# Rate-Adaptive Coded Modulation with Geometrically-shaped Constellations

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***Abstract*—Information-theoretic metrics are used to design rate-adaptive coded modulation based on geometrically-shaped constellations with soft- and hard-decision FEC. Numerical results show that an $8\%$ reach extension can be achieved with flexible data rates and transmission distances.**

***Index Terms*—Achievable information rates, coded modulation, geometric shaping, LDPC codes, staircase codes.**

## I. Introduction

Modern fiber optical communication systems require higher data rates to support the Internet's exponential traffic growth. Coded modulation—a combination of high-order modulation formats and forward error correction (FEC) [1], [2]—is a key technique to increase spectral efficiency and data rates in fiber optical systems. With the advent of modern FEC, the design and performance evaluation of such systems is nowadays based on achievable information rates (AIRs) [3]–[5].

Advanced FEC and modulation formats have been investigated as a means to reduce the gap to the channel capacity. Binary FEC comes in two flavors: hard-decision (HD) and soft-decision (SD) FEC. HD-FEC decoders use binary representations of bits, while SD-FEC decoders use more accurate information of bits: "soft information", also known as logarithmic likelihood ratios. Modern SD-FEC such as low-density parity-check (LDPC) codes offer a signal-to-noise ratio (SNR) improvement of about 1–2 dB compared with HD-FEC codes of the same rate. However, for applications with strict latency and complexity requirements (e.g., short reach), HD-FEC codes are an excellent alternative. Staircase codes (SCCs) [6] are a family of popular high-performance HD-FEC codes. Recently, low-complexity concatenated FEC schemes have been studied to combine the advantages of soft- and hard-decision decoders [7].

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Probabilistic and geometric shaping [8]–[13] have been studied to increase the gain for different transmission distance applications. The popularity of probabilistic shaping in the fiber optical community comes partially from its rate adaptivity. Here we show that rate adaptivity can also be achieved with geometrically-shaped (GS) constellations combined with variable-rate FEC. In particular, we adapt the set of GS constellation from [14], [15] to a multi-span wavelength-division multiplexing (WDM) optical fiber system and present AIRs and post-FEC BER results. It is shown that multiple line rates between 330 and 500 Gbps can be obtained by combining a small number of FEC rates and GS constellations. Additionally, reach increases of up to $8\%$ are demonstrated.

## II. AIRs, Model and Geometrical Shaping

The most popular AIR for (binary) SD-FEC is the generalized mutual information (GMI), which is suitable for bit-interleaved coded modulation (BICM) [2], [4]:

$$\mathrm{GMI}_{\mathrm{SD}} \triangleq \sum_{i=1}^{m} I(B_i;\boldsymbol{Y}) = \sum_{i=1}^{m} \mathbb{E}\left[\log_2 \frac{f_{\boldsymbol{Y}|B_i}(\boldsymbol{Y}|B_i)}{f_{\boldsymbol{Y}}(\boldsymbol{Y})}\right], \quad (1)$$

where the bits mapped to the channel input $\boldsymbol{X}$ are represented by the random variables $B_1$,$B_2$,...,$B_m$, $m = \log_2 M$ is number of bits per constellation point, and $f_{\boldsymbol{Y}|B_i}(\boldsymbol{Y}|B_i)$ is the channel law. When an HD-FEC is employed, the AIR is given by [3]

$$\mathrm{GMI}_{\mathrm{HD}} \triangleq m(1 - H(\mathrm{BER})), \quad (2)$$

where $H(\cdot)$ is the binary entropy function and BER is the average bit error rate.

The system model of the optical coded modulation system under consideration is shown in Fig. 1. We use an optical fiber link comprising multiple standard single-mode fibers (SSMFs) with $\alpha = 0.2$ dB/km, $D = 17$ ps/nm/km, $\gamma = 1.2$ $(\mathrm{W{\cdot}km})^{-1}$. A dual-polarization multi-span WDM system with 11 co-propagating channels was transmitted at a symbol rate of 45 GBaud, a WDM spacing of 50 GHz and a root-raised-cosine (RRC) filter roll-off factor of 0.1. Each SSMF span of length 80 km was followed by an erbium-doped fiber amplifier (EDFA) with a noise figure of 5 dB. Polarization mode dispersion (PMD) was not considered. The information bits of each polarization are independently encoded. The modulator

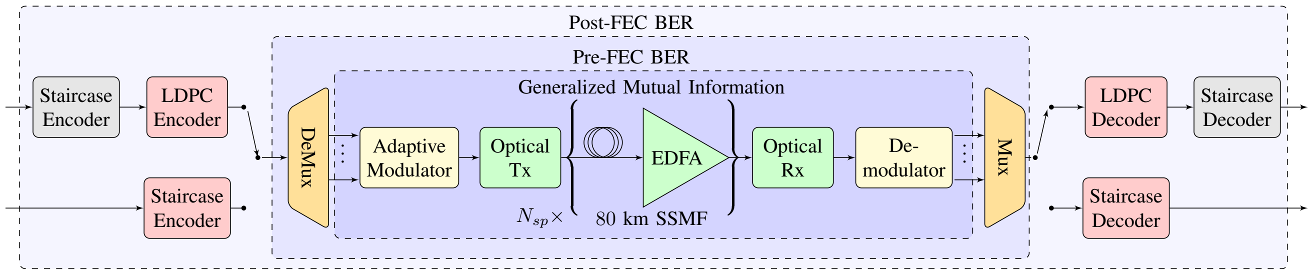


Fig. 1: Block diagram of the coded modulation system with adaptive geometrically-shaped modulation and HD-FEC/SD-FEC.

maps the encoded bits either to QAM or to GS symbols. At the receiver side, only the center WDM channel is bandpass-filtered and processed into the digital domain with ideal digital chromatic dispersion compensation and RRC matched filter. HD-GMI and SD-GMI are calculated for predicting the performance of HD-FEC and SD-FEC, resp. In addition, pre-FEC BER and post-FEC BER are measured to verify the performance of the coded modulation scheme. Two types of FEC are considered in this paper (see Fig. 1), one is concatenated FEC with LDPC as inner code and SCC as outer code, another one is HD-FEC with a single SCC.

We use geometrically-shaped, GMI-optimized (under a power constraint), 64-point modulation formats from [14], [15]. In the numerical simulations, three modulation formats and their corresponding labelings are used. These three formats are optimal for SNR=$\{15, 17, 19\}$ dB, and they are used at the adaptive modulator and demodulator for bit-symbol mapping and demapping, resp.

## III. Numerical Results

### A. Generalized Mutual Information Results

In Fig. 2 (top), two sets of results are shown for AIRs versus transmission distance (at optimum launch power). The first set (solid curves) show the SD-GMI vs. transmission distance for polarization-multiplexed (PM)-64QAM (black) and the three optimized PM-GS64 modulation formats (colors). In order to highlight the performance of each modulation format, we only show the GMI curves at their optimal transmission distance region. The three PM-GS64 constellations provide reach extensions of 80 km, 120 km, 200 km and 320 km with respect to the SD-GMI of PM-64QAM, at GMIs of 11.1 bits/sym, 10.4 bits/sym, 9.4 bits/sym, and 8.5 bits/sym, resp. The second set (dashed curves) show the HD-GMI vs. transmission distance for PM-64QAM and the two optimized PM-GS64 modulation formats.

As shown in Fig. 2, the reach increase difference between SD-GMI (solid lines) and HD-GMI (dashed lines) is relatively small for short distances. This is because the the impact of distortions is small in this region. On the other hand, for long distances, SD-FEC provides large gains with respect to HD-FEC. The results in Fig. 2 can therefore be used to decide which combination of FEC and modulation format to use. In particular, for short reach, the combination of HD-FEC and one PM-GS64 constellation offers a good complexity-gain trade-off. For long haul, PM-GS64 should be used with SD-FEC, providing gains of up to 320 km. The constellation from [14], [15] (red) is shown to offer gains for any distance above 1760 km.

### B. Post-FEC BER Results

In order to verify the HD-GMI results, we implemented staircase codes for short transmission distance and compare the post-SCC BER of PM-64QAM and PM-GS64. Bose-Chaudhuri-Hocquenghem (BCH) codes are used as the component codes of SCCs. The parameters of BCH codes are given by using a triple $(n_c, k_c, t)$, where $n_c$ is the codeword length, $k_c$ is the information length, and $t$ is the error-correcting capability. Then, the code rate of SCCs is given by $R_s = 2k_c/n_c - 1$. Here, two BCH codes $(504, 485, 2)$ and $(228, 209, 2)$ are considered. These parameters are obtained by shortening the extended BCH code $(512, 493, 2)$ by 8 and 284 bits, resp. These two BCH codes result in SCC rates $R_s^1 = 0.92$ and $R_s^2 = 0.83$, resp., and SCC block dimensions of $252 \times 252$ and $114 \times 114$, resp.

For verifying SD-GMI results, we concatenated LDPC codes with code rates $R_l = \{0.9, 0.83, 0.75, 0.66\}$ (as inner code) and a SCC with code rate $R_s = 0.92$ (as outer code). No interleaving is used between the codes. The LDPC codes are the ones from the DVB-S2 standard, with a block length of $N = 64800$ bits, and 50 decoding iterations. The decoded bits from the LDPC decoder are used as input to the staircase decoder. The FEC and modulation parameters are listed in Table I. This table shows that line rates of 333 Gbps, 375 Gbps, 416 Gbps, 450 Gbps, and 500 Gbps can be obtained.

TABLE I: FEC and modulation parameters

| Modulation | PM-64QAM / PM-GS64 | | | | | |
|---|---|---|---|---|---|---|
| HD-FEC | SCC | | | | | |
| $R_s$ | 233/252 | 95/114 | 233/252 | | | |
| SD-FEC | - | - | LDPC, $N = 64800$ | | | |
| $R_l$ | - | - | 9/10 | 5/6 | 3/4 | 2/3 |
| Total $R = R_s \cdot R_l$ | 0.92 | 0.83 | 0.83 | 0.77 | 0.69 | 0.62 |
| Bit rate (Gbps) | 540 | | | | | |
| Info. rate (Gbps) | 500 | 450 | 450 | 416 | 375 | 333 |

In Fig. 2 (bottom), post-FEC BER performance is shown for PM-64QAM and the three PM-GS64 modulation formats at different line rates. The error-free transmission reach extension of PM-GS64 with HD-FEC (SCC) at 500 Gbps and 450 Gbps, is measured to be up to 80 km. For PM-GS64 and SD-FEC (LDPC+SCC), we observe a shaping gain of up to 320 km compared to PM-64QAM. The transmission reach increase of $8\%$ is in excellent agreement with the prediction of the GMI curves in Fig. 2 (top).

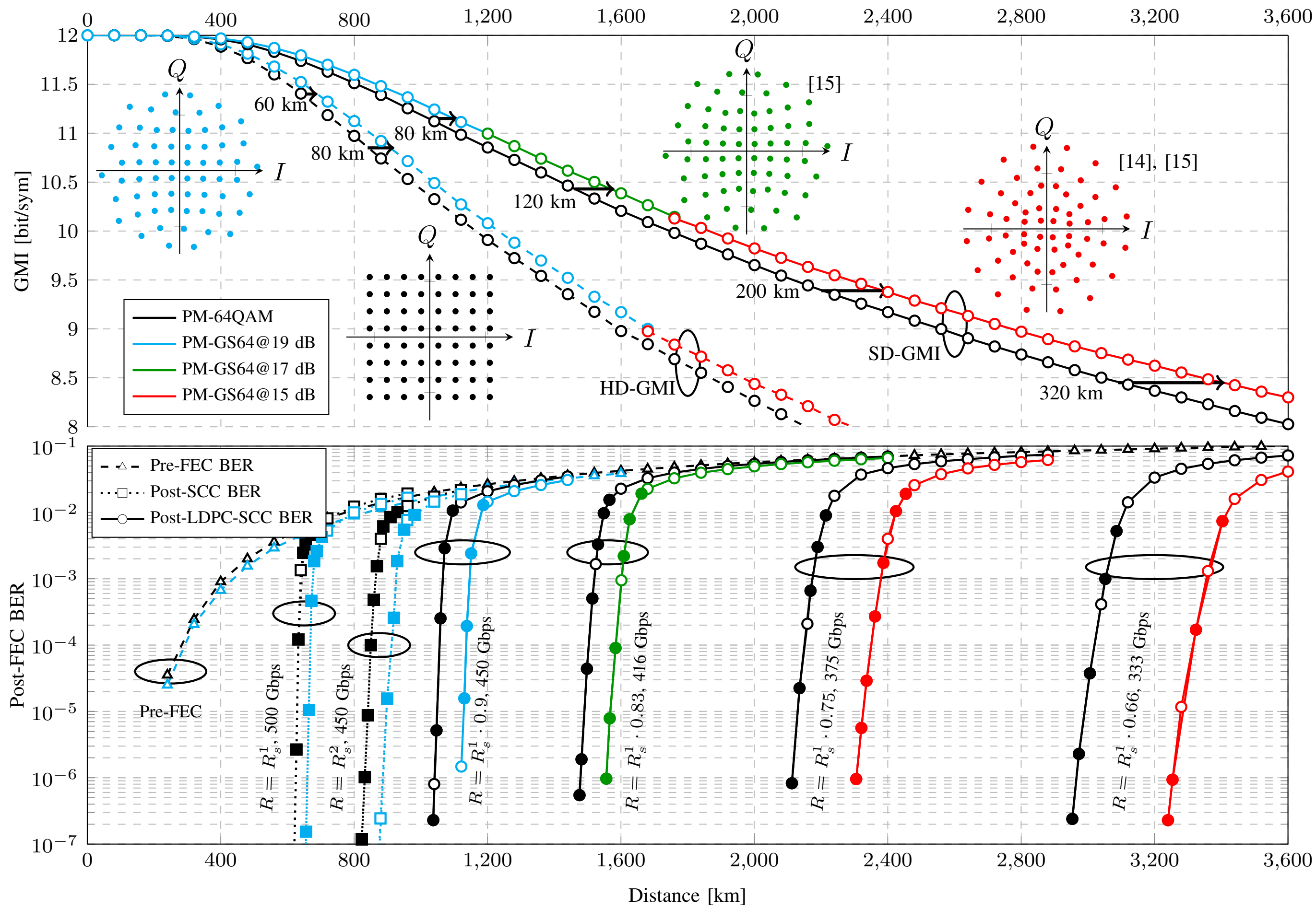


Fig. 2: Top: GMI as function of the transmission distance for different GS modulation formats with 64 points. Insets: the optimized formats for SNR= $\{15, 17, 19\}$ dB. Bottom: Post-FEC BER as a function of the transmission distance for different GS modulation formats and FEC codes. White markers indicate the BERs after the EDFAs. Filled markers show BER between two amplifiers (obtained by noise loading). SCC rates are $R_s^1 = 0.92$ and $R_s^2 = 0.83$.

## IV. Conclusions

In this paper, we presented a rate-adaptive coded modulation scheme based on geometric shaping. The analysis was performed both in terms of achievable information rates and post-FEC BER. One of the advantages of the analyzed rate-adaptive scheme is its low implementation complexity, as only the mapping and demapping functions of the transponder need to be modified. In this paper, we showed that rate adaptivity can be obtained by using three geometrically-shaped constellations. By combining these constellations with two HD-FEC and four SD-FEC codes, multiple line rates between 333 Gbps and 500 Gbps can be obtained. Furthermore, reach increases for a wide range of distances (from 600 km to 3300 km) were reported.

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