

RESTORING THE LIMINAL: A CRITIQUE OF ROTMAN'S SCHEMA OF MATHEMATICAL SUBJECTIVITY

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Rotman has specified a triadic conception of the mathematical subject, drawing inspiration from Peirce's semiotics. This paper interrogates Rotman's discussion by imposing a different theoretical framework recontextualised by Dowling from Hjelmslev. The argument is that by separating out dimensions of expression and content a fully relational account of subjectivity can be achieved; one in which the necessary pedagogic paths and their difficulties in the formation of specialised subjects are specifiable with some texture. It is argued that to become subject requires moments of liminality; with all that this entails about the significance of ritualising action, and the perils of avoiding a necessary symbolic violence in education. The approach does not grant semiotic privilege to the discursive, so satisfying one of Rotman's principle concerns: the embodied nature of subjectivity. The resulting framework is complementary to Rotman. It provides a basis for considering the apprenticeship and distribution of the subject in the socio-political context of mathematics education. It seeks to do this without falling back on an ontologised notion of the value of school mathematics in-itself – one conceived somehow to lie outside of its socio-cultural-historical instantiation. A connecting thread is the recontextualisation of Hjelmslev by Deleuze and Guattari, whose work is significant to a reading of Rotman. Yet the framework also offers distance from a current trend in the mathematics education literature: one in which the names, if not the substance, of Rotman, Deleuze and Guattari are heavily recruited to substitute, for a proper relationality, an (ultimately humanistic and liberatory) material ontology.

Key Words: Deleuze and Guattari, Dowling, Hjelmslev's semiotics, liminality, recontextualisation, Rotman, Social Activity Method, mathematical subjectivity, semiotics in mathematics education research.

Abbreviations used in the text:	< >	marks words initially to be considered as expression rather than content.
	:	relation of opposition
	::	as
	~	not
	I +/-	strong or weak institutionalisation
	DS +/-	strong or weak discursive saturation
	A _i	a specialized social activity, i.
	E	expression
	C	content
	s _i	a subject position in A _i

INTRODUCTION

The objective of this paper is to consider the nature of subjectivity in mathematics education. A contrast is drawn between Brian Rotman's philosophical-semiotic approach and Paul Dowling's semiotically informed sociological work. Both are concerned with the social, and its history, and take a processual view that focuses on *activity* (what people do rather than what they are). They are both concerned with the ways in which semiosis (the cultural making of sense through the composing of signs) is recruited in the formation of *social* action. However, this paper argues that the philosophical antecedents recruited by Rotman, with a focus here particularly on his engagement with Deleuze and Guattari, are less productive for engaging with social process. This as opposed to their possible virtues in a more strictly philosophical essaying of (albeit processual and determinedly anti-Platonist) metaphysics. In particular, it is argued that some recent literature on school mathematics that draws explicitly on Rotman, and also Deleuze and Guattari, despite its overt concern with the sociality of the activity concerned, tends to collapse necessary sociological distinctions.

The stance of this paper upholds the idea that a great deal needs to be said about the ways in which social alliance is established, and then more or less secured under perpetual threat of dissolution. In terms of *subjectivity* this is a question of how people are called into acting with some solidarity as (more or less) valued *persons* who can recognise each other in common purpose, how that identification is continually (re)avowed, and the processes under which it sometimes falls apart. The (re)production of subjectivity is a central concern of sociologically informed educational research. Often this has the purpose of revealing injustice (for example, inequality of access). However, post Foucault, the research concern is less that students' imaginations are repressed by monotonous practices such as the recitation of times-tables, or unfairly treated in their mathematical interactions according to broad categories of the social such as class or gender, than to ask about the conditions of possibility of becoming subject *at all*. To seek their liberation *as such* would beg the question of how they were being constituted as if *outside* social action (Allen, 2014). It would also leave the

elementary content of school mathematics – and its haphazard history – unexamined; as if incidental to the question of the subject, looking only at how its transmission had been biased.

From a sociological regard, education is better seen as offering identities, to those who are drawn into its practices, on the basis of selected cultural arbitrariness (Bourdieu & Passeron, 1990) – those that have no foundation in nature. Their acceptance or rejection is a matter of sociological enquiry. Certainly, these practices, at any given time and place, normalise categories such as gender in particular ways. They also consecrate legitimation. To examine the formation of subjectivity, a Weberian insight into its (re)production is then indispensable: the ways in which identities *self-posit* their legitimacy or values as they participate in the practices offered to them by school.

But this process is socio-cultural: self-positing makes sense as a (re)production of practices in which sense is recognised inter-subjectively: there is no avoiding the circularity here. From a sociological regard, specialised disciplinary activities such as school mathematics offer symbolic *resources* for the interpretative process of identification/subjectivation in action – the semiotic basis of the socio-cultural – and they distribute *position* in terms of who can say, think or do what (Dowling, 1998, 2009) in the realisation of each practice.

To interrogate processes of subjectivation in education therefore requires an organisational language capable of mapping the socio-cultural *specificity* of school disciplines to explain the particular attraction of mathematics. Dowling (2013) has suggested that an appropriate way to achieve this is to adopt a fully relational approach to mapping emergent strategic action, a process he has called *social activity method*, and this is taken up in what follows.

A DEFORMANCE OF ROTMAN'S SCHEMA OF MATHEMATICAL SUBJECTIVITY

Rotman (1993, 2000, 2008, 2012) has produced a triadic schema of mathematical subjectivity for the practice of *mathematics research*. As a theory of subjectivity this will initially be taken on its own terms before considering the framework in the context of apprenticeship to *school mathematics*. Rotman's schema is explicitly by way of analogy with – not derivation from – Peirce's semiotics. In summarizing this we will already introduce our own emphasis, inevitably deforming both Rotman and Peirce but in a way which endeavours to make our principles explicit.

For Peirce, semiosis occurs as interpretants are recruited to mediate the pragmatic action of sense-making: a sign stands for something other than itself. Semiosis is thorough-going: there is no limit to the potential taking up of sign; but this is always-already through the processual and transformative effect of some *other* sign – Peirce considered here as the first great American philosopher of process metaphysics (Rescher, 1996: 14). The temporality of the subject who finds themselves as becoming-self through this taking up (for example, enabled to make judgements) is constituted through *types of sign*: index (the present startling moment), icon (remembrance of things past), and symbol (trajectory of future meaning¹).

¹ The emphasis on temporalisation (significant for the discussion of apprenticeship below) is explicit in Peirce – icons work as resemblance or recalled likeness (Peirce, 1998: 13), an index has some immediate “real connection” with the object it stands for (*ibid.*, 14) and symbols act to canalise what *will be* a regularity established by them for their object in the “indefinite future” (*ibid.*, 274).

The ontologising tendency of Peirce's typology of signs runs counter to his otherwise relational approach. But it does not threaten the fundamental insight that sense-making is (to escape Peirce's own terms but not his spirit) a *process of recontextualisation* (the unavoidable selection of interpretants as one social activity (re)produces itself through its regard on another), and that sense must therefore be grasped abductively as contexts provide new occasions for deciding meaning; their contingencies defeating pure forms of both deductive and inductive logic.

Rotman's appropriation of this is essentially twofold. He establishes a prevailing trinity, and puts emphasis on indirection. Subjectivity is then dismantled as something no longer self-possessed, autonomous and disembodied. Rotman (and the schema runs through consistently from his early to his later work) achieves this by demarcating three "material arenas of operation" of the mathematician, "a mortal Person, a virtual agent, and a semiotic Subject" (2008: 130):

The mathematical *person* subjectively situated in language is the one who imagines, makes judgments, tells stories, and has intuitions, hunches, and motives; the mathematical *agent*, imagined by the Person, is a formal construct which executes ideal actions and lacks any capacity to attach meaning to the signs which control its narratives; and mediating between them as their interface, is the mathematical *subject*, who embodies the materiality of the symbolic apparatus that writes and is written by mathematical thought. (Rotman, 2008: 130, original emphasis and capitals).

Thus the Person could be asked to count off natural numbers. As a child they may have found themselves taught a number-line with ticks for each number – a material historical and cultural semiotic Subject-apparatus to be reproduced in their counting. But the apparatus itself cannot count: the Person must project an automaton, agent of a mechanical process (Rotman, 1993: 99). The agent's actions in some sense realise the line. Such realisation then points both ways: it is in the making of the line that the sense of what it is about materialises (as something otherwise merely there as marks on a page becomes *zuhandenheit*). The Person, of course, must then ventriloquize the dumb agent, speaking out its movement from number to number, accepting praise or correction, acceptance or rejection, in their dialogic quest to become mathematical Subject – one who can follow the right line.

For Rotman, then, much of the art of doing mathematics research lies in finding an appropriate agent or "skeletal diagram" (2000: 14) – this is metaphorical territory – with which to *experiment* in the semiotic space opened up by the Subject. A key opposition is then introduced. The **Platonic Subject's** agent is presented by classical mathematics as being able to count forever, as if disembodied. Contrary to this, the **material Subject** emerges as a realisation of *virtual embodiment*: of finding appropriate gestures or points of view. If the Platonic Subject is iconoclastic, a virtually embodied Subject would recognise the "*necessity*" (Rotman, 2000: 60) of diagrammatic modes of semiosis – their rejection a denial of the *material-semiotic* constitution of formal symbols in the making of inter-subjective sense in favour of timeless Platonic certainties.

In general, the Person (always to be considered as an inter-subjective phenomenon) must be convinced by the personification of the agent enough to reactivate and thus extend the Subject "Code" of mathematics. The development of mathematical thought is then a "waking dream" (Rotman, 1993: 9): one that needs to be *persuasive* both by rigorous conformity to the Code and by the integrity of a "resemblance" of agent and Person. How well one remembers the frustration in compulsory maths classes when no such resemblance could be found.

* * *

The critical intent of Rotman's schema is well known: to challenge the granting of unlimited potential to the agent in classical mathematics. *This* agent stands accused, as it were, of pretending to be a real ghost in the machine – one capable of infinite work – confounding the Person into giving up their corporeal vitality and playfulness in exchange for timeless Platonic certainties. The classical agent is an avatar for nothing but dead labour floating in the icy waters of our current self-evidences². There is no historical spirit in these bones.

We certainly do not want to criticize either Rotman's attack on Platonism or his Peirce-inspired emphasis on indirection in the movement of sense-making. The dismantling of self-evident and substantive subjectivity in favour of a consideration of the transformations between the articulating positions in Rotman's schema produces an instructive move towards a social-semiotic relationality. Such a move is necessary but very frequently repressed, perhaps particularly so in educational research where shibboleths founded on the attainment of transferable <knowledge> predominate. The teacherly subject so often demands to exercise their care for the one-who-should-know, and the researcher would optimize such a relation – much mischief entails. The detailed composition of Rotman's target includes the *chosism* of set theoretical approaches where “objects are primary, relations between them secondary” (Rotman, 2012: 249), the psychologism of imagining an internally self-sufficient subject (*ibid.*, 2012: 250), and the absolutism (and, one might add, secular puritanism) of demanding “abstract, linear, logico-syntactical language” (*ibid.*, 2012: 252) to the *exclusion* of the non-discursive in mathematical argument (especially diagrams).

All this adds up to a philosophy of some power, one that insists on process rather than identity (of objects, of people) and the *historicity* of human action. Accepting this, our concern is to recontextualise some of Rotman's themes into a different theoretical framework; one that, in turn, recontextualises Hjelmslev's rather than Peirce's semiotics – for an exuberant introduction see Deleuze & Guattari, 1988: Plateau 3; or, more soberly, Barthes (1967); Eco (1984). Our argument is that Rotman's philosophy of subjectivity tends to collapse some distinctions made possible by a more tightly textured social-semiotic machinery. This is particularly the case, we suggest, when it comes to giving attention to issues of apprenticeship. What follows claims to provide a more productive schema of subjectivity with which to address such issues in a socio-political framework of pedagogy.

Dowling (2009, 2013) has put emphasis on a deformance of Hjelmslev's separation of planes of expression (E) and content (C) by dimensioning them according to a specifically *social* polarity:

² For his attack on Platonism Rotman draws on Bernays (1935). Bernays argues that the expression <whole number> can *neither* point to a totality of essential contents <the complete – yet somehow infinite - numbers as such> *nor* to (what would have to be an infinitely determined) essences (or just clear ideas) <intuitively given in evidence>. Hacking (2014: 224f), in an extended discussion, notes that Bernays founds his critique of Platonism (here restricted to one of the class of whole numbers) on phenomenological presuppositions – the relation between the subject and the mathematical content of their thought. Numbers eventually get too large for reflective clarity by that subject. Both Rotman and the approach adopted in this paper move beyond phenomenology to consider the socio-semiotic constitution of stabilised regions of sense.

whether or not their regularities are *specific* to the practice (such as mathematics) or available elsewhere (in other specialised practices or in the quotidian). By integrating the production of cultural signs with the social processes of action in which they are recruited, there is a significant move beyond purely philosophical or semiotic description:

Domain of Action Schema		
Expression (E)	Content (C)	
	I+	I-
I+	<i>esoteric domain</i>	<i>descriptive domain</i>
I-	<i>expressive domain</i>	<i>public domain</i>

Figure 1 *Domains of Action Schema* from Dowling (2009: 206)

As illustrated in Figure 1, establishing an analytical orthogonality produces four distinct domains in which specialised social action (with its own expression and content) can be described. One take on this schema is that it provides a map of the ways in which a subject-to-be of the specialised activity can relate to other activities in which they are already subject. To provide a path of apprenticeship for the would-be mathematical subject, a regularity of practice or institutionalisation (I) is required in which their agency, ultimately, can be realised. This involves the selection of specialised or quotidian expression from elsewhere (I-) to point to the practice's own specialised (I+) content. The *expressive domain* thus regulates the tropic composition of the activity – where expression is separated from its accustomed content in favour of new signification; for example, in the adoption of key metaphors. An established subject has competence of the discursive expression of the activity: I+ expression and I+ content – the *esoteric domain*. Someone new to the activity only has available signs interpretable on principles located elsewhere; the *public domain*. As new esoteric expression is introduced, here the specialised terms of mathematics, its unfamiliarity may be resolved by the apprentice attaching to it sense from outside the practice: the *descriptive domain*.

In what follows we want to interpret this schema as providing a map of the process of subjectivation in specialised social activities. This will be read as involving an *essential liminality*. From the point of view of subjectivity, in the expressive domain pedagogic action is *anaphoric*: reactivating sedimented subjective sense in a newly specialised context. The effect is retroactive and transformative on the symbolic resources consciously or non-consciously available to the would-be subject from their prior (social) history. This process of recontextualisation, in which the subject is wrenched from elsewhere through an appropriative transformation to a new regard, necessarily involves moments where the apprentice is neither one thing nor the other.

The make-up of an I+ activity also extends to its regard on other activities: I+ expression pointing to I- content (the descriptive domain). Here pedagogic action is *cataphoric*: esoteric expression may be introduced but not achieve I+ content for the would-be subject until after much disorientation, sweat and tears. This is the second domain of liminality. The subject must gradually resolve esoteric

expression to the sense it has that is recognised by competent members of the activity; at first, their would-be articulations are only simulacra.

The public domain too may involve the liminal³; but in a *suspension* of apprenticeship. Attracted to the subject, with so much fearful anticipation, access to the esoteric domain may nonetheless be denied: frequently in school the futility is resolved through the dull routine of establishing spurious use-values for the quotidian subject's future life (Dowling, 1998).

The schema of Figure 1 thus moves beyond Rotman precisely through the expanding power of a diagram: there could never be *three*⁴ modes of subjectivity and they are not best in a relational, anti-essentialist, framework typologised as avatars such as the Person's "agent" but, rather, as strategic *modes of action*.

* * *

Some advantages of this approach can be highlighted. First, a purely semiotic point, but one that has far-reaching implications for what one might mean by a 'relational' point of view. The transformation of Hjelmslev frees the analysis from the somewhat atomistic Saussurian sign. If one has given up with Saussure the idea of types of sign in favour of a focus on the difference that establishes relations between signs, then no residue of sense can be allowed to accrue to 'a sign'. But although signs are established as nominal in a Saussurian approach as flip sides of signifier and signified, and never contain sense in themselves, the *unit of description* is still atomized to relations of difference *between signs*. Contrary to this, in Hjelmslev's framework, the orthogonality of expression and content enables a semantics of *sign-functions* rather than signs: one that allows consideration of multiple layers of the flow of signification (Eco, 1984). Thus, if E is expression and C content, the orthogonal relation (E : C) *is always potentially recursive as new expression* ((E : C) : C) & etc.

Second, *nothing limits the text of the social to language*. Rotman's attack on alphabeticism can be read in this way – the insistence that embodiment is best described as a social-semiotic process in the constitution of the mathematical Subject. But great care is needed here. In some of the associated literature there is a rejection of semiotics *qua* linguistics, i.e. as if only concerned with abstract or formal signs in, for example, the influential work of Barad (2007:132ff.). The Platonic Subject is here dispelled *by rejecting semiosis as the basis of description* in favour of a material ontology.

³ This term is meant in its full anthropological sense, for example as in Turner (1967). For a recent review of the productivity of the term see Thomassen (2014). However, in anthropology the focus has been on subjectivation through processes of traditional formal ritual. That it is necessary to consider such matters in the secularised and more discursive arena of mathematics education has been observed, in quite general terms, by McCloskey (2014).

⁴ Peirce is nonetheless an important antecedent of Figure 1. At a very general level of description there is a correlation that runs expressive :: icon (recognitions), descriptive :: symbol (sense to come), public :: index (current presence stabilised on common sense). But this is to recontextualise from signs-in-themselves to the relationality of Hjelmslev's semiotics. In what follows the principal modalities of the *esoteric* domain are further examined and their semiotic action proves multiple as is the case at more local levels of description in each of the other domains.

But this is to confuse principles of description with the (notional) realities of action. For example, the gestures of a child in a mathematics classroom. Despite being *embodied* such action is still *semiotic* – that is, its content is a matter of interpretation. The classroom carries an insistence that that action be regulated by the esoteric domain even when its semiosis is constituted in the other domains. There is no foundation in action that is not so regulated. This is also clearly the case for the mathematics education researcher who must describe action discursively according to their principles of research not those of the classroom. It is also true for the interpretations that constitute social action more generally, for example if the classroom insistence is rejected by the pupil. From these multiple regards there is no thing, gesture or body *as such* – the description of what matters, including that of the material, concerns the process of recontextualisation, the interpretative appropriation, by which social activity is stabilised.

The placeholder ‘embodiment’, used by Rotman to mark the beginning of a semiotic investigation by reconnecting with the significance of gestures, *does no work* in such description. Specifying a distributed sense of heterogeneous embodiment interweaved with the materiality of things (*qua* actants and so forth) is all well and good in terms of breaking with substantialist categories but leaves *open* the principles by which the ‘relations’ within such a distribution can be rationalised as research.

Recent work in the philosophy of mathematics education has tended to elide this difference – see for example de Freitas and Sinclair (2014) whose “inclusive materialism” draws considerably on Barad. They provide powerful critiques of the ontologising tendency of run-away formal languages, and take care to avoid any misreading that the body is being hypostatized in their materialism. They are also clear about the need to encompass language as an artefact of the socially and materially distributed body. An integration should therefore be possible that, following Châtelet (2000), marks the *intrinsic* role of embodied movements such as gesture in mathematical innovation. But the putative *ontology* of this integration, “the power and efficacy of a body in relation to mathematics [that] must be understood as distributed across an assemblage of heterogeneous relations” as Rotman (2014: xv) puts it in his Foreword to their book, provides no access to a relational description of social activity. To put emphasis on activity from the point of view of the material object is one thing, but developing a technology of relational description quite another.

In this book, we ask: What happens to our understanding of learning when we consider the cube itself as not merely passive, but actively involved in the assembling of meaning? How might we conceptualize a material agent or recognize the way that degrees of agency saturate the situation and all its ‘actants’? ... Distributing the agency across the cube situation demands that we rethink the borders of the body, the nature of matter, and the ontology of mathematics. What if we consider the assemblage of child-cube-concept to be a *body* that emerges at that instant? (de Freitas & Sinclair, 2014: 24)

The theoretical burden that is put on “assembling” here is considerable (as we discuss further in the next section) – we might call the child-cube-concept an assemblage, and we might call it a body, but then what?

One cannot escape the conclusion that *both* idealism (that the subject is just an effect of language) *and* distributed corporealism (the idea that it is the body of humans-with-things that matters, that materializes, the subject) resist the principle of recontextualisation. The latter by appeal to the *being*

of the “relations” between Subjects-objects-ideas. And there is a reason for this: once the descriptive power of ‘top down’ approaches has been (rightly) dismantled there is still the requirement to organise a description that is capable of a tightly woven and detailed account of what happens in the social. Appeals to the body – its essential founding role and enduring vitality, its Maffesolian *puissance* – do not provide any such thing however “heterogeneous” (Bennett, 2010: 23) the result is allowed to be.

To counter this, in what follows we will make extensive use of a key distinction introduced by Dowling (1998: 138). Some sense-making is bound to the circumstances of its articulation (for example, gesture): what Dowling has labelled *low discursive saturation*, or DS-. Other sense-making is regulated by explicitly discursive principles (for example, the standard curriculum content of school mathematics): *high discursive saturation*, or DS+. Yet *both* terms index semiotic process – in their description (and, by constituting itself as research, the description here is DS+) questions of ontology simply do not arise.

Third, the method of arranging (at least two) independent oppositions regarding appropriative action orthogonally⁵ – here the cultural appropriations of expression and content – is generalisable to any aspect of the social (Dowling, 2009). The result is a description of strategic *modes* of action in the formation of subjectivity rather than avatars such as Rotman’s “agent”. What materializes *as description* – artefacts of the DS+ principles of writing research – are the modes in which that experience is institutionalised (we return to this below).

The elements of Rotman’s approach can now be briefly dismantled and reassembled. This must entail abandoning the *content* of Rotman’s terms. The starting point is a deformation of Althusser’s (1971) well known characterisation of subjectivising process. Let A_i represent institutionalisations (I+) of differently principled social activities (these are also available in the quotidian, e.g. the difficulties to be overcome in learning to tie one’s shoelaces). The person is a distribution of subjectivities able to articulate (speak, gesture, touch) strategic modes in terms of the (often tacit) recognition and realisation principles of each A_i of which they are subject, s_i . Access to the significations of some new activity A_j is only possible on the basis of metonymic continuity with those already achieved. The Subject of A_j , to be denoted S_j , – now understood as the virtuality of the new activity’s currently constituted esoteric domain – must weaken the institutionalisation (I-) of both E and C so that actual subjects, s_j , can find themselves (although still ambiguously) in terms of their antecedent s_i . This s_j can initially *act* in relation to S_j only in terms already familiar to the person whose flesh has been called into this new being (already dreamt of, but an-other not a ‘metacode’ - contrary to Rotman, 1993: 70). This in the *public* domain. The prior gestural and tropic experiences of the person are then fundamental to expressive domain apprenticeship to what will become esoteric I+ E and C in A_j .

To be productive in the arena of Education Studies a theory of subjectivity must encompass issues of how the subject comes to be. We have already seen how, for Rotman (2000: 19), the agent must in some sense “resemble” the Subject, and the Subject the Person. There must be a “narrative” of

⁵ Rotman (1993: 34) notes that the deployment of binary oppositions tends to privilege one pole over another – but it is not enough simply to speak vaguely of “overturning” the resulting hierarchy. Arranging binaries orthogonally (so that there are always *four* rather than *two* modes of semiosis) is a step towards this.

recognisability. At root, then, Rotman's notion of subjectivity relies on the same unresolved idea of resemblance as Althusser's notion of interpellation. To become subject in Rotman's schema seems to require clairvoyance: I am expected to know when *I* have been called out. To be sure, a move from A_i to A_j requires the Lacanian/Althusserian 180 degree turn necessary for interpellation into a new esoteric domain. But how does the subject come to recognise that they have been called within a specialised social activity? This cannot simply be unmediated mimeticism⁶ – to suppose that would be to claim the existence of recognition rules *prior* to the activity concerned (Heath, 1981:103). The solution is to consider pedagogy through the I- (in expression and/or content)⁷.

ASSEMBLING ROTMAN'S AGENT

As Ernest (1998) has emphasised, what is brought together to constitute social recognition in the specialism of mathematics will change over time and the practice comprises many prisms, not just, for example, its formal proofs. So one needs a placeholder for current practice that does not prejudice the question of change and which does not hypostasize the formally discursive. The obvious choices are *motley* (Wittgenstein) or *assemblage* (Deleuze and Guattari). We will consider Dowling's use of "assemblage" in this section as it permits a detailed description of issues raised by Rotman's characterisation of the relation between Subject and agent. Dowling (2013) explicitly recontextualises the term from Turnbull (2000) who in turn draws on Deleuze. However, to recontextualise means just that – here, the creative misprision of a philosophical language as a social-semiotics. The matter deserves further attention because, as discussed above, it has become key to the new materialisms of mathematics education research. Although not himself using this particular term in his account of subjectivity, Rotman too makes an explicit deformation of some related Deleuzian vocabulary in forming his semiotic materialism and we consider this in what follows.

In Education Studies the use of "assemblage" has become something of a fashion but it tends to be used in ways where it stands for itself, as it were, leaving the luggage of its genealogy (the unexpected and contingent history of its recontextualisations) behind (Buchanan, 2015). The term then arrives with no means of support – turning it into an archetypal floating signifier (see below). Often, precisely, it now stands *in place of* a properly theorised account. This is a pity, because as Buchanan discusses and particularly saliently for the theme of this paper, the original formulation (*agencement*) has the particular virtue of putting emphasis not only on the process of arrangement and thus ordering and disordering of action (disordering because an arrangement can include blockages or jams to action) but also on *what is constrained and enabled of subjectivation* within activity. The Deleuzian term makes play with the complex factors constituting (quite possibly dysfunctionally in *that* sense of complex as Buchanan (2015: 385) points out) what we can recognise as who we are.

Buchanan (2015: 390) argues that in Deleuze and Guattari the assemblage is constituted as an ongoing interplay of content (embodied action, designated things) and expression (affectual

⁶ See particularly Derrida's (1981[1972]: 36) attack on "mimetologism".

⁷ Related to this, Rotman's privileging of the pragmatic is secured on a resistance to "ideology": but then a critique of Platonism risks falling back on the assumption of a reality (or of a truth) already-known with which to dispel it.

intensities, valued symbols) in which action and value are intertwined. But this is to mark only one articulation (and here, of course, transformation) of Hjelmslev space to further a conception of processual action in which self-identification takes place. Here is another:

Modality of Esoteric Domain Strategy		
Mode of Action	Semiotic Mode	
	Discursive	Non-Discursive
Interpretative	<i>theorem</i>	<i>template</i>
Procedural	<i>procedure</i>	<i>operational matrix</i>

Figure 2 *Modality of Esoteric Domain Strategy* from Dowling (2013)

In Dowling’s schema (Figure 2) of the assemblage of the esoteric domain⁸, two dimensions are identified. Action may be either procedural or interpretative. Independent of this, the semiotic mode may be either discursive or non-discursive.

Thus, if we consider S_j to be actualized by the work of s_j then we can discern *two* modes of Rotman’s ‘agent’. For Rotman, the agent is “simply required to behave according to a prior pattern” (2000: 14). But Dowling’s schema allows us to see the resulting opposition with great clarity. The Platonic Subject’s iconoclastic nature means that *its* agent is a reduction to *procedure* in Figure 2. This Subject rejects the non-discursive as incidental to pure formality. The finitist *material* Subject, on the contrary, privileges the possibility of strategy in *non-discursive semiotic modes*, rejecting the purely discursive as a hypostatization: *its* agent proceeds in the material.

But there is a slide here: the *primary* mode of the material Subject’s agent is to act in the procedurally non-discursive; for example, the embodied relation to equipment that lends structuration to action such as the experimentation made possible by computers given so much emphasis in Rotman (2008) – what Dowling has called the *operational matrix*. *This* agent is materially distributed in the technologies to hand. It is as if modalities of *procedure* and *theorem* are then to become *subordinate* to the non-discursive to defend the “materially embodied” finitist position.

Contrary to this biasing, the schema in Figure 2 simply places emphasis on the role of all four strategic modes in the composition of a given mathematical subject. To speak of the *embodiment* of that subject then means no more than to describe the strategic modes that organise *action* in this specialised social activity. *Any given body-subject is constituted by the full relationality of the four modalities composing the assemblage of A_j .* It is one thing to insist on the significance of non-discursive semiotic modes, quite another to use this to smuggle in an assumption of their foundational and “ontological” importance.

⁸ We have only deployed the generalizing version of this schema in this paper.

PIVOTAL MODES OF EXPRESSION

We want now to consider in more detail a further element of Rotman’s description of mathematical subjectivity, his emphasis on the key role of “meta-signifiers”. These are crucial in Rotman’s account of the *development* of subjectivity so are relevant both for a discussion of mathematical subjectivity and of apprenticeship. The objective here, once more, is to recontextualise Rotman’s semiotics into social activity method. In line with the principles argued for above, rather than typologise ‘kinds of signifiers’ we will consider *modes of action* in which expression works in pivotal ways. Again, the intent is to discuss the transformation of the subject of action rather than their simple repositioning.

Figure 3 organises four different *modes of pivotal expression* discussed in the semiotic literature. The figure follows the same principles as Figures 1 and 2: that is, the independent dimensions are composed of oppositions rather than continua and relate to *action* rather than the classification of things. We will refer to schemas satisfying these conditions as pure or *p-schemas*. By schema we simply mean a diagram that relies on some orthogonality to establish oppositional relationality: thus the many familiar 2x2 spaces that appear in a wide variety of social research. Many schemas have an apparent family resemblance to p-schemas but do not satisfy these conditions; they often, for example, look to classify typologies rather than modes of action⁹. *It is on the productivity of the sense of <relationality> established in p-schemas that we want to insist.*

Modes of Pivotal Expression		
	Equilibration	
Activity Content	Stabilising	De-Stabilising
Determining	<i>anchoring</i>	<i>meta-signifying</i>
Non-Determining	<i>floating</i>	<i>emptying</i>

Figure 3 *Modes of Pivotal Expression*

In the specifically semiotic context of Figure 3, the relationality at stake is based on the presumption, as already discussed above, that it generally makes little sense to hive off ‘signifiers’ as such – atomistic components of expression such as a single word. To do so would collapse Hjelmslev space and risk introducing the Lacanian idea of signifiers somehow having a life of their own (Eco, 1984: 134f). Expression is, rather, considered here to be always-already interweaved in its institutionalised content. It is not, to take Rotman’s paradigmatic example, the *meta-signifying* expression <0> that *itself* does the work of moving things on mathematically. Rather the expression allows the pivoting of the *content* of mathematical practice in particular ways by already forming a relation with an existing esoteric domain in the development of new sign-functions.

⁹ For a detailed discussion see Whiteman & Dudley-Smith (in preparation).

What is more, salient for the discussion of corporeal realism above, we take this mode to be the force of Châtelet's (2000: 10) discussion of "hingeing points" to obtain a Deleuzian "fold". Châtelet urges that these are constituted particularly by some kinds of diagrams – the "natural accomplice[s] of thought experiments" (*ibid.* 11). They permit a de-stabilisation of (or line of flight from) discursive norms. The requirement is that the diagram must synchronize a gesture – as "a figure of pure exuberance" (*ibid.* 24) – as a touch of the mathematical imagination¹⁰.

But there are other modes. As is well known, the idea of a *point de capiton* was introduced by Lacan (1993: 268). The Lacanian precedence of signifier over signified, and the sliding of the former, suggests the importance of quilting points through which, in the contingencies of any particular text, sense is (at least momentarily) stabilised. However, it is clear that, in fact, what Lacan isolates as "signifiers" are granted recognition rules only in the institutionalisation of their *signifieds* (Eco, 1984). And here, in the weaved texture of realised expression *and* content, certainly nothing is fixed. Expression does not naturally map to some established order of sense so that the subject, as it were, really knows where they are. Rather such order is equilibrated semiotically. Slides, sometimes avalanches, of sense will take place. The Lacanian *point* can therefore be recontextualised as *mode*: the autonomy of any I+ social activity requires *anchoring* (DS+, DS-) to grant temporary stability. Determination of content to the point of self-evidence thus acts as a gravitational centre in the nebula¹¹ of semiosis.

To renew the emphasis introduced above, such *anchoring does not have to be established through words*: for example, in the maths classroom some teachers are adept at establishing a specific rhythm in the recitation of times-tables (Leroi-Gourhan, 1993). Thus memorised in the descriptive domain (the child has the esoteric expression but no sense of its signifieds), it may be some time before the child adds content to their participation in the esoteric domain of multiplication. And other senses such as taste and smell may anchor a memory that is always-already collective (for a powerful evocation see Serres, 2008: 157).

Signifying action can also equilibrate through expression involved in the *non*-determination of content; the floating signifiers, introduced by Levi-Strauss (1987). Here expression (DS- or DS+) is cut out from the fabric of discussion to stand as something in-itself; it then has no content. Here the *absence of content itself serves to stabilise the esoteric domain* by reducing it to an institutionalisation requiring little work for the would-be subject. This *floating* strategy plays with great force in the marketing required to achieve popularisation (now seemingly the requirement of so many academic journals – for example, dropping in the name of a famous author as an alibi for having accomplished theoretical work). Beyond perfectly useful verbal memes that stabilise phatic relations, there are, unfortunately, no shortage of both quotidian and specialised examples of this mode. In the constellation of the authenticity (Adorno, 1973) of school mathematics the current UK floating jargon seems to be composed of <standards>, <global competitiveness>, <productivity> and a notion of <literacy> reduced to use value.

¹⁰ One exemplary means to such "disorientation" is the orthogonality of binary oppositions captured in Grassmann's schema (Châtelet, 2000: 111), an early (1844) antecedent of the technology of p-schemas deployed in this paper.

¹¹ The metaphor is Lotman's (2001).

This leaves one pivotal textual strategy in Figure 2 as yet unexplored: the case where disequilibrating action utilises expression that is *not* determined in content and thus cannot itself move the structuration of the esoteric domain forwards. Here expression is *emptying* of the differential structuration of the esoteric domain. The idea is recontextualised from Laclau’s notion of an “empty signifier ... something which points, from within the process of signification, to the discursive presence of its own limits” (1996: 36). This is pivotal expression that indicates the determinate *indetermination* of constituted sense – the impossibility of an ultimate ground.

This schema can now shed some light on Rotman’s (2000) project of a semiotics of mathematics. In Rotman’s (1987) discussion of the introduction of expression for zero in the number system rather than being *floating* or *anchoring* the signifier <0> was initially *de-stabilising*. It clearly cannot denote an act of counting, and yet it has determinate *content* only within the connotations of a *reconfigured* number system. The introduction of the expression destabilises the established sense of number in the movement to a new esoteric domain. In terms of the Domains of Action Schema introduced above (Figure 1) the situation is given by Figure 3 below; see also Burke (2016) and Dowling (2009: 236).

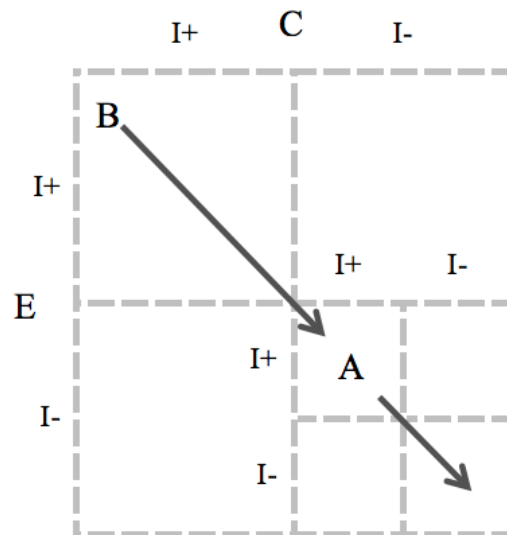


Figure 4: Invention Schema (from Burke, 2016)

We have labelled Figure 4 as a schema describing *invention*. This can be thought of as either the work of developing a specialised activity such as mathematics, or, as is the focus here, the process in which a social activity is communicated to those who would acquire its esoteric domain¹². A given (relatively synchronised) state of an activity allows the formation of a regard from point A in the esoteric domain. It is through making that domain *public* that development proceeds: organised by a new esoteric domain with a regard from point B. Thus what was esoteric at A becomes self-evident (or literalised) from B.

A number of features of this schema deserve emphasis. First (signified by the dotted lines) no social activity is closed to its outside. Apprentices must be found and they will need expressive domain

¹² See Ernest (1998: 75) on Wittgenstein’s portrayal of mathematicians as inventors. The schema may, we hope, recall a *mise-en-abyme* effect.

entry points. Second, the space is fundamentally *dialogic*¹³. An s_j has no simultaneity with another. Nor can they (rationally) assume to be S_j . The process of maintaining A_j requires the antagonism of inter-action. Initially I- innovations must struggle to find recognition to count as I+ invention. Third, the schema does not privilege the discursive (let alone formal discursive action such as proof). Fourth, the schema describes the process of change: there is no implication of an origin, or founding A. There is simply *always-already* a process of recontextualising what is esoteric from A to what is to become B. The arrows in Figure 3 should be thought of as a *flow*.

Thus, to interrogate one of Rotman's key examples, the meta-signifying action that develops the mathematics of the number-line *reassembles* the esoteric domain *and thus what is constituted in the regard of mathematics as identifiable*. The *autonomy* (Rotman, 1987: 28) of signifying action generates new objects that are alien to those denoted in the public domain. Number as graspable counting (deictic, indexical) must then give way when the system is reorganized around the new semiotics of zero: precisely not countable.

To make zero the origin of number is to claim for all numbers, including the unit, the status of free, unreferenced signs. Not signs *of something*, not *arithmoi*, certainly not real collections, and not abstractions of 'units' considered somehow as external and prior to numbers, but signs produced by and within arithmetical notation. (Rotman, 1987: 29)

In generalising mode, the subject of the new esoteric domain (at B) drives out the old counting subject – and even in localising mode (for example, numerical instantiations of algebraic equations) cannot return to the old subjectivity (Rotman, 1987: 32). In some sense, that subjectivity (its history) no longer exists but has been transformed. Rotman describes how the development of such semiotic autonomy through an inaugural meta-signification has enabled a plethora of new subjects since the Renaissance. To add two salient examples from the C20th, one thinks particularly of Malevitch and Schoenberg: the *absence* of representation and tonality then allows a *retroactive* regard on earlier painting or music (for example, one can then hear considerable dissolution of tonality in, say, <Brahms>).

THE DOUBLE ARTICULATION OF THE MATHEMATICAL SUBJECT

In his account of the material Subject, Rotman (2000, 2008) draws explicit inspiration from a (creative) misreading of Deleuze & Guattari. It is to this that we now want to turn. However, there is one feature of their philosophy that Rotman does not recruit in his own work, and this absence, we argue, forecloses some key aspects of subjectivity when considered in its social context.

In both the apparatus presented here and for Deleuze and Guattari, the flow of semiotic process is *doubly articulated* (1988: ch.4, drawing on Hjelmslev) and multiply so at different points in the hierarchy of sense making. As Badir (2006) discusses, by putting attention on oppositions such as expression : content, denotative : ~denotative, process : system, and so forth, Hjelmslev's schematisation of sense-making puts attention on its *dynamic* movement. These productive tensions

¹³ The dialogic nature of mathematical activity has been given emphasis by Ernest (1998: 162ff.) who points out that this is often hidden "under its monological appearance" (*ibid.* 173). Rotman (2000: 7ff.) also puts emphasis on this. A striking aspect of Villani's (2015) recent account of practising mathematics is the emphasis on *frequently de-stabilising* conversational immersion (by email, at institutes of advanced study, and with the subjects of past practice).

(one can hardly say ‘dialectic’ in this context) are echoed in Deleuze and Guattari’s many oppositions such as that between the molar and the molecular and their many re-workings (each involving, no doubt, a slight displacement). But, as introduced above, in moving to social description some transformation of both Hjelt’s semiotics and Deleuze and Guattari’s philosophy becomes necessary.

First, in recontextualising this principle in terms of a specifically *social*-semiotics the orthogonality of expression and content has been reconfigured in terms of institutionalisation (Figure 1).

Second, a subject of school mathematics may achieve context dependent (DS-) or context independent (DS+) content in their specialised social action. There is then a *single* surface of description encompassing the material-discursive – if this is a “body” in de Freitas and Sinclair’s sense (*op. cit.*) then it is certainly one “without organs”. The child in the mathematics classroom is situated in the affectual-movement-embodied DS-, *and* in the alienated-abstract of the DS+ (at least if they are being taught anything). Such double articulation then doubles up over and over again: the child subjectivated in disciplinary or non-disciplinary action, and in I+ or I- disciplinary mathematical action, in discursive or non-discursive modes of the esoteric domain. Recovering the gestural, the embodied process of thinking, is, then, certainly a necessary move to counter mind-body, subject-object dualisms: but we think this is better read as a socio-semiosis not an alternative ontology.

For Rotman (1993: 7-8) the official “code” of classical mathematics is idealistic precisely in eviscerating any overt signifier of the indexical in the resulting Subject. In Deleuze and Guattari (1988) the surface of description intertwines three key oppositions; thus smooth : striated :: deterritorialisation : territorialisation :: plane of consistency : plane of organisation¹⁴. Our reading is that any process philosophy, in so far as it wants to engage with readers, has to articulate the diachronic by writing itself in the synchronic. Much then depends on what the writing sets out to do. In his recontextualisation of Deleuze and Guattari, Rotman tends to privilege the first against the second term: thus the affectual, non-linear, and distributed self is opposed to the discursive, linear, centred self. The emphasis extends the opposition smooth : striated to :: the numbering number : the numbered number (Rotman, 2000: 139). The former provides the war machine Rotman needs to counter the infinitist transcendentalist state-supporting disembodied (ac)counting synchronisations of striated mathematics. For example, the *expression* of integer counting may not change in the move to the smooth (Rotman, 2000: 150); but true content must be performed by a non-transcendental and embodied rather than ghostly agent. This duality of counting content is the *anchoring* strategy of Rotman’s materialism:

Evidently, State and nomad counting give rise, when arbitrarily prolonged, to two kinds of infinite progressions: the infinity of classical mathematics synonymous with the endlessness of the so-called natural numbers; and unbounded iteration [Rotman’s neologism], the continued movement along a line drawn from point to point by this very movement of the Nomad. The first, top-down, from a transcendently external viewpoint; the second, bottom-up, from an always prior and intrinsic materiality. (Rotman, 2000: 147)

All this is a necessary counter, no doubt, to non-processual thinking. But when writing about a social activity such as mathematics this risks a category mistake. To put the matter bluntly, thinking

¹⁴ For the latter see Deleuze & Guattari (1988: 266).

about the plane of consistency demands a metaphysics, description of the plane of human organisation demands a sociology. It is not enough simply to identify the *existence* of the state apparatuses. As should perhaps be more often noticed in the burgeoning Deleuzian literature, Deleuze himself often recruits Foucault when discussing these (see especially, and in an overtly Hjelmslevian context, Deleuze & Guattari, 1988: 66ff.). As an analytic move in which to write about social formations such as mathematics teaching, privileging the metaphysics of embodied affects is not *in itself* productive. This is not at all to legitimise control forms of organisation or to downplay the DS- in the process of institutionalisation.

Rather, one might see subjectivity as an emergence from a process of capture (Deleuze & Guattari, 1988: 40) that is best thought of as operating according to principles of appropriative transformation in the formation of alliance (and thus opposition). Such principles produce *organisation* that the technology of p-schemas can map as strategic modes in the socio-cultural. It is thus that description of the process of capture (the double articulation that composes the striated, the territorialised), with its associated construction of a specific socio-cultural milieu, becomes possible

Third, p-schemas for socio-cultural action describe an <intentionality> that is also doubly articulated¹⁵. Here the ‘aboutness’ of <intentionality> becomes the meta-signifying *strategic mode*, displacing the standard phenomenological (as well as quotidian) content of this expression.

This has far reaching consequences. A good way to see this is that it is one of the few points of contact between Deleuze’s and Wittgenstein’s philosophies. In his interpretation of Wittgenstein’s social philosophy of mathematics, Ernest makes the essential point that a language game “woven into our forms of life” (1998: 72, see also 88) is not a conventionalism. The point is the same as for those who would read <intentionality> with quotidian content as autonomous subjective decision, as ‘intent’. But one needs to be precise about the texture of what is replacing such atomism. Here Dowling’s technology reformulates the phenomenological notion of strategy to one constrained and enabled by institutionalisation upon which the subject is emergent, a subject whose actions within Merleau-Ponty’s (1967:168) “lines of force” are reorganised and transformed by p-schemas¹⁶.

Yet, and only at first glance paradoxically, the referent ‘the body’ is then as redundant as the ‘autonomous self’ in description (not of course in its practices). It, together with the multiplicity of things and mathematical concepts with which it is “assembled”, properly disappears *as such* precisely in the articulation of its *modes of action*. The great challenge is then to describe such modes without falling back on *anchoring* or *floating* pivotal expression (however distributed). The technology of p-schemas is one way to achieve this. As we will now discuss, an alienating move of this kind is necessary – one generally not recognised in the mathematics education literature. The

¹⁵ See Dowling (2009, 2013) and, for the DS-, Dudley-Smith (2015a) for a large number of such schemas.

¹⁶ There is a misrecognition in de Freitas and Sinclair’s (2014: 23f) discussion of Merleau-Ponty (and his followers) in which it is asserted that phenomenology puts “over-emphasis on human intentionality”. But a particular version of philosophical intentionality (one precisely opposed to psychologistic languages of ‘intent’) is the *ground* of Merleau-Ponty’s philosophy, one that stands *counter to* quotidian notions of intention and paves the way to (if not arriving at) the social-semiotic interpretation of <strategy> used in this paper. Through a deformation of the quotidian (to accommodate better the process of inter-subjectivity) it forms the foundation for a discussion of relationality *qua* (for example) lines of force – one that eviscerates the very subject-object dichotomy that de Freitas and Sinclair claim to have as their own target. Of course a materialist philosophy must deny the Husserlian lineage that identifies with precision the poverty of the “mathematization of nature” (Woodruff Smith, 1995: 325) where, in some way, mathematical concepts are mythologized as “partaking of the physical world” (de Freitas and Sinclair, 2014: 6).

corporeal turn tends to resist such alienation in its pursuit of the ontology of the body: but description cannot recover any such thing.

Rotman (2000: 111) indexes Varela, Thompson and Rosch's (1993) justly celebrated mind-embodied-in-action thesis. He takes from this the argument that embodied enaction is best read as fundamentally *localising* to the situated context of that action *from which* generalising strategies may be possible (see also Rotman, 1993: 147). There is so frequently in mathematics a loss of deixis. This takes the form, to put it in the organisational language developed here, of the DS+ supplanting the DS- (for example through iconoclasm). To combat this, Rotman puts emphasis on the ways in which "code" (the purity of symbolic argument) and "meta-code" (all that is impure but involved in the production of sense) are intertwined – to cut away the meta-code would be to deprive the *practice* of mathematics of its source of energy (Rotman, 1993: 70). Such a deprivation leads to the Subject of mathematics becoming idealised as an ahistorical restriction of the Person, requiring the avatar of the Agent to materialise operations.

All well and good, and one could add Varela's emphasis on autonomy and self-reference as institutionalised socio-semiotic systems self-organise in response to the contingencies they encounter in historical process. Contrary to this good sense, much of mathematics does indeed impose a "top down" approach; as if mathematical concepts stand outside history and, as it were, have a prior organisation that is only to be discovered. Clearly this is the case for Platonist interpretations of mathematics. It is in reaction against such interpretations that the corporeal turn in mathematics education, often drawing on Rotman, has signed its warrant.

However, it is noticeable that Rotman does not engage with Rosch's semiotics (the significance of prototypes) in terms of organising his description, so the nature of autopoietic action in mathematics goes unexamined¹⁷. The problem with this disregard is that it risks 'the embodied mind' becoming something as ontologising as the (other) worlds Roman rejects. Rotman's own approach is to resist this by putting emphasis on the dream-like quality of mathematics as "inscriptional fantasy" (2000: 122). This is to counter Platonism by limiting description to improvisation – one dream (always so very personal to the one who dreams) recoding another. Yet, a *rationalisation*, a DS+ regard on a DS- practice in the sense developed by Dowling (2013), and one shareable as a matter of social description, whilst *organised from the regard of a specialised esoteric domain*, is not in itself either 'top down' or Platonist. For example, it seems entirely possible to describe deixic semiotic systems in the generalising language of p-schemas. Intertwining of the DS+ and DS- yes; hypostatization of bodily knowledge no.

This issue has been a familiar story in both philosophy and sociology. For an exceptionally clear philosophical critique of privileging the body as ground see Hintikka (1974: 80ff). And (as also discussed in Dudley-Smith, 2015a), the move to establish "relational" sociology by cutting through high level abstractions, whilst absolutely necessary, is highly unstable. One central strand of the approach is the "embodied turn" in social theory – now echoed in mathematics education research. Yet 'the body' does not provide anything other than a placeholder or it risks becoming a first cause. The focus on embodied interaction places attention laudably on the nitty-gritty of the socio-cultural; but no amount of observations of, or participation in, fleshwork will generate an academic

¹⁷ For a powerful semiotic interpretation of Rosch descended from Eco's semiotics, see Violi (2001).

description however close to, or empathetic with, the researcher is to the embodiments concerned. Perhaps it is for this reason that those who take this approach spend so much time looking for ontological justification – for example in the philosophy of critical realism (Shilling, 2005), or in the “agential realist ontology” of Barad (2008: 137).

DISTRIBUTED MATHEMATICAL SUBJECTIVITY

Rotman (2008) has put emphasis in his more recent work on the *distributed* nature of subjectivity. His focus is on the new embodiments enabled by technology – ones where a profusion of images, connectivity and hypertext enable those subjects to escape the hegemony and monovalence of linear alphabetic inscription. Rotman’s requirement of finitism in mathematics is met through the insistence that the Agent instantiated by the mathematical Subject is just such an embodiment. The Subject can then realise the multiplicity of the virtual by adopting discrete, computerised, and experimental method.

The advent of an electronically modified Agent destabilizes the triadic assemblage of actors that has held the pure ‘mathematician’ in place since classical times. In the wake of this empirical – that is, impure – Agent comes a digitally refigured Person who can intuit, imagine and recognize the new sorts of mathematical objects, simulations, and iterations, delivered by computation; and at the same time a digitally refigured Subject is required with the language and writing system – the appropriate mathematical ‘software’ – necessary to mediate the traffic between Person and Agent. (Rotman, 2008: 67)

The Platonic (and, one notes, the Peircean triadic) imaginary gives way to a flesh whose cybernetic proprioception realises true rather than phantasmic imagination.

As Rotman acknowledges, undoubtedly there is potential here for distraction, imaginary identifications and fragmentation – this is much discussed and is familiar to every teacher in an ICT enabled and mobile-phone permitted school setting. But the fundamental message is liberatory: freed from Platonic conceit, the Person can develop as an embodied emergence of new Subjects.

However, the distinction between the ideal and embodied (of course conceived of as an attack on dualism) carries a heavy burden here. It is as if the DS+ of continuity, explicit principle and formalised mathematics *stands for* the Platonic. In opposition to this the contextualised DS- of limited energy where diagrams rule. Yet the DS+ cannot itself be blamed for Platonism - and indeed, of course, Rotman’s books are all *argued* in a recognisably linear and alphabetic presentation. *It is precisely because the DS+ has reflexive properties of meta-signification (Table 2) that the Platonistic tendencies of hypostatized conceptual entities are visible at all*¹⁸. In the apprenticeship of mathematical (or any other specialised discursive practice) an alienation of

¹⁸ This is of particular concern with respect to Rotman’s critical discussion of linguistics as denying the significance of embodiment. Jakobson and Halle (1956, discussed by Rotman, 2008: 86) provided the crucial development of a model of parallel rather than serial signification, and Benveniste (Rotman, 2008: 108) a dismantling of the grammatical self-evidence of the subject – what were these if not moves towards the social semiotics to which Rotman aspires? If language has always-already been appropriated for purposes of social control (who could deny this?) then it is *also* a resource by which such control can be challenged. The nominalising moves of linguistics (and of structuralist semiotics in the 1960s) had a critical intent very similar to that expressed by Rotman. The situation is even more embarrassing with respect to Derrida: Rotman (2008: 124) cites the latter’s *il n’y a pas de hors-texte* as a “body-annihilating” move. But Derrida nowhere proposes an ontology of text: “To allege that there is no absolute outside of the text is not to postulate some ideal immanence, the incessant reconstitution of writing’s relation to itself.” (Derrida, 1981 [1972]: 35).

quotidian notions of, for example, counting is *necessary* – as Rotman (1987) describes so well – but at least in part this *must* take the form of a discursive principling (or rationalisation). As discussed above, there is no necessary reason why DS+ discursive principling should exclude the productivity of the DS- even if, in practice, it often does.

It is then clear that the call to embodiment risks hypostatizing the body *and thus foreclosing the interplay of DS+ and DS- modes*. A socio-semiotic framework *requires* that a description of embodied work is mediated rather than standing for-itself. There is considerable irony here: although of contrary intent, privileging a return to the body in school contexts will suggest to some the kind of Piagetian liberalism that Rotman (1977) so forcibly rejects.

It seems essential, therefore, to recontextualise the idea of distributed subjectivity without losing sight of the principles (*both* DS- and DS+) that grant regularity to specialised social activities such as mathematics. We will discuss this only with regard to three aspects of pedagogy made visible by the domains of action schema.

A. Pedagogic Distribution

Figure 5 returns to the Domain of Action Schema discussed above. Rotman’s Person, the inhabitant of the meta-code, occupies the public domain. *The other domains are specialised by the code*: the expressive domain, in particular, recruits metaphors from elsewhere but the selection principles are determined by the Subject in the pedagogy of its subjects *in the esoteric domain*. Only in the public domain is the subject not alienated from their accustomed sense, yet content here too is *selected*. Thus the Person may find themselves stranded if routes of apprenticeship through the expressive and descriptive domains are not provided (Dowling, 1998).

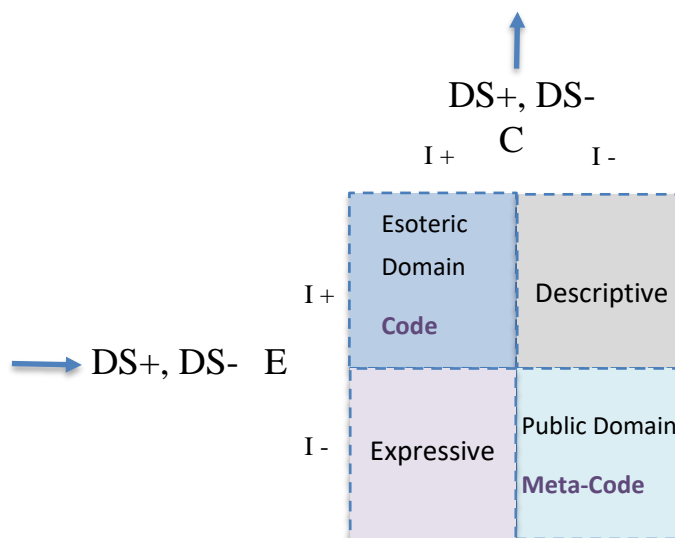


Figure 5: Distribution of Subjectivity - SAM and Rotman Compared

Figure 5, to be considered as a flow of social-semiosis, suggests that in social-semiotic terms, Rotman has reduced the distribution of subjectivity to the public and esoteric domains. The anaphoric subject and the cataphoric subject described above – and an essential liminality occasioned by the symbolic violence necessary in any institutionalisation – are left unrecognised. In Rotman’s framework there is just the specialised esoteric domain subject and the common subject

of the public domain: knowledge versus ignorance. At some point the student learns to articulate within the esoteric domain but this involves a jump, a mythologised mimesis free of the need to follow a path of semiotic transformation.

It deserves note that in professional mathematical activity esoteric domain activity cannot be totalised as the practice is *open* in the precise sense that new *meta-signifying moves are allowed to happen*. In school mathematics – as in school disciplines generally – the esoteric domain tends to be closed for the purposes of curriculum and control. Strangely, and unethically, school mathematics offers a subjectivity that can aspire to be the Subject of the activity: in mathematical practice such an assumption would rightly be regarded as pure hubris.

B. Distribution of Contestation

Subjectivity may also be distributed in a state of *ambivalence*. We mean this in Bauman's (1993) sense of an ambivalence of classification. Figure 6 (in homage to Althusser) formalises this as a 180° anti-clockwise rotation of the domains of action schema.

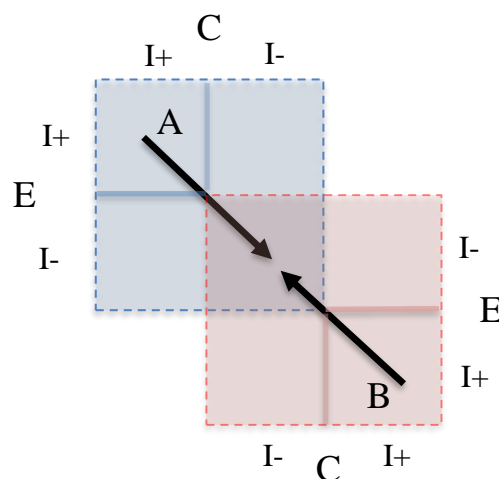


Figure 6: Contestation Schema

There has long been an undercurrent in mathematics education research worried about the status of school mathematics versus the mathematics that people can be observed to use in practice. Perhaps school mathematics trains people in principles that they will *not* use as successful practitioners. The *ur-text* is of course Lave (1988), but one could equally recontextualise Rotman's own example: from the DS- haptic organisation of counting when utilising the procedural non-discursive of an abacus to the DS+ organisation of mathematical number (Rotman, 1987: 13). The schema in Figure 6 is reproduced from Dudley-Smith (2015a).

Even where it escapes monologism, pedagogy is not a particularly intrinsically friendly dialogue. The symbolic violence required to get the subject to switch esoteric domains from A to B might be quite considerable – much depends on the prior dispositions of the would-be subject of B. Only the public domain can be a matter of pure seduction. Liminality is necessary because the esoteric domain of B cannot be instantly acquired. The drama is that there is every opportunity for pedagogic action to fail, for the self-positing investments to be lost.

C. Distribution of Competence

A further question concerns the distribution of subjectivity *within* the esoteric domain. From such a position the subject can form a regard on the other domains of *this* practice as well as on *other* practices. We can identify two opposed basic processes of subjectivity in the esoteric domain of social activity of *school* mathematics (to be sure the second of these would not be recognized in mathematics research).

First, a subject able to command an admixture of the different modalities of the esoteric domain. Although not working in the same theoretical framework, Norma Presmeg has pinpointed with precision the requirement: in her terms, a “facility” to “[convert] fluently between graphical and algebraic registers” (2008: 110). We can recontextualise this as the achievement of a competence to flexibly use the strategies of the esoteric domain assemblage. As Presmeg recounts, students often get stuck in a single modality, unable to exploit the synergies of mixing modes. And this is the case *even for the closed practices* – those that forbid new meta-signification – *of school mathematics*¹⁹.

Second, a subject only provided with a limited repertoire of modalities. In a school context, students might seek, and teachers agree to provide, ‘how to do that’ – where ‘that’ is examined. The apparatus of league tables and international comparisons tends to reinforce such subjectivity.

CONCLUSION: PEDAGOGIC AND SOCIAL CONSEQUENCES

Roman’s materialism would be uninteresting if it was overtly a call to the body as such:

... the power and efficacy of a body in relation to mathematics must be understood as distributed across an assemblage of heterogeneous relations, a posthumanist understanding not to be identified with the capacity that is ‘localized in a human body or in a collective produced (only) by human efforts’.
(Rotman, 2014: xv)

Nonetheless, the argument of this article has been that *in itself* a philosophical materialism of this kind is of little use when attempting a description of social process. In the end, the interrogation boils down to what one might mean by “relations” when agreed (as both the approaches considered here are) that the social is profoundly heterogeneous and that humanist models of thought have had their day.

Writing within the organisational language of social activity method, we have argued that the relations that specify *action* (that is, to be clear, *socially embodied action*; but that is ultimately to constitute a pleonasm) are best considered as strategic-modes through the descriptive technology of p-schemas elaborated above. *This* is how one might come to describe subjectivity as being composed and “distributed”. One hopes that putting things this way marks Rotman’s “posthuman” uncompromisingly – this seems necessary because, despite Rotman’s warning, the experience “localized in a human body” is a deeply held humanist conviction: as such, the signifier <body> will always be likely to evoke what Rotman is rightly seeking to overcome.

¹⁹ A generalisation of this first notion of competence would point to the *analogical method* involved. Rotman (1987) illustrates such a method in his discussion of the recruitment of the expression <zero> across contexts, each producing analogical content – in mathematics, money, perspective drawing, and Shakespeare.

A few aspects of the discussion can be highlighted in conclusion. First, the vexed question of epistemology and ontology when regarding an activity from a socio-semiotic theoretical framework. Second, a summary of the “theoretical networking” relations discussed above between Rotman and the principles articulated here. Third, the pedagogic consequences of p-schemas given the modes of subjectivity they identify.

1. *Ontology and Epistemology.* This paper has looked at the constitution of mathematical subjectivity. Subjectivity has been described by the theoretical framework as 1. The coordination of strategic action; where such action has the specific sense given by p-schemas *and no other sense* and 2. The self-positing and legitimation of that action under the regulative gaze of the (virtual) Subject of that action. To be sure, other statements may be warranted by other frameworks; but there is a necessary limitation in *any* such framework. This description has had nothing to say regarding either ontology (the ‘is-ness’ of the cultural objects constituted in mathematical action) or epistemology (the modality of truth, for example criteria for successful proof) of such objects. It is the case that a socio-semiotic regard can point to the strategies that claim ontological or epistemological warrant for the activity – for example, in the case of school mathematics the claim that the discipline has a use-value in *differently principled social activities* such as shopping. The resulting mythologizing (Dowling, 1998) would seem to be generally denied. However, the approach has nothing to say about questions of the reality of mathematical objects (whether formalist, intuitionist, Platonist, or socially constructed) – the principles of description have no purchase on them. This is not even an agnosticism: there is simply no contact with such questions²⁰. They may be of interest elsewhere, but not here.

It is therefore of some concern that the new materialism of mathematics education puts so much emphasis on ontology. To deploy Rotman’s own argument, *gesture* is, in social activity, *a mediated action*. If gesture matters in the mathematics classroom it is not its being that is at stake but its recontextualisation: it is always-already a semiotic phenomenon. To describe it in empirical settings therefore requires a technology oriented to semiotic process not philosophical ontology.

2. *Theoretical Networking.* There has been considerable recent discussion of the potential for ‘theoretical networking’ between theories of mathematics education – see Radford (2008, 2014); Prediger & Bikner-Ahsbals (2014); Dudley-Smith (2015b) – and where the word theory itself is contested.

Figure 7 draws together some points of contact between the two organisational languages presented here: However, and as formalized in Dudley-Smith (2015b), the ‘connectivity’ here is semiotically

²⁰ *From a semiotic regard*, the one upheld here and by Rotman, there cannot be unmediated access to objects ‘in themselves’ (either as real or ideal). Rotman (2000: 31) deploys this as one argument against Platonism, repeated in Rotman (2014: xiii). But such arguments can always be thrown back (for example, as in Bhaskar’s critical realism) as only succeeding against *naïve* ontologies i.e. as projecting a denotative notion of the real (or ideal) that the realist or idealist rejects. This issue is therefore not decidable from a semiotic framework (or perhaps from any other).

understood as the transformative, and necessarily deformative, action of one organised regard upon another.

Rotman	Social Activity Method
Interweaving of Code and Meta-Code (1993: 77). The Subject as restricted Person	Recontextualisation by esoteric domain of public, expressive and descriptive domains. The subject as one position within the esoteric Subject
Person	Embodied realisation of contested public domains
Subject – classical reduction Subject – re-embodied	Idealized purity of thought Modalities of esoteric domain assemblage, restoration of the non-discursive
Agent – classical reduction, “mechanical” (1993: 91) dis-embodied and “infinitary” (1993: 93) Agent – re-embodied, finitist	Procedurally discursive action (modality of the assemblage) DS- in DS+, interplay of the modalities of the esoteric domain assemblage
Meta-signifier	Pivotal expression. Invention diagram – esoteric content pushed to public domain status
System	Institutionalisation – the regularity of who can say, think or do what

Figure 7: Subjectivity in Rotman and in SAM

3. *Failed Pedagogic Action.* The schemas presented above suggest a number of pedagogic traps for the would-be subject of school mathematics.

First, I+ principles may be withheld from those regarded as ‘less able’ on an alibi of providing use-values (Dowling, 2013). These subjects remain doomed to the public domain.

Second, routes through the expressive and descriptive domain may themselves entrap the apprentice. Particularly in school contexts, the metaphors offered to pupils in the expressive domain, sufficient for examination success at a given stage, may prove unhelpful in their longer term apprenticeship. In the descriptive domain the provision of I+ expression, necessary to apprentice the subject into the esoteric vocabulary and grammar of the activity, may become reified in terms of its initially necessarily I- content. This is particularly true when school subjects become commodified for examination purposes where recitation of esoteric expression is deemed adequate for a pass, or even a high grade.

Third, even within the esoteric domain, procedural rather than interpretative modes may be hypostatized, and/or non-discursive modes marked as ‘not proper maths’ despite convincing

accounts that they are intrinsic to the formation of the Subject (see especially Châtelet, 2000, 2006 and Kvasz, 2008).

Fourth, and of most general significance, the formation of the esoteric subject may insinuate an image of thought. Here, despite our critique above, we stand with Rotman. School risks contributing to the naturalisation of the sense of space-time for the one who counts: linearity, unboundedness, and sovereign disembodied intelligence on the misrecognition of an ahistorical and eternal Subject (Rotman 1993: 74) standing outside of the contingencies of the socio-semiotic and of the inter-subjective work that forges our potential futures and freedom. In school this is accomplished through a taboo on open disciplines. Meta-signifying and emptying expression are excommunicated in terms of floating and fixing forms of equilibration.

In the UK school system (itself always-already on the point of breakdown to the extent of manifest dysfunctionality – so many suddenly ‘failing’ schools) mathematics education has achieved disciplinary hegemony. It achieves this, first, by *excluding*. Esoteric school mathematics, by claiming an objective measure of intelligence, and thus stupidity, identifies an elite. Second, by *including*. School mathematics *qua* a literacy conflated with numeracy, made compulsory for all, is seen as meeting the vital need, as the press and government see it, of not failing in international league tables, or, as the current Education Secretary has said, of “winning the global race”.

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