PIONEERS IN MATHEMATICAL GEOLOGY

DANIEL F. MERRIAM[1] AND RICHARD J. HOWARTH[2]

[1]*Kansas Geological Survey, University of Kansas, Lawrence, Kansas 66047, USA*
dmerriam@kgs.ku.edu
[2]*Department of Earth Sciences, University College London, Gower Street, London WC1E 6BT, UK*
r.howarth@ucl.ac.uk

ABSTRACT

Statistical and mathematical techniques have been used in the earth sciences for about one-hundred years, but only after the introduction of the electronic computer in the mid-Twentieth Century did a revolution in the science take place. The story of the quantification of geology is best told through the works of those who fostered the dramatic change. Here is chronicled the contributions of six pioneers in numerical geology in short exposés by authors close to, and knowledgeable of, the people and their work. The pioneers include F. Chayes (American), J. C. Griffiths (American/Welsh), W. C. Krumbein (American/German), G. Matheron (French), R. A. Reyment (Australian/Swede), and A. B. Vistelius (Russian). These magnificent six also played a major role in forming the International Association for Mathematical Geology in 1968 at the International Geological Congress in Prague, Czechoslovakia.

INTRODUCTION

History is a relative concept including everything from yesterday back to the 'big bang.' It is a story, a chronological record, or a record of past events. Here, we highlight six scientists and their contributions that had major effects in the later half of the Twentieth Century on change of a discipline—geology.

These six scientists were quantitatively inclined and for a variety of reasons from early in their careers they used numbers. In the beginning they accomplished their objectives using a slide rule, then a calculator, and finally transferred their computations to the computer when it became available. These scientists, as many others with similar interests, generally have their work grouped loosely under the heading of geomathematics or mathematical geology.

As will be seen from the biographies of these pioneers of mathematical geology presented here, they contributed heavily to what is known as the quantitative revolution in geology.

These papers are by no means inclusive, but review the work of William Christian Krumbein (American of German descent), John Cedric Griffiths (American of Welsh descent), Andrei Borisovich Vistelius (Russian), Felix Chayes (American), Georges Matheron (French), and Richard Arthur Reyment (Australian of Swedish descent); only Reyment remains active today. Each provided his own expertise in effecting the change that accounted for the leap forward in the science, and each had a profound effect on his students and colleagues. In addition, they were instrumental in founding a scientific society that promoted quantitative aspects that formed the basis for this revolution.

As pointed out by Krumbein (Krumbein, 1960, p. 83), in an article aimed at interesting contemporary statisticians in the problems posed by analysis of geological data, although many applications of statistics in geology are straightforward, particular problems are posed by "severe sampling restrictions in some geological studies, the multiplicity of variables in even the seemingly simplest

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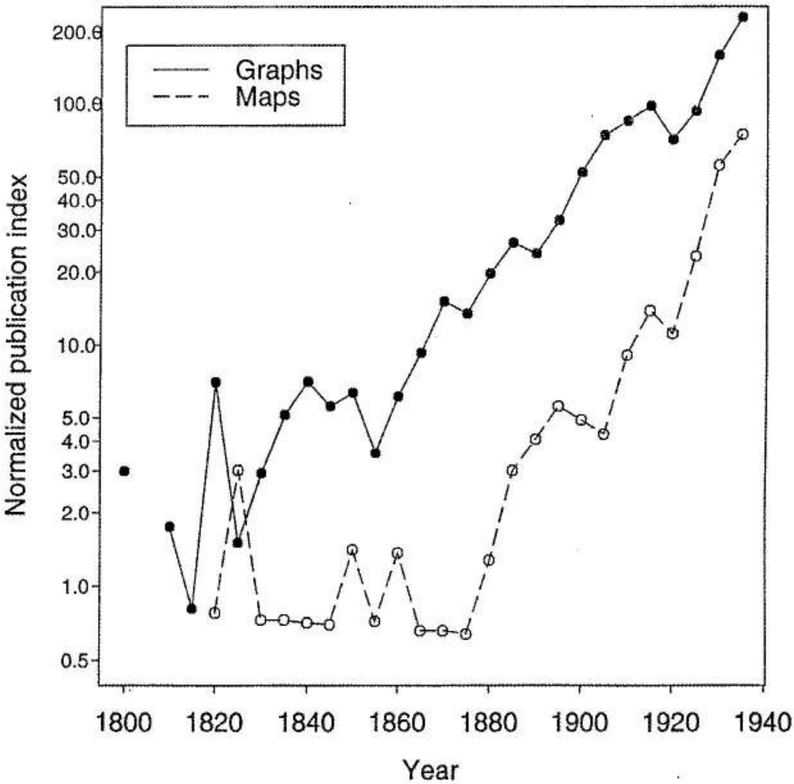


Figure 1. Relative numbers of graphs (n = 1492) and statistical maps (n = 236) showing either values at the data points (point-value); symbols at the data points (point-symbol); or isolines ('contours') which appeared in articles on geological subjects in a total of 103 serial publications (1800–1935). Frequency of occurrence is normalized relative to increasing numbers of serial publications with time.[3] Gap corresponds to interval in which no examples were found.

geological situations, and the high 'noise level' [variability] of some geological data."

We thought it appropriate not only to document the contributions to the discipline of these six, but to note their role in the founding of the International Association for Mathematical Geology (IAMG). These papers on the history of mathematical geology were first presented at the IAMG 2003 meeting in Portsmouth, England, in recognition of the thirty-fifth anniversary of the founding of the IAMG at the International Geological Congress in Prague in 1968.

SET THE STAGE

Prior to the Second World War, relatively little use was made of statistics, or for that matter mathematics, outside a few subdisciplines such as geophysics and geohydrology. Geologists, however, have used various forms of graphics from early in the Nineteenth Century (Figure 1; Howarth, 2002). Near the turn of the Twentieth Century they regularly employed statistical graphics (histograms, bar

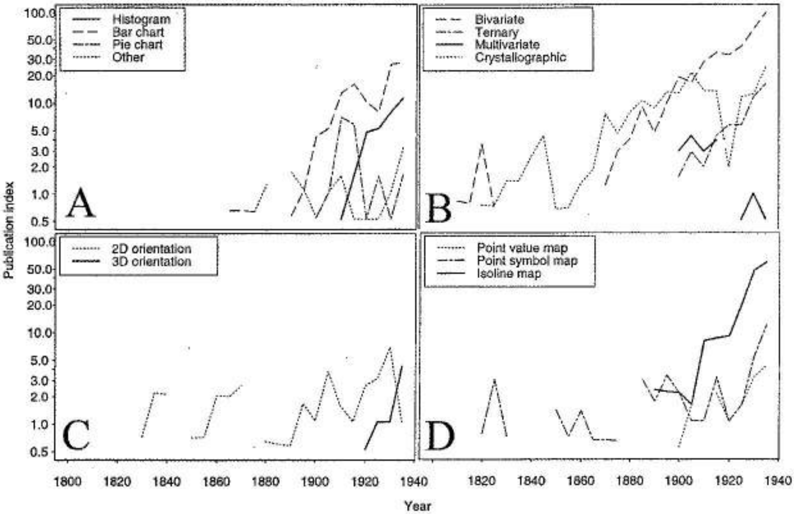


Figure 2. Early use of statistical graphics in the earth sciences: A, histograms, bar and pie charts; B, bivariate plots, ternary diagrams, multivariate symbols (e.g., star plots) and gnomonic or stereographic projections of crystallographic data); C, two-dimensional orientation data (e.g., rose diagrams, etc.) and three-dimensional orientation data (stereographic projections); and D, point-value, point-symbol, and isoline maps. Frequency of occurrence is normalized relative to increasing numbers of serial publications with time.[3] Gaps correspond to intervals in which no examples were found.

and pie charts, etc.) to summarize data and help present their story (Figure 2), regression analysis (i.e., fitting a functional equation to data) also began to play an increasingly important role (Howarth, 2001). All of this changed, of course, with introduction of the computer to the academic and commercial worlds (Merriam, 2004). The earliest geophysical publication to use results obtained with a digital computer is believed to be that of Simpson (1954) in the United States of America, closely followed by a geological application to facies analysis (Krumbein and Sloss, 1958); computer facilities did not become available to geologists in Europe or Russia (Vistelius, 1967) until the early 1960s, and in China in the 1970s (Liu and Li, 1983). Although geologists as a group were slow to take up quantitative methods, a few numerically oriented ones saw the potential and utilized this new time-saver and mind-extender (Figure 3). It is to these foresighted pioneers that we render tribute here.

In the earth sciences, statistical graphics, regression methods, and a variety of other quantitative techniques have been utilized on a limited scale, where the computations were possible either by hand or with a calculator, for many decades (Figure 4). With the introduction of electronic computers in the mid-1950s, these techniques became applied more widely and their use was accelerated with in-

[3] See Richard J. Howarth, From graphical display to dynamic model: Mathematical geology in the earth sciences in the nineteenth and twentieth centuries, in: *The Earth Inside and Out: Some Major Contributions to Geology in the Twentieth Century*, ed. David R. Oldroyd (London: Geological Society of London, Special Publication 192, 2002), 59–97, on 87 for derivation of the normalised publication index.

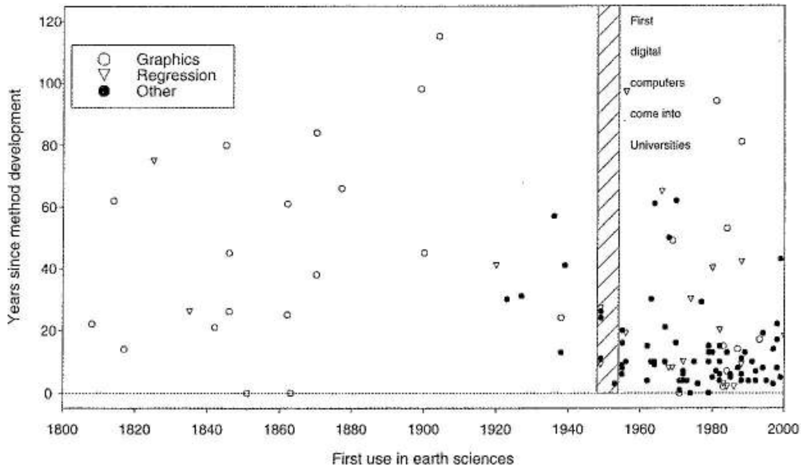
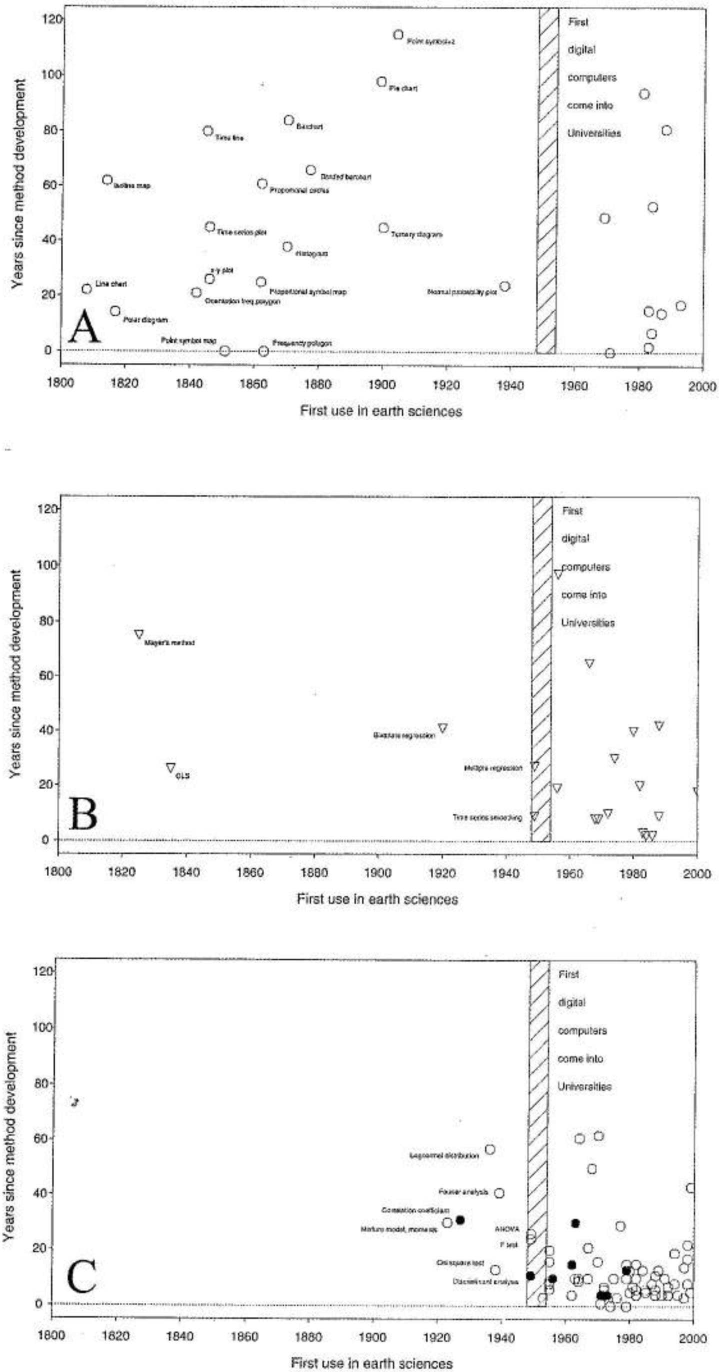


Figure 3. Synoptic view from 1800 to 2000 of the time following the original development of specific statistical techniques in the fields of graphic displays (open circle), regression (triangle) and other methods (solid circle) to their first published use in the earth sciences. Note the generally more prolific method development and generally shorter take-up times following the introduction of computers. For detail, see Figs. 4 and 5.

creased computing power of the hardware and availability of software (Figure 5) and continues today (Merriam 1981, 2004; Howarth, 2001, 2002).

In the pre-computer era, contributions by statisticians or mathematicians working with geologic data helped to encourage the application of quantitative methods more widely among sections of the geological community. Examples include the classic investigations by the English founder of the science of statistics, Karl Pearson (1857–1936), e.g., Pearson (1894, 1895, 1896, 1898), which encouraged the study of biometrics; the American statistician Churchill Eisenhart (1913–1994) showed how the chi-square test could be used to determine whether two samples could have come from the same parent material (Eisenhart, 1935); the Dutch mathematician and astronomer Jacobus Cornelius Kapteyn (1851–1922) discussed the application of the lognormal distribution in biology (Kapteyn, 1903); and the Russian mathematician and statistician Andrey Nikolaevich Kolmogorov (1903–1987) showed its application to the distribution of particle sizes during crushing (Kolmogorov, 1941).

Within the geological community, early paleobiometric studies were often directed at the evolution of invertebrate fossils as an aid to determining stratigraphic position. For example, echinoids in the English Chalk (Rowe, 1899) by the medical practitioner and amateur stratigrapher Arthur Walton Rowe (1858–1926); the Jurassic ammonite *Kosmoceras* (Brinkmann, 1929) by the German geologist Roland Brinkmann (1898–1995); bivalves (Trueman, 1922, 1930) by the English paleontologist and stratigrapher Sir Arthur Elijah Trueman (1894–1956); and microfossils (Schmid, 1934) by the Swiss stratigrapher and micropaleontologist Kurt Schmid (1906–1957). The rising interest in sedimentary rocks and their properties also provided opportunity. The American geologist Chester Keeler Wentworth (1891–1969) made many pioneering studies in the mechanical composition of sediments, establishing a commonly used size-grade scale and pebble shape (Wentworth, 1922a, 1922b, 1929); C. J. C. Ewing (1931) discussed operator variation in heavy-mineral separations; and a study of subsampling meth-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Years since method development
First use in earth sciences
First
digital
computers
come into
Universities
Point symbol+z
Pie chart
Barchart
Time line
Divided barchart
Proportional circles
Time series plot
Ternary diagram
Histogram
x-y plot
Line chart
Proportional symbol map
Orientation freq polygon
Normal probability plot
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Fourier analysis
ANOVA
F test
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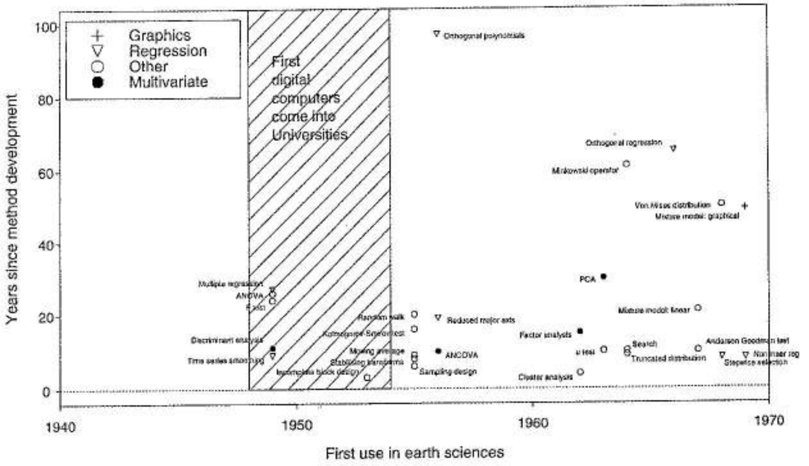

Figure 5. Time to first use in the earth sciences of specific statistcal techniques between 1940 and 1970. Major change came with introduction of the computer in the 1950s.

ods (Otto, 1933, 1937) by the American soil scientist George H. Otto (1909–2000) was probably the first to apply techniques developed by the American statistician Walter Andrew Shewhart (1891–1967) for statistical quality-control of industrial products (Shewhart 1931) to a geological problem. In geochemistry, the English civil engineer and geologist Wilfred Alfred Richardson (1887–1965) was the first to apply a method developed by Pearson to the decomposition of a bimodal normal distribution into its two subpopulations (Richardson, 1923). In Russia, the petrologist and geochemist Franz Youlievich Loewinson-Lessing (1861–1939) was an early user of statistical methods (Loewinson-Lessing, 1926) to analyse geochemical data. By 1939, the geologist and mineralogist Nikolai Konstantinovich Razumovsky (1893–1966) had recognized the utility of the log-normal distribution for description of metal grades in placer gold deposits; to metal concentrations in ore deposits; and its wide application to frequency distributions in petrology and geochemistry (Razumovsky, 1940, 1941).

Nevertheless, the few early English-language texts made only a slight impression on geology in general. Looking back, it is obvious that Bill Krumbein's early work was definitive and his textbook *Manual of Sedimentary Petrography* (Krumbein and Pettijohn, 1938), written in collaboration with the geologist and sedimentary petrologist Francis J. Pettijohn (1904–1999), was a landmark publication. In 1962, Robert L. Miller and James S. Kahn introduced to the geologic public their text on *Statistical Analysis in the Geological Sciences,* but this again had relatively little outward effect within a geological community rooted in traditions of qualitative description. With this background and the ever-increasing pervasion of computers in science, the stage was set for what might be termed a 'georevolution'.

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Figure 4. Time to first use in the earth sciences of specific statistical techniques: A, statistical graphics; B, regression methods; and C, other methods. Solid circles in C are multivariate techniques; other symbols as Fig. 3.

MAJOR PLAYERS

William Christian Krumbein (1902–1979) was the son of German immigrants who became a business major and later studied geology at the University of Chicago. His interest in sedimentary geology, piqued by Pettijohn, was a subject he recognized with a potential for the application of statistics. He also came under the influence of Marion King Hubbert (1903–1989), another stalwart in numerical geology, and with his background in numbers, Krumbein took to the analysis of sediments and sedimentary particles. His early work was all but ignored, except by a few numerically inclined sedimentologists. Later, he delved into a variety of quantitative methods that included descriptive statistics, Latin square experiments, regression analysis, Markov chains, and probabilistic modeling as applied to geological problems. His book, written in collaboration with the statistician Franklin A. Graybill in 1965, *An Introduction to Statistical Models,* put him far in the forefront of quantitative sedimentology and led to his designation as the 'Father of Computer Geology'.

John Cedric Griffiths (1912–1992), 'Dean of Geomathematicians', was born in Wales and immigrated to the U.S. just after World War II. He was educated at the University of Wales and Imperial College (London, England) before accepting a position as a sedimentary petrologist for an oil company in the British West Indies. Griff joined the faculty at Pennsylvania State University in 1947, where he began his extraordinarily productive career in characterizing properties of sedimentary rocks. He published his landmark book, *Scientific Method in Analysis of Sediments,* in 1967. He championed the development of a geologic metalanguage for which problems could be defined and an appropriate solution be achieved. His forte was in developing a philosophical method to integrate data-gathering techniques and statistical analysis, especially in sedimentary petrography. He adapted military search theory to the exploration of mineral deposits, which led him to his seminal study on the prediction of the 'unit regional value' of an area.

Andrei Borisovich Vistelius (1915–1995), a true pioneer in quantitative geology, was founder of the Russian school of mathematical geology. Although he used computers, he restricted the subject of mathematical geology, as he termed it, to the application of stochastic methods to solve geological problems; he considered computers only as a means to an end. Educated in Leningrad (St. Petersburg) before and during World War II, he created a center for mathematical geology in the U.S.S.R. and all of Eastern Europe. His ideas were summarized in his book on *Principles of Mathematical Geology* (1980), translated from the Russian and published in English in 1992. Unfortunately, little is known yet today of the work of Russian mathematical geologists outside of Eastern Europe except by a few specialists in the field. Vistelius essentially was a theoretician, but he was concerned in particular with mineral resources. His early work was in calculating mineral-deposit reserves, the theoretical development of mineral resources, and on risk prediction. Vistelius was one of the first to apply standard statistical techniques in geology, but his greatest contribution was being a proponent and using quantitative hypothesis testing in geology.

Felix Chayes (1916–1993), quantitative petrologist deluxe, was born in New York City and obtained his education at New York University and Columbia University. He came under the influence of the igneous petrologist Samuel James Shand (1882–1957) and this led to Chayes's career as a petrologist. Completing his studies at Columbia Chayes was employed in industry, then the U.S. Bureau of Mines, the Manhattan Project at the Massachusetts Institute of Technology, and in 1947 he joined the Geophysical Laboratory of the Carnegie Institution of Washington. His book on *Ratio Correlation,* published in 1971, challenged an important enigma in petrology and he continued with studies on constant-sum percentage data. His research was exploration into uncharted areas for knowledge's sake, and he frowned on programmed science directed for specific needs. Chayes's work has been described as being concerned with small-sample statistics and bases of petrographic inference.

Georges Matheron (1930–2000), 'Père de la Géostatistique', was a mining engineer who founded the French school of spatial statistics which became known as 'geostatistics', initially developed as an aid to improved valuation of ore deposits. Educated at the École Polytechnique and École Normale Supérieure des Mines de Paris, France, he began his illustrious career with the BRGM (French Geological Survey) in Algeria. He became interested in regionalized random variables through the work of statisticians Henri J. De Wijs (1951, 1953) in the Netherlands and Herbert Simon Sichel (1947, 1952) and the mining (financial) engineer Daniel ('Danie') G. Krige (b. 1919) in South Africa (Krige, 1951), and built on their foundation. Matheron, a modest and unassuming person who preferred to work in isolation, created and spent most of his career at the Centre de Morphologie Mathématique in Fontainebleau. His ideas on spatial analysis were presented in his *Traité de Géostatistique Appliquée,* published in two parts in 1962 and 1963. Most of his contributions were written in French and published internally through the Centre and were thus initially known only by a few specialized workers. However, through their contributions his students and colleagues disseminated knowledge of his work widely throughout the world. The first international meeting on the subject was a NATO Advanced Study Institute, held in 1975 in Rome, Italy, on the topic *Advanced Geostatistics in the Mining Industry* (Guarascio, David, and Huijbregts, 1976). Led by Matheron and many of his ex-students from Fontainebleau, it could be said to mark the emergence of geostatistics as a new field of statistical endeavor, whose applications have today spread far beyond geoscience applications.

Richard Arthur Reyment (b. 1926), quantitative paleobiologist extraordinaire, is the 'Father of the IAMG', founded at the ill-fated International Geological Congress in Prague in 1968. Reyment was born in Australia but eventually immigrated to Sweden after an extensive career in West Africa. He was educated in Australia and Sweden in his forte of paleontology. His early studies on random events, multivariate morphometrics, and statistical analysis in geology and biology naturally led him into the quantitative aspect of his chosen profession. He has made major contributions to several areas, including post-mortem distribution of cephalopod shells, morphometrics, and evolution of the South Atlantic Ocean. He has published numerous books, culminating in his collaboration with Enrico Savazzi on *Aspects of Multivariate Statistical Analysis in Geology,* published in 1999. He realized that an association of workers with a common interest would be desirable to share ideas and results. Thus, his concept of the IAMG, his effort to organize the Association, and seeing his ideas realized was a major step in his professional career.

THE TIE THAT BINDS

The history of the International Association for Mathematical Geology (IAMG) has been chronicled briefly for the tenth and twenty-fifth anniversaries (Merriam, 1978; Reyment, 1993). The thirty-fifth anniversary, however, presented a special occasion to honor some of the avant-garde founders of not only the IAMG, but the entire discipline of quantitative geology. The results of that symposium are presented here.

Krumbein, Griffiths, Vistelius, Chayes, Matheron, and Reyment were in the vanguard of the quantification of geology. Their students or coworkers benefitted immeasurably from their products and teaching. All but Felix Chayes were university professors who produced a bevy of disciples who furthered the cause and distributed the good word to the geologic masses. These apprentices, plus the acceptance of the computer, assured the quantitative revolution that changed geology forever. In years to come, the second generation who carried on the work and spread the word to assure the acceptance of the new approach will be honored just as the first ones are here.

All six of the group have been involved with the IAMG since its founding; all were charter members of the Association. Vistelius and Reyment were presidents in 1968–1972 and 1972–1976, respectively; Krumbein, Griffiths, Vistelius, and Chayes all have awards in their name; all were recipients of the Krumbein Medal (of course, with exception of its namesake, although Matheron respectively declined to accept his, as he did all honours); all were active in IAMG scientific activities.

Never in such a short time, have so few contributed so much toward change in a scientific discipline; they ushered in Information Technology (IT), the communication revolution, and the computer age in geology.

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