The Effect of Modified Dispersion Relation on Dumb Holes

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Abstract

In this paper, we will deform the usual energy-momentum dispersion relation of a photon gas at an intermediate scale between the Planck and electroweak scales. We will demonstrate that such a deformation can have non-trivial effects on the physics of dumb holes. So, motivated by the physics of dumb holes, we will first analyse the effect of such a deformation on thermodynamics. Then we observe that the velocity of sound also gets modified due to such a modification of the thermodynamics. This changes the position of the horizon of a dumb holes, and the analogous Hawking radiation from a dumb hole. Therefore, dumb holes can be used to measure the deformation of the usual energy-momentum dispersion relation.

1 Introduction

In this paper, we will analyze the deformation of the usual energy-momentum dispersion relation. However, instead of deforming the usual energy-momentum dispersion relation

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at Planck scale, we will deform it at an intermediate scale between the Planck and electroweak scales. Such a deformation will modify the physics at energy scales much lower than the Planck scale. We will demonstrate that it will modify the physics of a dumb hole, as it will modify the velocity of sound. So, we will use this modified physics of dumb hole to test the deformation of the usual energy-momentum dispersion relation. We will study and propose a test for the deformation of the usual energy-momentum dispersion relation, as such a deformation is well motivated from various different theoretical considerations [1, 2, 3, 4, 5]. In fact, as the Lorentz symmetry fixes the form of the usual energy-momentum dispersion relation, so the usual energy-momentum dispersion relation will get deformed by the breaking of the Lorentz symmetry. Even though the Lorentz symmetry is one of the most important symmetries in nature, there are strong indication from various approaches to quantum gravity that it might only be broken in the UV limit. It has been observed that such deformation of the usual energy momentum dispersion relation in the UV limit (due to the breaking of Lorentz symmetry) occurs in the discrete spacetime [6], spacetime foam [7], spin-network in loop quantum gravity (LQG) [8], non-commutative geometry [9, 10], ghost condensation [11], Horava-Lifshitz gravity [12, 13] and in some modified teleparallel theories of gravity [14, 15, 16, 17]. The existence of a modified energy-momentum dispersion relation has lead to the development of doubly special relativity (DSR) [18], where non-linear Lorentz transformation are used to incorporate a maximum energy scale in the theory. The generalization of such a theory to curved spacetime has led to the development of gravity's rainbow [19].

It is possible for the usual energy-momentum dispersion to get deformed in string theory due the breaking of Lorentz symmetry, which can occur because of an unstable perturbative string vacuum [20]. In string field theory, the tachyon field can have the wrong sign for its mass squared. This makes the perturbative string vacuum unstable. This theory is ill-defined when the vacuum expectation value of the tachyon field is infinite. However, it is possible to obtain a finite negative vacuum expectation value for the tachyon field. This will break the Lorentz symmetry as it will make the coefficient of the quadratic term for the massless vector field to become nonzero and negative. It is known that supergravity theories can be obtained as the low energy effective action for string theory. However, it is possible to construct a gravitational version of the Higgs mechanism in supergravity theories, and this can also break the Lorentz symmetry in string theory [21]. It may be noted that black branes in type IIB string theory have also been used to break the Lorentz symmetry in string theory [22]. This is done by analysing the moduli stabilization using a KKLT-type moduli potential in type IIB warped flux compactification. It has been observed that a Higgs phase for gravity exists when all moduli are stabilized. This breaks the Lorentz symmetry in type IIB string theory. Compactification in string field theory has also been used for breaking the Lorentz symmetry in string theory [23]. Thus, there is a strong motivation to study the modified energy-momentum dispersion even in string theory.

Even though there is a strong motivation to study the deformation of the usual energy-momentum dispersion relation, such a deformation is usually studied at Planck scale, and so it cannot be tested using low energy experiments. However, it has been argued that quantum gravitational effects can occur at an intermediate scale between the Planck and electroweak scales, and such quantum gravitational effects can have low energy consiquences [24, 25]. The quantum gravitational effects at such an intermediate scale occur due to the generalized uncertainty principle [24, 25], and generalized uncertainty principle is related to deformed energy-momentum dispersion relation [26, 27], so the deformation of the usual usual energy-momentum dispersion relation can also occur at such an intermediate scale. In fact, as quantum gravitational effects are one of the main motivation to study Lorentz symmetry breaking, it is possible that the Lorentz symmetry can break at an intermediate scale between the Planck and electroweak scales. In this case, the scale at which the Lorentz symmetry breaks will be bounded by the present day experimental results. In fact, cosmic rays can be used for analyzing the breaking of Lorentz symmetry, as they can be used to set upper limits on the energy at which quantum gravitational effects can occur. It is possible to use the Greisen-Zatsepin-Kuzmin limit (GZK limit) to argue for the existence of a bound on the deformation of the usual energy-momentum dispersion relation [28, 29]. It may be noted that even the Pierre Auger Collaboration and the High Resolution Fly's Eye (HiRes) experiment have reconfirmed earlier results of the GZK cutoff, thus suggesting the deformation of the usual energy-momentum dispersion relation in the UV limit [30]. The hard spectra from gamma-ray burster's also suggests that the usual energy-momentum dispersion relation will get deformed in the UV limit [31]. So, these experimental results can be used to obtain a bound on the scale at which such a UV modification of the usual energy-momentum dispersion relation can occur.

Now as the usual energy-momentum dispersion relation can be deformed at an intermediate scale between the Planck and electroweak scales, it is possible to test such effects. Therefore, in this paper, we propose to test such a deformation using the modification to velocity of sound in a photon gas produced from such a deformation. The velocity of sound in the gas of photons depends on the thermodynamic theory of the photon gas [32], and the deformation of the usual energy-momentum dispersion relation will also deform the thermodynamics of this system [33, 34, 35, 36, 37]. We will explicitly calculate this change in the velocity of sound because of a deformation of the energy-momentum dispersion relation.

Such a correction to the velocity of sound can be measured using a dumb hole, as the horizon of phonons in a dumb hole depends on the local velocity of sound [38, 39], and so a change in the velocity of sound, which change the geometry of a dumb hole. It may be noted that such sonic black holes have been studied in Bose-Einstein condensates [40, 41, 42]. The Hawking radiation from such sonic black holes has been studied [43]. It was demonstrated that for such sonic black holes, quasi-particle are radiated with a thermal distribution, and this distribution is in exact agreement with the distribution produced by the Hawking radiation. The gravity dual of dumb holes has also been analysed using the fluid-gravity correspondence [44]. As the physics of a dumb holes depends on the geometry of a dumb hole, which in turn depends on the local velocity of sound, it is possible to study the corrections to the physics of a dumb hole from the corrected velocity of sound. This can be measured experimentally, and so it can be used to test the deformation of the energy-momentum dispersion relation.

2 Modified Dispersion Relation

As the velocity of sound depends on the thermodynamics of the system, we will first have to analyze the deformation of the thermodynamics produced by the deformation of the usual energy-momentum dispersion relation. In this section, we will analyze the effect of this deformation on the partition function. It may be noted that the deformation of the thermodynamics of a photon gas from the deformed energy-momentum dispersion relation has already been studied (see [33, 34, 35, 36, 37]). However, this has been studied using a modified dispersion relation proposed by Maguejo and Smolin (MS) [45, 46]. The MS deformation of the usual energy-momentum dispersion relation is given by $E^2 - p^2 = m^2(1-\kappa^{-1}E)^2$, where κ is the deformation parameter. In this paper, we will use a different form of the modified dispersion relation (MDR). This dispersion relation is given by [47, 48]

$$E^2 = p^2(1+\lambda E)^2 + m^2\,, \tag{1}$$

where λ is the deformation parameter. Note that we used natural units $c = h = 1$. It may be noted that the thermodynamics in a high temperature limit of this modified dispersion relation has been studied in [33]. In the following, we will analyse the effect of this deformation on the thermodynamics, without taking any such high temperature limit. Furthermore, we will assume that this modification occurs at an intermediate scale between the Planck and electroweak scales, so this can have interesting consequences for dumb holes. Now for photon gas ($m = 0$), Eq. (1) becomes

$$E = p(1+\lambda E)\,. \tag{2}$$

Parameter λ is related with maximum momentum $p_{\max} = 1/\lambda$. It may be noted that this maximum momentum is related to a Planckian momentum in the usual approach, and in the limit $\lambda \to 0$, we obtain the usual energy-momentum dispersion relation $E = p$.
The usual single particle partition function $Z_1(T,V)$ can be written as

$$Z_1(T,V) = 4\pi V \int_0^{\frac{1}{\lambda}} p^2 e^{-\beta E} dp\,. \tag{3}$$

where $\beta = 1/(k_B T)$, k_B is the Boltzmann constant and T is the temperature of the particle. By solving Eq. (2) for the momentum p, one gets $p = E/(1+\lambda E)$. Now by replacing that expression in the above integral, the partition function for one single particle becomes

$$Z_1(T,V) = 4\pi V \int_0^{\infty} \frac{E^2 e^{-\beta E}}{(1+\lambda E)^4} dE\,. \tag{4}$$

The above integral cannot be computed analytically. However, it may be noted that this partition function has a intrinsic cut-off at the order λ. Thus one can rewrite the above integral as follows

$$Z_1(T,V) \;=\; 4\pi V \left(\int_0^{\frac{1}{\lambda}} \frac{E^2 e^{-\beta E}}{(1+\lambda E)^4} dE + \int_{\frac{1}{\lambda}}^{\infty} \frac{E^2 e^{-\beta E}}{(1+\lambda E)^4} dE \right)\,. \tag{5}$$

We proceed to compute the integrals in the above expression. Since $\lambda E \leq 1$ in the first integral, by a power series expansion in λE, the first integral in Eq. (5) can be written as

$$\int_0^{\frac{1}{\lambda}} \frac{E^2 e^{-\beta E}}{(1+\lambda E)^4} dE = \int_0^{\frac{1}{\lambda}} E^2 e^{-\beta E} dE - 4\lambda \int_0^{\frac{1}{\lambda}} E^3 e^{-\beta E} dE + 10\lambda^2 \int_0^{\frac{1}{\lambda}} E^4 e^{-\beta E} dE + \mathcal{O}(\lambda^3). \tag{6}$$

The integrals appearing in the above expression have the general form

$$I_n = \int_0^{\frac{1}{\lambda}} E^n e^{-\beta E} dE\,, \quad n = 2, 3, 4. \tag{7}$$

The integrals can be computed, and up to second order in λ, we obtain

$$\begin{aligned}\int_0^{\frac{1}{\lambda}} E^2(1 - 4\lambda E + 10\lambda^2 E^2) e^{-\beta E} dE &= -\frac{e^{-\frac{\beta}{\lambda}}}{\beta^3}\left(7\frac{\beta^2}{\lambda^2} + 30\frac{\beta}{\lambda} + 98 + 216\frac{\lambda}{\beta} + 240\frac{\lambda^2}{\beta^2}\right) \\ &\quad + \frac{2}{\beta^3}\left(1 - \frac{12\lambda}{\beta} + \frac{120\lambda^2}{\beta^2}\right) + \mathcal{O}(\lambda^3).\end{aligned} \tag{8}$$

For the second integral in Eq. (5), $\lambda E \geq 1$. Hence, we carry out a power series expansion in $1/(\lambda E)$, which upto second order in λ yields

$$\begin{aligned}\frac{1}{\lambda^4}\int_{\frac{1}{\lambda}}^{\infty} \frac{e^{-\beta E}}{E^2}\left(1 - \frac{4}{\lambda E} + \frac{10}{\lambda^2 E^2}\right) dE &= \frac{e^{-\frac{\beta}{\lambda}}}{\beta^3}\left(7\frac{\beta^2}{\lambda^2} - 30\frac{\beta}{\lambda} + 158 - 984\frac{\lambda}{\beta} + 7040\frac{\lambda^2}{\beta^2}\right) \\ &\quad + \mathcal{O}(\lambda^3)\,.\end{aligned} \tag{9}$$

By substituting the expansions of the integrals given in Eq. (8) and Eq. (9) into Eq. (5), we obtain the partition function (up to second order in λ),

$$Z_1(T,V) = 4\pi V\left[\frac{2}{\beta^3}\left(1 - \frac{12\lambda}{\beta} + \frac{120\lambda^2}{\beta^2}\right) - 60\frac{e^{-\frac{\beta}{\lambda}}}{\beta^3}\left(\frac{\beta}{\lambda} - 1 + 20\frac{\lambda}{\beta} - 114\frac{\lambda^2}{\beta^2}\right)\right] + \mathcal{O}(\lambda^3). \tag{10}$$

Finally, for a N classical particle system, the partition function $Z_N(V,T)$, can be written as

$$Z_N(T,V) = \frac{1}{N!}[Z_1(T,V)]^N\,. \tag{11}$$

Thus, we have obtained the expression for the partition function for the deformed photon gas. Now we will be able to analyse the effect of such a deformation on the thermodynamics of a photon gas.

3 Thermodynamics of Photon Gas

In this section, we will analyse certain aspects of thermodynamics of photon gas by asuming a modified energy-momentum dispersion relation [35]. The thermodynamics of a photon gas depend on the usual energy-momentum dispersion relation, so a deformation of the usual energy-momentum dispersion relation will deform the thermodynamics of the photon gas. It may be noted that such a modified thermodynamics has been studied in the high temperature limit [33]. However, in this section, we will analyse the deformed thermodynamics without using any such limit. This will be important, as we will use this deformed thermodynamics to dumb holes. However, dumb holes are usually obtained in Bose-Einstein condensates [40, 41], and these condensates become unstable at high temperatures. Thus, we will first have to analyse the effect of such a deformation without taking any high temperature limit. The free energy of the photon gas is given by

$$F = -k_B T \ln(Z_N(T,V)) \,. \tag{12}$$

Now, by using Eqs. (10) and (11), we obtain

$$F = -Nk_BT\Big[1+\ln\Big\{\Big(\frac{4\pi V k_B^3 T^3}{N}\Big)Q(\lambda,T)\Big\}\Big] + \mathcal{O}(\lambda^3) \,, \tag{13}$$

where for simplicity we have defined the function

$$\begin{aligned} Q(\lambda,T) &= 2 - 24\lambda k_B T + 240\lambda^2 {k_B}^2 T^2 \\ &\quad -60 e^{-\frac{1}{\lambda k_B T}}\Big(\frac{1}{\lambda k_B T} - 1 + 20\lambda k_B T - 114\lambda^2 {k_B}^2 T^2\Big) \,. \end{aligned} \tag{14}$$

The above quantity was defined to avoid long equations since it will also appear in the next thermodynamics quantities.
The expression of the free energy can be used to calculate the pressure,

$$P = -\left(\frac{\partial F}{\partial V}\right) = \frac{Nk_BT}{V} \,, \tag{15}$$

so that the equation of state $PV = Nk_BT$ is not deformed by the deformation of the energy-momentum dispersion relation.
The entropy of the photon gas $S = (\partial F/\partial T)_V$ can now be written as

$$\begin{aligned} S &= Nk_B\Big[4+\ln\Big\{\Big(\frac{4\pi V k_B^3 T^3}{N}\Big)Q(\lambda,T)\Big\} \\ &\quad -\Big\{60 e^{-\frac{1}{\lambda k_B T}}\Big(\frac{1}{\lambda^2 {k_B}^2 T^2} - \frac{2}{\lambda k_B T} + 20 - 94\lambda k_B T - 228\lambda^2 {k_B}^2 T^2\Big) \\ &\quad +24\lambda k_B T - 480\lambda^2 {k_B}^2 T^2\Big\}Q(\lambda,T)^{-1}\Big] + \mathcal{O}(\lambda^3) \,. \end{aligned} \tag{16}$$

The relation between internal energy, free energy and entropy of this deformed photon gas $U = F + TS$, can be used to obtain,

$$\begin{aligned} U &= Nk_BT\Big[3 - \Big\{60 e^{-\frac{1}{\lambda k_B T}}\Big(\frac{1}{\lambda^2 {k_B}^2 T^2} - \frac{2}{\lambda k_B T} + 20 - 94\lambda k_B T - 228\lambda^2 {k_B}^2 T^2\Big) \\ &\quad +24\lambda k_B T - 480\lambda^2 {k_B}^2 T^2\Big\}Q(\lambda,T)^{-1}\Big] + \mathcal{O}(\lambda^3) \,. \end{aligned} \tag{17}$$

As the density $\rho = U/V$ is obtained from internal energy and volume, we will have

$$\begin{aligned} P &= \Big(\frac{Nk_BT}{3V}\Big)Q(\lambda,T)^{-1}\Big[60 e^{-\frac{1}{\lambda k_B T}}\Big(\frac{1}{\lambda^2 {k_B}^2 T^2} - \frac{2}{\lambda k_B T} + 20 - 94\lambda k_B T - 228\lambda^2 {k_B}^2 T^2\Big) \\ &\quad 24\lambda k_B T - 480\lambda^2 {k_B}^2 T^2\Big] + \frac{\rho}{3} + \mathcal{O}(\lambda^3) \,. \end{aligned} \tag{18}$$

The specific heat of this photon gas $C_v = (\partial U/\partial T)_V$ can be written as

$$C_v = Nk_B(3 + C'Q(\lambda,T)^{-2}) + \mathcal{O}(\lambda^3) \,, \tag{19}$$

where

$$C' = (-96\lambda k_B T + 3456\lambda^2 {k_B}^2 T^2) - 60 e^{-\frac{1}{\lambda k_B T}} \Big(\frac{2}{\lambda^3 {k_B}^3 T^3} - \frac{30}{\lambda^2 {k_B}^2 T^2} + \frac{400}{\lambda k_B T} - 2452 + 9812\lambda k_B T$$
$$-34956\lambda^2 {k_B}^2 T^2 \Big) + 3600 e^{-\frac{2}{\lambda k_B T}} \Big(- \frac{1}{\lambda^2 {k_B}^2 T^2} + 80 - 1180\lambda k_B T + 1084\lambda^2 {k_B}^2 T^2 \Big) + \mathcal{O}(\lambda^3) \ . \quad (20)$$

Figures 1-3 represent the entropy, internal energy and specific heat versus the temperature plotting different dispersion models. In these plots, we have chosen the following values: $k_B = 1, \lambda = 10^{-4}, N = 100000, V = 0.01$. It may be noted that here we use $\lambda = 10^{-4}$ to study effects of such deformations, and in the next section, we will discuss the physically values for such a deformation. It may be noted that at low temperatures, the entropy of the photon gas is the same for special relativity (SR), the model used by Magueijo-Smolin (MS) and the modified dispersion relation (MDR) used in this paper. However, the entropy of MS is less than SR, and bigger than MDR at higher temperatures. The internal energy for SR is bigger than MDR and MS at high temperatures. However, the internal energy of MS is initially higher than MDR and then it becomes equal to MDR at about $T = 4000$. After that, the internal energy becomes smaller than MDR. The specific heat in SR is constant and it is bigger than the specific heat for both MDR and MS models. The specific heat in MS is also constant for low temperature, and then it starts to reduce as the temperature increases. However, the specific heat for MS remains higher than MDR till about $T = 2000$, then it becomes equal to MDR at $T = 2000$. Finally, the specific heat becomes less than MDR. Hence, we observe that the thermodynamic properties depend critically on the modification of the usual energy-momentum dispersion relation.

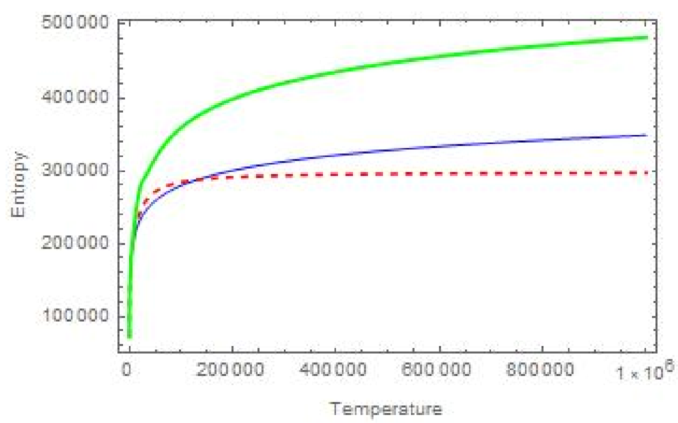

Figure 1: **Entropy S as a function of the Temperature T for three cases**: Special Relativity (SR), Magueijo-Smolin (MS) model and Modified Dispersion Relation (MDR). Here upper curve (green line) represents MDR model, middle curve (blue line) represents SR and lower curve (red line) represents MS model.

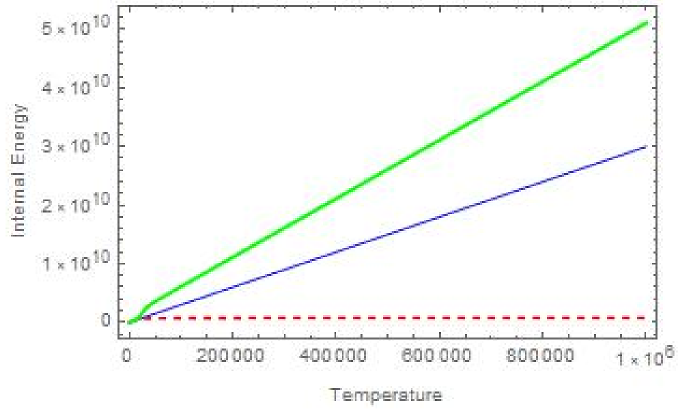

Figure 2: **Internal energy as a function of the Temperature T for three cases**: Special Relativity (SR), Magueijo-Smolin (MS) model and Modified Dispersion Relation (MDR).

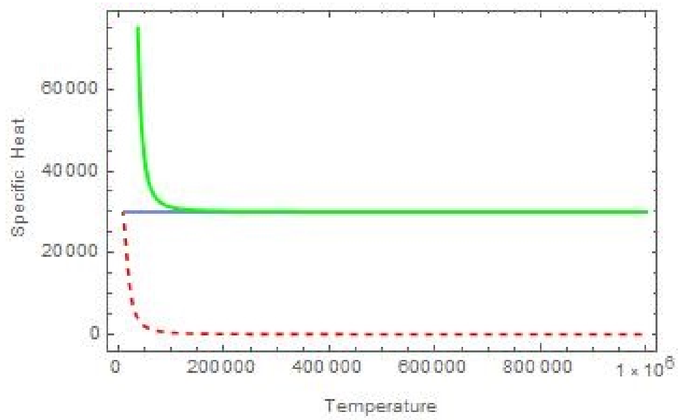

Figure 3: **Specific heat as a function of the Temperature T for three cases**: Special Relativity (SR), Magueijo-Smolin (MS) model and Modified Dispersion Relation (MDR).

4 Dumb Hole

In the previous section, we assumed the value $\lambda = 10^{-4}$. However, actually such a deformation from quantum gravity is expected to occur at Planck scale, and for a deformation at such a scale, the deviation from the original results at $T \sim 10^3$ would be very small. It is possible to assume such a deformation to the energy-momentum dispersion relation could occur at an intermediate scale between the Planck and electroweak scales [24, 25, 49]. In fact, the Lorentz violation breaking has been studied with optical ring cavity, and it has been demonstrated that such a breaking can occur only at a scale 10^{-14} [50]. Even though this scale is still much larger than Planck scale, it is still too small to directly observe the corrections to the thermodynamics from modified energy dispersion relation. So, we need a very sensitive system to measure such effects. We will propose that dumb holes will form such a system. A dumb hole (also called as a sonic black hole), is an geometric object which traps phonons (sound perturbations), just as a black hole traps photons [38, 39]. The geometry of a dumb hole depends on the velocity of sound, so a deformation of the velocity of sound will deform the geometry of a dumb hole.

However, the trapping of sound by the horizon of the dumb hole is a very sensitive process, and even for a very small change in the velocity of sound, the dumb hole would not form. Hence, if the velocity of sound changes even by a very small amount, the original horizon will not be able to trap sound, and we will not get the dumb hole geometry. Therefore, we propose that dumb holes can be used to analyze the small differences in the velocity of sound produced by a deformation of energy-momentum dispersion relation.

Now to demonstrate that, we will first have to analyze the effects of such a deformation of the energy-momentum dispersion relation on the velocity of sound. So, first we will calculate such effects on the velocity of sound. It is known that the velocity of sound depends on the thermodynamics of photon gas [32], so that the velocity of sound should be deformed because of the deformation of thermodynamics. The velocity of sound can be written as

$$v \;=\; \Big(\frac{E}{\rho}\Big)^{1/2}, \tag{21}$$

where $\rho = U/V$, and the Young modulus E is given by

$$E \;=\; -V\frac{dP}{dV}. \tag{22}$$

Here V and P are the volume and pressure of the photon gas. In our case, we have a relationship between P and V for this deformed photon gas given by

$$P = \frac{\rho}{3} - \left(\frac{Nk_BT}{3V}\right)\alpha(\lambda,\beta)\,, \tag{23}$$

where α is a long function of λ and β defined as

$$\begin{aligned}\alpha(\lambda,\beta) \;=\; & -\Big\{60e^{-\frac{1}{\lambda k_BT}}\Big(\frac{1}{\lambda^2{k_B}^2T^2} - \frac{2}{\lambda k_BT} + 20 - 94\lambda k_BT - 228\lambda^2{k_B}^2T^2\Big)\\ & +24\lambda k_BT - 480\lambda^2{k_B}^2T^2\Big\}Q(\lambda,T)^{-1}\,. \end{aligned}\tag{24}$$

Thus, by using Eqs. (22) and (21), the Young modulus and the velocity of sound can be written as

$$E \;=\; \frac{\rho}{3} - \Big(\frac{Nk_BT}{3V}\Big)\alpha(\lambda,\beta)\;, \tag{25}$$

$$v \;=\; \sqrt{\frac{1}{3}\Big[1-\frac{Nk_BT}{\rho V}\alpha(\lambda,\beta)\Big]} = \sqrt{\frac{1}{3}\Big[1-\frac{P}{\rho}\alpha(\lambda,\beta)\Big]}\,, \tag{26}$$

where in the second equation we have used $PV = Nk_BT$. As v depends on λ and β, we will write v as $v(\lambda,\beta)$. This will be important as we will need this to show that the dumb hole horizon also depends on λ and β.

By using Eq. (23) (and again using $PV = Nk_BT$), we have that the pressure is

$$P = \frac{\rho}{3+\alpha(\lambda,\beta)}\,. \tag{27}$$

Using the above relation in Eq. (26), we finally obtain that the velocity of sound is given by

$$v(\lambda,\beta) = \frac{1}{\sqrt{3+\alpha(\lambda,\beta)}}\,. \tag{28}$$

This is the expression for the modified velocity of sound in photon gas. It may be noted that this modification of the velocity of sound is due to the deformation of the energy-momentum dispersion relation and it will have interesting astrophysical application, as it will effect the accretion of CMB photons around these primordial black holes [67, 68]. It is reassuring to note that the above expression reduces to the usual result in the $\lambda \to 0$ limit [32]

$$v = \frac{1}{\sqrt{3}}\,. \tag{29}$$

Remark that the velocity of sound depends critically on the temperature due to a deformation of the usual energy-momentum dispersion relation. Such a modification to the velocity of sound becomes significant at higher temperatures. Further, the velocity of sound is constant when the usual energy-momentum dispersion relation is used. Thus, we can use this modified expression for the velocity of sound to analyse the physical effects of the deformed energy-momentum dispersion relation. Fig. 4 shows the velocity of sound versus Temperature for Special Relativity (which assumes the standard energy-momentum dispersion) and MDR.

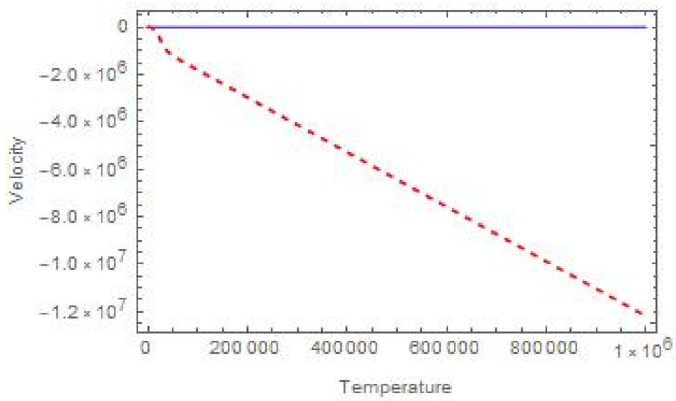

Figure 4: **Velocity of sound as a function of the Temperature** T **for two cases**: Special Relativity (SR) and Modified Dispersion Relation (MDR). Here lower curve(red line) represents MDR, upper curve(blue line) represents SR.

Now after we have analysed the effect of the modified dispersion relation on the velocity of sound, we can analyse the effect of such a deformation on dumb holes [38, 39]. The deformed energy-momentum tensor and the current for a perfect fluid, can be written as

$$\begin{aligned} T^{\mu\nu} &= Pg^{\mu\nu} + (\rho + P)u^\mu u^\nu \\ &= \frac{\rho}{3+\alpha(\lambda,\beta)}\Big[g^{\mu\nu} + (4+\alpha(\lambda,\beta))u^\mu u^\nu\Big]\,, \end{aligned} \tag{30}$$

$$j_i^\mu = q_i u^\mu\,, \tag{31}$$

where q_i are the conserved charges and u^μ is the four velocity which satisfies $u^\mu u_\mu = -1$. It may be noted that this four velocity satisfies a similar relation to the original four velocity [44]. These two quantities are conserved as $\nabla_\mu T^{\mu\nu} = 0$ and $\nabla_\mu j_i^\mu = 0$. Now, we can define

$$\bar{\mathcal{T}}^4 = \frac{\rho}{3+\alpha(\lambda,\beta)} = P\,. \tag{32}$$

Clearly, if we set $\alpha = 0$, the standard equation of state is recovered. Moreover, if one assumes that $\alpha(\lambda,\beta) \ll 1$, and one expands the above expression up to first order in α, one gets

$$\begin{aligned} P &= \frac{\rho}{3} - \frac{\alpha\rho}{9} + \mathcal{O}(\alpha^2) \\ &= P_0 - \frac{\alpha\rho}{9} + \mathcal{O}(\alpha^2) \\ &= \mathcal{T}^4 - \frac{\alpha\rho}{9} + \mathcal{O}(\alpha^2)\,, \end{aligned} \tag{33}$$

where $P_0 = \rho/3$ and $\mathcal{T}^4 = P_0$ are the quantities computed in the standard theory which assumes the usual energy-momentum dispersion. Therefore, up to first order in α, we have that the pressure differs by $-\alpha\rho/9$.

We can project the energy-momentum conservation law onto the four velocity as $u_\nu \nabla_\mu T^{\mu\nu} = 0$, yielding

$$\nabla_\mu \Big(\bar{\mathcal{T}}^3 u^\mu \Big) = 0 \,. \tag{34}$$

Now, by projecting the energy-momentum tensor onto the projector $h^\lambda_\nu = \delta^\lambda_\nu + u^\lambda u_\nu$, we obtain

$$\nabla_\mu \Big(\bar{\mathcal{T}} u_\nu \Big) \;\; = \;\; \nabla_\nu \Big(\bar{\mathcal{T}} u_\mu \Big) \,, \tag{35}$$

where we have assumed that the fluid satisfies $h^\lambda_\mu h^\sigma_\nu (\partial_\lambda u_\sigma - \partial_\sigma u_\lambda) \equiv 0$. This kind of fluids are called irrational since they do not have a vorticity term. The above equation allows us to define a potential such as

$$\bar{\mathcal{T}} u_\mu = \partial_\mu \phi \,, \tag{36}$$

and hence by using $u_\mu u^\mu = -1$, we also get

$$\bar{\mathcal{T}}^2 = -(\partial_\mu \phi)(\partial^\mu \phi) \,. \tag{37}$$

We have all the ingredients to study isentropic sound waves for this kind of fluid. To do this, we need to take small variations of the velocity potential $\phi \to \phi + \delta\phi$. By doing that, all our other quantities also are deformed as follows

$$u^\mu \;\; \to \;\; u^\mu + \delta u^\mu \,, \tag{38}$$

$$q_i \;\; \to \;\; q_i + \delta q_i \,, \tag{39}$$

$$\bar{\mathcal{T}} \;\; \to \;\; \bar{\mathcal{T}} + \delta \bar{\mathcal{T}} \,. \tag{40}$$

If we replace these quantities in (36), we find the following expression

$$u^\mu \delta \bar{\mathcal{T}} + \bar{\mathcal{T}} \delta u^\mu \;\; = \;\; \partial^\mu \delta\phi \,. \tag{41}$$

Now, if we use that $u_\mu \delta u^\mu = 0$, we obtain $\bar{\mathcal{T}} \delta u^\mu = h^{\mu\nu} \partial_\nu (\delta\phi)$, and $\delta\bar{\mathcal{T}} = -u^\mu \partial_\mu \delta\phi$. By using these expressions in the above equation, we obtain

$$\partial_\mu \Big[\sqrt{-g} \bar{\mathcal{T}}^2 (g^{\mu\nu} - 2 u^\mu u^\nu) \partial_\nu \Big] (\delta\phi) = 0 \,. \tag{42}$$

Now we can write the acoustic metric as

$$G_{\mu\nu} = \sqrt{3} \bar{\mathcal{T}}^2 (g_{\mu\nu} + \frac{2}{3} u_\mu u_\nu) \,, \tag{43}$$

and then we can rewrite Eq. (42) as a Klein-Gordon massless scalar field equation, yielding

$$\partial_\mu \Big[\sqrt{-G} G^{\mu\nu} \partial_\nu \Big] (\delta\phi) = 0 \,. \tag{44}$$

The metric $G^{\mu\nu}$ describes the acoustic metric of the dumb hole, and such a metric has been constructed for a dumb hole with the usual energy-momentum dispersion relation [44]. It may be noted that it is possible to construct a similar metric using the deformed energy-momentum dispersion relation, and changing the the time-coordinate as

$$d\tau = dt + \frac{\frac{3}{2}\gamma(\lambda,\beta)^2 v_i}{1 - \frac{2}{3}\gamma(\lambda,\beta)^2} dx^i \,, \tag{45}$$

where $\gamma(\lambda, \beta)$ is the Lorentz like factor for this metric, in which the velocity of light is replaced by the velocity of sound [44]. It may be noted that for the deformed case, the velocity of sound is a function of λ and β, and γ is the Lorentz like factor with velocity of sound as the limiting velocity, so the deformed γ will also be a function of λ and β. Thus, following the calculations done for the original dumb holes [44], the corresponding metric for the dumb holes in the deformed theory can be written as

$$d\tilde{s}^2 = \sqrt{3}\bar{\mathcal{T}}^2 \left\{ -\Big(1 - \frac{2}{3}\gamma(\lambda,\beta)^2\Big) d\tau^2 + \left(g_{ij} + \frac{\frac{2}{3}\gamma(\lambda,\beta)^2}{1 - \frac{2}{3}\gamma(\lambda,\beta)^2} v_i v_j \right) dx^i dx^j \right\} . \tag{46}$$

Now this metric has a horizon at $\gamma(\lambda, \beta) = \sqrt{3/2}$, and this horizon also is a function of λ and β. The horizon of a dumb hole by definition traps the sound. However, as the velocity of sound is a function of λ and β, the original horizon is not the horizon in the deformed theory, as it cannot trap the sound in the deformed theory. Now for a given value of λ, the horizon is a function of temperature, as the sound is also a function of temperature. However, for a given temperature and a given value of λ, this theory has a well defined fixed horizon. So, the geometry of the dumb holes get deformed due to a deformation of the energy-momentum dispersion relation. Such a deformation of the geometry of dumb holes can have observational consequences, and it would be interesting to analyse such consequences. In fact, one such observational consequence occurs due to the analogous Hawking radiation from a dumb hole. This dumb hole can emit Hawking radiation at a temperature [43, 51]

$$T = \frac{\kappa}{2\pi} , \tag{47}$$

where κ is the analogous surface gravity of this dumb hole. It would be possible to measure this analogous Hawking radiation and observe at what values the dumb hole form, as the Hawking radiation will only occur when a dumb hole forms. However, formation of the dumb hole gets modified by the deformation of the energy-momentum dispersion relation, so we can use the Hawking radiation to measure this deformation. In fact, as the formation of a dumb hole is very sensitive to even small changes in the velocity of sound, this system can detect such small changes in the velocity of sound, which might not be observed by various other effects. So, a dumb hole can be used to measure deformation of the usual energy-momentum dispersion relation.

5 Conclusion

In this paper, we have analysed the deformation of the usual energy-momentum dispersion relation. We deformed the usual energy-momentum dispersion relation at an intermediate scale between the Planck and electroweak scales. We have investigated the effect of such a deformation on the physics of a photon gas. It was observed that the deformation of the usual energy-momentum dispersion relation will deform the partition function for the photon gas. It was also demonstrated that the deformation of the partition function deformed the thermodynamics of the photon gas. Therefore, we performed a derivation of the corrections to various thermodynamic quantities from the deformation of the usual energy-momentum dispersion relation. The analysis we performed in this paper was more general than the previous works, where high temperature limit was taken [33]. Note that since we were interested on studying dumb holes and they are usually constructed using Bose-Einstein condensates [40, 41], and such condensates can become unstable at high temperature, we had to analyse the modification to the thermodynamics without taking such a high temperature limit. Furthermore, in this work a different form of modified dispersion relation (MDR) was used than the MS model. It may be noted that these corrections only become significant in the UV limit of the theory, and the usual thermodynamics is recovered in the IR limit of the theory. This is because the corrections depend on the factor λ, and in the IR limit $\lambda \to 0$, as in the IR limit the deformed energy-momentum dispersion relation reduces to the usual energy-momentum dispersion relation. Thus, in the IR limit, the correction terms for the modified thermodynamics vanish, and we recover the usual thermodynamics. However, the correction terms cannot be neglected in the UV limit of this theory. As the velocity of sound depends on the thermodynamics of the photon gas [32], a deformation of this thermodynamics also modified the expression for the velocity of sound. In this paper, we have explicitly observed the effect of deforming the usual energy-momentum dispersion relation on the velocity of sound. Furthermore, as the horizon of a dumb hole depends on the local velocity of sound, a change in the local velocity of sound would change the horizon of a dumb hole. We have thus analysed the effect of the deformation of the usual energy-momentum dispersion relation on the geometry of a dumb hole. This deformation of the horizon of a dumb hole can also deform the Hawking radiation and this deformation in the Hawking radiation can be detected. This can be used to test the breaking of the Lorentz symmetry at a scale between the electroweak and Planck scales. Thus, in this paper, it has been argued that the geometry of a dumb hole is non-trivially deformed by the deformation of the usual energy-momentum dispersion relation, and dumb holes can be in turn used to test such deformations of the usual energy-momentum dispersion relation.

The velocity of sound fixes the value of the sonic points, and the sonic points are important for many important process like such accretion around a black hole [52, 53, 54, 55]. The accretion around a black hole was first studied by Bondi in the Newtonian framework [56], and so it is called Bondi accretion. The relativistic generalization of the Bondi type accretion has been used for analysing accretion around a black hole [57, 58]. It has been demonstrated that radiative processes can have interesting effects for accretion around a black hole [59, 60, 61] or other spherically symmetric space-times [62]. The accretion also gets effected from rotation of a black hole [63]. It has been observed that the cosmological constant can also effect the accretion around a black hole [64, 65]. It has also been demonstrated that there is a correspondence between sonic points of ideal photon gas and photon spheres, and this correspondence has been used to study the accretion of ideal photon gas [66]. In fact, it is possible for that the primordial black holes would be formed at early stages of the evolution of this universe, and the photon gas of CMB photon would couple to such primordial black holes. Therefore, the accretion of photons would occur around such primordial black holes. In fact, such accretion of CMB photons around primordial black holes has already been discussed [67, 68]. In this analysis, it was observed that the accretion of CMB photons around these primordial black holes depends on the velocity of sound. This is because the velocity of sound fixes the value of the sonic points, and the accretion of the CMB photons depend on the transonic solution across the sonic point. As it was demonstrated in this paper, that the velocity of sound changes due to a deformation of the usual energy-momentum dispersion relation, so, the results of this paper can have direct application for analysing the accretion of CMB photons around these primordial black holes.

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