Atmospheric Laser Propagation at Near Infrared

by

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Thesis Presented For Examination
For The Degree of Ph.D. at the
University Of London

Dept. Electronic and Electrical Engineering

University College London

February 1996
Acknowledgements

To Professor Roy Cole for his support, expert supervision and encouragement all through the course of this work.

To my brother Lourenço for always keeping me company, even when we were an ocean apart.

To Sue for making me more real.

To the late Mr. John Chaimowicz, who designed and built the 0.83µm equipment and to Mr. Les Reardon, whose help was invaluable to the development of this work.

To British Telecom Research Laboratories for the lease of the 1.55µm equipment.

To the Brazilian National Research Council (CNPq) for financial support.
Abstract

The main aim of this project is to investigate the characteristics of the propagation of laser beams through the atmosphere at the wavelengths of 0.83 and 1.55\mu m across a 4km path in central London, a densely urbanised terrain.

The thesis reports practical measurements of the propagation characteristics in clear air and in also in rain and mist conditions. Meteorological measurements of rainfall rate and wind speed were also made to supplement the propagation measurements. Measurements were also made at 1.55\mu m with the transmitter modulated at 155Mbits/s.

The analysis of the results is divided into four sections. (1) The statistical and spectral analysis of amplitude scintillations and angle of arrival in dry weather, (2) in rain, (3) modelling of the rainfall induced attenuation of the received optical power and (4) the performance evaluation of the prototype of a 155Mbits/s digital free space optical communications system operating at 1.55\mu m.

Results for studies (1) and (2) indicate that the gamma is the best distribution for the received amplitude scintillations for varying turbulence conditions in clear air and also in rain conditions. The normal distribution is the best fit for angle of arrival data, regardless of the strength of turbulence and weather. Level crossing statistics of amplitude scintillations are presented for clear air conditions. Spectral analysis of the scintillations and angle of arrival showed results that confirm theoretical assumptions in dry but not in rain conditions. No evidence of saturation of scintillations was found.

In study (3), cumulative distributions of attenuation and rainfall for a year of measurements are presented and the rainfall induced attenuation is evaluated.
Statistical comparison of the performances of both wavelengths in rain are shown and indicate that the 0.83\textmu m system has a slightly better overall performance.

For study (4), reliability and quality analysis results are presented. Availability figures show values as high as 94.5\%. Analysis of bit error rates using level crossing statistics showed an approximate overall 92\%-95\% probability of error-free operation for the system.
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Chapter 1

General Introduction and Theoretical Background
1.1-PART 1- GENERAL INTRODUCTION

1.1.1-OBJECTIVES OF THIS WORK.

This work is concerned with the experimental study of the propagation of laser beams through the urban boundary layer on a 4 km path across central London. The wavelengths used are 0.83μm and 1.55μm chosen basically for being inside two water vapour transmission windows. These two wavelengths have not yet been used in propagation experiments.

The research is divided in four branches:

1-Statistical and spectral analysis of the received amplitude scintillations and angle of arrival in clear air conditions.

2-Statistical and spectral analysis of the received amplitude scintillations and angle of arrival in rain.

3-Study of the effects of hydrometeors, particularly rain and mist on the received optical power.

4-Evaluation of the availability and quality of a digital optical communication system operating at 155Mbits/s and using the 1.55μm link.

As will be seen in more detail later on in this chapter, all four studies are highly original not only because laser propagation experiments in a city environment are virtually non-existent in the literature but also because of the number of questions raised by the International Telecommunications Union in their resolution ITU-R5 which have direct connections with the results obtained from the experimental data. The International Telecommunications Union (ITU) is the United Nations consultative organisation on normalisation and standardisation and research of telecommunication issues. Its reports, documents and recommendations give guidelines and set objectives to research in telecommunications. The ITU Radiocommunication Assembly, Geneva
(1992), produced Resolution ITU-R5, which is entitled “Work programme of Radiocommunication study groups for 1993-1995”. In this Resolution, a series of questions are posed to the eleven Radiocommunication study groups. These questions are in fact requests for research in various topics, which are classified according to the level of priority and urgency. The work presented in this thesis covers a total of 25 questions spread over six different study groups. Examples of questions posed to the radio wave propagation study group (study group 3) are the gathering of radiometeorological data required for the planning of terrestrial and space communication systems, propagation data and prediction methods required for terrestrial line-of-sight systems and compilation of new measurements and data banks.

1.1.2-THE ATTRACTIONS BEHIND THE USE OF FREE SPACE OPTICAL COMMUNICATION SYSTEMS.

Free space optical communication systems offer several advantages over guided optical systems and other wavelengths:

1-More agility. Optical systems can overcome difficult terrain constraints, such as rocks, water, or other situations where cabling installation is particularly troublesome.

2-Low power consumption. Only 0.4W are required for a single audio channel [Chaimowicz and Cole, 1989], which allows the use of solar or common batteries to power the system.

3-The hardware is smaller, lighter and potentially cheaper than equivalents in microwaves. This makes the system easy to install and excellent for short term installations. During the experimental phase of this thesis, a group of three people was capable of setting up in less than 30 minutes two 108 metres links, using 3mm apertures.
4-Large bandwidth. An optical system can transmit wide band signals. A prototype by Canon [Canon Technical manual, 1993] can transmit signals with bandwidths up to 500MHz. This wide bands enables, among other uses, relaying of TV, teleconferencing or high speed data transmission.

5-Immunity from electromagnetic noise. Optical signals suffer little interference from electromagnetic sources while causing none themselves.

6-Freedom from licensing constraints.

7-Compatibility with distribution fibre technology. There is a growing interest in D fibre technology or holographic gratings which could form the basis for the final transmitter to “floodlight” several premises.

The main argument against the use of free space optical systems is their sensitivity to the propagating medium, mainly to the effects of turbulence and hydrometeors. Even so, as will be demonstrated along the course of this thesis and has been verified by other researchers like Chu and Hogg, 1968, optical systems can still compare favourably with microwaves and millimetre waves in conditions of attenuation by rain and fog.

1.1.3-CHAPTER SUMMARIES

Chapter one concludes with a review of the theoretical background necessary for the understanding of the experimental results. The theoretical review is divided in four sections. The first is a description of the atmosphere as a propagating medium for electromagnetic waves at optical wavelengths, starting with a description of the Navier-Stokes equations, which are the basic set of equations of fluid dynamics. This introduction demonstrates the pseudo-random characteristics of atmospheric turbulence and proves the need of the use of probabilistic models for characterising the refractive index fluctuations. Relevant issues on atmospheric physics follow. This section ends with the
stochastic representation of random functions which is basic for the understanding and use of Fourier techniques for the spectral analysis of turbulence. Models for the turbulence spectrum are presented and discussed.

The second section presents the solution of Maxwell's equations for a random medium. The statistical and spectral properties of intensity scintillations and angle of arrival are developed. Further issues relevant to understanding free space optical propagation in non-ideal conditions, such as effects of aperture averaging and saturation of scintillations in strong turbulence are shown at the end of the section. The third theoretical section presents a brief review of the literature with emphasis on previously obtained experimental results. Finally, in the fourth and last section, a concise account of the turbulence characteristics of the urban boundary layer is presented. Results from experimental works by atmospheric scientists are shown and their relevance to this work is discussed.

Chapter two presents a description of the experimental equipment. The description is divided into seven parts: the propagation path, the 0.83 μm system hardware, the 1.55 μm system hardware, the 155 Mbits/s digital system, rainfall measurements, data logging and storage and the effects of building movements on the received signal.

The presentation of the experimental results of this thesis begins in Chapter three, with the statistical and spectral analysis of received signal scintillations and angle of arrival for about 18 months of propagation measurements in clear air conditions. A theoretical model for the atmosphere, proposed by Phillips and Andrews, 1982, which the authors affirm is valid for all turbulence conditions, is presented and tested against measured scintillations. Results for the distribution of scintillations and angle of arrival are presented for a wide range of values of the strength of turbulence. The data bank obtained for scintillations and angle of arrival is in itself an original result not only because of the nature of the propagation path but also because of the length of the experimental period. The use of nonparametric tests for ascertaining the true
nature of the obtained distributions is rarely seen in the literature. Experimental results on the statistical characteristics of angle of arrival are, to this author’s knowledge, unprecedented. The level crossing problem for the scintillations is explored and results compared with theoretical predictions for microwaves. The connection between the level crossing probability density function of the scintillations and the bit error rate on an optical digital system (shown in chapter six) has never been experimentally explored. Typical measured spectral power density functions for scintillations and angle of arrival are presented and compared with theoretical predictions. The spectral analysis can provide unprecedented backup for atmospheric sciences studies about the turbulence characteristics within the urban boundary layer.

Chapter four presents the statistical and spectral analysis of scintillations and angle of arrival in rain conditions. All the experimental results presented in this chapter are, to this author’s knowledge, absolutely original. As a theoretical preamble, initially the Mie theory of light scattering by a dielectric sphere is shown, followed by the solution of the Maxwell equations for a medium containing a dynamic distribution of scatterers. The theoretical forms for the spectra of scintillations and angle of arrival in rain are presented. The probability density function estimates for scintillations and angle of arrival are presented for a range of rainfall rates from 0.1 to 12.0mm/h. The distributions in rain are compared to distributions obtained in dry weather by means of percentile-percentile plots. Spectral analysis for scintillations and angle of arrival are performed and compared with theoretical predictions.

Chapter five is an extension of the statistical analysis of laser propagation through rain with a focus on the effect of hydrometeors, particularly rain and mist, on the received power level. So far in the literature, only one paper concerning laser light attenuation by precipitation in a city environment has been written by Chu and Hogg in 1968. More recently, Gibbins et al. (1989) and Maitra and Gibbins (1995) have produced papers on laser attenuation by
rain for a 500 metres short path over plane terrain. These measurements of the received power on both 0.83 and 1.55μm systems are denoted long term measurements, if compared with an usual 15 or 12.5 seconds run typical of scintillations and angle of arrival. Rain events usually last for periods of time of hours and logging systems have to be set accordingly. Results of the cumulative distribution of attenuation for one year of measurements are presented. Combination of these results with the cumulative distribution of rainfall on the same period aim to obtain a power-law-type relationship of the \( A = a^R^b \) kind between attenuation and rainfall rate. Comparisons between the performances of the 0.83 and 1.55μm links are presented. Wavelength diversity experiments such as this have not been reported in the literature.

Chapter six shows new results on the performance evaluation of a 155Mbits/second digital link at 1.55μm, using reliability analysis and level crossing techniques. The reliability analysis will ascertain the availability of the link based on the presence or absence of the clock signal at the receiver, while the level crossing techniques is used to find the statistics of the bit error rate based on the statistics of the received signal amplitude fluctuations. Initially, the fundamentals of reliability analysis are presented, followed by the description of the relationship between bit error rate and the received signal. The descriptive statistics and probability density estimates of the times to the onset of a clock outage and the times for clock recovery are presented and correlations with the strength of turbulence are attempted. Markov methods are applied as a means for the calculation of the state transition probabilities and forecasting of state changes between the clock being received and being absent. Availability estimation for the system with relation to the strength of turbulence are shown last in the reliability analysis section. The level crossing probabilities are obtained for a range of refractive index structure parameter \( C_n^2 \) values from 0.5 to \( 10*10^{14} \text{ m}^{-2/3} \) and bit error rate statistics are presented for the first time.
1.2-PART 2-THE ATMOSPHERE AS A PROPAGATING MEDIUM.

1.2.1-INTRODUCTION.

An ideal atmosphere is defined as an isotropic and homogeneous mass of gases. In an ideal atmosphere, the refractive index is a constant so that a light beam propagating through a parcel of that atmosphere would experience no diffraction or scattering. Air flows would behave as laminar flows, i.e., every part of the stream travels with the same speed. In essence, the fluid acts like a large pack. The real model is quite different. Energy inputs cause shear forces to act upon this laminar flow upsetting the continuity of its motion by accelerating different sections of the big pack. These eddies, being smaller than the original pack, have smaller masses and move with faster speeds than the larger volume so that the total momentum is conserved. These eddies distribute the surplus energy, which has been introduced into the system, and restore the system to equilibrium by colliding elastically with each other until a size limit is reached in which the viscous effects become significant and the eddies dissipate due to friction (in the sense that a equilibrium temperature between these small cells and the mean laminar flow volume is achieved). According to the law of the conservation of heat (to be shown later) a continuous input of energy to an open system such as the atmosphere would create a flow of heat which affects the air density. The air density is a directly contributing factor to the value of the refractive index. In an atmosphere not in equilibrium, the refractive index will not be a constant, but will vary both in space and in time. These non-equilibrium conditions are referred to as turbulence. Understanding the mechanisms of turbulence is a very important part of understanding the propagation of light in the atmosphere.
1.2.2-FUNDAMENTAL EQUATIONS OF FLUID DYNAMICS.

The basic set of equations which control the movement of a fluid parcel are the Navier-Stokes equations. These are the equivalent of Newton's Second Law:

\[
\rho \left( \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = \rho f_i - \frac{\partial p}{\partial x_i} + \mu \nabla^2 u_i 
\]  \hspace{1cm} (1.1)

In the above equation:

- \( p \): pressure field.
- \( u_i \): \( i \)-th component of the velocities field.
- \( \rho \): density
- \( \mu \): dynamic viscosity.
- \( f_i \): \( i \)-th component of the actuating force field.

This version of the Navier-Stokes equations has already been simplified, supposing that air is an incompressible fluid. This is a very large assumption, but without which, the original set of equations would be much more complicated to solve. Yet, in this form (1.1), there are some difficulties in the analytical treatment. The set is non-linear, coupled, with boundary conditions that are themselves non-linear and do not conform to any of the known standard formats of partial differential equations (elliptic, hyperbolic or parabolic), for which closed analytical solutions are well known. The drawbacks caused by the coupling and the non-conformity compared with the standards can be overcome by appropriate procedures which are beyond the scope of this work, but could be found in the works of Stanisic [1988] and Fleagle and Businger [1980]. It is, however, the non-linearity in both the set and the boundary conditions which pose the most difficult problem to be
overcome. In conditions near a thermodynamic equilibrium, linearisation of (1.1) is possible and a closed solution can be found. Away from equilibrium, the non-linearity makes unique solutions impossible. The multitude of possible solutions gives the problem of solving (1.1) random characteristics and turbulence itself can be then considered as a stochastic process. In this context, the refractive index, among other state variables such as density, pressure, wind speed and temperature also becomes a random variable.

1.2.3-REYNOLDS AVERAGING TECHNIQUE

A commonly used mathematical tool to facilitate the handling of random variables (described in the paragraph above) is the Reynolds averaging technique. This considers the random variable as the sum of a deterministic part and a random fluctuation. The deterministic part is the original random variable's mean value and the fluctuation is considered to have zero mean and to be much smaller than the deterministic value. For this model to be applicable it is then necessary for the mean value of the original random variable to be constant over a sufficiently large period of time, in effect, the turbulence has to be stationary both in time and space. Space stationarity implies homogeneity and isotropy, at least over the region in which the measurements are taking place. Formally, for the refractive index, density, pressure, wind speed and temperature, the Reynolds averaging gives:

\[
\begin{align*}
  n &= \bar{n} + n' \\
  \rho &= \bar{\rho} + \rho' \\
  p &= \bar{p} + p' \\
  u_i &= \bar{u}_i + u_i' \\
  T &= \bar{T} + T'
\end{align*}
\]

(1.2)

The primes indicate the random fluctuations and the over-bars the mean value of the original variable, which is defined, for a given quantity \( s \) as:
Where $T$ is the sampling, or measurement, time.

1.2.4-POTENTIAL TEMPERATURE AND THE STABILITY OF THE ATMOSPHERE.

The already mentioned non-linearity of the Navier-Stokes equations causes pressure, temperature and water vapour pressure to undergo irregular variations that do not necessarily follow the velocity components of the turbulent motions. However, some variables are able to conserve their characteristics inside a volume of space as it moves about in a turbulent field. Quantities such as these are termed conservative additives. If these variables also do not exchange energy with the turbulence, they are also called passive, which makes them conservative passive additive quantities. Potential temperature and specific humidity are the two conservative additive variables relevant to the study of light propagation through atmosphere, because of their relationship with density, as will become clearer in the following sections. For the moment, the expression for potential temperature and specific humidity will be derived and conditions for the stability of the atmosphere given the behaviour of these variables will be shown.

The first necessary approximation to the atmospheric turbulence model is to consider it as an isentropic process. This means that no heat is added or withdrawn from an air packet as it moves through turbulence and also that the entropy is kept constant. Turbulence is hence assumed to be an adiabatic reversible thermodynamic process.

The starting point to obtain the form for the potential temperature is the first law of thermodynamics for an isentropic process:

$$\bar{s} = \frac{1}{T} \int_{0}^{T} s \, dt$$

(1.3)
\[ c_v dT + pd\alpha = 0 \]  \hspace{1cm} (1.4)

where \( c_v \) is the specific heat at constant volume, \( T \) is temperature, \( p \) is pressure and \( \alpha \) is specific volume. Algebraic manipulation leads to the following equation [Fleagle and Businger, 1980]:

\[ TP^{-R_v/c_p} = \text{constant} \]  \hspace{1cm} (1.5)

The constant can be evaluated for a particular state of the system. If an air packet is brought isentropically from a pressure point \( p \) to, say, the surface (where the pressure value, \( p_0 \), will be taken as a reference), equation (1.5) would give a value for the temperature known as the potential temperature, \( \theta \):

\[ \theta = T \left( \frac{p_0}{p} \right)^{R_v/c_p} \]  \hspace{1cm} (1.6)

The potential temperature is a very useful parameter to assess the stability of the atmosphere. It can be proved that (see [Fleagle and Businger, 1980]), for a stable atmosphere, the potential temperature should increase with height.

The laws of thermodynamics give explicitly the interrelation between the so called state variables. Of particular interest for this work is the relationship between variations in temperature and variations in density and also the influence of moisture in both temperature and density. The following sections are devoted to develop these relations.

1.2.5-THE RELATION BETWEEN TEMPERATURE AND DENSITY VARIATIONS.

The equation of state for an ideal gas can be written as:
\[ p = R_m \rho T \]  

(1.7)

In a turbulent medium, using Reynolds averaging, this equation becomes:

\[ \bar{p} + p' = R_m (\bar{\rho} + \rho')(\bar{T} + T') \]  

(1.8)

Equation (1.8) has to be further manipulated to show more clearly the density variations dependence on the temperature variations. Applying the average operator to both sides of (1.8) the following expression is obtained:

\[ \bar{p} = R_m \bar{\rho} \bar{T} \left( I + \frac{\rho'T'}{\bar{\rho}\bar{T}} \right) \]  

(1.9)

In the atmosphere [Fleagle and Businger, 1980] \(|\frac{\rho'}{\rho}| < 10^{-2}\) and \(|\frac{T'}{T}| < 10^{-2}\). Therefore, the last term in (1.9) is very small compared to unity and (1.9) becomes:

\[ \bar{p} = R_m \bar{\rho} \bar{T} \]  

(1.10)

Dividing (1.10) per (1.9) gives:

\[ \frac{p'}{\bar{p}} = \frac{\rho'}{\bar{\rho}} + \frac{T'}{\bar{T}} \]  

(1.11)

Noting that [Fleagle and Businger] \(|\frac{\rho'}{\rho}| << |\frac{\rho'}{\bar{\rho}}| or |\frac{T'}{\bar{T}}|\), (1.11) can be written in the form:

\[ \frac{\rho'}{\bar{\rho}} = -\frac{T'}{\bar{T}} \]  

(1.12)

Equation (1.12) means that thermodynamic processes in the lower part of the atmosphere (surface layer) are essentially at constant pressure (surface pressure). This characteristic makes surface pressure an ideal reference point to which the variations of the other state variables can be referred.
This state of affairs is somewhat more complicated when the effects of water vapour are concerned. Moisture in the atmosphere will cause changes not only in density but also in temperature, as the following section will demonstrate.

1.2.6-WATER VAPOR EFFECTS IN A TURBULENT ATMOSPHERE.

Starting with the Clausius-Clapeyron equation, which relates the variation of the saturation pressure with temperature, as a substance changes phase from liquid to gas:

$$\frac{dp_s}{dT} = \frac{L}{T(\alpha_{gas} - \alpha_{liq})}$$  \hspace{1cm} (1.13)

In the above equation:

- $\alpha_{liq}, \alpha_{gas}$: specific volume of the liquid and gaseous phases.
- $p_s$: specific pressure.
- $T$: temperature.
- $L$: latent heat of vaporisation.

This equation can be applied to water vapour, by replacing $p_s$ by $e_s$, the saturation pressure of water vapour. Furthermore, in the case of normal conditions of pressure and temperature in the atmosphere (1000mb, 25°C), the specific volume of the gaseous phase for water is much higher than the specific volume of the liquid phase. The combination of (1.13) with the equation of state(1.7) gives:

$$\frac{de_s}{e_s} = \frac{L}{R_{m,water}} \cdot \frac{dT}{T^2}$$  \hspace{1cm} (1.14)
Integrating (1.14) and substituting the typical values $L=2500 \times 10^6 \text{J/kg}$, $R_{m,\text{water}}=461.7 \text{J/kg K}$ and $P=0.611 \text{kPa}$, equation (1.14) becomes:

$$\log_{10} e_s = 11.40 - 2353/T$$

Equation (1.15) provides a way to evaluate the water vapour saturation pressure given a certain temperature, and is rather accurate [Fleagle and Businger, 1980]. This result can then be used with a measurement of relative humidity to give the water vapour pressure:

$$r = \frac{e}{e_s} \therefore e = r \cdot e_s$$

From the above two equations, the temperature for the moist air has a different value than the temperature for dry air for the same value of pressure. This so-called virtual temperature is expressed by:

$$T_v \equiv (1 + 0.61q)T$$

In (1.17), $q$ is the specific humidity, defined as the mass of water vapour per unit mass of air. It is related to the water vapour pressure in the following manner:

$$r = \frac{e}{e_s} \equiv \frac{q}{q_s}$$

The virtual temperature accounts for changes in the air density due to humidity. The specific heats at constant pressure and constant volume are also affected by humidity, as the following equations show:

$$c_p = (1 + 0.90q)c_{p,\text{dry air}}$$
$$c_v = (1 + 1.02q)c_{v,\text{dry air}}$$

The effects in the density become explicit if it is noted that:

$$R_{m,\text{moist air}} = c_p - c_v$$
For the sake of illustration, the values of $c_{p, \text{dry air}}$ and $c_{v, \text{dry air}}$ are, respectively, $1.004 \times 10^3$ J/kg K and $0.717 \times 10^3$ J/kg K.

Substituting this result into the equation of state (1.7) in a situation of constant pressure and temperature, the variations in the density of air become a function of the variations in the specific water vapour in the atmosphere:

$$\rho'_{\text{moist air}} = -0.6 \rho_{\text{dry air}} q'$$

Equation (1.21) ends the presentation of the relations between variations in temperature and water vapour and variations in density. It is safe to say that any electromagnetic wave whose propagation is affected by changes in the profile of the refractive index and consequently in the profile of density will be affected by variations in temperature and water vapour pressure (or, equivalently, in relative humidity.)

The main advantage in using a stochastic approach to turbulence is the possibility of the use of Fourier techniques to describe the statistical characteristics of turbulence and also the several variables involved in the problem, mainly temperature and refractive index. The following sections present mathematical introductions to the spectral analysis of turbulence. First, the introduction to the use of Fourier techniques to represent stochastic functions, then on to the concept of structure functions and to the Kolmogorov's spectral theory of turbulence. Kolmogorov innovated the study of turbulent motion, as will be seen, by developing the concept of structure functions as a means of obtaining a spatial correspondent to the time autocorrelation function.

1.2.7-SPECTRAL REPRESENTATION OF STOCHASTIC FUNCTIONS

The Fourier transform of a stochastic process $x(t)$ is also a stochastic process $X(\omega)$ given by:
\[ X(\omega) = \int_{-\infty}^{\infty} x(t) \exp(-j\omega t) dt \quad (1.22) \]

This integral is understood as an mean square limit. The inversion integral is:

\[ x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \exp(-j\omega t) d\omega \quad (1.23) \]

The autocorrelation function of \( X(\omega) \) is a Fourier transform in two dimensions:

\[ \Gamma(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R(t_1, t_2) \exp[-j(ut_1 + vt_2)] dt_1 dt_2 \quad (1.24) \]

of the autocorrelation \( R(t_1, t_2) \) of \( x(t) \). It is also valid to say that:

\[ E[X(u)X^*(v)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E[x(t_1)x^*(t_2)] \exp[-j(ut_1 + vt_2)] dt_1 dt_2 \quad (1.25) \]

Comparing (1.25) with (1.24) it can be written that:

\[ E[X(u)X^*(v)] = \Gamma(u, -v) \quad (1.26) \]

Supposing that the process \( x(t) \) is wide sense stationary with autocorrelation \( R(t_1, t_2) = R(t_1,-t_2) \) and power spectrum \( S(\omega) \), then the following theorem arises:

\[ \Gamma(u, v) = 2\pi S(\omega) \delta(u + v) \quad (1.27) \]

The proof begins by making \( t_1 = t_2 + \tau \). Then, from the definition of \( \Gamma(u,v) \) in equation (1.27):

\[ \Gamma(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R(t_1, t_2) \exp[-j(ut_1 + vt_2)] dt_1 dt_2 = \]

\[ = \int_{-\infty}^{\infty} \exp[-j(u + v)\tau] \int_{-\infty}^{\infty} R(\tau) \exp(-j\tau) d\tau d\tau \quad (1.28) \]

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But \( \int_{-\infty}^{\infty} R(\tau) \exp(-j\omega\tau) d\tau = S(U) \), the power spectrum of \( x(u) \) and, because

\[ \int_{-\infty}^{\infty} \exp(-j\omega t) dt = 2\pi S(\omega), \]
equation (1.28) becomes (1.27) and the theorem is proved. According to (1.27), the Fourier transform of a stochastic process is therefore a non-stationary white noise with mean power \( 2\pi S(U) \). Expressing

\( X(\omega) = A(\omega) + jB(\omega) \), for a real process it is valid to write that \( A(\omega) = A(-\omega) \) and \( B(\omega) = B(-\omega) \). Consequently, \( \mathbb{E}[x(u),x(v)] = \Gamma(u,v) \) and thus:

\[
x(t) = \frac{1}{\pi} \int_{0}^{\infty} A(\omega) \cos \omega t d\omega - \frac{1}{\pi} \int_{0}^{\infty} B(\omega) \sin \omega t d\omega
\] (1.29)

It suffices, then, to calculate the spectra of real processes for the closed positive semiplane, that is, for \( \omega \geq 0 \).

1.2.8-THE CONCEPT OF STRUCTURE FUNCTION.

The majority of stochastic processes found in nature cannot in formal terms be classified as stationary or homogeneous, because, in the long term, the mean values of the process both in time and space cannot be regarded as constant. However, in practice, many of these processes such as wind speed [Fleagle and Businger, 1980] are able to hold their stationarity and homogeneity over a certain length of time or spatial distance. Besides, if a stochastic process \( f(\vec{r}) \) is inhomogeneous, the difference between the values of the process at two different points is homogeneous. It can be proved by first establishing this difference:

\[
f(\vec{r}_1) - f(\vec{r}_2) = f(\vec{r} + \vec{d}) - f(\vec{r})
\] (1.30)

For a stochastic process to be homogeneous, the mean value of (1.30) must be a constant. Taking the derivative of the mean value:
It comes from (1.31) that the mean value of (1.30) is a constant, furthermore, from (1.31):

\[ E[f(\bar{r})] = c_1 \bar{r} + c_2 \]  

(1.32)

Substituting (1.32) in (1.30) results

\[ E[f(\bar{r} - \bar{r}_d) - f(\bar{r})] = c_1 r_d \]  

(1.33)

Taking the ensemble average of the square of the magnitude of the right side of (1.30) and using (1.32), the following relationship can be derived:

\[ D_f(\bar{r}_d) = E\left[|f(\bar{r} + \bar{r}_d) - f(\bar{r})|^2\right] \]  

(1.34)

Equation (1.34) defines the structure function. If (1.33) and (1.34) hold, then the function \( f(\bar{r}) \) is said to be a stochastic process with stationary increments.

The same approach is valid, in the time domain, considering two instants in time instead of two points in space and following the same steps.

It is always possible to retrieve the autocorrelation function in space and in time from the structure function, by expanding and evaluating the argument from the expected value in (1.34). In the more general case in the time domain, this relationship becomes:

\[ D_f(\tau) = B_f(t + \tau, t) + B_f(t, t + \tau) - B_f(t + \tau, t) - B_f(t, t + \tau) \]  

(1.35)

If the process is stationary and real, (1.35) simplifies to:

\[ D_f(\tau) = 2\left[B_f(0) - B_f(\tau)\right] \]  

(1.36)
1.2.9-RELATIONSHIP BETWEEN THE STRUCTURE FUNCTION OF TEMPERATURE AND REFRACTIVE INDEX.

Variations in the refractive index have a direct impact on the propagation medium and therefore on the statistical and spectral characteristics of the received signal. Variations of the refractive index are not as trivial to measure as variations in temperature and therefore, it is useful to obtain a relationship between the structure functions of temperature and refractive index (and consequently, between the structure parameters). The starting point is the structure function of the refractive index:

$$D_n(\rho) = \left( \langle n(r) - n(r + \rho) \rangle \right)^2 = C_n^2 \rho^{2/3} \quad (1.37)$$

From the equation of state, it can be written that:

$$N = N_0 \frac{p}{p_0} \frac{T_0}{T} \quad (1.38)$$

where $N_0$, $p_0$ and $T_0$ are the values of the refractivity, pressure and temperature evaluated at ground level.

Using the assumption that the variations on the refractivity are due mostly to variations in temperature, application of Reynolds averaging to equation (1.38) leads to:

$$N = \langle N \rangle + N' = \langle N \rangle \left( 1 - \frac{T'}{\langle T \rangle} \right) \quad (1.39)$$

Using equation (1.39) in (1.37) it is possible to obtain the relationship between the structure functions of temperature and refractive index:

$$D_n(\rho) = 10^{-12} \cdot \frac{\langle N \rangle^2}{\langle T \rangle^2} D_T(\rho) \quad (1.40)$$
Using equation (1.37) and (1.40), the relationship between the structure parameters of refractive index and temperature is obtained:

\[ C_n^2 = 78 \cdot \left( \frac{\langle p \rangle}{\langle T \rangle^2} \right)^2 C_T^2 \]  \hspace{1cm} (1.41)

1.2.10-SPECTRAL ANALYSIS OF TURBULENCE.

The choice of spectral analysis to explain the solutions of the Navier-Stokes equations comes from the coupling between the fluid elements (or eddies) in the turbulent flow. Each and every eddy is affected by and affects the movement of all the others. In this sense, if a probability distribution function for a specific eddy is to be calculated, it has to take into consideration the distributions for all other eddies. This is a procedure of very little practical value. The use of the structure function, however, brings about the possibility of the calculation of the autocorrelation function and from it, the spectral power density function. It is a necessary condition, for the use of the Fourier transform to obtain the spectrum, that the stochastic process involved is stationary. Real turbulent motions are not homogeneous and isotropic, because it would imply that no deterministic boundary conditions are stipulated in the flow [Stanisic 1988]. Kolmogorov [1990] recognised this difficulty and devised a way to counteract it by the means of a mathematical construction, by which a local homogeneous and isotropic domain is guaranteed to exist within any turbulent flow, therefore allowing the use of the Fourier techniques. The development of the Kolmogorov's spectrum of turbulence is the aim of the following sections.
1.2.10.1-KOLMOGOROV'S THEORY OF TURBULENCE.

Formally, Kolmogorov started his approach by defining a fixed frame of reference \((x_1, x_2, x_3)\) as shown in figure 1.1, below.

![Geometrical description of the coordinate system.](image)

Figure 1.1: Geometrical description of the coordinate system.

The velocity at any given point in the space \(x_i = (x_1, x_2, x_3)\) and time can be referred to that frame in the form:

\[
v_i(P(x_i,t)) = v_i(x_i,t)
\]  

It is then assumed that the velocity components are random variables and that the energy of the flow is finite, i.e., the mean square values of speeds and accelerations are bounded. Taking an arbitrary point \(P_0(x_{i,0}, t_0)\), within a locally homogeneous and isotropic domain \(G\) (shown in figure 1.1) moving with speed \(v_i(P_0) = v_{i,0}\), a new coordinate system \((y_1, y_2, y_3)\) is placed within \(G\) in such a way that its relation to the \((x_1, x_2, x_3)\) system is:

\[
y_i = x_i - [x_{i,0} + v_{i,0}(t-t_0)]
\]

Since \(y_i\) depends on the random speed \(v_{i,0}\) this variable is itself random. The mean speed or the mean flow is considered to be zero (\(v_{i,0}\) is equally probable...
to be positive or negative). This assumption allows any absolute velocity $v_{i,k}[P_{i,k}]$ within $G$ to be written relative to the velocity at the point $P_{i,0}$:

$$v_{i,k} = v_{i,0} + w_{i,k} \quad (1.44)$$

In the above equation $w_{i,k}$ is the relative speed of the point $P_{i,k}$ with respect to the point $P_{i,0}$. Solving (1.43) for the relative speed:

$$w_{i,k} = v_{i,k} - v_i \quad (1.45)$$

The variable $w_{i,k}$ is stationary because it is referred to a locally homogeneous and isotropic domain. The probability distribution of $w_{i,k}$, considering a 3-D system containing $n$ points in a $3n$-D function, $F_n$, of the parameters $x_{i,0}$, $t_0$, $v_{i,0}$, $y_{i,k}$ and $(t_k-t_0)$. In a locally homogeneous domain, $F_n$ must be independent of $x_{i,0}$, $t_0$ and $v_{i,0}$, provided all points $P_k$ are situated in $G$. Turbulence is considered to be locally isotropic if it is homogeneous and the distribution law is invariant for rotations and reflections with respect to the coordinate system.

The development of the expressions for the first and second moments of $w_{i,k}$ will be omitted (can be found in [Stanisic, 1991]). However, it is important to derive the necessary conditions for the power spectrum to exist.

Kolmogorov's model for the energy transfer in a turbulent flux states that for high values of the Reynolds number, incoming external energy affects the mean flow. The energy flow continues as eddies of larger scale sizes interact with eddies of smaller scale sizes until a limit is reached where the eddies are so small that viscous effects become relevant and energy is dissipated into heat. The set of wave numbers for which viscous effects can be neglected is called the inertial subrange. Its upper limit is denominated the outer scale of turbulence (usually represented in the literature as $L_0$). The lower limit is called the inner scale of the turbulence (usually represented as $l_0$). Typical values for the outer and inner scales can be of the order of hundreds of meters and
centimetres respectively. Sizes below the inner scale are said to be in the
dissipation range, while sizes larger than the outer scale are in the input range.

If the scale sizes are \( l_i \), \( i \) being the order of the eddy, the corresponding
Reynolds numbers are:

\[
Re_i = \frac{l_i v_i}{v}
\]  (1.46)

In the above equation, \( v \) is the kinematic viscosity, \( l_i \) is the scale size and \( v_i \) the
air parcel speed.

The Reynolds number decreases as the index \( i \) increases. Mathematically,
equation (1.46) represents a convergent sequence and therefore it is legitimate
to assert the existence of an energy spectrum density function and a power
spectrum density function.

1.2.11-THE TURBULENCE SPECTRUM IN THE SUBRANGES.

To calculate the turbulence spectrum, it is first necessary to recall the results of
sections 1.2.7 and 1.2.8 above. The usual approach to the formal calculation of
the Kolmogorov spectrum of turbulence [Tatarski, 1961] consists in following
Obukhov’s dimensional analysis and arriving at an expression for the structure
function in the different subranges. Obukhov obtained an expression for the
structure function in the dissipation range and used asymptotic analysis to find
an expression for values within the inertial subrange. The development will be
omitted at this level and can be found in [Tatarski]. The forms of the structure
function for both regions are presented below. Firstly, for scale sizes in the
dissipation range, the structure function is

\[
D_n(r) = C_n l_0^{2/3} \left( \frac{r}{l_0} \right) \text{ for } r < l_0
\]  (1.47)
It is noticeable that this form is dependent on the value of the inner scale size.
For the inertial subrange, the structure function is not dependent on the inner scale:

\[ D_n(r) = C_n^2 r^{2/3} \text{ for } l_0 < r < L_0 \quad (1.48) \]

The structure constant, \( C_n^2 \), is a measure of the strength of the turbulent motions. Typical values range from \( 10^{-14} \text{ m}^{2/3} \) for strong turbulence to \( 10^{-18} \text{ m}^{2/3} \) for weak turbulence.

As seen elsewhere, the structure function can be used to calculate the correlation function. The physical model of the turbulence states that eddies with sizes below the inner scale are virtually non-existent (the eddies have been dissipated by friction) and the spectrum for that region will be negligibly small. For the inertial subrange, the form of the spectrum is dependent only on the rate of energy transfer and the kinematic viscosity. It is a condition of quasi-static equilibrium, which characterises isotropic turbulence as an organised system, even though seemingly chaotic in nature.

The form of the Kolmogorov's spectrum for the inertial subrange is independent of the value of the inner scale and can be written in the form:

\[ \Phi_n(\kappa) = 0.033 C_n^2 \kappa^{-11/3} \quad (1.49) \]

To include the conditions on the dissipation range, Tatarski (1961) suggested the following form for the turbulence spectrum:

\[ \Phi_n(\kappa) = 0.033 C_n^2 \kappa^{-11/3} \exp\left(\frac{-\kappa}{(5.921 l_0)^2}\right) \quad (1.50) \]

To account for the influence of scale sizes greater than the outer scale, von Karman proposed a modified form for the spectrum within the input range. This name is due to the fact that the energy enters the system at very low wave...
numbers, i.e., affecting the mean (slowly varying) flow. Von Karman's spectrum can be written:

\[ \Phi_n(\kappa) = 0.033C_n^2 \left( \kappa^2 + \frac{1}{L_0} \right)^{-11/3} \]  \hspace{1cm} (1.51)

Although both equations (1.50) and (1.51) are adequate representatives of the spectra within their validity ranges, there is still the problem of the singularity at the origin in Tatarski model. To overcome that mathematical difficulty, Strohbehn (1968) proposed an alternative form for the turbulence spectrum, valid for all subranges:

\[ \Phi_n(\kappa) = \alpha \left[ \exp \left( -\frac{\kappa^2}{\kappa_m^2} \right) \right] \left( 1 + \kappa^2 L_0^2 \right)^{11/6} \]  \hspace{1cm} (1.52)

The constant \( \alpha \) is given by:

\[ \alpha = \frac{E[n^2]}{\pi^{3/2}} \frac{L_0^3 \Gamma(11/6)}{\Gamma(1/3)} C(\kappa_m L_0) \]  \hspace{1cm} (1.53)

\[ C(\kappa_m L_0) \approx \left[ 1 + \frac{\Gamma(11/6)}{\Gamma(1/3)} \frac{\Gamma(-1/3)}{\Gamma(3/2)} \left( \kappa_m L_0 \right)^{-2/3} \right]^{-1} \]

The factor \( \kappa_m \) is given by: \( \kappa_m = \frac{2\pi}{L_0} \).
1.3-PART 3-SOLUTION OF THE MAXWELL EQUATIONS FOR A RANDOM MEDIUM

1.3.1-SOLVING THE MAXWELL EQUATIONS FOR A RANDOM MEDIUM USING THE RYTOV METHOD.

The earliest attempts to study the propagation of electromagnetic waves through the turbulent atmosphere were carried out by mainly by astronomers, who were interested in studying the phase fluctuations and amplitude scintillations of starlight. These researchers used a perturbation method to solve the Maxwell equations called the Born approximation. The Born approximation is basically a 2-ray model based on a single scattering assumption. For this reason, the results obtained using the Born approximation had a limited range of validity and were not appropriate to deal with long paths, in which multiple scattering effects become relevant. In the beginning of the sixties, Russian researchers (Tatarski, 1959), (Chernov, 1960) addressed the problem of multiple scattering using a method that is similar to the Born approximation in that it is a perturbation technique, but applied to a transformation of the scalar wave equation. This method is called the Rytov's method. Rytov's method was initially believed to be valid for all turbulence conditions, but further analysis of experimental results from American researchers (e.g. Strohbehn, 1969) cast doubts on the initial claims that the Rytov method would extend the Born approximation and geometrical optics to all turbulence regimes. It has been verified (Fante, 1975, Lawrence and Strohbehn, 1970) that the Rytov method breaks down in very strong turbulence and the variance of the scintillations saturates. The phenomenon of saturation is explored at the end of this section. If saturation does not occur, the Rytov method is still the best method of solution for the Maxwell equations in a turbulent atmosphere.

Following the analysis by Tatarski (1959), as presented by Fante (1975) and Lawrence and Strohbehn (1970), the solution of the Maxwell equations using
the Rytov method begins with the expression for the Maxwell wave equations for a narrow band beam:

\[ \nabla \times \vec{E}(\vec{r}) = j\omega \mu_0 \vec{H}(\vec{r}) \]
\[ \nabla \times \vec{H}(\vec{r}) = -j\omega \varepsilon_0 n^2(\vec{r})\vec{E}(\vec{r}) \]  

A narrow band beam has such a small spread in frequency space that \( \partial^2 \vec{E}/\partial t^2 \) can be replaced by \( \omega^2 n^2(\vec{r})\vec{E} \). After some algebraic manipulation, the Maxwell equations become:

\[ \nabla \times \nabla \times \vec{E}(\vec{r}) = \omega^2 \mu_0 \varepsilon_0 n^2(\vec{r})\vec{E}(\vec{r}) \]  

Further development of equation (1.55) gives:

\[ \nabla^2 \vec{E}(\vec{r}) + \omega^2 \mu_0 \varepsilon_0 n^2(\vec{r})\vec{E}(\vec{r}) - 2\nabla \left( \frac{\nabla n}{n} \right) = 0 \]  

It can be found in Hill and Clifford (1970) that the last term on the left of (1.56) is at least 160 dB smaller than the other terms and can be neglected.

Assuming a propagation along the x-axis, the y component of the electric field satisfies:

\[ \left( \nabla^2 + k_0^2 n^2(\vec{r}) \right)E_y(\vec{r}) = 0 \]  

The refractive index can be expressed as the sum of a mean value and a random fluctuation (see Reynolds averaging in chapter 1). For air, the mean value will be approximated to 1 and therefore, the random refractive index is

\[ n = 1 + n' \]  

The use of Reynolds averaging permits the space dependence to be dropped.

\[ \left[ \nabla^2 + k_0^2 (1 + n')^2 \right]E_y(\vec{r}) = 0 \]
Expanding the binomial on the right side of (1.59) gives:

\[(1 + n')^2 = 1 + 2n' + n'^2\]  (1.60)

The refractive index fluctuates a few parts in a million [Fleagle and Businger, 1980]. Therefore the fluctuations squared are much smaller than the fluctuations themselves and can be neglected. Substituting equation (1.60) into equation (1.59), the following is obtained:

\[\nabla^2 E_y + k_0^2 E_y = -k_0^2 n'E_y\]  (1.61)

The method used to solve the above equation was devised by Rytov. Rytov’s method is based on the method of small perturbations. Instead of expanding the solution field in a convergent series, i.e., \(E_y = \tilde{E}_0 + \tilde{E}_1 + \ldots + \tilde{E}_N\), the method proposes the following transformation:

\(\tilde{E}_y = \exp(\psi_0 + \psi_1 + \ldots + \psi_N) = \exp(\chi + j\delta)\)  (1.62)

This method simplifies the procedure of obtaining amplitude and phase fluctuations and is valid for a broad range of turbulence regimes. Besides, this method does away with the vector equations and transforms the problem into a scalar one.

Algebraic manipulations of equation (1.61) and the use of (1.62) result in the following:

\[\nabla^2 \psi + \nabla\psi \cdot \nabla\psi + k_0^2 (1 + n') = 0\]  (1.63)

Equation (1.63) is known as the Riccati equation.

Substituting equation (1.62) into (1.63) and expanding leads to:
\[ \nabla^2 \psi_0 + \nabla \psi_0 \cdot \nabla \psi_0 + k_0^2 (1 + n')^2 = 0 \]
\[ \nabla^2 \psi_1 + 2(\nabla \psi_0 \cdot \nabla \psi_1) + 2k_0^2 (1 + n')^2 n' = 0 \]
\[ \vdots \]
\[ \nabla^2 \psi_p + 2(\nabla \psi_0 \cdot \nabla \psi_p) = -\sum_{j=1}^{p-1} \nabla \psi_j \cdot \nabla \psi_{p-j} \]  

(1.64)

In the case of weak turbulence, the expansion equation (1.64) can be halted at the second term [Ishimaru, 1978], [Chernov, 1960]. In this case, \(|\nabla \psi_1| \ll |\nabla \psi_0|\) and the solution is a superposition of the solution of a free space equation (first equation in the set of equations (1.64)) and the solution of the second equation in (1.64). The second equation in (1.64) can be solved by the use of Green functions. The solution \(\psi_0\) can be considered as the deterministic part of the solution, while the solution \(\psi_1\) is the random fluctuation. The solution \(\psi_0\) comes from the vacuum equation:

\[ \nabla^2 \psi_0 + (\nabla \psi_0)^2 + k^2 = 0 \]  

(1.65)

The solution for \(\psi_1\) comes from:

\[ \psi_1(\vec{r}) = 2k_0^2 \int \frac{G(\vec{r} - \vec{r'}) E_0(\vec{r'})}{E_0(\vec{r}) n'(\vec{r})} dV \]  

(1.66)

For a plane wave travelling along the x-axis, the solution of the first equation on the set (1.64) is:

\[ E_0(\vec{r}) = \exp(jk_0x) \]  

(1.67)

Substituting (1.67) into (1.66) gives:

\[ \psi_1(L, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{k_0^2}{2\pi(L-x')} \exp \left[ j \frac{k_0}{2} \frac{(y-y')^2 + (z-z')^2}{L-x'} \right] dx' dy' dz' \]  

(1.68)
Therefore, the complete solution for the Ricatti equation, for the weak turbulence case is:

\[ E(\bar{r}) = E_0(\bar{r}) \exp[\psi_1(\bar{r})] \]  \quad (1.69)

Where \( E_0(\bar{r}) = \exp(\psi_0) \).

1.3.2-FLUCTUATIONS OF THE LOG-AMPLITUDE AND PHASE.

Going back to the Rytov solution for weak turbulence in equation (1.64), the expressions for the logamplitude and phase fluctuations of the electromagnetic wave can be obtained in the following way; Firstly, the solution field is expressed in terms of the phase and amplitude of the homogeneous and inhomogeneous solutions:

\[
E_y(\bar{r}) = A(\bar{r}) \exp[jS(\bar{r})] \]
\[
E_0(\bar{r}) = A_0(\bar{r}) \exp[jS_0(\bar{r})] \]  \quad (1.70)

The superposition yields:

\[
\psi_1(\bar{r}) = \chi + jS_1(\bar{r}) = \ln(A/A_0) + j(S - S_0) \]  \quad (1.71)

The real part of \( \psi_1 \) is called the logamplitude and represents the random fluctuations of the logarithm of the amplitude of the solution wave. The imaginary part, \( S_1 \), is the representative of the random phase fluctuations of the solution wave. The description of the statistical characteristics of these two fundamental variables can be obtained either in the time or the frequency domain. In the time domain the statistical characterization is done by the study of the autocorrelation functions, while in the frequency domain, the characterization is by the power spectrum density functions. It is also possible to obtain some kind of relationship between the statistics of the logamplitude and phase fluctuations and the statistics of the fluctuations of the refractive
index. The preferred choice of approach in the literature is the spectral one. The use of Fourier techniques allows mathematical simplifications and opens vast analytical possibilities.

Recalling the final solution for the Riccati equation:

\[ \psi_1 = \int \frac{2k_0^2 G(\vec{r} - \vec{r}')}{E_0(\vec{r})} E_0(\vec{r}') n'(\vec{r}') dV' \]  

(1.72)

The spectrum of the refractive index was calculated in section 1.2.9. As previously, the use of the Wiener-Khintchine to provide a spectral representation to the fluctuations of the refractive index leads to:

\[ n'(x, \vec{p}) = \int dN'(x, \vec{k}) \exp(j\vec{k} \cdot \vec{p}) \]

\[ \vec{k} = k_x \hat{y} + k_z \hat{z} \]

\[ \vec{p} = y\hat{y} + z\hat{z} \]  

(1.73)

Substituting (1.68) into (1.67) and expanding the triple integral in the rectangular co-ordinates, it is possible to calculate the Fourier transform of equation (1.67):

\[ \Psi_1(L, \vec{p}) = \int_0^L dx' \int \exp(j\vec{k} \cdot \vec{p}) H(L - x', \vec{k}) dN'(x', \vec{k}) \]  

(1.74)

where \( H(L - x', \vec{k}) \) is the Fourier transform of \( h(L - x', \vec{k}) \). The function \( h \) is given by:

\[ h(\vec{r} - \vec{r}') = 2k_0^2 \frac{G(\vec{r} - \vec{r}')}{E_0(\vec{r})} E_0(\vec{r}') \]  

(1.75)

And the Fourier transform of (1.75) is:
\[
H(L - x', \kappa) = \int dp \exp(-j \vec{k} \cdot \vec{p}) h(L - x', \vec{p}) = j k_0 \exp \left[ -j \frac{(L - x') \cdot \kappa}{2k_0} \right] \tag{1.76}
\]

where \( dp = dydz \) and \( \kappa^2 = |\kappa|^2 = k_y + k_z \).

The process of extracting the phase and logamplitude spectra from (1.74) begins with the use of the following relationships:

\[
\chi(L, \vec{p}) = \frac{1}{2} \left[ \Psi_1(L, \vec{p}) + \Psi_1^*(L, \vec{p}) \right] \\
S_1(L, \vec{p}) = \frac{1}{2j} \left[ \Psi_1(L, \vec{p}) - \Psi_1^*(L, \vec{p}) \right] \tag{1.77}
\]

In (1.77) the star superscript denotes the complex conjugate.

Because the refractive index fluctuations form a real stochastic process, it is valid to say that:

\[
dN'(x', \kappa) = dN'(x', -\kappa) \tag{1.78}
\]

Using this identity, the complex conjugate can be written as:

\[
\Psi_0^*(L, \vec{p}) = \int_0^L \int_0^\infty \int_{-\infty}^{\infty} \exp(i \vec{k} \cdot \vec{p}) H^*(L - x', -\kappa) dN'(x', \kappa) \tag{1.79}
\]

With this equation, the solution (1.74) and the relationships (1.77) the spectra of the logamplitudes and phase fluctuations can be written as:

\[
\chi(L, \vec{p}) = \int_0^L \int_0^\infty \int_{-\infty}^{\infty} \exp(i \vec{k} \cdot \vec{p}) \text{Re}[H(L - x', \kappa)] dN'(x', \kappa) \\
S_1(L, \vec{p}) = \int_0^L \int_0^\infty \int_{-\infty}^{\infty} \exp(i \vec{k} \cdot \vec{p}) \text{Im}[H(L - x', \kappa)] dN'(x', \kappa) \tag{1.80}
\]

Where:
Equations (1.80) and (1.81) give a full spectral representation of the logamplitude and phase fluctuations in relation to the fluctuations of the refractive index.

In the time domain, the autocorrelation functions of the logamplitude and phase fluctuations can be found by:

\[ B^{(L,P)}_{\bar{\rho}}(L, \bar{\rho}_1, \bar{\rho}_2) = \mathbb{E}[\chi(L, \bar{\rho}_1)\chi(L, \bar{\rho}_2)] = \mathbb{E}[\chi(L, \bar{\rho}_1)\chi^*(L, \bar{\rho}_2)] \]  

The evaluation of (1.83) involves the calculation of the spectral 3-D tensor of \( \chi \). This calculation was performed in section 1.2.7 when the structure function of the refractive index was obtained. Therefore, the mathematical steps to obtain the autocorrelation functions of the logamplitude and phase fluctuations will be omitted, and any details can be found in section 1.2.7.

The autocorrelation function for the logamplitude fluctuations is

\[ B^{(L,\rho)}_{\chi}(L,\rho) = (2\pi)^2 \int_0^L d\xi \int_0^\infty dk \kappa J_0(\kappa \rho) \left\{ \text{Re}[H(L - \xi, \kappa)] \right\}^2 \Phi_n(\kappa) \]  

The autocorrelation function of the phase fluctuations is calculated in an identical manner and has the same form as (1.83), only replacing the real part of \( H(L - \xi, \kappa) \) for the imaginary part.
\[ D^\chi(L,\rho) = E\left\{ [\chi(L,\rho_1) - \chi(L,\rho_2)]^2 \right\} \]
\[ D_{S_i}(L,\rho) = E\left\{ [S_i(L,\rho_1) - S_i(L,\rho_2)]^2 \right\} \]  

(1.84)

In essence, the operations used to calculate the structure functions are the same used in the previous section to find the autocorrelations. This is due to the fact that the expansion of the argument of the expected values will lead to the expression for the autocorrelation functions. In this way, the structure functions for the logamplitude and phase fluctuations are:

\[ D^\chi(L,\rho) = 8\pi^2 \int_0^L \int_0^\infty d\xi \int_0^\infty d\kappa [1 - J_0(\kappa \rho)] \left\{ \text{Re}[H(L - \xi, \kappa)] \right\}^2 \Phi_n(\kappa) \]
\[ D_{S_i}(L,\rho) = 8\pi^2 \int_0^L \int_0^\infty d\xi \int_0^\infty d\kappa [1 - J_0(\kappa \rho)] \left\{ \text{Im}[H(L - \xi, \kappa)] \right\}^2 \Phi_n(\kappa) \]  

(1.85)

It is evident that the presence of the spectrum of the refractive index occurs in the expressions for the spectra, autocorrelation functions and structure functions. Therefore, substituting the forms for \( \Phi_n \) presented in part 2, it is possible to obtain the final forms of the aforementioned functions of the logamplitude and phase fluctuations.

The variance of the logamplitude and phase fluctuations are obtained by evaluating (1.85) for \( \rho = 0 \). The logamplitude variance is

\[ \sigma^2_{\chi} = 8\pi^2 k_0^2 L \int_0^\infty \left( 1 - \frac{k_0}{\kappa^2 L} \sin \frac{\kappa^2 L}{k_0} \right) \Phi_n(\kappa) \kappa d\kappa \]  

(1.86)

If the von Karman spectrum (1.51) is substituted in (1.86), the variance takes its final form:

\[ \sigma^2_{\chi} = 1.23 \cdot C_n^2 k_0^{7/6} L^{11/6} \quad \text{for} \quad \frac{12}{\lambda} \ll L \]  

(1.87)
1.3.3-TEMPORAL POWER SPECTRUM OF THE LOGAMPLITUDE.

The frequency spectrum of the logamplitudes is caused by changes in the mean quantities and direction of the mean flow. This is equivalent to say (Fante, 1975) that the fluctuating part of the refractive index (or more strictly, its autocorrelation function) is a function of the wind speed only. This assumption is known as Taylor's frozen turbulence theory. For the frozen turbulence model, the frequency spectrum of the logamplitudes can be written as:

\[
W_X(f) = 4 \int_0^\infty \cos(2\pi ft) R_X(v_\tau \tau) d\tau
\]  (1.88)

Using the Tattarski spectrum, the asymptotic forms for the logamplitude spectrum are:

\[
W_X(f) = 1.14 \sigma_X^2 \frac{\Omega^{-8/3}}{f_0} \quad \Omega > 1
\]  (1.89)

\[
W_X(f) = 0.44 \frac{\sigma_X^2}{f_0} \quad \Omega \ll 1
\]

In the above equation, \( \Omega = f/f_0 \), \( f_0 \) being the cut-off frequency defined as:

\[
f_0 = \frac{v_t}{(2\pi\lambda L)^{1/2}}
\]  (1.90)

where \( v_t \) is the transverse wind speed and \( L \) is the path length. The cut-off frequency is the point in the spectral density function where the curve departs from the plane shape typical of the lower frequencies.
1.3.4-TEMPORAL POWER SPECTRUM OF ANGLE OF ARRIVAL.

To calculate the power spectrum of the angle of arrival, it is first necessary to find the spectral density function of the phase difference between two points. Initially, the spectrum of phase fluctuation is found.

1.3.4.1-TEMPORAL POWER SPECTRUM OF PHASE FLUCTUATIONS.

The procedure to calculate the spectrum of the phase fluctuations is similar to that used for the logamplitude fluctuations. The difference is that, because the phase fluctuation is a spatial variable, the spectrum has to be calculated from the structure function. The frequency spectrum is obtained from the structure function of the phase by replacing the spatial variable in the argument by the product of the wind speed and time in the form:

\[ D_{S_1}(v \tau) = 2 \int_0^\infty [1 - \cos(2\pi f\tau)] W_{S_1}(f) df \quad (1.91) \]

Using the form of the structure function presented in (1.91), it can be shown that [Fante, 1975] the asymptotic forms of the phase spectrum are:

\[ W_{S_1}(f) = 3.28 \cdot 10^{-2} C_n^2 k^2 L v_1^{5/3} f^{-8/3} \quad \Omega << 1 \]
\[ W_{S_1}(f) = 1.64 \cdot 10^{-2} C_n^2 k^2 L v_1^{5/3} f^{-8/3} \quad \Omega >> 1 \quad (1.92) \]
1.3.4.2-TEMPORAL POWER SPECTRUM OF THE PHASE DIFFERENCE BETWEEN TWO POINTS.

The next step in finding the spectral density function of angle of arrival is to calculate the spectrum of the phase difference between two points in space. This has to do with the definition of the angle of arrival which is the angle that the wave front makes with the normal to the receiver aperture. The phase difference across an aperture of diameter $b$ is defined in terms of the angle of arrival $\alpha$ as:

$$\Delta S_1 = kb \sin \alpha \approx kb\alpha \quad (1.93)$$

Using (1.93) the spectral characteristics of the angle of arrival can be extracted from the phase difference spectrum. The phase difference is defined as:

$$\Delta S_1(\bar{r}, t) = S_1(\bar{r}, t) - S_1(\bar{r} + \bar{\rho}, t) \quad (1.94)$$

The covariance function of the phase difference can be obtained from the structure function of the phase by:

$$B_{\Delta S_1}(\tau) = \frac{1}{2} \left[ D_{S_1}(\rho - v_t \tau) + D_{S_1}(\rho + v_t \tau) - 2D_{S_1}(v_t \tau) \right] \quad (1.95)$$

If $v_t$ and $\rho$ are perpendicular, then it is possible to write [Lawrence and Strohbehn, 1970]:

$$W_{\Delta S_1}(f) = 4 \sin^2 \left( \frac{\pi \rho f}{v_t} \right) W_{S_1}(f) \quad (1.96)$$

Using the definition (1.93) and making $\rho=b$, the spectrum of angle of arrival can be written as:

$$W_{\alpha}(f) = \frac{1}{k^2} W_{\Delta S_1}(f) \quad (1.97)$$
The spectrum of angle of arrival has the same asymptotic properties of the phase spectrum.

The variance of the angle of arrival can be obtained from (1.97) and is written [Lawrence and Strohbehn, 1970]:

\[ \sigma_{\alpha}^2 = 1.05 \cdot b^{-1/3} \cdot L \cdot C_n^2 \]  (1.98)

where \( b \) is the telescope aperture diameter, \( L \) is the path length and \( C_n^2 \) is the refractive index structure parameter.

1.3.5-STATISTICAL CHARACTERISTICS OF INTENSITY SCINTILLATIONS AND ANGLE OF ARRIVAL.

Remembering the solution for the received field using the Rytov method, it can be written, for the logamplitude and phase fluctuations:

\[ \chi = \chi_0 + \nu \chi_1 + \nu^2 \chi_2 + ... \]
\[ S = S_0 + \nu S_1 + \nu^2 S_2 + ... \]  (1.99)

The logamplitude \( \chi \) is the natural logarithm of the amplitude. Applying the central limit theorem to the summation in equation (1.99), it is possible to affirm that the logamplitude and the phase must follow a Gaussian distribution. If the logamplitude follows a Gaussian law, the amplitude must follow a lognormal distribution.

The solution (1.99) is based in the weak turbulence assumption. In this case, the scattering of the incoming wave is directed forward and is contained in a very small cone. The effects of the medium in this situation are multiplicative, as can be seen from the figure below:
1.3.6-APERTURE AVERAGING.

To explain the concept of aperture averaging, it is first necessary to remember that the solutions for the wave equation found in equation (1.69) assume that the receiver has a point aperture, i.e., the diameter of the aperture is much smaller than the size of the first Fresnel zone. For a point detector, the intensity scintillations are correlated over the aperture. If the aperture size is larger than the first Fresnel zone size, the scintillations will be uncorrelated and therefore, the received intensity scintillations will be averaged over the size of the aperture. Because of this averaging, the variance of the intensity scintillations for a finite aperture is smaller than for a point aperture. The aperture averaging factor is defined as the ratio between the normalized variance of the intensity scintillations from a receiver with a finite diameter $b$ and the normalized variance of the intensity scintillations from a point aperture.

The aperture averaging factor is mathematically defined [Churnside, 1982] as:
Andrews, 1982, calculated the aperture averaging factor for a range of aperture
diameters and inner scale values. In this work, the aperture averaging factors
are restrained to the cases with large values of the ratio between the aperture
diameter and the inner scale size. For plane and spherical waves, the aperture
averaging factor is given by:

\[ A = \frac{16}{\pi b^2} \int_0^b R_1(\rho) \left[ \cos^{-1} \left( \frac{\rho}{b} \right) - \frac{\rho}{b} \left( 1 - \frac{\rho^2}{b^2} \right)^{1/2} \right] \rho \, d\rho \quad (1.100) \]

Andrews, 1982, calculated the aperture averaging factor for a range of aperture
diameters and inner scale values. In this work, the aperture averaging factors
are restrained to the cases with large values of the ratio between the aperture
diameter and the inner scale size. For plane and spherical waves, the aperture
averaging factor is given by:

\[ A = 0.453 \left( \frac{b}{l_0} \right)^{-7/3} - 0.215 \left( \frac{b}{l_0} \right)^{-3} \quad \text{for plane waves and } \frac{b}{l_0} > 0.5 \]

\[ A = \left[ 1 + 0.387 \left( \frac{b}{l_0} \right) \right]^{-7/3} \quad \text{for spherical waves and } \frac{b}{l_0} > 2 \quad (1.101) \]

1.3.7-SATURATION EFFECTS ON SCINTILLATION.

Saturation of scintillations is a phenomenon intrinsically connected with strong
turbulence. It has been observed by several authors (e.g. Clifford et al., 1974,
Ochs et al., 1976) that, in conditions of very strong turbulence, the linear
relationship between the variance of the received logamplitude and the
refractive index structure parameter \( C_n^2 \) (equation 1.86) breaks down. In other
words, the variance of the scintillations saturates at a given value and no longer
increases with \( C_n^2 \). A rather comprehensive paper on the subject of saturation is
the 1974 paper by Clifford et al.. The aspects of the saturation theory of direct
interest to this work is the most effective eddy size for producing scintillations
in weak turbulence and how saturation alters this most effective size. The
second aspect is the experimentally verified fact that the angle of arrival is not
affected by saturation.

It has been verified experimentally (Clifford et al., 1974, Hill and Clifford,
1981) that the weak turbulence assumption breaks down for values of the
variance of the logamplitude higher than 0.3. Following the method of Clifford et al., 1974, the spatial correlation function was used to indicate the presence of large and small scale structures in the scintillation pattern, hence the residual correlation at larger lags. To better illustrate the action of the eddies to produce scintillations, the following geometry is proposed (Figure 1.3); the source S emits a spherical wave of wavelength $\lambda$ which encounters a turbulent eddy of diameter $2l$ at a distance $z$ from the source. This eddy produces a diffraction pattern, observed at the receiver at $L$, whose size and contrast are determined by $z$, $L$, $\lambda$ and the refractive index fluctuation $n'$ of the eddy.

![Figure 1.3: Geometry of the most effective scintillation-producing eddy model.](image)

At the receiver position, the most severe scintillations will be caused when the direct and diffracted ray combine destructively. This happens when the difference between the length of the path covered by the diffracted and direct rays is $\lambda/2$. This condition leads to a value of the eddy radius of:

$$l \approx \sqrt{\left(\frac{z}{L}\right)^2 + 1 - \left(\frac{z}{L}\right)^2} \cdot \sqrt{\lambda L} \quad (1.102)$$

Equation (1.102) shows that the most effective eddies for producing scintillations are Fresnel-zone size eddies. As turbulence increases, multiple scattering effects cause the light to become spatially incoherent. Fresnel size
eddies are no longer illuminated coherently and therefore they are rendered ineffective in the production of scintillations because their diffraction patterns become increasingly blurred. The most effective eddies are now those with sizes equal or smaller than the spatial coherence length, because they are the ones being illuminated coherently.

As seen before in section 1.3.6, if the receiver aperture is larger than the Fresnel zone size, the scintillations become uncorrelated over the aperture. This causes the onset of averaging process for the scintillations which causes the variance to be reduced. Making use of this phenomenon, Ochs et al., 1976, devised an instrument to measure the path averaged transverse wind speed based on scintillation measurements. To avoid saturation effects, which would impair the instrument performance, the authors point out that, because of aperture averaging, a detector and transmitter with aperture diameters larger than a Fresnel zone size would be blind to eddies smaller than the Fresnel zone size. Since such eddies are the most effective in producing scintillations, this system would be nearly immune to the effects of highly integrated refractive index turbulence. In this work, the apertures used are 138mm in diameter and the Fresnel zone size is 58.3mm. Being larger than the Fresnel zone size, the system used in this work would be saturation resistant.

Effects of saturation are caused by eddies with sizes comparable with the coherence length. The angle of arrival fluctuations are caused by large scale eddy sizes. Therefore, the angle of arrival should not experience saturation, unless in the highly unlikely event of the large scale sizes approaching Fresnel zone sizes (Ochs et al., 1976). The value of the outer scale size in the experimental path used in this work has been measured by others (e.g. Siqueira, 1989). Values found were in the range from 10 to 1750 metres, which are much larger than the aperture diameter used in the measurements in this work. Investigation of the occurrence of saturation on the path and the test for the angle of arrival immunity to saturation will be investigated in chapter 3.
1.4-PART 4- A BRIEF HISTORY OF THE RESEARCH ON LIGHT AND LASER PROPAGATION THROUGH A TURBULENT ATMOSPHERE.

1.4.1-INTRODUCTION.

In this section, a brief account of the history of the research on light propagation through the turbulent atmosphere is presented. Summarised descriptions of the research works and results are separated into three time periods: until 1960, from 1960 to 1975 and from 1975 to 1995. It has been a common factor among almost all of the work produced in this area that the experiments are conducted under conditions of homogeneous turbulence. This is because of the need to produce confirmation of a fundamental theoretical framework.

1.4.2-UNTIL 1960.

Research on light propagation in a random medium was conducted mainly by astronomers. The research interest was focused on the study of the effects of the turbulent atmosphere on the phase fluctuations and amplitude scintillations of starlight. A solution for the Maxwell equations is obtained using the perturbation method called the Born approximation, which uses the assumption of single scattering. This approach is valid only for short paths and weak turbulence and is therefore inappropriate to deal with multiple scattering conditions which are bound to occur in strong turbulence or long paths.

1.4.3-FROM 1960 TO 1975.

Russian researchers, such as Tatarski (1961- and later in a 1971 book) and Chernov (1960) approached the problem of solving the Maxwell equations for an infinite plane wave propagating in a random medium by using a perturbation
technique based on the Rylov method. The Rylov method is supposed to be valid for all turbulence conditions since it is based on a multi-scattering assumption. Spectral analysis of phase and amplitude fluctuations yield asymptotic behaviours which are independent of frequency for the low frequency part of the spectrum and follow a \(-8/3\) power-law for the high frequency part of the spectrum. Application of the central limit theorem predicts a Gaussian distribution for the phase fluctuations and a lognormal distribution for the amplitude fluctuations.

American researchers began applying the Rylov method to extend the results of Tatarski and others to beam waves (e.g. Fried and Seidman, 1967) and spherical waves (e.g. Clifford, 1971). Results for beam waves demonstrate the mathematical complexities of the treatment of beamed laser propagation. Fried and Seidman found a simplification for the treatment of beam waves which consists in expressing the variance of the amplitude scintillations as a combination of the variance of a spherical wave and a factor which is a function of the beam size at the receiver. Results for spherical waves are similar to the results for plane waves, with differences only in the coefficients of the spectra.

Doubts arose about the initial range of validity of the Rylov method since the phenomenon of saturation of scintillations is observed (e.g. Deitz and Wright, 1969). The Rylov method is said to break down for highly turbulent conditions, with the variance of the scintillations no longer being linearly dependent on the refractive index structure parameter but converging to a constant value. The problem of saturation is still not fully understood today. However, Clifford et al. (1974) obtained a theoretical expression for the autocovariance function of the scintillations in saturated conditions. The authors use geometrical considerations to prove that a system that uses aperture sizes larger than the Fresnel zone size would be virtually immune to saturation effects. Fante (1975) showed that the scintillations become a wide sense Markov process in saturated conditions.
conditions. Several works are dedicated to determining the probability distribution of the scintillations in saturated conditions (e.g. Ochs and Lawrence, 1969, deWolf, 1967), but there is no definitive agreement.

During this period several comprehensive review papers were published, such as Strohbehn (1969), Lawrence and Strohbehn (1970) and Fante (1975).

To the author's knowledge, in this period only two papers are concerned with the laser propagation in the urban boundary layer. The first paper is by Bertolotti et al. (1974). The authors study the phase structure function on a 3.5km path over Florence. Their results show that the spectrum of phase fluctuations have a faster decrease with frequency than is expected for homogeneous turbulence. The authors hypothesise that this difference is due to possible irregularities in the shape of the structure function of temperature in the urban environment. The second paper by Chu and Hogg (1968) is concerned with effects of precipitation on laser propagation on a 2.6km path over suburban Vancouver. Their results show the overwhelming influence of mist on the attenuation. Also, the authors point out the practical difficulties of measuring rainfall rate in a city. In addition, the errors that are introduced on the results by the inhomogeneity of rain make it impractical to use extremely complicated drop size distributions in the calculation of the attenuation.

Angle of arrival is only theoretically explored (e.g. Lawrence and Strohbehn, 1970) as a complement to the study of phase fluctuations. Askenov (as quoted by Churnside and Lataitis, 1987) demonstrates that the angle of arrival is not affected by saturation effects and that the probability distribution remains Gaussian regardless of the strength of turbulence. The spectral asymptotes follow the same laws as those for amplitude scintillations.
1.4.4-FROM 1975 TO 1995.

The period 1960-1975 saw the basic theoretical framework produced for laser propagation through the turbulent atmosphere. Contemporary works are mainly concerned with the topics that are still in dispute such as the study of saturation and the distribution of scintillations in strong turbulence e.g. Phillips and Andrews (1982), but are still conducted over controlled homogeneous turbulence conditions. Other subjects of theoretical interest such as aperture averaging are fully explored, (e.g. Azar et al, 1979, Churnside, 1991). In these papers, approximate expressions for aperture averaging factor are obtained for a wide range of turbulence conditions.

The level crossing problem for the scintillations is theoretically explored, e.g. Beckman (1967) and Yura and McKinley (1987). All the authors, however, use approximations to calculate the level crossing probability distribution function based on a lognormal distribution for the amplitude scintillations, which may not be valid for large integrated path turbulence, where saturated conditions are bound to occur.

Angle of arrival was more widely explored in this period, but is still mostly theoretically, e.g. Dunphy and Kerr (1977), Ziad et al. (1992) and Yura and Travis, (1985) with the few experimental works still being conducted under controlled homogeneous turbulence conditions (e.g. Churnside and Lataitis, 1987).

As far as precipitation effects are concerned, Wang and Clifford (1975) give a thorough theoretical analysis of rainfall-induced scintillations. The authors predict that the spectrum of amplitude scintillations should have a bandwidth of more than 1000Hz. This bandwidth should decrease with rainfall rate because of the decrease in the number of small drops in heavy rain. Due to the movement of the raindrops in the wind, there should be a movement of the fringes of interference between the incident wave and the spherical waves.
generated by scattering in the drops. This movement should cause bumps in the spectral curve, which should be more pronounced in light rain due to the higher concentration of small drops, which are more efficient in producing scintillations. Ishimaru (1978) also explored this problem and, even though using a more approximate approach, predicted the spread in the spectral bandwidth with rain. An experimental work in millimetre waves by Hill et al. (1989) found evidence of the widening of the scintillation spectrum in rain, but not to the levels predicted theoretically. The authors attributed this to aperture averaging effects. In the optical case, aperture averaging is unavoidable, since to be able to see the scintillations caused by raindrops (some of which being fractions of a millimetre in diameter) the aperture should be smaller than the drops. This is absolutely impractical, since reduction in the size of the aperture severely reduces the amount of received power.

Rainfall attenuation is explored by Mitra and Gibbins (1995) and Gibbins et al. (1989). In both papers, the authors produce a model for the drop size distribution based on measurements of attenuation in a 500m path.

1.5-PART 5: THE TURBULENCE CONDITIONS IN THE URBAN BOUNDARY LAYER.

1.5.1-INTRODUCTION

The framework theory for laser propagation though the atmosphere was derived in parts 2 and 3 of this chapter under the assumptions of local homogeneity and unsaturated turbulence. How the theoretical approach changes for conditions of highly inhomogeneous turbulence and saturated conditions is still the subject of discussion among the atmospheric laser propagation community. It is, however, intuitive to assume that the combined influence of the different structures with different heights and albedo that compose the city environment are going to create propagation conditions that are far from ideal, i.e., isotropic and
homogeneous. The next section will concisely explore, in the light of studies on turbulence within a city environment, if and how the ideal conditions are changed by the influence of the urban environment.

1.5.2-URBAN ATMOSPHERIC SUBLAYERS.

The atmospheric sublayers in a city environment can be divided in two main sections: the urban canopy and the roughness sublayer. The urban canopy is the region close to the ground and the streets (the urban canyon), where the terrain can be considered roughly homogeneous. The roughness sublayer is the layer in which roughness elements such as buildings and trees have a direct effect on the flow. The flow in this layer is complex because it is mechanically and thermally influenced by proximity to canopy elements, Oikawa and Meng (1995).

Jackson (1978) measured the height of the urban roughness sublayer and found that it extends up to three times the average building height. Therefore, it is reasonable to assume that massive turbulence exists over urban areas.

1.5.3-BURST WIND MOVEMENTS IN AN URBAN ENVIRONMENT

Rotach (1993) observed that momentum transport within the two urban sublayers occurred through bursts, i.e. intermittent, wind movements. These burst movements are divided into sweep and ejection movements. Sweep movements are characterised by high speed winds from above moving towards the surface and sweeping cold air from above the street canopy. This movement generates a negative temperature gradient. Ejection movements are characterised by low speed winds moving upwards and ejecting warm air from the canopy causing a positive temperature gradient.
It has been verified, e.g. Lykossov and Wamser (1995) and Oikawa and Meng, 1995 that these movements are periodic and that ejection events happen much more frequently than sweep events. In addition, the burst winds cause the probability distribution of vertical and longitudinal velocities to be skewed [Oikawa and Meng]. The strong and constant turbulent conditions over a city destroy strong temperature gradients. Therefore the urban atmosphere, being well mixed, does not experience a strong diurnal change in stability [Oikawa and Meng].

1.5.4-SPECTRAL CHARACTERISTICS OF THE URBAN ATMOSPHERE.

It has been verified by several authors, e.g. Oke (1987), Roth and Oke (1993) and Oikawa and Meng that the temperature, velocity and energy spectra within a city follow approximately the same behaviour as expected for weak homogeneous turbulence.

Spectral estimates for velocity and temperature fluctuations showed dips and bumps, Roth and Oke and Oikawa and Meng, which are indication of the presence of localised large Reynolds stresses caused by sweep and ejection winds. Results from different locations (Roth and Oke in Vancouver, Canada compared to Oikawa and Meng in Sapporo, Japan) showed dips and bumps at about the same frequency.

Oikawa and Meng point out that spectral data obtained in the roughness sublayer are very limited in the literature and reiterate the need for extending the existing data bank.
Chapter 2

Description of the Experimental Equipment
2.1-INTRODUCTION TO THE GENERAL DESCRIPTION OF THE EXPERIMENTAL ARRANGEMENT.

This chapter is dedicated to the description of the propagation path and experimental equipment used in this work. These descriptions are divided in six parts:

1-THE PROPAGATION PATH
2-THE 0.83μm SYSTEM HARDWARE.
3-THE 1.55μm SYSTEM HARDWARE.
4-THE 155Mbits/s DIGITAL SYSTEM.
5-RAINFALL MEASUREMENTS.
6-DATA LOGGING AND STORAGE.
7-INVESTIGATION OF THE EFFECTS OF BUILDING MOVEMENTS ON THE RECEIVED POWER.

The circuitry on both links have been thoroughly tested for linearity of amplifiers, etc..., calibration of instruments and conversion curves, such as the curve which relates the spot position on the photodetector and the value of the angle of arrival. This curve is shown later in this chapter. The complete results from these tests together with detailed circuit diagrams can be found in Luthra, 1995.
2.2-THE PROPAGATION PATH.

The 0.83 µm and the 1.55 µm transmitters are located in the 12th floor plant room at the Department of Electronic and Electrical Engineering, Imperial College (I.C.), South Kensington. The 0.83 µm and the 1.55 µm receivers are located in the 11th floor at the Department of Electronic and Electrical Engineering, U.C.L. The 4km path across central London is orientated 239.1° from the north at an altitude of approximately 50m. The smallest roof clearance has been estimated at 10 m. Approximately one-third of the path is over Hyde park while the rest is over part of London’s west end and other heavily urbanised areas. A map of the path is shown in Figure 2.1. The arrow connects the transmitter and receiver ends of the path:
Figure 2.1: Map of the experimental path.

The UCL Photogrammetry and Surveying Department provided a profile of the propagation path, which can be seen in Figure 2.2:
In Figure 2.2 the x-axis units is percentage of path length. The y-axis unit is related to the scale of the x-axis in the following way: 1cm on the x-axis
corresponds to 40m and one unit on the y-axis corresponds to 10 times the x-axis scale.

A panoramic view of the propagation path as seen from the transmitter site at Imperial College Engineering Building is shown in Figure 2.3. In the foreground left hand side is the 1.55μm transmitter. On the right hand side is the 0.83μm transmitter. In the middle, but closer to Imperial College, the trees in Hyde park can be seen. BT tower is in the background and the UCL Engineering Building is to the right of the tower.
Figure 2.3: Panoramic view of the propagation path from the site of the transmitters
2.3 - THE 0.83 \( \mu \text{m} \) SYSTEM.

2.3.1 - ELECTRONIC AND MECHANICAL DESIGN.

The transmitter terminal at Imperial College has been designed for automatically switching itself on and off. This is achieved through means of a programmable time switch. The transmitter is operational between the hours of 0900 and 1830. The transmitter uses a Spectral Diode Labs GaAlAs semiconductor (s/c) pig-tailed laser; SLD-2100-E2 and emits continuous i-r pulses of duration 300 ns at a pulse repetition frequency of 2 kHz. This corresponds to a duty factor of \( 6 \times 10^{-4} \), i.e. 0.06\%, ensuring long laser life time. It has a mean transmit power of 0.3 mW and a peak power of 500 mW. The wavelength of the laser is 830\( \pm 10 \) nm, which is within the second line of the first atmospheric window.

2.3.2 - THE 0.83\( \mu \text{m} \) TRANSMITTER.

The transmitter is located on the moving head of a high quality, heavy duty photographic tripod or theodolite stand. This is shown in Figure 2.4. Coarse movements of the transmitter direction are obtained by means of a pan-and-tilt mechanism of the tripod/theodolite stand. A spring-loaded micrometer allows finer adjustments to be made in the azimuth plane. Vertical movements are made via a gravity assisted "Screwjack. The fibre tip of the laser output fibre is located in the focal plane of the transmitter lens. The position of the fibre tip can be adjusted by a micrometer \( x, y, z \) “Matlock” translation stage. This allows movements in the \( x, y, z \) directions of up to 1.1mm. The fibre is clamped in a brass ferrule which is located in the transition stage.
Figure 2.4: Schematics of the 0.83μm system telescope transmitter.

The block diagram of the transmitter is shown in Figure 2.5. This consists of the telescope, the fibre cable, the electronics and a power meter ("sneaker") which is used to monitor the output power immediate in front of the telescope aperture.

Figure 2.5: Block diagram of the 0.83μm system transmitter.
2.3.3-OPTICAL DESIGN OF THE 0.83μm TRANSMITTER

The function of the transmitter antenna optics is to collect the largest possible proportion of the available source power and to collimate it into a beam with the highest possible degree of parallelism. A high radiation gathering capability dictates the use of a lens with a large numerical aperture, whilst good collimation demands optics with a small numerical aperture (NA). With the fibre optic feeders used in this project, however, the situation is different: there is no virtue in increasing the numerical aperture of the optics \((\text{NA})_o\) beyond that of the fibre \((\text{NA})_f\), which thus becomes the natural limit for \((\text{NA})_o\). In fact, it is advisable to make \((\text{NA})_o\) some 10% to 20% lower than \((\text{NA})_f\) to allow for contingencies such as imperfect feeder, imperfect perpendicularity of fibre tip endings to their axes and manufacturing spreads of \((\text{NA})_f\). In the present design: \((\text{NA})_o = 0.68 (\text{NA})_f\). A plane-convex singlet lens 138 mm in diameter and focal length of 250 mm has been chosen for economic reasons. This choice had the unfortunate consequence of introducing an appreciable amount of spherical aberration. All the lenses in the system have multi-layer anti-reflection coating on both faces. This measure reduces the optical losses of each beam by 9%.

The fibre termination of a laser ("pigtail") is responsible for the guided propagation of the laser radiation and is therefore a constituent part of the transmitter optics. The through losses of the present day splices lie in the range 0.5 dB – 1.5 dB. A removable sighting telescope used for link alignment rests on two saddles, one horizontally the other vertically adjustable, for making its’ axis parallel to that of the optogun.

The 0.83μm radiation is guided through a step index (0.3 N.A.) multimode optical fibre which forms a fibre pigtail. The fibre tip is placed in the focal plane of the transmitter optics. The fibre is made of fused silica. The core is 100μm in diameter, and the fibre is 140μm in diameter including cladding. The fibre is protected by a plastic jacket giving an overall diameter of 0.5 mm. The normal pigtail length is 1 m. Thus the output from the fibre is a spatially
incoherent symmetrical beam. Optically, this must be treated as a spatially incoherent, but a temporally coherent source with an aperture equal to the fibre core diameter (Spectral Diode Labs 1988). A typical beam profile is shown in Figure 2.6.

Figure 2.6: Beam profile of the 0.83µm laser.

The choice of lasers with a high peak power capability has been dictated by the desire of achieving a 4km range with good radiation penetration in adverse atmospheric conditions. Consequently, a laser drive giving currents of the order of an ampere was necessary. A dedicated pulsed current source capable of continuous adjustment 0-1.6A output was available. The nominal drive current used was 1.5A. The transmitter driver circuit delivers a clean, flat topped negative pulse of length 100ns to 500ns when driven by a pulse generator. In practice, a pulse of 300ns was used.
2.3.4-RADIATION PATTERN OF THE 0.83µm TRANSMITTER

In order to obtain the radiation pattern of the 0.83µm transmitter, the following experiment was carried out. The system was aligned for maximum power and the 0.83 and 1.55µm transmitters were tilted sideways and up and down. Since both lasers are fed to circular fibres, the beams are considered to be symmetrical and only the horizontal profile is shown here. One additional reason for the choice of the horizontal plane is the fact that the 0.83µm system has a micrometer to adjust the horizontal tilt but a graded screw, which is much less accurate, to adjust the vertical tilt. The obtained radiation profile is presented in Figure 2.7:

![Radiation Pattern Graph]

Figure 2.7: Radiation pattern of the 0.83 µm system transmitter.

2.3.5-EYE SAFETY CONSIDERATIONS.

In the interest of range, high radiance sources had to be chosen for the transmitter antenna. Care had to be taken, however, to ensure the eye safety of
both the informed equipment operator(s), and casual passers-by. The solution adopted consists of using high peak/low average power lasers. The semiconductor laser type SLD-2100-E2, satisfied the conditions for Class 3A operation when the peak power was reduced to 300 mW. Safety factors were calculated from the viewpoint of both the energy contents of a single pulse and the average spatial power density (irradiance) of the radiation.

2.3.6-THE 0.83\textmu m RECEIVER.

The opto-mechanical design of the receiver antenna is similar to that of the transmitter. A commercial continuous centroid spot position detector (United Detector Technology PIN-SC4D) located in the focal plane of the receiving optics produces four outputs: two azimuth and two vertical currents. Only two of these outputs, the vertical ones, were usually used. A mechanical 90 degree rotation permits a quick change from azimuth to vertical measurements. Two transimpedance amplifiers convert the two outputs into two corresponding voltages for processing. These voltages are processed to produce the sum output which corresponds to the received signal strength and the difference output which gives an indication of the angle of arrival of the incoming beam. The exact value of the angle of arrival is obtained by taking the quotient of the difference and the sum signals. Figure 2.8 shows a block diagram of the receiver.
The description of the components shown in Figure 2.8 will begin with the scalers. The scalers amplify the received signal prior to further processing, keeping the signal within the linear range of the stretchers. One output from the scalers goes to a fast sum box, which combines the two vertical outputs from the PIN diode quadrant detector and allows the received pulses to be observed on an oscilloscope. The other output from the scalers goes to the pulse stretchers. The need for the use of a device, which stretches the received pulses in time by 5000 times without change in amplitude, is due to the fact that the 300ns pulses are too fast for recording in magnetic media. The pulses are also too short for reasonably affordable A/D converters. By using the stretchers, the pulses become almost continuous, with each stretched pulse lasting until the beginning of the next one, when the amplitude is updated. Because of the low bandwidth of the signal fluctuations, the Nyquist sampling condition is amply satisfied at all times, making the created waveform replicate faithfully the envelope of the received pulse train. The two outputs from the stretchers are input to the slow sum/difference box, which performs the sum and difference of
the two stretched pulses. The sum and difference outputs from the slow sum/difference box go to the quotient box, which gives an indication of the angle of arrival. This indication is represented by a parameter denominated position. This parameter had to be related to the actual value of the angle of arrival in radians and therefore a calibration curve was obtained. Because of the importance of the angle of arrival in this work, next section is dedicated to describing the calibration procedure.

2.3.7-ANGLE OF ARRIVAL CALIBRATION PROCEDURE.

The spot position detector senses the centroid of a light spot and provides continuous analogue outputs as the spot traverses the active area. Dual axis position sensors provide both x and y axis position information. This is a planar diffused PIN diode with an active area of 5 mm². Typical position linearity over the central 25% and 75% area of the device are 5 % and 20% respectively. The maximum allowable power density is 10 mW/cm².

Only the two vertical outputs of the detector were used in practice, thus providing spot position detection in the vertical plane. Before the detector could be used, a calibration which relates the incident spot position to angle was needed. This is different for each detector purchased. In order to do this, a mock propagation experiment was set up in the laboratory under controlled conditions. An 830nm laser internally pulsed by a 300ns pulse with a p.r.f of 2kHz was fired at the receiver in a collimated beam, the voltage amplitude of the pulse was fixed at 1V. Vertical outputs of the PIN photodiode were then fed into the circuitry which calculates the sum and difference of the two signals. The value of AOA corresponds to the difference divided by the sum, ie. normalised to the signal strength. This is the output voltage known as "Position" and is defined as:
where \( V_{1,2} \) are the input voltages to the Quotient box. The output voltage range is \( \pm 10\) V and the stated rms. noise for 10V full scale is 2mV. The quotient box multiplies by 10 in order to amplify the signal when the difference, \( V_1 - V_2 \), is small.

In order to start the calibration procedure, the zero position needed to be located. This was achieved by aligning the receiver, via a micrometer, in order to yield a zero quotient value on the laboratory oscilloscope. Once this had been found a careful note was made of the micrometer position which could then be moved to rotate the receiver in known amounts to yield both negative and positive quotient values. From simple geometry, a knowledge of the focal length of the receiver lens and the graduated micrometer movements, the corresponding angles were calculated. The micrometer was scanned over 5 mm and the corresponding values of quotient, which varied from +4.75V to -5.75V, noted. The smallest micrometer division was 0.01mm. The graph of "Position" against micrometer movement is shown in Figure 2.9:
The micrometer reading from 3.0 to 4.0 represents 0.5 cm. The TX was fixed and the RX was moved over 5.75 mm and the position o/p was linear. Outside of these regions non-linearities take over.

\[ y = -37.273 + 10.501x \]
\[ R^2 = 0.985 \]

Figure 2.9: Angle of arrival calibration curve.

The graph is fairly linear over the entire region scanned and the gradient interpolated to be 10.501 V/micrometer unit. With the micrometer used, each whole division corresponds to exactly 5 mm, thus the gradient of the graph becomes 2.1002 V mm\(^{-1}\). The focal length of the receiver lens is 330 mm. The corresponding angle is given by the expression: \( \theta = \tan^{-1}(d/f_{RX}) \approx (d/f_{RX}) \) for \( d \ll f_{RX} \), where \( d \) is the micrometer displacement and \( f_{RX} \) the receiver focal length. Thus with \( d = 5.75 \) mm and \( f_{RX} = 330 \) mm, this gives \( \theta = 1.742 \times 10^{-2} \) rads. A total detector movement of 5.75 mm results in an angle of 1.742 \times 10^{-2} rads, this corresponds to 3.03 \times 10^{-3} rads/mm. Therefore, the Gradient [radsV\(^{-1}\)] = 3.03 \times 10^{-3} \text{ rads mm}^{-1} / 2.1002 \text{Vmm}^{-1}, which gives 1.443 \times 10^{-3} \text{ radsV}^{-1}. Therefore, to convert a value of "Position" in volts to a corresponding angle in microradians, the following expression can be used:

\[
\text{AOA(\mu radians)} = 144.3 \times \text{position(V)} \quad (2.2)
\]

The division by 10 is necessary due to the Quotient electronics as can be seen by equation (2.1).
The difference output from the slow sum/difference box also feeds into a
difference meter and is also displayed on a small viewer screen. The sum
output is fed into a sum meter and also to an extra slow filter, which is a low
pass filter with cut-off at 0.85Hz. The output from the extra slow filter is a
measure of the time averaged received power. This extra slow filter allows the
recording of the received power on magnetic media.

2.3.8-THE 0.83 μm RECEIVER OPTICAL DESIGN AND CIRCUITRY.

Figure 2.10 shows a picture of the 0.83μm receiver:

The function of the receiver antenna is to collect as large a portion of the
transmitters' radiation as possible, irrespective of its continually fluctuating
angle of arrival, and to project it onto as small an area as possible in the focal
plane of the photoreceiver. The choice of the antenna aperture is subject to
constructional and economical constraints. Making it equal to that of the
transmitter antenna seemed an all-round sound engineering solution. It remained then to choose the antenna angle of acceptance (or field of view FOV) and the size of the photoreceiver. However, a large angle of acceptance of an antenna may have lead to an increased susceptibility to collecting spurious back ground radiation. Also, the surface area of the photoreceiver should be kept reasonably small, in order to restrict the value of its junction capacitance and thereby to ensure a sufficiently broad band-width of the link to receive the short pulses.

The optics consisted of plane-convex singlet lens of BK-7 glass, type 01-LPX-289 having a diameter of 140 mm (clear aperture CL = 138mm) and a focal length of 330mm. All lenses of the system had a multilayer anti-reflection coating on both faces giving a 9% saving in the power budget.

The photodetector is a 5-terminal silicon PIN photodiode type SC4-D (United Detector Technology, USA) with a rise time of 70 nsec, capable of centroid spot detection in two orthogonal directions.

Ingress of unwanted background radiation (diffuse sky irradiance, direct sunshine etc), which would otherwise be severe for this antenna owing to its large angle of acceptance, is largely eliminated through the combined use of spatial and chromatic filtering. For the former, a set of absorbent baffles is mounted inside the optogun in order to eliminate side lobes of the antenna. For the latter, an interference filter λ=830nm±10nm mounted in front of the photoreceiver reduces very effectively the ingress of radiation from broadband sources. The use of spatial-chromatic filtering enables link operation despite quasi-horizontal (at sunset) sun rays entering the antenna aperture at zero-angle incidence.
2.3.9-RADIATION PATTERN OF THE 0.83\,\mu m RECEIVER.

The procedure to obtain the radiation pattern for the 0.83\,\mu m receiver is analogous to the one used to obtain the radiation pattern for the 0.83\,\mu m transmitter. The link was aligned for maximum received power and the receiver was rotated in the azimuthal plane. The radiation pattern obtained is shown in Figure 2.11:

![Radiation pattern of the 0.83\,\mu m receiver.](image)

Figure 2.11: Radiation pattern of the 0.83\,\mu m receiver.

2.4-THE 1.55\,\mu m SYSTEM.

The 1.55\,\mu m system electronics and antennas were provided by researchers of British Telecom Research Laboratories. The transmitter and receiver antennas are modified astronomical telescopes.
2.4.1-THE 1.55\,\mu m TRANSMITTER.

Figure 2.12 shows a schematic diagram of the 1.55\,\mu m link. The transmitter consists of a Fabry-Perot 1535nm pigtailed semiconductor laser, an erbium fibre amplifier, an isolator and a transmit telescope. The Fabry-Perot laser has a mean operating wavelength of 1535nm and can be modulated up to 1Gbit/s directly from a digital modulation unit. The output of the laser, typically 0dBm mean, is fed into an erbium power amplifier which is co- and contra- pumped at 1480nm by high power semiconductor lasers. The amplifier has a maximum mean output power of +14dBm into a standard single mode fibre and is followed by an isolator which is required to prevent self-lasing but has a loss of 1dB. The transmitter telescope is of Schmidt-Cassegrain design with a 2m focal length, 20cm aperture and the cleaved fibre end is located in its focal plane. Despite having gold coated mirrors, the telescope transmitted power is 3dB below that in the fibre due to the mismatch between the telescope numerical aperture, 0.05, and that of the fibre, 0.1, and the obstruction caused by the secondary mirror.
Alignment of the telescope is achieved by means of angle adjusting micrometers with a resolution of better than 50μrad, or 20cm at 4km. A boresighted sightscope, with a 900mm focal length, is attached to the transmitter telescope as an aid to alignment. The beam produced by the telescope is diffraction limited. Beamwidth measurements show the radiation is broadened by atmospheric turbulence to a diameter of between 0.9 and 2m at 4km. Figure 2.13 is a photograph of the transmitter unit. The transmitter is located alongside the 0.83μm system on the 12th floor (50m high) in the plant room of the Engineering Department of Imperial College and the receiver is in an 11th floor laboratory of the Department of Electrical and Electronic Engineering, University College London.
2.4.2-RADIATION PATTERN OF THE 1.55\textmu m TRANSMITTER.

The radiation pattern of the 1.55\textmu m transmitter was obtained in the same way as the radiation pattern of the 0.83\textmu m transmitter. The radiation diagram is presented in Figure 2.14:
2.4.3-THE 1.55μm RECEIVER.

The receiver, shown in Figure 2.15, consists of an identical telescope to that of the transmitter with a 3dB optical beam splitter located 10cm before the focal plane, providing two ports for experimentation. At the first port an optical power meter is located which either logs atmospheric attenuation, via a computer system, or is used to measure the average power during high-speed data transmission experiments. Port number two contains either the high speed data receiver or a Ge quadrant photodiode which can be used to measure intensity and angle of arrival fluctuations. The optical power meter uses a 2mm diameter diode and a sensitivity of -81dBm when the source is chopped at 270 Hz, or -61dBm for CW signal. The Ge detector has a 5mm$^2$ total area, a sensitivity of better than -60dBm and a bandwidth of 1kHz. An optical absorption long-pass filter with a 1μm cut-off is used to reduce the ambient background light collected by the telescope. The optical loss of the telescope beam splitter and filter is 5dB. As an additional aid to alignment, an eye piece
may slotted into the receiver telescope to view a light source beacon from the transmitter room.

**Figure 2.15:** The 1.55\(\mu\)m receiver.

Figure 2.16 shows the receiver optics for the 1.55 \(\mu\)m link.

**Figure 2.16:** The 1.55\(\mu\)m receiver optics.
2.4.4-RADIATION PATTERN OF THE 1.55μm RECEIVER

Figure 2.17 presents the measured radiation pattern of the 1.55μm receiver system:

![Radiation pattern](image.png)

Figure 2.17: Radiation pattern of the 1.55μm system receiver.

2.4.5-EYE SAFETY CONSIDERATIONS.

The system has the following characteristics: wavelength 1535nm, peak power 50mW, average power 25mW, pulse repetition frequency 270Hz and a 314cm² beam area emitted from telescope. The 1.55μm link is normally operated at 2mW peak and 270Hz for scintillation experiments and the higher power level is used for high-speed transmission.

If only the hazard outside the transmitter room is considered then the worst case scenario would be someone close to the window looking in with binoculars. The standard states that such instruments may be assumed to have...
an aperture of up to 80mm and concentrate all the light falling onto them into a 1mm² area. If the power is spread uniformly over the telescope aperture of 200mm then the power entering the binoculars is: \( 25\text{mW} \times \frac{80}{200} = 4\text{mW} \).

The only potential hazard in the transmitter room is if the fibre from the amplifier to the telescope is broken. Unless an optical instrument was used to view the fibre tip, which is a highly unlikely hypothesis, then at the near point (100mm) the fibre would only produce about 0.2mW mm⁻² and would be Class 1 as far as eye safety is concerned. The fibre output would be a skin hazard if placed in direct contact with the skin since 25mW would be delivered to an area of very much less than 1mm². This would exceed the skin damage limits by about 25 times. In practice, however, the only effect, if any, of such an exposure would be a very mild burn.

Changes to BS 9172 have been implemented which will increase the accessible exposure limit for wavelengths greater than 1400nm to 10mW for expose times between 10s and 30000s. Given the change it is hard to see how a transmitter with 25mW mean power could represent any realistic danger. The only hazard remains a broken fibre in the transmitter room.

2.4.6-DIGITAL LINK SPECIFICATIONS.

A digital communications system operating at 1.55μm at a speed of 155Mbits/s was designed by British Telecom Research and Procurement Division and was tested for performance in this work (chapter 6). The complete, detailed description of the 155.52 Mbits/s system can be found in [McCullagh, 1992]. In this section, a summarised description of the system circuitry and modulation characteristics is presented.

As seen in section 2.4.3, the current generated at the PIN diode by the incoming light at the receiver reaches the preamplifier and is transformed into voltage...
levels. A postamplifier boosts the signal while acting as an automatic gain control device. This minimises the effects of amplitude scintillations caused by the propagating medium and allows the comparator to operate more efficiently. A predetection filter serves to control the jitter and optimise the SNR. This stage is shown in 2.18.

The clock extraction and decision circuitry are shown in Figure 2.19. The clock extraction procedure is based on non-linear processing in which a spectral line is generated at a frequency equal to the bit rate. This frequency is then narrowband filtered and the clock extracted. The clock and the input data are fed into a flip-flop and the final regenerated data is obtained.

![Figure 2.18: Receiver circuitry - from the telescope to the high speed comparator.](image)
The clock is fed into a Schottky clamping circuit. The clamp switches between one of two distinct values representing the presence or absence of the clock signal. As will be seen in chapter 6, the knowledge of the statistics of the periods of time that the clock was present or absent can provide information about the availability of the digital system operating at optical frequencies through the atmosphere.

2.4.6.1-MODULATION SCHEME: ON-OFF KEYING NONRETURN TO ZERO PULSE CODE MODULATION WITH INTENSITY MODULATION AND DIRECT DETECTION - PCM-IM/DD (OOK-NRZ).

Pulse modulation is the modulation scheme in which some parameter of a pulse train is varied in accordance with the message signal. In digital pulse modulation, the signal is represented in a form that is discrete both in time and amplitude, permitting the transmission in a digital form as a sequence of coded
pulses. The digital pulse modulation scheme used in the system is the pulse-code modulation (PCM). In PCM a message signal is represented as a sequence of coded pulses, which is accomplished by representing the signal in a form which is discrete both in time and amplitude (or intensity). To represent binary digits electrically, the transmitter uses a on-off nonreturn to zero (On-Off NRZ) line code. NRZ is the most common mode of binary transmission, where the 1 is represented by a positive pulse and the 0 is represented by the absence of the pulse. The scheme is called non return to zero because the signal does not return to zero between subsequent pulses, as shown in Figure 2.20:

![Diagram of the On-Off keying nonreturn to zero line code scheme.](image)

2.4.6.2-NOISE CONSIDERATIONS AND BIT ERROR RATE.

The regenerated PCM wave should ideally be a clear sequence of pulses 1 and 0, as seen in Figure 2.20. However, what happens in practice is that noise is introduced into this waveform. This is a result of the presence of channel noise and interference, which cause the decision mechanism to make errors. The rate at which error bits occur in the output of the decision mechanism is called the bit error rate (BER).
2.5-RAINFALL MEASUREMENTS.

A tipping bucket rain gauge with an integral logger Casella London Ltd, provides measurements of rainfall. This is secured to the roof of 66–72 Gower St., which is a building located about 100 metres from the receivers site. The raingauge is mounted on a heavy stand in order to keep it steady in strong wind conditions. The tipping bucket can be seen in its position on the roof of 66-72 Gower Street at the slightly to the left centre of Figure 2.21:

![Raingauge](image)

Figure 2.21: Tipping bucket location on top of 66-72 Gower Street.

The tipping bucket rain gauge is based on the divided bucket mechanism in which a pre-determined amount of water is collected in the bucket which, when full, tips over. The tip is logged in real time on a cassette which has a memory capacity of 32 K. The logger cassette can contain about 6000 minutes of rainfall data. Data is downloaded from the cassette via a portable computer and is then transferred to a larger unit, where the data is stored and some of the statistical analysis is carried out. The interior of the raingauge is shown in Figure 2.22:
The bucket size is 0.1mm and has a reported accuracy of 1%. The real time clock accuracy is stated at ± 1 minute per month. The finest time resolution of the tipping bucket is 1 minute so, for example, if 5 ticks are reported between 13:45 and 13:46, then the rain-rate would be 0.5 mm/min.

2.6-DATA LOGGING DEVICES AND PROCEDURES.

As mentioned earlier, data obtained from the receivers circuitry can be stored on magnetic tape. The tape recorder used is a 7 speed RACAL STORE 4FM. Its signal bandwidth extends from DC to 20kHz according to the recording
speed used. The speed mostly used in the current experiments was 7.5in/s, which corresponds to a bandwidth of 313Hz. This value is sufficiently large for monitoring both amplitude scintillations and angle of arrival as well as long time power measurements.

Propagation data could also be read directly into a computer by the means of virtual instrumentation provided by the software package LABVIEW. The interface between the computer and the receivers circuitry or the tape recorder is provided by a GPIB A/D converter board. Due to hardware memory constraints on the recipient computer only a total maximum of 64000 data points could be stored in the machine buffers at each run, which in the case of a 4 channel recording and a factor of 3 on the buffer size, would mean that only a maximum of 5000 points could be stored for each channel.

The optical power meter attached to the output of the receiver telescope on the 1.55μm link is logged by a HP9125 series computer which provides chart printouts and screen monitoring.

2.7-ANALYSIS OF BUILDING STRUCTURAL MOVEMENTS AT THE TRANSMITTER AND RECEIVER SITES AND THEIR EFFECTS ON THE RECEIVED POWER LEVELS.

An operational problem which occurred during the operation of the two laser systems is the effect of a variation in received power identified as building movement at the transmitter site. Methods have been devised to counter this effect. The chart records of the received signal at 1.55μm showed large cyclical variations, mainly during clear sunny days. The received power level suffered a gradual attenuation, usually starting about sunrise. The attenuation reached a maximum and then returned gradually to a level different to the initial level. This could be higher or lower. Usually the link required realignment after this
phenomenon. An example of the received power record during a typical event is shown in Figure 2.23:

![Figure 2.23: Received signal on the 1.55μm showing signs of periodic behaviour-29 May. 94.](image)

Analysis of the records showed an approximate 4 hours cycle which needed to be identified if long term measurements were to be made.

A process which can be separated into a short-term random part and a long-term deterministic trend is called an error model. It is clear from Figure 2.23 that the received power curve is a superposition of a long term trend and a faster ripple. In general, trends can be considered harmonic or polynomial according to their behaviour with time. The trend shown in Figure 2.23 is a typical representative of the building movement power variations and is considered to be a polynomial. In the case of such a trend being identified in the received power record, the procedure to remove it is to fit a polynomial to the received power curve and subtract it from the original data. This procedure, although causing some degree of smoothing on the fast random fluctuations, is
chosen as the best compromise between simplicity and precision. Fortunately, such building movements did not usually occur during rainy days, therefore attenuation events were rarely affected by the effect.

To illustrate the procedure of detrending by means of polynomial fitting, a curve was fitted to a record obtained on 26 March 1994, a clear sunny day which became overcast by mid afternoon. Figure 2.24 show the original received power record for the 1.55μm system:

![Figure 2.24: Received power for the 1.55μm system under conditions of building movement induced power variations-26 March 1994.](image)

Investigation of the choice of the polynomial degree showed that in most cases a higher order value than three would not improve the detrending effect. In figure 2.25, the power curve shown in Figure 2.24 is accompanied by the third order polynomial:
Figure 2.25: Received power and fitted third degree polynomial.

The effect of subtracting the effect of the trend is shown in Figures 2.26 and 2.27. The former shows the difference between the original received signal and the fitted polynomial:
26 March 1994

Figure 2.26: Detrended received power using the third degree polynomial shown in Figure 5.3.

The effect of the detrending on the autocorrelation function is shown in Figure 2.27:

Figure 2.27: Autocorrelation function of received power on 26 Mar. 94 showing original and detrended versions.
The effect of the trend removal is shown in the smoothing of the oscillations on the autocorrelation function of the original received power curve, indicating that the oscillatory trend has been attenuated. It can also be seen from Figure 2.27 that the differences between the effects of using a third or fourth degree polynomial is negligible. Although this characteristic was generalised, the actual shapes of the polynomials, i.e., the values of the coefficients, varied considerably from one day to another, which made it impossible to obtain a generalised model. Therefore, the polynomials would have to be evaluated for each occasion the trend was observed. In any case, the residual oscillation would still produce a relatively large error in the estimation of loss.

Simultaneous measurements of the received power of the 0.83\(\mu\)m and the 1.55\(\mu\)m systems revealed that this phenomenon affected the links differently. Figure 2.28 presents an event for the 1.55\(\mu\)m signal together with the simultaneous power level of the 0.83\(\mu\)m signal. Theoretically [Ishimaru, 1981] the effects of the random variations of the index of refraction of the atmosphere are not frequency selective, i.e., degradations in signal level at these two wavelengths are supposed to be correlated, if only atmospheric effects are considered. That is not what is being seen in Figure 2.28. Oscillations in power superimposed on a slope for the 1.55\(\mu\)m in contrast to a rather flat received signal for the 0.83\(\mu\)m having a much smaller slope. It is not clear, however, what causes the more rapid fluctuations on the 1.55\(\mu\)m received signal.
To estimate the cause of this effect the radiation patterns of both the transmitter and receiver of both for the 0.83µm and 1.55µm systems were measured. This was done by lining up the transmitters and receivers for maximum received power. To measure the radiation pattern of a transmitter, it was rotated vertically and horizontally and the angle plotted as a function of received power. This was repeated for the receiver to obtain the receiver polar diagram. In practice, the 0.83µm system only had a precision micrometer in the horizontal plane. In the case of the 1.55µm system, the vertical adjustment micrometer suffered from an “hysteresis” effect and the horizontal adjustment was also used in practice. This was acceptable because the beams are symmetrical.

The 1.55 µm receiver polar diagram is much narrower than that of the 0.83µm and therefore slight movements of the building would affect both systems differently. A comparison of the patterns for both links is shown in Figure 2.29:
Figure 2.29: Radiation patterns of both receivers.

In figure 2.29 it can be seen that the 0.83μm radiation pattern is much broader than that of the 1.55μm. Movement of the receiver is therefore more likely to affect the 1.55μm system and any effect will be much larger than for the 0.83μm system. The effect of building movement is illustrated in Figure 2.30:
Trigonometry gives the amount of building displacement (negative values represent contraction of the front wall and positive values expansion) given the radiation pattern. Figure 2.31 shows the radiation patterns of the receivers in terms of necessary building movement to produce the reduction in the received signal:

Figure 2.31: Amount of building displacement per received signal level. Movements at RX.
Inspection of the figure 2.31 shows that, for a reduction of about 25dB in the received signal for the 1.55μm system, the building has to be displaced by about 200 millimetres, while, as far as the 0.83 system is concerned, the building has to move by twice this much. The flat top portion of the 0.83 micrometres diagram guarantees a much larger stable range. A drop of 5 dB would require a displacement 6 times larger for the 0.83μm system than for the 1.55μm. Movements of the UCL building, therefore, cause insignificant variation in the 0.83μm received signal and is likely to be very small even for a 10mm movement at 1.55μm.

Movements at the transmitter end, however, proved to be more critical. The radiation patterns, which were presented earlier in this chapter are reproduced in Figures 2.32 and 2.33:
Figure 2.33: Radiation pattern of the 1.55 μm system transmitter.

The amount of building displacement at Imperial College for both links is shown in figures 2.34 and 2.35:

Figure 2.34: Amount of building displacement per signal level. Movements at 0.83μm TX.
For the 0.83\(\mu\)m transmitter, a reduction of 10dB in the received signal requires a movement of about 8 to 10 millimetres of the building structure. A drop of 3dB needs a 5 to 6 millimetres displacement. The diagram relative to the 1.55\(\mu\)m is presented in figure 2.35:

![Diagram showing the relationship between building displacement and received signal level for the 1.55\(\mu\)m TX.](image)

Figure 2.35: Amount of building displacement per signal level. Movements at 1.55\(\mu\)m TX.

For this system, an 8 to 10 millimetres movement at Imperial College would cause a decrease in signal level of about 30dB (3 times higher than the 0.83\(\mu\)m); 5mm, about 15dB (5 times higher than the 0.83\(\mu\)m).

This difference in the amount of signal loss caused by the same amount of building movement on both systems is responsible for the uncorrelated behaviours. The quantification of this effect would require a structural analysis on both buildings which is clearly far beyond the scope of this work.
It is interesting to note that the loss of signal attributed to building movement could never be restored by realignment of the transmitter in just a single plane, i.e., elevation or azimuth. It was always necessary to adjust in both planes which infers a twisting movement of the building. It was also observed that the effect was negligible in wet weather and when there was cloud cover. The effect was maximum in clear sunny weather. It can be speculated that the heating of the front of the building by the sun in the morning was responsible for the effects which were observed.

Other possibilities to try to account for this behaviour were investigated, e.g., movement of the bench on which the transmitters were mounted due to changes in temperature, movements of the mounts of the transmitter telescopes. The temperature of these structures were monitored over a period of time to try to detect any cyclical change in temperature, but no evidence was found for any movement.
Chapter 3

3.1-INTRODUCTION

This chapter is dedicated to the statistical and spectral analysis of scintillations and angle-of-arrival. Initially, a theoretical background is presented, followed by the presentation of the results. The results presented in this chapter are for the 0.83μm system only. The number of data files gathered during the measurement phase of this work makes it impossible to present them all here. Instead, typical examples are taken from chosen sets. The samples are coded and their description is shown in Table 3.1:

<table>
<thead>
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<th>DATE</th>
<th>SAMPLE</th>
<th>( C_n^2 ) ( \times 10^{-14} \text{m}^{-2/3} )</th>
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<th>TEMP. (C)</th>
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</tr>
<tr>
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Table 3.1: Description of the samples used in the examples presented in this chapter.

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</tr>
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</tr>
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</tr>
<tr>
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<td>14:15</td>
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</tr>
</tbody>
</table>

3.2-PRELIMINARY THEORETICAL CONSIDERATIONS

Although a large number of scientific studies throughout the years have been devoted to the subject of the identification of the statistical characteristics of the received signal amplitude fluctuations, there is no agreement as to which is the best probability distribution fit. To quote from the conclusions of Strohbehn et al., (1975), "In summary, it is clear that as yet there are no well-accepted theoretical predictions for the probability distributions of an optical signal in the saturation region." By saturation region, the authors mean the turbulent conditions for which multiple scattering effects become important. This is particularly true for very long
paths. The path in the present work is not only long but extends through a very complex terrain, where turbulence is bound to be inhomogeneous.

Thinking along these lines, the best model for a probability distribution for the scintillations should take into consideration all effects concerned with a generalised long path subject to all kinds of turbulent conditions. A model like this is discussed at length by Phillips and Andrews (1982). They model the atmosphere as a series of thick slabs, much larger than the outer scale of turbulence, as shown in Figure 3.1, below:

![Generalised multi-scattering propagation model](image)

Figure 3.1: Generalised multi-scattering propagation model.

The specular component in the model shown in Figure 3.1 represents the effects of forward scattering caused by outer scale effects. The diffuse component is generated by the beam broadening and subsequent multiple-scattering by off-axis eddies. Applying the central limit theorem to the specular component the resulting
distribution is found to be lognormal, while applying the central limit theorem to
the specular and diffuse component gives a Rayleigh distribution.

Phillips and Andrews (1982) propose a single model for both components. The
authors suggest the use of the Nakagami-m distribution for both specular and
diffuse components. The Nakagami-m distribution can be written:

\[ p(x) = \frac{2m^m x^{2m-1}}{\Gamma(m) \langle x^2 \rangle^m} \exp \left( \frac{-mx^2}{\langle x^2 \rangle} \right) \quad x > 0 \quad (3.1) \]

The Rayleigh distribution is a special case of equation (3.1) for \( m=1 \). Strohbehn et
al. (1975) suggest that the use of the gamma distribution as a very good substitute
for the Nakagami-m distribution.

Although the lognormal distribution is not a special case of (3.1), Phillips and
Andrews (1982) found little difference between numerical values predicted by the
normalised moments of the lognormal and Nakagami-m distributions. The
Nakagami-m distribution has the advantage of being more tractable mathematically
than the lognormal.

The Nakagami-m distribution can therefore be considered as a model for the
statistical characteristics of the received amplitude which is valid for all turbulent
conditions. For weak turbulence the Nakagami-m distribution reduces to the
lognormal, while in strong turbulence, where multiple scattering effects are
dominant, the Nakagami-m tends to the Rayleigh distribution. Using the results of
Strohbehn et al (1975) for hypothesis testing purposes in the intermediate
turbulence range, the gamma distribution will be used.
3.3-ESTIMATION OF THE PROBABILITY DENSITY FUNCTION OF SCINTILLATIONS.

This section presents results of a statistical analysis of scintillations on a complex urbanised terrain, which is a good representation of the real conditions in which a free space optical communications system would operate. The system is located at a height above ground of some 50m. This height should be enough to provide conditions of locally homogeneous turbulence for the periods of time during which data samples are taken. Usually, the results files consist of 5000 or 6000 data points sampled at a frequency of 400 Hz. These values were chosen according to the results of Ben-Yosef and Goldern (1982) to assure independence between data points and temporal stationarity of the sample, which is analogous to spatial homogeneity. Initially it is necessary to determine which method is to be used in determining the distribution of the scintillations. There are two main methods used in the literature for the estimation of the probability density function. First there is the moment method, in which theoretical moments are plotted against the measured moments of the distribution. Secondly, there is the application of goodness-of-fit tests to the maximum likelihood estimation of the density function based on the measured data files.

The moment method has been used by several authors. However, Ben-Yosef and Goldern suggest that, within reasonable measurement times, i.e. guaranteeing independence between points and stationarity, the moment method leads to an almost universal form for the scintillation distribution, which is independent of the strength of turbulence. The authors also point out that the moment method is highly sensitive to extreme values of scintillations which occur for very low percentages of time. These surges may cause the variance of the estimate to become very large.
Goodness-of-fit methods are less sensitive to extreme measurements and are quite reliable. Nowadays, with the development of powerful statistical software packs, the application of these tests has become quite trivial. This is the reason why the direct estimation of the pdf from the data and the application of a goodness-of-fit method to test the obtained distribution is used here. The goodness-of-fit test used is the nonparametric Kolmogorov-Smirnov test. The K-S test is a powerful tool which can be applied to any distribution without any \textit{a priori} specifications. More details about the K-S test can be found in Rice, (1978). It suffices for now to say that, for a very large number of points ($n \gg 100$), the 5\% confidence limit for the null hypothesis to be accepted is given by $d_i=1.36/\sqrt{n}$. For 5000 and 6000 points this limit is respectively 0.0192 and 0.0176.

With this introduction, it is now possible to start presenting the results. To cover a wide range of climatic and turbulent conditions, the results were chosen from measurement sessions realised for different seasons of 1994. Because each experimental run has 5000 or 6000 points sampled at 400Hz, the sampling time intervals are 12.5 and 15 seconds respectively. The measurement sessions varied in duration from half an hour to some 11 hours, which gives an idea of the amount of data files obtained. Certainly, it is not possible to present results from all files in here. Therefore the graphs presented in the following sections are to be understood as illustrative examples of the overall results.

3.4-CHOOSING THE BEST DISTRIBUTION FOR THE SCINTILLATIONS.

According to the work of Phillips and Andrews (1982) and Strohbehn et al. (1975) the measured distributions are tested for lognormal, gamma (for the Nakagami-m) and Rayleigh distributions. For the Rayleigh distribution only, the null hypothesis
is that the distribution of the squared amplitudes (irradiance) follows an exponential law.

The results here indicate that the gamma distribution is the best fit for the scintillations, regardless of the received signal variance (which is an indication of the strength of turbulence). The gamma distribution was accepted for over 80% of the analysed data files. Figure 3.2 shows some examples of the preponderance of the gamma distribution. In this Figure, illustrative examples of the obtained probability density functions of the received signal amplitude fluctuations are taken from samples with corresponding $C_n^2$ values of 3.10, 12.76, 66.20 and 121.37($10^{-14}$ m$^{-2/3}$).

![Figure 3.2: Histograms and estimated pdf's of scintillations for 10 May 94, 14 Jun. 94, 22 Apr. 94 and 01 Jun. 1994. Cn2 values respectively 3.10, 12.76, 66.20 and 121.37 ($10^{-14}$ m$^{-2/3}$). Gamma hypothesis accepted at 5% confidence level.](image-url)
All these examples consist of 5000 points, which yields a limit d statistic of 0.0192. All 4 data files can be considered to be gamma distributed with a 5% level.

On some occasions, it was not possible to distinguish between the gamma and Rayleigh distributions, which is a strong indication of the presence of the Nakagami-m distribution. An example of such an occasion is shown in Figure 3.3:

Figure 3.3: Comparison between gamma and exponential (for Rayleigh) distributions. Example from 14 Jul. 94, Cn² value is 99.77*10¹⁴ m⁻²/³. Both distributions were accepted at 5% confidence level.

The Kolmogorov-Smirnov statistic is an indication of the maximum absolute difference between the measured and estimated densities. In this case, the exponential distribution has a slightly larger d statistic, which indicates a worse fit. The overall behaviour of data sets which present competitive values of the Kolmogorov-Smirnov statistic for the gamma and exponential (for Rayleigh) hypothesis is to have a larger value of the K-S statistic for the exponential hypothesis. This situation is only found for very high values of values of $C_n^2$, say, above $20*10^{-14}$ m⁻²/³.

There are occasions in which the Rayleigh distribution is accepted while the gamma distribution is rejected. This situation only also happens for high values of $C_n^2$, as can be seen in the example presented in Figure 3.4 for a data set from 01 Jun. 94. The value of $C_n^2$ is 104.97*10⁻¹⁴ m⁻²/³.

Chapter 3 .104.
Comparing the values of the Kolmogorov-Smirnov statistics shown in figures 3.2 to 3.4 it is clear that the differences between the gamma and Rayleigh (represented by the exponential distribution for the irradiance) distributions are sometimes so difficult to see and even the goodness-of-fit test is not capable of distinguishing between the two, as can be seen from Figure 3.3. Formally, however, since it appears to be more uniformly applicable, the gamma distribution can be considered to give a better overall description of the distributions regardless of the strength of turbulence for a very complex path such as the used in this work. In conclusion, the experimental results point towards the use of the Nakagami-m distribution as a universal model for the distribution of the scintillations, as theoretically proposed by Phillips and Andrews (1982).

The lognormal distribution was not accepted in any of the analysed samples, which is an indication of multiple scattering. This is expected because of the nature of the path. Figures 3.5 and 3.6 show comparative examples of the differences between the gamma and lognormal distribution:
In these cases, the d statistic for the gamma are respectively 2 and 3 times lower than the lognormal, which is typically the ratio for the rest of the data sets. This result may prove to be an obstacle for the use of the level crossing formulas for normally distributed variables, e.g. the logamplitude. The level crossing problem gives information about the behaviour of a stochastic process in time, such as the frequency of crossings of a determined level and the mean time duration of events of a given level being exceeded. These pieces of information can be used in the calculation of the cumulative distribution or, as will be seen in chapter 6, as measures of the BER in a digital transmission.

Chapter 3.106.
3.5-THE LEVEL CROSSING PROBABILITY FOR SCINTILLATIONS.

The level crossing probability is defined as the probability that a stochastic process will exceed a certain level as it evolves with time. The theory behind the level crossing problem can be found in two very comprehensive works: Papoulis (1978) and Blake and Lindsay (1973).

It is very important to point out from the start that the level crossing problem can be very simple to handle if the process follows a normal probability law, [Papoulis], [Blake and Lindsay]. Otherwise, the computations become rather complex. Most authors use the assumption that the scintillations are lognormal and so the logamplitudes should follow a normal law.

In their work, Phillips and Andrews (1982) affirm that, for weak turbulence, the Nakagami-m distribution can be used instead of the lognormal with negligible discrepancies. In the last section it was shown that the distribution which best fits the data, regardless of the turbulence conditions, is the gamma distribution. Strohbehn et al. (1975) affirm that the gamma distribution is a very good substitute for the Nakagami-m. Therefore, if these assumptions all hold, it should be possible to use the expressions for the logamplitude in the level crossing problem for scintillations with a gamma distribution. To test this possibility, the scintillations are transformed into normalised irradiance, according to the level crossing work of Yura and McKinley (1983). The normalised irradiance is defined as:

$$I_N = 20 \log \left( \frac{A}{A_0} \right)$$

(3.2)

and is measured in dB. The advantage of the use of this notation is that it is now possible to identify fades and surges within the received signal.

Chapter 3 .107.
Before examining the results, it is necessary to explain how the level crossing data is extracted from the received signal amplitude fluctuations. Below is a figure showing the relevant parameters:

![Figure 3.7: The concepts in the level crossing problem.](image)

From Figure 3.7 it can be seen that the stochastic process $x(t)$ crosses the level $L_n$ four times and the process stays above $L_n$ for a total time of $\tau_1 + \tau_2$. The cumulative level crossing probability is then estimated by $(\tau_1 + \tau_2)/T$, the mean time duration is $(\tau_1 + \tau_2)/2$, the number of crossings is four and the density of crossings is $4/T$. All results presented below were calculated using these concepts.

3.5.1-PRESENTATION OF RESULTS.

Figure 3.8 shows the level crossing probability density functions (PDF) measured against the normalised irradiance in dB for measurement sessions realised throughout the year of 1994.
From the overall results, for which the graphs in Figure 3.8 are typical examples, it can be seen that the PDF's are usually a combination of two segments with different slopes corresponding to normalised irradiance values lower and higher than about -10dB. For lower values of the turbulence strength, the deep fade region has a characteristic slope of about 7.5 dB/decade. This slope gradually changes towards a asymptotic value of 5 dB/decade as the value of $C_n^2$ increases. The low fades-surges segment presents a overall 10 dB/decade behaviour regardless of the turbulence conditions.

It was also found that there is a tendency for the fades to be more numerous than the surges. In all the PDF's, the signal, in relation to the mean stays below 0 dB for more than 50% of the time. Values of fades reached 35dB with a minimum recorded of 3dB. The mean value of the maximum fade was about 18 dB. Surges
had a maximum of 13 dB with a mean value for the maximum surge of about 11 dB. Although high values of fades were recorded, it is important to point out that these fades usually last for a very short period of time, hence the rather low percentages of time the signal was below these very low signal levels. This characteristic can be seen from the graphs of the number of times a certain level is crossed and mainly from the mean time duration of an event that a given fading level is exceeded. Examples of such graphs are presented in Figures 3.8 to 3.12.

According to the report 338-5, 1989 from the International Telecommunications Union (ITU), for microwave propagation, the number of times a certain level of amplitude is crossed should follow a lognormal law. In Figure 3.9, examples of the measured number of times a value of the normalised irradiance was crossed are shown:

![Figure 3.9](image)

Figure 3.9: Number of times a level of normalised irradiance was crossed. The data used here is the same as presented in Figure 3.8.
From the overall results, it could be observed that the number of times a level of normalised irradiance is crossed seems to follow a general law. Examples of such distributions can be seen in Figure 3.9. The data sets are the same as used in Figure 3.8. Note that there is a difference in $C_n^2$ of some 34 times between the smallest value presented ($3.10 \times 10^{-14} \text{ m}^{-2/3}$) and the highest ($102.44 \times 10^{-14} \text{ m}^{-2/3}$) without any change in the form of the graph. This bell shaped curve with a long tail is characteristically gaussian, which indicates that the number of times a level of amplitude is crossed is lognormal. This is the same as the ITU prediction for microwave wavelengths. Some typical results for the mean time duration that an event at a certain fading level is exceeded is presented in figures 3.10 to 3.12:

![Graph](image)

Figure 3.10: Mean time duration a fading level is exceeded-10 May 94, sample A. The $C_n^2$ value is $3.10 \times 10^{-14} \text{ m}^{-2/3}$. The correlation coefficient for the fit is 0.98.
In every case, the mean time duration of a fade follows an exponential law of the kind:

$$ \overline{t} = k_1 \exp(k_2 \cdot l_N) $$ (3.3)
The obtained parameters $k_1$ and $k_2$ appear to be insensitive to the turbulence conditions. From the overall results, for which Figures 3.10 to 3.12 are illustrative examples, it was found that, for a change of some 40 times on the value of $C_n^2$, the values of $k_1$ and $k_2$ remained approximately 0.02 and 0.01 respectively. This exponential behaviour and the parameters immunity to propagation conditions are features predicted for line-of-sight microwave propagation by the ITU model presented in Report 338-5, 1989.

3.6-MODELLING THE LEVEL CROSSING RESULTS BASED ON A LOGNORMAL DISTRIBUTION FOR THE SIGNAL AMPLITUDES.

The results of the level crossing studies presented above were obtained without any *a priori* considerations about the statistical distribution of the received amplitudes. As pointed out by Blake and Lindsay (1973) it is only for very special cases that the level crossing problem can have closed expressions for modelling the level crossing probability distribution function, i.e., the number of times that a certain level was crossed and the mean time duration of events. One of these cases is the case in which the amplitudes are lognormally distributed and therefore the logarithm of these amplitudes is normally distributed. Yura and McKinley (1983) propose expressions for the level crossing probability density, in which the contribution of fades and surges are separated. The expressions are presented, first for the fades and then for the surges. Note that $A$ is the amplitude of the received signal and $F_0$ is the fade level in dB and $\chi$ is the logamplitude:

$$P(A \leq F_0) = \left\{ 1 + \text{erf} \left[ \frac{-0.23F_0 + 0.5\sigma^2}{\sqrt{2}\sigma}\right] \right\}$$  (3.4)

If a surge level $S_0$ is concerned, then the expression (3.4) becomes:
The expression for the fades was tested against the results obtained for several sets of measurements which showed gamma distribution characteristics. In all cases, the theoretical Yura and Mc.Kinley model (Y&McK) gave good approximations to fades up to 5 dB and overestimated the measured values for the values below 5 dB. The discrepancies grow larger as the fades increase. This is illustrated in Figure 3.13 and Table 3.1, for fades down to 10 dB, for the 22 Apr. 94, sample E2. In Table 3.1 it can be seen that the discrepancy between the theoretical and measured values increases from about zero for low fades to about 100 times for the higher values. The discrepancy is due to the nature of the scintillation distribution. The hypothesis of a theoretical model based on a lognormal distribution for the amplitudes is therefore rejected. The experimental model found in section 3.5.1 and illustrated in Figure 3.8 appears to be the best choice.

![Figure 3.13: Level crossing probability for fades-22 Apr. 94, sample E2.](image)

\[
P(A \geq S_0) = \left\{ \begin{array}{l l}
1 - \text{erf} \left[ \frac{0.23S_0 + 0.5\sigma_\chi}{\sqrt{2}\sigma_\chi} \right] \\
\end{array} \right.
\] (3.5)
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</tr>
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<tr>
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</tr>
<tr>
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</tr>
<tr>
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<td>0.001086</td>
</tr>
<tr>
<td>-20</td>
<td>0.04</td>
<td>0.000421</td>
</tr>
<tr>
<td>-21</td>
<td>0.02</td>
<td>0.000157</td>
</tr>
</tbody>
</table>

Table 3.2: Percentage of time the amplitude fell below the normalised irradiance $NI$ - 22 Apr. 94, sample E2. Second column is measured, third column is the Yura-McKinley model predictions.

The experimental model presents approximately a 5 dB/decade slope for the fades and a slope of 10 dB per decade for the surges. Also, the fades are more numerous than the surges. This situation is typical and can be seen better in a boxplot of the received log-amplitudes. A boxplot is a graphical description of the data distribution spread and shape. The box contains data from the 25% to the 75% percentiles and the lines define the 1% to 99% percentiles. The points outside the line are denominated outliers and represent the extreme negative and positive log-amplitudes measured. A typical example of the presence of more negative than positive outliers can be seen in Figure 3.14. This example comes from a
measurements session carried out during 02 March 1994. The range of $C_n^2$ values are close to the overall mean value of about $50*10^{-14}$ m$^{-2/3}$ (more details about $C_n^2$ can be seen in Table 3.3, section 3.8.2).

![Boxplots of logamplitude values showing the preponderance of fades over surges. 02 Mar. 94. $Cn^2$ values fluctuate slightly around $50*10^{-14}$ m$^{-2/3}$.](image)

**Figure 3.14:** Boxplots of logamplitude values showing the preponderance of fades over surges. 02 Mar. 94. $Cn^2$ values fluctuate slightly around $50*10^{-14}$ m$^{-2/3}$.

### 3.7-SPECTRAL ANALYSIS OF SCINTILLATIONS.

The spectral analysis of the time series of the received amplitude scintillations is presented in the next sections. To test the stationarity of the data, the Kruskal-Wallis ANOVA test was employed and more details can be seen in appendix A. The analysed files have a duration of 12.5 seconds, containing 5000 points sampled at 400Hz. The spectral analysis will begin by the study of the autocorrelation function of the scintillations. Theoretically, (see, for instance, Fante, 1973) the autocorrelation function of the scintillations should present, for weak homogeneous turbulence, a temporal behaviour that should be of the $t^k$ type, where $k$ is real parameter. If $k=1$, then the process can be considered to be a wide
sense Markov process. The future probabilistic behaviour of a Markov process is uniquely determined by its present state. This kind of process is called memoryless. The determination of the Markovian characteristics of the process is related to the range of validity of the Rytov method. Fante, 1973, theoretically proposed that, for a long distance path where the integrated turbulence is very strong, saturation of the scintillations is sure to happen and therefore the Rytov method loses its validity. For such high turbulence conditions, the autocorrelation function should become an exponential of the $\exp(k_1 t)$ form, instead of a slower decaying $\exp(k_1 t^{k_2})$.

To test the Markovian hypothesis, curves of the $\exp(k_1 t^{k_2})$ form were fitted to the autocorrelation estimates for a wide range of the index of refraction structure parameter $C_n^2$. Illustrative examples for values of $C_n^2$ varying from about 3 to $100 \times 10^{-14} \text{m}^{-2/3}$ are shown in section 3.7.1.

3.7.1-AUTOCORRELATION FUNCTION ESTIMATION FOR THE SCINTILLATIONS.

Figures 3.15 and 3.16 present two examples of typical autocorrelation functions obtained from scintillations time series. The circles on Figures 3.15 and 3.16 highlight the $t^k$ behaviour that is theoretically predicted for weak homogenous turbulence. Results such as the ones obtained in this work, confirming that the weak homogeneous turbulence spectral approximations are still valid for the urban boundary layer using optical methods are original. Results from urban turbulence characterisation experiments have only been obtained so far by conventional methods using meteorological measurements (see, for instance Rotach, 1993). Oikawa and Meng (1995) also performed experiments on urban turbulence using...
meteorological measurements in suburban Tokyo and pointed out the difficulty of setting up these kinds of turbulence experiments in a city (placing equipment, for instance, is particularly difficult) and emphasise the need for more experimental data from other climates. The results from both Rotach (1993) and Oikawa and Meng (1995) show that the spectra of temperature, wind speed and energy within a city follow the theoretical expectations for weak homogenous turbulence. For more details about the spectral characteristics of meteorological variables see, for instance, Stanisic (1991).

![Figure 3.15: Autocorrelation function estimate-10 May 94, sample A. Cn2 value is $3.10 \times 10^{-14} \text{m}^{-2/3}$](image1)

![Figure 3.16: Autocorrelation function estimate-22 Apr. 94-Sample E2. Cn2 value is $66.20 \times 10^{-14} \text{m}^{-2/3}$](image2)
Another typical characteristic of the autocorrelation function estimates obtained in this work is what Dunphy and Kerr (1973) defined as “residual correlation” at long lags. The authors attribute this residual correlation to effects of multiple scattering, which are due to occur for long paths and large integrated turbulence.

As mentioned in section 3.7, exponential functions of the form \( \exp(k_1 t^{k_2}) \) were fitted to the autocorrelation functions. Results indicate that the exponent \( k_2 \) was generally close to 1 for a wide range of \( C_n^2 \) values. Figures 3.17 and 3.18 show the fitted exponential curves for the examples presented in Figures 3.15 and 3.16, together with the obtained values of the parameters \( k_1 \) and \( k_2 \).

![Figure 3.17: Autocorrelation function estimate and \( \exp(k_1 t^{k_2}) \) fit. The fit has a 99.7% correlation coefficient. 10 May 94-sample A.](image)

10 May 94
\[ C_n^2 = 3.10 \times 10^{-14} \text{ m}^{-2/3} \]
\[ k_1 = 104.8 \]
\[ k_2 = 1.07 \]
3.7.2-ESTIMATION OF THE SPECTRAL POWER DENSITY FUNCTION OF THE AMPLITUDE SCINTILLATIONS.

In this section, the evaluation of the spectral power density functions of the amplitude scintillations will be performed. The theory (e.g. Clifford, 1971) for weak homogeneous turbulence, as seen in chapter 2, predicts an asymptotic \(-8/3\) power law for the high frequency part of the spectrum. For the lower frequencies, the spectrum should be independent of the frequency. The results for the spectral analysis of amplitude scintillations presented here are, to the authors knowledge, the first to be presented for turbulent conditions within a city environment.

It has to be noted that the aperture size used in the receiver (138mm) is about 2.4 times larger than the Fresnel zone (58.3mm). The large size of the aperture causes the scintillations to be averaged over the aperture size. This is the phenomenon of aperture averaging, explored in chapter 1. Because of the averaging effects, the obtained scintillations spectra will have a somewhat narrower bandwidth that would be expected from a point detector. The use of a large aperture is necessary in such a long distance path if any meaningful amount of power is to be efficiently received. On the other hand, aperture averaging can be used to protect the system.
against saturation. It was also seen in chapter 1 that a system that uses a large enough aperture can be virtually blind to saturation effects as indeed the system used in this work has been found to be. Experimental results exploring the possibility of saturation on the link are presented later in this chapter.

Figures 3.19 and 3.20 show the corresponding spectra obtained for the data used in obtaining the autocorrelation functions shown in figures 3.17 and 3.18. Although only two sets of measurements are presented here, they are typical examples of the results obtained for some other 72 sets obtained at different times of the year. A uniformity in the results is expected from equations 1.36. These equations show that the only meteorological variable affecting the spectral density function is the wind speed. Wind speeds during the measurement sessions were typically light and therefore it was not expected that the spectral densities would differ radically from one set of measurements to another as far as the cut-off frequency is concerned. This is indeed seen in figures 3.19 and 3.20, where the normalised spectrum (normalised in respect to the value at the origin) is modified by multiplying the abscissa by the frequency. This procedure allows a better identification of the cut-off frequency, which is the location of the maximum value of the modified spectrum where the low and high frequency asymptotes intersect.
Figure 3.19: Normalised modified spectrum of logamplitude scintillations - 10 May 94, sample A.

Figure 3.20: Normalised modified spectrum of logamplitude scintillations - 22 Apr. 94, sample A.

The cut-off frequencies, i.e., the frequencies above which the spectrum tends asymptotically to the -8/3 power-law behaviour are respectively 12.9 and 21.2 Hz for the spectra shown in figures 3.19 and 3.20. Cut-off frequencies varied from 10 to 35 Hz for the other analysed files. The curves showed the $f^{8/3}$ high frequency and the $f^0$ low frequency behaviour, which become $f^1$ and $f^{5/3}$ in the modified spectra. It has been observed by several authors, e.g., Roth and Oke (1993) and
Oikawa and Meng (1995) that the spectra of temperature, wind speed and energy in a city environment follows the theoretical expectations for homogeneous turbulence. It is therefore, reasonable to assume that the spectral power density function of scintillations (and angle of arrival) should also follow theoretical expectation for homogeneous turbulence. Another indication of the influence of the urban characteristics on the spectra of the scintillations are represented by the dips and bumps at frequencies below 100Hz. It is the first time the occurrence of this phenomenon has been reported on the spectra of the intensity scintillations, and its cause is probably due to similar irregularities observed on the spectra of temperature and wind speed within an urban environment (Kalogiros and Helmis, 1995; Oikawa and Meng, 1995). These dips and bumps on the spectra of temperature and wind speed are said to be due to the influence of burst wind motions, which are typical of city environments. More details of the influence of an urban environment on the characteristics of turbulence can be seen in chapter 1.

To better illustrate the presence of dips and bumps on the spectrum of intensity scintillations, Figures 3.21 and 3.22 show a typical scintillation spectrum and a spectrum of temperature fluctuations obtained on the same path by Sarma (1985). Both curves present the bumps and dips and are typical examples of the influence of highly localised Reynolds stresses (combination of horizontal and burst wind movements), which are typical of city environments.
Figure 3.21 Normalised spectrum of logamplitude scintillations - 22 Apr. 94, sample A1.

Figure 3.22: Spectrum of temperature fluctuations measured in the experimental path by Sarma (1985).
3.8-DESCRIPTIVE STATISTICS OF ANGLE OF ARRIVAL.

3.8.1-INTRODUCTION

One of the original results that came out of this thesis is a data bank of angle of arrival fluctuations on a free space optical communications link. Beam wander is believed by several authors, for instance Chiba (1971), to be the determining factor in accessing the quality and reliability of an optical communications system. Large values of angle of arrival (AOA) may cause the laser spot to escape the detector's area and the signal is then lost.

The data bank formed during the experimental phase of this work consists of sets of measurements taken during the period September 1993-November 1994 in dry and rain conditions. The files contain 5000 points sampled at 400Hz. This data bank is in itself an original result from this thesis, since, to the author's knowledge, experimental results of angle of arrival for large integrated path turbulence are not to be found in the literature. The AOA in rain conditions is one of the subjects discussed in chapter 4. These results are also believed to be unprecedented. Because the AOA was always measured simultaneously with the amplitude scintillations, a description of the experimental sessions can be found in section 3.3. The experimental sessions were usually some three to five hours long (for onsite direct measurement using the VI package, as described in chapter 2), reaching some 11 hours (for continuous tape recording). This gives an indication of the number of files that could be obtained from the measurement sessions. The number of data files obtained makes presentation of all results here impossible. As mentioned before for the scintillation results, only illustrative examples will be shown here. Each and every result shown in the next sections is a typical representative of the information it presents.
The variance of angle of arrival can be used to calculate the strength of turbulence. As seen in chapter 2, the strength of turbulence is related to the variance of the AOA in the form, Lawrence and Strohbehn (1970):

$$\sigma_{\text{AOA}}^2 = 1.05L \cdot b^{-1/3} C_n^2$$ (3.6)

In equation 3.6, $b$ is the telescope diameter and $L$ is the path length. Because the AOA is primarily influenced by the outer scale of turbulence, which is much larger than the receiver aperture, the AOA suffers no aperture averaging effects. This validates the use of the AOA variance in equation (3.6).

3.8.2-RESULTS FOR THE DESCRIPTIVE STATISTICS OF AOA.

Averaged descriptive statistics for the AOA, i.e., averages over the results from 150 sets of measurements are presented below in Table 3.3.

<table>
<thead>
<tr>
<th>DESCRIPTIVE STATISTICS</th>
<th>AVERAGE VALUE</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>STANDARD DEVIATION (µrad)</td>
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<tr>
<td>MINIMUM (µrad)</td>
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</tr>
<tr>
<td>MAXIMUM (µrad)</td>
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</tr>
<tr>
<td>RANGE (µrad)</td>
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</tr>
<tr>
<td>VARIANCE (µrad²)</td>
<td>4513.49</td>
</tr>
</tbody>
</table>
Table 3.3: Averaged descriptive statistics for AOA. Data from 150 sets of AOA measurements taken between September 1993 and November 1994.

| $C_n^2 \times 10^{-14} \text{ m}^{-2/3}$ | 54.20 |

Results from Table 3.3 can be compared with results from Cameron et al. (1990). The Cameron experiment involved a 143.5km link at 2400m of height and at 0.83μm. The authors found a standard deviation of 19.8μm during daytime. Gurvich (as quoted by Lawrence and Strohbehn, 1970) obtained maximum standard deviation values of 15 to 45μrad for a 6500m path. No mention of the wavelength or the nature of the path was made.

Applying the geometrical approximation, AOA = spot displacement/focal length and using the results from table 3.3 it will be noted that the mean value of the spot displacement is $19.4 \times 10^{-6} \times 0.33 = 6.4μm$. The angle of arrival values vary between -290.3μrad and 216.5μrad, which give a range of spot displacement values between -95.8μm and 71.5μm. Combining the results for the mean and range of angle of arrival values, and considering that the detector is a 5mm$^2$ square, it can be concluded that the spot does not escape from the detector.

3.9-ESTIMATION OF THE PROBABILITY DENSITY FUNCTION OF ANGLE-OF-ARRIVAL.

It is universally accepted in the literature that the phase fluctuations of an electromagnetic wave propagating through the atmosphere should follow a Gaussian distribution law. This comes from the central limit theorem. Since the angle of arrival has a linear relationship with the phase difference between two points, as seen in equation 2.40, it should also follow a Gaussian distribution.
Unlike the scintillations, the distribution of which vary according to the turbulence conditions, the AOA distribution holds for all turbulent regimes [Lawrence and Strohbehn, 1971]. To test this assumption typical examples of AOA distributions for several turbulence regimes and times of the year are presented below. A summary of the basic descriptive statistics can be seen in the normal probability plot graph. The normal probability plot is used mainly to provide information about the extreme values of the distribution. These extreme values can cause formal goodness of fit tests to fail. This was explained earlier on this chapter in section 3.1. Since the files consists of 5000 points, the Kolmogorov-Smirnov (K-S) test statistic, \( d \), is 0.0192. Estimates with K-S statistic lower than 0.0192 are accepted as being of the hypothetical distribution proposed. The hypothetical distribution tested here is the normal, with parameters calculated from the data using the maximum likelihood estimation technique.

![Normal probability plot and probability density function estimate of AOA.](image)

Figure 3.23: Normal probability plot and probability density function estimate of AOA. Structure parameter is $1.66 \times 10^{-14} \text{m}^{-2/3}$, standard deviation is $11.65 \mu \text{rad}$ and K-S parameter $d$ is 0.008 (12 Nov. 93 - sample A).
Figure 3.24: Normal probability plot and probability density function estimate of AOA. Structure parameter is $3.10 \times 10^{-14} \text{m}^{-2}$, standard deviation is $16.06 \mu\text{rad}$ and K-S parameter $d$ is 0.010 (10 May 94 - sample A).

Figure 3.25: Normal probability plot and probability density function estimate of AOA. Structure parameter is $50.10 \times 10^{-14} \text{m}^{-2}$, standard deviation is $64.63 \mu\text{rad}$ and K-S parameter $d$ is 0.010 (02 Mar 94 - sample B1).

Figure 3.26: Normal probability plot and probability density function estimate of AOA. Structure parameter is $101.00 \times 10^{-14} \text{m}^{-2}$, standard deviation is $91.94 \mu\text{rad}$ and K-S parameter $d$ is 0.019 (22 Apr. 94 - sample A1).
From the graphs on Figures 3.23 to 3.27 it can be seen that the distribution remained Gaussian for a change on the structure constant $C_n^2$ of about 80 times, i.e. from 1.66 to 131.37 ($10^{-14}$m$^{-2/3}$). This confirms the initial assumption that the form of the AOA distribution is relatively immune to turbulence conditions. This confirms the theoretical work of Aksenov et al, as quoted by Churnside and Lataitis (1987) and also by Churnside and Lataitis. These authors showed that there is no difference in the theory for the AOA in weak or strong turbulence.

A certain amount of asymmetry is observed in the histograms presented in Figures 3.23 to 3.27. This is probably due to the effects of burst wind movements (more details in appendix B). These wind movements cause the distribution of longitudinal and vertical wind speeds to become skewed [Oikawa and Meng, 1995]. This skewness of the wind speed distributions will be carried onto the distributions of angle of arrival since its behaviour is directly connected with the behaviour of the outer scale of turbulence.

The histograms asymmetry is represented by the depart from the Gaussian trendline on the lower and upper tails of the normal probability plots shown in the right hand side of Figures 3.22 to 3.27. This asymmetry increases as the turbulence...
grows. Although these extreme values may cause the goodness of fit test to give a test statistic very close to the rejection limit, as is the case for Figure 3.26, the middle section of all distributions follow perfectly the Gaussian trendline. The central tendency of the distributions and the dispersion characteristics of the AOA distributions can be better ascertained by the use of the Box-Whiskers plot. This plot was used in section 3.6 for the analysis of scintillation outliers and is explained there. The Box-Whiskers plots compare the extremes in turbulence conditions. First for the data presented in Figures 3.23 and 3.27 and next for the data presented in Figures 3.24 and 3.26. The $C_n^2$ values (in $10^{-14} m^{-2/3}$) are respectively 1.66 and 131.37 (Figure 3.28) and 3.10 and 101.00 (Figure 3.29).

![Figure 3.28: Comparison between Box-Whiskers plots of AOA for data files from 12 Nov. 93 and 01 Jun. 94, corresponding to $C_n^2$ values of 1.66 and 131.37 ($10^{-14} m^{-2/3}$).](image)

![Figure 3.29: Comparison between Box-Whiskers plots of AOA for data files from 10 May 94 and 22 Apr. 94, corresponding to $C_n^2$ values of 3.10 and 101.00 ($10^{-14} m^{-2/3}$).](image)

The first feature that can be seen from the graphs in Figures 3.28 and 3.29 is that the range of angles increase by a factor of more than 10 from the low turbulence to
the high turbulence plots. This is evidence of the change of the dominant propagating mechanism between weak and intermediate-to-strong turbulence. In weak turbulence, the main mechanism is forward scattering. The angular spread is not very large and hence the shorter range of angle. In stronger turbulence, multiple scattering becomes increasingly dominant and so is scattering by off-axis eddies. Careful inspection of Figures 3.28 and 3.29 also shows that the interquartile range, which is the region between the 25% and 75% quartiles, of the higher turbulence examples (graphs on the right hand side of the Figures 3.28 and 3.29), is about the same size of the 98% range of the lower turbulence examples. This indicates a composition of effects happening for the higher turbulence AOA. Forward scattering being responsible by the concentration of angles in the centre of the distribution and multiple scattering, responsible for the existence of extreme angles. This is in accord with the atmospheric model of Phillips and Andrews (1981) shown in Figure 3.1, section 3.2 and corroborates the results found for the scintillation.

The presence of outliers is represented by the points outside the lines that limit the location of 98% of the data points. The outliers exist independently of turbulence strength and can be of values of more than twice the minimum angle within the 98% range. Ben-Yosef and Goldern (1981) mentioned the high degree of influence that outlier scintillation points have on the precision of the moment method, used for the determination of the statistical characteristics of the received amplitude variations. Results indicate that the same assumption is valid for the AOA. Therefore the best way to determine its distribution is the combination of non-parametric estimation, via histograms and maximum likelihood estimation of the distribution parameters, and non-parametric goodness-of-fit tests.

As a summary for this section, it can be said that the Gaussian distribution holds for the AOA even when the values of the structure parameter vary by 80 times.
This confirms the theoretical predictions. Increase in turbulence strength causes slight deviation from the normal in the distribution extremes by the addition of very high angles. This deviation, however, is not strong enough to cause the K-S test to fail. The extreme angles are produced by multiple scattering effects. Forward scattering effects in low turbulence cause shrinkage in the range of AOA values in comparison with higher turbulence results, which illustrates the change in propagation characteristics predicted by Phillips and Andrews (1982). Due to the existence of such high AOA values, the moment method seems to be inadequate for the statistical characterisation of the AOA. This result extends the results obtained by Ben-Yosef and Goldern for the scintillation.

3.10-VARIATION OF $C_n^2$ WITH TIME OF DAY.

Before introducing the spectral analysis of angle-of-arrival, examples of daily variations of the strength of turbulence will be presented. This is an extension of the statistical description of AOA and provides a broader view of the time variation of the statistical characteristics of AOA throughout the day. Because, for optical wavelengths, the $C_n^2$ is dependent mostly on the temperature variations, it is expected that the daily curve should have a peak at the point of maximum solar irradiance, around midday and late morning-early afternoon. This value decreasing as the day progresses. The results here can be compared, for instance, to the works of Dunphy and Kerr (1973), Dowling and Livingston (1973), Bertolotti et al. (1974) and Bouricious and Clifford (1970).

The daily variation of the strength of turbulence is calculated using the variance of the angle of arrival, which does not suffer either from aperture averaging or saturation effects. By comparing the temporal variation of $C_n^2$ with the corresponding variation of the received amplitude variance, the effects of
saturation can be identified. Examples of daily variation of $C_n^2$ with corresponding variation of the received amplitude variance can be seen in Figures 3.30 and 3.31. The data is relative to the measurement sessions carried out on 22 April and 01 June of 1994. The former is characterised by a mild mean temperature of 19°C, clear sky and low winds. The latter is characterised by a warm 25°C, periods of partially covered alternating with clear sky conditions and low winds.

Figure 3.30: Daily variation of $C_n^2$ and received amplitude variance. Data from 22 Apr. 94, temperature 19°C, sunny with clear sky and low wind.
The points in Figures 3.30 and 3.31 are calculated from 12.5 seconds of measurements starting at the time indicated on the ordinate. From Figure 3.30, which is an example of the variation of $C_n^2$ during a mild spring day, it can be seen that the amplitude variance follows closely the behaviour of the strength of turbulence variations. The $C_n^2$ variations followed the expected decline with decrease in solar irradiance.

It can be observed that the values of $C_n^2$ shown in Figures 3.30 and 3.31 are concentrated in a rather short range as the time of day passes. This is expected in an urban environment [Rotach, 1993] since the constant high turbulence over a city mixes the air and prevents the occurrence of strong temperature gradients. This short range of $C_n^2$ values was also observed in the work of Bertolotti et al., (1974).

The example regarding a warm day shows signs of poorly developed turbulence near 13:00 with high variation of the received amplitude variance. This is probably due to a combination of low wind and high radiation from roof tops along the path.
Similar behaviour was observed by Dunphy and Kerr (1973) on their 6 km path. As the day progresses, the variance of the received amplitude tends to follow the strength of turbulence. This was observed by Dunphy and Kerr (1973) and Bertolotti et al. (1974). No sign of saturation was found in any of the long term measurement results. This result is explained in the light of the results obtained by Ochs et. al. (1976). Their paper is concerned with devising an saturation-free optical scintillometer. It is explained there that a system that is saturation-free should be able to avoid “seeing” eddies of the size of the first Fresnel zone. The receiver should have an aperture larger than the first Fresnel zone size, so that the effects of such eddies are smoothed out by aperture averaging and thus their capacity for generating saturation effects is not seen. For the scintillometer to work the source has to be spatially incoherent, as the source used in this work. See specification given in chapter 1. Therefore, the system in this present work can be considered to be insensitive to saturation effects.

3.11-SPECTRAL ANALYSIS OF AOA.

The last part in this chapter concerns the spectral analysis of AOA. The spectral estimation of angle of arrival will follow the same structure as used with the scintillations in section 3.7. This study, which is, to the author’s knowledge, not to be found in the literature, can provide information that can help the design of tracking systems to reacquire incoming wandering beams. According to Lawrence and Strohbehn (1970), the response time of such tracking system should be no less than the inverse of the higher end of the interval:

\[
\frac{0.01V}{2\pi b} \leq f \leq \frac{10V}{2\pi b}
\]  

(3.7)
where $V$ is the transverse wind speed and $b$ is the telescope diameter. According to Fante (1975), the interval presented in (3.7) defines the spectral range which contains all AOA fluctuations. The response time is therefore:

$$t_R \approx \frac{1}{f_{\text{max}}} \approx \frac{2\pi b}{10V} \quad (3.8)$$

3.11.1-ESTIMATION OF THE AUTOCORRELATION FUNCTION OF AOA.

Figure 3.32 shows 4 examples of autocorrelation function estimates for the angle of arrival. The $C_n^2$ values range from $2.03 \times 10^{-14}$ m$^{-2/3}$.

![Figure 3.32: Autocorrelation function estimates for AOA. The data is from 12 Nov. 93, 22 Apr. 94, 02 Mar. 94 and 01 Jun. 94. Cn2 values are respectively 2.03, 28.92, 66.20 and 121.37 ($*10^{-14}$ m$^{-2/3}$).](image)

The autocorrelation estimates of angle of arrival generally possess the same generalised $t^k$-type of temporal behaviour observed for the scintillations. The slow decay for lower lags is highlighted by the circles in Figure 3.32. This type of
Markovian characterisation of the time series of angle of arrival is, to the author’s knowledge, unprecedented in the literature.

The bump close to lag zero can be seen distinctly in the autocorrelation functions from low $C_n^2$ values. As the strength of turbulence increases, the correlation between the points close to lag zero decrease and the decay becomes much steeper. According to Equation (1.58), the only factor affecting the spectrum is the wind speed. It cannot, however, alter the shape of the spectral curve but only modify the position of the cut-off frequency, which is the frequency for which the low and high frequency asymptotes intersect. The shape of the autocorrelation curves, indeed, possesses an overall exponential format with high correlation coefficients, as can be seen in Figures 3.33 to 3.36:

Figure 3.33: Fitted autocorrelation function of angle of arrival. Sample B from 12 Nov. 93. Correlation coefficient is 99.93%.
Figure 3.34: Fitted autocorrelation function of angle of arrival. Sample B2 from 22 Apr. 94. Correlation coefficient is 99.93%.

Figure 3.35: Fitted autocorrelation function of angle of arrival. Sample E2 from 02 Mar 94. Correlation coefficient is 99.69%.
Figure 3.36: Fitted autocorrelation function of angle of arrival. Sample H from 01 Jun. 94. Correlation coefficient is 99.96%.

Unlike the scintillations, the angle of arrival autocorrelation functions have an overall behaviour that cannot be considered to be of the exp(-kt) type, regardless of the strength of turbulence. This may be due to the large size of the outer scale of turbulence on the experimental path. Siqueira (1989) has measured the values and obtained outer scale sizes varying from 10 to 1750 metres. Since the Markovian autocorrelation function is typical of saturated conditions, it is not expected that the angle of arrival should be a Markov process. This is because saturation of angle of arrival would only happen if the size of the outer scale became much smaller than the aperture size (Ochs et al., 1976). For a receiver aperture of 0.138 metres it is readily concluded from the results of Siqueira that the angle of arrival is not subject to saturation. Therefore the angle of arrival is not a Markov process in the wide sense.
3.11.2-SPECTRAL ESTIMATION OF THE AOA.

The angle of arrival spectral density function should theoretically follow a \(-8/3\) slope for the high frequency asymptote and be independent of frequency for the low frequency part of the spectrum. This is the case for the overall data, regardless of time of year or time of day, since the most influential meteorological parameter to the spectrum is the wind speed. In fact, the spectrum of AOA can be used to find an estimate for the mean wind speed along the path, by using equation (3.7). Initially, examples of the spectral estimates conformity with the asymptotic theoretical expectations are presented in Figures 3.37 and 3.38, using the same modified spectrum presented in Figure 3.19:

![Graph](image-url)

**Figure 3.37:** Modified normalised spectral density function of AOA-10 May 94, sample A. Cn\(^2\) value is \(3.10 \times 10^{-14} \text{ m}^{-2/3}\).
It is generally observed from the obtained spectra that the theoretical asymptotic behaviour for homogeneous turbulence is present regardless of the strength of turbulence. As mentioned for the scintillation spectra, it is expected that the angle of arrival spectral estimates in a city environment should follow the expected behaviour for homogeneous turbulence since it has been observed [Roth and Oke, 1993] and [Oikawa and Meng, 1995] that the spectra of temperature, wind speed and energy in an urban environment follow the expected behaviour for homogeneous turbulence.

It is also important to note the bumps and dips located below 100Hz. These bumps and dips are to be found in the great majority of the obtained AOA and intensity scintillations spectra and, as seen in section 3.7.2, and may be related to similar behaviour observed experimentally by several authors (e.g. Oikawa and Meng, 1995) in the spectra of temperature and wind speed in a city environment. Figure 3.39 shows the bumps and dips on a typical angle of arrival spectrum.
The cut-off frequencies are in the range of 8 to 25 Hz. The particular examples in Figures 3.37 and 3.38 have cut-off frequencies at 10.7 and 16.2 Hz, respectively. According to Fante (1975), the cut-off frequency can be used as the upper limit in (3.7), and therefore, estimates for the path averaged wind speed can be calculated. Using the range of 8 to 25 Hz, the range of averaged wind speeds is 0.69 and 2.16 m/s. For the examples in Figures 3.37 and 3.38 the corresponding wind speeds are 0.93 and 1.41 m/s. These values cannot be verified due to the impossibility of placing wind sensors along the path, but nevertheless are in reasonable agreement with observations at the time of the measurements. Using equation 3.8 and the higher value of the cut-off frequency, the response time of a tracking device operating in this path should be no less than $t_R \approx 1/f_c \approx 0.04$ s.
Chapter 4

Statistical and Spectral Analysis Of Line-Of-Sight Laser Propagation Through Rain

Part 1: Intensity Scintillations
And Angle Of Arrival
4.1-INTRODUCTION.

The next two sections are a theoretical introduction to the statistical analysis of the experimental data on the propagation of laser light through precipitation. First, the problem of the scattering of an electromagnetic wave by a single dielectric sphere will be explored using an approach known as the Mie solution. The Mie solution provides fundamental insight on the problem of laser propagation through a distribution of particles. The Rytov solution for the wave equation for propagation through a distribution of particles is studied in section 4.2. Using the same approach as seen in chapter 2, the spectral characteristics of the scintillations and phase fluctuations are predicted.

4.2-SCATTERING OF LIGHT BY A DIELECTRIC SPHERE OF ARBITRARY SIZE- THE MIE SOLUTION.

The problem of scattering of light by homogeneous, dielectric spheres, can only be solved by the formal solution of the Maxwell equations in spherical coordinates. In spherical coordinates, the use of Hertz vectors, allow any electromagnetic field to be expressed in terms of two scalar functions, $\pi_1$ and $\pi_2$, which are the radial components of the electric and magnetic Hertz vectors [Ishimaru, 1991]:

$$\vec{\pi}_e = \pi_1 \hat{r} \quad \text{and} \quad \vec{\pi}_m = \pi_2 \hat{r} \quad (4.1)$$

Using these scalar functions, the wave equation can be written in the following form:

outside the sphere: \( (\nabla^2 + k^2)\pi = 0 \) 
inside the sphere: \( (\nabla^2 + k^2 m^2)\pi = 0 \) \( (4.2) \)

Using (4.2) the electric field can be written as:
\[ \overline{E} = \nabla \times \nabla \times \left(r \pi_1 \mathbf{f} \right) + j \omega \mu_0 \nabla \times \left(r \pi_2 \mathbf{f} \right) \] (4.3)

Equations (4.2) are separable in spherical coordinates. The general solution is of the form:

\[ \pi = X_1(r)X_2(\theta)X_3(\phi) \] (4.4)

The steps to the solution of (4.2) are omitted here and can be found in [Ishimaru, 1991] or [van de Hulst, 1981]. The development following is for the electric field. Details on the solution for the magnetic field can be found in the references given above. The spherical harmonic representation of the incident wave is:

\[ r \cdot \pi_i = \frac{1}{k} \sum_{m=1}^{\infty} \frac{(-j)^{m-1}(2m+1)}{m(m+1)} \Psi_m(k_2r) P^1_m(\cos \theta) \cos \phi \] (4.5)

Solution (4.5) is said to be the spherical harmonic representation because it involves the use of spherical Bessel functions \( \Psi_m(x) \), which are related to the ordinary Bessel function \( J_{m+1/2}(x) \) by:

\[ \Psi_m(k_2r) = x_jm(x) = \sqrt{\frac{\pi x}{2}} J_{m+1/2}(x) \] (4.6)

The spherical harmonic form of the scattered wave is given by:

\[ r \cdot \pi_s = \frac{(-j)}{k} \sum_{m=1}^{\infty} \frac{(-j)^{m-1}(2m+1)}{m(m+1)} a_n c_m(k_2r) P^1_m(\cos \theta) \cos \phi \] (4.7)

where, again, the spherical Bessel function \( c_m(x) \) can be expressed in terms of the ordinary Bessel function \( H_{m+1/2}(x) \) by:

\[ c_m(x) = x h_{m}^{(2)}(x) = \sqrt{\frac{\pi x}{2}} H_{m+1/2}^{(2)}(x) \] (4.8)

For the far field:

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\[ \zeta_m(k_2 r) \rightarrow j^{m+1} \exp(-jk_2 r) \quad (4.9) \]

and therefore, the scattered field becomes:

\[ r \cdot \pi_s \rightarrow \frac{\exp(-jk_2 r)}{\frac{k_2^2}{2}} \sum_{m=1}^{\infty} \frac{2m+1}{m(m+1)} a_m p_m^1(\cos \theta) \cos \phi \quad (4.10) \]

The coefficients \( a_m \) can be obtained from the boundary conditions, given a specific problem.

To account for the effect of a distribution of particles, a quantity of interest is the total cross section. The definition of the total cross section involves the use of the forward scattering theorem:

\[ \sigma_t = \frac{-4\pi}{k_2} \hat{i} \cdot \hat{s} \Im \left[ f(\hat{i}, \hat{i}) \right] \quad (4.11) \]

In the above equation:

\( \hat{i}, \hat{s} \): are unit vectors in the direction of the incident and scattered waves, respectively. In the present case \( \hat{i} = \hat{z} \).

\( f(\hat{i}, \hat{i}) \): is the scattering amplitude in the forward direction.

The scattering amplitude is given by:

\[ \mathcal{E}_s = f(\hat{i}, \hat{i}) \frac{\exp(-jkR)}{R} \quad (4.12) \]

4.3-PROPAGATION THROUGH A DISTRIBUTION OF PARTICLES.

An electromagnetic wave propagating through a distribution of particles experiences scattering. The scattered wave forms an incoherent field which interacts with the incident coherent field. If the intensity of the incident
coherent field is $I_c$ and the intensity of the scattered incoherent field is $I_i$, the overall intensity $I_t$ can be written as:

$$I_t = I_c + I_i \quad (4.13)$$

The coherent intensity suffers scattering and absorption and, as it propagates through the distribution of particles, it decreases exponentially in the form:

$$I_c = \exp(-\rho \sigma_t z) \quad (4.14)$$

where:

$\rho$: density of the particles

$\sigma_t$: total cross section, defined in (4.11)

Substituting the total cross section by the absorption cross section in (4.14), it is possible to obtain the expression for the overall intensity. Subtracting the coherent intensity from the overall intensity, the scattered incoherent intensity can be found:

$$I_i = \exp(-\rho \sigma_t z) \left[ \exp(\rho \sigma_s z) - 1 \right] \quad (4.15)$$

From (4.15) it can be seen that, if the factor $\rho \sigma_t z$ is much smaller than unity, the incoherent intensity is much smaller than the coherent intensity and the field can be written as a sum of its mean value and an incoherent fluctuation:

$$u(\vec{r}_1) = \langle u(\vec{r}_1) \rangle + u_f(\vec{r}_1) \quad (4.16)$$

The average field is taken from the coherent intensity:

$$\langle u(\vec{r}_1) \rangle = \exp\left(jkL - \frac{\gamma}{2}\right) \quad (4.17)$$

The factor $\gamma$ is called the optical distance, which is defined as:
\[ \gamma = \int_{0}^{L} \rho \sigma_t dz \tag{4.18} \]

The scattered field is a function of the scattering amplitude, as seen in (4.12). Beginning from the contribution of a single particle \(dV\) located at \(P_0=(x',y',L)\), the scattering field can be written as:

\[ u_f(\bar{r}) = \frac{f(\delta, \hat{z})}{R_1} \exp \left( jkz' + jkR_1 - \frac{\gamma_0}{2} \right) \tag{4.19} \]

The field is measured at \(\bar{r} = (x_1,0,L)\). The optical distances \(\gamma_0\) and \(\gamma_1\) are given by:

\[ \gamma_0 = \int_{0}^{L} \rho \sigma_t dz \quad \text{and} \quad \gamma_1 = \int_{0}^{R_1} \rho dR \tag{4.20} \]

The term \(R_1\) is the distance between \(P_0\) and \(\bar{r}\).

4.4-SOLUTIONS TO THE WAVE EQUATION FOR PROPAGATION THROUGH A DISTRIBUTION OF PARTICLES.

In this section, the wave equation is solved using the Rytov method introduced in chapter 1. This method assumes a condition of multiple scattering, which is a reasonable assumption for propagation through a general distribution of particles.

The usual representation of the received field is a sum of a mean field and a random fluctuation:

\[ u(\bar{r},t) = \langle u(\bar{r},t) \rangle + u'(\bar{r},t) \tag{4.21} \]
The Rytov method is well known in the literature (e.g. [Tattarski, 1962]). The method consists of writing the received field as a function of a “random” complex phase $\psi$:

$$u(\vec{r}, t) = \langle u(\vec{r}, t) \rangle \exp[\psi(\vec{r}, t)] \quad (4.22)$$

The phase $\psi$ can be expanded in the form:

$$\psi(\vec{r}, t) = \chi(\vec{r}, t) + S_1(\vec{r}, t) \quad (4.23)$$

The parameter $\psi(\vec{r}, t)$ represents the logamplitude fluctuations. The logamplitude fluctuations are defined as the natural logarithm of the ratio between the amplitude of the received field and the amplitude of the mean field. The parameter $S_1(\vec{r}, t)$ is the phase difference between the received field and the mean field.

Using (4.23) in the wave equation transforms it into a nonlinear Ricatti equation in $\psi$, the solution to which [Tattarski, 1962] gives:

$$u(\vec{r}, t) = \langle u(\vec{r}, t) \rangle \exp[\frac{u'(\vec{r}, t)}{\langle u(\vec{r}, t) \rangle}] \quad (4.24)$$

Comparing (4.24) and (4.22), the random phase can be written:

$$\psi(\vec{r}, t) = \frac{u'(\vec{r}, t)}{\langle u(\vec{r}, t) \rangle} \quad (4.25)$$

Using the geometry presented in figure 4.1:
it is possible to show that [Ishimaru, 1978]:

\[ \psi(\hat{\mathbf{r}}) = \int_V \frac{|f(\hat{\mathbf{r}}, \hat{z})|}{R} \exp \left[ jkR(1 - \cos \theta) - \rho \sigma_t R(1 - \cos \theta)/2 + kd \sin \theta \cos \phi /2 \right] \rho dV \] (4.26)

According to figure 4.1: \( dV = R^2 dR \sin \theta d\theta d\phi \) and \( \hat{\mathbf{r}}_1 = (d/2, 0, L) \). The function \( f(\hat{\mathbf{r}}, \hat{z}) \) is the scattering amplitude and the parameters \( \rho \) and \( \sigma_t \) are respectively the particle number density and the total cross section.

From (4.24), the logamplitude fluctuations can be written:

\[ \chi(\hat{\mathbf{r}}_1) = \frac{1}{2} \left[ \psi(\hat{\mathbf{r}}_1) + \psi^*(\hat{\mathbf{r}}_1) \right] \] (4.27)

Substituting (4.26) in (4.27) the following expression for the logamplitude is obtained:

\[ \chi(\hat{\mathbf{r}}_1) = \int_V \frac{|f(\hat{\mathbf{r}}, \hat{z})|}{R} \cos \left[ kR(1 - \cos \theta) \right] \exp \left[ -\rho \sigma_t (1 - \cos \theta)/2 - j kd \cos \phi \sin \theta /2 \right] \] (4.28)

From this expression the autocorrelation function and the temporal frequency spectrum can be found. Following the development in Ishimaru (1978) and
using the approximation for particles much larger than the wavelength, the autocorrelation function and temporal frequency spectrum can be written:

$$B_{\chi}(\tau) = \frac{\rho \sigma_s L}{2} \exp \left[ -\frac{(k v_t \tau)^2}{4\alpha} \right]$$ \hspace{1cm} (4.29)

$$W_{\chi}(\omega) = 2\rho \sigma_s L \left[ \frac{\pi \alpha_p}{(kv_t)^2} \right]^{1/2} \exp \left[ -\frac{\alpha_p \omega^2}{(kv_t)^2} \right]$$ \hspace{1cm} (4.30)

To include effects of a distribution of raindrop sizes, the mean number density, the scattering cross section, the factor $\alpha_p$ and the terminal velocity $v_t$ become functions of the raindrop diameter. Because the drop size is much larger than the wavelength, the scattering cross section can be approximated by $\sigma_s = 2\pi a^2$.

The spectrum in equation (4.30) becomes:

$$W_{\chi}(\omega) = \int_0^{\infty} n(a) \delta^2 \, da \int \frac{\pi \alpha_p (a)}{(kv_t(a))^2} \frac{1}{2} \exp \left[ -\frac{\alpha_p (a) \omega^2}{(kv_t(a))^2} \right]$$ \hspace{1cm} (4.31)

In the above equation, $n(a) \, da$ is the differential particle number density function. The integral $\int_0^{\infty} n(a) \delta^2 \, da$ gives an estimate of the attenuation of the incident wave due to scattering from all raindrops. The parameter $\alpha_p$ is a raindrop size related parameter given by $\alpha_p = 2.77/(0.5\lambda/a)^2$, $L$ is the path length, $k$ is the wave number and $\lambda$ is the wavelength. Substituting $n(a)$ in (4.31) by a distribution of drop sizes, it is possible to find the final form of the logamplitude spectrum. If the Marshall-Palmer distribution:

$$n(a) = n_0 \exp(-\alpha a)$$ \hspace{1cm} (4.32)

where $n_0=8*10^6$ (m$^{-3}$), $\alpha=8200p^{-0.21}$ (m$^{-1}$) and $p$ is the rainfall rate in mm/h is used, then the spectrum in (4.31) becomes:

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\[ W_x(\omega) = 4\pi L(\pi A)^{1/2} \frac{\Gamma(7/2)}{(\alpha + A\omega^2)^{7/2}}, \quad A = 6.7 \times 10^{-6} \quad (4.33) \]

Ishimaru (1978) calculates the bandwidth of this spectrum to be about 2.5kHz.

4.5-EXPERIMENTAL RESULTS - PRELIMINARY CONSIDERATIONS.

The measurements of intensity scintillations and angle of arrival in rain were taken during the period between October 1993 and November 1994. The wavelength used was 0.83\( \mu \)m.

For the sake of uniformity, the data files contained 5000 points, which according to Ben-Yosef and Goldern (1982) is sufficient to assure an unbiased estimation of the moments of the distribution. The sampling frequency was a matter of more careful consideration. This is because, for measurements of intensity scintillations and angle of arrival, Ben-Yosef and Goldern suggest the use of a sampling frequency above 200Hz. For rain conditions Wang and Clifford, (1975) suggest the use of a frequency above 1000Hz. The reason for this latter choice is to make it possible to differentiate between the effects of rain and ordinary turbulence in the spectrum. Theoretically, [Wang and Clifford], the effects of rain on the spectrum should become evident at high spectral frequencies. Due to the finite size of the apertures on the 0.83\( \mu \)m system used in this work, it is expected that aperture averaging effects will prevent the observation of such a phenomenon. There is no practical way to counter this effect for this particular path, since in making the aperture sufficiently small, the amount of power received would be too small for any meaningful readings to be taken. It was observed during aperture averaging experiments in this work, that even for clear day conditions, the power received would be severely impaired if the aperture was reduced to values much smaller than the first Fresnel zone size (58.3 mm). A possible suggestion for a
continuation of this work would be to establish a much shorter link that would work with very small receiver apertures.

The sampling frequency chosen is the same as that used in the experiments described in chapter 3, i.e., 400 Hz. This frequency is within the range of options suggested by Ben-Yosef and Goldern to assure that the sampled data points are independent and identical distributed (i.i.d.) random variables.

The rainfall rate measurements were made using a tipping bucket raingauge (described in chapter 2), located close to the receiver end of the path. The gauge has a resolution of 0.1 mm and an integration time of 1 minute. The nature of the propagation path (over a city centre) makes it impossible to place an array of raingauges along the path. Therefore, all the rainfall rate measurements are related to values at the receiver. Because the scintillations are influenced by the rainfall throughout the path, the results may sometimes be affected by inhomogeneities in the rain profile. In their classic work, Chu and Hogg (1968) used a 2.6 km experimental path over sparsely urbanized terrain with 3 raingauges spaced along it. The results showed that, even with three raingauges, the results presented discrepancies, which the authors attributed to the inhomogeneity of rain and to the fact that raindrops are not actually spherical, but have an oblate spheroid shape.

The experimental results are divided into results for the intensity scintillations and results for angle of arrival.

4.6-STATISTICS OF THE LOGAMPLITUDE SCINTILLATIONS.

4.6.1-INTRODUCTION.

Files were formed with data recorded for a range of rainfall rates varying from 0.10 to 12.0 mm/h. Although the number of hours of measurements taken (approximately 18 hours with data stored in magnetic tape and computer floppy
discs) and the amount of obtained files are not as large as the values for the scintillations and angle of arrival in clear conditions, a considerable data bank could be formed. For this reason, and as in chapter 3, the graphs presented in the next sections should be considered as typical examples of the information they present.

4.6.2-PROBABILITY DENSITY FUNCTION ESTIMATION FOR THE SCINTILLATIONS IN RAIN.

As Chu and Hogg (1968) and Wang and Clifford (1975) pointed out, the effects of forward scattering are enhanced in rain. Therefore, if the general propagation model of Phillips and Andrews (1982) described in section 3.2 holds for rain conditions, then the lognormal distribution should “perform” better in rain than in dry conditions, since the enhancement of forward scattering would enhance the specular component on Phillips and Andrews model. The current results show that, although a lognormal distribution is sometimes accepted at the 5% confidence level, the gamma distribution is generally the best fit, as observed in the clear air case. However, the dominance of the gamma distribution is not as complete as in the clear air case. On some occasions, although the shape of the histogram resembles a gamma density, the hypothesis failed the Kolmogorov-Smirnov test. It can then be concluded that the general theoretical model of Phillips and Andrews would seem to be appropriate for rain conditions as well as for dry weather. To illustrate the results summarised above, examples of the dominance of the gamma distribution are presented in Figures 4.2 to 4.5, for various rainfall rates:
Figure 4.2: Histogram and estimated pdf of scintillations in rain-22 Mar. 94. The rainfall rate is 0.10mm/h.

Figure 4.3: Histogram and estimated pdf of scintillations in rain-03 Jun. 94. The rainfall rate is 0.14mm/h.
Figure 4.4: Histogram and estimated pdf of scintillations in rain-01 Feb. 94. The rainfall rate is 2.0mm/h.

Figure 4.5: Histogram and estimated pdf of scintillations in rain-25 Mar. 94. The rainfall rate is 6.0mm/h.

To illustrate the presence of the lognormal distribution, Figures 4.6 and 4.7 show comparisons between lognormal and gamma distribution estimates for identical samples:

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Figure 4.6: Comparison between lognormal and gamma hypothesis for a sample from 25 Feb. 94. The rainfall rate is 0.13mm/h. Both hypothesis were accepted at 5% confidence level.

Figure 4.7: Comparison between lognormal and gamma hypothesis for a sample from 25 Oct. 94. The rainfall rate is 2.0mm/h. Both hypothesis were accepted at 5% confidence level.

In Figures 4.6 and 4.7 it will be noticed that, although the lognormal is formally accepted, it always has a higher value of the Kolmogorov-Smirnov test statistic. This was generally found to be true.

The fact that the lognormal distribution is accepted on some occasions indicates that the turbulence conditions over the path approach the characteristics of the weak turbulence case. This rainfall induced weakening of the strength of turbulence is explained by Lykossov and Wamser (1995). The authors verified experimentally that the major effect of suspended particles in the atmosphere (rain drops as a particular case) is to reduce intense small-scale turbulence and to cause a decrease in turbulence production in general.

The raindrops are kept suspended and in motion by the action of ejection wind motions. The atmosphere spends energy by interaction with the drops that otherwise would be used to generate turbulence. The combined effects of wind...
motions and rainfall affect the scintillations, by affecting the small-scale turbulence and the angle of arrival, by the temperature gradients caused by the ejection wind movements. These meteorological characteristics are typical of urban environments and therefore make the results of this thesis distinct from the usual controlled conditions in which some previous experiments have been performed.

4.6.3-COMPARISON BETWEEN LOG-AMPLITUDE DISTRIBUTION ESTIMATES IN RAIN AND IN DRY CONDITIONS.

Having analysed the logamplitude distributions for the given rainrates separately, a comparative study of the relationships between the distributions in rain and dry conditions is presented. The method used is the percentile-percentile plot. In this plot, the percentiles of one distribution are plotted against the percentiles of another. The curve fitted to the graph can provide an insight into the relationships between the distributions. The hypothesis here is that the relationship between the distributions is multiplicative and additive, i.e., the curve is a straight line of the form \( \text{percentiles of dist.1} = k_1 \times \text{percentiles of dist.2} + \text{intercept} \). In this fashion, the relationship between the logamplitude distributions in different rainfall regimes and in dry conditions can be described as:

\[
G(x) = F\left(\frac{x}{c} - h\right) \tag{4.34}
\]

Which means that the relationship between the percentiles is:

\[
x_p = cy_p + h \tag{4.35}
\]

The linear regression was performed and the results are presented in tables 4.1 to 4.3. The percentiles of the distributions in rain are the averaged percentiles of distributions from samples recorded for different rainfall rates. This
procedure is valid because the data sets are independent of each other. The percentiles for the distributions of the logamplitudes in dry conditions were calculated from the average of the percentiles of data files recorded from sessions in the spring of 1994.

Tables 4.1, 4.2 and 4.3, present the estimated values of the intercept (the additive factor) and the multiplicative factor $K_1$. The p-level statistic is a measure of the accuracy of the estimate. A p-level of zero means an estimate that is precise with probability 1.

\[
\text{Corr. Coef} = .9979
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated Value</th>
<th>Std.Err. of Estimated</th>
<th>p-level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-1.73936</td>
<td>0.003822</td>
<td>2.80E-45</td>
</tr>
<tr>
<td>$K_1$</td>
<td>0.356599</td>
<td>0.004995</td>
<td>1.56E-27</td>
</tr>
</tbody>
</table>

Table 4.1: Regression analysis for the percentile-percentile plot. Comparison between distributions in dry conditions and in rain conditions at 0.10mm/h.

\[
\text{Corr. Coef} = .9970
\]

<table>
<thead>
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</tr>
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<tr>
<td>Intercept</td>
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<td>0.005288</td>
<td>3.67E-28</td>
</tr>
<tr>
<td>$K_1$</td>
<td>0.418386</td>
<td>0.006911</td>
<td>5.75E-26</td>
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</tbody>
</table>

Table 4.2: Regression analysis for the percentile-percentile plot. Comparison between distributions in dry conditions and in rain conditions at 1.0mm/h.

\[
\text{Corr. Coef} = .9827
\]

<table>
<thead>
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<th>Parameter</th>
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<th>Std.Err. of Estimated</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
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</tr>
<tr>
<td>$K_1$</td>
<td>0.404117</td>
<td>0.016252</td>
<td>1.35E-27</td>
</tr>
</tbody>
</table>

Table 4.3: Regression analysis for the percentile-percentile plot. Comparison between distributions in dry conditions and in rain conditions at 12.0mm/h.
The correlation coefficients are approximately one for regression shown in Tables 4.1 to 4.3, which means the fits were almost perfect.

The graphs presenting the fitted and observed values are presented in Figures 4.8 to 4.10:

$pct\ 0.1\text{mm/h} = 0.356599\cdot\text{pctdry} - 1.7394$

Figure 4.8: Percentile-percentile plot of the logamplitude distributions. Comparison between scintillations in rain at 0.1mm/h and scintillation in dry conditions.
\[ \text{pct } 1.0\text{mm/h} = 0.418386 \times \text{pct dry} - 0.40325 \]

Figure 4.9: Percentile-percentile plot of the logamplitude distributions. Comparison between scintillation in rain at 1.0mm/h and scintillation in dry conditions.

\[ \text{pct } 12.0\text{mm/h} = 0.404117 \times \text{pct dry} - 4.5314 \]

Figure 4.10: Percentile-percentile plot of the logamplitude distributions. Comparison between scintillation in rain at 12.0mm/h and scintillation in dry conditions.

The results from tables 4.1 to 4.3 and from Figures 4.8 to 4.10 indicate that the received amplitude distributions in rain and in clear air are generally related by
\( P_{\text{rain}}(A) = 0.4 * P_{\text{dry}}(A) + \text{Mean Value of Logamplitude} \). This result indicates that the received amplitude statistics are dependent much more on the atmospheric turbulence than on the characteristics of the rain. The effects of rainfall are to decrease the production of turbulence by reducing the temperature and momentum exchanges.

4.7-TEMPORAL AUTOCORRELATION AND FREQUENCY SPECTRA OF THE LOGAMPLITUDE FLUCTUATIONS.

The next step in the statistical analysis of the logamplitude fluctuations in rain is concerned with the time series analysis of the sampled received signal. The theoretical approach is explained in the beginning of the chapter and the results to be sought are the autocorrelation functions and spectra of the scintillations for different rainrates.

4.7.1-AUTOCORRELATION FUNCTION ESTIMATES OF THE LOGAMPLITUDE FLUCTUATIONS.

Autocorrelation function estimates were obtained for a wide range of rainfall rates. The obtained curves showed the same \( \exp(k_1 t^{k_2}) \), \( k_2 \neq 1 \), as generally observed for the autocorrelation function of scintillations in clear air conditions (section 3.7.1). The bump becomes less prominent as the rainfall rate increased. Figures 4.11 to 4.13 show examples of autocorrelation functions and the bump close to zero lag are shown. The bump is not noticed on the 12.0mm/h rainfall rate.
Figure 4.11: Autocorrelation function estimate for scintillations in rain. The circle highlights the bump close to zero lag-25 Feb. 94, 0.13mm/h.

Figure 4.12: Autocorrelation function estimate for scintillations in rain. The circle highlights the bump close to zero lag-25 Oct. 94, 2.0mm/h.
Using the same approach as in the clear air case (presented in chapter 3), exponential curves of the form $\exp(k_1 t^{k_2})$ were fitted to the autocorrelation functions. Figures 4.14 to 4.16 present examples of the obtained fits for the examples shown in Figures 4.11 to 4.13:

Figure 4.13: Autocorrelation function estimate for scintillations in rain. The circle highlights the bump close to zero lag - 25 Mar. 94, 12.0 mm/h.

Figure 4.14: Autocorrelation function estimate (and fitted $\exp(k_1 t^{k_2})$ curve) of scintillations in rain - 25 Feb. 94, 0.13 mm/h.
Figure 4.15: Autocorrelation function estimate (and fitted $\exp(k_1t^2)$ curve) for scintillations in rain - 25 Oct. 94, 2.0 mm/h.

Figure 4.16: Autocorrelation function estimate (and fitted $\exp(k_1t^2)$ curve) for scintillations in rain - 25 Mar. 94, 12.0 mm/h.
The overall results show that the time exponent was contained within a range varying from 1.10 and 1.35. If this result is compared with the exponents found for the clear air case, which were close to one for a wide range of values of $C_n^2$, it can be verified that the rain caused a reduction effect on the turbulence. As it was seen in section 3.7.1, it is theoretically predicted (Fante, 1975) that, for large integrated turbulence the scintillations should approach a wide sense Markov process. A wide sense Markov process should have an autocorrelation function of the $\exp(-kt)$ form.

4.7.2-SPECTRAL ANALYSIS OF LOGAMPLITUDE FLUCTUATIONS.

In this section, spectral power density function estimates of amplitude scintillations in rain are presented for the first time in the literature (to this author’s knowledge). The theoretical approaches used by Wang and Clifford (1975) and Ishimaru (1978) lead to a spectral power density function for the scintillations which is a combination of effects from the clear air spectrum of turbulence and the rainfall induced spectrum. Because the raindrops are much smaller than the inner scale sizes, which are in the order of few centimetres (Luthra, 1995), the spectrum of scintillations in rain should have a much broader bandwidth (few kHz, depending on the form of the drop size distribution) than it is theoretically expected in clear air. However, for a receiver to be able to “see” the whole rainfall induced scintillations spectrum, the aperture size should be at the same size as the smallest raindrop. Otherwise, the scintillations would be averaged over the aperture and the bandwidth would be shortened by aperture averaging. The smallest raindrop should have a diameter of a few microns, which makes it virtually impossible for any practical system to be able to observe the full spectral bandwidth. Therefore, the most influential contribution to the spectra of scintillations in rain will come from the atmospheric turbulence.
Results, however, show indications of some theoretical expectations of Wang and Clifford (1975). One approximation used in developing the general equation for the scintillations spectrum in rain, equation (4.30), considers that the drop falls vertically and the wind speed is zero. In practice, there is always the influence of wind movements in all directions. Changes in wind direction and intensity causes movements of the fringes of interference between the incident wave and the spherical wave fronts generated by the scattering by the drops. These movements will affect the shape of the spectral estimates by giving the spectrum a smoothed oscillatory behaviour [Wang and Clifford, 1975], superimposed to the usual power law decline caused by the turbulent atmosphere. This theoretically predicted effect was observed, as can be seen in the following examples:

![Figure 4.17: Spectral density function estimate of logamplitude scintillations for 0.13mm/h rainfall rate. Sample from 25 Feb. 94.](image-url)
Figure 4.18: Spectral density function estimate of logamplitude scintillations for 2.0mm/h rainfall rate.

Another theoretical expectation is that the lower the rainfall rate, the wider the spectrum [Wang and Clifford, 1975]. This is because of the larger concentration of smaller drops at the lower rainfall rates. The small drops, having sizes closer to the wavelength size, are those which contribute the most to the logamplitude scintillations as they are the most likely to be coherently illuminated. There is an indication that this theoretical assumption is justified in the results for the autocorrelation function estimates. It was seen in section 4.7.1, Figures 4.14 to 4.16, that the autocorrelation function estimates decline more slowly as the rainfall rate increases.

To eliminate the variable nature of the received power, the spectra were normalized to the value at zero. Initially, it was necessary to find an estimate for the cut-off frequency, i.e., the frequency above which the spectrum tends asymptotically to a given exponent. This was achieved by the used of the modified normalised spectrum, introduced in chapter 3. Below, in Figures 4.19 to 4.22, examples of the use of the modified normalised spectrum for scintillations in rain are shown for the autocorrelation function estimates.
produced in Figures 4.14 to 4.16 together with a spectrum from a sample taken
during a 0.43 mm/h rainfall rate:

Figure 4.19: Modified normalised spectrum of logamplitude scintillations in 0.13 mm/h rain.
Sample from 25 Feb. 94.

Figure 4.20: Modified normalised spectrum of logamplitude scintillations in 0.43 mm/h rain.
Sample from 01 Feb. 94.
Figure 4.21: Modified normalised spectrum of logamplitude scintillations in 2.0 mm/h rain. Sample from 25 Oct. 94.

Figure 4.22: Modified normalised spectrum of logamplitude scintillations in 6.0 mm/h rain. Sample from 25 Mar. 94.
The cut-off frequencies are, for these examples, respectively, 10.1, 16.8, 18.5 and 9Hz. The overall range of values of the cut-off frequencies is between 5 and 25 Hz.

Due to the inhomogeneity of the rain and also from the large influence of wind movements on the spectrum, the effect of bandwidth decrease with rainfall cannot be clearly observed in the cutoff frequencies.

However, one effect that may be a sign of bandwidth increase could be identified in the results. It will be noticed from Figures 4.19 to 4.22 as well as for the overall results, that the decay with frequency is initially flatter than the \(-8/3\) law \((-5/3\) on the modified spectrum) typical of scintillations in clear conditions, while the independence of frequency for the low frequency part of the spectrum was generally observed. Hill et al. (1989) verified the frequency widening of the spectrum in rain for millimetre waves and also noticed the appearance of secondary and even tertiary cut-off frequencies. To clearly identify this phenomenon in the results obtained in this work, nonlinear regression techniques were employed to fit power law curves to different parts of the spectrum.

The spectral density to be fitted is of the form:

\[ W_X(f) = k_1 \cdot f^{k_2} \]  

(4.36)

The first step in this analysis will find the regression curve (4.36) for frequencies above 10Hz. The choice of this frequency is arbitrary and was based in observation of obtained spectra. Examples of the obtained regression curves can be seen in Figures 4.23 to 4.25:
Figure 4.23: Observed (dots) and fitted spectra of logamplitude scintillations. Rainfall rate is 0.13mm/h. Sample from 25 Feb. 94.

Figure 4.24: Observed (dots) and fitted spectra of logamplitude scintillations. Rainfall rate is 0.13mm/h. Sample from 01 Feb. 94.
The spectra showed an overall tendency to depart from the initial flatter decay after a secondary cutoff frequency. The secondary cut-off frequency had an overall range of values between approximately 30 and approximately 55 Hz, with no correlation with rainfall found.

This "break" in the spectral densities was not observed on the files related to the higher rainfall rates. In Figures 4.26 and 4.27 are two examples, for 6.0 and 12.0 mm/h:
The intermediate slope for the lower rainfall rates is, on average over the analysed data files, equal to approximately -5/3. The slope for the higher...
rainfall rate is, on average a little less than -8/3, which is the expected slope for the scintillation spectrum in clear air and homogeneous turbulence conditions. The obtained -8/3 slope is a clear indication of the overall dominance of turbulence effects over the rainfall induced effects.

The second step in the curve fitting analysis of the obtained scintillations spectra is to fit similar generalised power-law curves to the part of the spectra above the secondary cutoff frequency. Results indicate that the slope is, in average also close to -8/3, as the following examples in Figures 4.28 and 4.29 show:

![Graph](image)

**Figure 4.28:** Observed (dots) and fitted spectra of logamplitude scintillations for the high frequency portion of the spectrum. Rainfall rate is 0.13mm/h. Sample from 25 Feb. 94.
Again, the overall asymptotic -8/3 slope found for the lower rainfall rates indicate the dominance of turbulence induced effects over rainfall induced effects on the spectral densities.

4.7.2.1-TIME EVOLUTION OF THE AMPLITUDE SCINTILLATIONS SPECTRA.

Before passing on to the statistical and spectral analysis of angle of arrival, an illustrative example of how quickly changeable are the turbulence characteristics during a rainfall event is presented. Figure 4.30 shows the obtained spectra for samples of received amplitude scintillations taken at subsequent intervals of about 15 seconds. The variation on the shapes and position of the spectral bumps is evident:
4.8-DESCRIPTIVE STATISTICS OF ANGLE OF ARRIVAL.

Angle of arrival measurements were always taken simultaneously with the logamplitude scintillations. Therefore, the files were recorded on the same date and time as those used in the statistical analysis of the logamplitudes. The descriptive statistics of AOA are averaged within each rainfall rate group. To give a general view of the variation of the averaged descriptive statistics with rainfall intensity, some examples are summarised below in table 4.4:

<table>
<thead>
<tr>
<th>Rainrate (mm/h)</th>
<th>Mean</th>
<th>Median</th>
<th>Min.</th>
<th>Max.</th>
<th>Range</th>
<th>Var.</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.17</td>
<td>-26.55</td>
<td>-29.9</td>
<td>-107.3</td>
<td>65.489</td>
<td>172.74</td>
<td>935.22</td>
<td>30.581</td>
</tr>
<tr>
<td>0.55</td>
<td>6.4686</td>
<td>6.4435</td>
<td>-65.84</td>
<td>80.661</td>
<td>146.5</td>
<td>409.39</td>
<td>20.233</td>
</tr>
<tr>
<td>1.2</td>
<td>4.4851</td>
<td>3.932</td>
<td>-54.63</td>
<td>76.026</td>
<td>130.66</td>
<td>367.97</td>
<td>19.183</td>
</tr>
<tr>
<td>3</td>
<td>-3.28</td>
<td>-3.34</td>
<td>-6.99</td>
<td>1.16</td>
<td>8.15</td>
<td>1.41</td>
<td>1.2613</td>
</tr>
<tr>
<td>6</td>
<td>-2.947</td>
<td>-2.965</td>
<td>-6.766</td>
<td>1.999</td>
<td>8.7646</td>
<td>1.5908</td>
<td>1.2613</td>
</tr>
<tr>
<td>12</td>
<td>-2.45</td>
<td>-2.50</td>
<td>-6.26</td>
<td>2.92</td>
<td>9.18</td>
<td>1.73</td>
<td>1.32</td>
</tr>
</tbody>
</table>

Table 4.4: Averaged descriptive statistics for some chosen rainfall rates. The mean, median, minimum, maximum, range and standard deviation values are measured in \( \mu \)radians. The variance is measured in \( \mu \)radians\(^2\).
It can be seen from table 4.4, that there is a marked decrease in range and in variance as the rainfall increases. This may be because of the enhancement of forward scattering for large raindrops, which are present in heavier rain in much larger quantities. The lower variances are caused by the cooler conditions and the suspended raindrops in the atmosphere in heavier rain, which contribute to smoother temperature gradients.

4.9-HISTOGRAMS AND PROBABILITY DENSITY ESTIMATES FOR THE AOA.

Theoretically, in multiple scattering conditions, the phase of the received signal is the sum of a very large number of components. Applying the central limit theorem, the phase must follow a gaussian distribution. The AOA is directly proportional to the phase difference between two points, therefore, the AOA must also follow the same distribution law as the phase. Figures 4.31 to 4.34 are examples of histograms for AOA measured during rainfall regimes of 0.17, 1.2, 3.0 and 12.0mm/h. To help in the analysis of the quality of the estimation, normal probability plots are placed alongside the histograms and the estimated density functions.

Figure 4.31: Histogram, estimated pdf and normal probability plot of AOA. Rainfall rate is 0.17mm/h. Sample from 14 Nov. 94.
The great majority of AOA distributions for rainfall rates lower than 3.0mm/h comfortably pass the Kolmogorov-Smirnov test. The distributions, however, possess a certain degree of asymmetry which is not sufficient to affect the results of the goodness-of-fit test. The normal probability plots generally
showed perfect agreement with the Gaussian trend. These characteristics are illustrated in Figures 4.31 and 4.32.

As the rainfall rate increases, the AOA distributions have more difficulty in passing the Kolmogorov-Smirnov test. However, over 70% of the analysed files passed the test in spite of noticeable degrees of asymmetry, as illustrated by Figures 4.33 and 4.34, although the Kolmogorov-Smirnov statistic is relatively high. The normal probability plots show the appearance of deviations from the Gaussian trend at the extremities of the distributions. A similar effect was observed in clear air conditions with increase in strength of the turbulence. The outlier values of AOA do influence the result of the goodness-of-fit test, if the deviations from the Gaussian trend are sufficiently large. It can be seen from the normal probability plots that the central part of the distributions follows the normal trend extremely well. This is typical and happens for all data files, regardless of rainfall intensity. This result indicates that other goodness-of-fit tests, like the normal probability plot, have to be employed in parallel with the Kolmogorov-Smirnov test if spurious outlier effects are to be explicitly identified.

This asymmetry of the histograms was also observed in clear air conditions and is associated with the skewness on the distributions of wind speed caused by the effects of the ejection and sweep wind movements [Oikawa and Meng, 1995]. More details about ejection and sweep wind movements, together with other issues related to the effect of urban environments on the characteristics of turbulence can be found in part three of chapter 1.

The averaged descriptive statistics and the values of the ordinate axis from the examples shown in Figures 4.31 to 4.34, indicate that the range of values of angle-of-arrival is reduced as the rain intensity increases. To give a graphical representation of this phenomenon, examples of the linear regression analysis of percentile-percentile plots comparing percentiles for several rainfall rate values and in dry conditions are presented in tables 4.4 to 4.8. The AOA files in
clear air were taken from measurements on the 22nd of April 1994. The regression line is of the form percentiles of AOA (in rain) = A * percentiles of AOA (clear air) + B. The theoretical description of the percentile-percentile plot technique can be seen in section 4.5.

**Table 4.5:** Regression analysis for the percentile-percentile plot. Comparison between AOA distributions in a rainfall rate of 0.17 mm/h and in dry conditions.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.3739</td>
<td>-18.0715</td>
<td>.99436</td>
</tr>
<tr>
<td>Std.Err.</td>
<td>0.0085</td>
<td>0.9526</td>
<td></td>
</tr>
<tr>
<td>p-level</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.6:** Regression analysis for the percentile-percentile plot. Comparison between AOA distributions in a rainfall rate of 0.55 mm/h and in dry conditions.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.2737</td>
<td>13.0412</td>
<td>.99761</td>
</tr>
<tr>
<td>Std.Err.</td>
<td>0.004</td>
<td>0.4534</td>
<td></td>
</tr>
<tr>
<td>p-level</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.7:** Regression analysis for the percentile-percentile plot. Comparison between AOA distributions in a rainfall rate of 1.2 mm/h and in dry conditions.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.2565</td>
<td>10.8647</td>
<td>.99727</td>
</tr>
<tr>
<td>Std.Err.</td>
<td>0.0041</td>
<td>0.4542</td>
<td></td>
</tr>
<tr>
<td>p-level</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.8:** Regression analysis for the percentile-percentile plot. Comparison between AOA distributions in a rainfall rate of 3.0 mm/h and in dry conditions.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.0156</td>
<td>-2.8939</td>
<td>.99650</td>
</tr>
<tr>
<td>Std.Err.</td>
<td>0.0003</td>
<td>0.0313</td>
<td></td>
</tr>
<tr>
<td>p-level</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
Table 4.9: Regression analysis for the percentile-percentile plot. Comparison between AOA distributions in a rainfall rate of 12.0 mm/h and in dry conditions.

The variation in the multiplicative factor A is evidence of the shrinkage of the AOA ranges with rainfall rate in relation with values in dry conditions. However, for rainfall rates higher than 3.0mm/h the decrease in range stops. The value of B is connected to the mean value of the angle of arrival and corresponds to the relative deviation in the mean value of angle of arrival in rain from the mean value of the angle of arrival in clear air. However, uncertainties about the alignment of the link, and consequently about the mean value of the angle of arrival, make the interpretation of this parameter of little relevance. The important result that comes from this regression analysis is the fact that the percentiles are related through a straight line. The high correlation coefficient and the zero value of the p-level statistic attest the goodness of the fit. The straight line shows that there is no qualitative difference between the distributions of AOA in rain or dry conditions, i.e., the rain does not make the distribution of angle of arrival deviate from the Gaussian distribution that is typical of clear air conditions. The rain does, however, decrease the range of angles by decreasing turbulence production and focusing the light by enhancing forward scattering. Figure 4.35 shows the results from the regression analysis:
Figure 4.35: Comparison of percentile-percentile plots of angle of arrival distributions in clear air and in rain for rainfall rate values of 0.17, 1.2, 3.0 and 12.0 mm/h.

In Figure 4.35, the reduction in the range of angles with increasing rainfall is represented by the decrease in the slope of the straight lines. To this author's knowledge, this is the first time these results are presented in the literature.

4.10-TEMPORAL AUTOCORRELATION AND FREQUENCY POWER SPECTRAL DENSITY FUNCTIONS OF AOA.

The spectral analysis of the AOA variations in rain will follow that of the scintillation. A generalised model of the form $a.c.f(rainrate) = \exp[k_i \times time^k]$ will be fitted to the autocorrelation function estimates. Presented below in Figures 4.36 to 4.39 are typical autocorrelation functions from different rainfall rates which illustrate the overall behavior:
Figure 4.36: Autocorrelation function estimate for the AOA in rain. Rainfall is 0.17 mm/h. Sample from 14 Nov. 94. The correlation coefficient is 99.87%.

Figure 4.37: Autocorrelation function estimate for the AOA in rain. Rainfall is 1.2 mm/h. Sample from 08 Jun. 94. The correlation coefficient is 99.67%.
The overall values of the exponent $k_1$ usually decrease with the rainfall rate, which indicates that the spectral bandwidth decreases as well.

The values of the time exponent $k_2$ occupy a narrow range of values, never lower than 1.2 and never higher than 1.4. This range is narrower than the range obtained for the autocorrelation function of angle of arrival in clear air, as shown in chapter 3. Remembering that the Markovian characteristics of the scintillations or angle of arrival are linked to the occurrence of saturation [Fante, 1975], it can be said that the angle of arrival in rain is not subjected to saturation. This indicates that outer scale size is not dramatically reduced by the rain.

4.11-SPECTRAL ESTIMATION FOR THE AOA.

The spectral estimates of AOA in rain are calculated in the same fashion as the estimates for the scintillation. To test the asymptotic behaviour at both the low
and high frequency parts of the spectrum, the modified normalised spectrum was employed. Since AOA suffers no aperture averaging effects the AOA spectrum is free of the constraints of the scintillation spectrum. Ishimaru (1978) points out that, if the drop size is smaller than the first Fresnel zone size, than the form of the theoretical autocorrelation function for phase and amplitude scintillations is the same and therefore so are the spectra. As seen in chapter 2 and in Lawrence and Strohbehn (1970), the AOA spectrum has the same asymptotic behaviour as the phase spectrum. In Figures 4.39 to 4.41 three examples of typical AOA spectra are shown for rainfall rates of 0.17, 1.2 and 12.0 mm/h.

Figure 4.39: Modified normalised spectrum of AOA in rain. Rainfall is 0.17 mm/h. Sample from 14 Nov. 94.
The spectra generally agree with the theoretical $f^0$ and $f^{8/3}$ asymptotes, characteristic of clear weather. The bandwidths decrease with rainfall and the cut-off frequencies are in the range 5 to 27Hz.
As occurred with the scintillation spectra, there are indications of secondary cut-off frequencies. Using the same approach as for the scintillation, the starting point of 10 Hz was used. In Figures 4.42 to 4.44 three examples of fitted power law spectra are presented.

Figure 4.42: Observed (dots) and fitted spectra of AOA in rain. Rainfall rate is 0.17 mm/h. Sample from 14 Nov. 94.

Figure 4.43: Observed (dots) and fitted spectra of AOA in rain. Rainfall rate is 1.2 mm/h. Sample from 08 Jun. 94.
The secondary cut-off frequency decreases with rainfall rate, which is an indication of the reduction in bandwidth with rainfall predicted for scintillations by Wang and Clifford (1975). The cut-off frequency is typically between 50 and 65 Hz for rainfall rates lower than 1.0 mm/h, between 40 and 55 Hz for rainfall rates from 1.2 to 3.0 mm/h and from 25 to 45 Hz for the rainfall rates above 3.0 mm/h. The power law exponents are typically about -2/3 for rainfall rates lower than 1.0 mm/h, -4/3 for the range between 1.2 and 3.0 mm/h and -5/3 for rainfall rates above 3.0 mm/h.

The overall behaviour of the spectra, regardless of rainfall intensity, for frequencies above the secondary cut-off is to approach the -8/3 law, typical of clear air conditions. Figures 4.45 to 4.47 show the fitted power law spectrum for the region beyond the secondary cut-off for the examples presented in Figures 4.42 to 4.44:
Figure 4.45: Observed (dots) and fitted spectra of AOA in rain for the higher frequency portion of the spectrum. Rainfall rate is 0.17 mm/h. Sample from 14 Nov. 94.

Figure 4.46: Observed (dots) and fitted spectra of AOA in rain for the higher frequency portion of the spectrum. Rainfall rate is 1.2 mm/h. Sample from 08 Jun. 94.
4.12-COMPARING SPECTRA OF AMPLITUDE SCINTILLATIONS AND ANGLE OF ARRIVAL IN RAIN.

The spectral analyses of amplitude scintillations and angle of arrival in rain have shown that the obtained experimental spectra generally possess the expected theoretical behaviour for homogeneous turbulence. The similarities in the spectral densities of amplitude and angle of arrival are expected theoretically [Ishimaru, 1978] since the size of the drops is usually much larger than the wavelength. This results were attributed to the well mixed air that is typical of the highly turbulent urban boundary layer which makes the turbulence effects dominant over the rainfall. The experimental spectra also show features, like bumps and dips that are expected theoretically [Wang and Clifford, 1975]. It was verified, however, that the bumps and dips usually happen at a lower frequency for the amplitude and at a higher frequency for the angle of arrival, as the examples in Figures 4.48 to 4.50 illustrate:
Figure 4.48: Comparison between spectra of amplitude scintillations and angle of arrival in rain for a rainfall rate value of 0.17mm/h.

Figure 4.49: Comparison between spectra of amplitude scintillations and angle of arrival in rain for a rainfall rate value of 1.2mm/h.
The reason for the bumps on the amplitude scintillations spectra being located at lower frequencies than the bumps on the spectra of angle of arrival can only be hypothesised, since there is no developed theoretical framework to explain it. According to the results from Rotach (1993), Oikawa and Meng (1995) and Kanda and Hino (1994), roughness elements such as buildings cause a quicker decrease in the size of the outer scale than it would be otherwise expected in an open environment. The outer scale decreases at this faster rate until it reaches the size of the roughness element [Kanda and Hino, 1994] and from there on the rate of decrease in size slows down. The quicker decrease in the size of the outer scale would cause the first large bump in the scintillations spectra. The presence of a large number of large eddies moving around would cause the bumps at high frequencies on the angle of arrival spectra in the same way as the moving raindrops cause the bumps on the amplitude spectra. This combination of effects of rainfall and turbulence is to be considered typical of the urban boundary layer and consists on a rather original find.
Chapter 5

Statistical and Spectral Analysis of Line-of-Sight Laser Propagation Through Rain

Part 2: Rainfall Induced Attenuation
5.1-INTRODUCTION.

This chapter presents results for attenuation studies on 0.83 and 1.55μm for 12 months of measurements from the autumn of 1993. As seen in chapter 2, the rainfall rate is measured by a tipping bucket raingauge located on the roof of 66-72 Gower street, which is about 100 metres from the receiver site. Therefore, the rainfall measurements have to be understood as point values at the receiver. Because of the highly localised nature of this measurement, a point-to-point correlation in time between attenuation and rainfall is most of the times impossible to obtain with an acceptable level of precision. A statistical approach is therefore chosen, where the distributions of rainfall and attenuation over a period of time will be compared. For this statistical correlation study, the choice of using only the results from the 1.55μm system is based on the larger amount of attenuation data at that wavelength. However, although it is not correct to correlate rainfall and attenuation in a point to point basis, results show a good agreement between simultaneous values.

Due to the lack of operational means, it is also not possible to separate effects of mist and rain from the attenuation records, and results should be understood as being a combination of the effects of these two mechanisms. Comparisons between events of attenuation on both wavelengths are possible for discrete events. Comparison with a similar work by Chu and Hogg (1968) is made.

5.2-CUMULATIVE PROBABILITY DENSITY FUNCTION OF ATTENUATION.

The theoretical approach to the problem of rainfall induced attenuation of the received signal at optical wavelengths was discussed in the early sections of chapter 4. This chapter will only present the results and compare with theoretical expectations and other experimental results.
The cumulative probability distribution function is estimated by dividing the
total time that a certain level of attenuation was exceeded by the total
measurement period. One year of attenuation measurements on the 1.55 μm
system, from the autumn of 1993 is used to obtain the estimates. For seasonal
comparison, the results are grouped in four three month intervals. In
quantitative terms, spring 1994 was the worst of all seasons in rainfall. Table
5.1 shows the measurement times involved in this analysis. Records lost due to
misalignment and other factors are listed as losses in time in the table. The total
recorded time is the difference between the total time and the losses in time and
the percentage of rainfall time is obtained from the recorded time.

Time is Measured in Hours

<table>
<thead>
<tr>
<th>Season</th>
<th>Total Time</th>
<th>Losses (time)</th>
<th>Total Time Logged</th>
<th>Rainfall Time</th>
<th>% Logged Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autumn 93</td>
<td>2184</td>
<td>109</td>
<td>2075</td>
<td>220</td>
<td>10.6</td>
</tr>
<tr>
<td>Winter 93/94</td>
<td>2160</td>
<td>415</td>
<td>1745</td>
<td>182</td>
<td>10.4</td>
</tr>
<tr>
<td>Spring 94</td>
<td>2208</td>
<td>219</td>
<td>1989</td>
<td>242</td>
<td>12.2</td>
</tr>
<tr>
<td>Summer 94</td>
<td>2208</td>
<td>279</td>
<td>1929</td>
<td>215</td>
<td>11.2</td>
</tr>
</tbody>
</table>

| Total      | 8760       | 1022          | 7738              | 859           | 11.1          |

Table 5.1: Rainfall measurements time table

The difference between the seasons is very small. Spring was the worst season
for rainfall over the period covered, followed closely by summer, autumn and
winter.

The attenuation values used in the calculations presented in this and the
following sections represent the net decrease in the received power in relation
to the signal received in clear air conditions. In this analysis, an attenuation
event is implicitly connected with a rainfall event. The corrections for forward
scattering suggested by Chu and Hogg, 1968 were not performed because, for
the system designer, the net attenuation is the important parameter.
Figures 5.1 to 5.4 show the cumulative distribution functions of attenuation from the autumn of 1993 to the summer of 1994, separated into four quarters:

- Figure 5.1: Cumulative distribution function of attenuation: Sep.-Oct.-Nov. 1993 (autumn).
- Figure 5.2: Cumulative distribution function of attenuation: Dec. 93-Jan.-Feb. 94 (winter).

Chapter 5
Comparing the distributions shown in Figures 5.1 to 5.4 it can be seen that the winter of 1993-1994 had the highest percentages of time for the highest attenuation levels. On the other hand, the season which had the highest
concentration of rainfall events (spring 94) was less affected by attenuation. The difference between the worst season for rainfall (spring 1994) and for attenuation (winter 1993-1994) is probably due to the influence of mist, which is a phenomenon much more associated with the cooler seasons. It can also be seen from Figures 5.1 to 5.4 that the for the smaller percentages of time (<1%) have a -2/3 (dB/km)/decade slope for the winter, autumn and summer. Spring has a slightly greater slope of -1 (dB/km)/decade. Attenuation levels have a maximum value of 13 dB/km, which compares with the value of 20 dB/km obtained by Chu and Hogg (1968) on a 0.63μm, 2.6km path in suburban Vancouver, Canada. Chu and Hogg point out that their result is less than would be expected from a millimetre wave system operating under the same conditions, which is a result in favour of the use of this technology.

The distribution of attenuation for the whole period can be seen in Figure 5.5:

![Figure 5.5: Cumulative distribution function of attenuation - autumn 1993 to autumn 1994.](attachment:image.png)
The curves of attenuation distributions presented so far considered only events of attenuation with rainfall present. Although, since it is not possible in this work to separate events of “pure” rainfall from events of rainfall accompanied by mist the attenuation values inevitably include a contribution from mist. However, for design purposes, a curve of attenuation corresponding to the combined effects of all kinds of hydrometeors is presented below in Figure 5.6:

![Graph](image)

**Figure 5.6: Cumulative distribution function of hydrometeor-induced attenuation - autumn 1993 to summer 1994.**

The slope for the lower percentages of time (<1%) follows a -1/2 slope. It will be noted that the contribution of events of mist only induced attenuation is not very large. This is due to the already mentioned fact that the rain is usually accompanied by mist. It is also interesting to note that the mist contributions seem to cause a decline in the slope of the low percentages of time. Figure 5.3 shows the cumulative distribution of attenuation for the spring 94, which has a slope of -1(dB/km)/decade. On many occasions during the spring 94, the
rainfall events witnessed from the laboratory, usually heavy and localised, were not accompanied by mist.

5.3-RAINFALL INDUCED ATTENUATION

5.3.1-INTRODUCTORY REMARKS.

Olsen et al. (1978) did extensive work on the $A = a \cdot R^b$ type relation between rainfall rate $R$ and attenuation $A$, covering a range of frequencies from 1 to 1000GHz. Theoretically [Olsen et al.], the expression $A = a \cdot R^b$ is exact for optical wavelengths. By exact, it is meant that the parameters $a$ and $b$ are independent of frequency and other factors such as rain temperature. Olsen et al. showed that the estimates for $a$ and $b$ converge as the frequency increases towards 1000GHz to the values of 2.0 for $a$ and 0.61 for $b$. These values are obtained considering a Marshall-Palmer distribution for the raindrop sizes. Other distributions [Olsen et al.] yield values for $a$ between 0.9 and 2.9 and for $b$ between 0.61 and 0.77.

Although a good visual correlation between rainfall and attenuation was generally observed in the events recorded, localised correlations between rainfall rate and attenuation for specific events is not always possible, due to the point nature of the rainfall measurements, which make path averaging of the rainfall impossible. To try to screen out effects of inhomogeneity in the rain, a statistical approach seems to be a better choice for treating the problem. Comparing the percentiles of the cumulative distributions of rainfall rate and attenuation and looking for a statistical correlation rather than a point one, is the technique used in this work.

It is necessary to reiterate at this point that it was not possible to separate the effects of mist and rain from the measurements. On many occasions that a rain event was witnessed from the laboratory, mist could be observed.
To illustrate the good visual correlation between rainfall rate and attenuation, Figures 5.7 and 5.8 show examples of events of attenuation registered on the 1.55μm system with corresponding rainfall measurements:

Figure 5.7: Attenuation event on the 1.55μm system with simultaneous measurements of rainfall rate. Date: 12 Jan. 1994.

Figure 5.8: Attenuation event on the 1.55μm system with simultaneous measurements of rainfall rate. Date: 03 Mar. 1994.
Figures 5.7 and 5.8 illustrate two main kinds of attenuation events observed on the 1.55μm system. In Figure 5.7, from January 94, it can be seen that the rain almost coincides with the attenuation. This indicates a rain cell moving from the receiver, where the raingauge is located, towards the transmitter.

Figure 5.8 presents a typical case - which is 5 to 1 more common than the case presented in Figure 5.7 - in which the attenuation precedes the rainfall rate measurement in time. This is a sign of movement of the rain cell from the transmitter towards the receiver.

Factors other than rainfall contributing for the attenuation can be identified from a comparison between the levels of attenuation and corresponding measured rainfall in Figures 5.7 and 5.8. In Figure 5.7, a rainfall rate of 3mm/h corresponds to a value of attenuation of about 2 dB/km. On the other hand, in Figure 5.8, the same 3mm/h corresponds to a value of attenuation of 10dB/km. This difference is almost certainly due to differences in the thickness of mist.

5.3.2-CALCULATING THE PARAMETERS $a$ AND $b$.

In order to accomplish the statistical correlation between attenuation and rainfall rate, it is necessary to obtain the cumulative probability distribution function of rainfall and attenuation. The seasonal distributions of attenuations are presented in Figures 5.1 to 5.4. Figures 5.9 to 5.12 show the corresponding seasonal distributions of rainfall rates:
Figure 5.9: Cumulative probability distribution function of rainfall: Sep.-Oct.-Nov. 93 (autumn).

Figure 5.10: Cumulative probability distribution function of rainfall: Dec. 93-Jan.-Feb. 94 (winter).
Figure 5.11: Cumulative probability distribution function of rainfall: Mar.-Apr.-May. 94 (spring).

Figure 5.12: Cumulative probability distribution function of rainfall: Jun.-Jul.-Aug. 94 (summer).
The distributions shown in Figures 5.9 to 5.12 present the same general form. It can be seen that the great majority of rain events have rainfall rates below 5 mm/h. The highest rainfall observed in this study is 54mm/h.

The distribution of rainfall rate for the whole period can be seen in Figure 5.13:

![Cumulative probability distribution function of rainfall - autumn 1993 to autumn 1994.](image)

From the distributions of rainfall and attenuation, the percentiles can be calculated. It is not possible however to take the percentiles directly from the obtained distributions because the initially chosen bins did not correspond to whole percentile values. Instead, regression techniques were employed and curves fitted to the distributions. For the attenuation curves, the best regression found is a piecewise straight line of the form:

\[
A = \begin{cases} 
  a_1 + b_1 P & A \leq B_p \\
  a_2 + b_2 P & A > B_p 
\end{cases} \quad (5.1)
\]
with $P$ being the desired percentile and $B_p$ is the breakpoint. An exponential curve of the form $R = k_1 \cdot \exp(k_2 \cdot P)$ is found to be the best overall fit for the rainfall distribution. Tables 5.2 and 5.3 present the regression results for the distributions of attenuation and rainfall:

<table>
<thead>
<tr>
<th>SEASON</th>
<th>$b_1$</th>
<th>$a_1$</th>
<th>$b_2$</th>
<th>$a_2$</th>
<th>$B_p$</th>
<th>$r^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUTUMN 93</td>
<td>18.23</td>
<td>-9.06</td>
<td>9.32</td>
<td>-1.51</td>
<td>6.1</td>
<td>0.98</td>
</tr>
<tr>
<td>WINTER 93-94</td>
<td>20.79</td>
<td>-10.16</td>
<td>9.63</td>
<td>-1.57</td>
<td>6.59</td>
<td>0.99</td>
</tr>
<tr>
<td>SPRING 94</td>
<td>12.02</td>
<td>-5.71</td>
<td>9.45</td>
<td>-2.63</td>
<td>5.37</td>
<td>0.97</td>
</tr>
<tr>
<td>SUMMER 94</td>
<td>47.93</td>
<td>-23.87</td>
<td>7.39</td>
<td>-1.2</td>
<td>4.27</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Table 5.2 Regression analysis results for the piecewise-linear fit of the attenuation distributions.

<table>
<thead>
<tr>
<th>SEASON</th>
<th>$K_1$</th>
<th>$K_2$</th>
<th>$r^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUTUMN 93</td>
<td>17.28</td>
<td>-0.09</td>
<td>0.94</td>
</tr>
<tr>
<td>WINTER 93-94</td>
<td>20.57</td>
<td>-0.06</td>
<td>0.78</td>
</tr>
<tr>
<td>SPRING 94</td>
<td>21.67</td>
<td>-0.08</td>
<td>0.80</td>
</tr>
<tr>
<td>SUMMER 94</td>
<td>24.72</td>
<td>-0.07</td>
<td>0.79</td>
</tr>
</tbody>
</table>

Table 5.3 Regression analysis results for the exponential fit of rainfall distributions.

Relatively high correlation coefficients attest the overall goodness of the fits. The percentiles of the distributions can now be calculated and the corresponding values of rainfall and attenuation compared. Results for the seasonal comparisons are presented below in Figures 5.14 to 5.17:
Figure 5.14: Percentile-percentile plot of attenuation versus rainfall rate-autumn 93.

Figure 5.15: Percentile-percentile plot of attenuation versus rainfall rate-winter 93-94.
The $aR^b$ power law is verified for all seasons, which is a direct result from the choice of fits for the distributions. Although the correlation coefficient for the summer is the lowest of all, the fit is accepted at a 5% confidence level. The
percentile-percentile plot for the whole period, using data from Figures 5.5 and 5.13 can be seen in Figure 5.18:

\[
A = 3.749 R^{0.3586} \\
\text{correl. coef.} = 0.9173
\]

Figure 5.18: Percentile-percentile plot of attenuation versus rainfall rate - autumn 1993 to summer 1994.

For the whole period, the obtained values of \(a\) and \(b\) were 3.75 and 0.36. The values obtained above for the parameters \(a\) and \(b\) are different to the theoretical values of 2.0 and 0.61. This discrepancy is to be expected because the results are not comparable. The attenuation measured in this work is not only due to rainfall but includes the very practical case of the presence of mist. Also, the single measurement of rainfall rate is not necessarily indicative of the rainfall rate along the path. Maitra and Gibbins, 1995, comment that departures from the values obtained by Olsen et al., 1978 have been observed in rainfall attenuation for frequencies above 40GHz. These discrepancies are said to be due to the Olsen et al. choice of the Laws and Parsons as the drop size distribution used in calculating the attenuation. The Laws and Parsons
distribution is considered to underestimate the number of small drops, which are the most influential in causing attenuation at optical wavelengths. Maitra and Gibbins also mention the shortcomings of using a point measurement, rather than path average, of the rainfall, which is evidently the case on a complex path such as the one used in this work. It was not practical, however, to place an array of raingauges along the path and therefore the rainfall measurements in this work have to be considered to be point measurements.

5.4-COMPARISON BETWEEN THE PERFORMANCES OF THE 0.83 AND 1.55μm SYSTEMS IN RAIN.

Before presenting the results, it is necessary to reiterate the fact that both systems have different logging systems. The 1.55μm design averages the received power over 2.5 minutes in order to save computational space. The logging system of the 0.83μm gives a point measurement of the received power over 1 second. These procedures will make the 1.55μm received power graphs look smoother than the 0.83μm. In addition, correlations in time between the two signals will be impaired by the averaging. Because of these characteristics, a statistical comparison is applied rather than a point to point correlation, as has been done for the comparison between attenuation and rainfall. In addition, as seen in chapter 2, there is an amount of fluctuation on the 1.55μm received signal around the mean, which could not be explained, but is bound to affect the results somewhat. Three typical examples of the performance of both wavelengths in rain are presented in Figures 5.19 to 5.24. Figures 5.19 and 5.20 show the received power in both links and the corresponding rainfall rate for the 01 Feb. 94. Figures 5.21 and 5.22 show the same information for the 14 Apr. 94.
Figure 5.19: Received power levels on the 0.83 and 1.55 µm systems-01 Feb. 94.

Figure 5.20: Rainfall rate measured on 01 Feb. 94
Figure 5.21: Received power levels on the 0.83 and 1.55μm systems - 14 Apr. 94.

Figure 5.22: Rainfall rate measured on 14 Apr. 94.

It can be seen from Figures 5.19 and 5.21 that the correlation between the two received power curves is noticeable and this is a general trend for all observed...
simultaneous power records for both links. There is a small difference in the
effects of attenuation between the two systems, which has to be attributed to
the already mentioned fast power fluctuations on the 1.55\mu m system. On the
whole, however, it is clear that the two systems are similarly affected by the
rain. This is theoretically expected for the case of scattering by very large
spheres since the scattering cross section is in this case independent of the
wavelength. In order to clearly specify the relationship between the attenuation
suffered by both systems, a statistical correlation based on the percentile-
percentile plot is used.

The number of events collected for the 0.83\mu m system was considerably
smaller than for the 1.55\mu m because of the manual operation of the 0.83\mu m
logging system. Therefore, it was not possible to form complete seasonal
distributions for the attenuation. The majority of events recorded on the 0.83\mu m
happened on the spring of 1994, which was the worst rain season but the best
season for attenuation figures. Therefore, the commulative distribution on the
0.83\mu m was calculated for this season only and compared with the 1.55\mu m
distribution. For the sake of consistency, 1.55\mu m attenuation events that
happened during the night and early morning were not included in the analysis.
It is therefore expected that the relatively small number of events analysed may
cause some degree of bias in the obtained distributions. The two distributions
are presented below in Figure 5.23:
It can be seen from Figure 5.23 that the 0.83μm system appears to be less affected by attenuation than the 1.55μm. Both distributions seem to have the same shape, which indicates that both systems are governed by the same distribution law, which is expected since the attenuation in both systems is not wavelength selective. To allow a better comparison between the distributions, the percentile-percentile plot technique is employed. The percentile-percentile plot comparison procedure is the same used in the rainfall induced attenuation study. Initially, a piecewise straight line was fitted to the distributions and these approximations were used to build the percentile-percentile plot. Figure 5.24 shows the piecewise linear fits to both distributions:
Figure 5.24: Fitted cumulative distributions of attenuation for both systems.

The piecewise regressions have correlation coefficients of 99.0% and 99.1% for the 0.83μm and 1.55μm distributions respectively. There is a increasing discrepancy on the values of the corresponding percentiles for the two systems distributions, with the 1.55μm system distribution presenting a higher attenuation value for the same percentile. This discrepancy varies from about 2 to 4 dB, which is the value of the standard deviation of the fast power variations noticed on the 1.55μm system. There is also the problem of the relatively small population of events recorded for the 0.83μm in relation to the 1.55μm system. Nevertheless, this result suggests that both systems have similar performance in rain.
Chapter 6

6.1-INTRODUCTION.

In this chapter, reliability analysis and level crossing techniques will be used in order to characterize the performance of a 155 Mbits/second digital free space optical communications link operating at 1.55μm. Because of the great instability of the received signal, caused by atmospheric turbulence, it was not possible to install a BER meter at reception and therefore, the designers of the British Telecom Research Laboratories had to implement a Schottki clamp circuit to provide a method of acknowledging clock reception. Without an explicit measure of the BER on the system, the performance evaluation of the experimental digital link requires the use of alternative methods. Firstly, the presence or absence of the clock at the receiver is used to help obtain the reliability assessment of the system. Secondly, the level crossing theory is applied to the received scintillations to try to model the statistics of the BER. High BER is generally associated with burst errors generated by fades in the received signal below the detector sensitivity. Approximately, the amount of time the scintillations cause the received signal to fall below the receiver sensitivity can be considered as the amount of time the BER exceeded its maximum acceptable limit. The mathematical theory of the level crossing techniques was presented in chapter 3.

6.2-RELIABILITY ANALYSIS FUNDAMENTALS.

The next sections present a brief account of the basic mathematical statistics involved with reliability analysis.
6.2.1-INTRODUCTION.

Reliability is defined as the study of the failure of systems. In the context of this work, reliability is a quantitative measure of how long the clock signal is present or absent from the received signal on the 1.55 μm digital link operating at 155Mbits/second. Factors like the distribution of the time durations of the events of the clock being present or absent at the receiver and also the number of outages are among the possible measures of the reliability of this digital system.

6.2.2-REPAIRABLE AND NONREPAIRABLE SYSTEMS.

In reliability analysis, systems are divided into two basic categories. Systems which need to be completely replaced, or have part(s) replaced by a new one(s) when failure occurs and systems which can repaired and put back to work. There are distinct differences in the statistical analysis of both classes, and detailed description of the differences can be found in Crawder et al. (1991) and Asher and Feingold (1984).

Although failures and repairs are concepts akin to a piece of machinery, for which reliability concepts are evidently and intuitively applicable, outages and recoveries of the clock can be considered as characteristic of a repairable system. Therefore, although being a somewhat abstract concept, the digital link will be considered as a repairable system.
6.2.3-BASIC STATISTICAL CONCEPTS IN RELIABILITY.

6.2.3.1-DEFINITION OF A SURVIVAL DISTRIBUTION.

The basic probabilistic concept in reliability theory is the survival distribution function, defined as:

\[ S_X(x) = P\{X > x\} \quad (6.1) \]

This function is related to the cumulative distribution function \( F_X(x) = P\{X \leq x\} \) as:

\[ S_X(x) = 1 - F_X(x) \quad (6.2) \]

Because survival functions are concerned with time durations, \( X \) cannot be negative and so \( S_X(0) = 1 \). In this fashion, the relationship between the density function of \( X \) and its survival distribution is:

\[ f_X(x) = -\frac{dS_X(x)}{dx} \quad (6.3) \]

Inspection of equation (6.3) shows that the density \( f_X \) represents the instantaneous rate of failure. Another important failure rate is the relative instantaneous failure rate or, as known in the literature, the hazard rate function (HRF):

\[ \lambda_X(x) = \frac{f_X(x)}{S_X(x)} = -\frac{d \log S_X(x)}{dx} \quad (6.4) \]

6.2.3.2-CONDITIONAL PROBABILITIES OF FAILURE AND CENTRAL RATE.

The conditional probability of failure in the time interval \((x, x+t)\), given operational at time \(x\) is denoted by \(q(x, x+t)\):
From equation (6.4), it is possible to rewrite (6.5) in the form:

\[ q(x, x + t) = \frac{S_X(x) - S_X(x + t)}{S_X(x)} \]  

(6.6)

The conditional probability of "surviving" the time interval \( x \) to \( x + t \) given operational at time \( x \) is:

\[ p(x, x + t) = 1 - q(x, x + t) = \frac{S_X(x + t)}{S_X(x)} \]  

(6.7)

6.2.3.3-SURVIVAL DISTRIBUTIONS.

Random time intervals are intrinsically positive random variables. Therefore, the probability distribution functions which might represent the behaviour of survival times have to be also strictly positive. Among the standard survival distributions are the exponential, Gompertz and Weibull distributions. The exponential density and survival distribution are:

\[ f_X(x) = \lambda \exp(-\lambda x), \quad \lambda > 0, \quad x > 0 \]  

(6.8)

\[ S_X(x) = \exp(-\lambda x) \]  

(6.9)

The mean and variance can be written as:

\[ m_X = \frac{1}{\lambda}, \quad \var(X) = \frac{1}{\lambda^2} \]  

(6.10)

The conditional distribution of failure within a period \( \delta \) is:
\[ q(x, x+\delta) = 1 - \exp(-\lambda \delta) \] (6.11)

The exponential distribution has a constant hazard function.

The Gompertz survival distribution is written as

\[ S_X(x) = \exp\left(\frac{-R}{a} [1 - \exp(ax)]\right) \quad x > 0, \quad R > 0, \quad a > 0 \] (6.12)

The hazard rate function is:

\[ \lambda_X(x) = R \cdot \exp(ax) \] (6.13)

Finally, the expression for the Weibull distribution:

\[ S_X(x) = \exp\left(\frac{(x-\xi)}{\theta}^c\right) \quad x > \xi, \quad \theta > 0, \quad c > 0 \] (6.14)

The hazard rate is

\[ \lambda_X(x|X > \xi) = \frac{c}{\theta^c} (x - \xi)^{c-1}. \] (6.15)

6.2.4-RATE OF OCCURRENCE OF FAILURES (ROCOF).

An important measure in the reliability of repairable systems is the rate of occurrence of failures, or ROCOF. The ROCOF is defined as the time derivative of the expected number of failures in the interval \([0,t]\). A ROCOF with an upward concavity characterises a sad system, while a downward concave ROCOF characterises a happy system. Happy and sad systems are terms used in reliability analysis and processes such as these are denominated non-homogeneous Poisson process. A ROCOF with a linear behaviour, i.e., no apparent concavity represents a homogeneous Poisson process and a system with constant hazard rate. An estimator of the ROCOF can be written as:
\[
\text{ROCOF}(t) = \frac{\# \text{ failures in } [t, t + \delta t]}{\delta t} \quad (6.16)
\]

It is possible to use a parametric method to model the ROCOF based on the assumption that the occurrence of failures in time form a point process that can be considered to be non-homogeneous Poisson processes. The only difference between non-homogeneous Poisson processes and normal Poisson processes is that non-homogeneous Poisson processes have a time-varying hazard function.

6.3-APPLICATION OF LEVEL CROSSING TECHNIQUES TO THE EVALUATION OF THE STATISTICS OF THE BER.

The theories involved with the level crossing problem were presented in chapter 3. The use of the techniques in the evaluation of the bit error rates is based on the relationship between the received signal and the occurrence of burst errors, so called because they happen for a very short period of time. It is assumed that the bit error rate increases dramatically as the received signal is attenuated towards the detector sensitivity limit. For the 1.55\textmu m, the BER per signal level curve [McCullagh, 1992] is presented below in Figure 6.1:
It can be seen from the graph that the BER increases by a factor of $10^7$ for a fade value of 5 dB below receiver sensitivity. British Telecom technical specs allow a 10dB fading margin. This value will be used as a limit for the allowed scintillation induced fading for which the BER is considered to be above $10^{-9}$, i.e., error-free operation.

With this boundary in mind, the level crossing techniques can be applied to evaluate the percentage of time during a transmission of a given duration, for which the BER exceeded acceptable levels.
6.4-EXPERIMENTAL RESULTS.

6.4.1-INTRODUCTION.

The performance assessment of the digital system begins with the formation of the data bank. Samples were taken from about 40 hours of recorded experimental sessions over the period March 1994-June 1994. Each sample taken from the recorded signal is 5000 points long sampled at 400 Hz, giving a total run time of 12.5 seconds. Three channels are recorded and contain the Schottki diode circuitry output, the amplitude scintillations and the AOA.

6.4.2-CHOOSING THE BEST DISTRIBUTION FOR THE TIME DURATIONS OF CLOCK ON AND CLOCK OUTAGE EVENTS.

The first step in the statistical characterisation of the reliability data is the choice of a survival distribution for the time duration of clock present and clock outage events. To follow the terminology usually applied in reliability the clock present events will be called time to failure (TTF) and the clock outages will be denominated time to repair (TTR). The three distributions presented in section 6.2.3.3 are tested against the TTF and TTR measured from the Schottki diode circuitry output.

Results indicate an overall tendency towards the exponential distribution, regardless of the strength of turbulence. In the experimental sessions, the highest value of $C_n^2$ measured was $15 \times 10^{-14} \text{ m}^{-2/3}$ and the lowest was $0.86 \times 10^{-14} \text{ m}^{-2/3}$. To allow comparison with the strength of turbulence, the presentation of results is divided in three categories, corresponding to the ranges of $C_n^2$ values lower than 2, from 2 to 5 and above $5 \times 10^{-14} \text{ m}^{-2/3}$.

In Figures 6.2 to 6.4 examples of results from experiments having mean $C_n^2$ values of 1.25, 4.0 and 10 ($10^{-14} \text{ m}^{-2/3}$) are presented. The parameter lambda is the parameter of the fitted exponential distribution:
Figure 6.2: Exponential hypothesis for the time to failure distribution. Data from 18 Apr. 94, Cn2 is $1.36 \times 10^{-14}$ m$^{-2/3}$.

Figure 6.3: Exponential hypothesis for the time to failure distribution. Data from 01 Mar. 94, Cn2 is $4.11 \times 10^{-14}$ m$^{-2/3}$.
Figure 6.4: Exponential hypothesis for the time to failure distribution. Data from 14 Jun. 94, Cn2 is $10.00 \times 10^{-14} \text{ m}^{-2/3}$.

The overall values of the mean time to failure (MTTF) from the analysis of 100 data sets from each of the $C_n^2$ categories are:

<table>
<thead>
<tr>
<th>$C_n^2$ CATEGORY ($10^{-14} \text{ m}^{-2/3}$)</th>
<th>MTTF (ms) aver. of 100 samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>lower than 2</td>
<td>224.7</td>
</tr>
<tr>
<td>between 2 and 5</td>
<td>180.6</td>
</tr>
<tr>
<td>between 5 and 15</td>
<td>184.6</td>
</tr>
</tbody>
</table>

Table 6.1: Mean time to failure (MTTF) per $C_n^2$ category. Values are averages of 100 samples.

The results presented in table 6.1 are expected in the sense that the time to failure seems to decrease as the atmosphere becomes more turbulent, which
indicates that the clock takes longer to be lost in lower than in higher turbulence conditions.

To give a practical idea of what could be transmitted at 155Mbits/s within the times shown in table 6.1, a particular illustrative example is taken from Gowar (1993). In table 6.2, below, the approximate number of words coded in ASCII that could be transmitted during the times presented in table 6.1 is shown. Considering that a symbol in ASCII has seven bits and the average number of letters in a word is five, the information content presented in 100000 words coded in ASCII (a 250 page book) is 3.5Mbits.

<table>
<thead>
<tr>
<th>$C_n^2$ CATEGORY ($10^{-14}$m^{-2/3})</th>
<th>Approx. number of words</th>
</tr>
</thead>
<tbody>
<tr>
<td>lower than 2</td>
<td>995000</td>
</tr>
<tr>
<td>between 2 and 5</td>
<td>800000</td>
</tr>
<tr>
<td>between 5 and 15</td>
<td>820000</td>
</tr>
</tbody>
</table>

Table 6.2: Number of words in ASCII that could be transmitted given the time durations presented in table 6.1.

As can be seen from table 6.2, for the best transmission conditions, i.e., the highest MTTF, the amount of words that could be transmitted would compile a 2500 pages book (equivalent to approximately 4 copies of Gowar’s text book).

The exponential distribution is the best fit for the TTF. The other two distributions had much worse performance and on the few occasions (7%) in which the fit to the distributions was accepted (together with the exponential hypothesis), the shapes of the Weibull and Gompertz distributions were clearly exponential.
Results also indicate that the exponential distribution is the best fit for the clock outage times (TTR). In this case, the Weibull and Gompertz distributions were rejected in 97% of the cases. Figure 6.5 to Figure 6.7 show examples of some typical TTR density estimates:

**Figure 6.5:** Exponential hypothesis for the time to recover distribution. Data from 18 Apr. 94, Cn2 is $1.36 \times 10^{-14} \text{ m}^{-2/3}$.

**Figure 6.6:** Exponential hypothesis for the time to recover distribution. Data from 01 Mar. 94, Cn2 is $4.11 \times 10^{-14} \text{ m}^{-2/3}$.
Figure 6.7: Exponential hypothesis for the time to recover distribution. Data from 01 Mar. 94, Cn2 is $10.00 \times 10^{-14}$ m$^{-2/3}$.

Results for the mean time to recover (MTTR) averaged from 100 samples can be seen in table 6.3, below:

<table>
<thead>
<tr>
<th>$C_n^2$ CATEGORY ($10^{-14}$ m$^{-2/3}$)</th>
<th>MTTR (ms) aver. of 100 samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>lower than 2</td>
<td>13.2</td>
</tr>
<tr>
<td>between 2 and 5</td>
<td>22.2</td>
</tr>
<tr>
<td>between 5 and 15</td>
<td>14.6</td>
</tr>
</tbody>
</table>

Table 6.3: Mean time to recover (MTTR) per $C_n^2$ category. Values are averages of 100 samples.

The results from table 6.3, show is an increase in the mean time to recover with the strength of turbulence. However, this increase is not linear with the values.
of $C_n^2$. Clearly, the current results show no specific trend, i.e., the mean time to recover and failure do not appear to be dependent on $C_n^2$ over the range of values measured.

6.4.3-ESTIMATION OF THE RATE OF OCCURRENCE OF FAILURES.

As an effect of the exponential distribution for the TTF, the hazard rate is constant (see section 6.1). This means that the occurrence of failures is random and the system experiences no deterioration or improvement with time. This is expected because of the stationary random nature of atmospheric turbulence. Because the hazard rate is constant, the ROCOF will be a straight line. This behaviour for the ROCOF was observed for 99.3% of the analysed runs, as the examples presented below, for each of the $C_n^2$ categories, illustrate:

![Graph showing cumulative number of failures against time for a run on 11 Apr. 94. $Cn^2$ value is $1.51 \times 10^{-14}$ m$^{-2/3}$.](image)

Figure 6.8: Plot of cumulative number of failures against time for a run on 11 Apr. 94. $Cn^2$ value is $1.51 \times 10^{-14}$ m$^{2/3}$. 
Figure 6.9: Plot of cumulative number of failures against time for a run on 07 Jun. 94. Cn2 value is $4.11 \times 10^{-14} \text{ m}^{-2/3}$.

Figure 6.10: Plot of cumulative number of failures against time for a run on 23 Jun. 94. Cn2 value is $11.46 \times 10^{-14} \text{ m}^{-2/3}$.
Table 6.4 presents the 100 samples average of the values of the ROCOF for the three bands of $C_n^2$:

<table>
<thead>
<tr>
<th>$C_n^2$ CATEGORY ($10^{-14} \text{m}^{-23}$)</th>
<th>ROCOF (failures/s) aver. of 100 samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>lower than 2</td>
<td>5.6</td>
</tr>
<tr>
<td>between 2 and 5</td>
<td>7.66</td>
</tr>
<tr>
<td>between 5 and 15</td>
<td>7.14</td>
</tr>
</tbody>
</table>

Table 6.4: Rate of occurrence of failures per $C_n^2$ category. Values are averages of 100 samples.

The ROCOF appears to have the same uncorrelated behaviour with the strength of turbulence as the mean time to failure and recover. The relationship between the ROCOF and the MTTF can be seen in Figures 6.14-6.16, below:
Figure 6.11: Graph of rate of occurrence of failures vs. mean time to failure for values of $C_n^2 < 2 \times 10^{-14} m^{-2/3}$.

Figure 6.12: Graph of rate of occurrence of failures vs. mean time to failure or the interval $2 < C_n^2 < 5 (10^{-14} m^{-2/3})$. 

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The relationship between the rate of occurrence of failures and the mean time to failure follows a power law. The exponents increase towards unity as the strength of turbulence increases, which indicates that for high turbulence the ROCOF is approximately proportional to the inverse of the MTTF.

6.4.4-FORECASTING RELIABILITY USING MARKOV METHODS.

Another consequence of the exponential distribution for the TTF and TTR is that the reliability can be evaluated by Markov methods. A stochastic system with constant hazard rate is stationary and memoryless, i.e., the future of the stochastic process is decoupled from its past. This is the definition of a Markov process.

The digital system can be modelled as a two state Markov process, also known as a birth and death process. The transition probabilities can be inferred from
the MTTF and the MTTR. A state diagram for the digital system is shown in Figure 6.17:

![State transition diagram for the digital link system.](image)

The transition probabilities can be calculated from the MTTF and MTTR by the solution of the Chapman-Kolmogorov equations [Ramakumar, 1993], [Syski, 1989]. The transition probabilities are a function of time in the form:

\[
P_{on\rightarrow on}(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} \exp[-(\lambda + \mu)t] \quad (6.17)
\]

\[
P_{out\rightarrow on}(t) = \frac{\mu}{\lambda + \mu} - \frac{\mu}{\lambda + \mu} \exp[-(\lambda + \mu)t] \quad (6.18)
\]

\[
P_{on\rightarrow out}(t) = \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} \exp[-(\lambda + \mu)t] \quad (6.19)
\]

\[
P_{out\rightarrow out}(t) = \frac{\lambda}{\lambda + \mu} + \frac{\mu}{\lambda + \mu} \exp[-(\lambda + \mu)t] \quad (6.20)
\]

In the above equations (6.17-6.20) 1/\lambda is the MTTF and is 1/\mu the MTTR.

Using the MTTF and MTTR obtained from the 100 samples for each C_n^2 band, it is possible to build the time-varying state transition probabilities. Below in
The time-varying transition probabilities are presented according to the value of the refractive index structure parameter:

Table 6.5: Time-varying state transition probabilities for the interval $C_n^2 < 2 \times 10^{-14}$ m$^{-2/3}$.

<table>
<thead>
<tr>
<th>$C_n^2$</th>
<th>$p_{out \rightarrow out}(t)$</th>
<th>$p_{out \rightarrow on}(t)$</th>
<th>$p_{on \rightarrow out}(t)$</th>
<th>$p_{on \rightarrow on}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; 2 \times 10^{-14}$</td>
<td>0.151 + 0.849 \exp[-0.126 \times t(\text{ms})]</td>
<td>0.849 - 0.849 \exp[-0.126 \times t(\text{ms})]</td>
<td>0.151 - 0.151 \exp[-0.126 \times t(\text{ms})]</td>
<td>0.849 + 0.151 \exp[-0.126 \times t(\text{ms})]</td>
</tr>
</tbody>
</table>

Table 6.6: Time-varying state transition probabilities for the interval $2 \times 10^{-14} < C_n^2 < 5 \times 10^{-14}$ m$^{-2/3}$.

<table>
<thead>
<tr>
<th>$2 &lt; C_n^2 &lt; 5$</th>
<th>$p_{out \rightarrow out}(t)$</th>
<th>$p_{out \rightarrow on}(t)$</th>
<th>$p_{on \rightarrow out}(t)$</th>
<th>$p_{on \rightarrow on}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \times 10^{-14}$</td>
<td>0.109 + 0.891 \exp[-0.051 \times t(\text{ms})]</td>
<td>0.891 - 0.891 \exp[-0.051 \times t(\text{ms})]</td>
<td>0.109 - 0.109 \exp[-0.051 \times t(\text{ms})]</td>
<td>0.891 + 0.109 \exp[-0.051 \times t(\text{ms})]</td>
</tr>
</tbody>
</table>

Table 6.7: Time-varying state transition probabilities for the interval $5 \times 10^{-14} < C_n^2 < 15 \times 10^{-14}$ m$^{-2/3}$.

<table>
<thead>
<tr>
<th>$5 &lt; C_n^2 &lt; 15$</th>
<th>$p_{out \rightarrow out}(t)$</th>
<th>$p_{out \rightarrow on}(t)$</th>
<th>$p_{on \rightarrow out}(t)$</th>
<th>$p_{on \rightarrow on}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \times 10^{-14}$</td>
<td>0.073 + 0.927 \exp[-0.074 \times t(\text{ms})]</td>
<td>0.927 - 0.927 \exp[-0.074 \times t(\text{ms})]</td>
<td>0.073 - 0.073 \exp[-0.074 \times t(\text{ms})]</td>
<td>0.927 + 0.073 \exp[-0.074 \times t(\text{ms})]</td>
</tr>
</tbody>
</table>

The corresponding graphs of the "out to on" and "on to out" transition probabilities presented in tables 6.4 to 6.6 are shown below in Figures 6.18 to 6.20:
Figure 6.15: Graphs of the time-varying out to on and on to out transition probabilities for the interval $Cn2 < 2 \times 10^{-14} m^{-2/3}$.

Figure 6.16: Graphs of the time-varying out to on and on to out transition probabilities for the interval $2 \times 10^{-14} m^{-2/3} < Cn2 < 5 \times 10^{-14} m^{-2/3}$.
Figure 6.17: Graphs of the time-varying out to on and on to out transition probabilities for the interval $5 \times 10^{-14} \text{m}^{2/3} < Cn^2 < 15 \times 10^{-14} \text{m}^{2/3}$.

These curves can be used to inform the system operator what is the probability that the clock would be recovered within the next $t$ milliseconds, in case the clock is out at time zero, and what is the probability that the clock would be lost within the next $t$ milliseconds, given it being present at time zero.

6.4.5-AVAILABILITY CALCULATIONS.

With the overall averages of the MTTF and MTTR calculated in section 6.4.2, it is possible to calculate the system availability using the following expression [Ramakumar, 1993]:

$$A = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}} \quad (6.21)$$

Using equation (6.21) and the values obtained in section 6.4.2, the overall average availability of the system in the three $C_n^2$ ranges are summarised in table 6.8:

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<table>
<thead>
<tr>
<th>$C_n^2$ CATEGORY ($10^{-14}m^{-2/3}$)</th>
<th>Mean Availability (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>lower than 2</td>
<td>94.5</td>
</tr>
<tr>
<td>between 2 and 5</td>
<td>89.1</td>
</tr>
<tr>
<td>between 5 and 15</td>
<td>92.7</td>
</tr>
</tbody>
</table>

Table 6.8: Availability figures for the digital system as a function of the strength of the turbulence.

The availability values decrease slightly with $C_n^2$, as a consequence of the behaviour of the MTTF, which also decreases somewhat with the strength of turbulence. These figures can be considered promising, given the characteristics of the propagation path and the chosen amplitude modulation scheme. In a shorter path - more likely to be used in practice - and the use of a modulation scheme less affected by the atmosphere would help increase the availability values.

Using the definition of availability as the probability of continually receiving the clock for a certain period of time, the time varying availability can be understood as the on to on transition probability in equation 6.17. The availability calculated from equation 6.21 is called the steady state availability and is understood as the on to on transition probability for a value of time tending to infinity.

6.4.6-CONCLUSIONS FROM THE RELIABILITY ANALYSIS.

As a final summary, the reliability analysis of the digital system showed that the TTF and TTR are exponentially distributed, and therefore the hazard rate is
constant. The MTTF and the MTTR presented an uncorrelated behaviour with \( C_n^2 \). The ROCOF showed an increase with the strength of turbulence. Mean availability values were 94.5%, 89.1% and 92.7% for the three \( C_n^2 \) bands shown in table 6.8. The use of a more robust modulation scheme such as pulse position modulation or subcarrier modulation would improve the performance.

6.4.7-LEVEL CROSSING TECHNIQUES AND BER EVALUATION.

The level crossing problem for the scintillations on the 0.83\( \mu \text{m} \) system was presented in chapter 3. Results were obtained for a variety of \( C_n^2 \) values from about 1 to about \( 120*10^{-14} \, \text{m}^{-2/3} \).

Although the reliability analysis provides information about time duration of clock outages and times to outages, in other words, the availability of the system, the analytical techniques cannot provide information about the statistics of the bit error rate. Although it is reasonable to assume that the periods of clock outages can be directly linked to the value of the BER, it can happen that the BER is acceptable with the clock not present [Costa, E, Silva Mello, L and Dhein, N, 1995] and vice-versa.

In Figure 3.8 in chapter 3, it can be seen that the 10 dB attenuation level is located on the part of the curve with a overall 10dB/decade slope, regardless of the strength of the turbulence. To clarify this view, two examples from the obtained level crossing cumulative distribution are presented below in Figures 6.21 and 6.22 (reproduced from chapter 3):
Figure 6.18: Level crossing cumulative probability distribution for a sample from 10 May 94. Cn2 value is $3.10 \times 10^{-14} \text{m}^{-2/3}$.

Figure 6.19: Level crossing cumulative probability distribution for a sample from 22 Apr. 94. Cn2 value is $66.20 \times 10^{-14} \text{m}^{-2/3}$. 

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Analysing the overall results it can be observed that the 10dB attenuation (-10dB value of the normalised irradiance) was exceeded by 5 to 8% of the time of the runs (approximately 1.5 seconds) regardless of the strength of the turbulence. This uniformity is due to the experimentally observed fact that the number of events and the mean time duration of events that a certain level was crossed have the same overall form, regardless of the strength of the turbulence. Examples of the similarities between the number of events and mean time duration against the received normalised irradiance in dB are shown in Figures 6.23 to Figure 6.26. More details about the samples presented in the figures can be found in table 3.1. It was generally observed that there is virtually no difference between the number of times the level of -10 dB was crossed regardless of the $C_n^2$ value. From the examples presented in Figures 6.23 and 6.24 it can be seen that the value is about 70 to 75 times. The mean duration curves have exponential formats, with coefficients that are approximately constant with the strength of turbulence. The multiplicative coefficient was found to be approximately 0.02 and the coefficient of the argument of the exponential was found to be approximately 0.1. Using these values the mean time duration of the events the received normalised irradiance level was below -10dB is approximately 550ms.
Figure 6.20: Plot of number of events a certain level of the received normalised irradiance was crossed. Sample from the 10 May 94, $C_n^2$ value is $3.10 \times 10^{-14}$ m$^{2/3}$.

Figure 6.21: Plot of number of events a certain level of the received normalised irradiance was crossed. Sample from the 22 Apr. 94, $C_n^2$ value is $66.20 \times 10^{-14}$ m$^{2/3}$. 
Figure 6.22: Mean time duration a certain level of the received irradiance was exceeded. Sample from 10 May 94, Cn2 is 3.10*10^-14 m^-2/3.

\[ mn \text{ time}(s) = 0.018*\exp[0.104NI \text{ (dB)}] \]

10 May 94-sample A

\[ Cn2=3.10*10^{-14} m^{-2/3} \]

Figure 6.23: Mean time duration a certain level of the received irradiance was exceeded. Sample from 22 Apr. 94, Cn2 is 66.20*10^-14 m^-2/3.

\[ mn \text{ time}(s) = 0.022*\exp[0.080NI \text{ (dB)}] \]

22 Apr 94-sample E2

\[ Cn2=66.20*10^{-14} m^{-2/3} \]
6.4.8-CONCLUSIONS FROM THE BER ASSESSMENT.

The percentage of time that the normalised irradiance exceeded the -10dB limit of the detector sensitivity is found to be independent of the strength of turbulence, with values concentrated within the range 5 to 8%. This is due to the experimentally found fact that the number of level crosses and the mean time duration of an event of the normalised irradiance being below a certain level have forms that are independent of the strength of turbulence. The overall averaged number of crosses of the -10dB level is found to be 70 to 75 and the mean time duration, 7.35 ms. The product of the mean time duration with the number of times the level of -10dB was crossed gives a total time of approximately 552ms.
Chapter 7

Final Conclusions and Suggestions for Further Developments
7.1-INTRODUCTION TO THE SUMMARY OF THE MAIN CONCLUSIONS OF THIS THESIS.

This thesis has studied the statistical and spectral characteristics of laser propagation at the wavelengths of 0.83\(\mu\)m and 1.55\(\mu\)m along a 4.1km path extending across central London. This is a densely urbanised terrain, where the turbulence is likely to be inhomogeneous and where the classic theories of light propagation in the atmosphere, based on conditions of homogeneous, isotropic and weak turbulence are not intuitively applicable. Studies carried out on such a long and complex path are essential if a free space optical communications system is ever to use such a path. The conditions in which the system used in this work is operating can be considered as worst case scenario since it is probable that practical systems would use much shorter paths.

7.2-SUMMARY OF THE CONCLUSIONS FROM CHAPTER 3.

7.2.1-INTENSITY SCINTILLATIONS

Intensity scintillations were found to follow a gamma distribution law for a range of values of the refractive index structure parameter, \(C_n^2\), varying from about \(1*10^{-14}\) m\(^{-2/3}\) to \(120*10^{-14}\) m\(^{-2/3}\). On occasions, some distributions fitted a Rayleigh distribution for values of \(C_n^2\) above \(20*10^{-14}\) m\(^{-2/3}\). The lognormal distribution, assumed by many works, and widely accepted as the theoretical hypothesis for the distribution of amplitude scintillations, was not verified in any of the analysed data sets, which indicates that the single scattering assumption cannot be applied on this path.

New results (to the author's knowledge) for the level crossing statistics of amplitude scintillations showed agreement with theoretical assumptions developed for microwave propagation. The mean time duration of events during which a level of the normalised irradiance was crossed showed an exponential
behaviour which was found to be insensitive to the strength of turbulence. The graphs of the number of events a certain level of normalised irradiance was crossed showed a uniform behaviour regardless of the value of the structure parameter. The level crossing probability distribution function displayed two distinct slopes for the deep and low fade regions. The slope for the low fades presented an approximate 10dB/decade behaviour which was insensitive to turbulence strength. The slope for the deep fade region showed a 7.5dB/decade for low values of $C_n^2$. This slope approached a constant value of 5dB/decade as the strength of turbulence increased. A constant slope would indicate very weak turbulence and a Gaussian distribution. This was never observed on this path.

Spectral analysis of the amplitude scintillations indicated that the amplitude scintillations cannot be approximated as a wide sense Markov process. The shape of the autocorrelation function presented a slight bump close to zero lag, which is an indication of high correlation between the first lags. This “memory” is not a characteristic of a wide sense Markov process. According to Fante (1975), this is a sign that the scintillations do not saturate, which is to be expected because of the large diameter of the receiver aperture. The spectral densities were in conformity with the theoretical low and high frequency asymptotes of 0 and -8/3, with observed cut-off frequencies in the range 10 to 35Hz, depending on the wind speed. Bumps and dips were observed on the spectral curves, which are attributed to similar observations on temperature and wind speed spectra over the urban boundary layer (e.g. Oikawa and Meng, 1995, and Kalogiros and Helmis, 1995). These bumps and dips are thought to be due to localised Reynolds stresses, i.e., to the combined influence of vertical and horizontal burst wind motions and roughness elements of the city environment such as buildings.
7.2.2-ANGLE OF ARRIVAL.

Angle of arrival distributions in clear air were found to be Gaussian, regardless of the strength of turbulence, which is in agreement with theoretical expectations (Churnside, 1992). There were slight deviations from the Gaussian trend at the lower and upper tails of the distributions for high values of the turbulence strength. This is thought to be due to enhancement of scattered contributions from off-axis eddies, which produce spurious extreme angles. Studies involving the statistical analysis of angle of arrival distributions, including extreme value analysis is another original contribution. Ranges of angles of scatter increased with the strength of turbulence. For values of $C_n^2$ below $2 \times 10^{-14} \, \text{m}^{-2/3}$ the averaged range (over the entire data sets belonging to this range of $C_n^2$) was approximately $\pm 20 \mu\text{rad}$. The range increased to approximately $\pm 300 \mu\text{rad}$ for values of $C_n^2$ above $70 \times 10^{-14} \, \text{m}^{-2/3}$. From the average variance of angle of arrival, the average value of $C_n^2$ was found to be $54.20 \times 10^{-14} \, \text{m}^{-2/3}$. For the range of values of the angle of arrival it was observed that the spot never escaped the detector. Angle of arrival variations, therefore, caused no fading.

Variation of $C_n^2$, obtained from angle of arrival measurements, with time of day showed agreement with results from several researchers, with a peak at midday and a decrease in value towards the evening. Comparison with the simultaneously measured amplitude variance demonstrated that the system is saturation-free, which is due to the large receiver aperture. The small variation (typically lower than 40% between maximum and minimum values) of $C_n^2$ during the day confirms experimental evidence of the short variation of the strength of turbulence on the urban boundary layer (Oikawa and Meng, 1995).

Results from the autocorrelation estimates of the AOA indicated that the angle of arrival time series cannot be considered as a wide sense Markov process.
This is an expected result given the fact that angle of arrival is not likely to experience saturation because of the typical dimensions of the outer scale (tens to hundreds of metres) of turbulence on the experimental path. The spectral estimates displayed the theoretically expected (for homogeneous turbulence) low and high frequency asymptotes of $f^0$ and $f^{-8/3}$. The obtained spectra also contained bumps and dips which are connected to similar behaviour of temperature and wind speed spectra measured in the urban boundary layer.

Cut-off frequencies measured were concentrated in the range 8 to 25Hz, which corresponds to values of path averaged wind speed estimated to be 0.69 to 2.16m/s. Using the higher value found for the cut-off frequency as the worst case, and using equation 3.8, the response time of a beam tracking device designed to operate on the experimental path used in this work should be about 0.04 seconds.

7.3-SUMMARY OF THE CONCLUSIONS FROM CHAPTER 4.

7.3.1-INTENSITY SCINTILLATIONS.

New statistical analysis of amplitude scintillations in rain was performed for a range of rainfall rate values from 0.1 to 12.0 mm/h. The gamma distribution was found to be the best overall fit to the data sets. The lognormal distribution was observed on a few occasions, which would be explained by (a), enhanced forward scattering according to the atmospheric model of Phillips and Andrews (1982), and (b), the reduction in turbulence production caused by cooler conditions in rain and the presence of suspended particles in the atmosphere, according to similar conclusions from Lykossov and Wamser (1995).

Comparisons between amplitude scintillations in rain and dry conditions were performed using percentile-percentile plots. Results indicated that the percentiles were related by a straight line of the $(ax + b)$ type.
multiplicative factor $a$ was independent of the rainfall rate, which indicated that turbulence effects are dominant over rainfall-induced effects on the determination of the shape of the amplitude scintillations density function in rain. The factor $b$ was found to represent the attenuation of the received power measured during the run.

The estimation of the autocorrelation function of the amplitude scintillations in rain lead to the conclusion that the stochastic process cannot be considered to be wide sense Markov. The power-law coefficients obtained were about 10 to 20% higher than the values obtained in clear air. This is a consequence of the calming effect of rainfall on turbulence production.

The spectral power density function estimates presented bumps that are theoretically expected from movements of the fringes of interference between the incident wave and the spherical scattered waves from the raindrops. This effect has never been reported before. The spectral bandwidth of scintillations in rain should be theoretically about 3kHz, varying according to the distribution of the raindrop sizes (Ishimaru, 1978). This was not observed experimentally due to aperture averaging effects. Signs of bandwidth increase in rain are present in the form of an apparent secondary cut-off frequency verified in most spectra. This secondary cut-off frequency was identified in millimetre waves by Hill et al. (1989). The asymptotic behaviour of the spectral estimates for low and high frequencies showed agreement with theoretical assumptions for homogeneous turbulence. This is an indication that, for a large diameter receiver aperture, turbulence effects are dominant over rainfall-induced effects.

There is additional evidence of turbulence effects in the form of bumps at low frequencies, which indicate a faster rate of destruction of outer scale sizes than predicted by the spectral models of Tatarski (1971).

Primary cut-off frequencies (due to wind speed) were in the range 5 to 25Hz, while secondary cut-off frequencies (thought to be caused by the rain) were in
the range 30 to 55Hz. No correlation between rainfall rate and these values of
the cut-off frequency was found.

7.3.2-ANGLE OF ARRIVAL.

The descriptive statistics of angle of arrival in rain showed a marked decrease
in range for an increase in rainfall. Angular ranges varied from approximately \( \pm 80\mu \mathrm{radians} \) for a rainfall rate of \( 0.17\mathrm{mm/h} \) decreasing with increasing rainfall
rate to \( \pm 4.5\mu \mathrm{radians} \) at \( 12.0\mathrm{mm/h} \). Estimates for the probability distribution
showed the Gaussian distribution as the best fit. The distributions exhibited
increased degrees of asymmetry as the rainfall increased, which could be
ascertained from the deviations from the Gaussian trend on the normal
probability plots. Comparison between distributions of AOA in rain and dry
conditions demonstrated the shrinkage in the range of angle of arrival with
rainfall rate. This effect is attributed to enhanced focusing of the beam by
forward scattering and the decrease in turbulence caused by the rain.

Autocorrelation function estimates indicated that the angle of arrival in rain
cannot be considered as a wide sense Markov process.

Spectral analysis showed that the low frequency asymptotes were in conformity
with theory, the high frequency asymptotes were approximately \(-8/3\), which is
an indication of the dominance of turbulence-induced effects. Bandwidth
spread was observed, with evidence of two cut-off frequencies. Primary cut-off
frequencies were in the range 5 to 27Hz and secondary cut-off frequencies
were in the range 50 to 65Hz for rainfall rates lower than \( 1.0\mathrm{mm/h} \), 40 to 65 for
the 1.2 to 3mm/h range and 25 to 45Hz for rainfall values above 3mm/h. This
result is an indication of bandwidth reduction with rainfall that was
theoretically predicted by Wang and Clifford (1975). Bumps and dips on the
spectral curves were observed at higher frequencies than for the amplitude
scintillations spectra, which is an indication of a faster rate of destruction of the
outer scale sizes, probably by the interaction of outer scale size eddies with

roughness elements, like buildings for instance, present in the roughness sublayer of the urban boundary layer. This effect was predicted by Kana and Hino (1995).

7.4-SUMMARY OF THE CONCLUSIONS FROM CHAPTER 5.

The practical impossibility of separating effects of mist occurring simultaneously with rainfall made the results for the cumulative distribution function of rainfall-induced attenuation represent a combination of effects of mist and rain. The cumulative probability distribution function of rainfall (with mist) attenuation was obtained for one year of received power measurements from the autumn of 1993. The winter of 1993-4 was the worst season for attenuation, over the whole period, while the spring of 1994 was found to be the best. The lower percentages of time part of the distribution presented a -1dB/km*decade for the spring of 1994 and a general -2/3dB/km*decade for the other seasons as well as for the curve for the whole 12 months period. Recorded attenuation events due to mist without the presence of rainfall were included in the whole rainfall (with mist) attenuation curve and the lower percentages of time slope was found to be -1/2(dB/km)/decade. The introduction of more mist seemed to cause the slope of the lower percentages of time to decrease slightly.

Although good correlation could be seen in the graphs of attenuation and rainfall rate, differences in time correlation made point to point analysis impractical. A statistical correlation study was carried out. The rainfall rate distributions were calculated, curves were fitted to the obtained distributions of rainfall rate (with mist) attenuation. The percentiles obtained from distributions fitted to the empirical distributions were compared. A power law relationship of the kind $\text{Attenuation}=a*\text{rainfall rate}^b$, was calculated for each season as well as for the whole 12 months period.
The values obtained for the parameters \(a\) and \(b\) were lower than theoretically expected from the results from Olsen et al. (1978). This discrepancy is to be expected because the results are not comparable. The attenuation measured in this work is not only due to rainfall but includes the very practical case of the presence of mist. Also, the single measurement of rainfall rate is not necessarily indicative of the rainfall rate along the path. Maitra and Gibbins (1995), comment that departures from the values obtained by Olsen et al. (1978) have been observed in rainfall attenuation for frequencies above 40GHz. These discrepancies are said to be due to the Olsen et al. choice of the Laws and Parsons drop size distribution used in calculating the attenuation. The Laws and Parsons distribution is considered to underestimate the number of small drops, which are the most influential in causing attenuation at optical wavelengths. Maitra and Gibbins also mention the shortcomings of using a point measurement, rather than path average, of the rainfall, which is evidently the case on a complex path such as the one used in this work. However, it was not practical to place an array of raingauges along the path.

Statistical comparisons between the performances of the 0.83\(\mu\)m and the 1.55\(\mu\)m systems showed that, although both systems appear to have the same type of distribution law for the attenuation levels, the 0.83\(\mu\)m system was generally less affected by rainfall induced attenuation than the 1.55\(\mu\)m system. Percentile-percentile plots of fitted distributions showed that the 0.83\(\mu\)m system suffers 2 to 4 dB less attenuation than the 1.55\(\mu\)m system. This is probably due to observed fluctuations on the received 1.55\(\mu\)m power that are superimposed to the attenuation values. The cause of these fluctuations could not be found.
The new (to this author’s knowledge) reliability analysis for the prototype 155Mbits/s digital link operating at 1.55\,\mu m showed that the distribution of times to clock outages (TTF) and times to clock recovery (TTR) are exponential, which gives the process Markovian characteristics. Results were grouped into three categories, according to the values of $C_n^2$ observed during the measurement sessions (lower than $2\times10^{-14}\,\text{m}^{-2/3}$, between $2\times10^{-14}\,\text{m}^{-2/3}$ and $5\times10^{-14}\,\text{m}^{-2/3}$ and between $5\times10^{-14}\,\text{m}^{-2/3}$ and $15\times10^{-14}\,\text{m}^{-2/3}$). The averaged TTF values for each category, over the ensemble of data sets analysed, were respectively 224.7, 180.6 and 184.6\,ms. These results were expected in the sense that the time to failure seemed to decrease as the atmosphere became more turbulent, which indicates that the clock takes longer to be lost in lower than in higher turbulence conditions. This decrease did not appear to be correlated to the refractive index structure parameter.

The averaged values of the time to failure (TTR) were respectively 13.2, 22.2 and 14.6\,ms. The results, showed an increase in the mean time to recover with the strength of turbulence. However, this increase is not linear with the values of $C_n^2$. Clearly, the current results show no specific trend, i.e., the mean time to recover and failure do not appear to be dependent on $C_n^2$ over the range of values measured.

The rate of occurrence of failures (ROCOF) curves were obtained and presented an overall straight line behaviour, which was expected from the exponential distribution of the time to failure (TTF) values and the consequent constant hazard rate associated with the exponential distribution. The ROCOF showed an increase with the strength of turbulence. Averaged values of the ROCOF over the analysed data sets for the three $C_n^2$ bands were respectively 5.6, 7.7 and 7.1 failures per second. The ROCOF was found to be
approximately proportional to the inverse of the mean time to failure (MTTF) for values of $C_n^2$ higher than $5 \times 10^{-14} m^{2/3}$.

Markov state transition diagram models were obtained from the average values of the MTTF and mean time to recover (MTTR). These models can be used to forecast the behaviour of the clock availability in time.

Availability calculations based on the mean TTF and mean TTR (MTTF and MTTR) showed that for the three categories of $C_n^2$ the respective availability values were 94.5%, 89.1% and 92.7%.

Application of level crossing techniques to the evaluation of the bit error rate (BER) was based on the direct relationship between the BER and the received power. Assuming a fading margin of 10dB, and an approximately instantaneous jump in the BER value from error free to total loss, the level crossing probability distribution function of the normalised irradiance can provide the value of the probability of the normalised received power being below -10dB. This is the value of the probability that the BER would become unacceptable. Values obtained showed that the probability that the BER would be unacceptable was approximately in the range 5 to 8 percent.

7.6-SUGGESTIONS FOR FURTHER DEVELOPMENTS IN THE LINE OF THIS WORK.

Although two wavelengths were used in this work, the differences in design between the two systems were too large for comparative studies to be made. It is suggested that a multi-wavelength experiment using the same optics and circuitry in both systems would provide information on wavelength-diversity schemes for optical communication systems that can be used in design of practical systems, for instance a longer wavelength to be used in clear air and a
shorter one to be used in rain. A system such as this was proposed by Chu and Hogg (1968).

Parallel beams at the same wavelength with variable separation would allow studies of coherence length statistics on a very complex path.

In this parallel beam arrangement, varying the aperture of one link while keeping the other aperture fixed would, theoretically, allow the estimation of the inner scale of turbulence from angle of arrival measurements, according to the work of Ziad et al. (1994).

Still using the two parallel beam arrangement, if one of the links is placed above the other, it is possible to screen out the effects of turbulence and study in more depth the effects of rain in the amplitude scintillations (Wang and Clifford, 1975). Using two parallel beams vertically positioned scintillations (Wang and Clifford), it is theoretically possible to perform remote sensing of path averaged rainfall parameters such as mean drop size and terminal raindrop velocity. A shorter link would allow the use of small apertures, in order to avoid aperture averaging interference, and therefore a more detailed spectral analysis could be performed.

Experimenting with different modulation schemes would allow an investigation of which should be the best modulation scheme to be used with the free space digital link. A shorter path within a city is suggested as a better testing ground for the digital system.
Appendix A

Testing the Stationarity of the Data
A.1-THE KRUSKAL-WALLIS ANALYSIS OF VARIANCE TEST.

It is a necessary condition for the calculation of the spectral power density function that the stochastic process involved is stationary. There are numerous techniques to ascertain the stationarity of a process (see for instance Kokoska and Nevison (1989)). The technique chosen here is the Kruskal-Wallis analysis of variance test. The test is described as follows:

Assumption: Let there be \( k > 2 \) independent random samples from continuous distributions, \( n_i, \ i = 1, 2, \ldots, k \), be the number of observations in each sample, and
\[
n = n_1 + n_2 + \ldots + n_k.
\]

Hypothesis Test:

Null Hypothesis: the \( k \) samples are from identical populations.

Alternative Hypothesis: at least two of the populations differ in the mean.

Test Statistic:
\[
H = \frac{12}{n(n+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(n+1),
\]
where \( R_i \) is the total of ranks in the \( i \)-th sample.

Rejection region: \( H \geq \chi_{\alpha, k-1}^2 \). For a 5% confidence level and 5000 points, \( \chi_{0.05, 3}^2 = 77.93 \)

In order to apply the test, the data files were divided into four segments, i.e., \( k = 4 \) and then STATISTICA statistical software was used to calculate the test statistic. The program would give a table output such as presented in tables A.1 and A.2. The data files used contain 5000 amplitude scintillations points. The files were recorded on 01 Jun. 1994, during strong turbulence conditions, with \( C_n^2 \) values respectively 98.22 and 121.37*10^{-14} m^{-2/3}. Table A.1 shows an example of a data file which failed the test and table A.2 shows an example of
a data file that passed the test. Incidentally, the file used to obtain table A.2 is used as an example in several analyses carried out in chapter 3.

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<tr>
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<td>1250</td>
</tr>
<tr>
<td>Group 2</td>
<td>1</td>
<td>1250</td>
</tr>
<tr>
<td>Group 3</td>
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</tr>
<tr>
<td>Group 4</td>
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Table A.1: Typical FAILED result from Kruskal-Wallis ANOVA by ranks test.

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</tbody>
</table>

Table A.2 Typical PASSED result from Kruskal-Wallis ANOVA by ranks test.

As can be inferred from table A.1, the value of the H statistic is 104.75, which is higher than the rejection limit. Therefore, the null hypothesis is rejected and this data is not stationary at 5% confidence.

On the other hand, from table A.2., it can be seen that the H statistic is 6.45, which is lower than the rejection limit. Therefore, the null hypothesis is accepted at 5% and the data can be considered stationary at that confidence level.
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References .266.


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