Virtual Spatial Diversity Antenna for GNSS-based Mobile Positioning in the Harsh Environments

Hoda Tahami, *School of Civil and Construction Engineering, Oregon State University*
Anahid Basiri, *Centre for Advanced Spatial Analysis, University College London*
Terry Moore, *Nottingham Geospatial Institute, University of Nottingham*
Jihye Park, *School of Civil and Construction Engineering, Oregon State University*
Lukasz Bonenberg, *Nottingham Geospatial Institute, University of Nottingham*

BIOGRAPHY (IES)

Hoda Tahami is a PhD student in Geomatics program at Oregon State University. She received her Master of Science in Geospatial Information System with mainly focus on web and mobile GIS applications. Her PhD research interests cover Precise Point Positioning technique in GNSS positioning and navigation. Her ongoing research topics include utilizing GNSS for atmospheric monitoring and GNSS positioning in the harsh environment.

Dr. Anahid Basiri is a lecturer in UCL Spatial Data Science and Visualization Centre for Advanced Spatial Analysis. She works on the future applications of Location Based Services (LBS), its current challenges and potential solutions. Her interests mainly involve Spatial Data Science, Location Based Services, Crowdsourced Geospatial Data and Geospatial Data Quality.

Professor Terry Moore is the Director of the Nottingham Geospatial Institute (NGI) at the University of Nottingham and as such has overall responsibility for all the activities of this world leading postgraduate research and teaching institute. He also leads the Transport Technologies Research Priority Area, across the University. Professor Terry Moore was promoted to the UK's first Chair of Satellite Navigation in 2001 and has extensive research experience in a range of subjects including satellite navigation and positioning, navigation sensor integration, geodesy and orbit determination. He has taken a leading role in national and European initiatives aimed at integrating academic research and teaching activities in GNSS, and interacting closely with industry. He is the founding Director of GRACE, the GNSS Research and Applications Centre of Excellence, which was jointly funded by the University of Nottingham and East Midlands Development Agency. He is a Fellow and a Member of Council of both the Royal Institute of Navigation (RIN) and the (US) Institute of Navigation (ION).

Dr. Jihye Park’s research interests cover Global Navigation Satellite System (GNSS) positioning/navigation and GNSS remote sensing. GNSS positioning/navigation includes topics on Real Time Kinematic (RTK), Network RTK (NRTK), Precise Point Positioning (PPP), and Inertial Navigation System (INS). Park focuses on advanced algorithms that are used to improve the performance of positioning and navigation systems under harsh environments. GNSS is also capable of observing the atmosphere, referred to as GNSS Remote sensing. Dr. Park's ongoing research topics include utilizing GNSS to monitor ionospheric disturbances to observe geophysical events such as natural hazards or artificial explosions.

Dr Lukasz Bonenberg is a Senior Experimental Officer at the University of Nottingham, responsible for teaching and research in the area of precise positioning, navigation and GNSS. Lukasz received his PhD from the University of Nottingham, looking at the deep integration of GPS and terrestrial pseudolite signals. Lukasz participated in the UK and European research projects looking at precise positioning, navigation and sensor fusion. His current research interest includes higher accuracy high integrity navigation, application of raw GNSS observation's from Android devices and sensor fusion. He is the member of GNSS Raw Measurements Task Force, coordinated by the European GNSS Agency (GSA). He is Member of the Institute of Navigation, Corporate Member of Chartered Institute of Civil Engineering Surveyors and Member of Association for Geoinformatics, GeoIT and Navigation.

ABSTRACT

Global Navigation Satellite Systems (GNSS) is the most commonly used positioning technology for many Location Based Services (LBS) including navigation. However, GNSS applications are limited to outdoors as GNSS signals can get blocked and attenuated inside or between buildings, making positioning unreliable, inaccurate or impossible. Blockage of GNSS signals may result in the lack of availability of the minimum of four satellites in-view at each epoch, for a single constellation GNSS positioning, and consequently, lead to a failure in the continuity of the positioning service. This is a particularly common issue in urban canyon and indoors. This paper proposes and implements a framework to handle this challenge by virtually distribute the antenna in space and time and accumulate the measurements while adding some unknowns to solve the synchronization and the position solution. To test the proposed technique, Virtual Spatial Diversity Antenna (VSDA), raw GNSS measurements are captured using an Android 7.1.1. running smartphone over the period of forty-seven minutes. Then the observations are accumulated until the number of measurements outcounts the number of unknown, i.e. three position components plus the number of epochs. The results of the GPS-only measurements for a relatively limited period of 47 minutes, where satellite geometry may not significantly change, prove the feasibility of solving position solution in presence of fewer than four satellites at each epoch using VSDA scheme, and achieving the horizontal and vertical accuracy of 47.16 m and 68.45m, respectively.

INTRODUCTION

Although Global Navigation Satellite Systems (GNSS) is the most widely used positioning technology many applications and services, it may not be applied for indoor or urban canyons use (Seco-Granados et al., 2012) with the same level of reliability, continuity and accuracy. In such ‘harsh environment’ GNSS signals can be blocked, attenuated and reflected. Blockage of the signals can result in the epochs where fewer than four satellites, as the minimum number of single constellation observations for GNSS positioning, are in view and so, GNSS blockage can make positioning impossible. This is commonly happening also in dense urban canyons and indoors so that the users may need to use other technologies, e.g. technologies based on Wi-Fi, inertial systems, Bluetooth and mobile networks (Basiri at al., 2017). However, these techniques do not yet provide a globally available, free of charge, privacy preserving, accurate, and stand-alone positioning service like GNSS does in outdoors. In addition, some may require new infrastructure, device modifications, and/or prior data capture, such as mapping. In this regard, this paper suggests a novel GNSS-only technique to address the challenge of positioning in harsh environment where fewer than four satellites are in view.

The proposed technique is to virtually distribute the antenna in space and/or time to receive to be able to accumulate the GNSS measurements until the number of observations in total outcounts the number of unknowns, which is no longer four. The synchronization between epochs is resolved by adding an unknown for each additional set of observations at each added epoch. So, the proposed technique, Virtual Spatial Diversity Antenna (VSDA), is based on the accumulation of measurements at an epoch, even if the number of observation is fewer than four, and adding to the number of the unknowns at each iteration until the position is solvable. The technique is tested using a mobile phone as this is the most commonly used positioning device in the harsh environments such as urban canyons and indoors and also GNSS raw measurements are now accessible. Having Android 7 released, the android.gsm.location API allowed a direct access to the decoded navigation messages, clock, carrier and code measurements (Banville et al., 2016). Access to these measurements enables some techniques, such as VSDA, to be implemented on mobile devices that are part of the users' everyday lives.

To test the VSDA scheme, forty-seven minute long GNSS raw data are captured and with only two sets of measurements, i.e. two epochs, that are accumulated the position solution become solvable. The achievable accuracy is highly correlated with the geometry of the satellites at each epoch, variety and diversity of the satellites in view throughout the observations, and quality (e.g. accuracy) of the measurements. This particular test achieves the horizontal and vertical accuracy of 47.16 m and 68.45m, respectively. While it is notable that the position is solvable for a location where no epochs with enough number of satellites in view is available, to enable the traditional positioning techniques to localize the receiver, this is particularly important to note that the achievable accuracy could potentially improve in light of better geometry (both in space and time) of the measurements, more diverse combination of satellites, and/or larger number of measurements in total which may lead to higher degree of freedom.

This paper is structured as follows; section two explains the preliminaries, including the mathematical framework of the VSDA scheme and also the raw GNSS measurements. Section three discusses the experiments and then finally the results are presented.

PRELIMINARIES

Mobile positioning using GNSS, solely, could be limited to outdoor use due to several issues including the unavailability and the quality of the measurements. Earlier research identified that mobile phone's IFA (planar inverted-F antenna) offers a poor multipath suppression, visualizing in poor carrier performance and degraded signal to noise ratio (SNR) (Pesyna et al. 2014, Humphreys et al., 2016). Riley et al. (2017) demonstrated that differential techniques could be successfully applied in right conditions while Odolinski & Teunissen,2018 confirmed that a solution is even possible during ionospheric disturbance periods by using multi-epoch model.

Mobile phones now can provide with higher sensitivity (Diggelen, 2009), which translate to signal availability even inside the buildings, and access to the raw measurements. This paper utilizes these and proposes a novel technique to localize the mobile devices, solely based on GNSS, for the “harsh environment” and “difficult scenarios”, such as indoors and urban canyons, where not enough number of satellites may being view.

- ### VIRTUAL SPATIAL DIVERSITY ANTENNA

To test the VSDA scheme, forty-seven minute long GNSS raw data are captured and with only two sets of measurements, i.e. two epochs, that are GNSS signal blockage can result in the lack of availability of the minimum four required satellites in view at each epoch, for the single constellation. In order to address this challenging issue, which is common in dense urban areas, Sadrieh (2011) and Zhang et al. (2014) suggested the spatial diversity system. Their proposed system uses multiple antennas, that are physically distributed but electronically synchronized using wires and cables, in the form of one antenna diversity system. The diversity system combines the pseudo-range observations from the multiple antennas. Therefore, the location can be calculated when the summation of the captured satellites by two (or more) separate antennas is more than four, which is the number of the unknown for single constellation positioning. However, this approach may not be applicable in real-world applications as (a) it may be too expensive to have two or more receiving antennas linked together at the same time. Also, (b) most of the LBS applications are run on mobile devices which cannot be modified easily by the users. And even if it was possible to create a network of several antennas embedded in one mobile device, (c) due to the small size of mobile devices, it is the most likely to view exactly the same satellites by two (or more) antennas at the same time.

This paper proposes a technique to virtually generate a spatial diversity antenna by accumulating raw GNSS measurements to overcome the limitation of the inadequate number of satellites in view. The proposed methodology is based on adding all stored raw measurements at one epoch (even if the number of observation is fewer than four) to another set of observations at another epoch (and/or in another nearby location where is close enough to be assumed as the same location, depending the required accuracy) which can have fewer than four satellites in view. Each iteration may add one new unknown variable due to the synchronization between the satellite clocks and the receiver's at that epoch. The number of the unknowns in multiple epochs is no longer four since each epoch adds one more clock offset, therefore in total, at least n+3 observations are required, where "n" is the number of epochs. The process of accumulation of the measurements (and so adding to the number of unknowns) continues until the total number of observations outcounts the total number of unknowns. An example scenario is discussed here, more fully. As shown in equation 1, in the first epoch there are only three measurements and so position solution is not solvable. Adding the second epoch (at time t2) will increase the number of measurements to 6, however, it also adds one unknown due to clock offset, allowing one degree of freedom. This could be improved if the third epoch, with three measurements and one new unknown, is also added. Note that none of these epochs could solve the position solution on individually but if accumulated, as long as the measurements are independent enough, the covariance matrix will no longer become singular. As explained, VSDA makes the observations virtually synchronized by adding an additional unknown.

$$
X = \begin{bmatrix} X \\ Y \\ Z \\ clk_{t_1} \\ clk_{t_2} \\ clk_{t_3} \end{bmatrix}
\quad
A = \begin{bmatrix} 111100 \\ 111100 \\ 111100 \\ \\ 111010 \\ 111010 \\ 111010 \\ \\ 111001 \\ 111001 \\ 111001 \end{bmatrix}
\begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} t_1 \\ \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} t_2 \\ \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} t_3 \end{matrix}
\quad
Y = \begin{bmatrix} L_{t_1,G_{01}} \\ L_{t_1,G_{02}} \\ L_{t_1,G_{03}} \\ \\ L_{t_2,G_{01}} \\ L_{t_2,G_{02}} \\ L_{t_2,G_{03}} \\ \\ L_{t_3,G_{01}} \\ L_{t_3,G_{02}} \\ L_{t_3,G_{03}} \end{bmatrix}
$$

(eq. 1)

The position solution for this example can be calculated using the regular least square equation (eq. 1) as shown below

$$\hat{x} = (A^T P A)^{-1} A^T P y$$

(eq. 2)

Where y is the observation matrix, x is the unknown matrix, A is the design matrix and P is the weight matrix. The weight matrix can be obtained from the pseudorange noise level that is accessible from raw GNSS measurements, which is explained in detailed in the next subsection.

VSDA ON MOBILE DEVICES

The access to raw GNSS data on Android running devices enabled several applications that required raw measurements, including the proposed VSDA scheme. The GNSS raw measurements are contained in the GnssClock and GnssMeasurement software classes, which are described in the android location APIs . Google has released the GNSS Logger application along with its source code that is used to retrieve the raw GNSS measurement in the current study. Depending on the device manufacturer/type, raw GNSS measurements can include Pseudorange and pseudorange rate, navigation messages, accumulated delta range or carrier and hardware (HW) clock. The used test device limits the quality of the carrier phase measurements (Humphreys et al, 2016; Riley et al. 2017) making pseudorange measurements most appropriate to use for this study.

The retrieved raw measurements are then used to be combined in the virtual spatial diversity antenna VSDA algorithm to calculate the user position. Figure 1 describes the workflow of the implementation of the virtual spatial diversity antenna algorithm using raw GNSS measurements coming from the mobile device.

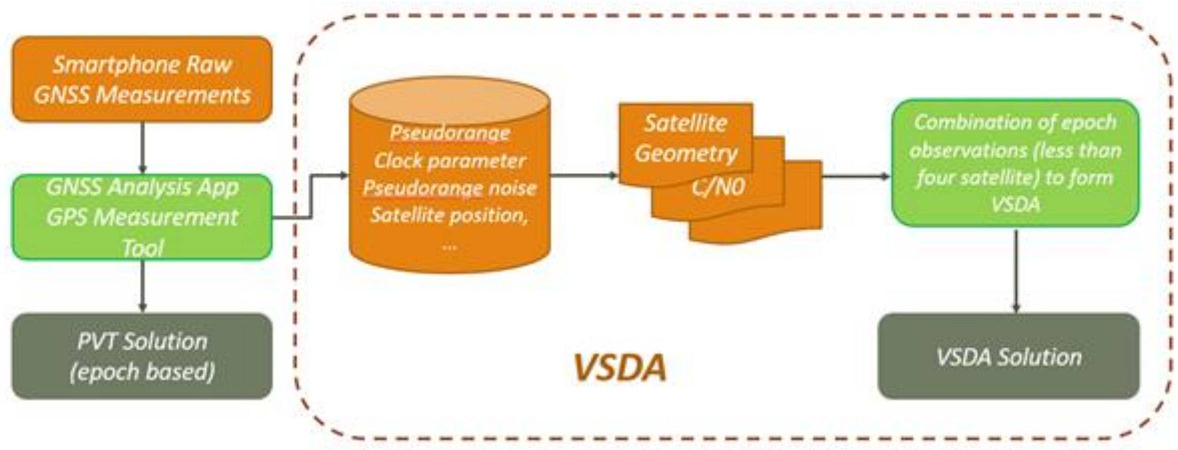

Figure 1: Virtual Spatial Diversity Antenna process

In figure 1, the left side of the flowchart represents how Google GNSS Analysis application (or the released GPS measurement tool) uses the Android GNSS raw measurements in an epoch based solution to calculate the position. While GNSS Analysis application functions as long as four or more satellites are in view at each epoch, the VSDA can accumulate the raw measurements (even if fewer than the minimum required number of satellites are in view at each epoch). VSDA forms a matrix of the observations, accumulating the measurements at different epochs, to calculate the three components of location (x,y,z) and the all the clock offsets. Presumably, the achievable accuracy of the position solution is highly correlated with the quality of input raw measurements. Therefore, for showing the feasibility of implementing the algorithm, a number of factors are considered to define a combination setting for raw measurements accumulation. The factors can include the satellite geometry, data continuity for specific satellite Pseudo Random Noise (PRN) over time, and the carrier to noise ratio of signals for each satellite PRN.

EXPERIMENT DESCRIPTION

In this study, we conducted an experiment to (a) examine the feasibility and prove the concept of the virtual diversity antenna and (b) estimate the achievable accuracy of the proposed technique. This experiment is based on the single constellation measurements, GPS only, to simplify the scenarios and avoid extra unknown due to the synchronisation between multi-constellation systems. The GPS-only measurements are captured using an S8 Samsung smartphone (Model: SM-G950F) running Android 7.1.1. This device enables logging of pseudorange measurements on the L1 signal of GPS without the duty cycle. A forty-seven-minute long data capture phase on 3rd of September 2018 was conducted based on a known location on top of the roof of Nottingham Geospatial Institute (52º 57’ 6.9192’’ N, 1º 11’ 2.7686’’ W, 91.209 m ETRS89 Geodetic coordinates). At top of the building with a few obstacles around, obviously, numerous epochs contain more than four satellites in view. Therefore, a GPS denied scenario has been simulated by withdrawing some of the measurements at each epoch making them fewer than four.

- **GNSS DATA PROCESSING**

The collected data were processed by GPS measurement tool released by Google Android to derive the pseudorange from the received satellite time. To test the quality of the measurements coming from the smartphone, the pseudorange availability in the observation time has been provided. Figure 3 shows each raw pseudorange measurements over the 47-minute period of data capture. The figure 3 shows that several satellites cannot be captured measured. Those measurements or at least the discontinuous part of pseudoranges will not be considered in the processing phase of the study.

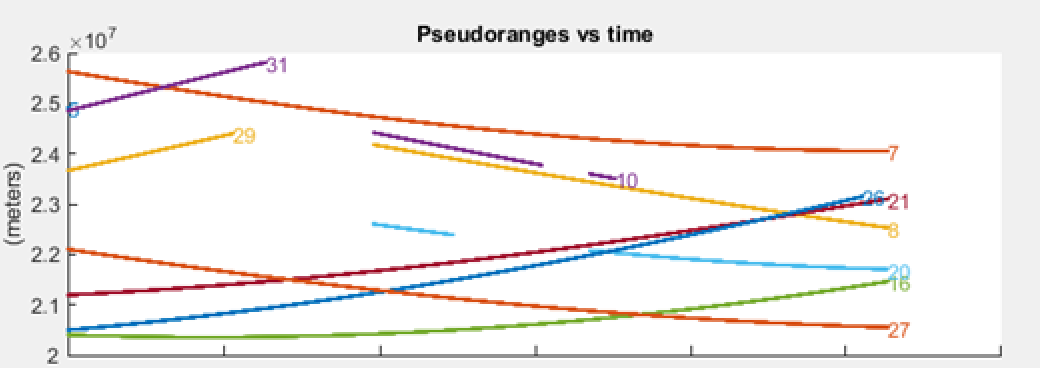

Figure 2: Pseudo range vs time

The GNSS antenna embedded in the smartphone uses linear polarisation, making it especially susceptible to multipath effects. Multipath is a result of from GNSS signals bouncing off the ground or surrounding features before reaching the antenna. The level of noise in GNSS receivers may determine how precisely pseudorange can be measured (Banville et al., 2016). In addition, the variation in the level of noise can depend on the satellite transmitter power output and the differences in the satellite and the receiver antennas gain with respect to the elevation and azimuth angles. This can be also due to losses engendered by the other satellites in view and the various blocks of satellites (Langley 2000). Therefore, to test the VSDA scheme, the direct and noisier signals should be distinguished. The level of noise in the observations is measured by the signal-to-noise ratio (SNR), where S is the power of the received signal and N the power of noise. It is also possible to describe the signal level against the noise level, the carrier-to-noise-density ratio, C/N0, which represents the SNR in a 1Hz-bandwidth. Figure 3 shows the carrier-to-noise density (C/N0) for the received signal.

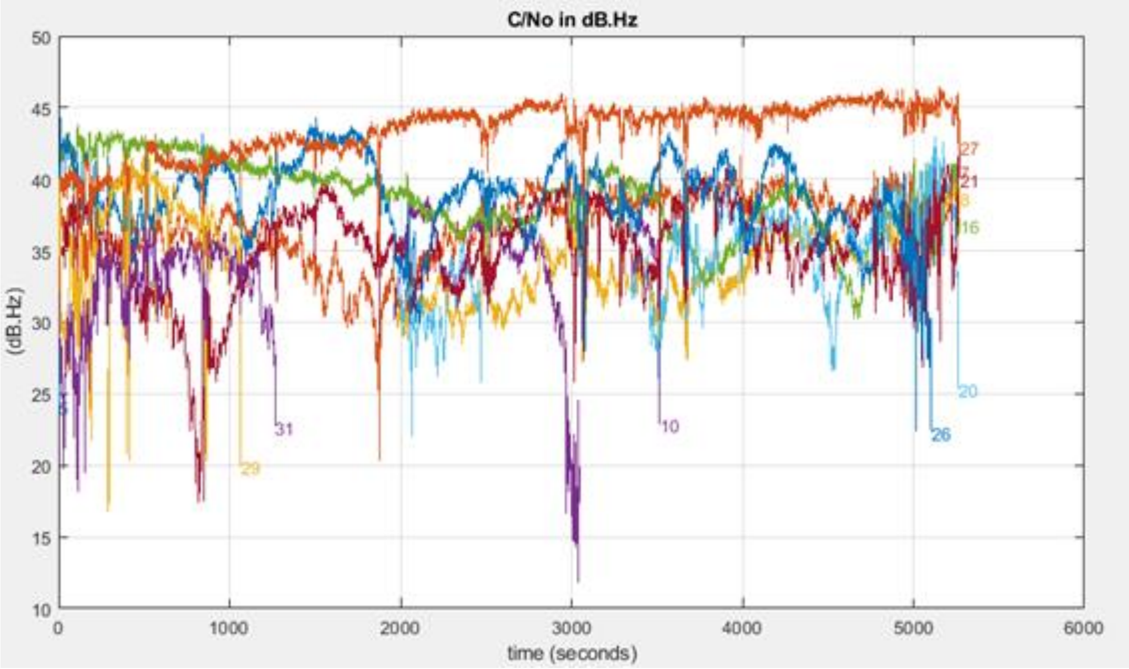

Figure 3: Signal to noise ratio for tracked GPS satellites

For comparing the result with the achievable accuracy in ‘normal’ scenario, where position solution is solvable thanks to the availability of more than four satellites in view at each epoch, first, the achievable accuracy from all the measurements, i.e. 160652 raw measurements from 5273 epochs, are calculated. Among these, the measurements from the satellites with the low S/N ratio and low Dilution Of Precision (GDOP) are removed. Figure 4 shows the location of the satellites in view at the receiver point during the observation period. The GPS satellites with the elevation angle below 5 degrees are not considered in the processing. This is mainly to avoid receiving the signals potentially reflected from the ground or surrounding features. Differences in the elevation angle of satellites above the horizon typically explain the differences of C/N0 values among satellites (Banville et al., 2016). The C/N0 values measured for those satellites are between 10 to 25 dB-Hz, which may degrade the quality of the measurements and so are removed.

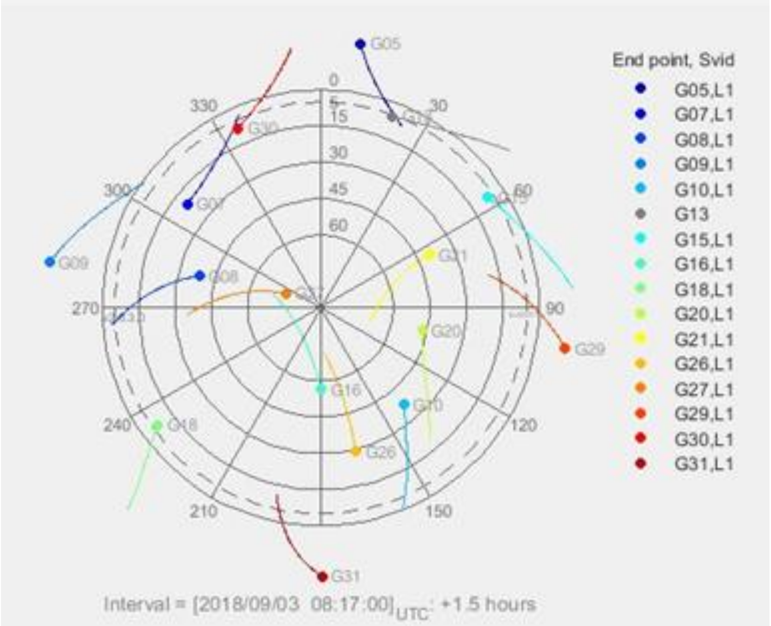

Figure 4: Sky plot showing the geometry of the GPS satellites in view

To generate a GPS denied scenario, some of the observations from each epoch should be removed, and otherwise, more than four measurements are available. This is done in the way that the remaining measurements provide with the best Geometric GDOP and higher carrier to noise ratio. To do so, the sky plot, signal to noise ratio and the signal strengths are used to select the best satellite for GPS denied scenario processing. Figure 5 illustrates the four satellites with the highest signal strengths. Among GPS satellites recorded by the smartphone, G27, G26, G16 and G07 are considered as the top four satellites providing with the highest signal strengths. However, in some cases, the combination of the satellites with the highest signal strength may not be an ideal case due to the satellite geometry. For example, accumulating of the observations from G07, G16 and G27 leads to a lower HDOP value. This echoes the importance of optimizing both criteria for selecting the measurements.

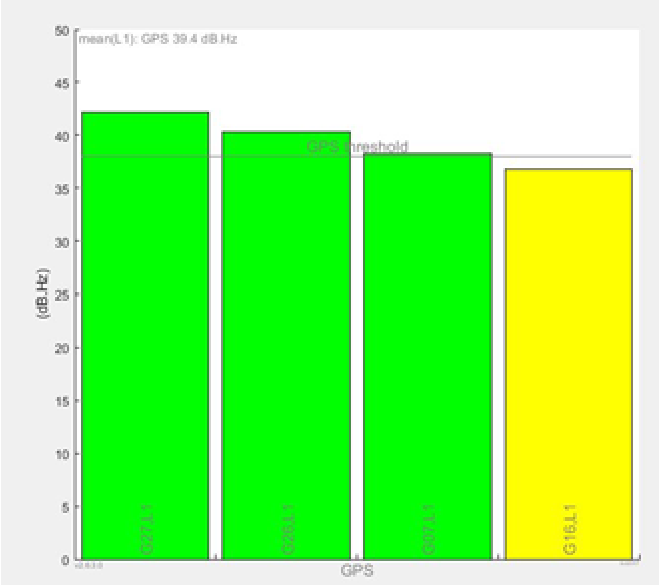

Figure 5: Satellites with the higher signal strengths

RESULTS AND DISCUSSION

As explained in section 3, the proof of concept and the estimation of the achievable accuracy of the proposed virtual spatial diversity antenna technique, are conducted and examined at different simulated GPS denied scenarios. Based on the satellite selection criteria, i.e. satellite geometry at each epoch, independence/correlation of the measurements at the end of accumulation process, carrier to noise ratio, and the signal strength, three satellites of G08, G20 and G27 are considered to be used for simulating the GPS denied scenarios. The achievable accuracy from these simulated scenarios should be compared against the achievable accuracy if all the measurements are taken into account. While this is simply to show how VSDA performs in comparison with the actual dataset, in GNSS (partially) denied areas there is no position solution and so any accuracy could be better than nothing if the uncertainties associated with the results are explained to the users. The actual data set, with all the satellites in-view taken into account for positioning, the horizontal errors range between 1.6m and 5.5m while the vertical error range between 10.1m and 60.1m with the confidence interval of (50%, 95%), see figure 6 and 7.

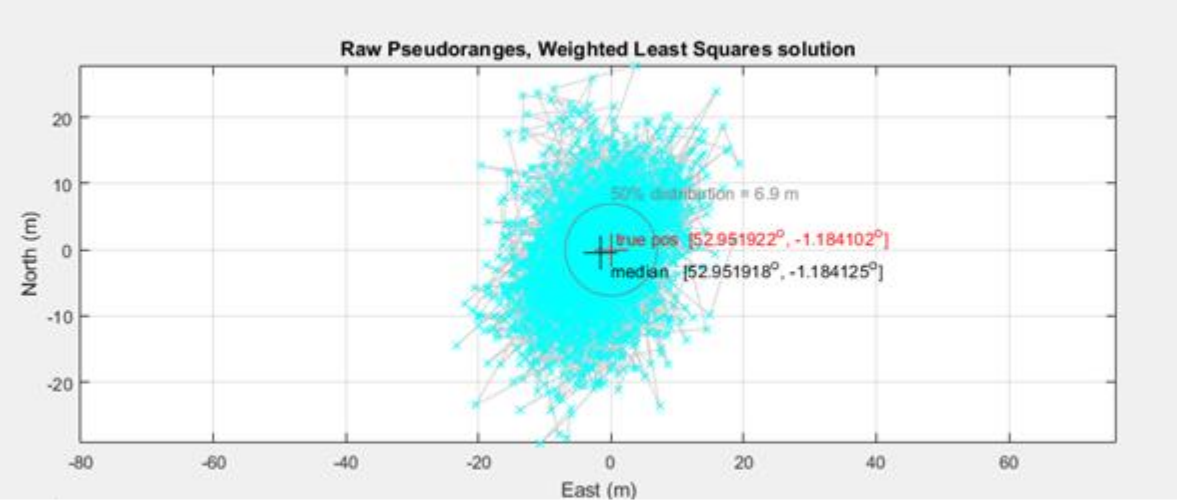

Figure 6: Weighted least square solution: Processing all available raw pseudo ranges.

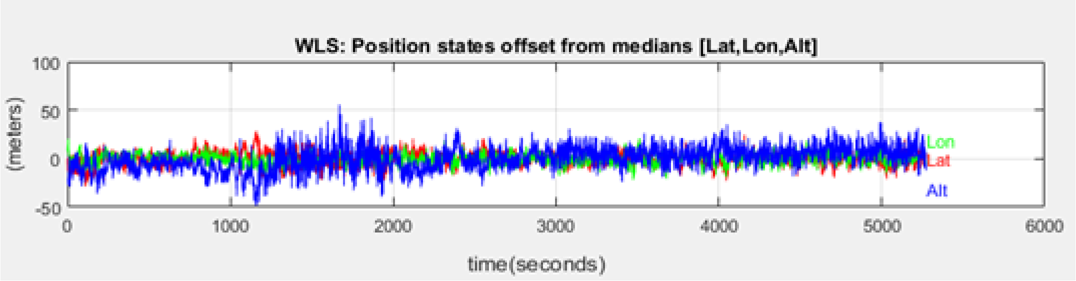

Figure 7: Weighted least square solution offset from median

Figure 6 and 7 show the achievable accuracy from the sequential Least Square solution. While this may seem to be a fair benchmark accuracy as the position solution is usually calculated in this way, VSDA technique is based on adding the measurements of epochs, and so it may be useful to calculate and compare VSDA's accuracy with the accuracy of position solution taking into account all the observations. This may seem, methodology-wise, a more justifiable and perhaps better comparison. Considering all 160652 pseudorange observations together to solve the 5237 unknowns, i.e. 5234 unknowns for the clock offsets and three location components (x,y,z), could provide the horizontal and vertical accuracies of is 5.8m and 1m, respectively. Note that every original epoch contains more than four observations and so each can independently calculate the position. However, the accumulation of all observations may not necessarily provide with a much better accuracy level in comparison with the case of having only a few epochs. This could be due to having linearly dependent equations as there are several almost repetitive measurements, which potentially can result in the singularity of the matrix. The geometry of the satellites and so the pseudorange measurements may not change dramatically in a short time interval, e.g. over a few minutes. This can make the observations (highly) dependent failing to provide a practical higher degree of freedom. Selecting the epochs that are temporally further from each other to be accumulated and formed the observation matrix can potentially avoid the singularity and provide with more independent observations from the very same set of satellites.

Figure 8 shows the locations of the selected satellite used to simulate the GPS denied scenario at three epochs that are being accumulated. As it is shown in figure 8, thanks to the time interval between the epochs, satellites G08, G20 and G27 have moved considerably enough to make the independent for the VSDA solution.

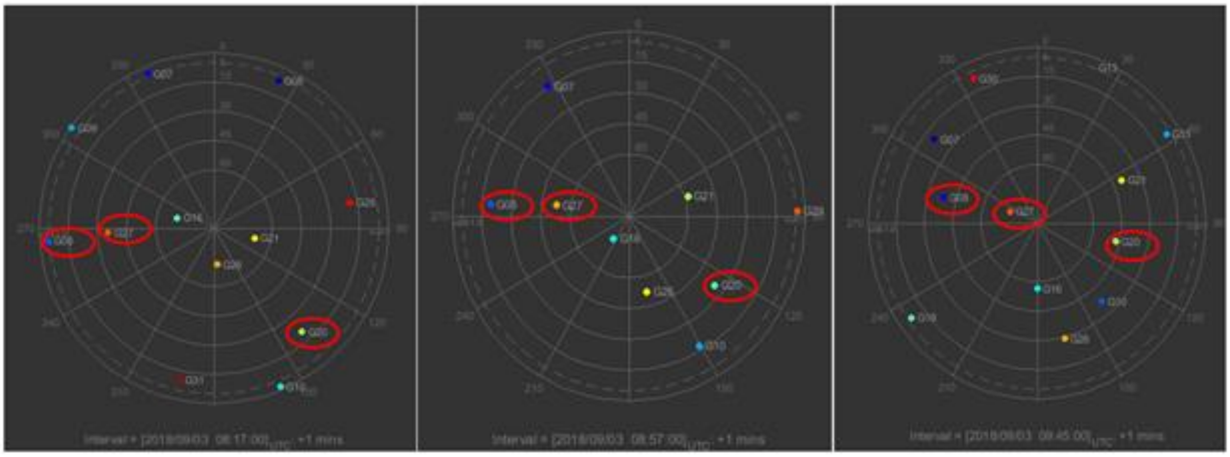

Figure 8: Variation of Satellite geometry for selected epochs in GPS denied scenario

However, in order to make use of all the observations contribute to the experiment, the selected epochs are separately used to calculate the position solution of the same point , i.e., by GPS measurement tool.

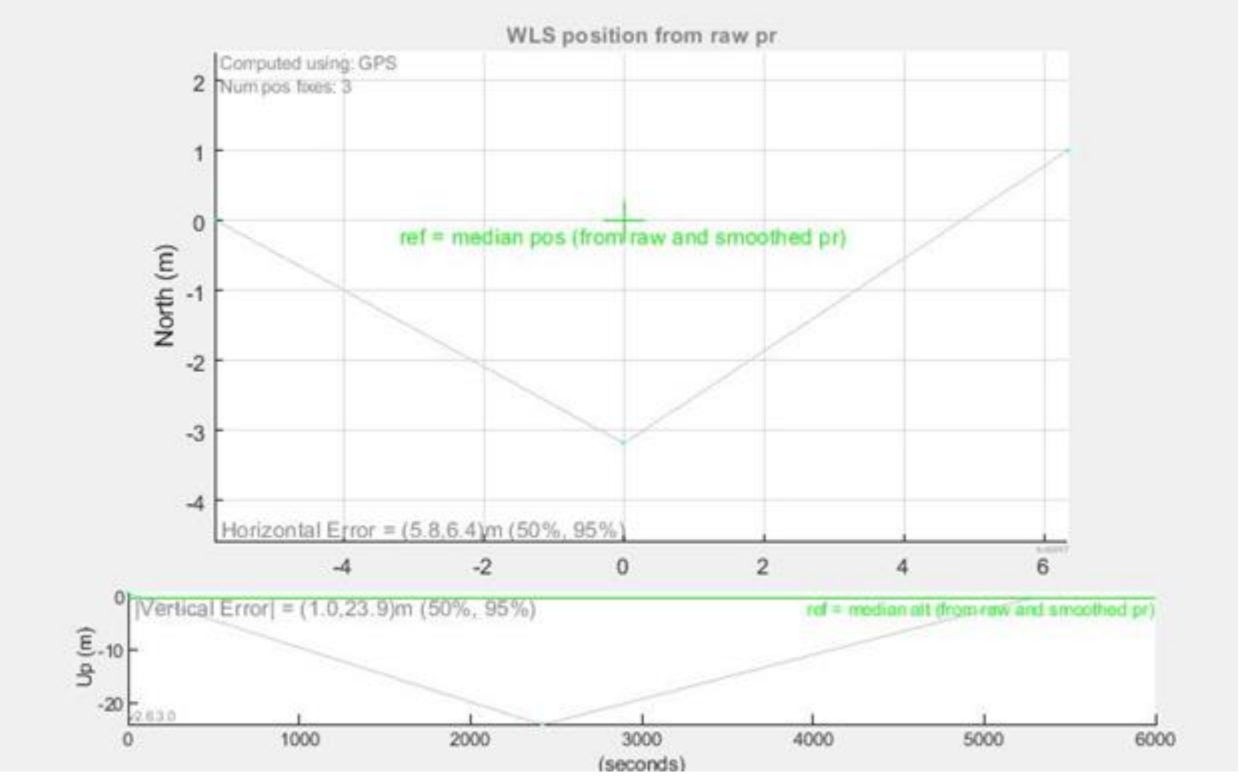

Figure 9: Weighted least square (WLS) result: total number of observation epochs and corresponding measurements in each epoch (more than four measurements in each epoch: 3 epochs in total)

Figure 9 shows the results of the VSDA technique for the simulated GPS denied scenario. While neither of epochs can provide a position solution independently, the accumulation of the measurements at all three epochs, shown in Figure 8, can solve the position solution. Epochs with only one pseudorange measurement may not contribute to the design matrix as they add one equation while adding one unknown as well to solve the clock offset between the mobile phone receiver and the satellite clock. For the experiment is conducted with three epochs, each having only three measurements, as shown in figure 8, and so the total nine pseudorange measurements that can maintain the independence of the observations. The VSDA's design matrix has 9 rows and 6 columns and the results of VSDA's Least Square solution are provided in table 1. Among the different sets of GDOP and C/N which have been tested in GPS denied scenario, the results for the highest and lowest GDOP and C/N have been presented as scenario 1 and 2 respectively.

Table 1: Comparison of position error for different combinations of 3 satellites in each epoch (3 epochs in total)

			Confidence Interval 50 %		Confidence Interval 95 %	
	GDOP	C/N0 (dB)	Hz Error (m)	V Error (m)	Hz Error (m)	V Error (m)
Scenario1	<2	> 38	47.16	68.45	72.83	80.12
Scenario2	<6	>34	106.64	142.22	166.25	203.15

Depending on the criteria for satellite selection (such as satellite geometry and carrier to noise ratio) and the selected epochs, the horizontal and vertical accuracies may vary. Therefore, a solid investigation seems to help to find the best balance of the criteria and so to select the most optimum set of observations to achieve the highest accuracy.

CONCLUSION

Blockage of GNSS signals can result in epochs with fewer than four satellites in view and so make positioning impossible. This is commonly happening also in dense urban canyons and indoors: This paper proposed and implemented a technique based on the accumulation of measurements at different epoch and solving synchronization by adding an unknown to address such challenge. The proposed technique virtually distributes the receiving antenna in space and time and so enables position solution to be solvable. The paper conducted a proof of concept for the virtual spatial diversity antenna technique and measured the achievable accuracy in two scenarios. The position solution from only three epochs is shown to reach 47.16 m and 68.45m horizontally and vertically (respectively). These may be enough or give a better initial estimation of location for many location-based services and applications, which require the users' positional data to function, while none of the epochs can calculate a position solution independently due to the blockage of the signals.

The presented scenario is solely based on the pseudorange measurements from a single constellation, GPS only. It is expected that the achievable accuracy would improve by including carrier phase measurements. The carrier phase measurements and also multi-GNSS observations are made available by more recent versions of Android running mobile devices. While multi-GNSS may add some additional unknowns, it may provide with the better geometry of the satellites and so improve the accuracy. Additional error sources affecting GNSS observations should also be taken into account to improve the achievable accuracy. Such error sources include the tropospheric and Ionospheric delay.

ACKNOWLEDGMENTS

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