Introduction to the papers of TWG16: Learning Mathematics with Technology and Other Resources

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The use of technology and other resources for mathematical learning is a current issue in the field of mathematics education and lags behind the rapid advances in Information and Communication Technology. Technological developments offer opportunities, which are not straightforward to exploit in regular teaching. In CERME10 TWG16, the recent research findings, issues and future questions have been explored and discussed in detail. In this introductory chapter, we will outline the scope and focus of the work, describe the results with respect to existing questions, and identify upcoming topics as well as missing topics that might set the agenda for future work in this domain.

Keywords: Digital resources, mathematical learning, educational technologies.

Scope and focus of the Working Group

In recent years, discussions within the CERME-technology-group have confirmed the relevance of Information and Communication Technology (ICT) for the learning of mathematics. ICT provides a range of resources, such as software, handheld devices and online classroom activities. This range of resources has been compared to non-digital resources, such as textbooks, worksheets and other types of tools and manipulatives. The impact of both digital and non-digital resources on mathematical learning has been of great interest to our working group. The scope of this working group was to explore and discuss opportunities and possibilities, as well as challenges and limitations, of technological resources for student learning. We wanted to establish an overview of the current state of the art in the use of technology in mathematics education, including both practice-oriented experiences and research-based evidence, as seen from an international perspective and with a focus on student learning, as well as to suggest important trends for technology-rich mathematics education in the future, including a research agenda. TWG 15 is closely related to this theme, but focuses on the teachers' roles and practices.

In the pre-conference call for papers and poster proposals, theoretical, methodological, empirical or developmental contributions were particularly welcomed on the following topics:

- Analyses of the impact of using digital and non-digital technology on students' learning;
- New forms of digital resources, including mobile devices and dynamic e-textbooks;
- Digital assessment of and for learning;
- E-learning, blended mathematics education and (Massive Open) Online Courses for mathematics;
- Influence and use of social media in students' perception of learning mathematics;
- Promoting communication and collaborative work between students through ICT;
- Using ICT for out-of-school informal mathematics learning;
- Examples of the use of technologies devoted to the support of students with disabilities.

This introduction provides an overview of the 24 presented papers and 6 posters and the discussions in TWG16 building up on theories and past research on digital technologies and other resources for mathematical learning. We especially refer to the CERME history of this technology group and consider the results of the 2017 conference as a continuation of the background, aims and scope of the conferences since 1999 (Trgalova, Clark-Wilson & Weigand, to appear). To do so, we will first address "old" questions, then describe upcoming topics, and close off with topics we missed.

Taking up "old" questions

Some contributions to TWG16 continued the discussion on topics that had been addressed in the past, such as the potential of digital tools to evoke the dynamical aspect of manipulating objects within a digital tool, functional thinking, and the use of e-books.

Interactivity, dynamics and multiple representations

Since the early years of using digital technologies in mathematics education in the 1970s and 1980s, interactivity, dynamic and multiple representations played an important role in developing new strategies for understanding mathematical concepts. Dynamic manipulations were prominently present in dragging opportunities in Dynamic Geometry Systems (e.g. Leung, 2008). Digital technologies created easy access to multiple representations and interactions between the user and the software (e.g. Noss & Hoyles, 1996; Moreno-Armella, Hegedus & Kaput, 2008). On a more elaborated level, the interactions between the *knowledge*, the *tool* and the *learner* built three main aspects of digital technologies and were also strongly represented in TWG16 of CERME 10.

Dynamic digital tools can promote conceptual understanding (e.g. Drijvers, 2015) and potentially support low-achieving students. An example is the interactive environment presented by Swidan, Daher and Darawsha, to support the learning of the concept of equivalent equations. An applet gives the possibility to work with numerical, algebraic and/or graphical representations. Moreover, a pan balance represents enactive experiments with "weights" and a slider allows to dynamically change the x-values. The idea is to represent enactive actions and to allow students to work with a visual mediator while changing mathematical objects. The difficulties, limits and obstacles of working with multiple representations are also highlighted. Low-achieving students, for example, can become overwhelmed when faced with a large number of representations, which may prevent their progress. The consequence is *not* to avoid working with multiple representations, but to create didactical reflected learning environments with a successive introduction of multiple representations and reciprocal interpretation of the transition between these representations.

Functional thinking

Another "old" question concerns the prototypical dynamic view of functions while filling bowls with water and asking for the height of water in a bowl as a function of the volume of water in the bowl (Carlson et al., 2002). Lisarelli's contribution to TWG16 involved the outcomes of investigating different dragging modalities in the frame of the above-mentioned problem, as shown in Figure 1. Users had to be familiar with different kinds of dragging possibilities: (quasi) continuous dragging, discrete dragging (e.g. if only natural numbers are allowed), or impossible dragging, (i.e. where the user tries to drag a dependent point). She argued for the importance of recognizing the aim for a specific type of dragging and considering whether it is a random movement, a movement for testing possibilities or a guided dragging to reach a special configuration. Such a classification of dragging modalities gives the possibility to observe, describe and analyze students' processes involved in the exploration and solution of dynamic problem solving activities. This example shows clearly a digital tool as a medium, which is – or mediates – between the user and the mathematical concepts.

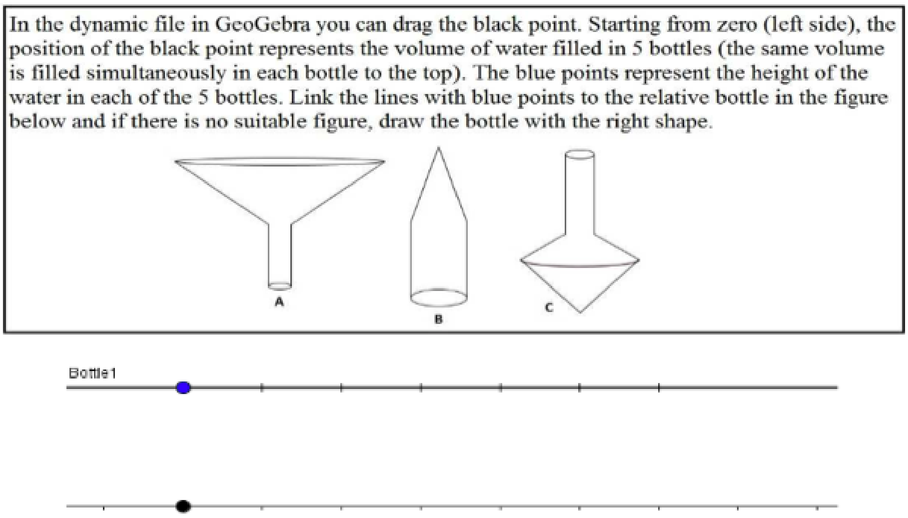

Figure 1. The Bottle Problem task and its dynamic representation

The interactive worksheets presented by Lindenbauer and Lavicza focus on functional thinking through a situational model (the area of a triangle) and a related graphical representation. The explanation and interpretation of the graphical representation is – especially for lower achieving students – challenging and as the author stated, the help of the teacher may be crucial. These graphical representations allow students to reflect on what the impact of moving the point on the x-axis is by showing the small or big changes to the area of a triangle. Such an approach provides students with an intuitive access to the concept of rate of change.

E-books

A great variety of digital books or e-books for classroom use exists. Such books may be more or less extended versions of the traditional schoolbooks, including dynamic activities and in-built assessments (Gueudet et al., 2017). The "Creative Electronic Book on Reflection" presented by Geraniou and Mavrikis allows students to explore mathematics situations individually and interactively, and it also encourages them to reflect on their actions while they are exploring and solving mathematical tasks. A key role in the students' reflection is played by the so-called "bridging activities" which emphasize the mathematics integrated into the book. As claimed by the authors, the design and evaluation of such interactive learning environments, learning paths or trajectories and the promotion of their wider use in classrooms is a new challenge.

Theories

The discussion on theoretical approaches regarding digital technologies for mathematical learning is also an on-going one within CERME (Trgalova, Clark-Wilson & Weigand 2017). There are some well-developed and experimentally confirmed theories like semiotic mediation (Bartolini Bussi & Mariotti, 2008), instrumental genesis (Trouche, 2004) or the documentational approach (Gueudet & Trouche, 2009), which are also used in many papers and discussions in TWG16. Murphy and Calder, for example, applied a framework including social semiotics and multimodality to interpret screen casts of students working in a problem-solving application on an ipad, to understand the learning that took place.

In spite of theoretical developments in the field (e.g., see Monaghan, Trouche & Borwein, 2017), Schacht's was the only contribution to TWG16 that paid attention to a new theoretical field. Taking an inferential perspective, he investigated the relationships between mathematical and tool language while working with digital technologies and the transition – or non-transition – from one to the other. He showed how the way in which this transition can be accomplished can have implications on the individual concept formation processes. He especially emphasized the meaning – but also the obstacles – of the transition in the language use (by students) from a tool-oriented language to a mathematical-oriented language. The philosophical discourse about the concept of "digital" (see Galloway, 2014) - "Any discourse that produces or maintains differences between two or more elements can be labelled digital" (Schacht) - might give orientation also in the evaluation of the language transfer in mathematics education.

Upcoming topics

The continuous development of technological tools, which are used both in and out of school, requires us to address old questions under a new perspective. On one hand, this new perspective has to consider new developments in hardware (tablets, smartphones) but also in software (social media, cloud computing). On the other hand, we have to consider new developments in society, science and (mathematics) education, for example with respect of online communication without any limitations in time and space. Goals in education have to be continually rethought and evaluated.

3D-geometry

Regarding the future development and progress of our working group, there are different topics for which we see the potential for further investigations. Kynigos and Zantzos presented a study, during which students were asked to construct the shortest path between two points on a cylindrical surface. To solve the problem, they had to see the relationship between 3D- and 2D-geometry and activated the "old idea" of a turtle geometry which allowed access to difficult concepts like the curvature of a special surface.

MOOCs and new kinds of e-learning

A second aspect is the meaning and the impact on mathematical learning of free available massive open online courses (MOOCs). Khan Academy[1] offers a free tool that allows teachers to monitor students' activity and provide them with feedback and guidance. Vančura used this tool at a Czech high school to provide feedback for students' homework. The investigation showed that weak knowledge of the English language might not be a barrier for students. Vančura also sees the danger of using such courses just for the training of algorithms without developing knowledge of underlying mathematical concepts.

Gray, Lindstrøm and Vestli also used the Khan Academy (KA) tool for pre-service teachers in mathematics who were allowed to substitute their compulsory mathematics assignment with exercises

[1] www.khanacademy.org/ (06.04.2017)

in KA. They compared their results with those of a control group, learning in the traditional way. At the end, there was no statistically significant difference in the performance of the two groups.

It is an open question whether MOOCs or SPOCs[2] will have an influence on the teaching and learning at schools and universities. Nevertheless, identifying good ways of e-learning will remain important, whether open resources on the internet or special courses integrated in learning management systems are used.

Tablets

Since the very first CERME conference, an important question has always been what kind of interactions take place between the tool and the learner. The goal has always been to bring the individual into the centre of learning. Digital technologies can mediate between mathematics and understanding. Nowadays, the relatively straightforward and intuitive use of digital technologies in the form of laptops and smartphones gives users the chance to not put too much emphasis on the technical aspects of the tool, but to concentrate on the learning. Palha and Koopman created the tablet-driven project Interactive Virtual Math: a tool to support self-construction of graphs through dynamical relations. The aim of the project is to develop a visualization tool that supports students' learning and relational understanding of graphical situations. The medium – here a tablet – allows the students to "draw" graphs using a finger, a digital pen or a mouse, to ask for help and to compare their own solution to the expected solution. According to the authors, this tool has the potential to help students understand functional relationships, but more importantly, allows the students to work on their own, experiment, create self-productions and reflect on them. Until now the authors only evaluated their tool in a small qualitative study.

Tablets will be important tools in the years to come. With multi-touch technologies, gestures have become an essential feature of user interface. The relation between touching and meaning-making might become more important. De Freitas and Sinclair used multi-touch technology and tangible gestures with young children to promote counting on and with fingers. These children used their fingers – one after another – while counting sequentially, they used their fingers simultaneously to represent numbers and they left a trace on the screen with one or more fingers. With the touchscreen interface, and particularly the multi-touch actions, they see the hand involved in a process of communicating and a process of inventing and interacting. "We interpret these speculative comments as an indication that the future of the gesturing hand in relation to new media may involve all sorts of surprises, and that perhaps even pre-school children may count 'on their hands' to 100 as they engage with these media" (De Freitas and Sinclair).

Smartphones

Nur Cahyono and Ludwig used smartphones to help students engage in meaningful mathematical activities. A math trail is a walk in which mathematics is explored in the environment by following a planned route and solving outdoor mathematical tasks related to what is encountered along the path. In the MathCityMap-Project students are confronted with special situations and questions along the path, supported by a GPS-enabled mobile phone app. Students were intrinsically and extrinsically

[2] Small Private Online Courses

motivated and engaged in this project. Moreover, they got to know more about their environment and model problems related to it.

Digital games

Computer game characteristics could also be exploited for the purpose of mathematical learning. As an example, Gjovik and Kohanova developed a mobile app on the topic of linear functions. The mobile phone is a tool we can expect to see more in mathematics education as learning becomes further individualized and online. In the "Lucky Hockey" game students have to strike a hockey puck along a straight line by entering a linear expression. Prior knowledge concerning the properties of linear functions is required when playing this game and in order to identify the path of the puck so that it hits the coins. The results of this project especially concerning the long term effect have not been satisfactory. The authors conclude that it might be difficult to make applications that facilitate exploration and discovery while doing mobile learning. It might be more effective if quite narrow mathematical topics are used. The concept of linear function might already be a too elaborate topic.

There are many questions around the use of games in mathematics classrooms which still need to be examined. How do we integrate games into the curriculum? When do students play these games? Is the motivation to play these games just an initial effect? What is the impact on students' learning and understanding? How sustainable is that knowledge over time?

Computational thinking

Robots are starting to play a more important role in our daily life. Robot competitions are quite popular in schools, but these activities usually take place outside regular lessons. The control of the robots, e.g. while walking through a labyrinth, needs algorithmic thinking similar to the turtle geometry of the 1980s. Seymour Papert (1980) originally created the label "computational thinking", but nowadays this concept has a much wider scope: it includes collecting, analysing and visualizing data, programming, creating computational models, and understanding relationships in systems. Broley, Buteau and Muller presented a model of computational thinking practices based on Weintrop et al.'s (2016) taxonomy for computational thinking in mathematics and science practices. The authors ask for further clarification of this concept and ways to integrate it into mathematics lessons.

Missing topics

If we compare the TWG16 call for proposals with the actual contributions made by the participants, we see some interesting gaps. Firstly, no attention was paid to digital assessment of and for the learning of mathematics. There are on one hand questions concerning written (final) examinations: Which technologies are allowed? Which tools are needed (Drijvers et al., 2016)? Which tasks are appropriate? How do students report their thinking? On the other hand, the question of how formative assessment might be a means to develop student competences is also of interest (Beck, 2017; Black & Wiliam, 2009). These topics have been addressed in some aspects in TWG 15 and in more detail in TWG21 on assessment.

Moreover, the topics of e-learning, blended mathematics education and (Massive Open) Online Courses for mathematics may set the agenda for CERME11. This includes issues such as personalized and adaptive learning, and the design of online feedback for students. The opportunities and constraints of using social media in students' perception of mathematics and their learning have also been absent, as was the case for the intriguing topic of virtual and augmented reality. Examples of the use of technologies devoted to the support of students with disabilities have not been addressed either.

With respect to the methodologies in the reported studies, the focus was on small-scale qualitative studies, whereas large-scale experimental studies were not presented. Even if the latter may have pitfalls, the field might benefit from an integration of both qualitative and quantitative approaches, so as to gain sustainability and applicable knowledge on how mathematical learning can benefit from the interaction with digital resources.

Concluding remarks

Digital technologies are now an element across all CERME groups (e.g., see Ferrara & Ferrari, TWG24; Hogstad, Norbert Isabwe & Vos, TWG14; Montone, Faggiano & Mariotti, TWG4). This indicates how digital tools permeate the mathematics education research landscape and have gained legitimacy across the field. In today's mathematics classrooms, different types of digital technologies are integrated in daily practice: interactive whiteboards, tablets, notebooks, graphing calculators with and without CAS. We have noticed a significant gap between research findings and mathematics teaching and learning practices in the regular classroom. The overall impression is that we cannot yet speak of a sustainable change through the use of digital technology, scaled up beyond the incidental level. We should acknowledge that integrating digital tools in a way that is beneficial to student learning is not as straightforward as we might have thought some decades ago. Thus, a specific working group on digital tools in mathematics education is appropriate within the frame of CERME, even if the impact of technological developments is hard to isolate from its context and from the topics central to other CERME working groups. A TWG dedicated to this issue could make a distinct contribution to important questions on the future of mathematics education.

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