Cognitive Factors Predicting Variation in Arithmetic Performance

Imogen Sinead Anne Long

University College London

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Declaration

I, Imogen Long, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Abstract

Children begin to develop mathematical knowledge from a young age in varying degrees. For this reason, it is important to identify which cognitive skills are foundations to later mathematical knowledge and which are the most important for numerical development. This thesis focuses on some of the earliest cognitive processes involved in arithmetic (addition and subtraction) abilities in typically developing children between the ages of 4 and 8. Arithmetic abilities are an important outcome for children of this age and is predictive of later, more advanced arithmetic and mathematical skills. Therefore, identifying foundational factors is important for educational practices and for theories of typical and atypical arithmetic development.

Three studies focused on three cognitive factors: sensori-motor skills including finger awareness, pattern understanding and symbolic number knowledge. The first two predictors were examined whilst controlling for important predictors of arithmetic including age, number knowledge, executive function and spatial skills. We showed firstly that sensori-motor skills are less important predictors of arithmetic than number knowledge and counting. Next, the potential causal relationship between number knowledge and arithmetic was examined via a training study. Our findings suggest that training in number knowledge can improve numerical and arithmetic outcomes, although we failed to reach significance due to a lack of power. Finally, we examined pattern understanding using a large patterning battery and are the first study to show that different pattern tasks (numbers, letters, shapes and objects) load onto one factor. Moreover, this factor is a unique and significant predictor of arithmetic. Together, these studies help to outline the shape of arithmetic development in typically developing children.

Impact statement

The studies presented within this thesis have provided an outline for some of the cognitive factors which may be implicated in arithmetic development in children. This is important for academic theories of typical and atypical numerical development, as well as being important for educational practices.

Throughout the studies presented, the research techniques and tasks show good reliability and are methodologically sound. One study has been published in the Journal of Experimental Child Psychology. The study was deemed appropriate for publication due to importance of a new and reliable finger gnosis task which highlights the importance of including reliable tasks in research. Moreover, the task was used in collaboration with academics in Australia investigating arithmetic development and culminated in a second paper, under review. The impact of this study is that researchers have a greater understanding of how finger awareness may (or may not) be important in numerical development.

In educational research it is important to identify factors which can be examined and trained in school-aged children. In one study, pattern understanding was identified as an important factor for arithmetic development. This is a skill that is already taught in schools despite a need for more scientific evidence to support the relationship with arithmetic. Therefore, the evidence we report can provide some support for the potential usefulness of teaching pattern skills at schools. Importantly, the study identified all types of pattern tasks across a range of stimuli (numbers, letters, shapes and objects) as important predictors of arithmetic. Therefore, teachers can teach pattern understanding with a range of different tasks which may increase attention and interest; an important consideration for educators and cognitive researchers alike.

Overall, the studies presented in this thesis have important implications for both academic research and educational practices. The findings have been presented at conferences and to teachers at schools (via posters and oral presentations). This shows how research can make an impact in the wider society and may lead to changes or improvements in educational practices.

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Chapter 1. Thesis overview

We use numbers every day and in all aspects of life, and a basic understanding of symbols and quantities is required to complete simple tasks such as telling the time, paying for groceries, getting on the right bus and counting change. For this reason, numerical understanding of Arabic digits and associated quantities is a critical part of development, and a focus of early education. These foundational numerical skills are crucial for mastering arithmetic (addition, subtraction, multiplication and division) which, in turn are a critical educational outcome (Duncan et al., 2007).

This thesis presents three studies which examine arithmetic development in children. Each study aims to identify which cognitive skills may be important factors in arithmetic development for children between four and eight years old. The three studies are described in full in Chapters 3, 4 and 5 and an overview of these studies is provided later in this chapter.

1.1 Early arithmetic

Early arithmetic is considered to include addition, subtraction, multiplication and division. Children will follow a basic learning trajectory of simple addition (which is akin to counting) and simple subtraction, first orally and then with symbols. Only after several years of formal education will children tackle multiplication and division. As the research presented for this thesis lies in early numerical development, addition and subtraction skills will be examined. The outline of how these skills develop will now be presented.

1.1.1 Counting

Early addition skills are essentially an extension of counting. For example, in the sum 2+1 children simply need to count on from 2 to reach the answer. Therefore, understanding how counting develops is an important part of understanding early arithmetic.

Children will typically learn the counting sequence once they have acquired some language and will quickly become proficient, at least to the number ten. In a series of studies examining how children develop counting skills, Gelman and Gallistel (1986) suggest that learning progresses through “how to count” principles. Initially, children may simply recite the count sequence, before understanding that effective counting relies upon the following principles: one-to-one principle (count each item only once), stable ordering (count words appear in the same order, “one, two, three…”), cardinality principle (the last number counted is equal to the total number in the set), order irrelevance (count items in any order) and item-kind irrelevance (all objects can be a count entity).

Knowledge of the counting principles is demonstrated in a range of studies which use counting violation methods. For example, in Gelman, Meck and Merkin (1986) children from 3- to 5- years of age watched a puppet counting a number of objects correctly or incorrectly. Even younger children were reported to identify the correct and incorrect trials, showing high sensitivity to counting errors and knowledge of the one-to-one and cardinality principles. In a further study, the same children exhibited knowledge of the order irrelevance principle in a “Doesn’t Matter” task, in which they were able to correctly count items in an unconventional order, such as by starting from the middle of a row rather than the edge. Evidence such as this has led some theorists to conclude that children have an inherent understanding of the basic counting principles (Gelman and Gallistel, 1986; Gelman & Meck, 1983; 1986).

Inherent knowledge has been disputed by some because of evidence showing that children need to first learn to count before mastering the counting principles. For example, Wynn (1990) used three different counting tasks to examine the cardinality principle in 2-3 year old children. Although able to count objects well, some younger children failed to demonstrate the same knowledge when counting actions and sounds. Moreover, the same children failed to provide a puppet with the correct number of toys in a later “give-N” task with younger children simply grabbing a seemingly random number of objects. In all tasks, only children over 3.5 years of age were able to show consistent counting knowledge for larger digits (up to six items). Evidence such as this has led Rittle-Johnson and Siegler (1998) to propose that counting principles develop through practice, and thus argue that procedural knowledge (the ability to solve problems) is required before conceptual knowledge (understanding principles that govern the counting domain) can develop.

Regardless of the way that children learn the counting principles, these skills are essential as a foundation for the development of basic addition and subtraction (see

Koponen et al., 2013; 2016). After all, simple addition is essentially an extension of counting skills, particularly in the early developmental stages when children will use count objects to summate different amounts.

1.1.2 Basic addition

Children may begin to perform simple addition sums as soon as they learn the count sequence (Bisanz, Sherman, Rasmussen, & Ho, 2005). Initially, basic sums may be demonstrated verbally and with the assistance of objects, such as toys or bricks. Once a child enters formal education there is a focus on progressing from a verbal number system to one which includes symbolic knowledge. Learning to associate number names with the associated Arabic digit appears to be relatively straightforward, although studies examining this are fewer than those examining the verbal number system (Fayol & Seron, 2005).

Children are required to learn to associate the sound a number makes with the written code, known as the Arabic digit (e.g. 4). During this learning process, the child must also learn to associate the magnitude that these figures represent. Simply learning the way that numbers are written appears to be relatively straight forward for most children at school. However, learning the associated magnitudes appears to take a bit longer and is a more complex and potentially more critical aspect of number knowledge. Learning symbolic number skills is a crucial component in learning arithmetic because most educators will require children to read and write basic sums within the first few years of formal schooling. The following chapter (Chapter 2) is dedicated to describing the process of symbolic number learning and evaluating evidence which examines how this skill may be implicated in numerical and arithmetic development.

When computing verbal or written sums, children appear to use a variety of different methods, all of which rely on children understanding the counting principles (Siegler & Jenkins, 2014). Let us take an example of the sum: 2 + 3. Younger children may use the simplest, *sum* or *count-all* method which involves counting all the objects and applying the cardinality principle (last object counted is total number in sequence). A more mature approach would be to use a *count-on* principle which would involve counting on from two (*count-from-first* approach), or more effective, counting on from three (*min* approach). The *min* approach is the most effective method because it involves the least counting, and Siegler and Jenkins (1989) report that it is used by around nine percent of 4-5 year-olds. The authors report that most children of this age (34%) will use the simple sum approach (e.g. count two, then count three and then add together). Other approaches include retrieval (22%), finger recognition (11%) and guessing (2%). Retrieval is possible once children have practised addition and subtraction sums such that they are able to store these sums in memory and therefore retrieve rather than compute the answer.

Overall, what appears to be important in arithmetic development is that children are able to recite the count sequence and learn how to count objects. Children can then learn basic addition and subtraction before committing basic sums to memory through practice. The exact way that a child computes a sum will vary and will become more efficient with increased practice and age. Memory is implicated in basic computation both via the long-term store (once children have sufficient experience) and the short-term store during the time of computation (see Section 2.3.1 for a fuller discussion of memory).

1.1.3 Mathematics disorder

Mathematics disorder, also known as developmental dyscalculia, are terms used to describe those who fail to learn arithmetic and numerical skills as expected. The Diagnostic and Statistical Manual of Mental Disorders (4^{th} addition) and the World Health Organisation define mathematics disorder as a failure to grasp numerical concepts and in particular arithmetic despite adequate schooling and intelligence. It appears in around 3-6% of the population, although this number differs depending on the research and definition used.

Alongside a difficulty in learning basic arithmetic, children with mathematics disorder will typically use immature strategies to complete calculations and will be slower than peers to commit sums to memory. This may manifest in low arithmetic fluency scores due to an inability to rapidly solve calculations, or a failure to learn the sums correctly. Importantly, it appears that numerical difficulties persist despite adequate intelligence, and therefore this cannot be the only cause of mathematics disorder, although intelligence is implicated in numerical learning (see Butterworth, 2009 for a review). Knowledge of mathematics disorder is in its infancy in comparison to other developmental disorders such as dyslexia, and considerably more work needs to be done with both typical and atypical populations to better understand the nature and causes of this disorder.

1.2 Overview of the three studies

Understanding how children develop basic arithmetic abilities is critical for researchers because it is an important milestone in early education. Although research into numerical learning continues to progress, there are still many unanswered questions.

One of the challenges for researchers examining arithmetic development is that there are multiple ways in which children are able to compute a basic sum. Take for instance the sum 3+4. To reach the answer 7 children can use one of many methods including (but not restricted to): counting all individually, counting on from 3, counting on from 4, using memory/learning by rote. Alongside this is the challenge of understanding which skills are most critical and the time at which these are most prevalent. For example, we know that children are required to have a basic understanding of the count sequence to compute basic addition and subtraction, and an understanding of Arabic digits for written sums, but how important are these skills compared to more general abilities including attention, memory and spatial skills? More complex still is understanding how children will go on to develop more advanced arithmetic abilities such as multiplication and division and understanding how early number skills relate to these later abilities, and more advanced skills such as geometry and algebra.

The studies presented in this thesis aim to examine which of these skills may be most important in the development of arithmetic.

1.2.1 Study 1: Sensori-motor skills

The first correlational study (discussed in full in Chapter 3) examines how some sensori-motor abilities are related to arithmetic. Sensori-motor skills relate to how well children can combine sensory and motor information and some researchers have examined the potential relationship between some sensori-motor skills and arithmetic in young children (e.g. Fayol, Barrouillet, & Marinthe, 1998; Noël, 2005).

One sensori-motor measure that is common across these studies is finger gnosis, the ability (or inability) to identify which finger is pressed without visual assistance. One potential reason for a relationship with finger gnosis (and other sensori-motor skills) and arithmetic is that fingers are used functionally in learning to count (a functional account – see Butterworth, 1999). Another potential explanation for a relationship is the localizationist account (see Noël, 2005) which proposes that some sensori-motor abilities, including finger gnosis, are associated with similar neural regions as arithmetic and other numerical abilities.

A primary focus for the first study was to create a methodologically sound research design, largely because of limitations in previous evidence. To do this, previous measures were adapted to increase reliability and reduce numerical confounds. Moreover, a range of additional numerical tasks were included in the test battery to act as control measures (an important part of correlational designs and something rarely addressed in previous research). In the study, children underwent tests on a range of sensori-motor skills, including finger gnosis, alongside a range of numerical-based outcome measures including an arithmetic fluency task, a dot counting task and measures of magnitude comparison. Previous studies have generally failed to control for such a wide range of skills. Our results clearly showed, in contrast to some previous evidence, that the sensori-motor skills were less well correlated with arithmetic than numerical abilities. It is likely that the stronger relationship reported in previous evidence is due, in part, to the failure to control for other predictors of arithmetic.

The most important factors in predicting variation in arithmetic were shown to be numerical measures, specifically counting and symbolic magnitude comparison which remained as unique and significant predictors after controlling for age. Our timed counting task required children to count dots and write the associated Arabic digit, and the finding that counting and arithmetic were strongly linked was not surprising given that early arithmetic is essentially an extension of the counting principles (however, see a discussion of potential limitations of the dot counting task in Section 6.2).

Our symbolic number comparison task required children to identify which of two Arabic digits (numbers 1-9) is larger. Performance on this task correlated well with arithmetic, in line with previous evidence (see Section 2.2.1). This finding was the basis to investigate symbolic number skills further, and the potential causal relationship with arithmetic.

1.2.2 Study 2: Training in symbolic number knowledge

A training study was run to examine whether training in symbolic number skills can transfer to arithmetic. The study is described in full in Chapter 4.

In the training sessions, children played a computerised game (played on a tablet). The game was invented by researchers in the Department of Language and Cognition at University College London (Newton, Bruce & Donlan, 2016). Based on observation of the common challenges encountered in number sequences tasks in adults with aphasia (De Luccia & Ortiz, 2014) and children with Developmental language Disorders (Donlan, Cowan, Newton & Lloyd, 2007), the software was developed to address these issues in both populations. The game was funded by 'seedcorn' and organised by UCL and a useable prototype was developed in conjunction with the gaming software company SoftV.

Players in the game create numerical and/or alphabetical sequences. The numerical sequences game involves number knowledge learning (number identification and ordering) and therefore was deemed an appropriate training design to examine potential transfer to arithmetic. Other children in the study either played the letter sequences game (experimental control) or were a business-as-usual group (no training). Groups were based on randomised stratified sampling of pre- training arithmetic score. Having three groups was beneficial as it allows us to understand better the value of the game itself rather than a) effects found due to attention from an experimenter or b) general learning within the study period. One limitation of having three conditions is that this limits the sample sizes for each group which reduces power.

To evaluate the effectiveness of the training game, we compared performance of the number game condition to the baseline condition across all post-training scores (controlling for pre-training scores). Similarly, we measured the difference between the letter training game and baseline control. The results showed that those in the number training condition scored more highly than the baseline condition on numerical scores post-training. Conversely, those in the letter training group showed no difference in numerical post-training scores compared to those in the baseline condition. These results allude to a specific effect of the number game on numerical and arithmetic outcomes. However, no improvements were at significant levels, which is likely due to a lack of power as the sample size was relatively small and the training time relatively short. A full discussion of the limitations of power, including the reasons why we chose the training time and sample size are presented in Section 4.4.

Alongside the issue of power, there were some practical limitations to the study. The primary concern was that children did not enjoy playing the game. This is likely because the first two levels of the game were very challenging; it had been developed such that the player is required to have a good grasp of number knowledge in order to complete the opening level and therefore those children of lower ability struggled to the play the game and found it demotivating. Other children reported finding the game repetitive and boring, even when capable of playing it. Moreover, the letter game was not, as originally thought, of equivalent difficulty to the number game. This is because children of this age are not taught number and letter names at equivalent time points (see Section 4.3). These limitations (discussed in full in Section 4.4) led to a decision that the design of the game and the pre- and post- tests were not appropriate for children of this age group and therefore adaptations would need to be made before any additional data were collected.

Due to limited time and finances, it was not possible to adapt the game within the timeframe of my PhD and therefore this study was deemed to be a pilot and another avenue was explored for a final study. The training design is currently being prepared for roll-out out on a larger scale with changes to the game in development to make it more appropriate for children of this age. The study was instrumental in my understanding of some of the practical elements that are found when conducting training studies and when testing young children.

1.2.3 Study 3: Pattern understanding

For the final study (presented in full in Chapter 5) we considered other ways that symbolic number knowledge is involved in studies examining arithmetic development. Typically, this is via a range of overt symbolic number knowledge tasks such as number reading, number identification or symbolic comparison (as we had used in my first two studies). However, it also appeared in the literature in a more surprising way, through patterning.

Pattern tests commonly require children to identify the missing number at the end of a sequence using either repeating (*1,2,1,2__*) or increasing (*1,3,5,7__*) patterns. Researchers have shown that combined scores on these two types of pattern test correlate with arithmetic (e.g. Lee et al., 2012; Schmerold, 2015). However, the vast majority of these studies do not control for number knowledge despite number knowledge being implicated in understanding number patterns. It is arguable that repeating number patterns do not rely on number knowledge as children may use perceptual skills to identify the correct response. However, increasing patterns *do* require number knowledge. For example, to identify what comes next in the following pattern *3,6,9,12* children must be able to identify the number that they are reading and know the number sequence. This is symbolic number knowledge but measured with the addition of pattern understanding and therefore leading to the question: is it symbolic number knowledge that is predicting arithmetic here, or is it patterning skills? A failure to control for number knowledge in previous literature means this question has been largely unanswered.

To examine whether patterning is important in predicting arithmetic, rather than symbolic number knowledge, we designed a pattern understanding battery which included a range of stimuli (numbers, letters, objects and shapes) and different pattern types (increasing, repeating and rotating). The pattern battery was large enough (120 items in total) to allow for analyses across the different stimuli. Importantly, symbolic number knowledge and other numerical skills were controlled to assess whether a potential relationship was driven by number skills or pattern abilities. Moreover, other control measures were included in the design such as spatial skills and executive function. These are domain-general skills associated with both patterning and arithmetic (see Section 2.3). In keeping with the first correlational study, a range of important factors were controlled to create a methodologically sound design and to allow conclusions to be drawn about the specific relationship between patterning and arithmetic.

The results showed that pattern tasks remained significantly correlated with arithmetic after controlling for a range of skills including number knowledge. To investigate how the patterning stimuli were related to each other, an exploratory factor analysis was conducted which showed that across the different stimuli, pattern tasks could be identified as a single factor. This factor remained a significant predictor of arithmetic after controlling for a range of numerical and other predictors including executive function and spatial skills. This finding suggests something specific about the relationship between patterning and arithmetic, although the reasons for this relationship are not clear. It is possible that patterning is a domain-general skill which relates to arithmetic and other developmental milestones, including reading. Alternatively, patterning may be specifically related to arithmetic because arithmetic may, in part, require an understanding of patterns: e.g. 2+3 is the same as 3+2. These potential reasons are discussed in detail in Section 5.4.3.

Across all three studies, we investigated a range of numerical predictors of arithmetic and, in the pattern study, some domain-general predictors of arithmetic. Symbolic number knowledge was measured across all three studies and was consistently well correlated with arithmetic. Number knowledge remained as a unique predictor of arithmetic in Study 1 and was the focus of the training in Study 2.

1.3 Use of number tasks across the studies

Across all three studies a range of arithmetic and number tasks were chosen. These were chosen predominantly because they were tasks used in previous similar studies or because they are considered relevant skills in theories of arithmetic development (see Section 1.4).

1.3.1 Arithmetic

Fluency (timed) addition and subtraction tasks were used to measure arithmetic across all three studies. Timed tasks measure the fluency of arithmetic and reduce the chances of ceiling scores. This is particularly important for the first and third studies when children (aged 5-7) made few errors. In the second study (where children were in the first year of formal schooling and aged 4-5) the length of time given for the addition task was greater, and subtraction was not measured. This difference reflects the importance of designing measures which are appropriate for the ages of the children in the study.

1.3.2 Counting

Counting was measured in all three studies. In the second study, counting was measured via a rote counting task in which children were required to count up from one and stopped when they made a mistake (maximum count to 50). A more complex dot counting task was used in the first and third study as the children were older. In these studies, children were required to count the total number of dots presented and write or verbalise the response within a given time-frame. One of the reasons for using a fluency task is that children of this age (5-7) are generally very good at counting and therefore timed tasks will reduce ceiling scores.

1.3.3 Symbolic number knowledge

Number knowledge is considered a critical part of numerical development (Merkley & Ansari, 2016) and therefore was measured in each of the studies conducted. In the first study, number knowledge was measured via a symbolic magnitude comparison task: compare two digits and identify the larger. This requires an understanding of number identification (knowing the associated sound for an Arabic digit) alongside an understanding of the magnitude or order in which digits appear. One reason for using this task in the first study was an interest in examining whether approximate (dots) or symbolic (digits) comparison tasks are more highly correlated with arithmetic.

In the second and third studies, symbolic number skills were measured via number identification and number reading tasks. The former required children to identify a number (from an option of four potential responses) and in the latter to read a digit shown to them. Numbers used in these tests were 1-1000. One benefit of measuring symbolic number knowledge this way is that it uses a wider range of numbers than a symbolic comparison task which relies only on the digits 1-9. The identification and reading tasks do not require as much of an understanding of magnitude as a symbolic comparison task likely does, although some understanding of place value is required to read and identify larger numbers which reflects, in turn, some understanding of number ordering and magnitude.

1.4 Theories of numerical development

Early arithmetic skills require children to understand the verbal count sequence, the order in which numbers appear and the related quantities, or magnitudes of the associated digits. Alongside this is the need to understand the Arabic digit in written form and associate this with the verbal code. These skills are conveyed in Dehaene's 'Triple Code Model of Numerical Processing' (e.g. Dehaene, 2001) which is one of the more prominent theories of numerical processing.

In Dehaene's model, numbers are processed according to three mental representations of a number: Visual Number Form, Verbal Number Form and Analogue Number Form. The way that these three processes interact is depicted in Figure 1.1.

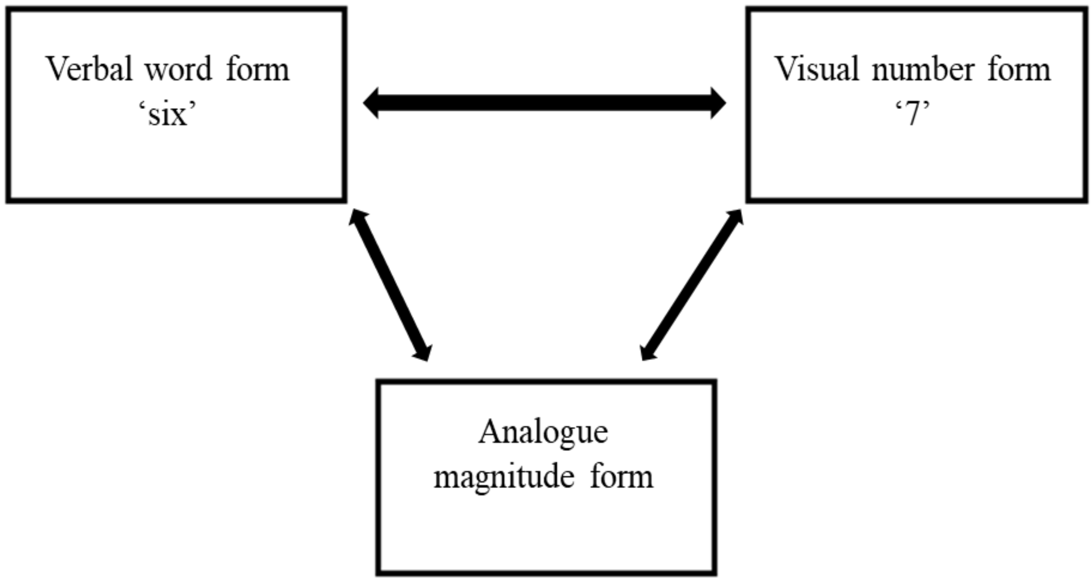

Figure 1.1 A simplified version of Dehaene's Triple Code Model Processing (Dehaene & Cohen, 1995).

Dehaene's core number model argues that the verbal form is linked to language abilities and the visual form linked to visuospatial skills. These two processes are linked to the third component, analogue magnitude. All three processes are related to one another and therefore may be examined via similar tasks which assess all components, but which may draw more strongly on one. For example, approximate comparison tasks are a good measure of the analogue magnitude, counting tasks can be for the verbal form and written arithmetic measure the visual form. The analogue magnitude is considered the most important component because without having a developed concept of the magnitude (quantity meaning) of a number, processing of digits, including arithmetic, is not possible.

This simplistic model is used for both adults and children although some argue these processes must be different. One prominent development theory by Carey (2009) suggests that number skills are learnt in qualitative stages in which the child will develop the conceptual understanding of number over time (see also Sarnecka, 2015). In this view, the innate analogue representation of number is linked to the exact number system via counting and culturally specific learning (e.g. the Arabic digit system and English language) which results over time in a full mental representation of number. What is evident (from different research studies and in contrasting theories) is that children improve the ability to understand, represent and manipulate numbers with age. Therefore, age is an important consideration in the studies presented in this thesis.

So far, the position assumed is that children are required to understand numbers (i.e. conceptual knowledge) in order to solve basic sums. This view does not consider learning arithmetic by rote, which may be one ways that addition and subtraction sums are computed. Integrated theory (Siegler & Braithwaite, 2017) argues that arithmetic facts and procedures are learnt best when children have a complete understanding of number, and that this understanding of number will then, in turn, lead to a better understanding of later numerical skills such as fractions and algebra. This perception is critical for the studies presented in this thesis which, as with the other theories, assume that children need an understanding of digits and magnitude in order to calculate sums. Evidence that this is important comes from studies which examine arithmetic errors. For instance, Siegler and Braithwaite (2017) show that children are more likely to calculate an incorrect answer which is close in size to the correct answer. Also, they show that children are quicker to identify an answer shown as incorrect when the answer is further from the correct answer (e.g. 5+3=15 is quicker to respond “incorrect” than 5+3=9). This thesis takes the view that a full understanding of number is critical and aims to further investigate the skills which are most important for this learning.

It is also important to discuss the role of other factors in learning arithmetic, such as environment and domain-general skills, in the early years. These may act as important factors in predicting how well children learn to associate numbers with magnitudes, and thus show number knowledge (e.g. Merkley & Ansari, 2016). Many studies and indeed theories of numerical development fail to include the importance of domain-general predictors and the relative contribution of general skills is not well known.

This is something that is specifically looked at in the final study undertaken and discussed in more detail in Section 2.3.

1.5 Summary

This introductory chapter firstly outlined the typical trajectory of arithmetic development which includes learning the basic count sequence and developing basic addition and subtraction skills. To learn more complex addition and subtraction (arithmetic) children must develop an understanding of number knowledge which is the identification, order and magnitude of a number. Some theorists suggest that number knowledge and counting are innate skills, with others suggesting practice and exposure is important for development. Either way, it appears that these skills are critical for the development of arithmetic. The next chapter evaluates previous evidence investigating a link between magnitude knowledge, number skills and arithmetic.

Chapter 2. Predictors of arithmetic

This chapter evaluates evidence examining the role of some potentially important predictors of arithmetic, including numerical (domain-specific) predictors and non-numerical (domain-general) predictors. Numerical predictors include preverbal and approximate number skills, as well as symbolic number knowledge. The domain-general predictors discussed are memory, executive function and spatial skills. All of the factors that are discussed in this chapter will feature across the three studies that were conducted for the purposes of this thesis.

2.1 Preverbal numerical skills

Children appear to possess some numerical skills before being able to speak and one suggestion is that preverbal numerical skills are the foundation to later numerical abilities, and thus children with poor basic number perception will go on to develop poor arithmetic (e.g. Dehaene, 2010). As basic number perception abilities do not rely on exact counting skills, this system is commonly referred to as an approximate number system, and a common way to measure it is through nonsymbolic magnitude comparison tasks.

Evidence for an innate approximate number system comes from infant and animal studies which support the existence of some basic numerical processing abilities, including the ability to discriminate between different quantities and show sensitivity to numerical manipulations (see Dehaene, 1997 for a review). Animal research has demonstrated this through both observational evidence (animals instinctively pick the larger of two selections of food, Kilian, Yaman, von Feren & Gunturkun, 2003; West & Young, 2002) and examples from animals who have been trained on basic numerical tasks. For instance, Church and Meck (1984) trained rats to press specific levers depending on the number of flashes from a light. Furthering this, chimpanzees have been shown to possess more advanced numerical discrimination abilities, comparable to addition. Rumbaugh, Savage-Rumbaugh and Hegel (1987) gave two chimpanzees the choice of taking chocolates from two trays containing two piles of chocolate. Rather than using basic perceptual skills, the chimpanzees had to sum the number of chocolates in each pile, and on each tray, to identify which tray contained the most chocolate. As both chimps reliably chose the tray with the most (total) chocolates, the authors suggest these animals possess some basic calculation abilities. This ability has been reported in other studies in which chimps have been trained in basic counting and addition (see Boysen & Berntson, 1989). Such clear demonstrations of numerical aptitude make sense from an evolutionary perspective and has led some to suggest that some basic numerical abilities, particularly numerical discrimination, are a primitive skill (e.g. Butterworth, 1999; Dehaene, 1997).

Infant studies provide evidence that parallel the findings from animal studies, with particularly robust evidence of early approximate number discrimination abilities. For example, Xu & Spelke (2000) demonstrated numerical discrimination in 6-month-old infants (*N* = 16) whereby infants would habituate (become bored) after repeated exposure to 8 dots, but then dishabituated (showed interest) when 16 dots were presented. Similar findings are reported for discrimination between 16 and 32 dots (Xu, Spelke, & Goddard, 2005) and, importantly, both tests controlled for surface area differences, suggesting the discriminations are numerical in nature. However, infants do not appear able to distinguish between 8 and 12 dots suggesting a numerical discrimination ability for a 2:1, but not 3:1, ratio (Xu & Spelke, 2000). Similar findings have been reported from studies using cross-modal discrimination and numerical change detection (Izard, Sann, Spelke, & Streri, 2009; Starr, Libertus, & Brannon, 2013a).

Going one step further, Wynn (1992) suggests that infants possess some basic arithmetic abilities for dealing with small numbers of items. In the study, 5-month-old infants (*N* = 32) were shown basic (1+1 or 2-1) addition and subtraction sums played out with small toy objects. Infants sat in front of a small theatre set-up and observed one toy placed on the “stage”. Then, a screen was added, and a second toy was placed behind the screen. In the impossible events, the screen was removed and only one toy remained. In possible events, the screen was removed, and two objects remained as expected. Wynn found that infants looked longer at the impossible events rather than possible events and surmised that these longer looking times, indicating surprise, suggest that infants as young as five months have some understanding of number quantity which is greater than simply discrimination, and perhaps evidence of a basic exact numerical system (Wynn, 1992).

Evidence of these basic numerical skills in young infants supports the claim that infants are born with an innate number system (Dehaene, 1997). What is less clear is if these early numerical abilities are important predictors of the development of later numerical and arithmetic skills. Typically, evidence for this comes from approximate number discrimination tasks which do not require exact understanding of numerosity. In approximate magnitude comparison tasks, the participant is required to determine which of two nonsymbolic (e.g. dot) sets has the larger numerosity. As this does not require verbal information, and does not rely on any counting abilities, it is considered to assess early approximate number skills. Conversely, symbolic magnitude tasks (choose the larger of two Arabic digits) are a comparable measure used to assess magnitude abilities relying on knowledge of the symbolic (and exact) number system.

2.1.1 Approximate comparison tasks

In a typical approximate number discrimination task, a participant is presented with two dot sets on the left and right side of a screen and is required to identify which dot set contains the larger number of dots. It is important that the dots are not counted, but rather that the discrimination is determined using an approximate judgement (quick reaction times can provide support of this). The difficulty of the task is manipulated in two main ways. Firstly, if the overall quantity of dots in each array is increased, the task is more challenging. For example, choosing the larger of 10 or 20 dots is more difficult than choosing the larger of 5 or 10 dots. Secondly, the ratio difference between the dot arrays can be manipulated, with more challenging discrimination involving larger ratios. For example, a 12 to 9 dot ratio is 0.75 which is harder than a 6 versus 12 dot task which has a smaller ratio of 0.5.

It is also important to control for surface area which should be matched across the two arrays. Clearly, a larger dot array with a larger surface area is easier to discriminate than two dot arrays with equal area but different numerosity. Although most studies have attempted to control for this, there is debate about how possible it is to measure dot discrimination without including confounds such as dot size, density and area. For example, Gebuis and Reynvoet (2011) showed that number of items in a set cannot be extracted without the use of visual cues and suggest that approximate number processing is a challenge to measure. Despite this, approximate number tasks have remained a common way of assessing numerical discrimination within the literature, although controlling for potential confounds is clearly an important consideration.

The dot discrimination score is typically based on overall accuracy, for example gain one point for each correct discrimination. A reaction time may be included as evidence that the task was completed without counting (counting will take longer than an approximate discrimination). Other researchers have used a Weber fraction (*w*) ratio to determine the difficulty of the discrimination task. This is the ratio between the two digits (with a ratio of 1:2 digits being easier to discriminate than 13:14, for example). The Weber fraction can be applied to many stimuli (e.g. sound, light intensity) and has been argued to provide a precise representation of approximate number acuity. At some point, an individual will reach a ratio at which they can no longer discriminate between the two digits and importantly, *w* appears to increase over time as children become more accurate and better able to distinguish between greater ratios and smaller numerical distance (Halberda, Mazzocco, & Feigenson, 2008). This finding has implications for the approximate number sense theory as it shows that this system can change, develop and improve with age which may be linked to developing later arithmetic.

2.1.2 Symbolic comparison tasks

Symbolic comparison tasks are much the same as approximate tasks except that participants are required to identify the larger of two Arabic digits. Typically, numbers used will be between 1 and 9 (inclusive) and the numbers will be presented in the same size. As with approximate number tasks, the ratio and number discrimination will affect the difficulty of the task, and performance is related to the level of difficulty in children and adults. One limitation of these tasks is that they cannot be presented to young or preverbal children as this task requires the child to have some understanding of Arabic digits and their associated magnitudes.

2.1.3 A relationship with arithmetic?

There is evidence to suggest that both approximate and symbolic comparison tasks correlate with arithmetic, although for some time, the relationship between approximate comparison tasks and arithmetic dominated the literature. Many researchers found evidence that early approximate comparison abilities (between the ages of 2-5) relate to later arithmetic, with some suggesting it acts as a foundation for later numerical abilities (e.g. Feigenson, Libertus, & Halberda, 2013; Gilmore, McCarthy, & Spelke, 2010; Halberda et al., 2008; Libertus, Feigenson, & Halberda, 2011; 2013; Lyons & Beilock, 2011; Marle, Chu, Li, & Geary, 2014; Mazzocco, Feigenson, & Halberda, 2011; Piazza et al., 2010).

In one longitudinal design, Mazzocco, Feigenson and Halberda (2011) found that an approximate comparison score at 3-years-old predicted standardised maths scores (Test of Early Mathematics Ability; TEMA; Ginsburg & Baroody, 2003) three years later (N = 17). Starr, Libertus and Brannon (2013b) reported that numerical discrimination ability at 6-months of age (N = 48) correlated with math score (TEMA) three years later (r = 0.28, p < .03) predicting a small but unique proportion of the variance in arithmetic (β = .31, p = 0.02). Similarly, Ceulemans et al. (2015) found that approximate number abilities at 18 months (N = 31) correlated with arithmetic and counting at 24 months (r = .31, r = .38, respectively), although infant number discrimination at 8-months and arithmetic at 48-months were not correlated.

Although these findings appear to show a relationship between early approximate number abilities and later arithmetic, the strength of this relationship is not clear. For example, when Starr, Libertus, & Brannon (2013b) controlled for IQ, the strength of the relationship was weakened. Approximate number abilities and IQ accounted for only a small variation in arithmetic (R^2 = 28%) and IQ was the stronger unique predictor of later numerical abilities (β = .45, p = .001) (although see a critical review of this study by Cragg & Inglis, 2013). Ceulemans et al. (2015) report no unique linear relationship between infant number discrimination and later exact number abilities, when IQ was controlled. A meta-analysis by Chen and Li (2014) reports that in cross-sectional studies (36 samples) approximate skills and arithmetic show a weak correlation (r = .20) and longitudinal studies (6 samples) revealed a similarly small prospective relationship (r =.24). Moreover, much of the positive literature supporting a relationship suffers from small sample sizes and a failure to include important numerical and cognitive predictors of arithmetic, such as symbolic number knowledge.

A recent meta-analysis by Schneider and colleagues (2016) reports that of 45 articles (284 effect sizes and 17,201 participants) approximate comparison tasks were correlated with arithmetic, but effect sizes were lower for approximate (r = .24, 95% *CI* [.20, .28]) than symbolic (r = .30, 95% *CI* [.24, .36]) tasks. Furthermore, when symbolic number skills are controlled, both the correlational and longitudinal relationship between approximate number tasks and arithmetic is weakened (Castronovo & Göbel, 2012; Clarke & Shinn, 2004; Fuhs & McNeil, 2013; Göbel, Watson, Lervåg, & Hulme, 2014; Holloway & Ansari, 2009; Long et al., 2016; Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Sasanguie, De Smedt, Defever, & Reynvoet, 2012; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013; Xenidou-Dervoua et al., 2017).

Göbel et al. (2014) examined the predictive power of the approximate number system (measured by an approximate comparison task) in 173 children in the first year of school. The results showed that although approximate comparison correlated with arithmetic, it was not a unique and significant predictor of arithmetic 11 months later when other factors such as symbolic number knowledge (e.g. number identification) were controlled. Similarly, Lyons et al. (2014) examined the role of number knowledge and approximate comparison in predicting arithmetic in 1,391 children in the first six years of school. Although correlated, approximate comparison was not a unique or significant predictor of arithmetic across any of the year groups when other symbolic number skills were considered. Furthermore, Xenidou-Dervoua et al. (2017) found in a large sample of kindergarten children (N = 326) that approximate comparison was a weaker longitudinal predictor of later maths achievement (including arithmetic) than symbolic comparison over three years. Importantly, these studies have large sample sizes and control for other important predictors of arithmetic (e.g. working memory and IQ) which is commonly lacking for studies supporting a role of approximate number abilities in predicting arithmetic.

If approximate number sense is the foundation to later arithmetic, it is arguable that training in discrimination abilities should improve later arithmetic. There is some evidence for this claim with one training study finding evidence that training children in approximate tasks can improve exact number skills and arithmetic (Hyde, Khanum, & Spelke, 2014) and another reports improvements in adult participants (Park & Brannon, 2014). However, a recent study trained children in both approximate number skills and exact number skills and found that only those children trained in exact symbolic number skills, or exact and dot but not dot alone improved significantly on arithmetic outcomes (Honoré & Noël, 2016). This is discussed in more detail in Chapter 4.

Taken together, it seems that although correlated with arithmetic, approximate number skills appear to play a less important role than some other predictors of arithmetic.

2.2 Symbolic number knowledge

Symbolic magnitude comparison tasks rely on children's ability to identify two numbers and know which of these numbers is larger. These skills are part of number knowledge defined by Merkley and Ansari (2016) as a combination of understanding the identity (symbol 4 is verbal code "four"), the cardinality (4 represents four objects) and the ordinality (4 comes before 5 and after 3) of a number (see Figure 2.1).

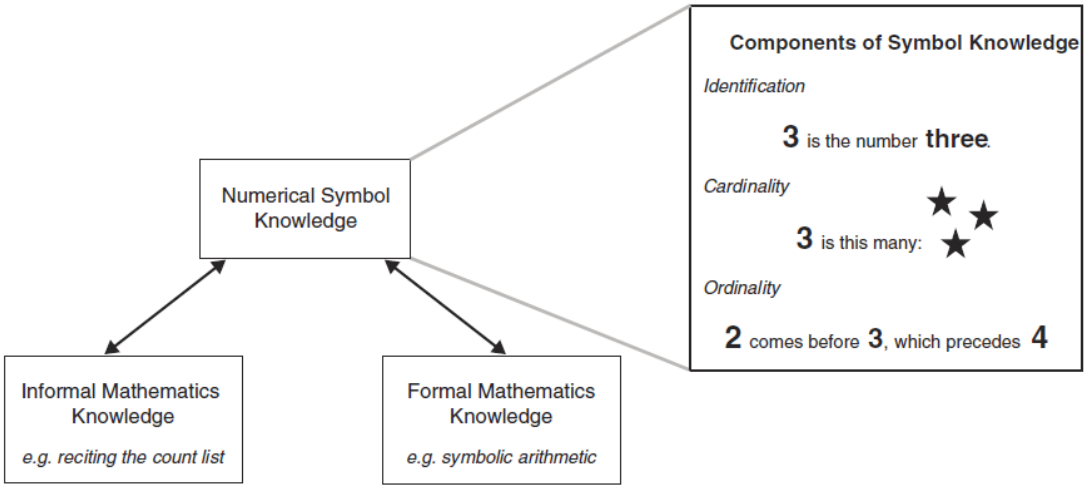

Figure 2.1 Components of number knowledge taken from Merkley & Ansari (2016).

The authors argue that symbolic number knowledge is not fully acquired until children have mastered cardinality which is considered the ability to associate the magnitude of a number with its verbal code. This is mirrored in the counting development literature which suggests that more advanced counting skills, relying on an understanding of cardinality, are more important for predicting later number abilities than basic rote counting (e.g. knowing the count sequence) (e.g. Nguyen et al., 2016).

Symbolic number knowledge is typically measured via tasks which require the child to identify Arabic digits, and/or the cardinality of a digit. Symbolic magnitude comparison tasks, akin to approximate comparison tasks, are perhaps the most common method used to assess number knowledge. Symbolic comparison tasks rely on children understanding the identity and order or magnitude of a number, typically for Arabic digits 1-9. Larger numbers can be tested via number identification tasks in which children identify which number matches that read aloud by an experimenter. The options will typically be similar to the target number (e.g. 71 or 171 for target 17). This relies on advanced number identification including the understanding of place value. Finally, ordinality can be measured via number sequences tasks. For example, identify if numerical sequences are presented in the correct order or fill in the missing number from a sequence.

2.2.1 The relationship between symbolic skills and arithmetic

Symbolic number measures (however tested) appear to be stronger predictors of arithmetic than approximate number abilities (e.g. Göbel et al., 2014; Lyons et al., 2014; Schneider et al., 2016; Xenidou-Dervoua et al., 2017). Additionally, these skills remain robust predictors of arithmetic after controlling for other cognitive factors implicated in arithmetic such as general cognitive skills including executive function and reading abilities (Durand, Hulme, Larkin, & Snowling, 2005; Lyons et al., 2014; Vanbinst, Ansari, Ghesquière, & De Smedt, 2016).

Vanbinst et al. (2016) tested 74 children at the start of third grade and again one year later. Symbolic skills (measured via a symbolic comparison task) uniquely predicted 16% of the variation in arithmetic (β = .41, p < .001), considerably more than either reading (β = -.22, p - .05, unique R^2 = 4%) or non-verbal IQ (β = -.19, p = .07, unique R^2 = 3%). Interestingly, the predictive power of symbolic comparison in arithmetic was similar to that of phonological awareness in reading (unique R^2 =16%) suggesting that the ability to identify digits and associate with a magnitude should be a focus of educational practices and interventions, as phonological awareness has been for reading development (see Hulme & Snowling, 2009).

Merkley and Ansari (2016) propose that early exact number skills are the foundation to later arithmetic, and it is this system which should be the focus of numerical development literature. Some convincing evidence for this proposal comes from studies examining early number and general cognitive skills, which support a critical role for these early symbolic number skills in predicting later arithmetic. Duncan et al. (2007) used six longitudinal data sets to examine the predictive nature of school entry level abilities including numerical knowledge, reading, attention and domain-general cognitive abilities (e.g. social skills). Across all six studies, children's numerical abilities at the start of school were shown to significantly predict later arithmetic, and a meta-analysis of these findings showed this was the single most important predictor of later achievement (followed by reading and then attention).

Similarly, Nguyen et al. (2016) examined performance in number skills at preschool (N = 1,375) and compared to mathematical performance (including arithmetic) in the fifth grade. Number knowledge tasks (measured via tasks assessing knowledge of counting and cardinality principles) were the best longitudinal predictors of later mathematical achievement (β = .42, SE = .04, p < .001) after controlling for other numerical skills (patterning tasks and geometry measures) and domain-general measures (e.g. socio-economic status; SES). Entry level exact numerical skills, measured by tasks assessing knowledge of the counting and cardinality principles, appear to be the most important foundation to later numerical skills (Koponen, Aunola, Ahonen, & Nurmi, 2007; Merkley & Ansari, 2016; Nguyen et al., 2016; Torbeyns, Gilmore, & Verschaffel, 2015) and, importantly, play a more critical role than other cognitive abilities (including literacy and attention) and general predictors of academic achievement (e.g. SES and home environment). Moreover, as the relationship between early counting abilities and later arithmetic appears to be mediated via symbolic number knowledge (Purpura, Baroody & Lonigan, 2013) it is proposed that this ability should be the focus of developmental research.

One outstanding question is why there is variation in the development of symbolic number knowledge. One possible suggestion is that the innate number system includes exact, as well as approximate, number skills and that it is this system which leads directly to the development of later exact number skills (e.g. Wynn, 1998). Evidence for this comes from studies suggesting that, with exception of the digit 0, children appear to learn to associate Arabic digits and magnitude relatively easily (Hughes, 1986; Wellman & Miller, 1986). Learning to represent 'zero' as a quantity appears to be problematic, even for adults who show longer reading times for this digit (Brysbaert, 1995). Wynn (1998) suggests that children learn symbolic and magnitude associations via the innate core number system which does not encompass the number zero, as this represents 'nothing' and thus is not represented in the system.

An alternative suggestion is that children may learn to associate numbers and quantities through practice with physical objects (Hughes, 1986). As zero cannot be represented by a physical quantity, children may struggle to learn what this number represents. This is perhaps a more convincing argument, considering the lack of evidence for a direct relationship between preverbal exact skills and later arithmetic. Moreover, children's knowledge of numbers in pre-school is largely related to parental income and socio-economic status (Cirino, 2011; Chu, van Marle, & Geary, 2016; Duncan et al., 2006; Fuchs, Geary, Fuchs, Compton, & Hamlett 2016; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; Welsh, Nix, Blair, Bierman, & Nelson, 2010) suggesting that understanding symbolic number is based upon a combination of general cognitive skills, home environment and teaching.

Whether symbolic number knowledge is learned through an innate number system, or via practice and exposure, it is clear from the literature that early knowledge of symbols appears to be a strong predictor of arithmetic, and relatively more important than other cognitive skills (such as reading or intelligence).

2.3 Domain-general predictors of arithmetic

So far, the cognitive predictors that have been discussed in relation to the development of arithmetic have been domain-specific predictors. These skills (for example magnitude comparison and number knowledge) are considered domain-specific because performance may affect numerical abilities but might not affect performance in other academic skills. On the other hand, researchers have investigated the role of domain-general predictors and number development (e.g. Duncan et al., 2007). Domain-general skills such as intelligence and memory are considered to affect more than one academic attainment skill, for instance numerical and literacy skills. Memory, executive function and spatial awareness are domain-general skills commonly associated with arithmetic development. These skills will be first described, then the way to measure these variables will be explained before finally discussing the relationship these have with arithmetic.

2.3.1 Memory

Memory is very important for both computing and storing calculations. Older children will typically commit arithmetic sums (including addition, subtraction and multiplication) to long-term memory through practice and exposure. For arithmetic problems which have not been learned and stored in long-term memory, children must use working memory to compute an answer (Adams & Hitch, 1997).

Baddeley (1992) defines working memory as a temporary storage system responsible for performance of cognitive tasks. Working memory is further divided into three interacting subsystems of a central executive (a central processing system) which is further separated into two “slave systems”, an articulatory or phonological loop (responsible for storing phonological information) and visuo-spatial scratch-pad (for visual/spatial information). An important characteristic of working memory is that the system is limited in capacity and information stored only temporarily.

Working memory is thought to be required for the performance of many cognitive skills including reading, problem solving and general learning. Importantly, it is closely linked to arithmetic. Take, for example, the sum 5 + 4. For young children who have not yet learned the answer to this, they must place these numbers in short-term memory and then process the answer. It is the coupling of short-term storage and simultaneous processing which is a critical component of the working memory system. Solving the sum may involve phonological information if the numbers are presented orally and/or visual information if the numbers are presented in written or symbolic form. According to Baddeley’s model, the related subsystems will deal with the relevant information (e.g. articulatory loop for a verbal sum) and interact with the central executive to solve the problem.

Others have commented that the visuo-spatial sketch-pad, which may deal with spatial awareness, could be involved if children are using spatial skills to solve the calculation (see Section 2.4). Working memory is also considered an important part of executive function, a general term for different processes involved in attention, inhibition and control (see Section 2.3.2).

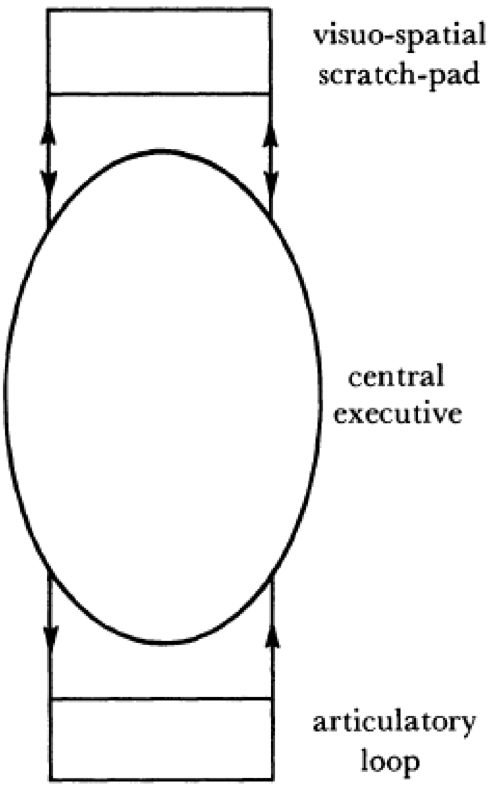

Figure 2.2 A simplified representation of the working memory model taken from Baddeley (1992).

Working memory capacity can be measured using tasks that require children to store and actively manipulate information. For example, repeating a set of digits or words backwards and increasing the number of items in the sequence to increase difficulty. Skills such as this are strongly linked to arithmetic ability (e.g. Gathercole, Pickering, Knight, & Stegmann, 2004; Mazzocco & Kover, 2007; Van der Ven, Kroesbergen, Boom, & Leseman, 2012) even when controlling for IQ (Alloway & Alloway, 2010). This skill is regularly included in arithmetic research and is an important construct to consider when examining which cognitive skills are most important in the development of arithmetic.

2.3.2 Executive Function

Executive Function is a term given to cognitive skills relating to the conscious control of thoughts and actions including basic processes of memory (particularly working memory), inhibitory control, attention and planning (see Figure 2.3). Researchers have suggested that important executive function skills form part of a central executive component of working memory (see Baddeley, 1992) and evidence has supported a relationship between these various working memory measures and arithmetic development (Blair & Razza, 2007; Bull, Espy & Wiebe, 2008; Bull & Lee, 2014; Chu et al., 2016). Bull and Lee (2014) suggest that executive function may be related to high academic achievement because children with good executive function may show good school-related behaviours including good attention in class, high focus on working activities and ability to refrain from disruptive behaviour in a classroom.

Despite a growth of interest in executive function in recent years, it is still an ill-defined construct and there continues to be discrepancies in the way that executive function is defined and measured (Zelazo & Müller, 2002). Moreover, there exists some debate within the literature about the true nature of executive function, with some researchers arguing that these skills are in fact separate constructs rather than a unitary factor (e.g. van der Sluis, de Jong, & van der Leij, 2007).

In numerical development literature, executive function is commonly (although not consistently) defined as a measure encompassing three main skills: inhibition (overriding a dominant response), shifting (switching flexibly between tasks and rules) and updating (monitoring the contents from working memory). Other skills also examined can include selective attention, planning, goal setting and self-regulation.

Working memory is an active part of the memory system, responsible for temporarily storing information and mentally working on and processing the information. It has been implicated in executive function since the earliest definitions and remains a common measure in more recent executive function tasks. Working memory and updating are clearly linked to arithmetic because in order to compute a calculation, an individual needs to consider the sum (phonologically or orthographically) and actively hold the digits in memory whilst retrieving results. Inhibition may be important for ensuring the correct use of a calculation method. For example, a child may be experienced in addition and thus have to inhibit this response for a less well practiced subtraction question. For more advanced number skills, inhibition of irrelevant information may be important, for example selecting appropriate information from a word problem.

Shifting skills are useful in a range of numerical processing tasks including computing different types of arithmetic sum (e.g. moving quickly from addition to subtraction) and moving between different forms of notation (verbal and orthographic).

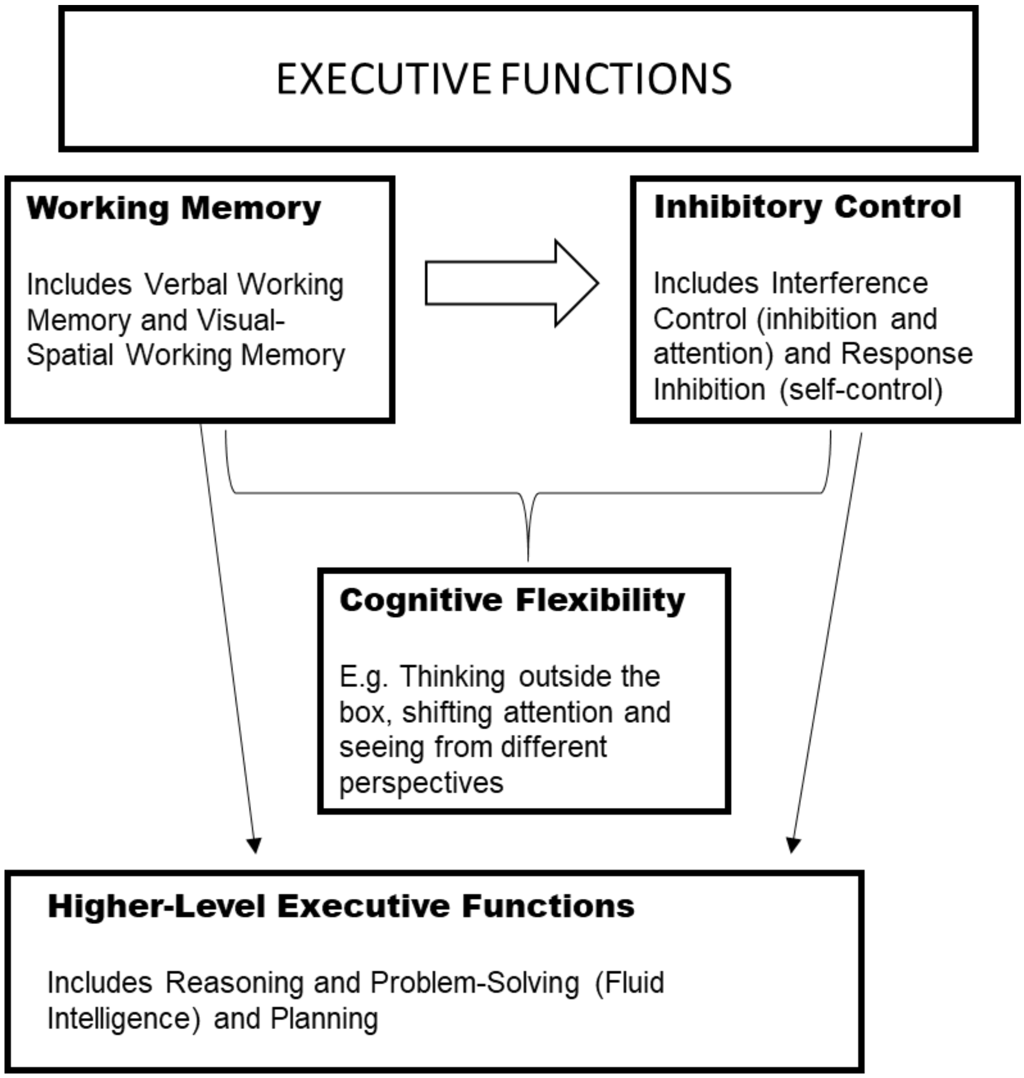

Figure 2.3 Simplified model adapted from (Diamond, 2013). Executive function and related terms.

The tasks used to measure these constructs in children vary across different studies, although there are a number of key tasks that are commonly used in the literature. Updating is often measured using a working memory task, for instance a backward digit or word span task (e.g. Wechsler Adult Intelligence Scale; WAIS-III; Wechsler, 1997). A simple forward digit or word span task is a good measure of a child's short-term memory (simply repeat the words you hear in increasing number of words within an item). However, as working memory and updating is assessing the ability to process information stored in a short-term (limited capacity) store, a backward span task (repeat the words/digits heard backwards) requires the use of processing upon the information held. Note that the use of a backward digit task may confound results for numerical development, and therefore a backward word span is a preferred working memory task within this literature.

Inhibition is commonly measured via a Stroop test (Stroop, 1935) or Head Toes Knees and Shoulders Task (HTKS; e.g. Burrage et al., 2008). Both of these assess the child's ability to inhibit an automatic (or dominant response). The HTKS task is suitable for young children and it requires them to control the dominant reaction when asked to "touch your head" by doing the opposite and touching their toes (or similarly for knees and shoulders). A common measure for a shifting task is the Wisconsin Card Sorting Task (e.g. Struss et al., 2000) whereby children learn to associate a rule with a set of cards but then this rule is switched, and the participant must deduce what the new rule is. For younger children, similar but simpler card sorting tasks have been used to measure the ability of a child to switch between different perspectives (e.g. shifting task; Dick, 2014).

Developmental studies have examined if these separate measures relate to a unitary underlying construct, using factor analysis. There is evidence to support this, and moreover to show that this factor correlates with general learning attainment including arithmetic (Bull et al., 2011; Bull & Scerif, 2001; Hughes, Ensor, Wilson, & Graham, 2009). However, some studies have suggested that these skills may be tapping into slightly different constructs, with a possible-two factor model; inhibition and shifting as one factor and working memory as a second (Lee et al., 2011; Van der Ven et al., 2012). For example, Miyake et al. (2000) showed through exploratory factor analysis that although correlated, three executive function tasks measuring shifting, updating and inhibition are clearly separable to one another. This finding suggests that it is important to consider the unity and diversity of executive functions. One potential reason for the different findings within the literature may be partly due to the measures used to examine executive function and the ages of the children within the study. In general, researchers examining numerical development assess executive function as one main factor although it is not completely clear if this is consistently the case (Fuhs, Hornburg, & McNeil, 2016).

2.3.3 Relationship with arithmetic

There is mounting evidence that some, or all of these executive function measures relate strongly with arithmetic and numerical development. Moreover, children with mathematical learning disorder have weaker executive function abilities (see Geary, 2004). In typically developing populations, a number of studies have reported concurrent and longitudinal correlations between executive function abilities and arithmetic (Blair & Razza, 2007; Bull, Espy, & Wiebe, 2008; Bull & Lee, 2014; Brock, Rimm-Kaufman, Nathanson, & Grimm, 2009; Chu et al., 2016; Fuhs et al., 2016; Mazzocco & Kover, 2007; Moll et al., 2015; Nesbitt, Farran, & Fuhs, 2015; Welsh et al., 2010).

For example, Chu et al. (2016) measured performance on executive function (card sorting task; Conflict Executive Function scale; Beck et al., 2011) and intelligence (Weschler Test of Intelligence) alongside domain-specific skills (various tests assessing number knowledge, counting and calculation) in 100 children in preschool (mean age = 3 years 10 months) then again 18 months later. Performance in executive function at time 1 and time 2 correlated significantly with mathematics achievement (WIAT Numerical Operations) ($r = .26$ and $r = .35$, respectively) and predicted growth in the domain-specific skills, which in turn were predictive of later mathematical abilities. Similarly, Moll et al. (2015) provided evidence for a strong relationship between executive function (measured by inhibition and selective attention tasks) and preschool verbal number skills, which in turn were predictive of later arithmetic abilities ($N = 93$). Executive function skills at time 1 (when children were 3 years 9 months) correlated significantly with arithmetic score at time 3, two years later (addition: $r = .28$, subtraction $r = .32$) and this relationship was mediated via preschool number skills which was a significant and unique predictor of arithmetic.

A meta-analysis by Friso-van den Bos, van der Ven, Kroesbergen and van Luit (2013) reports that executive function measures involving working memory, updating, inhibition and shifting, are associated with mathematical performance across 111 studies (16,921 participants between 4 and 12 years of age). Working memory components were best associated with arithmetic, with significant correlations remaining for all measures including inhibition and shifting ($r = .27$ for 131 correlations, 29 studies; $r = .28$ for 94 correlations, 18 studies) and the greatest overall relationship between updating and arithmetic (via weighted mean correlation coefficients). However, there appears to be no good evidence that training in working memory improves arithmetic, as reported in a meta-analysis of 30 randomised control trials with 30 group comparisons (Melby-Lervåg & Hulme, 2013).

In summary, there appears to be sufficient evidence to support a role of executive function in arithmetic development, although more work is required to support the relationship between different executive skills. We used executive function tasks (updating, shifting and inhibition) as control measures in a study examining the relationship between patterning and arithmetic (reported in Chapter 4) due to the evidence that these skills play a role in the development of arithmetic.

2.4 Spatial abilities

Spatial abilities are cognitive skills involving the ability to mentally manipulate spatial information and use visual imagery (Uttal et al., 2013). From a young age, children will play with blocks and other physical objects that involve spatial skills (Rittle-Johnson et al., 2018). These skills will improve over time and can be improved through increased play with toys, shapes and objects (Jirout & Newcombe, 2015; Levine, Huttenlocher, Taylor, & Langrock, 1999).

Children are typically assessed on a range of spatial skills involving spatial visualization (imaging and transforming mental information), form perception (copying and distinguishing different shapes or symbols) and visual-spatial working memory (holding locations in working memory). As with executive function, there are different tasks used to measure these spatial skills. Spatial visualisation involves manipulating objects within space and is commonly assessed using tasks such as a mental rotation task (e.g. which of these images is a rotation of the first) or the block design task which uses physical items (e.g. WISC-IV: Wechsler, 2008). Form perception is the ability to recognise and distinguish different shapes from one another and can be measured with tasks requiring a child to separate a shape into different components (e.g. hidden shapes task; Smith & Lord, 2002). Visuospatial working memory tasks may involve holding shapes in memory, for instance copying a figure or line drawing (e.g. Beery & Beery, 2010) or remembering where an item last appeared on a screen (e.g. Kaufman & Kaufman, 1983).

An important area of investigation, as with executive function tasks, is how different types of spatial skills are related and whether they form one factor or several separate factors. Hambrick, Kane and Engle (2005) argue that spatial abilities reflect separate components involving executive control, verbal and visual abilities. On the other hand researchers have examined a range of spatial skills and found that these can form one unitary factor, suggesting any difference with previous evidence may be related to the ages of children measured or types of tasks used (e.g. Mix et al., 2016). As with executive function, there is mixed evidence regarding the nature and relationship between different spatial skills, with debate surrounding the extent to which these skills can be called one factor.

2.4.1 Spatial skills and arithmetic

Spatial skills have been linked with mathematical abilities in a range of studies examining children and adults (see Mix & Cheng, 2012 for a review). Additionally, children with mathematics disorder perform significantly worse on visual-perceptual skills (Geary, 1993). Verdine, Irwin, Golinkoff, & Hirsh-Pasek (2014) show that spatial skills remain a unique and significant longitudinal predictor of mathematical abilities ($R^2 = .15$, $p < .001$) after controlling for executive function and verbal abilities ($N = 44$). Gunderson, Ramirez, Beilock, & Levine (2012) provide evidence that spatial skills in Grade 1 predict number line estimations and approximate calculations one year later. This suggests that the ability to spatially represent digits may help in the development of later numerical abilities.

This finding has also been reported for exact calculation skills (Mix & Cheng, 2012). Other studies examining this relationship have reported strong and unique correlations between arithmetic and various spatial abilities, including visuomotor skills, mental rotation and figure copying (Ansari et al., 2003; Gunderson et al., 2012; Rittle-Johnson et al., 2018). Additionally, there is evidence for transfer to arithmetic after training in spatial skills (Cheng & Mix, 2014; Hawes, Moss, Caswell, & Poliszczuk, 2015; Lowrie, Logan, & Ramful, 2017).

Spatial skills are linked to arithmetic and numerical abilities for a number of potential reasons. A dominant theoretical perspective is that numerical and mathematical thinking is governed by spatial representations, for example storing information via a mental number line or holding information about locations and quantities in space (e.g. Lakoff & Núñez, 2000). Children and adults appear to represent smaller numbers on the left side and larger numbers on the right with robust evidence from studies examining the SNARC effect; people are quicker to identify a smaller number with their left hand and larger number with their right (Berch, Foley, Hill, & Ryan, 1999).

In development, a child who is able to associate numbers to a mental number line in this way may develop stronger associations between numbers and their associated magnitudes; a critical component of numerical development (Mix & Cheng, 2012). Additionally, studies have shown that manipulating the spatial configuration of a calculation (e.g. distance between the Arabic digits in 3+4 x 2) can affect the way people will complete the sum (Fisher, Borchert, & Bassok, 2011; Landy & Goldstone, 2007). Evidence examining the use of a mental abacus has shown that children who employ this technique are able to perform complicated calculations more rapidly than children who have no formal training (Uttal, 2000). This suggests that providing a mental model for the calculation is beneficial for strong maths performance. A second theoretical (although not mutually exclusive) proposal is that spatial skills activate the same neural regions as numerical abilities, with a particular focus on the parietal cortex with evidence supporting brain activation in these regions in functional imaging studies (see Hubbard, Piazza, Pinel, & Dehaene, 2005)

Overall, there appears to be strong evidence for a relationship between spatial skills and numerical development, and it is worth considering these within studies examining arithmetic outcomes. We included spatial abilities as a control measure in our study examining patterning skills (presented in Chapter 5). This was a particularly important control measure for this study as our pattern stimuli include some spatial information within the rotating items.

2.5 Summary

This chapter has outlined some important numerical and domain-general skills which may be important in arithmetic development. Each of these skills are presented in some way in the three studies that were conducted for the purpose of this thesis to examine arithmetic development in children. Each study is now presented in turn including a detailed introduction which evaluates evidence related to the study in question. After the method and results section a discussion outlines these findings in relation to previous evidence with a general discussion presented in the final chapter (Chapter 6).

Chapter 3. Study 1: Finger gnosis and other sensori-motor skills as predictors of arithmetic development

This chapter reports a study conducted for this thesis which investigates the relative contribution of finger gnosis, and other sensori-motor abilities, as predictors of arithmetic ability. Parts of this chapter are published in Long et al. (2016).

3.1 Gerstmann's syndrome

Gerstmann's syndrome was first described in the 1900s by Josef Gerstmann who stated that patients with acquired brain disorders were exhibiting a cluster of symptoms. The damage was to the inferior parietal lobe and involved the co-occurrence of four deficits: acalculia (problems with numerical information, including calculation), finger gnosia (difficulty distinguishing and naming fingers), left-right disorientation (inability to label left and right on own and others' bodies) and handwriting difficulties (Gerstmann, 1940). For many years the nature and interpretation of the syndrome has been debated with some researchers claiming there is evidence for such a syndrome in both adults and children, particularly from imaging data (e.g. Benson & Geschwind, 1970; Geschwind, 1974; Mayer et al., 1999; Mazzoni, Pardossi, Cantini, Giorgetti, & Arena, 1990), and others continuing to debate the nature and interpretation of the syndrome (e.g. Benton, 1961, 1977; Miller & Hynd, 2004; Poeck & Orgass, 1966).

Despite the mixed evidence for Gerstmann (and Developmental Gerstmann) syndrome, the possible co-occurrence of these symptoms has implications for numerical and arithmetic, development. Some researchers have examined the potential link between fingers and arithmetic by testing finger awareness; the ability to identify which fingers are pressed without visual assistance. A typical way to measure this is to have children place their hands in a box such that their fingers are out of their sight, and an experimenter press one or two fingers on one hand and the child use the index finger of the other hand to identify the finger(s) pressed. This is similar to the way that Gerstmann would have tested finger gnosia in his patients.

Two main theories have been proposed to suggest why a relationship between fingers and arithmetic exists. The first, originally proposed by Gerstmann, is a localizationist account, which suggests that calculation and sensori-motor abilities (e.g. left-right orientation, finger and body-part awareness) may coincide due to the proximity of brain regions involved in these tasks (see Pinel, Dehaene, Riviere, & LeBihan, 2001). The second is a functional account which proposes that fingers are linked to arithmetic via the functional use of fingers in learning to count (e.g. Butterworth, 1999). The localization account could explain both acquired and developmental Gerstmann's Syndrome, both of which could be characterised by loss of, or lack of development of, a brain region (Dehaene, Piazza, Pinel, & Cohen, 2003). Gerstmann later extended this theory to postulate a functional role of the fingers in learning to count and arithmetic (see Lebrun, 2005). The relationship between fingers and counting has been well documented (Butterworth, 1999) and this has prompted a number of studies with children examining the possible role of finger gnosis in the development of arithmetic, and other number skills.

3.1.1 Functional account

One of the methods that children use to complete simple addition and subtraction sums, particularly in early development is finger counting (e.g. Siegler & Jenkins, 1989). Fingers are particularly useful in counting because both use a base-ten principle, helping children to develop an understanding of the numerical system. Children will begin to use fingers in learning to count from a young age, and before formal instruction of arithmetic has begun (Butterworth, 1999). Moreover, children with mathematical learning difficulties will often show immature finger counting strategies and use fingers to assist in counting for longer than their typical peers (Butterworth, Varma & Laurillard, 2011). Not only are fingers important in early number skills, but most adults will continue to use fingers for a range of tasks such as counting the days of the week or ticking off tasks in a check-list. The combination of this with other evidence has led Butterworth (1999) to argue that fingers are more than just a practical tool in numerical development and proposes that the functional use of fingers mediates the relationship between counting and arithmetic. Evidence supporting a relationship between finger gnosis and arithmetic may suggest that there is some functional link between using the fingers and learning to compute sums.

3.1.2 Localizationist account

Finger awareness is a skill which relies in part on sensori-motor information (the ability to associate sensory and motor functions). For example, to accurately identify a finger pressed, children must associate the sensation of the physical contact and the motor information relating to the finger and hand. This ability, along with other sensori-motor skills have been associated with the parietal lobe, a brain region commonly associated with numerical and arithmetic tasks (e.g. Dehaene et al., 2003). As the brain regions associated with finger awareness, and other sensori-motor skills, have been implicated in numerical development, it is possible that some damage to this region (either in adulthood or developmentally) may provide a reason for the connection between these skills.

3.2 Sensori-motor abilities and arithmetic – the evidence

A handful of studies have attempted to examine the relationship between finger awareness (and other Gerstmann symptoms or sensori-motor skills) and arithmetic. In one such study, Noël (2005) assessed if finger awareness, and other Gerstmann symptoms measured in 45 children in the first year of school (mean age = 6 years 10 months) predicted numerical skills including arithmetic 15 months later (N = 41). Children were assessed on the Gerstmann symptoms finger gnosis and left-right orientation at time 1, and handwriting and constructional apraxia (an inability to build, draw or assemble objects) at time 2. Constructional apraxia, measured via a block design task, was included due to previous links with potential cases of developmental Gerstmann syndrome (Benton, 1977). In the finger gnosis task, children were required to identify, without visual assistance, which of their finger(s) were pressed by an examiner, and then point to the finger(s) that had been touched. Left-right orientation measured the child's ability to identify left and right on both their own, and the experimenter's body.

Numerical tests at time 2 were symbolic and approximate magnitude comparison, number writing, subitizing, counting and an addition fluency task. Noël (2005) reports that the four Gerstmann symptoms were moderately or strongly correlated with numerical error score at time 2: finger gnosis and left-right orientation at time 1 (r = .48 and r = .34, respectively), and handwriting and block design at time 2 (r = .43 and r = .44, respectively). Conversely, word reading, and processing speed did not correlate with Gerstmann's symptoms nor numerical scores. Regression analysis showed that 46% of the variation in numerical error score at time 2 was explained by finger gnosis at time 1, handwriting and block design.

These results suggest that symptoms of Gerstmann’s syndrome may be implicated in the development of numerical, and arithmetic skills. As both finger gnosis and left-right orientation appear to play similar roles in predicting numerical abilities, it is not possible to identify if a localizationist, or functional account may explain such a relationship. However, before considering why the relationship exists, there are important limitations to consider. Firstly, the reliability of the finger gnosis task is questioned (exact reliability scores are not given) because at time 1 children were performing at ceiling on half of the trials in the finger gnosis task. Another potential problem is the small sample size (N = 41). In addition, the regression model does not account for the possible effects of age (children in this study ranged from 5 to 7 years). Finally, a “numerical skills” factor was created from the numerical tasks which does not represent typical or conventional measures of children’s arithmetic ability. Rather, the measure includes a range of numerical abilities from approximate numerical skills (e.g. subitizing and approximate comparison) to exact number abilities (counting and symbolic comparison) and number writing. Therefore, the exact relationship between Gerstmann symptoms and arithmetic is not known.

Interestingly, a later study by Penner-Wilger and colleagues (2007) used the same finger gnosis task on a larger group of children (N = 146) and found only a weak correlation (r = .16) between finger gnosis and arithmetic whilst reporting no longitudinal relationship between finger gnosis and calculation in a follow-up study (Penner-Wilger et al., 2009). Together, these results call into question the conclusions of Noël (2005) and cannot provide direct support for a functional account, which would predict a strong relationship between finger gnosis and arithmetic (which is either not supported or not shown). Furthermore, a training design by Gracia-Bafalluy and Noël (2008) examined a causal link by training children in finger gnosis but failed to find any difference in post-training addition scores compared to the control group. As this study suffered from a small sample size (N = 47) and failed to account for baseline differences between the groups (see Fischer, 2010) and therefore provides no convincing evidence for a causal role of finger gnosis in arithmetic. In summary, there is at best limited evidence for a functional relationship between finger awareness and arithmetic from a variety of concurrent, longitudinal and training designs.

An alternative explanation for the clustering of Gerstmann symptoms, is a localization account which would argue that the symptoms rely on similar brain regions. One potential way to assess this is to consider Gerstmann symptoms alongside other cognitive abilities which may rely on similar brain regions. Fayol, Barrouillet, and Marinthe (1998) examined a range of tasks considered to rely on adjacent parts of the parietal lobe (a brain area implicated in Gerstmann's syndrome and numerical abilities). Alongside calculation, four "neuropsychological" or sensori-motor tasks, were assessed. These were finger gnosis (identify fingers pressed), simultagnosia (identify body-parts pressed) and graphisthesia (identify shapes traced on back of hand) all of which rely on the integration of sensory information from the body. The authors assessed these measures, alongside numerical skills, in 189 children in the first year of school (mean age 5 years 9 months) and then again eight months later (N = 177). The sensori-motor skills (measured as a combined score) were reported to correlate well with numerical abilities at time 1 (r = .50, partial R^2 = .39) and time 2 (r = .47, partial R^2 = .36). However, at time 2, general intelligence correlated almost equivalently with the sensori-motor tasks (r = .44) and better explained numerical abilities than the sensori-motor tasks. Moreover, the numerical abilities construct comprised five numerical tasks (number writing, number sequencing, counting, calculation, and word problems).

As with Noël (2005) this is not a standard arithmetic measure but involved considerably more numerical knowledge than a basic calculation task. Indeed, the unique relationship between calculation and sensori-motor skills was far weaker (r = .23) than that of the combined numerical score. Finally, it is worth considering that the neuropsychological score (the combined performance on all sensori-motor tasks) which was associated with numerical abilities contained numerical processing. For example, the graphisthesia measure was the ability to identify shapes drawn on the back of the hand. These shapes included Arabic digits and numerical signs (e.g. +, 4) and in the finger gnosis test, children were asked to state which finger was pressed using an assigned number 1-5. Although minimal, it is possible that children with mathematical learning difficulties may have performed poorly on these tasks due to demands on number knowledge.

So far, the papers reviewed that have claimed to find evidence for a relationship between finger gnosis (and other sensori-motor skills) and arithmetic show important limitations (e.g. Fayol, Barrouillet, and Marinthe, 1998; Noël, 2005). Moreover, the findings have not been supported by other studies with similar aims (e.g. Penner-Wilger et al. 2007; 2009). One study that overcame some common limitations (e.g. small sample sizes and a failure to control for predictors of arithmetic) by Wasner, Nuerk, Martignon, Roesch and Moeller (2016) reports lower correlations between finger gnosis and arithmetic than previous findings. Participants were 321 German children in the first year of school (mean age = 6 years 6 months) examined on finger gnosis, arithmetic (addition and subtraction sums) numerical skills (symbolic-nonsymbolic mapping, magnitude comparison and number ordering) and general cognitive abilities (intelligence, short-term memory, spatial skills). Finger gnosis correlated significantly with both addition and subtraction (r = .23 and r = .24, respectively). However, a hierarchical regression showed that finger gnosis, alongside unique predictors of arithmetic (age, numerical abilities and memory) predicted only 1-2% of the unique variance in arithmetic which arguably is of no clinical significance. However, the finger gnosis task used shows low reliability (Cronbach's alpha = .55) which will limit the ability of this task to correlate with other measures.

Despite, this potential limitation, another recent study found no predictive relationship of finger awareness in arithmetic in 76 children (mean age = 8.67 years) after controlling for age and working memory (Newman, 2016). These studies support our view that with the inclusion of established predictors of arithmetic, finger gnosis will play, at best, a minor role in arithmetic development. As neither of these more recent studies included other sensori-motor skills, more evidence is needed to assess the relationship between these skills and arithmetic.

Overall, there is at best weak evidence that sensori-motor skills and finger gnosis are related to the development of arithmetic skills. Studies reporting such a relationship show limitations such as small sample sizes and a failure to account for other predictors of arithmetic including counting and magnitude comparison. One study which reports a very small unique relationship between finger gnosis and arithmetic

(Wasner et al., 2016) overcomes some of these limitations by using a large sample size (N = 321) and controlling for known predictors of arithmetic. However, the finger gnosis task they used had low reliability (Cronbach's alpha = .55) and this study did not include other sensori-motor skills, which have previously been linked to numerical abilities.

We conducted a study to overcome such limitations, including using a large sample size (N = 204) and controlling for important skills such as age counting and magnitude comparison. Additionally, we included a finger gnosis task with good reliability to increase the potential correlation with other measures, alongside a graphisthesia measure, like that in Fayol et al. (1998) but without any numerical processing, and the left-right orientation measure as used in Noël (2005) to investigate the relationship between different sensori-motor skills and arithmetic.

3.3 Method

3.3.1 Participants

A total of 204 children (103 boys, 101 girls) unselected for ability participated in the study with an average age of 7 years 1 month (*SD* = 9.42 months, range 5;6 – 8;8 [years; months]): 130 children from two schools in London, England and 74 children from one school in Brisbane, Australia. Children in England were tested on all measures, had an average age of 6 years 2 months (*SD* = 6.67 months, range = 5;6–7;8) and were in Year 1 (n =77) and Year 2 (n = 53)**.** Children in Australia were tested on finger gnosis and numerical measures, but not other sensori-motor measures, and were in Year 2 with an average age of 7 years 11 months (*SD* = 3.88, range = 7;4 – 8;8)[1]. Children of this age were chosen for consistency with previous similar studies. The schools provided consent for all children in a year group to participate and parents were given the option to opt-out or withdraw their child from the study at any time. Full ethical approval was provided by The University of College London Ethics Committee and the Australian Catholic University Ethics Committee.

[1] Note that Australian children showed equivalent scores for arithmetic and other outcomes as those in Year 2 (equivalent age) of the UK sample.

3.3.2 Test and procedures

Children in the UK were assessed on numerical skills (arithmetic, magnitude comparison tasks and counting) sensori-motor skills (finger gnosis, left-right orientation and graphisthesia) and receptive vocabulary. Children in Australia were tested on the numerical tests and the finger gnosis measures only.

The numerical and vocabulary measures were administered in groups whereby all children in a class completed the tests at the same time with instruction from the lead researcher. Teachers and teaching assistants supervised the sessions to ensure all children were concentrating and answering the questions appropriately. Sensori-motor skills were measured individually by the lead researcher and by trained research assistants. These sessions took place in an unused classroom at the school and took a total of 20-30 minutes per child. The order in which the sensori-motor tests were administered was randomised across participants. Children were awarded a sticker for their participation.

Sensori-motor tasks

The sensori-motor tasks, including finger gnosis, were based largely on those used in previous studies but were adapted to improve reliability (e.g. increase number of stimuli) and reduce cognitive confounds (e.g. remove numerical processing).

Finger Gnosis. The finger gnosis test was based on Noël (2005) and assessed a child's ability to identify the finger(s) pressed without visual assistance. It was adapted to include a larger number of finger presses to increase reliability. Children placed their hands palm down into a box that covered their hands but allowed the experimenter to see them. The experimenter applied light pressure to the child's finger between the nail and finger joint using a stylus. The child pointed to the finger(s) pressed using the index finger of the other hand whilst their hands remained out of their sight.

The test was first administered on the left hand and then repeated with the child's right hand. There were 25 trials for each hand with one point awarded for each correct trial. No feedback was provided apart from the first five practice items in which single finger presses were administered. The first five test items involved individual finger presses, the second 10 trials involved two different fingers being pressed in succession and the final 10 trials involved two different fingers being pressed simultaneously. In the simultaneous trials, children were awarded one point for correctly identifying both fingers but were not awarded a point if they only identified one finger correctly. Similarly, for successive trials, children were awarded a point for correctly identifying both fingers that were pressed although the correct identification did not have to be in the correct order. The maximum possible score on the task was 50.

Left-right orientation. The task used was the same as that used in Noël (2005) and aimed to measure the child's knowledge of left and right. There were two components to the task: colour and words which were identical apart from the lack of verbal confounds in the colour version of the task. All children completed the colour task first in which the experimenter tied different coloured paper to their wrists and feet. Facing away from the child, the experimenter asked the child to raise a coloured arm or leg (e.g. "raise your green arm like me"). Then the experimenter repeated this but facing the child. One mark was awarded if the child raised the same body part as the experimenter making a total score of 8 (four points possible when facing away from the child and four points when facing towards the child). Next, the children completed the same task, but the colours were replaced with left or right (e.g. "raise your right leg like me"). Again, one point was awarded for correct responses, with a total of 16 points for both parts of this task. Children were not given feedback and there were no practice items.

Graphisthesia. This task measured the child's ability to identify shapes through touch and was adapted from Fayol et al. (1998) to include more shapes and trials and remove numerical confounds. Children closed their eyes and the experimenter traced a shape onto the back of their hand with the tip of a pen. In front of the child was a piece of A4 paper showing ten shapes and the child had to point to the shape that they thought had been traced on their hand. Shapes were simple and verbal such as a square, heart, star and balloon and unlike in Fayol et al. (1998) there were no shapes that could be related to numerical learning (e.g. 4, +, -). Shapes were first traced onto the child's dominant hand and then the procedure repeated with the other hand. There were ten trials for each hand and one mark awarded for each correct answer; a maximum possible score of 20.

Arithmetic and number processing tasks

Number tasks were measured using the Test of Basic Arithmetic and Number Skills (TOBANS; Brigstocke, Moll & Hulme, 2016). This standardised measure requires children to complete simple numerical subtests under timed conditions. All children were tested in a group setting and were provided with a booklet to write their responses. Subtests were presented in the following order: addition, subtraction, symbolic magnitude comparison, approximate magnitude comparison and dot counting. For each subtest, there were three practice items for which the experimenter provided the answer once children had attempted the questions.

Arithmetic. One minute was given for each of the addition and subtraction tasks and 30 seconds for all other tasks. Children were instructed to answer as many questions as possible before the instructor said “stop”. Teaching assistants were in the classroom to ensure children were following instructions appropriately and understood the tasks. Arithmetic was a combined score from the addition and subtraction subtest which included items with sums less than 10 (e.g. 3+5 and 7 -3).

Number knowledge. Approximate and symbolic magnitude judgement tasks were used to assess numerical knowledge. The symbolic task used the digits 1-9 and the approximate task used dots. In the symbolic task, children were instructed to circle the larger of two numbers and in the approximate task, children were instructed to identify the more numerous group of dots from a series of stimulus pairs. Separate scores were created as the total number of correct items for each of these subtests.

Counting. Counting was assessed through a dot counting task in which children had to count the number of dots (up to 20) in a series of displays and write the associated Arabic numeral. Children scored one point for each correct written answer.

Vocabulary

Receptive vocabulary was assessed using a group-administered test adapted from the British Picture Vocabulary Scale (Dunn & Dunn, 2009). Four pictures were projected onto the classroom whiteboard and children were asked to identify the picture that matched a target word spoken by the experimenter. The items were graded in difficulty and children responded by marking the correct picture in their response booklet. There were two practice items in which feedback was given, followed by 33 test items.

3.4 Results

The primary aim of the study was to assess the contribution of finger gnosis, and other sensori-motor skills, as predictors of arithmetic skill. This was assessed through simple and partial correlations (controlling for age) and follow-up regression analyses whereby known predictors of arithmetic (e.g. counting and magnitude comparison tasks) were controlled. All data analyses were conducted in Stata (Version 14.0).

Descriptive statistics and reliabilities for all tasks are shown in Table 3.1. Tests showed good distributions (no ceiling or floor effects) and good reliabilities. As expected, the improved finger gnosis test showed better reliability (Cronbach's alpha = .74) than previous similar measures. Note that a small minority of children (n = 8) scored 0 on one or more of the following tests: dot counting (1 child), approximate comparison (4 children), symbolic comparison (1 child) and vocabulary (2 children). It is likely that these children had difficulty in following the instructions for these group-administered tasks. All children were retained in our analyses but dropping these eight cases made no difference to the pattern of effects reported. Some children were absent during the individual or group testing periods. Missing cases were dealt with by casewise deletion.

3.4.1 Correlations

The Pearson correlations (and partial correlations controlling for age) are shown in Table 3.2. Age was strongly correlated with all cognitive measures, apart from left-right orientation, and for this reason it was critical to control for it in the analyses. Importantly, although finger gnosis correlated with arithmetic (r = .43) this was greatly reduced once age was controlled (r = .12). The other two sensori-motor skills, left-right orientation and graphisthesia showed slightly better correlations with arithmetic (partial: r = .29 and r =.25, respectively) however, arithmetic was better correlated with approximate magnitude comparison (partial r = .34), and strongly correlated with symbolic magnitude comparison (partial r = .49) and dot counting (partial r = .58).

Unexpectedly, finger gnosis showed a moderate correlation (r = .38) with nonsymbolic magnitude comparison after the effects of age were controlled and vocabulary was only found to moderately correlate with arithmetic (r = .24).

Table 3.1 Descriptive statistics (means, standard deviations, and ranges), reliabilities and 95% confidence intervals for all measures.

	N	Mean (*SD*)	Reliability	Range	95% CI's
Finger (/50)	188	31.49 (7.95)	.74[1]	14-50	[30.35 – 32.63]
Left (/25)		15.83 (4.31)	---	7-25	[15.21 – 15.45]
Right (/25)		15.66 (4.54)	---	0-25	[15.01 – 15.31]
Single (/10)		9.41 (.97)	---	5-10	[9.27 – 9.55]
Succ (/20)		10.55 (4.07)	---	2-20	[9.96 – 11.13]
Sim (/20)		11.53 (4.15)	---	2-20	[10.94 – 12.13]
Left-right (/16)	111	9.63 (2.73)	.80[1]	4-16	[9.12 – 10.14]
Colour (/8)		4.89 (1.51)	---	2-8	[4.61 – 5.18]
Verbal (/8)		4.74 (1.79)	---	0-8	[4.40 – 5.07]
Graph (/20)	114	8.11 (3.21)	.64[1]	2-18	[7.51 – 8.70]
Right (/10)		3.84 (1.63)	---	0-8	[3.54 – 4.14]
Left (/10)		4.26 (2.05)	---	0-10	[3.88 – 4.64]
Calculation	197	18.99 (11.17)	.89[2]	0-55	[17.43 – 20.56]
Addition		11.28 (7.15)	---	0-34	[10.27-12.84]
Subtraction		7.72 (4.70)	---	0-24	[7.06-8.38]
Dot comp	197	11.50 (7.51)	.72[2]	0-32	[10.44 – 12.55]
Digit comp	197	19.63 (6.78)	.80[2]	0-38	[18.68 – 20.59]
Dot count	197	9.65 (3.53)	.79[2]	0-19	[9.15 – 10.15]
Vocab (/33)	197	21.04 (5.78)	.75[2]	0-31	[20.22 – 21.85]

Notes: CI, confidence interval. Finger = finger gnosis; Suc = successive; Sim = simultaneous; Left-right = left-right orientation; Graph = graphisthesia

[1] Cronbach's Alpha for children in the UK (*n* = 130)

[2] Test-retest reliabilities taken from the standardised TOBANS manual.

3.4.2 Finger gnosis, sensori-motor skills and arithmetic

The partial correlation between finger gnosis and calculation was *r* = .12 (95% confidence interval (CI) [-0.03, 0.26]) which is a very weak effect that is far from significant even in this large sample. We also assessed whether finger gnosis might be more closely related to calculation skills in younger than older children, as this group may use fingers more to assist with calculation. However, a median split of the sample indicated that the partial correlation between finger gnosis and calculation controlling for age was similar in the younger (*r* = .13 [95% *CI* -0.09; 0.33]; mean age 77 months, range 66 to 85 months) and older halves of the sample (r = .15 [95% *CI* -0.051; 0.339]; mean age 93 months, range 86 to 104 months).

It is worth noting that the putative role of finger gnosis in the development of arithmetic has often been related to its possible role in the development of counting skills (Butterworth, 1999). However, the partial correlation between finger gnosis and counting controlling for age was also negligible (r = .10 [95% *CI* -0.051; 0.238]).

Even though finger gnosis was only weakly correlated to arithmetic, both left-right orientation and graphisthesia were shown to correlate moderately with arithmetic, after age was controlled. The partial correlation for left-right orientation was r = .29, p = .002 and for graphisthesia was r = .25, p = .009. To assess the relative importance of these measures against other predictors of arithmetic, multiple regression models were used.

3.4.3 Predictors of calculation ability

We used hierarchical regression to model the relationships between calculation ability and the remaining cognitive measures that were significant correlates of calculation after age had been controlled (left-right orientation, graphisthesia, nonsymbolic magnitude comparison, symbolic magnitude comparison, dot counting and vocabulary).

Table 3.2 Correlations between measures.

	1	2	3	4	5	6	7	8
1. Finger gnosis	-	-.02	.23*	.12	.38**	.06	.10	.05
2. Left-right	-.02	-	.24*	.29*	.10	.25*	.17	.07
3. Graphisthesia	.23*	.29*	-	.25*	.16	.15	.18	.17
4. Arithmetic	.43**	.34*	.43**	-	.34**	.49**	.58**	.24*
5. Approximate comparison	.61**	.16	.27*	.61**	-	.50**	.36**	.26*
6. Symbolic comparison	.33**	.29*	.28*	.62**	.64**	-	.46**	.28*
7. Dot counting	.42**	.24*	.36**	.73**	.63**	.61**	-	.24*
8. Vocabulary	.43**	.15	.37**	.55**	.63**	.50**	.55**	-
9. Age	.58**	.17	.42**	.59**	.71**	.47**	.63**	.69**

Notes: Left-right = left-right orientation; Arithmetic = addition and subtraction summed scores. Partial correlations controlling for age are above the diagonal and simple correlations below the diagonal. * $p < .05$; ** $p < .001$.

In the first stage of the model, we entered age as this was strongly correlated with calculation ($r = .60$, $p < .001$) and is a well-established predictor of arithmetic (e.g. Durand et al., 2005). Age explained 35% of variation in arithmetic. In the next stage of the model, we entered nonsymbolic magnitude comparison, symbolic magnitude comparison, dot counting and vocabulary simultaneously as predictors. Approximate comparison and vocabulary were not significant predictors of arithmetic and were dropped from the model, leaving 61% of the variation in arithmetic explained by age symbolic comparison and dot counting (dot counting: unique $R^2 = .11$, $p < 0.001$; symbolic comparison, unique $R^2 = .04$, $p < 0.001$); together these two predictors accounted for 26% of the variance in calculation scores. Finally, we entered left-right orientation, graphisthesia and finger gnosis into the model. Only left-right orientation remained a significant predictor of arithmetic explaining an additional 2% of the variation ($R^2 = .02$, $p < 0.001$). When graphisthesia and left-right orientation were dropped from the model, finger gnosis accounted for just 1.4% of the variance, in line with Wasner et al. (2016).

In summary, both symbolic magnitude comparison and dot counting explained large proportions of unique variance in arithmetic whereas nonsymbolic magnitude comparison and vocabulary were not found to predict unique variation in arithmetic. Despite left-right orientation remaining significant in the final model, the relationship was small, accounting for only 2% of the variance in arithmetic. Neither finger gnosis nor graphisthesia explained any additional variation in arithmetic and it is likely that any such relationship between these sensori-motor measures and arithmetic are due to the substantial shared variance with age.

3.5 Discussion

In this study, we examined how a range of cognitive predictors relate to arithmetic. We showed that sensori-motor skills, including finger awareness, are less well correlated with arithmetic than numerical abilities. These findings contradict some previous evidence supporting a relationship between sensori-motor skills and arithmetic (Fayol et al., 1998; Gracia-Bafalluy & Noël, 2008; Noël, 2005). However, these studies suffer from small sample sizes, measures with low reliability and a failure to control for important predictors of arithmetic (e.g. age, counting and magnitude comparison). More recent evidence, which overcomes such limitations, is in line with our findings (Newman, 2016; Wasner et al., 2016).

3.5.1 Sensori-motor skills and arithmetic

The first, and major finding is that finger gnosis does not remain a strong correlate of arithmetic after age is controlled. It is important to consider age within the analyses because age is both a strong correlate of arithmetic and shows variance within our study (children ranged from 5-8 years across two year groups). Importantly, one study which suggests a strong correlation between finger gnosis and arithmetic (Noël, 2005) did not control for age, despite examining children with an age range of 5-7 years. Our results are in line with Newman (2016) who showed that before age, finger gnosis and arithmetic correlated significantly (r = .36) at similar levels to our study (r = .43). However, after controlling for age, finger gnosis correlated with arithmetic non-significantly and at negligible levels (r = .20) akin to our findings (r = .12). Wasner et al. (2016) do not provide partial correlations with age between finger gnosis and arithmetic.

The correlations with addition (r = .23) and subtraction (r = .24) are small (although significant) but when age is controlled in a hierarchical regression, the relationship between finger gnosis and arithmetic scores is reduced significantly (explaining only 1-2% of variation). This is in line with our finding that finger gnosis predicts (a non-significant) 1% of the variation in arithmetic, when other important factors are controlled. Together, this suggests a weak and clinically insignificant relationship between finger gnosis and arithmetic.

Our second finding concerns the role of two sensori-motor skills in arithmetic development. We find that left-right orientation and graphisthesia were small but significant correlates of arithmetic after controlling for age (r = .29, r = .25) but only left-right orientation remained a unique and significant predictor after controlling for other predictors of arithmetic (R^2 = .02). Previous studies supporting a relationship between sensori-motor skills and arithmetic are limited by a failure to control for other predictors, including numerical abilities (e.g. Noël, 2005) and the inclusion of numerical processing in the sensori-motor tasks (e.g. Fayol et al., 1998). As we found no strong relationship between the sensori-motor skills and arithmetic, we cannot support a localizationist account. Rather, we tentatively suggest that the small relationship in arithmetic explained by left-right orientation is due to verbal abilities; our left-right orientation task contained considerable verbal processing and arithmetic is linked with verbal skills (Durand et al., 2005), whereas our finger gnosis and graphisthesia tasks relied on minimal verbal information.

3.5.2 The functional and localizationist accounts

Sensori-motor skills were first linked to arithmetic via Gerstmann's syndrome, which is argued to be a cluster of symptoms involving skills relying on the integration of motor and sensory skills (Lebrun, 2005). There are two dominant theories attempting to explain the co-occurrence of these symptoms; a functional account and localizationist account. In the functional account, theorists propose that finger awareness (a sensori-motor skill) and arithmetic are related due to the role of using fingers in counting and addition and subtraction (Butterworth, 1999). In this account, finger awareness would be better correlated with counting and arithmetic than other sensori-motor skills. Alternatively, the localizationist account proposes that sensori-motor skills and other symptoms of Gerstmann's syndrome are correlated with arithmetic due to the proximity of brain regions associated with these skills (e.g. Dehaene, Piazza, Pinel, & Cohen, 2003). In this account, any sensori-motor ability relying on the parietal lobe will be related to numerical and calculation abilities.

We aimed to assess these theories by including finger gnosis and sensori-motor skills including graphisthesia (argued to rely on the parietal lobe) and left-right orientation (a Gerstmann's syndrome). Previous studies have not included all three factors within one design. Moreover, finger gnosis tasks used in previous studies have shown low reliability (Noël, 2005; Wasner et al., 2016) and one study examining graphisthesia included verbal and numerical information within it. Our study used a reliable finger gnosis task (Cronbach's alpha = .74), removed verbal and numerical confounds in the graphisthesia task, and included other Gerstmann syndrome tasks to directly compare the relationship between these measures.

In our study, we found no evidence to support a functional account and limited evidence to support a localizationist account. The three skills were moderately and significantly correlated with arithmetic (*r*s = .34-.43) but these were reduced to a weak correlation after age was controlled for left-right orientation and graphisthesia (*r*s = .29; .25, respectively) and non-significant and negligible correlation with finger gnosis (r = .12, $p < .05$). Moreover, regression analyses showed that graphisthesia does not play a unique role in predicting arithmetic once other important factors (age, counting and symbolic number comparison) are included, and left-right orientation is a weak correlate of arithmetic.

All three skills were entered into the model and only left-right orientation remained a significant predictor, explaining an additional 2% of the variation in arithmetic. Importantly, our counting and comparison tasks showed good reliability and a good range of scores, and the finding that these exact skills are important predictors of arithmetic is in line with previous evidence (see Section 2.2.1). This suggests that our study is well placed to find a relationship between sensori-motor skills and arithmetic, should one exist, and therefore we argue that there is limited evidence to support either a functional or localizationist account of the relationship between sensori-motor skills and arithmetic development.

We found that left-right orientation remained a significant unique predictor of arithmetic after controlling for exact number abilities (R^2 = .20). Although this is a small and likely clinically insignificant effect, it is worth considering what this means in regard to the localizationist theory. This theory proposes that skills relying on the parietal lobe, for instance Gerstmann symptoms, will predict arithmetic which also relies on the parietal lobe (see Dehaene et al., 2003). However, we found that the correlations between left-right orientation and finger gnosis and graphisthesia, also proposed to rely on the parietal lobe (Fayol et al., 1998; Noël, 2005) were weak.

We tentatively suggest that it is possible that verbal abilities may explain in part the correlations seen between left-right orientation and arithmetic, as both of these rely on verbal abilities (Durand et al., 2005). The left-right orientation task involved considerable verbal information, whereas the graphisthesia task (point to objects) and the finger gnosis task (point to fingers) required less verbal information. However, we found weak relationships between the sensori-motor skills and vocabulary once age was controlled (although vocabulary was moderately and significantly correlated with arithmetic; partial r = .24). Further research should consider including spatial awareness and non-verbal IQ measures to assess why such a relationship may exist.

We suggest that previous evidence which supports a relationship between finger gnosis and other sensori-motor skills is limited, explaining the difference in findings from some previous studies and our study. For example, Noël (2005) fails to account for the effect of age in a study which had variation within ages of the children and therefore likely over estimates the correlation between finger gnosis and left-right orientation. Our study highlights the importance of including age as a control measure due to the significant correlation with arithmetic (age explained 35% of the unique variation in arithmetic scores). Gracia-Bafalluy and Noël (2008) report a significant improvement in arithmetic scores after finger gnosis training, although this study had a small sample size (N = 33) and no baseline scores for the groups were reported (see Fischer, 2010). Fayol et al. (1998) report a relationship between a composite score of sensori-motor skills including graphisthesia and finger gnosis. However, this battery included some numerical processing which may have confounded the results, and the arithmetic measure was a composite of numerical tasks including approximate number abilities and complex verbal addition tasks (e.g. “Jack has 6 green pencils, 4 red pencils and 5 blue pencils. How many pencils has he got?” p. 66). Moreover, when the “pure” arithmetic task (addition and subtraction scores) was compared to the neuropsychological composite score, the relationship with arithmetic was far weaker than it was to the numerical composite.

More recent evidence, which has overcome such limitations, fails to report a significant and important relationship between finger gnosis and arithmetic. For example, Wasner et al. (2016) showed that finger gnosis explained 1-2% of the variation in arithmetic (N = 321) after controlling for age, number skills and memory; the same level we showed. Penner-Wilger et al. (2007) showed that finger gnosis and arithmetic was weakly correlated (r = .16, N = 146) which is equivalent to the correlation we found between finger gnosis and arithmetic. Similarly, Newman (2016) found no evidence for a correlation in a sample of 76 children once age was controlled. None of these studies have examined how sensori-motor skills, other than finger gnosis, relate to arithmetic and therefore we are the first to show that neither finger gnosis, nor other sensori-motor skills, appear to play an important role in arithmetic development.

3.5.3 Numerical measures

In line with other studies, our data showed that symbolic number comparison and dot counting were important predictors of arithmetic (together explaining 26% of the variation). Our approximate magnitude comparison task did not remain a unique predictor after controlling for these abilities. Both of these findings are in line with considerable evidence supporting the role of exact and symbolic number skills in arithmetic abilities (see Schneider et al., 2016 for a meta-analysis).

Symbolic magnitude comparison and counting rely on exact number abilities and counting abilities. Counting is clearly related to arithmetic: Basic addition and subtraction relies on counting abilities (see Section 1.1). Symbolic number skills are likely related to arithmetic because for symbolic comparison the child is required to have number knowledge which is the ability to identify the numbers, know the magnitude of the numbers and/or the order the numbers come in. Arithmetic also relies on number knowledge, as to compute written arithmetic sums a child must understand the relationship between the orthography, verbal code and magnitude of the number.

Several studies have linked these skills and, importantly, recently evidence has suggested that symbolic number comparison is more important in arithmetic than approximate number skills (e.g. Lyons et al., 2014; Schneider et al., 2016; Xenidou-Dervoua et al., 2017). Our data supports a greater role of symbolic number skills than approximate in this age group as approximate comparison was a weaker correlate with arithmetic. This suggests that, for children in the first years of formal education, exact number skills are a better predictor of arithmetic than approximate number skills. We discuss findings concerning a causal relationship in Chapter 4.

Our finding that approximate and symbolic number tasks are related to one another, but that symbolic number skills are more strongly linked to arithmetic, has implications for theories of numerical development. This finding does not support the innate core number hypothesis (See Section 1.4) which proposes that approximate skills will relate to later arithmetic. However, more recently, researchers have proposed that this innate skill may have some influence over the way that children learn symbolic numbers which, in turn, affects arithmetic. Our data could support this, although longitudinal evidence would be required to confirm this. For instance, by showing that early approximate skills and later arithmetic are mediated by symbolic number, as shown in some recent evidence (Marle, Chu, Li, & Geary, 2014).

One unexpected finding from the current study was that finger gnosis correlated moderately to strongly with nonsymbolic magnitude comparison (r = .61; after age was controlled, r = .38). This is a finding that deserves replication, although we may tentatively suggest that inhibitory control may be a third variable driving the relationship, as this has shown to relate to magnitude comparison (Fuhs & McNeil, 2013) and may potentially relate to finger gnosis, although further examination is required to support this suggestion.

3.5.4 Limitations

One possible limitation of the current study is that it examined children who had generally mastered basic addition and subtraction skills and therefore may be too old to identify any relationship that may exist in younger children. We chose children between 5-7 years old to match the age of children in previous studies. Newman (2016) found that the relationship between finger gnosis and arithmetic was stronger, although still non-significant, in younger children compared to older children. Moreover, our study showed that the youngest children (50% split) showed a slightly higher, although still small and non-significant, relationship with arithmetic (r = .17,

$p > .05$) compared to the full sample. Further research examining children closer to the start of formal education is needed to examine this question.

In summary, we found no support for the suggestion that finger gnosis, or other sensori-motor skills, are related in any important way to arithmetic development. On the other hand there is evidence to support a relationship between number skills and arithmetic, providing a rationale for the next study which further examined the causal relationship between these skills. We therefore argue that sensori-motor skills are less important in numerical development than exact number abilities in children aged 5-7 years, and therefore suggest that educational practices focus more closely on number knowledge learning (e.g. identifying numbers and knowing the magnitude and order; Merkley & Ansari, 2016) than on sensori-motor skills.

Chapter 4. Study 2: Training early number knowledge using an app-based game. The transfer to arithmetic and numerical skills.

In the first study, symbolic magnitude comparison and counting were good correlates and unique predictors of arithmetic. From this finding arose an interest in examining the causal relationship between symbolic number knowledge and arithmetic. A game that had been previously developed (see Section 1.2.2) was used to design a symbolic number knowledge training study. When playing the game, the player is required to create numerical sequences whilst hearing the sound of the digit pressed. This utilises two key components of symbolic number knowledge: understanding ordinality (creating sequences) whilst reinforcing the identity of Arabic digits (by hearing the sound of the number as they pressed to choose it).

Number knowledge is a critical part of early numerical development and is taught from a young age in formal education. Critical to mastering basic addition and subtraction sums is the ability to identify written numbers and understand numerical order. There is now robust evidence linking number knowledge to arithmetic skills in typically developing children from both correlational and longitudinal studies (e.g. Göbel et al., 2014; Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Nguyen et al., 2016; Vanbinst et al., 2016; Xenidou-Dervoua et al., 2017). This evidence provides rationale for developing training designs to assess if training in number knowledge can lead to improvements in arithmetic.

4.1.1 Training symbolic number knowledge

Much of the earliest training evidence comes from Siegler and Ramani (Ramani & Siegler, 2008; Siegler & Ramani, 2008; Siegler & Ramani, 2009) who showed, using a linear board game, that number knowledge can be taught. The linear game involved naming and recognising the numbers 1-10. Children played individually with an experimenter, and took turns rolling a 2-sided die (numbers 1 and 2) before moving the corresponding number of spaces up the board whilst naming the numbers moved aloud. The game allowed children to practice identifying numbers (identification) and reinforced knowledge of the order in which numbers appear (ordinality). Children in a control group played an equivalent game using colours rather than numbers.

In Ramani and Siegler (2008) 124 children aged 5 years 4 months were recruited from Head Start centres and randomly assigned to either a number game condition (n = 68) or colour game condition (n = 56). Children from these centres, who are from low-income families, were chosen due to the relatively poor performance in early numerical skills (see Jordan & Levine, 2009, for a review). After playing the game for a total of one hour, children were tested on counting, number line estimation, symbolic comparison, and number identification for the numbers 1-10. Those children in the number game condition improved more than those in the colour condition in all measures from pre-test (session one) to post-test (session four) and follow-up (session five). Improvements were all significant with medium to large effect sizes (for pre-test to post-test and pre-test to follow-up: counting, d = 0.65 and d = 0.69; line estimation, d = 0.99, d = 0.59; symbolic comparison, d = 0.79 and d = 0.57; number identification, d = 0.44 and d = 0.63). Similar findings were reported by Siegler and Ramani (2008, 2009) in two other training studies using the same board game. Taken together, these findings suggest that when trained in basic number knowledge, children from low-income families can improve on numerical tasks.

These few training studies are promising as they support a causal relationship between number knowledge and numerical skills. However, only children from low-income families benefited from playing the number game. Children from middle- to upper-class backgrounds failed to improve on post-training measures (Siegler & Ramani, 2008). Additionally, children in Ramani & Siegler (2008) were only tested on their knowledge of the digits 1-10, i.e. the digits they were trained on and therefore the transfer to other numbers is not known. Finally, in all but one training study, the authors failed to include arithmetic as an outcome measure. When arithmetic was included as an outcome (Siegler & Ramani, 2009) children were trained on the board game and had additional training in arithmetic. Therefore, this additional direct training in arithmetic may be the reason for increased arithmetic performance rather than training in number knowledge. Resultantly, the relationship between training in number knowledge and arithmetic outcome is not known from these studies.

Some subsequent training studies assessed transfer to arithmetic after more extensive training in number knowledge. For example, Jordan and colleagues (2012) randomly assigned 128 children in Kindergarten from low-income families to number training

(n = 42), language training (n = 42) or an untreated control group (n = 44). Training sessions for the intervention groups were conducted in small groups for 30 minutes over 24 sessions. Children in the number training group played the linear board game (as in Ramani & Siegler, 2008) as well as practicing other number skills, including numbers outside the 1-10 range. The skills practiced were number recognition, number sequencing, verbal subitizing, finger use in counting, number writing, and verbal and nonverbal addition and subtraction sums. The language intervention focused on story books and vocabulary. All children were assessed pre- and post-training on number knowledge skills including counting, number identification, story addition problems and nonverbal calculation.

Performance of the number training and language training groups were compared to an untreated control group, and larger gains were reported on all number measures for the number training group compared to untreated controls (overall average effect size for significant gains; d = 1.12) compared to no change for language group versus controls (overall average effect size for significant gains; d = -0.18). Particularly impressive were the gains found for verbal calculation; children in the number sense group showed large gains compared to untreated control group (d = 2.64) remaining at eight-weeks post-test (d = 2.27). Conversely, for verbal calculation, children in the language group did not improve significantly at post-test compared to those in the untreated condition (d = -.23) thus suggesting that the number intervention was instrumental in improving scores on verbal arithmetic questions. However, the same gains were not found for nonverbal arithmetic, in which children in the number training group showed small and non-significant gains compared to those in the untreated control group (d = .19).

In a follow-up training study, the primary aim was to improve non-verbal arithmetic abilities in a similar group of children (Dyson, Jordan and Glutting, 2013). The intervention was adapted slightly by increasing focus on the relationship between story problems (e.g. “Jane has three apples and picks two more, how many will she have?”) and the symbolic representation of numbers. Children were encouraged to choose symbolic numbers that matched the numbers used in the story to help them solve addition and subtraction problems. Other aspects of the intervention remained the same. Three sessions a week for 8-weeks were given to low-income Kindergarten children split into either a number sense group (n = 65) or "business as usual" group (n = 56). The number training group improved significantly more than those in the control group at post-test in number skills, including verbal and nonverbal calculation: written calculation (R^2 = .20, p < .01), story problems (R^2 = .30, p < .001) (pre-test number scores were controlled). However, the non-verbal calculation effect was not significant at six-weeks post intervention.

These findings suggest that through intensive training on number knowledge and calculation, and particularly the relationship between Arabic numerals and verbal codes, children can improve their calculation skills. A clear limitation of this study is a failure to include a treated control condition, and therefore, it is not clear if the improvements in the number group are merely due to extra attention. Additionally, for both this study and Jordan et al. (2012) the training included practice of calculation and therefore, it is not clear what the specific role of training in number knowledge is in transfer to arithmetic.

The training studies discussed so far provide initial evidence that children from low-income families can improve in arithmetic and other number skills, including number recognition and number knowledge after symbolic number training. A clear limitation of these studies is that the training included components of the outcome measures, for example training children in addition and then examining changes to addition scores. Moreover, these studies focus on children from low-income families, and thus there is a lack of evidence from studies training children from a range of demographic backgrounds.

A study by Honoré and Noël (2016) sought to overcome these limitations by examining children from a range of demographic backgrounds, and testing transfer to arithmetic whilst not including this as part of the training. Fifty-six preschool children (mean age 5 years 9 months) from a middle/high social class were trained for ten 30-minute sessions in symbolic number skills (n = 19) approximate number skills (n = 19) or non-numerical training (control condition; n = 18). Number training groups played two games during each session, either a comparison game or a line estimation game. For the comparison game, those in the symbolic training condition were shown three bags of equal physical size, but with three different Arabic digits printed on the front alongside three toy animals of different sizes. Children were told to give the bag with the highest digit to the biggest animal (or smallest digit to smallest animal) and were provided with feedback. The approximate training was equivalent but used magnitude representations (dots) rather than Arabic numerals to represent the size of the bag. Numbers used were 1-9 for small sizes and 10-19 for more difficult levels.

For the number line game, children were told to position an Arabic digit (or dot collection) on a number line between 1-10 (first level) or 1-20 (second level). Children played both games on a laptop with little input from the experimenter apart from encouragement to continue with the game. Arithmetic was measured immediately before and after training using a word addition task in which child chose answers in an Arabic digit or dot representation (depending on group). After controlling for pre-test scores, the authors report that children in the symbolic number group improved significantly in addition score at post-test compared to those in the other groups, with a large effect size (Pearson's correlation coefficient effect size; $r = .72$). Those in the approximate training group did not show significant improvements in arithmetic score after training. This study is therefore the first to show that training in symbolic number knowledge (without training in arithmetic) can lead to significant improvements in arithmetic performance.

Also, of interest from this study is the use of a computer-based training design. Studies using a computer-based design have potential advantages over traditional interventions. Firstly, interacting with a computer (rather than experimenter) allows more children to be trained at once. Secondly, it may enable children to play a game at home, and therefore play for longer periods of time than is available during the school day. Finally, computer games, particularly those with advanced graphics, may result in training which is interesting and more engaging for the child; an important consideration as maximum motivation is important for training.

Sella, Tressoldi, Lucangeli and Zorzi (2016) used a computerised design to examine changes in number skills after 23 preschool children played the Italian "Number Race" game (Wilson et al., 2006) which involved practising counting and calculation. Children played the game for approximately six hours, a long training time made possible as the game was played in large groups (each child had a computer and played without much input from the experimenter). Children in the training group ($n = 23$) showed improvements above those in an untreated control condition ($n = 22$) in some numerical tasks, including calculation. Mental calculation error scores were significantly lower for children in the training group compared to the control group, with a large effect (improvement index = 44). However, as with previous training studies, the game included calculation as part of the training. Moreover, when Obersteiner, Reiss and Ufer (2013) adapted the Number Race game to remove the calculation component of the game different outcomes were observed.

The authors examined the role of exact and approximate number training in numerical skills and arithmetic. Those in the exact training condition (n = 39) had to identify which of two dot sets matched the symbolic number presented (it is not reported what range of numbers were used). The authors report significant improvements in numerical skills for those in the exact number training study. However, no significant effects were found in arithmetic (p > .05, partial η^2 = .003). Despite these mixed findings, the implication of using a computerised task is twofold; both computer-based studies report high motivation through computerised learning and additionally show that it is possible to train children for large periods of time via group training and limited input from an experimenter.

In summary, there are a handful of studies which have aimed to improve arithmetic and numerical abilities in young children through training symbolic number knowledge. Of these, some report improvements in arithmetic after training (e.g. Dyson et al., 2013; Honoré & Noël, 2016; Jordan et al., 2012; Ramani & Siegler, 2009; Sella et al., 2016). However, the majority of these studies included calculation as part of the training, and only Honoré & Noël (2016) report significant improvements in arithmetic scores after number training not involving a calculation component. We therefore conducted a training study with typically developing children at the start of formal education to examine if training in number skills can improve arithmetic abilities.

4.1.2 The present study

The present study assesses whether training in number knowledge (but not arithmetic) improves calculation and other numerical abilities in children. Children in Reception, the first year of formal schooling in England, played a computerised game on a tablet every day for three weeks. The game required children to create numerical sequences and in doing so, children practiced (and learnt) the ordinality of numbers. As the sound of the digit pressed was presented orally (through headphones) the game also trained children in the association between the Arabic digit and verbal representation (identification). As the study involves training in basic number skills, children in the first year of formal schooling were chosen to participate as they will not yet have mastered basic number skills (as children in the second year of formal schooling may have done, for example). We compared changes to numerical outcome measures for three groups; experimental training group (numbers game), a treated control group (equivalent game with letter rather than number sequences) and a "business-as-usual" control group.

We aim to show that children in the number training group will improve in numerical measures, including arithmetic, compared to the untreated control group. Conversely, we do not expect that those in the letter training group will show meaningful differences in numerical measures at post-training when compared to the untreated control group. Additionally, we do not expect to find meaningful improvements in literacy-based outcomes for children trained in number knowledge compared to the untreated control group.

4.2 Method

4.2.1 Participants

Participants were children from three Reception classes at a school in South West London, England. All children in the year participated unless the parent/guardian opted out, or the teacher identified them as having a learning disorder (e.g. autism spectrum disorder). Ethical approval granted by University College London Ethics Committee. Six parents chose for their children to opt-out of the study, and three children were identified as having a learning need, resulting in the recruitment of 81 children (31 boys, 50 girls) in the initial testing stage. Three children left the school during the study (two from letter training group and one from numbers training group), so the final sample was 78 children (29 boys, 49 girls) with a mean age of 4 years 8 months (*SD* = 3.69 months, range = 4;3 – 5;4 [years; months]).

Children were tested on outcome measures immediately before training at time 1 (t1) and immediately after training at time 2 (t2). Using stratified random assignment of addition score at t1, children were split evenly into three training groups; Number

Training (n = 26), Letter Training (n = 25) and Untreated Control (n = 27). A one-way ANOVA (Analysis of Variance) test with between-subjects factor Group (number, letter, control) showed that the average addition scores at pre-test for number training (mean = 3.04, SD = 2.69) letter training (mean = 2.64, SD = 2.36) and untreated control (mean = 2.93, SD = 2.84) were equivalent across the groups ($F_{(2,75)}$ = .15, p = .86). Similarly, age was similar across groups: number training (mean = 57.62 months, SD = 3.89), letter training (mean = 56.32, SD = 3.45) and untreated control (mean = 57.41, SD = 3.74) with no significant differences found ($F_{(2, 75)}$ = .90, p = .41).

4.2.2 Measures

Participants were tested individually at t1 and t2 on a range of numerical and literacy measures. For all tests, feedback was provided for practice items but not for test items, although encouragement was given by the experimenters throughout. The individual testing sessions were run by a lead experimenter and two trained research assistants, with each session lasting no longer than 40 minutes. Pre-tests were administered during a three-week period at the end of the first term of school. In the new school term, the intervention was carried out (lasting three weeks) and t2 tests were administered over three weeks immediately following training. The research assistants were blind to group membership, but the lead researcher who also administered all training sessions had knowledge of the groups.

Number measures

Arithmetic. Arithmetic was measured via an addition task. Children were given a maximum of three minutes to solve basic addition questions, spoken aloud by the experimenter. After two practice questions, the first 13 questions were addends below 10 (e.g. 1+3, 7+2) and the final seven questions were addends above and including 10 (e.g. 5+5, 6+8). Children were encouraged to use their fingers to compute the sums. One point was awarded for each correctly written answer. The experimenter discontinued testing after five consecutive errors. A maximum of 20 marks were available.

Number identification. To assess number knowledge, children were asked to identify numbers spoken by the experimenter. On A4 pieces of paper, children were shown a grid with four numbers; three distractors and one target. Distractor items were similar to the target number for example, target number 7 had distractors 706, 17 and 70. Target numbers were single, double and triple digits ranging from 2-807 and were presented in ‘Comic Sans’ size 72 font. There were three practice items and 20 test items with one point awarded for each correct answer.

Number reading. Number knowledge was also assessed via a number reading task. Children were asked to read numbers aloud. Numbers were single, double and triple digits ranging from 1-437. All numbers were presented on A4 paper in bold ‘Calibri’ font, size 330. There were three practice items and 20 test items with one mark awarded for each.

Number writing. Children were instructed to write 15 numbers, including the numbers 1-10 and five larger numbers (range 15-100). There were 15 test items with one point awarded for each correct response. Mirror writing was allowed.

Counting. Rote counting was measured via an oral count-on task. Children were asked to count from one and were stopped when they made a mistake, or when they reached 50. The score was the number the child counted to without mistakes (maximum score was 50).

Literacy Measures

Letter-sound knowledge. Children’s knowledge of letter sounds was assessed using a subtest from the York Assessment for Reading Comprehension (YARC; Hulme et al., 2009). Children were presented with single letters and digraphs and asked to say aloud the sound that the letter(s) makes. One point was given for each correct response, and therefore a maximum of 17 points available.

Word reading. Examined using a subtest from the YARC, children were asked to read single words presented to them. Words increased in difficulty and experimenter stopped testing after ten consecutive errors. One point was given for each correct answer, and therefore a maximum of 30 points available.

Rapid Automatized Naming (RAN). The letter and number subtests from the Comprehensive Test of Phonological Processing (Wagner, Torgesen, Rashotte, & Pearson, 2013) were used to assess children’s ability to rapidly name letters and numbers. Children first practiced naming the numbers (4 8 7 2 5 3) and letters (a t s k c n) which were used in the test. Those children who were able to name the letters and numbers went on to take the test (note that letter names and not letter sounds were required). In both tests, 4 X 9 arrays of the letters or numbers were presented on A4 paper and children were asked to read the numbers/letters aloud as quickly as possible. If children made more than 4 errors, they were discontinued from testing (and given a score of 0). Other children progressed to Test 2 which involved the same numbers and letters presented in a different order. Total score for children who completed both tests was the combination of total time taken at Test 1 and Test 2 (seconds).

Sequences

Number sequences. Children were instructed to fill in the blanks for ten numerical sequences (e.g. 1 __ 3 __ 5 __). Numbers ranged from 1-101, with larger numbers used in later test items. Each sequence contained two or three blank items, and a child was awarded one point for correctly filling in all blank items (maximum 10 points). Numbers were presented in Calibri size 24. There was one practice item and the experimenter discontinued after two consecutive errors.

Letter sequences. The letter sequences task was the same as the number sequence task but used letters, rather than numbers. Sequences were generally matched across tasks (e.g. 1_3_5_ became A_C_E_) apart from the final three items which used numbers above 26 and therefore letters at the end of the alphabet were used. Letters were presented in capitals, Calibri size 24. Again, there was one practice item and ten test items with a maximum of 10 points available.

4.2.3 Training procedure

All children in the training groups played the assigned training game for a total of 60 minutes over a period of 15 consecutive days. Sessions lasted between five and seven minutes. Most children played the game over 12 consecutive days, although those who were absent caught up during the additional sessions. For training sessions, children were taken to an unused classroom in groups of five. Each child played the game on a tablet (Kindle Fire, 8-inch display) and wore over-ear headphones for the duration of the session. In the first training session, the experimenter showed all children how to play the game and allowed children to practice with feedback. For the remainder of the training sessions, the experimenter provided support if the child was stuck, for example prompting the child to recite the number sequence or alphabet.

All children received a similar level of help and most children in the numbers training group required little help from the experimenter after the third session, although some children in the letters group continued to require some assistance throughout the training period.

In the number sequences game, children were shown Arabic digits (numbers ranged from 1-10). The aim of the game was to create numerical sequences by joining adjacent numbers within an array (see Figure 4.1).

Figure 4.1 Example array in the numbers game (left) and letters game (right) with a correct sequence created.

Sequences were required to be at least three digits long and could start from any number. Every time the child pressed a square, they would hear the number or letter through their headphones. To submit a sequence the child pressed the final digit twice and received written positive feedback on the screen (e.g. “well done” or “fantastic”). Incorrect sequences turned purple (rather than blue for correct) and a prompt “press to correct” was shown. The letter sequences game was equivalent but used letters A-J rather than numbers 1-10).

All children started at Level 1 of the game and could progress to higher levels if they successfully completed the requirements of the level within the time given during the session. For each level the total number of moves varied, and the length of each required sequence differed. For example, to pass Level 1, players were given a total of 17 moves and required to make a minimum of two sequences at five digits long (e.g. 1-2-3-4-5) and two sequences containing at least six digits (e.g. 3-4-5-6-7-8). Higher levels required a greater number of longer sequences, or longer sequences within a varied number of moves. Additionally, the array size increased in higher levels. Levels 1 and 2 had arrays of 9 blocks, Levels 3 and 4 had arrays of 16 blocks and Level 5 had 36 (see Figure 4.2). The experimenter noted the level each child was on at the end of the training session and children started the next training session at the beginning of this level.

Figure 4.2 Example of screen for Number Game Level 3 (left) and Letters Game Level 5 (right).

4.3 Results

Outcome measures showed good distributions without floor or ceiling effects. However, the subtests of Rapid Naming were problematic for some children who did not know letter or number names. We removed the letter rapid naming task from analysis as only 10 children were able to complete this measure at pre-test. Performance on the number naming task was better with 58 children completing the test at t1 and 72 children completing at t2. We therefore included number naming in the analyses.

4.3.1 Effectiveness of the training games

We examined how well children played the number and letter games by assessing the number of children who progressed to higher levels of each game during the training period. The numbers game was considered to be successfully understood as 18 children (69%) progressed above Level 1; nine children were at Level 2 at the end of training, six children were at Level 3, two children were at Level 4 and one child was at Level 5. The eight children still at Level 1 at the end of the training period likely reflects a lack of basic number knowledge or an inability to complete level requirements within the time-frame of a session. For example, the experimenter noted that of these children the majority could create sequences, but the sequences were not of the required length to progress to the next level.

In the letters game fewer children progressed from Level 1; 19 children (76%) remained at Level 1 at the end of the training period, three children progressed to Level 2, one to Level 3 and two to Level 5. This indicates that the letters game was more difficult than the numbers game. Additionally, the experimenter noted that children required more assistance than those in the numbers condition. Despite the relative differences, we deem the training effective as most children were able to play the game and create alphabetic sequences. The issue appears to be a failure to create long enough sequences to pass the level within the timeframe of each session. This suggests that if the game is remodelled, earlier levels should have simpler requirements to ensure levels are passed. The lower abilities in the letter group likely reflects limited formal teaching of letter knowledge compared to letter-sound knowledge or number knowledge at this school; this was also reflected in the poorer scores in the letter naming task compared to number naming and letter-sound knowledge.

Table 4.1 Means, standard deviations, ranges and effect sizes on all outcome measures at t1 and t2 for all groups.

		Number group (*n* = 26)		Letter group (*n* = 25)		Control group (*n* = 27)		Cohen's *d*	
	Reliability	*M*	SD	*M*	SD	*M*	SD	Number vs. control	Letter vs. control
Age (months)	---								
t1		57.62	3.89	56.32	3.44	57.41	3.73		
t2		59.59	3.85	58.68	3.50	59.81	3.61	---	---
Addition (/20)	.59[a]								
t1		3.04	2.69	2.64	2.31	2.93	2.84		
t2		4.92	3.61	3.40	2.92	3.93	3.06	.29	-.09
Number sequence (/10)	.59[a]								
t1		1.62	2.14	1.28	1.59	2.59	2.32		
t2		3.77	2.25	2.92	2.40	4.26	2.89	.20	-.01
Rote counting (/50)	.52[b]								
t1		23.38	11.44	24.80	11.62	30.48	15.71		
t2		33.69	14.82	29.04	12.68	33.74	13.03	.51	.07
Number identification (/20)	.59[a]								
t1		9.46	1.84	9.20	3.00	10.63	3.51		
t2		11.42	2.67	10.24	2.96	11.44	2.72	.43	.08

Number writing (/15)	.80[a]								
t1		8.23	3.43	6.76	4.19	9.56	2.69		
t2		10.85	2.54	9.56	3.08	11.26	2.46	.33	.35
RAN[+1] (secs)	.69[a]								
t1		62.29	41.24	67.28	46.40	71.44	44.38		
t2		74.85	31.24	69.64	34.58	72.15	35.02	.35	.04
Number reading (/20)	.88[b]								
t1		11.81	3.31	10.40	4.12	12.19	3.83		
t2		14.08	2.64	12.88	3.44	14.30	2.48	.05	.11
Letter sequence (/10)	.63[a]								
t1		1.00	1.62	.92	1.78	1.22	1.72		
t2		1.85	1.93	2.52	1.87	2.19	2.24	-.06	.33
Word reading (/30)	.83[b]								
t1		5.08	4.17	3.80	4.51	4.74	7.06		
t2		9.92	5.31	8.24	6.00	10.96	7.08	-.23	-.29
LSK (/17)	.64[a]								
t1		10.62	2.67	10.12	4.01	10.33	2.81		
t2		14.46	1.73	14.28	2.72	14.67	1.88	-.22	-.06

[a] Cronbach's alpha internal reliability at time 1
[b]Test/retest reliability
[+]Higher scores indicate slower time, [1]Results for those who completed the task; numbers group (*n* = 26), letters group (*n* = 22), control group (*n* = 24)

4.3.2 Group analyses

Table 4.2 reports the raw scores for the three groups on measures at t1 and t2. As is expected, higher scores (or faster reaction times) are reported at t2 in all measures.

To examine if the changes in performance differed across groups, we conducted regression (ANCOVA) models in Stata (Version 14.0). For each measure at post-test, the same measure at pre-test was used as a covariate. The difference between each of the training groups (number, letter) and the control group were represented by two dummy codes. To check the homogeneity of regression slopes, initial models included the two group by covariate interaction terms. In no case did these interaction terms approach being significant, and they were therefore dropped from the models reported here.

The unstandardized coefficients and 95% confidence intervals for number training and letter training versus the control are reported in Table 4.2. These unstandardized coefficients represent the difference in marginal means between groups at post-test after controlling for any differences on the same measure at pre-test. Positive scores represent a higher score for the intervention (number of letter) group compared to the control group.

Numerical measures

Children in the number training group showed improvements compared to the untreated control group for the following numerical measures: addition, number sequences, counting, number identification, number writing and rapid number naming (ds = .20 - .51; unstandardized coefficient > 0). Number reading showed negligible improvements (d < .10). Despite good effect sizes in the predicted direction, none of the differences were found to be significant. Children in the letter training group conversely failed to show gains in numerical measures (unstandardized coefficient < 0) for all number measures except number writing and rapid number naming. Rapid number naming showed a negligible effect and number writing showed a small effect (d = .35). As with the number training group, no significant effects were found.

Table 4.2 Unstandardized coefficients (*B*) for number versus control and letters versus control and 95% confidence intervals (*CI*) for pre- and post-training.

	Number vs. control	**Letter vs. control**	**95% *CI***	
	B	B	Pre-training	Post-training
Addition	.92	-.32	[2.28 - 3.46]	[3.36 – 4.81]
Number sequence	.20	-.41	[1.37 – 2.32]	[3.09 – 4.24]
Counting	.20	-1.63	[23.29 – 29.30]	[29.16 –35.27]
Number ID	.64	-.40	[9.13 – 10.44]	[10.42 –11.68]
Number writing	.40	.02	[7.40 – 9.03]	[9.95 –11.19]
Number naming	4.76	5.06	[85.00 – 95.93]	[71.94 –84.59]
Number reading	.02	-.31	[10.63 – 12.34]	[13.12 –14.42]
Letter sequences	-.17	.56	[.67 – 1.43]	[1.72 –2.63]
Word reading	-1.36	-1.83	[3.33 – 5.77]	[8.34 -11.14]
Letter-sound knowledge	-.33	-.30	[9.64 – 11.07]	[14.00 –14.95]

Literacy measures

We did not expect children trained in number knowledge to improve on literacy measures compared to the untreated control group. This expectation was confirmed (see Table 4.2). Children trained in letter knowledge improved slightly in letter sequences compared to untreated controls (d = .33; unstandardized coefficient > 0). The other literacy measures did not show such improvements (unstandardized coefficient < 0).

4.4 Discussion

The primary aim of this study was to assess if training in number knowledge can lead to improvements in arithmetic (addition) score. Children in the first year of formal schooling (aged 4-5) played a computerised number knowledge game which trained them in number identification (recognising and associating the visual and verbal components of Arabic digits) and ordinality (the order of the digits 1-10). These are two important components of symbolic number knowledge (see Merkley & Ansari, 2016).

As predicted, children who played the number game showed greater gains in arithmetic, and other numerical measures, compared to children who did not play the game (untreated control group). This improvement was unique to numerical measures, as children trained in number knowledge did not improve in literacy-based measures compared to the untreated control group. Moreover, children trained in an equivalent game using letters (rather than numbers) did not show improvements in numerical outcomes compared to the untreated control group. These results support previous training studies which suggest that training in number knowledge can improve numerical skills, including arithmetic (e.g. Honoré & Noël, 2016; Jordan et al., 2009; Ramani & Siegler, 2008). However, despite initial findings supporting our hypotheses, none of the gains were statistically significant. This reflects the small sample size and resulting low statistical power of this study.

4.4.1 Our study lacks power

The major limitation of our study is that we have low power to find a significant effect should it exist. Our sample size is relatively small compared to previous similar studies. For example, Ramani and Siegler (2008) who used a similar training design to our study, had significant gains in numerical measures for 124 children (62 per group). Similarly, Jordan and colleagues (2012) had 44 children in each training group (N = 132). Our study had 79 children in total (n = 25-27 per group) which is considerably smaller than these two studies. Our sample size was influenced by practical considerations such as time available to test and train children within a year group. In the school there were 90 children in the year group, so we had the maximum number of children sign up that was possible within the timeframe. Practical limitations are discussed further in Section 6.4.

A second potential issue was the length of intervention. Our training period was short, at only one hour of game play for each child. Again, this time was chosen for practical reasons (see Section 6.4) and was less time compared to other similar computerised interventions in which children played for eight hours (Sella et al., 2016) or five hours (Honoré & Noël, 2016). Our one hour training time matched the time that the children in Ramani and Siegler (2008) played the numbers game. However, children in that study played the game one-on-one with an experimenter rather than in a group setting and therefore it is possible that within the first hour of training, children playing individually may gain more than children playing in a group.

Finally, there were some important limitations associated with the outcome measures used at pre- and post- analysis which would need to be changed if running the study again. Firstly, the arithmetic measure had very low scores (pre-training mean score = 2.87, post-mean score = 4.09) and this is likely to be because children of this young age (who have only been in formal education for a few months) are at the early stages of arithmetic learning. Moreover, the test required children to write the Arabic digit as an answer, rather than state it aloud. It was noted by researchers that children were sometimes able to speak aloud the answer to the question, but writing was not possible or was problematic (although mirror writing was allowed to help control for some issues here). Writing may be difficult due to limited motor and processing abilities in children of this age group. Children of this age were not fluent in their arithmetic and were working out each answer in turn. The processing power required to not only calculate the addition question but also hold it in memory whilst preparing to write the answer was challenging for many of the children. Moreover, simply holding the pencil and the motor skills require to write the Arabic digit (as well as remembering how it is written) was problematic for children of this age range. A similar finding can be reported from the number and letter sequence tests which again required children to write a response. One error in the preparation of this study was a lack of insight into the poor written abilities of children on this young age. Arguably a better test for children aged 4-5 would be one that required children to state the answer aloud, rather than write it and this observation highlights the importance of creating suitable tests for children of different ages.

If the present analysis showed good results in the predicted direction despite low power, a key question is: why not collect more data? A power analysis revealed that 180 additional participants would be required in order to reach the expected power and although this is theoretically possible to do, there were a number of very important reasons that this study was not extended, and more data were not collected. As presented in the overview chapter (Section 1.2.2), reasons for this included time and resource limitations, issues with the difficulty of the game meaning that many children did not enjoy playing the game, and a control group that was not equivalent to the experimental group (the letter game was harder than the number game). These reasons are presented in more detail in Section 6.4 which is devoted to discussing practical limitations of this type of research.

4.4.2 Theoretical issues

One potential reason for the lack of significance in our study is that training in symbolic number knowledge does not lead to an improvement in arithmetic because the two are not causally linked. To our knowledge the only which reports significant improvements in arithmetic after a symbolic number intervention, and which did not include calculation as part of the training design, is by Honoré and Noël (2016). In line with our study, the games they used supported the development of symbolic number knowledge (enhancing the relationship between the visual digit and verbal number). However, there are a few potentially crucial differences between their design and ours which are worth mentioning.

Firstly, their study used the numbers 1-20, whereas the present design was limited to the numbers 1-10. It is possible that this limited range of numbers is not sufficient to transfer to arithmetic (note that in Ramani & Siegler, 2008 which used the numbers 1-10, arithmetic was not an outcome measure). Secondly, and perhaps more importantly, the theoretical motivation behind the games in Honoré & Noël (2016) was different to that of the present study. In their study, children played a game involving number comparison and number line estimation, both of which focus on the magnitude and cardinality of numbers.

In the present design, the primary focus was on the identification and ordering of numbers; understanding the magnitude of numbers was not crucial to succeeding in the game. The magnitude of numbers is considered a critical part of number knowledge (Merkley & Ansari, 2016) and it is possible that in order to develop arithmetic skills, children need to understand the relative size of numbers rather than just reciting the number sequence. Indeed, in a cross-sectional study of 1,391 children, Lyons and colleagues (2014) report that symbolic number comparison is a more important predictor of arithmetic at the start of formal schooling than number ordering. Number ordering was a non-significant unique predictor of arithmetic (partial r-value = -.092, p = .20, d = -.18), whereas number comparison was a significant and unique predictor (partial r-value = .29, p < .001, d = .60). Therefore, it may be critical to consider the magnitude of numbers within a number knowledge training intervention to see benefits to post-training arithmetic scores.

4.4.3 Training in letter knowledge

A second aim of the study was to examine the changes in outcome measures for those trained in letter knowledge. As predicted, children in this training group showed limited improvement in number knowledge, similar to children in the untreated condition and less than those in the number training condition. Similarly, children in the letter training group showed greater gains in letter sequences at post-test than those in the untreated control group, whereas number training did not lead to any improvement in this measure. However, as with the numerical measures, this difference was not significant, and together with the other results suggests that the study had low power to detect an effect, should it exist.

4.4.4 Why are symbolic number skills related to arithmetic?

Despite not finding significant gains in our study, we propose that symbolic number skills *are* critical for numerical development. Our results were in the predicted direction and, with greater power, we suggest significant differences would have been reported. Additionally, there appears to be some evidence from training studies that it is possible to improve arithmetic through training in symbolic number knowledge (e.g. Honoré & Noël, 2016) and ample evidence from correlational and longitudinal studies supporting a non-casual but robust link (see Section 2.1.3).

The most obvious reason for a relationship is that without the ability to identify digits alongside and understand the count sequence and cardinality, it is not possible to perform arithmetic. Verbal sums and counting do not require written knowledge but in formal education, arithmetic is measured from an early age using Arabic symbols. Thus, a child is required to understand how much an Arabic digit represents in order to count-on to another digit or compute simple sums, and it may be this understanding that leads to the transition from informal to formal numerical knowledge (Merkley & Ansari, 2016).

4.4.5 Conclusions

In summary, this second study aimed to provide evidence that training in number knowledge can lead to improvements in arithmetic. Although children in our number training group improved on number measures (but not literacy measures) compared to an untreated control group, effect sizes were generally medium to small and none of the effects were statistically significant. This study had low statistical power and therefore, to reach significance, we suggest running the study with more participants or a longer training time (or both). However, due to practical limitations, as outlined above, this was not possible in our study, highlighting the importance of time and resource implications when conducting a training study. These findings are discussed in more detail in the final discussion chapter.

Chapter 5. Study 3: The role of patterning skills in predicting later arithmetic abilities

This chapter reports a study of the relationships between patterning and arithmetic skills in typically developing children. This study is reported in part in Long & Burgoyne (submitted manuscript). Patterns are sequences of items that follow a predictable rule (e.g. 112112, 13579) and became of interest due to the role of number knowledge in pattern tasks (see Section 1.2.3). Children's understanding of patterns is commonly assessed using alphanumeric (numbers, letters) or nonalphanumeric (e.g. shapes, objects) stimuli.

Pattern understanding (patterning skills) is commonly taught in early education, and evidence suggests that these skills are correlated with arithmetic (Lee et al., 2011, 2012; Pasnak, 2017; Rittle-Johnson, Fyfe, Hofer, & Farran, 2017; Schmerold, 2015; VanDerHeyden et al., 2011; Warren & Miller, 2013). However, the majority of the studies examining patterning and arithmetic have used pattern tests which involve number stimuli but have failed to control for number knowledge. Therefore, the relationship with arithmetic may be in part due to symbolic number knowledge which we know is an important component of arithmetic learning. Moreover, the majority of studies have failed to include other predictors of arithmetic, such as executive function and spatial awareness, which may also correlate with patterning. Finally, studies have used a variety of pattern tasks (e.g. repeating and increasing) with various stimuli (e.g. numbers or objects) without a full understanding of how these relate to one another, and to arithmetic.

In the present study, we aimed to examine how pattern skills relate to one another across different stimuli (numbers, letters, shapes and objects) and types of pattern task (e.g. repeating, increasing and rotating). Secondly, we wanted to examine if one or all of these pattern tests correlate with arithmetic, whilst considering other potential factors including numerical skills and domain-general predictors.

5.1 Patterning skills

In recent years, researchers have become interested in how pattern understanding may predict variation in arithmetic. Patterns are sequences of items that follow a predictable rule (e.g. 112, 1357) and children's understanding of patterns is commonly assessed using alphanumeric (numbers, letters) or nonalphanumeric (e.g. shapes, objects) stimuli. In early years' education children will typically be exposed to some simple patterns: repeating (e.g. abab) and increasing (e.g. 1234). In the United Kingdom, children are taught basic patterning in the first three years of formal education and it is considered an important aspect of numerical development (UK Department of Education, 2017). However, research examining the exact role of patterning in later arithmetic is in its infancy and more evidence is required to clarify the nature and strength of the relationship between these skills.

5.1.1 Development of patterning

The very earliest patterns that children become familiar with are repeating patterns, often presented using coloured blocks or objects. Most children aged four are able to duplicate these types of patterns (Clements, Sarama, & Liu, 2008; Papic, Mulligan, & Mitchelmore, 2011; Starkey, Klein, & Wakeley, 2004). Rittle-Johnson, Fyfe, Loehr and Miller (2015) used a coloured block task and showed that children aged four (N = 66) are also able to abstract repeating items. Abstraction requires the child to create the same pattern they see (e.g. *red-blue-red-blue*) using different items (e.g. *yellow-green-yellow-green*). Around three quarters of children in this study were able to copy repeating patterns, and half were able to abstract, suggesting variability in pattern understanding at this young age. For the half who show abstraction abilities, it suggests some advanced pattern understanding as abstraction requires understanding of the structure of a pattern, rather than merely using perceptual abilities to copy it. Rittle-Johnson et al. (2015) suggest that this advanced ability to manipulate and form a mental representation of patterns is related to more advanced reasoning skills, and perhaps arithmetic.

It is important to consider the way that patterns are taught because research has suggested that simply asking children to recreate patterns is not allowing them to understand the pattern in a sophisticated way (Rittle-Johnson et al., 2015; Rittle-Johnson, Fyfe, McLean, & McEldoon, 2013). Therefore, research examining pattern development should focus on abstraction and more advanced patterns such as increasing (e.g. 1357) and rotating patterns.

5.1.2 Patterning and arithmetic

Studies which have aimed to assess the relationship between patterning and arithmetic have typically looked at children from the ages of 5-8 and used both alphanumeric (numbers and letters) and non-alphanumeric (e.g. colours, objects or shapes) patterning tasks. The exact relationship between these skills and arithmetic is not clear, although it is logical to assume that alphanumeric tasks are better correlated with arithmetic than nonalphanumeric tasks (Burgoyne, Witteveen, Tolan, Malone, & Hulme, 2017).

The simplest patterning tasks require children to complete sequences which appear in a repeated format with more complex tasks requiring children to complete increasing or rotating items. It is possible that patterning tasks relate to numerical development as many early number abilities involve predictable sequences. For example, counting in 2s involves an alternating pattern of 0s and 2s and the ability to generalise underlying relationships for early arithmetic (e.g. 1+2 is the same as 2+1) is a key concept which may act as a foundation to later arithmetic ability (Burgoyne et al., 2017). However, as research into patterning and arithmetic is in its infancy, the underlying mechanisms are not completely clear.

Correlational evidence

Evidence supporting a concurrent correlation between patterning and arithmetic in children comes from a number of studies. For example, Warren and Miller (2013) examined performance on a simple repeating patterns task involving numbers in 230 children aged 5 years 9 months. Mathematics was measured via a general maths test which examined basic number knowledge (14 questions) and some advanced mathematical knowledge including probabilities and geometry (5 questions). Patterning scores correlated with general maths score (r = .44, p < .05) and explained unique variation in maths score, after controlling for language abilities (patterning unique beta = .24, p < .05; language unique beta = .52, p < .05). Together, language and patterning scores explained 43% of the variation in general math abilities (note that no unique R-squared values were provided). Despite controlling for language, the authors failed to control for other predictors of arithmetic, such as executive function which has been linked to both maths and patterning abilities (e.g. Bull & Lee, 2014; Lee et al., 2012; Miller, Rittle-Johnson, Loehr, & Fyfe, 2016).

A study by Lee, Ng, Bull, Pe, & Ho (2011) used a number patterning task whilst controlling for executive function in a correlational design involving 151 children (mean age = 10 years 1 month). Number patterns correlated strongly with maths ability, measured via three numerical tasks (rs = .42-.60, $p < .001$). Structural equation modelling showed that understanding of number patterns, arithmetic and working memory (an executive function measure) explained 63% of the variation in algebra, with number patterns remaining a unique predictor of algebra score concurrently ($\beta = .04$, $p < .01$) and when children were tested again one year later ($\beta = .31$, $p <.001$). For this study algebra, rather than arithmetic, was the main outcome measure (possibly because children were older in this study). Taken together, these studies suggest a role of patterning in maths skills after controlling for predictors of numerical abilities (including language or executive function). However, an important caveat is that the patterning tasks used included numbers, and therefore the relationship between pattering and numerical skills may be simply explained by number knowledge.

Schmerold (2015) used an extensive patterning task to assess the relationship with math ability and executive functioning in 74 children in the first grade (mean age = 7 years 2 months). Children completed repeating, increasing and rotating pattern tasks which used numbers, letters, shapes, pictures and object stimuli (a total of 48 patterning questions). As is commonly seen in patterning tasks, missing items were presented at the beginning, middle and end of a sequence and children were asked to identify the correct option from four possible answers. Maths abilities were measured using the Woodcock-Johnson Test of Cognitive Abilities III (W-J Test) (Woodcock, McGrew and Mather, 2001) and included applied problems, quantitative concepts and number series. Executive functioning was measured via working memory, cognitive flexibility and inhibition.

In line with previous evidence, patterning was strongly correlated with math score for the three math measures (r = .52-.54, $p < .001$). Patterning was also correlated with cognitive flexibility and working memory ($r = .41$, $p < .001$ and $r = .23$, $p < .05$, respectively) although not inhibition ($r = .17$, $p > .05$). Together, the correlated measures were entered into hierarchical regression and explained 40% of the variation in applied problems and quantitative concepts, with patterning explaining 29% of the variation in number series. Patterning remained a unique and significant predictor in all three models (β = .18, .08 and .11, respectively). Importantly, the patterning task used in this study used numbers and other stimuli and therefore this relationship cannot be explained purely by number knowledge. However, the results are not fully replicated in a study by Lee et al. (2012) who reports similar correlations between patterning (numbers and shapes test) and arithmetic (number patterns: $r = .46$, $p < .001$; shape patterns: $r = .25$, $p < .05$) but no unique relationship once executive functioning is controlled.

One reason for this difference may be due to methodology of the patterning tasks for which there were several differences across the two studies. Schmerold (2015) used a patterning task in which children chose one of four options to complete the pattern, as is seen for other pattern measures. Lee et al. (2012) created a combined pattern score based on the child's ability to choose the correct response and their ability to describe the rules governing the pattern, a process which may involve a number of different factors to choosing a missing piece. Secondly, the stimuli in Lee et al. (2012) were manipulated according to size and relative position of the shape but not in Schmerold (2015). Finally, only repeating shapes and numbers were examined in Lee et al. rather than range stimuli and pattern types used in Schmerold. These different methods may be in part the reason for the difference in results, although more evidence from other studies is required to confirm that patterning plays a unique role in number skills.

In summary, there are a number of studies which show that patterning and arithmetic are concurrently correlated and some evidence to support a unique relationship between these variables. What is not clear from these studies is whether pattern abilities predict later arithmetic, and therefore whether these two factors are causally linked.

Longitudinal evidence

Some longitudinal evidence points to a relationship between alphanumeric patterns and later numerical abilities. For example, Pasnak et al. (2016) explored the longitudinal relationship between increasing patterns and later arithmetic. Nighty-six children in Grade 1 (mean age 6 years 6 months) were examined on increasing pattern ability, maths score (W-J III) and reading ability at two time-points (beginning and end of school year). In the patterning task, children were presented with letter and number sequences that increased by one, two, or three numbers or letters. Children selected the correct answer from a choice of four responses. The authors used a time-lag correlation design which compares performances across tests at the two time points to indicate a direction of any significant relationship. Number patterning at time 1 was a significant correlate of math score at time 2 (r = .39, p < .01) although the converse was not true (maths at time 2 to number patterns at time 1: r = -.05, p < .05) thus indicating that earlier patterning may lead to later numerical ability (difference in correlation coefficients is significant: z = 3.47, p < .01). These findings mirror those of VanDerHeyden et al. (2011) who showed using a similar patterning task (but repeating rather than increasing) that number patterning correlates longitudinally with addition (r = .42, p < .05). Unfortunately, both studies failed to control for known predictors of arithmetic and therefore the unique relationship between these skills is not known. Moreover, as the relationship between patterning and maths abilities focuses on numerical patterns the relationship reported may be due largely to number knowledge.

One study which assessed nonalphanumeric patterning as a predictor of arithmetic is by Rittle-Johnson, Fyfe, Hofer and Farran (2017). In this large-scale longitudinal study, several predictors of mathematical achievement were examined in 517 low-income children aged 4-11. In the study, children were assessed at four time-points: beginning of pre-school year (mean age = 4 years 5 months) end of pre-school year, end of kindergarten and end of first grade (aged 7) and maths achievement was measured again four years later at time 5 when children were aged 11. Patterning was measured via a repeating nonalphanumeric task whereby children were required to choose the correct coloured cube to complete a repeating sequence presented with coloured cubes (e.g. red-blue-red-blue). Children were assessed on a range of numerical cognitive measures at several time points including general maths achievement, calculation, nonsymbolic comparison, counting, number identification and shape completion. Language measures, reading and general cognitive abilities (e.g. working ability in the classroom) were also taken at the four time-points.

Patterning at the end of first grade was a unique and significant predictor of maths achievement five years later (β = .08, SE = .04, p < .05) therefore suggesting an important role for early nonalphanumeric repeating patterning skills in later arithmetic. However, the patterning task was not reliable and despite controlling for many known predictors of arithmetic, the authors did not include executive function or intelligence measures, which are strong predictors of both patterning and arithmetic.

In a later study, Rittle-Johnson, Zippert, and Boice (2018) used a reliable patterning task (Cronbach's alpha = .83) and controlled for important predictors of arithmetic including spatial awareness, working memory and general cognitive skills. Seventy-three children were assessed at time 1 (mean age = 4 years 7 months) on repeating number patterns and general maths skills (Research-Based Early Mathematics Assessment; Weiland et al., 2012). Patterning score was significantly correlated with both concurrent mathematical skills, and maths performance measured 6.8 months later ($r = .64$, $r = .65$, respectively). Moreover, when other important factors of arithmetic were controlled (e.g. spatial skills, verbal skills and working memory) patterning remained a unique predictor of maths achievement (time 1: $\beta = .30$, $p = .01$; time 2: $\beta = .40$, $p < .001$). Spatial skills correlated significantly to both patterning score ($r = .38$) and maths scores (time 1: $r = .61$; time 2: $r = .59$). What is not known from this study is how different types of patterning task correlate with one another and with arithmetic, as the authors focused upon one type of task (repeating) with one type of stimuli (numbers).

5.1.3 Teaching patterning

Two studies have attempted to examine if teaching patterning can lead to changes in patterning and arithmetic. In Hendricks, Trueblood, and Pasnak (2007) 62 first grade children (mean age = 7 years 1 month) who performed poorly in class tests were randomly assigned to a pattern training group ($n = 33$) or control training group ($n = 29$). Training was conducted in small groups for 15 minutes every day on alternating weeks for 6 months. Those in the patterning training condition practiced creating repeating and increasing patterns with numbers, letters, shapes and objects. The patterns were presented in physical form (felt board and stickers) and on a computer. Those in the control group were trained in school subjects.

Patterning score at post-training was higher for those trained in patterning than those in the control group, and patterning score correlated with arithmetic ($r = .40$, $p < .01$). However, children trained in patterning did not score significantly higher in later arithmetic than those in the control group ($t = .39$, $p <.05$). Importantly, as with other studies, patterning score correlated well with a range of language measures, and therefore it is possible that intelligence may be explaining the relationship between patterning and numerical abilities, although the authors did not control for intelligence levels. This highlights a problem as is consistent throughout the literature; a failure to control for important predictors of patterning, language and arithmetic abilities.

In a later and similar training study, Kidd et al. (2013) examined how training in patterning affected performance on patterning tasks and mathematical skills for children from low income families aged 6 years 8 months. Children who scored in the bottom (40%) in their class for a pattern task were eligible for the study (N = 140) and were split randomly into four training groups according to the class they were in (2 from each class into each group): pattern training, mathematics training, reading training and social studies training. All children were trained for 45 minutes per week (over three sessions) for 6.5 months. Similarly to Hendricks et al. (2007), the patterning training consisted of practising with physical and computerised repeating and increasing patterns using numbers, letters, colours, objects and shapes. Immediately after training, children took the Woodcock-Johnson (W-J III) which included patterning, general mathematical tests and reading tests.

Analysis of variance (ANOVA) tests were conducted to compare the difference in Woodcock-Johnson scores across groups and showed that children trained in patterning scored significantly higher in the patterning subtest than those in the other groups (A priori Least Significant Differences test were significant for patterning vs. other groups). However, scores were not significantly higher for the patterning group versus control groups in any maths or reading subcomponent. Therefore, as with Hendricks et al. (2007) it appears that although possible to improve patterning skills, training in patterning does not appear to transfer to arithmetic.

Similar findings are reported in a later study by the same research group (Kidd et al., 2014) who trained 120 children (aged 6 years 5 months). In this sample (also low income children who scored poorly on a patterning screen test) children were randomly assigned according to classroom (again, two from each class to each group) into the same groups as used in Kidd et al. (2013). The patterning training now included a wider range of patterning tasks including random repeating, progressive increasing and rotating items for shapes, colours, letters and numbers. Children trained in patterning and maths skills scored significantly higher than other groups (language and social skills) in maths outcomes (W-J 18A and B). However, as pre-test scores were not taken this study lacks power. Aside from the patterning screening, only measures after training were assessed, and similarly pre-training scores were not included in the analysis in Kidd et al. (2013). Therefore, despite providing initial evidence that patterning skills can be trained in young children, the causal relationship with number skills and arithmetic is not clear.

5.1.4 The present chapter

Despite some initial evidence suggesting a relationship between patterning and arithmetic, many of these studies have used tests which involve number stimuli. Therefore, a relationship between understanding of numeric patterns (e.g. 1122) and arithmetic would not be surprising since it may reflect no more than Arabic number knowledge. Moreover, most of these studies have failed to include known predictors of arithmetic such as number skills or other predictors which may be related to both pattering and arithmetic, such as executive function and spatial awareness.

In this study, we have two main aims. Firstly, to assess how different types of patterns (repeating, increasing and rotating) compare to one another, and how this relationship compares across different stimuli (numbers, letters, shapes and objects). Secondly, to assess if patterning skills are predictive of arithmetic ability once known predictors of arithmetic are controlled.

Children completed a large battery of pattern tasks including different pattern types (repeating, increasing and rotating) for numbers, letters, shapes and objects (120 items). This is by far the most extensive patterning battery to date and should give a clearer indication of the way that different types of pattern task correlate with one another, and with arithmetic. As with many other similar studies, children will be shown a pattern (four stimuli with the fifth missing) and choose the answer from four possible options. We will examine how the different patterning tasks correlate with arithmetic controlling for known predictors of arithmetic including age, executive functioning and spatial awareness. This should provide a clearer indication of the unique relationship between these measures. We predict that patterning tasks will correlate with and predict unique variance in arithmetic.

5.2 Method

5.2.1 Participants

Seventy-nine children (43 boys, 36 girls) were recruited from two primary schools in London and were tested at the start of the school year. The average age was 6 years 1 month (*SD* = 7.34, range = 5;0 – 7;2 [years; months]) and children were in Year 1 (*n* = 41, mean age = 5;7, *SD* = 3.45 months, range = 5;0 – 6;8) or Year 2 (*n* = 38, mean = 6; 8, *SD* = 3.84 months, range = 6;1 – 7;2). Children were recruited from a North London academy (*n* = 25) and participated if parents opted the child in to the study. The remaining children (*n* =54) were recruited from a free-school in West London and all children within a year group or class participated unless the parent opted out. Full ethical approval was provided by The University College London Ethics Committee.

5.2.2 Design and procedure

All children were assessed on measures of the following constructs: pattern completion, arithmetic, counting, number knowledge, executive functioning, spatial awareness and non-verbal IQ. Tests were conducted individually or in a group setting. The individual test measures were split across two testing sessions lasting a maximum of 30 minutes each. In one session, children completed the following five tasks; two patterning tasks (numbers and objects), arithmetic (addition and subtraction), backward span and visual search. In the other session, children were assessed on the remaining four tasks; two patterning tasks (letter and shape), dot counting and cognitive flexibility. Sessions were randomised across participants. After all individual test items were collected, children completed the remaining measures of number knowledge, spatial awareness and nonverbal IQ in a group setting. Children completed these tests as a whole class in their normal classroom during one session which took a total of one hour. During each session, one teacher or teaching assistant was present to ensure all children were answering questions appropriately. All data were collected within six weeks of the initial testing date.

Patterning tasks

Pattern understanding was assessed using twelve subtests which varied stimulus type (numbers, letters, shapes and objects) and pattern type (repeating, rotating and increasing). Each subtest included 30 items which were 2 dimensional; 10 repeating, 10 rotating and 10 increasing items. For each item, children were shown a number, letter, shape or object which followed a predictable repeating pattern (10 items e.g. circle, square, circle, square; 1, 2, 1, 2). See Figure 5.1 as an example of a simple repeating number pattern. The first five rotating items were rotations of 90 degrees and the final five were rotations of 45 degrees. The stimuli rotated in the left and right direction and had different starting directions (e.g. upright or upside-down). Increasing patterns differed slightly across alphanumeric and non-alphanumeric stimuli. For letters and numbers, the patterns increased based on the ordinal position of the digit (e.g. 1357 or aceg) whereas objects and shape patterns used increasing geometric figures (unit identification) or relative size (proportion). See Appendix 1 for detailed descriptions of each of the items within each subtest.

Shape patterns used verbal shapes: triangles, squares, circles, rectangles, hearts and half-moon. Object patterns used a range of animals or objects (e.g. house, tree). All object and shape items used were familiar to children of this age. The final (i.e. fifth) item in each pattern was missing; children were asked to point to one of four alternatives to complete the pattern. For repeating trials, one distractor item appeared in the sequence and the other two distractor items were stimuli used in the other trials but not in the current trial (e.g. square-triangle-square-triangle; distractors: triangle, heart, circle). For increasing trials, one distractor was an item which appeared in the sequence, one was an impossible choice (i.e. could not appear in the sequence), and the third was an item which would appear later in the sequence (i.e. after the fifth item). For the first five rotating trials (item rotated by 90˚), the distraction items were items which existed within the sequence. For the next five rotating trials (item rotated by 45˚), two distractors were an item within the sequence, and one distractor was the same item but rotated in a way that was not presented in the sequence.

Test items increased in difficulty; the first five repeating patterns used an ABAB structure, and the final five used an ABBA/AABA structure. This was consistent across all stimuli. Increasing items increased in difficulty by involving larger numbers (e.g. 13,15,17,19) and/or a larger distance between items in the sequence (e.g. f, h, j, l). Similarly, object and shape increasing items increased in difficulty by including more complex rules between each item. For instance, unit identification could be based on one dimension (add/remove a unit), two dimensions (add/remove a unit of different colours) or three dimensions (add/remove more than one unit of different colours). Additionally, the trials become more difficult due to the distractor items which, on increasing trials, were more closely matched to the target item. For example, one distractor may be the sixth item in the trial, or the 'impossible' distractor may be only one unit different to the target item.

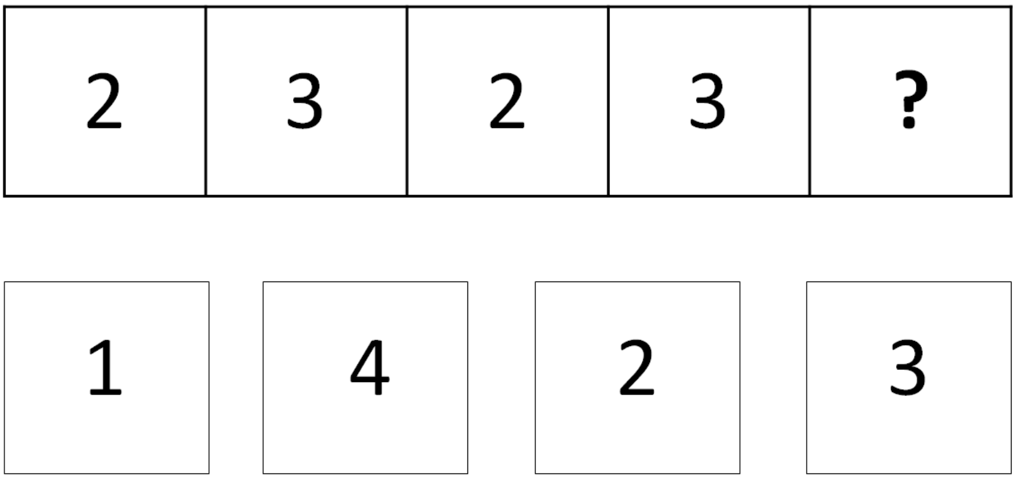

Figure 5.1 An example of a repeating pattern number question in the patterning numbers test.

To ensure that children were not simply looking at the final item in the sequence (for instance responding: 8 as the response for the pattern 1,3,5,7), children were given two practice trials for objects and shapes, and three practice trials for numbers and letters (see Appendix) during which the experimenter encouraged the child to vocalise the items in the sequence whilst pointing to each item in turn. Feedback was given in these trials if the child got the answer wrong. In the experimental trials, children were encouraged to point to the items in turn and vocalise the pattern when possible (experimenter said "point and say what you see and tell me what comes next"). Children were given no feedback on these trials. A discontinuation rule of three consecutive errors on repeating, rotating and increasing patterns was used, i.e. if a child made three consecutive errors on the repeating patterns, the experimenter discontinued that part of the test and administered the first increasing pattern item. One point was awarded for each correct answer. Scores on all items within a subtest were summed to create a single score for each pattern subtest.

Arithmetic and number processing tasks

Arithmetic. Arithmetic was measured via addition and subtraction fluency tests in which children were given two minutes to complete up to 15 addition questions followed by two minutes to complete up to 10 subtractions questions. Children completed two practice questions for each subtest in which the experimenter demonstrated using their fingers and corrected any mistakes the child made. For test items, the experimenter read aloud each question and noted down the verbal response given by the child. No feedback was given but children were reminded to use their fingers to help them answer questions. One point was awarded for each correct answer, making a maximum of 15 points for addition, and 10 for subtraction.

Dot counting. Counting ability was measured using an adjusted version of the dot counting subcomponent of the Test of Basic Arithmetic and Number Skills (TOBANS; Brigstocke et al., 2016). Dots were presented within a box and children were required to name aloud the total number of dots in each box. After three practice questions, children were given 30 seconds answer as many questions as possible. Within each question the number of dots ranged from 2-19. The experimenter wrote down the verbal response the child gave, and one mark was awarded for each correct answer.

Number identification. Number knowledge was measured in a group setting using a number identification task. Children were presented with an answer booklet and told that for each question they would need to circle the number read aloud by the experimenter. For each test item, children were given four possible answers, all of which were similar to the test item (e.g. target number is 28 and distractor items are 82, 208 and 20). Target numbers ranged from 7-807. During two practice items, feedback was given, and the experimenter and teaching staff checked that children understood the instructions. Following this there were 14 test items with one mark awarded for each correct response.

Executive functioning

Executive functioning was assessed with four measures.

Backward span. Working memory was measured via the backward span subtest of the Wechsler Adult Intelligence Scale – Third Revision (WAIS-III; Wechsler, 1997). Children were told to repeat words in the opposite order to which they heard them.

After two practice items in which feedback was given, there were eight experimental items in which words increased in length by one each time (starting with two words and ending in eight words) and two trials per item. Children were discontinued if they failed to correctly repeat both trials within an item. One point was awarded for each correct trial, making a total of 16 points.

Head Toes Knees Shoulders (HTKS). Inhibition was assessed via the HTKS task (Burrage et al., 2008). Children were first told to touch their head shoulders knees and toes, then were told that they would have to do the opposite of what the experimenter told them, “We’re going to be silly! If I say touch your head, you should actually touch your toes.” The first part of the test included head and toes only. Children only completed test items if they were successful across two training questions and four practice questions, with a maximum of three corrections allowed. There were then ten test items which consisted of five ‘touch your head’ questions and five ‘touch your toes’ questions. Two points were awarded for a correct response, one point was awarded if the child made an initial incorrect response but then corrected themselves, and 0 points awarded for an incorrect or no response. A total of 20 points were available for the first part of the test, and if children scored five or more points they proceeded to the final part of the test. In the final part, children were told to touch their shoulders and their knees. Then were told, “now we’re going to be silly again, and do the opposite of what I say”. After one training question (touch your knees) and four practice questions (knees and shoulders) children were given ten test questions which included touching the head, shoulders, knees and toes. No feedback was given, and scoring was the same as the first part of the test, with a total of 20 points possible for the second part of the test and 40 points possible in total for both parts of the test.

Selective Attention. In the Visual Search task (Breckenridge, 2007) children were told to find red apples. In the demonstration trial, the experimenter presented a piece of A4 laminated paper showing a red apple, white apple and red strawberry. The experimenter indicated that the child should find red apples and drew a line through the red apple to indicate how the child could identify them. The child then practiced identifying apples before being shown the test page containing a 9 X 10 array of red apples (N = 17), white apples (N = 36) and red strawberries (N = 37). Children were given one minute to find as many red apples as they could. Score was an efficiency rating comprised of total red apples found minus total number of errors (identifying strawberries or white apples) divided by total time taken.

Cognitive Flexibility. Shifting was measured via a card matching task, adapted from Dick (2014). In this task, children were expected to find similarities between cards presented on A4 paper. The cards shared similar features of colour, shape, size or number of items. Cognitive flexibility was required because children needed to find one common feature across two cards (e.g. colour) and then shift to another common feature (e.g. size). For example, as shown in Figure 5.3, children should state that cards 1 and 2 are the same because they are both red, and cards 2 and 3 are the same as they are both boats.

There were four parts to the test: demonstration trial, criteria trials, 2-Match trials and 3-Match trials. First in the demonstration trial, children were shown four cards made from two sets of identical pictures (see Figure 5.2). The experimenter familiarised the child with the language used in the task stating: "These two cards [points to cards 1&3] are the same because they both have one big blue rabbit. And these two cards [points to cards 2&4] are the same because they both have three small red roses". Following the demonstration trial, there were two criteria trials in which the experimenter described two similar cards and asked the child to state why the other two cards on the page were similar. If the child failed to describe similarity between the cards for both criteria items they were discontinued from testing. Those who successfully described the cards in both or one of the trials advanced to 2-Match questions.

For 2-Match questions, there were three cards on each page; two were the same for one reason and two the same for another (see Figure 5.3). After one demonstration trial, children completed six test items and were asked to "point to two cards which are the same and tell me why they are the same". If the child correctly identified two similar cards they were asked to "point to two other cards which are the same but for a different reason". Children were awarded one point each time they correctly identified two matching cards and described why they were the same. There were 2 points available for each test item and a total of 12 points for the 2-Match condition.

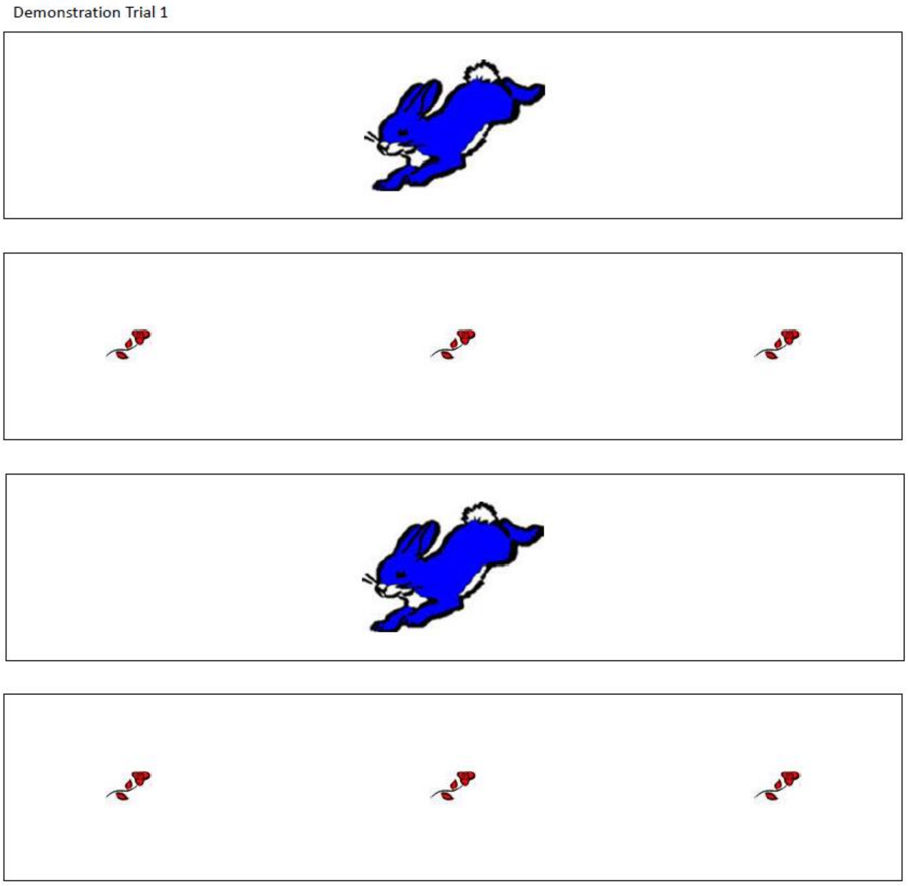

Figure 5.2 Demonstration trial.

If children scored the maximum two points for three or more items, they proceeded to the 3-Match condition. In the 3-Match condition there were three similarities between the three cards (e.g. two cards had the same colour, two had the same size, two had the same quantity, or two were the same object). Again, there was a demonstration trial followed by six experimental trials. Marking was the same as in the 2-Match condition, although 3 points were available for each item making a total of 18 points. In total there were 30 points for this task.

Non-verbal IQ

Non-verbal intelligence was measured using an adapted version on the Raven's Progressive Matrices (Raven, Court & Raven, 1995). In this group test, children in a classroom were shown on a whiteboard different coloured patterns with a piece missing. The experimenter explained that children were to find the missing piece from four options presented below the test item. In the answer booklet, children circled the option they thought was the missing piece. After three practice questions, the experimenter moved through each question in turn. A total of 12 marks were available.

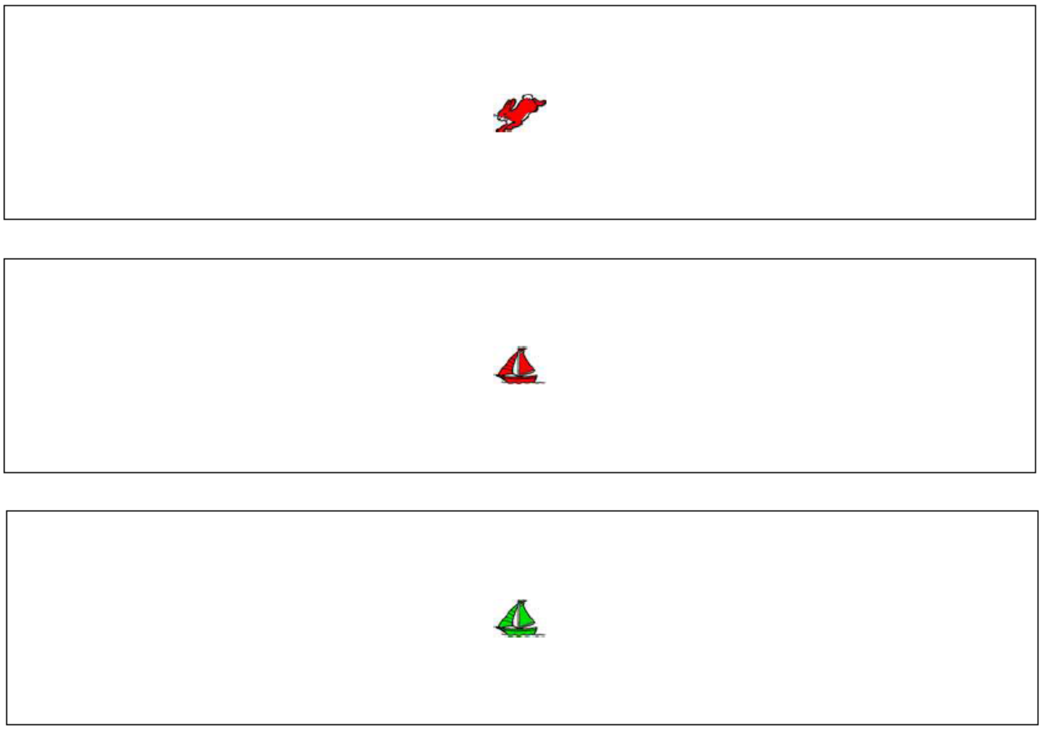

Figure 5.3 Example of a 2-Match task.

Spatial reasoning

Spatial awareness was measured in a group setting via a Spatial Reasoning subtest of the standardised Non-Verbal Reasoning Test suitable for children between 5:04–7:11 years (Smith & Lord, 2002). Children were presented with a question and answer booklet and guided through the Windows, Hidden Shapes and Stacks sections by the experimenter. For each section, the experimenter first demonstrated the task, then children answered two practice questions with feedback before starting the timed test items. A total of two minutes was given for each part of the test.

The windows questions assessed a child's ability to switch perspective and rotate items. Each test item showed an image of a "view from the street". The children's task was to choose one of four options which showed the "view from inside the shop" (see Figure 5.4). There were 12 test items, and therefore a maximum score of 12.

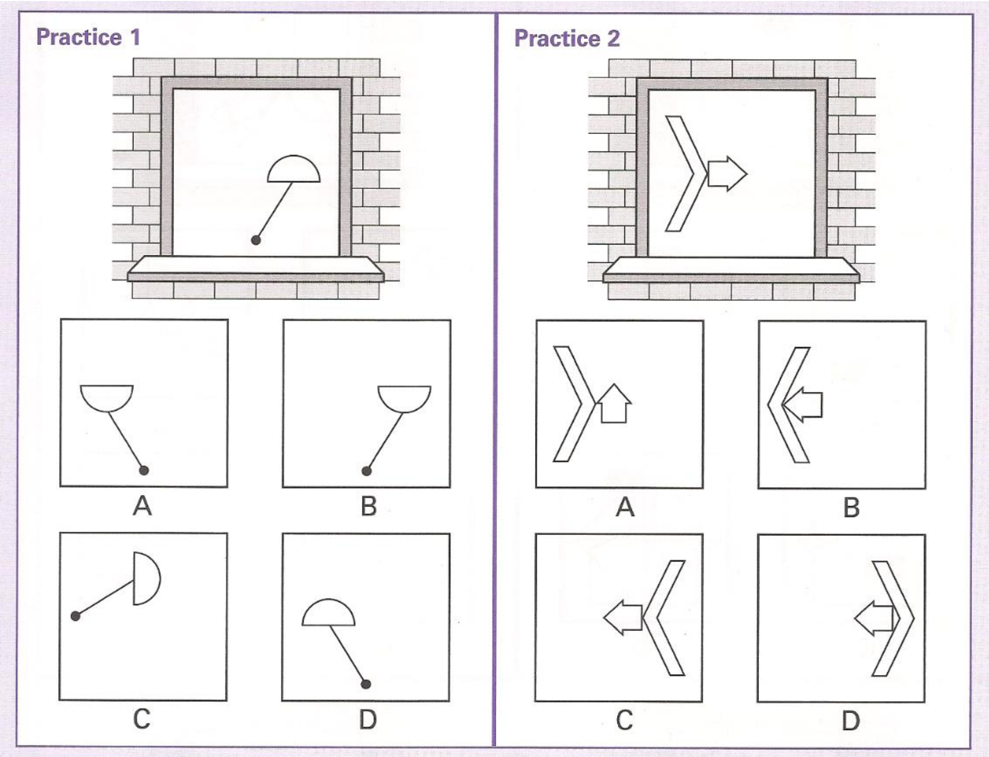

Figure 5.4 Example of a windows task.

Next, children completed the hidden shapes test which accessed the ability to identify shapes hidden within a larger shape. In the practice trials, children were shown four different shapes which were made from smaller shapes and labelled A-D. The experimenter demonstrated that for each test item, children would be asked to look at a smaller shape and identify which of the larger shapes (A-D) the smaller shape was "hidden in" (see Figure 5.5). Children were told that the hidden piece would be exactly the same in the larger shape, for example the same size and the same way around. After the practice items, children completed the test items. There were three sets, each contained one set of large shapes (A-D) and six associated hidden shapes; a total of 18 marks available.

The stacks test assessed the child's ability to visualise the location of shapes. In each test, one shape was stacked in various orientations and children were required to identify which of the shapes was at the bottom of the stack (see Figure 5.6). In each test there were four or five shapes piled together. After the demonstration and practice questions there were 16 test items.

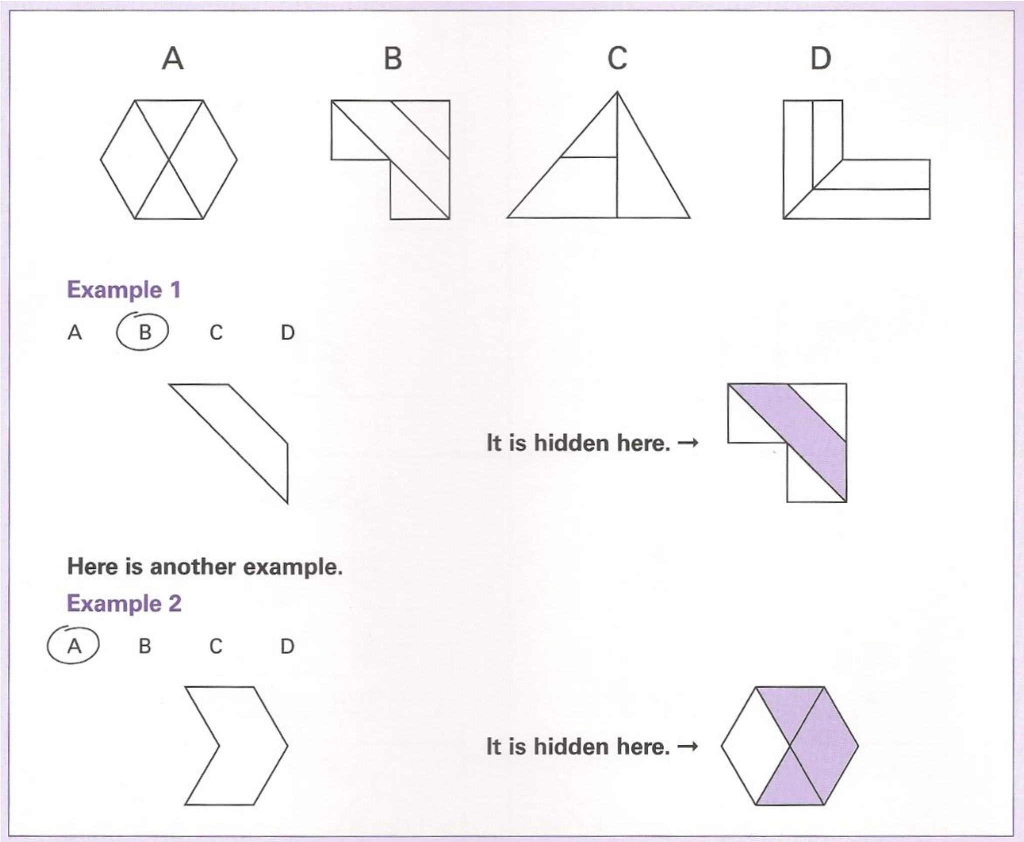

Figure 5.5 Hidden shapes test.

5.3 Results

A concurrent correlational design was used to assess the relationship between patterning and arithmetic. We then used a regression model to ascertain the strength of this relationship whilst considering other known predictors of arithmetic. An additional aim of the study was to examine how different types of patterning task relate to each other and we report these findings based on correlations and factor analyses. All data was examined using Stata (Version 14.0).

Descriptive statistics and reliabilities for all tasks are shown in Table 5.1. All tasks showed good reliabilities and distribution of scores, with no ceiling or floor effects. Importantly, children showed a good range of scores on the patterning tasks, and these measures had good reliability.

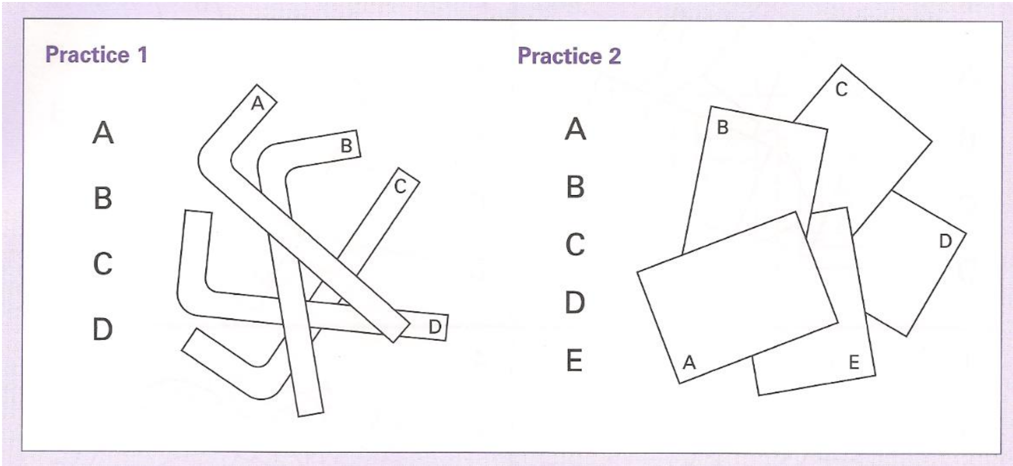

Figure 5.6 Example of the stacks test.

5.3.1 Patterning tasks

To assess differences in performance on the patterning tasks, we conducted a 2-way repeated measures Analysis of Variance (ANOVA) with stimulus type (Numbers, Letters, Objects, Shapes) and pattern type (Repeating, Rotating, Increasing) as independent variables (see Table 5.1).

There was a significant main effect of pattern stimuli, $F_{(3,234)} = 4.41$, $p = .005$, partial $\eta^2 = .053$. Post-hoc analyses using Bonferroni corrections revealed that the letter tasks were harder than the number tasks (mean difference = .51, $p = .006$) but other tasks did not differ significantly from each other.

The main effect of pattern type was also significant, $F_{(2,156)} = 420.39$, $p < .001$, partial $\eta^2 = .843$. Post-hoc analyses revealed that repeating items were significantly easier than rotating items (mean difference = 5.58, $p < .001$) and repeating item were significantly easier than increasing items (mean difference = 5.38, $p < .001$). Increasing items were not significantly different to rotation items (mean difference = .20, $p < .05$).

The interaction between pattern stimuli and pattern type was significant, $F_{(1,78)} = 22.06$, $p < .001$, partial $\eta^2 = .220$. This interaction appears to reflect the fact that for Number and Object patterns, rotation items are more difficult than increasing patterns, whereas for Letter and Shape patterns, rotation items are easier than the increasing patterns.

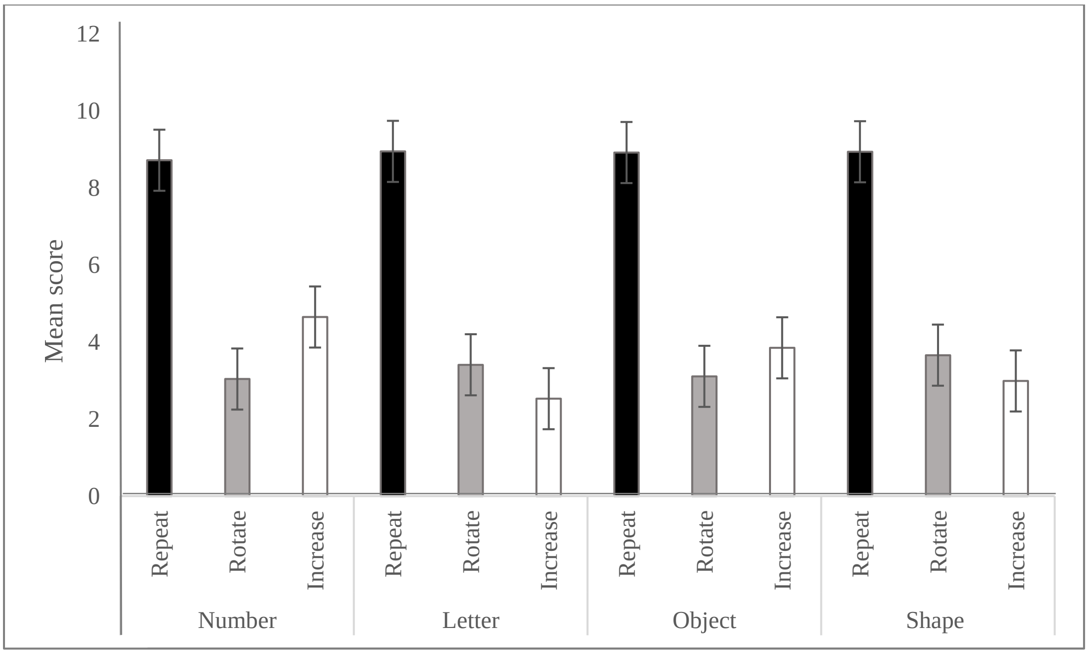

Figure 5.7 Mean levels of performance on pattern tasks (error bars represent standard error).

5.3.2 Correlations

Pearson correlations (and partial correlations controlling for age) between all measures are shown in Table 5.2. We controlled for age because there were a range of ages (children were from two year groups) and age correlated significantly with all measures (*r*s = .24 - .57, $p < .05$). We created a 'calculation' score based upon the combined score of addition and subtraction (both of which correlated strongly with each other, $r = .66$, $p < .001$).

As predicted, pattern tasks were well correlated with calculation (combined score on addition and subtraction task) after age was controlled (partial *r*s = .35 - .45, $p < .05$). Similarly, all other measures were significant correlates of calculation after age was controlled (*r*s = .29-.50, *p*s < .05). Importantly, our patterning tasks were well correlated with each other (partial *r*s = .63 - .75).

Table 5.1 Descriptive statistics, reliabilities (α) and 95% confidence intervals for all measures.

	N	Mean (*SD*)	Reliability	Range	95% CI
Age (months)	79	73.43 (7.34)	---	60-86	[71.79-75.07]
Patterning					
Numbers (30)	79	16.28 (5.35)	.88	7-29	[15.08-17.48]
Repeating (10)		8.72 (1.69)	.72	3-10	[8.34-9.10]
Rotation (10)		3.04 (2.64)	.86	0-10	[2.45-3.63]
Increasing (10)		4.65 (2.58)	.83	0-10	[4.07-5.22]
Letters (30)	79	14.86 (4.99)	.87	3-28	[13.74-15.98]
Repeating (10)		8.95 (1.77)	.80	1-10	[8.55-9.35]
Rotation (10)		3.41 (2.88)	.88	0-10	[2.76-4.05]
Increasing (10)		2.53 (2.18)	.81	0-9	[2.04-3.02]
Objects (30)	79	16.14 (5.12)	.86	7-29	[14.99-17.29]
Repeating (10)		8.92 (1.55)	.53	0-10	[8.58-9.27]
Rotation (10)		3.11 (2.80)	.87	0-9	[2.49-3.74]
Increasing (10)		3.85 (2.70)	.81	0-10	[3.24-4.45]
Shapes (30)	79	15.61 (5.15)	.86	7-29	[14.45-16.76]
Repeating (10)		8.94 (1.35)	.53	4-10	[8.63-9.24]
Rotation (10)		3.66 (2.59)	.83	0-10	[3.08-4.24]
Increasing (10)		2.99 (2.36)	.73	0-9	[2.46-3.52]
Number measures					
Calculation (25)	79	16.58 (6.44)	---	1-25	[15.14-18.02]
Addition (15)		10.00 (4.04)	.90	0-15	[9.09-10.91]
Subtraction (10)		6.58 (3.00)	.85	1-10	[5.91-7.25]
Number ID (14)	74	11.47 (2.05)	.64	5-14	[11.00-11.95]
Dot counting	79	9.39 (2.93)	---	2-17	[8.74-10.05]
Executive functioning					
Cognitive flexibility (30)	79	11.15 (9.22)	.95	0-30	[9.08-13.22]
Backwards span (16)	79	4.89 (1.37)	---	0-9	[4.58-5.19]
Visual search (secs)	79	.27 (.08)	---	-.07-.57	[.26-.29]
HTKS (40)	79	29.49 (9.99)	.94	0-40	[27.25-31.73]
Non-verbal IQ					
Non-verbal IQ (12)	72	6.54 (3.75)	.90	0-12	[5.66-7.42]
Spatial					
Spatial awareness (46)	72	15.35 (8.73)	---	0-36	[13.30-17.40]
Windows (12)		3.81 (2.62)	---	0-10	[3.19-4.42]
Hidden shapes (18)		4.47 (3.50)	---	0-12	[3.65-5.29]
Stacks (16)		7.07 (5.83)	---	0-16	[5.70-8.44]

5.3.3 Modelling relationships between measures

To assess the relationship between different measures and arithmetic, we first used factor analysis on each set of measures to derive factor scores. These analyses were conducted separately on the different theoretically defined sets of measures, because of the small sample size. The factor analyses allowed us to derive factor scores to represent the error free common variance from the different sets of measures. For each set of measures, there was clear evidence of a dominant single factor that captured a high proportion of common variance between the measures.

We chose not to create a factor score for “number knowledge” because our measures of number identification and dot counting are theoretically different constructs; number identification relies on knowledge of Arabic symbols and dot counting relies on the count sequence but no Arabic number knowledge. These two measures were therefore retained as separate measures in the analyses.

The first step of exploratory factor analysis is to compare the correlations between measures of a factor. According to Field (2009) if correlations between measures are within r = .3-.8, it is then possible to assess if these measures load onto one factor. Correlations that are too high indicate a single measure, and correlations that are too low indicate different measures. For all of our measures we found that correlations were within the acceptable range: patterning tasks; rs = .44 - .80 (number, letter, object, shape and colour), arithmetic; r = .66 (addition and subtraction) and executive function measures; rs = .28-.43 (backward span, visual search, HTKS and cognitive flexibility). We therefore derived factor scores for each group of measures.

Pattern factor

As correlations were strong between patterning measures (rs = .56 - .80) we assessed if the tasks loaded onto a single factor using factor analysis (principle axis factoring). A single pattern understanding factor was derived from four measures (number, letter, object and shape patterns) which explained 73% of the variance (factor loadings on the pattern factor are reported in Table 5.3).

Table 5.2 Correlations among measures.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1. Number patterns	-	.68**	.76**	.75**	.43**	.38*	.41**	.46**	.28*	.16	.45**	.37*	.48**	.43**	.38**
2. Letter patterns	.63**	-	.74**	.77**	.53**	.50**	.46**	.41*	.39**	.33*	.56**	.46**	.44**	.48**	.42**
3. Object patterns	.72**	.69**	-	.80**	.50**	.48**	.41**	.37*	.27*	.26*	.44**	.39**	.47**	.46**	.41**
4. Shape patterns	.71**	.73**	.75**	-	.49**	.33**	.47**	.43**	.34*	.26*	.44**	.42**	.44**	.51**	.44**
5. Calculation score	.35*	.45**	.41**	.40**	-	.94**	.88**	.55**	.50**	.45**	.46**	.58**	.40**	.54**	.60**
6. Addition	.30*	.43**	.40**	.34*	.93**	--	.66**	.53**	.54**	.42**	.45**	.59**	.35*	.51**	.60**
7. Subtraction	.33*	.37**	.32*	.38**	.88**	.62**	--	.47**	.34**	.40*	.36**	.45**	.38**	.46**	.47**
8. Number ID	.36*	.27*	.22	.22	.46**	.45**	.36*	-	.50**	.33**	.44**	.63**	.45**	.39**	.49**
9. Dot counting	.18	.29*	.15	.23*	.42**	.48**	.24*	.39**	-	.30*	.42**	.46**	26*	.38*	.44**
10. Visual search	.03	.21	.13	.11	.35*	.33*	.31*	.16	.18	-	.29*	.32*	.31*	.45**	.48**
11. Backward span	.40**	.52**	.38**	.30*	.40**	.40**	.31*	.38**	.37**	.22	-	.43**	.28*	.41**	.38*
12. HTKS	.27*	.36*	.28*	.30*	.50**	.52**	.36*	.52**	.39**	.20	.37**	-	.34*	.36*	.55**
13. Shifting Task	.39**	.34*	.37**	.32*	.29*	.26*	.28*	.31*	.13	.19	.21	.22	-	.35*	.48**
14. Non-verbal IQ	.34*	.40**	.37*	.42**	.46**	.44**	.38*	.23	.27*	.36*	.34*	.23	.24*	-	.52**
15. Spatial	.29*	.32*	.30*	.32*	.53**	.53**	.39*	.35*	.34*	.38**	.32*	.45**	.39**	.44**	-
16. Age	.35*	.39**	.40**	.43**	.37**	.34*	.35*	.57**	.36*	.38**	.24*	.40**	.40**	.38*	.40**

Notes: Partial correlations controlling for age are below the diagonal and simple correlations above the diagonal.
* $p < .05$, **$p<.001$.

These factor loadings are all relatively high and relatively uniform. The first factor extracted had a large eigenvalue (2.93) with other eigenvalues being trivial in size. This analysis therefore gives strong support to the view that the different patterning measures can all be seen as tapping a unitary pattern understanding factor.

Table 5.3 Factor loadings for patterning factor.

	Factor Loadings	Uniqueness
Number	.83	.32
Letter	.83	.31
Object	.88	.23
Shape	.89	.21

Arithmetic factor

The addition and subtraction tasks were well correlated (r = .66, p < .001) and as addition and subtraction are the main components of early arithmetic, we used factor analysis to derive an arithmetic factor. The factor explained 55% of the variance accounted for. These factor loadings are relatively high, and the first factor extracted had a moderate eigenvalue (1.10) with the other eigenvalue negligible in size. The factor loadings are reported in Table 5.4.

Table 5.4 Factor loadings for arithmetic factor.

	Factor Loadings	Uniqueness
Addition	.74	.45
Subtraction	.74	.45

Executive function factor

Correlations between the four executive function tasks (backwards span, visual search, cognitive flexibility and HTKS) were moderate (rs = .28 - .43). As it is common in the literature to assess executive function as one measure, we assessed if these tasks load onto a single factor. The executive function factor explained 34% of the variance. The factor loadings on the executive function factor are reported in Table 5.5.

These factor loadings are moderate in size and relatively uniform. The first factor extracted had a moderate eigenvalue (1.37) with other eigenvalues being trivial in size.

This analysis gives support to the view that the different executive function measure can be seen to tap into an executive function factor, however, the loadings are lower than the pattern and arithmetic factors.

Table 5.5 Factor loadings for executive function.

	Factor Loadings	Uniqueness
Backwards Span	.60	.64
Visual Search	.62	.62
HTKS	.45	.80
Shift task	.66	.57

5.3.4 Predictors of arithmetic

We used a regression model to evaluate which measures were unique predictors of arithmetic. In these models, we used factor scores saved from the factor analyses reported above. Age was retained in all models to control for age effects.

In the first stage, we entered all measures (age, pattern factor, executive function factor, number identification, dot counting, nonverbal IQ and spatial awareness) simultaneously as predictors of arithmetic. We then removed iteratively the least significant predictor from the model continuing until all retained predictors were significant. In the final model, executive function, patterning and spatial awareness were significant predictors of arithmetic, explaining 53% of the variation in arithmetic. Together, executive functioning and patterning explained 17% of the variation in arithmetic once spatial awareness and age were controlled (unique $R^2 = .17$, $p < .001$).

In summary patterning, executive function and spatial awareness explained unique variation in arithmetic scores whereas number identification, dot counting, and nonverbal IQ did not (see Figure 5.8).

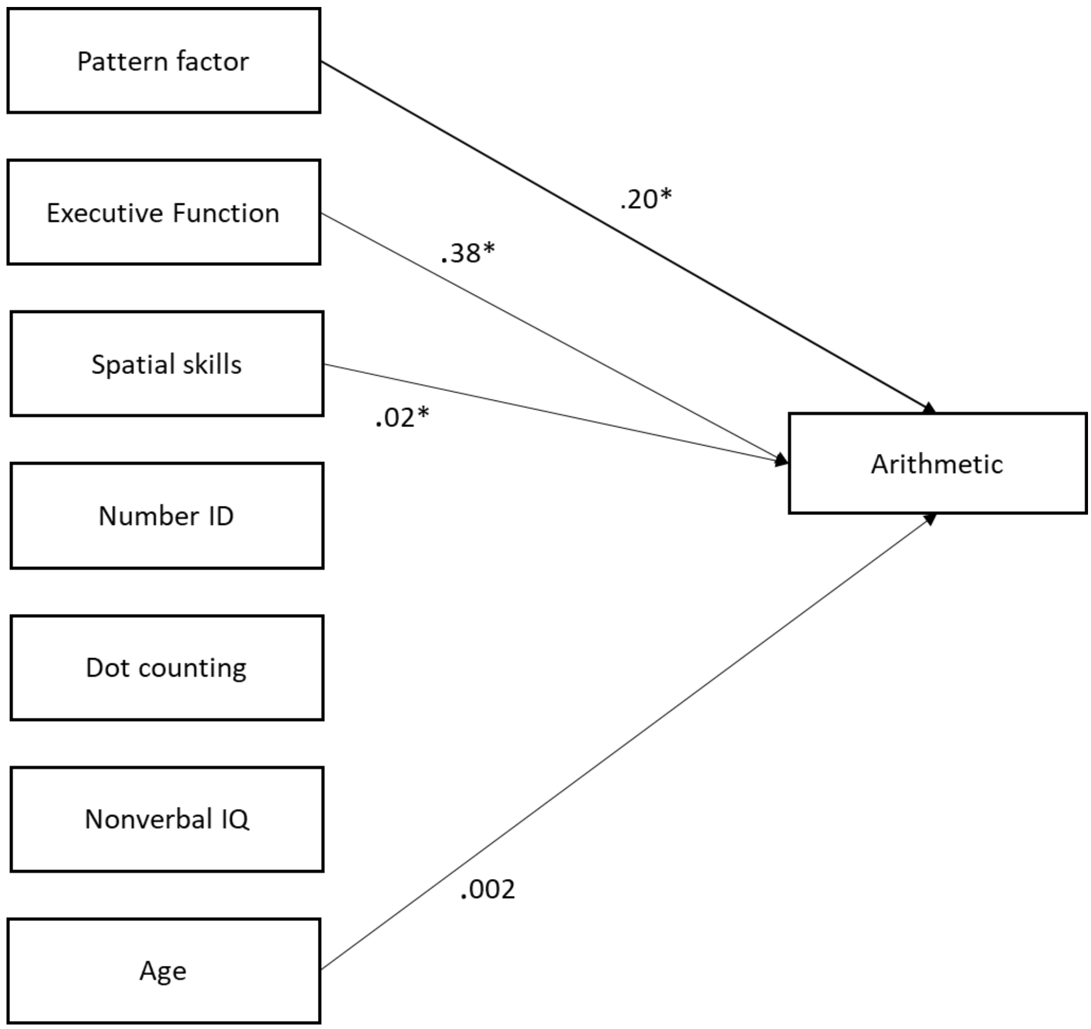

Figure 5.8 Path diagram showing significant and unique predictors of arithmetic (age is not significant but is retained in the model). * *p* < .05.

5.4 Discussion

In this study, we aimed to assess how different patterning tasks relate to one another, and how patterning relates to arithmetic. Firstly, and in line with our prediction, we found that pattern tasks correlate with arithmetic. Four pattern tasks (numbers, letters, objects and shapes) can be defined by one unitary factor which explains unique variation in arithmetic even after controlling for executive function, spatial skills and number knowledge. Secondly, we compared different types of patterning task (repeating, rotating and increasing) and showed that there is some variability in difficulty across task type and stimuli. Repeating patterns were shown to be the easiest and rotating patterns the most difficult type of task. Moreover, we found that letter tasks were significantly more difficult than number tasks, but no other significant differences were found with other measures (objects and shapes).

Our first and main finding that pattern tasks correlate with and explain unique variance in arithmetic is in line with previous evidence (e.g. Lee et al., 2011; Lee et al., 2012; Pasnak, 2017; Rittle-Johnson et al., 2017; Schmerold, 2015; VanDerHeyden et al., 2011; Warren & Miller, 2013). Previous studies have typically assessed patterning using a small number of tests and have not investigated the relationship between different types of patterning task and arithmetic. To do this, we tested children on a range of pattern tasks with different items (repeating, rotating, increasing) and different stimuli (numbers, letters, shapes, objects) and compared the relationship between different types of patterning task. We found that the different tasks can be loaded onto one factor and therefore are represented by one unitary factor. Moreover, this factor predicts unique variation in arithmetic.

Secondly, we show that repeating patterns are the easiest type of pattern task, being significantly easier than increasing and rotating items across all stimuli. These findings are in line with previous evidence showing that children are capable of completing repeating pattern tasks before increasing patterns (Rittle-Johnson, Fyfe, McLean, & McEldoon, 2013). This is likely because, to complete a repeating pattern (e.g. 1212, 1121) children do not need to abstract a rule but can use basic perceptual skills to identify the correct response. We found that rotating items were the most difficult task for number and object stimuli whereas increasing items were the most difficult task for letter and shape patterns. We suggest that increasing patterns for numbers are simpler than rotating items because children are used to increasing number patterns (e.g. 1357) but less familiar with rotating patterns. On the other hand, children are less exposed to alphabetic patterns (e.g. aceg) in school which may make the increasing letter patterns more difficult.

Finally, we show that pattern tasks are, alongside executive function and spatial skills, a unique predictor of arithmetic. Some potential reasons for this relationship include a domain-specific role of patterning in arithmetic or domain-general role of patterning in arithmetic. A domain-specific role may relate to patterns which exist in numbers, for instance an understanding that 2+1 is the same as 1+2 or that 2+4 is the same as 2+2+2. A domain-general role of patterning may be that general abstraction and manipulation skills that are required to complete pattern tasks are also important for the development of arithmetic and other developmental skills, such as reading. These suggestions do not need to be mutually exclusive and may well both be viable for explaining the association between patterning and arithmetic. Future evidence should use different types of pattern task with more outcome measures (e.g. reading fluency) to examine these possible theories, both of which are discussed in more detail later in this chapter.

5.4.1 Pattern tasks

An important aim of our study was to investigate the relationship between different types of pattern tasks. Previous studies examining patterning have often included a small battery consisting of either repeating, increasing or rotating items for numbers and letters (e.g. Pasnak, 2017; Rittle-Johnson et al., 2017; VanDerHeyden et al., 2011; Warren & Miller, 2013). One study by Schmerold (2015) used a wider range of pattern tasks but provided a composite across all tasks, therefore the individual relationships between different pattern types (e.g. repeating, rotating) and different stimuli (e.g. numbers, colours) cannot be determined.

In our study, we created separate tests for different types of patterning across different stimuli. We found that performance on these subtests were highly correlated with one another, and exploratory factor analysis showed that one factor adequately captured the shared variance between these different measures. This is consistent with other evidence as previous studies have shown similar correlations between patterning tasks and arithmetic regardless of the type of patterning used.

Secondly, we attempted to examine the different levels of difficulty across task type and stimuli. We found, in line with previous evidence, that repeating tasks are simpler than increasing tasks (Rittle-Johnson, Fyfe, McLean, & McEldoon, 2013). The difficulty of the increasing and rotating items varied across the stimuli. For instance, increasing items (e.g. 1357) were easier than rotating items for number and object stimuli, although the opposite effect was found for letter and shape stimuli.

We can tentatively suggest that this pattern was found for number and letter patterns because children are exposed to number sequences at school, and therefore may have found the increasing items more familiar than for letter items which are less well studied; children do not generally get exposed to the alphabetic sequence aceg whereas they are exposed to number sequences such as 1357. Similarly, children may be exposed to rotating shapes (e.g. triangles and hearts) more commonly than rotating objects (e.g. animal pictures). This may in part explain why the rotating items were simpler for shapes than objects, although more research is needed to further support this finding and explain this effect. One potential suggestion is that the letter and shape tasks were presented in the same testing period whereas number and objects were tested in a different session. However, this effect is unlikely related to the testing session because each session was randomised across participants. Overall, we can suggest that increasing and rotating items are more difficult than repeating items, which may have implications for the design of pattern stimuli.

5.4.2 Pattern understanding predicts unique variation in arithmetic

The second aim of the study was to examine how performance on patterning tasks correlated with arithmetic. We derived factor scores for patterning, executive function and arithmetic. In line with previous evidence, we found that patterning remained a unique predictor of arithmetic once all other variables were controlled, suggesting that patterning is an important factor in the development of arithmetic skills (Lee et al., 2011; Lee et al., 2012; Pasnak, 2017; Rittle-Johnson et al., 2017; Schmerold, 2015; VanDerHeyden et al., 2011). We extend previous knowledge because we used a large battery of pattern tasks with good reliabilities and controlled for a wider range of numerical confounds than previous studies. For example, Rittle-Johnson et al. (2017) found that performance on a patterning task (colours and repeating patterns) correlated longitudinally with arithmetic measures five years later (rs = 31 to .38; R squared 9.6% to 14.4%) and explained unique variation once other important factors were controlled ($\beta = .08$, $SE = .04$, $p < .05$). However, the patterning task had low reliability ($\alpha = .56$) and consisted of only repeating and coloured stimuli.

In our study, the patterning tasks generally had good reliability ($\alpha = .53\text{-}.88$) and we are able to show that similar correlations with arithmetic remain for all patterning types and multiple stimuli. Similarly, Warren and Miller (2013) found that a patterning score correlated concurrently with arithmetic ($r = .44$, $p < .05$) at similar levels to our study. However, only number patterns were used without controlling for number knowledge and therefore the relationship between patterning skills (and not number knowledge) and arithmetic is not known.

Our findings are directly in line with evidence from Schmerold (2015) who showed, using a large patterning battery, relying on numerical and non-numerical pattern stimuli, that patterning (a combined score from all tasks) correlated well with arithmetic (r = .52-.54) and at similar levels to arithmetic as our patterning tasks (r = .43-.53). Importantly, this study also controlled for executive function which was similarly correlated to patterning and arithmetic, as in our study.

5.4.3 Why does pattern understanding predict arithmetic?

The cognitive mechanisms underlying the relationship between patterning and arithmetic are not completely clear, although two potential reasons for the relationship are proposed. The first concerns the fact that basic arithmetic and counting skills rely on patterns. Take, for example, the sum 3+5. A child with good arithmetic abilities will understand that this is the same sum as 5+3, and additionally that the latter is quicker to compute than the former (counting on from 5 is quicker than counting on from 3). Counting also involves patterns, for example counting in 2's or 5's requires the child to understand a pattern which describes how numbers increase. Pattern understanding may help the child to develop an understanding of these links which may lead to improved arithmetic.

A second possible reason for the relationship is that pattern understanding, particularly for more difficult patterns, involves the manipulation and abstraction of information. In simple pattern tasks such as repeating patterns, children may be able to complete the pattern using visual representations and without understanding the underlying structure of the pattern. However, in increasing patterns, the child must predict the final pattern item by abstracting the underlying principles of the pattern. Arithmetic, and particularly more advanced or rapid arithmetic, also relies on the child understanding how numbers relate to one another. This may suggest that pattern understanding is related to arithmetic via the understanding of patterns within arithmetic. This general ability to identify patterns may be linked to other developmental skills, such as reading where, for example, children with good pattern understanding may be able to link certain sounds to letters. As a result, patterning may be considered as a domain-general skill and other researchers have suggested that patterns are important for the development of broader cognitive abilities and fluid intelligence which may help to develop not just arithmetic but other cognitive abilities, including reading (e.g. Burgoyne et al., 2017). As our study did not measure other developmental abilities, it is not possible to tease apart this claim although it is suggested that understanding how numbers relate to one another (i.e. showing a good understanding of patterning) will be important for more than just numerical abilities. Future studies should attempt to examine this further by measuring a range of educational outcomes alongside measures of patterning.

The suggestion that patterning is a domain-general skill is interesting as findings from our study suggest that domain-general skills are more important in predicting variation in arithmetic than number skills are. One suggestion is that the age of children is an important consideration here (discussed more in Section 6.3). For instance, for younger children, numerical abilities may be more important than for older children when executive function and domain-general skills may become more important (as basic number knowledge has been mastered). Future studies should attempt to examine how more advanced numerical abilities relate to arithmetic and other domain-general skills in this age group.

5.4.4 Executive function and spatial skills

One important aspect of our study was that we controlled for domain-general skills which may be related to both patterning and arithmetic. We found, in line with other studies, that executive function and spatial awareness are good predictors of arithmetic (see Section 2.3). As these tasks do not rely directly on any numerical abilities, it is suggested that they are domain-general factors which may also predict other cognitive and academic skills such as reading.

Executive function tasks rely on attention and working memory, which are linked strongly to arithmetic and classroom achievement (Duncan et al., 2007). A child with good attention will focus in class and therefore process information presented by a teacher. They may therefore be able to learn the required processes for arithmetic better than a child with poor attention. Similarly, executive function tasks rely heavily on working memory and this skill is related to arithmetic; addition and subtraction rely on the child processing information within short-term memory (see Section 2.3.1).

One consideration is how to teach executive function. It is not as simple to teach as number knowledge or patterning, which can be clearly taught using pencil-and-paper and traditional teaching methods. Moreover, there is debate within the literature surrounding the way that executive function tasks relate to one another and whether these are one underlying construct (e.g. Miyake et al., 2000). We found partial evidence that the tasks we used form one factor, although the correlations between our measures were weaker than some other measures we examined. Further studies may wish to focus on the relationship between different executive function measures and examine whether some, or all types of task may be better predictors of arithmetic.

Spatial awareness was found to predict arithmetic in our study, above that of patterning. This suggests that although related (patterning and spatial tasks were significantly correlated; partial *r*s = .29 - .32) these factors play, at least in part, separate roles in predicting variance in arithmetic. There are a number of potential reasons for the relationship between spatial skills and arithmetic. Children may represent numbers via a mental number line (refer to SNARC effect; Berch et al., 1999) or process numerical information using the same neural pathways as spatial information (see Hubbard et al., 2005). Our study was not placed to distinguish between these different potential mechanisms for this relationship and further studies are needed to clarify this. For example, by investigating the best ways to teach spatial skills to children, which, akin to executive function, warrants investigation.

5.4.5 Limitations and further work

One limitation of the patterning study is that our correlation does not provide evidence for the direction of the relationship between patterning and arithmetic. Future research should use longitudinal designs to examine the direction of the relationship, and training studies to test whether there is a causal link. Before there is strong evidence of a causal relationship between these factors, it is not possible to recommend that educational programmes contain pattern skills to enhance arithmetic abilities, although we may suggest that continuing to include patterning in early education will not disadvantage children.

One other limitation of our study is that we used just one form of pattern task in which children were required to find the final item in the pattern. This type of pattern task has been used in previous studies (e.g. Lee et al., 2011; Schmerold, 2015) but one consideration is that the pattern stimuli may have, in some way, contained some numerical information. Take, for instance, the shape increasing stimuli. To respond to one of these items (e.g. total number of triangles within the group increasing), children can count to find the answer. Similarly, in the object increasing stimuli, children were sometimes able to count the number of objects (e.g. increasing carriages on a train). Even for the increasing number and letter items, children were required to count the digits (e.g. 1357__) or count the number of letters missing within the sequence (e.g. DGJM__). Therefore, although potentially a minimal impact, it is possible that children were using counting knowledge to respond to some of the items.

One possible outcome of this is that the relationship between pattern understanding and arithmetic is driven in part by numerical and counting skills which are involved in some of the pattern stimuli. Future studies should try to eliminate any numerical skills involved in the pattern stimuli, particularly those which are not alphanumeric. One way to do this is to ask children to recreate a pattern using different stimuli (referred to as abstraction) which therefore does not require any counting, but which shows an understanding of the underlying pattern (as an increasing task would).

5.4.6 Conclusion

In summary, the study used a large battery to assess pattern understanding in children aged 5-7 years. We found, in accordance with previous evidence, that patterning tasks (regardless of stimuli used or type of patterning test) are well correlated with arithmetic. This relationship remains once age, executive function and spatial awareness skills are considered, which together explain 52% of the variation in arithmetic. Further research should examine the longitudinal association between patterning and arithmetic to further understand the direction of this relationship.

Chapter 6. General discussion

6.1 Key findings

This thesis has examined a range of cognitive factors which may be important for arithmetic development in children. Three studies were conducted to examine how different factors relate to arithmetic abilities in children between 4 and 8 years old (the first three years of formal schooling in the UK). In an initial study, the relationship between arithmetic and sensori-motor skills including finger awareness was examined. Next, the potentially causal relationship between number knowledge and arithmetic was assessed using a training study. Finally, we evaluated how pattern tasks relate to one another, and to arithmetic. Our main findings were as follows:

1) Finger gnosis, and other sensori-motor abilities are less important predictors of arithmetic than numerical abilities, including number knowledge.
2) Children trained in number knowledge showed trends to improve in arithmetic and number skills compared to control groups, but these effects were not statistically significant.
3) Pattern understanding is a significant predictor of arithmetic, and future research should focus on the potential longitudinal and causal relationship between these factors.

6.1.1 Sensori-motor skills

To examine the potential relationship between sensori-motor skills and arithmetic we conducted a correlational study. Our data showed that finger awareness and other sensori-motor abilities were moderately correlated with arithmetic, but this reduced to non-significant and negligible levels when age and other numerical skills are controlled. Some studies which have supported a relationship between sensori-motor abilities and mathematical learning are limited by small sample sizes and a failure to control for other known predictors (e.g. Gracia-Bafalluy & Noël, 2008; Noël, 2005). When we overcame these limitations, symbolic magnitude comparison and counting were shown to be unique and important predictors of arithmetic, in line with considerable evidence supporting a role of symbolic number skills in arithmetic development (Durand et al., 2005; Lyons et al., 2014; Vanbinst et al., 2016).

One of the most important aspects of this study was that we controlled for other predictors of arithmetic and used reliable measures. Controlling for important factors is a critical component of correlational designs, and something which is commonly lacking in the literature. A failure to control for known predictors can lead to overestimations of the role of other potential factors.

The finding that symbolic number comparison (which measures symbolic number knowledge) and counting were important predictors of arithmetic led to the inception and development of the second study which focused on the causal link between symbolic number knowledge and arithmetic.

6.1.2 Symbolic number training

Merkley and Ansari (2016) define symbolic number knowledge as the ability to identify digits and understand the magnitude and ordinality of numbers. One way to examine this is via symbolic magnitude comparison tasks, requiring the child to identify which of two digits is larger, with evidence supporting strong relationships between this skill and later arithmetic (see Schneider et al., 2016). Additionally, studies can use number identification and number ordinality tasks. One critical aspect of which task to choose to in order to examine number knowledge is the age of the children. For example, for younger children tasks may focus on the count sequence for smaller numbers (i.e. numbers 1-10). As children get more proficient in learning Arabic digits, number identification tasks may be used alongside tasks which assess magnitude understanding (e.g. symbolic comparison tasks). For older children, tasks which rely on an understanding of place-value may be the best measure of number knowledge.

Symbolic number knowledge has been shown to relate with arithmetic and our study investigating sensori-motor skills supported this. We showed that a large proportion of the variance in arithmetic is explained by counting and symbolic number comparison tasks. We examined a potential causal relationship between symbolic number knowledge and arithmetic through a training study. Children were trained in number knowledge via a game computerised App. The training consisted of playing a numbers game in which children were required to create numerical sequences (reinforcing ordinality) whilst hearing the sound of a number when pressing a digit (reinforcing identification).

We showed (non-significant) gains in the predicted direction; children in the number condition improved more than an untreated control group at post-training on number outcomes, including arithmetic. Equivalent scores were seen for literacy outcomes. Additionally, children in the letter training (control training group), who played an equivalent game with letters, did not improve more than the untreated control group on numerical scores but did show some improvements on some literacy measures. Although these findings did not reach significance, they are a good indication that training in number knowledge may have a positive impact on arithmetic abilities. This is in line with previous studies which have shown that training in number knowledge can improve arithmetic and numerical outcome scores (e.g. Honoré & Noël, 2016; Jordan et al., 2009; Ramani & Siegler, 2008).

The likely reason for lack of significance is argued to be due in part to a lack of power. The potential reasons for this are initially discussed in Chapter 4 and include low power to find an effect, should it exist, due to too few participants, too short training time and limitations in the measures used (see Section 4.4.1). Additionally there are practical limitations including that children did not enjoy playing the game and the game was difficult for this age group. One of the important findings from this study is that designing a methodologically sound training study is very challenging. The training itself needs to be designed with practical considerations. For example, if children are trained in a group (as was the procedure in our training study) this increases the number of children who can play the game within the training time which is positive for the practicality of time implications. However, one negative of group training is that it is necessary to ensure that the game is engaging enough so that they can play without the need for one-on-one instruction and encouragement from the experimenter.

We chose a group training design because it limits the overall time that children are taken out of the classroom for training. In our study, groups of five or six children were taken out of classroom during free-play as it was important for teachers that children were not missing lesson time each day. Due to extra pressure on teachers to ensure that children are reaching key milestones in the first years of formal education, it is possible that teachers do not want children to miss lesson time. One possible alternative is to have the training as part of the normal lesson, or have training conducted by teaching assistants in the classroom. This would reduce the amount of time that children are missing lessons. Again, if the study is re-run on a greater scale, this could be an important component of the training design.

The study was eye-opening to the many considerations that are important to consider when conducting training studies. Although more data could have been collected to reach power, the practical considerations were deemed too significant to warrant additional data collection (further discussion of practicalities are discussed in Section 6.4). As a result, the study is now being prepared in a different way for roll-out on a greater level with some practical elements reconsidered. For example, the training game has been redeveloped to ensure easier starting levels. Additionally, teaching assistants will roll out the training, rather than an experimenter.

6.1.3 Patterning

The final study involved pattern understanding which is gaining interest with researchers examining a potential link between this and arithmetic. Many pattern tasks require children to use number knowledge (e.g. complete the following sequence: 1,3,5,7__) however, many studies in literature fail to control for number knowledge and therefore we identified a need to examine these potential relationships further.

We created a large pattern battery and conducted a correlational study to examine the relationship between pattern tasks and arithmetic. Number knowledge and a range of other predictors (including executive function and spatial skills) were controlled. The findings support recent evidence showing that pattern understanding is well correlated with arithmetic and predicts unique variation in arithmetic after controlling for better-established predictors. An additional aim of the study was to examine how different types of pattern task relate to one another. Previous studies have either combined pattern tasks or focused on one type. In our study we were able to compare different pattern types (e.g. repeating, rotating or increasing) and different pattern stimuli (e.g. numbers, letters, objects, shapes). We showed that these reflect a single factor which predicts unique variation in arithmetic.

Interestingly, number tasks did not remain as unique predictors of arithmetic after controlling for domain-general skills. Therefore, there is a need to consider whether domain-general abilities are better predictors of arithmetic in this age range (ages 5-7) compared to domain-specific skills.

6.2 The importance of symbolic number knowledge

We measured symbolic number knowledge across all three studies. In the first study examining sensori-motor skills, symbolic knowledge was measured via a symbolic comparison task which relies on the ability to know the magnitude or order associated with a number (i.e. is 9 bigger than 3). This remained a significant and unique predictor of arithmetic. As with previous studies, this was a stronger predictor of arithmetic than approximate magnitude comparison (a similar task but using dots rather than digits; see Section 2.1.3).

In the second study, the potential causal relationship between symbolic number knowledge and arithmetic was measured with promising (albeit not significant) results which suggested that, with greater power, evidence for a causal link may have been shown. The study used a symbolic number training task in which children were required to create numerical sequences of Arabic digits whilst hearing the sound associated with the digit. This training did not include any specific magnitude training, for instance but rather trained in identification and ordinality. The game (which was already designed; see Section 1.2.3) was deemed appropriate for children in the first year of formal schooling. At this age children require more training on the more basic aspects of number learning, including the order in which numbers appear and the relationship between the sound and Arabic digit. One interesting avenue would be to examine whether the addition of magnitude training in this study would have led to stronger effects on arithmetic outcomes, as has been shown in one training study which included magnitude training (Honoré & Noël, 2016).

In the final study, symbolic number knowledge was examined and found to correlate well with arithmetic (r = .55, p < .001). However, it was not a unique predictor of arithmetic in the final model after controlling for patterning skills, executive function and spatial awareness. There are a number of potential reasons for this. Firstly, domain-general predictors of arithmetic may be more important than symbolic number skills in this age group (ages 5-7). This may be because children have largely mastered basic number knowledge skills, and therefore other abilities (memory, attention etc.) may be more important for predicting arithmetic fluency. To assess this, pattern tests could be used on younger children and we may predict that number skills are a better predictor in this age range. Alternatively, it may be that the pattern tasks required number identification and number counting skills (because they were largely increasing patterns that increased by a set number of items each time) and therefore the number skills that were predicting arithmetic were tied up in the pattern task. To assess this claim, different types of pattern task could be used which reduce potential numerical confounds (see Section 4.4).

We suggest that different types of symbolic number knowledge are more critical in different stages of development. In early development, identification and simple number ordering may be the best outcome measure. Later in development, understanding of magnitude may be more critical. Once symbolic number knowledge has been mastered, other skills (such as executive function) may be more important in predicting arithmetic scores.

Evidence for this proposal comes from our third study in which number knowledge did not remain a unique predictor of arithmetic once we controlled for domain-general skills. We found that executive function and spatial awareness (alongside patterning) remained unique predictors of arithmetic once number skills were controlled. This suggests that in this age group (children were 5-7 years old) domain-general skills are more important predictors of arithmetic than number abilities, particularly if patterning is deemed a domain-general skill. One limitation is that our dot counting task (used in the first and third study) may not be a pure measure of counting. As the task is timed, it is possible that this particular dot counting task is a measure of executive function (attention and inhibition). As executive function was controlled for in the third study, we may have been capturing the skills measured in the dot counting task within our executive function battery. Perhaps for younger children, with poorer counting skills, the dot counting task is a purer measure of cardinality, whereas older children it is measuring different skills (depending on the type of task used). Therefore, it is important for future studies to examine executive function versus number knowledge as predictors of arithmetic in older children.

This potential limitation aside, the role of executive function in predicting arithmetic warrants further investigation. Additionally, it is important to examine the ways that domain-general skills can be measured and taught, and the ways in which these abilities may be more and less important at different stages of development. For instance, it may be that in older stages of development executive function (e.g.

memory and attention) abilities may become more important in arithmetic variation when compared to number skills. This could be because numerical abilities (basic symbolic number knowledge for instance) may have been mastered therefore giving more precedence to general abilities such as attention. For younger children, we may suggest that number skills are critical in predicting variation in basic arithmetic, as without an understanding of number knowledge, children would not be able to compute basic sums.

Further studies should continue to better understand the way that number knowledge and other general skills relate to one another and to arithmetic and continue to understand when number knowledge skills (and which type) are most important in developing arithmetic understanding.

6.3 Theories of numerical development

The findings of the studies in this thesis can be applied to theories of numerical development. Perhaps the clearest finding is that our evidence challenges the suggestion that the innate magnitude analogue system is the core component of later numerical development. One aspect of the core number theory (Dehaene, 2001) is that approximate number skills are the foundation to later numerical abilities (although there are other components to this theory which we are not directly contesting). Our first study showed that, although correlated with arithmetic, approximate number skills were not as important as symbolic number knowledge in predicting variation in arithmetic. This finding challenges the claim that early approximate skills are the foundation to later arithmetic although it is possible that the approximate number system is related to some exact number skills which are related to arithmetic.

Our finding that symbolic number skills are important in predicting arithmetic is in line with theories supporting a critical role in the understanding of the magnitude, visual and verbal number form. For example, Merkley and Ansari (2016) argue that number knowledge is the most critical aspect of numerical development, alongside a range of other environmental and domain-general skills and our studies support this suggestion.

One important consideration for numerical development is how children develop an understanding of the association between cardinality, ordinality and magnitude. Is, for instance, magnitude understanding learned in tandem with ordinality and the count sequence? Or, is magnitude a separate component of learning which comes later in development and is distinct from the other aspects (although related)? Carey (2011) proposes that learning takes place through qualitative learning stages and that children's ability to understand the count sequence (which is in part linked to the core number system) is the first step in associating numbers with the magnitude.

Other studies support the suggestion that different types of numerical understanding task are more important for arithmetic at different stages of development (e.g. Lyons et al., 2014). Understanding the developmental trajectory is challenging. For instance, understanding of magnitude for smaller numbers (e.g. numbers 1-10) may develop before the identification or larger numbers (e.g. 1023) although number identification for smaller numbers (e.g. 1-5) may develop before magnitude understanding. It is therefore proposed that number learning is a dynamic process with a complex interaction and further research should attempt to better understand this process in order to inform educators and teachers about the best ways to develop numerical understanding in children.

One important component for theories of arithmetic development is the role of non-numerical predictors. It is important to develop is a framework for the role of domain-general and environmental factors involved in arithmetic learning. How, for instance, does the amount of learning at home change the way that children develop an understanding of numbers and arithmetic? Additionally, how do intelligence, attention, memory and other general skills relate to learning? In our final study, we showed that executive function, spatial skills and patterning were all important components of arithmetic and interestingly that these were stronger predictors than the numerical tasks. These findings highlight the importance in 1) finding the correct number tasks for the age range (for instance, would measuring magnitude knowledge, rather than identification, have led to a stronger role for number skills in this case?) and 2) including non-numerical learning in theories of arithmetic development. This is difficult, and at present there is a lack of evidence for the relative strength of these skills. It is proposed here that environmental aspects, along with domain-general skills, are critical considerations for theories. Therefore tasks measuring arithmetic development should include these factors in children of younger ages to access how number abilities and general skills relate to one another, and to arithmetic.

6.4 Practical limitations of developmental research

One of the key limitations of the training study was that we failed to show significant differences in scores, despite effects in the predicted direction. This was likely due to a lack of power with a sample size increase of a further 180 participants required as shown in a power analysis. Due to time and logistical constraints, no additional data were collected. These considerations are important for future researchers designing training studies, and warrant consideration for all research involving children.

Core factors were time and resource implications involved in collecting more data. The initial data collection of ~80 participants took considerable time and resources. These included school recruitment, and time and money for hiring research assistants to assist with collecting pre- and post- data. Moreover, it was important to consider the time that children would be out of class, as children were required to leave class every day to complete the training. As teachers did not want children missing lesson time, there was limited time within the day to train children (only during free-play or break-time). An estimated minimum of 18 months to collect the additional data was theoretically possible, but clearly a challenge given the time remaining for my PhD. These considerations are demonstrative of some of the practical challenges of conducting training studies with reasonable power. The time and resource implications are much greater than for a correlational or longitudinal design (when children are only tested once, for example) and demonstrate that training studies require meticulous planning, agreeable schools/teachers/parents and considerable resources. This may, I believe, have some part to play in why there is limited evidence from training studies for symbolic number knowledge.

The evidence presented in the introduction of Chapter 4 highlights that there are only a few studies that have published evidence of training in number knowledge. Additionally, the published studies are limited in their evidence for transfer to arithmetic and number skills, with only one showing positive transfer to arithmetic. One important reason for the lack of positive evidence may be that any additional studies that have been conducted, failed (like ours) to reach significance or were not able to find appropriate resources to conduct the study in the first place. This is an important consideration for many researchers who are planning training studies. One way to get around this may be to have teachers implement the training rather than experimenters. This would mean that the training could be conducted as part of the school-day for the child, avoiding children leaving the learning environment. A second possibility to overcome is using games or computerised training that children can complete alone, or at home. This was one of the reasons that we used a computerised App, as it might have been possible to role this out to be an at-home study (or free-play time training game) whereby researchers were not required for the daily running of the programme. However, further exploring this route was deemed problematic because of additional limitations that were highlight during the study.

During training, it became apparent that children did not enjoy playing the training games, and it took considerable effort to keep children focused on the game during the 5-10 minutes that they played. Ideally, children would have played the game for 15 minutes each day, but it became clear that children were unwilling to do this, and 5-10 minutes was the maximum time that they could keep their attention. One reason for this might be the age of the children (who were only four years old). Testing older children is problematic as it is likely that they will have fully developed symbolic number knowledge skills and therefore the training would not have led to any improvements in that skill. Another reason is the game itself, which was either too challenging leading to distraction, or too easy leading to boredom.

The game was designed in such a way that the first level required more number knowledge than was anticipated. In order to pass the level, children were required to create numerical sequences of differing lengths (e.g. create a sequence which includes 6 numbers; e.g. 2-7/4-9) and this meant that if the child had limited number knowledge of the digits 1-10, they may have struggled to create sequences. Rather, the game should have been progressive in training, starting with simple tasks of creating sequences of smaller length. This would have reinforced learning and enabled the children to progress with the game without feeling demoralised. Stopping children from getting bored is perhaps more challenging, as it was noted that children of all abilities were bored whilst playing the game. Anecdotally, older children appear to be better at focusing during testing and enjoy research activities more. This highlights the importance of creating engaging tasks and games, particularly for younger children along with the need for age-appropriate tests during developmental research.

After pre-training testing, it became apparent that children of this age are less proficient in letter knowledge than in number knowledge. At the start of formal education in the UK, children focus on letter sounds rather than letter knowledge. This was clearly evident from our findings in letter sound, rather than letter knowledge scores (Section 4.3). Therefore, the control condition was not of equal difficulty to the number condition (as demonstrated in the number of children who reached past the first level). This raised questions about the usefulness of the control game in the present design, which together with the above limitations led to the conclusion that, in its present form, the game was not ready to be rolled out to more participants.

This study highlights some of the many challenges that come with running a training study for young children. These implications should be considered by researchers who are planning training studies and hopefully will help to identify and anticipate key problem areas during the planning stages of a study.

6.5 Educational implications

Our results have implications for educational practices and theories of development. The findings, coupled with previous evidence, suggest that symbolic number understanding should be a prominent part of early formal education. Patterns, which currently form part of early formal education in the UK, appear to be implicated in arithmetic development and therefore are a potentially important part of early development (although we did not learn the direction of such a relationship from our study). Finally, we can suggest that educators should not focus on sensori-motor skills as a critical part of education for children within the second and third years of school. Despite fingers potentially being a useful tool for counting and early arithmetic, we did not find evidence that finger awareness is important.

We also assessed executive function and spatial awareness as control measures in one study and found that they are correlated with arithmetic. Future research may wish to examine the best ways to teach these skills to young children as they appear to be important predictors of arithmetic, although may be more challenging skills to teach to children than some other abilities such as number knowledge or patterning. Focusing on teaching skills to young children which are shown to be important in predicting variation in arithmetic may help to reduce the number of children who face difficulties in learning arithmetic, and thus reduce incidences of mathematical learning disorder.

6.6 Concluding thoughts

The data reported in this thesis add to the growing body of evidence examining the relationship between cognitive skills and arithmetic development in young typically developing children. We have identified pattern understanding as an important factor in the development of arithmetic, and this finding warrants future research which should examine the potential causal relationship. We highlight the importance of including known predictors of arithmetic in such future studies, as a failure to do so can result in overestimating the relationship between factors (as found in the sensori-motor study). Our training study, which failed to find a statistically significant effects highlights the importance of power within a study, and the difficulty of obtaining large sample sizes in child development studies (particularly when considering multiple groups within one study).

Overall, we have examined the literature and produced methodologically sound research to examine important questions. The results are important because they provide evidence for a number of key cognitive factors that may or may not be involved in arithmetic development. For example, we have provided clear evidence that finger awareness and other sensori-motor skills are less important than more exact number abilities such as symbolic number comparison and dot counting in children aged 5-7. Moreover, we have shown that it is possible to improve number skills in children through symbolic number learning whilst highlighting the difficulties of running effective and powerful training studies. Finally, we have shown that pattern understanding across multiple measures may have an underlying and common factor which predicts variation in arithmetic.

Across these three studies we consistently examined symbolic number knowledge and propose that it is an important factor to consider in arithmetic development. However, these findings highlight the importance of controlling for other factors within research designs. Additionally, we propose that the age of children are an important consideration for future research as this may determine which skills are most important for different stages of development. For example, number knowledge may most important in predicting variation in arithmetic in younger children. Other skills such as domain-general skills may become more important in later stages of development when basic number skills have been mastered. Further research should continue to assess arithmetic development and the cognitive factors implicated in order to further understand how children learn arithmetic and inform theories of development and educational practices.

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Appendix 1. Pattern Tests: Item Descriptions

Subtest: Number Patterns

Item number	Item description
Practice 1	Repeating pattern, 2 items
Practice 2	Repeating pattern, 3 items
Practice 3	Increasing pattern, linear sequence
Items 1-10	**Repeating items**
Item 1	2-item: 2 3 2 3
Item 2	2-item: 6 3 6 3
Item 3	2-item: 3 5 3 5
Item 4	2-item: 6 10 6 10
Item 5	2-item: 8 7 8 7
Item 6	3-item: 3 3 7 3
Item 7	3-item: 9 4 4 9
Item 8	3-item: 4 4 6 4
Item 9	3-item: 3 3 4 3
Item 10	3-item: 1 8 8 1
Items 11-20	**Rotation**
Item 11	Number 5, rotation 90˚right (starting 1)
Item 12	Number 3, rotation 90˚right (starting 1)
Item 13	Number 7, rotation 90˚ left (starting 5)
Item 14	Number 2, rotation 90˚ right, (starting 7)
Item 15	Number 4, rotation 90˚ right, (starting 3)
Item 16	Number 3, rotation 45˚ right, (starting 1)
Item 17	Number 4, rotation 45˚ left, (starting 5)
Item 18	Number 2, rotation 45˚ right, (starting 7)
Item 19	Number 1, rotation 45˚ left, (starting 1)
Item 20	Number 3, rotation 45˚ left, (starting 8)
Items 21-30	**Increasing items**
Item 21	Linear sequence: 3 4 5 6
Item 22	Linear sequence: 10 11 12 13
Item 23	Skip 1: 1 3 5 7
Item 24	Linear sequence: 14 15 16 17
Item 25	Skip 1: 4 6 8 10
Item 26	Skip 2: 1 4 7 10
Item 27	Skip 2: 13 15 17 19
Item 28	Linear sequence: 19 20 21 22
Item 29	Skip 1: 17 19 21 23
Item 30	Skip 2: 4 7 10 13

Notes. Starting positions 1,3,5,7 are 3 ω ɛ ო respectively, with starting positions 2,4,6,8 sitting in 45 degrees in between.

Subtest: Letter Patterns

Item number	Item description
Practice 1	Repeating pattern, 2 items
Practice 2	Repeating pattern, 3 items
Practice 3	Increasing pattern, linear sequence
Items 1-10	**Repeating items**
Item 1	2-item: a b a b
Item 2	2-item: b c b c
Item 3	2-item: s p s p
Item 4	2-item: c e c e
Item 5	2-item: f j f j
Item 6	3-item: e e f e
Item 7	3-item: d a a d
Item 8	3-item: t t v t
Item 9	3-item: x x z x
Item 10	3-item: y w w y
Items 11-20	**Rotation items**
Item 11	Letter a, rotation 90˚ left (starting 1)
Item 12	Letter f, rotation 90˚ left (starting 1)
Item 13	Letter k, rotation 90˚ right (starting 3)
Item 14	Letter y, rotation 90˚ left (starting 3)
Item 15	Letter h, rotation 90˚ right (starting 7)
Item 16	Letter a, rotation 45˚ right, (starting 1)
Item 17	Letter y, rotation 45˚ left, (starting 1)
Item 18	Letter a, rotation 45˚ right, (starting 3)
Item 19	Letter k, rotation 45˚ left, (starting 4)
Item 20	Letter j, rotation 45˚ left, (starting 5)
Items 21-30	**Increasing items**
Item 21	Linear sequence: c d e f
Item 22	Linear sequence: i j k l
Item 23	Skip 1: a c e g
Item 24	Linear sequence: o p q r
Item 25	Skip 1: f h j l
Item 26	Skip 2: a d g j
Item 27	Skip 1: n p r t
Item 28	Linear sequence: u v w x
Item 29	Skip 1: o q s u
Item 30	Skip 2: e h k n

Subtest: Shape Patterns

Item number	Item description
Practice 1	Repeating pattern, 2 items
Practice 2	Repeating pattern, 3 items
Items 1-10	**Repeating items**
Item 1	2-item: triangle (tr), circle (ci), tr ci
Item 2	2-item, numbers: square (sq), tr sq tr
Item 3	2-item: ci sq ci sq
Item 4	2-item: star (st), ci, st, ci
Item 5	2-item: tr, sq, tr, sq
Item 6	3-item: sq, tr, tr, sq
Item 7	3-item: ci ci sq ci
Item 8	3-item: tr tr ci tr
Item 9	3-item: tr st st tr
Item 10	3-item: st ci ci st
Items 11-20	**Rotation**
Item 11	Triangle, rotation 90˚ right (starting 1)
Item 12	Heart, rotation 90˚ left (starting 1)
Item 13	Triangle, rotation 90˚ left (starting 5)
Item 14	Heart, rotation 90˚ right (starting 3)
Item 15	Half-moon, rotation 90˚ left (starting 3)
Item 16	Heart, rotation 45˚ right (starting 1)
Item 17	Triangle, rotation 45˚ left (starting 1)
Item 18	Triangle, rotation 45˚ right (starting 7)
Item 19	Half-moon, rotation 45˚ right (starting 1)
Item 20	Heart, rotation 45˚ right (starting 6)
Items 21-30	**Unit increase**
Item 21	Cross made of individual squares, remove two blocks (from top, then sides)
Item 22	One triangle, add one triangle each time
Item 23	Two blocks, add two blocks each time
Item 24	One rectangle, add circle, add rectangle, add circle
Item 25	Two blue squares, add one green square, add two blue squares, add one green square
Item 26	One circle, add two circles, then one circle, then two circles
Item 27	Five triangles joined together, remove one triangle each time
Item 28	The triangles and 10 circles, remove two circles, remove one triangle, remove two circles,
Item 29	One square, add two squares, add two squares, add one square
Item 30	Five circles and four ellipses, remove one ellipse, remove one circle, remove one ellipses.

Subtest: Object Patterns

Item number	Item description
Practice 1	Repeating pattern, 2 items
Practice 2	Repeating pattern, 3 items
Items 1-10	**Repeating items**
Item 1	2-item: elephant, bird, elephant, bird
Item 2	2-item: dog snake dog snake
Item 3	2-item: bee dog bee dog
Item 4	2-item: bird horse bird horse
Item 5	2-item: snake bird snake bird
Item 6	3-item: lion lion bee lion
Item 7	3-item: snake bird bird snake
Item 8	3-item: lion lion horse lion
Item 9	3-item: dog dog bird dog
Item 10	3-item: butterfly bee bee butterfly
Items 11-20	**Rotation**
Item 11	House, rotation 90˚ right (starting 1)
Item 12	Tree, rotation 90˚ left (starting 7)
Item 13	House, rotation 90˚ right (starting 3)
Item 14	Mouse, rotation 90˚ left (starting 3)
Item 15	Cat, rotation 90˚ right (starting 7)
Item 16	House, rotation 45˚ right (starting 1)
Item 17	Cat, rotation 45˚ right (starting 1)
Item 18	Mouse, rotation 45˚ left (starting 3)
Item 19	Cat, rotation 45˚ right (starting 2)
Item 20	Tree, rotation 45˚ left (starting 5)
Items 21-30	**Relative size and unit change**
Item 21	Tree decreasing in size
Item 22	Pizza decreasing by one slice (adjacent)
Item 23	House increasing in size
Item 24	Balloon increasing in size
Item 25	Cake decreasing by one layer
Item 26	Mouse decreasing in size
Item 27	Train with carriages, reducing by one carriage at a time
Item 28	Pizza decreasing by one slice (opposite)
Item 29	House unit increase (one block with door, add windows, add a block, add windows)
Item 30	Cupcakes with add one layer at a time in pyramid shape