Fair Profit Distribution in Multi-echelon Supply Chains via Transfer Prices

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Abstract

The total profit maximisation of a supply chain network may result in an uneven and impractical profit distribution among the members. This work addresses the fair profit distribution within a multi-echelon supply chain using transfer prices. A mixed integer linear programming (MILP) model framework is proposed for the optimal production, distribution and capacity planning of a supply chain network of an active ingredient (AI), consisting of AI plants, formulation plants and markets. The transfer prices of the AI from AI plants to formulation plants, and those of products from formulation plants to markets are to be optimised. The proportional and max-min fairness criteria are adopted to define fair profit distributions. Considering bargaining powers of supply chain members, game theoretic solution approaches are developed for fair solutions using Nash bargaining and lexicographic maximin principles. Especially, a hierarchical approach is developed to obtain an approximate optimal fair solution efficiently. The applicability and efficiency of the proposed approaches are demonstrated by two examples, including a real world agrochemical supply chain network.

Keywords: Supply chains, fair profit distribution, transfer price, mixed integer linear programming, lexicographic maximin approach, Nash bargaining approach

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1. Introduction

Over the past decades, supply chains have been reshaped into complex networks involving suppliers, production sites, distribution facilities and markets. The optimisation of supply chains in the process industry has received extensive attention in the literature (Grossmann, 2005; Shah, 2005; Papageorgiou, 2009; Barbosa-Póvoa, 2012). Many supply chain optimisation models and approaches just consider one supply chain as a whole (Tsiakis & Papageorgiou, 2008; Sousa et al., 2011; Longinidis & Georgiadis, 2011; Liu et al., 2012; Cardoso et al., 2013; Muñoz et al., 2015; Gaur et al., 2017). However, in real practice, there are conflicting interests of individual supply chain members that all aim to pursue the most profit and benefit for themselves, which need to be taken into account for optimisation. When the total profit of a supply chain is maximised to enhance its performance, the profit of the whole supply chain is usually distributed to its members in an uneven way, which could lead to negative impacts, including dissatisfaction of members, instability of systems and coalition, disadvantage in competitive edge, loss of markets, increasing costs and reduction in revenue. Thus, a fairer profit distribution is preferred to maintain stability and competitiveness of supply chain networks.

The fairness issues have been widely investigated in some fields, e.g. welfare economics (Varian, 1975; Fleurbaey, 2008), telecommunications (Jain et al., 1984; Mazumdar et al., 1991), and supply chain contracting (Cui et al., 2007; Katok & Pavlov, 2013; Ho et al., 2014). The concept, perception and interpretation of fairness vary depending on problems and people involved, and there is no single fairness criterion applicable to all problems. In the literature, there are two widely accepted fairness criteria: proportional fairness and max-min fairness (Bertsimas et al., 2011), which satisfy a set of generally agreed axioms for ideal fairness criterion. This paper aims to develop an optimisation framework for fair profit distribution among supply chain members using transfer prices under these two fairness criteria.

Transfer price is generally referred to as the intra-company price that a selling department, division or subsidiary of a company charges for a product or service supplied to a buying department, division, or subsidiary of the same company (Abdallah, 1989; Pfeiffer, 1999). However, recently the terminology has been extended to inter-company payments in decentralised supply chain networks

(Gjerdrum et al., 2002; Lakhal, 2006). In this work, the considered multi-echelon supply chain members are regarded as independent profit centres that can determine their transfer prices. The transfer pricing has been used as an income-shifting mechanism to the subsidiaries in lower tax countries so as to increase their after-tax profits (O'Connor, 1997; Vidal & Goetschalckx, 2001; Shunko & Gavirneni, 2007; Miller & de Matta, 2008; Ernst & Young, 2013). In the meantime, it is also important to determine profit-based incentives for members or divisions involved in a supply chain network (Lakhal, 2006; Shunko et al., 2014; Hammami & Frein, 2014; Liu et al., 2015). Although, in the international trading, transfer pricing is controversial and under restriction and scrutiny from many regulations, rules and guidelines of fiscal agencies and governments to avoid the manipulation of transfer prices (Mehafdi, 2000), companies usually still have the flexibility to determine the transfer price level from some range of values within given limits (Vidal & Goetschalckx, 2001). There is a range of acceptable transfer prices, instead of only a single transfer price, allowed by the Organization for the Economic Cooperation and Development (OECD) and the United States (Markham, 2005).

Research studies and papers are emerging on the use of transfer price to distribute profit fairly in supply chains. Mixed integer nonlinear programming (MINLP) models were developed by Gjerdrum et al. (2001, 2002), where Nash bargaining approach was used for fair profit distribution in a multi-enterprise supply chain. Chen et al. (2003) developed a fuzzy-based decision model for multiobjective optimisation of the production and distribution planning of a multi-echelon supply chain network, considering profit maximisation, customer service level minimisation, and ensuring fair profit distribution. Then, Chen & Lee (2004) extended the above work for the uncertainties of both demand and price. A cooperative game constructed by Rosenthal (2008) fairly allocated the net profit using transfer prices, considering both perfect information and asymmetric information environment. Leng & Parlar (2012) developed a cooperative game to determine optimal transfer prices for fair profit allocation within a two-echelon supply chain with one upstream division and multiple downstream ones. Yue & You (2014) developed an MINLP model for profit allocation strategy using material transfer prices and revenue share policies of cellulosic bioethanol supply chains. Recently, Liu et al. (2016) proposed a mixed integer linear programming (MILP) model for optimal fair transfer prices of a two-echelon supply chain. However, this work did not clearly justify the criterion of fair solutions and consider the different negotiation abilities of the supply chain members.

In this work, we aim to develop an optimisation-based fair profit distribution framework for integrated production, distribution and capacity planning of multi-echelon supply chains, by extending the work of Liu et al. (2016). In order to reflect real practice, different bargaining powers of supply chain members are considered. Here, we use two well accepted fairness criteria, proportional and max-min fairness, to define fair profit distribution. It is beyond the scope of this work to investigate the negotiation process of fairness criterion, and examine whether a fairness criterion is accepted by all members of the supply chain. To find fair solutions under these two fairness criteria, two game theoretic principles, Nash bargaining and lexicographic maximin principles, are adopted to develop solutions approaches for fair profit distribution. To overcome the computational difficulties of large problem instances, a tailored efficient solution approach based on the classic lexicographic maximin approach is developed for max-min fair profit distribution. Overall, the main novel contributions of this paper can be summarised as follows:

- An optimisation model is developed for production, distribution and capacity planning of three-echelon supply chain networks;
- Two fairness criteria are defined for supply chain profit distribution;
- Bargaining powers of supply chain members are considered under the two fairness criteria;
- MILP-based solution approaches are developed using literature game theoretic principles under the two fairness criteria;
- A tailored hierarchical solution approach is proposed for max-min fair solutions, with advantage in efficiency for large instances;
- Two examples, including a large real world supply chain network in agrochemical industry, are investigated.

The rest of this paper is organised as follows. Section 2 provides the problem statement, while the mathematical formulation of the proposed model is described in Section 3. The definitions of fairness criteria and the development of fair solution approaches in Section 4. In Section 5, two examples are described, followed by the computational results and discussion in Section 6. Finally, some concluding remarks are drawn in Section 7.

2. Problem Statement

In this work, a three-echelon supply chain network of one active ingredient (AI) in a process industry, such as pharmaceutical, and chemical industry, is considered, consisting of AI plants, formulation plants and markets, as illustrated in Figure 1. At the primary manufacturing stage, one AI, which is the substance biologically or chemically active within the products and is the specific component responsible for the desired effect of the products, e.g. drugs and pesticides, considered in the problem. The considered AI, a low-volume high-value product, is produced centrally in few AI plants, and then shipped to different formulation plants, where different final products are produced in secondary manufacture, according to different recipes, formulation, packaging and labelling requirements. It is assumed that each formulation plant can only be able to produce the products belonging to certain product groups, depending on its production capability. At last, final products are shipped to various markets for sales to customers. Considering the potential advantages of certain supply chain members to attain higher profits than others, it is assumed that the involved supply chain members may have different bargaining powers.

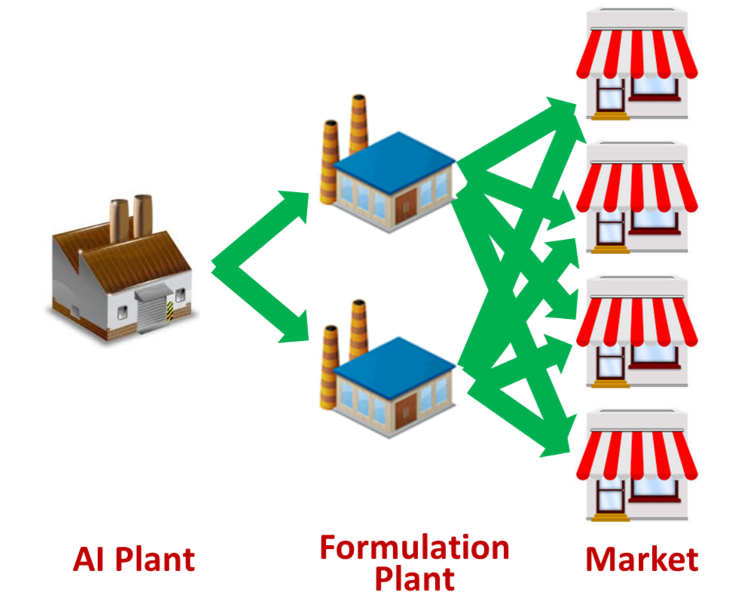

Figure 1: Supply chain network example of an AI.

The production, distribution and capacity planning of this three-echelon supply chain network is addressed in this work. It is assumed that the existing capacities of the AI plants and formulation plants cannot satisfy rapidly increased demand. Thus, capacity increment strategies are to be optimised, as well as production and distribution planning decisions.

The division of costs among supply chain members can be much different, depending on contracts and agreements on transportation responsibilities. In this work, we consider all trades are on Ex Works basis (International Chamber of Commerce, 2010), in which all costs and risks involved in taking the goods from the seller's premises are the obligation of the buyer or customer (Monczka et al., 2011). Note that the proposed model in this work can be easily modified to accommodate other trade responsibilities. The AI plants are responsible for AI's raw materials cost, AI production cost, AI inventory cost, and capital investment cost. The costs of formulation plants include the payments to AI plants to purchase the AI, raw materials cost, product formulation cost, inventory costs of both AI and products, AI transportation cost from AI plants to formulation plants, capital investment cost, and duties paid for AI importation. Each market pays for the purchase of products to formulation plants, product inventory, product transportation from formulation plants, duties for product importation, and unfulfilled demand.

The revenues of the AI and formulation plants come from the transfer payments from formulation plants and markets, respectively. Thus, transfer prices of the AI from AI plants to formulation plants, and those of products from formulation plants to markets, have significant effects on the profit of each member in the supply chain network. The transfer pricing decisions are to be optimised in this problem, based on given discrete penitential price levels and the bargaining powers of supply chain members. Meanwhile, it is assumed that the final selling prices of the products at markets are known.

In this problem, we aim to achieve a fair profit distribution among the supply chain members. Here, to define a *fair* profit distribution, we adopt two broadly accepted fairness criteria: proportional fairness and max-min fairness. In this work, a proportionally fair profit distribution is the one that any profit transfer leads to no increase in total proportional change of profit, while in a max-min fair profit distribution, any feasible profit increase of one member reduces the profit of an equal or less profitable member.

In summary, this optimisation problem can be described as follows:

Given are:

- supply chain network of an AI, consisting of AI plants, formulation plants, and markets;
- products and their products groups;
- capabilities and capacities of AI and formulation plants;
- product demands at markets;
- unit consumption of AI consumption for each product formulation;
- fixed and variable costs of AI production, product formulation, and transportation;
- transportation times of AI and products;
- unit raw materials cost, inventory cost, duties of AI and products;
- unit capital cost for capacity expansion at AI and formulation plants;

- minimum and maximum AI/product production and transportation flows;
- lost sales penalty of products;
- potential transfer price levels of AI and products;
- selling prices of products at markets;
- bargaining powers of supply chain members;

to determine:

- transfer prices of AI and products;
- productions of AI and products;
- distribution flows of AI and products;
- capacity increments of AI and formulation plants;
- inventory levels of AI and products at plants and markets;
- sales of products at markets;

so as to maximise the total profit of the supply chain network with a fair distribution to its members, under proportional and max-min fairness .

3. Mathematical Formulation

The proposed optimisation model for the fair profit distribution problem is extended from a literature supply chain optimisation model (Liu & Papageorgiou, 2013), which only considered total cost of the whole supply chain as objective function, and ignored transfer prices between supply chain members. In this work, considering a three-echelon supply chain, consisting of AI plants, formulation plants, and markets, each member is an individual profit centre, and its profit is aimed to be optimised to achieve a fair solution distribution through transfer prices. The definitions of all mathematical symbols used are given in the Nomenclature.

3.1. Production Constraints

If the AI is produced at AI plant a during time period t, its production amount, P^A_{at}, is restricted by given minimum ($MinP^A_a$) and maximum ($MaxP^A_a$) production limits, which are determined by production rates and operating time limits of the machines.

$$MinP^A_a \cdot W^A_{at} \leq P^A_{at} \leq MaxP^A_a \cdot W^A_{at}, \quad \forall a, t \tag{1}$$

where W^A_{at} is a binary variable indicating whether the AI is produced at AI plant a in time period t.

Similarly, the production amount of product i produced at formulation plant j during time period t, P^P_{ijt}, is limited by upper and lower bounds ($MinP^P_{ij}$ and $MaxP^P_{ij}$). By using the binary variable W^P_{ijt} (=1 if the product i produced at formulation plant j during time period t), we have the following constraints:

$$MinP^P_{ij} \cdot W^P_{ijt} \leq P^P_{ijt} \leq MaxP^P_{ij} \cdot W^P_{ijt}, \quad \forall j, g \in G_j, i \in \bar{I}_g, t \tag{2}$$

where G_j refers to the set of product groups g that can be formulated in plant j, and $\bar{I}_g$ indicates the set of products belonging to product group g.

3.2. Capacity Constraints

The total production at each AI and formulation plant is limited by its existing capacity (Cap^A_a for AI plant a and Cap^F_j for formulation plant j), plus any corresponding capacity increment (ΔCap^A_a and ΔCap^F_j, respectively), which are to be optimised.

$$P^A_{at} \leq Cap^A_a + \Delta Cap^A_a, \quad \forall a, t \tag{3}$$

$$\sum_{g \in G_j} \sum_{i \in \bar{I}_g} P^P_{ijt} \leq Cap^F_j + \Delta Cap^F_j, \quad \forall j, t \tag{4}$$

Note that some capacity expansion strategies (Liu & Papageorgiou, 2013) can be addressed by this model with additional constraints.

3.3. Flow Constraints

When there exists a shipment of the AI from AI plant a to formulation plant j during time period t, i.e. binary variable $Y^A_{ajt} = 1$, the shipped flow amount (F^A_{ajt}) cannot go beyond its minimum ($MinF^A_{aj}$) and maximum ($MaxF^A_{aj}$) limits.

$$MinF^A_{aj} \cdot Y^A_{ajt} \leq F^A_{ajt} \leq MaxF^A_{aj} \cdot Y^A_{ajt}, \quad \forall a, j, t \tag{5}$$

Similarly, for the transported amount of product i from formulation plant j to market k during time period t, F^P_{ijkt}, the following constraints are proposed:

$$MinF^P_{ijk} \cdot Y^P_{ijkt} \leq F^P_{ijkt} \leq MaxF^P_{ijk} \cdot Y^P_{ijkt}, \quad \forall j, k, g \in G_j, i \in \bar{I}_g \cap I_k, t \tag{6}$$

where binary variable Y^P_{ijkt} is 1 if product i is shipped from formulation plant j to market k during time period t; $MaxF^P_{ijk}$ and $MinF^P_{ijk}$ are corresponding maximum and minimum limits, respectively; and I_k represents the set of products sold in market k.

3.4. Inventory Constraints

The inventory considered in this problem includes AI inventory at both AI plants and formulation plants, and product inventory at both formulation plants and markets, as illustrated in Figure (2)

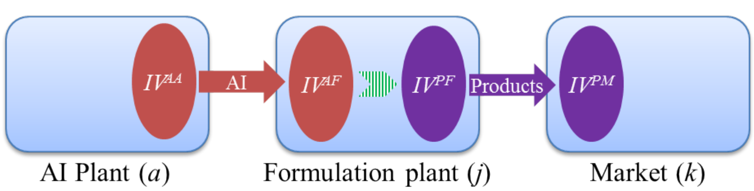

Figure 2: Illustration of the inventory of the AI and products.

The AI inventory at AI plant a in time period t (IV_{at}^{AA}) is equal to the inventory in the previous time period ($IV_{a,t-1}^{AA}$), plus local AI production amount (P_{at}^{A}), minus total outgoing AI flows to formulation plants (F_{ajt}^{A}):

$$IV_{at}^{AA} = IV_{a,t-1}^{AA} + P_{at}^{A} - \sum_{j} F_{ajt}^{A}, \quad \forall a, t \tag{7}$$

The AI inventory at formulation plant j at the end of each time period (IV_{jt}^{AF}) equals the AI inventory at the end of the previous time period ($IV_{j,t-1}^{AF}$), plus arriving AI flows from AI plants ($F_{aj,t-\bar{\tau}_{aj}}^{A}$), minus AI consumption for product production (P_{ijkt}^{P}):

$$IV_{jt}^{AF} = IV_{j,t-1}^{AF} + \sum_{a} F_{aj,t-\bar{\tau}_{aj}}^{A} - \sum_{i \in \bar{I}_g} \sum_{g \in G_j} \beta_{ij} \cdot P_{ijt}^{P}, \quad \forall j, t \tag{8}$$

where $\bar{\tau}_{aj}$ refers to the transportation time from AI plant a to formulation plant j

The inventory of product i at formulation plant j by the end of each time period (IV_{ijt}^{PF}) is calculated by its inventory at the end of the previous time period ($IV_{ij,t-1}^{PF}$), product production (P_{ijt}^{P}) and the total outgoing flows to the markets (F_{ijkt}^{P}) in that time period:

$$IV_{ijt}^{PF} = IV_{ij,t-1}^{PF} + P_{ijt}^{P} - \sum_{k \in K_i} F_{ijkt}^{P}, \quad \forall j, g \in G_j, i \in \bar{I}_g, t \tag{9}$$

where K_i indicates the set of markets that sell product i.

Similarly, the inventory of product i at market k in time period t (IV_{ijt}^{PM}) is equal to the product inventory in the previous time period ($IV_{ij,t-1}^{PM}$), plus any incoming flows from formulation plants ($F_{ijk,t-\tau_{jk}}^{P}$), minus local sales in the same time period (S_{ikt}) :

$$IV_{ikt}^{PM} = IV_{ik,t-1}^{PM} + \sum_{j \in J_g} \sum_{g \in \bar{G}_i} F_{ijk,t-\tau_{jk}}^{P} - S_{ikt}, \quad \forall k, i \in I_k, t \tag{10}$$

where τ_{jk} refers to the transportation time between formulation plant j and market k; $\bar{G}_i$ indicates the set of product groups including product i; J_g expresses the set of formulation plants capable for the production of product group g.

The above inventory is limited by corresponding lower and upper bounds, where upper bounds are often resulted from storage capacities, and lower bounds are usually considered as safety stocks for demand uncertainty.

$$MinIV_a^{AA} \leq IV_{at}^{AA} \leq MaxIV_a^{AA}, \quad \forall a, t \tag{11}$$

$$MinIV_j^{AF} \leq IV_{jt}^{AF} \leq MaxIV_j^{AF}, \quad \forall j, t \tag{12}$$

$$MinIV_{ij}^{PF} \leq IV_{ijt}^{PF} \leq MaxIV_{ij}^{PF}, \quad \forall j, g \in G_j, i \in \bar{I}_g, t \tag{13}$$

$$MinIV_{ik}^{PM} \leq IV_{ikt}^{PM} \leq MaxIV_{ik}^{PM}, \quad \forall k, i \in I_k, t \tag{14}$$

3.5. Transfer Price Constraints

Transfer prices of the AI between AI plants and formulation plants (TP_a^A), as well as those of products between formulation plants and markets (TP_{ij}^P), are considered as decision variables. It is assumed that each AI plant charges the same transfer price to all formulation plants. Similarly, for each product, each formulation plant sets a single transfer price to all markets. In addition, the determined transfer prices do not change throughout the considered planning horizon. Following the work of Gjerdrum et al. (2001), a set of candidate transfer price levels of the AI and products are set by each member within the range allowed by the rules and regulations of authorities, which is reasonable in real practice. Here, only one transfer price level of the AI (TPL_{al}^A) can be chosen by each AI plant, if the production occurs there. Similarly, only one transfer price level of each product (TPL_{ijl}^P) is selected by each formulation plant producing it. In the following constraints, binary variables O_{al}^A and O_{ijl}^P indicate whether price level l is selected or not, and E_{al}^A and E_{ijl}^P are for the production allocation of the AI and products, respectively.

$$TP_a^A = \sum_l TPL_{al}^A \cdot O_{al}^A, \quad \forall a \tag{15}$$

$$TP_{ij}^P = \sum_l TPL_{ijl}^P \cdot O_{ijl}^P, \quad \forall j, g \in G_j, i \in \bar{I}_g \tag{16}$$

$$\sum_l O_{al}^A = E_a^A, \quad \forall a \tag{17}$$

$$\sum_l O_{ijl}^P = E_{ij}^P, \quad \forall j, g \in G_j, i \in \bar{I}_g \tag{18}$$

3.6. Lost Sales Constraints

The lost sales (LS_{ikt}) of product i in market k in each time period is the difference between corresponding demand (D_{ikt}) and sales amount (S_{ikt}):

$$LS_{ikt} = D_{ikt} - S_{ikt}, \quad \forall k, i \in I_k, t \tag{19}$$

3.7. Logical Constraints

If AI plant a is not chosen for AI production, its production amount is always zero.

$$\sum_t W_{at}^A \leq \mid t \mid \cdot E_a^A, \quad \forall a \tag{20}$$

Similarly, if a link from AI plant a to formulation plant j is not allocated for AI shipment, there is no AI flow on this link in all time periods.

$$\sum_t Y_{ajt}^A \leq \mid t \mid \cdot X_{aj}^A, \quad \forall a, j \tag{21}$$

When AI plant a is not allocated for AI production, the link from AI plant a to any formulation plant j is not used for AI shipment.

$$\sum_{j} X_{aj}^{A} \leq \mid j \mid \cdot E_{a}^{A}, \quad \forall a \tag{22}$$

If formulation plant j is not allocated for the production of product i, there is no production of product i at formulation plant j.

$$\sum_{t} W_{ijt}^{P} \leq \mid t \mid \cdot E_{ij}^{P}, \quad \forall j, g \in G_j, i \in \bar{I}_g \tag{23}$$

When a link from formulation plant j to market k is not allocated for the shipment of product i , the flow of product i on this link is zero in all time period t.

$$\sum_{t} Y_{ijkt}^{P} \leq \mid t \mid \cdot X_{ijk}^{P}, \quad \forall j, k, g \in G_j, i \in \bar{I}_g \cap I_k \tag{24}$$

If formulation plant j is not used to produce product i, then there is no flow of product i from formulation plant j to any market k.

$$\sum_{k \in K_i} X_{ijk}^{P} \leq \mid k \mid \cdot E_{ij}^{P}, \quad \forall j, g \in G_j, i \in \bar{I}_g \tag{25}$$

3.8. Profit Constraints

The total profit of the supply chain network is the sum of the profits of AI plants (Pr_a^A), formulation plants (Pr_j^F) and markets (Pr_k^M).

$$TotalPr = \sum_{a} Pr_a^A + \sum_{j} Pr_j^F + \sum_{k} Pr_k^M \tag{26}$$

Next, the profit of each supply chain member is formulated in the following subsections.

3.8.1. AI plants

At an AI plant, its revenue (Re_a^A) is the total transfer payment from formulation plants, i.e., the AI transfer prices multiplied by corresponding flows between the AI plant and formulation plants:

$$Re_a^A = \sum_t \sum_j TP_a^A \cdot F_{ajt}^A, \quad \forall a \tag{27}$$

Note that Eq. (27) is a nonlinear equation, which is reformulated to be linear below using auxiliary variables and constraints and Eq. (15):

$$\overline{OF}_{ajlt}^A \leq MaxF_{aj}^A \cdot O_{al}^A, \quad \forall a, j, l, t \tag{28}$$

$$\sum_l \overline{OF}_{ajlt}^A = F_{ajt}^A, \quad \forall a, j, t \tag{29}$$

$$Re_a^A = \sum_t \sum_j \sum_l TPL_{al}^A \cdot \overline{OF}_{ajlt}^A, \quad \forall a \tag{30}$$

The costs of AI plants include raw materials cost (RMC_a^A), production cost (PC_a^A), inventory cost (IVC_a^A), and capital investment cost (CIC_a^A):

$$RMC_a^A = \sum_t MC_a^A \cdot P_{at}^A, \quad \forall a \tag{31}$$

$$PC_a^A = FPC_a^A \cdot E_a^A + \sum_t VPC_a^A \cdot P_{at}^A, \quad \forall a \tag{32}$$

$$IVC_a^A = \sum_t IC_a^{AA} \cdot IV_{at}^{AA}, \quad \forall a \tag{33}$$

$$CIC_a^A = crf \cdot CC_a^A \cdot \Delta Cap_a^A, \quad \forall a \tag{34}$$

The profit of an AI plant (Pr_a^A) is equal to its revenue minus all above costs.

$$Pr_a^A = Re_a^A - RMC_a^A - PC_a^A - IVC_a^A - CIC_a^A, \quad \forall a \tag{35}$$

3.8.2. Formulation plants

The revenue of a formulation plant (Re_j^F) include all transfer payments from markets, determined by product transfer prices and corresponding flows from this formulation plant to markets:

$$Re_j^F = \sum_t \sum_{g \in G_j} \sum_k \sum_{i \in \bar{I}_g \cap I_k} TP_{ijk}^P \cdot F_{ijkt}^P, \quad \forall j \tag{36}$$

Similar to Eq. (27), the nonlinear Eq. (36) can be linearised as follows:

$$\overline{OF}_{ijklt}^P \leq MaxF_{ijk}^P \cdot O_{ijl}^P, \quad \forall j, k, g \in G_j, i \in \bar{I}_g \cap I_k, l, t \tag{37}$$

$$\sum_l \overline{OF}_{ijklt}^P = F_{ijkt}^P, \quad \forall j, k, g \in G_j, i \in \bar{I}_g \cap I_k, t \tag{38}$$

$$Re_j^F = \sum_t \sum_k \sum_{g \in G_j} \sum_{i \in \bar{I}_g \cap I_k} \sum_l TPL_{ijl}^P \cdot \overline{OF}_{ijklt}^P, \quad \forall j \tag{39}$$

The costs incurred at formulation plants include transfer payment cost to AI plants (TPC_j^F), raw materials cost (RMC_j^F), formulation cost (FOC_j^F), inventory cost (IVC_j^F), AI transportation cost (TRC_j^F), capital investment cost (CIC_j^F) and duties (DUC_j^F).

$$TPC_j^F = \sum_t \sum_a \sum_l TPL_{al}^A \cdot \overline{OF}_{ajlt}^A, \quad \forall j \tag{40}$$

$$RMC_j^F = \sum_t \sum_{g \in G_j} \sum_{i \in \bar{I}_g} MC_{ij}^P \cdot P_{ijt}^P, \quad \forall j \tag{41}$$

$$FOC_j^F = \sum_{g \in G_j} \sum_{i \in \bar{I}_g} FFC_{ij}^P \cdot E_{ij}^P + \sum_t \sum_{g \in G_j} \sum_{i \in \bar{I}_g} VFC_{ij}^P \cdot P_{ijt}^P, \quad \forall j \tag{42}$$

$$IVC_j^F = \sum_t IC_j^{AF} \cdot IV_{jt}^{AF} + \sum_t \sum_{g \in G_j} \sum_{i \in \bar{I}_g} IC_{ij}^{PF} \cdot IV_{ijt}^{PF}, \quad \forall j \tag{43}$$

$$TRC_j^F = \sum_a FTC_{aj}^A \cdot X_{aj}^A + \sum_t \sum_a VTC_{aj}^A \cdot F_{ajt}^A, \quad \forall j \tag{44}$$

$$CIC_j^F = crf \cdot CC_j^F \cdot \Delta Cap_j^F, \quad \forall j \tag{45}$$

$$DUC_j^F = \sum_t \sum_a \sum_l DC_{aj}^A \cdot TPL_{al}^A \cdot \overline{OF}_{ajlt}^A, \quad \forall j \tag{46}$$

Then, the profit of a formulation plant (Pr_j^F) is as follows:

$$Pr_j^F = Re_j^F - TPC_j^F - RMC_j^F - FOC_j^F - IVC_j^F - TRC_j^F - CIC_j^F - DUC_j^F, \quad \forall j \tag{47}$$

3.8.3. Markets

The revenue of each market (Re_k^M) is given by the selling prices of product i in market k (V_{ik}) and corresponding sales amounts:

$$Re_k^M = \sum_t \sum_{i \in I_k} V_{ik} \cdot S_{ikt}, \quad \forall k \tag{48}$$

The costs incurred at each market include transfer payment cost to formulation plants (TPC_k^M), inventory cost (IVC_k^M), product transportation cost (TRC_k^M), duties (DUC_k^M) and lost sales cost (LSC_k^M).

$$TPC_k^M = \sum_t \sum_j \sum_{g \in G_j} \sum_{i \in \bar{I}_g \cap I_k} \sum_l TPL_{ijl}^P \cdot \overline{OF}_{ijklt}^P, \quad \forall k \tag{49}$$

$$IVC_k^M = \sum_t \sum_{i \in I_k} IC_{ik}^{PM} \cdot IV_{ikt}^{PM}, \quad \forall k \tag{50}$$

$$TRC_k^M = \sum_j \sum_{g \in G_j} \sum_{i \in \bar{I}_g \cap I_k} FTC_{ijk}^P \cdot X_{ijk}^P + \sum_t \sum_j \sum_{g \in G_j} \sum_{i \in \bar{I}_g \cap I_k} VTC_{ijk}^P \cdot F_{ijkt}^P, \quad \forall k \tag{51}$$

$$DUC_k^M = \sum_t \sum_j \sum_{g \in G_j} \sum_{i \in \bar{I}_g \cap I_k} \sum_l DC_{ijk}^P \cdot TPL_{ijl}^P \cdot \overline{OF}_{ijklt}^P, \quad \forall k \tag{52}$$

$$LSC_k^M = \sum_t \sum_{i \in I_k} PC_{ik} \cdot LS_{ikt}, \quad \forall k \tag{53}$$

Then, the profit of each market (Pr_k^M) is given by Eq. (54):

$$Pr_k^M = Re_k^M - TPC_k^M - IVC_k^M - TRC_k^M - DUC_k^M - LSC_k^M, \quad \forall k \tag{54}$$

Considering the maximisation of the total supply chain profit, the MILP model (denoted as MaxTotProf), consisting of Eq. (26) as objective function and Eqs. (1)–(25), (28)–(35), (37)–(54) as constraints, is to be solved. To achieve a fair profit distribution strategy with the maximum total profit, some solution approaches are developed in the next section.

4. Fair Solution Approaches

In this section, we first discuss the two fairness criteria considered in this work, based on literature fairness schemes. Then, game theoretical solution approaches are developed for fair solutions under the two fairness criteria.

4.1. Fairness Criteria

There exists extensive literature work on the fairness of allocation problems, mostly in the fields of economics, social science, and engineering. Although there is no unique fairness criterion wholly recognised, due to widely distinctive problem characteristics and interpretation of fairness, most fairness criteria are based on some general theories of justice and equity, including, but not limited to, Aristotles equity principle, classical utilitarianism, and Rawlsian justice (Sen, 1973; Young, 1995). In this work, we focus on two widely accepted and extensive studied fairness criteria: *proportional* fairness and *max-min* fairness (Kelly et al., 1998; Bonald et al., 2006; Bertsimas et al., 2011).

Proportional fairness considers an allocation between multiple players to be fair, if the total proportional change of utility is no greater than zero when compared to any other feasible allocations. This criterion was firstly proposed by Kelly (1997) based on changing rate control for elastic traffic in computer network services, and was widely studied afterwards. It can be regarded as an extension of Nash solution for a two-person bargaining game (Nash, 1950), simultaneously satisfying four axioms for fairness criterion, including Pareto optimality, symmetry, affine invariance, and independence of irrelevant alternatives (Conley & Wilkie, 1996). Considering a proportionally fair profit distribution in this problem, we take into account the excess profit, i.e. the profit higher than a specific minimum acceptable profit level, defined by $Pr_n - MinPr_n$, where Pr_n is the profit earned by supply chain member n ($= Pr_a^A$, Pr_j^F and Pr_k^M when $n = a$, j and k, respectively), and $MinPr_n$ is the minimum acceptable profit of supply chain member n. A proportionally fair profit distribution is defined as a profit distribution that any profit transfer does not benefit total proportional change of excess profit, i.e.:

$$\sum_n \frac{Pr_n - Pr_n^{pf}}{Pr_n^{pf} - MinPr_n} \leq 0 \tag{55}$$

where Pr_n^{pf} is the optimal profit of member n in a proportionally fair profit distribution. A proportionally fair solution can be achieved using the Nash bargaining approach, which will be described later in this section.

Max-min fairness is based on the work of Rawlsian justice (Rawls, 1971) and Kalai-Smorodinsky bargaining solution (Kalai & Smorodinsky, 1975). An allocation among multiple players is regarded to be max-min fair, if any increase in the allocation of one player can result in the allocation decrease of another player with an equal or less allocation. Max-min fairness satisfies the axioms of Pareto optimality, symmetry, affine invariance, and monotonicity. Originally proposed for a two-person game, it is usually operated in a normalised system in which all players have the same maximum achievable utility. Under max-min fairness, a fair allocation solution is achieved by maximising the lowest utility first, then the second lowest utility, and so on. The max-min fairness has a broad and extensive application in the area of networking and telecommunications (Bertsekas & Gallager, 1987; Luss, 1999). In this work, the scaled profit of each member is considered under max-min fairness, which is obtained by considering the maximum and minimum profit bounds of each member:

$$\overline{Pr}_n = \frac{Pr_n - MinPr_n}{MaxPr_n - MinPr_n}, \quad \forall n \tag{56}$$

where $MaxPr_n$ is the maximum profit of supply chain member n. A max-min fair profit distribution is the one that any scaled profit increase of one member cause the scaled profit decrease of another member with the same or less scaled profit. Lexicographic maximin approach can be used to obtain such max-min fair solution, whose details can be found later.

Next, we developed solutions approaches for this fair profit distribution problem, using Nash bargaining and lexicographic maximin principles, under proportional and max-min fairness criteria, respectively.

4.2. Nash Bargaining Approach

The Nash bargaining approach is applied to obtain the proportionally fair solutions. Originated from the two-player bargaining game, the Nash bargaining solution is defined as the maximiser of Nash product, i.e. the product of the two players' payoffs (Nash, 1950). The Nash bargaining approach has been used to achieve fair solutions in various areas (Gjerdrum et al., 2001, 2002; Chéron, 2002; Han et al., 2005; Hanany & Gerchak, 2008; Zhang et al., 2009; Ma et al., 2012; Yu et al., 2012; Zhang et al., 2013, 2017a). In our supply chain optimisation problem, a fair Nash bargaining solution can be obtained by maximising the product of each member's excess profit. Thus, taking into account the bargaining powers of the supply chain members, the Nash bargaining solution can be obtained by optimising the following objective function:

$$\Psi = \prod_n (Pr_n - MinPr_n)^{\alpha_n} \tag{57}$$

where α_n is the indicator of the bargaining power of member n.

By applying the logarithmic operation, the above nonlinear objective function (Eq. 57) can be rewritten as the following one:

$$\ln \Psi = \sum_n \alpha_n \cdot \ln(Pr_n - MinPr_n) \tag{58}$$

Eq. (58) is still a nonlinear function. To linearise it, the separable programming technique (Gjerdrum et al., 2001) was applied. Then, the continuous strictly convex function $\ln \Psi$ can be approximated by a piecewise linear function $\ln \overline{\Psi}$ with Q grid points, as follows:

$$\ln \overline{\Psi} = \sum_n \sum_{q=1}^{Q} \alpha_n \cdot \mu_q \cdot \ln(Pr_{nq} - MinPr_n) \tag{59}$$

where Pr_{nq} expresses the profit of supply chain member n at grid point q, and variables $\mu_q \geq 0$ is a SOS2 variable, allowing vales at only two adjacent grid points to be non-zero, and satisfying the following constraint:

$$\sum_{q=1}^{Q} \mu_q = 1 \tag{60}$$

Thus, Pr_n can be expressed as below:

$$Pr_n = \sum_{q=1}^{Q} \mu_q \cdot Pr_{nq}, \quad \forall n \tag{61}$$

Overall, the developed MILP model, denoted as Nash, includes Eq. (59) as objective function, and Eqs. (1)–(25), (28)–(35), (37)–(54), (60) and (61) as constraints. The solution of the above MILP model can be regarded as a close approximation to the optimal Nash bargaining solution (Gjerdrum et al., 2001), as well as a proportionally fair solution.

4.3. Lexicographic Maximin Approach

Lexicographic maximin (or minimax) approach is a game theoretic approach for max-min fair solutions with wide applications (Klein et al., 1992; Ogryczak, 1997; Ogryczak et al., 2003; Erkut et al., 2008; Wang et al., 2008; Luss, 2010; Liu & Papageorgiou, 2013; Sawik, 2014; Zhang et al., 2014, 2017b). A lexicographic maximin problem can be defined as follows:

$$\text{Leximax}_{\boldsymbol{x}\in\Omega}\Theta(\boldsymbol{f}(\boldsymbol{x})) \tag{62}$$

where $\boldsymbol{f}(\boldsymbol{x}) = \{f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \dots, f_N(\boldsymbol{x})\}$ is a vector function on the decision space $\boldsymbol{x} \in \Omega$, and $\Theta : \mathbb{R}^N \to \mathbb{R}^N$ is a mapping function that re-rank the components of vector in a nondecreasing order, such that $\Theta(\boldsymbol{f}(\boldsymbol{x})) = (\theta_1(\boldsymbol{f}(\boldsymbol{x})), \theta_2(\boldsymbol{f}(\boldsymbol{x})), \dots, \theta_N(\boldsymbol{f}(\boldsymbol{x})))$, where $\theta_1(\boldsymbol{f}(\boldsymbol{x})) \leq \theta_2(\boldsymbol{f}(\boldsymbol{x})) \leq \cdots \leq \theta_N(\boldsymbol{f}(\boldsymbol{x}))$, and there exists an permutation π of set $\{1, \dots, N\}$ such that $\theta_n(\boldsymbol{f}(\boldsymbol{x})) = f_{\pi(n)}(\boldsymbol{x})$.

Taking bargaining powers into account, the ratio of scaled profit to bargain power of each member is maximised for this problem. Thus, the lexicographic maximin problem for max-min fair profit distribution can be described as follows:

$$\begin{aligned} &\text{Leximax } \Theta(\overline{Pr}_n/\alpha_n) \\ &\quad s.t. \quad \text{Eqs. } (1)-(25),\ (28)-(35),\ (37)-(54) \text{ and } (56) \end{aligned} \tag{63}$$

4.3.1. Lexicographic Maximisation Model

According to Ogryczak et al. (2005), the lexicographic maximin optimisation problem (Eq. 63) can be transformed into a lexicographic maximisation problem, denoted by LexiMax in this paper, whose solution process involves solving a series of MILP models iteratively:

$$\begin{aligned}
&\text{Leximax } \{\lambda_1 - \sum_n d_{1n}, \dots, \lambda_N - \frac{1}{N} \cdot \sum_n d_{Nn}\} \\
&s.t. \quad \lambda_p - d_{pn} \leq \overline{Pr}_n/\alpha_n, \quad \forall p = 1, \dots, N, n \\
&\qquad \lambda_p \in \mathbb{R}, \quad \forall p = 1, \dots, N \\
&\qquad d_{pn} \geq 0, \quad \forall p = 1, \dots, N, n \\
&\qquad \text{Eqs. } (1) - (25),\ (28) - (35),\ (37) - (54) \text{ and } (56)
\end{aligned} \tag{64}$$

4.3.2. An Alternative Hierarchical Approach

Due to the involvement of multiple iterations and increasing model sizes, the above lexicographic maximisation approach for large supply chain networks requires high computational expense to solve. To overcome the computational difficulties, an efficient tailored hierarchical approach is developed to obtain an approximate optimal fair solution, which denoted as hLexi. In the proposed hLexi approach, an aggregated static lexicographic maximin problem is solved first as a lexicographic maximisation problem for fair profit distribution to obtain the optimal scaled profit of member n, $\overline{Pr}_n^*$. This aggregated model ignores the time discretisation and only considers the aggregated decisions of total productions, flows and sales in the planning horizon. The details of the developed aggregated model are presented in the Appendix.

Next, a detailed dynamic optimisation model is solved by maximising total profit. This model is extended from MaxTotProf model by including two additional constraints: Eq. (56) and a profit ratio limit constraint, in which the profit distributions obtained by the aggregated model are considered as the limits, i.e. the relative ratio of the scaled profits of a pair of members, n and n', is restricted by their ratio obtained in the aggregated model, with an allowed deviation δ, as follows:

$$(1-\delta) \cdot \frac{\overline{Pr}_n^*}{\overline{Pr}_{n'}^*} \cdot \overline{Pr}_{n'} \leq \overline{Pr}_n \leq (1+\delta) \cdot \frac{\overline{Pr}_n^*}{\overline{Pr}_{n'}^*} \cdot \overline{Pr}_{n'}, \quad \forall n, n' > n \tag{65}$$

The solution procedure of the hierarchical approach hLexi is given below:

Step 1: Solve the proposed aggregated model using lexicographic maximin approach, and obtain the optimal scaled profit earned by each member, $\overline{Pr}_n^*$;

Step 2: Solve a detailed MILP model for total profit maximisation with the fixed values of $\overline{Pr}_n^*$.

Overall, the above three game theoretic approaches were used for the optimal profit distribution problem. The Nash bargaining approach using separable programming (Nash) is implemented for a proportionally fair profit distribution, and the lexicographic maximin approach through lexicographic maximisation model (LexiMax) and the proposed hierarchical approach (hLexi) are used for a max-min fair profit distribution.

5. Examples

In this section, two examples are presented to demonstrate the applicability of the developed fair decision framework. Example 1 is a small illustrative example to justify the fair profit distributions obtained by the proposed approaches and the roles of transfer prices. Example 2 is based on a real world case study in agrochemical industry to demonstrate applicability of the proposed approaches at real practice. Here, cu is used as the currency unit and mu as the mass unit.

5.1. Example 1

Example 1 is a supply chain network consisting of one AI plant (A1), two formulation plants (F1–F2), and four market regions (M1–M4), as shown in Figure 1. This example considers eight products (P1–P8), with the first four in one product group (G1) produced by formulation plant F1 and the last four in another product group (G2) produced by formulation plant F2. The planning horizon is divided into 6 time periods (T1–T6), with each lasting for 2 months. The unit AI consumptions of product production are given in Table 1.

Table 1: Unit consumption of the AI in product formulation of Example 1

	P1	P2	P3	P4	P5	P6	P7	P8
F1	0.23	0.25	0.22	0.20	-	-	-	-
F2	-	-	-	-	0.18	0.27	0.28	0.13

In this example, there are 10 discrete potential transfer price levels (L1–L10) of the AI and each product. The given discrete potential transfer price levels of each product are the same at all formulation plants. The selling prices of final products at markets are assumed to be constant in the planning horizon, which are shown in Table 2.

Table 2: Product selling prices (cu/mu) in Example 1.

	M1	M2	M3	M4
P1	180	160	170	180
P2	200	180	220	190
P3	240	230	200	210
P4	190	200	170	190
P5	180	210	190	200
P6	220	240	210	210
P7	150	180	200	190
P8	220	200	210	200

5.2. Example 2

Example 2 considers a real world supply chain network in the agrochemical industry, based on the example in (Liu & Papageorgiou, 2013). It consists of one AI plant (A1), eight formulation plants (F1–F8) and ten market regions (M1–M10). There are 32 products (P1–P32) in 10 product groups (G1–G10). The planning horizon is divided into 52 weekly time periods (T1–T52). The capabilities of the formulation plants are shown in Table 3.

Table 3: Capabilities and capacities (mu) of the formulation plants in Example 2.

Product group (product)	Formulation plant							
	F1	F2	F3	F4	F5	F6	F7	F8
G1 (P1–P4)	-	-	-	✓*	-	✓	-	✓
G2 (P5–P6)	✓	✓	-	-	-	-	✓	-
G3 (P7–P10)	-	✓	✓	✓	✓	-	✓	✓
G4 (P11–P14)	✓	-	-	-	✓	✓	-	-
G5 (P15–P20)	-	✓	-	-	-	✓	-	-
G6 (P21–P22)	-	✓	-	-	✓	-	-	-
G7 (P23–P25)	✓	-	✓	✓	-	-	-	✓
G8 (P26–P27)	-	-	-	-	-	✓	✓	-
G9 (P28–P30)	-	-	✓	-	✓	-	-	-
G10 (P31–P32)	✓	-	-	-	-	✓	-	✓

*The formlation plant is able to produce corresponding product groups.

Similar to Example 1, there exit 10 transfer price levels (L1–L10) for the AI and each product. Note each product is sold only in some markets with different selling prices. The annual demand and selling price of each product in each market is given in Figures 3 and 4, where the most demanded products are P2 and P3, while markets M2 and M4 have the most potential sales.

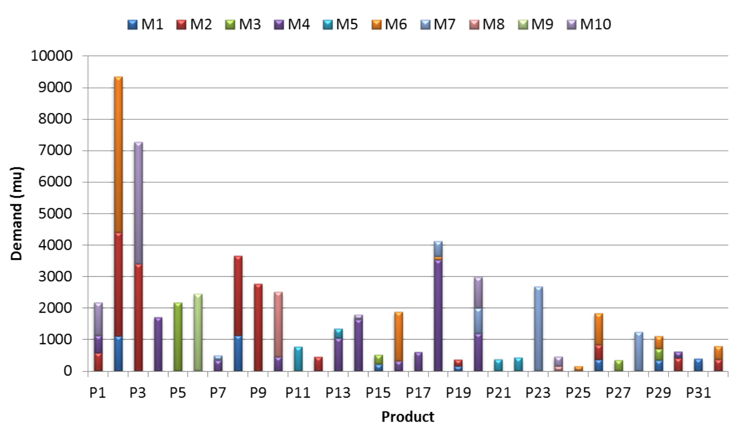

Figure 3: Product demands at each market in Example 2.

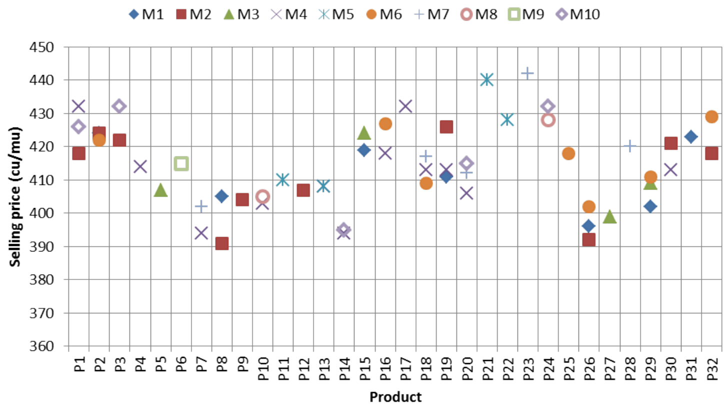

Figure 4: Selling prices of the products at each market in Example 2.

In both Examples 1 and 2, the maximum profit limit of each member was achieved by optimising its own profit only, while the minimum acceptable profit was set to a certain percentage of the maximum profit (30% in Example 1 and 10% in Example 2).

6. Computational Results and Discussion

In this section, three game theoretic approaches are applied to the two examples presented above, including the MILP model for Nash bargaining solution (Nash), lexicographic maximisation model (LexiMax) and hierarchical lexicographic approach (hLexi). The obtained fair solutions are compared with the solution of MaxTotProf model for total profit maximisation to demonstrate their advantages.

All implementations in this paper were done in GAMS 24.5 (GAMS Development Corporation, 2015) on a 64-bit Windows 7 based machine with 3.00 GHz Intel Core i5-3330 processor and 8.0 GB RAM, using CPLEX MILP solver with four threads. The optimality gap was set to 1%. A CPU limit of 10000s was used for each single MILP model run. The value of δ used in the hLexi approach is 20%, unless specified otherwise.

6.1. Example 1

In Example 1, two scenarios with different bargaining powers are investigated to examine the impact of bargaining powers. The model statistics of different approaches for Example 1 are presented in Table 4. All approaches have similar numbers of equations and variables, except the aggregated model at Step 1 of hLexi approach, which has one order of magnitude fewer equations and variables, and results in much shorter computational time.

Table 4: Model statistics in Example 1

	MaxTotProf	Nash	LexiMax	hLexi
No. of equations	2326	2341	2410*	500**/2382***
No. of continuous variables	1740	1811	1881*	504**/1754***
No. of discrete variables	307	307	307*	143**/307***

*Last iteration; **Aggregated model at the last iteration of Step 1; ***Detailed model at Step 2.

6.1.1. Scenario 1

In Scenario 1, it is assumed that all supply chain members have the same bargaining power. Without loss of generality, we let $\alpha_n = 1$. The obtained profits of all members are shown in Table 5, which also demonstrates the effect of fairness criterion on profit distribution. The total profit achieved by MaxTotProf model is the highest. However, the profits of formulation plants are negative, and the profits of markets reach the maximum limits, implying that the profit distribution is very uneven. Meanwhile, for other game theoretic solution approaches, the obtained total profit is only 3-4% less than the maximum total profit, while the profits of all supply chain members are within the same scale, varying between 23.0 and 55.0 thousand cu. Comparing the optimal solutions of Nash and LexiMax approaches, only formulation plant F2 earns more profit in the lexicographic maximin optimum, than in the Nash optimum, due to its higher maximum profit bound than others, which is taken into account in scaled profits.

Table 5: Optimal profits of supply chain members in Scenario 1 of Example 1, in cu.

	MaxTotProf	Nash	LexiMax	hLexi
A1	20.9	25.2	23.0	23.0
F1	-4.7	43.0	39.9	39.9
F2	-3.5	38.6	54.2	55.0
M1	59.6	35.0	33.0	33.0
M2	88.2	52.9	48.9	46.2
M3	61.9	37.1	34.7	35.9
M4	44.5	26.7	24.8	24.7
Total profit (thousand cu)	266.8	258.6	258.6	257.7
CPU (s)	0.3	3.9	78.7	2.7 (1.2*/1.5**)

*Step 1; **Step 2.

To further demonstrate the fairness of the obtained profit distributions, they are compared under two fairness criteria: proportional fairness and max-min fairness. The proportional fairness is demonstrated by excess profit ($Pr_n - MinPr_n$) in Figure 5, showing that the fluctuations of excess profits by Nash approach are much smaller than those by MaxTotProf model. The max-min fairness is illustrated using scaled profit ($\overline{Pr}_n$) in Figure 6, which demonstrates more fluctuations of scaled profit by MaxTotProf model. Especially, the minimum scaled profits of all members in the solutions of LexiMax and hLexi approaches are significantly higher than that of MaxTotProf model, and therefore their profit distributions much max-min fairer.

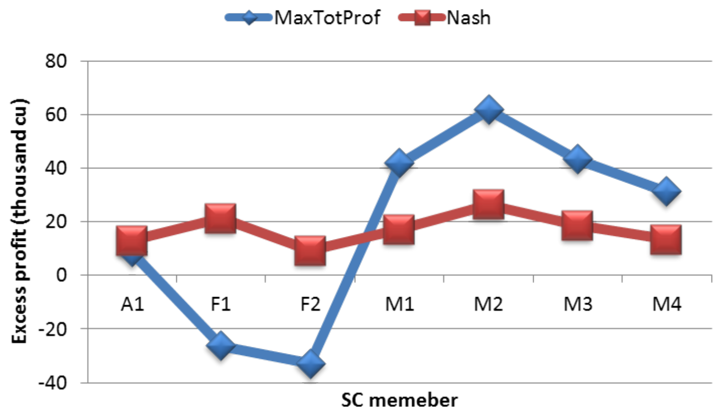

Figure 5: Proportional fairness: excess profits, $Pr_n - MinPr_n$, by MaxTotProf and Nash approaches in Example 1, in thousand cu.

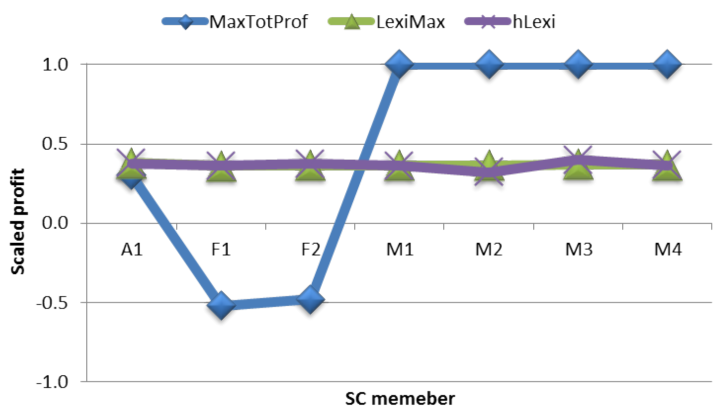

Figure 6: Max-min fairness: scaled profits, $\overline{Pr_n}$, by MaxTotProf, LexiMax and hLexi approaches in Example 1.

In order to measure the fairness of the obtained profit distributions, we introduce the coefficient of variation as a fairness index (FI) of profit distribution. The coefficient of variation is the ratio of standard deviation to mean, expressed as a percentage, which has been widely used in the literature as a fairness measure. Here, a lower value of FI indicates a fairer profit distribution. Table 6 shows the values of FI of the obtained profit distributions by different approaches. Under proportional fairness, FI indicates the coefficient of variation of excess profits, while under max-min fairness, FI represents the coefficient of variation of scaled profits. In terms of proportional fairness, Nash approach reduces the FI values to 31.4%, from 185.0% obtained by MaxTotProf model. Meanwhile, the FI value under max-min fairness obtained by LexiMax approach is two orders of magnitude lower than that maximising total profit. Such result shows that Nash and LexiMax approaches can effectively achieve proportionally and max-min fairer solutions, respectively. Comparing the profit distributions of LexiMax and hLexi approaches, it can be observed that hLexi approach can provide a close approximation to the max-min fair solution with one order of magnitude saving in computational time.

Table 6: FI of profit distributions in Scenario 1 of Example 1

Proportional fairness		Max-min fairness		
MaxTotProf	Nash	MaxTotProf	LexiMax	hLexi
185.0%	31.4%	139.6%	1.6%	6.2%

The optimal transfer price levels in the fair solutions are shown in Figure 7. When maximising total profit, lower transfer prices are preferred in order to minimise cost of duties. Thus, transfer price level M1 is selected for the AI and all products, causing low or negative profits to plant A1 and all formulation plants, and high profits to all markets. In the solutions of three game theoretic approaches, the selected AI transfer price levels becomes higher to increase the profit of A1. In addition, the product transfer prices are also increased, even to the highest level M10 for some products, which results in higher profits of formulation plants and lower profits of markets. It can be noticed that different solution approaches might choose different transfer prices for some products. The results demonstrate that the transfer pricing decisions have an important role in balancing profits of supply chain members and achieving a fair profit distribution.

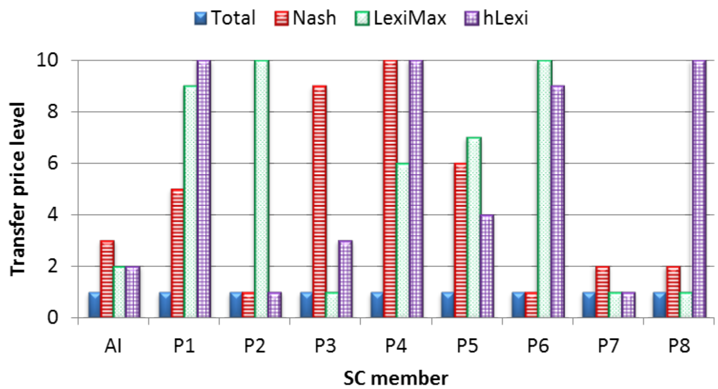

Figure 7: Optimal transfer price levels selected by different approaches in Scenario 1 of Example 1.

6.1.2. Scenario 2

In Scenario 2, the supply chain members do not have the same bargaining power. In order to focus on the effect of bargaining powers, the AI plant is assumed to have a lower bargaining power ($\alpha_{A1} = 0.5$) than others ($\alpha_{n \neq A1} = 1$). Similarly, the profit of each member is given in Table 7, in which all three game theoretic approaches obtain quite even profit distributions with up to 5% total profit loss. Comparing Tables 7 with 5, A1 earns 9–22% less actual profit in Scenario 2 than in Scenarios 1, due to its lower bargaining power, while most other members benefit from their relative higher bargaining powers, receiving up to 11% higher profit. The hLexi approach still provides a close approximation to the optimal max-min fair solution, taking significantly less computational time, compared to LexiMax approach.

Table 7: Optimal fair profits of supply chain members in Scenario 2 of Example 1, in cu.

	Nash	LexiMax	hLexi
A1	20.9	20.8	18.0
F1	43.3	40.5	38.5
F2	42.4	54.8	53.2
M1	35.7	33.4	35.2
M2	52.9	49.5	51.3
M3	37.1	34.7	35.2
M4	26.7	25.0	24.3
Total profit (cu)	259.1	258.7	255.6
CPU (s)	2.4	163.8	2.6 (1.4*/1.2**)

*Step 1; **Step 2.

The main driver of the obtained fair profit distribution is transfer pricing strategy, as shown in Figure 8. Due to lower bargaining power of A1, AI transfer price to formulation plants is reduced to the lowest level, M1, from levels M2 and M3 in Scenario 1. Thus, A1 receives less transfer payments from formulation plants, and therefore earns a lower profit. Moreover, although Nash approach and the other two lexicographic optimisation-based approaches are under different fairness criteria, all of them reduce the AI transfer price as a result of A1's lower bargaining power.

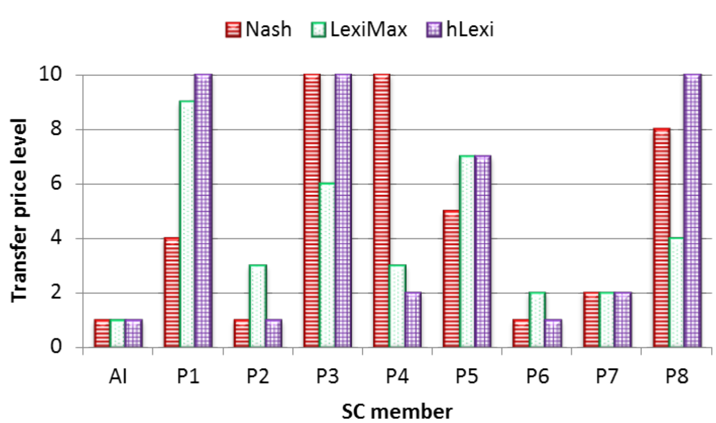

Figure 8: Optimal transfer price levels selected by different approaches in Scenario 2 of Example 1.

6.2. Example 2

In Example 2, all members are assumed to have the same bargaining power ($\alpha_n = 1$). The model statistics of Example 2 are presented in Table 8. Compared to Example 1, the model sizes in Example 2 are tens of times larger, and therefore the models are much more computationally difficult.

Table 8: Model statistics in Example 2

	MaxTotProf	Nash	LexiMax	hLexi
No of equations	152439	152478	152895*	3507**/152819***
No of continuous variables	114975	115166	137224*	3056**/115013***
No of discrete variables	15671	15671	15671*	1445**/15671***

*Last iteration; **Aggregated model at the last iteration of Step 1; ***Detailed model at Step 2.

6.2.1. Optimal Solutions

The computational results of Example 2 using the game theoretic approaches are presented in Table 9. The obtained excess profits and scaled profits are demonstrated and compared in Figures 9 and 10, respectively, and the FI values of profit distributions are given in Table 10. As the same as Example 1, MaxTotProf model cannot achieve a fair profit distribution, in which all markets earn negative profits, while only plants have positive profits. Meanwhile, the solution of hLexi approach is much max-min fairer, as the obtained scaled profits are all positive. Nash approach obtains the optimal solution within 1.3 hours, which has a much proportionally fairer profit distribution, with one order of magnitude reduction of FI than MaxTotProf model. Meanwhile, LexiMax approach fails to find a feasible solution at the 4th iteration of lexicographic loop within the given CPU limit, and therefore terminates without any solution returned after over 11-hour computation. However, the proposed hierarchical approach, hLexi, finds a solution within about 1.5 hours, taking a computational effort similar to Nash approach. The obtained solution also significantly reduces FI value by one order of magnitude from that by MaxTotProf model. Moreover, total profit of the two fair solutions are within 3% of the maximum total profit obtained. Thus, the proposed hierarchical approach successfully obtains a fair solution, regarded as a close approximation to the optimal lexicographic maximin and max-min fair solution, with high total profit and computational efficiency.

Table 9: Optimal profits of supply chain members in Example 2, in cu.

	MaxTotProf	Nash	LexiMax	hLexi
A1	-408.1	590.9	-	217.4
F1	599.3	780.6	-	1016.2
F2	1178.5	1430.6	-	1691.8
F3	1377.4	987.5	-	1269.6
F4	2175.8	2121.4	-	2711.9
F5	80.0	1018.4	-	1176.0
F6	1355.7	2263.9	-	2572.1
F7	644.8	971.7	-	1249.6
F8	340.3	1840.0	-	2113.5
M1	766.4	560.4	-	254.6
M2	3153.1	1072.3	-	930.5
M3	753.3	530.5	-	207.3
M4	1649.9	746.7	-	817.7
M5	295.2	393.6	-	136.4
M6	1228.4	625.7	-	498.9
M7	1231.8	480.3	-	358.9
M8	-33.1	481.1	-	145.5
M9	577.7	553.2	-	156.9
M10	1359.5	487.0	-	407.9
Total profit (cu)	18325.3	17935.9	-*	17932.8
CPU (s)	27.4	4446.5	40000.0	5447.6 (36.9**/5410.7***)

*No solution returned; **Step 1; ***Step 2.

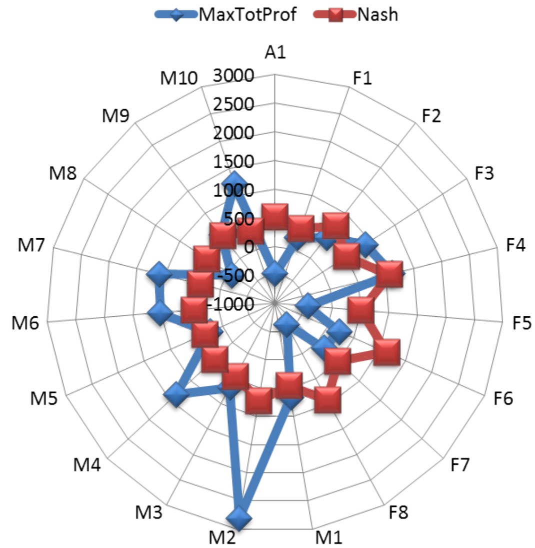

Figure 9: Excess profits, $Pr_n - MinPr_n$, obtained by MaxTotProf and Nash approaches in Example 2, in thousand cu.

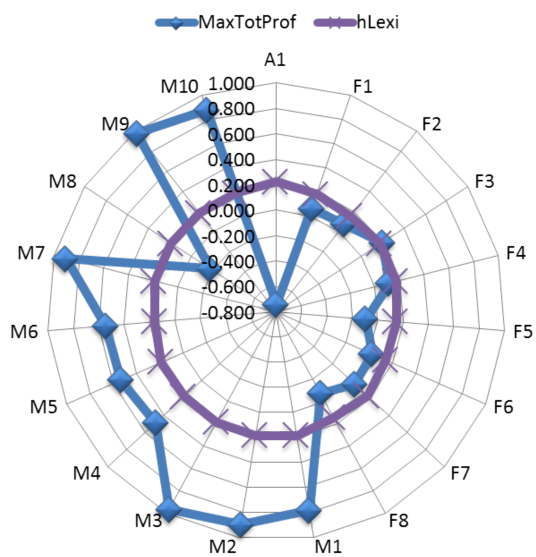

Figure 10: Scaled profits, $\overline{Pr}_n$, obtained by MaxTotProf and hLexi approaches in Example 2.

Table 10: FI of profit distributions in Example 2

Proportional fairness		Max-min fairness	
MaxTotProf	Nash	MaxTotProf	hLexi
132.4%	41.3%	140.1%	11.2%

Similar to Example 1, transfer price plays an important role in distributing profit more fairly. Different from MaxTotProf model, which chooses the lowest transfer price level for the AI and all the products, the obtained fair solutions use higher transfer prices. For example, in the max-min fair solution by hLexi approach, level M6 is selected as AI transfer price, which makes A1 earn a positive profit. Also, higher transfer prices of some products are chosen at formulation plants, e.g., P1 and P31, whose transfer prices are set to level M10 at all three formulation plants where they are produced. To earn a higher profit, F5, F6 and F8 assign transfer price level M10 to over three quarters of the products they produced, resulting in the most significant profit increases, compared to MaxTotProf solution. At the same time, the profit of F3 decreases, because it charged the lowest average transfer price, and paid a higher AI transfer price.

The optimal max-min fair transfer payments between supply chain members by hLexi approach are visualised using Circos in Figure 11. In this figure, all 19 members of the supply chain network are arranged circularly, and each one is represented by a colour and the label outside. The coloured links connect two members and illustrate the transfer payments between them. The links are in colour of the members who make the payments, and the width of each link is proportional to the transfer payment amount. On the outside of the ideogram, the cost and revenue from transfer payments of each member and their total amount are represented by three tracks, respectively, with percentage labels. In Figure 11, it can be observed that there are payments from all formulations to the AI plant, and the largest payment comes from F4, around a quarter of the total revenue of A1. As to the payments between formulation plants and markets, not every formulation plant is paid by all markets, and only four markets (M1, M2, M6 and M7) purchase products from all formulation plants. The links with the largest payment include M4→F6, M6→F8, and M2→F4.

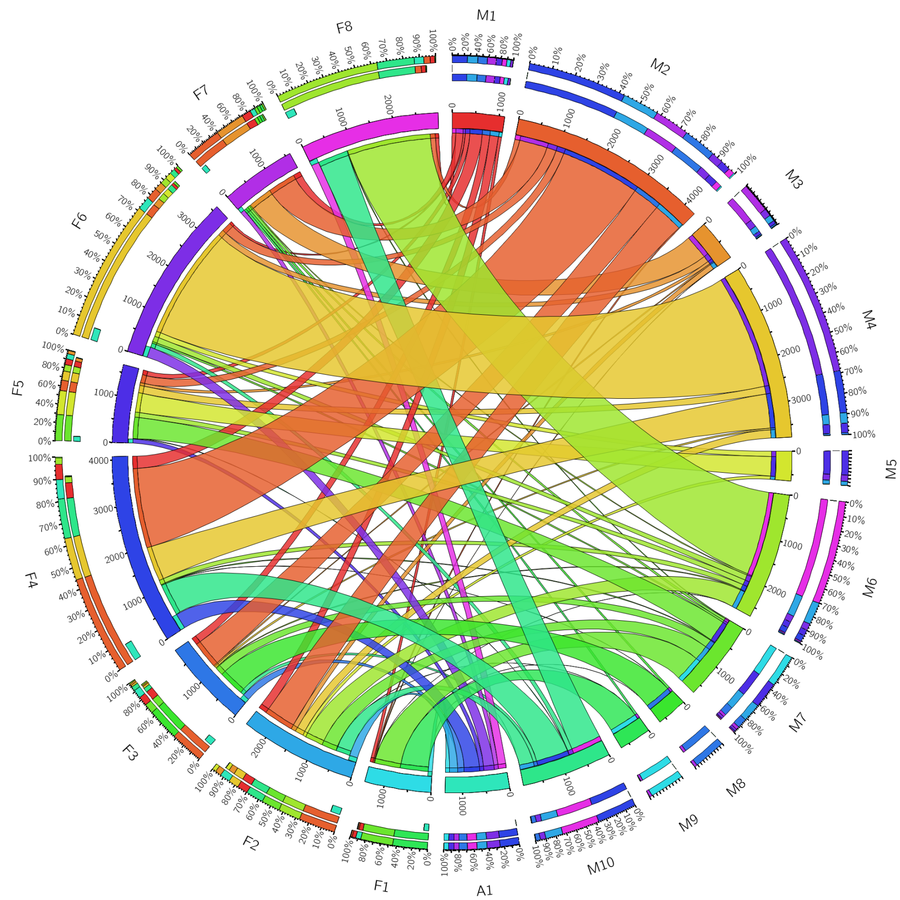

Figure 11: Optimal transfer payments by hLexi approach in Example 2, in thousand cu.

Figure 12 illustrates the optimal production and flows by hLexi approach using Sankey diagram. In this Sankey diagram, the four layers represent AI plants, formulation plants, products and markets. Each supply chain member and each product is represented by a block and a colour, and is labelled outside the block. The number after label represents the total production (for AI and formulation plant) or sales (for market) of each member, or the total shipped amount of each product. The links between AI plants between formulation plants represent AI flows, while the links between formulation plants and products, as well as between products and markets, illustrate product flows between formulation plants and markets, in colour of corresponding products. Comparing Figures

11 and 12, there are some interesting findings to link the optimal transfer payments and flows. In Figure 12, there is large AI flow to F4, which is consistent with the high transfer payments from F4 to A1 in Figure 11. There are also large product flows between F6 and M4, especially for products P13, P14, P18 and P20. Such fact explains over 70% of the revenue of F6 from M4, as illustrated in Figure 11. Similar cases can be found at markets M8 and M9, whose most payments and flows are to a single formulation plant. In all formulation plants, F4 and F6 earn the highest total revenues (Figure 11), as a result of their high productions than others (Figure 12), which lead to their relative higher profits (Table 9). Meanwhile, M2 and M4 make the largest payments to formulation plants (Figure 11), mainly because of their higher demands and sales (Figure 12). Similar results can also be observed in the optimal proportionally fair solution by Nash approach.

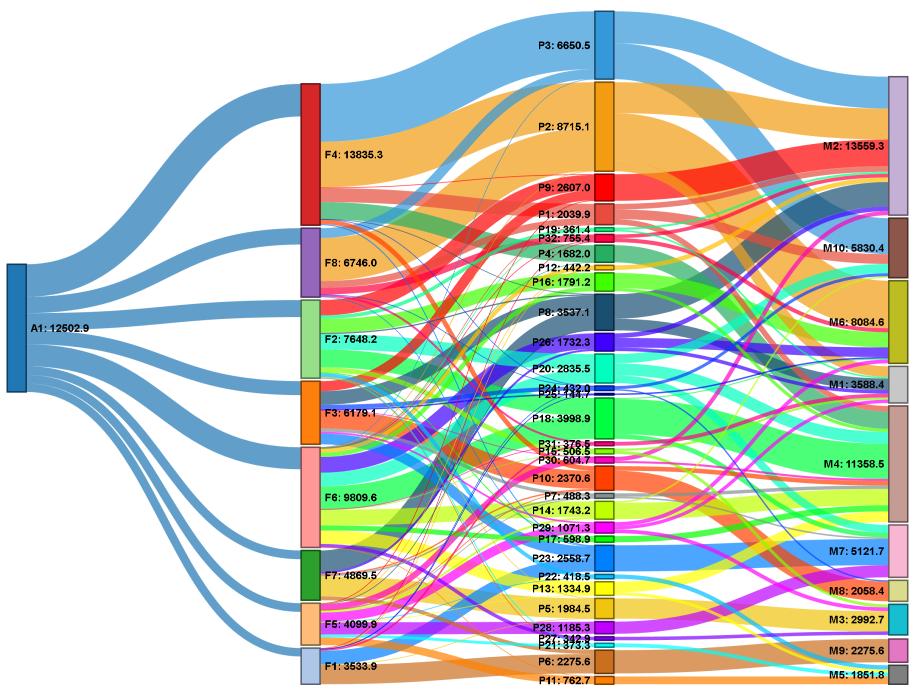

Figure 12: Optimal production and flows by hLexi approach in Example 2, in mu.

6.2.2. Sensitivity Analysis

Next, we further investigated the proposed hLexi approach, by analysing the effects of the value of parameter δ, the allowed deviation of pairwise relative profit ratios at Step 2 from those obtained at Step 1, on profit distribution and computational time. Here, we tested three values of δ: 10%, 20% and 30%. Table 11 summarises the max-min fair solutions of Example 2 by hLexi approach using these three values of δ. It is obvious that with a decreasing value of δ, the obtained profit distribution tends to be max-min fairer with smaller FI values. On the one side, for smaller value of δ (10%), due to the constraint (Eq. 65) is tighter, it is more computationally difficult to find the optimal solution at Step 2. On the other side, when the value of δ is higher (30%), as larger feasible region is generated by Eq. (65), more computational time is needed to search for the optimal solution at Step 2. Total profits become higher with increasing values of δ in Table 11. However, for a even higher value of $\delta > 30\%$, the model at Step 2 might not be able to find the optimal solution within given CPU time limit, and then the achieved solution could have a lower total profit and provide less proportionally fair distribution. It can be seen that 20% is able to achieve a good balance between fairness and computational expense. Note that similar results can be found in Example 1 as well.

Table 11: Sensitivity analysis on the value of δ in Example 2

δ	10%	20%	30%
Total profit (thousand cu)	17917.9	17932.8	17941.7
FI of max-min fairness	8.9%	11.2%	13.5%
CPU at Step 2 (s)	10476.5	5410.7	6981.7

7. Conclusions

This work addressed the fair profit distribution problem within a three-echelon supply chain network in the process industry, consisting of AI plants, formulation plants and markets with different bargaining powers. An MILP-based decision framework has been developed for production, distribution and capacity planning of the supply chain network. To achieve a fair profit distribution among all supply chain members involved, game theoretic approaches using Nash bargaining and lexicographic maximin principles were adopted under two different fairness criteria, proportional and max-min fairness. Especially, a tailored computationally efficient hierarchical approach has been proposed for max-min fair solutions. Two examples were examined, and computational results showed that both Nash bargaining and lexicographic maximin approachs can achieve fairer profit distribution, compared to what obtained by maximising total profit, in terms of proportional and max-min fairness, respectively. The effects of bargaining powers of supply chain members on profit distribution were studied. For large instances where the classic iterative lexicographic maximisation approach is highly time consuming, the proposed hierarchical approach is able to find good approximate optimal max-min fair profit distributions with much less computational efforts. At last, through sensitivity analysis, the values of an important parameter in the proposed hierarchical approach was investigated.

As to the future research directions, uncertainties of product demands can be considered and incorporated into the optimisation. In addition, the competition between products in the same group at markets can be studied. More detailed planning and scheduling decisions at plants can also be considered (Liu et al., 2008, 2010a,b, 2012). This work can be further extended to global supply chain networks, considering additional features, e.g., different tax rates and exchange rates.

Acknowledgement

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Appendix

The aggregated model solved at Step 1 of the proposed hierarchical approach, hLexi, is presented here. The static aggregated model determines the aggregated decisions of total production, flow and sales without considering time discretisation. As only the total mass balance is considered, all inventory-related equations and variables are not included in this model. The constraints of the aggregated model are as follows:

$$AP^A_a \leq ACap^A_a + \Delta ACap^A_a, \quad \forall a, t \tag{A.1}$$

$$\sum_{g \in G_j} \sum_{i \in \bar{I}_g} AP^P_{ij} \leq ACap^F_j + \Delta ACap^F_j, \quad \forall j \tag{A.2}$$

$$MinAF^A_{aj} \cdot X^A_{aj} \leq AF^A_{ajt} \leq MaxAF^A_{aj} \cdot X^A_{aj}, \quad \forall a, j \tag{A.3}$$

$$MinAF^P_{ijk} \cdot X^P_{ijk} \leq AF^P_{ijkt} \leq MaxAF^P_{ijk} \cdot X^P_{ijk}, \quad \forall j, k, g \in G_j, i \in \bar{I}_g \cap I_k \tag{A.4}$$

$$AP^A_a = \sum_j AF^A_{aj}, \quad \forall a \tag{A.5}$$

$$\sum_a AF^A_{aj} = \sum_{i \in \bar{I}_g} \sum_{g \in G_j} \beta_{ij} \cdot AP^P_{ij}, \quad \forall j \tag{A.6}$$

$$AP^P_{ij} = \sum_{k \in K_i} AF^P_{ijk}, \quad \forall j, g \in G_j, i \in \bar{I}_g \tag{A.7}$$

$$\sum_{j \in J_g} \sum_{g \in \bar{G}_i} AF^P_{ijk} = AS_{ik}, \quad \forall k, i \in I_k \tag{A.8}$$

$$TP^A_a = \sum_l TPL^A_{al} \cdot O^A_{al}, \quad \forall a \tag{A.9}$$

$$TP^P_{ij} = \sum_l TPL^P_{ijl} \cdot O^P_{ijl}, \quad \forall j, g \in G_j, i \in \bar{I}_g \tag{A.10}$$

$$\sum_{l} O^{A}_{al} = 1, \quad \forall a \tag{A.11}$$

$$\sum_{l} O^{P}_{ijl} = 1, \quad \forall j, g \in G_j, i \in \bar{I}_g \tag{A.12}$$

$$ALS_{ik} = AD_{ik} - AS_{ik}, \quad \forall k, i \in I_k \tag{A.13}$$

$$\sum_{j} X^{A}_{aj} \leq \mid j \mid \cdot E^{A}_{a}, \quad \forall a \tag{A.14}$$

$$\sum_{k \in K_i} X^{P}_{ijk} \leq \mid k \mid \cdot E^{P}_{ij}, \quad \forall j, g \in G_j, i \in \bar{I}_g \tag{A.15}$$

$$\overline{OAF}^{A}_{ajl} \leq MaxAF^{A}_{aj} \cdot O^{A}_{al}, \quad \forall a, j, l \tag{A.16}$$

$$\sum_{l} \overline{OAF}^{A}_{ajl} = AF^{A}_{aj}, \quad \forall a, j \tag{A.17}$$

$$Re^{A}_{a} = \sum_{j} \sum_{l} TPL^{A}_{al} \cdot \overline{OAF}^{A}_{ajl}, \quad \forall a \tag{A.18}$$

$$RMC^{A}_{a} = MC^{A}_{a} \cdot AP^{A}_{at}, \quad \forall a \tag{A.19}$$

$$PC^{A}_{a} = FPC^{A}_{a} \cdot E^{A}_{a} + VPC^{A}_{a} \cdot AP^{A}_{a}, \quad \forall a \tag{A.20}$$

$$CIC^{A}_{a} = crf \cdot CC^{A}_{a} \cdot \Delta ACap^{A}_{a}, \quad \forall a \tag{A.21}$$

$$Pr_a^A = Re_a^A - RMC_a^A - PC_a^A - CIC_a^A, \quad \forall a \tag{A.22}$$

$$\overline{OAF}_{ijkl}^P \leq MaxAF_{ijk}^P \cdot O_{ijl}^P, \quad \forall j, k, g \in G_j, i \in \bar{I}_g \cap I_k, l \tag{A.23}$$

$$\sum_l \overline{OAF}_{ijkl}^P = AF_{ijk}^P, \quad \forall j, k, g \in G_j, i \in \bar{I}_g \cap I_k \tag{A.24}$$

$$Re_j^F = \sum_k \sum_{g \in G_j} \sum_{i \in \bar{I}_g \cap I_k} \sum_l TPL_{ijl}^P \cdot \overline{OAF}_{ijkl}^P, \quad \forall j \tag{A.25}$$

$$PUC_j^F = \sum_a \sum_l TPL_{al}^A \cdot \overline{OAF}_{ajl}^A, \quad \forall j \tag{A.26}$$

$$RMC_j^F = \sum_{g \in G_j} \sum_{i \in \bar{I}_g} MC_{ij}^P \cdot AP_{ij}^P, \quad \forall j \tag{A.27}$$

$$FOC_j^F = \sum_{g \in G_j} \sum_{i \in \bar{I}_g} FFC_{ij}^P \cdot E_{ij}^P + \sum_{g \in G_j} \sum_{i \in \bar{I}_g} VFC_{ij}^P \cdot AP_{ijt}^P, \quad \forall j \tag{A.28}$$

$$TRC_j^F = \sum_a FTC_{aj}^A \cdot X_{aj}^A + \sum_a VTC_{aj}^A \cdot AF_{aj}^A, \quad \forall j \tag{A.29}$$

$$CIC_j^F = crf \cdot CC_j^F \cdot \Delta ACap_j^F, \quad \forall j \tag{A.30}$$

$$DUC_j^F = \sum_a \sum_l DC_{aj}^A \cdot TPL_{al}^A \cdot \overline{OAF}_{ajl}^A, \quad \forall j \tag{A.31}$$

$$Pr_j^F = Re_j^F - PUC_j^F - RMC_j^F - FOC_j^F - TRC_j^F - CIC_j^F - DUC_j^F, \quad \forall j \tag{A.32}$$

$$Re_k^M = \sum_{i \in I_k} V_{ik} \cdot AS_{ik}, \quad \forall k \tag{A.33}$$

$$PUC_k^M = \sum_j \sum_{g \in G_j} \sum_{i \in \bar{I}_g \cap I_k} \sum_l TPL_{ijl}^P \cdot \overline{OAF}^A P_{ijkl}, \quad \forall k \tag{A.34}$$

$$TRC_k^M = \sum_j \sum_{g \in G_j} \sum_{i \in \bar{I}_g \cap I_k} FTC_{ijk}^P \cdot X_{ijk}^P + \sum_j \sum_{g \in G_j} \sum_{i \in \bar{I}_g \cap I_k} VTC_{ijk}^P \cdot AF_{ijk}^P, \quad \forall k \tag{A.35}$$

$$DUC_k^M = \sum_j \sum_{g \in G_j} \sum_{i \in \bar{I}_g \cap I_k} \sum_l DC_{ijk}^P \cdot TPL_{ijl}^P \cdot \overline{OF}^A P_{ijkl}, \quad \forall k \tag{A.36}$$

$$LSC_k^M = \sum_{i \in I_k} PC_{ik} \cdot ALS_{ik}, \quad \forall k \tag{A.37}$$

$$Pr_k^M = Re_k^M - PUC_k^M - TRC_k^M - DUC_k^M - LSC_k^M, \quad \forall k \tag{A.38}$$

Given the above constraints, the following lexicographic maixmisation problem is solved at Step 1 of hLexi approach:

$$\begin{aligned} &\text{Leximax } \{\lambda_1 - \sum_n d_{1n}, \dots, \lambda_N - \frac{1}{N} \cdot \sum_n d_{Nn}\} \\ &s.t. \quad \lambda_p - d_{pn} \le \overline{Pr}_n / \alpha_n, \quad \forall p = 1, \dots, N, n \\ &\qquad \lambda_p \in \mathbb{R}, \quad \forall p = 1, \dots, N \\ &\qquad d_{pn} \ge 0, \quad \forall p = 1, \dots, N, n \\ &\qquad \text{Eqs. } (A.1) - (A.38) \text{ and } (56) \end{aligned} \tag{A.39}$$

Nomenclature

Indices

a	AI plant

g	product group
i	product
j	formulation plant
k	market region
l	transfer price level
n, n'	supply chain member, including a, j and k
p	index used in the dual model
t	time period

Sets

$\bar{G}_i$	set of the product group that product i belongs to
G_j	set of product groups suitable for formulation plant j
$\bar{I}_g$	set of products belong to group g
I_k	set of products sold in market k
K_i	set of markets selling product i

Parameters

$ACap_a^A$	aggregated capacity of AI plant a
$ACap_j^F$	aggregated capacity of formulation plant j
AD_{jk}	aggregated demand of product i at market k in time period j
Cap_a^A	capacity of AI plant a
Cap_j^F	capacity of formulation plant j
CC_a^A	unit capital cost for capacity expansion at AI plant a
CC_j^F	unit capital cost for capacity expansion at formulation plant j
crf	capital recovery factor
D_{jkt}	demand of product i at market k in time period j
DC_{aj}^A	unit duties of the AI from AI plant a to formulation plant j
DC_{ijk}^P	unit duties of product i from formulation plant j to market k

FFC_{ij}^{P}	fixed formulation cost of product i at formulation plant j
FPC_{a}^{A}	fixed production cost of AI at AI plant a
FTC_{aj}^{A}	fixed transportation cost of AI from AI plant a to formulation plant j
FTC_{ijk}^{P}	fixed transportation cost of product i from formulation plant j to market k
IC_{a}^{AA}	unit inventory cost of the AI at AI plant a
IC_{ij}^{AF}	unit inventory cost of the AI at formulation plant j
IC_{ij}^{PF}	unit inventory cost of product i at formulation plant j
IC_{ik}^{PM}	unit inventory cost of product i at market k
$MaxAF_{aj}^{A}$	maximum aggregated AI flow from AI plant a to formulation plant j
$MaxAF_{ijk}^{F}$	maximum aggregated flow of product i from formulation plant j to in k
$MaxAP_{a}^{A}$	maximum aggregated AI production at AI plant a
$MaxAP_{ij}^{F}$	maximum aggregated production of product i at formulation plant j
$MaxF_{aj}^{A}$	maximum AI flow from AI plant a to formulation plant j in each time period
$MaxF_{ijk}^{F}$	maximum flow of product i from formulation plant j to in k in each time period
$MaxIV_{a}^{AA}$	maximum inventory of the AI at AI plant a in each time period
$MaxIV_{j}^{AF}$	maximum inventory of the AI at formulation plant j in each time period
$MaxIV_{ij}^{PF}$	maximum inventory of product i at formulation plant j in each time period
$MaxIV_{ik}^{PM}$	maximum inventory of product i at market k in each time period
$MaxP_{a}^{A}$	maximum AI production at AI plant a in each time period
$MaxP_{ij}^{F}$	maximum production of product i at formulation plant j in each time period
$MaxPr_{n}$	maximum profit of supply chain member n
$MinAF_{aj}^{A}$	minimum aggregated AI flow from AI plant a to formulation plant j
$MinAF_{ijk}^{F}$	minimum aggregated flow of product i from formulation plant j to in k
$MinAP_{a}^{A}$	minimum aggregated AI production at AI plant a
$MinAP_{ij}^{F}$	minimum aggregated production of product i at formulation plant j
$MinF_{aj}^{A}$	minimum AI flow from AI plant a to formulation plant j in each time period

$MinF^{F}_{ijk}$	minimum flow of product i from formulation plant j to in k in each time period
$MinIV^{AA}_{a}$	minimum inventory of the AI at AI plant a in each time period
$MinIV^{AF}_{j}$	minimum inventory of the AI at formulation plant j in each time period
$MinIV^{PF}_{ij}$	minimum inventory of product i at formulation plant j in each time period
$MinIV^{PM}_{ik}$	minimum inventory of product i at market k in each time period
$MinP^{A}_{a}$	minimum AI production at AI plant a in each time period
$MinP^{F}_{ij}$	minimum production of product i at formulation plant j in each time period
$MinPr_{n}$	minimum profit of supply chain member n
MC^{A}_{a}	unit material cost of AI at AI plant a
MC^{P}_{ij}	unit material cost of product i at formulation plant j
PC_{ik}	lost sale penalty of product i at market k
TPL^{A}_{al}	transfer price level l of AI at AI plant a
TPL^{P}_{ijl}	transfer price level l of product i at formulation plant j
V_{ik}	selling price of product i at market k
VFC^{P}_{ij}	unit variable formulation cost of product i at formulation plant j
VPC^{A}_{a}	unit variable production cost of the AI at AI plant a
VTC^{A}_{aj}	unit variable transportation cost of the AI from AI plant a to formulation plant j
VTC^{P}_{ijk}	unit variable transportation cost of product i from formulation plant j to market k
α_{n}	bargaining power of supply chain member n
β_{ij}	unit AI consumption of product i formulation in formulation plant j
δ	allowed deviation of the relative profit ratio in hLeix approach
τ_{jk}	transportation time from formulation plant j to market k
$\bar{\tau}_{aj}$	transportation time from AI plant a to formulation plant j

Continuous Variables

AF^{A}_{aj}	aggregated flow of AI from AI plant a to formulation plant j

AF^P_{ijk}	aggregated flow of product i from formulation plant j to market k
ALS_{ik}	aggregated lost sales of product i at market k
AP^A_a	aggregated production of the AI at AI plant a
AP^P_{ij}	aggregated production of product i at formulation plant j
AS_{ik}	aggregated sales of product i at market k in time period t
CIC^A_a	capital investment cost of AI plant a
CIC^F_j	capital investment cost of formulation plant j
DUC^F_j	duties of AI paid by formulation plant j
DUC^M_k	duties of products paid by market k
F^A_{ajt}	flow of AI sent from AI plant a to formulation plant j in time period t
F^P_{ijkt}	flow of product i sent from formulation plant j to market k in time period t
FOC^F_j	formulation cost of formulation plant j
IV^{AA}_{at}	inventory of the AI at AI plant a in time period t
IV^{AF}_{jt}	inventory of the AI at formulation plant j in time period t
IV^{PF}_{ijt}	inventory of product i at formulation plant j in time period t
IV^{PM}_{ikt}	inventory of product i at market k in time period t
IVC^A_a	inventory cost of AI plant a
IVC^F_j	inventory cost of formulation plant j
IVC^M_k	inventory cost of market k
LS_{ikt}	lost sales of product i at market k in time period t
LSC^M_k	lost sales cost of products at market k
$\overline{OAF}^A_{ajl}$	auxiliary variable, $\equiv O^A_{al} \cdot AF^A_{aj}$
$\overline{OAF}^P_{ijkl}$	auxiliary variable, $\equiv O^P_{ijl} \cdot AF^P_{ijk}$
$\overline{OF}^A_{ajlt}$	auxiliary variable, $\equiv O^A_{al} \cdot F^A_{ajt}$
$\overline{OF}^P_{ijklt}$	auxiliary variable, $\equiv O^P_{ijl} \cdot F^P_{ijkt}$
P^A_{at}	production of the AI at AI plant a in time period t
P^P_{ijt}	production of product i at formulation plant j in time period t
PC^A_a	production cost of AI plant a

Pr_n	profit of supply chain member n
Pr_a^A	profit of AI plant a
Pr_j^F	profit of formulation plant j
Pr_k^M	profit of market k
Re_a^A	revenue of AI plant a
Re_j^F	revenue of formulation plant j
Re_k^M	revenue of market k
RMC_a^A	raw materials cost of AI plant a
RMC_j^F	raw materials cost of formulation plant j
S_{ikt}	sales of product i at market k in time period t
TP_a^A	transfer price of the AI from AI plant a
TP_{ij}^P	transfer price of product i from formulation plant j
TPC_j^F	transfer payment cost of formulation plant j
TPC_k^M	transfer payment cost of market k
TRC_j^F	transportation cost paid by formulation plant j
TRC_k^M	transportation cost paid by market k
$TotalPr$	total profit of the supply chain
$\Delta ACap_a^A$	aggregated capacity increment of AI plant a
$\Delta ACap_j^F$	aggregated capacity increment of formulation plant j
ΔCap_a^A	capacity increment of AI plant a
ΔCap_j^F	capacity increment of formulation plant j
λ_p, d_{pn}	variables in the dual model

Binary Variables

E_a^A	1 if the AI is produced at AI plant a; 0 otherwise
E_{ij}^P	1 if product i is produced at formulation plant j; 0 otherwise
O_{al}^A	1 if transfer price level l is selected for the AI formulated at AI plant a; 0 otherwise

O^P_{ijl}	1 if transfer price level l is selected for product i formulated at formulation plant j; 0 otherwise
W^A_{at}	1 if the AI is produced at AI plant a in time period t; 0 otherwise
W^P_{ijt}	1 if product i is produced at formulation plant j in time period t; 0 otherwise
X^A_{aj}	1 if the AI is shipped from AI plant a to formulation plant j; 0 otherwise
X^P_{ijk}	1 if product i is shipped from formulation plant j to market k; 0 otherwise
Y^A_{ajt}	1 if the AI is shipped from AI plant a to formulation plant j in time period t; 0 otherwise
Y^P_{ijkt}	1 if product i is shipped from formulation plant j to market k in time period t; 0 otherwise

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