Visible Maths Pedagogy Project:

'Visible pedagogies and equitable outcomes in school mathematics'

Report from Year 2 (October 2018 to July 2019)

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1. About the project

1.1. Context:

The Visible Maths Pedagogy Project continued into the second year in Stoke Newington School with the same team of researchers, Pete, Alba and Tiago (see VMP project report year 1 section 1.1).

The mathematics department extended the use of mixed attainment groupings in mathematics classes into Year 8 in 2018-19, with plans to extend this to 9 in the following year. However, due to pressure from parents, a strategic decision was taken to have a top set in Year 8 with all other groups 'mixed attainment' (Year 7 was fully mixed attainment). The department continued to meet regularly to collaboratively plan lessons. The two teacher researchers (TRs), Alba and Tiago, focused on their Year 8 classes for the research project. There was some restructuring of the mixed attainment groups between Year 7 and Year 8, therefore the two Year 8 groups involved in the study contained some (but not all) students who were involved in the project tin the previous year.

1.2. Aims:

The focus of the project was the same as in the previous year, i.e. to develop strategies teachers can use to make their pedagogy more visible, and help all students recognise their intentions as teachers, when adopting progressive approaches to teaching secondary mathematics. In so doing, the project aimed to explore strategies that may be effective in reducing the large gaps in mathematics achievement existing between children from different socio-economic groups.

The research questions were:

- Which teaching strategies are successful in helping students develop their ability to decipher the recognition and realisation rules of the mathematics classroom?
- What impact do these strategies have on students' mathematical achievement and engagement, particularly for those from disadvantaged backgrounds?

The project remained small-scale however a small grant had been secured from 'Let Teachers Shine' (£3000) to enable the project to continue into its second year. This grant was limited to funding the professional development aspect of the project rather than the research. The project continued to develop and refine research tools tried out in the first year with a view to generating a model that could be used by other academic and teacher researchers to transform mathematics classroom practice in other contexts. These research tools included maintaining research journals, conducting peer observations, surveys and interviews of students and using video recordings of lessons to stimulate a critical reflection on practice (video-stimulated reflection).

1.3. Background:

See VMP project report year 1 section 1.3

1.4. Methodological approach:

We used a similar methodological approach to year 1 (see VMP project report year 1 section 1.4).

1.5. Research group meetings held during 2018-19

There was an initial meeting held on 12th October 2018, which focused on reflecting on the development of the project in 2017-18 and looking ahead to 2018-19. We agreed that Pete would identify literature on video-stimulated recall to be read and presented at the next meeting. We decided to work with Alba and Tiago's Year 8 classes and to identify 3 students in each class (form those identified as Pupil Premium) to be the target group that would be interviewed individually following the research lessons.

The following eight research group meetings (each approximately one hour in duration and held in the school – except where indicated otherwise) took place during the second year of the project (note they have been numbered from 8 to 15 to avoid confusion with research group meetings during year 1):

1.5.1. Meeting 8 (22nd November 2018)

This was a half-day meeting held at UCL Institute of Education during which researchers each presented a paper for discussion (chosen by Pete) on the use of videos to stimulate reflection: Coles (2013), Geiger et al. (2016), Sherin & Dyer (2017) and Hollingsworth & Clarke (2017) – see Section 1.9 below. Pete also led a professional development session on video-stimulated reflection (VSR), critical reflection on practice and collecting feedback through surveys. The follow-up discussion led to the establishing of a protocol to be used for VSR:

- Video two Year 8 lessons (one for Tiago, one for Alba) in which strategies are tried out (camera placed at back of room).
- Tiago/Alba to peer observe each other's lessons peer observer to start video recording and generate a timeline of key events.
- Tiago and Alba to review both videos independently in the week following lessons;
- Tiago to select two short clips to facilitate discussion of strategies, one from his own lesson and one from Alba's.
- Similarly, Alba to select two short clips to facilitate discussion of strategies, one from her own lesson and one from Tiago's.
- Meeting to start with Tiago and Alba introducing the two clips they have chosen.
- Pete to facilitate critically reflective discussion focusing on evaluation of strategies.
- Use timelines to identify other parts of the lesson that Pete/Tiago/Alba think might be of interest in evaluating strategies.
- The protocol is intended to provide focus whilst maintaining flexibility.

Pete also presented initial findings of the analysis of the findings from year 1 of the project for further discussion during the meeting.

1.5.2. Meeting 9 (5th December 2018)

The researchers discussed the progressive pedagogies (and their rationale) to be used during the videoed research lessons on 11th and 12th December as part of the 3rd action research cycle. We planned the strategies to be used alongside the rich teaching activities to make the pedagogy more visible. We finalised the format, questions and timings for the surveys and interviews. It was decided to administer the surveys before the research lessons on 11th December.

1.5.3. Meeting 10 (16th January 2019)

The researchers used the VSR protocol to reflect on and evaluate the research lesson.

1.5.4. Meeting 11 (11th March 2019)

This was a half-day meeting held at UCL Institute of Education during which TRs each presented a paper for discussion (chosen by Pete) on teachers critically reflecting on, and transforming, their practice (and how academics can support them in doing this): Liu (2015), Hand (2012) – see Section 1.9 below. The researchers then discussed the implications of the papers for the design of the project and the development of the research tools, i.e. research journals, conducting surveys and interviews, and using videos to stimulate reflection. Pete led a brief discussion on how Nvivo was used in the data analysis of surveys and interviews in year 1 of the project.

1.5.5. Meeting 12 (20th March 2019)

The researchers discussed the progressive pedagogies (and their rationale) to be used during the videoed research lessons on 26th and 27th March as part of the 4th action research cycle. We planned the strategies to be used alongside the rich teaching activities to make the pedagogy more visible. We finalised the format, questions and timings for the surveys and interviews. It was decided that the interviews would be completed as soon as possible after the research lesson but the surveys (which were the same as for cycle 3) would be completed after the meeting on 3rd April.

1.5.6. Meeting 13 (3rd April 2019)

The researchers used the VSR protocol to reflect on and evaluate the research lesson.

1.5.7. Meeting 14 (1st May 2019)

The researchers reflected on the methods and research tools used in cycle 4.

1.5.8. Meeting 15 (8th July 2019)

This was a half-day meeting held at UCL Institute of Education. The researchers reflected on the project as a whole and reviewed the findings from year 1. We planned how to disseminate the findings through publications and conferences/seminars. We also considered how we might take the work of the project forward in future years.

1.6. The strategies

There were three strategies tried out during the second year of the project with the aim of making progressive pedagogies more visible to learners:

1.6.1. The 'model solution' strategy

Students have a go at a problem on factorising/substitution, on their own at first, and present their solutions to others. The teacher then facilitates a whole class discussion to agree a model solution and the reasons that make this a good solution. The model solution is then copied down by students and used for reference in solving a series of similar problems and presenting 'good' solutions. The rationale is to encourage students to share ideas with each other (and hence learn from each other), appreciate multiple ways of solving a problem, compare the merits of different methods (recognising differences and similarities), develop students' agency in negotiating a model solution, develop skills in communicating mathematical reasoning, develop students' independence in solving problems.

1.6.2. The 'boxing up' strategy

The teacher gave students a series of generic prompt questions (in boxes printed on laminated card – shown in Figure 1 below) to use when solving a series of probability problems. For this lesson, the focus was on two questions (in the green box): 'What is the question asking me? What information do I already have?' The teacher facilitated an early discussion with students on the reasons for using the 'boxing up' questions using the following questions: 'Why is this useful? What does this question allow you to do?' This was followed up later with the 'card sort' strategy (see 1.6.3 below) – with statements tailored to the 'boxing up' strategy. The rationale for the 'boxing up' strategy was to develop metacognitive skills, i.e. provide support in decoding the question, encourage students to spend time planning rather than jumping in, provide a structure that students can apply to solve other problems.



Figure 1: The 'boxing up' questions

Acknowledgment:

Note this strategy draws on the work of Zeb Friedman (who devised the 'Talk for writing' strategy) and Helen Hindle (who developed its use in the classroom) – further details can be found on the following website: <u>https://www.growthmindsetmaths.com/talk-for-writing-boxing-up.html</u>

1.6.3. The 'card sort' strategy

Students are given a card sort with a series of statements representing possible intentions of the teacher. Some statements represent a primary purpose for using a particular teaching approach, others are valid reasons but not identified as primary by the teacher, and others are invalid reasons. Students are asked to discuss in groups and arrange the cards in order, with the cards at the top that best describe the reasons why the teacher chose to use a particular teaching approach. The card sort can be used with any teaching approach. In this case, it was used together with the 'model solution' strategy in cycle 3 and with the 'Boxing up' strategy in cycle 4. The rationale for the card sort is to prompt a discussion about, and make explicit, the intentions of the teacher.

For the 'model solution' strategy, the statements in the 'card sort' were tailored directly to the rationale for using the strategy. Students were given the following instructions/questions and provided with the set of statements shown in Table 1 below:

Why do you think your teacher asked you to do the following?

- Get the class to come up with a model solution to the first problem.
- Copy down the model solution and use it to solve the other problems.

Cut out the pieces below and choose the ones that you think most closely match the teacher's reasons. Arrange them in order with the statements you think most closely reflect the teacher's reasons at the top.

A) So you can compare the advantages and disadvantages of different methods.	B) So you can see where you went wrong and correct your work.
C) So that you learn how to work well with other students in a group.	D) So you can create your own resources that will help you solve future problems on your own.
E) So you can work through all the problems more quickly.	F) So you can share your ideas with other students (and hence learn from each other).
G) So you are able to discuss and agree on the best solution.	 H) So you can recognise similarities and differences between different problems.
J) So that you focus on your work and are not distracted by others.	K) So you can learn better how to communicate your ideas to other students.
L) So you can see how useful it is to record a model solution to use for future reference.	M) So you appreciate that there may be different ways of solving a problem.

Table 1: Statements used in card sort related to 'model solution' strategy

Note statements A, F, K, M above were considered primary purposes for getting the class to come up with a model solution to the first problem, statements D, H, L were considered primary purposes for copying down the model solution and using it to solve the other problems, B, C were considered as other potentially valid purposes for using the 'model solution' strategy, and E, J were considered as invalid purposes.

For the 'boxing up' strategy, the research group decided to reduce the number of statements and produce statements in the 'card sort' that might be more easily adapted to other teaching approaches/strategies. Students were given the following instructions and provided with the set of statements shown in Table 2 below:

- Why do you think your teacher asked you to think about the questions in the green box?
- Cut out the pieces below and choose the ones you think most closely match the teacher's reasons.
- Arrange them in order with those you think most closely reflect the teacher's reasons at the top.

A) So I can work through all the	B) So I can make a plan to help me
problems more quickly.	to solve a problem.
C) So I can share my ideas with	D) So I can focus on my work without being
other students.	distracted by others.
E) So I can recognise similarities and differences	F) So I can identify the key information
between problems.	in the question.

Table 2: Statements used in card sort related to 'boxing up' strategy

Note statements B and F above were considered primary purposes for using the 'boxing up' strategy, C, E were considered as other potentially valid purposes, and A, D were considered as invalid purposes.

1.7. Survey and interview schedules

The following questions were agreed by the researchers for use in the surveys (administered before the 3rd research lesson at the start of cycle 3 and after the 4th research lesson at the end of cycle 4) and in the interviews (conducted in the days immediately following the 3rd and 4th research lessons during cycles 3/4).

1.7.1. Survey questions

Students were advised to read the introductory paragraphs of the survey before completing it. These outlined the aims of the research project, how the survey responses would be anonymised and used in the research, and that students could leave the survey blank if they did not wish to take part. Note the first question was purely for the purpose of assigning pseudonyms if this was felt necessary. The second and third questions (1 and 2 on the survey) were the same as for the survey administered during cycle 1. The fourth and fifth questions (3 and 4 on the survey) were changed for cycles 3 and 4 to enable a better comparison to be made between the start and end of year 2 of the project and to focus on the recognition and realisation rules of the classroom (rather than the strategies).

- 'Are you male or female (tick box)'?
- 'How successful do you think you are in maths in general?' (students answer on scale of 1 to 5).
- 'How do you know?' (provide space to explain).
- 'What do you think you can do to be more successful in maths?'
- 'What does your teacher do to help you to be successful in maths?'

1.7.2. Interview questions

Before beginning the interviews, TRs first checked with students that they have had a chance to read the information leaflet. They were then given the opportunity to ask any questions they liked before starting. The following questions were designed as initial prompts to facilitate discussion between TRs and interviewees. In accordance with 'empathetic' interviewing, TRs were encouraged to ask follow-up questions to encourage students to expand on their ideas, e.g. 'That's interesting, tell me more about ...'

We agreed the following initial questions to be asked after research lesson 3 (note that the third and fourth questions relate directly to the rationale for using the 'model solution' and 'card sort' strategies):

- 1. Did you enjoy today's/yesterday's lesson? Why?
- 2. Did you notice anything different about today's lesson?
- 3. Why do you think I got the class to come up with a model solution to the first problem?
- 4. Why do you think I asked you to copy down the model solution and use it to solve the other problems?
- 5. How well did you do in today's lesson? How do you know?

We agreed the following initial questions to be asked after research lesson 4 (note that the third, fourth and fifth questions relate directly to the rationale for using the 'boxing up' and 'card sort' strategies):

- 1. Did you enjoy today's/yesterday's lesson? Why?
- 2. Did you notice anything different about today's lesson?
- 3. Why do you think I asked you to think about the questions in the green box? [note – have a copy of the questions handy to refer to]
- 4. How might this strategy be useful to you outside the maths classroom?
- 5. What did you think about the idea of using the card sort?
- 6. How well did you do in today's lesson? How do you know?

1.8. Data collection and analysis

The data collected was similar to that in year 1, i.e. written responses from the student surveys, audiorecordings of interviews conducted by TRs with students, and audio-recordings of the discussions between the three researchers during research group meetings. The audio-recordings of student interviews were transcribed and fully anonymised, with pseudonyms being used for students and any third parties throughout the data analysis. Selected extracts from the audio-recordings of research group meetings were transcribed, i.e. those considered most relevant to the thematic analysis.

The coding themes from year 1 were used as a starting point for the thematic analyses of the data in year 2 (see VMP project report year 1 section 1.8). NVivo software was again used to code the data and to facilitate the thematic analyses. A summary of findings, along with more detailed findings (and supporting evidence) from each thematic analysis, are included in subsequent sections of this report.

1.9. References to theory underlying research:

These references are in addition to those in VMP project report year 1 section 1.9:

Coles, A. (2013). Using video for professional development: The role of the discussion facilitator. *Journal of Mathematics Teacher Education*, *16*(*3*), 165-184.

Geiger, V., Muir, T., & Lamb, J. (2016). Video-stimulated recall as a catalyst for teacher professional learning. *Journal of Mathematics Teacher Education, 19(5),* 457-475.

Hollingsworth, H., & Clarke, D. (2017). Video as a tool for focusing teacher self-reflection: Supporting and provoking teacher learning. *Journal of Mathematics Teacher Education*, 20(5), 457-475.

Hand, V. (2012). Seeing culture and power in mathematical learning: Toward a model of equitable instruction. *Educational Studies in Mathematics, 80(1/2),* 233-247.

Liu, K. (2015). Critical reflection as a framework for transformative learning in teacher education. *Educational review, 67(2),* 135-157.

Sherin, M., & Dyer, E. (2017). Mathematics teachers' self-captured video and opportunities for learning. *Journal of Mathematics Teacher Education, 20(5),* 477-495.

2. Summary of findings from thematic analyses

This section includes a summary of the themes emerging from each of the following thematic analyses (with references where appropriate to the findings from cycles 1 and 2):

- the student surveys administered before the third research lesson in cycle 3 and after the fourth research lesson in cycle 4 (see Section 3);
- the interviews of target students conducted after the third and fourth research lessons in cycles 3 and 4 (see Section 4);
- the eight research group meetings held during the second year (relating to cycles 3 and 4) of the project (see Section 5).

Note that more detailed findings (and supporting evidence) from each thematic analysis can be found in Sections 3, 4 and 5 of this report.

2.1. Thematic analysis of student surveys:

- The responses to question 1 were very similar in cycles 3 and 4, with *most students feeling* comfortable with their level of success in maths in general. The proportion of students in cycles 3 and 4 reporting that they were 'very' or 'quite' successful in maths (38% in both) was lower than in cycle 1 (53%) although this might be attributable to a chance of wording in the question or a restructuring of the groups between Year 7 and Year 8.
- The responses to question 2 suggested most students in both classes exhibited a traditional view of success in mathematics. In cycles 3 and 4, the three most common reasons students gave for explaining their success in maths were getting answers correct in class (the second most popular reason given in cycle 1), scores achieved on assessments (which didn't feature in cycle 1, perhaps because the question was worded differently) and finding the work easy (also common in cycle 1).
- The most popular reason given by students in cycle 1 (completing a large amount of work) became decreasingly popular, whilst grades and scores became increasingly popular (perhaps reflecting a greater emphasis placed on assessments as students progress through their secondary schooling).
 - > The following proportion of students attributed their success to getting correct answers: 20% in cycle 1; 43% in cycle 3; 16% in cycle 4.
 - The following proportion of students attributed their success to grades or scores on assessments:
 0% in cycle 1; 21% in cycle 3; 36% in cycle 4.
 - > The following proportion of students attributed their success to finding the work easy: 16% in cycle 1; 17% in cycle 3; 36% in cycle 4.
 - > The following proportion of students attributed their success to completing a large amount of work: 27% in cycle 1; 17% in cycle 3; 7% in cycle 4.
- In their responses to question 3, most students gave specific suggestions for what they could do to be more successful in maths, the most popular being doing more homework, revising and practising in areas of weakness (perhaps reflecting a growing concern with assessments). There was an apparent increase in students' self-efficacy between cycles 3 and 4 with an increase in suggestions involving taking responsibility for their own actions, e.g. doing more work/trying harder, concentrating more, focusing more, persevering, listening more carefully. There were surprisingly few responses relating to being more independent and taking on more challenge.
- When students were asked in question 4 how they thought their teacher helped them to be successful, the most common responses suggested a heavy reliance on teachers to help them understand, e.g. those relating to the teacher explaining the work fully and helping students when stuck. There was

some evidence that a handful of students were beginning to appreciate the teacher's efforts in encouraging them to be more independent and to engage more with progressive pedagogies, e.g. in references to the 'model solution, and 'boxing up' strategies in cycle 4. Some responses (6 in cycle 3 and 5 in cycle 4) suggested students were not able to articulate what the teacher did to help them to be successful.

2.2. Thematic analysis of interviews:

Note that there were six students interviewed in each cycle.

- Students developed a greater awareness of the different progressive teaching strategies used by the teacher as part of the project there was a significant increase in awareness between cycles 3 and 4.
 - > In cycle 3, only **one** student described features of the 'model solution' strategy (without prompting) when asked what they noticed that was different about the lesson (this was comparable to cycle 2, e.g. only **one** student described features of the 'scribing' strategy).
 - > In cycle 4, **four** students described features of the 'boxing up' strategy (without prompting) when asked what they noticed that was different about the lesson.
 - > In cycle 3, **two** students described features of the 'card sort' strategy (without prompting) when asked what they noticed that was different about the lesson.
 - > In cycle 4, **three** students described features of the 'card sort' strategy and **two** mentioned it without referring to its features (without prompting) when asked what they noticed that was different about the lesson.
- Students developed an *increasing appreciation of the teachers' intentions* (pedagogical rationale) when using the '*model solution' and 'boxing up'* strategies and showed increasing clarity in articulating the primary purposes of these (and other similar) strategies over the course of cycles 2, 3 and 4.
- Over the course of cycles 2, 3 and 4, there was a *lessening tendency to refer to other potentially valid purposes* for the teaching strategies not considered primary or invalid purposes.
 - > Note (for comparison) in cycle 2, **four**, **three** and **three** students articulated reasons closely reflecting the primary purposes of the 'scribing', 'annotating' and 'classifying' strategy respectively.
 - > In cycle 3, **three** students articulated 3 (of the 5) primary purposes of the 'model solution' strategy, i.e. engaging with other students' ideas, communicating mathematical reasoning, solving problems independently in future.
 - > Of the remaining **three** students, **two** articulated one of these primary purposes of the strategy and all **three** articulated another valid purpose not considered primary.
 - In cycle 4, all six students articulated 2 (of the 3) primary purposes of the 'boxing up' strategy, i.e. identifying key information in the question and either developing a generic problem-solving strategy (5 students) or spending more time planning (1 student). Two of these students also articulated another valid reason not considered primary.
- The use of the 'card sort' strategy appeared to facilitate a significantly greater appreciation amongst students of the teachers' pedagogical rationale (referred to above). This was apparent in the responses of students relating to the reasons for using the 'model solution' and 'boxing up' strategies.
 - > Students were not asked about the purpose of using the card sort. However, in Cycle 4 (only), they were asked what they thought about using the card sort.
 - > In response to this question, all **six** students referred to the primary purposes for using the 'boxing up' strategy (which are implicit in the rationale for using the card sort).
 - > In cycle 4, only **one** student (Keira) articulated a primary purpose for using the card sort, **one** student gave a valid purpose not considered primary, and **two** students gave an invalid purpose.

- The growing appreciation of the pedagogical rationale for the 'model solution' and 'boxing up' strategies reflected a *more general increase in awareness of the rationale behind progressive pedagogies* (apparent in students' responses). These included:
 - > Focusing on the development of generic skills needed for students to solve mathematical problems independently;
 - > Considering multiple ways of solving problems and comparing/evaluating different methods;
 - > Collaborative approaches to learning involving students sharing/discussing ideas with each other and presenting/explaining their mathematical reasoning to others.
- In cycles 3 and 4, all six students claimed they enjoyed the research lessons (as was the case in cycle 2). However, students showed an increasing tendency in cycles 3 and 4 to attribute their enjoyment of the lesson to collaborative working (3 students), other progressive teaching approaches (3 students), overcoming challenging problems they encountered in class (3 students), and achieving greater levels of understanding (2 students).
 - In cycle 2, the most common reasons given for enjoying the research lesson were finding the work easy/familiar (3 students), collaborative working (3 students), other progressive teaching approaches (2 students), overcoming challenging problems they encountered in class (2 students).
 - Note one of the two most common reasons given in cycle 2 (finding the work easy/familiar) was not given as a reason by students in cycles 3 and 4. One of the reasons suggested in cycles 3 and 4 (achieving greater levels of understanding) was not given as a reason by students in cycle 2.
- In cycles 3 and 4, all six students believed they were successful in the research lessons (as was the case in cycle 2). The reasons were given were quite similar across cycles 2, 3 and 4, although there was a small move away from attributing success in the lesson to getting correct answers towards attributing success to effort/engagement and working well with others.
 - > In cycles 3 and 4, the same **three** students attributed their success to achieving a higher level of understanding supported by getting answers correct (compared to **four** students in cycle 2).
 - > In cycle 3, **two** students attributed their success to their own effort and engagement with tasks and, compared with **three** students in cycle 4 (which included the same 2 students).
 - > In cycle 3, **one** student attributed his success to working well with others, compared to **two** students in cycle 4 (and **one** in cycle 2).
 - > In cycle 3, **one** student attributed her success to engaging with progressive teaching strategies, compared to **one** student in cycle 4 (and **one** in cycle 2).
- In cycles 3 and 4, there seemed to be a small shift away from the preoccupation with testing amongst students that was apparent in cycle 2.
 - > In each of cycles 3 and 4, **three** students referred to preparing for tests in their responses to interview questions (representing **four** different students across both cycles 3 and 4).
 - > In cycle 2, all **six** students demonstrated a preoccupation with testing by their responses.
- Two of the target students in cycles 3 and 4 (*Neal and Keira*) had also been target students in cycle 2 (they were the only two target students in cycles 3 and 4 to have also participated in the project in cycles 1 and 2). In cycles 3 and 4, *these two students stood out from the other four target students* in terms of the level of *awareness of the rationale behind progressive teaching approaches* that they demonstrated. This was not the case to the same extent in cycle 2, suggesting that their longer-term involvement with the project had a particularly positive impact on them.
 - > In cycle 3, **Neal and Keira** were the only two students to describe significant features of the 'model solution' strategy; they were two of the three students who described at least 3 primary purposes of the strategy;

- > In cycle 4, **Neal and Keira** were two of the four students who described significant features of the 'boxing up' strategy; **Neal** was the only student to describe the third purpose of the strategy, i.e. to encourage students to spend time planning how to solve a problem
- > **Keira** was the only student who managed to articulate a primary purpose for using the 'card sort' strategy in cycles 3 and 4 (demonstrating exceptional insight into its pedagogical rationale).
- > **Neil and Keira** were the only two students not to refer to tests in explaining the reasons for their success in mathematics.
- In cycle 2, Neal and Keira were two of the four students (along with Mary and Sophia) who articulated a primary purpose for 2 out of the 3 strategies, with the other two students (Ennis and Marcus) each doing so for 1 out of the 3 strategies. They were, however, the only two students to demonstrate a wider view of success in mathematics (incorporating working well with others and engaging with progressive pedagogies rather than completing questions correctly).

2.3. Thematic analysis of research group meetings

The thematic analysis of the research group meetings was divided up according to five coding themes as follows. Most themes were also identified from the analysis of research group meetings during the first year of the project. *New themes are identified in italics.* The references are to the sections in Section 5 where supporting evidence can be found.

2.3.1. Project research tools:

- Teacher researchers (TRs) significantly developed their use of a range of research tools, e.g. surveys, interviews, video-stimulated reflection, and these were seen as complementary. Effective use of a research 'tool-kit', where the combination was seen as more powerful than merely a collection of research tools, to evaluate success of project strategies. Surveys and interviews, as well as videos, proved invaluable for stimulating critically reflective discussions. (5.1.2, 5.1.3, 5.1.4, 5.1.5)
- The surveys were particularly useful for evaluating the impact of the project on all students, i.e. their general awareness of the teacher's pedagogical rationale and their appreciation of what it means to be successful in mathematics. (5.1.2)
- The interviews were particularly useful for evaluating the impact of the project on target (disadvantaged) students, i.e. their awareness of the pedagogical rationale of specific teaching approaches and their appreciation of how successful they were in the research lessons. (5.1.3)
- The researchers agreed, developed and refined detailed protocols for conducting video-stimulated reflection by drawing on relevant research literature and through evaluating their own experiences of trying it out. Key features include the timeline generated by a peer observer, pre-selection of short video clips (by teacher and peer observer), justifying the choice of these in relation to the aims of the research, the role of the external partner in facilitating a discussion focusing firstly on a descriptive phase, before moving onto an interpretive phase. (5.1.4, 5.1.5, 5.3.3, 5.4.2)
- TRs took a lead in designing and administering surveys and interviews, making effective use of 'empathetic' interviewing, and their knowledge of students, to enable more meaningful stories to be constructed. This was seen as enhancing the trustworthiness of the findings, since students were more comfortable being interviewed by their own teachers due to trust and rapport already established. (5.1.3, 5.3.2, 5.4.1, 5.4.2, 5.4.3)
- Through explicit focus on designing and conducting surveys and interviews, and video-stimulated reflection, TRs began to incorporate the collection of evidence to evaluate teaching and learning into their routine practice in the classroom, drawing on research tools, albeit in less formal ways. (5.4.6)

2.3.2. Strategies tried out:

- The strategies focused on making clear the benefits of a range of progressive teaching and learning approaches including: working collaboratively, listening/explaining/justifying to other students, developing mathematical reasoning, identifying/learning from errors, identifying/addressing misconceptions, drawing out ambiguities, precise use of mathematical language, classifying/sorting problems, *developing independent problem-solving skills, developing metacognitive skills*. It is noticeable how this list reflects Swan's (2006) 'collaborative orientation' towards teaching and learning mathematics. (5.2.1, 5.2.2, 5.2.3)
- Researchers were able to clearly articulate their rationale for using various progressive pedagogies and this was clearly reflected in the strategies. (5.2.1, 5.2.2)
- All strategies included (as a core element) a discussion with students of the teacher's pedagogical rationale, typically posing questions such as 'Why do you think that I ...?' in order to prompt discussion. (5.2.1, 5.2.2)
- The card sort strategy was particularly effective in prompting discussions around the teachers' pedagogical rationale and assessing students' understanding of this. (5.2.3, 5.5.6)
- There was a growing awareness amongst TRs of the need to moderate discussions around their pedagogical rationale by developing a balance between directly articulating their intentions and prompting discussions around what these might be. (5.2.3, 5.3.5, 5.4.4)
- A clearer distinction emerged between the progressive teaching approaches and the strategies for making the pedagogical rationale more explicit, with adaptations of the former, e.g. as in the 'model solution' and 'boxing up' approaches, facilitating the latter, e.g. the teacher explaining/asking why they adopted these approaches and using the card sort to facilitate a discussion around these. (5.2.1, 5.2.2, 5.2.3)

2.3.3. Critical research processes:

- The external stimulus provided by PW prompted critically reflective discussions of TRs' existing practice – this stimulus included posing probing questions *relating to TR's pedagogical rationale behind teaching approaches* and consistently relating ideas, suggestions and findings back to the theory and aims of the research project. (5.2.1, 5.2.2, 5.2.3, 5.3.3, 5.4.8)
- The research articles that TRs presented/discussed (selected by PW), and previous theory they had engaged with, facilitated critical reflection on existing practice and the research tools developed in the project. (5.3.2, 5.3.3)
- An alternative vision of practice, informed by the research literature, was clearly articulated and was used to focus the researchers' planning and evaluation of strategies to try out. (5.3.6)
- The TR's detailed knowledge of their students and the classroom context meant that constraints on developing practice, and implementing the alternative vision, were identified and addressed and that a pragmatic approach was adopted to conducting the research. (5.3.6)
- The researchers' critical reflection on practice was supported by using a range of triangulating evidence from videos, surveys, interviews and research journals. (5.1.2, 5.1.3, 5.1.4, 5.1.5, 5.3.8)
- The video-stimulated reflection enabled TRs to view their own teaching from different perspectives, i.e. that of the students, and to notice things it was not possible to attend to whilst teaching. (5.1.5, 5.3.9)
- TRs' engagement in the review and evaluation of the research design and analytical framework enabled them to develop a deeper understanding of research processes and methodology. (5.3.3, 5.4.1, 5.4.2, 5.4.3, 5.4.5)

2.3.4. Participatory Action Research characteristics:

- Collaborative relationships between researchers were apparent in the status deliberately afforded to TRs, e.g. in maintaining control over the videoing of lessons and ownership of the video recordings (5.1.5)
- The research project was participatory in that TRs played a leading role and took their own initiative in the design and implementation of the research tools and the *design, planning and* evaluation of the strategies. (5.1.2, 5.1.3, 5.1.4, 5.1.5, 5.2.1, 5.2.2, 5.2.3, 5.3.5, 5.4.2, 5.4.3, 5.4.4)
- The researchers examined in detail power relationships existing between all participants in the research, e.g. between TRs and PW, between TRs and students. (5.4.1, 5.4.2)
- Trialling and evaluating the strategies had a significant impact on TRs' classroom practice with many of the ideas taken up and used routinely with a range of classes. (5.4.6, 5.4.7)
- TRs developed a greater critical understanding of their own practice, becoming more aware of the rationale behind progressive pedagogies and beginning to question/critique previous assumptions, e.g. about what success in school mathematics entails. (5.3.9, 5.4.7)
- Through evaluating the effectiveness of the strategies, TRs developed a clear vision of how they might embed some of the ideas in their own practice (and that of others in the department) in order to make their pedagogy more visible to learners. (5.4.4, 5.4.6, 5.4.7, 5.4.8)

2.3.5. Impact on students:

- The following student outcomes of the project were seen as inter-connected and inseparable each reinforced and was reinforced by the others: greater appreciation of the teacher's pedagogical rationale ⇔ greater appreciation of what it means to be successful ⇔ responding in more appropriate ways to progressive pedagogies ⇔ greater levels of engagement of students with strategies ⇔ greater levels of engagement of students with strategies ⇔ greater levels of engagement of students with strategies ⇔ greater levels of engagement of students with strategies ⇔ greater levels of engagement of students with progressive pedagogies. (5.4.6, 5.5.1, 5.5.2, 5.5.3)
- There was evidence that disadvantaged students, and others who were previously prone towards misinterpreting the teachers' intentions, benefited significantly from the research project in terms of increasing their engagement with, and becoming more aware of how to respond appropriately to, progressive pedagogies. (5.5.1, 5.5.2, 5.5.3, 5.5.7)
- TRs and students initially had highly contrasting views of what success in school mathematics entails both moved towards more of a shared understanding of what success in school mathematics looks like. (5.5.3)
- Students' understanding of mathematical success moved significantly towards recognising the importance of engaging with progressive pedagogies whilst at the same time appreciating the need to prepare for tests. (5.5.3)

3. Thematic analysis of student surveys (cycles 3 and 4)

The surveys were given to students in both classes to complete 2-3 days before the third research lesson in cycle 3 and 8-9 days after the fourth research lesson on cycle 4. There were no noticeable differences in the patterns of responses between the two Year 8 groups. Similar research lessons were taught by Alba and Tiago, which were planned collaboratively and were based on the same content from the scheme of work, the same teaching approaches and the same strategies for making pedagogy visible. The responses from the two classes have therefore been combined in the following analysis.

3.0. Coding scheme and categorised responses for student surveys

For survey responses in cycles 3 and 4 (cycle 1 responses included for comparison)

*Note the 'Number of responses' refers to the number to which each code was applied. For cycle 1 there was a total of 45 responses to the survey, for cycle 3 there were 42 responses (43 surveys competed but one discounted), and for cycle 4 there were 45 responses

Note responses from cycle 1 were for a group of mixed-attainment Year 7 students. Responses from cycles 3 and 4 were from the same two classes of Year 8 'mixed-attainment' students – these contained some students from the Year 7 classes but there was a significant difference in that the Year 8 cohort had been reorganised (due to parental pressure) with a distinct top set and the remaining groups in 'mixed-attainment' groups.

Code/category	Description	Number of responses*		nses*
		Cycle 1	Cycle 3	Cycle 4
Question 1: How succ	essful do you think you are in maths in general?	(cycles 3 ar	nd 4)	
(in cycle 1 this questic	on was: 'How well do you think you've done in to	day's math	s lesson?')	
1	Not successful (well) at all	1	0	1
2	Not successful (well)	5	2	3
3	ОК	15	23	24
4	Quite successful (well)	18	14	15
5	Very successful (well)	6	2	2

Note that students were also asked to indicate whether they are male/female at the start of the survey.

Question 2: How do y	ou know?			
The following categor	ies are grouped as 'Students' dispositions toward	ds learning		
Good behaviour	Student attributes success to their own good	2	1	1
	behaviour.			
High Effort	Student attributes success to their own high	2	2	2
	levels of effort.			
Participation	Student attributes success to their own high	7	2	2
	levels of participation/focus/concentration.			
Working with others	Student attributes success to working well	5	0	0
	with other students.			
Lack of participation	Student attributes lack of success to their own	2	0	0
	low levels of participation.			

Poor behaviour	Student attributes lack of success to their own	6	0	0
	poor behaviour.			
Willing to question	Student attributes success to their own		1	1
	willingness to ask questions when stuck.			
Enjoy learning	Student attributes success to their own		1	1
	enjoyment of learning.			
The following categor	ies are grouped as 'Students' judgements about	their work	output':	
Answered questions	Student attributes success to answering	4	0	0
	teachers' questions correctly.			
Correct answers	Student attributes (lack of) success to getting	9	18	7
	(in)correct answers to questions completed			
	independently.			
Work completed	Student attributes success to completing a	12	7	3
	large amount of work.			
Real life application	Student attributes success to being able to		1	0
	apply maths learnt to real life problems.			
Grades and scores	Student attributes success to the grades or		9	16
	scores they receive on assessments.			
Explained to others	Student attributes success to being in a	0	0	1
	position to help/explain work to others.			
The following categor	ies are grouped as 'Students' perceptions about	how others	s saw them	' :
Teacher approval	Student attributes success to the approval or	5	9	5
	praise they receive from the teacher.			
Lack of ability	Student attributes success to their own level	0	0	1
	of ability.			
Lack of ability	Student attributes lack of success to their own	1	2	1
	lack of ability.			
The following categor	ies are grouped as 'Students' judgements about	their level	of understa	inding':
Found work easy	Student attributes success to finding the work	7	7	16
	easy.			
Found work difficult	Student attributes lack of success to finding	4	6	9
	the work difficult.			
Needed help	Student attributes lack of success to needing	1	0	2
	to ask others for help.			

Question 3: What do y	Question 3: What do you think you can do to be more successful in maths?			
General comments on	getting better without strategy:			
Be more successful	Get better/be more successful		2	1
Do better in tests	Do better in tests/be in top set		0	3
Understand more	Understand more/get more questions correct		1	0
Be more confident	Be more confident in giving answers		2	1
Important to do well	Appreciation of importance of doing well		1	0
General strategies for improving:				
Do more work	Do more work/study harder/try harder in class		4	9
Focus more	Focus/concentrate/engage more in class		1	3

More homework	Do more homework or practice at home		11	12
Specific strategies linked to individual study skills:				
Do more revision	Revise more/look back over work/record		10	9
	notes for future reference			
Practise weaknesses	Go over things/practise/research in weaker		6	4
	areas			
Take more care	Take more time/care over work		1	0
Be more efficient	Solve problems more quickly/more		2	1
	efficiently/not copy everything down			
Specific strategies link	ed to problem-solving skills:			
More care reading	Read question more carefully/several times		2	2
Key information	Identify key information in questions		1	1
Different methods	Explore different methods		0	1
Show working out	Show working out		2	0
Specific strategies link	ed to drawing on others for help:			
Ask others for help	Ask partner/other students for help		0	3
Ask teacher for help	Ask teacher for help/to explain		4	5
More challenge	Try/ask for more challenging questions		3	2
Contribute more	Contribute more to class discussions		2	3
Specific strategies link	ed to behaviour/dispositions:			
Concentrate more	Concentrate more/avoid being distracted by		4	10
	others			
Listen carefully	Listen more carefully/pay closer attention		6	7
Keep calm	Try not to panic/keep calm		1	1
Persevere	Persevere/don't give up		0	3
More independent	Be more independent/try questions on own		2	1
Seating arrangement	Think about seating arrangement		2	1

Question 4: What doe	Question 4: What does your teacher do to help you to be successful in maths?				
General strategies rela	ating to teaching approach:				
Negative response	Left blank or trivial/negative response, e.g.		6	5	
	'teaches', 'not much'				
More interactive	Makes the lesson more interactive/answers		2	5	
	students' questions				
More enjoyable	Makes maths activities fun and enjoyable		0	1	
Encourages talk	Encourages students to talk to each other		0	4	
	(think-pair-share)				
General strategies rela	ating to student engagement:				
Provides support	Provides support and guidance		1	3	
Speaks calmly	Is kind/talks to students calmly/doesn't shout		2	1	
Encourages students	Offers encouragement and praise/pushes		1	2	
	students to work harder				
Gives students time	Gives students time to fully understand		4	1	
Specific strategies relating to whole class input:					

Worked examples	Does worked examples before students tackle	4	6
Explains fully	Explains the work fully/sten-by-sten to	10	9
	students		5
Multiple methods	Explains more than one method	0	2
Worked solutions	Works through solutions with the whole class	2	
Shares difficulties	Shares/discusses difficulties students	1	
	experience with the whole class	_	
Key information	Uses glossaries/gets students to copy	8	3
	down/reminds students of key information		
Specific strategies rela	ating to independent work:		I
Allows practice	Lets students practise/get on with activities	2	0
Circulates class	Circulates the class to see who needs help	5	4
Asks rich questions	Checks students understand/asks questions to	3	6
•	help students understand		
Helps when stuck	Spends time helping individual students who	11	9
	are not understanding a question.		
Breaks down work	Breaks work down into smaller chunks when	0	3
	needed		
Re-reads questions	Re-reads question when students are stuck	0	1
Written feedback	Gives students written feedback on their work	0	1
Student reflection	Gets students to reflect on what they find	2	1
	difficult/uses scaling for understanding		
Extensions tasks	Gives extension tasks to those who have	1	1
	finished		
Specific strategies rela	ating to work outside classroom:		
Additional support	Provides additional support outside lessons	2	0
Sets homework	Sets homework related to class work	0	2
Monitors homework	Keeps students back if they don't do	0	1
	homework		
Specific strategies trie	d out through project:		
Model solution	Encourages students to generate/agree a	0	1
	model solution		
Boxing up strategy	Uses 'boxing up' strategy to help students	0	2
	understand what questions are asking		

3.1. Survey question 1 – 'how successful?'

Students were asked to respond to the question *"How successful do you think you are in maths in general? On a scale of 1 (Not successful at all) to 5 (Very successful)."*

The responses from the cycle 3 surveys were very similar to those from the cycle 4 surveys. Just over half of the students chose 3 (on the scale of 1 to 5), which might be interpreted as 'OK'. Close to a third of the students chose 4, which might be interpreted as 'quite successful'. The vast majority of student therefore chose either 3 or 4 on the surveys, suggesting that they were at least comfortable with their general level of success in mathematics. Very few students (2 out of 42 for cycle 3 and 4 out of 45 for cycle 4) chose 1 or

2, which suggests very few students were concerned about their success. Only 2 students considered themselves to be 'very successful' on each survey.

The proportion of students who thought they were 'very successful' or 'quite successful' in mathematics was 38% for cycles 3 and 4. For cycle 1, the proportion of students who thought they did 'very well' or 'quite well' in the mathematics lesson was 53%. This apparent drop from 53% to 38% might be for several reasons. When considering general success in mathematics, rather than success in a particular lesson, students might take into account a wider range of factors, e.g. results of other periodic or end-of-year assessments. It might be attributable to the re-organisation of the groups, in particular the introduction of a top set in Year 8, which resulted in the Year 8 'mixed-attainment' groups losing some of their most confident students. Alternatively, it might suggest a general deterioration of students' confidence levels over the course of their secondary education, in which case the similarity in responses between cycles 3 and 4 might suggest the work conducted through the project may have halted this process.

3.2. Survey question 2 – 'attributing success'

In a follow-up to question 1 ("How successful do you think you are in maths in general?"), students were asked to respond to the question "How do you know?".

The two most common reasons given by students to justify their level of success were getting answers correct or incorrect in class (particularly so in cycle 3 and less so in cycle 4), and the grades and scores achieved on assessments (particularly so in cycle 4 and less so in cycle 3). This reflected the responses from cycle 1, although grades and scores were not mentioned as the judgement was made about success in one particular lesson. Grades and scores appear to be an increasingly important factor for students in determining their success in mathematics

Examples from cycle 3 include:

I don't get REALLY bad test results and REALLY good test results. My confidence in maths is probably average.

Examples from cycle 4 include:

I know because I fully understand the work I'm doing. In a majority of the work I do and starters (in general). When I'm doing a question, and we mark it, I get the majority correct. However, I don't always do as well in test, as I prefer class work.

It was also common for students to attribute their level of success to the degree of understanding, i.e. either finding the class work easy or difficult (in some cases students referred to some work being difficult and other work being easy to justify their success being 'OK'). This reason was also evident in cycle 1 and appears to become more dominant from cycle 3 to cycle 4. This apparent increase over cycles 3 and 4 (e.g. finding the work easy increased from 7 to 16) is mirrored by a decline in students attributing their success to correct answers (from 18 to 7). This suggests students have a growing appreciation of the importance of understanding rather than merely obtaining correct answers. This appreciation was often combined with a recognition of the importance of tests.

Examples from cycle 3 include:

Because I understand most of the tasks we do pretty easily and sometimes finish quickly.

I know because ever since primary I was never that good at maths but there's always some things I understand and a lot of things I don't.

Examples from cycle 4 include:

Because I know that I'm not the smartest in the class but this doesn't mean I'm terrible at maths because most of the time I understand the work very well. So I feel like I'm in the middle because there are things I do find difficult.

Sometimes I understand things in detail but with others I find them a lot harder to understand. So I would say I am in the middle, not brilliant but not terrible, as I get some questions correct and some wrong.

Because of my test scores and results. And what my progress check says. I understand class work well.

The amount of work completed and the level of teacher approval were less common but still significant responses (as they were in cycle 1), although both dropped over the course of cycles 3 and 4 (from 7 to 3 and from 9 to 5 respectively), perhaps reflecting growing self-confidence and less concern for how others perceived them. Both reasons were often referred to in conjunction with reference to test scores and levels of understanding.

Examples from cycle 3 include:

Because of my test scores and things like my progress check. But also class work, results, and whether I answer a question correctly or not.

Because I don't struggle on most subjects. However, when it's about something I'm not so sure about, I can get things wrong.

I know because of the grades I get in my test. If I'm successful I know it because I get the questions correct.

Examples from cycle 4 include:

Because in class I find the work helpful and sometimes challenging but I can answer the questions. My grades are good as well and my teacher tells me that I am doing well.

It was noticeable that students were much less likely (than in cycle 1) to attribute their success in cycles 3 and 4 to what might be described as 'general dispositions towards learning', such as effort, behaviour, participation and working well with other students. This might suggest a more nuanced appreciation of what it means to be successful which allows students to be more specific in their reasons.

There were some responses made by individual students which reflected a broader view of what success in mathematics might look like, such as willingness to ask questions, making use of help, challenging yourself, being independent, and helping others.

Examples from cycle 3 include:

I always contribute in class and always ask questions if I don't understand a question.

Examples from cycle 4 include:

I am OK at maths. Sometimes I do need help with the work but with help I do get there in the end. So everything is good in maths.

I have increased my confidence in maths so I try to challenge myself. When we do tasks that we mark I usually get them all correct.

Also I'm very independent in maths and find it easy when it comes to doing the independent work.

I usually get developing on tests which means I do understand it but not fully. I also ask questions when I'm stuck which helps.

I get alright grades and I'm able to help other people.

3.3. Survey question 3 – 'being more successful'

Students were asked to respond to the question "What do you think you can do to be more successful in maths?"

Most students were able to come up with specific strategies for improving and generalised statements such as 'do better' or 'understand more' (which might be expected) were quite rare, with only 5 such responses in each of cycle 3 and 4 (although in cycle 4, 3 of these were moving towards being more specific, i.e. 'do better in tests').

Examples from cycle 3 include:

Engage in the lesson.

I think maybe if I became a bit more confident in answering questions then I would do better.

The most common strategies given for improving relate to doing more homework (11/12 responses in cycles 3/4), revision (10/9 responses in cycles 3/4), and practice in areas of weakness (6/4 responses in cycles 3/4), perhaps reflecting students' recognition of the importance of tests and assessments.

Examples from cycle 3 include:

To be more successful I would do more notes and easier to understand if I were to look back in my book for revision and or homework. Doing things like colour to make important info stand out and pop more.

Examples from cycle 4 include:

Whenever we learn a lesson I can look back at it and research the things I don't understand.

I think I could do 20-30 mins of revision each day, getting information from my exercise book, online maths apps and more.

Practice and revise what you learnt in the lesson so you don't forget. Go through the things you don't understand.

There appeared to be a general increase in self-efficacy of students between cycles 3 and 4 with more strategies offered for becoming more successful which involved students taking responsibility for their actions, e.g. doing more work/trying harder (increase from 4 responses in cycle 3 to 9 responses in cycle 4), concentrating more/avoiding being distracted by others (increase from 4 to 10), focusing more (increase from 1 to 3), persevering (increase from 0 to 3), listening more carefully (minimal increase from 6 to 7).

Examples from cycle 3 include:

Listen more and don't get distracted.

Examples from cycle 4 include:

I think I should carry on listening carefully to the information Ms says and make sure that I fully understand it. If I don't then I should ask Ms to explain it again. Make sure to stay positive while doing questions, and even if I don't fully understand the question, I should try and do it anyway.

To be more successful in maths I think that I could try a bit harder rather than just giving up all the time. So I think I could persevere a lot more and also try a lot harder on the homework instead of just rushing on in to get it finished.

I don't really know. Maybe by working harder and doing much more homework.

However, in contrast, there appeared to be surprisingly few responses related to being more independent (2/1 responses in cycles 3/4) and taking on higher levels of challenge (3/2 responses in cycles 3/4).

Examples from cycle 3 include:

Try to be more independent and show more working out for the questions.

To do more harder questions and try to revise on what we learnt.

Examples from cycle 4 include:

I need to ask for extensions!

Listen more. Also be more confident in maths when giving answers. Make sure I take down notes in my book so I have everything I need when it comes to independent work. To read worded problems more carefully.

Try to be more independent and to do the questions on my own.

Responses relating to working with others were less common although still significant in number, e.g. asking the teacher for help (4/5 responses in cycles 3/4) and contributing more to class discussions (2/3 responses in cycles 3/4). There were 3 responses relating to asking other students for help in cycle 4 (compared to 0 in cycle 3).

Examples from cycle 3 include:

I can work out more problems and I can also ask my teacher if I don't understand.

If I talk more/put my hand up I could get better. This would help because if I talk about it I may understand more.

Examples from cycle 4 include:

I could try not to get distracted and I could try to answer more questions when we are discussing as a class.

I think if I ask when I need more help more often and contribute more.

If I don't understand call Miss to help me out. Or ask my partner next to me or people around me.

... and maybe sit next to people who are good at maths so they could help me when the teacher is not available.

I also think that I could ask my partner for help a bit more.

There were a small number of responses relating to arrange of specific strategies for solving problems, e.g. showing working out, different methods, key information, taking more care of being more efficient (with little difference between the two cycles).

Examples from cycle 3 include:

I think I can try and focus on what the question is asking or saying by re-reading the question to help me fully understand. I can also ask for more help and assistance.

Do some more examples and learn to write faster as sometimes there is not much time.

Listen more in class and find more efficient ways to work out answers to the questions. Also be more confident in my answers. Also show my working out more.

Examples from cycle 4 include:

Work harder. Check out different methods.

3.4. Survey question 4 – 'help from the teacher'

Students were asked to respond to the question "What does your teacher do to help you to be successful in maths?"

The two most common strategies suggested by students were related to the teacher explaining the work fully (10/9 responses in cycles 3/4) and helping students when they were stuck (11/9 responses in cycles 3/4), suggesting students generally relied heavily on teachers for helping them to understand.

Examples from cycle 3 include:

When a student asks for help she explains to the whole class which I am happy about.

My teacher always helps me for when I need help and also explains the questions and shows examples.

My teacher helps me to be more successful in maths by going through it with us and making sure that everyone knows it before he continues.

Examples from cycle 4 include:

He always explains the question if I don't understand and shows us how to work it out.

My teacher helps me in maths by answering my questions when I'm stuck or struggling. My maths teacher also always explains the work very clearly.

This apparent reliance on the teacher was reinforced by more specific teacher strategies suggested by students, i.e. using worked examples (4/6 responses in cycles 3/4), worked solutions (2/1 responses in cycles 3/4), providing key information for students to copy for reference (8/3 responses in cycles 3/4), and circulating the class to see who needed help (5/4 responses in cycles 3/4).

Examples from cycle 3 include:

She also always answers a few questions for us, before we do the work by ourselves, in case there's any people who still may not understand.

Always gives examples on the board so we can copy down and look back at.

If I ever get confused in something I put my hand up and my teacher explains to me how to sum and I get less confused than before.

Examples from cycle 4 include:

When there is work, to do the first question with us so that we're 100% sure we understand how to do them all.

Also she goes around and helps as many people as she can.

There was some evidence that some students became more aware of the teachers' efforts to promote discursive and collaborative learning during cycles 3 and 4, e.g. in attempting to make lessons more interactive (increase from 2 responses in cycle 3 to 5 responses in cycle 4), encouraging more talk between students (increase from 0 to 4) and encouraging students to engage with more than one method (increase from 0 to 2).

Examples from cycle 4 include:

She includes us when we do our work all together as a class.

We do think pair share.

He makes me talk to my partner

When there is a method and she always does both even if people understand one. She is very helpful and makes everyone understand.

There was also some evidence that some students began to recognise teachers' attempts to encourage them to be more independent, e.g. by helping students break down questions into smaller chunks when stuck (increase from 0 responses in cycle 3 to 3 responses in cycle 4), re-reading the question to students (increase from 0 to 1), and asking students questions to help them understand (increase from 3 to 6). In contrast there was less recognition of teachers' giving students' time to fully understand (decrease from 4 responses in cycle 3 to 1 response in cycle 4) and allowing students to get on with practice activities (decrease from 2 to 0).

Examples from cycle 3 include:

We stay on a topic for a good amount of time so the topic gets stuck in our head and we learn more in depth in the topic.

He gives us stuff to practice and asks us questions in class.

Examples from cycle 4 include:

Our teacher double checks with us to see if we understand and also if we need help he breaks it down even more.

She helps me break down the question if I don't understand it

Re-reads the question I am usually stuck on.

My teacher asks us questions (randomly) which is helpful to find out if the person is understanding.

My teacher asks me questions that takes time to think about.

There were only 3 references in cycle 4 to the strategies used for making pedagogy more visible (2 for 'boxing up', 1 for generating a model solution, and 0 for the card sort). There were a significant number of responses (6/5 in cycles 3/4) which demonstrated students were not able to articulate what the teacher does to help the students be more successful in mathematics.

Examples from cycle 3 include:

Not much.

Make me successful.

Examples from cycle 4 include:

He teaches us methods like the boxing out method to assure that we have an understanding of what the question is asking us.

My teacher prints out sheets with questions to think about while solving a problem.

Also she helps us come up with model solution which really helps in maths.

She helps me be successful and learn.

There were some responses relating to the teacher's calm (2/1 responses in cycles 3/4), supportive (1/3 responses in cycles 3/4), and encouraging (1/2 responses in cycles 3/4) manner towards students (rather than to specific strategies employed).

Examples from cycle 3 include:

Ms is very good at talking calmly and slowly, therefore people who struggle more can understand better and take in all the information thoroughly.

He helps by pushing me to do more and to get more stuff.

Examples from cycle 4 include:

Gives us support and advice.

My teacher says that I'm doing well so it gives me more confidence in my work.

4. Thematic analysis of student interviews (cycles 3 and 4)

4.0. Coding scheme for student interviews

*Note the 'Number of references' refers to the number of extracts of text to which each code was applied. The number in brackets refers to the number of interviewees for whom each code was applied to at least one response.

Code	Description	Number of		
		references		
		(sources)*		
Teaching approaches (progressive pedagogy)				
Rich questioning	Student describes teachers' use of questioning to draw out	4 (1)		
	understanding.			
Problem-solving	Student describes the use of open-ended problems during	3 (3)		
	the lesson.			
Connecting topics	Student makes connections between different	0 (0)		
	mathematical topics.			
Focus on methods	Student refers to focusing on methods for solving	22 (6)		
	problems rather than correct answers.			
Explanation	Student refers to explaining and justifying solutions.	9 (4)		
Collaborative work	Student refers to working collaboratively with other	19 (9)		
	students.			
Misconceptions	Student describes the use of misconceptions and errors to	0 (0)		
	develop understanding.			
Problem-solving	Student refers to strategies that can be employed in	38 (7)		
strategies	solving problems.			
Teaching strategies (vis	Teaching strategies (visible pedagogy)			
Model solution	Student refers to features of the 'Model solution' strategy	49 (6)		
	(students present their own solutions before class agrees			
	model solution to use in solving further problems).			
Boxing up	Student refers to features of the 'Boxing up' strategy	78 (6)		
	(students consider two metacognitive questions before			
	starting to solve problems).			
Card sort	Student refers to features of the 'Card sort' strategy	46 (10)		
	(students arrange/discuss statements relating to reasons			
	why teacher might have chosen the teaching approach).			

Students' experiences of learning				
Satisfaction	Student expresses satisfaction with their learning during the lesson.	27 (8)		
Challenge	Student describes experiencing challenge during the lesson.	15 (7)		
Lack of challenge	Student describes experiencing a lack of challenge during the lesson.	8 (3)		
Frustration	Student expresses frustration with their learning during the lesson.	0 (0)		
Fear/anxiety	Student describes experiencing fear or anxiety during the lesson.	2 (2)		
Familiarity	Student refers to being familiar with the content of the lesson.	0 (0)		
Confusion	Student describes experiencing confusion during the lesson.	2 (1)		
New content	Student describes experiencing new content with which they are not familiar.	3 (3)		
Dispositions towards learning mathematics				
Enjoyment	Student articulates reasons for enjoying the lesson.	42 (12)		
Lack of enjoyment	Student articulates reasons for not enjoying the lesson.	0 (0)		
Perseverance	Student refers to own perseverance during the lesson.	7 (5)		
Motivation	Student refers to own motivation during the lesson.	2 (2)		
Independence	Student refers to own independence in learning.	26 (10)		
Empathy with others	Student exhibits empathy towards other learners.	8 (6)		
Shared responsibility	Student describes sharing responsibility for others' learning.	18 (6)		
Recognition rules		L		
Notices strategy	Student recalls the main strategy used by the teacher during the lesson.	30 (11)		
Fails to notice strategy	Student does not recall the main strategy used by the teacher during the lesson.	3 (2)		

Primary purpose	Student articulates a primary purpose (as identified by the	54 (12)
	teacher) for using the teaching approach in question.	
Valid purpose	Student articulates a valid purpose (not identified as	30 (9)
	primary by the teacher) for using the teaching approach in question.	
		2 (1)
Invalid purpose	Student articulates an invalid purpose for using the	9 (4)
	teaching approach in question.	
Realisation rules		
Successful	Student articulates reasons why they consider themselves	28 (12)
	successful during lesson.	
Unsuccessful	Student articulates reasons why they consider themselves	0 (0)
	unsuccessful during lesson.	
Appropriate behaviour	Student refers to behaviour which they consider	2 (2)
	appropriate during lesson.	
Better prepared for	Student refers to being better prepared for tests or	9 (6)
tests	examinations as a result of learning during lessons.	

4.1. Appreciation of the rationale behind specific teaching strategies

Alba and Tiago used three strategies for making their pedagogy more visible during the third and fourth action research cycles: 'model solution' was used during cycle 3, 'boxing up' was used during cycle 4, and 'card sort' was used in both cycles 3 and 4 (alongside the other two strategies). Note that these strategies are numbered as strategies 6, 7 and 8 (following on from the five strategies used during cycles 1 and 2).

4.1.1. The 'model solution' strategy (strategy 6)

Background/rationale for the strategy:

In the cycle 3 research lesson, students were given an initial problem on factorising/substitution, which they were asked to complete first on their own before presenting their solutions to others (similar to the 'think-pair-share' teaching approach used in cycles 1 and 2). The first of the strategies involved the teacher facilitating a whole class discussion to agree a 'model solution' to the initial problem and the reasons that make this a good solution. The teacher then asked students to copy down this model solution and use it for reference in solving a series of similar problems and presenting 'good' solutions.

The rationale/purpose for using this strategy (as discussed by Alba/Tiago/Pete in Meeting 9) was to:

- Encourage students to share ideas with each other (and hence learn from each other).
- Enable students to appreciate multiple ways of solving a problem and to compare the merits of different methods (recognising differences and similarities).
- Develop students' agency in negotiating a model solution.
- Develop skills in communicating mathematical reasoning.
- Develop students' independence in solving problems.

When asked whether they noticed anything different about the lesson, **only one of the six** students (**Keira**) described significant features of the 'model solution' strategy used by the teacher:

And we had to work in partners, like, why we did the model solution, and like what it would help us with, why we had to do it. (Keira, Interview 1, #10)

After further prompting, **Neal** mentioned some features related to the strategy:

You made us do that [looking at ways to find the perimeter] together, with our partners. And then we all worked it out as, like, a class. (Neal, Interview 1, #48)

Nasri referred to the strategy but did not describe any of its features:

Because we did fun activities, like ... the modelling thing, I forgot what it's called. (Nasri, Interview 1, #8)

None of the other three students referred to the 'model solution' strategy.

After being reminded about the strategy, students were then asked why they thought the teacher asked them to agree (as a class) a model solution and to use it in solving other problems.

Three out of six students (**Neal, Simon and Keira**) described at least three of the primary purposes for using the 'model solution' strategy. They recognised how discussing and comparing different methods enabled them to engage with other students' ideas:

And then you could all explain on how you got that ... how you got that and how you got it correct. (Neal, Interview 1, #62)

Um, so we could see which answer's better, like, which method is better and which one is quicker as well. (Simon, Interview 1, #28)

Discussed it as a class ... Because we came up with one, like, we put our hand up, and we chose which one was more efficient to use. And ... yeah, I like the fact that we did that because it's easier to discuss which one's more efficient. So then it gives people ideas like, yeah, that one would be more efficient, it's more quicker. (Keira, Interview 1, #22)

These three students also described how the strategy could help students communicate their mathematical reasoning:

And then someone who has the correct answer could explain, like, how they got the answer, and put it into, like, more detail. (Neal, Interview 1, #66)

Because if you can't explain it then other people aren't going to know how to do it. (Simon, Interview 1, #40)

So, like, um ... if we're working in partners, and it's like we're deciding on a method to use, we can say 'this one is more efficient to use because of this' and, like, we can ... it's easier for us to communicate in, like, a sense that, like ... (Keira, Interview 1, #48)

And not just, like, writing it down. Like we can say, like, 'why are we using this method? Why are we thinking it's the right one?' and stuff like that. (Keira, Interview 1, #52)

The same three students appreciated how the strategy could help them to solve similar problems independently in future:

So, you know ... if you get something like similar, like, you could just flip back and like check 'Oh, how did you do that? How do I do ... how do I answer the question?' Yeah. (Neal, Interview 1, #68)

Oh yeah. And when we're doing, like, the end-of-year tests ... we can see what we've done before. If we need to do that again, then we know where it is. (Simon, Interview 1, #48)

Say if we forget, like, because it's a new topic, so if we forget how to work out the answers, we can just look back in our book, like a couple of days, and see that 'yep, this is going to ... this is the method, this is how we're supposed to do it'. And, like, it will help you. (Keira, Interview 1, #56)

Two of the other three students (**Tom and Nasri**) described only one of the primary purposes of the strategy (comparing different methods, and solving problems independently, respectively):

So you could see each different answer and how they compare. And, like, how you did it, with the working out. (Tom, Interview 1, #34)

Maybe, like, if one day you decide to come up with a question like that, like, just randomly, we could, like, refer back to our books, like go and look back in our books to see how we could, like ... how we work it out. And then maybe we could answer your question. (Nasri, Interview 1, #42)

Anna was the only student out of the six not to articulate a primary purpose for the strategy.

Whilst **three out of six** students (**Tom, Nasri and Anna**) demonstrated only a limited appreciation of the primary purposes of the strategy, they were also able to provide other valid purposes for the strategy (relating to remembering methods or the convenience of focusing on a single method):

So we didn't forget how to do it. And then you could either use one way or the other way to solve out the other problems. (Tom, Interview 1, #38)

Maybe it would be difficult if we did it, like, differently because, like, everyone would have different methods. So, if we ... if everyone did the same method, it would have been easier. (Nasri, Interview 1, #46)

It's just, like, it's a more quicker way, rather than thinking, like, 'which one should I do?' I just know I can do that one. Because I know it's the best solution to finding the problem. (Anna, Interview 1, #32)

Simon was **the only student out of the six** to suggest a purpose that might be described as invalid, i.e. solving problems quickly:

We can, like, we can move onto different things faster. (Simon, Interview 1, #34)

None of the six students mentioned the primary purpose of the strategy to develop students' agency in negotiating a model solution (discussed by the research group during Meeting 9).

4.1.2. The 'boxing up' strategy (strategy 7)

Background/rationale for the strategy:

In the cycle 4 research lesson, students were given a series of problems to solve on probability. The teacher also gave students a series of generic prompt questions printed on laminated card (see Figure 1 in Section 1.6.2) to use when solving problems. These were divided into four coloured boxes (hence the name of the

strategy). The 'boxing up' strategy involved the teacher asking students to focus on the first (green) box which contained two questions: 'What is the question asking me? What information do I already have?' The teacher facilitated an early discussion with students on the reasons for using the 'boxing up' questions using the following prompts: 'Why is this useful? What does this question allow you to do?'

The rationale/purpose for using this strategy (as discussed by Alba/Tiago/Pete in Meeting 12) was to develop students' metacognitive skills, in particular to:

- Provide support to students in decoding a problem-solving question.
- Encourage students to spend time planning how to solve the problem rather than 'jumping in'.
- Provide a generic structure/strategy that students can apply in solving other problems.

When asked whether they noticed anything different about the lesson, **four of the six** students (**Neal**, **Nasri, Tom and Keira**) described significant features of the 'boxing up' strategy used by the teacher:

Because it wasn't, like, one of our usual lessons ... we did the work that we usually do, but then we had to, like, show what's the most important thing to use. Like planning's very important ... (Neal, Interview 2, #6)

Because we got to use something we never used before, which is the block. And you can write on it to help you with your answers. (Nasri, Interview 2, #5) ...

There's questions on it like ... 'What's the main part of the question?' And then, like, it also gives you questions on it like 'What's the main idea of the question?'. (Nasri, Interview 2, #7)

Um ... it was about, like, making a plan. There's, like, different ways, like making a plan, listing outcomes, and all these ways to do it. Like, you try and find the easiest way. (Tom, Interview 2, #12)

Um, so, when you were just asking us questions, like, about the 'boxing up' ... the 'boxing up' method, I think it was called. (Keira, Interview 2, #10) ... And you introduced the ['boxing up' method] ... I think the question was something about key

information. (Keira, Interview 2, #24)

Simon initially focused his responses to the question on the new topic (tree diagrams) he had not met before. After further prompting, **Simon** mentioned some features related to the strategy:

Like, breaking it down. (Simon, Interview 2, #26) ... Um, like ... the green box method. (Simon, Interview 2, #28)

Anna was the only one of the six students who did not recall anything about the strategy:

It was, yeah, basically like usual lessons, sort of. (Anna, Interview 2, #26)

Students were then shown the laminated card with the 'boxing up' questions, to remind them about the strategy, and then asked why they thought the teacher asked them to think about these questions.

All six of the students articulated two out of the three primary purposes of the 'boxing up' strategy (discussed by the research group during Meeting 12).

All six students described how the strategy enabled them to identify the key information in the question needed to solve the problem:

Yeah. So, then, like, what information you already have is, like, the main point as in ... It's like you have to find the key points of the word equation, as in ... you could have a word equation

like 'Charlie took away two sweets ... Charlie had four sweets and took away two. So you know that he had four sweets, and then you know he took away two, so those are the key points of the word equation. (Neal, Interview 2, #14)

And for what the question is asking me, what you could do is ... You could write the ... the part in bold, or like highlight the key part of the question, like ... What it's telling you, like, if its ratio or what's the ratio? the problem of it. (Nasri, Interview 2, #29)

And all the information you have, you have to underline it. And then, with that key information, you have to work from there slowly. (Tom, Interview 2, #24)

Um, like, highlight key information, and get rid of the irrelevant information. (Simon, Interview 2, #34)

So, when you first look at a questions, the first thing I do is I'll read it, and then I scan and look for the most key information. So I'd look for the numbers and then I'd look for ... at the, kind of, at the end bit, what the question's asking me to do. So then I also underline. (Keira, Interview 2, #36)

So rather than, like, just focusing on what other things we can do, focus on, like, what it's asking us to do. Like focus on the key information. (Anna, Interview 2, #30)

Five of the six students (**all except Neal**) described how the strategy provided them with a generic approach that they could apply in solving other problems:

Um ... it could help me in other lessons if I'm stuck, like, if I'm stuck and I need help and the teacher's busy, and there's no-one to help me, I could, like, try using my own method of doing it Like using the questions to help me. (Nasri, Interview 2, #33)

Um ... So you, like you're doing a paper, yeah? And, like, you don't know the answer, you have to, like, think for a moment what the question was asking you. (Tom, Interview 2, #24)

Like, cos, on different ... maybe on different lessons, you might have something similar, and then you're kind of stuck. And then, cos you're not sure, like, what to do, then it helps like highlight the key information, which is how you do it. (Simon, Interview 2, #38)

It helped you more to think about the question more. And so, like, say if you're stuck, and you're like 'OK, so I don't know what the question's asking me', you can look at the green box and see: 'OK, look what I have already, what is it ... what is the question about? What is the important key information about the question?' (Keira, Interview 2, #50)

Um ... maybe, like, if there are any other questions in, like, other classes, like word problems, then you could use that with those as well. (Anna, Interview 2, #38)

Neal was the only **one of the six** students to mention the third of the three primary purposes of the 'boxing up' strategy, i.e. to encourage students to spend time planning how to solve a problem before jumping in:

Like planning's very important ... you always have to plan before you start the work because then you, like, don't rush through it quickly. Because it wouldn't really, like, clearly make sense to you. And, like ... planning it will be ... making sure that everything's correct ... so that you know you're going to do it right, equations and stuff. (Neal, Interview 2, #6)

Most of the responses to the question were related to the primary purposes although some were related to other valid purposes not considered primary, e.g. completing problems quicker or more efficiently (which was articulated by **two of the six** students, **Tom and Simon**):

Because if you know what the question's asking you, and you do what the question's asking you, that's all you have to write. You don't have to add on, like, any additional information which means you waste time and stuff. (Tom, Interview 2, #20)

Cos it helps me to remind myself to do this, and then it will make me do the question faster. (Simon, Interview 2, #50)

None of the six students suggested a purpose that might be described as invalid.

4.1.3. The 'card sort' strategy (strategy 8)

Background/rationale for the strategy:

This strategy was used as a follow-up strategy to the 'model solution' and 'boxing up' strategies. Students were given a 'card sort' with a series of statements representing possible intentions of the teacher. Some statements were chosen to represent a primary purpose for using a particular teaching approach or strategy, i.e. those identified by the research group as part of their rationale in planning the approach/strategy (for the 'model solution' and 'boxing up' strategies, these relate to the bulleted points in sections 4.1.1 and 4.1.2). Other statements were chosen as potentially valid reasons, but not those identified as primary by the research group, and others were chosen as invalid reasons. Students were asked to discuss in groups and arrange the cards in order, with the cards at the top that best describe the reasons why the teacher chose to use a particular teaching approach/strategy.

For the 'model solution' strategy, the statements in the 'card sort' were tailored directly to the rationale for using the strategy. For the 'boxing up' strategy, the research group decided to reduce the number of statements and produce statements in the 'card sort' that might be more easily adapted to other teaching approaches/strategies. The statements included in the card sorts, and the instructions that accompanied them, can be found in Section 1.6.3.

The rationale/purpose for using the 'card sort' strategy (as discussed by Alba/Tiago/Pete in Meetings 2 and 5) was to:

- Prompt a discussion about the intentions of the teacher.
- Make the rationale of the teacher in using specific teaching approaches/strategies more explicit.
- Develop students' awareness of the teacher's pedagogical rationale.

The 'card sort' strategy was used in research lessons in both cycles 3 and 4, primarily as a way of making explicit the rationale behind using the 'model strategy' and 'boxing up' strategies respectively. When asked whether they noticed anything different about the lesson, some students mentioned the card sort as well as (or sometimes instead of) referring to the other strategies.

In cycle 3, **two of the six** students (**Nasri and Keira**) described significant features of the 'card sort' strategy used by the teacher (note Keira was the only one of the six students to notice the 'model solution' strategy without further prompting – see section 4.1.1):

Um, like, it's where we had a big piece of paper, and then we had to, like, cut up them. We had to place them in the right order, like, if you think one's the most important, or if it's least important. (Nasri, Interview 1, #10)

With the A3 piece of paper as well, when we had to, yeah like, choose which one ... well not choose but, like, put the statements in the two, like, two bits ... And then put which one was,

like, most important for, like, why is it important to do the model solution and what it would help us with. (Keira, Interview 1, #32)

After further prompting, **Tom** mentioned some features related to the strategy:

Er ... ranking in order ... to the importance of what we do in class. (Tom, Interview 1, #24)

None of the other three students referred to the 'card sort' strategy in cycle 3. **Two of the six** students (**Simon and Anna**) did not mention either the 'model solution' strategy or the 'card sort' strategy in cycle 3.

In cycle 4, **three out of the six** students (**Neal, Tom and Keira**) described significant features of the 'card sort' strategy used by the teacher (note all three of these students, as well as Nasri, noticed the 'boxing up' strategy without further prompting – see section 4.1.2):

Yeah. At the end, you made us, like ... you gave us, like, these cards and made us put it in, like, order. As in: What's the most important? What's the least important? (Neal, Interview 2, #10)

And the ranking things were, like, 'share ideas', like 'get distracted easily', like which ones you need to more focus on. So you have to rank the top to, like, which should do the best. And to, like, which you should do, but the most important thing to do. (Tom, Interview 2, #12)

And with the cards, we haven't done that in, like, a while. So then you introduced it again to us. And you asked us about what we think were the most relevant, at the top, and the least relevant of the questions ... of the statements. (Keira, Interview 2, #28)

Two of the six students (**Simon and Anna**) referred to the 'card sort' strategy without describing any of its features:

I liked, like, the cards bit at the end. (Simon, Interview 2, #12)

Er ... it was, like, interesting, cos like we did the cards, which makes it more like, like we got to do more ... not really like ... how do I explain it? (Anna, Interview 2, #8)

Only **one of the six** students (**Nasri**) failed to mention the 'card sort' strategy in cycle 4 (although she did describe significant features of its use in cycle 3). **All six** students referred to either the 'boxing up' strategy or the 'cord sort' strategy (or both) in cycle 4.

In cycle 3, students were not asked to explain the rationale behind the 'card sort' strategy. However, when **Keira** was asked to explain why she found the card sort enjoyable, she was able to articulate a primary purpose for its use:

It was, like, because it helps us, like, understand more why we do it, because some people just ... some people would be like 'Ah, what is this going to help us with in the future?', and stuff like this. And it would just help you, like, understand more, and like why we're doing this, why we're learning this, why it's going to help us. (Keira, Interview 1, #16)

In cycle 4, after being asked why they thought the teacher used the 'boxing up' strategy, students were asked what they thought about the idea of using the card sort. **Keira** was the only **one of the six** students who articulated a primary reason for using the 'card sort' strategy. She gave a detailed account of this primary purpose when the teacher prompted her to expand further on her initial responses:

I liked it. Because it made me, um ... think about what, um ... what is the purpose ... what is the purpose of what we're doing? (Keira, Interview 2, #62) ... So we can see, um, which one makes more sense. Cos some of ... they did have ... all of them I
think have something to do with the 'boxing up' method. However, we look at the one that's the most crucial. (Keira, Interview 2, #70) ...

To show us ... to show us, um ... well, to show us what you think, like ... want us maybe to start doing more? (Keira, Interview 2, #78)

One out of the six students (**Anna**) provided a potentially valid purpose for using the 'card sort' strategy related to distinguishing between valid and invalid reasons for using the 'boxing up' method:

Yeah, I thought, like, a good idea, cos it's like showing you what's less irrelevant ... more relevant, and what's less relevant. (Anna, Interview 2, #50)

Two out of the six students (**Nasri and Tom**) gave an invalid purpose for using the 'card sort' strategy. **Nasri** suggested the card sort was used as a self-assessment tool for students to evaluate their level of understanding:

Um ... so if we didn't probably have the cards, maybe people didn't really understand it. But then if, like, we had the cards, people can say what they understand, and like, what they didn't really understand ... and why or why not they understand. (Nasri, Interview 2, #45)

Tom appeared to contradict his earlier statement (Interview 2, #12 – see above) by suggesting that all statements were equally important and it was only their chronology that mattered:

Um ... it makes people think, like, 'this is what you have to do first'. And we're like 'don't forget about the last things but make sure you do the top things first'. And then you get down to the bottom, they're all equally important. (Tom, Interview 2, #34)

Keira demonstrated exceptional (and unexpected) insight in making reference to a higher level of thinking in relation to the teacher's pedagogical rationale for using the card sort. Note that **all six** students were able to articulate two of the three primary purposes for using the 'boxing up' strategy (see Section 4.1.2) and **the other five** students made reference to these purposes when asked about the card sort (which is implicit in the pedagogical rationale for using the 'card sort' strategy).

4.2. General appreciation of the rationale behind progressive pedagogies

As well as articulating some of the reasons why they thought the teachers used the three strategies trialled in cycles 3 and 4, students also demonstrated a more general appreciation of the pedagogic rationale behind various teaching approaches/strategies that the research group might characterise as 'progressive'. Each of these is dealt with in turn below.

4.2.1. Problem-solving skills

Both the 'model solution' strategy (cycle 3 – see Section 4.1.1 above) and the 'boxing up' strategy (cycle 4 – see Section 4.1.2 above) focused to a large extent on developing generic problem-solving skills that students could apply across different mathematical domains. In cycle 3, all references to problem-solving skills were directly related to the 'model solution' strategy. In cycle 4, whilst most of the references to problem-solving skills, related to the 'Boxing up' strategy, **four of the six** students (**Nasri, Simon, Keira and Anna**) demonstrated a wider appreciation of the value of problem-solving skills in becoming more independent in tackling challenging problems:

Um, because ... because it can help you with your answer and, like, help you solve the question. For instance, if you're [referring to the teacher] busy with someone, and your partner doesn't know, you can try doing it yourself. (Nasri, Interview 2, #23) It could give you like ... like a word problem, and then it says, like, lots of things that you don't need. There might be a few red herrings, and then you can just get rid of those in the question. (Simon, Interview 2, #42)

Cos, like, some questions can be challenging to find what the question's asking you, so it could be, like ... instead of just giving you a sum, it's telling you ... and it's put in a more worded way. (Keira, Interview 2, #32)

Cos they, like, give you information that maybe can make you, like, think it's a really hard question when you read it. Like, makes you panic, it's like, you don't know what to write, maybe, because it would be too hard to answer. (Anna, Interview 2, #40)

4.2.2. Mathematical reasoning

The 'model solution' strategy (cycle 3 – see Section 4.1.1 above) appeared to generate significant opportunities for students to consider different methods for solving a problem and to explain/present their mathematical reasoning to others.

As well as referring to this strategy, **four of the six** students (**Tom, Simon, Keira and Anna**) recognised the more general need to focus on the different methods that could be used to solve problems (see the evidence presented in Section 4.1.1 as well as the following example):

It's just, like, it's a more quicker way, rather than thinking, like, 'which one should I do?' I just know I can do that one. Because I know it's the best solution to finding the problem. (Anna, Interview 1, #32) – quote already used elsewhere

As well as referring to the 'model solution' strategy, **four of the six** students (**Neal, Tom, Simon and Keira**) demonstrated a wider appreciation of the value of students explaining different methods to each other (see the evidence presented in Section 4.1.1 as well as the following examples):

So, like, everyone could understand which one, like ... um ... well like, so everyone can know the right one ... (Neal, Interview 1, #60)

Because somebody could just say that 'something, then something, now this bit', nobody would understand why. So they need to be able to explain that as well. (Simon, Interview 1, #52)

To a lesser extent the 'boxing up' strategy also generated opportunities for students to demonstrate an appreciation of the importance of mathematical reasoning, for example **Keira** recognised the need to focus on methods used in solving problems (see also evidence presented in Section 4.1.2):

And, like, to think more about the questions and why we're actually doing ... um, the methods we do. (Keira, Interview 2, #8)

4.2.3. Working collaboratively

Both the 'model solution' strategy (cycle 3 – see Section 4.1.1 above) and the 'boxing up' strategy (cycle 4 – see Section 4.1.2 above) involved students discussing in pairs, small groups and as a whole class. **Five out of six** students (**all except Nasri**) referred to the benefits of collaborative working in cycle 3 and, of these five, four students (all except Anna) also made reference to these in cycle 4 (see the evidence presented in Sections 4.1.1 and 4.1.2 as well as the following examples):

Imagine if your partner knows it but then you don't know it ... and then, like, you'd just been like struggling on it. And you [referring to the teacher] said you can talk to your partner if you want some help. And then, yeah, it helped a lot as well. (Neal, Interview 1, #14) Because I knew some of the answers, and then, when it came to matching up, I could easily do it. It was, like ... some of them were hard to do but some of them were easy. And the hard ones, my partner helped me, kind of, with it. (Tom, Interview 1, #10)

And we think ... we discuss with our partner ... which one do we think is the most relevant, the most crucial? (Keira, Interview 2, #72)

4.3 Enjoyment of learning maths

All six students responded positively in both cycles 3 and 4 when they were asked whether they enjoyed the research lesson. When asked to give reasons why they enjoyed the lessons, students provided a range of reasons.

Three out of six students (**Simon, Keira and Anna**) claimed that they enjoyed the research lesson in cycle 3 because of opportunities provided by the teacher to work collaboratively (see the evidence presented in Section 4.2.3 as well as the following examples):

Because we got to discuss as a class how we do things differently, and which one is the best solution. (Simon, Interview 1, #10) ...

Because, it wasn't just like you was independent, it's like you can talk to other people and discuss different methods. (Simon, Interview 1, #12)

Um ... I enjoyed when we did ... when we, like, with the A3 piece of paper, and we had to work in partners, like, why we did the model solution, and like what it would help us with, why we had to do it. Keira, Interview 1, #10) – most of quote already used elsewhere

I like, like for me, finding the pieces, which was interesting, because you get to actually do, like, working out and stuff with our partner. (Anna, Interview 1, #14)

Simon also referred to collaborative work as the main reason he enjoyed the research lesson in cycle 4:

Because I prefer to do things with, like, team work, rather than independent. And then that was, like, doing it with another person, so that's team work. (Simon, Interview 2, #54)

Three of the six students (**Nasri, Neal and Anna**) described how they enjoyed the research lesson because it involved a different type of activity to what they encountered in 'normal' lessons:

Because sometimes we, like, just do book work. And, like, sometimes we like have fun, like ... Sometimes we make posters ... to help us with, like, revision. (Nasri, Interview 1, #12) ... Because we've never really done anything like that [the card sort] before in lessons, that's why. (Nasri, Interview 1, #14)

Because it wasn't, like, one of our usual lessons. (Neal, Interview 2, #6)

Like, rather than just writing lots of stuff, like we could do more activities (Anna, Interview 2, #10)

Three out of six students (Neal, Tom and Keira) claimed to enjoy the high level of challenge they experienced during the lesson:

Because ... I've never really seen that before. I've, like, usually just seen like the shapes which have numbers around them, not like, 'c+12' or something like that. Yeah. (Neal, Interview 1, #8)

Cos, some of the questions were hard, and some of the questions were easy. (Tom, Interview 2, #4)

I enjoyed it because um ... like, for me, I like learning about, like ... I like getting pushed. So when you were asking us questions, and it, like, it helped us, like, push ourselves. (Keira, Interview 2, #8)

Responses **from Tom and Anna** suggested **these two** students attributed their enjoyment to overcoming challenging problems and understanding the work:

Um, I enjoyed answering the questions ... because they were challenging but, like, if you work it out the proper way, then it was easy. (Tom, Interview 1, #16)

I just, like, understood the work well. So I was enjoying, like, that I knew what to do. (Anna, Interview 2, #16)

4.4 Success in learning maths

When asked how well they did in the research lesson in cycle 3, **all six** students appeared satisfied (first **four** responses below) or very satisfied (last **two** responses below) with their level of success:

Um, I did alright, yeah. (Neal, Interview 1, #70) Yeah. I did, like, 7 out of 10. (Tom, Interview 1, #42) I think I did quite well. (Keira, Interview 1, #68) I think I did OK. (Anna, Interview 1, #54) Um ... I think that I did really well. (Nasri, Interview 1, #54) Um, I think that me and my partner did really good. (Simon, Interview 1, #52)

The most common reason given by students (**three out of six** – **Neal, Keira and Anna**) for their success in the research lesson in cycle 3 was a high level of understanding supported by getting correct answers:

Some questions were quite hard and, like, sometimes you could get like one answer, but then you have to, like, simplify it even more, so you get it. (Neal, Interview 1, #70)

Um, I know I did well because, like ... when I was figuring it out, like ... and then you were going through it on the board, I was getting the answers right. And ... like, when you were telling us stuff, I was understanding it as well. So, like, I was understanding it really well. (Keira, Interview 1, #72)

Yeah, I think I understood the work, like what you were saying. And so I would probably remember, like, what you were saying. (Anna, Interview 1, #54)

Two out of six students (Nasri and Tom) related their success to their own effort and engagement with classroom tasks:

I did my work, like, I only got to do two questions because I was stuck on one of them. And, like, I answered lots of questions, and I was engaging with the class. (Nasri, Interview 1, #54)

Because I did try but, at the same time, I was talking, but I did my work. (Tom, Interview 1, #46)

Simon was the only **one of the six** to also attribute his success in cycle 3 to managing to work well with other students:

Because we was talking a lot and we was, like, doing that as quickly as we could. But we was re-checking it, to see if it was right. (Simon, Interview 1, #52)

Keira offered a second reason for her success which was closely related to the project's aims of developing a greater appreciation amongst students of the teacher's pedagogical rationale:

I learnt, like, more understanding of why we do, like, why we write down stuff in our book. And, like, if you do working out, and we discuss it, then we write it down. I understand why we do that more. (Keira, Interview 1, #68)

When asked how well they did in the research lesson in cycle 4, **all six** students appeared satisfied (first **three** responses below) or very satisfied (last **three** responses below) with their level of success (this was similar to the levels of success they reported in cycle 3):

Um ... I did alright. (Neal, Interview 2, #50) I did alright. (Tom, Interview 2, #40) So it went well I guess. (Anna, Interview 2, #64) I did good. (Nasri, Interview 2, #51) Um ... I think I did pretty good. (Simon, Interview 2, #62) I think I did rea... um, pretty well. (Keira, Interview 2, #84)

One of the most common reason given by students (**three out of six** – **Neal, Keira and Anna**) for their success in the research lesson in cycle 4 was (as in cycle 3) a high level of understanding supported by getting correct answers (note this was the same three students as in cycle 3):

And ... making sure I understood the question ... find out all the key points (Neal, Interview 2, #50)

Um, because when you were marking the questions, I was ... getting it, like, correct. And, when you would ask us questions, and I'd put my hand up and answer them, you would, like, tell me 'well done'. (Keira, Interview 2, #86)

Yeah, I understood the questions and, like, I knew the answers. I worked it out ... the tree diagrams. (Anna, Interview 2, #64)

The other **three of the six** students (**Nasri, Tom and Simon**) related their success in cycle 4 to their own effort and engagement with classroom tasks (note Nasri and Tom were also the only two students to give this reason in cycle 3):

And I wrote down all the answers, and then I got to the final answer, with the card. (Nasri, Interview 2, #53)

Um ... I answered some questions. And then I tried to work out the harder ones, but then, if I couldn't work them out, then I just left them, until ... I did the other questions, then I had to come back to them all. (Tom, Interview 2, #44)

And I completed a whole page. (Simon, Interview 2, #62)

Two of the six students (**Neal and Simon**) also attributed their success in cycle 4 to managing to work well with other students (note Simon was the only student to give this reason in cycle 3):

And, like, getting help from my partner as well, helped me with the work a bit. (Neal, Interview 2, #50)

And, yeah, and me and my partner agreed very easily. (Simon, Interview 2, #62)

Nasri offered a second reason for her success which was related to engaging successfully with progressive pedagogies, in this case the 'boxing up' strategy:

Um ... because I used the card [with 'boxing up' questions] to help me. Yeah, like, on one of the questions, I didn't understand, and I used the card. (Nasri, Interview 2, #53)

4.5 Preparation for tests

During both cycles 3 and 4, **four out of the six** students (**Nasri, Tom, Simon and Anna**) referred to preparing for tests when asked about the purpose of the two strategies. When students were asked about why they thought their teacher had used the 'model solution' strategy, **Nasri, Simon and Anna** mentioned tests in their responses:

Because, like, if you're doing revision, you could ... it could help you, like, if you're doing a test, it could help you, like get well on your test. (Nasri, Interview 1, #40)

Because, like, in GCSE exams, you're going to have a time-limit to do your test. So if, like, you need to finish the question, but you're using the method that takes too long, then you might not get the marks for that. But if you just finish, using the quicker method, then you could get the marks for it. (Simon, Interview 1, #32)

Well, like, in case we forget it. At any point, like, when we're trying to revise, we know it's right there and you can, like, book back at it. It helps. (Anna, Interview 1, #36)

When asked where else they might use the 'boxing up' method outside the maths classroom, **Tom, Simon** and **Anna** made reference to tests:

Um ... So you, like you're doing a paper, yeah? And, like, you don't know the answer, you have to, like, think for a moment what the question was asking you. And all the information you have, you have to underline it. And then, with that key information, you have to work from there slowly. (Tom, Interview 2, #24) – quote used already elsewhere

Um, well, like, in let's say, like, tests like science or English when you're … it could, like … it could give you like … like a word problem, and then it says, like, lots of things that you don't need. There might be a few red herrings, and then you can just get rid of those in the question. (Simon, Interview 2, #42) – most of quote used already elsewhere

Yeah, I guess I could ... for like science, and things like that, in tests maybe in future, like GCSEs. (Anna, Interview 2, #72)

Two of the six students (**Neal and Keira**) did not mention preparation for tests in their interview responses to any of the questions in cycles 3 and 4.

4.6 Solidarity between learners

One of the primary purposes of the 'model solution' strategy in cycle 3 was to encourage students to engage with others' ideas. Students' responses relating to this purpose (see Section 4.1.1), together with those relating to students' appreciation of the benefits of working collaboratively (see Section 4.2.3), resonated closely with the notion of solidarity between learners. This manifested itself in students exhibiting empathy towards other learners and a sense of shared responsibility for learning (both closely related to each other).

In cycle 3, three of the six students (Neal, Nasri and Simon) demonstrated solidarity with other learners.

Neal, Nasri and Simon exhibited empathy with other learners by recognising that they may have their own valuable contributions to make:

Like, some people could have different, like, reasons why ... because, like, not everyone thinks the same. So, like, you could have your own reason, whereas someone else could have their own reason for their own solution. (Neal, Interview 1, #58)

Because it might be, like, easier for them to help them. So, like ... if someone did the other one, but then they wasn't sure how to do it ... And the one we were using, they knew how to do that, they could have done that, so it would be easier for them. (Nasri, Interview 1, #48)

And we was comparing our ideas. So ... and then we would discuss it to see if one person thinks the other thing. And then we'd see what ... what is, like, the most understandable answer. (Simon, Interview 1, #52)

Neal and Simon demonstrated a shared responsibility for learning by emphasising the value in sharing ideas with others:

Because, like, working together as a class, it's just not like you working by yourself ... you're getting your own answer but then your answer is like ... not correct ... And then someone who has the correct answer could explain, like, how they got the answer. (Neal, Interview 1, #66)

Um ... because everybody else might think differently. And then that one person, like, could think of a new strategy (Simon, Interview 1, #38)

In cycle 4, four of the six students (Nasri, Tom, Simon and Keira) demonstrated solidarity with other learners.

Nasri, Tom and Simon exhibited empathy with other learners in highlighting the value of helping them:

Yeah, it could be useful for other people too, because, like, they might need help in the questions, and they may not understand it properly. And they can, like, use that to help them understand it. (Nasri, Interview 2, #41)

Um ... Say, like, someone's asking you a question, then don't give them, like, the straight answer, give them, like, an explanation, like what they need to do next. (Tom, Interview 2, #30)

Because I liked doing the independent work, and helping my partner as well. (Simon, Interview 2, #8)

Simon and Keira demonstrated a shared responsibility for learning in emphasising the importance of agreeing with others:

Cos it's like ... like agreeing with our partner and seeing what, like, what we should do ... to agree. (Simon, Interview 2, #14)

And with the, um ... when we did the card thing, and ... when you put your hand ... when I put my hand up, and I said what I thought was most relevant, like, people agreed, as well. (Keira, Interview 2, #86)

5. Thematic analysis of research group meetings (2018-19)

5.0. Coding theme used for thematic analysis of research group meetings

The following five coding themes were used for the thematic analysis of the research group meetings. Note 'RGM' is used throughout as an abbreviation for 'Research group meeting'.

5.0.1. Project research tools

- Research journals
- Student surveys
- Student interviews
- Peer observations
- Video-stimulated reflection

5.0.2. Strategies tried out

Cycle 3/4 strategies:

- Model solution
- Boxing up
- Card sort

5.0.3. Critical research processes

a) Pedagogical imagination:

- Reflect on own practice
- Reflect on existing practice
- Relate theory to practice
- Consider alternative practice

b) Practical organisation:

- Negotiate/design strategy to try out
- Consider constraints on practice
- Address constraints on practice

c) Explorative reasoning:

- Plan how to use evidence
- Use evidence to evaluate strategies
- Relate evaluation to own practice
- Relate evaluation to existing practice
- Relate evaluation to alternative practice

5.0.4. PAR characteristics

a) Collaborative:

- Consideration of relationships/roles
- Evaluation of relationships/roles

b) Participatory:

- TRs involved in design of research
- TRs involved in evaluation of research
- TRs involved in evaluation of strategies

• TRs involved in analysis of data

c) Positive social change/Impact on TRs:

- Impact on TRs' classroom practice
- Impact on TRs' thinking/beliefs
- Impact on TRs' agency/efficacy

5.0.5. Impact on students

- Appreciation of pedagogical rationale
- Engagement with project strategies
- Appreciation of how to achieve success
- Engagement with progressive pedagogies
- Appropriateness of responses to tasks
- Responses/engagement of all students
- Responses/engagement of target students

5.1. Project research tools

The following evidence was used in the thematic analysis of the research group meetings during the second year of the project (Meetings 8 to 15).

Note the references identify the research group meeting and the extracts of dialogue used in the thematic analysis, e.g. (RGM9, #20-22) refers to the 20th, 21st and 22nd extracts of dialogue used from Meeting 9.

5.1.1. Research journals

There was no additional evidence of significance to add to that from the first year of the project.

5.1.2. Student surveys

The researchers discuss, negotiate and agree changes to the survey for cycles 3 and 4, enabling better comparisons of students' general notions of success before and after the project:

- PW: This will come out of the data analysis ... What I was really aware of was ... we had a survey at the start and interviews at the end. And they were two very different things and it was really difficult to get a comparison out of it. But this time we're going to do interviews after the first lesson and interviews after the second lesson, with the same students, hopefully. So, seeing that, I think, will give us a much stronger feeling of what the development is, yeah? And I guess the same with the surveys, are you going to do surveys with both lessons as well?
- TC: It depends on how we structure ... I think, for it to happen, it needs to happen in the same lesson, they need to do it straight away. ...
- PW: The other option would be to do some sort of survey before you start with the class. And then do a survey right at the end. And, if the questions were simply around things like "How successful were you in today's lesson? How do you know?"
- AF: That might be a good idea.
- *PW: So, it's shorter. And then compare at the end and see if, by doing these sorts of things, they've become more aware of how successful they are.*
- TC: But they wouldn't necessarily have memory of the first lesson?
- PW: No, it would be more general, "How successful are you in maths? How do you know?" ...
- TC: The other one [question] could be with regards to the visibility of the pedagogies.
- PW: So, it could be something like: "How does your teacher help you to be successful?" ...

- AF: And maybe another one is "What tools do you have to help you be more successful?" And that takes it away from the teacher, more on themselves and whether they've been able to pick it up.
- TC: Do you think ... this needs to be re-phrased ... "Does your teacher make it clear how ... to be successful?" or "Does your teacher make it clear why you're using an activity?" ...
- AF: It's OK to be general, basically, because we want to capture what they know without too much leading questions from us. And to see if they're able to distinguish by the end of the year, by the end of the cycles, what we do mean by that. Because that's part of what we're trying to achieve, is being really visible, being really explicit to begin with but then, kind of, toning that down and allowing them to see meaning and to read intention, eventually, right?

(RGM8, #51-62)

5.1.3. Student interviews

The researchers review the questions from the cycle 2 interviews and agree interview questions for cycle 3:

- *PW: On the last page, these are the questions we had from the first interview. Do you want to do something similar where maybe questions 1 and 2 might be the same?*
- *AF: Yeah, they would still be applicable.*
- *PW: ... We could ask them that question about how successful they were ...*
- TC: That's the last question, "How well did you do?" ...
- PW: OK, so we could have 1, 2 and 6 again ...
- AF: Yeah
- TC: And then the other ones would be more specific to this?
- *PW: Yeah, and it would be literally asking the same questions that we asked them when we were getting them to do the card sort [followed by agreement].*

(RGM9, #64-71)

The researchers discuss, negotiate and agree the questions to be used in the cycle 4 interviews:

- *PW: I still think we should have the thing about enjoying because there are things that are, sort of, coming out.*
- AF: I think so.
- *PW: I'm not sure that "Did you notice anything different?" is still relevant.*
- AF: I don't know that it is ... I mean it's still quite different as in we don't use the card sorts in most lessons, or?
- TC: Yes. and it's also a way of prompting them to start talking about structures.
- *PW: OK, so leave that in there, yeah? So, obviously now we need: "Why do you think ...?" Now we just need to go back to the same question we used on there [question 3 from cycle 3 interviews] ... So, question 4 will be: "How well did you do in today's lesson? How do you know?" That will still be an important question to ask. Is there anything else we want as an additional question?*
- TC: I was thinking how can we relate this to the papers that we read, how can we use the interviews to ... connect culture and maths? ...
- PW: So, what about: "Do you think that might help you outside school?" or something ...?
- AF: I like the sound of that, something ... "How might this be useful in school, or outside of it, or beyond?"
- PW: ... Why don't you just say "... outside of maths lessons"? ...
- AF: I was also thinking, it would be interesting to gauge from them how useful they find the card sort ... so something about the card sort, maybe being in there? ... Because, if they're engaging with the card sort, and they understand the pedagogies, and our point for using the pedagogies, then it might be interesting to see ...
- *PW: How about something like: "What did you think about using the ...?"*

• AF: "... the card sort?" Something open like that.

TC recounts his experiences of conducting interviews:

• TC: When I was doing one of the interviews with Neal, he was taking long pauses, basically, to think. Very long pauses, sometimes like 30 seconds ... I didn't want to influence the response and I was just giving him time. Sometimes I would say things like 'it's fine', I would say 'brilliant' quite a lot, it appears that I did say that.

(RGM14, #7)

The researchers discuss the potential benefits of teachers conducting surveys and interviews themselves:

- PW: And I think one of the things that's interesting in the project is getting you two to do the interviews, and to collect the surveys ... Because you know them, you can put them at ease, you can support them. You know, when they're taking 20 or 30 seconds ... if that was with a stranger, they might be a lot more uncomfortable in that situation, thinking "Oh, I don't know what to say, I don't know what to say!" Whereas with you, you can just say "It's OK, take your time". And that's more likely to have an effect, because you know them, you have a relationship with them. And you will know when that person is just thinking, and when they're feeling really uncomfortable inside because they don't know what to say.
- AF: ... I had the same situation with one of my students ... he wasn't able to articulate the boxing up method ... And you know, as a teacher, then ... you kind of make a decision about whether you want to prompt at that point, or whether you want to ask the question in a different way, or whether you just accept that they've not got that ... Even if the interview does go in a way that's, let's say, where we ask a question just to prompt them a little bit more, we can kind of reflect on it and go; "Actually I did prompt them ... and, without my prompting, they wouldn't have got there". ...
- *PW: ...* But the evidence is still there in the interview, cos all the dialogue is there, that they didn't get it first time round ... To some extent it's more important, once you've reminded them about it [the strategy], do they know why you did it? ... It's quite interesting whether they can remember it, because that does show how much of an impact it had on them, and whether they saw that as an important part of the lesson.

(RGM14, #8-10)

5.1.4. Peer observation

The researchers discuss the relative merits of pre-selecting clips to focus on before the VSR discussion or allowing more flexibility to select clips during the discussion itself (making use of the timeline):

- TC: I've thought about this ... I think it introduces a bias on the discussions, and it's a limitation on what we are commenting, if I select. Because it's just one person thinking about what we want to address ... It might be interesting if we pre-select the moments that we want to see and maybe open that, I wouldn't mind that. In terms of practicalities, I don't think that it's possible, given our time constraint, to actually watch the whole video and decide in the moment, which moments to analyse. ... Instead, ... if we're peer observing, we could pre-select some moments, and then go and watch the video, which I would be happy to share ... and decide which moments, to have like two each, to discuss. And that would be more practical and would fit into an hour probably ... both videos. And would provide a focus... If we give a rationale for the analysis before: "we're going to focus on the strategies, or we're going to focus on these for now".
- AF: I think there's pros and cons to doing both. So, my paper [Coles (2013)] will go into exactly that in a minute ... I completely agree, I think it's best to pick the moments and then let the discussion be open about what is key about that clip, or that section. Because when you frame it too much, when you preselect beforehand what the topic is that you want to discuss ... there's pros and cons to that; the pro is that you're maybe having a more effective discussion and you're getting less side-tracked, depending on

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the experience of the teacher and their awareness, their ability to have that conscious awareness of what they need to be working on. But, if that's there, if they have that consciousness, you can just have a more effective discussion, get straight to the point. So, it depends on the context. But the ideal is that ... you don't want to have it super-defined. You want it to be as much about where this discussion could take us, because it's a learning process, and acting on that, on those thoughts, and literature and discussions, so that you can improve your practice, if that's want you want to do.

• PW: I think what we've developed is something a bit different, which does a bit of both. Actually, by chance really, the timeline has been really useful for facilitating these discussions. ... When people have looked at their video, they've thought about: "Well, let's start the conversation by looking at these two clips" ... And as the discussion goes on, everyone's got the timeline in front of them, they can sort of say: "Well, yeah, to understand this a bit more, why don't we look at this bit of the clip?" And they look at the timeline and it's then easy to locate the bit of the video you want to watch. So, it's leaving open the option of actually looking at something else. But I think we started off with people having a pretty good idea of what they were going to start off looking [at], maybe it wasn't presented in advance ... This idea of timeline I think really, kind of, retains that idea of flexibility, and it's something we could put forward as an idea. ... I think what's important is that we maybe do decide some protocols for how we're going to do it.

(RGM8, #6-8)

5.1.5. Video-stimulated reflection

PW raises the issue of who owns the videos (which was discussed in the papers):

• PW: I think it's quite important that I don't have the videos and that that's not part of the data analysis. Because, as soon as you do that, this idea that somebody else is going to have you on camera and they're going to be coding what you do, I think that would, kind of, be quite constraining.

(RGM8, #30)

AF relates discussion of papers to her own previous experiences of conducting VSR as part of the project:

- AF: I think what we've been doing maybe, consciously or unconsciously, has been actually quite good so far ... it feels like what you just presented to us Pete, all the benefits that video-stimulated discussion seems to have is what we have benefitted from too. It's had a massive impact, I think, in how I see and observe how the students do, how I present it to the students.
- *PW: So, is the thing to try and capture the protocols of what we've been doing, first?*
- *AF: I think that would be a good idea.*

(RGM8, #31-33)

The researchers continue to discuss, negotiate and agree the protocols for the VSR meetings:

- TC: So, we might want to do this differently, it's just a suggestion, of having access to the video ... and each of us could pick a point or two from the lesson with a focus on the strategy ... I like the idea of doing a narrative of what happened ...
- AF: Yeah, I like the idea of that ...
- TC: ... and then moving on to evaluating. And when we're doing the evaluating, that's where it becomes important, the language that we use ...
- AF: So, having each other pick the key points?
- TC: ... So, I would pick one point from my lesson ... to think OK, this was one of the strategies ... now I'm thinking I'm going to describe what is happening there, and then open for an evaluation of what happened. And you [Pete] would do the same thing for the same video, choose a 3 or 4 minute [clip], where a strategy is being used, could be the same.
- *PW: Right ... OK, now I think that's a big change, cos now you're saying you're going to give me the video.*

- TC: But I was going to ask you, is that a problem? ...
- PW: What I thought you were going to suggest, and maybe this is a better alternative ... I'm still a bit wary about me having the video ... we haven't actually got ethical approval for that either, by the way ... But how about this ...? You [Tiago] pick something from your lesson and something from Alba's lesson
- AF: I like that ...
- PW: A little narrative about each clip. Alba picks something from your [Tiago's] lesson and from her own lesson, and that's what we use for the basis of the discussion. You might have picked the same clip, but that's fine, because you might have a different narrative.
- AF: It opens the discussion up about why the other person may have picked that. ...
- TC: But I'm just thinking, would it not be useful to have it open, because that's the meaningful critique where we're all collaborating in an inquiry on ...
- *PW:* But isn't that what the timeline does? You're going to do the timeline as well, and you're going to observe each other's lessons?
- AF: Yes, peer observations ...
- *PW: I mean, I think this timeline is something we've come across, perhaps by chance, it's something that I did when I came to observe, but it seems quite powerful. So, maybe that's the way of keeping the flexibility, to have the four clips, with the narrative, but then the timeline then allows us to broaden out that discussion.*
- AF: ... I think us both having access to the video beforehand ... it would open up the discussion maybe in interesting ways that I'm happy to explore ...
- *PW:* But you don't necessarily have to talk to each other about it, you just need to share the video, write a narrative, and then that's what we focus the discussion on in the meeting.

(RGM8, #34-50)

PW reminds researchers of the VSR protocols, the focus of the discussions and the rationale for the strategies:

- *PW:* I've sent you these before, but I'll just give you a copy of these notes ... The second and third bullet points remind us what the questions were, that the students had [from the 'boxing up' strategy], and what the reasons were that we articulated for using the pedagogy and for then using the strategy. So, that might be quite a handy thing to have for reference ...
- I think last time we looked at Alba's lesson first, is that right? So, shall we look at Tiago's lesson first this time? And we'll follow the same protocol where Tiago chooses a clip, Alba chooses a clip, and we use that to prompt discussion. And the discussion is about 'how effective was our strategy?' And the strategy was ... flagging up earlier about the boxing up [strategy], but then using the card sort to consolidate their understanding of your rationale at the end of the lesson. And the pedagogy, that the rationale was for, was this idea of using the two green questions to get them to think about a structure for solving problems that can be used with other problems, spending time planning how to solve a problem rather than jumping straight in to solving it, and giving them help in decoding the question. (RGM13, #1)

TC explains his rationale for choosing the video clip from his lesson:

• TC: I was going to go for 9 minutes, reasons for using boxing up method [reading from timeline] ... that's one of the things that I think is worth discussing, about how this is introduced. Because I, kind of, I was trying to make it very explicit, what the reasons were, when I was showing them. But then I did a think-pair-share, just to see what other reasons that could come up, and I don't think that was necessary. Either one or the other, not both.

(RGM13, #2)

AF explains her reason for choosing the video clip from TC's lesson:

• AF: I chose that because I thought that ... it just showed very, very clearly how well they'd engaged with the purpose of the green box questions, and they understood the reasons very clearly. They understood that they were all good reasons, but not necessarily the reasons we'd use the green box, which is, you know, some really higher-level thinking.

(RGM13, #13)

The researchers discuss how VSR enables them to view the lesson from the students' perspective:

- AF: It's amazing, I think, how much you can find what you'd want to develop, or change or test, when you watch yourself. That potential for improvement feels so much bigger when you watch yourself, I feel like ...
- TC: But also ... I think that, not only, it allows me to see myself teaching, but also it puts me in the perspective of a student. So, sometimes I feel like it's taking too long. But now I'm seeing it from ... how do they, students perceive that? ... I'm watching someone ... myself ... teaching. So, I can see, OK, this felt too long. And this is how it feels, how it looks, or how it feels to ...
- AF: ... to them, to students as well.

(RGM13, #17-19)

 \rightarrow See also Section 5.3.3 for how researchers used research literature to inform use of VSR.

5.2. Strategies tried out

5.2.1. Model solution (cycle 3)

The researchers consider some possible reasons behind the 'model solution' strategy:

- TC: So, the discussion here about the hidden pedagogy is "What is ... is it important to have something written down in your book? Is that going to be used later?" ...
- AF: Do students use the notes in their books? Do they do it automatically but then never really look back at it? ...
- TC: Wouldn't it be nice for students to know that, actually, the teacher is asking me to have these notes because ... she's thinking that I'm going to use this later. Do they know that? Do they make that link? I doubt they do.

(RGM9, #20-22)

PW summarises what the researchers have agreed about the 'model solution' strategy. He asks some questions to help finalise the details and to clarify the ways in which it relates to progressive pedagogy:

- *PW: So, let's try and really narrow down what it is we're asking them to do. We're asking them to write something in their books. What is it we're asking them to write?*
- AF: ... Model examples ... if they have one clear, consistent example ...
- TC: ... write one example and not three ...
- AF: ... if you have that conversation about "Why am I asking you to have one?" Because you then have one clear example when you need it ...
- PW: We're going to draw attention to one thing we ask them to do ... as long as you're clear that you call it something ... the 'model example' ... And when we refer back to it, the question can be "Why did we think it was important for you to write down the model example?" ... So, what I'm not clear about is ... are you going to give them a problem and ask them to come up, first of all with their own solutions, and then come to some sort of agreement as a class what a model solution might look like, and then get them to copy to that? Are you going to give them an example and ask them to copy that as a reference point?
- AF: ... We haven't discussed that ...

- TC: We haven't agreed ...
- AF: But I think Pete has a good point in that if you get them to engage with the question first, before you just show them, then that might be a better way of, kind of, getting them to read it, absorb it, and then engage with the question, have a go ...
- PW: And it might be ... Cos we're trying to do progressive pedagogies as well. To some extent, "here's an example, copy it down in your book for reference" is kind of ... I'm not saying it's not a good strategy, but the project is about how actually students struggle when the pedagogies are a bit more open-ended and more progressive, cos they're less-clearly framed. So maybe the idea of a problem, and then as a class agreeing a model solution and then everybody write that down ... for use in the rest of the lesson ...
- *PW: So, they would all come up with their own solution. Maybe one group presents it to the class and then the other groups use their solutions to suggest improvements until ... it could be like crossing it out ... until you've got something that the whole class agrees is a model solution. That is a progressive approach, yeah, because it's taking on board lots of different ideas.*
- AF: Working together, it's collaborative ...
- *PW: It's not having a pre-conceived idea of what the model solution will be, from the teacher. So, it's sort of agency from the students.*

(RGM9, #26-37)

PW prompts researchers to consider and articulate the rationale behind the 'model solution' strategy:

- PW: So, what is the rationale for doing this?
- AF: I guess the benefits, what we want to get the students to engage with are something that they can refer back to ... for revision, homework, general learning ...
- TC: But it's also engaging ... it's, kind of, thinking that what takes place in a lesson has a value, so there is a value of copying something down if we do revise using our books ... it's respecting the fact that they are doing that as an investment for later on.
- *PW: I think these are two different pedagogical things we're doing. We're generating a model solution, and we're getting them to copy it, and then use it. ...*
- TC: I think the second one is less progressive ... the copying ...
- AF: But I suppose, the act of it isn't progressive, but what it enables you to do, which is give you that independence. Say you have a similar problem in the future, you then have that tool right there ... to try and solve the problem independently ... Finding similarities between problems and thinking "Ah, yes, I do have that example that will help me".
- PW: I think that's, what you just said there, is a good rationale, recognising similarity between problems and the solutions. ... And the generating the model solution in the first place, what's the rationale behind that?
- AF: I guess that's just about making it very clear how to lay out your work, because they will come up with different ideas ...
- PW: So, sharing different ideas ...
- AF: So, the sharing of different ways of solving the problems.
- *PW: ... And then, in doing that, you're comparing advantages and disadvantages ...*
- *AF: ... of different methods, exactly. And coming up with one clear way, but meanwhile appreciating that there might be more than one way. ...*
- PW: But it's negotiating ...
- AF: ... one clear way ...
- *PW: ... the agreed solution ... And it's about developing communication skills, ways of communicating mathematics.*

(RGM9, #38-52)

AF summarises how she and TC applied the 'model solution' strategy in the lesson:

• AF: So, what's the actual perimeter of the shape? We sort of had a couple of ways that you could be approaching this in mind when we formed the problem ... Then we went into discussing, negotiating different methods and then coming up with one method for each of these parts; writing that down clearly on the board; asking them to then copy that down; having a brief discussion at that stage, both of us did, about why it's important that we do come up with a model solution, and why is it important that we then copy it down?

(RGM10, #1)

5.2.2. Boxing up (cycle 4)

The researchers discuss further and refine their ideas for using the 'boxing up' strategy:

- PW: So, you're going to flag up that "There is a strategy going on in this lesson where we are going to help you to understand why we are doing this ... we're going to come back to that with the card sort. And now we're going to give you ... we're going to use something today to help you learn and then we're going to talk about why we've used that. ... And the thing we're going to use is these questions which help you to see how to go about tackling a problem."
- AF: Yes, decoding the question.
- TC: I wouldn't even show them the other four ['boxing up' questions] for now ... I don't know, what do you think?
- AF: I mean, I think it's OK to show them. We can say "This is what we're going to work on today, and we will reinforce the other steps ... even more at another time".

(RGM12, #10-13)

The researchers discuss the rationale for using the 'boxing up' strategy:

- PW: So, what do you see as the primary reasons for doing it? ...
- AF: One of the main rationales is, yes, providing a structure. And they are applicable to any problemsolving questions and they will help me with anything. It's understanding the question, that's the main rationale, and if I can understand the question then I can then translate that into a correct calculation, if I have the content knowledge. ... In maths, sometimes the questions are complex enough that you need to plan what you're going to do, and this is what this allows you to do.

(RGM12, #14-15)

5.2.3. Card sort (cycles 3 and 4)

The researchers discuss using the card sort to address students misunderstanding the teachers' intentions:

- AF: I want to come back to this, I want to, sort of, clarify it a bit more ... because the common points that came up from those surveys, that students were sort of misunderstanding, you know, the whole idea of "Miss is asking me this question ..." ... you know, "she's checking" ...
- *PW:* So that's a strategy we're going to use to make whatever pedagogy we use more explicit? And, I think it relates to my idea I had about getting them to do some sort of card sort, where on the card sort would be the reasons why the teacher might have done this, and then they, sort of, have to prioritise which ones they think the teacher had most in mind, or something?
- *AF: Yes, that could be good. That would address that.*

(RGM9, #23-25)

The researchers discuss how to use the card sort to facilitate a discussion about the rationale behind the 'model solution' strategy:

• *PW: So, the strategy then ... cos that's the pedagogy ... will our strategy be some sort of ... well, do you like the idea of the card sort? Maybe based on the rationale you just talked about here? ... So, there*

might be two questions, you might say "I want you all, in your groups, to sort these cards according to ..."

- *AF: ... the order of importance?*
- PW: Yeah, the order of importance for generating the model solution ... And then "I want you to sort them now in terms of copying it and applying it to other problems. What was the purpose of doing that?" ...
- AF: I like the sound of that ... The only potential issue from that is ... is it a bit leading because the strategies will be visible to them? Do we want to have any type of discussion before the card sorts to gauge whether they understand the point of why we're doing what we're doing, from themselves, completely coming from them, first?
- *PW: Well, some of the strategies could be genuine strategies that we think are important, and others could be ones ...*
- AF: ... "That Miss is checking that I know what I'm doing" ... or something generic that they tend to respond, maybe ...
- *PW: Yes, but nothing too obvious. I could try and think ... I could look back at what's come up, what they've suggested in the past, the common things they come up with, which aren't necessarily relevant ... So, there might be, maybe half of them are valid, and half not valid.*
- AF: That sounds good, yeah.
- PW: And it would be, like, a time in the lesson when you would just have that discussion. The question might be ... "Talk about ... the main reasons for me asking you to generate a model solution". And then the second question: "Think about the main reasons why I asked you to copy the model solution down in your book and use it to solve some other problems". How does that sound?
- AF: That would be nice.
- TC: I think it would, yeah.

(RGM9, #53-63)

The researchers clarified how the discussion that took place after the 'model solution' strategy related to the 'card sort' strategy:

- *PW: So, there was some discussion through the lesson about why you were using the teaching approaches*
- AF: In the starter, just after the starter ...
- *PW: ... particularly the first approach of generating a model solution?*
- TC: Why is it important? What are the advantages? We didn't use exactly the same language ... Why is it beneficial?
- *PW: So, the card sort was just reinforcing that at a later stage? They'd already engaged in that discussion. And it was, kind of, partly assessing?*
- TC: It was, kind of, gauging ... what did they get from the ... were the pedagogies visible, made visible to them, earlier on, or not?
- *PW:* But it was more assessing the extent to which they were aware of that? And then there was ... wider discussion followed it as well, yeah?
- *AF: Yes. It was a more in-depth discussion after the card sort.*

(RGM10, #2-9)

AF reflects briefly on using the 'card sort' strategy, highlighting some confusion amongst students in reflecting on both teaching approaches at the same time:

• AF: My feel was, before looking at the video, that they had engaged with the activity [card sort], most of them. Some got further than others. There was an understanding ... it was tricky for them, initially, to ... I had to be very clear in my instructions in saying "we're first going to look at placing reasons under the

'why we did a model solution'". Just to coordinate between the two, because I felt they [struggled] to do both at the same time.

- *PW: So, there was a little bit of confusion between the two teaching strategies?*
- AF: There was a little bit, yeah.

(RGM10, #10-12)

The researchers discuss their primary reasons for using the 'boxing up' strategy to include as statements in the card sort. Doing so prompts them to reflect more critically on their reasons for using progressive pedagogies:

- *PW: So, we talked about having six or eight [statements] only? Maybe six is enough? Two that are primary reasons, two that are not necessarily primary but might be valid reasons, and two that are red herrings? Yeah?*
- AF: Sounds good.
- *PW: So, let's just remind ourselves, what were the reasons for using them? To provide a ...?*
- AF: ... a clear structure ...
- TC: ... should we say 'organise'? ... to organise our answers? ... I'm just thinking about the language ... I would just keep it to something like "make a plan" ...
- PW: ... So, here it says "So you can ..." Should it say: "So I can ..." or just "To make ..." or?
- AF: I think it's nice for them to relate to it as in "I", don't you think? ...
- *PW: ... If it starts with "So I can ..." then it's really making it clear it's about how it's going to help you ...*
- TC: For me ... I'm thinking about some students who don't have the reading skills, or they just see there's too much text, so they're not going to engage with that. ...
- *PW:* What we could do ... they [the statements] could look like this [PW writes as he speaks] ... they could all say "So I can ..." and then it just says here "... make a plan to help me solve a problem". So, they all start off with "So I can ..." on the top line, then the thing that it's helping them to do is underneath.
- AF: OK, that's a good idea. and then it's less words, yeah.
- TC: It's more clear.
- *PW: So, we've got one. What's the other primary reason for doing this? ...*
- AF: ... "it helps me identify the key information" ...
- PW: So, in terms of the red herrings, I mean the ones we came across before were this idea about doing things more quickly and getting the right answer ... So, these are the primary ones, these are the invalid, and we need two that are partially valid, or I would just call them 'valid' ...
- TC: I like this one ... "so you can recognise similarities and differences between different problems".
- AF: Yeah, I like that one, cos it's not wrong. ...
- TC: Very often, what happens is that when we come up ... it's a part of the process that we think about things that we use, not knowing exactly what the reasons are for using them. And then the reasons emerge from these discussions.

(RGM12, #20-37)

 \rightarrow See also Section 5.4.4 for researchers' evaluations of the use of the card sort strategy.

5.3. Critical research processes

Note that the three critical research processes are based on those identified in Skovsmose and Borba's (2004) critical model of Participatory Action Research. These are pedagogical imagination, practical organisation and explorative reasoning.

a) Pedagogical Imagination

5.3.1. Reflect on own practice (Pedagogical Imagination)

→ See Sections 5.3.2, 5.3.3, 5.3.6, 5.3.9 and 5.4.6

5.3.2. Reflect on existing practice (Pedagogical Imagination)

AF questions the extent to which we have addressed historical and cultural contexts (raised in the Liu, 2015 paper) in the VMP project.

- AF: Do we look at the historical and cultural context? We talk about, you know, addressing problems, making pedagogy more visible to students. When we talk about PPI students often being the ones that don't ... and there's obviously a lot of historical and cultural context there, maybe?
- PW: Maybe the cultural context there is an awareness of that link between disadvantage and attainment, and what might cause it and all the, sort of, things we've looked at ... some of the other theories [e.g. Bourdieu] that might help to gain that sort of awareness.
- AF: I mean it could be that, as well, yes.

(RGM11, #15-17)

TC presents the Hand (2012) paper to others, and uses this (and references to other research, i.e. Mason, Freire) to pose a question from a perspective of equity about the extent to which we take on board the cultural context of disadvantaged students in interviews:

- TC: This idea of dispositions ... Teachers are predisposed to some ... they bring their backgrounds. And therefore, they can notice in the way that Mason says of sensing, or seeing, or perceiving, what happens in a classroom. And that can inform their actions ... Should we use video-stimulated reflection to aid noticing of issues of equity, inequity, with our groups? Are we doing that to some extent? Should we include this in the analysis of interviews? ... Also, I was thinking about our target students and what are ... why they act and interpret things in a certain way. And the questions that we ask them, are they ... do they reflect their culture? Are they fair on them, in a way? Should we be thinking, when formulating questions, of making them, fair? I was thinking about, sometimes when I'm interviewing a student, I feel the need of ... giving a bit of context. And that's because I'm not entirely sure that the students read it in the way that we wrote it ...
- *AF: Yeah, that was my concern when I was talking about giving things historical and cultural context earlier, sometimes. And discussing that when we plan the lessons, the cycles.*
- TC: So, it's kind of agreeing, in a Freirean way, of agreeing the terms of our discussion ... we should use the terminology that they would use ... that's their way of naming things ...
- AF: Linguistics should be aligned, yeah.
- *PW:* Reading the interviews, that's kind of what happens when ... Because sometimes you get ... one of you asks a question, and the response shows the child has misinterpreted your question. So, you rephrase it, and you give them more context. And you're in the best position to know whether that's just because they're giving the answer we didn't want or whether they haven't really understood the question, because you know them better. And this could be one of the advantages of getting teachers themselves to do the interviews. They can interpret when the question has been understood, whether they need to re-phrase it.

(RGM11, #18-22)

5.3.3. Relate theory to practice (Pedagogical Imagination)

Having presented the Geiger et al (2016) paper to others, TC poses some questions about how the use of VSR in the paper might inform the research group's use of VSR:

• TC: How might this research inform our use of VSR? So, I'm just going to lead with some questions ... So, the use of cycles and formats, it's different to what we've been doing. Do we want to carry on planning a lesson and then videoing that and doing VSR independently, or do we want to change? Should we use this sort of language and concepts [in the research paper] to document our changes? Could we do that? ... How soon after the lesson should the VSR session occur? Or before, like one of the case studies [in the research paper]? It seems that they did the VSR on the day, right after the lesson was taught - might not be practical for us. But it's then quite present, both the memory of the lesson, and the video there just brings confirmation. And that also explains, because the projects [in the research paper] were very, particularly the first one, was very short in duration, so just five weeks. And what should be the role of the facilitator? ... Because we decided, when we did the second VSR, which moments to stop and to talk about. Do we want to open that for others ... open for discussion? What should be the role of the critical friend (aka us)? So, do we offer support? We're doing the same lessons. We have similar classes. What sort of critique do we want to offer each other? Should we be having VSR sessions together or separate? ... Should we pre-select the moments we want to visualise and discuss? Or let others open up for critique? Is it practical?

(RGM8, #4)

Alba responds to TC's questions:

• AF: I mean these are all very good questions that we've probably thought about at some point, maybe some of them, already. This sort of brings it into perspective a bit more.

(RGM8, #5)

Having presented the Coles (2013) paper to others, AF poses some questions about the facilitator's role and how this is quite directive in the paper:

AF: Is the role [of the facilitator] too descriptive in the way he [Coles] is describing it? What kind of discussion do we want? What sort of outcomes do we want? I guess knowing what sort of discussion we want, kind of allows us to go back and talk about the role of the facilitator more clearly? Do we want it to be clear and set out? Or do we want it to be completely open? Depends on the type of discussion we want. ... And is it possible to try and define it [role of facilitator] ... if it is dependent on context, like he [Coles] tries to? ... He's using 3 to 4-minute clips. In our discussions, if we were to do this in future ... I was convinced by that ... Having a two-phased discussion. Do we want to take that forward? ... He [Coles] put quite a lot of focus on phase 1, describing what you see, before moving onto evaluating. ... Are small clips better than whole class observations? Pros and cons to it because, like we just mentioned before, having a small clip can over-simplify the situation, having something else you can refer back to for context might be helpful, so having that whole timeline, even if you don't watch the whole video, can be helpful.

(RGM8, #17)

The researchers compare and critique the two papers (Coles, 2013; Geiger et al, 2016) discussing what they have in common (e.g. making the distinction between a narrative and evaluative discourse) and differences between them (e.g. extent to which the facilitator guides the discussion):

- TC: The way it links to the focus of the paper I presented [Geiger et al] ... it intersects in some ways, like the moving from a narrative discourse to an evaluative discourse, he [Coles] seems to make that point, having those two phases ...
- PW: What's common between the two papers is just how powerful video-stimulated 'whatever you're going to call it' will be. That comes across in the two papers I'm going to talk about as well. ... In the paper you [Tiago] presented, it was almost like ... you were doing the 'video-stimulated recall' one ... video was a way of remembering what happened in the lesson, whereas here, I think, Coles says it kind of gives you much more than that. It gives you a different view. And, actually having a camera at the

back of the class is completely the opposite perspective in terms of just physical view that you get as a teacher. So that gives you something else. But being able to hone-in on the type of discussion, things you can't think about when you're actually in the act of teaching, is really powerful. But I just worry that ... I think he actually wants to be quite controlling as the facilitator.

- AF: This was what my problem with it was, it's really interesting the way he zones into it ... he makes the role of the facilitator quite powerful, basically ... It's a good consideration to have because it can steer the conversation ... but I wonder if he does try too much to steer it ...
- TC: The first paper [Geiger et al], the use of VSR ... the purpose is, it's a way of remembering, but also to enable self-reflection. To remove the element of an external observer that comes with notes and steers the discussion in that way ... to provoke change that is coming from within, from the teachers. ... There's a move from a narrative discourse, saying what is happening ... and then moving on to a more evaluative discourse ... it's done by the teacher that taught the lesson, whereas the second one [Coles paper], it seems like it's guided, and it's a joint effort. ...
- AF: It sounds like they're both agreeing on that descriptive first phase, where you are just agreeing on what's happened. And the importance of that in not falling into the traps of being negative, maybe, or being too interpretative straight away, too soon. He's [Coles] quite guided in the second phase, he talks about the importance of guiding it in the second phase ... in the way he talks about it, it is more controlling, but it isn't negatively controlling ... he's just making sure that there is a purpose to it and it is effective. ...
- TC: Oh, yeah, it's a positive agenda ...

(RGM8, #18-23)

PW raises some issues from the Sherin & Dyer (2017) paper that relate to the VMP Project (e.g. multiple lenses for viewing videos, when to start and stop the video and how the use of video encourages teachers to make their pedagogy more visible):

PW: And then they say teachers view the video through a different lens to the researchers. ... That worries me a bit, because, to some extent, 'yes', because they're your students ... But to other extent, you know, if you've talked about it and you have a shared purpose, you know, you've also got a certain amount shared when you're thinking about the video again through a lens. It flagged up to what extent is the research collaborative. And I quess the more collaborative it would be, maybe, the more you would have in common in terms of the lens and the perspective, what you're viewing the video. They say "asking teachers to locate key moments [by starting and stopping the video] made them more aware of whether this [mathematical thinking] was happening" ... My question is can you really focus on your teaching strategy if you're thinking about when to start and stop a video? ... This does seem relevant that, because you are, kind of, aware that you wanted to catch this thing on video, you were unconsciously doing things to make it more visible. So, if they wanted to capture mathematical thinking, they would have to bring it out into the open, which they wouldn't normally do. And that, kind of, is relevant to us, as our purpose is actually to bring these things out into the open ... Videoing is really useful for us because, just the act of having a video camera there, kind of makes you think "this needs to be seen". It needs to be seen because we want the students to understand it, but also, we want to capture it on the camera.

(RGM8, #27)

PW raises some issues from the Hollingsworth & Clarke (2017) paper that relate to the VMP Project (e.g. extent to which project is critical):

• *PW: I thought it was a little bit leading, it was kind of: "this is good teaching, which bit of this do you want to choose?" But that was because the project was about how do you want to develop your practice. It wasn't about "here's something that we want to tackle, how are we going to do it?" Which is what, I think, makes our project a bit more critical, and less just about reflective practice.*

• TC: It's more collaborative practice.

Having presented the Liu (2015) paper to others, AF relates the notion of using artefacts to facilitate reflection to the use of video clips in the VMP project:

- AF: So these prospective teachers were using reflective journals ... self-selected artefacts, they sort of chose as ways of showing how what they are doing is changing their journey, their developments as a teacher ... so she said because both of these are quite potentially biased ways of showing how you're developing as a teacher, you know, you are selecting what you deem to be worthy of selection ... these reflection-based actions as a result of reflective thinking are the most, sort of, important and significant way of deeming whether you are truly reflecting on your practice, because 1) they produce evidence, and 2) you've got to be fully honest with yourself because the evidence sort of speaks for itself and it requires you to, sort of, deeply think about what you practise. ... The way we do this is we select specific clips from our lessons to evaluate, when we have our, sort of, phase three meetings.
- PW: So that's like, the clip is like an artefact, in some way?
- AF: It's like an artefact ...
- TC: It's evidence, or ...?
- AF: It's evidence ... it's self-selected but also observed by someone else.
- TC: It's objective, because it's something that is open for ...
- AF: It's open for discussion ... We were both present in each other's lessons, and we have a discussion about "Oh yes, that clip is a meaningful clip", so we've got two perspectives on it.

(RGM11, #1-7)

5.3.4. Consider alternative practice (Pedagogical Imagination)

There was no additional evidence of significance to add to that from the first year of the project.

b) Practical Organisation

5.3.5. Negotiate/design strategy to try out (Practical Organisation)

AF and TC recount a discussion they had previously about possible strategies to try out. One idea evolves into the 'model solution' strategy:

- AF: What else did we talk about?
- TC: So, one of the things we talked about is about writing examples. Kind of, "Why do I ask you to write an example down? Why don't I ask you to write all of the examples down?" ... The thinking behind this is 'why do teachers, when they plan lessons, what do they think the book should look like?' ...
- PW: You mean like a model solution or something?
- TC: Yes ...
- AF: We were talking about how we'd address this differently, so for example, I was saying with the Year 8 group, I'm normally very explicit or very direct about when I want them to copy an example ... So, it's, sort of, making those expectations very explicit, and also having the conversation about "Why do I tell you [to copy it down] ...?"
- *PW: Writing down some mathematics, why do we get them to write mathematics?*
- TC: But not all, it's, kind of, like "Why is it that I ask you to have one example and not all of the examples I have on the board?" ...

(RGM9, #13-19)

TC/AF explain the 'boxing up' strategy they came across at a recent conference (which the whole maths department is now using) and the reason they would like to use it in cycle 4:

- TC: So, we were thinking about ... what is it that our students find difficult when going through these problems on probability? And what would we do to make them understand better? How can we minimise those difficulties? So that would be our pedagogies ... We thought about using this system, which is something called 'boxing up', from Helen Hindle ... So, the pedagogy would be getting students to ask themselves two questions from a problem: 'What is the question asking me?' and 'What information do I already have?' ...
- AF: And we thought this was quite a useful thing to do with something like probability because the problems are often worded and quite ... and there's distracting information in the problems. ...
- TC: We would do that [get students to answer the other questions from the 'boxing up' strategy] anyway but not centre discussion around all of it. ...

(RGM12, #1-3)

The researchers discuss how to adapt the 'card sort' strategy to use alongside the 'boxing up' strategy in cycle 4:

- TC: We also talked about how we would do this [card sort]. We were suggesting, kind of, having a discussion initially about what was going to happen, making reference to the previous use of the cards, and letting them know something was going to happen and at the end of the lesson there would be a discussion using the cards. So, they would be ready for it. ... What's different from the very last cycle ...
- AF: ... We had the discussion [in cycle 3] but we didn't say "This is why we're having the discussion" ...
- TC: ... and why it's important ... The lesson would have a moment at the beginning when we would show the cards ...
- AF: Yeah, we could have a picture of "this is what we will be doing at the end. This is the reason why we're doing it". ...
- *PW:* And the reason why you're doing it, what are you going to say to them about that? "Because we want you to understand our rationale for our decisions about teaching approaches"?
- TC: Yes, cos we're making it very clear before they actually engage with the task.

(RGM12, #4-9)

The researchers consider how to make their pedagogical rationale clear though the discussion accompanying the 'boxing up' strategy:

- *PW:* At that point [introducing the 'boxing up' strategy], are you going to have a discussion about why, you know ... what's this pedagogy for?
- *AF: ... Yes, we have to, I suppose, if we want to make it completely explicit and visible.*
- *PW:* And you talked about the balance between letting them speculate what the rationale might be, but then actually making clear what your rationale actually is in using this.
- *AF: ...* We might want to do a think-pair-share and let them ... ask them what they think our intention is, before we then make that clear, just to see what kind of discussion that generates.

(RGM12, #16-19)

5.3.6. Consider/address constraints on practice (Practical Organisation)

TC raises the question of what Hand's (2012) vision of equitable maths teaching might look like in practice and the extent to which such a vision is feasible within the constraints of the current schooling system:

TC: She [Hand] claims she was studying examples of classrooms where equitable mathematics
instruction is taking place. And I'd like to see this, what does it look like? Because I see loads of
opportunities with my classes. Just focusing on my Year 8, I think sometimes I could give them more
room, and sometimes I do ... I mean, what would that look like in a classroom? And what am I
jeopardising ... Where do I feel that I could be losing control? ... What could be disrespectful? Do they
know what to do with that, when they are given that power? Are they mature enough to respect it?

- AF: How do you set them up for that power?
- TC: And how do you do that in the context of a school that does not necessarily do that across the board? Are you not sabotaging yourself by compromising some rules, and some boundaries?
- AF: They're very good questions.
- PW: But isn't that, again, that's what our project is about, isn't it? If you're giving them that space, are some of them more predisposed to use it, because of ... their middle-class background, they know how to take advantage of that. And, I guess this [Hand (2012)] is saying equitable teaching requires you to give them that space. And what we're saying is here's a way of making sure that everybody takes up that space to the same extent ...
- AF: We're sort of saying these are some tools that could do that.
- TC: OK, we are doing that ... but in some way we are reproducing power hierarchies ... So, we're kind of saying you can go until here. We're creating this space by using progressive pedagogies and using discussion and thinking about strategies to make our pedagogy more visible, but where does it end? And what they are talking [about] here seems like it's much more ...
- AF: ... [more] expansive.
- TC: Because, on one hand, we are saying "OK, do this", because we believe in this ... but, on the other hand, we're restraining them.
- AF: Because we're doing it in the context of a school, with its own set of rules ...
- PW: But if we're thinking "Well, this is talking about some sort of idealistic teaching", that does happen in some places like Steiner Schools ... but who goes to those Steiner Schools, is it, kind of, disadvantaged, working-class kids? No. it's not. So, I guess we're saying, within the current constraints of the system, this is something that can have a big impact, at least, in making things a bit better.
- TC: It's a compromise ... Definitely, I think it would have to be.

(RGM11, #23-34)

The researchers discuss a potential constraint, i.e. that the cover to enable TC to cover AF's lesson could not be confirmed until the day of the research lesson, and how to address it:

- PW: Are we still on for the videos happening on the 11th and 12th?
- AF: We are. Tiago has had to ask for cover to observe my lesson and that hasn't been exactly confirmed. They've said he's going to have to wait to see on the day if he's on the cover board. And if he isn't, then he can't be covered. So, we're going to have to look within the department maybe, to see if someone else can cover as back-up ... just as a favour.
- TC: That's one of the practicalities ...
- *PW: I think if it works out you can't get the cover, I think it would be better to postpone until you can, which would be a shame.*
- TC: We would plan another lesson ... We would abandon the progressive pedagogies and do something else instead ...
- *PW: You could have a back-up lesson planned and then leave that lesson you had planned for something else.*

(RGM9, #1-6)

c) Explorative Reasoning

5.3.7. Plan how to use evidence (Explorative Reasoning)

There was no additional evidence of significance to add to that from the first year of the project.

5.3.8. Use evidence to evaluate strategies (Explorative Reasoning)

The researchers discuss the extent to which the pedagogical rationale was made explicit to students:

- *PW:* Watching that, it strikes me as quite clear that, you know, you have made your reasons clear, you have come to a conclusion, you haven't left it hanging.
- AF: ... I had a similar approach where I had the discussion with the class, but Tiago had a much more explicit discussion with the class ... I think that's really good, because then they were really clear on what they were doing from the beginning, and what was the most useful reason ... And you used language from the cards in your explanations, which related the cards to that they were doing throughout, I thought. And when you came back to them at the end, it felt like full circle.
- TC: I agree. That was my intention, to make it very, very explicit. And to have a discussion and see what they were thinking. I think that it took too long.

(RGM13, #3-5)

AF describes how a combination of different types of evidence from surveys, interviews and videos helps to evaluate the strategies:

• AF: I think you get different perspectives. You get the whole class perspective with the surveys. And then you, kind of, have a chance with the audio interviews to delve deeper with those key students. And then, obviously, the video-stimulated reflections, and peer observations, and the feedback and the evaluation that we have through that, allows us to see it from that perspective. So, I think having so many different perspectives to look at it from allows us to get the bigger picture, and see how effective the strategy, the pedagogies, have been in getting it across.

(RGM14, #11)

→ See also Sections 5.5.1 and 5.5.3 for how researchers used evidence from the interviews to evaluate the success of the 'boxing up' strategy.

5.3.9. Relate evaluation to own/existing practice (Explorative Reasoning)

TC prompts a discussion about how seeing the video provides a different perspective on the lesson for the teacher:

- TC: Teaching this lesson, I felt much better about teaching it than the previous one, from cycle 3. Because I was much more clear about what I wanted to do at each stage, and I could see it happening. However, looking at it now, I realise that it was ... it's weird cos ... there's the perception you have from teaching, then there's the perception you have from seeing you teaching, which is different ...
- *PW: What is it about seeing the video that makes you think differently about what happened in the lesson.*
- TC: So, in this particular case, I felt, looking at the time that it took, I would like to do it in a much quicker way ... there's a better way, I think, of doing that, it doesn't need to be so time consuming ...
- PW: But what you're saying is that, looking back at the video, makes you more aware of the timing, the amount of time that you spent on doing this, the amount of time students spent talking, and things like that? Which you wouldn't necessarily be aware of at the time, cos you can't judge time when you're in the moment.
- TC: ... There was enough time to have all the discussions, but less time for them to be doing. The way I like my lessons to be is more of them doing, and less of me talking, and this was the other way around. Because I really wanted to ... make that message really clear ...
- *AF* ... But it just meant that you really embedded the ideas, and I think that probably had an impact at the end when you did the card sort, actually, for the better.
- TC: Maybe, now thinking about it ...

(RGM13, #6-12)

5.3.10. Relate evaluation to alternative practice (Explorative Reasoning)

 \rightarrow See Sections 5.4.6 and 5.4.7

5.4. Participatory Action Research characteristics

Note the characteristics of PAR used in the analysis are derived from Brydon-Miller and Maguire (2009). They are collaborative, participatory and bringing about positive social change (in this case impact on TRs).

a) Collaborative

5.4.1. Consideration/evaluation of relationships/roles

The researchers discuss the importance of considering power relationships between researchers occupying different roles outside the research study:

- *PW: I think you have to put his [Coles'] study into context. He was the head of maths, working with his department ... his agenda is to get the best out of everybody so they can be good teachers ...*
- AF: It's for teacher development, yes.
- *PW:* Our context is slightly different because, together, we're looking at this thing. And we're kind of saying, well, you know: "I can't do this without your expert knowledge of the classroom, you can't do it without my expert knowledge of research methods, together we're looking at this idea". And, therefore there is less of an agenda, which is very difficult to break out from. If your head of department was trying to do this research, you would not be able to relate to them in the same way you can relate to me. You could argue that, in some ways, it was problematic that, historically, I've been in a situation where I've had some sort of power over Tiago, you know, as a tutor. There's still that relationship where I had an input into whether Tiago passed the course or not. That's a power relationship. I think it's important to be open about this. It was easier, I think, because there's been a break of a year and you [Tiago] came to me and said: "I want to do some research" and I didn't really still see you in that way. Whereas if I was doing some research with one of my tutees right now, it would be quite hard not to be affected by that. So, if you're a head of department trying to do some research with your department, can you really dissociate yourself from being the head of department?

(RGM8, #24-26)

The researchers discuss the power relations existing between teachers and students in interviews:

• TC: I think it's just also a golden opportunity I'm just thinking, that you have your non-dominant students sitting in front of you [in the interviews] ... The aim of being evaluative, I'm completely aligned with that. I'm just thinking, could we take advantage, or make the most, of that opportunity to begin to create some space for dialogue there as well ... we are interviewing them, that's again a position of power, isn't it? ... you, asking the questions ... that's not promoting dialogue, in that sense.

(RGM11, #43)

PW prompts a discussion about the way he selected papers for TC/AF to read and present:

- PW: So, what about the process itself of me suggesting readings that I think are relevant ...? So, clearly there's a bit of, you know, me leading here, in terms of being the outside person who maybe has some expertise in research and things ... suggesting some things for you to read, for you to then to discuss the implications for the project? It's a very different model to how teachers would normally be exposed to research, where somebody else would, kind of, produce a digest, a simplified version of the research for them, and say "OK, this is what the research says, now get on with it!" ... So, what do you think about the whole process of me suggesting papers, you actually engaging with research papers like this, to try and inform the project?
- TC: ... Is it not, kind of, opening it up by saying: "Read this, because I think this might be connected to what we are doing". And then, when we are doing that ... we are going to make our own links with the project, we're going to see it through our lenses, which may not be the same as you envisaged when you chose the papers ... I sometimes ask myself "How did Pete choose these two papers, what is the

reason?" ... it is interesting to think about in this, and to approach it in this way, rather than having a digest of research, and saying ... Because that's a position of authority, whereas here, is a position of guidance.

- PW: ... What do you [Alba] think about the process?
- AF: ... The way I see it is we're given a paper ... the only sort of link we're given is "how does this relate to what we're doing?" But how you find your way there is entirely up to you. And the links that you make are entirely up to you. And I think that's quite personal and you're bringing what you ... maybe there's things that stand out to me or I care more about than [other people].

(RGM11, #44-47)

PW explains his rationale for choosing research papers for others to present:

- TC: How are these papers selected?
- *PW:* ... If you think back to the first meeting, those first three were very much related to the theoretical framework behind the whole idea for the project. The second two papers were very much about ... methods really, because it was about video-stimulated recall. And that had a much more clearly defined aim of promoting some discussion around how we might do it, without necessarily having in mind what the outcomes might be ... I had no idea when we started how we would end up using videos. We seem to have come up with something which is very effective and quite powerful. With these [papers] ... I couldn't find anything [about] ... how teachers might do their own interviewing or surveys, so I thought "Well what's kind of underlying what the aim of doing those surveys and interviews were?" And it's, kind of, about generating evidence to help teachers reflect critically on their own practice. So, I thought why not find some things that are about critical reflection and transforming practice. And these happen to be some things I've read. They might not be the best ones, but they just happen to be things I've come across. And they're quite prac[tical] ... Even though they might sound quite theoretical, they are talking about working with teachers, at some point.

(RGM11, #48-49)

PW prompts a discussion about whether teachers conducting interviews might potentially result in bias:

- *PW: If I was playing Devil's advocate, I'd say: "OK, you've got a good relationship with these students, they're buying into the project, so does that make them more prone to say things are more successful than they really are, because they want to say things to please you?"*
- AF: There's definitely a sense of they want to say the right thing, but I don't know how you would distinguish between 'this is what they know' and 'this is what they're saying'.
- *PW: It's whether they genuinely believe what they're saying or whether they actually think something different, but they're saying what they think you want to hear.*
- AF: The thing is, because they explain themselves quite well ... in the interviews. And they were able to reason and back up their answer, and go into quite a lot of detail ...
- *PW: So, you think the detail they went into would have made it quite difficult for them to keep up a pretence of something different to what they actually thought?*
- *AF: Yeah. Yeah, I thought that it was quite genuine.*

(RGM14, #16-21)

→ See also Section 5.1.3 for how researchers considered the potential benefits of teachers conducting the interviews themselves (rather than outside researchers).

b) Participatory

5.4.2. Teacher Researchers involved in design of research

The researchers discuss further the use of the timeline and the role of the facilitator in facilitating the VSR discussion and the need for a protocol:

- *TC:* I agree with the idea of the timeline being a strong idea.
- *PW: Cos we'll have particular questions we're looking at, and to some extent it's more obvious which bits of the video we need to look at, because we're looking at strategies and it will be when those strategies are the focus of the lesson.*
- TC: But what is different is that, if all of us can choose points in the timeline to focus on, and that's what can open, in terms of ... gain, I think. Because what I might think is important might be different from the other person's analysis.
- AF: Well this is where the role of the facilitator ... comes in. I think Pete is, sort of, leading on it and I think we're, sort of, sharing it as well. ... Do we individually focus on what we perceive to be important from the videos? Or is it that we all agree that the focus is wherever the strategy comes into the lesson? Or a bit of both? Because they could be different points, what you might deem important might be related to the strategy, but it might be additional to the strategy as well.
- TC: I think it makes sense to focus on the strategies and to know where they happen ... if the strategy is in one moment in the lesson, then it makes sense. But having to analyse all of the moments where the strategy is implemented ...?
- AF: But I think this is where developing like a protocol is important. Because there is so much room for deviation. This is where, like, the focus is important ...
- TC: What I thought is if we kind of looked at each aspect from a 'what went well' and 'even better if' ... that's probably more supportive and reflective at the same time.
- AF: I agree.

(RGM8, #9-16)

PW gives TC and AF an opportunity to comment on the survey questions, as previously promised, before finalising:

- *PW:* And are you OK with the questions as they are? ... The way they are, they're not tailored to a specific strategy now, so it makes more sense in terms of evaluating the success of the project as a whole.
- TC: They look good. How much time would we give them, roughly?
- PW: Well, last time 10 minutes was a bit short, wasn't it? 15 minutes maybe? You could always do it, if you don't know exactly how long it's going to take, do it at the start of the lesson ... I wasn't sure if you wanted to change the questions ...
- AF: No, they look good ...
- *PW: But I didn't want to make assumptions, because you said something about looking at the questions again ...*
- TC: Yes ... I think they look good.

(RGM9, #7-12)

→ See also Section 5.1.3 for how researchers for how teacher researchers played an important role in the design of the interview questions.

5.4.3. Teacher Researchers involved in evaluation of research

Having presented the Liu (2015) paper to others, AF relates the iterative reflection process described in the paper to the development of the 'card sort' strategy:

- AF: Role of reflection in teacher research ...
- TC: So, going back, like an iterative process?
- AF: Exactly, this iterative process of planning, acting, observing, reflecting, which is what she's done, in that study, with the teacher educators and prospective teachers, but this time in a teacher research context.

- TC: Which is kind of what we decided to do after the last cycle, when we tried the cards, and we thought "OK, next time we should still use the cards because there's a lot we can do there", but think of another way ...
- AF: Another way, whether it's less cards, or ... Yes.
- TC: There was some element of reflection there, together, and then action. Is that it?
- AF: Yes. Yeah, yeah, that's what she means ... Which is what we've been doing before. But I think we really hummed into it in the last cycle in that there was something we felt we could improve upon and change for next time.

(RGM11, #8-14)

The researchers continue to discuss the potential benefits of teachers conducting surveys and interviews themselves:

- *TC:* Your question, if it would be any different if it was done by an outside researcher ... than actually doing the survey ourselves ...?
- *PW: ... The interviews, perhaps more so. Because you know the students better. You know what happened in the lesson better, because you were there. So, does that make the interviews more valid? Or does it make it less valid in that you might bias what they come up with?*
- TC: I think that ... because we planned the lesson, we planned the questions for the interview, so they're more meaningful for us, because we're thinking "What is it that we want to get?" If it's someone outside ... looking perhaps at the video and thinking "What is it that I would like to ask now the students?" Depending on a certain focus, that could be also very valid and more objective. But less meaningful for us, perhaps, because it's not coming from us. ... But I think the fact that we are there ... there's a different relationship with the student than someone outside, it's going to be a different interview. I think they're going to be more keen on answering. Also, they will feel they kind of owe you that ...
- AF: Yeah, I think that makes a massive difference. I think it makes it more valid ... If someone from the outside saw the video ... and came in and asked those questions, the students didn't know this person, I don't know, I don't think they would respond in as an engaged way. I think they know we really buy into what we're doing, because we're in there every single lesson, and so they buy into it more. And so there's this, I feel, cycle of "We really want to answer the questions and do that question justice, as fully as we can". And because there's a rapport and trust there, I feel they fully engage with the interview, as much as they can. ... But [it's] also about how engaged I think they feel with the project. And I think, because they are feeling engaged with the project, and because they are feeling part of it ... they are then able to, some of the time or a lot of the time, give answers that are quite intricate and quite elaborate.

(RGM14, #12-15)

5.4.4. Teacher Researchers involved in evaluation of strategies

AF reflects further on the 'card sort' strategy, emphasising that it relies on students having engaged with the progressive pedagogies first:

• *AF: I think ... those two at the front hadn't done very much, they'd only managed to get a few reasons. But that I don't think reflects that they weren't able to get through all of them, or reason through all of the work. That's just their pace of work ... The starter took about half an hour ... coming up with all the methods ... We brought loads of misconceptions to light, about algebra, that maybe we weren't expecting ... We sort of had to take the time to get those embedded before we could, kind of, have them feel as if they're successful, and they're understanding what they're doing. And have them negotiate the method and come up with it ... feel it was their choice and their ownership, and that takes time, exactly. So, that group didn't have that many. But most groups, there were a couple of groups that finished ... I think probably four or five that had finished. But others probably had most reasons in place, they'd made a decision.* The researchers reflect further on the card sort and how it reinforces the earlier discussions around the 'model solution' strategy:

- AF: They engaged really well at this point, considering how much talking there had been in the lesson ... When we had a discussion the first time, which was much earlier ... at 23 minutes [referring to timeline produced by Tiago], I ask the class what their preferred way of answering the question is ... 26.55, highlights it is important that students write down ... why it's beneficial to have a model solution in your books ... it was a brief conversation ... but I think that was helpful getting them to think what we were trying to do. I don't think this would have been as useful, this conversation at the end, if we didn't have this here. Because this triggered what we were aiming, with this lesson, to do with the students and what we wanted them to get out of it ... I think the card sort, what it did, is it provided more reasons, maybe reasons they hadn't thought about, why we were doing each thing ...
- TC: I think it's also the reflection point, isn't it? So, you're having those ... plenty of discussions about the negotiation that was taking place, and reflecting about why is it important to do this in this way? Why is it important to have a model solution? And that happened at the right time. If you'd left that to the end ... they might not associate to that anymore. You're, kind of, raising the flag there, and then later on, you're just: "OK, we've had the discussion, what are the reasons?" Kind of, just reinforcing it.
- AF: "You've had time now to ...". Yes.

(RGM10, #20-22)

The researchers reflect on the confusion caused by the complex way in which the card sort was used:

- TC: And they totally understood that [discussing the rationale for agreeing a model solution as part of the first part of the card sort]. When we get to the second part, I think some of them may have interpreted ... "OK, am I allowed to move all the reasons to the other side? Is it reset, and start again? Or is it OK, which ones stay here, which ones stay there?" I think that's really complicated, for the first time that they're doing it. Also, I think the reasons are very worded. So, I know some of my students will have difficulties. ... So, I think, narrowing it down, reducing the size of the problem, focusing on one strategy per lesson ...
- AF: I think we anticipated beforehand, when we planned the lesson, that it would be difficult ... because there's so much literacy there, there's so much comprehension, that that in itself might take time and we might have questions about it ... Actually, the best thing would have been to just have "Why did we do a model solution?" And just have reasons appropriate to that, and reasons not ... and fewer of them.
- TC: And reading ... I would really, now, love to read this [the statements] with them first, so I would like to read the reasons, and then they would have a go ... clearing out any words that they don't get ... and I think it's useful to make the point that some of them [reasons] don't apply.

(RGM10, #29-31)

There was consensus that the card sort was a worthwhile strategy that should be refined for future use:

- PW: Do you think the activity [card sort] has got potential to use again, but in a simpler form?
- AF: Absolutely.
- TC: I do, I think it's really good ... I really like the metacognition involved, like thinking about what happened and reflection.

(RGM10, #32-34)

The researchers reflect on the extent to which they should explain their pedagogical rationale directly or allow it to emerge through discussion:

- AF: You [Tiago] said: "What are the advantages [of using a model solution]?" And that was really nice, cos that was more open. Whereas, I said: "Why is it beneficial?" ... it's more leading, it's beneficial, can we identify the ways it is beneficial? ... There's benefits to both questions.
- TC: I think it is. And that's one of the things from discussions that we've had in the meantime. We think, in lessons such as this one, where there's so much to discuss, at some point, we feel that we have to give the answer, and not have a discussion ... if our aim is to try and make this approach visible to them through this strategy, maybe we need to do less of that [discussion] and just tell them sometimes, "actually this is the mistake you're making" ... choosing your battles, basically.
- PW: I guess there is an argument to say, if you just tell them something, it might just go in one ear and out the other. So, if you've asked them to think about something and grapple with it themselves first, and then you moderate that by saying what you think, then they're more likely to absorb it. Like you did with that plenary discussion, where they were clearly coming up with the ideas, but you were commenting on them. And you were, kind of, subtly saying they whether they were right or not. You know the answers, and they know that you know the answers. They're trying to guess what you were thinking. And so, you can say: "That's interesting. No, I wasn't actually thinking about it helping you to work quickly, because I don't think that's necessarily the most important thing about working in maths" ...

(RGM10, #35-37)

5.4.5. Teacher Researchers involved in analysis of data

TC/AF pose some questions to prompt a discussion about the thematic data analysis:

- TC: I'm just curious about how ... looking for themes ... but what are the themes, and how are they ...?
- AF: How do you decide which themes to focus on at one time? ... I guess one of the critiques could be you've been selective about what you've chosen to focus on, whereas this is very thorough, and kind of focused on everything.
- *PW: Well, everything within our theoretical framework.*
- AF: Do you look at every possible permutation? Or do you focus on the ones you think will give you meaningful findings? Significant findings? How do you decide?
- PW: Yeah, it's more the second one. But sometimes, in looking for those, you find other connections that you hadn't anticipated finding. A lot of them just come out from clicking on a code and re-reading everything that's been coded in that way. And so, some you'll see that every student is saying the same thing.

(RGM11, #50-54)

PW suggests ways that TC/AF can critique data analysis to enable him to construct a more robust argument for how the evidence supports the findings:

• PW: But maybe the first thing to do might be to see how I've explained it and whether you can follow what I've explained. And if you can't you say "No, that's not clear. And I can come back and explain it more clearly. And that will be a good challenge for me to write it in such a way that I can make it clear, how I went from transcripts to, sort of, describing the themes.

(RGM11, #55)

AF/TC pose a series of questions during PW's detailed presentation of the initial data analysis and findings from the first year of the project:

- TC: I was going to ask if these labels or categories [codes] are standard ... the ones from 3 [critical research processes] and 4 [PAR characteristics], or ... if you came up with them from the process that Skovsmose and others have theorised?
- *PW: No, I came up with this coding ... based on the theory. As far as I'm aware, nobody else has used the same scheme.*

• TC: I'm just thinking if there's someone else in the world doing something similar, based on the same work, would they be likely to use the same wording? So, we can create links with what other people are doing?

(RGM15, #1-3)

• AF: And would you, from that, because it tells you on the side the number of references, or the number of times it's [the code] come up, would you infer anything from that? Would you say: "Oh, that comes up quite a lot so there's evidence of ..."?

(RGM15, #4)

• TC: Can you not, as you are coding, do that analysis? As you're reading, could you say: "Oh, this is only one meaningful reference. All of these ones were just helping to elicit the discussion, or ..."?

(RGM15, #5)

• AF: So, the analysis you would make then, based on these reference points, is qualitative analysis. Is that right?

(RGM15, #6)

• TC: I should ask ... As you go along, is there a way in the NVivo software to record the things that are coming out from your reading and re-reading the codings? Or do you go to a Word document?

(RGM15, #7)

c) Positive social change/impact on teacher researchers

5.4.6. Impact on Teacher Researchers' classroom practice

AF explains how she has become less worried about devoting classroom time to discussing her pedagogical rationale:

- *PW: I think you were both worried about that, right at the beginning of last year, that would they just see this as a waste of time, and "we want to get on with our learning". But they're not seeing it like that?*
- AF: ... Now we've had a period of time to put this in place, you don't have to be doing it every lesson. Once you've really invested their time in that lesson, the next lesson I only need to mention "now we are copying down a model solution", and maybe, here and there, question them on why that's important, just to refresh that. But I don't need to be doing a half-an-hour starter to reinforce it ... But I see the value in it, definitely, more than enough to want to keep doing it.

(RGM10, #27-28)

The researchers discuss the impact of their involvement in the project on their classroom practice:

- PW: Do you think this involvement in this project is encouraging you to do that more, to sort of consider, you know, the 'why are we doing this?' more often? And perhaps routinely in your teaching, not just in the lessons we're focusing on for the research project?
- AF: It's something that I think we're encouraged to do from our training [professional development], why we do what we do. But I think because we are so involved in the project, it does ... what we're trying to do with the students has, sort, ended up happening to us, it becomes part of our pedagogy and our practice. I thought that it had become part of my practice a lot more ...
- *PW: ... Like you said you're talking to them more about why we're doing this, and you're annotating things in the same way, is that something that you may be more likely to have done, having been involved in thinking about your pedagogy in this way?*
- AF: Definitely more consistently using this approach, these strategies in all my classes ...
- TC: I agree. I'm more aware of the things that I'm doing, and what I'm constantly asking: "Do they understand why I'm doing this?" ... And, as Alba said, I also export things from this group to others.

The researchers discuss the impact of the VMP project on the extent to which they share their pedagogical rationale with students:

- *PW: So, do you think being part of the project has made you more disposed towards discussing 'why'* [you adopt particular teaching approaches] with them [students]?
- *AF: Yes, I think it's become a much more emphatic part of the lesson, and a much more consistent part of the lesson, than it was before.*
- TC: For me, I would say that I was already predisposed to justifying, for reasons of engagement, because I don't have that intuition of just going: "Ah, I will just give my rules" ...
- *PW: So, you might be more predisposed towards doing it, but were you doing it as much, is the other question?*
- TC: I think what changed with the project is becoming more aware, and noticing more ...
- AF: I think it's worth mentioning ... this is my sixth year of teaching, so I think of myself as still 'new' and developing a lot all the time ... that there's this sort of, happening at the same time, you know, learning with the project, but also just learning as being 'the teacher' ... through just normal classroom practice, and they sort of both reinforce each other ... they're very inter-linked.
- *PW:* And it's hard to separate what's because of the project and what would naturally happen to you as you get more experience as a teacher.
- AF: Exactly ... exactly.

(RGM11, #35-42)

5.4.7. Impact on Teacher Researchers' thinking/beliefs

The researchers discuss the impact of their involvement in the project on their planning and thinking around teaching and learning mathematics:

- *PW*: Does this, sort of, getting the students to consider what they think are the most important reasons for using a particular teaching approach, ... in doing that, does it actually make you think more about 'why am I doing this?' at the planning stage? ...
- TC: ... What this does is making us be more aware of that process, thinking that now 'what do we want?' Make that a fundamental step. What do we want them to get form this lesson? And in which way? What skills do we want them to develop? Is it working collaboratively? Why?
- AF: And we're also, not just in that lesson, built what habits do we want them to build in the long-term, whether that's working with other people, whether that's how they write the work down, whether that's the conversations that they have in discussion with me or with others. They're just habits, that maybe they're not always conscious things that they're doing, they're not always conscious about. But they're ... hopefully these strategies are making them more aware of, by the questions that we ask, or by the tools that we're using in class. Yeah, it's definitely making me think, often on the spot about: "Have they got this? How do I check that they've got this?" ...
- TC: All of the ... approaches, the pedagogies that we are using, for this lesson, are coming from things that we already were doing. And what we are doing is going through this process of" Do we know if they're getting this?" We're questioning: "We've been using this? Do they understand why we do this? ... "Do they need to understand why we do this? Would that be beneficial for them to engage more?" ... So, what this does is to make us be more aware, and then to decide how are we going to get that?
- AF: It is like a kind of assessment tool, as well ... you're gauging their understanding of your intentions. So, not their understanding necessarily of maths ... you're assessing their understanding of your intentions. And having that conversation, kind of, gives you an understanding of their understanding.

(RGM10, #43-47)

PW prompts a discussion about how making pedagogy more visible helps teachers to articulate their pedagogical rationale more clearly:

- PW: It does strike me ... we've talked about one, sort of, benefit of this project being that, in order to assess whether they [students] have the same idea of the purpose of the teaching approach as you do, you need to be really clear in advance what your rationale is for using it ... I think that comes across very clearly in that discussion [in the video clip], that you are able to articulate a much ... probably a much stronger rationale for why you're doing something, than would normally happen in a maths lesson.
- AF: Yes. Because it was so considered, so well-considered before, through our planning, through our intentions, through wanting a certain result from it, from them, as well ... I think it also makes the case for keeping it simple. Because the aim in itself, you know, trying to develop their metacognition skills is so 'high-order thinking' that it is necessary, always, to keep it simple.

(RGM13, #21-22)

5.4.8. Impact on Teacher Researchers' agency/efficacy

Becoming more aware of own practice and how it relates to development of project.

- AF: But I was thinking, we've also got a much better perspective of what we're doing with the project now we've had a whole year. And hopefully by then [VMP workshop to be run at upcoming Mixed Attainment Maths conference] we would have gone through the Let Teachers Shine [professional development workshop], and our conversations would have advanced ...
- TC: There's more things to talk about, yeah.
- *AF: I think just that in itself, talking about the project with more confidence, and knowing that's it's having an impact, that will hopefully help enhance it as well.*

(RGM8, #1-3)

AF describes how seeing the initial findings helps her to appreciate how much has been achieved through the project:

• AF: It's very interesting to see this, what, a year and a half, nearly two years now, down the line. Because it feels like, I don't know for you Tiago, or you Pete, but it feels like we've only had these conversations a few weeks ago, easily, now that I'm reading it. It must be very interesting for you [Pete] to look back at it. But I think it's very good to have it for us as well. Just to keep track of how far things have moved along.

(RGM15, #8)

AF/TC describe how seeing the initial findings helps them to reflect on where they are now and appreciate how much their practice has moved forward:

- PW: One of the reasons for doing this [presenting the findings] was, kind of, to get your response and see if ... from the limited amount that you've actually been able to see, does it feel ... "Yeah, that kind of ..."?
- AF: What you've picked out are the key elements, I feel like, of what happened in that first year. And I think it's also brought out to me how much more we've been able to build on it this year, as well. Like refine the model of professional development and come up with a protocol. And really zone into reasons and rationales behind the pedagogies and make them explicit amongst us. So that, then, we can go back to the classroom, make them explicit to the students, in return. I think those are some things that we've grown this year.
- TC: I agree. Having this document, just helps us reflect and go back. And this is from two years ago, or a year ago, so it gives us an overview. And also, it's interesting to see it through your eyes, like having access to the themes you pick up. Because they're not necessarily the ones that we pick up.

(RGM15, #9-11)

AF/TC describe how seeing the initial findings helps them see the bigger picture relating to the research processes and the model of professional development:

- *PW: I'm hoping that it's not just my take on this coming through, it's just what I've managed to do on behalf of the three of us. Because I've had the time to, sort of, go through the data.*
- AF: Yes ... I mean I definitely would pick out similar themes. Because you have the bigger picture, because you can see the data holistically. And it's really interesting to see it being picked out because ... I think, whilst I had all these things in mind, maybe they weren't all as clear as they are now that they're all laid out in front of me. And patterns are easier to spot when someone's collected all the relevant data.
- TC: And also, we have dual roles, basically ... so we are teacher researchers, in a way, so we have to set ourselves in different modes when we're thinking from, as Alba said, the bigger picture. It's different what emerges from that ... it's much easier and much more tempting for us to just zone into the pedagogy, than so much engaging with the critical research processes, or thinking about 'what was the importance of using video-stimulated recall?' And all of the history of that. It's only, I mean, having access to the data, or to the recordings of our interviews, makes it possible to actually think and see what was said when the discussions were taking place ... It's useful for us, as Alba said, to see the progress. But also, to think about things that we're not constantly reflecting [on] ... we are more on a layer of the classroom and not so much thinking about the research ... the professional development model.

(RGM15, #12-14)

5.5. Impact on students

5.5.1. Appreciation of pedagogical rationale

TC suggests the interviews provide evidence that students understood the rationale behind the boxing up strategy:

• TC: My three interviewees were very focused on the idea of 'making a plan' and that kind of confirms that ... I think I made that very explicit in the lesson and they got the idea. And the interviews just show that ... They appreciate that the boxing up [strategy] is a good way of stopping for a moment and making a plan ...

(RGM14, #1)

The researchers discuss evidence that suggests students are developing a greater awareness of the pedagogical rationale:

- *PW: So, do you think there was any evidence that the strategy helped them to develop that awareness of what the green box questions were all about?*
- TC: I think there was evidence of it making it explicit, more from what we saw from the video recordings than what we can deduce from the interview transcripts. Some of them make explicit mention of that in the surveys as well, so they know that it was something new ... and they remember these ideas.

(RGM14, #5-6)

The researchers continue to discuss evidence that suggests students are developing a greater awareness of the pedagogical rationale:

- PW: Coming back to that first question, what do they [interviews] actually tell us about the strategy? ...
- AF: I was quite impressed with, actually, how much they were able to explain. I think ... mine were able to say 'picking out key information, quite a lot. I think that's probably reflective of what I said during the lesson, as well. I think I said that quite a lot. So, I wonder, again, if they just said that because I said that

quite a lot, and I made that super explicit, or if they really did truly get that? My feeling is that they did get that.

- *PW: So, by you making it more explicit, they came away from that lesson understanding more what the purpose of the green box questions was?*
- AF: Exactly. So, I thought that was quite clear.

5.5.2. Engagement with project strategies

AF describes how students were receptive to spending time on discussing the card sort:

- AF: With my group, and I think in a sense the same with your [Tiago's] group, I didn't get the sense that they got frustrated at any moment. If anything, I was the one that was conscious of where they needed to go and the time ... With my group, this year in particular, they're very receptive, and very good at absorbing it ...
- PW: And they saw it as a valid and valuable thing to be doing? ...
- AF: Absolutely valuable.

TC highlights how one particular student engages with the 'boxing up' strategy:

- TC: I was also very surprised that there was such a consistent answer from them. And I really liked what [Student A] said at the back. He's made incredible progress. He is doing really well, generally. He is buying in with all of these strategies. He said ... it shows for the least important reason to be 'not doing the answers quickly' ...
- PW: Is he the one that says "It's not a race"?
- TC: "It's not a race ... you have to think." ... He used the boxing up card really well.

(RGM13, #14-16)

(RGM10, #24-26)

5.5.3. Appreciation of how to achieve success

The researchers reflect on the card sort and how it facilitated a discussion about what success looks like:

- AF: You [Tiago] went around quite a bit, what did you think?
- TC: Yeah, I think that every group engaged with the task. And they found ... chose some reasons, maybe not all ... I agree, that group at the front ... I think they spent some time talking about something else. And that's why, probably, he identified 'so you can work through the problems more quickly' as one of the reasons, which is invalid for both approaches.
- *PW:* That sort of goes back to this common thing that comes up about success in mathematics is working quickly and getting it correct, isn't it?
- AF: [Student A] said something similar, when I asked him. He said: "So you work well with other students". And it's going back to that, kind of, default of ... 'it's the teacher checking that I'm doing the right thing, whether I'm listening, whether I'm working well', rather than thinking about 'how does this specifically help us with what we're doing? What's the specific aim here?' ...
- TC: But I think that second student also mentioned that reason, and you challenged that. And I think that's a good way of managing the discussion: "Is it really important?" ...
- AF: I wasn't sure about how far to go with the challenging of that, But I thought, you know, "this is the time", if we want them to walk away understanding that, and making it clearer, then it is important that we challenge it. But we also want them to be thinking about it. So, I'm trying to do it in a way that's challenging it but helping him come up with it.

(RGM10, #14-19)

The researchers discuss how students' notion of success appears to be shifting away from simply working quickly:

(RGM14, #22-25)
- TC: They also seem to engage with the idea of not rushing. So, they've put that as the least important reason.
- *PW:* Which is quite a contrast to some of the things they were saying earlier, in the surveys ... success is all about getting through lots of questions correctly, they seem to be moving away from that, slightly, here.
- TC: Yeah, I agree. ... The interview was useful to confirm that they understood these reasons.

(RGM14, #2-4)

The researchers discuss how students' notion of success appears to be moving towards overcoming challenge:

- AF: In the surveys, what was the other question that we asked?
- PW: How successful are you? How do you know?
- AF: How successful? Most of them said things like: "Because, when she did questions on the board, it was similar to mine". ... Some of them said: "It was a really hard question and I was able to do it with the green box method", which was quite nice, because that's not something that I don't think [was] mentioned in the last [survey] ...
- PW: They seem to be welcoming challenge a bit more and seeing success more as overcoming challenging problems. Whereas, before, they were articulating success in that: "If I didn't experience any difficulty at all, that meant I was successful".

(RGM14, #26-29)

5.5.4. Engagement with progressive pedagogies

 \rightarrow See Sections 5.5.2, 5.5.3 and 5.5.7

5.5.5. Appropriateness of responses to tasks

 \rightarrow See Sections 5.5.2, 5.5.3 and 5.5.7

5.5.6. Responses/engagement of all students

AF reflects on how the card sort enables more students to engage with discussions around the pedagogical rationale:

• AF: And also, the nature of the card sort, it allows them to be working at their own pace a little bit more. Whereas as the questioning and answering, even if I'm doing targeted questioning, it's very 'on the spot' and they're not necessarily having time to think about it. Whereas, working with their partner, it's a little bit more relaxed and they can think, they can have a conversation between them.

(RGM10, #23)

5.5.7. Responses/engagement of target students

AF highlights how individual students engaged with strategies that encourage them to consider the teacher's pedagogical rationale:

• AF: So, I thought it was a really productive chat, conversation with the class. Like [Student B], for example, he's an EAL student, and for him to be able to say ... "pick out relevant information" ... one of the reasons he struggles in tests is he doesn't understand what the question's asking him, even though his maths is actually very, very good. So, for him to realise that importance is quite encouraging, to know that, OK, this could be useful to him ... And [Student C] ... he's very bright ... and I think that this sort of activity, these sort of strategies that we're doing, are perfect for students like him. Because, he's so easily distracted. And I feel like, for these types of activities, you can get their 'buy-in' even more ... when they see why you're doing what you do, they're much more likely to be on board with you ... You can see him really listening during the activities like this.

The researchers discuss the rationale behind helping students appreciate their pedagogical rationale:

- AF: I picked her ... because [Student D] is someone who works very, very hard, but she often misses the point with the work, with the maths, as well. And I wanted to gauge, you know, whether she had picked it up.
- *PW: So, she's one of the people who would be most likely to misinterpret ...?*
- AF: ... to misinterpret it. And she had. And the way she reasoned it was: "Well, because if I'm not distracted then I can work independently. And if I can work independently then I can get the question done on my own. And that's a strength." And she did change her mind, part way through the questioning, and said: "To make a plan for myself" ... it was the one that she changed her mind to, being the most important. But I think that shows the benefit of having that discussion afterwards, even after you feel you've made those reasons very explicit. ...
- TC: As we said, there may be other valid reasons, that are not the main reasons that we used ... and they may well choose those ones, it's fine ...
- *PW: I think [student D] ... she kind of had her own reasons, but she did seem to know what your reasons were at the same time, which is the point of the project, isn't it? That will help them to get a better idea of what success looks like, because "that's what the teacher is aiming for me to do".*

(RGM13, #23-27)