Foreign Body Detection in Food Materials using Compton Scattered X-Rays

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Abstract

This thesis investigated the application of X-ray Compton scattering to the problem of foreign body detection in food. The methods used were analytical modelling, simulation and experiment.

A criterion was defined for detectability, and a model was developed for predicting the minimum time required for detection. The model was used to predict the smallest detectable cubes of air, glass, plastic and steel. Simulations and experiments were performed on voids and glass in polystyrene phantoms, water, coffee and muesli. Backscatter was used to detect bones in chicken meat. The effects of geometry and multiple scatter on contrast, signal-to-noise, and detection time were simulated. Compton scatter was compared with transmission, and the effect of inhomogeneity was modelled. Spectral shape was investigated as a means of foreign body detection.

A signal-to-noise ratio of 7.4 was required for foreign body detection in food. A 0.46 cm cube of glass or a 1.19 cm cube of polystyrene were detectable in a 10 cm cube of water in one second. The minimum time to scan a whole sample varied as the 7th power of the foreign body size, and the 5th power of the sample size. Compton scatter inspection produced higher contrasts than transmission, but required longer measurement times because of the low number of photon counts. Compton scatter inspection of whole samples was very slow compared to production line speeds in the food industry. There was potential for Compton scatter in applications which did not require whole-sample scanning, such as surface inspection. There was also potential in the inspection of inhomogeneous samples. The multiple scatter fraction varied from 25% to 55% for 2 to 10 cm cubes of water, but did not have a large effect on the detection time. The spectral shape gave good contrasts and signal-to-noise ratios in the detection of chicken bones.
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Chapter 1

Introduction
1.1 Introduction

1.1.1 Foreign bodies in food

The problem of foreign bodies

Contamination by foreign bodies is a major problem for the food industry. Food manufactures in the UK are required to exercise due diligence in preventing contamination. They must have a system of HACCP (Hazard Analysis Critical Control Points) for dealing with safety-critical steps in production and must take appropriate steps to prevent foreign bodies entering the product (Holgate 1999). Despite the best efforts of manufacturers, it has been estimated that foreign bodies were the biggest cause of food industry prosecutions in the UK from 1988 to 1994 (Graves et al. 1998). A survey of foreign bodies in food in the UK has been given by Wallin & Haycock (1998a), and includes chapters on prevention, legislation and methods of detection. Similar statistics for the US (Hyman et al. 1993) showed that foreign bodies were responsible for 25% of food complaints in 1989, and that 14% of these incidents had resulted in illness or injury.

The most common contaminants, and the most likely to cause injury, were metal, glass, bone fragments and stones. Insects, rubber and plastic were also common. Other potential contaminants were string, hair pits, wood and paper. The rate of contamination can be extremely high. In one US study, (Schatzki et al. 1995), in which foreign bodies were to be added to processed meat, it was reported that 4.5% of the samples already contained bone fragments, and 0.5% contained pieces of metal.

Glass in food

The most commonly-reported and dangerous object found in food by US consumers in 1989 was glass, accounting for about 40% of the cases of injury (Hyman et al. 1993). Fragments of glass accounted for 10% of foreign body prosecutions in the UK from 1988 to 1994 (Graves et al. 1998). Glass fragments have been found in foods as diverse as pancakes (Wilkinson 1999), lager (Holgate 1999), processed meat (Schatzki et al. 1995) and yoghurt (Canadian Food Inspection Agency 1999).

Improvements in the inspection of food for glass would be valuable to the food industry. In addition, most methods for detecting glass fragments, particularly radiographic techniques, would also be applicable to the detection of stones in agricultural produce, because most types of stone are similar in composition to glass, and the density of stones, in the range 2.21 - 2.98 g cm$^{-3}$ (Slight 1966), is similar to that of glass at 2.60 g cm$^{-3}$ (Tennent 1978).
Bones in meat

Bone fragments in meat, particularly poultry, are a major problem in the food industry, whether in processed products or high-quality breast meat. It has been estimated that 6% of breast meat is trimmed off and assigned to cheaper products because of bone slivers and blemishes (Raj 1999). In the US, where 15 million tonnes of poultry are produced per year, 41% of which consists of nominally boneless products (Tao & Ibarra 2000), bones are the third most common cause of complaint (Smith 1999), accounting for two thirds of insurance claims and lawsuits against the poultry industry (Smith 1998).

Bone fragments may occur due to misaligned cutting blades shaving pieces from the skeleton, or they may be due to bones which were already broken before or during slaughter. In the UK, Gregory & Wilkins (1989) found that 98% of battery hens had broken bones after slaughter, whilst in the US, the risk of bone fragments has contributed to a total collapse in the market for spent laying hens (Brown 1993). Smith (2001) reported finding an incidence of approximately three defects, mostly broken bones, for every two birds at a commercial broiler processor, and approximately 3 bone fragments for every 10,000 breast fillets. Clavicles (wishbones) accounted for 64% of the fragments in the major muscle (fillets) and 95% in the minor muscle (tenders). Current radiographic methods often fail to detect small bone fragments because they are masked by the inhomogeneity of the product.

1.1.2 Methods of foreign body detection

Main technologies for NDT in food

The main technologies currently used in the food industry for foreign body detection are magnets, metal detectors, X-rays and optical methods (Wallin & Haycock 1998b).

Optical methods of detection rely on there being a visible difference, such as colour or shape, between a contaminant and the product. If an image of the product is produced, it can be analysed by human eye or by machine vision. In most cases, these methods cannot be applied to the packaged product, and are restricted to the detection of surface defects, or the removal of foreign bodies in loose produce, such as stones in rice.

Magnets and metal detectors are very commonly used in food inspection. Metal can occur in various forms, such as foil (Martens et al. 1993), wire (Penman et al. 1992), needles (Wallin & Haycock 1998a), or blades, bolts and nails (Hyman et al. 1993). Contamination can arise from machinery, packaging or even dental fillings (Holgate 1999). Modern permanent magnets are strong enough to pull very fine particles of ferrous metal out of moving food streams, and are very cheap and
robust. Metal detectors consist of balanced induction coils which are sensitive to changes caused by ferrous and non-ferrous metals, although stainless steel is particularly hard to detect. The construction must protect the coils against vibration, temperature changes and electrical interference. The resolution can be less than 1 mm, but depends on the size and water content of the sample and the shape and orientation of the foreign body (Wallin & Haycock 1998c).

Emerging technologies

Ultrasound discriminates between materials by differences in their acoustic properties, which gives it the potential to detect materials such as hard plastics in food which are difficult to detect by other methods. Problems with the technique include temperature dependence, attenuation in inhomogeneous materials, and the need to maintain a good acoustic contact with the sample (Haeggstrom & Luukkanala 2001).

NMR is a familiar diagnostic tool in chemistry and medicine, in which hydrogen nuclei are revealed by their resonant absorption of radio waves in a magnetic field. The application of NMR to on-line food inspection has attracted a small amount of research interest, for example, a fast measurement of the oil-to-water ratio of avocados (Chen et al. 1996), but the magnetic fields required are so powerful that the cost has prevented any take-up by industry.

Beta backscatter has been used to measure the strength of eggshells (Finney 1978).

Perhaps the newest of the emerging technologies is terahertz imaging, which uses the long-neglected part of the electromagnetic spectrum between the infrared and microwave regions. Many materials are semi-transparent at these wavelengths, and recent advances in the production of such waves has enabled researchers to produce images of teeth, and to distinguish between different tissue types in pork (Knott 1999, Arnone et al. 2000). This could become an important inspection tool if sources and detectors become sufficiently cheap for commercial use.

1.1.3 X-ray inspection

X-ray inspection is a well-established technology with numerous applications in food and agriculture. X-rays have been used to assess the quality of apples (Schatzki et al. 1997) and pistachio nuts (Casasent et al. 1996), to measure the maturity of lettuces (Lenker & Adrian 1971), and to detect broken pits in peaches (Han et al. 1992), weevils in grain (Keagy & Schatzki 1993) and voids in potatoes (Finney & Norris 1978). X-rays are very important for detecting foreign bodies such as glass in jars (Dykes 1985) or steel wire in cans (Penman et al. 1992), and have also been applied to the detection of bones and other foreign bodies in meat.
Schatzki et al. (1995) added bone fragments to meat products on an inspection line, and found that the human eye was able to detect pieces of bone 0.2 cm thick in X-ray images of meat packages. They also observed that 0.5% of the samples during the study already contained large pieces of bone. In other work, bones 0.5 cm thick were detected in lunch boxes containing sliced meat (Gupta 1995).

There has been much research into automated inspection of X-ray images, using image analysis techniques. Patel et al. (1996) used texture analysis and neural networks to detect foreign objects such as stones in highly inhomogeneous images of frozen peas and sweetcorn.

All these examples used the conventional technology of transmission radiography, in which X-rays are projected directly through the sample onto an imaging device. Regions of the sample with high density or atomic number, such as bone, glass or metal, absorb more X-rays, and cast a shadow on the image. The main problem with transmission radiography is that it can be difficult to achieve enough image contrast for the foreign body to be detectable, especially if the foreign body is small, or similar in density to the sample, or if the sample is inhomogeneous. The detection of bones in poultry meat, such as chicken breasts, is difficult because the variable thickness of the meat creates features in the image which can obscure small bones. Tao & Ibarra (2000) suggested using a laser to make an independent measurement of the meat thickness, and were able to compensate for a uniform wedge-shaped variation in thickness.

1.1.4 X-ray scattering

There has been some research interest in using alternative X-ray technologies, particularly X-ray scattering, in an attempt to obtain better contrasts in foreign body detection. In a short review of photon scattering, Speller & Horrocks (1991) indicated that scattered photons, which had long been regarded as a mere nuisance in medical imaging, carried potentially useful information about the scattering material.

There are two kinds of scatter: coherent and Compton. Coherent, or Rayleigh, scatter occurs at low energies and low angles of scatter, and forms interference spectra when photons scatter elastically from whole atoms or molecules. It can discriminate between substances of similar density and atomic number, and has been used to produce high-contrast images of plastics and bone in foods such as chocolate, cheese and meat (Luggar et al. 1993, Martens et al. 1993, Luggar & Gilboy 1994). As an imaging technique, coherent scatter is slow, and requires low energies which are strongly attenuated by thick samples, but there remain potential applications in measuring quality factors such as the oil content of produce (Luggar et al. 1999). There is much current interest in coherent scatter in the medical field (Royle & Speller 1995, Newton et al. 1992, Evans et al. 1991). A review of coherent scatter has been given by Harding & Kosanetzky (1987).
Compton scatter is the scatter of a photon from a single electron, and occurs at high energies and all angles. There has been much interest in using Compton scatter as a diagnostic tool in medicine, because it is possible to place a detector at any angle to the beam, and to count the scatter from a small volume element in the patient or sample. The amount of scatter from a voxel is proportional to the density, so by scanning the position of the scatter voxel, a density map of the sample can be produced. There are a number of potential advantages of this technique over conventional transmission imaging: the contrast is greater because the detector only sees photons which have actually interacted with the sample; the incident energy can be higher, giving better penetration of the sample or packaging; and the image can be 3-D, allowing features to be resolved which would be lost in a 2-D projection. In medicine, the technique of Compton scatter densitometry, in which the scatter and transmission measurements are combined to give a density measurement, has been used to measure the density of bone (Mooney et al. 1996, Huddleston et al. 1979) and lung tissue (Hanson et al. 1984) in vivo. An excellent description of Compton scatter densitometry and its sources of error is given in Duke & Hanson (1984). There has been recent interest in the application of Compton scatter techniques to food inspection (Bull et al. 1997). Elder (1998) used backscatter to monitor the salt content of potato crisps.

The main limitation of Compton scatter as a whole-sample inspection technique is that the number of scattered photons is small compared to the number transmitted. This causes problems because the counting of photons is subject to statistical noise, which is large when the number of counts is small. Long inspection times are therefore required to acquire enough photons to perceive features against the noise background.

The inspection times are less, the more powerful the X-ray source; A brief survey of X-ray equipment manufacturers showed that, at the time of writing, the most powerful sources available for industrial use have a maximum output power of about 4.2 kW, or 30 mA of beam current at 140 kV. Inspection times are also less if there is a priori knowledge of the position of the foreign body, for example, if the foreign body occurs only at the centre or near to the surface of the sample.

1.1.5 X-ray backscatter

A major advantage of detection and imaging with Compton scatter is the wide choice of detector position. The backscatter geometry, in which the source and detector are on the same side of the sample, has been utilised in many industrial applications, because it can be used to inspect surfaces such as aircraft fuselages (Lawson 1993, Lawson 1995) where transmission imaging is not possible. Philips have incorporated a source and detector into a handheld device called ComScan (Harding & Kosanetzky 1989, Niemann & Zahorodny 1998) which has been widely used. A review of Compton scatter techniques and a description of ComScan has been given by Harding (1997). Backscatter has been used for the inspection
of steel reinforced rubber domes (Ham et al. 1996), the imaging of hearts in vivo (Lamser et al. 1990, Herr et al. 1994), and the detection of explosives in baggage (Annis 1991). Ong et al. (1994) inspected oil pipelines using a technique called 'transscatter', which used a backscatter geometry to obtain a transmission measurement of the pipe wall by using the oil beneath as a uniform scattering medium. Backscatter has been applied to mine detection (Keshavmurthy et al. 1996, Campbell & Jacobs 1992), using multiple scatter to measure a larger volume of ground than would have been possible with single scatter alone.

### 1.1.6 Aims and contents

The aim of this work was to investigate the use of Compton scatter for foreign body detection in the food industry. Since the main limiting factor for Compton scatter inspection was the statistical noise caused by low photon fluxes, much of the work was concerned with modelling the number of counts required to detect a foreign body in a sample, measuring contrasts, and inferring the length of time required to scan the sample. Comparisons are drawn with transmission imaging wherever possible. The thesis consists of the following sections of work:

**Chapter 2** This gives a brief introduction to the physics of the X-ray interactions which are most important in this work: photoelectric absorption and Compton scatter.

**Chapter 3** This chapter is concerned with analytical modelling of the concepts related to inspection and detectability of foreign bodies.

- **Section 3.1** This is the most important modelling work in the thesis, in which the detectability of a foreign body is expressed in mathematical terms, an expression is developed for the number of scattered counts detected from sample, and the expressions are used to infer the minimum time required to detect a foreign body in a sample.

- **Section 3.2** A simple model of sample inhomogeneity is developed, and the effect of inhomogeneity on foreign body detection is discussed, for both Compton scatter and transmission.

- **Section 3.3** This section considers how the shape of the scattered spectrum might be used as additional information for foreign body detection.

- **Section 3.4** The spatial resolution and detector solid angle are recurrent and important parameters of the detection systems in this work. In this section, the spatial resolution is defined, and equations are derived for calculating the resolution and solid angle from the collimation geometry.

**Chapter 4** This chapter describes the equipment and software used in the work.
Chapter 5 This chapter describes two pieces of work using Monte Carlo simulation, the aims of which were to investigate the effect of multiple scatter and geometric parameters on foreign body detection.

Section 5.1 This describes simulations of Compton scatter detection of voids in polystyrene spheres, representing air-filled voids in produce such as pears and potatoes. The aim of this section was to investigate multiple scatter and geometry effects.

Section 5.3 The experience gained in Monte Carlo simulation in the previous section was applied to the more realistic problem of detecting glass in water, using a polychromatic source. The effects of multiple scatter and geometry were investigated in greater detail.

Chapter 6 This chapter describes two sets of experimental work which demonstrated the detection of glass in foods. Small pieces of glass were detected in water (a homogeneous material), coffee (a less dense material) and muesli (a highly inhomogeneous material).

Section 6.1 This describes preliminary experiments to detect a fragment of glass in water, coffee and muesli using Compton scatter. Useful results were obtained within the limitations of the X-ray system.

Section 6.3 This describes further experiments on glass detection in water, coffee and muesli, using more powerful X-ray equipment.

Chapter 7 Results from previous analysis and experiment showed that Compton scatter was likely to be too slow for the inspection of whole food samples, but that it retained potential in applications such as surface inspection. One such application was the detection of bones in chicken meat. This describes experiments investigating the detection of surface bone fragments in chicken meat using a backscatter geometry.

Chapter 8 This chapter gives detailed conclusions, section by section, followed by overall conclusions for the whole work, and a summary of possible further work arising from this thesis.
Chapter 2

Physics of X-ray Interactions
2.1 Interactions of X-rays with matter

There are four important ways in which X-rays interact with matter: photoelectric absorption, Compton scatter, Rayleigh scatter and pair production. Of these, only the first two are of interest in this work, and the physics of these interactions is introduced in this chapter.

Rayleigh, or coherent, scatter, is predominantly a low-energy, low-angle interaction, which occurs when the wavelength of a photon is sufficiently large that the photon can scatter from the charge distribution of a whole atom, molecule or crystal lattice, producing interference spectra. In the energy ranges used in this work (10-140 keV) the number of scattered counts is negligible for angles greater than about 6 degrees, and the only effect of this interaction in this work is its small contribution to the total attenuation coefficient (see later).

The process of pair production, in which a photon interacts with an atomic nucleus to create an electron-positron pair, cannot occur unless the incident photon energy is higher than twice the rest mass of the electron, and is not possible in the energy range considered in this work.

Introductions to the physics of X-rays interactions may be found in Attix (1986a) and Knoll (1989).

2.1.1 Theory of Photoelectric absorption

Photoelectric cross section

Photoelectric absorption is the complete absorption of an incident photon by an atom. The photon interacts with an electron in a low-energy orbital, usually the K or L shell, which is expelled from the atom. The cross section per atom, $\sigma \tau$, is a highly complicated function of the atomic number, $Z$, and the photon energy $E$, but to a reasonable approximation, it can be stated that:

$$\sigma \tau \propto Z^4 E^{-3}$$  \hspace{1cm} (2.1)

The rapid decrease of $\sigma \tau$ with energy is not a smooth function, but is interrupted by large step increases. These occur because the photon cannot interact with an electron unless its energy is greater than or equal to the electron binding energy. When the energy becomes equal to a binding energy, the electrons in the corresponding shell can contribute to the cross section, causing a sudden increase. The step changes are called K- or L-edges, depending on the electron shell involved.
Mass attenuation coefficient

At the macroscopic level, the probability of a photon being absorbed depends on a quantity called the attenuation coefficient, which is derived from $\alpha \tau$ as follows:

If a photon is incident on a thin sheet of matter, of thickness $dz$, the probability of interaction, $dN/N$, is given by

$$
\frac{dN}{N} = \frac{\rho_a \alpha \tau}{\rho} \text{d}z
$$

where $dN$ is the number of photons absorbed from an incident total $N$, $\rho_a$ is the number of atoms per unit volume, $\rho$ is the material density in g cm\(^{-3}\), $N_A$ is the Avogadro constant, and $A$ is the atomic mass in grams of the material. The photoelectric attenuation coefficient $\tau$, in cm\(^{-1}\), is defined by:

$$
\frac{dN}{N} = \tau \text{d}z
$$

which rearranging, gives:

$$
\tau = \frac{N_A}{Z} \left( \frac{Z}{A} \right) \rho \alpha \tau
$$

This is usually tabulated as the mass attenuation coefficient, $(\tau/\rho)$, in cm\(^2\) g\(^{-1}\). The ratio $Z/A$ is close to 0.5 for all elements except hydrogen, hence by Eqn. 2.1 and Eqn. 2.4, it is approximately the case that

$$
\tau \propto Z^2 E^{-3} \rho
$$

2.1.2 Theory of Compton Scattering

Kinematics of Compton Scatter

Compton scatter is the inelastic scatter of a photon from an electron. An incident photon of energy $h\nu$ scatters from an electron, which is assumed to be stationary, that is, its kinetic energy is negligible compared to $h\nu$. After scattering, the photon has energy $h\nu'$, and its direction is deflected through an angle $\theta$.

Conservation of energy and momentum show that the energy $h\nu'$ of the scattered photon is related to the scattering angle $\theta$ by

$$
h\nu' = \frac{m_0 c^2}{(1 - \cos \theta) + 1/\alpha}
$$
where \( m_0c^2 \) is the rest energy of the electron, equal to 511 keV, and
\[
\alpha = \frac{h\nu}{m_0c^2}
\]  
(2.7)
The kinematics can also be expressed in terms of the ratio of the scattered energy and the incident energy,
\[
\frac{h\nu'}{h\nu} = \frac{1}{1 + \alpha(1 - \cos \theta)}
\]  
(2.8)
or in terms of the change in wavelength,
\[
\Delta \lambda = \lambda' - \lambda = \frac{hc}{m_0c^2}(1 - \cos \theta)
\]  
(2.9)
The kinetic energy \( T_e \) of the recoiling electron is given by
\[
T_e = h\nu - h\nu'
\]  
(2.10)
and the recoil direction \( \theta_e \) of the electron is given by
\[
\cot(\theta_e) = (1 + \alpha) \tan(\frac{\theta}{2})
\]  
(2.11)
The variation of scattered energy with scattering angle is shown in Fig. 2.1. It can be seen that more energy is lost the greater the angle of scatter, and that the fraction of the incident energy lost by the photon increases as the incident energy increases.

Scattering Cross Section

The probability of scatter at a particular angle \( \theta \) is governed by the Klein-Nishina equation, which gives the differential scattering cross section per electron \( \frac{d\sigma}{d\Omega} \) in \( \text{cm}^2 \) per electron per steradian as
\[
\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2} \left( \frac{h\nu'}{h\nu} \right)^2 \left( \frac{h\nu}{h\nu'} + \frac{h\nu'}{h\nu} - \sin^2 \theta \right)
\]  
(2.12)
where \( r_e \) is the classical radius of the electron. It can be seen from Eqn. 2.8 that at low energy
\[
\frac{h\nu'}{h\nu} \to 1 \quad \text{as} \quad \alpha \to 0
\]  
(2.13)
and therefore the Klein-Nishina equation, Eqn. 2.12, reduces to the classical Thomson scatter equation
\[
\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2} (1 + \cos^2 \theta)
\]  
(2.14)
Figure 2.1: Plots of Compton scattered photon energy against scattering angle for a range of incident energies: —, 10 keV; —, 20 keV; —, 50 keV; —, 100 keV; —, 150 keV

The variation of scattering cross section with angle is shown in Fig. 2.2. The cross section is greatest for small angles, and is minimum at 90 degrees. It can be seen that as the energy increases, the cross section decreases, and large scatter angles become relatively less probable compared to forward scatter.

**Compton scatter coefficient**

For a photon in a bulk material, the probability of a Compton event occurring in a small distance $dz$ is given by $\sigma dz$, where $\sigma$ is the Compton scatter coefficient in cm$^{-1}$. By analogy with Eqns. 2.2 - 2.4, it can shown that $\sigma$ is related to $\varrho \sigma$ by the following:

$$\sigma = N_A \left( \frac{Z}{A} \right) \varrho \sigma$$  \hspace{1cm} (2.15)

The ratio $Z/A$ is close to 0.5 for all elements except hydrogen, so $\sigma$ varies very little with $Z$. The scattering cross section also varies little with energy in the range used in this work. Hence, the Compton coefficient is approximately proportional to the density of the material, with little dependence on $E$ and $Z$. 
2.1.3 Exponential attenuation law

The coefficients \( \tau \), and \( \sigma \) are respectively the probabilities of photoelectric interaction and Compton interaction per unit length through a thin slice of material. If all interactions are considered, then by analogy with Eqn. 2.15, the total attenuation coefficient \( \mu \) is defined by:

\[
\frac{dN}{N} = \mu \, dz
\]

where \( \mu \) is the sum of the coefficients for each interaction:

\[
\mu = \tau + \sigma + \sigma_{coh} + \cdots
\]

Integration of Eqn. 2.16 leads to the familiar law of exponential attenuation, which gives the number of photons \( N \) transmitted through a thickness \( z \), given an initial number \( N_i \),

\[
N = N_i \exp(-\mu z)
\]
Chapter 3

Analytical Modelling of Compton Scatter Inspection
3.1 Criteria for Foreign Body Detection

3.1.1 Definitions of contrast and signal-to-noise ratio

Let the expected number of photon counts, in the absence of any foreign body, be defined as $N_{\text{samp}}$, and the expected number of photon counts, when a foreign body is present, be defined as $N_{fb}$.

Let the contrast $C$, between the foreign body and the background sample, be defined as

$$C = \frac{N_{fb} - N_{\text{samp}}}{N_{\text{samp}}}$$ (3.1)

The interactions of photons, from production to detection, are countable, random events, governed by Poisson statistics (Barrett & Swindell 1981a, Brookes & Dick 1974a). In the Poisson distribution, the variance is always equal to the mean; hence, if the expected number of counts recorded by a detector is $N$, the standard deviation, or Poisson error $\epsilon$ in the count is given by

$$\epsilon = \pm \sqrt{N}$$ (3.2)

The Poisson noise is present whenever photons are counted, even if there are no other noise sources in the system, so it represents a minimum noise level against which the contrast due to a foreign body must be detected.

If the contrast is to be considered detectable, the difference between $N_{fb}$ and $N_{\text{samp}}$ must exceed the noise level by a sufficient margin. The detectability of a foreign body is determined by the basic signal-to-noise ratio $S$, which is defined, for the purposes of this work, as

$$S = \frac{N_{fb} - N_{\text{samp}}}{\sqrt{N_{\text{samp}}}}$$ (3.3)

where the numerator is the signal, given by the difference in the counts, and the denominator is the Poisson noise. To avoid confusion, it should be noted that this definition of the signal-to-noise ratio, or SNR, is not the same as that used in some signal processing texts, where it represents the ratio of power contained in the Fourier spectra. By substitution of Eqn. 3.1 in Eqn. 3.3, it can be shown that:

$$S = C\sqrt{N_{\text{samp}}}$$ (3.4)

Hence, the signal-to-noise ratio is proportional to the contrast, and increases as the square root of the number of counts.
3.1.2 Detectability criterion

In order to detect a foreign body, an operator must be applied to the data, and its output compared to a threshold level, such that the foreign body is ‘detected’ if the output exceeds the threshold.

Let $S_{th}$ be the minimum signal-to-noise ratio required for a foreign body to be detectable. This must be large enough such that a threshold can be set with acceptable rates of positive and negative errors. Hence, for a foreign body to be detectable:

\[ S \geq S_{th} \]  \hspace{2cm} (3.5)

or, rearranging, the number of counts must satisfy the inequality:

\[ N_{\text{samp}} \geq \frac{S_{th}^2}{C^2} \]  \hspace{2cm} (3.6)

In disciplines such as astronomy, microscopy and radiography, which are concerned with detecting signals from a small number of photons, Eqn. 3.5 is commonly referred to as the “Rose criterion” for detectability, after Rose (1973), in which the signal-to-noise ratio threshold for imaging devices was considered.

Consider the simple case in which the expected counts from the sample and foreign body are respectively $N_{\text{samp}}$ and $N_{fb}$, and a threshold level of counts $N_{th}$ is to be set. This is illustrated with synthetic data in Fig. 3.1.

![Figure 3.1: Synthetic data showing levels corresponding to background sample $N_{\text{samp}}$, foreign body $N_{fb}$, and detection threshold $N_{th}$](image)
A false positive occurs if the noise level about \( N_{\text{samp}} \) rises above \( N_{\text{th}} \) when no foreign body is present, and a false negative occurs when the noise level about \( N_{fb} \) falls below \( N_{\text{th}} \), causing the foreign body to be missed. The probabilities of each type of error depend on \( S_{fp} \) and \( S_{fn} \), respectively the signal-to-noise ratios for false positives and negatives. These are given by

\[
S_{fp} = \frac{(N_{\text{th}} - N_{\text{amp}})}{\sqrt{N_{\text{amp}}}} \quad (3.7)
\]
\[
S_{fn} = \frac{(N_{fb} - N_{\text{th}})}{\sqrt{N_{\text{amp}}}}
\]

and comparison with Eqn. 3.3 shows that

\[
S = S_{fp} + S_{fn} \quad (3.8)
\]

The minimum acceptable values for \( S_{fp} \) and \( S_{fn} \) depend on the application, but they are in general not identical. In the inspection of whole food samples for foreign bodies, \( S_{fp} \) must be at least five; the very low probability, about 1 in 3 million, of exceeding 5 standard deviations is necessary because of the large number of voxels per sample which must be scanned. For example, a limit of only four standard deviations would produce an average of one false alarm every 32000 voxels, which, at a resolution of 1000 voxels per sample, would result in 3% of the samples being rejected. To limit the number of false negatives to the point at which only 1% of the foreign bodies fail to be detected, \( S_{fn} \) must be approximately 2.4.

The relationship between standard deviation and probability is taken from normal distribution tables in Brookes & Dick (1974b).

This results in the following value as the criterion for detectability:

\[
S_{\text{th}} = 5 + 2.4 = 7.4 \quad (3.9)
\]

This value was robust against large changes in the number of readings per sample, and was therefore considered to be a reasonable threshold for all the detection problems in this work. For simplicity, it was been assumed in Eqn. 3.7 that the contrast was sufficiently small that both \( N_{\text{amp}} \) and \( N_{fb} \) experienced the same Poisson error. As it stands, the criterion of Eqn. 3.9 allows slightly more false negatives or positives to occur when the contrast is large, but the difference is not great enough to justify the extra complication.

### 3.1.3 Detection operators

**Detection by Second Differential**

The value of 7.4 in Eqn. 3.9 only depended on statistical tables and reasonable estimates of acceptable error rates. However, the signal-to-noise ratio on the
left-hand side is likely to depend on the choice of operator used to extract the foreign-body peak from the data. In the example given in Fig. 3.1, the sample and foreign-body levels were known constants, so no operator needed to be applied to the data. In the case where \( N_{\text{samp}} \) varies, it might not be possible to set a global threshold on the raw counts, and a local operator might be required. Such a local operator would have its own Poisson error, and hence its own signal-to-noise ratio.

The simplest local operator for detecting an isolated peak in the data is the second differential. If a count \( N(z) \) is to be compared with its neighbours, \( N(z-h) \) and \( N(z+h) \), the second differential \( N''(z) \) is given by:

\[
N''(z) = \frac{N(z-h) - 2N(z) + N(z+h)}{h^2}
\]  
(3.10)

and it can be shown that the expected value \( N''_{fb} \) of \( N''(z) \), when \( N(z) \) is equal to \( N_{fb} \) and its neighbours equal to \( N_{\text{samp}} \), is given by:

\[
N''_{fb} = -\frac{2}{h^2} CN_{\text{samp}}
\]  
(3.11)

The background Poisson noise \( \epsilon''_{\text{samp}} \), is the standard error in \( N(z) \) when no foreign body is present, and is given by:

\[
\epsilon''_{\text{samp}} = \frac{\sqrt{6}}{h^2} \sqrt{N_{\text{samp}}}
\]  
(3.12)

The signal-to-noise ratio \( S_{1D} \) for the one-dimensional second differential is the ratio of \( N''_{fb} \) to \( \epsilon''_{\text{samp}} \), and is given by:

\[
S_{1D} = \frac{2}{\sqrt{6}} C \sqrt{N_{\text{samp}}}
\]  
(3.13)

\[
\approx \frac{2}{\sqrt{6}} S
\]

The factor of 0.82 arises because the operator includes the uncertain measurements of the local background at \( z \pm h \), whereas \( S \) as defined in Eqn. 3.3 is an idealised quantity which assumes that the background is exactly known. If a three-dimensional Laplacian is used, with six neighbouring points instead of two, it can be shown that the signal-to-noise ratio becomes:

\[
S_{3D} = \frac{6}{\sqrt{42}} S
\]  
(3.14)

\[
\approx 0.93 S
\]
The effect of the choice of operator on the signal-to-noise is fairly small, and the detectability criterion given in Eqn. 3.5 and Eqn. 3.9 will be used for the remainder of the work, in order to preserve the independence of the argument from the choice of operator.

Second Differential of the Logarithm

The problem with using the second differential of the count $N''(z)$ as a detection operator is that the expected output $N''_{fb}$, as given by Eqn. 3.11, is proportional to $N_{samp}$, making it difficult to set a detection threshold if $N_{samp}$ varies across the sample. The problem can be eliminated by defining the log-count $f(z)$ as

$$f(z) = \ln(N(z)) \quad (3.15)$$

and taking the second differential $f''(z)$, given by:

$$f''(z) = \frac{f(z - h) - 2f(z) + f(z + h)}{h^2} \quad (3.16)$$

It can be shown that the expected value $f''_{fb}$ of $f''(z)$, due to a foreign body, is independent of $N_{samp}$, and is given by:

$$f''_{fb} = -\frac{2}{h^2} \ln(1 + C) \quad (3.17)$$

The use of logarithms also has the advantage that, because $N(z)$ is attenuated exponentially with depth through the sample, $f(z)$ tends to be a linear, or nearly linear, function of z.

The standard error $\zeta(z)$ in $f(z)$ is given by:

$$\zeta(z) = \frac{1}{\sqrt{N(z)}} \quad (3.18)$$

and the standard error $\zeta''(z)$ in $f''(z)$ is given by:

$$\zeta''(z) = \frac{\sqrt{6}}{h^2} \frac{1}{\sqrt{N(z)}} \quad (3.19)$$

However, the probability distribution of $f(z)$ is no longer Poisson, because $f(z)$ is the logarithm of a Poisson variable, so care must be taken in applying the detectability criterion of Eqn. 3.9 with this standard error. A naive use of $\zeta$ would suggest a threshold value for $f$ given by:

$$f_{th} = f_{samp} + 7.4 \zeta$$

but this does not correspond to 7.4 standard deviations of the original variable $N$. For example, with $N_{samp} = 500$, Eqn. 3.19 gives $f_{th} = 6.55$, corresponding
to $N = 696$, which is 8.8 standard deviations from $N_{\text{samp}}$. This is much improved by adding a correction term, such that the 'addition' of $k$ standard deviations to $f_{\text{samp}}$ is given by

$$f_k = f_{\text{samp}} + k\zeta - \frac{1}{2}(k\zeta)^2 \quad (3.21)$$

which, for $k = 7.4$, corresponds to 7.4 standard deviations of $N$, with an accuracy of at least 10% if $N > 200$, and at least 1% if $N > 2000$.

Similarly, the corrected addition of $k$ standard deviations to $f''_{\text{samp}}$ is given by

$$f''_k = f''_{\text{samp}} + k\zeta'' - \frac{1}{2}(k\zeta'')^2 \quad (3.22)$$

Hence it is possible to define a signal-to-noise ratio $S_{LN}$ for the second differential of the log-count, which is given by

$$S_{LN} = \frac{f''_{\text{fb}} - f''_{\text{samp}}}{\zeta''} \quad (3.23)$$

which can be used in Eqn. 3.9 provided that the standard errors are added to $f_{\text{samp}}$ using the corrected formula of Eqn. 3.22.

It can be shown that

$$S_{LN} = -\frac{2}{\sqrt{6}} \ln(1 + C)\sqrt{N_{\text{samp}}} \approx -0.82C\sqrt{N_{\text{samp}}} = -0.82S \quad (3.24)$$

where $S$ is the basic signal-to-noise ratio of Eqn. 3.3.

The different operators which have been suggested in this section for locating a peak in the data all have similar signal-to-noise ratios to the basic SNR given in Eqn. 3.3. Notably, the three-point second differential has a signal-to-noise ratio which is only 18% less than the basic SNR, which compares the peak with an exactly-known background.

### 3.1.4 Calculation of number of counts

As seen by Eqn. 3.4, the signal-to-noise ratio is proportional to the square root of the number of photons counted. This section derives an expression for the number of photons detected in a simple Compton scatter geometry.

The geometry of a Compton inspection system is shown in Fig. 3.2, the purpose of which is to scan the volume of a sample for foreign inclusions. The sample is assumed to be cubic, with linear dimension $L_{\text{samp}}$, and is divided into voxels with dimensions $\Delta x$, $\Delta y$ and $\Delta z$. A polychromatic source $S$, at a distance $R_i$ from the sample, is collimated into a thin beam, and a detector $D$ is placed at ninety degrees to the beam, collimated so as to collect the scattered photons from a small section of the beam. The scatter volume is the voxel formed by the intersection of
the incident beam and the collimated field-of-view of the detector. The number of counts from a voxel is proportional to its Compton scatter coefficient, which is approximately proportional to the physical density of the voxel. Hence, by scanning the incident beam in the \((x, y)\) plane, and the detector in the \((x, z)\) plane, a 3D picture can be built up of the density variations in the sample. It is difficult to infer the absolute density of a voxel from the photon counts, because the signal is affected by attenuation and multiple scatter. However, it is still possible to detect a foreign inclusion from the contrast between a voxel and its local neighbours.

Consider a voxel at position \((x, y, z)\) in the sample, where the coordinates are measured relative to the point \(P\). Let the fluence rate of the source be \(F_i\) photons \(s^{-1}sr^{-1}mA^{-1}\), where the \(mA^{-1}\) term refers to the operating current of the source. If there was no attenuation, the flux \(n_i\) incident on the voxel, in photons \(mA^{-1}s^{-1}\),
would be given by

\[ n_i = \frac{F_i \Delta x \Delta y}{(R_i + z)^2} \]  \hspace{1cm} (3.25)

In reaching the voxel, the beam is attenuated by a factor \( \exp(-\mu_{\text{samp}} z) \), where \( \mu_{\text{samp}} \) is the attenuation coefficient of the bulk of the sample. More precisely, if the attenuation coefficient \( \mu_{\text{vox}} \) of the voxel itself is significantly different from that of the sample, a self-attenuation term \( \exp(-\Delta \mu \Delta l) \) also needs to be included, where \( \Delta \mu \) is the difference in attenuation coefficients, given by

\[ \Delta \mu = \mu_{\text{vox}} - \mu_{\text{samp}} \]  \hspace{1cm} (3.26)

and \( \Delta l \) is the path length through the voxel, given by

\[ \Delta l = (\Delta z + \Delta y)/2 \]  \hspace{1cm} (3.27)

The probability of an incident photon undergoing Compton scatter from the voxel is given by \( \sigma_{\text{vox}} \Delta z \), where \( \sigma_{\text{vox}} \) is the Compton scatter coefficient of the voxel, and the remaining attenuation between the source and the detector is \( \exp(-\mu_{\text{samp}} y) \). Hence, the number of photons per second \( n_d \) which reach the voxel, scatter from it, and escape from the sample in the direction of the detector, in photons \( \text{mA}^{-1}\text{s}^{-1} \), is given by

\[ n_d = n_i \sigma_{\text{vox}} \Delta z \exp[-\mu_{\text{samp}}(z + y)] \exp[-\Delta \mu \Delta l] \frac{\Omega}{4\pi} \kappa_1 \kappa_2 \]  \hspace{1cm} (3.28)

\[ = \frac{F_i \Delta x \Delta y \Delta z}{(R_i + z)^2} \sigma_{\text{vox}} \exp[-\mu_{\text{samp}}(z + y)] \exp[-\Delta \mu \Delta l] \frac{\Omega}{4\pi} \kappa_1 \kappa_2 \]

where \( \Omega \) is the solid angle subtended by the detector at \((x, y, z)\), \( \kappa_1 \) is the efficiency of the detector at recording the photons which pass through it, and \( \kappa_2 \) is a correction factor for the variation of the Compton scattering probability with scattering angle. \( \kappa_2 \) is calculated for a given angle using the Klein-Nishina equation (see Section 2.1.2), which gives the cross section of the Compton interaction between photons and electrons in terms of the scattering angle and the incident energy. For a scattering angle of ninety degrees, and for the incident energy range 10-100 keV,

\[ \kappa_2 = 0.745 \pm 0.015 \]  \hspace{1cm} (3.29)

The value of less than unity indicates that fewer photons are scattered at ninety degrees than would have been the case for an isotropic distribution.
Contrast

In Eqn. 3.28, the only terms which depend on the voxel itself are the self-attenuation \( \exp[-\Delta \mu \Delta l] \), and the scatter coefficient \( \sigma_{\text{vox}} \), which is approximately proportional to the voxel density, as explained in Section 2.1.2. Hence, the contrast between counts from the voxel and the sample is given by

\[
C = \frac{\mu_{\text{vox}} \exp[-\Delta \mu \Delta l] - \mu_{\text{samp}}}{\mu_{\text{samp}}} \quad (3.30)
\]

or if \( \Delta \mu \Delta l \) is small,

\[
C = \frac{\mu_{\text{vox}} - \mu_{\text{samp}}}{\mu_{\text{samp}}} \approx \frac{\Delta \rho}{\rho_{\text{samp}}} \quad (3.31)
\]

where \( \Delta \rho \) is the difference in density. Thus, the contrast measured from the scattered counts is predicted to be approximately equal to the intrinsic contrast between the densities. This is compared to transmission contrast later in Section 3.2.2.

### 3.1.5 Calculation of time required for detection

If the whole sample is to be scanned, then either the time available or the solid angle must be divided amongst the voxels. The number \( J \) of voxels per sample is given by

\[
J = \frac{L_{\text{samp}}^3}{\Delta x \Delta y \Delta z} \quad (3.32)
\]

If the time taken to scan the whole sample is \( T_{\text{samp}} \), the time \( T_{\text{vox}} \) available to scan each voxel is given by

\[
T_{\text{vox}} = T_{\text{samp}} / J \quad (3.33)
\]

Therefore, for a voxel in the centre of the sample, where \( x = y = z = \frac{L_{\text{samp}}}{2} \), and given a source current \( I \), the number of counts per voxel \( N_{\text{vox}} \) in photons voxel\(^{-1}\) is given by

\[
N_{\text{vox}} = n_d I T_{\text{vox}} = n_d I T_{\text{samp}} / J \quad (3.34)
\]
The dependence on the resolution is very strong, as \( N_{\text{vox}} \) varies as the square of the voxel volume \( \Delta x \Delta y \Delta z \), or the sixth power of the linear dimension. This means that, if the time \( T_{\text{samp}} \) for scanning the whole sample is constant, and assuming the voxel to be cubic, the number of photons detected per voxel is reduced by a factor of 64 for every two-fold refinement of the resolution.

The minimum value of \( N_{\text{vox}} \) for detection of a foreign body with contrast \( C \) is given by substituting the inequality Eqn. 3.6 into Eqn. 3.34, replacing \( N_{\text{vox}} \) with \( N_{\text{samp}} \). This can be rearranged to give the minimum times \( T_{\text{vox}} \) and \( T_{\text{samp}} \) required for detection:

\[
T_{\text{vox}} > 7.4^2 \frac{1}{n_d I C^2}
\]

\[
T_{\text{samp}} > 7.4^2 J \frac{1}{n_d I C^2}
\]

where the value 7.4 has been substituted for \( S_{\text{th}} \) from Eqn. 3.9, \( n_d \) is calculated from Eqn. 3.28 and \( J \) is calculated from Eqn. 3.32.

**Maximum solid angle**

The maximum solid angle possible is less than \( 4\pi \) because of the need for collimation; for every piece of the detector solid angle which is dedicated to a voxel length \( \Delta z \) of the incident beam, the photons from the other voxels in the beam must be stopped from hitting the detector. Hence each piece of detector can only view a fraction \( \Delta z / L_{\text{samp}} \) of the beam length, with the rest of the photons being lost by collimation. Therefore, the maximum possible value of \( \Omega \) is given by

\[
\Omega_{\text{max}} = \frac{\Delta z}{L_{\text{samp}}} \frac{4\pi}{\Omega}
\]

Of course, it is not possible in practice to completely fill \( 4\pi \) steradians with detectors and collimators; the sample and the X-ray beam must enter and leave
the inspection space somehow. It is estimated that the largest reasonable solid angle is approximately:

$$\Omega_r = \frac{\Delta x \Delta y \Delta z}{L_{samp}} \times 0.5$$  \hspace{1cm} (3.39)

Scanning rate

It is sometimes useful to represent $T_{samp}$ as the inverse quantity, $U$, which will be called the scanning rate. $U$ is the maximum number of samples per second, per steradian of detector, in which the foreign body can be detected with an SNR of 7.4. $U$ is given by

$$U = \frac{C^2}{7.4^2 I} \times$$

$$F_i(\Delta x \Delta y \Delta z)^2 \sigma_{voz} \frac{\exp[-\mu_{samp} L_{samp}]}{L_{samp}^3 (R_i + L_{samp}/2)^2} \exp[-\Delta \mu \Delta l] \times \frac{\kappa_1 \kappa_2}{4\pi}$$  \hspace{1cm} (3.40)

The quantity $U \Omega$ gives the number of samples per second, where $\Omega$ is the solid angle of the detector.

Size of foreign bodies detectable in one second

The theory developed in this section can now be used to predict the detectability of foreign bodies in samples, expressed in terms of the smallest foreign body detectable in one second. Eqn. 3.40 was calculated for various foreign bodies and samples and the smallest foreign body size for which $U \Omega$ was greater than one sample per second was recorded. The results are tabulated in Table 3.1. The source fluence rate was $F_i = 5.16 \times 10^{12}$ photons mA$^{-1}$s$^{-1}$sr$^{-1}$ (see Table B.2), for a 140 kVp tungsten target source, the source distance $R_i$ was 20 cm, and the source current $I$ was 30 mA. The Klein-Nishina correction $\kappa_2$ was 0.74 (see Table B.2) and the values for the attenuation and scatter coefficients were taken at the mean incident energy of 67 keV (see Table B.1). The solid angle of detector was the maximum reasonable value, $\Omega_r$ from Eqn. 3.39. The scattering angle was 90 degrees, and the detected energy window was 40-110 keV. The sample material was water in these examples. For glass and air-filled voids, foreign body sizes from 0.13 cm upwards were detectable, depending on the sample size. The results for glass and voids were very similar, because the density contrasts of air and glass in water were similar. Plastics, with much lower density contrasts with water, were much harder to detect, with even a 1 cm cube of polystyrene being undetectable in 10 cm of water in one second. The values for stainless steel
(18% Cr, 8% Ni, density 7.93 g cm\(^{-3}\)) (Tennent 1978) are given for interest, and suggest that stainless steel is similar to glass in terms of detectability. However, further work is required to ascertain the validity of this conclusion; the values were calculated on the assumption that the X-rays passed through the centre of the foreign body, from which the contrast was greatly reduced by attenuation, and took no account of the contribution of photons scattering from close to the surface. Meanwhile, the similarity of steel to glass across a range of sizes, 0.1 to 0.4 cm, appears to support the conclusion.

Table 3.1: Predicted minimum size of foreign body detectable in a water sample, scanning at one sample per second: source is 140 kVp, 30 mA at 20 cm distance, dimensions are in cm

<table>
<thead>
<tr>
<th>foreign body</th>
<th>sample size, cm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>void</td>
<td>0.13</td>
</tr>
<tr>
<td>polystyrene</td>
<td>0.38</td>
</tr>
<tr>
<td>perspex</td>
<td>0.24</td>
</tr>
<tr>
<td>glass</td>
<td>0.13</td>
</tr>
<tr>
<td>stainless steel</td>
<td>0.11</td>
</tr>
</tbody>
</table>

3.1.6 Simplifying Assumptions

Various simplifying assumptions were made in deriving the Eqn. 3.28 for the number of detected photons.

Compton shift in energy

The first assumption was that the attenuation coefficients were the same for the scattered and incident beams. This is not strictly true, since the scattered beam has a lower energy than the incident beam, and therefore suffers greater attenuation.

The Compton scattered energies at 90 degrees are tabulated in Table 3.2, with the corresponding total attenuation coefficients for water. The difference in attenuation coefficient between the incident and scattered energies is larger at low energies, despite the small loss in energy; the loss of 1 keV at 20 keV is much more significant than a 30 keV loss at 140 keV.

The consequence of the change in attenuation depends on the exit path length. Over a 5 cm path length, the correction required to the detected counts is only about 5-10% for energies greater than 40 keV (although it rises steeply for lower energies), so this correction has been ignored in the rest of this work.
Table 3.2: Compton scattered energies at 90 degrees, and corresponding attenuation coefficients in water

<table>
<thead>
<tr>
<th>incident energy, keV</th>
<th>scattered energy, keV</th>
<th>attenuation coefficient at incident energy, cm$^{-1}$</th>
<th>attenuation coefficient at scattered energy, cm$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>19</td>
<td>0.810</td>
<td>0.913</td>
</tr>
<tr>
<td>30</td>
<td>28.5</td>
<td>0.376</td>
<td>0.406</td>
</tr>
<tr>
<td>40</td>
<td>37</td>
<td>0.268</td>
<td>0.289</td>
</tr>
<tr>
<td>50</td>
<td>45.5</td>
<td>0.227</td>
<td>0.242</td>
</tr>
<tr>
<td>60</td>
<td>53.5</td>
<td>0.206</td>
<td>0.218</td>
</tr>
<tr>
<td>70</td>
<td>61.5</td>
<td>0.193</td>
<td>0.204</td>
</tr>
<tr>
<td>80</td>
<td>69</td>
<td>0.184</td>
<td>0.194</td>
</tr>
<tr>
<td>90</td>
<td>76.5</td>
<td>0.177</td>
<td>0.187</td>
</tr>
<tr>
<td>100</td>
<td>83.5</td>
<td>0.171</td>
<td>0.181</td>
</tr>
<tr>
<td>110</td>
<td>90.5</td>
<td>0.166</td>
<td>0.176</td>
</tr>
<tr>
<td>120</td>
<td>97</td>
<td>0.161</td>
<td>0.172</td>
</tr>
<tr>
<td>130</td>
<td>103.5</td>
<td>0.157</td>
<td>0.169</td>
</tr>
<tr>
<td>140</td>
<td>110</td>
<td>0.154</td>
<td>0.166</td>
</tr>
</tbody>
</table>

Incident energy

The calculation of photon counts in Eqn. 3.28 assumed that the attenuation coefficients were constant, with any variation with energy being neglected. However, with a polychromatic beam, the coefficients would be expected to vary across the spectrum, so that the number of photon counts would be correctly calculated only by integrating Eqn. 3.28 over all the incident energies. Using Eqn. 3.28 to calculate photon counts for polychromatic sources, an average incident energy must be chosen at which the attenuation coefficients are representative of the whole spectrum. In this work, the incident energy used was that which corresponded (i.e. before the Compton shift) to the mean of the spectrum after scattering and detection. This was a slightly arbitrary way to calculate a representative incident energy, because it did not attempt to weight the energies in terms of their relative contribution to the attenuation coefficients. Ideally, what is required is not the mean energy, but the energy which gives the mean attenuation. However, this is more complex to calculate, and the result would depend on the dimensions of the sample; the approach used in this work has the advantages of simplicity and sample-independence.

Beam hardening

It was assumed in the analytical model that the incident beam was monochromatic. For a polychromatic source, a single energy representative of the spectrum
was calculated, as described in Section 3.1.6. One effect which was ignored was that of beam hardening, in which the mean energy of a beam increases as it passes through a sample, due to the greater attenuation of the lower energies in the spectrum. The effect is greater, the greater the thickness of the sample. A brief simulation using the IPEM spectrum processor (IPEM 1997) (see later, Section 4.4), showed that for a 140 kVp beam with 1mm Al filtration, 10cm of water increased the mean energy of the whole spectrum from 54 to 68 keV. Hence, beam hardening causes the representative energy of the beam to be underestimated, and therefore acts in the opposite sense to the Compton shift (see Section 3.1.6).

Multiple scatter

The largest simplification in Eqn. 3.28 is that it only considers single scatter. Single scatter can only reach the detector from the scatter voxel formed from the intersection of the incident beam and the detector field-of-view. However, photons which scatter more than once in the sample can reach the detector from any point in the sample which intersects the detector field-of-view. From the point of view of foreign body detection, multiple scatter acts as a noise signal which is added to the single scatter. This adds to the Poisson noise and reduces the contrast, increasing the time required to detect a foreign body.

If the multiple scatter fraction $f_m$ is defined as the ratio of the multiple scatter $N_m$ to the total scatter $N$:

$$f_m = \frac{N_m}{N} \quad (3.41)$$

then it is easily shown from Eqn. 3.3 that the SNR in the presence of multiple scatter is proportional to the square root of $1 - f_m$, and therefore, that the time required for detection is related to the multiple scatter by

$$T_{\text{samp}} \propto \frac{1}{1 - f_m} \quad (3.42)$$

It has been assumed that the multiple scatter flux is constant across the neighbourhood of the scatter voxel.

The number of multiple scatter photons is difficult to model analytically, and hard to measure experimentally, when there is no way to distinguish the single from the multiple scatter at the detector. Multiple scatter is most easily investigated (for any specific geometry) by Monte Carlo simulation, for which it is a trivial task to count the number of scatters made by a photon en route to the detector. One of the objectives of the simulations in this work is to investigate the effect of multiple scatter on the detectability of foreign bodies.
Location of representative voxel

It was assumed in Eqn. 3.35 that the scatter voxel at the centre of the sample was representative of the whole sample. However, this is not strictly true, because the attenuation path length varies with position. The effect of this depends on how the sample is scanned: if the scanning time $T_{vox}$ is the same for all voxels, then $T_{vox}$ must be large enough for sufficient photons to be collected from the most attenuated voxel at $Q$ in Fig. 3.2, so the scanning time is a function of the worst voxel, rather than the central one; but if the scanning mechanism allows $T_{vox}$ to vary such that the number of detected counts per voxel is constant, then the average time per voxel will be a mean across the sample, with the actual mean value occurring for a voxel somewhere close to the centre. In this work, it has been assumed for convenience that the time required per sample is a function of the centre voxel.

Focal spot size and primary collimation

The manner in which the incident beam is produced has been neglected so far. It has been assumed that the X-rays have been produced in a small focal spot and have been collimated into a narrow beam by a primary collimator, the details of which are unimportant. In particular, It has been assumed in this work that the size of the source focal spot is negligible, and therefore that the number of incident photons can be calculated from Eqn. 3.25, which simply multiplies the fluence rate $F_i$ in photons mA$^{-1}$.s$^{-1}$.sr$^{-1}$by the solid angle subtended by the scatter voxel at the source. However, if the size of the focal spot size is significant compared to that of the incident beam, the fluence rate across the incident beam will no longer be uniform, but will consist of an umbra and penumbra. Not all of the photons produced in the focal spot will reach the umbra of the beam, and some parts of the focal spot might not be able to "see" the scatter voxel through the primary collimation at all. If the spot is large, a significant fraction of $F_i$ may be lost in the primary collimation, causing Eqn. 3.25 to overestimate the number of incident photons.

Inhomogeneity

It has been assumed so far that the sample is homogeneous, except for one voxel which may or may not contain a foreign body. The effect of sample inhomogeneity is treated in Section 3.2.2.
3.2 Transmission Imaging and Inhomogeneities

3.2.1 Transmission contrast

A transmission geometry is shown in Fig. 3.3, in which two identical beams are attenuated through a length $L_{\text{samp}}$. The number of incident photons is $N_i$, and the number of photons $N$, or $N'$, depending on whether the path passes through the test voxel $\Delta z$, as shown.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3_3.pdf}
\caption{Transmission X-rays through a sample; $N_i$ incident number of photons; $N$, number of photons transmitted through homogeneous path; $N'$, number of photons transmitted through path containing test voxel; $L_{\text{samp}}$, thickness of sample; $\Delta z$, thickness of voxel}
\end{figure}

By Eqn. 2.18, the transmitted numbers of photons are given by

$$N = N_i \exp(-\mu_{\text{samp}}L_{\text{samp}}) \tag{3.43}$$

and

$$N' = N_i \exp(-\mu_{\text{samp}}L_{\text{samp}}) \exp(-\Delta \mu \Delta z) \tag{3.44}$$
where $\Delta \mu$ is the difference in the attenuation coefficients of the voxel and the sample. Therefore the perceived contrast $C$ between the two beams is:

$$C = \frac{N' - N}{N} = \exp(-\Delta \mu \Delta z) - 1 \approx -\Delta \mu \Delta z$$

(3.45)

or

$$C \approx \frac{-\Delta \mu}{\mu_{\text{samp}} \Delta z}$$

(3.46)

which is the intrinsic contrast $\Delta \mu/\mu_{\text{samp}}$ between the materials, multiplied by the term $\mu_{\text{samp}} \Delta z$, which is small. This may be compared with the contrast obtained using Compton scatter, in Eqn. 3.31, which is approximately equal to the intrinsic contrast. This shows that transmission imaging is a low-contrast imaging technique compared to Compton scatter. The transmission contrast can be increased by reducing the incident energy, but this is limited by the need to penetrate the sample.

However, it has been shown (Harding 1997), that the advantage belongs to transmission imaging when the signal-to-noise ratio is considered, because the number of scattered photons is so small. Harding (1997) showed that given a single thin-beam measurement in transmission and scatter modes, and given equal path lengths through the sample, the SNR for transmission is greater than that for scattering by a factor of $\sqrt{L_{\text{samp}} \sigma}$. This only becomes less than unity for thin samples (less than 5 cm of water), so the Compton scatter measurement will always require more time than transmission. This result is easily derived from Eqn. 3.4.

### 3.2.2 Sample inhomogeneity

The modelling of foreign body detection so far, particularly the comparison of scatter and transmission methods, of Harding (1997), has assumed that the sample is homogeneous, and that Poisson statistics are the only source of noise. However, in an inhomogeneous sample, variance in the signal comes from the sample itself, and is not affected by the number of photon counts. Real foodstuffs are often highly inhomogeneous, and it is important to consider the effect of this on the signal-to-noise ratio.

A very simple model of an inhomogeneous sample is one which is divided into cubic volume elements, of dimension $\Delta z$, with the density $\rho_j$ of each varying independently by $\pm \Delta \rho$ about the mean sample density $\rho_{\text{samp}}$. Assuming that the attenuation coefficient of each voxel $\mu_j$ is proportional to $\rho_j$, then the number of photons $N$ transmitted in a thin beam through a sample of diameter $L_{\text{samp}}$ is
given by

\[ N = N_i \exp(-\Delta z \sum_{j=1}^{K} \mu_j) = N_i \exp(-\Delta z \frac{\mu_{\text{samp}}}{\rho_{\text{samp}}} \sum_{j=1}^{K} \rho_j) \]  

(3.47)

where \( N_i \) is the number of incident photons, \( K = \frac{L_{\text{samp}}}{\Delta z} \) is the number of voxels in the beam path, and \( \mu_{\text{samp}} \) is the mean attenuation coefficient of the sample. Since the variation in \( \Sigma \rho_j \) is \( \pm \rho \sqrt{K} \), it can be shown that the variation \( dN \) in \( N \) is given by

\[
(dN/N)_t = \mu_{\text{samp}} \Delta z \sqrt{K} \frac{d\rho}{\rho_{\text{samp}}} \\
= \mu_{\text{samp}} \sqrt{\frac{L_{\text{samp}}}{\Delta z}} \rho \frac{d\rho}{\rho_{\text{samp}}} 
\]  

(3.48)

The quantity \( (dN/N)_t \) is the standard deviation of the signal due to inhomogeneity in the transmitted path. If the transmitted and scattered path lengths are the same, \( dN \) is the same for both. The relative error \( dN/N \) does not depend on the number of counts.

The scattered signal also suffers a relative error, due to the variation in the voxel from which it scatters, given by \( d\rho/\rho_{\text{samp}} \). This is typically much greater than the relative error in Eqn. 3.48, so the scattered path relative error is

\[
(dN/N)_s = d\rho/\rho_{\text{samp}} 
\]  

(3.49)

If the signal to be detected is a small change \( \Delta N \) in \( N \) photons, then the signal-to-noise ratio is given by

\[
R = \frac{\Delta N}{dN} = \frac{\Delta N/N}{dN/N} = \frac{C}{dN/N} 
\]  

(3.50)

Where \( C \) is the contrast, given by Eqn. 3.1, and \( dN/N \) is the inhomogeneity noise given by Eqn. 3.48 or Eqn. 3.49 for the transmitted and scattered measurements respectively. Using Eqn. 3.45 for the transmitted contrast, and Eqn. 3.31 for the scattered contrast, it can be shown that the SNR is greater for the scattered measurement by a factor of \( \sqrt{K} \), provided that the inhomogeneity dominates over the Poisson noise. This is a simple model, but it shows that the greater contrast of scatter imaging can be an important advantage over transmission imaging in inhomogeneous samples.
3.3 Spectral Shape

Most solid state and scintillation X-ray detectors are able to record the energy of the photons as they are counted, and the shape of the resulting spectrum is a potential source of information, independent of the total counts, which could be used to detect foreign bodies. This section considers the use of the spectral shape for detection.

3.3.1 Mean spectral energy

The simplest measure of spectral shape which is independent of the total counts is the mean energy of the spectrum. As X-rays penetrate further into a sample, the mean energy of the spectrum increases, because the lower energies suffer greater attenuation - an effect known as beam hardening (Barrett & Swindell 1981b). The effect increases with atomic number and density. For example, the total attenuation coefficient of crown glass (Weast & Astle 1980), calculated in the range 20-150 keV, (NIST 1990) corresponds approximately to that of a pure element with atomic number 13.1. The effective atomic number of water, similarly calculated, is approximately 7.69. The relatively high atomic number and density of glass will harden an X-ray beam more rapidly with depth than the sample material, and the effect should be visible in the recorded spectra. If there are $N_j$ photon counts of energy $E_j$ in bin $j$, the mean energy is given by:

$$
\bar{E} = \frac{\sum N_j E_j}{N}
$$

(3.51)

where $N$ is the total number of counts, and the summation is over all bins in the energy range of interest. It can be shown that the error $\eta$ in $\bar{E}$, given the Poisson error formula of Eqn. 3.2 for $N_j$, and a standard deviation of $\sigma_E$ for the spectral shape, is given by:

$$
\eta = \frac{\sigma_E}{\sqrt{N}}
$$

(3.52)

This is very similar to Eqn. 3.2, with an additional factor of $\sigma_E$ in the numerator.

3.3.2 Slope and Intercept

The mean energy of the spectrum depends on the density and atomic number of the attenuation path, and the total counts at the detector depend on both the attenuation path and the density of the scatter voxel. It would be of interest if the spectral shape could be used to distinguish between the effect of the attenuation path and the scatter voxel.
It can be shown that, given an incident spectrum \( N_i(E) \) and a detected spectrum \( N(E) \), and given that the Compton coefficient \( \sigma \) is constant with energy, and the photoelectric coefficient \( \tau \) is proportional to \( E^{-3} \), (see Eqn. 2.5), then a plot of \( \ln(N(E)/N_i(E)) \) against \( E^{-3} \) should be a straight line. The intercept of this line is a function of \( \sigma \) of the scatter voxel, and the slope is a function of the atomic number, length and density of the attenuation path.

If the log-ratio of the scattered spectrum to the incident spectrum can be modelled by a linear plot against \( E^{-3} \), it follows that the change in the spectrum is defined by only two variables, and therefore that the whole scattered spectrum can be constructed from the incident spectrum plus readings at two energies.

It is also noted that when this decomposition of the spectrum was attempted for the first time, an ordinary least-squares fit was applied to the data, but extremely noisy results were obtained. This was because the ends of the spectrum, which exerted the largest leverage on the line, had low numbers of counts, and hence had large Poisson errors. A correctly weighted least-squares fit produced much better results. An example may be found much later in the thesis, in Fig. 7.23, in which the noise at the extreme ends of the spectrum is evident.
3.4 Collimation and Resolution

In Section 3.1, the detectability of a foreign body was analysed in a 90 degree scatter geometry, shown in Fig. 3.2. The dimensions of the scatter voxel (i.e. the spatial resolution of the system) and the solid angle of the detector were important factors in determining the detectability of a foreign body. This section grew from the need for (a) a consistent definition of spatial resolution, and (b) a consistent set of equations for calculating the resolution and detector solid angle from the collimation geometry.

In this section, equations are derived for the resolution and solid angle of a collimated detector placed at 90 degrees to a thin beam, as in the scatter geometry of Fig. 3.2. It has been assumed throughout that the collimators are thin and stop all photons. The section is entirely the work of the author. A completely general theory of collimation, in terms of linear systems theory, may be found in Barrett & Swindell (1981c), but the simple collimation arrangements used in this work are most easily described by the straight-line geometry as described in this section.

The following types of collimation are considered:

- A thin rectangular slot is considered in Section 3.4.1.
- A wide rectangular slot is considered in Section 3.4.2.
- A rectangular or cylindrical tube is considered in Section 3.4.3.
- The placement of multiple scatter baffles is considered in Section 3.4.4.

3.4.1 Collimation by a thin rectangular slot

A simple collimation geometry is shown in Fig. 3.4. A beam of X-rays is incident in the $z$ direction, and is viewed from the $x$ direction by a detector of width $W_d$. The geometry is assumed to be infinite in the $y$ direction. The detector views the scatter from the beam through a slot of width $h$ in the collimator plane. The collimator plane is a distance $R_s$ from the beam, and the detector is $R_d$ from the collimator. The graph shows the response $\theta$ of the detector to the beam, where $\theta(z)$ is the angle of detector in the $(x, z)$ plane visible from the point $z$ on the beam.

If $h \ll R_s$, the response $\theta(z)$ is trapezoidal, as shown. Within the width $W_u$ of the umbra, each point on the beam sees the same angle of detector. Beyond the umbra, the response falls off linearly, reaching zero at the full width $W_f$. The nominal width $W_n$, which is the pinhole image of the detector cast through the centre of the slot, and the width $h_s$ of the image of the slot on the beam, are also indicated.
Figure 3.4: *Collimation by thin slot*
Let the magnification $M$ be defined as

$$M = \frac{R_s}{R_d} \tag{3.53}$$

Then by similar triangles, it is easily shown that the nominal width $W_0$ of the response is given by

$$W_0 = MW_d \tag{3.54}$$

and the width of the image of the slot $h_s$ is given by

$$h_s = (1 + M)h \tag{3.55}$$

The umbra width is given by

$$W_u = W_0 - h_s = MW_d - (1 + M)h \tag{3.56}$$

and the full width is given by

$$W_f = W_0 + h_s = MW_d + (1 + M)h \tag{3.57}$$

A thin slot may be defined by $h$ being small enough such that Eqn. 3.56 satisfies the inequality

$$W_u \geq 0 \tag{3.58}$$

If Eqn. 3.58 is not satisfied, then a set of equations for a wide slot must be applied, which is the subject of Section 3.4.2.

The maximum angle, $\theta_{\text{max}}$, is subtended by the detector at a point on the beam at the centre of the response, and is given by

$$\theta_{\text{max}} = \min\left(\frac{h}{R_s}, \frac{W_d}{R_s + R_d}\right) \tag{3.59}$$

However, the inequality in Eqn. 3.58 means that the $h$ term in Eqn. 3.59 is always smaller than or equal to the $W_d$ term, so the expression can be simplified to

$$\theta_{\text{max}} = \frac{h}{R_s} \tag{3.60}$$

The mean angle $\bar{\theta}$ subtended by the detector, integrated over the full width $W_f$ is given by

$$\bar{\theta} = \frac{W_0}{W_f} \theta_{\text{max}} \tag{3.61}$$

If $W_f$ is fixed, and $h$ and $W_d$ are allowed to vary, it can be shown that both $\theta_{\text{max}}$ and $\bar{\theta}$ are maximised when $h$ is as large as possible, that is the optimum slot size $\hat{h}$ is given by

$$\hat{h} = \frac{W_f}{2(1 + M)} \tag{3.62}$$
and if $W_0$ is fixed, the optimum slot size $h$ is given by

$$h = \frac{W_0}{(1 + M)}$$

(3.63)

The two simplest ways of defining the resolution of the system are $W_{HM}$, the width at half-maximum of the response, and the full-width $W_{FW}$. In the thin slot geometry in this section, these correspond respectively to

$$W_{HM} = W_0$$
$$W_{FW} = W_f$$

(3.64)

There are advantages to using either definition. $W_{HM}$ is convenient to measure directly, but it allows photon counts to reach the detector which are outside the nominal voxel width, whereas $W_{FW}$ includes all the photons which can hit the detector, but it is much harder to measure the zero-to-zero width in a practical system.

It is useful to note that the fraction of counts which are outside $W_0$ in this slot geometry is given by

$$f_{out} = \frac{h_s/4}{W_0} = \frac{1}{4} \left( \frac{1}{1 + M} \right) \frac{h}{W_d}$$

(3.65)

Thus, the number of counts arriving from outside the nominal voxel will be small if $h \ll W_d$.

In the rest of this work, both definitions of resolution will be used as convenient, and care will be taken to indicate which is being used. Other definitions, such as the widths which encompass 2/3 or 90% of the detected counts, are possible, but will not be discussed here.

### 3.4.2 Collimation by a wide rectangular slot

In this section, the case of a wide slot is considered, in which the inequality Eqn. 3.58 does not hold. This is illustrated in Fig. 3.5. If the magnification $M$ is still defined by Eqn. 3.53 then the width of the slot image $h_s$ is still given by Eqn. 3.55, but the width $W_u$ of the umbra is now given by

$$W_u = h_s - MW_d$$

(3.66)

and the full width $W_f$ is now given by

$$W_f = h_s + MW_d$$

(3.67)
The full-width at half maximum $W_{HM}$ is the mean of the umbra width and the full width, and is given by

$$W_{HM} = h_s$$ \hspace{1cm} (3.68)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.5.png}
\caption{Collimation by wide slot}
\end{figure}

### 3.4.3 Collimation by a straight tube

In this section, the collimation properties of a straight tube are calculated. The straight tube is a special case of the wide slot, in which $W_d = h$. This is illustrated
in Fig. 3.6. Inspection of Eqn. 3.56 shows that the inequality no longer holds, so the wide slot equations in Section 3.4.2 apply.

The response is trapezoidal, as before, with an umbra width $W_u$ now given by

$$W_u = h$$

(3.69)

The full width $W_f$ is given by

$$W_f = (1 + 2M)h$$

(3.70)
and the width at half height \( W_{HM} \) is the mean of \( W_u \) and \( W_f \), and is given by

\[
W_{HM} = (1 + M)h
\]  
(3.71)

The maximum angle \( \theta_{\text{max}} \) subtended by the detector is

\[
\theta_{\text{max}} = h/(R_d + R_s)
\]  
(3.72)

and the mean angle \( \bar{\theta} \) integrated over the full width is

\[
\bar{\theta} = \frac{W_{HM}}{W_f} \theta_{\text{max}} = \frac{h}{R_d + 2R_s}
\]  
(3.73)

If the tube is square in cross section, the mean solid angle \( \bar{\Omega} \) subtended by the detector at the beam is given by

\[
\bar{\Omega} = \bar{\theta}^2
\]  
(3.74)

and if the tube is circular in cross section, with diameter \( h \), the mean solid angle is given by

\[
\bar{\Omega} = \pi \bar{\theta}^2/4
\]  
(3.75)

### 3.4.4 Multiple Scatter and Collimator Slot Height

The geometry described in Section 3.4.1 is shown in Fig. 3.7 in a plane perpendicular to the incident beam. The heights of the detector, collimator slot and incident beam in the \( y \) direction are respectively \( Y_d \), \( Y_c \) and \( Y_i \). In Fig. 3.7, the single scatter photons originate only from the cross section of the incident beam, but the multiple scatter can originate from anywhere in the sample, and can reach the detector from any point in the trapezium ABCD. There is an optimum height for \( Y_c \), at which the whole beam is just visible to the detector. If \( Y_c \) is smaller than this value, some of the single scatter is lost; if it is bigger, the multiple scatter is increased with no increase in the single scatter. It is easily shown that the optimum value for \( Y_c \) is given by:

\[
Y_c = \frac{R_d Y_i + R_s Y_d}{R_d + R_s} = \frac{Y_i + MY_d}{1 + M}
\]  
(3.76)

where \( M \) is the magnification, as given by Eqn. 3.54.
Figure 3.7: Slot collimation: view perpendicular to incident beam, showing optimum height of collimator slot for maximising the ratio of single to multiple scatter at the detector.
Chapter 4

Description of Equipment and Software
4.1 X-ray Hardware: BBSRC Silsoe Research Institute

The experiments described in Section 6.1 were carried out using an X-Tek Benchtop X-ray machine at BBSRC Silsoe Research Institute (SRI). The unit was a radiologically sealed cabinet, approximately 100 cm wide by 90 cm high by 60 cm deep. The source was a tungsten target microfocus tube with a maximum operating power of 100 kVp at 1 mA. Transmission images could be captured with an imaging system, comprising a CsI phosphor screen, an image intensifier and a video camera, mounted at the opposite end of the cabinet, but this facility was not used except as a tool for aligning the geometry. Scattered photons were counted using an Amptek XR-100T solid state silicon detector. The depletion zone of this detector was 0.03 cm thick. The efficiency of the detector was calculated using the XGAM program (see Section 4.4), assuming a density of 2.3 g cm\(^{-3}\) for silicon (Tennent 1978), and is shown in Fig. 4.1 over the range 20-100 keV. The efficiency at 20 keV was 26%, falling to only 1.3% at 100 keV.

![Figure 4.1: Calculated efficiency of silicon detector](image)

The detector was powered and cooled by an Amptek PX2T amplifier. Cooling was necessary because of thermal electrons reaching the conduction band at room temperature, producing a dark current. The pulses due to detected photons were shaped and amplified by the Amptek, and passed to a 486 PC running Windows 3.1, where they were binned into spectra by a Visual Basic program created by the electronics group at SRI.
4.2 X-ray Hardware: University College London

The experiments described in Section 6.3 and Section 7.1 were carried out using an X-ray source in the Department of Medical Physics at University College London (UCL). The X-ray source at UCL was a tungsten target, with filtration equivalent to 1 mm Al, and an anode angle of 20 degrees (R.D. Speller, pers. comm.). The primary collimation was through a vertical rectangular slot in a block of aluminium. The width of the slot was adjustable, and the height was 2.5 cm, producing a beam 4.0 cm high at a distance of 16.5 cm, where the sample was located. A laser beam through the slot showed the precise position of the X-ray beam in the x direction, and allowed the y position to be calculated.

The detector was an EG&G Ortec GLP 25300/13-P, consisting of a solid state disc of high purity germanium (HpGe), 2.5 cm in diameter and 1.3 cm thick. The detector was cooled by a large liquid nitrogen cryostat. The efficiency was calculated using the XGAM program (see Section 4.4), assuming a density of 5.4 g cm\(^{-3}\) for germanium (Tennent 1978), and is shown in Fig. 4.2 over the range 10-140 keV. The efficiency was greater than 99\% for energies up to 93 keV, falling to 96\% at 110 keV, and 86\% at 140 keV.

![Figure 4.2: Calculated efficiency of germanium detector](image)

X-rays were incident on the detector disc through a rectangular slot in a lead jacket, 0.14 cm wide by 2.5 cm in height. Each recorded photon produced an electronic pulse which was counted and assigned to a spectrum channel by an EG&G Ortec 92X multi-channel analyser (MCA). The MCA was calibrated by
exposing the detector to a $^{241}$Am gamma source, with an energy peak at 59.54 keV. The energy resolution, determined by the FWHM of the $^{241}$Am peak, was typically about 0.40 keV.

Transmission images were produced using film or a dpiX FlashScan 30 portable Amorphous silicon detector, both of which were used in this work.

### 4.3 The EGS4 Code for Monte Carlo Simulation

The EGS4 Code was a program for Monte Carlo simulation of the transport of photons and charged particles in matter. A more detailed description is given later in Section 5.1.1. The Monte Carlo simulations carried out in this work were performed using EGS4 for Unix, versions 3.0 and 3.2 (Nelson & Rogers 1988, Nelson et al. 1985). At the time of writing, these versions and a new version, EGS4nrc, were available on-line from the National Research Council of Canada (NRC) at [http://www.sao.nrc.ca/inms/irs](http://www.sao.nrc.ca/inms/irs). The simulations in Section 5.1 were carried out using version 3.0, and those in Section 5.3 were carried out using version 3.2. Most of the processing was performed on a Sun Ultra60 Sparc processor, running SunOs 5.6, at SRI. With both CPU's of this system running simulations in parallel, it was possible to process the histories of approximately $10^9$ photons per hour through a fairly simple geometry. Additional parallel processing power was gained, in some simulations, by using similar but slower processors at UCL.

The EGS4 code is a package which is used world-wide to simulate the interactions of photons, electrons and positrons with matter. It is a Monte Carlo simulation: that is, random number generation is used to determine the path taken by each particle through a user-defined geometry. The aim is to simulate a sufficiently large number of particles for meaningful statistics to be extracted, such as the fraction of particles reaching a detector region. Further details on the Monte Carlo process are given later in Section 5.1.1.

The main advantages of simulation are twofold: firstly, that the results are very clean, eliminating experimental problems such as errors in the geometry, and noise photons scattering from the walls and ceiling; and secondly, that it is possible to separate multiple from single scatter, which is hard to do in a real experiment. The main disadvantages of EGS4 are firstly, its lack of speed, requiring hours to simulate the number of photons produced per second by a real source; and secondly, its cumbersome programming language, which contains many pitfalls for the novice user, and requires much effort to produce error-free code.

EGS4 3.2 is programmed using a pseudo-FORTRAN language called MORTRAN 3, which compiles into FORTRAN 77. Library functions can be written by the user in either language, or, if speed is important, as in-line macros. Some EGS4 utilities and macros were written during the course of this work, such as a function
for sampling from a polychromatic incident spectrum, and a macro for representing cuboidal regions.

4.4 Miscellaneous Software

All the attenuation coefficients used in this work were calculated using the well-known XGAM package (NIST 1990), which is a DOS program for calculating cross sections and attenuation coefficients of elements and compounds.

Another useful software tool used in this work was the IPEM Spectrum Processor (IPEM 1997). This is available on CD-ROM for Windows NT, and calculates the spectra of X-ray sources, given the target, kVp and filtration. It was used to calculate the number of photons per second in an incident beam, to generate the input spectra for simulation work, and to calculate the mean energies of spectra when required.

Most of the data in this work were processed using Microsoft Excel 97. Various C programs were written during course of the work, such as utilities to calculate Compton scattered energies and Klein-Nishina cross sections, to convert the Maestro 92X output to ASCII, to convert the 16-bit images produced by the amorphous silicon detector to a manageable 8-bit format, and to perform some of the data processing which was beyond the capabilities of Excel.
Chapter 5

Monte Carlo Simulations
5.1 Monte Carlo Simulations: Detection of Voids

The aim of this section was to investigate the detectability of voids in an object, paying particular attention to the effect of multiple scatter. The samples in this work were polystyrene spheres, chosen because of the similarity of polystyrene in density and atomic number to typical food materials. The inclusions were voids, chosen because they represented an extreme of contrast with the sample material, and can occur as defects in produce such as pears (Schreven et al. 1998) and potatoes (Finney & Norris 1978). The sizes of the samples and voids covered a range typical of food and agricultural produce. A computer simulation was used to measure the effect of sample size, beam size and incident energy on the detectability of voids in homogeneous polystyrene spheres. The contrast and signal-to-noise ratio were measured and compared with calculations for conventional transmission imaging. The contribution of multiple scatter, which causes loss of contrast, and which cannot normally be distinguished from single scatter in a real situation, was examined by the computer simulation. The time required to detect a void in a sample using a typical X-ray source was estimated.

5.1.1 Monte Carlo simulation using EGS4

The computer simulations used in this research were performed using the EGS4 code, version 3.0 (Nelson & Rogers 1988), which has been briefly described in Section 4.3. The code used the Monte Carlo method, in which large numbers of individual photons were tracked as they passed through a simulated experimental set-up. The path of each photon was determined by random number generation, to simulate the stochastic nature of real x-ray interactions.

The principles of Monte Carlo simulation in EGS4 are described in detail in Nelson et al. (1985). The user establishes a geometry which specifies the dimensions of all the media involved, and the locations at which particles are to be created, detected or discarded. In this work, the particles were always photons; recoil electrons from Compton scatter events were immediately discarded to save computing time, and the energy was too low for positrons to be created. Each photon history consisted of the route taken through the geometry by a single photon, from creation to detection or discard.

The Monte Carlo method in EGS4 proceeds as follows. First, the mean free path $\lambda$ (by definition, the mean distance to the next interaction) is calculated for the medium in which the particle is currently located. It can be shown for photons that $\lambda = 1/\mu$, where $\mu$ is the attenuation coefficient. Second, EGS4 calculates the number of mean free paths, $K_\lambda$, to move the particle to the next interaction. The correct probability distribution for $K_\lambda$ is modelled by sampling according to

$$K_\lambda = -\ln(r)$$ (5.1)
where \( r \) is a uniform random variable in the range \((0,1)\). This gives a distance \( \lambda K_\lambda \) to the next interaction. Finally, \( \lambda K_\lambda \) is compared to the distance to the next region boundary in the geometry, and the particle is moved either to the interaction or the boundary, whichever is closer. The appropriate processing takes place to deal with the interaction or the change of region, and then returns to the first step to begin again. Processing continues until the particle is absorbed by an interaction, or reaches a discard region. If the particle history ends in a detector region, it is recorded before discarding.

When a sufficiently large number of particle histories have been processed, statistics can be extracted from the recorded data, such as the fraction of incident photons which reach a detector region. Perhaps the greatest advantage of simulation, as opposed to experiment, is that the exact route taken by each particle from source to detector can be recorded. This enables statistics on the number and distribution of different interactions to be computed, and in particular, allows the single-scatter photons to be distinguished from those which reach the detector by multiple scattering.

### 5.1.2 Simulated geometry

The geometry used in the simulations is shown in Figs. 5.1 - 5.2. The incident beam was mono-energetic, parallel, circular in cross section, uniform in intensity, and centred about the z axis. The sample was a homogeneous polystyrene sphere containing a spherical void at the centre. The scatter was counted by a set of collimator arrays, one of which is shown in Fig. 5.1. Each collimator in the array had a square cross section with dimensions \( a \) and \( b \), as indicated. In order to reduce the computation time, advantage was taken of the axially-symmetric geometry by repeating the detector array many times, creating a complete cylinder around the z axis of the sample, as shown in Fig. 5.2. The collimator dimension \( b \) was selected such that the imaged voxel size corresponded to the width of the primary x-ray beam at the centre of the sample, and the dimension \( a \) was set to the same, to keep the collimator square. The number of arrays \( K_c \) which fitted around the cylinder is shown in Table 5.1. The number of collimators per array in the z direction was not important, but was sufficient to image the whole diameter of the sample. The output of each simulation was a plot of counts against z, divided into single and multiple scatter components.

The simulation was performed with different combinations of beam diameter \( D_i \), incident energy \( E \), sample diameter \( D_{samp} \) and void size \( D_{void} \), as shown in Table 5.1. For each of the two beam sizes, all 27 combinations of \( E, D_{samp} \) and \( D_{void} \) were simulated. The total number of incident photons \( N_i \) was \( 10^8 \) in each simulation.

**Table 5.2** gives the spatial resolution of the detectors on the beam, in terms of the full width, \( W_f \) and the full-width at half maximum, \( W_{HM} \), as calculated.
Figure 5.1: Geometry used in simulation: cross section showing sample with central void, and single array of collimators; a and b, dimensions of collimators; $R_d$, length of collimator tube; $R_s$, distance of collimator from beam; $D_i$, $D_{\text{samp}}$ and $D_{\text{void}}$, diameters of beam, sample and void.
Figure 5.2: Geometry used in simulation: perspective view showing sample and cylindrical configuration of collimator arrays; a, dimension of collimator

Table 5.1: Parameters used in simulations of void detection

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Set 1 (thin beam)</th>
<th>Set 2 (thick beam)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam diameter ($D_i$), cm</td>
<td></td>
<td>0.24</td>
<td>0.50</td>
</tr>
<tr>
<td>Length of collimator ($R_d$), cm</td>
<td></td>
<td>14.0</td>
<td>14.0</td>
</tr>
<tr>
<td>Distance to sample ($R_s$), cm</td>
<td></td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Collimator dimension (a), cm</td>
<td></td>
<td>0.089</td>
<td>0.200</td>
</tr>
<tr>
<td>Collimator dimension (b), cm</td>
<td></td>
<td>0.089</td>
<td>0.200</td>
</tr>
<tr>
<td>No. of collimator arrays ($K_c$)</td>
<td></td>
<td>706</td>
<td>314</td>
</tr>
<tr>
<td>Beam energy ($E$), keV</td>
<td></td>
<td>20 50 100</td>
<td>20 50 100</td>
</tr>
<tr>
<td>Sample diameter ($D_{samp}$), cm</td>
<td></td>
<td>4.0 8.0 16.0</td>
<td>4.0 8.0 16.0</td>
</tr>
<tr>
<td>Void diameter ($D_{void}$), cm</td>
<td></td>
<td>0.2 0.5 1.0</td>
<td>0.2 0.5 1.0</td>
</tr>
<tr>
<td>Number of incident photons ($N_i$)</td>
<td></td>
<td>$1 \times 10^8$</td>
<td></td>
</tr>
</tbody>
</table>
from Eqns. 3.70 - 3.71. Also shown is the total solid angle subtended at each voxel by the ring of detectors focused on it. The solid angles were calculated by Eqns. 3.73 - 3.74, and multiplied by the number of detectors $K_c$ in the ring.

Table 5.2: Resolution and solid angles in simulations of void detection

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Set 1 (thin beam)</th>
<th>Set 2 (thick beam)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full width ($W_f$), cm</td>
<td>0.22</td>
<td>0.49</td>
</tr>
<tr>
<td>Full width at half max ($W_{HM}$), cm</td>
<td>0.15</td>
<td>0.34</td>
</tr>
<tr>
<td>Solid angle per collimator ring ($\Omega$), sr</td>
<td>$4.8 \times 10^{-3}$</td>
<td>$1.1 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

The increase in the solid angle per ring was due to the increase in the $z$ dimension $b$ of the collimators, since the increase in $a$ was cancelled out by the corresponding reduction in the number of collimators per ring.

It was stated in Section 4.3 that EGS4 contains pitfalls for the novice user. One such problem concerned the default random number generator, which repeated with a period which was much shorter than the number of incident photons. As a result, the initial simulations showed levels of noise which were visibly much greater than the predicted Poisson noise, and which did not decrease when the number of counts increased. The problem was easily solved by replacing the default random number macro with another from the same EGS4 package, which had a period of $1 \times 10^{24}$. The noise levels in the simulations then decreased to the levels expected.
5.2 Simulated Detection of Voids: Results

This section presents the results of the simulations described in Section 5.1. A typical output from the simulation is shown in Fig. 5.3. This shows the output of the simulation for the thin beam, \( E = 20 \text{ keV} \), \( D_{samp} = 4.0 \text{ cm} \) and \( D_{void} = 0.2 \text{ cm} \). The upper curve shows the single scatter \( N_s(z) \), and the lower curve the multiple scatter \( N_m(z) \). The exponential shape of the single scatter curve was caused by the attenuation of the incident beam from one side of the sample to the other. The reduction in the single scatter counts at \( z = 0 \), as a result of the central void, can be clearly seen. The multiple scatter component was much less affected by the central void, showing that a substantial proportion of the multiple scatter events originated from outside of the primary scatter voxel.

![Profile of scattered counts, showing void at centre;](image)

The contrasts obtained from the various combinations of incident energy, sample size, void size and beam geometry are shown in Table 5.3. For example, the contrast at 50 keV, with a void size of 0.2 cm, a sample size of 16 cm and a thin beam, was 0.37. The contrast was defined by Eqn. 3.1, with \( N_f \) being the total count \( N(0) \) at the central position \( z = 0 \), and \( N_{samp} \) being the (geometric) interpolation of two nearby counts \( N(-0.6) \) and \( N(0.6) \). These were the closest points to the centre which were beyond the influence of the largest void. The geometric mean was used because of the exponential shape of the curve. For comparison, the predicted contrasts of the voids in a transmission image were
also calculated, using *Eqn. 3.46*, and are shown in **Table 5.3**.

**Table 5.3: Simulations of void detection: contrast obtained for different combinations of energy, void size, sample size and beam geometry**

<table>
<thead>
<tr>
<th>Beam energy, keV</th>
<th>Sample size, cm</th>
<th>Void size, cm</th>
<th>Thin beam</th>
<th>Thick beam</th>
<th>Transmission</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>4.0</td>
<td>0.2 0.5 1.0</td>
<td>0.12 0.75 0.95</td>
<td>0.12 0.78 0.94</td>
<td>0.12 0.76 0.89</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>0.62 0.85 0.96</td>
<td>0.03 0.72 0.86</td>
<td>0.05 0.73 0.86</td>
<td>0.12 0.64 0.80</td>
</tr>
<tr>
<td></td>
<td>16.0</td>
<td>- - -</td>
<td>0.85 0.66 0.48</td>
<td>0.85 0.66 0.48</td>
<td>0.85 0.66 0.48</td>
</tr>
<tr>
<td>50</td>
<td>4.0</td>
<td>0.60 0.96 0.98</td>
<td>0.12 0.78 0.94</td>
<td>0.07 0.79 0.95</td>
<td>0.07 0.79 0.95</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>0.44 0.94 0.95</td>
<td>0.05 0.73 0.86</td>
<td>0.04 0.71 0.80</td>
<td>0.04 0.71 0.80</td>
</tr>
<tr>
<td></td>
<td>16.0</td>
<td>0.54 0.98 0.88</td>
<td>0.12 0.64 0.80</td>
<td>0.12 0.64 0.80</td>
<td>0.12 0.64 0.80</td>
</tr>
<tr>
<td>100</td>
<td>4.0</td>
<td>0.58 0.96 0.99</td>
<td>0.07 0.79 0.95</td>
<td>0.07 0.79 0.95</td>
<td>0.07 0.79 0.95</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>0.64 0.95 0.95</td>
<td>0.10 0.76 0.91</td>
<td>0.10 0.76 0.91</td>
<td>0.10 0.76 0.91</td>
</tr>
<tr>
<td></td>
<td>16.0</td>
<td>0.47 0.87 0.89</td>
<td>0.63 0.81 0.81</td>
<td>0.63 0.81 0.81</td>
<td>0.63 0.81 0.81</td>
</tr>
</tbody>
</table>

The contrasts obtained from scattering were much greater than those calculated for transmission. This was because, in scatter imaging, all the detected photons (except the multiple scatter) have interacted with the imaged voxel, but in transmission imaging, most of the detected photons have passed through the sample without interacting, so the change in signal due to a void appears as a much smaller percentage of the total count. The main factor affecting the scatter contrast was the size of the void relative to the beam. The contrast was reduced when the volume occupied by the void was smaller than the primary scatter volume seen by the collimator. The incident energy and the sample size had little or no effect on the contrast.

The corresponding signal-to-noise ratios were calculated using *Eqn. 3.3*, and are shown in **Table 5.4**. The signal-to-noise ratio was a function of the contrast and the statistical noise in the number of counts, as described in **Section 3.1.1**. For the Compton scatter geometries, the SNR increased as the energy increased. The 50 keV energy was much better than 20 keV, although there was little further improvement between 50 and 100 keV. The improvement was due not to any change in the scattering probability, since the Compton coefficient varies little with energy in this range, but rather due to the increased number of photons penetrating and escaping from the sample at the higher energies. For transmission, the SNR was best at low energy, as reported in previous work (Zwiggelaar et al. 1997). In both scatter and transmission, the effect of increasing the sample size was to reduce the number of photons reaching the detector, decreasing the SNR. The signal-to-noise ratios calculated for transmission were better than...
those obtained from scattering, despite the relatively poor contrasts, due to the much greater number of counts. This is consistent with the arguments presented in Section 3.2.

Table 5.4: Simulations of void detection: signal-to-noise ratios obtained for different combinations of energy, void size, sample size and beam geometry

<table>
<thead>
<tr>
<th>Beam energy, keV</th>
<th>Sample size, cm</th>
<th>Void size, cm</th>
<th>Signal-to-noise ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Thin beam</td>
<td>Thick beam</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.2 0.5 1.0</td>
<td>0.2 0.5 1.0</td>
</tr>
<tr>
<td>20</td>
<td>4.0</td>
<td>10.6 17.1 17.2</td>
<td>5.2 31.7 40.2</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>4.3 5.3 7.1</td>
<td>- - 11.7 14.1</td>
</tr>
<tr>
<td></td>
<td>16.0</td>
<td>- - -</td>
<td>2.2 2.0 0.95</td>
</tr>
<tr>
<td>50</td>
<td>4.0</td>
<td>14.8 23.9 25.6</td>
<td>7.0 47.4 54.3</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>6.9 15.0 15.9</td>
<td>1.8 30.0 33.1</td>
</tr>
<tr>
<td></td>
<td>16.0</td>
<td>3.6 7.3 6.1</td>
<td>2.1 11.4 13.6</td>
</tr>
<tr>
<td>100</td>
<td>4.0</td>
<td>14.1 23.8 25.9</td>
<td>4.1 47.6 53.7</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>11.8 17.2 16.4</td>
<td>4.4 32.5 37</td>
</tr>
<tr>
<td></td>
<td>16.0</td>
<td>4 7.3 7.7</td>
<td>0 14.0 17.0</td>
</tr>
</tbody>
</table>

The signal-to-noise ratios were used to estimate the length of time required to scan a whole sample for a void, using a typical X-ray source, which was taken to be a tungsten target, at 100 kVp. The fluence $F_i$ from such a source, over the range 20-100 keV, was $F_i = 5 \times 10^{12}$ photons mA$^{-1}$s$^{-1}$, sr$^{-1}$ (IPEM 1997). Assuming a distance of 20 cm from the source to the centre of the sample, and given the beam diameters 0.24 and 0.50 cm from Table 5.1, Eqn. 3.25 gives an incident flux of $n_i = 5.7 \times 10^8$ photons mA$^{-1}$s$^{-1}$ in the case of the thin beam, and $n_i = 2.5 \times 10^8$ photons mA$^{-1}$s$^{-1}$ in the case of the thick beam. Clearly, the time required was inversely proportional to $n_i$.

A signal-to-noise ratio of 7.4 (see Eqn. 3.9) was required to detect the void. Given that the SNR was proportional to the square root of the number of counts (see Eqn. 3.4), and therefore to the square root of the time taken, the time required to detect a void was inversely proportional to the square of the corresponding SNR, $S$, in Table 5.4.

The number of linescans which had to be performed to scan the whole volume, and hence the time required, was equal to the squared ratio of the sample and beam diameters. Putting all these factors together gives the required time $T_{samp}$ as

$$T_{samp} = \frac{1 \times 10^8}{n_i} \left( \frac{7.4}{S} \right)^2 \left( \frac{D_{samp}}{D_{void}} \right)^2$$

(5.2)
The estimated whole-sample measurement times are shown in Table 5.5.

Table 5.5: Simulations of void detection: Estimated time required to scan samples using a source emitting $5 \times 10^{12}$ photons $s^{-1}sr^{-1}$ at 20 cm from centre of sample. *Thin beam*: $5.7 \times 10^9$ incident photons; *Thick beam*: $= 2.5 \times 10^9$ incident photons

<table>
<thead>
<tr>
<th>Beam energy, keV</th>
<th>Sample size, cm</th>
<th>Void size, cm</th>
<th>Time, s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Thin beam</td>
<td>Thick beam</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.2 0.5 1.0</td>
<td>0.2 0.5 1.0</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td>24 9.2 9.1</td>
<td>5.3 0.14 0.088</td>
<td></td>
</tr>
<tr>
<td>8.0</td>
<td>580 380 213</td>
<td>- 4.2 2.9</td>
<td></td>
</tr>
<tr>
<td>16.0</td>
<td>- - -</td>
<td>470 570 2500</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.2 0.5 1.0</td>
<td>0.2 0.5 1.0</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td>12 4.7 4.1</td>
<td>2.9 0.064 0.048</td>
<td></td>
</tr>
<tr>
<td>8.0</td>
<td>230 50 43</td>
<td>180 0.63 0.52</td>
<td></td>
</tr>
<tr>
<td>16.0</td>
<td>3300 808 1200</td>
<td>520 18 12</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.2 0.5 1.0</td>
<td>0.2 0.5 1.0</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td>14 4.7 4.0</td>
<td>8.5 0.063 0.050</td>
<td></td>
</tr>
<tr>
<td>8.0</td>
<td>77 36 40</td>
<td>30 0.54 0.42</td>
<td></td>
</tr>
<tr>
<td>16.0</td>
<td>2700 808 726</td>
<td>- 12 7.9</td>
<td></td>
</tr>
</tbody>
</table>

The voids took less time to detect, the larger the void and the smaller the sample. The predicted detection times decreased by a factor of at least two for an increase in incident energy from 20 keV to 50 keV, but there was little difference between 50 and 100 keV, not surprisingly, since the attenuation coefficient in between 50-100 keV was dominated by the Compton coefficient, which was fairly constant over this range. The results in Table 5.5 show that, given a measurement time of 1 s, the 0.2 cm void was not detectable in any of the samples, and none of the voids were detectable in the 16 cm sample. The 0.5 and 1.0 cm voids were detectable in 1 s in the 4.0 and 8.0 cm samples, given a suitably high incident energy.

There were several competing effects caused by a change of beam size. When the beam size was increased, the contrast decreased because the void occupied a smaller fraction of the scatter voxel. However, the SNR increased due to the increased number of photons in the beam, and the number of linescans necessary to scan the sample decreased. The solid angle of each collimated detector increased as the beam width increased, to accept photons from the larger scatter volume. The contrasts in Table 5.3 show that the beam size had a large effect on the contrast of the smallest void, because the void was smaller than the beams, but little or no effect on the contrast of the 0.5 and 1.0 cm voids. Not surprisingly, the increase in beam size led to a large reduction in the measurement times for the 0.5 cm and 1.0 cm voids. There was also a smaller improvement in measurement times, even for the 0.2 cm void, although this was not a very consistent result, presumably because the contrast was so close to zero.
It must be noted that these were results for an idealised geometry which all the photons scattered at 90 degrees were detected, but that in reality, many photons would be lost through gaps in the collimator array and inefficient detectors, making the measurement times longer. Improvements could be made only by increasing the power of the source, or by somehow increasing the number of photons captured. The Compton scatter probability is a minimum at 90 degrees, so a smaller scattering angle might be an improvement. In some applications, in which defects always appear in the same place in the sample, such as core or surface defects in fruit, it might not be necessary to scan the entire sample, which would greatly reduce the inspection time.

Table 5.6 shows the fraction of the total count which was due to multiple scatter at the \( z = 0 \) position over the centre of the sample. In cases where the number of scatter events are very small (where attenuation is high) and the SNR is high (see Table 5.3) the tabulated multiple scatter ratios show some spurious peaks. However, in general the multiple scatter fraction does not exceed 26% of the total in the thin beam case, or 35% in the thick beam case. This amount of multiple scatter would be a serious problem if the aim had been to measure the absolute density of the scatter volume from the number of counts, but the effect on the detection times, as calculated from Eqn. 3.42 was not significant in this application.

Table 5.6: Simulations of void detection: Multiple scatter fraction at centre of sample

<table>
<thead>
<tr>
<th>Beam energy, keV</th>
<th>Sample size, cm</th>
<th>Void size, cm</th>
<th>Multiple scatter fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>20</td>
<td>4.0</td>
<td>0.12</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>0.08</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>16.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>50</td>
<td>4.0</td>
<td>0.08</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>0.17</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>16.0</td>
<td>0.26</td>
<td>0.02</td>
</tr>
<tr>
<td>100</td>
<td>4.0</td>
<td>0.08</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>0.09</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>16.0</td>
<td>0.14</td>
<td>0.13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Beam energy, keV</th>
<th>Sample size, cm</th>
<th>Void size, cm</th>
<th>Multiple scatter fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>20</td>
<td>4.0</td>
<td>0.27</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>0.31</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>16.0</td>
<td>0.00</td>
<td>0.23</td>
</tr>
<tr>
<td>50</td>
<td>4.0</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>0.28</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>16.0</td>
<td>0.35</td>
<td>0.29</td>
</tr>
<tr>
<td>100</td>
<td>4.0</td>
<td>0.18</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>8.0</td>
<td>0.22</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>16.0</td>
<td>0.35</td>
<td>0.26</td>
</tr>
</tbody>
</table>
5.3 Monte Carlo Simulations: Detection of Glass

This section describes a set of simulations carried out to investigate the detectability of foreign bodies in more detail than had been possible in Section 5.1, and to simulate detection with a more realistic, polychromatic beam.

5.3.1 Description of simulations

The simulations were performed using EGS4 3.2, which has already been described in Section 5.1. The experience with EGS4 gained from Section 5.1 showed that simplicity was desirable, from the point of view of computing speed and the reliability of the code. Hence, a much simpler geometry was created, shown in Figs. 5.4 - 5.5, consisting of a cubic sample, containing a cubic foreign body, with the incident beam and two collimated detectors placed as shown. The samples were cubic, to enable comparisons to be drawn between the simulations and later experiments, with the cubic regions being represented in EGS4 by a macro written for this work called CUBOID. The beam was cubic in cross section, with width \( W_i \), and was non-diverging. The collimator slots were vertical, with height \( Y_c \), and width \( h \). Two detectors were used, because the geometry was bilaterally symmetric.

In each simulation, a number of incident photons \( N_i \) was chosen which gave enough detected counts \( N \). Each simulation required hours or days to run, depending on the value of \( N_i \). A simulation consisted of the sample being moved in the z direction, in discrete steps, with \( N_i \) incident photon histories being processed at each position. The photons reaching the detector were sorted into single and multiple scatter, and the spectra were recorded.

Composition of samples

The sample in each case consisted of water, and the foreign body consisted of crown glass. The composition of crown glass by weight is \( SiO_2 \) (75%), \( Na_2O \) (9%), \( K_2O \) (11%), and \( CaO \) (5%) (Weast & Astle 1980), or oxygen (45.6%), sodium (6.7%), silicon (35.1%), potassium (9.1%) and calcium (3.5%). Using atomic weight data from Tennent (1978), the formula for crown glass can be shown to be approximately \( Si_{1248}Na_{290}K_{234}Ca_{89}O_{2847} \). The density of glass is 2.6 g cm\(^{-3}\) (Tennent 1978). This formula was used as input to the EGS4 utility called PEGS4, which created the file of attenuation coefficients required by EGS4.

Simulation of polychromatic beam

The simulated beam spectrum was that of a tungsten target at 140 kVp, with 1mm Be filtration and a 20 degree anode angle. The data for this were produced
SUBROUTINE HOWFAR;
" describes a sample cube containing an inclusion cube
" photons are discarded in rgns 3 or 4 if they hit the coll. planes
" photons are recorded if they reach rgn 5
" "
" 5
" "
" ~---[2]-------- --------------- "
" "
" "
" "
" 4
" "
" "
" "
" ~---[1]-------- --------------- "
" "
" "
" "
" "
" --- ------------ "
" | | 2 | "
" | | ---- | "
" | | 1 | "
" "
" "
" "
" "
" "
" ~---|2|-------- "
" "
" "
" "
" "
" "
" "
" ~---|3|-------- "
" "
" "
" "
" "
" "
" "
" ~---|4|-------- "
" "
" "
" "
" "
" "
" "
" "
" "
" "
" "
" "
" "
" Figure 5.4: Geometry used in simulation, showing depiction of geometry in EGS4 subroutine HOWFAR
Figure 5.5: Geometry used in simulation, showing layout and dimensions
using the IPEM CD-ROM (IPEM 1997), previously described in Section 4.4.
(The simulated filtration was not quite the same as the UCL source described
in Section 4.2, but the difference was negligible in the energy window used
here). Various methods were considered for sampling from the spectrum. It is
possible to create a histogram which reproduces the spectrum when sampled by a
uniform random deviate, but this was considered too complicated to program in
the time available. A much simpler method of sampling using a uniform deviate
is to use a Monte Carlo algorithm which samples from the enclosing rectangle
of the spectrum, accepting only those samples which fall inside the area of the
spectrum. This method is neat, but is inefficient if the spectrum contains sharp
peaks, and may require some pre-flattening of the spectrum to reduce the amount
of empty space in the enclosing rectangle. However, random sampling of the
incident spectrum was not necessary in this application, because the detected
photons were all scattered, so the random sampling was performed by the scatter
events themselves. Furthermore, the number of incident photons was so large (of
the order of \(10^9\)) compared to the number of spectral bins (about 280) that the
difference between random and non-random sampling was negligible. Hence the
spectrum was sampled in a deterministic single pass through the spectrum, with
exactly the nominal number of photons being created for each bin, and the total
over the spectrum summing to \(N_i\).

Sets of simulations

The simulations were arranged in sets, each of which showed the effect of varying
one parameter about a nominal geometry, the parameters of which are given
in Table 5.7. The solid angle of the combined detectors was two times that
calculated from the equations in Section 3.4.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample composition</td>
<td>water</td>
</tr>
<tr>
<td>Foreign body composition</td>
<td>crown glass</td>
</tr>
<tr>
<td>Beam energy, kVp</td>
<td>140</td>
</tr>
<tr>
<td>Sample size ( (L_{samp}) ), cm</td>
<td>5.0</td>
</tr>
<tr>
<td>Foreign body size ( (L_{fb}) ), cm</td>
<td>0.4</td>
</tr>
<tr>
<td>Incident beam width ( (W_i) ), cm</td>
<td>0.4</td>
</tr>
<tr>
<td>Detector width ( (W_d) ), cm</td>
<td>0.37</td>
</tr>
<tr>
<td>Detector height ( (Y_d) ), cm</td>
<td>5.0</td>
</tr>
<tr>
<td>Collimator slot width ( (h) ), cm</td>
<td>0.13</td>
</tr>
<tr>
<td>Collimator slot height ( (Y_c) ), cm</td>
<td>2.01</td>
</tr>
<tr>
<td>Collimator-beam distance ( (R_s) ), cm</td>
<td>15.0</td>
</tr>
<tr>
<td>Collimator-detector distance ( (R_d) ), cm</td>
<td>28.0</td>
</tr>
<tr>
<td>Solid angle of detector ( (\Omega) ), sr</td>
<td>(1.15 \times 10^{-3})</td>
</tr>
</tbody>
</table>
Set 1 showed the effect of changing the sample size. Sample sizes ranging from 2.0 to 10.0 cm were simulated. Set 2 showed the effect of varying the foreign body size. Cubes of glass ranging from 0.2 to 0.8 cm were simulated. Set 3 varied the beam size. Beam sizes ranging from 0.1 to 0.8 cm were simulated. When the beam size was varied, the dimensions of the exit collimator and detector were also varied, such that the resolution (full width) in the z direction was always the same as the beam size. The dimensions corresponding to each beam width are given in Table 5.8. It can be confirmed using Eqn. 3.57 that the dimensions in Table 5.8 gave the same full-width resolution as the beam width. The height $Y_c$ of the collimator slot was varied with the beam size in accordance with Fig. 3.7 and Eqn. 3.76, such that the $y$ resolution matched that of the beam width. The solid angle of detector, as calculated using Eqn. 3.61 is also shown.

Table 5.8: Simulations of glass detection: collimator dimensions and solid angles for different beam widths

<table>
<thead>
<tr>
<th>beam width $(W_i)$, cm</th>
<th>detector width $(W_d)$, cm</th>
<th>slot width $(h)$, cm</th>
<th>slot height $(Y_c)$, cm</th>
<th>detector solid angle $(\Omega)$, sr</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.09</td>
<td>0.03</td>
<td>1.81</td>
<td>$2.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.2</td>
<td>0.19</td>
<td>0.07</td>
<td>1.87</td>
<td>$5.9 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.4</td>
<td>0.37</td>
<td>0.13</td>
<td>2.01</td>
<td>$1.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.6</td>
<td>0.56</td>
<td>0.20</td>
<td>2.14</td>
<td>$1.9 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.8</td>
<td>0.75</td>
<td>0.26</td>
<td>2.27</td>
<td>$2.6 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
5.4 Simulated Detection of Glass: Results

This section presents the results of the simulations described in Section 5.3. Note that the plots of X-ray counts in this section, such as Fig. 5.6, were plotted against the sample position. Hence, the counts increased from left to right across the graph, although the beam was incident from the left, as shown in Fig. 5.5.

5.4.1 Output of nominal simulation

The output of the simulation for the nominal geometry, (a 5.0 cm sample, a 0.4 cm cube of glass, and a 0.4 cm beam) is shown in Fig. 5.6. In all the simulations, the beam energy was 140 kVp, and the counts were recorded in a 40-110 keV window. The single scatter increased as the z position of the sample increased, because the attenuation path through the water decreased as the sample moved away from the source, but the multiple scatter was relatively constant across the whole sample. The peak in the single scatter due to the glass was clearly visible. A much smaller peak occurred in the multiple scatter, but it was not known whether this was caused by the presence of the glass or not. The error bars were too small to indicate in Fig. 5.6, but the standard Poisson error was equal to the square root of the counts.

![Figure 5.6: EGS4 Simulation of glass in water: counts obtained from nominal geometry; 140 kVp, 40-110 keV window; 5.0 cm sample, 0.4 cm glass, 0.4 cm beam; 5.5 x 10^9 incident photons; 1.15 x 10^3 sr of detector; — , single scatter; -- o -- , multiple scatter](image-url)
Normalisation of incident counts

The number of incident photons was not the same in each simulation, so the counts were 'normalised' to enable them to be compared on an equal basis. The normalisation procedure was to multiply all the counts by a constant, to give the result which would have been obtained in the same time with the same source. The normalised source was a 140 kVp tungsten source, with 1mm Be filtration, 20 cm from the centre of the sample. The source current was 30 mA, this being the maximum which could reasonably be expected from an industrial source at the time of writing. The fluence rate from this source was $9.17 \times 10^8$ photons cm$^{-2}$mA$^{-1}$s$^{-1}$at 75 cm (IPEM 1997), or $5.16 \times 10^{12}$ photons mA$^{-1}$s$^{-1}$sr$^{-1}$, integrated over the incident energy window 43-140 keV. (about 40% of the total spectrum). The value is taken from Table B.2. The number of incident photons in a real incident beam would also be proportional to the cross sectional area of the beam. Hence, the normalised number of incident photons (assuming one second per reading) was given by

$$N_i^{\text{norm}} = 5.16 \times 10^{12} \times 30 \times \left(\frac{W_i}{20}\right)^2$$  \hspace{1cm} (5.3)

To normalise the results, the detected counts were multiplied by $N_i^{\text{norm}}/N_i$, and the derived quantities, particularly the SNR, were based on the normalised value. The experimental errors were, however, proportional to the Poisson errors of the actual simulated counts - not the normalised counts.

5.4.2 Effect of sample size

The first set of simulations investigated the effect of sample size, and are shown together in Fig. 5.7. The counts were normalised, and were plotted as logarithms, to keep the scale manageable.

The effect of the sample size on the number of detected counts, the contrast, the SNR and the multiple scatter fraction are shown in Figs. 5.8 - 5.12. Fig. 5.13 shows the estimated number of samples per second, per sr of detector, which could be scanned, given the SNR's obtained in the simulations. It was considered more convenient to represent the scan time required per sample, previously derived in Section 3.1.5 and Section 5.2), as the inverse quantity, samples per second, because it allowed the number to be expressed per second per steradian of detector, rather than the more esoteric unit of steradian seconds.

As the sample size increased, the following effects were observed:

- The number of detected counts (see Fig. 5.8) decreased and was consistent with proportionality to $\exp(-\mu_{\text{ samp}} L_{\text{ samp}})$. 

The contrast decreased from about 90% to about 60% (see Fig. 5.9). This was entirely a multiple scatter effect, as can be seen from Fig. 5.10, which showed that the contrast from the single scatter component was constant.

The SNR decreased (see Fig. 5.11), due to decreasing contrast and count numbers.

The multiple scatter fraction increased from about 25% to 55% (see Fig. 5.12). The ratio of multiple to single scatter was approximately proportional to the linear dimension of the sample.

The estimated number of samples which it was possible to scan per sr per second decreased rapidly, from about 9000 to 10 (see Fig. 5.13). The decrease was approximately proportional to \( \exp\left(-\mu_{samp}L_{samp}/L_{samp}^3\right) \), which was similar to the relationship in Eqn. 3.40. The difference was probably due to multiple scatter.
Figure 5.8: *EGS4* Simulations of glass in water: effect of varying sample size on number of detected counts; 140 kVp, 0.4 cm glass, 0.4 cm beam

Figure 5.9: *EGS4* Simulations of glass in water: effect of varying sample size on contrast; 140 kVp, 0.4 cm glass, 0.4 cm beam
Figure 5.10: EGS4 Simulations of glass in water: effect of varying sample size on contrast from single scatter only; 140 kVp, 0.4 cm glass, 0.4 cm beam

Figure 5.11: EGS4 Simulations of glass in water: effect of varying sample size on signal-to-noise ratio; 140 kVp, 0.4 cm glass, 0.4 cm beam
Figure 5.12: EGS4 Simulations of glass in water: effect of varying sample size on multiple scatter fraction; 140 kVp, 0.4 cm glass, 0.4 cm beam

Figure 5.13: EGS4 Simulations of glass in water: effect of varying sample size on scanning rate; 140 kVp, 0.4 cm glass, 0.4 cm beam
5.4.3 Effect of foreign body size

The simulations computed for varying foreign body size are shown together in Fig. 5.14. The main effect which can be seen in Fig. 5.14 is the increasing attenuation shadow of the glass as the foreign body increases in size.

![Diagram](image)

Figure 5.14: *EGS4 Simulations of glass in water: effect of varying foreign body size; 140 kVp, 5.0 cm sample, 0.4 cm beam* — , Δz = 0.2 cm; — , Δz = 0.28 cm; — , Δz = 0.4 cm; — , Δz = 0.57 cm; — , Δz = 0.8 cm

The effect of increasing foreign body size is shown in Figs. 5.15 - 5.19. The following effects were observed as the foreign body size increased:

- The number of counts decreased (see Fig. 5.15), but only by a small fraction. This was an artefact caused by interpolating across the glass to infer what the number of counts at the centre would have been in the absence of the glass. This caused the interpolated value to be affected by the attenuation shadow of the glass.

- The contrast increased (see Fig. 5.16) until the foreign body size matched the beam size, after which point it remained constant. This was not surprising, since the contrast depended largely on the percentage of the scatter voxel which was occupied by glass.

- The SNR was proportional to the contrast (see Fig. 5.17), as expected given that the counts were almost constant.
- The multiple scatter fraction changed little (see Fig. 5.18), varying from about 37% to 44%.
- The scanning rate (see Fig. 5.19) behaved in the same way as the contrast and SNR, increasing from about 10 to 200 samples s\(^{-1}\)sr\(^{-1}\). There was no further increase in the scanning rate for foreign body sizes larger than the beam.

![Graph showing the effect of varying foreign body size on the number of detected counts.](image)

Figure 5.15: EGS4 Simulations of glass in water: effect of varying foreign body size on number of detected counts; 140 kVp, 5.0 cm sample, 0.4 cm beam
Figure 5.16: EGS4 Simulations of glass in water: effect of varying foreign body size on contrast; 140 kVp, 5.0 cm sample, 0.4 cm beam.

Figure 5.17: EGS4 Simulations of glass in water: effect of varying foreign body size on signal-to-noise ratio; 140 kVp, 5.0 cm sample, 0.4 cm beam.
Figure 5.18: EGS4 Simulations of glass in water: effect of varying foreign body size on multiple scatter fraction; 140 kVp, 5.0 cm sample, 0.4 cm beam

Figure 5.19: EGS4 Simulations of glass in water: effect of varying foreign body size on scanning rate; 140 kVp, 5.0 cm sample, 0.4 cm beam
5.4.4 Effect of beam size

The set of simulations showing the effect of beam size is shown in Fig. 5.20.

Figure 5.20: EGS4 Simulations of glass in water: effect of varying beam size; 140 kVp, 0.4 cm glass, 5.0 cm sample. — — , \( W_i = 0.1 \) cm; — — , \( W_i = 0.2 \) cm; — — , \( W_i = 0.4 \) cm; — — , \( W_i = 0.6 \) cm; — — , \( W_i = 0.8 \) cm

The effect of increasing the beam size is shown in Figs. 5.21 - 5.25. The following effects were observed as the foreign body size increased:

- The number of counts (see Fig. 5.21) increased over several orders of magnitude, and was proportional to the fourth power of the beam size. This was expected, since the voxel volume was proportional to the cube of the beam size, and the solid angle of detector was also proportional to the beam size in this set of simulations.

- The contrast (see Fig. 5.22) was constant for beam sizes smaller than the foreign body, but decreased rapidly for larger beam sizes.

- The SNR (see Fig. 5.23) was largest for a beam size equal to that of the foreign body. It fell away rapidly for smaller sizes, and less rapidly for larger sizes.

- The multiple scatter fraction (see Fig. 5.24) was not changed significantly by the change in beam size. This was expected because even the largest beam size occupied less than 3% of the sample volume, so all the beam sizes approximated to an infinitesimally thin beam.

- The scanning rate per second per steradian (see Fig. 5.25) reached a maximum when the beam size was the same as the foreign body size, and re-
mained constant for larger sizes. This showed that the beam size needed to be greater than or equal to the foreign body size to maximise the scanning rate.

Figure 5.21: EGS4 Simulations of glass in water: effect of varying beam size on number of detected counts; 140 kVp, 0.4 cm glass, 5.0 cm sample
Figure 5.22: EGS4 Simulations of glass in water: effect of varying beam size on contrast; 140 kVp, 0.4 cm glass, 5.0 cm sample

Figure 5.23: EGS4 Simulations of glass in water: effect of varying beam size on signal-to-noise ratio; 140 kVp, 0.4 cm glass, 5.0 cm sample
Figure 5.24: EGS4 Simulations of glass in water: effect of varying beam size on multiple scatter fraction; 140 kVp, 0.4 cm glass, 5.0 cm sample

Figure 5.25: EGS4 Simulations of glass in water: effect of varying beam size on scanning rate; 140 kVp, 0.4 cm glass, 5.0 cm sample
Apparent contradiction

The results of Section 5.4.3 (see Fig. 5.19) showed that for a fixed beam size, the fastest scanning rates were for foreign bodies which were greater than or equal to the beam size. However, the results of Section 5.4.4 (see Fig. 5.25) showed that for a fixed foreign body size, the scanning rates were fastest for beam sizes which were greater than or equal to the foreign body size. These conclusions appear to contradict each other, but this is not the case, as can be seen by Fig. 5.26, which shows a sketch of the scanning rate plotted along the two profiles: one with a fixed beam size, the other with a fixed foreign body size. It is perfectly possible for a function to exist with this shape, and the results taken together suggest that the scanning rate \( U \) was an increasing function of the minimum of \( W_i \) and \( L_{fb} \), that is:

\[
U = U[\min(W_i, L_{fb})]
\]  
(5.4)

Hence, to obtain the fastest scanning rate, the beam size should be at least as large as the smallest foreign body size expected in the sample. The scanning rate is dictated by the smallest foreign body, and the detection of larger foreign bodies must be easier than the smallest, regardless of the beam size.

5.4.5 Size of glass detectable

This section considers the minimum time \( T_{samp} \) to detect a piece of glass with a signal-to-noise ratio of 7.4. If the scanning time is \( U \), and the maximum reasonable solid angle of detector \( \Omega_r \) is given by Eqn. 3.39, then

\[
T_{samp} = \frac{1}{U \Omega_r}
\]  
(5.5)

Values for \( T_{samp} \), obtained from the simulations in which the beam size and glass sizes were equal, are shown in Table 5.9.

Interpolation of the values given in Table 5.9 predicted that a 0.4 cm cube of glass would be detectable in 1 s in a 7.8 cm cube of water, and that a 0.27 cm cube of glass would be detectable in one second in a 5 cm sample. The time required increased as the 5.2th power of the sample size, and the inverse 6.8th power of the foreign body size. The time required was very sensitive to the size of the glass: 20 minutes were required to detect a 0.1 cm cube of glass in a 5 cm sample. These results may be compared to the analytical predictions for glass in water, shown in Table 3.1. The simulations and analytical argument gave similar predictions for the sizes of glass detectable, the difference being due to the presence of multiple scatter in the simulations.
Figure 5.26: EGS4 Simulations of glass in water: effect of varying foreign body or beam size on scanning rate; 140 kVp, 0.4 cm glass, 5.0 cm sample

Table 5.9: Minimum scan times for glass detection, as predicted by simulation; source 140 kVp, 30 mA, 20 cm distance

<table>
<thead>
<tr>
<th>size of sample, cm</th>
<th>size of glass, cm</th>
<th>minimum scan time, s</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.4</td>
<td>$1.1 \times 10^{-3}$</td>
</tr>
<tr>
<td>2.7</td>
<td>0.4</td>
<td>$4.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>3.4</td>
<td>0.4</td>
<td>$1.2 \times 10^{-2}$</td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>0.11</td>
</tr>
<tr>
<td>7.5</td>
<td>0.4</td>
<td>0.89</td>
</tr>
<tr>
<td>10</td>
<td>0.4</td>
<td>4.4</td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
<td>1300</td>
</tr>
<tr>
<td>5</td>
<td>0.2</td>
<td>6.4</td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>0.11</td>
</tr>
</tbody>
</table>
5.4.6 Comparison with theory

The number of detected counts in the simulations were compared with the prediction of Eqn. 3.28. The nominal geometry (0.4 cm of glass), with dimensions given in Table 5.7, was simulated with $5.5 \times 10^9$ incident photons, of which 40% were in the energy window for detection (see Section 5.4.1). The total number of single scatter counts from the simulation was $6235 \pm 80$, compared to a predicted value of 7363 from Eqn. 3.28, a disagreement of 18%. Some of the disagreement could be accounted for by the effect of the Compton shift in energy, which was discussed in Section 3.1.6. For example, if the attenuation coefficients were used which corresponded to the scattered energy of 59 keV, instead of the incident energy of 67 keV, Eqn. 3.28 predicted 6690 counts, which was much closer to the simulation result.

Some of the disagreement appeared to be due to attenuation through the glass, since the simulation and analysis were in much closer agreement for the 0.1 cm glass simulation, and was probably due to 67 keV being an over-estimate of the incident energy in Eqn. 3.28. This problem has been discussed in Section 3.1.6.

A small fraction of the difference (about 1.5%) would have been due to the scatter voxel in the simulation not being completely square, so that some of the corners of the glass cube would not have been visible to the detector.
Chapter 6

Experiments: Detection of Glass in Food
6.1 Detection of Glass: Preliminary Experiments

This section describes experiments performed at Silsoe Research Institute, the purpose of which was to demonstrate the detection of glass in food materials. Three experiments were performed in which the Compton scatter was measured from samples of water, instant coffee and muesli containing a fragment of glass.

6.1.1 Experimental details

The X-ray equipment used in these experiments has been described in Section 4.1. The geometry is shown in Fig. 6.1, and the values of the dimensions are given in Table 6.1. The dimensions were the same in each experiment, except for minor variations in the sample depth \(y\). The value for the solid angle, \(\Omega\), was calculated from Eqn. 3.75.

Table 6.1: Preliminary experiments in glass detection: Values of Parameters in Experimental Geometry

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source to sample distance ((R_s)), cm</td>
<td>8.8</td>
</tr>
<tr>
<td>Beam to collimator distance ((R_d)), cm</td>
<td>6.0</td>
</tr>
<tr>
<td>Length of collimator ((R_d)), cm</td>
<td>7.0</td>
</tr>
<tr>
<td>Size of sample ((L_{samp})), cm</td>
<td>10.0</td>
</tr>
<tr>
<td>Thickness of perspex ((W_{px})), cm</td>
<td>0.6</td>
</tr>
<tr>
<td>Size of glass in (x) dimension ((\Delta x)), cm</td>
<td>0.44</td>
</tr>
<tr>
<td>Size of glass in (y) dimension ((\Delta y)), cm</td>
<td>0.44</td>
</tr>
<tr>
<td>Size of glass in (z) dimension ((\Delta z)), cm</td>
<td>0.4</td>
</tr>
<tr>
<td>Depth of glass in sample ((y)), cm</td>
<td>1.4</td>
</tr>
<tr>
<td>Position of glass in sample ((z)), cm</td>
<td>2.2</td>
</tr>
<tr>
<td>Solid angle subtended by detector ((\Omega)), sr</td>
<td>(4.9 \times 10^{-5})</td>
</tr>
</tbody>
</table>

The source was a tungsten target X-ray tube, operated at 100 kVp and 1 mA. The collimated incident beam had a diameter of 0.5 cm at a distance of 10 cm. A solid state detector consisting of 0.03 cm of silicon, was positioned above the sample and collimated to collect the scatter from a small length of the beam. The diameter of the detector collimator was 0.15 cm, giving a total field of view of approximately 0.4 cm at the distance of the incident beam. In each experiment, the detector was scanned in the \(z\) direction, in steps of 0.1 cm, and the number of counts detected in 25 s was recorded at each position. The container was made of perspex and was 0.6 cm thick. For each measurement, the energy spectrum was recorded, and thresholded to remove the low-energy noise in the detector-amplifier system. The counts used in this work were all total counts between 20 and 100 keV.
Figure 6.1: Experimental Compton scatter geometry; $R_i$, source to sample distance; $R_s$, beam to exit collimator distance; $R_d$, length of collimator; $W_{px}$, thickness of perspex container; $L_{samp}$, size of sample; $y$ and $z$, coordinates; $\Delta z$, voxel size; $\Omega$, solid angle subtended by detector.
The samples used were water, instant coffee and muesli. Water was used as an example of the large number of foods, such as milk, fruit juice, olive oil, fruit, vegetables, fish and meat with similar density and composition (Lewis 1987). Coffee was used as an example of a low-density foodstuff, and muesli as an example of an inhomogeneous material. The density of the instant coffee and the muesli were respectively measured to be $0.279 \pm 0.004$ and $0.45 \pm 0.005$ g cm$^{-3}$. The muesli was a supermarket brand: Tesco 'New Crispy Muesli', consisting of 74% cereal flakes and 26% dried fruit and nuts.

The foreign object used in the experiments was a piece of glass, roughly cuboid in shape, with dimensions $\Delta x = 0.8$, $\Delta y = 0.5$ and $\Delta z = 0.4$ cm. The glass was positioned such that it occupied the whole cross section of the incident beam. This alignment of the glass with the beam in the $(x,y)$ plane was easily performed by observing the transmission image of the beam. The detector was aligned with the beam by moving it in the $x$ direction and counting the scatter from a water surface; the detector was considered to be directly above the beam at the position of maximum counts. The detector position in the $z$ direction, relative to the sample position, was determined in a similar manner, by observing the scattered counts from the tip of an aluminium pointer, placed at a known position. The $z$ position of the glass was only known approximately, because it was not held rigidly in the sample.

Four experiments were carried out: the first measured the counts from a sample containing only water; the second from a water sample containing a glass fragment; the third from a sample of instant coffee containing glass; and the fourth from a glass fragment in a sample of muesli. In the water-only experiment, the logarithm of the counts was plotted against the displacement of the detector along the $z$ axis. The slope of the line was determined, and compared with theory. In the experiments in which a glass fragment was present, the log-counts were plotted against the $z$ displacement, and values were calculated for the contrast and signal-to-noise ratio of the glass in each medium. The energy spectra of the scattered photons were collected at the same time as the photons were counted, and the mean energies of the spectra, in the energy window 20-100 keV, were calculated.
6.2 Detection of Glass: Results of Preliminary Experiments

This section presents the results of the glass detection experiments described in Section 6.1.

6.2.1 Detection from scattered counts

The first experiment measured the counts from a sample of water containing no glass, the results of which are shown in Fig. 6.2. The exposure time of each reading was 50s in this case. As expected, this graph was a straight line, with a slope equal to the attenuation coefficient of water. A minor deviation in the straightness of the line was probably due to amplifier drift. The calibration from detected pulse height to keV was carefully monitored in the three detection experiments.

Figure 6.2: Experimental Compton scatter profile for water; ______, measured data; ______, regression line through data; bars indicate one standard error

The logarithmic profiles obtained for glass in water, coffee and muesli, are shown in Fig. 6.3. The corresponding Poisson errors were calculated using Eqn. 3.2. In each case, the glass was positioned within about 0.1 cm of \( z = 2.2 \) cm. The increased scatter over the glass is seen in each profile. The peaks were asymmetric, rising slowly on the source side of the glass and falling more steeply on the far side, because the scatter peak was superimposed on the attenuation gradient. In the water sample, the rising edge of the peak was almost entirely cancelled out by
the attenuation, but the falling edge dropped steeply into the attenuation shadow on the far side of the glass. The coffee profile showed a large rise and fall across the glass, with very little attenuation from the coffee itself. The peak due to the glass was easily seen in the muesli, but the scatter from the background sample was irregular, due to the inhomogeneity of the muesli.

The second differentials of the experimental profiles, shown in Fig. 6.4, were calculated using Eqn. 3.16, using a sampling interval \( h \) equal to 0.4 cm, and the corresponding Poisson errors were calculated using Eqn. 3.19.

The values obtained in each experiment at the position \( z_0 \) for the number of counts, the contrast and the signal-to-noise ratio are given in Table 6.2. The contrasts were calculated by substituting \( f''(z_0) \) into Eqn. 3.17 and rearranging. The values of the signal-to-noise ratio all exceeded the value of 7.4 (see Eqn. 3.9 which was required for the glass to be considered detectable.

<table>
<thead>
<tr>
<th>sample</th>
<th>counts</th>
<th>contrast, percent</th>
<th>signal-to-noise ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>water/glass</td>
<td>1029</td>
<td>50</td>
<td>10.1</td>
</tr>
<tr>
<td>coffee/glass</td>
<td>1022</td>
<td>270</td>
<td>23.4</td>
</tr>
<tr>
<td>muesli/glass</td>
<td>1021</td>
<td>260</td>
<td>24.3</td>
</tr>
</tbody>
</table>

### 6.2.2 Comparison with theory

The experimental results were obtained in a time of \( T_{vox} = 25 \) s voxel\(^{-1} \), using the small and inefficient silicon detector described in Section 4.1.

The number of counts obtained in the water-only experiment were compared with the prediction from Eqn. 3.28. The values used in this calculation are given in Table 6.3. Most of these are obtained from Table 6.1, assuming for simplicity that the voxel and glass dimensions were all equal to 0.4 cm. The incident energy was taken to be 32 keV (see Table B.1). In addition, the fluence rate of the source, \( F_i \), the detector efficiency \( \kappa_1 \) and the Klein-Nishina correction \( \kappa_2 \) were all taken from Table B.2. The attenuation coefficients at 32 keV were taken from Table C.1. The coefficients for the voxel, were in this case, the same as for water, since no foreign body was present. Note that the value of \( z \) used for calculation was offset from the definition of \( z \) in Fig. 6.1 to account for the thickness of the perspex.

The number of counts predicted by Eqn. 3.28 for the position \( z = 2.2 \) was 1244, and the actual number of counts on the regression line in Fig. 6.2 was 1457, so the actual and predicted values differed by only 15%. The extra counts in the real experiment could have been ascribed to multiple scatter, but the difference
Figure 6.3: Experimental Compton scatter profile for a glass fragment in various samples: (a) water; (b) instant coffee; (c) muesli; bars indicate one standard error.
Figure 6.4: Second differential of Compton scatter profile for a glass fragment in various samples: (a) water; (b) instant coffee; (c) muesli; bars indicate one standard error
Table 6.3: Preliminary experiments in glass detection: values used to calculate theoretical number of counts

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength of X-ray source ($F_1$), photons mA$^{-1}$sr$^{-1}$s$^{-1}$</td>
<td>$5.16 \times 10^{12}$</td>
</tr>
<tr>
<td>Voxel size ($\Delta x, \Delta y, \Delta z$), cm</td>
<td>0.4</td>
</tr>
<tr>
<td>Source to sample distance ($R_i$)</td>
<td>8.8</td>
</tr>
<tr>
<td>Position of voxel ($z$), cm</td>
<td>2.8</td>
</tr>
<tr>
<td>Depth in sample ($y$), cm</td>
<td>1.4</td>
</tr>
<tr>
<td>Compton coefficient of water at 32 keV ($\sigma_{samp}$), cm$^{-1}$</td>
<td>0.183</td>
</tr>
<tr>
<td>Attenuation coefficient of water at 32 keV ($\mu_{samp}$), cm$^{-1}$</td>
<td>0.343</td>
</tr>
<tr>
<td>Solid angle subtended by detector ($\Omega$), sr</td>
<td>$4.9 \times 10^{-5}$</td>
</tr>
<tr>
<td>Efficiency of detector ($\kappa_1$)</td>
<td>0.08</td>
</tr>
<tr>
<td>Correction factor for non-isotropic scatter ($\kappa_2$)</td>
<td>0.75</td>
</tr>
<tr>
<td>Time for one reading ($T_{\text{avg}}$), s</td>
<td>50</td>
</tr>
<tr>
<td>Source current ($I$), mA</td>
<td>1</td>
</tr>
</tbody>
</table>

was as likely to have been caused by any number of uncertainties in the geometry. For example, the difference was equivalent to an error of less than 0.01 cm in the diameter of the exit collimator.

Having shown that the experiment and theory were in close agreement for the number of detected photons, the contrast was considered. From Eqn. 3.31, the respective densities of glass and water of 2.6 and 1.0 (Tennent 1978) give an expected contrast of 160%. This compared with the actual value 50% obtained for glass in water (see Table 6.2). The main reasons for the much lower contrast obtained the experiment were as follows:

- The Compton coefficients for glass and water are not in the same ratios as their respective densities. Water has a higher value for $\sigma/\rho$ than glass, because of its hydrogen content (see Section 2.1.2), so the ratio of the coefficients is 2.2 at 32 keV (see Appendix C), and the expected contrast is 120%.

- There was a large amount of self-attenuation through the glass. This was evident from the asymmetry of the scatter 'peak' in Fig. 6.3. The self-attenuation in the $z$ direction was probably not important because the loss of contrast on the source side would have been balanced by an equal gain on the far side of the glass. However, the attenuation in the $y$ direction was significant; at 32 keV, the attenuation through $\Delta y/2$ would have been 0.51, which would leave only 34% contrast.
6.2.3 Detection using mean energy

For each sample, as the detector was scanned in z, the mean energy $\bar{E}(z)$ of the scattered spectrum was calculated in the range 20-100 keV. The energy profile for the water-only sample is shown in Fig. 6.5. The rate of beam hardening was approximately 0.9 keV cm$^{-1}$. The energy profile for glass in water, and its first differential, are shown in Fig. 6.6. The rate of beam hardening increased where the detector passed over the glass. As with the log-count profiles, the contrast and the signal-to-noise ratio were evaluated from the differential profile. The contrast was calculated by analogy with Eqn. 3.1, with the water-only rate of 0.9 keV cm$^{-1}$ being substituted for $N_{\text{samp}}$ and the peak rate $\bar{E}(z_0)$ for $N_{\text{ph}}$. This gave a contrast of 9, which was larger than that obtained from the total photon counts. However, the Poisson noise for each sample was much larger, because the value of $\sigma_E$ in Eqn. 3.52 was approximately 12.8, which produced a more noisy profile than the log-count profile in Fig. 6.4. The signal-to-noise ratio was calculated to be 3.3.

![Graph showing mean energy of spectrum scattered from water with bars indicating one standard error.](image)

Figure 6.5: Mean energy of spectrum scattered from water; bars indicate one standard error

The next higher-order measures of standard deviation and skew were also applied to the spectra, but there were no features that could be perceived above the noise.

In the glass-in-water experiment, the signal-to-noise ratio of the mean energy measurement was 3.3, compared to 10.1 for the photon count measurement shown in Table 6.2 for the same experiment. Since Eqn. 3.2 shows that the noise varies as the square root of the number of photons, this means that a measurement performed using only the spectral shape would require at least 9 times as
Figure 6.6: Mean energy of spectrum scattered from glass in water: (a) mean energy; (b) first differential with respect to position; bars indicate one standard error
long to achieve the same signal-to-noise ratio as a photon-counting measurement. Therefore, although the mean energy could in principle be used to detect a foreign object such as glass in water, it is probably of little practical use compared to the photon counting measurement, at least not in a time-critical application. However, the high contrast obtained suggests that the shape of the energy spectrum would probably be useful when used in a transmission geometry, where the number of photon counts would not be a limiting factor.

6.2.4 Limitations of X-ray system

The low energy of the SRI X-ray system (that is, the effective energy of the combined source and detector) made Compton scatter inspection difficult, because the scatter contrast was largely obscured by attenuation effects. The relatively low source current and the small size and inefficiency of the detector caused difficulty in obtaining enough photon counts for accurate readings. In addition, the detector system tended to overheat if used for more than a few hours continuously, limiting the length of the scans which were possible. Further work was required with a more powerful source and detector to properly investigate the Compton scatter detection of glass.
6.3 Detection of Glass: Further Experiments

This section describes a set of further experiments, numbered G1-G9, on the detection of glass in food materials. The experiments, carried out at UCL were similar to those carried out at SRI, which were described in Section 6.1, with various improvements: the source was more powerful, and the detector more efficient, allowing more counts to be collected in the time available; the source was accurately calibrated; and the dimensions of the geometry, particularly those of the fragments of glass, were more accurately defined.

6.3.1 X-ray Geometry

The geometry used in this work is shown in Fig. 6.7. A beam of X-rays was incident on a sample through a collimating slot. The beam was of rectangular cross section and of height $Y_i$. The backscattered X-rays from the sample were collimated through the exit slot of the collimator and counted by a HpGe detector, placed as shown. The height of the detector was $Y_d$. The collimator consisted of a sheet of lead 0.2 cm thick, stiffened by 0.4 cm of steel.

![Figure 6.7: Compton scatter geometry: 90 degree scatter, perspective view](image-url)
A plan view of the geometry is shown in Fig. 6.8. The width of the incident beam was $W_i$. X-rays were scattered from the sample at an angle of 90 degrees, through a thin slot in the collimator, and counted by a detector of width $W_d$. The purpose of this geometry was to count the photons scattered from the small volume element, or scatter voxel, which was formed by the intersection of the incident and scattered beams. This is shown shaded in Fig. 6.8. Single scatter photons, that is, those which scatter only once, can only reach the detector if they do so within the scatter voxel, although there is an additional flux caused by multiple scatter, which can reach the detector from a wider region of the sample. The distances from the detector and the scatter voxel to the collimator slot were respectively $R_d$ and $R_s$. The alignment of the geometry was performed by eye, with the aid of a laser spot.

Figure 6.8: Compton scatter geometry: 90 degree scatter, plan view

The spatial resolution was equal to the incident beam dimensions in the $x$ and
$y$ dimensions, and was defined as the full width $W_f$ of the detector response in the $z$ dimension. The full width was calculated from the geometry according to Eqn. 3.57. The geometry was set in each experiment to give equal resolution in each dimension, i.e. the scatter voxel was cubic. The geometry parameters for each experiment are shown in Table 6.5.

6.3.2 Description of samples

The sample holder was the same perspex box which was described in Section 6.1, 0.6 cm thick, with internal dimensions $10 \times 10 \times 10$ cm. In each experiment, the bulk of the sample consisted of either water, instant coffee or muesli, filled to a depth of 7.0 cm. It was not possible to fill the sample holder completely because the micrometer mount holding the sample was unable to support the weight of $10 \text{ cm}^3$ of water.

Fragments of glass were obtained from a broken coffee jar, and were filed into cubes with dimensions 0.20 and 0.40 cm. The weight of the 0.20 cm cube was 0.023 g, which was consistent with the nominal dimension, given a density of $2.6 \text{ g cm}^{-3}$. In each sample, a fragment of glass was mounted approximately in the centre of the cube, and held in place by being glued to the end of a vertical perspex rod. The rod was 0.3 cm thick, and was assumed to be neither large enough nor dense enough to significantly effect the results. The bulk material of the sample, water, coffee or muesli, was added around the glass, to a depth of 7.0 cm.

The dimensions and compositions of the samples are listed, for each experiment, in Table 6.4.

6.3.3 Description of X-ray Source and Detector

The UCL X-ray source and detector used in this work have already been described in Section 4.2. The apparatus was located in a laboratory, with the source control unit in the adjacent room. The sample was mounted on a motorised platform for remote operation, but this proved to be unusable because it created severe electrical interference in the X-ray detector. Hence the sample was moved by hand, with the $z$ position being measured by a micrometer, and the $x$ position by a steel rule. This necessitated switching off the beam and entering the laboratory in order to move the sample, which was not entirely satisfactory because it raised doubts over the consistency of the beam. However, the beam power and the repeatability of the readings were monitored during the experiments, and no problems appeared to arise due to this mode of operation.
6.3.4 Experiments

The compositions of the samples are given in Table 6.4, and the dimensions of the geometry are given in Table 6.5. The densities for coffee and muesli given in Table 6.4 were obtained by weighing the samples.

Table 6.4: Glass experiments: composition of samples

<table>
<thead>
<tr>
<th>expt. no.</th>
<th>size of glass, cm</th>
<th>resolution, cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>water, density = 1.0 g cm(^{-3})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G1</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>G4</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>G7</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>coffee, density = 0.26 ± 0.02 g cm(^{-3})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>G5</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>G8</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>muesli, density = 0.49 ± 0.03 g cm(^{-3})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G3</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>G6</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>G9</td>
<td>0.4</td>
<td>0.4</td>
</tr>
</tbody>
</table>

The incident collimator slot was adjusted in \(x\) and collimated in \(y\) in order to produce the appropriate beam dimensions at the distance of the glass fragment from the source.

The geometry was initially aligned to determine the position of the glass fragment relative to the X-ray beam. Alignment in the \(x\) direction consisted of moving the sample such that the glass was in the laser spot. Alignment in the \(z\) direction was performed by lining up the glass, the exit collimator slot and the detector by eye. In the \(y\) direction, alignment was more difficult, because the laser spot did not mark the \(y\) position accurately, and consisted of moving the sample, empty except for the glass, vertically until the maximum number of scatter counts was obtained, showing that the glass was in the beam.

Having aligned the geometry, multiple scatter baffles were added to the exit collimator. These were extra strips of lead restricting the height \(Y_c\) of the slot, so as to cut out as much multiple scatter as possible whilst leaving the detector with a full view of the beam. This geometry has been discussed in Section 3.4.4 and is shown in Fig. 3.7. The positions of the baffles were set by eye. The resulting values of \(Y_c\), given in Table 6.5, were slightly larger than the optimum values as calculated by Eqn. 3.76.
Table 6.5: Glass experiments: geometry parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value, cm</th>
<th>Expt. no.</th>
<th>G1-G3</th>
<th>G4-G6</th>
<th>G7-G9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width of incident beam ((W_i))</td>
<td>0.2</td>
<td></td>
<td>0.2</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>Width of detector slot ((W_d))</td>
<td>0.23</td>
<td></td>
<td>0.23</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>Width of collimator slot ((h))</td>
<td>0.05</td>
<td></td>
<td>0.05</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Detector-collimator distance ((R_d))</td>
<td>12.5</td>
<td></td>
<td>12.5</td>
<td>12.5</td>
<td></td>
</tr>
<tr>
<td>Beam-collimator distance ((R_s))</td>
<td>6.7</td>
<td></td>
<td>6.7</td>
<td>6.7</td>
<td></td>
</tr>
<tr>
<td>Height of incident beam ((Y_i))</td>
<td>0.2</td>
<td></td>
<td>0.2</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>Height of collimator slot ((Y_c))</td>
<td>1.2</td>
<td></td>
<td>1.3</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>Height of detector ((Y_d))</td>
<td>2.5</td>
<td></td>
<td>2.5</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>Full width resolution ((W_f))</td>
<td>0.20</td>
<td></td>
<td>0.20</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>Solid angle ((\Omega), \text{sr})</td>
<td>5.98 \times 10^{-4}</td>
<td></td>
<td>5.98 \times 10^{-4}</td>
<td>1.20 \times 10^{-4}</td>
<td></td>
</tr>
</tbody>
</table>

The experiment consisted of a series of scans, each of which was with \(x\) fixed, and the sample being moved in the \(z\) direction. At each \(z\) position, the X-ray beam was turned on, and the scattered spectrum was counted for a specified time interval.

Additional measurements and calibration readings

Various additional measurements were also performed to ensure the reliability of the data. At the start of each experiment, a \(^{241}\)Am gamma source was placed in front of the detector, and its spectrum was measured. The main peak of the spectrum, at 59.54 keV, was used to determine the constant of proportionality between the MCA channel numbers and the photon energy, and the FWHM of the peak was checked to ensure that the energy resolution was within the expected range. Also preceding each experiment, a noise spectrum was recorded. This was a spectrum taken with the sample absent, in order to measure the number of counts which were scattering into the detector from the laboratory surroundings.

It was observed that there was a small drift in the counts from the beginning to the end of each experiment; a reading taken from the same position would record one or two percent more counts at the end of each day than at the beginning. This was never explained, but was probably connected with a gradual rise in ambient temperature caused by the operation of the source. During each experiment, frequent calibration readings were taken from a single selected point in the sample, situated far from the glass fragment. A regression line was plotted through these readings, and was used to correct the experimental data for the drift. The data were further corrected by subtraction of the initial noise spectrum.

An ion chamber was used to independently monitor the incident beam during
the experiments, but it did not reveal any significant change in the beam power which would have accounted for the drift in the counts.

Temperature readings were taken at frequent intervals from the ambient air, and from the casing of the X-ray source.

Various noise spectra were taken, to determine the number of counts which were not due to scattering from the sample. These were readings which were taken with either no sample, or an empty sample in the beam.

6.3.5 Comparison Simulations

Three EGS4 simulations were performed, to simulate the glass-in-water experiments as closely as possible. These were identical to the simulations already described in Section 5.3, except that the dimensions and positions of the exit collimators were those of the experiments in Table 6.5, and the $y$ dimension of the cuboid was reduced to 7 cm, as in the experiments.

6.3.6 Transmission Images

Transmission images of three cubes of glass, 0.2, 0.3 and 0.4 cm in size, in samples of water, coffee and muesli, were captured using the FlashScan amorphous silicon detector described in Section 4.2. The images were captured at 40 kVp with varying source currents and exposure times.
6.4 Detection of Glass: Results of Further Experiments

This section gives the results of the experiments described in Section 6.3 which measured the ninety-degree scatter from samples containing fragments of glass.

6.4.1 Results of calibration readings

As described in Section 6.3.4, frequent readings were taken from the same point in the sample for calibration purposes. The calibration readings for experiments G1-G9 are shown in Figs. 6.9 - 6.17. These were plotted against the index of the reading, that is, the order in which the readings were taken. The reading index was proportional to the total length of time for which the beam had been operated since the start of the experiment. The size of the drift from start to finish varied from 0.1% on some days to 8% on others. There did not appear to be any systematic reason why the drift should have been large on some days and not on others. The size of the drift did not appear to be related to the rise in temperature, the composition of the sample or the use/non-use of the beam on the preceding day. The drift did not appear to be permanent, since the counts were observed to return to the starting value overnight. The most likely explanation for the drift was a change in the beam current, but the beam power, measured using an ion chamber, was monitored throughout the experiments and showed no change. Despite the lack of explanation for the drift, the regression lines fitted well to the data. The residual errors were consistent with the Poisson error bars shown, with one third of the readings beyond one standard deviation and only one beyond two standard deviations. This justified the assumption made throughout this work that the statistical Poisson error was the only significant source of random noise in the counts.
Figure 6.9: Experiment G1, calibration counts; (a) 10-40 keV window; (b) 40-110 keV window; — , counts; — , regression line

Figure 6.10: Experiment G2, calibration counts; (a) 10-40 keV window; (b) 40-110 keV window — , counts; — , regression line

Figure 6.11: Experiment G3, calibration counts; (a) 10-40 keV window; (b) 40-110 keV window — , counts; — , regression line
Figure 6.12: Experiment G4, calibration counts; (a) 10-40 keV window; (b) 40-110 keV window, counts; — , regression line

Figure 6.14: Experiment G6, calibration counts; (a) 10-40 keV window; (b) 40-110 keV window, counts; — , regression line

Figure 6.13: Experiment G5, calibration counts; (a) 10-40 keV window; (b) 40-110 keV window, counts; — , regression line

Figure 6.15: Experiment G7, calibration counts; (a) 10-40 keV window; (b) 40-110 keV window, counts; — , regression line
Figure 6.16: Experiment G8, calibration counts; (a) 10-40 keV window; (b) 40-110 keV window, counts; — — , regression line.

Figure 6.17: Experiment G9, calibration counts; (a) 10-40 keV window; (b) 40-110 keV window, counts; — — , regression line.
6.4.2 Scans of samples

The counts obtained from the experiments, corrected for the calibration drift, are shown in Figs. 6.18 - 6.26. Offsets have been added to the counts to make the figures more readable, and their values are indicated in the figure captions. In each experiment, as $z$ increased, the sample moved further from the source, and as $x$ increased, the sample moved closer to the detector (see Fig. 6.8). The $x$ coordinate increased from the top to the bottom of each chart shown.

Fig. 6.18 shows four profiles through the 0.2 cm cube of glass in water, at 0.2 cm resolution. The first profile, at $x = 0.0$ cm, showed a small dip due to the attenuation of the exit beam through the glass. The other profiles showed prominent Compton scatter peaks. The results were similar in both energy windows.

Fig. 6.19 shows a scan through 0.2 cm of glass in coffee. The Compton scatter peak was very prominent. An interesting feature of this profile was the scatter from the coffee between 0.3 and 2.3 cm, which showed 5 remarkably regular oscillations with a period of 0.4 cm. Repeat readings confirmed that the pattern was real. This was believed to have been caused by a regular alignment of coffee granules. A photograph of the coffee sample, shown in Fig. 6.27 showed some examples where the granules were linearly aligned.

Fig. 6.20 shows scans through 0.2 cm of glass in muesli. The peak from the glass at $x = 0.0$ was smaller in this case than the feature at $x = 1.4$, which was a raisin, deliberately placed at that point in the sample. The size of the raisin peak was surprising, since the raisin was not expected to be very dense. It did not appear to be as attenuating as the glass, judging from the difference between the two profiles. The raisin peak might have been made larger by coinciding with a gap in the attenuation path, but it was impossible to tell without further scans.

The profiles for the 0.4 cm cube of glass, at 0.2 mm resolution, shown in Figs. 6.21 - 6.23, were similar to those in Figs. 6.18 - 6.20. An anomalously high reading in the water profile, at $x = 0.25$, $z = 0.2$ in Fig. 6.21 was not explained. In the muesli profiles, in Fig. 6.23, a bump was observed behind the glass, at $x = 0.6$, $z = 0.4$. Careful excavation of the sample after the experiment revealed this to have been a sunflower seed.
Figure 6.18: Experiment G1, detected counts, 2mm glass in water, 2mm resolution; (a) 10-40 keV window; — , \( x = -0.1 \), offset = 0; — , \( x = 0.0 \), offset = -150; — , \( x = 0.1 \), offset = -300; , \( x = 0.5 \), offset = -450
(b) 40-110 keV window — , \( x = -0.1 \), offset = 0; — , \( x = 0.0 \), offset = -650; — , \( x = 0.1 \), offset = -1300; — , \( x = 0.5 \), offset = -1950
Figure 6.19: Experiment G2, detected counts, 2mm glass in coffee, 2mm resolution; (a) 10-40 keV window; (b) 40-110 keV window; — , $x = 0.0$.
Figure 6.20: Experiment G3, detected counts, 2mm glass in muesli, 2mm resolution; (a) 10-40 keV window; (b) 40-110 keV window; — , \( x = 0.0 \); — , \( x = 0.2 \)
The scans of the 0.4 cm glass at 0.4 cm resolution were similar to those preceding. These are shown in Figs. 6.24 - 6.26.
Figure 6.21: Experiment G₄, detected counts, 4mm glass in water, 2mm resolution; (a) 10-40 keV window; (b) 40-110 keV window; — — — , $x = 0.0$, offset = 0; — — — , $x = 0.25$, offset = -200; — — — , $x = 0.6$, offset = -400
Figure 6.22: Experiment G5, detected counts, 4mm glass in coffee, 2mm resolution; (a) 10-40 keV window; (b) 40-110 keV window; \( x = 0.0 \); \( x = 0.6 \)
Figure 6.23: Experiment G6, detected counts, 4mm glass in muesli, 2mm resolution; (a) 10-40 keV window; (b) 40-110 keV window; — , $x = 0.0$; — , $x = 0.6$
Figure 6.24: Experiment G7, detected counts, 4mm glass in water, 4mm resolution; (a) 10-40 keV window; (b) 40-110 keV window; ——, $x = 0.0$, offset $= 0$; ———, $x = 0.4$, offset $= -500$; ———, $x = 0.8$, offset $= -1000$
Figure 6.25: Experiment G8, detected counts, 4mm glass in coffee, 4mm resolution; (a) 10-40 keV window; (b) 40-110 keV window; 
- - - - , $x = 0.0$, offset = 0; 
- - - - - , $x = 0.4$, offset = -2000; 
- - - - - - - , $x = 0.8$, offset = -4000
Figure 6.26: Experiment G9, detected counts, 4mm glass in muesli, 4mm resolution; (a) 10-40 keV window; (b) 40-110 keV window; — x = 0.0; — x = 0.4; — x = 0.8
Figure 6.27: Photograph of coffee sample, showing examples (ringed) of linearly-aligned granules. The internal width of the sample is 10 cm.
6.4.3 Contrasts and signal-to-noise ratios

The contrasts and signal-to-noise ratios obtained from the sample scans are shown for the 10-40 keV window in Table 6.6 and for the 40-110 keV window in Table 6.7. The normalised SNR’s were necessary to enable the results to be compared, because the experiments used different sampling times and source currents. The normalised SNR’s were an estimate of what would have been obtained using a 30 mA source in one second, and were calculated on the assumption that the SNR was proportional to the square root of the product of the current and the sampling time.

Table 6.6: Results of Glass Experiments: Contrasts and Signal-to-Noise Ratios, 10-40 keV Window

<table>
<thead>
<tr>
<th>expt. no.</th>
<th>contrast, percent</th>
<th>signal-to-noise ratio (expt)</th>
<th>signal-to-noise ratio (normalised to 30 mA s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G1</td>
<td>19 ± 4</td>
<td>6.1</td>
<td>0.9</td>
</tr>
<tr>
<td>G4</td>
<td>32 ± 5</td>
<td>9.0</td>
<td>1.3</td>
</tr>
<tr>
<td>G7</td>
<td>28 ± 2</td>
<td>17.2</td>
<td>5.4</td>
</tr>
<tr>
<td>coffee</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G2</td>
<td>286 ± 7</td>
<td>129.1</td>
<td>13.2</td>
</tr>
<tr>
<td>G5</td>
<td>323 ± 10</td>
<td>120.2</td>
<td>13.4</td>
</tr>
<tr>
<td>G8</td>
<td>346 ± 6</td>
<td>210.1</td>
<td>66.5</td>
</tr>
<tr>
<td>muesli</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G3</td>
<td>148 ± 6</td>
<td>56.6</td>
<td>6.3</td>
</tr>
<tr>
<td>G6</td>
<td>374 ± 18</td>
<td>83.5</td>
<td>13.2</td>
</tr>
<tr>
<td>G9</td>
<td>122 ± 3</td>
<td>89.8</td>
<td>28.4</td>
</tr>
</tbody>
</table>

The contrasts in the low-energy window were similar to those in the high-energy window. The contrast was largest in coffee, smaller in muesli, and smallest in water, but none of the contrasts were as large as the density contrast between the materials, which were (using the densities from Table 6.4) 160% for glass in water, 900% for glass in coffee, and 430% for glass in muesli. There was a large increase in the contrast when the size of the glass was increased from 0.2 cm to 0.4 cm, whilst the beam size remained at 0.2 cm. This suggested that the scatter voxel was either slightly larger than 0.2 cm, or that it was slightly misaligned. The difference between the 25% and 46% contrasts for glass in water were equivalent to an increase of about 0.05 cm in the cube dimension.
Table 6.7: Results of Glass Experiments: Contrasts and Signal-to-Noise Ratios, 40-110 keV Window

<table>
<thead>
<tr>
<th>expt. no.</th>
<th>contrast, percent</th>
<th>signal-to-noise ratio (expt)</th>
<th>signal-to-noise ratio (normalised to 30 mA s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G1</td>
<td>25 ± 2</td>
<td>16.5</td>
<td>2.4</td>
</tr>
<tr>
<td>G4</td>
<td>46 ± 3</td>
<td>28.5</td>
<td>4.1</td>
</tr>
<tr>
<td>G7</td>
<td>47 ± 1</td>
<td>55.9</td>
<td>17.7</td>
</tr>
<tr>
<td>coffee</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G2</td>
<td>246 ± 4</td>
<td>200.0</td>
<td>20.4</td>
</tr>
<tr>
<td>G5</td>
<td>355 ± 6</td>
<td>241.1</td>
<td>27.0</td>
</tr>
<tr>
<td>G8</td>
<td>422 ± 4</td>
<td>415.9</td>
<td>131.5</td>
</tr>
<tr>
<td>muesli</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G3</td>
<td>143 ± 3</td>
<td>95.7</td>
<td>10.7</td>
</tr>
<tr>
<td>G6</td>
<td>364 ± 9</td>
<td>154.6</td>
<td>24.4</td>
</tr>
<tr>
<td>G9</td>
<td>144 ± 2</td>
<td>184.2</td>
<td>58.2</td>
</tr>
</tbody>
</table>
6.4.4 Comparison with simulations

The results of the simulations of the glass-in-water experiments are shown in Figs. 6.28 - 6.30. It can be seen that the multiple scatter was approximately half the total signal. The contrasts and multiple scatter fractions are shown in Table 6.8. The total contrasts of 67%, 61% and 64% may be compared with the experimental contrasts of 25%, 46% and 47% from Table 6.7.

It will be recalled from Section 6.2.2 that the expected contrast for glass in water at 32 keV was 120%, when the actual ratio of the Compton coefficients was used, instead of the densities. The experiments and simulations in this section were performed at an energy equivalent to 67 keV (see Table B.1), which gave an expected contrast of 127%, from the coefficients in Tables C.1 - C.2. The simulations showed that the single scatter contrast was only slightly less than this, and that most of the loss in contrast was due to multiple scatter.

The experimental contrast was much less for the 0.2 cube of glass, probably due to errors in the size and position of the scatter voxel. The contrasts for the other experiments were similar to but less than the simulations.

The number of detected X-rays from experiment G7 (0.4 cm of glass in water) was compared with the simulation of the experiment, and found to be less than predicted by the simulation by a factor of 4. Some of the disagreement was due to the primary collimator block, which (with hindsight) was set to a slot width which was too narrow, giving a resolution in the x direction of about 0.3 cm. A
Figure 6.29: EGS4 simulation of experiment G4, 0.4 cm glass, 0.2 cm beam; -- o --, multiple scatter; — - - - - - , total scatter

Figure 6.30: EGS4 simulation of experiment G7, 0.4 cm glass, 0.4 cm beam; -- o --, multiple scatter; — - - - - - , total scatter
Table 6.8: Contrasts and multiple scatter from simulations of experiments (40-110 keV)

<table>
<thead>
<tr>
<th>expt. no.</th>
<th>contrast (single scatter), percent</th>
<th>contrast (total scatter), percent</th>
<th>multiple scatter fraction, percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>110 ± 10</td>
<td>67 ± 6</td>
<td>46 ± 1</td>
</tr>
<tr>
<td>G4</td>
<td>102 ± 10</td>
<td>61 ± 6</td>
<td>49 ± 1</td>
</tr>
<tr>
<td>G7</td>
<td>109 ± 8</td>
<td>64 ± 4</td>
<td>52 ± 1</td>
</tr>
</tbody>
</table>

Radiographic measurement of the beam width would have been useful to confirm this. The bulk of the discrepancy was believed to have been due to the finite width of the source focal spot, which caused losses in the primary collimator which were not accounted for by simulation. This has been discussed in Section 3.1.6.
6.4.5 Mean spectral energy

The mean spectral energies are shown in Figs. 6.31 - 6.39, and the contrasts and SNR's are given in Table 6.9. The contrasts were larger, the lower the density of the sample material, as expected. In water, the only clear contrast was obtained with the 0.4 cm glass and 0.4 cm beam. The only convincing contrast in coffee was for experiment G8, since experiment G2 produced a negative contrast, and the contrast in experiment G5 was only based on one data point. The contrasts in the muesli were masked by the sample inhomogeneity. The normalised SNR’s were much smaller than those obtained from the 40-100 keV counts, by a factor of 6 in water, 16 in coffee, and 12 for muesli.

Table 6.9: Results of Glass Experiments: Contrasts and Signal-to-Noise Ratios, Mean Energy

<table>
<thead>
<tr>
<th>expt. no.</th>
<th>contrast, percent</th>
<th>signal-to-noise ratio (expt)</th>
<th>signal-to-noise ratio (normalised to 30 mA s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G1</td>
<td>0.7</td>
<td>1.4</td>
<td>0.20</td>
</tr>
<tr>
<td>G4</td>
<td>1.6</td>
<td>3.3</td>
<td>0.48</td>
</tr>
<tr>
<td>G7</td>
<td>2.1</td>
<td>8.7</td>
<td>2.76</td>
</tr>
<tr>
<td>coffee</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G2</td>
<td>-3.5</td>
<td>-9.3</td>
<td>-1.0</td>
</tr>
<tr>
<td>G5</td>
<td>6.4</td>
<td>14.2</td>
<td>1.6</td>
</tr>
<tr>
<td>G8</td>
<td>8.0</td>
<td>26.3</td>
<td>8.3</td>
</tr>
<tr>
<td>muesli</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G3</td>
<td>2.9</td>
<td>7.2</td>
<td>0.8</td>
</tr>
<tr>
<td>G6</td>
<td>4.2</td>
<td>6.8</td>
<td>1.1</td>
</tr>
<tr>
<td>G9</td>
<td>3.7</td>
<td>15.4</td>
<td>4.9</td>
</tr>
</tbody>
</table>
Figure 6.31: Experiment G1, mean energy of spectrum, 2mm glass in water, 2mm resolution; — — , $x = -0.1$, offset = 0; — — — — , $x = 0.0$, offset = 1.2; — — — — , $x = 0.1$, offset = 2.4; — — — — , $x = 0.5$, offset = 3.6;

Figure 6.32: Experiment G2, mean energy of spectrum, 2mm glass in coffee, 2mm resolution; — — — — , $x = 0.0$
Figure 6.33: Experiment G3, mean energy of spectrum, 2mm glass in muesli, 2mm resolution; — — , x = 0.0; — o — , x = 0.2

Figure 6.34: Experiment G4, mean energy of spectrum, 4mm glass in water, 2mm resolution; — — , x = 0.0, offset = 0; — — , x = 0.25, offset = 1.5; — — , x = 0.6, offset = 3.0
Figure 6.35: Experiment G5, mean energy of spectrum, 4mm glass in coffee, 2mm resolution; — , x = 0.0; — — , x = 0.6

Figure 6.36: Experiment G6, mean energy of spectrum, 4mm glass in muesli, 2mm resolution; — — , x = 0.0; — — — , x = 0.6
Figure 6.37: Experiment G7, mean energy of spectrum, 4mm glass in water, 4mm resolution; \( - - \), \( x = 0.0 \), offset = 0; \( \circ \circ \), \( x = 0.4 \), offset = 1.0; \( \bullet \bullet \), \( x = 0.8 \), offset = 2.0

Figure 6.38: Experiment G8, mean energy of spectrum, 4mm glass in coffee, 4mm resolution; (a) 10-40 keV window; (b) 40-110 keV window; \( - - \), \( x = 0.0 \), offset = 0; \( \circ \circ \), \( x = 0.4 \), offset = 1.5; \( \bullet \bullet \), \( x = 0.8 \), offset = 3.0
Figure 6.39: Experiment G9, mean energy of spectrum, 4mm glass in muesli, 4mm resolution; — , \( x = 0.0 \); — , \( x = 0.4 \); — , \( x = 0.8 \)
6.4.6 Comparison with transmission images

Transmission images of glass in water, coffee and muesli are shown in Figs. 6.40 - 6.42. The dimensions of the cubes of glass were 0.2, 0.3 and 0.4 cm. For each image, the contrasts of the glass cubes were measured. The inhomogeneity of the sample was the ratio of the standard deviation of the pixel grey levels to the mean grey level. The SNR was the ratio of the contrast to the inhomogeneity, or the ratio of the change in grey level to the sample standard deviation. For example, the grey level difference between the 0.2 cm piece of glass and the coffee was 5.4 times the standard deviation of the pixels. The figures for the water image were not reliable because the image appeared to contain digital noise and some linear artefacts, which suggested that the detector was not functioning correctly. The other images appeared to be clean. The SNR’s for the muesli sample were low enough to suggest that none of the pieces of glass were detectable, because the grey levels of the glass cubes were only 1.4-2.3 standard deviations below the sample average. However, although the 0.2 cm piece was indeed undetectable by thresholding, it was possible to set a threshold which divided the 0.3 and 0.4 cm glass cubes from the sample. This was possible because the pixel distribution was non-Gaussian and highly skewed, and showed that the mean and standard deviation did not fully describe the inhomogeneity.

The contrasts obtained in the scatter experiments were much greater than the transmission contrasts. It was not possible to compare the SNR’s because the transmission contrasts were compared to the inhomogeneity noise, whereas the scatter SNR’s were based only on the Poisson noise. Insufficient data were available to determine the statistics of the scatter from the sample, but it would be of interest in further work to compare the inhomogeneity effects in both modes of detection.
Figure 6.40: *Transmission image: glass in water sample, 40 kVp*
Figure 6.41: Transmission image: glass in coffee sample, 40 kVp
Figure 6.42: Transmission image: glass in muesli sample, 40 kVp
Table 6.10: Transmission images of glass in food: contrast, inhomogeneity and signal-to-noise ratio

<table>
<thead>
<tr>
<th>size of glass, cm</th>
<th>contrast, percent</th>
<th>inhomogeneity of sample, percent</th>
<th>signal-to-noise ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>water</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>14</td>
<td>5</td>
<td>3.0</td>
</tr>
<tr>
<td>0.3</td>
<td>19</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>22</td>
<td>4.7</td>
<td></td>
</tr>
<tr>
<td><strong>coffee</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>32</td>
<td>6</td>
<td>5.4</td>
</tr>
<tr>
<td>0.3</td>
<td>46</td>
<td>7.9</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>50</td>
<td>8.6</td>
<td></td>
</tr>
<tr>
<td><strong>muesli</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>34</td>
<td>24</td>
<td>1.4</td>
</tr>
<tr>
<td>0.3</td>
<td>48</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>55</td>
<td>2.3</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 7

Experiments: Backscatter Detection of Bone Fragments in Chicken Meat
7.1 Detection of Bone Fragments: Experiments

A study of Compton scatter inspection would not be complete without consideration of the backscatter geometry, which has been successfully applied to a very wide range of industrial problems, as described in Section 1.1.5. The results of previous chapters (see Table 3.1, Table 5.5 and Table 5.9) have so far shown that detection times for whole-sample scans tend to be slow, even compared to a generous inspection time of one second, and that the bulk scanning method is probably too slow for this type of food inspection. However, there remain applications in which the whole sample does not need to be scanned, because there is a priori knowledge about the location of the foreign body. One such application is the detection of bone fragments in fresh poultry meat (see Section 1.1.1). This application is well-suited to the backscatter geometry because the bones tend to occur on or near the surface of the meat.

This section describes a series of experiments to measure the detectability of near-surface bone fragments in chicken breasts, using Compton backscatter.

7.1.1 Introduction to X-ray Backscatter

A simple geometry for the detection of Compton scattered photons is shown in Fig. 7.1, in which an incident beam and a detector are focused on a scattering point B in a sample. The incident beam is attenuated from A to B according to Eqn. 2.18, is scattered at B with a probability proportional to the Compton coefficient \( \sigma \), and is attenuated again on the exit path from B to C, before reaching the detector at D. Hence the number of photons reaching the detector is proportional to the density at B, and to the total attenuation along the paths AB and BC. If the attenuation effects are small, scanning the point B in x and z produces a density map of the sample.

Attenuation and scatter also have effects on the spectrum of energies arriving at the detector. Photons lose energy when they are Compton scattered, depending on the incident energy and the angle of scatter. For example, an incident spectrum in the range 10-140 keV is compressed into the range 10-95 keV after scattering through 135° (see Table A.1). Attenuation causes the effect known as beam hardening, in which the greater attenuation of the low energy photons causes the mean energy of the spectrum to increase.

7.1.2 Measurement of bone and flesh densities

The density of chicken bone is lighter than the standard value of 1.85 g cm\(^{-3}\) (Attix 1986b) for human cortical bone, mainly because broiler chickens are slaughtered
Figure 7.1: Compton backscatter geometry

at 34-42 days old (D.B. Tinker, pers. comm.) before their bones are fully mineralised. No values for the density of fresh chicken bone were found in the literature, so the value was determined by experiment.

A total of 8 chickens (Tesco British class A, fresh, medium, 1.5 kg) were purchased from a supermarket, and the clavicles (wishbones) were removed. Each clavicle was cleaned of any remaining meat, and the top and tips were discarded, leaving two pieces of bone, approximately 3.0 cm in length. These bones were each split in two longitudinally, as shown in Fig. 7.2, and the blood or soft tissue inside was scraped out gently with a modelling knife.

A standard Archimedes method was used to determine the density. This was carried out by the staff of the analytical chemistry lab at SRI. The bones were sealed with a measured quantity of wax in order to prevent them drying out in air, or absorbing water during immersion, and were loosely tied together in pairs, each pair consisting of the two half-fragments from one or other arm of the clavicle. Each pair was weighed, and its volume was determined by measuring its weight displacement in a beaker of water. The density was calculated by dividing the weight by the volume. Care was taken not to trap air bubbles in the tied fragments during immersion. The results obtained for the chicken bones are given in Table 7.4. The mean density was 1.32 g cm$^{-3}$.

The bone masses were accurate to $\pm 5 \times 10^{-4}$ g, and the volumes to about $\pm 5 \times 10^{-4}$
Figure 7.2: Preparation of chicken bone samples for density measurements. (a) clavicle showing cutting points. (b) top and tips of clavicle removed. (c) bones split longitudinally.
cm$^3$, giving an experimental error in each density measurement of approximately ±0.006g cm$^{-3}$. The standard deviation of the results within the same chicken was ±0.016g cm$^{-3}$, which was similar to the estimated experimental error. The standard deviation of the results between chickens was ±0.050g cm$^{-3}$.

A single pair of chicken breast fillets were purchased from the same supermarket, were divided into 10 chunks of about 30 g each, and were analysed in the same way. The results are shown in Table 7.5. The mean density was 1.07 g cm$^{-3}$, with a very small standard deviation. The pieces included some small strips of fat, but the 30 g size was large enough to average out most of the small-scale density variations.

The low density of the bone fragments indicated that care was needed in choosing a suitable material as a phantom for chicken meat. For example, perspex was unsuitable, as its density of 1.2 g cm$^{-3}$ was more similar to the bone than to the meat. Polystyrene was a better choice, as its density of 1.05 g cm$^{-3}$ (Tennent 1978) was very close to that of the flesh. The nominal figure for the density of polystyrene was confirmed by weighing cuboidal samples.

7.1.3 X-ray Geometry

The backscatter geometry used in this work is shown in Fig. 7.3. A beam of X-rays was incident on a sample through a collimating slot. The beam was of rectangular cross section and of height $Y_i$. The backscattered X-rays from the sample were collimated through the exit slot of the collimator and counted by a HpGe detector, placed as shown. The height of the detector was $Y_d$. The collimator consisted of a sheet of lead 0.2 cm thick, stiffened by 0.4 cm of steel. Photographs of the experiments are shown in Figs. 7.4 - 7.5.

A plan view of the geometry is shown in Fig. 7.7. The width of the incident beam was $W_i$. X-rays were scattered from the sample at an angle $\alpha$, through a thin slot in the collimator, and counted by a detector of width $W_d$. The purpose of this geometry was to count the photons scattered from the small volume element, or scatter voxel, which was formed by the intersection of the incident and scattered beams. This is shown shaded in Fig. 7.7. Single scatter photons, that is, those which scatter only once, can only reach the detector if they do so within the scatter voxel, although there is an additional flux caused by multiple scatter, which can reach the detector from a wider region of the sample. The distances from the detector and the scatter voxel to the collimator slot were respectively $R_d$ and $R_s$. The alignment of the geometry was performed by eye, with the aid of a laser spot.

It can be shown by geometry that the $z$ dimension $W_s$ of the scatter voxel is given
Figure 7.3: Backscatter geometry: perspective view
Figure 7.4: Photograph of backscatter experiment: front view
Figure 7.5: Photograph of backscatter experiment: rear view
Figure 7.6: Photograph of backscatter experiment: close-up of sample
Figure 7.7: Backscatter geometry: plan view
by
\[ W_z = \frac{R_z W_d}{R_d \sin(\alpha)} \] (7.1)

The spatial resolution of the geometry depended on the size of the scatter voxel. The resolution in \( x \) was simply the beam width \( W_i \), but the resolution in \( z \) was not quite as straightforward to calculate, because of the rhomboid shape of the voxel, and the projective geometry. The resolution was defined as the full-width at half-maximum (FWHM) of the response of the detector, plotted as a function of the depth \( z \). In the experiments presented here, the \( z \) dimension \( W_z \) of the rhomboid was smaller than the beam width \( W_i \) so that the resolution was approximately equal to \( W_i \), and varied little with \( W_z \). The calculations agreed with measurements of the FWHM obtained by scattering from a lead surface (see later, Section 7.1.6).

### 7.1.4 Description of Samples

Two sets of experiments were performed in this work. The first set used phantoms constructed from chicken bones inserted into polystyrene blocks. The second set used real pieces of chicken breast containing bone fragments just below the surface. The structure of the polystyrene phantoms is shown in Fig. 7.8. As explained in Section 7.1.2, polystyrene was chosen because it was similar in density and atomic number to chicken flesh, and would therefore have similar radiological properties. Each phantom consisted of a polystyrene block with a surface slot, into which a known mass of chicken bone fragments were packed. The remaining air in the slot was displaced by water, and a thin polystyrene cover was glued onto the surface to seal the contents. The samples in the second set of experiments consisted of pieces of chicken breast, approximately 2 cm thick and cut to approximately 5 cm x 5 cm square. A cut was made in the chicken surface with a sharp knife, and a long, thin fragment of bone was inserted into the cut. The cut was slightly oblique, so that the bone could be pushed beneath the surface, to a depth of between 0.1 and 0.2 cm. The whole sample was wrapped in thin plastic food-wrap, for ease of handling, and to prevent dessication during the experiment. In all cases the bone fragments were prepared by the same procedure as described in Section 7.1.2.

### 7.1.5 Description of X-ray Source and Detector

The equipment used in this experiment has already been described in Section 4.2, the only difference being that the apparatus was arranged in a backscatter geometry. The primary collimation was through a vertical rectangular slot in a block of aluminium. The width of the slot was adjustable, and the height was 2.5 cm, producing a beam 4.0 cm high at a distance of 16.5 cm, where the
Figure 7.8: Chicken meat phantom, consisting of polystyrene block and cover with slot to accommodate bone fragments. (a) perspective view, (b) side view
sample was located. Radiographs of the beam were taken at distances of 59.3 cm and 16.5 cm from the primary collimator, the latter distance being the position of the sample. These are shown in Figs. 7.9 - 7.10, and confirmed the position and dimensions of the beam.

Figure 7.9: Radiograph of incident beam, 30 kVp, 1 mA, 2s exposure, 59.3 cm from exit slot of primary collimator; cross marks position of laser spot

7.1.6 Experiments

Experiment 1 was a preliminary experiment to demonstrate that the system could locate an air-filled slot in a polystyrene phantom, experiments 2 and 3 measured the backscatter from chicken bones in polystyrene phantoms, and experiments 4, 5 and 6 measured the backscatter from bone fragments in chicken meat. The
Figure 7.10: Radiograph of incident beam, 30 kVp, 2 mA, 2 s exposure, 16.5 cm from exit slot of primary collimator; cross marks position of laser spot
scatter experiments were followed by a set of transmission images of the phantoms and of some bone fragments in a chicken breast.

The parameters for the beam and the geometry are given in Table 7.1, the dimensions of the polystyrene phantoms are given in Table 7.2, and the masses of the bone fragments used in each experiment are given in Table 7.3.

Table 7.1: Beam and Geometry Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expt no.</td>
<td>1 2 3 4 5 6</td>
</tr>
<tr>
<td>Beam voltage, kVp</td>
<td>140</td>
</tr>
<tr>
<td>Beam current, mA</td>
<td>2 7 6 7 6 6</td>
</tr>
<tr>
<td>Sample time, s</td>
<td>30 120 100 120 120 120</td>
</tr>
<tr>
<td>Height of beam (Yi), cm</td>
<td>2.1</td>
</tr>
<tr>
<td>Height of detector (Yd), cm</td>
<td>2.5</td>
</tr>
<tr>
<td>Width of incident beam (Wi), cm</td>
<td>0.2 0.1 0.1 0.1 0.1 0.1</td>
</tr>
<tr>
<td>Width of detector (Wd), cm</td>
<td>0.3 0.14 0.14 0.14 0.14 0.14</td>
</tr>
<tr>
<td>Sample - collimator distance (Ra), cm</td>
<td>21.3</td>
</tr>
<tr>
<td>Detector - collimator distance (Rd), cm</td>
<td>4.5</td>
</tr>
<tr>
<td>Width of collimator slot (h), cm</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 7.2: Dimensions of polystyrene phantoms

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value, cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expt no.</td>
<td>1 2 3</td>
</tr>
<tr>
<td>Width of sample (Wa)</td>
<td>5.5</td>
</tr>
<tr>
<td>Length of sample (La)</td>
<td>5.5</td>
</tr>
<tr>
<td>Depth of sample cover (Da)</td>
<td>0.11</td>
</tr>
<tr>
<td>Depth of sample bulk (Db)</td>
<td>0.56</td>
</tr>
<tr>
<td>Width of slot (WB)</td>
<td>0.2</td>
</tr>
<tr>
<td>Length of slot (LB)</td>
<td>2.2</td>
</tr>
<tr>
<td>Depth of slot (Db)</td>
<td>0.1 0.1 0.2</td>
</tr>
<tr>
<td>Volume of slot, cm³</td>
<td>0.043 0.043 0.086</td>
</tr>
</tbody>
</table>

In each experiment, the sample was mounted on a perspex back plate 0.5 cm thick. The z position of the sample (see Fig. 7.7) was fixed, the y position was adjusted such that the centres of the bone and the incident beam were the same. The sample was then scanned in the x direction, with measurements of the backscatter being made at regular intervals. These scans were repeated for different values of z.
Table 7.3: Masses of bone fragments

<table>
<thead>
<tr>
<th>Expt no.</th>
<th>Bone mass, g</th>
<th>Estimated fill fraction</th>
<th>Estimated slot density, g cm⁻³</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(air only)</td>
<td>100%</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>0.036</td>
<td>63%</td>
<td>1.20</td>
</tr>
<tr>
<td>3</td>
<td>0.053</td>
<td>47%</td>
<td>1.15</td>
</tr>
<tr>
<td>4</td>
<td>0.060</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.031</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.020</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The nominal positions of the bone fragments were determined, in the \( x \) and \( y \) directions, by aligning the bone with the laser spot. The \( z \) position, i.e. that of the sample at which the bone was in the scatter voxel, was found as follows. A lead surface was placed on the front of a polystyrene sample, and the backscatter was measured while the sample was moved in the \( z \) direction. The attenuation coefficient of lead is so large that most of the scatter comes from a layer only 0.01 cm deep, so the peak scatter count accurately showed the point at which the surface intersected the scatter voxel. The position at which the bone would intersect this position was inferred by subtracting the known thicknesses of lead and polystyrene, giving a \( z \) position accurate to within about 0.05 cm. The \( z \) position of the bone was more difficult to determine in the meat samples, because the bone was barely visible when inside the flesh, and the thickness of the meat could only be estimated to within 0.5 cm. Location in the \( z \) direction consisted of searching for the position of the meat surface, at which the scatter would fall towards zero, and subtracting the estimated depth of the bone, giving an accuracy of about 0.2 cm.

The FWHM of the lead-surface scatter was used as a direct measurement of the depth resolution, which was found to agree with the geometric calculations to within 0.01 cm.

The same precautionary measurements as in Section 6.3.4 were taken during the experiments: calibration with a \(^{241}\text{Am}\) source; calibration readings to correct for drift; noise spectra; ion chamber readings; and temperature measurements.
7.2 Detection of Bone Fragments: Results

7.2.1 Densities of chicken bone and meat

The results for the densities of chicken bone are shown in Table 7.4 and those for breast meat are shown in Table 7.5.

<table>
<thead>
<tr>
<th>Chicken no.</th>
<th>bone weight (g)</th>
<th>bone volume (cm³)</th>
<th>bone density (g cm⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1750</td>
<td>0.1408</td>
<td>1.24</td>
</tr>
<tr>
<td>1</td>
<td>0.1679</td>
<td>0.1383</td>
<td>1.21</td>
</tr>
<tr>
<td>2</td>
<td>0.1427</td>
<td>0.1025</td>
<td>1.39</td>
</tr>
<tr>
<td>2</td>
<td>0.1760</td>
<td>0.1306</td>
<td>1.35</td>
</tr>
<tr>
<td>3</td>
<td>0.1787</td>
<td>0.1370</td>
<td>1.30</td>
</tr>
<tr>
<td>3</td>
<td>0.2041</td>
<td>0.1599</td>
<td>1.28</td>
</tr>
<tr>
<td>4</td>
<td>0.1775</td>
<td>0.1365</td>
<td>1.30</td>
</tr>
<tr>
<td>4</td>
<td>0.1618</td>
<td>0.1235</td>
<td>1.31</td>
</tr>
<tr>
<td>5</td>
<td>0.1858</td>
<td>0.1363</td>
<td>1.36</td>
</tr>
<tr>
<td>5</td>
<td>0.2295</td>
<td>0.1770</td>
<td>1.30</td>
</tr>
<tr>
<td>6</td>
<td>0.1718</td>
<td>0.1234</td>
<td>1.39</td>
</tr>
<tr>
<td>6</td>
<td>0.2061</td>
<td>0.1483</td>
<td>1.39</td>
</tr>
<tr>
<td>7</td>
<td>0.2394</td>
<td>0.1818</td>
<td>1.32</td>
</tr>
<tr>
<td>7</td>
<td>0.1547</td>
<td>0.1161</td>
<td>1.33</td>
</tr>
<tr>
<td>8</td>
<td>0.1758</td>
<td>0.1320</td>
<td>1.33</td>
</tr>
<tr>
<td>8</td>
<td>0.1966</td>
<td>0.1457</td>
<td>1.35</td>
</tr>
</tbody>
</table>

**mean** 1.32

**s.d.** 0.05

7.2.2 Calibration readings

The calibration readings were treated in the same way as those in the glass detection experiments (see Figs. 6.9 - 6.17). The new calibrations drifted in a very similar way, and it would be tedious to display them at this point. As before, the drift was accounted for by linear regression, and the residuals were consistent with Poisson errors. The calibrations were particularly important in these experiments because they showed that repeatable readings were possible in a real meat sample.
Table 7.5: Densities of chicken flesh

<table>
<thead>
<tr>
<th>sample no.</th>
<th>weight (g)</th>
<th>volume (cm³)</th>
<th>density (g cm⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>33.329</td>
<td>31.133</td>
<td>1.07</td>
</tr>
<tr>
<td>2</td>
<td>33.227</td>
<td>30.976</td>
<td>1.07</td>
</tr>
<tr>
<td>3</td>
<td>37.274</td>
<td>34.811</td>
<td>1.07</td>
</tr>
<tr>
<td>4</td>
<td>26.527</td>
<td>24.774</td>
<td>1.07</td>
</tr>
<tr>
<td>5</td>
<td>25.103</td>
<td>23.412</td>
<td>1.07</td>
</tr>
<tr>
<td>6</td>
<td>18.566</td>
<td>17.348</td>
<td>1.07</td>
</tr>
<tr>
<td>7</td>
<td>27.964</td>
<td>26.352</td>
<td>1.06</td>
</tr>
<tr>
<td>8</td>
<td>20.466</td>
<td>19.228</td>
<td>1.06</td>
</tr>
<tr>
<td>9</td>
<td>31.021</td>
<td>29.450</td>
<td>1.05</td>
</tr>
<tr>
<td>10</td>
<td>23.290</td>
<td>21.795</td>
<td>1.07</td>
</tr>
</tbody>
</table>

mean: 1.07  
s.d.: 0.007

7.2.3 Windowed Counts

For each scattered spectrum, the total counts were divided into two energy windows: a low-energy window from 10-40 keV, and a high-energy window from 40-95 keV. Examination of the spectra showed that the bone fragments showed a positive contrast against the polystyrene above 40 keV, but only a small or negative contrast below 40 keV. The plots of these windowed counts against sample position are shown in Figs. 7.11 - 7.16. In every experiment, the nominal position of the slot or the bone was at \((z, x) = (0, 0)\).

The counts from experiment 1, the air-filled slot phantom, are shown in Fig. 7.11. The plot shows a single scan, with the slot passing through the scatter voxel at \((0, 0)\). A large negative contrast was obtained, due to the absence of scattering material in the slot. The counts did not fall to zero at the minimum, showing that there was a significant number of counts originating from outside the slot. This was partly due to multiple scatter, and partly due to the resolution of the scatter voxel being larger than the depth of the slot. The resolution was reduced to 0.1 cm in the subsequent experiments.

The counts from experiments 2 and 3, the bone-polystyrene phantoms, are shown in Figs. 7.12 - 7.13. Each line shows a single scan of the sample in the \(x\) direction, with the depth \(z\) fixed. For \(z\) greater than zero, the scatter voxel was ‘in front of’ the bone (nearer to the surface); for \(z\) less than zero, it was ‘behind’ the bone. The charts can be visualised as a scan of the scatter voxel through the sample, with the surface at the top, the deepest scan at the bottom, and the bone near the centre. Offsets have been added to each line (see figure captions) in order to make the charts readable.

The results were qualitatively similar in each experiment: the counts in the high-
Figure 7.11: Experiment 1, air-polystyrene phantom, detected counts; (a) 10-40 keV window; (b) 40-95 keV window
Figure 7.12: Experiment 2, bone-polystyrene phantom, detected counts; (a) 10-40 keV window; (b) 40-95 keV window; 

- , $z = -0.20$, offset = 0; 
- , $z = -0.15$, offset = 3000; 
- , $z = -0.10$, offset = 6000; 
- , $z = -0.05$, offset = 9000; 
- , $z = 0.00$, offset = 12000; 
- , $z = 0.05$, offset = 15000; 
- , $z = 0.10$, offset = 18000; 
- , $z = 0.15$, offset = 28000; 
- , $z = 0.20$, offset = 53000; 
- , $z = 0.25$, offset = 68000.
Figure 7.13: Experiment 3, bone-polystyrene phantom, detected counts; (a) 10-40 keV window; (b) 40-95 keV window; ––– , $z = -0.20$, offset = 0; ––– , $z = -0.10$, offset = 1000; ––– , $z = 0.0$, offset = 2000; ––– , $z = 0.10$, offset = 7000; ––– , $z = 0.15$, offset = 12000
energy window produced a peak close to the nominal bone position. The counts in the low-energy window showed a much smaller corresponding peak, but also showed attenuation effects. The clearest example of this was seen in the $z = -0.20$ scan of Fig. 7.12(a), in which two attenuation dips were observed. Referring back to Fig. 7.1, these were caused by the bone passing across the incident path AB, and then through the exit path BC. As $z$ increased, the incident dip remained directly beneath the bone, and the exit dip converged diagonally towards it, as expected from the geometry. The 10-40 keV window was, in effect, viewing the bone by the transscatter method of Ong et al. (1994) (see Section 1.1.5). Experiment 3 produced a much stronger Compton peak in Fig. 7.12(b) than that of experiment 2 in Fig. 7.13(b), due to the greater mass of bone in the slot. In experiment 3, the exit dip was much stronger than the incident dip; this was probably due to a diagonal alignment of the bone fragments in the sample slot.

The results of the bone/meat experiments 4-6 are shown in Figs. 7.14 - 7.16. In the 40-95 keV window, Compton peaks were observed, as in the phantom experiments, but were also adjacent to dips in the counts. The reason for these dips was not clear, but they might have been due to surface irregularities or air gaps caused when the meat was cut to insert the bone. The peaks were not as clearly defined as in the polystyrene phantoms.
Figure 7.14: Experiment 4, bone-meat sample, detected counts; (a) 10-40 keV window; (b) 40-95 keV window; — , z = 0.20, offset = 0; — , z = 0.35, offset = 3000; — , z = 0.45, offset = 6000; — , z = 0.50, offset = 9000; — , z = 0.60, offset = 12000; — , z = 0.65, offset = 15000; — , z = 0.10, offset = 18000
Figure 7.15: Experiment 5, bone-meat sample, detected counts; (a) 10-40 keV window; (b) 40-95 keV window; — , z = 0.0, offset = 0; — , z = 0.10, offset = 3000; — , z = 0.15, offset = 7000; — , z = 0.20, offset = 9000; — , z = 0.25, offset = 11000
Figure 7.16: Experiment 6, bone-meat sample, detected counts; (a) 10-40 keV window; (b) 40-95 keV window; — , $z = 0.20$, offset = 0; — , $z = 0.35$, offset = 0; — , $z = 0.45$, offset = 2000; — , $z = 0.50$, offset = 4000; — , $z = 0.60$, offset = 7000; — , $z = 0.65$, offset = 10000; — , $z = 0.10$, offset = 13000
7.2.4 Mean Energies

The mean energies of the spectra were calculated, and are shown in Fig. 7.17 and Figs. 7.18 - 7.22. The mean energies from experiment 1, shown in Fig. 7.17, were initially surprising, because the mean increased, rather than decreased, over the slot. However, because the slot was filled with air, the spectrum was that of the remaining photons, which had not scattered from the slot. These would have included multiple scatter, with longer path lengths than the direct scatter, leading to a higher average energy.

![Graph showing mean energy of spectrum across different x coordinates](image)

**Figure 7.17: Experiment 1, air-polystyrene phantom, mean energy of spectrum**

All the experiments with chicken bones showed strong positive contrasts in the mean energy. Double peaks were seen when the scatter voxel was behind the bone, due to attenuation through the bone on the incident and exit paths. The peaks converged on the bone position as the depth was changed, as with the attenuation dips in the low-energy counts (see Section 7.2.3).

The mean energy was useful in resolving ambiguities such as the unexpected dips in experiments 4-6. These were all associated with increases in the mean energy, indicating that they were due to increased attenuation above the bone, rather than a void in the scattering.
Figure 7.18: Experiment 2, bone-polystyrene phantom, mean energy of spectrum;

- - , z = -0.20, offset = 0; — — , z = -0.15, offset = -0.8; — — — , z = -0.10, offset = -1.6; — — — , z = -0.05, offset = -2.4; — — — , z = 0.00, offset = -3.2; — — — , z = 0.05, offset = -4.0; — — — , z = 0.10, offset = -4.8; — — — , z = 0.15, offset = -5.6; -- -- -- , z = 0.20, offset = -9.4; -- -- -- , z = 0.25, offset = -20.2
Figure 7.19: Experiment 3, bone-polystyrene phantom, mean energy of spectrum; 

- , $z = -0.20$, offset = 0; 
- , $z = -0.10$, offset = -1.5; 
- , $z = 0.0$, offset = -1.5; 
- , $z = 0.10$, offset = -1.5; 
- , $z = 0.15$, offset = -1.5
Figure 7.20: Experiment 4, bone-meat sample, mean energy of spectrum; — , $z = 0.20$, offset = 0; ———, $z = 0.35$, offset = -0.8; ———, $z = 0.45$, offset = -1.6; ——, $z = 0.50$, offset = -2.4; ———, $z = 0.60$, offset = -3.2; ——, $z = 0.65$, offset = -4.0; —— ——, $z = 0.10$, offset = -4.8
Figure 7.21: Experiment 5, bone-meat sample, mean energy of spectrum; — , z = 0.0, offset = 0; —— , z = 0.10, offset = -0.8; —— , z = 0.15, offset = -1.6; —— , z = 0.20, offset = -2.4; —— , z = 0.25, offset = -3.2
Figure 7.22: Experiment 6, bone-meat sample, mean energy of spectrum; , z = 0.20, offset = 0; , z = 0.35, offset = 0; , z = 0.45, offset = 0; , z = 0.50, offset = -0.1; , z = 0.60, offset = -0.2; , z = 0.65, offset = -0.4; , z = 0.10, offset = -0.4
7.2.5 Contrasts and Signal-to-Noise Ratios

The important factors in measuring the detectability of the bone fragments were the contrast and the signal-to-noise ratio (SNR). The contrast was calculated for each scan by locating the maximum deviation from the background level, and taking the ratio of the difference to the background at that point. The background level was a linear regression through the readings which were far enough from the bone to be unaffected by it. The tabulated contrasts were the maxima over all the scans in each experiment. The SNR was the ratio of the difference at the maximum-contrast point to the Poisson error in the background count. The SNR depended on the number of photon counts, but the experiments were performed with various combinations of source current and sampling times. Therefore, to enable comparisons between experiments with different source currents and sampling times, the SNR's were normalised to what could have been attained with a 30 mA source and a 1 s sampling time, as in Section 5.4 and Section 6.4.

The contrasts and SNR's for the 10-40 keV counts are shown in Table 7.6. These were the largest negative contrasts, because the attenuation effects were of more interest in this energy range than the small positive contrasts. The contrasts increased with bone mass, and similar-sized bones gave similar contrasts in the polystyrene and meat samples; that is, experiments 2 and 5 produced similar results, as did experiments 3 and 4. All the normalised SNR's were above or near the detection threshold value of 7.4. A regression through the values indicated that fragments of approximately 25 mg and above were detectable in this mode at 30 mA s.

<table>
<thead>
<tr>
<th>Expt no.</th>
<th>mass of bone, g</th>
<th>z, cm</th>
<th>x, cm</th>
<th>Contrast, %</th>
<th>SNR</th>
<th>Normalised SNR at 30 mAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(air)</td>
<td>0.0</td>
<td>0.0</td>
<td>-38.5</td>
<td>64.0</td>
<td>32.0</td>
</tr>
<tr>
<td>2</td>
<td>0.036</td>
<td>-0.05</td>
<td>0.0</td>
<td>-18.7</td>
<td>38.1</td>
<td>7.2</td>
</tr>
<tr>
<td>3</td>
<td>0.053</td>
<td>-0.1</td>
<td>0.1</td>
<td>-25.8</td>
<td>52.4</td>
<td>11.7</td>
</tr>
<tr>
<td>4</td>
<td>0.060</td>
<td>0.45</td>
<td>-0.15</td>
<td>-17.7</td>
<td>38.2</td>
<td>7.2</td>
</tr>
<tr>
<td>5</td>
<td>0.031</td>
<td>0.15</td>
<td>0.0</td>
<td>-20.5</td>
<td>46.6</td>
<td>9.5</td>
</tr>
<tr>
<td>6</td>
<td>0.020</td>
<td>-0.2</td>
<td>-0.2</td>
<td>-13.1</td>
<td>28.4</td>
<td>5.8</td>
</tr>
</tbody>
</table>

The results for the 40-95 keV counts are shown in Table 7.7, where the maximum positive contrasts have been used. The contrasts ranged from 4.2% to 7.0 %, and the SNR's ranged from 1.6 to 2.5, except for experiment 3, which produced a contrast of 14.7%, and an SNR of 5.6. The contrast in experiment 2 was less than the 14% between the slot density and that of the polystyrene (see Table 7.3), but the contrast in experiment 3 was unexpectedly high, given that the density contrast with the polystyrene was only 10%. The SNR's were above the detection
threshold of 7.4 in the experiments, but were not large enough to be detectable at 30 mA s, ranging from 1.6 to 5.6. Given that the SNR was proportional to the square root of the time, it was estimated that the order of 10 seconds per reading, at 30 mA would be required to detect a 60 mg bone fragment in this energy range.

Table 7.7: Contrast and SNR of bone fragments: 40-95 keV window

<table>
<thead>
<tr>
<th>Expt no.</th>
<th>mass of bone, g</th>
<th>z, cm</th>
<th>x, cm</th>
<th>Contrast, %</th>
<th>SNR</th>
<th>Normalised SNR at 30 mAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (air)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-39.1</td>
<td>51.1</td>
<td>25.6</td>
</tr>
<tr>
<td>2</td>
<td>0.036</td>
<td>0.05</td>
<td>0.0</td>
<td>7.0</td>
<td>10.6</td>
<td>2.0</td>
</tr>
<tr>
<td>3</td>
<td>0.053</td>
<td>0.0</td>
<td>0.0</td>
<td>14.7</td>
<td>24.9</td>
<td>5.6</td>
</tr>
<tr>
<td>4</td>
<td>0.060</td>
<td>0.6</td>
<td>-0.15</td>
<td>5.4</td>
<td>10.8</td>
<td>2.0</td>
</tr>
<tr>
<td>5</td>
<td>0.031</td>
<td>0.25</td>
<td>-0.1</td>
<td>6.1</td>
<td>12.1</td>
<td>2.5</td>
</tr>
<tr>
<td>6</td>
<td>0.020</td>
<td>-0.25</td>
<td>-0.3</td>
<td>4.2</td>
<td>7.8</td>
<td>1.6</td>
</tr>
</tbody>
</table>

The contrasts and SNR's obtained from the mean spectral energy are shown in Table 7.8. The contrasts ranged from 1.7% to 6.7%, and the SNR's from 3.0 to 11.0 at 30 mA s. These were less than those obtained from the 10-40 keV counts, and more than those from the 40-95 keV counts.

Table 7.8: Contrast and SNR of bone fragments: mean spectral energy

<table>
<thead>
<tr>
<th>Expt no.</th>
<th>mass of bone, g</th>
<th>z, cm</th>
<th>x, cm</th>
<th>Contrast, %</th>
<th>SNR</th>
<th>Normalised SNR at 30 mAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (air)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>2.9</td>
<td>14.9</td>
<td>7.4</td>
</tr>
<tr>
<td>2</td>
<td>0.036</td>
<td>-0.05</td>
<td>0.0</td>
<td>4.3</td>
<td>31.9</td>
<td>6.0</td>
</tr>
<tr>
<td>3</td>
<td>0.053</td>
<td>-0.1</td>
<td>0.1</td>
<td>6.7</td>
<td>49.2</td>
<td>11.0</td>
</tr>
<tr>
<td>4</td>
<td>0.060</td>
<td>0.35</td>
<td>0.0</td>
<td>3.1</td>
<td>25.9</td>
<td>4.9</td>
</tr>
<tr>
<td>5</td>
<td>0.031</td>
<td>0.15</td>
<td>0.0</td>
<td>2.8</td>
<td>24.6</td>
<td>5.0</td>
</tr>
<tr>
<td>6</td>
<td>0.020</td>
<td>-0.3</td>
<td>-0.25</td>
<td>1.7</td>
<td>14.4</td>
<td>3.0</td>
</tr>
</tbody>
</table>

The SNR's obtained from the 40-95 keV Compton scatter window were on average about one-third of those obtained from the 10-40 keV attenuation window, and about half of those obtained from the mean energy of the spectrum. The signal-to-noise ratios showed that bone fragments of 25 mg or more were detectable in the backscatter mode, given one second per reading at 30 mA, but that this was mainly due to the low-energy attenuation properties of the bone, rather than the high-energy Compton scattering properties. Given that the attenuation properties were the most important, further work was required to determine whether there was any advantage in the backscatter geometry over conventional transmission imaging.
The maximum contrast in the 10-40 keV window tended to occur about 0.1 cm behind the maximum in the 40-95 keV.

**7.2.6 Slope and intercept of spectrum**

In Section 3.3.2, a method was discussed for plotting a regression line through the scattered spectrum in such a way that the intercept depended on the properties of the scatter voxel, and the slope depended on the properties of the attenuation path. This decomposition of the spectrum was performed for the scattered spectra. An example of the fit is shown in Fig. 7.23, in which a typical scattered spectrum was normalised by division by the mean incident spectrum (averaged over all the spectra for experiment 14), and a (weighted) regression line was fitted through a plot of the log-spectrum versus the inverse third power of the energy. The plot is shown against energy, for clarity, and the fit appeared to be good.

![Figure 7.23: Fitting of model to scattered spectrum; the spectrum has been normalised by division by the mean incident spectrum; _____, logarithm of spectrum; _____, fitted model](image)

The results of applying this to experiment 2 are shown in Fig. 7.24. Comparison with Fig. 7.12 showed that the slope gave very similar information to the counts in the 10-40 keV window, and the intercept gave information very similar to those in the 40-95 keV window. The contrasts and SNR's were slightly larger for Fig. 7.24(b) than for Fig. 7.12(b). This was a typical result for all the experiments in this section. The slightly greater contrast hardly justified the
greater complexity of the technique. The choice of the two energy windows in
this geometry closely matched the more complicated decomposition

### 7.2.7 Transmission Images

For comparison with the scatter measurements, transmission images were taken of
the polystyrene phantoms, shown in Fig. 7.25, and of a chicken breast containing
bone fragments of various sizes, shown in Fig. 7.26. The masses of the bones
and the image contrasts obtained are given in Table 7.9 In general, the more
massive bones produced the greatest contrasts, although the smallest bone in
Fig. 7.26 produced contrasts ranging from -8% to -29% due to the variation in
thickness. Comparing the transmission contrasts against those obtained from the
10-40 keV counts for similar-sized bones (Section 7.2.3), the latter were greater
by a factor of up to two, because the scatter measurements traversed the bone
twice: once in the incident direction, and again in the exit direction (AB and BC
in Fig. 7.1).

<table>
<thead>
<tr>
<th>Bone</th>
<th>Mass, mg</th>
<th>Contrast, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 7.25 (a)</td>
<td>36</td>
<td>-10</td>
</tr>
<tr>
<td>Fig. 7.25 (b)</td>
<td>53</td>
<td>-18</td>
</tr>
<tr>
<td>Fig. 7.26 (top)</td>
<td>575</td>
<td>-25</td>
</tr>
<tr>
<td>Fig. 7.26 (second from top)</td>
<td>64</td>
<td>-15</td>
</tr>
<tr>
<td>Fig. 7.26 (third from top)</td>
<td>45</td>
<td>-8</td>
</tr>
<tr>
<td>Fig. 7.26 (bottom)</td>
<td>34</td>
<td>-8 (-29)</td>
</tr>
</tbody>
</table>
Figure 7.24: Experiment 2, bone-polystyrene phantom, spectral shape; (a) slope of spectrum; ——, \( z = -0.20 \), offset = 0; ——, \( z = -0.15 \), offset = 1000; ——, \( z = -0.10 \), offset = 2000; ——, \( z = -0.05 \), offset = 3000; ——, \( z = 0.00 \), offset = 4000; ——, \( z = 0.05 \), offset = 5000; ——, \( z = 0.10 \), offset = 6000; ——, \( z = 0.15 \), offset = 7000; ——, \( z = 0.20 \), offset = 8000; ——, \( z = 0.25 \), offset = 10500; (b) intercept of spectrum; ——, \( z = -0.20 \), offset = 0; ——, \( z = -0.15 \), offset = 0; ——, \( z = -0.10 \), offset = 0.01; ——, \( z = -0.05 \), offset = 0.05; ——, \( z = 0.00 \), offset = 0.09; ——, \( z = 0.05 \), offset = 0.13; ——, \( z = 0.10 \), offset = 0.18; ——, \( z = 0.15 \), offset = 0.43; ——, \( z = 0.20 \), offset = 1.28; ——, \( z = 0.25 \), offset = 2.58;
Figure 7.25: Transmission images of phantoms; (a) Experiment 2 phantom, 60 kVp; (b) Experiment 3 phantom, 40 kVp
Figure 7.26: Transmission images of bones in chicken breast, 40 kVp; (top) half clavicle, 575 mg; (second from top) fragment, 64 mg; (third from top) fragment, 45 mg; (bottom) fragment, 34 mg
Chapter 8

Conclusions
8.1 Conclusions

This section consists of the following.

- Detailed conclusions drawn from each section of the thesis are given in Section 8.1.1.
- A summary of the thesis and its main conclusions can be found in Section 8.1.2.
- A description of potential further-work arising from the thesis is given in Section 8.1.3.

8.1.1 Conclusions by section

Modelling

A simple model was developed for the number of scattered counts detected from a sample, and the time required to detect a foreign body in the sample.

It was shown that a signal-to-noise ratio of at least 7.4 was required for a foreign body to be considered detectable in a food inspection application.

The signal-to-noise ratio was not strongly affected by the choice of operator for extracting foreign-body peaks from scan data, but a correction factor was necessary when adding standard deviations to logarithmic data.

The number of counts detected from a scatter voxel was predicted to be proportional to the inverse sixth power of the linear voxel dimension.

The Compton shift in the scattered energy causes a reduction in the number of scattered photons which is no greater than 10% above 40 keV, for a 5 cm path length through water.

The model was used to predict the minimum foreign body size detectable in water, given one second in which to scan the sample. For glass, a 0.13 cm cube was predicted to be detectable in a 2 cm cube of water, and a 0.46 cm cube in a 10 cm cube of water. The predictions for air-filled voids were similar to those for glass. For plastics, only much larger cubes were detectable, for example, 0.80 cm of perspex or 1.19 cm of polystyrene in 10 cm of water.

Stainless steel was predicted to be similar to glass in terms of detectability, but the validity of the model was uncertain for such a highly-attenuating material, and further work was required to confirm this conclusion.

An equation was derived for the effect of multiple scatter on the detection time. The detection time was predicted to be inversely proportional to the square root of one minus the multiple scatter fraction.
In homogeneous samples, Compton scatter was predicted to give lower signal-to-noise ratios (and hence longer detection times) than an equivalent transmission measurement, because of the low number of photon counts. However, a simple model predicted that Compton scatter could perform better than transmission imaging in inhomogeneous samples.

**Simulation of void detection**

The use of Compton scatter photons for the detection of voids in polystyrene spheres was investigated by Monte Carlo simulation. The contrast, signal to noise ratio, multiple scatter fraction and minimum detection time were measured for various combinations of beam energy and sample dimensions.

The contrast obtained from scattering was much greater than that which was predicted for transmission imaging. However, the signal-to-noise ratio was worse because of the low number of counts.

The Compton scatter contrast was determined by the size of the void relative to the beam size. The incident energy and sample size had little or no effect.

The signal-to-noise ratio for Compton scatter measurements decreased with increasing sample size and decreasing void size. An incident energy of 50 keV gave much higher signal-to-noise ratios than 20 keV, but there was little further improvement from a 100 keV beam.

The minimum time required to detect a void was calculated, assuming the use of a typical x-ray source, that all the photons scattered at 90 degrees were captured, and that the whole sample volume was inspected. The voids measuring 0.5 and 1.0 cm in diameter were predicted to be detectable in 1 s in the 4 cm and 8 cm diameter samples, but the 0.2 cm void was not detectable in 1 s in any of the samples, and none of the voids were detectable in 1 s in the 16 cm sample.

When the beam was smaller than or equal to the void size, The effect of increasing the beam size was to greatly reduce the predicted detection time, but the detection time was independent of the beam size when the beam was larger than the void.

The multiple scatter fraction of the signal was no greater than 35%, and had only a small effect on the time required to detect the voids.

**Simulations of glass in water**

Monte Carlo simulations were carried out to investigate the detection of foreign bodies. The nominal geometry which was used was a 0.4 cm cube of glass in a 5.0 cm cube of water, scanned with a 0.4 cm square incident beam. The source was 140kVp and 30 mA at a distance of 20cm.
The sample size, foreign body size and beam size were varied about the nominal geometry in order to investigate the effects on the detectability of the glass.

When the sample size increased in the range 2 - 10 cm, the multiple scatter fraction increased from 25% to 55%, and the contrast decreased from 90% to 60%, entirely due to the multiple scatter. The estimated scanning rate, i.e. the maximum number of samples in which the glass was detectable, per second per steradian of detector, decreased from about 9000 to 10.

When the foreign body size increased in the range 0.1 - 0.8 cm, the contrast and the scanning rate both increased until the glass was the same size as the beam, but was constant for sizes greater than or equal to the beam.

When the beam size increased in the range 0.1 - 0.8 cm, the detected counts increased as the fourth power of the beam size. The signal-to-noise ratio was maximum when the beam size and foreign body size were equal. The contrast and scanning rate increased until the beam size was equal to the foreign body size, but remained constant for larger beam sizes.

The results for the varying beam size and foreign body size, taken together, suggested that the scanning rate was a function of the minimum of the beam size and the foreign body size.

In order to maximise the scanning rate for foreign body detection, the beam size needed to be greater than or equal to the size of the smallest expected foreign body in the sample.

The simulations predicted that, given a 140 kVp, 30 mA source at 20 cm distance, the maximum reasonable solid angle of detector and a one second scan time for the whole sample, a 0.4 cm cube of glass was detectable in a 7.8 cm cube of water, and a 0.27 cm cube was detectable in a 5 cm cube of water. The time required increased as the 5.2th power of the sample size, and the inverse 6.8th power of the foreign body size. The time required was very sensitive to the size of the glass: 20 minutes were required to detect a 0.1 cm cube of glass in a 5 cm sample.

The simulation results were compared with the predictions of analytical modelling. The number of detected counts predicted by analytical modelling was 18% higher than the simulation of the nominal geometry. Some of the disagreement was due to the Compton shift in energy, which was not considered in the analytical model, and the remainder was probably due to errors introduced by approximating the incident spectrum as a single energy.

Experiments: glass in food (SRI)

Experiments were conducted which demonstrated the detection of glass in water, instant coffee and muesli from the Compton scattered photons. The contrasts
obtained for a 0.4 cm piece of glass in these media were respectively 50%, 270% and 260%.

The mean energy of the scattered photons was also used to detect glass in water. A contrast value of 900% was obtained for a 0.4 cm piece of glass in water. However the noise level of each mean energy measurement was approximately 12.8 times that of the corresponding photon count measurement, which made the mean energy less useful than the total count for detection. The high contrast obtained from the mean energy measurement suggested that this would be a useful technique for glass detection in the transmission mode, where the photon counts were not a limiting factor.

No detectable signal could be discerned above the noise for measurements of the standard deviation and skew of the spectra.

A model for the number of detected photons predicted the values obtained by experiment to within 15%.

The experimental contrast of glass in water was very low compared to the density contrast of the materials. This was mainly due to self-attenuation through the glass, and was caused by the low energy of the X-ray system.

**Experiments: glass in food (UCL)**

Experiments were performed to measure the scatter contrast of 0.2 and 0.4 cm cubes of glass in samples of water, coffee an muesli.

A small drift in the X-ray system was corrected for by linear regression, and the residuals were consistent with the Poisson error being the only significant source of random noise in the experiments.

Energy windows of 10-40 and 40-110 keV were used, and similar contrasts were observed in both.

The largest contrasts obtained were 47%, 422% and 364% for glass in water, coffee and muesli respectively. The contrasts were smaller for the 0.2 cm cube of glass than the 0.4 cm, probably due to a mismatch in the sizes or positions of the glass and the scatter voxel.

None of the contrasts obtained were as large as the density contrast between the materials.

Simulations of the experiments showed that approximately half the scatter from the water sample would have been multiple scatter, and that this was the main reason for the contrasts being lower than the density contrasts.

The use of the mean spectral energy for detection was examined, but contrasts were poor, and the signal-to-noise ratios were 6 to 16 times less than those obtained from the counts alone.
The contrasts obtained from the scatter measurements were much greater than transmission image contrasts.

The standard deviation of the grey levels in the transmission images was used as a measure of the sample inhomogeneity, but did not accurately predict the detectability of the glass cubes, because the grey level distribution was not Gaussian.

Further work was required to characterise the inhomogeneity of samples and its effects on scatter and transmission imaging.

The number of detected photons for 0.4 cm of glass in water was four times less than predicted by simulation. This disagreement was believed to have been due to the simulation failing to account for the finite size of the focal spot, and the resultant losses in the primary collimator.

Experiments: backscatter bone detection (UCL)

X-ray backscatter was used to detect near-surface fragments of clavicle, 2 cm long and 20-60 mg in weight, in polystyrene phantoms and in samples of breast meat.

The densities of fragments of chicken clavicle were 1.32 ± 0.05 g cm\(^{-3}\), and the densities of samples of chicken breast meat were 1.07 ± 0.007 g cm\(^{-3}\).

The detected counts in a 10-40 keV energy window showed little positive contrast, but strong negative contrasts due to attenuation. A characteristic double dip in the signal was observed when the scatter voxel was focused behind the bone, caused by the bone passing across the incident and exit paths of the beam.

In the 10-40 keV window, the contrasts were greater than those obtained in transmission images for bones of similar weight, because the scattered beam was able to pass through the bone twice.

The signal-to-noise ratios obtained in the 10-40 keV window were large enough to detect about 25 mg of bone with a 30 mA source at 1 reading per second.

Bones greater than 25 mg were detectable at 30 mAs using backscatter, but mainly by virtue of their attenuation contrast, rather than by Compton scatter contrast. However, the increased contrast of the Compton scatter measurement was a significant advantage over transmission imaging.

The detected counts in a 40-95 keV energy window showed peaks in the backscatter when the scan passed across a bone fragment. The contrasts were similar to those in transmission images, but the signal-to-noise ratios were not sufficient for the bones to be detectable with a 30 mA source at 1 reading per second. It was estimated that 30 mA for 10 seconds per reading would be required to detect 60 mg of bone in this energy range.
The mean spectral energy increased when the attenuation path passed through a bone fragment. The contrasts and signal-to-noise ratios were intermediate between those of the low- and high-energy windows.

The contrasts obtained from polystyrene phantoms were similar to those obtained from breast meat samples. However, the meat samples showed some additional sharp dips in the counts which may have been artefacts of the sample preparation.

Further work was required to develop differential operators to combine all the information available from the scans at different energies.

The use of two energy ranges yielded complementary information about the sample, and would be useful in samples of variable thickness, whether in scattering or dual-energy transmission.

A technique of decomposing the scattered spectrum into scatter and attenuation components gave similar results to those obtained from the counts in the 10-40 and 40-95 keV energy windows, but at the cost of much greater complexity.

The contrasts in this work were calculated only in the direction parallel to the sample surface, and neglected any contrast in the depth direction, in order to avoid confusion between Compton and attenuation effects. Improvements in the SNR's could be achieved by using a differential operator which combined the contrasts in both directions, and across the different energy windows.

Changes in the thickness of the meat make bones hard to detect in transmission images. Further work was required to assess how variations in the thickness would affect the scattering results. The use of two energy windows, or the mean energy, yielded information about scattering and attenuation, indicating that dual-energy measurements would be very useful in compensating for meat thickness, whether in a backscatter or a transmission mode.

8.1.2 Summary and Overall Conclusions

This thesis investigated the application of Compton scattering to the problem of detecting foreign bodies in food. The methods used were analytical modelling, Monte Carlo simulation and experiment.

A criterion was defined for the detectability of foreign bodies in samples, and an analytical model was developed for predicting the time required to scan samples for foreign bodies using Compton scatter. The model was used to predict the smallest cubes of air, glass, plastic and steel which were detectable in one second in water.

Simulations were performed, demonstrating the detection of voids in plastic and of glass in water, the former representing air-spaces in produce such as pears and potatoes, and the latter representing a glass in homogeneous food samples.
Experiments were performed, demonstrating the detection of glass in water, coffee and muesli.

Compton backscatter was applied to the important application area of bone detection in chicken meat.

Various quantities were predicted by the simulations and measured by the experiments, including the number of detected photons, the foreign body contrast and signal-to-noise ratio. The effect of varying some of the geometry parameters was investigated by simulation.

Simulations were used to predict the quantity of multiple scatter, and its effect on the detectability of foreign bodies.

Compton scatter inspection was compared with transmission imaging, and the effect of sample inhomogeneity on both inspection methods was modelled.

The shape of the scattered spectrum was investigated as a means of foreign body detection.

It was shown that a signal-to-noise ratio of at least 7.4 was required for a foreign body to be considered detectable in a food inspection application.

The analytical model was used to predict the minimum foreign body size detectable in water, given one second in which to scan the sample. For example, a 0.46 cm cube of glass or a 1.19 cm cube of polystyrene was predicted to be detectable in a 10 cm cube of water.

The time required to scan a whole sample varied approximately as the 7th power of the foreign body size, and the 5th power of the sample size.

For foreign body detection, the contrasts obtained from Compton scatter inspection were greater than those obtained from transmission images. However, Compton scatter inspection required longer measurement times because of the low number of photon counts.

When applied to whole samples, Compton scatter inspection was very slow compared to production line speeds in the food industry, even for high density contaminants such as glass, and given the most favourable assumptions regarding sources and detectors available at the time of writing.

There was potential for Compton scatter inspection in applications which did not require whole-sample scanning, such as surface inspection. There was also potential in the inspection of inhomogeneous samples.

It was shown that Compton backscatter could be used to detect bone fragments in the surface of chicken meat. Bones as small as 25 mg were detectable at the rate of one reading per second. Detection was mainly due to attenuation properties, but the contrast was up to twice as great as that obtained from transmission imaging.
A simple model suggested that Compton scatter could perform better than transmission imaging at detecting foreign bodies in inhomogeneous samples. However, further work was required to refine the model to predict the statistics of real samples.

The effect of multiple scatter was investigated using Monte Carlo simulation. The multiple scatter fraction varied from 25% to 55% for cubes of water ranging in size from 2 to 10 cm, but did not have a large effect on the time required to scan a sample.

The use of the shape of the scattered spectrum for foreign body detection was investigated. The results were mixed, with some experiments showing strong contrasts in the spectral shape, and others not. The mean spectral energy gave good contrasts and signal-to-noise ratios in the detection of chicken bones.

8.1.3 Further work

The analytical model, the simulations and the experiments were in reasonable agreement with each other. However, inaccuracies occurred in the predictions because of uncertainty in some of the input parameters. Further work is required in calculating the incident energy which is used to represent a polychromatic incident spectrum. Further work is also required in calculating the fluence rate of the incident beam, to take into account the effect of the focal spot and the primary collimator.

The analytical model was cautiously applied to the detection of stainless steel, but further work is required to confirm its validity when applied to such a strongly attenuating material.

It would be of interest to develop detection operators for foreign bodies which (a) utilise all of the neighbouring voxels in 3-D instead of only two in a linear scan, (b) utilise information from the scattered spectral shape, and (c) apply image analysis techniques to incorporate shape information, such as straight edges, into detection algorithms.

A simple model predicted that Compton scatter inspection could detect foreign bodies in inhomogeneous samples which could not be detected by transmission imaging. Further work is required, using analysis, simulation and experiment, to characterise the radiographic properties of inhomogeneous foods, particularly the statistical distribution of grey levels in transmission images. Comparison between Compton scatter inspection and transmission methods such as stereo or CT would also be required.

Further work is required in the application of Compton backscatter to chicken bone detection, using real samples from a processing line, in order to be free of any sample preparation artefacts. The ranges of size and surface depth for the
bone fragments would be measured, the effect of variations in the meat thickness on detection would be assessed, and methods of compensation for thickness variations would be proposed.

The two energy windows, 10-40 and 40-100 keV, in which the scatter was detected gave very similar results to each other when applied to 90 degree scatter from glass in water, but produced very different results from each other when applied to backscatter from bones in chicken. More work is required to explain why this should have been so, and to develop a model for selecting the best ranges for the energy windows.

Some experiments showed that useful detection contrast could be extracted from the mean energy of the scattered spectrum; others showed little or no contrast information in the spectral shape. Further work is required to predict the conditions under which the spectral shape would be useful.
Appendix A

Compton scattered energies
This section tabulates the energies of Compton scattered photons for various incident energies and scatter angles.

The incident spectra of the sources are shifted in energy by Compton scattering, according to Eqn. 2.8. The Compton shifted energies for photons in the incident energy range 10-105 keV are shown in Table A.1 for scatter angles of 45, 90 and 135 degrees. Table A.2 shows the converse: the incident energies which correspond to energies detected after Compton scattering at the same angles.

Table A.1: Compton scattered energies for scatter angles of 45, 90 and 135 degrees

<table>
<thead>
<tr>
<th>Incident energy, keV</th>
<th>Scattered energy, keV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>45 degrees</td>
</tr>
<tr>
<td>10</td>
<td>9.94</td>
</tr>
<tr>
<td>20</td>
<td>19.77</td>
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<td>30</td>
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<tr>
<td>40</td>
<td>39.1</td>
</tr>
<tr>
<td>50</td>
<td>48.61</td>
</tr>
<tr>
<td>60</td>
<td>58.01</td>
</tr>
<tr>
<td>70</td>
<td>67.3</td>
</tr>
<tr>
<td>80</td>
<td>76.49</td>
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<tr>
<td>90</td>
<td>85.59</td>
</tr>
<tr>
<td>100</td>
<td>94.58</td>
</tr>
<tr>
<td>110</td>
<td>103.48</td>
</tr>
<tr>
<td>120</td>
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<tr>
<td>130</td>
<td>120.99</td>
</tr>
<tr>
<td>140</td>
<td>129.6</td>
</tr>
</tbody>
</table>
Table A.2: Incident energies corresponding to energies detected after Compton scattering, for scatter angles of 45, 90 and 135 degrees

<table>
<thead>
<tr>
<th>Scattered energy, keV</th>
<th>Incident energy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>45 degrees</td>
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<tr>
<td>10</td>
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<td>40.94</td>
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<td>51.48</td>
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<td>72.93</td>
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<td>83.84</td>
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<tr>
<td>90</td>
<td>94.9</td>
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<tr>
<td>100</td>
<td>106.08</td>
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<td>110</td>
<td>117.4</td>
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<td>120</td>
<td>128.86</td>
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<tr>
<td>130</td>
<td>140.47</td>
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</tbody>
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Appendix B

Beam energies
This section calculates various mean energies and correction factors for the sources and geometries used in this work.

The simulation work in Section 5.1 used mono-energetic beams at 20, 50 and 100 keV.

The experimental work described in Section 6.1 used the SRI equipment described in Section 4.1 at 100 kVp, the scattering angle used was 90 degrees, and the detected energy window was 20-100 keV.

The experimental work described in Section 6.3 and Section 7.1 used the UCL equipment described in Section 4.2 at 140 kVp. In the glass detection experiments, the scatter angle was 90 degrees, and the detected energy windows were 10-40 and 40-110 keV. In the backscatter bone-detection experiments, the scatter angle was 135 degrees, and the detected energy windows were 10-40 and 40-95 keV.

The simulation work in Section 5.3 simulated the UCL source at 140 kVp and a 90 degree scatter angle, with a detected energy window of 40-110 keV. There was a small difference between the UCL source and the simulated source with respect to the filtration: the UCL source was filtered by the equivalent of 1mm Al (see Section 4.2), but the simulated spectrum was filtered by 1mm Be. The values tabulated in this section (see later) were calculated for the Be filtration. The only significant difference between the spectra was that the Al filtration reduced the calculated fluence rate in the 10-40 keV window to \(3 \times 10^{12} \text{s}^{-1}\text{mA}^{-1}\text{sr}^{-1}\).

The effect of the detector efficiency on the X-ray spectrum is shown in Fig. B.1 for the SRI source and detector. The incident spectrum was 100 kVp, calculated using the NIST CD-ROM described in Section 4.4. This spectrum was scattered at 90 degrees, calculated using Eqn. 2.8. Finally, the detected spectrum was calculated, which was the product of the scattered spectrum and the efficiency curve in Fig. 4.1. In the range 20-100 keV, the mean detected energy was 30 keV, and the mean efficiency was 8.0%.

The mean detected energies were calculated in the same way for all the sources, detectors and geometries used in the work. The mean detected energies and the corresponding incident energies (i.e. before Compton scatter) are shown in Table B.1.

The two correction factors \(\kappa_1\) and \(\kappa_2\), used in Eqn. 3.28 could now be calculated. The mean efficiency \(\kappa_1\) of the detectors was calculated as the ratio of the total detected to the total scattered over the appropriate energy window. The Klein-Nishina correction factor \(\kappa_2\) was calculated for each combination of energy and scattering angle. These are shown in Table B.2. Also listed in Table B.2 is the beam fluence rate, as calculated from the NIST CD-ROM (see Section 4.4).
Figure B.1: Spectrum of tungsten source at 100 kVp; (a) incident spectrum; (b) spectrum scattered at 90 degrees; (c) scattered spectrum detected by 0.03 cm silicon detector.
Table B.1: *Mean detected energy of beams*

<table>
<thead>
<tr>
<th>source</th>
<th>detected energy window, keV</th>
<th>mean detected energy</th>
<th>mean incident energy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>mono-energetic beams, 90 degree scatter</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mono 20 keV</td>
<td>-</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>mono 50 keV</td>
<td>-</td>
<td>46</td>
<td>50</td>
</tr>
<tr>
<td>mono 100 keV</td>
<td>-</td>
<td>84</td>
<td>100</td>
</tr>
<tr>
<td><strong>0.03 cm Si, 90 degree scatter</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100 kVp</td>
<td>20-100</td>
<td>30</td>
<td>32</td>
</tr>
<tr>
<td><strong>1.3 cm Ge, 90 degree scatter</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>140 kVp</td>
<td>10-40</td>
<td>24</td>
<td>25</td>
</tr>
<tr>
<td>140 kVp</td>
<td>40-110</td>
<td>59</td>
<td>67</td>
</tr>
<tr>
<td><strong>1.3 cm Ge, 135 degree scatter</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>140 kVp</td>
<td>10-40</td>
<td>24</td>
<td>26</td>
</tr>
<tr>
<td>140 kVp</td>
<td>40-95</td>
<td>56</td>
<td>69</td>
</tr>
</tbody>
</table>
Table B.2: Correction factors for beams

<table>
<thead>
<tr>
<th>source</th>
<th>detected energy window, keV</th>
<th>beam fluence rate ($F_1$), $s^{-1}mA^{-1}sr^{-1}$</th>
<th>detector efficiency ($\kappa_1$), percent</th>
<th>Klein-Nishina correction ($\kappa_2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>mono-energetic beams, 90 degree scatter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mono 20 keV</td>
<td>-</td>
<td>$5 \times 10^{12}$</td>
<td>100</td>
<td>0.75</td>
</tr>
<tr>
<td>mono 50 keV</td>
<td>-</td>
<td>$5 \times 10^{12}$</td>
<td>100</td>
<td>0.74</td>
</tr>
<tr>
<td>mono 100 keV</td>
<td>-</td>
<td>$5 \times 10^{12}$</td>
<td>100</td>
<td>0.73</td>
</tr>
<tr>
<td><strong>0.03 cm Si, 90 degree scatter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100 kV</td>
<td>20-100</td>
<td>$5 \times 10^{12}$</td>
<td>8.0</td>
<td>0.75</td>
</tr>
<tr>
<td><strong>1.3 cm Ge, 90 degree scatter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>140 kV</td>
<td>10-40</td>
<td>$6 \times 10^{12}$ *</td>
<td>100</td>
<td>0.75</td>
</tr>
<tr>
<td>140 kV</td>
<td>40-110</td>
<td>$5.16 \times 10^{12}$</td>
<td>100</td>
<td>0.74</td>
</tr>
<tr>
<td><strong>1.3 cm Ge, 135 degree scatter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>140 kV</td>
<td>10-40</td>
<td>$6 \times 10^{12}$ *</td>
<td>100</td>
<td>1.05</td>
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<tr>
<td>140 kV</td>
<td>40-95</td>
<td>$5 \times 10^{12}$ *</td>
<td>100</td>
<td>0.96</td>
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</table>

* $3 \times 10^{12}$ for 1mm Al filtration (*UCL source*)
Appendix C

Attenuation coefficients
This section lists the attenuation coefficients for water and glass in the range 10-150 keV. The coefficients were calculated using the XGAM program (NIST 1990) (see Section 4.4). The densities were taken from Tennent (1978) and the composition of crown glass was taken from Weast & Astle (1980) (see Section 5.3).

Table C.1: Attenuation coefficients of water
density: 1.0 g cm$^{-3}$

<table>
<thead>
<tr>
<th>Energy, keV</th>
<th>Compton scatter coefficient ($\sigma$), cm$^{-1}$</th>
<th>Photoelectric attenuation coefficient ($\tau$), cm$^{-1}$</th>
<th>Total attenuation coefficient ($\mu$), cm$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.155</td>
<td>4.94</td>
<td>5.33</td>
</tr>
<tr>
<td>20</td>
<td>0.177</td>
<td>0.544</td>
<td>0.81</td>
</tr>
<tr>
<td>25</td>
<td>0.181</td>
<td>0.264</td>
<td>0.508</td>
</tr>
<tr>
<td>26</td>
<td>0.182</td>
<td>0.232</td>
<td>0.473</td>
</tr>
<tr>
<td>30</td>
<td>0.183</td>
<td>0.146</td>
<td>0.376</td>
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<tr>
<td>32</td>
<td>0.183</td>
<td>0.118</td>
<td>0.343</td>
</tr>
<tr>
<td>40</td>
<td>0.183</td>
<td>0.0568</td>
<td>0.268</td>
</tr>
<tr>
<td>50</td>
<td>0.18</td>
<td>0.0272</td>
<td>0.227</td>
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<tr>
<td>60</td>
<td>0.177</td>
<td>0.0149</td>
<td>0.206</td>
</tr>
<tr>
<td>67</td>
<td>0.175</td>
<td>0.0104</td>
<td>0.196</td>
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<tr>
<td>69</td>
<td>0.174</td>
<td>0.00941</td>
<td>0.194</td>
</tr>
<tr>
<td>70</td>
<td>0.173</td>
<td>0.00897</td>
<td>0.193</td>
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<tr>
<td>80</td>
<td>0.17</td>
<td>0.00577</td>
<td>0.184</td>
</tr>
<tr>
<td>90</td>
<td>0.166</td>
<td>0.00391</td>
<td>0.177</td>
</tr>
<tr>
<td>100</td>
<td>0.163</td>
<td>0.00276</td>
<td>0.171</td>
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<tr>
<td>110</td>
<td>0.159</td>
<td>0.00202</td>
<td>0.166</td>
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<tr>
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<td>0.156</td>
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<td>0.161</td>
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<td>130</td>
<td>0.153</td>
<td>0.00117</td>
<td>0.157</td>
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<tr>
<td>140</td>
<td>0.15</td>
<td>0.000915</td>
<td>0.154</td>
</tr>
<tr>
<td>150</td>
<td>0.147</td>
<td>0.000731</td>
<td>0.151</td>
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</table>
Table C.2: Attenuation coefficients of crown glass

density: $2.6 \, \text{g cm}^{-3}$

<table>
<thead>
<tr>
<th>Energy, keV</th>
<th>Compton scatter coefficient ($\sigma$), cm$^{-1}$</th>
<th>Photoelectric attenuation coefficient ($\tau$), cm$^{-1}$</th>
<th>Total attenuation coefficient ($\mu$), cm$^{-1}$</th>
</tr>
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<tbody>
<tr>
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<td>0.3042 66.56</td>
<td>68.12</td>
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<tr>
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<td>0.377    8.398</td>
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<tr>
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<td>0.3926   4.238</td>
<td>4.966</td>
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<tr>
<td>26</td>
<td>0.3952   3.77</td>
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<td>30</td>
<td>0.4004   2.4154</td>
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</tr>
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<td>0.403    1.976</td>
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<td></td>
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<tr>
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<td></td>
</tr>
<tr>
<td>50</td>
<td>0.4056   0.4862</td>
<td>1.001</td>
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<td>0.6214</td>
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<td>0.4108</td>
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<td>0.3406   0.014768</td>
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Appendix D

Notation
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<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>A</td>
<td>atomic mass in grams</td>
<td>g mol⁻¹</td>
</tr>
<tr>
<td>a</td>
<td>dimension of collimator</td>
<td>cm</td>
</tr>
<tr>
<td>b</td>
<td>dimension of collimator</td>
<td>cm</td>
</tr>
<tr>
<td>C</td>
<td>contrast</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>depth or diameter</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>energy</td>
<td>keV</td>
</tr>
<tr>
<td>$F_i$</td>
<td>fluence rate of source</td>
<td>photons s⁻¹sr⁻¹mA⁻¹</td>
</tr>
<tr>
<td>f</td>
<td>natural logarithm of $N$ (log-count)</td>
<td>ln(photons)</td>
</tr>
<tr>
<td>$f_k$</td>
<td>value of $f$ at $k$ standard deviations from $f_{samp}$</td>
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</tr>
<tr>
<td>$f_m$</td>
<td>multiple scatter fraction</td>
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</tr>
<tr>
<td>$f_{out}$</td>
<td>fraction of counts outside nominal width</td>
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</tr>
<tr>
<td>h</td>
<td>small displacement</td>
<td>cm</td>
</tr>
<tr>
<td>$\hbar\nu$</td>
<td>energy of incident photon</td>
<td>keV</td>
</tr>
<tr>
<td>$\hbar\nu'$</td>
<td>energy of scattered photon</td>
<td>keV</td>
</tr>
<tr>
<td>I</td>
<td>source current</td>
<td>mA</td>
</tr>
<tr>
<td>J</td>
<td>number of voxels per sample</td>
<td>voxels per sample</td>
</tr>
<tr>
<td>$K_\lambda$</td>
<td>number of mean free paths</td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>no. of standard deviations</td>
<td>standard deviations</td>
</tr>
<tr>
<td>L</td>
<td>length</td>
<td>cm</td>
</tr>
<tr>
<td>M</td>
<td>magnification</td>
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</tr>
<tr>
<td>N</td>
<td>number of photon counts</td>
<td>photons</td>
</tr>
<tr>
<td>$N_{vox}$</td>
<td>number of photons per voxel</td>
<td>photons voxel⁻¹</td>
</tr>
<tr>
<td>n</td>
<td>number of photons per unit time, current or solid angle</td>
<td>s⁻¹, mA⁻¹ or sr⁻¹</td>
</tr>
<tr>
<td>$n_d$</td>
<td>detected photon flux</td>
<td>photons mA⁻¹s⁻¹</td>
</tr>
<tr>
<td>$n_i$</td>
<td>incident photon flux</td>
<td>photons mA⁻¹s⁻¹</td>
</tr>
<tr>
<td>symbol</td>
<td>description</td>
<td>unit</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>$R$</td>
<td>distance</td>
<td>cm</td>
</tr>
<tr>
<td>$R_d$</td>
<td>distance from detector to exit collimator</td>
<td>cm</td>
</tr>
<tr>
<td>$R_i$</td>
<td>distance from source to sample</td>
<td>cm</td>
</tr>
<tr>
<td>$R_s$</td>
<td>distance from beam to exit collimator</td>
<td>cm</td>
</tr>
<tr>
<td>$r$</td>
<td>uniform random variable in range $(0,1)$</td>
<td></td>
</tr>
<tr>
<td>$S$</td>
<td>signal-to-noise ratio (SNR)</td>
<td>standard deviations</td>
</tr>
<tr>
<td>$S_{1D}$</td>
<td>SNR for 1-D second differential</td>
<td>standard deviations</td>
</tr>
<tr>
<td>$S_{3D}$</td>
<td>SNR for 3-D second differential</td>
<td>standard deviations</td>
</tr>
<tr>
<td>$S_{LN}$</td>
<td>SNR for second differential of logarithm</td>
<td>standard deviations</td>
</tr>
<tr>
<td>$T_e$</td>
<td>recoil kinetic energy of electron</td>
<td>keV</td>
</tr>
<tr>
<td>$T_{samp}$</td>
<td>time to scan sample</td>
<td>s sample$^{-1}$</td>
</tr>
<tr>
<td>$T_{vox}$</td>
<td>time to scan voxel</td>
<td>s voxel$^{-1}$</td>
</tr>
<tr>
<td>$W$</td>
<td>width</td>
<td>cm</td>
</tr>
<tr>
<td>$x$</td>
<td>$x$ coordinate</td>
<td>cm</td>
</tr>
<tr>
<td>$Y$</td>
<td>height in $y$ direction</td>
<td>cm</td>
</tr>
<tr>
<td>$y$</td>
<td>$y$ coordinate</td>
<td>cm</td>
</tr>
<tr>
<td>$Z$</td>
<td>atomic number</td>
<td></td>
</tr>
<tr>
<td>$z$</td>
<td>$z$ coordinate</td>
<td>cm</td>
</tr>
</tbody>
</table>
### Notation

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>ratio of $h\nu$ to rest energy of electron</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>$x$ dimension of scatter voxel</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>$y$ dimension of scatter voxel</td>
</tr>
<tr>
<td>$\Delta z$</td>
<td>$z$ dimension of scatter voxel</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Poisson error in $N$</td>
</tr>
<tr>
<td>$\epsilon''$</td>
<td>Poisson error in $N''$</td>
</tr>
<tr>
<td>ζ</td>
<td>Poisson error in $f$</td>
</tr>
<tr>
<td>ζ''</td>
<td>Poisson error in $f''$</td>
</tr>
<tr>
<td>θ</td>
<td>scattering angle of photon</td>
</tr>
<tr>
<td>$\theta_e$</td>
<td>recoil direction of electron</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>correction factor for detector efficiency</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>correction factor for scattering probability</td>
</tr>
<tr>
<td>λ</td>
<td>wavelength of incident photon</td>
</tr>
<tr>
<td>or</td>
<td>mean free path in medium</td>
</tr>
<tr>
<td>$\mu$</td>
<td>total attenuation coefficient</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Compton scatter coefficient</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>Compton scattering cross section per electron</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>photo-electric cross section per atom</td>
</tr>
<tr>
<td>φ</td>
<td>angle subtended by slot</td>
</tr>
<tr>
<td>Ω</td>
<td>solid angle</td>
</tr>
<tr>
<td>$\Omega_r$</td>
<td>maximum reasonable solid angle</td>
</tr>
</tbody>
</table>

- α cm
- $\Delta x$ cm
- $\Delta y$ cm
- $\Delta z$ cm
- $\epsilon$ photons
- $\epsilon''$ photons cm$^{-2}$
- ζ cm$^{-2}$
- ζ'' cm$^{-2}$
- θ rad
- $\theta_e$ rad
- $\kappa_1$
- $\kappa_2$
- λ nm
- $\mu$ cm$^{-1}$
- $\rho$ g cm$^{-3}$
- $\sigma$ cm$^{-1}$
- $\sigma_e$ cm$^2$ electron$^{-1}$
- $\sigma_a$ cm$^2$ atom$^{-1}$
- φ rad
- Ω sr
- $\Omega_r$ sr
### Notation

#### physical constants
- $h$ : Planck's constant times the speed of light
  - $1.24 \text{ keV nm}$
- $m_0 c^2$ : rest energy of electron
  - $511 \text{ keV}$
- $N_A$ : Avogadro constant
  - $6.02 \times 10^{23} \text{ atoms mol}^{-1}$
- $r_e$ : classical radius of the electron
  - $2.82 \times 10^{-13} \text{ cm}$

#### subscripts
- $c$ : exit collimator
- $d$ : detector
- $e$ : electron
- $f$ : full width
- $fb$ : foreign body
- $fn$ : false negative
- $fp$ : false positive
- $FW$ : full width
- $HM$ : full-width at half-maximum
- $i$ : incident beam or source
- $j$ : $j$th voxel or spectral bin
- $m$ : multiple scatter
- $max$ : maximum
- $min$ : minimum
- $samp$ : sample
- $th$ : threshold value
- $u$ : umbra width
- $0$ : nominal value

#### diacritics
- $a'$ : differential or modified value
- $a''$ : second differential
- $\bar{a}$ : mean value
- $\hat{a}$ : optimum value
- $a(z)$ : value at position $z$
Appendix E

Publications Arising
The following publications have arisen as a result of this work.

The work on simulating the detection of voids (see Section 5.1) has been published as McFarlane, N.J.B. and Bull, C.R. et. al. (2000), 'The potential for Compton scattered X-rays in food inspection: The effect of multiple scatter and sample inhomogeneity', *Journal of Agricultural Engineering Research* 75(3), 265-274.

The initial work on glass detection (see Section 6.1) has been published as McFarlane, N.J.B. and Bull, C.R. et. al. (2001), 'Time constraints on glass detection in food materials using Compton scattered X-rays', *Journal of Agricultural Engineering Research* 79(4), 407-418.

The work on chicken bone detection (see Section 7.1) has been submitted for publication as McFarlane, N.J.B. and Speller, R.D. et. al. (2001), 'Detection of bone fragments in chicken meat using X-ray backscatter', *Biosystems Engineering* (formerly *Journal of Agricultural Engineering Research*) (submitted, July 2001).
References


NIST (1990), *XGAM: X-Ray and Gamma Ray Attenuation Coefficients and Cross Sections Database*, *Standard Reference Database 8, Version 2.0*, National Institute of Standards and Technology (NIST), USA.


