Alan Wilson - Contributions to Research on Population and Migration

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ABSTRACT

This article reviews Alan Wilson's research on population and migration in the 1970s and the 2010s, which supplements his principal contribution – mathematical modelling of urban and regional systems. In the 1970s, drawing on input-output models of economies and working with Philip Rees, Wilson established the accounting basis for Andrei Rogers' multi-regional projection model, adding international migration. Innovative methods were developed to complete demographic accounts, where there were data gaps. In the 2010s, working with Adam Dennett, Wilson systematised methods for estimating migration flows between regions in Europe, employing his family of spatial interaction models. The key aim of both research strands was to ensure that no information was ignored to ensure consistency in population and migration models. The influence of Wilson's contributions to research on population and migration is traced through a survey of subsequent research.

KEYWORDS

The work of Professor Sir Alan Wilson; models of urban and regional systems; demographic accounts; demographic projection models; migration estimation; forecasting migration

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Introduction

This paper is part of a special issue of *Interdisciplinary Science Reviews* dedicated to the research career of Professor Sir Alan Wilson. Alan Wilson has a background in mathematical physics which he has married with geographical skills to pursue, over five decades, an investigation into the principles under-pinning urban and regional systems and their evolution over time. His 1974 book *Urban and Regional Models in Geography and Planning* (Wilson 1974a) can be read as a blueprint for this research effort and his 2016 book on *Global Dynamics* (Wilson 2016) as the most recent output. The aim of this review is to describe and evaluate contributions in two sections of this grand scheme, namely demographic and migration models. The first author (Rees) collaborated with Alan Wilson in an early stage of his academic career on the first topic while the second author (Dennett) collaborated at a later stage on the second topic.

These were two periods in Alan's career in which he had time to work outside his wider responsibilities as Vice-Chancellor or leader of national initiatives such as the Turing Institute. Alan was an excellent collaborator and mentor, expert in planning research and outputs, asking the key questions and enthusing his co-authors to do innovative work. There was a fruitful division of labour in which Alan focussed on frameworks and mathematical models, while his collaborators concentrated on supplying the models with data and developing the computational code to generate results, which were jointly interpreted and written for publication.

Figure 1 helps situate the work on population and migration in the wider framework of urban and regional systems. The figure expands a system diagram in Wilson (1974) to represent two points in time (the LH and RH columns of modules) connected by a set of change processes (the middle column). The panels represent the demand side (top) and supply side (bottom) of an urban and regional system. The figure shows the way intra-urban structure and activities are related to city wide attributes and to how cities themselves are linked to regional contexts (e.g. the urban commuting field) and national structures (e.g. north-south divides, the openness of the economy to world trade). The figure is organized into shaded panels. The top and bottom panels refer to the physical and natural environment. The second and third panels (buff) represent the demographic side of the system, people living in residential dwellings. The fifth and sixth panels (grey) represent the economic system of jobs and workplaces. The daily interactions within nations, regions, cities and (intra-urban) zones between demand and supply sides are shown in the middle panel (pink). The interactions take place over transport and communications networks and involve person trips or goods deliveries or online transactions. The vertical arrows show the connections between modules. The environment connects to all demographic and economic modules.

[Figure 1 here]

Key to the demand side of urban systems are people, grouped into households or communal establishments. The supply side consists of individual enterprises and private and public organizations which supply goods and services to the population and employ them in activities. The two sectors of the urban system are connected by transport networks over which people make trips and communication networks over which messages for organizing transactions are sent.

In Figure 1 the middle column of modules connects population or economic structures at the start of a time interval with those at the end. The change module contains the *demographic* processes of fertility and mortality with *migration* specifically identified. The next section of the paper considers Alan Wilson's thinking about urban and regional systems and his work on input-output models. The latter informed his understanding of demographic processes, which are discussed in the third section of the paper. His work on migration models is reviewed in the fourth section. The final section of the paper assembles take-home messages from Alan Wilson's work.

The Context for the Work on Population and Migration

In this section, we provide the context for Alan Wilson's demographic contributions by discussing his agenda for general urban and regional models and his elucidation of input-output models. Throughout his research career, Alan Wilson has looked at the big picture. He has asked "how can we represent and model the structure and dynamics of whole systems". Wilson (1974) sets out a blueprint for a comprehensive model of how cities and regions function and might evolve and how models (or sub-models) might inform planning of new infrastructure. In Part III of the book, he describes Demographic Models (Chapter 7), Models of Urban and Regional Economies (Chapter 8), Transport Models (Chapter 9), The Spatial Distribution of Activities (Chapter 10) and Comprehensive Models (Chapter 10). Elaborating, extending and exploring this vision of the urban and regional system has been his life-long research agenda.

Here, the models covered in Chapter 8 are briefly described because they informed Wilson's contributions to research on Population and Migration. Isard and colleagues wrote in the 1950s on regional economic systems and inter-regional input-output analysis (Isard et al. 1960). This work generalised the work of Leontief (1936, 1966) from national to regional systems. Wilson (1974a) sets out an accessible account of the input-output model which was included in his mathematics text book (Wilson and Kirkby 1980). In the early 1990s Wilson collaborated with doctoral student Yu-xian Jin and colleague Christine Leigh to develop input-output models for multi-spatial urban systems (Jin and Wilson 1991, Jin and Wilson 1993, Jin, Leigh and Wilson 1991, Wilson and Jin 1992). These models are positioned in a structure that resembles a Russian *matryoshka* (nesting) doll. In its most elaborate form, the model comprises zones nesting inside cities, which nest inside regions alongside other cities and rural areas, which in turn nest inside a country. The model contains an economic input-output model (MULIO), a socio-demographic model (SOCDEM) and a housing stock model (HOSTOC), all linked. More recently, Wilson has worked with colleagues at the Centre for Advanced Spatial Analysis (CASA) at University College London to expand the input-output model to a set of 35 countries (Levy et al. 2014, Levy et al. 2016). Model inputs are drawn from the World Input-Output Database (WIOD) constructed by Timmer et al. (2015). The value of the inter-country input-output table is illustrated for the automobile industry, showing its growing dependence on supply and demand chains connected with China's growing consumer market and car production. In 2018, Germany experienced two quarters of GDP decrease, attributed in part to the slowdown of China's car market, alongside the home-grown challenge of replacing diesel powered cars. Countries and industries across the globe are highly connected.

Table 1 presents an input-output table for a single country, with definitions of variables and subscripts. The variables indicate the money value of transactions. The product, X, for a region and an industrial sector is supplied as intermediate inputs to industry sectors, satisfies final demand (mainly households), is exported as product or invested outside the country. The input-output table must balance, so that the product of a sector m must be sum of intermediate inputs to all sectors of the products of sector *m*, final demand for sector *m* products, exports of sector *m* products and investments of sector m capital outside the country:

$$\sum_{n=1}^{s} Z_{mn} + Y_m + E_m + M_m = X_m \qquad (1).$$

If all inputs in a column, including labour can be measured using the same units, then the column elements sum to total inputs to a sector in a region. To turn this into a model that can predict impacts of change in final demand, we need to convert cell entries in Table 1 into technical coefficients, a_{mn}, which measure the amount of product of sector *m* needed per unit product of sector *n*:

$$a_{mn} = Z_{mn}/X_n \qquad (2).$$

[Table 1 here]

The first term in equation (1) can be replaced by a model, the coefficients multiplied by the product of sector n, so that accounting equation (1) becomes:

$$\sum_{n=1}^{s} a_{mn}X_n + Y_m + E_m + M_m = X_m \qquad (3).$$

Re-arranging, the equation can be written as:

$$\sum_{n=1}^{s}(\delta_{mn} - a_{mn})X_n = (Y_m + E_m + M_m) \qquad (4)$$

where δ_{mn} is a Kronecker delta that equals 1 if *m* = *n*, but 0 otherwise. Given knowledge of the technical coefficients and assuming constancy over time, we can work out the required inputs for a given set of final demand plus exports and outward investments. Defining Y'_m as domestic plus overseas final demand, $(Y_m + E_m + M_m)$, and arranging terms in matrix notation where $\boldsymbol{I}$ is the identity matrix and $\boldsymbol{A}$ is an *s*×*s* matrix of technical coefficients, $\boldsymbol{x}$ is a vector of total product and $\boldsymbol{y}'$ a vector of final plus external demand, equation (4) can be written as

$$(\boldsymbol{I} - \boldsymbol{A}) \quad \boldsymbol{x} = \boldsymbol{y}' \qquad (5).$$

By pre-multiplying by $(\boldsymbol{I} - \boldsymbol{A})^{-1}$, we obtain a solution for the output across all sectors to satisfy final plus external demand by sector:

$$\boldsymbol{x} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{y}' \qquad (6).$$

All quantities in the input-output table must be measured using the same units. Money satisfies this condition. Wilson (1974, pp.114-115) shows how prices for units of product can be introduced into the system. It is then possible to analyse the consequences of increased or decreased demands on the product needed for industrial sectors or to compute the consequences of trade shocks such as a new post-Brexit trading relationship between the UK and EU or to assess the result of cartel-imposed quotas on key commodities such as oil. However, forecasting changes in the technical coefficients is challenging.

Input-output tables are easiest to implement at country scale where cross-border commodity, service and capital flows into and out the country are monitored. The problem of where product of a firm should be assigned across a set of activity locations, present when developing regional input-output tables, can be avoided. However, Table 2 shows that there have been many attempts at constructing input-output tables for different spatial systems: the single region, multi-regional systems, spatially nested systems or multi-country tables, to each of which Alan Wilson has contributed. We note too that input-output analysis is being increasingly used in environmental analysis to project future greenhouse gas and air pollution outputs (Shmelev 2012).

[Table 2 here]

Contributions to Demography

Alan Wilson's contribution to demography was to demonstrate that standards of consistency and comprehensiveness used in national economic accounts (Stone and Stone 1961), national input-output tables (Leontief 1966) and regional input-output models (Isard et al. 1960) could be combined with multi-regional population models (Rogers 1968) to produce better population projection models. Rogers had built his first multi-regional projection models with no immigration or emigration terms, although he did later add these later. Wilson realised from his knowledge of economic accounts and input-output analysis that closed systems like this were unrealistic. Always a systems thinker, he wanted to do something better on the population side. In 1970 Wilson moved to take up a chair at the University of Leeds and was joined by Rees as a new junior lecturer. He invited Rees to work with him to merge these frameworks to develop better models of population change. Rees supplied ideas for estimating the variables not (then) available in official demographic statistics. The under-pinning philosophy was that if the model demanded a variable for which data were not available in current population statistics, the researcher should make the best guestimate possible, using plausible assumptions or borrowing information from higher spatial units.

The development of population accounts

Between 1973 and 1976, Wilson and Rees wrote six joint papers on demographic accounting and modelling (Rees and Wilson 1973, 1975a and 1975b; Wilson and Rees 1974b and 1976), knowledge from which was synthesized and improved in a 1977 book, *Spatial Population Analysis*. Theory was illustrated numerically in the book using a mythical population system, called Middle Earth after the setting for Tolkien's Trilogy, *The Lord of the Rings*. Rees and Wilson (1973) defined the basic form of demographic accounts, their relationship with available statistics and methods for estimating missing transitions from vital event counts. Wilson and Rees (1974b) added age-sex groups to the accounting framework. This was a vital element for projecting populations because of heterogeneity of fertility, mortality and migration across age (the core of standard demography). Flows were divided by populations to create rates. These rates were multiplied by populations in projections to generate forecast flows.

Note that the variables used in these projection computations depended on the type of demographic accounts developed. Two types were recognized: transition-based accounts or movement-based (Rees 1985, Rees and Willekens 1986). Rees and Wilson (1975b) linked demographic accounts with classic life tables and multi-regional life tables. The latter were being developed at the time by Rogers (1973) and Rogers and Ledent (1976). Finally, Rees and Wilson (1975a) compared the Rees-Wilson framework with the model specifications of Rogers (1968) and Stone (1965, 1971). Bridges were built between the Rogers notation for population change, in which the variables occupying tables, matrices and vectors were made explicit, and the work of Stone, which used a very different matrix and vector notation without explicit representation of contents. In effect, Rees and Wilson wrote a Rosetta tablet to enable demographers and economists to converse. The meaning of open and closed accounts as used by Stone and as used in the Wilson-Rees accounts was clarified.

The accounting framework for representing population change, as developed by Wilson and Rees, is set out in Table 3, along with variable and subscript definitions. Table 3 contains variables which are classified by states at the start and end of a time interval which are termed "transitions". Later collaborations between the "Rogers team" and the "Wilson team" (Ledent 1978, Ledent and Rees 1980, Ledent and Rees 1986, Rees 1985, Rees and Willekens 1986) identified an alternative accounting framework which uses event variables, described as movements, shown in Table 4. Both frameworks use regional deaths and births counts or estimates. Transition accounts employ migration data derived from censuses or surveys, which ask retrospective questions about the previous residence of survivors. Non-surviving migrants are absent from official data and must be estimated (Rees and Wilson 1977). Movement accounts employ counts of migration events, captured in population or administrative registers. Both accounting frameworks require international migration data which derive from a variety of sources and are more prone to error. The variables in the diagonal of the LH and RH sub-matrices of Table 3 are people who stay in the same region, either surviving or not. The R variables in the diagonal of Table 4 are residual terms required to balance the accounts, rather than person events. Which framework should be used in compiling population estimates or projections? The answer is that both are suitable. The choice will depend on available data, with European countries with good population and event registers preferring the movement framework with extra-European countries using the transition framework, as censuses or surveys are the best sources of information in the absence of good migration registers. Some Nordic countries are blessed with complete life history data on their population and have the luxury of being able to implement either type of demographic account.

[Table 3 about here]

[Table 4 about here]

For a set of population change tables to constitute demographic accounts, it is necessary that the component terms sum to the known fixed totals. The accounting equations are presented here for a typical period-cohort. The equations for the new-born period cohort (not presented here) differ slightly in that the "initial population" consists of babies born in the time interval rather than the population at the start of the time interval.

The flows of people out of a region *i* must total the start population. For the transition accounts, the constraint equation is:

$$T_x^{eisi} + T_x^{eisj} + T_x^{eisr} + T_x^{eidi} + T_x^{eidj} + T_x^{eidr} = T_x^{ei+} \quad (7),$$

and, from the movement accounts, the constraint equation for the start population is:

$$R_x^i + M_x^{ij} + E_x^i + D_x^i = P_x^{Si} \quad (8).$$

For final (existing) population, from the transition accounts, the constraint equation is:

$$T_x^{eisi} + T_x^{ejsi} + T_x^{ersi} = T_x^{e+si} \quad (9),$$

and, from the movement accounts, the constraint equation is:

$$R_x^i + M_x^{ji} + I_x^i = P_x^{Fi} \quad (10).$$

Equivalent equations can be written for region *j*, the rest of the country, and region *r*, the rest of the world. This framework can be expanded to many regions within a country or to many countries within the world.

Note that the constraint variables are the same in both types of accounts, although represented using different notations. Some variables in the account tables must be estimated. Examples are migrants who die during the time interval, e.g. T_x^{eidj} in the transition accounts or the residual balance term in the movement accounts, e.g. R_x^i. If the constraint equations are not satisfied immediately, adjustments can be made using iterative proportional fitting.

From demographic accounts to population projection models

How are demographic accounts used to support population projections? Figure 2 shows the steps needed to construct a demographic projection model. The first step is to identify the system to be modelled, which will depend on the purpose of the projection exercise. It is necessary to choose the spatial units, the ages and whether a single sex or two sex model is appropriate in order to construct a cohort-component model. The time intervals and age intervals must match. It is useful also to add additional classifications of the population, depending on how much influence these have on demographic behaviour. Lutz et al. (2014) make a compelling case for the inclusion of the population's future educational attainment. Rees et al. (2017a, 2018) demonstrate what difference ethnicity makes in a UK projection. Once the system of interest is specified, it is necessary to assemble the necessary component measures, which may be available from national or international statistical databases or estimates must be made (Lutz et al. 2014; Rees et al. 2017a). Then a decision must be taken about whether the component estimates are better used in transition or movement accounts. Usually, not all component measures align with one schema or the other, and "bridging" estimates must be made. An example is the need to adjust UK internal migration data between local authorities from a transition measure generated in the NHS register analysis to a movement measure by multiplication of a ratio derived from a legacy database which holds both measures.

[Figure 2 about here]

The population accounts contain flow data (transitions or movements), vital events (deaths, births) and stock data (populations). These must be turned into intensities, either transition proportions or occurrence-exposure rates. Transition flows are turned into proportions, *t*, by division by the origin region populations. For example, the proportion of people existing in region *i* who migrate to region j and survive there at the end of the time interval is:

$$t_x^{eisj} = T_x^{eisi} / T_x^{ei++} \quad (11),$$

and similarly, for persons who do not survive:

$$t_x^{eidj} = T_x^{eidj} / T_x^{ei++} \quad (12).$$

The equivalent proportions for new-borns involve division by the total of births rather than population. Transition proportions must add to 1, by definition, if the accounts are properly specified.

Occurrence-exposure rates are defined for internal migration as:

$$m_x^{ij} = M_x^{ij} / P_x^{ARi} \quad (13).$$

For emigration the rate is defined thus:

$$e_x^{i} = E_x^{i} / P_x^{ARi} \quad (14).$$

For mortality the rate is defined as:

$$d_x^{i} = D_x^{i} / P_x^{ARi} \quad (15).$$

The denominator, P_x^{ARi}, is an estimate of the person-time of exposure or population-at-risk. Rates of immigration, effectively rates of emigration from the rest of the world, can be defined using the population of the rest of the world as P_x^{ARr}. However, this is rarely done in national projections because a rapidly growing rest-of-the world population multiplied by a constant immigration rate leads to huge increases in immigration over time.

To be useful in projection models, occurrence-exposure rates must be converted into transition probabilities. The population-at-risk can be estimated, assuming linear change in the time interval, as the average of start and finish populations:

$$P_x^{ARi} = ½\, (P_x^{Si} + P_x^{Fi}) \quad (16).$$

However, we do not know the value of P_x^{Fi} at the start of the calculation. Two approaches are possible: we start by setting the population-at-risk equal to the start population, compute the first approximation of the final population and then substitute this new value into equation (16). The calculation continues iteratively until the change in final population is smaller than a threshold difference, when the iteration is terminated. This is the method originally set out in Rees and Wilson (1973). An alternative method was developed by Rogers and Ledent (1976) employing a matrix format and a matrix inverse, which has the advantage of avoiding iteration.

Movement intensities are employed in equations to project the regional populations. Here we follow the exposition of Willekens and Drewe (1984). The end of interval population of region i is given as:

$$P_x^{Fi} = P_x^{Si} - [\sum_{j \neq i} m_x^{ij} + e_x^i + d_x^i] \times ½[P_x^{Si} + P_x^{Fi}] + \sum_{j \neq i} m_x^{ji} \times ½[P_x^{Sj} + P_x^{Fj}] + I_x^i \quad (17).$$

Multiplying through we obtain:

$$P_x^{Fi} = P_x^{Si} - ½[\sum_{j \neq i} m_x^{ij} + e_x^i + d_x^i] P_x^{Si} - ½[\sum_{j \neq i} m_x^{ij} + e_x^i + d_x^i] P_x^{Fi}$$
$$+ ½ \sum_{j \neq i} m_x^{ji} \; P_x^{Sj} + ½ \sum_{j \neq i} m_x^{ji} \; P_x^{Fj} + I_x^i \quad (18).$$

Re-arranging all final population terms on the LH side and start population terms on the RH side, we obtain:

$$[1 + ½ \left[\sum_{j \neq i} m_x^{ij} + e_x^i + d_x^i \right]] P_x^{Fi} - ½ \sum_{j \neq i} m_x^{ji} \; P_x^{Fj} =$$
$$[1 - ½[\sum_{j \neq i} m_x^{ij} + e_x^i + d_x^i]] P_x^{Si} + ½ \sum_{j \neq i} m_x^{ji} \; P_x^{Sj} + I_x^i \quad (19.)$$

This equation for a region i can be written in matrix format to apply to N regions by defining a matrix ***M*** as follows (Rogers and Ledent 1976, Willekens and Drewe 1984):

$\boldsymbol{M_x}$ =

$$\begin{bmatrix} m_x^{11} & -m_x^{21} & \cdots & -m_x^{N1} \\ -m_x^{12} & m_x^{22} & \cdots & -m_x^{N2} \\ \vdots & \vdots & \vdots & \vdots \\ -m_x^{1N} & -m_x^{2N} & \cdots & m_x^{NN} \end{bmatrix}$$

If (20)

where terms in the diagonal are $\left[\sum_{j \neq i} m_x^{ij} + e_x^i + d_x^i \right]$, the off-diagonal elements are migration rates between regions, which are transposed and made negative. Then the system for equations for N regions is:

$$[\boldsymbol{I} + ½\boldsymbol{M_x}]\boldsymbol{P_x^F} = [\boldsymbol{I} - ½\boldsymbol{M_x}]\boldsymbol{P_x^S} + \boldsymbol{A}_x \quad (21)$$

where $\boldsymbol{A}$ is a matrix of immigration flows by age (additions to the population from other countries). If we pre-multiply both sides by the inverse of $[\boldsymbol{I} + ½\boldsymbol{M_x}]$, we obtain the projection equation:

$$\boldsymbol{P_x^F} = [\boldsymbol{I} + ½\boldsymbol{M_x}]^{-\mathbf{1}}\,[\boldsymbol{I} - ½\boldsymbol{M_x}]\boldsymbol{P_x^S} + \,[\boldsymbol{I} + ½\boldsymbol{M_x}]^{-\mathbf{1}}\boldsymbol{A}_x \qquad (22).$$

Occurrence-exposure rates have now been turned into transition probabilities for use in the multistate projection model. The treatment of age transitions in matrices is given in most texts: people move from one age to the next in each time interval. Wilson (1974) discussed how models might be constructed when age and time intervals are not equal. For small areas, population data may only be available for broad ages. However, it is easier to estimate the allocations from broad ages to uniform ages matching the time interval than apply these methods. Wilson also describes the continuous variable model, which yields useful theoretical results such as stable populations (populations growing in the long run at the same rate in each population region or group). The assumption underlying stable population theory is that rates will be constant. However, in most applied projections efforts are made to model how rates might change in the next few years before adopting the constant assumption for the long run.

Subsequent progress in demographic projections and unsolved problems

Alan Wilson's contributions to demographic research can be evaluated by looking at further work in the field to detect influence. Table 5 selects some 21st century examples. We hope that Alan Wilson will recognise some fragments of the DNA of his classic papers in the table.

[Table 5 here]

Panel 5A selects some examples of demographic methods applied to generate knowledge needed in other fields, just as Wilson's work in spatial interaction modelling produced insights into retailing systems and location theory. Rees et al. (2013) make a modest attempt at projecting the impact of population change on the future labour supply, population health and household change for Northern England and implications for employment, transport, health and housing policies. Kupiszewski (2013) generates population forecasts for a set of European countries using a hierarchical multi-regional model (MULTIPOLES) and adds labour force participation, generating economic dependency rates which provide a better tool for assessing the challenge of population ageing. Nawaz et al. (2019) use ethnic population projections to generate forecasts of domestic water demand for London and the

Thames Valley. These results informed Thames Water's 2019 Water Resource Strategy. Work by Cafaro and Dérer (2018) projects future carbon dioxide emissions associated with population projections for EU states under different international migration scenarios.

Panel 5B gives examples of projections that add further dimensions to conventional models. Coleman (2006) reviews the results of nativity (country of birth) projections by national statistical agencies in Europe. Rees et al. (2017a) develop ethnic group population projections to enrich local area forecasts in the UK, arguing that these change future population numbers in areas of minority ethnic concentration. A Mid (Soft Brexit) scenario shows that the future ethnic composition will be very different: White groups will make up 70% of the 2061 population compared with 87% in 2011 while Black and Asian Minority Ethnic (BAME) groups will represent 30% compared with 13% in 2011. BAME groups will contribute 81% of change 2011-61. Another important innovation is the incorporation by Lutz et al (2014) of educational attainment forecasts in a multidimensional projection of the populations of all nation states. Because increasing education in developing countries lowers fertility, these Wittgenstein Centre projections result in a significantly smaller world population than forecast in recent UN projections.

The examples in Panel C of Table 5 illustrate the growing demand of policy makers for policy responsive projections. In a study funded by the European ESPON programme, Rees et al. (2012) link four policy scenarios to assumptions for future fertility, mortality, intra- and extra-Europe migration to generate population futures. Two scenarios in which environmental and economic challenges are met see the EU population rise to circa 600 million in 2050, while in the two scenarios where these challenges are not met, total EU population stagnates at circa 500 million. In all scenarios there are large differences across EU member states between substantial population decline in Romania and substantial population increase in the UK. Abel et al. (2016) ask the question "what if all countries achieve the Sustainable Development Goals of the UN, what would be the population outcomes". Their results show that the range between their alternative specifications using the Lutz et al. (2014) model is both lower and narrower than the 20 to 80 percentile range around the latest UN projection. The key mechanisms are achievements of education and health goals. Finally, Lomax et al. (2019) explore the consequences of different versions of Brexit (the nature of which is still uncertain at time of writing) on international migration and population change for the UK's ethnic groups. The growth in some minority groups is suppressed under Soft and Hard Brexits, but for other groups past immigration has created the demographic potential for continued further growth.

Table 5D provides some examples of sensitivity testing of projection methods. Rees et al. (2018) exposes the difficulties of evaluating the claims of competing projections, unless sensitivity analysis is run with a super-model that enables the impact of model design to be tested. Wilson (T) (2015, 2018) tests alternative models in controlled experiments with different designs. However, there is much still to learn from sensitivity testing.

Figure 1, based on Wilson's insight into urban and regional systems, includes environment as a determining influence. There are many examples which assess the impact of different climate change scenarios on population re-distribution and migration. However, the articulation to countries, cities and regions of the *Limits to Growth* model (Meadows et al. 1972, 2005), which builds in a population-environment feedback loop, is in its infancy.

Contribution to migration studies

Here we discuss Alan Wilson's contribution to migration studies in the context of other research in the field. Table 6 gathers together in Panel A the Wilson and Dennett publications that focus on using spatial interaction models to improve *migration estimation*. Other panels provide a context for the Dennett and Wilson work by discussing contributions to the study of *migration heterogeneity* by groups, achievements and issues in *migration measurement* and approaches to *modelling and projecting migration flows*.

[Table 6 about here]

Alan Wilson's contribution to migration studies derives from his adaption of entropy maximizing theory in Physics (Wilson 1970) to generate a family of spatial interaction models, SIMs (Wilson 1971). The models seek to find the most probable distribution of flows between origins and destinations, dependent on available information that could be used as constraints. The initial applications developed were for modelling journeys to work in cities and for journeys to shop. This work is discussed elsewhere in this special issue of *Interdisciplinary Science Reviews*. The models were applied to understanding migration by a Leeds colleague, John Stillwell, first in his doctoral thesis and then in subsequent journal papers (Stillwell 1978, Stillwell et al. 2016). As with many methodologies, it turned out that the same results could be derived through use of contingency table analysis (Bishop et al. 1975), log-linear models (Willekens 1983) or Poisson regression (Flowerdew and Aitken 1982).

While regression frameworks for fitting and calibrating SIMs predominate in the migration modelling literature, probably due to their ease of implementation in standard statistical software packages, a key feature of the Wilson spatial interaction models was that the methods could be used to model the interaction term experimenting with power, exponential or more complex functions to find the best-fit parameter – something less easy to achieve using standard statistical software. Various extensions to Wilson's original SIM family have found application in migration contexts with, for example, Pooler (1993, 1994) exploring the improvements offered to estimates of inter-provincial Canadian migration by relaxing origin and destination constraints to ranges rather than integer values. Stillwell et al. (2016) used the methods to calibrate the friction of distance for a set of country internal migration tables as part of the IMAGE project (Bell et al. 2015). The Wilson framework has informed the design of explanatory models of migration flows. For example, Fotheringham et al. (2004) used a model that predicted out-migrations from origin zones in England combined with a model that predicted destinations given origins. This model used an extensive set of predictor variables and generated regression parameters for each origin zone. However, many models of internal migration still use the title "gravity model" and a careful reading is needed to ascertain whether outputs are consistent with inputs, as required in each member of the Wilson family of SIMs.

Wilson returned to the application of his SIMs in a collaboration with Adam Dennett after moving to University College London. At Leeds in 2006-2010, Dennett worked with Stillwell on his thesis on internal migration in Britain using classification methods. In the final stages, he worked on estimating internal migration between NUTS2 regions in the UK (Dennett and Rees 2010) as input to the DEMIFER project which developed scenario projections of the population of European regions (de Beer et al 2010). When Dennett moved to University College London, he began a collaboration with Wilson which involved applying SIM methods to Europe-wide inter-regional migration (Dennett and Wilson 2011, 2013, 2016). The research question asked was: "how could region to region flows across member states be estimated in order to describe the changing patterns of migration across the EU?" Answers to this question would help monitor progress towards two European Union goals: "offer freedom, security and justice without internal borders" and "enhance economic, social and territorial cohesion and solidarity among EU countries" (EU 2018). That this work came to fruition in 2016 when UK voters chose to leave in EU in a June Referendum is ironic.

Dennett and Wilson start with the inputs used in the DEMIFER projections, a set of intra-country/inter-regional flows, generally available from national registers or censuses, except in the UK where Dennett and Rees (2010) had to make estimates based on partial official data, combined with estimates of migration flows between EU states produced in the MIMOSA project (Raymer et al. 2011). The task was to produce EU wide inter-regional migration estimates from these inputs, using different SIM models and assumptions. Six alternative models were carefully specified and used to generate flow estimates, calibrating the distance decay functions and generating fits using the known intra-country/inter-regional flows. The DEMIFER model was extended to identify and estimate inter-regional migration flows that crossed international borders. The assumption that the distribution of internal inflows and outflows could be used to allocate inter-country flows to regions in each country was tested. It was found necessary to allow for a capital city effect, where international migration is more concentrated on the capital than is internal migration. Dennett and Wilson (2016) provide an interesting peek at the results of the work. The field of migration estimation has moved on since their paper: for example, the MIMOSA estimates of EU inter-country flows were updated in Raymer et al. (2013) in the IMEM project which used a Bayesian statistical framework in which prior beliefs are confronted with available data and in which prediction intervals are derived. However, the principle of utilising constraints operating at different levels of a geographical hierarchy to generate the best estimates conforming to all known information about a spatial system is still to be fully explored. The new extended family of spatial interaction models developed by Dennett and Wilson still offers the tantalising prospect of a global sub-national migration model.

Subsequent progress in migration estimation and unsolved problems

Dennett (2016) has done further work on migration estimation at the world scale. Up to 2013, all analysis of world migration depended on use of net migration estimates generated by the UN through subtracting natural increase from total population change and on the use of population tables, derived mainly from censuses, for countries classifying the population by country of birth. The net measure provides little information about inflows and outflows because there are an infinite number of pairs of statistics that yield the same net figure. The migrant stock tables provide information on cumulative lifetime migration on migrants surviving to the time of the census or survey.

In 2013 Guy Abel published an important paper which estimated inter-census migration flows from the pairs of successive migrant stock tables, imputing the migration flows that would be needed to link a table of populations classified by country of residence and country of birth at one census into the same table at a subsequent census (Abel 2013). Abel's work "solved" a problem which migration researchers had been struggling with since Ravenstein (1885) based his "Laws of Migration" on the study of inter-county migrant stock tables for the British Isles. Dennett (2016) compares the Abel estimates with the MIMOSA estimates and finds them to under-estimate flows substantially. He then employs a simple alternative to the Abel methodology that applies destination conditional probabilities derived from the migrant stock data or from UN net balances to total migrant estimates. For European flows he shows that this does improve the estimates but also demonstrates that mis-estimates occur when international migration along any origin to destination channel is characterised by a wave of large flows in one era followed by diminution in another, a common phenomenon.

Recent work by Azose and Raftery (2018) takes another approach to the issue by developing a model that estimates return and repeat migration. Return migration is not recorded in the lifetime migration tables for those who have returned to their country of birth. Where migration flows are temporary and circular these will not appear in the migrant stock tables. Repeat migrants will be captured but only one of their migrations in the inter-census interval will count. Azose and Raftery claim to have achieved "genuine" estimates of total global migration though they recognise that 5-year transition and 1-year transition measures are different. Their method makes estimates of 1-year migration consistent with the UN definition. Dyrting (2018) has also developed a method for modelling return and repeat migrants.

Panel B of Table 6 lists examples of migration studies which estimate differences in migration propensity by group dimensions beyond location, age and sex. Sander et al. (2014) explain how the Abel and Sander (2014) estimates are assigned educational attainment grades for use in the Lutz et al. (2014) projections though the assignments do need improvement (Rees 2019b). Bernard and Bell (2018) use IPUMS survey data for 56 countries covering 65% of the world population to assess differentials in migration rates by educational grade. After controlling for a set of relevant individual level characteristics, they find ratios to migration by persons without schooling to be 1.1, 1.2 and 2.3 for people with primary, secondary and tertiary education respectively. Darlington-Pollock et al. (2018) use a 5% sample of individual data from the 2011 UK census to compute probabilities of 1-year migration by nine ethnicities, two nativities (UK-born, Foreign-Born), three "working" ages and shorter and longer distances migrated. In general, Black and Asian Minority Ethnic (BAME) groups migrate less than White, except for the Chinese. Pakistani and Bangladeshi origin individuals have rates only one third of those of the White British.

Panel 6C lists papers that tackle measurement issues. Papers by Bell et al. (2015), Stillwell et al. (2016) and Rees et al. (2017b) report on results of the IMAGE project to derive harmonized measures of internal migration for countries containing 80% of the world's population. The harmonized measures control for the effect of the number of spatial units on crude measures so that countries with big regions can be compared with countries with small regions. This work, led by Martin Bell at the University of Queensland, has provided internationally comparable measures of internal migration for the first time. However, harmonization over time interval of measurement was not attempted. Countries were compared in separate 1-year and 5-year groups. The methods used by Dyrting (2018) have the potential for supplying this missing ingredient. The paper by Ledent (1980) is included because it shows, using US and Sweden data, that inter-regional migration probabilities differ when the migration is a return to region of birth. Ideally, this heterogeneity should be built into population projections.

The final panel, 6D, lists research on modelling and projecting migration flows that might be included in population projections. Plane (1993) demonstrates that, in a multi-regional model, if constant propensities of migration are used, the regional shares converge on a stable distribution which may be implausible. Statistics Canada took this property of Markov Chains seriously and made adjustments to provide negative feedback (Dion 2014). Rees et al. (2011) experimented with a model that adopted flow number assumptions for immigration and rate assumptions for emigration which had the effect of reducing net international migration. Although Bijak (2012) recommended this model to the UK Office for National Statistics, the original authors were more sceptical. A full family of possible models is presented in Rees et al. (2015) but has yet to be subjected to experimental sensitivity analysis. Sander et al. (2014) present an interesting analysis based on a meta-expert focus group and a survey of experts about the likely future of international migration flows in different world regions. However, although these judgements were carefully quantified, they were not used to determine the future migration assumptions. The Kim and Cohen (2010) paper shows how explanatory variables can be introduced into a gravity-regression model for a sub-set of reliable inter-country migration flows. This experience would be invaluable should any research team take on the larger task of doing this for the whole world, once estimates of international migration flows have been agreed as reliable. The final paper by Shen (2017) is included to remind researchers to pay attention to the impedance term in a migration SIM. Shen found that in his models of Chinese inter-provincial migration more than 50% of error could be attributed to the impedance term.

Conclusions

Alan Wilson has had an outstanding career as an innovative thinker in the field of urban and regional systems analysis. His main strength, illustrated in his work on demographic and migration models is exceptionally clear conceptual thinking which he turns into formal mathematical models, always described with care, as is illustrated by his papers reviewed in this article. He understands the value of sound theory, under-pinned by the proper counting of events and transitions, the proper representation of system states (regions, groups, time). He is also very aware of the need for empirical testing, conducted usually in collaboration with colleagues. For these colleagues, he has been an exceptional motivator, giving gentle advice that ensures a better joint product.

Although not part of his core research agenda, Wilson has contributed to the specification of the demographic accounts upon which measurement and projection of population change and its components depend. His invention, the family of spatial interaction models, has become a powerful tool in estimating migration flows in complex systems.

We have attempted to trace progress in these two field of population and migration studies from the 1970s to the 2010s, by summarising recent contributions. These show what two of his collaborators have been engaged with and which important methodological problems have been solved and which remain to be untangled. At the start of the article we promised messages from Alan Wilson’s population and migration work for readers to take away. These are as follows:

- Spend time developing the *theoretical framework* for your investigation and derive formal models of the interrelationships between determinants and outcomes.
- Use *mathematics* to provide rigour for your models. The mathematics involved is standard; it is application to new demographic and mobility issues that creates the innovation.
- Developing the correct *notation* to match the theoretical framework is probably the key to success rather than equation design, which some one else has already done. To understand other people's models, you need to be able to translate between notations.
- Populate your models with *data* and with *assumptions*. The assumptions need to be tested through historical analysis, but users will want a view of the future as well. The uncertainties need to be identified, using a variety of scenario or statistical methods.
- Pursue research questions in a *team* that brings together experts in theory, in understanding data and in coding to build effective and useful models of the world we live in, so the best decisions can be taken to improve or save that world.

Notes on Contributors

Philip Rees is Professor Emeritus at the University of Leeds. He has taught population geography and researched demographic issues since 1970. His current interest focuses on the future of the UK's populations by ethnicity at national and local scales. Recent papers include 'The impacts of international migration on the UK's ethnic populations' (*Journal of Ethnic and Migration Studies*, 2019), 'Projections of domestic water demand over the long-term: A case study of London and the Thames Valley' (*Journal of Water Resources Planning and Management,* 2019), 'Evaluation of sub-national population projections: A case study for London and the Thames Valley' (*Applied Spatial Analysis and Policy*, 2018) and 'The impact of internal migration on population redistribution: An international comparison' (*Population, Space and Place*, 2017).

Adam Dennett is Associate Professor at the Bartlett Centre for Advanced Spatial Analysis, University College, London, where he is the Director. His main interests include migration studies (modelling migration using spatial interaction models), beer and brewing geographies, longitudinal analysis and urban analytics. He works on research projects in areas as diverse as building stock, crowd-sourcing, residential mobilities, urban health and inequalities and housing analysis. Tying all these themes together are an interest in quantitative data on human geographical activities and appropriate quantitative methods to analyse them. Recent papers include 'Modelling population flows using spatial interaction models' (*Australian Population Studies* 2018), 'New forms of data for understanding urban activity in developing countries' (*Applied Spatial Analysis and Policy*, 2018) and 'The geography of London's recent beer brewing revolution' (*The Geographical Journal*, 2017).

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Table 1. A set of multiregional input-output accounts for a single country

Sector	1	…	n	…	s	Final Demand	Exports	Invest (out)	Total Outputs
Sector 1	Z_{11}	…	Z_{1n}	…	Z_{1s}	Y_1	E_1	M_1	X_1
:	:		:		:	:	:	:	:
Sector m	Z_{m1}	…	Z_{mn}	…	Z_{ms}	Y_m	E_m	M_m	X_m
:	:		:		:	:	:	:	:
Sector s	Z_{s1}	…	Z_{sn}	…	Z_{ss}	Y_s	E_s	M_s	X_s
Labour	L_1	…	L_n	…	L_s	0	0	0	L_+
Imports	I_1	…	I_n	…	I_s	0	0	0	I_+
Invest (in)	N_1	…	N_n	…	N_s	0	0	0	N_+
Total Inputs	T_1	…	T_n	…	T_s	Y_+	E_+	M_+	G_+

Source: Adapted from Wilson (1974).
Definitions of terms:

Z = Product inputs from one sector to another
Y = Final Demand = product consumed by households
E = Exports
M = Outward Investments
X = Total Outputs/products
Sector subscripts = m origin sector, n destination sector
L = Labour inputs
I = Imports
N = Inward Investments
T = Total inputs
G = Grand Total Inputs/Outputs
+ =sum over subscript

Table 2. A family of input-output models

Spatial specification	Example publications
Single Region	Jin, Leigh, Wilson 1991
Multiple Regions	Isard et al. 1960, 1998
Nested Country-Region-City/Rural-Zones	Jin and Wilson, 1991, 1993
Single Country	Leontief 1936
Multiple Countries	Leontief 1974, Timmer et al. 2015, Levy et al. 2014, 2016
Environmental Analysis	Shmelev, S. 2012

Source: Elaborated from Wilson (1974), Jin and Wilson (1993) and Levy et al. (2016)

Table 3. Demographic accounts using transition data for a typical period-cohort

Exist in:	Survive in: Region i	Region j	Region r	Die in: Region i	Region j	Region r	Totals
Region i	T_x^{eisi}	T_x^{eisj}	T_x^{eisr}	T_x^{eidi}	T_x^{eidj}	T_x^{eidr}	T_x^{ei++}
Region j	T_x^{ejsi}	T_x^{ejsj}	T_x^{ejsr}	T_x^{ejdi}	T_x^{ejdj}	T_x^{ejdr}	T_x^{ej++}
Region r	T_x^{ersi}	T_x^{ersj}	T_x^{ersr}	T_x^{erdi}	T_x^{erdj}	T_x^{erdj}	T_x^{er++}
Totals	T_x^{e+si}	T_x^{e+sj}	T_x^{e+sr}	T_x^{e+di}	T_x^{e+dj}	T_x^{e+dj}	T_x^{e+++}

Source: Adapted from Rees (2019a)

Notes:

Locations: region *i* = region of interest, region *j* = rest of the country, region *r* = rest of the world

Variables, subscripts: *T* = transitions, *e* = existence at start, *s* = survival at finish, *d* = death in interval, *b* = birth in interval

Ages: *x* = period cohort. For the new-born period-cohort, *b* indicating birth in a time interval is substituted for e and the row totals refer to births in the region rather than populations.

Table 4. Demographic accounts using movement data for a typical period-cohort

Start-state:	End-state: Region i	Region j	Region r	Death	Totals
Region i	R_x^i	M_x^{ij}	E_x^i	D_x^i	P_x^{Si}
Region j	M_x^{ji}	R_x^j	E_x^j	D_x^j	P_x^{Sj}
Region r	I_x^i	I_x^j	R_x^r	D_x^r	P_x^{Sr}
Totals	P_x^{Fi}	P_x^{Fj}	P_x^{Fr}	D_x^+	G_x^+

Source: Adapted from Rees (2019a)

Notes:

Locations: region *i* = region of interest, region *j* = rest of the country, region *r* = rest of the world

Variables: *R* = accounting Residual, *M* = internal migration, *E* = emigration, *I* = immigration, *D* =death, P^S = start population, P^F = final population, *G* = grand total of movements.

Ages: *x* = period cohort. For the new-born period-cohort, *b* indicating birth in a time interval is substituted for x and the row totals refer to births in the region rather than populations.

Table 5. Recent developments in demographic models for projecting populations

Ref.	Topic/Example Paper	Description
A	**Forecasting Outcomes**	How future populations change future activities
	Rees et al. (2013)	Develops labour force, health and households from local area projected populations for local areas in N. England
	Kupiszewski (2013)	Projects populations and labour force with economic dependency ratios for set of European countries
	Nawaz et al. (2019)	Builds household and water demand forecasts for the Thames Water region in UK
	Cafaro and Dérer (2018)	Forecasts "carbon footprints" of the population of EU countries based on scenario projections
B	**Adding Groups**	How further heterogeneity changes future populations
	Coleman (2006)	Review official nativity projections for selected European countries
	Lutz et al. (2014)	Develops forecasts of educational attainment for most countries of the world and embeds them in multi-dimensional population projections
	Rees et (2017a)	Produces population projections by ethnic group all local areas in United Kingdom, based on innovative estimation of components of change by ethnicity ETHPOP model)
C	**Using Policy Scenarios**	How policy choices change future populations
	Rees et al. (2012)	Designs four scenarios base on economic/environmental policies and social policies determining inequalities to create assumptions for projections for 31 EU + EEA countries and 287 NUTS2 regions (DEMIFER model)
	Abel et al. (2016)	Using the Lutz et al. projection model with new assumptions conditional on attainment of UN Sustainable Development Goals 2015-2030
	Lomax et al. (2019)	The impacts of alternative "Brexits" on ethnic group populations via different international migration assumptions using ETHPOP model
D	**Sensitivity Testing**	How model choices as well as assumptions change future populations
	Wilson, T. (2015)	Evaluation of simple models for small area forecasts in a controlled data/methods "laboratory"
	Wilson, T. (2018)	Evaluation of simple methods for regional mortality forecasts in a controlled data/methods "laboratory"
	Rees et al. (2018)	Classifies different methods of comparing projections, identifying why sensitivity testing is best

Table 6. Recent development in estimating, modelling and projecting migration flows

Ref.	Topic/Example Paper	Description
A	**Migration Estimation**	
	Dennett and Rees (2010)	Develops integrated estimates of internal migration between NUTS2 regions in the UK (DEMIFER project)
	Raymer et al. (2011)	Develops methods of estimating migration between EU countries through quality grading and modelling (MIMOSA project)
	Dennett and Wilson (2011, 2013, 2016)	Adapts Wilson SIM models to estimate inter-regional migration between EU NUTS 2 regions as used in DEMIFER project to achieve consistency
	Raymer et al. (2013)	Adds Bayesian statistical inference to MIMOSA methods and computes uncertainty bounds for measures
	Abel (2013)	Develops a new method for estimating bi-lateral migration flows using migrant stock data for destinations by country of birth
	Abel and Sander (2014)	Explains the Abel 2013 method and describes flows using new graphical methods, showing global migration flows volumes are growing with population but rates are not increasing
	Dennett (2016)	Investigates errors in Abel estimates and proposed plausible methods for correction
	Dennett and Mateos (2016)	Reviews policy work on international migration
	Abel (2018)	Extends the time series of global migration flows from 1960 to 2015 and adds gender
	Azose and Raftery (2018)	Extends the Abel model to estimate return and repeat migration and confidence intervals using Bayesian statistical methods
B	**Migration Heterogeneity**	
	Sander et al. (2014)	Describes how educational attainment is estimated for bi-lateral international migration flows in WIC model
	Bernard and Bell (2018)	Establishes that education has a significant effect on rates of internal migration, after controlling for other factors
	Darlington-Pollock et al. (2018)	Analyses differences in internal migration rates over different distance bands in the UK by ethnicity, age and nativity
C	**Migration Measurement**	
	Ledent (1980)	Shows that return migration to birthplace influences migration transitions and alters projections and multi-regional life tables
	Bell et al. (2015)	Summary of the data and methods used to harmonize internal migration data across countries (IMAGE project)
	Stillwell et al. (2016)	Demonstrates the relationship of migration and distance (IMAGE project)
	Rees et al. (2017)	Results for impact measures of migration across countries (IMAGE project)
	Dyrting (2018)	Proposes a method to translate between one-year and five-migration probabilities
D	**Modelling and Projecting Migration Flows**	
	Plane (1993)	Demonstrates that a constant assumption leads to implausible regional population distribution
	Fotheringham et al. (2004)	Uses a combined production and distribution model to predict internal migration flows for the UK using numerous determinants
	Kim and Cohen (2010)	Develops an explanatory gravity model for a UN set of flows
	Dion (2014)	Uses a method to adjust transition probabilities to deal with the Plane problem (Statistics Canada)
	Rees et al. (2011)	Projects ethnic populations using assumptions for immigration and emigration flows or inflows and emigration rates
	Bijak (2012)	Review of methods for projecting gross flows rather than net flows for the Office for National Statistics (UK)
	Sander, Abel and Riosmena (2014)	Presents expert and meta-expert views on migration assumptions
	Rees et al. (2015)	Reviews methods for projecting international migration for National Records (Scotland)
	Shen (2017)	Establishes the high share of error due to the interaction cost term in a gravity model of inter-province migration in China

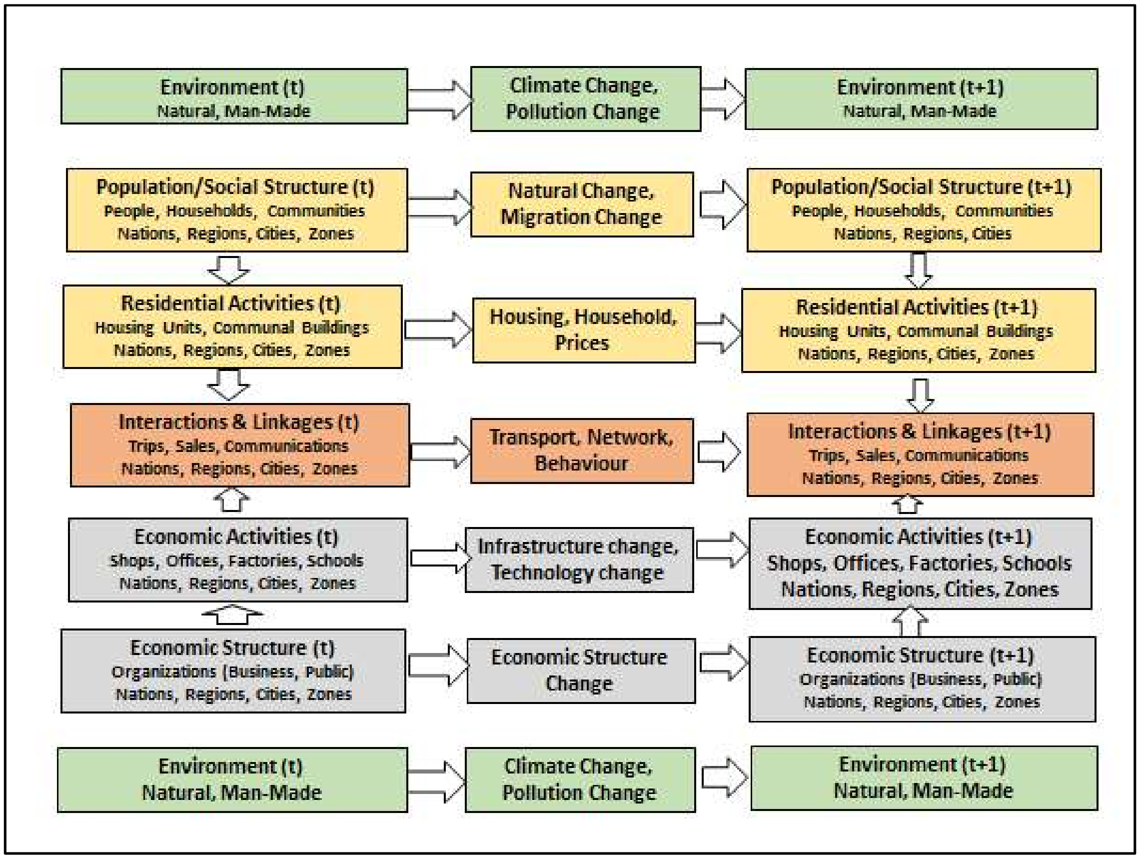

Figure 1. A diagram for an urban and regional system based on Wilson (1974, Figures 3.1, 3.2)

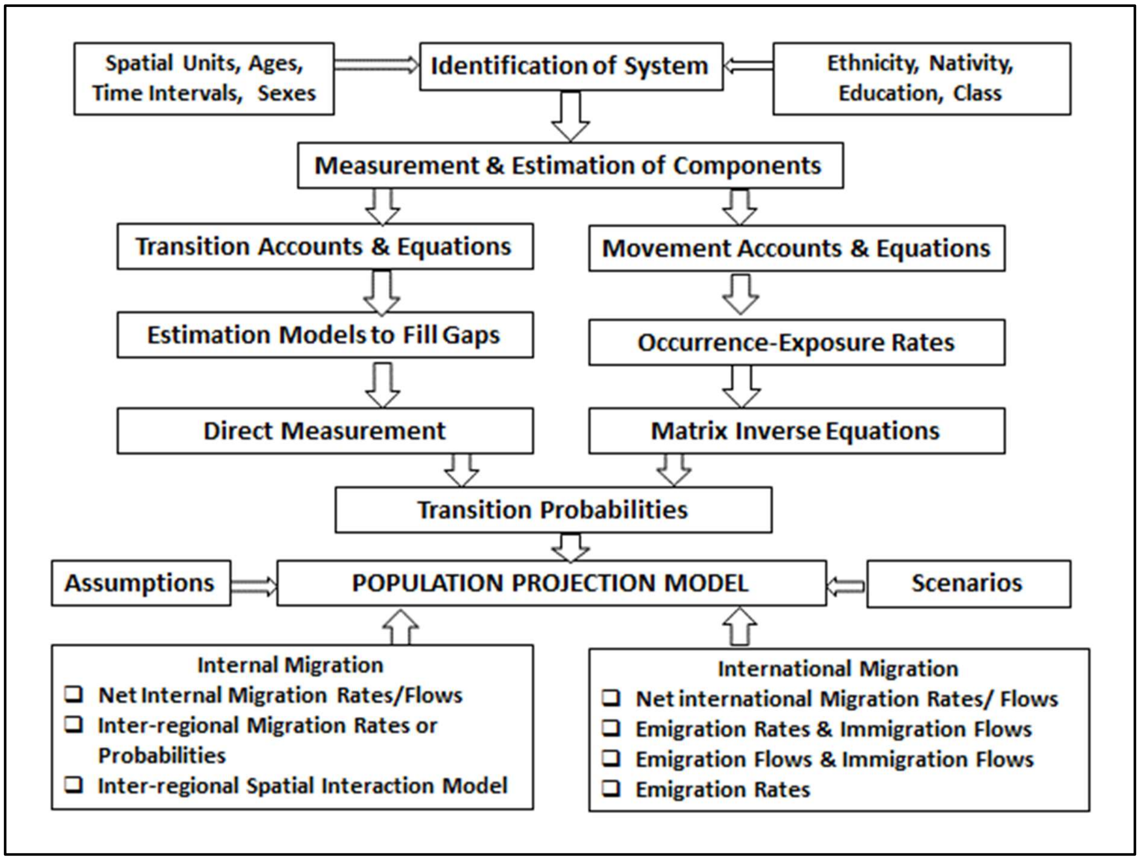

Figure 2. Steps in designing a population projection model
Source: Authors' elaboration of Willekens and Drewe (1984), Fig.15.1.