Prescription Smothers Creativity in Mathematics Education

Richard Cowley (2010)

Abstract

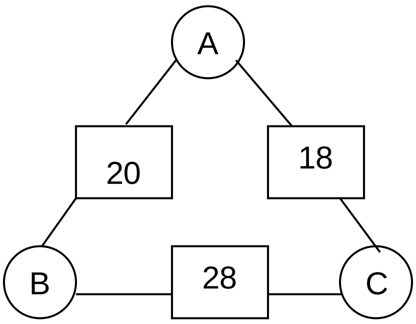
In this paper, I elaborate on a presentation I made to a Westminster Education Forum Seminar Reviewing the Maths Curriculum - part of the Westminster Education Forum National Curriculum Seminar Series. I draw on my experiences of posing one problem to children and young people in secondary schools and to well trained mathematicians in pre-service mathematics teacher education to illustrate how and argue that an overprescribed classroom practice cannot meet the stated aims of the UK National Curriculum (2007). Paulo Friere provides the inspiration for me to envisage an alternative practice.

An example of prescription

In the supplement of examples of the Key Stage 3 National Strategy Framework for teaching mathematics: Years 7, 8 and 9, (DfEE, 2001, henceforth referred to as the Framework) on page 123; under Algebra: Equations, formulae and identities; we have, "As outcomes, Year 8 pupils should, for example: …consolidate forming and solving linear equations with an unknown on one side." There is the following example.

In an arithmagon, the number in a square is the sum of the numbers in the two circles on either side of it.

In this triangular arithmagon, what could the numbers A, B and C be?



Over the last three years, I have presented this problem to over 100 trained mathematicians in pre-service mathematics teacher education. I present it as shown above; projected onto a whiteboard. I always demonstrate how arithmagons work by drawing an empty arithmagon, placing the numbers 1, 2 and 3 in the circles and then demonstrating how to get the numbers in the squares by adding: 1 + 2 = 3; 1 + 3 = 4; 2 + 3 = 5; writing the answers in the squares as I go. I then explain the problem set by pointing at the letters and numbers saying, "…so A + B = 20, A + C = 18 and B + C = 28; you have to work out the values of A, B and C."

A hypnotist might claim I am using autosuggestion here and I would be inclined to agree. Explaining the problem in this way, I am strongly prompting a particular method of solution that I know the students are familiar with. The problem is set in the context of pre-service secondary mathematics teacher education and solving simultaneous equations is nearly as infamous as Pythagoras' Theorem in that part of the school curriculum. It is no surprise, then that the vast majority (over 90%) choose to use simultaneous equations. A few students sheepishly use a trial and improvement method, which they are normally slightly embarrassed about and a few simply write down the answer by inspection; their intuitive feel for numbers being so powerful it proves to be quite difficult for them to explain how they did it. Here is the kind of method used by most.

Set up equations (1), (2) and (3) from the arithmagon:	A + C =	18	(1)
	A + B =	20	(2)
	B + C =	28	(3)
From (1):	C =	18 - A	(4)
From (2):	B =	20 - A	(5)
Substituting (4) and (5) into (3):	18 - A + 20 - A =	28	
Rearranging:	38 - 28 =	2A	
Solving:	5 =	A	(6)
Substituting (6) into (1):	5 + C =	18	
Solving:	C =	13	
Substituting (6) into (2):	5 + B =	20	
Solving	B =	15	
Therefore A = 5, B = 15 and C = 13			

Prescription

The Framework offers arithmagons as a context for forming and solving linear equations, but not simultaneous equations and this is a surprise to most students in pre-service mathematics teacher education. Some students are disappointed in themselves because they see the Framework method as more efficient. This is the method illustrated.

> Let x stand for the number in the top circle. Form expressions for the numbers in the other circles, (20-x) and (18-x). Then form an equation in x and solve it.

$$(20 - x) + (18 - x) = 28$$

$$38 - 2x = 28$$

$$2x = 10$$

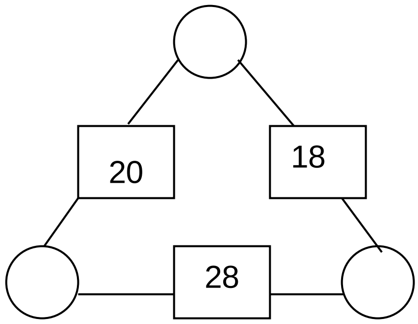
$$x = 5$$

So A = 5, B = 15, C = 13.

It is fair to say that the Framework does not suggest that arithmagons should only be solved in this way; however, allowing solution by simultaneous equations is not the issue. Even if learners are given the freedom to solve their own way; if later they are expected to adopt an apparently standard method, then their own mathematics is diminished; school maths becomes the oppressive practice of education that Freire (1970) described as an act of depositing rather than communicating, turning students into receptacles to be filled. He called this the “banking” concept of education. Oppression is a strong word with a long history of being used to describe the plight of humans suffering serious social injustice. How can a mathematics classroom in an economically developed country be described as oppressive? This idea may seem ridiculous or even cause insult to those who have suffered serious injustices such as slavery or torture. I mean oppressive in its literal and less rhetorical sense as authoritarian and weighing heavily on the spirit; the practice of school maths in the UK has been experienced in this way for some years (Boaler & Greeno, 2000), (Mendick, 2005) and (Buxton, 1981).

Prescription smothers creativity

As a school maths teacher, I discovered arithmagons and found them to be a rich source of fun with maths. I cannot recall the original source now but it was probably a colleague.



One pupil of mine presented with this problem without an explanation of how to solve it noticed that the difference between 20 and 18 is 2 so the difference between the base numbers must be 2 with the one on the left being greatest. For that pupil, the new problem was to find two numbers with a sum of 28 and difference of 2. The base-numbers were found but not the apex-number. For me, the fact that the apex-number was not evaluated was a lovely part of the explanation; that it was trivial to find the apex-number once the base-numbers were found was so clear to this problem solver there was no need to bother; a pure focus on the algebraic structure; a perspective missing from the systematic methods applied by trained mathematicians sometimes stunned by this elegance.

Pre-service teacher education

Through our dialogue on this problem, we are able to consider the nature of mathematical practice, critically examine the prestige of each method and notice some interesting connections between them. This facilitates a potentially revolutionary pedagogy of pre-service mathematics teacher education but does not reach into the school classroom. The teachers are liberated but not their pupils.

Conclusion

The prescribed and privileged methods (forming and solving a single linear equation or forming and solving simultaneous equations) are justified because they are assessed in high stakes testing at a key decision point in the lives of most of our children, when they take their GCSE maths exam at the end of year 11. For most learners, whether children or adults, this is the only reason for knowing how to do this. Back in 1982, the Cockroft Report (paragraph 77) noted the use of mathematics in everyday life.

> "It is not normally necessary to transform a formula; any form which is likely to be required will be available or can be looked up. Nor is it necessary to remove brackets, simplify expressions or solve simultaneous or quadratic equations, although algebra of this kind is sometimes encountered on courses at further education colleges. Solution of linear equations is required very occasionally." (Cockroft, 1982).

There has been no major change in this situation; however, according to our National Curriculum, there are other benefits to mathematics.

> "Mathematical thinking is important for all members of a modern society as a habit of mind …; and for personal decision-making…
>
> "Mathematics equips pupils with uniquely powerful ways to describe, analyse and change the world. It can stimulate moments of pleasure and wonder for all pupils when they solve a problem for the first time, discover a more elegant solution, or notice hidden connections…
>
> "Mathematics is a creative discipline… Mathematics has developed over time as a means of solving problems and also for its own sake." (DCSF, 2007, 139).

A "habit of mind" can take any form but which of the methods or practices of learning mathematics discussed here fits the bill? There is only one that involves "decision-making" or might develop a "habit of mind" to "analyse" or allows learners to "solve a problem." In the case of both prescribed methods, the problem has already been solved, an algorithm devised and then translated into an algebraic system. The presentation of the problem without a method of solution is the only road to mathematics as a "creative discipline."

My conclusion could be that the prescribed practice of learning methods for high stakes examinations should be replaced by a more liberated practice of opportunities to develop mathematics to solve problems but it is not. Paulo Friere has the answer.

> "Whereas the banking method directly or indirectly reinforces men's (sic) fatalistic perception of their situation, the problem-posing method presents this very situation to them as a problem." (Friere, 1970, 66)

So my conclusion is that there should be time for the girls and boys in our mathematics classrooms to critically examine the practices expected of them: the preparation for high stakes testing, the prescribed methods and the open-ended problem solving; and for them to transform their situation by making it the object of their analysis.

References

Boaler, J. and Greeno, J. G. (2000): "Identity, Agency and Knowing in Mathematics Worlds." In Jo Boaler (Editor) Multiple Perspectives on Mathematics Teaching and Learning. Ablex Publishing: Westport, CT. (pp 171-200)

Buxton, L. (1981): *Do you panic about maths?* Heinemann Educational Books.

Cockroft, W H (1982): *The Cockroft Report Mathematics Counts* HMSO. Viewed online at http://www.dg.dial.pipex.com/documents/docs1/cockcroft.shtml on 21 December 2009. Part of The History of Education in England www.dg.dial.pipex.com

DCSF (2007): Mathematics KS3 Programme of Study. In the National Curriculum. Qualifications and Curriculum Authority. URL: www.qca.org.uk/curriculum

Friere, P. (1970) *Pedagogy of the Oppressed.* First published by The Continuum Publishing Company. Published by Penguin Books in 1996.

Mendick, H. (2005) 'A beautiful myth? The gendering of being/doing 'good at maths'', Gender and Education, 17:2, 203 — 219