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Doctoral Thesis

Essays in Sovereign Debt and Default

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I, Ming Qiu, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Abstract

This thesis consists of three papers on sovereign debt and default. Chapter 1 studies public provision of liquidity in a model of public and private linkages that allows partial sovereign default. Entrepreneurs use their holdings of domestic bonds as liquidity stock to carry out investment and the bond default risk arises from the government's trade-off between private consumption and public expenditure. The model features a feedback loop between aggregate investment and debt sustainability. An adverse productivity shock reduces the government's tax base and hence its ability to repay. The initial decrease in bond price shrinks the economy's liquidity stock and leads to more projects being liquidated. Lower tax base, in turn, reduces bond price further.

Chapter 2 analyses the impact of disaster risk on risk premium of debt issued by emerging economies. I distinguish between "natural" and "economic" disasters based on the output dynamics prior to disaster occurrence. My empirical estimation results show that a sample of thirteen emerging countries are subject to economic disasters and the probabilities of disaster occurrence in those economies are positively correlated with their interest spreads. This is consistent with the theoretical prediction of a model constructed to compare economies with natural and economic disaster risks.

Chapter 3 relaxes an assumption made in previous works on optimal policy that the government has perfect knowledge of states in the economy and considers a model of optimal provision of liquidity when the government only has partial information. I present solutions to the full information and partial information cases.

Impact Statement

This thesis presents results that are of academic interests and have policy implications. Chapter one, *Public Provision of Liquidity with Default Risk*, is motivated by the prominent feature in the ongoing European debt crisis that the co-movement of corporate and sovereign CDS spreads in many peripheral countries. It draws questions as to whether there is a "vicious circle" in which weakening solvency of the governments would jeopardise domestic firms' balance sheets and firm distress would imperil public debt sustainability. My model formulates a two-way link between public and private sectors and shows that this link makes an economy susceptible to changes in exogenous shock. A change in the productivity shock affects the bond price more than proportionately. This result addresses the need for government policies to break the "vicious circle" which has been at the centre of the recent debate of banking union creation by the European Union.

In Chapter two, *Sovereign Debt and Disaster Risk in Emerging Economies*, by distinguishing natural and economic disasters, I show that risk premium of debt issued by emerging economies is sensitive to the type of disaster risk an economy is subject to. The paper also discussed two *ex post* state contingency measures. It is shown that welfare loss following default is reduced as creditors regain a fraction of the defaulted debt and debtors obtain access to the credit market through renegotiation over debt reduction. Upon the occurrence of a natural disaster, debt extension offers an effective means to improve debtors' welfare and since it is only available to debtors with no default history, it disciplines debt repayment for countries prone to disasters.

The third chapter, *Optimal Liquidity Provision with Partial Information*, addresses the issue of partial information governments often face in practice when setting optimal policies, but rarely discussed in previous works. The model shows that with partial information, a non-standard optimisation problem arises and the model has the scope for welfare comparison between full and partial information. It enables me to answer the question of whether lower availability of information increases welfare or not.

Acknowledgements

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Introduction

This thesis consists of three chapters on sovereign debt and default. The opening chapter, *Public Provision of Liquidity with Default Risk*, is the strongest chapter among all. The co-movement of sovereign default risk and private insolvency risk have been observed in many crisis episodes and it is particularly potent in the recent European crisis. I study public provision of liquidity in a model of public and private linkages. Entrepreneurs use their holdings of domestic bonds as liquidity stock to carry out investment and the bond default risk arises from the government's trade-off between private consumption and public expenditure. The model features a feedback loop between aggregate investment and debt sustainability. An adverse productivity shock reduces the government's tax base and hence its ability to repay. The initial decrease in bond price shrinks the economy's liquidity stock and leads to more projects being liquidated. Lower tax base, in turn, reduces bond price further.

The paper also studies partial default in the context of a two-way link between public and private sectors. Contrary to previous works treating sovereign defaults as binary events, countries often carry on borrowing at crisis with debt arrears. I show that absent other costs, it is strictly welfare-improving for the government to borrow using domestic currency denominated debt as it can adjust the repayment fraction in accordance with domestic productivity shocks. In cases where the government cannot directly choose the repayment fraction, e.g. when all debt is denominated in foreign currency, it can partially achieve outcomes of partial default via debt renegotiation. I also discuss a case of debt forgiveness where creditors may find it beneficial to forgive part of the debt before the government decides on default or repay. The feature of feedback loop makes the bond price particularly sensitive to changes in level of debt and therefore makes it more likely that creditors engage in debt forgiveness.

The second chapter, *Sovereign Debt and Disaster Risk in Emerging Countries*, analyses the impact of disaster risk on risk premium of emerging economies' debt. Emerging economies' external borrowing is usually associated with high and volatile interest spreads and constrained access to international credit market at times of crisis. Conventional views have focused on countries' proneness to disasters, occurrence of which leads to inevitable defaults. In the paper, I account for the transitional path of output leading to a disaster and not only for its occurrence. Specifically, I distinguish between "natural" and "economic" disasters based on the output dynamics prior to their occurrence.

The analysis is carried out in two steps. First, using output data of thirteen emerging economies from Latin America and Southeast Asia, I estimate a panel fixed effect Markov-switching model. The estimation identifies three distinct income states and the results show that countries are subject to economic disasters where the likelihood of disaster occurrence is higher in a bad state compared to a good one. I then computed disaster probabilities for those countries and interest rate measures on their external debt and find a positive relationship between the two. In the second step, I derive theoretical predictions in a model of sovereign debt and default distinguishing the two types of disaster risk. I make two sets of comparisons: 1) an economy without disaster risk (the benchmark economy) vs. one with an economic disaster risk; 2) benchmark economy vs. one with a natural disaster risk. Two patterns of impact on default risk emerge from the comparative statics. The presence of an economic disaster risk increases an economy's default risk whereas a natural disaster risk reduces it. The theoretical prediction of the model is consistent with the empirical results in the first step.

The paper also discusses two measures of *ex post* state contingency - debt renegotiation and debt extension. Welfare loss following default is reduced via renegotiation as creditors regain a fraction of the defaulted debt and debtors obtain access to the credit market. Debt extension enables debtors, with no default history, to postpone their debt repayment when a natural disaster occurs, until the economy recovers. Debt extension improves debtors welfare in disasters and disciplines debt repayment for countries prone to disasters.

Chapter 3, *Optimal Liquidity Provision with Partial Information*, studies optimal policy with partial information in a model of public provision of liquidity. Contrary to the assumption commonly made in previous works that the government has perfect knowledge of the underlying states of the economy, in practice they are often only able to learn about the states via the outcomes generated by those unknown states. I construct a model of optimal public provision of liquidity in the presence of two uncertainties. The government sets the optimal debt repayment fraction based on private sector's investment. The investment which is a function of both shocks calls for the government to react in opposite directions, depending on the source of the shock. This gives an interesting setting to analyse optimal policy with partial information. Using the General Signal Extraction method from Marcet et al.(2016), I present a solution of optimal liquidity provision with partial information. My ultimate aim is to evaluate the welfare implications of the lower availability of information in optimal policy. Further works would follow after this submission.

Chapter 1

Public Provision of Liquidity with Default Risk

1.1 Introduction

Sovereign default crises frequently coincide with liquidity and financial crises. Using long historical time series for a large range of countries, Reinhart and Rogoff (1999) document that linkages between sovereign and financial crises frequently recurred. While this phenomenon is usually associated with emerging economies, a prominent feature in the ongoing European debt crisis is the co-movement of corporate and sovereign CDS spreads in many peripheral countries. It draws questions as to whether there is a "vicious circle" in which weakening solvency of the governments would jeopardise domestic firms' balance sheets and firm distress would imperil public debt sustainability. It is also seen in the European crisis that Greece ceased interest repayment on parts of its sovereign debt and carried over its debt in arrears until IMF and ECB became involved in Greek debt restructuring. As documented by Arellano, Mateos-Planas and Rios-Rull (2013), contrary to the conventional view that regard sovereign defaults as binary events where countries either repay or default in full, sovereign defaults are always partial. In this paper, I study the two phenomena by formulating a two-way link between public and private sectors in a model featuring partial government default.

The model builds on two assumptions. Firstly, government debt represents a source of liquidity for the private sector. Specifically, government bonds are the only storage vehicle for private sector to refinance their future liquidity shock (cf. Holmstrom and Tirole, 1998). Secondly, upon default, the government cannot discriminate between domestic and foreign bondholders.

The above two assumptions have the following implications. The government's default decision (or repayment fraction in the case of partial default) involves a trade-off between private sector's consumption and public expenditure. On one hand, sovereign default prevents a transfer of domestic resources to external investors and enables the government to rebate tax revenue to private sector in the form of public expenditure while, on the other, it reduces private consumption of entrepreneurs whose bondholding is positive in the event of default. This opposing effect gives rise to a positive level of debt even in the absence of default penalties. The model also assumes a concave entrepreneurial preference over public expenditure, implying that the government is more likely to repay when the realisation of productivity shock is high.

By pricing public liquidity ex-ante (that is, with investor expectations about the government's default decision), the model generates a feedback loop between public and private sectors and induces an amplification mechanism. As entrepreneurs store liquidity in the form of sovereign bonds, sovereign debt sustainability is reduced for two reasons when the economy is hit by an adverse productivity shock. First, there is a direct negative effect of the shock on reducing private sector's production and therefore the government's fiscal revenue. Second, there is an indirect effect of the shock. An initial fall in bond price reduces private sector's liquidity stock. It makes more projects liquidate and shrinks the government's tax base. This, in turn, impairs the government's balance sheet and bond price drops even further.

This amplification effect is, however, sensitive to underlying distributions of liquidity and productivity shocks, as well as the amount of government debt. Without restricting the distributions, there might be multiple equilibrium bond prices. The multiplicity emerges as the government makes its default decision *after* bonds are traded in the market. This timing assumption, as shared by papers on self-fulfilling debt crises, e.g. Cole and Kehoe (1999), implies that bond prices reflect investors' expectations of the government's ability to repay and more than one solution are possible at equilibrium.

A common counterargument is that allowing domestic entrepreneurs to trade in foreign riskless bonds would release the domestic economy from the "vicious loop" and alleviate the need for government intervention of liquidity provision. While foreign bonds represent a perfect substitute for domestic government bonds, it is not necessarily the case that entrepreneurs would prefer foreign bonds over domestic ones. The intuition is that amplification mechanism generates a high fraction of projects to be refinanced when the state is good. Depending on the underlying distributions, the benefit of a disproportionately high level of liquidity threshold in an upturn may outweignt the loss from holding a defaultable bond and thus pushes the expected return of domestic bond holding above that of foreign riskless bonds.

Having established the linkage between sovereign defaults and liquidity crises, the paper explores measures that allow ex post state contingency - partial default, debt renegotiation and debt forgiveness. I model partial default as the government can optimally choose the fraction of debt that it repays and debt renegotiation as the government engage in renegotiation with creditors to decide the ratio of debt it needs to repay. It can be shown that absent other costs, it is strictly welfare-improving for the government to borrow using domestic currency denominated debt as it can adjust the repayment fraction in accordance with domestic productivity shocks. In cases where the government cannot directly choose the repayment fraction, e.g. when all debt is denominated in foreign currency, it can partially achieve outcomes of partial default via debt renegotiation. I discuss a case of debt forgiveness where creditors may find it beneficial to forgive part of the debt *before* the government decides on default or repay. The feature of feedback loop makes the bond price particularly sensitive to changes in level of debt and therefore makes it more likely that creditors engage in debt forgiveness.

This paper relates to a few strands of research. It is related to an early paper by Calvo in 1988 that studies government's optimal policy problem where partial debt repudiation is allowed. When the optimal taxation is high relative to government expenditure and the amount of debt, the model exhibits multiple equilibrium solution where the government can either default or partially repudiate. A few other papers study the interaction between sovereigns and private sectors. A recent working paper by Arellano, Bai and Bocola analyse the recessionary effects of sovereign default risk in a model with firm heterogeneity. Another working paper by Kaas, Mellert and Scholl also constructs a dynamic feedback mechanism between sovereign and private default risks to account for the counter-cyclicality of sovereign and private risk premia. While their papers mainly focus on the output impact of the feedback mechanism, my paper looks at how government's trade-off between private consumption and public expenditure impacts on the liquidity stock in the presence of the interaction.

My paper is also related to research concerning sovereign debt repayment under the assumption of non-discriminatory default. Broner and Ventura (2011) construct a model where a default on foreigners disrupts risk sharing among domestic residents. Brutti (2009) presents a setting related to mine, where default diminishes firms' ability to insure against idiosyncratic shocks. Based on their assumption and endogenous default cost, I take their model further by looking at an amplification mechanism which offers an explanation for the joint vulnerability between sovereigns and private sectors.

The rest of this paper is organised as follows. In Section 2, I outline the model and define the equilibrium. Section 3 illustrates the feedback mechanism. In Section 4, I discuss the partial default case, as well as debt renegotiation and debt forgiveness. Section 5 concludes and discusses potential extensions to the model.

1.2 Model

I consider the following economy. There are two dates, $t = 0, 1$, and $t = 0$ consists of two sub-periods, $t = 0$ and $t = 0.5$. There is an all-purpose good at every date and there is no storage technology.

The economy is populated by three types of agents: a domestic government, a continuum of measure one of domestic entrepreneurs and international investors.

Uncertainty is resolved over time. At $t = 0.5$, domestic entrepreneurs' idiosyncratic liquidity shock, ρ_0, is realised, as well as μ, part of the aggregate productivity shock. μ determines the bond price at $t = 0.5$ and the liquidity stock in the economy.

1.2.1 Setup

Private agents: domestic entrepreneurs and international investors. Each entrepreneur is endowed with one unit of domestic bond and an investment project of fixed size I at $t = 0$. The project requires an input of I units of good at $t = 0$ and an additional uncertain amount of $\rho_0 I$ at $t = 0.5$. ρ_0 is a liquidity shock which has a continuous probability distribution function $f(\cdot)$ with $\rho_0 \in [0, \bar{\rho}_0]$. If $\rho_0 I$ is paid, the project continues and a final payoff of zI is realised at $t = 1$. If $\rho_0 I$ is not paid, the project liquidates and yields zero[1]. An initial investment of I is irreversible[2].

z is an aggregate productivity shock consisting of two parts: $z = \mu + \epsilon$. The uncertainty over z is resolved sequentially: μ is realised at $t = 0$ and ϵ is realised at $t = 1$. $\mu \sim U[\underline{\mu}, \bar{\mu}]$, $\epsilon \sim h(\cdot)$, $\epsilon \in [0, \infty)$. The expected value of z at $t = 0.5$ depends on the realisation of μ. The government lays a flat-rate tax τ on projects' investment returns at $t = 1$. τ is fixed and exogenously given.

Entrepreneurs derive utility from both private consumption and government expenditure. A representative entrepreneur at date t maximises $V_t^E =$

[1]The liquidity problem in this setting can be considered as a special case of Holmstrom and Tirole (1998): entrepreneurs' pledgeability ratio of future investment return is zero.

[2]Relaxing the irreversibility of investment does not change the result as the risk neutral and deep-pocket international investors are the marginal traders in domestic bond market.

$\mathbb{E}_t[C_1^E + \log(G_1)]$ where C_1^E and G_1 denote the entrepreneur's consumption and government expenditure at $t = 1$. For simplicity, I assume that the future discount rate equals one for all agents in the economy.

International investors have a large endowment in every period. Their expected utility at date t is $V_t^F = \mathbb{E}_t\left(\sum_{s=t}^{1} C_t^F\right)$, and so the international rate of interest is zero. Investors are indifferent between consuming today or any time in the future.

The domestic government The government has some legacy bond of amount B_0. I assume that $B_0 > 1$ so that the marginal trader in the domestic bond market is a risk-neutral international investor. In the basic model, the government makes a binary decision of default or repay at date 1 to maximise V_t^E. The decision of default involves a trade-off between domestic entrepreneurs' consumption of private and public goods. By defaulting, the domestic economy does not transfer resources to outside investors and the government is able to transfer the entire taxation to the entrepreneurs. However, since the government cannot discriminate between foreign and domestic bond holders, a default on bonds reduces the consumption of domestic entrepreneurs whose bondholing is still positive after refinancing the liquidity shock at $t = 0.5$. The concavity of entrepreneurs' utility of public expenditure implies that the government is more likely to repay when the productivity shock is higher and therefore the resources to repay are higher.

I denote q_0 and $q_{0.5}(\mu)$ the bond prices at date 0 and date 0.5 respectively. D_1 is the gov't's default decision at date 1. $D_1 \in \{0, 1\}$, where $D_1 = 1$ for default. I assume that $\underline{\mu}$ is high enough that $(1-\tau)(\underline{\mu} + \mathbb{E}(\epsilon)) \geq \bar{\rho}_0$, implying that entrepreneurs always want to continue their projects if they can. The fraction of projects refinanced is denoted as $\hat{\rho}_0$.

The amount of liquidity stock at $t = 1$ is determined by the realisation of μ. The government's default decision D_1 depends on its tax base which is determined by both aggregate productivity shock z as well as the fraction of

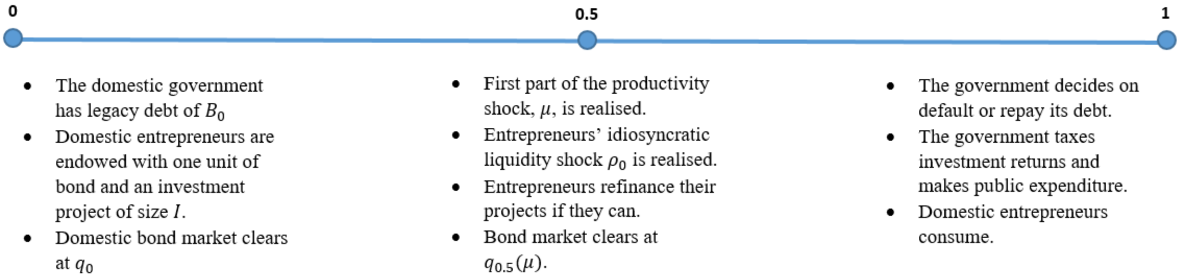


Figure 1.1: Timeline.

projects refinanced $\hat{\rho}_0$. Figure 1.1 depicts the timeline.

1.2.2 Equilibrium

Definition. *An equilibrium is prices $\{q_0^*, q_{0.5}^*(\mu)\}$, an allocation C_1^{E*} and government policies $\{G_1^*, D_1^*(z) \in \{0,1\}\}$, such that: (i) given $\{q_0^*, q_{0.5}^*(\mu)\}$ and $\{G_1^*, D_1^*(z)\}$, C_1^{E*} maximises entrepreneurs' expected utility; (ii) given $\{q_0^*, q_{0.5}^*(\mu)\}$, $\{G_1^*, D_1^*(z)\}$ solves the government's optimisation problem and (iii) $\{q_0^*, q_{0.5}^*(\mu)\}$ reflect the domestic bond's conditional default probability and are consistent with international investors' zero expected profits.*

The equilibrium can be computed by backward induction. I characterise the agents' problem at $t = 1$ for a given realisation of z and a given liquidity threshold $\hat{\rho}_0$.

1.2.2.1 $t = 1$

At $t = 1$, aggregate productivity shock z is fully realised. A fraction $F(\hat{\rho}_0)$ of projects continue to $t = 1$. The government decides on D_1 to solve the following problem:

$$\begin{aligned} &\max_{C_1^E, G_1, D_1} && C_1^E + \log(G_1) \\ s.t. \quad &C_1^E && = \int_0^{\hat{\rho}_0} [(1-\tau)zI - \rho_0 I] dF(\rho_0) + 1_{\{D_1=0\}} \end{aligned}$$

$$G_1 \qquad = \int_0^{\hat{\rho_0}} \tau z I dF(\rho_0) - B_0 \cdot 1_{\{D_1=0\}}$$

Solving the above gives a minimum level of z, $\tilde{z}$, below which the government defaults:

$$\tilde{z} = \left(\frac{1}{e-1}+1\right)\left(\frac{1}{F\left(\hat{\rho_0}\right)}\right)\frac{B_0}{\tau I}$$

It then follows that:

$$G_1^* = \begin{cases} \int_0^{\hat{\rho_0}} \tau z I dF(\rho_0) & \text{for} \quad z < \tilde{z} \\ \int_0^{\hat{\rho_0}} \tau z I dF(\rho_0) - B_0 & \text{for} \quad z \geq \tilde{z} \end{cases}$$

and

$$C_1^{E*} = \begin{cases} \int_0^{\hat{\rho_0}} [(1-\tau)zI - \rho_0 I] dF(\rho_0) & \text{for} \quad z < \tilde{z} \\ \int_0^{\hat{\rho_0}} [(1-\tau)zI - \rho_0 I] dF(\rho_0) + 1 & \text{for} \quad z \geq \tilde{z} \end{cases}$$

$\tilde{z}$ demonstrates that the government is more likely to default when the resources to repay is low (a low level of $\hat{\rho_0}$) and when its debt burden is high (a high level of B_0).

1.2.2.2 $t = 0$

Because the marginal investor in domestic bonds is a risk-neutral international investor, the prices of domestic bonds at $t = 0$ and $t = 0.5$ reflect the relevant conditional default probability:

$$q_{0.5}(\mu) = 1 - H(\tilde{z} - \mu) = 1 - H\left(\left(\frac{1}{e-1}+1\right)\left(\frac{1}{F\left(\hat{\rho_0}\right)}\right)\frac{B_0}{\tau I} - \mu\right) \tag{1.1}$$

$$q_0 = \int q_{0.5}(\mu) dU(\mu) \tag{1.2}$$

where $H(\cdot)$ is the conditional CDF of ϵ. I further assume that entrepreneurs receive a small amount of endowment δI when their liquidity shocks are realised, such that $\hat{\rho}_0 = \delta + \frac{q_{0.5}}{I}$. This assumption ensures that the fraction of projects that realise their returns is always positive, even at the lowest level of q_0, $q_0 = 0$. In combination with two other restrictions I introduce later, they act as sufficient conditions to ensure that there exists a unique positive solution to (1.1).

1.3 Public and private feedback loop

I now show how a shock to the government's balance sheet is amplified through a feedback loop: a low realisation of productivity shock reduces the government's ability to repay and implies a lower level of the bond price at $t = 0.5$. This in turn compresses the economy's liquidity stock and leads to more projects being liquidated. A lower tax base further deteriorates the government's ability to repay, and so forth. I then demonstrate a scenario featuring multiple equilibrium bond prices at $t = 0.5$ in which bond price depends on investors' expectation of the repayment likelihood.

1.3.1 Amplification mechanism

The feedback loop can be shown via the following fixed-point equation for the date 0.5 price of domestic bonds

$$q_{0.5}(\mu) = 1 - H(\tilde{z}(\mu))$$

where

$$\tilde{z}(\mu) = \left(\frac{1}{e-1} + 1\right)\left(\frac{1}{F\left(\hat{\rho}_0\right)}\right)\frac{B_0}{\tau I} - \mu$$

Using the implicit function theorem, I derive the following comparative statics result.

Proposition 1 (Feedback Loop) *The sensitivity of date-1 bond price $q_{0.5}(\mu)$ to the realisation of μ is given by*

$$\frac{dq^*_{0.5}(\mu)}{d\mu} = \frac{\frac{\partial H(\tilde{z})}{\partial \mu}}{1 - h(\tilde{z})\left(\frac{B_0}{\tau I^2}\right)\frac{f\left(\frac{q^*_{0.5}}{I}+\delta\right)}{F^2\left(\frac{q^*_{0.5}}{I}+\delta\right)}\left(\frac{1}{e-1}+1\right)} \tag{1.3}$$

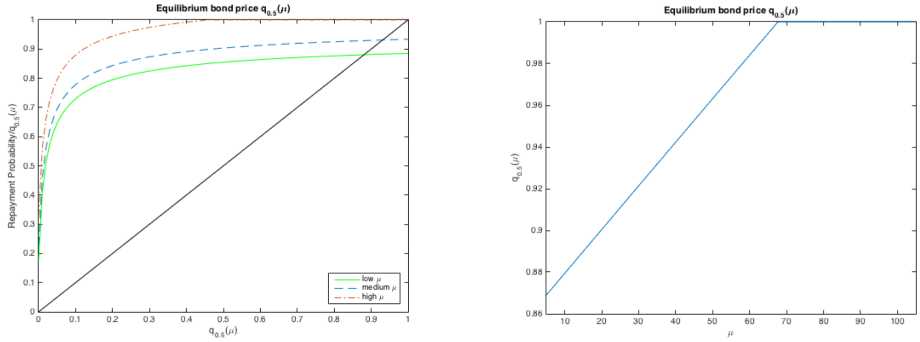
To ensure that there exists a unique solution to (1.1), in addition to the assumption made in the previous section ($\hat{\rho}_0$ is always positive regardless of the value of $q_{0.5}$), I impose the two further restrictions: (i) $\frac{e}{e-1}\frac{1}{F(\delta)}\frac{B_0}{\tau I} - \mu < \bar{\epsilon}$; and (ii) the density distribution of ϵ, h, is non-decreasing, and F is log-concave. The first restriction ensures that the RHS of (1.1) > 0 when $q_{0.5} = 0$ and the second restriction implies that $\frac{f}{F}$ is decreasing and therefore the RHS of (1.1) is concave. It follows that the numerator of (1.3) is positive and less than one. The numerator captures the direct impact of the change in μ on the date 0.5 bond price $q_{0.5}(\mu)$. The denominator is positive because of the restrictions. It takes the form of a multiplier, which represents the indirect impact of μ on the bond price $q^*_{0.5}$ through the change in the price at which entrepreneurs liquidate their bond holdings. The multiplier is higher, the larger the amount of government's outstanding debt at $t = 0$, B_0. This multiplier demonstrates the feedback loop between sovereigns and private sectors as an amplification mechanism: an increase in the default probability reduces $q_{0.5}(\mu)$ which reduces the entrepreneurs' liquidity stock and more liquidation occurs. Lower tax base reduces the bond price further etc. ad infinitum. Numerical illustrations of the comparative statics are in Figure 1.2.

1.3.1.1 Multiple equilibrium bond prices

In this section, I discuss the model results by relaxing the restrictions made in the previous section.

By relaxing the restrictions on distributions h and f, the concavity of the RHS of (*) is no longer guaranteed and there may exist multiple equilibrium bond prices at $t = 0.5$. I present a numerical example below.

There exist three equilibrium bond prices in this example. $q^*_0 = 0$ is the



(a) $q^*_{0.5}(\mu)$ at μ levels $\mu = \underline{\mu}, \frac{\bar{\mu}}{4}, \frac{3\bar{\mu}}{4}$ **(b)** $q^*_{0.5}(\mu)$

Figure 1.2: Equilibrium bond price $q^*_0(\mu)$ for $B_0 = 5.42$, $I = 1$ and $\tau = 0.2$; ρ_0 follows a Weilbull distribution with scale parameter 1 and shape parameter 0.5

trivial equilibrium bond price. The other two equilibrium prices correspond to different investor expectations. At point A, investors anticipate that a low fraction of projects are refinanced and continued to realisation, and they are only willing to pay a low price for the bonds and therefore the liquidity stocks are low. Few entrepreneurs can refinance their projects and expected date 1 tax revenue is low. A low level of expected tax revenue, in turn, makes the government more likely to default and therefore investors' expectation of low bond price is fulfilled in equilibrium. The same reasoning follows for point B where investors coordinate to a high expectation of the government's ability to repay. Between points A and B, only point B is a locally stable equilibrium one and exhibit the amplification mechanism shown in the previous section. This is shown in Fig. 1.3 as the level of μ varies: $q^*_0(\mu)$ rises at the locally stable point as μ but falls at the locally unstable point.

It is also interesting to note that the result of multiple equilibrium prices is sensitive to the amount of legacy debt, B_0. I illustrate the comparative statics in Fig. 1.4. At a low level of B_0, the extreme case of $q^*_{0.5} = 1$ is achieved at the locally stable equilibrium point. Depending on investors' expectation, the government is either able to fully repay its debt or pay very little of it. As the amount of B_0 increases, the locally stable $q^*_{0.5}$ falls but the locally unstable bond price increases. At a high level of B_0, the government cannot repay and

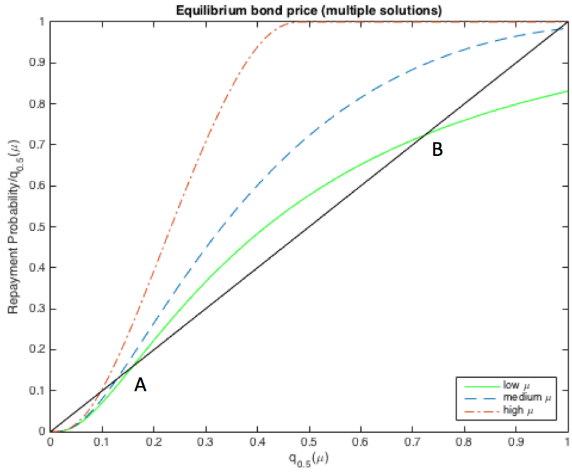


Figure 1.3: Multiple solutions of equilibrium bond price $q_{0.5}(\mu)$ for $ln(\rho_0) \sim \mathcal{N}(4.5, 0.5)$ and $\epsilon \sim U[0,1]$.

therefore the only equilibrium point is $q^*_{0.5} = 0$.

1.3.2 Discussion

A natural counterargument that arises is that allowing private sector to trade in foreign riskless bonds could break the ”vicious loop” and alleviate the need for government intervention in the economy. Yet it is unclear whether entrepreneurs would strictly prefer to hold foreign riskless bonds if they can do so. The intuition is that amplification mechanism generates large swings in domestic bond price at $t = 0.5$ and therefore in liquidity thresholds. Depending on the underlying distributions, the benefit of a disproportionately high level of liquidity threshold in an upturn may outweigh the loss from holding a defaultable bond and pushes the expected return of domestic bond holding above that of foreign riskless bonds. It is illustrative to compare the expected net return of holding defaultable domestic bonds and holding foreign riskless bonds.

Suppose a representative entrepreneur has an endowment of 1 at $t = 0$.

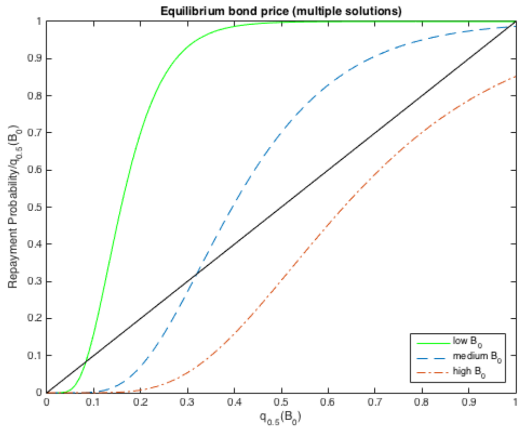


Figure 1.4: Multiple solutions of equilibrium bond price $q_{0.5}(B_0)$ for $\mu = \bar{\mu}/2$ and $B_0 = 2, 5, 8$.

Purchasing domestic bonds yields:

$$R_0^d(B_0) = -1 + \int_\epsilon \int_\mu \left(\int_0^{\hat{\rho_0}^d} \left((1-\tau)z - \rho_0\right) I dF(\rho_0) + \frac{1}{q_0^d(B_0)} \int_{\hat{\rho_0}^d}^{\bar{\rho_0}} q_{0.5}^d(\mu; B_0) dF(\rho_0) \right) dU(\mu) dH(\epsilon)$$

where $q_{0.5}^d$ is $t = 0.5$ prices of domestic bond, $q_0^d = \int_\mu q_{0.5}^d(\mu)$ and $\hat{\rho_0}^d$ is the liquidity threshold of refinancing using domestic bond at $t = 0.5$, $\hat{\rho_0}^d = \frac{q_{0.5}^d(\mu)}{q_0^d I}$. The first term in the integral bracket is entrepreneur's expected return when he can refinance his investment project and the second term is the expected return from holding $\frac{1}{q_0^d}$ units of domestic bond until $t = 1$ when liquidity shock is too large to refinance.

$$R_0^f = -1 + \int_\epsilon \int_\mu \left(\int_0^{\hat{\rho_0}^f} \left((1-\tau)z - \rho_0\right) I dF(\rho_0) + \int_{\hat{\rho_0}^f}^{\bar{\rho_0}} 1 dF(\rho_0) \right) dU(\mu) dH(\epsilon)$$

where $\hat{\rho_0}^f$ is the liquidity threshold of refinancing using foreign bond at $t = 0.5$, $\hat{\rho_0}^f = \frac{1}{I}$.

The feedback loop implies that for domestic bonds, at some $\mu > 0$, the liquidity threshold $\hat{\rho_0}^d = \frac{q_{0.5}^d(\mu)}{q_0^d I}$ is greater than $\hat{\rho_0}^f > \frac{1}{I}$ and larger the amplification effect, higher the $\hat{\rho_0}^d$ relative to $\hat{\rho_0}^f$.

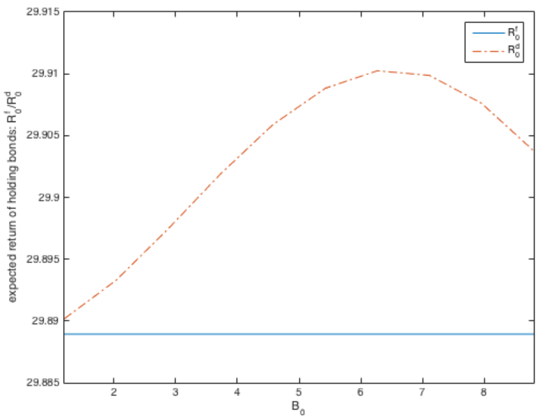


Figure 1.5: Expected net return from bond holding. R_0^f: net expected return from foreign riskless bond holding; R_0^d: net expected return from domestic bond holding.

I illustrate an example where $R_0^d > R_0^f$ as B_0 varies in Fig.1.5.

1.4 Partial default

1.4.1 $t = 1$

In this section, I extend the model by allowing the government to partially default on its debt[3].

At $t = 1$, given $\hat{\rho}_0$, the government decide on the fraction, γ_1, of legacy debt B_0 to repay. It sets γ_1 to solve:

$$
\begin{aligned}
& \max_{C_1^E, G_1, \gamma_1 \in [0,1]} && C_1^E + \log(G_1) \\
s.t. \quad & C_1^E && = \int_0^{\hat{\rho}_0} [(1-\tau)zI - \rho_0 I] dF(\rho_0) + \gamma_1 \cdot 1 \\
& G_1 && = \int_0^{\hat{\rho}_0} \tau z I dF(\rho_0) - B_0 \cdot \gamma_1
\end{aligned}
$$

[3] All other model set ups are the same as in the previous sections

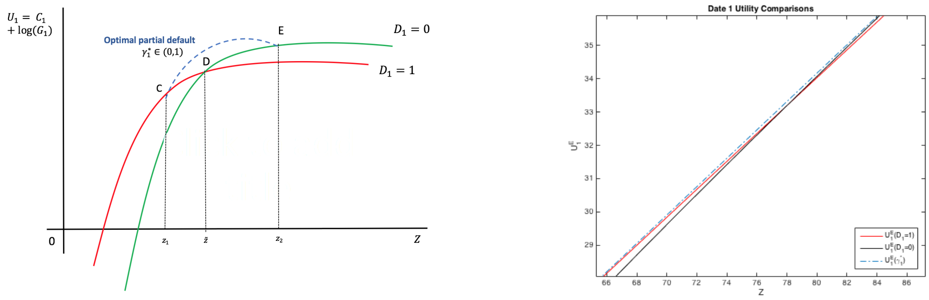
The optimal partial default schedule is:

$$\gamma_1^*(z; B_0, \hat{\rho}_0) = \begin{cases} 0 & \text{for} \quad z \in [0, z_1) \\ \frac{\tau z I F(\hat{\rho}_0)}{B_0} - 1 & \text{for} \quad z \in [z_1, z_2] \\ 1 & \text{for} \quad z \in (z_2, \bar{z}] \end{cases}$$

where $z_1 = \frac{B_0}{\tau I F(\hat{\rho}_0)}$, $z_2 = \frac{2B_0}{\tau I F(\hat{\rho}_0)}$ and $\bar{z}$ is the upper limit of z. z_1 represents the threshold level of z under which the government optimally chooses to default and z_2 represents the threshold level of z above which the government optimally chooses to repay in full. z_1 and z_2 are positively related to B_0 and inversely related to $\hat{\rho}_0$, implying that a high level of legacy debt increases the government's likelihood of defaulting whereas a high liquidity threshold increases the government's tax base and increases its likelihood of repay. If the value of B_0 is so high that $z_1 \geq \bar{z}$, the government would choose to default regardless of the realisation of productivity shock. For z_2 values such that $z_2 < \bar{z}$, there is a strictly positive range of z that the optimal repayment fraction, $\gamma_1^* \in (0, 1)$.

The option of partial default extends the government's budget set and gives it more flexibility in trading off between private consumption and public expenditure at $t = 1$. A graphical illustration and a numerical example are provided in Fig. 1.6. Figure 1.6 demonstrates that in the absence of partial default, entrepreneurs optimally chooses to default for $z < \tilde{z}$ and repay for $z \geq \tilde{z}$. This is implied by the concavity of entrepreneurs' utility in public expenditure, G_1. As in the standard model of full default, the option of default expands the government's budget set and improves its welfare by increasing it for low values of z. By expanding the notion of default further, partial default essentially extends the budget set further. The dashed line connecting points C and E in the figure is the upper contour for all possible utilities with partial default for $z \in [z_1, z_2]$.

Outright default and full repayment ($D_1 \in \{0, 1\}$) are nested in the partial



(a) Graphical illustration

(b) Numerical example

Figure 1.6: Entreprenerus' $t=1$ utility U_1^E for (i) $D_1=1$;(ii) $D_1=0$;(iii) $\gamma_1^* \in [0,1]$

default model. Partial default is strictly welfare-improving for the government because in the range of z where the government chooses $\gamma_1^* \in (0,1)$, outright default or full repayment is always an option. For levels of z above z_1, the government optimally chooses default for $z \in [z_1, \left(\frac{1}{e-1}+1\right)\left(\frac{1}{F(\hat{\rho}_0)}\right)\frac{B_0}{\tau I}]$ if partial default is not available, but this is not an optimal choice under partial default as $\frac{\partial U_1^E(\gamma_1; z\in[z_1,\tilde{z}])}{\partial \gamma_1}|_{\gamma_1=0} > 0$, denoting $\tilde{z} = \left(\frac{1}{e-1}+1\right)\left(\frac{1}{F(\hat{\rho}_0)}\right)\frac{B_0}{\tau I}$. Likewise, for levels of z below z_2, the government optimally chooses to repay for $z \in [\tilde{z}, z_2]$, but it is not an optimal choice under default as $\frac{\partial U_1^E(\gamma_1; z\in[\tilde{z},z_2])}{\partial \gamma_1}|_{\gamma_1=1} < 0$.

This implies that absent other costs of partial default, the government always prefers issuing debt denominated in domestic currency as it can adjust the repayment fraction corresponding to the domestic productivity shocks.

1.4.2 $t = 0$

Denote $\tilde{q}_0$ as $t=0$ bond price. Given the optimal schedule for partial default, γ_1^*,

$$
\begin{aligned}
\tilde{q}_0 &= \left(\frac{\mu}{X}-1\right)[H(2X-\mu)-H(X-\mu)] \\
&+ \frac{1}{X}\int_{X-\mu}^{2X-\mu} \epsilon dH(\epsilon) + 1 - H(2X-\mu)
\end{aligned}
$$

where $X = \frac{B_0}{\tau I F(\frac{\tilde{q}_0}{I})}$, $H(\cdot)$ is the CDF of ϵ that realises at $t=1$.

Under the same sufficient conditions as in Section 1.3.1, the amplification mechanism is still present:

$$\frac{d\tilde{q_0^*}(\mu)}{d\mu} = \frac{\frac{1}{X}[H(2X-\mu) - H(X-\mu)]}{1 - \frac{1}{X^2}\frac{\partial X}{\partial \tilde{q}_0}\left(\mu[H(2X-\mu) - H(X-\mu)] - \int_{X-\mu}^{2X-\mu} \epsilon dH(\epsilon)\right)}$$

And the denominator is positive and smaller than one.

1.4.3 Debt renegotiation

I show in this section that in the case that the government cannot directly choose the fraction of debt it repays, perhaps because the debt is issued in foreign currency due to inflation credibility concern, partial default can still be achieved, though not entirely, if creditors are willing to engage in debt renegotiation with the government once z is realised at $t = 1$.

Assume that creditors can perfectly coordinate. The renegotiation arrangement is as follows. Upon the realisation of z, the two sides renegotiate to decide fraction α of legacy debt B_0 the government repays, in a Nash bargaining fashion. The government's bargaining power is represented by λ and creditors' by $1-\lambda$, $\lambda \in [0,1]$. The value of their outside options is the value of their $t = 1$ utility without the option of renegotiation. The surplus of renegotiation is non-negative for levels of z at which the government optimally chooses to default in the absence of renegotiation option, $z \in [z_1, \tilde{z}]$.

The optimal repayment fraction α solves:

$$\begin{aligned} \max_{\alpha\in[0,1]} \quad & \left(U_1^E(\alpha; z) - U_1^E(D_1 = 0; z)\right)^{\lambda} \cdot \left(U_1^F(\alpha; z) - U_1^F(D_1 = 0; z)\right)^{1-\lambda} \\ s.t. \quad & U_1^E(\alpha; z) - U_1^E(D_1 = 0; z) \geq 0 \\ & U_1^F(\alpha; z) - U_1^F(D_1 = 0; z) \geq 0 \end{aligned}$$

When $\lambda = 1$, $\alpha^* = \gamma_1^*$. At the other extreme, $\lambda = 0$ (creditors have all the bargaining power), $\alpha^* = 1$. At its interior solution, α^* satisfies:

$$\log(1 - \frac{\alpha^* B_0}{X}) = \alpha^* \left((1 + \frac{1}{\lambda - 1})(1 - \frac{B_0}{X - \alpha^* B_0}) - 1\right)$$

where $X = \frac{B_0}{\tau I F(\frac{\tilde{q_0}}{I})}$.

As in standard models of debt renegotiation upon default, such as Yue (2009), it can be shown that the optimal debt repayment fraction α^* increases in z but decreases in B_0. In addition, α^* is monotonically decreasing in λ: $\alpha^* = 1$ when $\lambda = 0$ and it decreases to γ_1^* when $\lambda = 1$.

The above results imply that when the government's instruments are restricted to full default or repay, optimal partial default can be achieved via debt renegotiation with creditors under two conditions: 1) $z \leq \tilde{z}$ and 2) the government has all the bargaining power $\lambda = 1$. As λ falls, the government repays more of its legacy debt.

1.4.4 Debt forgiveness

In contrast to debt renegotiation discussed above, mutually beneficial negotiation can also take place at $t = 0.5$. I discuss a case of debt forgiveness in this section.

I model $t = 0.5$ debt forgiveness as follows. Assume that after μ is observed at $t = 0.5$, bondholders can forgive some of the legacy debt to an arbitrary amount $\hat{B}_0 \leq B_0$, before the government makes its default/repay decision. A mutually beneficial negotiation can take place between the legacy creditors and the government when some debt forgiveness can improve the utilities of the creditors. I further assume that legacy creditors are able to coordinate and have all the bargaining power. They collectively make a take-it-or-leave-it offer to the government. Domestic entrepreneurs' interests are perfectly aligned with foreign creditors so I take their decisions and foreign investors' as a whole.

Legacy creditors engage in debt forgiveness $\bar{B}_0(\mu) < B_0$ when legacy debt B_0 is on the decreasing ("bad") side of the legacy Laffer curve $q_{0.5}(\mu; \hat{B}_0)\hat{B}_0$, where $\bar{B}_0(\mu)$ is the peak of the legacy creditors' Laffer curve: $\frac{d(q_{0.5}(\mu;\hat{B}_0)\hat{B}_0)}{d\hat{B}_0}|_{\hat{B}_0=B_0} <$

0. The effects of debt forgiveness ($d\hat{B}_0 < 0$) are illustrated below:

$$\frac{d(q_{0.5}(\mu;\hat{B}_0)\hat{B}_0)}{d\hat{B}_0} = q_{0.5}(\mu;\hat{B}_0) - \hat{B}_0 \cdot \frac{\frac{\partial H(\tilde{z})}{\partial \mu} \cdot C}{1 - \frac{\partial H(\tilde{z})}{\partial \mu}\left(\frac{\hat{B}_0}{I}\right)\frac{f\left(\frac{q_{0.5}}{I}+\delta\right)}{F\left(\frac{q_{0.5}}{I}+\delta\right)} \cdot C} \tag{1.4}$$

where $\tilde{z} = \left(\frac{1}{e-1}+1\right)\left(\frac{1}{F\left(\frac{q_{0.5}}{I}+\delta\right)}\right)\frac{\hat{B}_0}{\tau I} - \mu$ and $C = \frac{1}{\tau I}\frac{e}{e-1}\frac{1}{F\left(\frac{q_{0.5}}{I}+\delta\right)}$.

$d\hat{B}_0 < 0$ has two opposing effects on the value of $q_{0.5}(\mu;\hat{B}_0)\hat{B}_0$: the first term of RHS of (1.4) represents the direct quantity-of-debt effect where $d\hat{B}_0 < 0$ contributes negatively to $q_0(\mu;\hat{B}_0)\hat{B}_0$. The second term of RHS represents the indirect price-of-debt effect where $d\hat{B}_0 < 0$ contributes positively to $q_{0.5}(\mu;\hat{B}_0)\hat{B}_0$. The second effect is stronger, the more elastic is the price $q_{0.5}(\mu;\hat{B}_0)$ to $\hat{B}_0$. The feedback loop makes the price-of-debt effect more potent without affecting the first effect. Therefore pushing the economy towards the decreasing part of the legacy Laffer curve $q_{0.5}(\mu;\hat{B}_0)\hat{B}_0$.

It can be shown that the peak of the legacy Laffer curve, $\bar{B}_0(\mu)$, is decreasing in μ: $\frac{d\bar{B}_0(\mu)}{d\mu} < 0$ implying that worse states are associated with more debt forgiveness. The results so far can be summarised in the proposition below.

Proposition 2 (Creditors' Laffer Curve and Debt Forgiveness) *Suppose that there are realisations of μ where debt forgiveness takes place so that $B_0(\mu) = \bar{B}_0(\mu)$. At those realisations, the amount of $t = 0.5$ debt after debt forgiveness is at the peak $\bar{B}_0(\mu)$ of the Laffer curve $q_{0.5}(\mu;\hat{B}_0)\hat{B}_0$. $\bar{B}_0(\mu)$ is increasing in μ.*

The proof is given in the Appendix **??**.

1.5 Conclusion

Motivated by the observation of the co-movement of sovereign default risk and private insolvency risk in many crisis episodes, this paper proposes a two-way link between public and private sectors in a model featuring partial government default. Entrepreneurs use their holdings of domestic bonds as liquidity stock to carry out investment and the bond default risk arises from the government's trade-off between private consumption and public expenditure. The model generates a feedback loop where a productivity shock impacts on bond price more than proportionally. The paper also discusses measures of state contingency - partial default, debt renegotiation and debt forgiveness.

The model can be extended in several ways. Another possibility for ex post state contingency would be in terms of taxation. An endogenous taxation rate gives the government an additional policy instrument in trading off the benefit and cost of public default. Relaxing the restriction of exogenous tax rate also alters entrepreneurs' refinancing decision in the intermediate period. It creates another channel through which the productivity shock μ is linked to liquidity threshold $\hat{\rho}_0$ and may further enhance the amplification effect. Given a low realisation of μ, if the government optimally responds to the low aggregate productivity shock by increasing tax rate at $t = 1$, anticipating that, liquidity threshold $\hat{\rho}_0$ would be low as more entrepreneurs opt not to refinance. As a higher fraction of entrepreneurs decides to liquidate their projects and consume their bondholdings at the end period, the government has a higher incentive to increase tax rate to avoid a costly public default.

As the model setup assumes the exogenous holding of bonds by the entrepreneurs, it is natural to relax this assumption and let entrepreneurs optimally choose their asset portfolios consisting of (i) domestic and foreign bonds, or (ii) bonds and other forms of asset, e.g. bank deposits. In (i), foreign bonds are risk-free and acts as a perfect substitute for domestic bonds. I have discussed in the paper a case where entrepreneurs do not necessarily prefer the risk-less foreign bonds due to the feedback mechanism between domestic bond price and aggregate investment. Strategic complementarity might arise from entrepreneurs' optimal choice of domestic bonds. Higher holding of domestic bonds increases the bond price because the government is more likely to repay and in the presence of the feedback loop, a representative entrepreneur's marginal value of holding a domestic bond might increase with the aggregate level of domestic holding. Sufficiently strong strategic complementarity gives rise to multiple equilibria - one equilibrium with high holding of domestic bonds and low holding of foreign bonds, and the other equilibrium with the opposite.

One may also let entrepreneurs (or firms in the case (ii)) optimally choose between public provision of liquidity, bonds, and private provision, bank deposits. I outline a model setup as follows. The government issues risk free bonds at date 0, backed by taxable income from households at date 1. Firms are endowed with initial capital and goods and money demand arises as a fraction of firms incur an investment opportunity at $t = 0.5$ and requires additional investment input. Firms carry liquidity by holding deposits and bonds. At $t = 0$, banks issue deposits that are short-term risk-free debt and extend loans to firms that are backed by firms' capital as collateral. Firms are willing to accept a return of deposit lower than their time discount rate, i.e. paying a positive premium, because of the marginal value of investment return. Bonds are perfect substitutes of deposits and therefore firms are also willing to pay a premium for them. To make the model more interesting and address the need for public provision, one could further extend the model by adding frictions in the private asset market, e.g. search frictions as in Cui (2016, 2018), such that the private provision of liquidity is insufficient to achieve efficient level of investment. The above extensions form part of my planned future work.

Chapter 2

Sovereign Debt and Disaster Risk in Emerging Countries

2.1 Introduction

Emerging economies' external borrowing is usually associated with high and volatile interest spreads and constrained access to international credit market at times of crisis. Alongside other factors, conventional views have attributed this stylised fact to emerging countries' proneness to disasters, occurrence of which lead to inevitable defaults. In this paper, I account for the transitional path of output leading to a disaster and not only for its occurrence. Specifically, I distinguish between "natural" and "economic" disasters based on the output dynamics prior to their occurrence. My empirical estimation results show that a range of emerging countries are subject to economic disasters and probability of disaster occurrence is positively correlated with countries' interest spreads. This is consistent with the theoretical prediction of a model constructed to compare economies with natural and economic disaster risks that the presence of economic disaster increases default risk and constrains external borrowing.

My research design is motivated by the differences between natural and economic disasters. Natural disasters, such as tsunamis and hurricanes, occur unexpectedly overtime with instantaneous impact on countries' output. Their occurrences are unrelated to countries' economic fundamentals. In contrast, economic disasters, such as the Great recession and Latin American debt crisis in the 1980s, are usually preceded by consecutive periods of low incomes and they take longer to recover. Their occurrences often reflect weak economic fundamentals. Although both types of disasters lead to defaults *ex post* if the impact on output is sufficiently severe, the underlying causes and recovery length imply different reputation costs and borrowing schedules in non-disaster times.

I proceed in two steps. First, I analyse output dynamics of emerging economies using 13 countries from Latin American (LA) and Southeast Asian (SeA) regions. To determine the type of disaster accounting for the large output contractions in those economies, I estimate a panel fixed effect Markov-switching model. The estimation identifies three distinct income states and the results show that countries are subject to economic disasters where the likelihood of disaster occurrence is higher in a bad state compared to a good one. I then computed disaster probabilities for those countries and interest rate measures on their external debt and find a positive relationship between the two. Higher the likelihood that a disaster occurs, more costly it is to borrow externally.

In a second step, I derive theoretical predictions in a model of sovereign debt and default distinguishing the two types of disaster risk. The calibration of the two disaster risks differs in their transitional dynamics but the size of output contraction in the disaster state is the same for both types. Two comparisons are made: 1) an economy without disaster risk (the benchmark economy) vs. one with an economic disaster risk; 2) benchmark economy vs. one with a natural disaster risk. Two patterns of impact on debt riskiness emerge from the comparative statics. When compared with the benchmark economy, the presence of an economic disaster risk increases an economy's default risk for every income shock. The economy's financial contract is more stringent with lower bond price and the interest rate charged for every loan size is higher. In a recession, facing a likelihood of disaster occurrence in the following period, a debtor's borrowing is severely constrained that for some specifications, no risky borrowing exists on the equilibrium path. In contrast, the economy with a natural disaster risk is associated with lower default risk. Bond prices are higher and the interest rate charged is lower. The economy is able to borrow more and has looser borrowing limit.

In the model, the presence of a disaster shock changes an economy's output process in three ways. Firstly, it makes a country poorer in terms of mean output level; secondly the output is more volatile; and thirdly the country faces a likelihood of transiting to the disaster state in the following period which may or may not relate to its current period of income, depending on the type of disaster. These features of the income process increase a risk-averse borrower's incentive to insure *ex ante*, but they also make the country more likely to default *ex post*. The presence of economic disaster risk shortens the sequence of bad shocks needed for an economy to default and when coupled with income persistence, makes debt particularly risky in a bad state. Natural disaster risk, on the other hand, gives the government a strong incentive to oblige its debt contract so that it can trade with international investors and build up assets for insurance.

I extend the model to discuss two measures of *ex post* state contingency - debt renegotiation and debt extension. Upon default, debtors and creditors engage in debt renegotiation. Welfare loss following default is reduced as creditors regain a fraction of the defaulted debt and debtors obtain access to the credit market through renegotiation over debt reduction. Debt extension enables debtors, with no default history, to postpone their debt repayment when a natural disaster occurs, until the economy recovers. They still have access to the credit market during a disaster phase. Debt extension improves debtors' welfare in disasters and disciplines debt repayment for countries prone to disasters.

My paper is related to two strands of works. The first analyses emerging economies' sovereign debt risk as driven by income fluctuations (e.g. Aguiar &

Gopinath (2007) and Arellano (2008)). With a combination of income persistence and incomplete asset markets, their papers are able to generate countercylicality of debt interest rate. I take their models further and look at how an additional disaster state with two different transitional dynamics impact on a debtor's default risk and borrowing shedules in normal times. The second strand of works use disaster risks to improve asset price evaluation (e.g. Barro (2006), Barro et al.(2013) and Gourio (2008, 2015)). While they focus on credit spreads on corporate debt and stock market returns, I find disaster risk relevant in explaining sovereign debt spreads across a range of emerging economies.

The rest of this paper is organised as follows. Section 2.2 analyses a sample of emerging economies' output dynamics using a panel Markov-switching model and section 2.3 relates those economies' disaster probabilities with their interest rate spreads. Section 2.4 and section 2.5 present the model and derive its theoretical predictions. Section 2.6 and section 2.7 extend the model and present its quantitative analysis. Section 2.8 concludes by discussing potential extensions to the model.

2.2 Empirical estimation of income processes

My aim is to determine the type of disaster shocks (natural or economic) causing the extreme events that occur in emerging countries. To this end, I estimate a fixed effect panel Markov-Switching model of three states, using output data of emerging countries from two regions: Latin America (LA) and Southeast Asia (SeA). Countries selected in each panel are known to have experienced episodes of severe output disruptions over time. A panel Markov-switching model is chosen for estimation because I consider the output process to be driven by an unobserved random variable s_t which represents the states at which output fluctuates. The dynamics of s_t are captured by the transitional probabilities. If countries are subject to natural disaster shocks, estimated results would show similar likelihoods of output transiting from a boom to a disaster state and that from a recession to a disaster state. Economic disasters, on the other hand, would generate a higher probability of output moving from a recession to a disaster state than that from a boom.

Section 2.2.1 describes the data used and Section 2.2.2 explains the estimation of income process. Section 2.2.3 presents the estimation results.

2.2.1 Data

Output data are obtained from Barro and Ursua (2010)'s dataset. The original dataset includes both consumption and GDP per capita data for 41 developed and developing countries over a long time period. I select the following countries for estimation: SeA - Taiwan, Malaysia, Philippines, Sri Lanka, Indonesia and Korea; and LA - Argentina, Brazil, Mexico, Peru, Uruguay and Chile.

Output data for each selected country (in indices relative to 2006) are filtered with a linear trend. To ensure continuous data series for estimation, I drop points where there are gaps succeeding them. Data is pooled in each panel to increase the number of rare disaster episodes and make the estimations more efficient.

2.2.2 A fixed effect panel Markov-switching model

In this section, I describe the estimation of a fixed effect Markov-switching model of three states using a Bayesian inference approach. Bayesian approach is chosen because inference for latent variable s_t requires simulation based methods, which can be naturally included in a Bayesian framework.

I categories three income states corresponding to disaster (state 1), low-income (state 2) and high income (state 3). The states differ in mean and standard deviation. States 2 and 3 can be regarded as "normal" states across which output fluctuates over time. State 1 occurs with rarity and has exceptionally large impacts on both output level and volatility. Within each state, income fluctuates in an AR(1) manner. I assume that countries in the same panel share the same transitional dynamics: transitional probabilities and auto-regressive coefficient are the same across countries, but they differ in their means and standard deviations of incomes.

To determine the type of disaster shock accounting for the severe output drops of the selected emerging countries, the parameters of interest are the transitional probabilities of income from normal (low income or high income) to disaster state.

I follow Hamilton (1994) in modelling the income process as:

$$y_{k,t} - \mu_{k,S_t} = \phi_1(y_{k,t-1} - \mu_{S_{k,t-1}}) + \epsilon_{k,t}, \quad \epsilon_{k,t} \sim N(0, \sigma^2_{k,S_t}) \tag{2.1}$$

where $y_{k,t}$ denotes the deviation from trend at t for country k in the panel, ϕ_1 is the persistence parameter in the AR(1) process and μ_{k,S_t} and σ_{k,S_t} are individual's state-dependent mean and standard deviation of output. For each country k, its μ_{S_t} and σ_{S_t} are modelled as:

$$\mu_{S_t} = \mu_1 S_{1t} + \mu_2 S_{2t} + \mu_3 S_{3t} \tag{2.2}$$

$$\sigma^2_{S_t} = \sigma^2_1 S_{1t} + \sigma^2_2 S_{2t} + \sigma^2_3 S_{3t} \tag{2.3}$$

where the indicator variable $S_{jt} = 1$, if $S_t = j$ and $S_{jt} = 0$ otherwise, $j = 1, 2, 3$.

The transitional probabilities shared across countries in the same panel are:

$$p_{ij} = Pr[S_t = j | S_{t-1} = i], \quad \textstyle\sum_{j=1}^{3} p_{ij} = 1$$

p_{21} and p_{31} indicate how the economy transits from normal income states (S_2 or S_3) to the disaster state S_1. If $p_{21} > p_{31}$, it implies that the economy is subject to economic disasters whereas if $p_{21} \approx p_{31}$, the economy is subject to natural disasters. In addition, to characterise the states of boom, recession and disaster, I impose two restrictions on the mean and variance of income: 1) $\mu_1 < \mu_2 < \mu_3$; and 2) $\sigma_{S_j} > 0$, $j = 1, 2, 3$.

The parameters to be generated are as follows: for each $k \in \{1, 2, .., N\}$,

$$\boldsymbol{S_{k,1:T_k}} = (S_{k,1}, S_{k,2}, ..., S_{k,T_k})'$$

$$\boldsymbol{\sigma_k} = (\sigma_{k,1}, \sigma_{k,2}, \sigma_{k,3})'$$
$$\boldsymbol{p} = (p_{11,12}\ , p_{21}, p_{22}, p_{31,32}\ )'$$
$$\boldsymbol{\mu_k} = (\mu_{k,1}, \mu_{k,2}, \mu_{k,3})'$$
$$\phi_1$$

where $S_{k,t}, t = 1, ..., T$ denotes country k's state at each period. $(\boldsymbol{S_{k,1:T}}, \boldsymbol{\sigma_k}, \boldsymbol{\mu_k})$ are generated using individual country's data whereas $(\boldsymbol{p}, \phi_1)$ are generated using data across the panel. Define $\boldsymbol{\theta_k} = (\boldsymbol{S_{k,1:T}}, \boldsymbol{\sigma_k}, \boldsymbol{p}, \boldsymbol{\mu_k}, \phi_1)$.

Posterior estimates are obtained by iterating a Gibbs sampling algorithm. For each sample i out of M and each country k, I derive a posterior distribution of $\boldsymbol{\theta^{i+1}_{k,j}}$ for parameter j, given all the other parameters generated from previous sample $\boldsymbol{\theta^{i}_{k,-j}}$. M is set to 12000 and the first 2000 samples are discarded to avoid the dependence on the initial conditions of the sampler. The remaining samples are thinned down by a factor of five such that posterior samplers are less dependent. I explain in details the derivation of posterior distributions of $\boldsymbol{S_{k,1:T}}$, $\boldsymbol{\mu_k}$, $\boldsymbol{\sigma^2_k}$, ϕ_1 and $p_{i,j}$ in Appendix B.1.

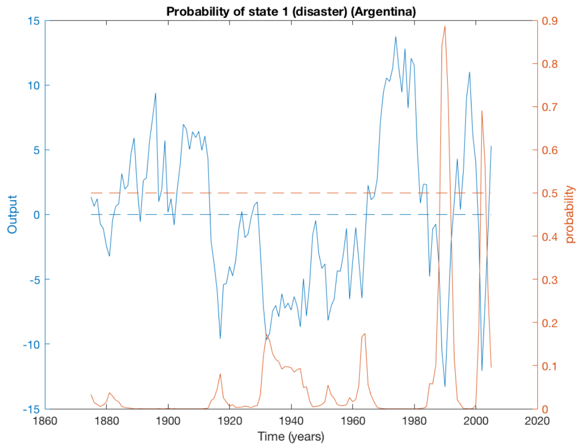
2.2.3 Estimation results

Although the method outlined in section 2.2.2 is not limited by the number of countries in a panel, in practice, as the number of countries increases, the heterogeneity across them reduces the accuracy of estimation of common parameters (ϕ_1 and $p_{i,j}$) and increases computation time substantially[1]. I divide my estimation of SeA countries into panels of four and two - one panel for relatively less developed ones: Indonesia, Malaysia, Philippines and Sri Lanka; and the other panel for developed ones: Taiwan and South Korea. For LA countries, panels of four (Argentina, Venezuela, Brazil and Chile) and three (Mexico, Peru and Uruguay) are estimated. As an example, I present estimated parameters and smoothed probabilities for Argentina where the results are estimated along with three other LA countries in a panel of four. This is

[1]Computation time is increased substantially because of the rejection sampling employed.

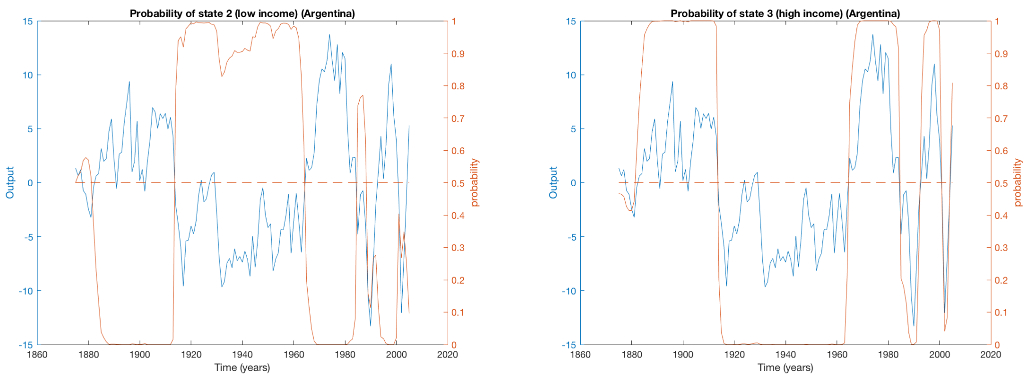
followed by a collection of transitional probabilities across the whole sample.

Figure 2.1: Probability of state 1 at each year, Argentina



Notes: $Pr(S_t = 1)$

Figure 2.2: Probability of state 2 and 3 at each year, Argentina



Notes: $Pr(S_t = 2)$, $Pr(S_t = 3)$

Argentina. Argentina has been subject to volatile output swings and it defaulted several times in history, notably in 1982-1983 and again in 2001-2005. This is reflected in Figures 2.1 and 2.2. The panel Markov-switching model identifies three regimes: disaster ($S_t = 1$), low income ($S_t = 2$) and high income ($S_t = 3$). In Fig.2.1, the model identifies an occurrence of disasters in mid-1980 and 2001, which are the times of LA debt crisis and Argentina's

Table 2.1: Estimated parameters (Argentina, 1875 - 2005)

Parameter	mean	sd	median	5% quantile	95% quantile
p_{11}	0.728887	0.141387	0.759213	0.486048	0.904333
p_{12}	0.135756	0.145329	0.081538	0.002677	0.435985
p_{21}	0.040422	0.025246	0.035862	0.008291	0.085575
p_{22}	0.939495	0.031846	0.944873	0.881613	0.978951
p_{31}	0.010221	0.010794	0.006384	0.000401	0.032852
p_{32}	0.037158	0.018527	0.033588	0.012966	0.073368
ϕ_1	0.370164	0.030546	0.370368	0.320418	0.421555
σ_1^2	22.051021	16.319644	18.198170	6.491548	54.030846
σ_2^2	6.887824	1.693807	6.658978	4.466252	9.947404
σ_3^2	11.116174	2.581112	10.724636	7.620075	16.034300
μ_1	-8.749646	0.283867	-8.745265	-9.225563	-8.307831
μ_2	-4.237918	0.372941	-4.233660	-4.849827	-3.628902
μ_3	4.192339	0.390070	4.168699	3.505045	4.862351

largest default in history. The smoothed probabilities for those two points in time are well above the 0.5 threshold. In other times, Argentinean output fluctuates across the low-income and high-income regimes, as shown in Fig.2.2, with roughly the same duration of time spent in each.

As presented in Table 2.1, estimates of the mean income levels and volatility differ substantially in the three regimes. Disaster state fluctuates around a very low level of mean income, with a much higher volatility. The panel Markov-switching model estimates show that the probability that a disaster state will be followed by another year of disaster is $p_{11} = 0.73$, so that this state will persist on average for $\frac{1}{1-p_{11}} = 3.7$ years. This is in line with the duration of debt crisis in Argentina. The other two normal states are more persistent, $p_{22} = 0.94$ and $p_{33} = 0.96$. The difference in persistence between the disaster regime and the other two normal regimes can also be seen graphically in Fig.2.2. p_{21} and p_{31} represent the probabilities that Argentina transits from a low or high income state to a disaster state. With p_{21} being four times as large as p_{31}, it shows that Argentina is much more likely to enter a disaster state if it is at a low income state.

Transitional probabilities. In Table 2.2, I present estimated transitional probabilities of p_{21} and p_{31} for the panels. Across the four panels, estimates of p_{21} are all greater than p_{31}, indicating that it is more likely for those countries to transit to a disaster when they are in a contractionary regime. Thus, according to the criterion set out in section 2.2.2, countries in the sample are subject to economic disasters. Both LA and SeA countries are frequently subject to regimes changes, financial crisis and trade sanctions, and therefore their output dynamics show a pattern as characterised by an economic disaster. The rest of estimated parameters and graphical illustrations of smoothed probabilities are in Appendix B.2.

Table 2.2: Estimated transitional probabilities, four panels

Parameter	mean	sd	median	5% quantile	95% quantile
LA panel 1					
p_{21}	0.040422	0.025246	0.035862	0.008291	0.085575
p_{31}	0.010221	0.010794	0.006384	0.000401	0.032852
LA panel 2					
p_{21}	0.025686	0.015615	0.022985	0.006937	0.055908
p_{31}	0.010073	0.007607	0.008574	0.000728	0.023529
SeA panel 1					
p_{21}	0.059492	0.033581	0.055333	0.015364	0.123716
p_{31}	0.017146	0.017776	0.011512	0.000413	0.053161
SeA panel 2					
p_{21}	0.098022	0.137142	0.045012	0.001606	0.371231
p_{31}	0.045211	0.041521	0.033909	0.001452	0.129815

2.3 Disaster probabilities and real interest rate

Having estimated the output dynamics of countries from Latin American and Southeast Asian regions, I establish an empirical association between countries' probabilities of disaster occurrence and their real interest rates in this section.

Disaster probabilities. To reflect countries' exposure to disasters overtime, I use long time annual series GDP per capita constructed in Barro and Ursua (2010) to calculate disaster probabilities[2]. In total, I have seven LA countries: Argentina, Brazil, Chile, Mexico, Peru, Uruguay, and Venezuela and six SeA countries (Indonesia, Malaysia, Philippines, South Korea, Sri Lanka, and Taiwan).

The disaster probabilities are calculated following Barro (2006) who uses peak-trough measurement of sizes of macroeconomic contractions. Proportionate decreases in GDP per capita are computed peak to trough over time, and only declines by 10% or greater were considered to be disaster episodes. For example, using the above method I identify eight disaster episodes from 1895 to 2008 in Argentina, three of which are associated with Argentinean sovereign crisis: 1958-1959 (-10.1%)[3], 1980-1982 (-11.1%) and 1998-2002 (-22%). Disaster probabilities are calculated by dividing the number of years a country is in disaster episodes by the normalcy years (subtracting the disaster years from the total number of observations). Details are included in Appendix B.3.

The average disaster probability calculated for SeA countries ranges between 1.79% and 5.41% with an average of 3.42%. Latin American countries are subject to large output contractions almost as twice as likely as their SeA counterparts, ranging between 3.03% and 11.43% with an average of 6.46%. The contrast between countries from the two regions is reflected in their real interest rate as in Figure 2.3.

Real interest rates. The interest rate measures the riskiness of an emerging country's external debt and I use two selection criteria in choosing the measure. Firstly the interest rates should be denominated in US dollars. Given the large swings in inflation rates in many emerging economies, it is difficult to attain

[2]Latin American countries' data spans from 1895 to 2009 whereas SeA countries' data is from 1911 to 2009.

[3]-10.1% is the percentage output drop from 1958 to 1959

a measure of domestic expected inflation to construct the real interest rate. Secondly the interest rate should reflect the true intertemporal cost faced by those economies and I use secondary market prices of emerging economies' bond. As pointed out by Neumeyer and Perrieri (2005), interest rate data on new loans denominated in U.S. dollars is not useful because during the financial crises most of the new borrowing of emerging countries is through official institutions, e.g. IMF, which do not reflect the true borrowing cost over time. Based on those two selection criteria, I use expected three-month interest rates as measure of real interest rate and it is J.P. Morgan's EMBI Global Stripped Spread measure (EMBI SSPRD)[4] + US 90 days T-Bill rate - US GDP deflator inflation rate. For countries not included in EMBI Global Index[5], I use other measures such as 90-days commercial paper rate for expected three-month interest rates. Details are included in Appendix B.3.

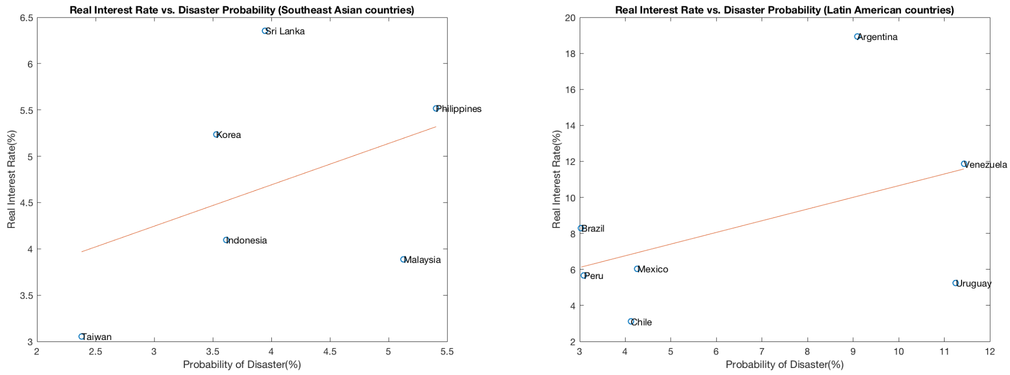
Although the first sets of J.P. Morgan's EMBI Global spread measures were only constructed in 1993 whereas the computation of disaster probabilities relies on long time series of over a hundred years, one can still establish a relationship between countries' disaster risk and their borrowing costs as the EMBI spread measures already embody a historical premium of emerging economies' credit risk.

Figure 2.3 plots the real interest rate on emerging countries' external debt against their disaster probabilities. The data plot displays a positive relationship for both SeA countries and LA countries, with a correlation of 0.41 for SeA, 0.46 for LA and 0.56 for both. SeA countries are less prone to disasters than LA ones and their real interest rates are also much lower, as shown in Figure 2.4. In particular, disaster probabilities of Argentina and Venezuela are the highest among the 16 countries in the sample and the risk premium on their government debt are also the highest. The sample statistics

[4]JP Morgan's EMBI Global family includes two relevant data types: Blended and Stripped Yield Spread (SSPRD). SSPRD differs from the more standard 'blended' spread because the values of any collateralized flows are stripped from the bond. SSPRD reflects the risk premium of emerging economies' bond.

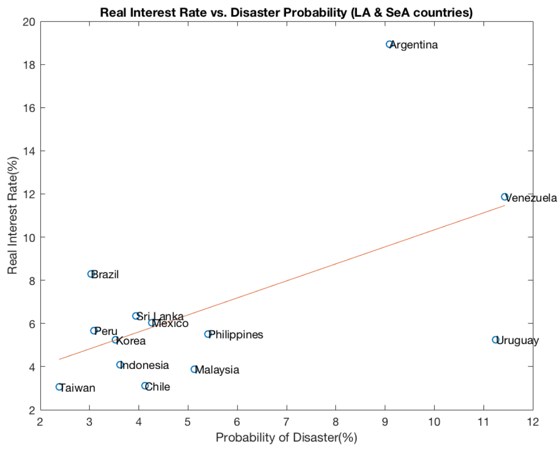
[5]Thailand and Taiwan are not included in EMBI Global Index.

Figure 2.3: Disaster probability and interest spread



(a) SeA countries **(b)** LA countries

Figure 2.4: Disaster probability and spread: SeA and LA countries



suggests that there is a positive relationship countries' exposure to disaster risk and their borrowing costs. Higher the likelihood that a disaster occurs, the higher the risk premium on the economy's external debt.

2.4 The Model

In this section, I study a small open economy model of sovereign default, based on Arellano (2008). I assume that debt is non-state contingent, that lenders are competitive and risk-neutral and that sovereign cannot commit to repay its debt. In this setting, default risk is reflected in bond price and it would also impact on countries' access to the credit market.

2.4.1 Model Environment

Households. There is a continuum of measure of one of infinitely lived households in the economy4 They have rational expectations and maximise their expected discounted lifetime utility. Households' preferences are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $0 < \beta < 1$ is the discount factor, c_t is the consumption in period t and $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ is the period utility function. $u(.)$ is increasing, strictly concave and satisfies the inada condition.

Households can neither save nor borrow. They receive a stochastic stream of exogenous endowment y_t which is non-storable. y_t is drawn from a compact set $Y = [\underline{y}, \overline{y}] \subset \mathbb{R}_+$. It is assumed to follow a Markov process with its probability distribution function of y_t conditional on the previous realisation y_{t-1} given by $f_y(y_t|y_{t-1})$.

The government. The government is assumed to be benevolent and maximises the utility of households. It has access to the international credit market and can trade in one-period bonds B' at price $q(B', y)$ to smooth households' consumption. The government also decides whether to repay or default on its debt. The bond price $q(B', y)$ depends on next period's debt obligation B' and current period endowment y because default probability depends on both. B' can be either ≥ 0 or < 0. If B' is positive, it means that the government enters a contract where it pays an amount of $q(B', y)B'$ at the present period in exchange for receiving B' units of goods next period. If B' is negative, it means that the government borrows an amount of $-q(B', y)B'$ now and pays back $-B'$ amount of goods next period, should it choose to repay. The proceedings from the government's international credit trading are rebated to households entirely in a lump sum fashion.

If the government chooses to repay its debts, the resource constraint for the economy is the following:

$$c = y + B - q(B', y)B'$$

Since the asset in the model, one-period bond, is non-contingent, the fluctuations in endowment induced by y cannot be insured away with the set of bonds available. In this setting, default acts as an *ex post* state contingency measure and expands the households' budget constraint.

The default costs is modelled as follows. If the government defaults, current debts are lifted from the government's budget constraint and it is excluded from the international financial market. The government remains in financial autarky for a stochastic number of periods and re-enters financial market with an exogenous probability. Default cost also includes a direct output element:

$$c = y^{def}$$

where $y^{def} = h(y) \leq y$ and $h(y)$ is an increasing function.

The government enters the period with B units of assets, income shock y. At the beginning of the period, sets of $(B,\ y)$ determine the government's resources to roll over the debt. It then decides whether to default or repay its debt. If it decides to repay, the bond price schedule $q(B', y)$ is taken as given, the government chooses B' subject to its budget constraint. Creditors then take q as given and choose B'. If it decides to default, it is excluded from financial market for a stochastic number of periods. Consumption c takes place.

Denote $\nu^0(B, y)$ the value of the government that has the option to default and that starts the current period with assets B and endowment y. Let $\nu^d(y)$ denote the value of a government which is excluded from international financial market and $\nu^c(B, y)$ the value of a government that has access to international financial markets. Note that the autarky value is independent of B because the country's debt is erased in default. Given (B, y), the government has first to decide whether to honour its debt or not:

$$\nu^0(B, y) = \max_{(c,d)} \left(\nu^c(B, y), \nu^d(y)\right)$$

The value of autarky, $\nu^d(y)$, is given as:

$$\nu^d(y) = u(y^{def}) + \beta \int_{y'} [\theta \nu^0(0, y') + (1 - \theta)\nu^d(y')] f(y', y) dy'$$

where θ is the exogenous probability that the government can re-enter the international credit market and it determines the length of periods that the government is excluded from the market. If the government is allowed to borrow and lend again, it will start with $B = 0$, and hence $\nu^0(0, y')$. Otherwise it remains in a financial autarky, the value of which is given by $\nu^d(y)$.

When the government chooses to remain in the debt contract, the value conditional on repay is as follows:

$$\nu^c(B, y) = \max_{(B')} u(y + B - q(B', y)B') + \beta \int_{y'} v^0(B', y')f(y', y)dy'$$

In addition, non-Ponzi schemes condition needs to be satisfied: $B' \geq -Z$.

Given a level of assets B, let $D(B)$ denote the set of y's for which the government finds it optimal to default:

$$D(B) = y \in Y : \nu^c(B, y) \leq \nu^d(y)$$

Foreign lenders. There are a large number of identical risk neutral foreign lenders who enter the market freely. A lender purchases b' bonds at the price $q(B', y)$ and receives b' in the following period unless the government defaults. It is assumed that in the lenders' asset portfolio, their alternative investment is a risk-free asset with a real return $1 + r$. An individual lender's expected present value, Π, from purchasing the government bonds is therefore:

$$\Pi = -q(B', y)b' + \mathbb{E}\tfrac{1-D'}{1+r}b'$$

Free entry implies that:

$$q(B', y) = \tfrac{1-\delta}{1+r}$$

where δ denotes the probability of default in the subsequent period.

2.4.2 Equilibrium

Let $s = \{B, y\}$ be the aggregate states for the economy.

Definition. The *recursive equilibrium* for this economy is defined as a set of policy functions for (i) consumption $c(s)$; (ii) government's asset holdings $B'(s)$, and default sets $D(B)$; and (iii) the price function for bonds $q(B', y)$ such that:

1. Taking as given the government policies, households' consumption $c(s)$ satisfies the resource constraint.

2. Taking as given the bond price function $q(B', y)$, the government's policy functions $B'(s)$ and default sets $D(B)$ satisfy the government optimisation problem.

3. Bonds prices $q(B', y)$ reflect the government's default probabilities and are consistent with creditors' expected zero profits.

Default probabilities $\delta(B', y)$ is linked to the default sets $D(B')$ by:

$$\delta(B', y) = \int_{D(B')} f(y', y) dy'$$

2.5 Quantitative Analysis

In this section I analyse the properties of the model by solving numerically a calibrated model. I calibrate the model to represent an emerging economy prone to disaster risks and carry out two sets of comparisons - 1) benchmark economy vs. an economy with an economic disaster risk; and 2) benchmark economy vs. an economy with a natural disaster risk. Two distinct patterns of impact from the two types of disaster risk are shown in the results section. In comparison with the benchmark economy, the presence of an economic disaster risk increases bonds' risk premium and the economy is associated with more stringent financial contract. In contrast, the presence of a natural disaster risk reduces an economy's default risk and entails higher bond price schedule and higher borrowing.

Calibration. I calibrate the model parameters from two sources: estimated parameter values from section 2.2.3 and Barro (2006)'s data on a wide range of countries' disaster episodes. Argentina experienced one of the largest defaults in history in 2002 and its sovereign bond spread has historically been high and volatile which exemplifies an emerging economy prone to high default risk.

One period is a quarter. The preferences and default costs are specified as:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{2.4}$$

$$h(y) = \begin{cases} \hat{y} & \text{if } y > \hat{y} \\ y & \text{if } y \leq \hat{y} \end{cases} \tag{2.5}$$

Equation (2.5) shows that output loss from default are asymmetric. This specification makes the value of autarky less sensitive to changes in endowment shocks and extends the range of debt B' that carry positive but finite default premium.

The stochastic output process consists of normal and disaster states. I follow Arellano (2008) in calibrating the normal income states to Argentina's quarterly real GDP from Q1 1980 to Q4 2003 from the Ministry of Finance (MECON). The real GDP is assumed to follow a log-normal AR(1) process as in (2.6). Tauchen and Hussy (1991) procedure[6] is employed to compute the endowment grid and Markov chain with 21 states. The 21 endowment states are percentage deviation from the linear trend with a mean value of zero.

$$log(y_t) = \rho log(y_{t-1}) + \varepsilon_t^y, \quad \text{with } E(\varepsilon^y) = 0 \text{ and } E(\varepsilon^2) = \eta_y^2 \tag{2.6}$$

Disaster risk is modelled by including an additional state of extremely low income, representing a disaster state. The addition of a disaster state extends the range of endowment fluctuations and enables the quantitative model to generate impacts of disaster risk. I use estimated parameter values

[6] The code is taken from Martin Flodon's teaching page (http://www2.hhs.se/personal/floden/Code.htm).

of Argentina to calibrate an output process with economics disaster risk and I calibrate parameters associated with natural disaster risk according to Barro (2006). To isolate the effect of disaster risk from the size of output contraction upon disaster occurrence, I calibrate income level of the disaster state to a single value for the two types of disaster. I set this value to represent a -40 % deviation from mean output[7], which is in line with the mean of the contraction sizes adjusted for trend growth as estimated by Barro (2006), 35%.

Using data covering 35 (OECD and non-OECD) countries during twentieth century, Barro (2006) records events which lead to more than 15 percent declines in real GDP. Those events include both natural disasters and economic crises. There are 60 such events for 35 countries over 100 years. Thus, the probability of one country entering into a 15 percent or greater event is 1.71% per year (0.43% per quarter). Natural disasters, such as hurricanes and earthquakes, occur unexpectedly overtime and the disaster likelihood is unrelated to current income shocks. I model the probability of an economy transiting to the disaster state as independent of previous income shocks:

$$P_{i,d} = Pr(y_t = y^d | y_{t-1} = y_i) = Pr(y_t = y^d | y_{t-1} = y_j), \quad \forall i, j = 1, 2, ..., 21, i \neq j$$

where $P_{i,d}$ denotes the transitional probability and y^d denotes the disaster income state. $y^d < y_1 < ... < y_{21}$. The transitional probability is set as 0.43%. Natural disasters are also known to be short-lived and I calibrate the disaster persistence to represent an instantaneous event. $Pr(y_t = y^d | y_{t-1} = y^d)$ is set as 0.01, corresponding to an on average $\frac{1}{1-0.01} = 1.01$ periods that an economy stays in a disaster state. Once the economy exits a disaster state, I assume that the economy returns to its mean output level - $Pr(y_t =$ mean-output state$|y_{t-1} = y^d) = 1 - p_{d,d}$.

Economic disasters, as caused by economic crisis commonly seen in Latin America during the 1980s - 2000s, are charaterised by a downward income

[7]The income level of disaster state is set such that one can find income states corresponding to the same percentage deviations from mean outputs in both the benchmark and the disaster risk models

trajectory. When the government finds it difficult to rollover its foreign debt, debt risk premium rises and as output falls further, sovereign debt crisis turns into an economic crisis at which output plummets. In section 2.2.3, data is on yearly frequency. The estimation results show that Argentina transits from a recession to a disaster state with a probability of 0.04. I assign this value to $Pr(y_t = y^d | y_{t-1} = y_r)$ where y_r denotes the income state representing a -5% deviation from mean. In the model, one period corresponds to a quarter and $Pr(y_t = y^d | y_{t-1} = y_r)$ on quarterly frequency should be greater than 0.04. As shown later in the analysis, the model is unable to generate any positive amount of borrowing for $Pr(y_t = y^d | y_{t-1} = y_r) > 0.04$. I postpone the discussion until later and carry on the analysis with a calibration of 0.04.

In section 2.2.3, disaster persistence is estimated as $p_{1,1} = 0.7$, corresponding to an on average 3.3 years (13.2 quarters) of remaining in a disaster state. I calibrate the disaster persistence in the model to $1 - \frac{1}{13.2} = 0.92$. The persistence reflects the dynamics associated with an economic disaster episode that disasters unfold over several years. The rest of the transitional probabilities is calibrated in the following way:

$$Pr(y_t = y^d | y_{t-1} = y_i) > Pr(y_t = y^d | y_{t-1} = y_j), \quad \forall i, j = 1, 2, ..., 21, i < j \tag{2.7}$$

Lower the income state, higher the likelihood that the economy transits to a disaster state in the subsequent period.

Table 2.3 summarises the parameter values.

Results (ii): economic disasters. In this section, I analyse policy functions of the model solved and compares the model statistics with a benchmark model without disaster risk.

Figure 2.5 shows the bond price schedule and the equilibrium interest rate the borrower faces in models with and without economic disaster risk. The comparison is made using income shocks that are 3% above their mean

Table 2.3: Model calibration

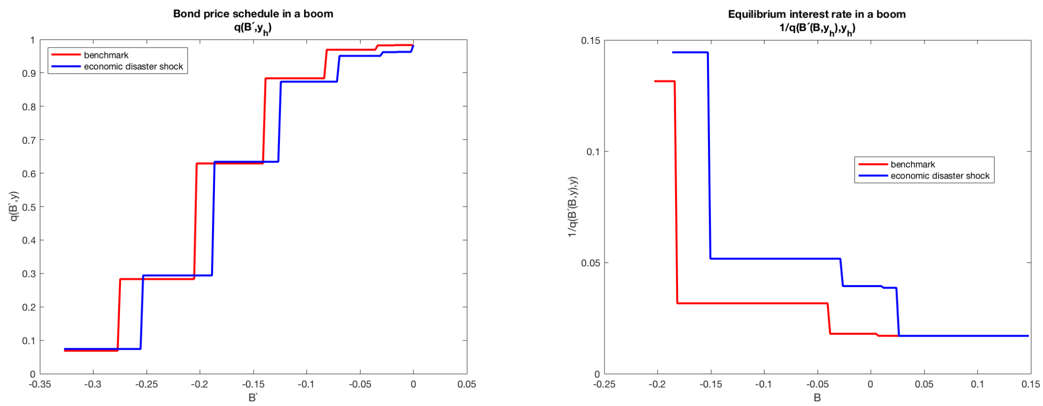
Parameter	Description	Value	Source
r	Risk free interest rate (%)	1.7	Arellano (2008)
σ	Risk aversion	2	Standard
β	Discount factor	0.953	Arellano (2008)
θ	Probability of re-entry	0.282	Arellano (2008)
$\hat{y}$	Output costs	$0.969E(y)$	Arellano (2008)
Endowment process			
ρ	Stochastic structure	0.945	Arellano (2008)
η	Stochastic structure	0.025	Arellano (2008)
Disaster shock			
y^d	Disaster magnitude (%)	-0.40	Barro (2006)
$p_{d,d}$ (natural disaster)	Disaster persistence	0.01	Estimation
$p_{i,d}$ (natural disaster)	Transitional probability (%)	0.4	Barro (2006)
$p_{d,d}$ (economic disaster)	Disaster persistence	0.91	Estimation
$p_{r,d}$ (economic disaster)	Transitional probability (%)	4	Estimation

incomes, which represent a boom. I choose to base the comparison on the same income shock in relative terms, rather than on absolute terms. The reason is that the economy with an economic disaster risk has an additional low income state and the 3% shock in the benchmark would imply a higher level of income in relative terms, in the economic disaster economy.

The left panel of figure 2.5 plots the price schedule, which constitutes the set of contracts $\{q(B', y), B'\}$ the borrower can choose from in both economies at every period. Bond prices are an increasing function of assets implying that low asset position (large levels of debt) carry higher interest rates. It is shown that for the same asset, B' (reported as a ratio of mean output), the economy with economic disaster risk is associated with more constraining financial contracts. The bond price is lower and the interest rate charged for every loan size is higher.

The right panel of the figure shows the interest rate $\frac{1}{q(B',y)}$ both economies pay along the equilibrium path in state $\{B, y\}$ given their choices of borrowing $B'(B, y)$. If assets relative to output is below -0.17, the borrower defaults in the economy with an economic disaster risk while she continues borrowing in the benchmark economy. The comparison shows that in the presence of an economic disaster risk, the borrower pays higher interest rates in equilibrium and has stricter borrowing limits than in the economy without: $\bar{B}^b(y_h) < \bar{B}^e(y_h)$.

Figure 2.5: Bond price and equilibrium interest rate in a boom: economic disaster

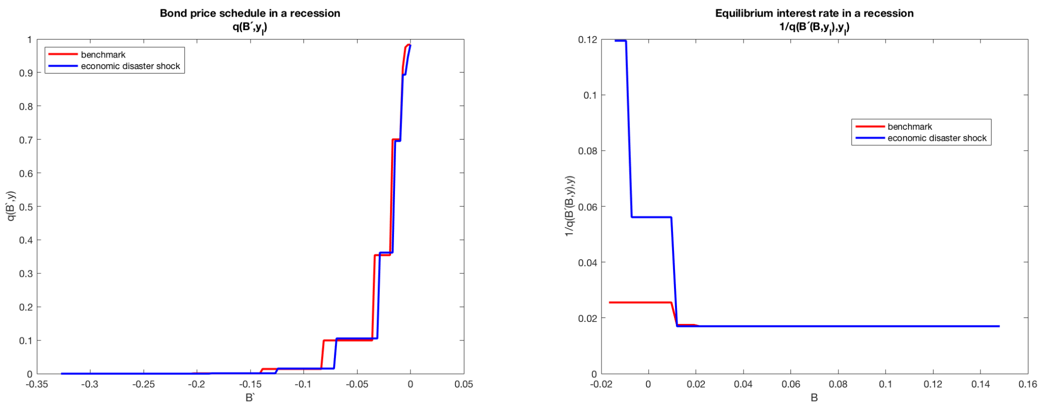


The result of higher default risk holds if the economies are in a recession. Figure 2.6 compares the bond price schedule and equilibrium interest rate in the two economies given a shock -5% below their mean incomes. As in the case of a boom, the economy with an economic disaster risk faces lower bond price schedules and higher interest rates for every loan size. In the right panel of figure 2.6, the debtor faces an interest rate which exceeds that in the benchmark model by a level of 10%. The maximum amount of debt a debtor can borrow in a recession is also lower.

Across figures 2.5 and 2.6, the policy functions illustrate two points. Firstly, within the same economy, the model generates a counter-cyclical interest rates and borrowing constraints, with booms associated with more lenient financial contracts than recessions. The reason is that default is preferable when income is low and given that shocks are persistent, a low shock today means that the shock will likely be low again tomorrow. This causes a debtor to default for even a small amount of debt in recessions.

Secondly, across the economies, the presence of an economic disaster risk increases a debtor's default risk and makes his financial contracts more stringent. An economic disaster shock alters the income process in three ways: firstly it makes a country poorer in terms of mean output level; secondly the output is more volatile; and thirdly when the country is in a recession, compared to a boom, the country faces a higher likelihood of transiting to a disaster state in the following period. These features of the income process increase the risk-averse borrower's incentive to insure *ex ante*, but they also make the country more likely to default *ex post*. The transitional dynamics of an economic disaster creates a downward trajectory towards default once a recession occurs. A recession today means that there is a 4% likelihood of disaster occurring next period, and given that income shocks are persistent, if a disaster does not occur in the following period, it is likely to be in a recession again facing the same risk of disaster. The coupling of income persistence and disaster transitional dynamics makes a borrower's debt particularly risky in a low income state.

Figure 2.6: Bond price and equilibrium interest rate in a recession: economic disaster



I now illustrate the two policy functions of the government in Figure 2.7. The left panel presents the savings policy function $B'(B, y)$ conditional on not defaulting as a function of assets B for a high y shock. Savings B' and assets B are reported as percentage of mean output. When the asset-output ratio, B, is above -0.15, the economic disaster economy borrows less than the benchmark economy because the interest rates on these loans are higher. As its wealth becomes positive, the disaster economy saves more than the benchmark economy, implying a higher incentive to insure. What is puzzling is the disaster economy's borrowing at $B < -0.15$. Its borrowing limit is higher than the benchmark economy, but it borrows more heavily until B reaches the -0.15 threshold.

The second policy the borrower has is to default or not. Figure 2.7 shows comparison of the value of the option to default or repay, $\nu^o(B, y)$, as a function of assets B for a high income shock in both economies. For a given output shock, the economy finds it optimal to default below an asset threshold. In the right panel, in the benchmark model, default is chosen for assets less than -20.5% of mean output when y is 3% above the mean. In the model with economic disaster risk, the default threshold is higher - -18.6% of mean output.

Figure 2.7: Savings and value functions in a boom: economic disaster

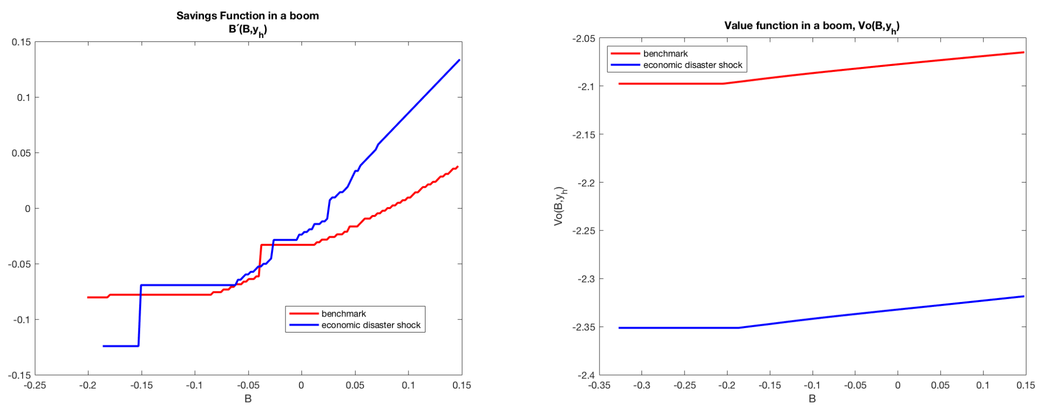
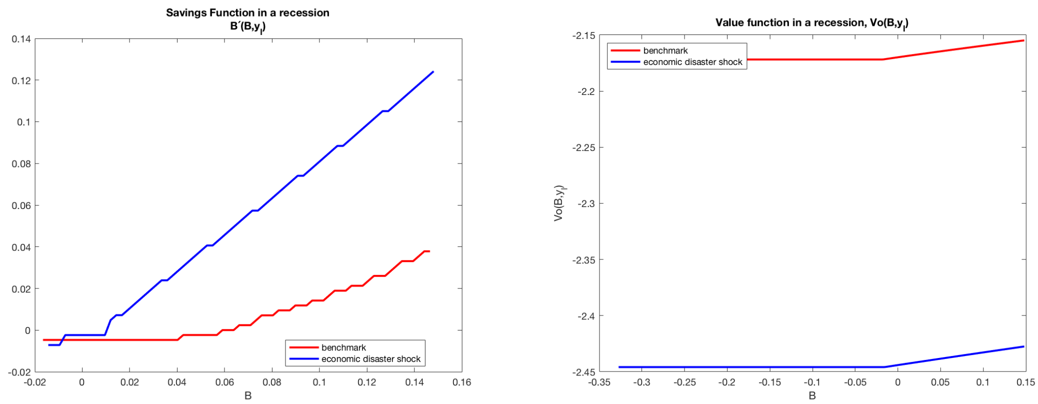


Figure 2.8 presents the two policy functions given a low income shock. Similarly to the savings function with a high income shock, left panel of figure 2.8 shows that compared to the benchmark economy, the disaster economy has higher borrowing limit. The amount of borrowing is lower for $B > -0.01$ and savings is higher when wealth is positive. When wealth is small below -0.01, the disaster economy borrows more heavily than the benchmark economy. In the right panel, given an income shock -5% below the mean output, the benchmark economy defaults for assets less than -1.91% of mean output and the economy with economic disaster risk defaults for a higher ratio of -1.67%.

Within the same economy, the counter-cyclical default risk generates a lower default threshold in booms than in recessions. Across the economies, given the same income shock, the presence of an economic disaster makes the borrower default for higher level of assets. Since the borrowing is more expensive with a disaster risk, the economy borrows less when its wealth is above a negative threshold and it saves more when its wealth is positive. When its asset is below a certain negative threshold, the disaster economy borrows more even though its borrowing cost is higher. This is at odds with the more stringent borrowing conditions it has.

Figure 2.8: Savings and value functions in a recession: economic disaster



I now turn to compare the simulated statistics of the models. Table 2.4 reports mean, standard deviation, correlation of the statistics and mean percentage deviations for the two models. In Arellano (2008), the model is simulated over time to find 100 default events. 74 observations are extracted before the default event and mean statistics are reported from these 100 samples. I follow the same simulation procedure in order to compare the statistics with the benchmark model. I conduct 50000 simulations of the model economy with 152 periods in each simulation. 74 observations are extracted prior to the default event in the stationary distribution to compute the statistics.

Both models are able to generate a higher volatility of consumption relative to income and countercyclical interest rates. The presence of an economic disaster risk increases both consumption and output standard deviations. The output contraction during default episodes is also larger. These are obvious

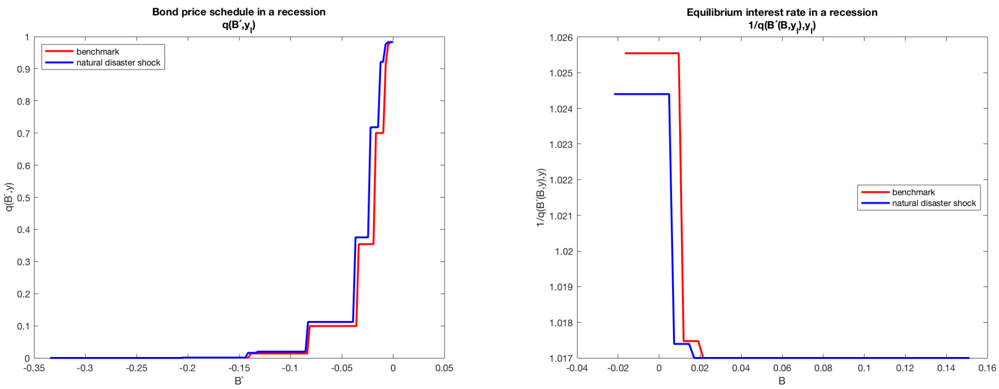
Table 2.4: Business cycle statistics comparison: benchmark and economic disaster

	Benchmark	Economic disaster
Consumption sd. (%)	6.56	10.3
Consumption sd./output sd.	1.07	1.04
Trade balance sd. (%)	1.30	1.34
Output sd. (%)	6.16	9.92
Bond spreads sd. (%)	9.30	9.43
Corr (consumption, output)	0.98	0.98
Corr (trade balance, spreads)	0.55	0.58
Corr (spreads, output)	-0.30	-0.17
Output drop during default episodes[8]	9.12	32.0
Mean debt (percent output) (%)	5.00	5.18
Mean spread (%)	4.00	5.00

results following an additional state of extremely low income. The economic disaster model generates a higher level of mean spread, but the level of mean debt is similar in both models. As shown in the savings function, though the disaster economy has a higher borrowing limit, it borrows a larger amount when its asset is below a certain negative threshold. The heavier borrowing for the low asset position counteracts the impact of higher borrowing limit and as a whole the disaster economy has a mean debt level similar to the benchmark economy.

[8]Measured as mean percentage deviation from mean income in default.

Figure 2.9: Bond price and equilibrium interest rate in a recession: natural disaster



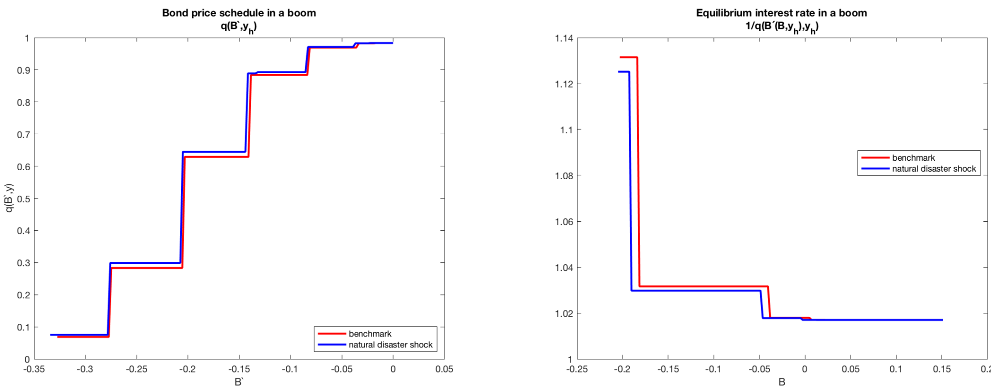
Results (i): natural disasters. In this section, I analyse policy functions of the model with a natural disaster risk and compares them with the benchmark model.

Figure 2.9 plots the bond price schedule and the equilibrium interest rate the borrower faces in both economies, given a negative income shock (-5% deviation from the economies' perspective mean incomes). In the left panel, the bond price $q(B', y)$ is higher on every loan size in the economy with a natural disaster risk, implying a lower default risk. The right panel of figure 2.9 shows the interest rate, $\frac{1}{q(B',y_l)}$, the economy pays along the equilibrium path in state $\{B, y_l\}$ given its choice of borrowing $B'(B, y_l)$. In contrast to the case of economic disaster risk, the economy with a natural disaster risk pays a lower interest rate on its equilibrium path. The right panel also shows that in the benchmark economy, the borrower defaults for assets ratio below -0.02, whereas in the economy with a natural disaster risk, it can still borrow.

The size of output contraction upon disaster occurrence is the same across both types of disasters. Similar to the case of economic disaster, a natural disaster shock reduces the mean output level and increases output volatility. These features increase the risk-averse borrower's incentive to insure *ex ante*, but they also make the country more likely to default *ex post*. An output process with a natural disaster risk differs from an economic disaster risk in two aspects. Firstly the probability that a normal income state is followed by a disaster state is smaller and each state of normal income has the same likelihood of transiting to the disaster state. Secondly the disaster state is less persistent and returns to its mean output level once it exits the disaster state.

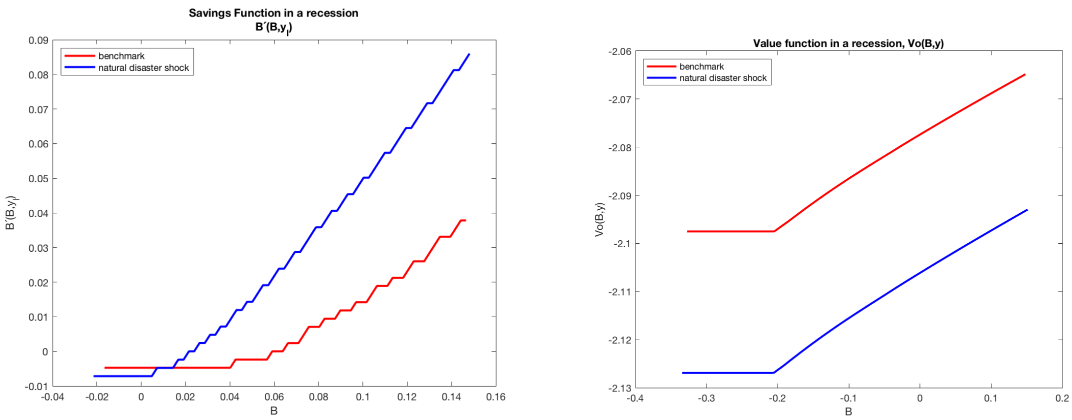
The above differences in output dynamics not only make a natural disaster economy less risky than the one with an economic disaster risk, they have also generated lower default risk than in the benchmark economy. Despite an additional state of large output contraction, the *ex ante* insurance effect generated by the presence of a natural disaster shock outweigh the *ex post* higher default risk. Facing a constant probability of a disaster occurrence in the subsequent period, the government has a strong incentive to oblige its debt contract so that it can engage with the international financial market and build up assets for insurance against the dire state. I illustrate the same disaster impact for a positive income shock y_h, in figure 2.10.

Figure 2.10: Bond price and equilibrium interest rate in a boom: natural disaster



I further demonstrate the impact of natural disaster shock in figure 2.11. The left panel of Figure 2.11 plots the savings function $B'(B, y)$ against current asset levels conditional on not defaulting, given a negative output shock y_l. Savings B' and assets B are reported as percentage of mean output. It shows that when wealth is negative, the borrower borrows more with a looser borrowing limit in the presence of a natural disaster risk whereas when wealth is positive, it saves more. The larger amount of borrowing in a recession is en-

Figure 2.11: Savings function and value functions in a recession: natural disaster



abled by the more lenient financial contract the debtor faces. When the wealth improves, the strong insurance motive drives the higher savings to build up its assets.

The right panel of Figure 2.11 shows the value of the option to default or repay, $\nu^o(B, y)$, as a function of assets B for a low y shock. In contrast to the comparative statics in the model with an economic disaster risk, the economy with a natural disaster risk defaults at a lower asset threshold. In the benchmark economy, the borrower defaults at an asset to output ratio of -1.91% whereas in the economy with natural disaster risk, it defaults at a ratio of -2.2%.

The impact of natural disaster risk on savings functions and default threshold is illustrated in figure 2.12 for a positive income shock, y_h. The model generates the same default threshold in both economies, but as shown in the left panel, the presence of a natural disaster risk makes the debt cheaper and enables the economy to borrow more when wealth is small. For an asset ratio above -0.1, the natural disaster economy borrows less and when wealth is positive, it saves more.

I now compare the simulated statistics of the models. Table 2.5 reports

[9] Measured as mean percentage deviation from mean income in default.

Figure 2.12: Savings function and value functions in a boom: natural disaster

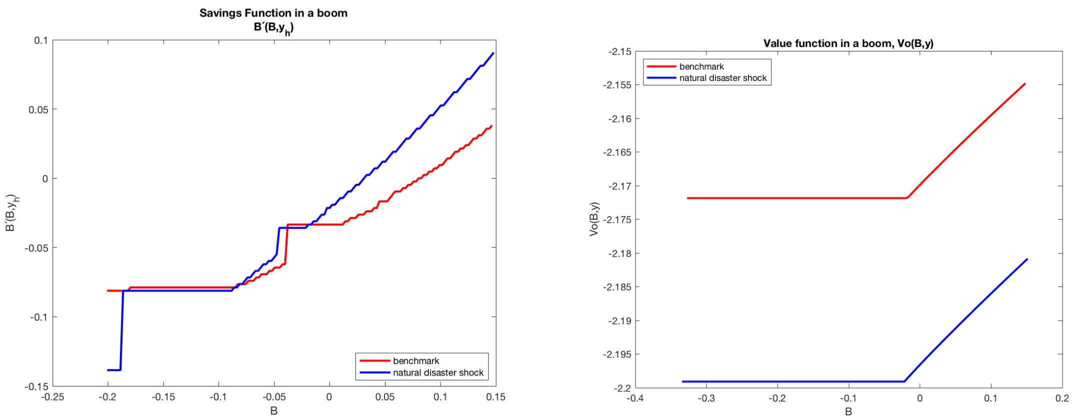


Table 2.5: Business cycle statistics comparison: benchmark and natural disaster

	Benchmark	Natural disaster
Consumption sd. (%)	6.56	7.41
Consumption sd./output sd.	1.07	1.01
Trade balance sd. (%)	1.30	1.38
Output sd. (%)	6.16	7.57
Corr (consumption, output)	0.98	0.98
Corr (trade balance, spreads)	0.55	0.32
Corr (spreads, output)	-0.30	-0.41
Output drop during default episodes[9]	9.12	9.36
Bond Spreads Std. Dev. (%)	9.30	8.13
Mean debt (percent output) (%)	5.00	5.18
Mean spread (%)	4.00	2.94

mean, standard deviation, correlation of the statistics and mean percentage deviations for the two models. The simulation procedure is the same as in the case of economic disaster risk. Comparing with the benchmark model, the presence of a natural disaster risk increases standard deviations of output and consumption, and lowers the size of output contraction during default episodes. These features of difference are shared by the model with an economic disaster risk. In contrast to the economic disaster model, the one with a natural disaster risk generates a lower level of spread (2.94% vs. 4.00%). The mean debt ratio, however, is similar in both economies.

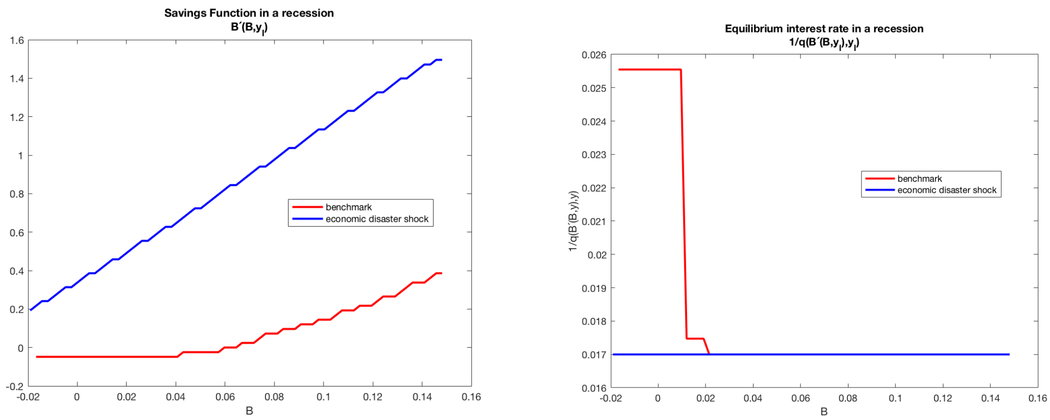
Discussion. The model has a few limitations. In the economy with an economic disaster risk, a calibration of $p_{r,d} = 0.04$ generates a positive amount of borrowing in a recession. However, as illustrated in fig.2.13, the debt risk is sensitive to the value of $p_{r,d}$ and a slight increase makes the debt so risky that no borrowing contract exists for any negative amount of asset. This restricts the ability of the model to generate useful comparative statics for economies with higher disaster risk. Interestingly, the model is relatively insensitive to the calibration of disaster persistence. Changing the value of $p_{d,d}$ from 0.9 to 0.7 yields similar statistics on mean debt and mean spread in the simulation exercise.

The calibration of the natural disaster risk is limited by the availability of data on natural disaster events and the calibration in this section can be seen as somewhat arbitrary. The result of lower default risk is sensitive to values of 1) disaster persistence, $p_{d,d}$; and 2) the probability at which a disaster state is followed by a normal income state. These parameters determine the default risk of debt in the natural disaster economy. The model is relatively insensitive to the probability that a normal state is followed by a disaster state. As one increases the disaster probability of 0.4% to 1%, the default risk is still lower than the benchmark economy.

Although the comparative statics demonstrate that when compared to the benchmark economy, the debt limit is stricter in the case of economic disaster risk and looser in the case of natural disaster risk, both disaster models are unable to generate mean debt-output ratio at levels different from the benchmark economy. In the case of economic disaster, though the economy defaults at a higher level of asset, it also borrows more heavily when its wealth is below a negative threshold. These two features are at odds with each other as the former is consistent with a higher default risk in the presence of an economic disaster risk but the latter contradicts it. Further work is needed to understand the causes.

As in Arellano (2008), the model also produces a level of mean spread

Figure 2.13: Savings function and equilibrium interest rate in a recession: economic disaster



that is much lower than the mean spread in Argentina. In the presence of an economic disaster risk, interest rate is higher on every size of loans but it still falls short of Argentina's mean spread of 10.25%.

Despite the shortcomings, the comparative statics illustrate that given the same level of income at each state, the model is capable of generating two patterns of impact from a natural disaster risk and an economic disaster risk. Importantly, the higher default risk in the presence of an economic disaster risk is consistent with the empirical relationship between disaster probability and interest rate in section 2.3.

2.6 Extension: debt renegotiation and debt extension in disaster state

In this section, I extend the model to feature debt renegotiation upon default and debt extension when natural disaster occurs. In the model outlined in previous sections, given the non state-contingent contract, default is costly for both debtors and creditors. Once countries default on their debt, creditors lose their debt claims and debtors are excluded from the international finance market and remain in autarky for an exogenous period of time. If a default is caused by a disaster shock, the welfare loss on debtor's side is particularly severe.

I construct an alternative debt contract to improve debtor and creditor's welfare based on the original one-period zero-coupon debt contract. In addition to complete default, it entails two forms of state contingency: debt renegotiation and debt extension. Debtors and creditors engage in debt renegotiation upon default in a Nash bargaining fashion. Welfare loss following default is reduced as creditors regain a fraction of the defaulted debt and debtors obtain access to the credit market through renegotiation over debt reduction. Debt extension during a disaster phase means that if debtors choose to stay in the contract, they can postpone their debt repayment until the economy recovers and they still have access to credit market while in a disaster.

2.6.1 The model environment

Households. There is a continuum of measure of one of infinitely lived households in the economy. Their preferences are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

Each period, households are subject to two types of exogenous shocks: an endowment shock y_t and a natural disaster shock z_t, and the value of their current income is denoted as $\delta^{z_t} y(t)$. The endowment shock y_t is a stochastic stream of non-storable consumption good. y_t is drawn from a compact set $Y = [\underline{y}, \bar{y}] \in R_+$. The probability distribution function of a shock y_t conditional on the previous realisation y_{t-1} is $\mu_y(y_t|y_{t-1})$.

For the natural disaster shock, I model it differently from the previous sections. z_t takes two values: 0 and 1. z_t follows a Markov process independent of y_t. When $z_t = 1$, a natural disaster occurs, and the economy incurs a reduction of its income proportional to y_t. Its current income becomes δy. When $z = 0$, the economy is in the non-disaster states. The probability distribution function of a shock z_t conditional on the previous realisation z_{t-1} is $f_z(z_t|z_{t-1})$. In this way, the impact of disaster shock is dependent on the current economic condition of the debtor which is usually the case in practice. It would also imply that the option of obliging a contract with debt extension is more valuable to a country in a recession than in a boom.

The government. In this economy, the government is benevolent and maximises the representative household's expected lifetime utility. The government is not obliged to repay its debt and may choose to default if the value of default exceeds that of repayment. I introduce a discrete variable $h \in \{0, 1\}$ denoting a country's credit history. When $h = 0$, a country has a good credit record with no unresolved default. A good credit record gives the country two benefits. Firstly, if it defaults on its debt $b < 0$, the debt is reduced to a fraction through renegotiation. Secondly, for a debtor experiencing a disaster shock ($z_t = 1$), it gives the debtor the benefit of debt extension. If the debtor defaults on their debt, the credit record h deteriorates to $h = 1$ next period.

If a country has a bad credit history $h = 1$ and unpaid debt $b < 0$, it has an unsolved default. The country is excluded from the credit market and incurs a direct output cost. Immediately following a default, creditors and debtors engage in a Nash bargaining game to renegotiate on the fraction of the defaulted debt the debtor needs to pay back. The fraction depends on the defaulted debt b and the current endowment shock (y or δy in the case of disaster) because those two variables determine the surplus of creditors and debtors. The debtor country then starts paying back the reduced debt but it cannot borrow additional amount while being excluded from the credit market. Once it pays back all the reduced debt, it regains access to the market. When $h = 1$, repaying the debt out of the endowment is very costly for a country in a disaster state.

Each period, the government makes default decision $D = 0/1$ and decides on its asset position for the next period, given the current endowment shocks y and z and asset position b. Let $v(b, h, y, z) : L \to R$ denote the value function for the country which has the debt obligation b from last period, starts current period with the credit record h and faces the current endowment shocks y and z, where $L = B \times 0 \times Y \times Z \cup \underline{B} \times 1 \times Y \times Z$. The space of bond price functions is $Q = \{q|q(b, y, z) : B \times Y \times Z \to [0, 1/(1+r)]\}$ where the highest bond price is obtained when $b \geq 0$. The space of debt recovery schedules is $A = \{\alpha|\alpha(b, y, z) : B \times Y \times Z \to [0, 1]\}$. Given $q \subset Q$ and $\alpha \subset A$, the country solves its optimisation problem.

I now define each value function in the recursive equilibrium. When $h = 0$, the country has a good credit record. If $z = 0$, it faces a decision of honouring the debt obligation or defaulting. If it chooses to repay, it determines its next-period asset b' and consumes. The good credit record carries over to next period $h' = 0$. If it defaults, it is immediately excluded from the credit market and its credit record deteriorates to 1 next period $h' = 1$. The country's debt is reduced to $\alpha(b, y, z = 0)b$ as a result of debt renegotiation.

If $z = 1$, a disaster occurs. For a country with good credit record, $h = 0$, debt extension is available. By staying in the contract, the country's previous debt obligation is postponed until it exits the disaster state and it can issue new bonds during disaster phase at a risk free rate r to smooth consumption. If the country chooses to default in a disaster phase, the same procedure follows the case when $z = 0$. The debt is reduced to $\alpha(b, y, z = 1)b$.

In comparison to previous section's model, the country's default decision now depends on two additional variables: the debt repayment schedules and the disaster shock z. The debt repayment schedules $\alpha(b, y, z)$ affect the country's ex-ante default incentive as it determines how much debt it has to repay following default and it is itself dependent on z. z affects the value of autarky in a disaster state and value of repayment entailing debt postponement and debt extension. Given $h = 0$ and (b, y, z), the value function is:

$$\nu(b, 0, y, z) = \max\left(\nu^r(b, 0, y, z), \nu^d(b, 0, y, z)\right)$$

Where $\nu^r(b, 0, y, z = 0)$ is:

$$\nu^r(b, 0, y, z = 0) = \max_{c, b' \in B : c + q(b', y)b' = y + b} u(c) + \beta \int_{Y,Z} \nu(b', 0, y', z') d\mu_y(y'|y) df_z(z'|z)$$

and $\nu^r(b, 0, \delta y, z = 1)$ is:

$$\nu^r(b, 0, \delta y, z = 1) = \max_{c, b' \in B : c + \frac{b' - b}{1+r} = \delta y} u(c) + \beta \int_{Y,Z} \nu(b', 0, y', z') d\mu_y(y'|y) df_z(z'|z)$$

The value of default $\nu^d(b, 0, y, z)$ is:

$$\nu^d(b, 0, y, z) = u((1 - \lambda)\delta^z y) + \beta \int_{Y,Z} \nu(\alpha(b, y, z)b, 1, y', z') d\mu_y(y'|y) df_z(z'|z)$$

When $h = 1$, the country has a bad credit record and it is excluded from credit market. It cannot borrow new debt and incurs a direct output cost λy. To regain access to the market, it needs to pay back the renegotiated debt $\alpha(b, y, z)b$ at an interest rate of $1 + r$. The credit record h remains bad as long as there is unpaid debt arrears. The value function for $h = 1$ is:

$$\nu(b, 1, y, z) = \max_{c, b' \in [b, 0] : c + \frac{b'}{1+r} = (1-\lambda)\delta^z y + b} u(c) + \beta \int_{Y,Z} \nu(b', 0, y', z') d\mu_y(y'|y) df_z(z'|z)$$

The exclusion from the credit market puts a lower bound on the country's asset position next period b' and the country will eventually repay the unsolved debt $\alpha(b, y, z)b$ as the endowment shocks improve.

I follow Yue (2009) in modelling the debt renegotiation process. Both creditors and a debtor country engage in a Nash bargaining game following a default. It is assumed that there is only one round of bargaining for one default.

The value of such an agreement to a debtor country comes from the fact that their debt is reduced to a fraction and they would be able to return to credit market after a finite period of time. If the debtor chooses not to renegotiate, it will be in permanent autarky:

$$\nu^{aut}(y,z) = u((1-\lambda)\delta^z y)) + \beta \int\limits_{Y,Z} \nu^{aut}(y',z')d\mu_y(y'|y)df_z(z'|z)$$

Given endowment shocks (y,z), defaulted debt b and the debt recovery rate a, the debtor's surplus from renegotiation is:

$$\Delta^B(a;b,y,z) = [u(\delta^z y) + \beta \int\limits_{Y,Z} \nu(ab,1,y',z')d\mu_y(y'|y)df_z(z'|z)] - \nu^{aut}(y,z)$$

For the creditor, the gain from renegotiation is the present value of reduced debt:

$$\Delta^L(a;b,y,z) = -\frac{ab}{1+r}$$

As in all Nash bargaining games, the outcome depends on the bargaining power of perspective party, which I denote as θ. θ is assumed to be between 0 and 1 to ensure that there is a unique optimum for each pair of b and y.

Given the debt b and endowment shocks (y,z), the debt recovery rate $\alpha(b,y,z) \in A$ solves the following bargaining problem:

$$\begin{aligned} \alpha(b,y,z) \quad &= \arg\max_{\alpha\in[0,1]} [(\Delta^B(a;b,y,z))^{\theta}(\Delta^L(a;b,y,z))^{1-\theta}] \\ s.t. \quad \Delta^B(a;b,y,z) \quad &\geq 0; \\ \Delta^L(a;b,y,z) \quad &\geq 0 \end{aligned}$$

The renegotiation outcome is welfare-enhancing because $\alpha(b,y,z)$ is restricted such that the surplus of both parties is nonnegative. It provides better insurance to the country if it decides to default.

International investors. There are a large number of risk-neutral international investors and they have perfect information of the endowment shocks and asset position of the country that they trade with. The asset market is incomplete - only one-period zero-coupon bonds are traded. The face value of the debt is denoted as b and the set of bond face values is $B = [b_{min}, b_{max}] \in R$, where $b_{min} \leq 0 \leq b_{max}$. The investors have access to borrowing and lending at a risk-free interest rate r and they are always commit to repay their debt.

I define investors' maximisation problem when $z = 0$. When $z = 1$, the extended lending is rolled over at the risk-free interest rate as part of the contract. Foreign investors are risk neutral and therefore they are willing to purchase the country's asset claim b' (debt claim if $b' < 0$; savings if $b' > 0$) if it maximises their expected profit, given by:

$$\pi(b', y) = \frac{[1 - p(b', y) + p(b', y)\gamma(b', y)]}{1 + r}(-b') - q(b', y)(-b')$$

where $p(b', y)$ is the probability of default for a country with asset position b' and endowment shock y. $\gamma(b', y)$ is the expected recovery rate, given by the expected proportion of defaulted debt that the creditors can reclaim, conditional on default.

Since the debt market is competitive, creditors can only earn zero expected profit in equilibrium. This gives a bond price schedule as:

$$q(b', y) = \frac{1 - p(b', y)[1 - \gamma(b', y)]}{1 + r}$$

The bond price compensates the creditors for the default risk and debt restructuring. For every unit of defaulted debt, creditors expect to recover a fraction of $\gamma(b', y)$.

2.6.1.1 Recursive Equilibrium

I define a stationary recursive equilibrium in the model economy:

Definition. A *recursive equilbrium* is a set of functions for *(i)* the country's value function $\nu^*(b, h, y, z)$, asset holdings $b'^*(b, h, y, z)$, default set $D^*(b)$, consumption $c^*(b, h, y, z)$, *(ii)* recovery rate $\alpha^*(b, y, z)$, and *(iii)* pricing function

$q^*(b', y)$ such that:

1. Given the bond price function $q^*(b', y)$ and debt recovery rate $\alpha^*(b, y, z)$, the value function $\nu^*(b, h, y, z)$, asset holdings $b'^*(b, h, y, z)$, consumption $c^*(b, h, y, z)$, and default set $D^*(b)$ satisfy the country's optimisation problem.

2. Given the bond price function $q^*(b', y)$ and value function $\nu^*(b, h, y, z)$, the recovery rate $\alpha^*(b, y, z)$ solves the debt renegotiation problem.

3. Given the recovery rate $\alpha^*(b, y, z)$, the bond price function $q^*(b', y)$ satisfies the zero expected profit condition for foreign ivnestors, where the default probability $p^*(b', y)$ and expected recovery rate $\gamma^*(b', y)$ are consistent with the country's default policy and renegotiation agreement.

In equilibrium, the default probability $p^*(b', y)$ is:

$$p^*(b', y) = \int\limits_{D^*(b')} d\mu_y(y'|y)$$

Where the default set is defined as:

$$D(b) = y \in Y : \nu^r(b, 0, y) \leq \nu^d(b, 0, y)$$

The default set is defined for y only. This is because the extended debt when $z = 1$, debt is priced at risk free rate and there is no q involved.

The expected recovery rate $\gamma * (b, y)$ in equilibrium is determined by:

$$\gamma^*(b', y) = \frac{\int\limits_{D^*(b')} \frac{\alpha^*(b', y', z')}{1+r} d\mu_y(y'|y)}{p^*(b', y)}$$

2.7 Quantitative analysis

In this section, I calibrate the model to analyse quantitatively the impact of endowment shocks on sovereign debt.

2.7.1 Calibration

I calibrate the model based on Yue (2009). One period is a quarter. The utility function has the CRRA form:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$$

where the risk coefficient σ is set to be 2. The risk-free interest rate is 1%, which is the average quarterly interest rate on 3-month US treasury bills. The output loss parameter λ is set to 2% as in Aguiar and Gopinath (2006).

The endowment process is calibrated to the Argentina's quarterly real GDP for 1980Q1 and 2003Q4 from the Ministry of Finance (MECON). Aguiar and Gopinath (2006) find that shocks to the trend of output growth play a much more important role than standard transitory shocks in explaining default frequencies in emerging economies and accounting for fluctuations in business cycles. Following Yue (2009), I use a similar output process as in Aguiar and Gopinath and model the output growth rate as an AR(1) process:

$$\log g_t = (1-\rho_g)\log(1+\mu_g) + \rho_g \log g_{t-1} + \epsilon_t^g, \quad \epsilon_t^g \sim^{iid} N(0, \sigma_g^2)$$

where growth rate is $g_t = \frac{y_t}{y_{t-1}}$, growth shock is ϵ_t^g, and $log(1+\mu_g)$ is the expected log gross growth rate. The endowment process is estimated to match the average growth rate, as well as the standard deviation and autocorrelation of HP detrended output. As the endowment shock y_t in the model embodies a stochastic trend, the model economy is nonstationary. I re-define the value function, bond price function and debt recovery schedule using the detrended variables as in Aguiar and Gopinath (2007). The calibration results are listed in Table 2.6.

The disaster related parameters δ, $P_{z=0,z'=1}$ and $P_{z=1,z'=1}$ measure the deviation from trend growth, probability of disaster occurrence and disaster persistence. For comparative statics, I assign values to those parameters to characterise the following types of disasters. δ is set to be 0.99 (-1% deviation

Table 2.6: Calibration of extended model

Paramter	Value	
Risk Aversion	$\sigma = 2$	
Risk Free Interest Rate	$r = 1\%$	US Treasury-bill interest rates
Output Loss in Default	$\lambda = 2\%$	Sturzenegger (2002)
Average Output Growth	$\mu_g = 0.42\%$	Argentina average growth
Endowment Growth Process	$\sigma_g = 2.53\%$, $\rho_g = 0.41$	Argentina's GDP
Calibration	Value	Target Statistics
Time Discount Factor	$\beta = 0.72$	2.78% default frequency
Bargaining Power	$\theta = 0.72$	27% debt recovery rate
Disaster		
Size of contraction	$\delta = \{0.99, 0.95\}$	
Prob. of occurrence	$P_{z=0,z'=1} = \{0.01, 0.1\}$	
Persistence	$P_{z=1,z'=1} = \{0.1, 0.5\}$	

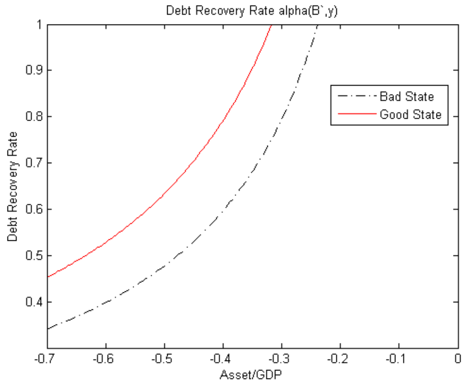
from trend growth) for a small natural disaster and 0.95 (-5% deviation) for a large one. I set the $P(z_t = 1|z_t = 0)$ as 1% for an economy resistant to natural disasters and 10% for an economy prone to natural disasters. For countries such as Indonesia and Philippines, a probability of 10% is not unreasonable. For the persistence, I set $P(z_t = 1|z_t = 1)$ as 0.1 for a short-lived disaster shock. The probability suggests that the disaster shock lasts for 1.1 periods on average. I set the persistence parameter as 0.5 for a persistent disaster shock which lasts for 2 periods on average.

2.7.2 Simulation results

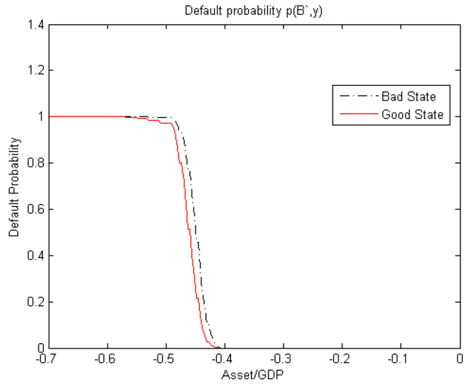
In this section, I start with the equilibrium debt recovery schedule and show the additional state contingency generated by debt renegotiation. I then examine how countries' default probability is affected by the characteristics of a disaster shock (size, persistence and probability of occurrence).

Figure 2.14 plots the equilibrium debt recovery schedule when $z = 0$ in the left panel and default probability $p(b', y)$ in the right panel. The debt recovery rate is higher for a defaulter with a good economic shock and vice

Figure 2.14: Recovery rate and default probability with debt renegotiation



(a) Recovery rate $\alpha(b, y, z = 0)$



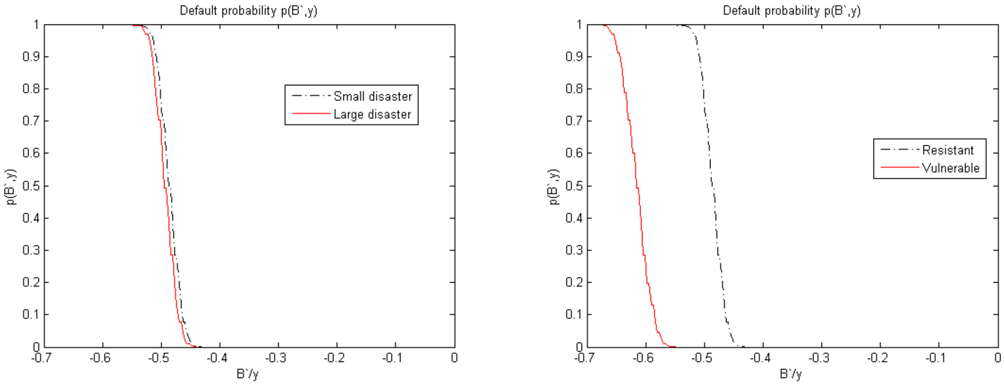
(b) Default Probability $p(b', y)$

versa. This is because the equilibrium debt recovery rate is determined by both creditor's and debtor's surplus but only the debtor's surplus is dependent on the endowment shock. When the defaulting country is in a boom, it has more resources to repay its debt than in a recession and the efficient outcome is to repay a higher fraction of its debt.Thus, debt renegotiation provides additional state contingency. The equilibrium debt recovery schedule displays the same property when $z = 1$.

The default probability schedule shows that for low levels of debt ($b > -0.4$), despite the scope of debt reduction generated by renegotiation, debtors still choose not to default. Once defaulted, the debtor government's credit record deteriorates and exclusion from credit market results. As in Arellano (2008)'s model, a low income shock is associated with a higher default probability because default is an instrument of the government to insure themselves when the time is bad.

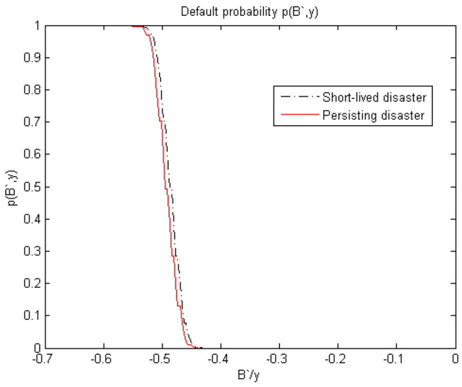
Figure 2.15 and Figure 2.16 illustrate the comparative statics of default probability for different types of disasters. In Fig.2.15, a country facing a likelihood of incurring large disasters has more incentive to oblige its debt contract and therefore has lower default probability. The higher repayment incentive comes from the fact that honouring the contract gives the debtors the benefit of postponing their debt payment and continuing borrowing at risk free

Figure 2.15: Default probability comparisons: small and large output contractions; frequent and rare occurrence



(a) $p(b', y)$: small vs. large **(b)** $p(b', y)$: frequent vs. rare disasters

Figure 2.16: Default probability comparison: short-lived vs. persistent disaster



interest rate during a disaster phase and therefore it is more valuable to those prone to large disasters. For lower default probability of a debtor vulnerable to disasters and a debtor facing persistent disasters, the same reasoning follows.

In models with only one-period state non-contingent debt contract, default is the only instrument for state contingency for the debtor when the income shock is low. To avoid getting into the autarky associated with disaster state, the debtor countries have to build up assets in a costly way by giving up consumption in non-disaster states, and is often constrained to do6 so due to countercyclical interest rate schedules. With debt postponement and debt extension, they can obtain insurance against disaster shocks at risk-free rate and therefore the incentive to stay in the contract is higher the larger the cost of autarky and market exclusion associated with defaulting in disaster state.

2.8 Conclusion

In this paper, I account for the output dynamics prior to disaster occurrence to study the impact of disaster risk on default risk. I distinguish between two types of disaster risk, natural and economic. Using output data of a range of emerging economies, my empirical estimation finds that those countries are subject to economic disasters and there is a positive correlation between disaster probabilities and debtors' borrowing costs. This is consistent with the theoretical prediction of the model constructed to compare the impacts of natural and economic disaster risks on borrower countries' default risk.

As discussed in Section 2.5, the model's ability to generate impact of natural disaster risk is limited by the data availability to calibrate transitional probabilities associated with a natural disaster. The model's quantitative performance also hinges on its capability to generate positive debt in recessions for economies with high disaster risk. Further works are needed on the model. I also outline briefly below three directions for future extensions.

First, disaster events can be analysed in more details to assess their underlying causes and severity so as to study the long-term implications on countries' borrowing costs and market access. Second, in practice, countries do not necessarily default in debt crisis but engage in lengthy process of debt renegotiation and restructuring. There have been extensive empirical works (e.g. Cruces and Trebesch (2013), Das et al.(2012)) studying aspects of debt restructuring but none looks at the difference between debt restructuring following natural and economic disasters.

Third, the impact of natural disasters is often limited and does not always lead to severe output contraction and therefore sovereign default. Small countries, such as the Caribbean ones, are vulnerable to incidences of hurricanes and provide good examples for analysis. The default history[10] of nine

[10]Source: Standard & Poor's. See Appendix B.4 for details.

Caribbean countries[11] records eleven incidents of default but only two of them are caused by natural disasters[12]: Grenada was hit by hurricanes in 2004 and 2005 and it was in default 2004-2005; Belize was hit by hurricanes in 2005 and 2007 and defaulted in 2006 and 2007. If those Caribbean countries are similar in their economic structures, why do some contain the impact of hurricanes better than the others? And how does the hurricane occurrence impact on those countries' bond spreads? To answer those questions, one would need to collect and construct more detailed statistics for those countries around the times of default and disasters.

[11] Trinidad and Tobago, Jamaica, Grenada, Belize, Antigua, Dominica and Dominican Republic.

[12] For each default preceded by a natural disaster, I checked the Google News Archive for the cause of default. I can only find evidence for Grenanda and Belize.

Chapter 3

Optimal Liquidity Provision with Partial Information

3.1 Introduction

Governments often face difficulties setting optimal policy when the underlying states of the economy (e.g. booms or recessions) cannot be directly observed. Their learning about the states can only be done through the observation of outcomes from those unknown states, e.g. unemployment and investment. In this paper, I relax an assumption made in previous works on optimal policy that the government has perfect knowledge of shocks in the economy and consider a model of optimal provision of liquidity when the government only has partial information. It cannot observe the two shocks in the economy but only the outcome of private sector's investment choice.

The outcome of investment which is a function of both shocks calls for the government to react in opposite directions, depending on the source of the shock. This gives an interesting setting to analyse optimal policy with partial information. Using the General Signal Extraction (GSE hereafter) method from Marcet et al. (2016), I present a solution of optimal liquidity provision with partial information.

Ultimately, I would like to evaluate the welfare implications of the lower availability of information in optimal policy. Many papers have studied environments where contrary to conventional wisdom, more information does not necessarily increase welfare. My model has the scope for welfare comparison between full and partial information. When a government lacks means to distinguish the two shocks, learning through the private sector's outcome might increase overall welfare. I outline the future steps in the last section.

3.2 Model

I consider the following economy. There are two dates, $t = 0, 1$, and $t = 0$ consists of two sub-dates, $t = 0$ and $t = 0.5$. There is an all-purpose good at every date and there is no storage technology. The economy is populated by three types of agents: a domestic government, a continuum of measure one of domestic entrepreneurs and international investors.

There are two uncertainties in this economy: 1) a preference shock to entrepreneurs' utility of private consumption at $t = 0$, θ_0, and 2) stochastic return of entrepreneurs' capital investment, z, at $t = 1$. At $t = 0.5$, both shocks are realised. The realisations are fully observed by the entrepreneurs and investors, but not by the government. The government optimally sets its policy on an endogenous signal - the investment undertaken by entrepreneurs at $t = 0.5$.

3.2.1 Setup

Domestic entrepreneurs. There are two types of entrepreneurs of fractions λ and $1 - \lambda$. Both are endowed with one unit of bond and an investment project of fixed size I at $t = 0$. One unit of investment yields a stochastic return z at $t = 1$. z is a productivity shock with a continuous probability distribution function $f_z(\cdot)$, $z \in [0, \bar{z}]$. The government lays a flat rate tax τ_1 at $t = 1$ on entrepreneurs' investment returns. τ_1 is fixed and exogenously given. Fraction λ of entrepreneurs derive utility from private consumption at two dates and government expenditure at $t = 1$, whereas fraction $1 - \lambda$ entrepreneurs only consume at $t = 1$. Their expected utility at date 0 is given as $U^E_{t,\lambda} = \mathbb{E}\left(\theta_0 u(c^E_0) + u(c^E_1) + \nu(G_1)\right)$ and $U^E_{t,1-\lambda} = \mathbb{E}\left(u(c^E_1) + \nu(G_1)\right)$. $u' \geq 0, u'' \leq 0, \nu' > 0, \nu'' < 0$.

At $t = 0.5$, the fraction λ of entrepreneurs incur an investment opportunity each, which requires additional investment input. An addition of i_0 units of investment input produces $F(i_0)$ units of return at $t = 1$. $F(\cdot)$ is a concave function. Uncertainties z and θ_0 are realised and fully observed by the entrepreneurs when they make their investment decisions. When θ_0 is high, entrepreneurs are more impatient and less willing to invest. In order to invest, entrepreneurs would have to sell their holding of bonds. I assume that bonds cannot be sold in fraction.

In this economy, the government acts as a Stackleberg leader. At $t = 0.5$, it sets two policies which take effect at $t = 1$: 1) the fraction of debt, $\gamma_1 \in [0, 1]$ that it repays at $t = 1$, and 2) public expenditure, G_1. I assume that the government can commit to its choice of γ_1 and G_1 at $t = 1$.

Define A the set of shocks z and θ_0, $A \equiv (z, \theta_0)$ and Ψ the space of possible values of A. Entrepreneurs are assumed to know that fiscal policy is given by two functions $(\tilde{\gamma}_1, \tilde{G}_1)$: $\Psi \to R^2$. Upon the realisation of A, entrepreneurs choose $(c^E_{0,\lambda}, c^E_{1,\lambda}, c^E_{1,1-\lambda}, i_0) : \Psi \to R^4$ knowing the fiscal policy and the bond price $q^B_0 : \Psi \to R$. The λ fraction of entrepreneurs solve the following maximisation problem at $t = 0.5$:

$$\max_{c^E_{0,\lambda}, c^E_{1,\lambda}, i_0} \quad \theta_0 u(c^E_{0,\lambda}) + u(c^E_{1,\lambda}) \tag{3.1}$$

$$s.t. \quad c^E_{0,\lambda} + i_0 \leq \tilde{\gamma}_1 \tag{3.2}$$

$$c^E_{1,\lambda} = (1 - \tau_1)(1 + F(i_0))zI \tag{3.3}$$

The optimisation problem for the fraction $1 - \lambda$ of entrepreneurs is:

$$\max_{c^E_{1,1-\lambda}} \quad u(c^E_{1,1-\lambda}) \tag{3.4}$$

$$s.t. \quad c^E_{1,\lambda} = (1 - \tilde{\tau}_1)zI + \tilde{\gamma}_1 \tag{3.5}$$

Obviously, the solution of the entrepreneurs' problem in the above setup is the same as the non-stochastic model where A is known. Uncertainty only plays a role in the government's problem.

International investors. Investors have a large endowment in every period. Their expected utility at date t is $V_t^F = \mathbb{E}_t\left(\sum_{s=t}^{1} C_t^F\right)$, and therefore the international rate of interest is zero. Investors also observe A.

The government. The government has some legacy bond of amount B_0. I assume that $B_0 > 1$ so that the marginal trader in the domestic bond market is a risk-neutral international investor. The government can partially default on its debt and decides on the repayment fraction γ_1 and public expenditure G_1 at $t = 0.5$ to maximise aggregate utilities of entrepreneurs. In contrast to private agents, the government cannot observe A but only the investment undertaken by entrepreneurs at $t = 0.5$[1]. It can commit to γ_1 and G_1. When the government makes its decision on repayment fraction, it is essentially determining the amount of liquidity provision in the economy. A higher repayment fraction reduces the resources available to distribute to domestic agents, but it increases the liquidity holding of the entrepreneurs who want to invest and it also increases the consumption of the other entrepreneurs who still hold one unit of bond at $t = 1$.

3.2.2 Competitive equilibrium

Definition 1 *A* ***competitive equilibrium*** *is a fiscal policy ($\tilde{\gamma}_1$,$\tilde{G}_1$), price q_0^B, and allocations ($c_{0,\lambda}^E, c_{1,\lambda}^E, c_{1,1-\lambda}^E, i_0$) such that when entrepreneurs take ($\tilde{\gamma}_1$,$\tilde{G}_1$,q_0^B) as given, the allocations maximise the entrepreneurs' utility $U_{t,\lambda}^E$ and $U_{t,1-\lambda}^E$ subject to (3.2),(3.3) and (3.5). Bond market clear.*

[1] I provide a justification for this assumption - one may consider a market in which full information revelation is exclusive to participating agents (including international investors) only.

Definition 1 implies that for all realisation of A,

$$\frac{u'_\lambda(c_1^E)}{\theta_0 u'_\lambda(c_0^E)}(1-\tilde{\tau}_1)F'(i_0)zI = 1 \tag{3.6}$$

Bond market clearance implies that: $q_0^B = \gamma_1$.

3.2.3 Ramsey equilibrium

In this section, I provide a definition of Ramsey equilibrium when the government decides on optimal liquidity provision under partial information, based on Marcet et al.(2016). I assume that the repayment fraction of debt at $t = 0.5$ has to be set before both z and θ_0 shocks are known but after observing a signal s, which is chosen as entrepreneurs' investment in this model. Before proceeding to the definition of a Partial information Ramsey equilibrium, I provide a definition of Full information Ramsey equilibrium below.

Definition 2 *A* ***Full Information Ramsey equilibrium is a fiscal policy*** **(**γ_1**,**G_1**)** *that achieves the highest aggregate utility,* $U_t^E = \lambda U_{t,\lambda}^E + (1-\lambda)U_{t,1-\lambda}^E$*, when allocations are determined in a competitive equilibrium.*

Definition 3 *A* ***Partial Information Ramsey Equilibrium*** *when government observes a signal* s *is a Full Information Ramsey equilibrium satisfying*

1. γ_1 *is measurable with respect to* s
2. *fiscal policy (*γ_1*,*G_1*) achieves the highest utility from among all equilibria satisfying 1.*

Following Marcet et al. (2016), restriction 1 can be expressed as the Partial information Ramsey equilibrium having to satisfy:

$$\gamma_1 = \mathbf{\Delta}(s) \quad \text{for all} \quad A \in \Psi, \text{for some function} \quad \mathbf{\Delta} : R \to R. \tag{3.7}$$

Interesting results arise when restriction 1 prevents the Partial information Ramsey equilibrium from achieving the Full information version. Whether private agents know that 3.7 holds does not alter the government's Ramsey problem. Entrepreneurs are atomistic and take as given the liquidity provision and public expenditure that arise from the equilibrium.

I derive the Full information Ramsey equilibrium using the primal approach. Denote private agents' optimal choice of i_0 as $i_0 = h(\gamma_1, \theta_0, z)$. At $t = 0.5$, the government chooses γ_1 that solves:

$$\begin{aligned} \max_{\gamma_1, G_1} \quad & U_0^E = \lambda u(c_{0,\lambda}^E) + \lambda u(c_{1,\lambda}^E) + (1-\lambda) u(c_{1,1-\lambda}^E) + \nu(G_1) \\ s.t. \quad & c_{0,\lambda}^E = \gamma_1 - i_0 \\ & c_{1,\lambda}^E = (1-\tau_1)(1+F(i_0))zI \\ & c_{1,1-\lambda}^E = (1-\tau_1)zI + \gamma_1 \\ & G_1 = \tau_1(1+F(i_0))zI - \gamma_1 B_0 \\ & i_0 = h(\gamma_1, \theta_0, z) \end{aligned}$$

And it can be reduced to:

$$\begin{aligned} \max_{\gamma_1 : \Psi \to R} \quad & U_0^E(i_0; A) \\ s.t. \quad & i_0 = h(\gamma_1, \theta_0, z) \end{aligned}$$

Following definition 3, a Partial information Ramsey equilibrium solves

$$\max_{\Delta : R \to R} \quad U_0^E(i_0; A) \tag{3.8}$$

$$s.t. \quad i_0 = h(\gamma_1, \theta_0, z) \tag{3.9}$$

$$\gamma_1 = \Delta(i_0) \tag{3.10}$$

In contrast to the standard optimsation problem, in the presence of partial information, one looks for an optimal function Δ instead of an optimal point. I use optimal control under GSE as introduced in Marcet et al.(2016) to solve the optimisation problem. The solution to the problem of optimal liquidity provision is in the next section.

3.3 Optimal liquidity provision with GSE

I illustrate a solution of the optimisation problem in (3.8). I consider the following specifications. $u(c) = c$, $\nu(G_1) = \log(G_1)$ and $F(i_0) = \frac{m}{1-\alpha} i_0^{1-\alpha}$. Parameter m is used to adjust the value of $F(i_0)$. For simplicity, $I = 1$. Denote $\tilde{\theta_0}$ and $\tilde{z}$ as the means of θ_0 and z shocks. Let θ_0 and z be uniformly distributed as: $\theta_0 \sim U[\underline{\theta_0}, \bar{\theta_0}]$; $z \sim U[\underline{z}, \bar{z}]$, where the supports of both shocks are a range of $+/-$ 10% from the mean. For the numerical example, $\tilde{\theta_0} = 1$ and $\tilde{z} = 3$.

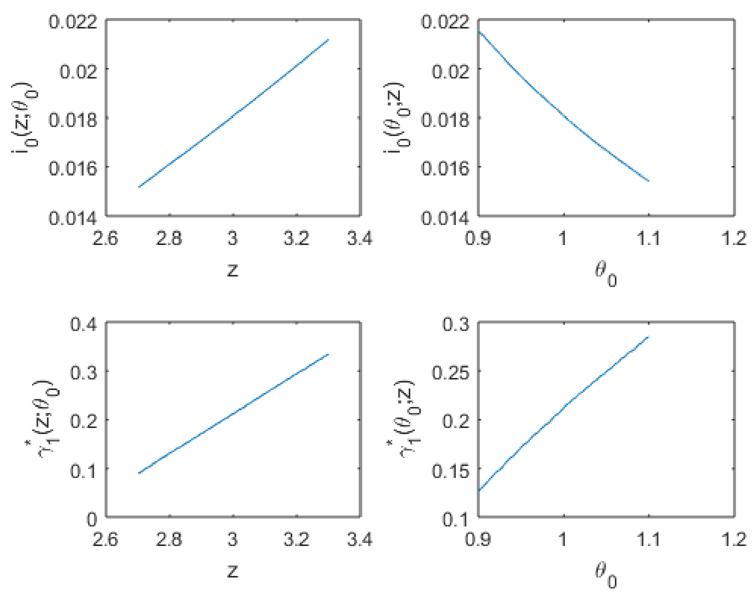
Under the above preference and production function, equilibrium condition (3.6) becomes:

$$i_0^* = \left(\frac{(1-\tau_1)mz}{\theta_0} \right)^{\frac{1}{\alpha}} \tag{3.11}$$

Optimal investment is increasing in productivity shock z, but decreasing in the preference shock θ_0. When entrepreneurs make their investment decisions at $t = 0.5$, they are evaluating the payoffs between consuming the payoff from bond sale now and invest now and consume the return from investing later at $t = 1$. For sufficiently large values of z or small values of θ_0, optimal investment could reach a corner solution - $i_0^* = \gamma_1$ or $i_0^* = 0$ for the opposite. In the numerical example, I focus on the interior solution case.

In Figure 3.1, I show how investment and repayment fraction change with the two different shocks under full information. On the left side of the figure, optimal investment and repayment fractions are plotted keeping θ_0 constant and equal to its mean. Both investment and repayment fraction are shown to be increasing in z. On the right side, z is kept as constant and equal to

Figure 3.1: Investment and repayment fraction



its mean and the figures show that investment falls with θ_0 but repayment fraction increases with θ_0.

When partial information is introduced where only investment can be observed by the government, an interesting case arises for analysing optimal policy. If the government sees an increase in investment, it can either be caused by an increase in z or a decrease in θ_0. Under full information, this calls for opposite directions of reaction by the government: if the increase in investment is driven by an increase in z, the optimal repayment should increase; if it is driven by a decrease in θ_0, γ_1^* should decrease. Under partial information, by observing a certain value of i_0 and deciding on the liquidity provision γ_1, the government cannot infer the value of the shocks separately.

The policy functions demonstrated in Figure 3.1 is sensitive to the calibration of parameters. Using implicit function theorem, I state below the sufficient conditions under which a change in i_0 calls for government reaction in opposite directions.

Under the sufficient conditions:

$$
\begin{aligned}
\tau_1 z &> B_0 && (3.12)\\
\frac{\lambda}{1-\lambda} &> \frac{1-\alpha}{\theta_0(2\alpha-1)} && (3.13)
\end{aligned}
$$

One obtains:

$$
\begin{aligned}
\frac{d\gamma_1^*}{dz} &> 0\\
\frac{d\gamma_1^*}{d\theta_0} &> 0
\end{aligned}
$$

A higher level of productivity shock improves the government's resources and enables it to repay more of its debt. A higher preference shock makes entrepreneurs' date 0 consumption more valuable and this has two implications for the government's optimal choice of γ_1. On one hand, the government would like to set a higher repayment fraction enabling entrepreneurs to consume more when it is more valuable, but at the same time the high preference shock reduces entrepreneurs' investment and therefore the government's tax revenue for debt repayment. Calibration of parameters and range of productivity shock satisfying the sufficient conditions in Proposition 1 represent an economy where the debt burden is not heavy that government can pay off its debt without using the tax revenue on additional investment made (condition (12)) and within the population of domestic agents, more of them consume at both periods and therefore value date 0 consumption (condition (13)).

Under partial information, for an intermediate value of investment, there is a continuum of realisations of (z, θ_0) consistent with the observation of i_0 and a policy Δ. The government cannot identify z and θ_0 separately and therefore it cannot choose the policy under full information. I modify the generic method of solving models of optimal policy with partial information provided in Marcet et al. (2017) to compute the solution. The algorithm is explained in Appendix C.

Figure 3.2 presents the optimal policy under partial information. The bounded area is the set of all equilibrium pairs $(i_0^{FI}, \gamma_1^{FI})$ that could have been realised under full information. The dashed red line is the optimal policy under partial information $\mathbf{\Delta}^*$. There is full revelation of shocks at the extreme values of the signal i_0. As i_0 is increasing in z and decreasing in θ_0, denote $\underline{i_0}$ and $\bar{i_0}$ as the extreme values of i_0 in the partial information solution. One has the following results:

$$\begin{aligned} \underline{i_0} &= I(\mathbf{\Delta}^*; \bar{\theta_0}, \underline{z}) = I^{FI}(\bar{\theta_0}, \underline{z}) \\ \bar{i_0} &= I(\mathbf{\Delta}^*; \underline{\theta_0}, \bar{z}) = I^{FI}(\underline{\theta_0}, \bar{z}) \end{aligned}$$

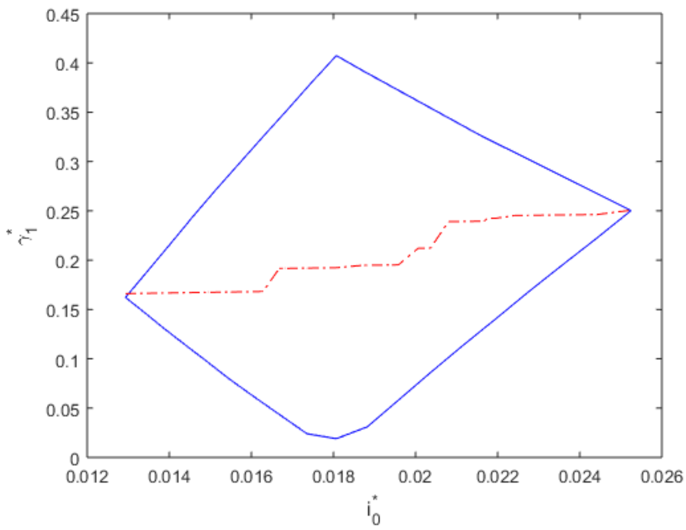
For intermediate values of shocks, the partial information solution is in the interval $(\underline{i_0}, \bar{i_0})$.

For low levels of investment, the government is confident that preference shock is high, so it keeps the repayment fraction low. The lowest investment realisation leads to the full revelation equilibrium for $(\bar{\theta_0}, \underline{z})$. As investment increases, higher i_0 signals lower expected preference and therefore repayment fraction starts to decrease. As investment goes up to a point that the set of admissible θ_0's conditional on i_0 is the whole set $[\underline{\theta_0}, \bar{\theta_0}]$. From that point on, further increase in investment must be explained by an increase in z, which calls for higher debt repayment and the optimal policy changes slope. It continues until the highest level of θ_0 is ruled out and optimal repayment fraction changes direction again. The process repeats another time until investment reaches the other full revelation point $\bar{i_0} = I^{FI}(\underline{\theta_0}, \bar{z})$.

3.4 Next steps and conclusion

The ultimate aim of this paper is to evaluate the welfare of domestic agents under partial information in a model of optimal public provision of liquidity. To achieve that end, I have provided solutions to a simple model under both full information and partial information. Further works need to be done on the partial information solution since I find that the current solution is not robust

Figure 3.2: Investment and repayment fraction under partial information



to alternative starting value used in the computation algorithm. I outline the next steps to be taken below.

First, I would like to compare the optimal policy under partial information with that under full information. With partial information solutions represented in Figure 3.2, a comparison is difficult. I would like to compute γ_1^{FI} and γ_1^{PI} as illustrated in Figure 3.1. The government, of course, does not observe z and θ_0 directly, but one can see if γ_1^{PI} deviates from the linear policies under full information. I expect to see non-linearities in γ_1^{PI}.

Second, aggregate utilities under full information and partial information, $U_t^{E,FI}$ and $U_t^{E,PI}$ are to be computed and compared. It is not necessarily true that more information is better, as shown in other studies. When a government lacks means to distinguish the two exogenous shocks, learning through the private sector's outcome might increase overall welfare.

Thirdly, in the current model setup, tax rate τ_1 is set as fixed and exogenous. An alternative modelling choice can add tax rate to the set of optimal policy at date 0: (τ_1, γ_1, G_1). The assumption on the government's commitment to the fiscal instruments matters. If the government can commit to both tax rate and repayment fraction, there would be an infinite number of pairs (τ_1, γ_1) that a Ramsey planner can choose from. In this setting, under partial information, τ_1 enters the entrepreneurs' equilibrium condition 3.6 and makes i_0 an endogenous signal. $\mathbf{\Delta}$ changes τ_1, which in turn, changes i_0 which constitutes the argument of $\mathbf{\Delta}(s)$.

The above setup would be more workable if one assumes that the government can only commit to τ_1 at date 0 (as in arguably the case in practice where tax rate is set one period ahead), but can change γ_1 after learning the realisation of shocks at date 1. This reduces the optimisation problem to choosing one instrument, τ_1 at date 0. This setup would provide richer mechanism than the current one. For example, by setting τ_1, the government alters the private sector investment and therefore its tax base. Tax base changes its debt repayment γ_1 which in turn influences private sector's investment choice.

Appendix A

Appendix to Public Provision of Liquidity with Default Risk

A.1 Proof of Proposition 2

To show that $\bar{B}_0(\mu)$ increases in μ, I expand $\frac{d(q_{0.5}(\mu;\hat{B}_0)\hat{B}_0)}{d\hat{B}_0}|_{\hat{B}_0=\bar{B}_0} = 0$ below:

$$\frac{d(q_{0.5}(\mu;\bar{B}_0)\bar{B}_0)}{d\bar{B}_0} = q_{0.5}(\mu;\bar{B}_0) - \bar{B}_0 \cdot \frac{\frac{\partial H(\tilde{z})}{\partial \mu} \cdot \frac{1}{\tau I}\frac{e}{e-1}\frac{1}{F\left(\frac{q_{0.5}}{I}+\delta\right)}}{1 - \frac{\partial H(\tilde{z})}{\partial \mu}\left(\frac{\bar{B}_0}{I}\right)\frac{f\left(\frac{q_{0.5}}{I}+\delta\right)}{F\left(\frac{q_{0.5}}{I}+\delta\right)} \cdot \frac{1}{\tau I}\frac{e}{e-1}\frac{1}{F\left(\frac{q_{0.5}}{I}+\delta\right)}} = 0$$

Using implicit function theorem, let $L = \frac{d(q_{0.5}(\mu;\bar{B}_0)\bar{B}_0)}{d\bar{B}_0} = 0$.

$$\begin{aligned}
\frac{\partial L}{\partial \bar{B}_0} &= \frac{\partial q_{0.5}}{\partial \bar{B}_0} \\
&\quad - \frac{\left(1-\frac{e}{e-1}\frac{1}{\tau I^2}\frac{f(\frac{q_{0.5}}{I}+\delta)}{F(\frac{q_{0.5}}{I}+\delta)}Y(\bar{B}_0,\mu)\right)\frac{e}{e-1}\frac{1}{\tau I}}{\left(1-\frac{e}{e-1}\frac{1}{\tau I^2}\frac{f(\frac{q_{0.5}}{I}+\delta)}{F(\frac{q_{0.5}}{I}+\delta)}Y(\bar{B}_0,\mu)\right)^2} \\
&\quad - \frac{\left(\frac{e}{e-1}\right)^2\frac{1}{\tau^2 I^3}Y(\bar{B}_0,\mu)\left(\frac{\partial Y(\bar{B}_0,\mu)}{\partial \bar{B}_0}\frac{f(\frac{q_{0.5}}{I}+\delta)}{F(\frac{q_{0.5}}{I}+\delta)}+Y(\bar{B}_0,\mu)\frac{\partial\frac{f(\frac{q_{0.5}}{I}+\delta)}{F(\frac{q_{0.5}}{I}+\delta)}}{\partial \bar{B}_0}\right)}{\left(1-\frac{e}{e-1}\frac{1}{\tau I^2}\frac{f(\frac{q_{0.5}}{I}+\delta)}{F(\frac{q_{0.5}}{I}+\delta)}Y(\bar{B}_0,\mu)\right)^2}
\end{aligned}$$

where $Y(\bar{B}_0,\mu) = h(\tilde{z}(\bar{B}_0,\mu)\frac{\bar{B}_0}{F(\frac{q_{0.5}}{I}+\delta)}$.

The followings are known:

$$\begin{aligned}
\frac{\partial q_{0.5}}{\partial \bar{B}_0} &< 0 \\
\frac{\partial \frac{f(\frac{q_{0.5}}{I}+\delta)}{F(\frac{q_{0.5}}{I}+\delta)}}{\partial \bar{B}_0} &> 0 \\
\frac{\partial h(\tilde{z})}{\partial \bar{B}_0} &> 0
\end{aligned}$$

$$\frac{\partial Y(\bar{B}_0,\mu)}{\partial \bar{B}_0} = \frac{\partial h(\tilde{z})}{\partial \bar{B}_0}\frac{\bar{B}_0}{F(\frac{q_{0.5}}{I}+\delta)} + h(\tilde{z})\left(\bar{B}_0 - \frac{f(\frac{q_{0.5}}{I}+\delta)}{F(\frac{q_{0.5}}{I}+\delta)}\frac{\partial q_0}{\bar{B}_0}\frac{1}{I} + \frac{1}{F(\frac{q_{0.5}}{I}+\delta)}\right) > 0$$

Hence $\frac{\partial L}{\partial \bar{B}_0} < 0$.

$$\begin{aligned}
\frac{\partial L}{\partial \mu} = \frac{\partial q_0}{\partial \mu} & \\
- & \frac{\left(1-\frac{e}{e-1}\frac{1}{\tau I^2}\frac{f(\frac{q_{0.5}}{I}+\delta)}{F(\frac{q_{0.5}}{I}+\delta)}X(\bar{B}_0,\mu)\right)\left(\frac{e}{e-1}\right)^2\frac{1}{\tau I}\frac{\partial X(\bar{B}_0,\mu)}{\partial \mu}}{\left(1-\frac{e}{e-1}\frac{1}{\tau I^2}\frac{f(\frac{q_{0.5}}{I}+\delta)}{F(\frac{q_{0.5}}{I}+\delta)}X(\bar{B}_0,\mu)\right)^2} \\
- & \frac{\frac{e}{e-1}\frac{1}{\tau^2 I^3}X(\bar{B}_0,\mu)\left(\frac{\partial X(\bar{B}_0,\mu)}{\partial \mu}\frac{f(\frac{q_{0.5}}{I}+\delta)}{F(\frac{q_{0.5}}{I}+\delta)} + X(\bar{B}_0,\mu)\frac{\partial \frac{f(\frac{q_{0.5}}{I}+\delta)}{F(\frac{q_{0.5}}{I}+\delta)}}{\partial \mu}\right)}{\left(1-\frac{e}{e-1}\frac{1}{\tau I^2}\frac{f(\frac{q_{0.5}}{I}+\delta)}{F(\frac{q_{0.5}}{I}+\delta)}X(\bar{B}_0,\mu)\right)^2}
\end{aligned}$$

$\frac{\partial X(\bar{B}_0,\mu)}{\partial \mu} < 0$ as q_0 increases with μ and $\tilde{z}$ decreases with μ. Therefore $\frac{L}{\partial \mu} > 0$.

Finally, $\frac{d\bar{B}_0}{d\mu} = -\frac{\partial L/\partial \mu}{\partial L/\partial \bar{B}_0} > 0$.

Appendix B

Appendix to Sovereign Debt and Disaster Risk in Emerging Countries

B.1 Bayesian estimation with Gibbs sampling

The Bayesian estimation approach is based on Chapter 9 of Kim and Nelson (1999) and modified for estimating a panel Markov-switching model.

Generation of $\boldsymbol{\mu}^{i+1}$. I assume that the prior μ_{S_t} are jointly distributed as:

$$\tilde{\boldsymbol{\mu}} \sim N(\boldsymbol{a_0}, \boldsymbol{A_0}) \tag{B.1}$$

where $\tilde{\boldsymbol{\mu}}$ denote the prior vector of means $(\mu_1, \mu_2, \mu_3)'$, $\boldsymbol{a_0}$ is chosen as the three quantiles of the distribution of output data and $\boldsymbol{A_0}$ controls the tightness of prior distribution.

Conditional on parameters $\boldsymbol{\sigma^i}$, ϕ^i and $\boldsymbol{S^i_{1:T}}$ derived from the last simulation i and the output series $\boldsymbol{y_{1:T}}$, and denoting $\tilde{\boldsymbol{y}}_{\boldsymbol{1:T}}$ as $\frac{y_t - \phi_1^i y_{t-1}}{\sigma^i_{S_t}}$, $\tilde{\boldsymbol{S}}_{\boldsymbol{1:T}}$ as $\frac{S^i_{j,t} - \phi_1^i S_{j,t-1}}{\sigma^i_{S_t}}$, the posterior distribution for $(\mu_1, \mu_2, \mu_3)'$ is derived as:

$$\boldsymbol{\mu}^{i+1} | \boldsymbol{\sigma^i}, \phi_1^i, \boldsymbol{S^i_{1:T}}, \boldsymbol{y^i_{1:T}} \sim N(\boldsymbol{a_1}, \boldsymbol{A_1}) \tag{B.2}$$

where

$$\begin{aligned}\boldsymbol{a_1} &= \left(\boldsymbol{A_0}^{-1} + \boldsymbol{\tilde{S}^i_{1:T}}'\boldsymbol{\tilde{S}^i_{1:T}}\right)^{-1}\left(\boldsymbol{A_0}^{-1}\boldsymbol{a_0} + \boldsymbol{\tilde{S}^i_{1:T}}'\boldsymbol{\tilde{y}_{1:T}}\right)\\ \boldsymbol{A_1} &= \left(\boldsymbol{A_0}^{-1} + \boldsymbol{\tilde{S}^i_{1:T}}'\boldsymbol{\tilde{S}^i_{1:T}}\right)\end{aligned}$$

Generation of $\boldsymbol{\sigma^{i+1}}$. $\sigma^2_{S_t}$ defined in (2.3) can be expressed as:

$$\begin{aligned}\sigma^2_t &= \sigma^2_1(1+S_{2,t}h_2)(1+S_{3,t}h_3)\\ \sigma^2_2 &= \sigma^2_1(1+h_2)\\ \sigma^2_3 &= \sigma^2_1(1+h_3)\end{aligned}$$

Conditional on h_2 and h_3,

$$y^*_{1,t} = \frac{y_t - \phi_1 y_{t-1}}{\sqrt{(1+S_{2,t}h_2)(1+S_{3,t}h_3)}} \sim^{i.i.d.} N(0,\sigma^2_1)$$

I assume that prior value of σ^2_1 follows an inverse gamma distribution: $\tilde{\sigma}^2 \sim IG(\frac{\nu_1}{2},\frac{\delta_1}{2})$, and the posterior value of σ^2_1 is generated from:

$$\sigma_1^{i+1^2}|\boldsymbol{y^i_{1:T}},\boldsymbol{S^i_{1:T}},h^i_2,h^i_3 \sim IG\left(\frac{\nu_1+T}{2},\frac{\delta_1+\sum_{t=1}^T {y^*_{1,t}}^2}{2}\right) \tag{B.3}$$

where the parameters ν_1 and δ_1 of the prior distribution are set as zeros.

To generate σ^2_2, I similarly have:

$$y^*_{2,t} = \frac{y_t - \phi_1 y_{t-1}}{\sqrt{\sigma^2_1(1+S_{3,t}h_3)}} \sim^{i.i.d.} N(0,1+h_2)$$

and with a prior distribution for $1+h_2$ as $1+\tilde{h}_2 \sim IG(\frac{\nu_2}{2},\frac{\delta_2}{2})$, the posterior value of h_2 is generated from:

$$h_2^{i+1}|\boldsymbol{y^i_{1:T}},\boldsymbol{S^i_{1:T}},\sigma_1^{i+1},h^i_3 \sim IG\left(\frac{\nu_2+T_2}{2},\frac{\delta_2+\sum_{t=1}^{T_2} {y^*_{2,t}}^2}{2}\right)$$

where ν_2,δ_2 are the parameters of prior distribution of $1+h_2$ and T_2 is the number of time periods where $S_{2,t} = 1$.

To generate σ_3^2, I have:

$$y_{3,t}^* = \frac{y_t - \phi_1 y_{t-1}}{\sqrt{\sigma_1^2(1 + S_{2,t}h_2)}} \sim^{i.i.d.} N(0, 1 + h_3)$$

and with a prior distribution for $1 + h_3$ as $1 + \tilde{h}_3 \sim IG(\frac{\nu_3}{2}, \frac{\delta_3}{2})$, the posterior value of h_3 is generated from:

$$h_3^{i+1} | \boldsymbol{y^i_{1:T}}, \boldsymbol{S^i_{1:T}}, \sigma_1^{i+1}, h_2^{i+1} \sim IG\left(\frac{\nu_3 + T_3}{2}, \frac{\delta_3 + \sum_{t=1}^{T_3} {y_{3,t}^*}^2}{2}\right)$$

where ν_3, δ_3 are the parameters of prior distribution of $1 + h_3$ and T_3 is the number of time periods where $S_{3,t} = 1$.

Generation of $\boldsymbol{\phi_1^{i+1}}$. Define $y_{k,t}^* = \frac{y_{k,t} - \mu_{k,S_t}}{\sigma_{k,S_t}}$, $y_{k,t-1}^* = \frac{y_{k,t-1} - \mu_{S_{k,t-1}}}{\sigma_{S_{k,t-1}}}$ and $v_{k,t} = \frac{\epsilon_{k,t}}{\sigma_{k,S_t}}$. And stack $y_{k,t}^*$ up across the k's and define it as $\boldsymbol{y_t^*}$ and similarly for $\boldsymbol{y_{t-1}^*}$, $\boldsymbol{S_t}$. Rewrite (2.1) as:

$$\boldsymbol{y_t^*} = \phi_1 \boldsymbol{y_{t-1}^*} + \boldsymbol{v_t}, \quad \boldsymbol{v_t} \sim N(\mathbf{0}, I_k)$$

where $\mathbf{0}$ denotes a vector of N zeros. I assume that prior value of ϕ_1 follows a normal distribution: $\tilde{\phi}_1 \sim N(b_0, B_0)$, and the posterior value of ϕ_1 is generated from:

$$\phi_1^{i+1} | \boldsymbol{y^i_{1:T}}, \boldsymbol{S^i_{1:T}}, \boldsymbol{\sigma^i}, \boldsymbol{\mu^i} \sim N(b_1, B_1)$$

where b_0 and B_0 are prior parameters and

$$\begin{aligned} b_1 &= \left(B_0^{-1} + \boldsymbol{y_{1:T-1}}'\boldsymbol{y_{1:T-1}}\right)^{-1}\left(B_0^{-1} b_0 + \boldsymbol{y_{1:T-1}}'\boldsymbol{y_{2:T}}\right) \\ B_1 &= \left(B_0^{-1} + \boldsymbol{y_{1:T-1}}'\boldsymbol{y_{1:T-1}}\right)^{-1} \end{aligned}$$

Rejection sampling is used to make sure that the solution to $1 - \phi_1^{i+1} L = 0$ lies outside the unit circle.

Generation of $p_{i,j}$. Given the generated states across N countries, $\boldsymbol{S_{k,1:t}}$, define $n_{ij}, i,j = 1,2,3$, be the total number of transitions from state $S_{t-1} = i$ to $S_{t-1} = j, t = 2,3,...,T$, summed across countries in the same panel. Define $\bar{p}_{ii} = Pr[S_t \neq j|S_{t-1} = i], i = 1,2,3$, and $\bar{p}_{ij} = Pr[S_t = j|S_{t-1} = i, S_t \neq i]$, for $i \neq j$. I then have $p_{ij} = Pr[S_t = j|S_{t-1} = i] = \bar{p}_{ij} \times (1 - p_{ii})$ for $j \neq i$. Similarly, define $\bar{n}_{ii}$ to be the number of transitions from state $S_{t-1} = i$ to $S_{t-1} \neq i$.

I use beta distributions as conjugate priors, it can be shown that the posterior distributions of p_{ii} are given by

$$p_{ii}|\boldsymbol{S_{1:T}} \sim beta(u_{ii} + n_{ii}, \bar{u}_{ii} + \bar{n}_{ii}), i = 1,2,3$$

where u_{ii} and $\bar{u}_{ii}$ are prior parameters.

Generation of $\boldsymbol{S_{k,1:t}}$. Conditional on $\boldsymbol{\sigma^{i2}}$, $\boldsymbol{p^i}$, $\boldsymbol{\mu^i}$, ϕ_1^i, and $\boldsymbol{y_{1:T}}$, for each country k, I simulate S_T from the distribution below:

$$g(S_T|\boldsymbol{y_{1:T}}) = g(S_T|\boldsymbol{y_{1:T}}) \prod_{t=1}^{T-1} g(S_t|S_{t+1}, \boldsymbol{y_{1:t}})$$

I firstly run Hamilton's (1989) basic filter to get $g(S_t|\boldsymbol{y_{1:t}})$ and $g(S_t|\boldsymbol{y_{1:t-1}})$, for $t = 1,2,...,T$. The last iteration of the filter gives $g(S_T|\boldsymbol{y_{1:T}}, \boldsymbol{p}, \boldsymbol{\sigma}^2)$, from which S_T is generated. Then I successively generate S_t from $g(S_t|S_{t+1}, \boldsymbol{y_{1:t}})$, for $t = T-1, T-2, ..., 1$, using the following results:

$$g(S_T|S_{t+1}, \boldsymbol{y_{1:T}}) \propto g(S_{t+1}|S_t) g(S_t|\boldsymbol{y_{1:t}})$$

$Pr[S_t = 1|S_{t+1}, \boldsymbol{y_{1:t}}]$ is calculated in the following way:

$$Pr[S_t = 1|S_{t+1}, \tilde{y}_t] = \frac{g(S_{t+1}|S_t = 1) g(S_t = 1|\tilde{y}_t)}{\sum_{j=1}^{3} g(S_{t+1}|S_t = j) g(S_t = j|\tilde{y}_t)}$$

A random number is generated from the uniform distribution. If the generated number is less than or equal to $Pr[S_t = 1|S_{t+1}, \boldsymbol{y_{1:t}}]$, I set $S_t = 1$; if it is greater than $Pr[S_t = 1|S_{t+1}, \boldsymbol{y_{1:t}}]$, another random number is generated. If the second number is less than or equal to $Pr[S_t = 1|S_{t+1}, \boldsymbol{y_{1:t}}, S_t \neq 1]$, $S_t = 2$; if it is greater than $Pr[S_t = 2|S_{t+1}, \boldsymbol{y_{1:t}}, S_t \neq 1]$, $S_t = 3$. $Pr[S_t = 1|S_{t+1}, \tilde{y}_t, S_t \neq 1]$ are calculated in the following way:

$$Pr[S_t = 2|S_{t+1}, \boldsymbol{y_{1:t}}, S_t \neq 1] = \frac{g(S_{t+1}|S_t = 2)g(S_t = 2|\boldsymbol{y_{1:t}})}{\sum_{j=2}^{3} g(S_{t+1}|S_t = j)g(S_t = j|\boldsymbol{y_{1:t}})}$$

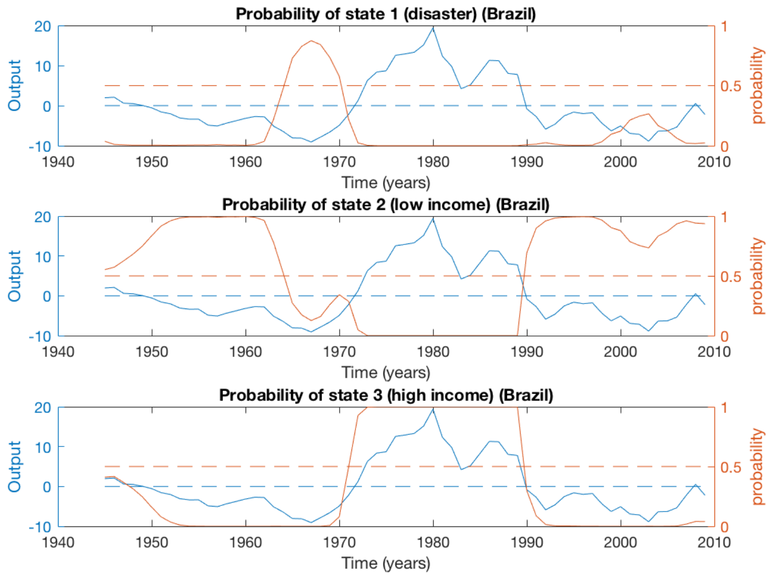
B.2 Estimation results

1. Brazil.

Table B.1: Estimated parameters (Brazil, 1945 - 2009)

Parameter	mean	sd	median	5% quantile	95% quantile
p_{11}	0.728887	0.141387	0.759213	0.486048	0.904333
p_{12}	0.135756	0.145329	0.081538	0.002677	0.435985
p_{21}	0.040422	0.025246	0.035862	0.008291	0.085575
p_{22}	0.939495	0.031846	0.944873	0.881613	0.978951
p_{31}	0.010221	0.010794	0.006384	0.000401	0.032852
p_{32}	0.037158	0.018527	0.033588	0.012966	0.073368
ϕ_1	0.370164	0.030546	0.370368	0.320418	0.421555
σ_1^2	6.414134	6.292715	4.432543	1.391758	18.633876
σ_2^2	4.018540	1.426046	3.796280	2.154069	6.520515
σ_3^2	23.997155	8.159759	22.713105	13.738077	39.457394
μ_1	-7.584279	0.296020	-7.570505	-8.080096	-7.096872
μ_2	-4.280542	0.394558	-4.286068	-4.944133	-3.640736
μ_3	4.282467	0.464177	4.286314	3.576509	5.103643

Figure B.1: Probability of states at each year, Brazil

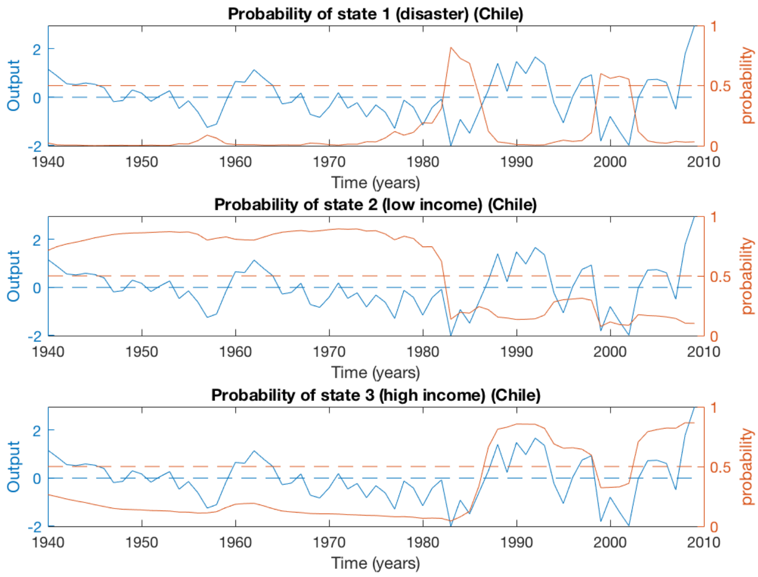


2. Chile.

Table B.2: Estimated parameters (Chile, 1940 - 2009)

Parameter	mean	sd	median	5% quantile	95% quantile
p_{11}	0.728887	0.141387	0.759213	0.486048	0.904333
p_{12}	0.135756	0.145329	0.081538	0.002677	0.435985
p_{21}	0.040422	0.025246	0.035862	0.008291	0.085575
p_{22}	0.939495	0.031846	0.944873	0.881613	0.978951
p_{31}	0.010221	0.010794	0.006384	0.000401	0.032852
p_{32}	0.037158	0.018527	0.033588	0.012966	0.073368
ϕ_1	0.370164	0.030546	0.370368	0.320418	0.421555
σ_1^2	1.543236	2.738226	1.006580	0.369746	3.844717
σ_2^2	0.503845	0.929971	0.278716	0.168746	1.517001
σ_3^2	0.945906	0.564323	0.863329	0.229492	1.956045
μ_1	-1.438168	0.232917	-1.426634	-1.831018	-1.090818
μ_2	-0.174735	0.183912	-0.153095	-0.467052	0.048582
μ_3	0.609089	0.354523	0.615046	-0.021963	1.203319

Figure B.2: Probability of states at each year, Chile

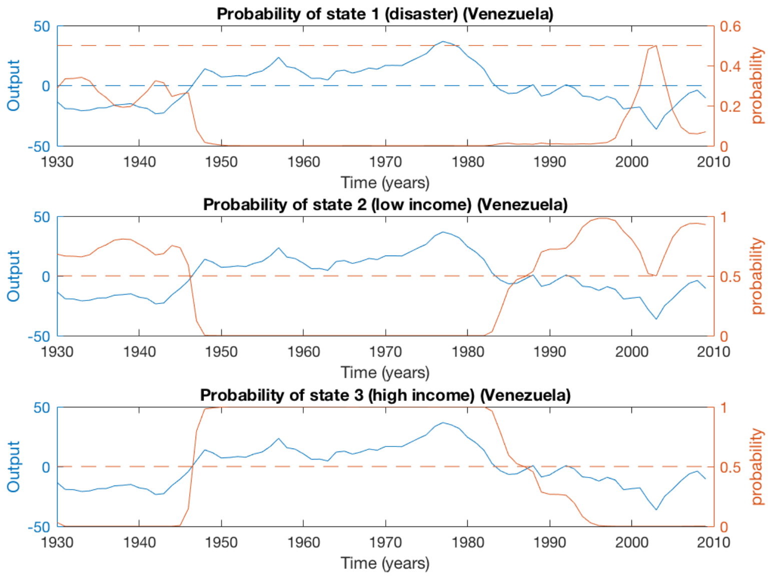


3. Venezuela.

Table B.3: Estimated parameters (Venezuela, 1930 - 2009)

Parameter	mean	sd	median	5% quantile	95% quantile
p_{11}	0.728887	0.141387	0.759213	0.486048	0.904333
p_{12}	0.135756	0.145329	0.081538	0.002677	0.435985
p_{21}	0.040422	0.025246	0.035862	0.008291	0.085575
p_{22}	0.939495	0.031846	0.944873	0.881613	0.978951
p_{31}	0.010221	0.010794	0.006384	0.000401	0.032852
p_{32}	0.037158	0.018527	0.033588	0.012966	0.073368
ϕ_1	0.370164	0.030546	0.370368	0.320418	0.421555
σ_1^2	74.746877	109.155743	46.196812	7.358384	227.587010
σ_2^2	43.129168	16.334854	40.949688	20.130019	74.855609
σ_3^2	55.588268	16.450569	53.644346	32.921862	85.552819
μ_1	-23.267432	0.327219	-23.268398	-23.804766	-22.759592
μ_2	-15.475671	0.457613	-15.468791	-16.229055	-14.727637
μ_3	12.908788	0.460875	12.921584	12.106424	13.697952

Figure B.3: Probability of states at each year, Venezuela

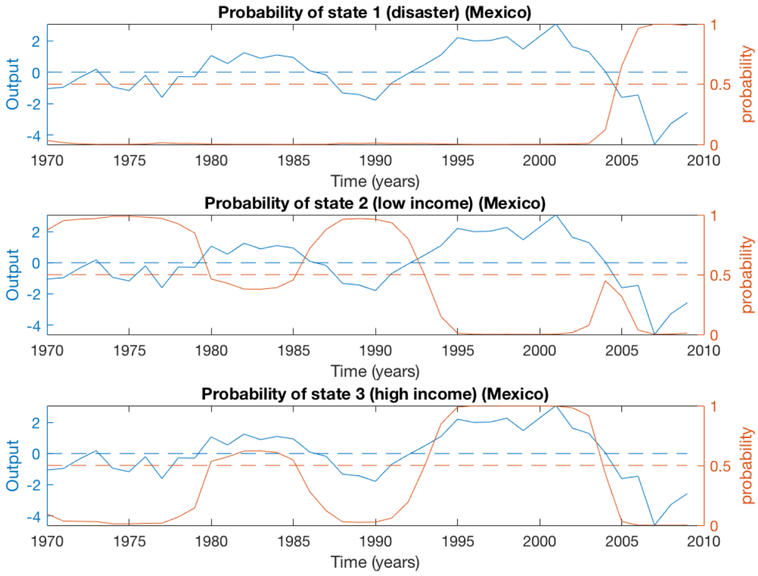


4. Mexico.

Table B.4: Estimated parameters (Mexico, 1970 - 2009)

Parameter	mean	sd	median	5% quantile	95% quantile
p_{11}	0.902993	0.060677	0.915906	0.782760	0.979531
p_{12}	0.049560	0.051298	0.030384	0.001248	0.160797
p_{21}	0.025686	0.015615	0.022985	0.006937	0.055908
p_{22}	0.914105	0.040240	0.919362	0.837007	0.967141
p_{31}	0.010073	0.007607	0.008574	0.000728	0.023529
p_{32}	0.057174	0.034890	0.052893	0.009239	0.120327
ϕ_1	0.327128	0.033794	0.327327	0.270133	0.381994
σ_1^2	3.644131	5.918705	2.177635	0.743993	11.142587
σ_2^2	0.719472	0.400741	0.626036	0.321516	1.428337
σ_3^2	0.690588	0.505396	0.565033	0.289139	1.524799
μ_1	-2.957387	0.249528	-2.946769	-3.382718	-2.570864
μ_2	-0.569781	0.305552	-0.544437	-1.085227	-0.124543
μ_3	1.362128	0.343144	1.367243	0.823326	1.940827

Figure B.4: Probability of states at each year, Mexico

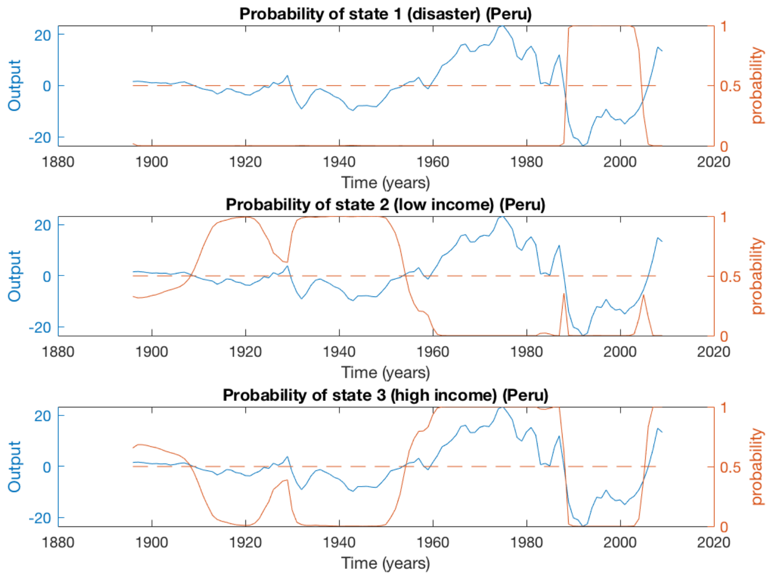


5. Peru.

Table B.5: Estimated parameters (Peru, 1896 - 2009)

Parameter	mean	sd	median	5% quantile	95% quantile
p_{11}	0.902993	0.060677	0.915906	0.782760	0.979531
p_{12}	0.049560	0.051298	0.030384	0.001248	0.160797
p_{21}	0.025686	0.015615	0.022985	0.006937	0.055908
p_{22}	0.914105	0.040240	0.919362	0.837007	0.967141
p_{31}	0.010073	0.007607	0.008574	0.000728	0.023529
p_{32}	0.057174	0.034890	0.052893	0.009239	0.120327
ϕ_1	0.327128	0.033794	0.327327	0.270133	0.381994
σ_1^2	15.584703	6.306003	14.514151	8.281644	27.092901
σ_2^2	7.245207	1.987854	7.034582	4.520300	10.729232
σ_3^2	43.753817	13.229356	41.196595	27.868516	66.599504
μ_1	-14.596170	0.322332	-14.599124	-15.124313	-14.086079
μ_2	-4.660646	0.491682	-4.658589	-5.472831	-3.863717
μ_3	3.409620	0.438981	3.392699	2.676589	4.127463

Figure B.5: Probability of states at each year, Peru

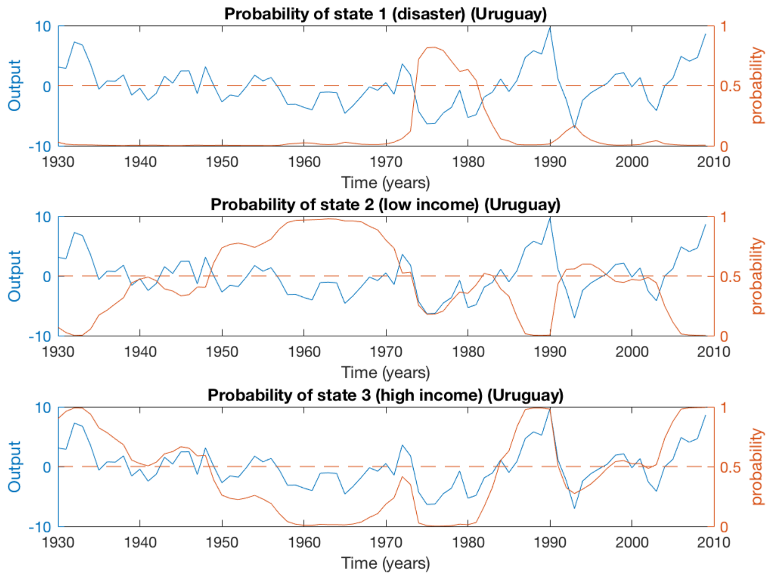


6. Uruguay.

Table B.6: Estimated parameters (Uruguay, 1930 - 2009)

Parameter	mean	sd	median	5% quantile	95% quantile
p_{11}	0.902993	0.060677	0.915906	0.782760	0.979531
p_{12}	0.049560	0.051298	0.030384	0.001248	0.160797
p_{21}	0.025686	0.015615	0.022985	0.006937	0.055908
p_{22}	0.914105	0.040240	0.919362	0.837007	0.967141
p_{31}	0.010073	0.007607	0.008574	0.000728	0.023529
p_{32}	0.057174	0.034890	0.052893	0.009239	0.120327
ϕ_1	0.327128	0.033794	0.327327	0.270133	0.381994
σ_1^2	9.102411	13.097919	5.533730	1.680441	26.945148
σ_2^2	4.433267	1.515332	4.237683	2.306150	7.031176
σ_3^2	8.395016	3.472055	7.648684	4.820553	14.306609
μ_1	-5.001851	0.319784	-5.013139	-5.482517	-4.474105
μ_2	-1.827111	0.457536	-1.801355	-2.595507	-1.117367
μ_3	2.030654	0.449129	1.989048	1.327146	2.808533

Figure B.6: Probability of states at each year, Uruguay

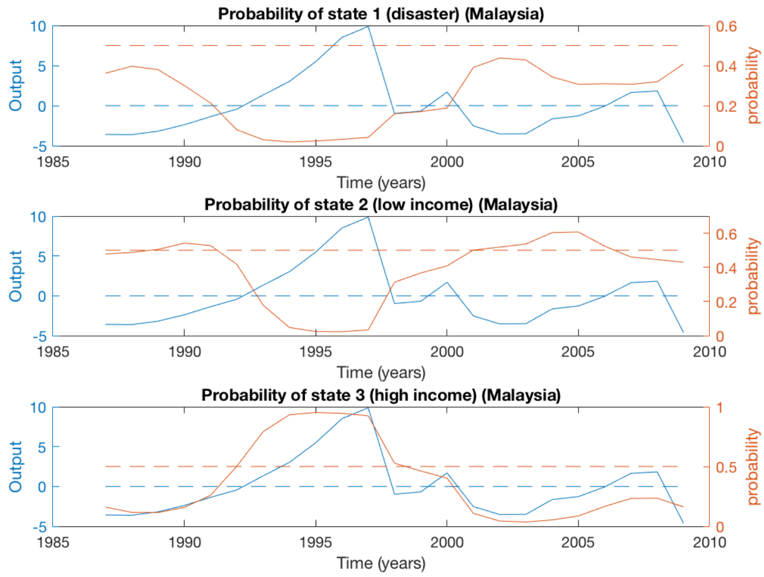


7. Malaysia.

Table B.7: Estimated parameters (Malaysia, 1987 - 2009)

Parameter	mean	sd	median	5% quantile	95% quantile
p_{11}	0.808889	0.090067	0.819447	0.646901	0.936625
p_{12}	0.165280	0.083891	0.155519	0.047480	0.317285
p_{21}	0.059492	0.033581	0.055333	0.015364	0.123716
p_{22}	0.862152	0.047863	0.867946	0.773734	0.930215
p_{31}	0.017146	0.017776	0.011512	0.000413	0.053161
p_{32}	0.047353	0.030199	0.041646	0.009264	0.100226
ϕ_1	0.279531	0.031970	0.280444	0.226858	0.328441
σ_1^2	13.321138	26.495686	6.167186	0.987987	48.361299
σ_2^2	9.690507	18.877162	5.565169	1.434158	24.273739
σ_3^2	20.693669	19.273075	14.355126	5.779241	59.790055
μ_1	-4.207508	0.330181	-4.197886	-4.767780	-3.679464
μ_2	-2.890829	0.451123	-2.910432	-3.644217	-2.120241
μ_3	1.923255	0.448920	1.925930	1.171657	2.666477

Figure B.7: Probability of states at each year, Malaysia

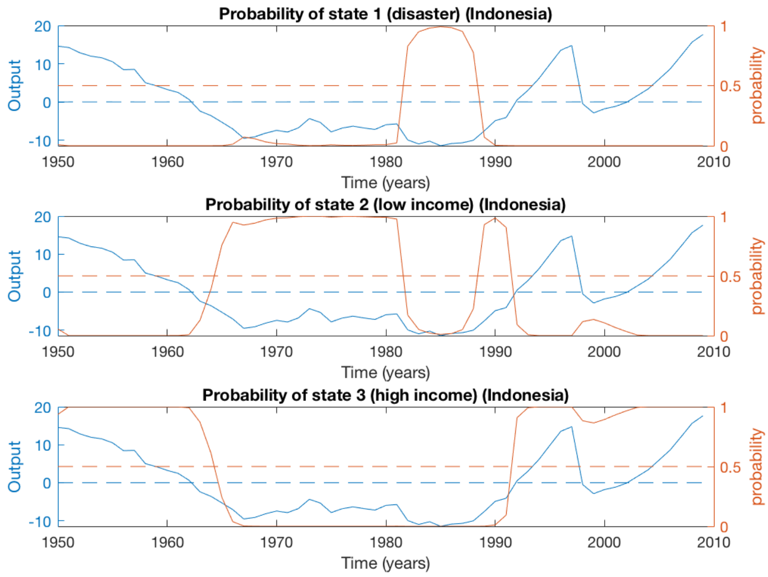


8. Indonesia.

Table B.8: Estimated parameters (Indonesia, 1950 - 2009)

Parameter	mean	sd	median	5% quantile	95% quantile
p_{11}	0.808889	0.090067	0.819447	0.646901	0.936625
p_{12}	0.165280	0.083891	0.155519	0.047480	0.317285
p_{21}	0.059492	0.033581	0.055333	0.015364	0.123716
p_{22}	0.862152	0.047863	0.867946	0.773734	0.930215
p_{31}	0.017146	0.017776	0.011512	0.000413	0.053161
p_{32}	0.047353	0.030199	0.041646	0.009264	0.100226
ϕ_1	0.279531	0.031970	0.280444	0.226858	0.328441
σ_1^2	1.677672	1.811435	1.085616	0.325953	4.842564
σ_2^2	5.574559	3.138059	5.083631	1.869066	11.374770
σ_3^2	24.184371	6.886989	23.198267	14.895282	36.487002
μ_1	-10.712061	0.262498	-10.710684	-11.148488	-10.266970
μ_2	-6.916182	0.371359	-6.912489	-7.552766	-6.303546
μ_3	6.235343	0.412971	6.238661	5.553619	6.942358

Figure B.8: Probability of states at each year, Indonesia

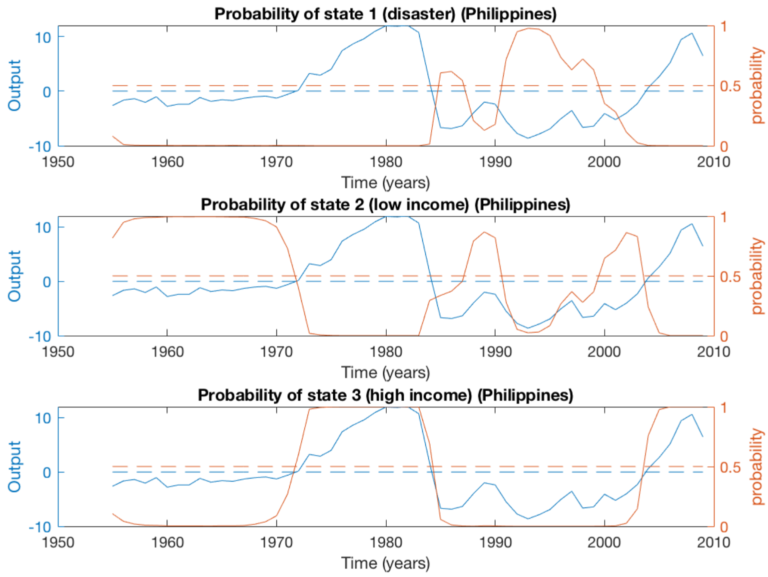


9. Philippines.

Table B.9: Estimated parameters (Philippines, 1955 - 2009)

Parameter	mean	sd	median	5% quantile	95% quantile
p_{11}	0.808889	0.090067	0.819447	0.646901	0.936625
p_{12}	0.165280	0.083891	0.155519	0.047480	0.317285
p_{21}	0.059492	0.033581	0.055333	0.015364	0.123716
p_{22}	0.862152	0.047863	0.867946	0.773734	0.930215
p_{31}	0.017146	0.017776	0.011512	0.000413	0.053161
p_{32}	0.047353	0.030199	0.041646	0.009264	0.100226
ϕ_1	0.279531	0.031970	0.280444	0.226858	0.328441
σ_1^2	4.883467	4.350206	3.563437	1.306509	13.932540
σ_2^2	3.297393	1.458989	3.066372	1.506499	6.212805
σ_3^2	17.642406	6.591136	16.473205	9.601207	31.072090
μ_1	-7.135848	0.319609	-7.141538	-7.648723	-6.610819
μ_2	-3.091021	0.438102	-3.112963	-3.781616	-2.350155
μ_3	3.781165	0.449679	3.785710	3.042017	4.528828

Figure B.9: Probability of states at each year, Philippines

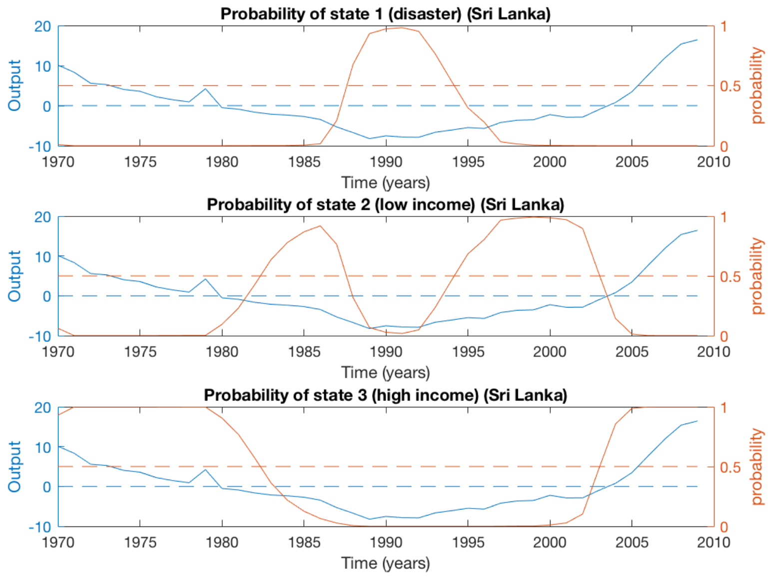


10. Sri Lanka.

Table B.10: Estimated parameters (Sri Lanka, 1970 - 2009)

Parameter	mean	sd	median	5% quantile	95% quantile
p_{11}	0.808889	0.090067	0.819447	0.646901	0.936625
p_{12}	0.165280	0.083891	0.155519	0.047480	0.317285
p_{21}	0.059492	0.033581	0.055333	0.015364	0.123716
p_{22}	0.862152	0.047863	0.867946	0.773734	0.930215
p_{31}	0.017146	0.017776	0.011512	0.000413	0.053161
p_{32}	0.047353	0.030199	0.041646	0.009264	0.100226
ϕ_1	0.279531	0.031970	0.280444	0.226858	0.328441
σ_1^2	1.413288	1.860556	0.920577	0.269515	3.877951
σ_2^2	5.376895	3.664558	4.563012	1.859504	11.158757
σ_3^2	19.767660	7.670207	18.289268	10.790329	33.808168
μ_2	-4.702176	0.406669	-4.712975	-5.331483	-3.994254
μ_3	3.972427	0.400380	3.958461	3.298074	4.618057

Figure B.10: Probability of states at each year, Sri Lanka

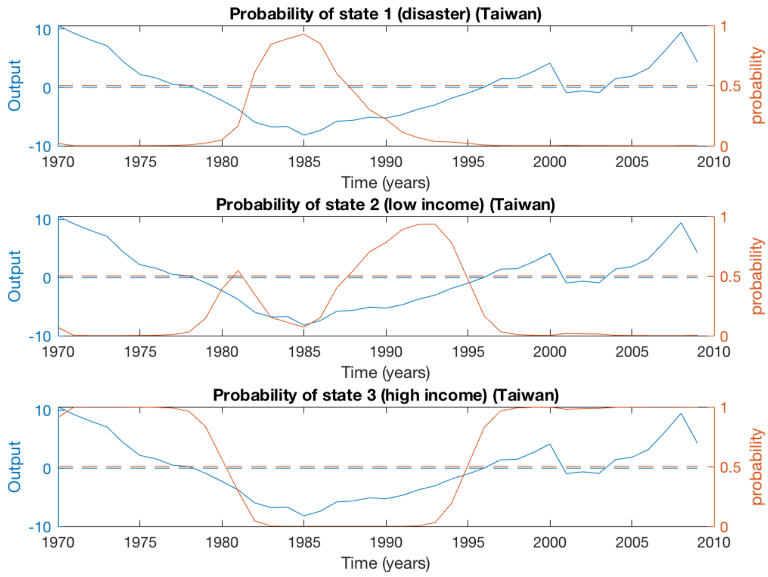


11. Taiwan.

Table B.11: Estimated parameters (Taiwan, 1970 - 2009)

Parameter	mean	sd	median	5% quantile	95% quantile
p_{11}	0.691368	0.219888	0.751107	0.161139	0.939955
p_{12}	0.236003	0.197399	0.176731	0.028880	0.645492
p_{21}	0.098022	0.137142	0.045012	0.001606	0.371231
p_{22}	0.778495	0.168709	0.821949	0.444849	0.947661
p_{31}	0.045211	0.041521	0.033909	0.001452	0.129815
p_{32}	0.043850	0.045297	0.029399	0.001287	0.131994
ϕ_1	0.229615	0.030925	0.230607	0.180687	0.281339
σ_1^2	4.253260	6.821728	2.331062	0.544508	14.538130
σ_2^2	5.003647	3.438395	4.152461	1.566193	11.382806
σ_3^2	8.140088	2.786327	7.588439	4.774904	13.302362
μ_1	-7.204513	0.295976	-7.199415	-7.693560	-6.709297
μ_2	-4.423518	0.419220	-4.434049	-5.121414	-3.694326
μ_3	2.794887	0.397480	2.795177	2.175986	3.443325

Figure B.11: Probability of states at each year, Taiwan

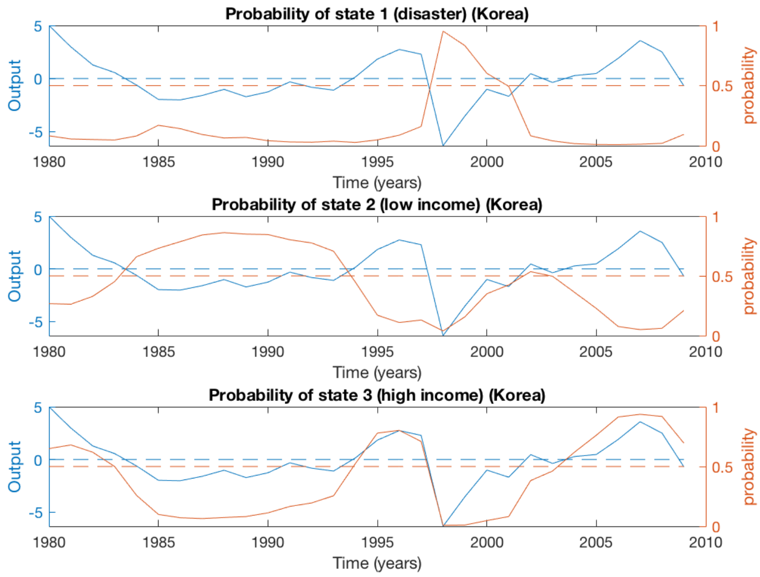


12. South Korea

Table B.12: Estimated parameters (Korea, 1980 - 2009)

Parameter	mean	sd	median	5% quantile	95% quantile
p_{11}	0.691368	0.219888	0.751107	0.161139	0.939955
p_{12}	0.236003	0.197399	0.176731	0.028880	0.645492
p_{21}	0.098022	0.137142	0.045012	0.001606	0.371231
p_{22}	0.778495	0.168709	0.821949	0.444849	0.947661
p_{31}	0.045211	0.041521	0.033909	0.001452	0.129815
p_{32}	0.043850	0.045297	0.029399	0.001287	0.131994
ϕ_1	0.229615	0.030925	0.230607	0.180687	0.281339
σ_1^2	17.628738	31.508888	7.119980	1.684514	74.980141
σ_2^2	4.198148	6.217772	2.316147	0.544390	14.029112
σ_3^2	3.836999	4.254526	2.429464	0.657330	11.048795
μ_1	-4.809709	0.305519	-4.823973	-5.320926	-4.311179
μ_2	-1.170295	0.419183	-1.167228	-1.861230	-0.461697
μ_3	1.528147	0.437100	1.538447	0.818019	2.234836

Figure B.12: Probability of states at each year, South Korea



B.3 Real interest rate and disaster probability

Real interest rates. I calculate the real interest rates of 16 countries in the sample using J.P. Morgan's EMBI Global Stripped Spread (SSPRD): real interest rate = SSPRD + US T-Bill rate (90 days) - US GDP deflator inflation. The real interest rates of Taiwan is calculated by: commerical paper rate (90 days) - countries' GDP deflator inflation. The EMBI Global data of Korea is constant from 2004 Q2 onwards, indicating a problem of data availability. I calculate Korea's interest rate from 2004 Q2 to 2013 Q1 as: interbank rate (90 days) - country's GDP deflator inflation. I list statistics related to the real interest rates in Table B.13.

Table B.13: Real Interest Rate on External Debt (%)

	Mean (%)	Periods
SeA countries		
Malaysia	3.89	Q4 1996 - Q1 2013
Indonesia	4.10	Q4 2004 - Q1 2013
Philippines	5.52	Q1 1998 - Q1 2003
South Korea	5.24	Q1 1994 - Q4 2013
Sri Lanka	6.35	Q1 2008 - Q1 2013
Taiwan	3.06	Q1 1994 - Q1 2013
LA countries		
Argentina	18.9	Q1 1994 - Q1 2013
Brazil	8.29	Q2 1994 - Q1 2013
Chile	3.11	Q2 1994 - Q1 2013
Mexico	6.04	Q1 1994 - Q1 2013
Peru	5.67	Q2 1997 - Q1 2013
Uruguay	5.26	Q2 2001 - Q1 2013
Venezuela	11.86	Q1 1994 - Q1 2013

Disaster probabilities. Disaster probabilities for the 16 countries are calculated according to Barro (2006). I list the probabilities in Table B.14.

B.4 Collection of defaults for Caribbean countries

Table B.14: Disaster probabilities

	Probability (%)
SeA countries (1911 - 2009)	
Malaysia	5.13
Indonesia	3.61
Philippines	5.41
South Korea	3.53
Sri Lanka	3.95
Taiwan	2.38
LA countries (1895 - 2009)	
Argentina	9.09
Brazil	3.03
Chile	4.12
Mexico	4.26
Peru	3.09
Uruguay	11.25
Venezuela	11.43

Table B.15: Years in Default: Caribbean countries, 1975-2012

Issuer	Foreign currency bank debt
Trinidad and Tobago	1988-1989
Jamaica	1978-1979; 1981-1985;1987-1993; 2010
Grenada	2004-2005
Belize	2006-2007
Antigua and Barbuda	1996-2006
Dominica	2003-2005
Dominican republic	1982-1994;2005

Appendix C

Appendix to Optimal Provision of Liquidity with Partial Information

I briefly explain the algorithm used to compute the partial information solution. The algorithm follows from Marcet et al.(2016).

The optimality condition of a partial information equilibrium can be summarised by the following integral:

$$\int_{\Theta_2(i_0)} \frac{U^*_{\gamma_1}}{h^*_z} f_{(z,\theta_0)} \left(\boldsymbol{A}^*(i_0,\theta_0),\theta_0\right) d\theta_0 = 0 \quad \text{for all} \quad s \in \boldsymbol{S^*} \tag{C.1}$$

where $U^*_{\gamma_1}$ is the partial derivative of the objective function U^E w.r.t γ_1, evaluated at $(i_0,\ \boldsymbol{\Delta}(i_0))$; h^*_z is the partial derivative of private agent's optimal condition w.r.t shock z, evaluated at $(i_0,\ \boldsymbol{\Delta}(i_0))$; $\Theta_2(i_0) = \{\theta_0 : i_0 = h(z,\theta_0) \quad \text{for some} \quad (z,\theta_0) \in \Psi\}$; and $z = \boldsymbol{A}^*(i_0,\theta_0)$.

Given a candidate function $\tilde{\boldsymbol{\Delta}}$, one can find the pairs of (z,θ_0) such that $\gamma_1 = \tilde{\boldsymbol{\Delta}}(i_0)$. Then evaluate all the functions involved in the integrand. If the integrand computed is not zero, iterate on $\tilde{\boldsymbol{\Delta}}$ until equation (C.1) holds.

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