INNOVATION AND THE PRINCIPLES OF
PRODUCT DIFFERENTIATION

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To Sandra and my Parents
ABSTRACT

Over the last two decades considerable attention has been devoted to two areas of microeconomic theory: the Theory of Innovation and the Theory of Product Differentiation. These have developed largely independent of one another. The Theory of Innovation has focused mainly on process innovation. Whenever it has considered product innovation, it has taken the amount of product improvement, and hence the degree of product differentiation, to be linked very mechanically to the process of innovation. By contrast, the Theory of Product Differentiation has assumed that firms can costlessly introduce whatever new products they wish, and has examined whether the resulting equilibrium satisfies the principle of maximal differentiation or the principle of minimal differentiation.

The aim of this Thesis is to investigate the principles of product differentiation in the context of models which takes seriously the idea that new products arise as part of an explicit process of costly innovation. The novel feature of the Thesis is that within this process firms can exercise some degree of choice over the level of differentiation. In such a framework there are two channels through which innovation impacts on the differentiation decision. The first is that, given the stochastic nature of innovation, outcomes of the innovation process are often asymmetric. This contrasts with the assumption of symmetry typically made in the product differentiation literature. It is shown that recognition of these asymmetries can lead to firms pursuing a strategy of minimal differentiation in contexts where, had they been asymmetric, they would have pursued a maximal differentiation strategy. The second channel comes through spillovers. It is shown that these may lead to greater differentiation than would arise with no spillovers.

The Thesis considers both cooperative and non-cooperative equilibria, and shows that cooperative equilibria always results in greater differentiation.
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DECLARATION

No part of this thesis has been presented to the University for any degree.
CHAPTER 1: INTRODUCTION
1.1. Introduction

Over the last decade there has been an upsurge of interest in issues dealing with product differentiation, strategic R&D, innovation and endogenous growth. Authors addressing this type of subjects consider a great deal of aspects and assume the most varied hypotheses, in an attempt to get closer to reality, explain observed behaviour, advise in firms' innovative activities and predict the consequences of innovation policy. The emergence of such studies and analyses was probably motivated by the fast pace that technological development has been experiencing in the past years and resulted in a great development of the economic literature dealing with innovation and product differentiation, establishing a Theory of Innovation and a Theory of Product Differentiation.

With respect to the latter, the typical concern of such theory is to give an answer to the question of how similar or different products should be. This has to take into account the strategic implications and consequences that the differentiation choice will bring: How will rival firms react? What will be the consequences for competition in the product market and for market power? How will consumers choose? ... The answer to these questions is, just by itself, important and proof of this is the vast range of applications it has been given: the results emerging from the Theory of Product Differentiation have been extensively used in fields as different as marketing, political/ideological positioning and church and sect behaviour.

There are at least two typical characteristics that seem to be common to the enormous literature arising from it. The first is the generalised consideration of symmetric firms in the analysis; the second is the assumption that firms are able to differentiate products as they want and there are usually no constraints in reaching the desired product differentiation level. Basically, no matter how firms decide to differentiate products, all possibilities are available, not considering the question of possible financial or technological constraints.

With respect to the Theory of Innovation, the importance of innovation to an economy can be summarised, according to Beath, Katsoulacos and Ulph (93), by the
following three reasons: investment in R&D is the motor driving technical progress which will eventually lead to economic growth; new and improved products, resulting from innovative activities, will have a positive effect on the living standards; besides price, there are several factors that contribute to the trade performance of a country (for instance, product quality), and some of those can be influenced through R&D and innovative activities.

However, when pursuing innovation, firms are not typically concerned with the aspects mentioned above. What most firms seek is profit and strategic advantage. The first reason is easily understandable: by creating a new/improved product or by finding a cheaper way to produce it, profits will increase. The second reason is linked with the fear of being left behind by rival firms that succeeded in creating a better product or can now produce it at a lower cost and are therefore able to charge lower prices. In both cases the firm that did not innovate will see its demand reduced. Moreover, if firms do not invest in R&D they might not develop their capability of absorption of information externally available. In order to avoid such situation, firms allocate resources to the R&D activity expecting to be successful in innovating or at least to learn something about them that might propitiate a future innovation.

From the literature addressing these issues it can be easily observed that the Theory of Innovation was developed in two directions: Process Innovation and Product Innovation. Within each of these fields the main concerns evidenced by the authors are often linked with issues like the level of R&D expenditure that is made and its comparison to what be socially desirable, the timing of innovation and its diffusion and financing.

This type of analysis is obviously important and unquestionably useful, however it seems that the Theory of Innovation is somewhat self-constrained and its inter-relations with other fields of the economic knowledge have been under explored. This characteristic of the Theory of Innovation is shared by the Theory of Product Differentiation, which is in most cases also viewed as an independent and fully compartmentalised theory.

\(^{1}\) See for instance “An Economic Theory of Church Location”, Barros P. and Garoupa, N. (97)
In this Thesis a bridge between these two theories is built. There is no doubt that the issues dealt by the Theory of Product Differentiation are intimately related to the innovative activity. The most obvious connection is that in order to differentiate products, usually some work of product creation or development has to be made and this activity concerns directly to the Theory of Innovation, namely to the aspects associated with product innovation. Therefore it is somewhat surprising that the study and analysis of links connecting these two theories and its consequences has not been given more attention in the literature.

The first point this Thesis aims to make is that there is an obvious connection between the Theory of Product Differentiation and the Theory of Innovation, which has not been explored.

The pertinent question to ask after having recognised that the two theories may be intimately related is if this fact will, in anyway, affect the results established in both theories. If the answer to this question is negative, then we can continue treating these theories as independent in spite of the fact that they may be functionally inter-related. However, this does not seem to be the case.

In fact, the consideration that the Theory of Product Differentiation and the Theory of Innovation are inter-related and that the specificities of the innovative activity and product differentiation decision may affect each other, will bring new aspects into the analysis. One of such aspects and the one that is probably the major contribution of this Thesis to the economic literature is the consideration of the possibility of asymmetric firms when analysing the decision of product differentiation.

Why the consideration of asymmetric firms?
How is it related to the Theory of Innovation?
What implications can it have for the Theory of Product Differentiation?

The answer to these three questions is given next.

It is a widely accepted fact that the innovative activity has as its intrinsic characteristic a great deal of uncertainty. When starting research engineers, chemists and even academic researchers (economists included) do not generally know what the
outcome of that activity will be. A big breakthrough may be made, but the possibility of getting nowhere is also present. This reality is often acknowledged in the literature of the Theory of Innovation, and is usually formalised by the introduction of probabilities associated with each possible outcome of the R&D activity.

This uncertainty feature makes possible that the outcomes of R&D processes undertaken by two separate firms can be different, even if the efforts made by those firms was equal. Given this, and as a result of an ongoing R&D activity firms may become asymmetric if initially they started in a symmetric position. Thus, when looking into the future, firms have to consider that one of the possible scenarios they may find themselves in is one of asymmetry.

Moreover, asymmetry may be viewed not only as a “fatality”, as a natural consequence of the R&D process they have engaged but also as something that firms pursue. Firms may feel encouraged to do everything at their reach in order to gain some type of advantage over rivals.

Having acknowledged the fact that differentiating products has usually behind it some innovative activity, when creating, improving or modifying products firms should not forget the implications that Innovation may bring into the field of the Product Differentiation Theory. One of those implications is, as mentioned before, that firms may become asymmetric, given the intrinsic nature of the R&D activity.

Therefore, when studying the decision of product differentiation we should allow for the existence of asymmetric firms. Besides being an easily observed fact that in practice and in most industries firms are asymmetric, the R&D activity behind product differentiation can be accounted as one of the sources for its existence.

Having answered to the first two questions (Why the consideration of asymmetric firms? How is it related to the Theory of Innovation?), it remains to explain what are the possible implications that the assumption of asymmetric firms may have on the Theory of Product Differentiation and some of the established benchmarks.

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2 Other sources for asymmetry include financial constraints, management, size and capacity.
3 John Beath and Yannis Katsoulacos (91) and Anderson, de Palma and Thisse (92) provide a comprehensive study on the subject.
In order to do so and understand how the consideration of asymmetric firms may change the results obtained in the existing literature of product differentiation, it is worthwhile to go back to Ferreira (95). This work studies the optimal degree of product differentiation (from the firm's point of view) when firms are asymmetric and was partially motivated by the non-existence of literature that considered such realistic hypothesis. It should be noticed that the use of the symmetric firms' assumption is frequently justified in the literature by mathematical tractability reasons and not that it constitutes a good approximation to reality – in practice, symmetric firms seldom exist.

In that work it is shown that the decision to differentiate products may differ substantially whether we consider symmetric or asymmetric firms. Moreover, it is also shown that the relative magnitude of the asymmetry plays an important role in this decision. As such, the results presented by authors addressing this type of issues, and that typically assume the existence of symmetric firms, may only be considered valid to the case of symmetric firms.

The basic conclusions that can be drawn from Ferreira (95) are that if firms are symmetric then the choice will be to maximise product differentiation – this result is identical to those obtained by several authors (namely d'Aspremont, Gabszewicz and Thisse (79) in a context of symmetric firms, although in a different framework. When firms are assumed to be asymmetric (for instance as a consequence of different R&D outcomes), that work predicts that the firm that succeeded in innovating will want to minimise product differentiation (as long as its advantage over the rival is relatively large), whereas the firm that did not innovate will seek to maximise product differentiation.

The basic intuition behind such results is that if firms were symmetric and decided to minimise product differentiation, this would cause an intense competition in the product market, which would lower prices and equally damage both firms. By choosing to maximise product differentiation firms would gain some market power, allowing higher prices to be charged and thus increasing profits.

In the case of asymmetric firms, besides the incentive firms have to maximise product differentiation mentioned in the previous paragraph, there is another argument that encourages the innovating firm to move in the opposite direction, i.e. to minimise product differentiation. By having some advantage in the amount of progress made, this firm may find profitable to produce a similar product, but with
relatively higher quality (conferred by its advantage). Despite increasing the competition in the product market, the innovating firm will have the preference of most consumers, as its product is relatively better. This way, in the limit, the non-innovating firm may be driven out of the market – this is why this firm tries to differentiate its product as much as possible, and gain some market power.

Notice that, despite the fact that the analysis in Ferreira (95) was done under a different context (in that case there was certainty about the R&D outcome: only after knowing the result of the R&D activity would firms decide the desired level of product differentiation), the intuition behind the conclusions obtained can be carried out to different frameworks. In particular, it may still be valid in a context of uncertainty, where firms have to decide, a priori, the level of product differentiation (which in turn will determine the choice of the research path adopted), before knowing the outcome of the R&D process. Therefore, in such context, the choice of the product differentiation level is made in the ignorance of the R&D activity outcome. Firms will then be in a situation where they do not know if in the future they will keep their symmetry or will become asymmetric. Thus, we have what can be called a “potential asymmetry”.

Given all that was mentioned above, the starting and fundamental point of this Thesis is then, by establishing a link between the Theory of Innovation and the Theory of Product Differentiation, the study of the choice of product differentiation in a context of innovative uncertainty, that may or may not create a situation of asymmetry between firms. This constitutes a step towards reality adherence and will contribute to the explanation of some firms’ observed behaviour.

Having established and explained what is the core and motivations of the present research work it should be said that this type of analysis, despite being important just by itself and on its own, can be used as background in the study of related subjects. On this, and recognising yet another possible connection between the Theory of Innovation and the Theory of Product Differentiation, in a second phase of this Thesis, the notion of spillovers will be added to the research work described above.
In order to understand this other link that may exist connecting those two theories it is worthwhile to make a brief incursion in the Theory of Innovation and understand what it has to say concerning the existence of spillovers.

In practice, technological progress (either in process innovation and product innovation) can be viewed as a good with two specific characteristics: non-rivalry and partial excludability (Romer (90)). By definition, we say that a good is non-rival if a firm or person’s consumption does not reduce or excludes its use by other persons or firms. Excludability refers to the possibility of excluding economic agents from the consumption/use of that good.

Romer (90) argues that technology, being itself a form of knowledge, can be considered as non-rival as the cost of replicating or imitating is usually trivial when compared with the cost of creating it in the first place. Why then should technology be considered a partial-excludable good? In order to protect and incentive the entrepreneurs that wish to undertake R&D activity, mechanisms that try to defend property rights, like the use of patents, were created. This is a clear attempt of making the outcome of R&D activity more appropriable, allowing it to have an excludability characteristic. However, in most cases, these mechanisms are not full proof. For instance, the degree of protection supplied by patents is limited, as is the period of time for which they are assigned. If this was true in 1962, when Arrow stressed the fact that intangibles like information or knowledge are bound to be revealed (at least partially) as the innovator has to disclose information in order to get the property rights, it is even more true in the present days where technological alternatives are more developed and abound.

Therefore, given that scientific and technological improvements cannot be fully appropriated by the inventor, spillovers are a reality affecting not only firms that undertake R&D activities, but also every firm in the market or that considers entering it.

Following d’Aspremont and Jacquemin (90), the size of spillovers is determined by many factors, which may be grouped in three types:

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4 Commonly known as the Hirshleifer problem.
(1) Those related with the nature of research: if the research reaches particular and product specific results these may be only applied to that particular product, with no application to other products, whereas if the results are general they may have a vast range of applications.

(2) When firms cooperate it is natural that the spillover among them be higher than the spillover between the firms that are in a Research Joint Venture (RJV) and those that are left out of the agreement and among these.

(3) Those related with the different degrees of product differentiation - if products are homogeneous it is easy to use the R&D output of other firms, but if products are highly differentiated then it is less probable that the spillovers between the firms that produce them will be high.

In spite of the fact that these three factors can easily be considered a triviality or almost common sense, the reality is that the literature acknowledges the existence of spillovers and that its dimension may depend on research agreements or the degree of specificity of the results obtained, but fails to relate the magnitude of spillovers with the degree of product differentiation.

The second link connecting the Theory of Innovation and the Theory of Product Differentiation becomes now evident. The innovative activity behind any choice of product differentiation will generate spillovers whose magnitude may depend on the level of product differentiation chosen. The less differentiated one firm chooses its product to be, the more spillovers rival firms will be able to absorb, allowing them to improve their products. On the other hand, the firm that chooses to produce a less differentiated product will be also able to receive more spillovers from other firms. Of course that, if the firm’s choice is to maximise product differentiation, then all that was mentioned in the last paragraph goes in the opposite direction.

By establishing such type of relation between the magnitude of spillovers and the degree of product differentiation we have obtained the notion of what might be called endogenous spillovers.

This question gains even more interest and importance as firms may become asymmetric after the R&D process. In such case, and intuitively speaking, a firm that believes that it will not innovate may find profitable to minimise product differentiation, as it will allow her to receive more spillovers and improve its product. Notice that this type of incentives was not present when spillovers were assumed not
A firm that has a high probability of success in innovating may now not be so willing to minimise product differentiation as it will give away a large amount of spillovers, truncating some extend the advantage obtained by the innovation. In the absence of spillovers this type of strategic considerations was also not present.

Thus, the decision to differentiate products will have to be carefully analysed and acquires another dimension of strategic behaviour, as it will determine the level of spillovers received and given in a context of uncertainty about the future symmetry or asymmetry of firms, which in turn will affect the degree of product differentiation.

To formalise and model the notion of endogenous spillovers is a task that can be performed in an almost unlimited number of ways. In this Thesis only a few will be presented and they mainly aim at conveying the idea that the less differentiated products are, the larger the magnitude of spillovers will be, and vice-versa.

This feature, emanating from a link between the Theory of Innovation and the Theory of Product Differentiation, and that builds itself on the previously presented connection of those two theories, constitutes another source of making the model under analysis more realistic.

Having explained the main points studied in this Thesis, it is useful as well as informative to situate the present research work in the context of what of most importance was written on the area. This will allow a better understanding of its motivations and contributions to the subject. For this reason a Literature Review follows.

1.2. Literature Review

The theoretical research on Product Differentiation was pioneered by Harold Hotelling in 1929, with the seminal paper “Stability in Competition”. Hotelling built a model in which firms can choose a location along a line of finite length l - the “Linear City”. Given the assumptions of symmetric firms and linear costs of transportation, this author showed that the equilibrium location for firms is precisely the centre of the line. All other locations do not constitute Nash equilibrium since there is always one
firm that has an incentive to change its strategy/location. This conclusion was named the “Principle of Minimum Differentiation”.

Although, somewhat intuitive\(^5\), in 1979 d’Aspremont, Gabszewicz and Thisse proved that this principle was not correct. Using the same framework, these authors argue that if two firms are located on the same spot on the line, a price war will start (by Bertrand competition), driving prices to the marginal cost and profits to zero, being equally damaging for both firms. Such situation is hardly an equilibrium as firms have the chance of making strictly positive profits just by moving away from each other. Modifying some of Hotelling’s assumptions (namely the structure of transportation costs) these authors showed that the equilibrium that emerges is one in which firms choose to be located at opposite extremes of the line, establishing the “Maximum Differentiation Principle”. Osborne and Pitchick (87), making use of the findings on the use of mixed strategies in Dasgupta and Maskin (86) also conclude that firms will not choose to minimise product differentiation. The subgame-perfect equilibrium found is shown to be unique.

Neven (85), setting up a framework close to Hotelling’s, rules out the equilibrium existence problem (pointed out by d’Aspremont et al. (79)), and shows that the Maximum Differentiation Principle holds. The key for this existence result is in some of the assumptions made that provide a useful concavity in the profit function.

Notice however, that the conclusions stated by these authors hinge on a number of restrictive assumptions, namely consumers uniformly distributed (as is usual in linear bounded market models), homogeneous consumers\(^6\), perfect information and symmetric firms. The relaxation of these assumptions can drastically modify the results obtained as shown in a comprehensive survey in Beath and Katsoulacos (91). De Palma et al. (85) show that the principle of minimum product differentiation can be valid if it is assumed that consumers are sufficiently heterogeneous and that, despite knowing the valuation distribution across a population, firms cannot predict with certainty the product choice of a given

\(^5\) It is reasonable to think that firms would want to move towards the centre since the competition usually takes place in the space between the two firms. If this space is reduced, firms will increase their “backyard” where they do not have to compete for consumers.

\(^6\) Intended as: consumers on the same location place the same valuation on a given product.
consumer. A similar result was obtained by Stahl and Varaiya (78). These authors defend that the tendency for agglomeration is due to uncertainty about the distribution of demand. By locating near to existing firms, a new entrant knows that at least some demand for that specific product must exist - "information externalities".

The introduction of consumer uncertainty about which products are available and where they can be found, in presence of significant fixed search costs can also restore the Principle of Minimum Differentiation as shown by Stahl (82).

It is also fairly obvious to understand that a non-uniform distribution of consumers causing tastes to be relatively identical concerning the characteristics of a product may lead firms to produce similar goods in order to meet demand (this factor may dominate the disadvantages of increased competition), restoring (at least partially) the Minimum Differentiation Principle.

All this work was undertaken, using as underlying assumption and for mathematical tractability reasons, a one-dimensional notion of space (or product).

However most products are intrinsically multi-dimensional. This becomes even more evident in the marketing of new products: a firm entering a market usually tries to convey the image of having improved, changed or introduced at least one characteristic to the product. This provides a strong argument to the use of a multi-dimensional framework when studying product differentiation. Tabuchi (94) recognises this evidence and affirms that all the conclusions obtained using a one-dimensional product can only be considered valid and important if they hold when generalising to higher dimensional products. According to Tabuchi (94), and considering a two-dimensional product space, this is not the case. This author states that firms will want to maximise product differentiation in one characteristic while minimising it in the other characteristic.

The results obtained were later corroborated by Irmen and Thisse (98), who generalise the analysis to an n-dimensional product. In their paper "Hotelling Was Almost Right", they allow each product characteristic to be weighted differently from the others and conclude that firms will minimise product differentiation in all but one characteristic - the dominant characteristic\(^7\). With such result price equilibrium is not a problem insofar as maximising product differentiation in one characteristic is

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\(^7\) The dominant characteristic is intended to be the characteristic to which consumers attach a higher weight. If all the weights are equal then it is stated that there are n local equilibria in which firms
enough to relax price competition. The example given to support the result obtained refers to the international magazines Newsweek and Time. Basically, it is argued that these magazines minimise the differentiation in several characteristics (layout, content, distribution and price), but differentiate themselves with respect to the cover story, which is believed to be the dominant characteristic.

In what respects the acknowledgement that spillovers are a reality, there exists a vast literature that studies how these may determine or influence the several dimensions that constitute a firm's life.

In what follows only a few examples will be cited. Hartwick (84), d'Aspremont and Jacquemin (88), Spence (84), Cohen and Levinthal (89), Levin (88), Steurs (95) are authors that consider the existence of spillovers in their work, and are concerned with the consequences they may have on the level of R&D spending. On this issue empirical studies have also been done by Bernstein (89), Bernstein and Nadiri (89) and Griliches (79) and (91), just to mention a few. The effects of spillovers on market equilibrium and profitability were studied by Simpson and Vonortas (94), Klette (96) and Geroski, Machin and Reenen (93), and their importance in patent races was analysed by Fraja (93).

Some departures from the line of work that considers the existence of spillovers mentioned above were probably triggered by d'Aspremont and Jacquemin (88), (90). The works that will be mentioned next consider the possibility of spillovers between firms in a Research Joint Venture (RJV) to be higher than the spillovers between non-cooperating firms. For that reason they are intimately linked with the second argument pointed out by d'Aspremont and Jacquemin (90) for why models should allow different magnitudes for spillovers.

In 1998 Beath et al. (98)* extend the d'Aspremont and Jacquemin model by considering that R&D is made in two steps. First, firms have to invest in R&D in order to create knowledge. In the second step this knowledge is used to reduce production costs. Given this setup, there are two instances in which spillovers may occur. Considering that is possible to transfer information in the RJV, these authors

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* Also a Discussion Paper, University of Bristol 1990.
find that this constitutes an additional advantage for the cooperating firms, that was not taken into account by d'Aspremont and Jacquemin (88). So, besides internalising the positive effects of spillovers, we may assist to the elimination of needless duplication.

Kamien et al. (92) analyse different settings for competition and cooperation, taking into account that when information is shared, the spillover rate is higher. This is studied in an oligopoly with differentiated products and Bertrand competition in the output market. The conclusions reached can be summarised as follows: RJV competition (R&D decision is independent, information is shared) leads to high prices and low R&D; RJV cartelization (R&D is co-ordinated) is the best form of cooperation for society since prices are relatively low and firms' profits are higher when compared with other setups. Another conclusion highlighted by these authors reinforces one known advantage of RJV - if there is a minimum efficient scale to research investment then, the cost sharing in RJVs may allow for R&D that would not be pursued by firms alone.

Motta (92) analyses another type of innovation - instead of process innovation he looks at product innovation so that R&D may give rise to products with different levels of quality. He considers a model of vertical product differentiation where research will affect the product's quality. Once more, the conclusions obtained by d'Aspremont and Jacquemin (88) are reinforced: if the spillover rate is high enough then the cooperative solution will generate a higher level of welfare than the non-cooperative solution. He also addresses the question of the number of firms that may exist in the market with and without cooperation. It is found that the existence of cooperation allows more firms to exist in the market producing, and as a consequence of the increased competition, prices will fall and hence increase consumers' welfare. Also analysing questions concerned with the number of firms, but now in the RJV, Poyago-Theotoky (95) finds that the market solution may not achieve the optimal RJV dimension, depending on the degree of spillovers.

Poyago-Theotoky (94) considers a framework that includes horizontal and vertical product differentiation - products have different characteristics and, for each characteristic different quality levels are allowed. When firms compete in R&D there is a possibility of generating a "super-product" that has all the characteristics available in the market. Under the assumptions considered for competition in the product market, it is shown that RJVs can be a way of increasing welfare, when compared to
the non-cooperative setup. Moreover, when firms cooperate in R&D, the welfare level reached by a socially managed economy is always higher.

1.3. Main Contributions

From this Literature Review (that despite not being complete, it mentions some of the most important issues dealt in the area of product differentiation and on the subject of spillovers) it is easily understood that the relation between the Theory of Innovation and the Theory of Product Differentiation is not being conveniently explored. This may account for the fact that both the assumption of asymmetric firms and the consideration of endogenous spillovers, in such context, are being ignored.

This Thesis aims precisely at, by building a bridge between those two theories, filling that gap in the literature.

It should also be said that, besides those two aspects that have been forgotten and that this research work introduces in the literature, others are that deserve our attention insofar they contribute for a more accurate depiction of reality.

From the literature review above we can perceive a set of assumptions, explicitly or implicitly made throughout those authors' work that, if modified, would constitute another step towards reality adherence. This Thesis also aims to change some of those assumptions.

The first modification that will be introduced is related to the way products are usually modelled in the existing literature on the Theory of Product Differentiation. Apart from a few exceptions of which Poyago-Theotoky (94) and Ulph and Owen (94) are example, the consideration of products that can be simultaneously vertically and horizontally differentiated is by no means a widespread assumption. The typical hypothesis made by authors is to consider one and only one type of product differentiation (vertical or horizontal). In spite of this, most commodities, besides possessing a set of characteristics that can be horizontally differentiated, usually have some characteristics that are vertically differentiated. In almost all products available in the market, we can find at least one characteristic for which all consumers agree over the preference ordering. In this Thesis it will be considered that products are both
vertically and horizontally differentiated. As a natural consequence of this assumption, the product under analysis will have to be multi-dimensional (i.e. it will be characterised by at least two characteristics). This feature will also add to a better representation of reality.

The second modification made with respect to what is usual in the literature addressing this type of subjects finds its origins in the close relation that the Theory of Innovation and the Theory of Product Differentiation unquestionably should have, some aspects of which were already stressed out. The latter theory usually does not devote a great attention to technological or financial constraints. In the generality of the research works concerning product differentiation, firms are assumed to be able to differentiate products as they want. When the optimal level of product differentiation is theoretically found, the question of how much it will cost to develop a product with those characteristics or even if it is technologically feasible is typically left for the engineers to solve or, in the best of the scenarios, to the Theory of Innovation to answer. However, having recognised the intimate relation between those two theories, it would certainly benefit the analysis of the choice of the optimal level of product differentiation to also consider, for instance, the related technological and cost issues through the eyes of both theories.

Having this in mind, and addressing in particular the question of costs, it is assumed throughout this Thesis that the innovative activity leading to product differentiation has costs associated. Just by observing the world that surrounds us we can perceive that this is not an academic assumption but a reality affecting most industries (e.g. pharmaceutical industry).

If it is true that the R&D processes may be very expensive it is not less true that the resources allocated to that activity are scarce, and thus cannot be allocated to the development of every product characteristic in infinite amounts. In most cases, firms have a limited R&D budget, which they have to carefully assign to each product characteristic. Notice that the consideration of such financial constraints introduces an opportunity cost associated with the decision of product differentiation: given a limited R&D budget, if a firm invests it totally in the improvement of a given product characteristic, the other characteristics will not have resources to be developed.
As was already explained, innovation is crucial not only for a firm but also for an economy in general, however, if it is misguided it may benefit no one and constitute a waste of valuable and scarce resources. The role of the Theory of Product Differentiation is then, having in mind these financial constraints, to point out the "right" aspects or product characteristics where innovation should take place so that firms could concentrate its R&D efforts on those, and thus reach the level of product differentiation that maximises profits.

Having explained the main contributions and motivations of this Thesis and having placed them in the context of the existing literature, it is now the moment to describe the organisation and the contents of each chapter.

1.4. Outline

Chapter 2 serves the purpose of introducing the basic model that will be utilised throughout the whole Thesis. The model is a variant of the one used by Ulph and Owen (94), and Poyago-Theotoky (94) and encapsulates all the assumptions and features that have been mentioned before and that try to approximate this simplified representation of reality to reality itself. The description made within this chapter will present the basic properties of the model that are common to all chapters. However, in each of the other chapters the formalisation of the model will be enriched and, in some way, completed according to the specific assumptions made and issues dealt in that particular chapter.

It is worth mentioning that, in spite of the fact that virtually all results and conclusions obtained in the course of the present research work can be directly drawn from the particular model presented, some of them can be easily derived using a much more general framework, where it is only required the satisfaction of a few reasonable and intuitive assumptions. As the Thesis develops, this will become obvious by the nature of the analytical proofs presented that do not need to resort to the particular model presented in the second chapter. However, given that in some cases these proofs are not totally conclusive, the use of the model, either analytically or through numerical simulations, will prove to be a valuable tool in obtaining the results.
At this point it is worthwhile to mention some aspects of the analysis is made within each chapter and that will persist throughout the whole Thesis:

* The first concerns the fact that the object of this research work is to study the decision of product differentiation acknowledging the inter-relations that exist between the Theory of Innovation and the Theory of Product Differentiation. As already was stressed, this may imply the introduction of an uncertainty feature (that has its origin in the Theory of Innovation) that may lead or not to the existence of asymmetric firms, when analysing the firms' desired level of product differentiation. In spite of the unquestionable existence of uncertainty about firms' future symmetry or asymmetry, in every chapter (except the one where the model is presented), a prior study will be made considering a context of certainty. In such context the cases of firm symmetry and asymmetry will be separately studied in a situation where firms have full information about the R&D outcome. This, besides providing valuable insights to the analysis and understanding of the results obtained in a context of R&D outcome uncertainty, where firms do not know whether their symmetry will be kept or not, that is, where firms arecontemplating a situation of "potential asymmetry", will also help to better realise the impact of such assumption.

* The second aspect that will be common to the analysis made in every chapter (except chapter 2) of this Thesis is that both the non-cooperative and cooperative cases will be studied. The consideration of the latter is related with yet another link that exists connecting the Theory of Innovation and the Theory of Product Differentiation. In spite of the growing importance that has been given to issues concerning cooperation between firms at the R&D level in the economic literature, authors usually focus their attention in factors like the level of R&D expenditure or stability and enforcement of the agreements. Issues like the effects that this type of cooperation may have on the decision of product differentiation seem to have been forgotten in most cases, if not all. Also, the practice of technological and competition policies apparently do not take into account the consequences to product differentiation of measures that try to
incentive Research Joint Ventures or other types of R&D agreements\(^9\). Despite this, the degree of product differentiation, variety and quality available to the consumer has obvious implications to its welfare. Proof of this is for instance Spence (75), (76), Lancaster (79) and Ana \textit{et al.} (93), just to mention a few. So, in the sense that cooperating in R&D may affect the product differentiation decision, and this may have consequences at the consumer welfare level, R&D cooperation is an issue that is worth being studied.

In this Thesis only the consequences of cooperating in R&D on product differentiation will be studied, as welfare considerations are beyond the scope of this research work. However, this may constitute a springboard to the analysis of welfare aspects. Within this second aspect it is also important to define what is the underlying notion of firms cooperating in R&D activities that will be utilised. It will be assumed that when firms cooperate in R&D they will try to maximise expected joint profits by organising the relevant aspects of innovation together (i.e. coordinate research design). Thus, when introducing the notion of spillovers into the model (Chapter 4), in no case it will be considered that, just because firms cooperate, the spillover level will be exogenously larger relatively to the level it would assume in a non-cooperative setup. This contrasts with what is considered by several authors already mentioned in the Literature Review. The reason for this is that the coordination of research design between firms is not a sufficient condition to guarantee that information about any technological progress or discovery will be shared by all firms. Therefore, the magnitude of spillovers (when they are considered to exist in the model) will remain endogenous and dependent of the degree of product differentiation chosen.

Moreover, it is implicitly assumed that any agreement reached by firms is fully enforceable, with respect to the choice of research design and information sharing. This assumption is by no means a guarantee that in

\(^9\) For instance, article 85 of the Treaty of Rome prohibits all agreements between firms, except those that improve the production or distribution of goods or promote technical and economical progress.
practice these type of agreements are implementable, however, we can observe some examples of successful cooperation in R&D. Thus, despite knowing that situations like the one depicted by the cooperative equilibrium may occur in reality, there is still some research work to undertake concerning agreement implementability.

* The third aspect that is worth to mention before the remainder of the Thesis structure and organisation is presented concerns the problem of price equilibrium existence.

Ever since Hotelling (29) did not pay enough attention to this aspect and consequently reached an incorrect conclusion, that authors concerned with product differentiation issues try to guarantee the existence of such equilibrium in their work. This is done in the most varied ways, either in pure and in mixed strategies (Gabszewicz and Thisse (86) provide a survey on the subject). For this work in particular, the problem of price equilibrium existence is not the central one, and for that reason we will not be spending much time analysing it. However, if the conclusions obtained are to be valid then, behind the analysis presented, the existence of price equilibrium must be ensured. This is done in two levels. The first one relates to the particular model utilised. In the model, the utility function for consumers that is used generates an iso-elastic demand function, which allows an easy computation of the price equilibrium, demonstrating that it exists. The second level at which price equilibrium is guaranteed to exist is in a broader context, thus not limiting the equilibrium existence to this particular model. This is done with reference to the valuable work of Caplin and Nalebuff (91), that establish the general conditions under which there exists a pure-strategy price equilibrium for any number of firms producing any set of products. Moreover, the possibility of multi-characteristic products has also been contemplated by these authors. Given this, and whenever the conclusions drawn in this work are obtained for a wider class of models than the one that will be presented in the next chapter, price equilibrium is guaranteed

This constitutes a clear encouragement to R&D agreements. Notice however that no constraints on the level of product differentiation resulting from such agreements are made.
to exist, provided that the conditions stated be Caplin and Nalebuff (91) are satisfied.

The third chapter of the Thesis analyses the question of product differentiation, acknowledging the inter-relations that exist between the Theory of Innovation and the Theory of Product Differentiation, and that were already mentioned, in a model that incorporates all the changes and assumptions previously referred. This specific chapter will focus on the consequences of the consideration of the fact that firms may not keep their symmetry and may become asymmetric as a result of an R&D process (that has some uncertainty associated) to the firms' choice of the level of product differentiation.

It will be proved that if firms are assured to keep their symmetry after the R&D process, then they will choose to maximise product differentiation. If firms contemplate some degree of uncertainty about the outcome of the R&D activity (or are assured that asymmetry will arise from that outcome), then it will be shown that firms will be willing to minimise product differentiation, provided that the potential (or effective) asymmetry degree reached is large enough.

With respect to the cooperative setup, where firms co-ordinate the research design, the results obtained show that firms will decide for a situation of maximum product differentiation, which will soften market competition thus allowing higher profits.

Chapters 4, 5 and 6 maintain the structure and the type of problems studied in the third chapter, adding to it a new and realistic feature: endogenous spillovers. The fact that the basic model utilised is the same, either in absence or presence of spillovers will allow an easy perception of the impact that the consideration of spillovers may have in the decision of product differentiation.

As was already stressed, the idea that firms share (voluntarily or involuntarily) information and knowledge is not new, having Arrow (62) pointed out this fact. It is also not original the perception that the degree of product differentiation might affect the spillover magnitude (d'Aspremont and Jacquemin (90)). In spite of this, this relation and inter-dependence, taking into account all the strategic behaviour that may arise due to its consideration has not yet been addressed in the literature. So, the
innovative character of chapters 4, 5 and 6 lies on the fact that this type of issues, despite of being important and realistic, have not been conveniently explored.

The interesting thing about the introduction of spillovers as a function of the degree of product differentiation chosen by firms, is that new forces and incentives come into play when the decision to differentiate products or not is taking place. In the third chapter it will be shown that firms may choose to minimise product differentiation so that competition in the product market is increased which may benefit the firm that obtains some type of relative advantage from the R&D activity. However, in presence of spillovers, this result could be reversed insofar as by minimising product differentiation, the firm in the advantageous position would also be giving away to its rival a high level of spillovers, thus truncating the advantage obtained from the successful R&D process. Therefore, it may be the case that the optimal strategy is no longer to minimise product differentiation. On the other hand, as we are contemplating a scenario of uncertainty relatively to the R&D outcome, a firm may have incentive to minimise product differentiation, as, if it does not succeed in the R&D activity, it will at least receive a large amount of spillovers, thus enabling this firm to make some progress.

It is this type of problems and interactions that will be studied in chapters 4, 5 and 6. In each of these chapters an alternative formulation for spillovers will be used, as will be explained next.

In the fourth chapter it is considered a simple formulation for spillovers, where the level of product differentiation chosen by firms will only determine the magnitude of spillovers. The way spillovers are allocated between the two dimensions that will characterise the product is up to an exogenous variable. It is worthwhile to stress at this point that whenever spillover existence is assumed, these spillovers will not be considered as characteristic specific. This means that even if a firm focuses all its R&D activity in only one product characteristic and innovates in that particular characteristic, the spillovers generated may benefit the rival firm not only in the innovated characteristic but also in the other product characteristic. In practice we often observe situations that are in accordance with this assumption. Just to mention a couple of examples we might refer to the pharmaceutical industry or even the economic academic research. With respect to the former example, a laboratory that
finds a new way to synthesise *Ibuprofen* may be helping other laboratories (through spillovers) not only in the production of *Ibuprofen* but also in the creation of other chemicals, by the use of similar techniques or methodologies. Other example may be extracted from the economic repository of knowledge. For instance, the researcher that, while investigating the Theory of the Consumer, established the notion of Marginal Rate of Substitution certainly helped other researchers in the development of Consumer Theory. Notice however that the benefits of such innovation did not stop there and were extremely useful in developing the Theory of the Firm and in creating concepts like the Technical Marginal Rate of Substitution, by using a similar methodology or concepts.

Thus, in both examples cited, in spite of the possible specialisation of the R&D activity, its spillovers are not specific to the initial object of improvement. Obviously that the consideration of spillovers that are not characteristic specific constitutes only an assumption that depicts the reality of some industries or situations but it is not the only one that could be made. A possible alternative assumption could be to consider characteristic specific spillovers, which means that any innovation in a given product characteristic could only benefit other firms in developing that particular product characteristic. It should be noted that this is an assumption that might be applicable in some industries or R&D processes and for that reason it constitutes a possible direction for further research.

Within this context several assumptions can be made about how the spillovers flow from one firm to the other and who is eligible to absorb spillovers. The more relevant hypotheses (given their reality adherence) will be dealt within this chapter.

Firstly it will be assumed that spillovers only occur between firms that succeeded in the R&D activity. This conveys the idea that only by being successful in R&D can firms acquire the necessary knowledge to be able to absorb and use the information spillovered by rival firms. The alternative assumption that can be made is that, besides the existence of spillovers between successful firms in R&D, spillovers may also occur from the innovating firms to the firms that did not innovate.

This latter assumption will be further dissected for the sake of a more rigorous representation of reality. In order to understand how the model works, in a first instance, it will be analysed the case where the magnitude of spillovers is the same, whether we are dealing with spillovers between innovating firms or spillovers from
successful to non-successful firms. Once this case is understood a more realistic setup will be studied, by considering that the magnitude of spillovers between innovating firms is larger than the spillovers that may exist from innovating to non-innovating firms. The idea behind this assumption is related with the perception that if a firm was able to innovate, then it developed a set of information and/or techniques that will facilitate the absorption of spillovers. The absorption of spillovers by a non-innovating firm will not be so large, insofar it did not acquired the instruments that will ease the understanding and application of knowledge flowing from rival firms. This situation can be compared to that of an illiterate person (non-successful in a learning process) versus a literate person (successful in the “R&D process”). If someone writes a book the knowledge it might encapsulate will be more easily learned by the literate person than by the illiterate one. The latter may only grasp some vague ideas, as its inability to read does not allow him to go further.

Chapter 5 builds on the previous one, but a new formulation for spillovers will be introduced. When we observe what happens in reality concerning spillovers (if not, by common sense), we can easily reach the conclusion that a firm that has been mainly researching on one specific characteristic of the product will probably be more capable of absorbing spillovers in that characteristic than in any other. Even if the firm in question did not succeed in improving that particular characteristic, it developed an amount of knowledge that may facilitate the incorporation of some discovery made by another firm. Therefore it seems reasonable to assume that spillovers may not be assigned exogenously to each characteristic as was considered in the fourth chapter, but that they will be allocated according to the research design chosen by the firm. For this reason, a larger share of total spillovers should probably be devoted to the more researched characteristic.

The question that now is under analysis is then, “Will the fact that when firms decide the desired level of product differentiation, they will be simultaneously deciding on the allocation of spillovers between characteristics, affect the product differentiation decision?”

In such setup, a firm devoting all its R&D efforts to a particular characteristic, besides the advances its own R&D may bring to that characteristic, has to take into account that more improvements may be made as the firm capacity to absorb spillovers is concentrated in that characteristic. Thus, a firm may have an incentive to
reduce its R&D activities in that direction as it would still have a large quality advance in this characteristic, while devoting a little more attention to the other characteristic, which, in turn, would also benefit from spillovers.

In order to understand the motivation lying behind chapter 6, it is important to clarify that spillovers may have its origins in two distinct sources. According to Beath, Poyago-Theotoky and Ulph (97) or Katsoulacos and Ulph (97) spillovers depend on the capacity to adapt other firms' research in own benefit and the amount of information sharing. The former has been dealt with in chapters 4 and 5, and is related with the choice that has to be made before the R&D activity is undertaken: “research design stage”. The latter source of spillovers will be analysed in the sixth chapter and concerns the fact that after firms know that succeeded in the R&D activity, they can control to some extent what information will be passed (spillovered) to rival firms.

Again, this constitutes another step towards a more realistic approach of spillovers, as both types of spillovers may co-exist in real life.

Having made this distinction, it will be shown that, despite the fact that firms may have some control over the spillover level (other than the choice of research design), they may still choose to maximise product differentiation when deciding the research design (ex-ante). This automatically implies that spillovers will not exist, thus making irrelevant the use of instruments that could control their magnitude. Therefore, the additional control of the spillover level might not add any novelty to the product differentiation decision. Another somewhat surprising effect, that has already been observed by Katsoulacos and Ulph (97) is that, even when a cooperative setup is analysed, the conclusion above can still be obtained. This contradicts the somehow established idea that if firms cooperate they will always choose to share all the information.
2.1. Introduction

The basic framework considered was originally inspired by Ulph and Owen (94) in *Racing in Two Dimensions*. These authors allow for four possible situations (*full dominance; partial dominance; no dominance; product specialisation*), though for the purposes of this work, only the product specialisation case will be considered. The reason for this simplification is explained by three factors: first, and as was already mentioned, most firms usually try to present themselves as having some kind of advantage in (at least) one characteristic over their rivals; secondly, it will become clear that for the purposes of this work the consideration of this case is enough; and finally, the model will become much more tractable.

The model represents an industry where \(n+2^1\) firms exist, but only two undertake R&D activities, and are therefore the motor of innovation. The remaining \(n\) firms are assumed to be able to systematically upgrade the quality level of each product characteristic to the lowest level that the innovating firms exhibit.

It is considered that the product produced by these \(n+2\) firms incorporates two fundamental characteristics vertically and horizontally differentiated, which can be improved through R&D activity.

The assumption of considering only a two-dimensional product may seem, at a first glance, somewhat restrictive and unrealistic, as in practice products have usually more than two characteristics. Moreover, as Tabuchi (94) and Irmen and Thisse (98) point out, the dimensional question may constitute a problem to the generalisation of results to higher dimension products.

However, and as was stressed by Miller (56), individuals just consider a sub-set of product characteristics as they are not able to process all the information. According to this author, seven is the "magical" number of characteristics that consumers are capable of evaluating, so it might be irrelevant to consider in the analysis a large number of product characteristics. Still, by generally using a two-dimensional product, one might ask how general can the conclusions drawn be. In this matter it should be
noticed that even Tabuchi (94) considered a two-dimensional product, and when Irmen and Thisse generalised the problem to an n-dimensional product, the results obtained by Tabuchi remained valid. Therefore, it seems that if any difference is going to exist in the results by considering products with different dimensions, they are more likely to occur when comparing the results obtained with one-dimensional products with higher dimension products, than when considering comparisons between different multi-dimension products. Moreover, prior studies utilising the same type of model considered throughout this thesis have shown that the conclusions obtained for a higher number of firms and product characteristics remain, in qualitative terms, valid. Still, this generalisation constitutes a direction for further research.

Denoting the two innovating (specialist) firms by L and R, it will be considered, without any loss of generality, that firm L is specialised in the first characteristic and firm R in the second. This means that firm L will have a higher quality level in the first characteristic, while firm R’s product has an higher quality level in the second characteristic, relatively to its direct rival – firm L. From what was stated until now, it can be easily understood that there will be three varieties of the same product available to the consumer: one produced by firm L, other produced by firm R and the last one produced by the \( n \) non-innovating firms.

Labelling the good produced by the \( n \) non-innovating firms by good 0\(^2\), the one produced by firm L as good 1, and the good produced by firm R as good 2, we can now introduce the notion of the characteristic vector. This vector expresses in terms of some quality unit, the quality level of a product in each of its characteristics. Thus, it can be represented by \((c^i, d^i)\), where the first (respectively second) co-ordinate represents the quality level of the first (respectively second) characteristic that good \( i \) embodies (with \( i = 0,1,2 \)).

It will prove to be very useful to measure the quality levels of product 1 and 2 in each characteristic relatively to those in good 0. If we define \( \alpha^i = c^i - c^0 \) and similarly \( \beta^i = d^i - d^0 \), with \( i = 1, 2 \), we will have the “characteristic gaps” of good \( i \).

\(^1\) The consideration of \( n+2 \) firms is related to the original model from which this particular model was adapted, which, besides helping mathematical tractability, also adds more realism to the setup.
relative to good 0. Notice that, given the way specialisation was defined and the process by which the \( n \) non-innovating firms improve its product, we will surely have the following: \( c^1 > c^2 = c^0 \) and \( d^2 > d^1 = d^0 \). Therefore, and by definition, \( \alpha^2 = \beta^1 = 0 \) and \( \alpha^1 > 0, \beta^2 > 0 \).

This provides the space for some simplification in the notation that will be made now and will remain valid throughout this work. Let us then define \( \alpha (\alpha > 0) \) as the quality advantage that the product produced by firm L has on the first characteristic, and \( \beta (\beta > 0) \) the quality gap that product 2 has on the second product characteristic. The way firms can influence the values of \( \alpha \) and \( \beta \) may differ according to the setup considered, and will be explained in detail at the beginning of each chapter. For now, only the aspects relating to demand and product market equilibrium will be addressed, as the main present concern is with presenting the more general features of the model that will persist throughout the whole Thesis.

The ensuing closely follows Ulph (91) and Ulph and Owen (94) and does not constitute an original contribution to the literature. With that said, and for the sake of the good understanding of the model utilised in the analysis and its mechanisms, its presentation will still be made.

2.2. Demand, Prices and Profit

In order to define the demand functions for each good we need to know how consumers will value each good. Since this is a model of vertical and horizontal differentiation, individuals have to express their preferences over the different quality levels of the same characteristic and, simultaneously, between different characteristics. It is assumed that the overall quality of a particular good can be expressed by,

\[
q^i(v,w) = \exp(vc^i + wd^i) \quad i = 0, 1, 2.
\]

\(^2\) Also called “basic good”, by Ulph and Owen (94).
where \( v \) and \( w \) are the weights assigned by a consumer to each characteristic and are i.i.d. in the population with density functions \( e^v \) and \( e^w \).

Having expressed a two-dimensional vector as a scalar, it is now easier to write the consumers’ preferences over the goods.

Each consumer has income \( M \) and the following utility function:

\[
U = \log(z) + \log\left(\sum_{i=0}^{2} q^i x^i\right)
\]

where \( z \) is the expenditure in all other goods, and \( x^i \) is the quantity each consumer buys of good \( i \).

Insofar as the three goods in question are perfect substitutes, consumers will buy just one of them. Which good to buy is a decision that depends on the price that the consumer has to pay per unit of (perceived) quality, i.e., the good chosen will be the one for which \( \frac{p^i}{q^i(v,w)} \) (quality-adjusted price) is minimised. It is now easy to show that each consumer will buy \( \frac{M}{2p^i} \) units of the good for which the quality-adjusted price is the lowest.

In order to find the total demand faced by each firm, we have to know which consumers are indifferent between goods: 0 and 1, 0 and 2, and 1 and 2. For those indifferent between good 0 and good 1 the following equality must be true:

\[
\frac{p^0}{q^0(v,w)} = \frac{p^1}{q^1(v,w)} \iff \frac{p^0}{\exp(vc^0 + wd^0)} = \frac{p^1}{\exp(vc^1 + wd^1)}
\]

Taking into account that \( (c^i - c^0) = \alpha \) and that \( (d^i - d^0) = 0 \), this can be simplified to

\[
v = \overline{v} = \frac{1}{\alpha} \log\left(\frac{p^1}{p^0}\right)
\]

where \( \overline{v} \) represents the weight that these individuals attach to the first characteristic.

---

\(^3\) The respective cumulative distribution functions are \( F(v) = 1 - e^v \) and \( F(w) = 1 - e^w \).
Following the same kind of reasoning, the individuals who are indifferent between buying good 0 and good 2, are those for whom

\[ w = \frac{1}{\beta} \log \left( \frac{p^2}{p^0} \right). \]

Similarly, those who are indifferent between good 1 and good 2 are defined by

\[ w = \frac{\alpha}{\beta} v + \frac{1}{\beta} \log \left( \frac{p^2}{p^1} \right). \]

We can place all this information into a figure that allows an easier identification of the combinations of \( v \) and \( w \) for which a consumer will buy a particular good.

From now on it will be assumed that the price charged by the non-innovating firms \( (p^0) \) is 1. This is a simplifying assumption that will not affect the results.

Integrating up the densities over the areas where good 1 is the most preferred, we obtain the share of the population that will buy that good. It can be shown that the referred share will be:
As we would expect, \( S'(\alpha, \beta) \) is increasing in \( \alpha \), i.e., the market share of firm L increases with the characteristic gap of good 1.

Multiplying the individual demand for good 1 by the share of consumers that buy this good we will get the total demand faced by firm L:

\[
D'(p^1, p^2; \alpha, \beta) = (p^1)^{-\frac{1}{\alpha}} \left[ 1 - \frac{\beta}{\alpha + \beta} (p^2)^{-\frac{1}{\beta}} \right] \frac{M}{2}.
\]

Notice that this demand has constant own-price elasticity equal to \( 1 + \frac{1}{\alpha} \), which will be useful to find the equilibrium prices of good 1 and good 2. Similar expressions and conclusions can be found for good 2.

Given that firms compete a la Bertrand, the equilibrium prices will be:

\[
p^1 = 1 + \alpha, \quad p^2 = 1 + \beta.
\]

Now we have all the necessary elements to write the equilibrium profits as a function of the quality gaps, which in turn depend on the differentiation degree to be chosen by the firms.

It is obvious that all firms that produce good 0 will make zero profits, as there are \( n \) firms producing exactly the same good, facing the same costs and competing a la Bertrand. As it was assumed the price of the basic good (good 0) to be 1, so must be the unit cost of production.

The equilibrium profits of firm L\(^4\) will be:

\[
\prod^L(\alpha, \beta) = \alpha (1 + \alpha) \left[ 1 - \frac{\beta}{\alpha + \beta} (1 + \beta)^{-\frac{1}{\beta}} \right] \frac{M}{2}
\]

\(^4\) For firm R the profit expression is similar - it is just a question of switching \( \alpha \) for \( \beta \).
If we denote \((i + \alpha)^{\frac{i}{\alpha}}\) by \(\psi(\alpha)\), and \((1 + \beta)^{\frac{i}{\beta}}\) by \(\psi(\beta)\)\(^5\), we can write the profit function for firm L as

\[\Pi^L(\alpha, \beta) = \frac{\alpha}{1 + \alpha} \psi(\alpha) \left[ 1 - \frac{\beta}{\alpha + \beta} \psi(\beta) \right] \frac{M}{2}\]

The term \(M/2\) on the profit function will be ignored from now on, insofar as we are not interested in the level of the profits itself, but on the choice of \(\alpha\) and \(\beta\) that maximise profits. Obviously, this assumption will not affect the results or the conclusions.

Having obtained the profit expression for the firms in question, we can now rapidly observe some features of its behaviour. As would be expected, profits are increasing in the characteristic gap in which firms have the advantage (specialisation characteristic), but decreasing in the other firm’s characteristic gap.

Analytically this can be seen by computing the partial derivatives of profits with respect to \(\alpha\) and \(\beta\). For firm L we will get the following expressions:

\[
\frac{\partial \Pi^L}{\partial \alpha} = \frac{\psi(\alpha)}{(1 + \alpha)^2} \left[ 1 - \frac{\beta}{\alpha + \beta} \psi(\beta) \right] + \frac{\alpha}{1 + \alpha} \psi'(\alpha) \left[ 1 - \frac{\beta}{\alpha + \beta} \psi(\beta) \right] + \frac{\alpha}{1 + \alpha} \frac{\beta}{(\alpha + \beta)^2} \psi(\alpha) \psi(\beta) > 0; \\
\frac{\partial \Pi^L}{\partial \beta} = \left[ \frac{\alpha}{1 + \alpha} \frac{\beta}{\alpha + \beta} \psi(\alpha) \psi'(\beta) + \frac{\alpha}{1 + \alpha} \frac{\beta}{(\alpha + \beta)^2} \psi(\alpha) \psi(\beta) \right] < 0
\]

Another feature that comes out naturally from this model and that is fairly intuitive and realistic has to do with what will be called “the direct effect” and “the indirect effect”. We would expect that if two firms make the same amount of technological progress in their respective specialisation characteristics, and the firms

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\(^5\) It can be easily shown that these functions have the following properties:

\[0 < \psi(x) < 1; \quad \psi'(x) > 0; \quad \psi''(x) < 0, \quad \forall x > 0\]
themselves had initially the same gap advantage\textsuperscript{6}, then both firms would still benefit from the innovation. Despite the fact that the rival firm, by increasing the quality of its specialisation characteristic, has a negative impact on the profit level of the firm under analysis, this is more than compensated by this firm advance in its own specialisation characteristic. This is the same as saying that the “direct effect” dominates the “indirect effect”.

This intuitive feature can be easily seen to exist in the model described above, by adding \( \frac{\partial \Pi^L}{\partial \alpha} + \frac{\partial \Pi^L}{\partial \beta} \), and simultaneously imposing that \( \alpha = \beta \):

\[
\frac{\partial \Pi^L}{\partial \alpha} + \frac{\partial \Pi^L}{\partial \beta} = \frac{\psi(\alpha)}{(1 + \alpha)^2} \left[ 1 - \frac{\beta}{\alpha + \beta} \psi(\beta) \right] + \frac{\alpha}{1 + \alpha} \frac{(\beta - \alpha)}{(\alpha + \beta)^2} \psi(\alpha) \psi(\beta) + \\
+ \frac{\alpha}{1 + \alpha} \psi'(\alpha) \left[ 1 - \frac{\beta}{\alpha + \beta} \psi(\beta) \right] \left[ 1 + \frac{\psi'(\alpha)}{\psi(\beta)} \frac{\psi'(\beta)}{\psi(\alpha)} \right].
\]

When we consider \( \alpha = \beta \), this expression simplifies to

\[
\frac{\partial \Pi^L}{\partial \alpha} + \frac{\partial \Pi^L}{\partial \beta} = \frac{\psi(\alpha)}{(1 + \alpha)^2} \left[ 1 - \frac{1}{2} \psi(\alpha) \right] + \frac{\alpha}{1 + \alpha} \psi'(\alpha) \left[ 1 - \psi(\alpha) \right]
\]

which is clearly positive, thus implying what we would intuitively expect.

Having presented the basic framework and some of its properties under which the questions and problems put forward in the previous chapter will be analysed, we will now proceed with their study.

\[\text{where } x \text{ stands for either } \alpha \text{ or } \beta.\]

\[\text{\textsuperscript{6} Obviously in the respective specialisation characteristics.}\]

\[\text{\textsuperscript{7} For firm R similar expressions will hold.}\]
CHAPTER 3 - PRODUCT DIFFERENTIATION WITH ASYMMETRIC FIRMS
3.1. Introduction

As was stressed in the introductory chapter, despite the fact that the Theory of Innovation and the Theory of Product Differentiation have been treated in the literature as two fully independent and compartmentalised theories, the inter-relations that exist between them are unquestionable.

The most evident connection is that, in order to differentiate products an innovative activity is usually required. Therefore, the Theory of Product Differentiation should, in some sense, have as background the Theory of Innovation.

Having acknowledged this, it is important to identify the links that connect these two theories, and exploit the new considerations and consequences that each theory will bring into play and that might affect the analysis and conclusions of the related theory.

One of the results that might be considered a consequence of the uncertainty feature generally associated with the innovative activity is the emergence of asymmetries between firms. Even if firms start from a perfectly symmetric position, investing the same amounts in R&D, devoting the same effort to that activity and choosing the same research design, there is a set of random factors that may determine different R&D outcomes for the firms. If this is the case, then the firms will become asymmetric. Insofar that this is a natural consequence of the innovative activity (a consequence that is often pursued by firms), the existence of asymmetric firms (or potentially asymmetric) should be considered when analysing the decision of product differentiation.

In spite of the fact that what was mentioned in the previous paragraph is almost a triviality, the economic literature dealing with this type of issues typically fails to consider the existence of asymmetric (or potentially asymmetric) firms. The reason for this gap in the literature seems to usually be justified by mathematical tractability reasons, and not that the consideration of such assumption will add new realistic features to the economic knowledge on the subject. Proof of this is that the consideration of asymmetric firms, besides giving a more authentic depiction of reality, also introduces into the analysis new incentives that are not present when the assumption of symmetric firms is made. So, along with the incentives to maximise
product differentiation, which will reduce competition, allow firms some market power and permits the practice of higher prices, the consideration of firm asymmetry will introduce a new class of incentives that may encourage firms to minimise product differentiation.

The economic intuition behind this statement was already explained in the first chapter: a firm, having some type of advantage over its rival, may find it to be profitable to minimise product differentiation and increase competition in the product market. The idea is that, by using this advantage (in product quality) it will capture consumers that previously bought the product from another firm, leaving the competition with a reduced demand and eventually driving its rival out of the market. As was pointed out by several authors, namely d'Aspremont, Gabszewicz and Thisse (79), such strategy will not be considered optimal in the context of symmetric firms, as the increased competition would be equally damaging for both firms.

This is also the result obtained by Tabuchi (94) and Irmen and Thisse (98), which, assuming a multi-dimensional product and symmetric firms, state that, in spite of minimising product differentiation in all but one characteristic, firms would have to maximise product differentiation in one product characteristic in order to relax price competition. The work undertaken by these authors constitutes an important contribute to the theory of product differentiation. However, there is still some room for improvement insofar as the possibility of asymmetric firms is not taken into account.

The main objective of this chapter is then to study how the presence of effective or potential firm asymmetry will affect the choice of the product differentiation level.

Along with the consideration of asymmetric firms, this chapter will also introduce into the analysis two other features, not contemplated by the above-mentioned authors, but which surely add to the model reality. These were already described in the previous chapter and concern the consideration of a product that can be horizontally and vertically differentiated, and the fact that R&D resources are usually scarce and thus, its allocation to the development of each product characteristic (i.e. definition of the research design, that will ultimately determine the degree of product differentiation) has to be given serious thought.
With respect to the first feature mentioned in the last paragraph, its introduction has to do with the fact that most commodities available in the market can be differentiated both horizontally and vertically. If it is true that for some product characteristics, consumers as a whole cannot reach an agreement to what is the preferred version of the good (horizontal differentiation), it is not less true that in other characteristics everybody agrees over the most preferred version of the commodity in question (vertical differentiation). So, the introduction of such a feature in the model, in the way that was already described in the previous chapter, will benefit reality adherence.

Concerning the R&D process itself, besides the fact that it is considered that there is a certain degree of uncertainty attached to its outcome, it will also be assumed that it can be more complex than the one implicitly present in the work of many authors. As was stressed earlier in this Thesis, the costs that the R&D activity involves are usually significant to the firm's life, but also limited. Therefore, in respect to the definition of research design and thus product differentiation, the management of such activity may be of primordial importance. Given the scarcity of resources, not all product characteristics will be able to be developed as optimally as one would like. It is then crucial to allocate these limited resources to the development of the particular product characteristics that will allow higher profit levels to be achieved. This choice has an implicit opportunity cost, as the resources that are devoted to the development of one product characteristic will not be available to the development of other characteristics.

This constitutes yet another feature of the Theory of Innovation whose importance goes beyond this theory and may influence related theories, as is the case of the Theory of Product Differentiation. Notice that, in what concerns this latter theory, the literature usually assumes that whatever the degree of product differentiation that is considered to be optimal, it is always possible to be reached not taking into account the financial constraints that usually firms face. The consideration of these constraints may undoubtedly affect the conclusions obtained with respect to the degree of product differentiation as not all "product profiles" will be feasible.
Although the model that will be used was presented in the previous chapter, and that many of the features described earlier can clearly be seen to emanate from the model itself, it is still necessary to develop the model so that it accommodates the assumption on the scarcity of R&D resources. Given this, we will now elaborate on the specifics of the R&D process and the way it is linked with the decision of product differentiation.

3.2. Developing the Model

Recalling the notion of characteristic gap vector, and the assumption that firms initially start from a symmetric position, it will be assumed that each firm starts with an advantage $S$ (measured in some quality unit) in its specialisation characteristic. Then, we can define the initial characteristic gap vector as:

$$(\alpha_0, \beta_0) = (S, S)$$

This kind of formulation is already introducing asymmetry in the model, although it does so in a symmetric way: firms start with the same amount of advantage, but in different characteristics.

The way the model is set up could be viewed as being a two-stage game. In the first stage firms decide how much they are willing to spend on the R&D activity. This may determine the probability of success of each firm in that activity, giving R&D a realistic uncertainty feature. As this is not the central point of study in this particular research work, it will be assumed that the probabilities of success in R&D are exogenously given, thus this first stage of the model will not be dealt within this Thesis. Moreover, it will be assumed that the probability of success in that activity is equal to both firms in both product characteristics, and will be represented by $p$. At this point of the exposition, it is worthwhile to point out that it would certainly be more realistic to consider different probabilities of success for each characteristic. However, this would imply the introduction of more parameters into the model, create
new incentives to allocate R&D resources to a particular characteristic and increase its complexity. If, in any case, we would like to know what would be the impact of having different probabilities of success, intuitively speaking, it is fairly easy to understand that the results obtained with this simplified version would change, so that higher R&D investment is made on the characteristics with higher probability of success. Despite this, this remark still constitutes a direction for further research.

In the second stage of the game, firms have to decide the degree of product differentiation they want. The way this is formalised is described next.

Let us assume that firms have an R&D budget, which they have to allocate between characteristics. If the whole budget is devoted to just one characteristic, then the potential improvement that can be made in that characteristic is given by \( g \cdot S \), where \( g \) represents the proportion of total initial gap that is potentially available for discovery. In order to keep things simple, we may assume that if a firm invests \( X\% \) of its R&D budget in a particular characteristic, then the potential improvement that can be made on that characteristic is proportional to \( X\% \cdot g \cdot S \). The choice of \( X \) will define the research design firms want to adopt, which implicitly expresses the firms desire to differentiate or not.

Thus, the decision in this second stage of the game will be:

- \( a \) = percentage of the R&D budget that firm L devotes to the 2nd characteristic
- \( b \) = percentage of the R&D budget that firm R devotes to the 1st characteristic

The differentiation choice will be observed in the research design that firms will want to adopt. If a firm chooses to spend a large proportion of its R&D budget in developing the characteristic in which it is not specialised, then we can conclude that this firm is trying to make its product more alike to its rival's. This translates into a minimum differentiation strategy. On the other hand, if this firm allocates a great deal of its R&D budget to its own specialisation characteristic, this signifies that the firm in

---

1. The strategic considerations made earlier about the incentives to differentiate products or not, which is the basic question of interest in this Thesis, would have to be weighted with other considerations that have to do with probabilities relative magnitude.
2. Obviously \( g > 0 \).
question wants to have its product as differentiated as possible giving rise to a maximum differentiation result.

Having all this defined we can write the gap values for each characteristic and for each firm, conditional on its own success and the success of its rival. As it is obvious, each firm starts with its own initial advantage in the specialisation characteristic (S), to which it has to add the progress made by itself on that characteristic, minus the progress the rival has made in that same characteristic. Therefore the gap expressions are given by:

- If firm both firms are successful:
  \[\alpha_s = S + S_g(1 - a) - S_g b\]
  \[\beta_s = S - S_g a + S_g(1 - b)\]

- If neither firm is successful:
  \[\alpha_n = S\]
  \[\beta_n = S\]

- If only firm L is successful:
  \[\alpha_L = S + S_g(1 - a)\]
  \[\beta_L = S - S_g a\]

- If only firm R is successful:
  \[\alpha_R = S - S_g b\]
  \[\beta_R = S + S_g(1 - b)\]

Notice that firms start from a similar initial position where they both have the same “potential improvement available” which implies the existence of symmetric firms. However, after the uncertainty of the R&D is resolved, firms may end up getting different outcomes from this activity (hence different gaps), which will introduce the asymmetry between firms and will have consequences in the competition in the product market.

As *ex ante* firms are symmetric, the whole analysis will be undertaken considering just firm L. This is made in order to avoid repetition, but it will not affect
the generality of the results obtained. Obviously, all that will be said about firm L also applies to firm R, whenever this firm finds itself in a similar context.

3.3. Results

As was mentioned when describing the organisation and structure of this Thesis, both the non-cooperative and the cooperative setups will be analysed. First the non-cooperative case is presented, and, after this is done, the cooperative situation will be studied. This will allow a better understanding of the consequences that cooperation in R&D will have on the degree of product differentiation chosen.

3.3.1. The Non-Cooperative Equilibrium

Given all that was mentioned before, it is not surprising that the expected profit expression firm L is interested in maximising is the following:

\[
\pi_L^* = p^2 \pi_L(\alpha_b, \beta_b) + (1 - p) p \pi_L(\alpha_L, \beta_L) + p(1 - p) \pi_L(\alpha_R, \beta_R) + (1 - p)^2 \pi_L(\alpha_s, \beta_s)
\]

An important point to note is that if a firm is not successful in the R&D activity, the term containing the respective gaps will not include that firms' decision variable. The rationale for this is the following: if the firm didn't make a discovery, then whatever the research path that might have been chosen it would be inconsequential for the characteristic gaps. This way, when computing a firm's optimal strategy we will only be concerned with the possibility of that firm being successful.

So, firm L's expected profit, conditional on being successful, can be expressed as:

\[
\pi_L^* = p \pi_L(\alpha_b, \beta_b) + (1 - p) \pi_L(\alpha_L, \beta_L)
\]

where \( p \) now represents the conditional probability of the rival's success in R&D, conditional on the success in that activity of the firm in question.
In order to proceed with the analysis, and to make it more understandable, we will start by analysing the expression above term-by-term, which in practice may be viewed as assuming that the R&D activity has no uncertainty associated. This will help us to understand the forces at work behind each term. The more realistic case of R&D uncertainty will then be a “weighted average” of the forces implied by those terms.

3.3.1.1. Certainty Case

In the study of this case, it will be considered that firms know, before even starting the R&D activity, if they will be successful. Moreover, firms also have complete information about what the outcome of the rival firm’s R&D process will be. As was mentioned earlier, the aim of analysing this context is not a wish to depict reality, but to understand each element that composes it.

3.3.1.1.1. The “Both Firms Innovate” Term

The relevant profit expression to be maximised, from firm L’s point of view is given by:

\[ \text{Max}_{a} \pi_{s}(\alpha_{s}, \beta_{s}) = \pi_{s}[S + Sg(1 - a) - Sbg, S + Sg(1 - b) - Sga] \]

The first order condition, with respect to firm L’s decision variable, \( a \), will then be:

\[ \frac{\partial \pi_{L}}{\partial a} = \frac{\partial \pi_{L}}{\partial \alpha_{b}} \frac{\partial \alpha_{b}}{\partial a} + \frac{\partial \pi_{L}}{\partial \beta_{b}} \frac{\partial \beta_{b}}{\partial a} = 0 \]

where:

\[ \frac{\partial \alpha_{b}}{\partial a} = \frac{\partial \beta_{b}}{\partial a} = -Sg < 0 , \text{ and } \alpha_{b} = \beta_{b}. \]

Therefore

\[ \frac{\partial \pi_{L}}{\partial a} = \left( \frac{\partial \pi_{L}}{\partial \alpha_{b}} + \frac{\partial \pi_{L}}{\partial \beta_{b}} \right) \frac{\partial \alpha_{b}}{\partial a} \]

\[ > 0 \quad < 0 \quad < 0 \]
As proved earlier, when the model was introduced, the direct effect dominates the indirect effect, which is to say \( \frac{\partial \pi^L}{\partial \alpha_b} > \frac{\partial \pi^L}{\partial \beta_b} \), so the term in brackets is positive. Notice that this is only valid because we know that \( \alpha_b = \beta_b \). This results in \( \frac{\partial \pi^L}{\partial a} \) being negative. Therefore, firm L will opt for the minimum value of \( a \) possible \((a=0)\) in order to maximise its profit level, which, in economic terms, means that all of the R&D potential will be devoted to its own specialisation characteristic. This obviously implies a clear wish to maximise product differentiation. If firms chose to produce products as similar as possible \((a=I)\) they would be fomenting a fierce competition in the product market that would be equally damaging for both firms. This is in accordance with the type of arguments used by d’Aspremont et al. (79) to justify the maximum product differentiation result obtained in the one-dimensional model. Notice that by maximising product differentiation firms will avoid competition, gain some market power and will set higher prices. This can easily be seen by looking at the expression for the price equilibrium, computed in the introductory chapter:

\[
p^L = 1 + \alpha
\]

The higher the characteristic gap in the specialisation characteristic, the higher the price that will be set. As \( \alpha \) depends negatively of \( a \), the lower the value of \( a \), the higher will be \( \alpha \) and hence the price. Not gaining any additional demand by lowering the prices\(^3\), the best possible strategy is to try to guarantee some market power so that higher prices can be charged. This can be done by differentiating products as much as possible.

So, this term represents a force pushing the result towards a Maximum Differentiation outcome.

This contrasts with the results presented by Irmen and Thisse (98), which, for a symmetric firm setup, defend minimisation of product differentiation in all characteristics, except one. The reasons for this difference in the results obtained are intimately linked with one assumption that was made in this work and that is not present in Irmen and Thisse (98). In this Thesis it is explicitly assumed that R&D

\(^3\) The rival firm would do the same, and each firm’s demand would remain constant.
resources are scarce, thus if a firm wishes to develop one particular product characteristic it can do so at the cost of not developing the other characteristic. The choice of the firm is considered to be made in a way that it has to allocate all its R&D resources to one characteristic or the other, or as a linear combination to both characteristics. Therefore, a result of Max-Min as the one obtained by the above-mentioned authors, cannot, strictly speaking, be an outcome in the context considered for this research work. If a firm wants to develop its own specialisation characteristic and directs all its R&D effort to that objective (thus maximising product differentiation in that particular characteristic), it will not have any resources left to improve on the other product characteristic. For this reason, minimising product differentiation in the rival's specialisation characteristic while maximising the differentiation in its own specialisation characteristic is something that is out of reach in the setup considered. Notice however that this setup is flexible enough to allow a strategy somewhat similar to the one pointed out as optimal by Irmen and Thisse (98). In fact, the possibility of developing the specialisation characteristic whilst improving on the other characteristic is contemplated in this model, but it cannot be taken to the extreme of Max-Min. Such strategy can be depicted in this model by considering, for instance, a value of \(a\) equal to \(\frac{1}{2}\). In this case, the R&D effort would be equally divided by the two product characteristics, causing the differentiation to increase in the specialisation characteristic and decrease in the other characteristic. However, strictly speaking, it would not be correct to say that this strategy maximises product differentiation in one characteristic and minimises it in the other.

Despite all that was said concerning the work of Irmen and Thisse (98), it should be noted that it can be valid, namely when considering products for which the R&D costs (or the costs of changing product characteristics) are neglectable. This seems to be the case in the example about the international magazines “Newsweek” and “Time” presented by those authors.
3.3.1.1.2. The “Only Firm L Innovates” Term

In this case, one and only one firm will succeed when pursuing the R&D activity, which means that firms know with certainty that after the R&D process is concluded they will become asymmetric in a way that will affect competitiveness.

The relevant profit expression to be maximised is given by the following expression:

\[ \max_\pi \pi_L(\alpha_L, \beta_L) = \pi_L[S + Sg(1 - a) - Sbg, S + Sg(1 - b) - Sga] \]

Similarly to what was done in the previous case, the first order condition, computed with respect to firm L’s decision variable is:

\[ \frac{\partial \pi_L}{\partial \alpha_L} = \frac{\partial \pi_L}{\partial \alpha_L} \frac{\partial \alpha_L}{\partial a} + \frac{\partial \pi_L}{\partial \beta_L} \frac{\partial \beta_L}{\partial a} = 0 \]

where: \( \frac{\partial \alpha_L}{\partial a} = \frac{\partial \beta_L}{\partial a} = -Sg < 0 \), and \( \alpha_L \geq \beta_L \).

As was made for the case of “Both Firms Innovate”, the above expression simplifies to:

\[ \frac{\partial \pi_L}{\partial \alpha_L} = \left( \frac{\partial \pi_L}{\partial \alpha_L} + \frac{\partial \pi_L}{\partial \beta_L} \right) \frac{\partial \alpha_L}{\partial a} > 0 \quad < 0 \quad < 0 \]

However, now nothing can be said about the sign of the term in brackets, except that it may be positive or negative depending on the relative magnitudes of \( \alpha \) and \( \beta \). Despite the fact that the own gap effect is stronger for the same gap size, now we have \( \alpha_L \geq \beta_L \), and, as a model feature, decreasing returns on the gap size. So, it may well be that the second term in brackets dominates the first, resulting in a negative sum. If this is to be the case, then we will get: \( \frac{\partial \pi_L}{\partial a} > 0 \), and consequently the optimal value for \( a \) will be 1. Obviously it would imply a Minimum Product Differentiation result. However, this is not guaranteed to happen and it only may occur if the value of
\( \alpha \) is relatively larger than the value of \( \beta \). Otherwise we would get a Maximum Product Differentiation result as before.

Keeping in mind that the relative magnitude of \( \alpha \) and \( \beta \) may be decisive in the determination of the sign of \( \frac{\partial \pi^L}{\partial a} \), we can resort to graphical analysis in order to observe the behaviour of the profit function.

Assuming that \( S \) is equal to 1, and letting \( a \) and \( g \) vary between 0 and 1, the shape of the profit function, for the case where only one firm innovates, is as presented in figure 3.1.

![Figure 3.1. - Firm L's profit as a function of \( a \) and \( g \), considering \( S=1 \).](image)

As we can easily observe, the optimal value of \( a \) is zero, pointing to a Maximum Differentiation result. We can also confirm what we would intuitively expect: the profit level increases with the potential improvement available (\( g \)). Just by itself this does not provide interesting results, however, and making use of the theoretical analysis made earlier, we already know that the magnitude of the asymmetry plays an important role. Therefore, if we allow gap magnitudes to be relatively larger, we can easily see that interesting results will start to emerge. This can be done by assuming higher values for the scaling factor, \( S \).

Several values for \( S \) were considered. When these values are relatively low (e.g. \( S=1.5 \)), the conclusions do not change - a Maximum Differentiation result is obtained (figure 3.2). However, for values of \( S \) like 5 or 20 (figures 3.3 and 3.4), the conclusions are quite different:
We can easily observe from the plots that the shape of the profit function has changed. This is much more noticeable if we consider a large value for $S$ ($S=20$). From the 3D plots we can perceive that the profit function is increasing in $g$ at a decreasing rate for relatively small values of $a$, and is increasing in $g$ at an increasing rate for large values of $a$. Given these concavities and convexities we can conclude that, for small values of $g$, the optimal research design is one in which products will become as differentiated as possible sustaining the Maximum Differentiation Principle, whereas in the case of a large $g$, firm L will choose to minimise differentiation ($\alpha=1$) giving rise to a Minimum Differentiation Principle (if the asymmetry is large enough).
A table containing the results of numerical simulations\textsuperscript{4} for the cases $S=1.5$, $S=5$ and $S=20$ follows showing that the Maximum Differentiation Principle does not hold in presence of sufficiently asymmetric firms:

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$g$ & $S=1.5$ & $S=5$ & $S=20$ \\
\hline
.1 & 0 & 0 & 0 \\
.2 & 0 & 0 & 1 \\
.3 & 0 & 0 & 1 \\
.4 & 0 & 0 & 1 \\
.5 & 0 & 0 & 1 \\
.6 & 0 & 0 & 1 \\
.7 & 0 & .967 & 1 \\
.8 & 0 & 1 & 1 \\
.9 & 0 & 1 & 1 \\
\hline
\end{tabular}
\caption{Optimal choice of $a$, for different levels of $g$ and $S$}
\end{table}

The plots of the relevant profit function and table 3.1, give an idea of how the optimal value of $a$ changes with $g$ and $S$. When we consider $S=20$, we could set a decision turning point at $g=2$, i.e., for $g<2$ the optimal $a = 0$; for $g\geq 2$, the optimal $a$ is strictly positive.

However, if the value of $S$ considered is 5, in order for firm L to decide to lower the product differentiation the value of $g$ must be higher ($g > .6$). This reveals some degree of substitutability between the variables $S$ and $g$, which, qualitatively affects the gap dimension in the same way.

From the results above we can conclude that firms are willing to minimise product differentiation provided that:

\begin{itemize}
\item[i)] they know that will succeed alone in the R&D activity;
\item[ii)] the initial quality advantage in the specialisation characteristic ($S$) is sufficiently large;
\end{itemize}

\textsuperscript{4}The programs supporting this and all other numerical simulations presented in this Thesis can be made available upon request.
iii) the percentage of improvement that can be made from the initial gap advantage \( (g) \) is large enough.

This suggests that firms have to first feel comfortable, have a relatively large advantage in its own specialisation characteristic and not feel threatened by the possibility of the rival firm truncating that advantage \( ^5 \), before starting to venture in other areas in which the rival is specialised.

Given this, the idea that the assumption of symmetric firms could be conditioning the results is proven to be true. By guaranteeing that only one firm will succeed in the R&D activity, thus assuring that firms will become asymmetric, we can see that the decision to differentiate products is changed whenever the degree of asymmetry is sufficiently large. Moreover, we can also observe that as we increase the size of the asymmetry (values of \( g \) and \( S \)), the willingness of the successful firm to produce similar products is also increased.

Having understood the behaviour of each term, the analysis of the uncertainty case is now somewhat simplified: we now know that the term representing the case where both firms succeed in R&D pushes the result towards a maximum product differentiation decision, while the term depicting the case of a sole innovator points to a minimum product differentiation decision, provided that the degree of asymmetry is large enough, and to maximum differentiation otherwise. As the expected profit function for the R&D uncertainty setup is a weighted average of the two terms, we expect the results to be somewhere between those obtained for the case where both firms innovate and the case where only one firm succeeds. Notice however that this expectation is by no means certain to become true, as we can have one term dominating over the other (obviously for values of \( p \) different from 0 and 1).
3.3.1.2. Uncertainty Case

In this case firms will have to consider the possibility of being sole innovators along with the possibility of being successful together with its rival. From the analysis made in the previous sections we already know in which direction each term of the expected profit function pushes the result. The outcome of these two forces working simultaneously depends not only on the relative strength of each term alone but also on the way a firm weights the possibility of being successful alone or together with its rival. This latter effect is expressed on the value of the conditional probability $p$.

As stated previously, the relevant expected profit function, conditional on the success of the firm in question, is given by the following expression:

$$\pi'_L = p\pi_L(\alpha_b, \beta_b) + (1 - p)\pi_L(\alpha_L, \beta_L)$$

Given the complexity that would result if we opted to study the above function analytically, numerical simulations were run in order to observe its behaviour and determine the value of $\alpha$ which maximises the expected profit level.

In order to run the numerical simulation, the value of $S$ considered was 20, as, from the previous section, it was observed that the degree of (potential) asymmetry was an important factor that, if constrained, could condition the results. Therefore, a relatively large value of $S$ will contemplate a sufficiently wide range of possible asymmetry magnitudes, thus revealing the whole sort of obtainable results.

The values of $g$ considered for the purpose of the simulation are between 0,1 and 0,5, inclusive. The idea behind the choice of such range is that, on one side, a value of $g = 0$ would not make sense, as $g$ is the quality development measured as a percentage of the initial gap advantage ($S$). Therefore a value of 0 would mean that no improvement would be made, even if a firm succeeds in innovating. On the other hand, values of $g$ larger than 0,5 would not bring anything new or relevant to the analysis.

---

5 As would happen if the rival firm also succeeded in R&D and had allocated some effort to the development of the other firm's specialisation characteristic.
6 Moreover larger values for $g$ would cause some problems of equilibrium existence in future chapters, where, through the presence of spillovers, a large amount of improvement might lead to leapfrogging, which is not contemplated in the framework utilised.
An increase in $g$ would mean that the advances made could be larger, thus increasing the degree of potential asymmetry (if only one firm innovates). In this matter, the consideration of larger values for $g$ would solely be replicating the role of $S$, which already contemplates the possibility of large asymmetries. As was observed before there is a certain degree of substitutability between these two variables.

With respect to the values considered for $p$ in the numerical simulation, it is obvious that, whenever $p = 0$ we will be facing a situation where there is only one firm succeeding in R&D, whereas whenever $p = 1$ we have the certainty case where the two firms innovate. Both of these cases were already analysed in the previous sections. For this reason the extreme values of $p = 0$ and $p = 1$, will be left out of this simulation.

For the range of values considered for the variables $p$, $g$ and $S$ the symmetric Nash equilibrium was computed, giving rise to the following equilibrium values for $a$:

<table>
<thead>
<tr>
<th>$g$</th>
<th>$p$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>.2</td>
<td>1</td>
<td>.319</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>.3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>812</td>
<td>199</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>.4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>.853</td>
<td>.453</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>.5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>.967</td>
<td>.485</td>
<td>.363</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.2. Equilibrium values of $a (= b)$, for different pairs $(g, p)$, considering $S=20$

From the table we can easily observe that the basic conclusions drawn for the certainty case where only one firm succeeded in R&D, can be stated for this more realistic uncertainty case.

Notice however that the tendency to differentiate the products less, is now, as expected, somewhat softened by the “averaging” with a term that always pushes the results towards Maximum Product Differentiation (the case where both firms succeed in innovating), specially for large values of $p$.

The higher the value of $p$, the more we weight the situation where both firms succeed in innovating, therefore increasing the likelihood of ending up in a symmetry.

---

7 Remember that firms start from a symmetric context in spite of contemplating a potential asymmetric situation for the future.
context. Faced with this prospect, firms wish to differentiate their products more. The reason for such behaviour was already explained and has to do with the fact that, if the probability of keeping the symmetry is large, then firms wish to differentiate products as much as possible in order to avoid price competition. By decreasing $p$ the probability of firms becoming asymmetric increases which gives the incentive to minimise product differentiation, dominating the "less competition" argument, as they are more likely to succeed alone\(^8\). Therefore, if $p$ is relatively small, firms will value more the possibility of ending with a product that is not far behind in terms of quality in the rival's specialisation characteristic while keeping a comfortable advantage in its own specialisation characteristic. So, the disadvantages of increased price competition are more than compensated by the increase in demand. There will be a class of consumers that are willing to change its product choice: if before they bought firm R’s product, they may now find firm L’s product more attractive. By losing a little in quality terms on the second product characteristic, they will more than offset that loss with the great quality of the product in the first characteristic.

Notice as well that the asymmetry size still plays an important role in the decision to differentiate or not when R&D uncertainty is present. By increasing the value of $g$ (or $S$), the potential asymmetry will be larger, increasing the firms willingness to produce similar products.

It is also interesting to examine the trade-off that exists between $p$ and $g$. We can see from the table that the larger the value of $p$, the larger the value of $g$ needed to trigger a less differentiation decision. A higher value for $p$ implies a lower probability of an asymmetric outcome, thus reducing the incentives to minimise product differentiation. If we want firms to differentiate less, then we have to “compensate” them for the loss on the probability of being sole innovators. This can be done by increasing the potential asymmetry size (increasing the value of $g$).

\(^8\) As we have seen before the term representing asymmetric firms pushes the result towards minimum differentiation provided the asymmetry degree is large enough.
An example that can be given to illustrate a similar situation to the one presented in the model and that, to some extend, supports the conclusions obtained, relates to a well-known situation in the car manufacturer industry. Generally speaking, this industry fits the characteristics presented at the beginning of this work: there is firm asymmetry; the outcome of R&D is uncertain; R&D budgets are limited and car manufacturers have to decide which car characteristics to develop, knowing beforehand that it will be impossible to improve all of them by a desired amount, given the budget constraint.

The Swedish car manufacturer Volvo has grown a world-wide reputation for manufacturing safe cars. For many years, a great deal of the Volvo R&D efforts were directed to the development of new and improved safety features, most of the times overlooking aesthetics issues. Only in a recent past did this car manufacturer decide to make Volvos more “pleasant to the eye”. This change in the R&D strategy may be explained by the model and results presented above. The results obtained suggest that firms have to first feel comfortable and have a relatively large advantage in its own specialisation characteristic (when compared to the rival’s quality gap), probably growing and maintaining some kind of reputation (as safety is for Volvo vehicles). Only after this stage will firms venture in other areas, in which rivals (e.g. BMW) are specialised. Consequently for Volvo, this meant the recent attempts to make them more attractive by developing some “sporty” features.

3.3.2. The Cooperative Equilibrium

The research concerning cooperation in the R&D activity despite being vast usually focuses on issues like the level of R&D expenditure that is done or the implementability and stability of such agreements. Topics like the impact that this type of cooperation might have on the level of product differentiation chosen by firms are typically left out of the analysis. The knowledge of such aspects grows in importance as R&D cooperation is one of the few types of cooperation that is allowed and even encouraged by competition policies. This becomes even more relevant when we recognise that the level of product differentiation and variety of products available to the consumer affect their welfare, as it is well documented in the literature.
At this point, and before starting the study of the cooperative equilibrium it is worthwhile to recall what notion of R&D cooperation will be used. It will be assumed that, by cooperating in R&D, firms will aim at maximising the expected joint profit, coordinating, for that purpose, the research design. An assumption that is made throughout this Thesis and whenever the cooperative equilibrium is studied, is that the agreement reached by firms with respect to the research design chosen is fully enforceable. In spite of the fact that we can imagine situations where it would be profitable for one firm not to respect the agreement, this work will not address this type of issues. The focus will be put on the product differentiation decision, leaving the questions about implementability and enforcement as a direction to further research.

When firms coordinate their research design in order to maximise the expected joint profit, they will have to choose the values of $a$ and $b$ that maximise the following expression:

$$\pi_L^* + \pi_R^* = p^2 \pi_L(a_b, \beta_b) + (1 - p)p \pi_L(\alpha_L \beta_L) + p(1 - p)\pi_L(\alpha_R \beta_R) + (1 - p)^2 \pi_L(\alpha_n \beta_n) + p^2 \pi_R(a_b, \beta_b) + (1 - p)p \pi_R(\alpha_L \beta_L) + p(1 - p)\pi_R(\alpha_R \beta_R) + (1 - p)^2 \pi_R(\alpha_n \beta_n)$$

Rearranging the above expression we will obtain:

$$\pi_L^* + \pi_R^* = p^2 [\pi_L(\alpha_b, \beta_b) + \pi_R(\alpha_b, \beta_b)] + (1 - p)p[\pi_L(\alpha_L \beta_L) + \pi_R(\alpha_L \beta_L)] + p(1 - p)[\pi_L(\alpha_R \beta_R) + \pi_R(\alpha_R \beta_R)] + (1 - p)^2 [\pi_L(\alpha_n \beta_n) + \pi_R(\alpha_n \beta_n)]$$

where the quality gaps are the same as defined before.

In order to analyse the equilibrium level of product differentiation, and following the same structure used for the non-cooperative situation, a term-by-term analysis follows. As already explained this is equivalent to the study of a case where firms have complete information about the R&D outcome: certainty case. Once this is done, and having understood the forces behind each term of the above expression, the analysis of the whole expression will take place, corresponding to the more realistic case of R&D uncertainty.
3.3.2.1. Certainty Case

3.3.2.1.1. The "Both Firms Innovate" Term

In this case the aim is to maximise the expression: \( \pi_L(\alpha_b, \beta_b) + \pi_R(\alpha_b, \beta_b) \), by choosing the equilibrium values of \( a \) and \( b \).

Computing the first derivative of the above expression with respect to \( a \), we obtain:

\[
\frac{\partial(\pi_L + \pi_R)}{\partial a} = \frac{\partial \pi_L}{\partial a} \frac{\partial \alpha_b}{\partial a} + \frac{\partial \pi_L}{\partial \beta_b} \frac{\partial \beta_b}{\partial a} + \frac{\partial \pi_R}{\partial a} \frac{\partial \alpha_b}{\partial a} + \frac{\partial \pi_R}{\partial \beta_b} \frac{\partial \beta_b}{\partial a}
\]

It is now useful to remember the following: \( \frac{\partial \alpha_b}{\partial a} = -\frac{\partial \beta_b}{\partial a} = -sg < 0 \). This is trivially derived from the quality gap expressions already presented. On the other hand, we also know, from the chapter where the model was explained, that: \( \frac{\partial \pi_L}{\partial \alpha_b} > 0; \frac{\partial \pi_L}{\partial \beta_b} < 0; \frac{\partial \pi_R}{\partial \alpha_b} < 0; \frac{\partial \pi_R}{\partial \beta_b} > 0. \)

Keeping this in mind, the expression above can be re-written as:

\[
\frac{\partial(\pi_L + \pi_R)}{\partial a} = \left[ \left( \frac{\partial \pi_L}{\partial \alpha_b} + \frac{\partial \pi_L}{\partial \beta_b} \right) + \left( \frac{\partial \pi_R}{\partial \alpha_b} + \frac{\partial \pi_R}{\partial \beta_b} \right) \right] \frac{\partial \alpha_b}{\partial a}.
\]

It was already proven in the previous chapter that the two terms in round brackets are positive. This has been called the dominance of the direct effect over the indirect effect. Thus the resulting sum of what is in square brackets is undoubtedly positive. Given that \( \frac{\partial \alpha_b}{\partial a} \) is negative, we will obtain that the derivative \( \frac{\partial(\pi_L + \pi_R)}{\partial a} \) is always negative. This result implies that the optimal value for \( a \) is zero. Given the firms' symmetry, this result can be generalised to firm R and its decision variable, \( b \).

This means that, if firms are sure that both will succeed in the R&D activity and will keep their symmetry, then the best strategy for the research design coordination is
to go for Maximum Product Differentiation. The intuition behind this result is simple and just points to the fact that firms will want to avoid any kind of competition in the product market by differentiating products as much as possible.

Therefore, in a context of certainty about the success of both firms in the R&D activity, the fact that firms cooperate in R&D brings nothing new to the product differentiation decision when compared with the non-cooperative setup, with the same type of reasons justifying this choice. The decision will then be to always maximise product differentiation.

3.3.2.1.2. The “Only firm L Innovates” Term

In this case we want to find the value of $a$ that maximises the term that represents the case where only firm L succeeds in R&D, i.e., $\pi_L (\alpha_L, \beta_L) + \pi_R (\alpha_L, \beta_L)$. Given the symmetric set-up, and the fact that in such a case the decision variable of the firm that did not innovate is not present in the expression (given the quality gap definitions considered), the analysis for the case where firm R is the sole innovator is similar. Thus, in order to avoid unnecessary repetition, we should keep in mind that all conclusions drawn with respect to firm L’s decision variable, are valid for firm R whenever this firm finds itself in a similar situation.

The research design chosen by firms (in coordination) that will lead to the maximisation of joint profits can be found by the computation of the first derivative of $\pi_L (\alpha_L, \beta_L) + \pi_R (\alpha_L, \beta_L)$, with respect to $a$. Notice that, as already mentioned, the decision variable $b$ is not present in the expression and, as such, its choice is irrelevant. This stems from the fact that, if firm R did not innovate, the research design chosen previously will not have any consequences on the profit level.

The first order condition for this maximisation problem can be expressed as:

$$\frac{\partial (\pi_L + \pi_R)}{\partial a} = \frac{\partial \pi_L}{\partial \alpha_L} \frac{\partial \alpha_L}{\partial a} + \frac{\partial \pi_L}{\partial \beta_L} \frac{\partial \beta_L}{\partial a} + \frac{\partial \pi_R}{\partial \alpha_L} \frac{\partial \alpha_L}{\partial a} + \frac{\partial \pi_R}{\partial \beta_L} \frac{\partial \beta_L}{\partial a} = 0$$
As in the previous case, despite the fact that the quality gaps are different, we obtain that: \( \frac{\partial \alpha_{L}}{\partial a} = \frac{\partial \beta_{L}}{\partial a} = -Sg < 0 \). Thus, the above expression simplifies to:

\[
\frac{\partial (\pi_{L} + \pi_{R})}{\partial a} = \left[ \frac{\partial \pi_{L}}{\partial \alpha_{L}} + \frac{\partial \pi_{L}}{\partial \beta_{L}} + \frac{\partial \pi_{R}}{\partial \alpha_{L}} + \frac{\partial \pi_{R}}{\partial \beta_{L}} \right] \frac{\partial \alpha_{L}}{\partial a}
\]

Given the knowledge that we have about the sign of the derivatives in square brackets, it is not straightforward to draw any conclusion about the sign of \( \frac{\partial (\pi_{L} + \pi_{R})}{\partial a} \). However, we can, in some way, simplify the analysis of the sum in the square brackets by dividing that expression in two terms. Let us start with the term

\[
\frac{\partial \pi_{L}}{\partial \alpha_{L}} + \frac{\partial \pi_{R}}{\partial \alpha_{L}}.
\]

Recalling the expressions obtained for these partial derivatives in the chapter where the model was introduced, it is easy to show that this sum will result in:

\[
\psi(\alpha) \left[ \frac{1 - \beta}{\alpha + \beta} \psi(\beta) \right] + \left[ \frac{\alpha(\alpha - \beta \psi(\beta)) + (\alpha \beta + \alpha^{2} \beta + \alpha \beta^{2})(1 - \psi(\beta))}{(1 + \alpha)(1 + \beta)(\alpha + \beta)} \right] \psi'(\alpha) + \left( \frac{\alpha}{1 + \alpha} - \frac{\beta}{1 + \beta} \right) \frac{\beta}{(\alpha + \beta)^{2}} \psi(\alpha) \psi(\beta)
\]

This expression is undoubtedly positive since, as we know, firm L is the sole innovator implying that \( \alpha_{L} > \beta_{L} \). This assures that each of the three terms in the expression are positive.

We have then guaranteed that the sum \( \frac{\partial \pi_{L}}{\partial \alpha_{L}} + \frac{\partial \pi_{R}}{\partial \alpha_{L}} \) is positive. It remains now to show that the term \( \frac{\partial \pi_{L}}{\partial \beta_{L}} + \frac{\partial \pi_{R}}{\partial \beta_{L}} \) is also positive. Once this is done it will be straightforward to reach a conclusion about the sign of \( \frac{\partial (\pi_{L} + \pi_{R})}{\partial a} \), and the optimal value of \( \alpha \) to be chosen.
With respect to the analysis of the sign that \( \frac{\partial \pi_L}{\partial \beta_L} + \frac{\partial \pi_R}{\partial \beta_L} \) assumes, the analytical study of the term proves to be very complex not enabling any conclusions from the enormous expression obtained. However, in spite of this difficulty, the use of graphical analysis and numerical simulation are useful and conclusive about the sign of the sum in question.

From the 3D plots of \( \frac{\partial \pi_L}{\partial \beta_L} + \frac{\partial \pi_R}{\partial \beta_L} \), we can easily observe that this sum is always positive, whatever the values of \( \alpha_L \) and \( \beta_L \).

![3D plot of sum](image)

**Figure 3.5. First derivative of joint profit with respect to \( \beta_L \).**

We can also see, by changing the variation ranges of \( \alpha_L \) and \( \beta_L \), that the value of the sum tends to zero as the dimension of the quality gaps increases, but it will never be negative.

Therefore, being \( \frac{\partial \pi_L}{\partial \alpha_L} + \frac{\partial \pi_R}{\partial \alpha_L} \) and \( \frac{\partial \pi_L}{\partial \beta_L} + \frac{\partial \pi_R}{\partial \beta_L} \) always positive, so is its sum.

Then we can conclude that \( \frac{\partial (\pi_L + \pi_R)}{\partial \alpha} = \left[ \frac{\partial \pi_L}{\partial \alpha_L} + \frac{\partial \pi_L}{\partial \alpha_L} + \frac{\partial \pi_R}{\partial \alpha_L} + \frac{\partial \pi_R}{\partial \alpha_L} \right] \frac{\partial \alpha}{\partial \alpha} \) is always negative, as the term in square brackets is positive and \( \frac{\partial \alpha_L}{\partial \alpha} \) is negative.

---

\(^9\) This can also be seen by using numerical simulation.
In economic terms this means that if firms cooperate in deciding the research design in order to maximise joint profits, knowing in advance that only one firm will succeed in innovating, then they will choose to maximise product differentiation. This strategy aims at reducing the level of competition that will follow in the product market and that can damage both firms.

From this analysis we can conclude that, by considering a cooperative agreement, the incentive an innovative firm might have in order to differentiate less and attract consumers that before bought the rival's product, thus increasing its own profit, is dominated by the consideration of the rival firm's profit, that, having not innovated, will see part of its profits transferred to the rival firm. In other words, this means that the gain the innovating firm will have by differentiating less is smaller than the loss inflicted in the non-innovating firm. The reason for such fact is that, in spite of the demand transference that would occur towards the product produced by the innovating firm might not affect the aggregate profit level, the fall in price due to the increased product competition resulting from such strategy would have negative effects. Thus, when firms cooperate and consider the joint profit as the object to be maximised, they will prefer to produce products as differentiated as possible so that, in aggregate terms, there is a gain.

From this analysis, the question that can now be put forward is: What is the interest of a firm, knowing that it will be the sole innovator, in engaging in this type of agreement? As was seen before, this firm might have more to gain if product differentiation is minimised. Then we would be facing problems of agreement enforcement, which are beyond the scope of this Thesis. This is a fact; however, the knowledge of such conclusions can be helpful for policy making with respect to the innovative activity and others.

Moreover, this result is extremely helpful when studying the situation of R&D uncertainty, insofar as by knowing the forces implied by each of the terms of the expected profit function

\[ \pi^*_L + \pi^*_R = p^2[\pi_L(\alpha, \beta) + \pi_R(\alpha, \beta)] + (1-p)p[\pi_L(\alpha_L, \beta_L) + \pi_R(\alpha_R, \beta_R)] + \]

\[ p(1-p)[\pi_L(\alpha, \beta) + \pi_R(\alpha, \beta)] + (1-p)^2[\pi_L(\alpha, \beta) + \pi_R(\alpha, \beta)] \]

we can, in this case, obtain conclusions for the decision of product differentiation, when firms are faced with uncertainty about the R&D outcome.
3.3.2.2. Uncertainty Case

The analysis of this case is now simplified by the term-by-term study done before. Being the relevant expression to be maximised

\[ \pi_L^* + \pi_R^* = p^2 [\pi_L(\alpha, \beta_L) + \pi_R(\alpha, \beta_R)] + (1-p)p[\pi_L(\alpha_L, \beta_L) + \pi_R(\alpha_L, \beta_L)] + \\
(1-p)^2 [\pi_L(\alpha_n, \beta_n) + \pi_R(\alpha_n, \beta_n)] \]

we already know that the first and second terms point to a maximum product differentiation result. With respect to the third term, it was already mentioned that, given the symmetry with the term representing the case of firm L being the sole innovator, the result implied by this term will be the same as the one presented for the second term. For this reason we can conclude that, when firm R is the sole innovator, the best strategy in terms of research design will be to maximise product differentiation. In what concerns the last term of the expression above there is nothing to say insofar that there are no decision variables present. This last term represents a situation where there are no innovators, and no progress will be made by the firms. Thus, the choice of the research design is irrelevant for the joint profit level.

Given that the first three terms point out for the same result and the fourth term is irrelevant for profit maximisation purposes, we can then conclude that, when firms cooperate in R&D the optimal strategy will be to maximise the degree of product differentiation. For that reason, the decision variables for the definition of the research design will assume the value 0 (a=b=0).

When comparing the cooperative situation with the case where firms act individualistically, we can observe that the joint definition of the research design will have an impact on the decision of product differentiation. If, by acting non-cooperatively firms would like, in some cases, to differentiate less their product, when considering the rival's profit as being part of the object to be maximised, then the decision will be to produce products as differentiated as possible. This type of strategy
will enable firms to establish some market power, lower competition in the product market and charge higher prices, allowing higher profit levels to be reached.

The divergence of results between the cooperative and non-cooperative setups finds its roots in the possible situation of a firm becoming the sole innovator. When firms do not coordinate research design and therefore are not worried about the rival's profit level, it is the possibility of a firm obtaining quality advantage over the other that motivates and encourages a lower differentiation of products. Notice that, if a firm, already having a comfortable advantage in its own specialisation characteristic, decides to invest all its R&D budget in the rival's specialisation characteristic, then, if successful, it will end up producing a product that in quality gap terms still maintains a large lead in the specialisation characteristic while narrowing the quality gap in the other product characteristic. That advantage, if achieved, will enable the capture of more consumers that are willing to lose a "small quantity of quality" in one product characteristic in order to get a "large quantity of quality" in the other characteristic. Thus, a larger profit level will be reached, at the expenses of the rival's profit level.

This type of incentives to differentiate less is not present in a situation where firms coordinate research design and value the other firm's profit. The reason for this is that, the potential gain an innovating firm might have by differentiating less is not enough to compensate the loss in profits the non-innovating firm would have if such a strategy were adopted. Thus, in this situation, the best thing to do is to prevent inflicting losses to the other firm, which might occur if an asymmetric situation emerges from the uncertain R&D process. This is done by maximising the level of product differentiation.

3.4. Conclusion

In this chapter the symmetric firms' assumption that is made in the generality of studies on product differentiation is dropped. Alongside it is considered that the product has more than one dimension and can be horizontally and vertically differentiated in an attempt to make the model more realistic. Within this aim, it is also implicitly assumed that there is an R&D budget constraint, which prevents firms from
developing each product characteristic as much as they would like. Therefore, firms have to decide how to allocate the R&D efforts to the improvement of each product characteristic, which, implicitly, will determine the degree of product differentiation.

The introduction of asymmetry in the model considered is made using the possibility that after an R&D process (which may involve uncertainty), firms (may) become asymmetric. This is done in two ways: by considering a deterministic R&D process that leads with certainty to asymmetric firms (effective asymmetry) and by assuming uncertainty in the R&D process which creates potential asymmetry. In the latter case, firms are just previewing the possibility of becoming asymmetric. This will be very important for firms as it is quite different for them to compete in the product market having product (quality) advantage, not having any kind of advantage or disadvantage, or having product disadvantage.

Given firms’ certainties or beliefs about the R&D outcome, they have to decide if they want to direct their research towards the development of a product similar to its rival - Minimum Differentiation- or to the development of a totally different product - Maximum Differentiation, (but within the definition of the market).

In a non-cooperative setup, it was shown that if the assumptions made coincide with those of d’Aspremont et al. (79) (although in a different framework), the results obtained by those authors (Maximum Differentiation Principle) hold. In such case firms have no chance of becoming asymmetric (deterministic R&D with both firms succeeding). However, if we relax the symmetry assumption and consider that after the R&D process one firm will have advantage over the other for sure, firms previewing this situation, may choose to differentiate less. The crucial factor is the magnitude of the potential asymmetry. Only when the potential improvement that can be made is large enough, do firms choose to differentiate less. Moreover, after a certain threshold on the dimension of the mentioned improvement, we can even talk of a “Principle of Minimum Differentiation”. This is due to the fact that its own product has (or may have in the future) a much higher quality in its specialisation characteristic and has the possibility to close the gap in the rival’s specialisation characteristic. This way, the innovating firm may threaten the non-innovating firm as consumers will prefer the product of the former - consumers are willing to give up a small amount of quality in
one of the characteristics if the quality gains in the other characteristic are sufficiently large.

The conclusions obtained by Irmen and Thisse (98) are also shown not to be robust to the relaxation of assumptions like: symmetric firms, only horizontal differentiation and no significant costs in changing the product. However, in this case, the different framework utilised may account for some of the differences in the results obtained.

The main conclusions for the non-cooperative setup are that the higher the probability of ending up in an asymmetric situation and/or the larger the size of potential asymmetry, the more willing firms are to minimise product differentiation. These results are easily understandable if we keep in mind the conclusions obtained for the case where R&D was deterministic. Now we are just weighting the forces implied by each of the situations (certain symmetry and certain asymmetry) by the probability of them occurring.

With respect to the choice of the level of product differentiation when firms coordinate the research design in order to maximise the (expected) joint profit, it was shown that, both in the certainty case and the uncertainty case, firms will prefer to maximise product differentiation. This result can be explained using two types of arguments. The first relates to the possibility of both firms being successful in the innovative activity. If this is to be the case, then, given the assumptions made in the model, firm symmetry will be kept. This means that if firms chose to differentiate the products less, then competition in the product market would be increased, which would not benefit any of the firms, as prices had to be lowered as would profits. Therefore, the best strategy that can be adopted is to produce products as differentiated as possible allowing the exercise of some market power, which will lead to higher prices and profits.

The second argument is linked with the possibility that a firm has of being the only innovator. In non-cooperative circumstances it was shown that this firm might feel encouraged to differentiate the products less in order to partially capture the rival’s demand. By doing so, it would increase its profit level with the loss due to the lower prices charged (consequence of the increased competition in the product market)
being more than compensated by the gain of part of the other firm's demand. This would unquestionably cause profit loss to the non-innovating firm, which would have to charge a lower price to a reduced demand. Considering a cooperative setup, where firms give importance to the rival's profit level, this gain and loss has to be weighted. From the analysis it becomes clear that the loss inflicted in the non-innovating firm by pursuing such a strategy is greater than the potential gain obtained by the firm that succeeded in R&D. Given this, it comes out that the research design that maximises the (expected) joint profit is one where firms try to differentiate their products as much as possible, so that competition is avoided and there is no prejudicial profit transference.

The differences found in the conclusions obtained in the cooperative and non-cooperative setups show that the organisation of R&D activities is not innocuous with respect to the degree of product differentiation chosen by firms. Insofar as the variety of one product available to the consumer is important to its welfare, competition policies that usually support R&D cooperation having welfare considerations in mind should also consider this additional aspect that has not yet been conveniently dealt with in the literature.
CHAPTER 4 - PRODUCT DIFFERENTIATION WITH SPILLOVERS EXOGENOUSLY ALLOCATED TO PRODUCT CHARACTERISTICS
4.1. Introduction

Having established a first link connecting the Theory of Innovation and the Theory of Product Differentiation and explained the importance of the consideration of firm asymmetry when studying the latter, which resulted in a set of conclusions different from the ones usually presented in the literature, it is now time to find new connections between those two theories that might be relevant for the analysis of both, without forgetting the contribution of the previous chapter.

As explained in Chapter 1, the existence of knowledge/information spillovers between firms is a reality that is already part of the Theory of Innovation. The fact is not new and has been acknowledged by authors like Arrow (62) or Romer (90). The particular nature of technological progress (either in process innovation and product innovation) allows us to characterise it as a non-rival partially excludable good. This means that the consumption of such good by someone does not preclude the use of that same good by another, and that, in spite of the attempts made in order to increase the appropriability of technological progress\(^1\), the total exclusion from consumption is in most cases impossible. For this contributes undoubtedly the fast pace of technological development that we have been experiencing in the past decades, which offers a great deal of information and an uncountable number of alternatives for producing a given good. This way the protection conferred by patents and similar mechanisms is often insufficient, allowing firms to replicate or imitate the rival's product. Even if this is not the case, when a firm is successful in some R&D process its results and information about them are rapidly available to all firms in the market or those that consider entering it.

This supports the fact that spillovers do exist. Acknowledging this, several authors have studied the impact that spillovers might have in the innovative activity. As mentioned earlier their concerns are usually centred in the level of R&D expenditure that is made when spillovers are significant\(^2\). The results obtained of under or over-expenditure when compared to what is socially desirable has given rise to a line of

\(^1\) As patents are an example.
\(^2\) Other topics addressed include, for instance, the optimal timing for innovation.
work that tries to solve the appropriability problem by using mechanisms like patents or the encouragement of cooperative agreements that internalise the spillovers.

In spite of the attention devoted to this topic, the study of the consequences of its existence is usually limited to the Theory of Innovation, not considering the impact that it may have in related theories as is the case of the Theory of Product Differentiation.

D'Aspremont and Jacquemin (90) have implicitly identified this question when affirming that the size of spillovers is related, among other factors, to the degree of differentiation that products present. The intuition behind this is easily understandable: when products are similar it is more likely that the R&D activity made by one firm will benefit other firms producing products alike; on the other hand, if products are highly differentiated, the R&D results of a firm may only benefit themselves as the rivals' products share a minimum of similarities and may not be able to incorporate any findings of the innovating firm. It is then a fact that the innovative activity that is usually behind the choice of product differentiation will generate spillovers, whose magnitude may be directly related to the degree of product differentiation chosen. Once this link is established, it is obvious that the decision of product differentiation will have to take into account a whole new range of strategic questions.

In the previous chapter we have seen that, in the absence of spillovers, if firms were both sure to innovate they would choose to maximise product differentiation in order to avoid competition in the product market. However in the presence of spillovers, new incentives emerge. A firm, knowing that the rival will succeed in the R&D activity, can also benefit from its success if the products produced are similar and, as a consequence, a larger amount of spillovers is transferred. This may encourage a firm to differentiate less. On the other hand, by differentiating less an innovative firm would allow a larger amount of spillovers to be transferred to the rival, thus truncating its own advantage conferred by its success in R&D. This constitutes an incentive to maximise product differentiation. These two new incentives have to be weighted with the existing incentive to differentiate as much as possible so that competition in the product market is minimised.
The reasoning made for the case where both firms are successful in the R&D activity can also be made for the case where only one firm succeeds or for the case where there is uncertainty about the R&D outcome. In such cases, the analysis of the decision of product differentiation is even more interesting insofar as we are allowing the possibility of (potential) asymmetry between firms. Within this context, each firm has to contemplate the hypothesis of being the only not to innovate. This would put the firm in question in a disadvantageous position as it would not be capable of improving the quality of its own product whilst the rival firm would produce a good that is more developed, and thus would be more attractive to some consumers. If spillovers are known to exist in the industry under study, the non-innovating firm can feel encouraged to produce a less differentiated product so that it can absorb more spillovers, which would contribute to the development of its product, not allowing the innovating firm to have such a large advantage. Notice that this incentive to differentiate less co-exists with the willingness to maximise product differentiation already analysed in the previous chapter aiming at decreasing competition. Thus, in the presence of spillovers, a sole non-innovating firm has to consider these two effects when deciding which research design to adopt and hence the degree of product differentiation.

Concerning the sole innovator, there are also new aspects to consider when spillovers are present. It is already known that in the absence of spillovers and when the (potential) asymmetry is sufficiently large, this firm may choose to differentiate less in order to attract consumers away from the rival firm. If pursuing such strategy in a situation where spillovers exist, the firm that succeeded in the R&D activity would also be contributing for a larger transference of information/knowledge to the rival firm, which would truncate its own quality gap advantage with negative consequences for its profit level. This may lead the innovating firm to change its choice of product differentiation as the share of consumers attracted from the non-innovating firm would not be as large given that, despite not innovating, the rival firm would be able to improve its product and shorten the quality gap in the rival’s specialisation characteristic and/or increase the gap in its own specialisation characteristic. Then, besides the already studied reasons to minimise product differentiation, the innovating firm now has a new argument to consider and that may push the decision towards a
maximum differentiation result. Therefore, a sole innovator has to take into account
the following aspects when deciding the level of product differentiation:

1. Maximising product differentiation would decrease competition in the product
market;
2. Maximising product differentiation would reduce the amount of spillovers given
away to the rival firm thus enabling the innovating firm to appropriate a larger
amount of the benefits conferred by the discovery;
3. Minimising product differentiation would attract consumers who before had
bought the rival’s product and are now willing to buy the innovating firm’s
product as it has almost the same quality in one characteristic (the rival’s
specialisation characteristic) and a relatively larger amount of quality in the
innovator specialisation characteristic.

When we acknowledge the relation between the magnitude of spillovers and
the degree of product differentiation and realise that they are endogenously inter-
dependent, as explained above, giving rise to new aspects of the firms’ strategic
behaviour, we have what from now on will be called endogenous spillovers. The
presence of these may change the results obtained in chapter 3 as was intuitively
explained in the previous paragraphs.

The aim of this chapter is then to study the impact that the consideration of
endogenous spillovers will have on the decision of product differentiation. Given that
the model and structure that will be used for this purpose is the same as the one in the
previous chapter, the comparison of results is facilitated and any difference in the
results obtained can surely be attributed to the presence of such spillovers.

Before starting with the analysis it is worthwhile to briefly mention what notion
of spillovers will be utilised and how this chapter is organised. It will be assumed that,
as explained above, the level of product differentiation chosen (or the research design
adopted) will be directly related to the magnitude of spillovers: the more differentiatied
the firms choose their products to be, the less spillovers will exist. The question that
now can be put forward is how will these spillovers be distributed between the two
characteristics that identify the product. Several assumptions can be made about this, however and in order to keep things simple and the conclusions obtained more easily understandable, we will start by considering one of the simplest formulations and then, in subsequent chapters, its complexity and reality adherence will be gradually built. So, in the present chapter, it will be assumed that, once the magnitude of spillovers is implicitly decided by the choice of the research design, its allocation between the two product characteristics is made exogenously according to a parameter $\sigma$.

Another useful clarification at this point is the explanation of who is "eligible" to generate spillovers and/or to be the recipient of such spillovers. The answer to this question is not unique and a multiplicity of assumptions could be made. This chapter will only deal with the ones that, given our common sense, we would expect (or assume are more reasonable) to occur in the reality. Once again, the modelisation of this aspect will be made gradually, starting with simpler hypotheses and increasing its complexity through the chapter.

The starting assumption will convey the idea that only firms that innovate will be able to give and receive spillovers. This implicitly says that a non-successful firm in R&D will not be able to generate spillovers as it did not produce any new information or knowledge, and also that it won't be capable of using the information that flows from innovative firms as its failure in R&D did not allow it to develop the means to handle it. Only by being successful in innovating do firms acquire the necessary knowledge to absorb the information spillovered by well-succeeded rivals.

The alternative assumption that will be considered is that, besides the existence of spillovers between successful firms in R&D, they may also occur from innovating firms to non-innovating firms. This seems a more reasonable assumption to make as, in practice, we observe firms that did not contribute in any way to the generation or development of knowledge replicating or imitating the rival's product.

Concerning the assumption presented in the previous paragraph, its study will be made in two steps. Firstly it will be assumed that the magnitude of spillovers is the same, whether they exist between innovating firms or from firms that succeeded in innovating to non-innovating firms. In a later stage, a perhaps more appealing hypothesis will be analysed: it will be considered that the spillovers between firms that succeeded in R&D are larger than the spillovers that flow from innovating firms to
firms that did not innovate. The intuition behind this assumption is related with the perception that if a firm was able to innovate, then it developed a set of information and/or techniques that will facilitate the absorption of spillovers. As non-innovating firms were not capable of acquiring the instruments that would ease the understanding and utilisation of knowledge flowing from innovating firms, the assimilation of spillovers will not be as large.

The structure of the present chapter will then be as follows: Firstly the changes that have to be made in the original model so that it accommodates the notion of endogenous spillovers will be explicated. Once this is done, the results for each hypothesis contemplated will be computed, which will originate three sections. In section A, the assumption that spillovers only occur between successful firms will be studied. Within this section both the non-cooperative and cooperative setups are analysed and for each setup the certainty and uncertainty cases are presented. Section B will have a similar structure, having as background hypothesis that spillovers occur both between firms that succeeded in R&D and from innovating firms to firms that did not innovate. In a final section, an extension of section B will be introduced where it will be considered that spillovers that occur between successful firms have a larger magnitude than the spillovers flowing between firms with different R&D outcomes.

4.2. The Model

The changes that have to be made in the model so that it encapsulates the existence of endogenous spillovers will only affect the quality gap definitions, and not the model itself as presented in the second chapter.

With respect to this, it is useful to remember that when a firm engages in the R&D activity and invests all its R&D budget in the development of a particular product characteristic, the amount of improvement that will be brought to that characteristic, if the firm in question is successful in the R&D process, is given by $g*S$. According to the research design chosen, this potential improvement will be allocated between the two product characteristics. Thus, any innovating firm will produce $g*S$ of new and useful knowledge or information, which means that this is the maximum
amount of spillovers that can theoretically occur, as it would not make sense for a firm to give away more knowledge than the one it produced. Given the way we want the model to behave, where the spillover level depends directly on the degree of product differentiation, it will be assumed that the maximum amount of spillovers that might occur will in fact take place whenever firms choose a research design that implies minimum product differentiation. For this reason, when firms choose $a=1$ and $b=1$, which in product terms translates into minimum product differentiation, the spillover magnitude will be maximal, i.e. $g*S$. On the other hand, it will also be assumed that if the degree of product differentiation chosen by firms is the largest possible ($a=0$ and $b=0$), then spillovers will be 0. These simplifying assumptions, despite the fact of not having any empirical evidence to support them, but that convey the general idea about the relation between amount of spillovers and degree of product differentiation, can be easily modelled by the following "spillover magnitude function": $g*S*(\frac{a+b}{2})$. This function determines, for every possible combination of research designs chosen by the firms, the magnitude of spillovers that will take place.

Having defined the amount of spillovers that will be transferred from one firm to the other, it is now necessary to define how those spillovers will be allocated between the two product characteristics.

As a first step in the study (which in the following chapters will be developed), we will presently assume that the spillover allocation will be decided by an exogenous parameter, $\sigma$. This parameter represents the percentage of total spillovers that will be given away by an innovating firm in its specialisation characteristic. Obviously, $(1-\sigma)$ of the total spillover will be allocated to the other characteristic. From this, and in order to re-define the quality gaps, whenever spillovers exist it will be necessary to subtract $\sigma*g*S*(\frac{a+b}{2})$ to the quality gap of the firm’s specialisation characteristic and add $(1-\sigma)*g*S*(\frac{a+b}{2})$ to the quality gap of the rival firm.

This simple formulation could easily be made more complex by assigning different values of $\sigma$ to each firm. However, given the initial symmetry of firms it does not seem to make great sense to do it, moreover new and interesting insights would
not be brought into the analysis. Thus, it will be assumed that the parameter $\sigma$ will be the same for both firms.

The study of the decision of product differentiation according to the assumptions just presented will be made for different hypotheses about who generates and receives the spillovers, as already mentioned. To each hypothesis will correspond a particular definition of spillovers that encapsulates the idea conveyed by the hypothesis in question. The first hypothesis to be analysed depicts the case where spillovers only occur between firms that succeeded in the R&D activity. In a second section it will be considered that spillovers also exist from innovating to non-innovating firms. With respect to this, an extension will be made to consider the situation where spillovers between innovating firms are larger than spillovers from firms that succeeded in R&D to non-successful firms.

4.3. Results

Section A — Spillovers only exist between innovating firms

In order to introduce this assumption into the model, the only thing that has to be changed is the quality gap definition associated with the situation where both firms innovate. All the remaining gap definitions will be unchanged. Thus, and according to the assumptions presented before, the relevant definitions for the quality gaps will be:

- If firm both firms are successful:
  $\alpha_b = S + Sg(1 - a) - Sgb - \sigma \frac{a+b}{2} Sg + (1 - \sigma) \frac{a+b}{2} Sg$
  $\beta_b = S - Sga + Sg(1 - b) + (1 - \sigma) \frac{a+b}{2} Sg - \sigma \frac{a+b}{2} Sg$

- If neither firm is successful:
  $\alpha_n = S$
  $\beta_n = S$
- If only firm L is successful:
  \[ \alpha_L = S + Sg(1 - a) \]
  \[ \beta_L = S - Sga \]

- If only firm R is successful:
  \[ \alpha_R = S - Sgb \]
  \[ \beta_R = S + Sg(1 - b) \]

Following the structure utilised in chapter 3, and for the same reasons presented then, the analysis will start with the study of the non-cooperative case and will then proceed to the case where firms coordinate the research design.

4.3.A.1. The Non-Cooperative Equilibrium

Using the above expressions, the expected profit function firm L will be interested in maximising is:

\[ \pi_L^* = p^2 \pi_L(\alpha_L, \beta_L) + (1 - p)p \pi_L(\alpha_L, \beta_L) + p(1 - p)\pi_L(\alpha_R, \beta_R) + (1 - p)^2 \pi_L(\alpha_n, \beta_n) \]

Once again, similarly to what was done in the previous chapter, we can easily observe that the only relevant terms for the purpose of profit maximisation are those that represent a situation where firm L was successful in R&D. All the others do not include firm L’s decision variable, \( a \). This stems from the fact that if a firm did not innovate, the research design chosen will be of no consequence to the profit level. Thus we will only be interested in the first two terms of the above expression, which can be seen as firm L’s expected profit, conditional on firm L’s success in innovating.

Given this, the expression to be maximised will then be:

\[ \pi_L^* = p \pi_L(\alpha_L, \beta_L) + (1 - p)\pi_L(\alpha_L, \beta_L) \]

where \( p \) now represents the conditional probability of the rival’s success in R&D, conditional on the success in that activity of the firm in question.

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3 As stated previously and given firms’ symmetry, a similar expression can be written for firm R.
As was done in the previous chapter and in order to make the analysis more easily understandable, we will start by analysing the expression above term-by-term, which in practice may be viewed as assuming that the R&D activity has no uncertainty associated. This will help us to understand the forces at work behind each term. After this is done, the more realistic case of uncertainty about the R&D process outcome will be presented.

4.3.A.1.1. Certainty Case

In the study of this case, it will be considered, as before, that firms have complete information about the outcome of the R&D processes (own and rival’s).

4.3.A.1.1.1. The “Both Firms Innovate” Term

The term that represents this situation, from firm L’s point of view, and that we want to maximise is given by:

\[ \pi_L(a_b, \beta_b) = \]

\[ = \pi_L \left[ 1 + g(1-a) - bg - \sigma \frac{a+b}{2} + (1-\sigma) \frac{a+b}{2} \right] \]

The first order condition, with respect to firm L’s decision variable, \( a_b \), will then be:

\[ \frac{\partial \pi_L}{\partial a} = \frac{\partial \pi_L}{\partial a_b} \cdot \frac{\partial a_b}{\partial a} + \frac{\partial \pi_L}{\partial \beta_b} \cdot \frac{\partial \beta_b}{\partial a} = 0 \]

where:

\[ \frac{\partial a_b}{\partial a} = \frac{\partial \beta_b}{\partial a} = -Sg + \frac{Sg(1-2\sigma)}{2} < 0 \]

and \( \alpha_b = \beta_b \).

\[ \frac{\partial \pi_L}{\partial a} = \left( \frac{\partial \pi_L}{\partial a_b} + \frac{\partial \pi_L}{\partial \beta_b} \right) \cdot \frac{\partial a_b}{\partial a} \]

Therefore

\[ > 0 \quad < 0 \quad < 0 \]
As proved when the model was introduced, the direct effect dominates the indirect effect whenever $\alpha_b = \beta_b$, which is the same thing to say that $\frac{\partial \pi^L}{\partial \alpha_b} > \frac{\partial \pi^L}{\partial \beta_b}$. This allows us to conclude that the term in brackets is positive. This results in $\frac{\partial \pi^L}{\partial a}$ being negative. Therefore, firm L will opt for the minimum value of $a$ available ($a=0$) in order to maximise its profit level, which, in economic terms, means that all of the R&D potential will be devoted to its own specialisation characteristic. This obviously implies a clear wish to maximise product differentiation. Given the firms' symmetry that is assured to continue existing, firm R will adopt a similar strategy and will set $b=0$. If firms chose to produce products as similar as possible ($a=b=1$) this would result in a damaging competition in the product market, as was also concluded for the case where spillovers were considered not to exist.

From this analysis, we obtain that the introduction of endogenous spillovers will not change the strategy chosen by firms - the choice of maximum product differentiation will be maintained. Thus, the gain a firm might have by differentiating less in order to be able to absorb a larger amount of spillovers is not enough to compensate the losses inflicted by the simultaneous giving away of more spillovers and the increased competition that will exist in prices as a consequence of the products being less differentiated.

4.3.A.1.1.2. The “Only Firm L Innovates” Term

By looking at the term that is under analysis and the relevant quality gap expressions, we can easily observe that the introduction of spillovers only between firms that succeeded in R&D will not change anything when compared to the case where spillovers were assumed not to exist. For this reason all the conclusions drawn in section 3.1.1.2. of the previous chapter will remain unchanged. Briefly, it was concluded that innovating firms might want to minimise product differentiation provided that the asymmetry between firms is large enough. If the degree of
asymmetry to be reached is considered insufficient, then the strategy chosen will be one of maximum product differentiation.

Having said this, and observed that the conclusions obtained with these type of spillovers in a context of certainty do not change when compared with the situation of spillover absence, one might feel tempted to conclude that no changes would also occur for the case where the outcome of the R&D activity is uncertain. However, this conclusion would not be correct since, in spite of no changes occurred in the term where a sole firm is the innovator and in the results for the case where both firms succeed in R&D, the latter might enclose a different “relative willingness to maximise product differentiation”. In fact, this is the case, as it will be shown when the situation of R&D uncertainty that now follows is presented.

4.3.A.1.2. Uncertainty Case

The objective function to be maximised is given by the following expression, of which we already know the “behaviour” of each term:

\[ \pi^e_L = p\pi_L(\alpha_b, \beta_b) + (1 - p)\pi_L(\alpha_L, \beta_L) \]

The results will provide the equilibrium research design that firms will adopt in order to maximise expected profit. This choice is made before firms know if they will be successful or not in R&D, but being aware that if both firms succeed, a certain amount of information will be shared according to the degree of product differentiation chosen.

Intuitively speaking we expect the outcome of such maximisation to be somewhere in between the results obtained for each term, as in the uncertainty case we are faced with a weighted average of the two terms. The relevant question to be answered at the present point is how the results of such maximisation will compare to the ones obtained in absence of spillovers. The answer will provide an analysis of the impact, if any, that the introduction of endogenous spillovers between innovating firms will have on the decision of product differentiation.
Given the complexity of a complete analytical analysis, that was increased due to a more complex quality gap definition and that may not allow any conclusions to be drawn, the study of the questions presented in the last paragraph will be made through the use of numerical simulation. The parameters considered for the simulation will be the same as the ones utilised in the previous chapter so that a direct comparison of the results can be easily made. Thus, this exercise will utilise a value of S=20, and, as before, will allow g to vary between 0 and 0.5 and p between 0.1 and 0.9.

With respect to the new exogenous parameter that was introduced, \( \sigma \), and which is “responsible” for allocating spillovers between the two product characteristics, it is worthwhile to look at it carefully and think what may be considered reasonable values. By definition, \( \sigma \) is the percentage of the total spillover that a firm will give away in its own specialisation characteristic to the rival firm. As such, according to economic intuition, if not common sense, in a case where spillovers are exogenously allocated, it seems more sensible and reasonable to consider that a firm gives away more information about things that it knows best, that is, things in which it is specialised. For this reason, the values of \( \sigma \) that will be considered for the purpose of the simulation will have to be larger (or equal) than 0.5. In particular, the values of \( \sigma \) utilised in the simulation will be: \( \sigma = 0.5 \) and \( \sigma = 0.9 \). The consideration of such values will also allow us to draw some conclusions about the way the exogenous allocation of spillovers might affect the decision of product differentiation, enabling us to infer what would happen if different values of \( \sigma \) were assumed.

The results are presented in the following tables:

\[ \sigma = 0.5 \]

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Table 4.1. Equilibrium values of \( a (=b) \), for different pairs \((g, p)\), considering S=20 and \( \sigma = 0.5 \).
\( \sigma = 0,9 \)

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Table 4.2. Equilibrium values of \( a (=b) \), for different pairs \((g, p)\), considering \( S=20 \) and \( \sigma = 0,9 \).

From the analysis of the tables several conclusions can be drawn. The first concerns that, qualitatively, the effects of the variables \( g \) and \( p \) on the equilibrium values for the research design did not change when compared with the case where spillovers were assumed not to exist. Basically, the larger the value of \( g \), the more firms are willing to produce less differentiated products, as the level of potential firm asymmetry increases with that variable. To this general conclusion there are however two exceptions presented in table 4.2, in the light shaded cells. There we observe that when the value of \( g \) increases the willingness to minimise product differentiation is decreased (equilibrium values of \( a \) and \( b \) decrease). Given that the only change that was introduced in the model concerns the term representing the case where both firms innovated, the reason for this occurrence is certainly connected with it. This fact can be explained by studying the impact that \( g \) has on the dimension of the quality gaps \( \alpha_b \) and \( \beta_b \), in particular for the equilibrium values of the endogenous variables and the values considered for the exogenous variables.

So, being \( \frac{\partial \alpha_b}{\partial g} = (1 - a - b)S - \frac{a + b}{2} (2\sigma - 1)S \)

\( ^4 \) it is straightforward to show that this derivative is negative for large values of \( a \) and \( b \) and of \( \sigma \). This is the case for the situations presented in table 4.2 as being exceptions. Thus, an increase in \( g \) would decrease the quality gap. This also happens in the case of \( \sigma=0,5 \), when the equilibrium values of \( a \) and \( b \) are, for instance, equal to 1. However we did not get a change in the results then. This stems from the fact that the impact that \( g \) has on the quality gap can be considered mild when compared with the consequences it has when a larger value for \( \sigma \) is considered. From the derivative we can see that if \( \sigma=0,5 \) the second term on

\( ^4 \) The derivative with respect to \( \beta_b \) is similar.
the right hand side is null, but if it is larger than 0,5 it will be negative boosting the
effects that a change in g has on $\alpha_g$. Given this, the increase in g in such context will
surely decrease the quality gap dimension by a reasonable amount, thus firms may feel
tempted to overcome this situation by reducing the willingness to differentiate the
products less. This action would have a direct impact in the gap magnitude through the
own success of R&D (that would be designed so that each firm invests a little more in
the development of its specialisation characteristic) and would also indirectly affect the
gap dimension via reduction in the spillovers. Notice that this latter effect is much
more evident for larger values of c as it represents the percentage of improvement that
is spillovered in the specialisation characteristic. For instance, a value of $c=0.9$ in a
symmetric situation, as this is, would cause a net give away of 80% of $S_g \frac{a+b}{2}$ as
spillovers in the specialisation characteristic. Another factor that might help to explain
the exception in the results obtained draws directly from what was mentioned above
about the diminishing of the gap dimension and the way consumers perceive the
products. We know that even in the case a firm is always willing to maximise product
differentiation, there are different “degrees of willingness” to do it. If the profit
maximisation for the case where both firms innovate was not constrained to values of a
and b between 0 and 1, we might have found equilibrium values of -2 or -8, with the
latter representing a higher willingness to differentiate products. It is also
understandable that, even maximising product differentiation, at the eyes of the
consumer products will appear to have different degrees of differentiation according to
the quality gap dimension. Thus, if products have a small quality gap in the
specialisation characteristic, consumers might view them as almost identical, which
would increase firm competition. On the other hand, if gap magnitude is relatively
large, consumers will have no problems in considering them totally differentiated
products, which will soften competition. Having said this, the change in the strategy
observed in the light shaded cells, might be due to a increased willingness to
differentiate more products as a consequence of smaller quality gaps and more
competition. This means that the term representing the case where both firms succeed
in R&D gained relative importance and is no longer totally dominated by the term
depicting the situation of a sole innovator. This results in smaller equilibrium values of
$a$ and $b$. A question that might be asked now is why this just happens for $p=.1$ and $p=.2$. The answer is once again related with the derivative

$$\frac{\partial a}{\partial g} = (1 - a - b)S - \frac{a + b}{2}(2\sigma - 1)S.$$ 

Despite the fact that the value of $\sigma$ is still large, the equilibrium values of $a$ and $b$ are no longer as large\(^5\) (before the increase in $g$), thus reducing the impact that $g$ has on the quality gaps. For this reason it is no longer necessary to increase product differentiation when $g$ increases.

With respect to the conclusions about the effect that the variable $p$ has on the equilibrium values of $a$ and $b$, it is straightforward to see that the larger the value of $p$ the more firms want to differentiate their products, as the likelihood of firms keeping their symmetry after the R&D process is increased. The explanation for these facts was already presented in the previous chapter, and in order to avoid unnecessary repetition of arguments, these will not be mentioned again.

A second conclusion that the tables allow us to observe is that the results obtained and its comparison to the case of spillover absence depends crucially on the value chosen for the exogenous variable that allocates the total spillovers between the two product characteristics. When spillovers between innovating firms are equally allocated to the development of each characteristic then no changes occur relative to the case where spillovers were not present. This becomes obvious when we look carefully to the gap definitions and consider a value of $\sigma=0.5$. By setting such a value for this variable the spillovers received in each characteristic are equal in magnitude to the spillovers given away in that same characteristic, thus cancelling the spillover effect, and every thing remains as before. So, in quality terms each firm will keep its advantage in spite of being exchanging information/knowledge with the rival. However, if the value chosen for variable $\sigma$ is larger than 0.5, then the results obtained will change as table 4.2 shows, which leads us to ask what is the effect that this exogenous variable has on the decision to differentiate products. The answer is given next.

\(^5\) Notice that higher values of $p$ imply a higher likelihood of a symmetric outcome and for that reason firms are not so willing to differentiate products less, which means a choice of lower values of $a$ and $b$. 

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It can be easily seen that the more spillovers a firm transfers in its own specialisation characteristic to the rival (larger values of $\sigma$), the more willing firms are to increase the degree of product differentiation. This stems directly from the observation that the equilibrium values of $a$ and $b$ decrease as $\sigma$ increases. The reason for this fact may, in some way, be related to the conclusions reached in the previous chapter. Then, it was observed that firms needed to build a comfortable quality advantage in the specialisation characteristic before venturing in the rival’s specialisation characteristic. In this case a similar result seems to apply as firms, by giving away more spillovers in their own specialisation characteristic than those they receive, see that their advantage in that characteristic is truncated and, as a result, firms are not willing to decrease product differentiation.

As the only change that was made in the model concerns the term representing the case where both firms innovate, it must be this term that is responsible for the changes in the results. This means that, even if firms are symmetric and always willing to maximise product differentiation, their disposition to do it has different levels. Thus, if the symmetric gap advantage is not so large, their inclination to produce products as different as possible is larger. This is surely related to the fact that, if the quality gaps firms present are not very large, even though firms choose maximum product differentiation, in the eyes of the consumer they may be viewed as almost similar products, thus increasing competition in the product market. On the other hand, if the quality gaps are relatively large, from the consumers’ point of view, they will be seen as more differentiated products and competition between products may not be so intense. This explains the observed results that the larger the value of $\sigma$ considered, the more willing firms are to differentiate less their products.

4.3.A.2. The Cooperative Equilibrium

In this sub-section it will be studied how the existence of endogenous spillovers between innovating firms will affect the decision of product differentiation when firms
coordinate the research design in order to maximise the expected joint profit. The relevant expression firms want to maximise is given by:

\[
\pi^*_L + \pi^*_R = p^2 \pi_L(a_b, \beta_b) + (1 - p) p \pi_L(a_L, \beta_L) + p(1 - p) \pi_L(a_R, \beta_R) + (1 - p)^2 \pi_L(a_n, \beta_n) + p^2 \pi_R(a_b, \beta_b) + (1 - p) p \pi_R(a_L, \beta_L) + p(1 - p) \pi_R(a_R, \beta_R) + (1 - p)^2 \pi_R(a_n, \beta_n)
\]

Rearranging the above expression we will obtain:

\[
\pi^*_L + \pi^*_R = p^2 [\pi_L(a_b, \beta_b) + \pi_R(a_b, \beta_b)] + (1 - p) p [\pi_L(a_L, \beta_L) + \pi_R(a_L, \beta_L)] + p(1 - p) [\pi_L(a_R, \beta_R) + \pi_R(a_R, \beta_R)] + (1 - p)^2 [\pi_L(a_n, \beta_n) + \pi_R(a_n, \beta_n)]
\]

where the quality gaps are the ones defined at the beginning of section A.

Similarly to what was done before, a term-by-term analysis will precede the study of the more realistic case of uncertainty about the outcome of the R&D activity. As previously mentioned this is equivalent to the analysis of a situation where firms have complete information about the R&D outcome, which puts firms in a context of certainty.

4.3.A.2.1. Certainty Case

4.3.A.2.1.1. The “Both Firms Innovate” Term

The term that now is under consideration is: \( \pi_L(a_b, \beta_b) + \pi_R(a_b, \beta_b) \). By choosing together the values of \( a \) and \( b \) firms will try to maximise its value.

Computing the first derivative of the above expression with respect to \( a \), we obtain:

\[
\frac{\partial (\pi_L + \pi_R)}{\partial a} = \frac{\partial \pi_L}{\partial a} \frac{\partial a}{\partial a} + \frac{\partial \pi_L}{\partial \beta_b} \frac{\partial \beta_b}{\partial a} + \frac{\partial \pi_R}{\partial a} \frac{\partial a}{\partial a} + \frac{\partial \pi_R}{\partial \beta_b} \frac{\partial \beta_b}{\partial a}
\]
Computing the derivatives \( \frac{\partial \alpha_b}{\partial a} \) and \( \frac{\partial \beta_b}{\partial a} \) is useful in the determination of the sign of the above expression. Given firms' symmetry it is not necessary to compute those with respect to firm R's decision variable, \( b \).

It is straightforward to show that \( \frac{\partial \alpha_b}{\partial a} = \frac{\partial \beta_b}{\partial a} = -S_g + S_g \frac{(1-2\sigma)}{2} < 0 \). On the other hand, we also know, from the chapter where the model was explained, that:

\[
\frac{\partial \pi_L}{\partial \alpha_b} > 0; \quad \frac{\partial \pi_R}{\partial \alpha_b} < 0; \quad \frac{\partial \pi_L}{\partial \beta_b} < 0; \quad \frac{\partial \pi_R}{\partial \beta_b} > 0,
\]

whatever the gap definitions utilised.

Keeping this in mind, the expression above can be re-written as:

\[
\frac{\partial (\pi_L + \pi_R)}{\partial a} = \left( \frac{\partial \pi_L}{\partial \alpha_b} + \frac{\partial \pi_L}{\partial \beta_b} \right) \frac{\partial \alpha_b}{\partial a} + \left( \frac{\partial \pi_R}{\partial \alpha_b} + \frac{\partial \pi_R}{\partial \beta_b} \right) \frac{\partial \beta_b}{\partial a}.
\]

It was already proven in the previous chapter that the two terms in round brackets are positive - this has been called the dominance of the direct effect over the indirect effect. Thus the resulting sum of what is in square brackets is undoubtedly positive. Given that \( \frac{\partial \alpha_b}{\partial a} \) is negative, we will obtain that the derivative \( \frac{\partial (\pi_L + \pi_R)}{\partial a} \) is always negative. This result implies that the optimal value for \( a \) is zero. Given firms' symmetry this result can be generalised to firm R, and its decision variable, \( b \).

This means that the introduction of endogenous spillovers between cooperative innovating firms do no change the results obtained for the case of spillover absence. Therefore, if firms are sure that both will succeed in the R&D activity and exchange spillovers in a way that will keep their symmetry, then the best strategy for the research design coordination is to go for Maximum Product Differentiation. The intuition behind this result is simple and is related to the arguments of competition in the product market already explained in the previous chapter. Therefore, in qualitative terms, the consideration of endogenous spillovers in a context of certainty about the success of both firms in the R&D activity with research design coordination brings nothing new to the product differentiation decision when compared with the non-cooperative setup or the cooperative setup without spillovers, with the same type of reasons justifying this choice. The decision will then be to always maximise product differentiation.
4.3.A.2.1.2. The "Only firm L Innovates" Term

As the hypothesis of endogenous spillovers was introduced only for the case where both firms were successful in the R&D activity, no change in the term \( \pi_L(\alpha_L, \beta_L) + \pi_R(\alpha_L, \beta_L) \) occurs. For this reason, the analysis that was made in the previous chapter for this situation remains unchanged as well as the results obtained.

From section 3.2.1.2 of the third chapter, we already know that, in such case, firms will adopt a maximum product differentiation strategy and will set \( a = b = 0 \).

4.3.A.2.2. Uncertainty Case

Once again the analysis of this case is now simplified by the previous term-by-term study. As the relevant expression to be maximised is

\[
\pi^e_L + \pi^e_R = p^2 [\pi_L(\alpha, \beta_L) + \pi_R(\alpha, \beta_R)] + (1-p)p[\pi_L(\alpha_L, \beta_L) + \pi_R(\alpha_L, \beta_L)] + p(1-p)[\pi_L(\alpha_R, \beta_R) + \pi_R(\alpha_R, \beta_R)] + (1-p)^2[\pi_L(\alpha_n, \beta_n) + \pi_R(\alpha_n, \beta_n)]
\]

we already know that all the terms but the last point to a maximum product differentiation result. With respect to \( \pi_L(\alpha_n, \beta_n) + \pi_R(\alpha_n, \beta_n) \) it can be easily seen that no decision variables are contained in it given the gap definitions, thus this term is not relevant for expected joint profit maximisation purposes.

Given that the first three terms point to the same result and the fourth term is irrelevant, we can then conclude that, when firms cooperate in R&D even with spillovers between innovating firms, the optimal strategy will be to maximise the degree of product differentiation. For that reason, the decision variables for the definition of the research design will assume the value 0 (\( a = b = 0 \)).

The arguments for such a result are similar to the ones already presented for a similar situation in absence of spillovers. So, in order to avoid unnecessary repetition, we will refrain from mentioning them again.
Section B – Spillovers occur between innovating firms and from innovating to firms that did not innovate

The novelty in this section when compared with the previous is that now, besides considering that firms that were successful in R&D exchange information/knowledge among them, it is also assumed that spillovers also occur from innovating to non-innovating firms. The reasons leading to the consideration of such an assumption were already presented when this chapter was introduced.

In order to contemplate this additional assumption in the model, the only thing that has to be changed relatively to section A is the quality gap definition associated with the situation where there is only one firm that innovates. Thus, according to the explanations given in the introduction to this chapter, the relevant definitions for the quality gaps will be:

- If firm both firms are successful:
  \[ \alpha_b = S + Sg(1-a) - Sgb - \sigma \frac{a+b}{2} Sg \]
  \[ \beta_b = S - Sga + Sg(1-b) + (1-\sigma) \frac{a+b}{2} Sg \]

- If neither firm is successful:
  \[ \alpha_n = S \]
  \[ \beta_n = S \]

- If only firm L is successful:
  \[ \alpha_L = S + Sg(1-a) - \sigma \frac{a+b}{2} Sg \]
  \[ \beta_L = S - Sga + (1-\sigma) \frac{a+b}{2} Sg \]

- If only firm R is successful:
  \[ \alpha_R = S - Sgb +(1-\sigma) \frac{a+b}{2} Sg \]
  \[ \beta_R = S + Sg(1-b) - \sigma \frac{a+b}{2} Sg \]

It should be noticed that now, in the presence of spillovers from innovating to non-innovating firms, even if a firm was not successful in the R&D activity (and the
rival is), the research design chosen will be important and the variable that translates it for the non-successful firm will appear in the gap definition insofar as it will contribute to the determination of the spillover magnitude. The idea behind this hypothesis is that, even not being successful, if a firm devotes its resources to the development of a product similar to the one produced by its successful rival, it will acquire a higher capability of absorption of the information/knowledge generated by the other firm. So, in spite of not being able to achieve and obtain the information by itself, it developed the instruments that ease the understanding of the rival’s discovery. Obviously, if a non-innovating firm allocates its R&D resources to the development of a product that shares a minimum of similarities with the rival’s product, the probability of the firm in question to be able to understand and use the other firm’s discovery is much lower.

This could be compared to the case of two students in which one chooses to study algebra and succeeds in demonstrating a theorem. If the second student also chooses to study algebra, the chances that he will be able to understand and replicate the theorem demonstration are much higher than if he opted to study poetry. The bottom line is that, even not being successful, there is a learning process underlying the R&D activity that will be useful if the research path would lead to a product similar to the one of a R&D successful rival.

Following the structure utilised in section A, and to allow a direct comparison between these alternative assumptions and between these and the case of spillover absence, the analysis undertaken within this section will follow a similar pattern.

4.3.B.1. The Non-Cooperative Equilibrium

Considering the above expressions for the quality gaps, the expected profit function firm L will be interested in maximising is:

\[ \pi^*_L = P^2 \pi_L(\alpha_b, \beta_b) + (1 - p)p\pi_L(\alpha_L, \beta_L) + p(1 - p)\pi_L(\alpha_R, \beta_R) + (1 - p)^2 \pi_L(\alpha_n, \beta_n) \]

Notice that now, as opposed to what was done for the case where spillovers were absent or were considered to exist only between innovating firms, all

\footnote{As stated previously and given firms' symmetry, a similar expression can be written for firm R.}
the terms of firm L's expected profit function (except the last, for obvious reasons) contain its decision variable, $a$. The explanation for this fact was already given when the gap definitions were presented. Therefore, the reasoning made in the previous chapter and section, that firms would solely be concerned with the possibility of R&D success, which permitted the study of only the first two terms of the expected profit expression, is no longer valid. Given this, when maximising firm L's expected profit all the terms (except the last, which is a constant) will have to be considered.

As usual a term-by-term analysis will precede the study of the more realistic case where it is assumed that the R&D activity has associated an uncertainty feature. Once again, this term-by-term analysis will be undertaken considering that firms have full information and know with certainty what the R&D outcome will be.

4.3.B.1.1. Certainty Case

4.3.B.1.1.1. The "Both Firms Innovate" Term

As the assumption of endogenous spillovers existence between innovating firms was already introduced in section $A$, the analysis of the product differentiation decision made in sub-section 4.3.A.1.1. applies entirely to the present case. Nothing was changed in the term under study with the introduction of spillovers from innovating to non-innovating firms. For that reason the conclusions obtained in the sub-section mentioned, that state a result of maximum product differentiation, are maintained.

4.3.B.1.1.2. The "Only Firm R Innovates" Term

For reasons that later will become obvious, the order by which the analysis of each term will be done is reversed relatively to the order of appearance in the expected profit expression.

The aim is then to know how a firm that did not succeed in its R&D activity will choose its research design, knowing in advance that it will not innovate while the
rival will. The dilemma this firm faces can be summarised as follows: if this firm chooses to maximise product differentiation, then it will avoid intense competition in the product market but will not be able to absorb spillovers and improve the quality of its product. On the other hand, by minimising product differentiation the firm will increase the spillover magnitude but will be more exposed to competition with a rival that was able to develop its product.

In order to see which strategy the sole non-innovator (firm L, in this case) will choose, the profit expression to be maximised is condensed in $\pi_L(\alpha_R, \beta_R)$ with

$$\alpha_R = S - Sgb + (1 - \sigma) \frac{a + b}{2} Sg \quad \text{and} \quad \beta_R = S + Sg(1 - b) - \sigma \frac{a + b}{2} Sg.$$ 

Considering the first order condition $\frac{\partial \pi_L(\alpha_R, \beta_R)}{\partial \alpha} = \frac{\partial \pi_L}{\partial \alpha_R} + \frac{\partial \pi_L}{\partial \beta_R} \frac{\partial \beta_R}{\partial \alpha} = 0$ and from the partial derivatives $\frac{\partial \alpha_R}{\partial \alpha} = (1 - \sigma) \frac{Sg}{2} > 0$, $\frac{\partial \beta_R}{\partial \alpha} = -\frac{\sigma Sg}{2} < 0$ and $\frac{\partial \pi_L}{\partial \alpha_R} > 0$, $\frac{\partial \pi_L}{\partial \beta_R} < 0$ it is straightforward to show that $\frac{\partial \pi_L(\alpha_R, \beta_R)}{\partial \alpha}$ is undoubtedly positive. Given this, the optimal choice of $\alpha$ will be its maximum possible value, i.e., 1. This unequivocal result is due to the fact that now an increase in $\alpha$ will simultaneously increase the quality gap in the own specialisation characteristic and decrease the rival firm’s advantage in the other product characteristic. For the firm in question, this constitutes only advantages. Notice that in all the other situations studied before, this was never the case: either the variable was not present in the expression or its increase would reduce both quality gaps.

In terms of the decision of product differentiation by the firm that did not innovate, this translates into the adoption of a strategy that implies the production of less differentiated products. By doing so, this firm will be able to absorb a larger amount of spillovers and thus increase its product quality. From this result, we can also conclude that this effect is stronger than the fear of more intense competition in the product market and the losses it may bring to the profit level.
4.3.B.1.3. The "Only Firm L Innovates" Term

In this case, the firm in question, firm L, will be the sole innovator and it has this knowledge. When deciding what research design to choose, and consequently the level of product differentiation, this firm has to weigh the argument that in the absence of spillovers motivated an option of less differentiated products for sufficiently large asymmetries, with the incentives that it has to differentiate products more. The latter may be summarised by the following two reasons. The first reason is not new and concerns the avoidance of intense competition generated if the products produced by the two firms were similar. This, we have already seen, despite being a strong argument, may be supplanted by considerations of increased demand caused by the production of a similar product but with higher quality in one of the characteristics. The second reason emerges from the consideration of the existence of spillovers from innovating to non-innovating firms. In particular, a firm that succeeds alone in the R&D activity and is aware of the fact that the rival firm will choose to minimise product differentiation in order to capture as much spillovers as possible (as seen in section 4.3.B.1.1.2) may find that it is no longer profitable to produce less differentiated goods. This stems from the fact that, given the existence of spillovers and the rival's option to lower product differentiation, the degree of asymmetry reached by innovating alone will not be as large, as the other firm will also be able to improve its product thus truncating the advantage conferred by innovation. So, in order to minimise the transference of knowledge and increase the appropriability of any discovery, the innovating firm may feel compelled to change its strategy when compared to the case of spillover absence.

Having explained the arguments in favour and against a strategy of minimum and maximum product differentiation, the firm's choice will result, once again, from the maximisation of $\pi_L(\alpha_L, \beta_L)$, with the gap expressions being given by:

$$
\alpha_L = S + Sg(1-a) - \sigma \frac{a+b}{2} Sg \\
\beta_L = S - Sga + (1-\sigma) \frac{a+b}{2} Sg
$$
As the analytical study of the profit expression did not provide conclusive results, it was necessary to resort to graphical analysis and numerical simulation. Given that the results obtained from the numerical simulation for different parameters values always pointed to the same optimal value of \( a \), it would constitute a redundancy to present it. Instead, the behaviour of the profit expression will be analysed graphically. For this purpose it was considered several values for the parameters \( \sigma (0.5 \leq \sigma \leq 1) \) and \( S \). It can be shown that the shape of the profit function does not qualitatively change with the values chosen, which indicates a certain robustness in the conclusion. The parameters considered for the plots presented were: \( \sigma = 0.5 \) and \( \sigma = 0.9 \), and in each case \( S \) always equal to 20.

Figure 4.1. Profit function for \( \sigma = 0.5 \) and \( S = 20 \)
From the 3D plots above of the profit function, we can easily observe that the profit level is maximised for the minimum value of $a$ possible ($a=0$), whatever the value of $g$ considered. Notice that it was considered that the value of $b$ would be 1, as was concluded in the previous section and firms have complete information, not only about the R&D outcome but also of the rival’s profit function. Thus firm L when solving its profit maximisation problem knows that the non-innovating firm will choose to minimise product differentiation. We have then reached a result affirming that the non-innovating firm will want to maximise the incoming spillovers even though it means more competition. On the opposite side, the firm that succeeded in the R&D activity, faced with the rival’s choice, will want to increase the appropriability of its discoveries, which means the minimisation of spillovers. This is achieved through a choice of maximisation of product differentiation.

The incentives studied in the previous chapter (absence of spillovers) to produce less differentiated products seem to have been dominated by arguments of softening competition and also minimisation of knowledge transference. Therefore, we may conclude that the non-innovating firm just by choosing to minimise product differentiation and increase the spillover magnitude will not allow the innovating firm to feel comfortably ahead in its specialisation characteristic preventing it from assuming the (somewhat) aggressive behaviour observed in chapter 3. Basically it can be said that with such decision from the firm that did not succeed in the R&D process, it is impossible to generate the asymmetry degree that would incentive the innovating firm to also differentiate the products less. The explanation for this resides in the fact...
that for firms to be more asymmetric, a larger breakthrough has to be made as a result of R&D, but this immediately implies that more knowledge will be transmitted as spillovers, since these are also a function of the "innovation size".

Also from the plot other interesting and intuitive conclusions can be drawn. For instance, if firm L chose a large value for \( a \) then its profit level would be decreasing in \( g \). This means that the innovating firm may even loose profit opportunity if the R&D success is large (i.e. the percentage of initial quality advantage that is available for improvement is larger). The reason for this fact is linked to the truncation of the advantage of the innovating firm that is directly related to the improvement available by discovery and the spillovers magnitude.

On the other hand, we can observe that the profit increases with \( g \) for small values of \( a \). The explanation for this lies in the greater ability that firm L has to appropriate its R&D results, which is negatively correlated with the spillover magnitude. If the latter is small, then it becomes profitable to "discover on a larger scale", as knowledge transference and advantage truncation will not be as large.

4.3.B.1.2. Uncertainty Case

The objective function to be maximised is given by the following expression, of which we already know how each term individually behaves:

\[
\pi_L^\varepsilon = p^2 \pi_L(\alpha_b, \beta_b) + (1-p)p\pi_L(\alpha_L, \beta_L) + p(1-p)\pi_L(\alpha_R, \beta_R) + (1-p)^2 \pi_L(\alpha_n, \beta_n)
\]

The results obtained will constitute the equilibrium values for \( a \) and \( b \) chosen by firms contemplating four possible future situations, knowing in advance that spillovers will be given away by firms that succeeded in the R&D activity, and that the magnitude of these might be influenced by the research design chosen.

In such context, it will be interesting to analyse two questions. The first is how the results obtained will compare to the case presented in section 4.3.A.1.2 where spillovers only occurred between innovating firms. This will provide some insights to the effects that the existence of spillovers from innovating to non-innovating firms has
on the product differentiation decision. The second question relates to the comparison of results between the case of total spillover absence with the present case where it is sufficient to have only one firm succeeding in R&D in order to have spillovers.

In order to obtain results that permit the clarification of such questions, given the complexity that an analytical study would imply (in some cases not allowing the achievement of conclusions) it was necessary to resort to numerical simulation. The results were obtained considering the usual values for the parameters (σ, S, g and p) enabling in this way a direct comparison with the conclusions drawn before. The tables with the equilibrium values for the different cases considered follow:

\( \sigma = 0.5 \)

<table>
<thead>
<tr>
<th>g</th>
<th>p</th>
<th>.1</th>
<th>.2</th>
<th>.3</th>
<th>.4</th>
<th>.5</th>
<th>.6</th>
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<th>.8</th>
<th>.9</th>
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Table 4.3. Equilibrium values of \( a = b \), for different pairs \((g, p)\), considering \( S = 20 \) and \( \sigma = 0.5 \).

\( \sigma = 0.9 \)

<table>
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<th>.3</th>
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Table 4.4. Equilibrium values of \( a = b \), for different pairs \((g, p)\), considering \( S = 20 \) and \( \sigma = 0.9 \).

Once again, the usual conclusions about the effects that the variables \( g \) and \( p \) have on the decision of product differentiation remain valid. Essentially these state that the larger the value of \( g \), and thus the larger the magnitude of the potential asymmetry, the more willing firms are to differentiate their products less. On the other hand, the higher the probability of both firms being simultaneously successful in R&D (higher value of \( p \)), the more likely firms are to keep their symmetry and therefore, in order to avoid competition, the disposition to increase product differentiation is larger.
The second conclusion that might be drawn concerns the effect that variable \( \sigma \) has on the equilibrium values for the research design. It is easily seen that the magnitude of this variable affects the decision of product differentiation in the same way it did in the case where spillovers only existed between firms that innovated. So, the larger the percentage of total spillovers that is given away in the own specialisation characteristic the more willing firms are to differentiate products more. The reasons that motivate this result were already explained in section A and remain valid for the present case.

When comparing the situation where spillovers only occurred between innovating firms with the circumstances under analysis now (comparison of table 4.1 with table 4.3, and table 4.2 with table 4.4) the immediate conclusion is that, whatever the value of \( \sigma \) considered, the consideration that spillovers also exist between a firm that succeeded in R&D and a non-successful firm will motivate firms to differentiate their products more. This result is easily detected as, for every pair \((g, p)\) that we may consider, the equilibrium values obtained in the case of spillovers also occurring between firms with different R&D outcomes are never superior to the ones reached in a situation where spillovers only existed between innovating firms. In essence, the change in the results when compared with the previous section is due to the terms representing the situations of only one firm being successful when pursuing the R&D activity, as these were the only terms modified. Having established that these two terms are the ones responsible for any changes, we may conclude that the incentives a firm has to minimise product differentiation when it believes that it will be the only one not to innovate are dominated by the motivations a sole innovator has to maximise product differentiation. This stems from the findings of the term-by-term analysis. It remains to be explained why is the term in favour of maximising product differentiation dominating the term pointing to a minimum differentiation result causing, in aggregate, a lower willingness to produce less differentiated products. The justification for this is in the fact that the variable choice involved in the latter will only affect the quality gap dimension through the co-joint determination of the spillover magnitude. On the other hand, if a firm is the sole innovator its choice of the research design variable will have a double impact on the gap dimension: the decision of allocation of R&D resources between product characteristics, which constitutes a major source of product
improvement; and the effect on spillovers as mentioned for the non-innovating rival. So, when balancing only those two terms, the importance of the value chosen for the research design in the case where the firm is the only one to innovate is larger than the research design choice of a non-innovating firm that will only affect the spillover magnitude.

Having established the comparison of the results obtained in section B with those of section A, and previously compared the latter with the conclusions reached for the case of spillover absence, the differences in the choice of R&D design for a situation where spillovers are considered not to exist and the situation where spillovers exist between innovating firms as well as from innovating to non-innovating firms are obvious. The basic conclusion that might be drawn is that, in an uncertainty context about the R&D outcome, the introduction of spillovers will strengthen the incentives firms have to differentiate their products more. The reason lying behind this result is linked with the fact that firms, by sharing the discoveries made, will contribute to reduce the advantage in terms of quality gaps that the successful R&D activity would bring. Thus, in the case where a firm is the sole innovator, its superiority will be truncated by the giving away of spillovers, not allowing the firm in question to build a comfortable advantage over the rival that would motivate a more aggressive strategy like the minimisation of product differentiation. Moreover, even in the case where both firms succeed in the innovative activity, which we know will always cause firms to maximise product differentiation, the existence of endogenous spillovers increases even more the willingness to pursue such strategy. Once again, it is the reduction in the quality gap magnitude caused by spillovers that explains this behaviour. The point is that even if product differentiation is already maximised, a reduction in the gap dimension will make the products more similar thus increasing the market competition. Therefore, the determination to maximise product differentiation is re-enforced.

Given this, firms will want to reduce the spillover magnitude, and, at the same time, try to increase its market power by differentiating their products more.
4.3.B.2. The Cooperative Equilibrium

As seen before, when firms cooperate they try to organise the relevant aspects of research design so that the expected joint profit is maximised. This implies the maximisation of the following expression:

\[ \pi_L^* + \pi_R^* = p^3 [\pi_L(\alpha_b, \beta_b) + \pi_R(\alpha_b, \beta_b)] + (1 - p) p[\pi_L(\alpha_L, \beta_L) + \pi_R(\alpha_L, \beta_L)] + p(1 - p) [\pi_L(\alpha_R, \beta_R) + \pi_R(\alpha_R, \beta_R)] + (1 - p)^2 [\pi_L(\alpha_n, \beta_n) + \pi_R(\alpha_n, \beta_n)] \]

being the quality gaps the same as defined at the beginning of section B.

In order to understand how this profit expression behaves, and following the procedure used on other occasions, a term-by-term analysis will precede the study of the expression as a whole. Once again, the former will be equivalent to the study of a situation where firms have full information about the R&D outcome (certainty case) and the latter corresponds to the more realistic case of uncertainty about the success in that activity.

4.3.B.2.1. Certainty Case

4.3.B.2.1.1. The “Both Firms Innovate” Term

When comparing the term \( \pi_L(\alpha_b, \beta_b) + \pi_R(\alpha_b, \beta_b) \) with the one analysed in section 4.3.A.2.1, it can be easily observed that they are the same, as the gap definitions relevant for the term in question did not change from the previous section. This means that all the conclusions reached then can be entirely applied to this section. Basically it was found that firms would want to maximise product differentiation, and that the introduction of the parameter \( \sigma \) did not change the results in qualitative terms.
4.3.B.2.1.2. The “Only Firm R Innovates” Term

Before starting with the study of the term $\pi_L(\alpha_R, \beta_R) + \pi_R(\alpha_R, \beta_R)$ with the relevant gap expressions, it should be noticed that the conclusions obtained from its analysis will have to be similar to the ones that would be reached from the examination of the term representing the case of being firm L the only to innovate, i.e. $\pi_L(\alpha_L, \beta_L) + \pi_R(\alpha_L, \beta_L)$. Therefore, the title of this section could be “the case where only one firm innovates”, irrespective of which one does it. Given this, it will be sufficient to analyse just one term and then extrapolate the conclusions to the other, avoiding this way unnecessary repetition.

The particular case under analysis gains added interest if we remember the conclusions obtained for each term individually in a similar context of R&D certainty. It was observed that, if a firm was the only not to innovate, then it would prefer to minimise product differentiation in order to increase the amount of spillovers absorbed. On the other hand, the sole innovator, knowing that the strategy that would best suit the rival would be the one just described, will want to maximise product differentiation so that the magnitude of spillovers is reduced.

The question now is to know how these two antagonistic forces will behave in the search for the maximal joint profit.

Once again the method of study utilised comprises numerical simulation and graphical analysis, as a complete analytical treatment of the expression $\pi_L(\alpha_R, \beta_R) + \pi_R(\alpha_R, \beta_R)$ easily collapses into something from which it is not possible to draw any conclusions.

Numerical simulations were run considering different values for the parameters $\sigma$ and $g$. The usual value of $S=20$ was assumed. It is worthwhile explaining at this point how the numerical simulations were programmed. For this particular case and for the particular parameter values assumed, it was computed for each value of $b$ (in intervals of 0,01, from 0 to 1, incl.) the value of variable $a$ that would maximise the value of the joint profit. After this was done, it was a question of observing what combination of $a$ and $b$ maximised the profit level. The combination providing the
higher profit would be the research design chosen by firms coordinating their R&D activity. In order to avoid a tiresome presentation of series of numbers, a short summary of some of the simulations, containing the most important results follows.

<table>
<thead>
<tr>
<th>$\sigma = 0.5$</th>
<th>$g = 0.1$</th>
<th>Joint Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal $a$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
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</tr>
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</tr>
</tbody>
</table>

Table 4.5. Optimal choice of $a$ and $b$ ($\sigma = 0.5$, $g = 0.1$)

<table>
<thead>
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<th>$\sigma = 0.5$</th>
<th>$g = 0.3$</th>
<th>Joint Profit</th>
</tr>
</thead>
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<tr>
<td>Optimal $a$</td>
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Table 4.6. Optimal choice of $a$ and $b$ ($\sigma = 0.5$, $g = 0.3$)

<table>
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<th>$\sigma = 0.5$</th>
<th>$g = 0.5$</th>
<th>Joint Profit</th>
</tr>
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Table 4.7. Optimal choice of $a$ and $b$ ($\sigma = 0.5$, $g = 0.5$)

From the three tables above we may observe that, whatever the value of variable $b$, the best response in order to maximise joint profit is to choose $a = 0$. Once this is known we just have to choose the value of variable $b$, that together with $a = 0$ allows firms to reach a higher joint profit level. In the tables, this is represented in the shadowed areas. So, we may conclude that, if total spillovers are equally allocated
between the two product characteristics ($\sigma = 0.5$), then the decision made in a non-cooperative context will not change when an agreement to coordinate research design is made. The best strategy for firms is to adopt the following research design: the firm that succeeded in R&D should maximise product differentiation whereas the non-innovating firm will maximise joint profit by minimising product differentiation.

Implicitly this result is stating that the gains a non-innovating firm has by minimising product differentiation and consequently absorbing more spillovers are larger than the loss inflicted in the innovating firm due to that information sharing and subsequent truncation of gap advantage. However, it should be noticed that this might be true only because the amount of spillovers given away in the own specialisation characteristic is not substantially large. A pertinent question would be to ask what would happen to the decision of product differentiation if the innovating firm, instead of giving away 50% of total spillovers in its specialisation characteristic, transferred for instance 90%? This would surely cause a larger loss in the gap advantage of the innovating firm which may dominate any gains by the non-innovating firm for not being left so behind in that characteristic. In such case it would benefit both firms (as we are concerned with the joint profit level) to minimise spillovers as much as possible so that the advantage brought by innovation translates into profit. In order to verify the validity of this argument the same numerical simulations were run considering a value for $\sigma$ equal to 0.9. All the other parameters remain the same in order to facilitate the comparison of results.

<table>
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Table 4.8. Optimal choice of $a$ and $b$ ($\sigma = 0.9, g = 0.1$)  
Table 4.9. Optimal choice of $a$ and $b$ ($\sigma = 0.9, g = 0.3$)
From the above tables we can see that there was a change in the results caused by the consideration of a larger value for $\sigma$. Now, for each value of $b$, the best response in terms of joint profit is to choose $a = 0$. Having established this and comparing the profit levels obtained from all the combinations ($a$, $b$), the conclusion reached is that the research design strategy that maximises joint profit is to manufacture products as differentiated as possible, irrespective of firm’s R&D outcome.

This confirms the intuition that if spillovers are large in the innovating firm’s specialisation characteristic, then firms will not be able to completely seize the profit opportunity the R&D discovery confers. In essence, if spillovers are large, firms will be less asymmetric as the success of one firm is also the development of the other, and this will increase competition in the product market where no type of agreement was considered to exist. Thus, by transferring larger amounts of knowledge, which may seem a “noble attitude”, firms would in fact be harming or in some way cannibalising themselves, as competition would be more intense. The best option, therefore, is to maximise product differentiation, so that the R&D discovery is fully appropriated allowing the effective improvement of the specialisation characteristic and generating profits.

The interesting thing to note is that when spillovers in the innovator’s specialisation characteristic are small, then some knowledge transference is beneficial...
in aggregate terms. Therefore we may conclude that there is some threshold value for information sharing beyond which it is no longer profitable to transfer that knowledge.

In order to better visualise the results and understand the behaviour of the joint profit function behind these results, a brief graphical presentation will be made.

The first plot shows how the expression \( \pi_L(\alpha_R, \beta_R) + \pi_R(\alpha_R, \beta_R) \) behaves assuming the following values for the parameters: \( S = 20; g = 0.3; \sigma = 0.5 \).

From the image it is easily seen that the optimal choice in terms of variable \( b \) is to set it equal to 0. This means that the firm that innovated should choose a research design that implies maximum product differentiation. However, the graphic is not so clear about the optimal value for \( \alpha \). In order to analyse this, a two dimensional plot was done, considering that \( b \) will be equal to 0, and keeping the parameter values constant:
As is now evident, the larger the value of \( a \) the higher the profit level will be. Thus, the best option in order to maximise joint profit in case one firm innovates and the rival does not, it is for the innovator to maximise product differentiation and the unsuccessful rival to differentiate less the products, capturing (small) spillovers.

Also graphically, we can observe the changes that occur in the decision to differentiate products when the amount of spillovers transferred in the own specialisation characteristic by the innovator is large. To see this, a value of \( \sigma = 0.9 \) was considered in the drawing of the following plot (the other parameter values remain the same as before).
Once again the optimal choice for the firm that innovated is obvious: $b = 0$ maximises the joint profit. With respect to the non-innovating firm the choice is not visually evident, in spite of the fact that, for large values of $b$, we can observe that low values of $a$ will allow higher profit levels to be reached. Using a two-dimensional plot will help in the visualisation of the optimal value for variable $a$, knowing that $b$ will be 0.

![Profit graph](image)

Figure 4.6. Profit function for $\sigma=0.9$; $g=0.3$; $S=20$ and $b=0=0$

Now it becomes clear that the research design choice for the non-innovating firm that maximises joint profit is the one implying maximum product differentiation ($\sigma=0$). Thus, as seen before in the numerical simulation analysis, if spillovers are large in the specialisation characteristic of the innovator, the profit level will be maximised through the production of products as differentiated as possible.

To complete the analysis, and from the results of the numerical simulation, it should be noted that the equilibrium profit level is still increasing in the variable $g$ as one would expect: the progress and development that firms are allowed to make through R&D is positively correlated with the profit level that might be reached.
4.3.B.2.1.3. The “Only Firm L Innovates” Term

As mentioned at the beginning of the previous sub-section the symmetry between the case studied before and the present one allow us to extrapolate the conclusions reached then, to the situation considered in this sub-section. The identity of the firms is irrelevant, and the conclusions about the optimal strategy to follow in what concerns product differentiation decisions only depends on the fact if we are considering a firm that innovated or not. Thus, in order to avoid unnecessary repetition the analysis and conclusions will not be restated.

4.3.B.2.2. Uncertainty Case

The analysis of the decision of product differentiation in a context where firms do not know beforehand what the outcome of the R&D activity will be, but are interested in organising together the relevant aspects of research design in order to maximise the expected joint profit, implies the study of the following expression:

\[
\pi^*_L + \pi^*_R = p^2 [\pi_L(\alpha_b, \beta_b) + \pi_R(\alpha_b, \beta_b)] + (1 - p)p[\pi_L(\alpha_L, \beta_L) + \pi_R(\alpha_L, \beta_L)] + p(1 - p)[\pi_L(\alpha_R, \beta_R) + \pi_R(\alpha_R, \beta_R)] + (1 - p)^2 [\pi_L(\alpha_n, \beta_n) + \pi_R(\alpha_n, \beta_n)]
\]

As one might expect, the mathematical examination of the above profit function is no easy task, not enabling the achievement of conclusive results given the complexity of the terms involved. Once more, the use of numerical simulation and graphical analysis will prove to be a useful and enlightening tool. Given the nature of the results obtained it would be tiresome to present the series of numbers generated by the numerical simulation, and so an elucidating graphical analysis will be presented. Notice however that the pictures shown do not constitute a full proof of the conclusions that will be presented but only examples supporting and illustrating the results achieved. The parameter values chosen for this purpose were picked randomly,
and, given that the shape of the profit function evidenced in the plots does not change significantly with the values of $g$, $p$ or $\sigma$, only a few figures will be displayed.

Figure 4.7. Joint expected profit considering: $S=20; g=.1; p=.1; \sigma=.5$

Figure 4.8. Joint expected profit considering: $S=20; g=.5; p=.1; \sigma=.9$
From the observation of the sample of plots above, it can clearly be seen that the profit level is maximised in every case for values of $a = b = 0$. This clearly points to the results evidenced by the numerical simulation that firms, when facing the context that is presently under analysis, will find that the best strategy is to maximise product differentiation. So, even if in certain states of nature (R&D outcomes) firms would have preferred to follow a different strategy (as seen for the case where only one firm innovated and spillovers were equally allocated between the product characteristics) when faced with the whole range of possible R&D outcomes, the term that pushes the
result towards a maximum product differentiation result dominates. This may be implicitly saying that firms “fear” much more a symmetric R&D outcome (that would keep firms’ symmetry) than are willing to risk the adoption of a different strategy envisaging a possible future asymmetric situation. So, the loss of profit opportunity if firms choose to maximise product differentiation and the R&D outcome is asymmetric (which in a certainty context could imply a different optimal strategy) is lower than would be if firms “bet” on a future asymmetry (and act accordingly), but the R&D outcome dictates that the symmetry will be kept.

4.3.B.3. Extending the Model

Having analysed in the previous section the case concerning spillover occurrence between innovating firms and from innovating to non-innovating firms, an interesting question that might be asked is the following: “How would the decision to differentiate products change if we considered that the total magnitude of spillovers is larger in the case where these existed between firms that innovated?” This translates to a perhaps more realistic case: by innovating, firms acquire the necessary tools to the understanding and adoption of the rival’s discovery, not ruling out the possibility that non-innovating firms may also be capable of spillover absorption, albeit on a smaller scale. In practice, we observe firms that, even though not innovating, are capable of using the knowledge generated by others. Obviously, the fact that these firms were not the creators of the new information may, in some cases, constitute a handicap. This could be justified by the fact that the non-success in R&D did not allow the firm in question to fully develop the instruments that would allow the capture of more spillovers.

The answer to the question put forward in the previous paragraph is already implicitly given by the comparison of the results obtained in the analysis made of the two distinct cases considered so far in this chapter: spillovers only occur between firms that innovate and spillovers occur between innovating firms and from innovating to non-innovating firms. Obviously these represent the two extremes of the situation presently investigated. In the former case, the magnitude of spillovers from firms that
succeeded in R&D could not be smaller, and in the latter the spillover magnitude is the same, whether we are dealing with spillovers between innovative firms or spillovers from successful to unsuccessful firms in that activity.

From the observation of the results provided by these alternative hypothesis we may conclude that as the magnitude of spillovers between firms with different R&D outcomes increases, the willingness to minimise product differentiation associated with the term representing the case of a sole innovator decreases. This results in the production of more differentiated products, both in the certainty case, where firms know in advance which firm will innovate, and in the uncertainty case.

In this sub-section a systematic presentation as the one done before will not be undertaken, as the conclusions can be intuit from previous work. In order to illustrate it, some of the results of a numerical simulation for a non-cooperative equilibrium that considers that spillovers from innovating to non-innovating firms are smaller than those between successful firms will be presented and compared with the values obtained in the other sections in similar contexts.

With the purpose of modelling the situation considered in this extension, the gap definitions will have to suffer a minor change. It will be necessary to introduce a new parameter that represents the percentage of total spillovers that will be captured by a non-innovating firm – this will be called δ. Thus, the new quality gap definitions will be given by:

- If firm both firms are successful:

\[
\alpha_s = S + Sg(1-a) - Sgb - \sigma \frac{a+b}{2} Sg + (1-\sigma) \frac{a+b}{2} Sg
\]

\[
\beta_s = S - Sga + Sg(1-b) + (1-\sigma) \frac{a+b}{2} Sg - \sigma \frac{a+b}{2} Sg
\]

- If neither firm is successful:

\[
\alpha_n = S
\]

\[
\beta_n = S
\]
- If only firm L is successful:

\[ \alpha_L = S + Sg(1-a) - \delta \sigma \frac{a+b}{2} Sg \]

\[ \beta_L = S - Sga + \delta (1-\sigma) \frac{a+b}{2} Sg \]

- If only firm R is successful:

\[ \alpha_R = S - Sgb + \delta (1-\sigma) \frac{a+b}{2} Sg \]

\[ \beta_R = S + Sg(1-b) - \delta \sigma \frac{a+b}{2} Sg \]

Obviously, a situation of total spillover absence between firms that did innovate and non-innovating firms implies \( \delta=0 \), whereas if we consider the same spillover magnitude from innovating firms to innovating and non-innovating firms, the value of \( \delta \) is one.

For the numerical simulation presented below, it was considered that \( \delta=0.5 \). This means that spillovers between successful firms will be twice as much as spillovers from innovating to non-innovating firms. There is no economic reason behind the choice of such value and the parameter could obviously assume other values, however, for this particular case it seems to be an interesting value as it is halfway between the two extreme cases already analysed.

The results will be presented next. In order to facilitate the comparison of these with the values obtained in the extreme cases already studied, the tables associated with the three situations will be displayed.

\[ \sigma = 0.5, \delta = 0 \]

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Table 4.11. Equilibrium values of \( a (=b) \), for different pairs \((g, p)\), considering \( S=20 \) and \( \sigma = 0.5 \), in a situation where spillovers only occur between innovating firms.
\[ \sigma = 0.5, \delta = 0.5 \]

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Table 4.12. Equilibrium values of \( a (=b) \), for different pairs \((g, p)\), considering \( S=20 \) and \( \sigma = 0.5 \), in a situation where spillovers occurring between innovating firms are twice than those from innovating to non-innovating firms.

\[ \sigma = 0.5, \delta = 1 \]

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Table 4.13. Equilibrium values of \( a (=b) \), for different pairs \((g, p)\), considering \( S=20 \) and \( \sigma = 0.5 \), in a situation where spillovers have the same magnitude when they occur between innovating firms and from innovating to non-innovating firms.

As one would expect the equilibrium values for the product differentiation decision when we consider a value of \( \delta=0.5 \), are somewhere in between the two extreme cases studied previously.

From the observation of the results we can then conclude that the larger the relative magnitude of spillovers from innovating to non-innovating firms when compared to the spillover magnitude between innovating firms, the more willing firms are to differentiate their products. The explanation of this fact is related to the arguments already put forward when justifying a decreased willingness to produce less differentiated products: the existence of spillovers will cause a truncation in the advantage an innovating firm has, which was the motor driving this type of firms to opt for a less differentiation solution. Recalling the conclusions obtained in the third chapter of this Thesis, we know that firms need to have a comfortable advantage in their specialisation characteristic before venturing in the rival’s specialisation characteristic. The existence of spillovers will not allow the innovating firm to be so far
ahead its rival, not giving it the "confidence" to produce similar products and face intense competition in the product market. Thus, the larger the spillover magnitude, the more willing firms are to differentiate their products and establish some market power.

4.4. Conclusion

This chapter builds on the previous one, to which, along with the consideration of the possibility of existence of asymmetric firms, it is added the assumption that spillovers between firms may be present.

The existence of spillovers has been widely acknowledged in the literature, namely the one related with the Theory of Innovation. The main object of study of authors addressing the topic concern issues like the level of R&D expenditure that is made, the optimal timing for innovation or the design of mechanisms with the purpose of increasing the appropriability of the outcome of the innovative activity.

However, the study of the impact that spillovers might have on the decisions of product differentiation has not been given enough attention. This aspect grows in significance as we accept that the Theory of Innovation and the Theory of Product Differentiation are intrinsically and strategically inter-related. As was pointed out in the previous chapter, being the innovative activity usually behind the differentiation of products, we should identify the features of each theory that might be carried to the other and affect the conclusions obtained. The consideration of spillovers is one of such aspects that being a characteristic of most innovative processes should be taken into account when analysing the product differentiation decision insofar as it might affect the results reached. Moreover, as d’Aspremont and Jacquemin (90) have stated, the magnitude of spillovers itself can be affected, among other factors, by the degree of product differentiation. It is intuitive and reasonable to say that, in most cases, the more differentiated products are, the lower the spillovers each firm will be able to absorb as the similarities products share are fewer. Obviously the reverse is also true. The acknowledgement of this fact may constitute the source of a deeper inter-relation
between the two theories in question, raising strategic questions about the optimal degree of product differentiation.

In this chapter it is shown that the consideration of this type of spillovers (endogenous spillovers) does in fact change the results concerning the decision of product differentiation. In order to show this, it is assumed that there exists an exogenous variable, $\sigma$, which represents the percentage of total spillovers that will be given away by a innovating firm in its specialisation characteristic. This parameter thus determines how spillovers are allocated to the improvement of each of the product characteristics.

Several scenarios were considered within the framework presented above:

A) Spillovers only exist between innovating firms;

B) Spillovers exist not only between innovating firms but also from innovating to non-innovating firms:
   B.1) Spillover magnitude is the same whether the spillovers are between innovating firms or from innovating to non-innovating firms;
   B.2) Spillovers between innovating firms are larger than spillover from innovating to non-innovating firms.

Whatever the scenario considered, it was seen that whenever firms act non-cooperatively and are certain that both will innovate, then the results obtained in absence of spillovers apply: firms will choose to maximise product differentiation as their symmetry will surely be kept and in such a situation, competition in the product market could be very harmful for both firms.

If we maintain the context above but instead consider that firms know with certainty that only one firm will innovate (and firms know who will be the innovator) then it is necessary to distinguish scenario A from scenario B. In the former, no changes occur relative to the results obtained for the same type of situation in the previous chapter. This obviously has to do with the fact that despite the spillovers that were introduced, these only occur between innovating firms, which is not the case when there is a sole innovator. When we analyse the same situation under scenario B,
then results change dramatically when compared with the case where spillovers were not present. The conclusion obtained was that the innovating firm would want to maximise product differentiation, whereas the non-innovating firm would opt for a research design that would lead to less differentiated products. Notice that in the absence of spillovers, we obtained a result that could be considered the reverse: the innovating firm might want to minimise product differentiation (provided the asymmetry was large enough), and the decision of the non-innovating firm was irrelevant. This difference in strategies can only be accounted for by the existence of spillovers. When these exist, a non-innovating firm will try to minimise product differentiation so that it maximises the amount of spillovers absorbed. This is the only chance this firm has to improve its product. In this case the non-innovating firm will not be afraid of increased competition as on one hand it knows that the innovating firm will maximise product differentiation, thus softening the competition while on the other hand, the gains of improving the product compensates any loss that might be inflicted through more competition. The innovating firm, by involuntarily sharing its discovery with its rival will see its advantage truncated. This will not confer it the comfortable lead that would allow a more aggressive strategy as would be the minimisation of product differentiation.

With respect to the case where it is assumed that the R&D process has uncertainty associated and therefore firms lack knowledge on what the outcome of the innovative activity will be, the qualitative results concerning the effect that the parameters \( g \) and \( p \) have on the decision of product differentiation do not change relatively to what was found in chapter 3. Briefly, it was observed that the higher the value of \( g \) and the lower the value of \( p \), the more willing would firms be to differentiate more their products. This result is not new and was already explained in the previous chapter. Another conclusion that might be drawn from both scenarios analysed is related to the impact that the parameter \( \sigma \) has on the decision to differentiate products. In both cases, it was observed that an increase in \( \sigma \) would increase the willingness to maximise product differentiation. The explanation for this fact is again related to the magnitude of the asymmetry between firms. A high value of \( \sigma \) means that an innovating firm will be giving away a large amount of spillovers in its specialisation characteristic thus not allowing itself to build the necessary advantage to pursue a
minimum product differentiation strategy. Thus the larger the value of \(\sigma\), the more
difficult it will be to generate the degree of firm asymmetry that might lead to the
production of more similar products.

It is the same order of arguments that provides the reasons for the difference in
results when comparing scenario A with scenario B: in an uncertainty situation where
firms act non-cooperatively, the introduction of spillovers from innovating to non-
innovating firms will encourage them to differentiate their products more. Basically,
the consideration of such spillovers will once again shorten the advantage a innovating
firm has over a non-innovating firm in the eventuality of an asymmetric R&D outcome,
which, added to the desire of maximising product differentiation in case both firms
innovate will result in a decision of more differentiated products.

Concerning the cooperative equilibrium in a context where firms are certain
that both will innovate, the decision will be to maximise product differentiation, as was
the case when spillovers were not present. Thus one might say that the introduction of
spillovers in this setup did not change the results. However, if we compare scenario A
with scenario B in a situation where firms are sure that only one will innovate, then we
can observe that the results are different. In the case where we consider spillovers to
eXist only between innovating firms, the conclusions remain obviously the same relative
to what was observed in the third chapter for a similar situation but in total absence of
spillovers. It should be noticed that the introduction of spillovers only between
innovating firms will not affect in anyway the results for the case where it is known for
sure that only one firm succeeds in the R&D activity: spillovers will not be present in
such a situation, as they were not in the previous chapter. Notice however that this is
not the case in scenario B. In such a scenario, spillovers do exist from innovating to
non-innovating firms and therefore will affect the case where there is a sole innovator.
The results obtained show that for relatively small values of \(\sigma\) \((\sigma=0.5)\), the non-
innovating firm will choose to minimise product differentiation whereas the innovating
firm will opt to maximise product differentiation. This conclusion is altered if the
percentage of spillovers given away in the specialisation characteristic of the
innovating firm is large (large value of \(\sigma\)). In such case we can observe that both firms
will choose to maximise product differentiation. This means that for small values of \(\sigma\),
the gains a non-innovating firm has by minimising product differentiation, and thus absorbing more spillovers, are larger than the loss inflicted in the innovating firm due to the information sharing and subsequent truncation of gap advantage. This result is changed when the amount of spillovers given away in the innovating firm specialisation characteristic is significantly large. In such cases, the loss in the gap advantage of the innovating firm will dominate the eventual gains the non-innovating firm would have if product differentiation was minimised and more spillovers were absorbed.

The bottom line is that if spillovers are large in the innovating firm's specialisation characteristic, then firms will not be able to completely seize the profit opportunity conferred by the innovation. The idea is that if spillovers are large, then the gap magnitude will be decreased which may be viewed as a decrease in the degree of firm asymmetry insofar as the innovation of one firm is also the development of the product of its rival. As seen in other occasions, the small degree of firm asymmetry will be prejudicial to firm's profit as competition in the product market intensifies. Thus, the logical result when large amounts of knowledge are transferred is to maximise product differentiation and keep firm asymmetry.

With respect to a cooperative situation where firms do not know what the outcome of the R&D activity will be, it was observed that firms, both in scenario A and scenario B will adopt a strategy of maximum product differentiation. When comparing the results obtained in such contexts with those obtained for the case of total spillover absence, we can observe that the introduction of spillovers did not change the decision of product differentiation. The reasons for this are the same as the ones presented in the previous chapter and relate to competition issues and the attainment of market power.

To conclude it should be noted that the observation of the two scenarios presented above constitute the extreme cases of a perhaps more realistic situation in which spillovers from innovating to non-innovating firms do exist but are relatively smaller than the spillovers between innovating firms. From this and the use of numerical simulation, it was possible to re-enforce the result that the larger the amount of spillovers (in this case, from innovating to non-innovating firms), the less willing
firms are to minimise product differentiation. The explanation for this may be found above when scenario A and B were compared.
CHAPTER 5 – PRODUCT DIFFERENTIATION WITH SPILLOVERS ENDOGENOUSLY ALLOCATED TO PRODUCT CHARACTERISTICS
5.1. Introduction

The acknowledgement of the inter-relations existing between the Theory of Innovation and the Theory of Product Differentiation led us to consider and conclude that some of the particularities of the innovative theory, as are the uncertainty associated to most R&D processes (which could cause firm asymmetry) and the presence of endogenous spillovers, should be taken into account when studying the product differentiation choice. The proof of the necessity of the consideration of such features is given in the preceding two chapters where it is shown that the decision to differentiate products may change when compared to established benchmarks as hypotheses like firm asymmetry and/or endogenous spillovers were introduced into the analysis.

With respect to the assumption of endogenous spillovers’ existence, it was mentioned in the previous chapter that there is a number of ways of modelling its presence. The background idea to be transmitted, also stated by d’Aspremont and Jacquemin (90), is that the more differentiated products are, the less knowledge/information will spillover from one firm to the other, and vice-versa, but the variations that can be done around this theme are uncountable.

In chapter 4 some of these variations have already been analysed, and mainly concerned the different assumptions made about who was eligible to absorb the spillovers generated by a firm that succeeded in the R&D activity and the magnitude of those spillovers. However, some simplifying assumptions were made at the time, whose purpose was not to closely depict reality but to, in some way, isolate some features of the model and understand its implications to the decision of product differentiation. One of these assumptions is materialised in the introduction of the parameter $\alpha$. This exogenous variable served the purpose of allocating the total amount of spillovers ($\sum g \frac{a+b}{2}$) between the two characteristics that define the product. By using this setup, the research design choice (that implicitly determines the desired level of product differentiation) would only affect spillovers in their magnitude, not having any effect on the amount of spillovers that will go to each product characteristic (remember that spillovers are assumed to be non-characteristic specific).
Thus, the decision of the level of product differentiation was studied considering that it would only determine the spillover size, without introducing other considerations that, despite being more realistic, would cause the analysis to be more complex and would not clarify which aspects are responsible for which results.

Having isolated and understood the impact that the choice of research design has on the determination of spillover magnitude and its consequences to the desired level of product differentiation, we are now in a position to introduce new features to the model, that will add to its reality adherence, without the fear of losing information about which assumptions cause which effects.

In the present chapter, the assumption on spillover allocation between product characteristics will be altered to another that is more intuitive and in accordance to what might be called "common sense". With respect to this, one would expect, and may observe in most industries, that a firm that has been mainly researching on a specific product characteristic will probably be more capable of absorbing spillovers in that particular characteristic than in any other. Even if the firm in question failed to innovate, the effort made in the R&D activity can constitute a learning process that translates into useful knowledge which may facilitate the incorporation of some discovery made by another firm. As an example we can mention the pharmaceutical industry: if a laboratory spends most of its resources investigating the properties of Ibuprofen, it will probably be more prepared to adopt any innovation made by other laboratories that relate to that chemical than developments concerning other chemicals. Therefore it seems reasonable to assume that spillovers are not exogenously assigned to each product characteristic as was considered in the fourth chapter, but that its allocation depends on the research design chosen by the firm. For this reason, a larger share of total spillovers should probably be devoted to the characteristic the firm researched more intensively.

Having put forward some considerations about the assumption of spillover allocation between characteristics and proposed what can be considered a more realistic way to model it, it is worthwhile to intuitively point out and understand the consequences that the new assumption might have on the firms' decision of product differentiation.
The basic change that is made to the model when compared with the previous chapter is that the choice of the research design will not only determine the total magnitude of spillovers, but also the way these spillovers are allocated between characteristics. Thus, the choice of \( a \) and \( b \) will play a double role in the model. The question that might be asked now is if this second attribution of the research design will change in any way the results obtained in the preceding chapters. This is the same as asking if the fact that when firms decide the desired level of product differentiation, they will also be determining the allocation of spillovers between characteristics, will affect the product differentiation decision.

In such a setup, a firm devoting all its R&D resources to a particular product characteristic, besides the developments its own innovative activity may bring to that characteristic, has to take into account that more improvements may be made in that attribute insofar as the firm's capacity to absorb spillovers is concentrated in that aspect. Thus, given this management of spillovers, which re-enforces the decisions concerning characteristic improvement, a firm might find itself "over-developing" one product feature, relative to the other product characteristic. This might encourage a firm initially devoting all its R&D efforts to just one characteristic, to reduce its R&D activities in that direction as spillovers can also help in its development. Also, the R&D resources freed can be diverted to the development of the other product characteristic, which in turn will also benefit from spillovers. Obviously this is not the only type of reasonable strategy available to the firms. For instance, a firm may seize the spillover re-enforcement in order to build a even larger advantage in its specialisation characteristic hoping to attract consumers with the product's high quality in that characteristic.

Another interesting question that might be raised is how the decision of a firm that knows that it will be the sole non-innovator is going to change with this type of spillovers. In the previous chapter it was shown that a firm in such context would prefer to minimise product differentiation in order to maximise spillovers, which would be exogenously allocated between product characteristics. This meant that the non-innovating firm could also benefit from spillovers in its specialisation characteristic. In the present situation, a decision to minimise product differentiation means equally that the spillover magnitude will be maximised, but it also implies that spillovers will only
occur in the non-specialisation characteristic. This obviously contributes in making products more similar, and consequently will increase market competition. Thus, a firm in such a context might want to change its strategy when compared to the framework analysed in chapter 4.

Clearly, the examples given of the existence of new incentives for firms that might alter their strategies concerning product differentiation are proof that a study of this type of situation is worthwhile, not only because it may affect the conclusions obtained, but also and essentially because they aim at depicting reality more closely.

In the next section, the changes that are necessary to make in the model so that it incorporates this alternative kind of spillovers, are presented. In a third section the results obtained will be shown and explained. A final section concludes.

5.2. The Model

The model that will be used in the analysis of the question put forward when introducing this chapter is the same utilised in the preceding chapters. The only alteration that has to be done so that it encapsulates the particular assumption about spillover allocation between product characteristics relates to the quality gap expressions that will have to be redefined. Before this is done, it is worthwhile to think objectively and question which other assumptions analysed in the previous chapter (about who is eligible to receive spillovers) should be considered in the present study.

One option available is to follow the same structure presented in chapter 4 and extensively analyse the situations: a) where spillovers only occur between innovating firms; b) where spillovers occur not only between innovating firms but also from innovating to non-innovating firms. Within the latter hypothesis, consideration could also be given to the case where spillovers have the same magnitude irrespective of the R&D success of the firm receiving the spillovers and the case where spillovers between innovating and non-innovating firms are smaller than the spillovers among innovating firms. In spite of the completeness of such an option, it would be somewhat tiresome and repetitive to undertake such type of analysis insofar as the main effects caused by each of the above assumptions have already been identified and explained. The
introduction of a different assumption for spillover allocation between product characteristics would, in some cases, change the numerical results but it surely would not alter qualitatively the effects of the introduction of spillovers from innovating to non-innovating firms or the consideration of different spillover magnitudes. Thus, no added insights would be gained by adopting such methodology.

Given what was mentioned in the previous paragraph, this chapter will concentrate on the analysis of the study of only one situation, in the certainty that the results for the other possible spillover assumptions follow the rationale and reasoning of the preceding chapter. The concrete situation that will be assumed for the study of this alternative hypothesis of spillover allocation is the case where spillovers occur not only between innovating firms, but also from innovating to non-innovating firms. It is also considered that their magnitude is the same irrespective of the R&D outcome of the receiving firm. By studying such a case and using the conclusions of chapter 4, we can easily deduce that both a lower(ed) spillover magnitude or its total absence from innovating to non-innovating firms would increase the willingness to differentiate the products more in comparison to the results that will be displayed for the particular case under study.

Having established the assumption that will be considered in this chapter about who is eligible to receive spillovers and their magnitude, we can now present and explain the necessary changes to the quality gap definitions so that it incorporates the new mechanism of spillover allocation between product characteristics.

According to what was mentioned in the introduction to this chapter, the basic idea to be transmitted is that a firm will be, in principle, more capable of absorbing spillovers in the product characteristic for which it has done more research work. Therefore, the amount of spillovers received in each product characteristic should be a direct function of the research design chosen. Making (partial) use of the spillover specification utilised in the fourth chapter, we already know that the total amount of spillovers generated will be given by $Sg \frac{a+b}{2}$, which has to be divided by the two product characteristics according to the research design. If a firm devotes all or the majority of its R&D efforts to the development of a given characteristic, then, in
conformity to the idea we wish to express about spillover allocation, the major share of spillovers should be allocated to that characteristic.

At this point a simplifying assumption will be introduced: it will be assumed that, in case a firm spends all its R&D resources in the development of just one characteristic, then spillovers will be totally directed to that particular characteristic. Moreover, the spillover distribution for the two characteristics will be a simple linear function of the research design choice variable. This means that, if a firm wishes to spend 75% of its R&D resources in the development of a particular characteristic, then 75% of the total spillovers will be assigned to that characteristic and the remaining 25% to the other.

This assumption does not try to depict reality, but only not to introduce a greater complexity into the model and also to analyse an extreme situation, which is typically useful in economic theory for the characterisation of intermediate situations.

So, in accordance to all that was previously mentioned and making use of the same type of reasoning utilised in the preceding chapters when defining the quality gaps, these can be expressed for the situation under analysis as follows:

- If both firms are successful:
  \[
  \alpha_b = S + Sg(1-a) - Sgb - b \frac{a+b}{2} Sg + (1-a) \frac{a+b}{2} Sg \\
  \beta_b = S - Sga + Sg(1-b) + (1-b) \frac{a+b}{2} Sg - a \frac{a+b}{2} Sg
  \]

- If neither firm is successful:
  \[
  \alpha_n = S \\
  \beta_n = S
  \]

- If only firm L is successful:
  \[
  \alpha_L = S + Sg(1-a) - b \frac{a+b}{2} Sg \\
  \beta_L = S - Sga + (1-b) \frac{a+b}{2} Sg
  \]
- If only firm R is successful:

\[
\begin{align*}
\alpha_R &= S - Sgb + (1 - a) \frac{a + b}{2} Sg \\
\beta_R &= S + Sg(1 - b) - a \frac{a + b}{2} Sg
\end{align*}
\]

Having presented and justified the assumptions that will be considered throughout this chapter and explained the necessary modifications that had to be made to the model so that it incorporates the above-mentioned assumptions, it is now possible to analyse the product differentiation decision. This analysis will be undertaken following the same type of methodology and organisation utilised in previous chapters. This will contribute to a better understanding of the consequences that the particular assumptions made in this chapter will have insofar as it will allow a direct comparison with the results obtained previously. Therefore we will begin by analysing the non-cooperative setup, and later we will proceed to the study of the cooperative setup. In both cases, the certainty and uncertainty contexts will be considered.

5.3. Results

5.3.1. The Non-Cooperative Equilibrium

As usual, and given the firms’ initial symmetry, the study of the product differentiation decision will be undertaken only from the point of view of one firm (firm L), knowing that the conclusions reached will also be valid to the other firm (firm R) whenever it finds itself in a similar situation.

Considering the expressions presented above for the quality gaps, the expected profit function firm L will want to maximise is\(^1\):

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\(^1\) As stated previously and given firms’ symmetry, a similar expression can be written for firm R.
\[ \pi_L^* = p^2 \pi_L(\alpha_b, \beta_b) + (1-p)p \pi_L(\alpha_L, \beta_L) + p(1-p)\pi_L(\alpha_R, \beta_R) + (1-p)^2 \pi_L(\alpha_n, \beta_n) \]

In spite of the fact that this expression is already familiar, it should be noted that it is a general expression, which might represent distinct situations according to the definition of quality gaps utilised. As seen before, that definition is crucial and may significantly change the results obtained. This serves as a reminder that quality gaps or, more specifically, spillovers may be modelled in a number of different ways with different conclusions for the product differentiation decision. However, this Thesis does not aim at covering all the possibilities – its objective is to show that elements like firm asymmetry or spillovers play an important role in the Theory of Product Differentiation (as it is related to the Theory of Innovation) and to reach results and conclusions for what might be considered the more appealing situations.

As usual, a term-by-term analysis will precede the study of the more realistic case of uncertainty about the outcome of the R&D activity.

5.3.1.1. Certainty Case

5.3.1.1.1. The "Both Firms Innovate" Term

This depicts a situation where firms know in advance that both will be successful in innovating, thus the profit expression firm L will be interested in maximising is given by: \( \pi_L(\alpha_b, \beta_b) \), with \( \alpha_b \) and \( \beta_b \) defined above.

Computing the first derivative of this expression with respect to firm L's decision variable \( (a) \) we will obtain:

\[ \frac{\partial \pi_L}{\partial a} = \frac{\partial \pi_L}{\partial \alpha_b} \frac{\partial \alpha_b}{\partial a} + \frac{\partial \pi_L}{\partial \beta_b} \frac{\partial \beta_b}{\partial a} \]

where:

\[ \frac{\partial \alpha_b}{\partial a} = \frac{\partial \beta_b}{\partial a} = -S_g \left( \frac{1}{2} + a + b \right) < 0. \]

Simplifying the expression of the profit derivative it collapses into:

\[ \frac{\partial \pi_L}{\partial a} = -S_g \left( \frac{1}{2} + a + b \right) < 0. \]
From what was shown in chapter 3, we already know what was then called the domination of the direct effect over the indirect effect, that is\[ \frac{\partial \pi^L}{\partial a} = \left( \frac{\partial \pi^L}{\partial \alpha_b} + \frac{\partial \pi^L}{\partial \beta_b} \right) \frac{\partial \alpha_b}{\partial a} > 0 < 0 < 0 \]

\( \alpha_b = \beta_b \), which is the case in the present situation. For this reason the term in brackets is positive which makes the whole expression of \( \frac{\partial \pi^L}{\partial a} \) to be unequivocally negative. Mathematically speaking it means that the lowest possible value of \( a \) will maximise \( \pi^L \). Thus, firm L will choose a value of \( a \) equal to zero and this translates into a decision of maximum product differentiation. So, the firm in question will devote all its R&D efforts and also all the benefits that come from the rival firm in the form of spillovers to the development of the product characteristic in which it is already specialised.

Once again, the reasons that motivate such decision are linked with the desire to avoid competition and establish some market power. As such, it seems that any incentive an innovating firm might have to differentiate products a little less and to be able not only to self-improve but also to benefit from the rival’s discovery and direct it to the other characteristic, are not enough to overcome the fear of market competition. This is understandable insofar as firms start from an initial symmetric position and know in advance that the symmetry will be kept, making the results of competition to be equally harmful for both firms.

So, this term still represents a force pushing the result towards a Maximum Differentiation outcome, in spite of any incentives that might have softened (obviously not enough) a firms’ determination to pursue such strategy.
5.3.1.1.2. The “Only Firm R Innovates” Term

In this case, firms are sure that only one of them will innovate, and the identity of the innovator is known. This means that firms are aware that asymmetry will arise and have to take this into account when deciding the level of product differentiation. The analysis will start with the study of the product differentiation decision for firm L in the case firm R is the only one to innovate.

The relevant profit expression to be maximised is given by the following expression:

\[ \max_a \pi_L (\alpha_R, \beta_R) = \pi_L \left[ S - Sg b + Sg \frac{a + b}{2} (1 - a), S + Sg (1 - b) - Sg \frac{a + b}{2} a \right] \]

In order to find the argument that maximises firm L’s profit it is helpful to compute its first derivative with respect to \( a \):

\[ \frac{\partial \pi^L}{\partial a} = \frac{\partial \pi^L}{\partial \alpha_R} \frac{\partial \alpha_R}{\partial a} + \frac{\partial \pi^L}{\partial \beta_R} \frac{\partial \beta_R}{\partial a} \]

It is straightforward to show that the derivatives of the quality gap expressions with respect to \( a \), are the following:

\[ \frac{\partial \alpha_R}{\partial a} = -Sg \frac{a + b}{2} + (1 - a) \frac{Sg}{2} \quad \text{and} \quad \frac{\partial \beta_R}{\partial a} = -Sg \frac{a + b}{2} - a \frac{Sg}{2} \].

Whilst the latter is undoubtedly negative, the former may assume negative or positive values, depending on the sign of \((1/2 - a - b/2)\). Given the uncertainty about this and despite knowing the signs of all other terms involved in the expression of \( \frac{\partial \pi^L}{\partial a} \) it is impossible to tell if it will be positive or negative. Thus, any conclusions about the choice of the degree of product differentiation by the only firm that did not to innovate cannot be drawn using such methodology.

When we compare this case with the similar situation analysed in the previous chapter (that considered a different assumption about spillover allocation), we observe that where before there was absolute certainty that the best strategy would be to minimise product differentiation, now this result, if emerging, is not so evident. The
main cause for this lies precisely in the derivative $\frac{\partial \alpha_R}{\partial a}$ that in chapter 4 was unequivocally positive but now its sign is undetermined. This stems from the fact that before, an increase in the degree of product similitude (increase in $a$) would have as only effect the enlargement of the spillover magnitude, which was beneficial to both product characteristics insofar as spillover allocation was exogenously determined. In the present case, by minimising product differentiation, the non-innovating firm will still be able to increase spillover magnitude, but added to this and given the double role that the research design now plays, the characteristic in which this firm is specialised may see its development decreased as a lower percentage of total spillovers will be allocated to that characteristic. So, in spite of the increase of total spillover magnitude, there is also a relative decrease in its allocation to the first product characteristic, thus, in net terms, it is not certain that this characteristic will benefit from lowering product differentiation and the consequent increase of spillovers. On the other hand, a strategy of minimum product differentiation would also be allocating more spillovers to the characteristic for which the non-innovating firm is not specialised, which might increase competition in the product market. This is the main cause for not obtaining a clear-cut result as the one achieved in the previous chapter for a similar context.

Notice that the difficulty in drawing any conclusion analytically emerges from the complexity of comparing the relative magnitudes of $\frac{\partial \pi^L}{\partial \alpha_R} \frac{\partial \alpha_R}{\partial a}$ and $\frac{\partial \pi^L}{\partial \beta_R} \frac{\partial \beta_R}{\partial a}$. If this was known, we might go a little further in the results obtained through this method. However, this does not constitute a great obstacle as there are alternative ways that allow the achievement of conclusions. One of these methods is the use of graphical analysis that will prove to be very conclusive about the optimal strategy a non-innovating firm should adopt concerning the degree of product differentiation.

The following pictures represent the behaviour of firm L's profit expression when this firm is the only not to innovate. The usual value of $S=20$ was considered to allow comparisons with the cases previously studied. Different values for $g$ were assumed ($g=0.1, g=0.3, g=0.5$).
As can be easily understood from the three plots presented, the value of $a$ that maximises firm L’s profit is $a = 1$, whatever the value of $g$ considered, and the strategy chosen by the innovating firm. In this respect, it is interesting to notice that not only does the non-innovating firm want to minimise product differentiation, but also this firm would like the innovating firm to follow a similar strategy – this would cause the
total magnitude of spillovers to increase even more, which is the only source of product improvement for the firm in question.

Thus, we still maintain the decision of minimum product differentiation found in chapter 4 for a similar context. It can then be concluded that a firm that knows it will not innovate, will try to develop its product by maximising the possible level of spillovers. This type of free-rider behaviour compensates for any disadvantages (as increased market competition is an example) that might be encountered by producing a product that is more similar to the rival’s.

Notice however, that the fact the analytical proof of the result was not possible to achieve, (which was due to the double role the choice of the research design now has and which has brought uncertainty about the effects a minimum differentiation strategy would have on the specialisation characteristic of a non-innovating firm) may be interpreted as a weakening in the determination to minimise product differentiation found in the previous chapter. So, in spite of the fact that the final result is the same, we may intuit changes in the firm’s incentives that might affect the results when all terms of the expected profit expression are analysed simultaneously in the uncertainty case.

5.3.1.1.3. The “Only Firm L Innovates” Term

In this case, firms are sure that only one of them will be the innovator, whose identity is known. This means that firms are aware that asymmetry will arise and have to take this into account when deciding the level of product differentiation.

The relevant profit expression for firm L to maximise is given by the following expression:

\[
\max_{a} \pi_L(\alpha_L, \beta_L) = \pi_L \left[ S + Sg(1 - a) - Sgb \frac{a + b}{2}, S - Sga + Sg \frac{a + b}{2} (1 - b) \right]
\]
In order to write the first order condition, it is useful to look at the first derivative of the profit expression with respect to $a$:

$$\frac{\partial \pi^L}{\partial a} = \frac{\partial \pi^L}{\partial \alpha_L} \frac{\partial \alpha_L}{\partial a} + \frac{\partial \pi^L}{\partial \beta_L} \frac{\partial \beta_L}{\partial a}$$

The derivatives of the quality gap expressions with respect to $a$, can be easily computed: $\frac{\partial \alpha_L}{\partial a} = -Sg - \frac{Sgb}{2}$ and $\frac{\partial \beta_L}{\partial a} = -Sg + \frac{Sg(1-b)}{2}$. Both derivatives are clearly negative. Adding this to our previous knowledge of the sign of $\frac{\partial \pi^L}{\partial \alpha_L}$ and $\frac{\partial \pi^L}{\partial \beta_L}$, the sign of the whole expression of $\frac{\partial \pi^L}{\partial a}$ is impossible to determine. This constitutes an obstacle in obtaining conclusions about the choice of the degree of product differentiation by the only firm to innovate. Again, alternative methods of reaching results will have to be utilised. Perhaps the most informative and interesting is once more the graphical analysis, which gives exactly the same results as numerical simulation methods but whose presentation is probably less tiresome.

In the analysis it will be considered the usual value of $S=20$ and $g$ between 0 and 0,5. This will allow a direct comparison with the results obtained in previous chapters.

It should be noticed that, relatively to the previous section, we have now more information. The results obtained in section 5.3.1.1.2. constitute information about the behaviour of the non-innovating firm that is common knowledge. Therefore, when deciding the level of desired product differentiation, the innovating firm will have to take into account that its non-innovating rival will choose to minimise product differentiation, whatever the innovating firm chooses to do.
By observing the plot it is straightforward to conclude that, whatever the value of $g$, the optimal research design for firm L is one of maximum product differentiation as the value of $\alpha$ that maximises the profit level is 0. The reasons that led firm L to opt for such strategy are linked with the fact that its rival is already minimising product differentiation and thus trying to maximise the spillover magnitude. This will cause the innovating firm to not be capable of appropriating enough of its R&D success, truncating the gap advantage that the innovation could confer. As such, firm L will not be able to establish a sufficiently large lead that would allow it to follow a more aggressive strategy, by improving in the rival’s specialisation characteristic, and capture new consumers from rival’s clients.

Before going into the study of the case where R&D uncertainty is present, it may be interesting to notice how the profit level changes with the value of $g$ for a given choice of research design ($\alpha$). It can be seen from the plot above that, for large values of $\alpha$, an increase in the innovation magnitude ($g$) affects negatively the innovating firm. On the other hand the same increase in $g$ may be beneficial for the innovating firm as long as it chooses to differentiate the products enough (small values of $\alpha$). The explanation for this fact lies completely in the effects that spillover magnitude might have on the innovating firm: if a larger breakthrough may be made, the firm that succeeds in achieving it may loose profits if it chooses to minimise product differentiation, as larger spillovers will exist and more improvements will be transmitted to the rival firm. However, in the case where the choice of product differentiation is one that differentiates products enough, then the degree of
appropriability will be high and the innovating firm will benefit more from the innovation not giving away so much information to the rival.

Notice that these conclusions are not totally new: the results obtained in a similar situation but for a different spillover formulation in the previous chapter also pointed in the same direction. Given this, we may say that the introduction of this second role for the research design choice did not change the results quantitatively. Thus, at a first glance, it may seem that when firms decide the total spillover magnitude, they implicitly determine its allocation between product characteristics, bringing nothing new into the analysis. We should recall, however, the analysis of section 5.3.1.1.2. where, comparatively to the fourth chapter, the results were not crystal clear to obtain, denoting that some minor changes might have occurred but were not enough to change the results. This may cause that, in spite of no changes being observed in the conclusions obtained for each case (term) individually, when we analyse a situation where all terms come into play simultaneously (R&D uncertainty), the conclusions might be altered.

5.3.1.2. Uncertainty Case

In the introduction to this chapter it was already stressed that the new formulation for spillovers (the choice of research design determines not only spillover magnitude but also its division between characteristics) may bring new incentives and motivations for firms' behaviour. Despite this, the basic conclusions drawn in the previous chapter remained valid for each particular case of R&D outcome in a situation of certainty. It is now time to verify if, in an uncertainty context (where all terms are considered together), the conclusions are affected in anyway.

The objective function is a weighted average of the terms previously analysed, plus a term depicting the situation where firms are not capable of innovating. This latter term was not explicitly analysed, but as there are no innovators and consequently no spillovers, the choice of research design is totally irrelevant. This can also be seen in the absence of the choice variables (a and b) from the quality gap expressions. Thus,
firm L will want to maximise the following expected profit expression by choosing the
value of $a$:

$$
\pi^*_L = p^2 \pi_L(\alpha_s, \beta_s) + (1 - p)p \pi_L(\alpha_L, \beta_L) + p(1 - p)\pi_L(\alpha_R, \beta_R) + (1 - p)^2 \pi_L(\alpha_n, \beta_n)
$$

Given the previous experience in maximising each term individually or the same
expression with different gap definitions, it is not surprising that an analytical study of
the above expression rapidly becomes extremely complex constituting an obstacle to
the achievement of results. Thus, alternative methods will have to be used if we want
to learn something about the decision of product differentiation in the present context.
Similarly to what was done in other chapters a clarifying numerical simulation will be
presented, considering the usual parameter values ($S=20$, $0.1<g<0.5$, $0.1<p<0.9$). The
table with the equilibrium values for the different parameter values analysed follows:

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<th>$g$</th>
<th>$p$</th>
<th>.1</th>
<th>.2</th>
<th>.3</th>
<th>.4</th>
<th>.5</th>
<th>.6</th>
<th>.7</th>
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</tbody>
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Table 5.1. Equilibrium values of $a (=b)$, for different pairs ($g$, $p$), considering $S=20$

From the observation of the values displayed in the table above, one can
perceive that the general qualitative conclusions reached in preceding chapters about
the impact that the parameters $p$ and $g$ have on the decision of product differentiation
remain valid. These conclusions are in respect to the fact that the smaller the value of $p$
and the larger the value of $g$, the more willing firms are to differentiate the products
less. The reasons behind this have already been discussed and explained when this type
of results was first obtained, so we will refrain from doing it again in order to avoid
unnecessary repetition.

More interesting is undoubtedly the comparison of the table above with the
results presented for a similar context except that the decision of research design only
affected the spillover magnitude and not its allocation. In order to facilitate the
comparison, the tables presented in chapter 4, section 4.B.1.2. (Tables 4.3 and 4.4) will
now be reproduced.
\[ \sigma = 0.5 \]

<table>
<thead>
<tr>
<th>g</th>
<th>p</th>
<th>.1</th>
<th>.2</th>
<th>.3</th>
<th>.4</th>
<th>.5</th>
<th>.6</th>
<th>.7</th>
<th>.8</th>
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<tbody>
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</tr>
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<td>.4</td>
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<td>.589</td>
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<td>.471</td>
<td>.389</td>
<td>.282</td>
<td>.127</td>
<td>0</td>
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</tr>
</tbody>
</table>

Table 4.3. (previous chapter) Equilibrium values of \( a (= b) \), for different pairs \((g, p)\), considering \( S = 20 \) and \( \sigma = 0.5 \).

\[ \sigma = 0.9 \]

<table>
<thead>
<tr>
<th>g</th>
<th>p</th>
<th>.1</th>
<th>.2</th>
<th>.3</th>
<th>.4</th>
<th>.5</th>
<th>.6</th>
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<th>.8</th>
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<tbody>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<tr>
<td>.2</td>
<td></td>
<td>0</td>
<td>0</td>
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<td>0</td>
</tr>
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<td></td>
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<td>.355</td>
<td>.278</td>
<td>.185</td>
<td>.064</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.4. (previous chapter) Equilibrium values of \( a (= b) \), for different pairs \((g, p)\), considering \( S = 20 \) and \( \sigma = 0.9 \).

From the comparison of the values found with those in the tables above (both in the case \( \sigma = 0.5 \) and \( \sigma = 0.9 \)), we can observe that the introduction of a second role for the research design (spillover allocation) has changed the results. When the research design is also responsible for spillover allocation between product characteristics, the values obtained may be considered to be somewhat less extreme. By this it is meant that where before firms adopted a maximum product differentiation strategy, now in some cases the strategy is not so radical (examples of this are the pairs \( g, p = (0.1,0.1) \), \( (0.5,0.8) \) or \( (0.3,0.7) \)). So, it can be said that we do not need relatively high values of \( g \) any longer in order to trigger a decision of differentiating the products less. Moreover, on the other extreme, we can observe that the equilibrium values of \( a (= b) \) do not go beyond 0.352 whereas, with the first assumption on spillover allocation they might reach values of 0.636.

From this we may conclude that the amplitude of values assumed by the decision variables is more condensed, showing some moderation in the strategies chosen. Recalling the options and motivations available to the firms when this new type of spillover allocation was introduced into the model, it is clear that the incentives against the adoption of radical strategies have dominated all others.
Thus, given the mechanism of allocating spillovers assumed, firms will try not "over-develop" the product in just one characteristic, as would be the case if they chose a maximum differentiation strategy. In such a case there would be an accumulation of potential improvement in just one characteristic originated by self-innovation and spillovers that would be totally directed to that characteristic. From the results it becomes clear that having a product with one super-characteristic and another that is much less developed is not as profitable as having what might be called a better-balanced product. So it pays to reduce R&D activities in just one direction and divert the resources freed to the development of the other product characteristic, which in turn will also benefit from spillovers. The profitability of such behaviour finds its sources in the fact that in case a firm becomes the sole innovator, it will not be so damaging for its advantage in the specialisation characteristic to share some knowledge. This happens as the majority of the transferred information will be used by the rival firm in its own specialisation characteristic, whereas with the previous assumption the amount of knowledge shared would be utilised, in the best of scenarios, equally in both characteristics.

Having studied the case of non-cooperative equilibrium it is now time to analyse what happens to the level of desired product differentiation when firms coordinate their research design.

5.3.2. The Cooperative Equilibrium

In such case, firms will try to maximise the expected joint profit by organising together the relevant aspects of the R&D activity. So, the expression both firms are interested in maximising by choosing the values of $a$ and $b$ is the following:

$$
\pi_L^e + \pi_R^e = p^2[\pi_L(\alpha_b, \beta_b) + \pi_R(\alpha_b, \beta_b)] + (1 - p)p[\pi_L(\alpha_L, \beta_L) + \pi_R(\alpha_L, \beta_L)] + p(1 - p)[\pi_L(\alpha_R, \beta_R) + \pi_R(\alpha_R, \beta_R)] + (1 - p)^2[\pi_L(\alpha_n, \beta_n) + \pi_R(\alpha_n, \beta_n)]
$$
where the quality gaps are the same as defined when the alterations to the model were introduced.

Following the same procedure utilised in previous chapters, the analysis will begin with the study of the decision of product differentiation in a context of certainty. This will correspond to a term-by-term analysis that will be helpful in the understanding of the more realistic case of R&D uncertainty.

5.3.2.1. Certainty Case

5.3.2.1.1. The "Both Firms Innovate" Term

The expression to be maximised by jointly choosing the values of \( a \) and \( b \) is given by: \( \pi_L(\alpha_b, \beta_b) + \pi_R(\alpha_b, \beta_b) \).

Computing the first derivative of the expression above with respect to \( a \) will give:

\[
\frac{\partial(\pi_L + \pi_R)}{\partial a} = \frac{\partial \pi_L}{\partial \alpha_b} \frac{\partial \alpha_b}{\partial a} + \frac{\partial \pi_L}{\partial \beta_b} \frac{\partial \beta_b}{\partial a} + \frac{\partial \pi_R}{\partial \alpha_b} \frac{\partial \alpha_b}{\partial a} + \frac{\partial \pi_R}{\partial \beta_b} \frac{\partial \beta_b}{\partial a}
\]

It is now useful to recall that \( \frac{\partial \alpha_b}{\partial a} = \frac{\partial \beta_b}{\partial a} = -Sg \left( \frac{1}{2} + a + b \right) < 0 \) in order to determine the sign of \( \frac{\partial(\pi_L + \pi_R)}{\partial a} \). We also know from the way the whole model was setup that, whatever the gap definitions considered, we will have: \( \frac{\partial \pi_L}{\partial \alpha_b} > 0; \)

\( \frac{\partial \pi_L}{\partial \beta_b} < 0; \frac{\partial \pi_R}{\partial \alpha_b} < 0; \frac{\partial \pi_R}{\partial \beta_b} > 0. \)

Keeping this in mind, the expression above can be re-written as:

\[
\frac{\partial(\pi_L + \pi_R)}{\partial a} = \left[ \left( \frac{\partial \pi_L}{\partial \alpha_b} + \frac{\partial \pi_L}{\partial \beta_b} \right) + \left( \frac{\partial \pi_R}{\partial \alpha_b} + \frac{\partial \pi_R}{\partial \beta_b} \right) \right] \frac{\partial \alpha_b}{\partial a}.
\]

\(^2\) In the case \( \sigma=0.9 \), the innovating firm would be loosing much more in its own specialisation characteristic.
As proven in chapter 3, the direct effect dominates the indirect effect, which makes, for $\alpha = \beta$, the two terms in round brackets to be positive. This has been called the dominance of the direct effect over the indirect effect. Given that $\frac{\partial \alpha}{\partial a}$ is negative and the resulting sum of what is in square brackets is undoubtedly positive, the derivative $\frac{\partial (\pi_L + \pi_R)}{\partial a}$ is always negative. This result implies that the value of $a$ that maximises the joint profit level in this situation is the lowest possible, i.e. zero. Given the firms' symmetry, this conclusion can be generalised to firm R, and its decision variable, $b$.

In economic terms, this means that when both firms know that they will succeed in the R&D activity, the best strategy for the research design coordination is to produce products as differentiated as possible – maximum product differentiation. This type of result is not new in a context where firms are certain that both will innovate, thus, the introduction of a second role for the choice of research design did not bring significant alterations to the way firms decide. The reasons behind such behaviour are mainly linked with market competition issues that have already been extensively explained in previous chapters.

5.3.2.1.2. The “Only Firm R Innovates” Term

The analysis of the case where firm R is the only firm to succeed in the R&D activity will give the optimal research design that will be chosen both for the innovating and the non-innovating firm. As the firms' identity is not important, the results obtained in this section can be more generally applied to the case where there is a sole innovator, thus permitting the case of firm L being the only firm to innovate not to be analysed. So, any conclusions reached with respect to firm R and firm L, decisions on product differentiation will be valid for any innovating and non-innovating firm, respectively, whatever their identity.

In order to proceed with the study of this case it should be noticed that in such context the mathematical analysis of the expression $\pi_L(\alpha, \beta) + \pi_R(\alpha, \beta)$ rapidly
becomes very complex, therefore not enabling conclusions to be drawn. Therefore, the alternative method of numerical simulation will be used. Given the similarities in the setup and the situation considered with section 4.B.2.1.2. of the previous chapter, the same type of programming for the simulation will be utilised. Mainly this aims at, for parameter values of $S=20$ and $g$ equal to 0.1, 0.3 and 0.5 (which allow comparison with chapter 4), to find for each value of variable $b$ (within the interval 0 to 1, with increases of 0.01) the value of $a$ that maximises the profit level. Then, it is only a question of finding the pair $(a, b)$ that results in a higher profit level – this will be the choice emerging from the research design coordination. As the presentation of the outputs from the numerical simulation are quite long, a short summary with the main results obtained will be displayed:

<table>
<thead>
<tr>
<th>$g=0.1$</th>
<th>Optimal $a$</th>
<th>Joint Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b=0$</td>
<td>0.3</td>
<td>0.93639306</td>
</tr>
<tr>
<td>$0.1$</td>
<td>0.2</td>
<td>0.93630101</td>
</tr>
<tr>
<td>$0.2$</td>
<td>0.2</td>
<td>0.93576244</td>
</tr>
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</tr>
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<td>0.93422144</td>
</tr>
<tr>
<td>$0.6$</td>
<td>0</td>
<td>0.93371977</td>
</tr>
<tr>
<td>$0.7$</td>
<td>0</td>
<td>0.93320797</td>
</tr>
<tr>
<td>$0.8$</td>
<td>0</td>
<td>0.93268857</td>
</tr>
<tr>
<td>$0.9$</td>
<td>0</td>
<td>0.93216139</td>
</tr>
<tr>
<td>$1$</td>
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<td>0.93162627</td>
</tr>
</tbody>
</table>

Table 5.2. Optimal choice of $a$ and $b$ ($g=0.1$)

<table>
<thead>
<tr>
<th>$g=0.3$</th>
<th>Optimal $a$</th>
<th>Joint Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b=0$</td>
<td>0.3</td>
<td>0.94272540</td>
</tr>
<tr>
<td>$0.1$</td>
<td>0.2</td>
<td>0.94137711</td>
</tr>
<tr>
<td>$0.2$</td>
<td>0.2</td>
<td>0.93988249</td>
</tr>
<tr>
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<td>0.93859721</td>
</tr>
<tr>
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</tr>
<tr>
<td>$0.6$</td>
<td>0</td>
<td>0.93429186</td>
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<tr>
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<td>0</td>
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</tr>
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</tr>
<tr>
<td>$1$</td>
<td>0</td>
<td>0.92776681</td>
</tr>
</tbody>
</table>

Table 5.3. Optimal choice of $a$ and $b$ ($g=0.3$)

<table>
<thead>
<tr>
<th>$g=0.5$</th>
<th>Optimal $a$</th>
<th>Joint Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b=0$</td>
<td>0.3</td>
<td>0.94759523</td>
</tr>
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<td>0.93933548</td>
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</tr>
<tr>
<td>$1$</td>
<td>0</td>
<td>0.92338616</td>
</tr>
</tbody>
</table>

Table 5.4. Optimal choice of $a$ and $b$ ($g=0.5$)
For the sake of exactness it should be mentioned that the actual pair of values found in the numerical simulation that maximises the joint profit is \( b = 0 \) and \( a = 0.26 \), however for the purpose of simplifying the displaying of results, an approximation to the first decimal case was utilised.

An alternative, and perhaps more appealing, way of finding and confirming the results obtained through numerical simulation is to use graphical analysis. The plots of the joint profit expression will be shown for the same parameter values utilised in the simulation:

Figure 5.5. Joint profit level when there is a sole innovator and \( g = .1 \)

Figure 5.6. Joint profit level when there is a sole innovator and \( g = .3 \)
As is evidenced by the shadowed cells in the tables above and from the plots presented, the optimal research design is one in which the innovating firm chooses to maximise product differentiation, while the non-innovating firm opts to devote around 26% of its R&D resources to the development of the characteristic in which it is not specialised. Similarly to what was observed in the previous chapter for the same circumstances, the magnitude of parameter $g$ does not seem to play a determinant role in the decision of product differentiation. Notice however that this new way of spillover allocation brought some changes to that decision. In chapter 4 we concluded that when firms cooperate (and there is certainty about the R&D outcome so that it is known in advance that only one firm will innovate), if spillovers were exogenously and equally allocated between characteristics, then the innovating firm would adopt a maximum differentiation strategy while the non-innovating firm would choose to minimise product differentiation. In case the amount of spillovers transferred were largely allocated to the characteristic the innovating firm is specialised in, then both firms would prefer to maximise product differentiation. In the present situation we can see that the strategy of the innovating firm has not changed: it will always choose to maximise product differentiation. The best strategy that the non-innovating firm should follow, however, is different: it is best not to go to extremes or being radical. So, this firm will adopt a moderate strategy and choose $a = 0.26$. Such a value is understandable insofar as it allows the non-innovating firm to relatively develop both product characteristics\(^3\), while not creating a menace to its rival’s profits, as the

\(^3\) As spillovers will not be zero and these will be allocated to both characteristics.
amount this firm will give away in its specialisation characteristic may be considered to be not very large. This explains why in the present situation, a non-innovating firm would choose a strictly positive value for its decision variable whereas in the case where spillovers were exogenously and largely allocated to the innovating firm specialisation characteristic, the unsuccessful firm would choose to maximise product differentiation.

It remains to be explained why, in the case spillovers were equally allocated between characteristics, the non-innovating firm would adopt a minimum differentiation strategy and with the new mechanism is far from doing that. The reason behind it is that in the previous chapter the only concern the firms had was with spillover magnitude, thus it could be profitable to minimise product differentiation so that spillovers were increased, as long as the advantage of the innovating firm was not threatened. In the situation now under analysis, firms' decisions on research design will not only determine spillover magnitude but also its allocation. With respect to the latter, where before this was exogenously set, the firms are now free to choose it and weigh the relative importance of spillover allocation with its total magnitude. From the results obtained now, we can perceive that in such case spillover allocation is perhaps more important than its total size. So, firms prefer to have less spillovers and have them more efficiently allocated than to have large spillovers but be bound by exogenous rules of allocating spillovers. This can be seen by comparing the profit level values obtained for the different mechanisms of spillover allocation studied. Higher profit levels are obtained for the case where spillover allocation is endogenously determined.

5.3.2.1.3. The “Only Firm L Innovates” Term

As explained when presenting the case of firm R being the only firm to innovate, given firm symmetry and the unimportance of their identity, the conclusions obtained then apply entirely to this sub-section. Thus we can restate that the innovating firm will adopt a maximum differentiation strategy whereas the non-innovating firm

---

"It’s not only the size of the cake that matters but also how it is divided"
will invest 26% of its R&D resources to the development of the characteristic in which it is not specialised.

5.3.2.2. Uncertainty Case

The object of study now is how firms will jointly decide to differentiate products when there is uncertainty about who will innovate, if anyone. Firms will have to choose the values of $a$ and $b$ that maximise the following expected joint profit expression:

$$\pi^*_L + \pi^*_R = p^2[\pi_L(\alpha, \beta^L) + \pi_R(\alpha, \beta^R)] + (1-p)^2[\pi_L(\alpha^L, \beta_L) + \pi_R(\alpha^L, \beta_L)] + \frac{p(1-p)}{2}[\pi_L(\alpha, \beta^L) + \pi_R(\alpha, \beta^R)] + \frac{(1-p)^2}{2}[\pi_L(\alpha^L, \beta_L) + \pi_R(\alpha^L, \beta_L)]$$

It constitutes no surprise that the analysis of such expression will not be possible to undertake by using an analytical methodology, given the complexity it involves. Moreover, in spite of the fact that numerical simulations are a useful tool in obtaining results, in the present case and taking into account the nature of the results involved, it would be tiresome and monotonous to present the results of such methodology. Instead, graphical analysis will be utilised, not with the intention of constituting proof but only to illustrate the typical behaviour of the expected joint profit expression. Similarly to what was done in the fourth chapter, the value of parameter $S$ will be 20, and the values of the remaining parameters ($g$ and $p$) will be picked to confer them some representativity.

![Figure 5.8. Expected joint profit for S=20, g=0.2 and p=0.3](image)
From the plots it is visually clear that if firms coordinate their research design, the decision will not differ from the conclusions achieved for the same context of
uncertainty and cooperation but with other assumptions about spillovers. Once again the reasons that explain such behaviour can be found in the fact that by cooperating, firms not only care about their own profit but also about their rival's. So, if in a non-cooperative situation firms may consider to produce less differentiated products having in perspective a potential disadvantageous asymmetry and try to gain from spillovers\(^5\), when firms cooperate these gains might not be enough to compensate the potential losses inflicted in the rival. Thus, in benefit of the aggregate expected profit, individual profit opportunities will be sacrificed.

5.4. Conclusion

Chapter 5 analyses the decision of product differentiation taking into consideration that the choice of the research design might not only determine the spillover magnitude but also the spillover allocation between product characteristics. The basic idea that the new spillover definition tries to convey is that, even if a firm does not succeed in the R&D activity, its efforts in R&D may be determinant for the amount of knowledge absorbed incoming from innovating firms and also for the development of a particular product characteristic using the same source of information. Moreover, by devoting a greater share of the R&D resources to the improvement of a particular characteristic, this makes that characteristic the principal beneficiary of any spillovers. This translates into what might be considered almost common knowledge in some industries: if a firm researches intensively on one product feature it will be more capable of adapting any innovation made by a rival to that particular characteristic. Obviously, this assumption (as the one utilised in chapter 4) implies that spillovers are not characteristic specific, which is a fact that can be easily observed in reality\(^6\). Examples of this were already presented previously and cover areas as different as the pharmaceutical industry or academic research.

\(^5\) By developing more their product and truncating the innovator's advantage.
\(^6\) Evidently alternative assumptions that imply the existence of characteristic specific spillovers could be made, and examples of this also abound in reality. This might constitute a direction for further research.
The consideration of this second role for the research design choice if innocuous for the results of the case where both firms innovate (both cooperatively and non-cooperatively) or for a situation of R&D uncertainty where firms act cooperatively, did in fact alter the conclusions obtained for the remaining cases. The general conclusion that might be applied to these is that more moderation and less radicalism is implicitly or explicitly present in the decision of product differentiation. This typically means that where before firms were willing to adopt solutions of great product differentiation or produce products relatively similar to each other, now firms prefer, respectively, to produce less differentiated products or to increase the degree of differentiation.

This type of result in the non-cooperative setup stems from the fact that non-innovating firms will try to develop both product characteristics, which is only possible in such context by choosing a research design that aims at improving both characteristics. Notice however that in spite of this not being clearly visible in a certainty context, it is implicitly there, as was shown by the type of analysis required. When firms cooperate in a context of certainty, the present chapter has proven that the mentioned wish of moderation is also present: the choice of the non-innovating firm will not be 0 or 1, as was when spillovers were exogenously allocated but instead an intermediate value. If uncertainty is introduced in a situation where there is coordination of research design, then, even if firms have incentives to differentiate their products less expecting a future disadvantageous asymmetry, these will be usually dominated by other considerations. These include the reduction of the potential competition that firms might face in the market should a symmetric outcome emerge or the willingness to minimise spillovers and subsequent advantage truncation that can cause a relatively large loss of profits for the innovating firm.
CHAPTER 6 - PRODUCT DIFFERENTIATION IN PRESENCE OF TWO-STAGE SPILLOVERS
6.1. Introduction

As from the third chapter that the basic model presented in chapter 2 has been gradually developed with the purpose of, by acknowledging the inter-relations between the Theory of Innovation and the Theory of Product Differentiation, making it more realistic. Obviously, the space for improvement is far from exhausted, and the assumptions that could be made to increase the adherence of the model to reality are uncountable. In particular, and as has already been admitted in previous chapters, the hypotheses concerning spillover existence and its concrete characterisation are not unique and often vary according to the specific R&D process under analysis.

With this in mind, the study of the product differentiation decision may benefit from a broadening of the horizons with respect, once again, to the Theory of Innovation and what it has to say about spillovers. Following this line of thought, it is worthwhile to mention the clarification made by Beath, Poyago-Theotoky and Ulph (97) or Katsoulacos and Ulph (97) on the source and nature of spillovers. These authors state that there are two factors that may affect the amount of spillovers occurring between two firms. The first is connected to the idea of the "adaptability" of the R&D activity outcome, that is, the "capacity" of a firm to benefit from an innovation made by its rival. The significance of this factor to the decision of product differentiation has been studied in chapters 4 and 5, and relates to the choice that has to be made before the R&D activity is undertaken: "research design stage". The second factor concerns the fact that, after any innovation is achieved, the innovating firm can control to some extent what information will be passed (spillovered) to rival firms; this has been called the decision about "information sharing". This latter source of spillovers is the object of study of this sixth chapter, cumulatively with the adaptability question already analysed.

This constitutes another step towards a more realistic approach to spillover existence, as the knowledge transferred from one firm to the other often depends not only on the "adaptability degree" but also of the willingness of firms to share information.
It could now be asked how this second instrument to control the spillover level\(^2\) is going to be utilised and how it might affect the decision of product differentiation. The answer is somewhat intuitive. In previous chapters it was observed that firms might choose to maximise product differentiation so that the spillover level was set to the minimum possible, not allowing its rival to fully benefit from it and avoiding the truncation of the advantage conferred by the discovery. This was, for instance, the case of an innovating firm acting non-cooperatively in case it was the only one to succeed in the R&D activity. In the presence of a second instrument to control for the spillover level, the innovating firm will not be forced to use the research design with that purpose, and can devote it exclusively to choose the desired level of product differentiation. On the other hand, a non-innovating firm will incur in the possibility of not being capable of absorbing any spillovers, in spite of the fact that the innovating firm might have chosen to minimise product differentiation (whilst deciding not to share any information). So, a decision of less differentiated products by a non-innovating firm may bring no benefits in terms of spillovers (as it did in some situations analysed in chapter 4), but it will still increase market competition, which might be hazardous to its profit level. Therefore, the motivation for a non-innovating firm to minimise product differentiation seems to have vanished.

Making use of what was concluded in the preceding chapters, one is in a position of making an educated guess about the firms’ decision concerning the use of this new instrument of spillover control. It is probably not unexpected (and it will be proved) that, in the case that there is a sole innovator, the non-innovating firm would like its rival to share all the information, whereas the innovator will be of the opposite opinion. With such a setup it will be interesting to study how these firms will behave in a cooperative situation, where diametrically different interests will have to be weighted in order to maximise joint profit.

From what was mentioned in the previous paragraphs, we can see that the introduction of this second nature or source of spillovers is a topic that is worthwhile

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1 This control might, for instance, take the form of patents and its various possible degrees of protection, or measures of industrial anti-espionage.
2 The first instrument already analysed is the choice of the research design.
to be analysed, insofar as the firms’ incentives and motivations concerning the product
differentiation decision might have changed. So, an explanation follows in the next
section of the modifications that have to be made to the model so that it
accommodates this new instrument of spillover control. A third section, which follows
a similar structure of preceding chapters, will present the results for the decision of
product differentiation (and the level of information sharing). A final section
concludes.

6.2. The Model

The introduction of the new instrument that controls the level of information
firms are willing to share will not imply major changes to the model utilised. The basic
framework that has been utilised throughout this Thesis will remain unchanged (as
presented in chapter 2), being only necessary to alter the gap definitions.

Before presenting the new gap expressions, it is worthwhile to explain the
rationale and assumptions that will be involved in their construction.

First of all, it should be stressed that this chapter not only uses the model
information provided in the second chapter, but also maintains several assumptions of
anterior chapters.

Directly emanating from chapter 4 and in what concerns the amount of
spillovers and its relation with the research design firms choose, the total magnitude of
spillovers will still be expressed by $Sg \frac{a + b}{2}$.

Given that now it is assumed that firms can control the amount of information
shared, it is over the expression above that their decision will be reflected. Therefore, it
is necessary to define a variable $\theta_i$ ($i = L, R$) which represents the percentage of total
potential\(^3\) spillovers that innovating firm $i$ allows to be passed to its rival. It will be
assumed, with the intention of simplifying the analysis, that $0 \leq \theta_i \leq 1$. Notice that this

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\(^3\) Notice that now the magnitude of total spillovers may not be effectively transferred, as firms can
control the amount that is passed to the rival. So we have a total potential spillover that will only be
equal to the total effective spillover if firms choose $\theta=1$. 

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is not necessarily true in reality, as firms may not have total control over the information or knowledge they produce and that is passed to rival firms. In practice this means that there may exist a lower limit to the value of $\theta$ which represents the unavoidable involuntary spillovers, as acknowledged by Arrow (62). For the purposes of this Thesis however, the above assumption will still be made as, given the nature of the results obtained and the set of conclusions reached in previous chapters relating spillover dimension with product differentiation choice, either it is innocuous or it will be relatively easy to reach a conclusion about the consequences of imposing a lower bound to the value of $\theta$.

So, in spite of the fact that spillovers may potentially amount to $Sg\frac{a+b}{2}$, in terms of effective information sharing their magnitude, it will be given by $Sg\frac{a+b}{2}\theta_i$.

To simplify the analysis it will be assumed that the mechanisms of spillover control do not involve significant costs. Evidently this is not the case in many situations, however the analysis of the implications of such costs to the decision of information sharing is beyond the scope of this Thesis. Regarding this, the existing economic literature on the value of such mechanisms (like patents and their protection degree) may complement this analysis, which can be viewed as a good direction for further research.

Similarly to what was done in chapter 5, the present chapter will only analyse the situation where spillovers occur not only between innovating firms but also from innovating to non-innovating firms, being the spillover magnitude independent of the R&D success of the firm absorbing it. The alternative assumptions on this subject made in chapter 4 will not be studied since the main consequences of their consideration have already been identified and explained in that chapter. With that information, it can be easily deduced how the results of the present chapter would change if an alternative assumption about whom is eligible to receive spillovers and its magnitude was considered.

In regards to the allocation of spillovers between product characteristics, it will be assumed, as in chapter 5, that the research design chosen by the firm absorbing the
spillovers plays an important role. This assumption will be maintained so that the analysis is gradually and cumulatively built, in an attempt to get a closer depiction of reality. Thus, $Sg \frac{a+b}{2} \theta_j$ is the amount of spillovers firm $j$ allows to be passed to its rival, of which the rival allocates $(1-i)\%$ to its specialisation characteristic increasing its quality gap, whilst $Sg \frac{a+b}{2} \theta_j i$ will serve to improve the other characteristic, reducing its disadvantage in that characteristic (where $i = a$, $b$, and $j = R$, $L$ whenever $i = a$, $b$, respectively).

Using the explanations above, it should be straightforward to write the quality gap expressions as follows:

- If firm both firms are successful:

  $\alpha_b = S + Sg(1-a) - Sgb - b \frac{a+b}{2} Sg\theta_L + (1-a) \frac{a+b}{2} Sg\theta_R$

  $\beta_b = S - Sga + Sg(1-b) + (1-b) \frac{a+b}{2} Sg\theta_L - a \frac{a+b}{2} Sg\theta_R$

- If neither firm is successful:

  $\alpha_n = S$

  $\beta_n = S$

- If only firm L is successful:

  $\alpha_L = S + Sg(1-a) - b \frac{a+b}{2} Sg\theta_L$

  $\beta_L = S - Sga + (1-b) \frac{a+b}{2} Sg\theta_L$

- If only firm R is successful:

  $\alpha_R = S - Sgb + (1-a) \frac{a+b}{2} Sg\theta_R$

  $\beta_R = S + Sg(1-b) - a \frac{a+b}{2} Sg\theta_R$

With the new gap expressions defined and explained so that they encapsulate the assumptions presented in the introduction to this chapter, we can now proceed with the analysis of the product differentiation decision under the novel circumstances.
As usual, the methodology and structure used follow closely what was done in other chapters. Once again, this aims at easing the understanding of the effects that the particular assumptions in this chapter will have, insofar as it will allow a direct comparison with the results obtained previously.

6.3. Results

6.3.1. The Non-Cooperative Equilibrium

As usual, the non-cooperative analysis will be undertaken considering the point of view of firm L, in the certainty that the results obtained also apply to firm R, whenever it finds itself in a similar situation.

The relevant expected profit expression that firm L will want to see maximised is already familiar:

\[ \pi^*_L = p^2\pi_L(\alpha, \beta) + (1 - p)p\pi_L(\alpha_L, \beta_L) + p(1 - p)\pi_L(\alpha_R, \beta_R) + (1 - p)^2\pi_L(\alpha', \beta') \]

With respect to the maximisation of such profit expression and given the introduction of the new spillover control instrument, it is worthwhile to remember the timing and sequence of the decisions a firm has to make.

Firms have to intervene in two distinct occasions: the first is before the R&D activity takes place – research design stage; the second if after the results of the R&D process are known – information sharing stage. In the research design stage firms will have to decide how they would like to differentiate their products by choosing the variables \( a \) and \( b \). Afterwards, in the information sharing stage, firms will be called to decide upon the value of \( \theta \) (\( i = L, R \)), which is the percentage of total potential spillovers firm \( i \) allows to effectively pass to its rival, conditional on the R&D outcome. Therefore this takes the form of a 2-stage game, which has to be solved in the reverse order stages occur so that the sub-game perfect equilibrium is obtained.
6.3.1.1. Certainty Case

6.3.1.1.1. The "Both Firms Innovate" Term

The term of interest from the point of view of firm L, depicting a situation where both firms succeed in innovating, is given by: \( \pi_L(\alpha_b, \beta_b) \), with \( \alpha_b \) and \( \beta_b \) defined above. As the last stage must be the first to be analysed, firm L will have to choose the value of \( \theta_L \) that maximises that expression. In order to do so, it is useful to compute the first derivative of \( \pi_L(\alpha_b, \beta_b) \) with respect to \( \theta_L \). This will give:

\[
\frac{d}{d\theta_L} \pi_L = \frac{\partial}{\partial \sigma_b} \pi_L \left( \frac{\partial \alpha_b}{\partial \theta_L} \right) + \frac{\partial}{\partial \theta_b} \pi_L \left( \frac{\partial \beta_b}{\partial \theta_L} \right)
\]

where: \( \frac{\partial \alpha_b}{\partial \theta_L} = -Sg \left( \frac{a + b}{2} \right) \leq 0 \) and \( \frac{\partial \beta_b}{\partial \theta_L} = Sg \left( \frac{a + b}{2} (1 - b) \right) \geq 0 \).

Given what we know about the sign of the profit derivative with respect to the quality gaps, it is straightforward to conclude that \( \frac{d\pi_L}{d\theta_L} \) is negative. This leads us to the conclusion that firm L will set \( \theta_L = 0 \), which means that, in case both firms innovate, no information about the discovery will be shared, independently of the research design that might have been chosen in the previous stage. As firms are symmetric in this situation, a similar strategy will be followed by firm R. Therefore, we may conclude that this mechanism will be devoted, in the present circumstances, to the elimination of spillovers. One might think that such strategy aims at giving firms the freedom to produce less differentiated products without the worries of simultaneously giving away information to the rival. However, this is not necessarily true. By solving this 2-stage game backwards, firms will realise that spillovers will not take place, and will choose their research design aware of that fact. This is clearly the situation analysed in the third chapter (where spillovers were considered not to exist), and thus the conclusions obtained then apply to the solution of the first stage of the game. In that chapter it was proven for similar circumstances that firms would adopt a maximum differentiation strategy. Therefore, this must also be the solution for the first stage of
this game. So, when choosing the research design and knowing that no information will be spillovered, firms will still prefer to not generate any knowledge that might be useful to its rival and truncate their gap advantage. Given this, we may conclude that the objective of not sharing any information is not to confer protection to a minimum (or lower) product differentiation strategy, as this will not be pursued.

Moreover, it is evident that, given the strategy chosen for the first stage of the game \((a=b=0)\), the choice of \(\theta_i\) in the second stage is irrelevant, as the “adaptability issue” pre-empted the question of “information sharing”.

The conclusion is then that the fact of firms having an additional mechanism to control for spillovers brings no new incentives to the analysis. In fact, such a result might be intuit by comparing the results of chapters 3 and 4. If it was true that in the fourth chapter, firms might be maximising product differentiation and that such strategy eliminated spillovers, it is not less true that the purpose of that choice was not intended (at least primarily) at avoiding spillovers, as a similar strategy was followed in absence of spillovers (chapter 3). The reasons that motivate the choice of maximising product differentiation are also similar to those that led firms to do it in the third chapter, and are linked with the wish of establishing some market power and avoid competition.

6.3.1.1.2. The “Only Firm L Innovates” Term

In case firm L is the only firm to innovate, and this information is available before the R&D process takes place, in what concerns the information sharing decision, it is only this firm that has to decide what it is willing to share with its rival. Notice that, the relevant term firm L would like to see maximised is

\[
\pi_L(\alpha_L, \beta_L) = \pi_L \left[ S + S(1-a) - Sg \frac{a+b}{2} b \theta_L, S - Sga + Sg \frac{a+b}{2} (1-b) \theta_L \right]
\]

It can be observed that \(\theta_R\) is not present in the profit expression. This stems from the fact that firm R did not generate new information and thus it has nothing to share with firm L.
As the second stage is the first to be solved, it is helpful to compute the first derivative of the expression above with respect to $\theta_L$:

$$
\frac{\partial \pi^L}{\partial \theta_L} = \frac{\partial \pi^L}{\partial \alpha_L} \frac{\partial \alpha_L}{\partial \theta_L} + \frac{\partial \pi^L}{\partial \beta_L} \frac{\partial \beta_L}{\partial \theta_L}
$$

It is straightforward to show that the derivatives of the quality gap expressions with respect to $\theta_L$ are the following:

$$
\frac{\partial \alpha_L}{\partial \theta_L} = -SGb \frac{a+b}{2} \leq 0 \quad \text{and} \quad \frac{\partial \beta_L}{\partial \theta_L} = SG(1-b) \frac{a+b}{2} \geq 0.
$$

Given our previous knowledge of the sign of the remaining terms involved in $\frac{\partial \pi^L}{\partial \theta_L}$, it is obvious that this derivative will certainly be non-positive. This means that the value of $\theta_L$ that maximises firm L's profit is zero. So, once again, we conclude that the innovating firm will not want to share any information with its rival after a discovery is made.

Taking into account the decision on the level of information sharing by the innovating firm, it is evident that we will once again fall in the same situation as the one depicted in the third chapter where spillovers were considered not to exist. Therefore, the solution for the first stage of the game (research design choice) will have to be the same as the one found in that chapter. At the time it was concluded, that if firms knew an asymmetric situation would arise from the different R&D outcomes, the innovating firm would try to produce less differentiated products, provided the asymmetry was sufficiently large. It is interesting to notice that the introduction of the mechanism that controls the level of information sharing is now useful to the innovating firm, insofar as we know that in presence of spillovers this firm would adopt a maximum differentiation strategy. So, by choosing to share nothing, the innovating firm will be protected against the negative effects identified in the fourth chapter (truncation of gap advantage) which constituted an incentive to maximise product differentiation in such circumstances. Having neutralised the spillovers, this firm is again willing to pursue a more aggressive strategy and produce less differentiated products. The reasons leading to this type of behaviour are the same as those explained in chapter 3.
6.3.1.1.3. The “Only Firm R Innovates” Term

In a situation where firm L is the only firm not to innovate, the second stage of
the game does not exist for this firm. Notice however that, given firms’ symmetry and
certainty about the R&D outcome, the non-innovating firm knows what strategy will
be chosen by its innovating rival. Taking this into consideration, firm L knows that its
decision about the research design is irrelevant, as spillovers will not exist. This can be
easily observed by looking at the gap expressions firm L is confronted with, where its
choice variables \(a\) and \(\theta_L\) will not be present as a consequence of firm R choosing
\(\theta_R = 0\): \(\alpha_R = S - Sgb\) and \(\beta_R = S + Sg(1 - b)\).

As a curiosity it might however be interesting to analyse what strategy the non-innovating firm would prefer its innovating rival to follow respecting to the level of
information sharing. This can be done by maximising the non-innovating firm’s profit
expression with respect to its rival information-sharing variable. In the present case this
means: Max \(\pi_L(\alpha_R, \beta_R)\) with respect to \(\theta_R\). By computing the first derivative of this
profit expression with respect to \(\theta_R\), we will obtain:

\[
\frac{\partial \pi_L}{\partial \theta_R} = \frac{\partial \pi_L}{\partial \alpha_R} \frac{\partial \alpha_R}{\partial \theta_R} + \frac{\partial \pi_L}{\partial \beta_R} \frac{\partial \beta_R}{\partial \theta_R}
\]

It is straightforward to show that:

\[
\frac{\partial \alpha_R}{\partial \theta_R} = Sg \frac{a + b}{2} (1 - a)
\]

\[
\frac{\partial \beta_R}{\partial \theta_R} = -Sga \frac{a + b}{2}.
\]

It is obvious that the former is positive and the latter negative, which, jointly with our knowledge of how profit responds to changes in the quality
gaps, makes \(\frac{\partial \pi_L}{\partial \theta_R}\) positive. This means that the non-innovating firm would like its
rival to share all the knowledge possible, which is what one would expect as it is the
only source of product improvement for the non-innovating firm and of not being left
so behind in quality terms. If the innovating firm chose to share information about its
discovery, it would only be truncating its own gap advantage whilst allowing its rival
to increase its lead in the other product characteristic. This would surely benefit the
non-innovating firm, but is certainly not a rational strategy for an innovating firm as it
has nothing to gain and only to lose with it: it gives away knowledge without receiving
anything back.

In spite of the fact that what the analysis made in the previous paragraphs
constitutes only a curiosity, it is already pointing to the fact that when firms cooperate,
there will be antagonistic incentives respecting the choice of the information sharing
level, which increases the interest of the analysis of the cooperative setup.

6.3.1.2. Uncertainty Case

When the outcome of the R&D activity is not known, all the terms previously
analysed come into play simultaneously. So, from the point of view of firm L, the
expected profit expression it wants to maximise by choosing $a$ and $\theta_L$ is the following:

$$\pi^*_L = p^2 \pi_L(\alpha, \beta) + (1 - p) p \pi_L(\alpha, \beta) + p(1 - p) \pi_L(\alpha, \beta) + (1 - p)^2 \pi_L(\alpha, \beta)$$

Once again the analysis of the two stages that compose this game has to be
done in the reverse order that they occur in reality. With respect to the solution of the
second stage of the game, it is helpful to recall the results of the term-by-term study
undertaken in the previous sections. All the terms involved in the profit expression
above pointed to the fact that either the firm in question would prefer not to share any
information with its rival or, in case the firm did not innovate, firm L would not have
to decide on the value of $\theta_L$ as it had no new information to share. From this
observation we may then conclude that, in a situation of uncertainty about the R&D
outcome, whenever firm L is called to decide upon the value of $\theta_L$, it will set it to zero
not sharing any information and thus avoiding any spillover possibility.

The reasoning above can easily be confirmed by computing the first derivative
of the expected profit expression with respect to $\theta_L$:
Given our previous knowledge of the partial derivatives involved, it becomes evident that the two terms on the right hand side of the expression above are negative, which makes the whole expression negative. So, firm L will choose not to share any information with its rival. Notice that, given firms' symmetry, the same conclusion is valid for firm R and the choice of $\theta_R$.

Having said this, the immediate consequence is that no spillovers will occur, even in case firms opted to minimise product differentiation in the first stage of the game. Being aware of this fact, the decision of the desired level of product differentiation (choice of research design) will follow closely the analysis undertaken in the third chapter, as the present situation collapsed in a setup of total spillover absence. For this reason, the motivations that led firms to choose the research design presented in chapter 3 (in the relevant section), apply entirely now. The only result that perhaps deserves a comment is the choice made on the second stage, but again, it should be obvious that if firms by sharing information only lose gap advantage without receiving anything in exchange, then the reasonable strategy to follow is to not allow spillover existence.

As mentioned before, the results obtained in a non-cooperative context, in spite of not providing unexpected conclusions (if we take into account the findings of the preceding chapters), increase the interest of analysing firms' behaviour in a situation where both the research design and the level of information sharing are decided so that joint profit is maximised. This stems from the fact that in the non-cooperative setup, we identified diametrically opposed incentives concerning the choice of the information sharing level.

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4 The derivative of the two last terms of the expected profit expression with respect to $\theta_L$ is zero as this variable is not present in any of them.
6.3.2. The Cooperative Equilibrium

In such case, firms will try to maximise the expected joint profit by organising together the relevant aspects of the R&D activity. This includes not only the choice of the research design, but also the decision of the level of information they will share in case of a discovery. At this point it should be re-stated that considerations about enforcement (and, in the present circumstances, monitoring) of the agreement reached by firms, is beyond the scope of this Thesis. In spite of that, these considerations might constitute a good direction for further research. Meanwhile it will be assumed that firms have a genuine interest in maximising joint profit and will not defect from the agreement.

In order to pursue this objective, firms will have to choose the values of $a$, $b$, $\theta_L$ and $\theta_R$ so that the following expression reaches its highest possible value:

$$\pi_L^* + \pi_R^* = p^2[\pi_L(\alpha_b, \beta_b) + \pi_R(\alpha_b, \beta_b)] + (1 - p) p[\pi_L(\alpha_L, \beta_L) + \pi_R(\alpha_L, \beta_L)] + p(1 - p)[\pi_L(\alpha_R, \beta_R) + \pi_R(\alpha_R, \beta_R)] + (1 - p)^2[\pi_L(\alpha_n, \beta_n) + \pi_R(\alpha_n, \beta_n)]$$

As in previous chapters the decision involving the above variables will firstly be studied in a context of R&D certainty.

6.3.2.1. Certainty Case

6.3.2.1.1. The “Both Firms Innovate” Term

The relevant term to be maximised by jointly choosing the values of $a$, $b$, $\theta_L$, and $\theta_R$ is $\pi_L(\alpha_b, \beta_b) + \pi_R(\alpha_b, \beta_b)$.

Computing the first derivative of the expression above with respect to $\theta_L$ will give:

$$\frac{\partial(\pi_L + \pi_R)}{\partial \theta_L} = \frac{\partial \pi_L}{\partial \alpha_b} \frac{\partial \alpha_b}{\partial \theta_L} + \frac{\partial \pi_L}{\partial \beta_b} \frac{\partial \beta_b}{\partial \theta_L} + \frac{\partial \pi_R}{\partial \alpha_b} \frac{\partial \alpha_b}{\partial \theta_L} + \frac{\partial \pi_R}{\partial \beta_b} \frac{\partial \beta_b}{\partial \theta_L}.$$
A similar expression may be found by computing the first derivative of the same term with respect to \( \theta_R \). Recalling our previous knowledge of the partial derivatives involved in the expression above, we can observe that the first two terms on the right hand side are negative whereas the last two are positive. One might be tempted at this point to say that \( \alpha_s = \beta_s \) and then invoke the domination of the direct effect over the indirect effect to conclude that the whole derivative is negative and thus the optimal value of \( \theta_L \) (and similarly \( \theta_R \)) would be 0. Notice however that, by coordinating the R&D activity, firms may find profitable for just one firm to share the knowledge, and so we would have \( \theta_L \) different from \( \theta_R \). This is not an affirmation that firms have any type of incentives to follow such strategy, but is one possibility available to them. If this is the case, then it would no longer be true that \( \alpha_s = \beta_s \) which makes it impossible to say whether the direct effect dominates the indirect effect or not. Given this fact, the analytical study of this situation becomes more complex and we need to resort to alternative methods of obtaining the results.

As there are four endogenous variables involved, the use of graphical analysis is unrealisable. We are then left with numerical simulation methods. Noticing that firms have to jointly decide the values of \( a, b, \theta_L \) and \( \theta_R \), the solution for both stages of the game can be (and probably will be) found simultaneously. So, the program written to numerically simulate the results, tries to capture the combination of values of \( a, b, \theta_L \) and \( \theta_R \) that maximises the profit level. Obviously, it would be unfeasible to simulate all the possible values for each variable within the interval \([0,1]\), and for that reason the program analysed all the possible combinations of those variables considering they assume values to the second decimal case (i.e. increments of 0.01). The output of the numerical simulation may be summarised by the following table, which just displays the results of the possible combinations when the variables in question assume the extreme values:

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\(^5\) Several values of \( g \) were considered in the simulation, however this variable does not seem to be relevant for the choice of the decision variables, as it was not in similar circumstances with different assumptions about spillovers.
From the observation of table 6.1 it is straightforward to conclude that the variable values in the shadowed cells are those that maximise the joint profit level, whatever the value of $g$ considered. In spite of the fact that the value of this variable might be relevant for the profit level itself, it has no impact in the choice of the decision variables.

Not only by closely checking the complete output of the numerical simulation but also by looking at the values involved, it becomes obvious that the relevant decision in such a setup is the choice of the research design. With respect to this stage, the results show that the optimal value for $a$ and $b$ is zero, which creates from the beginning a situation of maximum product differentiation. As such, and given the hypothesis of the model, there will be no chance for the creation spillovers (adaptability is non-existent). Therefore the choice of the level of information sharing is irrelevant. This explains why in the table summarising the results we get the same

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<th>$\theta_R$</th>
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Table 6.1. Joint profit level for different values of $a$, $b$, $\theta_L$ and $\theta_R$.
profit level for of \( a = b = 0 \), irrespective of the values of \( \theta_L \) and \( \theta_R \). The decision at the research design stage pre-empts the information sharing stage.

This type of result cannot be considered a novelty if we take into account the conclusions and explanations of previous chapters when analysing a similar context. Notice however that, with the introduction of the new mechanism for spillover control, we can conclude that in spite of the benefits spillovers may cause to the firm that receives it (and that makes this firm willing for its to rival share all the information), these are not enough to compensate the loss in profit the firm generating those spillovers would have if information was shared\(^6\). So, in aggregate terms, it is not profitable to share information, despite the fact that individually each firm would profit if its rival allowed the spillover to occur.

6.3.2.1.2. The “Only Firm L Innovates” Term

In this case firms will be interested in maximising \( \pi_L(\alpha_L, \beta_L) + \pi_R(\alpha_L, \beta_L) \) and have to jointly decide the values of \( a, b \) and \( \theta_L \) in conformity with such an objective. As firm R will not innovate, firms do not need to decide on the optimal level of information this firm will share, as it will not generate any. From what was learned when the certainty case was analysed, it is not surprising that the achievement of conclusions using exclusively analytical methods will not be feasible. This can be easily seen in the expression of the first derivative of the joint profit expression with respect to \( \theta_L \):

\[
\frac{\partial(\pi_L + \pi_R)}{\partial \theta_L} = \frac{\partial \pi_L}{\partial \alpha_L} \frac{\partial \alpha_L}{\partial \theta_L} + \frac{\partial \pi_L}{\partial \beta_L} \frac{\partial \beta_L}{\partial \theta_L} + \frac{\partial \pi_R}{\partial \alpha_L} \frac{\partial \alpha_L}{\partial \theta_L} + \frac{\partial \pi_R}{\partial \beta_L} \frac{\partial \beta_L}{\partial \theta_L}
\]

The first two terms of the expression are negative and the last two are positive. This depicts the opposite incentives firms have concerning the level of information sharing: the innovating firm prefers not to share any information whereas the non-innovating firm would benefit from the spillovers. As we cannot clearly conclude which effect dominates from the profit expression and respective derivative, an

\(^6\) Losses would be caused by decrease in the specialisation characteristic quality gap and/or the increase of the rival’s advantage in its specialisation characteristic.
alternative method of analysis will be utilised. Numerical simulations were run and graphical analysis will also be used. As both methods provide the same results, the optimal values for the endogenous variables reached in the numerical simulation will be mentioned and re-enforced with some of the plots of the graphical analysis, which are perhaps more revealing and appealing.

The program used to run the numerical simulation was similar to the one utilised in the previous section: it finds the values for the combination of variables $a$, $b$ and $\theta_L$ that maximises the joint profit. This was done considering the usual value of $S = 20$, and $g = 0.1, 0.2, 0.3, 0.4, 0.5$. The output of the numerical simulation pointed to the fact that, whatever the value of $g$ considered, the optimal combination of variables was: $a = 0; b = 0.26; \theta_L = 1$.

Before analysing these results it is also useful to look at some plots of the joint profit level:

![Picture of joint profit graph]

From the plot above it can easily be observed that the optimal value of $a$ is zero, implying a profit level of 0.93664. Notice however that the choice of $b$ becomes irrelevant as $\theta_L = 0$ does not allow the sharing of information and thus, the choice of the research design by the non-innovating firm is not an issue. Obviously, if spillovers are allowed, the variable $b$ may become an important piece in the profit maximisation process. The next plot shows us exactly this, as a value of $\theta_L = 1$ is considered:
Again it is unequivocal that the optimal value of $a$ is zero, but now the profit level is sensitive to the choice of $b$. This becomes more visible as we set $a$ to zero and observe how the profit function reacts to changes in $b$:

Now it is obvious that the optimal value of $b$, given the parameter values considered, is around 0,26. This enables firms to reach a joint profit of 0,93684.

Another conclusion that might be drawn from the observation of the two 3D plots above and that is fully confirmed by the results of the numerical simulation, concerns the comparison of the profit levels achieved for different values of $\theta_L$. The higher value of $\theta_L$ provided a larger profit, indicating that it is profitable for both firms that the innovating firm shares information with its rival. In order to transmit the perception that this type of results is obtained whatever the value of $g$ considered, the plots above will be repeated considering a value of $g = 0,5$.  

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Thus we can see that in terms of qualitative results, the alteration of the value of $g$ did not imply any major changes.
Having presented the results, it is now the moment to find an explanation for them. With that aim it is worthwhile to recall the conclusions obtained in the previous chapter, section 5.3.2.1.2. In that occasion the results were the same as those displayed above with just one difference: in the previous chapter firms did not have the chance to control spillovers with a second instrument, and so they were forced to share all the information that firms had the capacity to adapt. It now seems that it is in firms' best interest to share information and thus, the imposition considered in the preceding chapter that all information had to be shared, is not harmful and it is even desirable in these particular circumstances. As in chapter 5, the fact that firms coordinate research design and that there is a commitment that the strategy previously chosen will be followed, causes the innovating firm to be willing to share all the information, in opposition to the non-cooperative solution. The basis for this option is that this firm trusts that its rival will not minimise product differentiation (thus not increasing the total amount of spillovers by much) and will choose to allocate around 26% of total spillovers to the innovating firm's specialisation characteristic. As such, the innovating firm will not lose a significant amount of its lead in that characteristic and allows the rival firm to make some progress in the other product characteristic. Therefore, the gains of the non-innovating firm, by being able to absorb some spillovers (and allocating them in a particular way), are more than enough to compensate any loss in the innovating firm's profit. This implicitly means that relatively small transfers of gap magnitude may be beneficial, but as the transference size grows it can be harmful in aggregate terms.

The explanation for this lies in the particular profit expression utilised throughout the whole Thesis, but whose properties can be more widely applied as most of them can be considered almost common sense. One of these regards the sensitiveness of the profit level to changes in gap magnitude. It is not unreasonable to admit that there are decreasing returns in the magnitude of the specialisation characteristic gap. As such, and keeping in mind that \( \alpha_L > \beta_L \), an increase in \( \beta_L \) may have a greater positive effect on firm R's profit than the negative effect, the decrease of \( \alpha_L \) will have on firm L's profit. Notice however that when the gap magnitudes are

\[ \]

\[ ^7 \] Obviously one also has to consider the consequences that the increase of \( \beta_L \) will have on firm L's profit (which are negative) and the effects the decrease of \( \alpha_L \) will have on the profit of firm R.
not very different, this feature may not take place, which makes such movement unprofitable.

6.3.2.1.3. The “Only Firm R Innovates” Term

The analysis undertook in the previous section was made considering the situation where firm L was the only firm to innovate, however, given the way the model was setup it is easily observable that firms’ identity are once again irrelevant and the conclusions obtained in the previous section can be generally applied to the case where there is only one innovator. Obviously, the results concerning firm L are applied to the innovating firm, and the choices of firm R are in respect to the decisions of the non-innovating firm, irrespective of their identity.

6.3.2.2. Uncertainty Case

As it is usual in this case, firms will have no information on the outcome of the R&D activity and as such will have to take into account simultaneously all the scenarios they may find themselves in after the R&D uncertainty is resolved. Contemplating this, they will have to choose the values of \( a, b, \theta_L \) and \( \theta_R \) that maximise the following expected joint profit expression:

\[
\pi^*_L + \pi^*_R = p^2[\pi_L(\alpha_b, \beta_b) + \pi_R(\alpha_b, \beta_b)] + (1 - p)p[\pi_L(\alpha_L, \beta_L) + \pi_R(\alpha_L, \beta_L)] + \\
p(1 - p)[\pi_L(\alpha_R, \beta_R) + \pi_R(\alpha_R, \beta_R)] + (1 - p)^2[\pi_L(\alpha_n, \beta_n) + \pi_R(\alpha_n, \beta_n)]
\]

As it is probably evident by now, a complete analytical analysis of the above expression will not be feasible, forcing the use of alternative methods in order to achieve results. Given the number of endogenous variables involved, the utilisation of

However, even if it is the case that the latter are dominated by the former, the results point to the fact that these are also dominated by the consequences of the alteration of gap magnitude in the profit of the firm that is specialised in that characteristic. This points to the idea that firms’ profits are more sensitive to the own specialisation gap than to changes in the rival’s specialisation gap.
graphical analysis does not constitute an alternative, as we are limited to three dimensions. Therefore numerical simulation will be used.

The aim is to find the combination of values of those four endogenous variables that achieve the highest profit level possible. As firms coordinate their choices in this respect, the result will certainly be the strategy chosen by firms.

Given that a complete presentation of the output would probably be tiresome and tedious the results that will be displayed concern all the possible combinations of variable values, only admitting that these assume extreme values. Notice however that the simulation was run considering that each variable could assume any value within the interval [0,1] up to two decimal cases, but no valuable insights were gained.

The usual value of $S=20$ was considered. The values of the remaining parameters were chosen by their representativity of other values they might assume.

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Table 6.2. Joint profit level for different values of $a$, $b$, $\theta_L$ and $\theta_R$, considering different pairs of $(g, p)$.
As was the case when the cooperative equilibrium was studied in a context where both firms were sure that they would innovate, now the conclusions that we obtain from the observation of the table above are similar. The basic result is that, when firms cooperate and there is uncertainty involved about the R&D outcome, the best strategy to follow is, at the research design stage, to choose to produce products as differentiated as possible. A natural consequence of such a strategy is, given the way the model was setup, that no spillovers will exist as firms capacity to adapt the knowledge flowing from the rival firm is null. Thus any decision concerning the level of information to be shared on the second stage of the game is totally irrelevant, since there is no useful information to be shared. This can be easily seen in the tables above as the highest profit level is reached whenever $a = b = 0$, whatever the values of $\theta_L$ and $\theta_R$. Once again, the choice made in the first stage of the game pre-empted the second stage of the game.

The conclusion above is not surprising if we recall the results obtained when firms cooperated but the second instrument for spillover control was not available – chapter 5. With regards to the cooperative equilibrium, if we compare the present with the preceding chapter we can conclude from the term-by-term analysis that either this second instrument is irrelevant or it serves the purpose of guaranteeing that all information is shared, as in the fifth chapter. This points to the fact that the coordination of research design is sufficient for firms to maximise the profit level, not being necessary the introduction of a new instrument that facilitates the reduction of information sharing.

Notice that the result obtained does not mean that firms will be automatically willing to share all the information just because they are coordinating research design. In fact, the decision of adaptability (research design choice) implicitly conveys the idea that firms want to avoid spillovers, however in order to do so they do not need to resort to a second instrument of spillover control. This result corroborates to some extent the results obtained by Katsoulacos and Ulph (97) which conclude, in a different framework, that a cooperative agreement in R&D does not imply full information sharing.
Before concluding this chapter it is perhaps worthwhile to make a brief comment on the assumption utilised throughout the present chapter that firms could totally control the level of spillovers they pass to rival firms. It was noted, when the changes to the model were introduced, that in spite of in practice there may exist a limit to the extend firms control the spillovers, this would not be taken into consideration in order to keep the analysis simple. One might ask now, in the presence of the results obtained and in light of the conclusions of previous chapters, what consequences could have the consideration that there is a minimum amount of involuntary spillovers, which firms cannot avoid.

The possible changes to the results reached in this chapter can be easily grasped and may be grouped in two types:

1. The cases where the research design choice implied maximum product differentiation and therefore those variables \((a\) and \(b\)) were set to zero, and the cases where, in spite of \(a\) or \(b\) being strictly positive, the level of information firms are willing to spillover was set to its maximum \((\theta = 1)\).

2. The cases where \(a\) or/and \(b\) were strictly positive (choice of lower product differentiation at the research design stage) and firms chose not to allow any spillovers by setting \(\theta = 0\).

When the results obtained in a particular situation fall in the first category, it is fairly easy to observe that the imposition of a lower limit to spillover existence is innocuous. This stems from the fact that either the choice at the research design stage was one of maximum product differentiation thus pre-eliminating any spillover possibility (the value of \(\theta\) becomes irrelevant as there is no information to be shared), or it is in the firms best interest for knowledge transference to be at its maximum level \((\theta = 1)\) and for that reason any lower bound for involuntary spillovers does not constitute a constraint.

If however the results obtained are of the second type, then a little more attention has to be put into the analysis. By setting \(\theta\) to zero and then choosing a research design that would generate spillovers, firms are making use of the power they
have to exogenously control spillovers in order to adopt a strategy of less product
differentiation while avoiding the impact this would have on knowledge transference.
This was the case of the non-cooperative situations analysed where firms were
(potentially) asymmetric. In these contexts the spillover control instrument plays an
important role as can be inferred by comparing the results obtained in the relevant
sections of this chapter with those of chapter 5, where firms were bound to transfer all
"adaptable" information/knowledge. Given this we should expect the conclusions to
change if a strictly positive lower limit to $\theta$ is to be considered. Notice however that, if
such assumption is made, the results obtained both in chapter 5 and in chapter 4 make
the alterations to the conclusions of the present chapter somewhat predictable.

The analysis of the changes in the results in a non-cooperative setup in case
only one firm innovates can be enlightening. When firms have no ex-post control over
the information that is passed to the rival (as in the previous chapter), the innovating
firm will choose to differentiate its product as much as possible and its rival will try to
minimise product differentiation in search for spillovers. However, if spillovers are
non-existent (because $\theta$ will be set to 0), the innovating firm might want to produce
less differentiated products, whereas the research design of the non-innovating firm
becomes irrelevant. Thus we may conclude that the consideration of spillovers
discourages a firm that was successful in the R&D activity to differentiate its product
less, as it would be giving away more spillovers. Such a conclusion is also supported
by the results obtained in the fourth chapter where different spillover magnitudes were
considered to exist, although in a different framework. Given this we can deduce that
as the level of involuntary spillovers increases (as it would if a lower bound to $\theta$ was
considered) the innovating firm will be more willing to differentiate products more in
order to increase the appropriability of its discovery, and the non-innovating firm will
try to capture the spillovers by differentiating less. So, there must exist a threshold
value for the lower limit of $\theta$ for which the decision of the innovating firm concerning
product differentiation is reversed.

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8 This would imply that the lower limit of $\theta$ is no longer 0, but a strictly positive value, possibly
exogonously given.
9 The cases where only one firm innovates for sure or the outcome of the R&D activity is uncertain
and thus might generate an asymmetric situation.
10 In the relevant sub-sections.
Keeping in mind that it is the situation of firm asymmetry that explains the strictly positive values obtained when uncertainty about the R&D activity is considered, the differences in the results that would be obtained for the uncertainty case if a lower bound to $\theta$ was assumed, follow a reasoning similar to the one just explained.

6.4. Conclusion

This chapter studies the effects on the decision of product differentiation of the introduction of a mechanism that allows firms to control the amount of information they are willing to share (in case they make a discovery). It also analyses how this new mechanism will be utilised by firms. Therefore there will be two distinct phases in which firms will be called to decide. The first is at the research design stage, which happens before the R&D activity takes place, and where firms have to decide how they would like to improve their products - this will determine their capacity of spillover absorption\textsuperscript{11}. The second stage occurs when R&D uncertainty is resolved and the innovating firms have to choose the amount of information to share with the rival. So, the novelty in this sixth chapter is the consideration of a new stage that also affects spillover magnitude but that is not directly linked with the choice of research design - this was inspired by Beath, Poyago-Theotoky and Ulph (97) and Katsoulacos and Ulph (97), when these authors describe the nature and source of spillovers.

In order to gradually increase the model’s reality adherence, this new feature is studied cumulatively with many others introduced and analysed in preceding chapters.

The results obtained can be grouped into two types, depending on whether we are considering a non-cooperative or a cooperative setup. With respect to the former, we can conclude that in case both firms are sure to innovate the introduction of a mechanism of spillover control becomes irrelevant as firms prefer to maintain their decision of maximising product differentiation at the research design stage. This option obviously pre-empt the second stage of the game as spillovers will not exist and thus

\textsuperscript{11} This stage has been the object of analysis of preceding chapters.
there is no "adaptable" information to be shared. So, the choice of the information sharing level is irrelevant. If this new instrument is useless in a situation where firms know beforehand that their symmetry will be kept, this is not the case when firms know that an asymmetric outcome will arise from the R&D activity. In such circumstances it is somewhat intuitive that the innovating firm (the only one that has the power to decide how much information to share) will neutralise spillovers by choosing not to share any information. The rationale behind this is that if the innovating firm chose to share some information it would only be truncating its own gap advantage whilst allowing its rival to increase its own gap. Thus, it would be giving away something without receiving anything in exchange. Notice however, as a curiosity, that the non-innovating firm has the opposite incentive, i.e., it would like the innovating firm to share all the information as this is the only way it has to improve its product and not be left so behind its rival.

Keeping in mind the results obtained in a certainty context it is evident that in an uncertainty context, firms' decisions will not change: firms will always choose not to share information as information sharing brings no advantages to the innovating firm and can in fact be harmful. Given this type of solution for the second stage of the game, firms are aware that spillovers are out of the picture. As such, their behaviour concerning research design choice will follow exactly the conclusions obtained in the third chapter where it was assumed total spillover absence.

When it comes to the study of the cooperative setup it can be considered that the second instrument of spillover control introduced in this chapter is useless. When we compare the results obtained with those of the previous chapter, where information sharing was exogenously set to its maximum, we observe that the same type of results is achieved. This is due basically to two facts: either firms choose a research design that automatically rules out spillover existence (certainty context/ both firms are sure to innovate; uncertainty context) which makes the second stage irrelevant, or the level of information sharing will be freely set by firms to its maximum, which mimics the circumstances of chapter 5, implying the same results for the research design stage. Therefore, we may conclude that in a cooperative context, the permission given to firms to control for the level of information sharing (as opposed to a situation of forced full information sharing) is redundant, as firms by coordinating the relevant aspects of
R&D activity will freely choose to share all the information and achieve the same profit levels. This occurs as cooperation implies the commitment the firms have in maximising joint profit and abiding by the strategy chosen to do it. For this reason, firms do not need to reduce the level of information sharing as what is spillovered can be perfectly controlled through the research design choice. In particular this means that, for instance, in spite of the fact of not being profitable enough to allow a great spillover magnitude, this can be set to its optimum size just by correctly deciding the research design that will cause such desirable spillovers to occur and simultaneously allowing them to be totally transferred.
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