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Dotty triangles: two different approaches to analysing young children's responses to a pattern replication activity

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This article presents results from an exploratory study into developing pattern awareness with children aged 3 to 5, following the work of Mulligan and Mitchelmore (2009) on Awareness of Mathematical Pattern and Structure (AMPS). When the children copied a 6-dot triangular pattern, we similarly found diverse responses, which we analysed using the AMPS levels and then Biggs and Collis' SOLO taxonomy. The latter approach revealed that children responded to up to 5 elements in the pattern. This approach allowed us to identify positively the beginning stages of structural understanding, when children recognised 1 or 2 elements of the pattern. It also emphasised the challenge that the apparently simple task of copying an image can present to young children.

Keywords: patterns; early years mathematics; SOLO taxonomy

Introduction

As humans we are particularly prone to search for regularity and patterns in our environment: for example, in music we find pleasure in listening to notes arranged in a predictable manner and with a regular rhythm, whereas we tend to dislike random sounds (Orton, 1999). But it is in mathematics where pattern comes to the fore, with mathematics referred to as the 'science of patterns' because it involves the search for, construction and communication of patterns and regularity (Smith, 2003).

Young children's pattern awareness has recently been linked to general mathematical competence and to be predictive of later achievement. Mulligan and Mitchelmore (2009) identified that young children had different levels of Awareness of Mathematical Pattern and Structure (AMPS) which was consistent across pattern types (repeating, spatial, growing) and modes (spatial regularity, colour, shape, number). Children with higher levels of AMPS tended to also perform better in other measures of mathematics (Mulligan & Mitchelmore, 2009). Rittle-Johnson, Fyfe, Hofer and Farran (2017) identified patterning at age 5 (assessed in terms of repeating patterns) as predictive of mathematics achievement at age 11. Furthermore, Papic, Mulligan and Mitchelmore (2011) found that the AMPS levels are not immutable: with focused teaching, pre-school children's AMPS scores could be improved, with positive effects on their mathematics, particularly with regard to number and pre-algebraic thinking. Recent studies have also shown that teaching pattern awareness can have particular benefits for the mathematics of low achieving or disadvantaged children (Papic et al., 2011; Kidd et al., 2014). However, Kidd et al. (2014) also conclude that the mechanisms whereby pattern instruction helps maths performance are currently unknown. Rittle-Johnson et al. (2017), while recommending a greater focus on patterning in pre-school and the early primary grades, also argue that 'patterning knowledge requires more attention in theories of mathematical development' (p. 12), and that more reliable and appropriate assessments are needed.

We consequently became interested in developing ways of teaching pattern awareness in order to enhance young children's mathematics. We also felt that pattern as a mathematical topic would be likely to appeal to the various talents and interests of both early years teachers and children. Early years teachers in England are required to teach pattern: it is included within the spatial mathematics goal for five-year-olds (DfE, 2017). However, there is no clear progression in learning, as it is subsequently included in the national curriculum first within Numbers and then within Geometry (DfE, 2013). It therefore seemed a relevant and potentially fruitful area for collaborative enquiry with teachers.

Our study originated from an Australian research programme which developed a pattern awareness assessment (PASA: Mulligan, Mitchelmore & Stephanou, 2015) and teaching programmes (PASMAP: Mulligan & Mitchelmore, in press; Papic, et al., 2011). We used and adapted several tasks from the PASA assessment and then used this information to develop an intervention. In this article we focus on one task from the assessment which involved copying and extending a 6-dot triangular pattern. We found that children gave a surprising variety of responses that did not fit easily with the AMPS levels. We therefore reanalysed the children's responses with the SOLO taxonomy (Biggs & Collis, 1982). Hence, the research question we examined in this study was: What affordances do the two frameworks—AMPS and the SOLO taxonomy—offer when analysing young children's response to copying a 6-dot triangle?

Literature review and theoretical framework

Pattern may be defined in many ways, but mathematical patterns must involve some kind of regularity (Orton,1999). Papic et al. (2011) regard pattern as including 'any replicable regularity', which may include 'simple repetition' (p. 238) or 'consistent relations' between elements (p. 240). The Erikson Early Math Collaborative (2018) define pattern as a sequence with a rule.

Developing pattern awareness is considered important because it develops mathematical thinking: recognising pattern structure involves the analysis and simplification of complex information, focusing on mathematical relationships while ignoring other features (Rittle-Johnson et al., 2013). Even spatial single object patterns, such as a triangle, give an opportunity for abstracting and generalising: 'the aim is to find consistent relations within specific categories of geometrical shape' (Papic et al., 2011, p. 240). This type of pattern includes arrangements of dots on a dice, recognition of which develops important subitising skills (Sarama & Clements, 2009). According to Mitchelmore and Mulligan (2009), AMPS also includes a motivational tendency to seek and analyse patterns.

According to the AMPS framework, children with low levels of pattern awareness may recognise features of a pattern but not the way they are organised, whereas those with high levels will recognise and generalise the pattern structure to other contexts. The definition of the AMPS levels was partly derived from Biggs and Collis' (1982) generic SOLO taxonomy (Mulligan & Mitchelmore, 2009), which analyses the quality of children's learning on a particular task. The SOLO taxonomy proposes five possible progressive levels of responses to a task: when the child does not give an appropriate response to the task, their response is at the *pre-structural* level. At the *uni-structural* level the child only focuses on one aspect of the task, whereas at the *multi-structural* level the child focuses on several relevant aspects of the task but treats them as if they are independent. At the *relational* level, the child has integrated all the aspects of the task into a coherent whole and at the *extended abstract* level the child can generalise the knowledge to a new topic. Other research has shown that 4- to 6-year-old children transition from focusing on only one aspect of a task to coordinating their attention on two dimensions (Case & Okamoto, 1996), or from a *uni-structural* understanding of tasks to a *multi-structural* understanding of tasks. This implies that children's pattern awareness may relate to more generalised measures of learning quality: we therefore decided to use both measures to analyse children's responses, to see what insights this gave us as to their interpretation of the pattern.

Methods

Participants and Setting

All of the participants came from four schools in an inner-borough in London, UK. All of these schools had pupils from a wide variety of minority ethnic groups, with a higher proportion than average who spoke English as an additional language (EAL). There were 26 children aged between 36 and 62 months at the beginning of this study, with fourteen of them in reception and twelve in nursery. Fourteen were girls and twelve were boys. Fourteen of the children were identified by their teachers as high achieving in mathematics and twelve as low achieving. There were 15 children aged between 43 and 69 months during the post-assessment period of this study, with five children in reception and ten in nursery. Nine were girls and six were boys. Ten had been identified by their teacher as high-achieving in mathematics and five as low-achieving. Twelve children were unavailable for reassessment for a variety of reasons. One child with special educational needs only participated in the post-assessment because of communication difficulties at the beginning of the year.

Procedures

Either the teachers or the researchers conducted pre- and post-assessments (derived from Mulligan et al., 2015); these included copying and extending an ABC pattern, creating an AB border pattern, copying and extending a triangular pattern, and subitising eight dots. While our main study included an intervention (for intervention activities, see Gifford, 2017) to help the children improve their understanding of pattern, this article will focus on how the children copied the triangular pattern (see Figure 1).

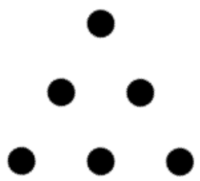

Figure 1: Triangular pattern that the children were asked to copy

In the original PASA assessment (Mulligan et al., 2015) the children were asked to draw the image of 6 dots that they had only seen for 2 seconds, and then asked to extend the pattern. As we were working with younger children we decided to follow the protocol in Papic, Mulligan and Mitchelmore (2011), so asked the children to copy the pattern in front of them.

Data Analysis

The data comprised the children's drawings and notes of what they said as they were asked to copy the triangular pattern. Responses were initially assigned levels using Mulligan et al.'s (2015) criteria. As we reviewed the children's varied responses to the problem, we noticed that these did not fit easily with Mulligan et al.'s (2015) levels and so we reanalysed the data using the SOLO taxonomy (Biggs & Collis, 1982). We found the new levels more closely reflected what these young children were producing; we then chose exemplars to illustrate these.

Results

Mulligan et al.'s (2015) scale

We found that few children could accurately copy the 6-dot pattern (see Table 1). Only 19% of the children could do this at pre-assessment and 33% at post-assessment.

Table 1: Pre- and post-assessment levels in comparison with Mulligan et al.'s (2015) levels

Levels	**Pre-assessment % of 26 children (actual numbers in brackets)**	**Post-assessment % of 15 children (actual numbers in brackets)**
1 Pre-structural: Does not copy the given pattern	23 (6)	7 (1)
2 Emergent: Draws a triangular group of dots not arranged in rows	50 (13)	53 (8)
3 Partial: Draws a triangular group of dots not correctly arranged in rows	8 (2)	7 (1)
4 Structural: Draws a correct copy but an incorrect extension	19 (5)	33 (5)
5 Advanced: Draws and extends the pattern correctly	0 (0)	0 (0)

We found that, using Mulligan et al.'s (2015) classification scale, most children were classified as either pre-structural or emergent at both assessment points (19 children or 73% at pre-assessment and (9 children or 60% at post-assessment: see Table 1). Despite these similar classifications we saw patterns of finer gradations within the children's responses that we thought would give us more information. For example, using Mulligan et al.'s (2015) scale both the child who just scribbled and the child who drew six lines were classified as pre-structural (see Table 2), even though the latter had clearly responded to the image.

SOLO taxonomy

We subsequently reanalysed the data by looking at the number of pattern elements the children represented and using the SOLO taxonomy (Biggs & Collis, 1982). There were five possible elements to the pattern: the shape of the dots, the numerosity of six, the triangular shape, equal spacing and rows (see Table 2). We numbered the pre-structural level as 0, for children who either made no response to the prompt, scribbled, or wrote something unrelated to the prompt (e.g. writing the numerals 1-8). Children at the uni-

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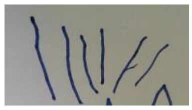
structural level (1) represented only one element of the pattern, either the dots, the number, or the triangle shape. Children at the multi-structural level (2) focused on two elements of the pattern, which were either dots and the number, dots and the shape, or rows and the dots. Children who focused on three elements of the pattern—either dots, spacing and shape; dots, number and rows; or dots, rows and shape—were assigned level 3, which we later decided was transitional. At the relational level (4) children represented at least four elements and correctly replicated the image. However, we noticed they had produced this in different ways. Some children put dots along the sides of the triangle, some placed the dots in rows, and one placed dots at the corners and then put dots at the mid-points of the sides. We noted that the six-dot arrangement could be seen as six dots forming the sides of a triangular space, rather than as three rows, of one, two and three dots.

Table 2: Elements of the pattern represented by the children

Revised Levels	**Descriptor**	**Examples**
0 Pre-structural	The child makes no response, scribbles, or writes something unrelated to the prompt.	
1 Uni-structural	The child focuses on one element of the pattern, either the dots, the number, or the triangle shape.	 **Triangles**  **Six** lines
2 Multi-structural	The child focuses on two elements of the pattern, either dots and number, dots and shape, or rows and dots.	 **Dots** in a **triangle**
3 Transitional	The child focuses on three elements of the pattern: either dots, spacing and shape; dots, number and rows; or dots, rows and shape.	 **6 dots** drawn in **rows**
4 Relational	The child produces at least four elements of the pattern, either placing the dots as sides of the triangle or in rows.	 **6 dots** drawn as sides of the **triangle** with roughly **equal spacing.**  **6-dot triangular** pattern built up **row by row**.

Re-analysis of our data allowed us to discriminate children's responses in greater detail and more positively, especially at the lower levels. Rather than 23% being assessed as pre-structural and 50% as emergent, the children's responses were distributed across three levels (see Table 3), identifying their attention to one or two structural features of the pattern.

In both scales there is an interesting dip at level 3, suggesting that there is not a gradual progression in the number of elements that children notice. This is likely to be because level 3 is a transitional state, which occurs when children are beginning to focus on the relationships between the elements but have not yet coordinated all the relationships needed to see the pattern as a whole (Biggs and Collis, 1982). Siegler (2006) regards this as a fleeting but vital state for learning, as it occurs only while cognitive change is happening.

Table 3: Percentage of children in each category

Percentage of children numbers in brackets	**0 Pre-structural**	**1 Uni-structural**	**2 Multi-structural**	**3 Transitional**	**4 Relational**
Pre-assessment % (26)	12 (3)	27 (7)	35 (9)	12 (3)	15 (4)
Post-assessment % (15)	0 (0)	7 (1)	47 (7)	20 (3)	27 (4)

Discussion

While Mulligan et al.'s (2015) classification scale was a useful starting point for analysing children's responses to the 6-dot triangular pattern, it was not sufficient. There was a need for a finer-grained scale, particularly at the lower end of the scale where the AMPS scale grouped children who were beginning to show appreciation of the structure of the pattern with those who demonstrated no understanding of structure.

We suggest that the 6-dot triangular pattern was difficult for the young children to copy because they see it as composed of five separate elements: dots, number, triangular shape, equal spacing and rows. When there are multiple features to focus on, young children have to decide where and how to focus their attention. Some children can only focus on and represent one element at a time, some can integrate 2 or 3 elements but only a few can relate all the elements to produce a whole that resembles the original. This supports Papic and Mulligan's (2005) finding that some children can only see one element in a pattern whereas others can spot multi-modal patterns. One interesting finding was that, whereas we had expected children to interpret the image as a growing pattern, they saw a different, but equally valid structure, interpreting it as a spatial single object, or an empty triangle.

Using the SOLO taxonomy (Biggs & Collis, 1982) rather than Mulligan et al.'s (2015) classification scale we can see more developmental growth because it allows for more discrimination at the lower levels. It is plain that this taxonomy is a potentially useful way of measuring young children's quality of learning: There was a good spread of responses across the first four levels of the taxonomy with a dip in the transitional level suggesting a cognitive change in learning to relate the various elements together. While none of our children attained the top extended abstract level, there was sufficient discrimination between the levels to comment on the quality of the children's understanding.

The fact that the SOLO taxonomy (Biggs & Collis, 1982) was a good model in this instance implies that children's developing pattern awareness may reflect a more generic development in the number of elements they can pay attention to at once and in their appreciation of complex images. It highlights young children's difficulties in focusing on an image as a whole, requiring them to synthesise multiple elements. However, pattern contexts provide an appropriate level of complexity to discriminate levels of learning among young children and can give us insights into their development and the way they interpret such images.

References

Biggs, J. B., & Collis, K. (1982). *Evaluating the Quality of Learning: the SOLO taxonomy*. New York, NY: Academic Press.

Case, R., & Okamoto, Y. (1996). The role of central conceptual structures in the development of children's thought. *Monographs for the Society for Research in Child Development*, *61*(1-2).

Department for Education. (2013). *Mathematics Programmes of Study: Key Stages 1 and 2 National Curriculum in England.* Retrieved from https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/335158/PRIMARY_national_curriculum_-_Mathematics_220714.pdf

Department for Education. (2017). *Early Years Foundation Stage Statutory Framework.* Retrieved from https://www.gov.uk/government/publications/early-years-foundation-stage-framework--2

Erikson Early Math Collaborative. (2018). *Patterns are sequences (repeating or growing) governed by a rule; they exist both in the world and in mathematics*. Retrieved from http://earlymath.erikson.edu/big-ideas/patterns-are-sequences-repeating-or-growing-governed-by-a-rule-they-exist-both-in-the-world-and-in-mathematics-pre-elementary-1st-grade-math-activities/

Gifford, S. (2017). *Developing pattern awareness with young children*. Retrieved from https://nrich.maths.org/13362

Kidd, J. K., Pasnak, R., Gadzichowski, K. M., Gallington, D. A., McKnight, P., Boyer, C. E., & Carlson, A. (2014). Instructing first-grade children on patterning improves reading and mathematics. *Early Education & Development*, 25, 134–151. https://doi.org/10.1080/10409289.2013.794448

Mulligan, J. T., Mitchelmore, M.C., & Stephanou, A. (2015). *Pattern and Structure Assessment: An assessment program for early mathematics (F-2).* Victoria, Australia: Australian Council for Educational Research.

Mulligan, J.T., & Mitchelmore, M. (In press). *Pattern and Structure Mathematics Awareness Progamme (F-1).* Victoria, Australia: Australian Council for Educational Research.

Mulligan, J.T., & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development, *Mathematics Education Research Journal*, *21*(2), 33–49.

Orton, A. (1999). Preface. In A. Orton (Ed.) *Pattern in the teaching and learning of mathematics* (pp. vii-viii). London, UK: Cassell.

Papic, M., & Mulligan, J. (2005, July). *Preschoolers' mathematical patterning.* Paper presented at the Mathematics Education Research Group of Australasia Conference, Melbourne. Retrieved from https://www.researchonline.mq.edu.au/vital/access/manager/Repository/mq:17263?queryType=vitalDismax&query=preschoolers+mathematical.

Papic, M., Mulligan, J., & Mitchelmore, M. (2011). Assessing the development of pre-schoolers' mathematical patterning. *Journal for Research in Mathematics Education*, *42*(3), 237-268.

Rittle-Johnson, B., Fyfe, E.R., McLean, L.E., & McEldoon, K.L. (2013). Emerging understanding of patterning in 4 year olds. *Journal of Cognition and Development*, *14*(3), 376–396. https://doi.org/10.1080/15248372.2012.689897

Rittle-Johnson, B., Fyfe, E.R., Hofer, K.G., & Farran, D.C. (2017). Early Math Trajectories: Low-Income Children's Mathematics Knowledge From Ages 4 to 11, *Child Development*, *88*(5), 1727-1742.

Sarama, J., & Clements, D. (2009). *Early childhood math education research: learning trajectories for young children.* Abingdon, Oxon: Routledge.

Siegler, R. S. (2006). Micro-genetic analyses of learning. In D. Kuhn, & R. Siegler (Eds.) *Handbook of Child Psychology: Volume 2: Cognition, Perception, and Language* (6th ed., pp. 464-510). Hoboken, NJ: John Wiley & Sons Inc. https://doi.org/10.1002/9780470147658.chpsy0211

Smith, E. (2003). Stasis and change: Integrating patterns, functions, and algebra throughout the k-12 curriculum. In J. Kilpatrick, W.G. Martin, & D. Schifter (Eds.) *A Research Companion to Principles and Standards for School Mathematics* (pp. 136-150). Reston, VA: The National Council of Teachers of Mathematics.