A Scalable Performance-Complexity Trade-off for Full Duplex Beamforming

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***Abstract*—In this paper, we consider a multiuser single-cell transmission facilitated by a full duplex (FD) base station (BS). We formulate two multi-objective optimization problems (MOOPs) via the weighted Tchebycheff method to jointly minimize the two desirable system design objectives namely the total downlink and uplink transmit power. In the first MOOP, multiuser interference is suppressed while in the second MOOP, multiuser interference is rather exploited. In order to solve the non-convex MOOPs, we propose a two-step iterative algorithm to optimize jointly the receive beamformer, transmit beamformer and uplink transmit power, respectively. Simulation results show the proposed scheme achieves a scalable performance-complexity trade-off that allows performance improvements compared to existing solutions.**

***Index Terms*—full-duplex, multi-objective optimization, beamforming.**

I. Introduction

Full duplex (FD), being one of the key solutions for the next generation 5G communication systems [1] is in the forefront of research attention. This is due to their potential to drastically improve the spectral efficiency over half duplex (HD) communication systems. However, for many years the implementation of FD systems has been considered impractical due to the presence of self-interference (SI). Hence, major efforts have been made in order to reduce the effect of SI [2], [3]. With the success in most of the proposed SI mitigation techniques over the years, other issues such as, protocol and resource allocation designs need to be re-investigated with respect to FD transmission.

Many of the FD beamforming and resource allocation solutions build upon the existing HD beamforming solutions in the literature. These include relay topologies, base station topologies and bidirectional topologies [4]. In [3], a FD bidirectional communication systems that uses pilot-aided channel estimates to perform transmit beamforming, receive beamforming, and interference cancellation was investigated. [5] studied the performance of a FD bidirectional system and a FD relay system with multiple-input multiple-output (MIMO) in terms of maximum system ergodic mutual information with imperfect channel state information (CSI). In [6], the ergodic capacity of a FD relays was investigated, where the FD relay harvests a portion of the received signal from the source node to power itself. Similarly, [7] studies the joint optimization problem of two-way relay beamforming with energy harvesting to maximize the achievable sum rate of the FD system.

On the other hand, in terms of power minimization, it has been shown in [8]–[10] that with the knowledge of the users' data symbols and the CSI at the base station (BS), the interference can be classified into constructive and destructive interference. Further findings in [11]–[20] show that tremendous power gains can be achieved by exploiting the constructive interference based on symbol level optimization for both PSK and QAM modulations. However, these findings are all based on MISO HD systems.

Most relevant to the focus of this paper are [21]–[23]. The authors in [21] investigated the power efficient resource allocation for a MU-MIMO FD system. They proposed a multi-objective optimization problem (MOOP) to study the total uplink and downlink transmit power minimization problems jointly via the weighed Tchebycheff method. They extended their work to a robust and secure FD system model in the presence of roaming users (eavesdroppers) in [22]. In [23], the concept of interference exploitation in multiuser FD systems was presented, where the knowledge of the downlink data symbols at the FD BS was utilized to constructively exploit interference rather than suppress it. In addition to the significant power savings achieved compared to the existing solutions, it was shown that although the concept of constructive interference was applied for downlink transmission, the benefits extend to uplink transmission. The above works limit the optimization to the downlink beamforming vectors and the uplink power allocation, employing a closed form zero forcing (ZF) detector for the uplink. In this work we aim to further increase the power savings in multiuser FD communication systems via an iterative algorithm that jointly optimizes the receive beamformer, transmit beamformer and the uplink power, respectively, which increases the degrees of freedom in the optimization, and allows a scalable performance-complexity trade-off.

II. System Model

We consider a FD multiuser communication system as shown in Fig. 1. The system consists of a FD radio BS with N antennas serving single-antenna K HD downlink users and J HD uplink users. Let $\mathbf{h}_i \in \mathbb{C}^{N\times 1}$ be the channel vector between the FD radio BS and the *i-th* downlink user, and $\mathbf{f}_j \in \mathbb{C}^{N\times 1}$ be the channel vector between the FD radio BS and the *j-th*

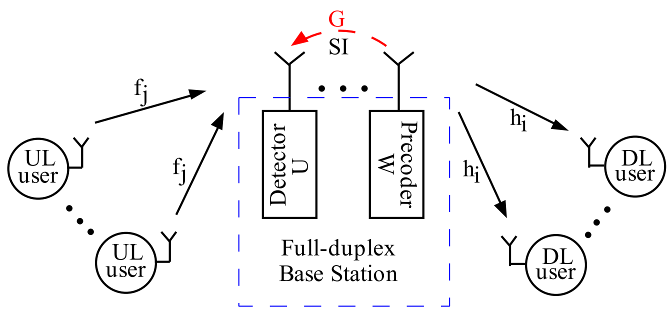

Fig. 1. System model with a FD radio BS with N antennas, K HD downlink users and J HD uplink users.

uplink user. We denote the transmit signal vector from the FD radio BS to the *i-th* downlink user as

$$\mathbf{t}_i = \mathbf{w}_i d_i, \tag{1}$$

where $\mathbf{w}_i \in \mathbb{C}^{N\times 1}$ and d_i denote the beamforming vector and the data symbol for the *i-th* downlink user. The received signal at the *i-th* downlink user is

$$y_i = \mathbf{h}_i^H \mathbf{t}_i + \sum_{k\neq i}^{K} \mathbf{h}_i^H \mathbf{t}_k + n_i, \tag{2}$$

where $n_i \sim \mathcal{CN}\left(0, \sigma_i^2\right)$ represents the additive white Gaussian noise (AWGN) at the *i-th* downlink user. The first term in (2) represents the desired signal while the second term is the multiuser interference signal. For ease of exposition and since the uplink interference cannot be exploited in the style of interference exploitation that we present in this paper due to the absence of the knowledge of the uplink data at the FD BS, we neglect the uplink-to-downlink interference in our system model [24]. In practice, this may be due to the weak uplink-to-downlink user channels due to physical obstructions and shadowing effects, or due to a dedicated overlaid interference avoidance scheme such as the one in [25]. Accordingly, the explicit interference terms can be avoided for simplicity, or assumed incorporated in the downlink users' noise term.

The received signal from the J uplink users at the FD radio BS is

$$\mathbf{y}^{BS} = \sum_{j=1}^{J} \sqrt{P}_j \mathbf{f}_j x_j + \mathbf{G} \sum_{k=1}^{K} \mathbf{t}_k + \mathbf{z}, \tag{3}$$

where P_j and x_j denotes the uplink transmit power and the data symbol from the *j-th* uplink user respectively. The vector $\mathbf{z} \sim \mathcal{CN}(0, \sigma_N^2)$ represents the AWGN at the FD radio BS. The matrix $\mathbf{G} \in \mathbb{C}^{N\times N}$ denotes the self-interference (SI) channel at the FD radio BS, and hence the second term in (3) accounts for the SI. In the literature, different SI mitigation techniques have been proposed in addition to digital cancellation [2], [3] to reduce the effect of self-interference though the SI cannot be canceled completely due to the limited dynamic range at the receiver [3]. In order to isolate our proposed scheme from the specific implementation of a SI mitigation technique, we consider the worst-case performance based on deterministic model to model the residual-SI channel after cancellation, that is known imperfectly at the BS such that $\mathbf{G} = \check{\mathbf{G}} + \Delta\mathbf{G}$, where $\check{\mathbf{G}}$ denotes the SI channel estimate known to the FD BS and $\Delta\mathbf{G}$ represents the SI channel uncertainties, which are assumed to be bounded such that $\|\Delta\mathbf{G}\|^2 \leq \epsilon_G^2$, for some $\epsilon_G \geq 0$.

Based on the uplink and downlink received signals, we define the signal-to-interference plus noise ratio (SINR) at the *i-th* downlink user and at the FD radio BS respectively as

$$\mathrm{SINR}_i^{DL} = \frac{|\ \mathbf{h}_i^H \mathbf{w}_i\ |^2}{\sum_{k\neq i}^{K} |\ \mathbf{h}_i^H \mathbf{w}_k\ |^2 + \sigma_i^2}, \tag{4}$$

$$\mathrm{SINR}_j^{UL} = \frac{P_j\ |\ \mathbf{f}_j^H \mathbf{u}_j\ |^2}{\sum_{n\neq j}^{J} P_n\ |\ \mathbf{f}_n^H \mathbf{u}_j\ |^2 + \sum_{k=1}^{K} |\ \mathbf{u}_j^H \mathbf{G} \mathbf{w}_k\ |^2 + \sigma_N^2 \|\mathbf{u}_j\|^2}, \tag{5}$$

where $\mathbf{u}_j \in^{N\times 1}$ is the receive beamforming vector for detecting the received symbol from the *j-th* uplink user.

We note that in this paper, following [21], [22], we assume perfect channel state information (CSI) for the uplink and downlink channels.

III. MOOP Formulation for Interference Cancellation

In this section, we study the two desirable system objectives, namely the total downlink and uplink transmit power, jointly through the multi-objective optimization problem (MOOP) formulation, subject to the SINR constraints (4) and (5), where interference is suppressed. Complementary to existing MOOP approaches for FD that assume perfect knowledge of the SI channel, here we present a robust optimization. The MOOP is employed when there is need to study jointly the trade-off between two desirable objectives via the concept of Pareto optimality. A point is said to be Pareto optimal if there is no other point that improves any of the objectives without decreasing the others [26]. In [26], a survey of multi-objective optimization methods in engineering was presented. By using the weighted Tchebycheff method [26] which can achieve the complete Pareto optimal set with lower computational complexity. Hence, the MOOP that jointly minimizes the total downlink and uplink transmit power for the considered FD system can be formulated following [21], [22] as

$$\begin{aligned}
\mathcal{P}1: \quad & \min_{\mathbf{w}_i, P_j, \mathbf{u}_j} \quad \max_{a=1,2} \left\{\lambda_a \left(R_a^* - R_a\right)\right\} \\
\text{s.t.} \quad & \mathrm{A1}: \frac{|\ \mathbf{h}_i^H \mathbf{w}_i\ |^2}{\sum_{k\neq i}^{K} |\ \mathbf{h}_i^H \mathbf{w}_k\ |^2 + \sigma_i^2} \geq \Gamma_i^{DL}, \forall i, \\
& \mathrm{A2}: \frac{P_j\ |\ \mathbf{f}_j^H \mathbf{u}_j\ |^2}{I_j + \sigma_N^2 \|\mathbf{u}_j\|^2} \geq \Gamma_j^{UL}, \forall\, \|\Delta\mathbf{G}\|^2 \leq \epsilon_G^2, \forall j,
\end{aligned} \tag{6}$$

where, $I_j = \sum_{n\neq j}^{J} P_n\ |\ \mathbf{f}_n^H \mathbf{u}_j\ |^2 + \sum_{k=1}^{K} |\ \mathbf{u}_j^H (\check{\mathbf{G}} + \Delta\mathbf{G}) \mathbf{w}_k\ |^2$ and we define Γ_i^{DL} and Γ_j^{UL} as the minimum required SINRs for the *i-th* downlink user and the *j-th* uplink user, respectively. We denote $R_1 = \sum_{i=1}^{K} \|\mathbf{w}_i\|^2$ and $R_2 = \sum_{j=1}^{J} P_j$ as the two desirable system objectives, and R_1^* and R_2^* are constants representing their optimal values, respectively. The variable $\lambda_a \geq 0$, $\sum \lambda_a = 1$, specifies the priority given to the *a-th* objective. By varying λ_a we can obtain the complete Pareto optimal set. Problem $\mathcal{P}1$ is a non-convex problem due to the SINR constraints A1 and A2. To tackle the non-convexity, we transform $\mathcal{P}1$ as a semi-definite program (SDP) problem given by

$$\widetilde{\mathcal{P}1}: \min_{\mathbf{W}_i, P_j, \mathbf{U}_j} \max_{a=1,2} \left\{\lambda_a \left(R_a^* - R_a\right)\right\}$$

s.t.

$$\begin{aligned}
&\widetilde{\text{A1}}: \frac{\text{Tr}\left\{\mathbf{H}_i^H \mathbf{W}_i\right\}}{\text{Tr}\left\{\mathbf{H}_i^H \mathbf{W}_k\right\} + \sigma_i^2} \geq \Gamma_i^{DL}, \forall i,\\
&\widetilde{\text{A2a}}: \frac{P_j \text{Tr}\left\{\mathbf{F}_j^H \mathbf{U}_j\right\}}{\sum_{n\neq j}^{J} P_n \text{Tr}\left\{\mathbf{F}_n^H \mathbf{U}_j\right\} + s_j^{SI} + \sigma_N^2 \text{Tr}\left\{\mathbf{U}_j\right\}} \geq \Gamma_j^{UL}, \forall j\\
&\widetilde{\text{A2b}}: \text{Tr}\left\{\left(\check{\mathbf{G}} + \Delta\mathbf{G}\right) \sum_{k=1}^{K} \mathbf{W}_k \left(\check{\mathbf{G}} + \Delta\mathbf{G}\right)^H \mathbf{U}_j\right\} \leq s_j^{SI}, \forall j,\\
&\forall \left\|\Delta\mathbf{G}\right\|^2 \leq \epsilon_G^2, \forall j,
\end{aligned} \tag{7}$$

where, $\mathbf{W}_k = \mathbf{w}_k \mathbf{w}_k^H, \mathbf{F}_j = \mathbf{f}_j \mathbf{f}_j^H$ and $\mathbf{U}_j = \mathbf{u}_j \mathbf{u}_j^H$. Here, we rewrite constraint A2 into two constraints by introducing slack variables $s_j^{SI} > 0, \forall j$, respectively. Next, we review the definitions of the S-procedure for completeness.

Lemma 1. (S-procedure [27]): Let $g_l(\mathbf{x})$, $l = 1, 2$, be defined as

$$g_l(\mathbf{x}) = \mathbf{x}^H \mathbf{A}_l \mathbf{x} + 2Re\left\{\mathbf{b}_l^H \mathbf{x}\right\} + c_l,$$

where $\mathbf{A}_l \in \mathbb{C}^{n\times n}, \mathbf{b}_l \in \mathbb{C}^n$ and $c_l \in \mathbb{R}$. Then, the implication of $g_1(\mathbf{x}) \geq 0 \Rightarrow g_2(\mathbf{x}) \geq 0$ holds if and only if there exists a $\lambda \geq 0$ such that

$$\lambda \begin{bmatrix} \mathbf{A}_1 & \mathbf{b}_1 \\ \mathbf{b}_1^H & c_1 \end{bmatrix} - \begin{bmatrix} \mathbf{A}_2 & \mathbf{b}_2 \\ \mathbf{b}_2^H & c_2 \end{bmatrix} \succeq 0,$$

provided there exists a point $\hat{\mathbf{x}}$ with $g_1(\hat{\mathbf{x}}) > 0$.

Following Lemma 1 and by using the fact that $\text{Tr}\left\{\mathbf{ABCD}\right\} = \text{vec}\left(\mathbf{A}^H\right)^H \left(\mathbf{D}^H \otimes \mathbf{B}\right) \text{vec}\left(\mathbf{C}\right)$, $(\widetilde{\text{A2b}})$ can be expanded as

$$\Delta\mathbf{g}^H \left(\mathbf{U}_j \otimes \sum_{k=1}^{K} \mathbf{W}_k\right) \Delta\mathbf{g} + 2Re\left\{\check{\mathbf{g}}^H \left(\mathbf{U}_j \otimes \sum_{k=1}^{K} \mathbf{W}_k\right) \Delta\mathbf{g}\right\}$$
$$+ \check{\mathbf{g}}^H \left(\mathbf{U}_j \otimes \sum_{k=1}^{K} \mathbf{W}_k\right) \check{\mathbf{g}} + \sigma_N^2 \text{Tr}\left\{\mathbf{U}_j\right\} - s_j^{SI} \leq 0, \forall j, \tag{8a}$$

$$\Delta\mathbf{g}^H \mathbf{I} \Delta\mathbf{g} - \epsilon_G^2 \leq 0. \tag{8b}$$

We define $\check{\mathbf{g}} = \text{vec}\left(\check{\mathbf{G}}^H\right)$ and $\Delta\mathbf{g} = \text{vec}\left(\Delta\mathbf{G}^H\right)$ where, $\text{vec}\left(\cdot\right)$ stacks the columns of a matrix into a vector and $\otimes$ stands for Kronecker product. Hence, according to Lemma 1, (8a) and (8b) hold if and only if there exist a $\rho \geq 0$ such that

$$\begin{bmatrix} \rho\mathbf{I} - \mathbf{Z}_j & -\mathbf{Z}_j \check{\mathbf{g}} \\ -\check{\mathbf{g}}^H \mathbf{Z}_j & s_j^{SI} - \check{\mathbf{g}}^H \mathbf{Z}_j \check{\mathbf{g}} - \rho\epsilon_G^2 \end{bmatrix} \succeq 0, \forall j, \tag{9}$$

where, $\mathbf{Z}_j = \left(\mathbf{U}_j \otimes \sum_{k=1}^{K} \mathbf{W}_k\right)$. Hence, the transformed $\mathcal{P}1$ is given as

$$\begin{aligned}
\mathcal{P}2: &\min_{\mathbf{W}_i, P_j, \mathbf{U}_j, t, \rho, s_j^{SI}} \quad t\\
\text{s.t. } &\widetilde{\text{A1}}: \text{Tr}\left\{\mathbf{H}_i^H \mathbf{W}_i\right\} - \Gamma_i^{DL} \text{Tr}\left\{\mathbf{H}_i^H \mathbf{W}_k\right\} \geq \Gamma_i^{DL} \sigma_i^2, \forall i,\\
&\widetilde{\text{A2a}}: P_j \text{Tr}\left\{\mathbf{F}_j^H \mathbf{U}_j\right\} - \Gamma_j^{UL} \left(\sum_{n\neq j}^{J} P_n \text{Tr}\left\{\mathbf{F}_n^H \mathbf{U}_j\right\} + s_j^{SI}\right)\\
&\qquad \geq \Gamma_j^{UL} \sigma_N^2 \text{Tr}\left\{\mathbf{U}_j\right\}, \forall j,\\
&\widetilde{\text{A2b}}: \begin{bmatrix} \rho\mathbf{I} - \mathbf{Z}_j & -\mathbf{Z}_j \check{\mathbf{g}} \\ -\check{\mathbf{g}}^H \mathbf{Z}_j & s_j^{SI} - \check{\mathbf{g}}^H \mathbf{Z}_j \check{\mathbf{g}} - \rho\epsilon_G^2 \end{bmatrix} \succeq 0, \forall j,\\
&\text{A3}: \lambda_a \left(R_a^* - R_a\right) \leq t, \forall a \in \{1, 2\},\\
&\text{A4}: \mathbf{W}_i \succeq 0, \forall i, \quad \text{A5}: \mathbf{U}_j \succeq 0, \forall j.
\end{aligned} \tag{10}$$

where, t is an auxiliary variable.

The problem $\mathcal{P}2$ is still non-convex due to the joint optimization of P_j and $\mathbf{U}_j$ in constraint $\widetilde{\text{A2a}}$. It is very difficult to obtain a closed-form solution that jointly optimizes $\mathbf{W}_i$, P_j and $\mathbf{U}_j, \forall i, j$, hence, to solve $\mathcal{P}2$, we propose a two-step iterative process. In the first step, we initialize a feasible $\mathbf{U}_j$ to solve problem $\mathcal{P}2$ obtaining the optimal values of $\mathbf{W}_i$ and P_j, using CVX [28]. In the second step, we solve problem $\mathcal{P}3$ below using the solution from the first step to obtain the optimal $\mathbf{U}_j^*$. We summarize the overall procedure in Algorithm 1.

Algorithm 1 Procedure for solving the problem $\mathcal{P}2$

1: **Input** : $\mathbf{h}_i, \mathbf{f}_j, \check{\mathbf{G}}, \Gamma_j^{DL}, \Gamma_j^{UL}, \sigma_i, \sigma_N$.
2: *Initialise:* $n = 0, \mathbf{U}_\mathbf{j}^{(\mathbf{0})} = \mathbf{u}_j \mathbf{u}_j^H$,
repeat,
3: $n = n + 1$,
4: solve the problem $\mathcal{P}2$ to obtain $\mathbf{W}_i$ and P_j,
5: use the solution from step 4 to solve the problem $\mathcal{P}3$ to obtain $\mathbf{U}_\mathbf{j}^{(\mathbf{n})}$,
6: until convergence.
7: **Output** : $\mathbf{W}_i^*$ and $P_j^*, \forall i, j$.

$$\begin{aligned}
\mathcal{P}3: &\min_{\mathbf{U}_j, \tau, \rho, s_j^{SI}} \tau\\
\text{s.t. } &\widetilde{\text{A2a}}, \quad \widetilde{\text{A2b}},\\
&\mathbf{U}_j \succeq 0, \forall j,\\
&\lambda_a \left(R_a^* - R_a\right) \leq \tau, \forall a \in \{1, 2\}.
\end{aligned} \tag{11}$$

In problem $\mathcal{P}3$, we ignore constraint $\widetilde{\text{A1}}$ since it is independent of $\mathbf{U}_j$. Note that the problem $\mathcal{P}2$ and $\mathcal{P}3$ are relaxed form of $\mathcal{P}1$. While it is difficult to prove the rank-one solution, we have observed over multiple simulations, problem $\mathcal{P}2$ and $\mathcal{P}3$ always return rank-one solution $(\mathbf{W}_i, \forall i)$ and $(\mathbf{U}_j, \forall j)$. Nevertheless, in the unlikely case of a non rank-one solution the optimal solutions can always be obtained by randomization technique as in [29], such that $\mathbf{W}_i = \mathbf{w}_i \mathbf{w}_i^H, \forall i$ and $\mathbf{U}_j = \mathbf{u}_j \mathbf{u}_j^H, \forall j$, respectively.

IV. MOOP based on Constructive Interference

In this section, we study the considered FD system design by exploiting the interference rather than suppressing it as in the conventional cases. To be precise, constructive interference (CI) is the interference that pushes the received signal further into the detection region of the constellation and away from the detection threshold [12]. This concept has been thoroughly studied in the literature for both PSK and QAM modulation. We refer the reader to [11], [12] for further details of this topic. Motivated by this idea, here, we apply this concept to the MOOP formulation in Section III for PSK modulations. We note that constructive interference is only applied to the downlink users and not the uplink users following that only the prior knowledge of the CSI and users' data symbols for the downlink users are available at the BS. Nevertheless, we show in the following that power savings can be obtained for both uplink and downlink transmission, by means of the MOOP design.

As detailed in [12], for any given modulation order (M)-PSK constellation point, to guarantee CI, the received signal (y_i) must fall within the CI region of the constellation. The size of the region is determined by $\theta = \pm\frac{\pi}{M}$, which is the maximum angle shift of the CI region for a modulation order M. Accordingly, it was shown in [12] that the downlink SINR constraint that guarantees CI at the *i-th* downlink user is given by

$$\left|\Im\left(\mathbf{h}_i^H \sum_{k=1}^{K} \mathbf{w}_k e^{j(\phi_k-\phi_i)}\right)\right| \leq \left(\Re\left(\mathbf{h}_i^H \sum_{k=1}^{K} \mathbf{w}_k e^{j(\phi_k-\phi_i)}\right) - \sqrt{\Gamma_i^{DL}\sigma_i^2}\right)\tan\theta, \quad (12)$$

where the unit symbol d_i for the *i-th* downlink user is represented as de^{ϕ_i} with a phase angle of ϕ_i.

Based on (12), we can modify the SINR constraints for the downlink users to accommodate for CI. The MOOP based on CI can be expressed as

$$\begin{aligned}
\mathcal{P}4:\ &\min_{\mathbf{w}_i, P_j, \mathbf{u}_j, t}\ t\\
\text{s.t. B1}:\ &\left|\Im\left(\mathbf{h}_i^H \sum_{k=1}^{K} \mathbf{w}_k e^{j(\phi_k-\phi_i)}\right)\right| \leq \left(\Re\left(\mathbf{h}_i^H \sum_{k=1}^{K} \mathbf{w}_k e^{j(\phi_k-\phi_i)}\right) - \sqrt{\Gamma_i^{DL}\sigma_i^2}\right)\tan\theta, \forall i,\\
\text{B2}:\ &\frac{P_j\left|\mathbf{f}_j^H\mathbf{u}_j\right|^2}{I_j^{PSK} + \sum_{k=1}^{K}\left|\mathbf{u}_j^H(\check{\mathbf{G}}+\Delta\mathbf{G})\mathbf{w}_k e^{j(\phi_k-\phi_1)}\right|^2} \geq \Gamma_j^{UL},\\
&\forall \|\Delta\mathbf{G}\|^2 \leq \epsilon_G^2, \forall j,\\
\text{B3}:\ &\lambda_a\left(R_a^* - R_a(\mathbf{w}_i, P_j)\right) \leq t, \forall a \in \{1,2\},
\end{aligned} \quad (13)$$

where t is an auxiliary variable, and $I_j^{PSK} = \sum_{n\neq j}^{J} P_n\left|\mathbf{f}_n^H\mathbf{u}_j\right|^2 + \sigma_N^2\|\mathbf{u}_j\|^2$. Here $R_1 = \left\|\sum_{k=1}^{K}\mathbf{w}_k e^{j(\phi_k-\phi_1)}\right\|^2$ and $R_2 = \sum_{j=1}^{J} P_j$.

Note that, w.r.t. [23] here we include the receive beamforming in the MOOP instead of assuming a ZF receiver. It can be observed that, due to the substitution of the conventional downlink SINR constraint with the CI SINR constraint, the constraint B1 is now convex. Furthermore, by introducing slack variables $s_j^{SI} > 0$, constraint B2 can be rewritten as the following two constraints,

$$P_j\left|\mathbf{f}_j^H\mathbf{u}_j\right|^2 - \Gamma_j^{UL}\left(\sum_{n\neq j}^{J} P_n\left|\mathbf{f}_n^H\mathbf{u}_j\right|^2 + s_j^{SI} + \sigma_N^2\|\mathbf{u}_j\|^2\right) \geq 0, \forall j, \quad (14)$$

$$\sum_{k=1}^{K}\left|\mathbf{u}_j^H(\check{\mathbf{G}}+\Delta\mathbf{G})\mathbf{w}_k e^{j(\phi_k-\phi_1)}\right|^2 \leq s_j^{SI}, \forall\ \|\Delta\mathbf{G}\|^2 \leq \epsilon_G^2, \forall j. \quad (15)$$

Notice that (15) can be guaranteed by the following modified constraint

$$\max_{\|\Delta\mathbf{G}\|^2\leq\epsilon_G^2} \quad \sum_{k=1}^{K}\left|\mathbf{u}_j^H(\check{\mathbf{G}}+\Delta\mathbf{G})\mathbf{w}_k e^{j(\phi_k-\phi_1)}\right|^2 \leq s_j^{SI}, \forall j, \quad (16)$$

By using the fact that $\|\mathbf{x}+\mathbf{y}\|^2 \leq (\|\mathbf{x}\|+\|\mathbf{y}\|)^2$, (16) can always be guaranteed by the following constraint

$$\max_{\|\Delta\mathbf{G}\|^2\leq\epsilon_G^2} \sum_{k=1}^{K}\left(\left|\mathbf{u}_j^H\check{\mathbf{G}}\mathbf{w}_k e^{j(\phi_k-\phi_1)}\right| + \left|\mathbf{u}_j^H\Delta\mathbf{G}\mathbf{w}_k e^{j(\phi_k-\phi_1)}\right|\right)^2 \leq s_j^{SI}, \forall j, \quad (17)$$

whose worst-case formulation is given by

$$\sum_{k=1}^{K}\left(\left|\mathbf{u}_j^H\check{\mathbf{G}}\mathbf{w}_k e^{j(\phi_k-\phi_1)}\right| + \epsilon_G\left\|\mathbf{u}_j^H\right\|\left\|\mathbf{w}_k e^{j(\phi_k-\phi_1)}\right\|\right)^2 \leq s_j^{SI}, \forall j. \quad (18)$$

Hence, the MOOP based on CI can be expressed as

$$\begin{aligned}
\mathcal{P}5:\ &\min_{\mathbf{w}_i, P_j, \mathbf{u}_j, t, s_j^{SI}}\ t\\
\text{s.t.}\ &\\
\text{B1}:\ &\left|\Im\left(\mathbf{h}_i^H \sum_{k=1}^{K} \mathbf{w}_k e^{j(\phi_k-\phi_i)}\right)\right| \leq \left(\Re\left(\mathbf{h}_i^H \sum_{k=1}^{K} \mathbf{w}_k e^{j(\phi_k-\phi_i)}\right) - \sqrt{\Gamma_i^{DL}\sigma_i^2}\right)\tan\theta, \forall i,\\
\widetilde{\text{B2a}}:\ &P_j\left|\mathbf{f}_j^H\mathbf{u}_j\right|^2 \geq \Gamma_j^{UL}\left(\sum_{n\neq j}^{J} P_n\left|\mathbf{f}_n^H\mathbf{u}_j\right|^2 + s_j^{SI} + \sigma_N^2\|\mathbf{u}_j\|^2\right), \forall j,\\
\widetilde{\text{B2b}}:\ &\sum_{k=1}^{K}\left(\left|\mathbf{u}_j^H\check{\mathbf{G}}\mathbf{w}_k e^{j(\phi_k-\phi_1)}\right| + \epsilon_G\left|\mathbf{u}_j^H\mathbf{w}_k e^{j(\phi_k-\phi_1)}\right|\right)^2 \leq s_j^{SI}, \forall j,\\
\text{B3}:\ &\lambda_a\left(R_a^* - R_a\right) \leq t, \forall a \in \{1,2\}.
\end{aligned} \quad (19)$$

Problem $\mathcal{P}5$ is still non-convex due to the joint optimization of P_j and $\mathbf{u}_j$ in constraint $\widetilde{\text{B2a}}$. Hence, we propose a two-step iterative process to jointly optimize $\mathbf{w}_i$, P_j and $\mathbf{u}_j, \forall i, j$. In the first step, we initialize a feasible $\mathbf{u}_j, \forall j$ to solve $\mathcal{P}5$ to obtain the optimal values of $\mathbf{w}_i$ and P_j using CVX [28]. Then we use the values obtained from the first step to solve $\mathcal{P}6$ below to obtain the optimal values of the $\mathbf{U}_j{}^*, \forall j$. The iterative process is summarized in Algorithm 2.

$$\begin{aligned}
\mathcal{P}6: \quad & \min_{\mathbf{U}_j,\tau,\rho,s_j^{SI}} \quad \tau \\
\text{s.t.} \quad & P_j \mathrm{Tr}\left\{\mathbf{F}_j^H \mathbf{U}_j\right\} - \Gamma_j^{UL}\left(\sum_{n\neq j}^{J} P_n \mathrm{Tr}\left\{\mathbf{F}_n^H \mathbf{U}_j\right\} + s_j^{SI}\right) \\
& \qquad\qquad\qquad\qquad \geq \Gamma_j^{UL}\sigma_N^2 \mathrm{Tr}\left\{\mathbf{U}_j\right\}, \forall j, \\
& \begin{bmatrix} \rho\mathbf{I} - \mathbf{Z}_j & -\mathbf{Z}_j\breve{\mathbf{g}} \\ -\breve{\mathbf{g}}^H\mathbf{Z}_j & s_j^{SI} - \breve{\mathbf{g}}^H\mathbf{Z}_j\breve{\mathbf{g}} - \rho\epsilon_G^2 \end{bmatrix} \geq 0, \forall j, \\
& \lambda_a\left(R_a^* - R_a\right) \leq \tau, \forall a \in \{1,2\}, \mathbf{U}_j \geq 0, \forall j.
\end{aligned} \tag{20}$$

Algorithm 2 Procedure for solving the problem $\mathcal{P}5$

1: **Input** : $\mathbf{h}_i, \mathbf{f}_j, \breve{\mathbf{G}}, \Gamma_j^{DL}, \Gamma_j^{UL}, \sigma_i, \sigma_N$.
2: *Initialize:* $n = 0, \mathbf{u}_j$,
repeat,
3: $n = n + 1$,
4: solve the problem $\mathcal{P}5$ to obtain $\mathbf{w}_i$ and P_j,
5: use the solution from step 4 to solve the problem $\mathcal{P}6$ to obtain $\mathbf{U}_\mathbf{j}^{(\mathbf{n})}$,
6: until convergence.
7: **Output** : $\mathbf{w}_i^*$ and $P_j^*, \forall i, j$.

Note that the optimal $\mathbf{u}_j^*$ can be obtained by randomization as in [29].

V. Results

In this section, we investigate the performance of our proposed iterative schemes through Monte Carlo simulations. We consider the system with the FD BS at the center of a cell with $N = 4$ antennas, each for transmitting and receiving. We assume $K = J = 4$ downlink and uplink users, are randomly and uniformly distributed. We model the channels to the uplink and downlink users as Rayleigh fading. The SI channel is modeled as Rician fading channel with Rician factor 5dB. Systems with QPSK modulation are considered while it is clear that the benefit extends to any higher order modulation. For comparison in every scenario, we compare the iterative approach with the approach in [21]–[23] for both CI and conventional cases, where ZF beamforming is used for the detection of uplink signals at the FD BS. We consider $\epsilon_G^2 = 0.1, \sigma_i^2 = -20\text{dB}, \forall i, \sigma_N^2 = -30\text{dB}, \forall j$ and $\Gamma_j^{\text{UL}} = 0\text{dB}, \forall j$, respectively. In addition, we select $\lambda_1 = 0.9$ and $\lambda_2 = 0.1$, which gives higher priority to the downlink users, respectively, though it is intuitive that the gains of the proposed schemes extend to other simulation set-ups and parameters.

In Fig. 2, we show the average power consumption versus the minimum required downlink SINR (Γ_i^{DL}). It can be observed that both the uplink and downlink power consumption increase with increase in Γ_i^{DL}. This is because an increase in the downlink SINR requirement translates to increase in downlink transmit power and hence increase in the SI power. Therefore, the uplink users have to transmit with a higher power to meet their QoS requirement (Γ_j^{UL}). However, we can still see power savings of up to 4.5dB and 0.5dB for the uplink and downlink users, respectively, for the proposed iterative schemes compared to the baseline ZF schemes in [21]–[23]. In addition, we note that while CI is applied to only the downlink users, more power is saved for the uplink users than the downlink users. This is because with CI the total downlink transmit power is reduced and this directly reduces the residual SI power at the FD BS. Therefore, the constructive interference power has been traded off for both uplink and downlink power savings.

Fig. 3 illustrates a direct performance-complexity comparison in the form of average power consumption versus the resulting complexity in terms of maximum number of iterations in Algorithms 1-2. We can clearly see that the proposed approach provides a scalable trade-off which allows power gains w.r.t. baseline schemes in [21]–[23] at a controllable complexity increase.

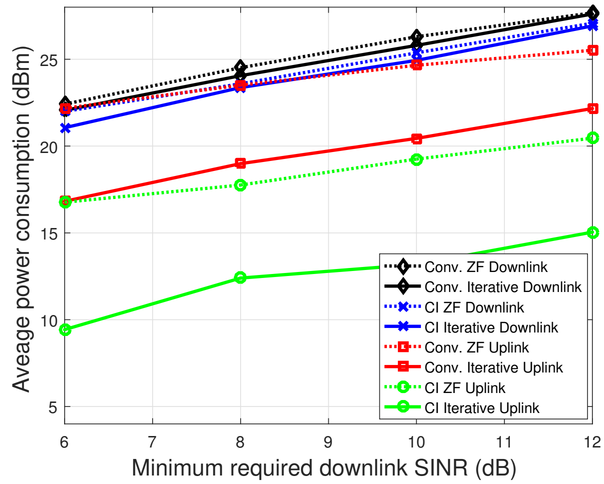

Fig. 2. Average power consumption versus the minimum required downlink SINR given $N = K = J = 4$ for QPSK modulation.

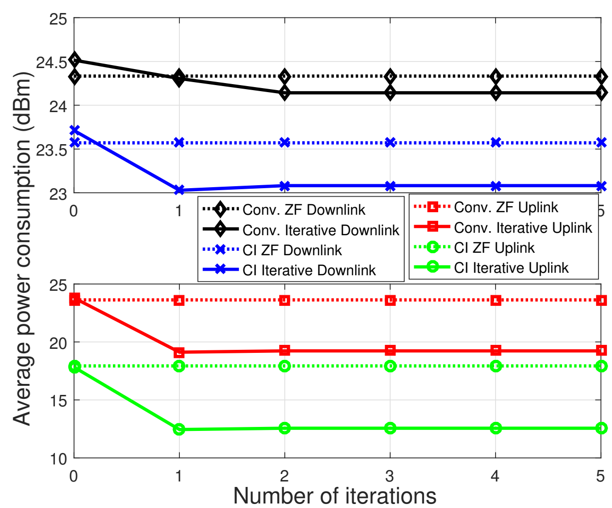

Fig. 3. Average power consumption versus number of iterations given $N = K = J = 4$ and $\Gamma^{\text{DL}} = 8\text{dB}$, for QPSK modulation.

VI. Conclusion

In this paper we proposed a two-step iterative algorithm for FD transmission. The algorithm jointly optimizes the transmit beamformer, receive beamformer and uplink transmit power, providing a step forward from [21]–[23] in the literature. Simulations results show power savings compared to the ZF approaches in both conventional and CI cases at the expense of a scalable complexity increase.

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