Default Cycles*

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Abstract

Recessions are often accompanied by spikes of corporate default and credit spreads. This paper develops a tractable macroeconomic model in which credit spreads reflect the fundamental corporate default risk as well as an excess premium which responds to variation in self-fulfilling beliefs about credit conditions. The model is calibrated to evaluate the macroeconomic impact of belief shocks in comparison to standard fundamental shocks. Changes in credit market expectations generate sizable countercyclical responses of default and spreads together with endogenously persistent credit cycles, accounting for most of the volatility of corporate default and close to 40% of output growth volatility.

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1. Introduction

Many recessions are accompanied by substantial increases of corporate default rates and credit spreads, together with declines of business credit. On the one hand, corporate defaults are clustered over prolonged episodes which can only partly be explained by observable firm-specific or macroeconomic variables, but are driven by unobserved factors that are correlated across firms and over time.[1] On the other hand, credit spreads tend to lead the cycle and are not fully accounted for by expected default from firm-level data. Indeed, less than half of the volatility of credit spreads can be explained by expected default losses; instead, the "excess premium" on corporate credit has the strongest impact on investment and output growth (cf. Gilchrist and Zakrajšek, 2012).

This paper uses a tractable macroeconomic model to examine the joint dynamics of firm default, credit spreads, and output. The distinguishing feature is that corporate default harms the access to future credit which punishes the defaulting borrower on top of any net worth losses resulting from liquidation or reorganization procedures. An important consequence of this feature is that default incentives are susceptible to variations in self-fulfilling beliefs over future credit conditions. We argue that the magnitude of such belief shocks can be inferred from the excess premium of the credit spread in the data. This permits us to evaluate the quantitative contribution of belief shocks to the business cycle, in comparison to standard (fundamental) financial and productivity shocks.

To illustrate the main idea, we present in Section 2 a simple partial-equilibrium model of default by firms with limited commitment. Credit contracts specify the amount of debt and the interest spread, both of which depend on the value that borrowing firms attach to future credit market access. This credit market value is a forward-looking variable which potentially responds to self-fulfilling expectations. A well-functioning credit market with a low spread and a low default rate is highly valuable for borrowing firms, which makes credit contracts with few defaults self-enforcing. Conversely, a weak credit market with a higher interest rate and more default is valued less by firms, and therefore cannot sustain credit contracts that prevent high default rates.

In Section 3, we extend the illustrative model to a tractable general-equilibrium model in order to

[1] See, e.g., Duffie et al. (2009) and Giesecke et al. (2011).

examine the respective roles of self-fulfilling beliefs (sunspots) and fundamental shocks for the dynamics of default, the spread, and their relationships with the aggregate economy. As in the simple model, corporate default rates depend on the value that borrowers attach to credit market access which responds to changes in self-fulfilling beliefs.

The credit spread, i.e., the difference between the borrowing rate and the safe interest rate, includes two terms. One is a predicted component reflecting the expected default losses explained by fundamentals. The other component is the excess interest premium which accounts for the so-called "credit spread puzzle" according to which actual credit spreads can be far from expected default losses.[2] As we show in this paper, the excess interest premium partly reflects variations of self-fulfilling beliefs about credit market conditions, but it may also depend on an exogenous component, which we refer to as "intermediation costs" (that may also capture unmodelled risk premia).

Firms in our macroeconomic model differ in productivity and in their access to the credit market. Therefore, aggregate productivity depends endogenously on the allocation of capital among firms which itself reacts to current credit market conditions and past default events. When credit is tightened or when more firms defaulted in the past, less capital is operated by the most productive firms so that aggregate productivity and output fall. If a firm opts for default, a fraction of the outstanding debt can be recovered by creditors. In line with the evidence, we allow for aggregate (procyclical) fluctuations of the recovery rate which have a direct impact on corporate default and on the predicted component of the credit spread.

Our closed-form default decisions lead to exact aggregation and allow for quantitative analyses via standard methods. Section 4 calibrates this model to the U.S. economy in order to examine the respective contributions of belief shocks, and fundamental shocks to aggregate productivity, recovery rates, and intermediation costs. A key feature is that aggregate risk matters for corporate leverage and the credit spread. For this reason, we log-linearize the model around the *risky steady state* (cf. Coeurdacier et al., 2011) which describes a stationary model solution that takes aggregate risk into account. Given a level of intermediation costs in steady state, the excess interest premium in the data can be used to pin down the variance of belief shocks. The remaining parameters of the (fundamental) shock processes in the model are estimated to reflect the joint dynamics of the spread, recovery, and output growth.

[2] See, e.g., Elton et al. (2001) and Huang and Huang (2012).

We find that shocks to self-fulfilling beliefs are crucial for the credit cycle, explaining 56% of the variation in default and 31% of credit spread fluctuations. Differently from (financial) shocks to the recovery rate or intermediation costs, belief shocks generate countercyclical spreads and default, as well as sizable and persistent drops of credit, output and TFP growth. The belief channel accounts for almost 40 percent of the variation in output growth.

Different experiments further demonstrate that endogenous default is crucial: If we ignore belief shocks and default in the estimation, the likelihood of the model drops substantially. Likewise, if we set default to zero and let exogenous collateral shocks generate the same debt dynamics as in the benchmark, the model fails to generate sizable and persistent credit and business cycles.

Related Literature. Our work relates to a number of recent contributions analyzing the macroeconomic role of credit spreads and firm default. Building on Bernanke et al. (1999), Christiano et al. (2014) show that risk shocks in a quantitative business-cycle model not only generate countercyclical spreads but also account for a large fraction of macroeconomic fluctuations. Miao and Wang (2010) include defaultable debt in a macroeconomic model with financial shocks to the recovery rate, finding that credit spreads are countercyclical and lead output and stock returns. Gourio (2013) argues that the time-varying risk of rare depressions (disaster risk) generates a plausible volatility of credit spreads, which amplifies macroeconomic fluctuations. Similarly to Hennessy and Whited (2007), Gomes and Schmid (2012), and Khan et al. (2016), aggregate factor productivity in our model is an endogenous variable that responds to financial conditions and default.

In previous papers, corporate default can also respond to expectations because firm value is a forward-looking variable. The difference is that they do not focus on reputation losses of default, which is the key feature for multiplicity in our model (see more below). Default in our model entails a loss of reputation, resulting in harmed access to credit in subsequent years. This feature makes credit contracts responsive to credit market expectations and thereby to self-fulfilling beliefs.[3] Such stochastic variation of beliefs may capture some aspects of systematic risks or disaster risks. In addition, we introduce a new approach to measure the belief risks inferred from the default and the spread data, and this approach still permits quantitative analyses using standard perturbation routines.

[3] Benhabib et al. (2018) introduce adverse selection in credit markets, featuring countercyclical credit spreads and pro-cyclical TFP with self-fulfilling expectations, but default disappears in equilibrium once reputation is introduced.

Our mechanism also introduces another propagation channel. A one-time negative belief shock induces a persistent tightening of credit, together with a rise of default on impact. In the following years, fewer firms have access to credit, and those that have access face tighter constraints. Both features give rise to persistent drops in aggregate productivity and output growth.

Self-fulfilling beliefs matter precisely because default is punished by the (temporary or permanent) exclusion from borrowing in future periods, which makes the value of credit market access a forward-looking variable. Early contributions on limited commitment, such as Eaton and Gersovitz (1981), do not consider the possibility of multiple equilibria by *assuming* that the borrowers' Bellman equation has a unique solution.[4] Our illustrative example of Section 2 shows that this assumption is not always valid. This finding is not entirely new (albeit often overlooked in the literature on limited commitment): Alvarez and Jermann (2000, 2001) show that the value function operator in a limited commitment economy is not a contraction so that multiple equilibria can arise. Building on Bulow and Rogoff (1989), Hellwig and Lorenzoni (2009) demonstrate that credit with limited commitment is equivalent to a bubble on an outside asset.[5]

Closely related to our paper is Azariadis et al. (2016) who consider the role of sunspot shocks in a model of unsecured firm credit with limited commitment. There are two major differences. First, Azariadis et al. (2016) have no default in equilibrium so that credit spreads are zero, while our credit contract allow lenders to adjust leverage and interest rate simultaneously.[6] We show that these features are quite important to account for the persistent dynamics of default, spread, leverage, productivity, and output. Second, in our quantitative analysis we utilize the property that the risky steady state depends on the variance of belief shocks. This permits a new empirical strategy which identifies the size of belief shocks from the credit spread data. Building on this feature, we estimate the model using perturbation methods (around the risky steady state) and the Kalman filter. Because of endogenous default, we find that recovery shocks are much less important compared to Azariadis et al. (2016).

[4] See the text after equation (5) in Eaton and Gersovitz (1981, p. 291).

[5] Further, Bethune et al. (2018) shows that multiple and periodic equilibria are possible in a matching model with credit subject to limited commitment constraints, and Krueger and Uhlig (2018) show that multiple equilibria can arise in a general-equilibrium model with one-sided commitment.

[6] This also applies to all papers cited in the previous paragraph. Likewise, the wider literature on self-fulfilling expectations in macroeconomics with financial frictions (e.g., Harrison and Weder, 2013; Benhabib and Wang, 2013; Liu and Wang, 2014; Gu et al., 2013) does not feature equilibrium default.

In this regard, our approach is complementary to the literature on sentiments with imperfect information and "correlated equilibria" (e.g., Benhabib et al., 2013, 2015; Angeletos and La'O, 2013), in which the variance of sentiments is uniquely determined. Our approach may be easier for a quantitative estimation exercise as (aggregate) belief shocks only have to satisfy a mild restriction.[7]

The co-existence of equilibria with high (low) interest rates and high (low) default rates relates to a literature on self-fulfilling sovereign debt crises. In an essentially static model, Calvo (1988) shows how multiple equilibria emerge from a positive feedback between interest rates and debt levels.[8] Besides the focus on corporate default, our mechanism for multiplicity is different from this literature by emphasizing the role of expected credit conditions.

2. An Illustrative Partial-Equilibrium Model

We present here a simple partial-equilibrium model in order to show analytically how self-fulfilling expectations can induce fluctuations of corporate default and credit spreads. In Section 3, this model is extended to a richer macroeconomic environment with aggregate shocks.

2.1. The Setup

Consider a continuum of firms with limited commitment living through infinitely many discrete periods $t \geq 0$. Each firm starts in $t = 0$ with given net worth ω_0 and has access to a safe linear technology which transforms k units of investment in t into Πk units of output in $t+1$. Firms may obtain one-period credit from perfectly competitive and risk-neutral investors who have a safe outside investment opportunity at gross return $\bar{R} < \Pi$. Both $\bar{R}$ and Π are exogenous in this section.

Firm owners are risk-averse and maximize the discounted expected utility

$$\mathbb{E}_0 \sum_{t \geq 0} \beta^t \left[(1-\beta) \log c_t - \mathbb{I}_{\{\text{Default in } t\}} \eta_t \right] , \tag{1}$$

where c_t is the dividend payout in period t, $\beta < 1$ is the discount factor, $\mathbb{I}$ is an indicator function, and η_t is a default cost that materializes *only when the firm defaults* in period t. The default cost is idiosyncratic

[7] Specifically, sunspot belief shocks are uncorrelated random variables with conditional mean zero.

[8] Further contributions on self-fulfilling debt crises in dynamic models of sovereign debt are Cole and Kehoe (2000), Bocola and Dovis (2016), and Lorenzoni and Werning (2019).

and stochastic: with probability p it is zero, otherwise it is $\Delta > 0$. After paying this cost, a defaulting firm continues to operate without access to credit in all future periods.

In every period, investors offer one-period credit contracts, specifying the gross interest rate R and the amount of credit b. Competition between investors implies that the offered contracts (R, b) maximize the borrower's utility subject to the investors' participation constraint. The latter requires that the expected return equals the outside return $\bar{R}$ per unit of debt. In recursive notation, a firm's value $V(\omega)$ depends on the firm's net worth ω and satisfies the Bellman equation

$$V(\omega) = \max_{c,s,(R,b)} \left\{ (1-\beta)\log(c) + \beta \mathbb{E} \max\left[V(\omega'), V^d(\omega'_d) - \eta' \right] \right\} \text{ s.t.} \tag{2}$$

$$c = \omega - s \ , \ \omega' = \Pi(s+b) - Rb \ , \ \omega'_d = \Pi(s+b) \ , \text{ and}$$

$$\bar{R}b = \mathbb{E}(Rb) = \begin{cases} Rb & \text{if } V(\omega') \geq V^d(\omega'_d) \ ; \\ (1-p)Rb & \text{if } V^d(\omega'_d) > V(\omega') \geq V^d(\omega'_d) - \Delta \ ; \\ 0 & \text{else.} \end{cases} \tag{3}$$

The firm chooses dividend payout c, savings s, and a credit contract (R, b), subject to the investors' participation constraint (3). Next period, she can choose to repay Rb with continuation net worth ω'; alternatively, she chooses to default with continuation net worth ω'_d. The second maximization in (2) expresses the optimal default choice at the beginning of the next period after realization of the default cost $\eta' \in \{0, \Delta\}$. The participation constraint (3) captures three possible outcomes. In the first case, the firm repays for any realization of the default loss and investors are fully repaid Rb. In the second case, the firm only repays when the default loss is positive, which is reflected in the expected payment $(1-p)Rb$. In the third case, the firm defaults with certainty.

After a default, the firm is punished by exclusion from future credit: $V^d(.)$ is the utility value of a self-financing firm with a default history. The firm saves s and earns Πs next period. We have

$$V^d(\omega) = \max_s \{ (1-\beta)\log(\omega - s) + \beta V^d(\Pi s) \} \ . \tag{4}$$

We briefly discuss how this setup differs from the modeling of corporate default in other macroeconomic models of the literature cited in the introduction. Most importantly, our credit contract captures the reputation loss of a default, which results in harmed access to credit in the aftermath of a default.[9]

[9]For instance, after a reorganization (such as Chapter 11 of the U.S. Bankruptcy Code) the bankrupt firm continues

It is precisely this feature that gives rise to a role for self-fulfilling expectations in corporate default, which is absent when default decisions are independent of the firm's subsequent borrowing conditions like in previous papers on firm default. We elaborate this idea in this section, while the extended general-equilibrium model in Section 3 further features productivity shocks, the partial recovery of loans, and temporary rather than permanent credit exclusion. Another, less important feature is that firms have concave utility which either represents a preference for dividend smoothing or adjustment costs of net equity payout; cf. Lintner (1956) and Jermann and Quadrini (2012). Lastly, the default utility cost with log utility ensures that there is a closed form solution for binary choice (cf. Cui, 2017) which can be shown to be equivalent to a proportional loss of net worth. Therefore, the default costs are meant to capture costs arising from liquidation or reorganization procedures.

2.2. Self-Fulfilling Expectations

Appendix A.2 shows that all firms save $s = \beta\omega$ and that value functions take the simple forms

$$V(\omega) = \log(\omega) + \bar{V} \qquad \text{and} \qquad V^d(\omega) = \log(\omega) + \bar{V}^d\,, \tag{5}$$

where $\bar{V}$ and $\bar{V}^d$ are independent of net worth. This result comes from the constant-returns-to-scale technology and log utility. The implication is that the size of the firm does not affect the firm's default decision, while its leverage does.

Write $v \equiv \bar{V} - \bar{V}^d$ to express the surplus value of access to credit. This is a forward-looking endogenous variable reflecting *expected credit conditions* which is a key determinant of the optimal credit contract (R, b) specified in Proposition 1 below. If v is larger than a threshold value $\bar{v}$, the optimal credit contract has no default: The threat of market exclusion is so strong that investors offer a large credit volume at a low interest rate, and even firms with zero default cost decide to stay solvent. Conversely, for values of v smaller than $\bar{v}$, the credit contract entails equilibrium default of a fraction p of firms with zero default costs, which is reflected in an interest premium.

Proposition 1. *Suppose that the parameter condition*

$$\frac{(e^{\Delta}-1)(1-p)}{e^{\Delta}-1+p} < \frac{\bar{R}}{\Pi} < \frac{(e^{(1-p)\Delta}-e^{-p\Delta})(1-p)}{e^{(1-p)\Delta}-1} \tag{6}$$

operation but may find it difficult to obtain loans. If a sole proprietor of a firm opts for liquidation (Chapter 7), the owner can start a new business but retains a negative mark on the personal credit report.

holds. There exists a threshold value $\bar{v} \in (0, v^{\max})$ *with* $v^{\max} \equiv \log(\Pi/(\Pi - \bar{R}))$*, such that*

(i) *If* $v \in [\bar{v}, v^{\max})$*,* $(R, b) = (\bar{R}, b(s))$ *with* $b(s) = s\frac{\Pi(1-e^{-v})}{\bar{R}-\Pi(1-e^{-v})}$ *and no default.*

(ii) *If* $v \in [0, \bar{v})$*,* $(R, b) = (\bar{R}/(1-p), b(s))$ *with* $b(s) = s\frac{\Pi(1-p)(1-e^{-v-\Delta})}{\bar{R}-\Pi(1-p)(1-e^{-v-\Delta})}$ *and default rate* $p > 0$*.*

The proofs of this and all following propositions are contained in Appendix A. The parameter condition (6) says that the ratio of Π (the firm investment return) to $\bar{R}$ (the risk-free return) must be in a certain range to obtain default and no-default outcomes for different values of expected credit conditions v. This range depends on the distribution of default costs characterized by Δ and p. If the firm's productivity is very high, the firm is desperate for credit so that the good outcome (i) arises regardless of the value of v. Conversely, when the investment return is very low, lenders anticipate that some default occurs, leading to outcome (ii) for all values of v. Furthermore, feasible solutions require that debt is finite which necessitates $v < v^{\max}$ (or $\bar{R} - \Pi(1 - e^{-v}) > 0$).

While v determines the *current* credit volume and interest rate via Proposition 1, v itself depends on *future* states of the credit market. We show that there is a role for self-fulfilling expectations, giving rise to multiple stationary equilibria and sunspot cycles with time-varying default rates and credit spreads. Consider first a *stationary equilibrium* in which v is constant over time. A stationary value v^* is the solution of a fixed-point equation that maps next period's expected credit conditions into today's credit conditions. To derive this equation, take the difference between Bellman equations (2) and (4). Utilizing the functional forms (5) for $V(\omega)$, $V^d(\omega)$, $s = \beta\omega$, as well as Proposition 1, we have:

Proposition 2. *The stationary equilibrium value* $v^* = \bar{V} - \bar{V}^d$ *solves the fixed-point equation*

$$v^* = f(v^*) \equiv \begin{cases} \beta \log\left[\frac{\bar{R}}{\bar{R}-\Pi(1-e^{-v^*})}\right] & \text{if } v^* \geq \bar{v}\ ; \\ \beta\Big\{\log\left[\frac{\bar{R}}{\bar{R}-\Pi(1-p)(1-e^{-v^*-\Delta})}\right] - (1-p)\Delta\Big\} & \text{if } v^* < \bar{v}\ . \end{cases}$$

Suppose that parameters satisfy condition (6) and

$$\Big(\frac{\bar{R}}{\bar{R} - \Pi(1 - e^{-\bar{v}})}\Big)^{\beta} < \frac{\Pi[1 - (1-p)e^{-p\Delta}]}{\Pi - \bar{R} + e^{(1-p)\Delta}(\bar{R} - \Pi(1-p))}\ . \tag{7}$$

Then, there are two stationary equilibria $v^D < v^N$ *such that default rates and interest spreads are positive at* v^D *and zero at* v^N*.*

Any solution of $v^* = f(v^*)$ constitutes a stationary equilibrium. Under the parameter conditions of Proposition 2, f is increasing and continuous and it satisfies $f(0) > 0$, $f(v) \to \infty$ for $v \to v^{\max}$, and

$f(\bar{v}) < \bar{v}$. Therefore, there is one equilibrium at $v^* = v^D < \bar{v}$ which involves default and a positive interest spread, and another equilibrium at $v^* = v^N > \bar{v}$ which has no default and a zero spread (see Figure 1).[10] This reflects the fact that the value function is not a contraction when the continuation value affects borrowing, i.e., with endogenous limited commitment constraints.

– Insert Figure 1 here –

The implication of Proposition 2 is that the state of the credit market is a matter of self-fulfilling expectations. A well-functioning credit market with a low interest rate and a low default rate is highly valuable for firms, and this high valuation makes credit contracts without default self-enforcing. Conversely, a weak credit market with a higher interest rate and more default is valued less by the firms, and therefore it cannot sustain credit contracts that prevent default.

With a similar intuition, this economy not only permits multiple steady states but also sunspot cycles which fluctuate perpetually between periods with positive spreads and default and periods with zero spreads and no default. These fluctuations are induced by self-fulfilling changes in beliefs about expected credit market conditions.[11]

Proposition 3. *Under the conditions of Proposition 2, there exists a stochastic equilibrium in which the economy alternates between states with positive default* $v_1 < \bar{v}$ *and states without default* $v_2 > \bar{v}$ *with symmetric transition probability* $\pi \in (0, 1)$.

A reputation loss arising from default, whose value is reflected by the endogenous forward-looking variable v, is essential for the multiplicity results of Propositions 2 and 3. Conditional on v, equilibrium in the credit market, as characterized by Proposition 1, is unique. This distinguishes our result from others where multiplicity of credit-market equilibria arises in static models such as, e.g., Calvo (1988).

3. The Macroeconomic Model

To study the relevance of belief shocks for business-cycle dynamics, we extend the model of the previous section to a general equilibrium economy with additional features: (i) the safe interest rate is determined

[10] If the parameter condition (7) (which is equivalent to $f(\bar{v}) < \bar{v}$) fails, there exist at most two equilibria with default, or at most two equilibria without default. Since function f is convex and kinks upwards at $\bar{v}$, there cannot be more than two stationary equilibria.

[11] The proof rests on a continuity argument with multiple steady states (cf. Chiappori and Guesnerie (1991)).

in equilibrium; (ii) lenders can recover some of their exposure in default events; (iii) defaulters are not permanently excluded; (iv) due to idiosyncratic productivity shocks the state of the credit market impacts aggregate factor productivity; (v) aggregate shocks are introduced to study business-cycle implications: these include fundamental shocks to productivity and financial variables, as well as belief shocks which impact credit market expectations.

3.1. The Setup

Firms and Workers A continuum of firm owners (with measure one) have the same objective as in (1). The idiosyncratic default loss η is distributed with cumulative distribution function $G(.)$ with no mass points. Differently from the model of the previous section, a continuous distribution helps to generate a continuous variation of default rates in response to aggregate shocks.

A firm with capital k_t and labor ℓ_t produces output y_t (to be used for consumption and investment purposes) according to the technology

$$y_t = (z_t k_t)^\alpha (A_t \ell_t)^{1-\alpha}$$

with capital share $\alpha \in (0, 1)$. A_t grows over time and is subject to aggregate risk. Firms can have high or low idiosyncratic capital productivity z_t. Specifically, a firm has high productivity z^H with probability π and low productivity $z^L < z^H$ with probability $1 - \pi$. To simplify algebra, we assume that the capital productivity shock affects the stock of capital (rather than the capital service), so that the firm's capital stock at the end of the period is $(1 - \delta)z_t k_t$, where δ is the depreciation rate.[12]

The economy further includes a unit mass of hand-to-mouth workers who supply labor l_t and consume labor earnings $c_t^w = w_t l_t$. Their preferences are represented by a modified Greenwood-Hercowitz-Huffman utility function $u\left(c_t^w - A_t \frac{\kappa l_t^{1+\nu}}{1+\nu}\right)$ to adapt to a growing economy, where u is increasing and concave, and κ, $\nu > 0$.[13] Labor supply satisfies the relation

$$w_t/A_t = \kappa l_t^\nu \,. \tag{8}$$

That workers are hand-to-mouth consumers is not a strong restriction but follows from imposing a zero borrowing constraint on workers: if workers have the same discount factor β as firm owners, they do not

[12] These features of idiosyncratic shocks can be relaxed at the cost of introducing more state variables. See Appendix B.
[13] The reason behind is that technological growth also increases the quality of leisure time (see Mertens and Ravn, 2011).

save in the steady-state equilibrium in which the gross interest rate satisfies $\bar{R}_t < 1/\beta$, so that workers' consumption equals labor income in all periods.[14]

Consider a firm operating the capital stock k_t. In the labor market, the firm hires workers at the competitive wage rate w_t. This leads to labor demand which is proportional to the firm's effective capital input $z_t k_t$, so that the firm's net worth (before debt repayment) is $\Pi_t z_t k_t$, where the gross return per efficiency unit of capital is (see Appendix A.4)

$$\Pi_t \equiv \alpha \left[\frac{(1-\alpha)A_t}{w_t} \right]^{\frac{1-\alpha}{\alpha}} + 1 - \delta \,. \tag{9}$$

Credit Market In the type of equilibrium we study, the credit market channels funds from low-productivity firms (lenders) to high-productivity firms (borrowers). Competitive, risk-neutral financial intermediaries pool the savings of lenders, taking the safe lending rate $\bar{R}_t$ as given, and offer credit contracts to borrowers. Credit contracts take the form (R_t, b_t), where R_t is the gross borrowing rate, and b_t is the firm's debt.

As before, the debt level in the optimal contract is proportional to the firm's internal funds (equity). Moreover, because all borrowing firms face the same ex-ante default incentives, the debt-to-equity ratio for all borrowing firms is the same and only depends on the aggregate state. This implies that the equilibrium contract can be written as (R_t, θ_t) where θ_t is the common debt-to-equity ratio for all borrowers.

If a firm borrows in period t and defaults in period $t+1$, creditors can recover the fraction λ_{t+1} of the debt exposure (possibly through seizing some of the collateral assets). This recovery parameter can be subject to "financial shocks," to be understood as disturbances to the collateral value or to the cost of liquidation.[15] The defaulting firm keeps the remaining part of the net worth but carries a default flag which temporarily prevents access to credit.[16] In any period following default, the default flag disappears with probability ψ in which case the firm regains full access to credit.

The (gross) credit spread, i.e. $R_t/\bar{R}_t$, reflects not only the expected default cost but also includes an "excess bond premium" term which is unrelated to the fundamental default risk and which may represent investor sentiments or risk appetite (cf. Gilchrist and Zakrajšek, 2012). One part of the excess premium

[14] This standard argument extends to a stochastic equilibrium around the steady state as long as shocks are not too large.

[15] See e.g. Jermann and Quadrini (2012) for similar modeling approaches. See Chen (2010) for cyclical recovery rates.

[16] Hence, after a liquidation or reorganization, the firm owners continue to operate a business (cf. footnote 9). We abstract from the entry and exit of firms.

comes from beliefs and will be discussed later. The other part stands for intermediation costs Φ_t that intermediaries must pay per unit of debt. Such costs may thus include risk or insurance premia against aggregate default risk.[17] Φ_t is exogenous and may be subject to shocks.

Timing Timing within each period is as follows. First, the aggregate state defined as $X_t \equiv (A_t, \lambda_t, \Phi_t, \varepsilon_t^b)$ realizes. The first three components are the fundamental parameters described above which follow a Markov process. ε_t^b is a belief shock (specified below) which is uncorrelated over time. Next to X_t, idiosyncratic default costs η realize. Second, indebted firms either repay their debt or opt for default. Firms with a default history lose the default flag with probability ψ. Third, firms learn their current idiosyncratic productivity $z_t \in \{z^L, z^H\}$ and make borrowing and production decisions.

Specifically, a firm with net worth ω_t (after debt repayment or default) chooses dividends c_t, capital k_t and debt b_t (or savings, if negative) to maximize the firm owner's utility as in (1), subject to the budget constraint $c_t + k_t - b_t = \omega_t$. All firms can save ($b_t \leq 0$) at gross interest rate $\bar{R}_t$. Firms with credit market access can also borrow $b_t \geq 0$ at borrowing rate R_t. Next period, after realization of aggregate shocks and idiosyncratic default costs, the net worth is $\omega_{t+1} = \Pi_t z_t k_t - \max(R_t b_t, 0) - \min(\bar{R}_t b_t, 0)$ if the firm does not default. If the firm defaults on debt $b_t > 0$, net worth after partial recovery of debt is $\omega_{t+1} = \Pi_t z_t k_t - \lambda_{t+1} R_t b_t$.

3.2. Credit Contracts

This section derives the optimal credit contract that intermediaries offer to all borrowing firms, conditional on the aggregate state X_t. Write credit contracts in the form (ρ_t, θ_t) where $\rho_t = R_t / \left(z^H \Pi_t\right)$ is the (gross) interest rate relative to the borrower's capital return and θ_t is the debt-to-equity ratio. If an intermediary offers contract (ρ_t, θ_t), it pools idiosyncratic default risk over many borrowers, anticipating that the ex-post (stochastic) default rate is $G(\tilde{\eta}_{t+1})$, where the threshold value of default costs is denoted by $\tilde{\eta}_{t+1}$. This threshold is the outcome of default decisions at the beginning of $t+1$. It depends both on the terms of the contract (ρ_t, θ_t) and on the realization of aggregate shocks at the beginning of $t+1$.

[17] Although intermediaries insure lenders against *idiosyncratic* default risks, they cannot insure themselves against the *aggregate* component of default risk. The latter may be obtained from unmodeled (foreign) insurance companies selling credit default swaps (cf. Jeske et al., 2013). In the absence of such insurance, intermediaries could not offer a safe rate to depositors in combination with standard credit contracts, but need to offer risky securities to lenders to fund credit to risky borrowers.

Consider a borrowing firm that signs a credit contract (ρ_t, θ_t) in period t. With equity equal to $\omega_t - c_t$, this firm borrows $b_t = \theta_t(\omega_t - c_t)$. At the beginning of the next period, net worth is

$$\omega_{t+1} = z^H \Pi_t (1 + \theta_t - \theta_t \rho_t)(\omega_t - c_t) \ ,$$

if the firm repays; under default, the firm's net worth is

$$\omega^d_{t+1} = z^H \Pi_t (1 + \theta_t - \lambda_{t+1} \theta_t \rho_t)(\omega_t - c_t) \ .$$

Let $V(\omega_{t+1}; X_{t+1})$ and $V^d(\omega^d_{t+1}; X_{t+1})$ denote the continuation values under repayment and default, respectively. In period t, the firm chooses dividends c_t and a credit contract to maximize

$$(1-\beta) \log c_t + \beta \mathbb{E}_t \max \left[V\left(\omega_{t+1}; X_{t+1}\right), V^d\left(\omega^d_{t+1}; X_{t+1}\right) - \eta_{t+1} \right] ,$$

where $\mathbb{E}_t$ is over the realization of the aggregate state X_{t+1} and the idiosyncratic default cost η_{t+1}.

Appendix A.6 shows that value functions take the form $V^{(d)}(\omega; X_t) = \log(\omega) + \bar{V}^{(d)}(X_t)$, implying that all borrowers and lenders save fraction β of their net worth, i.e. $c_t = (1-\beta)\omega_t$. Write $v_t \equiv \bar{V}(X_t) - \bar{V}^d(X_t)$ to denote the surplus value of a clean credit record. As in Section 2, v_t reflects *expected credit conditions*. Then, the objective of a borrowing firm can be rewritten as[18]

$$\mathbb{E}_t \max \Big[\log\left(1 + \theta_t - \theta_t \rho_t\right), \log\left(1 + \theta_t - \lambda_{t+1}\theta_t \rho_t\right) - \eta_{t+1} - v_{t+1} \Big] . \tag{10}$$

Notice that the borrowing firm will be indifferent between repaying and defaulting when $\eta_{t+1} = \tilde{\eta}_{t+1}$ which makes the two utilities in the max operator of (10) the same. Therefore, the ex-post default threshold level $\tilde{\eta}_{t+1}$ can be expressed as

$$\tilde{\eta}_{t+1} = \log\left(\frac{1 + \theta_t - \lambda_{t+1}\theta_t \rho_t}{1 + \theta_t - \theta_t \rho_t} \right) - v_{t+1} \ , \tag{11}$$

such that the borrower defaults if and only if the default cost is $\eta_{t+1} < \tilde{\eta}_{t+1}$. This threshold varies with the terms of the contract (ρ_t, θ_t) and it declines in next period's recovery rate λ_{t+1} and in the expected credit conditions variable v_{t+1}. That is, a lower recovery rate or a decline in expected credit conditions lead to an increase of default, given the credit contract.

To issue credit, financial intermediaries raise funds from lenders who receive the gross saving interest rate $\bar{R}_t$ in period $t+1$. Intermediaries further need to pay the intermediation cost Φ_t per unit of debt.

[18] The terms $(1-\beta)\log c_t$ and $\beta[\log(z^H \Pi_t(\omega_t - c_t)) + \mathbb{E}_t \bar{V}(X_{t+1})]$ are irrelevant for the maximization over credit contracts $\{(\rho_t, \theta_t)\}$ and hence cancel out.

Competition drives expected intermediary profits to zero, which implies

$$\bar{\rho}_t(1+\Phi_t) = \rho_t \mathbb{E}_t \left[1 - G(\tilde{\eta}_{t+1}) + G(\tilde{\eta}_{t+1})\lambda_{t+1}\right] \,, \tag{12}$$

where $\bar{\rho}_t \equiv \bar{R}_t/(z^H \Pi_t)$ is the safe interest rate normalized by the borrowers' capital return. The right-hand side of (12) is the expected revenue per unit of debt (again relative to $z^H \Pi_t$). In default events, which occur with probability $G(\tilde{\eta}_{t+1})$, intermediaries recover fraction λ_{t+1} of debt.

Under perfect competition, the contract (ρ_t, θ_t) maximizes borrowers' expected utility in (10), subject to the zero-profit condition for intermediaries (12), taking the ex-post default threshold given by (11). We characterize the optimal contract via the first-order condition of this problem as follows:[19]

Proposition 4. *Given ($\bar{\rho}_t$, Φ_t), the optimal credit contract (ρ_t, θ_t) in period t satisfies (12) and*

$$(1+\theta_t)\frac{1-\rho_t}{\rho_t} = \mathbb{E}_t \left\{ \frac{(\lambda_{t+1}-1)(1+\theta_t) + \rho_t\theta_t(1-\lambda_{t+1})}{1+\theta_t-\rho_t\theta_t} \left[G(\tilde{\eta}_{t+1}) - \frac{\rho_t\theta_t G'(\tilde{\eta}_{t+1})}{\bar{\rho}_t(1+\Phi_t)} \right] \right\} \,. \tag{13}$$

The first-order condition (13) captures the basic trade-off that borrowers face: they gain from higher leverage, which generates a higher incentive to default, but they dislike the higher spread as a result. As in the model of Section 2, the measure of expected credit conditions v_t depends itself on the state of the credit market, satisfying the following recursive equation (see Appendix A.7 for the derivation):

$$v_t = \beta\pi \mathbb{E}_t \left[\log\left(1+\theta_t - \lambda_{t+1}\rho_t\theta_t\right) - \tilde{\eta}_{t+1}[1-G(\tilde{\eta}_{t+1})] - \int_{-\infty}^{\tilde{\eta}_{t+1}} \eta \, dG(\eta) \right] + \beta(1-\psi-\pi)\mathbb{E}_t\left[v_{t+1}\right]. \tag{14}$$

The value of access to the credit market in period t includes two terms. First, with probability π the firm is a borrower in which case it benefits from higher leverage θ_t and lower relative interest rate ρ_t, whereas a higher expected default threshold $\tilde{\eta}_{t+1}$ reduces the value of borrowing. Second, the term $\beta(1-\psi-\pi)\mathbb{E}_t v_{t+1}$ captures the discounted value of credit market access from period $t+1$ onward. The forward-looking equation (14) is key for the possibility of self-fulfilling beliefs. Similarly to the logic of Proposition 2 and Figure 1 in the previous section, there is a positive relationship (a dynamic complementarity) between future credit conditions and today's value.

To elaborate on this, rewrite equation (14) as $v_t = \mathbb{E}_t f(\tilde{X}_t, \tilde{X}_{t+1}, v_{t+1}) - \mathbb{E}_t \varepsilon^b_{t+1}$, where $\tilde{X}_t = (A_t, \lambda_t, \Phi_t)$ is the fundamental state vector, and i.i.d. ε^b_{t+1} is a random variable ("belief shocks") with

[19] In our parameterizations with normally distributed default costs we verify that the second-order condition is also satisfied and that the solution is indeed a global maximum.

mean zero and variance σ_b^2. If the function f is monotonically increasing in v_{t+1}, this equation can be inverted into $v_{t+1} = \tilde{f}(\tilde{X}_t, \tilde{X}_{t+1}, v_t + \varepsilon_{t+1}^b)$, where $\tilde{f}(X_1, X_2, .)$ is the inverse of $f(X_1, X_2, .)$. If the steady state is locally indeterminate, this forward solution of equation (14) is a stationary process, implying that v_t can be treated as a *predetermined* variable which is subject to changes in self-fulfilling beliefs in period $t+1$. Therefore, v_{t+1} is going to be persistent and is affected by belief shocks ε_{t+1}^b, as well as changes in recovery and intermediation costs. Note that belief shocks must be uncorrelated in order to be self-fulfilling. The realization ε_{t+1}^b alters expected credit conditions v_{t+1} which, in turn, impacts the default threshold in period $t+1$ via equation (11). Moreover, from an ex-ante perspective, the variance of belief shocks has a positive impact on the credit spread via the credit contract. We use this idea in the next section to quantify this variance on the basis of an empirical measure of credit spread.

3.3. Equilibrium

In the competitive equilibrium, firms and intermediaries behave optimally as specified above, and the capital and labor markets are in equilibrium.

Consider first the capital market. We focus on an equilibrium where the safe interest rate is identical to the capital productivity of unproductive firms, i.e. $\bar{R}_t = z^L \Pi_t$. Then some capital is used in low-productivity firms which in turn implies that total factor productivity (TFP), formally defined in Appendix C.1, responds endogenously to the state of the credit market.[20] Such an equilibrium requires that the savings of unproductive firms are not smaller than the demand for capital from borrowing firms.

Let $f_t \in [0, 1]$ denote the fraction of aggregate net worth Ω_t *owned by firms with access to credit*. The demand for credit is $\theta_t f_t \pi \beta \Omega_t$: All firms save fraction β of their net worth, fraction $f_t \pi$ of these firms want to borrow and have access to credit, and they borrow θ_t per unit of equity. When $\bar{R}_t = z^L \Pi_t$, the supply of capital is identical to the savings of unproductive firms which is $(1 - \pi)\beta\Omega_t$. Therefore, the capital market is in equilibrium if

$$\bar{\rho}_t = \frac{z^L \Pi_t}{z^H \Pi_t} = \frac{z^L}{z^H} \quad \text{and} \quad \theta_t f_t \pi \leq (1 - \pi)\,. \tag{15}$$

Consider next the labor market. Labor demand of any firm is proportional to the efficiency units of

[20] $\bar{R}_t$ cannot fall below this value because then all firms want to borrow and no one saves. Equilibria with $\bar{R}_t > z^L \Pi_t$ (and an efficient allocation of capital among firms) are possible if credit constraints are soft enough.

capital: $\ell = zk[(1-\alpha)A_t^{1-\alpha}/w_t]^{1/\alpha}$. The capital stock operated by productive firms is $\beta\pi\Omega_t\Big[f_t(1+\theta_t)+1-f_t\Big]$. That is, savings of productive firms in period t are $\beta\pi\Omega_t$; fraction f_t of this is owned by borrowing firms whose capital is $1+\theta_t$ per unit of equity, and fraction $1-f_t$ is owned by firms without access to credit whose capital is all internally funded. The capital stock operated by unproductive firms is $\beta\Omega_t\left[(1-\pi)-\pi f_t\theta_t\right]$. That is, these firms use the fraction of savings not invested in the capital market for production. With labor supply given by (8), the real wage that clears the labor market satisfies

$$\left(\frac{w_t/A_t}{\kappa}\right)^{1/\nu} = \left[\frac{(1-\alpha)A_t^{1-\alpha}}{w_t}\right]^{\frac{1}{\alpha}} \beta\Omega_t\left\{z^L\Big[(1-\pi)-\pi f_t\theta_t\Big] + z^H\pi\Big[f_t(1+\theta_t)+1-f_t\Big]\right\}. \quad (16)$$

It remains to describe the evolution of aggregate net worth Ω_t and the fraction f_t. We first discuss Ω_t. In period t, all firms save fraction β of their net worth. Fraction $1-\pi$ are unproductive and earn return $\bar{R}_t = z^H\Pi_t\bar{\rho}_t$. Fraction πf_t of aggregate savings is invested by borrowing firms of which fraction $1-G(\tilde{\eta}_{t+1})$ do not default and $G(\tilde{\eta}_{t+1})$ default in $t+1$. Fraction $\pi(1-f_t)$ of aggregate savings is invested by productive firms without credit market access who earn return $z^H\Pi_t$. The aggregate net worth in period $t+1$ is thus

$$\begin{aligned}\Omega_{t+1} = {} & \beta z^H\Pi_t\Omega_t\Big\{(1-\pi)\bar{\rho}_t + \pi f_t\left[1-G(\tilde{\eta}_{t+1})\right](1+\theta_t-\theta_t\rho_t) \\ & + \pi f_t G(\tilde{\eta}_{t+1})\left[1+\theta_t-\lambda_{t+1}\theta_t\rho_t\right] + \pi(1-f_t)\Big\}. \quad (17)\end{aligned}$$

Now consider f_t. Fraction $1-\pi$ of these firms earn $\bar{\rho}_t z^H\Pi_t$, and fraction $\pi(1-G(\tilde{\eta}_{t+1}))$ of firms borrow and do not default, earning return $(1+\theta_t-\theta_t\rho_t)\,z^H\Pi_t$. All these firms retain access to the credit market in the next period. Fraction $1-f_t$ of net worth is owned by firms without access to credit in period t. They earn $\bar{\rho}_t z^H\Pi_t$ with probability $1-\pi$, and $z^H\Pi_t$ with probability π, and they regain access to the credit market with probability ψ. Adding up the net worth of all these firms gives the net worth of firms with credit market access in period $t+1$,

$$f_{t+1}\Omega_{t+1} = \beta z^H\Pi_t\Omega_t\Big\{(1-\pi)f_t\bar{\rho}_t + \pi f_t\left[1-G(\tilde{\eta}_{t+1})\right](1+\theta_t-\theta_t\rho_t) + (1-f_t)\psi[(1-\pi)\bar{\rho}_t+\pi]\Big\}. \quad (18)$$

Definition 1. *Given an initial state (f_{-1}, Ω_{-1}) and an exogenous stochastic process for the aggregate state vector $X_t = (A_t, \lambda_t, \Phi_t, \varepsilon_t^b)$, a competitive equilibrium is a mapping (f_{t-1}, Ω_{t-1}, X_t) $\mapsto$ (f_t, Ω_t, X_{t+1}), together with the profit rate Π_t, the wage rate w_t, the credit contract (ρ_t, θ_t), the safe interest rate $\bar{\rho}_t$ (relative to $z^H\Pi_t$), the default threshold $\tilde{\eta}_t$, and the surplus of credit market access v_t, as functions of*

(f_{t-1}, Ω_{t-1}, X_t), such that: (i) firms make optimal labor hiring, savings and borrowing decisions, and borrowing firms decide optimally about default, i.e., (9), (11), and (14) hold; (ii) financial intermediaries make zero expected profits by offering standard debt contracts to borrowers and safe interest rates to lenders, i.e., (12) and (13) hold; (iii) the capital and the labor market are in equilibrium, i.e., (15) and (16) hold; (iv) aggregate net worth Ω_t and the fraction f_t evolve according to (17) and (18).

The shock processes are specified as follows. Productivity A_t grows according to a unit root process $\log A_t = \log A_{t-1} + \mu_t^A$ with stationary trend growth μ_t^A. Following common practice, AR(1) processes are imposed for μ_t^A, λ_t, and Φ_t as

$$\mu_t^A - \mu^A = \rho_A \left(\mu_{t-1}^A - \mu^A\right) + \varepsilon_t^A \ ,$$

$$\lambda_t - \lambda = \rho_\lambda \left(\lambda_{t-1} - \lambda\right) + \varepsilon_t^\lambda \ ,$$

$$\log(1+\Phi_t) - \log(1+\Phi) = \rho_\Phi \left[\log(1+\Phi_{t-1}) - \log(1+\Phi)\right] + \varepsilon_t^\Phi \ ,$$

where μ^A, λ, and Φ are steady-state parameters, ρ_A, ρ_λ, ρ_Φ are persistence parameters, and ε_t^A, ε_t^λ, and ε_t^Φ are i.i.d. normally distributed with mean zero and variances σ_A^2, σ_λ^2 and σ_Φ^2. These random variables are called below "productivity shocks," "recovery shocks," and "intermediation shocks," respectively. Belief shocks ε_t^b are normally distributed with variance σ_b^2. Note again that these are self-fulfilling shocks which are i.i.d. innovations to the endogenously persistent credit market expectations variable v_t. With this structure, our model captures three types of financial shocks: recovery shocks are essentially credit demand shocks, while intermediation shocks affect the intermediaries' willingness to supply credit. Belief shocks could reflect both credit demand and supply, as they determine credit market expectations of borrowers, lenders, and intermediaries.

4. Quantitative Analysis

Now we explore the quantitative implications of the macroeconomic model. The model is made stationary by dividing the wage rate and the capital stock by A_t; see Appendix C.1 for the equilibrium conditions. As in the illustrative model of Section 2, the macroeconomic model typically generates two steady states, one of which is locally indeterminate and hence susceptible to belief shocks. We focus on local dynamics around this steady state which features a higher v and therefore a larger volume of credit than the other (determinate) steady state.

The model is calibrated to suitable long-run targets for the U.S. economy. We use the concept of a *risky steady state* (cf. Coeurdacier et al., 2011). This is a stationary equilibrium given that all shock realizations are zero, while aggregate risk is still taken into account by agents. In our model, the presence of risk has an impact on credit contracts (interest rates and leverage) via equations (12) and (13). In particular, risky recovery and beliefs affect the credit spread in our model. Differently from traditional quantitative analysis, this approach is useful to identify belief shocks from credit spread data. This procedure is explained in Section 4.1, before we describe the data, the calibration, and the results.

4.1. The Credit Spread and Belief Risk

As is well known, credit spreads are usually larger than the realized default costs. The gap is referred to as the excess bond premium. Compared to a standard model with exogenous collateral constraints, the application of a risky steady state is particularly useful to extract information on belief variation from credit spread and default data.

Let us first define the credit spread and the excess bond premium in the model. Define the expected default threshold at time t, $\tilde{\eta}^e_t \equiv \mathbb{E}_t[\tilde{\eta}_{t+1}]$. The expected default rate is approximately[21]

$$\mathbb{E}_t[G(\tilde{\eta}_{t+1})] \approx \mathbb{E}_t[G(\tilde{\eta}^e_t - \varepsilon^b_{t+1} - \tilde{\xi}_t \varepsilon^{\lambda}_{t+1})] \approx G(\tilde{\eta}^e_t) + \frac{G''(\tilde{\eta}^e_t)}{2}\left(\sigma_b^2 + \tilde{\xi}_t^2 \sigma_\lambda^2\right)$$

where $\tilde{\xi}_t \equiv \frac{\theta_t \rho_t}{1+\theta_t - \lambda_t \theta_t \rho_t}$ summarizes the effect from today's leverage, spread, and recovery on default threshold tomorrow. That is, the expected default rate is approximately the default rate evaluated at the expected default threshold $G(\tilde{\eta}^e)$, adjusted by the variances of belief and recovery shocks if the default cost function $G(.)$ is non-linear. This adjustment results from the fact that the realized default threshold tomorrow $\tilde{\eta}_{t+1}$ is equal to the expected threshold $\tilde{\eta}^e$ disturbed by belief and recovery shocks. In the steady state, the expected default rate $\mathbb{E}[G(\tilde{\eta})]$ is larger than the average default rate $G(\tilde{\eta}^e)$ if there are either belief shocks or recovery shocks and if $G''(\tilde{\eta}^e) > 0$ (which follows from Jensen's inequality).

Now, utilizing the approximated expected default losses from equation (12) above, the (gross) credit spread in the model $\Delta_t = \rho_t/\bar{\rho}_t$, from the lender's zero-profit condition, can be approximated as (see

[21] This equation approximates the default threshold $\tilde{\eta}_{t+1}$ from equation (11) up to the first order around $\tilde{\eta}^e_t$. In Appendix C.2, we actually approximate to the second order, but quantitatively this generates negligible differences.

Appendix C.2 for details):

$$\begin{aligned}
\Delta_t &\equiv (1+\Phi_t)\Big\{1-\mathbb{E}_t\left[(1-\lambda_{t+1})G(\tilde{\eta}_{t+1})\right]\Big\}^{-1} \qquad (19)\\
&\approx (1+\Phi_t)\left\{1-[1-\rho_\lambda\lambda_t-(1-\rho_\lambda)\lambda]\left[G(\tilde{\eta}^e_t)+\frac{G''(\tilde{\eta}^e_t)}{2}\left(\sigma_b^2+\tilde{\xi}_t^2\sigma_\lambda^2\right)\right]-G'(\tilde{\eta}^e_t)\tilde{\xi}_t\sigma_\lambda^2\right\}^{-1}.
\end{aligned}$$

Intuitively, the risks of belief variation and recovery variation are going to be priced in the credit spread so that the variances σ_b^2 and σ_λ^2 increase Δ_t.

We can decompose the total credit spread into one component reflecting the predicted default losses based on *fundamental* information, and another residual component, the excess bond premium. The first or the predicted component is calculated similarly as in (19), but ignoring the non-fundamental belief risk and setting intermediation costs to zero, i.e.,

$$\begin{aligned}
\tilde{\Delta}_t &\equiv \Big\{1-\mathbb{E}_t\left[(1-\lambda_{t+1})G(\tilde{\eta}_{t+1})|\varepsilon^b_{t+1}=0\right]\Big\}^{-1}\\
&= \left\{1-[1-\rho_\lambda\lambda_t-(1-\rho_\lambda)\lambda]\left[G(\tilde{\eta}^e_t)+\frac{G''(\tilde{\eta}^e_t)}{2}\tilde{\xi}_t^2\sigma_\lambda^2\right]-G'(\tilde{\eta}^e_t)\tilde{\xi}_t\sigma_\lambda^2\right\}^{-1}.
\end{aligned}$$

The second component or the excess bond premium is then defined as $\Delta_t/\tilde{\Delta}_t$. Two factors contribute to the excess bond premium: (1) the intermediation costs Φ_t, which is exogenous to the model, and (2) the remaining part that is contributed by the belief risk measured by $\sigma_b > 0$.

Belief shocks are systematic shocks which are hard to be detected from fundamental information on corporate balance sheets and cash flows. Therefore, these shocks are taken into account by investors (hence they are reflected in the credit spread), while expected default losses *in practice* are calculated on the basis of fundamental information only. For this reason, belief shocks are crucial for explaining the part of the spread that is not explained by fundamental default costs. Note, however, that not only belief shocks will shift the excess bond premium over the cycle, but any type of aggregate shocks that moves the default threshold $\tilde{\eta}_e$ will move the excess bond premium when σ_b is positive.

Given a variance of beliefs σ_b^2 and a variance of recovery shocks σ_λ^2, the fixed point of the approximated equilibrium system is the risky steady state of our model. As a comparison, the traditional deterministic steady state is the fixed point when $\sigma_b = \sigma_\lambda = 0$. The concept of a risky steady state is crucial to identify the variance of beliefs. Specifically, given calibrated values for leverage θ, the average default rate $G(\tilde{\eta}^e)$, the average credit spread Δ, intermediation costs Φ, and a calibrated functional form for G, as well as the average recovery rate λ and recovery risk σ_λ^2 estimated from financial data, we calculate the steady-state values of $\tilde{\xi}$ and $\tilde{\eta}^e$. Then, σ_b^2 is uniquely identified by the credit spread equation (19).

Equivalently, the size of the belief risk is pinned down by the size of the excess bond premium not explained by Φ since the expected default cost term $\tilde{\Delta}$ is already determined from the other parameters in the steady state. Simply put, lenders charge a premium for the belief risk. We will see that the model dynamics around the risky steady state can generate quantitatively significant excess bond premium fluctuations (arising from beliefs) that have been shown as an important predictor for the business cycle (Gilchrist and Zakrajšek, 2012).

4.2. Data

We use the recovery rate and the all-rated default rate from Moody's rated corporate bonds, covering the period 1982–2016, all in percentage terms, and we use the credit spread index developed by Gilchrist and Zakrajšek (2012), representative for the full corporate bond market. Moody's data are obtained from the 2016 annual report published by Moody's Investors Service. The recovery rate is measured by the post-default bond price for one dollar repayment. Regarding the spread series, we consider annual averages of the monthly series, updated until 2016.[22] Output is defined as business output in the U.S. national accounts, as our model describes a closed economy without government. Output growth refers to the growth rate of U.S. real per capita output after using the population growth rate.

– Insert Table 1 here –

Table 1 shows the basic statistics of these four variables. The sample means of the credit spread, the recovery rate, and the default rate are 2%, 41.8%, and 1.58%, respectively. As a back-of-the-envelope calculation, the average spread (2%) is more than twice of the average default cost (i.e., 1.58% $\times$ (1 - 41.8%) = 0.92%) which suggests that the excess bond premium accounts for a large fraction of the spread. In terms of cyclical dynamics, the spread and the default rate are highly positively correlated, and both of them are countercyclical. The recovery rate is highly negatively correlated with the default rate, but much less with the credit spread and it is mildly procyclical.

Time series of the three variables are shown in Figure 2. Evidently, the default rate spikes up in all three recessions since 1982, and most strongly during the Great Recession. The recovery rate reaches a

[22] See Simon Gilchrist's website: http://people.bu.edu/sgilchri/Data/data.htm

trough during each recession. Interestingly, however, the credit spread does not increase during the 1991 recession; this further motivates the need to explore the distinct roles of the credit spread and corporate default for macroeconomic dynamics.

– Insert Figure 2 here –

4.3. Parametrization

Given that we consider annual default and recovery rates, we calibrate the model at annual frequency. Table 2 summarizes all parameter choices.

– Insert Table 2 here –

The following parameters can be externally calibrated without solving the model. Directly calibrated are $\alpha = 0.33$ (capital share), $\delta = 0.1$ (annual depreciation rate), and $\psi = 0.1$ which implies a ten-year exclusion period.[23] The labor supply elasticity is set to $1/\nu = 1.5$, a conventional number. We then set $\kappa = 2$ by arbitrarily normalizing the steady-state labor supply at 1/3. The fraction of financially constrained firms is set to $\pi = 0.3$ (see, e.g., Almeida et al., 2004). The mean growth rate of labor efficiency is set to $\mu^A = 1.7\%$, the data average for output growth in Table 1.

The steady-state recovery rate λ is set to the data average 41.8%, and the two parameters for the AR(1) process for λ_t are directly estimated via ordinary least squares (OLS) from the empirical series. To reduce the complexity of estimation, the steady-state level of intermediation costs $\Phi \geq 0$ is not directly estimated. Instead, we experiment with different values of Φ and compare different likelihoods of the model (see Table 4 below). It turns out that $\Phi = 0.05\%$ generates the highest likelihood, so this value is chosen in our benchmark calibration.

Other parameters are either related to the risky steady state or to the dynamics around it. We begin with those parameters which are chosen to ensure that the risky steady state generates certain long-run targets. It is supposed that the default cost distribution function $G(.)$ is normal with mean μ and variance σ^2. Given that the variance of recovery shocks σ_λ is already estimated, five parameters remain to be

[23] This corresponds to the bankruptcy flag for sole proprietors (or for partnerships with personal liabilities) filing for bankruptcy under Chapter 7 of the U.S. Bankruptcy Code. Note that the choice of ψ does not matter much as the parameters governing the $G(.)$ will adjust in calibration if ψ changes.

calibrated jointly to match the following targets in the risky steady state: (i) the capital-output ratio $K/Y = 1.5$; (ii) the credit-output ratio $B/Y = 0.82$, based on all (non-financial) firm credit 1982–2016; (iii) a 1.58% default rate so that $G(\tilde{\eta}^e) = 1.58\%$; (iv) a 2% credit spread (see Table 1); a leverage ratio equal to $\theta = 1.95$ which is in line with the choice of $\pi = 0.3$.[24] These targets identify the five parameters β, μ, σ, z^H, and the belief variance σ_b^2 uniquely according to Section 4.1. z^L is normalized such that average capital productivity equals one.

Finally, the model is log-linearized around the risky steady state and the system is expressed in a Kalman filter form. We use the four time series data described above. Since the recovery process and the variance of belief shocks have been calculated, only the intermediation cost and aggregate productivity processes are estimated. We compute the log-likelihood of observing the period-t data conditioning on past data, and we calculate the total log-likelihood of observing the whole sample. Then, the parameters are estimated by maximizing the total log-likelihood. The estimates of the standard deviations of the two shocks are significantly different from zero. The estimate of the persistence of intermediation shocks is significantly positive (with a t statistics above two), while the one for productivity growth is not.

4.4. Quantitative Results

We first show the smoothed shocks from the maximum likelihood estimation. Then, we illustrate impulse responses after feeding in one standard deviation innovations for each of the shocks, and finally we present variance decompositions of several variables of interest.

Estimated Shocks Once the model is estimated through the Kalman filter, underlying shocks that generate the same observations as in the data can be backed out. This exercise is done through the Kalman smoother that uses information of the whole sample to infer the states in each date. All four estimated shocks (their mean levels at each date and normalized by their respective standard deviations, which will be used in later analysis) are plotted in Figure 3. Note that positive innovations to intermediation costs are recessionary shocks.

– Insert Figure 3 here –

[24] This value of θ corresponds to the 85th percentile of debt-to-equity ratios in COMPUSTAT, hence is the median of the 30 percent highest debt-to-equity ratios in these data.

Through the lens of our model, the 2007-2009 Great Recession is indeed special compared to the previous ones. It has a combination of a deep fall in recovery, a sharp deterioration of expected credit conditions (reflecting a six percentage-points increase of the default probability), and the recession is led by a larger-than-usual intermediation shock (corresponding to a 260 basis points rise of the credit spread). Exogenous aggregate productivity growth also falls during this period.

The Great Recession features a large liquidity and pledgeability drop of financial assets, which is captured in our model by the negative shocks to λ (i.e., a fall in recovery ability). Note also the positive shocks to λ in the years prior to the Great Recession which may mirror the real-estate boom and the surge of collateral assets in this period, leading to a higher-than-usual recovery ability. After the recession, recovery rises for some period, possibly reflecting the asset-purchase programs implemented by the Federal Reserve in 2009-2010.

We observe a deterioration of expected credit conditions (i.e. negative belief shocks) prior to all three recessions since 1982, which go hand in hand with spikes of corporate default in these episodes. Intermediation costs rise during the 2001 and 2008/09 recessions, generating sharp increases of the excess bond premium. On the other hand, the credit spread does not go up during the 1991 recession, despite a significant increase of the default rate. This is mirrored in the absence of positive intermediation shocks in this period.

Impulse Responses By construction, the model fed with the estimated shocks generates the observed credit spread, recovery rate, default rate, and output growth. In order to understand the transmission mechanism, we plot impulse response functions after each of the four shocks in Figures 4 and 5.

– Insert Figure 4 here –

A negative one standard-deviation innovation to the recovery rate (recovery shock) gives borrowers more incentive to default on impact (year 0), resulting in a 1.6 percentage-point spike of the default rate. After the initial shock, lenders tighten credit substantially (7% fall in leverage). This brings down the default incentive from year 1, and the default rate falls even slightly below the steady-state level, which causes a modest decline of the credit spread. The temporary tightening of credit in combination with a larger number of firms without access to credit reduces aggregate TFP and output growth. Yet leverage fully recovers from year three onward after which the growth rates of TFP and output turn positive again.

Similarly to recovery shocks, a one-time adverse belief shock generates an increase of the default rate by 1.7% on impact. However, there are two crucial differences.

The first is much stronger persistence: Due to weakened credit expectations, the default rate remains above the steady-state level after the initial shock and leverage drops persistently. Therefore, the misallocation of credit and the resulting declines of TFP growth and output growth are long-lasting. Intuitively, the persistent deterioration of credit market conditions is a necessary feature of self-fulfilling expectations: The credit market value v_t must decline for an extended period such that a *one-time* belief shock can be self-confirming.

The other important difference is that the belief shock triggers an increase of the credit spread by 15 basis points, about one quarter of which comes from the excess bond premium. Together with the persistent rise of the default rate, both the expected default losses and the excess bond premium (via the belief channel) go up. Hence, the belief shock generates a positive co-movement between the spread and default, as opposed to what comes out of recovery shocks (and intermediation shocks shown below in Figure 5).

– Insert Figure 5 here –

In response to a rise of intermediation costs alone, the credit spread increases significantly. In contrast to the response to an adverse belief shock, the default rate *falls* persistently.[25] The explanation is that higher intermediation costs reduce on impact the supply of credit available to productive firms. This is why leverage falls by 1.8% in year 0, reducing incentives to default in the next year. The credit market value v goes up (not shown) and because it is persistent, the default rate remains below the steady-state level, despite the larger credit spread. In turn, leverage rises quickly after the initial shock which justifies the persistent rise of expected credit conditions measured by v. Of course, the initial reduction in leverage contracts output growth, though only by 0.2 percentage points. Rising leverage from year two onward quickly dampens the initial contractionary effect.

Finally, a negative shock to productivity growth generates a substantial drop of TFP growth, which is not persistent as the estimated persistence parameter ρ_A is rather small. Productivity shocks do not

[25] Because of falling default, the excess bond premium turns out to be seven basis points higher than the credit spread on impact, implying that the belief channel in this case reduces the spread.

affect the spread and the default rate since there is no link between aggregate labor efficiency (via the aggregate capital return Π_t) and credit contracts in our model (see Proposition 4).

Variance Decomposition We now examine how much of the variation in the data can be separately explained by each of the four shocks. Note again that there are three distinct financial shocks, namely recovery shocks, belief shocks and intermediation shocks, next to productivity shocks. Table 3 shows how these shocks account for the dynamics of several outcome variables.

The variation of the default rate is explained mainly by two financial shocks: belief shocks (56%) and recovery shocks (43%) since both of them change default incentives on impact. In line with the insight from the impulse response functions, credit-spread fluctuations are mainly explained by intermediation shocks which have a direct impact on the spread (67%), but also by belief shocks which move the spread significantly (31%).

– Insert Table 3 here –

Regarding output growth, the belief channel plays the most important role of all financial shocks, accounting for 40% of the variation. Financial shocks together contribute to 45% of output growth variations. There are two channels through which the credit flow impacts output dynamics. On the one hand, the credit flow affects the capital allocation among productive and unproductive firms. This is the *productivity effect* of credit. On the other hand, the credit flow also affects the firms' aggregate demand for capital and labor, and therefore aggregate production. This is the *factor effect* of the credit flow. To shed light on these two effects, we show how much of the variation of debt growth and TFP growth in the model can be explained by each shock in the last two rows of Table 3. Endogenous fluctuation of productivity growth due to the credit allocation is about 18%, while almost all debt growth can be attributed to financial shocks. In other words, financial shocks generate some variation in TFP growth but they mostly affect the firms' factor demands.

4.5. Further Discussion

As explained before, default and spread data are used to quantify the contributions of the three different financial shocks in our model. To what extent these features provide indeed useful information for the business cycle must be properly assessed. In this section, several experiments are implemented to highlight the importance of endogenous default and credit spreads for aggregate dynamics.

As a first robustness exercise, the model is re-estimated by setting the steady-state value of intermediation costs Φ to different numbers while keeping all other parameters unchanged. Table 4 presents the model likelihoods. One can see that the model performs best at the baseline $\Phi = 0.05\%$. Intuitively, although larger values of Φ can directly match the excess bond premium in steady state, they reduce the importance of belief shocks for the overall credit cycle, which limits the model's ability to generate plausible fluctuations in default, leverage, and output, in line with the impulse responses shown before.

– Insert Table 4 here –

Stochastic intermediation costs are important for our model to account for the full credit spread dynamics (Table 3). As these costs have no direct immediate counterpart in the data, we also consider an alternative estimation of our model in which these shocks are absent and the variation in the spread is treated as an observation error. Thus, this exercise keeps the original data used for the baseline estimation, but avoids the stochastic singularity in the estimation. Column two of Table 5 shows that the model likelihood drops in comparison to that of the baseline. This is a fair comparison, as the observational error biases in favor of the model against the data. However, the falling likelihood suggests that using the spread data in the structural estimation (instead of treating the variation as an observational error) improves the fit of the model. Variance decompositions, in particular the impact of belief shocks and recovery shocks on credit markets and output, are similar to those of the baseline estimation.[26]

Next, we assess the importance of endogenous default by comparing a model with exogenous leverage θ_t which can be interpreted as an exogenous collateral constraint. For instance, if lenders can seize an exogenous fraction of existing capital, lending will be restricted by an exogenous debt-to-equity ratio. In this alternative model, there is no default and the credit spread is zero, while all other features are kept the same. We use the same calibration targets including the same steady-state level of θ. Then, an AR(1) process is estimated for θ_t using the debt growth generated from the baseline estimation, and the drift process for aggregate productivity growth is estimated using output growth data. The second column of

[26] Recall that intermediation cost shocks generate small reactions of output growth compared to belief shocks and recovery shocks. Default falls together with output growth in Figure 5, which is at odds with data.

Table 5 shows that this collateral-constraint model has a significantly lower likelihood compared to the baseline model. Again, this is a fair comparison because both models use the same observations.

– Insert Table 5 here –

Although the model with exogenous collateral constraints generates the same debt growth and output growth, financial shocks become much less important for the business cycle as compared to the baseline. We find that exogenous shocks to θ_t can explain about 10% of variation in output growth and 1.5% of TFP growth, compared to about 45% and 18% in Table 3. Since both models generate the same dynamics of debt (leverage), the difference is explained by the impact of endogenous default on the time-varying group of firms without access to credit. That is, in the baseline model with default, the fraction of firms with access to credit f_t varies in response to financial shocks, which affects both aggregate debt and the allocation of resources among heterogeneous firms.

Finally, we consider a version of our model without belief shocks. In particular, the steady-state value of Φ is set to the upper bound (0.378%) such that intermediation costs account fully for the excess bond premium. We then also add an observational error to default and re-estimate the model. Besides a fall of the likelihood (about 20%), this alternative model fails on two dimensions. First, the correlation between the default rate and output growth is -0.14 as compared to -0.54 in the data (and in the baseline model). The reason can be seen from the dynamics of default implied by the impulse response functions. Only belief shocks generate a persistent rise of default together with a decline of output growth, while the other recessionary shocks can only generate a temporary rise of default, or even a decline of the default rate. Second, the large positive correlation between default and spread falls by 50 percent, mainly because none of the other shocks generates a positive co-movement between the spread and default.

On the basis of all these experiments, we conclude that using the default data and the spread data in the estimation improves our understanding of financial shocks to generate persistent and deep credit and business cycles. Future research could replace standard financial frictions by ours in a medium scale dynamic stochastic general equilibrium model (such as Christiano et al. (2014)). This may further help understand how default rates and credit spreads identify the belief channel, and could shed some light on understanding volatile asset price fluctuations.

5. Conclusions

Variations in expected credit conditions can affect incentives to default and thus take an impact on credit spreads and leverage. We develop this idea and apply it to a macroeconomic model in order to explore the respective roles of belief shocks, fundamental financial shocks, and aggregate productivity shocks for the business cycle. We show how the variance of belief shocks can be parameterized on the basis of the excess credit spread evaluated at the risky steady state. Compared to fundamental financial shocks that directly affect the recovery ability or the credit spread, belief shocks generate a persistent credit cycle and counter-cyclical dynamics of default and the credit spread, and they contribute significantly to the variation of output growth.

On the theoretical side, an interesting avenue for further research is the examination of long-term debt for the impact of self-fulfilling beliefs on default rates. One may conjecture that strategic default incentives are less sensitive to market expectations when borrowers hold long-term debt. Nevertheless, the ability of firms to roll over long-term debt may react to investors' sentiments, as is known from the literature on sovereign debt cited in the introduction.

Regarding policy implications, government policies that alter belief variations could strongly affect economic activity, both in the *long run* (once we take into account belief risks in shaping credit contracts) and over the business cycle.

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Table 1: **Descriptive Statistics**

Correlation	Spread	Recovery Rate	Default Rate	Output Growth
Spread	1	-0.39	0.64	-0.58
Recovery Rate	-	1	-0.76	0.33
Default Rate	-	-	1	-0.54
Output Growth	-	-	-	1
Mean (%)	2.01	41.77	1.58	1.70
Std dev. (%)	0.86	8.94	1.05	1.90

Table 2: **Parameters (Risky Steady State)**

Parameter	Value	Explanation	Target / T statistics (std errors)
α	0.3300	Capital income share	Exogenous
δ	0.1000	Depreciation rate	Exogenous
κ	2.0031	Disutility parameter of labor supply	$\ell = 1/3$
ν	0.6667	Macro labor supply elasticity	$1/\nu = 1.50$
β	0.9663	Discount factor	Capital-to-output ratio 1.5
π	0.3000	Fraction of constrained firms	Almeida et al. (2004)
ψ	0.1000	10-year default flag	Exogenous
μ^A	0.0170	Steady-state output growth	Table 1
λ	0.4177	Steady-state recovery rate	Table 1
Φ	0.05%	Steady-state intermediation costs	Exogenous
σ_b	0.0624	Std. dev. of belief shocks	Credit spread 2%
z^H	1.0230	High productivity	Leverage $\theta = 1.95$
z^L	0.8696	Low productivity	Normalization
μ	0.3543	Mean of η	Default rate 1.58%
σ	0.1359	Std. dev. of η	Debt-to-output ratio 82.5%
ρ_λ	0.5920	Persistence of recovery shocks	OLS estimates
σ_λ	0.0728	Std. dev. of recovery shocks	OLS estimates
ρ_Φ	0.6250	Persistence of intermediation shocks	4.88 (0.13)
σ_Φ	0.0079	Std. dev. of intermediation shocks	8.33 (0.0009)
ρ_A	0.2025	Persistence of productivity shocks	1.03 (0.20)
σ_A	0.0374	Std. dev. of productivity shocks	8.20 (0.0046)

Table 3: **Variance Decomposition in Percents**

	Exogenous Shocks to				
	Intermediation (1)	Recovery (2)	Beliefs (3)	Productivity growth (4)	All financial shocks (1) + (2) + (3)
Credit spread	67.24	1.93	30.83	0	100
Default rate	0.78	43.12	56.10	0	100
Output growth	2.33	2.80	39.58	55.29	44.71
Debt growth	1.92	31.46	66.37	0.25	99.75
TFP growth	0.70	1.12	15.84	82.34	17.66

Table 4: **Log-likelihoods of Different Parametrizations**

Baseline	$\Phi = 0$	$\Phi = 0.10\%$	$\Phi = 0.20\%$	$\Phi = 0.30\%$
354.4	354.2	354.2	350.7	216.9

Table 5: **Log-likelihoods of Different Versions of the Model**

Baseline	Observational error on the spread	Exogenous collateral constraint	$\Phi = 0.38\%$ + Observational error on default
354.4	341.3	308.0	282.9

Figure 1: **Co-Existence of Default and No-Default Equilibria**

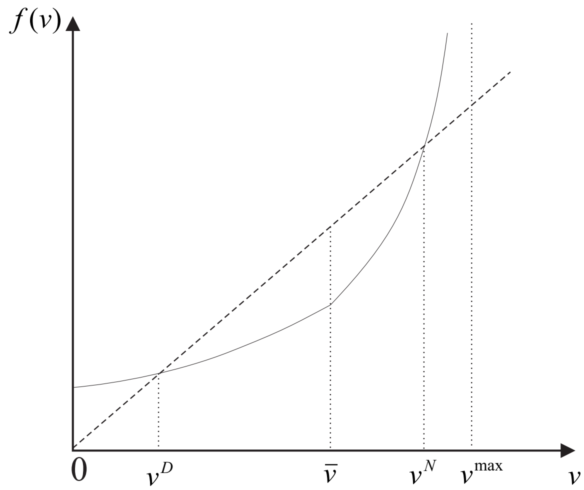
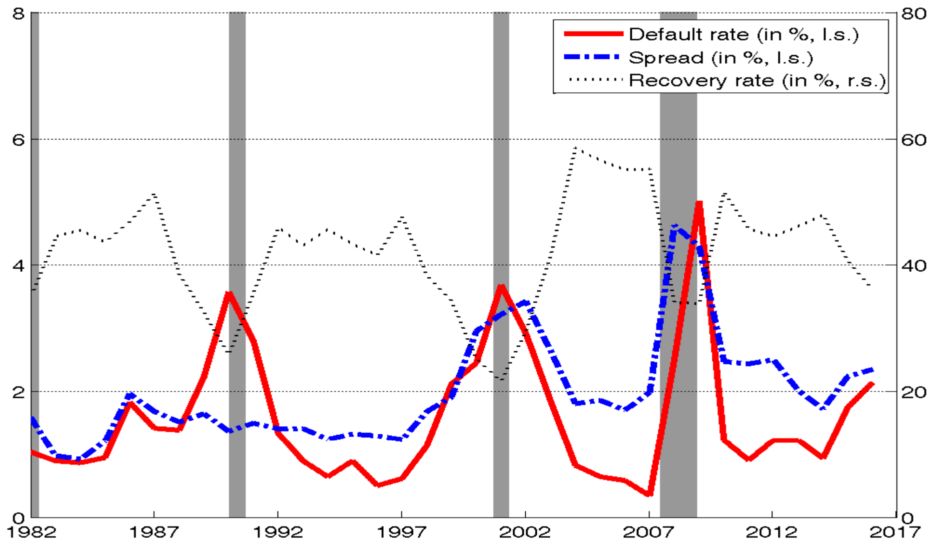
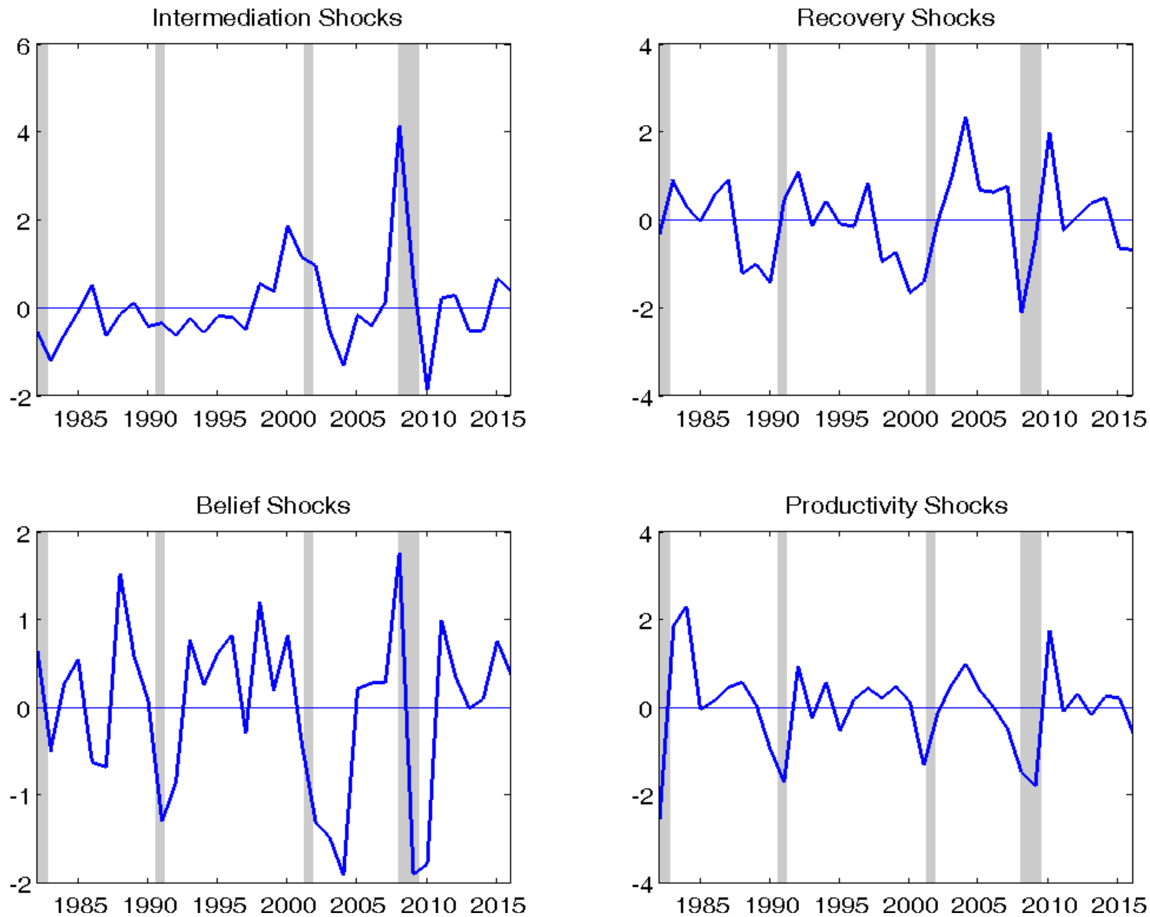


Figure 2: **Default Rate, Credit Spread, and Recovery Rate**



Note: Shaded areas are NBER dated recessions.

Figure 3: **Estimated Shocks at the Mean Levels**



Note: All shocks are normalized by their respective standard deviations. Shaded areas are NBER dated recessions.

Figure 4: **Impulse Responses to Recovery and Beliefs Shocks**

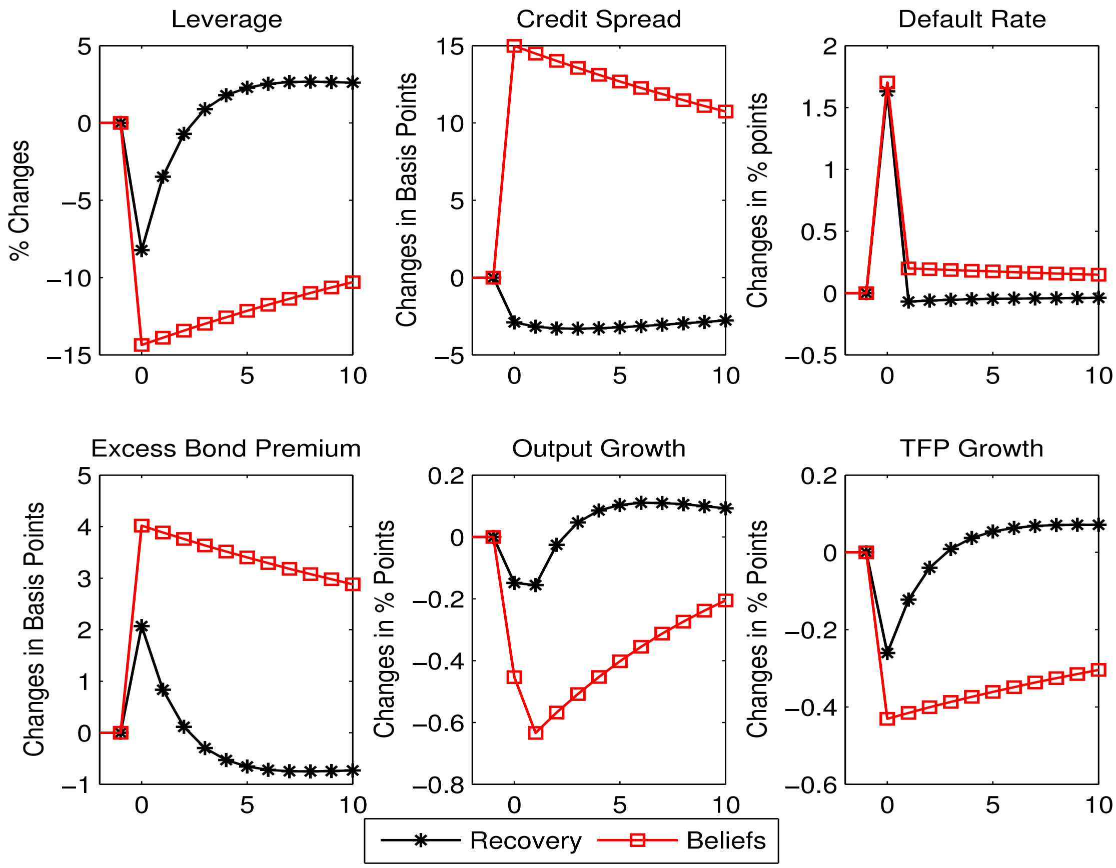


Figure 5: **Impulse Responses to Intermediation and Productivity Shocks**