or at least significantly reduce, the complex unmodelled systematic errors present within the GPS datasets as seen by the numerous artefacts within the true and formal error time series. Clearly the nature of the relationships between the variances and covariances of phase-filtered differentially corrected pseudoranges are more complex than any of the models tested here.

8.2 RECOMMENDATIONS FOR FURTHER RESEARCH

The findings of this research suggest that in order to obtain a more realistic DGPS stochastic model, it is necessary to investigate and develop additional stochastic functions to better represent the complex relationships between the variances and covariances of phase-filtered differentially corrected code pseudoranges. This should lead to a reduction in the impact of the remaining high-frequency unmodelled systematic errors on both the position solutions and their quality measures.

In this section, recommendations are made with the fundamental aim of improving the performance of the stochastic modelling techniques through the implementation of new and revised models. Prior to these recommendations, a number of points are raised
Conclusions and Recommendations for Further Work

Concerned with improving the accuracy of differential code positioning. As the stochastic model and its a-priori variances used are directly related to the pre-processing activities and functional model, improvements in positioning accuracy must be acknowledged when making use of quality measure fidelity statistics for evaluation purposes.

8.2.1 Pre-processing and Functional Modelling

With the IGEB’s modernisation of GPS underway, in particular their Accuracy Improvement Initiative (AII), an improvement should be duly noticed with their efforts to reduce the GPS clock and ephemeris contribution to UERE by 50% to 1.25 metres [Shaw et al, (2000)]. The quality of predicted orbit and clock correction products has been noted and exploited successfully by many groups for post-processing activities (cf. §2.1.2). It would be interesting to investigate any benefits of using the now official IGS ultra-rapid orbit and clock products to reprocess the North Sea datasets.

The addition of two new civilian C/A-code signals to future GPS satellites; the L2 signal in 2003 [Van Dierendonck and Hegarty, (2000)], and the ‘safety of life’ signal L5 in 2005 [Shaw et al, (2000)], will afford further improvements in accuracy. Civilian users will then be able to calculate accurate real-time corrections for the ionospheric delay and improve the accuracy of their position solutions, without the need for signal reconstruction and correlation procedures to overcome Anti-Spoofing.

The continued research into receiver technologies, in particular signal processing and multipath mitigation algorithms, means that the overall quality of GPS observables is constantly improving, and will be seen through the reduction of code multipath and receiver noise errors on undifferenced measurements.

Improved Atmospheric Modelling

As the empirical Klobuchar model is based on solar flux measurements over the previous five days, it cannot always accurately reflect the current state of the ionosphere. Therefore concerted investigations should be made into the determination of real-time ionospheric delays, in particular through the use of dual-frequency measurements. If only single-frequency measurements were considered in further work, additional means of modelling the ionospheric biases should be sought. A number of possibilities follow.
Recommendations for Further Research

- Given the improved resolution afforded to C/A-code observables [Leick, (1995) and Shaw et al, (2000)], determine TEC counts directly from the single-frequency measurements.
- Incorporate, into the research software, values of the ionospheric delay from the near real-time ionospheric maps created by GIBS [GIBS, (2000)] and JPL [JPL Ionosphere Working Group, (1999)].
- Make use of the rapid IONEX products created by CODE which have been shown to perform significantly better, for the single-frequency user, than the GPS broadcast coefficients [AIUB-CODE, (2000b)].

All three approaches would afford (near) real-time estimates of the single-frequency ionospheric delays in the region of each observing receiver. However ideally, dual-frequency measurements would be available and used to determine the ionospheric delay, especially given the recent focus of interest regarding the validity and accuracy of the IONO coefficients contained within the BRDC navigation message [Hernandez, (2000)]. Alternative ionosphere models could be considered such as the Bent model.

In terms of further research into tropospheric modelling, the performance of more recent and more sophisticated tropospheric models should be considered in place of the refined Saastamoinen model and cosecant mapping functions used in this research. For example, the use of more advanced mapping functions, such as the cosecant-squared mapping function [Springer, (1997)] and Neill’s mapping function [Neill, (1996)], should be investigated to facilitate the improved modelling of path delays for lower elevation angle satellites. Both of these mapping function examples are used by IGS Analysis Centres [Springer, (1997)].

Multiple Ramps within the Single-frequency Carrier-phase Filtering Algorithm

One major problem encountered in single-frequency filtering algorithms is that the presence of a signal interruption or cycle slip enforces a phase-filter reset on a particular satellite, and all information corresponding to that phase-filtered pseudorange, and its increased weight, is lost. The code and carrier-filtered pseudoranges then have equal weight. Consequently the user sees a decrease in the DGPS positioning accuracy at that epoch and, depending on the stochastic model applied, the formal precision estimators also (particularly noticeable for the PENC datasets). In order to reduce these effects, ramping techniques could be incorporated into the single-frequency algorithm that would reduce the
positional glitches once $N_{\text{max}}$ is reached, preferably multiple ramps say at 10% intervals of $N_{\text{max}}$. Ideally only dual-frequency processing should be used meaning that multiple ramping routines would be unnecessary.

**Automatic Cycle Slip Detection Routines**

Given the influence of cycle slips on the adaptive multipath variances and the positioning accuracy, the use of a fully automatic algorithm for the detection and possible repair of cycle slips should be considered. With the successful correction of cycle slips, smaller and less erratic step functions would be seen within the multipath variance estimates. One example of a detection routine that could be incorporated within the phase-filtering module is the acknowledged TurboEdit algorithm as developed by Blewitt [1990]. Incorporating or designing such a routine would depend on whether the application required positions in real-time or from post-processing.

**Use of Precise Observables and Linear Combinations of Observables**

An immediate focus of future research must be the inclusion of L2 observations into the positioning model, not only in terms of the precise codes (P1 and P2), but also with consideration for the proposed C/A-code on L2 [Van Dierendonck and Hegarty, (2000)]. The subsequent availability of dual-frequency observations would allow the determination of linearly combined observables such as the ionosphere-free code pseudorange and the code multipath observable (akin to the phase-filter corrections). These observables would not possess ionospheric delays or code multipath but greater levels of noise from the combination process meaning further investigations would be necessary with regards to:

- means of obtaining the P2 observable from receivers using codeless and cross-correlated measurements [Ray et al, (1999a)],
- the inter-frequency correlations between the code observables (C1, P1 and P2), and between the phase observables (L1 and L2) [Tiberius [2000] and Bona and Tiberius (2000)]; neither party found any explicit correlations between the code and carrier-phase measurements, and
- the propagation of measurement noise into observables formed from linear combinations of dual-frequency measurements.
Recommendations for Further Research

Accuracy of Multipath Estimates

As the current phase-filtering routine may be effecting some systematic biases into the filtered measurements and error time series, the assumption that the phase-filter corrections are equivalent to the code multipath should be validated as follows:

- Use linear combinations of dual-frequency observables to determine the true code multipath errors or make use of an off-the-shelf program such as teqc [Estey, (2000)].
- Were this a real-time model, then routines to model the code multipath as a function of the signal-to-noise ratio as in Comp and Axelrad [1996], and Barnes et al [1998] could be used.

8.2.2 Stochastic Modelling

Given the apparent failure of the stochastic models designed in this research, the determination of true error sizes and behaviours would permit their verification and assist in their subsequent evolution.

Empirical Approach to Reverse Engineering Studies

It would be beneficial to carry out a number of studies to determine how certain GPS parameters change over time including:

- the true ranges and thus the true double-difference errors,
- the true variances and covariances,
- the differential errors within the satellite orbits, and the ionospheric and tropospheric delays.

Analysis of these parameters and their trends over long periods of time, e.g. several days at high data rates, would provide information as to how these errors behave and how a truth covariance matrix responds to their variations. With this information, it would be possible to interpolate both functional and stochastic parameters for one or more mobile receivers within a reference station network. A number of temporal functions could be created that model the differential errors and change in stochastic quantities for a number of epochs in advance.

Referring to the spatial decorrelation functions, a similar routine could be created to interpolate the double-difference errors at the mobile receiver from the true double-
Conclusions and Recommendations for Further Work

difference ionospheric and tropospheric delays calculated between each receiver pair. A
temporal function based on the rate of change of these errors at the reference stations could
be used to determine the precisions of these quantities through a moving average function.
It is believed that the implementation of such functions would introduce more time-varying
quantities into the stochastic model and consequently the quality measures providing a
means for validating the stochastic models designed in this research.

It would be interesting as a further research task to implement this truth trajectory
information to investigate the performance of this research’s stochastic models and those
designed specifically for use in dynamic positioning applications. The kinematic data
collected with the Leica MC1000 during the North Sea campaign could be processed with
the data from the nearest reference station DELF and used to determine a truth trajectory
for the vessel albeit in a simulated real-time mode (cf. §6.1.4). Further studies could be
carried out where more complex mapping functions are evaluated in terms of their overall
accuracy and repeatability.

Revised Atmospheric Modelling

The use of ionosphere-free code pseudoranges and more advanced ionospheric and
tropospheric delay models will assist in the estimation of more accurate position solutions.
The stochastic model should then be amended to model any residual errors from new
functions added to the pre-processing routines and functional model. One such example
would be to use a variant of Equation 4-18 to model residual ionospheric noises when
processing with ionosphere-free code pseudoranges [Euler and Ziegler, (2000)]. A revised
SDF algorithm could use the ionospheric delay estimates calculated from dual-frequency
observations or revised single-frequency models, to provide a-priori vertical delay
estimates to determine ionospheric gradients along the baseline vectors.

McGraw et al [2000] believe that any temporal correlation within the residual
ionospheric delay can be modelled as a function of the phase-filtering time constant, $N_{max}$
(cf. §3.2.3). With the operation of a single-frequency filter parallel to a dual-frequency
filter for each observing receiver, it may be possible to estimate the ionospheric
divergence in near real-time and then downweight the single-frequency filtered
pseudoranges accordingly.


Recommendations for Further Research

**Code Multipath Modelling Routines**

It has been seen that high-frequency code multipath cannot be modelled through elevation angle functions alone, although a general trend is followed. Any future research should investigate the elevation-dependent multipath function as featured McGraw et al [2000] and the concept of scaling the combined variance estimate by the satellite elevation angle. Even though satellite clock errors are usually regarded as being independent of both receiver type and satellite elevation angle, Raquet and Lachapelle [2000] obtained satisfactory results by applying an overall mapping function to the variances estimated by the NetAdjust algorithm (cf. §4.2.3).

**Moving Average Routines**

There is no disputing that the use of information from consecutive epochs has been beneficial for improving positional accuracy and quality measures through the phase-filtering, multipath modelling and unit variance scaling factor routines. Further research needs to be carried out into the propagation and accumulation of temporal errors within these moving window routines, and the possibility of conflicts between the moving window methodologies and explicit temporal correlations within the GPS data (based on the findings of the reverse engineering studies).

Given that the artefacts remaining within the error time series have typical periods of 100-200 seconds, further studies into the optimal filter duration are needed. It may be possible to implement two different length filters that run concurrently for both the phase-filter standard deviations and unit variance routines. Specifying a longer sample size would provide more information on the lower-frequency trends should a signal interruption occur, the new multipath variance afforded by the phase-filter for that epoch will yield a sizeable step function in the filter correction time series. A shorter sample size would effect a smaller step function although the variances would exhibit more variation from the higher-frequency residual multipath trends. For example, Han [1997] specifies a variable window length of 120-300 seconds for the scaling factors present within his post-processing and real-time stochastic model estimation routines (cf. §4.2.2).

With largely different window lengths, this dual-filter routine should pick-up information on both high and low-frequency multipath missed by the 100 second phase-filter in pre-processing. The high-pass filter would contain the high-frequency (relative) variations within the dataset and the low-pass filter would afford details of the absolute multipath variance. The revised real-time multipath variance estimate would then be the
sum of both filters. The sensitivity of the multipath variance and unit variance moving average routines and their respective lengths must be considered, possibly even made dynamic following periods of satellite geometry change, as suggested by Wang [2000].

**Modelling of Temporal Correlations within the GPS Code Pseudorange Datasets**

As there have been no explicit temporal functions included in this research's algorithms, it would be interesting to investigate the performance of time-varying functions derived from the findings of the reverse engineering studies. This would essentially afford real-time analysis of differentially corrected pseudoranges for the determination of the elements of a time-varying covariance matrix that 'truly' describes the precision of the data used. That is, mimicking, through the covariance matrix of undifferenced observations, the actual errors within the system. The use of simulated GPS data contaminated progressively with predefined systematic errors would be very useful to ascertain the sensitivity of new and existing stochastic functions.

The temporal nature of the ionosphere must be taken into account if the research software is not transmogrified into a dual-frequency system. An immediate yet basic function to model the variation of the a-priori vertical ionospheric delay from its peak at around 1400LT each day to its nadir approximately 12 hours later as well as the transition in between, could be a cyclic function with a period of 24 hours. It would require an amplitude based on the ionospheric delay computed from the linear combinations or estimated from real-time ionosphere maps. Another temporal ionospheric function could be created in conjunction with the parallel single and dual filters described earlier in the cycle slip detection routines.

Functions to represent the temporal correlation within the differential tropospheric delays caused by the passage of weather fronts over GPS reference stations could be included with reference to the advanced empirical models researched in [Gregorius, (1998)] and Gregorius and Blewitt, (1998)].

An amendment must be made to represent, perhaps dynamically, the receiver noise associated with each satellite's pseudorange measurement, be it unfiltered or linearly combined, similar to those used in Roberts and Cross [1993]. The approximate quality of the code observables could be based initially on that signal-in-space range error included within the broadcast navigation message, and estimates of the combined observable measurement noises from error propagation calculations. In future studies, the distinct differences in observation noises from different receivers must be investigated and, if
Recommendations for Further Research

possible, generalised according to each popular receiver model [Bona and Tiberius, (2000)]. This could be done in conjunction with stochastic modelling of the code pseudorange errors against elevation as in Jin [1996] and Han [1997a; 1997b]. Latency effects could also be evaluated with the deliberate injection of latencies into the processing software.

Mutual At-receiver Correlations

The improvement in quality measure fidelity for single-baselines as effected by the mutual covariance terms was positive, however it is necessary to investigate this module’s failing with respect to the multiple-baseline networks. Consideration must also be given to the presence of negative correlations between the observations at each receiver and the algorithmic format that such considerations would take. As an optimal value was not found for the coefficient of mutual correlation, it may be the case that the mutual correlation between two rising, or setting, satellites is positive, and negative when one satellite is rising and the other falling in elevation. The dynamic estimation of the correlation coefficient could be based on the differential elevation angle between two observing satellites; additional dynamic terms reflecting the difference in azimuth between the two satellites could also be added.

With this research’s findings that an elevation-weight model can provide adequate performance for the single-baseline datasets, it would be interesting to evaluate the incorporation of modified covariance terms within the elevation-weight model; no atmospheric variance terms would be included, only elevation angles and a (dynamic) correlation coefficient.

Differential Azimuth Studies

It must be noted that the broadcast ionosphere model contains terms for the azimuth angles of each satellite observed by a receiver whilst the refined Saastamoinen tropospheric correction model does not, and at present, there are no azimuthal terms included in the stochastic functions. If two observed satellites are both at 45° elevation relative to the user, it is assumed that atmospheric errors are effecting similar delays and fully correlated; however this correlation is also dependent on the azimuth of the satellites to each other. In fact, the signals observed from two satellites with 180° difference in azimuth will be passing through very different sections of the ionosphere and troposphere. The mutual covariance algorithm should include additional terms for the azimuth of the
Conclusions and Recommendations for Further Work

satellites; a proposed differential azimuth term for the correlation between atmospheric variances would decrease from a differential azimuth of $0^\circ$ to a maximum of $180^\circ$, and thus afford a minimum correlation between their measurements.

A means of assessing the sensitivity of the stochastic modelling algorithms, in particular the mutual at-receiver covariance terms, would be to evaluate the performance of these algorithms through reverse-engineered point positioning studies. Specific stochastic models could be incorporated into the pre-processing routine to calculate the reference receiver SPP prior to the main double-difference routine and would aid in determining an optimal mutual correlation algorithm whilst evaluating the use of dynamic correlation coefficients.

**Statistical Testing**

Once the unmodelled systematic errors have been removed through revised pre-processing and functional routines, the sensitivity of the stochastic modelling algorithms can be reassessed as to the accurate representation of the real variations within the GPS observations, and their subsequent effect on the adjustment. All future positioning models should contain routines capable of identifying outlying observations as well as statistical and reliability tests as suggested within the UKOOA P2/94 guidelines (cf. §3.2) [Ashkenazi et al, (1994) and Roberts et al, (1997)].

With the mitigation of outlying observations and some of the more significant systematic errors from the study datasets using the statistical testing and linear combinations respectively, it would be interesting to re-evaluate the performance of the quality assessment routines, and particularly the chi-squared test routine. From the tests conducted in §5.6, it could be seen that some normal distribution trends were present in the histograms of Figure 5-10, albeit offset by some systematic biases. With the mitigation of those biases affecting the formal error estimates, more integrity would be afforded to the chi-squared test than at present. In turn, a more objective assessment of the distribution of the quality measures would be possible.

The Position Confidence Analysis routine should also be amended to break down the current ratio of wrongly assigned solutions and classify the number of pessimistic and optimistic solutions - analysis of these solutions may afford a better understanding of the fidelity of the quality measures yielded by more sophisticated stochastic modelling routines.
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A.1 THE GLOBAL POSITIONING SYSTEM (GPS)

This Appendix section contains some basic background on GPS, the measurements it provides and means of modelling them, to refresh the reader of concepts of particular interest within this research.

GPS is a space-based radionavigation system capable of providing all-weather instantaneous three-dimensional positioning and timing 24 hours per day. Developed and operated by the United States’ Departments of Defense and Transport (DoD and DoT respectively), it comprises, at the time of writing, a constellation of 29 satellites distributed within six orbital planes at an altitude of approximately 20,200 kilometres [USCG-NAVCEN, (2001)]. Their orbital configuration is such that it affords four visible satellites to be seen from any point on the Earth's surface at any time. Although primarily intended as a military navigation system in the 1970s, the majority of the user base now consists of civilians. With the development of geodetic receivers and specialised processing algorithms, civilian users can carry out highly accurate navigation and surveying tasks using these same GPS satellites.

All GPS satellites transmit carrier signals at two distinct L-band microwave frequencies, the primary signal at 1575.42 MHz (L1) and the secondary signal at 1227.60 MHz (L2). Atomic clocks aboard each satellite produce the fundamental L-band frequency at 10.23 MHz. The L1 and L2 carrier frequencies are generated by multiplying the fundamental frequency by 154 and 120, respectively. From these signals, a GPS receiver can obtain two fundamental observable types; the code pseudorange observable and the carrier-phase observable. These observables are basically range measurements derived from the measured time or phase differences based on a comparison between actual signals and receiver generated signals.
Appendix A – The Global Positioning System (GPS)

The relationships of the GPS observables to the fundamental GPS frequency can be seen in Figure A-1. The primary L1 carrier wave is modulated with two pseudorandom noise (PRN) ranging codes: the 1 millisecond C/A-code with a chipping rate of about 1.023 MHz, and a week-long segment of the P-code with a chipping rate of 10.23 MHz. The L2 carrier is modulated by the P-code but not yet with the C/A-code (cf. §8.2.1). Each carrier is also modulated with a navigation message containing up-to-date predictions of satellite metadata; in particular, the ephemeris parameters describing the predicted satellite positions, and the predicted satellite clock correction terms (cf. §A.6.1.3).

A.1.1 GPS Measurements

A.1.1.1 Code Pseudorange Observable

The basic code pseudorange is a measure of the distance between the GPS satellite at the time of transmission and the user's receiver at the time of receipt. However as the satellite and receiver are never perfectly synchronised to true GPS time, there are some small clock biases associated with these range measurements. Unfortunately, the satellite and receiver clock oscillators are not perfect and do not run exactly at the fundamental GPS frequency. Deviations from this frequency propagate directly into the code and carrier measurements, and therefore the positions derived from them. Ranges that contain such clock bias offsets are slightly longer or shorter than the true geometric range, and hence are called pseudoranges.
Appendix A – The Global Positioning System (GPS)

This pseudorange distance measurement is based on the correlation of a satellite-transmitted code and a local receiver’s reference code that has not been corrected for synchronisation errors between the transmitter and receiver clocks. The actual pseudorange observation $P$ from satellite $i$ to receiver $A$ can be written as:

$$P_A^i = \rho_A^i + c\tau_A - c\tau^i$$  \hspace{1cm} \text{Equation A-1}

where $\rho_A$: range from receiver $A$, at receive time $t_A$, to satellite $i$, at transmit time $t^i$

$c$: speed of light - around 299,792,458 ms\(^{-1}\)

$\tau_A$: receiver clock bias

$\tau^i$: satellite clock bias

As the C/A-code repeats every millisecond corresponding to a range error of around 300 kilometres, there is an ambiguity associated with C/A-code pseudoranges. This can be easily resolved during processing, for example by the introduction of approximate receiver co-ordinates (to a few hundred metres) within the WGS-84 datum.

A.1.1.2 Carrier-phase Observable

The carrier-phase observable is derived from the measurement of the difference between the phase of the signal arriving from the GPS satellite and the phase of the replica signal generated locally by the receiver. Due to its shorter wavelength and higher resolution, the carrier-phase pseudorange observable can be thought of as a more precise code pseudorange albeit with an additional bias due to the carrier-phase integer ambiguity $N$.

Phase, $\varphi$, or angle of rotation, is the measure in units of cycles of an electromagnetic (EM) wave which, after conversion into appropriate units, can provide a measure of GPS signal travel time. The direct measurement consists of a phase reading of the fractional part of the whole (integer) number of cycles in the range between the satellite and the receiver. Unfortunately, the receiver has no knowledge of the number of whole signal wavelengths when it acquires the satellite’s signal, but is able to keep count of the integer number of wavelengths to be added or subtracted as the receiver-to-satellite range changes. This initial whole number of cycles, referred to as the integer ambiguity $N$, must be
resolved in order to determine the range between the receiver and the satellite. With the correct ambiguity resolved, ranges can be determined to millimetric accuracy and consequently, the carrier-phase observables are used in higher precision surveying applications.

The carrier-phase measurements, normally output in units of cycles, are multiplied by their respective nominal wavelengths to obtain phase pseudoranges (in metres):

\[
\lambda \Phi_A^i = \rho_A^i - \lambda N + c \tau_A - c \tau^i
\]  

Equation A-2

where

- \( \Phi \): phase pseudorange from satellite \( i \) to receiver \( A \) in units of metres
- \( \rho \): range calculated from the true signal travel time
- \( \tau_A \): clock bias of receiver \( A \)
- \( \tau^i \): clock bias of satellite \( i \)
- \( N \): carrier-phase integer ambiguity
- \( \lambda \): wavelength of the respective carrier signal (L1 or L2)

The phase pseudorange observable above only differs from the code pseudorange (cf. Equation A-1) by integer multiples of wavelengths, \( \lambda N \), relating to the integer ambiguities and the wavelength of the observable used.

A.1.1.3 Navigation Message

A navigation message data stream is transmitted by each satellite at a rate of 50 bps on both frequencies. The 25 subframes within the message contain information essential for successful GPS satellite tracking by a user receiver as in Table A-1 [Leick, (1995)]:

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Appendix A – The Global Positioning System (GPS)

<table>
<thead>
<tr>
<th>SUBFRAME</th>
<th>SUBFRAME CONTENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Predicted satellite clock corrections and reference biases</td>
</tr>
<tr>
<td>2 &amp; 3</td>
<td>Transmitting satellite's orbital parameters</td>
</tr>
<tr>
<td>4</td>
<td>Ionospheric correction terms (8 coefficients); Coefficients to convert GPS Time to UTC; Almanac data for SV25 and higher</td>
</tr>
<tr>
<td>5</td>
<td>Almanac data for SV1 through SV24</td>
</tr>
</tbody>
</table>

Table A-1 Contents of the Broadcast GPS Navigation Message

With the information contained in this navigation message and the signal from at least one GPS satellite, a user's receiver can obtain the ephemeris and clock details of all other satellites within the operating constellation. Also included are flags relating to the health status of these satellites (for quality control purposes), terms describing the ionosphere's current behaviour and coefficients allowing the conversion of GPS Time to Universal Time Co-ordinated (UTC).

The accuracy and currency of the navigation messages are monitored by the GPS control segment ensuring that the clock and orbit predictions do not degrade significantly between navigation message uploads. Coefficients describing each satellite’s Keplerian elements are estimated at 2 hour intervals with updated orbit and clock corrections being uploaded to each satellite at least once a day [Shaw et al, (2000)].

A.1.1.4 Doppler Observable

This observable is derived from the ratio of the transmitted and received GPS frequencies and is directly related to the topocentric range rate of the satellite integrated over two epochs [Leick, (1995)]. As the name suggests, it is based on the Doppler effect, whereby the motion of the transmitting satellite relative to the observing receiver effects a shift in the frequency of the transmitted signal as experienced by the receiver.

Although the concepts of carrier-phase and Doppler are similar in the sense of physics, different processing approaches are necessary [Leick, (1995)]. Carrier-phase processing normally involves an instantaneous epoch measurement, whereas Doppler needs to be integrated, i.e. considered over two epochs [Thomson, (1996)].
Appendix A – The Global Positioning System (GPS)

A.1.2 Modelling GPS Observations

Taking the fundamental observables discussed in §1.2, and knowledge of those errors biasing the GPS measurements, it is possible to relate them to the point by means of an observation equation. These equations then form the basic variable within the least squares position computation and are detailed within the following sections.

A.1.2.1 Code Pseudorange Observation Equation

The observation equation for a code pseudorange on the L1 frequency is:

\[ P_A' = \rho_A' + d\tau_A' - c\tau_A' + I_A' + Z_A' + m_{pA}' + e_A' \]  

Equation A-3

where

- \( P \): pseudorange observation between satellite \( i \) and receiver \( A \)
- \( \rho \): true geometric range computed from \( |x_i - x_A| \), \( x_i \) is the satellite position vector, \( x_A \) is the station position vector
- \( d\rho \): effect of ephemeris errors, including SA effects
- \( t_A' \): receiver clock error, including SA effects
- \( t_i' \): satellite clock error, including SA effects
- \( I \): ionospheric delay (the positive sign indicates a group delay on the code signal as it passes through the ionosphere)
- \( Z \): delay due to tropospheric refraction
- \( m_{pA} \): multipath bias on the pseudorange measurement
- \( e \): pseudorange measurement noise

A.1.2.2 Carrier-phase Observation Equation

The carrier-phase observation equation is similar to that of the code pseudorange (cf. Equation A-3) with the exception of different size multipath and receiver noise biases, and a term relating to the integer ambiguity as can be seen in Equation A-4.

\[ \Phi_A' = \rho_A' + d\phi_A' + c\tau_A' - c\tau_i' - I_A' + Z_A' + m_{pA}' + B_A' + e_A' \]  

Equation A-4

where

- \( m_{pA} \): multipath bias on the carrier-phase
- \( B \): carrier-phase bias consisting of:
• an integer carrier-phase ambiguity $N$
• instrument biases at satellite and receiver

e: carrier-phase measurement noise

All parameters in Equations A-3 and A-4 have been defined in metres and with respect to GPS Time. Note that the ionospheric delay $I$ for carrier-phase observations is the same magnitude as for the pseudorange ionospheric delay but with an opposite sign; this is because the phase is advanced as the GPS signal passes through the ionosphere [Leick, (1995)].

A.1.3 GPS Bibliography

There are a number of comprehensive sources of information on the Global Positioning System and its architecture. The most official is the '1999 Federal Radionavigation Plan' (FRP) document as written by the joint owners of GPS, the United States Departments of Transport and Defense, [US DoT and DoD, (1999)]. Less formal reading can be found in Ackroyd and Lorimer [1990] or the Geographer’s Crafts Project website at the University of Colorado at Boulder [Dana, (2000)]. At the time of writing, a more recent FRP document has not yet been released [Casswell, (2000)]. Messages regarding the activity status of the GPS space and control segments can be found on the United States Coast Guard Navigation Center website [USCG, (2000)], and the United States Navy website [US Navy, (2000)].
Appendix B – Pre-processed Estimates of GPS Receiver Clocks

**Figure B-1** Estimates of the GPS Receiver Clock for Station BEDS as determined from the Single Point Position Computations during Pre-processing (Ashtech Z-FX)

**Figure B-2** Estimates of the GPS Receiver Clock for Station DELF as determined from the Single Point Position Computations during Pre-processing (Trimble 4000SSe)
Appendix B – Pre-processed Estimates of GPS Receiver Clocks

Figure B-3  Estimates of the GPS Receiver Clock for Station HANN as determined from the Single Point Position Computations during Pre-processing (Ashtech Z-XII)

Figure B-4  Estimates of the GPS Receiver Clock for Station LOND as determined from the Single Point Position Computations during Pre-processing (Leica MC1000)
Appendix B – Pre-processed Estimates of GPS Receiver Clocks

Figure B-5 Estimates of the GPS Receiver Clock for Station DELF as determined from the Single Point Position Computations during Pre-processing (Ashtech Z-XII)

Figure B-6 Estimates of the GPS Receiver Clock for Station PENC as determined from the Single Point Position Computations during Pre-processing (Trimble 4000SSe)
<table>
<thead>
<tr>
<th>CANDIDATE MODEL</th>
<th>MEAN TRUE ERROR (m)</th>
<th>MEAN FORMAL ERROR (95%) (m)</th>
<th>MEAN 95% PCA (%)</th>
<th>MEAN UNIT VARIANCE</th>
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<td>North  East Height Vector</td>
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### Figure C.2: Summary of Results for the 397 Kilometre Single-Baseline BEIDS-LONDON

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<th>CANDIDATE MODEL</th>
<th>MEAN TRUE ERROR (m)</th>
<th>MEAN FORMAL ERROR (95%) (m)</th>
<th>MEAN 95% PCA (%)</th>
<th>MEAN UNIT VARIANCE</th>
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<td>East</td>
<td>Height</td>
<td>Vector</td>
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<td></td>
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<tr>
<td>OH_scaled_elev</td>
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<td>0.15</td>
<td>1.21</td>
<td>1.43</td>
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<tr>
<td>OH_vars_CMV_no_corr</td>
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<td>0.163</td>
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<td>1.722</td>
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<tr>
<td>OH_vars_CMV_corr-1</td>
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<td>0.163</td>
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<td>1.723</td>
</tr>
<tr>
<td>OH_vars_CMV_corr-5</td>
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<td>1.723</td>
</tr>
<tr>
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<td>0.166</td>
<td>1.364</td>
<td>1.724</td>
</tr>
<tr>
<td>OH_vars_AMV_no_corr</td>
<td>1.070</td>
<td>0.186</td>
<td>1.374</td>
<td>1.751</td>
</tr>
<tr>
<td>OH_vars_AMV_corr-1</td>
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<td>0.188</td>
<td>1.373</td>
<td>1.751</td>
</tr>
<tr>
<td>OH_vars_AMV_corr-5</td>
<td>1.074</td>
<td>0.195</td>
<td>1.370</td>
<td>1.752</td>
</tr>
<tr>
<td>OH_vars_AMV_corr-9</td>
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<td>0.205</td>
<td>1.368</td>
<td>1.753</td>
</tr>
<tr>
<td>OH_vars_TMV_no_corr</td>
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<td>0.186</td>
<td>1.374</td>
<td>1.751</td>
</tr>
<tr>
<td>OH_vars_TMV_corr-1</td>
<td>1.071</td>
<td>0.188</td>
<td>1.373</td>
<td>1.751</td>
</tr>
<tr>
<td>OH_vars_TMV_corr-5</td>
<td>1.074</td>
<td>0.195</td>
<td>1.370</td>
<td>1.752</td>
</tr>
<tr>
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<td>0.206</td>
<td>1.368</td>
<td>1.753</td>
</tr>
<tr>
<td>OH_vars_ATMV_no_corr</td>
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<td>0.195</td>
<td>1.370</td>
<td>1.748</td>
</tr>
<tr>
<td>OH_vars_ATMV_corr-1</td>
<td>1.070</td>
<td>0.187</td>
<td>1.369</td>
<td>1.748</td>
</tr>
<tr>
<td>OH_vars_ATMV_corr-5</td>
<td>1.073</td>
<td>0.194</td>
<td>1.366</td>
<td>1.748</td>
</tr>
<tr>
<td>OH_vars_ATMV_corr-9</td>
<td>1.076</td>
<td>0.203</td>
<td>1.364</td>
<td>1.749</td>
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<td>0.154</td>
<td>1.214</td>
<td>1.438</td>
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<td>0.154</td>
<td>1.208</td>
<td>1.430</td>
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<td>1.409</td>
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<td>1.461</td>
</tr>
<tr>
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<td>0.778</td>
<td>0.150</td>
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<td>1.441</td>
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<tr>
<td>OH_vcv_AMV_corr-1</td>
<td>0.802</td>
<td>0.147</td>
<td>1.049</td>
<td>1.329</td>
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<tr>
<td>OH_vcv_AMV_corr-5</td>
<td>0.760</td>
<td>0.153</td>
<td>1.222</td>
<td>1.447</td>
</tr>
<tr>
<td>OH_vcv_ATMV_no_corr</td>
<td>0.760</td>
<td>0.153</td>
<td>1.193</td>
<td>1.423</td>
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<tr>
<td>OH_vcv_ATMV_corr-1</td>
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<td>1.003</td>
<td>1.273</td>
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<tr>
<td>OH_vcv_ATMV_corr-5</td>
<td>0.769</td>
<td>0.153</td>
<td>1.005</td>
<td>1.275</td>
</tr>
</tbody>
</table>
CANDIDATE
MODEL
00

MEAN TRUE ERROR (m)
North

c/3

!

0.454

i

0. 249

0.521

0 0 s c a le d 1

0.4 54

i

0.2 49

0.521

i

0. 204

0 0 s c a le d elev

0.3 34

:

0.204

0.3 92

0 0 vars CMV no corr

0.4 44

;

0. 232

0.479

0 0 vars CMV corrO-1

0.4 44

:

0.232

0.479

0 0 vars CMV corrO-5

0 .4 45

;

0.233

0.479

0 0 vars CMV corrO-9

0.4 46

:

0. 234

0.479

0 0 vars AMV no corr

0.4 50

i

0. 242

0.5 04

0.735
i 0.735
0.554
; 0.554
; 0.693
: 0.693
; 0.694
: 0.695
; 0.718
; 0.718
: 0.721
: 0.724
: 0.718
: 0.718
: 0.721
: 0.724
1 0.718
0.718
0.721
; 0.723
0.555
0.554
0.553

0 0 vars AMV corrO-1

0.4 50

i

0.243

0.5 04

0 0 _ v a rs_AMV_c orrO-5

0.4 52

:

0.2 45

0.505

0 0 vars AMV corrO-9

0 .4 54

i

0.247

0 .5 07

0 0 vars TMV no corr

0.4 50

i

0. 24 2

0.5 04

0 0 vars TMV corrO-1

0.4 50

i

0. 24 3

0.5 04

0O_vars_TMV_corrO-5

0.4 52

i

0. 24 5

0.5 06

0 0 vars TMV corrO-9

0.4 54

;

0. 247

0.507

0 0 vars ATMV no corr

0 .4 50

i

0. 24 2

0.5 04

0 0 vars ATMV corrO-1

0 .4 50

i

0.243

0.504

0 0 vars ATMV corrO-5

0.4 52

i

0.245

0.505

0 0 vars ATMV corrO-9

0.4 53

;

0.247

0.5 07

0 0 vcv CMV no corr

0.3 35

i

0. 204

0. 392

0 0 vcv CMV corrO-1

0.3 35

1

0.204

0.391

0 0 vcv CMV corrO-5

0.3 36

0.2 04

0.389

BQ vcv CMI/ conO-9

—

i
;

0 0 vcv AMV no corr

0.3 45

i

0.206

0. 392

i

0 0 vcv AMV corrO-1

0.3 48

;

0. 207

0.3 86

i

0 0 vcv AMV corrO-5

0.388

:

0.211

0.3 82

;

0 0 vcv TMV no corr

0.3 42

1

0. 205

0.3 94

;

ï

0 0 vcv TMV corrO-1

0.3 45

;

0. 205

0.389

:

0 0 vcv TMV corrO-5

0.3 82

i

0. 209

0.3 99

;

§
>

North

East

; Heiqht i

0.861

;

2 .495

:

1.092

0.861

i

2.4 95

:

0. 929

0.626

i

1.7 67

:

0.929

0.626

:

1. 767

■

2.350

:

1.092

;

1.055
1.055

0.807

;

2. 357

;

1.058

0.812

i

2.386

:

0. 817

:

2.416

;

0.843

:

2. 449

;

0. 845

;

2.4 59

i

0. 853

;

2.501

i

0.862

:

2. 54 6

:

1.061

;

1. 080
1.081
1. 086

;

1. 092
1. 080

;

1.081

0.843

2.449

0.8 45

i

2.4 59

;

1. 086

0.8 53

I

2.5 02

;

1.092

0.862

;

2. 547

i

1. 080

0.843

;

2. 449

i

2. 459

;

:

2.5 02

:

1.081

^

0.845

1.086

:

0. 853

1.092

:

0. 862

;

2.5 47

;

0. 929

.

0.627

:

1.7 73

:

0.6 28

;

1.7 95

:

0.631

;

1. 889

i

1.8 17

:

0. 930
0. 932

;

Vector

MEAN 95% PCA (%)
Heiqht i

Plan

4.5

3.4

4.5

3.4

4.0

3.9

2.092

4.0

3.9

2.705
2.733
2.762
2.806
2.816
2.857
2.901
2.806
2.816
2.858
2.902
2.806
2.816
2.858
2.902
2.098
2.117
2.199

4.8

4.2

4.8

4.2

4.8

4.2

4.5

3.8

4.6

3.8

4.7

3.6

4.8

3.5

4.5

3.8

4.6

3.8

4.7

3.6

4.8

3.5

4.5

3.8

4.6

3.8

4.7

3.6

2.856

4.8

3.5

4.0

4.0

4.1

3.9

4.3

3.9

2.144
2.256
3.139

4.4

4.4

4.6

Vector

0.561
0.559
0.584
0.561
0.559
0.591

MEAN UNIT
VARIANCE
0 .2830

4.9
4.4
4.4

0 .0943

5.0
5.0
5.0
4.9
4.9
4.9
5.0
4.9
4.9
4.9
5.0
4.9
4.9
4.9
5.0
4.5
4.5
4.6

0.0 0 0 2

0.0411
0 .0137

0.643

0. 944

0. 649

:

1.943

;

2.8 94

i

0. 984

:

0.713

:

0.931

!

0.631

i

0. 964

;

0.689

:

2. 982

;

2.122
2.236
3.209

0.931

;

0.631

;

1.7 98

!

2.121

0. 934

0.636

1.799

:

1.929

:

0 .0002
0 .0 0 0 3
0 .0 0 0 3
0 .0 0 0 3
0 .0003
0 .0003
0 .0003
0 .0003
0 .0003
0 .0003
0 .0003
0 .0003
0 .0003

0.3 42

i

0. 205

0.3 93

;

0 0 vcv ATMV corrO-1

0.3 45

:

0.205

0.389

i

0 0 vcv ATMV corrO-5

0.381

i

0.208

0 .3 95

;

8 0 vcv

coirO-9

0.560
0.559
0.587

0.934
0.964

0.690

i

2. 97 5

i

3.203

q

is
I
t
M

0 .0 0 1 3
0 .0014

0 .0069

4.4

0 .0 0 8 0

I

5.0

4.6

5.0

0 .0208

0
1/5

4.2

3.9
3.9

5.0

4.2

4.5
4.7
5.0

0 .0 0 7 3

4.4

4.2

4.0
4.1

5.0

4.2

4.5
4.7
5.0

0 .0073

4.4

0 .0 0 1 5

0 .0 0 8 6
0 .0254

8 0 vc v 7MI/ co/rO-9
0 0 vcv ATMV no corr

n

0 .0 0 0 2

-

0. 940

t

4.6
5.0

—

—

%
CL

C/5

MEAN FORMAL ERROR (95%) (m)

Vector

;

BO vcv AMV corr{>-9

I

U)
t

Heiqht i

default 1

0 0 default elev

i

East

0 .0085
0 .0253

hj

1
I


### Figure C-5: Summary of Results for the 1164 Kilometer Single-Baseline DEL-FENC

**CANDIDATE** | **MEAN TRUE ERROR (m)** | **MEAN FORMAL ERROR (95%) (m)** | **MEAN 95% PCA (%)** | **MEAN UNIT**
--- | --- | --- | --- | ---
DP default | 0.647 | 0.488 | 1.133 | 0.511
DP default, no corr | 0.511 | 0.488 | 1.133 | 0.511

**MODEL** | **North East** | **Height** | **Vector**
--- | --- | --- | ---
North | East | Height |
--- | --- | --- | ---
North East | 0.133 | 0.487 | 1.133 | 0.481
North East | 0.133 | 0.487 | 1.133 | 0.481

**CANDIDATE** | **MEAN UNIT**
--- | ---
DP default | 0.848 | 0.606
DP default, no corr | 0.848 | 0.606

**PLAN** | **VARIANCE**
--- | ---
North East | 0.848 | 0.606
North East | 0.848 | 0.606

**PLAN** | **VARIANCE**
--- | ---
North East | 0.848 | 0.606
North East | 0.848 | 0.606

**PLAN** | **VARIANCE**
--- | ---
North East | 0.848 | 0.606
North East | 0.848 | 0.606
<table>
<thead>
<tr>
<th>CANDIDATE</th>
<th>MEAN TRUE ERROR (m)</th>
<th>MEAN FORMAL ERROR (95%) (m)</th>
<th>MEAN 95% PCA (%)</th>
<th>MEAN UNIT VARIANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MODEL</td>
<td>North</td>
<td>East</td>
<td>Height</td>
<td>Vector</td>
</tr>
<tr>
<td>BLH-DELF_default</td>
<td>0.224</td>
<td>0.149</td>
<td>0.489</td>
<td>0.057</td>
</tr>
<tr>
<td>BLH-DELF_default</td>
<td>0.298</td>
<td>0.165</td>
<td>0.537</td>
<td>0.035</td>
</tr>
<tr>
<td>BLH-DELF_var_CMV_no_corr</td>
<td>0.202</td>
<td>0.149</td>
<td>0.512</td>
<td>0.050</td>
</tr>
<tr>
<td>BLH-DELF_var_CMV_corr-0.1</td>
<td>0.202</td>
<td>0.149</td>
<td>0.512</td>
<td>0.050</td>
</tr>
<tr>
<td>BLH-DELF_var_CMV_corr-0.5</td>
<td>0.202</td>
<td>0.149</td>
<td>0.513</td>
<td>0.051</td>
</tr>
<tr>
<td>BLH-DELF_var_CMV_corr-0.9</td>
<td>0.202</td>
<td>0.150</td>
<td>0.514</td>
<td>0.052</td>
</tr>
<tr>
<td>BLH-DELF_var_AMV_no_corr</td>
<td>0.143</td>
<td>0.134</td>
<td>0.593</td>
<td>0.093</td>
</tr>
<tr>
<td>BLH-DELF_var_AMV_corr-0.1</td>
<td>0.143</td>
<td>0.134</td>
<td>0.593</td>
<td>0.093</td>
</tr>
<tr>
<td>BLH-DELF_var_AMV_corr-0.5</td>
<td>0.144</td>
<td>0.135</td>
<td>0.592</td>
<td>0.096</td>
</tr>
<tr>
<td>BLH-DELF_var_AMV_corr-0.9</td>
<td>0.144</td>
<td>0.136</td>
<td>0.594</td>
<td>0.098</td>
</tr>
<tr>
<td>BLH-DELF_var_TMV_no_corr</td>
<td>0.166</td>
<td>0.130</td>
<td>0.546</td>
<td>0.080</td>
</tr>
<tr>
<td>BLH-DELF_var_TMV_corr-0.1</td>
<td>0.166</td>
<td>0.130</td>
<td>0.547</td>
<td>0.080</td>
</tr>
<tr>
<td>BLH-DELF_var_TMV_corr-0.5</td>
<td>0.166</td>
<td>0.130</td>
<td>0.548</td>
<td>0.080</td>
</tr>
<tr>
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<td>0.131</td>
<td>0.550</td>
<td>0.090</td>
</tr>
<tr>
<td>BLH-DELF_var_AMV_corr-0.1</td>
<td>0.166</td>
<td>0.130</td>
<td>0.546</td>
<td>0.085</td>
</tr>
<tr>
<td>BLH-DELF_var_AMV_corr-0.5</td>
<td>0.166</td>
<td>0.130</td>
<td>0.547</td>
<td>0.085</td>
</tr>
<tr>
<td>BLH-DELF_var_AMV_corr-0.9</td>
<td>0.166</td>
<td>0.130</td>
<td>0.548</td>
<td>0.087</td>
</tr>
<tr>
<td>BLH-DELF_vcv_CMV_no_corr</td>
<td>0.148</td>
<td>0.119</td>
<td>0.569</td>
<td>0.600</td>
</tr>
<tr>
<td>BLH-DELF_vcv_CMV_corr-1</td>
<td>0.148</td>
<td>0.119</td>
<td>0.570</td>
<td>0.601</td>
</tr>
<tr>
<td>BLH-DELF_vcv_AMV_no_corr</td>
<td>0.272</td>
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<td>0.718</td>
<td>0.772</td>
</tr>
<tr>
<td>BLH-DELF_vcv_AMV_corr-1</td>
<td>0.303</td>
<td>0.084</td>
<td>0.795</td>
<td>0.855</td>
</tr>
<tr>
<td>BLH-DELF_vcv_TMV_no_corr</td>
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<td>0.080</td>
<td>0.685</td>
<td>0.742</td>
</tr>
<tr>
<td>BLH-DELF_vcv_TMV_corr-1</td>
<td>0.312</td>
<td>0.088</td>
<td>0.779</td>
<td>0.844</td>
</tr>
<tr>
<td>BLH-DELF_vcv_AMV_no_corr</td>
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<td>0.080</td>
<td>0.689</td>
<td>0.744</td>
</tr>
<tr>
<td>BLH-DELF_vcv_AMV_corr-0.1</td>
<td>0.311</td>
<td>0.088</td>
<td>0.780</td>
<td>0.844</td>
</tr>
<tr>
<td>CANDIDATE MODEL</td>
<td>MEAN TRUE ERROR (m)</td>
<td>MEAN FORMAL ERROR (95%) (m)</td>
<td>MEAN 95% PCA (%)</td>
<td>MEAN UNIT VARIANCE</td>
</tr>
<tr>
<td>-----------------</td>
<td>---------------------</td>
<td>-----------------------------</td>
<td>------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td></td>
<td>North</td>
<td>East</td>
<td>Height</td>
<td>Vector</td>
</tr>
<tr>
<td>OLP-HANN_default</td>
<td>0.408</td>
<td>0.236</td>
<td>0.959</td>
<td>1.059</td>
</tr>
<tr>
<td>OLP-HANN_default_elev</td>
<td>0.544</td>
<td>0.329</td>
<td>1.242</td>
<td>1.395</td>
</tr>
<tr>
<td>OLP-HANN_vars_CMV_no_corr</td>
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<td>0.201</td>
<td>0.925</td>
<td>1.019</td>
</tr>
<tr>
<td>OLP-HANN_vars_CMV_corr-0.1</td>
<td>0.378</td>
<td>0.201</td>
<td>0.924</td>
<td>1.018</td>
</tr>
<tr>
<td>OLP-HANN_vars_CMV_corr-0.5</td>
<td>0.379</td>
<td>0.201</td>
<td>0.924</td>
<td>1.018</td>
</tr>
<tr>
<td>OLP-HANN_vars_CMV_corr-0.9</td>
<td>0.380</td>
<td>0.201</td>
<td>0.924</td>
<td>1.018</td>
</tr>
<tr>
<td>OLP-HANN_vars_AMV_no_corr</td>
<td>0.261</td>
<td>0.136</td>
<td>0.834</td>
<td>0.884</td>
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<tr>
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Appendix D – Summary of Time Series Plots for the Single-Baseline LOND-DELF (316 kilometres)
LOND-DELF: True and Formal Error Vectors afforded by a User-defined Variance Model with AMV and No Mutual At-receiver Correlations

LOND-DELF: True and Formal Error Vectors afforded by a User-defined Variance Model with CMV and Mutual At-receiver Correlations at 10%

LOND-DELF: True and Formal Error Vectors afforded by a User-defined Variance Model with CMV and Mutual At-receiver Correlations at 50%

LOND-DELF: True and Formal Error Vectors afforded by a User-defined Variance Model with CMV and Mutual At-receiver Correlations at 90%
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(316 kilometres)
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(316 kilometres)
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(316 kilometres)
LOND-DELF: True and Formal Error Vectors afforded by a Variance-covariance Model with ATMV and No Mutual At-receiver Correlations

Epochs (seconds)

True Error — Formal Error

LOND-DELF: True and Formal Error Vectors afforded by a Variance-covariance Model with ATMV and Mutual At-receiver Correlations at 10%

Epochs (seconds)

True Error — Formal Error

10 kilometers
Appendix E – Summary of Time Series Plots for the Single-Baseline BEDS-LOND (397 kilometres)
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(397 kilometres)
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(397 kilometres)
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(397 kilometres)
Appendix F – Summary of Time Series Plots for the Single-Baseline ONSA-HANN (575 kilometres)
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(575 kilometres)
Appendix G – Summary of Time Series Plots for the Single-Baseline BEDS-ONSA (881 kilometres)
Appendix G – Summary of Time Series Plots for the Single-Baseline BEDS-ONSA

(881 kilometres)
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(881 kilometres)
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(881 kilometres)
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Appendix G – Summary of Time Series Plots for the Single-Baseline BEDS-ONSA

(881 kilometres)
Appendix H – Summary of Time Series Plots for the Single-Baseline DELF-PENC (1164 kilometres)
Appendix H – Summary of Time Series Plots for the Single-Baseline DELF-PENC

(1164 kilometres)
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(1164 kilometres)
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(1164 kilometres)
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(1164 kilometres)
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Appendix I – Summary of Time Series Plots for the Multiple-Baseline Network BLH-DELF
Appendix I – Summary of Time Series Plots for the Multiple-Baseline Network

BLH-DELF
Appendix I – Summary of Time Series Plots for the Multiple-Baseline Network

BLH-DELF
Appendix J – Summary of Time Series Plots for the Multiple-Baseline Network OLP-HANN
Appendix J – Summary of Time Series Plots for the Multiple-Baseline Network

OLP-HANN

![Graphs showing time series plots for OLP-HANN model with different error variances and correlations.](image-url)
Appendix J – Summary of Time Series Plots for the Multiple-Baseline Network

OLP-HANN

![Graphs representing time series plots for the Multiple-Baseline Network with OLP-HANN model.](image-url)
Appendix K – Summary Tables of Chi-squared Test Statistics from All Long-Range Positioning Studies

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<th>CANDIDATE CHI-SQUARED TEST STATISTICS</th>
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<th>Height</th>
<th>Plan</th>
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Figure K-1 Summary Table of Chi-squared Test Statistics for the 316 kilometre Single-Baseline LOND-DELF
## Appendix K – Summary Tables of Chi-squared Test Statistics from All Long-Range Positioning Studies

### CANDIDATE CHI-SQUARED TEST STATISTICS

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**Figure K-2** Summary Table of Chi-squared Test Statistics for the 397 kilometre Single-Baseline BEDS-LOND
### Appendix K – Summary Tables of Chi-squared Test Statistics from All Long-Range Positioning Studies

#### Figure K-3 Summary Table of Chi-squared Test Statistics for the 575 kilometre Single-Baseline ONSA-HANN

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Figure K-3 Summary Table of Chi-squared Test Statistics for the 575 kilometre Single-Baseline ONSA-HANN
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Figure K-4 Summary Table of Chi-squared Test Statistics for the 881 kilometre Single-Baseline BEDS-ONSA

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### Appendix K – Summary Tables of Chi-squared Test Statistics from All Long-Range Positioning Studies

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**Figure K-5** Summary Table of Chi-squared Test Statistics for the 1164 kilometre Single-Baseline DELF-PENC
### Appendix K – Summary Tables of Chi-squared Test Statistics from All Long-Range Positioning Studies

#### Figure K-6 Summary Table of Chi-squared Test Statistics for the Multiple-Baseline Network BEDS+LOND+HANN-DELF (Mean Baseline Length ~401 kilometres)

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Appendix K – Summary Tables of Chi-squared Test Statistics from All Long-Range Positioning Studies

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**Figure K-7** Summary Table of Chi-squared Test Statistics for the Multiple-Baseline Network ONSA+LOND+PENC-HANN (Mean Baseline Length ~704 kilometres)