Modelling patient flow and outcomes in community healthcare — a fluid approximation of a stochastic queueing system

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In recent decades, an ambition of healthcare policy has been to deliver more care in the community sector [6].

- Diverse range of services, operating in different physical locations
- Common for patients to use a range of services which they may re-use
- Considered to be crucial in meeting the current and future challenges that face modern health care services [3]

**Challenge:** how to organise and deliver these services given: physical distribution, patients using multiple services, increased referrals, case mix, and long term care requirements [7].
Quality measurement in community healthcare

▶ Operational capability - waiting time, queue length, resource utilisation, capacity

▶ Outcome measures - aspects of a patient’s health or experience, influenced by a care interaction
  ▶ i.e. measurable behaviours, opinions, medical characteristics or health status
  ▶ Used to monitor and evaluate the progression of patients and the quality of care received

Increasingly used by managers, clinicians and commissioners to inform quality improvement [2].
Uses in healthcare:

- Regularly evaluated as periodically calculated proportions
  - Misses time dependent variability in output of service and outcomes achieved
- Often considered in isolation from other services
- Misleading when considering dynamic patient flow of stochastic healthcare systems

Common modelling assumptions:

- Operational improvements positively affect outcomes
- Uniform patients
Rationale and aims

Understand dynamics of patient flow and use of patient outcomes in evaluating community healthcare

1. Unify two perspectives of quality in a single modelling framework
   ▶ Account for flow of patients in different health states
   ▶ Transition in health throughout queueing process important to understanding demand for and effect of system

2. Establish a concept of the flow of outcomes - how individual services contribute to a system’s performance
   ▶ Flow: bottlenecks, required capacity, waiting times
   ▶ Outcomes: how they accrue over time through a combination of services
Dynamics of patient flow in community healthcare

- Multiple services of varying configuration
- Patients reusing the same services - either sequentially or after care within another service
- Possibility of patients abandoning queue - impatient; seek healthcare in a non-community setting
- Patients arrive in different health states
  - Different capacity to benefit/resource requirement
  - Health may improve or decline throughout
- Time dependent demand

Traditional methods do not cope well with these dynamics - computationally expensive
We present the application of a deterministic fluid and diffusion approximation for a stochastic queueing system. A novel application and extension of work by both S Ding et al. (2015)[1] and A Mandelbaum et al. (1998, 2002) [4, 5].

- Network of multiple services
- Health states - different parameters
- Application of diffusion equation
The system - a series of stochastic processes

Introduction

Fluid approximation

Simulations

Discussion and conclusion
The system - a series of stochastic processes

Number of servers split across $A$ parallel queues
For analytical tractability, patients served first come first serve in parallel queues according to health state, \( a \in A \). Capacity \( C_{a,i}(t) \) allocated to each queue at time \( t \), with \( \sum_{a \in A} C_{a,i}(t) = c_i(t) \).

Capacity \( C_{a,i}(u, Z(u)) \) is given by:

\[
C_{a,i}(u, Z(u)) = \begin{cases} 
\frac{c_i Z_{a,Q,i}(u)}{\sum_{b=1}^{A} Z_{b,Q,i}(u)}, & \text{if } c_i < \sum_{b=1}^{A} Z_{b,Q,i}(u) > 0 \\
0, & \text{otherwise}
\end{cases}
\]

(1)

\[
C_{a,i}(u, Z(u)) = \begin{cases} 
\frac{c_i \mu_{a,i} Z_{a,Q,i}(u)}{\sum_{b=1}^{A} \mu_{b} Z_{b,Q,i}(u)}, & \text{if } c_i < \sum_{b=1}^{A} Z_{b,Q,i}(u) > 0 \\
0, & \text{otherwise}
\end{cases}
\]

(2)

\[
C_{a,i}(u, Z(u)) = \frac{Z_{a,Q,i}(0) + \lambda_{a,i}}{\sum_{b=1}^{A} Z_{b,Q,i}(0) + \lambda_{b,i}} \times c_i
\]

(3)
Work within Skorokhod space with J1 metric: intuitively provides wiggle space within both space and time.

A natural and convenient formalism for describing trajectories of stochastic processes that may admit discontinuities, i.e. trajectories of Poisson processes.

Consider a sequence of models where the \( n \)-th model denoted by the superscript \( (n) \) has an arrival rate of \( \lambda_{a,i}n \) for new patients in health state \( a \) and the total number of servers is \( nc_i \). The scaled fluid process is defined as:

\[
\tilde{Z}_{a,m,i}(t) := \frac{Z_{a,m,i}^{(n)}(t)}{n},
\]

where \( a \in \{1, \ldots, A\} \), \( i = 1, \ldots, J \) and \( m \in \{Q, R, F, O, D, L\} \).
Within this space and scaling the system for $n \rightarrow \infty$, we can define the fluid limit:

$$z_{a,Q,i}(t) = z_{a,Q,i}(0) + \lambda_{a,i}t + \sum_{b=1}^{A} s_{b,a,R,i} \delta_{b,R,i} \int_{0}^{t} z_{b,R,i}(u) \, du$$

$$+ \sum_{b=1}^{A} s_{b,a,F,i} \delta_{b,F,i} \int_{0}^{t} z_{b,F,i}(u) \, du$$

$$+ \sum_{b=1}^{A} s_{b,a,O,i} \delta_{b,O,i} \int_{0}^{t} z_{b,O,i}(u) \, du$$

$$- \mu_{a,i} \int_{0}^{t} \min(z_{a,Q,i}(u), C_{a,i}(u, z(u))) \, du$$

$$- \theta_{a,i} \int_{0}^{t} (z_{a,Q,i}(u) - C_{a,i}(u, z(u)))^+ \, du$$
Fluid approximation of patient flow

\[ z_{a,R,i}(t) = z_{a,R,i}(0) - \delta_{a,R,i} \int_0^t z_{a,R,i}(u) \, du \]

\[ + p_{a,i} \sum_{b=1}^A s_{b,a,i} \theta_{b,i} \int_0^t (z_{b,Q,i}(u) - C_{b,i}(u, z(u)))^+ \, du \]

\[ z_{a,F,i}(t) = z_{a,F,i}(0) - \delta_{a,F,i} \int_0^t z_{a,F,i}(u) \, du \]

\[ + r_{a,i} \sum_{b=1}^A s_{b,a,s,i} \mu_{b,i} \min(z_{b,Q,i}(u), C_{b,i}(u, z(u))) \, du \]
Fluid approximation of patient flow

\[ z_{a,O,i}(t) = z_{a,O,i}(0) - \delta_{a,O,i} \int_0^t z_{a,O,i}(u) \, du \]

\[ + \sum_{j=1; \, j \neq i}^J \sum_{b=1}^A s_{b,a,s,j} r_{a,j,i} \int_0^t \mu_{b,j} \min \left( z_{b,Q,j}(u), C_{b,j}(u, z(u)) \right) \, du \]

\[ z_{a,L,i}(t) = z_{a,L,i}(0) \]

\[ + (1 - p_{a,i}) \sum_{b=1}^A s_{b,a,l,i} \theta_{b,i} \int_0^t \left( z_{b,Q,i}(u) - C_{b,i}(u, z(u)) \right)^+ \, du \]

\[ z_{a,D,i}(t) = z_{a,D,i}(0) + r_{a,i,D} \sum_{b=1}^A s_{b,a,s,i} \mu_{b,i} \int_0^t C_{b,i}(u, z(u)) \, du \]
Analytical expressions cannot be found for the above, however can be solved iteratively using common numerical schemes.

- Rewrite
  
  \[ z_{a,Q,i}(t), z_{a,R,i}(t), z_{a,F,i}(t), z_{a,O,i}(t), \quad i = 1, \ldots, J, \quad a = 1, \ldots, A \]
  
  as
  
  \[ z(t) = \phi(z(t)) \]

- Let \( z^{(0)}(0) = 0 \)

- Calculate \( z^{(k+1)} = \phi(z^{(k)}), \quad k = 0, 1, \ldots \) using a common numerical scheme

- Stop when difference between \( z^{(k+1)} \) and \( z^{(k)} \) is deemed sufficiently small
Computed in MATLAB, produced a Discrete Event Simulation for the basic stochastic system and extension with health states

- Compare fluid model to simulation
- Basic model - rebook and follow up [1]
  - Explore parameter space for community healthcare
  - Triangulation of models
- Extended to include health states - single service
  - Assess accuracy of extension
Effective traffic intensity: \( \hat{\rho} = \frac{\lambda}{c\mu(1-q)} \)
Simulations - Basic case
Larger probability of re-use
Simulations - Introduction of outcomes

Introduction

Fluid approximation

Simulations

Discussion and conclusion
Simulations - System with outcomes
Time dependent behaviour - arrival spikes

Introduction
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Measures from fluid approximation of patient flow

- Number of patients, over time, in:
  - Services (or orbits)
  - Health states

- Waiting times, number in the system, waiting time distribution - using Erlang A or R, Virtual Waiting Time [5]
  - Per service

- Production of outcomes: number of patients discharge from a service/system in a given health state over time

- Loss over time
Types of analyses

- Capacity allocation - i.e. optimisation:
  1. Balance across queues for equitable wait times
  2. Reduce net loss
  3. Prevent resource intensive re-joins

- Production of outcomes:
  1. Begin to understand how a network of services work together to "produce" patients with good health
  2. Seek balance across multiple services
     - Identify bottlenecks
     - Holistic view of operational measures
Limitations and future work

Limitations:

- A deterministic analogue of a stochastic system
- Errors for none heavily loaded systems
- Less accurate for smaller systems

Future directions:

1. Joint use of services - could an extension capture this?
2. Can the patient flow and outcomes of patients with multiple morbidities be informatively modelled?
3. Combining with optimisation methods - can useful information be gained for service planning?
Extending a fluid approximation of stochastic systems to a network of services, including patient health, is beneficial since:

- Model key dynamics of community healthcare
- Overcomes computational burden and time expense of other methods
- Provides time dependent analysis of system outputs
[1] Ding S, Remerova M, van der Mei RD, and Zwart B.
Fluid approximation of a call center model with redials and reconnects.

Managing quality in community health care services.

[3] Ham C, Dixon A, and Brooke B.
Transforming the delivery of health and social care: the case for fundamental change.

Strong approximations for markovian service networks.

Queue lengths and waiting times for multiserver queues with abandonment and retrials.

Evidence: Getting out of hospital?
2011.

A systematic literature review of operational research methods for modelling patient flow and outcomes within community healthcare and other settings.
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Thank you for your attention
Are there any questions?