Reduced Complexity Maximum Likelihood Detector for DFT-s-SEFDM Systems

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***Abstract*—In this paper, we report on the design of a Complexity-Reduced Maximum Likelihood (CRML) detector for DFT-spread Spectrally Efficient Frequency Division Multiplexing (DFT-s-SEFDM) systems. DFT-s-SEFDM systems are similar to DFT-spread Orthogonal Frequency Division Multiplexing (DFT-s-OFDM) systems, yet offer improved spectral efficiency. Simulation results demonstrate that the CRML detector can achieve the same bit error rate (BER) performance as the ML detector in DFT-s-SEFDM systems at reduced computational complexity. Specifically, compared to a conventional ML detector, it is shown that CRML can decrease the search region by up to 2^M times where M denotes the constellation cardinality. Depending on parameter configuration, CRML can offer up to two orders of magnitude improvement in execution runtime performance. CRML is best-suited to applications with small system sizes, for example, in narrowband Internet of Things (NB-IoT) networks.**

***Keywords*—SEFDM, DFT-s-SEFDM, BER, maximum-likelihood (ML), reduced complexity detector, NB-IoT.**

I. Introduction

June 2018 saw the completion of stage 3 of Release 15 of the 3GPP specification which define the new set of 5th Generation (5G) communication standards [1]. Multiple technologies have been proposed to enable 5G communications including new waveforms for improving spectrum efficiency with respect to the well-established Orthogonal Frequency Division Multiplexing (OFDM). Examples of new waveform candidates include Faster-than-Nyquist Signaling (FTN), Generalized Frequency Division Multiplexing (GFDM) and Spectrally Efficient Frequency Division Multiplexing (SEFDM) amongst many others [2].

Here, we introduce a variant of SEFDM (SEFDM was first proposed in 2003 [3]), which we term Discrete Fourier Transform spread SEFDM (DFT-s-SEFDM). DFT-s-SEFDM offers similar benefits to Single Carrier Frequency Division Multiple Access (SC-FDMA), in terms of lower Peak-to-Average Power Ratio (PAPR) [4] when compared to their counterparts SEFDM and OFDMA, respectively. A special case of DFT-s-SEFDM termed Zero Headed DFT-s-SEFDM (ZH-DFT-s-SEFDM) was first proposed in 2017 [5] and employs a Zero Forcing (ZF) detector. While an increase in the number of zeros inserted in the head improves the performance of the ZF detector, this comes at the expense of reduced bandwidth efficiency and increased PAPR.

The primary challenge in SEFDM systems is the self-created inter-carrier interference (ICI), as a result of reducing the spacing between the subcarriers beyond the orthogonality limit defined for OFDM systems. To maintain an acceptable bit error rate (BER) at tangible computational complexity, a number of techniques have been proposed for recovering SEFDM signals with Sphere Decoding (SD) offering near Maximum Likelihood (ML) performance [6].

More recently, Huang et al. [7] achieved rates of up to 20 Gb/s in optical SEFDM using a cascaded binary-phase-shift-keying iterative detection (CBID) algorithm in conjunction with a fixed sphere decoder (FSD). At the same time, Rashich et al. [8] designed an FFT-based Trellis Receiver to recover SEFDM signals having hundreds of subcarriers.

Reduced complexity ML techniques have been applied in Multiple Input Multiple Output (MIMO) systems [9] [10]. The work by Kim et al. [10] uses a minimum mean square error (MMSE) based pre-detection approach to attain quasi-optimal BER performance. This is achieved by reducing the search space via separation of the MIMO channel into distinct sub-channels.

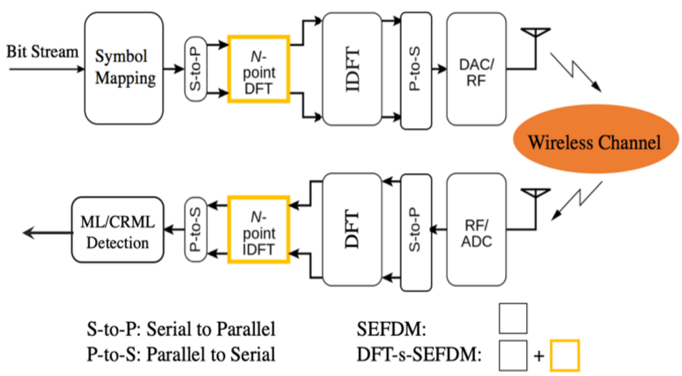

Fig. 1. High-level block diagram of SEFDM and DFT-s-SEFDM systems.

The contributions of this paper may be summarized as follows:

- Our system model considers the general case of DFT-s-SEFDM systems, rather than the specific ZH-DFT-s-SEFDM scheme.
- Inspired by the work in MIMO systems, we propose a new Complexity-Reduced ML (CRML) detector, which can achieve the same BER performance as the ML detector at significantly reduced computational complexity.

This paper is organized as follows: Section II describes the theory and model for CRML. Section III presents results acquired through simulation and compares the performance of CRML to the conventional ML detector. Finally, Section IV concludes this paper.

II. SYSTEM MODEL AND THEORY

A simplified transceiver architecture of SEFDM and DFT-s-SEFDM systems is depicted in Fig 1. In SEFDM, the message signal after SEFDM modulation may be expressed as [11]

$$s(t) = \frac{1}{\sqrt{T}}\sum_{k=0}^{N-1} S_k e^{jw_k t},\ \ t \in [-\infty, +\infty] \quad (1)$$

where s_k denotes the k^{th} modulating symbol, w_k represents the k^{th} angular carrier frequency ($w_k = 2\pi f_k$), N denotes the number of subcarriers in a single SEFDM symbol, and T denotes the symbol period. By setting $t = nT_s$, where T_s denotes the sampling period and n denotes the sampling index varying from 0 to $+\infty$, and $w_k = 2\pi k D_f \alpha$, where $D_f = f_k - f_{k-1}$ represents the frequency spacing, α denotes the bandwidth compression varying from 0 to 1, the discrete SEFDM signal is then given by

$$s(n) = \frac{1}{N}\sum_{k=0}^{N-1} S_k e^{\frac{j2\pi k n \alpha}{N}} \quad (2)$$

When $n = 0$, the first sample $s(0)$ becomes

$$s(0) = \frac{1}{N}\sum_{k=0}^{N-1} S_k e^{j0} = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} S_k \quad (3)$$

On the contrary, in DFT-s-SEFDM, the symbols undergo DFT spreading prior to SEFDM modulation. Hence, the discrete message signal after DFT spreading is given by

$$s'(n) = \sum_{k=0}^{N-1} S_k e^{\frac{-j2\pi k n}{N}} \quad (4)$$

Subsequently, after SEFDM modulation, the signal becomes

$$s(n) = \frac{1}{N}\sum_{k=0}^{N-1} s'_k e^{\frac{j2\pi k n \alpha}{N}},\ (s'_k = s'(k)) \quad (5)$$

The first sample $s(0)$ is then equal to

$$s(0) = \frac{1}{N}\sum_{k=0}^{N-1} s'_k e^{j0} = \frac{1}{N}\sum_{k=0}^{N-1} s'_k$$

Expanding the term $s(0)$ gives:

$$s(0) = \frac{1}{N}(s'_0 + s'_1 + \cdots + s'_{N-2} + s'_{N-1})$$

$$= \frac{1}{N}\left(s'(0) + s'(1) + \cdots + s'(N-2) + s'(N-1)\right)$$

$$= \frac{1}{N}(\sum_{k=0}^{N-1} S_k e^{\frac{-j2\pi k 0}{N}} + \sum_{k=0}^{N-1} S_k e^{\frac{-j2\pi k 1}{N}} + \cdots +$$

$$\sum_{k=0}^{N-1} S_k e^{\frac{-j2\pi k(N-2)}{N}} + \sum_{k=0}^{N-1} S_k e^{\frac{-j2\pi k(N-1)}{N}})$$

$$= \frac{1}{N}(NS_0 + S_1\sum_{i=0}^{N-1} e^{\frac{-j2\pi i * 1}{N}} + S_2\sum_{i=0}^{N-1} e^{\frac{-j2\pi i * 2}{N}} +$$

$$\cdots + S_{N-2}\sum_{i=0}^{N-1} e^{\frac{-j2\pi i *(N-2)}{N}} + S_{N-1}\sum_{i=0}^{N-1} e^{\frac{-j2\pi i *(N-1)}{N}})$$

Using the result of the geometric sequence:

$$\sum_{i=0}^{N-1} e^{\frac{-j2\pi i * k}{N}} = \frac{1*(1-e^{-j2\pi * k})}{1-e^{\frac{-j2\pi * k}{N}}} = \frac{1*(1-1)}{1-e^{\frac{-j2\pi * k}{N}}} = 0$$

leads to: $s(0) = \frac{1}{N} * NS_0 = S_0$ (6)

Equation 6 proves that the first sample $s(0)$ is equal to the first message symbol S_0 and independent of the frequency spacing. Therefore, the design idea is to recover the first sample and then use this known information to decrease the search region of the ML detector. Each message symbol has 2^M possible values, where M denotes the constellation cardinality. Furthermore in this work, we assume that Quadrature Amplitude Modulation (QAM) is employed. Hence, in DFT-s-SEFDM systems, the number of all possible complex values for $s(0)$ is given by

$$P = 2^M \quad (7)$$

If we denote S_0 by $A + jB$ then A and B can each take one of $2^{\frac{M}{2}}$ possible values given by

$$V = \{V_1, V_2, \ldots V_{2^{M-1}}\} \rightarrow$$

$$V = \{-2^{\frac{M}{2}} + 1, -2^{\frac{M}{2}} + 3, \ldots 2^{\frac{M}{2}} - 1\} \quad (8)$$

Suppose that the signal is transmitted through an Additive White Gaussian Noise (AWGN) channel. The first sample at the receiver may then be expressed as

$$y(0) = s(0) + n_0 = A + Bj + n_0 = C + Dj \quad (9)$$

where n_0 denotes the noise power. If we denote the recovered signal by $Y(0)$, define an index $I \in [0{:}1{:}2^{M-1}+1]$ and a function O where

$$O(I) = \begin{cases} +\infty, & I = 0 \\ V_I, & I \in [1{:}1{:}2^{M-1}] \\ -\infty, & I = 2^{M-1}+1 \end{cases} \tag{10}$$

Then

$Real\big(Y(0)\big) = O(I)$, for values of I that satisfy

$$\frac{O(I-1)+O(I)}{2} > C \geq \frac{O(I)+O(I+1)}{2}$$

$Imag\big(Y(0)\big) = O(I)$, for values of I that satisfy

$$\frac{O(I-1)+O(I)}{2} > D \geq \frac{O(I)+O(I+1)}{2} \tag{11}$$

In a conventional ML detector, all the possible transmitted data streams are tested and the one that has the maximum probability of correctness is selected [10]. There are 2^{M*N} possible data streams in total and in matrix format these data streams are given by

$$\overrightarrow{D_n} = [S_{n,1} \cdots S_{n,N}]$$

$$\boldsymbol{D} = \begin{bmatrix} \overrightarrow{D_1} \\ \vdots \\ \overrightarrow{D_{2^{M*N}}} \end{bmatrix} = \begin{bmatrix} S_{1,1} & \cdots & S_{1,N} \\ \vdots & \ddots & \vdots \\ S_{2^{M*N},1} & \cdots & S_{2^{M*N},N} \end{bmatrix} \tag{12}$$

where $\boldsymbol{D}$ represents all the possible transmitted message signals, $S_{a,b} \in S_n$, and a, b represent the row and column of matrix **D**, respectively.

In the CRML detector, the data streams that satisfy (6) are tested. This means that the vectors that satisfy the following equation are selected

$$\overrightarrow{D_m}(1) = S_{m,1} = Y(0) \tag{13}$$

In DFT-s-SEFDM systems, Matrix **D** contains 2^{M*N} possible vectors $\overrightarrow{D_n}$. Using the conventional ML detector, all the 2^{M*N} possible vectors need to be tested. However, using the CRML detector, only $2^{M*N}/2^M = 2^{M*N-M}$ possible vectors can satisfy (13) and hence only these vectors are tested. Thereby, the search region is decreased.

In OFDM systems, parallel data symbols are modulated at different subcarrier frequencies. The summation of multiple subcarriers at a particular instance in time may lead to high peak to average power ratio (PAPR). Given a time domain sequence s = {s[n]}, its PAPR will be equal to

$$PAPR = \frac{max\,(|s|^2)}{E\,(|s|^2)} \tag{14}$$

where $E(\cdot)$ denotes the expected value.

The complementary cumulative distribution function (CCDF) is applied to analyze the PAPR. The CCDF represents the probability of PAPR being higher than a specified value $PAPR_0$, that is

$$CCDF = Prob(PAPR > PAPR_0) \tag{15}$$

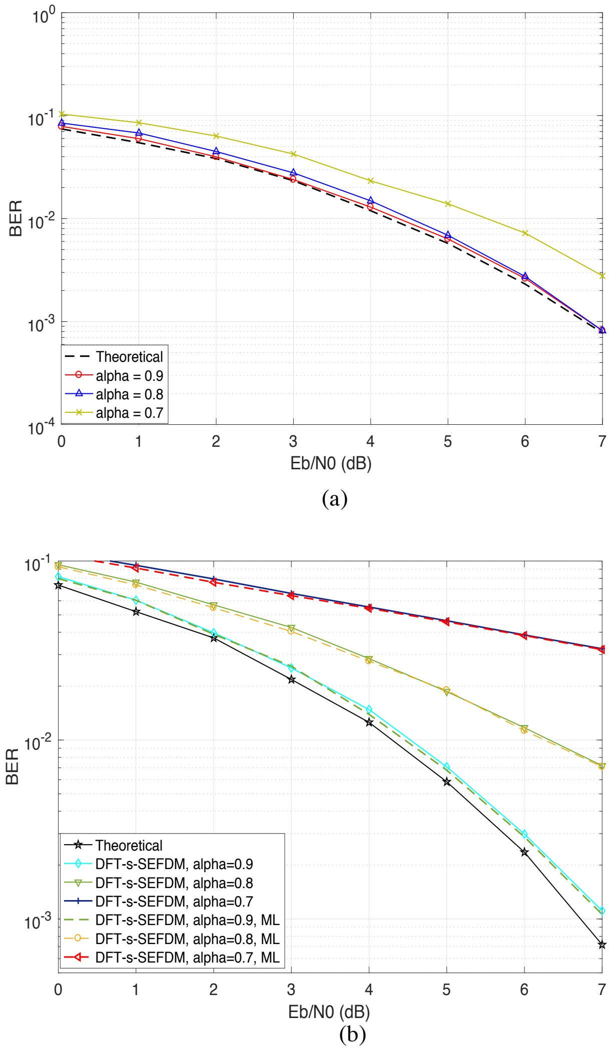

Fig. 2. BER for a) SEFDM systems, b) DFT-s-SEFDM systems in an AWGN channel (N = 4, 4-QAM).

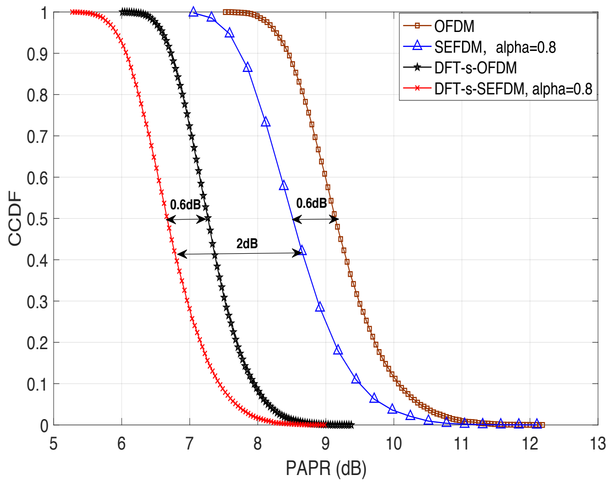

Fig. 3. PAPR comparison of the different techniques (N=1024, 16-QAM).

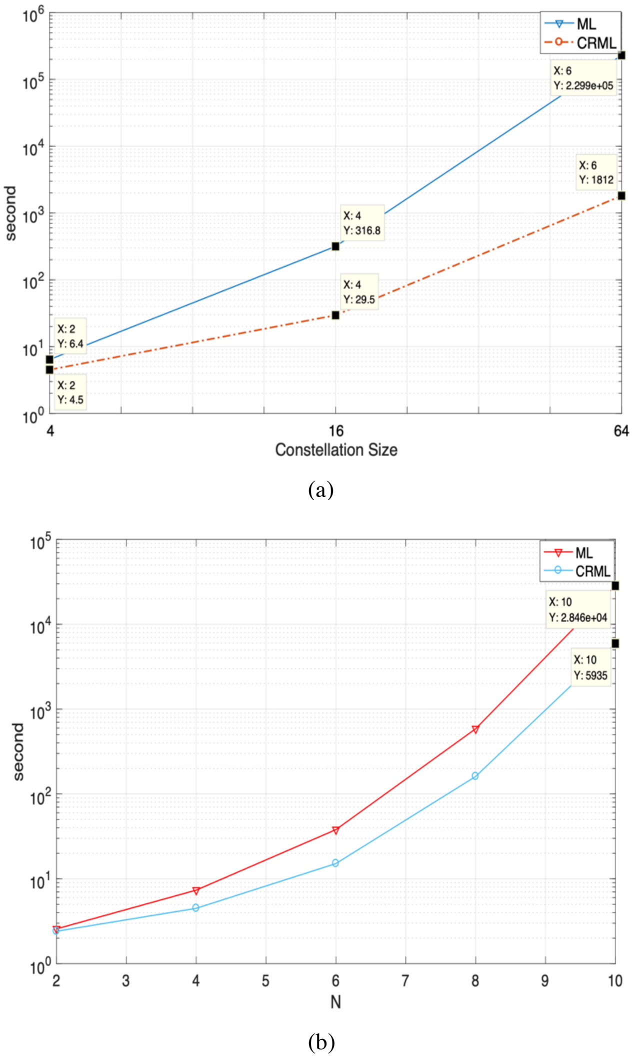

Fig.4 Runtime for CRML and ML detectors to recover the same number of bits for a) different constellation cardinalities (α = 0.8, N = 4), b) different number of subcarriers (α = 0.8, 4-QAM).

III. Simulation Results

Simulation was carried out assuming an AWGN channel with QAM modulation for different system sizes and bandwidth compression factors.

Fig. 2(a) shows that for bandwidth compression factors up to 20% ($\alpha \geq 0.8$), SEFDM has commensurate BER performance to OFDM when ML is employed. Fig. 2(b) provides evidence that the proposed CRML detector can achieve the same BER performance as the conventional ML detector in DFT-s-SEFDM systems.

In terms of PAPR, Fig. 3 shows that DFT-s-SEFDM provides the best performance offering a reduction of almost 2 dB with respect to SEFDM and 0.6 dB with respect to DTF-s-OFDM, for a large number of sub-carriers (N = 1024) and typical bandwidth compression (α = 0.8).

To evaluate the computational complexity of the CRML detector, we compare its execution runtime to that of the ML detector. The computer used to run the simulations was a MacBook Pro (MJLT2CH/A) and the results are plotted in Fig. 4. Fig. 4(a) compares the runtime for different constellation schemes namely 4QAM, 16QAM and 64QAM. From Fig. 4(a), it is clear that CRML offers performance gains with increasing M. For M = 6 (64-QAM), CRML is nearly 127 times (22990/1812 ≅ 127) faster compared to ML, that is to say two order of magnitude faster.

Fig. 4(b) shows the runtime for different system sizes by averaging over five random runs. Once again, CRML executes faster compared to conventional ML with this improvement in speed increasing in proportion to the size of the system.

To conclude, it is worth noting that the proposed CRML detector would be well-suited to applications requiring a small number of subcarriers. A topical example are narrowband Internet of Things (NB-IoT) systems, which only require up to 12 subcarriers and could use DFT-s-SEFDM modulation to further increase transmission speed and prolong battery life [12] [13].

IV. Conclusion

In this paper, a reduced complexity ML detector for DFT-s-SEFDM systems is proposed. DFT-s-SEFDM reduces PAPR by almost 2 dB compared to SEFDM and over 2 dB compared to OFDM. Hence, using DFT-s-SEFDM in conjunction with CRML, considerable gains in overall system performance can be achieved.

We show that by pre-detecting the first sample of the transmitted time-domain signal, CRML can decrease the search region by up to 2^M times with respect to conventional ML detection, thereby reducing computational complexity.

The performance of CRML is evaluated through simulation considering an AWGN channel and assuming that QAM modulation is employed. Results demonstrate that CRML can provide the same BER performance as ML detection when considering DFT-s-SEFDM systems. In terms of runtime performance, CRML overall executes faster compared to ML. In particular, for large constellation sizes, such as 64-QAM, CRML can offer up to two orders of magnitude improvement in execution speed.

It is well known that ML has non-polynomial (NP-hard) complexity. Future work will therefore look at combining the CRML concept with other reduced complexity detection techniques, for example, sphere decoding.

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