A Study of Active Mode-Locking of External Cavity Semiconductor Lasers

by

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Abstract

This thesis describes an experimental and theoretical investigation of active mode-locking of semiconductor lasers. The lasers used in this project belong to the class of long wavelength, buried heterostructure devices.

Integrated optical modulators used in this project were designed and fabricated at UCL. The waveguides were formed by titanium indiffusion into a lithium niobate substrate. The travelling-wave electrodes were devised as coplanar waveguides and asymmetric coplanar lines.

Short optical pulse generation using composite cavity InGaAsP - LiNbO₃ lasers was demonstrated. Three different configurations were tested and mode-locking by laser current modulation, AM and FM mode-locking were demonstrated. These structures were compared to bulk external cavities incorporating a plane mirror or a grating. The best performance was achieved from the external grating laser followed by a semiconductor optical amplifier. Optical pulses shorter than 10ps and having a peak power of 5mW in the fibre were generated.

The theoretical work was based on the frequency-domain formulation of mode-locking of monolithic external cavity semiconductor lasers. The equations which describe mode-locking by current modulation, intracavity phase and amplitude modulation were derived.

Numerical simulations were performed for gain-switching of a solitary laser and the equivalent monolithic external cavity laser. The frequency domain formulation of active mode-locking was tested numerically utilising twenty five modes and a parabolic loss profile. Mode-locking by current modulation was systematically investigated and AM, FM mode-locking and FM laser operation have been demonstrated.
Zivki
Momcilu
i
Tanji
Acknowledgements

I wish to express my sincere gratitude to Professor M G F Wilson for providing me with this project and guiding me towards its completion. I would also like to thank Professors D E N Davies, G Parry and Mr J W Arterton who supported my case and allowed me to stay at UCL during times of uncertainty.

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### Chapter 9
List of Symbols

Semiconductor Laser

electric field vector $\vec{E}$
conductivity of the medium $\sigma$
vacuum permittivity $\varepsilon_0$
speed of light in vacuum $c$
induced electric polarisation $\vec{P}$
complex envelope of the field $\tilde{E}$
complex envelope of the polarisation $\tilde{P}$
optical angular frequency $\omega$
optical frequency $v$
vacuum wavelength $\lambda$
vacuum wave number $k_0$
susceptibility of the medium $\chi$
susceptibility in absence of pumping $\chi_0$
susceptibility due to external pumping $\chi_p$
complex dielectric constant $\varepsilon$
complex propagation constant $\beta$
complex index of refraction $\tilde{\mu}$
refractive index of the medium $\mu$
background refractive index $\mu_b$
refractive index contribution due to pumping $\Delta\mu_p$
gen net gain coefficient $g$
gen net absorption coefficient $\alpha$
confinement factor $\Gamma$
mirror reflectivity $R$
mirror loss $\alpha_m$
internal loss $\alpha_{\text{int}}$
length of the laser $L$
cavity resonance frequency $\nu_i$
longitudinal mode spacing $\Omega$
group refractive index $\mu_g$
<table>
<thead>
<tr>
<th>Term</th>
<th>Symbol</th>
</tr>
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<tbody>
<tr>
<td>gain coefficient</td>
<td>$a$</td>
</tr>
<tr>
<td>carrier density</td>
<td>$n$</td>
</tr>
<tr>
<td>carrier density for transparency</td>
<td>$n_0$</td>
</tr>
<tr>
<td>constant ($\Delta \mu_p$)</td>
<td>$b$</td>
</tr>
<tr>
<td>linewidth enhancement factor</td>
<td>$\beta_c$</td>
</tr>
<tr>
<td>carrier diffusion</td>
<td>$D$</td>
</tr>
<tr>
<td>current density</td>
<td>$J$</td>
</tr>
<tr>
<td>charge of electron</td>
<td>$q_e$</td>
</tr>
<tr>
<td>laser active area volume</td>
<td>$V$</td>
</tr>
<tr>
<td>thickness of the active area</td>
<td>$d$</td>
</tr>
<tr>
<td>width of the active area</td>
<td>$w$</td>
</tr>
<tr>
<td>total recombination rate</td>
<td>$R(n)$</td>
</tr>
<tr>
<td>non-radiative recombination factor</td>
<td>$A_{nr}$</td>
</tr>
<tr>
<td>radiative recombination factor</td>
<td>$B$</td>
</tr>
<tr>
<td>Auger recombination factor</td>
<td>$C$</td>
</tr>
<tr>
<td>photon density</td>
<td>$N_{ph}$</td>
</tr>
<tr>
<td>stimulated recombination rate</td>
<td>$R_{st}$</td>
</tr>
<tr>
<td>complex envelope of the field</td>
<td>$E$</td>
</tr>
<tr>
<td>longitudinal cavity resonance</td>
<td>$\Omega_0$</td>
</tr>
<tr>
<td>gain constant</td>
<td>$G$</td>
</tr>
<tr>
<td>loss constant</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>photon lifetime</td>
<td>$\tau_p$</td>
</tr>
<tr>
<td>envelope of the field (real)</td>
<td>$A$</td>
</tr>
<tr>
<td>phase of the field</td>
<td>$\phi$</td>
</tr>
<tr>
<td>photon number</td>
<td>$P$</td>
</tr>
<tr>
<td>average rate of spontaneous emission</td>
<td>$R_{sp}$</td>
</tr>
<tr>
<td>output power</td>
<td>$P_{out}$</td>
</tr>
<tr>
<td>carrier number</td>
<td>$N$</td>
</tr>
<tr>
<td>current</td>
<td>$I$</td>
</tr>
<tr>
<td>carrier recombination rate</td>
<td>$\gamma_e$</td>
</tr>
<tr>
<td>spontaneous carrier lifetime</td>
<td>$\tau_e$</td>
</tr>
<tr>
<td>spontaneous emission factor</td>
<td>$\beta_{sp}$</td>
</tr>
</tbody>
</table>
### External Cavity Laser

- Linewidth of a single-mode laser: $\Delta v_{\text{sol}}$
- Linewidth of an external cavity laser: $\Delta v_{\text{ext}}$
- Length of a solitary laser: $L_L$
- Length of the cavity: $L$
- Single-pass optical coupling efficiency: $\eta$
- Threshold current: $I_{\text{th}}$
- Reflectivity of AR coating: $R_{\text{ar}}$
- External reflectivity: $R_{\text{ext}}$
- Modulating frequency: $f$
- Bias current: $I_b$
- RF power: $P_{\text{rf}}$

### Mode-Locking

- Complex amplitude of the mode: $E_i$
- Mode amplitude (real): $A_i$
- Photon number: $P_i$
- Phase of the mode: $\phi_i$
- Lasing angular frequency: $\omega_0$
- Modulating angular frequency: $\omega_M$
- Longitudinal mode spacing: $\Omega$
- Longitudinal cavity resonances: $\Omega_i$
- Central Fabry-Perot frequency: $\Omega_0$
- Lateral mode distribution: $\xi$
- Transverse mode distribution: $\zeta$
- Spatially averaged dielectric constant: $(\varepsilon)$
- Mode index: $\mu$
- Mode absorption index: $\alpha$
- DC component of the carrier number: $N_b$
- Fourier component of the carrier number: $N_q$
- Coupling loss coefficient: $\alpha_c$
- Gain constant: $B_o$
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$L$</td>
<td>length of the cavity</td>
</tr>
<tr>
<td>$L_L$</td>
<td>length of the lasing area</td>
</tr>
<tr>
<td>$L_M$</td>
<td>length of the modulator</td>
</tr>
<tr>
<td>$\gamma_m$</td>
<td>modified distributed mirror loss</td>
</tr>
<tr>
<td>$\Delta_q$</td>
<td>coupling coefficient</td>
</tr>
<tr>
<td>$I$</td>
<td>current</td>
</tr>
<tr>
<td>$G'$</td>
<td>modified gain expression</td>
</tr>
<tr>
<td>$\beta_{si}$</td>
<td>self saturation, cross saturation coefficients</td>
</tr>
<tr>
<td>$R_{sp}$</td>
<td>average rate of spontaneous emission</td>
</tr>
<tr>
<td>$\omega_{sp}$</td>
<td>frequency shift due to the average rate of spontaneous emission</td>
</tr>
<tr>
<td>$\Delta\varepsilon_{FM}$</td>
<td>phase modulator contribution to $\varepsilon$</td>
</tr>
<tr>
<td>$\Delta\mu_M$</td>
<td>refractive index perturbation</td>
</tr>
<tr>
<td>$\varphi_{max}$</td>
<td>modulation depth of the phase modulator</td>
</tr>
<tr>
<td>$\Delta\varepsilon_{FM}$</td>
<td>FM coupling coefficient</td>
</tr>
<tr>
<td>$\Delta\alpha_{AM}$</td>
<td>amplitude modulator contribution to $\varepsilon$</td>
</tr>
<tr>
<td>$\Delta\alpha_{AM}$</td>
<td>loss perturbation due to modulation</td>
</tr>
<tr>
<td>$\Delta\varepsilon_{AM}$</td>
<td>coupling loss AM</td>
</tr>
<tr>
<td>$\Delta\alpha_{AM}$</td>
<td>coupling coefficient AM</td>
</tr>
<tr>
<td>$\delta_M$</td>
<td>spatial variation of the modulating signal</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>distance from the mirror</td>
</tr>
<tr>
<td>$\mathcal{M}$</td>
<td>intracavity modulating efficiency (modulus)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>intracavity modulating efficiency (phase)</td>
</tr>
</tbody>
</table>

Lithium Niobate Modulators

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$\mu_o$</td>
<td>ordinary index</td>
</tr>
<tr>
<td>$\mu_e$</td>
<td>extraordinary index</td>
</tr>
<tr>
<td>$\Delta\mu_o$</td>
<td>change of the ordinary index</td>
</tr>
<tr>
<td>$\Delta\mu_e$</td>
<td>change of the extraordinary index</td>
</tr>
<tr>
<td>$r_{ij}$</td>
<td>electro-optic coefficients</td>
</tr>
<tr>
<td>$E$</td>
<td>applied field</td>
</tr>
<tr>
<td>$\psi'$</td>
<td>driving voltage</td>
</tr>
<tr>
<td>$G$</td>
<td>interelectrode gap</td>
</tr>
</tbody>
</table>
overlap between el. field and opt. mode
interaction length
induced phase shift
output intensity
maximum light intensity (in phase)
phase shift difference
directional coupler factor
difference in eff. prop. constants
electrical bandwidth
electrode resistance
electrode capacitance
microwave loss coefficient

Γ_m
L_m
Δβ L_m
I_{out}
I_{out(max)}
Δφ
p
Δβ
Δf
R
C
a'_o
Chapter 1

Introduction

The field of short optical pulse generation using semiconductor lasers was created in the late sixties and early seventies to exploit the main advantages of the semiconductor medium namely small size, electrical pumping and potential for integration with other optical and electronic components. In the late seventies, when reliable operation of several laser structures was achieved, the field received a sudden impetus by the demonstration of active mode-locking and gain-switching resulting in optical pulses shorter than 20ps. Since then, considerable effort has been directed towards short pulse generation using techniques of mode-locking and gain switching. Until the end of the eighties, advances were restricted to improved laser structures (low parasitics structures, distributed feedback lasers and multiple quantum well devices) rather than radical improvement in the techniques of short pulse generation. The second major development in the field was made in the late eighties by the introduction of fibre amplifiers. It was realised that semiconductor laser diodes can serve as optical pulse generators in future soliton systems and also play a dominant role in optical pumping of fibre amplifiers. The concept of short pulse generation was enriched because of the inclusion of self-phase modulation and dispersion into practical systems which consist of laser diodes, fibre amplifiers and dispersion shifted fibres.

The consequence of the soliton system race was that the largest telecommunication research institutes (ATT&Bell, NTT, BTRL) have concentrated on generation of transform limited pulses using semiconductor lasers. Initial activities were aimed at active mode-locking using bulk or hybrid cavities or alternatively at gain-switching in conjunction with spectral filtering. Longer term projects were aimed at
monolithic lasers utilising concepts of active and hybrid mode-locking, as
the integration of lasers, waveguides and Bragg filters was already in an
advanced state because of the work directed towards narrow linewidth
sources and frequency tunable lasers. At present, the work on active
mode-locking of monolithic lasers seems to be directed towards longer
structures (5-10 GHz round-trip frequencies), high-frequency modulation
capability of the laser section and the optimisation of the Bragg section.1
Regarding passive and hybrid mode-locking the effort is concentrated on
the design of saturable absorbers in multiple quantum well structures.2
Colliding pulse mode-locking has also been demonstrated in monolithic
configurations and it resulted in sub-picosecond optical pulses.3

The main disadvantage of mode-locked semiconductor lasers is their
moderate output power. The power levels can be increased by the use of
external semiconductor lasers amplifiers or fibre amplifiers. An
alternative approach to increasing the optical power of the mode-locked
semiconductor laser is the use of a vertical cavity semiconductor laser
instead of edge-emitting laser structure. The optically pumped vertical
cavity semiconductor lasers in an external cavity configuration can
produce sub-picosecond optical pulses. Moreover, additional pulse
shortening can then be achieved using standard pulse compression
techniques and multiple fibre amplifier stages. In this manner pulses of
15fs at optical wavelength of 1.5μm have been produced and they represent
the shortest pulses achieved utilising a semiconductor laser medium.4

Picosecond optical pulses produced by mode-locking of various types of
laser systems have found applications in the study of ultrafast
phenomena in physics, chemistry and biology. The small size, low power
requirement and ease of use of gain switched and monolithic mode-locked
semiconductor lasers allows the possibility of applications that would not
be practical using other laser systems. Actively mode-locked lasers have
better phase noise performance than gain-switched lasers and that would
be an advantage in most of the applications.
One of the most promising applications of mode-locked semiconductor lasers is for high-speed time division multiplexed telecommunications systems. On the transmitting end, a mode-locked semiconductor laser is used as a source of nearly transform-limited optical pulses for soliton generation. The outputs of several of these mode-locked lasers can then be interleaved to form a very high data rate channel with very small amount of timing jitter. Moreover, there is the potential of integrating the modulator and the optical pulse source on the same chip. At the receiving end, mode-locked lasers can be used to demultiplex the received data stream using four-wave mixing effects in optical fibres or in conjunction with nonlinear loop mirror switching.

Mode-locked semiconductor lasers have also found applications in opto-electronic measurement systems. These applications include impulse response testing of photodiodes, high resolution optical time-domain interferometry and electro-optic sampling of electronic signals.

The project described in this thesis was proposed in 1987 and it started in 1988. The project was funded by National Physical Laboratory, Teddington and our objective was to generate 10ps optical pulses for the calibration of fast photodetectors. The proposed method was active mode-locking of a composite cavity InGaAsP-LiNbO₃ laser utilising an intracavity modulator and the project was entitled 'Mode-Locked Laser using Integrated Optics'. This approach was initially adopted because at that time UCL had fabrication facilities for lithium niobate waveguide devices and fast semiconductor lasers were not commercially available.

In the jargon of mode-locking the proposed project could be called AM and FM mode-locking of the composite cavity semiconductor lasers (AM refers to intracavity amplitude modulation, FM to intracavity phase modulation). The theory of AM and FM mode-locking of the inhomogeneously and homogeneously broadened lasers with long relaxation times was formulated in the late sixties, following the experimental demonstrations in He-Ne and Nd-Yag lasers.
When we started this project there was no reported demonstration of FM mode locking of semiconductor lasers. The best example of AM mode-locking was a twin-section AlGaAs diode laser within an external cavity incorporating a bulk grating generating 8ps pulses. The other reported configuration, on which our project proposal was based was a composite cavity InGaAsP-LiNbO₃ laser incorporating a high speed directional coupler switch, which produced 22ps pulses. The state of the art of active mode-locking by current modulation was a fast InGaAsP laser within a bulk external cavity producing multiple subpicosecond pulses at the repetition rate of 16GHz.

In the first and second year of the project we concentrated on the experimental work: setting up a bulk external cavity laser, fabrication of titanium indiffused lithium niobate waveguides and the fabrication of both intensity and phase modulators. At the beginning of the third year (September 1990) we had demonstrated short pulse generation by both intracavity intensity and phase modulation (pulses of the order of 45-50 ps) and by mode-locking of a composite InGaAsP - LiNbO₃ laser by current modulation (pulses in the range of 30-35 ps). With the same laser, used in a bulk external cavity, incorporating a plane mirror we produced 27ps pulses at a much lower bias current. At that time we made a decision to stop further fabrication of lithium niobate modulators and assemble an external grating laser in order to fulfil our research contract. We planned to use the remaining time in order to develop a numerical model that would unite our experimental attempts. Ultimately, we concluded our experimental work by generating optical pulses shorter than 10 ps using an external grating laser followed by a semiconductor laser amplifier. In our theoretical investigation, we performed numerical simulations of active mode-locking of monolithic external cavity lasers by current, amplitude and phase modulation. The model was based on a frequency-domain formulation using multi-mode field equations.

This thesis comprises nine chapters, the first being this introduction.
Because of the broad scope of the material presented in this thesis, Chapters 2, 3, and 4 are introductory in their character.

The basic theory concerning a semiconductor laser is presented in Chapter 2. The classical description of the laser field in conjunction with the phenomenological gain model is given. Emission characteristics of a semiconductor laser are outlined. The rate equations for the laser field and carrier population are introduced. Finally, the optical feedback effects and the external cavity operation of a semiconductor laser are discussed.

General methods for short optical pulse generation in semiconductor lasers are outlined in the first section of the Chapter 3. In the second section, the experimental results related to different methods of pulse generation are reviewed, with the emphasis placed on active mode-locking. In the last section, a brief discussion on short pulse generation is presented.

An introduction to integrated optic lithium niobate modulators is given in Chapter 4. Waveguides, the electro-optic effect, different modulator configurations and high-frequency design requirements are discussed.

The possibilities of composite-cavity InGaAsP-LiNbO₃ lasers are discussed in Chapter 5. The technical requirements of composite-cavity lasers such as optical coupling, antireflective coatings, and mirrors, are considered. Experimental results on short optical pulse generation using different mode-locking configurations are presented in the second section of the chapter. Our experiments have included modulation of the laser current as well as intracavity intensity and phase modulation.

Our experimental work concerning active mode-locking using bulk external cavities is presented in Chapter 6. The external mirror configurations were tested at UCL, while the more complete measurements of the external grating laser followed by a semiconductor optical amplifier were performed at NPL.
Time-domain and frequency-domain theories of active mode-locking are examined in the first section of Chapter 7. This is followed by a review of the theory of active mode-locking of semiconductor lasers. In the second section of Chapter 7, the frequency-domain formulation of active mode-locking of monolithic semiconductor lasers is derived. We obtained three similar sets of equations for mode-locking by current modulation, AM and FM mode-locking. Chapter 7 is concluded with a comment on the intracavity modulation efficiency of lumped and travelling-wave modulators.

Numerical simulations of gain-switching of a solitary and monolithic external cavity semiconductor laser are presented in the first section of Chapter 8. The aim was to illustrate the behaviour of the averaged, single-mode rate equations. In the second section, the results of numerical simulations of active mode-locking by current modulation are presented. We have investigated the evolution of mode-locking, detuning of the modulating frequency, variations of the bias and RF drive, and role of the linewidth enhancement factor. The results obtained for FM mode-locking presented in the third section of the chapter, show two distinct types of behaviour. FM-laser operation is also demonstrated in our numerical model for FM mode-locking. Finally, a set of simulations of AM mode-locking is presented.

Conclusions from the work as a whole are described in Chapter 9, together with directions for future work.
Chapter 2

Semiconductor Laser

Since the first demonstration of lasing in a semiconductor medium in 1962, there has been enormous progress in both the theory and the manufacture of semiconductor lasers optimised for different applications. Using different semiconductor materials, lasing has been achieved over a very broad range of frequencies, from near ultraviolet to far infrared. Numerous books have been written on the subject of semiconductor lasers. The standard textbooks date from the late seventies and early eighties\(^1,2,3\) and deal with basic device physics. More recent books are concerned with more specific topic such as long wavelength lasers\(^4,5\), modulation and noise\(^6\) and single-frequency structures\(^7\).

In this thesis the introduction to semiconductor lasers will be presented from the point of view of laser user, rather than the laser designer. Moreover, the emphasis will be placed on time-dependent phenomena, which are pertinent to gain-switching and mode-locking. The lasers used in this project belong to the class of devices designed for optical communications systems - 1.55µm index-guiding (buried heterostructure) InGaAsP lasers. The basic theoretical formulation and notation that we adopt are that of Agrawal's textbook\(^4\) on long wavelength semiconductor lasers.

The major part of this chapter (Section 2.1) is devoted to basic theory of semiconductor lasers introducing the classical field description, threshold condition, longitudinal modes, gain and stimulated emission, waveguide modes, and the emission characteristics. Section 2.2 deals with the single-mode rate equations and includes the definition of several important entities. In the final section of this chapter the concept of external cavity operation of a semiconductor laser is introduced.
2.1 Basic Concepts

In semiconductor lasers a semiconductor material is electrically pumped using a forward-biased p-n diode structure. Charge carriers, which are injected into a thin active layer, provide the necessary gain at optical frequencies. A resonator formed by the two cleaved facets of the semiconductor gain medium can produce a sufficient amount of optical feedback. The strength of the pumping of a semiconductor laser is determined by the injected current density. The threshold condition for the build-up of laser oscillation is reached when the gain overcomes the total cavity loss. Any further increase in injected current density above its threshold value leads to laser radiation dominated by stimulated emission.

In this section, our intention is to provide a brief description of the elementary concepts related to understanding of the semiconductor laser dynamics and mode-locking. We start with the classical field description.

2.1.1 Classical Field Description

The classical description of optical phenomena is based upon Maxwell’s equations. The propagation of an electromagnetic wave through an atomic medium is described by the wave equation. The aim of this subsection is to introduce the wave equation and optical parameters such as the dielectric constant, the refractive index and the absorption coefficient which determine the propagation characteristics.

Starting from Maxwell’s equations a wave equation of the following form can be derived:

\[ \nabla^2 \vec{E} - \frac{\sigma}{\varepsilon_0 c^2} \frac{\partial \vec{E}}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 \vec{D}}{\partial t^2} \]  

(2.1)
In this equation $\vec{E}$ and $\vec{P}$ represent the electric field vector and induced electric polarisation vector respectively; $\varepsilon_0$ is the vacuum permittivity, $c$ is the speed of light and $\sigma$ is the conductivity of the medium. The wave equation (2.1) is valid for arbitrary time-varying fields. For optical fields with harmonic time variation we write

$$\vec{E}(x,y,z,t) = \text{Re} [ \vec{E}(x,y,z) e^{j\omega t}]$$

$$\vec{P}(x,y,z,t) = \text{Re} [ \vec{P}(x,y,z) e^{j\omega t}]$$

where $\omega = 2\pi v$ is the angular frequency and $v = c/\lambda$ is the oscillation frequency of the optical field at the vacuum wavelength $\lambda$. Substituting Eqs. 2.2 and 2.3 in Eq. 2.1 we obtain:

$$\nabla^2 \vec{E} + k_0^2 (1 + \frac{j\sigma}{\varepsilon_0 \omega}) \vec{E} = -\frac{k_0^2}{\varepsilon_0} \vec{P}$$

where $k_0 = \omega/c = 2\pi/\lambda$ is the vacuum wave number.

Under steady-state conditions the response of the medium to the electric field is governed by the susceptibility $\chi$ defined by $\vec{P} = \varepsilon_0 \chi(\omega) \vec{E}$. The susceptibility can be decomposed into two parts: $\chi_0$, the medium susceptibility in the absence of external pumping and $\chi_p$ related to the strength of pumping. Both $\chi_0$ and $\chi_p$ are complex and frequency-dependent.

Equation 2.4 can be re-written as the time-independent wave equation of the form:

$$\nabla^2 \vec{E} + \varepsilon k_0^2 \vec{E} = 0$$

where we have introduced the complex dielectric constant defined as

$$\varepsilon = \varepsilon' + j\varepsilon'' = 1 + \chi_0 + \chi_p + j\frac{\sigma}{\varepsilon_0 \omega}$$

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The time-independent wave equation can be used to obtain the spatial mode structure of the optical field in a practical device. In the simplest case, the plane-wave solutions of Eq. 2.5 can be treated. The propagation characteristics of a plane wave in a medium are usually described in terms of two optical constants, the index of refraction \( \mu \) and the absorption constant \( \alpha \). If we consider a plane wave propagating in the \( z \) direction

\[
\vec{E} = \vec{E}_o \exp(j\beta z)
\]

(2.7)

where \( \vec{E}_o \) is the constant amplitude and the complex propagation constant is given as

\[
\beta = k\delta \varepsilon \delta = k\mu
\]

(2.8)

and where \( \mu \) is the complex refractive index usually written as

\[
\mu = \mu + j\frac{\alpha}{2k_o}
\]

(2.9)

Assuming that \( \alpha \ll \mu k_o \), using \( \varepsilon = \mu^2 \) and equating the real and imaginary parts, following relations can be obtained:

\[
\mu = \sqrt{1 + \text{Re}(\chi_o) + \text{Re}(\chi_d)}
\]

(2.10)

\[
\alpha = \frac{k_o \varepsilon''}{\mu} = \frac{k_o}{\mu} \left[ \text{Im}(\chi_o + \chi_d) + \frac{\sigma}{\varepsilon_o} \right]
\]

(2.11)

2.1.2 Threshold Condition and Longitudinal Modes

The plane wave solution of the wave equation can be used to obtain an estimate of the laser frequency and the optical gain required for the onset of lasing. In a semiconductor laser of length \( L \), shown schematically in Fig. 2.1, the thin central region provides the optical gain and cleaved facets form a Fabry-Perot cavity to provide optical feedback. The optical field in plane wave approximation is given by Eq. 2.7. The expression for
Fig. 2.1 Schematic illustration of a semiconductor laser and its Fabry-Perot cavity

Fig. 2.2 Gain profile and longitudinal modes of a semiconductor laser. The threshold is reached when the gain of the dominant mode equals loss.
the complex propagation constant can be obtained by combining Eq. 2.8 and 2.9:

$$\beta = \mu k_o + j\frac{\chi}{2}.$$  \hspace{1cm} (2.12)

If Re($\chi_p$) is small (Eq. 2.10), the refractive index can be approximately expressed as

$$\mu \equiv \mu_b + \Delta\mu_p$$  \hspace{1cm} (2.13)

where $\mu_b = \sqrt{1 + \text{Re}(\chi)}$ and $\Delta\mu_p \equiv \frac{\text{Re}(\chi_p)}{2\mu_b}$

$\mu_b$ is the background index of the unpumped material and $\Delta\mu_p$ is contribution due to the presence of charge carriers. $\Delta\mu_p$ is negative and although it is often less than 1% of $\mu_b$, it significantly affects the semiconductor laser behaviour.

The absorption coefficient $\alpha$ has three contributions: the material absorption $\text{Im}(\chi_o)$, its reduction with external pumping $\text{Im}(\chi_p)$ and the $\sigma/e_0\omega$ term which accounts for other internal losses such as free-carrier absorption and scattering. It is often convenient to introduce the net gain $g$ defined as

$$g = -\frac{k_o}{\mu_b} \text{Im}(\chi_o + \chi_p)$$  \hspace{1cm} (2.14)

and to express the net absorption coefficient as

$$\alpha = -\Gamma g + \alpha_{\text{int}}$$  \hspace{1cm} (2.15)

In this expression $\Gamma$ represents the confinement factor, which is the fraction of the optical mode energy contained in the active region.
To obtain the threshold condition, it is required that the optical field should reproduce itself after each round-trip under steady-state condition. The threshold condition is:

$$\sqrt{R_1 R_2} e^{2\pi L} = 1$$

(2.16)

Equating the real and imaginary parts of Eq. 2.16 and using Eq. 2.12 following expressions can be obtained:

$$\sqrt{R_1 R_2} e^{-\alpha L} = 1$$

(2.17)

$$\sin(2\mu k_o L) = 0$$

(2.18)

The first expression (Eq. 2.17) gives the threshold gain. It can be written in the form:

$$\Gamma g = \alpha_m + \alpha_{\text{int}}$$

(2.19)

where $\alpha_m$ represents the mirror loss expressed as

$$\alpha_m = \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right)$$

(2.20)

The physical meaning of the threshold gain equation (Eq. 2.19) is that the gain due to external pumping must balance the total losses. The effect of spontaneous emission has been ignored in this simple analysis.

The imaginary part of the threshold equation (Eq. 2.18) can be used to obtain the lasing frequency. Eq. 2.18 has multiple solutions of the form

$$2\mu k_o L = 2i\pi$$

(2.21)

where $i$ is an integer. Using $k_o = 2\pi v/c$, the lasing frequency $v$ is

$$v = v_1 = \frac{ic}{2\mu L}$$

(2.22)
The laser tends to oscillate at a frequency that coincides with a longitudinal mode supported by the FP cavity. In the case of homogeneous broadening only one longitudinal mode, whose frequency nearly coincides with the peak gain, reaches threshold (Fig. 2.2) and the laser also remains single-moded above threshold.

In a semiconductor laser, the refractive index $\mu$ varies with frequency, and the longitudinal mode spacing $\Omega$ is given as

$$\Omega = \frac{c}{2\mu_g L} \tag{2.23}$$

where $\mu_g = \mu + \nu \frac{\partial \mu}{\partial \nu}$ represents the group refractive index.

2.1.3 Gain and Stimulated Emission

In semiclassical laser theory the medium response is governed by the polarization induced by the optical field and leads to a susceptibility $\chi$ defined by the expression $\vec{P} = \varepsilon_0 \chi(\omega) \vec{E}$. The induced polarization is given in terms of density-matrix and dipole-moment operators. The decay mechanisms include intraband processes that occur at a time scale of $\sim 0.1\text{ps}$ and interband processes at a time scale of few ns. Intraband processes are electron-electron scattering and electron-phonon scattering while interband processes include radiative recombination (spontaneous and stimulated emission) and nonradiative recombination (Auger process). The analysis of a semiconductor laser using a rigorous semiclassical approach is very complex.

A phenomenological approach, which is based on the observation that the gain is almost a linear function of the injected carrier density, is generally used for describing a semiconductor laser. Therefore, the gain can be approximated as
\[ g(n) = a(n-n_0) \quad (2.24) \]

where \( a \) is the gain coefficient and \( n_0 \) is the carrier density required to achieve transparency (\( a n_0 \) is the absorption coefficient of the unpumped material). The refractive index is also assumed to vary linearly with the carrier density

\[ \Delta \mu_p = b n \quad (2.25) \]

where \( b = \frac{\partial \mu}{\partial n} \) and is often determined experimentally.

Comparing the Eqs. 2.24 and 2.25 to the Eqs. 2.13 and 2.14 it can be seen that the variation of the complex susceptibility \( \chi_p \) is also a linear function of the carrier density

\[ \chi_p = \mu b \left( 2b - jk_o \right) n \quad (2.26) \]

A very important parameter for the semiconductor laser modelling is defined as:

\[ \beta_c = \frac{\text{Re}(\chi_p)}{\text{Im}(\chi_p)} = \frac{-2k_o b}{a} = 2k_o \left( \frac{\partial \mu}{\partial n} \right) \left( \frac{\partial g}{\partial n} \right) \quad (2.27) \]

and it is called the linewidth enhancement factor or antiguiding parameter.

The carrier density \( n \) is related to the pump parameter, current density \( J \), through the rate equation that incorporates all the mechanisms by which the carriers are generated and annihilated inside the active region:

\[ \frac{\partial n}{\partial t} = D(V^2 n) + \frac{J}{q \varepsilon d} \cdot R(n) \quad (2.28) \]
In the carrier density rate equation (Eq. 2.28) the first term accounts for carrier diffusion and $D$ is the diffusion coefficient. The second term describes external pumping which is inversely proportional to the magnitude of the electron charge $q_e$ and the active layer thickness $d$. The last term, $R(n)$, represents the carrier loss due to radiative and nonradiative recombination.

In the case of strongly index-guiding lasers the diffusion term can be neglected because the dimensions of the active region are often small compared to the diffusion length. In this case the steady-state can be written as

$$J = q_e d R(n) \quad (2.29)$$

The carrier recombination rate is often expressed as

$$R(n) = A_{nr} n + B n^2 + C n^3 + R_{st} N_{ph} \quad (2.30)$$

The first term represents non-radiative recombination such as the recombination at defects. The quadratic term $B n^2$ is due to spontaneous radiative recombination. The cubic term $C n^3$ is due to Auger recombination and it is particularly important for long-wavelength lasers. The last term $R_{st} N_{ph}$ is due to stimulated recombination which leads to lasing action and it is proportional to the photon density $N_{ph}$ with the net rate

$$R_{st} = \frac{c}{\mu_g} g(n) \quad (2.31)$$

2.1.4 Waveguide Modes

The light emitted by a semiconductor laser has finite transverse dimensions because it originates from the thin active region, which provides the gain via stimulated emission. The output is in the form of a narrow beam with an elliptic cross-section. The field distribution across
the beam is dependent on the laser structure.

In heterostructure semiconductor lasers the field confinement in the transverse direction, perpendicular to the junction plane, occurs through dielectric waveguiding due to the refractive index discontinuity between the active and cladding layers. Field confinement in the lateral direction, parallel to the junction plane depends on the laser structure. In gain-guided lasers, the variation of the optical gain (due to carrier diffusion) confines the optical mode. Index-guiding lasers have a built-in refractive index step that confines the mode. A schematic cross-section of a buried heterostructure laser, which is a typical example of a strongly index guiding laser, is shown in Fig. 2.3. Besides providing the optical confinement, the buried heterostructure laser incorporates p-n-p blocking structures which create the current confinement.

The most common method for solving transverse and lateral field distributions of an index guided semiconductor laser is the effective index method. Instead of solving the time-independent two-dimensional wave equation (Eq. 2.5), the problem is split into two one-dimensional parts whose solutions are relatively easy to obtain. The effects of gain and loss are treated as perturbations of the lossless case. The mode propagation constant is given by

\[ \beta = \tilde{\mu} k_0 + j \tilde{\alpha} \frac{\omega}{c} \]  

(2.32)

where \( \tilde{\mu} \) and \( \tilde{\alpha} \) represent the refractive index and the absorption coefficient of the mode supported by rectangular waveguide of width \( w \) and thickness \( d \).

2.1.5 Emission Characteristics

Emission characteristics used to characterise the performance of a semiconductor laser are the light-current, spatial-mode, spectral and transient (dynamic) characteristics.
Fig. 2.3 Schematic diagram of a buried heterostructure laser. The refractive index discontinuity between the active and cladding layers is providing mode confinement through the total internal reflection.

Fig. 2.4 Schematic light-current curve of a buried heterostructure laser. Characteristic regions are threshold, linear region and power saturation.
Typical light-current characteristic of an index-guided long-wavelength laser is shown in Fig 2.4. Three regions of the light-current curve correspond to below threshold operation dominated by spontaneous emission, linear above-threshold region dominated by stimulated emission and the power saturation region. The output power saturation can be caused by the increase of leakage currents, Auger recombination and internal loss. The slope of $\frac{dP_{out}}{dI}$ is a measure of the device efficiency.

The dimensions of the elliptical spot and its divergence angles, both parallel and perpendicular to the junction plane, are important beam parameters associated with the laser mode. In the case of the index-guided laser the near field distribution can be expressed as a Gaussian, and from this the far-field pattern can be easily obtained. The situation is more complicated for narrow-stripe gain-guided lasers where the lateral far field takes the form of a twin-lobe pattern.

In optical communication applications, the power spectrum of a semiconductor laser is an important device characteristic. Below threshold, the output is determined by spontaneous emission with a spectrum of the order of 30nm. Above threshold, the longitudinal mode closest to the gain peak increases in power, while the power in the remaining side peaks saturates. The gain profile of the semiconductor laser is homogeneously broadened and the multi-mode behaviour is attributed to spatial-hole burning, spectral-hole burning and a high rate of spontaneous emission entering the laser mode. A parameter used to describe the spectral purity of a semiconductor laser is the mode-suppression ratio (MSR). MSR is defined as a ratio of the main-mode power to the power carried by the most intense side mode. For some optical communication applications, MSRs in excess of 20dB are needed, and they can be achieved with distributed-feedback semiconductor laser configurations. Another quantity of practical interest is the spectral width of a single longitudinal mode which is caused by the quantum
fluctuations associated with the process of spontaneous emission. Spectral linewidth is typically of the order of 100MHz and is inversely proportional to the laser power.

In many applications, particularly in optical fibre communications, the device current is modulated periodically and the laser output takes a pulsed form. The laser responses to a step increase of current with a turn-on delay of few nanoseconds, followed by the rapid increase in laser power and oscillations of several nanoseconds before attaining a steady-state value. The frequency of relaxation oscillations is governed by the nonlinear dynamics of the photon-carrier interactions. The relaxation oscillation frequency is typically in the lower GHz range and increases with an increase of the laser output power.

Under high-frequency direct modulation, the spectral side modes are considerably enhanced compared to those in the CW case. Moreover, the linewidth of an individual longitudinal mode increases. The line broadening is related to the carrier-induced refractive index change and leads to frequency chirping.

2.2 Rate Equations

Rate equations provide the standard way of investigating static, spectral and dynamic characteristics of a semiconductor laser. In general, they are multi-mode equations, but in most cases a single-longitudinal mode treatment is sufficient. Rate equations can be formulated either for the optical field, in cases when both the amplitude and phase information are needed, or just for the photon density or number.

The field rate equation can be derived from the wave equation using the adiabatic approximation. The meaning of this approximation is that the material response is instantaneous, which is certainly valid on timescales of 1ps or longer. The optical field can be expanded in terms of normal modes and the slowly varying envelope approximation is applied.
to eliminate second order derivatives and products of first order
derivatives. This procedure is employed in Chapter 7 to derive the
equations of mode-locking. A further approximation which is usually
applied is to replace localised mirror losses by a distributed mirror loss.
Applying these approximations to the case of a single-mode laser, a field
rate equation can be derived as

$$\frac{dE}{dt} = j\frac{\mu}{\mu_g}(\omega - \Omega_o)E + \frac{1}{2}(G - \gamma)(1 - j\beta_c)E$$ (2.33)

In this equation $E$ represents the slowly varying complex mode
amplitude. $\mu_g$ is is the group index corresponding to the mode index $\mu$. $\Omega_o$
is the cavity-resonance frequency and $\omega$ is the lasing frequency. $\beta_c$ is the
linewidth enhancement factor. The net rate of stimulated emission and
photon decay rate are defined as

$$G = \Gamma v_g$$ (2.34)
$$\gamma = v_g(\alpha_m + \alpha_{int}) = 1/\tau_p$$ (2.35)

where $\tau_p$ represents the photon lifetime and $v_g$ is the group velocity.

The amplitude and phase rate equations can be obtained if the field
equation is separated into its real and imaginary parts using

$$E = Ae^{j\phi}$$ (2.36)

The amplitude and phase rate equations are

$$\frac{\partial A}{\partial t} = \frac{1}{2}(G - \gamma)A$$ (2.37)
$$\frac{\partial \phi}{\partial t} = -\frac{\mu}{\mu_g}(\omega - \Omega_o) + \frac{1}{2} \beta_c(G - \gamma)$$ (2.38)

The amplitude rate equation is more often written in terms of the photon
number defined as
\[ P = \frac{\varepsilon d^2}{2\hbar} \int |E|^2 \, dV \] (2.39)

where \( \hbar \omega \) is the photon energy and \( V \) is the volume of the active layer. For the photon number we can write following rate equation

\[ \frac{\partial P}{\partial t} = (G - \gamma)A + R_{sp} \] (2.40)

The last term \( R_{sp} \) represents the rate at which spontaneously emitted photons are added to the total photon population. The output power from a facet, \( P_{out} \), is related to the photon number \( P \) by the following relation

\[ P_{out} = \frac{1}{2} \hbar \nu g \gamma_m P \] (2.41)

where \( \nu g \gamma_m \) represents the rate at which photons of energy \( \hbar \omega \) escape through the two facets.

The carrier rate equation (Eq. 2.28) can be expressed in terms of the carrier number inside the active layer defined as

\[ N = \int n \, dV = nV \] (2.42)

where the volume \( V \) is \( V = L \times W \times d \) (length \( x \) width \( x \) thickness). If the effect of carrier diffusion is neglected, the carrier number rate equation becomes

\[ \frac{\partial N}{\partial t} = \frac{1}{q_e} - \gamma_e - GP \] (2.43)

where

\[ \gamma_e = A_{nr} + Bn + Cn^2 = 1/\tau_e \] (2.44)
\( \gamma_e \) represents the carrier recombination rate and is used to define the spontaneous carrier lifetime \( \tau_e \).

The last term in the photon number rate equation (Eq. 2.40), \( R_{sp} \) can be defined as

\[
R_{sp} = \beta_{sp} B \frac{N^2}{V}
\]  

(2.45)

where \( \beta_{sp} \) is called the spontaneous emission factor and \( B \) is the radiative recombination coefficient.

Equations 2.40, 2.38 and 2.43 are the single-mode rate equations. They are deterministic equations because the spontaneous emission noise is represented by an average value. These deterministic equations can be used for studying the steady-state characteristics and transient behaviour of single-mode or multi-modeled semiconductor lasers. However, to model spontaneous emission effects such as intensity noise and linewidth it is necessary to include stochastic noise sources in the rate equations. Regarding dynamic behaviour, spontaneous emission also plays an important role in mode-locking and is the source of timing jitter in gain-switched lasers.

2.3 External Cavity Semiconductor Laser

The behaviour of continuous-wave and modulated semiconductor lasers is affected by reflection feedback. Much research has been devoted to studying both the merits and demerits of external optical feedback. Improved mode selection of Fabry-Perot lasers\(^8\), linewidth narrowing and tuning\(^9\) and reduced chirping\(^10\) can be mentioned as advantages of the external cavity structures. Whereas, in a certain feedback region, an instability named 'coherence collapse' can occur\(^11\); moreover, tilting of the external mirror (asymmetric feedback) can lead to an optically chaotic
emission\textsuperscript{12}.

The operation of a semiconductor laser with an external reflector depends on both the magnitude of the reflection and the length of the external cavity. The CW performance of a distributed-feedback laser exposed to reflection feedback has been classified into five regimes\textsuperscript{13}:

1) At the lowest levels of feedback the laser operation is stable but the amount of line narrowing depends on the phase of the reflected light. Linewidth reduction has been observed for a feedback level of -80dB.

2) At a feedback level which depends on the distance to the external reflector (-70dB for a 40cm cavity), mode-hopping between external cavity modes has been detected.

3) In the range from -45dB to -40dB stable operation and line narrowing have been observed, independent of the feedback phase.

4) For optical feedback levels from -40dB to -10dB linewidth broadening, by several orders of magnitude, has been observed. This phenomenon is called 'coherence collapse' and it is accompanied by a large increase of intensity noise.

5) For strong feedback levels stable single-mode operation with a narrow linewidth has been observed. In this region, the stability of operation is not affected by changes in the external cavity length.

Generally, external cavity devices should be designed to operate in the strong feedback regime. In the case of bulk external cavities an anti-reflective coating should be deposited on one semiconductor laser facet in order to improve the coupling efficiency.

One of the most important criteria for semiconductor lasers in high performance coherent communications systems is the narrow linewidth.
The linewidth of a single-mode laser diode is described by the modified Schawlow-Townes formula:

\[
\Delta v_{\text{sol}} = \frac{R_{sp}}{4\pi P} \left(1+\beta_c^2\right) \tag{2.46}
\]

where \(R_{sp}\) is the average spontaneous emission rate into the lasing mode, \(P\) is the total lasing photon number in the semiconductor cavity and \(\beta_c\) is the linewidth enhancement factor. The linewidth of an external cavity laser can be expressed as:

\[
\Delta v_{\text{ecl}} = \frac{\Delta v_{\text{sol}}}{\left(1+\eta\frac{L_L}{L}\right)^2} \tag{2.47}
\]

\(L\) and \(L_L\) are the optical lengths of the external cavity and solitary laser, \(\eta\) is the effective single-pass coupling efficiency. In reality, the linewidth of an external cavity laser is inversely proportional \(L^2\) below a certain value (~15 cm) above which it reaches a saturated value. One possible explanation of linewidth saturation is the effect of 1/f noise.

The longitudinal mode separation in long external cavity lasers is of the order of 1 GHz and it is necessary to include a frequency selective element in the cavity in order to prevent mode-hopping. The most common way is to use a grating instead of the plane mirror. The external grating configuration has the additional benefit of a broad tuning range. The linewidth of 2 kHz and a coarse tuning of 90 nm have been reported for the 18 cm long cavity. A more compact, fixed wavelength configuration can be formed using a distributed-feedback laser with the fibre-extended cavity. For such a device the linewidth of 70 kHz has been reported for a 5.5 cm cavity. Monolithic external cavity lasers having sub-MHz linewidth have been demonstrated using semiconductor structures shorter than 2 mm.

The theoretical analysis of external cavity semiconductor lasers is complicated by multiple reflections. Most theoretical investigations
reported in the literature deal with the case of weak feedback, which strictly speaking, is not valid for an external cavity laser. In the weak feedback approximation a single reflection is added to the semiconductor laser field rate-equation\(^{19}\). The effects of strong feedback can be treated by introducing a frequency dependent amplitude reflectivity\(^{20}\) into the rate equation for the laser field. Another approach is to treat a composite cavity system in which separate field equations for active and passive cavity are written\(^{21}\).
Chapter 3

Optical Pulse Generation Using Semiconductor Lasers

The methods of short pulse generation using semiconductor lasers fall into two main categories: the first relies on switching of the gain or cavity Q, and the second is by mode-locking. The purpose of this chapter is to introduce the basic concepts related to picosecond optical pulse generation, to review the major developments in the field and to provide a short discussion on the advantages of the various methods.

3.1.1 Gain Switching

Gain-Switching is a method which utilises the large-signal modulation response of a laser diode. The optical response to a current pulse takes the form of a narrow pulse followed by 'ringing' which is related to damped relaxation oscillations. With the proper choice of electrical parameters namely, bias current, pulse amplitude and pulse width, the subsequent peaks can be suppressed and the output becomes a single gain-switched pulse.

An example of the damped relaxation oscillations of the laser, in response to the current step, is shown in Fig. 3.1. Numerical simulations were based upon single-mode rate equations, and included typical parameter values (Appendix 1) of a long wavelength buried heterostructure laser. The same values are used in our simulations of active mode-locking of monolithic external cavity lasers, presented in Chapter 8.

Gain-switched pulse shapes are presented in Fig. 3.2. In this simulation the laser is biased slightly above threshold and modulated by large signal current at 1GHz. In practice, the length of pulses is determined by the amplitude and width of the current pulse and the laser ability to respond
Fig. 3.1 Relaxation oscillations of the photon number (3.1.1) and carrier number (3.1.2) in response to a 30mA current step at $t=0$
Fig. 3.2  Gain-Switching of a solitary BH laser: 
\( I_b = 20 \text{mA}, I_{rf} = 40 \text{mA}, f = 1 \text{GHz} \)
to it. At high frequencies (GHz range), the laser is usually driven by a large-signal sinusoidal current. At lower repetition rates, step-recovery diodes are used to produce a pulsed electrical drive. The modulation response of the laser diode is determined by the intrinsic modulation response and electrical parasitic elements. The modulation bandwidth for the first case (small-signal relaxation oscillation frequency) is usually defined as

\[ f_r = \frac{1}{2\pi} \sqrt{\frac{G \cdot P_{out}}{\tau_p}} \]  

(3.1)

where G represents the differential gain, \( P_{out} \) is the laser output power and \( \tau_p \) is the photon lifetime. The buried heterostructure laser used in our model has a relaxation oscillation frequency in the range of 3GHz (Fig. 3.1). The simplest circuit representing the laser electrical parasitics consists of an inductance (representing bonding wire), in series with a parallel combination of the laser capacitance and internal resistance. For high frequency modulation, a very short bonding wire or a mesh should be used. At frequencies above 5GHz, relevant to active mode-locking of monolithic external cavity lasers, low capacitance structures have a significant advantage over buried heterostructure lasers.

Pulses produced by gain-switching are chirped and so the time-bandwidth product is typically much larger than the Fourier transform limit. Chirping is caused by the refractive index dependence on the carrier concentration. In general, the chirp is nonlinear but in most cases it can be regarded as linear over the central portion of the pulse. In the linear approximation, the lasing wavelength increases linearly over the duration of the pulse, and so the chirp is often called red chirp.

The variation of the lasing wavelength can be directly computed from the field rate equations (Eq. 2.39, 2.37 and 2.42). Analytic expressions can be derived from the field rate equations assuming a linear chirp and a particular pulse shape. For a Gaussian optical pulse, the time-bandwidth
product increases by the factor \( \sqrt{1+\beta_c^2} \) where \( \beta_c \) is the linewidth enhancement factor\(^2\).\(^7\).

One important feature of linearly chirped pulses is that they can be compressed by propagation in a dispersive media. For red chirp, the pulse compression can be achieved by propagation in a normally dispersive media.

In our short discussion we have assumed single-mode laser operation. It is important to note however, that all buried heterostructure lasers are multi-moded under large-signal modulation. This is a serious drawback in most applications involving propagation in single-mode fibre, including pulse compression. For this reason, most gain-switched laser systems are based on distributed feedback (DFB) lasers. These devices, which incorporate a frequency selective element, are single-moded for a continuous wave operation. Though at present, under gain-switched condition, most commercially available DFB lasers operate multi-moded. Typical pulse widths of gain-switched DFB's are in the region of 20-40ps.

3.1.2 Q Switching

Q-switching is based on varying the cavity loss rather than modulating the laser gain (gain-switching). The Q-factor of the laser cavity is reduced to allow a higher carrier density to build up. When the carrier density has reached a value considerably above the threshold value for the normal laser Q, the loss is abruptly reduced to its initial value and an intense optical pulse is emitted. For a continuously pumped laser it is a requirement that the Q is switched back to its initial value over a period which is short compared to carrier relaxation time.

Q-switching can be accomplished by the means of a fast intracavity loss modulator (active Q-switching) or by an intracavity saturable absorber (passive Q-switching). An example of Q-switched operation of a long, monolithic external cavity semiconductor laser including a short
Fig. 3.3 Q-switching of a monolithic external cavity laser containing a short loss modulator; $I_b=50\text{mA}$, mirror reflectivity variation ($R_{on}/R_{off}$) $\sim 20\text{dB}$. The time sequences are modified distributed mirror loss, given by Eq. 7.22, (Fig. 3.3.1), photon number (3.3.2) and carrier number (3.3.3).
amplitude modulator is shown in Fig. 3.3. Time sequences of the loss coefficient, photon number and carrier concentration are used to illustrate the concept of Q-switching, described in the introductory paragraph. The displayed Q-switched pulse has typical characteristics of a gain-switched pulse: a fast rise time and a slower decay time.

From the theory of Q-switching based on single-mode rate equations, the peak power of the Q-switched pulse is proportional to the inversion ratio (maximum carrier density/threshold carrier density) and is inversely proportional to the photon lifetime. Since the photon lifetime is directly proportional to the length of the laser cavity, more intense pulses are produced by shorter laser structures.

3.1.3 Mode-Locking

Mode-locking is achieved by introducing a mechanism inside the laser cavity to cause the longitudinal modes to interact with one another, locking them in phase. The result of mode-locking is a train of pulses, separated by the round-trip time of the cavity. The concept of mode-locking is illustrated in Fig. 3.4.

In the absence of noise, the total optical field of a multi-mode laser, represented as a function of time, can be written as:

\[ \mathcal{E}(t) = \sum_{m} \tilde{E}_m e^{j\omega_0 t} = \sum_{m} E_m e^{j[(\omega_0 + m\Omega)t + \phi_m]} \]  \hspace{1cm} (3.2)

where \( \omega_0 \) represents the optical frequency of the central mode, \( \Omega \) is the mode spacing frequency, and \( \phi_m \) is the phase of the \( m^{th} \) mode. \( \mathcal{E}(t) \) is a periodic function of time, and the period is

\[ T = \frac{2L}{c} \]  \hspace{1cm} (3.3)

where \( L \) is the length of the cavity, and \( c \) is the speed of light. The term mode-locking is normally reserved for case in which the number of the
Fig. 3.4 Concept of mode-locking
a) mode-locking configuration
b) frequency-domain representation: gain curve and cavity modes
c) laser output in time domain
modes is \( \geq 3 \) and where phases of the modes are locked in such a way as to produce the highest peak amplitude of the field. One set of phases \( \varphi_m \) which satisfy the mode-locking condition is when all \( \varphi_m = 0 \). In this case, for \( M \) modes, assuming equal mode amplitudes and taking \( E_m = 1 \), the output field can be written as:

\[
E(t) = \sum_{-(M-1)/2}^{(M-1)/2} e^{j(\omega_m + m\Omega)t} = e^{j\omega_{st}} \frac{\sin M\Omega t}{\sin \frac{\Omega t}{2}}
\]  

(3.4)

The output power is

\[
P_{\text{out}} \sim |E(t)|^2 = \frac{\sin^2 M\Omega t}{\sin^2 \frac{\Omega t}{2}}
\]  

(3.5)

The output power is a periodic function. The peak power is \( M \) times larger than the total average power. The individual pulse width, defined as the time from the peak to the first zero is given as:

\[
\tau = \frac{T}{M}
\]  

(3.6)

From Fig. 3.4b we can see that ideally, the number of oscillating modes can be estimated as

\[
M = \frac{\Omega_G}{\Omega}
\]  

(3.7)

where \( \Omega_G \) represents the width of the laser gain spectrum. From equations 3.6 and 3.7 it can be seen that in this ideal case, the pulse width is then the inverse of the gain linewidth. Moreover, for a top-hat spectrum (equal mode amplitudes), the time-bandwidth product of the pulse is equal to unity.
Thus far, mode-locking has been treated only as a function of time. Treated in time and space, mode-locking gives rise to optical packets that travel back and forth between the mirrors with the velocity of light (Fig. 3.4.1). The pulsation period $T$ corresponds simply to the time interval between the arrival of pulses at the mirror. The spatial length of each pulse is given by the product of its time duration and its velocity.

The two main mechanisms used to obtain mode-locking are described as either active or passive. In active mode-locking the gain or loss of the cavity is modulated at the round-trip frequency of the cavity, so that each longitudinal mode is affected by the modulation sidebands of its neighbours. Alternatively, active mode-locking can be achieved by intracavity phase modulation (FM mode-locking). The simplest way of achieving active mode-locking in semiconductor lasers is by laser current modulation (gain modulation).

In passive mode-locking a saturable absorber is placed in external cavity. This is a material that absorbs strongly at a low intensity, but as the intensity builds up the absorption decreases, so that at high intensity the absorption is small. As the laser is not modulated, the pulses build up from random fluctuations which are selectively transmitted through the absorber and amplified in the gain medium. The shortest mode-locked pulses are produced by a technique known as colliding-pulse mode locking. In this method the saturable absorber is placed in a ring cavity (or Fabry-perot cavity) and two counter-propagating waves collide in it and thereby increase the effective saturation efficiency. Finally, the combination of active and passive mode locking is called hybrid mode-locking.

The typical length of a semiconductor laser is approximately 300$\mu$m and the intermodal frequency spacing is beyond 100 GHz. An extended laser cavity is generally used to reduce the round-trip frequencies to a few GHz or even a few tens of GHz. The minimum attainable pulse width for InGaAsP lasers as predicted from the width of the gain spectrum is of the
order of 50fs. In practice, the best results for pure mode-locking are of the order of 1ps.

3.2 Selected Review of Experimental Results

A full report on short optical pulse generation using semiconductor lasers would contain hundreds of references. The aim of our selected review is to complement the previous part of this chapter which dealt with basic concepts of short pulse generation. Special attention is devoted to active-mode locking of bulk and hybrid external cavity configurations - a topic which is particularly relevant to the experimental work contained in this thesis.

3.2.1 Gain-Switching

The first indication of short pulse generation using directly modulated semiconductor laser was reported in 1970. The potential for utilising this technique was examined in 1980, when the exact pulse widths were measured by streak cameras and autocorrelators. It was found that the pulse widths were comparable to those obtained by active mode-locking. Since then, there have been numerous reports describing gain-switching of various types of semiconductor lasers resulting in pulses of ~ 20ps. The shortest reported pulses obtained by gain-switching are 1.3ps. However, these pulses were produced by optical switching of the laser gain (MQW laser), and since the main advantage of gain-switching is its simplicity, we will refer to the results obtained by electrical pumping. Pulses produced by conventional gain-switching of Fabry-Perot lasers are seldom shorter than 10ps. The disadvantage of Fabry-Perot type lasers is that they are multi-moded under large-signal modulation. Gain-switching of distributed feedback lasers typically results in pulses of the order of 30 ps. The spectral properties of these pulses are better than of those produced by Fabry-Perot lasers. The pulses are red chirped, and in some cases a true single-mode operation under modulation has been observed.
At 1.3 μm, the compression of red chirped pulses can be achieved by propagation through a 1.5 μm dispersion shifted fibre\(^6\). Utilising this method pulses of 2ps\(^7\) and 3ps\(^8\) have been produced. These results were achieved using a standard comb generators for gain-switching of DFB lasers and in conjunction with propagation through several kilometers of dispersion shifted fibre. An alternative means of linear compression is a grating-pair compressor. Using this system, pulses of 5.3ps have been produced\(^9\).

The most powerful method for compressing optical pulses is based upon nonlinear pulse propagation. The compression is accomplished in three stages\(^10\). In the first stage, pulses produced by a mode-locked laser are amplified. In the second stage, these pulses are propagated through an optical fibre and a strong blue chirp is imposed as a result of self-phase modulation. Finally, the pulses are compressed in a grating pair as a result of anomalous dispersion. Soliton compression\(^11\) is a method in which the second and third stage are combined by pulse propagation in a fibre that exhibits anomalous dispersion. This method has attracted attention since the development of laser diode pumped erbium fibre amplifiers. Pulses shorter than 200fs have been generated from a 1.55μm gain-switched DFB followed by three amplification and compression stages\(^12\).

3.2.2 Q-Switching

Active Q-switching of semiconductor lasers has been utilised less often than gain-switching because of both the complicated structures that are needed and the multi-mode operation. Q-switching has been implemented in quaternary lasers by a monolithically integrated absorption modulator\(^13\). It has been also demonstrated in GaAs multiple quantum well lasers resulting in 18ps optical pulses at a 3 GHz repetition rate\(^14\).

Passive Q-switching of semiconductor lasers has been investigated since late sixties\(^15\). It provides the least expensive way of producing pulses of
the order of 100ps, because expensive driving circuits (RF sources and amplifiers) are not needed. However, the major obstacle to this method is the fabrication of a reliable saturable absorber. Nevertheless, multiple optical pulses of 5ps at a repetition rate of 18 GHz have been reported for a three-section AlGaAs laser. Finally, the combination of gain-switching (an active drive) and passive Q-switching (saturable absorber) can result in pulses of 12ps. In this configuration (Ref.17), the single-mode operation was achieved by injection-locking.

3.2.3 Active Mode-Locking

The standard way of active mode-locking is to modulate an external cavity laser at the round-trip frequency of the composite cavity. The first demonstrations of proper active mode-locking of AlGaAs and InGaAsP lasers was reported in 1978. The external cavities were formed by spherical mirrors and pulses of the order of 20ps were achieved. The lasers used in these experiments were uncoated. Autocorrelation measurements revealed multiple noise bursts separated by the round-trip of the laser diode cavity. Coherent pulses were produced using an antireflective coated laser diode placed in an external cavity containing a plane mirror. Subsequently, a Fabry-Perot etalon was used to select a single cluster of external cavity modes and bandwidth-limited pulses of 16ps were obtained for an angled-stripe mode-locked laser. With this approach, a complete suppression of internal mode structure of the laser diode was achieved by the combination of antireflective coatings and angled stripe contacts. Active mode-locking was also reported for ring cavity configuration, and a comparison between the linear and ring cavity operation was made. In all these experiments a large-signal RF modulation of the laser current was used. An alternative to this type of the electrical drive is to use a step-recovery diode.

Active mode-locking by current modulation has been reported for a variety of external cavity configurations including graded-index lenses, single-mode fibres and a variety of external gratings in both bulk and hybrid
forms. This last group includes a small packaged bulk grating\textsuperscript{27}, a silicon chip Bragg reflector\textsuperscript{28}, and a fibre Bragg reflector\textsuperscript{29}.

In laboratory conditions, the preferred configuration is an external cavity containing a bulk external grating because it provides stable single-mode operation with narrow linewidth, broad tunability and in some cases transform-limited pulses as short as 5ps\textsuperscript{30}. Moreover, a bulk external-cavity laser, when tuned to shorter wavelengths, can produce shorter pulses due to the larger differential gain\textsuperscript{31}. In another bulk configuration, the intracavity chirp compensation using a Gires-Tournois interferometer has been implemented\textsuperscript{32} resulting in pulses of 4.6 ps. An external compression of the pulses produced by bulk external grating lasers in a single-mode fibre\textsuperscript{33} and in a grating pair\textsuperscript{34} resulted in pulses of the order of 4ps.

The shortest pulses produced by an actively mode-locked external cavity laser are 0.58ps\textsuperscript{35}. These pulses were produced by a fast semiconductor laser (14 GHz bandwidth) at a repetition rate of 16GHz. However, in this case instead of a single pulse there is a sequence of multiple pulses separated by the round-trip time of the laser diode. This multiple pulsing represents the limit of short pulse generation using external cavities and it is due to imperfect anti-reflective coating. A theoretical treatment based on travelling-wave rate equations, predicts that the reflectivity should be reduced to $10^{-5}$ in order to eliminate this effect\textsuperscript{36}.

AM mode-locking of semiconductor lasers has received considerably less attention than mode-locking by direct current modulation. The main reason for this is the need for a more complex configuration in the case of AM mode-locking. A composite-cavity Ti:LiNbO$_3$-InGaAsP/InP laser\textsuperscript{37} utilising high-speed, travelling-wave directional coupler switch is the example of a hybrid cavity approach to AM mode-locking. Pulses shorter than 22ps at repetition rates up to 7.2GHz were produced by sinusoidally driving the directional coupler switch.
Pulses with durations as short as 8 ps have been produced by an external cavity semiconductor laser with an integrated modulator section. The potential of this configuration was realised after the observations of ultra-short multiple pulses caused by non-perfect antireflective coatings. In this configuration, multiple pulsing can be suppressed if the modulator is driven by a short electrical pulse generator and if the length of the lasing area is selected in a way that the second pulse arrives at the modulator section when it is off. Nearly-transform limited pulses of 1.4 ps were generated utilising this concept. This represents the shortest single pulses generated from an actively mode-locked external-cavity semiconductor laser ever reported.

Finally, in the literature, FM mode-locking appears to be the least successful method of mode-locking of semiconductor lasers. The first demonstration of FM mode-locking of semiconductor lasers, utilised an external cavity and AlGaAs phase modulator and resulted in detector-limited pulses of 200 ps (deconvolved ~ 150 ps). The only other reported configuration consisted of a tunable distributed Bragg reflector laser placed in an external cavity containing a plane mirror. Measured pulse widths varied from 158 ps at the fundamental frequency of 300 MHz to 24 ps achieved at the fifth harmonic.

3.2.4 Passive and Hybrid Mode-locking

Passive mode-locking of an external-cavity semiconductor lasers was reported in 1980. In this report, pulses of 5 ps were obtained using an aged buried heterostructure laser. This was followed by demonstrations of sub-picosecond pulse generation using proton bombared and aged lasers. Colliding pulse mode-locking of a proton bombarded laser in a ring configuration resulted in pulses of 0.56 ps. Colliding pulse mode-locking was also attempted in a Fabry-Perot cavity, and pulses shorter than a picosecond were obtained. In all cases of sub-picosecond pulse generation, multiple pulse bursts were produced as a consequence of non-perfect anti-
reflective coatings of the laser diodes.

Passive mode-locking has also been achieved using the excitonic absorption feature of multiple quantum well samples. Chirped mode-locked pulses of 2.4 ps were compressed to 0.83 ps by a grating pair with negative group velocity dispersion.

Semiconductor lasers can also be passively mode-locked by dividing the semiconductor cavity into two electrically isolated sections, with one biased to provide controlled introduction of saturable absorption. The reduction of differential gain at higher carrier density in quantum well materials makes them more suitable for passive mode-locking. A source of 4.5 ps pulses has been built by placing a two section multiple quantum well laser into an external grating cavity.

Finally, the shortest pulses obtained from the system based on an external cavity, mode-locked semiconductor laser are 200 fs. In this configuration, the hybrid mode-locking of an angled-stripe semiconductor amplifier in combination with a proton-bombarded multiple quantum well absorber resulted in chirped pulses of 10 ps. These mode-locked pulses were amplified using semiconductor laser amplifier and then compressed by a grating pair.

3.2.5 Mode-locking of Monolithic Semiconductor Lasers

Recent advances in monolithic integration techniques have resulted in fabrication of several long cavity mode-locked lasers. The first reported structure consisted of a laser with a passive integrated waveguide. This laser was actively mode-locked by current modulation at 40 GHz generating 4.4 ps pulses. Hybrid mode-locking of a structure containing the gain section, active waveguide and a saturable absorber resulted in 1.4 ps at a repetition rate of 15 GHz. Bandwidth-limited pulses with durations of 1.4 ps were generated by colliding-pulse mode-locking at 32 GHz. The cavity of this multiple quantum well monolithic laser was
divided into five sections: the two end sections near the facets were modulators, the center section was a saturable absorber and the remaining two sections were active waveguides. Passive colliding pulse mode-locking was achieved in a three section device which contained a saturable absorber in the centre and gain sections on the sides. With longer devices of this type pulses of 1.1ps at 40GHz were produced.

The most advanced monolithic structure for pure active mode-locking consists of a 5.5mm long extended-cavity laser with a passive waveguide and a Bragg reflector. Transform-limited pulses of 20ps were obtained at a repetition rate of 8 GHz. Active mode-locking was also reported for a monolithic ring laser and pulses of 27ps at a repetition rate of 9 GHz were achieved.

3.3 Discussion

In the last section, the literature on short pulse generation using semiconductor lasers was reviewed. This subject included a review of gain-switching, Q-switching and active, passive and hybrid mode-locking in external cavities and monolithic configurations. Compression methods using linear and nonlinear fibre propagation were also mentioned.

From the point of view of the laser user there are two possible approaches for short optical pulse generation. The first one is gain-switching and the second one is active mode-locking in an external cavity. Advantages and disadvantages of these configurations are listed in Table 3.1. In practice, for most of the applications, a distributed feedback laser is preferred to a Fabry-Perot structure in gain switching because of the single mode (or nearly single mode) operation. In mode-locking, a high quality anti-reflective coating is needed on one facet of the laser. Regarding external cavity configurations, the use of a grating leads to mode-locking of a single cluster of modes and results in nearly transform-limited pulses. Typically, gain-switching results in pulses of 20-30ps while active mode-locking produces pulses of the order of 20ps. The advantages of
Gain-switched DFB laser | Mode-locked BH laser external grating cavity
---|---
Simple structure | Low chirp
Variable repetition rate | Low timing jitter
Wavelength tunable
High jitter | Complex structure
High chirp | Low output power
Fixed wavelength | Fixed repetition rate

**Table 3.1** Comparison between a gain-switched DFB laser and an actively mode-locked external grating semiconductor laser

<table>
<thead>
<tr>
<th>configuration</th>
<th>pulse width (ps)</th>
<th>spectral width (GHz)</th>
<th>peak power (mW)</th>
<th>rep. rate (GHz)</th>
<th>timing jitter (ps)</th>
<th>ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>gain-switched</td>
<td>17</td>
<td>194</td>
<td>30</td>
<td>0.1</td>
<td>na exp &gt;5</td>
<td>7</td>
</tr>
<tr>
<td>+fibre comp.</td>
<td>3</td>
<td>294</td>
<td>100</td>
<td>0.01</td>
<td>&gt;5</td>
<td>8</td>
</tr>
<tr>
<td>act. M-L</td>
<td>1.4</td>
<td>350</td>
<td>200</td>
<td>3</td>
<td>0.25</td>
<td>39</td>
</tr>
<tr>
<td>external cav.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>act. M-L</td>
<td>20</td>
<td>17</td>
<td>na exp&lt;20</td>
<td>8</td>
<td>(&lt;1)</td>
<td>56 (58)</td>
</tr>
<tr>
<td>monolithic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pass. M-L</td>
<td>2.5</td>
<td>860</td>
<td>500</td>
<td>5</td>
<td>12.5</td>
<td>58</td>
</tr>
<tr>
<td>external cav.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 3.2** State of the art performance at repetition rates below 10GHz
gain-switched lasers are their simplicity, variable repetition rate, and higher peak powers. Actively mode-locked laser have better spectral properties, lower timing jitter and can provide wavelength tunability in excess of 50nm.

State of the art performance in short pulse generation at repetition rates below 10GHz is presented in Table 3.2. The results presented in this table refer to single optical pulse operation as opposed to bursts of sub-picosecond pulses contained within an envelope which is usually broader than 10ps\textsuperscript{35,43}.

It can be seen from Table 3.2 that active mode-locking leads to pulses of the best spectral properties and smallest timing jitter. In terms of peak power, our third entry from Table 3.2 (AM mode-locked laser) is superior to a typical actively mode-locked laser which produces less than 10mW.

Monolithic actively mode-locked lasers are at a relatively early stage of development, and it is expected that the pulse widths will be reduced in the near future. However, it is important to note that the pulse timing jitter of monolithic lasers is larger than of external cavity lasers. Passive mode-locking can produce short, chirped pulses of large peak powers but the range of applications is quite restricted because of the lack of synchronisation with electrical signals. At present, hybrid mode-locking attracts significant attention as a method which brings the best of two worlds: synchronisation and low jitter due to the active drive, and shorter pulses and higher powers owing to saturable absorption effects.

As alluded to, the main advantage of gain-switching is its simplicity. Compressed pulses of gain-switched distributed-feedback laser are often used in laboratory measurements. Their only disadvantages are a relatively large timing jitter which is a characteristics of gain-switching, and the distorted pulse tails due to compression.
Chapter 4

Lithium Niobate Modulators

As a research field, lithium niobate integrated optic technology has reached a very mature state in the last decade. Most effort was directed towards devices for optical communication systems such as fast intensity and phase modulators, switch arrays, wavelength multiplexers, frequency shifters and polarisation controllers. Additionally, a variety of devices have been demonstrated for signal processing and sensing applications. More recently, there has also been some interest in the investigation of non-linear optical phenomena in lithium niobate waveguides as well as lasing in neodymium and erbium doped structures.

The aim of this short chapter is to introduce the basic concepts and design strategies of lithium niobate integrated modulators. Fabrication techniques and waveguide characteristics are presented in the first section, followed by the brief description of the electro-optic effect in lithium niobate. Integrated optic phase and intensity modulator configurations are described in the third section. In the concluding part, the high frequency operation of the modulators is discussed.

4.1 Waveguides

Integrated optic lithium niobate modulators are essentially electro-optically controlled single-mode channel waveguide devices. The waveguides are formed by increasing the refractive index of the substrate within a define volume through the addition of dopants. There are two standard techniques for waveguide fabrication in lithium niobate: titanium indiffusion and proton exchange. Both of these techniques are relatively simple compared to waveguide fabrication in semiconductor
materials.

In titanium indiffusion technology waveguide stripes or more complicated patterns are defined using photolitographic technique. Titanium is deposited over the entire crystal by RF sputtering, electron beam deposition or thermal evaporation. Subsequently, the liftoff technique (immersing in photoresist solvent) is used to remove photoresist and unwanted titanium. The crystal is then placed in a diffusion furnace at temperatures of 980°-1050°C for typical diffusion times of 4-10 hours.

The indiffusion of titanium stripes is often accompanied by a parasitic film waveguide at the surface of the crystal due to an increase of the extraordinary index $\mu_e$. This is attributed to the outdiffusion of $O_2/Li_2O$. Various means for the suppression of this parasitic film waveguide have been proposed. At UCL we have investigated two techniques for outdiffusion suppression: wet diffusion, and long dry diffusion in an atmosphere rich in $Li_2O$. The fabrication techniques that we have employed are described in more detail in Appendix 2.

Titanium diffusion allows the fabrication of optical waveguides supporting both TE and TM modes. Waveguide losses below 0.1dB/cm have been reported$^5$. The mode size of waveguides can be tailored by varying the fabrication parameters, and total fibre-waveguide-fibre device insertion losses of 1dB have been reported$^6$. However, because of relatively small refractive index changes ($\Delta\mu_e<0.04$, $\Delta\mu_0<0.02$), branching angles of Y-junctions have to be small and bend's radii large resulting in device sizes of several centimeters.

The photorefractive effect or optical damage in terminology of integrated optics severely limits the optical power in titanium indiffused waveguides at shorter wavelengths (0.8µ) to several microwatts only. At 1.3-1.5 µm no degradation has been observed up to levels of tens of miliwatts$^7$, so in that respect titanium indiffused waveguide devices are well suited to long wavelength optical communication systems.
The proton exchange technique is based on an ion-exchange reaction between \( \text{Li}^+ \) from the \( \text{LiNbO}_3 \) substrate and \( \text{H}^+ \) from some appropriate source. Proton exchange can be achieved by immersing the lithium niobate substrate in benzoic acid at temperatures in a range between 150\(^\circ\) to 250\(^\circ\). The immersion times vary with temperature and can be from minutes at higher temperatures to hours at lower temperatures.

Proton exchanged waveguides have a step refractive index profile with a large, positive extraordinary change \( (\Delta \mu_e < 0.12) \) and small negative ordinary refractive index change \( (\Delta \mu_o < -0.04) \). Due to this inherent anisotropy, proton exchanged waveguides support only TM modes in \( z \)-cut material and TE modes in \( x \) and \( y \)-cut materials. Because of the better optical confinement proton exchanged waveguides are more suited to grating based devices, waveguide bends and ring resonators than titanium diffused waveguides. However, until recently proton exchange waveguides suffered from a number of disadvantages including large propagation losses and a much reduced electro-optic activity.

Two methods for improving the proton exchange technique have been established. The first one is based on the use of buffered benzoic acid and results in long immersion times. The second, more practical method is annealing and it involves heating the substrate after the proton exchange. Waveguides produced by proton exchange and annealing can exhibit low propagation loss (0.15dB/cm), can be well matched to single-mode fibre (total fibre-to-fibre insertion loss of 1.2dB at 0.8\( \mu \)m) and can have totally restored electro-optic coefficients\(^8\).

4.2 Electro-Optic Effect in Lithium Niobate

The linear electro-optic (Pockels) effect, which is the basis for active waveguide device control, provides a change in refractive index proportional to the applied electric field. The linear change in the coefficients of the index ellipsoid due to an applied electric field \( (E_j) \) along
the principal crystal axes is

\[ \Delta \left( \frac{1}{\mu^2} \right) = \sum_{j=1}^{3} r_{ij} E_j \]  

(4.1)

or alternatively

\[ (\Delta \mu)_i = -\frac{\mu^3}{2} \sum_{j=1}^{3} r_{ij} E_j \]  

(4.2)

where \( i = 1,6 \) and \( r_{ij} \) is a 6x3 electro-optic tensor. This relation can be contracted to 3x3 matrix form:

\[
\Delta \mu_{ij} = -\frac{\mu^3}{2} \begin{vmatrix}
-r_{22} E_y + r_{13} E_z & r_{22} E_x & r_{51} E_x \\
 r_{22} E_x & r_{22} E_y + r_{13} E_z & r_{51} E_y \\
 r_{51} E_x & r_{51} E_y & r_{33} E_z
\end{vmatrix}
\]  

(4.3)

where \( \mu \) is either ordinary or extraordinary value. For lithium niobate at 24.5°C and at \( \lambda = 1.439 \mu \text{m} \), \( \mu_0 = 2.151 \) and \( \mu_e = 2.143 \).

Utilization of diagonal matrix elements results in an index and phase change for the proper incident field orientation. The off diagonal elements represent electro-optically induced conversion or mixing between orthogonal polarization components.

For the purpose of modulation it is desirable to utilise the strongest electro-optic coefficient \( r_{33} \) (\( \sim 30.8 \times 10^{-12} \text{mV}^{-1} \)). In this case

\[ \Delta \mu_e = -\frac{\mu^3}{2} r_{33} E_z \]  

(4.4)

Figure 4.1 shows the standard cuts of crystal and optical polarisations employed for most practical cases in which \( r_{33} \) is utilized.
Figure 4.1  Electrode and waveguide configurations in lithium niobate which utilise the maximal electrooptic coefficient $r_{33}$

a) X-cut LiNbO$_3$ for TE polarized light: modulators
b) Z-cut LiNbO$_3$ for TM polarized light: directional couplers and phase reversal modulators
4.3 Phase and Intensity Modulators

An integrated optic phase modulator is shown in Figure 4.2a. Electrodes are placed on the sides of the waveguide in case of X or Y-cut lithium niobate or on the top of the waveguide in case of Z-cut material. In the latter case, an insulating buffer layer is required to eliminate loss to TM polarization.

In an integrated optic modulator, the effective electro-optically induced index change of the optical waveguide mode can be written as

$$\Delta \mu(\mathcal{V}) = \frac{\epsilon_0^3}{2r_{33}} \frac{\mathcal{V}}{G} \Gamma_m$$  \hspace{1cm} (4.5)

where $\mathcal{V}$ is the driving voltage, $G$ is the interelectrode gap and $\Gamma_m$ is the overlap integral between the applied electric field and the optical mode (typically 0.3-0.5). The total phase shift over the interaction length $L_m$ is then

$$\Delta \beta L_m = -\pi \frac{\mu^3}{2} r_{33} \Gamma_m \frac{\mathcal{V} L_m}{G \lambda}$$  \hspace{1cm} (4.6)

For a phase shift of $\pi$, a voltage of nearly 10V is required if $\lambda=1.5\mu$m, $L_m=1$cm, $G=10\mu$m and $\Gamma_m=0.5$. This gives an indication of the relative weakness of the electro-optic effect which imposes further restrictions on the minimal size of lithium niobate modulators. For the same applied voltage, the resulting phase shift is reduced at high frequencies. The bandwidth of a modulator will be introduced and discussed in the final section of this chapter.

An intensity modulator is shown in Figure 4.2b. It consists of a Y-junction splitter, phase shifting electrodes and Y-junction recombiner. If the optical paths of two arms are equal, two components combine in phase and continue to propagate in the output waveguide. For a $\pi$ phase shift.
Figure 4.2 Standard modulator configurations
a) Phase Modulator
b) Mach-Zehnder Intensity Modulator
c) Directional Coupler Intensity Modulator/Switch
introduced by electro-optic effect the combined mode is not supported by the output single-mode waveguide, and the light is radiated into the substrate. A three electrode configuration can be used to achieve push-pull operation in which the required voltage is halved with respect to the case of a single branch phase modulator. In the case of a push-pull intensity modulator, the variation of the output intensity is given by the relation

\[ I_{\text{out}} = I_{\text{out}(\text{max})} \cos^2 (\Delta\phi/2) \]  

(4.7)

where \( \Delta\phi \) is the difference between the phase shifts introduced in each arm, and \( I_{\text{out}(\text{max})} \) is the output light intensity when the two waves recombine in phase. If we neglect Y-junction and waveguide propagation losses, \( I_{\text{out}(\text{max})} \) equals the input intensity. From the Equation 4.7 it can be seen that the linear region of operation is positioned around the intensity \( I_{\text{out}} = I_{\text{out}(\text{max})}/2 \).

A directional coupler switch/modulator (Fig.4.2c) consists of two parallel waveguides separated by a small distance so that the light propagating in one guide can couple into the other. A voltage applied across one of the waveguides changes its propagation characteristics and modifies the coupling between the two guides. For a length of interaction equal to one coupling length, the light initially launched in one guide comes out from the other guide when no voltage is applied. On application of an electric field, the light can be switched back to the original waveguide. The DC characteristic of a directional coupler, at the output of the second guide, can be expressed as

\[ I_{\text{out}} = I_{\text{out}(\text{max})} \frac{\sin^2(p\pi/2)}{p^2} \]  

(4.8)

where \( p = \sqrt{\left(\Delta\beta L_m/\pi\right)^2 + 1} \), \( \Delta\beta \) is the difference in the effective propagation between the waveguides and \( L_m \) is the length of the coupling region. As
for a Mach-Zehnder modulator, the linear region is centered at 
$I_{out} = I_{out(max)}/2$.

### 4.4 High Frequency Operation

The lumped electrodes, shown in Fig. 4.3a, act as a capacitor in the electrical circuit. If the electrode length is small compared to the RF wavelength and if the static resistance is not too large, the frequency of operation of the modulator is limited by its RC constant:

$$\Delta f = \frac{1}{\pi R C}$$  \hspace{1cm} (4.9)

where $\Delta f$ is the 3dB bandwidth and $R$ is usually a 50Ω termination. As the capacitance of the electrode is directly proportional to the length there is a clear trade off between drive voltage and bandwidth. Theoretically, the bandwidth length product for lumped modulators is approximately 2 GHz cm.

The travelling wave electrode structure is shown in Fig. 4.3b. The electrode is the continuation of the driving transmission line and the speed in the lossless case is limited by the difference in transit time for the optical and modulating RF waves.

Two types of electrode geometries are commonly used for high frequency operation: coplanar waveguide for push-pull intensity modulators and asymmetric coplanar strip electrodes for phase modulators. The characteristic impedance of these structures is dependent on the electrode gap to width ratio and is typically 20-40Ω.

In lithium niobate the effective refractive indices for the optical and microwave signals are 2.2 and 4.2 respectively, so the optical front will not see the same microwave induced index as it travels along the structure. For a sufficiently large electrode length or modulating frequency a complete cancellation of modulation may occur. The
Figure 4.3  Standard electrode configurations
a) Lumped electrode structure
b) Travelling-wave electrode structure
normalised frequency response of a lossless and properly terminated modulator is given by the following expression:

\[ \Phi_{\text{rel}} = \frac{\sin \Delta_{\text{mod}}}{\Delta_{\text{mod}}} \]  

(4.10)

where

\[ \Delta_{\text{mod}} = \omega_{\text{mod}} |\mu_{\text{opt}} - \mu_{\text{mod}}| \frac{L_m}{2c} \]  

(4.11)

and \( \omega_{\text{mod}} \) is the modulating angular frequency, \( \mu_{\text{opt}} \) is the optical refractive index, \( \mu_{\text{mod}} \) is the microwave effective index, \( L_m \) is length of the interaction region and \( c \) is the velocity of light. The bandwidth length product limit due to velocity mismatch is of the order of 9 GHz cm.

The high frequency operation is also restricted by electrode loss. The microwave loss is dominated by skin depth and can be expressed as \( a' = a'_0 r^{1/2} \) in dB/cm where \( a'_0 \) depends upon electrode and conductivity. For an often reported experimental value of \( a'_0 = 1 \text{dB/cm(GHz)}^{1/2} \) the bandwidth length product limit due to microwave loss only is around 15 GHz cm.

In order to design an efficient modulator for high frequency operation it is necessary to combat electrode loss and velocity mismatch. To reduce loss it is common to employ highly conductive and thick electrodes such as electroplated gold. Regarding velocity mismatch, the simplest way to achieve high frequency operation is to use short electrode structure. Modulation up to 40 GHz was measured using a 2.5 mm directional coupler switch\(^9\). However, in this case, the switching voltage was 25V. A more efficient strategy to combat velocity mismatch is to employ a transmission line with a reduced effective index such as a coplanar waveguide with shielding ground plane\(^10\) or extremely thick electrode\(^11\). Another approach is the use of phase reversal electrodes\(^12\).

Resistively terminated travelling-wave structures enable broad-band
high-frequency operation. With a reactive termination a resonant structure results. Devices can be made to operate at high frequencies over a narrow band with reduced driving voltage\textsuperscript{13}.

In conclusion, a broadband modulator designed for a frequency range between 1 and 10 GHz would be a travelling-wave structure with an electrode length in between 5 and 10mm. For applications where the travelling-wave modulator is placed in an optical resonator it is important to notice that the modulation frequency response is different for backward and forward propagating optical waves. This difference is due to an increased velocity mismatch for the case of contradirectional propagation of the microwave and optical signals. To illustrate this effect, the normalised frequency responses of a 1cm long travelling-wave lithium niobate modulator, including velocity mismatch and electrode attenuation are presented in Fig. 4.4. The frequency response is normalised to the lossless case.
Fig. 4.4 Normalised frequency response of a travelling-wave modulator with a 1cm electrode for codirectional optical and microwave (4.4.1) and contradirectional case (4.4.2).

(courtesy of Dr C. Watson, UCL)
Chapter 5

Short Pulse Generation Using Composite Cavity InGaAsP-LiNbO₃ Lasers

The aim of this chapter is to discuss the operation of composite cavity InGaAsP-LiNbO₃ lasers and to describe our experimental work on short optical pulse generation using these lasers. In this chapter we discuss continuous wave operation and wavelength tuning of these lasers (subsection 5.1.1). This is followed by a short discussion on AM, FM modelocking and FM-laser operation in subsection 5.1.2. The first section of this chapter is concluded by the consideration of several technical aspects of the composite cavity, namely optical coupling, antireflective coatings and mirrors (subsection 5.1.3). The second section of Chapter 5 deals with the experimental work. Short pulse generation has been achieved by current modulation of the laser and both intracavity amplitude and phase modulation using lithium niobate modulators (subsections 5.2.3, 5.2.4, 5.2.5 respectively).

5.1 Lithium Niobate External-Cavity Laser

Integrated-optics channel waveguide devices in lithium niobate can be used to form a hybrid external cavity for a semiconductor laser. Due to the increased length of the laser cavity, these structures have the potential for both very narrow emission linewidth and round-trip frequencies suitable for active mode-locking. The electro-optic effect, when used in combination with different waveguide structures in lithium niobate can provide both the control of continuous-wave and modulated operation of the composite-cavity laser. Moreover, several functions can be integrated on the same lithium niobate chip allowing either intracavity control or providing external control (such as time multiplexing or encoding).
Technical requirements for an external cavity structure include the deposition of an anti-reflective coating on both the laser and the lithium niobate chip together with the deposition of a mirror on the other facet of the lithium niobate chip. Moreover, an additional optical coupling element may be needed if there is a significant mismatch between the optical modes of the semiconductor laser and lithium niobate channel waveguide.

5.1.1 Continuous-Wave Operation

Narrowing of the laser linewidth by means of an external cavity has been discussed in Chapter 2 (section 2.3). Equation 2.46 can be used to estimate the amount of the linewidth narrowing in the strong feedback regime. For a composite cavity laser incorporating a 3 cm long lithium niobate chip, and assuming efficient optical coupling, the linewidth approximately 100kHz should be obtainable. An important feature of lithium niobate external cavity configurations is that they can provide tuning of the lasing frequency in addition to the linewidth narrowing.

A single-frequency semiconductor laser can be thermally tuned with a rate of 10 GHz/°C; the speed being dictated by the thermal time constant of both the laser and the mount. The lasing frequency is also dependent on the bias current. Above threshold, the behaviour is dominated by heating and a change in the lasing frequency of 1.5 GHz/mA has been reported for distributed-feedback lasers\(^2\). Below threshold, the emission spectrum is influenced by a combination of both the carrier density induced refractive index change and the heating effects. Temperature and current tuning of a semiconductor laser are accompanied by changes in the output power.

In recent years considerable effort has been devoted to the development of tunable multi-segment distributed-Bragg reflector lasers. These lasers have all the benefits related to monolithic devices: good reliability, low costs and reduced intra-cavity reflections. The most common configuration is a three section distributed-Bragg reflector laser which
has separate lasing, tuning and Bragg sections. A device that has a total tuning range of 10nm and a continuous range of 4.4nm (550GHz) has been reported\(^3\). The linewidth of multi-segment distributed-Bragg reflector lasers is dependent on the lasing frequency and is typically in the range from 5 to 25 MHz\(^1\).

In applications where narrow linewidth is required, a tunable external cavity laser represents the most convenient choice. A bulk configuration which consists of AR-coated laser, lens and grating seems to be a standard narrow linewidth source for laboratory PSK coherent communication experiments. A commercially available laser of this type\(^4\) can provide a linewidth smaller than 100kHz, coarse tuning range of 50nm and continuous tuning of 50GHz.

Two tunable composite cavity InP-LiNbO\(_3\) lasers have been reported in the literature\(^5,6\). The earlier reported configuration consisted of a directional coupler filter used in conjunction with additional phase shifters. A coarse tuning range of 20nm and continuous tuning of 500 MHz were achieved using this configuration. The second configuration utilised a wavelength tunable TE-TM polarisation converter. A total tuning range of 7nm and continuous tuning of 1 GHz were demonstrated with this laser. The linewidths of both reported lithium niobate composite cavity lasers were narrower than 100kHz. The output power was 300\(\mu\)W for the directional coupler configuration and 1mW for the TE-TM converter structure. The main advantage of electro-optically controlled external cavity lasers over bulk external grating configurations is their tuning speed which can be of the order of 10-100 MHz for a conventional electrode structure.

The simplest tunable lithium niobate composite cavity consists of a semiconductor laser coupled to a lithium niobate phase modulator. The change of the lasing frequency \(\delta v\) due to the induced double-pass phase shift \(\delta f_m\) is given as
\[
\frac{\delta v}{\Delta v} = \frac{\delta \phi_m}{2\pi} \quad (5.1)
\]

In Equation 5.1, \( \Delta v \) represents the longitudinal mode spacing. It can be seen that one-way phase shift of \( \pi \) can produce a change of the lasing frequency equal to one mode spacing. The phase shift of \( \pi \) can be produced by a voltage of the order of 10V for a 1cm electrode (Eq. 4.6). Lithium niobate composite cavity lasers can be designed to have the external cavity mode separation frequencies in the range 1-4GHz and a continuous tuning range of the same order should be possible. In practice, the continuous tuning range is smaller than the external cavity mode spacing because of effects related to imperfect anti-reflective coatings. Tuning speed depends on both the electrode structure and its length. For traveling-wave electrodes it is in the GHz range.

Another relatively simple configuration, suitable for implementation in composite cavity lithium niobate lasers is an interferometric structure based on the Michelson interferometer\(^7\). This configuration exhibits improved mode-selection properties and extended tuning ranges compared to ordinary Fabry-Perot resonator configurations. The structure is based on a Y-junction with different arm lengths. The effective reflectivity of this configuration is a sinusoidal function with a period inversely proportional to the branch length difference. If two separate electro-optic phase shifters are applied to the two arms of the interferometer, either continuous tuning or mode-switching can be accomplished depending on the mutual orientation of the applied electric fields.

5.1.2 AM, FM Mode-Locking and FM-Laser Operation

Methods of short pulse generation using semiconductor lasers were discussed in Chapter 3. It was shown that Q-switching and AM mode-locking require an intracavity loss modulator while for FM mode-locking an intracavity phase modulator is needed. Lithium niobate modulators
have the speed required for mode-locking. Moreover, there is a potential for the investigation of harmonic mode-locking at frequencies up to 20GHz with conventional, but relatively inefficient modulator structures. However, because of the relatively long photon lifetimes and the low Q-factors of the cavity in the on-state, composite cavity InGaAsP-LiNbO₃ lasers are not particularly well suited for Q-switching applications.

AM mode-locking using a lithium niobate composite cavity laser has been reported in the literature. The composite cavity laser consisted of an InGaAsP laser coupled to a lithium niobate directional coupler switch by a lensed optical fibre. Mode-locking was achieved at repetition rates up to 7.2GHz and pulses shorter than 22ps were measured using a fast photodetector and a sampling oscilloscope. The average optical power of the laser was 2mW implying a peak power of 50mW. The time-bandwidth product and coherence properties of the pulses were not reported.

Mode-locking of external cavity semiconductor lasers by current modulation can result in optical pulses of the order of 5-10ps. The main disadvantage of the system is the low optical output power. Typical average power of the external grating configuration is of the order of 0.1mW. One potential advantage of AM mode-locking over mode-locking by current modulation is the stronger mode-coupling due to the larger modulation depths that can be achieved for the loss modulation than for carrier modulation. Stronger mode-coupling could lead to shorter pulses and increased powers provided both good optical coupling can be obtained and intracavity reflections can be suppressed.

Mode-locking of a semiconductor laser by current modulation is a combination of AM and FM mode-locking because of the dependence of the refractive index on the carrier density. Mode-locked pulses are chirped, although the amount of chirp is smaller than in the case of directly modulated solitary laser. The chirping properties of the pulses produced by AM mode-locking of semiconductor lasers have not been reported in the literature. However it is clear that these pulses are chirped because of the
linewidth enhancement factor. The important question is whether this chirp is linear and therefore more suitable for compression.

FM mode-locking of semiconductor lasers has received considerably less attention than other types of mode-locking. One of the reasons for this is that FM mode-locking can result in three states of pulsed operation. One state is excited when the timing of the pulse, during its passage through the phase modulator, coincides with the maximum of phase modulation. The second state corresponds to the case when the pulse is formed under the minimum of phase deviation. Finally, there is a possibility of exciting these two states simultaneously so forming a pulse train of double the normal repetition rate. In practice the transition between these states can be achieved by slight detuning of the modulating frequency or by the adjustment of the external cavity.

FM-laser operation takes place when the modulating frequency of the intracavity phase modulator is detuned from the round-trip frequency. The difference between FM mode-locking and FM-laser operation is that the first one is a pulsed mode of operation while the second one is a constant amplitude, but frequency-swept type of operation. FM-laser operation of Nd:YAG lasers can result in very broad FM spectra of the order of $120\text{GHz}$. FM-laser operation has been observed in InGaAsP lasers. An antireflective coated laser was placed in a bulk external cavity, biased well above threshold, and modulated with the small signal current in the vicinity of the round-trip frequency. FM bandwidths of the order of $9\text{GHz}$ were observed for a modulating frequency of $500\text{MHz}$. FM-laser operation was accompanied by a parasitic amplitude modulation in the range from 1 to $10\%$.

Integrated optic lithium niobate modulators can provide pure loss and phase modulation so allowing the investigation of both AM and FM mode-locking as well as FM-laser operation of semiconductor lasers. High-
frequency modulators can be designed as bulk, travelling-wave or standing-wave modulators (Chapter 4). Bulk modulators have to be very short and inefficient if designed for operation above 2 GHz. Therefore, there is no potential for additional pulse shortening due to an increase of the mode-locking repetition rate. Standing-wave modulators in lithium niobate offer very efficient narrow-band modulation at high frequencies in the range from 10 to 30 GHz. However, in this case, the efficient modulation can be obtained only for a set of harmonics. The main reason why we decided to use the travelling-wave structures is they can provide an efficient modulation in the range from DC to 10GHz. This allows the flexibility of changing the composite-cavity length and therefore the means of intracavity optical coupling between the laser and the modulator. Additionally, it would be possible to investigate harmonic mode-locking with any chosen composite-cavity configuration.

Travelling-wave lithium niobate modulators fill a significant part of the composite laser cavity. In this respect, InGaAsP-LiNbO₃ lasers are different from the common mode-locking configurations consisting of a long external cavity and a relatively short modulator. A brief discussion of the intracavity modulation efficiency of both bulk and travelling-wave modulators is presented in Chapter 7.

5.1.3 Technical Aspects of the Composite Cavity

In the last section of Chapter 2, the external cavity operation and feedback regimes of a semiconductor laser were discussed. We noted that linewidth narrowing and stable operation can be achieved for either very low feedback levels (~50dB) or for high levels of the order of ~10dB. In the first case, the linewidth is influenced by the phase of the returned light¹²,¹³, and this regime is similar to coherent light injection. External cavity devices are in general designed to operate in the strong feedback regime in which the stability of the system is not affected by the position of the external reflector.
Active mode-locking of lasers with short carrier lifetimes (e.g. semiconductor laser) and long carrier lifetimes (e.g. Nd:YAG laser) differ in respect to the pulse shaping mechanisms. In the case of AM mode-locking of a Nd:YAG laser, the circulating optical pulse is sharpened only by the amplitude modulator while in the case of a semiconductor laser it is additionally sharpened by the dynamic gain saturation. In order to utilise this additional pulse sharpening mechanism in semiconductor lasers it is necessary to achieve a sufficient level of optical feedback. Theoretically, a double-pass coupling efficiency of the order of 10% is needed for efficient mode-locking according to a treatment based on travelling-wave rate equations\(^{12}\).

The quality of the anti-reflective coating usually proves to be limiting factor on the pulse width in actively mode-locked systems utilising a bulk external cavity. A theoretical treatment based on traveling-wave field equations\(^{13}\) predicts pulse widths longer than 20ps for parasitic reflectivities of the order of 1%. In the case of composite-cavity InGaAsP-LiNbO\(_3\) lasers an additional antireflective coating on one lithium niobate facet is required. Even in the simplest case, in which the semiconductor laser is directly coupled to lithium niobate waveguide, composite cavity effects may limit the mode-locking performance.

5.1.3.1 Optical Coupling

Direct coupling, which is also known as butt or end-fire coupling, represents the simplest way of forming a composite cavity InGaAsP-LiNbO\(_3\) laser. The coupling efficiency can be calculated from the overlap between the laser and waveguide mode distributions. If these distributions are assumed to be Gaussian, the maximum coupling efficiency, assuming both perfect alignment and zero separation between the laser and waveguide, can be expressed as:
Fig. 5.1 Intracavity optical coupling schemes
a) direct coupling
b) coupling via antireflective coated graded index lens
c) scheme utilising antireflective coated lensed fibre
where \( w_{lx}, w_{ly}, w_{gx} \) and \( w_{gy} \) are half-widths of the Gaussian fields at 1/e points. Typical buried heterostructure semiconductor laser can have a mode-size of the order of 0.8 x 1.3 \( \mu \text{m} \). The mode size of titanium indiffused waveguides is bigger though it can be tailored to some extent. For most applications, the waveguides are designed for efficient fibre-coupling. Despite the fact that lithium niobate waveguide mode shapes are elliptic, fibre to waveguide coupling efficiencies of the order of 0.5 dB can be achieved. In terms of modulation efficiency, tightly confined waveguides have an advantage over the large mode sizes optimised for fibre coupling because of the increased overlap between the guided optical and the applied modulating field. A mode-size of 2.9 x 5.5 \( \mu \text{m} \) has been reported for TE propagation in X-cut titanium indiffused lithium niobate waveguide. Using Eq. 5.2 and the above data for a buried heterostructure laser and lithium niobate waveguide, a one-way coupling efficiency of 20% can be deduced. This would imply an estimation of 4% for the light returned back into the semiconductor laser due to mode mismatch only. This mode-mismatch can be improved by using proton exchange technology, though in this case an increase in the propagation loss and possible degradation of the electro-optic coefficients may result.

The most efficient coupling between a semiconductor laser and titanium indiffused lithium niobate waveguide can be achieved using a lensed single-mode fibre. A coupling efficiency of the order of 50% can be expected if the waveguide mode size is well matched to a fibre mode size. However, the difficulty of this approach is obtaining a low-loss and permanent connection between the single-mode fibre and the lithium niobate waveguide.

An alternative way of coupling between the laser and the lithium niobate waveguide would be to use a lens as a coupling element. Anti-reflective coated spherical lenses are often used for laser diode to single-mode

\[
k = 4 \frac{w_{lx}w_{gx}}{w_{lx}^2 + w_{gx}^2} \frac{w_{ly}w_{gy}}{w_{ly}^2 + w_{gy}^2}
\]
fibre coupling. Coupling efficiencies of the order of 3dB have been reported for a double lens arrangement and 4dB for a single lens\(^{14}\). Anti-reflective coated graded-index (GRIN or SELFOC) lenses are also used for semiconductor laser to fibre coupling. Coupling efficiencies better than 3dB have been reported for plano-convex graded-index lenses in the single lens configuration\(^{17}\).

5.1.3.2 Antireflective Coatings

The main drawback of the hybrid cavity approach to mode-locking is the need for deposition of high quality anti-reflective coatings on all reflective surfaces within the extended cavity. In the case of a composite cavity InGaAsP-LiNbO\(_3\) laser, anti-reflective coatings should be deposited on one semiconductor laser facet and one lithium niobate facet. Commercially available spherical and graded-index lenses are usually AR-coated and they represent a convenient choice for the optical coupling element. If a lensed fibre is to be used as a coupling element, there is a need for the deposition of an anti-reflective coating on both the tip of the lens and the lithium niobate chip at the interface (Fig.5.1c).

The deposition of anti-reflective coatings on semiconductor laser facets has been thoroughly investigated because of the importance of travelling-wave semiconductor optical amplifiers and their potential for applications in optical communication systems. Reflectivities smaller than 0.1%, in the range of 40nm, have been achieved by the proper design of a single-layer coating and a thorough control of fabrication parameters\(^{18}\). In the plane-wave approximation the thickness of the coating and its refractive index are given as

\[
d_{ar} = 2(m+1) \frac{\lambda}{4} \quad m=0, 1, 2, ...
\]

\[
\mu_{ar} = \sqrt{\mu}
\]

where \(\lambda\) represents the lasing wavelength and \(\mu\) is the refractive index of the semiconductor laser. However, if even lower reflectivities are to be
achieved, the optimal thickness and refractive index of the coating have to be calculated as functions of the laser mode profile. Additionally, there is a need for in situ monitoring during the actual deposition of the anti-reflective coatings. Alternatively, very low effective reflectivities can be achieved using angled-stripe lasers and anti-reflective coatings of lesser quality\textsuperscript{19}. Indeed, effective facet reflectivities of the order of 0.05\% have been demonstrated using a laser with angled facets (7° incidence angle) and anti-reflective coatings having reflectivities exceeding 1\%.

Single-mode semiconductor lasers are often used in conjunction with fibre pigtailed integrated optic lithium niobate modulators for laboratory optical communications experiments. Reflections of the order of 4\% are created at the fibre to lithium niobate interface and these reflections can significantly degrade the laser performance (Chapter 2.3). The best way of protecting the laser from these spurious reflections is by using an optical isolator. As an alternative method, the antireflective coating of the lithium niobate devices has been investigated and reflections smaller than -35dB (0.03\%) have been obtained using single-layer yttrium oxide films\textsuperscript{20}.

5.1.3.3 Mirrors

A composite cavity InGaAsP-LiNbO$_3$ laser is bounded by two mirrors, one on the semiconductor laser side, and the other on the lithium niobate chip. The output of the composite cavity laser can be taken either from the semiconductor or lithium niobate side. The advantage of taking the output from the lithium niobate side is that more efficient coupling into a single-mode fibre can be achieved. In this case, a semi-reflective mirror should be formed on the lithium niobate substrate, and an additional mirror should be deposited on the semiconductor laser chip to compensate for the relatively large total loss of the composite-cavity laser. A simpler solution would be to take the output from the semiconductor laser cleaved facet, and to deposit a highly reflective mirror on the lithium niobate chip.

High reflectivity mirrors can be formed either by metal evaporation or by
deposition of a multi-layer dielectric coating. Gold and silver are suitable for the mirror deposition because of the reflectivities which are larger than 95% at 1.5μm. Multi-layer dielectric coatings can have reflectivities in excess of 99%. Additionally, dielectric mirrors have a very small transmission loss, so any residual light can be used for the alignment of the composite-cavity laser.

A high reflectivity multi-layer dielectric coating is usually designed as a stack composed of quarter-wave layers of alternately high and low index materials. For an odd number of layers used if the substrate refractive index is low, the reflectivity of the m-layer stack is given as

\[
R = \left(\frac{\mu_o - \left(\frac{\mu_{\text{high}}}{\mu_{\text{low}}}\right)^{m-1} \frac{\mu_{\text{high}}^2}{\mu_{\text{sub}}}}{\mu_o + \left(\frac{\mu_{\text{high}}}{\mu_{\text{low}}}\right)^{m-1} \frac{\mu_{\text{high}}^2}{\mu_{\text{sub}}}}\right)^2
\]  

(5.5)

where \(\mu_o, \mu_{\text{sub}}, \mu_{\text{high}}\) and \(\mu_{\text{low}}\) represent the refractive indexes of the environment, substrate, and high and low index materials used in the stack, respectively. According to Eq. 5.5 the reflectivity increases if a larger number of layers is used or if the ratio of \(\mu_{\text{high}}\) to \(\mu_{\text{low}}\) is increased by choosing a different combination of materials.

The reflectivity of the waveguide-mirror combination is very sensitive to an angular offset from the right angle. The importance of the mirror tilt has been examined in both for composite cavity InGaAsP-LiNbO\(_3\)\(^5\) and extended fibre cavities\(^21\). Excess loss due to the mirror tilt can be estimated from several theoretical formulations reported in the literature\(^22,16\). According to Ref. 22, at 1.5μm the mirror tilt should be smaller than 0.5° if the excess loss is to be kept below 1dB.
5.2 Experimental Work

Our experimental work on composite-cavity InGaAsP-LiNbO$_3$ lasers includes the design and fabrication of both intensity and phase modulators in lithium niobate, as well as testing of composite-cavity structures utilising these components. Short optical pulse generation has been accomplished using three different types of active mode-locking namely AM, FM and mode-locking by current modulation. However, because of large intra-cavity losses and pronounced parasitic reflectivities, lithium niobate composite-cavity lasers have exhibited inferior performance compared to their bulk external cavity counterparts (Chapter 6).

In this project we decided to use titanium indiffused lithium niobate waveguides. This choice was dictated by the time-scale of the project and the previous research conducted in Integrated Optics group at UCL. Both intracavity intensity and phase modulators were conceived as travelling-wave structures because of their broad-band high frequency operation. The electrode interaction length of 1 cm was used in order to achieve efficient modulation at frequencies up to 10 GHz. The electrode structures were based on previous UCL designs. A new set of masks was designed and the fabrication of several intensity and phase modulators was accomplished utilising fabrication techniques developed at UCL. Although the fabrication of lithium niobate devices took nearly two years to complete, for the sake of brevity, only a short summary of the fabrication steps and the fabrication parameters are presented in Appendix 2.

The semiconductor lasers used in this project were supplied by BTRL, Ipswich. These lasers were 1.55μm buried heterostructures with a modulation ability in the 1-2 GHz range. Broadband anti-reflective coatings with reflectivities lower than 0.5% were applied to one laser facet. These lasers were not suitable for direct coupling because the antireflective coated facet was not aligned to the edge of the heat sink. For
direct coupling, the distance between the laser and waveguide should be ideally kept below 10μm. In the package that we used, the laser facet was separated from the edge of the heat sink by 200μm.

The optical coupling element used in our experiments was a commercially available graded-index lens. The lens had a pitch of 0.29 and it was antireflective coated to 1.5%. Anti-reflective coatings and mirrors on the lithium niobate were designed and fabricated at Kendall & Hyde, Basingstoke. The anti-reflective coating was designed as a single-layer SiO₂ coating having a reflectivity of 0.3%. The mirror was conceived as a multi-layer stack of TiO₂/SiO₂ with the reflectivity of 99%.

5.2.1 Experimental Set-Up

The experimental set-up used in our short pulse generation experiments is schematically presented in Fig. 5.2. Laser bias current was provided from the ILX-3270B semiconductor laser power supply. The microwave synthesiser was a Systron Doner model 1618 having a frequency range from 2-18 GHz. The amplifier used in our experiments was a Mini-Circuits ZHL-42 providing 800mW of microwave power over the range 0.7 to 4.2 GHz. A bias-T circuit was used in order to combine the microwave and DC signals for driving the semiconductor laser or lithium niobate modulator (the second case is presented in Fig. 5.2).

Our composite-cavity lasers were assembled on an optical table using high precision micropositioners. The GRIN lens positioner had piezoelectric translators with a range of 10 microns and accuracy of 0.1μm.

The semiconductor laser was mounted on a copper heat sink and the RF connection was made using a semi-rigid coaxial cable with an SMA connector on one end. A series resistor of 47Ω was used in order to reduce microwave reflections and protect the microwave amplifier. The laser package was placed on a micropositioner with tilt control. The output of the semiconductor laser was taken using a piece of a cleaved single-mode
Fig. 5.2 Mode-locking Experimental Arrangement
fibre. The reflectivity of the fibre to air interface was 4% and the fibre was slightly tilted in order to reduce the backreflection into the laser. The coupling efficiency between the laser and the cleaved fibre was estimated to be of the order of 10-15%.

The fast photodetector used in experimental set-up was a Plessey photodiode with a bandwidth of 14GHz. The oscilloscope comprised a Tektronix 7854 mainframe, a sampling unit 7S11, a sampling head S4 and sampling sweep unit 7T11A. The response time of the measurement system was estimated to be of the order 42ps.

An infra-red camera was used for the alignment of the composite cavity system. A small amount of light, leaking through the dielectric mirror on lithium niobate was imaged on the camera vidicon plate using a microscope objective. The initial stage of the alignment procedure involved launching the laser light into the lithium niobate waveguide; while the second phase of the alignment involved adjusting the tilt of the laser in order to couple the reflected light back into the laser active area.

Ideally, the alignment of the composite cavity laser should be monitored using either an optical spectrum analyser or a fast photodetector in conjunction with an amplifier and a microwave spectrum analyser. We used a scanning Fabry-Perot analyser and observed unstable operation and mode-hopping both for lithium niobate and bulk external cavities. Assuming a well aligned cavity, the mode-hopping could be attributed to the lack of optical isolation and the introduction of time-varying reflection if the scanning Fabry-Perot is used.

5.2.2.1 Estimation of the Intracavity Optical Feedback Strength

An estimation of the optical feedback into the semiconductor laser in an external cavity configuration can be obtained from the difference in threshold current of the laser with and without external cavity\textsuperscript{23}. A schematic drawing of the external cavity laser is presented in Fig. 5.3. A
semiconductor laser of the length L and the internal loss \( \alpha \) has mirror reflectivities \( R_1 \) and \( R_2 \). The amount of optical feedback \( x \) (Fig. 5.3) is determined by the coupling and reflection losses of the external cavity. Assuming a linear variation of the threshold current with the total optical loss, the threshold currents of the solitary and external cavity laser can be expressed as

\[
I_{th} \sim 2\alpha L - \ln R_1 - \ln R_2 \tag{5.6}
\]

\[
I_{th}^* \sim 2\alpha L - \ln R_1 - \ln [R_2 + (1 - R_2)x] \tag{5.7}
\]

Using Eqs. 5.6 and 5.7, the relative change in threshold current due to the feedback from the external cavity \( x \) can be written as

\[
\frac{\Delta I_{th}}{I_{th}} = \frac{I_{th} - I_{th}^*}{I_{th}} = \frac{\ln \left( 1 + x \frac{1 - R_2}{R_2} \right)}{2\alpha L - \ln R_1 R_2} \tag{5.8}
\]

Equation 5.8 is plotted in Fig. 5.4 for the following parameter values: \( \alpha L = 0.8 \), \( R_1 = 32\% \), \( R_2 = 0.5\% \) and \( x \) in the range from 0 to 30%.

5.2.2.2 Estimation of the Quality of Anti-reflective Coating on the Laser Facet

The reflectivity of antireflective coated semiconductor laser facet can be estimated from the optical power measurements of the solitary laser. The reflectivity of the coated facet \( R_2 \) can be calculated from the ratio of differential quantum efficiencies of the coated and uncoated laser facet\(^{24} \). A consequence of the different reflectivities of laser facets is that the output power at each facet is different. If the output powers of the two facets are measured for the same value of bias current, and if the measurement is taken from the linear portion of \( L/I \) curve, then following formula can be used:
Fig. 5.3  Schematic diagram of an external cavity laser

Fig. 5.4  Estimation of the optical feedback from the relative change of threshold current of the solitary and external cavity laser
The implication of Eq. 5.9 is that for a 1% antireflective coating on one facet, the ratio of output powers from the coated and uncoated facet would be of the order of 8.

5.2.3 Mode-Locking Utilising Laser Current Modulation

Short pulse generation was accomplished using three different InGaAsP-LiNbO$_3$ configurations whose schematics are presented in Fig. 5.5. The first configuration is a directly modulated semiconductor laser coupled to a short waveguide in lithium niobate (Fig. 5.5.a). The second configuration comprises a travelling wave Y-junction intensity modulator in lithium niobate (Fig. 5.5b) while the third is based on a travelling-wave phase modulator (Fig. 5.5c).

The main aim of using the laser coupled to the short waveguide was to estimate the minimum intracavity loss which could be achieved with a composite cavity lithium niobate configuration. The length of the lithium niobate sample was chosen to accommodate a short traveling-wave electrode which would be used in the second generation of our devices. Additionally, a set of waveguides was available on the sample offering the possibility of choosing the best combination of waveguide, anti-reflective coating and dielectric mirror. The advantage of using the laser current modulation was that a direct comparison could be made using the same laser in both lithium niobate and bulk external cavity.

The semiconductor laser used in this experiment was the 1.55μm, BTRL buried heterostructure laser (ser. no. 10306, wafer 936B). The laser was anti-reflective coated on one facet. The reflectivity of the coating was estimated to be better than 0.3% using light-current curves measured by BTRL and Eq. 5.9.
Fig. 5.5 Three mode-locking InGaAsP-LiNbO₃ configurations
a) semiconductor laser current modulation  
b) intracavity amplitude modulation  
c) intracavity phase modulation
The straight waveguides were fabricated on 1.2 cm long, X-cut, Y-propagating lithium niobate sample using a long titanium diffusion in dry oxygen. Silicon dioxide buffer layer of the thickness of 0.2 μm was deposited on the top of the sample. An anti-reflective coating and a dielectric mirror were deposited on two opposite edges of lithium niobate sample, perpendicular to the waveguides. Under visual inspection, it was noticed that the coatings were not uniform in the area close to the edges of the sample.

A composite InGaAsP-LiNbO₃ cavity was set-up using a Melles Griot graded index lens. A significant leakage of the light through the dielectric mirror was observed for several waveguides on the sample. For the best waveguide-mirror combination a reduction of the threshold current from 65 to 42 mA was measured. The optical feedback due to the lithium niobate external cavity was estimated to be of the order of 6-7% (Eq. 5.8) corresponding to ~12 dB of the intracavity loss. This loss comprised of an optical coupling loss of the order of 10 dB, waveguide propagation loss of the order of 1 dB, and combined mirror leakage / tilt loss also of the order of 1 dB.

For short pulse generation, the semiconductor laser was biased above threshold and driven with 800 mW of RF power in the vicinity of 3.8 GHz. The best pulses were achieved using the DC bias of 90 mA (Fig. 5.6). A pulse width of 35 ps was estimated from an averaged oscilloscope trace of 55 ps. The output power measured from the fibre was 250 μW. Double pulses were observed for lower bias currents and detuned frequencies (relative to the frequency at which we detected the shortest pulses). The double pulses were probably caused by the parasitic reflection from the antireflective coated lithium niobate facet.
**Fig. 5.6** Mode-locking of the InGaAsP-LiNbO$_3$ laser by current modulation: sampling oscilloscope traces

a) modulating frequency 3.814 GHz (hor. scale 100ps/div)

b) modulating frequency 3.817 GHz (hor. scale 20ps/div)
5.2.4 AM Mode-Locking Experiment

The composite cavity configuration used in this experiment is shown in Fig. 5.5.b. The same laser was used as in section 5.2.3. The modulator was designed as a travelling wave Y-junction modulator. Effectively, within the laser cavity it acted as a Mach-Zehnder interferometer structure. The waveguides were formed using a long, dry diffusion on X-cut, Y-propagating lithium niobate. The Y-junction was formed using a branching angle of 0.5° in order to provide low-loss splitting and recombination. The distance between the branching waveguides was 35µm, large enough to prevent any optical coupling. A 200nm SiO₂ buffer layer was deposited on the top of the waveguides in order to reduce the electrode loading loss. The electrodes were designed as a coplanar waveguide structure allowing push-pull operation of the intensity modulator. An interaction length of 1cm was chosen as a compromise between high-frequency capability and efficient modulation. The widths of the central electrode and the gap were 20µm and 15µm leading to the impedance of 35Ω. The electrodes were fabricated using chrome and gold evaporation and electroplated to the thickness of 1.1µm. The measured DC resistance of the central electrode was 11Ω. This modulator was fixed to an aluminium holder and electrical connection was made with SMA connectors. The electrical return loss of the modulator was of the order of -10dB in the frequency range from 3-18GHz. Two resonances of the order of -6 to -7dB have been detected below 3GHz and were probably caused by the modulator package.

When the modulator was biased for maximum transmission, the threshold current of the of the composite-cavity configuration was 52mA. This implies the optical feedback of the order of 2% or the intracavity loss of 17dB. The leakage of light from the dielectric mirror was quite high, of the order of 3dB. The coupling loss was expected to be around 10dB, leaving 4dB for the waveguide propagation and splitting losses.

The composite cavity laser had a switching voltage of 2.75 V, which was
due to double-pass phase shift of the push-pull electrode structure. Regarding the efficiency of the modulator, this would correspond to a $V_\pi$ of 5.5 V for a push-pull intensity modulator or 11 V for an equivalent phase modulator. The ON/OFF power ratio of the composite cavity laser was 1.6 (2dB).

The round-trip frequency of the composite cavity was estimated to be of the order of 1.8GHz. Mode-locking was attempted at the second harmonic of the round-trip frequency. The lithium niobate modulator was driven with 800mW of power in the vicinity of 3.6 GHz. Pulsing was observed when the modulator was biased for minimum transmission. In order to measure the pulse width we had to increase the bias current of the laser up to 100 mA. The shortest pulses were 65 ps on the oscilloscope or around 50 ps deconvolved. The average optical power, measured from the fibre was 50 $\mu$W. One sampling oscilloscope trace, obtained for a 100mA bias and full available RF power at 3.535 GHz is presented in Fig. 5.7.

2.5 FM Mode-Locking experiment

The composite cavity structure utilising a lithium niobate phase modulator is shown in Fig. 5.5.c. The semiconductor laser used in this experiment was a BTRL device (ser. no. 10315, wafer 936B). In every respect it was similar to the laser used in the previous experiments, apart from the anti-reflective coating which had an estimated reflectivity of 0.4%. Waveguides for the phase modulator were fabricated using long titanium diffusion in dry oxygen atmosphere on X-cut, Y-propagating lithium niobate. No buffer layer was used on this sample. The electrodes were formed as an asymmetric coplanar line. The interaction length was 1cm and the $V_\pi$ of the device was estimated to be 12V. The electrode width and gap were 16$\mu$m and 8$\mu$m respectively, leading to the impedance of 39Ω. The electrodes were electroplated to 1.2$\mu$m and the DC resistance of 8.5Ω was measured for the narrow strip.
Fig. 5.7 AM mode-locking of the composite cavity InGaAsP-LiNbO$_3$ laser: sampling oscilloscope trace
modulating frequency 3.535 GHz
horizontal scale 100ps/div, vertical scale 2mV/div
The modulator was attached to an aluminium holder. SMA connectors were used to form an electrical connection. The electrical return loss of the packaged modulator was measured in the region from 1-18 GHz. It was found to be better than -9dB up to 9GHz. The antireflective coatings and dielectric mirrors on this device differed from our previous lithium niobate samples in that prior to polishing, two small lithium niobate blocks were glued on the top of the sample, above the edges of the sample to be coated. Our aim was to provide a uniform surface for coating, both below and above the waveguide. However, even in this case, non-uniform coatings were obtained.

A composite cavity was formed using a graded-index lens as a coupling element between the semiconductor laser and the modulator. For the best optical alignment, the threshold current was reduced from 65 mA to 50 mA. The optical feedback due to the lithium niobate cavity was estimated to be around 3%. This corresponded to 15 dB of intracavity loss. Assuming 10 dB of the optical coupling loss, the waveguide loss and mirror leakage/tilt were estimated to be 3 dB and 2 dB respectively.

The round-trip frequency of the composite cavity laser was estimated to be in the vicinity of 1.9 GHz. Mode-locking was attempted on the second harmonic. The phase modulator was driven with 400mW of the RF signal and the laser was biased at 85-90 mA. Pulses of 60ps (~45ps deconvolved) were observed around 3.9 GHz. The average optical power, measured from the output fibre was 150 μW. A sampling oscilloscope trace of FM mode-locked pulses are shown in Fig. 5.8. The pulses were placed on the top of a relatively small DC pedestal. Double pulsing was observed with the ratio of the main pulse to the secondary of approximately 3:1. This effect could not be suppressed by small detuning of the modulating frequency, and it seemed that it was a composite Fabry-Perot cavity effect.

Using a different microwave synthesiser (HP 8462B) within the experimental set-up, mode-locking at the round-trip frequency of the composite cavity was attempted. Pulses of 130ps were obtained at the
**Fig. 5.8** FM mode-locking of the composite cavity InGaAsP-LiNbO$_3$ laser: sampling oscilloscope trace
modulating frequency 3.915 GHz
horizontal scale 100ps/div, vertical scale 2mV/div
frequency of 2 GHz. As in the previous case, we observed double pulsing; though in this case, FM mode-locking resulted in the formation of two pulse trains of different average pulsewidth.

Finally, an attempt was made to mode-lock the composite cavity configuration by modulating the laser instead of the lithium niobate modulator. Double pulses of the order of 60ps, and separated by ~100ps were generated at 1.9GHz. The separation between these pulses corresponds to the round-trip time of the cavity formed by the laser chip, GRIN lens and the front facet of the LiNbO$_3$ chip (Fig. 5.5c). The ratio of the main pulse to the secondary was approximately 2:1. This result for mode-locking by current modulation of the InGaAsP-LiNbO$_3$ laser confirmed the existence of composite cavity effects in our FM mode-locking system.
Chapter 6

Active Mode-Locking Using Bulk External Cavities

This chapter is devoted to a description of our experimental work on active mode-locking of semiconductor lasers within several bulk external cavity configurations. This work has spanned a period of three years and during this period our experiments have evolved greatly. This chapter is divided into two sections. The first section describes the work which was accomplished at UCL. Subsection 6.1.1 presents the experimental work which is directly compatible with our lithium niobate work (Chapter 5). The experimental work presented in subsection 6.1.2 was performed using an improved experimental set-up.

The second section of this chapter deals with the experimental work conducted at NPL, Teddington. The principle benefit of working at NPL was that the measurement facilities included an HP lightwave analyser and a fast sampling oscilloscope. This apparatus enabled us to measure exact round-trip frequencies as well as to monitor the stability of mode-locked operation. The later capability is particularly relevant to investigations of both frequency detuned behaviour and harmonic mode-locking.

Our short pulse generator installed at NPL consists of an actively mode-locked external grating laser in conjunction with a semiconductor optical amplifier. The semiconductor laser used within this external cavity belongs to a more advanced generation of buried heterostructure BTRL devices. It has a lower threshold current and higher modulation capability compared to the lasers which were used for our lithium niobate composite cavity work. Our best experiments resulted in pulses of 7-8ps within the used set-up. We believe even shorter pulses are possible with our set-up if the packaging of the laser is modified for high-frequency
operation.

6.1 External Mirror Configurations

Several external cavity configurations incorporating a plane mirror were investigated during the course of our experimental work at UCL. As alluded to, the first configuration we present may be compared with our lithium niobate composite cavity work. Autocorrelation measurements were performed in addition to the estimation of the pulsewidth from sampling oscilloscope traces. The second configuration resulted from our attempts to measure the spectrum of a mode-locked laser using a scanning Fabry-Perot resonator. Despite the limited finesse of the FP instrument, we managed to resolve individual external cavity modes. This was achieved using a modified external cavity configuration.

6.1.1 First Experimental Configuration

Our first experimental configuration is shown in Fig. 6.1, and is similar to the experimental set-up similar employed in Chapter 5.2. The laser is the BTRL device (ser. no. 10306) which was used for mode-locking by current modulation and AM mode-locking of composite cavity lithium niobate structures. The graded-index lens has a pitch of 0.23 and is antireflective coated to 1.5%. The external mirror is an aluminium coated, plane mirror. The output of the mode-locked laser is taken from the non-coated semiconductor laser facet via cleaved fibre end. The output is split using a 3dB single-mode fibre coupler. One branch is fed to the photodiode and the sampling oscilloscope, as described in the previous chapter. The light emerging from the other branch of the coupler is collimated using a microscope objective and fed into an Inrad 5-14LD autocorrelator. This autocorrelator is a slow-scan device with a linear mirror motion. The sensitivity of the autocorrelator which is utilising a 5mm thick LiNbO₃ crystal, is specified to be 1mW² (average power x peak power).
Fig. 6.1 Mode-locking experimental arrangement
The external cavity configuration was 7.5cm long, and the corresponding round-trip frequency was in the vicinity of 2GHz. The threshold current of the external cavity laser was 28mA. Reduction of the threshold current from 65mA (solitary laser) to 28mA corresponded to a level of optical feedback of the order of 29%. This bulk external cavity configuration was superior to our lithium niobate composite cavities in both optical coupling and intra-cavity losses. For the sake of comparison, the levels of optical feedback achieved for lithium niobate configurations utilizing the short waveguide, intensity and phase modulator were 7%, 2% and 3% respectively.

For mode-locked operation, the laser was biased at or above threshold, and driven with full RF power (800mW) in the vicinity of the round-trip frequency. The equipment used for driving the laser is listed in Chapter 5. The shortest pulses, of the order of 50ps on the scope, were detected when the laser was biased just above the threshold, at around 30mA. The peak voltage, which was observed on the sampling oscilloscope was higher than the corresponding peaks obtained for AM and FM mode locking using a bias in the range from 85 to 100mA. Increasing the bias from 30 to 40 mA in this configuration lead to a doubling of the peak power and to an increase of the average power in the fibre up to 200μW, on account of the slight broadening of the pulses. Further increase of the bias, up to 50mA, was leading to significant pulse distortion. As in the case of lithium niobate configurations, we were not equipped to measure the exact round-trip frequency, so no attempt was made to investigate frequency detuned behaviour.

The autocorrelation trace, obtained at the bias of 40mA is presented in Fig. 6.2. The full width at half maximum of the trace is 38ps. Assuming a Gaussian pulse shape this corresponds to a theoretical pulsewidth of 27ps. We were not able to produce satisfactory autocorrelation traces in the region of the laser threshold, because of the limited sensitivity of the instrument.
Fig. 6.2 Mode-locking of the bulk external cavity laser, 1st configuration (no intra-cavity filtering):
autocorrelation trace
f=2.017GHz, I_{dc}=40mA, P_{rf}=800mW
full scan 152ps, FWHM=38ps (Gaussian 27ps)
A coherence spike, which is an indication of partial mode-locking, was observed in all our measurements. Theoretically, the coherence spike can be explained using the concept of a spectrally filtered thermal radiation modulated by a temporal pulse envelope. In practice, partial mode-locking is a consequence of independent mode-locking of several clusters of external cavity modes due to composite cavity Fabry-Perot effects.

In our experiment, mode-locking was also achieved at the second harmonic of the round-trip frequency (approximately 4GHz). The laser was biased at 60mA for the autocorrelation measurements. Traces of 40ps were obtained and they suggest a pulsewidth of 29ps on the assumption of a Gaussian pulse shape.

6.1.2 Second Experimental Configuration

Our second external cavity configuration and measurement set-up are presented in Fig. 6.3. The semiconductor laser used in this experiment was the BTRL device (ser. no. 10315) which was also used for FM mode-locking of the lithium niobate composite cavity. The external cavity was formed from an antireflective coated spherical lens of the diameter of 2mm, and a plane mirror. The main advantage of the spherical lens over the graded index lens used in our previous experiments was a much smaller reflectivity of ~0.25%. An uncoated glass etalon of thickness of 0.25mm was used to provide some bandwidth control which was needed to suppress cluster mode-locking. The output of the laser was taken by a BT&D single-mode lensed fibre. The fibre was spliced to a polarisation insensitive optical isolator, BT&D OIC1100-1550, which provided 40dB of isolation. The overall coupling efficiency from the laser to the measurement system was more than doubled, despite the insertion loss of the isolator of 1.5dB.

The photodiode used in this experiment was a BTRL device boxed by NPL. The bandwidth of this device was 22GHz. The sampling oscilloscope comprised a Tektronix 7854 mainframe and 7T11A, 7S11 and S4 units,
Fig. 6.3 Mode-locking experimental arrangement: spectrum measurement
also used in our previous experiments. The response time of this measurement system was estimated to be around 31ps.

The scanning Fabry-Perot resonator, Tec-Optics FRI-25, was set-up using a 1.52 μm He-Ne laser. The best finesse which was achieved with this optical spectrum analyser was 45. Our aim was to resolve the external cavity modes, so the instrument was initially set-up with a free spectral range of 35GHz. During the course of the experiment the free spectral range was increased to 60GHz, while still resolving the external cavity modes. We have also used larger free spectral ranges to display only the laser diode modes.

The external cavity was set-up to be 7.5 cm long, and having a round-trip frequency of ~2GHz. The threshold current was 32mA, implying an optical feedback of around 18%. For comparison, the external cavity utilising the graded-index lens had an estimated feedback of 29%. Single-mode operation of the external cavity laser was achieved using an intra-cavity étalon. For mode-locking, the laser was biased at 34mA and driven with 800mW of RF power at ~2GHz. The average output power, measured from output of the fibre coupler was around 50μW.

The shortest pulses achieved using this experimental arrangement were of the order of 35ps on the oscilloscope, or around 16ps deconvolved. These pulses were approximately 10ps shorter than the pulses produced by our first experimental set-up (Chapter 5.1.1). We believe the major factor for pulse shortening was the reduction of external reflections, achieved by the utilisation of the lensed fibre and optical isolator. Though the reduction of intracavity reflections (spherical lens instead of graded-index lens) and the inclusion of intra-cavity filtering were also of importance. Using the same semiconductor laser within an external cavity containing a graded index lens, pulses of 37ps pulses (20ps deconvolved) were produced.

In this experiment our aim was to measure the spectrum of the mode-locked laser. This was achieved only in the case of frequency detuned
Fig. 6.4 Mode-locking of the bulk external cavity laser, 2nd configuration (with intra-cavity filtering):

a) sampling oscilloscope trace
b) scanning Fabry-Perot resonator trace
detuned operation $f=2.000\text{GHz}$ (shortest pulses at $f=2.015\text{GHz}$)
operation, which resulted in broader pulses. For example, one sampling oscilloscope trace and the corresponding spectrum for mode-locking at 2GHz are presented in Fig. 6.4. Time-bandwidth product of the order of 0.6 can be estimated for pulses of the order of 40ps (51ps on the oscilloscope).

For shorter pulses, the external cavity modes could not be properly resolved and the measured spectrum would become nearly flat. We did not manage to significantly improve the measurement with the increase of the free spectral range of the instrument up to 65GHz.

6.2 External Grating Configuration

Preliminary experimental work accomplished at NPL, Teddington consisted of testing several external cavity configurations which utilised different grating and optical coupling elements, as well as two types of electrical drive for the semiconductor laser. Our final configuration consisted of a mode-locked laser followed by a semiconductor optical amplifier. The amplifier was used to provide sufficient optical power for testing of a fast GTE photodiode.

6.2.1 Initial Experiments

Our initial experiment involving the choice of electrical drive was performed using the 1.57μm BTRL, buried heterostructure laser (ser.no. 21924, wafer 1140). Compared to those employed in our previous experiments, this laser was a more advanced structure having threshold current of around 45mA and modulation bandwidth of the order of 4GHz. The laser was anti-reflective coated on one facet to approximately 0.3%. The external cavity was formed using a graded-index lens, pitch 0.23 and a Milton-Roy grating, 600l/mm blazed for 1.6μm. The output of the laser was taken using a lensed fibre, optical isolator and fibre coupler (as presented in Fig. 6.3). Optical pulses generated by this mode-locked laser were measured using a BT&D photodiode in combination with a Tektronix mainframe CSA 803 and SD-26 sampling head. The response time of the
system was estimated to be around 25.5ps. The output was also monitored on an HP Lightwave Analyser (HP70000+70810A). The external cavity was 15cm long and the corresponding round-trip frequency was 1GHz. For the sinusoidal laser drive, a Wiltron sweeper model 6659B was used in conjunction with a Mini-Circuits amplifier and microwave isolator. Pulses of 37ps on the oscilloscope, or 27ps deconvolved, were produced. For a pulsed drive an HP33005C step recovery diode was used. This device generated 40ps pulses electrical pulses into a 50Ω termination. Optical pulses produced by the mode-locked laser driven with this step recovery diode were 37ps, as in the case of the sinusoidal drive. The same external cavity laser was also tested for harmonic mode-locking at 2GHz using the Systron Donner microwave synthesiser, and pulses of 35ps (24ps deconvolved) were obtained.

In an attempt to produce shorter pulses, we assembled the best laser we had within an identical external cavity. This laser was a BTRL buried heterostructure (ser. no. 27989, wafer no. 1140). It was anti-reflective coated on one facet to 0.1% and it had a threshold current of 30mA. The external cavity was set-up for a fundamental mode-locking frequency slightly above 2GHz. For mode-locked operation, the shortest pulses were measured as 30ps (16ps after deconvolution). Mode-locking was also attempted for the configuration which utilised a spherical lens instead of the GRIN lens. Pulses of 28ps (11.5ps deconvolved) were observed for a cavity containing a 3mm spherical lens. The estimated optical bandwidth of this external grating configuration was 135GHz.

The optimal external cavity configuration was selected for our final experimental set-up which is presented in Fig. 6.5. A series of autocorrelation measurements was performed for different combinations of spherical lenses and gratings. In these experiments complete suppression of the autocorrelation spike was achieved for an external cavity containing a 2mm lens and grating with 1200 lines per millimeter, indicating excellent mode-locking.
Fig. 6.5 Mode-locking experimental arrangement (NPL)
6.2.2 Final configuration

Our final experimental configuration (Fig. 6.5) consisted of a mode-locked external grating laser used in conjunction with the travelling-wave semiconductor optical amplifier. Several configurations of this type have been reported in the literature. They were used for the investigation of gain compression of semiconductor optical amplifiers for both DC and pulsed electrical drives. They were also implemented to study chirping and compression of optical pulses. Finally, the use of a semiconductor optical amplifier to stabilise and shape the pulses produced by a mode-locked semiconductor laser has been investigated and reported in Ref. 6.

6.2.1 Description of the System

The external cavity laser consists of the BTRL laser (ser. no. 27989) AR-coated to 0.1%, Melles Griot 2mm coupling sphere AR-coated to 0.25% and a Milton-Roy grating utilising 1200l/mm and blazed for 1.4μm. The optical bandwidth of this external grating configuration is estimated to be 100GHz. The laser heat sink is mounted on a Marlow Peltier cooler, model MI1023T-02AC. The microwave connection to the laser is provided by a connectorised semi-rigid coaxial cable bonded to the laser submount using a silver loaded epoxy. All mechanical components are assembled on a Photon Control bench rail providing variable cavity length from 3.75 to 15cm (corresponding to 4GHz and 1GHz repetition rates respectively).

The output of the laser is taken by an AR-coated lensed fibre, obtained from BTRL, and spliced to a BT&D optical isolator model OIC1100-1500. The fibre coupler is a Sifam 22S15A50, providing equal splitting between two branches. One output arm of the coupler is spliced to a Diamond connector, providing the direct output from the external cavity laser, while the other arm is spliced to a BT&D SOA 3100-1550 travelling-wave semiconductor optical amplifier. The amplifier is specified to provide 12dB gain for TE and 7dB for TM mode when biased at 100mA. In our set-up it is driven by the ILX LDX-3412 laser supply.
For mode-locked operation, the laser is driven using the Systron Donner 1618 synthesiser in conjunction with a Mini-Circuits ZHL-4240 amplifier and an MI F7117-05 microwave isolator. The microwave drive is combined with the DC bias from ILX LDX-3207B using an HP 33150 bias-T.

Amplified pulses from the mode-locked laser were measured using the Inrad 5-14-LD autocorrelator. The HP Lightwave Analyser (HP70000+70810A) was employed to display the RF spectra of mode-locked optical pulses before and after amplification. GTE photodiode with a bandwidth of 40GHz and the Tektronix CSA 803 with SD-32 sampling head (50GHz bandwidth) were used to display the pulses.

6.2.2 Characterization of the System

The external cavity system was tested for two cavity lengths whose round-trip frequencies were in the vicinity of 2GHz and 4GHz. The external cavity were set-up using the lightwave analyser to monitor the noise spectra of the laser. For optimal alignment, the noise components at the round-trip frequency and its harmonics are completely suppressed for a broad range of the bias current. We did not have the equipment to measure the lasing frequency of the external cavity laser. The solitary semiconductor laser was presumed to lase at ~ 1.57μm. The external cavity laser was expected to have a tuning range of the order of 40nm.

In our experiments there was a need to detune the lasing frequency of the external cavity configuration in order to obtain maximal pulse amplification. From the BTRL data sheets, the gain peak of the amplifier was positioned around 1.535μm at the bias of 100mA. This implied that the laser should be detuned to the shorter wavelength side of its gain spectrum. Detuning of the lasing frequency towards shorter wavelengths can result in an increase in the differential gain and a decrease in the linewidth enhancement factor of the semiconductor laser. There is also some experimental evidence which was reported in the literature to
indicate that this type of detuning can lead to pulse shortening in mode-locking\(^8\).

A set of light / current curves presenting the operation of the solitary and external cavity laser is presented in Fig. 6.6. The optical power of the external cavity laser is plotted for both the DC and combined DC and RF drive (mode-locking) of the laser. The lasing frequency of the external cavity laser was detuned for maximal pulse amplification. The optical feedback was estimated to be in the region between 5 and 7%.

6.2.2.1 4 GHz External Cavity

The external cavity laser was set-up with a cavity length corresponding to a 3.85 GHz round-trip frequency. The semiconductor laser was driven with a DC current of 30mA and RF signal of approximately 0.8W. The semiconductor optical amplifier was biased at 100mA. The best autocorrelation trace of the amplified optical signal is presented in Fig. 6.7. The full width half maximum of the trace is 11.5ps implying the pulsewidth of 8.2ps for a Gaussian pulse shape or 7.4ps for sech\(^2\) pulse. The absence of the autocorrelation spike gives an indication of proper mode-locking. A comparison can be made with Fig. 6.2 which represents the case of partial mode-locking. No autocorrelation measurements have been obtained from the direct output of the mode-locked laser due to the limited sensitivity of the autocorrelator.

Typically, autocorrelation traces were 12 to 12.5ps long. They corresponded to the averaged oscilloscope traces of 16ps using the GTE photodiode, SD-32 sampling head and the sampling head extender. Lightwave analyser traces of mode-locked pulses before and after the amplifier are shown in Fig. 6.8a & 6.8b respectively. The average optical power measured from the output fibre was 16 and 400\(\mu\)W respectively. The first value implies an average output power of the mode-locked laser of around 100\(\mu\)W if the coupling, insertion loss and splitting loss are taken into account.
Fig. 6.6 Light / current characteristics of the solitary laser, DC driven and mode-locked external cavity laser obtained from the output fibre. External cavity configuration adjusted for maximal amplification.
Fig. 6.7  Autocorrelation trace - amplified output
mode-locking frequency 3.85 GHz
autocorrelator settings: scanning range 102ps
delay rate 0.2ps/s, RC constant 1.0s
Fig. 6.8  HP lightwave analyser traces: mode-locking at 3.85GHz

a) direct output of the mode-locked laser
b) amplified output
The effect of detuning the modulating frequency on the pulsewidth, measured from the amplified output is presented in Fig. 6.9a. The shortest pulses were produced for small positive detuning, up to 5MHz. The laser was more sensitive to negative detuning, and the pulsewidth would double for the reduction of modulating frequency of 20MHz with respect to the round-trip frequency.

6.2.2.2 2 GHz External Cavity

The mode-locking experiment was repeated using a longer cavity with a round-trip frequency of 2.23GHz. The laser was biased at 20mA and driven with an RF signal of 0.8W. The optical amplifier was biased at 100mA, as in our previous experiment. Sampling oscilloscope traces shorter than 16ps were measured for the amplified output. A typical trace is presented in Fig. 6.10. The diagram shows a full width at a half maximum of 15.4ps. The equivalent deconvolved pulsewidth is 8.2ps. The average optical power measured from the direct and amplified outputs were 12.5 and 250μW respectively. In this experiment there was some uncertainty in the determination of the exact round-trip frequency of the external cavity laser. However, the frequency detuning curve, presented in Fig. 6.9b, has the same trend as the curve obtained for the shorter cavity (Fig. 6.9a).

Mode-locking was also achieved when the laser was driven at the second harmonic of the round-trip frequency (4.46GHz). In this case the DC bias of the laser was increased to 30mA. The measured pulses were nearly identical to the pulses produced by mode-locking of the shorter external cavity structure. However, it seemed that the operation was more sensitive to the cavity misalignment because of the appearance of 2.23GHz components in the RF spectrum. This effect was more pronounced for both positive and negative detuning, and sometimes the RF spectra which resembled fundamental mode-locking at 2.23GHz were observed. Pulse instabilities in harmonically driven external cavity semiconductor lasers
Fig. 6.9 Detuning of the mode-locking frequency: variation of the pulsewidth measured from the amplified output

a) mode-locking at 3.85GHz
b) mode-locking at 2.23GHz

two averaging programs used: Tektronix & wfmparas (NPL)
Fig. 6.10 Sampling oscilloscope trace: amplified output of the mode-locked laser detected by the 40 GHz GTE photodiode
as well as period doubling (only for positive frequency detuning) have been observed and reported in the literature\textsuperscript{9,10}.
Chapter 7

Theory of Active Mode-Locking of Semiconductor Lasers

The theory of active mode-locking of lasers has been a research topic for longer than twenty five years. The theory of active mode-locking was first formulated for both homogeneously and inhomogeneously broadened lasers modulated by intracavity phase or intensity modulators in late sixties and early seventies. Two approaches were utilised, one in the frequency domain in which the optical field was expressed in terms of normal modes, and in the time-domain where a self-reproducing pulse profile was assumed. From the mid seventies, the time-domain formulation based on rate equations has been used for the theoretical modelling of active mode-locking of dye lasers. Although the early history of active mode-locking of semiconductor lasers was dominated by the self-reproducing profile approach, the rate equation formulations became prevalent in mid eighties. The majority of publications regarding the active mode-locking of semiconductor lasers deal with time-domain formulations and nearly all of them are concerned with mode-locking by current modulation.

The theoretical work presented in this thesis is concerned with active mode-locking of monolithic external cavity semiconductor lasers. Our aim was to model both AM and FM mode-locking of semiconductor lasers as well as mode-locking by current modulation. The concept of composite cavity lithium niobate lasers has limited appeal since active mode-locking of monolithic lasers has already been accomplished in several research laboratories. Monolithic integration offers the potential for reducing intracavity reflections which can seriously affect mode-locking performance and which also make the analysis of mode-locking very complex.
This chapter is divided into two sections. The first subsection introduces basic principles of both the time and frequency domain approaches to mode-locking. A review of the major publications regarding the theory of active mode-locking of semiconductor lasers is presented in Section 7.1.2. Derivation of the frequency domain formulation based on the averaged rate equations applied to a monolithic external cavity semiconductor laser is presented in Section 7.2. The first three subsections (7.2.1, 7.2.2 and 7.2.3) are devoted to mode-locking by current, phase and amplitude modulation respectively. Results of numerical simulations based on these equations are presented in Chapter 8. The last subsection of this chapter (7.2.4) examines the application of intracavity travelling-wave modulators in active mode-locking. It is shown that there is an advantage to using a travelling-wave modulator for mode-locking provided that the modulator structure fills most of the cavity.

7.1 Theory of Active Mode-Locking

Mode-locking of lasers can be treated either in the time-domain or in the frequency domain, depending on the description of the optical field in the analysis. The aim of this introductory section is to introduce the concepts outlined above and to provide a review of the theory germane to the active mode-locking of semiconductor lasers.

7.1.1 Time and Frequency Descriptions of Active Mode-Locking

Early work on the theory of active mode-locking was performed in the frequency domain\(^1,2\) and it was based on the semiclassical description of the laser field\(^3\). Inside the laser cavity, the electric field can be written as a superposition of cavity modes

\[
E(z,t) = \sum_n E_n(t) e^{j\omega_n t} U_n(z) + \text{c.c.} \quad (7.1)
\]

where \(E_n(t)\) represents the complex slowly-varying amplitude of the \(n\)-th mode, \(\omega_n\) is the mode frequency and \(U_n(z)\) is the modal spatial
distribution. The cavity mode functions can be either standing or unidirectional waves, depending on the type of the laser resonator to be investigated. The mode frequency $\omega_n$ is related to the modulating frequency and in the general case is different from the cavity round-trip frequency. Using the frequency-domain formalism, a set of coupled, time-dependent equations for the slowly varying complex amplitudes can be derived from the wave equation. The coupling terms in these equations are dependent on the strength of modulation as well as relative position and length of the intracavity modulator.

The main advantage of the frequency-domain formalism is that a variety of mode-coupled and multimode laser phenomena can be modelled. In particular this formalism has been employed in the modelling of FM mode-locking and FM laser operation\(^1\) as well as AM mode-locking\(^2\). The disadvantage of the frequency-domain approach is that a large number of coupled differential equations is required for modelling mode-locked systems which generate picosecond pulses.

The time-domain approach, pioneered by Kuizenga and Siegman\(^4\), is based on a Gaussian pulse circulating in the laser cavity and the assumption that a self-consistent solution is reached after one round-trip. During one round-trip, the pulse propagates through the active medium and modulator. In order to keep the pulse shape Gaussian it is necessary to make approximations regarding the gain lineshape and modulation characteristic. The self-consistency requirement includes an additional phase shift to allow detuning of the modulating frequency. Using this method, simple analytic expressions for the pulsewidth, frequency chirp and bandwidth of the mode-locked pulses can be derived.

The time-domain approach allows a simple physical interpretation of mode-locking. In the steady state, pulse sharpening effects have to balance against pulse broadening effects. For example, in the case of AM mode-locking, the sharpening of the pulse due to passage through the modulator has to be compensated by the broadening caused by
transmission through the finite gain bandwidth active medium.

A comparison between the time and frequency approaches, applied to active mode-locking of homogeneously broadened lasers was presented by Haus\(^5\). His treatment employed an equivalent circuit picture in which mode-locking was treated as injection locking. The resulting difference equation, which is very similar to the equation obtained by summing all field equations in the standard frequency-domain approach\(^6\), provides a bridge to the time description of the mode-locking process. This difference equation can be replaced by the second order differential equation assuming large number of modes. The resulting equation is that of a quantum-mechanical harmonic oscillator. After performing the Fourier transform, the equivalent time-domain equation of mode-locking can be obtained.

7.1.2 Development of the Theory of Active Mode-Locking of Semiconductor Lasers

Although the first reported attempt to model active mode-locking of a semiconductor laser was based on numerical integration of the rate equations\(^7\), the self-reproducing pulse profile approach became the dominant one until the mid eighties. In a series of papers, Haus applied his method (Ref. 5) to the case of sinusoidal gain modulation of semiconductor lasers\(^8\), and it was followed by the criticism of the only published rate equations formulation\(^9\). The effect of spontaneous emission noise\(^10\) was also treated within the self reproducing profile framework and the series culminated with a review paper summarising the experimental and theoretical work conducted at MIT\(^11\). In the beginning of the eighties, Haus's approach was also combined with the rate equation for the carrier concentration\(^12\). The self reproducing profile philosophy was also used in conjunction with frequency domain approaches utilising the vector potentials\(^13\) and rate equations\(^14\). Later, the self reproducing profile was used in time-domain models involving rate equations\(^15,16\).
The validity of the self reproducing profile approach in the case of a laser medium of short recovery time was questioned by New and Catherall\textsuperscript{17} in 1984. They made a comparison between the self reproducing profile formulation and their numerical method for the case of a synchronously pumped dye laser and found significant discrepancies. The characteristic feature of their 'stepping' model was that no bandwidth limitation was needed for the theoretical formulation of mode-locking. This was followed by a publication by Demokan\textsuperscript{18}, based on the semiconductor laser travelling-wave rate equations for the photon density. Despite the gross overestimation of the average rate of spontaneous emission, the model produced asymmetrical pulse shapes often seen in experiments. Another contribution from Catherall and New\textsuperscript{19} stressed the importance of spontaneous emission. Although the numerical simulations they presented were describing synchronously pumped dye lasers, their results were relevant to the mode-locking of semiconductor lasers.

Since the late eighties, time-domain models have increasingly become the standard way of modelling active mode-locking of semiconductor lasers. Multiple subpicosecond pulses, observed in experiments, have also been generated numerically using a model based on the travelling-wave rate equations\textsuperscript{20}. The first numerical model to include the laser field was reported by Lowery\textsuperscript{21} and it was based on the transmission-line matrix method (TLM). This model was later expanded to include dispersive elements and chirp\textsuperscript{22} and was also used for modelling of integrated mode-locked structures\textsuperscript{23}. Time-domain models formulated for the laser field were also reported by Werner\textsuperscript{24}, Schell\textsuperscript{25} and New\textsuperscript{26}.

Recent developments in the frequency-domain approach include an analytic approach reported by Lau\textsuperscript{27} and a numerical model published by Hsu\textsuperscript{28}. Lau's publication deals with both active and passive mode-locking at high frequencies (~100GHz) and it is based on the small-signal analysis of mode-locking among three modes. Hsu has presented a numerical simulation of active mode-locking using a large number of modes (10 and 50) and together with spontaneous emission. However, the results
presented in the paper are more relevant to the statistics of mode-locking as opposed to an investigation of mode-locking itself. Hsu's paper is assuming the 'top hat' spectrum (gain profile), leading to sinc² pulses. No results were presented regarding the effects of frequency detuning, bias or RF current.

All the theoretical formulations mentioned in this short review deal with active mode-locking of semiconductor lasers by current modulation. To the best of our knowledge, no results on pure AM or FM mode-locking of semiconductor lasers have ever been reported. The only article that deals with the modelling of FM mode-locking of tunable distributed Bragg reflector laser within an external cavity (combination of AM and FM mode-locking)²⁹ is based on Haus's self reproducing profile approach.

7.2 Frequency-Domain Formulation of Active Mode-Locking of Semiconductor Lasers

The main advantage of the frequency domain approach to active mode-locking is that it allows a unified description of AM, FM and mode-locking by laser current modulation. Applied to semiconductor lasers, this approach leads to a set of multi-mode field equations together with a rate equation for the carrier number. The formulation is based on averaged rate equations and although the main approximation made in the derivation is an assumption of a slowly varying envelope, the range of validity is determined by averaging effects. The most obvious limitation is the neglect of the spatial variation of the gain which leads to the requirement that the gain region is short compared to the optical pulse. Also the variations of the carrier and photon populations have to be small along the length of the gain region.

The derivation of the model will be presented for the case of mode-locking by current modulation in order to emphasize the main approximations while keeping the expressions concise. A simple formulation for the gain saturation and a deterministic treatment of the effects of spontaneous
emission will be introduced. This will be followed by the inclusion of the terms that correspond to FM and AM mode-locking in subsequent sections. Finally, a brief comment regarding the intracavity modulation efficiency of both lumped and travelling-wave modulators will be made.

7.2.1 Active Mode-Locking by Current Modulation

The derivation of the frequency domain model is based on the procedures described in Siegman's\textsuperscript{30} and Agrawal's\textsuperscript{31} textbooks and using Hsu's\textsuperscript{28} notation. A schematic diagram of a monolithic external cavity semiconductor laser is presented in Fig. 7.1.a. The structure consists of a lasing segment and extended semiconductor waveguide. A proper mode-locking configuration would also include an intracavity filter (numerical simulation presented in Section 8.2).

If we assume an instantaneous material response, the wave equation can be written as

$$\nabla^2 \overline{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} (c \overline{E}) = 0$$

(7.2)

For a structure designed to support a single mode the total optical field inside the laser cavity can be expressed as

$$\overline{E}(x,y,z,t) = \frac{1}{2} \bar{x} \xi(x) \zeta(y) \sum_i E_i(t) e^{j\omega t} \sin k_i z + \text{c.c.}$$

(7.3)

where $E_i(t)$ represents the slowly varying component (complex value) and $e^{j\omega t}$ represents the fast optical oscillation. The spatial dependence of the mode profile is described by $\xi(x)$ and $\zeta(y)$ and $\sin k_i z$ is the longitudinal distribution of the $i$-th mode. $\bar{x}$ is a unit vector related to polarisation of the optical field.
Fig. 7.1 Schematic diagrams of monolithic external cavity semiconductor lasers
a) active mode-locking by current modulation (section 7.2.1)
b) FM mode-locking (section 7.2.2)
c) AM mode-locking (section 7.2.3)
The expansion presented in Eq. 7.3 is based upon the normal modes of a closed cavity assuming negligible diffraction loss. Although this is not the case for any practical laser system, a vast amount of work is based on this assumption including the major textbooks written by Haken\textsuperscript{32}, Sargent\textsuperscript{33} and Yariv\textsuperscript{34}. The validity of Eq. 7.3 is even more questionable in the case of a semiconductor laser which has a large output coupling. However, the rate equations theory presented in Agrawall's textbook on semiconductor lasers\textsuperscript{31} is also based on the expansion formulated by the Eq. 7.3.

The mode frequencies $\omega_i$, used in the expansion (Eq. 7.3) are determined from the lasing frequency $\omega_o$ and the modulating frequency $\omega_M$ using the relation

$$\omega_i = \omega_o + i\omega_M$$  \hfill (7.4)

These mode frequencies are in general different from the longitudinal cavity resonances $\Omega_i$ given as

$$\Omega_i = \Omega_o + i\Omega$$  \hfill (7.5)

where $\Omega_o$ represents the central Fabry-Perot frequency with the respect to the gain or loss profile and $\Omega$ is the longitudinal mode spacing.

In a dispersive media $\bar{\varepsilon} \vec{E}$ can be expanded in the following form

$$\bar{\varepsilon} \vec{E}(x,y,z,t) = \varepsilon \frac{1}{2} \xi(x)\xi(y) \sum_i E_i(t) e^{j\omega t} \sin k_z + j \frac{1}{2} \xi(x)\xi(y) \frac{\partial E_i}{\partial \omega} \sum_i \frac{\partial E_i}{\partial t} e^{j\omega t} \sin k_z + \text{c.c.}$$  \hfill (7.6)

and substituted into the wave equation (Eq. 7.2). Applying the slowly varying envelope approximation, the third order derivatives and products of the mode frequencies and second order derivatives can be neglected. Also a discrete mode frequency in one expression can be replaced by a continuous frequency assuming that the mode separation is much
smaller than the optical frequency. After the spatial integration, the following expression can be obtained

\[
\frac{2j\omega}{c} \left( \varepsilon + \frac{\omega}{2} \frac{\partial\varepsilon}{\partial\omega} \right) \sum_i \frac{\partial E_i}{\partial t} e^{-j2\pi f t} \sin k_i z + \sum_i \frac{\varepsilon_i}{c^2} E_i e^{-j2\pi f t} \sin k_i z \nonumber \\
- \sum_i k_i^2 E_i e^{-j2\pi f t} \sin k_i z = 0
\] (7.7)

This is the multi-mode generalization of the equation 6.2.5 from Agrawal's textbook. The spatially averaged dielectric constant is given as

\[
\langle \varepsilon \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \varepsilon(x, y) \xi^2(x) \eta^2(y) \, dx \, dy
\] (7.8)

where \( \xi(x) \) and \( \eta(y) \) are assumed to be properly normalized. According to Agrawal's analysis the averaged dielectric constant is approximately equal to the effective dielectric constant of the waveguide mode

\[
\langle \varepsilon \rangle = \bar{\mu}^2 + 2\Gamma \bar{\mu} \Delta \mu + j\Gamma \bar{\mu} \bar{\alpha} \frac{\mu}{k_o} \] (7.9)

In this expression \( k_o \) is the vacuum wave number, \( \Gamma \) is the confinement factor, \( \Delta \mu \) is the carrier induced refractive index change, \( \bar{\mu} \) is the mode index and \( \bar{\alpha} \) is the mode absorption coefficient expressed as

\[
\bar{\alpha} = -\Gamma g + \alpha_{\text{int}} + \alpha_m
\] (7.10)

where \( \alpha_{\text{int}} \) represents the total internal loss and \( \alpha_m \) is the distributed mirror loss. The expression for the effective dielectric constant (Eq. 7.9) is more often formulated using the linewidth enhancement factor \( \beta_c \)

\[
\langle \varepsilon \rangle = \bar{\mu}^2 - j\bar{\mu} \Gamma g \frac{\alpha_{\text{int}} - \alpha_m}{k_o} (1 - j\beta_c)
\] (7.11)

Finally, regarding Eq. 7.7, the wave number of the \( i \)-th mode, \( k_i \) should
Substituting Eq. 7.11 into Eq. 7.7, recognizing the group refractive index definition on the left hand side of Eq. 7.7 and neglecting the pumping contributions of the effective dielectric constant when multiplied by $\omega/c^2$ we can write the following set of equations:

$$
\sum_i \alpha_i \frac{\partial E_i}{\partial t} e^{j\omega t} \sin k_i z = \frac{j \mu}{2 \mu_g} \sum_i \Omega_i^2 E_i e^{j\omega t} \sin k_i z
$$

$$
- \frac{j \mu}{2 \mu_g} \sum_i \Omega_i^2 E_i e^{j\omega t} \sin k_i z
$$

$$
\frac{1}{2\mu_g} \sum_i \frac{\Gamma_{\alpha}}{k_0 V} (N-N_0)(1-j\beta_c) \alpha_i^2 E_i e^{j\omega t} \sin k_i z
$$

$$
- \frac{1}{2\mu_g} \sum_i \frac{\alpha_{im}+\alpha_m}{k_0} (1-j\beta_c) \alpha_i^2 E_i e^{j\omega t} \sin k_i z
$$

In Eq. 7.13 we have assumed the linear variation of gain with carrier density defined by Eq. 2.24. Regarding the notation, $N$ represents the carrier number and $N_0$ is the carrier number required for transparency, both defined inside the active region of the volume $V$. Also a group index $\mu_g$ corresponding to the mode index $\mu$ has been introduced.

In active mode-locking, modulation of the laser current at the round-trip frequency or in its vicinity, leads to the generation of short optical pulses. From Eq. 2.42 it can be seen that the carrier modulation depends both on the current modulation and the photon number inside the active region. Modulation at the round-trip frequency means that a synchronization between the forcing term due to laser current modulation and the arrival of the optical pulse, reflected from the external cavity can be reached. In this case it is expected that the spectrum of the carrier number will contain frequency components at the modulating frequency and its harmonics. The carrier number can be expressed as
\[ N(z,t) = N_0(z) + \sum_{q} N_q (e^{i\omega_q z} + e^{-i\omega_q z}) \quad q=1,...,Q \] (7.14)

where \( \omega_m \) is the modulation frequency, \( N_0 \) is the DC component of the carrier number and \( N_q \) is the Fourier component of the \( q \)-th harmonic. Coupling between the field component \( E_i \), \( E_{i-q} \) and \( E_{i+q} \) is achieved through the \( q \)-th Fourier component of the carrier number. Neglecting the population pulsations is the standard approximation used in the frequency domain modelling in late sixties. Within our framework this approximation would result in the neglect of a dynamic gain saturation. This approximation was also used by Lau, who performed a small-signal analysis of mode-locking using only three modes. In order to perform numerical simulations it is more convenient to keep the carrier number \( N \) in Eq.7.13 as a variable used in conjunction with a separate differential equation (Eq. 2.42), and to add the coupling contribution resulting from Eq. 7.14 and to include the coupling loss. The introduction of the coupling loss is required for the energy conservation of the system. This approach was used in Hsu's derivation of the frequency domain model. Following this procedure and including the group velocity \( v_g \), Eq. 7.13 can be written as

\[
\sum_{i} \omega_i \frac{\partial E_i}{\partial t} e^{i\omega t} \sin k_i z = \frac{j}{2} \sum \omega_i^2 E_i e^{i\omega t} \sin k_i z \\
- \frac{j}{2} \sum \Omega_i^2 E_i e^{i\omega t} \sin k_i z \\
+ \frac{1}{2} v_g \sum \frac{\Gamma_{\alpha}}{V} (N-N_0)(1-j\beta_c) \omega_i E_i e^{i\omega t} \sin k_i z \\
+ \frac{1}{2} v_g \sum \frac{\Gamma_{\alpha}}{V} \left[ \sum N_q (e^{i\omega_q z} + e^{-i\omega_q z}) (1-j\beta_c) \omega_i E_i e^{i\omega t} \sin k_i z \\
- \frac{1}{2} v_g \sum (\alpha_{int}+\alpha_m-\alpha_c)(1-j\beta_c) \omega_i E_i e^{i\omega t} \sin k_i z \right] \quad (7.15)
\]

In Eq.7.15, the coupling gain is presented by the term \( \frac{\Gamma_{\alpha}}{V} N_q \) and the coupling loss is expressed as \( \alpha_c \). After making the approximation,

\[ \omega_i^2 - \Omega_i^2 = 2\omega_i (\omega_i - \Omega_i) \]
and selecting the terms having $e^{j\omega t}$ dependence from the system described by Eq. 7.15 we can write

$$\frac{\partial E_i}{\partial t} \sin k_i z = j \frac{\mu}{\mu_g} (\omega - \Omega) E_i \sin k_i z$$

$$+ \frac{1}{2} B_d (N-N_0) (1-j\beta) E_i \sin k_i z$$

$$+ \frac{1}{2} B_d (1-j\beta) \sum_N N_q (E_{i+q} \sin k_{i+q} z + E_{i-q} \sin k_{i-q} z)$$

$$- \frac{1}{2} (1-j\beta) (\gamma_{int} + \gamma_m + \gamma_c) E_i \sin k_i z$$

(7.16)

where $\gamma_{int}$, $\gamma_m$ and $\gamma_c$ represent loss contributions, and are defined in the manner of Eq. 2.34. $B_o$ denotes the gain constant and is defined as

$$B_o = \frac{v_g \Gamma a}{V}$$

(7.17)

We should note that $B_o$ is assumed to be constant for all modes. In general, $B_o$ is a function of frequency if a realistic gain profile is included. However, in active mode-locking of semiconductor lasers the intracavity filters provide much stronger frequency selection than does the gain curvature, and so the introduction of a frequency dependent loss is of more significance. To simplify our considerations, we assume equal losses for all modes, and introduce a loss profile at a later stage.

Following the standard procedure, we multiply both sides of Eq. 7.16 by $\sin k_i z$ and integrate over the length of the whole cavity $L$. All the terms related to gain, loss and coupling except the distributed mirror loss are assumed to be localized within the lasing region of the length $L_L$. Supposing that $L_L << L$ we can evaluate the following integrals

$$\int_0^L \sin^2 k_i z \, dz = \frac{L}{2}$$

(7.18)
\[ \int_0^{L} \sin^2 k_i z \, dz = \frac{L}{2} \]  

(7.19)

\[ \int_0^{L} \sin k_i z \sin k_{i+q} z \, dz = \frac{L}{2} \]  

(7.20)

The last integral (Eq. 7.20) has an important implication. If the gain medium fills the whole cavity \( L \), then integral becomes the orthogonality statement and its value is zero. This means that no mode-locking will take place in a uniformly filled cavity. A further comment on Eq.7.20 will be presented in Section 7.2.4. Using Eq. 7.18, 19 and 20 we can finally write our basic mode-locking equation formulated in terms of the complex field inside the laser cavity,

\[
\frac{\partial E_i}{\partial t} = j \frac{\mu}{\mu_g} (\omega_i - \Omega_e) E_i + (1 - j \beta_c) \left( \frac{1}{2} \frac{L}{L} [B_q(N-N_q) - \gamma_{\text{int}} - \gamma_m] - 2 \sum_q \Delta_q \right) E_i \\
+ (1 - j \beta_c) \sum_q \Delta_q (E_{i-q} + E_{i+q})
\]  

(7.21)

In the mode-locking equation (Eq.7.21) the modified distributed mirror loss \( \gamma_m \) is given as

\[ \gamma_m = \frac{L}{L_c} \gamma_m = \frac{\nu_g}{2L_c} \ln \frac{1}{R_1 R_2} \]  

(7.22)

where \( R_1 \) and \( R_2 \) represent the reflectivities of the resonator mirrors. In the frequency domain approach, where losses are treated as distributed losses, the coupling loss between the lasing region and the waveguide can be incorporated as a reduced mirror reflectivity. Also a frequency dependent loss can be introduced through the expression for the mirror loss.

The coupling coefficient \( \Delta_q \) is defined as

\[ \Delta_q = \frac{1}{2} \frac{L}{L} B_q N_q \]  

(7.23)
where the gain constant $B_0$ and $N_q$ are defined in Eq. 7.17 and 7.14 respectively. The total coupling loss appearing in the mode-locking equations is set equal to the total coupling gain in order to maintain energy conservation. This condition can be written as

$$\frac{1}{2} \frac{L}{L} \gamma_c = 2 \sum_q \Delta_q \quad (7.24)$$

The system of multi-mode field equations described by Eq. 7.21 must be used in conjunction with a rate equation for the carrier population. The rate equation for the carrier number inside the active area can be expressed as

$$\frac{\partial N}{\partial t} = \frac{1}{q_e} N - \gamma_e N - G' \left( \sum_i E_i \sum_i E_i^* \right) \quad (7.25)$$

where the carrier recombination rate $\gamma_e$ is given as

$$\gamma_e = A_{nr} + B \frac{N}{V} + C \left( \frac{N}{V} \right)^2 \quad (7.26)$$

Equations 7.25 and 7.26 correspond to Eq. 2.42 and 2.43. The last term on the right hand side of Eq. 7.25 represents the coherent interaction among the laser modes which is characteristic for mode-locking. $G'$ is used to indicate that the gain factor has to be modified if the rate equation for the carrier number is to be written in terms of the laser field instead of the photon number.

At this point, it should be noted that the basic model for active mode-locking of semiconductor lasers is given by Equations 7.21, 7.25 and 7.26. To form a numerical model, and to include the simplest interpretation of spontaneous emission and gain saturation (non-linear gain) we will express the basic mode locking equation in terms of a photon number of the $i$-th mode $P_i$, and a corresponding phase factor $\phi_i$. As a first step we
can write the complex field $E_i$ as

$$E_i = A_i e^{j\phi_i} \quad (7.27)$$

where $A_i$ represents the real slowly varying amplitude of the mode and $\phi_i$ is the phase of the $i$-th mode. The time derivative of the complex field then becomes

$$\frac{\partial E_i}{\partial t} = \frac{\partial A_i}{\partial t} e^{j\phi_i} - j \frac{\partial \phi_i}{\partial t} A_i e^{j\phi_i} \quad (7.28)$$

The photon number of the mode $P_i$ is defined with respect to the complex amplitude of the mode $E_i$ using the Eq. 2.38. However, in order to reformulate the basic mode-locking equation (Eq. 7.21) it is sufficient to represent the photon number $P_i$ as

$$P_i = \text{const} \cdot A_i^2 \quad (7.29)$$

In this case the time derivative of the photon number of the mode can be written as

$$\frac{\partial P_i}{\partial t} = 2 \cdot \text{const} \cdot A_i \frac{\partial A_i}{\partial t} \quad (7.30)$$

Using Eq. 7.25, 26, 27 and 28, the basic mode locking equation is transformed into the following set of equations

$$\frac{\partial P_i}{\partial t} = \left[ \frac{L}{L} [B_d(N-N_d) - \gamma_m - \gamma_m] - 4 \sum_q \Delta_q \right] P_i$$

$$+ 2 \sum_q \Delta_q \sqrt{P_i P_{i+q}} \left[ \cos(\phi_{i+q} - \phi_i) - \beta_c \sin(\phi_{i+q} - \phi_i) \right]$$

$$+ 2 \sum_q \Delta_q \sqrt{P_i P_{i-q}} \left[ \cos(\phi_{i-q} - \phi_i) - \beta_c \sin(\phi_{i-q} - \phi_i) \right] \quad (7.31)$$
\[
\frac{\partial \phi_i}{\partial t} = -\frac{\mu}{\mu_g} (\Omega_i - \Omega) + \beta_c \left( \frac{1}{2} L \left[ B_d (N-N_d) - \gamma_{\text{int}} - \gamma_m \right] - 2 \sum_q \Delta_q \right) \\
+ \sum_q \Delta_q \sqrt{\frac{P_{i+q}}{P_i}} \left[ \beta_c \cos(\phi_{i+q} - \phi_i) + \sin(\phi_{i+q} - \phi_i) \right] \\
+ \sum_q \Delta_q \sqrt{\frac{P_{i-q}}{P_i}} \left[ \beta_c \cos(\phi_{i-q} - \phi_i) + \sin(\phi_{i-q} - \phi_i) \right]
\]  

Equations 7.31 and 7.32 represent the formulation of our basic mode-locking equation (Eq. 7.21) in terms of the photon number \( P_i \) and phase \( \phi_i \) of the i-th mode.

The inclusion of nonlinear gain is an important factor in rate equation analysis of semiconductor lasers. In most publications, nonlinear gain is attributed to spectral hole-burning or dynamic carrier heating. Several formulations exist for a gain saturation term \(^{35}\) and we will adopt the formulation presented in Ref. 36 and 28 and write it in our notation as

\[
G = B_d (N-N_d) - \sum_s \beta_{si} P_s
\]  

(7.33)

where \( \beta_{si} \) represent the self-saturation coefficient for \( s=i \) and cross-saturation coefficient for \( s\neq i \). It has to be noted that Eq. 7.33 describes the gain saturation in terms of the incoherent addition of the modal photon numbers. The formulation based on the total photon number, which is given as a coherent sum of the modal photon numbers, would be a more realistic description. However, with our method of numerical evaluation it was essential to treat the gain nonlinearity in terms of the incoherent addition of the modal photon numbers.

The last issue to be addressed in this section is the effect of spontaneous emission. Within a semi-classical treatment, fluctuations arising from spontaneous emission and from carrier generation / recombination are incorporated by adding a Langevin noise sources\(^{31}\) into the system of
equations formulated for the complex field (Eq. 7.21 and 25) or alternatively into the system consisting of Eq. 7.31, 32 and 25. This would lead to a stochastic formulation of the mode-locking process. An alternative description of mode locking is a deterministic representation, based on coherent injection of the average rate of spontaneous emission into the lasing mode. The later approach does not represent the real situation, as an increase in spontaneous emission results in shorter pulses and higher peak powers rather than increasing pulse to pulse amplitude fluctuations and jitter. Nevertheless, the deterministic representation is often used because it allows for both easy numerical integration and mode-locking start-up.

Regarding the introduction of the terms corresponding to the average rate of spontaneous emission into mode-locking equations for the photon number (Eq. 7.31) and phase (Eq. 7.32), two important points should be emphasised. First, the effect of the external cavity is to reduce the average rate of spontaneous emission into the lasing mode. According to Ref. 37, for a monolithic external cavity configuration the average rate of spontaneous emission can be expressed as

$$R_{sp} \approx \frac{L}{L} R_{sp(sol)}$$  \hspace{1cm} (7.34)

where $R_{sp(sol)}$ represents the average rate of spontaneous emission of the solitary laser and $L/L$ is the ratio of lengths of the solitary and external cavity laser. Using Eq. 2.44, we can rewrite the average rate of spontaneous emission of the external cavity laser as

$$R_{sp} = \frac{L}{L} \beta_{sp} \frac{N^2}{N}$$  \hspace{1cm} (7.35)

Second, the effect of the linewidth enhancement factor $\beta_c$ is to introduce an additional frequency shift into Eq. 7.32 which corresponds to the inclusion of the average rate of spontaneous emission into Eq. 7.31. Following the derivation presented in Ref. 38, we can write the frequency shift due to the average rate of spontaneous emission in the case of an
external cavity laser as

\[ \omega_{\text{sp}} = \frac{\beta_c}{2} \frac{R_{\text{sp}}}{P_i} \]  

(7.36)

From Eq. 7.36 it is apparent that the frequency shift of the i-th mode, \( \omega_{\text{sp}} \), is inversely proportional to the photon number of the mode. In mode locking, this has an important implication because if the same rate of spontaneous emission is assumed for all modes, the corresponding frequency shifts will differ under the assumption of any realistic gain or loss profile. To conclude this rather long section we will collect the equations which we have used in our numerical model:

\[
\begin{align*}
\frac{\partial P_i}{\partial t} & = \frac{L}{L} [B_d(N-N_0) - \sum_s \beta_{si} P_s - \gamma_{\text{int}} - \gamma_m] - 4 \sum_q \Delta_q \frac{\sqrt{P_i P_{i+q}}}{P_i} \left[ \cos(\phi_{i,q} - \phi_i) - \beta_c \sin(\phi_{i,q} - \phi_i) \right] \\
& + 2 \sum_q \Delta_q \sqrt{P_i P_{i+q}} \left[ \cos(\phi_{i+q} - \phi_i) - \beta_c \sin(\phi_{i+q} - \phi_i) \right] \\
& + R_{\text{sp}} \\
+ \frac{\partial \phi_i}{\partial t} & = -\frac{\mu}{\mu_g} (\alpha - \Omega_c) \\
& + \beta_c \frac{L}{2L} [B_d(N-N_0) - \sum_s \beta_{si} P_s - \gamma_{\text{int}} - \gamma_m] - 2 \sum_q \Delta_q \\
& + \sum_q \Delta_q \sqrt{P_{i+q} P_i} \left[ \beta_c \cos(\phi_{i+q} - \phi_i) + \sin(\phi_{i,q} - \phi_i) \right] \\
& + \sum_q \Delta_q \sqrt{P_{i+q} P_i} \left[ \beta_c \cos(\phi_{i+q} - \phi_i) + \sin(\phi_{i+q} - \phi_i) \right] \\
& + \frac{\beta_c}{2} \frac{R_{\text{sp}}}{P_i}
\end{align*}
\]

(7.37)

\[
\begin{align*}
\frac{\partial N}{\partial t} & = \frac{L}{Q_e} \left[ A_{nt} + B_N + C(N/N)^2 \right] N \\
& - [B_d(N-N_0) - \sum_s \beta_{si} P_s] \sum_i \sum_i \sqrt{P_i P_{i+s}} \cos(\omega M + \phi_{i+s} - \phi_i)
\end{align*}
\]

(7.38)

(7.39)
7.2.2 FM Mode-Locking

Equations 7.37, 38 and 39 describe our model of an actively mode-locked laser by current modulation. In order to include the effect of an intracavity phase modulator (Fig. 7.1.b) we will rewrite the expression for the effective dielectric constant (Eq. 7.11) as

\[
\varepsilon = \mu^2 - j\mu \frac{\Gamma g - \alpha_{\text{int}} - \alpha_m}{k_o} (1 - j\beta_c) + \Delta \varepsilon_{\text{FM}} \tag{7.40}
\]

where \(\Delta \varepsilon_{\text{FM}}\) represents the contribution of the phase modulator. This perturbation of the dielectric constant is localised within the region of the composite cavity occupied by the modulator. In the case of relatively weak phase modulation, the perturbation can be written as

\[
\Delta \varepsilon_{\text{FM}}(z) = 2\mu \Delta \mu M \cos \omega_M t = \mu \Delta \mu M (e^{-j\omega_M t} + e^{j\omega_M t}) \tag{7.41}
\]

where \(\Delta \mu M\) is the maximum refractive index change caused by the modulator. For an electro-optic modulator, the maximum refractive index change is given by Eq. 4.4 assuming maximum electro-optic efficiency. Following the procedure described in the previous section, we can substitute Eq. 7.41 and 7.42 into Eq. 7.7 and obtain an enhanced version of

\[
\frac{\partial E_i}{\partial t} \sin k_i z = \frac{j}{\mu g} (\omega_i - \Omega_i) E_i \sin k_i z
\]

\[
+ \frac{1}{2} B_d (N - N_c)(1 - j\beta_c) E_i \sin k_i z
\]

\[
+ \frac{1}{2} B_d (1 - j\beta_c) \sum_q N_q (E_{i+q} \sin k_{i+q} z + E_{i-q} \sin k_{i-q} z)
\]

\[
- \frac{1}{2} (1 - j\beta_c)(\gamma_{\text{int}} + \gamma_m + \gamma_c) E_i \sin k_i z
\]

\[
+ \frac{j}{2} \mu \Delta \mu M \omega_i (E_{i_{-1}} \sin k_{i_{-1}} z + E_{i_{+1}} \sin k_{i_{+1}} z)
\]

(7.42)

The last term of Eq. 7.42 represents the effect of any intracavity phase modulation. Integrating over the length of the cavity we obtain the set of
characteristic integrals described by Eq. 7.18, 19 and 20 in addition to the modulator overlap given by

$$ \int_{L-L_m}^L \sin ikz \sin ikz \, dz = \int_{L-L_m}^L \frac{1}{2} \cos \frac{\pi z}{L} \, dz = -\frac{L}{2\pi} \sin \frac{\pi L_m}{L} $$  \hspace{1cm} (7.43)$$

The basic equation for FM mode-locking, formulated for the complex slowly varying envelope of the mode can be expressed as

$$ \frac{\partial E_i}{\partial t} = j \frac{\mu}{\mu_g} (\omega - \Omega_e) E_i + (1 - j\beta_c) \left( \frac{1}{2} \frac{L}{L} [B_d N N_d - \gamma_{int} - \gamma_{m} ] - 2 \sum_q \Delta_q \right) E_i $$

$$ + (1 - j\beta_c) \sum_q \Delta_q (E_{i-q} + E_{i+q}) $$

$$ - j \frac{\Delta_{FM}}{L} (E_{i-1} + E_{i+1}) $$  \hspace{1cm} (7.44)$$

The coupling coefficient due to intracavity phase modulation \( \Delta_{FM} \) can be formulated either using the refractive index change \( \Delta \mu_M \) caused by the modulator or the modulation depth \( \phi_{max} \) in conjunction with the modulator overlap integral (Eq. 7.43). If we define the modulation depth as

$$ \phi_{max} = \Delta \beta L_m = \frac{\omega}{v_g} \Delta \mu_M L_m $$  \hspace{1cm} (7.45)$$

the expression for the coupling coefficient \( \Delta_{FM} \) can be written as

$$ \Delta_{FM} = \frac{1}{2} \frac{v_g}{v} \frac{\sin \frac{\pi L_m}{L}}{\frac{\pi L_m}{L}} $$  \hspace{1cm} (7.46)$$

Eq. 7.46 states that an intracavity modulation efficiency is a sinc function of the relative modulator length if the constant modulation depth is assumed. In the case of short cavity and electro-optic phase modulator it is particularly important to consider the modulator overlap, because the modulator has to be relatively long in order to be efficient (Eq. 7.45).

Regarding the mode-locking equation (Eq. 7.44), the term containing the
product $\Delta q E_{i+q}$ represents the coupling due to the carrier number modulation. In the FM mode-locking of semiconductor lasers, the carrier number modulation is a consequence of the dynamic gain saturation caused by individual optical pulses. Although the coupling terms due to carrier number modulation $\Delta q$ in Eq. 7.44 and Eq. 7.21 have the same mathematical definition, given by Eq. 7.14, they have different physical origins. In mode-locking by current modulation, the carrier number modulation is a combined effect of the forcing term due to current modulation and the dynamic gain saturation caused by individual optical pulses.

After expressing the mode-locking equation (Eq. 7.44) in terms of the photon number and phase of the mode and including the gain nonlinearity and spontaneous emission terms we write the following set of equations

$$\frac{\partial P_i}{\partial t} = \frac{L}{L} [B_d(N-N_d) \cdot \sum_s \beta_s P_s \cdot \gamma_{\text{int}} - \gamma_{\text{m}} - 4 \sum_q \Delta q] P_i$$

$$+ 2 \sum_q \Delta q \sqrt{P_i P_{i+q}} \left[ \cos(\phi_{i,q} - \phi_i) - \beta_c \sin(\phi_{i,q} - \phi_i) \right]$$

$$+ 2 \sum_q \Delta q \sqrt{P_i P_{i+q}} \left[ \cos(\phi_{i+q} - \phi_i) - \beta_c \sin(\phi_{i+q} - \phi_i) \right]$$

$$- 2 \frac{\Delta_{\text{FM}}}{L} \sqrt{P_{i-1}} \sin(\phi_{i-1} - \phi_i)$$

$$- 2 \frac{\Delta_{\text{FM}}}{L} \sqrt{P_{i+1}} \sin(\phi_{i+1} - \phi_i)$$

$$+ R_{\text{sp}} \quad (7.47)$$
\[ \frac{\partial \phi_i}{\partial t} = -\frac{\mu}{\mu_g} (\omega_i - \Omega_i) \]

\[ + \beta_c \left( \frac{L}{2L} [B_d N - N_o] - \sum_s \beta_s P_s - \gamma_{int} - \gamma_{m} \right) - 2 \sum_q \Delta_q \]

\[ + \sum_q \Delta_q \sqrt{\frac{P_{i-q}}{P_i}} \left[ \beta_c \cos(\phi_{i-q} - \phi_i) + \sin(\phi_{i-q} - \phi_i) \right] \]

\[ + \sum_q \Delta_q \sqrt{\frac{P_{i+q}}{P_i}} \left[ \beta_c \cos(\phi_{i+q} - \phi_i) + \sin(\phi_{i+q} - \phi_i) \right] \]

\[ + \frac{D_{FM}}{L} \sqrt{\frac{P_{i+1}}{P_i}} \cos(\phi_{i+1} - \phi_i) \]

Equations 7.47 and 7.48, used in conjunction with the rate equation for the carrier number, described by Eq. 7.39, represent our deterministic model of FM mode-locking of semiconductor lasers. Additionally, with these equations the transition from FM mode-locking to FM-laser operation can be investigated by varying the frequency detuning term in Eq. 7.48. The formulation of FM mode-locking (Eq. 7.47, 48 and 39) is very similar to the description of mode-locking by current modulation (Eq. 7.37, 38 and 39). The only differences are the inclusion of the coupling terms \( \Delta_{FM} \) into the model for FM mode-locking and the interpretation of the laser current \( I \) in the carrier number equation (Eq. 7.39). In FM mode-locking, the DC current is used, while in mode-locking by current modulation an RF component is added to the DC term.

7.2.3 AM Mode-Locking

For an AM mode-locking structure presented in Fig. 7.1.c, the effective dielectric constant can be expressed as

\[ \epsilon = \mu^2 - j\mu \frac{\Gamma g - \alpha_{int} - \alpha_m}{k_0} (1 - j\beta_c) + \Delta \epsilon_{AM} \] (7.49)
When compared to Eq. 7.11, the effective dielectric constant of the structure presented in Fig. 7.1c contains an additional contribution due to the loss of the modulator, denoted as $\Delta \varepsilon_{AM}$. In the case of a relatively weak loss modulation, the perturbation $\Delta \varepsilon_{AM}$, localised within the modulating element can be written as

$$
\Delta \varepsilon_{AM}(z) = \frac{j\mu}{k_0} 2\Delta \alpha_{AM}(1 + \cos\omega_M t) = \frac{j\mu}{k_0} [2\Delta \alpha_{AM} + \Delta \alpha_{AM}(e^{-j\omega_M t} + e^{j\omega_M t})] \quad (7.50)
$$

where $2\Delta \alpha_{AM}$ represents the amplitude of the loss perturbation. Following the procedure outlined in the previous sections we can write the following equation

$$
\begin{align*}
\frac{\partial E_i}{\partial t} \text{sink}_i z &= j \frac{\mu}{\mu_g} (\omega_i - \Omega_i) E_i \text{sink}_i z \\
&+ \frac{1}{2} B_i(N - N_p)(1 - j\beta_0) E_i \text{sink}_i z \\
&+ \frac{1}{2} B_i(1 - j\beta_0) \sum_q N_q (E_{i+q} \text{sink}_{i+q} z + E_{i-q} \text{sink}_{i-q} z) \\
&- \frac{1}{2} (1 - j\beta_0)(\gamma_{int} + \gamma_m + \gamma_c) E_i \text{sink}_i z \\
&- v g \Delta \alpha_{AM} E_i \text{sink}_i z \\
&- \frac{1}{2} v g \Delta \alpha_{AM} (E_{i-1} \text{sink}_{i-1} z + E_{i+1} \text{sink}_{i+1} z)
\end{align*}

\quad (7.51)
$$

Equation 7.51 can be compared to Eq. 7.16 (current modulation) and Eq. 7.17 (phase modulation). In this case, the terms in the last two lines represent the effect of intracavity loss modulation. Multiplying by $\text{sink}_i z$ and integrating over the length of the cavity, we obtain two integrals for the modulating element

$$
\int_{-L}^{L} \sin^2 k_i z \, dz = \frac{L_M}{2} \quad (7.52)
$$

$$
\int_{-L}^{L} \text{sink}_i z \text{sink}_{i\pm 1} z \, dz = \int_{-L}^{L} \frac{1}{2} \cos \pi z \, dz = - \frac{L}{2\pi} \sin \frac{\pi L M}{L} \quad (7.53)
$$

The basic AM mode-locking equation, formulated for a complex envelope
can be then written as

\[
\frac{\partial E_i}{\partial t} = j \frac{\mu}{\mu_g} (\omega_i - \Omega) E_i + (1 - j \beta_0) \left( \frac{1}{2} \frac{L}{L} \left[ B_{\text{d}} N - N_0 - \gamma_{\text{int}} - \gamma_m \right] - 2 \sum \Delta_q \right) E_i \\
+ (1 - j \beta_0) \sum_q \Delta_q (E_{i-q} + E_{i+q}) \\
- 2 \frac{\Delta_{\text{AM0}}}{L} E_i + \frac{\Delta_{\text{AM}}}{L} (E_{i-1} + E_{i+1})
\]  

(7.54)

where \( \Delta_{\text{AM0}} \) is the coupling loss (self-coupling term) due to the AM modulation and is given by

\[
\Delta_{\text{AM0}} = \frac{1}{2} v_g \Delta \alpha_{\text{AM}} L_M
\]  

(7.55)

\( \Delta_{\text{AM}} \) denotes the coupling coefficient (cross-coupling term) defined as

\[
\Delta_{\text{AM}} = \frac{1}{2} v_g \Delta \alpha_{\text{AM}} L_M \frac{\sin \frac{\pi L_M}{L}}{\frac{\pi L_M}{L}}
\]  

(7.56)

As in the case of \( \Delta_{\text{FM}} \) (Eq. 7.46) it can be seen that the coupling coefficient is a sinc function of the modulator length provided it is positioned at the end of the cavity.

If we express our basic equation for AM mode-locking (Eq. 7.54) in terms of the photon number and phase, include the gain nonlinearity and average rate of spontaneous emission we can obtain the following set of equations.
\[
\frac{\partial P_i}{\partial t} = \frac{L_b}{L} (B_d (N-N_0) - \sum_s \beta_s P_s - \gamma_{int} - \gamma_m) - 4 \sum_q \Delta_q P_i \\
+ 2 \sum_q \Delta_q \sqrt{P_i P_{i+q}} \left[ \cos(\phi_{i-q} - \phi_i) - \beta_c \sin(\phi_{i-q} - \phi_i) \right] \\
+ 2 \sum_q \Delta_q \sqrt{P_i P_{i+q}} \left[ \cos(\phi_{i+q} - \phi_i) - \beta_c \sin(\phi_{i+q} - \phi_i) \right] \\
- 4 \frac{\Delta_{AM0}}{L} P_i \\
+ 2 \frac{\Delta_{AM}}{L} \sqrt{P_i P_{i+1}} \cos(\phi_{i+1} - \phi_i) \\
+ 2 \frac{\Delta_{AM}}{L} \sqrt{P_i P_{i+1}} \cos(\phi_{i+1} - \phi_i) \\
+ R_{sp}
\]

\[
\frac{\partial \phi_i}{\partial t} = - \frac{\mu}{\mu_g} (\omega - \Omega) \\
+ \beta_c \frac{L_b}{2 L} (B_d (N-N_0) - \sum_s \beta_s P_s - \gamma_{int} - \gamma_m) - 2 \sum_q \Delta_q \\
+ \sum_q \Delta_q \sqrt{\frac{P_{i-q}}{P_i}} [\beta_c \cos(\phi_{i-q} - \phi_i) + \sin(\phi_{i-q} - \phi_i)] \\
+ \sum_q \Delta_q \sqrt{\frac{P_{i+q}}{P_i}} [\beta_c \cos(\phi_{i+q} - \phi_i) + \sin(\phi_{i+q} - \phi_i)] \\
+ \frac{\Delta_{AM}}{L} \sqrt{\frac{P_{i+1}}{P_i}} \sin(\phi_{i+1} - \phi_i) \\
+ \frac{\Delta_{AM}}{L} \sqrt{\frac{P_{i+1}}{P_i}} \sin(\phi_{i+1} - \phi_i) \\
+ \frac{\beta_c R_{sp}}{2 P_i}
\]

Equations 7.57, 58 and 39 represent the deterministic model of AM mode-locking of semiconductor lasers. It should be noted that if the distributed loss profile is included, the loss perturbation \(2\Delta_{\alpha AM}\) will be a function of the total loss of the mode as well as of the modulation amplitude. In that case the self-coupling and cross-coupling terms, \(\Delta_{AM0}\) and \(\Delta_{AM}\) respectively, will become mode-dependent.

As alluded to in the introductory section, the main advantage of the frequency domain approach to mode-locking is the variety of phenomena which can be modelled by very similar sets of equations. For example, in
addition to our basic models, systems of equations describing pure FM mode-locking (Eq. 7.47, 48, 39) and AM mode-locking (Eq. 7.57, 58, 39) can be combined to form a more general model of mode-locking by chirped loss modulation. Moreover, there is also a possibility of investigating mode-locking by combined current and phase modulation or by synchronous current and loss modulation.

7.2.4 Modulator Effects

In section 7.2.2 we dealt with a lumped phase modulator, positioned close to one mirror of the cavity. Since we conducted an FM mode-locking experiment utilising a long travelling-wave modulator, we will also apply the orthogonality integral (Eq. 7.43) assuming a travelling-wave excitation. To start, we will define an intracavity modulation efficiency as

\[ \frac{2}{L_M} \int_{Z_0}^{Z_0 + \frac{L_M}{2}} \delta_M(z) \sin k_i z \sin k_{i \pm 1} z \, dz \]  

(7.59)

where \( Z_0 \) represents the distance of the center of the modulator from the mirror. For simplicity, we can assume that \( Z_0 \) equals zero at the left mirror of the cavity (Fig. 7.1). \( \delta_M(z) \) is the spatial variation of the modulating signal which will become complex for the travelling-wave excitation. The intracavity modulation efficiency of the lumped modulator can be determined under the assumption that \( \delta_M(z) = 1 \) over the length of the modulator and \( \delta_M(z) = 0 \) elsewhere. Equation 7.59 can be written as

\[ \frac{2}{L_M} \int_{Z_0}^{Z_0 + \frac{L_M}{2}} \sin k_i z \sin k_{i \pm 1} z \, dz = \frac{2}{L_M} \int_{Z_0}^{Z_0 + \frac{L_M}{2}} \frac{1}{2} \cos \frac{\pi Z}{L} \, dz = \frac{\sin \frac{\pi L_M}{2L}}{\frac{\pi L_M}{2L}} \cos \frac{\pi Z_0}{L} \]  

(7.60)

where we have neglected the fast variation, as in previous cases. Equation 7.60 is a more general form of Eq. 7.43 and premultiplied by \( 2/L_M \). The difference in sign occurs as a result of the different positioning of the modulator (the opposite side of the cavity). The intracavity modulation
efficiency of the lumped modulator (Eq. 7.60) is plotted as a function of the relative modulator length in Fig. 7.2.1. It is assumed that one end of the modulator is positioned at the mirror of the laser cavity.

For a travelling-wave excitation, propagating in the positive direction, we can express the perturbation of the effective dielectric constant due to phase modulation as

\[
\Delta \varepsilon_{FM}(z) = 2\mu \Delta \mu \cos(\omega_M t - k_M z) = \mu \Delta \mu \left( e^{j \omega_M t} e^{j k_M z} + e^{j \omega_M t} e^{-j k_M z} \right)
\]  

(7.61)

Equation 7.61 is an extension of Eq. 7.41 which described action of a lumped modulator. In Eq. 7.61, \( k_M \) is the propagation constant of the modulating wave. Following the procedure described in the previous sections we can substitute Eq. 7.61 into Eq. 7.40, then incorporate the resulting equation into Eq. 7.7 and select the components having \( e^{j \omega t} \) dependence. The coupling coefficients into the \( i+1 \) and \( i-1 \) mode are complex for travelling-wave excitation, and they are determined by the following overlaps,

\[
\frac{2}{L_M} \int_{z_e - \frac{L_M}{2}}^{z_e + \frac{L_M}{2}} e^{j k_M z} \sin k_i z \sin k_{i\pm 1} z \, dz = \frac{2}{L_M} \int_{z_e - \frac{L_M}{2}}^{z_e + \frac{L_M}{2}} e^{j \xi z} \frac{1}{2} \cos \frac{\xi z}{L} \, dz = \frac{1}{2} \left[ 1 + \frac{\sin \pi L_M}{2 \pi L M} \left( \cos \frac{2\pi Z_0}{L} \pm j \sin \frac{2\pi Z_0}{L} \right) \right] = \mathcal{M} e^{j \psi}
\]  

(7.62)

In Eq.7.62, the same value of the propagation constant was assumed for the optical mode and modulating wave. We can regard the modulus of the overlap integral, \( \mathcal{M} \) as an indicator of the intracavity modulation efficiency. An additional phase shift \( +\psi \) will be introduced into the coupling coefficient for \( i-1 \) mode, and \( -\psi \) will be introduced for \( i+1 \) mode. The intracavity modulation efficiency of the travelling-wave modulator, \( \mathcal{M} \) is plotted in Fig. 7.2.2. When compared to the lumped modulator case presented in Fig.7.2.1, it can be seen that there is an advantage of using travelling-wave modulator if it fills most of the cavity.

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Fig. 7.2 Intracavity modulation efficiency as a function of the relative modulator length for the lumped modulator (7.2.1) and travelling-wave modulator with no velocity mismatch (7.2.2)
Chapter 8

Numerical Simulations of Active Mode-Locking of Monolithic External Cavity Semiconductor Lasers

In the previous chapter, a frequency domain theory for the treatment of several active mode-coupling phenomena was presented. It was shown that mode-locking by current modulation, AM, FM mode-locking and FM-laser operation can be described by using similar sets of multi-mode field equations in conjunction with the rate equation for the carrier population. In this chapter we present the results of our numerical simulations which are based on the systems of equations derived in Chapter 7. Our aim was to demonstrate a basic numerical model for active mode-coupling phenomena. We have performed a more detailed investigation of mode-locking by current modulation in order to compare our results with several published models based on travelling-wave rate equations. Unfortunately, we have not found any published data which could be used for a direct comparison with our AM, FM mode-locking and FM-laser simulations.

All numerical simulations of gain-switching and mode-locking presented in this chapter have been obtained using the same set of laser parameters. These parameters are listed in Appendix 1. Furthermore, in order to allow a direct comparison between gain-switching and several types of active mode-locking, all pulse sequences, shapes and spectra have been obtained using the same laser bias and RF current amplitude.

Numerical simulations of gain-switching of a solitary laser and its monolithic external cavity equivalent are presented in Section 8.1. These simulations were obtained using single-mode rate equations. Numerical simulations of mode-locking by current modulation of a monolithic external cavity laser are presented in Section 8.2. Through these
simulations we have addressed several problems including the evolution of mode-locking and the pulsewidth dependence on the driving conditions (frequency, DC bias and RF amplitude). Section 8.3 is devoted to FM mode-locking and FM-laser operation. In these simulations we have observed two distinct types of behaviour depending on the inclusion of the linewidth enhancement factor. Finally, this chapter is concluded in Section 8.4 with a numerical demonstration of AM mode-locking.

8.1 Averaged Single-Mode Rate Equations

The aim of this section is to present a simple numerical simulation of semiconductor laser dynamics using the averaged single-mode rate equations. Our main intention is to illustrate the difference between gain switching of a solitary laser and its monolithic cavity counterpart. In addition, we would like to show the two distinct regimes of operation of a monolithic external cavity laser: highly non-linear operation in the lower GHz range and the linear behaviour at higher frequencies (~10GHz).

Our numerical simulations of gain-switching were performed using Fortran 77 programs in conjunction with a NAG Runge-Kutta integration routine D02BBF. A step size of 1ps and the integration tolerance of $10^{-4}$ were utilised. The initial conditions for the photon and carrier number were $P=0$ and $N=0$.

8.1.1 Gain-Switching of the Solitary Laser

For the single-mode description of gain-switching we can use Eqs. 7.37 and 7.39. In these equations we set coupling terms to zero, $L_M/L$ to one and use a single-mode photon number instead of the coherent sum of mode amplitudes. The gain nonlinearity can be described by the same expression (Eq. 7.33) and in this case it is a function of the single-mode photon number. The forcing term for the current $I$ is introduced as a combination of the DC and RF components.
One numerical simulation of proper gain-switching is shown in Fig. 3.2 (Chapter 3). In this case, the laser is biased just above threshold and modulated by the RF signal at 1GHz. The resulting pulse (Fig. 3.2a) has a shape which is characteristic of gain-switching: a steep front edge followed a longer trailing edge. The full width at half maximum of the pulse is 43ps and the peak power is approximately 30mW.

The photon and carrier number traces, obtained at a bias of 30mA and RF amplitude of 45mA (these are the standard values in our mode-locking simulations) are presented in Fig. 8.1. It should be noted that the time sequence in Fig. 8.1 is two times longer than in Fig. 3.2. It can be seen that the increase in the laser bias has led to some pulse distortion: in Fig. 8.1.1 a pedestal follows the laser pulse. The FWHM of this gain-switched pulse is 37ps.

8.1.2 Gain-Switching of the External Cavity Laser

For the description of the external cavity laser, the averaging terms $L_M/L$ are introduced into the single-mode rate equations, as in the case of Eq. 7.37. It is important to note that the averaging terms have to be incorporated into the expressions for both stimulated and spontaneous emission. The steady-state photon and carrier numbers of the gain-switched monolithic external cavity laser (whose round-trip frequency is 10GHz) at 1GHz are shown in Fig. 8.2. These gain-switched pulses exhibit period doubling and have a pulsewidth of 170ps. In this simulation the laser is biased at 30mA and driven with the RF amplitude of 45mA.

Period doubling and chaos have been observed in the numerical investigations of the direct modulation of laser diodes$^{1,2}$. These two effects were reported both for the single-mode and multi-mode rate equation formulations$^1$. In the more recent publications it was argued that the nonlinear gain$^3$ and Auger recombination$^4$ can completely suppress the chaotic behaviour of the single-mode rate equations. Period doubling was also confirmed experimentally for the high-frequency modulation of
Photon Number (1 unit = 10^5)

Carrier Number (1 unit = 10^8)

Fig. 8.1 Gain-switching of the solitary laser; steady-state photon number (8.1.1) and carrier number (8.1.2)

I_b = 30mA, I_{rf} = 45mA, f = 1GHz
Fig. 8.2 Gain-switching of the external cavity laser; steady-state photon number (8.2.1) and carrier number (8.2.2)

$I_b=30\text{mA}, I_{rf}=45\text{mA}, f=1\text{GHz}$
Fig. 8.3 Gain-switching of the external cavity laser; steady-state photon number (8.3.1) and carrier number (8.3.2)
$I_b=30\, mA, I_{rf}=45\, mA, f=10\, GHz$
InGaAsP lasers. In our numerical simulations of gain-switching of the monolithic laser at 1GHz we have obtained both period doubling and quadrupling, depending on the modulation depth.

To illustrate the behaviour of the single-mode rate equations of the monolithic laser at the mode-locking frequency, we plot the steady-state photon and carrier number for 10GHz modulation (Fig. 8.3). It should be noted that in a real system mode-coupling would take place. Our intention is only to present the dynamics of gain-switching of the external cavity laser at high frequencies. From Fig. 8.3 it can be seen that although we apply a large amplitude current modulation, the response of the system is linear and the variations of the photon and carrier numbers are small.

8.2 Mode-Locking by Current Modulation

All numerical simulations of mode-locking by current modulation, FM and AM mode-locking were performed using Fortran 77 programs and NAG routines. Numerical integration of the mode-locking equations was accomplished using a variable-step Adams method, routine D02CBF. A step-size of 0.1ps was chosen for the data output and the integration tolerance was specified as $10^{-6}$. The averaged spectrum of mode-locked pulse sequences was calculated using Fast Fourier Transform routine, C06ECF. For the frequency detuned behaviour, which resulted in unstable pulse sequences, the correlation routine C06FKF was used for the evaluation of the average pulsewidth. Regarding the initial conditions, all our simulations were started with the normalised photon number of each mode equal to $10^{-4}$. The phase of each mode was selected using a random number generator G05DAF (uniform distribution in the range from $-\pi$ to $\pi$).

Equations 7.37, 7.38 and 7.39 have been used to formulate the numerical model for mode locking by current modulation. We have assumed a constant gain for all modes and a parabolic loss dependence with a bandwidth of 290GHz. We made two approximations in our numerical
model. First, the coupling coefficients are actually time dependent, whereas we introduced them as constants. The advantage of this approximation is that system of equations can be integrated efficiently over a long time sequence using optimised numerical routines. In addition, there is no need for the simultaneous FFT computation of the carrier spectrum. The second approximation is that we assume coupling to neighbouring modes only. The consequence of these two approximations is that the coupling coefficient can be computed from the multi-mode or even single-mode rate equations of the modulated external cavity laser.

In our programs, we have used the values of laser parameters presented in Appendix 1. Note that the photon number $P$, carrier number $N$ and time $t$ in Eqs. 7.37, 38 and 39 have been scaled by $10^{-5}$, $10^{-8}$ and $10^{10}$ respectively.

8.2.1 Numerical Simulations

Our program was tested using the same driving conditions as given in Ref. 7.28. For a flat gain profile and mode locking at 1GHz we obtained $\text{sinc}^2$ pulses. The pulses produced from our model look very similar to the pulses presented in Ref. 7.28, for both ten mode and fifty mode cases. However, there is one important difference. We use the formulation given by Equations 7.37, 38 and 39, in which the coupling coefficient $\Delta_q$ is real and positive, and introduce a current $I(t)$ into Eq. 7.39 of the form:

$$I(t) = I_b - I_r \sin \omega_m t$$  \hspace{1cm} (8.1)

According to our numerical simulations of direct modulation of the external-cavity laser, the carrier number $N$ is lagging the current $I$ for approximately $\pi/2$. The $-\sin \omega_m t$ variation of the current (Eq.8.1) then implies the $+\cos \omega_m t$ variation of the carrier number, leading to a real and positive $\Delta_q$ (Eqs. 7.14 and 7.23). In Reference 7.28, the time-varying part of the current in Eq.8.1 is introduced as $+\sin \omega_m t$. The mode-locked
pulses obtained in Ref. 7.28 do not represent a physical solution because each pulse is emitted just after the minimum value of the carrier number (instead of following the maximum).

In our numerical simulations of monolithic external cavity lasers we use twenty five laser modes within a frequency range of 250GHz or a wavelength span of approximately 2nm at 1.5μm. We assumed a parabolic loss with a bandwidth of 290GHz. This combination of modes and loss distribution leaves a sufficient leeway for the frequency shifts which can be observed in frequency detuned operation. In our case, the loss distribution represents the dominant control mechanism and the number of modes which are lasing is small.

A set of plots, obtained for mode-locking at the round-trip frequency of 10GHz, a bias current of 30mA and an RF current amplitude of 45mA is shown in Fig. 8.4. The threshold current of the monolithic laser is approximately 19mA. The spectrum, obtained by averaging over fifty pulses is shown in Fig. 8.4.1. As expected, the spectrum is asymmetric as a consequence of optical phase modulation due to the linewidth enhancement factor. The shape of the spectrum is in accordance with the experimental trace presented in Reference 3.56. Since our model neglects the spatial propagation of the pulse through the gain media, no amplifier effects are present, and the spectrum is not shifted with respect to the central frequency for zero detuning.

The photon number and carrier number are plotted for a sequence of fifty pulses in Figs. 8.4.2 and 8.4.3 respectively. At a wavelength of 1.5μm, the optical power, expressed in mW, is simply the photon number P in units of 10^5 multiplied by 2.5 (Eq. 2.41). The carrier number N corresponding to the threshold is approximately 2.15 (Fig. 8.4.3). One pulse and the corresponding carrier number waveform, extracted from Fig. 8.4.2 and 8.4.3 are presented in Figs. 8.4.4 and 8.4.5. We can see that the pulse shape is slightly asymmetric and its full width at half maximum is 11.5ps. The FWHM of the spectrum is 33.3GHz and the corresponding
Fig. 8.4 Mode-locking by current modulation; averaged spectrum over 50 pulses with center frequency at 130GHz (8.4.1), steady-state photon number (8.4.2) and carrier number (8.4.3) for a long pulse sequence, steady-state photon number (8.4.4) and carrier number (8.4.5) for an extracted pulse
$I_b=30\text{mA}, I_{rf}=45\text{mA}, f=10\text{GHz}$
Photon Number (1 unit=10^5)

Carrier Number (1 unit=10^8)
time-bandwidth product of the pulse is 0.38. The time-bandwidth product of the monolithic laser reported in Ref. 3.56 is 0.34. A bandwidth of the Bragg section of this reported monolithic laser is around 135GHz.

8.2.2 Evolution of Mode-Locking

The evolution of mode-locking in our numerical model is presented in Fig. 8.5. We used our standard initial conditions: photon numbers $P_i$ equal to $10^{-4}$ and random phases $\phi_i$. The development of the peak photon number during the first 50ns is shown in Fig. 8.5.1. The equivalent evolution of the average photon number (which we define as an incoherent sum of the modal photon numbers $P_i$) is displayed in Fig. 8.5.2. It can be seen that if the constant coupling coefficient, corresponding to the steady-state modulation depth of the carrier number is introduced, the evolution is dominated by the relaxation oscillation effects in the first 8ns and by mode-coupling effects afterwards. For the RF amplitude of 45mA, the steady-state is reached after 25ns. It should be noted that the time sequence of the peak photon number is affected by the choice of initial phases of the modes only in the first 7-8ns.

8.2.3 Detuning of the Modulating Frequency

Detuning of the modulating frequency is introduced by the first term of Equation 7.38. In our program, we assumed that the central mode of the field expansion coincides with a resonant frequency of the Fabry-Perot cavity, and that the optical frequency of the central mode also corresponds to the minimum of the parabolic loss distribution. These two approximations are commonly employed in numerical models of mode-locking based on a frequency-domain formulation.

In our model, detuning of the modulating frequency from the round-trip frequency results in unstable pulse sequences which exhibit a cyclic behaviour. At this stage it is not clear whether this cyclic behaviour occurs in the real system or whether it is figment of our model. In our
Fig. 8.5 Evolution of mode-locking by current modulation; initial conditions: zero amplitudes, random phases; peak photon number (8.5.1) and average (incoherent) photon number (8.5.2)

$I_b=30mA, I_{rf}=45mA, f=10GHz$
experiments with mode-locked external cavity lasers we have observed the
RF spectra which indicated cyclic instabilities. Moreover, other groups
have reported similar experimental observations.

The topic of pulse stability in mode-locked semiconductor lasers has been
addressed by Lowery in a series of publications \(^6,7,8\). The results obtained
from our frequency-domain model show the same trend as Lowery's
results which are based on a transmission line matrix method. As in
Lowery's publications, we define detuning of the modulating frequency \(\Delta f\)
as:

\[
\Delta f = f_{\text{mod}} - f_{\text{cav}} \tag{8.2}
\]

where \(f_{\text{mod}}\) is the modulating frequency and \(f_{\text{cav}}\) is the Fabry-Perot spacing
of the laser cavity. The general trend which we observe in our simulations
is that of positive detuning - the new pulse evolves from the front edge of
the old pulse. Conversely, for negative detuning, the new pulse develops
from the trailing edge of the previous pulse. The main effect of the
frequency selective element inside the laser cavity is to reduce these
instabilities.

The results which have been obtained for frequency detuned behaviour are
presented in Fig. 8.6. In Fig. 8.6.1 we show the average pulsewidth (over
50 pulses) calculated in the range from -200MHz to +200MHz. The shortest
pulses are produced for zero detuning. The pulses produced in the range
of approximately -150 to 140 MHz exhibit relatively small amplitude
variations and jitter. For larger detuning, the pulses start to break up and
the pulse evolution from the front edge (positive detuning) or the back edge
(negative detuning) becomes apparent. This is also accompanied by
creation of a DC pedestal on the average trace. The transition from the
first to the second regime is quite abrupt.

The frequency shift of the optical spectrum due to detuning of the
modulating frequency is shown in Fig. 8.6.2. It can be seen that if the
Fig. 8.6 Detuning of the modulating frequency; average pulsewidth (8.6.1) and optical frequency shift (8.6.2) versus modulating frequency. Zero detuning for \( f = 10.0 \text{GHz}, I_b = 30 \text{mA}, I_{rf} = 45 \text{mA} \)
amount of detuning is relatively small, the spectrum shifts towards higher optical frequencies for negative detuning, and towards lower optical frequencies for positive detuning. For larger detuning, when mode-locked pulses start to break up, the spectrum takes the shape of a distorted FM spectrum. In this regime the frequency shifts are smaller and they have the opposite sign if compared to the previous case.

In our model, the time-bandwidth product of mode-locked pulses is increased for frequency detuned operation. For a positive detuning of 150MHz, at which pulse breaking occurs, the time-bandwidth product is approximately four times larger than for the optimal mode-locking (zero detuning).

To the best of our knowledge, there are no reported data on experimental measurements of the detuned behaviour of monolithic external cavity lasers. However, a comparison can be made with Reference 6 which is a theoretical and experimental investigation of the stability and spectral behaviour of a mode-locked external grating semiconductor laser. The difference between our frequency-domain model and the transmission-line laser model (Ref. 6) is that we do not account for the spatial gain saturation of the laser. The consequence of this approximation is that no effects of self-phase modulation can be seen in our numerical results. The second difference between the two models is that our is deterministic, while Lowery's includes a random number generator to simulate the spontaneous emission noise.

Our result shown in Fig. 8.6.1, which represents the average pulsewidth versus detuning, is more symmetric than the equivalent measurement from Ref.6. In our simulations, the minimum pulsewidth is obtained for zero detuning, while in Ref.6 the shortest pulses are obtained with a small positive detuning (relative frequency increase of 0.25%). With regard to the optical frequency shift due to detuning (Fig. 8.6.2), we obtain the same trend as reported in Ref. 6, provided the frequency detuning is small.
A comparison also can be made with Kuizenga and Siegman's classic paper on AM and FM mode-locking of a homogeneous laser (Ref. 7.4). If we describe active mode-locking of a semiconductor laser as a combination of AM mode-locking due to gain modulation and FM mode-locking due to intracavity phase modulation, the frequency shift of the spectrum will be caused by FM-mode locking. In a semiconductor laser, a mode-locked pulse will be formed under minimum of phase variation and this solution corresponds to Kuizenga and Siegman's 'negative mode'. For small detuning, we obtain the same direction of the frequency shift as predicted in Ref. 7.4. based on the self-consistent pulse analysis.

In our detuning results we also observe the 'sideband flipping' reported by Schremer and Tang\(^9\). In Ref. 9, the frequency modulation of an external cavity semiconductor laser was investigated. An abrupt phase reversal of the optical signal was observed at a frequency which corresponds to the round-trip of the external cavity. Schremer and Tang used a theoretical formulation based on coherent feedback interference\(^10\) to explain this effect. As a large-signal effect, our mode-locking results show both the shift of the total spectrum and the 'sideband flipping'. Our mode-locked spectra obtained for +20MHz and -20MHz detuning are shown in Fig. 8.7, while the zero detuning case is presented in Fig. 8.4.1. Regarding the 'sideband flipping', our results show the same trend as the experimental measurements of the frequency modulated, anti-reflective coated semiconductor laser placed in an external cavity reported in Ref. 9.

8.2.4 Variation of the Bias Current

The dependence of the pulsewidth upon the bias current of the mode-locked laser is presented in Fig. 8.8.1. The shortest pulses are obtained when the laser is biased at threshold. Since we treat spontaneous emission noise as a coherent injection, a relatively large average rate of spontaneous emission can cause excessive pulse shortening at a lower laser bias. This is the main reason why we use the bias of 30mA in our simulations.
Fig. 8.7 Detuning of the modulating frequency - 'sideband flipping'; spectra obtained for +20MHz detuning (8.7.1) and -20MHz detuning (8.7.2). Center frequency of the spectrum 130GHz.

$I_p=30\text{mA}$, $I_{rf}=45\text{mA}$, $f_{rt}=10\text{GHz}$
Fig. 8.8 Variation of the bias current; pulsewidth (8.8.1) and peak photon number (8.8.2) versus bias current.
Laser threshold at $I_b=19$ mA, $I_{rf}=45$ mA, $f=10$ GHz
One drawback of our frequency-domain model is that the peak photon number increases linearly with the bias current. This dependence is shown in Fig. 8.8.2. From our experimental experience of an external grating semiconductor laser, we would expect significant pulse distortion with an increase of the bias current. Lowery's simulations of a distributed-Bragg reflector laser mode-locked at 30GHz\(^{11}\), based on the transmission-line matrix model, predict double pulses which we do not observe in our model. We believe that averaging, introduced by the terms \(L_L/L\) in Eq.7.37 has some influence on the behaviour of the system. It can be seen from Figs. 8.2 and 8.3 that the external cavity laser is much less responsive at 10GHz than at 1GHz and that its behaviour is linear even for the large-signal modulation that we use. The fact that we use a relatively weak expression for the gain nonlinearity (Eq. 7.33) is probably less important than the averaging effects.

8.2.5 Variation of the RF Current Amplitude

A change of RF amplitude affects both the forcing term in Eq. 7.39, and the value of the coupling coefficients used in Eqs. 7.37 and 38. The effects of variation of the RF amplitude are presented in Fig. 8.9. If the amplitude is varied from 5 to 90mA, the pulsewidth decreases from 19ps to around 10ps, as shown in Fig. 8.9.1. Using our deterministic model, we can investigate mode-locking with RF currents much smaller than 5mA, but in practice this regime would be seriously affected by noise. The variation of the peak photon number, when the RF amplitude is varied from 5 to 90mA, is shown in Fig. 8.9.2. The saturation of the peak power with the increase in RF drive, which we observe in our simulation, is expected in a real system.

8.2.6 Effects of the Linewidth Enhancement Factor

Thus far, all the simulations which we presented included a refractive index dependence on the carrier concentration. If we set the linewidth enhancement factor to zero, we obtain a symmetric spectrum (Fig.
Fig. 8.9 Variation of the RF current amplitude; pulsewidth (8.9.1) and peak photon number (8.9.2) versus amplitude of the sinusoidal drive. $I_b=30mA$, $f=10GHz$
8.10.1). The corresponding pulse shape is shown in Fig. 8.10.2. This pulse shape is nearly identical to one shown in Fig. 8.4.5 with $\beta_c=5$. The main effect of $\beta_c$ is in evolution of mode-locking. With $\beta_c=0$, the evolution of mode-locking is slower, and the steady-state is achieved after approximately 60ns. The difference between the pulses obtained for $\beta_c=5$ and $\beta_c=0$ becomes pronounced for lower values of the DC bias. For example, for a bias current of 20mA, the pulses become ~ 5% shorter if the linewidth enhancement factor is included.

With regard to detuning of the modulating frequency, no frequency shifts of the spectrum are generated when the linewidth enhancement factor is set to zero. According to our model, the effect of the linewidth enhancement factor can be seen in the dependence of the average pulse width on the modulating frequency (Fig. 8.6.1). If the linewidth enhancement factor is set to zero, the broadening of pulses with the increasing detuning is gradual, and there are no abrupt transitions at +/- 150MHz as shown in Fig. 8.6.1.

8.3 FM Mode-Locking

Equations 7.47, 48 and 39 have been used to form a numerical model of FM mode-locking. In our program, we have only used the coupling contribution of the carrier number variation introduced by Eq. 7.39 and set the corresponding coupling coefficient $\Delta_q$ in Eqs. 7.47 and 48 to zero. As in the case of mode-locking by current modulation we have utilised a constant gain coefficient for all modes and a parabolic loss dependence. The phase modulator section was assumed to be short compared to the length of the monolithic cavity and the intracavity modulation efficiency (Fig. 7.2.1) was set to one. In our simulations of FM mode-locking we observed two distinct types of behaviour dependent on the inclusion of the linewidth enhancement factor.
Fig. 8.10 Effects of the linewidth enhancement factor; spectrum (8.10.1) and pulseshape (8.10.2) for the linewidth enhancement factor set to zero. $I_b=30\,mA$, $I_{rf}=45\,mA$, $f=10\,GHz$
8.3.1 Numerical Simulation Including the Linewidth Enhancement Factor

In this simulation, we used twenty five modes, as in the case of mode-locking by current modulation. For the current \( I \) in Eq. 7.39, only a DC term was used. The coupling coefficient \( \Delta_{FM} \) in Eqs. 7.47 and 48 was calculated using Eq. 7.46 assuming a modulation depth of 0.1\( \pi \) (Appendix 1). Using our standard laser parameters, a bias \( I_b=30mA \) and a linewidth enhancement factor \( \beta_c=5 \), we obtained the results presented in Fig. 8.11. The averaged spectrum calculated over the sequence of fifty pulses is presented in Fig. 8.11.1. The pulse shape and the corresponding carrier number waveform are shown in Figs. 8.11.2 and 8.11.3 respectively. The pulses produced by FM mode-locking have the same shape as the pulses obtained by active mode-locking using current modulation (Fig. 8.4.4). For a modulation depth of 0.1\( \pi \), we obtained pulses with a pulsewidth of 12.7ps and time-bandwidth product of 0.38. The difference between FM mode-locking and mode-locking by current modulation can be seen in the carrier number waveform. From Fig. 8.11.3 it can be seen that in FM mode-locking the carrier number variation is only caused by the depletion due to the arrival of each individual pulse. In mode-locking by current modulation, the carrier number variation is determined by a combined effect of the forced modulation and depletion (Fig. 8.4.5).

One of the difficulties of a frequency-domain formulation of mode-locking is that the method does not allow easy physical interpretation of the process. In the time-domain, the steady-state of FM mode-locking is achieved when the chirp, introduced by the phase modulator, is balanced by dispersion. As mentioned in Chapter 5.1.2, two solutions exist for the case of FM mode-locking. One solution corresponds to the pulse formed under the maximum of the phase deviation of the modulator and the other solution is formed at the minimum of the modulation cycle. Within our model, it seems that the phase modulation caused by the linewidth enhancement factor enhances one solution and completely suppresses the other one.
Fig. 8.11  FM mode-locking including the linewidth enhancement factor $\beta_c=5$; averaged spectrum over 50 pulses with center frequency at 130GHz (8.11.1), steady-state photon number (8.11.2) and carrier number (8.11.3) versus time

$I_b=30mA$, $\varphi_{max}=0.1\pi$, $f=10GHz$
In our FM mode-locking experiment with InGaAsP-LiNbO$_3$ laser we observed both solutions excited simultaneously (Fig. 5.8). We are reluctant to use our experimental result in this discussion because of the pronounced composite-cavity effects of our hybrid cavity. Recently, an experimental investigation of pure FM mode-locking of an external cavity semiconductor laser was reported$^{12}$. The gain element was a two-section optical amplifier and the cavity was formed using an external grating. A surprising result of this paper, which received no comment, was that two different regimes of FM mode-locking were observed in two experiments utilising different laser chips. In both cases the same type of external cavity was used. In the first case, a single train of pulses was generated, while in the second case a double train of pulses (similar to our trace in Fig. 5.8) was produced.

It is certain that further experimental evidence is needed to justify the validity of our theoretical result. In the case of long external cavities the effects of the linewidth enhancement factor are reduced. If the level of the optical feedback is small then the combined effect of composite cavity, asymmetry of feedback, and spontaneous emission noise may become the dominant control mechanism of the mode-locked output.

8.3.2 Numerical Simulation Excluding the Linewidth Enhancement Factor

We have also performed numerical simulations of FM mode-locking assuming a zero value for the linewidth enhancement factor. Our results are presented in Fig. 8.12. The main characteristic of FM mode-locking with $\beta_c$=0 is double pulsing. This simultaneous excitation of two mode-locking solutions is shown in the calculated waveforms of the photon number (Fig. 8.12.2) and carrier number (Fig. 8.12.3). The contributions of the two pulse trains are resolved clearly in the overall spectrum shown in Fig. 8.12.1.
Fig. 8.12  FM mode-locking when the linewidth enhancement factor is set to zero ($\beta_c=0$); averaged spectrum over 50 pulses with center frequency at 130GHz (8.12.1), steady-state photon number (8.12.2) and carrier number (8.12.3) versus time

$I_b=30\text{mA}$, $\varphi_{\text{max}}=0.1\pi$, $f=10\text{GHz}$
The second characteristic of our result is that the pulses obtained for $\beta_c=0$ have a different shape than the equivalent pulses achieved for $\beta_c=5$. If the linewidth enhancement factor is set to zero, the pulses become slightly shorter (12.2ps) and the time-bandwidth product increases to approximately 0.63. According to Kuizenga and Siegman's time-domain theory\textsuperscript{[7,4]}, based on the self-consistent pulse analysis, the time-bandwidth product of a Gaussian pulse produced by FM mode-locking is 0.626.

Further investigation is needed in order to prove the validity of our numerical results produced for zero detuning. The first step would be to include the coupling coefficient due to the carrier number variation $\Delta_n$ in the program. However, if the program is kept in its original form, additional insight can be obtained if detuning of the modulating frequency is included.

8.3.3 FM-Laser Operation

A well known result of general laser theory is that sufficient detuning of the modulating frequency in FM mode-locking leads to a frequency swept mode of operation called FM-laser operation. We have discussed FM-laser operation in Chapter 5.1.2 and considered the use of a composite cavity InGaAsP-LiNbO$_3$ laser in order to demonstrate it.

In our numerical model we have observed two types of behaviour for FM-laser operation, dependent on the inclusion of the linewidth enhancement factor. Two spectra obtained by averaging over 500 pulses are shown in Fig. 8.13. Both spectra are obtained for positive detuning of 100MHz and our standard driving conditions. For $\beta_c=5$, the spectrum has a shape of a distorted FM spectrum (Fig. 8.13.1). A proper FM spectrum is obtained when the linewidth enhancement factor is set to zero (Fig. 8.13.2). As in the case of detuned operation of mode-locking by current modulation, unstable pulse sequences are achieved both for $\beta_c=0$ and $\beta_c=5$. For the same amount of detuning, the ratio of the pulse height to the DC pedestal is larger when the linewidth enhancement factor is included. In the
Fig. 8.13 FM-laser operation obtained for +100MHz detuning; averaged spectrum over 500 pulses with center frequency at 130GHz. Linewidth enhancement factor $\beta_C=5$ (8.13.1) and $\beta_C=0$ (8.13.2)

$I_b=30mA$, $\varphi_{\text{max}}=0.1\pi$, $f_{rt}=10$GHz
example presented in Fig. 8.13 this ratio is 2.7 for $\beta_c=5$, and 0.25 for $\beta_c=0$. This infers that the semiconductor laser is still strongly pulsing for 100MHz detuning if the linewidth enhancement factor is included. For the same amount of detuning, the fluctuations in the output power are relatively small if the linewidth enhancement factor is set to zero.

8.4 AM Mode-Locking

Finally, to conclude this chapter, we present two simulations of AM mode-locking. Equations 7.57, 58 and 39 have been used in conjunction with the parameter values listed in Appendix 1. As in the case of FM mode-locking, we have only utilised the coupling contribution of the carrier number variation represented by Eq. 7.39 and set $\Delta_q$ to zero. The loss modulator section was assumed to be short if compared to the length of the monolithic cavity. As in our previous simulations of active mode-locking, we have assumed a parabolic loss dependence. In the case of AM mode-locking, we have introduced a parabolic distribution of coupling coefficients (both $\Delta_{AM}$ and $\Delta_{AM0}$) to compensate for the different modulation depths of individual modes.

8.4.1 Numerical Simulation Including the Linewidth Enhancement Factor

In our numerical simulation of AM mode-locking we have used twenty five modes, a DC current of 30mA and the amplitude of RF variation of the mirror reflectivity of approximately 10%. Fig. 8.14. shows the results of this simulation. The first plot (Fig. 8.14.1) represents the spectrum which is calculated from a sequence of fifty pulses. Although we obtain a stable sequence of mode-locked pulses, the spectrum shows a small amount of broadening which is probably caused by the modal phase variations. Due to the inclusion of the linewidth enhancement factor, the spectrum is asymmetric. The pulse shape and the equivalent carrier variation are presented in Figs. 8.14.2 and 8.14.3. The pulses are 17.2ps long and the time-bandwidth product is 0.62.
Fig. 8.14 AM mode-locking with $\beta_c=5$; averaged spectrum over 50 pulses with center frequency at 130GHz (8.14.1), steady-state photon number (8.14.2) and carrier number (8.14.3) versus time
$\text{I}_b=30\text{mA}$, $\Delta R=10\%$, f=10GHz
8.4.2 Numerical Simulation Excluding the Linewidth Enhancement Factor

For the same driving conditions, but for the linewidth enhancement factor set to zero, we have obtained the spectrum shown in Fig. 8.15.1 and the pulse shape displayed in Fig. 8.15.2. As expected, the spectrum is symmetric. According to our simulation, the pulse width decreases to 8.6 ps if the linewidth enhancement factor is reset to zero. The time-bandwidth product is also reduced for $\beta_c = 0$ and is equal to 0.43. This time-bandwidth product is in close agreement with Kuizenga and Siegman's result\(^\text{7.4}\) for AM mode-locking of a homogeneous laser.

In this section, our aim was to demonstrate the basic numerical model formulated in Chapter 7.2.3. In our model for AM mode-locking, the difference between mode-locking with $\beta_c = 5$ and $\beta_c = 0$ is much more pronounced than in the case of current modulation (Section 8.2). The inclusion of the the coupling coefficient $\Delta_q$ is needed in order to provide an explanation for the cause of this difference.

8.5 Summary

In this chapter we presented numerical simulations of gain-switching and mode-locking of semiconductor lasers. The objective of our gain-switching simulations were to illustrate the behaviour of the averaged single-mode rate equations. In particular, we compared the operation of a solitary laser to its monolithic external cavity equivalent, and emphasised the reduced responsiveness of a monolithic laser modulated at high frequencies. Using our frequency-domain model, we demonstrated mode-locking by current modulation, AM, FM mode-locking and FM laser operation of monolithic lasers, and achieved very good consistency between all these simulations.
Fig. 8.15 AM mode-locking with $\beta_c=0$; averaged spectrum (8.15.1) and steady-state pulse shape (8.15.2)

$I_b=30\text{mA}, \Delta R=10\%, f=10\text{GHz}$
With regard to active mode-locking by current modulation we investigated the time evolution of mode-locking, and the effects of detuning of the modulating frequency and variation of the bias and RF current. The pulse shape and spectrum that we produced were in good agreement with experimental data reported in the literature. Detuning of the modulating frequency in our model resulted in cyclic pulse sequences, as predicted by a transmission-line matrix model. Our results for FM mode-locking and FM-laser operation show two distinct types of behaviour which are dependent upon the inclusion of the linewidth enhancement factor. According to our simulations, the linewidth enhancement factor provides a selection mechanism in FM mode-locking. In FM-laser operation, the inclusion of the linewidth enhancement factor results in the generation of distorted FM spectra.
Chapter 9

Conclusions and Future Work

This thesis has described an experimental and theoretical investigation of active mode-locking of semiconductor lasers. The project was undertaken in collaboration with National Physical Laboratory who defined the objective of the experimental work as the generation of 10ps optical pulses. This objective has been met and was accomplished using an actively mode-locked, external grating semiconductor laser in conjunction with a semiconductor optical amplifier. In addition, we have studied short pulse generation using composite cavity InGaAsP-LiNbO$_3$ lasers. In these structures, we have investigated mode-locking by current modulation, AM mode-locking, and for the first time in this type of configuration FM mode-locking. We have complemented our experimental investigations with a theoretical study of active mode-locking of semiconductor lasers using a frequency-domain formulation. We have performed numerous numerical simulations and demonstrated mode-locking by current modulation, AM, FM mode-locking and FM-laser operation of monolithic external cavity semiconductor lasers. To the best of our knowledge, this is the first unified description of active mode-coupling effects in semiconductor lasers.

To review our experimental work, we start with the fabrication of lithium niobate modulators. The waveguides were formed by titanium indiffusion into lithium niobate substrates. In order to prevent the outdiffusion of Li$_2$O we investigated both wet and dry diffusions. The latter method was used for the waveguide fabrication of our modulators. We fabricated a phase modulator by depositing a travelling-wave electrode structure on the waveguide substrate. For the intensity modulator, a Y-junction was utilised which acts as a Mach-Zehnder interferometer within a composite cavity. The electrodes of the intensity modulator were fabricated as a
travelling-wave push-pull configuration. Both modulators were designed as waveguide devices utilising optical TE-mode propagation. Anti-reflective coatings and mirrors were deposited on all the waveguide samples and modulators used in our experiments.

With regard to our composite-cavity InGaAsP-LiNbO₃ laser experiments, we have investigated several methods of optical coupling between the laser and lithium niobate waveguide. The most efficient external cavity operation was achieved using a graded-index lens as a coupling element. We also investigated both direct coupling and the use of spherical lenses. In these two cases we failed to produce proper external cavity operation.

Using three composite-cavity InGaAsP-LiNbO₃ configurations we demonstrated active mode-locking by current modulation, AM and FM mode-locking. Due to the low intracavity optical coupling efficiency and high parasitic reflections we obtained relatively broad pulses. We could not complete a more detailed experimental investigation of active mode-locking of InGaAsP-LiNbO₃ lasers because of the low optical power levels. Our experimental investigation of InGaAsP-LiNbO₃ lasers culminated in the following results:

1) InGaAsP-LiNbO₃ laser, current modulation 35ps
2) InGaAsP-LiNbO₃ laser, AM mode-locking 50ps
3) InGaAsP-LiNbO₃ laser, FM mode-locking 45ps

The results described above were obtained under consistent experimental conditions and using two very similar BTRL buried heterostructure lasers. These measurements were made using a fast photodiode / sampling oscilloscope combination.

Within the same measurement system we investigated a bulk external cavity laser and measured 27ps pulses. The bulk external cavity in this experiment consisted of a GRIN lens and a plane mirror. The best mode-locked pulses which we measured at UCL were 16ps. These 16ps pulses
were obtained from a modified bulk external cavity configuration using an improved measurement system which incorporated a lensed fibre and an optical isolator. The modified cavity consisted of a spherical lens, uncoated glass etalon and a plane mirror. Unfortunately, we were only able to measure the spectrum of broad mode-locked pulses (~40ps) due to the low finesse of the scanning Fabry-Perot resonator used in this experiment. The time-bandwidth product of the 40ps pulses was estimated to be 0.6.

The experimental work undertaken at NPL included the testing of several external grating configurations. We experimented with several gratings and lenses in order to find the optimal combination for mode-locking. In addition we experimented with two types of electrical drive. Surprisingly, we found little difference between the performance of the mode-locked systems when driven either with a step-recovery diode or a sinusoidal drive. Our final configuration, which is now used for calibration purposes at NPL, incorporates a mode-locked semiconductor laser and semiconductor optical amplifier. For this system we have measured 7.5ps pulses (for sech^2 pulse shape) after the optical amplifier and obtained 15.4ps sampling oscilloscope traces (Fig. 6.10).

With regard to the theoretical work, we have developed a frequency-domain model of active mode-locking of semiconductor lasers. Several frequency-domain formulations of active mode-locking of semiconductor lasers by current modulation have been reported in the literature. To the best of our knowledge, our attempt is the first to provide a unified description of mode-locking by current modulation, AM, FM mode-locking and FM-laser operation of semiconductor lasers. Our formulation is based on a set of multi-mode field equations, used in conjunction with the rate equation for the carrier number. We have also utilised the frequency-domain approach to show that long travelling-wave modulators can still provide sufficient intracavity modulation in AM and FM mode-locking when their lumped equivalents become inefficient.
Parasitic intracavity reflections are neglected in our model. These reflections can play an important role in mode-locking, and in practice they limit the minimum pulse widths in bulk external cavity lasers. Since the formulation that we use neglects the spatial variation of the carrier concentration, the validity of our model is probably restricted to pulses longer than 5ps, as in the case of ordinary averaged rate equations. The main advantage of the frequency-domain approach is that it provides unified description of mode-coupling effects caused by an active, forcing element.

A complete set of programs dealing with the dynamics of semiconductor lasers has been produced. These programs are written in Fortran 77 and they use NAG routines and GINO graphics. In the development of these programs, we started with a single-mode rate description of gain-switching (both solitary and external cavity lasers) and Q-switching. The next step was a multi-mode formulation of these effects. Our first numerical model for mode-locking by current modulation included a flat gain and loss profile and assumed 1GHz modulation. We have compared our simulations with Reference 2 and have found excellent agreement for both ten-mode and fifty-mode cases. We improved upon the model presented in Reference 2 in two ways. First, we modified the timing between the photon and carrier number variations by proper synchronisation of the forcing term due to current modulation and the equivalent coupling coefficient. Second, we introduced a parabolic loss which resulted in the generation of realistic pulse shapes and spectra. Regarding the second improvement, it was necessary to include an additional phase shift as an effect of the linewidth enhancement factor.

An investigation of active mode-locking of monolithic semiconductor lasers at 10 GHz was performed using twenty five modes and a parabolic loss distribution. The time evolution of mode-locking by current modulation was studied using our standard initial conditions, namely zero photon numbers and random phases. The effect of detuning of the modulating frequency and variation of the bias and RF current have been

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examined. Finally, the effect of the linewidth enhancement factor was investigated. We have also studied FM mode-locking of monolithic lasers. Two types of behaviour were observed, depending on the inclusion of the linewidth enhancement factor. For $\beta_c=0$ we obtained double pulsing - a characteristic which we had obtained in our experiment with composite cavity InGaAsP-LiNbO$_3$ lasers. A single sequence of mode-locked pulses was generated from our model for $\beta_c=5$. We have also demonstrated FM-laser operation of a monolithic semiconductor laser. As in the case of FM mode-locking, we observed two types of behaviour. For $\beta_c=0$ we obtained symmetric FM spectra, whereas with $\beta_c=5$ a distorted FM spectra and enhanced pulsation were obtained. We also tested our theoretical formulation of AM mode-locking and demonstrated mode-locked pulse sequences for both $\beta_c=5$ and $\beta_c=0$.

We have achieved good consistency between our numerical simulations of mode-locking by current modulation, AM, FM mode-locking and FM laser operation. Validation of our numerical simulations has proved difficult as very little has been reported on experimental measurements and theory of active mode-locking of monolithic semiconductor lasers. Most of our comparisons are made with the published experimental investigations of bulk external cavity lasers.

Our numerical results for active mode-locking by current modulation are in good agreement with the experimental traces produced by a monolithic ATT&Bell laser$^{3,56}$. Unfortunately, there are no reported experimental investigations of frequency detuning or the effect of variations of the bias and RF current to compare with our data.

Detuning of the modulating frequency in our model results in irregular, but cyclic pulse sequences. The jitter and amplitude variations are not very pronounced for small detuning. However, for large detuning ($>150\text{MHz}$), the pulses start to break-up. In the case of positive detuning, the new pulse evolves from the back edge of the old pulse. Conversely, for negative detuning, the new pulse develops from the trailing edge of the
previous pulse. Such behaviour is in agreement with the results obtained for an external cavity laser using a transmission-line laser model\textsuperscript{8,6}.

During the preparation of this thesis, a special issue of IEEE Journal of Quantum Electronics\textsuperscript{1} on ultra-fast optoelectronics which contained several invited articles on mode-locking of monolithic external cavity lasers was published. It seems that hybrid mode-locking and colliding-pulse monolithic structures are the current trend in mode-locking of semiconductor lasers. The research on pure active mode-locking we presented in this thesis belongs to the 'slow lane' of mode-locking and all the recommendations for the future work will be made regarding optical pulse generation on the time-scales longer than 5ps.

From our experimental experience we can not recommend integrated optic lithium niobate modulators for active mode-locking of semiconductor lasers. In a more advanced environment than ours, it would be possible to improve the optical coupling efficiency and the uniformity of the anti-reflective coatings and dielectric mirrors. However, even with these improvements, the fabrication of a frequency selective element within the cavity would still present a problem.

With regard to our theoretical work, a comparison with a formulation based on the travelling-wave rate equations is needed in order to establish the range of validity of the frequency-domain model. Improvement of our program would result in the inclusion of coupling coefficients due to variations in the carrier concentration in AM and FM mode-locking; and a more advanced version of the program would include the coupling to higher order modes in all three formulations of active-mode locking. Ultimately, time-dependent coupling coefficients could be introduced by performing a simultaneous computation of the Fourier coefficients of the carrier number.

A stochastic frequency-domain model can be formed by the inclusion of random noise sources. Within the frequency-domain formulation,
fluctuations arising from spontaneous emission and carrier shot noise can be incorporated by adding Langevin noise sources to the equations for the photon numbers, phases and carrier number\textsuperscript{2}.

Finally, the equations describing coupled-cavity mode-locking of semiconductor lasers could be derived using a scattering-matrix approach\textsuperscript{3}. In coupled-cavity mode-locking (or additive pulse mode-locking), two cavities of nearly identical lengths are used. This scheme has been extensively utilised in solid-state lasers which have long relaxation times. Coupled-cavity passive mode-locking of semiconductor lasers has been demonstrated\textsuperscript{4} and has resulted in the generation of 20ps optical pulses. An active scheme utilising a coupled-cavity has been proposed for self-stabilisation of synchronously mode-locked lasers\textsuperscript{5}. This active scheme could also be applied to semiconductor lasers and it could serve as a severe test of the frequency domain formulation.
References

Chapter 1


Chapter 2


vol. 56, pp. 3110-3115.


Chapter 3


25, pp. 621-622.


Chapter 4


Chapter 5


Chapter 6


Chapter 7


Chapter 8


Chapter 9


### Appendix 1

**Parameter Values for a 1.3-1.5μm BH Laser Structure**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Active region width</td>
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<tr>
<td>Active layer thickness</td>
<td>( d ) 0.2μm</td>
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<tr>
<td>Confinement factor</td>
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<tr>
<td>Effective mode index</td>
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<tr>
<td>Group mode index</td>
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<tr>
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<tr>
<td>Internal loss</td>
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<td>Gain constant</td>
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</tr>
<tr>
<td>Nonradiative recombination rate</td>
<td>( A_{nr} ) 1 x 10(^8)s(^{-1})</td>
</tr>
<tr>
<td>Radiative recombination coefficient</td>
<td>( B ) 1 x 10(^{-10})cm(^3)/s</td>
</tr>
<tr>
<td>Auger recombination coefficient</td>
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<tr>
<td>Threshold carrier population</td>
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<td>( \Delta_{AM/L} ) 9.9 x 10(^{-4})ps(^{-1})</td>
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</table>
Appendix 2

Fabrication of Integrated Optic Lithium Niobate Modulators

1. Fabrication of Titanium Indiffused Waveguides

Fabrication is accomplished using samples obtained from three inch, X-cut, optical grade wafers manufactured by Crystal Technology. All fabrication procedures are carried out in a clean room area (class 100). The first stage is cleaning of the substrate accomplished using solvents (trichlorethylene, acetone and iso-propyl alcohol) and acids (sulphuric acid/hydrogen peroxide and fuming nitric acid). After rinsing in de-ionised water and blow drying using dry nitrogen the sample is baked at 120°C.

A positive photoresist Hoechst AZ4110 is spun onto the surface of the sample at 5000rpm for 20 seconds. Following a soft bake at 85°C for 20 minutes, the coated surface is exposed to UV light (approximately 5 sec.) through a shadow mask manufactured by EBLF Rutherford. To improve lift-off, the surface of the photoresist is hardened in chlorobenzene for 20 minutes. The pattern is developed in a 1:3 solution of Shipley AZ351 developer, and the sample is rinsed in de-ionised water and dried. Titanium is then thermally evaporated over the entire surface of the sample in a low pressure chamber (10^-6 torr). The lift-off is accomplished in a bath of acetone which is agitated ultrasonically.

Diffusion of the titanium is performed in a thermal diffusion furnace at approximately 1050°C. Diffusion times between 8 and 11 hours are used. To suppress the outdiffusion, the samples are placed on the alumina tray covered with lithium niobate powder, and dry oxygen is fed to the furnace for the duration of the run.
2. Electrode Fabrication

The same cleaning and photolitographic stages are used for fabrication of the electrodes. Following thermal evaporation of a 10nm chrome layer and a 100nm gold layer, the lift-off is accomplished using the procedure described in the previous section. To prevent excessive RF attenuation of the drive signal, the gold metalisation is electroplated to the thickness of several microns using Engelhard potassium gold sulphite solution. In order to avoid lateral electroplating, a photoresist pattern is defined between the electrodes. If AZ4110 resist is used, the maximum thickness of the electroplated gold is limited to approximately 1.5µm.

3. Polishing

The end-faces of the samples are prepared by epoxying blocks of lithium niobate over input and output facets. The polishing is performed using a polyurethane wheel covered with a syton polishing liquid. To ensure that the end-faces are perpendicular to the propagation direction of the waveguides, a specially designed substrate holder is used.
4. Fabrication Parameters of Lithium Niobate Devices Used in Three Composite Cavity InGaAsP-LiNbO₃ Laser Configurations

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<th>Phase Modulator</th>
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<td>Ti thickness (nm)</td>
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<td>Diffusion temp (°C)</td>
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<td>Diffusion time (hrs)</td>
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<td>Measured resistance (Ω)</td>
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